THE IMPACT OF ADVERTISING AND USER-GENERATED CONTENT ON MEDIA BIAS

by

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This dissertation consists of two studies that investigate the impact of advertising and user-generated content on media bias. The first study analyzes how advertising revenues in addition to subscription revenues play a role in affecting the extent of media bias. When making advertising choices, advertisers evaluate both the size and the composition of the readership of the different outlets. The profile of the readers matters since advertisers wish to target readers who are likely to be receptive to their advertising messages. It is demonstrated that when advertising supplements subscription fees, it may serve as a polarizing or moderating force, contingent upon the extent of heterogeneity among advertisers in appealing to readers having different political preferences. When heterogeneity is large, each advertiser chooses a single outlet for placing ads (Single-Homing), and greater polarization arises in comparison to the case that media relies only on subscription fees for revenues. In contrast, when heterogeneity is small, each advertiser chooses to place ads in multiple outlets (Multi-Homing), and reduced polarization results. In the second study, a newspaper’s decision to expand its product line by adding an online edition that incorporates user-generated content and the impact of this decision on its slanting of news are investigated. It is demonstrated that adding an online edition results in reduced profits for competing newspapers in comparison to an environment in which they offer only print editions. However, at the equilibrium, each newspaper offers the online version in order to avoid losing market share to rivals.
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1.0 GENERAL INTRODUCTION

Gentzkow and Shapiro (2006) argue that a healthy news media is fundamental for a healthy democracy. It is important for public to obtain news that is objective. Yet, bias in news media is widely recognized (e.g., Groseclose and Miley 2005, and Hamilton 2004) and it often appears in forms of selective omission, and choice of words, and picking information sources to cite.

Previous research cited a variety of reasons for the existence of media bias, ranging from journalists’ desires to enhance their career opportunities (Baron 2006) to media’s incentive to increase audience ratings (Bernhardt, Krasa and Polborn 2008). In a recent paper by Mullainathan and Shleifer (2005), a link is established between subscription fees and media bias. By assuming that readers prefer news consistent with their beliefs and that newspapers can slant toward these beliefs, authors show that when the papers’ sole source of revenue is from subscription fees (i.e., price for news), newspapers slant news storeries toward extreme positions. Following Mullainathan and Shleifer (2005), Gentzkow and Shapiro (2006) and Xiang and Sarvary (2007) also argued that media bias results from newspapers’ desires to maximize subscription revenues. To extend and complement these earlier research, this dissertation provides two other explanations for why news media might be biased: the desire to maximize advertising revenues (in addition to subscription revenues), and extending product line to offer a user-generated content (UGC) enhanced version of a news product.
First, advertising revenues can play a significant role in determining the extent of bias because for many media outlets revenue comes from advertising as opposed to subscription fees. For example, newspapers are traditionally reported to earn 80% of total revenues from advertising and 20% from circulation (Sass 2009). Given this reality, in the first study, we extend the investigation of Mullainathan and Shleifer (2005) by recognizing that newspapers rely on revenues that accrue both from subscription fees paid by readers and advertising fees paid by advertisers. We investigate how the existence of these two sources of revenues affect the extent of bias in reporting that is selected by the media. Based on the correlation between the advertised product and the political opinions of a consumer, we allow the advertisements to have varying levels of effectiveness to enhance the probability of buying. An advertisement that reminds a consumer that the product is consistent with his political opinions may increase the likelihood that he will purchase the product.

Second, a newspaper’s decision to diversify its product mix to offer an online edition with UGC can also influence media bias. UGC is becoming increasingly more common in the news media, appearing in the forms of comments, blogs, photos, and video news. Such integration of user content is also known as “citizen journalism”. For example, in the print media, The New York Times and Wall Street journal allow readers to comment on the articles and create discussion groups in the online edition of the newspapers. In the broadcast media, CNN and Fox News have been disseminating news videos (‘I-reports’ and ‘U-reports’) that are submitted by their audience. Yet, for news companies the impact of extending their product line by including an online edition with UGC on media bias and profitability is unknown. To investigate this impact, we analyze an environment where only the online editions of newspapers offer UGC. Based on recent studies by Miller and Morrison (2009) and Morrison and Miller...
(2008), we assume that consumers with more extreme opinions have higher appreciation for the online editions because of UGC. As a result, diversification of the product mix leads to possible segmentation of readers according to their political opinions: readers who are moderates prefer the print edition of the newspaper and readers who are extremes subscribe to the online edition and can be active in generating content on the newspaper’s site.

In analyzing the impact of advertising and UGC on media bias, based on Mullainathan and Shleifer (2005), we form two key assumptions about the utility a consumer receives from reading news. First, it is assumed that readers prefer news consistent with their political opinions. Therefore, a reader of leftwing (rightwing) political opinions prefer to read news stories with leftwing (rightwing) orientation. Second, it is assumed that readers prefer news stories with less slant. For example, when news stories presented are equidistant from a reader’s political opinion, his utility from the story with lower slant is higher. These two assumptions together suggest that readers show a split personality and carry conflicting goals in acquiring news. Newspapers, in exchange, strategically slant toward these opinions.

Findings from the first study show that when newspapers rely both on advertising and subscription fees, advertising can serve as a polarizing or moderating force in affecting the reporting of newspapers through two effects. First, the “readership effect” enables the newspapers to charge higher advertising fees by reducing newspapers’ reliance on subscribers in favor of advertisers. As a result, newspapers can afford to have lower slant in their news reports and appeal to moderate readers and by doing so, offer a bigger readership to advertisers. However, in seeking to lure advertisers, the counter “incremental pricing effect” may arise when advertisers choose to Single-Home. Newspapers may have stronger incentives to polarize in order to alleviate price competition in both markets.
Findings from the second study show that segmentation of readers reduces the extent of bias in reporting of the print edition but intensifies the extent of bias of the online edition compared to an environment in which newspapers do not diversify their product mix. When UGC is added by readers to the online editions, each newspaper is indirectly forced by subscribers to offer two differentiated versions of its product. With this added differentiation, the profitability of the newspaper declines in comparison to an environment where it has the exclusive right to choose the bias of both editions.

Rest of the dissertation is organized as follows. In the following section, the influence of advertising revenues on media bias is modeled and explained. Subsequently, the model is extended to consider the influence of UGC when subscription is the main source of revenue. The proofs for all the proposition, corollaries and lemmas can be found in the Appendix.
2.0 THE IMPACT OF ADVERTISING ON MEDIA BIAS

2.1 INTRODUCTION

Bias in news media is well known (e.g., Groseclose and Milyo 2005, and Hamilton 2004) and can be defined as selective omission, choice of words and varying credibility ascribed to the primary source (Gentzkow and Shapiro 2006). In a recent paper by Mullainathan and Shleifer (MS 2005), a link is established between subscription fees and media bias. By assuming that readers prefer news consistent with their political opinions and that newspapers can slant toward these opinions, MS (2005) show that when the papers’ sole source of revenue is from subscription fees (i.e., price for news), they slant news toward extreme positions.

For many media outlets, however, 60% to 80% of total revenue stems from advertising (Strömberg 2004), as opposed to subscription. Thus, in this study, we aim to complement the work of MS (2005) by recognizing that newspapers rely on revenues that accrue both from subscription fees paid by readers and advertising fees paid by advertisers. We investigate how the existence of these two sources of revenue affect the extent of bias in reporting that is selected by the media.
In order to understand the role of advertising in determining the nature of competition between newspapers, we specify in the model the effectiveness of advertisements to enhance consumers’ probability of purchase. We argue that this effectiveness, for some products, may depend upon the political opinions of readers of the ads. It has been long established in the Consumer Behavior literature that products reflect a person’s self-concept (Belk 1988). They provide a way for a person to express her self-image, which may be strongly correlated with her political opinions. We introduce, therefore, a product specific variable that measures the extent to which political preferences play a role in enhancing consumers’ probability of purchase of the product. While for some products this measure is significant, for others it is trivial. For example, while “green” products, such as Toyota Prius, or Apple’s Mac computer may appeal more to liberals, “American” products, such as the Chevy Truck, may appeal more to conservative consumers. However, there are many products, such as automobile tires or insurance policies, for which political opinions do not affect consumers’ choices to a large extent. ¹ When political preferences play an important role in consumers’ purchase decisions, advertising the product can be effective if it targets the correct consumers. An advertisement that reminds the consumers that the product is consistent with their political opinions may increase the likelihood that they purchase the product.

Heterogeneity among advertisers with respect to the appeal of their products to consumers having different preferences is distributed in our model over a bounded interval. The length of this interval captures the extent of heterogeneity among advertisers, with longer

¹ The appeal parameter of the advertised product that we introduce in the model assumes a value in the vicinity of zero if political preferences do not play an important role in consumers’ purchase decisions.
intervals indicating significant differences in the appeal of products to liberal vs. conservative readers. In our model we show that the degree of heterogeneity among advertisers plays a role in determining whether advertisers choose to place ads with a single newspaper or with both newspapers. The literature on two-sided markets has referred to these two possible outcomes as Single and Double-Homing by advertisers, respectively (See Armstrong (2006), for instance.) While Single-Homing arises as the unique equilibrium when the extent of heterogeneity is large, Double-Homing arises when it is small.

We further investigate the manner in which the advertisers’ choice between the newspapers affects the slanting strategies of media outlets. We show that when newspapers rely both on advertising and subscription fees, advertising can serve as a polarizing or moderating force in affecting the reporting of newspapers through two effects. First, adding the advertising market implies that newspapers reduce their reliance on subscribers in favor of advertisers. As a result, they may choose less slanting in their reporting strategies to improve their appeal to moderate readers, and by doing so, offer a bigger readership to advertisers. This “readership effect” enables the newspapers to charge higher advertising fees.

However, in seeking to lure advertisers a second, counter effect may arise when advertisers choose to Single-Home. Specifically, when downward pressures on subscription fees arise due to reduced slanting of the newspapers, similar downward pressures on advertising fees appear, as well, as each newspaper attempts to defend its market share among advertisers. Hence newspapers may have stronger incentives to polarize in order to alleviate price competition in both markets. This “incremental pricing effect” to polarize is above and beyond the traditional attempt of companies to introduce product differentiation in order to soften price competition in a given market. Due to the two-sided markets we consider, polarization serves to soften price
competition in both markets. We demonstrate that at the Single-Homing equilibrium, the “incremental pricing effect” is stronger than the “readership effect”, thus leading to intensified bias in reporting. In contrast, at the equilibrium with Double-Homing the “readership effect” is the only force present, thus giving rise to reduced bias at the equilibrium.

There is a growing body of literature on media bias as implied by the media’s attempt to appeal to readers’ beliefs. In addition to MS (2005), Gentzkow and Shapiro (2006) and Xiang and Sarvary (2007) also investigate this kind of bias. In Gentzkow and Shapiro (2006) readers who are uncertain about the quality of an information source infer that the source is of higher quality if its reports are consistent with their prior expectations. Xiang and Sarvary assume that there are two types of consumers, those who enjoy reading news consistent with their political opinions and conscientious consumers who care only about the truth. This assumption is different from MS (2005) or our study, where each consumer values both some consistency with political opinions and accuracy. The reporting strategy of the newspapers depends then on the relative weights consumers assign to consistency with their political opinions vs. accuracy. In addition, these earlier studies on bias assume that the media’s sole source of revenue stems from selling news. In contrast, in the present study we allow the papers to earn revenues from advertising fees as well.

There are two recent papers that consider, like us, a media market with both advertising and subscription fees as sources of revenue. In Gabszewicz, Laussel and Sonnac (2002) and Ellman and Germano (2009), advertisers care only about the size and not the profile of the readership of each newspaper. This assumption is different from our setting, where advertisers wish to target audiences that are receptive to their advertising messages. This targeting objective of advertisers is pursued in Bergemann and Bonatti (2010) in an environment where the sole
source of revenues of media outlets is from advertising. In this recent study, the authors investigate how improvements in the targeting technology that is facilitated by online advertising affects the allocation of advertisements across different media and the equilibrium prices of advertising messages. The topic of targeted advertising is also investigated in Iyer, Soberman, and Villas-Boas (2005) in an environment where the firms themselves and not media outlets possess the targeting technology.

Another strand of literature related to our study deals with consumers who may choose one or two of competing products. In Sarvary and Parker (1997) consumers decide whether to rely on a single information source or to diversify their purchases to include competing sources. They show that the segmentation of consumers between those who purchase one or two sources of information depends upon the relative importance consumers assign to obtaining precise information. In Guo (2006), a similar diversification of the consumption bundle may arise when there is uncertainty about future preferences. Buying competing products simultaneously serves as “insurance” against such uncertainty. The main difference between our study and the previous two is our focus on competition between media outlets in two-sided markets instead of the one-sided framework considered in these studies.
2.2 THE MODEL

Consider a market with two newspapers, $i=1, 2$, a mass of $A$ advertisers and a mass of $M$ consumers, where $M_1$ of these consumers are subscribers to one of these two papers and $M_2$ are nonsubscribers. Newspapers provide news and print advertisements. By simultaneously operating in these two markets, newspapers have two potential sources of revenue: subscription fees ($P_i$) and advertising fees ($K_i$).

Each of the $M_1$ consumers reads either Newspaper 1 or 2 (but not both), and may buy products from the advertisers. We adapt the model developed by MS (2005) to capture the interaction between subscribers and newspapers. Specifically, when reading the newspaper, a subscriber receives information about a certain news item $t$, which is distributed according to $N(0, \sigma_t^2)$. Each consumer has some belief about the news item that is affected by her political opinion. We designate this political preference by $b$, and assume that the consumer believes the news item to be distributed according to $N(b, \sigma_t^2)$. In comparison to the true distribution of the news item, the consumer’s belief is biased. The political opinion parameter $b$ measures the extent and direction of this bias. It is uniformly distributed in the population of readers between $-b_0$ and $b_0$. For example, readers with beliefs closer to $-b_0$ can be considered liberals, and those in the proximity of $b_0$ can be considered conservatives.

Newspapers report news about $t$. They receive some data $d = t + \varepsilon$, where the random variable $\varepsilon$ is independently distributed of $t$ according to $\varepsilon \sim N(0, \sigma_\varepsilon^2)$. Note that the data received by the newspapers may be different since $t$ and $\varepsilon$ are random variables. Hence, $d \sim N(0, \sigma_d^2)$,
where $\sigma_d^2 = \sigma_i^2 + \sigma_e^2$. Newspapers may choose to report the data with slant $s_i$, so the reported news is $n_i = d + s_i$. Readers incur disutility when reading news inconsistent with their political opinions, as measured by the distance between the reported news and the readers’ opinions: $(n_i - b)^2$. Holding constant the extent of inconsistency with their opinions, they also prefer less slanting in the news. As in MS (2005), the overall utility of a reader is:

$$U_{b}^i = \bar{u} - \chi s_i^2 - \phi(n_i - b)^2 - P_i$$

Where $\bar{u}$ is the reservation price of the reader, $\chi$ calibrates her preference for reduced slant, and $\phi$ calibrates the reader’s preference for hearing news consistent with her political opinion. Note that the utility of the reader increases the smaller the slant $s_i$, and the smaller the discrepancy between the reader’s opinion $b$ and the reported news $n_i$.

Similar to MS (2005) we also focus on the characterization of the equilibrium with full coverage of the market and linear slanting strategies of the newspapers in the form $s_i(d) = \frac{\phi}{\phi + \chi} (B_i - d)$ with $B_i$ interpreted as a choice of location of newspaper $i$. In Appendix we show the optimality of linear slanting strategies when the newspapers’ sole source of revenue is from subscription fees. However, in our analysis, in which both advertising and subscription fees are sources of revenue, we implicitly assume that the linearity of slanting strategies is still valid. This location choice of the newspaper can be a point inside or outside of the interval $[-b_0, b_0]$ and reflect the newspaper’s political preference. Using $s_i(d)$, the study slants data toward its

\[ \text{Notice that there is no vertical differentiation between the newspapers in this setting (i.e., the accuracy of the data received by both newspapers is identical: } \sigma_{d_1}^2 = \sigma_{d_2}^2 = \sigma_d^2. \text{ In the Web Appendix, we demonstrate that our utility specification may also give rise to a tradeoff between vertical and horizontal differentiation. Specifically, when } \sigma_{d_1}^2 < \sigma_{d_2}^2, |B_1| < |B_2|. \]
preference $B_i$ when reporting news. Notice that the extent of slanting is an increasing function of $\phi$ and a decreasing function of $\chi$. Hence, as readers derive higher utility from hearing news consistent with their political opinions and reduce the importance placed on obtaining accurate information, newspapers choose greater slanting in their reporting. Without loss of generality, we assume that Newspaper 2 is located to the right of Newspaper 1 ($B_1 < B_2$). That is, while Newspaper 1 slants more to the left, Newspaper 2 slants more to the right.

Substituting the linear slanting strategies for $s_i$ and $n_i$ into Equation 1 and using the distributional properties of the random variable $d$ (specifically, that $Ed = 0$ and $Ed^2 = \sigma_d^2$), yields the expected payoff of a consumer having opinion $b$ at the time she chooses between the two newspapers. Note that at this time, the realizations of $d$ and $s_i(d)$ are yet to be determined due to the fluctuations of the data supporting news stories. At the time of the choice, the reader is aware only of the locations and fees chosen by the newspapers ($B_i$ and $P_i$) as well as her own political opinion $b$. Since the actual news may fluctuate depending upon the realization of $d$, in evaluating the utility she derives from subscribing to the papers the reader calculates expectation over all possible $d$ realizations in Equation 1. For Newspaper $i$ and reader of type $b$ this yields the following expected utility.

$$EU_b^i = \bar{u} - \frac{\phi^2}{\phi + \chi} (B_i - b)^2 - \frac{\chi \phi}{\phi + \chi} (b^2 + \sigma_d^2) - P_i.$$  

The consumer who is indifferent between the two newspapers satisfies the equation $EU_b = EU_b^2$. Solving this equation for $b$ yields:

$$b_{indiff} = \frac{\phi + \chi}{2\phi^2} \frac{(P_2 - P_1)}{(B_2 - B_1)} + \frac{B_1 + B_2}{2}.$$  

Given the expression derived for $b_{indiff}$, the papers’ subscription revenues are:

$$R_{1, sub} = M_1 P_1 \frac{b_0 + b_{indiff}}{2b_0} \text{ and } R_{2, sub} = M_1 P_2 \frac{b_0 - b_{indiff}}{2b_0}.$$
The population of advertisers is distributed according to the appeal of their products to consumers having conservative opinions, namely those situated in the positive segment of the distribution of opinions. We designate this appeal parameter by $\alpha$ and assume it is uniformly distributed on the interval $[-\alpha_0, \alpha_0], \alpha_0 \geq 0$. Negative values of $\alpha$ indicate products unappealing to conservative consumers with opinions in the range $[0, b_0]$, with more negative values indicating increased appeal to liberal consumers with opinions in the range $[-b_0, 0]$. Positive values of $\alpha$ indicate products having the opposite characteristics, with bigger positive values indicating increased appeal to conservatives. Products whose attractiveness to the consumer is unlikely to be determined by political opinions assume an $\alpha$ value in the neighborhood of zero. Given the above specification, the parameter $\alpha_0$ can be interpreted as reflecting the extent of heterogeneity of the appeal of different products to consumers with different political opinions.

We assume that in the absence of advertising each consumer has a certain probability of purchasing a product. This probability can be modified with advertising. The change in purchase probability for a given reader depends on the extent of compatibility between the political opinion of the reader (her location $b$) and the type of the product advertised (its appeal $\alpha$). When an ad is successfully targeted to enhance compatibility, the reader’s purchase probability of the advertised product increases. However, with lack of compatibility, her purchase probability might actually decrease. We designate by $E(\alpha, b)$ the incremental probability (positive or negative) when a reader of political preference $b$ is exposed to an ad related to product $\alpha$, and specify it as:

\[
E(\alpha, b) = \left( h_0 + \frac{\alpha b}{b_0} \right), \quad \text{where } h_0 > 0.
\]
Hence, the effectiveness of advertising is higher when political opinions are more consistent with the appeal parameter of the advertised product, measured by the term $ab$ in Equation 4. Note that the product $ab$ is positive for both liberal consumers of products having a negative measure of appeal $a$ and conservative consumers of products having a positive measure of appeal. The parameter $h_0$ is a measure of the basic effectiveness of advertising to increase consumers’ purchase probabilities. The change in the probability of purchase $E(a, b)$ depends also upon the extent of compatibility between the variables $b$ and $a$. For example, when a liberal consumer is exposed to an advertisement of a green product, this will cause an increase in her probability of purchasing this product that is above $h_0$, which is the basic increase in purchase probability when the consumer becomes aware of the product due to the advertisement. However, an extremely conservative consumer can respond very negatively to this product in which case the change in her purchase probability due to the advertisement $E(a, b)$ might even become negative. According to Equation 4, the change in the purchase probability for extreme products and consumers is larger than that for moderate products and consumers. As we mention later, when this feature of our model is not valid, some of our results may change, even though the strategic effects we identify will continue to operate.\(^3\)

The specification in (4) implies that an advertiser is likely to pursue two objectives in designing its advertising strategy: to obtain a large audience for its ads and to target an audience that is receptive to its advertising message. The first component of the advertising response

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\(^3\) Let $F_b$ denote the initial probability of purchase in the absence of advertising by an individual with opinion $b$ and $P_b$ denote the probability of purchase after advertising such that $P_b = F_b + E(a, b)$. In order to guarantee that $0 \leq P_b \leq 1$ we assume that $1 - F_b - a_0 \geq h_0 \geq a_0 - F_b$ and $F_b + a_0 < 1$. Note that these parameter restrictions do not conflict with those given in Lemma 1.
function motivates the large audience objective and the second motivates the targeting objective. Finally, for simplicity, we assume that advertising has the same effect on a subscriber and nonsubscribers with whom she shares information about advertised products. This assumption is reasonable since subscribers tend to communicate with friends and relatives who normally hold similar political opinions.

