FOOD DENSITY ESTIMATION USING FUZZY INFERENCE

by

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Submitted to the Graduate Faculty of
Swanson School of Engineering in partial fulfillment
of the requirements for the degree of
Master of Science in Electrical Engineering

University of Pittsburgh

2011
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This work presents a new fuzzy logic approach to food density estimation in supporting research in diet and nutrition. It has been a historical problem to measure people’s daily food intake in real life. Recent advances in electronic devices have provided novel tools for recording volumetric information of food, while the current food databases often list nutrients and calories in terms of gram weights instead of volumes. Thus, a density value, which connects the volume to weight, is required to use the existing databases when the volumetric information is unavailable. In this work, we approach the density estimation problem using fuzzy inference which “guesses” the food density by collecting and organizing relevant human knowledge about a food. French fries are taken as an example of this new approach. A Fuzzy Inference System (FIS) is constructed to estimate the bulk density of French fries under different cooking conditions. Our experimental results show that our FIS system built upon human knowledge about the frying time and temperature can accurately estimate the density of French fries under controlled conditions.
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PREFACE

I would like to thank my advisor, Dr. Mingui Sun for his support and guidance throughout this research. I also appreciate all my committee members, who provided me with valuable suggestions on my work. I would also like to thanks all my lab classmates for their support. This work is supported by NIH U01.
1.0 INTRODUCTION

1.1 BACKGROUND AND MOTIVATION

It has been a historical problem to accurately measure people’s daily food intake in real life. Recent advances in electronic devices make it possible for measurement of calories and nutrients using digital food images [1-3]. Digital pictures can be used to directly identify foods and provide volumetric information of foods by applying the algorithms from the literature [4, 5]. Eventually the calories and nutrients of food can be calculated out, based upon the United State Department of Agriculture (USDA) Food and Nutrient Database for Dietary Studies (FNDDS) [6-7]. However, in FNDDS, the calories and nutrients of many foods are given in terms of gram weights. Thus, a conversion factor between the volume and weight, the density, is required.

In general, density is the unit mass per unit volume. With respect to food type, there are several different food densities to define. These are: true density, substance density, particle density, apparent density, and bulk density respectively [8].

True density is the density of a pure substance calculated from its component densities with conservation of mass and volume. Substance density is the density calculated without pores left inside of the substance. Particle density is the density calculated without modifying the structure of the sample, in other words, all the pores inside of the substance are taken into account but not the external pores. Apparent density is the density calculated with all the pores of
the material included. Bulk density is the density of the sample when it is packed in bulk as shown below:

\[ p_{\text{bulk}} = \frac{\text{mass}}{\text{bulk volume}} \]  

(1.1)

Among all the different density types listed above, those that have been utilized most in food engineering are the true density, apparent density, and bulk density [9].

One method of calculating a true density of the food measurement is to obtain a pure sample volume through use of image processing. First, a food structure would be captured via microscope, X-ray, or Magnetic Resonance Imaging (MRI), as shown in Fig.1 [10]. Then the food volume, without any internal or external pores, would be calculated by applying image analysis onto the food picture.

![Figure 1. Food structures by using microscopy, X-ray and MRI respectively, (a) Microscopic image of bread, (b) X-ray image of bread, (c) MRI image of bread [10]](image)

Another approach for measuring food true density is to compress the food in a container with high pressure air as shown in Fig.2 [11]. By squeezing away the pores inside the food sample the pure volume can be measured, and thus true density obtained. The current available commercial equipment for such volume measurement is an air comparison pycnometer and a
helium pycnometer. This method can also be used for apparent food density calculation for certain foods with closed and rigid surfaces.

Figure 2. Container used for true density calculation[11]

True density is important in laboratory for studying food models. However, true density can hardly be applied to calculate food weight in this project because in most of the cases the pictures that were used for food volume estimations were captured by a normal digital camera, which indicates that the food volumes were measured with the internal and external pores included.

For apparent food density, beside the gas method mentioned previously, solid and liquid displacement methods can also be applied to determine the volume of food. For the liquid method, the food sample needs to be placed into an empty container, which would then be filled with water, toluene, or mercury, to the rim [8, 11]. The volume calculated by taking the container volume and subtracting the water volume in such a container is the food sample volume. Then the food density can be derived. In the solid displacement method, the volume of irregular solid food can be measured by canola seed displacement, sand displacement, or glass beads displacement methods.
For bulk density, as illustrated in the previous definition, as long as the volume of the container that is holding the food is measured, the density can be calculated by the weight being divided by the volume.

Apparent food density and bulk food density both are density types that can be used in our project because the volumes related to the two density types are the volumes that are calculated from the pictures of food. If the food in the picture was not in a container, the volume calculated would match the volume that is used for deriving apparent food density. On the other hand, if the food in the picture was held within a container, the volume calculated would be the volume of the container itself, and would match the bulk density to calculate the weight.

Except for those direct measurement methods that have been introduced above, it is believed that by knowing the components in the food and the cooking process (i.e. the recipe), the food density in general can be estimated. There are all kinds of cooking conditions and recipes for the same food. With different cooking processes and recipes, the densities of the same food could vary to a large degree. There have been several potential approaches discussed with regard to food density estimation [9, 12-14].

One of the approaches to estimate food density is to build a mathematical model that can describe the whole food physical and chemical changing process during cooking [8, 12, 14]. Food density can then be calculated from the model given certain parameters that relate to the cooking condition. However, such a computational approach is not straightforward. It is known that food preparation involves complex physical and chemical changes, which are not all well understood. Although information about the physical properties of many foods and food components has been documented in the food engineering literature [8, 9, 12], the information is still incomplete and may not be utilized directly or conveniently to estimate food density.
Even if a complete description of a certain food component is known, the description is usually in the form of mathematical expressions or partial differential equations (PDEs). Numerical solutions to PDEs are usually obtained by using the finite element method blocks called elements. This discretization process is called meshing. The PDEs describing the physical problem are approximated by a set of algebraic equations on the continuous solution over the domain of interest. The mesh building, initial condition setting, and the computational procedures are often time-consuming and expensive. In addition, some PDEs are numerically unstable, which requires smaller elements, compounding the computational problem.

Another method estimates food density via grouping [13]. Foods are divided into different groups based on their density. In this method whole food categories are separated into 4 groups with regard to their density, which are syrup (density 1.2 g/cm$^3$), water (density 1.0 g/cm$^3$), pure fat (density 0.8 g/cm$^3$), and highly aerated (density 0.5 g/cm$^3$) food respectively. Without too much extra effort needed, this method is easy to be realized. However, the grouping approach can only work in situations where a low precision is acceptable, and it excludes the cases that involve changes in cooking conditions.

