LESSON PLANNING AS A VEHICLE FOR DEVELOPING PRE-SERVICE SECONDARY TEACHERS' CAPACITY TO FOCUS ON STUDENTS' MATHEMATICAL THINKING

by

Elizabeth Koopman Hughes

B.S., Mathematics and Computer Science, Dickinson College, 1994

M.A.Ed., Mathematics Education, Wake Forest University, 1995

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This dissertation was presented

by

Elizabeth Koopman Hughes

It was defended on

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and approved by

Dr. Ellen Ansell

Dr. Gaea Leinhardt

Dr. Mary Kay Stein

Dr. Margaret S. Smith
Dissertation Director

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Elizabeth Koopman Hughes, Ed.D.

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This study investigated the extent to and ways in which attention to students' mathematical thinking was evident in the written lesson plans or lesson planning process of ten pre-service secondary mathematics teachers at various points during their teacher education program: prior to and immediately after participation in a course (the Teaching Lab) that emphasized students' mathematical thinking as a key element of planning, during teachers' first semester of their field experience as they planned lessons in their actual practice of teaching, and near the end of the first semester of their field experience as they planned lessons on demand and for university assignments.

With respect to learning from the Teaching Lab, the study shows that the teachers demonstrated significant growth on pre to post course measures in their ability to attend to students' thinking when planning a lesson on demand and for a university assignment. Furthermore, teachers continued to be able to apply these ideas when planning on demand and for university assignments several months later.

When investigating whether or not teachers would apply the ideas they had learned when planning in their own practice, the study suggests three findings. First, teachers' attention to students' thinking when planning lessons that used tasks with a high level of cognitive demand was not significantly different from their planning for a lesson on demand or the lesson plan they

produced for the Teaching Lab assignment. Furthermore, teachers were more likely to attend to students' thinking when planning a lesson that used a high-level task compared to a lesson that used a low-level task. Second, for some teachers, written lesson plans significantly under-represented their attention to students' thinking in their planning process. Finally, the study suggests that support from the mentor teacher and/or university supervisor may be an important factor in determining whether or not the teacher applies their knowledge of attention to student's thinking to their planning in practice.

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CHAPTER 1: THE RESEARCH PROBLEM

1.1 Introduction

For more than fifteen years now, mathematics education has been engaged in a reform movement. This reform was precipitated by the National Council of Teachers of Mathematics' publication of Standards documents (NCTM, 1989, 1991, 1995, 2000).[1] The Standards describe the kind of mathematical knowledge students need in order to be successful in life and the kind of teaching that needs to occur in order for students to develop mathematical understanding. The Standards and other documents (e.g., Hiebert *et al.*, 1997; National Research Council [NRC], 2001) describe what mathematics is important for students to know and understand and what it means to know and understand mathematics. These documents recommend that students should not only be proficient with the processes of calculating, labeling and defining but also need to be proficient in other mathematical processes, such as reasoning, communicating, conjecturing, and justifying.

Furthermore, research has shown that students learn what they have an opportunity to learn (Hiebert, 2003). Therefore it is important for standards-based instruction to provide students with opportunities to engage in mathematical reasoning, communicating, conjecturing, and justifying. Traditional instruction has rarely offered students these kinds of opportunities and the typical U.S. mathematics lesson can be described as being:

> Organized around two phases: an acquisition phase and an application phase. In the acquisition phase, the teacher demonstrates or leads a discussion on how to solve a sample problem. The aim is to clarify the steps in the procedure so that students will be able to execute the same procedure on their own. In the application phase, students practice using the procedure by solving problems similar to the sample problem (Stigler & Hiebert, 1997, p. 18)

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[1] The phrase "Standards" is used to capture the recommendations for K-12 curriculum, teaching, and assessment contained in the initial documents (NCTM, 1989, 1991, 1995) and in the revised document *Principles and Standards for School Mathematics* (NCTM, 2000).

Traditional instruction that follows this lesson format provides students with opportunities to become proficient in enacting procedures, however it does not offer them opportunities to develop conceptual ideas or to connect the procedures they are learning with the concepts that show why those procedures work (Hiebert, 2003).

By contrast, standards-based instruction provides students with opportunities to develop a deeper understanding of mathematics because it builds directly on students' prior knowledge, provides students with opportunities to do mathematics, focuses on the analysis of multiple representations or solution strategies, and presses students to provide explanations. This type of instruction is often referred to as "student-centered" because of the central role played by students in the classroom and its focus on students' thinking. This type of instruction also calls for a new role for the teacher. Effective standards-based instruction occurs when the teacher focuses on the important mathematical content and "takes sensitive account of students' current knowledge and ways of thinking as well as ways in which those develop" (NRC, 2001, p. 315). Thus knowledge of content and knowledge of students' mathematical thinking are both needed in order for teachers to effectively implement standards-based instruction. Although secondary pre-service teachers often bring the former (i.e., content knowledge) to the teacher education programs in which they enroll, they seldom bring the latter (i.e., knowledge of students' mathematical thinking). The purpose of this study is to investigate the ways in which pre-service secondary mathematics teachers' attention to students' mathematical thinking during lesson planning develops over the course of their first semester in a teacher education program. The following sections provide further detail on the research and framework that this study is built upon.

1.2 Focusing on Students' Mathematical Thinking

While there is wide agreement that to teach mathematics a teacher must have knowledge of content, research has emphasized, among other things, that teachers also need knowledge of their students as learners of mathematics (Ball, 1993; Ball & Bass, 2000; Ball, Lubienski, & Mewborn, 2001; Fennema & Franke, 1992; Shulman, 1986). That is, teachers should have knowledge of how students think about and learn specific mathematics content; including knowledge of how students acquire new mathematical content, the possible solution strategies or processes students might employ, and the likely preconceptions and misconceptions that students will have.

While some researchers have worked to identify the specific knowledge of students that teachers need to teach, others have focused on *how* teachers should use their knowledge of students to plan for and implement standards-based instruction (Ball, 2001; Lampert, 2001; Schifter, 2001; Stein, Engle, Hughes, & Smith, 2006). For example, Schifter (2001) provides a detailed set of actions in which teachers need to engage as part of their teaching practice that includes:

> (1) attending to the mathematics in what one's students are saying and doing, (2) assessing the mathematical validity of students' ideas, (3) listening for the sense in students' mathematical thinking even when something is amiss, and (4) identifying the conceptual issues the students are working on (p. 119).

Lampert (2001) and Stein and her colleagues (2006) describe how teachers make use of their knowledge of students as they monitor students' work on a mathematics problem, select students to share their solutions with the whole class and consider the order in which these solutions should be presented, and consider questions to ask in order to help students make connections between solutions and concepts. There is a growing consensus that in order for teachers to effectively implement standards-based instruction, they must have knowledge of their students' mathematical thinking, have methods for assessing and making sense of students' thinking during class, and be able to make critical decisions about using their students' thinking to facilitate a mathematically productive discussion.

Research has shown, however, that many U.S. teachers, particularly in elementary and middle schools, do not have content specific knowledge of students' mathematical thinking (e.g., Ball, Lubienski, & Mewborn, 2001; Ma, 1999). Furthermore, even teachers who do have some knowledge of students as learners of mathematics find it challenging to make use of that knowledge in the process of teaching (Ball, 2001; Ball, Lubienski, & Mewborn, 2001; Chazen & Ball, 1999; Lampert, 2001; Schifter, 2001; Schoenfeld, 1998; Sherin, 2002). That is, when a teacher is attempting to interweave the important mathematical content of the lesson with the students' current and projected mathematical thinking, particular challenges arise. One problem arises as teachers try to figure out what students actually understand about the mathematics in the lesson. For example,

> If teachers "fill in" and overinterpret what students know and can do, they may inappropriately credit students with a "right" answer. Consequently, they may attribute to students an understanding that students have not yet reached. If, however, teachers cannot hear "below the surface" features of children's talk and representations, they may miss the mark by considering a student wrong who has in fact an interesting idea or is carrying out a nonstandard procedure, but one with mathematical promise. Suspending one's desire for students to get answers right and thinking mathematically about what a child might mean are among the most difficult problems of teaching (Ball, 2001, p. 19).

Another critical challenge that mathematics teachers face when teaching is managing and using the multiple representations and solution strategies generated by students. For example, the sharing of students' various strategies to solving a problem can lead to nothing more than a 'show and tell' session. Rather than producing mathematical clarity and depth, the result is "cacophony" (Ball, 2001, p. 19). That is students merely see a lot of ways to solve the problem and are not pressed to make connections between the strategies or understand why they work.

Thus the challenge of finding ways to engage students in comparing multiple solution strategies and representations should not be underestimated. Furthermore, the problem of managing a class's mathematical progress such that individual students' ideas and questions are respected and used while at the same time moving the class along the intended mathematical trajectory of the lesson is a nontrivial problem of teaching (Ball, 2001; Hiebert *et al.*, 1997; Lampert, 2001; Leinhardt & Steele, 2005).

While there is evidence of teachers successfully making use of students' thinking in the process of their teaching, it is often seen in "expert" teachers (e.g., Ball, 1993; Lampert, 2001; Leinhardt, 1993; Schoenfeld, 1998; Schoenfeld, Minstrell, & van Zee, 2000). For beginning teachers, this can be a daunting, if not seemingly impossible, task (Heaton, 2000; Schoenfeld, 1998; Sherin, 2002; Zimmerlin & Nelson, 2000). Therefore, it is not surprising that research shows most U. S. teachers do not focus on students' mathematical thinking in the classroom in ways that enable them to implement standards-based instruction (National Center for Educational Statistics [NCES], 2003; Stigler & Hiebert, 1999; Weiss & Pasley, 2004).

1.3 Implementing Standards-Based Instruction in the United States

Even though reform efforts that emphasize the role of student thinking have been underway for almost 15 years, the majority of mathematics teachers in the United States do not effectively use students' thinking in ways that develop students' conceptual understandings in their teaching practice on a regular basis. For example, in an international comparison of instruction in eighth grade classrooms, Stigler and Hiebert (1999) found that less than 10% of the mathematics lessons in the U.S. were "student-controlled." By contrast, twice as many German lessons, and four times as many Japanese lessons were "student-controlled." "Student controlled" lessons were ones in which the students were responsible for working out various solution methods as opposed to the teacher providing a procedure that the students then executed. Therefore, in U.S. classrooms where teachers are doing the mathematics for their students rather then giving the responsibility of solving problems to the students there is very little student thinking about mathematics that teachers can access and make use of to orchestrate mathematically meaningful discussions.

Other research on the quality of instruction in U. S. classrooms indicates little progress has been made in implementing instruction that focuses on the development of students' mathematical thinking. In an analysis of a national K-12 sample of more than 350 mathematics and science lessons, researchers report that teachers were not making effective use of their students' thinking during lessons (Weiss, Pasley, Smith, Banilower, & Heck, 2003). In particular, the mean score for the teacher being able to adjust instruction according to the level of students' understanding was 2.42 on a scale from 1 to 5, with 5 being the highest rating. A mean score of 2.15 was reported for teachers' questioning enhancing the development of students' understanding. Similarly, overall quality of lesson implementation scores were poor, with only 15 percent of the lessons rated as high in quality, 27 percent medium, and 59 percent low in quality.

While there is agreement within the mathematics education community of the importance of teachers attending to and making instructional decisions based on students' mathematical thinking, there is a growing body of evidence, from large-scale international and national studies, that teachers in the United States are not effectively using students' mathematical thinking in ways that enable them to implement standards-based instruction. Thus there is a need for professional education that will develop teachers' capacity to focus attention on students' mathematical thinking during their instruction. The mathematics education community has responded to this call with various professional development initiatives.

1.4 Professional Development Initiatives

While evidence from international and national large-scale studies show few American mathematics teachers routinely use students' thinking in their teaching in order to improve students' conceptual understandings, smaller-scale studies indicate there are a variety of ways to increase teachers' knowledge of and focus on students' mathematical thinking. These include, but are not limited to, the following: (a) developing teachers' capacity to recognize and use research-based knowledge on students' cognition in a specific mathematical domain (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Fennema *et al.*, 1996; Swafford, Jones, & Thornton, 1997; Vacc & Bright, 1999; Warfield, 2001); (b) having teachers analyze students' written work in order to see potential in "wrong" answers and to develop a deeper understanding of students' mathematical thinking in a specific mathematical domain (Crespo, 2000; Franke & Kazemi, 2001; Kazemi & Franke, 2004; Little, Gearhart, Curry, & Kafka, 2003b); (c) having teachers analyze their own videotaped lessons for evidence of students' mathematical thinking in order to reframe pedagogical issues in terms of student thinking (Masingila & Doerr, 2002b; Sherin, 2001; Sherin & Han, 2004); (d) having teachers read and discuss cases of mathematics instruction (narrative or video) that make salient the importance of assessing and using students' mathematical thinking during a lesson (Barnett, 1998; Stein, Hughes, Engle, & Smith, 2003); and (e) developing teachers' capacity to attend to students' mathematical thinking through practice-based professional development experiences that incorporate multiple elements of the earlier programs (Hughes & Smith, 2004; Schifter, 1998; Stein *et al.*, 2006).

The changes in teachers' knowledge of students' thinking and teaching practices as a result of participating in professional development initiatives that focus on students as learners of mathematics have all been promising. One example of a project that showed positive growth in teachers' ability to focus on students is Warfield's (2001) study of a fifth grade teacher who had participated in the Cognitively Guided Instruction (CGI) project. Warfield concluded that:

> Teachers with knowledge of both research-based information on children's thinking and the mathematics they teach are able to: (a) pose questions that go beyond asking children to describe their solution strategies; (b) understand children's mathematical thinking that differs from what might be expected based on the research-based information on children's thinking; (c) critically examine children's thinking to determine if it is mathematically valid; and (d) use what they learn about their children's thinking to create tasks that enable children to extend their thinking (p. 151).

While it is encouraging to see professional development projects affecting teachers' practice in such a way, most of the research to date has been with K-5 practicing elementary teachers. Little research has investigated whether practicing or pre-service secondary teachers can learn to focus their teaching practices on students' mathematical thinking. Furthermore, many of the projects focused on students' thinking within a very specific mathematical domain, such as addition and subtraction of whole numbers, and leave unanswered the question of how to impact the teachers' practice for the entire curriculum they are responsible for teaching. In the next section, Lesson Study, an approach to professional development that is both focused on students' mathematical thinking and applicable to all aspects of the curriculum, will be explored.

1.5 Japanese Lesson Study

It is well documented that Japanese K-8 teachers have extensive knowledge of their students' mathematical thinking and effectively use it in their planning and teaching (e.g., Lewis & Tsuchida, 1998; Stigler & Hiebert, 1999; Yoshida, 1999). The change in Japan from a traditional mode of instruction to a student-centered approach to teaching has been attributed to the professional development program of "*jugyokenkyu"* or the direct English translation, "lesson study" (Lewis & Tsuchida, 1998). Japanese lesson study is a form of professional development that focuses teachers on planning, implementing, and reflecting on a single lesson. In lesson study, the planning of the lesson is critical. Significant time and effort is put in to collaboratively developing a lesson plan. Much of the content discussed and debated during the planning session is focused on students' mathematical thinking. According to Stigler & Hiebert (1999), Japanese teachers engage in detailed discussions of the following topics in the weeks spent planning the lesson:

- The problem with which the lesson would begin, including such details as the exact wording and numbers to be used.
- The materials students would be given to use in trying to solve the problem.
- The *anticipated solutions, thoughts, and responses that students might develop* as they struggled with the problem.
- The kinds of questions that could be asked to *promote student thinking* during the lesson, and the kinds of guidance that could be given to *students who showed one or another type of misconception* in their thinking.
- How to use the space on the chalkboard (Japanese teachers believe that organizing the chalkboard is a key ingredient to *organizing students' thinking* and understanding).
- How to apportion the fixed time of the lesson—about forty minutes—to different parts of the lesson.
- How to handle individual differences in level of mathematical preparation among the students.
- How to end the lesson – considered a key moment in which *students' understanding* can be advanced. (p. 17, *emphasis added*)

After a lesson plan has been agreed upon, one of the teachers from the collaborative team teaches the lesson while the remaining members of the team and others observe the lesson. Afterwards, a de-briefing session is held in which the lesson plan is critiqued. Based on the feedback from the lesson, the lesson plan is revised by the team of teachers and often is taught again by another teacher from the team.

Lesson study is very common in the Japanese education system for elementary and middle grades teachers. Research has shown that lesson study has improved Japanese science and mathematics education, the teachers' instruction, and the students' achievement (Lewis & Tsuchida, 1998; Watanabe, 2002; Yoshida, 1999). In an attempt to improve U.S. student achievement and teaching, one currently popular suggestion has been to adopt lesson study (Lewis, 2002; Stigler & Hiebert, 1999; Yoshida, 1999).

1.6 Lesson Study in the United States

Lesson study groups are popping up all over the United States, in at least 250 schools in 29 states (Lewis, Perry, & Hurd, 2004). They are usually created by a university teacher educator in collaboration with practicing K-8 teachers (Lewis, 2002). Some teacher education programs are even adapting lesson study. While there is considerable interest in lesson study as a model for professional development, there is little empirical research showing its effective implementation in the United States. In fact, the published research on lesson study in the United States is fraught with difficulties in its implementation (Fernandez, 2002; Fernandez, Cannon, & Chokshi, 2003; Wagner, 2003). For example, in an empirical study of 16 teachers (K-8) and administrators from an urban public school that engaged in lesson study with twelve Japanese teachers serving as coaches for the lesson study sessions, Fernandez, Cannon and Chokshi (2003) cite multiple examples where the American teachers were not discussing or grappling with issues focused on students' mathematical thinking, particularly in the planning process of their lesson. Instead, teachers focused on other aspects of the lesson (e.g., how to group students, the materials needed). The Japanese teachers, serving as coaches, posed questions or suggestions to focus the American teachers on specific issues related to students' mathematical thinking. As a result of the intervention by the coaches, the American teachers were able to discuss and make progress on incorporating students' mathematical thinking in their lesson plan. However, the teachers continually ran into limitations of their ability to really think deeply about students' mathematical thinking. Thus the American teachers could attend to students' mathematical thinking in their lesson planning discussions when pressed to do so from a coach and with some limitations. However, even after being pressed, they continued to need input from the Japanese coaches in order to attend to students' mathematical thinking and still did not spontaneously focus on students' thinking. Fernandez et al. (2003) argue that lesson study can be a format for professional development that helps American teachers focus on students' mathematical thinking. However, they also make clear the need for well-trained coaches to facilitate the lesson study discussions since teacher-led lesson study groups that do not have the aid of an expert facilitator would be unlikely to engage in the critical elements of focusing on students' mathematical thinking. This raises a question regarding the types of experiences that may help develop teachers' capacity to focus on students' mathematical thinking in a deep and meaningful way during the planning process.

1.7 This Study

A growing body of literature (e.g., Ball, Lubienski, & Mewborn, 2001; Lampert, 2001; NCTM, 1991; Stigler & Hiebert, 1999) has recognized the importance of teachers focusing on students' mathematical thinking in order to implement standards-based instruction. There is also evidence that focusing on students' mathematical thinking in the process of planning a lesson is a critical part of improving instruction and student achievement (e.g., Lampert, 2001; Lewis &

Tsuchida, 1998; Stigler & Hiebert, 1999). However, there is very little empirical research on the lesson planning process of mathematics teachers in the United States. Specifically, there is little data on the ways in which pre-service secondary mathematics teachers attend to students' mathematical thinking in their lesson planning. The study described herein focuses on planning practices of pre-service secondary mathematics teachers as they engage in a university teacher education program that emphasizes students' mathematical thinking as a critical and key component of the planning process. In particular, the study aims to answer the following research questions:

1. To what extent is attention to students' mathematical thinking evident in pre-service secondary mathematics teachers' written lesson plans or lesson planning process prior to and immediately after participation in a course that emphasizes students' mathematical thinking as a key element of planning?
2. To what extent is attention to students' mathematical thinking evident in pre-service secondary mathematics teachers' written lesson plans or lesson planning process during the first semester of their field experience?
3. To what extent does attention to students' mathematical thinking, as evident in pre-service secondary teachers' lesson planning, change over time?
4. In what ways does attention to students' mathematical thinking, as evident in pre-service secondary teachers' lesson planning, change over time?

The first two research questions seek to document the extent to which prospective secondary mathematics teachers attend to students' mathematical thinking as evident in their lesson planning prior to and immediately after participating in a course that focuses on students' mathematical thinking as well as during the first semester of their field experience. The third research question aims to document the extent to which teachers' attention to students' mathematical thinking, as evident in their lesson planning, has changed over time. Finally, the fourth research question seeks to identify patterns that emerge in the ways that teachers attend to students' mathematical thinking as evident in their lesson planning by providing qualitative portraits of these patterns that may explain changes in teachers' planning over time.

1.7.1 Significance of the Study

The study contributes to the literature base in several important ways. This study shows whether pre-service secondary mathematics teachers can learn to attend to students' mathematical thinking in their lesson planning and the ways in which that is evident in written lesson plans and in their thinking about a lesson. In addition, this study identifies whether any element of focusing on students' mathematical thinking is more or less likely to be evident in pre-service teachers' planning. This study documents whether teachers continue to apply what they have learned in their teacher education program or change their planning practices as they engage in the actual practice of teaching during their field experience. Furthermore, the elements of focusing on students' mathematical thinking that are more or less likely to be attended to in the actual practice of teaching are identified. The results of the study may benefit teacher educators as they design standards-based teacher education programs and professional development initiatives.

1.7.2 Limitations of the Study

There are several limitations to the proposed study. First, all participants in this study were enrolled in a graduate teacher education program at a large, urban university in the northeast United States that culminated in certification in 7-12 mathematics and a Master of Arts in Teaching degree. In order to be accepted into this master-level program, applicants needed to have a bachelor's degree in mathematics (or the equivalent) and a minimum QPA of 3.0. Thus these pre-service secondary teachers may not be representative of secondary pre-service teachers in the United States. In addition, the sample size is small (N=10) and there is no equivalent group that can be used for comparison. Therefore the results may not generalize to other teacher education programs. Furthermore, this study does not document the effects of the pre-service secondary teachers' attention to students' mathematical thinking in their lesson planning on their lesson implementation or how this impacts student-learning outcomes.

1.8 Organization of the Document

In the next chapter, three bodies of literature that are pertinent to the study are reviewed: professional development initiatives designed to increase teachers' use of students' mathematical thinking, the relationship between planning and teaching, and pre-service teacher education. In Chapter Three, the methodology for the study is described. Results of data analysis are reported in Chapter Four. Finally, the results of the study are briefly summarized and situated in the mathematics education literature in Chapter Five.

CHAPTER 2: REVIEW OF THE LITERATURE

2.1 Introduction

A growing body of literature has recognized the importance of teachers' content knowledge and knowledge of students' as learners in order to implement standards-based instruction (Ball, Lubienski, & Mewborn, 2001; National Council of Teachers of Mathematics, 1991; National Research Council, 2001). One way of conceptualizing the unique blend of mathematical and pedagogical knowledge is as *mathematics knowledge needed for teaching* (Ball, Bass, & Hill, 2004). Ball and colleagues identify two facets of knowledge needed for teaching: *Knowledge of mathematics and mathematical activities, and knowledge of mathematics for student learning*. Knowledge of mathematics for student learning entails the knowledge needed to analyze student work, the questions one might ask to advance a student's understanding, and the ways students might think about and do mathematical problems.

As teachers have tried to adapt their teaching in order to implement standards-based instruction, researchers have begun to document the complexities and challenges teachers face as they make use of their knowledge of mathematics for student learning in their classrooms (e.g., Ball, Lubienski, & Mewborn, 2001; Lampert, 2001; Schifter, 2001; Sherin, 2002). Research also indicates that many mathematics teachers in the United States do not focus on their students' mathematical thinking in their planning and teaching (e.g., Stigler & Hiebert, 1999; Weiss *et al.*, 2003).

By contrast, teachers in Japan do focus on students' mathematical thinking during instruction and this focus is a critical part of the lesson planning process (e.g., Fernandez & Yoshida, 2004; Lewis & Tsuchida, 1998; Stigler & Hiebert, 1999; Yoshida, 1999). Similar planning practices have been found when investigating expert teachers' implementation of standards-based instruction in the United States (e.g., Lampert, 2001; Leinhardt, 1993;

Schoenfeld, 1998). This raises the question regarding lesson planning as a vehicle for improved instructional practice with a focus on students' thinking for all teachers in the United States. However, there is little empirical research on the lesson planning process of mathematics teachers in the United States. Specifically, there is little data on the ways in which pre-service secondary mathematics teachers attend to students' mathematical thinking in their lesson planning. The study described herein, focuses on the planning practices of pre-service secondary mathematics teachers as they engage in a university education program that emphasizes attending to students' mathematical thinking as a critical element of the planning process.

In this chapter, three bodies of literature will be reviewed that are pertinent to this study. First, studies will be reviewed that have investigated the impact of professional development initiatives on teachers' attention to students' mathematical thinking and when available, the impact these efforts have had on student achievement. This will be followed by a description of research studies that have explored the relationship between planning and teaching as well as empirical evidence of teachers' planning practices. Then a framework for attending to students' mathematical thinking while planning a lesson, focused on the four key elements that have emerged from the literature, will be described. Finally, literature will be reviewed in order to describe what we currently understand about pre-service teacher education.

2.2 Professional Development Initiatives

Research suggests that there are a variety of ways to increase teachers' knowledge of and attention to students' mathematical thinking. These include but are not limited to the following: (a) developing teachers' capacity to recognize and use research-based knowledge on students' cognition in a specific mathematical domain (Carpenter *et al.*, 1989; Fennema *et al.*, 1996; Swafford, Jones, & Thornton, 1997; Vacc & Bright, 1999; Warfield, 2001); (b) having teachers analyze students' written work in order to see potential in "wrong" answers and to develop a deeper understanding of students' mathematical thinking in a specific mathematical domain (Crespo, 2000; Franke & Kazemi, 2001; Kazemi & Franke, 2004; Little, Gearhart, Curry, & Kafka, 2003a); (c) having teachers analyze videotaped lessons for evidence of students' mathematical thinking in order to reframe pedagogical issues in terms of student thinking (Masingila & Doerr, 2002a; Sherin, 2001; Sherin & Han, 2004); (d) having teachers read and discuss narrative cases of mathematics instruction that make salient the importance of assessing and using students' mathematical thinking during a lesson (Barnett, 1991, 1998; Stein *et al.*, 2003); (e) developing teachers' capacity to attend to students' mathematical thinking through practice-based professional development experiences that incorporate multiple elements of the earlier programs (Hughes & Smith, 2004; Schifter, 1998; Stein *et al.*, 2005); and (f) focusing teachers' attention on students' mathematical thinking, particularly in their lesson planning process by implementing the Japanese Lesson Study model in the United States (Fernandez, Cannon, & Chokshi, 2003; Wagner, 2003). The following sections will review studies that contribute to our understanding about the different ways in which professional development can increase teachers' attention to students' mathematical thinking. Thus, these sections provide a review of professional development efforts that specifically focus on increasing teachers' (in-service and pre-service) attention to and use of students' thinking and that speak to the variety of ways in which this type of professional development can be designed. While this is not an exhaustive review, it aims to be representative of the variety of ways in which professional development initiatives have tried to, and often successfully, increase teachers' attention to student thinking and the ways in which researchers have studied a professional development initiative's impact on teachers.

2.2.1 Research-Based Knowledge on Students' Mathematics Cognition

Professional Development for In-service Teachers. Cognitively Guided Instruction (CGI) is based on the premise that providing teachers with research-based knowledge on children's thinking in a specific mathematics domain will influence the teachers' instruction and their students' mathematics achievement. The intention is to help teachers build connections between a research-based model of students' thinking and their own students' thinking. As a result of making these connections, it is believed that teachers will spend more time having students solve problems, will be more likely to expect multiple solution strategies from their students, and will listen to their students in order to make sense of students' thinking. CGI provides some of the most extensive research on teachers' knowledge of and focus on students' mathematical thinking and its impact on instruction and student achievement to date.

In its initial work, CGI used extensive research on children's addition and subtraction concepts as the basis for teacher professional development. Teachers were taught to recognize a taxonomy of problem types of addition and subtraction word problems and to identify the various strategies children use in solving those problems and how the strategies build on each other. More recently, CGI has been used in the following domains: addition and subtraction of whole numbers, multiplication and division of whole numbers, place value, algebra, fractions, and geometry (Franke, Carpenter, Levi, & Fennema, 2001).

In an early study of CGI, results showed that teachers who participated in CGI changed their instructional practices and had higher student mathematical achievement than teachers not trained in CGI (Carpenter *et al.*, 1989). This study used an experimental design where 20 first grade teachers participated in a 4-week summer workshop to learn about CGI and another 20 first grade teachers served as a control group by participating in two 2-hour workshops about non-routine problem solving. All 40 teachers and their students were observed during mathematics instruction throughout the following school year. At the end of the year, teachers' knowledge of their students was measured by asking them how individual students in his or her class would solve a specific problem and if the student would get a correct answer. Teachers' predictions were then matched to their students' actual responses. Students were also given a standardized mathematics achievement pretest and posttest. Results indicated that CGI teachers listened to the strategies their students used to solve problems significantly more often than control teachers. CGI teachers also encouraged their students to use a variety of problem solving strategies more often than control teachers. CGI teachers taught problem solving significantly more and number facts significantly less than did control teachers. CGI teachers also knew more about individual students' problem solving strategies. Furthermore, students in CGI classes outperformed students in control classrooms in number fact knowledge and problem solving. Students in CGI classes also reported higher confidence in their problem solving abilities than students in control classrooms.

Similar results were found in a 4-year longitudinal study of CGI teachers and their students (Fennema *et al.*, 1996). Twenty-one elementary teachers (grades 1-3) took part in CGI professional development programs that focused on research-based knowledge of children's thinking in solving addition, subtraction, multiplication, and division word problems. These teachers' instructional practices and the achievement of their students were studied over a 4-year period. Changes in instructional practice were measured through observations and were categorized by levels of cognitively guided instruction (1 being the lowest and 4 the highest). There was significant improvement in the level of cognitively guided instruction used by teachers over the 4-year period. The instruction of 90% of the teachers became more cognitively guided and was categorized at Level 3 or higher by the end of the study. Teachers changed from demonstrating procedures to engaging students in a variety of problem-solving situations and encouraging them to talk about their mathematical thinking. Every teacher also showed increased student achievement in concepts and problem solving from the first year to the fourth year.

In a case study of a fifth grade teacher who had participated in CGI training, Warfield (2001) describes similar changes in the teacher's instruction. In particular, a more detailed description of the kind of teaching CGI teachers engage in led Warfield to conclude that:

> Teachers with knowledge of both research-based information on children's thinking and the mathematics they teach are able to: (a) pose questions that go beyond asking children to describe their solution strategies; (b) understand children's mathematical thinking that differs from what might be expected based on the research-based information on children's thinking; (c) critically examine children's thinking to determine if it is mathematically valid; and (d) use what they learn about their children's thinking to create tasks that enable children to extend their thinking (p. 151).

Swafford, Jones and Thornton (1997) build on the CGI premise that instruction can be improved by increasing teachers' knowledge of students' cognition in a specific mathematics content area, but differ from CGI efforts in grade level (middle-grade (4-8) teachers rather than elementary teachers (K-5)) and mathematics content focus (geometry rather than whole number operations). In their study, 49 middle-grade in-service teachers participated in a 4-week summer session that consisted of a mathematics content course in geometry and a weekly seminar that focused on the van Hiele model of cognitive development in geometry. As a part of this seminar, teachers had the option of interviewing a student at the grade level they teach or analyzing their textbooks by van Hiele levels. In addition to taking a pre- and post-test on geometry content, participants were asked to plan a lesson at the beginning and end of the summer session. The lesson-plan task provided teachers with a two-page geometry lesson from a textbook for their grade level. Teachers were then asked to imagine they were going to teach this lesson to their students and were given 20 minutes to write a lesson plan. Various aspects of the lesson plans were coded according to the van Hiele levels of geometric understanding (the van Hiele model has five sequentially ordered levels of geometric thinking with level 1 being the lowest and level 5 the highest). Results from the pre- and post- lesson-plan task indicated a significant change in teachers' goals and expectations for students, with respect to van Hiele levels. In particular, the initial plans showed 39% of the plans at the first level, 58% at the second level, and 2% at level three. By contrast, the second lesson plans showed 25% at the first level, 73% at the second level, and 3% at the third van Hiele level. Additionally, in the second lesson plan, teachers more frequently suggested giving a pre-assessment to determine what their students already knew about the topic. Although teachers were still in the lower levels of the van Hiele model (the highest score attained was level three out of five levels), they did show improvement in their goals and expectations of students in these geometry lessons.

During the following academic year, eight of these teachers were observed and videotaped teaching from three to five lessons in geometry and participated in interviews immediately after the observations. Researchers found that teachers

> (a) were spending more time and more quality time on geometry instruction; (b) were more willing to try new ideas and instructional approaches; (c) were more likely to engage in risk-taking that enhanced student learning; and (d) were more confident in their abilities to provoke and respond to higher levels of geometrical thinking (Swafford, Jones, & Thornton, 1997, p. 476).

These positive changes were influenced by teachers' increased geometry content knowledge and research-based knowledge of student cognition in geometry. Unfortunately, the researchers could not separate the effects of these two components in their influence on the teachers' instruction. The researchers claim that both the increase in content knowledge and the increase in knowledge of students' cognition played a role in improving the teachers' instruction.

This speaks to the importance of and interconnectedness of *knowledge of mathematics and mathematical activities* and *knowledge of mathematics for student learning* as encompassing the *knowledge needed for teaching* proposed by Ball, Bass, and Hill (2004).

Teacher Education for Pre-service Teachers. While most CGI studies have involved in-service elementary teachers, CGI training has been shown to be effective in changing pre-service teachers' practice as well. However, it also faces challenges specific to pre-service teacher education (Vacc & Bright, 1999). In a study of 34 pre-service elementary school teachers trained in CGI, Vacc and Bright found significant changes in the pre-service teachers' beliefs and perceptions about mathematics instruction. Two of the 34 pre-service teachers in the study (Helen and Andrea) were selected for an in-depth study of their student teaching practices. Both pre-service teachers had significantly changed their beliefs about mathematics instruction and believed students' mathematical thinking should be an important part of instruction. One difference in their student teaching experience was that Helen was placed with a mentor teacher who had been trained in CGI, whereas Andrea's mentor teacher had not been trained in CGI. During the student teaching experience, Helen exhibited signs of employing CGI principles in her teaching by basing her instruction on problem solving and facilitating student understanding through a high level of questioning. Furthermore, she exhibited these behaviors throughout her student teaching experience. On the other hand, Andrea's beginning lessons showed signs of employing CGI principles in her questioning techniques. However, as her student teaching experience progressed she moved to a more teacher directed style of instruction and her questions focused on correct answers and predetermined strategies for solving a problem. In interviews and reflections, Andrea continued to talk about wanting to focus on students' mathematical thinking and that questioning was the most effective way to find out what students were thinking. Unfortunately, Andrea's actual teaching practices did not exhibit this belief throughout her student teaching experience.

This study only looked at two students in-depth, therefore further studies need to be done before conclusions can be made about the effects of CGI on pre-service teachers. However, this raises some very important questions about the field placements universities make for their pre-service teachers. This study lends credence to the notion that it is important for pre-service teachers' mentor teachers' instruction to be aligned with that espoused by the teacher education program in which the pre-service teachers are enrolled.

Conclusions. The studies reviewed here indicate that providing teachers with research-based knowledge of students' mathematical thinking can change their instruction. Looking across these studies, there are consistencies in the kind of instruction CGI teachers, equipped with research-based knowledge of students' thinking, engaged in, including the following: listening to students' strategies to make sense of their thinking, encouraging and anticipating students' use of a variety of problem solving strategies, facilitating opportunities for students to articulate their mathematical thinking, and asking students questions to assess and advance their understanding. Furthermore, several of these studies go on to show that teachers' increased use of students' mathematical thinking during instruction impacts students' mathematics achievement in positive ways. This is positive evidence that the kind of teaching found in CGI classrooms, which is congruent with the teaching promoted by *Standards*, results in higher student achievement.

These studies, however, focus on narrow bands of content that have been thoroughly researched and practicing elementary and middle school teachers. Very little research exists related to pre-service teachers and there is no empirical evidence to date of the impact of providing secondary mathematics teachers with research-based knowledge of students' mathematical cognition.

2.2.2 Students' Mathematical Work

Professional Development for In-service Teachers. Kazemi and Franke (2004) describe a community of practice (teacher work groups) that enabled elementary in-service teachers to focus on the mathematical thinking of their own students. Four teacher work groups, each consisting of 12 teachers (K-5) met on a monthly basis. Each month the teachers were given a mathematics word problem to pose to their students. The teachers could change the numbers and the context of the problem but were not to change the structure of the problem. They then brought student work to the subsequent work group meeting where they shared their students' solutions, compared strategies used to solve the problem, and discussed how particular strategies were elicited and built on each other. The teachers were also asked to make sense of students' mathematical thinking. The authors worked with these teacher work groups over a four-year period and report important changes in the teachers' analysis of their students' work. The teachers became much better at detailing their students' mathematical thinking and began including the pedagogical practices that supported student thinking and the questions asked to elicit student thinking in their analyses.

In describing their engagement with teacher work groups, Franke and Kazemi (2001), argue that this work builds on the researchers' earlier involvement and understandings of CGI. They claim:

> Student thinking remains at the core of our CGI work. What has changed is how we conceptualize what it means to engage with student work, how we come to understand what teachers and students are learning, and how we create opportunities for teacher and student learning (p. 108).

As facilitators during teachers' discussions of student work, the researchers encouraged teachers to think about the relationships across strategies and highlighted the mathematical ideas students were developing; similar to knowledge that was involved in CGI training. Franke and Kazemi suggest this new approach to focusing on students' mathematical thinking is more situated in the practice of teachers (the student work comes from their own students) and thus offers teachers opportunities to ground theories of students' mathematical thinking in their actual practice. As a result, teachers' experienced generative growth and the teacher work groups continued two years after the researchers discontinued their participation.

Studies of other teacher groups that engage in examining student work have not been as well documented, but do support the notion that looking at student work yields benefits for teaching and learning (Little *et al.*, 2003b). The authors engaged in case studies of teacher groups working with three nationally recognized organizations. Specifically, an elementary school affiliated with the Harvard Project Zero; a middle school working with the Academy for Educational Development; and two high schools participating in the Coalition of Essential Schools. Three common elements were shared by the projects and sites: (1) bringing together teachers in order to focus on student learning and teaching practices; (2) getting student work on the table and into the conversation; and (3) structuring the conversations through the use of protocols designed to focus teachers on examining what the student work can tell teachers about student understanding and teaching practices. These protocols were particularly helpful in facilitating group discussions where everyone was required to participate and deepening the conversation beyond superficial examinations of student work. However, the effects of the session in which student work is analyzed on teachers' teaching practices were not investigated.

Teacher Education for Pre-service Teachers. Crespo (2000) describes an interesting way of providing pre-service elementary teachers with authentic student work to analyze and a safe and supportive environment in which they can play the role of teacher. The activities in Crespo's methods course include a pen-pal letter exchange between the pre-service teachers and fourth grade students. The pre-service teachers experienced a mathematics problem in their methods class, they then wrote to their pen-pal asking them to solve the same or similar problem. The fourth graders solved the problems and wrote letters with their work back to the pre-service teachers. The pre-service teachers analyzed the students' work and wrote responses back to the students. The fourth grade students and pre-service teachers both worked within groups of four so they could collaboratively read and write letters. For thirteen of the twenty pre-service teachers in her course, Crespo studied six mathematics letters received and written in conjunction with three common class problems they sent to the students, journals about their interactions with the students, and a case report written at the end of the course. Results of the study showed that in the beginning of the course, pre-service teachers tended to focus on the correctness of students' answers. They tended to accept correct answers as signs of students understanding the mathematics while wrong answers were taken as evidence of students' confusion or carelessness. The pre-service teachers did not look deeply into students' wrong answers to figure out what the students did and did not understand. The pre-service teachers were quick to draw conclusions in their analysis of students' work and did not explore or speculate about what the students' work might mean or suggest about their understanding of the mathematics. The pre-service teachers also made judgments about the student's mathematical abilities and attitudes toward mathematics based on the student's work. In later letter exchanges and journal entries (after the 5th week of an 11 week course), Crespo found evidence of a change in the pre-service teachers' focus in reviewing students' work, from correctness to looking for meaning. The pre-service teachers' interpretations were also not so quick and conclusive; instead they explored students' work for the meaning of the students' mathematical thinking and made speculations about students' mathematical understanding of a specific topic.

According to Crespo, there are several factors that influenced this change in the pre-service teachers: (a) pre-service teachers' own experiences with unfamiliar and difficult mathematical tasks in class; (b) face-to-face interviews with their student pen-pal which allowed them to stop making judgments about their ability and attitude towards math based on their work; and (c) the journal reflection assignments the pre-service teachers wrote and on which they received feedback seemed to push them to look deeper at the student work. Crespo also points out that the pen-pal system allowed the pre-service teachers time to reflect on students' work and discuss it with peers, as opposed to interviewing students or tutoring where on-line decisions need to be made. Therefore the pen-pal approach to analyzing student work provided pre-service teachers with an authentic experience, but also allowed for a safe and supportive environment for them to analyze student thinking.

<u>Conclusions.</u> The research related to professional development initiatives designed to have teachers analyze authentic student work provides evidence that this approach can increase teachers' capacity to make sense of students' mathematical thinking. In addition, there is evidence that teachers who engage in analyzing student work from their own classrooms also realize the importance of asking students questions to elicit their thinking during class. Similar to the CGI research, it is primarily limited to elementary teachers. Furthermore, there is not any empirical evidence investigating the impact of teachers' increased ability to make sense of student thinking on teachers' actual teaching practices or on student achievement.

2.2.3 Teachers' Videotaped Lessons

Professional Development for In-service Teachers. Sherin and Han (2004) describe how participation in a video club changed the professional visions of four middle school teachers. The teachers participated in seven monthly video club meetings over the course of a year. During video club meetings, teachers and a researcher/facilitator watched and discussed excerpts of videotapes from the teachers' classrooms. Initially, teachers' focused solely on the actions of the teacher, what pedagogical strategies the teacher used and what alternative strategies he could have used. The facilitator began working with the teachers to change their focus by posing a question that drew teachers' attention to what actually happened in the classroom and what the students were thinking mathematically. When analyzing the discussions from video club meetings, the authors report important shifts as the teachers began closely examining the student ideas that emerged during the lessons and the mathematics that was discussed. Discussions of students' thinking moved from teachers simply restating students' words to providing detailed analyses of students' mathematical thinking. Over the course of the year, teachers identified more complex issues related to students' thinking to examine. In addition, teachers began connecting their analyses of pedagogical issues with their ideas about students' mathematical thinking.

In a case study that examines the effects of the video club on one teacher (David Louis) from the Sherin and Han (2004) study, Sherin (2001) reports that David Louis' new vision of his videotaped teaching also influenced his classroom instruction. He began spending more time during instruction trying to understand his students' mathematical thinking. This resulted in a change in the questions he asked and more discussion of mathematical ideas among his students. This study provides evidence that having a teacher focus on his students' mathematical thinking during instruction results in teacher questions that elicit students' mathematical thinking and mathematical discussions amongst students; teaching that is aligned with standards-based instruction.

Teacher Education for Pre-service Teachers. Masingila and Doerr (2002) provide similar evidence of pre-service secondary teachers grappling with the challenges of trying to use student thinking while also attending to the mathematical goals of the lesson. In this study, nine pre-service secondary teachers participated in a weekly seminar during their student teaching experience. During five weeks near the end of the seminar class, multimedia cases of teachers' mathematics instruction were explored and discussed. Weekly journal assignments and a final paper assignment asked teachers to identify a specific issue that they had been working on in their own practice and that they thought was addressed in the case study teachers' practice as well. The authors explain that three categories emerged from the links pre-service teachers made between their own practice and the case teachers' practice, with the most prevalent category addressing issues related to teachers' attending to students' mathematical thinking. Masingila and Doerr argue that the multimedia cases enabled the pre-service teachers to delve more deeply into the complexities of teaching. In particular, the multi-media cases enabled teachers to grapple with issues that are important in attending to students' thinking: checking for students' mathematical understanding, promoting students' mathematical thinking through the use of appropriate questions, and using student responses in furthering the teacher's mathematical agenda. Furthermore, the assignments that asked teachers to trace an issue from the case studies through their own practice enabled the teachers to think more deeply about important issues in their own emerging practice.

Conclusions. While there is limited empirical research on professional development programs that employ the analyses of videotaped lessons, there is positive evidence that facilitated discussions can result in growth in teachers' capacity to attend to students' mathematical thinking. It appears that video of mathematics instruction can serve as an impetus for drawing teachers' attention to the importance of focusing on students' mathematical thinking. In particular, the importance of posing questions to elicit students' thinking and advance students' understanding and using students' responses to further the teacher's mathematical goal for a lesson. Furthermore, there is evidence that at least one case of this type of professional development positively affected a teacher's classroom instruction (Sherin, 2001).

2.2.4 Narrative Cases of Mathematics Instruction

Research on narrative case discussions highlights the importance of having a skilled facilitator if the intention is to have teachers attend to students' thinking within the case. For example, Barnett (1991) used a case that focused on fractions with four different groups of teachers in order to analyze the common themes that arose in the case discussions. The author did not have the goal of focusing on students' mathematical thinking, only to see what teachers did find to talk about in the case. The following thematic categories arose in the case discussion: (a) mathematical concepts, (b) mathematical relationships, (c) assessment, (d) language, (e) strategies for promoting mathematical thinking, (f) strategies for promoting understanding and meaning, and (g) strategies for fostering motivation and positive dispositions. The teachers did not spontaneously choose to focus on the mathematical thinking of the students in the case, but instead focused on the actions or strategies of the teacher.

There is evidence that well-facilitated case discussions have the potential to provide teachers with opportunities to attend to students' thinking. Barnett (1998) analyzed case discussions of the monthly meetings of three groups involving 27 elementary and middle school teachers. Barnett claims the cases were designed to bring forth a variety of issues, teaching strategies, and expose students' thinking about rational numbers (in all its complexity, rationality, and flaws). The facilitator in this study was skilled in leading case discussions and knowledgeable about the key mathematical and pedagogical issues that typically arise in the discussion of each case. Before each case discussion, teachers were asked to solve a mathematics problem that was taken from the case or mathematically related to the problem in the case. Teachers were also asked to consider the problem from a student's point of view and think about any misconceptions or difficulties students might have solving the problem and why. Barnett provides evidence of teachers paying attention to students' mathematical thinking and considering students' perspective when explaining various teaching options during case discussions.

Similar to Barnett (1998), Stein, Hughes, Engle, & Smith (2003) provide evidence that attention to students' mathematical thinking can be the focus of a case discussion. This study explores the role of a case discussion in fostering the learning of seventeen teachers enrolled in an advanced methods course (two in-service elementary, two in-service middle school, twelve pre-service elementary, and one pre-service secondary teachers). The study provides evidence that the case discussion provided teachers with an opportunity to learn the importance of attending to students' thinking and how to use students' thinking to orchestrate a mathematically productive discussion, by making these ideas public during the discussion. The authors also provide limited evidence that teachers actually learned from the experience, through analysis of teachers' written course assignment in which they analyzed the teaching of two case teachers. Since the teachers also had other opportunities to focus on students' thinking as part of a course (see Stein, Engle, Hughes, & Smith (2006) described below), teacher learning cannot be attributed solely to the discussion of a case of mathematics instruction.

Conclusions. The research on discussions of narrative cases of mathematics instruction provide some evidence of the potential of case discussions to focus teachers' attention on students' mathematical thinking and its importance in making instructional decisions. In addition, the research also makes clear the importance of having a facilitator who guides teachers to focus on students' thinking in order for the case discussion to reach its potential. While there is some evidence that case discussions can provide opportunities for teachers to learn to attend to students' thinking there is little evidence connecting teacher learning to case discussions. These studies suggest that cases may be an important component of professional development programs that use a variety of practice-based experiences for focusing teachers' attention on students' mathematical understanding.

