ESSAYS ON FINANCIAL MARKETS

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This dissertation consists of three essays, the first two on the foreign exchange market and the third on credit markets.

Chapter 2 examines empirically the exchange rate – interest differential relationship in a co-integration framework. We test and estimate an error correction model (ECM) for the currency pair US dollar and British pound at the daily frequency. The exchange rate and the interest differential are found to be co-integrated, and the parameters in the ECM exhibit signs that are consistent with market practitioners' observation concerning the relationship. The interest differential can thus be viewed as a long-run anchor for the two currencies' exchange rate.

In Chapter 3, we model the direct inter-dealer trading in the foreign exchange market as a two-stage, alternating-offer bilateral bargaining game in an asymmetric information environment. Under the naïve conjecture rule for updating the uninformed dealer's beliefs, there exists a unique perfect Bayesian equilibrium (PBE) of the game. The PBE outcome depends on the informed dealer's valuation of the future exchange rate. An alternative bargaining procedure is also considered which produces an identical PBE outcome.

In Chapter 4, we re-examine the problem of credit rationing by modeling the loan contracting problem between a monopolist lender and the borrowers as a two-stage screening game in which pre-contractural and post-contractural informational asymmetry are simultaneously present. The screening game has a unique subgame perfect equilibrium. Depending on the level of borrowers' prior debts, credit rationing may or may not occur. In the pooling equilibrium, there is no credit rationing and all borrowers get a loan whose size is socially efficient. Credit rationing arises in the semi-separating equilibrium in which high-debt borrowers do not obtain a loan from the lender.

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1.0 INTRODUCTION

Financial markets are a vital part of any well-functioning economy. The studies in this dissertation cover the foreign exchange market and credit markets, with a particular emphasis on the micro aspects of the markets.

The foreign exchange market has long been a subject of investigation in International Finance. Research in this area has turned up many puzzles that have so far not been resolved. Among those, the uncovered interest parity (UIP) puzzle has attracted much attention in the academic literature. In Chapter 2, we seek to contribute to the understanding of the UIP problem by looking at how foreign exchange market practitioners view the problem. Within the framework of co-integration, we examine empirically the interrelationship between the exchange rate and the interest differential for the currency pair US dollar and British pound at the daily frequency. Our results are used to confirm an observation made by the market practitioners concerning the exchange rate - interest differential relation as it relates to the UIP problem.

The study in Chapter 3 is a contribution to the literature on financial market microstructure. We model the direct inter-dealer foreign exchange trading using tools from game theory. In this respect, we adopt a bargaining approach that has hitherto not been pursued in the literature. Our micro-based investigation of the foreign exchange market provides a useful complement to the traditional macro approach to the exchange rate determination.

Chapter 4 deals with credit markets. The study is motivated by the observation that credit markets in less developed countries are rather opaque. We re-examine the problem of credit rationing in such an environment, by formulating the loan contracting problem between a monopolist lender and a pool of heterogeneous borrowers in rigorous game-theoretic terms. Conditions are derived under which credit rationing may or may not arise. The techniques developed in this study have potential applications in other studies of the principal-agent problem in which adverse selection and moral hazard might be simultaneously present.

2.0 A COINTEGRATION ANALYSIS OF THE EXCHANGE RATE-INTEREST DIFFERENTIAL RELATION: CONFIRMING THE MARKET PRACTITIONERS' VIEW

2.1 INTRODUCTION

That currency exchange rates are closely related to the interest rates of the two currencies involved is a common perception dating back to Keynes (1923). There have been numerous studies ever since that try to theorize and empirically test the various relationships between them, among which most notably is the interest parity hypothesis. It asserts that returns on interest-bearing assets denominated in different currencies should be equalized through speculative forces in the market. Two basic forms of interest parity have received much attention in the literature, namely, covered interest parity (CIP) and uncovered interest parity (UIP). CIP involves both the spot and forward foreign exchange markets and can be stated as follows. Let S_t be the spot exchange rate (domestic currency price of one unit of foreign currency), F_t be the one-period forward exchange rate (i.e., units of domestic currency to be delivered in exchange for one unit of foreign currency in the next period), and i_t and i_t^* the one-period domestic and foreign nominal interest rates respectively, all effective in period t. Then what CIP asserts is the equality

(2.1) $(1+i_t^*)F_t = (1+i_t)S_t$

CIP has been subjected to extensive empirical testing (e.g. Branston, 1969; Marston, 1976; Fratianni and Wakeman, 1982; Frenkel and Levich, 1975, 1977; Taylor, 1987, 1989). The consensus that emerges from the tests is that CIP holds reasonably well across different sampling periods and under various market conditions. This is hardly surprising given that, in practice, all banks engaging in foreign exchange activity quote forward prices according to the relation in (2.1). Setting forward prices in this manner essentially eliminates a source of covered arbitrage profit; the latter can be achieved via simultaneous, "round-trip" trades in the spot and forward markets when the equality in (2.1) is violated. Indeed, the cambist interpretation of CIP is that it is simply an accounting identity rather than something predicted by economic theory.

In sharp contrast, there is little consensus as to whether or not UIP holds, much less its explanation, despite the considerable effort devoted to the investigation of this problem in the literature. UIP postulates that market forces equilibrate the expected return on uncovered foreign currency investments to the return on comparable domestic currency investments, once both are converted into the same currency. Specifically, UIP hypothesis can be expressed as

$$(2.2) \qquad (1+i_t^*)E_t S_{t+1} = (1+i_t)S_t$$

where $E_t S_{t+1}$ denotes the market's expectation in period t of the spot exchange rate in period $t+1$. The left-hand side of (2.2) represents the *expected* one-period gross return from investing one unit of foreign currency in the foreign money market in period t, and the right-hand side is the gross return from first converting one unit of foreign currency into domestic currency in period t and then investing the proceeds in the domestic money market for one period; both are denominated in domestic currency. Denote by s_t the logarithm of S_t and (2.2) is approximately the same as

$$(2.2') \quad E_t s_{t+1} - s_t = i_t - i_t^*$$

Note that a key assumption in arriving at (2.2) is that investors are risk-neutral and hence their investment decisions are driven solely by the expected return without any consideration of the return's variance. UIP as expressed in $(2.2')$ is not directly testable however, since $E_t s_{t+1}$, the market's expectation of future spot rate is not observable in general. Empirical tests have commonly substituted s_{t+1}, the realized log spot rate, under the assumption that investors form expectations rationally based on relevant information available at the time of forecasting; that is, that investors do not make systematic forecast errors so that $s_{t+1} = E_t s_{t+1} + \varepsilon_{t+1}$, where ε_{t+1} is the white-noise forecast error. This combined with $(2.2')$ gives

$$(2.3) \quad s_{t+1} - s_t = i_t - i_t^* + \varepsilon_{t+1}$$

A large body of empirical work has been carried out to test the validity of UIP, typically employing the following specification form of regression equation

$$(2.4) \quad s_{t+1} - s_t = a + b(i_t - i_t^*) + \varepsilon_{t+1}$$

Under the joint assumption of risk-neutrality and rational expectations on the part of the investors (from both of which (2.3) follows), if UIP were to hold, the regression analysis should yield a slope coefficient $b = 1$.

Empirical evidence, however, has soundly rejected the null hypothesis of $b = 1$. It reveals that not only is b significantly different from the hypothesized value of 1, but it is more often than not negative. Froot and Thaler (1990) surveyed some 75 regression studies and reported an average b value of about -0.88. In another survey paper, Engel (1996) found that a b value between -4 and -3 seemed to be a representative result. Overall, the evidence against (2.3) is overwhelming.

Economists have not yet resolved the question of how to interpret the large and often significantly negative estimates of b, but have proposed two major possible explanations. The first explanation focuses on the existence of a foreign exchange risk premium. It argues that investors in the foreign exchange market are not totally risk-neutral – risk-neutrality being the assumption underlying the relation in (2.2) – but rather they are more accurately characterized as being risk-averse. In this light, the apparent discrepancy between $E_t s_{t+1} - s_t$ and $i_t - i_t^*$ can be explained by a risk premium: It is the existence of this risk premium that drives a wedge between the two quantities. Another line of explanation, while maintaining the risk-neutrality assumption, questions the other underlying assumption, namely, that the investors have rational expectations. If investors cannot be assumed to have rational expectations, then $(2.2')$ and (2.3) are not completely equivalent and hence the rejection of (2.3) is not necessarily evidence against $(2.2')$, the original UIP hypothesis. It thus attributes the rejection of (2.3) to the investors' expectational error, $s_{t+1} - E_t s_{t+1}$. In actuality, it may well be the case that the two explanations both have some merit in their own right and, when combined, may give a more accurate picture of the matter, but perhaps because of the difficulty involved in obtaining reliable data on market expectations for empirical work, whether UIP holds and, if it doesn't, what accounts for its failure remains an unresolved issue, despite the fact that UIP is a key assumption in many theoretical models of international economics.[1]

In this paper, we depart from the various theoretical standpoints adopted in the academic literature and instead look at how market practitioners see the UIP problem as it occurs in the foreign exchange market. After all, it is what those market players believe and how they act based on their beliefs that determine the exchange rates observed on the market. As it turns out, if one were to subscribe to their view about the exchange rate movements, the empirical failure of UIP would seem less of a puzzle than it has been for the economists. However, this paper, although closely related to it, is not an attempt to resolve the issue of UIP *per se*; rather its primary purpose is to empirically confirm a regularity observed in the actual market as it relates to the UIP, in a cointegration framework. The main contribution of the paper is the finding that there exists a long-run equilibrium relationship between the US dollar - British pound bilateral nominal exchange rate and the short-term nominal interest differential of the two currencies at the daily frequency, and that the nominal exchange rate adjusts through an error correction mechanism in response to deviations from the long-run equilibrium.

The remainder of the paper proceeds as follows. In Section 2, a comparison is made between how practitioners and economists view the UIP problem, as motivation for the paper. Section 3 presents a simple model of currency portfolio holdings which theoretically justifies the empirical work that follows. In Section 4, we empirically examine the US dollar – British pound bilateral exchange rate series in relation to the interest differential series in the framework of cointegration, where the associated ECM is derived and is subsequently used in an out-of-sample forecasting comparison with a first-difference VAR. Summary and conclusion are contained in Section 5.

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[1] See Lewis (1995) for a detailed account of the UIP problem.

2.2 DIFFERING VIEWS OF PRACTITIONERS AND ECONOMISTS ON THE UIP PROBLEM

2.2.1 An Explanation Based on Foreign Exchange Market Institution

The deviation from UIP implies that there are un-exploited profits in the market. This is most clearly seen from the following relation:

(2.5) $$s_{t+1} - s_t = a + b(f_t - s_t) + \varepsilon_{t+1}$$

which is derived from combining (2.4) and the log-linearized version of (2.1). Given the validity of CIP, (2.5) is equivalent to (2.4). A finding of negative values of *b* then means, for example, if the currency is traded at a forward discount in the current period (i.e. $f_t < s_t$), it tends to appreciate in the next period. In such a situation, buying forward in that currency would have yielded a profit. According to the efficient market hypothesis, however, this type of arbitrage profit should not exist, at least not for long, as speculative forces in the market would have eliminated it. Yet, deviations from UIP (or, equivalently, forward bias) persist and un-exploited profits do seem to exist. And this is what economists find difficult to explain.

While this phenomenon is seemingly at odds with the efficient market hypothesis from a theoretical point of view, practitioners seek to explain the puzzle by drawing on the institutional realities of speculation in the market. The foreign exchange market is a complex interconnected web of heterogeneous players. Lyons (2001) divides them into several categories: leveraged investors, unleveraged investors and nonfinancial corporations. He argues that unleveraged investors (e.g., insurance companies) and nonfinancial corporations in general do not engage in currency speculation, presumably due to their comparative disadvantage in the activity. Indeed, as McKinnon (1979) puts it, “the freeing of capital for speculative purposes in the foreign exchanges can hardly be given a high priority by big companies whose existence in the long run depends on their expertise in manufacturing and commerce.”

Leveraged investors (e.g., proprietary bank traders and hedge funds) who do have the potential and expertise in exploiting the forward bias, Lyons contends, allocate their speculative capital largely based on the Sharpe ratio.[2] The Sharpe ratio of currency strategies exploiting the forward bias, based on historical data, has been roughly 0.4 on an annual basis, similar to that of a naive strategy of buying and holding an equity index, and well below most institutions' minimum threshold for inducing capital allocation.[3] From the practitioners' perspective, then, the lack of speculative capital allocated to exploiting the forward bias leads to its persistence. Additional evidence supporting the seemingly very risk-averse behavior of the leveraged investors is provided by Carlson (1998) and Shleifer and Vishny (1997), both taking into account the institutional features of speculative activities as undertaken by bank traders and hedge fund managers.

Excluded from Lyons' description of foreign exchange market participants are the numerous individual investors who have to decide on where to park their idle money. Admittedly more comparatively disadvantaged, the individual investors are even less likely to speculate in the forward exchange market. Goodhart and Taylor (1992), after studying the minimal size of standard forward contracts and the associated transactions costs, conclude that the vast majority of individual wealth holders will be deterred from seeking to speculate in the market, given "reasonable" estimates of their coefficients of relative risk aversion. Indeed, "commercial banks have never made it easy for individuals who were not major depositors to take speculative positions" (McKinnon, 1979).

The overall picture that can be gleaned from the above institution-based descriptions is that most, if not all, participants in the foreign exchange market do not engage in the risky activities of forward speculation, which explains the persistence of the forward bias (or, equivalently, the deviation from UIP).

2.2.2 The Random Walk View of Exchange Rate Movements and its Implications

The empirical evidence cited in Section 1 reveals an important point regarding the relationship between the exchange rate change, $s_{t+1} - s_t$, and the two currencies' interest

[2] Sharpe ratio is defined as the ratio of a financial portfolio's excess return to its standard deviation over a specific period; see for example Bodie et al (1998).
[3] Based on Lyons' own discussions with proprietary traders at banks and hedge funds; see Lyons (2001).

differential, $i_t - i_t^*$. That is, a negative slope coefficient b in the regression equation (2.4) means that, on average, when the domestic currency's interest rate exceeds that of the foreign currency, the domestic currency tends to subsequently appreciate relative to the foreign currency, and vice versa. This is the exact opposite of how UIP predicts exchange rate should move, which makes the explanation even more difficult than if b were positive.

The conceptual argument for UIP is indeed quite simple. It is essentially one of (uncovered) arbitrage. A hypothetical situation depicted in Froot and Thaler (1990) illustrates this:

… suppose that the one-year (US) dollar interest rate is 10 percent, and that the comparable German mark interest rate is 7 percent…Risk neutral, rational investors then must expect the dollar to depreciate against the mark by 3 percent over the next year…If instead these investors expected a different rate of dollar depreciation, say 4 percent, they would all wish to borrow in dollars and lend in marks. Consequently, dollar interest rates would tend to rise and mark interest rates would tend to fall until the interest differential also became 4 percent.

The reasoning behind this perceived equality between the (unobservable) expected depreciation rate and the (observable) interest differential is a simple supply-demand analysis as applied to the money market: If the interest differential between US dollar and German mark is different from the expected depreciation rate, then borrowing and lending activities in the two currencies (which take place in the money market), motivated by the expectation of earning an arbitrage profit, will eventually bring the two back in line. Thus, the very fact that we observe a 3 percent interest differential, with the US dollar rate exceeding the German mark rate, implies that investors expect a 3 percent depreciation of US dollar. Note that a crucial assumption that underlies this reasoning is that market participants, based on all relevant information, believe that the exchange rate will move in a certain direction by a certain percentage in the next period.

If, however, the market participants view exchange rate change as inherently unpredictable, or more specifically, if they believe the exchange rate is as likely to go up as go down tomorrow, then their currency allocation pattern is likely to be rather different from that described in the above hypothetical example. That currency exchange rate follows a random walk process, whether perceived or actually so, is a well established proposition in the academic literature (e.g., Goodhart, 1988). Ever since the seminal finding of Meese and Rogoff (1983) that none of then existing structural models of exchange rate determination performed better than a random walk model in out-of-sample forecasting exercises, the random walk model has become a benchmark in the empirical literature against which many subsequent models are evaluated, and yet none of those models seem to have prevailed over the random walk model, at least over short time horizons (Rogoff, 2001). Moreover, as Adams and Chadha (1991) have demonstrated, although the actual exchange rate movement may have been determined by a structural model, it is still hard to distinguish from a random walk.

Whether the exchange rate movement is *perceived* by market participants as following a random walk is perhaps more important than whether it is actually so, to the extent that market expectation influences market outcome. Empirical studies that employ survey data on market participants' exchange rate expectations find that the hypothesis cannot be rejected that market participants form their expectations in a fashion that amounts to assuming a random walk model (Allen and Taylor, 1990; Frank and Froot, 1990).

Given that most investors do not take speculative positions in a particular currency as argued in the previous section, it seems plausible that those investors will place their currency investments as if they didn't take any view on the exchange rate movement. This is especially true of the individual investors who arguably lack the necessary expertise to make reliable predictions about the rate change that they can use to guide their investment decisions. Choie (1993) describes an attitude best characterized as "a bird in the hand is worth two in the bush," which he claims as being prevalent among currency investors. This attitude dictates that, in the face of uncertainty about future exchange rate movement, today's known interest differential outweighs tomorrow's unknown exchange rate change in determining the short-term flow of funds in the market. Regarding the relation between the exchange rate change and interest differential, he echoes an observation repeatedly made by currency traders "…as the interest rate differential between two currencies increases, the currency of the country with higher short-term interest rate appreciates relative to the other currency…" In light of the above described attitude, this observation can be explained by a simple supply-demand analysis, this time taking place in the foreign exchange market: The increased interest rate of a currency induces the investors to shift their portfolio holdings toward that currency and creates a higher demand for it in the foreign exchange market, leading to the subsequent appreciation of the currency. In a sense, this analysis is the flip-side of the same kind of analysis that takes place in the money market, in the above hypothetical example of Froot and Thaler: The supply-demand condition in the money market influences the price of money (i.e. the interest rate), while the supply-demand condition in the foreign exchange market influences the price between two monies (i.e. the exchange rate). Hopper (1997) notices a similar currency price movement pattern which he describes as an example of the technical trading rules used by some currency traders.

If the divergence of opinions among the economists and currency traders can be viewed as representing a tension between theory and practice, another group of practitioners, the central bankers, probably will sympathize more with the currency traders' views about the exchange rate – interest differential relationship than with the economists'. Monetary authorities have long been known to use the tool of interest rate to alter the path of future exchange rate: raising the domestic interest rate to defend a weak domestic currency and reducing it to resist an undesirable appreciation of the domestic currency. Here, again, the basic premise is that market participants tend to respond to the incentives created by the increased interest differential, by increasing their demand for the currency with a now-higher interest rate, which in turn raises it price, as long as there remains uncertainty in their assessment of the future exchange rate movement.