The payoff of an advertiser is measured by the average increase in the number of consumers likely to buy its product (average incremental probability times the mass of consumers $M$) net of the advertising fees paid to the newspapers. Hence, when an advertiser of appeal parameter $\alpha$ chooses to advertise only in Newspaper 1, its expected payoff as derived from the subscribers of Newspaper 1 is given as:

$$E_1(\alpha) = M \int_{-b_0}^{b_{indiff}} \frac{1}{2b_0} \left( h_0 + \frac{ab}{b_0} \right) db - K_1,$$

if it chooses to advertise only in Newspaper 2 its expected payoff is:

$$E_2(\alpha) = M \int_{b_{indiff}}^{b_0} \frac{1}{2b_0} \left( h_0 + \frac{ab}{b_0} \right) db - K_2,$$

and if it chooses to advertise in both papers its expected payoff is:

$$E_{12}(\alpha) = E_1(\alpha) + E_2(\alpha).$$

By choosing to advertise only in Newspaper 1, an advertiser recognizes that subscribers to this newspaper tend to have left leaning political opinions, lying in the interval $[-b_0, b_{indiff}]$ where $b_{indiff} = 0$ at the symmetric equilibrium (when $-B_1 = B_2 \geq 0$). For instance, if it advertises a green product ($\alpha < 0$) in Newspaper 1, it can expect a positive payoff if the advertising fee paid to the newspaper ($K_1$) is not too large, given that the average change in these readers’ purchase probability due the advertisement is positive (i.e., $\int_{-b_0}^{0} \frac{1}{2b_0} \left( h_0 + \frac{ab}{b_0} \right) db > 0$).
In contrast, by choosing to advertise only in Newspaper 2, the advertiser draws readers who have more right leaning opinions, in the interval \([b_{\text{indiff}}, b_0]\). In this case, even though these readers become aware of its product \((h_0 > 0)\), their political preferences are inconsistent with the product \((\alpha b \leq 0\) when \(\alpha < 0\) and \(b \in [0, b_0]\)), thus possibly leading to a negative expected payoff.

When advertising in both newspapers, an advertiser draws the entire population of readers. An advertiser chooses to advertise in a single newspaper \(i\) if \(E_i(\alpha) > E_{12}(\alpha)\) and \(E_i(\alpha) > 0\). From Equations 5-7 it follows that for this advertiser \(E_j(\alpha) < 0\) for \(j \neq i\), namely the added benefit from advertising in the second newspaper falls short of the fee newspaper \(j\) charges. This may happen if the advertiser’s product appeals mostly to readers having extreme political opinions. Advertising in a newspaper whose readership consists mostly of readers with opposing opinions in the political spectrum may not be worthwhile to the advertiser in this case. In contrast, an advertiser whose product’s appeal is not highly correlated with political preferences (having an appeal parameter in the neighborhood of zero) may advertise in both newspapers since the added benefit from advertising in each paper is likely to be positive for this advertiser, implying that \(E_{12}(\alpha) > E_i(\alpha), i = 1, 2\). The above discussion indicates that the population of advertisers can be segmented into at most three intervals as described in Figure 1.
Advertised products with appeal parameter less than $\hat{a}_1$ are advertised only in Newspaper 1 since the advertisers of these products try to target mostly liberals (from (6) $E_2(\alpha)$ is an increasing function of $\alpha$, thus if $E_2(\hat{a}_1) = 0$, $E_2(\alpha) < 0$ for all $\alpha < \hat{a}_1$). In contrast, those with appeal parameter bigger than $\hat{a}_2$ are advertised only in Newspaper 2, since advertisers wish to reach only conservative readers for such high values of appeal parameter (from (5) $E_1(\alpha)$ is a decreasing function of $\alpha$, thus if $E_1(\hat{a}_2) = 0$, $E_1(\alpha) < 0$ for all $\alpha > \hat{a}_2$). For intermediate values of $\alpha \in [\hat{a}_1, \hat{a}_2]$, advertisers choose to advertise in both newspapers (since both $E_1(\alpha)$ and $E_2(\alpha)$ are positive in this range). The number of segments in Figure 1 can be smaller than three. If $\hat{a}_1 \geq \hat{a}_2$, no advertiser chooses to advertise in both newspapers (referred to in the literature on two-sided markets as Single-Homing) and if $\hat{a}_1 = -\hat{a}_0$ and $\hat{a}_2 = \hat{a}_0$ all advertisers choose to advertise in both newspapers, (Double-Homing). Note, in particular that when $\alpha_0 = 0$, the mass of $A$ advertisers is located at $\alpha = 0$, and in this case, advertisers do not care about targeting. At the symmetric equilibrium, from Equations 5 and 6 each advertiser derives the net benefit of $\frac{Mh_0}{2} - K$ when placing an ad with either one of the newspapers. Double-Homing is obviously implied, given that both newspapers offer the same net benefit to each advertiser.
From Equations 5-7 we can derive the expressions for $\hat{a}_1$ and $\hat{a}_2$ as functions of the locations and advertising fees chosen by the newspapers as follows:

\begin{align*}
\hat{a}_1 &= \frac{2b_0}{b_0^+ + b_{\text{indif}}} \left( \frac{2b_0 K_2}{M(b_0 - b_{\text{indif}})} - h_0 \right), \\
\hat{a}_2 &= \frac{2b_0}{b_0^- - b_{\text{indif}}} \left( h_0 - \frac{2b_0 K_1}{M(b_0 + b_{\text{indif}})} \right).
\end{align*}

The appeal parameter $\hat{a}_1$ ($\hat{a}_2$) characterizes an advertiser who is indifferent between advertising in Newspaper 1(2) and advertising in both newspapers (i.e., $E_2(\hat{a}_1) = 0$ and $E_1(\hat{a}_2) = 0$).

In the Single-Homing equilibrium, the interior segment of Figure 1 disappears and the advertiser who is indifferent between Newspaper 1 and 2 can be derived from Equations 5 and 6 by solving for $\alpha$ in the equation $E_1(\alpha) = E_2(\alpha)$:

\begin{equation}
\alpha_{\text{indif}} = \frac{2b_0 b_{\text{indif}}}{(b_0^+ - b_{\text{indif}}^2)^2} h_0 - \frac{2b_0^2 (K_1 - K_2)}{(b_0^+ - b_{\text{indif}}^2)^2 M}.
\end{equation}

From Equation 9 we obtain the advertising revenues that accrue to the newspapers in the equilibrium with Single-Homing as follows:

\begin{align*}
R_{1,\text{adv}} &= AK_1 \frac{\alpha_0 + \alpha_{\text{indif}}}{2\alpha_0} \quad \text{and} \quad R_{2,\text{adv}} = AK_2 \frac{\alpha_0 - \alpha_{\text{indif}}}{2\alpha_0}.
\end{align*}

When some advertisers Double-Home, the segment of the market covered by Newspaper 1 is $(\alpha_0 + \hat{a}_2)/2\alpha_0$ and that covered by Newspaper 2 is $(\alpha_0 - \hat{a}_1)/2\alpha_0$. As a result, the advertising revenues of the newspapers are:

\begin{align*}
R_{1,\text{adv}} &= AK_1 \frac{\alpha_0 + \hat{a}_2}{2\alpha_0} \quad \text{and} \quad R_{2,\text{adv}} = AK_2 \frac{\alpha_0 - \hat{a}_1}{2\alpha_0}.
\end{align*}

In what follows we will derive symmetric equilibria with the market of advertisers fully covered. At such equilibria, $-\hat{a}_1 = \hat{a}_2 \geq 0$, and $-B_1 = B_2 \geq 0$. We will focus on two possible cases: equilibrium with Single-Homing, where each advertiser chooses to advertise in a single newspaper ($\hat{a}_1 = \hat{a}_2 = 0$ in Figure 1); and Double-Homing, where all advertisers choose to
Double-Home ($\tilde{\alpha}_1 = -\alpha_0$, $\tilde{\alpha}_2 = \alpha_0$). We formulate the decision process of the newspapers as a two stage game. In the first stage, each newspaper simultaneously announces a strategy $s_i(d)$ of how to report the news (its location $B_i$). In the second stage, the papers choose their prices $P_i$ and $K_i$ simultaneously. Subsequent to those two stages, advertisers choose where to advertise and readers decide to which newspaper to subscribe. Next, papers receive data $d$ and report news $d + s_i(d)$. Finally, consumers read the news, get exposed to the advertisements, and form new impressions of the advertised products.

Using this framework but with no advertising, MS (2005) show that the equilibrium locations of the newspapers are $B_1^{MS} = -3b_0/2$ and $B_2^{MS} = 3b_0/2$. Hence, with subscription fees being the only source of revenues of newspapers, extreme bias in reporting, to the right by Newspaper 2 and to the left by Newspaper 1, are chosen at the equilibrium. Such extreme differentiation in reporting alleviates the extent of competition on subscription fees. In what follows, we investigate how these equilibrium locations change if newspapers earn revenues from advertising as well.

It may be interesting to point out how bias in reporting as a vehicle to introduce differentiation between newspapers is different from other product features aimed at achieving horizontal differentiation. First, the utility of readers depends upon two different attributes of news reports, accuracy and consistency with political opinions, thus introducing potentially opportunities for both vertical and horizontal differentiation. While the location choice of each newspaper ($B_i$) is the vehicle to introduce horizontal differentiation, the weight assigned to this location in designing the slanting strategy (i.e., $\frac{\phi}{\phi + \chi}$) captures the relative importance of the vertical versus the horizontal attributes (i.e., accuracy vs. consistency with political opinions) in the utility function of the consumers. In particular, if the consumers’ appreciation for accuracy
(the vertical attribute) is infinite, the papers stop slanting the news and don’t use reporting bias for horizontal differentiation. Another aspect that distinguishes bias from traditional models of horizontal differentiation is that newspapers attempt to appeal to two different audiences, readers and advertisers. Hence, the positioning of each newspaper has implications for price competition in both markets. This contrasts with most models of product differentiation, where features are chosen by taking into account competition in a single consumer market.

2.3 ANALYSIS

When both subscription and advertising revenues are available, the objectives of the newspapers are:

Single-Homing ($\hat{\alpha}_1 = \hat{\alpha}_2 = 0$)

$$\pi_1 = A\frac{\alpha_0 + \alpha_{indiff}}{2\alpha_0}K_1 + M_1\frac{b_0 + b_{indiff}}{2b_0}P_1, \quad \pi_2 = A\frac{\alpha_0 - \alpha_{indiff}}{2\alpha_0}K_2 + M_1\frac{b_0 - b_{indiff}}{2b_0}P_2;$$

where $b_{indiff}$ and $\alpha_{indiff}$ are given in Equations 2 and 9, respectively.

Double-Homing ($\hat{\alpha}_1 = -\alpha_0, \hat{\alpha}_2 = \alpha_0$)

$$\pi_1 = AK_1 + M_1\frac{b_0 + b_{indiff}}{2b_0}P_1, \quad \pi_2 = AK_2 + M_1\frac{b_0 - b_{indiff}}{2b_0}P_2;$$

where $b_{indiff}$ is given by Equation 2.
The newspapers choose subscription and advertising fees in the second stage to maximize Objectives 12-13. When the newspapers locate symmetrically so that $-B_1 = B_2 = B$, the solution to the maximization is as follows:

**Single-Homing ($\hat{\alpha}_1 = \hat{\alpha}_2 = 0$)**

\[
P_{S}^{**} = \frac{4B\phi^2b_0}{\phi + \chi} - \frac{A h_0}{M_1}, \quad K_{S}^{**} = \frac{M \alpha_0}{2}.
\]

**Double-Homing ($\hat{\alpha}_1 = -\alpha_0$, $\hat{\alpha}_2 = \alpha_0$)**

\[
P_{D}^{**} = \frac{4B\phi^2b_0}{\phi + \chi} - \frac{Ah_0}{\frac{M_1}{M}}, \quad K_{D}^{**} = \frac{M}{2} \left[h_0 - \frac{\alpha_0}{2}\right].
\]

Hence, for a fixed symmetric choice of locations, subscription fees are higher if subscribers have greater preference for reports that are consistent with their political opinions (bigger $\phi$), smaller preference for accurate reporting (smaller $\chi$), and are more heterogeneous (bigger $b_0$). Subscription fees are also higher when the advertising market is smaller (smaller $A$), the relative size of the population of subscribers is bigger (bigger $M_1/M$), and the effectiveness of advertising declines (smaller $h_0$). In general, the more important advertising revenues in comparison to subscription revenues, the lower the fees newspapers charge to subscribers at the symmetric equilibrium.

Substituting the equilibrium advertising fees derived in Equations 14 and 15 back into Equation 8 implies different types of homing depending on the extent of heterogeneity among the advertisers (value of $\alpha_0$). While for large values ($\alpha_0 > 2h_0$), Single-Homing is the unique equilibrium, for small values ($\alpha_0 \leq 2h_0/3$), Double-Homing is the unique equilibrium. Note that between $2h_0/3$ and $2h_0$ there is an equilibrium in which while some advertisers Single-Home (place their ads in a single newspaper), others Double-Home (place ads in both outlets). As well, multiple equilibria may arise in this range (see Appendix for derivations.) As explained
earlier, advertisers in our environment care both about the number and profile of readers who are exposed to their ads. When heterogeneity among advertisers is significant, targeting readers who are compatible with advertised products is very important to the advertisers. Single-Homing is more successful than Double-Homing in achieving such targeting. In the absence of targeting, ads might reach consumers with extreme political opinions incompatible with the products advertised. When heterogeneity is large, such lack of targeting is especially costly for advertisers since the product $ab$ might assume very large negative values in Equation 4. To obtain the equilibrium locations chosen by the newspapers in the first stage, one has to solve first for the second stage fees, $P_l(B_i, B_j)$ and $K_l(B_i, B_j)$, as functions of arbitrary location choices selected in the first stage (not necessarily symmetric locations only). The second stage equilibrium strategies have to be substituted back into Equations 12-13 to obtain the first stage payoff functions of the newspapers.

Assuming the existence of an interior equilibrium, next we compare the locations selected at the symmetric equilibrium (designated by $B^{**}$) to those derived when newspapers obtain revenues from subscribers only (denoted as $-B_1^* = B_2^* = B^{MS}$). When there is no heterogeneity among advertisers, namely when $\alpha_0 = 0$, advertisers Double-Home and $B^{**} = B^{MS} = 3b_0/2$, meaning that bias remains unaffected when advertising is added as a source of revenue. However, when $\alpha_0 > 0$, adding advertising to supplement subscription fees may moderate or intensify bias. In Lemma 1, we first derive restrictions on the parameters of the model to guarantee that those regimes can be supported with positive streams of revenues from subscribers (namely that $B^{**} > 0$ and $P^{**} > 0$). For ease of presentation, we introduce a measure for the importance of advertising relative to subscription as a source of revenue for the papers, $T \equiv (AM/M_1)((\phi + \chi)/(8\phi^2))$, where $(AM/M_1)$ represents the size of the advertising market.
relative to the subscription market and \((\phi + \chi)/(8\phi^2)\) is a measure of the importance consumers attach to accuracy relative to consistency with their political opinions. If consumers attach great importance to accurate reporting (i.e., \((\phi + \chi)/\phi^2\) is large), the papers cannot charge high subscription fees. Hence, if either one of the two components of \(T\) increases, the subscription market loses its importance as a source of revenues relative to the advertising market.

**Lemma 1.** To ensure positive subscription prices and strict differentiation between newspapers (i.e., \(P^{**} > 0\) and \(B^{**} > 0\)):

(i) At the Single-Homing equilibrium: \(T < T^S_{\max} \equiv \frac{3b_0^2(9\alpha_0 - 4h_0)}{2h_0(9\alpha_0 - 2h_0)}\), and \(\alpha_0 > 2h_0\).

(ii) At the Double-Homing equilibrium: \(T < T^D_{\max} \equiv \frac{b_0^2(3h_0 - 2\alpha_0)}{2h_0(2h_0 - \alpha_0)}\), and \(\alpha_0 < 2h_0/3\).

Restricting attention to the regions specified in Lemma 1, we derive the optimal locations chosen by the newspapers at the symmetric equilibrium in Equations 16 and 17.

**Single-Homing**

\[
16 - B^{**}_1 = B^{**}_2 = B^{**}_S = \frac{3b_0 h_0}{4} + \frac{T_0}{2} \left(\frac{1}{3} + \frac{h_0}{3a_0}\right) + \sqrt{\left(\frac{3b_0}{4} + \frac{T_0}{4} \left(\frac{1}{2} + \frac{h_0}{3a_0}\right)^2 - \frac{4T_0^2}{3a_0}\right)}.
\]

**Double-Homing**

\[
17 - B^{**}_1 = B^{**}_2 = B^{**}_D = \frac{3b_0}{4} + T \frac{a_0}{2b_0} + \sqrt{\left(\frac{3b_0}{4} + T \frac{a_0}{2b_0}\right)^2 - 2T^2 a_0}.
\]

Proposition 1 follows from the expressions derived in Equations 16 and 17.
PROPOSITION 1. With both advertising and subscription fees contributing to the newspapers’ revenues,

(i) When heterogeneity among advertisers is sufficiently large ($\alpha_0 > 2h_0$):

Each advertiser chooses a single newspaper for placing its ads (Single-Homing), and newspapers introduce more bias in their reporting ($B^*_S > B^{MS}$). This bias increases as the importance of advertising as a source of revenue increases ($\frac{\partial B^*_S}{\partial T} > 0$).

(ii) When heterogeneity among advertisers is sufficiently small ($\alpha_0 < 2h_0/3$): Each advertiser chooses both newspapers for placing its ads (Double-Homing), and newspapers introduce less bias in their reporting ($B^{**}_D < B^{MS}$). This bias decreases as the importance of advertising as a source of revenue increases ($\frac{\partial B^{**}_D}{\partial T} < 0$).

To understand the results reported in Proposition 1, it is important to highlight the new effects influencing the location choice of the newspapers that arise when advertising is added as a source of revenues to supplement subscription fees. The first “readership effect” relates to the intensified incentives of each newspaper to increase its readership (for Newspaper 1 this means increasing $b_{indiff}$, and for Newspaper 2 decreasing it). Note that at the symmetric equilibrium (when $b_{indiff} = 0$) $\frac{\partial K^S_1}{\partial b_{indiff}} = \frac{Mh_0}{2b_0} > 0$ and $\frac{\partial K^D_2}{\partial b_{indiff}} = \frac{Mh_0}{2b_0} > 0$. Hence, irrespective of the type of homing, a newspaper that delivers a bigger readership can command a higher advertising fee

\[ K^S_1 = M \left( \alpha_0 \left( \frac{b_0^2 - b_{indiff}^2}{2b_0^2} \right) + \frac{b_{indiff}h_0}{3b_0} \right), \quad K^S_2 = M \left( \frac{\alpha_0 (b_0^2 - b_{indiff}^2)}{2b_0^2} - \frac{b_{indiff}h_0}{3b_0} \right), \quad \text{and} \quad K^D_1 = \frac{M(b_0 + b_{indiff})}{2b_0} \left( h_0 - \frac{\alpha_0 (b_0 - b_{indiff})}{2b_0} \right), \quad K^D_2 = \frac{M(b_0 - b_{indiff})}{2b_0} \left( h_0 - \frac{\alpha_0 (b_0 + b_{indiff})}{2b_0} \right). \]
from advertisers. This implies that each newspaper has extra incentives to move closer to its competitor’s location in order to increase its market share among readers (e.g., \( \frac{\partial \alpha_{indiff}}{\partial B_1} = \frac{1}{2} > 0 \) at symmetry, when \( P_1 = P_2 \)).

Adding advertising as a source of revenue introduces, though, a second counter force when advertisers Single-Home. We refer to this force as the “incremental pricing effect” to capture the idea that a change in a newspaper’s location does not only have a direct effect on the intensity of price competition in the subscription market but may also have an indirect, incremental effect on the intensity of price competition in the advertising market. Note that this effect does not exist in standard models of horizontal differentiation in which a change in location has implications on price competition in only one market. When a newspaper modifies its location and advertisers Single-Home, the competing newspaper may have to adjust its advertising fee in order to defend its market share among advertisers. For instance, when Newspaper 1 increases \( B_1 \), it moves closer to the location of Newspaper 2, and due to reduced differentiation, Newspaper 2 is forced to cut subscription fees. In addition, since the new, moderated location of Newspaper 1 offers a larger readership to advertisers, Newspaper 2 has to cut its advertising fee as well in order to defend its market share in the advertising market.  

The existence of this “incremental pricing effect” introduces, therefore, incentives for Newspaper 1 to polarize in order to discourage aggressive pricing by Newspaper 2. These incentives are stronger than in an environment where newspapers compete in a single, subscriber market because

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5 As Newspaper 1 increases its readership by increasing \( B_1 \), Newspaper 2 loses market share among advertisers since at the symmetric equilibrium \( \frac{\partial \alpha_{indiff}}{\partial B_1} = \frac{2h_0}{b_0} > 0 \). Thus, Newspaper 2 has an incentive to cut its advertising fee since \( \frac{\partial K_2}{\partial B_1} = -\frac{Mh_0}{6b_0} < 0 \) at symmetry.
Newspaper 2 is forced to cut both its advertising and subscription fees. According to part (i) of Proposition 1, the “incremental pricing effect” present at the Single-Homing equilibrium more than outweighs the objective of increasing readership, thus leading to intensified bias at the equilibrium when advertising is added as a source of revenues to augment subscription fees. Moreover, this bias increases as the importance of advertising as a source of revenue ($T$) increases. In contrast, according to part (ii) of the Proposition, at the equilibrium with Double-Homing, bias in reporting the news is reduced when advertising supplements subscription fees. At this type of equilibrium, the only additional effect that advertising introduces is the added objective of newspapers to offer bigger readerships to advertisers. Since the market share of each newspaper in the advertising market is fixed at 100% and the newspapers don’t need to defend their market shares among advertisers, the “incremental pricing effect” is non-existent in the Double-Homing environment. Note that the “readership effect” intensifies, in this case, when advertising is a more important source of revenue (large $T$). Figure 2 depicts the relationship between the equilibrium locations of the newspapers and the importance of advertising as a source of revenue to the newspapers, as reported in Proposition 1.
Figure 2: Equilibrium Locations as a Function of T

Note that with a different advertising response function, which implies that the change in purchase probability for moderate products and consumers is larger than that for extreme products and consumers, the readership effect will be stronger, since in this case, the moderate readers will be more valuable for the advertisers, and therefore the newspapers. We predict that while the results for Double-Homing reported in Proposition 1 will continue to hold in such an environment, the results for Single-Homing may change as the readership effect may outweigh the incremental pricing effect.

We can use the results reported in Proposition 1 to conjecture how the equilibrium is likely to change in case of less than full coverage of readers. At the Single-Homing equilibrium (when $\alpha_0$ is big) bias in reporting is significant. Hence, it is sensible that when the market is less than fully covered, it is consumers with moderate opinions in the neighborhood of $b=0$ who
choose to drop out of the market ($EU_b^i < 0$ for such consumers). As a result, the subscribers of each newspaper are fewer in number and have more extreme beliefs in comparison to a fully covered market. This new composition of subscribers reduces even further the benefit from Double-Homing. In the Appendix, we demonstrate that newspapers may have reduced incentives to polarize as a result of incomplete coverage of the subscriber market. In fact, when the reservation price of readers is relatively low and their valuation of accurate reporting is high, bias is more moderate than that derived in MS (i.e., smaller than $\frac{3}{2} \beta_0$) even though advertisers Single-Home. At the Double-Homing equilibrium (when $\alpha_0$ is small) bias is moderate. It is now consumers with very extreme opinions who are likely to drop out of the market. The population of subscribers becomes less heterogeneous, as a result, thus enhancing the benefit from Double-Homing. In the Appendix, we demonstrate, that in this case as well, incomplete coverage may moderate the extent of bias selected by the newspapers if the reservation price of readers (and their valuation of accuracy) is low (high), respectively.