2.2.5 Composite Practice-based Professional Development Program

Professional Development for In-service Teachers. Schifter (1998) describes some important changes in teachers' instruction as a result of being involved in a four-year professional development program. Thirty-six elementary teachers met for two-week summer institutes, biweekly after-school seminars, and one-on-one biweekly classroom visits. Schifter identifies some of the changes that occurred in the instruction of one of the teachers who participated in the program – Beth Keeney. Beth has begun starting her mathematics units by looking for the major conceptual issues on which her students need to work by analyzing her students' mathematical thinking. She identified these issues by collecting students' written work and carefully analyzing it and by listening to the whole-group discussions to follow how the students' ideas were developing. Once she had identified the conceptual issues the students needed to work through, she provided opportunities for her students to confront the issues and watched carefully to see which students had successfully worked through them and which ones needed further scaffolding.

Schifter claims that these positive changes in Beth Keeney's instruction were influenced by the activities she participated in during the professional development program. These activities included investigating students' mathematical thinking by: (a) analyzing other teachers' students; (b) studying videotapes of clinical interviews and classroom discourse; (c) analyzing students' written work; (d) reading research articles about students' mathematical thinking; (e) reading and discussing cases of mathematics instruction; and (f) engaging in "episode writing". Episode writing consists of writing a "2- to 5-page narrative that captured some aspect of the mathematical thinking of one or more students, using transcriptions of classroom dialogue or samples of students' written work" (p.80). These episode writing assignments allowed the teachers' own classrooms to become a major resource for learning about student thinking. These assignments were given twice in the first year and as a regular monthly assignment in the second and third years. The teachers would then meet to share and discuss their episodes. Project staff would then review the episodes and provide feedback to the teachers, often posing questions about the mathematical ideas on which the students were working.

Schifter argues that the discussion of students' mathematical thinking through the use of cases, research articles, videotaped lessons and interviews, and students' written work was a good way for teachers to begin to learn how to focus on students' mathematical thinking. However, Schifter also claims that it was important for the teachers to learn to listen and to hear their own students making sense of mathematics. She believes the writing episodes were an effective medium in allowing teachers to closely analyze their own students' mathematical thinking.

Professional Development for In-service and Pre-service Teachers. Stein, Engle, Hughes, and Smith (2006) propose a pedagogical model that specifies some key practices that teachers can learn in order to use student responses in mathematically productive discussions. The five practices are: anticipating students' mathematical responses, monitoring students as they work in order to make sense of their thinking, purposefully selecting student responses for public display, purposefully sequencing student responses, and connecting student responses. The authors describe the opportunities of seventeen teachers enrolled in an advanced methods course (two in-service elementary, two in-service middle school, twelve pre-service elementary, and one pre-service secondary) to learn how to use student responses in orchestrating a whole-class discussion. The course focused on proportional reasoning in the middle grades and was comprised of various experiences including: analyzing narrative and video cases of mathematics instruction, analyzing samples of student work, and engaging in planning activities. A discussion of a narrative case and planning assignment that focused teacher's attention on students' thinking were shown in particular to provide teachers with an opportunity to learn about the five practices. Some evidence of teachers actually learning the five practices as a result of engaging in these experiences is also provided.

Hughes and Smith (2004) describe a similarly designed course for twenty-one teachers that focused on algebra as the study of patterns and functions in the middle grades. The authors describe evidence of teachers' attending to student thinking in their planning through a written assignment where teachers were asked to describe a plan for a lesson and explicitly asked to consider students' thinking by using the Thinking Through a Lesson Protocol as a guide when planning. The Thinking Through a Lesson Protocol (TTLP) provides a series of questions for teachers to consider as they plan a lesson that focuses their attention on students' thinking throughout the lesson (see section 3.3.1 for a more detailed description or Appendix A for a copy of the TTLP). Additional lesson planning data came from a post-course interview in which teachers were asked to plan a lesson and then talk about their plans. In the prompted lesson planning assignment, 100% of the teachers anticipated students' solutions, 95% identified specific questions to ask students, and 68% described how they would orchestrate a class discussion that made connections between students' solutions. In the post-course interview on lesson planning, teachers were not explicitly asked to consider students' thinking in their lesson planning. Under these circumstances 43% of teachers anticipated students' solutions, only 29% identified specific questions to ask students, and a remarkable 86% showed how they would orchestrate a class discussion that made connections between students' solutions. Thus, teachers addressed key aspects of planning that focus on student thinking under both prompted and unprompted conditions. In addition, the TTLP may have drawn teachers' attention to the importance of focusing on student thinking in the process of planning. The teachers were then able to apply this knowledge to a subsequent lesson planning situation several weeks later.

In addition, a case study of one teacher who participated in the course investigated the relationship between the teacher's content knowledge and pedagogical content knowledge and the role the two play in planning a lesson (Hughes & Smith, 2004). Toward the end of the course, the teacher, Ursula Sinden, had planned a lesson in which she focused on students' thinking. In particular, she anticipated what students would do and provided specific questions to advance students' mathematical understanding. She also expected students to make connections between representations and provided specific questions intended to move students toward this mathematical goal. Through an analysis of pre-post written assessments, Hughes and Smith suggest that it was unlikely that Ursula could have written this lesson plan prior to the course because she could not make connections between representations herself, and struggled to make sense of students' solutions. However, Ursula's post-course assessment indicated an understanding of mathematics and students' solutions similar to her lesson planning assignment. Thus, Hughes and Smith provide evidence of teachers attending to students' thinking when planning a lesson as part of their participation in a practice-based course that provided teachers with multiple opportunities to attend to students' mathematical thinking. Furthermore, the study points to the importance of *knowledge of mathematics* and *knowledge of mathematics for student learning* in the process of lesson planning that attends to students' mathematical thinking.

<u>Conclusions.</u> Research on professional development initiatives that incorporate various experiences, many of which have been described individually earlier in this chapter, suggests that entire courses or programs can focus teachers' attention on students' thinking using a variety of artifacts (e.g., student work, narrative and video cases, etc.). Furthermore, this approach offers teachers with a variety of opportunities to realize the importance of and how to attend to students' thinking that is grounded in the actual work of teaching. This "practice-based" approach grounds teachers' learning experiences in the tasks, questions, and problems of practice, so that "teachers' everyday work becomes the source for constructive professional development" (Ball & Cohen, 1999). While Ball & Cohen espouse the potential of a "practice-based" approach to teacher education, there is currently limited evidence of this approach's impact on teacher learning and their classroom practices.

2.2.6 Lesson Study in the United States

In an attempt to improve U.S. student achievement and instruction, one currently popular suggestion has been to adopt Japanese Lesson Study (Lewis, 2002; Stigler & Hiebert, 1999; Yoshida, 1999). While there is considerable interest in lesson study as a model for professional development, there is little empirical research showing its effective implementation in the United States. Furthermore, the published research on lesson study in the United States is fraught with difficulties in its implementation (Fernandez, 2002), particularly in its purpose of increasing teachers' attention to students' thinking in their planning process (Fernandez, Cannon, & Chokshi, 2003; Wagner, 2003). Below is a review of research studies investigating the implementation of lesson study in the United States and its ability to improve teachers' attention to students' mathematical thinking.

<u>Lesson Study with In-service Teachers</u>. As summarized in Chapter One (section 1.5), Fernandez, Cannon and Chokshi (2003) describe an empirical study of 16 teachers (K-8) and administrators from an urban public school that engaged in lesson study with twelve Japanese teachers serving as coaches for the lesson study sessions. During the lesson study sessions, there are multiple examples where the American teachers were not discussing or grappling with issues focused on students' mathematical thinking, particularly in the planning process of their lesson. Instead, teachers focused on other aspects of the lesson (e.g., how to group students, the materials needed). The Japanese teachers, serving as coaches, posed questions or suggestions to focus the American teachers on specific issues related to students' mathematical thinking. As a result, the American teachers had a discussion and made progress towards incorporating students' mathematical thinking in their lesson plan. However, there were limitations in teachers' ability to think deeply about students' mathematical thinking. Thus the American

teachers could attend to students' mathematical thinking in their lesson planning discussions when pressed to do so from a coach but with some limitations. However, even after being pressed over the course of several planning sessions, they continued to need input from the Japanese coaches in order to attend to students' mathematical thinking and still did not spontaneously focus on students' thinking. Fernandez et al. (2003) argue that lesson study can be a format for professional development that helps American teachers focus on students' mathematical thinking. However, they also make clear the need for well-trained coaches to facilitate the lesson study discussions since teacher-led lesson study groups that do not have the aid of an expert facilitator would be unlikely to engage in the critical aspects of focusing on students' mathematical thinking.

Lesson Study with Pre-service Teachers. In Wagner's dissertation (2003), she investigates eighteen pre-service elementary teachers' (grades K-3) ideas of what factors are important in designing mathematics lessons prior to and during participation in lesson study. In addition, Wagner determines the ways in which the lesson study groups used students' thinking in designing mathematics lessons. In this study, the lesson study cycle consisted of five groups initially planning a lesson, one of the teachers implementing the lesson during a field experience while the others observed, and the teachers reflecting on the lesson and revising it. The lessons were taught and revised a total of three times.

Wagner reports that prior to participating in lesson study, teachers identified many factors that they perceived as important in designing a lesson, including aspects relevant to developing students' thinking. However, upon deeper analyses, the teachers' definitions of these aspects were procedural in nature and based on a fragile connection to building students' understanding. For example, fifteen of the eighteen teachers identified the selection and use of manipulatives as an important factor to consider in planning a lesson. However, thirteen of these fifteen teachers explained the reason for using a manipulative was to aid in the routinization of a standard procedure rather than to help students understand a particular concept. While managerial considerations remained important to teachers at all points prior to and during the lesson study cycle, teachers diminished their focus on teacher behaviors and began placing more emphasis on students' construction of knowledge during the lesson study cycle. Furthermore, prior to the lesson study experience, none of the teachers mentioned anticipating students' solution strategies as a factor to consider in lesson planning. In contrast, all of the lesson study groups identified anticipating students' solutions as important and attempted to do so in their lesson design discussions.

While Wagner did see evidence of increased attention to students' mathematical thinking in the design of lesson plans during participation in lesson study groups, there were differences in how the groups chose to use their knowledge of students in their actual lesson plans. Two lesson study groups became aware of a student misconception during the first implementation of their lesson plan. However, in both of these cases, the teachers made note of the misconception by adding it to their list of possible student strategies, but made no attempts to integrate this information into the subsequent implementations of their lessons. In the case of one lesson study group, the teachers did finally make use of students' thinking to substantively change their lesson plan, but this was only after making superficial changes after the first implementation and then finding students facing the same challenges during the second implementation. Thus, at least one group of teachers finally made use of students' thinking in their lesson design, but it took at least two iterations of the lesson study cycle for this to occur. Wagner describes a third instance of the ways in which teachers used students' thinking in their lesson study cycle as learning from mistakes and circumventing them in future lessons. This means that during implementations of the lesson student difficulties arose. In subsequent lesson designs, the teachers incorporated this information. However they did this to elicit correct answers, thus reducing the students' struggles and removing the challenges of the tasks. Therefore, Wagner's study provides evidence that a lesson study group has the potential to increase teachers' attention to students' thinking in the lesson planning process. However, this study also highlights the ways in which teachers may choose to use their new knowledge of students' thinking; ways that may be contrary to the intention of lesson study.

Conclusions. There is limited empirical research on the implementation of lesson study in the United States despite the wide interest in it as a model for professional development. Furthermore, the empirical evidence that does exist predominately involves elementary teachers. It appears that the positive impact of lesson study on teachers' ability to attend to students' thinking in important ways during the planning process faces some critical challenges (e.g., the need for skilled facilitators during lesson study sessions).

2.2.7 Conclusions

Table 2.1 summarizes research studies on professional development initiatives aimed at increasing teachers' attention to students' mathematical thinking that were reviewed. In particular, the table indicates the status of participants (in-service or pre-service teachers), whether they were elementary, middle school or secondary teachers, and whether or not the studies examined the effects of the professional development on teachers' classroom instruction.

Table 2.1 Summary of research studies on professional development programs

Type of PD	Study	In-service	Pre-service	Elem. K-5	M.S. 6-8	H.S. 9-12	Study Effect on Practice
Research-based info on children's cognition	Carpenter, et al., 1989	X		X			X
	Fennema, et al., 1996	X		X			X
	Swaford, Jones & Thornton, 1997	X		X	X		X
	Vacc & Bright, 1999		X	X			X
	Warfield, 2001	X		X			X
Student work	Crespo, 2000		X	X			
	Franke & Kazemi, 2001	X		X			
	Kazemi & Franke, 2004	X		X			
	Little, et al., 2003	X		X	X	X	
Video cases of teaching	Sherin, 2001	X			X		X
	Sherin & Han, 2004	X			X		
	Masingila & Doerr, 2002		X		X	X	
Narrative cases of teaching	Barnett, 1998	X		X	X		
	Barnett, 1991	X		X			
	Stein et al, 2003	X	X	X	X		
Multiple types of practice-based activities	Hughes & Smith, 2004	X	X	X	X	X	
	Schifter, 1998	X		X			X
	Stein et al., 2005	X	X	X	X		
Lesson Study	Fernandez, Cannon, & Chokshi, 2003	X		X	X		
	Wagner, 2003		X	X			

In synthesizing the studies that were reviewed, several interesting commonalities emerge. Many studies identified important ways in which teachers should attend to students thinking: (1) asking students questions to elicit their thinking in order to make sense of it (e.g., Fennema *et al.*, 1996; Kazemi & Franke, 2004; Masingila & Doerr, 2002a; Sherin, 2001; Vacc & Bright, 1999); (2) anticipating the variety of strategies students may use in solving a problem (e.g., Barnett, 1998; Carpenter *et al.*, 1989; Fennema *et al.*, 1996; Stein *et al.*, 2005; Wagner, 2003); (3) posing questions to elicit students' thinking and to advance their mathematical understanding (Fennema *et al.*, 1996; Kazemi & Franke, 2004; Masingila & Doerr, 2002a; Sherin, 2001; Vacc & Bright, 1999); and (4) understanding the mathematical concepts students will be developing during a lesson (e.g., Masingila & Doerr, 2002a; Schifter, 1998; Swafford, Jones, & Thornton, 1997; Warfield, 2001). The other lesson to be learned from the reviewed studies is the importance of a skilled facilitator in order for the professional development's potential to be realized (e.g., Barnett, 1991, 1998; Fernandez, Cannon, & Chokshi, 2003; Franke & Kazemi, 2001; Sherin & Han, 2004; Stein *et al.*, 2003; Wagner, 2003).

While it is encouraging to see professional development initiatives affecting teachers' practice in the positive ways described above, there are several limitations to the research that was reviewed. First, most of the research to date has been with in-service elementary teachers (e.g., Barnett, 1991; Carpenter *et al.*, 1989; Fennema *et al.*, 1996; Kazemi & Franke, 2004; Schifter, 1998). Little research has investigated whether pre-service secondary teachers can learn to focus their teaching practices on students' mathematical thinking (Hughes & Smith, 2004; Masingila & Doerr, 2002a). In addition, many of the professional development initiatives focused on students' thinking within a very specific mathematical domain do not measure the generalization to teachers' practice for the entire curriculum they are responsible for teaching (Barnett, 1991, 1998; Carpenter *et al.*, 1989; Fennema *et al.*, 1996; Hughes & Smith, 2004; Stein *et al.*, 2005; Stein *et al.*, 2003; Swafford, Jones, & Thornton, 1997; Vacc & Bright, 1999). Furthermore, with the exception of CGI studies, there is limited information about the effects of these professional development initiatives on teachers' actual practices of planning and teaching (Schifter, 1998; Sherin, 2001). Therefore, further research is needed on pre-service secondary mathematics teachers' attention to students' mathematical thinking and particularly on the ways in which they attend to students' thinking in their actual teaching practice during their field experience.

2.3 The Relationship Between Planning and Teaching

Though researchers such as Philip Jackson (1965) have long pointed to the importance and need for research that investigates teacher behavior in the pre-active setting, it was not until the 1970's and 1980's that researchers heeded this call. In 1977, when Clark and Yinger wrote their review of the literature on teacher thinking, there were less than five empirical studies of teacher planning on which to report. Much of the early work on teacher planning was done outside the realm of teachers' natural planning practices (e.g., situations where teachers had to come in to a lab to plan a lesson on a random topic to teach to a small set of students (often university students). The research on planning often focused on determining the type of planning that teachers engaged in and identifying models of planning and comparing the models to experienced teachers' actual planning. In addition, a limited quantity of research all indicated that there was a strong relationship between teachers' planning and their implementation of the lesson. In other words, teachers rarely changed their teaching from what they had planned, "even when instruction was going poorly" (Clark & Yinger, 1977, p. 293).

In a review of literature on teacher planning that is often cited, Clark and Peterson (1986) echoed the review nine years earlier from Clark and Yinger (1977). While Clark and Peterson had more empirical studies to review, their findings were similar to Clark and Yinger. They found that the research was almost exclusively descriptive and was based primarily on the planning practices of experienced elementary teachers. The research on teacher planning fell into three major categories. The first identified the types and functions of teacher planning. Results indicated that there are as many as eight different types of planning that teachers engage in during the school year (e.g., weekly, daily, long range, short range, yearly, term, unit, and lesson planning). They also provide evidence that teachers' written lesson plans seldom reflect the teachers' entire plan. Therefore, Morine-Dershimer (1977, 1979, as described in Clark & Peterson, 1986) coins the term "lesson image" as the comprehensive planning structure teachers have and that written plans are nested within these lesson images.

The second major category of research on teacher planning addresses the models that have been used to describe the process of planning (Clark & Peterson, 1986). Traditionally, the model taught for lesson planning was a linear model consisting of four steps: (1) specify objectives; (2) select learning activities; (3) organize learning activities; and (4) specify evaluation procedures. Research studies in the 1970's and 1980's focused on comparing the model teachers were taught to use and what experienced teachers actually did in their planning. The research reviewed by Clark and Peterson indicated that teachers do not usually plan in a linear fashion and that most often the objectives of the lesson are specified towards the end of their planning process rather than at the beginning. Therefore, Clark and Peterson concluded that this linear model is not the most accurate model of lesson planning. However, other models have not been systematically tested to determine if another model is better.

The third and final category identified by Clark and Peterson (1986) in their review of the literature on teacher planning tries to determine the relationship between teacher planning and the teacher's subsequent actions in the classroom. The research demonstrates that teachers' planning influences students' opportunities to learn, the content of instruction and sequence of topics, grouping for instruction, the general focus of classroom processes, as well as the time allocations for elementary school subject matter areas.

Since Clark and Peterson's (1986) review of the literature on teacher planning, there has been some research conducted specific to the planning processes of mathematics teachers that sheds light on the teachers' role of attending to students' mathematical thinking during the lesson planning process. This research will be reviewed in three sections. First research on Japanese teachers' planning and its impact on their instruction and student achievement will be described. Then research on American teachers that are implementing standards-based instruction will be explored, in the form of case-studies of expert teachers. Finally, comparisons between expert and novice teachers will shed light on the planning and teaching practices of pre-service mathematics teachers.

2.3.1 Japanese Teachers' Planning and Teaching

Much of what we know about Japanese teachers' planning comes from research studies examining the Japanese professional development program of "lesson study". As described briefly in Chapter One, Japanese lesson study is a form of professional development that focuses teachers on planning, implementing, and reflecting on a single lesson and is a common practice for elementary and middle school teachers. Significant time and effort is put in to collaboratively developing a lesson plan. Much of the content discussed and debated during the planning session is focused on students' mathematical thinking. According to Stigler & Hiebert (1999), Japanese teachers engage in detailed discussions of the following topics in the weeks spent planning the lesson:

- The problem with which the lesson would begin, including such details as the exact wording and numbers to be used.
- The materials students would be given to use in trying to solve the problem.
- The *anticipated solutions, thoughts, and responses that students might develop* as they struggled with the problem.

- The kinds of questions that could be asked to *promote student thinking* during the lesson, and the kinds of guidance that could be given to *students who showed one or another type of misconception* in their thinking.

- How to use the space on the chalkboard (Japanese teachers believe that organizing the chalkboard is a key ingredient to *organizing students' thinking* and understanding).

- How to apportion the fixed time of the lesson—about forty minutes—to different parts of the lesson.

- How to handle individual differences in level of mathematical preparation among the students.

- How to end the lesson – considered a key moment in which *students' understanding* can be advanced. (p. 17, *emphasis added*)

The lessons generated from these discussions are remarkably detailed. In describing the activities of the lesson, teachers often use a four-column chart. The first column outlines the learning activities or sequence of tasks as well as key questions the teacher has planned to ask students. The second column is made up of a range of expected student responses for each step of the lesson. The next column includes responses the teacher might make when dealing with students' reactions to each step of the lesson as well as reminders about why a certain step is important in the lesson and what mathematics it is intended to make salient for the students. Finally, the fourth column lists methods of evaluating the success of each step of the lesson. This includes specific things teachers' expect to see and hear that lets them know students are understanding the mathematics of the lesson (Fernandez & Chokshi, 2002; Fernandez & Yoshida, 2004).

Japanese teachers believe that creating such detailed lesson plans that focus on students' thinking is an important part of teaching. For example, Fernandez and Yoshida (2004) report that when they asked a teacher in their study about why Japanese teachers develop such detailed lesson plans, Ms. Tsukuda explained:

> … anticipating student solutions and how to react to them is excellent preparation for teaching a lesson. Feeling prepared helps allay the nervousness that the teacher doing the teaching is likely to experience. Second, these anticipations prepare the teacher for understanding the student responses and solutions that occur in the classroom and equip the teacher with appropriate reactions to these. Finally, providing this detail in the lesson plan prepares the teacher to be better able to make use of student responses to lead the class to the desired outcomes in terms of their thinking and understanding (p. 47-48).

According to Stigler and Hiebert (1999), "This kind of planning is decidedly intellectual in nature; these teachers are thinking deeply about the options available to them and the way the experiences they structure in their classrooms will facilitate students' understanding of mathematics" (p. 120). While Japanese teachers do not prepare such detailed written lesson plans for everyday lessons, the teachers believe that making them for lesson study sessions provide them with a good opportunity to think deeply about how students learn. Furthermore, Japanese teachers believe that all lesson planning, whether all the details are written down or not, should be planned from the perspective of students, a stance that they often refer to as "trying to see lessons with students' eyes" (Fernandez & Yoshida, 2004, p. 228). Furthermore, research has shown that lesson study has improved Japanese science and mathematics education; the teachers' instruction and the students' achievement (Lewis & Tsuchida, 1998; Watanabe, 2002; Yoshida, 1999).

While there have been investigations into the planning and teaching of typical Japanese elementary and middle grades teachers, research on American teachers' planning practices has been mostly limited to expert and novice teachers. However, there is a research study that has compared the planning and subsequent teaching of typical Japanese fifth grade teachers with that of typical fifth grade teachers in the United States (Stigler, Fernandez, & Yoshida, 1996). Analyses of two Japanese and two American lessons, one of each dealing with the area of a triangle and the concept of equivalent fractions provide distinctions between the planning and teaching of Japanese and American teachers. Comparing the written lesson plans of the Japanese and American teachers results in remarkable differences, in particular the fact that Japanese teachers place students' thinking at the core of their lesson planning and are far more detailed than the American teachers' written lesson plans that consist of outlines of the actions the teacher will take. (This focus on teacher actions rather than student thinking was echoed in the study of lesson study (Wagner, 2003) discussed earlier in this chapter.) In analyzing the subsequent implementation of these lesson plans, there are again marked differences between the Japanese and American lessons. In particular, Japanese lessons provided students with more opportunities to think during instruction, and created an environment in which students' thinking was valued and legitimized to a far greater degree than the American lessons.

Stigler, Fernandez, and Yoshida (1996) hypothesize reasons why there may be such differences between the Japanese and American teachers. One possibility stems from the resources available to Japanese teachers when they are planning a lesson and trying to anticipate students' responses. Japanese teachers do not have to come up with these on their own; reference books and publications describing students' thinking about all the topics in the mathematics curriculum are available. The authors note that when American elementary teachers have access to this same research-based knowledge of students' thinking, they too anticipate students' solutions in their planning and their instruction appears to be more like that of the Japanese teachers (e.g., the CGI studies described in section 2.3.1).

It appears that the elements of focus in Japanese lesson planning are the same ones advocated as necessary for standards-based instruction to occur in the United States. While focusing on student thinking may not be apparent in typical mathematics teachers' planning, it is evident in case studies of "expert" American teachers' planning and thinking during a lesson (e.g., Lampert, 2001; Schoenfeld, 1998).

2.3.2 Case Studies of Expert Teachers' Planning and Teaching

This section describes the case-studies of two "expert" teachers engaged in teaching mathematics. The "experts" in these studies are mathematics teacher educators who offer a window into their teaching of mathematics (Magdelene Lampert and Alan Schoenfeld). Both are engaged in the work of teaching mathematics in a way that would be called standards-based instruction (as described in Chapter One). The studies of their teaching provide insight into the planning in which they engage and the relationship between their planning and classroom instruction. Here are their stories.

Magdalene Lampert. Lampert explores the actions of a single teacher (herself), teaching mathematics to a class of fifth graders over an entire academic year (Lampert, 2001). In this book, Lampert attempts to identify the problems that must be addressed in the work of standards-based teaching. As a teacher, she identifies the following important actions that she takes in preparing for a lesson: solve the problem yourself, think about students' prior knowledge and how they might use it in thinking about this new idea, anticipate various strategies students of your age group might use to solve the problem, and anticipate where students might get stuck or distracted. According to Lampert, anticipating students' thinking is valuable because

> This kind of preparation showed me what words might be useful in talking about their solutions, as well as what drawings they or I might use to support their studies. To respond to their work in a thoughtful way, I needed to be able to anticipate what they might be able to do independently and where they would need information from me to proceed productively (p. 103).

Lampert uses all of this information in order to lay out the particular moves she will make and when she will make them. Throughout her thinking through a lesson, she focuses on the mathematics that is being worked on by the students.

Lampert contends that the work she does during planning the lesson directly impacts her implementation of the lesson. For example, the problem she faces in teaching while students work independently or in groups is that of guiding and inquiring into individuals' thinking. She argues that while her teaching is "constructed in response to whatever I saw or heard on the spot," (p. 123) her ability to make instructional decisions is because of the work she engaged in when planning the lesson (e.g., anticipating students' strategies).

Additional insight into Lampert's teaching is offered in an in-depth look at a 10-lesson unit on functions and graphs (Leinhardt & Steele, 2005). The purpose of the analysis of Lampert's teaching was to

> … explore some of the tools that Lampert uses to create a coherent and thorough exploration of the mathematics at hand, keeping the class on a bounded intellectual journey while still positioning herself to the side and consequently her students' ideas at the center of the class (p. 89)

Because Leinhardt had access to Lampert's notebooks, where she reflected on and wrote about what happened in the lessons and did most of her planning for the next day, as well as analysis of the actual classroom lessons, Leinhardt and Steele were able to shed light on the relationship between Lampert's planning, teaching, and reflecting on teaching. According to Leinhardt and Steele, Lampert expresses a clear mathematical trajectory for the unit as a whole and for each lesson as she plans. Lampert's, instructional decisions are based on students' prior knowledge and experiences, the mathematical trajectory, and students' current understandings of the mathematics at hand. In particular, the moves Lampert makes to direct the dialogue are based on her understanding of where students are and where she wants them to go mathematically. As described by Leinhardt and Steele, "her moves are subtle, raising to the front student comments that serve the direction in which she wants the dialogue to move" (p. 152). Leinhardt and Steele's description of Lampert's teaching practices provide further evidence of the importance of using students' thinking in orchestrating mathematically productive discussions. Their study also portrays the ways in which a teacher attends to students' thinking in their planning, teaching, and reflecting on teaching all work together in order to make it possible for a teacher to engage in the kind of standards-based teaching that Lampert does.

Alan Schoenfeld. Schoenfeld described his lesson image and enactment of the opening day of his undergraduate course in mathematical problem solving (1998). He designed the lesson and course himself, had taught it many times, and spent significant time reflecting on his teaching of the course. Schoenfeld's lesson image provides a strong sense of what he wanted to have happen during the first class and the things he wanted to do to ensure that it would happen. He makes a point of drawing attention to the extensive set of pedagogical knowledge, content knowledge, knowledge of students as learners, and support structures such as routines and scripts that he has accessible as resources. Therefore, during his planning, he spends limited yet efficient time reviewing the key points and concepts he wants to make salient for the students. He is able to anticipate students' responses to the mathematical problems he poses. While he does not necessarily know the order in which students will respond to the problem, he is fairly certain of the responses he will receive and knows what questions he will ask and how the students in turn will respond. Schoenfeld points out that while he does know how things are likely to unfold – down to an extremely fine level of detail, he explains that "it is not that I follow a rigid plan and coerce students into it; there are many branch points and contingencies. However, I know what most of them are likely to be" (p. 8). While Schoenfeld contends that lesson images vary widely from teacher to teacher, and from context to context for the same teacher, they play a major role in shaping what will actually take place in the classroom. Thus there is a very important relationship between planning and teaching.

2.3.3 Expert and Novice Teachers' Planning and Teaching

Leinhardt (1993) synthesized results from her research studies in order to identify overarching differences between twenty expert and novice teachers. One of the strands of teaching she identified as showing differences between experts and novice teachers is agendas. Teachers were asked prior to teaching a lesson "what are you planning to do today?". The response to this question is referred to as an agenda by Leinhardt and is defined as "an operational plan that is concise, focused, and descriptive of the general set of goals and actions in which the teacher intends to engage for the next 40 to 50 minutes" (p.19). The agendas of experts are different from the agendas of novice teachers. In particular, expert teachers anticipate difficulties students might have with the content of the lesson and attend to student thinking in order to assess the success of a particular lesson. Furthermore, experts appear to be capable of thinking of the lesson along two dimensions simultaneously. One dimension is the teacher's own actions and thoughts and the other is the students' thinking and understanding of the content. In contrast, novice teachers do not report prior to the lesson that they intend to monitor students' thinking, nor do they generally report after the lesson that they had attended to the thinking of their students. Furthermore, novices are not able to simultaneously attend to their own actions and students' thinking.

In addition, Leinhardt argues that teacher's agendas are closely related to the teacher's implementation of the lesson. Because expert teachers' agendas have a clear content-specific goal and anticipate students' work on the new idea, expert teachers' lessons often make use of students' responses in order to build new knowledge and remain focused toward the goal of the lesson. By contrast, novice teachers' agendas lack goals for the lesson and as a result, their lessons provide evidence that the decisions made during the lesson were not guided by the goals of the lesson. In addition, novice teachers' agendas and lessons were focused on the teachers' actions in the absence of students' thinking or understanding of the content.

Similar distinctions between expert and novice teachers are depicted in a series of articles by Schoenfeld and colleagues (Schoenfeld, Minstrell, & van Zee, 2000; Zimmerlin & Nelson, 2000). A similar portrait of planning and teaching, as detailed earlier for expert teachers, is described in a case study of an experienced teachers' planning and implementation of a standards-based physics lesson (Schoenfeld, Minstrell, & van Zee, 2000). In this case, the teacher (Minstrell) designed the lesson and had taught it many times before. Minstrell's planning for the lesson (or lesson image) as verbally described to the researchers, had clear content goals for student understanding, expectations for how he would interact with the students, the kinds of issues students would raise, and how he would react to those. In analyzing the implementation of the lesson, the researchers found that the lesson image greatly influenced the actual lesson and that "Minstrell's extensive set of pedagogical and content resources allowed him to proceed smoothly through what might otherwise become troubled waters" (p. 311).

The depiction of Minstrell's planning and teaching is in contrast to that of Nelson, a pre-service teacher planning and teaching a traditional Algebra 1 lesson (Zimmerlin & Nelson, 2000). Nelson's lesson image also included goals for the lesson, activities in which he planned to engage (including specific example problems to work through), as well as which problem he thought would be difficult for his students. While Nelson's actual lesson started out very similar to his plan, challenges arose when students responded differently than he had expected. Since Nelson's lesson image did not include a plan for dealing with this student response, he had to develop a new plan on the spot, drawing on his repertoire of pedagogical and content resources.

However, when Nelson was unable to produce a helpful explanation for students, it was clear to the researchers that Nelson did not have the same extensive set of resources that Minstrell relied on in his teaching.

Similar distinctions between expert and novice teachers in the planning and implementing of lessons occurred in an investigation contrasting two review lessons of two pre-service secondary mathematics teachers with those of their high school mentor teachers (Livingston & Borko, 1990). In this case, the review lessons were designed to review the material covered in the textbook chapter, in preparation for a chapter test the following day. The experienced mentor teachers' planning was brief and efficient. They reviewed notes and examples from the previous year's lesson in preparation for this lesson. While the written plans of the experienced teachers were sparse, upon interviewing the teachers it was evident that both were easily able to elaborate the content, identify the key lesson objectives, situate the material into a bigger mathematical picture, and anticipate students' questions and difficulties. During the implementation of these lessons, both experienced teachers were able to base the flow of the lesson on students' questions while at the same time covering essential concepts, their relationships, as well as warn students of common errors. The teachers' responses to students drew attention to concepts as well as procedures and provided a curricular big picture (Livingston & Borko, 1990).

By contrast, the pre-service teachers' written plans and verbal descriptions of their plans did not contain any of the aforementioned components (Livingston & Borko, 1990). Both teachers had planned carefully by working out problems in advance and presented accurate solutions to problems for which they had planned. However, their instruction focused on procedural knowledge and led students sequentially toward a problem's solution. Neither pre-service teacher made connections between problems or provided an overall framework in which to view the formulas and problems (as the experienced teachers had done). Furthermore, both teachers failed to anticipate students' difficulties and experienced challenges in generating examples or providing explanations for student responses that were unexpected. Both pre-service teachers also failed to cover all the essential concepts or strategies in their review lessons.

Livingston and Borko (1990) hypothesize that the pre-service teachers exhibited two major knowledge limitations that influenced their planning and implementation of the review lessons. First, they seemed to have limited knowledge of student learning in their subject area, including little knowledge of common misconceptions or the concepts with which students would have the most difficulty. They also failed to realize the importance for student understanding of communicating the big picture mathematically. Consequently they were more willing to let students dictate the content of the review lesson. The second knowledge limitation was in their ability to go beyond their personal or preferred ways of understanding and generate alternative explanations or representations for students. The authors conclude that these deficiencies are "reasonable, probably inevitable, characteristics of novice performance rather than the result of insufficient planning or inadequate academic preparation" (p. 385).

Borko and Livingston (1989) report similar results when investigating the planning and teaching of the same pre-service and mentor teacher dyads as studied in the Livingston and Borko (1990) study just described. However, this study differed in two ways: an additional pre-service and mentor teacher dyad teaching elementary school was included, and it investigated teachers' planning and teaching over a one week period. Thus the results and conclusions of the Livingston and Borko (1990) study described above are not specific to review lessons but are typical for these expert and novice teachers' planning and teaching practices.

2.3.4 Conclusions

The research reviewed here provides evidence that planning has an impact on the implementation of a lesson. Furthermore, these studies suggest that there are distinct differences between the planning practices of expert and pre-service (novice) mathematics teachers. The research appears to show that expert teachers have access to more robust pedagogical knowledge, content knowledge, knowledge of student learning, and support structures such as routines. In looking across the studies that were reviewed, there are commonalities that emerge in what is important to attend to in planning and teaching. Several studies identified important ways in which teachers should attend to students' thinking in their planning and teaching by: (1) understanding the mathematical concepts that will be developing during a lesson (e.g., Borko & Livingston, 1989; Fernandez & Yoshida, 2004; Lampert, 2001; Leinhardt, 1993; Leinhardt & Steele, 2005; Livingston & Borko, 1990; Schoenfeld, 1998; Schoenfeld, Minstrell, & van Zee, 2000; Stigler & Hiebert, 1999); (2) anticipating the variety of strategies students may use in solving a problem as well as the misconceptions or difficulties students may have (e.g., Borko & Livingston, 1989; Fernandez & Yoshida, 2004; Lampert, 2001; Leinhardt, 1993; Livingston & Borko, 1990; Schoenfeld, 1998; Schoenfeld, Minstrell, & van Zee, 2000; Stigler & Hiebert, 1999); and (3) asking questions to elicit students' thinking and advance students' understanding (e.g., Fernandez & Yoshida, 2004; Schoenfeld, 1998; Stigler & Hiebert, 1999).

2.4 Developing a Framework

In looking across the research on planning and teaching, four elements emerge as critical to teaching in ways that attend to students mathematical thinking: (1) identifying the mathematical goals of the lesson, (2) anticipating students' responses and possible misconceptions, (3) identifying specific questions that will assess or advance students' understanding while they work, and (4) orchestrating a whole-class discussion that builds on students' thinking and makes salient the mathematics of the lesson. These aspects are discussed below in more detail.

Identifying the mathematical goals of the lesson involves the teacher determining the specific mathematical concepts the students will engage with during the lesson, how the mathematics of this lesson connects to students' prior knowledge or experiences, and what mathematical understandings students will take with them from the lesson (Borko & Livingston, 1989; Fernandez & Yoshida, 2004; Hiebert *et al.*, 1997; Lampert, 2001; Leinhardt, 1993; Leinhardt & Steele, 2005; Livingston & Borko, 1990; Masingila & Doerr, 2002a; Schifter, 1998; Schoenfeld, 1998; Schoenfeld, Minstrell, & van Zee, 2000; Stigler & Hiebert, 1999; Swafford, Jones, & Thornton, 1997; Warfield, 2001).

Anticipating students' solutions and possible misconceptions means that teachers make an effort to consider how students might mathematically interpret a problem, the array of strategies – both correct and incorrect – they might use to tackle the problem, and how those strategies and interpretations might relate to the mathematical concepts, representations, and processes that the teacher would like his or her students to learn (Barnett, 1998; Borko & Livingston, 1989; Carpenter *et al.*, 1989; Fennema *et al.*, 1996; Fernandez & Yoshida, 2004; Lampert, 2001; Livingston & Borko, 1990; NRC, 2001; Schoenfeld, 1998; Schoenfeld, Minstrell, & van Zee, 2000; Stein *et al.*, 2005; Stigler & Hiebert, 1999; Wagner, 2003).

Identifying specific questions that will assess or advance students' understanding while they work entails teachers engaging with specific anticipated solution strategies and the mathematics embedded within them in order to determine specific questions that will elicit thinking and advance the student along the mathematical trajectory of the lesson (Fennema *et al.*, 1996; Fernandez & Yoshida, 2004; Kazemi & Franke, 2004; Masingila & Doerr, 2002a; Sherin, 2001; Stigler & Hiebert, 1999; Vacc & Bright, 1999; Yoshida, 1999).

Orchestrating a whole-class discussion that builds on students' thinking and makes salient the mathematics of the lesson involves teachers purposefully selecting student responses for public display (Lampert, 2001; Schoenfeld, 1998; Stein *et al.*, 2005; Stigler & Hiebert, 1999), determining the sequence of those responses (Lampert, 2001; Schoenfeld, 1998; Stein *et al.*, 2005; Stigler & Hiebert, 1999), and identifying the specific questions to ask students that will enable them to make connections between solutions and to mathematical concepts (Ball, 2001; Brendefur & Frykholm, 2000; Lampert, 2001; NRC, 2001; Schoenfeld, 1998; Stein *et al.*, 2005; Stigler & Hiebert, 1999).

Thus these four elements provide a framework for both creating opportunities for teacher to learn how to focus on students' thinking and evaluating what teachers learned from the opportunity to focus on student thinking. Although pre-service teachers may benefit from such experiences, making them part of pre-service teacher education will require significant changes in the way teachers are prepared.

2.5 Pre-service Teacher Education and Learning

Some teacher education programs, as reviewed earlier in this chapter, are beginning to focus on student thinking. Specifically, some pre-service teachers are being asked to attend to students' thinking by recognizing and using research-based knowledge of students' mathematical cognition (Vacc & Bright, 1999); analyzing student work (Crespo, 2000); analyzing video cases of mathematics instruction (Masingila & Doerr, 2002a); discussing narrative cases of mathematics instruction (Stein *et al.*, 2003); engaging in a variety of these practice-based experiences (Hughes & Smith, 2004; Stein *et al.*, 2005); and implementing lesson study (Wagner, 2003).

These experiences stand in contrast to what is known about traditional teacher education, which gives theoretical knowledge an elevated status, to be learned before engaging in the practice of teaching. In this model, effective instruction is defined as knowing how to apply theory to practice (Sykes & Byrd, 1992). Furthermore, in mathematics teacher education, the two facets of *knowledge needed for teaching* are often taught separately; *knowledge of mathematics* through content courses in mathematics departments and *knowledge of mathematics for student learning* through school of education methods courses (if at all). In this section, several literature reviews on teacher education will be discussed, so as to identify common themes and implications for the study described herein.

Borko and Putnam (1996) provide a review of literature on learning (in all subjects) within a cognitive psychology framework. They found teachers' knowledge and beliefs about students as learners to be an important issue for teacher education because beliefs about learners promoted in teacher education programs often differ, sometimes markedly, from many pre-service teachers' beliefs. Borko and Putnam conclude from reviewing studies in which teacher education programs tried to change and measure change in pre-service teachers' beliefs that pre-service teachers are resistant to change through instruction and experiences in traditional teacher education programs. Furthermore, according to Borko and Putman, another common message from the research is that, because of their prior beliefs, pre-service teachers may not see the relevance of the pedagogy courses, and therefore may not attend closely to the information and experiences offered in teacher education courses.

Borko and Putnam conclude that research suggests that pre-service teachers need to be placed in field experience settings that provide opportunities and support for them to teach in ways that are compatible with the goals and vision of the university teacher education program in which they are enrolled. Similarly, the support and feedback from mentor teachers and university supervisors should be compatible and mutually reinforcing with what teachers are learning in their teacher education courses.

In a review of 40 empirical studies on learning to teach, published between 1987 and 1991, Kagen (1992), reported similar conclusions to Borko and Putman. Teachers' prior beliefs about teaching and students, and experiences in classrooms as students appeared to determine what could be learned from their teacher education course work. Many pre-service teachers also believed that there was a lack of connection between the content of the university courses and the reality of classroom teaching. A resounding theme from the studies was that teacher education programs had little impact on teachers' prior beliefs. Another common theme that Kagen found was that novice teachers' growing knowledge of students as learners could and should be used to "challenge, mitigate, and reconstruct prior beliefs and images" (p. 142). Finally, Kagen's review of the literature indicated that novice teachers first needed to acquire knowledge of classroom procedures for handling class management and discipline. After these were in place, novices turned their attention to instruction and ultimately found standard routines for integrating instruction and management. Only when these were in place, could novices begin to focus on students' learning in the classroom. Most of the studies reviewed were with elementary teachers and very few investigated the planning process of the pre-service teachers or looked at secondary mathematics pre-service teachers.

Brown and Borko (1992) report similar results in their review of literature on what is entailed in becoming a mathematics teacher to Borko and Putman's (1990) findings across all subject. In other words, teacher education programs for mathematics teachers also had a limited affect on teachers' prior beliefs about teaching and learning and were not aligned with the experiences novice teachers had in their field placements. The authors acknowledged that becoming a mathematics teacher involves both general and mathematics-specific knowledge, however, most research to date has investigated generic pedagogical issues with very little research focused on mathematics-specific concerns. Brown and Borko also included a review of literature on pre-service teachers' planning that echoed the findings presented earlier in this chapter. However, they also concluded that the research done thus far focused on how novice and expert teachers think during their teaching. There were fewer studies about how teachers think about their current planning. Furthermore, there was no research addressing how teachers learn to plan.

Thus, an overarching theme in all of these reviews of literature was the importance of an alignment between teachers' field experiences, including mentor teacher, curriculum, and university supervisor, and the teaching espoused by the teacher education program in which the teacher is enrolled. A more recent case study by Vacc and Bright (1999), as described earlier, found that a pre-service teacher placed with a mentor teacher trained in CGI taught in a CGI manner throughout her student teaching experience. By contrast, another CGI trained pre-service teacher was assigned to a mentor teacher who had not been trained in CGI and taught in a traditional way. This pre-service teacher showed a continual decline in her teaching with respect to teaching in a CGI manner.

Thus it appears that not all field experiences offer equal opportunities for teachers to develop the knowledge that is valued by the teacher education programs in which they are enrolled. As Sykes and Byrd (1992) note: “if the aim of teacher education is a reformed practice that is not readily available, and if there is no reinforcing culture to support such practice, then the basic imagery of apprenticeships seems to break down” (p. 501).

In summary, mathematics teacher education is facing critical challenges that must be addressed in order to significantly improve the quality of teacher preparation: 1) explicitly confronting pre-service teachers’ beliefs about students as learners; 2) bridging theory and practice so as to help teachers see how theories are used in the actual work of teaching; 3) providing novice teachers with the support and opportunities they need to move beyond focusing on their own actions and classroom management to focusing on their students’ thinking; and 4) developing field experiences that provide a “reinforcing culture” to support standards-based instruction. A teacher education program that addressed these challenges would be in sharp contrast to traditional teacher education approaches and could be a location where student thinking could be attended to in authentic ways, as outlined in the framework presented earlier.

CHAPTER 3: METHODOLOGY

3.1 Introduction

This study sought to identify the extent to and the ways in which teachers' attention to students' mathematical thinking, as evident in their written lesson plans and verbal descriptions of planning, changed over time. To do this, the study documented the extent to and the ways in which attention to students' mathematical thinking was evident in pre-service secondary mathematics teachers' ("teachers") written lesson plans and their verbal descriptions of planning. The study investigated teachers' attention to students' mathematical thinking prior to and immediately after participating in a course that emphasized students' mathematical thinking as a key element of planning. Then, following the course, teachers' attention to students' mathematical thinking was investigated during the first semester of their field experience.

This study utilized a within-subjects design drawing on teachers' written lesson plans and interviews about lesson planning in order to describe the extent to and ways in which teachers attended to students' mathematical thinking at various times during their teacher education program. The study also made use of a pre- post- delayed-post test design in order to capture change over time in teachers' attention to students' mathematical thinking. The sections that follow describe the methodology for the study, beginning with a description of the participants and the teacher education program that emphasized students' mathematical thinking. Then the data sources are described and their relation to the research questions is identified. Finally, the ways in which the data were analyzed in order to answer the research questions is explained.

3.2 Participants

The participants in this study were ten pre-service secondary mathematics teachers who were enrolled in a post-baccalaureate teacher education program at a large, urban university in the northeast United States, that culminates in certification in secondary (7-12) mathematics and a Masters of Arts in Teaching degree. In order to be accepted into this master's-level program, applicants needed to have a bachelor's degree in mathematics (or its equivalent) and a minimum QPA of 3.0. In this program, teachers engaged in a year-long internship in a public school (grade 7-12) mathematics classroom as well as university coursework in the summer prior to their field experience and in the evenings during the school year. For the 2005-2006 academic year, sixteen people were admitted to the MAT program described above. Ten teachers completed all elements of the study and made up the participants in this study (two chose to not join the study initially; two dropped out of the teacher education program; and two chose to drop out of the study before completing the data collection).

3.3 Teacher Education Program

The teacher education program in which participants were enrolled emphasized the importance of attending to students mathematical thinking as teachers plan, teach and reflect on their lessons. This was primarily done through the use of frameworks and tools that focus teachers' attention on students' mathematical thinking. The program utilized *The Mathematical Tasks Framework* (MTF), described by Stein, Grover, and Henningsen (1996), as a guideline for thinking about planning, implementing, and reflecting on mathematics lessons. The MTF modeled the progression of a task through an instructional episode. The phases of the MTF (the task as it is written, the task as it is set up by the teacher, and the task as it is enacted by students and the teacher) constituted points at which the cognitive demands of a task are likely to be altered from its original form. The MTF provided a guideline for assessing the cognitive demands of a task, allowing teachers to categorize mathematical tasks as low-level or high-level. The MTF also has associated with it a set of classroom-based factors that influence students' opportunities to engage with the high-level cognitive demands of a task (Henningsen & Stein, 1997). In planning a lesson, the MTF provided a lens for determining which mathematical tasks to use in a lesson and for thinking about how the high-level demands of a task can be maintained throughout the lesson. The MTF also provided teachers with a common language to discuss, analyze, and reflect on mathematics lessons.

The secondary mathematics teacher education program also utilized a planning tool called *The Thinking Through a Lesson Protocol* (TTLP). The TTLP, as shown in Appendix A, draws on the questioning that occurs in Japanese lesson study, the Launch, Explore, Summarize questions from teachers' editions in the *Connected Mathematics Project* curriculum (e.g., Lappan, Fey, Fitzgerald, Friel, & Phillips, 2002) and research on mathematical tasks (e.g., the importance of high-level tasks and what it takes to keep tasks at a high level during implementation, as described in Stein, Grover, and Henningsen (1996) and Henningsen and Stein (1997)). The TTLP differed from typical lesson plan formats in several significant ways. First, a typical lesson plan format often had the following components: goals and objectives, materials, motivation, lesson procedure, and closure (Brahier, 2000). The lesson procedure generally indicated what the teacher would do and what students would do – rather than what students were thinking. In addition, there was little attention given to the interaction between a teacher and her students. The closure portion of the lesson often amounted to having students share solutions to a problem without indicating how each solution presented related to the goal of the lesson or to other solutions and/or to teachers telling students what it was they should have learned from the lesson.