2.3 A SIMPLE MODEL OF CURRENCY PORTFOLIO HOLDINGS

In the conventional approach to the UIP problem agents' expectations about future spot rate are assumed to be summarized in a single value, $E_t s_{t+1}$. Indeed, the relation $E_t s_{t+1} - s_t = i_t - i_t^*$ is what's termed the Uncovered Interest Parity. This characterization of market expectation suffers from its inability to allow for explicit modeling of the risk associated with the exchange rate. Though, in its attempt to explain the empirical failure of the UIP based on the risk argument, the conventional approach does introduce the notion of risk premium to account for the apparent discrepancy between $E_t s_{t+1} - s_t$ and $i_t - i_t^*$, it does so only in an indirect way; namely, the quantification of risk is not derived from the agent's utility maximizing behavior. Risk premium, which is believed to be the force driving a wedge between $E_t s_{t+1} - s_t$ and $i_t - i_t^*$ in this line of explanation, is a measure of compensation for the agent's willingness to bear the risk of future exchange rate change. This (subjective) risk, of course, arises from the fact that tomorrow's spot rate viewed from today is a random variable rather than a single value, from the agent's perspective. Hence, the quantity $E_t s_{t+1}$ - if it exists at all because it is unobservable anyway - is at best interpreted as a summary measure of the agent's expectations about tomorrow's spot rate; it fails to capture the full content of his expectations.

In the finance literature, by contrast, it is customary to model risk explicitly by assuming that investors view asset returns as following a certain fully specified stochastic process and derive asset allocation decisions from the more primitive principle of utility maximization. A classic example is Harry Markowitz's mean-variance analysis of portfolio choice and the related CAPM, in which investors are assumed to be maximizing a utility function defined over the mean and variance of asset returns (Markowitz, 1952).

Recall that our story of why interest differential change leads to exchange rate change hinges on the following assumption: In the face of uncertainty about tomorrow's exchange rate change which we characterize as following a random walk process, agents tend to shift their portfolio holdings toward the currency that earns a higher relative interest, while also taking into account the separate but related goal of risk diversification. In this section we develop a simple model based on agents' utility maximization that is capable of bolstering this assumption to some extent.

Consider a representative agent with period *t* domestic currency holding in the amount of m_t. Denote by s_t the spot exchange rate (amount of domestic currency per unit of foreign currency), by i_t the one-period domestic interest rate, and by i_t^* the one-period foreign interest rate, all effective in period *t*. The agent has a utility function $u(\cdot)$ exhibiting constant absolute risk aversion defined over his wealth measured in domestic currency: $u(W) = -e^{-\theta W}$ with $\theta > 0$.

The agent views the spot exchange rate in the next period, s_{t+1}, as essentially following a random walk process with a zero-mean normal error term, that is, $s_{t+1} = s_t + \varepsilon_{t+1}$ where $\varepsilon_{t+1} \sim N(0, \sigma^2)$, which implies that $s_{t+1} \sim N(s_t, \sigma^2)$. He maximizes his expected period $t+1$ utility by allocating funds in domestic and foreign currencies in the amount of m_{1t} and m_{2t} respectively, in period *t*. This yields the following budget constraint faced by the agent in period *t*:

(2.6) $m_t = m_{1t} + s_t m_{2t}$

His period $t+1$ wealth measured in domestic currency is given by

(2.7) $W_{t+1} = m_{1t}(1+i_t) + s_{t+1}m_{2t}(1+i_t^*)$

which upon substituting (6) gives

(2.8) $W_{t+1} = m_t(1+i_t) + m_{2t}\left(s_{t+1}(1+i_t^*) - s_t(1+i_t)\right)$

W_{t+1}, being a linear transformation of the normal random variable s_{t+1}, is itself a normal random variable distributed as $N\left(\mu_{W_{t+1}}, \sigma^2_{W_{t+1}}\right)$, where $\mu_{W_{t+1}} = m_t(1+i_t) + m_{2t}s_t(i_t^* - i_t)$ and $\sigma^2_{W_{t+1}} = m^2_{2t}(1+i_t^*)^2\sigma^2$. To simplify notation, let $A \equiv m_t(1+i_t)$, $B \equiv s_t(i_t^* - i_t)$ and $C \equiv (1+i_t^*)\sigma$ so that $\mu_{W_{t+1}} = A + Bm_{2t}$ and $\sigma^2_{W_{t+1}} = C^2 m^2_{2t}$.

With the above parameterizations, the agent's expected period $t+1$ utility can be written as

(2.9) $$E_t\left(-e^{-\theta W_{t+1}}\right) = -\int_{-\infty}^{\infty} e^{-\theta w} \frac{1}{\sqrt{2\pi}Cm_{2t}} e^{\frac{-(w-A-Bm_{2t})^2}{2C^2m^2_{2t}}} dw$$

Using a change of variable and the identity $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$, we get

(2.10) $$E_t\left(-e^{-\theta W_{t+1}}\right) = Ke^{\frac{1}{2}\theta^2 C^2 m^2_{2t} - \theta(A+Bm_{2t})}$$

where $K < 0$ is a constant which doesn't depend on m_{2t}. Differentiating the right hand side of (2.10) with respect to m_{2t} yields the condition for the maximization of $E_t\left(-e^{-\theta W_{t+1}}\right)$ as

(2.11) $$\frac{\partial E_t\left(-e^{-\theta W_{t+1}}\right)}{\partial m_{2t}}$$

$$= Ke^{-\theta A}\left(\theta^2 C^2 m_{2t} - \theta B\right)e^{\frac{1}{2}\theta^2 C^2 m_{2t}^2 - \theta B m_{2t}}$$

$$= 0$$

Solve for m_{2t} and we have

(2.12) $$m_{2t} = \frac{B}{\theta C^2} = \frac{s_t\left(i_t^* - i_t\right)}{\theta\sigma^2\left(1+i_t^*\right)^2}$$

This is the optimal amount of foreign currency the agent chooses to hold in his portfolio during period *t*.[4] It makes intuitive sense as the expression indicates that the higher the agent's risk aversion (a larger θ) and/or the higher the exchange rate's variation (a larger δ^2), the less the agent chooses to hold in the foreign currency.[5]

Taking partial derivatives of m_{2t} with respect to i_t^* and i_t respectively, we get $\frac{\partial m_{2t}}{\partial i_t^*} = \frac{s_t\theta\sigma^2\left(1+i_t\right)\left(1-i_t^*+i_t\right)}{\theta^2\sigma^4\left(1+i_t^*\right)^4} > 0$ and $\frac{\partial m_{2t}}{\partial i_t} = \frac{-s_t}{\theta\sigma^2\left(1+i_t^*\right)^2} < 0$, since $\theta > 0$, and $i_t^* < 1$ in general. Although a complete comparative statics result may well depend on the relative magnitudes of the relevant parameters in the model, there are at least two cases where an unambiguous relationship exists between the interest differential change and the change in the agent's currency holdings: Other things being equal, an increase in the foreign interest rate induces the agent to hold more foreign currency, and an increase in the domestic interest rate reduces his holdings of foreign currency and thus raises his holdings of domestic currency, if his holdings of the two currencies were both positive initially. While the model is admittedly a

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[4] Strictly speaking, $m_{2t} = \max\left\{0, \frac{s_t\left(i_t^* - i_t\right)}{\theta\sigma^2\left(1+i_t^*\right)^2}\right\}$.

[5] We must emphasize though that this is a consequence of our choice of the domestic currency as the currency of denomination in the agent's utility specification. On the other hand, it seems plausible to define domestic investors' utility function in terms of domestic rather than foreign currency.

highly ad hoc and simplistic one in that, for one thing, the results are not invariant to the choice of currency denomination in the agent's utility specification, the model nevertheless lends some support to our basic premise concerning the relationship between the interest differential change and the change in agents' currency portfolio holdings.

2.4 EMPIRICAL ANALYSIS

In this section we apply Johansen's multivariate cointegration techniques to investigate the interrelationship between the US dollar – British pound exchange rate and the interest differential of the two currencies. The analyses in the preceding sections suggest that these two variables cannot drift too far apart in the long-run, that is, they are tied together in a dynamic process, because a change in the interest differential tends to cause a corresponding change in the exchange rate. Thus the two variables exhibit a kind of co-movement which renders themselves amenable to an analysis in the cointegration framework.

In the traditional regression analysis, a high value of R^2 generally is indicative of a strong causal relationship between the independent and dependent variables. This is not the case though, if the variables involved are non-stationary, as demonstrated by Granger and Newbold (1974). In fact, a high R^2 in such a case might merely indicate a high contemporaneous correlation rather than a meaningful causal relation between the variables, a phenomenon Granger and Newbold call "spurious regression." In principle, non-stationary variables can be made stationary by differencing, and a regression analysis can then be conducted on the differenced variables. However, the resulting regression equation will only capture the short-run dynamics (i.e. the relationship between the variables' *changes* rather than that between their levels), and ignore possible long-run relationships that might still exist.

The concept of cointegration as elaborated by Engle and Granger (1987) deals with just such a problem. A non-stationary time series variable X_t is defined to be *integrated of order d*, denoted by $I(d)$, if it must be differenced *d* times in order to achieve stationarity. (X_t is also said to have *d* unit roots.) In general, a linear combination of two $I(d)$ variables, X_t and Y_t, is itself an $I(d)$ variable. If it happens that a particular linear combination $\alpha X_t + \beta Y_t$ is

$I(d-b)$ with $b>0$, then X_t and Y_t are said to be cointegrated. In most applied work, the variables of interest are usually $I(1)$, and cointegration then requires that a linear combination exist that is $I(0)$, i.e., stationary. A great advantage of the existence of a cointegration relationship is that it allows one to examine simultaneously the short-run dynamics as well as long-run relationships between the non-stationary variables, via the associated error correction mechanism (ECM).

2.4.1 Data

The daily US dollar – British pound bilateral exchange rates used in this study are taken from www.oanda.com for the period from January 2001 until December 2003. For the daily interest rates of the two currencies during the same period, we use the Eurocurrency rates, more specifically, the 3-month London InterBank Offer Rates (LIBOR), taken from www.economagic.com. Among the wide range of available interest rates on assets denominated in the two different currencies that can be of potential use for our study, Eurocurrency rates enjoy the highest degree of comparability, in terms of issues like regulation, tax, capital control and sovereign risk, compared with other types of interest rates such as those on government bonds of the respective countries. This rather homogeneous feature of the Eurocurrency rates makes them most suitable for our purpose. The Eurocurrency market typically offers instruments with maturities ranging from overnight to one year. We choose to use the 3-month rate because it is the most heavily traded maturity on the market (see for example Dacorogna et al, 2001). If there exists any systematic relationship at all between the exchange rate and the interest differential, such a relationship is more likely to be manifest in the 3-month rate than in other less heavily traded maturities.

2.4.2 Unit Root Tests

As a first step in the cointegration analysis stationarity tests, commonly known as unit root tests, are conducted on the log exchange rate series XR_t (expressed as dollar/pound) and the interest differential series ID_t (expressed as British rate minus US rate). This is accomplished by employing the augmented Dickey-Fuller (ADF) test which involves the following OLS regression:

(2.13) $$\Delta x_t = \alpha + \gamma t + \beta x_{t-1} + \sum_{i=1}^{p} \beta_i \Delta x_{t-i} + u_t$$

The null hypothesis of the ADF test is $\beta = 0$ (corresponding to x_t being non-stationary) and the alternative hypothesis is $\beta \neq 0$ (corresponding to x_t being stationary). Dickey and Fuller (1979,1981) show that the test statistic, τ, does *not* have the standard *t* distribution; critical values of τ under the null are tabulated in Fuller (1996).

A typical problem encountered in conducting the ADF test is the appropriate choice of *p*, the number of lagged differences of the variable to be included in the regression equation. It is not *a priori* clear what value of *p* should be used, since the underlying data generating process for the series is generally unknown. In fact, it is not uncommon for the ADF test to indicate a unit root for some lags but not for others (see, for example, Enders 2004). Since our focus here is on detecting whether a unit root is present in the time series and not on determining an appropriate univariate model for it, we choose to conduct the ADF tests using a variety of lag lengths, ranging from 0 to 20. This also has the benefit of ensuring a degree of robustness, if the results from different lag lengths agree with each other.

Table 2.1 reports the stationarity tests for the two series being studied. It provides strong evidence that both XR_t and ID_t series are $I(1)$: (a) For the original series, the null hypothesis that XR_t is non-stationary cannot be rejected at the 95% significance level in all 21 lag specifications, and the null hypothesis that ID_t is non-stationary cannot be rejected at the 95% significance level in 19 out of 21 lag specifications; (b) for both first-difference series, ΔXR_t and ΔID_t, the null hypothesis of non-stationarity can be rejected in favor of the alternative of stationarity for all 21 lags, at the 95% level. Overall, then, for our currency pair and the chosen sample period, it appears appropriate to treat XR_t and ID_t both as $I(1)$ series.

2.4.3 Applying the Johansen Procedure

Having established that XR_t and ID_t are both $I(1)$ we turn now to the test of cointegration between the two series. The purpose is to examine whether a stationary linear combination exists and, if so, to derive the associated error correction model.

2.4.3.1 Methodology

To test for cointegration between two series x_t and y_t, the original Engle-Granger methodology, after having ascertained that both series are $I(1)$, proceeds by estimating an OLS regression of the form:

(2.14) $y_t = a + bx_t + e_t$

The residual series, $\hat{e}_t$, from the above regression is then tested for stationarity. A cointegration relationship between x_t and y_t is obtained if $\hat{e}_t$ is determined to be stationary, with the associated cointegrating vector $(1, b)$ (Engle and Granger, 1987).

Johansen (1988) extends the Engle-Granger method to deal with situations where there is *any* number of variables and thus possibly multiple cointegrating vectors in the system,[6] based on a maximum likelihood estimation procedure (see also Johansen, 1991; Johansen and Juselius, 1990). There is some evidence that this method is more powerful at detecting a cointegration relationship among $I(1)$ variables than the Engle-Granger two-step method, especially in small samples.[7] Perhaps also due to the wide availability of software for its implementation,[8] the Johansen procedure has become the "method of choice" for cointegration analysis (Kennedy, 2003). We thus choose Johansen over Engle-Granger for our analysis of the bivariate system

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[6] In general, an *n*-variable system can have between *0* and $n-1$ independent cointegrating vectors. However, the cointegrating vectors themselves are not uniquely identified; only the linear space spanned by them, called the *cointegrating space*, is uniquely identified.
[7] MacDonald and Taylor (1994), for example, find non-cointegration in a monetary model of exchange rate determination using the Engle-Granger two-step procedure but find cointegration using the Johansen procedure, for the same sample of data.
[8] For example, SAS provides PROC VARMAX for cointegration analysis using the Johansen procedure; RATS has an add-on program CATS that is devoted to the Johansen procedure exclusively.

Table 2-1 ADF Tests for the Exchange Rate and Interest Differential Series

Lags	XR_t	ΔXR_t	ID_t	ΔID_t
0	$-2.72(0.2283)$	$-28.33^{*}(<0.0001)$	$-4.89^{*}(0.0004)$	$-21.81^{*}(<0.0001)$
1	$-2.74(0.2208)$	$-19.75^{*}(<0.0001)$	$-4.16^{*}(0.0054)$	$-18.44^{*}(<0.0001)$
2	$-2.61(0.2764)$	$-15.73^{*}(<0.0001)$	$-2.85(0.1784)$	$-16.61^{*}(<0.0001)$
3	$-2.83(0.1867)$	$-13.34^{*}(<0.0001)$	$-2.57(0.2947)$	$-14.65^{*}(<0.0001)$
4	$-2.92(0.1574)$	$-11.97^{*}(<0.0001)$	$-2.55(0.3032)$	$-12.63^{*}(<0.0001)$
5	$-2.86(0.1767)$	$-10.26^{*}(<0.0001)$	$-2.63(0.2655)$	$-10.35^{*}(<0.0001)$
6	$-2.93(0.1538)$	$-9.65^{*}(<0.0001)$	$-2.57(0.2930)$	$-10.20^{*}(<0.0001)$
7	$-2.89(0.1654)$	$-9.59^{*}(<0.0001)$	$-2.75(0.2175)$	$-9.50^{*}(<0.0001)$
8	$-2.98(0.1387)$	$-8.83^{*}(<0.0001)$	$-2.65(0.2597)$	$-8.31^{*}(<0.0001)$
9	$-2.75(0.2182)$	$-8.51^{*}(<0.0001)$	$-2.97(0.1409)$	$-7.66^{*}(<0.0001)$
10	$-2.71(0.2317)$	$-8.46^{*}(<0.0001)$	$-2.94(0.1499)$	$-7.43^{*}(<0.0001)$
11	$-2.58(0.2878)$	$-7.97^{*}(<0.0001)$	$-2.97(0.1409)$	$-7.24^{*}(<0.0001)$
12	$-2.65(0.2563)$	$-7.89^{*}(<0.0001)$	$-2.69(0.2405)$	$-7.25^{*}(<0.0001)$
13	$-2.62(0.2732)$	$-8.12^{*}(<0.0001)$	$-2.73(0.2255)$	$-7.24^{*}(<0.0001)$
14	$-2.35(0.4039)$	$-7.71^{*}(<0.0001)$	$-2.79(0.2017)$	$-6.79^{*}(<0.0001)$
15	$-2.42(0.3669)$	$-7.55^{*}(<0.0001)$	$-2.76(0.2141)$	$-6.61^{*}(<0.0001)$
16	$-2.47(0.3411)$	$-7.27^{*}(<0.0001)$	$-3.00(0.1340)$	$-6.25^{*}(<0.0001)$
17	$-2.31(0.4294)$	$-6.61^{*}(<0.0001)$	$-2.89(0.1656)$	$-5.86^{*}(<0.0001)$
18	$-2.51(0.3242)$	$-6.57^{*}(<0.0001)$	$-2.78(0.2055)$	$-5.70^{*}(<0.0001)$
19	$-2.45(0.3515)$	$-6.37^{*}(<0.0001)$	$-2.74(0.2203)$	$-5.59^{*}(<0.0001)$
20	$-2.43(0.3626)$	$-6.04^{*}(<0.0001)$	$-2.90(0.1623)$	$-5.96^{*}(<0.0001)$

Notes: a) SAS Version 8 was used to calculate the test statistics; b) numbers in parentheses are *p*-values associated with the test statistics; c) an asterisk (*) indicates significant at 95% level.

consisting of XR_t and ID_t, although both methods are applicable in the present context.

Denote $z_t = (z_{1t}, z_{2t}, ..., z_{nt})'$ where each z_{it} is an $I(1)$ variable. The Johansen procedure for testing cointegration among the components of z_t starts with the following vector autoregression (VAR):

(2.15) $z_t = A_0 + A_1 z_{t-1} + ... + A_p z_{t-p} + \varepsilon_t$

and rewrites it in the error correction form as:

(2.16) $\Delta z_t = \pi_0 + \pi z_{t-1} + \sum_{i=1}^{p-1} \pi_i \Delta z_{t-i} + \varepsilon_t$

where $\pi_0 = A_0$, $\pi = -I + \sum_{j=1}^{p} A_j$, $\pi_i = -\sum_{j=i+1}^{p} A_j$, and ε_t is a vector of iid Gaussian errors.