1.2 PREPOSED SOLUTION

In this work, we present a novel application of fuzzy inference system (FIS) to food density estimation. In the analytical perspective, the estimation of food density is an extremely difficult problem because of the large variety of foods and the variation of processing/cooking methods. However, a human, especially who cooks by him/herself, usually has good knowledge about the foods that he or she is familiar with, including its weight and size. Therefore, we approach the
density estimation problem using fuzzy logic which collects and organizes relevant human knowledge and uses it to “guess” the density value. As an initial work, we take a simple example to estimate the density of French fries using only two types of knowledge, frying temperature and frying time, based on intuitive concepts. Our experimental results have produced promising results. It will allow experienced people to communicate with the system interface and have their knowledge well understood and used. With this initial success, we will generalize our approach to other foods.

1.3 THESIS STRUCTURE

In Chapter 2 the fundamental and applications of fuzzy logic theory will be illustrated. Chapter 3 will introduce the build-up procedure of the FIS system in general, and derive the mathematical expression of the FIS system. In Chapter 4 we will take French fries as an example to construct a specific fuzzy logic based upon a food density estimation system that deals with French fry density. The results and the corresponding analysis will also be present in this chapter. The conclusion and future work will be indicated in Chapter 5.
This work utilizes the fuzzy logic theory for food density estimation. Fuzzy logic, which was first introduced by Zadeh [15], is a mathematical tool that manipulates intuitive, loosely defined, and inexact concepts. It can deal with reasoning that is unfixed or approximate instead of fixed and exact. In recent years, fuzzy logic has been widely applied to many fields including control, transportation, classification, and food processing [16-38].

2.1 FUZZY LOGIC THEORY

2.1.1 Basic Fuzzy Logic Concepts

Fuzzy logic is a theory and tool for knowledge representation and inference, which is different from traditional mathematical modeling. Most of human knowledge and reasoning, especially common sense reasoning, are approximate rather than exact in nature. Fuzzy logic is a kind of method to support knowledge of vague representation and fuzzy reasoning, which is more appropriate to depict nature. For example, when we cook, we would use “too much” or “a little” to describe the amount of oil which is the “fuzzy” description based on our knowledge, but how much is “too much”? Fuzzy logic can solve the problem by introducing the concept “fuzzy set”. The diagram shown in Fig.3 illustrates exactly how fuzzy logic works.
Following the arrows, the fuzzy logic system starts with numerical inputs, which will be “fuzzyfied” to match themselves with the input fuzzy sets based on the corresponding fuzzy membership function. The input fuzzy sets combined via logic operator (i.e. “and”, “or”) constitute the antecedent of the if-then fuzzy rules. Only two rules are utilized in this diagram. The fuzzy rules link the antecedent to the consequent, which are the output fuzzy sets. The consequents are also affected by the implication, which is used to reshape the output fuzzy set for each rule. The results or decisions from different fuzzy rules are summed up through aggregation. Aggregation is the process by which the fuzzy sets that represent the outputs of each rule are combined into a single fuzzy set. In last step, the aggregated result is defuzzified to a numerical output value, which is the final result of the whole system. It is the process of formulating the mapping from a given input to an output using fuzzy logic. The process involves all of fundamental fuzzy logic, which will be described in the following sections.
2.1.2 Fuzzy Set

A fuzzy set is a set that allows its elements to partially belong to the set, which indicates that the elements can in certain degree belong to the fuzzy set. While in a classical set, the elements were either in a set or out of the set, but they cannot partly belong to it. Such difference between a fuzzy set and a classical set enables fuzzy logic to handle questions without an absolutely yes or no answer.

Mathematically, a fuzzy set $A$ can be characterized by a set of ordered pair

$$A = \{x, \mu_A(x) \mid x \in X, \mu_A \in [0,1]\}$$

(2.1)

where $X$ is the universe of discourse, $x$ is the element in the universe of discourse $X$, and $\mu_A$ is the membership function (MF) of $A$ and defined over a universe of discourse $[15]$. MF exactly describes how the relevant element is mapped to a membership value with value range from 0 to 1. A membership value of 0 means the element doesn’t belong to the fuzzy set at all. A membership value of 1 indicates the corresponding element totally belongs to the fuzzy set. A membership value between 0 to 1 means the element is partially involved in the set in certain degree. One of the simplest and most common used membership functions is the triangular membership function. Three points are requested for forming a triangle membership function as shown in Fig.4 (a). Another widely used function is Gaussian membership function as show in Fig.4 (b). Two points are needed to determine a Gaussian membership function. There are several other kinds of membership functions such as trapezoidal membership function, generalized bell membership function, and so on. A wide range of membership functions can be chosen to fit specific applications.
2.1.3 Fuzzy Rule

If - then rules for the fuzzy inference system are used to connect the input and output fuzzy sets, which are the reflection and formulation of knowledge. Following is the general form of the single fuzzy if-then rule:

\[ \text{If } x \text{ belongs to } A \text{ then } y \text{ belongs to } B \quad (2.2) \]

the rule above consists of an antecedent part and a consequent part. With fuzzy operations, these if-then rules are utilized to establish the fuzzy inference system. Specifically, the antecedents of the rule will be transferred to a degree of weight value between 0 and 1, which can be taken as the degree of support for the corresponding rule. Based on degree of support the output fuzzy set is reshaped according to the implication method. Thus, the consequent of each fuzzy rule will transfer the reshaped fuzzy set to the output.

2.1.4 Fuzzy Operation Strategy

The fuzzy logic cannot function only with the input, output and fuzzy rules. It also includes logic operators, implication, and aggregation.
In general, the antecedent of a given rule has more than one part involved. The AND or OR operation, called logic operator, is utilized to combine all the parts in antecedent to a single number, which will then be applied to the output function. The intersection of the involved fuzzy sets, for example A and B, is specified in general by a binary. Any number of well-defined methods can fill in for the AND operation or the OR operation.

The fuzzy intersection AND of two set A and B refer to linguistic statement:

\[(x \text{ belongs to } A) \text{ AND } (y \text{ belongs to } B)\]  

(2.3)

For fuzzy logic intersection AND, the operator is known as triangular or T-norms. [40]. The new membership function after the AND operation is represented as:

\[\mu_{A \cap B}(x, y) = \mu_A(x) \mu_B(y)\]  

(2.4)

where * refers to the T-norm operator. The most commonly used method for fuzzy logic intersection is to take the minimum or product as shown in equation (2.5) and (2.6):

\[\mu_{A \cap B}(x, y) = \mu_A(x) \mu_B(y)\]  

(2.5)

\[\mu_{A \cap B}(x, y) = \min\{\mu_A(x), \mu_B(y)\}\]  

(2.6)

The Fuzzy union OR of two fuzzy set A and B refer to linguistic statement:

\[(x \text{ is } A) \text{ OR } (y \text{ is } B)\]  

(2.7)

It is mathematically calculated using T-conorms [40].