By contrast, the TTLP moves beyond the structural components of a typical lesson plan and provides the opportunity to focus on specific ways in which the teacher can advance students' mathematical thinking during a lesson. In particular, the TTLP provides a series of questions for the teacher to consider during planning that focuses on selecting and setting up a mathematical task (e.g., What do students already know? What will they learn? What ways might students solve the task? What misconceptions might students have?), supporting students' exploration of the task (e.g., What questions will you ask to focus students' thinking, assess students' understanding, and advance students' understanding of the mathematical ideas?), and orchestrating a whole-class sharing and discussion of the task (e.g., Which solution paths do you want to have shared during the class discussion? In what order should they be shared? In what ways will the order of the solution paths help students make connections between the strategies and mathematical ideas?).

The certification program in secondary mathematics includes four core methods courses: the Teaching Lab, Methods 1, Methods 2, and the Technology Workshop. Participants in the study took the Teaching Lab course in the summer prior to their field experience (a year-long teaching internship). The Methods 1, Methods 2, and Technology Workshop were taken concurrently during the fall semester. The Teaching Lab was taught by the researcher and was designed to introduce key frameworks and tools (e.g., *The Mathematical Tasks Framework* and the *Thinking Through a Lesson Protocol*), that were drawn on in subsequent courses and field work, and to provide students with peer-teaching opportunities. The Methods 1 and 2 courses built on and contextualized the tools and strategies developed in the Teaching Lab and broadened the base for instructional decision making by considering theories of learning, assessment, and curricular development. The Technology Workshop provided students opportunities to use electronic tools and resources that are accessible by calculators and computers. The framework of the MTF and tools, such as the TTLP, laid the groundwork for these courses. Other key elements of the program that offered teachers opportunities to attend to students' mathematical thinking were the analysis of student work and the analysis of cases of mathematics teaching (e.g., written narrative and video). The description of the Teaching Lab that follows highlights examples of opportunities teachers had to realize the importance of and how to attend to students' mathematical thinking.

3.3.1 Teaching Lab Course

The Teaching Lab was taught during six weeks in the summer of 2005 and consisted of bi-weekly class sessions, each lasting three hours in length. The author had multiple prior experiences in teaching this course. The textbook for the course (Stein, Smith, Henningsen, & Silver, 2000) served as a source of research and practical materials. In particular, the textbook provided research-based information on aspects of the *Mathematical Tasks Framework* (MTF): the model for the progression of a task through an instructional episode, a guideline for assessing the cognitive demands of a task (Stein, Grover, & Henningsen, 1996), and a set of classroom-based factors that influence students' opportunities to engage with the high-level cognitive demands of a task (Henningsen & Stein, 1997). In addition, the textbook provided narrative cases of mathematics instruction designed to make salient aspects of the MTF. Table 3.1 provides a summary of what teachers did during each session of the Teaching Lab as well as the out of class assignments with which they engaged. The highlighted cells indicate opportunities teachers had to explicitly learn about lesson planning. However, many of the other activities and assignments completed during the course supported teachers' ability to attend to students' thinking as they plan, teach and reflect on a lesson. For example, during the first three classes, teachers were introduced to the various aspects of the MTF, as laid out in their textbook. In particular, teachers engaged in activities designed to draw attention to the cognitive demands of mathematical tasks and to increase their capacity to categorize tasks as low- or high-level (e.g., Comparing two tasks in class #1; The Task Sort in class #2). The classroom-based factors associated with the maintenance or decline of a high-level task were introduced during the third class and were used as a lens for analyzing written narrative cases of mathematics teaching (e.g., Case “A” during class #3 and Case “C” during class #5). Many of these factors involved teachers attending to and making use of their students’ thinking, such as selecting tasks that build on students’ prior knowledge; scaffolding students’ learning; modeling high-level performance; and pressing students for explanation and meaning (Henningsen & Stein, 1997).

Table 3.1 Teaching Lab Activities for Summer 2005

Class #1	Class #2	Class #3	Class #4	Class #5	Class #6	Classes #7-12
Solve 2 tasks on same content – 1 High-level & 1 Low-level	Teachers present their classroom rules	Discuss narrative Case “A”: How did the teacher support and/or inhibit students’ learning?	Solve task used in narrative Case “C”	Introduce rubric for “grading” lesson plans – based on attending to all aspects of TTLP	Introduce tools for observing lessons and language for talking with a teacher about his/her instruction	Teachers teach the lessons they have planned for their PTR assignment to their peers
Compare the two tasks: How are they the same and/or different?	Categorize Mathematical Tasks: The Task Sort	Introduce Factors associated with maintenance and decline of high-level cognitive demands	Discuss cognitive demands of task from Case “C”	Discuss Case “C”: Similarities and differences between 2nd and 6th period. Do the differences matter?	Solve high-level Task “E”	Teachers reflect on selected video tape segments of their peer’s teaching
	Introduce Levels of Cognitive Demand and The Mathematical Tasks Framework	Discuss what should be in a lesson plan and introduce the TTLP	Writing mathematical goals for a lesson	Analyze student work from Task “D” as a supplement to teachers’ work on Part 1 of the TTAL assignment	Analyze the instructor’s “teaching moves” when solving Task “E”	

			Groups solve high-level task “D” and think of all the ways students may correctly and incorrectly work with this task (Part 1 of the TTAL assignment)		Analyze video of 9th grade classroom implementation of Task “E”	
Out of Class Assignment						
Read article about classroom rules and routines then design a classroom rules handout and be prepared to present it	Solve the task used in Case “A”	Write a lesson plan around the high-level Task “B” that addresses all elements of the TTLP	Read Case “C”: consider whether students in 2nd period engage with the task in the same way as students in 6th period	Revise the Explore Phase of your lesson plan for Task “B”. Write a reflection summarizing the revisions	Write a lesson plan around Task “D” (Part 2 of the TTAL assignment)	Write a lesson plan for the PTR assignment and a reflection paper based on the implementation of the lesson
	Read Case “A”: identify how the teacher supported and/or inhibited students’ learning		Turn in a high-level task to use for PTR assignment and a mathematical goal for the lesson			

The *Thinking Through a Lesson Protocol* (TTLP) was introduced to teachers during the third class as a tool to facilitate their lesson planning. As part of their coursework in the Teaching Lab, teachers planned three different lessons using the TTLP. It was expected that teachers would consider all questions contained in the TTLP in their written lesson plan assignments. Classes three, four, and five provided teachers with opportunities to focus on specific elements of attention to students' thinking in their lesson planning. These opportunities are highlighted in Table 3.1 and described in more detail below.

As an out of class assignment between classes three and four, teachers had their first opportunity to write a lesson plan using the TTLP as a guide in their planning. Teachers were given a mathematical task, "B", that was categorized as a high-level task and asked to plan a lesson around the task, using the TTLP as a guide. Since this was the first lesson plan that teachers were asked to write, it was expected that they would not attend to students' thinking in the level of detail that is suggested in the TTLP. Therefore, after teachers had attempted to use the TTLP in writing their first lesson plan, subsequent class time was spent focusing on two specific elements of attention to students' thinking when planning a lesson: writing a mathematical goal for the lesson based on students' developing an understanding of a concept and identifying questions to ask students while they work that will assess and advance their mathematical thinking.

During the fourth class teachers turned in their written lesson plans for Task "B" and engaged in a forty-minute activity around writing mathematical goals. In particular, teachers were asked to consider two versions of a lesson goal for Task "B":

> Version 1: Students will understand the relationship between the equation of a quadratic function and its graph.
>
> Version 2: Students will develop an understanding of how the coefficient of the squared term in an equation of a quadratic function affects the graph of the function. Specifically, students will demonstrate (through words and graphs) that the larger the coefficient the narrower the curve and that the closer the coefficient is to zero, the wider the curve.

Teachers were asked to consider the differences between these two goal statements and then discuss which goal would be more helpful in planning the lesson and why. The teachers concluded that version two would be more helpful in planning a lesson because it described the ways in which students would show their understanding which would then aid the teacher in selecting activities that offered students these opportunities.

During the fifth class, teachers were given a scoring rubric that the instructor would use in grading the lesson plans they turned in for their "Thinking Through a Lesson" and "Plan, Teach, and Reflect" assignments (both assignments are described in more detail below). The rubric (as shown in Appendix B) assigned point values based on the level of detail to which teachers attended for each of the elements in the TTLP as evident in their written lesson plan. In class, teachers engaged in a series of activities lasting seventy-five minutes, in which they focused on the "Student Exploration Phase" of the TTLP and rubric. In particular, teachers were given two sample lesson plans for Task "B" (these were two lesson plans the instructor selected from those that teachers had turned in during the previous class). Teachers considered how they would score these two lesson plans with respect to the rubric's five-point scale for identifying questions that focus, assess, and advance students' thinking. The rubric awarded points based on whether or not specific questions were listed that have the potential to focus, assess and advance students' thinking, whether the questions were tied to particular strategies or approaches students may be using, and whether or not the questions were related to the mathematical goal for the lesson. The two sample lesson plans were limited in the type of questions asked, the question were not tied to particular strategies, and were not clearly related to the mathematical goal of the lesson. Teachers were also asked to think about how they might revise the lesson plans such that they would meet the expectations laid out in the rubric. Finally, teachers were given a third sample lesson plan. This lesson plan was for Task "C" that had been solved during the previous class and was written as an example of a lesson plan that met all the expectations laid out in the TTLP (the lesson plan was written by experienced mathematics teacher educators at the university). This lesson plan made salient that when multiple strategies for solving a task were anticipated then specific questions that focus, assess and advance students' thinking could be tied to the specific strategies and furthermore could be related to the mathematical goal of the lesson. Finally, as an out of class assignment, teachers were then asked to revise the "Student Exploration Phase" of their own lesson plan for Task "B" and write a reflection summarizing the similarities and differences between their original plan and the revised plan. The revised plans teachers submitted at the next class were vastly improved, with many of them meeting the highest criteria score on the rubric for identifying questions that focus, assess, and advance students' mathematical thinking.

The "Thinking Through a Lesson" assignment, which was assigned during the fourth class and due at the sixth class, provided teachers with a second opportunity to produce a lesson plan that used the TTLP as a guide. For this assignment, teachers were again provided with a particular mathematical task, Task "D" (categorized as a high-level task), around which they were to plan a lesson. In preparing for this assignment, teachers were given opportunities in class to focus on anticipating students' correct and incorrect thinking about the task. In particular, during class session four, teachers were given forty-five minutes to work with a group of four peers to solve the task in as many ways as possible and to consider all the ways students' may correctly and incorrectly think about this task. In addition, during the fifth class, teachers again worked in small groups to analyze authentic work on Task "D" from a class of 7th grade students. In this way, teachers could supplement their own ideas about the correct and incorrect ways students might think about the task with authentic examples. Armed with multiple ways in which students' would think about the problem, each teacher then individually wrote a lesson plan around Task "D". These lesson plans were graded according to the scoring rubric described earlier. However, teachers did not receive feedback on the lesson plans for their “Thinking Through a Lesson” assignment prior to planning their third and final lesson plan for the course.

The third and final lesson that teachers planned was part of a “Plan, Teach, and Reflect” (PTR) assignment in which teachers planned a lesson around a high-level task of their choice, taught the lesson to their peers, and wrote a reflection paper based on the videotape of the lesson enactment. Class sessions seven through twelve were devoted to teachers teaching their lesson to their peers. Immediately following each lesson, participants in the lesson provided feedback to the teacher through the lens of the MTF (e.g., highlighting factors that may have led to the maintenance or decline of the high-level cognitive demands of the task). In addition, on three occasions, the instructor of the course selected a clip from a teacher’s videotape to engage the class in further reflection on instructional strategies. These reflection sessions focused on the “Sharing and Discussing” Phase of the lessons. For example, during the eighth class teachers were asked to review a video clip and consider the following questions: 1) what specific mathematical thinking/ideas/strategies were present during the discussion; 2) what sparked the mathematical discussions identified in question 1; and 3) who was doing the mathematical work (e.g., the teacher, a student, multiple students, all students)? The discussion around this video clip made salient that the teacher in the video had students share their thinking and asked questions that promoted mathematical thinking. However, these important “discussions” occurred only between the teacher and the student presenter while all other students in the class remained silent. Teachers in the Teaching Lab course then discussed ways in which they could build on individual students’ thinking in the whole-class discussion phase of the lesson while engaging all students in the mathematical discussion.

In summary, during the Teaching Lab course, teachers had multiple opportunities to learn about the importance of and how to attend to students' mathematical thinking in their lesson planning and to actually try it out in a lesson taught to their peers. Coursework in the fall semester built on the lesson planning foundation laid during the Teaching Lab course and offered teachers additional opportunities to engage with the TTLP in planning lessons. For example, as part of their fall coursework, teachers again had a Plan, Teach, and Reflect assignment. However, in this PTR, they taught the lesson to a class of students during their field experience rather than to their peers.

3.4 Data Sources

A variety of data sources (as shown in Table 3.2) were collected in order to investigate the four research questions: 1) To what extent is attention to students' mathematical thinking evident in teachers' written lesson plans or lesson planning process prior to and immediately after participation in a course that emphasizes students' mathematical thinking as a key element of planning?; 2) To what extent is attention to students' mathematical thinking evident in teacher's written lesson plans or lesson planning process during the first semester of their field experience?; 3) To what extent does attention to students' mathematical thinking, as evident in teacher's lesson planning, change over time?; and 4) In what ways does attention to students' mathematical thinking, as evident in teacher's lesson planning, change over time? The table makes clear the timing and frequency of data collection as well as the mapping of particular data sources to the research questions which they were used to address. All data were in the form of written or oral lesson plans or interviews about teachers' planning process. The data sources represented three different lesson planning processes: *Planning On Demand, Planning for a University Course Assignment,* and *Planning in the Actual Practice of Teaching*. Each occurred under different circumstances and had different purposes for accessing teachers' lesson planning processes.

Table 3.2 Correlation of Research Questions and Data Sources

Lesson Plans		Research Question Addressed			
Data Source	**Timing of Collection**	**1**	**2**	**3**	**4**
Planning On Demand					
Pledge Plans Task Written lesson plan & interview (PPint1)	1st week of TL course (June 2005)	X		X	X
Pledge Plans Task Written lesson plan & interview (PPint2)	Week after TL course ends (Aug. 2005)	X		X	X
Pledge Plans Task Written lesson plan & interview (PPint3)	Near the end of 1st semester of field experience (Dec. 2005)			X	X
Planning for a University Assignment					
Written lesson plan from PTR course assignment (PTR1)	Near end of TL course (July 2005)	X		X	X
Written lesson plan from PTR course assignment (PTR2)	Near end of Methods 1 course (Dec. 2005)			X	X
Planning in Practice					
Written lesson plans from field experience (LP)	Two per month for Oct. & Dec. 2005 and a set of 3 consecutive lessons in Nov. 2005		X	X	X
Interview about influences on planning & 1 LP from their field experience	Near the end of 1st semester of field experience (Dec. 2005)				X

3.4.1 Planning on Demand

The Pledge Plans Task interview was a semi-structured interview (see Appendix C for a copy of the interview protocol) that asked teachers to plan a lesson around The Pledge Plans Task (also included in Appendix C). Prior to the interview, teachers were asked to solve The Pledge Plans

Task and to identify the mathematical ideas the task had the potential to address. During the interview, teachers were given twenty minutes to write a lesson plan around the task intended for an 8th grade class during a unit on linear functions[2]. Teachers were asked to provide as much detail as possible in the written lesson plan and then to talk about their lesson plan. The interviews were semi-structured and allowed the interviewer to probe teachers' thinking. Probing was intended to press teachers to explain their thinking and clarify their responses. For example, questions such as, "Can you say more about that?", "What do you mean by…?", or "Can you say more about why you decided to…?" were used to understand teachers' planning process and to clarify their verbal descriptions of their planning. Also, if elements of the written lesson plan were not brought up by the teacher during the interview, then the interviewer probed by asking "I noticed you have (x) in your lesson plan here, can you tell me about that?".

Each teacher participated in the Pledge Plans Task interview at three different time points: (1) the beginning of their teacher education program, between classes 1 and 3 of the summer Teaching Lab course – June 2005 (denoted as PPint1), (2) immediately after the summer Teaching Lab course but before they began their field experience – August, 2005 (denoted as PPint2), and (3) near the end of the first semester of their field experience – December 2005 (denoted as PPint3). Teachers were permitted to write on the materials available at their interview (e.g., graph paper, blank paper, Pledge Plans Task, planning assignment sheet) and these materials were kept for analysis. Each interview was audio-taped and was between forty-five minutes to one hour in length. The interviews were conducted by either the researcher or a mathematics education doctoral student. They were transcribed and then verified by the

[2] During earlier research under ASTEROID project, it was determined that twenty minutes was a reasonable amount of time to get thoughts down on paper, given that teachers had ample time to explicate their written lesson plan during the interview.

researcher. The interviewers all had previous experience conducting interviews and attended an hour-long training session in which they became familiar with the interview protocol and came to consensus on the probing questions that were appropriate to ask during an interview. Clarifications resulting from the training session were subsequently added to the interview protocol.

The purpose of the Pledge Plans Task interviews conducted prior to and immediately after the Teaching Lab (PPint1 and PPint2) was to gain access to the pre-service teachers' lesson planning process through their written plans and verbal description about their lesson plan. These two interviews provided data for answering research question one; to what extent is attention to students' mathematical thinking evident in pre-service secondary mathematics teachers' written lesson plans or lesson planning process prior to and immediately after participation in a course that emphasizes students' mathematical thinking as a key element of planning. The third Pledge Plans Task interview (PPint3), conducted near the end of teachers' first semester of their field experience, was used in conjunction with the first two interviews in order to answer research question three; to what extent does attention to students' mathematical thinking, as evident in pre-service secondary teachers' lesson planning, change over time. By holding the task around which teachers were planning a lesson constant, comparisons of teachers' planning over time could be made. In addition, holding the task constant kept the math content stable and therefore teachers' own mathematical knowledge did not affect their ability to plan a lesson and attend to students' thinking about the task. Since it was difficult to have three mathematically equivalent tasks, using different tasks would have left open the possibility of individual teachers having more or less knowledge about a given task, thus affecting their lesson planning. One possible drawback of holding the task constant in the three interviews was that in subsequent interviews, teachers' prior experience in solving and planning a lesson around the task could account for improvement. The Pledge Plans Task interviews served as an indicator of teachers' ability to plan a lesson on demand and did not necessarily reflect what teachers were capable of doing when given extensive time to plan and were explicitly asked to attend to students' thinking in their planning or the planning teachers did in their actual teaching practice. Therefore additional data was necessary to access teachers' planning processes.

3.4.2 Planning for a University Assignment

As described earlier, teachers produced written lesson plans as part of their coursework for the secondary mathematics teacher education program. In particular, students engaged in a Plan, Teach, and Reflect (PTR) assignment during the Teaching Lab and also during Methods 1 in the fall semester. As described earlier, for the PTR assignment, teachers were asked to select a high-level mathematical task, plan a lesson around the task, teach the lesson, and then reflect on the enactment of the lesson. Teachers were explicitly told to use the TTLP as a guide when planning the lesson for their PTR assignments. Teachers began planning their lessons mid-way through the Teaching Lab and Methods 1 courses. For the Teaching Lab, teachers began submitting lesson plans for their PTR assignment after the sixth class session. As indicated in the earlier course description, instruction during the first five class sessions focused on the Mathematical Tasks Framework and lesson planning. During the remaining class sessions of the Teaching Lab, teachers taught the lesson to their peers and provided feedback to their peers on the implementation of their lessons. By contrast, in Methods 1 teachers taught the lesson to students in their field placement classroom. For this study, participants were asked to submit copies of the written lesson plans they produced as part of their PTR assignments during the Teaching Lab (PTR1) and in the fall semester for Methods 1 (PTR2). Because the PTR1 lesson plans were written after explicit instruction on lesson planning in the Teaching Lab was completed, PTR1 were considered a post-Teaching Lab measure.

The purpose of collecting the written lesson plans produced as part of the PTR assignments was to assess teachers' attention to students' mathematical thinking when explicitly told to use the TTLP as a guide in their planning process. Thus these lessons provided evidence of teachers' ability to attend to students' mathematical thinking when explicitly directed to do so at the end of the Teaching Lab, but before they begin their field experience, and near the end of the first semester of their field experience. However, the PTR lesson plans were not necessarily reflections of teachers' planning in their actual practice. Additional data was needed in order to access teachers' actual planning practices.

3.4.3 Planning in Practice

Information about teachers' lesson planning process during the first semester of the teachers' field experience was collected through written lesson plans and interviews about their lesson planning and provided data that was used to answer research question two (To what extent is attention to students' mathematical thinking evident in pre-service secondary mathematics teachers' written lesson plans or lesson planning process during the first semester of their field experience?). Furthermore, this data, in conjunction with the Pledge Plans Task interview data (PPint1, PPint2, and PPint3) and PTR1 and PTR2 lesson plans, was used to answer research question three and four (To what extent does attention to students' mathematical thinking, as evident in pre-service secondary teachers' lesson planning, change over time? And in what ways does attention to students' mathematical thinking, as evident in pre-service secondary teachers' lesson planning, change over time?).

Teachers were asked to submit copies of seven lesson plans during the first semester of their field experience. In particular, teachers submitted two lesson plans from the month of October, a set of three consecutive lesson plans from November, and two lesson plans from the month of December. Teachers also submitted copies of the mathematical problems used in each of the lesson plans they turned in to the researcher. Teachers selected a course for which they were responsible for planning lessons as their "focus course" for the study. All of the lesson plans they submitted were to come from the "focus course". For example, if a teacher was responsible for teaching three sections of Algebra and one section of Geometry, then they could submit seven Algebra lesson plans or seven Geometry lesson plans. If a particular lesson extended more than one day, the two days would be considered as one lesson. These lesson plans provided information about teachers' lesson planning and attention to students' mathematical thinking during their field experience.

In the beginning of December, each teacher participated in a final interview that consisted of four parts (see Appendix D for a copy of the interview protocol). Parts one and two asked teachers to talk about their lesson planning process in general (e.g., "What should be included in a lesson plan? and a series of questions about the things that might influence teachers' planning such as the textbook, mentor teacher, and university supervisor). Part three of the final interview was the Pledge Plans Task lesson-planning interview (PPint3) as described earlier. For part four of the final interview, each teacher brought a written lesson plan for a lesson that they had not yet taught as a focus lesson to discuss during the final interview (this written lesson plan also served as one of the written lesson plans they turned in for the month of December). The interview occurred prior to the enactment of the lesson so that the teacher could verbalize their planning process (rather than report what actually happened during the implementation of the lesson plan). As shown in the interview protocol, the interview was semi-structured and allowed the interviewer to probe teachers' thinking, as described earlier for the Pledge Plans Task Interview. Each interview was audio-taped and between 75-90 minutes in length. The interviews were conducted by either the researcher or a mathematics education doctoral student. They were transcribed and then verified by the researcher. The interviewers all had previous experience conducting interviews and attended a training session similar to the one described for the Pledge Plans Task interview.

The purpose of the written lesson plans and final interview part four around one of the lesson plans was to gain access to teachers' actual lesson planning process through their written and verbal descriptions of their lesson plans. These lesson plans and interview differed from the Pledge Plans Task interviews in that these lesson plans were authentic to the practice of the teacher. That is, these lesson plans were going to be implemented by the teacher in real classrooms, as opposed to the Pledge Plans Task interviews that asked teachers to imagine that they were going to teach the lesson to a class of eighth graders. Therefore, these written lesson plans and interviews provided access to teachers' planning process during their field experience. It is also worth noting that the earlier Pledge Plans Task interviews (PPint1 and PPint2) occurred when teachers were enrolled in the Teaching Lab course and the teachers may have been influenced by knowing what the instructor/researcher valued. However, during the field experience, the researcher had no authority over the teachers and it was expected that the values of the mentor teacher or university supervisor or the teacher's own beliefs would have a greater influence on the teacher's lesson planning process.

Through the final interview, aspects of the written plans that were unclear could be clarified and aspects of the teachers' planning that were not written down could be verbally described (see Schoenfeld, 1998 for a description of differences between written lesson plans and teachers' lesson images which contain detailed information not usually written down). Because interviewing teachers about their lesson planning is time-consuming, only a limited number of lessons could reasonably be discussed. However, by collecting teachers' written lesson plans for three consecutive lessons and two per month during October, November, and December, the researcher had access to multiple examples of the pre-service teachers' actual lesson plans. Because the interview was about one of the written lesson plans, analysis could show if there was a difference between the verbal descriptions of planning and the corresponding written lesson plan (see Data Analysis section later). This could then be taken into consideration when interpreting results of the analysis of all written lesson plans (e.g., analyses could indicate that teachers' written lesson plans under-represent their attention to students' mathematical thinking when planning a lesson).

In addition, the interviews provided information about possible factors that influenced teachers' attention to students' mathematical thinking in the lesson planning process. Specifically, teachers were asked to explain who or what may have influenced the instructional decisions they have made in their lesson planning. Specific prompts provided teachers with an opportunity to share factors that they perceived to have influenced their planning (i.e., "What role has the textbook played in your planning?, What role has your mentor teacher played in your planning?, What role has your university supervisor played in your planning?, and Are there any other factors that have influenced your planning?). These interview responses served as data for exploring possible explanations for changes in teachers' planning over time.

3.5 Coding

Data were coded to enable both quantitative and qualitative analyses. All data were scored by the researcher and a double-blind, stratified random sample of 20% of the data sources (22 written lesson plans and interview transcripts; 2 PPint1, 2 PPint2, 2 PPint3, 2 PTR1, 2 PTR2, 12 LP) was scored by a knowledgeable rater (i.e., a mathematics education doctoral student) to check the consistency of the scores and the reliability of the coding scheme. Agreement for individual elements of the scoring rubric ranged from 88% to 100% with overall agreement being 94.1%. Reliability measures are discussed more thoroughly within each element of the scoring rubric later in this section. In this section, the ways in which the data were coded is described. First, a scoring rubric for assessing teachers' attention to students' mathematical thinking as evident in their written lesson plans and interviews about lesson plans is described. This is followed by an explanation of a scoring rubric for assessing the level of cognitive demand of the mathematical tasks used in the lesson plan.

3.5.1 Assessing Teachers' Attention to Students' Mathematical Thinking

As described in Chapter Two, four elements of focusing on students' mathematical thinking have emerged from the literature as important in the lesson planning process: (1) identifying the mathematical goal of the lesson; (2) anticipating students' responses and possible misconceptions; (3) identifying specific questions that will assess or advance students' understanding while they work; and (4) specifying how to orchestrate a whole-class discussion that builds on students' thinking and makes salient the mathematics of the lesson. All lesson plan data (written lesson plans and interviews about lesson plans) were coded according to a scoring rubric that assessed teachers' attention to students' thinking with respect to the four key elements featured in the study. A description of the scoring used for each of the four elements of attention to students' thinking in the process of lesson planning follows. Table 3.3 contains summary descriptions of each score and Appendix E contains the scoring rubric used in the study as well as examples from lesson plans that are characteristic of each score.

Lesson planning element one: Identifying the mathematical goal of the lesson. Lesson planning element one involved the teacher determining the specific mathematical concepts with which students would engage during the lesson or what mathematical understandings students would take with them from the lesson. Specificity was important in identifying the mathematical goal so that it could guide the lesson plan and subsequent instruction. Goals should make clear the understanding(s) students would gain about mathematical concepts rather than skills students would exhibit or tasks they would accomplish. This element was coded on a three-point scale (0, 1, or 2 points). Agreement between the researcher and an independent rater was 97% for this element.

A score of 2 points was assigned when the teacher described specific mathematical concepts students would understand and what it meant to "understand" the particular concept. Mathematical goals that identified mathematical concepts of which students would gain understanding but did not fully explicate the particular understanding of the concept were coded with a score of 1 point. Similarly, goals that focused on skills that students would learn or things students would do in order to complete the task were also coded with a score of 1 point. When the teacher did not provide any information about the mathematical goals of the lesson, then the data was scored as 0 points, indicating that a mathematical goal was not evident.

Lesson planning element two: Anticipating students' thinking about a mathematical task. In lesson planning element two, teachers provided evidence that they considered how students might mathematically interpret a problem or the array of strategies – both correct and incorrect – that they might use to tackle the problem. Scoring for lesson planning element two consisted of two separate categories: 1) anticipating students' correct thinking and 2) anticipating students incorrect thinking. Each of these two sub-categories was scored on a four-point scale (0, 1, 2, or 3 points) and was dependent upon the specificity with which the teacher described the anticipated students' thinking and whether or not the teacher attempted to describe many ways in which a student may think about the problem (see Table 3.3 for summary descriptions of each score or Appendix E for examples coded at each score level). Agreement between the researcher and an independent rater was 95% for *anticipating students' correct thinking* and 100% for *Anticipating Students' Incorrect Thinking.*

Lesson planning element three: Identifying specific questions the teacher can ask that will assess or advance students' understanding while they work on a mathematical task. For lesson planning element three, each data source was coded along a three-point scale (0, 1, or 2 points) with respect to whether or not the teacher provided a specific example of a question to ask students that would either assess or advance students' mathematical understanding and the circumstances under which they would ask the question. Agreement between the researcher and an independent rater was 91% for this element.

In order to score 2 points for this element, teachers needed to provide an example of at least two specific questions to ask as well as the circumstances under which they would ask the questions. By contrast, a score of 1 point for element three occurred when a teacher provided at least one example of a specific question to ask students that would assess or advance their understanding, but did not identify the circumstance(s) under which the question would be asked or the circumstances were not based on students' mathematical thinking about the particular mathematical task involved in the lesson. A score of 0 points was given if the teacher did not provide any specific example questions that could be asked of students as they work (individually or in groups) on the mathematical task. Specific questions that were meant to be asked to the whole class did not count in this lesson planning element and were addressed in lesson planning element four, as described below. While the quality of the specific example questions is an important aspect in promoting student learning, the evaluation of the quality of the questions was not within the scope of this study and hence not a part of this coding scheme.

Lesson planning element four: Orchestrating a whole-class discussion that builds on students' thinking and makes salient the mathematics of the lesson. Lesson element four involved two important aspects of orchestrating a meaningful whole-class discussion: (1) evidence that the discussion was building on students' mathematical thinking and/or work in solving the problem; and (2) identifying specific questions that would enable students to make connections between solutions or make the mathematics of the lesson salient. Teachers could plan a discussion that was built on students' mathematical thinking in several ways: (a) purposefully selecting student solutions for public discussion; (b) determining the sequence in which those solutions should be discussed; or (c) identifying specific questions to ask students that explicitly referred to students thinking or work on the problem. Each of the two aspects (building on students' mathematical thinking and example questions that make the mathematics salient) was coded separately on a 3-point scale (0, 1, or 2 points) with respect to specificity. It was possible that a question posed in the lesson plan served dual purposes of building on students' work or thinking as well as making the mathematics of the lesson salient. Therefore it was possible that a particular question was included in determining both the *Building on Student Thinking* score and the *Making the Mathematics Salient* score. Agreement between the researcher and an independent rater was 91% for *building on student thinking* and 88% for *making the mathematics salient*.

In order to earn a score of 2 points for the element of specifying how to orchestrate a whole-class discussion that *Builds on Student Thinking*, the lesson plan must have identified specific questions that highlight the mathematics in a specific student solution. By contrast, a score of 1 point occurred when the teacher selected and/or sequenced students' solutions to be discussed but did not provide any specific questions to ask related to the student work, or identified a question to ask, but was vague about for which student solution the question was appropriate, or simply asked students to explain or share his/her solution without specific questions that highlight mathematical ideas. When evidence of building on student thinking did not exist, a score of 0 points was given.

Evidence of planning a whole-class discussion that had the potential to *Make the Mathematics of the Lesson Salient* at a score level of 2 points occurred when the teacher identified a series of specific questions that developed mathematical ideas. If the teacher identified questions that were vague or so few that a particular mathematical idea was not being well-developed then the lesson plan received a score of 1 point. A score of 1 points was also given to lesson plans that expressed specific mathematical ideas that the teacher wished to address in the discussion, but offered no specific questions to ask in order to achieve their mathematical intentions. A score of 0 points was given when evidence of thinking about making the mathematics of the lesson salient did not exist.

Summary. Each lesson plan (either written lesson plan or transcript of an interview about a lesson plan) was scored with respect to the six specific dimensions of focusing on students' mathematical thinking described above. Table 3.3 provides a scoring matrix summarizing the six dimensions that were coded and the possible scores assigned to each dimension. Each lesson plan also received an overall score for attention to students' thinking that was the sum of the scores earned for the six dimensions. Therefore each individual lesson plan could receive a maximum total score of 14 points.

Table 3.3 Scoring Matrix for Lesson Plans

Element of Attending to Students' Thinking	Score = 0	Score = 1	Score = 2	Score = 3
Mathematical Goal	A mathematica l goal does not exist	Vaguely describes concepts OR focuses on skills students will exhibit OR focuses on things students will do to complete the task	Specifies concepts and what it means to "understand" the concept	N/A
Anticipating Students' Correct Thinking	Evidence of anticipating students' correct thinking does not exist	Vaguely describes correct strategies/thinking students may use when working on the problem	Specifically describes at least one correct strategy/approach students may use when working on the problem. However, the strategies/approaches are limited and do not represent an attempt to describe the many ways in which students may solve the problem(s).	Specifically describes correct strategies/thinking students may use when working on the problem AND there is an attempt to identifying the many possible solution strategies or representations students may use
Anticipating Students' Incorrect Thinking	Evidence of anticipating students' incorrect thinking does not exist	Vaguely describes incorrect ways in which they may think about the problem	Specifically describes at least one incorrect way in which students may think about the problem or specific question students may ask or difficulty students may encounter as they work on the problem, however the challenges and misconceptions are limited and do not represent an attempt to describe the many challenges or misconceptions that students may have	Specifically describes incorrect ways in which students may think about the problem or specific questions students may ask or difficulties students may encounter as they work on the problem AND there is an attempt to identifying the many challenges or misconceptions students may encounter with the given mathematical task

Questions to Assess and Advance Students' Thinking	Specific example questions do not exist	Provides a specific example question to ask students but the circumstances under which the question is appropriate are not given, are not based on students' mathematical thinking about the problem, or only one circumstance based on students' mathematical thinking is present	Provides a specific example question to ask students AND the circumstances under which the question is appropriate (circumstances based on students' mathematical thinking about the problem). There must be at least two different circumstances based on students' mathematical thinking with a corresponding specific question(s)	N/A
Discussion Building on Students' Thinking	Evidence of building on student thinking does not exist	Selects and/or sequences students' solutions to be discussed but does not provide any specific questions to ask related to the student work OR identifies a question to ask, but is vague about for which student solution the question is appropriate, OR simply asks students to explain or share his/her solution without specific questions that highlight mathematical ideas	Identifies specific questions that highlight the mathematics in a specific student solution	N/A
Discussion Making the Mathematics Salient	Evidence of thinking about making the mathematics of the lesson salient does not exist	Identifies questions that are vague or so few that a particular mathematical idea is not being well-developed OR expresses specific mathematical ideas that they wish to address in the discussion, but offer no specific questions to ask in order to achieve their mathematical intentions	Identifies a series of specific questions that develop mathematical ideas	N/A

3.5.2 Assessing the Cognitive Demands of a Mathematical Task

Teachers were asked to provide all of the mathematical tasks that were featured in the lesson plans they submitted. A mathematical task, as defined by Stein, Grover, and Henningsen (1996), was a set of problems or single complex problem the purpose of which was to focus students' attention on a particular mathematical idea. The mathematical task that was determined to be the main instructional task for each of the submitted lesson plans was coded and analyzed. The main instructional task was identified by the following criteria: 1) it was identified in the teachers' lesson plan as the main instructional task; 2) it was described in the teachers' lesson plan as taking the largest amount of instructional time in the lesson; or 3) it was the central task for the lesson (i.e., not the warm-up problem or extension/homework problem). In most instances (54/60 lesson plans), the mathematical task was clearly identified on a separate sheet as a handout given to students or a copy of the students' textbook problems that were used in the lesson. However, there were six lesson plans (representing 10% of the data) in which the mathematical task was embedded in the lesson plan in the form of example problems and questions asked to students. Most tasks that were identified as the main instructional task were made up of a series of questions posed around a particular scenario or mathematical idea, rather than being a single mathematics problem with one question posed.

Each main instructional task (herein referred to as the task) was coded as either high-level or low-level according to the level of cognitive demand (i.e., "doing mathematics" and "procedures with connections" tasks would be classified as high-level; "procedures without connections" and "memorization" tasks would be classified as low-level) identified by Stein and colleagues (1996) (see the Task Analysis Guide in Appendix F) When a task was made up of multiple problems or questions, each problem or question was coded as high or low-level. The task earned a final score of high or low-level depending on the highest score of each individual problem (i.e., as long as one of the sub-questions was considered to be high-level than the task was determined to be high-level).

The mathematical task used in the Pledge Plans Task interview had been categorized as a high-level task by the researcher and other knowledgeable mathematics teacher educator prior to the study. The tasks used in the PTR lesson plans were all high-level tasks, as each teacher needed approval from the instructor of the course that the task was indeed high-level before the teacher could proceed in writing a lesson plan around the task. Therefore, only the tasks associated with the written lesson plans teachers created for their field experience needed to be categorized as low- or high-level tasks. The researcher scored all sixty of the main instructional tasks analyzed in this study. A knowledgeable rater (i.e., a doctoral student in mathematics education) independently scored a random sample of 20% of the tasks (12 tasks) to determine reliability. Exact agreement on the task level scores (i.e., low-level or high-level) between the researcher and the independent rater was 91.7% (11/12).

3.6 Analysis

All written lesson plans and interview transcripts were scored according to the scoring rubric described in the previous section. Thus, each lesson plan type (see Table 3.2) had an individual score for each of the six dimensions of focusing on students' mathematical thinking. In addition, each lesson plan also received an overall score for attention to students' mathematical thinking that was the sum of scores received for each of the six dimensions coded. Therefore the overall score for each lesson plan ranged from zero to fourteen. The following sections address how the results for the lesson plan coding were analyzed in order to answer the research questions.

3.6.1 Research Question One

In order to answer to what extent is attention to students' mathematical thinking evident in pre-service secondary mathematics teachers' written lesson plans or lesson planning process prior to and immediately after participation in a course that emphasizes students' mathematical thinking as a key element of planning (research question one), scores from the written lesson plans and interviews for the Pledge Plans Task completed by teachers during the first week of the Teaching Lab course (PPint1) and immediately following the course (PPint2) were analyzed. Descriptive statistics (e.g., means) were used in conjunction with qualitative descriptions to provide a measure of the extent to which teachers were attending to students' thinking with respect to the six dimensions for which the data was coded.

In addition, coding results from teachers' written lesson plans from their Plan, Teach, Reflect assignment turned in at the end of the Teaching Lab course (PTR1) were analyzed. These results provided a measure of what the teachers were able to do, with respect to attending to students' mathematical thinking, when they were explicitly told to use the TTLP as a guide in lesson planning. Similarities and differences between the results of the post-course lesson planning (PPint2 – where teachers did not have the TTLP as a guide) and the PTR lesson plan (PTR1 – where teachers were required to use the TTLP as a guide) were identified. The Wilcoxon paired ranks test was used to determine if there were significant differences between the two types of lesson planning vis-a-vis each of the six coded dimensions of attending to students' mathematical thinking.

3.6.2 Research Question Two

Answering to what extent is attention to students' mathematical thinking evident in pre-service secondary mathematics teachers' written lesson plans or lesson planning process during the first semester of their field experience (research question two) entailed analyzing the coding results of the written lesson plans teachers submitted during the first semester of their field experience. Teachers were asked to submit copies of seven lesson plans during the first semester of their field experience. In particular, teachers submitted two lesson plans from the month of October, a set of three consecutive lesson plans from November, and two lesson plans from the month of December. Teachers also submitted copies of the mathematical tasks used in each of the lesson plans they turned in to the researcher. Every teacher turned in the lesson plan they had created as part of their Plan, Teach, and Reflect assignment for the Methods 1 course (PTR2) as one of their seven lesson plans used in practice. Because the analysis showed that written lesson plans created for a university assignment differed significantly in their attention to students' thinking from the lesson plans teachers usually produced for their field experience, the teacher's PTR2 lesson plan was not included in the set of lesson plans considered data for teachers' planning during practice. Thus each teacher's set of lesson plans representing their planning during practice was comprised of a total of six written lesson plans, none of which was produced for their university assignment (PTR2). One teacher, Faith Norris was an exception. For her three consecutive November lesson plans, it was subsequently discovered that the first two lesson plans together made up the one PTR2 lesson plan she had turned in for Methods 1. Thus, by the rules described above, these two lesson plans should have been eliminated from Faith's set of lesson plans. However, Faith's PTR2 lesson plan was consistent in content and scores with the other four lesson plans she had submitted that used high-level tasks. Therefore, for the purposes of this study, Faith's PTR2 lesson plan (which was the same as the NovLP1 and NovLP2) was used as her NovLP1 and the third lesson plan that she had submitted was used as her NovLP2.

The average of an individual teacher's six lesson plans was calculated and provided a numerical snapshot of that teacher's attention to students' thinking in his or her planning from October to December of the first semester of the field experience. For teachers who seemed to be fairly consistent in their planning, particularly in their attention to students' thinking when planning, the average of their six lesson plans provided a fairly accurate account of their planning. However, for four teachers, the difference between their highest and lowest lesson plan scores was nine or more points. Further investigation indicated that the cognitive demands of the task used in the lesson was in fact related to teachers' attention to students' thinking in the corresponding lesson plan. Comparisons between an individual teacher's attention to students' thinking in written lesson plans that used low- and high-level tasks were made in order to determine whether teachers were planning differently for tasks based on their cognitive demands. In addition, a comparison of means of all the lesson plans that used a low-level task and the lesson plans that used high-level tasks for each of the lesson planning elements was made, using a t-Test for two samples assuming equal variance.

Comparisons of teachers' individual lesson plan scores as well as average lesson plan scores (separated for lesson plans that used low- and high-level tasks) were used to identify trends in teachers' attention to students' thinking as evident in each of the six elements of lesson planning that were addressed in the study. Analyses were also done to determine whether or not there was a relationship between teachers' attention to any two of the six elements of lesson planning that were addressed in the study (e.g., evidence could indicate that teachers who anticipated students' correct thinking were more likely to orchestrate a discussion that built on students' thinking).

Finally, an analysis was done to determine similarities and differences between written lesson plans and verbal descriptions of lesson plans. A comparison of written and verbal lesson plans was first done in terms of reliability; determining the consistency with which the coding of written lesson plans agreed with the coding of verbal descriptions of those same lesson plans. A Kappa coefficient was calculated as a measure of reliability. If the reliability is high ($K > .80$), then written and verbal descriptions of lesson plans can be seen as providing the same information regarding a teacher's planning process. However, if the reliability was low, this would suggest that teachers were more likely to verbalize and not write down the their thinking about students' mathematical thinking. Hence, relationships between written lesson plans and verbal descriptions of lesson plans were formally tested using the Wilcoxon paired ranks test.

3.6.3 Research Question Three

To what extent does attention to students' mathematical thinking, as evident in pre-service secondary teachers' lesson planning, change over time (research question three) was addressed by comparing teachers' scores for attention to students' mathematical thinking across three time points; prior to the Teaching Lab, immediately after the Teaching Lab but before the field experience began, and during the first semester of teachers' field experience as well as within and across the three types of lesson planning (on demand, for university assignment, and in practice). The Wilcoxon paired ranks test was used to compare teachers' attention to students' thinking between any two data points. Table 3.4 summarizes the data that were compared and rationale for the comparison with respect to the time period being compared and the type of lesson planning involved in the comparison.

Table 3.4 Summary of Data Comparisons

Data to Compare	Time Period	Type of Lesson Planning
PPint1 vs. PPint2 vs. PPint3	Describe change from Pre-Teaching Lab (June) to Post-Teaching Lab (Aug.) to Near the End of First Semester of the Field Experience (Dec.)	Describes changes in planning over time within one type of planning (on demand)
PTR1 vs. PTR2	Describe changes in planning from Post-Teaching Lab (Aug.) to Near the End of First Semester of the Field Experience (Dec.)	Describes changes in planning over time within one type of planning (university assignment)
PPint2 vs. PTR1 PPint3 vs. PTR2	Describe differences between types of planning at two separate time points: Post-Teaching Lab and Near the End of the First Semester of the Field Experience	Describe differences between planning on demand and planning for a university assignment
PPint1 vs. AvgLP-LL, AvgLP-HL PPint2 vs. AvgLP-LL, AvgLP-HL PPint3 vs. AvgLP-LL, AvgLP-HL	Describe changes in planning on demand from Pre-Teaching Lab, Post-Teaching Lab, and End of First Semester to planning in practice During the Field Experience	Describe differences between planning on demand and planning in practice
PTR1 vs. AvgLP-LL, AvgLP-HL PTR2 vs. AvgLP-LL, AvgLP-HL	Describe changes in planning for a university assignment from End of Teaching Lab, and End of First Semester to planning in practice During the Field Experience	Describe differences between planning for a university assignment and planning in practice

Comparisons between the Pledge Plans task lesson-planning interviews conducted prior to (PPint1) and immediately after (PPint2) teachers' participation in the Teaching Lab and near the end of the first semester of their field experience (PPint3) served as an indicator of the extent to which teachers' ability to attend to students' thinking when planning a lesson on demand changed over time. The Wilcoxon paired ranks test was used to determine if there were significant differences between two time points (Pre-Teaching Lab vs. Post-Teaching Lab and Post-Teaching Lab vs. Near the End of the First Semester of the Field Experience) with respect to each of the six coded dimensions of attention to students' mathematical thinking.

The lesson plans teachers produced as part of their "Plan, Teach, and Reflect" assignments for the Teaching Lab (PTR1) and the Methods 1 course in the fall (PTR2) represented teachers' ability to attend to students' thinking when explicitly asked to do so at two different time points in their teacher education program. Comparisons between the written lesson plans produced for PTR1 and PTR2 served as an indicator of the extent to which teachers' ability to attend to students' thinking when planning a lesson for a university assignment changed over time. The Wilcoxon paired ranks test was used to determine if there were significant differences between the PTR1 and PTR2 written lesson plans with respect to each of the six elements of attention to students' thinking.

Furthermore, comparisons were made between the three types of lesson plan data (i.e., planning on demand, planning for a university course assignment, and planning in the actual practice of teaching) to determine the extent to which teachers' attention to students' thinking differed when planning under different circumstances. In particular, distinct comparisons were made between the following: 1) teachers' planning on demand and for a university assignment, 2) teachers' planning on demand and in practice, and 3) teachers' planning for a university assignment and in practice.

First, comparisons were made between teachers' Pledge Plans interview lesson plans where teachers planned a lesson on demand and were not explicitly asked to attend to students' thinking in their planning and teachers' written lesson plans turned in as part of their "Plan, Teach, and Reflect" assignments in which teachers were explicitly asked to attend to students' thinking in their planning through use of the TTLP. The Wilcoxon paired ranks test was used to determine if there were significant differences between the two types of lesson planning vis-à-vis each of the six coded elements of attention to students' thinking by comparing PPint2 with PTR1 and PPint3 with PTR2.

Second, comparisons of teachers' planning in practice and their planning on demand, as represented by their planning during an interview around the Pledge Plans task that occurred at three different time points: at the beginning of the Teaching Lab (PPint1), immediately after the Teaching Lab (PPint2), and towards the end of the first semester of the teacher's field experience (PPint3) were made. Teachers' lesson planning in practice was represented by six lesson plans that each teacher submitted from their field experience. As described earlier, the cognitive demands of the task played a significant role in the level of attention teachers paid to students' thinking in their planning. Therefore, separate comparisons to teachers' lesson planning for lessons that used low- and high-level tasks (AvgLP-LL and AvgLP-HL, respectively) were needed. Again, the Wilcoxon paired ranks test was used to determine significant differences between the following: 1) PPint1 vs. AvgLP-LL and AvgLP-HL, 2) PPint2 vs. AvgLP-LL and AvgLP-HL, and 3) PPint3 vs. AvgLP-LL and AvgLP-HL.