The key insight of the Johansen testing procedure is that the rank of the matrix π is equal to the number of independent cointegrating vectors existing among the *n* component variables of z_t. The rank of π, in turn, is equal to the number of its non-zero eigenvalues. Denote the estimated eigenvalues of π in descending order as $\hat{\lambda}_1 \geq \hat{\lambda}_2 \geq ... \geq \hat{\lambda}_n$, some of which may be zero. Johansen (1988) proposes the following two test statistics:

(2.17) $\lambda_{trace}(r) = -T \sum_{j=r+1}^{n} \log(1 - \hat{\lambda}_j)$

and

(2.18) $\lambda_{\max}(r) = -T \log(1 - \hat{\lambda}_{r+1})$

where T is the number of observations in the data. The first, called the *trace test*, has as its null hypothesis that there are at most *r* cointegrating vectors, against the alternative that there are more than *r* cointegrating vectors in the system. The second, the *maximum eigenvalue test*, tests the null hypothesis of exactly *r* cointegrating vectors, against the alternative of exactly $r+1$ cointegrating vectors. Both tests have a non-standard distribution, whose critical values are found in Johansen and Juselius (1990).

Given that the $n \times n$ matrix π has rank *r*, it can be decomposed as $\pi = \alpha\beta'$, where α and β are both $n \times r$ matrices having full column/row rank, i.e., rank *r*. As such, β' is the *cointegrating matrix*: its *r* linearly independent rows correspond to the *r* independent cointegrating vectors; that is, each of the *r* component series of the vector $\beta' z_{t-1}$ is stationary. Thus the elements in each row of β' represent the *long-run parameters* because they link the variables in the long-run. And α is the matrix of the *speed of adjustment parameters*.

The decomposition $\pi = \alpha\beta'$ facilitates the interpretation of (2.16) as an error correction model. To see this, note the form of the *k*th equation in (2.16): the left-hand side consists of the single term Δz_{kt}, while the right-hand side, among other terms, contains a linear combination of the *r* stationary variables in $\beta' z_{t-1}$; the coefficients of this linear combination are exactly those from the *k*th row of α. If we think of the values of the *r* stationary variables in $\beta' z_{t-1}$ as deviations from the long-run equilibrium (or disequilibrium "errors") in the previous period $t-1$--- since we interpret those *r* stationary variables as representing the long-run equilibrium relationships among the *n* variables of z_t --- then these disequilibrium errors enter the equation for the *k*th variable z_{kt}, via the speed of adjustment parameters (i.e. elements of the *k*th row of matrix α), to make the necessary "correction" (i.e. adjustment) of this variable, toward a long-run equilibrium in the current period *t*. In fact, if *n* $I(1)$ variables are cointegrated, then there must exist an error correction representation for these variables in the form of (2.16), and vice versa. This result is known as the Granger representation Theorem.

A practical issue that arises when applying the Johansen procedure is the appropriate choice of the lag length *p* in the VAR model specification of (2.15). One way to handle this is by invoking some information criterion such as Akaike Information Criterion (AIC) which typically minimizes some function involving the sum of squared residuals, the number of parameters in the model and the number of observations used in the estimation.

Yet another issue involved in the Johansen procedure concerns the form of the deterministic term π_0 in (2.16): an unrestricted π_0, or a restricted π_0. In the former case, there is a linear trend in each of the variables of z_t, while in the latter, there is an intercept term in each of the *r* cointegrating vectors. Which specification is appropriate given the data can be determined by employing a likelihood ratio test as described in Johansen (1991) and Johansen and Juselius (1990).

Besides providing maximum likelihood estimation of α and β', the Johansen procedure also allows for testing hypotheses about the elements of α and β'. In particular, one is often interested in testing whether a particular row of α is zero. Denote the *k*th row of α as α_k, then the *k*th equation in (2.16) has the following form:

(2.19) $\Delta z_{kt} = \alpha_k \beta' z_{t-1} + \textit{other terms}$

If $\alpha_k = 0$, then the *k*th variable z_{kt} does not respond to those disequilibrium errors from the previous period $t-1$, and thus the remaining $n-1$ variables in z_t do all of the adjustment, through the error correction mechanism, to induce a long-run equilibrium in the current period *t*. In this case, z_{kt} is said to be *weakly exogenous*. A likelihood ratio test is available for testing the weak exogeneity of a particular variable in the system.

Various linear restrictions on β', the cointegrating matrix, can also be tested. In our analysis, we are interested in whether a particular variable in the system can be excluded from the cointegration relationships, or, equivalently, whether that variable's coefficient appearing in the cointegrating vectors is significantly differently from zero. Again, the Johansen procedure offers a likelihood ratio test for this purpose.

2.4.3.2 Results

Let $z_t = (XR_t, ID_t)'$ be the vector containing the two variables under investigation. Akaike Information Criterion (AIC) selects a lag length of $p = 4$ for the VAR model in (2.15). As a robustness check, we apply another criterion, Final Prediction Error Criterion of Hsiao (1979) to the system, which gives the same lag length for the model as AIC.

Results from both λ_{trace} and $\lambda_{\max}$ tests are reported in Table 2.2. As shown, λ_{trace} test rejects the null of $r = 0$ in favor of the alternative $r > 0$, but does not reject the null of $r \leq 1$, both at the 95% significance level. Although this result alone is sufficient to pin down the number of cointegrating vectors, results of the $\lambda_{\max}$ test provide further evidence: it rejects the null of $r = 0$ in favor of the alternative $r = 1$, but does not reject the null of $r = 1$, both at the 95% significance level. Thus, we conclude that the cointegration rank is one, i.e., there exists a single cointegration relationship between the two variables XR_t and ID_t.

Table 2-2 Trace and Maximum Eigenvalue Tests

Null Hypothesis	Alternative Hypothesis	Test Statistic	95% Critical Value
λ_{trace} tests			
$r = 0$	$r > 0$	19.09	15.34
$r \leq 1$	$r > 1$	0.45	3.84
$\lambda_{\max}$ tests			
$r = 0$	$r = 1$	18.63	14.07
$r = 1$	$r = 2$	0.45	3.76

Note: SAS Version 8 was used to calculate the test statistics.

Conditional on $r = 1$, a likelihood ratio test is performed for the restriction on π_0. This test, as shown by Johansen (1991), has a χ^2 distribution with $n - r = 1$ degree of freedom. The calculated test statistic is 5.44 with an associated *p*-value of 0.0196 (the 95% critical value for the χ^2 distribution with 1 degree of freedom is 3.84), so the null hypothesis that the deterministic term π_0 has a restricted form can be rejected at the 95% significance level, implying that the cointegrating vector does not contain an intercept and that (2.16) includes a nonzero constant term.

Having confirmed the existence of a single cointegrating vector and the form of the deterministic term in (2.16), maximum likelihood estimation produces the error correction models for the exchange rate series XR_t and the interest differential series ID_t respectively as:

$$\begin{aligned}(2.20)\quad \Delta XR_t &= -0.00046428 - 0.00318 ecm_{t-1}\\ &- 0.04756\Delta XR_{t-1} + 0.01526\Delta ID_{t-1}\\ &- 0.01415\Delta XR_{t-2} + 0.00202\Delta ID_{t-2}\\ &+ 0.00818\Delta XR_{t-3} + 0.00694\Delta ID_{t-3} + \varepsilon_{1t}\end{aligned}$$

and

$$\begin{aligned}(2.21)\quad \Delta ID_t &= -0.00653 + 0.01840 ecm_{t-1}\\ &- 0.07461\Delta XR_{t-1} + 0.13656\Delta ID_{t-1}\\ &+ 0.06794\Delta XR_{t-2} + 0.03686\Delta ID_{t-2}\\ &+ 0.15930\Delta XR_{t-3} - 0.02570\Delta ID_{t-3} + \varepsilon_{2t}\end{aligned}$$

where the error correction term is given by $ecm_{t-1} = XR_{t-1} - 0.30251 ID_{t-1}$. Thus, in the notation introduced above, $\alpha = (-0.00318, 0.01840)'$, which is the matrix of the speed of adjustment parameters, and $\beta' = (1, -0.30251)$, the cointegrating matrix.[9]

The likelihood ratio test for weak exogeneity, which Johansen (1991) shows is distributed as χ^2 (with one degree of freedom in the present case since there is one restriction on each α_k), yields test statistic values 8.40 and 8.06 for XR_t and ID_t respectively. Since both values exceed the 95% critical value of 3.84, the null hypothesis that XR_t (or ID_t) is weakly exogenous can be rejected. This means that *both* variables XR_t and ID_t respond to the disequilibrium error in making the adjustments toward the system's long-run equilibrium. Finally, likelihood ratio test for the significance of the ID_{t-1} coefficient in the above expression of ecm_{t-1}, which is also distributed as χ^2 with one degree of freedom, rejects the null hypothesis that this coefficient is zero, since the calculated value of the test is 15.39 which exceeds the 95% critical value of 3.84.

The adjustment parameters $\alpha = (-0.00318, 0.01840)'$ and the long-run parameters $\beta' = (1, -0.30251)$, taken together, have the "correct" signs. First, the negative coefficient in

[9] The cointegrating matrix (or space) is uniquely identified if the leading coefficients of all the cointegrating vectors are normalized to one.

(2.20) and the positive coefficient in (2.21) for the ecm_{t-1} term mean that when, for example, $ecm_{t-1} = XR_{t-1} - 0.30251 ID_{t-1} > 0$ and other things remain unchanged, XR_t decreases and ID_t increases so that the system is brought closer to its long-run equilibrium in period *t*, which is exactly how the error correction mechanism is supposed to work. Second, since ID_t is defined as the British interest rate minus the US interest rate and XR_t is expressed in dollar/pound, the positive coefficient for the ID_{t-1} term in (2.20) [i.e., (-0.00318)*(-0.30251)>0] implies that, other things being equal, an increase in the interest differential of the two currencies tends to cause a subsequent appreciation of the British pound. This is in agreement with how the currency traders observe the exchange rate moves in response to an interest differential change in the foreign exchange market, as elaborated in the preceding sections.

2.4.4 Forecasting Comparison with the Unrestricted VAR Model

As a last bit of the empirical analysis, we assess the adequacy of the above ECM model by comparing its forecasting ability with that of a comparable VAR model. The VAR to be considered is one based on the first differences of the variables, namely, $\Delta z_t = \left(\Delta XR_t, \Delta ID_t\right)'$ with the same lag structure as the ECM in (2.16), which we denote by DVAR. Note that this specification of DVAR makes it a model that is "nested" in the ECM model, since the latter has the form of a DVAR augmented with the error correction term πz_{t-1}. Thus, the marginal contribution of the error correction term can be assessed in term of the model's forecasting ability, which in turn can serve as an indication of the adequacy of the model specification.

For our purpose we only need the exchange rate equation from the DVAR, and it is given by the usual maximum likelihood estimation as:

$$
\begin{aligned}
(2.22)\quad \Delta XR_t &= 0.00015514 - 0.03859 \Delta XR_{t-1} + 0.01404 \Delta ID_{t-1} \\
&\quad - 0.00443 \Delta XR_{t-2} + 0.00037413 \Delta ID_{t-2} \\
&\quad + 0.01641 \Delta XR_{t-3} + 0.00492 \Delta ID_{t-3} + \eta_{1t}
\end{aligned}
$$

The forecasting comparison is conducted as follows. Taking as known the values of the exchange rate variable XR_t and the interest differential variable ID_t up to period $t-1$, the estimated models in (2.20) and (2.22) are used to forecast the exchange rate change in period *t*, respectively. The forecast error is defined as the difference between the actual change and the forecasted change from each model. We use the root mean square error (RMSE) as a criterion to judge the relative forecasting performance of the two models.

Recall that the ECM and DVAR were initially estimated for the sample period from January 2001 until December 2003, using the daily data. Our forecasting exercise essentially employs a one-step-ahead out-of-sample forecasting procedure, and uses daily data from the first two months of 2004 which have a total of 41 usable observations. Since 4 lagged values of each variable are needed to calculate the first RMSE, a total of 37 RMSEs are obtained for each of the two model specifications and are reported in Table 3.

The results in Table 2.3 show that the ECM model outperforms the DVAR model in 33 out of all 37 forecast periods, by the RMSE criterion. Presumably this difference in out-of-sample forecasting ability of the two models stems from the role played by the error correction term in the ECM specification. This suggests that there is likely considerable information contained in the disequilibrium error that is relevant for the exchange rate change, and also gives us additional confidence as to the adequacy of the ECM model in describing the dynamic process of the two variables under investigation.

Table 2-3 Forecasting Comparison between ECM and DVAR

Number of Periods	RMSE from ECM	RMSE from DVAR	Comparison
1	0.001952997	0.001825336	
2	0.005200512	0.005311347	*
3	0.006780523	0.006900219	*
4	0.005912105	0.006012086	*
5	0.00529591	0.005384015	*
6	0.005810611	0.005842727	*
7	0.005864123	0.005846308	
8	0.007390023	0.00728843	
9	0.007716713	0.007552234	
10	0.009583386	0.009592167	*
11	0.009321197	0.009347438	*
12	0.009235342	0.009267595	*
13	0.009484942	0.009511817	*
14	0.00934222	0.009345481	*
15	0.009171004	0.009201905	*
16	0.008931858	0.008948854	*
17	0.008704283	0.008712472	*
18	0.008463336	0.008468768	*
19	0.008240141	0.008248846	*
20	0.008299598	0.008332706	*
21	0.008121555	0.008149856	*
22	0.00793502	0.007962481	*
23	0.007832563	0.007868824	*
24	0.007802126	0.007849312	*
25	0.007779789	0.007832675	*
26	0.007900937	0.007956972	*
27	0.007753779	0.007808905	*
28	0.007651972	0.007703461	*
29	0.007528395	0.007581298	*
30	0.007505658	0.007566017	*
31	0.007565753	0.007616156	*
32	0.007459926	0.007512712	*
33	0.008032431	0.008059074	*
34	0.007927781	0.007960779	*
35	0.008080284	0.008133515	*
36	0.008182141	0.008220125	*
37	0.008116216	0.008148125	*

Note: An asterisk (*) denotes that the RMSE from ECM is smaller than the RMSE from DVAR.

2.5 CONCLUSION

Inspired by the reasoning behind the uncovered interest parity (UIP) condition in the theoretical literature and drawing on an observation made by the foreign exchange market practitioners, we have investigated in this paper the interrelationship between the US dollar – British pound exchange rate and the interest differential of the two currencies, at the daily frequency. The two variables are found to be cointegrated with cointegration rank one, and the associated error correction model (ECM) is derived whose parameters are demonstrated to have correct signs conforming to the empirical regularity observed by the market practitioners. The ECM is also shown to possess better out-of-sample forecasting ability compared with a comparable VAR model in first difference, thus confirming the important role played by the error correction term in the model.

The existence of a cointegration relationship means that the interest differential serves as an anchor for the two currencies' exchange rate in the long-run, which is the main thesis of this paper. In light of the connection between cointegration and common stochastic trends (Stock and Watson, 1988), one single cointegration relationship in a two-variable system implies that the exchange rate and the interest differential share one common stochastic trend; this common trend, itself non-stationary, can be interpreted as being determined jointly by the two countries' macroeconomic variables such as money supply and inflation rate.[10] Clearly, the exchange rate and the interest differential *both* are closely related to those macroeconomic variables. And this provides an alternative interpretation of the results in this paper.

The present work can be extended in at least two directions. First, we have only used the 3-month Eurocurrency rate in our analysis. Though it is argued that the 3-month rate seems to be the most appropriate maturity for our purpose, including other maturities in the ECM might have the potential to improve the model's forecasting ability, since those other rates may contain information relevant for the exchange rate that is absent in the 3-month rate. Another extension concerns the data frequency. By the very nature of the speculative activity in the foreign exchange market, the exchange rate should respond to the change in interest differential much more swiftly than at the daily frequency. Our analysis would be better carried out using higher

[10] At the daily frequency, these determinants are probably more appropriately interpreted as news or expectations about the macroeconomic variables.

frequency, preferably tick-by- tick, real time transaction data, but whether this is possible in the future depends on the availability of such data.

3.0 DIRECT INTER-DEALER TRADING IN THE FOREIGN EXCHANGE MARKET: A BARGAINING APPROACH

3.1 INTRODUCTION

Markets for financial assets are organized differently in terms of the specific trading mechanisms employed. While much of economics regards those trading mechanisms as largely irrelevant for the functioning of the market and generally relegates them to the economic “black box,” it seems evident that, in order to gain a deeper understanding of how financial markets function to allocate resources, one must consider the impact different trading rules have on issues like the price formation process. Research in the area of market microstructure represents an endeavor in this direction (O’Hara, 1995). Until recently most of the market microstructure literature has focused on the study of equity markets such as the New York Stock Exchange (NYSE), NASDAQ, and the London Stock Exchange (LSE).

With an average daily turnover volume of almost US $1.9 trillion and still growing, the foreign exchange market is by far the world’s largest financial market (BIS, 2004). Given its importance and the fact that it shares many common features with the equity markets, the foreign exchange market would seem a natural candidate for the study of financial market microstructure. However the foreign exchange rate, the relative price of two countries’ currencies, traditionally has been treated in economics as a macro phenomenon; that is, it is viewed as being determined by macroeconomic variables, the so-called “fundamentals” which among other things include GDP, money supply, interest rate and inflation rate in the two countries. Despite the considerable research effort over the past few decades devoted to explaining exchange rate movements via macro channels, it is now widely acknowledged that, by and large, the macro approach to exchange rate determination has been unsuccessful, especially over short time horizons. Perhaps the most severe indictment against this approach comes from the finding of Meese and Rogoff (1983) that macro-based models of exchange rate determination have inferior out-of-sample forecasting ability even compared with a naïve random walk model. Some twenty years later and many more papers written on the subject, the same author concludes that the failure of macro-based models is "still true" (Rogoff, 2001). This puzzling failure has led economists to suspect that they were probably looking in the wrong place for the "true" determinants of the exchange rate. To cite Flood and Taylor (1996), "Given the exhaustive interrogation of the macro fundamentals in this respect over the last twenty years, it would seem that our understanding of the short-run behavior of exchange rates is unlikely to be further enhanced by further examination of the macro fundamentals. And it is in this context that new work on the microstructure of the foreign exchange market seems both warranted and promising."

Lyons (2001) summarizes the work done so far on the foreign exchange market microstructure. Due to the similarity of the foreign exchange market to the equity markets, most theoretical models on the former are borrowed from the existing ones on the latter, though the empirical work uses distinctively foreign exchange data. The purpose of this paper is to contribute to the theoretical front of the microstructure literature by exploiting a distinctive feature of the direct inter-dealer foreign exchange trading. We model the trading process as a sequential, alternating-offer bilateral bargaining game between two dealers whose motives for the trading might differ. While almost all existing work on inter-dealer trading adopts an auction-based approach, we argue that it is only appropriate for modeling the *brokered* inter-dealer trading. Ours thus represents the first attempt to analyze an important segment of the inter-dealer market which has thus far left unmodeled in the literature, using a modeling apparatus that accords particularly well with the mechanism employed in the actual trading process.