The most commonly used method for fuzzy union is to take the maximum, which is the function:

\[\mu_{AUB}(x) = \max(\mu_A(x), \mu_B(x))\]  

(2.8)

The implication process is to use the degree of support for the entire rule to reshape the output fuzzy set. Consequent of a fuzzy rule assigns an entire fuzzy set to the output. Two
common methods are utilized for implication. One method is “minimum”, which truncates the output fuzzy set, and another one is product, which scales the output fuzzy set.

Each rule deduces a result. The decisions from all the rules must be combined following some rule in order to make a final decision. This process is called aggregation, by which each single output are integrated into a single fuzzy set. The possible approaches for aggregation are maximum, probabilistic, and sum (simply the sum of each rule’s output set).

2.1.5 Defuzzification

The output of each rule is fuzzy. Therefore fuzzy output needs to be converted into a scalar output quantity so that the nature of the action to be performed can be determined by the system. The process of converting the fuzzy output is called defuzzification. There are many defuzzification methods in use, such as maximum defuzzification technique, centroid defuzzification technique, and bisector defuzzification technique. Among these three, centroid defuzzification technique is the most popular one. Fig.5 shows the defuzzified value of the same fuzzy output with respect to different defuzzification strategies.

The maximum method includes LOM, MOM, and SOM methods respectively as shown in Fig.5. Maximum defuzzification methods pick the element with the highest membership function value as the output. It can be expressed as

$$\mu_A(x') \geq \mu_A(x) \quad \text{for all } x \in X$$  \hspace{1cm} (2.9)
Figure 5. Defuzzified value of the same fuzzy output by applying LOM (largest of maximum), MOM (middle of maximum), SOM (smallest of maximum), bisector and centroid defuzzification method[42]

where $x^*$ is the output value after defuzzification. This series of approach provides fast and accurate results when defuzzifing the peaked MF of the fuzzy set.

The centroid defuzzification method was first proposed by Sugeno in 1985. This method is also called center of area defuzzification and can be expressed as

$$x^* = \frac{\int \mu_i(x)x dx}{\int \mu_i(x)dx} \quad \text{for all } x \in X$$  \hspace{1cm} (2.10)

where $x^*$ is the output value after defuzzification. This is the most commonly used method and can give accurate results in general.

The bisector is the vertical line that separates the MF region into two equal areas. The expression for bisector defuzzification is

$$\int_{-\infty}^{x^*} \mu(x)dx = \int_{x^*}^{\infty} \mu(x)dx \quad \text{for all } x \in X$$  \hspace{1cm} (2.11)

where $x^*$ is the output value after defuzzification. In some cases it is coincident with the centroid line.
2.1.6 Summary

Fuzzy sets, fuzzy operations, if-then rules as well as defuzzification are the four basic parts that comprise fuzzy logic and are the key factors in Fuzzy Inference System (FIS). Chapter 3 will introduce the way to build the FIS for food density estimation, which essentially is to find proper fuzzy sets, define related fuzzy operations, derive if-then rules and choose a defuzzification strategy.

2.2 FUZZY LOGIC APPLICATIONS

In recent years, the number and variety of applications of fuzzy logic have increased significantly. In the nutrition and dietetic field, fuzzy logic was utilized in an educational software for nutrition, called Nutri-Expert, in order to train the end-users on choosing daily food intake. The study showed that the fuzzy logic based software led to great improvement in balancing meals [23-24, 26]. In another application, fuzzy logic was utilized to develop a computer-based program. The program can describe the different range of intakes of a nutrient, and calculates in what degree an individual meets the nutrient standard requirements with regard to the diet intake [38].

Applications of fuzzy logic theory to food engineering have been reported [17, 19-21, 23-30, 35-36]. In a fuzzy rice cooker [21], an optimum fuzzy method with genetic algorithm was applied to determine switching patterns of a heater in cooking cycles based on the amounts of rice and water. Fuzzy logic was also applied as a decision making support to grade apples. Factors such as the color size and defect of apples were selected as the input variables for apple
grading, the result from the fuzzy logic approach matched the human expert results well [25]. Another study investigated an approach to extract parameters from a fuzzy logic algorithm modeled using MATLAB's fuzzy logic toolbox. The fuzzy logic algorithm was used to control the cooking temperature and cooking time for products of varying size [36]. Other fuzzy logic applications to food engineering include predicting the crop yield [27], assisting food supply [29], and conducting crop production [30].

Fuzzy logic has been applied in many other industrial areas as well, including control engineering, automobile, and image processing [18, 31-35, 37]. In one study, fuzzy logic was used to control temperature. It was based on the use of a personal computer and an electro-pneumatic transducer to control an aseptic processing high-temperature short-time system. In that study, linguistic rules based fuzzy algorithms were applied to temperature control [37].
3.0 FUZZY INFERENCE SYSTEM IMPLEMENTATION

In this work, Fuzzy Inference system (FIS) is applied to estimate the densities of all types of food under different cooking conditions. The structure of food density FIS is shown in Fig.6, which has the following function blocks: input and output, fuzzy rules, logic operators, implication, aggregation, and defuzzification. A general three-step procedure was performed to build a well-targeted FIS system for food density estimation. Step one was to design input and output fuzzy sets and the correspondent membership functions. Step two set fuzzy inference rules. Step three dealt with FIS strategy design, including defining fuzzy logic operator, designing implication method, designing aggregation method, as well as choosing the defuzzification method. The detailed procedure for each step is elaborated in this chapter.

Figure 6. FIS architecture for food density estimation
3.1 FUZZY SETS DESIGN

The whole FIS system buildup starts with the design of an input and output fuzzy set. The input fuzzy set variables should be the factors that would affect the output variable. The output variable is what directly connects to the target of the analysis in the FIS. In this work, it’s the density of the food that we need to solve, hence food density is chosen as the output variable, which can be represented as $D$. Then the $i$th fuzzy set of the density output variable $D$ can be expressed as

$$D_i = \{ x, \mu_{D_i}(x) \mid x \in X, i \in [1, 2, \ldots, n] \}$$

(3.1)

where $\mu_{D_i}(x)$ refers to the MF of $D_i$. Instead of only one type of output variable, there are multiple input variables involved in FIS for food density estimation. Food density is affected by multiple factors in the cooking process. To simplify the whole FIS system, only the dominate factors were chosen as the input variables in our FIS. The dominate factors chosen vary depending upon the food that was analyzed.