Third, a comparison between teachers' planning in practice and their planning for university assignments in which they were explicitly asked to attend to students' thinking as they used the TTLP as a guide were made. The Wilcoxon paired ranks test was used to determine if there were significant differences between the two types of lesson planning vis-a-vis each of the six coded dimensions of attention to students' mathematical thinking through the following comparisons: 1) PTR1 vs. AvgLP-LL and AvgLP-HL and 2) PTR2 vs. AvgLP-LL and AvgLP-HL.

Finally, patterns were identified with respect to the extent to which teachers' attention to students' thinking, as evident in their lesson planning, changed over time. In other words, which teachers maintained, declined, or increased their attention to students' mathematical thinking during their field experience was determined. In addition, trends were identified in comparing teachers' lesson planning under different circumstances (i.e., on demand, for a university assignment, or in practice) with respect to each of the six elements of attention to students' thinking that were the focus of this study.

3.6.4 Research Question Four

In contrast to the quantitative analyses done to answer research questions one, two, and three, In what ways does attention to students' mathematical thinking, as evident in pre-service secondary teachers' lesson planning, change over time? (research question four) sought to provide additional insight into teachers' attention to students' thinking, as evident in their lesson planning. This was done in two distinct ways: 1) the trajectory that an individual teacher could traverse over time was explored rather than depicting the planning practices of the teachers as a group (as had been done when answering the previous three questions) and 2) a more robust portrait of a teacher's planning practices was described by selecting three teachers for whom a case study was written. Results from the analysis done to answer research question three were used to determine which teachers maintained, declined, or increased their attention to students' mathematical thinking during their field experience. Three teachers were selected to portray each of the three trajectories. Responses to questions asked during the final interview about teachers' planning as well as results from coding all of their lesson plans served as data for each teacher's case study.

3.7 Summary

The study used a within-subjects design to document the extent to and ways in which attention to students' mathematical thinking was evident in pre-service secondary mathematics teachers' written lesson plans and their verbal descriptions of planning. The study investigated teachers' attention to students' mathematical thinking prior to and immediately after participating in a course that emphasized students' mathematical thinking as a key element of planning. Then, following the course, teachers' attention to students' mathematical thinking was investigated during the first semester of their field experience. Thus, the study employed a pre-post- delayed post design in order to capture change over time in teachers' attention to students' mathematical thinking. The next chapter presents results of these data analyses.

CHAPTER 4: RESULTS

The results of the data analysis are reported in this chapter, organized into four sections that correspond to the four research questions presented in Chapter One. Section 4.1 describes the extent to which the ten pre-service mathematics teachers involved in the study attended to students' mathematical thinking in their written lesson plans and lesson planning process prior to and immediately after participation in a course that emphasizes students' mathematical thinking as a key element of planning (herein referred to as the Teaching Lab). Section 4.2 focuses on the extent to which teachers' attention to students' mathematical thinking is evident in their lesson planning during the first semester of their field experiences by presenting results from the analysis of lesson plans created by teachers for their daily practice of teaching. Section 4.3 describes the extent to which teachers' attention to students' mathematical thinking, as evident in their lesson planning, changed over time. Finally, Section 4.4 provides a more detailed picture of the ways in which individual teachers' attention to students' mathematical thinking, as evident in their lesson planning, changed over time, by presenting cases for three of the teachers.

4.1 Planning Prior To and Immediately After Participation in the Teaching Lab

The results presented in this section pertain to Research Question #1:

> To what extent is attention to students' mathematical thinking evident in pre-service secondary mathematics teachers' written lesson plans or lesson planning process prior to and immediately after participation in a course that emphasizes students' mathematical thinking as a key element of planning?

To determine the extent to which teachers attended to students' mathematical thinking prior to participation in the course, lesson plans produced by teachers during the "pledge plans task" lesson-planning interview conducted at the beginning of the course were scored. Teachers' written lesson plans created as part of the "Plan, Teach, and Reflect" (PTR1) assignment given during the course as well as the "pledge plans task" lesson-planning interview conducted at the end of the course were scored in order to determine the extent to which teachers attended to students' mathematical thinking after participating in a course that emphasized students' mathematical thinking as a key element of planning. The results of these analyses are presented in the remainder of this section.

4.1.1 Planning Prior To Participation in the Teaching Lab

The Pledge Plans lesson-planning interview conducted between the first and third class of the Teaching Lab (here after referred to as PPint1) serve as an indicator of teachers' ability to attend to students' mathematical thinking prior to participation in a course that focused on students' thinking as a key element of planning. In preparation for the PPint1, teachers were asked to solve the Pledge Plans task (shown in Appendix C) and to consider the mathematics involved in solving the task. During the interview, teachers were given twenty minutes to write a lesson plan for an 8^{th} grade algebra class around the Pledge Plans Task. Teachers were then asked to talk about the lesson plan they had devised. Transcripts of each teacher's interview were scored for the six dimensions pertaining to evidence of attention to students' thinking. It was possible for a teacher to score a maximum of 14 total points: 2 points for the Mathematical Goal of the lesson; 3 points for Anticipating Students' Correct Thinking; 3 points for Anticipating Students' Incorrect Thinking; 2 points for Identifying Questions that Assess and Advance Student's Thinking; 2 points for Orchestrating a Discussion that Builds on Students' Thinking; and 2 points for Orchestrating a Discussion that Makes the Mathematics Salient (The scoring rubric is provided in Appendix E). Table 4.1 displays the results for individual teachers' PPint1.

Table 4.1 Individual Teacher's Results from Pledge Plans Interview 1

Teacher (pseudonym)	*Goal*	*Anticipate S.T.*		*Questions*	*Discussion*		***TOTAL***
		Correct	*Incorrect*		*ST*	*Math*	
(Max. Poss.)	*(2)*	*(3)*	*(3)*	*(2)*	*(2)*	*(2)*	*(14)*
Carter, Anthony	0	1	0	0	0	1	2
Edwards, Kevin	0	2	2	0	1	2	7
Ingram, Alicia	0	1	1	0	0	2	4
Irving, Kaitlyn	0	1	0	0	0	1	2
Knight, Kyle	1	0	1	0	1	0	3
Mercer, Diana	0	0	0	1	1	0	2
Nichols, Keith	0	1	0	0	1	1	3
Norris, Faith	0	1	0	0	2	2	5
Thompson, Laura	1	1	0	0	1	1	4
Yinger, Brittany	0	1	0	1	1	2	5

Prior to participation in the Teaching Lab, the extent to which teachers' planning focused on students' mathematical thinking was limited. Teachers' scores were very low for their overall lesson plan as well as five of the six categories (the one exception was Orchestrating a Discussion that Makes the Mathematics Salient). These results indicate that when planning the lesson, teachers were not primarily focused on attending to students' mathematical thinking during the lesson. Also, because the scoring rubric was designed to discriminate between vague suggestions and specific plans, the scores in Table 4.1 indicate that teachers were lacking specificity in many categories (e.g., scoring a 1 rather than a 2 or 3) as well as not attending to some elements at all (i.e., scoring a 0). For example, eight of the ten teachers provided no Mathematical Goal for the lesson. Furthermore, only two of the ten teachers (DM and BY) Identified any Questions to Assess and Advance Students' Thinking as students worked on solving the task. Results for three of the categories indicated that teachers were somewhat attending to them in their planning, but lacked specificity. At least seven of the ten teachers scored at least one point in the categories of Anticipating Students' Correct Thinking, Orchestrating a Discussion that Builds on Students' Thinking, and Orchestrating a Discussion that Makes the Mathematics Salient. In many of the interviews, teachers attempted to Anticipate Students' Correct Thinking by suggesting various representations students may use in solving the problem (e.g., writing an equation, making a table or chart of values, graphing points on a coordinate plane). However, individual teachers rarely suggested all three of these representations. Thus, there was little evidence that teachers were attempting to think of the many ways students might solve the problem. Furthermore, the teachers did not offer more explicit details about how students might construct or use these representations. Teachers were also vague in their thoughts about how to Orchestrate a Whole Class Discussion that Built on Students' Thinking, as exemplified in the following interview excerpt:

> I think that each, let's say there is say five groups. Each group would, either they could stand up or come to the front and give their explanation as to why they did what they did and what their reason was. Each group would do that, and then once the groups were done, the teacher could comment on things that certain groups had done that were good, maybe point out something that none of the groups had done if it seemed important. And then the teacher would conclude the lesson with their thoughts on the scenario [KN PPint1, p. 4-5].

Here, Keith expresses an interest in having students present their solutions and feels the teacher should comment on the work of the students. However, Keith offers no details as to what exactly he expects students to be presenting and what specific questions he would ask students about the mathematics in their work. This was typical, in that many teachers suggested having students share their solutions, but offered little detail on what these solutions were or the specific questions the teacher might ask related to the presented solutions.

Four of the ten teachers scored two points for planning a Discussion Making the Mathematics Salient, indicating that they provided specific questions to ask the class that had the potential to make the mathematics in the lesson salient for students. This is shown in the following excerpt where Brittany Yinger suggested placing a large coordinate plane on the board and having three students come up and graph the three plans:

> Then from there, we would again just discuss as a class. What do you see from these graphs? What can you tell me about the slopes? About intercepts? Intersections? What do you see based on these three graphs? And from there I would kind of allow the students, you "So what do you guys think is best?" … Hopefully through this discussion we would be, I would focus on more on like the intersections. Look at these intersections. What do you think this intersection means? And that would kind of lead into almost a discussion of break-even points. These two lines or these two graphs cross at this point. What is that saying about the graphs on the left side? What is it saying about the graphs on the right side from the intersection? [BY PPint1, p.3]

In this example, Brittany described the mathematics she wanted to discuss with the students and provided a series of specific questions she would ask the class that have the potential to make the mathematics salient for the students.

While it is encouraging that some teachers were able to plan a lesson that specified how they would lead a discussion that made the mathematics of the lesson salient, the low scores earned by most teachers for their overall plan and in five of the six dimensions for attending to students' mathematical thinking indicated that teachers were not focused on these critical elements of planning upon entering their teacher education program.

4.1.2 Immediately After Participation in the Teaching Lab

In order to assess teachers' attention to students' mathematical thinking after participating in a course that focused on students' thinking as a key element of planning, two data sources were analyzed: 1) the written lesson plan each teacher created as part of a course assignment (PTR1) and 2) the Pledge Plans lesson-planning interview conducted during the two weeks immediately after teachers' participation in the Teaching Lab (PPint2). This section presents results for each of these data sources and an analysis across the two data sources in order to describe the extent to which teachers attended to students' mathematical thinking in their lesson planning after participation in the Teaching Lab.

PTR1. During the final six class sessions of the course, teachers completed a PTR assignment in which they: 1) created a written lesson plan that incorporated all elements of the Thinking Through a Lesson Protocol (TTLP, see Appendix A); 2) taught the lesson to their peers in the course; and 3) wrote a paper in which they reflected on their planning and teaching of the lesson. The written lesson plans teachers submitted as part of this PTR assignment serve as indicators of teachers' ability to attend to students' mathematical thinking during planning when explicitly asked to do so, since the TTLP specifically focused their attention on these important elements. In addition, as discussed in Chapter Three, teachers completed their PTR1 lesson plan after six weeks of instruction that explicitly dealt with lesson planning. Therefore, for purposes of this study, the PTR1 was considered a post-Teaching Lab measure. Table 4.2 reports individual teacher's scores on PTR1.

Table 4.2 Individual Teacher's Results from PTR1

Teacher (pseudonym)	*Goal*	*Anticipate S.T.*		*Questions*	*Discussion*		***TOTAL***
		Correct	*Incorrect*		*ST*	*Math*	
(Max. Poss.)	*(2)*	*(3)*	*(3)*	*(2)*	*(2)*	*(2)*	*(14)*
Carter, Anthony	1	0	1	0	1	1	4
Edwards, Kevin	1	3	2	2	2	1	11
Ingram, Alicia	1	1	1	1	1	1	6
Irving, Kaitlyn	1	1	0	2	1	1	6
Knight, Kyle	1	3	3	1	1	1	10
Mercer, Diana	2	3	3	2	2	2	14
Nichols, Keith	2	3	0	2	2	1	10
Norris, Faith	2	3	3	2	2	2	14
Thompson, Laura	2	3	3	2	1	2	13
Yinger, Brittany	1	0	0	2	2	2	7

In the written lesson plans that teachers produced for their PTR1 assignment, teachers showed some evidence of attending to students' thinking as indicated by non-zero scores and totals. Remarkably, two teachers (DM and FN) achieved the maximum score of 14 points on their PTR1 lesson plan. Furthermore, 60% of the teachers scored 10 or more *total* points for their PTR1 lesson plan. This indicates that after six class sessions in the Teaching Lab, the majority of the teachers were capable of focusing on students' mathematical thinking when explicitly asked to do so. Teachers' ability to attend to a particular element and to provide specificity in their lesson plans varied across the six dimensions. For example, all of the teachers included a Mathematical Goal for their lesson. However, 60% of the teachers wrote lesson goals that either focused on the actions the students would take in order to solve the problem or were vague in describing the conceptual understandings they expected students to gain from the lesson (i.e., scored 1 point). For example, in the following excerpt the teacher was focused on what the students would do in order to solve the problem rather than on the concepts students might learn from their engagement with the task:

> Objectives: To have students take two columns of data and interpret them into a meaningful linear equation which has a unique solution.
>
> Goals: Derive a linear equation from a collection of data points.
>
> Analyze and interpret data to provide meaningful solutions to said linear equations. [KE, PTR1]

While many of the lesson goals resembled the above example, four of the teachers were capable of writing a lesson goal at the highest level by the end of the Teaching Lab. Diana Mercer's lesson goal is an example of what these teachers were capable of producing and is shown below:

> Goals: Students will develop an understanding of exponential curves. More specifically, students will recognize that the relationships of both curves are not linear, and that they are both exponential. In addition, they will recognize that exponential graphs grow much faster than linear graphs. They will demonstrate their understanding through their words, choices, and graphs. By being able to understand how exponential curves look and act, they will be able to start learning about growth and decay and logarithmic problems in the future [DM, PTR1].

Diana's goal began with a vague statement about a concept students should understand and is the point where many teachers' lesson goals concluded. However, Diana went on to further explicate what it was about exponential curves that she wanted her students to understand from this lesson.

As evident in the PTR1 scores, teachers varied in their ability to Anticipate Students' Correct and Incorrect Thinking. For example, when Anticipating Students' Correct Thinking, 60% of the teachers scored the highest possible score of 3 points, meaning that they attempted to identify a number of ways in which students' might correctly think about the problem when solving it. These teachers provided explicit examples of different strategies involving a variety of representations that they believed students might use to solve the task. By contrast, two teachers (AI and KI) only suggested one or two possible ways students might solve the problem and provided no details in their descriptions (e.g., "find the point of intersection algebraically" – AI). Finally, two teachers (AC and BY) provided no evidence of Anticipating Students' Correct Thinking. Similar results were shown for Anticipating Students' Incorrect Thinking, with four teachers scoring the maximum 3 points and five teachers scoring only 1 or 0 points. Four of the six teachers who received the maximum score for Anticipating Students' Correct Thinking also received the maximum score for Anticipating Students' Incorrect Thinking. There was only one teacher (KE) who scored two points for Anticipating Students' Incorrect Thinking, meaning that he provided specific examples of students' incorrect thinking, but they were limited in their scope. For example, Kevin provided specific descriptions of students' misconceptions that were restricted to difficulty interpreting the graph (e.g. "confused about where the flat rate plan fits into the graph", and "confused about the month the solution lies in") and provided only vague suggestions of other possible difficulties students may encounter (e.g., "trying to find exact equation from the data" and "points are plotted, unsure where to go"). Therefore, teachers seemed to have either fully understood what it meant to Anticipate Students' Correct or Incorrect Thinking, by scoring the maximum three points, or they were really struggling with meeting these expectations as they wrote lesson plans for their PTR1 assignment.

Seven of the ten teachers Identified Specific Questions to Ask Students as they worked on the problem and described circumstances under which it would be appropriate to ask a student a question that were based on students' mathematical thinking. For example, Laura Thompson provided three distinct ways students might solve a geometric pattern task in which students were asked to: 1) describe a pattern you see in the cube buildings; 2) use your pattern to write an expression for the number of cubes in the nth building; 3) use your expression to find the number of cubes in the 5th building. Check your results by constructing the fifth building and counting the cubes and; 4) look for a different pattern in the buildings. Describe the pattern and use it to write a different expression for the number of cubes in the nth building. She then went on to link specific questions she could ask students who were "having problems generalizing an expression" to each of the three solution strategies. Finally, she identified possible challenges students might encounter as they answered each of the four questions posed in her task and identified specific questions to ask students in order to move them along, mathematically, as shown in the following excerpt:

> What do I do if students are having problems with 4 [writing a generalized expression for the number of cubes in the nth building]?
>
> -- Try to encourage students not to focus only on the number of cubes in each building. How is the building shaped? *~five arms coming out the sides;* What would the next building look like? Did changes occur to the shape?
>
> --How does the number of blocks change for each building? What type of relationship is this? *~linear*
>
> -- Try to break the building into separate pieces. How do each of these pieces change?

Laura Thompson's lesson plan was representative of those lesson plans scoring the maximum two points. There were two teachers (AI and KK) who provided a list of specific questions to ask students as they worked on the task, however neither teacher provided any explanation as to when they might deem it appropriate to ask a specific question. Finally, one teacher (AC) did not provide any specific questions to ask students as they worked. The "Exploration" phase of his lesson (where students worked on the problem) consisted of the following: "a) Students work on problem for 20 minutes in a group; b) I go around asking about different solutions have occurred and confirming assumptions." The majority of the teachers, however, attended to this element of students' thinking with a high level of specificity.

All teachers made attempts to attend to student thinking as they planned how to Orchestrate a Whole-Class Discussion that Built on Students' Thinking and Made the Mathematics of the lesson Salient (i.e., no one earned a score of zero points). However, as a group they were not as specific as they had been in Identifying Questions to Ask Students as they worked. In Orchestrating a Discussion that Built on Students' Thinking, only 50% of the teachers were able to identify the specific student solutions they wanted presented and specific questions to ask that were linked to a specific student solution. Diana Mercer's written lesson plan provides an example of what this kind of planning looks like. She explains that she wants the first presenter to be a student who used a table to solve the problem, followed by someone who used a graph. She offers the following rationale for presenting the table first:

> By presenting this first, students will be able to see where the exact numbers come from, how to arrive at them, and where the numbers start to grow. This is the approach that will give the correct answer, but will not necessarily help students understand that the relationship between the two is exponential. Students need to however, be able to understand where the data came from before considering how the graph will look and act.

She then provides the following set of questions associated with the table that will allow her to make sure students understand:

> On the second day, it says that Daniel will make $1800. I thought his salary was going to double. Wouldn't he then have $2000?
>
> What does this mean on the 10th day? Why are the numbers the same?
>
> What did your group conclude about this offer?

Diana continues to provide the same level of detail as she talked about having the graph presented. Her lesson plan is indicative of the content and level of specificity that 50% of the teachers proved capable of providing in their PTR1 lesson plans. The other half of the teachers made attempts to include student work in their whole class discussion, some even provided information about specific student solutions they wanted presented and in what order they should be presented. However, these teachers did not provide any specific questions to ask the class that were explicitly linked to the student's work. This lack of specificity can be seen in Alicia Ingram's lesson plan involving a task about planning a skating party in which two options are compared (both linear functions, one with a y-intercept of zero). The following is an excerpt from her written lesson plan where she has "scripted" what she will say to the class:

Discussion:

- My goals were to have you be able to make connections/ make sense out of the pairs of points and point of intersection in a real world problem situation.
- So what exactly is this point of intersection?
- Ok, now I need 1 person from each group to come up and show how their group found the point of intersection.
- Now that each group has shared can we all see how the point of intersection can be found/represented in different ways?
- I will ask students if they can see the importance of math in everyday life.

Similar results were found in teachers' Planning of a Whole-Class Discussion that Makes the Mathematics Salient. Only 40% of the teachers were able to provide a series of specific questions to ask the whole class that had the potential to make the mathematics salient for the students. Meanwhile the other 60% provided ideas (sometimes very specific) about the mathematics they wanted to discuss with their class, but did not provide details about how they would orchestrate such a mathematical discussion in the form of specific questions to ask students.

Therefore, after six class sessions in the Teaching Lab, half of the teachers were still struggling to provide specific details in their lesson plans when planning a Discussion that Built on Students' Thinking and Made the Mathematics Salient. It is important to note that as described in Chapter Three, there were specific in-class activities that focused on each of the other elements of attention to students' thinking during the first six class sessions of the Teaching Lab (i.e., prior to the PTR1 lesson plans being written). Prior to writing their PTR1 lesson plans, teachers did receive feedback on the first lesson plan they wrote in which they used the TTLP as a guide when planning. The elements involved in planning a whole-class discussion (i.e., building on students' thinking and making salient the mathematics of the lesson) were the focus of reflection sessions after the teachers had taught their lessons in the Teaching Lab. Because teachers had fewer opportunities in the Teaching Lab to improve their attention to students' thinking when Planning a Discussion that Built on Students' Thinking and Made the Mathematics Salient prior to writing their PTR1 lesson plans, it is reasonable to expect the teachers to score lower on these two elements than on the other four elements at that point in time.

PPint2. The second Pledge Plans lesson-planning interview was conducted during the two weeks immediately following teachers' participation in the Teaching Lab. The PPint2 data serve as an indicator of teachers' attention to students' mathematical thinking after participation in a course that focused on students' thinking as a key element of planning. As in PPint1, the teachers were not given a copy of the TTLP, nor were they asked to consider the TTLP when they planned their lesson. Therefore, the PPint2 serve as an indicator of what teachers attend to when asked to plan a lesson on demand. Table 4.3 reports individual teacher's scores from PPint2.

Table 4.3 Individual Teacher's Results form Pledge Plans Interview 2

Teacher (pseudonym)	*Goal*	*Anticipate S.T.*		*Questions*	*Discussion*		***TOTAL***
		Correct	*Incorrect*		*ST*	*Math*	
(Max. Poss.)	*(2)*	*(3)*	*(3)*	*(2)*	*(2)*	*(2)*	*(14)*
Carter, Anthony	0	3	0	0	1	0	4
Edwards, Kevin	1	2	0	2	2	2	9
Ingram, Alicia	1	2	0	2	1	2	8
Irving, Kaitlyn	1	1	0	1	1	2	6
Knight, Kyle	1	2	1	0	1	1	6
Mercer, Diana	1	3	1	2	2	2	11
Nichols, Keith	1	2	0	2	2	2	9
Norris, Faith	1	3	3	2	2	1	12
Thompson, Laura	1	1	0	0	1	1	4
Yinger, Brittany	1	1	0	1	2	2	7

Overall scores for PPint2 were relatively high for some teachers, although no one scored the maximum of 14 points. Two teachers (DM and FN) earned 11 and 12 points respectively, while the remaining 8 teachers all scored below 10 points on their PPint2. The groups' scores with respect to the six dimensions for attention to students' thinking showed some interesting patterns. For example, almost all of the teachers (90%) had a Mathematical Goal for their lesson. However, no one wrote a goal that was considered to be very specific about the mathematical concepts students would develop and what it meant to understand a specific concept. Five teachers expressed mathematical goals that described the actions students would take as they solved the problem, as depicted in the following excerpt:

> The goal was to take the three plans, develop a data set and then graph each plan according to the data set. And they're supposed to be able to take this information and make a decision on which one will give them the most money. So that's my goal. That they are able to, um, generate the graphs and make the decision based on the graphs [KN PPint2].

While four other teachers provided lesson goals that were vague in describing mathematical concepts students would develop during the lesson, as shown in the following:

> My goal here is to have students examine the effects of the slope of the line…Another one of my goals is just to, um, use the concept of slope in a real world situation so that it will hopefully increase the students' understanding of the meaning of slope [AI PPint2].

Thus in the PPint2, teachers were able to come up with Mathematical Goals for the lesson, however they either focused on skills students would use to solve the problem rather than concepts students may develop or they identified concepts students should understand as a result of the lesson but were not specific in describing what it would mean to understand a concept such as slope.

All of the teachers provided evidence of Anticipating Students' Correct Thinking. However, the teachers showed a mixture in their level of specificity and whether or not they attempted to think of the many ways students might think about the task. For example, only three teachers earned the maximum score, four teachers provided specific examples of ways in which students may think but not an attempt to think about the many ways in which students may tackle the problem, and another three teachers provided vague descriptions of students' strategies. These results stand in stark contrast to teachers' Anticipation of Students' Incorrect Thinking. 70% of the teachers provided no evidence of considering students' incorrect thinking, in the form of misconceptions, miscalculations, difficulties students may encounter, or questions students may ask. Two teachers (KK and DM) did provide limited attention to students' incorrect thinking in the form of vague statements such as "possible misconceptions I thought of were: students may not realize that each plan is a linear equation. They might not make that connection on how to actually create the equation with the information" [DM PPint2]. Only one teacher (FN) was specific and identified many possible incorrect ways students' might think about the problem.

Attention to students' thinking in the form of Identifying Specific Questions to Ask Students as they worked and the circumstances in which the specific question would be appropriate to ask a student was a struggle for half of the teachers. In particular, there were three teachers (AC, KK, and LT) who did not provide any evidence of thinking of specific questions to ask students as they worked or even that they should ask students questions as they worked. Two teachers (KI and BY) provided a list of specific questions to ask students without explanations as to when it would be appropriate to ask students these questions. When pressed by the interviewer to say more about when they might ask these questions, both teachers offered circumstances that were not based on students' mathematical thinking (e.g., "if the students finish early"). However, 50% of the teachers did earn the maximum score of two points. These teachers identified specific questions to ask students as well as circumstances under which they would ask them that were based on students' mathematical thinking. For example, Keith Nichols provided specific questions to ask students based on whether or not the student was having trouble getting started, had a table of data and was struggling with plotting points on a graph, or had a graph of the three plans and needed to be pushed in interpreting the graph in the context of the problem.

Individual teachers were quite attentive to students' thinking in Orchestrating a Discussion that Builds on Students' Thinking and Makes the Mathematics Salient. For example, 60% of the teachers were able to identify the specific student solutions they wanted presented and specific questions to ask that were linked to a specific student solution. The remaining teachers made attempts to include student work in their whole class discussion, some even providing information about specific student solutions they wanted presented and in what order they should be presented. However, these teachers did not provide any specific questions to ask the class that were explicitly linked to the student's work. With respect to Orchestrating a Discussion that Makes the Mathematics Salient, 70% of the teachers were able to provide a series of specific questions to ask the whole class that had the potential to make the mathematics salient for the students. Meanwhile the other 30% provided ideas (sometimes very specific) about the mathematics they wanted to discuss with their class, but did not provide details about how they would orchestrate such a mathematical discussion in the form of specific questions to ask students.

Summary. After participating in the Teaching lab, teachers were able to attend to students' mathematical thinking in a variety of ways, as evident in their PPint2 and PTR1 scores for the six elements that were the focus of this study. For example, 90% of the teachers now had a Mathematical Goal for their lesson with 40% of the teachers scoring the maximum 2 points for the Mathematical Goal on the PTR1. In addition, on the PTR1 and PPint2, eight of the ten teachers scored at least 1 point for the category of Identifying Questions that Assess and Advance Students' Thinking as they work with at least 50% of the teachers scoring 2 pts. Teachers also provided specificity in their lesson plans with respect to attending to students' mathematical thinking. For example, all ten teachers scored identically on PTR1 & PPint2 for Orchestrating a Discussion that Builds on Students' Thinking*,* with half scoring 1 pt and the other half scoring 2 pts. Thus teachers were poised to begin their field experience in the fall semester (one month later) with the knowledge necessary and ability to attend to students' mathematical thinking in their lesson planning practices.

4.2 Teachers' Planning in Practice

The results presented in this section pertain to Research Question #2:

> To what extent is attention to students' mathematical thinking evident in pre-service secondary mathematics teachers' written lesson plans or lesson planning process in the first semester of their field experience?

Each teacher submitted six lesson plans that they had created for and used in their classrooms during the first semester of their field experience (October – December). The analyses of these lesson plans served as an indicator of teachers' attention to students' mathematical thinking in their actual teaching practice. Teachers' lesson plans were individually scored using the same rubric used to score all lesson plan data sources (see Appendix D). In the initial analysis of results, the average of each teacher's six lesson plan scores was calculated. Table 4.4 reports individual teacher's average lesson plan scores as well as the mean for all teachers for each of the six dimensions of attention to students' thinking.

Table 4.4 Individual Teacher's Average Lesson Plan Scores from Their Field Experience

Teacher (pseudonym)	*Goal*	*Anticipate S.T.*		*Questions*	*Discussion*		***TOTAL***
		Correct	*Incorrect*		*ST*	*Math*	
(Max. Poss.)	*(2)*	*(3)*	*(3)*	*(2)*	*(2)*	*(2)*	*(14)*
Carter, Anthony	1.17	2.00	1.83	1.17	1.17	1.67	9
Edwards, Kevin	0.83	0.00	2.00	0.00	0.00	1.83	4.67
Ingram, Alicia	1.00	0.00	0.00	1.00	0.67	1.83	4.5
Irving, Kaitlyn	1.00	0.00	0.00	1.00	0.50	0.33	2.833
Knight, Kyle	1.00	0.00	0.00	0.00	0.00	0.67	1.67
Mercer, Diana	1.00	0.50	0.83	0.50	0.50	1.67	5
Nichols, Keith	0.83	0.50	0.00	1.00	1.00	1.67	5
Norris, Faith	1.17	2.50	2.50	1.67	1.50	2.00	11.33
Thompson, Laura	1.00	0.50	0.00	1.17	0.67	1.00	4.33
Yinger, Brittany	0.83	1.17	1.00	0.67	0.67	0.67	5
MEAN	**0.98**	**0.72**	**0.82**	**0.82**	**0.67**	**1.33**	**5.33**

The average of an individual teacher's six lesson plans provided a numerical snapshot of that teacher's attention to students' thinking in his or her planning during the first semester of the field experience. For teachers who seemed to be fairly consistent in their planning, particularly in their attention to students' thinking when planning, the average of their six lesson plans provided a fairly accurate representation of their planning. However, a few teachers had lesson plans with extremely different levels of attention to students' thinking. For example, Faith Norris had a lesson plan with a score of 14 points as well as a lesson plan with a score of 2 points. Table 4.5 shows the total score each teacher received for each of the six lesson plans, as well as the average and range for the six lesson plans.

Table 4.5 Individual Teacher's Lesson Plan Scores from Their Field Experience

Teacher (pseudonym)	LP1	LP2	LP3	LP4	LP5	LP6	AVG	Range
Carter, Anthony	8	12	10	9	8	7	9	8-12
Edwards, Kevin	6	5	6	5	2	4	4.67	2-6
Ingram, Alicia	6	5	4	4	3	5	4.5	3-6
Irving, Kaitlyn	2	2	3	3	3	4	2.833	2-4
Knight, Kyle	2	2	1	1	1	3	1.67	1-3
Mercer, Diana	3	13	3	3	4	4	5	3-13
Nichols, Keith	11	2	3	4	4	6	5	2-11
Norris, Faith	14	13	14	2	13	12	11.33	2-14
Thompson, Laura	5	3	5	7	3	3	4.33	3-7
Yinger, Brittany	6	4	0	8	10	2	5	0-10

For the four teachers that were highlighted in Table 4.5, the difference between their highest and lowest lesson plan scores was 9 or more points. By contrast, the difference between the highest and lowest lesson plan scores was 4 points or less for the other six teachers. Therefore, it appeared that some teachers were inconsistent in their attention to students' thinking across their written lesson plans. This raised the question of why these teachers might display such variation in their attention to students' thinking. An analysis of Faith Norris's lesson plans provided some insight. Faith Norris's LP4, which scored two points, involved a task that would be considered low-level in its cognitive demands of students. Faith's other five lesson plans (in which she scored between 12 and 14 points) all used tasks that were considered high-level in their cognitive demands. Further investigation was undertaken to determine if the cognitive demand level of the task used in the lessons was in fact related to teachers' attention to students' thinking in the corresponding lesson plan. The following section describes the analysis of the relationship between teachers' attention to students' thinking when planning and the cognitive demand level of the task used in the lesson.

4.2.1 Cognitive Demands of the Tasks

All of the teachers turned in a complete set of mathematical tasks associated with their written lesson plans. Each of these 60 tasks was classified as high-level or low-level according to the Task Analysis Guide (see Appendix F) (i.e., "doing mathematics" and "procedures with connections" tasks would be classified as high-level; "procedures without connections" and "memorization" tasks would be classified as low-level). As a result, 65% (39/60) of the tasks were classified as high-level and 35% (21/60) were considered to have cognitive demands that were low-level. Furthermore, eight of the ten teachers submitted lessons in which at least 4/6 of their tasks were classified as high-level (KK and DM were the two exceptions). There were three teachers who submitted lessons where all six tasks were categorized at the same level. Anthony Carter and Kaitlyn Irving both turned in lessons in which all six of the tasks were classified as high-level and Kyle Knight turned in six tasks that were all classified as low-level.

Each teacher's actual lesson plan scores and mean score disaggregated by the cognitive demand level of the task used in the lesson are shown in Table 4.6. The six teachers highlighted in the table had average lesson plan scores that were higher for their lesson plans using high-level tasks (AvgLP-HL) compared to their lessons using low-level tasks (AvgLP-LL). For the three teachers in bold print (DM, KN, and FN), their average lesson plan score for lessons using high-level tasks was at least twice that of their lesson plans that used low-level tasks. Therefore, for 60% of the teachers there is evidence that they were more likely to attend to students' thinking when planning a lesson that used a high-level task.

Table 4.6 Individual Teacher's Scores and Means for Lesson Plans with High-Level and Low-Level Tasks

Teacher (pseudonym)	Lesson Plans that used Low-Level Tasks		Lesson Plans that used High-Level Tasks	
	Actual LP Scores	AvgLP-LL	Actual LP Scores	AvgLP-HL
Carter, Anthony	None	N/A	8,12,10,9,8,7	9
Edwards, Kevin	6,5	5.5	6,5,2,4	4.25
Ingram, Alicia	3,4	3.5	6,5,4,5	5
Irving, Kaitlyn	None	N/A	2,2,3,3,3,4	2.833
Knight, Kyle	2,2,1,1,1,3	1.67	None	N/A
Mercer, Diana	3,3,4,4	3.5	3,13	8
Nichols, Keith	2,4	3	11,3,4,6	6
Norris, Faith	2	2	14,13,14,13,12	13.2
Thompson, Laura	3,3	3	5,5,7,3	5
Yinger, Brittany	0,8	4	6,4,10,2	5.5

Most of the teachers who varied in their attention to students' thinking based on the cognitive demands of the mathematical task were very aware that this was a factor influencing their planning and talked about this in their final interview. For example, five of these six teachers (LT being the exception) talked about writing two different types of lesson plans: longer, more detailed plans for "discovery" or "exploration" lessons and shorter plans for "lecture" or "directive" lessons. This is shown in the following excerpt from the final interview in December with Brittany Yinger:

Interviewer: Okay. For the second part of the interview I'd like to ask some questions that relate to your lesson planning in general and I'd like you to talk about the things that influence your planning. So I'll start by asking, "How do you decide what to include or not include in a lesson plan?".

B.Y.: I guess it depends on if I'm doing a more exploration type of day or a...or more like a lecture day.

Interviewer: Okay. Well maybe if you break that down and talk about each of those.

B.Y.: I think it just depends on the nature, of a lesson because, I mean, by this point, the kids know that there'll probably be like two or three days of actual exploration and then two or three days of time tying big ideas together, where they're just kinda sitting down.

Interviewer: Right.

B.Y.: Um...starting with the lecture, just making sure 45 minutes of engagement as far as there's not, I'm not gonna give them ten minutes to start their homework or, so there's always that opportunity for them to ask questions and the more questions and more practice.

Interviewer: Right.

B.Y.: Um... what was the question?

Interviewer: How do you decide what to include in it?

B.Y.: Um...if it's a lecture, (chuckle), I don't know, I just, I look at, I guess I look through the text and what, what their objectives are and what they feel are the main points. And then, obviously those are going to be included and then it's more of what more can I do from this two pages in the book that they gave me.

Interviewer: Right.

B.Y.: Uh, there's that and then also, again, misconceptions of where are the kids probably gonna go wrong. Misconceptions are probably the biggest one that I focus on. And then it's the time factor for me with just the lectures. How am I factoring this time? That's both the considerations.

Interviewer: Right.

B.Y.: As far as the exploration ones go um, again, keeping all the kids engaged, but as far as how I do decide what goes in it, um, I think it, it depends on the type of task that I'm going to be doing.

Interviewer: Okay.

B.Y.: If there's one that there's gonna be a lot of possible student work, I need to figure out how many possibilities there are going to be, which ones do I want to focus in on and how can I tie these together.

Interviewer: Okay.

B.Y.: If it's gonna be a task where I think that there's probably only gonna be one or two main ideas, I need to decide where do I want the larger discussion to go. And that also goes with if it's a, I won't say easier but if it's one that can be done quicker.

Interviewer: Mm..hmm.

B.Y.: by the students, again, what I'm gonna talk about for twenty minutes, fifteen to twenty minutes, deciding that how, where do I want it to go.

This excerpt showed how Brittany thinks about different things when she's planning a lesson for a "lecture day" versus an "exploration day." She did indicate that for a "lecture" she attended to students' thinking in the form of Anticipating Students' Incorrect Thinking. However, for the "exploration" lessons, she also attended to students' thinking by Anticipating Students' Correct

Thinking, and Orchestrating a Discussion that Builds on Students' Thinking and Makes the Mathematics Salient. In this excerpt, Brittany identified that there was a distinction between the type of lesson ("lecture" and "exploration") and the things she needed to consider when planning the lesson that was similar to views expressed by Alicia Ingram, Diana Mercer, Keith Nichols, and Faith Norris. While it was important to identify that individual teacher's attention to students' thinking was influenced by the level of cognitive demand of the task, it was also important to explore the relationship between cognitive demands of a task and each of the elements of attention to students' thinking in planning a lesson across the entire group of teachers.

Table 4.7 provides a comparison of means between all the lesson plans that used a low-level task and the lesson plans that used high-level tasks for each of the lesson planning elements. The lesson planning elements in bold print were found to have differences that were significant as a result of t-Tests for two samples assuming equal variance with $p < 0.05$ [two-tailed].

Table 4.7 Comparison of Means for Lesson Plans with High-Level and Low-Level Tasks

Lesson Planning Element	Mean for LP with Low-Level Task (n=21)	Mean for LP with High-Level Task (n=39)
Goal	0.857	1.051
Anticipating Students' Correct Thinking	0.190	1
Anticipating Students' Incorrect Thinking	0.476	1
Questions to Assess and Advance Students' Thinking	0.286	1.154
Discussion Building on Students' Thinking	0.095	0.974
Discussion Making the Mathematics Salient	1.143	1.385
Total Lesson Planning Score	3.048	6.564

There was a significant relationship between the cognitive demands of the task and the overall score for teachers' attention to students' thinking in the lesson plan for that task. Results of a t-

Test for two samples assuming equal variance indicated that the difference in means between total lesson plan scores for lessons using low-level versus high-level tasks was significant ($p < 0.001$ [two-tailed]). A higher level of attention to students' mathematical thinking, as evident in teachers' written plan, was related to the use of tasks with high cognitive demands. Furthermore, the relationship between the cognitive demand of the task and the six dimensions of attention to students' thinking proved to be significant for Anticipating Students' Correct Thinking ($p < 0.010$ [two-tailed]), Identifying Questions to Assess and Advance Students' Thinking ($p < 0.001$ [two-tailed]), and Orchestrating a Discussion that Builds on Students' Thinking ($p < 0.001$ [two-tailed]). It is important to note that while there was a significant relationship between the cognitive demand of the task and the extent to which a teacher attended to students' thinking in the written lesson plan, it is not a causal relationship (i.e., using a high-level task does not cause a teacher to attend to students' thinking). In fact one teacher, Kaitlyn Irving, used high-level tasks in all six of the lesson plans she submitted yet her attention to student thinking was consistently very limited (e.g., overall mean score was 2.83, the second lowest of all ten teachers). However, for many of the teachers, the cognitive demand of the task did play a role in their attention to students' thinking. This relationship is explored further when discussing the results of teachers' lesson plans with respect to each of the elements of attention to students' thinking in the following section.

4.2.2 Teachers' Attention to Each of the Elements of Lesson Planning

Figure 4.1displays the scores for the sixty lesson plans disaggregated by the six elements of attention to students' thinking that were the focus of this study. This is followed by descriptions of the results for each of the six elements of attention to students' thinking.

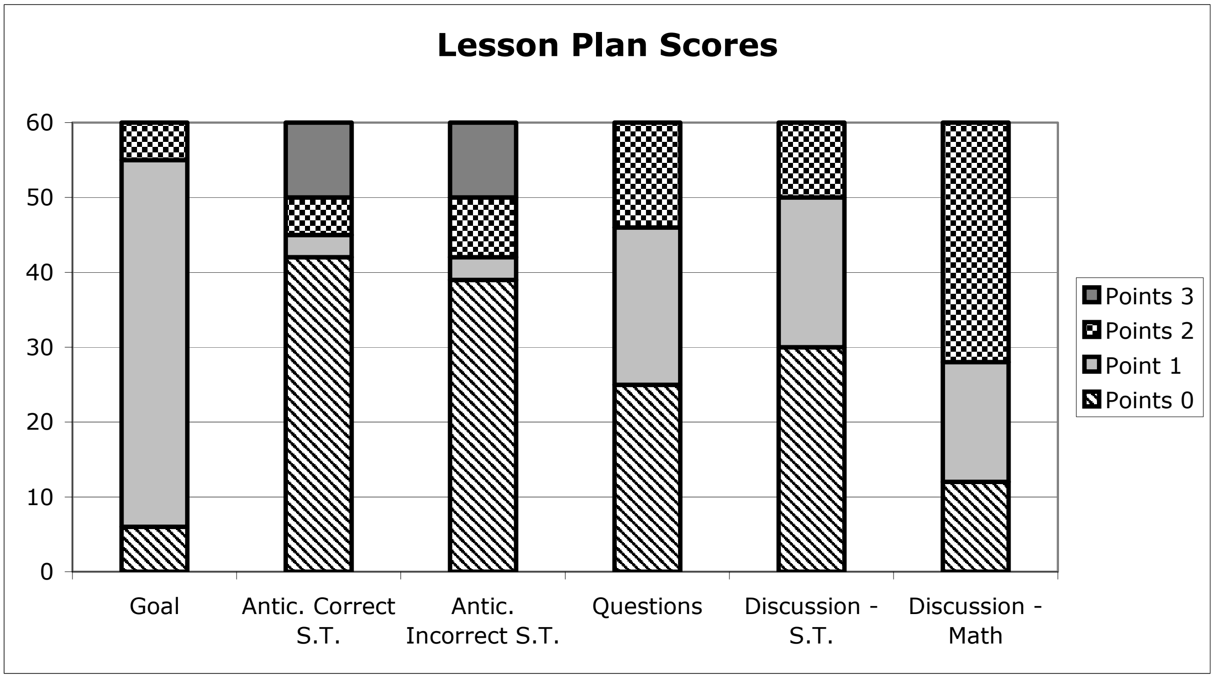


Figure 4.1 Lesson Plan Scores Disaggregated by Planning Element

Mathematical Goals for the Lesson. For the most part, teachers included a goal for the lesson in their written lesson plans and 90% (54/60, as shown in column 1 of Figure 4.1) of the lesson plans reflected this. There were two teachers (KE and FN) who submitted one lesson plan each that did not have any written goal and two other teachers (KN and BY) who submitted two lesson plans each without a written goal. The goals that teachers wrote on their lesson plans were mainly focused on procedural aspects of students' work during the lesson (e.g., "students will find an equation for the line") or were vague in describing the conceptual understanding students were to achieve from the lesson (e.g., "students will understand the concept of slope and y-intercept"). This was reflected in 82% (49/60, as shown in column 1 of Figure 4.1) of the lesson plans receiving a score of one point for the goal. The majority of the teachers (60%, as shown in column 1 of Figure 4.1) never wrote a goal for their lesson plan that earned a score of two points. Only five lesson plans (8%, as shown in column 1 of Figure 4.1) included goals that were specific in describing the understanding that the teacher wanted students to come to by the end of the lesson. An example of this higher level of specificity can be seen in the following excerpt from a written lesson plan for a Trigonometry lesson on amplitude and period of the sine function:

> Goals: 1) Students will notice changes in the sine graph when the A in Asin(t) and the b in sin(bt) change. 2) Students will make a generalization about how the magnitude of A in Asin(t) affects the amplitude of a sine function. Students will be able to graph f(t) = Asin(t). 3) Students will make a generalization about how the numerical value and sign of the b (the coefficient of t in a sine function) affects the period of a sine function. Students will be able to graph f(t) = sin(bt). [FN, Nov.LP1].

There were four teachers (AC, KN, FN, and BY) who wrote at least one lesson plan that had a goal statement that scored two points (one teacher (FN) submitted two such lesson plans). However, two of the teachers (KN and BY) who submitted a lesson with a very well-written, explicit goal were also the two teachers who submitted two lesson plans each with no goal. This apparent inconsistency in writing lesson goals was directly related to the cognitive demands of the task used in the lesson. In other words, for these two teachers, when they used a low level task, they did not write any mathematical goal for the lesson. By contrast, they were able to write a lesson goal in which they specifically described the conceptual understandings students were to develop during the lesson, when the lesson involved a high-level task. In fact, for the five lesson plans that scored two points for the lesson goal, all were associated with lesson plans that used a high-level task.

Anticipating Students' Correct and Incorrect Thinking. Teachers' attention to Anticipating Students' Correct and Incorrect Thinking about the mathematical task as evident in their written lesson plans was very limited. For example, as shown in columns two and three of Figure 4.1, 65% of the lesson plans displayed no evidence that the teacher had anticipated the ways in which students might think about the mathematics in the lesson (correctly or incorrectly). By contrast, 17% of the lesson plans showed many correct and incorrect ways students might think about the task in the lesson. 60% of the teachers were very consistent in whether or not they had evidence of anticipating students' thinking in their written plans. For example, three teachers never provided any evidence of Anticipating Students' Correct or Incorrect Thinking in their written lesson plans. While three other teachers consistently (in at least 5/6 of their lesson plans) scored two or three points for the elements of Anticipating Students' Correct or Incorrect Thinking. On a positive note, six of the ten teachers submitted at least one lesson plan in which they demonstrated an attempt to think about the many ways in which students would correctly or incorrectly think about the mathematics in the lesson. There also appeared to be a relationship between thinking about students' correct versus incorrect thinking. The majority of the time (in 78% of all lesson plans), teachers paid equal levels of attention to students' correct and incorrect thinking. In other words, for an individual lesson plan the score earned for Anticipating Students' Correct Thinking was the same as the score earned for Anticipating Students' Incorrect Thinking. However, there were some interesting exceptions to this finding. For example, one teacher (KE) never provided evidence of Anticipating Students' Correct Thinking (he often provided an answer key that had one correct answer for each problem, even when there were other possible ways of doing the problem), but always attended to the possible misconceptions and troubles students might have during the lesson.

As indicated earlier, there were significant differences in the level of attention to Anticipating Students' Correct Thinking between lessons using low and high level tasks. For example, for 90% (19/21) of the lessons that used a low-level task teachers provided no evidence of Anticipating Students' Correct Thinking. By contrast, this was true for only 59% (23/39) of the lessons that used a high-level task. It is also interesting to note that all (5/5) of the lesson plans that provided specific ways in which students would think about the problem (i.e., scored two points) and 90% (9/10) of the lesson plans that showed evidence of thinking about the many ways students' may solve the problem (i.e., scored three points) were associated with lessons that used high-level tasks. Thus, teachers were more likely to show evidence of Anticipating Students' Correct Thinking in their written lesson plans when they used a high-level task in the lesson.

Questions that Assess and Advance Students' Thinking. When it comes to teachers identifying specific questions to ask students that will assess and advance students' thinking as they work on the task, teachers fell into one of the following three categories: 1) teachers who did not identify specific questions to ask; 2) teachers who usually provided a list of specific questions to ask, but did not identify circumstances based on student's mathematical thinking under which the specific question should be asked; and 3) teachers who attended to this element depending on the cognitive demands of the task they were using in the lesson.