In contrast to the assumption of homogeneous dealers in most literature on the subject, which allows for the analysis to be focused on the risk-sharing aspect of the trading, we assume that dealers are heterogeneous; specifically, in a bilateral setting, one dealer is assumed to be a hedger and the other a speculator, with the latter possessing better information about the future exchange rate change. Still, we model both dealers as being risk averse so that risk-sharing is also a relevant element in the dealers' trading decisions. Thus, in our model, inter-dealer trades are motivated by both risk-sharing and information-exploitation considerations. We are able to completely characterize the perfect Bayesian equilibrium of the bargaining game which is unique under some plausible restriction on the rule for updating beliefs on the part of the uninformed dealer. The result highlights the important role of informational asymmetry in determining the outcome of the bargaining game. In some cases, trades may break down (i.e., dealers remain autarkic) if the informed dealer deems it unprofitable to engage in any trade with the uninformed dealer, even though she may potentially "skim" all the informational rents.

This paper is also a contribution to the large literature on bargaining with several novel features. First, unlike most existing papers where the agents involved in the bargaining have deterministic valuations of the underlying asset being bargained over, in the model of this paper, the bargainers' respective valuations of the asset's future payoff are stochastic, which is a necessary consequence of the uncertain nature of the currency exchange rate. Second – and this is related to the first – in the traditional divide-the-pie bargaining models with incomplete information, the incomplete information is with regard to the counterparty's valuation of the object, whereas in our model the incomplete information takes the form that the uninformed dealer is uncertain about what superior payoff-relevant information the informed dealer has. And third, we derive a procedure-invariant result, in contrast to other standard results in the literature where the outcome is generally dependent upon the bargaining procedure adopted.

In the next section, a brief descriptive account is provided of the foreign exchange market with emphasis on those aspects of the market that are relevant for the modeling of inter-dealer trading. We also review some related work in the area. Section 3 describes the bargaining model. In Section 4, we analyze the perfect Bayesian equilibrium of the model where two different bargaining procedures are considered and their outcomes compared. Section 5 concludes.

3.2 STRUCTURE OF THE FOREIGN EXCHANGE MARKET AND LITERATURE REVIEW[11]

Without a centralized exchange and with a decentralized global network of players, the foreign exchange market is essentially an over-the-counter (OTC) market. The players in this market, roughly classified, fall into one of the three types: customers, dealers, and inter-dealer brokers. And trades are correspondingly classified into three categories: customer-dealer, brokered inter-dealer, and direct inter-dealer, each having its own distinct characteristics.[12]

In the customer-dealer trading, dealers act as market-makers in the sense that they quote two-way bid-ask prices to their customers. Customers, whether retail or wholesale, in general do not have access to the brokers so that dealer banks are the only venues where customers can trade.[13] Order flows from customers, from the dealer's perspective, are largely stochastic in nature, which subsequently become an important source of the dealer's inventory imbalances. For the purpose of the present paper, the role of the customer-dealer trading is exogenous; bilateral inter-dealer trading occurs only after one dealer has transacted a deal with his own customer.

When a dealer receives an inventory shock as a result of customer trades, he has potentially two options to lay off his undesirable position in the inter-dealer market: through a broker or by trading directly with another dealer. In the brokered market, the broker acts as a go-between for the client dealers, and keeps a running list of limit orders from participating dealers. When a dealer contacts the broker, the latter will give him the best price available (highest bid price and/or lowest ask price) among all the limit orders outstanding. The dealer can then hit the bid, take the ask, join the bid and/or ask, or improve the bid and/or ask.[14] A noticeable feature of

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[11] In terms of instruments traded, the foreign exchange market comprises a spot market, a forward market, and markets for derivatives (futures, swaps and currency options). The description in this section will focus on the spot market only.

[12] Relative shares of these transaction categories by trading volume are: customer-dealer transactions account for approximately 25%, and inter-dealer transactions (both brokered and direct) for approximately 75% of total trading volume (Danielsson and Payne, 2002). Although constantly changing over time, Cross (1998) reports a share of 41% for the inter-dealer trading volume handled by brokers.

[13] As an empirical matter, foreign exchange dealers derive most of their trading profits from customer trades (Yao, 1998).

[14] Prior to 1990s, all brokering in the inter-dealer market was handled by the so-called voice-brokers through dedicated telephone lines connecting the broker and client dealers. The advent of electronic brokerage systems (e.g.,

this trading mechanism is that it involves a time dimension so that the market is run as a continuous rather than a batch auction.

A dealer may also directly contact another dealer to ask for quotes on the currency of interest. While technological advances have also enabled greater trading efficiency,[15] the bilateral, one-on-one nature of direct trading between dealers is preserved. The dealer being contacted quotes buy and sell prices for the amount requested, and the originating dealer may then decide whether to accept or pass the other's quotes. There is no negotiation or haggling between the two, but if the originating dealer passes the quoting dealer's offer and if the latter asks for a reciprocating quote, the former must oblige by providing his/her own quote to the counterparty, which is a convention in the dealer circle.

While the classification of instruments, players, and trade types in this market is fairly clear, the working of the foreign exchange market is not well understood and remains something of a mystery, from the theoretical point of view. Many authors have studied the inter-dealer trading focusing mainly on the equity markets which have a multiple-dealer structure such as NASDAQ and LSE.[16] They discuss among other issues motives for inter-dealer trading (e.g., Ho and Stoll, 1983; Volger, 1997; Werner, 1997; Reiss and Werner, 1998; and Viswanathan and Wang, 2004). Among those, risk-sharing is an important trading motive for the dealer because of the minimum quote size in the customer-dealer trades which at times can cause significant inventory imbalances for the dealer. When such imbalances arise a dealer seeks to unwind his undesired position in the inter-dealer market, resulting in the foreign exchange "hot-potato" (Lyons, 1997). To isolate the risk-sharing motive of trading from trading motives based on private information, dealers are often presumed to be homogeneous with respect to their valuation of the asset, as in Ho and Stoll (1983), Volger (1997), and Werner (1997). Other papers in this area dealing exclusively with the foreign exchange market include Lyons (1995, 1996a, 1996b) and Perraudin and Vitale (1995).

Regarding the modeling tools used in the theoretical literature on inter-dealer trading, most existing papers adopt an auction approach. For example, Werner (1997) uses a double

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EBS and Reuters Dealing 2000-2) has greatly improved price transparency and efficiency of order execution. However, the essential features of the brokered market as a continuous auction have largely remained unaltered.

[15] Reuters Dealing 2001-1, the direct inter-dealer counterpart of Reuters Dealing 2001-2, has replaced the traditional dedicated telephone lines between dealers that were in use prior to mid-1980s.

[16] NYSE's "specialist" system does not qualify as an example of multiple-dealer market since a particular stock is assigned to only one dealer who acts as a broker-dealer to make the market in that stock.

auction format in which dealers simultaneously post buy and sell orders of a fixed amount, and a single market-clearing price is then decided by a pre-specified rule. In the auction model of Volger (1997), dealers are allowed to submit to an auctioneer demand schedules which unlike Werner's single-price orders are a continuous set of price-quantity pairs. Here, again, orders are submitted simultaneously and the market clears at a single price. Viswanathan and Wang (2004) also employ an auction approach in their analysis of inter-dealer trading. All these papers represent an attempt to model the strategic interactions among dealers following their respective trading with customers. Due to the presence of an explicit auctioneer in their models, these studies are most appropriately viewed as modeling the *brokered* inter-dealer trading where the auctioneer plays the role of an inter-dealer broker. But they are not precisely consistent with the institutional realities of the brokered inter-dealer trading. In the actual brokered inter-dealer market, the matching of buy and sell orders from dealers is conducted as a *continuous* auction (see footnote 4), rather than a batch auction as described in the models of Werner, Volger, and Viswanathan and Wang. To the extent that all economic models abstract from reality to some degree, these auction models may be seen as a first approximation to the actual market setting. Nevertheless, the mechanics of direct inter-dealer trading are so conspicuously different from those of the brokered trading as to defy any meaningful modeling by the use of auction.

In a population context, Lyons (1997) develops a simultaneous trade model of the foreign exchange inter-dealer market. There all dealers are assumed to use identical quoting strategies, and a linear, symmetric, market-wide equilibrium price is derived as the outcome of dealers' collective strategic behavior. Though also qualifying as a strategic microstructural model, his model does not treat dealers' strategic interactions at the more micro level of individual transactions. Further, without any explicit reference to the specific mechanism used for the trading, the result should be viewed as describing generically the outcome emerging from both trading modes, brokered and direct. However, these two different types of inter-dealer trading clearly merit different treatments; after all, the very goal of the microstructure theory is to achieve a better understanding of the performance of the market, through a closer examination of the particulars of the trading environment which macro approaches traditionally ignore.

Auction and bargaining are among the most researched topics in game theory as two basic price-generating mechanisms in a strategic setting. Whereas auction is obviously an appropriate modeling tool in an environment where the outcome of a game is determined by the collective action of a large number of players, in a bilateral setting such as the foreign exchange direct inter-dealer trading, bargaining captures the rules of the game more accurately and thus is more suitable for the analysis.

Since the seminal paper by Rubinstein (1982) in which non-cooperative game theoretic concepts are elegantly applied to a bargaining situation, numerous papers have been written on the subject, and the theory of bargaining has found many applications in other fields of economics. Indeed, the literature is too extensive to allow for a comprehensive survey here.[17] Suffice it to mention here that research has been pursued along several lines, such as whether the model assumes symmetric or asymmetric information and whether one bargainer makes all the offers or offers are made alternately by both parties.

3.3 DIRECT INTER-DEALER TRADING AS A BARGAINING GAME

As noted above, while the brokered inter-dealer foreign exchange trading is best modeled as an auction game (batch or continuous), the mechanics of direct inter-dealer trading conform most closely to a bargaining game between two dealers. In the latter trading mode, the dealers contact each other, either by telephone or on screens electronically, asking for quotes on a currency they are interested in trading. A typical direct deal goes like this (Cross, 1998):

Dealer 1: "Spot dollar-swissie on ten dollars, please."

Dealer 2: "Dollar-swissie is 1.4585-90."

Upon receiving dealer 2's quotes, one of the following happens:

Either

Dealer 1: "I buy (sell)."

Dealer 2: "Done. I sell (buy) ten dollars at 1.4590 (1.4585)."

or

Dealer 1: "Pass."

[17] A concise introduction to the subject is given in Muthoo (1999); more technical coverage can be found in Roth (1985), Osborne and Rubinstein (1990), and Ausubel, Cramton and Deneckere (2002).

In the above conversation, "ten dollars" is shorthand in the profession for 10 million US dollars[18], and "1.4585-90" is to be understood as follows: Dealer 2 is willing to buy 10 million US dollars from dealer 1 at a price of 1.4585 US dollars per Swiss Franc (bid price), and is willing to sell the same amount to dealer 1 for 1.4590 US dollars per Swiss Franc (ask price). If dealer 1 "passes" dealer 2's quotes, then the trading protocol dictates that dealer 1 must reciprocate quotes to dealer 2 if the latter so asks.[19] At this stage, another conversation similar to the above one takes place with the roles of the two dealers reversed. If the second-stage offer of dealer 1 is not accepted by dealer 2, the trading session ends with no more conversation taking place.

Given this sequential, reciprocal nature of the direct inter-dealer trading, we model the trading process as a two-stage, alternating-offer bargaining game between the two dealers. We noted in the previous section that risk-sharing explains much of inter-dealer trading in the foreign exchange market. This explanation entails not only the assumption that dealers are all risk-averse, but also the assumption that dealers are homogeneous in their trading motives. However, the second assumption is not a realistic one. As Flood (1991) points out, "Research into the microstructure of the foreign exchange market should presume such heterogeneity among market-makers … Further, it is well known that 'taking a view,' that is, speculating on future prices, is routine for many participants. To omit this heterogeneity from a model is to ignore an important characteristic of the market." In view of these comments, the dealers in our model are assumed to possess differential information concerning the future exchange rate change.[20]

In the bargaining literature, it is a well known fact that outcomes of a bargaining game depend crucially on the relative bargaining power of the two parties. In the context of our model, such bargaining power takes the form of a dealer's relative degree of risk aversion as well as the superiority of one dealer's private information over that of another. To isolate the effect of

[18] In the direct inter-dealer trading, the quantity traded is usually one of a handful of "customary amounts." (Flood, 1991).

[19] Indeed, part of the advantage to a dealer of using brokered rather than direct trading is "the freedom not to quote to other market-makers on a reciprocal basis, which can be required in the direct market" (Flood, 1991).

[20] Such superior information of one dealer over another could be, for example, the result of in-house research; see Ito, Lyons and Melvin (1997) for empirical evidence on the existence of private information in the foreign exchange market.

private information on the bargaining outcome, we assume that the two dealers have the same degree of risk aversion with respect to the potential profit from trading.

The two dealers' preferences are identical, given by a simple form of mean-variance derived utility function: $u(\tilde{Z}) = E(\tilde{Z}) - Var(\tilde{Z})$, where $\tilde{Z}$ is dealer's stochastic trading profit.[21] The fact that dealers' utility depends negatively on the variance of the profit implies that dealers are risk averse. For concreteness and ease of exposition, the two traded currencies are called Euro and US dollar (USD), and the exchange rate is expressed in American terms, i.e., in amounts of USD per Euro. The argument in the dealers' utility function, $\tilde{Z}$, is measured in USD.[22] Further, throughout this paper, we express the exchange rate as a premium or a discount in reference to the prevailing rate in the customer-dealer market during the current period. For example, if the prevailing rate in the current period is 1.1720 USD/Euro, then an exchange rate of 1.1730 USD/Euro will be expressed as $r = 0.001$ (a premium), while an exchange rate of 1.1710 USD/Euro will be expressed as $r = -0.001$ (a discount). Expressing the exchange rate in this fashion allows us to more easily keep track of the trading profits and losses of dealers. Given that dealers typically trade with one another in standardized quantities (see footnote 8), we normalize, without loss of generality, the quantity traded to one Euro.[23]

Dealer 1, after having bought one Euro from his customer at the prevailing exchange rate in the customer-dealer market, wishes to lay off his position by contacting dealer 2 who may want to take a speculative position in Euro. The information structure of the game is as follows. It is common knowledge among all parties that next period's exchange rate, $\tilde{V}$, is uniformly distributed over the interval $[-v, v]$, which is all the information dealer 1 has *ex ante* regarding the future exchange rate change. Dealer 2 has better information than dealer 1 does, in that she knows that next period's exchange rate is uniformly distributed over $[-x, x]$, where $0 \le x \le v$ and is only privately known to dealer 2.[24] From dealer 1's perspective, x is a random variable, denoted by $\tilde{X}$. Without further information, his best guess about $\tilde{X}$ is that it is uniformly

[21] Our rationale for defining the utility function over dealers' profit rather than their total wealth partly comes from the celebrated prospect theory of utility, which argues that people tend to perceive changes (gains or losses) rather than absolute values in making economic decisions (Kahneman and Tversky, 1979).
[22] This is a plausible assumption if the two dealers are both US banks.
[23] The setup is thus similar to the one in the standard bargaining literature where agents bargain over the price of an indivisible object.
[24] These assumptions are consistent with the random walk view of exchange rate distributions; see, for example, Goodhart (1988).

distributed over its range, the interval $[0, v]$. Thus, in this setup, x is the realization of a random variable $\tilde{X}$ which is observed only by dealer 2; dealer 1 knows the distribution of $\tilde{X}$. We also adopt the convention in exchange rate economics to express changes in exchange rate in percentage terms so all values of v and x are restricted to lie between 0 and 1.

Though in practice a dealer always quotes two-way prices (bid and ask) as required by trading conventions, and the other dealer has the choice whether to buy at the ask price or sell at the bid price, our study will only concentrate on one side of the market. More specifically, we suppose that dealer 1 is only interested in laying off his position in Euro, while dealer 2 is only interested in taking a position in Euro; in other words, the two dealers' roles as a buyer or seller in the trade are unambiguously defined. Consequently, a dealer's quoting strategies in this game are restricted to a single bid or ask price but not both.

3.4 CHARACTERIZATION OF THE PERFECT BAYESIAN EQUILIBRIUM

In this section, we analyze the two-stage, alternating-offer bargaining game using as the solution concept perfect Bayesian equilibrium (PBE). PBE is an analytic tool designed for modeling dynamic games of incomplete information. A generalization of both the Bayesian-Nash equilibrium concept for static games of incomplete information and the subgame perfect equilibrium concept for dynamic games of complete information, PBE requires that players' strategies be sequentially rational given their beliefs, and that the beliefs be derived whenever possible from equilibrium strategies using Bayes rule.

A typical problem with most bargaining models of incomplete information is the existence of a large number of equilibria. In order to select some more plausible ones for the game, many authors have imposed various restrictions on the uninformed player's beliefs about his opponent's type after the latter's move. For example, Rubinstein (1985) introduces six different kinds of conjectures the uninformed player can potentially have about his opponent's type, some of which are quite ad hoc. For the present model, we impose a *naïve* conjecture rule on the part of the uninformed dealer: after receiving the informed dealer's quote r to buy Euro, the uninformed dealer's belief about his counterparty's valuation is updated to $\tilde{X} > r$. This simply says that the informed dealer's offer price to buy never exceeds her highest valuation of the currency.

We begin with the analysis of the bargaining game where dealer 1 (he, uninformed) approaches dealer 2 (she, informed) first for quotes on the USD-Euro exchange rate. Let r_1 denote dealer 2's quote to buy Euro in the first stage. Dealer 1, on receiving dealer 2's quote, can either accept or decline it. If he declines, he himself quotes a price of r_2 to sell Euro to his counterparty in the second stage, which dealer 2 then either accepts or declines.