For an unspecified food sample, the human cooking experience and knowledge would help to decide which factors are significant as the input variables. The input fuzzy set can be described as

$$C_i^t = \{ x, \mu_{C_i^t}(x) \mid x \in X, \mu_{C_i^t} \in [0, 1], t \in [1, 2, \ldots, T], i \in [1, 2, \ldots, n] \}$$

(3.2)

where $C_i^t$ indicates the $i$th fuzzy set of the $t$th type of the input variable. For example, if for one kind of food, the input variables are cooking temperature and cooking time, any set related to temperature could be defined as $C_i^1$ ($i \in [1, 2, \ldots, n]$), and any set related to time could be defined as $C_i^2$ ($i \in [1, 2, \ldots, n]$).
Once input and output fuzzy variables are determined, what follows next is to derive the corresponding MFs (i.e. $\mu_{C_i}$ and $\mu_{D_i}$) for each input or output variable. As listed in Chapter 2, there are a great number of curve functions that can be utilized as the membership function for input and output variables. Any function can be taken as the type of the input or output membership function. The most generic functions are triangular function and gauss function as shown below.

**(Triangular fuzzy membership function):**

$$
\mu_A(x) = \begin{cases} 
0 & x < a \\
\frac{x-a}{b-a} & a \leq x \leq b \\
\frac{c-x}{c-b} & b \leq x \leq c \\
0 & x > c 
\end{cases}
$$

where $a$, $b$, $c$ are the left base line point, vertex, and right base line point of the triangle membership function $\mu_A(x)$ respectively.

**(Gauss fuzzy membership function):**

$$
\mu_B(x) = e^{-\frac{(x-c)^2}{2\sigma^2}}
$$

where $c$ and $\sigma$ are the expectation and variation of $\mu_B(x)$ respectively.

It’s not sufficient to completely determine a membership function with only a function type. Additional information and support data should be provided to build a specific membership function, which is related to certain input or output variables. For instance, with a triangular membership function, both the vertex and the endpoint of the base line need to be determined. Pre-experiments are needed to get the necessary information or supporting data for determining the parameters of such functions. A series of key cooking conditions need to be set as the
cooking conditions in the pre-experiments. The food densities under different cooking conditions need to be recorded as well. Thus with the dominative cooking conditions, parameters of input variable membership function can be derived. In the meantime, with the recorded densities of each cooking condition, the parameters of the output variable membership function can also be solved.

### 3.2 DESIGN FUZZY RULES

Fuzzy rules contribute to the relationship between input and output fuzzy sets. The fuzzy rules are if-then logic judgment tools designed to connect the input and output in FIS. Information from previous pre-tests was utilized here to derive the rules.

A general fuzzy algorithm is composed of a set of fuzzy rules of the form:

\[
RULE_h: \quad IF(x_i \text{ belongs to } C_i^1 \text{ AND } \ldots \text{ AND } x_n \text{ belongs to } C_j^n) \quad \text{THEN}(y \text{ belongs to } D_k) W_h
\]  

(3.5)

where \( RULE_h \) is the \( h \)th fuzzy rule which connect a series of input fuzzy set (i.e. \( C_i^1 \ldots C_j^n \)) to the \( k \)th output fuzzy set \( D_k \). \( A \) refers to the input fuzzy set. \( C_j^n \) indicates the \( j \)th fuzzy set of the \( n \)th type of the input variable. The degree of confidence, or weight of the rule, with which the input fuzzy set \( C_j^n \) is related to food density fuzzy set \( D_k \) is given by \( W_h \) with \( W_h \in [0, 1] \). When \( W_h \) is zero, the rule is inactive and does not contribute to the output. Otherwise, it partially takes effect whenever its antecedent is activated to a degree greater than zero. Again, this weight value was also determined empirically using the knowledge gained during our experiments. Thus, the rule base is adjusted by the set of rule weight \( W_h \), which can be stored in a rule weight...
matrix \( W = \{ W_k \} \). The rule weights represent the expert’s knowledge about a particular process and also form a convenient set of parameters to obtain the fuzzy system from experimental data.

3.3 FIS OPERATION STRATEGY

The fuzzy inference system cannot function only with the input, output, and fuzzy rules. FIS operation strategy also needs to be designed in order to build up the whole system. In this section we will design logic operators, define implication method, and set aggregation approach as well.

The logic operators as mentioned in Chapter 2 are defined to specify the relationships among input variables, which are fuzzy intersection (e.g. AND operation) and fuzzy union (e.g. OR operation) respectively.

If there are two fuzzy input sets \( C_i^1 \) and \( C_i^2 \) involved in fuzzy rule \( RULE_h \), then the fuzzy intersection of these two sets refers to linguistic statement

\[
(x_i \text{ belongs to } C_i^1) \text{ AND } (x_2 \text{ belongs to } C_i^2)
\]  

(3.6)

where the superscript “1” and “2” of fuzzy set \( C \) indicates that the fuzzy sets belong to 1st type of input variable and 2nd type of input variable respectively. In our system, instead of “min”, algebraic product function was chosen as the fuzzy operation for AND, because the product operator allows error information to be back propagated, while the min operator acts as a truncation operation, which loses information contained in the original membership function \( \left\{ \mu_{C_i^1}(x_1), \mu_{C_i^2}(x_2) \right\} \). Therefore, we have:
\[ \mu_{C_i \cap C_j}(x_1, x_2) = \mu_{C_i}(x_1) \mu_{C_j}(x_2) \]  

(3.7)

where \( \mu_{C_i \cap C_j} \) indicates the integrated MF of fuzzy set \( C_i \) and \( C_j \) after intersection operation. \( x_1 \) and \( x_2 \) refer to the crispy input of two input variables.

Likewise, the fuzzy union (e.g. OR) of the two fuzzy set \( C_i \) and \( C_j \) refer to linguistic statement

\[ (X_1 \text{ is } C_i) \text{ OR } (X_2 \text{ is } C_j) \]  

(3.8)

The most commonly used method for fuzzy union is to take the maximum, which is

\[ \mu_{C_i \cup C_j}(x_1, x_2) = \max(\mu_{C_i}(x_1), \mu_{C_j}(x_2)) \]  

(3.9)

where \( \mu_{C_i \cup C_j} \) indicates the integrated MF of fuzzy set \( C_i \) and \( C_j \) after union operation. By logic operator, two or more MFs in antecedent of the rule are integrated as one MF with multiple variables.