\subsection*{2.4 CONCLUSION}

In this study we extend the work of MS (2005) by investigating media bias when advertising is added as a source of revenue to supplement subscription fees. We show that the additional advertising market introduces two counteracting effects on the behavior of newspapers. First, as newspapers attempt to increase their readership in order to attract advertisers, they moderate slanting in order to appeal to readers having moderate opinions. Second, when advertisers choose to Single-Home a second effect arises that may lead to greater
polarization in news reporting. If newspapers moderate bias in this case they are forced to compete more aggressively not only for subscribers, but for advertisers as well. Downward pressure on subscription as well as advertising fees follows. To avoid such intensified price competition, newspapers may choose to increase polarization. We demonstrate that when the heterogeneity among advertisers in appealing to consumers with different political preferences is significant, the attempt to alleviate price competition dominates, thus leading to greater polarization. When this heterogeneity is negligible, reduced polarization is predicted.
3.0 USER-GENERATED CONTENT AND BIAS IN NEWS MEDIA

3.1 INTRODUCTION

User-generated content (UGC) is increasingly common in the online economy, often appearing in forms of blogs, wikis, podcasts, pictures, videos and social networks (Lee 2008). In 2008, 42.8% of Internet users (82.5 million people) contributed to some form of UGC; and it is expected that this number will reach 51.8% by 2012 (114.5 million people) (Verna 2009). In the case of news media, use of websites to integrate user content has intensified. For example, the Wall Street Journal (WSJ), on its online version, offers readers the opportunity to add content under the section titled “Journal Community”. In this digital platform, readers create groups having particular interests (e.g. “The Mideast,” “The New Regulation Economy,” “American Views on European Politics”, etc.) and share opinions on the subject. In addition, using this platform, news readers can make comments or ask questions about stories published by WSJ journalists. The New York Times (NYT), on its digital version, publishes news stories and opinions of readers in the form of letters and op-eds, and has a separate ‘Public Editor’ assigned in charge of responding to comments and opinions of readers. CNN and Fox News have been broadcasting news videos (called ‘I-reports’ and ‘U-reports’) that are submitted by their audience. For these news companies, the impact of UGC on profitability is unknown, as it can be a substitute to the professionally prepared content. A report by Accenture confirms this concern.
by arguing that media owners see UGC as the biggest threat to the survival of their businesses (Accenture 2007). In this study we investigate a newspaper’s decision to extend its product line to include an online edition that incorporates UGC. Specifically, we are interested in the impact of this decision on bias of reported news as well as the role of UGC in determining the extent of this reporting bias and newspapers’ profits.

We demonstrate that the segmentation of readers reduces the extent of bias in reporting of the print edition but intensifies the extent of bias of the online edition. This intensified bias is mostly generated by the readers themselves as they add news stories and opinions to the online edition. In fact, we demonstrate that if newspapers could completely prevent readers from adding UGC to their online editions, they would choose bias to be identical in their print and online editions. In contrast, when UGC is added by readers to the online editions, each newspaper is indirectly forced by subscribers to offer two differentiated versions of its product. With this added differentiation, the profitability of the newspaper declines in comparison to an environment where it has the exclusive right to choose the bias of both editions.

In our model the main characteristic that distinguishes the online edition of a newspaper from its print edition is the ability of readers to add UGC to the former variant. We assume that this feature of the online edition is especially appreciated by readers who have extreme political opinions. We conjecture that such readers have a stronger desire to be heard and/or convince other readers of their views. This assumption is consistent with recent research in psychology that investigates how people’s opinions deviate from that of the average group member. Morrison and Miller (2008), for instance, show that people whose opinions are extreme in the direction of the norm that reflects the common attitudes of their group (e.g., liberal positions for college students), are more likely to express their opinions than moderates. As a result of the
added appreciation of some consumers for UGC, the diversification of the product mix leads to possible segmentation of readers according to their political opinions. Readers who are moderates prefer the print edition of the newspaper. In contrast, readers who are extreme in their opinions opt for the online edition and can be active in generating content on the newspaper’s site.  

It is noteworthy that the reduced profitability that is predicted in our model at the equilibrium when each newspaper adds an online version stems from two characteristics of our formulation. First, the extension of the product mix results in reduced bias of the print editions of the newspapers, translating to reduced product differentiation and intensified competition on subscription fees. Second, since the extent of slant of the online editions is partly determined by subscribers, the ability of the newspapers to extract consumer surplus via price discrimination is restricted. Several recent empirical findings in the literature support the reduced profitability our model predicts. In particular, Filistrucchi (2005) and Gentzkow (2007) find that adding an online edition:

\[ \rho = 0.54 \] for Letters to the Editor, and \[ \rho = 0.64 \] for online comments. In order to determine a political opinion rating for a reader, for each article we first calculated the average score of the two raters. Then, as some readers provided comments to more than one article, we calculated overall political rating of a reader by taking the average of her (rater-averaged) ratings across all of the articles. Using this procedure the obtained mean ratings were 5.27 and 4.82 for the online and print commentators, respectively. Further, the difference between these ratings was statistically significant (\( r_{2158} = 4.39, p < 0.01 \)).
online version reduces print sales and profits. Even though those studies demonstrate this finding in an environment where access to online content is free, our model predicts that profits decline even when newspapers charge for access to their online editions.

Given our goal of investigating the role of UGC in affecting political bias in news reporting, our model focuses primarily on the political opinions of readers as the sole determinant of their choice between the print and online editions. There are obviously many other attributes that distinguish consumers who prefer one edition over the other. Online users are likely to be younger or have higher valuation for the technological features provided by online newspapers (such as content sharing-digging, mobile applications, and so on). In an extension of our model we incorporate a second dimension of heterogeneity, unrelated to politics that differentiates among readers. We show that this additional heterogeneity leads to increased bias of the print edition and to a reduction of the average size of the online segment. Essentially, this additional heterogeneity moves the equilibrium closer to the outcome that arises when newspapers have full control over the attributes of both variants of their products.

To formulate the competition between the newspapers, we extend a model that was developed by Mullainathan and Shleifer (MS 2005). In this model, consumers prefer reading news consistent with their opinions and two newspapers can slant their reporting of the news towards these opinions. This assumption is consistent with recent experimental evidence of ideological selectivity in media use (see Iyengar and Hahn 2009). As in MS (2005), we assume that the only source of revenues of the newspapers is from subscription fees. Even though advertising is also an important source of revenue, in recent years newspapers have reduced their reliance on advertising, as more advertisers switch to Internet advertising. In 2009, the NYT reported, for instance, that its revenues from circulation surpassed advertising revenues for the
first time (Chittum 2009). We assume that the newspapers can charge subscription fees for both their print and online editions. The WSJ, for instance, has different subscription fees for print and online subscriptions, and the NYT has recently announced that it will start charging for access to its online edition in 2011.

Our study contributes to several strands of literature. First is the literature on media bias that is implied by the media’s attempt to appeal to consumers who have different opinions. Mullainathan and Shleifer (MS 2005) investigate the relationship between newspaper competition on subscription fees and such bias. In Gentzkow and Shapiro (2006) bias occurs since media firms slant their reports toward consumer priors in order to maintain reputation for high quality reporting. Xiang and Sarvary (2007) examine media bias in the presence of conscientious consumers who seek the truth. Finally, first study of the dissertation analyzes slanting in news media when advertisers wish to target readers who are receptive to their messages. None of these studies addresses, however, the question of how introducing an online edition to supplement a print edition is likely to affect the extent of slant in reporting of news.

The second strand of literature to which this study contributes deals with how competing sellers choose the breadth of their product lines in order to facilitate improved segmentation. Some of this literature assumes exogenous product attributes (e.g., Brander and Eaton 1984, Gilbert and Matutes 1993). Our work is more similar to the literature that examines product line rivalry when product characteristics are endogenously chosen (e.g., Katz 1984, Moorthy 1987, Champsaur and Rochet 1989, Desai 2001, Schmidt-Mohr and Villas-Boas 2008). In contrast to this literature on competitive product line design, in our study the enrichment of the product line occurs by active participation of customers in determining the (non-price) characteristics of the products included in the line.
Finally, our study contributes also to literature related to UGC. There has been significant amount of research that involves empirical measurement of the effects of UGC on sales (Chevalier and Mayzlin 2006, Liu 2006, Dhar and Chang 2009, and Zhu and Zhang 2010) and on other similar variables such as TV ratings (Godes and Mayzlin 2004) or new customer acquisition (Trusov et al. 2009). However, analytical work in this area has been limited. Further, all such work has addressed UGC in the context of the exchange of information about products among online readers (e.g., Mayzlin 2006, Chen and Xie 2008, Kuksov and Shachar 2010). In contrast, our research focuses on UGC in generating news reports online.

3.2 THE MODEL

Consider a market with two newspapers, \( i = 1, 2 \), where each can decide on whether to add an online version to supplement the print version of its publication. We assume that due to technological advancement, the online version facilitates far greater capabilities for the readers to add content to the publication than the print version. For simplicity, we assume that only the online version can incorporate readers’ input. We will refer to the activity of readers on the online version as User-Generated Content (UGC). We assume that the only source of revenues of the newspapers is from subscription fees, and that the unit cost of offering the print version is higher than the online version. We designate by \( c \) and \( \delta c \), with \( 0 < \delta < 1 \), the unit cost incurred by the newspaper to produce the print and online versions, respectively. The added cost of the print version may relate, for instance, to added distribution costs. Consumers choose whether to
subscribe to the print or online versions of one newspaper.\textsuperscript{7} We assume that customers who have extreme political opinions are likely to be attracted to the greater capabilities offered by the online version to share stories and opinions with other readers. Hence, in our model customers are segmented according to the intensity of their political opinions. Those who have more moderate opinions choose the print version, since they do not plan to engage in UGC, and those who have extreme opinions choose the online version since they value the UGC feature of this medium.\textsuperscript{8}

To capture the heterogeneity of customers according to political opinions we adopt the model developed by MS (2005). Specifically, there is a unit mass of consumers who are uniformly distributed according to their political opinions, designated by $b$, on the interval $[-b_0, b_0]$. Readers with left leaning opinions belong to the negative region of this interval and those with right leaning opinions belong to the positive region. Information about news items $t$ is normally distributed according to $N(0, \sigma_t^2)$. Newspapers provide the readers with news about $t$. A reader of type $b$, has prior beliefs about these news items that is normally distributed according to $N(b, \sigma_t^2)$. Hence, in comparison to the true distribution, readers have biased beliefs about the news, determined by their political opinions. The variable $b$ measures the extent to which the beliefs of the reader are biased relative to the true mean of the distribution of $t$.

\textsuperscript{7} In a recent study, Gentzkow (2007) investigates the newspaper market in Washington DC and demonstrates that the print and online versions of newspapers are considered substitutes rather than complementary goods by readers. This empirical finding lends some support to our formulation.

\textsuperscript{8} Note that in our model readers with extreme beliefs necessarily contribute to the online content. If the utility of readers included an additional argument that relates to benefit derived from influencing others, our assumption would be consistent with extremes deriving greater benefit than moderates, and therefore, opting to add content to the online editions. It would be interesting to address such an extension in future research.
Newspapers receive some data \( d_i = t + \varepsilon_i \), where the random variable \( \varepsilon_i \) is independently distributed of \( t \) according to \( \varepsilon_i \sim N(\theta, \sigma^2) \). Each newspaper \( i \) may choose to slant its reporting so that \( n_i = d_i + s_i \), where \( n_i \) is the reported news, and \( s_i \) is the slant in reporting. While the newspaper has full control over the extent of slanting of its print version, the slant of the online version may depend also on the UGC added by subscribers to this product. To allow for the possibility of different levels of slanting, we designate by \( s_i \) and \( s_i^o \) the slant of the print and online products, respectively, and similarly, by \( n_i \) and \( n_i^o \) the reported news in the two variants.

As in MS (2005), we assume that readers incur disutility when reading news inconsistent with their opinions, as measured by the distance between the reported news and the readers’ opinions: \( (n_i - b)^2 \) and \( (n_i^o - b)^2 \). As well, holding constant the extent of inconsistency with their opinions, readers dislike slanting. When Newspaper \( i \) chooses the subscription fees \( P_i \) and \( K_i \) for its print and online versions, the net utility of a consumer having opinion \( b \) is:

\[
U_b = \begin{cases} 
\bar{u} - \chi s_i^2 - \phi (n_i - b)^2 - P_i & \text{if she subscribes to print version of } i \\
\bar{u} - \chi (s_i^o)^2 - \phi (n_i^o - b)^2 - K_i & \text{if she subscribes to online version of } i
\end{cases}
\]

where \( \bar{u} \) stands for the reservation price of the reader, \( \phi > 0 \) calibrates her preference for hearing news consistent with her political opinions, and \( \chi > 0 \) calibrates her preference for reduced slant. Using the utility framework in (1), readers first choose a newspaper and then decide whether to subscribe to the online or print versions of the newspaper, while incorporating the fact that the online version includes UGC.

Similar to MS (2005), we assume full coverage of the market of consumers and focus on linear slanting strategies \( s_i(d_i) = \frac{\phi}{\phi + \chi} (B_i - d_i) \), where \( B_i \) is the location choice of the print version of Newspaper \( i \), and represents a focal point around which slanting of the news arises. This location choice can be a point inside or outside of the interval \([-b_0, b_0]\). By choosing
location $B_i$ the paper becomes more appealing to readers with opinions close to $B_i$. Notice that the extent of slanting decreases with $\chi$ and increases with $\phi$. Thus, as readers place more importance on receiving accurate information and less importance on hearing confirmatory news, newspapers choose lower slanting in their reporting$^9$. Without loss of generality, we assume that Newspaper 2 is located to right of Newspaper 1 ($B_1 < B_2$). That is, while Newspaper 1 has a left-wing slant, Newspaper 2 has a right-wing slant.

Note that in our formulation the slanting depends upon the ex-post realization of the data. However, the focal point $B_i$ is chosen ex-ante. Hence, when the data is very different from the ex-ante focal point chosen by the newspaper $i$, the extent of slanting is big. In contrast, if the data is close to the focal point, the extent of slanting is small.

Since subscribers to the online version are active in generating additional content, the political position of the online product reflects both the position of the newspaper and the UGC supplied by subscribers to this variant of the product. We designate the combined positioning of the online variant of Newspaper $i$ by $B_i^o$ and specify it as follows:

$$B_i^o = B_i + \alpha(E[b_i^o]) \quad 0 < \alpha < 1,$$

where $E[b_i^o]$ measures the mean opinion of subscribers to the online version of Newspaper $i$. $^10$ Hence, the modified location of the online version is the sum of the position chosen by the newspaper and the mean opinion of subscribers to this product multiplied by a positive fraction $\alpha$

$^9$ For instance, if $\chi > 2 \phi$ $ES_t < \frac{B_t}{3}$ and when $\chi \to \infty$ $ES_t = 0$.

$^10$ Note that the qualitative results of our analysis are likely to remain unchanged for a more general formulation, as long as $B_i^o$ is an increasing function of the location of the print version and the mean political beliefs of the readers who subscribe to the online edition.
that measures the extent of discretion of online readers to generate content online. The fact that \( \alpha < 1 \), reflects the sensible assumption that the effect of the newspaper itself in determining the positioning of the online variant is higher than that of its readers.

It is noteworthy that when readers make their choice among the different media, the realization of the data supporting the news stories (the random variable \( d_i \)) is yet to be determined. At the time the reader makes her choice, she is familiar with the subscription fees of the newspapers (\( P_i \) and \( K_i \)), their locations (\( B_i \) and \( B_2 \)), and her own political opinion \( b \). Hence, in comparing the different media, the reader evaluates her prior expected utility calculated from (1) by integrating over all possible realizations of the random variable \( d_i \) and using the distributional properties of \( d_i \) (namely, \( E[d_i] = 0 \) and \( \text{Var}[d_i] = \sigma_d^2 \)). Hence, we obtain:

\[
E[U_b] = \begin{cases} 
\bar{u} - \frac{\phi^2}{(\phi + \chi)} (B_i - b)^2 - \frac{\chi \phi}{(\phi + \chi)} (b^2 + \sigma_d^2) - P_i & \text{for print version of } i \\
\bar{u} - \frac{\phi^2}{(\phi + \chi)} (B_i^0 - b)^2 - \frac{\chi \phi}{(\phi + \chi)} (b^2 + \sigma_d^2) - K_i & \text{for online version of } i.
\end{cases}
\]

Our specification implies that when a newspaper decides to add an online variant to supplement its print version, it expands its product mix to consist of two products with differing levels of slanting in reporting. According to (2), the slanting in reporting is higher online due to the added input supplied by subscribers to this product. The expanded product mix is likely to support, therefore, improved segmentation of readers as described in Figure 3, when both newspapers offer an expanded product mix. Later we show that such segmentation can indeed exist.
In the Figure, readers having extreme political opinions ($b > \hat{b}_2$ and $b < \hat{b}_1$) choose to subscribe to one of the online products and those having moderate opinions ($\hat{b}_1 < b < \hat{b}_2$) choose to subscribe to one of the print products. Since slanting is more extreme online than in the print version, it is readers with extreme political opinions who self select to subscribe to the product that is more consistent with their extreme preferences. Moreover, since those subscribers choose to add UGC to the website of the newspaper, the modified location of the online version reflects the extreme opinions of these subscribers. Specifically,

$$ (4) \quad B_2^o = B_2 + \alpha \frac{(b_0 + \hat{b}_2)}{2} \quad \text{and} \quad B_1^o = B_1 + \alpha \frac{(-b_0 + \hat{b}_1)}{2}. $$

We model the game as consisting of three stages. In the first stage, the newspaper decides whether to supplement its print version with an online product. We designate this choice by $E_i$ and $NE_i$ when expanding and not expanding the product mix, respectively. In the second stage, each newspaper decides the political positioning of its print version, $B_i$. In the third stage, each newspaper chooses its subscription fees $P_i$ and $K_i$, where the latter choice is relevant only if an online version is added in the first stage. Following the three stages, consumers decide on their
subscription patterns (prior to the realization of \( d \)), newspapers gain access to news and report them according to the locations selected in Stage 2, and readers of the online version add UGC to the newspapers’ website.  

3.3 DERIVATION OF THE EQUILIBRIA

Contingent upon the expansion decision of the two newspapers in the first stage, four different possibilities may arise, as follows: \( \{E_1, E_2\} \), \( \{E_1, NE_2\} \), \( \{NE_1, E_2\} \), and \( \{NE_1, NE_2\} \). The last possibility refers to the case that both newspapers offer only the print version. This case has already been investigated in MS (2005). The authors find that the positioning of the newspapers when only a print version is offered by each is \( B_1 = -\frac{3}{2} b_0 \) and \( B_2 = \frac{3}{2} b_0 \). Such extreme positioning leads to greater differentiation between the newspapers and alleviated competition on subscription fees. In what follows, we characterize the remaining two cases: the symmetric case when both newspapers choose to add an online product and the asymmetric case when only one newspaper adds the online version.

\[ \]

\(^{11}\) Note that if the timing of the decisions is changed so that \( B_i \) were chosen first followed by the decision on whether to offer the online editions, there are circumstances under which the equilibrium that we derive in the next section would prevail. We provide this analysis in Appendix B.
3.3.1 Both Newspapers Add an Online Version

When both papers choose to add the online product, the segmentation of consumers is characterized in Figure 3. For simplicity, we will use the superscript \{E,E\} to characterize the equilibrium variables in this symmetric case. Considering the stage when consumers choose their subscription patterns, we start by identifying the threshold reader \(b_{\text{indiff}}^{E,E}\), the reader who is indifferent between the print editions of Newspapers 1 and 2, and \(\hat{b}_1^{E,E} (\hat{b}_2^{E,E})\), the readers who are indifferent between the print and online editions of Newspaper 1(2), respectively.

The marginal reader \(b_{\text{indiff}}^{E,E}\) has the same expected utility from subscribing to the print editions of Newspapers 1 and 2. That is, from (3):

\[
(5) \quad b_{\text{indiff}}^{E,E} = \frac{B_2^{E,E} + B_1^{E,E}}{2} + \frac{(P_2^{E,E} - P_1^{E,E}) \phi + \chi}{(B_2^{E,E} - B_1^{E,E}) 2\phi^2}.
\]

From (5), the location of the subscriber indifferent between Newspapers 1 and 2 is shifted away from the average locations of the two newspapers, \(\frac{(B_2^{E,E} + B_1^{E,E})}{2}\), in a manner dependent on the discrepancies between the fees charged for the print subscriptions.

Similarly, the location of the indifferent reader \(\hat{b}_i^{E,E}\) is a function of the locations of the online and print editions of newspaper \(i\) and the difference between the prices of these editions:

\[
(6) \quad \hat{b}_i^{E,E} = \frac{(B_i^{o,E})^{E,E} + B_i^{E,E}}{2} + \frac{(P_i^{E,E} - K_i^{E,E}) (\phi + \chi)}{(B_i^{E,E} - (B_i^{o,E})^{E,E}) 2\phi^2}, \quad i=1,2.
\]

Note that the right hand side of (6) is also a function of \(\hat{b}_i^{E,E}\) since the location of the online edition \((B_i^{o,E})^{E,E}\) is a function of \(\hat{b}_i^{E,E}\) from (4). Solving the system of equations (6) for \(\hat{b}_i^{E,E}\) in terms of the locations and fees of the newspapers yields:
In order to support the segmentation depicted in Figure 3, the solution for $\hat{b}_l^{E,E}$ should satisfy the inequalities $-b_0 < \hat{b}_1^{E,E} < b_{lndif}^{E,E} < \hat{b}_2^{E,E} < b_0$. From the expressions derived in (7) and (8), this may not necessarily be the case. In particular, when the print edition of the newspaper is significantly more expensive than the online edition ($ P_l^{E,E} \gg K_l^{E,E} $), $\hat{b}_1^{E,E}$ may be bigger and/or $\hat{b}_2^{E,E}$ may be smaller than $b_{lndif}^{E,E}$. Hence, the print edition may not attract any subscribers. This result is consistent with the experience of the NYT when it started to provide free access to its digital content in 2007, leading to a significant decline of the circulation of the newspaper. Given this experience, it announced that it will start charging for access to content online in 2011 (Clark 2010, Economist 2010).