There were two teachers (KE and KK) who never identified specific questions to ask as students worked on the task (category 1). For both of these teachers, there was no evidence that the cognitive demands of the task affected their attention to this element of planning. Kevin Edwards used four high-level tasks and two low-level tasks in his lesson plans, yet did not identify questions to ask students as they worked on the task in any of his written plans. Kyle Knight's lesson plans all used low-level tasks. Therefore there was no data available to determine whether or not he would have identified questions to ask students as they worked on high-level tasks. For both of these teachers, their lack of attention to this element of lesson planning may have had more to do with their style of instruction than their ability to attend to students' thinking when planning. In ten of the 12 lesson plans submitted by these two teachers, there was never any explicit mention of providing time for students to work on a mathematical task. The entire lesson was designed as a “whole class discussion”. For Kyle Knight, all his lesson plans consisted of going over the correct answers to a textbook activity and it was not clear in the written plans when or if students took time to work through the mathematics themselves. Kevin Edwards’ lesson plans indicated a series of example problems and specific questions to ask the class as they all “worked through the problems together”.

Three teachers consistently (in at least 5/6 of their individual lesson plans) provided a list of specific questions to ask students as they worked on a mathematical task (category 2). Two of these teachers, Kaitlyn Irving and Anthony Carter provided fairly extensive lists of questions (e.g., ten or more questions). For both of these teachers, all of their lesson plans involved a high-level task. The third teacher, Alicia Ingram, provided a limited list of questions (e.g., usually 2-4 questions) in her lesson plan, regardless of the level of the task. While one may be able to infer from the question when it would be appropriate to ask a student, none of these teachers provided written evidence of linking their specific questions to students’ mathematical thinking. That is, what response from a student would result in asking a specific question.

Five of the ten teachers (DM, KN, FN, LT, and BY) had inconsistencies in scores they received for this element of lesson planning, individually ranging from 0-2 points (category 3). Upon further analyses, it was found that their score was consistently (in at least 5/6 of their lesson plans for four of the teachers and in 4/6 lesson plans for LT) related to the cognitive demands of the task they used in the lesson. In other words, for these five teachers, a higher-level of specificity with respect to questions to ask students as they worked was evident in teachers’ written plans when a high-level task was used in the lesson. For example, when teachers used a task that was classified as low-level (11 lesson plans), 91% (10/11) of their lesson plans did not provide any questions to ask students as they worked (a score of 0 points).

By contrast, for 84% (16/19) of the high-level tasks these five teachers used, their lesson plans provided specific questions to ask students. In addition, for 63% (12/19) of the lessons that used high-level tasks these five teachers provided circumstances based on students' mathematical thinking under which it was appropriate to ask the specific question. Thus, for half of the teachers, it was evident that there was a relationship between Identifying Specific Questions to Ask Students as they worked and the cognitive demands of the task used in the lesson. In other words, half of the teachers were more likely to identify specific questions to assess and advance students' thinking when they used a high-level task.

Orchestrating a Discussion that Builds on Students' Thinking. When planning a whole class discussion, half of the lesson plans teachers submitted explicitly described ways in which the discussion would build on students' mathematical thinking (i.e., scoring 1 or 2 points, as shown in column 5 of Figure 4.1). In 33% of the lesson plans teachers either selected and/or sequenced students' solutions to be discussed but did not provide any specific questions to ask the class related to the specific student work or they identified specific questions to ask the class but were unclear about for which student solution the question was appropriate (i.e., earned a score of 1 point). Furthermore, 17% (10/60) of the lesson plans were explicit in identifying specific questions that highlight the mathematics in a specific student solution (i.e., scored 2 points).

When analyzing the lesson plans by teacher, some interesting results were found. There were two teachers who never attempted to build on students' thinking within a whole class discussion. These were the same two teachers (Kevin Edwards and Kyle Knight) who never included questions to ask students when they worked on the task. As described in the previous section, these teachers' lessons were focused on a style of instruction that did not provide students with time to solve mathematical problems. Therefore it is reasonable that the lesson plan would not include ways in which to build a discussion around students' work. By contrast, there was one teacher (Faith Norris) who 66% of the time planned her lesson with exceptional detail in selecting students' solutions, deciding in what order she would have students present their work, and identifying specific questions to ask each presenter and the class, related to a specific student solution. However, Faith was not alone in her ability to produce such a lesson plan. In fact 60% of the teachers produced at least one lesson plan that had this same level of detail in attending to students' thinking. Furthermore, 40% of the teachers made a relatively consistent attempt to build on students' thinking in the whole-class discussion they had planned (as evident by scoring at least one point on at least 4/6 of their individual lesson plans).

As identified earlier, there was also a significant relationship between teachers' attention to Orchestrating a Discussion that Builds on Students' Thinking in their written lesson plans and the cognitive demands of the task used in the lesson. This can be seen in the fact that 90% (19/21) of the lessons that used a task classified as low-level made no attempt to include students' thinking in the lesson plan (i.e., scored zero points for this element). In addition, for those lesson plans that provided specificity in the questions to ask the class related to specific student solutions (i.e., scoring two points for this element), 100% (10/10) were associated with the use of a high-level task. While there was a relationship between the cognitive demand of the task and the lesson plans' attention to students' thinking in the whole-class discussion, this was not a causal relationship. For example, 28% (11/39) of lessons that used a high-level task also made no attempt to include students' thinking in the lesson plan for the whole-class discussion (i.e., scored zero points for this element). However, it is clear that teachers were more likely to plan a lesson in which the discussion built on students' thinking when they used a high-level task.

Additional analysis indicated that there was a relationship between teachers' attention to Orchestrating a Discussion that Builds on Students' Thinking and teachers' attention to Anticipating Students' Correct Thinking. For example, 80% (8/10) of the lesson plans that received the maximum score of two points for orchestrating a discussion that builds on students' thinking also paid significant attention to Anticipating Students' Correct Thinking (i.e., scoring two or three points for that element). In addition, 89% (16/18) of the lesson plans that showed no evidence of building on students' thinking in the whole-class discussion also had no evidence of anticipating the correct ways in which students could think about the task. In other words, in order to plan a lesson in which the teacher specifies questions to ask the class and relates those questions to specific student solutions, the teacher may first need to anticipate the possible student solutions that could be produced during the lesson.

<u>Orchestrating a Discussion that Makes the Mathematics Salient</u>. Most teachers focused their attention on planning a discussion that could make the mathematics salient for the students. For example, eight of the ten teachers provided a series of specific questions to ask the class that highlighted key mathematical concepts and/or made connections between mathematical representations or concepts (i.e., scored the maximum two points for this element). These eight teachers achieved this level of specificity in the majority of their lesson plan (at least 4/6 lesson plans for each teacher). High levels of attention to building the mathematics of the lesson during a whole-class discussion was not surprising, since, as described earlier, these teachers were doing this in their pre- and post- Teaching Lab lesson plans (as evident in the PPint1 and PPint2 data sources described in Section 4.1)

4.2.3 Written Lesson Plans Versus Verbal Descriptions

The analysis presented earlier was based on six written lesson plans that the teachers chose to submit for the study from October to December. While there were many things that could be learned from analyzing teachers' written lesson plans, there was also a danger in relying solely on written lesson plans as a data source for describing teachers' planning practices (see Schoenfeld (1998) for a description of differences between written lesson plans and teachers' lesson images which contain detailed information not usually written down). In other words, as research has shown, teachers have a tendency to think about many things when planning a lesson that they do not ultimately put into a written lesson plan. Therefore, it was possible that the written lesson plans analyzed for this study actually under-represent teachers' attention to students' thinking in their planning. For the final interview conducted in early December, each teacher brought a written lesson plan for a lesson that they had not yet taught as a focus lesson to discuss during the interview (this written lesson plan also served as one of the written lesson plans they turned in for the month of December). The interview occurred prior to the teacher teaching the lesson so that the teacher could verbalize their planning process (rather than report what actually happened during the implementation of the lesson plan). Through this interview, aspects of the written plans that were unclear could be clarified and aspects of the teachers' planning that were not written down could be verbally described. Each written lesson plan and transcript of the interview about the lesson plan was coded using the same scoring rubric that was used for all lesson plan data (see Appendix E).

Table 4.8 and Table 4.9 provide the results of individual teacher's scores for their written lesson plan and their verbal description of their planning for that lesson respectively.

Table 4.8 Individual Teacher's Scores on Written Lesson Plan used in Final Interview

Teacher (pseudonym)	Goal	Anticipate S.T.		Questions	Discussion		**TOTAL**
		Correct	Incorrect		ST	Math	
(Max. Poss.)	(2)	(3)	(3)	(2)	(2)	(2)	(14)
Carter, Anthony	1	2	2	1	1	1	8
Edwards, Kevin	1	0	1	2	0	0	4
Ingram, Alicia	2	0	0	1	1	1	5
Irving, Kaitlyn	1	0	0	1	0	1	3
Knight, Kyle	1	0	0	0	0	2	3
Mercer, Diana	1	0	1	0	0	2	4
Nichols, Keith	0	0	0	2	1	1	4
Norris, Faith	1	3	3	2	2	2	13
Thompson, Laura	1	0	0	1	0	1	3
Yinger, Brittany	1	0	1	0	1	1	4

Table 4.9 Individual Teacher's Score on Verbal Description of LP in Final Interview

Teacher (pseudonym)	Goal	Anticipate S.T.		Questions	Discussion		**TOTAL**
		Correct	Incorrect		ST	Math	
(Max. Poss.)	(2)	(3)	(3)	(2)	(2)	(2)	(14)
Carter, Anthony	1	0	2	1	1	2	7
Edwards, Kevin	1	0	1	0	0	0	2
Ingram, Alicia	1	1	1	1	1	1	6
Irving, Kaitlyn	0	0	1	1	0	2	4
Knight, Kyle	1	0	1	0	0	2	4
Mercer, Diana	1	0	2	0	0	2	5
Nichols, Keith	0	3	3	2	2	2	12
Norris, Faith	1	3	3	2	2	2	13
Thompson, Laura	1	2	2	2	1	1	9
Yinger, Brittany	1	2	2	2	1	2	10

Comparisons between the total scores for the written lesson plan and the verbal description of the lesson plan indicated that seven of the ten teachers had higher scores for their verbal description of the lesson plan (for four of these seven teachers the difference was only 1 point, and for the other three teachers the difference was 6 points or more). However, when looking at the teachers as a group, differences between teachers' total scores on the written and verbal lesson plans were not significant (Wilcoxon Signed-Rank, $p < 0.13$). Similarly, there were no significant differences between teachers' scores on the written and verbal lesson plans with respect to five of the six elements of attention to students' thinking. Anticipating Students' Incorrect Thinking was the only element where significant differences were found for this group of teachers (Wilcoxon Signed-Rank, $p < 0.016$). Seven teachers showed evidence of thinking more about students' possible misconceptions than what they provided in their written lesson plans. Furthermore, during the final interview teachers were explicitly asked whether or not there were things they thought about when planning the lesson that were not present in their written plan and that five of the ten teachers claimed that yes, there were things they thought about but did not write down. For these five teachers, the most common thing that they cited as being left out of their lesson plans was their thinking about students' misconceptions.

While as a group, there was no evidence to support the notion that teachers' written lesson plans showed significantly less attention to students' thinking than what the teachers reported actually thinking about when planning there lessons, there were individual teachers for which this was true. For three teachers in particular (KN, LT and BY) there was a marked difference between their written lesson plans and their verbal descriptions of their lesson planning. For example, Keith's total score for his written plan was only 4 points compared to his verbal description of his plan, which yielded a score of 12 points. Laura and Brittany demonstrated similar results; scoring 3 and 4 points respectively on their written lesson plans and 9 and 10 point respectively on their verbal descriptions. All three teachers provided evidence that they had thought more (and in more detail) about Students' Correct and Incorrect Thinking than what was reflected in the written lesson plans. They often went from having zero evidence of thinking about students' correct or incorrect thinking in their written lesson plans to providing evidence of thinking about very specific ways in which students would think about the problem in their verbal descriptions of their planning for the lesson. Additionally, in Keith's case, his interview indicated that he had attempted to anticipate many ways his students may solve the problem, both incorrectly and correctly. It is worth noting that Keith and Laura were the only teachers who demonstrated that they had thought more about how to Orchestrate a Whole-Class Discussion that Builds on Students' Thinking than what was evident in their written plan.

Laura and Brittany were the only teachers who provided evidence of thinking more about specific Questions to Ask Students to Assess and Advance their Thinking as they work, and in particular, the teachers demonstrated that they had thought about circumstances that were based on students' mathematical thinking for deciding when the specific question would be appropriate to ask a student. There were three other teachers (AC, AI, and KI) who had listed specific questions in their written lesson plans. These three teachers were also asked by the interviewer to say more about these questions and when they might ask them. The responses provided by these three teachers did not indicate that they had thought about students' mathematical thinking as circumstances under which to ask the questions. Instead, these teachers responded with non-mathematical circumstances (e.g. "if the students finish early").

For those teachers who made an effort to think about students' thinking as they planned a lesson and then chose to not include evidence of this thinking in their written plan, the following excerpt provides some insight into their planning process:

> I have an overall idea of what I want to do by like looking at the lesson, looking at the task. I don't have to like go through it inch by inch. I have like an overall view and I can just kind of fill in and it's to the point where I don't need to actually – when I make a really detailed lesson plan I don't go through and read every question off the lesson plan. The questions are there to make me think while I'm planning it so I can give a good lesson when I'm teaching it. So I feel like I'm more prepared to just kind of plan ahead of time and just do the lesson without relying on, like without treating the lesson plan like a script [KN FinalInt]

In this quote, Keith Nichols also provides some insight into the ways in which his lesson planning has changed over time. The next section presents the results of investigating the extent to which teachers' attention to students' thinking may have changed over time.

4.3 The Extent to Which Teachers' Planning Changed Over Time

The results in this section pertain to Research Question #3:

> To what extent does attention to students' mathematical thinking, as evident in pre-service secondary teachers' lesson planning, change over time?

This section presents statistical comparisons between the various data sources in order to understand the extent to which teachers' attention to students' mathematical thinking may have changed over time. Table 4.10 summarizes the data that are compared in this section and the purpose for the comparison with respect to the time period being compared and the type of lesson planning involved in the comparison.

Table 4.10 Summary of Data Comparisons

Data to Compare	Time Period Compared	Type of Lesson Planning
PPint1 vs. PPint2 vs. PPint3 (Section 4.3.1)	Describe change from Pre-Teaching Lab (June) to Post-Teaching Lab (Aug.) to Near the End of First Semester of the Field Experience (Dec.)	Describe changes in planning over time within one type of planning (on demand)
PTR1 vs. PTR2 (Section 4.3.2)	Describe changes in planning from Post-Teaching Lab (Aug.) to Near the End of First Semester of the Field Experience (Dec.)	Describe changes in planning over time within one type of planning (university assignment)
PPint2 vs. PTR1 PPint3 vs. PTR2 (Section 4.3.3)	Describe differences between types of planning at two separate time points: Post-Teaching Lab and Near the End of the First Semester of the Field Experience	Describe differences between planning on demand and planning for a university assignment
PPint1 vs. AvgLP-LL, AvgLP-HL PPint2 vs. AvgLP-LL, AvgLP-HL PPint3 vs. AvgLP-LL, AvgLP-HL (Section 0)	Describe changes in planning on demand from Pre-Teaching Lab, Post-Teaching Lab, and End of First Semester to planning in practice During the Field Experience	Describe differences between planning on demand and planning in practice
PTR1 vs. AvgLP-LL, AvgLP-HL PTR2 vs. AvgLP-LL, AvgLP-HL (Section 4.3.5)	Describe changes in planning for a university assignment from End of Teaching Lab, and End of First Semester to planning in practice During the Field Experience	Describe differences between planning for a university assignment and planning in practice

4.3.1 Planning on Demand

Comparisons between the Pledge Plans task lesson-planning interviews conducted prior to (PPint1) and immediately after (PPint2) teachers' participation in the Teaching Lab and near the end of the first semester of their field experience (PPint3) served as an indicator of the extent to which teachers' ability to attend to students' thinking when planning a lesson on demand changed over time.

4.3.1.1 Beginning versus End of the Teaching Lab

Comparisons between PPint1 and PPint2 served as an indicator of the extent to which teachers' attention to students' thinking changed as a result of their participation in the Teaching Lab. When comparing PPint1 and PPint2, results of the Wilcoxon Signed-Rank tests for non-parametric, paired data indicated that teachers made significant gains in their attention to students' mathematical thinking in their lesson planning. Table 4.11 shows the p-value results of the Wilcoxon tests for all the categories and those that were significant ($p < 0.05$) are shown in bold print.

Table 4.11 Results of Wilcoxon Signed-Rank tests Comparing PPint1 and PPint2

Lesson Plan Element	p-value
Goal	**$P < 0.016$**
Anticipating Students' Correct Thinking	**$P < 0.031$**
Anticipating Students' Incorrect Thinking	$P < 0.875$
Questions to Assess and Advance Students' Thinking	**$P < 0.031$**
Discussion Building on Students' Thinking	**$P < 0.016$**
Discussion Making the Mathematics Salient	$P < 0.438$
Total Lesson Planning Score	**$P < 0.004$**

For the lesson-planning element of Anticipating Students' Incorrect Thinking, teachers remained at a fairly low score both prior to the course and immediately after (as evidenced in the PPint1 and PPint2 data sources). Two teachers (KE and AI) actually lowered their scores (e.g., from 2 points to 1 and from 1 point to 0). One teacher (FN) did make a vast improvement, scoring 0 points prior to the course and then scoring the maximum of 3 points after the course. The remainder of the teachers all maintained their scores from PPint1 to PPint2 (six teachers with a score of 0 and one teacher who maintained a score of 1 point). Therefore teachers were still paying very little attention to the incorrect ways students' may think about the Pledge Plans task after participation in the Teaching Lab. The study does not provide insight into the reasons why teachers may have low scores for this element. This lack of attention to incorrect methods may not be surprising since the Teaching Lab did not provide teachers with practical experience or research on the ways in which students think about the mathematical ideas present in the Pledge Plans task. In addition, it is possible that teachers' low scores for this element were a result of 1) having a limited ability to anticipate students' incorrect thinking for this particular task, 2) having a limited ability to anticipate students' incorrect thinking in general, 3) choosing not to attend to this element because they forgot or deemed it unimportant, or 4) some other reason.

By contrast, teachers remained at a moderately high level in the category of Orchestrating a Discussion that Makes the Mathematics Salient (with means of 1.2 for PPint1 and 1.5 for PPint2). Four of the ten teachers had identical scores on the PPint1 and PPint2 (three of those receiving the maximum score of 2 points). Two teachers declined by 1 point, while four teachers improved by 1 point. After participation in the Teaching Lab, 60% of the teachers scored the maximum of 2 points, indicating that they were very specific in their lesson plans about specific questions to ask students that would make the mathematics salient for the students. It is not unexpected that these pre-service teachers would enter the teacher education program with an ability to ask questions focused on the mathematics of the lesson as they all had bachelor's degrees in mathematics (or its equivalent) and many had served as teaching assistants in their undergraduate mathematics departments.

4.3.1.2 End of the Teaching Lab versus End of the First Semester of Their Field Experience

Table 4.12 and Table 4.13 provide individual teacher's scores from both the second and third Pledge Plan interview data sources, respectively.

Table 4.12 Individual Teacher's Scores from PPint2

Teacher (pseudonym)	*Goal*	*Anticipate S.T.*		*Questions*	*Discussion*		***TOTAL***
		Correct	*Incorrect*		*ST*	*Math*	
(Max. Poss.)	*(2)*	*(3)*	*(3)*	*(2)*	*(2)*	*(2)*	*(14)*
Carter, Anthony	0	3	0	0	1	0	4
Edwards, Kevin	1	2	0	2	2	2	9
Ingram, Alicia	1	2	0	2	1	2	8
Irving, Kaitlyn	1	1	0	1	1	2	6
Knight, Kyle	1	2	1	0	1	1	6
Mercer, Diana	1	3	1	2	2	2	11
Nichols, Keith	1	2	0	2	2	2	9
Norris, Faith	1	3	3	2	2	1	12
Thompson, Laura	1	1	0	0	1	1	4
Yinger, Brittany	1	1	0	1	2	2	7

Table 4.13 Individual Teacher's Scores from PPint3

Teacher (pseudonym)	*Goal*	*Anticipate S.T.*		*Questions*	*Discussion*		***TOTAL***
		Correct	*Incorrect*		*ST*	*Math*	
(Max. Poss.)	*(2)*	*(3)*	*(3)*	*(2)*	*(2)*	*(2)*	*(14)*
Carter, Anthony	1	2	0	2	1	2	8
Edwards, Kevin	2	1	3	2	0	1	9
Ingram, Alicia	2	1	1	1	2	2	9
Irving, Kaitlyn	0	2	0	1	2	2	7
Knight, Kyle	1	1	0	1	1	0	4
Mercer, Diana	1	1	0	1	2	2	7
Nichols, Keith	1	2	0	2	2	1	8
Norris, Faith	1	1	3	2	2	1	10
Thompson, Laura	2	2	0	0	0	2	6
Yinger, Brittany	1	1	0	1	2	2	7

There were only slight differences between PPint2 and PPint3 and teachers did not consistently score higher for one data source than the other. Therefore it was not surprising that there were no significant differences between the two data sources. Wilcoxon Signed-Rank comparisons between PPint2 (immediately after the Teaching Lab in early August) and PPint3 (towards end of the first semester of teachers' field experience in early December) indicated that teachers did not make significant changes when planning a lesson on demand in any of the lesson-planning elements that were the focus of this study between completion of the Teaching Lab and four months later, at the end of the first semester of their field experience. While teachers did have opportunities to plan lessons as part of their coursework during the fall semester, their coursework did not focus solely on lesson planning, as that was the focus topic of the Teaching Lab. Therefore, it is understandable that teachers did not show significant growth in their attention to students' thinking over this time period.

4.3.1.3 Summary of Changes Over Time within Teachers' Planning on Demand

After participating in the Teaching Lab, teachers showed significant growth in their ability to attend to students' thinking when planning a lesson on demand. In addition, teachers were able to attend to students' mathematical thinking in a variety of ways, as evident in their scores for the six elements that were the focus of this study. Thus teachers were poised to begin their field experience in the fall semester (one month later) with the knowledge and ability to attend to students' mathematical thinking in their lesson planning practices. Furthermore, near the end of the first semester of their field experience (early December), there was evidence that teachers had maintained their abilities to attend to students' thinking when planning on demand.

4.3.2 Planning for a University Assignment

The lesson plans teachers produced as part of their "Plan, Teach, and Reflect" assignments for the Teaching Lab (PTR1) and the Methods 1 course in the fall (PTR2) represented teachers' ability to attend to students' thinking when explicitly asked to do so at two different time points in their teacher education program. Because teachers were directed to use the TTLP in their planning, were given explicit feedback from the instructor and were going to be graded on the extent to which they adequately responded to the questions on the TTLP, these lesson plans represent a teacher's best effort to plan a lesson that attends to students' thinking.

Table 4.14 and Table 4.15 provide individual teacher's scores from both the PTR1 and PTR2 data sources, respectively.

Table 4.14 Individual Teacher's Scores from PTR1

Teacher (pseudonym)	*Goal*	*Anticipate S.T.*		*Questions*	*Discussion*		***TOTAL***
		Correct	*Incorrect*		*ST*	*Math*	
(Max. Poss.)	*(2)*	*(3)*	*(3)*	*(2)*	*(2)*	*(2)*	*(14)*
Carter, Anthony	1	0	1	0	1	1	4
Edwards, Kevin	1	3	2	2	2	1	11
Ingram, Alicia	1	1	1	1	1	1	6
Irving, Kaitlyn	1	1	0	2	1	1	6
Knight, Kyle	1	3	3	1	1	1	10
Mercer, Diana	2	3	3	2	2	2	14
Nichols, Keith	2	3	0	2	2	1	10
Norris, Faith	2	3	3	2	2	2	14
Thompson, Laura	2	3	3	2	1	2	13
Yinger, Brittany	1	0	0	2	2	2	7

Table 4.15 Individual Teacher's Scores from PTR2

Teacher (pseudonym)	*Goal*	*Anticipate S.T.*		*Questions*	*Discussion*		***TOTAL***
		Correct	*Incorrect*		*ST*	*Math*	
(Max. Poss.)	*(2)*	*(3)*	*(3)*	*(2)*	*(2)*	*(2)*	*(14)*
Carter, Anthony	1	2	2	2	2	1	10
Edwards, Kevin	2	0	2	2	2	2	10
Ingram, Alicia	2	0	0	1	1	2	6
Irving, Kaitlyn	1	0	2	2	2	2	9
Knight, Kyle	1	3	3	2	0	2	11
Mercer, Diana	2	3	3	2	2	2	14
Nichols, Keith	2	1	1	2	1	2	9
Norris, Faith	2	3	3	2	2	2	14
Thompson, Laura	2	3	3	2	2	2	14
Yinger, Brittany	1	1	1	2	1	2	8

It is interesting that for some teachers their total score on the PTR2 was slightly higher than their total score on the PTR1 lesson plan. However, there were no significant differences between the two data sources. Results of the Wilcoxon Signed-Rank tests for non-parametric, paired data indicated no significant differences between the total lesson plan scores for the PTR1 and PTR2

($p < 0.22$). There were also no significant differences for each of the six elements of lesson planning when comparing PTR1 and PTR2 scores. Thus teachers' ability to attend to students' thinking when explicitly asked to do so in their planning did not significantly change between completion of the Teaching Lab and the end of the first semester of their field experience (four months later). As mentioned earlier, teachers did have opportunities to plan lessons as part of their coursework during the fall semester. However, their coursework did not focus on lesson planning, as that was the focus topic of the Teaching Lab. In addition, there was little room for significant growth as teachers performed very well on their first "Plan, Teach, Reflect" assignment (PTR1). Therefore, it is expected that teachers would not show significant growth in their attention to students' thinking over this time period. Teachers did remain capable of producing lesson plans with a very high level of attention to students' thinking, as evidenced in their PTR2 scores. For example, every teacher had a Mathematical Goal for their lesson, with 6/10 teachers writing goals that specifically identified the mathematical understandings students would come to with regard to a mathematical concept. 90% of the teachers were capable of Identifying Specific Questions to Ask Students while they worked and also described circumstances based on students' mathematical thinking under which to ask the specific question. 90% of the teachers planned a whole-class discussion in which they explicitly described ways in which the Discussion would Build on Students' Thinking. Furthermore, 6/10 teachers' lesson plans were explicit in identifying specific questions that highlight the mathematics in a specific student solution.

4.3.3 Planning on Demand versus Planning for a University Assignment

There was data to support a comparison of teachers' planning on demand and for a university assignment at two different time points in their teacher education program: 1) towards the end of and immediately after teachers' participation in the Teaching Lab and 2) towards the end of the first semester of the teachers' field experience. Comparisons were made between teachers' Pledge Plans interview lesson plans where teachers planned a lesson on demand and were not explicitly asked to attend to students' thinking in their planning and teachers' written lesson plans turned in as part of their "Plan, Teach, and Reflect" assignments in which teachers were explicitly asked to attend to students' thinking in their planning through use of the TTLP.

4.3.3.1 Comparison Between PPint2 and PTR1

Immediately after participating in the Teaching Lab, teachers were interviewed about planning a lesson around the Pledge Plans task (July 29 – August 2). These interviews were the PPint2 data source and represented teachers' planning on demand after participating in the Teaching Lab. During the final six class sessions of the Teaching Lab (July 12 – 28), teachers turned in their written lesson plans for their "Plan, Teach, Reflect" assignments (PTR1). Thus PTR1 served as the data source that indicated teachers' planning for a university assignment near the end of the Teaching Lab. Table 4.16 provides descriptive statistics on scores from both the PTR1 and PPint2 data sources.

Table 4.16 Descriptive Statistics on Scores from PTR1 and Pledge Plans Interview 2

	Highest Possible Score	Mean Score (n=10)		Range of Scores	
		PTR1	PPint2	PTR1	PPint2
Total Lesson Planning Score	14	9.5	7.6	4-14	4-12
Goal	2	1.4	0.9	1-2	0-1
Anticipating Students' Correct Thinking	3	2	2	0-3	1-3
Anticipating Students' Incorrect Thinking	3	1.6	0.5	0-3	0-3
Questions to Assess and Advance Students' Thinking	2	1.6	1.2	0-2	0-2
Discussion Building on Students' Thinking	2	1.5	1.5	1-2	1-2
Discussion Making the Mathematics Salient	2	1.4	1.5	1-2	0-2

While the mean scores on the PTR1 are often slightly higher than the mean scores on the PPint2 lesson plans, there were no significant differences between the two data sources. Results of the Wilcoxon Signed-Rank tests for non-parametric, paired data indicated no significant differences between the total lesson plan scores for the PTR1 and PPint2 ($p < 0.08$). There were also no significant differences for five of the six categories when comparing PTR1 and PPint2 scores. The one exception was Anticipating Students' Incorrect Thinking (Wilcoxon Signed-Rank, $p < 0.03$), with teachers paying significantly more attention to this element in their PTR1 lesson plan compared to their Pledge Plans lesson plan (PPint2).

4.3.3.2 Comparison Between PPint3 and PTR2

Towards the end of the first semester of their field experience, teachers were interviewed for a third time about planning a lesson around the Pledge Plans task (December 1 - 8). These interviews were the PPint3 data source and represent teachers' planning on demand towards the end of the first semester of their field experience. During the latter half of the Methods 1 course (November 15 – December 15), teachers turned in their written lesson plans for their "Plan, Teach, Reflect" assignments (PTR2). Thus PTR2 served as the data source that indicated teachers' planning for a university assignment near the end of the first semester of their field experience. Table 4.17 provides descriptive statistics on scores from both the PTR1 and PPint2 data sources.

Table 4.17 Descriptive Statistics on Scores from PTR2 and Pledge Plans Interview 3

	Highest Possible Score	Mean Score (n=10)		Range of Scores	
		PTR2	PPint3	PTR2	PPint3
Total Lesson Planning Score	14	10.5	7.5	6-14	4-10
Goal	2	1.6	1.2	1-2	0-2
Anticipating Students' Correct Thinking	3	1.6	1.4	0-3	1-2
Anticipating Students' Incorrect Thinking	3	2.0	0.7	0-3	0-3
Questions to Assess and Advance Students' Thinking	2	1.9	1.3	1-2	0-2
Discussion Building on Students' Thinking	2	1.5	1.4	0-2	0-2
Discussion Making the Mathematics Salient	2	1.9	1.5	1-2	0-2

Results of the Wilcoxon Signed-Rank tests for non-parametric, paired data indicated a significant difference between the total lesson plan scores for the PPint3 and PTR2 ($p < 0.027$). In general, teachers paid more attention to students' thinking when planning their university assignment (PTR2) than when planning on demand (PPint3). Similar to the comparison between PPint2 and PTR1, there were no significant differences for five of the six categories when comparing PPint3 and PTR2 scores. The one exception was Anticipating Students' Incorrect Thinking (Wilcoxon Signed-Rank, $p < 0.039$), with teachers consistently paying significantly more attention to this element in their PTR2 lesson plan compared to their Pledge Plans lesson plan (PPint3). While 9/10 teachers scored a higher total lesson plan score on their PTR2 than on their PPint3, the ways in which they showed their increased attention varied across five of the six specific elements of attention to students' thinking. For example, with respect to identifying a Mathematical Goal for the lesson, 4/10 teachers showed a 1-point increase in their level of specificity when planning their university assignment lesson. While the remaining six teachers produced mathematical goals at the exact same level of specificity for both their PPint3 and PTR2 lesson plans. For the element of Identifying Questions to Assess and Advance Students' Thinking as they work, 9/10 teachers scored the maximum 2 points for their PTR2 lesson plan, compared to 4/10 teachers doing so for their PPint3 lesson plan. Similarly, 9/10 teachers scored the maximum 2 points for Orchestrating a Discussion that Makes the Mathematics Salient for their PTR2 lesson plan, compared to 6/10 teachers doing so for their PPint3 lesson plan. One reasonable explanation for teachers demonstrating more attention to students' thinking when planning their university assignment (PTR2) than when planning on demand (PPint3) is that when planning their university assignment, the teachers: 1) had extensive time to plan the lesson; 2) received feedback from their instructor on aspects of the TTLP on which they needed to provide more information in their written plan; and 3) knew they were going to receive a grade on the assignment based on their inclusion of elements of the TTLP.

4.3.3.3 Summary of Planning on Demand versus Planning for a University Assignment

Teachers demonstrated a higher level of attention to students' thinking when planning a lesson for the Methods 1 (PTR2) university assignment that explicitly asks them to do so than when they planned a lesson on demand (PPint3). When analyzing both university assignments (PTR1 and PTR2) in comparison to the lessons planned on demand (PPint2 and PPint3), there was evidence that teachers paid significantly more attention to only one of the six specific elements of attention to students' thinking when planning for a university assignment: Anticipating Students' Incorrect Thinking. For example, 80% of the university assignment lesson plans showed evidence of the teacher making an attempt to think of some ways in which students may struggle with the mathematics in the lesson. By contrast, in 70% of the lesson plans teachers created on demand there was no evidence that they had thought about any of the incorrect ways students may think about the problem(s) in the lesson (i.e., scored 0 points).

4.3.4 Planning on Demand versus Planning in Practice

This section presents comparisons of teachers' planning in practice and their planning on demand, as represented by their planning during an interview around the Pledge Plans task that occurred at three different time points: at the beginning of the Teaching Lab (PPint1), immediately after the Teaching Lab (PPint2), and towards the end of the first semester of the teacher's field experience (PPint3). Teachers' lesson planning in practice was represented by six lesson plans that each teacher submitted from their field experience. As described earlier, the cognitive demands of the task played a significant role in the level of attention teachers paid to students' thinking in their planning. Therefore, analysis presented in this section shows separate comparisons to teachers' lesson planning for lesson that used low- and high-level tasks (AvgLP-LL and AvgLP-HL, respectively).

4.3.4.1 Beginning of the Teaching Lab versus In Practice

This section presents results of comparisons between PPint1 and AvgLP-LL or AvgLP-HL data sources. The results were indicative of the similarities and differences between teachers' attention to students' thinking when planning a lesson on demand at the beginning of the Teaching Lab (June) and teachers' planning in practice, as evident in their written lesson plans from their field experience (October – December). Separate comparisons were made with respect to teachers' lesson planning in practice based on the cognitive demands of the task used in the lesson.

Lessons that used tasks with low cognitive demands. Teachers' planning for lessons that used low-level tasks was not significantly different from their planning at the beginning of the Teaching Lab with respect to teachers' overall attention to students' thinking (Wilcoxon Matched-Pairs Signed-Ranks, $p < 0.156$). There were also no significant differences with respect to five of the six elements of attention to students' thinking between teachers' planning with low-level tasks and their planning at the beginning of the Teaching Lab. Orchestrating a Discussion that Builds on Students' Thinking was the one exception (Wilcoxon Matched-Pairs Signed-Ranks, $p < 0.016$). Teachers paid less attention to Orchestrating a Discussion that Builds on Students' Thinking, as evident in their written lesson plans created during their field experience, than they did in their initial Pledge Plans interview conducted at the beginning of their teacher education program.

Lessons that used tasks with high cognitive demands. In contrast to the written lesson plans teachers produced for low-level tasks, their lessons for high-level tasks were significantly different from their planning at the beginning of the Teaching Lab with respect to their overall attention to students' thinking (Wilcoxon Matched-Pairs Signed-Ranks, $p < 0.039$). In general, teachers paid more attention to students' thinking when planning a lesson that used a high-level task during their field experience than they did in their initial Pledge Plans interview. This was particularly true for two of the six elements of attention to students' thinking: having a specific Mathematical Goal (Wilcoxon Matched-Pairs Signed-Ranks, $p < 0.008$) and Identifying Questions to Assess and Advance Students' Thinking as they work (Wilcoxon Matched-Pairs

Signed-Ranks, $p < 0.016$). For example, in the initial Pledge Plans lesson-planning interview, only one teacher had a mathematical goal for the lesson. By contrast, 92% (36/39) of the lesson plans written around high-level tasks during teachers' field experience had a specific mathematical goal for the lesson. In addition, for the lesson plans teachers created for the Pledge Plans task during their interview conducted at the beginning of the Teaching Lab, 80% of the lesson plans did not identify any specific questions to ask students as they worked. By contrast, only 18% (7/39) of the lesson plans written during the teachers' field experience that used high-level tasks did not identify any specific questions to ask students as they worked. Furthermore, 33% (13/39) of the lesson plans that used high-level tasks scored the maximum 2 points. In other words, there was evidence that the teacher had identified specific questions to ask students as they worked and the teacher had provided specific circumstances based on students' mathematical thinking under which it would be appropriate to ask the specific questions.

4.3.4.2 Immediately After the Teaching Lab versus In Practice

This section presents results of comparisons between PPint2 and AvgLP-LL or AvgLP-HL data sources. The results were indicative of the similarities and differences between teachers' attention to students' thinking when planning a lesson on demand immediately after participating in the Teaching Lab (early August) and teachers' planning in practice, as evident in their written lesson plans from their field experience (October – December). Separate comparisons were made with respect to teachers' lesson planning in practice based on the cognitive demands of the task used in the lesson.

<u>Lessons that used tasks with low cognitive demands.</u> Teachers' planning for lessons that used low-level tasks was significantly different from their planning immediately after participation in the Teaching Lab with respect to their overall attention to students' thinking

(Wilcoxon Matched-Pairs Signed-Ranks, $p < 0.008$). In general, teachers paid more attention to students' thinking when planning a lesson on demand immediately after participation in the Teaching Lab than they did when planning a lesson that used a low-level task during their field experience, as evident in their written lesson plans. This was particularly true for three of the six elements of attention to students' thinking: Anticipating Students' Correct Thinking (Wilcoxon Matched-Pairs Signed-Ranks, $p < 0.016$), Identifying Questions to Assess and Advance Students' Thinking as they work (Wilcoxon Matched-Pairs Signed-Ranks, $p < 0.047$), and Orchestrating a Discussion that Builds on Students' Thinking (Wilcoxon Matched-Pairs Signed-Ranks, $p < 0.008$). For example, in planning a lesson on demand immediately after participation in the Teaching Lab, all the teachers' lesson plans showed evidence of identifying at least some of the ways students may correctly think about the problem(s) in the lesson. By contrast, only 10% (2/21) of the written plans teachers' wrote for lessons that used a low-level task showed any evidence that the teacher had thought about ways in which students may correctly think about the problem(s) in the lesson. The exact same results were found with respect to teachers' attention to Orchestrating a Discussion that Builds on Students' Thinking (100% attending to this element in their second Pledge Plans interview and only 10% of the written lesson plans for low-level tasks showing any evidence of attention to this element).

Lessons that used tasks with high cognitive demands. In contrast to the written lesson plans teachers produced for low-level tasks, their lessons for high-level tasks were not significantly different from their planning immediately after participating in the Teaching Lab with respect to teachers' overall attention to students' thinking (Wilcoxon Matched-Pairs Signed-Ranks, $p < 0.25$). There were also no significant differences with respect to four of the six elements of attention to students' thinking between teachers' planning with high-level tasks and their planning immediately after the Teaching Lab. Anticipating Students' Correct Thinking (Wilcoxon Matched-Pairs Signed-Ranks, $p < 0.016$) and Orchestrating a Discussion that Builds on Students' Thinking (Wilcoxon Matched-Pairs Signed-Ranks, $p < 0.03$) were the exceptions. Teachers paid less attention to Anticipating Students' Correct Thinking and Orchestrating a Discussion that Builds on Students' Thinking, as evident in their written lesson plans created during their field experience regardless of the cognitive demands of the task used in the lesson, than they did in their second Pledge Plans interview conducted immediately after their participation in the Teaching Lab.

4.3.4.3 End of First Semester of Field Experience versus In Practice

This section presents results of comparisons between PPint3 and AvgLP-LL or AvgLP-HL data sources. The results were indicative of the similarities and differences between teachers' attention to students' thinking when planning a lesson on demand near the end of the first semester of their field experience (early December) and teachers' planning in practice, as evident in their written lesson plans from their field experience (October – December). Separate comparisons were made with respect to teachers' lesson planning in practice based on the cognitive demands of the task used in the lesson.

<u>Lessons that used tasks with low cognitive demands.</u> Results from comparing teachers' scores on their third Pledge Plans interview to their written plans for lessons with low-level tasks were identical to those reported earlier for the second Pledge Plans interview. In other words, teachers' planning for lessons that used low-level tasks was significantly different from their planning for the Pledge Plans task near the end of the first semester of their field experience with respect to their overall attention to students' thinking (Wilcoxon Matched-Pairs Signed-Ranks, $p < 0.008$). In general, teachers paid more attention to students' thinking when planning a lesson

on demand near the end of the first semester of their field experience than they did when planning a lesson that used a low-level task in practice during their field experience, as evident in their written lesson plans. As with the second Pledge Plans interview, this was particularly true for the same three elements of attention to students' thinking: Anticipating Students' Correct Thinking (Wilcoxon Matched-Pairs Signed-Ranks, $p < 0.008$), Identifying Questions to Assess and Advance Students' Thinking as they work (Wilcoxon Matched-Pairs Signed-Ranks, $p < 0.016$), and Orchestrating a Discussion that Builds on Students' Thinking (Wilcoxon Matched-Pairs Signed-Ranks, $p < 0.03$).

Lessons that used tasks with high cognitive demands. In contrast to the written lesson plans teachers produced in their practice for low-level tasks, their lessons for high-level tasks were not significantly different from their planning on demand for the Pledge Plans task near the end of the first semester of their field experience, with respect to teachers' overall attention to students' thinking (Wilcoxon Matched-Pairs Signed-Ranks, $p < 0.164$). There were also no significant differences with respect to any of the six elements of attention to students' thinking between teachers' written plans produced in practice for lessons that used high-level tasks and their planning on demand for the Pledge Plans task near the end of the first semester of teachers' field experience.

4.3.4.4 Summary of Planning on Demand versus Planning In Practice

Teachers' attention to students' thinking when planning a lesson that used a low-level task looked more like their planning at the beginning of the Teaching Lab, prior to any formal instruction about lesson planning and the importance of attending to students' thinking when planning. Overall, teachers paid much less attention to and provided much less specificity about students' mathematical thinking in their written lesson plans that used low-level tasks than they did in their second and third Pledge Plans interviews. It is possible that the differences between the Pledge Plans lesson plans (PPint2 and PPint3) and the AvgLP-LL exist because the Pledge Plans lesson plans used a high-level task compared to the AvgLP-LL lesson plans that used low-level tasks. Thus the results may say more about the differences between planning for a high-level versus low-level task than about the differences between planning on demand and planning in practice.

By contrast, teachers' planning for lessons that used a high-level task, showed evidence of teachers attending to students' thinking that was similar to what they demonstrated after participation in the Teaching Lab and near the end of the first semester of their field experience (as evident in their PPint2 and PPint3 lesson plans). In particular teachers tended to use relatively the same level of detail in their written plans for high-level tasks they used in their practice as they did when planning a lesson on demand with respect to four of the six elements of attention to students' thinking: having a Mathematical Goal for the lesson, Anticipating Students' Incorrect Thinking, Identifying Questions that Assess and Advance Students' Thinking as they work, and Orchestrating a whole-class Discussion that Makes the Mathematics Salient for students. There was some evidence that when compared to the second Pledge Plans interview in particular, teachers provided less specificity in their Anticipation of Students' Correct Thinking and in Orchestrating a Discussion that Builds on Students' Thinking. Thus, these two elements appear to be the ones that teachers continue to struggle with providing a high-level of specificity, as evident in the written lesson plans they create in their actual practice of teaching.

4.3.5 Planning for a University Assignment versus In Practice

This section presents comparisons of teachers' planning in practice and their planning for university assignments in which they were explicitly asked to attend to students' thinking as they used the TTLP as a guide. The PTR1 and PTR2 data sources served as indicators of teachers' attention to students' thinking when explicitly asked to do so for a university assignment at two different time points: during the last six class sessions of the Teaching Lab (July 12-28) and during the last half of Methods 1 (November 15 – December 15). Teachers' lesson planning in practice was represented by six lesson plans that each teacher submitted from their field experience. As described earlier, the cognitive demands of the task played a significant role in the level of attention teachers paid to students' thinking in their planning. Therefore, analysis presented in this section showed separate comparisons to teachers' lesson planning for lesson that used low- and high-level tasks (AvgLP-LL and AvgLP-HL, respectively).

Lessons that used tasks with low cognitive demands. Teachers' average written plans for lessons that used low-level tasks showed significantly less attention to students' thinking overall than when teachers were explicitly asked to attend to students' thinking in their PTR1 lesson plans (Wilcoxon Signed-Rank, $p < 0.008$) and PTR2 lesson plans (Wilcoxon Signed-Rank, $p < 0.008$). In particular, teachers showed a significant decrease in their attention to three of the six elements of lesson-planning focused on in the study: 1) Anticipating Students' Correct Thinking (Wilcoxon Signed-Rank, $p < 0.023$ (PTR1)); 2) Identifying Questions that Assess and Advance Students' Thinking as they work (Wilcoxon Signed-Rank, $p < 0.016$ (PTR1 & PTR2)); and 3) Orchestrating a Discussion that Builds on Students' Thinking (Wilcoxon Signed-Rank, $p < 0.008$ (PTR1), $p < 0.016$ (PTR2)). As mentioned earlier for the comparison between AvgLP-LL and the Pledge Plans lesson plans, the PTR lesson plans involved high-level tasks compared to the low-level tasks used in the AvgIP-LL lesson plans. Thus the results may say more about the differences between planning for a high-level versus low-level task than about the differences between planning for a university assignment and planning in practice.

Lessons that used tasks with high cognitive demands. Teachers also showed some evidence of paying less attention to students' thinking in their written plans for lessons that used high-level tasks than they did in their lesson plans for university assignments in which they were explicitly asked to do so. This difference was significant for the PTR2 assignment lesson plans generated in the fall Methods 1 course (Wilcoxon Signed-Rank, $p < 0.004$) but not significant for the PTR1 lesson plans from the Teaching Lab (Wilcoxon Signed-Rank, $p < 0.054$)[3]. In particular, teachers showed a significant decrease in their attention to three of the six elements of lesson-planning focused on in the study: 1) Anticipating Students' Incorrect Thinking (Wilcoxon Signed-Rank, $p < 0.016$ (PTR2)); 2) Identifying Questions that Assess and Advance Students' Thinking as they work (Wilcoxon Signed-Rank, $p < 0.016$ (PTR2)); and 3) Orchestrating Discussion that Builds on Students' Thinking (Wilcoxon Signed-Rank, $p < 0.031$ (PTR1), $p < 0.004$ (PTR2)).

4.3.6 Summary of Teachers' Change in Planning Over Time

Table 4.18 shows teachers' total lesson plan scores for all the data sources and is organized across time (pre-Teaching Lab, post-Teaching Lab, in practice, and near the end of the first semester of teachers' field experience).

[3] If one teacher had scored one more point on the PTR1, then there would have been a significant difference.

Table 4.18 Teachers' Total Scores for All Data Sources

	Pre-TL	Post-TL		In Practice		End 1st Sem. of Field Exp.	
	PPint1	PTR1	PPint2	AvgLP-LL	AvgLP-HL	PTR2	PPint3
Carter, Anthony	2	4	4	N/A	9	10	8
Edwards, Kevin	7	11	9	5.5	4.25	10	9
Ingram, Alicia	4	6	8	3.5	5	6	9
Irving, Kaitlyn	2	6	6	N/A	2.833	9	7
Knight, Kyle	3	10	6	1.67	N/A	11	4
Mercer, Diana	2	14	11	3.5	8	14	7
Nichols, Keith	3	10	9	3	6	9	8
Newman, Faith	5	14	12	2	13.2	14	10
Thompson, Laura	4	13	4	3	5	14	6
Yinger, Brittany	5	7	7	4	5.5	8	7
MEAN	3.7	9.5	7.6	3.05	6.56	10.5	7.5

Table 4.19, below, summarizes the results of comparisons made between pairs of data with respect to total lesson plan scores. The first row indicates the time period when the data was collected. The second row identifies all the data sources and the type of lesson plan (on demand, university assignment, or in practice). Each cell represents a comparison between two data sources with an "x" meaning no significant differences were found for the total lesson plan scores, and ">" or "<" indicating a significant difference between total lesson plan scores as well as the direction in which there was more or less attention to students' thinking. For example, the comparison of PPInt1 and PPInt2 (the ">" in row five and column two), indicates teachers' total

lesson plan attention to students' thinking on PPint2 was significantly greater than their total lesson plan attention to students' thinking on PPint1. A cell value "na" indicates a comparison was not made because it would not provide insight into teachers' planning for a particular type of lesson planning or their change over time.