To simplify the derivation process, we assumed the relationship between the fuzzy sets of the rule’s antecedent is either intersection or union. Hence the result after combining the fuzzy sets in the antecedent part can be mathematically expressed as

\[ \varphi_h(x) = \begin{cases} 
\kappa \mu_{C_i}(x_1) \mu_{C_j}(x_2) \cdots \mu_{C_t}(x_n) \\
(1 - \kappa) \max(\mu_{C_i}(x_1), \mu_{C_j}(x_2), \ldots, \mu_{C_t}(x_n)) 
\end{cases} \quad x = [x_1, x_2, \ldots, x_n]^T, \kappa \in \{0, 1\} \]  

(3.10)

where \( \mu_{C_i}(x_1), \mu_{C_j}(x_2), \ldots, \mu_{C_t}(x_n) \) refer to the MFs of fuzzy set that were involved in the antecedent of the \( h \)th rule. \( \kappa = 1 \) for intersection operation and \( \kappa = 0 \) for union operation.

As mentioned in Chapter 2, implication operators decide in which way the output fuzzy set is scaled or mapped based on the input fuzzy sets according to a fuzzy rule. In our case,
implication was defined as the product operation. With taking the rule weight $W$ into account, then the $h$th fuzzy rule can be mathematically described as

$$\psi_h(x, y) = \varphi_h(x) W_h \mu_{D_h}(y) \quad x = [x_1, x_2, \ldots, x_n]^T \quad y \in Y, \quad W_h \in [0, 1]$$  \hspace{1cm} (3.11)

where $\mu_{D_h}(y)$ refers to the MFs of food density fuzzy set $D_h$ that was involved in the consequence of the $h$th rule, and $W_h$ indicates the weight of the $h$th rule.

Aggregation is the process combining all food density results derived from the fuzzy rules. Several methods can be used in this combination [1]. In our work, we simply summed all results in the aggregation stage, which can be described as

$$f(x, y) = \sum_{h=1}^{n} \psi_h(x, y)$$  \hspace{1cm} (3.12)

where $f(x, y)$ refers to the aggregated fuzzy rule expression, and number $n$ indicates there are $n$ rules involved in the food density FIS.

Defuzzification converts a fuzzy output set to a crisp value as the numerical output of the FIS. As mentioned in Chapter 2, there are many methods to implement this conversion. We utilized the popular centroid method as this is the most commonly used technique. After centroid defuzzification, finally, the food density from FIS can be obtained, which can be expressed as:

$$y^* = \frac{\int f(x, y) \cdot y dy}{\int f(x, y) dy}$$  \hspace{1cm} (3.13)

Where $y^*$ is the defuzzified output, which is the food density estimated by FIS.
3.4 SUMMARY

In this chapter we discussed the significant factor of FIS for food density estimation and input and output fuzzy sets were defined in detail. Also, fuzzy rules for food density estimation were designed and the FIS scheme was discussed. In addition, the mathematical expression for food density was derived after following all the FIS setup.
In this work, frozen French fries were taken as an example to indicate the fuzzy logic based approach. In this method a general three-step procedure was performed to build a well targeted FIS to estimate French fry density. The FIS was realized by Matlab tool box, whose structure is shown in Fig.7.

**Figure 7.** FIS structure for French fry density estimation, input variables: time and temperature, output variable: French fry density

From the French fry density FIS structure we can see that the time and temperature were chosen as the input variables with Gauss function being the corresponding fuzzy set membership function. The output variable is the French fry density itself using triangular function as the membership function. Fuzzy rules that connect the input and output fuzzy sets would imply which kind of input fuzzy set to use ---time, temperature, or the combination of those two by logic operator---which would result in the kind of output fuzzy set---the density. All the output
fuzzy sets involved were aggregated. The defuzzification defuzzified the aggregated result to a crisp number, which was the estimated density by the FIS. The detailed setup strategy is introduced in the following sections.

### 4.1 EXPERIMENTS FOR DETERMINING FIS PARAMETERS

The knowledge about French fry density under different cooking conditions is based on the experiments. An experienced chef can tell well the fry density by knowing how it will be cooked, which allows him/her to easily build up the FIS for French fry density estimation. In this study, a series of pre-experiments were implemented to acquire the knowledge that is necessary for the French fry density estimation FIS buildup.

The whole experimental process is shown in Fig. 8. Specifically, as shown in Fig. 8(a), the type of French fry chosen to be used in this experiment was the Golden Crinkle frozen French fry produced by H.J. Heinz Company. A certain volume of fries were fired in an oil cauldron with all cooking conditions recorded in real time as shown in Fig. 8(b). Upon finishing cooking under this certain cooking condition, the fries were picked up and put onto a paper towel for three minutes in order to draw out the oil as shown in Fig. 8(c). Then the weight of the fries were measured as shown in Fig. 8 (d) and based upon this, the density was calculated and recorded. Each test, using the same cooking conditions was repeated three times; the average value of each test was taken as the final fry density for that specific cooking condition. Three cooking temperatures were chosen for this pre-examination: 130 ºC, 150 ºC, and 170 ºC respectively. Under each cooking temperature, 3 different cooking times were picked for the
total frying time. In total, there were 9 cooking conditions set for this pre-examination. All the results were utilized for the input/output fuzzy set design and the fuzzy rule design.

Figure 8. Experimental procedure: (a) type of French fry chosen for experiment, (b) French fry cooking process, (c) oil filtering for 3 minutes, (d) weighing the French fries, the container is 3 inches deep with a 5.75 inch diameter.
4.2 FUZZY SETS DESIGN FOR FRENCH FRY

The density of frying food is affected by multiple factors in the cooking process, among which, are time and temperature. As these are the top two major elements [39], they were chosen as the input variables of FIS. Based upon what we concluded from Chapter 3, the density of the French fry is the output of this system. There are multiple concepts used to describe the density of food such as true density, apparent density, bulk density and mixture density [8,10]. With regard to the requirement of the obesity project [1], we measure the bulk density of French fry, which is the density when particles are packed or stacked in bulk including void spaces. Based on daily French fry cooking experiments, the shape of the fuzzy sets of input and output were decided and are listed in Table 1 and Table 2 respectively. The “μ”, “σ” in Table 1 stand for the mean and the variance of Gaussian function respectively. The “a”, “b”, “c” in Table 2 refer to the left bottom point, vertex, and right bottom point of triangular function respectively.