It may be interesting to point out that the threshold consumers $\hat{b}_l^{E,E}$ play a dual role in our model. The first is the traditional role that exists in any environment with market segmentation. Specifically, these threshold levels designate consumers who are indifferent between two adjacent variants of a given product. The second role is new to our model, and relates to the active role that online subscribers play in determining the characteristics of the online variant of the product. According to (2), the political position of the online edition depends upon the composition of subscribers to this product. As the threshold levels $|\hat{b}_l^{E,E}|$ increase, the segment of consumers who choose the online subscription has more extreme political opinions, thus generating more extreme content online via the UGC. As a result, the slant in reporting of the online variant intensifies.
Further, note that in traditional models of horizontal product differentiation, when consumers cannot affect the characteristics of the different variants the threshold consumers that demarcate the different segments are given by equations similar to the system (6). However, in contrast to our setting, product characteristics (represented by \((B_i^0)^{E,E}\) for the online editions in our model) are considered exogenous by consumers in the traditional models. When UGC plays a role in affecting the slant of the online versions, \((B_i^0)^{E,E}\) is no longer considered exogenous by the readers. Instead, they are fully cognizant of the fact that when readers with more extreme political opinions subscribe to the online edition, the content of this edition becomes more politically biased. Readers use this information in deciding whether to choose between the print and online editions. Such considerations transform the system of equations (6) to the expressions for \(\tilde{b}_t^{E,E}\) in (7) and (8).

Given the locations of the indifferent consumers expressed in (5), (7) and (8), in stage three newspapers choose their subscription fees \(P_i^{E,E}\), \(K_i^{E,E}\) to maximize their profits as follows:

\[
\pi_1^{E,E} = \frac{1}{2b_0} \left( (b_0 + \tilde{b}_1^{E,E}) \left( K_1^{E,E} - c\delta \right) + \left( b_{indiff}^{E,E} - \tilde{b}_1^{E,E} \right) (P_1^{E,E} - c) \right),
\]

\[
\pi_2^{E,E} = \frac{1}{2b_0} \left( (b_0 - \tilde{b}_2^{E,E}) \left( K_2^{E,E} - c\delta \right) + \left( \tilde{b}_2^{E,E} - b_{indiff}^{E,E} \right) (P_2^{E,E} - c) \right).
\]

Optimizing (9) and (10) with respect to \(P_i^{E,E}\), yields the subscription fees of the print editions of both newspapers as functions of the choices made in the first two stages of the game:

\[
P_1^{E,E} - c = \frac{\phi^2}{(\phi + \chi)} \left( B_2^{E,E} - B_1^{E,E} \right) \left( \frac{(B_1^{E,E} + B_2^{E,E})}{3} + 2b_0 \right),
\]

\[
P_2^{E,E} - c = \frac{\phi^2}{(\phi + \chi)} \left( B_2^{E,E} - B_1^{E,E} \right) \left( -\frac{(B_1^{E,E} + B_2^{E,E})}{3} + 2b_0 \right).
\]
Optimizing (9) and (10) with respect to \( K_i^{E,E} \) yields:

\[
(12) \quad K_1^{E,E} - P_1^{E,E} = -c(1 - \delta) - \frac{\partial b_1^{E,E} + b_0}{\partial K_1^{E,E}},
\]

\[
K_2^{E,E} - P_2^{E,E} = -c(1 - \delta) + \frac{b_0 - b_2^{E,E}}{\partial K_2^{E,E}},
\]

where the expressions for \( \frac{\partial b_1^{E,E}}{\partial K_1^{E,E}} \) are derived from (7) and (8). Note that while the first term of the right hand side of (12) is negative, the second term is positive since \( \frac{\partial b_1^{E,E}}{\partial K_1^{E,E}} < 0 \) and \( \frac{\partial b_2^{E,E}}{\partial K_2^{E,E}} > 0 \). We will show, however, that the magnitude of the first term always dominates, thus yielding a lower subscription fee for the online than the print version due to the lower cost of producing the online variant. However, since the second term is positive, (12) implies that \( K_i^{E,E} - \delta c > P_i^{E,E} - c \). As a result, the profit margin of the online edition is higher than that of the print edition.

We can use (7) and (8) to express the relationship between \( K_i^{E,E} \) and \( P_i^{E,E} \) in terms of \( \tilde{b}_i^{E,E} \) and the location choices of the newspapers as follows:

\[
(13) \quad K_1^{E,E} - P_1^{E,E} = -c(1 - \delta) + \frac{\phi^2}{(\phi + \chi)} \left( \frac{\alpha(b_0 + \tilde{b}_1^{E,E})^2(2 - \alpha) + b_1^{E,E} + 2B_1^{E,E}}{2} \right).
\]

\[
K_2^{E,E} - P_2^{E,E} = -c(1 - \delta) + \frac{\phi^2}{(\phi + \chi)} \left( \frac{\alpha(b_0 - \tilde{b}_2^{E,E})^2(2 - \alpha) - b_2^{E,E} - 2B_2^{E,E}}{2} \right).
\]

It may be interesting to evaluate (11) and (13) at the symmetric equilibrium, when \( -B_1^{E,E} = B_2^{E,E} = B_{EE} \) and \( -\tilde{b}_1^{E,E} = \tilde{b}_2^{E,E} = \tilde{b}_{EE} \), because this type of equilibrium will be the main focus of this paper.
We obtain:

\[(14) \quad P_i^{E,E} - c = \frac{\phi^2}{(\phi + \chi)} 4B^{E,E} b_0,\]

\[(15) \quad K_i^{E,E} - c\delta = \frac{\phi^2}{(\phi + \chi)} \left( 4B^{E,E} b_0 + \frac{\alpha(b_0 - \bar{b}^{E,E})(2\alpha b_0 + (4\alpha - 2b^{E,E} - 2B^{E,E})}{2} \right).\]

The second term inside the parentheses of the expression of \((K_i^{E,E} - c\delta)\) measures the added markup that each newspaper may be able to derive due to the improved match between the added online variant and the preferences of online subscribers. Note that fees are higher when \(\phi\) increases, \(\chi\) decreases, and \(b_0\) or \(B^{E,E}\) increase. Hence, as subscribers care more about confirming reports, less about accuracy, and are more heterogeneous, competition on fees is alleviated. This is also the case when newspapers choose more slanted reporting.

Substituting (11) and (13) back into (9) and (10), yields the second stage payoff functions of the newspapers given that both chose to add the online option. Each newspaper chooses its location \(B_i^{E,E}\) to maximize this second stage payoff function. We illustrate this second stage optimization by considering only Newspaper 2. A similar approach is also valid for Newspaper 1. Using the Envelope Theorem in (10) when optimizing with respect to \(B_2^{E,E}\), we obtain:

\[(15) \quad \frac{\partial \pi_2^{E,E}}{\partial B_2^{E,E}} = \frac{\partial \pi_2^{E,E}}{\partial b_2^{E,E}} \frac{\partial b_2^{E,E}}{\partial B_2^{E,E}} + \frac{\partial \pi_2^{E,E}}{\partial b_{indiff}^{E,E}} \frac{\partial b_{indiff}^{E,E}}{\partial B_2^{E,E}} + \frac{\partial \pi_2^{E,E}}{\partial p_1^{E,E}} \frac{\partial p_1^{E,E}}{\partial B_2^{E,E}}.\]

A change in \(B_2^{E,E}\) has a direct effect on \(\pi_2^{E,E}\) via the expressions for \(b_{indiff}^{E,E}\) and \(\hat{b}_2^{E,E}\) in (5) and (8) and an indirect effect via the effect of Newspaper 2’s location on the print subscription fee of Newspaper 1, \(P_1^{E,E}\) (by the Envelope Theorem the effect on \(P_2^{E,E}\) and \(K_2^{E,E}\) vanishes and \(K_1^{E,E}\) does not affect \(\pi_2^{E,E}\) at all). We substitute from (5), (8), (11) and (13) in the
derivatives on the right hand side of (15) and evaluate the resulting expression at the symmetric equilibrium to obtain a relationship between $B_2^{E,E}$ and $\hat{b}_2^{E,E}$ as follows:

\[
(16) \quad -B_1^{E,E} = B_2^{E,E} = B^{E,E} = \frac{3}{4b_0} \left( \alpha \left( \hat{b}^{E,E} \right)^2 + (2 - \alpha) b_0^2 \right).
\]

It is easy to see from (16) that if segmentation arises, namely if $\hat{b}^{E,E} < b_0$, then $B^{E,E} < \frac{3b_0}{2}$. Hence, the positioning of the print version is less extreme in comparison to the case that newspapers do not add an online option (the case considered in MS (2005)). This result is not surprising given that each newspaper expanded its product mix to include a variant that is more politically extreme. It reduces, therefore, the slanting of the product that is chosen by the segment of the consumers who have moderate preferences.

To investigate whether a symmetric equilibrium with segmentation by both newspapers is feasible, we now use (16) to derive conditions under which there exists $\hat{b}^{E,E} \in (0, b_0)$. We designate by $\Delta U_p(b, \hat{b}^{E,E})$ the added utility that a reader having beliefs $b$ derives from the print over the online edition, given that the online segment comprises of readers in the interval $[\hat{b}^{E,E}, b_0]$. At the equilibrium with segmentation, $\Delta U_p^*(\hat{b}^{E,E}, \hat{b}^{E,E}) = 0$, namely the reader of type $\hat{b}^{E,E}$ is indifferent between the print and online editions. Moreover, for $0 < b < \hat{b}^{E,E}$, readers prefer the print version and $\Delta U_p(b, \hat{b}^{E,E}) > 0$, and for $\hat{b}^{E,E} < b \leq b_0$ readers prefer the online version and $\Delta U_p(b, \hat{b}^{E,E}) < 0$. We define by $T \triangleq c(1 - \delta) \frac{(\phi + \chi)}{2\phi^2}$, an adjusted cost advantage measure of the online over the print versions of the product. In Lemma 1 we present conditions on $\alpha$ and $T$, which satisfy these requirements on $\Delta U_p(b, \hat{b}^{E,E})$, and therefore support segmentation.
LEMMA 1.

(i) To support market segmentation at the symmetric equilibrium, \( 0 < \alpha < \alpha^* \approx 0.376 \) and \( LB_T < T < UB_T \), where

\[
LB_T \equiv \frac{\alpha}{2}(\alpha + 1)b_0^2 \quad \text{and} \quad UB_T \equiv \frac{(4-\alpha)(2(\alpha+2) - \sqrt{(4-\alpha)^2 - 3\alpha(2-\alpha)})^2}{72\alpha}b_0^2.
\]

(ii) Otherwise, when \( \alpha \geq \alpha^* \) or if \( T \leq LB_T \), only the print version of each newspaper can be supported at the equilibrium.

(iii) If \( \alpha < \alpha^* \) and \( T > UB_T \), only the online version of each newspaper can be supported at the equilibrium.

(iv) When segmentation can be supported, at the symmetric equilibrium: \( \frac{b_0}{2} < b_{E,E} < b_0 \).

Note that the extension of each newspaper’s product mix can be supported only if the extent of discretion of online readers to generate UGC is relatively moderate. Specifically, the relative control of online readers over the location of the online edition can be no more than 0.376 of the control of the newspaper itself. Even with such limited discretion awarded to readers, segmentation may still fail unless the adjusted cost advantage of the online version, \( T \), lies in the interval specified in part (i) of the Lemma. In particular, in the absence of any cost advantage, so that at \( T = 0 \), each newspaper will choose not to extend its market offering at the symmetric equilibrium. The adjusted cost advantage should be bigger than \( LB_T \), an expression that increases with \( \alpha \) and \( b_0 \). However, the print version of each newspaper might be cannibalized altogether if the cost advantage is extremely big. This happens when \( T \geq UB_T \), an expression that increases in \( \alpha \) and \( b_0 \), once again. The Lemma further demonstrates that, since
each newspaper loses control of the characteristics of its online version due to UGC, when segmentation can be supported, the size of the print segment is bigger than the size of the online segment.

Given the results reported in (16) and Lemma 1, we can now compare the extent of slanting in reporting when newspapers extend their product mix to the extent of slanting when only the print version is offered by each newspaper.

**PROPOSITION 1.** When both newspapers offer both print and online editions:

(i) \( \left( B^o \right)^{E,E} = B^{E,E} + \alpha \left( \frac{b_0 + b^{E,E}}{2} \right) > \frac{3b_0}{2} \).

(ii) \( B^{E,E} < \frac{3b_0}{2} \).

(iii) \( \frac{B^{E,E} b^{E,E} + (B^o)^{E,E} \left( b_0 - b^{E,E} \right)}{b_0} < \frac{3b_0}{2} \).

Recall that when only the print version is offered, \( B^{NE,NE} = \frac{3b_0}{2} \). Hence, extending the product mix to include an online version reduces reporting slant of the print version but increases the slant of the online version.\(^{12}\) In essence, product diversification facilitates obtaining a better match between the preferences of the readers and the variants of the products they choose to consume. According to part (iii) of the Proposition, however, the weighted average location of each newspaper, with weights determined by the relative market shares of the two editions, declines as a result of segmentation.

---

\(^{12}\) Restricting the expected slant of the print edition to the extreme location of the readers (i.e., \( \frac{\phi}{\phi+\chi} B^{E,E} < b_0 \)) will not change the equilibrium we derive as long as \( \phi < 2\chi \).
Next we investigate how UGC affects equilibrium locations and profits of the newspapers. Recall that $\alpha$ measures the extent of discretion of online readers to generate content online. Proposition 2 provides comparative statics with respect to $\alpha$.

**PROPOSITION 2.** *If both newspapers offer print and online subscription options:*

\[
\frac{\partial B^{E,E}}{\partial \alpha} > 0
\]

*and* \[
\frac{\partial \pi^{E,E}}{\partial \alpha} < 0.
\]

To understand the role of $\alpha$ in explaining the comparative statics reported in Proposition 2, recall from (2) that as $\alpha$ increases, the slanting in the online edition intensifies. Hence, readers with more extreme political opinions self select to subscribe to the online version when $\alpha$ increases ($\hat{b}^{E,E}$ increases).

Notice from (16) that the location $B^{E,E}$ of the print edition is an increasing function of $\hat{b}^{E,E}$. When $\hat{b}^{E,E}$ increases readers with more extreme political opinions generate UGC in the online edition. As a result, the online edition becomes more politically extreme, and so does the print edition, which competes against it. Since $B^{E,E}$ moves in the same direction as $\hat{b}^{E,E}$, $B^{E,E}$ increases with $\alpha$. The second part of the proposition states that as the discretion awarded to readers increases, the profits of the newspapers decrease. Bigger values of $\alpha$ translate to a more significant transfer of control from the newspaper to the readers themselves in determining the characteristics of the online version. Such a transfer of control leads to lower profits.
3.3.2 Only One Newspaper Adds an Online Version

In this section we consider the asymmetric case when only one newspaper extends its product mix. Without any loss of generality, we assume that Newspaper 2 offers both versions and Newspaper 1 offers the print version only. We use the superscripts \( NE, E \) to designate this case. Figure 4 depicts the segmentation of the market for such an asymmetric environment.

\[
\begin{array}{c|c|c}
\text{Print Subscribers} & \text{Print Subscribers} & \text{Online Subscribers} \\
\text{to Newspaper 1} & \text{to Newspaper 2} & \text{to Newspaper 2} \\
\hline
-b_0 & b_{\text{indiff}}^{NE,E} & \hat{b}_2^{NE,E} & b_0 \\
\end{array}
\]

**Figure 4**: Market Segmentation when only Newspaper 2 Extends Its Product Mix.

By using a similar approach as in the previous section, the relationships between the locations of the print versions and the threshold reader \( \hat{b}_2^{NE,E} \) can be derived as follows:

\[
B_1^{NE,E} = -\frac{3}{4} \left( b_0 + \sqrt{-\alpha \left( \hat{b}_2^{NE,E} \right)^2 + (1 + \alpha) b_0^2} \right),
\]

\[
B_2^{NE,E} = \frac{3}{4} \left( 5 b_0 - 3 \sqrt{-\alpha \left( \hat{b}_2^{NE,E} \right)^2 + (1 + \alpha) b_0^2} \right).
\]

Proposition 3 follows from equations (17) and (18).
PROPOSITION 3. When Newspaper 1 offers only the print edition and Newspaper 2 offers both the print and online editions:

\[(i) \quad B_1^{NE,E} < -\frac{3}{2}b_0, \quad B_2^{NE,E} < B^{E,E} < \frac{3}{2}b_0,\]

\[(ii) \quad b_{indiff}^{NE,E} = \frac{1}{2} \left( b_0 - \sqrt{-\alpha \left( \hat{b}_2^{NE,E} \right)^2 + (1 + \alpha)b_0^2} \right) < 0.\]

According to part (i) of Proposition 3, since Newspaper 2 extends its product mix while its competitor does not, it chooses to reduce the slanting of its print version below the level established when both newspapers extend their lines. Such a choice facilitates Newspaper 2 to steal market share from Newspaper 1. As a result, Newspaper 1 is forced to shift its location further to the left in order to differentiate itself from the print version of Newspaper 2. Part (ii) of the Proposition states, indeed, that when Newspaper 1 limits its product mix in comparison to Newspaper 2, it loses market share, and Newspaper 2 attracts more than 50% of the readers to one of its two editions.

3.3.3 Equilibrium Product line Extension Decision and the Role of UGC

With the characterization of the symmetric and asymmetric cases complete, we can now investigate whether segmenting the market by both newspapers corresponds to a Nash Equilibrium. In Proposition 4, we prove even a stronger result, namely that extending the product mix for each newspaper constitutes a dominant strategy. Unfortunately, in spite of being a dominant strategy, the equilibrium profits of the newspapers are lower with segmentation than if both offer only the print versions of their papers.
PROPOSITION 4.

(i) Offering both print and online versions is a dominant strategy for each newspaper. Specifically, for Newspaper 1 $\pi_1^{E,E} > \pi_1^{NE,E}$ and $\pi_1^{E,NE} > \pi_1^{NE,NE}$, and similarly for Newspaper 2.

(ii) In spite of being a dominant strategy, the profit of each newspaper are lower with segmentation than if both offer just print editions, namely $\pi_i^{NE,NE} > \pi_i^{E,E}$.

According to Proposition 4, competitive forces lead each newspaper to offer two different versions of the product. The resulting improved segmentation of consumers does not lead, however, to higher profits. There are two reasons why the profits of the newspapers decline with segmentation. First, note from (14) that equilibrium subscription fees decline when the newspapers reduce the slanting of their print editions. Because $B^{E,E} < B^{NE,NE}$, segmentation diminishes the extent of product differentiation between the print editions, and newspapers are forced to compete more aggressively for their print subscribers, thus leading to lower fees. Second, given that online subscribers are active in determining the extent of slant in the online editions, the ability of the newspapers to extract rents from consumers diminishes, as each newspaper loses some control over the attributes of its extended product line. In spite of the reduced profitability, though, each newspaper is forced to offer the online edition in order to prevent the rival from gaining market share.

The analysis so far has assumed that $\alpha$ is exogenous. In order to further explore the role of UGC we now extend our analysis to allow $\alpha_i$ to become a decision variable chosen simultaneously with the decision on whether to extend the product mix in Stage 1.
COROLLARY 1: If $\alpha_{i}, i = 1,2$ are chosen in the first stage together with the extension decision of the newspapers, each newspaper chooses a positive $\alpha_{i}$. The outcome $\alpha_{1} = \alpha_{2} = 0$ cannot correspond to an equilibrium even though the profits of each newspaper decline with the parameter $\alpha$.

Recall that according to Proposition 2, equilibrium profits in our model decline with $\alpha$. However, as Corollary 1 shows this does not mean that each newspaper chooses $\alpha_{i} = 0$ when it has the flexibility to vary $\alpha_{i}$. When the competitor of Newspaper $i$ chooses $\alpha_{j} = 0$, Newspaper $j$’s two editions are undifferentiated. Newspaper $i$ can then choose $\alpha_{i} > 0$ in response, and steal market share from $j$ by introducing two different variants of its product.

Next we consider an environment where each newspaper offers both editions but has full control over the positioning of the online edition. Essentially, the newspapers do not permit readers to add UGC online. Instead, newspaper $i$ has the exclusive rights to choose both $B_{i}$ and $B_{i}^{0}$. In Proposition 5 we report that in such an environment, each newspaper eliminates any product differentiation between its print and online editions, thus preventing further segmentation of its readers according to the intensity of their political opinions.

PROPOSITION 5: When newspapers have the exclusive rights to choose the positioning of both the print and online editions, at the equilibrium each newspaper does not introduce any differentiation in the location of the two editions (i.e., $B_{i} = B_{i}^{0}$).

In view of the result reported in Proposition 5 it is now easier to explain part (ii) of Proposition 4. Specifically, even though at the equilibrium both newspapers choose to extend their product lines by introducing online editions, their profits actually decline in comparison to
an environment where both offer only print editions. As readers become involved in generating UGC on the online versions, each newspaper is forced to de-facto offer two differently slanted versions of its product. According to Proposition 5, the newspapers would not choose to offer such differentiation if they could fully control the characteristics of both editions.

Notice that while according to Corollary 1 each newspaper chooses to differentiate its products with UGC ($\alpha_i > 0$ and therefore $B_i^o > B_i$), according to Proposition 5 it is never optimal for a paper to differentiate its products in an environment where readers are not permitted to add UGC to the online editions ($B_i^o = B_i$). The difference in these results is due to two opposing effects that extra segmentation introduces. The positive effect is the ability of the newspaper to steal market share from its rival by introducing a differentiated product. The negative effect is the intensified price competition that is implied by excessive segmentation. While in an environment with UGC the market share effect is dominant, in an environment without UGC the market share effect is weaker and the competitive effect dominates.

It is noteworthy that with UGC each newspaper is more limited in its ability to differentiate the online variant than when it has full control over the content of this edition. Hence, introducing the differentiated variant with UGC is an indirect commitment of the newspaper to keep the slanting of its online edition and therefore the competing print edition relatively moderate, thus squeezing the size of the audience that remains for the competing newspaper. In other words, in comparison to an environment in which the newspapers have full control over both editions the temptation to steal market share from the competitor is relatively strong in an environment with UGC, and thus $\alpha_i > 0$ in equilibrium.
3.4 CONCLUDING REMARKS

Over the past decade news media have been increasingly publishing opinions and news stories of their readers. This is facilitated by online editions which provide technical capabilities for readers to add their own content to the publications. We show that extending the product line to include online editions reduces the extent of slanting in reporting of the print edition but increases the extent of slant in the online edition. The increased slant of the online edition is primarily generated by the readers themselves who choose to add content to this variant of the product. In fact, we demonstrate that if newspapers had full control over the content of the online editions, they would choose the slant of their print and online editions to be identical. In contrast, when UGC is added by readers to the online editions, each newspaper is indirectly forced by subscribers to offer two differentiated versions of its product. We also find that as the extent of discretion of users to generate content online increases, newspapers become more polarized, yet their profits decline. The additional discretion awarded to users implies that the newspapers lose control over the attributes of their product lines, thus limiting their ability to extract rents from consumers. Hence, in spite of increased differentiation the profitability of the newspapers declines.