Table 4.19 Summary of Results of Comparisons Between Data Sources

	Pre-TL	Post-TL		In Practice- 1st Sem. of Field Exp		End 1st Sem. of Field Exp.	
	PPint1 *On Demand*	**PTR1** *Univ. Assign*	**PPint2** *On Demand*	**AvgLP-LL** *In Practice*	**AvgLP-HL** *In Practice*	**PTR2** *Univ. Assign*	**PPint3** *On Demand*
PPint1		na	<	x	<	na	<
PTR1	na		x	>	x	x	na
PPint2	>	x		>	x	na	x
AvgLP-LL	x	<	<		<	<	<
AvgLP- HL	>	x	x	>		<	x
PTR2	na	x	na	>	>		>
PPint3	>	na	x	>	x	<	

As a caveat, there were many comparisons made between data sources and therefore an increase in the number of significant differences that could occur from chance. In this study, there were 17 pairings of data sources and for each pair seven statistical comparisons were made (1 for each of the six elements and 1 for the total lesson plan score) for a total of 119 comparisons. As identified throughout section 4.3, this study found 42 of the comparisons to result in significant differences. Using a value of $p < 0.05$, one could expect 6 (119×0.05) of the comparisons to report a significant relationship by chance (Type I error). While it is difficult to isolate which results may be reporting false positives, the balance of this study shows real differences in teachers' attention to mathematical thinking in their lesson planning.

Together, Table 4.18 and Table 4.19 provide evidence that pre-service secondary mathematics teachers did learn to attend to students' mathematical thinking in their lesson planning. In particular, teachers demonstrated significant growth on pre to post course measures in their attention to students' thinking when planning a lesson on demand and for a university assignment. Specifically, teachers improved in their ability to attend to and specifically describe five of the six elements of attention to students' thinking that were the focus of the this study: Identifying a Mathematical Goal based on students' development of a mathematical concept, Anticipating Students' Correct and Incorrect Thinking, Identifying Specific Questions to Assess and Advance Students' Thinking as they work, and Orchestrating a whole-class Discussion that Builds on Students' Thinking. For the sixth element of attention to students' thinking, Orchestrating a whole-class Discussion that Makes Salient the Mathematics of the lesson, teachers entered the teacher education program with a fairly high-level of attention to this element and showed limited growth in their level of specificity when attending to this element. In addition, teachers maintained over time (from the end of their first summer course to the end of the first semester of their field experience, July to December) their attention to all these elements of attention to students' thinking when planning lessons on demand and for university assignments. Thus, after the Teaching Lab and during the first semester of their field experience, teachers had the ability to attend to students' thinking vis-à-vis the six elements of attention to students' thinking that were the focus of this study.

However, when it came to explicitly attending to students' thinking when planning lessons for their own teaching practice during the first semester of their field experience, evidence suggests that teachers either maintained, increased, or declined in their level of attention to students' thinking. Results indicated that for some of the teachers, their "decline" might not have been as serious as initially indicated. For example, for some teachers their attention to students' thinking differed dramatically depending on the cognitive demands of the task used in the lesson. Specifically, teachers paid more attention to students' thinking (much closer to the attention they paid when planning on demand or for university assignments) when planning a lesson that used a high-level task compared to a lesson that used a low-level task. In addition, for some teachers their written lesson plans significantly under-represented their attention to students' thinking in their planning process. In other words, these teachers were thinking about the elements of attention to students' thinking as they planned their lessons and then chose not to explicitly write all their thoughts down. This was particularly true for most teachers when Anticipating Students' Incorrect Thinking.

The study found compelling results with respect to each of the six elements of attention to students' thinking that were the focus of the study. For example, the majority of teachers were capable of writing Mathematical Goals for the lesson that focused on conceptual understanding and identified specifically what understanding the students would develop through the lesson when teachers wrote lesson plans for a university assignment. However, when planning a lesson on demand, their mathematical goals lacked the same specificity, instead referring to concepts in vague terms (e.g., "students will understand slope of a line"). When planning in practice, teachers' goals for their lessons tended to focus on skills or actions students would take in order to solve the problem(s) in the lesson rather than conceptual understandings students could develop. Overall, teachers' attention to Anticipating Students' Correct and Incorrect Thinking was limited, especially when planning a lesson in practice. However, it was interesting that teachers paid more attention to Anticipating Students' Correct Thinking when planning a lesson that used a high-level task. In addition, teachers' written lesson plans under-represented their attention to Anticipating Students' Incorrect Thinking. With respect to Identifying Questions to Assess and Advance Students' Thinking as they work, teachers showed evidence of providing high-levels of detail when planning a lesson for a university assignment. In practice, however, teachers' level of attention and in particular their level of specificity, was dependent on the cognitive demands of the task used in the lesson. In other words, for lessons that used high-level tasks, teachers were more likely to attend to this element and provide greater specificity. When planning to Orchestrate a Discussion that Builds on Students' Thinking, teachers showed evidence of being quite capable of attending to this with high-levels of specificity when planning on demand and for university assignments. When planning in practice, teachers' attention to this element was dependent on the cognitive demands of the task used in the lesson. Specifically, teachers were more likely to attend to this element and provide specific questions that were tied to specific student work when they used a high-level task in their lesson. Similarly, teachers were more likely to attend to this element and provide specificity when they had Anticipated Students' Correct Thinking about the task in the lesson. Finally, teachers continued to focus on and usually provided great detail in their plans to Orchestrate a whole-class Discussion that Makes Salient the Mathematics of the lesson in all three types of planning they used in this study. Thus, the study found that teachers were capable of learning the importance of and how to attend to students' thinking in a variety of ways (vis-à-vis the six elements that were the focus of this study), as evident in their planning of lessons on demand and for university assignments. Furthermore, many teachers used this knowledge in their practice, as evident in the written lesson plans they produced during the first semester of their field experience, particularly when planning a lesson around a high-level task.

4.4 Ways in Which a Teacher's Attention to Students' Thinking Changed Over Time

The results in this section pertain to Research Question #4:

> In what ways does attention to students' mathematical thinking, as evident in pre-service secondary teachers' lesson planning, change over time?

The previous sections of this chapter have presented results of teachers' attention to students' thinking when planning a lesson at various time points, using different types of planning (on demand, university assignments, and in practice), and how it may have changed over time. These results depicted the planning practices of the teachers as a group. By contrast, this section presents the trajectory over time for individual teachers. In general, there are four possible trajectories that a teacher could traverse during their experiences in the first six months of their teacher education program. The study indicated that when teachers entered the teacher education program, they knew very little about how to attend to students' thinking when planning a lesson. After participation in the Teaching Lab, it was expected that teachers would have either made great strides forward or still be struggling with how to explicitly attend to students' thinking in one's lesson plans. A teacher who was still struggling at the end of the Teaching Lab could then either continue to struggle throughout the first semester of their field experience (i.e., "never quite gets it") or the teacher could show improvement in their ability to attend to students' thinking (i.e., "a late bloomer"). For a teacher who had already shown an increased ability to attend to students' thinking when planning by the end of the Teaching Lab, the teacher could then take two different paths during their first semester of their field experience: decline in their attention to students' thinking when planning (i.e., "does not live up to potential") or maintain their high-level of attention to students' thinking (i.e., "can and does do it").

Given the level of support provided to teachers in the teacher education program with respect to the importance of and how to attend to students' thinking when planning a lesson, it was unlikely that a teacher would not show evidence of being capable of so doing (at least when explicitly asked to do so in a university assignment). And, in fact there were no teachers in this study that fit with the trajectory of "never quite gets it". The remainder of this section is used to present case stories for three individual teachers, each representing one of the three remaining trajectories: 1) “a late bloomer”, 2) “does not live up to his/her potential”, and 3) “can and does do it”.

4.4.1 “A Late Bloomer”: The Story of Anthony Carter

A comparison of total lesson plan scores (shown in Table 4.20) from pre- and post-Teaching Lab measurements (PPint1 versus PPint2 and PTR1) indicate that Anthony Carter improved very little in his ability to attend to students’ thinking after participating in the Teaching Lab. There is evidence that he was still struggling with the elements of how to focus on students’ mathematical thinking in the process of planning a lesson after participating in the Teaching Lab. However, by October, when he began submitting written lesson plans from his field experience, evidence suggests that Anthony figured out what it means to attend to students’ thinking and made that explicit in his written lesson plans. This section describes the trajectory of Anthony Carter’s growth over time in planning lessons that attend to students’ thinking.

Prior to his participation in the Teaching Lab, Anthony showed little evidence of attending to students’ mathematical thinking when planning a lesson on demand, as demonstrated in his initial Pledge Plans lesson-planning interview. His scores on PPint1 (see column two of Table 4.20) indicated that at the beginning of the Teaching Lab he showed no evidence of having an explicit Mathematical Goal for the lesson, Anticipating Students’ Incorrect Thinking, Identifying Questions to Assess and Advance Students’ Thinking as they worked, or Orchestrating a Whole-Class Discussion that Builds on Students’ Thinking.

Table 4.20 Anthony Carter's Scores for All Data Sources

Anthony Carter	**PPint1**	**PTR1**	**PPint2**	**AvgLP-HL**	**PTR2**	**PPint3**
Goal Max Score = 2	0	1	0	1.17	1	1
Anticipating Students' Correct Thinking Max Score = 3	1	0	3	2.0	2	2
Anticipating Students' Incorrect Thinking Max Score = 3	0	1	0	1.83	2	0
Questions to Assess and Advance Students' Thinking Max Score = 2	0	0	0	1.17	2	2
Discussion Building on Students' Thinking Max Score = 2	0	1	1	1.17	2	1
Discussion Making the Mathematics Salient Max Score = 2	1	1	0	1.67	1	2
Total Lesson Planning Score Max Score = 14	**2**	**4**	**4**	**9**	**10**	**8**

In his initial Pledge Plans interview, Anthony talked a lot about the mathematics of the task and how he solved it. He provided evidence of anticipating some of the correct ways in which students may approach solving the problem, as seen in the following excerpt:

> The plan that I have come up with for the students to solve this problem is of course first have the students read the problem to themselves, and then have the students identify all the important facts of the equation to not be intimidated by the words and just get the mathematical concepts that they need from the problem. So, as a teacher, certain things that I would be looking for is I want the students to identify that there are three separate plans, each that can be represented by an equation for each plan. I also want students to be able to convert the words that they are given into equations, so they see that Jeff's plan requires one dollar and fifty cents per kilometer, Annie's is four dollars plus seventy-five cents per kilometer, and Rachael's is two dollars and fifty cents per kilometer. I want students to then be able to see that each equation has kilometers in it and to be able to represent that by another variable that's simpler to use than just per

> kilometers. So I'd look for students to write X as the number of kilometers, some other variable they are comfortable with; that way they can rewrite all their equations with the same variable and it'd be much simpler. …And so I'd have them finally get to solving the equations, and I'd have them graph or tabulate what's the biggest to see if students did get paid by the kilometer, how much each plan would pay at the beginning, throughout, and at the end. I'd also have them see, interpret the data once they'd finished solving the equation, so I'd have them see that Rachael's pay is the most for finishing, but if the investor's not willing to pay, then they definitely can't use that plan [AC, PPint1].

In this excerpt, Anthony indicated that he thought students would write equations for each of the three plans, solve the equations, and create graphs or tables in order to determine which plan was the best. However, it was unclear at times whether or not the students would be working on these various strategies on their own, or whether he would be influencing their work (e.g., "And so I'd have them finally get to solving the equations, and I'd have them graph or tabulate…"). Throughout much of his lesson plan, it was not clear what the teacher would actually be doing and saying. Instead, Anthony spoke about solving the problem and the mathematics involved (e.g., realizing that the range of x, distance in km, was from zero to ten). When pressed, Anthony could provide some detail about how he might get the class to discuss a mathematical idea, as shown in the following excerpt:

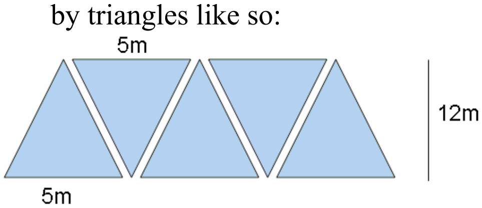
> *Interviewer:* And then so, for example, like you're talking about looking at, I think you had mentioned with the range that you want them to be looking at, and that it shouldn't be negative values or beyond the ten. And you were talking about graphs and tabular ways of doing that. How were you going to try to have that happen in the classroom?
>
> *Interviewee:* I think that as, when we start the problem and we write the equations, we'll have the equations just written as regular equations. And so probably the first time we graph the whole equation and just see what the whole equation looks like, and then identify at what point, if X equals this value that's outside of the range, how much money will they make? And then that'll show students that they're making negative amounts of money. Which means there isn't money, which is not the purpose, so that's not a relevant region. And then I ask if they go beyond this point, what does that represent? That was where they ran more than ten kilometers, but the race is only ten kilometers, so that's all they can run [AC, PPint1].

After participation in the Teaching Lab, Anthony showed limited improvement in his attention to students' thinking. For example, it was during the tenth of twelve class sessions that Anthony turned in his written lesson plan for his PTR1 assignment. Anthony created a task for this lesson (shown in Figure 4.2).

Advent Disaster

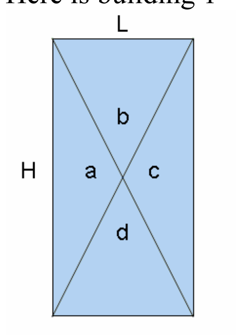
A co-worker of yours has just finished constructing a portion of a city but his city is not good. It will collapse unless you do something to fix it. The city section that needs to be fixed includes a bridge and 2 buildings. The building masses are 7500kg and 8400kg respectively.

a. The bridge must hold the mass of all the buildings plus 20,000kg for transport. It is constructed by triangles like so:



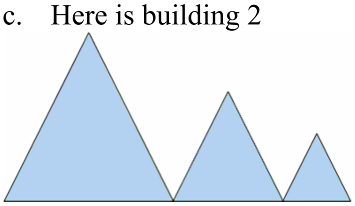
Each triangle can hold a mass of 1800kg. The original bridge has 5 triangles. Extend the bridge to hold the desired mass. (Do not redraw just state the number of triangles and if necessary the set up of the triangles.)

b. Here is building 1



1<M2A<1.2 and height at least twice the width. Note: M2A= Mass to area ratio. Deduce interior angles a, b, c, d such that no angle is less than a-30.

c. Here is building 2



The sum of the bases must not take more than half the length of the bridge. All three sections of the building (each triangle is a section) are similar. Specify all three sides of each section.

Figure 4.2 AC's PTR1 Task

His lesson plan was typed in the form of an outline that was double-spaced and two and one-half pages in length. He did have an explicit goal for this lesson:

> Students should be able to utilize ratios to compare two values to determine equivalencies. Students should think about open ended situations in multiple different ways and be able to compare the multiple different ways to solve the task. Students should be able to apply given conditions and make decisions on their own about picking values and then finding the logical values that go with it [AC, PTR1].

His goals, however, were focused on actions students should take in solving the problem, rather than on mathematical concepts students should be developing through the lesson. Although his goal statement made reference to the "multiple different ways to solve the task", his lesson plan did not include any evidence of him thinking about the specific ways students may work on the problem. His lesson plan did mention an assumption students might make that would cause them to struggle with the problem. However, that was the extent of his anticipation of students' thinking, as evident in the written plan. Similar to his initial Pledge Plans lesson plan, Anthony provided no Specific Questions to Ask Students as they worked on the task. He did have a "share and discuss" phase of his lesson plan (shown below) that showed very limited thoughts for leading a Discussion that Built on Students' Thinking and Made the Mathematics of the Lesson Salient.

> 4) Share and discuss (18 minutes)
>   a) Share the two solutions ask for justification and field questions
>   b) Have other students explain or rebuttal the person who is explaining critique with reason.
>   c) Begin discussion of degrees of freedom of an equation for tomorrow's class with equation example:
>   a. 3x+5y=7
>   b. 10x+15=9
>   c. Can you solve these equations? If so what is the answer if not why?
>   d) Discussion of degrees of freedom leads to systems of equations.
>   e) Assign homework [AC, PTR1]

Anthony's attention to students' thinking was also limited when planning on demand for his second Pledge Plans interview, conducted immediately after his participation in the Teaching Lab. For example, he provided no explicit Mathematical Goal for the lesson. Also similar to his initial Pledge Plans interview, he provided no evidence of Anticipating Students' Incorrect Thinking, or Identifying Questions to Assess and Advance Students' Thinking as they worked. However, after participating in the Teaching Lab, Anthony did show evidence of attempting to anticipate multiple ways students could correctly solve the problem, and did so in a rather methodical way, so as to cover all aspects of the problem. He also provided evidence of planning a Discussion that Built on the Students' Work by suggesting that students present their work in a specific order (e.g., starting with Annie's plan "because it's the best early on in the race", then having Jeff's plan presented, followed by Rachel's plan). However, he offered little detail other than "now students present their explanations and graphs and tables and whatever else they've used to work on the problem." Thus his second Pledge Plans lesson plan showed growth in that he now expressed an explicit interest in having a class discussion that built on students' work, although he lacked specificity in what questions he would ask about the students' work in order to make the mathematics salient.

In summary, after the Teaching Lab, unlike many of his peers, Anthony showed very limited growth in his attention to students' thinking when planning. His lack of improvement from PPint1 to PTR1 is shown in Figure 4.3. The figure also highlights the fact that Anthony had made less progress than other members of the class. He was struggling to write goals that were based on conceptual learning and was the only teacher who did not explicitly state a mathematical goal for the lesson in his Pledge Plans interview (PPint2). He demonstrated limitations and inconsistencies in Anticipating Students' Correct or Incorrect Thinking. He was now considering ways in which he could facilitate a Discussion that Built on Students' Thinking but was vague in specifying the questions he could ask students about the work that would Make the Mathematics of the Lesson Salient for the students. Finally, he showed no change in his attention to Identifying Questions to Ask Students to Assess and Advance their Thinking as they worked, indicating that at this point he did not yet know what that should look like in a lesson plan or that it was something he should include.

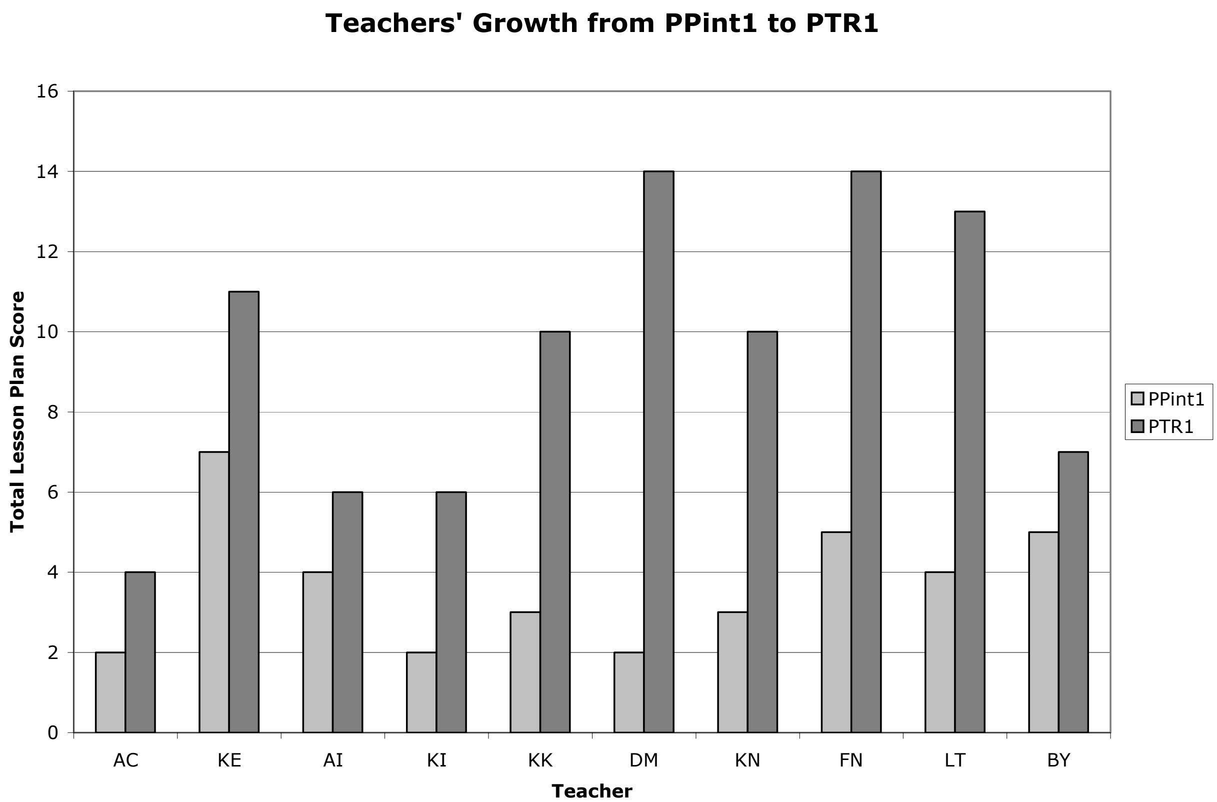


Figure 4.3 Teachers' Growth from PPint1 to PTR1

One might have anticipated that Anthony was not well-prepared to begin his field experience and would be unlikely to attend to students' thinking when planning lessons in his actual practice of teaching. However, as seen in column five of Table 4.20 (presented earlier in this section), Anthony's average lesson plan[4] scores showed significant growth in his attention to students' thinking. Table 4.21 provides the scores for each of his six lesson plans with respect to each of the elements of attention to students' thinking.

[4] Anthony's six lesson plans all used high-level tasks.

Table 4.21 Anthony Carter's Scores for Written Lesson Plans In Practice

Anthony Carter	**Oct LP1**	**Oct LP2**	**Nov LP1**	**Nov LP2**	**Dec LP1**	**Dec LP2**	**AvgLP-HL**
Goal Max Score = 2	1	2	1	1	1	1	1.17
Anticipating Students' Correct Thinking Max Score = 3	1	3	2	2	2	2	2.0
Anticipating Students' Incorrect Thinking Max Score = 3	2	3	2	2	2	0	1.83
Questions to Assess and Advance Students' Thinking Max Score = 2	2	1	1	1	1	1	1.17
Discussion Building on Students' Thinking Max Score = 2	0	2	2	1	1	1	1.17
Discussion Making the Mathematics Salient Max Score = 2	2	1	2	2	1	2	1.67
Total Lesson Planning Score Max Score = 14	**8**	**12**	**10**	**9**	**8**	**7**	**9**

Anthony submitted lesson plans for a geometry course that he was responsible for teaching at an urban high-school that used the Cognitive Tutor curriculum (five of the six tasks he submitted came from his textbook and one task was created by Anthony). Growth could be seen in many of the individual elements of attention to students' thinking from Anthony's post-Teaching Lab lesson planning to his written lesson plans produced for his field experience. For example, Anthony always included an explicit Mathematical Goal for his lessons. In most of his lesson plans, he was still writing goals that focused on students' skills or the actions they would take in order to solve the task. However, he had begun to include mathematical concepts in his goals as well (although in most lessons this was vague rather than specific about what it meant for students to understand a new concept). This could be exemplified in the following goal statement for a textbook task in which students derived the formula for the area of a circle

> Objective: Use students' knowledge of previous areas to derive the area of a circle. Help students gain a better understanding of measuring objects; in this case straightening the length around the circle. Review the previous area formulas and manipulating equations. Help students develop the habit of writing down everything given and unknown. Continue to relate everything that they are doing to the basics of mathematics [AC, Oct.LP1].

Anthony had also shown growth in his attention to Anticipating Students' Thinking. He was now consistently providing evidence of thinking of specific ways in which students' would correctly think about the problem as well as specific misconceptions and difficulties students might have (e.g., "Students will have the most trouble identifying the base. The side is not straight and by no means looks straight and that is what they look for in a base" [AC, Oct. LP1]). He often listed specific questions that he thought students might ask him as they worked on the problem and ran into specific troubles (e.g., How do we know that the area made by us will be the same as the area in the circle and does not change when we make the triangle? How doe we find the base and the height? [AC, Oct. LP1]).

The most dramatic change in Anthony's planning was the inclusion of specific Questions to Ask Students to Assess and Advance their Thinking as they worked on the task. He now consistently included a "Student Exploration" phase in each of his lesson plans which had a series of specific questions to ask students. After his October lesson plans, he sorted these questions under two headings: assessment questions and advancement questions. There were usually 7-10 specific questions under each of these headings. While he did not explicitly identify circumstances based on students' mathematical thinking under which to ask students these questions, the questions were so specific to the mathematics in the problem that it was reasonable to infer that he had thought about these circumstances when writing the questions.

With respect to Planning a Discussion that Built on Students' Thinking, Anthony now demonstrated that he was capable of planning this in great detail in two of his lesson plans (OctLP2, NovLP1). In other words, he specifically identified student solutions in what order they were to be presented. Furthermore, he identified specific questions related to each of the student solutions that he could ask in order to highlight important mathematical ideas in the solution. In his first November lesson plan, he even asked questions that required students to make comparisons between the various solutions presented. While Anthony demonstrated he was capable of Planning a Discussion that Built on Students' Thinking with a great deal of specificity, he did not consistently do this in his written lesson plans. The last three lesson plans that he submitted (NovLP2, DecLP1, DecLP2) were similar to his post-Teaching Lab planning (PPint2), in that he identified student work to share and the order in which to present them (although the order in which to share it was often just in the order of the text book questions). However, he did not provide any specific questions that were explicitly tied to a specific student solution. Instead, he provided more detail in the form of a series of specific questions to ask students that could Make the Mathematics of the Lesson Salient.

In summary, Anthony showed significant growth in his capacity to attend to students' thinking in his planning from the Teaching Lab to his planning in practice for his field experience. He always included Mathematical Goals for his lesson plans that focused on skills students would use in solving the problem as well as identifying concepts with which students would engage (although the exact understandings of the concept were often lacking specificity). He consistently Anticipated Students' Correct and Incorrect Thinking. Anthony showed the most dramatic improvement in his ability to include specific Questions to Ask Students to Assess and Advance their Thinking as they worked. He also showed an increased capacity to provide more specificity in Planning a Discussion that both Built on Students' Thinking and Made the Mathematics Salient. Anthony's attention to students' thinking when planning for high-level tasks in his practice showed not only a remarkable improvement from his post-Teaching Lab planning it represented some of the best planning for high-level tasks as seen in Figure 4.4. Anthony's third Pledge Plans lesson-planning interview and PTR2 assignment for his Methods 1 course, both occurring towards the end of the first semester of his field experience, showed similar results. Indicating that Anthony's ability to plan a lesson on demand and for a university assignment in which he was explicitly asked to attend to students' thinking were very similar in attention to students' thinking to the written lesson plans he was producing in his own teaching practice.

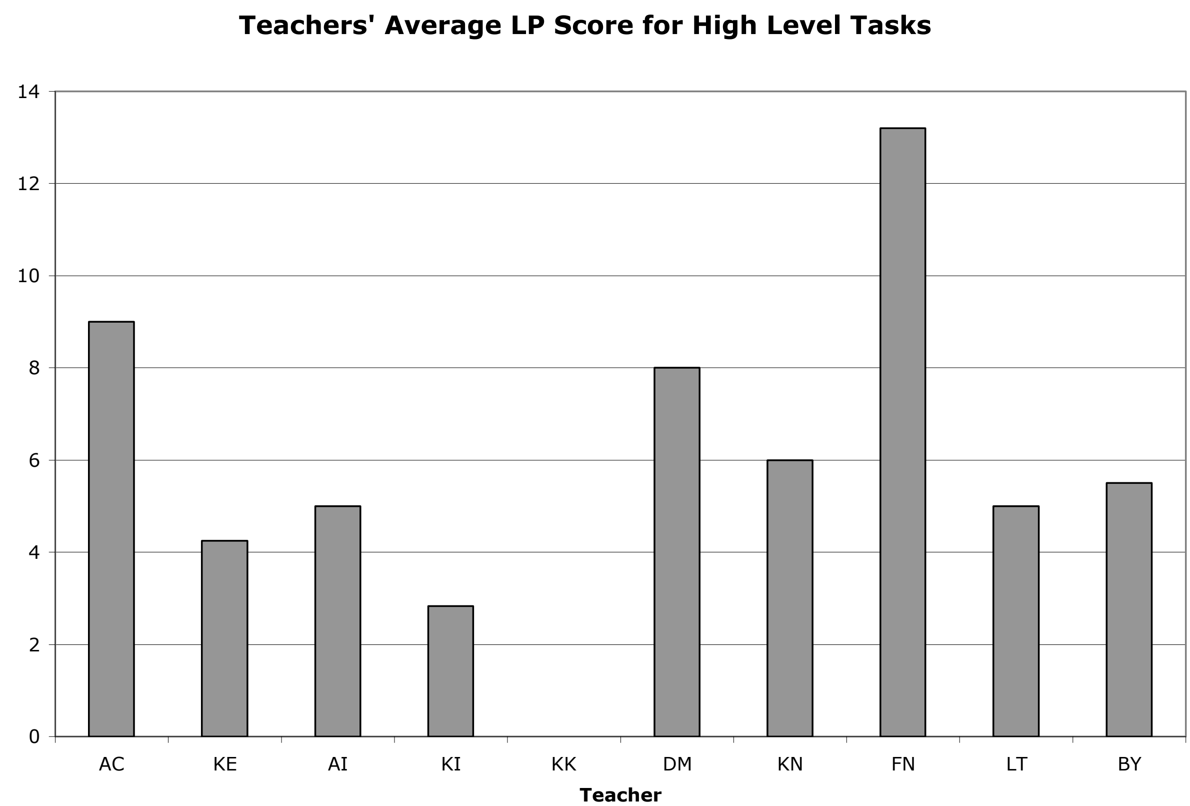


Figure 4.4 Teachers' Average Lesson Plan Score for High Level Tasks

In his final interview, Anthony talked about how his lesson planning had changed over time. He claimed that he had "been using the TTLP throughout but I think as time's gone on I've been getting closer to it." He specifically credits the second PTR assignment in his fall Methods 1 course as being a point where he could see that he had gotten closer to planning like the TTLP. Anthony also claimed that his mentor teacher played an important role in his planning, as seen in the following transcript excerpt:

> *Interviewee:* She does give me a lot of feedback prior to me giving a lesson. I show her my lesson plans and we discuss how exactly we're going to make the students engage, how many days it may take to do and because of the curriculum we only teach three days a week. They have two days in the computer lab so the students are at their own pace in the computer lab. But she gives me a lot of feedback on my lesson plans, typically introductions. We don't spend much time. We spend more time towards the share and discuss phase.
>
> *Interviewer:* Because that's where you spend more time discussing and talking about it. Okay.
>
> *Interviewee:* Yes.
>
> *Interviewer:* And what kinds of feedback does she usually give you related to that?
>
> *Interviewee:* Try to get the students more engaged, get more students coming up to do it instead. Spending less time at the front of the room. [AC, Final Interview]

It appeared that the combination of additional support from his mentor teacher and Methods 1 coursework aided Anthony in his increased ability to attend to students' thinking when planning. When pressed to say more about the ways in which his planning had changed over time, he responded:

> I think that it's just gotten more like the TTLP. I changed the template. I think one of the biggest changes is I put more questions in that I'm going to ask the students, that I plan to ask the students as opposed to I want to come up with a question that does this need. That's how I was doing it before sometimes. Here's

> something that may come up. I want a question for this to make this, address this need, but instead now I'm putting the exact question I want to ask. So that's helping me better prepare. Other changes, I think I'm putting more time in my share and discuss phase so it's getting a little more expansive and I think now that I'm thinking - getting better to my introduction and objective I'm getting better at ensuring the students do reach the objective. I've actually seen and proven it in some of my classes and some of my students now that we've been doing a little more share and discuss where they're more involved and I spend less time at the front of the room. So that's helped out a lot. Even some of my most disruptive students are benefiting from it. I'm noticing a vast improvement [AC, Final Interview]

Anthony had seen improved behavior and learning from his students as a result of his lesson planning attending to the elements suggested in the TTLP. In the final interview, he finished his thoughts about lesson planning with the following:

> Sometimes it's very time consuming, trying to write these lesson plans but it's very helpful. It really helps the lesson go a lot smoother and even not having it in front of me I think it really helps me focus my thinking which then it kind of helps me focus my students' thinking, which helps us get to an objective and leads to a better lesson [AC, Final Interview].

Therefore, it was apparent as to why Anthony wrote the detailed lesson plans that he did and spent so much time attending to students' thinking when planning his lessons – he was seeing better teaching and better student learning as a result of his efforts.

4.4.2 "Does Not Live Up To His Potential": The Story of Kevin Edwards

The lesson plan scores for Kevin Edwards' initial Pledge Plan interview (shown in column 2 of Table 4.22) indicated that he entered the teacher education program with some ability to attend to students' thinking when planning a lesson on demand. A comparison of *total lesson plan scores* from pre- and post- measures (PPint1 versus PPint2 and PTR1) indicated that after participating in the Teaching Lab, he showed improvement in his attention to students' thinking. There was evidence, in on demand and university assignment lesson plans, that he had knowledge of and could sufficiently implement the six elements of attention to students' thinking that were the focus of this study and that were emphasized in the Teaching Lab. However, the written lesson plans that Kevin submitted from his field experience suggested that he was not applying what he had learned in the Teaching Lab, with respect to attention to students' thinking, in his own planning practice. This section paints a portrait of the trajectory of Kevin Edwards' lesson planning over time with respect to his attention to students' thinking.

Table 4.22 Kevin Edwards' Scores for All Data Sources

Kevin Edwards	**PPint1**	**PTR1**	**PPint2**	**AvgLP-LL**	**AvgLP-HL**	**PTR2**	**PPint3**
Goal Max Score = 2	0	1	1	1.00	0.75	2	2
Anticipating Students' Correct Thinking Max Score = 3	2	3	2	0.00	0.00	0	1
Anticipating Students' Incorrect Thinking Max Score = 3	2	2	0	2.50	1.75	2	3
Questions to Assess and Advance Students' Thinking Max Score = 2	0	2	2	0.00	0.00	2	2
Discussion Building on Students' Thinking Max Score = 2	1	2	2	0.00	0.00	2	0
Discussion Making the Mathematics Salient Max Score = 2	2	1	2	2.0	1.75	2	1
Total Lesson Planning Score Max Score = 14	**7**	**11**	**9**	**5.5**	**4.25**	**10**	**9**

Prior to his participation in the Teaching Lab (see PPint1 scores in column two of Table 4.22) Kevin showed no evidence of having an explicit Mathematical Goal for the lesson or Identifying Questions to Assess and Advance Students' Thinking as they worked, when planning a lesson on demand. However, Kevin did show evidence of Anticipating Students' Thinking (both correct and incorrect), by making specific suggestions of some ways in which students might solve or struggle with the problem. This was illustrated in the following excerpt:

> I'm assuming they're probably gonna figure out the equation that I have written here, one point five X, two point five X. And this one might give 'em some trouble, the point seven five… Annie's might give 'em some trouble … It would throw them off, I think, because they said they're asking for a donation of four dollars, and then seventy-five cents per kilometer. I could see students saying, "Well, you know, seventy-five cents per kilometer, and then four dollars? And then I do seventy-five cents per kilometer and then four dollars again? [KE, PPint1].

It was clear in Kevin's initial Pledge Plans lesson plan that the teacher would lead the class through the process of solving the problem together, rather than giving students time to work on the problem themselves then using their work to orchestrate a whole-class discussion. During the interview, he essentially narrated what he thought the class discussion would be like, including what he would say and what the students would likely say. He provided many examples of specific questions he would ask the class that had the potential to Make Salient the Mathematics of the lesson. In addition, many of the questions were based on what he thought students would be thinking or how they would respond to the previous questions. Therefore, there was evidence that he was trying to Orchestrate a Discussion that Built on the Students' Thinking (or at least their responses to his series of questions). Kevin's planning was illustrated in the following excerpt (*emphasis* indicated places where Kevin was anticipating students' thinking and then posing questions to build on their thinking in response):

> Alright, so from our graphs now, we have this nice picture on the board or on the display projector, I said, "Which one, which plan is better?" *And many of them are probably going to look at it and say, "Well Rachael's is the best, then Jeff's, and then Annie's."* I said, "Well, that's very good." I say "that ten kilometers, being sure to emphasize at ten kilometers Rachael's is the best, then Jeff's and then Annie's. Why is that?" … Like if you say which plan is better, *I would assume a student would think it's a 10K race; everyone is going to complete the race*. 10K, oh 10K is not too bad if you're training, but I don't know if I could run a 10K right now. *But I'm sure that's probably where they would go. They would say okay, da da da da da, the end, 10K, this is the best*. And of course, that's right for that. I said, "Now, suppose that these people are out of shape, and people only run between say two kilometers and six kilometers. So what now, between here, which plans are the best? In fact, let's take these points of intersection right here, and say people only run four kilometers on average, and

then they all just drop off. They're smokers or whatever. What do you have then? Whose is the best?" [KE, PPint1, *emphasis added*].

Thus, prior to participating in the Teaching Lab, Kevin differed from his peers in showing evidence of substantial attention to students' thinking for three of the six elements that were the focus of this study: 1) Anticipating Students' Correct Thinking, 2) Anticipating Students' Incorrect Thinking, and 3) Orchestrating a Discussion that Makes Salient the Mathematics of the lesson. With respect to the other three elements of attention to students' thinking, Kevin was similar to his peers and showed little or no evidence of thinking about them when planning a lesson on demand. Figure 4.5 displays teachers' scores on PPint1 for the six elements of attention to students' thinking. Kevin was the only teacher who scored 2 points for Anticipating Students' Correct and Incorrect Thinking and was one of four teachers who scored 2 points for Orchestrating a Discussion that Makes Salient the Mathematics of the lesson.

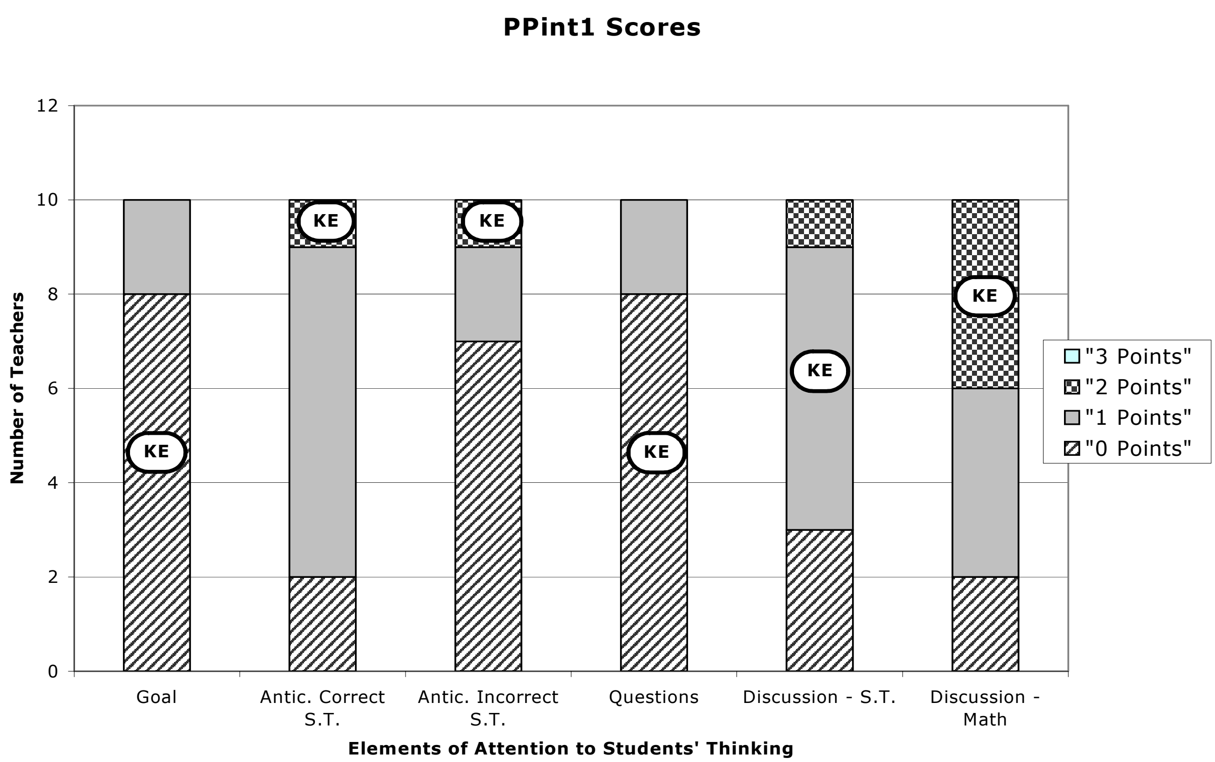


Figure 4.5 PPint1 Scores with Kevin Edwards Highlighted

After participation in the Teaching Lab, Kevin showed improvement in his attention to the three elements he had previously neglected: 1) having an explicit Mathematical Goal for the lesson, 2) Identifying Questions to Assess and Advance Students' Thinking as they worked, and 3) Orchestrating a Discussion that Built on Students' Thinking. For example, in both his PTR1 and PPint2 lesson plans, Kevin had an explicit Mathematical Goal for his lesson. However, both of these goals were not at the level of identifying a specific mathematical concept and explicating what it meant for students to develop an understanding of the concept. In his PTR1 written plan, his goal for the lesson focused on skills students would need to employ in order to solve the problem (e.g. "have students take two columns of date and interpret them into a meaningful linear equation which has a unique solution"). Whereas, in his PPint2 lesson plan, Kevin provided a mathematical goal that described a mathematical concept students would come to understand, but did not explicate exactly what understanding of the concept students would develop (e.g., "The mathematical goals are for students to see the relationship between a word problem, table generated from this word problem and a graph, see the relationship between these things"). Kevin's lesson plans had also significantly changed in the style of instruction they represented. In other words he now was using the "task setup", "student exploration", and "share and discuss" phases that were taught in the Teaching Lab rather than an approach where the teacher leads the whole class through the steps of solving the problem together. Thus the most dramatic change in Kevin's planning was seen in his inclusion of a "student exploration" phase where he Identified specific Questions to Assess and Advance Students' Thinking as they worked. In addition, he provided information about the circumstances under which he would deem the question appropriate to ask a student and these circumstances were based on students' mathematical thinking. For example, in his PPint2 lesson plan, he offered the following circumstances along with specific questions: "if they can't get started on generating a table", "if they get caught up in Annie's plan because she has, she starts off with $4", and "if they are having trouble creating a graph". In addition, Kevin showed improvement in his level of specificity when planning to Orchestrate a Discussion that Built on Students' Thinking, as illustrated in the following excerpt (*emphasis* indicated places where Kevin was anticipating students' thinking and then posing questions to build on their thinking in response or making use of students' work (e.g., the table or graph they had created)):

> *I'd bring up the table of the data first and ask them how they got their values and then they'd explain the process of … finding values*. And then I'd ask them if they noticed if any plans become better or worse than any other plan during this and, *hopefully, some of them will notice,* "*Oh! Well, look! Between two and three, between five and six, something happens between these plans. Something is going on."* And then I'd say, you know, "Did any of these plans decrease?" and *they'd say, "No, they're all increasing."* and I said, "Well, if they're all increasing and one all of sudden dips below another one, what does that mean? That means that some of them must be increasing at a greater rate than others, right? Something is going on because none of the numbers are standing still either. They're all moving up." And that would be a point that I would emphasize, that they're all increasing but some are increasing faster and slower than others. And "how do we know which ones are faster and slower? *Let's bring up the graph now"* and then we'd have this graph, … and say, "Okay, here are plans now...you got Rachel, Jeff and Annie. Here's that point between two and three, they intersect". (I'd say), "What about between five and six on the table?" [KE, PPint2, *emphasis added*].

With respect to the three elements of attention to students' thinking that Kevin had included in his initial Pledge Plans lesson plan (1) Anticipating Student's Correct Thinking, 2) Anticipating Students' Incorrect Thinking, and 3) Orchestrating a Discussion that Makes Salient the Mathematics of the lesson), he continued to include and provide specificity in his attention to these elements in his post-Teaching Lab lesson plans.

In summary, at the end of the Teaching Lab, Kevin showed growth in and was capable of attending to students' thinking when planning a lesson for a university assignment and on demand. There was evidence that Kevin was poised to begin his field experience with the knowledge necessary and ability to attend to students' mathematical thinking in his lesson planning practices.

Kevin submitted lesson plans from his field experience for an Algebra 2 course that he was teaching in a suburban high school. Kevin's average lesson plan scores for his written lesson plans that used either low- or high-level tasks indicated that Kevin was not applying what he had learned from the Teaching Lab, with respect to attention to students' thinking, to his own lesson planning practice (see column eight and nine in Table 4.23 below). Table 4.23 provides the scores for each of Kevin's six lesson plans with respect to each of the elements of attention to students' thinking.

Table 4.23 Kevin Edwards' Scores for Written Lesson Plans In Practice

Kevin Edwards	**Oct LP1**	**Oct LP2**	**Nov LP1**	**Nov LP2**	**Nov LP3**	**Dec LP2**	**AvgLP-LL**	**AvgLP-HL**
Level of Cognitive Demands of the Task	Low	Low	High	High	High	High		
Goal Max Score = 2	1	1	1	1	0	1	1.00	0.75
Anticipating Students' Correct Thinking Max Score = 3	0	0	0	0	0	0	0.00	0.00
Anticipating Students' Incorrect Thinking Max Score = 3	3	2	3	2	0	2	2.50	1.75
Questions to Assess and Advance Students' Thinking Max Score = 2	0	0	0	0	0	0	0.00	0.00
Discussion Building on Students' Thinking Max Score = 2	0	0	0	0	0	0	0.00	0.00
Discussion Making the Mathematics Salient Max Score = 2	2	2	2	2	2	1	2.0	1.75
Total Lesson Planning Score Max Score = 14	**6**	**5**	**6**	**5**	**2**	**4**	**5.5**	**4.25**

As shown in Table 4.23, Kevin produced written lesson plans that consistently attended to three of the elements of attention to students' thinking and neglected the other three elements. Unlike some of his peers, Kevin's lesson plans were consistent in their attention to students' thinking regardless of the level of cognitive demand of the task used in the lesson. In the six written lesson plans he submitted for the study (two that used a low-level task and four that used high-level tasks), he provided no evidence of Anticipating Students' Correct Thinking, Identifying Questions to Assess and Advance Students' Thinking as they worked, and Orchestrating a Discussion that Built on Students' Thinking. Kevin's lack of attention to the latter two elements of lesson planning (Identifying Questions and Orchestrating a Discussion that Built on Students' Thinking) may have had more to do with his style of instruction than his proclivity to attend to students' thinking when planning. For example, in five of the six lesson plans submitted there was never any explicit mention of providing time for students to work on a mathematical problem. The entire lesson was designed as a "whole class discussion" in which the teacher led students through the process of solving the problem. The lesson plans he submitted were typed, between two and four pages long, and contained the following sections: 1) "lesson objective", 2) "points of confusion" or "misunderstandings", 3) "intro", which consisted of example problems and questions to ask the class before beginning the main lesson, and 4) "main", where he provided problems to work out together with the class and the sequence of questions he would ask to lead students through the process.

For example, in Kevin's NovLP1 the task for the lesson involved six problems in which two matrices were given that were to be multiplied and students were asked, "Are you noticing any patterns related to the dimensions of the matrices in question?" Kevin states the objective of the lesson: "students will learn how to do matrix multiplication without their calculators." This goal focused on a skill that students would learn rather than a concept that students might come to understand through the lesson. Most of his lesson goals focused on skills with only one of the plans providing a vague description of a concept that students might grapple with in the lesson (e.g., "to have students understand the careful domain restrictions that we must make in a piecewise function" [KE, OctLP1]). Throughout his planning, as evident in his written plans, Kevin Anticipated Students' Incorrect thinking by providing specific examples of questions students might ask or specific mathematical ideas that may prove to be difficult for the students. For example, in his NovLP1 he provided the following "points of confusion":

> No doubt students will be confused about what size of matrices are allowed to be multiplied and what size matrix results (i.e., a *m x n* matrix can only be multiplied by an *n x p* matrix and the resulting matrix will be of dimension *m x p*). This will probably confuse students because in matrix addition, the matrices have to be the same size, and the resulting matrix is always the same dimension as the two being added. Another point of confusion will be the actual process of multiplication, students will no doubt have trouble keeping track of which row gets multiplied by which column, and where the resulting product goes in the final answer. I hope to remedy this by working with small examples first, and then progressing to larger examples [KE, NovLP1].