Table 1. Input Fuzzy Sets from Training Experiments

<table>
<thead>
<tr>
<th>Input Variable Type</th>
<th>Fuzzy Sets</th>
<th>Membership Functions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Function Type</td>
</tr>
<tr>
<td>Temperature (ºC)</td>
<td>Low Operating Temperature</td>
<td>Gaussian Function</td>
</tr>
<tr>
<td></td>
<td>Normal Operating Temperature</td>
<td>Gaussian Function</td>
</tr>
<tr>
<td></td>
<td>High Operating Temperature</td>
<td>Gaussian Function</td>
</tr>
<tr>
<td>Time (minute)</td>
<td>Short Frying Time</td>
<td>Gaussian Function</td>
</tr>
<tr>
<td></td>
<td>Normal Frying Time</td>
<td>Gaussian Function</td>
</tr>
<tr>
<td></td>
<td>long Frying time</td>
<td>Gaussian Function</td>
</tr>
</tbody>
</table>
Table 2. Output Fuzzy Sets from Training Experiments

<table>
<thead>
<tr>
<th>Output Variable Type</th>
<th>Fuzzy Sets</th>
<th>Membership Functions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Function Type</td>
</tr>
<tr>
<td>Fry Density</td>
<td>Very High</td>
<td>Triangular Function</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>Triangular Function</td>
</tr>
<tr>
<td></td>
<td>Over Normal</td>
<td>Triangular Function</td>
</tr>
<tr>
<td></td>
<td>Normal</td>
<td>Triangular Function</td>
</tr>
<tr>
<td></td>
<td>Under Normal</td>
<td>Triangular Function</td>
</tr>
<tr>
<td></td>
<td>Low</td>
<td>Triangular Function</td>
</tr>
<tr>
<td></td>
<td>Very Low</td>
<td>Triangular Function</td>
</tr>
</tbody>
</table>

Based on Table 1, the Membership Functions (MF) of the time variable are listed below:

\[
C_1^t(x_t) = e^{-\frac{(x_t-3)^2}{2\times0.9^2}} \\
C_2^t(x_t) = e^{-\frac{(x_t-5)^2}{2\times0.9^2}} \quad (x_t \in [3, 7]) \\
C_3^t(x_t) = e^{-\frac{(x_t-7)^2}{2\times0.9^2}}
\]  

(4.1)

where \(C_1^t\), \(C_2^t\) and \(C_3^t\) represent the short, normal, long fuzzy set MF of the time variable respectively.

The MFs of fuzzy set of the temperature variable are listed as:

\[
C_1^t(x_2) = e^{-\frac{(x_2-130)^2}{2\times6^2}} \\
C_2^t(x_2) = e^{-\frac{(x_2-150)^2}{2\times6^2}} \quad (x_2 \in [130, 170]) \\
C_3^t(x_2) = e^{-\frac{(x_2-170)^2}{2\times6^2}}
\]  

(4.2)
where \( C_1^2, C_2^2, C_3^2 \) represent the short, normal, long fuzzy set MF respectively. Based on Table 2

The MFs of fuzzy set of output variable food density are listed as:

\[
\begin{align*}
D_1(y) &= \text{Max}[\frac{y - 0.300}{0.300 - 0.300}, 0] \\
&\vdots \\
D_7(y) &= \text{Max}[\frac{y - 0.417}{0.440 - 0.417}, 0] \\
&\text{for } y \in [0.300, 0.440]
\end{align*}
\]  

where \( D_1, D_2, D_3, D_4, D_5, D_6, D_7 \) stand for the very high, high, somewhat high, normal, somewhat low, low, very low fuzzy set MFs respectively. In this manner, The inputs and outputs are fuzzified over all the qualifying MFs.

**4.3 IMPLEMENTATION OF FUZZY RULES**

Fuzzy rules are “if-then” rules that connect the inputs and outputs in the FIS. In order to gain experience in making French fries, nine actual cooking experiments were conducted with different cooking times and temperatures. The frying conditions of the pre-tests and corresponding density results are shown in Table 3.

Depending upon the density measured under its specified cooking condition fuzzy rules can be derived. For example, in pre-test 1, the cooking condition was set to 130\(^\circ\)C frying temperature (which belongs to the low temperature fuzzy set) and 3 minutes of cooking time (which belongs to the low time fuzzy set), the density in such conditions as these is 0.441 \( g \cdot cm^{-3} \).

This density not only belongs to the very high density output fuzzy set but also partially belongs to the high density output fuzzy set as the assignment of output fuzzy set in Table 2. Therefore,
the related output density set is “very high” as well as ‘high” as listed in Table 3. Thus, two fuzzy rules can be educed from pretest 1: The first is, “if time is low and temperature is low then density is very high”; the second is, “if time is low and temperature is low then density is high”. they are rules 1 and 2 in Table 4 respectively. Based on the pre-test results shown in Table 3, 18 rules were derived; these are listed in Table 4.

**Table 3. Training Table for Designing Fuzzy Rules**

<table>
<thead>
<tr>
<th>Test</th>
<th>Temp (°C)</th>
<th>Time (minute)</th>
<th>Food Density (g·cm⁻³)</th>
<th>Related Output Fuzzy Sets</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>130</td>
<td>3</td>
<td>0.441</td>
<td>Very High, High</td>
</tr>
<tr>
<td>2</td>
<td>130</td>
<td>5</td>
<td>0.413</td>
<td>High, Somewhat High</td>
</tr>
<tr>
<td>3</td>
<td>130</td>
<td>7</td>
<td>0.374</td>
<td>Normal, Somewhat High</td>
</tr>
<tr>
<td>4</td>
<td>150</td>
<td>3</td>
<td>0.389</td>
<td>Somewhat High, Normal</td>
</tr>
<tr>
<td>5</td>
<td>150</td>
<td>5</td>
<td>0.359</td>
<td>Normal, Now</td>
</tr>
<tr>
<td>6</td>
<td>150</td>
<td>7</td>
<td>0.330</td>
<td>Now, Somewhat Low</td>
</tr>
<tr>
<td>7</td>
<td>170</td>
<td>3</td>
<td>0.386</td>
<td>Normal, Somewhat High</td>
</tr>
<tr>
<td>8</td>
<td>170</td>
<td>5</td>
<td>0.330</td>
<td>Low, Somewhat Low</td>
</tr>
<tr>
<td>9</td>
<td>170</td>
<td>7</td>
<td>0.304</td>
<td>Very Low, Low</td>
</tr>
</tbody>
</table>

The weight of each rule in Table 4 specifies to what degree such a rule will take effect. The weight value is also determined by the pre-tests. Taking training test 5 as an example, the French fry density is 0.359 g·cm⁻³, which drops it into the range of “normal” density set (i.e. range from 0.3464 g·cm⁻³ to 0.3932 g·cm⁻³, center point: 0.3698 g·cm⁻³). The density set interval has already been divided into 4 sub-intervals, which are
The experience obtained during these experiments was translated into intuitive rules as shown in Table 4. A total of 18 rules are involved in this system producing outputs (fry densities) of VH (very high), H (high), SH (somewhat high), N (normal), SL (somewhat low), L (low), and VL (very low). The numerical value, ranging from 0 to 1, following each fuzzy set symbol provides a weight of each individual rule according to the importance of the rule in the inference system. Again, this weight was determined empirically using the knowledge gained during our experiments.