Note that our model assumes full coverage of the market of readers. If the market is less than fully covered when only print editions are offered, introducing online editions that are more politically biased due to UGC might lead to greater coverage. We demonstrate in the Appendix that this can enhance each newspaper’s profits. In addition, given our goal of investigating the role of UGC in affecting political bias in news reporting, our model focuses on the political opinions of readers as the sole determinant of their choice between the print and online editions. There are obviously many other attributes that distinguish consumers who prefer one edition
over the other. Online users are likely to be younger or have higher valuation for the technological features provided by online newspapers (such as content sharing-digging, mobile applications, and so on). In the Appendix, we incorporate a second dimension of heterogeneity, unrelated to politics that differentiates among readers. In this case we show that there is reduced tendency on the part of newspapers to rely on political beliefs when segmenting the market. As a result, the polarization of the newspapers moves closer to the outcome in an environment when only print editions are offered.
APPENDIX A: PROOFS

PROOFS FOR “THE IMPACT OF ADVERTISING ON MEDIA BIAS”

Derivations of Equations 16-17 and Proof of Lemma 1

(i) Single-Homing: Second stage prices are obtained by optimizing (12) with respect to $P_i$ and $K_i$ as follows:

\[ \frac{\partial \pi_1}{\partial P_1} = \frac{A}{2\alpha_0} \left[ \frac{\partial \alpha_{indiff}}{\partial b_{indiff}} \frac{\partial b_{indiff}}{\partial P_1} K_1 \right] + \frac{M_1}{2b_0} \left[ (b_0 + b_{indiff}) + \frac{\partial b_{indiff}}{\partial P_1} P_1 \right] = 0, \]

\[ \frac{\partial \pi_2}{\partial P_2} = \frac{A}{2\alpha_0} \left[ \frac{\partial \alpha_{indiff}}{\partial b_{indiff}} \frac{\partial b_{indiff}}{\partial P_2} K_2 \right] + \frac{M_1}{2b_0} \left[ (b_0 - b_{indiff}) - \frac{\partial b_{indiff}}{\partial P_2} P_2 \right] = 0, \]

\[ \frac{\partial \pi_1}{\partial K_1} = \frac{A}{2\alpha_0} \left[ \frac{\partial \alpha_{indiff}}{\partial K_1} + \left( \alpha_0 + \alpha_{indiff} \right) \right] = 0 \text{ and} \]

\[ \frac{\partial \pi_2}{\partial K_2} = \frac{A}{2\alpha_0} \left[ - \frac{\partial \alpha_{indiff}}{\partial K_2} K_2 + \left( \alpha_0 - \alpha_{indiff} \right) \right] = 0. \]

From (9):

\[ \frac{\partial \alpha_{indiff}}{\partial b_{indiff}} = \frac{2b_{indiff}}{b_0^2 - b_{indiff}^2} \alpha_{indiff} + \frac{2b_0 \phi}{b_0^2 - b_{indiff}^2}, \]

\[ \frac{\partial \alpha_{indiff}}{\partial K_1} = - \frac{2b_0^2}{(M_1 + M_2)(b_0^2 - b_{indiff}^2)}, \text{ and} \]

\[ \frac{\partial \alpha_{indiff}}{\partial K_2} = \frac{2b_0^2}{(M_1 + M_2)(b_0^2 - b_{indiff}^2)}. \]

From (2):

\[ \frac{\partial b_{indiff}}{\partial P_1} = \frac{\phi + \chi}{2\phi^2(B_1 - B_2)} \text{ and } \frac{\partial b_{indiff}}{\partial P_2} = \frac{\phi + \chi}{2\phi^2(B_2 - B_1)}. \]

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Substituting (2), (9), (A.5), (A.6) and (A.7), into the first order conditions (A.1)-(A.4), evaluating them at symmetry (\(-B_1 = B_2 = B\)), and solving for \(K_i\) and \(P_i\), we get \(P_i^{**}\) and \(K_i^{**}\) as given in (14). And substituting (A.6), (A.7) and (9) into (A.3) and (A.4) and solving for \(K_1\) and \(K_2\), one can get equilibrium advertising fees as a function of the locations:

\[
K_1^S = M \left( \frac{\alpha_0 (b_0^2 - b_{indiff}^2)}{2b_0^2} + \frac{b_{indiff}h_0}{3b_0} \right) , \quad K_2^S = M \left( \frac{\alpha_0 (b_0^2 - b_{indiff}^2)}{2b_0^2} - \frac{b_{indiff}h_0}{3b_0} \right)
\]

To obtain the equilibrium locations chosen by the newspapers in the first stage, one has to solve first for the second stage fees, \(P_i(B_i, B_j)\) and \(K_i(B_i, B_j)\) as functions of arbitrary location choices (not only symmetric). Substituting the equilibrium strategies back into (12), we obtain the first stage payoff functions designated as \(V_i(B_i, B_j)\). Differentiating with respect to the locations yields from the Envelope Theorem that:

\[
\frac{\partial V_i}{\partial B_i} = \frac{\partial \pi_i}{\partial B_i} + \frac{\partial \pi_i}{\partial P_j} \frac{\partial P_j}{\partial B_i} + \frac{\partial \pi_i}{\partial K_j} \frac{\partial K_j}{\partial B_i} = 0 \quad i,j = 1,2, \ i \neq j.
\]

To illustrate the derivation of the first stage equilibrium, we focus on the optimization of Newspaper 1. For this newspaper, the terms of (A.10) can be derived as follows:

\[
\frac{\partial \pi_1}{\partial B_1} = \frac{M_1}{2b_0} \left( \frac{\partial b_{indiff}}{\partial B_1} \right) P_1 + \frac{A}{2\alpha_0} \left( \frac{\partial \sigma_{indiff}}{\partial B_1} \right) K_1.
\]

\[
\frac{\partial \pi_1}{\partial P_2} = \frac{M_1}{2b_0} \left( \frac{\partial b_{indiff}}{\partial P_2} \frac{\partial P_2}{\partial B_1} \right) P_1 + \frac{A}{2\alpha_0} \left( \frac{\partial \sigma_{indiff}}{\partial b_{indiff}} \frac{\partial b_{indiff}}{\partial P_2} \frac{\partial P_2}{\partial B_1} \right) K_1 \text{ and}
\]

\[
\frac{\partial \pi_1}{\partial K_2} = \frac{A}{2\alpha_0} \left( \frac{\partial \sigma_{indiff}}{\partial K_2} \frac{\partial K_2}{\partial B_1} \right) K_1.
\]

While the expression for \(\frac{\partial K_2}{\partial B_1}\) in (A.13) can be directly derived from (A.9), to obtain the expression from \(\frac{\partial P_2}{\partial B_1}\) in (A.12), we need to utilize the Implicit Function Approach by totally differentiating the first order conditions (A.1) and (A.2) that determine subscription fees

\[
(\frac{\partial \pi_1}{\partial P_1} = 0 \text{ and } \frac{\partial \pi_2}{\partial P_2} = 0). \quad \text{We obtain:}
\]

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\[ dP_1 \left( \frac{\partial^2 \pi_1}{\partial P_1^2} \right) + dP_2 \left( \frac{\partial^2 \pi_1}{\partial P_1 \partial P_2} \right) + dB_1 \left( \frac{\partial^2 \pi_1}{\partial P_1 \partial B_1} \right) + dB_2 \left( \frac{\partial^2 \pi_1}{\partial P_2 \partial B_2} \right) = 0 \text{ and} \]

\[ dP_1 \left( \frac{\partial^2 \pi_2}{\partial P_1^2} \right) + dP_2 \left( \frac{\partial^2 \pi_2}{\partial P_1 \partial P_2} \right) + dB_1 \left( \frac{\partial^2 \pi_2}{\partial P_2 \partial B_1} \right) + dB_2 \left( \frac{\partial^2 \pi_2}{\partial P_2 \partial B_2} \right) = 0. \]

From (A.14) and (A.15):

\[ \left( \begin{array}{c} \frac{\partial P_1}{\partial B_1} \\ \frac{\partial P_1}{\partial B_2} \\ \frac{\partial P_2}{\partial B_1} \\ \frac{\partial P_2}{\partial B_2} \end{array} \right) = - \left( \begin{array}{cccc} \frac{\partial^2 \pi_1}{\partial P_1^2} & \frac{\partial^2 \pi_1}{\partial P_1 \partial P_2} & \frac{\partial^2 \pi_1}{\partial P_1 \partial B_1} & \frac{\partial^2 \pi_1}{\partial P_1 \partial B_2} \\ \frac{\partial^2 \pi_1}{\partial P_2 \partial P_1} & \frac{\partial^2 \pi_1}{\partial P_2^2} & \frac{\partial^2 \pi_1}{\partial P_2 \partial B_1} & \frac{\partial^2 \pi_1}{\partial P_2 \partial B_2} \\ \frac{\partial^2 \pi_2}{\partial P_1 \partial P_1} & \frac{\partial^2 \pi_2}{\partial P_1 \partial P_2} & \frac{\partial^2 \pi_2}{\partial P_1 \partial B_1} & \frac{\partial^2 \pi_2}{\partial P_1 \partial B_2} \\ \frac{\partial^2 \pi_2}{\partial P_2 \partial P_1} & \frac{\partial^2 \pi_2}{\partial P_2 \partial P_2} & \frac{\partial^2 \pi_2}{\partial P_2 \partial B_1} & \frac{\partial^2 \pi_2}{\partial P_2 \partial B_2} \end{array} \right)^{-1} \left( \begin{array}{c} \frac{\partial^2 \pi_1}{\partial P_1 \partial B_1} \\ \frac{\partial^2 \pi_1}{\partial P_1 \partial B_2} \\ \frac{\partial^2 \pi_2}{\partial P_2 \partial B_1} \\ \frac{\partial^2 \pi_2}{\partial P_2 \partial B_2} \end{array} \right). \]

Using (A.1) and (A.2) in evaluating (A.16) at the symmetric equilibrium yields:

\[ \left[ \frac{\partial P_1}{\partial B_1}, \frac{\partial P_1}{\partial B_2}, \frac{\partial P_2}{\partial B_1}, \frac{\partial P_2}{\partial B_2} \right] = \left[ \frac{-\phi + \chi}{2\phi^2 B} (2 - Z), \frac{-\phi + \chi}{2\phi^2 B} (1 - Z), \frac{-\phi + \chi}{2\phi^2 B} (1 - Z), \frac{-\phi + \chi}{2\phi^2 B} (2 - Z) \right] \left[ \begin{array}{cccc} \frac{-Y}{Y - W} & -Y + W \\ -Y - W & Y + W \end{array} \right]; \]

where \( Z \equiv \frac{A(\phi + \chi)h_0^2}{6M_1\phi^2 b_0 \alpha_0 B} \), \( Y \equiv 1 - \frac{AM(\phi + \chi)h_0^2}{6M_1 \alpha_0 b_0 \phi^2 B} \) and \( W \equiv -\frac{\phi + \chi}{4\phi^2 B^2} P^{**} - \frac{AM(\phi + \chi)h_0}{M_1 \phi^2 B^2}. \)

For second order condition, the determinant of the inverted matrix on the RHS of (A.17) should be positive implying that \( Z < 1.5 \). From (A.17), therefore:

\[ \frac{\partial P_2}{\partial B_1} = \frac{\partial P_1}{\partial B_1} = \frac{2\phi^2 B}{\phi + \chi} \left( W - \frac{Y}{3 - 2Z} \right). \]

We can now complete the characterization of the optimal location choice of Newspaper 1. Using (A.11) - (A.13), as well as the derivation for \( \frac{\partial K_2}{\partial B_1} \) from (A.9) and \( \frac{\partial P_2}{\partial B_1} \) from (A.18) in (A.10), we obtain at the symmetric equilibrium:

\[ \frac{\partial Y_1}{\partial B_1} = \frac{M_1}{4b_0} P^{**} + \frac{M_1}{2} \frac{\partial P_2}{\partial B_1} + \frac{A}{3\alpha_0} K^{**} h_0 = 0, \text{ where } \frac{\partial P_2}{\partial B_1} \text{ is given by (A.18).} \]
At the symmetric equilibrium when \(-B_1 = B_2 = B\), we obtain from (A.19) a quadratic equation as follows:

\[(A.20) \quad B^2 - B \left(\frac{3b_0}{2} + 2T \left(\frac{h_0}{3\alpha_0} + \frac{1}{2}\right)\right) + 4T \frac{h_0^2}{3\alpha_0} \left(1 + \frac{2Th_0}{3b_0^2}\right) = 0.\]

The two roots of this quadratic equation are:

\[B_*^* = \frac{3b_0}{4} + T \frac{h_0}{b_0} \left(\frac{1}{2} + \frac{h_0}{3\alpha_0}\right) \pm \sqrt{\Delta} ; \text{ where } \Delta \equiv \left(\frac{3b_0}{4} + T \frac{h_0}{b_0} \left(\frac{1}{2} + \frac{h_0}{3\alpha_0}\right)\right)^2 - 4T \frac{h_0^2}{3\alpha_0} \left(1 + \frac{2Th_0}{3b_0^2}\right).\]

Only the bigger root guarantees stability of reaction functions (i.e. \(\frac{\partial^2 v_1}{\partial B^2_i} < 0\)). As a result, the optimal location at the Single-Homing equilibrium is given in (16). Note that if \(\Delta < 0\) the quadratic expression (A.20) is positive for all values of \(B\). Hence, \(\frac{\partial v_1}{\partial B^*_i} > 0\) for all \(B\) and the optimal location is the corner solution \(B^* = 0\). Hence, \(B^* \neq 0\) if:

\[(A.21) \quad \Delta = \left(\frac{3b_0}{4} + T \frac{h_0}{b_0} \left(\frac{1}{2} + \frac{h_0}{3\alpha_0}\right)\right)^2 - 4T \frac{h_0^2}{3\alpha_0} \left(1 + \frac{2Th_0}{3b_0^2}\right) > 0.\]

Inequality (A.21) holds if:

\[(A.22) \quad T < \frac{3b_0^2\alpha_0}{2h_0(2h_0 - \alpha_0)}.\]

We next investigate the conditions under which \(P^*\) is positive. From (14):

\[(A.23) \quad P^* = \frac{4B\phi^2 b_0}{\phi^*X} - \frac{A M h_0}{M_1} = \frac{4\phi^2}{\phi^*X} (Bb_0 - 2Th_0).\]

\(P^* > 0\) implies \(B > 2 \frac{T h_0}{b_0}\) or equivalently from (16):

\[(A.24) \quad \sqrt{\left(\frac{3b_0}{4} + T \left(\frac{h_0}{3\alpha_0} + \frac{1}{2}\right)\right)^2 - 4T \frac{h_0^2}{3\alpha_0} \left(1 + \frac{2Th_0}{3b_0^2}\right)} > 2 \frac{\frac{T h_0}{b_0} - \frac{3b_0}{4}}{\frac{T h_0}{b_0} \left(\frac{1}{2} + \frac{h_0}{3\alpha_0}\right)}.\]
Given that the LHS is positive, there are two cases where this inequality can hold: when 
RHS is negative (Case 1) and when both sides are positive but the LHS is bigger (Case 2). Case 
1 implies that \( T < \frac{9b_0^2\alpha_0}{2h_0(9\alpha_0-2h_0)} \). For Case 2, squaring both sides of (A.24) and solving for \( T \) 
yields \( T < \frac{3b_0^2(9\alpha_0-4h_0)}{2h_0(9\alpha_0-2h_0)} \). Combining the two cases, yields that \( P^{**} > 0 \) if:

(A.25) \[ T < \frac{3b_0^2(9\alpha_0-4h_0)}{2h_0(9\alpha_0-2h_0)}. \]

Combining (A.25) and (A.22) yields the condition of part (i) of Lemma 1.

(ii) Double-Homing: Using a very similar approach to that developed when advertisers 
Single-Home, we obtain the following first order condition for the choice of location in the first 
stage.

(A.26) \[
\frac{d\pi_1}{dB_1} = A \left[ \frac{\partial K_1}{\partial b_{indiff}} \frac{\partial b_{indiff}}{\partial B_1} + \frac{\partial K_1}{\partial B_1} \right] + \frac{M_1P_1}{2b_0} \frac{\partial b_{indiff}}{\partial B_1} + \left[ \frac{A\partial K_1}{\partial b_{indiff}} \frac{\partial b_{indiff}}{\partial P_2} + \frac{M_1P_1}{2b_0} \frac{\partial b_{indiff}}{\partial P_2} \right] \frac{\partial P_2}{\partial B_1} = 0,
\]

where the expression for \( K_1 \), which follows from the maximization of (11) is:

(A.27) \[
K_1^D = \frac{M(b_0+b_{indiff})}{2b_0} \left( h_0 - \frac{\alpha_0(b_0-b_{indiff})}{2b_0} \right),
\]

\[
K_2^D = \frac{M(b_0-b_{indiff})}{2b_0} \left( h_0 - \frac{\alpha_0(b_0+b_{indiff})}{2b_0} \right).
\]

At the symmetric equilibrium (A.26) reduces to:

(A.28) \[
\frac{d\pi_1}{dB_1} = \frac{AMh_0}{4b_0} + \frac{M_1}{2} \left[ \frac{P}{2b_0} + \frac{\partial P_2}{\partial B_1} \right] = 0.
\]
To derive the expression for $\frac{\partial P_2}{\partial B_1}$, we have to use, once again, the Implicit Function Approach, by totally differentiating the first order condition for the subscription fees $P_1$. Those conditions are:

\begin{align*}
\frac{\partial \pi_1}{\partial P_1} &= M_1 \left( b_0 + b_{\text{indiff}} \right) - \frac{(\phi + \chi)}{2\phi^2(B_2-B_1)} \left\{ MA \left( h_0 + \frac{\alpha_0}{h_0} b_{\text{indiff}} \right) + M_1 P_1 \right\} = 0, \\
\frac{\partial \pi_2}{\partial P_2} &= M_1 \left( b_0 - b_{\text{indiff}} \right) - \frac{(\phi + \chi)}{2\phi^2(B_2-B_1)} \left\{ MA \left( h_0 - \frac{\alpha_0}{h_0} b_{\text{indiff}} \right) + M_1 P_2 \right\} = 0.
\end{align*}

Total differentiation of the first order conditions yields the following system of equations for $\frac{dP_1}{dB_1}$ and $\frac{dP_2}{dB_1}$:

\begin{equation}
\begin{bmatrix}
R - M_1 & -R + \frac{M_1}{2} \\
-R + \frac{M_1}{2} & R - M_1
\end{bmatrix}
\begin{bmatrix}
\frac{dP_1}{dB_1} \\
\frac{dP_2}{dB_1}
\end{bmatrix}
= \frac{M_1 b_0}{2B} - \frac{M_1}{2} + R
\end{equation}

where $R \equiv \frac{M_1 T \alpha_0}{b_0 B}$ and for second order conditions $R < \frac{3}{4} M_1$ or $T < \frac{3 b_0 B}{4 \alpha_0}$. Solving for $\frac{dP_2}{dB_1}$, we obtain:

\begin{equation}
\frac{dP_2}{dB_1} = \frac{2\phi^2}{(\phi + \chi)} \left[ -b_0 + \frac{b_0 T \alpha_0}{2} \right].
\end{equation}

Substituting back into (A.28), yields a quadratic equation in $B$ as follows:

\begin{equation}
B^2 - B \left( \frac{T \alpha_0}{b_0} + \frac{3}{2} b_0 \right) + 2T \alpha_0 = 0.
\end{equation}

There are two roots to this equation. However, only one satisfies also the condition for stability of reaction function. It is given in equation (17). The discriminant of the solution in (17) is positive if $T < \frac{b_0^2}{2\alpha_0}$. As well, to guarantee that $P^{**} > 0$, it follows from (15) that $T < \frac{B b_0}{2h_0}$.

Using the expression for $B$ from (17) in the last inequality, yields $T < \frac{(3h_0 - 2\alpha_0)b_0^2}{2h_0(2h_0 - \alpha_0)}$. This is a more demanding constraint than the one necessary to insure that the discriminant is positive, thus yielding part (ii) of Lemma 1.
Proof of Proposition 1

First note from (16) and (17) that \( B_S^{**} = B_D^{**} = \frac{3}{2} b_0 \) when \( T = 0 \). Differentiating the expressions of \( B_S^{**} \) and \( B_D^{**} \) with respect to \( T \) for the range of parameters that support each type of equilibrium yields \( \frac{\partial B_S^{**}}{\partial T} > 0 \) and \( \frac{\partial B_D^{**}}{\partial T} < 0 \). Hence, \( B_S^{**} > B^{MS} \) and \( B_D^{**} < B^{MS} \).

Regions of the Parameter \( \alpha_0 \) that Support Single and Double-Homing

(i) Single-Homing: To ensure that Single-Homing is an equilibrium, we use \( K_S^{**} \) from (14) in (8) and evaluate (8) at the symmetric equilibrium (i.e., \( b_{indif} = 0 \)) to obtain \( \hat{\alpha}_1 = 2(\alpha_0 - h_0) \) and \( \hat{\alpha}_2 = 2(h_0 - \alpha_0) \). To guarantee that the interior interval in Figure 1 disappears, we impose the restriction that \( \hat{\alpha}_1 \geq \hat{\alpha}_2 \), which happens when \( \alpha_0 \geq h_0 \).

(ii) Partial Double-Homing: In Partial Double-Homing equilibrium, in which some advertisers Double-Home and some Single-Home (i.e., \( \alpha_0 > \hat{\alpha}_2 > \hat{\alpha}_1 > -\alpha_0 \)), the newspapers choose subscription and advertising fees in the second stage to maximize the objectives:

\[
\pi_1 = A \frac{\alpha_0 + \hat{\alpha}_2}{2\alpha_0} K_1 + M_1 \frac{b_0 + b_{indif}}{2b_0} P_1 , \quad \pi_2 = A \frac{\alpha_0 - \hat{\alpha}_1}{2\alpha_0} K_2 + M_1 \frac{b_0 - b_{indif}}{2b_0} P_2 .
\]

When the newspapers locate symmetrically so that \(-B_1 = B_2 = B\), the solution to this maximization can be obtained as:

\[
P_D^{**} = \frac{4B\phi^2 b_0}{\phi + \chi} - \frac{Ah_0}{M_1} \frac{1}{2\alpha_0} [h_0 + \frac{\alpha_0}{2}] \quad \text{and} \quad K_D^{**} = \frac{M}{4} [h_0 + \frac{\alpha_0}{2}].
\]

Using \( K_D^{**} \) in (8) and evaluating it at the symmetry (i.e., \( b_{indif} = 0 \)) we obtain \( \hat{\alpha}_1 = \frac{\alpha_0}{2} - h_0 \) and \( \hat{\alpha}_2 = h_0 - \frac{\alpha_0}{2} \). Hence, \( \hat{\alpha}_2 \geq \hat{\alpha}_1 \) if \( \alpha_0 \leq 2h_0 \), and \( \hat{\alpha}_2 < \alpha_0 \) if \( \alpha_0 > \frac{2}{3} h_0 \).
(iii) *Double-Homing:* All advertisers will Double-Home when $\alpha_2 \geq \alpha_0$. It follows from part (ii) that this happens if $\alpha_0 \leq \frac{2}{3} h_0$.