Finally, Kevin's written lesson plans were predominately filled with almost a script-like series of questions he wanted to ask students as the class together progressed through solving the problem or set of problems. In this respect, Kevin provided a high-level of specificity in planning how he would Orchestrate a Whole-Class Discussion that Made Salient the Mathematics of the lesson. It is also important to note that the analysis of his written plan versus his verbal description of the plan (as described earlier for all teachers in section 4.2.3), Kevin did not show any evidence of paying more attention to students' thinking in his verbal lesson plan compared to his written lesson plan. Thus, it was likely that his written lesson plans were an accurate account of his thinking when planning the lesson.

In summary, in his lesson planning practices during his field experience, as evident in his written lesson plans, Kevin included only three of the six elements of attention to students' thinking: having a Mathematical Goal for the lesson, Anticipating Students' Incorrect Thinking, and Orchestrating a Discussion that Made Salient the Mathematics of the lesson. He completely neglected to Anticipate Student's Correct Thinking, Identify Questions to Assess and Advance Students' Thinking as they worked, and Orchestrate a Discussion that Built on Students' Thinking. In addition, his style of instruction was in contrast to the standards-based, student-centered, style that was emphasized in the Teaching Lab. This raised the question of what factors might have supported or inhibited Kevin's application of what he had learned in the Teaching Lab, with respect to attention to students' thinking when planning, to his own practice during his field experience?

In exploring possible explanations to this question, the data supported the investigation of three particular factors: 1) the teacher education program through his university coursework, 2) his mentor teacher, and 3) his university supervisor. There was evidence that the teacher education program had supported Kevin in initially learning about the importance of and how to attend to students' thinking when planning a lesson, as evident in his growth and relatively high scores for planning a lesson for a university assignment in the Teaching Lab and planning on demand for his second Pledge Plans interview. Furthermore, near the end of the first semester of his field experience, evidence from his PTR2 and PPint3 lesson plans indicate that Kevin was still quite capable of attending to students' thinking vis-à-vis all six of the elements of attention to students' thinking. Thus, his teacher education program, through his university coursework, appeared to support Kevin in his ability to understand and implement the elements of attention to students' thinking in his planning, when he was specifically asked to do so for a university assignment or when asked to plan a lesson on demand in an interview. However, there was evidence that the teacher education program had a limited affect on Kevin's own beliefs about teaching and planning, as illustrated in the following excerpt:

> I use to do pretty much what Pitt said to do, think of all the misunderstandings, think of ways to address all these different things, make everything explicit. Now I take on a more holistic, organic point of view. By that I mean what the older teachers have told me, have a goal for every day, like I want to do this by the end of the day. If they do more great, if they don't do this we're in trouble, and then have a goal for the week. So by the end of the week they should be able to do these series of things and see kind of how they're related. … So I'm getting better at, like I said looking at it more holistically and trying to get some sense of continuity to the lessons instead of you know trying to memorize every single way they could misunderstand and reading off these insanely long plans --… I think I've settled into a nice little format. Like I have my, you know I have, some things are bolded and some things are indented. I number my equations and things like that…
>
> I think Pitt should make it clear that the purpose of doing these very long lesson plans, and this is also from talking to other professors, teachers. Once you get a couple of years in your job, like you know one page is gonna hold everything you're gonna do for a week and a lot of teachers have showed me their plans, oh there it is. They said that the point of going through all this rigamarole is not just to make us jump through hoops. It really does make you think about how these kids are learning, especially if you're coming, if you don't have an ed. degree undergrad, it makes you think about the learning process, the level the kids are on. It makes you think through, it puts you more in their mindset and more and prepares you more for how you're going to run your class. You're usually more confident if you've thought things out beforehand. So they should make it clear that that's kind of why we're doing these things because in the beginning I was very frustrated and saying, you know I'm never going to do this once I get a real job. And then I realized oh, I still have to go through all these exercises [KE, Final Interview].

This excerpt suggested that Kevin was struggling with understanding the purpose of the TTLP and PTR assignments and whether or not it was truly going to help him in his teaching practice. As Kevin mentions in the above excerpt, the teachers in his school were not producing the kind of lesson plans that his university was asking him to produce. In his final interview, Kevin talked about the influence his mentor teacher and other teachers at his school as well as his university supervisor had on his planning practices. In the interview, there was additional evidence that he was trying to do what other teachers at his school were doing and that when he did so, they and his university supervisor approved. There was no evidence that the teachers in Kevin's field placement school or his university supervisor were supporting or were aligned with the teacher education program's lesson planning approach that emphasized attention to students' thinking through using the TTLP as a guide when planning. This was illustrated in the following excerpt:

> Yeah she's [university supervisor] pretty much been saying, as far as plans are considered, they're very, I don't feel comfortable saying that but like she's saying that they're really good, top-notch, wonderful, blah, blah, just implement them better. So I think from between supervisor, mentor and then other teachers that I talk to I've got the planning thing pretty much down pat [KE, Final Interview].

Thus it was possible to conclude that Kevin's limited attention to students' thinking, as evident in his written lesson plans created for his field experience, was not because he lacked the knowledge necessary or ability to attend to students' thinking when planning. In other words, his planning for the Pledge Plans interviews and university PTR assignments indicate Kevin was quite capable of attending to students' thinking vis-à-vis the six elements of attention to students' thinking that were the focus of this study. Rather, it appeared that Kevin's own beliefs about lesson planning contradicted what he was being taught at the university. Furthermore, the feedback he received for his lesson planning during his field experience, from his mentor teacher, veteran teachers at his school, and university supervisor, were more aligned with his own beliefs than with those of the university.

4.4.3 "Can and Does Do It": The Story of Faith Norris

A comparison of total lesson plan scores (shown in Table 4.24) from pre- to post-measures (PPint1 to PPint2 and PTR1) indicated that Faith Norris made dramatic improvements in her attention to students' thinking after participating in the Teaching Lab. In addition, evidence indicated that she fully understood and could implement the six elements of attention to students' thinking that were the focus of this study and that were emphasized in the Teaching Lab, when planning on demand and for university assignments. The written lesson plans that Faith submitted from her field experience suggested that she was applying what she had learned in the Teaching Lab, with respect to attention to students' thinking, in her own planning practice when planning lessons that used high-level tasks. This section portrays the trajectory of Faith Norris' lesson planning over time with respect to her attention to students' thinking.

Table 4.24 Faith Norris' Scores for All Data Sources

Faith Norris	**PPint1**	**PTR1**	**PPint2**	**AvgLP-LL**	**AvgLP-HL**	**PTR2**	**PPint3**
Goal Max Score = 2	0	2	1	0	1.17	2	1
Anticipating Students' Correct Thinking Max Score = 3	1	3	3	0	2.0	3	1
Anticipating Students' Incorrect Thinking Max Score = 3	0	3	3	0	1.83	3	3
Questions to Assess and Advance Students' Thinking Max Score = 2	0	2	2	0	1.17	2	2
Discussion Building on Students' Thinking Max Score = 2	2	2	2	0	1.17	2	2
Discussion Making the Mathematics Salient Max Score = 2	2	2	1	2	1.67	2	1
Total Lesson Planning Score Max Score = 14	**5**	**14**	**12**	**2**	**13.2**	**14**	**10**

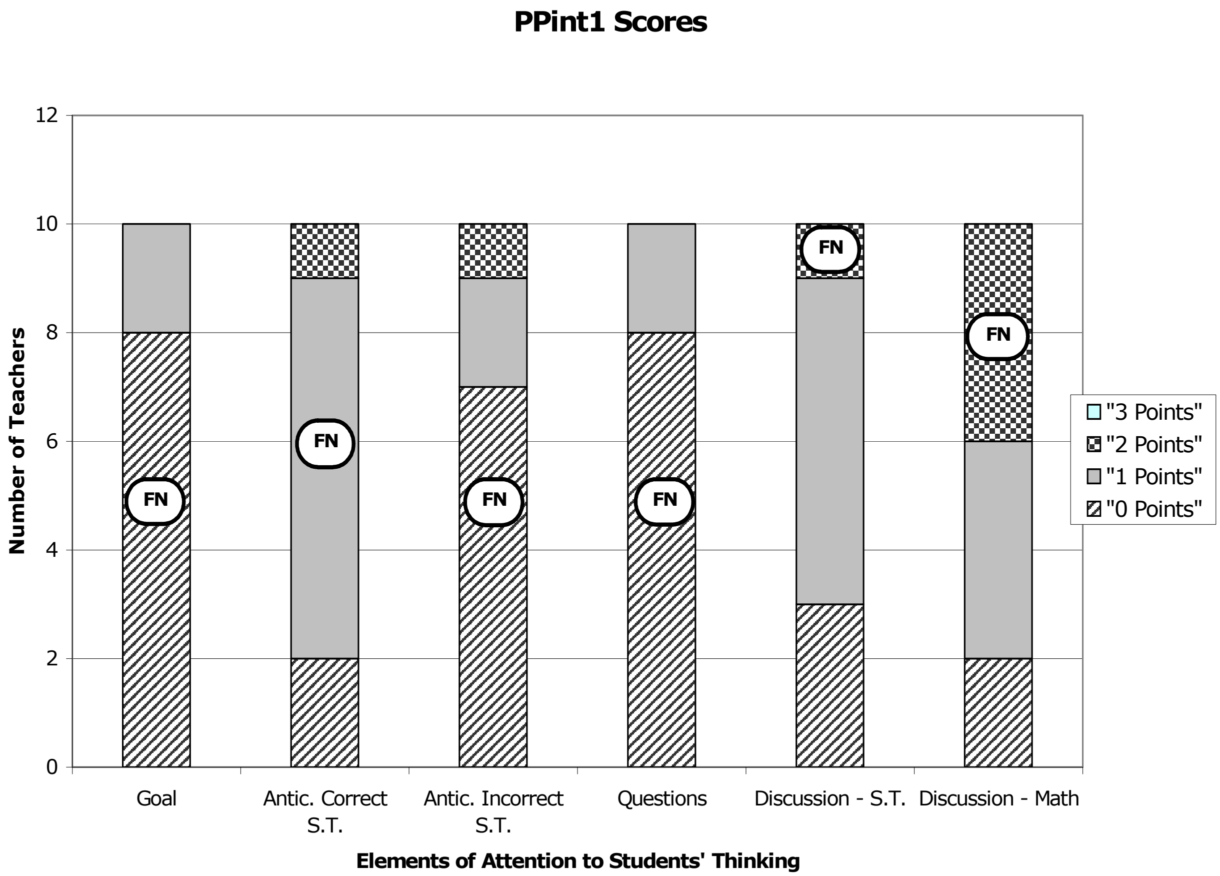


Figure 4.6 PPint1 Scores with Faith Norris Highlighted

Prior to her participation in the Teaching Lab, Faith demonstrated limited evidence of attention to students' mathematical thinking when planning a lesson on demand, as shown in her initial Pledge Plans lesson-planning interview (PPint1). Faith scored similarly to her peers on PPint1 for five of the six elements of attention to students' thinking, as shown in Figure 4.6. Her scores on PPint1 (see column two of Table 4.24) indicated that prior to the Teaching Lab she showed no evidence of having an explicit Mathematical Goal for the lesson, Anticipating Students' Incorrect Thinking, or Identifying Questions to Assess and Advance Students' Thinking as they worked. In her initial Pledge Plans interview, Faith did show limited evidence of Anticipating Students' Correct Thinking, by making vague suggestions of the ways in which students might solve the problem (e.g., “equations to represent the problem”, “solve for the intersection point of a line”, “graphs of the lines”). In a striking contrast to her peers prior to participating in the Teaching Lab (as shown in Figure 4.6), Faith had planned to orchestrate a whole-class discussion that built on students’ thinking, as illustrated in the following excerpt:

> And then the teacher asks one person to show the class how he or she came up with the equations to represent the problem, and then the teacher asks the class a question if the student hadn’t done so, like what the values of one point five, two point five, and point seven five represent in each equation. And also what four represents so that they go over slope and Y intercepts. And then another student shows how to solve for the intersection point of a line, just like algebraically, and then another student comes up and graphs the lines and explains how the best plan for the model depends on the number of kilometers. If like the value of X, which represents number of kilometers is less than the intersection point or greater than [FN, PPint1].

This excerpt provides evidence that Faith had planned the whole-class discussion to build on students’ thinking with specificity about 1) what student work she would like to have presented, and in what order (e.g., having equations presented first “and then another student shows how to solve for the intersection point of a line”) and 2) specific questions related to specific student work that she would ask the class in order to highlight mathematical ideas in the student work (e.g., questions to ask about the equations presented such as “what the values of one point five, two point five, and point seven five represent in each equation” which would highlight the value of the slope in the equation and that within the context of the problem these values represented the cost per km for each of the three pledge plans). In addition, Faith’s PPint1 showed evidence of her attention to Orchestrating a Discussion that Made Salient the Mathematics of the lesson. This could be seen in the earlier excerpt as well as the additional questions she said she would ask the class (e.g., “Does this model work if students have a different number of sponsors?”, “Can the lines have negative y-values in this problem?”). Thus, prior to participating in the Teaching Lab, Faith showed evidence of attention to students’ thinking for two of the six elements that were the focus of this study: Orchestrating a Discussion that 1) Builds on Students' Thinking and 2) Makes Salient the Mathematics of the lesson. With respect to the other four elements of attention to students' thinking, Faith showed little or no evidence of thinking about them when planning a lesson on demand.

After participation in the Teaching Lab, Faith showed marked improvement in her attention to the four elements she had previously neglected. Furthermore, she demonstrated a remarkable ability to provide a high level of specificity for all the elements of attention to students' thinking. For example, the written lesson plan that Faith turned in for her PTR1 assignment earned the maximum possible score for each of the elements of attention to students' mathematical thinking. Thus by the end of the Teaching Lab, Faith was capable of meeting the highest expectations of attention to students' thinking when planning a lesson in which she had ample time to plan and was explicitly asked to attend to students' thinking by using the TTLP as a guide when planning. It is also worth noting that she did seek feedback from the instructor when working on her PTR1. Thus her PTR1 score may reflect the feedback she received rather than her own ability to meet the highest expectations of attending to student thinking. However, her second Pledge Plans interview, conducted immediately after her participation in the Teaching Lab, echoed the capability she demonstrated in her PTR1 to attend to students' thinking when planning. For example, in her PPint2 lesson plan she earned the maximum score possible for four of the six elements of attention to students' thinking. The two exceptions were 1) having an explicit Mathematical Goal for the lesson that focused on the mathematical understanding students would develop and 2) Orchestrating a Discussion that Makes Salient the Mathematics of the lesson. For example, in her second Pledge Plans interview, Faith had an explicit Mathematical Goal:

> The goal is for students to be able to write equations of lines for the different plans and to determine the intersection points of these lines in order to determine which plan is best for different x values or different number of kilometers [FN, PPint2].

Unlike the goal she had written for her PTR1 lesson plan that focused on the conceptual understanding of students, this goal pointed out the actions students needed to take in order to solve the problem rather than the mathematical concepts students might develop through the lesson. The second element of attention to students' thinking that Faith showed limited attention to in her second Pledge Plans interview was Orchestrating a Discussion that Makes Salient the Mathematics of the lesson. Faith had spent the allotted twenty minutes focusing on many of the details associated with attending to students' thinking and had not completed her thoughts about planning the whole-class discussion. She articulated the mathematical ideas she hoped to get out of the discussion but could not think of specific questions to ask on demand during the interview.

In summary, after the Teaching Lab, Faith had shown tremendous growth and was capable of earning the maximum scores for attention to students' thinking when planning a lesson for her PTR1 assignment. In her second Pledge Plans interview, she displayed similar abilities in her attention to students' thinking. Therefore, Faith was poised to begin her field experience with the knowledge necessary and ability to attend to students' mathematical thinking in her lesson planning practices.

Faith submitted lesson plans for a pre-calculus course in a suburban high school. As seen below in the last column of Table 4.25, Faith's average lesson plan scores for her written lesson plans around high-level tasks echoed the exemplary attention to students' thinking that she had displayed after her participation in the Teaching Lab. However, her planning for a lesson that used a low-level task was quite different (see column five of Table 4.25 below). Table 4.25 provides the scores for each of Faith's six lesson plans with respect to each of the elements of attention to students' thinking. Her second lesson plan in November (Nov LP2 in column five) was the only lesson plan she submitted that used a low-level task.

Table 4.25 Faith Norris' Scores for Written Lesson Plans In Practice

Faith Norris	**Oct LP1**	**Oct LP2**	**Nov LP1**	**Nov LP2**	**Dec LP1**	**Dec LP2**	**AvgLP-LL**	**AvgLP-HL**
Level of Cognitive Demands of the Task	High	High	High	Low	High	High		
Goal Max Score = 2	2	1	2	0	1	1	0	1.17
Anticipating Students' Correct Thinking Max Score = 3	3	3	3	0	3	3	0	2.0
Anticipating Students' Incorrect Thinking Max Score = 3	3	3	3	0	3	3	0	1.83
Questions to Assess and Advance Students' Thinking Max Score = 2	2	2	2	0	2	2	0	1.17
Discussion Building on Students' Thinking Max Score = 2	2	2	2	0	2	1	0	1.17
Discussion Making the Mathematics Salient Max Score = 2	2	2	2	2	2	2	2	1.67
Total Lesson Planning Score Max Score = 14	**14**	**13**	**14**	**2**	**13**	**12**	**2**	**13.2**

In this set of six lesson plans, the NovLP2, that used a low-level task and received a low score for attention to students' thinking, appeared to be an anomaly in Faith's lesson planning practices. However, it is actually much more common in her actual practice. In her final interview, Faith talked about two different types of lesson plans that she writes: 1) a "really long lesson plan" once a week that usually covers two to three class days and served as an introduction and discovery time for a new mathematical idea and 2) "daily ones" that were much shorter, hand written, and usually came after the lessons covered in the long lesson plan and were meant to serve as review or lessons where students could apply and practice what they had recently learned. She claimed the one thing that she puts in all of her lesson plans is specific questions to ask students. In the following excerpt, Faith described her "daily lesson plans" and how they differ from the "really long ones":

> I might not necessarily separate them [questions] into different categories... Like I have on these, separated to like if students are having trouble getting started, if students are having trouble graphing, I wouldn't like differentiate the different types of questions usually... Usually I'll probably just separate them, into assessing and advancing questions. I don't go into detail a lot about my reasoning for the ordering during the share and discuss. Sometimes I might not even say "share and discuss" because I kind of know what I want to do. And I don't do possible misconceptions and possible errors all the time … Let's see, possible solutions, I probably wouldn't go through every possible solution, because usually like just based on their prior knowledge, I have a really good idea of what most of the students are gonna do… It just depends, like normally, I just have one long lesson plan per week and like I said, that's what I use, you know because a lot of the questions end up coming up again and again [FN, Final Interview].

Evidence of the two types of lesson plans described above is clear when comparing two consecutive lesson plans she turned in for November.

Faith submitted a "really long" lesson plan that was used for two class days (November 17 and 18)[5]. The written lesson plan was typed, approximately 13 pages long, and employed a variety of bulleted lists to organize the document. The 2-day lesson was around two problems:

> PROBLEM 1:
>
> A) Using a graphing calculator, plot the following pair of graphs on the same set of axes and then draw them on graph paper on the same set of axes. f(t) = sin(t) and g(t) = Asin(t)
>
> B) How does the constant A affect the sine curve? Graph g(t) = Asin(t)
>
> PROBLEM 2:
>
> A) Using a graphing calculator, plot the following pair of graphs on the same set of axes and then draw them on graph paper on the same set of axes. f(t) = sin(t) and g(t) = sin(bt)
>
> B) How does the constant b affect the sine curve? Graph g(t) = sin(bt)

Faith's written goals for the 2-day lesson were as follows:

> Goals: 1) Students will notice changes in the sine graph when the A in Asin(t) and the b in sin(bt) change. 2) Students will make a generalization about how the magnitude of A in Asin(t) affects the amplitude of a sine function. Students will be able to graph f(t) = Asin(t). 3) Students will make a generalization about how the numerical value and sign of the b (the coefficient of t in a sine function) affects the period of a sine function. Students will be able to graph f(t) = sin(bt). [FN, NovLP1].

Faith then Anticipated Students' Thinking by explicating multiple ways in which the students could use their graphing calculators, explore different values of A, and ways of showing that the amplitude is equal to the magnitude of A. She highlighted possible misconceptions or troubles students might encounter in general when working on each problem as well as specific difficulties related to the solution strategies she anticipated students using (e.g., "students may think that the amplitude is just A and not $|A|$ if they do not try negative values for A"). She

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[5] As indicated in Chapter 3, this was also her PTR2 lesson plan for Methods 1. However, other than being a few pages longer than her other "really long" plans, its content and scores were consistent with the 4 other "really long" plans she submitted.

provided the same level of detail when Anticipating Students' Correct and Incorrect Thinking for the second problem. For problem one and two together, her written plan provided three pages of information with respect to anticipating students' thinking about these two problems. For each of the two problems, Faith Identified Specific Questions to Ask Students as they worked that were targeted towards students' mathematical thinking (e.g., questions to ask if students were having trouble "getting started", "graphing", or "interpreting the graphs and summarizing"). Finally, Faith's written plan had many details about how she would Orchestrate a Discussion that Built on Students' Thinking and Made Salient the Mathematics of the lesson. For example with problem one, she first would have a student present the various values he/she tried for A in a graph then she would "ask the class what they noticed was the same or different between the graphs of sin(t), 2sin(t), .5sin(t), etc."[FN, NovLP1]. Then she would have a student present a "table for the different values of t and their corresponding function values for 2sin(t), .5sin(t), 3sin(t), etc.", and would ask, "what they noticed about the maximum and minimum values of the functions" [FN, NovLP1]. Faith then offered a series of questions in which she would "try to connect the function, the graph, and the table". Last, she would "have someone who has come to the conclusion that the amplitude of Asin(t) is equal to the magnitude of A present part of their solution on poster paper" [FN, NovLP1]. She would then "ask the students what the domain and range of the functions are and if either of them or both seem to be affected by the different values for A" and "how we can find some of the points for Asin(t) for different values of A using points that satisfy sin(t)" [FN, NovLP1].

As was illustrated with examples from Faith's NovLP1, in her "really long" lesson plans, she provided high-levels of specificity in her attention to students' thinking, with respect to all six elements that were the focus of this study. Four other written lesson plans submitted by Faith were also “really long” and mirrored the attention to students’ thinking that was shown in her NovLP1 (although they were somewhat shorter in length, ranging from 8-11 pages each). These “really long” lesson plans were in stark contrast to the “daily” lesson plan she submitted for her NovLP2. NovLP2 was designed for her class on November 21 (the class following the two class days described above in NovLP1). This lesson plan was handwritten and approximately three pages long. The written plan consisted of 1) a title, “Period of Sine 4.4”, 2) directions, “Graph 2 periods. Label adjusted quadrantal points. No calc!”, and 3) four functions (f(x) = sin2x; f(x) = sin(1/2x); f(x) = sin(3x); and f(x) = 2sin(2x)). For each of the four functions, Faith had a solution that consisted of a table of x and f(x) values and a graph of the function. She also had specific questions to ask students related to each function (e.g., “Is the function stretching/shrinking”, “by how much?”, “in which direction?”, and “what is the new period?”). In addition, she had a series of questions to ask after the fourth function that could summarize the mathematical ideas Faith wanted students to take from the review lesson. The lesson plan concluded with a list of 12 questions under the title “review for test”. With respect to the six elements of attention to students’ thinking, this written plan provided evidence that Faith had considered only one of the elements: Orchestrating a Discussion that Made Salient the Mathematics of the lesson. However, it is important to consider 1) the placement of this lesson plan immediately after the extensive lesson plan written for the previous two lessons and 2) Faith’s own description of how she uses these two different types of lesson plans (e.g., “I just have one long lesson plan per week and like I said, that’s what I use, you know because a lot of the questions end up coming up again and again” [FN, Final Interview]). Thus, it is possible to infer that Faith had thought about more of the elements of attention to students’ thinking than what was evident in her written plan.

In summary, Faith showed evidence of applying the knowledge and ability to attend to students' thinking that she had gained from her participation in the Teaching Lab to her lesson planning in practice during her field experience. She attended to students' thinking with great specificity and in a variety of ways, as evident in her high (often reaching the maximum) scores for the six elements of attention to students' thinking that were examined in this study. Faith's third Pledge Plans lesson-planning interview and PTR2 assignment for her Methods 1 course, both occurring towards the end of the first semester of her field experience, echoed the results of her post-Teaching Lab lesson plans and written lesson plans during her field experience.

In her final interview, Faith talked about some of the ways in which she was supported in her efforts to plan lessons that attended to students' thinking. For example, she said the TTLP had influenced her practice of lesson planning during her field experience in that "I am always constantly thinking of prior knowledge and the types of questions I can ask. So that's influenced that. And I am always thinking about what kind of questions I can use in the share and discuss and the exploration" [FN, Final Interview]. Faith also said that her mentor teacher played a role in her lesson planning. Specifically, her mentor teacher shared some of her own practical knowledge about students' mathematical thinking as well as ways in which Faith could alter her lesson plans in light of how students may react to the lesson. Her mentor teacher's influence was illustrated in the following excerpt:

> We always discuss stuff like, I'll be like, do you think this is okay? For this lesson that I'm going to do and you know if she thinks there's something I need to change, something I might add or take out that's better for the students that maybe I should say okay, like this is one of the things I need to remind them, "make sure your calculators are in radian mode", like that kind of stuff [FN, Final Interview].

However, it was Faith's university supervisor who seemed to have the most influence on her lesson planning. She often talked to her supervisor about her lessons prior to her supervisor coming to her school for an observation and conference as evident in her statement: "I actually called him the other night before he was coming for a lesson because I wanted some ideas…so I do talk to him about it. Or when I send him my lesson plan, he'll send me suggestions and we also discuss it before the lesson" [FN, Final Interview]. Furthermore, it appeared that some of the suggestions her supervisor offered about her lesson plans were explicitly related to elements of attention to students' thinking. For example, when talking about the ways in which her lesson planning had changed over the semester, she said she was "better at coming up with questions". When pressed to say more about this, it was apparent that she had been actively working with her supervisor on improving her ability to come up with questions in her lessons, as illustrated in the following excerpt:

> I think I'm better at, I mean especially during the exploration, I think I am because like if I see their work, I'm constantly thinking, okay I've got to assess them first and then I need to give them an advancing question. So, usually he's [supervisor] telling me okay why, you could just say "tell me about your graph" or, you know "what have you done so far?" So during that part of the lesson, I think I'm pretty good at that. But "share and discuss" is where I'm trying to work on more and more…Yeah like my last lesson plan I did, last week one of my goals I talked about with my supervisor was try to work on bringing it all together, at the end. So now I'm trying to think about that more. Like how can I conclude that better…I try to work on better questions for "share and discuss" rather than just procedural questions, so more open ones [FN, Final Interview].

Thus it may be reasonable to conclude that Faith's success in implementing what she had learned about lesson planning with respect to attention to students' thinking during the Teaching Lab was not only a result of her own abilities but also a result of being in an environment where her mentor teacher and especially her university supervisor were actively supporting her in her efforts to attend to students' thinking in her lesson planning.

CHAPTER 5: DISCUSSION

5.1 Importance of this Study

In chapter one, the argument was presented that a growing body of literature (e.g., Ball, Lubienski, & Mewborn, 2001; Lampert, 2001; NCTM, 1991; Stigler & Hiebert, 1999) has recognized the importance of teachers focusing on students' mathematical thinking in order to implement standards-based instruction. While focusing on students' mathematical thinking is identified as an important part of standards-based instruction, it is also acknowledged as very challenging for teachers (Ball, 2001; Ball, Lubienski, & Mewborn, 2001; Chazen & Ball, 1999; Lampert, 2001; Schifter, 2001; Schoenfeld, 1998; Sherin, 2002). When teachers do successfully make use of students' thinking in the process of their teaching, however, evidence suggests that these practices are most often seen in 1) "expert" teachers in the U.S. (e.g., Ball, 1993; Lampert, 2001; Leinhardt, 1993; Schoenfeld, 1998; Schoenfeld, Minstrell, & van Zee, 2000) or 2) Japanese elementary teachers involved in the professional development program of lesson study (e.g., Lewis & Tsuchida, 1998; Stigler & Hiebert, 1999; Yoshida, 1999). For beginning teachers, this can be a daunting, if not seemingly impossible, task (Heaton, 2000; Schoenfeld, 1998; Sherin, 2002; Wagner, 2003; Zimmerlin & Nelson, 2000). Therefore, it is not surprising that research showed most U. S. teachers do not focus on students' mathematical thinking in the classroom in ways that enable them to implement standards-based instruction (NCES, 2003; Stigler & Hiebert, 1999; Weiss & Pasley, 2004). Thus there is a need for professional education that will develop teachers' capacity to focus attention on students' mathematical thinking during their instruction. The mathematics education community has responded to this call with various professional development initiatives. While it is encouraging to see professional development projects affecting teachers' practice in a desired way, most of the research to date has been with K-5 practicing elementary teachers. Little research has investigated whether practicing or pre-service secondary teachers can learn to focus their teaching practices on students' mathematical thinking. One promising starting point for improving teachers' instruction and ultimately student achievement is in the development of teachers' lesson planning process. Specifically, there is evidence that focusing on students' mathematical thinking in the process of planning a lesson is a critical part of improving instruction and student achievement (e.g., Lampert, 2001; Lewis & Tsuchida, 1998; Stigler & Hiebert, 1999). However, there is very little empirical research on the lesson planning process of mathematics teachers in the United States. Specifically, there is little data on the ways in which pre-service secondary mathematics teachers attend to students' mathematical thinking in their lesson planning.

The purpose of this study was to investigate the ways in which pre-service secondary mathematics teachers' ability to attend to students' mathematical thinking during lesson planning developed over the course of their first semester in a teacher education program that emphasized students' mathematical thinking as a critical and key component of the planning process. In particular, the study investigated the extent to which attention to students' mathematical thinking was evident in the teachers' written lesson plans or lesson planning process at various points during their teacher education program: prior to and immediately after participation in a course that emphasized students' mathematical thinking as a key element of planning, during teachers' first semester of their field experience as they planned lessons in their actual practice of teaching, and near the end of the first semester of their field experience as they planned lessons on demand and for university assignments. The extent to and the ways in which teachers' attention to students' mathematical thinking, as evident in their lesson planning, changed over time was also explored.

The results of the study provide evidence of teachers' attention to students' thinking in their lesson planning with respect to two important aspects of the study: 1) teachers learning from a course (the Teaching Lab) that emphasized students' mathematical thinking as a key element of planning and 2) teachers applying the ideas they learned in their own practice. With respect to learning from the Teaching Lab, the study shows that pre-service secondary mathematics teachers did learn to attend to students' mathematical thinking, vis-à-vis the six elements that were the focus of this study, in their lesson planning. In particular, teachers demonstrate significant growth on pre to post course measures in their ability to attend to students' thinking when planning a lesson on demand and for a university assignment. In addition, teachers continued to be able to apply these ideas on demand and for university assignments several months later.

When investigating whether or not teachers would apply the ideas they had learned when planning in their own practice, the results suggest three compelling findings. First, evidence suggests that teachers attend to students' thinking, as evident in their written lesson plans, less in their daily planning than when planning on demand or for a university assignment. However, when the cognitive demands of the task were taken into consideration, teachers' attention to students' thinking when planning lessons that used high-level tasks was not significantly different from their attention to students' thinking when planning a lesson on demand or the PTR lesson plan they produced for the Teaching Lab. Furthermore, teachers were more likely to attend to students' thinking when planning a lesson that used a high-level task compared to a lesson that used a low-level task. Second, for some teachers, their written lesson plans significantly under-represent their attention to students' thinking in their planning process. In other words, these teachers were thinking about the elements of attention to students' thinking as they planned their lessons and then chose not to explicitly write all their thoughts down. Finally, the study suggests that support from the teacher's mentor teacher and/or university supervisor may be an important factor in determining whether or not the teacher applies their knowledge of attention to student's thinking to their planning in practice.

In summary, a growing body of literature (e.g., Ball, Lubienski, & Mewborn, 2001; Lampert, 2001; NCTM, 1991; Stigler & Hiebert, 1999) has recognized the importance of teachers focusing on students' mathematical thinking in order to implement standards-based instruction. Furthermore, as described in chapters one and two, there is also evidence that suggests that focusing on students' mathematical thinking in the process of planning a lesson is a critical part of improving instruction and student achievement (e.g., Lampert, 2001; Lewis & Tsuchida, 1998; Stigler & Hiebert, 1999). Therefore, based on evidence from prior research, teachers' improved attention to student's thinking when planning as identified in this study, holds potential for improving teachers' implementation of standards-based instruction and ultimately improving students' opportunities for learning.

5.2 Explanations and Implications of the Results

This section highlights some of the results found from the study and then draws on prior research to provide 1) some possible explanations for results, 2) a perspective on the findings with respect to research in the field of mathematics education, and 3) implications for teacher education programs.

5.2.1 Types of Planning and the Role of the Cognitive Demands of the Task

In this study three types of lesson planning are investigated: 1) planning a lesson on demand, 2) planning for a university assignment where teachers have extensive time to prepare the lesson plan, are explicitly asked to attend to students' mathematical thinking by using the

Thinking Through a Lesson Protocol (TTLP) as a guide, and are graded on their adequate response to the questions on the TTLP, and 3) planning in the actual practice of teaching. Results indicate there was very little difference between teachers' attention to students' thinking when planning a lesson on demand and for university assignments. Because of the similarity between teachers' planning under these two circumstances, it is possible to conclude that teachers have begun to internalize the elements of the TTLP that they had focused on during their lesson planning for the university assignments and are able to duplicate that level of attention to students' mathematical thinking when planning on demand. It is possible that teachers came to believe in the importance of attending to students' thinking when planning and thus their lesson planned on demand reflects their beliefs about what should be in a lesson plan. Another explanation is that teachers came to know what the university instructors and researcher value and therefore chose to produce it. Regardless of their motivation, when planning a lesson on demand, the teachers demonstrate that they understand how to and were able to attend to students' thinking in their lesson planning, without having the TTLP to look at as a guide.

When planning lessons in their own practice of teaching, results of this study indicate there is a significant relationship between teachers' attention to students' thinking and the level of cognitive demands of the task (as defined by Stein, Grove & Henningsen (1996)). In other words, teachers are more likely to attend to students' thinking when planning a lesson around a high-level task. Thus many teachers were applying in their own practice what they had learned at the university about attending to students' thinking when planning lessons around high-level tasks and not for lessons that use low-level tasks. This raises several questions: is the TTLP applicable as a guide in planning a lesson around a low-level task? Are there ways to attend to students' thinking when using a low-level task? Does this attention look different from the six elements of attention to students' thinking that were the focus of this study, and if so, in what ways? What purpose does attending to students' thinking serve when using a low-level task?

As Stein, Grover, and Henningsen (1996) point out, low-level tasks are not "bad" or "inappropriate" tasks. If, for example, the goal of a lesson is for students to memorize formulae or become more efficient in using a given procedure then tasks that require low levels of cognitive demands are appropriate. If on the other hand, the goal of a lesson is for students to think and reason mathematically, then the lesson needs to use a high-level task (Stein, Grover, & Henningsen, 1996). I would argue that the TTLP can be applied when planning a lesson that uses a low-level task, but that such planning may not be needed in the same way due to: 1) the ease/difficulty in attending to students' thinking, based on the complexity of the mathematical task; and 2) the potential pay off in terms of student learning. First, because a low-level task is constrained in the ways it can be solved and discussed, it would be easier to predict students' thinking and there would be fewer potential paths the lesson could take. Stein and her colleagues have argued that high-level tasks are more difficult to enact well because they are more unpredictable with respect to what the students' will think and do. Therefore, high-level tasks require teachers to anticipate students' thinking and plan appropriate responses more so than with low-level tasks (Henningsen & Stein, 1997; NCES, 2003; Weiss & Palsey, 2004). Second, the benefits of attending to students' thinking and costs of not attending to students' thinking with respect to student learning outcomes is quite different for low- and high-level tasks. For a lesson that uses a low-level task, the learning outcomes for students remain at a level of low cognitive demands (i.e., procedural rather than conceptual understanding) (Stein & Lane, 1996). However, for a lesson that uses a high-level task the learning outcomes for students have the potential to remain at a high-level (i.e. conceptual) or decline to a more procedural level of understanding, depending on the enactment of the lesson (Stein & Lane, 1996). In other words, if the teacher can implement the lesson in such a way that the high-level of cognitive demands of the task are maintained, then students are likely to come away from the lesson with a deeper, conceptual understanding of the mathematics they are learning. While, this study did not investigate the enactment of the planned lesson or the subsequent student learning, prior research has shown that planning a lesson in which teachers have paid attention to students' thinking is an important aspect of implementing a lesson in which students could develop conceptual understandings (e.g., Lampert, 2001; Lewis & Tsuchida, 1998; Stigler & Hiebert, 1999). Thus planning a lesson around a high-level task in which the teacher has attended to students' thinking serves as an indicator of the teachers' potential to implement the lesson in such a way that the cognitive demands of the task are maintained. In summary, spending time attending to students' thinking, vis-à-vis the six elements of attention to students' thinking that were the focus of this study, may be easier to do for a low-level task, however, it is not likely to have a significant impact on students' developing understanding. In contrast, attending to students' thinking when planning a lesson for a high-level task may be quite time-consuming and challenging, but has the potential to positively impact student learning outcomes.

Therefore, it is encouraging that when planning a lesson in practice that uses a high-level task, this study finds that teachers' attention to students' thinking mirrors in many ways their planning on demand and is only slightly less than their attention to students' thinking when planning for a university assignment. Furthermore, the significance of these novice teachers being able to attend to students' thinking in their practice of lesson planning should not be underestimated. As discussed in chapters one and two, attending to students' thinking is 1) an extremely challenging aspect of implementing standards-based instruction (e.g., Ball, 2001; Ball, Lubienski, & Mewborn, 2001; Chazen & Ball, 1999; Lampert, 2001; Schifter, 2001; Schoenfeld, 1998; Sherin, 2002) and 2) rarely seen in the teaching practices of novice teachers (e.g., Borko & Livingston, 1989; Heaton, 2000; Schoenfeld, 1998; Sherin, 2002; Wagner, 2003; Zimmerlin & Nelson, 2000).

Finally, as shown in this study, using a high-level task does not guarantee teachers will attend to students' thinking in their lesson plans. In addition, prior research has shown that using a high-level task does not necessarily mean that the teacher will maintain the cognitive demands of the task when implementing the lesson (e.g., NCES, 2003; Stein & Lane, 1996). However this study has shown that using a high-level task does make it more likely that teachers will attend to students' thinking and provide more specificity in their lesson planning. In addition, prior research has shown that planning a lesson in which teachers have paid attention to students' thinking is an important aspect of implementing a lesson in which students could develop conceptual understandings (e.g., Lampert, 2001; Lewis & Tsuchida, 1998; Stigler & Hiebert, 1999). Thus planning a lesson around a high-level task in which the teacher has attended to students' thinking may serve as an indicator of the teachers' potential to implement the lesson in such a way that the cognitive demands of the task could be maintained. Additional research is needed on the relationship between attending to students' thinking when planning a lesson around a high-level task and whether or not the cognitive demands of the task are maintained during enactment of that lesson.

In summary, this study adds to the fields' limited research on secondary mathematics pre-service teachers' lesson planning processes. The finding that these novice teachers can learn to attend to students' thinking when planning a lesson for a high-level task and furthermore can apply it to their own teaching practice during the first semester of their field experience are encouraging. The findings of this study suggest that focusing on teachers' ability to attend to students' thinking during the planning process for lessons that use high-level tasks is a critical and fruitful starting point for teacher education programs aimed at helping teachers develop the capacity to implement standards-based instruction. The factors that influence teachers' growth in their ability to attend to students' thinking and the subsequent application of their knowledge to lesson planning in their own practice is explored in more detail later in this section and have specific implications for teacher education programs. In addition, further investigation into the ways in which tasks influence teachers' attention to students' thinking is warranted. For example, is it easier for teachers to anticipate students' correct and incorrect responses for a "procedures with connections" task than a "doing mathematics" task (Stein, Grover, Henningsen, 1996)? Are some mathematical concepts more difficult to plan for with respect to attending to students' thinking? How does the teachers' own mathematical understanding and ability to solve the task in multiple ways affect their capacity to attend to students' thinking when planning?

5.2.2 Teachers' Written Lesson -Plans versus Their Thinking when Planning

Prior research has indicated that teachers' written lesson plans under-represent teachers' thinking when planning. In other words, much of what the teachers think about when planning a lesson does not make it into a written lesson plan (e.g., Clark & Peterson, 1986; Livingston & Borko, 1990; Schoenfeld, 1998). This study compared one written lesson plan per teacher to that teachers' verbal description of planning for that same lesson. Thus the findings may not necessarily apply to all written lesson plans the teachers submitted. However, the results were surprising when compared to previous studies. This study found that written lesson plans under-represented teachers' thinking for only 30% of the teachers, with respect to their attention to students' thinking vis-à-vis the six elements that were the focus of this study. For these three teachers, there was evidence that the teachers thought about students' thinking significantly more than what was evident in their written lesson plan. However, for the majority of teachers in this study, the only significant difference between teachers' attention to students' thinking when planning, as evident in their written plan, was their Anticipation of Students' Incorrect Thinking. When the teachers talked about the things that they thought about but do not include in their written plans, the following were most prevalent: 1) the tools and materials needed for the lesson, 2) how the students would work (e.g., independently, in small groups, in pairs, as a whole class), 3) the time allotted for various portions of the lesson, 4) students' prior knowledge and experiences, and 5) the possible misconceptions, questions, errors, or troubles students would have with the lesson. Hence, the teachers' written lesson plans did under-represent their thinking about the lesson, however little of what was missing in the written plan was related to students' thinking. While this study provides evidence that suggests, for most teachers, the attention to students' thinking as evident in their written lesson plan was likely an accurate account of their attention to students' thinking as they planned, there are limitations to this study. For example, teachers had the opportunity to expand on their written lesson plans with only one lesson plan. Furthermore, the level of the task used in these plans was not held constant. For example, seven of the ten teachers used a high-level task, while KE, KK, and DM used low-level tasks. Results may have been different for DM if she had brought in a lesson plan that used a high-level task for her interview since she was one of the teachers for whom task level made a significant difference in her attention to students' thinking[6]. Finally, the teachers were not equally pressed to elaborate on their thinking with respect to each of the six elements of attention to students' thinking. Therefore, it is not known whether or not they did not think about an element, only

[6] KE showed no evidence of planning differently for high- versus low-level tasks. KK only turned in lesson plans that used low-level tasks.

that they did not provide evidence of thinking about an element during their interview. Hence, additional research would be useful in validating the findings of this study.

5.2.3 Factors Influencing Teachers' Planning

While this study was not specifically designed to identify the factors influencing teachers' lesson planning, it does provide some information about the role the following played in teachers' attention to students' thinking as they planned: 1) teacher education coursework, 2) textbook/curriculum used in the field experience, and 3) mentor teacher and university supervisor.

<u>Teacher education coursework</u>. This study shows a relationship between teachers' attention to students' thinking when planning a lesson and the level of cognitive demands of the task used in the lesson. In other words, teachers are more likely to attend to students' thinking in their plans for lessons that use a high-level task. This in addition to the fact that teachers showed significant growth in their ability to attend to students' thinking when planning a lesson after participating in the Teaching Lab indicate that the opportunities teachers had in their teacher education program to learn about the importance of and how to attend to students' thinking when planning a lesson around a high-level task were effective. While further investigation into which particular activities best supported teachers' learning is needed, there is evidence to suggest that the teacher education program's 1) practice-based approach and 2) use of tools (e.g., MTF and TTLP) were effective in supporting teachers' learning. Practice-based professional learning tasks (Ball & Cohen, 1999; Smith, 2001) were employed throughout the teacher education program's coursework. For example, teachers had opportunities to engage in 1) analyzing authentic student work from a mathematical task in order to gain practical knowledge of students' correct and incorrect thinking about a task that could then be drawn upon when

planning a lesson around the same task and 2) analyzing cases (narrative and video) that depict episodes of mathematics instruction in which students' mathematical thinking is portrayed and the ways in which teachers' attention to students' thinking can help maintain the cognitive demands of a high-level task are made salient (see Chapter Three's description of the Teaching Lab course for more details on these and other activities). In addition, tools such as: 1) the *Mathematical Tasks Framework,* which focuses on the cognitive demands of tasks as they are selected and implemented in an instructional episode as well as the factors associated with the maintenance and decline of high-level cognitive demands (as depicted in Stein, et al., (2000) which served as the textbook for the Teaching Lab); and 2) the *Thinking Through a Lesson Protocol,* which provides teachers with a guide to focus on students' thinking when planning a lesson, were used to teach teachers' the importance of and how to attend to students' thinking when planning a lesson around a high-level task. Therefore this study adds to the literature showing practice-based professional learning tasks are effective in increasing teachers' ability to attend to students' thinking (e.g., Barnett, 1998; Crespo, 2000; Hughes & Smith, 2004; Kazemi & Franke, 2004; Schifter, 1998; Stein et al., 2006).

<u>Textbook/curriculum used in the field experience.</u> Since the results of this study indicate an important relationship between teachers' attention to students' thinking when planning and the level of cognitive demands of the task used in the lesson, it is reasonable to question whether the curriculum teachers used in their field placement influenced the frequency of high-level tasks used in their lessons and hence their attention to students' thinking when planning. In this study 60% of the tasks came from units in which a traditional curriculum[7] was the primary textbook for

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[7] The criteria used by Boston (2006) was used here to identify reform-oriented curriculum as a curriculum identified as "exemplary or promising curricula" by the U. S. Department of

the course. Five of the teachers used a traditional curriculum and three teachers used a reform curriculum as their primary textbook for their course. The remaining two teachers used units from a reform curriculum as well as units from a traditional curriculum in their courses. Table 5.1 displays the number of tasks that were high- and low-level in their cognitive demands and whether or not the task was from a unit in which the primary textbook used was a traditional or reform-oriented curriculum.

Table 5.1 Number of Tasks by Cognitive Demand Level and Type of Curriculum

	Reform Curriculum	Traditional Curriculum	TOTAL
High Level Tasks	20	19	39
Low Level Tasks	4	17	21
TOTAL	24	36	

The results shown in Table 5.1 suggest that teachers who use a reform-oriented curriculum were less likely to submit a written lesson plan that used a low-level task than those who use a traditional curriculum. For example, 83% (20/24) of the tasks that were from lessons in units that used a reform-oriented curriculum as the primary textbook were considered high-level tasks. By contrast, 53% (19/36) of the tasks from units using a traditional curriculum were high-level. It is also noteworthy that the lesson plans submitted that used high-level tasks were equally likely to be a part of a unit that used a traditional curriculum as one that used a reform-oriented curriculum. Therefore, using a traditional curriculum did not appear to keep teachers from using high-level tasks in their lessons. However, the results do suggest that teachers were more likely to use a high-level task in their lessons and therefore were more likely to attend to students' thinking in their lesson planning if they used a reform-oriented curriculum. However, this study was not specifically designed to identify the frequency with which teachers use high-level tasks.

Education (USDE, 1999) or were rated highly in the review of mathematics curricula conducted by the American Association for the Advancement of Sciences (AAAS, 2000).

In other words, because this study was not designed to investigate teachers' selection of high-level tasks, specific measures were not taken to ensure that the tasks teachers submitted with their lesson plans were representative of the tasks they usually used in their teaching practice. Therefore, further research is needed to verify the relationship between the frequency of high-level tasks used and the curriculum used by the teacher. In addition, future research could explore the ways in which teachers' curriculum materials did or did not support their attention to students' thinking when planning (e.g., Did the curriculum materials provide information about possible correct and incorrect student responses to anticipate?).