<table>
<thead>
<tr>
<th>Time</th>
<th>Low</th>
<th>Normal</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short</td>
<td>VH, 1</td>
<td>SH, 1</td>
<td>N, 0.5</td>
</tr>
<tr>
<td></td>
<td>H, 0.25</td>
<td>N, 0.25</td>
<td>SH, 0.25</td>
</tr>
<tr>
<td>Normal</td>
<td>HI, 1</td>
<td>N, 0.75</td>
<td>L, 0.75</td>
</tr>
<tr>
<td></td>
<td>SH, 0.25</td>
<td>L, 0.5</td>
<td>SL, 0.5</td>
</tr>
<tr>
<td>Long</td>
<td>N, 1</td>
<td>L, 0.75</td>
<td>VL, 1</td>
</tr>
<tr>
<td></td>
<td>SH, 0.25</td>
<td>SL, 0.5</td>
<td>L, 0.25</td>
</tr>
</tbody>
</table>
4.4 FIS OPERATION STRATEGY FOR FRENCH FRY DENSITY

As mentioned in Chapter 3, the fuzzy inference system cannot function only with the input, output, and fuzzy rules. The inference scheme also needs to be put in place to build the entire system. Specifically, the inference scheme setup refers to defining logic operators, designing implication, setting aggregation strategy and choosing a defuzzification approach.

As seen in Table 4, the antecedent of a given rule has two parts (i.e. time condition part and temperature condition part). In our system, only AND operator is involved and is defined as the product operation as mentioned in Chapter 3. Thus, the results of the antecedents will be the product of its two parts, which is:

\[ \phi_n(x_1, x_2) = \mu_{C_{i1}}(x_1) \cap \mu_{C_{i2}}(x_2) \quad i \in [1, 2, 3], \quad j \in [1, 2, 3], \quad h \in [1, 2...18] \]  

(4.4)

where \( \phi_n(x_1, x_2) \) represent the antecedence result of \( n \) th rule. \( \mu_{C_{i1}}(x_1), \mu_{C_{i2}}(x_2) \) refer to the MFs of input variables shown in equation (4.1), (4.2) respectively. \( x_1 \in (3, 7), x_2 \in (130, 170) \) stand for the ranges of time and temperature frying condition respectively.

For example, in rule 5, the “time is long” set is related to MF \( C_{i3}(x_1) \), and the “temperature is low” set is related to MF \( C_{i1}(x_2) \), therefore the result of its antecedent is

\[ \phi_5(x_1, x_2) = \mu_{C_{i1}}(x_1) \cap \mu_{C_{i2}}(x_2) \]  

(4.5)

Implication is implemented for each rule. The input for the implication process is a single number given by the antecedent, and the output is a fuzzy set base on the result of the antecedent. In this system, implication was defined as product operation which scales the output fuzzy set. Then by substituting equation (4.3) and (4.4), a corresponding density function for each rule was deducted as listed in equation (4.6).
\( \psi_h(x, y) = \mu_{C_1}(x_1) \cdot \mu_{C_2}(x_2) W_h \mu_{D_h}(y) \quad x = [x_1, x_2]^T \)  

(4.6)

where \( \psi_h \) indicates the implicated density function of \( h \) th rule. \( y \in [0.300, 0.440] \) refers to the range of French fry density. The rest of this equation is the same as that shown in equation (4.3) and (4.4).

Because decisions are based on the testing of all of the rules in a FIS, the rules must be combined in some manner in order to make a decision or estimation. Aggregation is the process by which the fuzzy sets that represent the outputs of each rule are combined into a single fuzzy set. Therefore, all the 18 functions in (4.6) need to be combined into one function. We set the aggregate method as sum, and therefore, equation (4.7) was derived as follows:

\[
f(x, y) = \sum_{h=1}^{18} \psi_h(x, y) \quad x = [x_1, x_2]^T, y = R
\]

(4.7)

Defuzzification is one of the most important processes in FIS. The final desired output of the system is generally a single number. However, as seen in equation (4.7), the aggregate of a fuzzy set under frying condition factor \( x_1, x_2 \), still encompasses a range of output values, and so must be defuzzified in order to get single output value from the set. As mentioned in Chapter 3, the centroid method was applied in this system following which the French fry density close form solution after defuzzification can be derived, which is shown in equation (4.8):

\[
y^* = \frac{\int f(x, y) \cdot y dy}{\int f(x, y) dy}
\]

(4.8)

Substituting (4.7) into (4.8), under boundary condition \( y \in [0.300, 0.440] \) we get:

\[
y^*(x_1, x_2) = \frac{\int_{0.300}^{0.440} \sum_{h=1}^{18} \psi_h(x_1, x_2, y) \cdot y dy}{\int_{0.300}^{0.440} \sum_{h=1}^{18} \psi_h(x_1, x_2, y) dy}
\]

(4.9)
Based on (4.9), given specific frying time $x_{1}^{*}$ and temperature $x_{2}^{*}$, a single number $y^{*}$ can be calculated, which is the estimated density of French fries inferred by FIS in such conditions. Thus, by implementing the above three steps, a French fry density estimation system is established based upon fuzzy logic.

4.5 EXPERIMENTAL RESULTS

The FIS in this work was implemented in Matlab. By applying all the settings mentioned in the above method sections, FIS for food density estimation was built. The input, output, and rules are shown in Fig.9.

Figure 9. Input, output and fuzzy rules, (a) input time variables, (b) input temperature variables, (c) output food density variables (d) fuzzy rules
Fig. 9 (a) and Fig. 9 (b) indicate the sets for the input time variable and input temperature variable respectively. Gaussian function is applied to build the input variable MFs. Each variable consists of three fuzzy sets as mentioned in the previous section. Fig. 9 (c) depicts the 7 fuzzy sets of output variable food density, whose MFs are all triangular functions. Fig. 9 (d) lists the total 18 if-then fuzzy rules, each of which was arranged a weight number as shown at the end to indicate the corresponding rule’s confidence level.

Fig. 10 indicates the complete fuzzy inference process. Information flows through the fuzzy inference diagram. The flow proceeds up from the inputs in the lower left, then across each rule row, and then down the rule outputs to finish in the lower right. This flow proceeds from linguistic variable fuzzification all the way through defuzzification of the aggregate output, to estimate out the crispy density value. As shown in Fig. 10, the yellow area in each fuzzy set depicts to which degree the input crispy value is taken as the member of the related fuzzy set. The red straight lines in the time and temperature column indicate the crispy time and temperature input value. The right column represents the fuzzy sets of the food density output variable. The blue area depicts to which degree the related density fuzzy set is taken into account based on the input and the very bottom row of the right column indicates the aggregated output fuzzy set, the red link of which refers to crispy density inferred by FIS after defuzzification.