The PBE of this bargaining game is derived by using the usual backward induction. Suppose dealer 1 declines dealer 2's first-stage offer r_1, and makes the counter-offer r_2 in the second stage. Since by staying autarkic dealer 2 can obtain an expected utility of at least zero, dealer 1's second-period quote r_2 will be accepted by dealer 2 if and only if the following holds:

$$(3.1) \qquad u(\tilde{U}_X - r_2) \geq 0$$

where $\tilde{U}_X$ is a random variable uniformly distributed on the interval $[-\tilde{X}, \tilde{X}]$. Given the distribution of $\tilde{U}_X$, we have

$$(3.2) \qquad E(\tilde{U}_X - r_2) = \frac{1}{2}[(-\tilde{X} - r_2) + (\tilde{X} - r_2)] = -r_2$$

and

$$(3.3) \qquad Var(\tilde{U}_X - r_2) = \frac{1}{12}[(\tilde{X} - r_2) - (-\tilde{X} - r_2)]^2 = \frac{1}{3}\tilde{X}^2$$

Note that from dealer 1's perspective $Var(\tilde{U}_X - r_2)$ is a random variable which reflects his uncertainty about dealer 2's information on next period's exchange rate. With these expressions (3.1) becomes

(3.4) $$-r_2 - \frac{1}{3}\tilde{X}^2 \geq 0$$

This is the condition for dealer 2's acceptance of dealer 1's offer r_2 in the second stage. Clearly, r_2 must be negative in order for (3.4) to hold. We can thus restrict our analysis to the case $r_2 \leq 0$. We have

(3.5) $$-r_2 - \frac{1}{3}\tilde{X}^2 \geq 0 \Leftrightarrow \tilde{X} \leq \sqrt{-3r_2}$$

Going back to dealer 2's first stage offer r_1, we consider two cases: $r_1 \geq 0$ and $r_1 < 0$. Given our proposed naïve conjecture on the part of dealer 1, the first case is informative about dealer 2's valuation; specifically, after receiving the quote $r_1 \geq 0$, dealer 1's conjecture about dealer 2's valuation is updated to $\tilde{X} \geq r_1$ so that, from dealer 1's perspective, $\tilde{X}$ is now uniformly distributed on the interval $[r_1, v]$. By offering r_2 to dealer 2 in the second stage, dealer 1 can get an expected utility of[25]

(3.6) $$\begin{aligned} A(r_2) &= \int_{r_1}^{\sqrt{-3r_2}} \frac{1}{v-r_1} \cdot r_2 dx + \int_{\sqrt{-3r_2}}^{v} \frac{1}{v-r_1}[-\frac{1}{12}(2x)^2]dx \\ &= \frac{1}{v-r_1} \cdot r_2(\sqrt{-3r_2} - r_1) - \frac{1}{9} \cdot \frac{1}{v-r_1}[v^3 - (\sqrt{-3r_2})^3] \\ &= \frac{1}{v-r_1}[-\frac{1}{9}v^3 - \frac{2\sqrt{3}}{3}(-r_2)^{\frac{3}{2}} - r_1 r_2] \end{aligned}$$

for which

(3.7) $$A'(r_2) = \frac{1}{v-r_1}[\sqrt{3}(-r_2)^{\frac{1}{2}} - r_1]$$

[25] If $\sqrt{-3r_2} < r_1$, then by (3.5) dealer 1's offer r_2 will never be accepted by dealer 2 so we may consider only the case $\sqrt{-3r_2} \geq r_1$. The first integral in (3.6) is dealer 1's expected utility from dealer 2 accepting r_2, and the second integral is dealer 1's expected utility from dealer 2 declining r_2.

and

$$(3.8) \qquad A''(r_2) = -\frac{\sqrt{3}}{2}\frac{1}{v-r_1}(-r_2)^{-\frac{1}{2}}$$

Solving for $A'(r_2) = 0$, we get $r_2 = -\frac{1}{3}r_1^{\,2}$. And since $A''(r_2) < 0$ we know that $A(r_2)$ has a maximum at $r_2 = -\frac{1}{3}r_1^{\,2}$; the maximum value is

$$(3.9) \qquad A_{\max} = A(-\frac{1}{3}r_1^{\,2}) = -\frac{1}{9}\cdot\frac{v^3 - r_1^{\,3}}{v-r_1}$$

which is always negative.

Now back to the point when dealer 2 has made an offer $r_1 \geq 0$ in the first stage and when dealer 1 has to decide whether to accept or decline this offer. Knowing that he will get a negative maximum expected utility (i.e., the $A_{\max}$ given above) if he declines it and thus has to make a counter-offer himself, dealer 1 will accept dealer 2's first-stage offer which gives him a positive expected utility equal to r_1. But then dealer 2's expected utility will be $-r_1 - \frac{1}{12}(2x)^2 < 0$, where x is the realization of $\tilde{X}$. For dealer 2, this outcome would be worse than if she had made a negative price offer r_1 , which will give her an expected utility of at least zero because she can always decline dealer 1's second-stage offer, even if r_1 is not accepted by dealer 1 in the first stage. Therefore, it is never dealer 2's best strategy to offer $r_1 \geq 0$ in the first stage. Hence, we may restrict our analysis to the case $r_1 < 0$ in searching for dealer 2' best strategy.

Again, suppose dealer 1 declines dealer 2's first-stage offer $r_1 < 0$. This offer is uninformative under deal 1's naïve conjecture in that, after receiving the offer, dealer 1's conjecture about dealer 2's valuation remains the same as the prior: $\tilde{X}$ is distributed uniformly on the interval $[0, v]$. By the condition in (3.5), if $\tilde{X} \in [0, \sqrt{-3r_2}]$, dealer 1's second-stage offer r_2 will be accepted by dealer 2, and hence his expected utility is

$$(3.10) \quad B(r_2) = \int_0^{\sqrt{-3r_2}} \frac{1}{v} \cdot r_2 dx = -\frac{\sqrt{3}}{v} \cdot (-r_2)^{\frac{3}{2}}$$

And for $\tilde{X} \in (\sqrt{-3r_2}, v]$, dealer 2 will decline his offer, so dealer 1's expected utility is

$$(3.11) \quad C(r_2) = \int_{\sqrt{-3r_2}}^{v} \frac{1}{v} \cdot [-\frac{1}{12}(2x)^2] dx = -\frac{1}{9}v^2 + \frac{\sqrt{3}}{3v}(-r_2)^{\frac{3}{2}}$$

Note that a rationale for the above calculations of dealer 1's expected utilities is that he *believes* dealer 2 has better information than he does about the future exchange rate. Dealer 1's expected utility from quoting r_2 to dealer 2 in the second stage is thus given by

$$(3.12) \quad D(r_2) = B(r_2) + C(r_2) = -\frac{1}{9}v^2 - \frac{2\sqrt{3}}{3v}(-r_2)^{\frac{3}{2}}$$

Clearly, $D(r_2)$ attains a maximum value of $-\frac{1}{9}v^2$ at $r_2 = 0$.

If dealer 2's first-period offer r_1 is such that it will be declined by dealer 1, then dealer 2's expected utility is zero which comes from her declining dealer 1's second-stage offer $r_2 = 0$ and remaining autarkic, since accepting $r_2 = 0$ in the second stage would get her a negative expected utility $-\frac{1}{12}(2x)^2 = -\frac{1}{3}x^2 < 0$. Consequently, if dealer 2 wants to get a higher expected utility by inducing dealer 1 to accept her offer in the first stage, she should offer at least $r_1 = -\frac{1}{9}v^2$. However, dealer 2 is willing to make such an offer if and only if she can get a positive expected utility from doing so, that is, if and only if

(3.13) $-r_1 - \frac{1}{12}(2x)^2 = -r_1 - \frac{1}{3}x^2 = \frac{1}{9}v^2 - \frac{1}{3}x^2 > 0$

or, equivalently, $x < \frac{v}{\sqrt{3}}$.

We summarize the above analysis in the following proposition.

Proposition 3.1. *In the two-stage, alternating-offer bargaining game between the informed and uninformed dealers where the informed dealer makes the first offer, if the informed dealer's valuation parameter,* x*, for next period's exchange rate is such that* $x < \frac{v}{\sqrt{3}}$*, an exchange of currencies occurs in which the uninformed dealer accepts the informed dealer's first-stage offer price* $r_1 = -\frac{1}{9}v^2$*; for other valuation parameters of the informed dealer (i.e., for* $x \geq \frac{v}{\sqrt{3}}$*) , there is no exchange between the two dealers.*

The no-trade result when $x \geq \frac{v}{\sqrt{3}}$ is noteworthy. Here the breakdown of trade is attributed to the informed dealer's valuation of the currency; it happens when her views of future exchange rate are such that the riskiness of the currency (indicated by a large x) outweighs the potential informational rents she might gain by trading with the uninformed dealer. On the other hand, the uninformed dealer in this model always has a hedging motive to trade although he believes that his counterparty has better information regarding future exchange rate than he does. This contrasts with the result in Milgrom and Stokey (1982) where, in equilibrium, it is the uninformed trader who forgoes trading with the informed trader to avoid being taken advantage of by the latter.

Research on bargaining has revealed that bargaining procedures generally matter for the bargaining outcome. In the actual foreign exchange inter-dealer trading, any dealer bank may initiate contact with another dealer bank to request quotes on a currency pair, so it is cannot be *a priori* specified which dealer should act as the initiator in a particular trade. In view of this fact, we consider next the alternative bargaining procedure where dealer 1 (he, uninformed) makes dealer 2 (she, informed) an offer first.

Again, let r_1 denote dealer 1's first-stage offer, and let r_2 denote dealer 2's counter-offer in the second stage if she declines dealer 1's offer in the first stage. It is straightforward to show that it is not dealer 2's best strategy to offer dealer 1 a positive r_2, because she could do (weakly) better by offering dealer 1 a negative r_2: in case of dealer 1 declining her offer, she gets a zero expected utility, and in case of dealer 1 accepting her offer, her expected utility is $-r_2 - \frac{1}{3}x^2$. As before, dealer 2's negative offer price r_2 is uninformative under dealer 1's naïve conjecture. Using analyses analogous to those above, it can be shown that dealer 2's second-stage offer r_2 will be accepted by dealer 1 if and only if $r_2 \geq -\frac{1}{9}v^2$, and that dealer 2 will make dealer 1 an offer $r_2 = -\frac{1}{9}v^2$ (which dealer 1 accepts) if and only if dealer 2's valuation of next period's exchange rate is such that $x < \frac{v}{\sqrt{3}}$. To summarize, in the equilibrium of the *subgame* following dealer 2's decline of dealer 1's first-stage offer, dealer 1's expected utility is $-\frac{1}{9}v^2$, and dealer 2's expected utility is $\frac{1}{9}v^2 - \frac{1}{3}x^2$ if $x < \frac{v}{\sqrt{3}}$ and is zero if $x \geq \frac{v}{\sqrt{3}}$.

Now back to the first stage of the game where dealer 1 makes the offer r_1 to dealer 2. It is not dealer 1's best strategy to offer $r_1 < -\frac{1}{9}v^2$, because there is a positive probability that dealer 2 will accept his offer,[26] and if dealer 2 accepts his offer dealer 1's expected utility is equal to r_1 which is strictly less than $-\frac{1}{9}v^2$, so by offering $r_1 < -\frac{1}{9}v^2$ in the first stage, dealer 1's expected utility from the *whole* game is strictly less than $-\frac{1}{9}v^2$. (Recall that dealer 1's expected utility from the subgame following dealer 2's decline is equal to $-\frac{1}{9}v^2$.) On the other hand, an

[26] In fact, this probability is greater than $\frac{1}{\sqrt{3}}$.

offer $r_1 > -\frac{1}{9}v^2$ by dealer 1 will not be accepted by dealer 2 because, regardless of whether $x < \frac{v}{\sqrt{3}}$ or $x \geq \frac{v}{\sqrt{3}}$, dealer 2 can do better by declining this offer and making her own offer in the second stage; consequently dealer 1's expected utility from offering $r_1 > -\frac{1}{9}v^2$ is $-\frac{1}{9}v^2$. And if dealer 1 offers $r_1 = -\frac{1}{9}v^2$, this will leave dealer 2 just indifferent between accepting and declining the offer, regardless of whether $x < \frac{v}{\sqrt{3}}$ or $x \geq \frac{v}{\sqrt{3}}$. Assuming a tie-breaking rule that dealers always trade in the first stage whenever they are indifferent between trading in both stages, the PBE outcome of the bargaining game can be stated as follows.

Proposition 3.2. *In the two-stage, alternating-offer bargaining game between the informed and uninformed dealers where the uninformed dealer makes the first offer, if the informed dealer's valuation parameter,* x *, for next period's exchange rate is such that* $x < \frac{v}{\sqrt{3}}$*, an exchange of currencies occurs in which the informed dealer accepts the uninformed dealer's first-stage offer price* $r_1 = -\frac{1}{9}v^2$*; for other valuation parameters of the informed dealer (i.e., for* $x \geq \frac{v}{\sqrt{3}}$*) , there is no exchange between the two dealers.*

This outcome of PBE is identical to the one in Proposition 3.1. Therefore, in contrast to some standard results in the bargaining literature where different procedures produce different outcomes (e.g. Rubinstein, 1982), we have obtained here a procedure-invariant result. A possible explanation is that informational asymmetry is a more important determinant of the bargaining outcome than the bargaining procedure. As before, it is still the informed dealer who forgoes the trading with the uninformed dealer when she perceives the currency as so risky that no trade with her counterparty will be profitable.

3.5 CONCLUSION

In this paper, we have modeled the direct inter-dealer trading in the foreign exchange market as a two-stage, alternating-offer bargaining game. By assuming heterogeneous and risk-averse dealers, our model incorporates two ingredients that are common for models of inter-dealer trading, namely, risk-sharing and speculation. In the unique perfect Bayesian equilibrium of the game, the bargaining outcome depends on the informed dealer's valuation of the currency's next-period payoff; there could be no trade between the two dealers if this valuation is too low, implying that the bargaining power lies entirely with the informed dealer. For the two alternative bargaining procedures considered, the equilibrium outcomes are identical. This result suggests that, in a bargaining situation characterized by asymmetric information, informational advantage might play a more important role than the bargaining procedure in determining the outcome, as the informed dealer in our model does not suffer the usual second-mover disadvantage when the uninformed dealer makes the first offer.

We can envision at least two extensions for the model's setup. First, we have assumed in our model a pre-specified role for the two dealers, namely, the uninformed dealer as the seller and the informed dealer as the buyer of a currency. This, in effect, leaves the dealer's bid-ask price spread out of the picture. A more general model should allow the role of a dealer to be determined endogenously, and thus allow the inter-dealer trading to take place in a two-way market as it does in reality. Second, although our model attempts to provide a strategic "building block" for the inter-dealer trading at the level of individual pair-wise transactions, a richer model can be constructed which "embeds" ours in a population context. Since the actual inter-dealer trading is genuinely a search and matching process, such a richer model may potentially make use of results from the extensive theoretical literature on search and dynamic matching (see, for example, Corbae, Temzelides and Wright, 2002, 2003; Shi, 1995; Rupert, Schindler and Wright, 2001). We see this as a promising direction for future research on the foreign exchange inter-dealer trading.

Finally we should mention that, although the bargaining model in this paper is placed in the context of the foreign exchange market, it is equally applicable to other securities markets such as the OTC markets for government and corporate bonds. In fact, as long as the trading is of a "direct" bilateral nature (i.e., not intermediated by a broker), and the roles of the traders are symmetric in the sense that both quote buy and sell prices to their counterparty, the particular characteristics of the instruments traded (whether stocks, currencies or bonds) are mostly irrelevant for the analysis. It is for this reason that the present paper can be considered a contribution to the broad literature on financial market microstructure, not just that of the foreign exchange market.

4.0 MORAL HAZARD CONTRACTING AND CREDIT RATIONING IN OPAQUE CREDIT MARKETS

4.1 INTRODUCTION

One of the main characteristics of the credit markets is that lenders and borrowers are constrained by asymmetric information. For example, borrowing firms usually have more and/or better knowledge about the projects to be funded than the lending banks do. Similarly, credit card holders tend to know more about their own financial status and earnings potential and thus more about their ability to repay their debts than the credit card companies. A typical consequence of this informational asymmetry is the occurrence of adverse selection. In addition, lenders are often unable to monitor the borrowers' actions that will affect the returns to them once a loan has been granted, presumably due to regulatory constraints as well as the high costs associated with monitoring. This is the moral hazard problem.

Among the many problems arising in the study of credit markets, the phenomenon of credit rationing has received much attention in the literature. Credit rationing refers to a situation where, among a population of observationally indistinguishable borrowers, some borrowers obtain a loan from the lender while others do not. Since the demand for credit of those who don't get the loan is not met, which implies the existence of excess demand in the market, this matter is seemingly at odds with the basic economic principle that excess demand should be eliminated by a price increase. In a classic paper, Stiglitz and Weiss (1981) seek to explain credit rationing by appealing to the adverse selection problem in the credit market. They consider a model in which the bank cannot observe the riskiness of a firm's project, and the return to the bank is affected by the possibility of firm's default. They argue that, under limited liability on the part of the firm, raising the interest rate of the loan does not necessarily increase the return to the bank. The reason is that, when facing a higher interest rate, only those firms with more risky projects will demand credit while the less risky ones will drop out of the market, because with the increased cost of loan the latter group see their projects as no longer profitable, but the more risky firms go on to purse their projects opportunistically due to their limited liability to the lender (that is, the more risky firms can expect to reap the larger return in case of project's success but does not bear the larger loss if the project fails.) Thus, the risk characteristics of the pool of borrowers demanding credit have been adversely altered; the average borrower becomes more risky with the increased interest rate, from the bank's point of view. And this consideration prevents the bank from raising interest rate to eliminate the excess demand.

A number of criticisms have been raised in the literature about the model of Stiglitz and Weiss. One of them concerns the form of the loan contract between the lender and the borrower. In their model, the loan contract is exogenously specified as a standard debt contract, which requires the borrower to repay a fixed pre-specified amount, and which is offered to all potential borrowers without discrimination. In practice, however, a lender will often try to find a way to sort out borrowers and offer differentiated loan contracts to different borrowers; that is, in an environment of asymmetric information, the lender will try to screen the borrowers if he cannot otherwise distinguish among them. This observation has led some researchers to further examine the question of under what circumstances credit rationing will or will not occur. For example, Bester (1985) explores the idea of screening by collateral and shows that credit rationing does not arise if loans are collateralized. But, the result in Besanko and Thakor (1987) seems to depict a different story where credit rationing returns to the scene, if risk is defined by first-order stochastic dominance rather than by mean-preserving spread as in the above paper of Bester, even though collateral is employed as the screening device. Indeed, the conclusion in the original paper of Stiglitz and Weiss is also reversed when the definition of risk in terms of mean-preserving spread is replaced by one in terms of first-order stochastic dominance (DeMeza and Webb, 1987).[27]

These controversies surrounding the existence of credit rationing in the credit markets illustrate an undesirable aspect of models of credit rationing, namely, the conclusions may depend sensitively on the specification of the model's environment in general. Moreover, in many such models no attempt is made to formulate the loan contracting problem in rigorous

[27] Williamson (1987) establishes the possibility of credit rationing without making explicit reference to a definition of risk; in his model, credit rationing is generated by the problem of costly state verification.

game theoretic terms. As pointed out by Hellwig (1987), once such an attempt is made, the conclusions again become sensitive to the particular structure of the game, and in some cases equilibrium may fail to exist. For example, the model of Bester (1985) simply assumes the existence of an equilibrium and the result of no credit rationing is derived under this assumption. Overall, then, it seems that the question of credit rationing deserves further examination under different institutional arrangements of the market.

In all the above papers, the occurrence of credit rationing is attributed to the existence in the credit market of borrowers' hidden information (e.g., the lender cannot observe the risk characteristics of the borrowers' projects). In another line of research on the problem, Bester and Hellwig (1987) consider the possibility of credit rationing as a result of borrowers' hidden action. The borrower can choose between a "good" and "bad" investment project, characterized by the projects' different return distributions (or riskiness), after having obtained the funds from the lender. With limited liability the borrower's project choice has an impact on the return to the lender, but the loan contract cannot prescribe the choice before the project is undertaken. Under such circumstances credit rationing again arises, this time as a consequence of the *post*-contractural informational asymmetry between the lender and the borrowers, as opposed to the above cases of borrowers' hidden information present at the time of contracting which represent the *pre*-contractural informational asymmetry.

In this paper, we study a hybrid model of the credit market in which pre-contractural and post-contractural informational asymmetry are *simultaneously* present. We reexamine the question of credit rationing in a richer setting characterized by both adverse selection and moral hazard, as compared with those found in the existing literature where the focus is typically on one single aspect of the informational asymmetry. Given the aforementioned sensitivity to the model specification of conclusions about credit rationing, such a re-examination may serve to contribute to our further understanding of the cause of the phenomenon.