From (i), (ii) and (iii) we conclude that while for $\alpha_0 > 2h_0$ Single-Homing is the unique equilibrium, for $\alpha_0 \leq \frac{2}{3} h_0$ Double-Homing is the unique equilibrium. It also follows from the above that between $h_0$ and $2h_0$ Single-Homing and Partial Double-Homing equilibria may co-exist.

**Optimality of Linear Decision Rules**

We show the optimality of linear slanting strategies when the newspapers’ sole source of revenue is subscription fees. To this end we first derive the first order conditions that follow from the newspapers’ first stage location choices without restricting the functional form of the slanting strategies $s_i(B_i, d), i = 1,2$. Then we show that a linear slanting rule satisfies these first order conditions.

The consumer who is indifferent between the two newspapers satisfies the equation $EU_b^1 = EU_b^2$ where $U_b^i$ is given by (1). Solving this equation and using the distributional properties of the random variable $d$ ($d \sim N(0, \sigma_d^2)$) and the uniform distribution of the parameter $b$, yields:

\[
(A.31) \quad b_{indif} = \frac{(P_2 - P_1)}{2\phi (E_s - E_{s_1})} + \frac{(\phi \chi (Es_2^2 - Es_1^2))}{2\phi (E_s - E_{s_1})} + \frac{E(d(s_2 - s_1))}{(E_s - E_{s_1})}.
\]
In the second stage, newspapers set their prices $P_1$ and $P_2$ to maximize (3). First order conditions for this maximization are:

\[
\frac{M_1}{2b_0} \left( b_{indif} + b_0 \right) - \frac{P_1}{2\phi(Es_2 - Es_1)} = 0,
\]

(A.32)

\[
\frac{M_1}{2b_0} \left( b_0 - b_{indif} \right) - \frac{P_2}{2\phi(Es_2 - Es_1)} = 0.
\]

(A.33)

At the symmetric equilibrium (i.e., $B_1 = B_2 = B, b_{indif} = 0$) the solution to (A.32) and (A.33) is:

\[
P_1 = P_2 = P^* = 2\phi(Es_2 - Es_1)b_0.
\]

(A.34)

To obtain the equilibrium locations chosen by the newspapers in the first stage, one has to solve first for the second stage fees, $P_i(B_i, B_j)$ as functions of arbitrary location choices (not only symmetric). Substituting the equilibrium strategies back into (3), we obtain the first stage payoff functions $V_i(B_i, B_j)$. Differentiating with respect to the locations yields from the Envelope Theorem that:

\[
\frac{\partial V_i}{\partial B_i} = \frac{\partial \pi_i}{\partial B_i} + \frac{\partial \pi_i}{\partial P_j} \frac{\partial P_j}{\partial B_i} = 0 \quad i, j = 1, 2, \; i \neq j.
\]

(A.35)

It follows from (A.35) and (3) that,

\[
\frac{\partial b_{indif}}{\partial B_1} + \frac{\partial b_{indif}}{\partial P_2} \frac{\partial P_2}{\partial B_1} = 0,
\]

(A.36)

\[
- \frac{\partial b_{indif}}{\partial B_2} - \frac{\partial b_{indif}}{\partial P_1} \frac{\partial P_1}{\partial B_2} = 0.
\]

(A.37)

From (A.31):

\[
\frac{\partial b_{indif}}{\partial P_1} = \frac{-1}{2\phi(\text{Es}_2 - \text{Es}_1)}
\]

(A.38)

\[
\frac{\partial b_{indif}}{\partial B_2} = \frac{-\left(P_2 - P_1\right)Es_2(B_2)}{2\phi(\text{Es}_2 - \text{Es}_1)^2} + \frac{\left(\phi + \chi\right)}{2\phi} \frac{2Es_2s_2(B_2)(\text{Es}_2 - \text{Es}_1) - Es_2(B_2)(Es_2^2 - Es_1^2)}{(\text{Es}_2 - \text{Es}_1)^2} + \frac{E[d(s_2(B_2)(Es_2 - Es_1) - Es_2(B_2)(s_2 - s_1))]}{(Es_2 - Es_1)^2}, \text{ where } s'_2(B_2) \equiv \frac{\partial s_2(B_2, d)}{\partial B_2}.
\]

(A.39)
To obtain $\frac{\partial P_1}{\partial B_2}$, we utilize, once again, the Implicit Function approach by totally differentiating the first order conditions (A.32) and (A.33), and solving the resulting system of equations using $P^*$ from (A.34). This procedure yields

$$\frac{\partial P_1}{\partial B_2} = \frac{2\phi(E_{s_2} - E_{s_1})}{3} \frac{\partial b_{indiff}}{\partial B_2} + 2\phi b_0 E_{s_2}'(B_2).$$

(A.40)

Suppose the firms use a linear decision rule:

$$s_i(B_i, d) = A_1 B_i + A_2 d,$$

then:

$$s_i'(B_i) = A_1,$$

(A.41)

$$E_{s_2} - E_{s_1} = A_1 (B_2 - B_1),$$

(A.42)

$$E_{s_2}^2 - E_{s_1}^2 = A_1^2 (B_2^2 - B_1^2).$$

(A.43)

(A.44)

Substituting (A.42)-(A.44) into (A.38)-(A.40), and using these in (A.37) at the symmetric equilibrium yields:

$$-(\frac{\phi + \chi}{3\phi}) A_1 + \frac{b_0}{2B} = 0.$$

(A.45)

From (A.45):

$$B = \frac{3}{2} b_0 \frac{\phi}{(\phi + \chi) A_1}.$$

(A.46)

Thus, the linear slanting rule (A.41) allows us to solve the first order condition in (A.37) and find an optimal location as given in (A.46). Since the newspapers are symmetric, the same rule also satisfies the first order condition for Newspaper 1 (i.e., (A.36)) as well.
We will now show that this rule can be expressed as \( s_i(d) = \frac{\phi}{(\phi + \chi)} (B_i - d) \) for newspaper \( i \). We will assume that Newspaper 1 follows such a decision rule and demonstrate that the best response of Newspaper 2 is to follow such a rule, as well. Specifically, assuming that 
\[ s_1(d) = \frac{\phi}{(\phi + \chi)} (B_1 - d) \]
we designate the linear rule followed by Newspaper 2 as \( s_2(d) = m + A_2 d \), where \( m \equiv A_1 B_2 \).

Given the assumed behavior of Newspaper 1, a consumer who chooses to subscribe to it derives the expected payoff: 
\[ EU^1_b = \bar{u} - \frac{\phi^2}{\phi + \chi} (B_1 - b)^2 - \frac{\chi \phi}{\phi + \chi} (b^2 + \sigma_d^2) - P_1. \]
Her payoff when choosing Newspaper 2 is:
\[ EU^2_b = \bar{u} - (\phi + \chi)(m^2 + A_2^2 \sigma_d^2) - \phi(b^2 + \sigma_d^2(2A_2 + 1) - 2bm) - P_2. \]

To find the consumer who is indifferent between the two newspapers we solve the equation \( EU^1_b = EU^2_b \) for \( b \) as follows:

\[
\text{(A.47)} \quad b_{\text{indiff}} = \frac{(\phi + \chi)m^2 - \frac{\phi^2}{\phi + \chi} B_1^2 + (\phi + \chi)\sigma_d^2 A_2 + \frac{\phi}{\phi + \chi}}{2\left(\frac{\phi}{\phi + \chi} b_1\right)}. 
\]

Assuming without loss of generality that \( m > \frac{\chi}{\phi + \chi} B_1 \), in the second stage the newspapers choose their subscription fees to maximize: \( \pi_1 = \frac{(b_0 + b_{\text{indiff}})}{2b_0} P_1 \) and \( \pi_2 = \frac{(b_0 - b_{\text{indiff}})}{2b_0} P_2 \). Note that \( m > \frac{\chi}{\phi + \chi} B_1 \) simply guarantees that Newspaper 2 serves the upper end of subscribers above \( b_{\text{indiff}} \) and Newspaper 1 serves the lower end. Once the coefficients are derived this assumption is indeed satisfied as \( B_2 > B_1 \). Optimizing with respect to \( P_1 \) and \( P_2 \), yields the second stage prices as a function of \( B_1, m \) and \( A_2 \) as follows:
It follows, therefore, that:

\[
\frac{\partial p_1}{\partial m} = 2\phi b_0 + \frac{2(\phi + \chi) m}{3}, \quad \frac{\partial p_1}{\partial A_2} = \frac{2(\phi + \chi) \sigma_d^2 (A_2 + \frac{\phi}{\phi + \chi})}{3}.
\]

When Newspaper 2 chooses its slanting strategy rule in stage 1, namely \( m \) and \( A_2 \), it optimizes its payoff function \( \pi_2 \), given the prices established subsequently in the second stage. Substituting for \( p_1(m, A_2, B_1) \) and \( p_2(m, A_2, B_1) \) back into the payoff functions, yields the first stage payoff function for Newspaper 2, \( V_2(m, A_2, B_1) \). Using the Envelope Theorem:

\[
(A.49) \quad \frac{\partial v_2}{\partial m} = -\frac{p_2}{2b_0} \left[ \frac{\partial b_{\text{indif}}}{\partial m} + \frac{\partial b_{\text{indif}}}{\partial p_1} \frac{\partial p_1}{\partial m} \right] = -\frac{p_2}{2b_0} \left[ -\frac{b_0 + \frac{2(\phi + \chi) m - b_{\text{indif}}}{m - \frac{\phi}{\phi + \chi} B_1} \right],
\]

\[
\frac{\partial v_2}{\partial A_2} = -\frac{p_2}{2b_0} \left[ \frac{\partial b_{\text{indif}}}{\partial A_2} + \frac{\partial b_{\text{indif}}}{\partial p_1} \frac{\partial p_1}{\partial A_2} \right] = -\frac{p_2}{3b_0} \left[ \frac{(\phi + \chi) \sigma_d^2 (A_2 + \frac{\phi}{\phi + \chi})}{m - \frac{\phi}{\phi + \chi} B_1} \right].
\]

From the second equation of (A.49), it follows that \( \frac{\partial v_2}{\partial A_2} = 0 \) when \( A_2 = -\frac{\phi}{\phi + \chi} \). Hence, the decision rule of Newspaper 2 \( s_2(B_2, d) = A_1 B_2 + A_2 d \) can be written as

\[
s_2(d) = \frac{\phi}{(\phi + \chi)} (B_2 - d) \text{ where } A_1 \text{ has been normalized to } -\frac{\phi}{(\phi + \chi)}. \]

To find the value of \( B_2 \), we further restrict our attention to symmetric Bayesian equilibria, which implies that \( b_{\text{indif}} = 0 \). Substituting into the first equation of (A.49), implies that \( m = \frac{3b_0}{2} \frac{\phi}{\phi + \chi} \), thus \( B_2 = \frac{3b_0}{2} \), and by the symmetry assumption, \( B_1 = -\frac{3b_0}{2} \).
Incomplete Coverage of the Subscribers Market

Single-Homing:

In this section, we demonstrate that when advertisers Single-Home and when the subscriber market is not covered as in Figure 5 there exist conditions under which \( B_2 < \frac{3}{2}b_0 \)

<table>
<thead>
<tr>
<th>Buy Newspaper 1</th>
<th>Do not Buy</th>
<th>Buy Newspaper 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-b_0)</td>
<td>(b_1)</td>
<td>(b_2)</td>
</tr>
<tr>
<td></td>
<td>(b_0)</td>
<td></td>
</tr>
</tbody>
</table>

**Figure 5:** Segmentation of the Subscriber Market

The reader who is indifferent between buying Newspaper 1 and not buying at all satisfies the equation:

\[
(A.50) \quad EU_{b^1} = 0 \text{ where} \\
EU_{b^1} = -\phi b^2 + \frac{2b\phi^2B_1}{\phi + \chi} + \bar{u} - P_1 - \frac{B_0^2\phi^2}{\phi + \chi} - \frac{\sigma^2\phi}{\phi + \chi} \quad \text{from (1).}
\]

The above equation has two roots:

\[
(A.51) \quad b_1 = \frac{\phi B_1}{\phi + \chi} \mp \frac{\sqrt{\frac{\phi^2 B_1^2}{(\phi + \chi)^2} + \frac{1}{\phi} (\bar{u} - P_1 - \frac{B_0^2\phi^2}{\phi + \chi} - \frac{\sigma^2\phi}{\phi + \chi})}}{\frac{\phi^2 B_1^2}{(\phi + \chi)^2}}.
\]
Notice that for the segmentation given in Figure A1 to hold we need \( \frac{\partial E U_{b}^{1}}{\partial b} = -2\phi b + \frac{2\phi^{2}B_{1}}{\phi + \chi} < 0 \) at \( b = b_{1} \) which implies:

\[
(A.52) \quad b_{1} > \frac{\phi B_{1}}{(\phi + \chi)}.
\]

Further at \( b = 0 \), we should have \( E U_{b}^{1} < 0 \) and at \( b = -b_{0} \), \( E U_{b}^{1} > 0 \), thus:

\[
(A.53) \quad \bar{u} - P_{1} - \frac{B_{1}^{2}\phi^{2}}{\phi + \chi} - \frac{\sigma_{A}^{2}\chi\phi}{\phi + \chi} < 0, \quad \text{and} \quad \bar{u} - P_{1} - \frac{B_{1}^{2}\phi^{2}}{\phi + \chi} - \frac{\sigma_{A}^{2}\chi\phi}{\phi + \chi} - \phi b_{0}^{2} - \frac{2b_{0}\phi^{2}B_{1}}{\phi + \chi} > 0.
\]

The root in (A.51) that satisfies (A.52) is:

\[
(A.54) \quad b_{1} = \frac{\phi B_{1}}{\phi + \chi} + \sqrt{\frac{\phi^{2}B_{1}^{2}}{(\phi + \chi)^{2}} + \frac{1}{\phi} \left( \bar{u} - P_{1} - \frac{B_{1}^{2}\phi^{2}}{\phi + \chi} - \frac{\sigma_{A}^{2}\chi\phi}{\phi + \chi} \right)}
\]

\[
= \frac{\phi B_{1}}{\phi + \chi} + \sqrt{\frac{\bar{u} - P_{1}}{\phi} - \frac{\chi}{\phi + \chi} \left( \sigma_{d}^{2} + \frac{B_{1}^{2}\phi}{\phi + \chi} \right)}.
\]

Similarly, the location of the reader who is indifferent between buying Newspaper 2 and not buying at all can be calculated as:

\[
(A.55) \quad b_{2} = \phi B_{2} \phi + \chi \phi - \sqrt{\frac{\bar{u} - P_{2}}{\phi} - \frac{\chi}{\phi + \chi} \left( \sigma_{d}^{2} + \frac{B_{2}^{2}\phi}{\phi + \chi} \right)},
\]

and \( 0 < b_{2} < b_{0} \) if

\[
(A.56) \quad \bar{u} - P_{2} - \frac{B_{2}^{2}\phi^{2}}{\phi + \chi} - \frac{\sigma_{A}^{2}\chi\phi}{\phi + \chi} < 0, \quad \text{and} \quad \bar{u} - P_{2} - \frac{B_{2}^{2}\phi^{2}}{\phi + \chi} - \frac{\sigma_{A}^{2}\chi\phi}{\phi + \chi} - \phi b_{0}^{2} - \frac{2b_{0}\phi^{2}B_{2}}{\phi + \chi} > 0.
\]

When an advertiser of appeal parameter \( \alpha \) chooses to advertise in Newspaper 1, its expected payoff is given as:

\[
(A.57) \quad E_{1}(\alpha) = M \int_{-b_{0}}^{b_{1}} \frac{1}{2b_{0}} \left( h_{0} + \frac{ab}{b_{0}} \right) db - K_{1}.
\]

If it chooses to advertise in Newspaper 2 its expected payoff is:

\[
(A.58) \quad E_{2}(\alpha) = M \int_{b_{2}}^{b_{0}} \frac{1}{2b_{0}} \left( h_{0} + \frac{ab}{b_{0}} \right) db - K_{2}.
\]
The advertiser who is indifferent between Newspaper 1 and 2 can be derived from (A.57) and (A.58) by solving for $\alpha$ in $E_1(\alpha) = E_2(\alpha)$:

\[(A.59) \quad \alpha_{\text{indif}} = \frac{2b_0 h_0 (b_1 + b_2)}{(2b_0^2 - b_1^2 - b_2^2)} + \frac{(K_2 - K_1)4b_0^2}{M(2b_0^2 - b_1^2 - b_2^2)}.\]

In the last stage the newspapers set their subscription and advertising fees to maximize their profits:

\[(A.60) \quad \pi_1 = A^\frac{a_0 + \alpha_{\text{indif}}}{2a_0} K_1 + M_1^\frac{b_0 + b_1}{2b_0} P_1, \quad \pi_2 = A^\frac{a_0 - \alpha_{\text{indif}}}{2a_0} K_2 + M_1^\frac{b_0 - b_2}{2b_0} P_2,\]

which yields the following first order conditions:

\[(A.61) \quad \frac{A}{2a_0} \left[ (\alpha_0 + \alpha_{\text{indif}}) - \frac{4b_0^2 K_1}{M(2b_0^2 - b_1^2 - b_2^2)} \right] = 0,\]

\[(A.62) \quad \frac{A}{2a_0} \left[ (\alpha_0 - \alpha_{\text{indif}}) - \frac{4b_0^2 K_2}{M(2b_0^2 - b_1^2 - b_2^2)} \right] = 0,\]

\[(A.63) \quad M_1 \left( \frac{b_1 + b_0}{2b_0} - \frac{P_1}{4b_0 \phi\left(\frac{\phi B_1}{\phi+\chi}\right)} \right) - \frac{2AK_1 b_0 h_0 (2b_0^2 + b_1^2 - b_2^2 + 2b_1 b_2)}{4a_0 \phi\left(2b_0^2 - b_1^2 - b_2^2\right)^2 (b_1 - \frac{\phi B_1}{\phi+\chi})} = 0,\]

\[(A.64) \quad M_1 \left( \frac{b_0 - b_2}{2b_0} - \frac{P_2}{4b_0 \phi\left(\frac{\phi B_2}{\phi+\chi}\right)} \right) - \frac{2AK_2 b_0 h_0 (2b_0^2 + b_1^2 - b_2^2 + 2b_1 b_2)}{4a_0 \phi\left(2b_0^2 - b_1^2 - b_2^2\right)^2 (\frac{\phi B_2}{\phi+\chi} - b_2)} = 0.\]

Simultaneously solving (A.61) and (A.62) we get:

\[(A.65) \quad K_1 = M \left( \frac{a_0 (2b_0^2 - b_1^2 - b_2^2)}{4b_0^2} + \frac{1}{6b_0} (b_1 + b_2) \right),\]

\[(A.66) \quad K_2 = M \left( \frac{a_0 (2b_0^2 - b_1^2 - b_2^2)}{4b_0^2} - \frac{1}{6b_0} (b_1 + b_2) \right).\]

Thus, at the symmetric equilibrium (i.e., $b_1 = -b_2$):

\[(A.67) \quad K^* = \frac{M(b_0^2 - b_2^2) a_0}{2b_0^2}.\]

Using (A.67) in (A.64) at the symmetric equilibrium yields:

\[(A.68) \quad P^* = 2\phi \left( \frac{\phi B_2}{\phi+\chi} - b_2 \right) (b_0 - b_2) - \frac{AM h_0}{2M_1}.\]
To obtain the equilibrium locations with incomplete coverage we differentiate the first stage payoff functions $V_i(B_i, B_j)$ with respect to the locations and use the Envelope Theorem:

$$\frac{\partial V_i}{\partial B_i} = \frac{\partial \pi_i}{\partial B_i} + \frac{\partial \pi_i}{\partial P_j} \frac{\partial P_j}{\partial B_i} + \frac{\partial \pi_i}{\partial K_j} \frac{\partial K_j}{\partial B_i} = 0. \quad i, j = 1, 2, \ i \neq j. \tag{A.69}$$

For Newspaper 2, it follows from (A.69) that,

$$\frac{\partial V_2}{\partial B_2} = \frac{\partial \pi_2}{\partial B_2} + \frac{\partial \pi_2}{\partial P_2} \frac{\partial P_2}{\partial B_2} + \frac{\partial \pi_2}{\partial K_1} \frac{\partial K_1}{\partial B_2} = 0 \tag{A.70}$$

Using the Implicit Function approach by totally differentiating the first order conditions (A.63) and (A.64) and solving the resulting system of equations using $K^*$ from (A.67) and $P^*$ from (A.68) at the symmetric equilibrium we obtain:

$$\frac{\partial P_2}{\partial B_2} = - R \frac{\partial K_1}{\partial B_2} \tag{A.71}$$

where

$$R = \frac{A \alpha h_0}{4 \alpha_0 \phi (b_0^2 - b_2^2) \left( \frac{2 \phi + \chi}{\phi + \chi} \right)} \left[ \frac{M_1 \left( \frac{2 \phi + \chi}{\phi + \chi} \right)}{4 b_0 \phi \left( \frac{2 \phi + \chi}{\phi + \chi} \right) b_2} + \frac{M \alpha \left( \frac{2 \phi + \chi}{\phi + \chi} \right) b_2}{b_0 (2 b_0^2 - b_2^2) \phi \left( \frac{2 \phi + \chi}{\phi + \chi} \right) b_2} \right].$$

From (12), (A.59), and (A.71), at the symmetric equilibrium (A.70) becomes:

$$\frac{\partial V_2}{\partial B_2} = - \frac{\partial P_2}{\partial B_2} \left[ \left( \frac{P^* M_1}{2 b_0} + \frac{M \alpha h_0}{4 b_0} \right) - \frac{\partial K_1}{\partial b_2} \frac{M \alpha}{4} \left( 2 - \frac{h_0 R}{2 b_0 \left( \frac{2 \phi + \chi}{\phi + \chi} \right) b_2} \right) \right] \tag{A.72}$$

From (A.55), $\frac{\partial P_2}{\partial B_2} > 0$. Notice that the first term inside the brackets in (A.72) is always positive and the second term is positive if $\frac{\partial K_1}{\partial b_2} < 0$ and if $b_2 > \frac{b_0 h_0}{2 \alpha_0}$. From (A.65) $\frac{\partial K_1}{\partial b_2} < 0$ if $b_2 > \frac{b_0 h_0}{3 \alpha_0}$. Therefore, if $b_2 > \frac{b_0 h_0}{2 \alpha_0}$, $\frac{\partial V_2}{\partial B_2} < 0$, and the newspaper will continue to reduce bias.

Since $\frac{\partial P_2}{\partial B_2} > 0$ it follows that bias will be reduced until $b_2 \leq \frac{b_0 h_0}{2 \alpha_0}$. Because at the Single-Homing equilibrium with complete coverage $\alpha_0 > h_0$, it follows that $b_2 < \frac{b_0}{2}$. Hence, incomplete
coverage of the type depicted in Figure 5, will never lead to an equilibrium where each newspaper covers less than half of the segment of readers who prefer its location best (for 1 this segment is $b < 0$, and for 2 this segment is $b > 0$).