Mentor teacher and university supervisor. Eight of the ten teachers in this study claimed that their mentor teacher influenced their lesson planning process beyond determining the pacing of lessons. Furthermore, four of these eight teachers indicated that their mentor teacher affects their planning with respect to at least one of the elements of attention to students' thinking that was the focus of this study. In addition, half of the teachers stated that their university supervisor influenced their lesson planning in ways that are aligned with the TTLP. This study echoes findings from prior research on teacher education programs that indicate the mentor teacher plays an influential role in the teaching practices of pre-service teachers (e.g., Borko & Mayfield, 1995; Mossgrove, 2006; Vacc & Bright, 1999). Specifically, this study found there is some evidence that pre-service teachers' attention to students' thinking when planning lessons around high-level tasks during their field experience was influenced by the mentor teacher and university supervisor. In other words, when the mentor teacher and/or university supervisor support the pre-service teacher in attending to students' thinking (as shown in the case of Anthony Carter and Faith Norris), the pre-service teacher is more likely to successfully attend to students' thinking when planning lessons for high-level tasks. In contrast, when the mentor teacher and university supervisor do not appear to be pressing the pre-service teacher to attend to students' thinking, as reported by Kevin Edwards, the pre-service teacher may be less likely to apply what he has learned through his university coursework to his own practice. This study was not specifically designed to measure the influence mentor teachers or university supervisors have on teachers' lesson planning and attention to students' thinking. Therefore further research is needed in order to clarify the impact mentor teachers and university supervisors might have on teachers' attention to students' thinking when planning a lesson. Furthermore, while this study suggests that the mentor teacher and university supervisor may influence pre-service teachers attention to students' thinking when planning for a lesson that uses a high-level task, it does not investigate their impact on pre-service teachers' attention to students' thinking when implementing a lesson and their ability to maintain the high-level cognitive demands of the task during the enactment of the lesson. However, in a recent case study of two pre-service teachers, one of which is also in this study (Keith Nichols), Mossgrove (2006) found that the mentor teacher and university supervisor played a significant role in the pre-service teachers' selection and implementation of high-level tasks. Specifically, Keith Nichols had support from his mentor teacher and university supervisor, in the form of "educative mentoring" (Feiman-Nemser, 2001), and was more likely to use high-level tasks in his lessons and to maintain the high-level cognitive demands of the task in his enactment of the lesson than the other pre-service teacher who did not receive such support from her mentor teacher and university supervisor. Additional research is needed in order to understand the influence mentor teachers and university supervisors have on teachers' attention to students' thinking when planning a lesson for a high-level task and ability to maintain the high-level cognitive demands of the task in the enactment of the lesson.

In summary, the results of this study indicate that a teacher education program that 1) emphasizes the importance of and how to attend to students' thinking when planning for lessons that use high-level tasks, 2) engages teachers in practice-based professional learning tasks, and 3) employs tools such as the *Mathematical Tasks Framework* as a framework for the courses and the *Thinking Through a Lesson Protocol* as a guide when planning is effective in supporting teachers' ability to attend to students' thinking when planning a lesson. When it comes to having teachers apply what they learn from their university coursework in their own practice during their field experience, the results of this study echo earlier research suggesting that an alignment between the university teacher education program and the mentor teacher and university supervisor within the field experience is helpful. The study also indicates the curriculum used by teachers in their field experience may affect their attention to students' thinking when planning a lesson by influencing the frequency with which they use high-level tasks in their lessons. Therefore, when teacher education programs are attempting to place pre-service teachers in field experiences that are aligned with the teacher education program, the mentor teacher and university supervisor as well as the curriculum used in the placement combine to make a difference.

5.3 Conclusions and Directions for Future Research

The results of this study are important for several reasons. First and foremost, teachers in the study improved their lesson-planning along dimensions that have been linked to improved implementation of standards-based instruction and to increased students' opportunities for learning. Following their participation in a teacher education course that emphasized attention to students' thinking as a key and critical component of lesson planning, teachers significantly improved in their capacity to attend to students' thinking when planning a lesson. Many of the teachers continued to apply their knowledge of attending to students' thinking when planning lessons in their actual practice of teaching during their field experience, particularly when planning lessons around tasks with high levels of cognitive demands for students. While prior research has shown a strong relationship between teachers' planning and the enactment of the lesson, this study provides no insight into what teachers actually do with the plans they create. Future research will endeavor to directly establish the link between teachers' attention to students' thinking when lesson-planning and teachers' implementation of standards-based instruction in their classrooms and ultimately to student learning outcomes.

Second, in order to further contribute to the field's understanding of mathematics teacher education, future research should explore the ways in which pre-service (and in-service) teachers can best be supported in their effort to attend to the six elements of attention to students' thinking when planning that were the focus of this study. In particular, it would be helpful to understand which specific activities in the teacher education program best supported the development of teachers' ability to attend to each of the six elements of attention to students' thinking. In addition, it is important to identify how best to support teachers in their actual practice of teaching, since this is a location for the potential decline in teachers' attention to students' thinking. This study suggest the mentor teacher and university supervisor may play an important role in supporting teachers' attention to students' thinking when planning during their field experience. Further research is needed to understand the role they play and the ways in which they can best support teachers during their field experience.

Appendix A: Thinking Through A Lesson Protocol

The main purpose of the Thinking Through a Lesson protocol is to prompt you in thinking deeply about a specific lesson that you will be teaching. The goal here is to move beyond the structural components associated with lesson planning (e.g., listing the materials you will need, describing the way students will be grouped, determining teacher actions during the lesson) to a deeper consideration of how you are going to advance students' mathematical understanding during the lesson. This is not to say that structural components of a lesson are not important, but rather that a focus on structural components alone is *not sufficient* to ensure that students learn mathematics.

Selecting and Setting up a Mathematical Task

- What are your goals for the lesson? What mathematical content and processes do you hope students will learn from their work on this task?

- In what ways does the task build on students' previous knowledge? What definitions, concepts, or ideas do students need to know in order to begin to work on the task?

- What are all the ways the task can be solved?
    - Which of these methods do you think your students will use?
    - What misconceptions might students have?
    - What errors might students make?

- How will you ensure that students remained engaged in the task?
    - What will you do if a student does not know how to begin to solve the task?
    - What will you do is a student finishes the task almost immediately and becomes bored or disruptive?
    - What will you do if students focus on non-mathematical aspects of the activity (e.g., spend most of their time making a beautiful poster of their work)?

- What are your expectations for students as they work on and complete this task?
    - What resources or tools will students have to use in their work?
    - How will the students work -- independently, in small groups, or in pairs -- to explore this task? How long will they work individually or in small groups/pairs? Will students be partnered in a specific way? If so in what way?
    - How will students record and report their work?

- How will you introduce students to the activity so as not to reduce the demands of the task? What will you hear that lets you know students understand the task?

Supporting Students' Exploration of the Task

- As students are working independently or in small groups:
  - What questions will you ask to focus their thinking?
  - What will you see or hear that lets you know how students are thinking about the mathematical ideas?
  - What questions will you ask to assess students' understanding of key mathematical ideas, problem solving strategies, or the representations?
  - What questions will you ask to advance students' understanding of the mathematical ideas?
  - What questions will you ask to encourage students to share their thinking with others or to assess their understanding of their peer's ideas?

Sharing and Discussing the Task

- Which solution paths do you want to have shared during the class discussion in order to accomplish the goals for the lesson?
  - Which will be shared first, second, etc.? Why?
  - In what ways will the order of the solution paths help students make connections between the strategies and mathematical ideas?

- What will you see or hear that lets you know that students in the class understand the mathematical ideas or problem-solving strategies that are being shared?

- How will you orchestrate the class discussion so that students:
  - make sense of the mathematical ideas being shared?
  - expand on, debate, and question the solutions being shared?
  - make connections between their solution strategy and the one shared?
  - look for patterns and form generalizations?

- What extensions to the task will you pose that will help students look for patterns, make connections or form a generalization?

TTLP was developed through the collaborative work of Margaret Smith, Victoria Bill and Elizabeth Hughes at the University of Pittsburgh as a tool to support teachers in their efforts to plan for and reflect on instruction.

Smith, M.S. & Bill, V. (2004, January). Thinking through A lesson: Collaborative lesson planning as a means for improving the quality of teaching. Presentation at the annual meeting of the Association of Mathematics Teacher Educators, San Diego, CA.

Hughes, E.K., & Smith, M.S. (2004, April). Thinking Through a Lesson: Lesson Planning as Evidence of and a Vehicle for Teacher Learning. Poster presented as part of a symposium, "Developing a Knowledge Base for Teaching: Learning Content and Pedagogy in a Course on Patterns and Functions " at the annual meeting of the American Educational Research Association, San Diego, CA.

Appendix B: Rubric for Grading Lesson Plans in the Teaching Lab

Solving the Task **3 Points**

3 pts Included solutions represent a range of approaches to the task, varying by representation or strategy where appropriate.
Solutions are fully developed and clear
Solutions include incorrect pathways/note possible misconceptions

2 pts Included solutions represent a range of approaches to the task, with some variation by representation or strategy where appropriate.
Solutions are fully developed and clear
Solutions include incorrect pathways/note possible misconceptions

1 pt Included solutions represent a narrow range of approaches to the task with little variation OR incorrect pathways/misconceptions are not included OR solutions are described in a general way rather than representing fully developed solutions

0 pts Solutions are not included

Mathematical Goal **2 Points**

2 pts An appropriate math goal is included and describes the specific mathematical ideas students should understand and what it means to "understand it"

1 pt An appropriate math goal is included however it is vague in describing the mathematical ideas students should understand or what it means to "understand"

0 pts Math goal is inappropriate (focuses on procedures or topics rather than concepts) or not included

Building on Prior Knowledge **2 Points**

2 pts Prior knowledge that students will have is identified and connected to the mathematical task and the mathematical goal

Questions are included that will help students access their appropriate prior knowledge

1 pt Prior knowledge that students will have is identified, but connections to the mathematical task and the mathematical goal are weak or unspecified OR

Questions that will help students access their appropriate prior knowledge are not included

0 pts No information about how the task builds on prior knowledge

Expectations for Students **1 Point**

1 pt Resources for students to use are identified
Grouping strategies/formats are specified and the means for reporting work is included

0 pt Either resources or grouping strategies and reporting are not included, or both are included and unclear OR no information about expectations for students

Task Setup **2 Points**

2 pts Information about how the teacher will set up the task is included This information is explicitly connected to maintaining a high level of cognitive demand for the task (e.g., a specific solution path is not provided, analogous examples are not demonstrated)

Information about what the teacher will ask and hear from students that lets her know her students understand what the task is asking is clearly provided

1 pt Information about how the teacher will set up the task is included but is not well-connected to maintaining a high level of cognitive demand for the task OR

Information about what the teacher will ask and hear from students that lets her know her students understand what the task is asking is unclear or not provided

0 pts No information about the task setup

Questions: Focus, Assess, Advance **4 Points**

4 pts A variety of questions are listed that have the potential to focus, assess, and advance student thinking
Questions are tied to particular strategies or approaches
Questions are clearly related to the target mathematical goal for the lesson

3 pts A variety of questions are listed that have the potential to focus, assess, and advance student thinking, but one category may be narrowly represented
Questions are loosely tied to particular strategies or approaches
Questions are related to the target mathematical goal for the lesson

2 pts A variety of questions are listed that have the potential to focus, assess, and advance student thinking, but one category is absent or multiple categories are narrowly represented
Questions are generally not tied to specific strategies
Questions are related to the target mathematical goal for the lesson

1 pt Questions are listed, but it is not clear how the questions have the potential to focus, assess, or advance student thinking
Questions are not tied to specific strategies
Questions are not clearly related to the target mathematical goal for the lesson

0 pts No questions are listed

Ensuring Student Engagement **2 Points**

2 pts Strategies are discussed that address what the teacher will do if students cannot begin the task, if they finish almost immediately, and if they focus on non-mathematical aspects of the task
Strategies presented are sufficiently open in that they do not reduce the demands of the task

1 pt One of the categories in score point 2 is not addressed, OR the strategies presented reduce the cognitive demands of the task OR it is unclear how the strategies will support students in their engagement with the task

0 pts Ensuring student engagement is not addressed

Selecting and Sequencing Student Responses **3 Points**

3 pts Specific student responses are identified for sharing during the Share & Discuss
A specific ordering for the sharing of responses is specified
Rationale for the selection and ordering is clearly stated and related to the development of students' mathematical understandings
Questions or issues relating to each response are included

2 pts Specific student responses are identified for sharing during the Share & Discuss
A specific ordering for the sharing of responses is specified
Rationale for the selection and ordering is stated and loosely related to the development of students' mathematical understandings
Questions or issues relating to some responses are included

1 pt Specific student responses are identified for sharing during the Share & Discuss
A specific ordering for the sharing of responses is specified
Rationale for the selection and ordering unclear
Questions or issues relating to some responses are included

0 pts Specific responses are not identified for the Share & Discuss phase

Connecting Ideas & Making Sense of the Mathematics **2 Points**

2 pts Specific questions or other comments are presented that connect the mathematical ideas in the shared responses
Connecting questions or comments align with the mathematical goal

1 pt Specific questions or other comments are presented that connect the mathematical ideas in the shared responses
Connecting questions or comments loosely align with the mathematical goal

0 pts No connecting ideas are presented

Students' Understanding of the Math Ideas **2 Points**

2 pts Specific words or work (things the teacher might see or hear) are identified that will help the teacher know if students are understanding the mathematical ideas

1 pt Vague descriptions of talk and work are presented that will help the teacher know if students are understanding the mathematical ideas

0 pts No information is given related to how the teacher will assess students' understandings of the mathematical ideas

Extending to the Next Day **2 Points**

2 pts A task or discussion is described for the next day's work that either promotes deeper engagement with the target mathematical goal for the lesson, or connects the understandings from the lesson to a new but related mathematical goal

1 pt A task or discussion is described for the next day's work, but it is unclear how this task promoted deeper engagement with the mathematical ideas or connects to a new mathematical goal

0 pts No information about the next day's work

TTAL Scoring Sheet

Teacher Name:

Task Used:

Category	Possible Points	Points Awarded
Solving the Task	3	
Mathematical Goal	2	
Building on Prior Knowledge	2	
Expectations for Students	1	
Task Setup	2	
Questions: Focus, Assess, Advance	4	
Ensuring Engagement	2	
Selecting & Sequencing	3	
Connecting Ideas & Making Sense of the Mathematics	2	
Students' Understanding of the Math Ideas	2	
Extending to the Next Day	2	
Total Points:	25	

Additional Comments:

Appendix C: Pledge Plans Task Interview Protocol

BEFORE you conduct the interview…

- Review the protocol and check batteries for the digital audio recorder.

DURING the interview…

- Phrases in bold should be read exactly as they appear. Phrases in italics are notes to the interviewer.
- When teachers are referring to items on handouts by pointing, referring to the item as 'this' or 'that', help them articulate what they're referring to (e.g., 'you're pointing to the graph?').
- Keep an eye on the batteries for the digital audio recorder and change when necessary.
- If the teacher uses a calculator, ask him or her to articulate the computations they performed on it.
- Take notes of important ideas the teacher brings up that you want to return to in the margins of the interview protocol.

AFTER the interview…

- Complete the cover sheet with information about the audio recorder, folder, and files corresponding to the recording (e.g., Recorder B, Folder A, Files 3 and 4) (note: a new file is started each time the recorder is turned off, so there should be two files associated with each interview).
- Label all sheets with the teacher's first and last name in the upper right hand corner.
- Place all materials back in the envelope and return the envelope to Elizabeth in the Math Project Office in 5519 WWPH.

WITHIN 12-24 HOURS AFTER the interview…

- Give the audio recorder to Elizabeth so that she can download the digital file to the computer, labeling with Teacher's Name and PPint1, PPint2, or PPint3

WITHIN A WEEK of the interview…

- Elizabeth will burn backup CDs of the digital audio files

This is (<u>YOUR NAME</u>) interviewing (<u>TEACHER'S NAME</u>) on (<u>DATE</u>) and this is the (<u>Pre-Course</u>, <u>Post-Course</u>, or <u>End of 1st semester of the field experience</u>) Pledge Plans Interview.

Thank you for participating in this interview. The purpose of this interview is for us to understand your current thinking on lesson planning.

Time when Interview was started: ___________

(Prior to the interview, teachers were given the Pledge Plans task and the following instructions: Please solve this problem before your interview and bring all of your work with you to the interview. Please identify the mathematical ideas the Pledge Plans Task has the potential to address.)

> **I'd like you to recall the Pledge Plans problem you were given and asked to solve before this interview. Today I would like for you to plan a lesson based on the Pledge Plans Task.** *(Hand out the <u>green sheet</u> that has directions for planning, several sheets of blank paper and a copy of the Pledge Plans Task if needed).*
>
> **This cover sheet tells you the specifics of what I would like you to do** *(read the instructions on the green sheet)***.**
>
> **In preparing the lesson, you are also free to modify the task to suit your needs. I'd like you to put as much detail in your written lesson plan as possible.**
>
> **I'm going to turn off the recorder and give you about 15 minutes to write your lesson plan. After that, I'd like you to walk me through the plan you have made.**
>
> **Do you have any questions?**

Time when tape was stopped: ___________

Time when tape was restarted: __________

Could you walk me through the plan you have come up with?

Probes: You should probe on anything related to the four key elements of planning that involve attending to students' thinking:
(1) identifying the mathematical goal of the lesson;
(2) anticipating students' responses and possible misconceptions;
(3) identifying specific questions that will assess or advance students' understanding; and
(4) specifying how students' solutions will be shared and discussed

Use the general probes below to offer teachers an opportunity to provide more specificity if they are thinking about, but do not specifically prompt them on any of these four planning elements.

Can you say more about (lesson element that is unclear)?
What do you mean by (term they used)?
Can you say more about why you decided to (decision that is interesting)?

If teacher made changes to the task, ask Why did you decide on these changes?

If aspects of the written lesson plan are not brought up by the teacher (e.g., they have a goal written on their lesson plan, but have not yet talked about the goal of the lesson) then ask about them…"I noticed you have (x) in your lesson plan here, can you tell me about that?

Finish with a general probe of **Is there anything else you would like to say about your lesson plan?**

Ok, great. Thank you very much for participating in this interview.

Interview end time: __________

Planning a Lesson Around the Pledge Plans Task

Imagine that you are working on a unit on linear functions with your Algebra students. You have selected the Pledge Plans Task to use in your next lesson.

Plan a lesson based on the Pledge Plans Task. Please describe your plan in as much detail as possible.

PLEDGE PLANS TASK[8]

Name ______________________________

Several students who are participating in a 10-kilometer walk-a-thon to raise money for charity need to decide on a plan for sponsors to pledge money for the walk-a-thon. Jeff thinks that $1.50 per kilometer would be an appropriate pledge. Rachel suggests $2.50 per kilometer because it would bring in more money. Annie says that if they ask for too much money, people won't agree to be sponsors; she suggests that they ask for a donation of $4.00 and then $0.75 per kilometer.

Which plan is better, Jeff's, Rachel's or Annie's? Explain your reasoning.

[8] The Pledge Plans is adapted from Navigating through Algebra in Grades 6-8 (2001), NCTM.

Appendix D: Final Interview Protocol
(December 2005)

The interview should last approximately 1.5 hours

BEFORE you conduct the interview…

- Review the protocol and check batteries for the digital audio recorder.

DURING the interview…

- Phrases in bold should be read exactly as they appear. Phrases in italics are notes to the interviewer.

- When teachers are referring to items on handouts by pointing, referring to the item as ‘this’ or ‘that’, help them articulate what they’re referring to (e.g., ‘you’re pointing to the graph?’).

- Keep an eye on the batteries for the digital audio recorder and change when necessary.

- Take notes of important ideas the teacher brings up that you want to return to in the margins of the interview protocol.

AT THE END of the interview…

- Give the teacher the money for this interview.

AFTER the interview…

- Complete the cover sheet with information about the audio recorder, folder, and files corresponding to the recording (e.g., Recorder B, Folder A, Files 3 and 4) (note: a new file is started each time the recorder is turned off, so there should be two files associated with each interview).

- Label all sheets that were written on with the teacher’s first and last name in the upper right hand corner.

- Place all materials back in the envelope and return the envelope to the Math Project Office in 5519 WWPH.

WITHIN 12-24 HOURS AFTER the interview…

- Download the digital file to the computer, labeling with Teacher’s Name and “Final Int”

WITHIN A WEEK of the interview…

- Burn backup CDs of the digital audio files

Final Interview

Teacher's Name: ________________________

Interviewer's Name: ______________________

Date of Interview: ______________________

Digital Recorder: ____

Folder: __

Files: __________

Time when Interview was started: ___________

This is (<u>YOUR NAME</u>) interviewing (<u>TEACHER'S NAME</u>) on (<u>DATE</u>) and this is the Final Interview.

Thank you for participating in this interview. The purpose of this interview is for us to understand your current thinking on lesson planning. There will be 4 parts to this interview.

<u>PART 1 – What should be in a lesson plan?:</u>

For the first part of the interview, I'd like you to take a minute to write down the things you believe you should think about when planning a mathematics lesson, or what you would include in a lesson plan for a mathematics class. Then I'm going to ask you to tell me about them.

You can write on this YELLOW sheet.
(Give the teacher about a minute or two to write down some thoughts – keep the recorder running, the purpose is to allow a brief moment for individual think time before they have to start talking).

Ok. I'd like for you to tell me about the things you believe you should think about when planning a mathematics lesson.
Use the general probes below to offer teachers an opportunity to provide more specificity or clarify their descriptions.
Can you say more about <u>(item that is unclear or brief)</u>?
What do you mean by <u>(term they used)</u>?

PART 2 – Discussing the teacher's lesson planning practices during the first semester of the internship:

For the second part of the interview, I'd like to ask some questions that relate to your lesson planning in general.

I'd like you to talk about the things that influence your planning.

So I'll start by asking, how do you decide what to include/not include in a lesson plan?

What role does your textbook or curriculum play in your planning? *(sub-prompts, if needed, may include: "How do you use your textbook or curriculum when you plan?", "Does the textbook or curriculum influence your planning in any way?, if so, in what ways?")*

What role does your mentor teacher play in your planning? *(sub-prompts, if needed, may include: "Do you discuss your lesson plans with your mentor teacher?", "Have you planned lessons together?", "What kinds of things have you discussed with your mentor teacher, with respect to lesson planning?")*

What role does your university supervisor play in your planning? *(sub-prompts, if needed, may include: "Do you discuss your lesson plans with your university supervisor?", "Have you planned lessons together?", "What kinds of things have you discussed with your university supervisor, with respect to lesson planning?)* **In what ways are the lesson plans you provide for your university supervisor similar and different from those you usually produce?**

What other things influence your planning. *Move on only after teachers have offered as many factors as they can.(these could include such things as: time constraints (either in the time they have to devote to planning or in the time they have to teach something), things they are learning/doing in their teacher education program, their beliefs about what it means to learn and do mathematics and about students, resources available, PSSA, parents, students, etc).*

Do you believe your planning has changed in any ways, over the course of this semester? *If yes, then* **Can you describe the ways in which your planning has changed?**

Are there any other ways in which you believe your planning has changed?

Finish with a general probe of **Is there anything else you would like to say about your lesson planning?**

PART 3 – Planning a Lesson around the Pledge Plans Task:

(Prior to the interview, teachers were given the Pledge Plans task (on green paper) and the following instructions: Please solve this problem before your interview and bring all of your work with you to the interview. Please identify the mathematical ideas the Pledge Plans Task has the potential to address.)

For the third part of the interview, I'd like you to recall the Pledge Plans problem that you were given and asked to solve before this interview. Today I would like for you to plan a lesson based on the Pledge Plans Task.

(Hand out the PURPLE sheet that has directions for planning, several sheets of blank paper and a copy of the Pledge Plans Task (if needed)).
This cover sheet tells you the specifics of what I would like you to do *(give teacher a minute to read the instructions on the purple sheet)***.**

In preparing the lesson, you are also free to modify the task to suit your needs. I'd like you to put as much detail in your written lesson plan as possible.

I'm going to turn off the recorder and give you about 15 minutes to write your lesson plan. After that, I'd like you to walk me through the plan you have made.

Do you have any questions?

Time when tape was stopped: ___________

(During this time, you should leave the room. In preparation for Part 4 of this interview, take this time to read over the teacher's lesson plan that they brought in with them).

Time when tape was restarted: ___________

Could you walk me through the plan you have come up with?

Probes: You should probe on anything related to the four key elements of planning that involve attending to students' thinking:
(1) identifying the mathematical goal of the lesson;
(2) anticipating students' responses and possible misconceptions;
(3) identifying specific questions that will assess or advance students' understanding; and
(4) specifying how students' solutions will be shared and discussed

Use the general probes below to offer teachers an opportunity to provide more specificity if they are thinking about one or more of these elements, but do not specifically prompt them on any of these four planning elements.

Can you say more about (lesson element that is unclear)?
What do you mean by (term they used)?
Can you say more about why you decided to (decision that is interesting)?

If teacher made changes to the task, ask Why did you decide on these changes?

If aspects of the written lesson plan are not brought up by the teacher (e.g., they have a goal written on their lesson plan, but have not yet talked about the goal of the lesson) then ask about them…"I noticed you have (x) in your lesson plan here, can you tell me about that?

Finish with a general probe of **Is there anything else you would like to say about your lesson plan?**

PART 4 – Talking about a lesson plan the teacher has written:

(Prior to the interview, teachers were asked to bring a copy of a lesson plan they have written but have not yet taught.)

For the fourth and final part of the interview, I'd like to discuss the lesson plan that you were asked to bring with you today. First I'd like to ask a few questions about the lesson and then I would like for you to talk in more detail about the lesson plan you've written.

What class/course is this lesson plans for? How many sections of the course do you teach? Which period(s)?
(be sure to get the Subject of the course (e.g., Algebra 2 Honors, Regular Geometry) & have them explain any descriptors, such as honors, regular, etc)

How long have you been teaching this course and section?
How long have you been making lesson plans for this course and section? *(Some teachers may have been teaching the course for a few months, but only recently been writing their own lesson plans (as opposed to "using" their mentor teacher's plans), so be sure to distinguish between teaching and planning)*

Earlier, you identified some things that influence your planning. I'd like to ask about the role they played in planning this specific lesson. For example,…*Referring to things the teacher identified in Part 2 of the interview that influence their planning, ask if these were factors present in planning this lesson by using the following prompts as appropriate:*

- **In what ways did you use your textbook in planning this lesson?**
- **Did you plan this lesson with your mentor teacher?**
- **Did you plan this lesson with your university supervisor?**
- **In what ways did you use the "Thinking Through a Lesson Protocol" in planning this lesson?**

When do you plan to teach this lesson? *(It is important that the teacher has not yet taught this lesson, but is planning to teach it in the future).*

Now I'd like you to walk me through this lesson plan, providing as much detail as possible about your thinking when you planned it?

Probes: You should probe on anything related to the four key elements of planning that involve attending to students' thinking:
(1) identifying the mathematical goal of the lesson;
(2) anticipating students' responses and possible misconceptions;
(3) identifying specific questions that will assess or advance students' understanding; and
(4) specifying how students' solutions will be shared and discussed

Use the general probes below to offer teachers an opportunity to provide more specificity if they are thinking about one or more of these elements, but do not specifically prompt them on any of these four planning elements.

Can you say more about (lesson element that is unclear)?
What do you mean by (term they used)?
Can you say more about why you decided to (decision that is interesting)?

If aspects of the written lesson plan are not brought up by the teacher (e.g., they have a goal written on their lesson plan, but have not yet talked about the goal of the lesson) then ask about them…"I noticed you have (x) in your lesson plan here, can you tell me about that?

Is there anything (else – *if appropriate)* **that you thought about in planning the lesson that is not included in your written lesson plan?**

In looking at the list you made earlier (on Yellow Paper) of the things you think you should think about when planning a lesson, I'd like you to talk about whether or not you think this lesson plan included all of the aspects you identified as important. Are there any aspects that are on the list that are missing from this lesson plan?

Now I'd like you to think about how typical this lesson plan is compared to other lesson plans you have been writing during your internship. We can divide this comparison into issues of content and format. By content, I mean the issues you address and the level of detail with which they are described. For example, in this lesson plan you have identified [e.g., ***goals ________________________]*** **and have provided** ***[e.g., specific questions ___________________________].*** **By format, I mean things such as structure or length. For example, in this plan you [e.g.,** ***used an outline format, have typed/handwritten it ___________________________]*** **and it is [*****x- ____]*** **pages long.**

With respect to CONTENT, how typical is this lesson plan on a scale of 1 to 5 with 1 being "atypical" and 5 being "very typical".

Please elaborate on the ways in which the CONTENT is ("atypical"*(if a 1 or 2)***, "typical"** *(if a 4 or 5)***, or "both atypical & typical"** *(if a 3)***).**

With respect to FORMAT, how typical is this lesson plan on a scale of 1 to 5 with 1 being "atypical" and 5 being "very typical".

Please elaborate on the ways in which the FORMAT is ("atypical"*(if a 1 or 2)***, "typical"** *(if a 4 or 5)***, or "both atypical & typical"** *(if a 3)***).**

The "Thinking Through a Lesson Protocol" was introduced in the Teaching Lab Course this summer and you are currently completing an assignment in Methods 1, which requires you to plan and teach a lesson using this tool.

What role, if any, has the "Thinking Through a Lesson Protocol" played in your planning beyond assignments for specific courses at the University?

Finish with a general probe of **Is there anything else you would like to say about your planning for this lesson or lesson planning in general?**

Ok, great. Thank you very much for participating in this interview.

Interview end time: ________

Appendix E: Scoring Rubric for Attention to Students' Thinking

The scoring rubric contains descriptions of each of the codes and examples that would be coded at each score level for each of the four elements. The examples come from several data sources. One source is written lesson plans produced by pre-service secondary mathematics teachers enrolled in the Teaching Lab course taught by the researcher in the summer of 2004 (one year prior to this study). During the first and last class of the Teaching Lab course, pre-service teachers were asked to write a detailed lesson plan for the Pledge Plans Task (initials and PPpre or PPpost at the end of an excerpt indicate who (pseudonym) made the statement and when). The other data come from a study of teacher learning in a methods course focused on algebra in the middle grades that was conducted under the auspices of the ASTEROID project[9]. The ASTEROID data sources, from which examples were taken, are transcribed interviews of pre-service and practicing teachers describing a lesson they planned around the Pledge Plans Task (as denoted by the teacher's initials and Int2Q3) and excerpts from teachers' "Thinking Through a Lesson" (TTAL) assignments in which they were asked to write a paper that described their plan for a lesson using the Thinking Through a Lesson Protocol as a guide for what to consider in the planning process (as denoted by the teacher's initials and TTALp.#).

Lesson planning element one: Identifying the mathematical goal of the lesson.

Lesson planning element one involves the teacher determining the specific mathematical concepts with which students will engage during the lesson or what mathematical understandings students will take with them from the lesson. Specificity is important in identifying the mathematical goal so that it can guide the lesson plan and subsequent instruction. Goals should

[9] ASTEROID (A study in teacher education: Research on instruction design) is an NSF sponsored research project (NSF Award #0101799), principal investigator Margaret S. Smith

make clear the understanding(s) students will gain about mathematical concepts rather than skills students will exhibit or tasks they will accomplish. This element is coded on a three-point scale (0, 1, or 2 points).

Scoring: 2 pts = specifies concepts & what it means to "understand" the concept

1 pt = vaguely described concepts OR skills students will exhibit OR things students will do to complete the task

0 pts = A mathematical goal Does Not Exist

A score of 2 points is assigned when the teacher describes specific mathematical concepts students will understand and what it means to "understand" the particular concept. For example, the following excerpt describes the specific mathematical concepts students should understand as a result of a lesson involving a geometric pattern task:

> I want my students to recognize a pattern and move beyond an additive, recursive way of generalizing it. I want them to think about the pattern multiplicatively and represent it with a formula that can be used to find any shape in the pattern. I hope that students recognize that the involved variables can represent any shape number or perimeter, with the second being a function of the first. My final goal is that students can utilize and interpret different representations of the contextual pattern, including equations, tables, and possibly, graphs (OY TTAL p.9-10).

This example would be coded with the highest score (2 points). The teacher explicates student understanding about variable, the multiplicative nature of the relationship, and generalizing a pattern with a formula.

Mathematical goals that identify mathematical concepts with which students will gain understanding but not fully explicate the particular understanding of the concept will be coded with a score of 1 point. Similarly, goals that focus on skills that students will learn or things students will do in order to complete the task will also be coded with a score of 1 point. For example, the following would be coded as lesson planning element one being somewhat present with a score of 1 point:

> The mathematical goals of the lesson is to successfully complete the task. Creating a table, graph, and equation successfully will show the students can represent the data different ways. To learn the form of a linear function and the idea of a y-intercept. The relationship of the graphs and why they look different (AH PPpre).

In this example the teacher has focused on the completion of the task and skills that students will exhibit (e.g., student can represent the data in different ways), while at the same time vaguely identifying some mathematical concepts with which students should grapple (e.g., the idea of a y-intercept and the relationship of the graphs and why they look different). This goal does not clearly describe what it is about the form of a linear function that students will learn, what specifically students will know about y-intercept, or what is entailed in looking at the "relationship of the graphs and why they look different." The following is another example that is coded as 1 point; "Goal: to have the students understand the slope of a line and y=mx+b form" (CM PPpre). Here the teacher has used a vague term, "understand", without specifying what it means to "understand" the slope of and line and the slope intercept form of an equation.

When the teacher does not provide any information about the mathematical goals of the lesson, then the data will be scored as 0, indicating that a mathematical goal is not evident.

<u>Lesson planning element two: Anticipating students' thinking about a mathematical task</u>.

In lesson planning element two, teachers provide evidence that they have considered how students might mathematically interpret a problem or the array of strategies – both correct and incorrect – that they might use to tackle the problem. Scoring for lesson planning element two consists of two separate categories: 1) *Anticipating Students' Correct Thinking* and 2) anticipating students incorrect thinking. Each category is scored on a four-point scale (0, 1, 2, or 3 points) and is dependent upon the specificity with which the teacher has described the anticipated students' thinking and whether or not the teacher has attempted to describe the many ways in which a student may think about the problem.

Scoring: Correct Student Thinking

3 pts = specifically describes correct strategies/thinking students may use when working on the problem AND there is an attempt to identify the many possible solution strategies or representations students may use

2 pts = specifically describes at least one correct strategy/approach students may use when working on the problem. However, the strategies/approaches are limited and do not represent an attempt to describe the many ways in which students may solve the problem(s).

1 pt = vaguely describes correct strategies/thinking students may use when working on the problem

0 pts = evidence of anticipating students' correct thinking Does Not Exist

Scoring: Incorrect Student Thinking or Possible Questions/Difficulties Students May Have

3 pts = specifically describes incorrect ways in which students may think about the problem or specific questions students may ask or difficulties students may encounter as they work on the problem AND there is an attempt to identify the many challenges or misconceptions students may encounter with the given mathematical task

2 pts = specifically describes at least one incorrect way in which students may think about the problem or specific question students may ask or difficulty students may encounter as they work on the problem, however the challenges and misconceptions are limited and do not represent an attempt to describe the many challenges or misconceptions that students may have

1 pt = vaguely describes incorrect ways in which they may think about the problem

0 pts = evidence of anticipating students' incorrect thinking Does Not Exist

In order to receive a *score of 3 points for Anticipating Students' Correct Thinking*, the teacher should provide specific descriptions of multiple solution strategies or approaches in solving the problem. For example, with the Pledge Plans Task, there would be evidence of thinking about the problem by writing equations (e.g., y=.75x + 4, y= 1.5x, and y=2.5x), using systems of equations and solving them algebraically (e.g., setting 2 equations equal to each other and solving), creating a data table (e.g., describing the values in the table or providing the table), creating a graph of each of the three plans (e.g., describing how students may create the graph (either from the equation or by plotting points from the table) and showing the graph or describing it), variety in the assignment of dependent and independent variables (e.g., the number of sponsors could be held constant at 1, the independent variable could be number of kilometers walked, and the independent variable could be the money earned from one sponsor or the number of kilometers walked could be held constant at 10km, the independent variable could be the number of sponsors and the dependent variable could be the total money earned). While the teacher does not need to specify each and every one of the correct student thinking identified above, the teacher should show evidence of anticipating many of these approaches rather than just one or two (e.g., equation and graph).

Evidence of *Anticipating Students' Incorrect Thinking at the level of a 3 point score* should indicate some breadth in thinking about that with which students' may struggle. For example, in the Pledge Plans Task, the following are some of the incorrect thinking or struggles students may encounter: difficulty writing an equation from the given problem or table of data, not understanding what to do with the +4 in Annie's equation, not identifying the assumptions they are making (e.g., does everyone have to walk the whole 10km?), difficulty finding and interpreting the points of intersection, understanding how the slope and y-intercept are represented in the equation, table, graph, and context of the problem. While the teacher does not need to specify each and every one of the ideas identified above, the teacher should show evidence of anticipating many of the difficulties students may encounter rather than just one or two.

When the teacher provides specific information about at least one form of correct or incorrect student thinking, but does not provide evidence of attempting to anticipate the many ways students may think, the data is scored as 2 points. For example, the following would be coded with a score of *2 points for anticipating incorrect student thinking* and is from an interview where the teacher indicated "Because, they [students] would probably know what to do with the seventy-five cents, and then not to know what to do with the four, because it's not like Jeff's and Rachel's equations" (NR Int2Q3). In this case, the teacher is anticipating a specific difficulty students may encounter when writing an equation. In order for the data to receive a score of 3 points, several other specific misconceptions or challenges would also need to be described.

By further contrast, if a teacher provides evidence of considering how students might think about the problem but is not specific in exactly what the student might do, say, or think, with respect to correct and incorrect student thinking, then it is coded with a score of 1 point. The following is an example that is coded as a *score of 1 point for incorrect student thinking and a score of 1 point for anticipating correct student thinking*: "Since it is a unit on linear functions, students shouldn't have a difficult time making the table and graphs. Difficulty may come in when analyzing data" (RM PPpost,). In this example the teacher has provided some insight into what elements of the task may and may not be troublesome for students, but is not specific about the misconceptions or difficulties student will have when analyzing the data or any details into the solution strategies that involve making tables and graphs. The following is another example of lesson plan element two being coded with a *score of 1 point for anticipating correct student thinking*: "So some, y'know, one- I'm sure at least one of the groups is gonna have made a chart"(BY Int2Q3). In this case, there is evidence of the teacher anticipating a specific representation of the functional relationship involved in the problem that students might use, a chart. However, the teacher does not provide specifics on how students might create or use the chart.

If teachers do not make any attempt to anticipate the responses or possible misconceptions students may have when engaging with a mathematical task or a teacher provides only "worked out solutions" or an "answer key" to the problem(s) (evidence that the teacher has thought about the correct answers, but not necessarily evidence that the teacher has considered how students might think about the task) then the data will be scored as 0 points.

<u>Lesson planning element three: Identifying specific questions the teacher can ask that will assess or advance students' understanding while they work on a mathematical task</u>.

For lesson planning element three, each data source is coded along a three-point scale (0, 1, or 2 points) with respect to whether or not the teacher provides a specific example of a question to ask students that will either assess or advance students' mathematical understanding and the circumstances under which they would ask the question.

Scoring: 2 pts = provides a specific example question to ask students AND the circumstances under which the question is appropriate (circumstances based on students' mathematical thinking about the problem). There must be at least two

different circumstances based on students' mathematical thinking with a corresponding specific question(s).

1 pt = provides a specific example question to ask students but the circumstances under which the question is appropriate are not given, are not based on students' mathematical thinking about the problem, or only one circumstance based on students' mathematical thinking is present

0 pts = Specific example questions Do Not Exist

In order to be coded as element three being fully present (a score of 2 points), teachers must provide an example of at least one specific question to ask as well as the circumstances under which they would ask the question. There must be at least two different circumstances based on students' mathematical thinking about the problem with a corresponding specific question(s), as seen in the following example:

> If students are having trouble creating the table, then ask them how they think the info should be organized? Ask what are we comparing? Ask how would you find the total amount owed if its a 10km walk?
>
> If students have trouble graphing, then ask what should we label the x and y axis with? How many lines should we have when we are finished? How would we find the (x,y) points for each plan? (i.e., (km$)) (CG PPpost)

In this excerpt, the teacher identifies a strategy or representation (i.e., crating a table and graphing) with which students may struggle, indicating circumstances, based on students' mathematical thinking about the task, under which to ask students questions. The teacher then suggests several specific questions pertaining to creating a table and graphing that may help students move forward. Because the teacher provides at least two different circumstances based on students' mathematical thinking and specific questions for each of the circumstances, this data source would be scored as 2 points. In identifying the circumstance(s) under which to ask specific questions, the teacher may refer to a particular portion of the mathematical task (e.g., question #2). Because the question that is referred to pertains to a particular mathematical question, the circumstance is considered to be based on the students' mathematical thinking (i.e., based on the idea that the student is struggling with the particular mathematics involved in the particular question).

By contrast, a code of element three being somewhat present (a score of 1 point) occurs when a teacher provides at least one example of a specific question to ask students that will assess or advance their understanding, but does not identify the circumstance(s) under which the question will be asked or the circumstances are not based on students' mathematical thinking about the particular mathematical task involved in the lesson. The following is an example of lesson planning element three coded with a score of 1 point:

> If students do not know how to get started, ask them to read the question and pick out the important information. How can you organize this information? How can you compare the plans? Describe what you notice about each graph. How can these graphs be related to the tables? Which plan do you feel is the best? (HS PPpost)

This example is the entire section of the lesson plan in which the students are working on the task. In this example, the teacher provides only one circumstance (based on students' mathematical thinking) under which to ask a specific question (i.e., "if students do not know how to get started"?), thus it would be scored as 1 point. The following is another example that would be scored as 1 point:

> I would ask students questions like: why does your graph start at 0? What happens when the points intersect from the graphs? What is going on before that point and after? How does your graph relate to your table? (TF PPpost)

In this example, the teacher offers multiple examples of specific questions to ask students. While the mathematics that the teacher wishes to make salient may be inferred from the questions posed, it is not clear when the teacher will deem it appropriate to ask any of these particular questions.

Specificity in providing information about questions to ask is very important, because coming up with a good question in the heat of the moment is quite challenging, especially for novice teachers. The following example contains evidence of a teacher providing a general or vague suggestion for the type of question to ask but does not provide a specific example question: “I will ask clarification questions related to the work they have done so far in order to understand their thought processes and gauge where I need to take them next” (DL TTALp. 4). It is doubtful that planning questions to ask at this level would be helpful when enacting the lesson. Therefore, this example would be coded as element three not being present and receive a score of 0 points. Data will receive a score of 0 points if the teacher does not provide any specific example questions that can be asked of students as they work (individually or in groups) on the mathematical task. Specific questions that are meant to be asked to the whole class do not count in this lesson planning element and are addressed in lesson planning element four, as described below. While the quality of the specific example questions is an important aspect in promoting student learning, the evaluation of the quality of the questions is not within the scope of this study and is not a part of this coding scheme.

<u>Lesson planning element four: Orchestrating a whole-class discussion that builds on students’ thinking and makes salient the mathematics of the lesson</u>.

Lesson element four involves two important aspects in orchestrating a meaningful whole-class discussion: (1) evidence that the discussion is building on students’ mathematical thinking and/or work in solving the problem; and (2) identifying specific example questions that will enable students to make connections between solutions or make the mathematics of the lesson salient. Teachers may plan a discussion that is built on students’ mathematical thinking in several ways: (a) purposefully selecting student solutions for public discussion, (b) determining the sequence in which those solutions should be discussed, or (c) identifying specific questions to ask students that explicitly refer to students thinking or work on the problem. Each of the two aspects (building on students' mathematical thinking and example questions that make the mathematics salient) will be coded on a 3-point scale (0, 1, or 2 points) with respect to specificity. It is possible that a question posed in the lesson plan may be serving dual purposes of building on students' work or thinking as well as making the mathematics of the lesson salient. Therefore it is possible that a question would be included in determining both the *Building on Student Thinking* score and the *Making the Mathematics Salient* score.

Scoring: Building on Student Thinking

2 pts = identifies specific questions that highlight the mathematics in a specific student solution

1 pt = selects and/or sequences students' solutions to be discussed but does not provide any specific questions to ask related to the student work OR identifies a question to ask, but is vague about for which student solution the question is appropriate, OR simply asks students to explain or share his/her solution without specific questions that highlight mathematical ideas

0 pts = evidence of building on student thinking Does Not Exist

Scoring: Making the Mathematics of the Lesson Salient

2 pts = Identifies a series of specific questions that develop mathematical ideas

1 pt = Identifies questions that are vague or so few that a particular mathematical idea is not being well-developed OR expresses specific mathematical ideas that they wish to address in the discussion, but offer no specific questions to ask in order to achieve their mathematical intentions

0 pts = evidence of thinking about making the mathematics of the lesson salient Does Not Exist

The following example provides evidence of a teacher considering the selection and sequence of students' solutions as well as providing specific questions to ask and would be scored a *2 points for building on students' thinking*:

> Groups will hang posters in front of room. Have a group explain how they came up with table. Have a group share their choice based on table. Have group share choice based on graph. How can you relate the tables to the graphs? Does everyone have the same choice? If not, explain why you chose. Did every group have the same reasoning? Can someone else in the class explain another groups' reasoning? What happens in each plan? (HS PPpost)

In this example the teacher has specified how students' solutions will be shared and provides specific questions referring to the students' work that highlight mathematical aspects of the work.

The following excerpt from a teacher's written lesson plan involves a series of questions pertaining to the Pledge Plans Task:

> Questions to ask class:
> - What is the \$4? How is it represented in the graph?
> - Why do the lines cross at the "origin"? How is that represented on the written equation?
> - If there was a \$18 minimum rather than \$4, what would the graph look like?
> - Looking at the graph predict what the amounts would be if they each walked 8km?
> - At which point is Rachel's better than Annie's – what about in relation to Jeff?
> - What is occurring at those points of intersection?(KC PPpost)

In this example the teacher describes a series of specific questions that may help students develop mathematical concepts related to graphing linear functions, such as the idea of y-intercept and how it is represented in a graph and equation as well as how changing its value may affect the graph of the function. This is an example that would earn a *score of 2 points for*

Making the Mathematics Salient. However, the questions in the excerpt do not make specific reference to students' work or thinking about these issues when solving the problem, thus for the category of *Building on Students' Thinking, this excerpt would be scored as 0 points*.

The following example would be coded as *1 point for Building on Student Thinking and 1 point for Making the Mathematics Salient:*

> Have a pair present their table and graph. Discuss the relationship between the info given, the table, and the graph. Discuss the relationship between the $ earned and distance walked in words. Then have one group present their formulas. Discuss the differences of Annie's plan and the implications of these differences on the table and graph (y-int, slope). Make connections between info, table, graphs, formulas, slope and y-intercepts (CG PPpost).

The example shows the teacher selecting different representations to be presented, the order in which to present them and describing the specific mathematics the teacher wants to make salient in the discussion. However, the teacher is not specific in *how* to orchestrate this discussion by providing the specific example questions to ask students.

For example, in the following excerpt, the teacher selects solution strategies that involve graphs and tables, however it is not clear how the teacher will offer students opportunities to make connections between these representations, thus earning a score of *1 point for building on student thinking*. This particular excerpt is so vague in describing the mathematics to be discussed that it would be scored as *0 points for Making the Mathematics Salient*.

> Have students share based on correctness and uniqueness of solutions. Choose different groups to present different parts of task and have them relate this to the graph/table. For this reason, graph and table should be part of every presentation so that it can be referred to. Discuss as many different possibilities to which is best as possible (EV PPpost).

Appendix F: Task Analysis Guide

Lower-Level Demands	Higher-Level Demands
<u>Memorization Tasks</u> • involve either reproducing previously learned facts, rules, formulae or definitions OR committing facts, rules, formulae or definitions to memory. • cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure. • are not ambiguous. Such tasks involve exact reproduction of previously seen material and what is to be reproduced is clearly and directly stated. • have no connection to the concepts or meaning that underlie the facts, rules, formulae or definitions being learned or reproduced.	<u>Procedures With Connections Tasks</u> • focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas. • suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts. • usually are represented in multiple ways (e.g., visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations helps to develop meaning. • require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.
<u>Procedures Without Connections Tasks</u> • are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task. • require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it. • have no connection to the concepts or meaning that underlie the procedure being used. • are focused on producing correct answers rather than developing mathematical understanding. • require no explanations or explanations that focus solely on describing the procedure that was used.	<u>Doing Mathematics Tasks</u> • require complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example). • require students to explore and understand the nature of mathematical concepts, processes, or relationships. • demand self-monitoring or self-regulation of one's own cognitive processes. • require students to access relevant knowledge and experiences and make appropriate use of them in working through the task. • require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions. • require considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process required.

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