For example, when a pair of inputs (i.e. time and temperature) are inserted, as show in Fig. 10 (time is 5 minutes and temperature is 150 °C), by rule, logic operator, and implication, then aggregated, and finally defuzzified, we can obtain a single number value, which is an estimated density of $0.356 \text{ g cm}^{-3}$. 
Figure 10. FIS process from input to output, each row refers to a fuzzy rule, the left two columns indicate the time and temperature variables respectively, red straight lines in the columns refer to the time and temperature inputs, the third column depicts the output fuzzy set, red straight line in the column refers to the density inferred by FIS.

Figure 11. Estimated French fry food density vs. time and temperature, the x, y and z axes are defined as the time, temperature and density respectively, time range is from 3 minutes to 7 minutes, temperature range is from 130 °C to 170 °C, French fry density range is from $0.32 \text{ g} \cdot \text{cm}^{-3}$ to $0.42 \text{ g} \cdot \text{cm}^{-3}$.

Fig.11 plots the 3D curved surface of the estimated food density from Matlab. The x, y and z axis are defined as the time, temperature and density respectively. For each operating condition, there is a specific density corresponding to it. As shown in Fig.11, the cooking temperature range is from 130 °C to 170 °C, while the cooking time range is from 3 minutes to 7 minutes. The density of French fries ranges from $0.32 \text{ g} \cdot \text{cm}^{-3}$ to $0.42 \text{ g} \cdot \text{cm}^{-3}$. From the result
shown from Fig.11, we can conclude that the density would decrease if the cooking temperature was increased, and the density would increase if the cooking time was extended, vice versa.

This result is reasonable based upon the nature of the French fry cooking process. When the frying temperature is fixed, the longer the cooking time, the more water inside the raw fry would be cooked out in the form of steam. At the same time, more oil is absorbed into the French fry. Both of these two factors result in decreasing French fry density. When frying time is fixed, the water-oil exchanging speeds up with the increasing of the frying temperature. Hence, the density of French fry becomes less. The highest temperature used in this work is 170 ºC. If the temperature keep increasing, the shield that covers the surface of the fry would be formed more easily and faster [39], which in turn will resist the water inside the raw fry from venting out, reducing the process of absorbing oil as well. Hence, the density of French fry would increase.

4.6 FIS RESULT EVALUATION

A series of validation tests were performed to inspect the fuzzy logic based food density estimation approach. Experiments were repeated three times under each cooking condition with respect to frying time and temperature. Specifically, the frying times used for the experiments were 3 minutes, 4 minutes, 5 minutes, 6 minutes and 7 minutes, respectively. For each frying time, the frying temperatures were changed from 130 ºC to 170 ºC in an interval of 10 ºC. The French fry densities estimated by FIS were compared with the densities measured from the real tests. The results are shown in Fig.12.
Figure 12. Estimated densities vs. measured densities with 3 to 7 minutes frying time at different temperatures, (a), (b), (c), (d), and (e) depict the comparisons of estimate values with real test values in frying time 3 minutes, 4 minutes, 5 minutes, 6 minutes, and 7 minutes respectively.

The mean estimate error of different frying temperatures in the 3 minutes frying time condition was 3.0% with a standard deviation of 1.1%. The real density was always higher than the estimated value. One of the reasons for this may be that the rules related to short time conditions did not match the realistic cases. The consequence of the rules linking itself to a food density set less than the real value degrades the food density.
In the 4 minutes frying time condition the mean estimate error was 1.92% with a standard deviation of 1.4%. These estimation results match the real test values precisely when the temperatures are 140°C and 160°C. When the cooking temperatures are 130°C and 170°C, the estimation results show a small error from the real values. However, for 150°C the estimate food density biases from the real value in a relatively large degree, which may be caused by the weight setup of the related rules.

In the 5 minutes condition, the mean estimate error was 2.3% with a standard deviation of 2.7%. All the estimated food densities fit the corresponding real values except for the result at 140°C, which shows a 6.5% error. The reason may be that 140°C is not one of the three original temperature conditions that were used to derive the system fuzzy rules.

The mean error in the 6 minutes condition was 2.3% with a standard deviation of 2.9%. As with the situation in 5 minutes, a 7.1% error happened at 140°C as well, which is also the biggest error of the whole FIS system. The reason for this may be that not only the temperature is not one of the three original temperature conditions that were used for fuzzy rules setup, but it also is not the time of the three original time conditions for the fuzzy rules setup.

The density results matched well in the 7 minutes frying condition for every single point with a 1.5% mean error and a 0.67% standard deviation. With longer time the fry density tended to stabilize under the same temperature cooking conditions, and there was not a significant contradiction involved among the related rules.

The overall mean estimate error is 2.2%, and the corresponding standard deviation is 1.9%. The biggest error comes out under the condition of 140 °C frying temperature and 6 minutes frying time, which is 7.1%.
4.7 SUMMARY

In this chapter FIS was implemented in Matlab for a French fry density estimation considering frying time and temperature as affected factors. French fry density was estimated under various frying conditions. Close form solution of FIS for French fry density was derived as well. A series of experiments to verify the FIS method was performed in which the actual densities were measured and compared against the fuzzy logic outputs. We compared results for different frying times between three and seven minutes with one-minute intervals and frying temperatures between 130°C and 170°C with 10°C intervals. The overall mean estimation error was 2.2% with a standard deviation of 1.9%. 
5.0 CONCLUSION AND FUTURE WORK

FIS system was developed to implement the expert’s skill to assess the food density assuming the recipe of such food is given. Setups for the primary factors of FIS for food density estimation are introduced in detail, including input and output fuzzy set definition, fuzzy rule designing, and FIS operation scheme designing. The close form for food density was derived after following all the FIS setup. The approach was applied to the estimation of fried French fry density given frying conditions for method verification. By taking frying time and temperature as input variables, French fry density was estimated under various frying conditions. A series of experiments were performed in which the actual densities were measured and compare against the fuzzy logic outputs. Fry density results obtained from FIS showed a good agreement with real test results. The overall mean estimation error was 2.2% with a standard deviation of 1.9%. FIS methods can be successfully applied to serve as a decision support technique in food density estimation.

In the current work, we only set up the food density FIS system for the French fry with simple input variables. Further research will be conducted to estimate densities of other foods using different knowledge and under more complex conditions. We also plan to add other methods such as density grouping into FIS food density estimation to simplify the system buildup process and improve accuracy of the results. Computer cooking will be studied as well. Based on FIS, a virtual cooking environment will be established on the computer. Instead of
actually cooking the food, people only need to input the recipe into the computer to see the end result following said recipe. FIS could play an important role as an intelligent assistant in the field of food industry.
BIBLIOGRAPHY