We now derive the conditions on the parameters of the model to guarantee that less than full coverage moderates the extent of bias at the equilibrium in comparison to $B^{MS} = \frac{3}{2} b_0$. To obtain the conditions we substitute the equilibrium price from (A.68) back into (A.55) and solve for $B_2$ in terms of $b_2$ as follows:

\[
(A.73) \quad B_2 = \sqrt{(b_0 - 2b_2)^2 + \frac{\phi + \chi}{\phi} \left( \frac{\bar{u}}{\phi} - \frac{\chi \sigma^2_d}{\phi + \chi} + \frac{AM h_0}{2M_1 \phi} + 2b_2 b_0 - 3b_2^2 \right) - (b_0 - 2b_2)}.
\]

It is very easy to show that $\frac{\partial B_2}{\partial b_2}$ in the above expression is positive when $b_2 < \frac{b_0}{2}$.

Evaluating the right-hand side of (A.73) at $b_2 = \frac{b_0}{2}$ yields therefore, that:

\[
(A.74) \quad B_2 < \sqrt{\frac{\phi + \chi}{\phi} \left( \frac{\bar{u}}{\phi} - \frac{\chi \sigma^2_d}{\phi + \chi} + \frac{AM h_0}{2M_1 \phi} + \frac{b_0^2}{4} \right)}.
\]

The right-hand side of (A.74) is smaller than $\frac{3}{2} b_0$ if:

\[
(A.75) \quad R \overset{\text{def}}{=} \frac{\bar{u}}{\phi} - \frac{\chi \sigma^2_d}{\phi + \chi} + \frac{AM h_0}{2M_1 \phi} < \frac{b_0^2}{4} \frac{\phi + \chi}{\phi + \chi}.
\]

Hence, as long as $R$ is sufficiently small (e.g., when the reservation price of readers $\bar{u}$ is low and their valuation of accurate reporting $\chi$ is high), incomplete coverage may yield moderation of bias below $B^{MS}$. Note that condition (A.75) does not necessarily contradict (A.53) and (A.56), conditions necessary to support the type of incomplete coverage we consider.
Double-Homing:

Suppose that the newspapers, this time, cover only the middle of the market (i.e., between $b_1$ and $b_2$ in Figure 6) and advertisers Double-Home.

<table>
<thead>
<tr>
<th>Do not Buy</th>
<th>Buy Newspaper 1</th>
<th>Buy Newspaper 2</th>
<th>Do not Buy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-b_0$</td>
<td>$b_1$</td>
<td>$b_{indif}$</td>
<td>$b_2$</td>
</tr>
</tbody>
</table>

Figure 6: Segmentation of the Subscriber Market

We demonstrate that at the limit as $b_1 \to -b_0$ and $b_2 \to b_0$, the newspapers may have incentives to moderate their bias in reporting in comparison to the full market coverage case that is analyzed in Section 2.4.

Again, the reader who is indifferent between buying Newspaper 1 and not buying at all satisfies equation (A.50). For the segmentation given in Figure 6 to hold we need:

$$\frac{\partial EU_{b1}}{\partial b} = -2\phi b + \frac{2b\phi^2B_1}{\phi+\chi} > 0 \text{ at } b = b_1 \text{ which implies:}$$

(A.76) \[ b_1 < \frac{\phi B_1}{(\phi+\chi)}. \]

Further at $b = 0$, we should have $EU_b^1 > 0$ and at $b = -b_0$, $EU_b^1 < 0$. The root in (A.51) that satisfies (A.76) is:

(A.77) \[ b_1 = \frac{\phi B_1}{\phi+\chi} - \frac{\phi^2 B_1^2}{(\phi+\chi)^2} + \frac{1}{\phi} \left( \frac{\bar{u} - P_1 - \frac{2\phi^2}{\phi+\chi} - \frac{\sigma_d^2}{\phi+\chi} \phi}{\phi+\chi} \right) = \frac{\phi B_1}{\phi+\chi} - \sqrt{\frac{\bar{u}-P_1}{\phi} - \frac{\chi}{\phi+\chi} \left( \frac{\sigma_d^2 + \frac{B_1^2}{\phi+\chi} \phi}{\phi+\chi} \right)}. \]
Similarly, the location of the reader who is indifferent between buying Newspaper 2 and not buying at all can be calculated as:

\[
\begin{align*}
(A.78) \quad b_2 &= \frac{\phi B_2}{\phi + \chi} + \sqrt{\frac{\bar{u} - P_2}{\phi} - \frac{\chi}{\phi + \chi} \left( \sigma^2 d + \frac{B_2^2}{\phi + \chi} \right)}.
\end{align*}
\]

One can show that in this case as well, the newspapers will choose to cover at least one half of the readers who prefer their locations best, namely \( b_2 > \frac{b_0}{2} \) and \( b_1 < -\frac{b_0}{2} \). Note from (A.78) that:

\[
\begin{align*}
(A.79) \quad \frac{\partial b_2}{\partial B_2} &= \left\{ \begin{array}{ll}
\frac{(b_2 - B_2)\sqrt{\chi}}{\phi + \chi} & > 0 \text{ if } b_2 > B_2 \\
\frac{b_2 - \frac{\chi B_2}{\phi + \chi}}{\phi + \chi} & < 0 \text{ if } b_2 < B_2.
\end{array} \right.
\end{align*}
\]

Equation (A.79) implies that newspapers may have incentives to moderate their bias, once again. The benefit to advertisers in this case can be derived as: 
\[
E_1(\alpha) = M \int_{b_1}^{b_{\text{indiff}}} \frac{1}{2b_0} \left( h_0 + \frac{ab}{b_0} \right) db - K_1,
\]
and 
\[
E_2(\alpha) = M \int_{b_{\text{indiff}}}^{b_2} \frac{1}{2b_0} \left( h_0 + \frac{ab}{b_0} \right) db - K_2.
\]

When advertisers Double-Home, advertising fees are determined by the requirement that 
\( E_1(\alpha_0) = 0 \) and \( E_2(-\alpha_0) = 0 \), thus yielding:

\[
(A.80) \quad K_1 = M \left( \frac{h_0(b_{\text{indiff}} - b_1)}{2b_0} + \frac{\alpha_0(b_{\text{indiff}}^2 - b_1^2)}{4b_0^2} \right),
\]

\[
(A.81) \quad K_2 = M \left( \frac{h_0(b_2 - b_{\text{indiff}})}{2b_0} - \frac{\alpha_0(b_2^2 - b_{\text{indiff}}^2)}{4b_0^2} \right).
\]

Newspapers’ profits when they cover only the middle of the market (and when advertisers Double-Home) are:

\[
(A.82) \quad \pi_1 = AK_1 + M_1 \frac{b_{\text{indiff}} - b_1}{2b_0} P_1, \quad \pi_2 = AK_2 + M_1 \frac{b_2 - b_{\text{indiff}}}{2b_0} P_2.
\]

where \( K_1 \) and \( K_2 \) are as given in (A.80) and (A.81).
Let $\pi_2^{cov}$ and $\pi_2^{non-cov}$ denote Newspaper 2’s profits when it covers and does not cover the market as given in (13) and (A.82), respectively. Then, when $b_1 \to -b_0$ and $b_2 \to b_0$:

(A.83) \[
\frac{\partial \pi_2^{non-cov}}{\partial b_2} - \frac{\partial \pi_2^{cov}}{\partial b_2} = \frac{1}{2b_0} \left( M_1 P_2 + AM (h_0 - \alpha_0) \right) \frac{\partial b_2}{\partial b_2}.
\]

Recall that at the Double-Homing equilibrium $h_0 > \alpha_0$. Hence, the sign of the above difference depends only on the sign of $\frac{\partial b_2}{\partial b_2}$. From (A.79), $\frac{\partial b_2}{\partial b_2} < 0$ if $b_2 < B_2$. This inequality is more likely when the variable $R$, defined in (A.75), is relatively small (e.g., when the reservation price of readers $\bar{u}$ is low and their valuation of accurate reporting $\chi$ is high). If the sign of (A.83) is negative, newspapers have incentives to moderate when $b_1 \to -b_0$ and $b_2 \to b_0$. Note that even when $\frac{\partial b_2}{\partial b_2} > 0$, in which case the newspapers have incentives to polarize, bias will at most be equal to $b_0$, because $\frac{\partial b_2}{\partial b_2} > 0$ when $B_2 < b_2$ and $b_2 \leq b_0$.

**Asymmetric Accuracy**

We investigate the impact of asymmetry in newspapers’ data accuracy on reporting bias when the papers’ sole source of revenue is subscription fees. Specifically, we assume that Newspaper 1 has access to more accurate data than Newspaper 2: $\sigma_{d_1}^2 < \sigma_{d_2}^2$. In this case location of reader who is indifferent between the two newspapers is:

(A.84) \[
b_{indiff} = \left( \frac{\phi + \chi}{2\phi^2} \frac{(P_2 - P_1)}{(B_2 - B_1)} \right) + \frac{B_1 + B_2}{2} + \frac{\chi}{2\phi} \frac{(\sigma_{d_2}^2 - \sigma_{d_1}^2)}{(B_2 - B_1)}.
\]
For the newspapers’ second stage pricing decisions, we optimize (3) with respect to $P_1$ and $P_2$ which yields the following first order conditions:

\begin{align*}
(A.85) \quad \frac{M_1}{2b_0} \left( b_{indif} + b_0 - \frac{(\phi + \chi)}{2\phi^2} \frac{P_1}{(B_2-B_1)} \right) &= 0, \\
(A.86) \quad \frac{M_1}{2b_0} \left( b_0 - b_{indif} - \frac{(\phi + \chi)}{2\phi^2} \frac{P_2}{(B_2-B_1)} \right) &= 0.
\end{align*}

Substituting (A.84) in (A.85) and (A.86) and simultaneously solving for $P_1$ and $P_2$ we get equilibrium subscription fees as functions of the newspapers locations:

\begin{align*}
(A.87) \quad P_1 &= \frac{(\phi \chi)(\sigma_{d_2}^2 - \sigma_{d_1}^2)}{3(\phi + \chi)} + \frac{\phi^2(B_2-B_1)}{(\phi + \chi)} \left( 2b_0 + \frac{B_1 + B_2}{3} \right), \\
(A.88) \quad P_2 &= \frac{-(\phi \chi)(\sigma_{d_2}^2 - \sigma_{d_1}^2)}{3(\phi + \chi)} + \frac{\phi^2(B_2-B_1)}{(\phi + \chi)} \left( 2b_0 - \frac{B_1 + B_2}{3} \right).
\end{align*}

Substituting these second stage equilibrium strategies $P_i(B_i, B_j)$ into (3) we obtain the first stage payoff functions $V_i(B_i, B_j)$. Differentiating with respect to the locations yields from the Envelope Theorem:

\begin{align*}
(A.89) \quad \frac{\partial V_i}{\partial B_i} = \frac{\partial \pi_i}{\partial B_i} + \frac{\partial \pi_i}{\partial P_j} \frac{\partial P_j}{\partial B_i} &= 0 \quad \text{for } i, j = 1, 2, \ i \neq j.
\end{align*}

It follows from (A.89) that,

\begin{align*}
(A.90) \quad \frac{\partial b_{indif}}{\partial B_1} + \frac{\partial b_{indif}}{\partial P_2} \frac{\partial P_2}{\partial B_1} &= 0, \\
(A.91) \quad -\frac{\partial b_{indif}}{\partial B_2} - \frac{\partial b_{indif}}{\partial P_1} \frac{\partial P_1}{\partial B_2} &= 0.
\end{align*}

Using (A.87), (A.88) and (A.84) to find the terms of (A.90) and (A.91) and solving them simultaneously for the newspapers’ first stage location choices yields:

\begin{align*}
(A.92) \quad B_1 &= -\frac{3}{2} b_0 + \frac{\chi(\sigma_{d_2}^2 - \sigma_{d_1}^2)}{6\phi b_0}, \\
(A.93) \quad B_2 &= \frac{3}{2} b_0 + \frac{\chi(\sigma_{d_2}^2 - \sigma_{d_1}^2)}{6\phi b_0}.
\end{align*}
The equilibrium locations derived illustrate that Newspaper 1, which has access to more accurate data, introduces less bias in reporting and the opposite is true about Newspaper 2, which has less precise data available. Hence, the utility formulation introduces some tradeoff between vertical differentiation (in our case precision of data) and horizontal differentiation (in our case bias in reporting). There may be different reasons why a newspaper has access to more accurate data, including lower cost of conducting investigations due to greater experience in investigative reporting. Hence, if $\sigma_{d_i}^2$ can be chosen endogenously, the newspaper facing lower cost will likely choose greater accuracy in gathering information.
Proof of Lemma 1:

In order to support segmentation the expression inside the radicals in (7) and (8) (the discriminants of the quadratic equations) have to be positive. From (8) for

$$\sqrt{(B_2^{E,E} + b_0)^2 - \frac{(4-\alpha)(p_2^{E,E} - K_2^{E,E})}{\alpha} \frac{(\phi + \chi)}{\phi^2}} > 0,$$

we need, therefore:

(B.1) \((4 - \alpha)\hat{b}_2^{E,E} - 2B_2^{E,E} + (2 - \alpha)b_0 > 0.\)

Using (16) in (B.1), the last inequality is satisfied when

$$\frac{(4-a) - \sqrt{(4-a)^2 - 6a + 3a^2}}{3a} b_0 < \hat{b}_2^{E,E} < \frac{(4-a) + \sqrt{(4-a)^2 - 6a + 3a^2}}{3a} b_0.$$ Notice that

$$\frac{(4-a) - \sqrt{(4-a)^2 - 6a + 3a^2}}{3a} b_0 > 0 \quad \text{and} \quad \frac{(4-a) + \sqrt{(4-a)^2 - 6a + 3a^2}}{3a} b_0 > b_0 \quad \text{for} \quad 0 < \alpha < 1. \quad \text{Thus,}$$

$$\frac{(4-a) - \sqrt{(4-a)^2 - 6a + 3a^2}}{3a} b_0 < \hat{b}_2^{E,E} < b_0.$$ The restrictions for Newspaper 1 are similarly found.

Thus, \(\frac{(4-a) - \sqrt{(4-a)^2 - 6a + 3a^2}}{3a} b_0 \equiv \hat{b}_E^{E,E} < \hat{b}_{E,E} < \overline{b}_{E,E} \equiv b_0.\)

We define \(H(\hat{b}_{E,E})\) as the function obtained by multiplying \(\Delta U_P^\ast(\hat{b}_{E,E}, \hat{b}_{E,E})\) by \(\frac{(\phi + \chi)}{2\phi^2},\)

then: \(H(\hat{b}_{E,E}) \equiv \frac{(\phi + \chi)}{2\phi^2} \Delta U_P^\ast(\hat{b}_{E,E}, \hat{b}_{E,E}) = \frac{\alpha}{8} \left(2\hat{b}_{E,E}(4B_{E,E} + \alpha b_0) + (4 - \alpha)\left(-3\hat{b}_{E,E}^2 + b_0^2\right)\right) - T = 0.\) Note that for \(\alpha \in [0,1],\) \(H(\cdot)\) is a concave function of \(\hat{b}_{E,E},\) implying that it can change sign from positive to negative at most once. To guarantee that a root to the equation
\( H(\hat{b}^{E,E}) = 0 \) exists, we investigate whether \( H(\cdot) \) changes its sign from positive to negative over the interval \((\hat{b}^{E,E}, \overline{b}^{E,E})\). Specifically, whether \( H(\hat{b}^{E,E}) > 0 \) and \( H(\overline{b}^{E,E}) < 0 \). Evaluating the function \( H(\cdot) \) at \( \hat{b}^{E,E} \) and \( \overline{b}^{E,E} \) yields:

\[
B.2 \quad H(\hat{b}^{E,E}) = \left(\frac{4-\alpha}{2(\alpha+2)-\sqrt{(4-\alpha)^2-3\alpha(2-\alpha)}}\right)^2 b_0^2 - T, \quad \text{and}
\]

\[
B.3 \quad H(\overline{b}^{E,E}) = \frac{\alpha}{2} (\alpha + 1) b_0^2 - T.
\]

Requiring that \( H(\hat{b}^{E,E}) > 0 \) and \( H(\overline{b}^{E,E}) < 0 \) yields:

\[
LB_T \equiv \frac{\alpha}{2} (\alpha + 1) b_0^2 < T < \frac{(4-\alpha)(2(\alpha+2)-\sqrt{(4-\alpha)^2-3\alpha(2-\alpha)})}{72\alpha} b_0^2 \equiv UB_T. \quad LB_T < UB_T \text{ if and only if } 0 < \alpha < \alpha^* \approx 0.376.
\]

Note that the function \( H(\cdot) \) is concave because

\[
\frac{\partial^2 H(\hat{b}^{E,E})}{\partial (\hat{b}^{E,E})^2} = \frac{\alpha}{4} \left( \frac{18\alpha}{b_0} \hat{b}^{E,E} - 3(4 - \alpha) \right) < 0 \quad \text{when } \alpha < \alpha^* \text{ and } \hat{b}^{E,E} < b_0.
\]

It obtains its maximum value in the range \([0, b_0]\) at \( \hat{b}^{E,E}_{\text{max}} \equiv \frac{(4-\alpha)-\sqrt{(4-\alpha)^2-8\alpha(3-\alpha)}}{6\alpha} b_0 \). It is easy to show that \( \hat{b}^{E,E} < \frac{b_0}{2} < \hat{b}^{E,E}_{\text{max}} < b_0 \). Because \( H(\hat{b}^{E,E}) > 0 \) from Lemma 1, it follows that \( H(\hat{b}^{E,E}_{\text{max}}) > 0 \) as well. The threshold reader who is indifferent between the print and the online editions satisfies the equation \( H(\hat{b}^{E,E}) = 0 \). It follows, therefore, that the root of the last equation is bigger than \( \frac{b_0}{2} \).

**Proof of Proposition 1:**

From (9) and (10) optimizing with respect to the subscription prices \( P_1^{E,E}, K_1^{E,E}, P_2^{E,E}, \) and \( K_2^{E,E} \) yields the following first order conditions:

\[
B.4 \quad \frac{\partial \pi_1^{E,E}}{\partial K_1^{E,E}} = \frac{1}{2b_0} \left( \left( \frac{\partial \hat{b}^{E,E}}{\partial K_1^{E,E}} \right) \left( (K_1^{E,E} - c\delta) - (P_1^{E,E} - c) \right) + \left( b_0 + \hat{b}_1^{E,E} \right) \right) = 0,
\]
(B.5) \[ \frac{\partial \pi_{1,EE}}{\partial P_1^{EE}} = \frac{1}{2b_0} \left( \left( \frac{\partial b_1^{EE}}{\partial P_1^{EE}} \right) \left( K_1^{EE} - c \delta \right) - (P_1^{EE} - c) \right) + \frac{\partial b_{indiff}^{EE}}{\partial P_1^{EE}} (P_1^{EE} - c) \right) = 0, \]

(B.6) \[ \frac{\partial \pi_{2,EE}}{\partial K_2^{EE}} = \frac{1}{2b_0} \left( - \left( \frac{\partial b_2^{EE}}{\partial K_2^{EE}} \right) \left( K_2^{EE} - c \delta \right) - (P_2^{EE} - c) \right) + (b_0 - \hat{b}_2^{EE}) \right) = 0, \]

(B.7) \[ \frac{\partial \pi_{2,EE}}{\partial P_2^{EE}} = \frac{1}{2b_0} \left( \left( \frac{\partial b_2^{EE}}{\partial P_2^{EE}} \right) \left( K_2^{EE} - c \delta \right) - (P_2^{EE} - c) \right) - \left( \frac{\partial b_{indiff}^{EE}}{\partial P_2^{EE}} \right) (P_2^{EE} - c) + \right) = 0. \]

According to (7) \( \frac{\partial b_1^{EE}}{\partial K_1^{EE}} = - \frac{\partial b_1^{EE}}{\partial P_1^{EE}}. \) Using this relation and summing (B.4) and (B.5), we get:

(B.8) \[ P_1^{EE} - c = - \left( b_0 + b_{indiff}^{EE} \right). \]

Substituting (5) into (B.8) we obtain:

(B.9) \[ P_1^{EE} = \frac{c}{2} - \frac{\phi^2}{\phi + \chi} \left( B_1^{EE} - B_2^{EE} \right) (b_0 + \frac{B_2^{EE} + B_1^{EE}}{2}) + \frac{P_2^{EE}}{2}. \]

Using a similar approach, the subscription fee for the print edition of Newspaper 2 can be derived as:

(B.10) \[ P_2^{EE} = \frac{c}{2} - \frac{\phi^2}{\phi + \chi} \left( B_1^{EE} - B_2^{EE} \right) (b_0 - \frac{B_2^{EE} + B_1^{EE}}{2}) + \frac{P_2^{EE}}{2}. \]

Solving (B.9) and (B.10) for \( P_1^{EE} \) and \( P_2^{EE} \) we get the expressions in (11). Rewriting (B.4) as

\[ \left( K_1^{EE} - c \delta \right) - (P_1^{EE} - c) = - \left( \frac{b_1^{EE} + b_0}{\partial \pi_{1,EE}} \right), \]and substituting

\[ \frac{\partial b_1^{EE}}{\partial K_1^{EE}} = \frac{\phi + \chi}{\phi^2} \frac{1}{(ab_0 - 2(B_1^{EE} + b_0) + b_1^{EE}) (4 - \alpha)} \]from (7), we obtain:
Using a similar approach for Newspaper 2 we find:

\[(B.12) \quad (K_2^{EE} - c\delta) - (P_2^{EE} - c) = \frac{\phi^2}{(\phi + \chi)} \frac{\alpha(b_0 - b_2^{EE})((2-\alpha)b_0 + b_2^{EE} - 2b_2^{EE})}{2}.\]

Substituting the equilibrium strategies back into the profit functions (9) and (10), we obtain the second stage profit function \(V_i^{EE}(B_i^{EE}, B_j^{EE})\). From the Envelope Theorem:

\[(B.13) \quad \frac{\partial V_i^{EE}}{\partial B_i^{EE}} = \frac{\partial \pi_i^{EE}}{\partial B_i^{EE}} + \frac{\partial \pi_i^{EE}}{\partial P_i^{EE}} \frac{\partial P_i^{EE}}{\partial B_i^{EE}} = 0,\]

\[(B.14) \quad \frac{\partial V_j^{EE}}{\partial B_j^{EE}} = \frac{\partial \pi_j^{EE}}{\partial B_j^{EE}} + \frac{\partial \pi_j^{EE}}{\partial P_j^{EE}} \frac{\partial P_j^{EE}}{\partial B_j^{EE}} = 0.\]