The particular form of pre-contratural informational asymmetry in our model is motivated by the observation that credit markets in many less developed countries are rather opaque in the sense that a lender may not easily observe a borrower's financial status when the borrower applies for a loan. This hidden information of the borrower, in the form of pre-existing debts, poses a risk on the lender since it affects the borrower's ability to repay the debt. We model the loan contracting problem between the lender and the borrower as a screening game in which the borrower not only self-selects the lender's contract offers but also subsequently chooses a level of work effort optimally. Since the return to the lender depends partially on the borrower's choice of work effort in a stochastic fashion, this generates endogenously the risk faced by the lender and, in this respect, our model is similar in spirit to that of Bester and Hellwig (1987) where the risk to the lender is also generated endogenously by the borrower's project choice. However, there is at least one important difference. Credit rationing in Bester and Hellwig's model arises because, at the lender's quoted interest rate, all borrowers demand a loan but the funds available to the lender are insufficient to satisfy all needs and consequently the lender must "somehow" ration the borrowers. In our environment, funds availability is not an issue because of the lender's equity funding mode, and credit rationing arises instead from the lender's perceiving the borrowers as too risky and thus forgoing the lending opportunity by quoting a prohibitively high interest rate. Also, in contrast to the use of collateral as the screening device in the papers mentioned above, the lender in our model uses the size of the loan to screen borrowers as collateral is no longer a feasible means with the existence of borrowers' prior debts. Depending on some parameter values of the model, credit rationing may or may not occur in the unique subgame perfect equilibrium of the screening game. When credit rationing does occur, it is the more risky (i.e., higher-debt) borrowers who drop out of the market as a result of self-selecting the loan contracts offered by the lender.

The study in this paper is also related to the literature on non-exclusive contracts (e.g., Bizer and DeMarzo, 1992, 1999; Bisin and Guaitoli, 2004; Bisin and Rampini, 2002; Kahn and Mookherjee, 1998; and Park, 2004). Among those Park's (2004) paper is closest to ours but with some differences. He considers the moral hazard contracting problem between one single borrower and a lender in a model in which the borrower decides on an interim wealth level which subsequently becomes his private information, through borrowing in an outside credit market, before contracting with the lender. In our model, a borrower's pre-existing debt is also her private information at the time of contracting with the lender; however, unlike the borrower in Park's model who borrows from the outside source for the purpose of smoothing consumption, the borrowers in our model try to obtain a loan from the lender in order to fund a productive project. Also, the focus of Park's paper is on incentive provision and not on credit rationing because it is not a relevant issue with just one single borrower, while the latter is the main focus of our paper as the lender faces a population of heterogeneous borrowers in our model.

The rest of the paper is organized as follows. Section 2 presents the formal model and describes some institutional details of the credit markets. Section 3 analyzes the subgame perfect equilibrium for the situation of symmetric information, as a preparation for and comparison with that in the situation of asymmetric information. We then examine in Section 4 the screening game where the subgame perfect equilibrium is completely characterized under different parameter values of the model, and its relation to credit rationing is analyzed. Conclusion and extensions are contained in Section 5.

4.2 THE MODEL

4.2.1 Opaque Credit Markets and the Coexistence of Multiple Debts

Borrowers in the credit markets often have multiple sources from which to obtain funds. The consumer credit market provides a good example. In the United States, it is common for consumers to hold several credit cards issued by different lending banks. Bisin and Guaitoli (2004) estimate that a typical American household has more than seven credit cards on average. Most credit card debts are unsecured and thus are subject to default. As documented by Petersen and Rajan (1994, 1995), small businesses are also frequently able to borrow from multiple lenders. In Europe, multiple sources of credit are even more prevalent (see, for example, Detragiache, Garella and Guiso, 2000). In all these instances, debt owed by a borrower to one lender can impose an externality on another lender, because the higher the borrower's total debt the higher the risk of default to a lender.

While it is practically impossible to restrict borrowers' access to multiple sources for loans, institutions have been created to alleviate the problem of informational asymmetry between the lenders and borrowers that may arise in their absence. Credit bureaus are such an example. Lenders participating as members of a credit bureau share information, for example, on their common debtors' financial status. Thus, when a consumer applies for a line of credit at a lender bank, her total indebtedness to date, among other things, will be checked by the bank using information gathered from various credit bureaus. If it is determined that her total debt is too high relative to her perceived ability to repay (e.g., as indicated by her annual income), the application will likely be turned down.

Although countries like the United States and Britain have a relatively mature system of consumer information sharing among lenders, in other countries like Belgium, Italy and Spain, such information sharing is minimal (Pagano and Jappelli, 1993). And in some less developed countries like China, such information sharing is virtually non-existent which partly accounts for the slow development of the consumer credit industry in these countries.[28] Further, in most countries, there are regulations credit bureaus must observe which forbid the collection of certain types of consumer information; for example, information on debts owed to friends, relatives and private money lenders is generally not collected by credit bureaus. Though in principle lenders themselves may try to find out about a borrower's private debts by incurring a cost, such cost may prove so prohibitively high as to render the practice impossible.[29]

In view of these institutional realities of the credit markets, we model the borrower's indebtedness at the time of applying for a new loan as simply unobservable to the lender, following Bizer and DeMarzo (1992). To avoid further complication of the analysis, we suppose in the event of a borrower's bankruptcy that she pays off her prior debts with whatever she is able, before repaying the new lender's loan. This can be justified if the prior debts are those owed to private lenders such as friends and relatives and the new lender is a credit card company, for example.[30]

4.2.2 The Loan Contract

The credit market consists of a monopolist lender and a large population of borrowers. The borrowers for some reason have incurred a certain amount of debt in the past, and are currently in a state of financial distress. They do, however, own a productive technology which,

[28] According to Iwasaki (2004), among all bank cards issued in China, only 5% are credit cards in the usual sense; the rest are what one will call debit cards in the US which require the card holder to have deposits in the bank as a collateral. See, also, Li et al (2005).

[29] The related literature on costly state verification addresses a similar point; see, for example, Townsend (1979), Gale and Hellwig (1985), and Williamson (1986, 1987).

[30] While in bankruptcy proceedings law prohibits a debtor from transferring her assets (e.g., giving them away as gifts), private debts generally retain priority in getting repaid over debts owed to public lenders.

if funded by appropriate amounts of investment, can produce sufficient output to pay off the existing debts. Borrowers differ in their indebtedness: some have a pre-existing debt d_L, while others have a pre-existing debt d_H, with $0 < d_L < d_H$. Borrowers are otherwise identical as described below, and they are henceforth referred to as *L*-borrowers and *H*-borrowers according to their pre-existing debt level respectively.

With a first-period investment $l \in [0, \infty)$, each borrower using her technology and an input of work effort $e \in [0,1)$ can produce a second-period output $F(l,e)$. Output increases with effort level in the sense of first-order stochastic dominance. Specifically, we assume that *F* has a multiplicative form: $F(l,e) = f(l)Z(e)$, where f satisfies $f(0) = 0$, $f'(0) = \infty$, $f' > 0$, $f'(\infty) = 0$ and $f'' < 0$, and $Z(e)$ is 0-1 random variable with $prob\{Z(e) = 1\} = e$, and $prob\{Z(e) = 0\} = 1 - e$. In other words, higher effort level leads to higher probability of success for any fixed amount of investment. Each borrower has the following utility function: $U(w,e) = u(w) - g(e)$, defined over her wealth *w* and effort level *e*, where *u* satisfies $u(0) = 0, u' > 0$ and $u'' < 0$, and g satisfies $g(0) = 0$, $g(1) = \infty$, $g'(0) = 0$, $g'' > 0$ and $g''' > 0$.[31] Thus the borrower is risk-averse in wealth and a higher effort level costs her more in utility.

The lender is risk-neutral who operates in the market attempting to maximize his expected profits. However, the lender himself does not have the funds to invest in the borrowers' technology; instead he raises the investment funds by issuing equity shares to his shareholders, in the amount of l_i, should he decide to invest that amount in the *i*-borrowers, $i = L, H$. It is assumed that returns from the two types of borrowers are stochastically independent. If the investment turns out successful, that is, if the gross return to the investment exceeds l_i then a fraction $1-\theta$ of the net profit goes to his share holders, and another fraction θ is retained by the lender as his compensation.

All parties have limited liability. First, the borrower assumes limited liability to the lender. That is, if the borrower goes bankrupt in the second period her final wealth is guaranteed no less than zero. Second, the lender has limited liability to the borrower's prior creditors: when the borrower fails to repay her prior debts the new lender cannot be held responsible to pay off those debts on behalf of the borrower. Finally, the lender's liability to his shareholders is also

[31] An example of g satisfying these properties is $g(x) = -\ln(1-x) - x$.

limited in that he cannot be penalized by a negative compensation, in case the gross return to an investment falls below the principal.

We model this investment process as a two-stage game as follows. In the first period, after having identified an investment opportunity, the lender raises funds in the amount of *l*. He then offers a loan contract, (*l*, *r*), to the borrower, where *l* is the size of the loan and *r* the interest rate. When offered a loan contract the borrower either declines the offer, in which case she has a final wealth of zero (she's also unable to repay her prior debt), and hence a zero utility in the second period; or she accepts the offer, in which case she heads off to work and privately chooses an effort level *e* to produce a second-period output. If she does accept the offer then, in the second period, she repays her debts subject to limited liability and the seniority rule for debt repayment (i.e., earlier debts get repaid before the later ones can.) For simplicity we adopt a tie-breaking rule by assuming that the borrower always declines a contract offer whenever she if indifferent between declining and accepting the offer, and the lender always chooses not to make an offer whenever he is indifferent between making and not making the offer.

4.3 SUBGAME PERFECT EQUILIBRIUM WITH OBSERVABLE DEBTS

We wish to characterize the subgame perfect equilibrium (SPE) of this contracting game. In this section, we first examine as a benchmark case the situation in which the borrower's private information about her prior debt is observable to the lender.

When a loan contract is offered to the borrower, she must decide whether to accept or decline the offer. The following lemma establishes the condition under which the borrower will accept the lender's loan contract.

Lemma 4.1. *An i-borrower will accept the loan contract* (l_i, r_i) *if and only if* $f(l_i) - d_i - (1+r_i)l_i > 0$. *When this condition is satisfied the borrower chooses an effort level* e_i *which is the unique solution to* $g'(e_i) = u(f(l_i) - d_i - (1+r_i)l_i)$.

Proof. It suffices to show: the borrower's maximum expected utility by accepting the loan contract (l_i, r_i) is strictly positive if and only if $f(l_i) - d_i - (1+r_i)l_i > 0$.

If the borrower accepts the contract (l_i, r_i) and chooses an effort level e_i, then her expected utility, under limited liability, is given by

$$E_i = E[u(\max\{f(l_i)Z(e_i) - d_i - (1+r_i)l_i, 0\}) - g(e_i) \mid e_i]$$
$$= e_i \cdot u(\max\{f(l_i) - d_i - (1+r_i)l_i, 0\}) - g(e_i)$$

If $f(l_i) - d_i - (1+r_i)l_i > 0$, then $E_i = e_i \cdot u(f(l_i) - d_i - (1+r_i)l_i) - g(e_i)$, and E_i attains its maximum at the solution, $\hat{e}_i$, to the equation

(4.1) $\quad g'(e_i) = u(f(l_i) - d_i - (1+r_i)l_i)$

with the maximum value being

(4.2) $\quad E_i^{\max} = \hat{e}_i \cdot u(f(l_i) - d_i - (1+r_i)l_i) - g(\hat{e}_i)$

It remains to show: (a) a solution to (4.1) exists and is unique, and (b) $E_i^{\max} > 0$.

Recall the properties of g: $g(0) = 0, g(1) = \infty, g'(0) = 0,$ and $g'' > 0$. It follows immediately that g' is strictly increasing and $g'(e) > 0$ for $0 < e < 1$. We show that g' is unbounded above. Suppose, to the contrary, that $g' \le M$ for some $M > 0$. Then, by the mean-value theorem, for any $0 < e < 1$, there exists $\hat{e}$, with $0 < \hat{e} < e$, such that $g(e) = g(e) - g(0) = g'(\hat{e}) \cdot (e - 0) = g'(\hat{e}) \cdot e$. But then $g(e) = g'(\hat{e}) \cdot e \le M \cdot 1 = M$ for any $0 < e < 1$, contradicting the assumption $g(1) = \infty$. Hence g' is strictly increasing and unbounded above, which implies that (4.1) has a unique solution $\hat{e}_i > 0$, by the intermediate-value theorem, the continuity of g', the assumption $g'(0) = 0$, and the fact that $u(f(l_i) - d_i - (1+r_i)l_i) > 0$ (which follows from the properties of u). This proves (a). To prove (b), use again the mean-value theorem. By this theorem, there exists $\tilde{e}$, with $0 < \tilde{e} < \hat{e}_i$, such that $g(\hat{e}_i) = g(\hat{e}_i) - g(0) = g'(\tilde{e}) \cdot (\hat{e}_i - 0) = g'(\tilde{e}) \cdot \hat{e}_i$ which, when substituted into (4.2), yields

$$E_i^{\max} = \hat{e}_i \cdot (u(f(l_i) - d_i - (1+r_i)l_i) - g'(\tilde{e})) = \hat{e}_i \cdot (g'(\hat{e}_i) - g'(\tilde{e})) > 0$$

using (4.1) and since $\hat{e}_i > 0$, $\tilde{e} < \hat{e}_i$, and g' is strictly increasing. This proves (b).

If $f(l_i) - d_i - (1+r_i)l_i \le 0$, then $E_i = -g(e_i)$, and the maximum of E_i is obtained at $e_i = 0$, with the maximum value being 0. The proof is thus complete. □

Having established the necessary and sufficient condition for the borrower's acceptance of a loan contract, we now turn to the analysis of the SPE of the screening game under various conditions concerning the parameter values of the model. But before proceeding we first establish another lemma and define two quantities, l^* and h^*, that will be needed frequently in later analysis.

Lemma 4.2. *There exists a unique* $l^* \in [0,\infty)$ *such that* $f(l^*) - l^* \geq f(l) - l$ *for any* $l \in [0,\infty)$. *Moreover* $l^* > 0, f'(l^*) = 1$ *and* $h^* \equiv f(l^*) - l^* > 0$.

Proof. Similar to that of Lemma 4.1 by noticing the properties of f: $f(0) = 0, f'(0) = \infty$, $f'(\infty) = 0$ and $f'' < 0$. □

As the pre-existing debts are observable the lender can perfectly discriminate the borrowers by offering each type of borrower a possibly different loan contract. And since the returns to the investments in the two borrower types are assumed to be independent, maximizing the lender's total expected profit is the same as maximizing his expected profit from each borrower type. The lender's expected profit from the *i*-borrower, if the latter accepts the contract (l_i, r_i) and chooses an effort level e_i as given in Lemma 4.1, is given by

(*) $$\pi_i = E[\max\{\theta(\min\{\max\{f(l_i)Z(e_i) - d_i, 0\}, (1+r_i)l_i\} - l_i), 0\} \mid e_i].$$

Note that the two max's in the above expression, in their order of appearance, reflect respectively the limited liability of the lender to his own equity shareholders and that to the borrower's prior creditors.

We derive the SPE for each of the following three cases in turn. Case 1: $h^* \leq d_L$; Case 2: $d_L < h^* \leq d_H$; and Case3: $d_H < h^*$.

<u>Case 1</u>: $h^* \leq d_L$. Then $h^* \leq d_i, i = L, H$. Since, by Lemma 4.2, l^* is the unique maximum for $f(l) - l$ and $h^* = f(l^*) - l^*$, we have, for any contract offer (l_i, r_i), that $f(l_i) - d_i - (1+r_i)l_i \leq f(l^*) - l^* - d_i - r_i l_i = h^* - d_i - r_i l_i < 0$. By Lemma 4.1, neither borrower type will accept the contract, and consequently the lender will make offer to neither borrower type. This leads to the following theorem.

Theorem 4.1. *If $h^* \le d_L$, then there exists a unique SPE of the game. In this SPE, the lender makes no contract offer to either borrower type; each borrower type receives a zero expected utility, and the lender earns a zero expected profit in the second period.*

<u>Case 2</u>: $d_L < h^* \le d_H$. First of all, the *H*-borrower will decline any offer from the lender, and thus the lender will make no contract offer to this borrower type in the SPE, as the analysis in Case 1 shows. As for the *L*-borrower, knowing that by offering a contract (l_L, r_L) with $f(l_L) - d_L - (1+r_L)l_L \le 0$ he will earn a zero profit in the second period because the *L*-borrower will decline such an offer, the lender will try to offer (l_L, r_L) such that $f(l_L) - d_L - (1+r_L)l_L > 0$, to get a positive second-period expected profit. Then, since $f(l_L) - d_L - (1+r_L)l_L > 0$ implies $f(l_L) - d_L > (1+r_L)l_L \ge 0$, the lender's expected profit π_L, from (*), is $\pi_L = \theta e_L r_L l_L$. The lender's problem is to maximize π_L, or equivalently $e_L r_L l_L$, subject to the *L*-borrower accepting the contract and her rule for choosing e_L as described in Lemma 4.1. That is, the lender solves the following maximization problem

Maximize $e_L r_L l_L$

s.t.

(4.3) $g'(e_L) = u(f(l_L) - d_L - (1+r_L)l_L)$

(4.4) $f(l_L) - d_L - (1+r_L)l_L > 0$

The conditions for this problem are

(4.5) $r_L l_L - \lambda g''(e_L) = 0$

(4.6) $e_L l_L - \lambda l_L u'(f(l_L) - d_L - (1+r_L)l_L) = 0$

(4.7) $e_L r_L - \lambda((1+r_L) - f'(l_L))u'(f(l_L) - d_L - (1+r_L)l_L) = 0$

where λ is the multiplier associated with the constraint (4.3), plus constraints (4.3) and (4.4). From (4.6) and (4.7) we get

(4.8) $f'(l_L) = 1$

which, by Lemma 4.2, implies $l_L = l^*$. And (4.5), (4.7) and (4.8) yield

(4.9) $e_L g''(e_L) = r_L l_L u'(f(l_L) - d_L - (1+r_L)l_L)$

Now, with $l_L = l^*$, the problem reduces to finding e_L, r_L which satisfy

(4.3′) $g'(e_L) = u(f(l^*) - d_L - (1+r_L)l^*)$

(4.9′) $e_L g''(e_L) = r_L l^* u'(f(l^*) - d_L - (1+r_L)l^*)$

(4.4′) $f(l^*) - d_L - (1+r_L)l^* > 0$

Writing $A \equiv f(l^*) - l^* - d_L$, (4.3′), (4.9′), (4.4′) become respectively

(4.3″) $g'(e_L) = u(A - r_L l^*)$

(4.9″) $e_L g''(e_L) = r_L l^* u'(A - r_L l^*)$

(4.4″) $A - r_L l^* > 0$

Note that $A > 0$, since $f(l^*) - l^* - d_L = h^* - d_L > 0$ by assumption.

Proposition 4.1. *There exists a unique solution* $(\bar{r}_L, \bar{e}_L)$ *that satisfies (4.3″), (4.9″) and (4.4″).*

Proof. It's clear that for each $0 \le r_L \le \frac{A}{l^*}$, there is a unique $e_L \in [0,1)$ that satisfies (4.3″).

Thus (4.3″) uniquely defines e_L as a function of r_L: $e_L = p(r_L)$ for $0 \le r_L \le \frac{A}{l^*}$. It is easily seen that p is a strictly decreasing function and is continuous, with $p(0) = g'^{-1}(u(A)) > 0$ and $p(\frac{A}{l^*}) = 0$.

In the same way (4.9″) uniquely defines e_L as a function of r_L: $e_L = q(r_L)$, where q is strictly increasing and continuous, with $q(0) = 0$ and $q(\frac{A}{l^*}) > 0$, since $u' > 0$ and $e_L g''(e_L)$ is increasing in e_L because $g''' > 0$.

Thus finding a solution to (4.3″), (4.9″) and (4.4″) reduces to finding (r_L, e_L) that satisfies $e_L = p(r_L)$, $e_L = q(r_L)$ as well as (4.4″). Consider $s(r_L) \equiv p(r_L) - q(r_L)$. Then s is continuous and decreasing in r_L, $s(0) = p(0) - q(0) > 0$, and $s(\frac{A}{l^*}) = p(\frac{A}{l^*}) - q(\frac{A}{l^*}) < 0$.

By the intermediate-value Theorem 4., there exists a unique $\bar{r}_L$ such that $0 < \bar{r}_L < \frac{A}{l^*}$ and

$s(\bar{r}_L) = 0$, or equivalently, $p(\bar{r}_L) = q(\bar{r}_L)$. Let $\bar{e}_L \equiv p(\bar{r}_L) = q(\bar{r}_L) > 0$. Then $(\bar{r}_L, \bar{e}_L)$ is indeed the unique solution to (4.3″) and (4. 9″). Since $0 < \bar{r}_L < \frac{A}{l^*}$, this solution satisfies (4.4″) also. □

We can now state the theorem for the SPE for the case $d_L < h^* \leq d_H$.

Theorem 4.2. *If $d_L < h^* \leq d_H$, then there exists a unique SPE of the game. In this SPE, the lender offers the H-borrower no contract and offers the L-borrower a contract (l_L, r_L) where $l_L = l^*$ and $r_L = r_L^*$, the unique solution to the equations (4.3′) and (4.9′); the L-borrower chooses an effort level e_L^*, the unique solution to (4.3′) and (4.9′); the L-borrower receives a positive expected utility, the H-borrower receives a zero utility and the lender earns a positive expected profit in the second period.*

<u>Case 3</u>: $d_H < h^*$. Then $d_i < h^*, i = L, H$. Since the lender can perfectly discriminate the two types of borrowers, his maximization problem consists of maximizing his expected profit from each borrower type. Clearly, the analysis for Case 2 carries over to the present case, and we can establish the following theorem.

Theorem 4.3. *If $d_H < h^*$, then there exists a unique SPE of the game. In this SPE, for $i = L, H$, the lender offers the i-borrower a contract (l_i, r_i), where $l_i = l^*$ and $r_i = r_i^*$, the unique solution to (4.3′) and (4.9′) with L replaced by i; the i-borrower chooses an effort level e_i^*, the unique solution to (4.3′) and (4.9′) with L replaced by i; the i-borrower receives a positive expected utility and the lender earns a positive expected profit in the second period.*

4.4 SCREENING WHEN DEBTS ARE UNOBSERVABLE

In this section we investigate the problem of screening when the borrowers' pre-existing debts are unobservable to the lender. Not knowing to which type a borrower belongs, the lender's best guess is that the borrower is as likely an *L*-borrower as an *H*-borrower, and a pure strategy for the lender consists of choosing two loan contacts (l', r') and (l'', r''), to be offered simultaneously to the borrowers. A pure strategy for a borrower, when facing the two contracts from the lender, is to decide whether to accept (l', r'), to accept (l'', r''), or to accept neither. If the borrower accepts one of the two contracts, she also needs to choose a corresponding effort level as part of her pure strategy. We assume that if a borrower is indifferent between two acceptable contracts she always accepts the one with the larger loan size. In this way, the borrowers self-select the lender's contract offers.

4.4.1 Subgame Perfect Equilibrium of the Screening Game

The three cases in the last section are again considered in turn, in our search for the SPE under asymmetric information. As it turns out, Cases 1 and 2 yield essentially the same SPE outcomes as those described in Theorems 4.1 and 4.2 respectively. To see this, consider for example Case 2 where $d_L < h^* \leq d_H$. Although he cannot observe the actual pre-existing debts of the borrowers, realizing that no contract will be accepted by the *H*-borrower, the lender's maximization problem in the current situation of asymmetric information is essentially the same as that in Section 3 for the same case $d_L < h^* \leq d_H$. More precisely, there is a unique SPE outcome of the game, in which the lender offers all borrowers the same two contracts, (l', r') and (l'', r''), where $l' = l^*$ and $r' = r_L^*$, the unique solution to (4.3′) and (4.9′), and (l'', r'') is any arbitrary loan contract satisfying $f(l'') - d_L - (1 + r'')l'' \leq 0$; the *L*-borrower accepts the contract (l', r') and the *H*-borrower accepts neither (l', r') nor (l'', r''); the *L*-borrower chooses an effort level e_L^*, the unique solution to (4.3′) and (4.9′); and the second-period payoffs to the three parties are the same as those in Theorem 4.2.

Therefore, $d_H < h^*$ is the more interesting case which might give rise to different SPE outcomes than those in Theorem 4.3, under the current condition of asymmetric information about the borrowers' pre-existing debts. We henceforth restrict our attention to this case.

Assume $d_H < h^*$. We first partition the lender's strategy space, $D = \{((l', r'), (l'', r''))\}$, into five disjoint subsets: $D = \bigcup_{j=1}^{5} D_j$, where

$D_1 = \{((l', r'), (l'', r'')) \mid$ The *H*-borrower accepts one of the two contracts and the *L*-borrower accepts neither$\}$

$D_2 = \{((l', r'), (l'', r'')) \mid$ The *L*-borrower accepts one of the two contracts, the *H*-borrower accepts the other, and $(l', r') \neq (l'', r'')\}$

$D_3 = \{((l', r'), (l'', r'')) \mid$ The *L*-borrower accepts one of the two contracts and the *H*-borrower accepts neither$\}$

$D_4 = \{((l', r'), (l'', r'')) \mid$ The *L*-borrower accepts neither and the *H*-borrower accepts neither$\}$

$D_5 = \{((l', r'), (l'', r'')) \mid$ The two borrower types accept the same one contract$\}$

Definition. An SPE of the game is called *fully-separating* if the lender's strategy in this SPE lies in D_2; it's called *semi-separating* if the lender's strategy lies in $D_1 \cup D_3$; and it's called *pooling* if the lender's strategy lies in D_5.

We show that some of the D_j's are actually empty and thus narrow down the search area for an SPE of the game.

Proposition 4.2. D_1 *is empty.*

Proof. Suppose D_1 is not empty. Let $((l', r'), (l'', r'')) \in D_1$. Without loss of generality suppose the *H*-borrower accepts (l', r'). By Lemma 4.1, it must be the case that $f(l') - d_H - (1 + r')l' > 0$. But then $f(l') - d_L - (1 + r')l' > 0$, which, by Lemma 4.1, implies that the *L*-borrower also accepts (l', r'). Hence $((l', r'), (l'', r'')) \notin D_1$, a contradiction. □

Proposition 4.3. *D_2 is empty.*

Proof. Suppose D_2 is not empty. Let $((l',r'),(l'',r'')) \in D_2$. Without loss of generality suppose the *L*-borrower accepts (l',r') and the *H*-borrower accepts (l'',r''). It follows by Lemma 4.1 that

(4.10) $f(l') - d_L - (1+r')l' > 0$

(4.11) $f(l'') - d_H - (1+r'')l'' > 0$

Let e_L, e_H be, respectively, the corresponding effort level choice of the *L*-borrower and the *H*-borrower. Then

$$v_L = e_L u(f(l') - d_L - (1+r')l') - g(e_L)$$

$$v_H = e_H u(f(l'') - d_H - (1+r'')l'') - g(e_H)$$

are the second-period expected utility for the *L*-borrower and the *H*-borrower respectively.

Using arguments analogous to those in the proof of Lemma 4.1, it's straightforward to establish the following result.

Result: Let $t_i(e) \equiv eB_i - g(e), i = 1,2$, be two functions defined on $0 \le e < 1$. Denote by T_i their corresponding maximum value. Then the following holds: for $B_1 \in (0,\infty)$, $T_1 > T_2$ if and only if $B_1 > B_2$; and $T_1 = T_2$ if and only if $B_1 = B_2$

Now consider what the *L*-borrower's expected utility would be if she, instead of (l',r'), were to choose (l'',r''). Let e_{LH} be her corresponding effort level choice. Then the *L*-borrower's second-period expected utility is given by

$$v_{LH} = e_{LH} u(f(l'') - d_L - (1+r'')l'') - g(e_{LH})$$

Similarly, if the *H*-borrower were to choose (l',r') instead of (l'',r''), her second-period expected utility would be

$$v_{HL} = e_{HL} u(f(l') - d_H - (1+r')l') - g(e_{HL})$$

where e_{HL} is her corresponding effort level choice when she chooses the contract (l',r').

The fact that the *L*-borrower accepts (l',r') rather than (l'',r'') means that either $v_L > v_{LH}$ or $v_L = v_{LH}$ and $l'' < l'$, which in turn means either

(*L*1) $f(l') - d_L - (1+r')l' > f(l'') - d_L - (1+r'')l''$

or

(L2) $\quad f(l') - d_L - (1+r')l' = f(l'') - d_L - (1+r'')l''$ and $l'' < l'$

by (10), the above *Result*, and the fact that u is a strictly increasing function.

Similarly, the fact that the H-borrower accepts (l'', r'') rather than (l', r') means either

(H1) $\quad f(l'') - d_H - (1+r'')l'' > f(l') - d_H - (1+r')l'$

or

(H2) $\quad f(l'') - d_H - (1+r'')l'' = f(l') - d_H - (1+r')l'$ and $l' < l''$

It is a straightforward exercise to show: (L1) and (H1) together yield a contradiction, so do (L1) and (H2) together, (L2) and (H1) together, and (L2) and (H2) together. These contradictions complete the proof of the proposition. □

Strictly speaking, we need to address the following question for the sake of completeness: which contract will a borrower accept if she is indifferent between two different contracts yet the two contracts have the same loan size? It can be easily shown that the borrower *cannot* be indifferent between two different contracts with the same loan size, given that both contracts are acceptable to her.

A corollary to Proposition 4.3 is that a fully-separating equilibrium does not exist, which we state as a theorem.

Theorem 4.4. *Assume* $d_H < h^*$. *In the contracting game in which the borrowers' preexisting debts are unobservable to the lender and the lender screens the borrowers, there exists no SPE that is fully-separating.*

Now that D_1 and D_2 are empty we may in the search for an SPE restrict our attention to $D_3 \cup D_4 \cup D_5$. However, an SPE cannot exist in which the lender's strategy is taken from D_4, for he can do strictly better (i.e., earn a positive vs. zero expected profit in the second period) by deviating to a strategy in D_3. (D_3 is not empty, since any contract pair $((l', r'), (l'', r''))$ satisfying $f(l') - d_L - (1+r')l' > 0$, $f(l') - d_H - (1+r')l' \le 0$ and $f(l'') - d_H - (1+r'')l'' \le 0$ is in D_3.) Thus the search area for an SPE is further reduced down to $D_3 \cup D_5$, from which the lender chooses his best strategy.

In what follows we examine the conditions under which a pooling or semi-separating SPE exists. For expositional convenience we collect here some of the functions, constraints, optimization problems and other quantities to be referenced frequently in later analysis.

Functions:

$$\phi(l, r, e_L, e_H) \equiv \frac{1}{2} e_L rl + \frac{1}{2} e_H rl$$

$$\varphi(l, r, e_L) \equiv \frac{1}{2} e_L rl$$

Constraints:

(4.12) $g'(e_L) = u(f(l) - d_L - (1+r)l)$

(4.13) $g'(e_H) = u(f(l) - d_H - (1+r)l)$

(4.14) $f(l) - d_H - (1+r)l > 0$

(4.15) $f(l) - d_H - (1+r)l \geq 0$

(4.16) $f(l) - d_H - (1+r)l \leq 0$

(4.17) $f(l) - d_L - (1+r)l > 0$

(4.18) $f(l) - d_H - (1+r)l = 0$

Optimization problems:

($P1$) *Maximize* $\phi(l, r, e_L, e_H)$ *s.t.* (4.12), (4.13) and (4.14)

($P2$) *Maximize* $\varphi(l, r, e_L)$ *s.t.* (4.12), (4.16) and (4.17)

($P3$) *Maximize* $\phi(l, r, e_L, e_H)$ *s.t.* (4.12), (4.13) and (4.15)

($P4$) *Maximize* $\phi(l, r, e_L, e_H)$ *s.t.* (4.12), (4.13) and (4.18)

($P5$) *Maximize* $\varphi(l, r, e_L)$ *s.t.* (4.12) and (4.18)

($P6$) *Maximize* $\varphi(l, r, e_L)$ *s.t.* (4.12)

Quantities

l^* is as defined in Lemma 4.1; e_L^*, r_L^* are the unique solution to the equations (4.3′) and (4.9′); e_H^*, r_H^* are the unique solution to the equations (4.3′) and (4.9′) with *L* replaced by *H*; and r^{**} is the unique optimum solution to problem (*P*1) as given in the following Lemma 4.3.

We also use the following notation: [*Maximize* $\Omega(x)$ *s.t.* (1), (2),…,(n)] denotes the maximum *value* of the corresponding maximization problem if an optimum solution to this problem exists; similarly for the notation [(*P*)] where (*P*) is the name of a maximization problem.

Using techniques similar to those in the proof of Proposition 4.1, the following lemma can be proved.

Lemma 4.3. *Assume* $d_H < h^*$. *The maximization problem (P1) has a unique optimum solution* $(l^*, r^{**}, e_L^{**}, e_H^{**})$. *Moreover* $\min\{r_L^*, r_H^*\} \le r^{**} \le \max\{r_L^*, r_H^*\}$.

With these preparations we are now in a position to characterize the pooling and semi-separating SPE of the game under asymmetric information. This is accomplished in Theorems 4.5 and 4.6. Recall that we are searching for the lender's best strategy in the set $D_3 \cup D_5$.

Theorem 4.5. *If* $f(l^*) - d_H - (1+\max\{r_L^*, r_H^*\})l^* > 0$, *then there exists a unique SPE outcome of the screening game, which is pooling. In the SPE, the lender offers both borrower types the same two contracts* (l', r') *and* (l'', r''), *where* $(l', r') = (l^*, r^{**})$ *and* (l'', r'') *is such that* $f(l'') - d_L - (1+r'')l'' \le 0$; *both borrower types accept the contract* (l', r'); *both borrower types receive a positive expected utility and the lender earns a positive expected profit in the second period.*

Proof. Consider first what is the best the lender can do, if his strategy choice is restricted to those in D_5. Since, in D_5, the two borrower types accept the same one contract, the lender need not worry about the other contract in his offer that is not accepted; he can make sure neither borrower type will accept the other contract (l'', r'') by, say, making $f(l'') - d_L - (1+r'')l'' \le 0$. Hence, noting that $f(l) - d_H - (1+r)l > 0$ implies $f(l) - d_L - (1+r)l > 0$, the lender's objective essentially is to solve problem (P1), if his strategy choice is restricted to D_5. Now, consider what is the best the lender can do, if his strategy choice is restricted to those in D_3. Again, the lender can just concentrate on the contract that is accepted by the L-borrower, by making the other contract (l'', r'') satisfy $f(l'') - d_L - (1 + r'')l'' \le 0$ so that no borrower type will accept this contract. So, essentially, the lender's objective is to solve problem (P2), if his strategy choice is restricted to D_3.

Under the following condition of the theorem:

(4.19) $\quad f(l^*) - d_H - (1 + \max\{r_L^*, r_H^*\})l^* > 0$

problem (P1) has a unique optimum solution as described in Lemma 4.3, and thus the value [(P1)] does exist. Consider the 4-tuple $(l^*, r_L^*, e_L^*, \hat{e}_H)$ where $\hat{e}_H$ is given by $g'(\hat{e}_H) = u(f(l^*) - d_H - (1 + r_L^*)l^*)$. Then $\hat{e}_H > 0$ by (4.19). It can be easily verified that , under (4.19), the above 4-tuple satisfy (4.12), (4.13) and (4.14). We thus get

$$\phi(l^*, r_L^*, e_L^*, \hat{e}_H) = \frac{1}{2} e_L^* r_L^* l^* + \frac{1}{2} \hat{e}_H r_L^* l^* > \frac{1}{2} e_L^* r_L^* l^*$$

and hence

(4.20) $\quad [(P1)] \ge \phi(l^*, r_L^*, e_L^*, \hat{e}_H) > \frac{1}{2} e_L^* r_L^* l^*$

On the other hand, we have, for any (l, r, e_L) satisfying (4.12), (4.16) and (4.17), that

(4.21) $\quad \varphi(l, r, e_L) \le$ [*Maximize* $\varphi(l, r, e_L)$ *s.t.* (4.12)] $= \frac{1}{2} e_L^* r_L^* l^*$

Here, the inequality in (4.21) results from dropping constraints (4.16) and (4.17) from problem (P2). It follows from (4.20) and (4.21) that, for any (l, r, e_L) satisfying (4.12), (4.16) and (4.17), $\varphi(l, r, e_L) < [(P1)]$. Hence, the lender's best strategy, under (4.19), lies in D_5 and in D_5 only. And Lemma 4.3 guarantees the existence of and gives a pooling SPE.□

Theorem 4.6. *Assume* $d_H < h^*$. *If* $f(l^*) - d_L - (1 + \min\{r_L^*, r_H^*\})l^* \le 0$, *then there exists a unique SPE outcome of the screening game, which is semi-separating. In the SPE, the lender offers both borrower types the same two contracts* (l', r') *and* (l'', r''), *where*

$l' = l^*, r' = \dfrac{h^* - d_H}{l^*}$*, and* (l'', r'') *is such that* $f(l'') - d_L - (1 + r'')l'' \le 0$*; the L-borrower accepts the contract* (l', r') *and the H-borrower accepts neither contract; the L-borrower receives a positive expected utility, the H-borrower receives a zero utility and the lender earns a positive expected profit in the second period.*

Proof. Again we consider the two maximization problems (P1) and (P2), under the condition of the current theorem:

(4.22) $f(l^*) - d_L - (1 + \min\{r_L^*, r_H^*\})l^* \le 0$

Consider (P1) first. If (P1) had an optimum solution, then the optimum solution should be $(l^*, r^{**}, e_L^{**}, e_H^{**})$ as given in Lemma 4.3. In particular it should satisfy (4.14), i.e.,

(4.23) $f(l^*) - d_H - (1 + r^{**})l^* > 0$

But (4.23) contradicts (4.22), since $\min\{r_L^*, r_H^*\} \le r^{**}$ and $d_L < d_H$. Therefore (P1) does not have an optimum solution under (4.22).

Now consider problem (P3), which is the same as (P1) except that the constraint (4.14) is replaced by (4.15). Since (P1) does not have an optimum solution, constraint (4.15) must be binding at any optimum solution of (P3). Thus (P3) is equivalent to (P4). For (P4), constraints (4.13) and (4.18) imply $e_H = 0$. Since $\phi(l, r, e_L, 0) = \varphi(l, r, e_L)$, we have

(4.24) [(P3)] = [(P4)] = [*Maximize* $\phi(l, r, e_L, 0)$ *s.t.* (4.12) and (4.18)] = [(P5)]

Because, as shown above, (P1) doesn't have an optimum solution, it must be the case that, for any (l, r, e_L, e_H) satisfying (4.12), (4.13) and (4.14),

(4.25) $\phi(l, r, e_L, e_H) < [(P3)]$

And (4.24), (4.25) imply that, for any (l, r, e_L, e_H) satisfying (4.12), (4.13) and (4.14),

(4.26) $\phi(l, r, e_L, e_H) < [(P5)]$

Next consider (P2). The optimum solution to (P6), (l^*, r_L^*), doesn't satisfy (4.17) because $f(l^*) - d_L - (1 + r_L^*)l^* \le 0$ by (4.22). Thus (4.16) must be binding at any optimum solution of (P2). But once (4.16) is binding (4.17) becomes extraneous. Hence, (P2) is equivalent to (P5). It can be easily shown that (P5) does indeed have an optimum solution

$(l^*, \tilde{r}_L, \tilde{e}_L)$ where $\tilde{r}_L = \dfrac{h^* - d_H}{l^*}$ and $\tilde{e}_L = g'^{-1}(u(d_H - d_L))$. This optimum solution of (P5) is also the optimum solution of (P2), and hence

(4.27) [(P5)] = [(P2)]

Summarizing, we have shown, for any (l, r, e_L, e_H) satisfying (4.12), (4.13) and (4.14), that $\phi(l, r, e_L, e_H) < [(P2)]$, by (4.26) and (4.27). From this we see that, under (4.22), the lender's best strategy lies in D_3 and in D_3 only, and is given by $(l^*, \tilde{r}_L)$ as defined above. This leads to the semi-separating SPE outcome as described in the theorem □

4.4.2 Discussion of the Results

The literature on credit rationing defines the phenomenon as a situation where some borrowers are unable to obtain a loan from a lender even though they are *ex ante* indistinguishable from those who do obtain a loan. So, the lender's no-offer of loan contracts to some of the borrowers when their pre-existing debts are observable (Theorems 4.1 and 4.2) does not constitute a case of credit rationing *per se*, because the borrowers can be distinguished *ex ante* from the lender's point of view. Credit rationing does occur in the situation described in Theorem 4.6, where information about the borrowers' pre-existing debts is asymmetric and the high-debt borrowers accept none of the lender's loan offers. Early literature explains the phenomenon of credit rationing by the availability doctrine, which says that credit is rationed because the lender's available funds are not sufficient to meet all borrowers' borrowing needs. This explanation does not apply in the current environment; funds shortage does not constrain the lender's ability to make loans because, with the equity funding mode, he can always raise enough funds if he sees profitable investment opportunities exist. Here credit rationing arises instead from the informational constraint the lender faces since he is unable to distinguish between low-risk and high-risk borrowers.

Comparing the SPE outcome in Theorem 4.3 with that in Theorem 4.5, we see that the existence of informational asymmetry in the current environment does not necessarily lead to an efficiency loss. In both SPE outcomes, both low-debt and high-debt borrowers obtain a level of investment funds equal to l^* which is socially efficient, for l^* is quantity that maximizes the total surplus per Lemma 4.2. Still, interest rates charged on the borrowers are different in the two cases; with observable debts each borrower type is charged a possibly different interest rate (r_L^* and r_H^* in Theorem 4.3), while with unobservable debts both borrower types get the same interest rate (r^{**} in Theorem 4.5). Since r^{**} lies between $\min\{r_L^*, r_H^*\}$ and $\max\{r_L^*, r_H^*\}$ (Lemma 4.3), one borrower type is made worse-off and the other borrower type better-off by the existence of informational asymmetry as they pay, respectively, a higher and lower interest rate than when the information is symmetric.

A loss of welfare does result though, when the borrowers' unobservable debts are too high as indicated by the condition $f(l^*) - d_L - (1 + \min\{r_L^*, r_H^*\})l^* \leq 0$ (Theorem 4.6) vs. the condition $f(l^*) - d_H - (1 + \max\{r_L^*, r_H^*\})l^* > 0$ (Theorem 4.5). Under the same conditions as those of Theorem 4.6, if existing debts were observable, the high-debt borrowers would have received a positive expected utility and the lender would have earned a positive expected profit on this borrower type (Theorem 4.3), as opposed to the zero-payoff, Pareto-inferior outcome in Theorem 4.6. Clearly, the no-investment in the high-debt borrowers is a consequence of the lender's inability to observe the indebtedness of this borrower type.

4.5 CONCLUSION

We have studied in this paper a hybrid model of adverse selection and moral hazard in the context of credit markets, and re-examined the problem of credit rationing. When the credit market is opaque as is typically the case in less developed countries where information sharing among lenders is non-existent, the unobservability of borrowers' financial status is a possible cause of credit rationing. If borrowers' indebtedness is high enough to pose a default risk on a lender, credit rationing arises as a result of the lender's trying to sort out borrowers of different risk characteristics. For certain parameter values, there exists a unique subgame perfect equilibrium in the screening game of our model where only low-debt borrowers obtain a loan from the lender.

From the methodological point of view, this paper contributes to the inventory of existing principal-agent models by exploring a richer setting characterized by both pre-contractual and post-contractural informational asymmetry. The non-existence of equilibrium is a typical problem associated with many competitive screening models. In our model of monopolistic screening, we are able to completely characterize the unique subgame perfect equilibrium of the screening game, and the techniques developed here have potential applicability in a variety of other related problems. As far as credit rationing is concerned, the present study helps in resolving the issue of its existence by formulating the problem in rigorous game-theoretic terms.

Several extensions and modifications might improve the present paper. First, note that there is a gap between the two cases in Theorem 4.5 and Theorem 4.6; those cases that lie between $f(l^*) - d_H - (1 + \max\{r_L^*, r_H^*\})l^* > 0$ and $f(l^*) - d_L - (1 + \min\{r_L^*, r_H^*\})l^* \leq 0$ are not considered. Second, we have assumed for the ease of analysis that outputs from the two borrower types' technology are stochastically independent. If we allow those to be correlated, then another kind of credit rationing is possible as the lender may then want to diversify the default risk by allocating funds between different borrower types (type II credit rationing), instead of granting the loan entirely to one borrower type and leaving the other borrower type with nothing (type I credit rationing). Finally, a model of competitive screening may be developed in which multiple lenders compete in the opaque credit market. We expect the analysis of such a model to be more involved than the current one and leave it as a topic for future research.

BIBLIOGRAPHY

Adams, C. and B. Chadha (1991) “On interpreting the random walk behavior of nominal and real exchange rates,” *International Monetary Fund Working Paper* 91/7, Washington, D.C.

Allen, H. and M. Taylor (1990) “Charts, noise and fundamentals in the London foreign exchange market,” *Economic Journal* 100 (supplement), 49-59

Ausubel, L., P. Cramton, and R. Deneckere (2002) “Bargaining with incomplete information,” in *Handbook of Game Theory*, Vol.3, edited by R. Aumann and S. Hart, Amsterdam: Elsevier

Besanko, D. and A. Thakor (1987) “Collateral and rationing: Sorting equilibria in monopolistic and competitive credit markets,” *International Economic Review* 28, 671-690

Bester, H. (1985) “Screening vs. rationing in credit markets with imperfect information,” *American Economic Review* 75, 850-55

Bester, H. and M. Hellwig (1987) “Moral hazard and equilibrium credit rationing,” in *Agency Theory, Information and Incentives*, edited by G. Bamberg and K. Spremann.

BIS (2004) *Triennial Central Bank Survey of Foreign Exchange and Derivatives Market Activity*, Bank for International Settlements

Bisin, A. and D. Guaitoli (2004) "Moral hazard and nonexclusive contracts," *RAND Journal of Economics* 35, 306-328

Bizer, D. and P. DeMarzo (1992) "Sequential banking," *Journal of Political Economy* 100, 41-61

Bizer, D. and P. DeMarzo (1999) "Optimal incentive contracts when agents can save, borrow, and default," *Journal of Financial Intermediation* 8, 241-269

Bisin, A. and A. Rampini (2002) "Exclusive contracts and the institution of bankruptcy," Mimeo, New York University and Northwestern University

Bodie, Z. et al (1998) *Essentials of Investments*, Irwin/McGraw-Hill

Branston, W.H. (1969) "The minimum covered interest differential needed for international arbitrage opportunities," *Journal of Political Economy* 77, 1028-1035

Carlson, J. (1998) "Risk aversion, foreign exchange speculation and gambler's ruin," *Economica* 65, 441-453

Choie, K. (1993) "Currency exchange rate forecast and interest rate differential," *Journal of Portfolio Management*, Winter 1993, 58-64

Corbae, D., T. Temzelides and R. Wright (2002) "Matching and money," *American Economic Review Papers and Proceedings* 92, 67-71

Corbae, D., T. Temzelides and R. Wright (2003) "Directed matching and monetary exchange," *Econometrica* 71, 731-756

Cross, S. (1998) *All about the Foreign Exchange Market in the United Sates*, Federal Reserve Bank of New York

Dacorogna, M. et al (2001*) An Introduction to High-Frequency Finance,* Academic Press

Danielssson, J. and R. Payne (2002) “Real trading patterns and prices in spot foreign exchange markets,” *Journal of International Money and Finance* 21, 203-222

DeMeza, D. and D. Webb (1987) “Too much investment: a problem of asymmetric information,” *Quarterly Journal of Economics* 102, 281-292

Detragiache, E., P. Garella, and L. Guiso (2000) “ Multiple versus single banking relationships,” *Journal of Finance* 55, 1133-1161

Dickey, D. and W. Fuller (1979) “Distribution of the estimates for autoregressive time series with a unit root,” *Journal of the American Statistical Association* 74, 427-431

Dickey, D. and W. Fuller (1981) “Likelihood ratio statistics for autoregressive time series with a unit root,” *Econometrica* 49, 1057-1072

Enders, W. (2004) *Applied Econometric Time Series*, 2nd edition, John Wiley &Sons

Engel, C. (1996) “The forward discount anomaly and the risk premium: A survey of recent evidence,” *Journal of Empirical Finance* 3, 123-192

Engle, R. and C. Granger (1987) “Cointegration and error-correction: Representation, estimation, and testing,” *Econometrica* 55, 251-276

Flood, M. (1991) “Microstructure theory and the foreign exchange market,” *Review*, Federal Reserve Bank of St. Louis, 73 (Nov/Dec), 52-70

Flood, R. and M. Taylor (1996) “Exchange rate economics: What’s wrong the conventional macro approach?,” in *The Microstructure of Foreign Exchange Markets*, edited by J. Frankel et al, University of Chicago Press

Fratianni, M. and I.M. Wakeman (1982) “The law of one price in the Eurocurrency market,” *Journal of International Money and Finance* 1, 307-323

Frankel, J. and K. Froot (1990) “Chartists, fundamentalists and the demand for dollars,” in Courakis, A.S. and Taylor, M.P. (eds.), *Private Behavior and Government Policy in Interdependent Economies*, Oxford University Press.

Frenkel, J.A. and R.M. Levich (1975) “Covered interest arbitrage: unexploited profits?” *Journal of Political Economy* 83, 325-338

Frenkel, J.A. and R.M. Levich (1977) “Transactions costs and interest arbitrage: tranquil versus turbulent periods,” *Journal of Political Economy* 85, 1209-1224

Froot, K and R. Thaler (1990) “Anomalies: Foreign exchange,” *Journal of Economic Perspectives* 4, 179-192

Fuller, W. (1996) *Introduction to Statistical Time Series*, New York: Wiley

Gale, D. and M. Hellwig (1985) “Incentive compatible debt contracts: The one-period problem,” *Review of Economic Studies* LII, 647-664

Goodhart, C. (1988) “The foreign exchange market: A random walk with a dragging anchor,” *Economica* 55, 437-460

Goodhart, C. and M. Taylor (1992) “Why don’t individuals speculate in the forward foreign exchange market?” *Scottish Journal of Political Economy* 39,1-13

Hellwig, M. (1987) “Some recent developments in the theory of competition in markets with adverse selection,” *European economic Review* 31, 319-325

Ho, T. and H. Stoll (1983) “The dynamics of dealer markets under competition,” *Journal of Finance* 38, 1053-1074

Hopper, G. (1997) “What determines the exchange rate: Economic factors or market sentiment?” Federal Reserve Bank of Philadelphia *Business Review*, September/October 1997

Hsiao, C. (1979) “Autoregressive modeling of Canadian money and income data,” *Journal of the American Statistical Association* 74, 553-560

Ito, T., R. Lyons, and M. Melvin (1998) “Is there private information in the FX market? The Tokyo experiment,” *Journal of Financial Intermediation* 53, 1111-1130

Iwasaki, K. (2004) “China’s budding credit card market,” *Japan Research Quarterly*, Summer, 2004

Johansen, S. (1988) “Statistical analysis of Cointegration Vectors,” *Journal of Economic Dynamics and Control* 12, 231-254

Johansen, S. (1991) “Estimation and hypothesis testing of cointegrating vectors in Guassian vector autoregressive models,” *Econometrica* 59, 1551-1580

Johansen, S. and K. Juselius (1990) “Maximum likelihood estimation and inference on cointegration with applications to demand for money,” *Oxford Bulletin of Economics and Statistics* 52, 169-209

Kahn, C. and D. Mookherjee (1998) “Competition and incentives with nonexclusive contracts,” *RAND Journal of Economics* 29, 443-465

Kahneman, D. and A. Tversky (1979) "Prospect theory: An analysis of decision under risk," *Econometrica* 47, 263-292

Kennedy, P. (2003) *A Guide to Econometrics*, 5th edition, The MIT Press

Keynes, John Maynard (1923) *A Tract on Monetary Reform*, London: Macmillan

Lewis, K. (1995) "Puzzles in international financial markets," in *Handbook of International Economics* Vol 3, G.Grossman and K.Rogoff (eds), North Holland: Amsterdam

Li, Yang et al (2005) "*Dianzi zhifu yu zhongguo jingji*" [Electronic Payments and Chinese Economy], report published by the Institute of Finance, Chinese Academy of Social Sciences, November, 2005

Lyons, R. (1995) "Tests of microstructural hypotheses in the foreign exchange market," *Journal of Financial Economics* 39, 321-351

Lyons, R. (1996a) "Optimal transparency in a dealer market with an application to foreign exchange," *Journal of Financial Intermediation* 5, 225-254

Lyons, R. (1996b) "Foreign exchange volume: Sound and fury signifying nothing?," in *The Microstructure of Foreign Exchange Markets,* edited by J. Frankel et al, University of Chicago Press

Lyons, R. (1997) "A simultaneous trade model of the foreign exchange hot potato," *Journal of International Economics* 42, 275-298

Lyons, R. (2001) *The Microstructure Approach to Exchange Rates*, The MIT Press

McKinnon, R. (1979) *Money in International Exchange: The Convertible Currency System*, Oxford University Press

MacDonald, R. and M. Taylor (1994) "The monetary model of exchange rate: long-run relationships, short term dynamics and how to beat a random walk," *Journal of International Money and Finance* 13, 276-290

Markowitz, H. (1952) "Portfolio selection," *Journal of Finance* 7, 77-91

Marston, R.C. (1976) "Interest arbitrage in the Eurocurrency markets," *European Economic Review* 7, 1-13

Meese, R and K. Rogoff (1983) "Empirical exchange models of the seventies: do they fit out-of-sample?," *Journal of International Economics* 14, 3-24

Milgrom, P. and N. Stokey (1982) "Information, trade, and common knowledge," *Journal of Economic Theory* 26, 17-27

Muthoo, A. (1999) *Bargaining Theory with Applications*, Cambridge University Press

O'Hara, M. (1995) *Market Microstructure Theory*, Blackwell Business, Cambridge, MA

Osborne, M. and A. Rubinstein (1990) *Bargaining and Markets*, Academic Press

Pagano, M. and T. Jappelli (1993) "Information sharing in credit markets," *Journal of Finance* 48, 1693-1718

Park, I.U. (2004) "Moral hazard contracting and private credit markets," *Econometrica* 72, 701-746

Perraudin, W. and P. Vitale (1996) "Interdealer trade and information flow in a decentralized foreign exchange market," in *The Microstructure of Foreign Exchange Markets*, edited by J. Frankel et al, University of Chicago Press

Petersen, M. and R. Rajan (1994) "The benefits of lending relationships: evidence from small business data," *Journal of Finance* 49, 3-37

Petersen, M. and R. Rajan (1995) "The effect of credit market competition on lending relationships," *Quarterly Journal of Economics* 110, 407-444

Reiss, P. and I. Werner (1998) "Does risk sharing motivate interdealer trading?," *Journal of Finance* 53, 1657-1704

Rogoff, K. (2001) "The failure of empirical exchange rate models: no longer new, but still true," www.economic-policy.org, No.1, October 2001

Roth, A. (1985) *Game Theoretic Models of Bargaining*, Cambridge University Press

Rubinstein, A. (1982) "Perfect equilibrium in a bargaining model," *Econometrica* 50, 97-109

Rubinstein, A. (1985) "Choice of conjectures in a bargaining game with incomplete information," in *Game Theoretic Models of Bargaining*, edited by A. Roth, Cambridge University Press

Rupert, P., M. Schindler and R. Wright (2001) "Generalized bargaining models of monetary exchange," *Journal of Monetary Economics* 48, 605-622

Shi, S. (1995) "Money and prices: A model of search and bargaining," *Journal of Economic Theory* 67, 467-496

Shleifer, A. and R. Vishny (1997) "The limits to arbitrage," *Journal of Finance* 52, 35-56

Stiglitz, J. and A. Weiss (1981) "Credit rationing in markets with imperfect information," *American Economic Review* 71, 393-409

Stock, J. and M. Watson (1988) "Testing for common trends," *Journal of the American Statistical Association* 83, 1097-1107

Taylor, M.P. (1987) "Covered interest parity: a high-frequency, high-quality data study," *Economica* 54, 429-438

Taylor, M.P. (1989) "Covered interest arbitrage and market turbulence," *Economic Journal* 99, 376-391

Townsend, R. (1979) "Optimal contracts and competitive markets with costly state verification," *Journal of Economic Theory* 21, 265-293

Viswanathan, S. and J. Wang (2004) "Inter-Dealer trading in Financial Markets," *Journal of Business* 77, 987-1040

Volger, K. (1997) "Risk allocation and inter-dealer trading," *European Economic Review* 41, 417-441

Werner, I. (1997) "A double auction model of interdealer trading," Stanford Business School Research Paper, no.1454

Williamson, S. (1986) "Costly monitoring, financial intermediation, and equilibrium credit rationing," *Journal of Monetary Economics* 18, 159-179

Williamson, S. (1987) "Costly monitoring, loan contracts, and equilibrium credit rationing," *Quarterly Journal of Economics* 102, 135-145

Yao, J. (1998) “Spread components and dealer profits in the interbank foreign exchange market,” New York University Salomon Center Working Paper #S-98-4