To illustrate the derivation of the equilibrium location choices, we focus on the optimization of Newspaper 2. From (10): \(\frac{\partial V_2^{EE}}{\partial B_2^{EE}} = \frac{1}{2b_0}\)

\[(B.15) \quad \left(- \frac{\partial b_2^{EE}}{\partial B_2^{EE}} \left((K_2^{EE} - c\delta) - (P_2^{EE} - c)\right) - \left(\frac{\partial b_{indiff}^{EE}}{\partial P_2^{EE}} \frac{\partial P_2^{EE}}{\partial B_2^{EE}}\right) (P_2^{EE} - c)\right) = 0\]

Using \(\frac{\partial b_{indiff}^{EE}}{\partial B_2^{EE}} = \frac{1}{2} + \frac{B_2^{EE} + B_1^{EE}}{3(B_2^{EE} - B_1^{EE})}\) and \(\frac{\partial b_2^{EE}}{\partial B_2^{EE}} = \frac{2}{(4-\alpha)} \left(1 + \frac{2(B_2^{EE} + b_0)}{(2-\alpha)b_0 + b_2^{EE} - 2b_2^{EE}}\right)\), from (5), (8), and (11) in (B.15) yields:
\[
\frac{\partial V_{1,E,E}}{\partial B_{2,E,E}} = 0 = \\
(B.16) \frac{\phi^2}{2b_0(\phi + \chi)} \left( -(b_0 + \hat{b}_2^{E,E}) \alpha (b_0 - \hat{b}_2^{E,E}) - \left( \frac{(B_1^{E,E} - 3B_2^{E,E} + 6b_0)}{6} \right) \left( \frac{(B_1^{E,E} + B_2^{E,E})}{3} \right) \right) \frac{2b_0}{13}.
\]

At the symmetric equilibrium, \(-B_1^{E,E} = B_2^{E,E}\), therefore, we can replace \(-B_1^{E,E}\) with \(B_2^{E,E}\) in (B.16) and solve for \(B_2^{E,E}\):

\[
(B.17) -B_1^{E,E} = B_2^{E,E} = \frac{3}{4b_0} \left( (2 - \alpha)b_0^2 + \alpha(\hat{b}_2^{E,E})^2 \right) = \frac{3}{2} b_0 - \frac{3\alpha}{4b_0} \left( b_0^2 - (\hat{b}_2^{E,E})^2 \right).
\]

Notice that \(\hat{b}_2^{E,E} < b_0\) implies \(-B_1^{E,E} = B_2^{E,E} \leq B^{E,E} < B^{NE,NE} = \frac{3}{2} b_0\). Further,

\[
(B^o)_{E,E} = B^{E,E} + \alpha \left( \frac{b_0 + \hat{b}_2^{E,E}}{2} \right) = \frac{3(2-\alpha)b_0}{4} + \frac{3\alpha(\hat{b}_2^{E,E})^2}{4b_0} + \alpha \left( \frac{b_0 + \hat{b}_2^{E,E}}{2} \right)
\]

\[
> \frac{3(2-\alpha)b_0}{4} + \frac{3\alpha b_0}{16} + \frac{3\alpha b_0}{16} = \frac{3(\beta + \alpha)b_0}{16} \geq B^{NE,NE} \geq \frac{3b_0}{2}, \text{ since } \hat{b}_2^{E,E} > b_0/2. \text{ Finally,}
\]

\[
\frac{b_0^{E,E} \hat{b}_2^{E,E} + (b_0^{E,E})(b_0 - \hat{b}_2^{E,E})}{b_0} = B^{E,E} + \left( \frac{b_0 - \hat{b}_2^{E,E}}{b_0} \right) \left( \alpha \frac{b_0 + \hat{b}_2^{E,E}}{2} \right)
\]

\[
= \frac{3b_0}{2} - \frac{\alpha}{4b_0} \left( b_0^2 - (\hat{b}_2^{E,E})^2 \right) < B^{NE,NE} = \frac{3b_0}{2}, \text{ since } b_0 > \hat{b}_2^{E,E}.
\]

---

\[\text{\footnotesize\textsuperscript{13} Note that in a similar fashion the first order condition with respect to } B_1^{E,E} \text{ can be derived as:}\]

\[
\frac{\partial V_{1,E,E}}{\partial B_{1,E,E}} = 0 = \frac{\phi^2}{2b_0(\phi + \chi)} \left( -(b_0 - \hat{b}_1^{E,E}) \alpha (b_0 + \hat{b}_1^{E,E}) + \left( \frac{3B_1^{E,E} - B_2^{E,E} + 6b_0}{6} \right) \left( \frac{(B_1^{E,E} + 2^{E,E})}{3} + 2b_0 \right) \right).
\]
Proof of Proposition 2:

First observe from (16) that \( \frac{\partial B_{E,E}}{\partial \hat{b}_{E,E}} = \frac{3a\hat{b}_{E,E}}{2b_0} > 0 \). From the proof of Lemma 1 we know \( \frac{H(\hat{b}_{E,E})}{\partial \hat{b}_{E,E}} < 0 \) when \( H(\hat{b}_{E,E}) = 0 \), given that \( H(\cdot) \) changes its sign from positive to negative at this point. Using the Implicit Function Theorem we can write:

\[
\frac{\partial \hat{b}_{E,E}}{\partial \zeta} = -\frac{\partial H(\hat{b}_{E,E})}{\partial \hat{b}_{E,E}}, \quad \zeta = b_0, \text{ or } \alpha. \quad \text{It is immediate from the expression derived for } H(\cdot) \text{ that}
\]

\[
gn \left\{ \frac{\partial \hat{b}_{E,E}}{\partial b_0} \right\} = sgn \left\{ \frac{\partial H(\hat{b}_{E,E})}{\partial b_0} \right\} = sgn \left\{ \alpha \frac{\phi^2(-3\alpha(\hat{b}_{E,E})^3 + 2(3-\alpha)\hat{b}_{E,E}b_0^2 + (4-\alpha)b_0^3)}{2b_0^5(\phi + \chi)} \right\} > 0. \quad \text{Thus, using}
\]

\[
\frac{\partial B_{E,E}}{\partial b_0} = \frac{3(2-\alpha)}{2} - \frac{3}{4b_0^2} \left( (2-\alpha)b_0^2 + \alpha(\hat{b}_{E,E})^2 \right) + \frac{3a\hat{b}_{E,E}}{2b_0} \frac{\partial \hat{b}_{E,E}}{\partial b_0} = \frac{3(2-\alpha)}{4} - \frac{3\alpha(\hat{b}_{E,E})^2}{4b_0^2} + \frac{3a\hat{b}_{E,E}}{2b_0} \frac{\partial \hat{b}_{E,E}}{\partial b_0} > 0.
\]

Again from (16), \( \frac{\partial B_{E,E}}{\partial \alpha} = \frac{3}{4b_0} \left( 2a\hat{b}_{E,E} \frac{\partial \hat{b}_{E,E}}{\partial \alpha} + (\hat{b}_{E,E})^2 - b_0^2 \right) \), which implies \( \frac{\partial \hat{b}_{E,E}}{\partial \alpha} = \frac{3}{4b_0} \left( \frac{\partial H(\hat{b}_{E,E})}{\partial \hat{b}_{E,E}} \right)^{-1} + \left( (\hat{b}_{E,E})^2 - b_0^2 \right) \). Substituting for \( \frac{\partial H(\hat{b}_{E,E})}{\partial \hat{b}_{E,E}} \) and \( \frac{\partial H(\hat{b}_{E,E})}{\partial \alpha} \) into this expression, we obtain:

\[
(B.18) \quad \frac{\partial B_{E,E}}{\partial \alpha} = -\frac{3}{4b_0} \left( 3a(\hat{b}_{E,E})^4 + 3a(\hat{b}_{E,E})^3 b_0 + (3(2+\alpha)\hat{b}_{E,E} - (2+\alpha)b_0)\hat{b}_{E,E}b_0^2 + 2(3-\alpha)b_0^3(\hat{b}_{E,E} - b_0) \right). \quad \text{Notice that the denominator in (B.18) is negative since } \frac{\partial H(\hat{b}_{E,E})}{\partial \hat{b}_{E,E}} < 0. \text{ In the numerator,}
\]

\[
3(2+\alpha)\hat{b}_{E,E} - (2+\alpha)b_0 > 0 \text{ since } \hat{b}_{E,E} > \frac{b_0}{2}. \text{ Thus, } \frac{\partial B_{E,E}}{\partial \alpha} > 0. \text{ As well, } sgn \left\{ \frac{\partial \hat{b}_{E,E}}{\partial \alpha} \right\} = sgn \left\{ \frac{\partial H(\hat{b}_{E,E})}{\partial \alpha} \right\} > 0 \text{ in the region } \hat{b}_{E,E} \in \left( \frac{b_0}{2}, b_0 \right).
\]
Finally, \( \frac{\partial \pi_2^{E,E}}{\partial \alpha} = \frac{1}{2b_0} \left( - \frac{\partial \tilde{b}_2^{E,E}}{\partial \alpha} \left( (K_2^{E,E} - c\delta) - (P_2^{E,E} - c) \right) \right) < 0 \) since \( \frac{\partial \tilde{b}_2^{E,E}}{\partial \alpha} > 0 \)

and \( (K_2^{E,E} - c\delta) - (P_2^{E,E} - c) = \frac{\alpha(b_0 - \tilde{b}_2^{E,E})}{(2 - \alpha)b_0 + \tilde{b}_2^{E,E}(4 - \alpha) - 2b_2^{E,E}} > 0 \) given that \( \tilde{b}_2^{E,E} > \frac{b_0}{2} \).

**Proof of Proposition 3:**

Using a similar approach, as in section 3.3.1 of the study, the expressions for the threshold consumers \( b_{indif}^{NE,E} \) and \( \tilde{b}_2^{NE,E} \) as functions of the decisions made by the newspapers in the three stages of the game are:

\[
\begin{align*}
\text{(B.19)} & \quad b_{indif}^{NE,E} = \frac{(b_1^{NE,E} + b_2^{NE,E})}{2} + \frac{(P_2^{NE,E} - P_1^{NE,E})}{(B_2^{NE,E} - B_1^{NE,E})} \frac{\phi + \chi}{2\phi^2}, \\
\text{(B.20)} & \quad \tilde{b}_2^{NE,E} = \frac{2b_2^{NE,E} - (2 - \alpha)b_0}{(4 - \alpha)} + 2\sqrt{\frac{(b_2^{NE,E} + b_0)^2 - \frac{(4 - \alpha)(P_2^{NE,E} - K_2^{NE,E})}{(4 - \alpha)}}{(4 - \alpha)}} \frac{\phi + \chi}{\phi^2}. 
\end{align*}
\]

The payoff functions of the newspapers are:

\[
\begin{align*}
\text{(B.21)} & \quad \pi_1^{NE,E} = \frac{(b_{indif}^{NE,E} + b_0)}{2b_0} (P_1^{NE,E} - c), \\
\text{(B.22)} & \quad \pi_2^{NE,E} = \frac{b_0 - \tilde{b}_2^{NE,E}}{2b_0} (K_2^{NE,E} - c\delta) + \frac{(P_2^{NE,E} - b_{indif}^{NE,E})}{2b_0} (P_2^{NE,E} - c). 
\end{align*}
\]

From (B.21) and (B.22) optimizing with respect to \( P_i^{NE,E}, i = 1,2 \), and \( K_2^{NE,E} \), yields expressions similar to those derived when both newspapers extend their product lines.

Specifically, similar expressions to (11) and (B.12), as follows:

\[
\begin{align*}
\text{(B.23)} & \quad P_1^{NE,E} - c = \frac{\phi^2}{(\phi + \chi)} \left( B_2^{NE,E} - B_1^{NE,E} \right) \left( \frac{(b_1^{NE,E} + b_2^{NE,E})}{3} + 2b_0 \right),
\end{align*}
\]
Substituting the equilibrium strategies back into the profit functions (B.21) and (B.22), we obtain the second stage profit function $V_{i}^{NE,E}(B_{i}^{NE,E}, B_{j}^{NE,E})$. Differentiating with respect to the locations yields from the Envelope Theorem that:

(B.26) \[ \frac{\partial V_{i}^{NE,E}}{\partial B_{1}^{NE,E}} + \frac{\partial V_{i}^{NE,E}}{\partial B_{2}^{NE,E}} = \frac{1}{2b_0} \left( \frac{\partial b_{indiff}^{NE,E}}{\partial B_{1}^{NE,E}} + \frac{\partial b_{indiff}^{NE,E}}{\partial B_{2}^{NE,E}} \right) (P_{1}^{NE,E} - c) = 0, \]

(B.27) \[ \frac{\partial V_{2}^{NE,E}}{\partial B_{1}^{NE,E}} + \frac{\partial V_{2}^{NE,E}}{\partial B_{2}^{NE,E}} = \frac{1}{2b_0} \left( -\frac{\partial b_{2}^{NE,E}}{\partial B_{1}^{NE,E}} \right) (K_{2}^{NE,E} - c\delta) - (P_{2}^{NE,E} - c) \]

Substituting (B.23) and (B.24) in (B.19) yields:

(B.28) \[ \frac{\partial b_{indiff}^{NE,E}}{\partial B_{1}^{NE,E}} = \frac{(-5B_{1}^{NE,E} + B_{2}^{NE,E})}{6(B_{2}^{NE,E} - B_{1}^{NE,E})}. \]

From (B.24):

(B.29) \[ \frac{\partial P_{1}^{NE,E}}{\partial B_{1}^{NE,E}} = \frac{\phi^2}{(\phi + \chi)} \left( \frac{2B_{1}^{NE,E}}{3} - 2b_0 \right). \]

Substituting (B.23), (B.28) and (B.29) into (B.26) we obtain:

(B.30) \[ B_{1}^{NE,E} = \frac{B_{2}^{NE,E}}{3} - 2b_0. \]
According to (B.19) \( \frac{\partial b_{indf}^{NE,E}}{\partial p_2^{NE,E}} = - \frac{\partial b_{indf}^{NE,E}}{\partial p_1^{NE,E}} \). Substituting in (B.27) the last relation, (B.24), (B.25), the fact that from (B.19) \( \frac{\partial b_{indf}^{NE,E}}{\partial p_1^{NE,E}} = \frac{(5b_{2,NE,E} - b_{1,NE,E})}{6(b_{1,NE,E} - b_{1,NE,E})} \) and \( \frac{\partial b_{indf}^{NE,E}}{\partial p_1^{NE,E}} = \frac{1}{2\phi^2} \frac{1}{(b_{2,NE,E} - b_{1,NE,E})} \), and from (B.23) \( \frac{\partial p_1^{NE,E}}{\partial B_{2,NE,E}} = \frac{2\phi^2}{(\phi + \chi)} \left( \frac{b_{2,NE,E} + 3b_0}{3} \right) \) yields a quadratic equation in \( B_{2,NE,E} \) as follows:

(B.31) \( - \frac{\phi + \chi}{2\phi^2} \left( \frac{1}{b_{2,NE,E} - b_{1,NE,E}} \right) + \frac{1}{162b_0(\phi + \chi)} \left( (16B_{2,NE,E})^2 - 120B_{2,NE,E}b_0 + 81\alpha \left( \hat{b}_{2,NE,E} \right)^2 + 9(16 - 9\alpha)b_0^2 \right) = 0. \)

Solving (B.31) for \( B_{2,NE,E} \) and choosing the root to ensure that \( \frac{\partial b_{2,NE,E}}{\partial b_{2,NE,E}} > 0 \), we obtain \( B_{2,NE,E} \) as given in (18). Substituting (18) in (B.30), we obtain \( B_{1,NE,E} \) as expressed in (17). From these solutions, it follows that \( B_{1,NE,E} < \frac{3}{2}b_0 \), and \( B_{2,NE,E} < \frac{3}{2}b_0 \), since \( \hat{b}_{2,NE,E} < b_0 \).

In order to demonstrate that \( B_{2,NE,E} < B_{E,E} \), we will first show that \( \hat{b}_{2,NE,E} < \hat{b}_{E,E} \). Note that the solution for \( \hat{b}_{2,NE,E} \) can be obtained implicitly as in the proof of Lemma 1 as follows:

\[
G \left( \hat{b}_{2,NE,E} \right) \overset{\text{def}}{=} \frac{\alpha}{8} \left( 2\hat{b}_{2,NE,E} \left( 4B_{2,NE,E} + \alpha b_0 \right) + (4 - \alpha) \left( -3 \left( \hat{b}_{2,NE,E} \right)^2 + b_0^2 \right) \right) - T = 0, \quad \text{(B.32)}
\]

where \( B_{2,NE,E} \) is expressed in terms of \( \hat{b}_{2,NE,E} \) as in (18). Notice that

\[
B_{E,E} = \frac{3}{4b_0} \left( \alpha \left( \hat{b}_{E,E} \right)^2 + (2 - \alpha)b_0^2 \right) > \frac{3}{4} \left( 5b_0 - 3 \sqrt{-\alpha \left( \hat{b}_{E,E} \right)^2 + (1 + \alpha)b_0^2} \right).
\]

Hence using the definition of function \( H(\cdot) \) from the proof of Lemma 1, \( 0 = H(\hat{b}_{E,E}) > G(\hat{b}_{E,E}) \). Since the function defined in (B.32) is negative when evaluated at \( \hat{b}_{E,E} \) and it should be equal to zero at \( \hat{b}_{2,NE,E} \), it follows that \( \hat{b}_{2,NE,E} < \hat{b}_{E,E} \). As a result, from (18) \( B_{2,NE,E} < \frac{3}{4} \left( 5b_0 - 3 \sqrt{-\alpha \left( \hat{b}_{E,E} \right)^2 + (1 + \alpha)b_0^2} \right) < B_{E,E} < \frac{3}{2}b_0. \)

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Finally, substituting (17), (18), (B.23) and (B.24) into (B.19) yields:

\[(B.33) \quad b_{\text{indif}}^{NE,E} = \frac{1}{2} \left( b_0 - \sqrt{-\alpha \left( \hat{b}_2^{NE,E} \right)^2 + (1 + \alpha) b_0^2} \right) < 0. \]

**Proof of Proposition 4:**

Substituting (17), (18), (B.23), and (B.33) into (B.21) yields:

\[(B.34) \quad \pi_1^{NE,E} = \frac{3\phi^2}{8b_0(\phi+\chi)} \left( 3b_0^2 - \sqrt{-\alpha \left( \hat{b}_2^{NE,E} \right)^2 + (1 + \alpha) b_0^2} \right)^3. \]

Similarly, substituting (14) and (16) into (9) we get:

\[(B.35) \quad \pi_1^{E,E} = \frac{\phi^2}{(\phi+\chi)} \left( 3b_0^2 - \frac{\alpha(b_0-\hat{b}_{E,E})(-3\alpha(\hat{b}_{E,E})^3+(8+\alpha)b_0(\hat{b}_{E,E})^2+(2+3\alpha)b_0^2\hat{b}_{E,E}+(14-\alpha)b_0^3)+3b_0\left(3b_0-\sqrt{-\alpha(\hat{b}_{E,E})^2+(1+\alpha)b_0^2}\right)^3}{8b_0^2} \right). \]

From (B.35) \( \pi_1^{E,E} < \pi_1^{NE,NE} = \frac{\phi^2}{(\phi+\chi)} 3b_0^2 \), and since the firms are symmetric \( \hat{b}_2^{NE,E} < \hat{b}_2^{NE,NE} \) as well. Since from the proof of Proposition 3, \( \hat{b}_2^{NE,E} < \hat{b}_{E,E} \), it follows that:

\[(B.36) \quad \pi_1^{E,E} - \pi_1^{NE,E} > \]

\[\phi^2 \left( 3b_0^2 - \frac{\alpha(b_0-\hat{b}_{E,E})(-3\alpha(\hat{b}_{E,E})^3+(8+\alpha)b_0(\hat{b}_{E,E})^2+(2+3\alpha)b_0^2\hat{b}_{E,E}+(14-\alpha)b_0^3)+3b_0\left(3b_0-\sqrt{-\alpha(\hat{b}_{E,E})^2+(1+\alpha)b_0^2}\right)^3}{8b_0^2} \right) \]

The second term subtracted inside the parenthesis of (B.36) is:

\[(B.37) \quad \frac{\alpha(b_0-\hat{b}_{E,E})(-3\alpha(\hat{b}_{E,E})^3+(8+\alpha)b_0(\hat{b}_{E,E})^2+(2+3\alpha)b_0^2\hat{b}_{E,E}+(14-\alpha)b_0^3)+3b_0\left(3b_0-\sqrt{-\alpha(\hat{b}_{E,E})^2+(1+\alpha)b_0^2}\right)^3}{8b_0^2}. \]
(B.37) decreases with \( \hat{b}^{E,E} \). Therefore at \( \hat{b}^{E,E} = 0 \), it obtains its maximum value of

\[
\frac{a(14-\alpha)+3(3-\sqrt{1+\alpha})}{8} b_0^2 \text{ which is less than } 3 b_0^2 \text{ when } \alpha < 0.376. \text{ Thus, } \pi_1^{E,E} - \pi_1^{NE,E} > 0.
\]

Note that \( \pi_2^{NE,E} = \frac{(b_0-\hat{b}_2^{NE,E})(K_2^{NE,E} - c \delta)}{2b_0} + \frac{(\hat{b}_2^{E,E}-b_{indiff}^{NE,E})}{2b_0} (P_2^{NE,E} - c) \)

\[
> \frac{(b_0-\hat{b}_2^{NE,E})}{2b_0} (P_2^{NE,E} - c) + \frac{(b_2^{NE,E}-b_{indiff}^{NE,E})}{2b_0} (P_2^{NE,E} - c) = \frac{(b_0-\hat{b}_{indiff}^{NE,E})}{2b_0} (P_2^{NE,E} - c)
\]

Substituting (17), (18), (B.24), and (B.33) into \( \frac{(b_0-\hat{b}_{indiff}^{NE,E})}{2b_0} (P_2^{NE,E} - c) \) implies \( \pi_2^{NE,E} \)

\[
\phi^2 \frac{3\left(3b_0-\sqrt{\alpha(b_2^{NE,E})^2+(1+\alpha)b_0^2}\right)^2}{(\phi+\chi)^2} > \pi_2^{NE,E} = \frac{\phi^2}{(\phi+\chi)^2} \frac{3b_0^2}{8b_0}.
\]

**Proof of Corollary 1:**

Notice that when \( \alpha_i = 0, B_i = B_i^0 \). Therefore, in our formulation, choosing \( \alpha_i = 0 \) is equivalent to not expanding the product line. From Proposition 4 we know that when one newspaper does not expand, the other chooses to expand. Hence, the outcome \( \alpha_1 = \alpha_2 = 0 \) cannot correspond to an equilibrium. We also know from Proposition 4 that when one newspaper expands, the other chooses to expand as well. Therefore, in equilibrium each newspaper chooses a positive \( \alpha_i \).

**Proof of Proposition 5:**

For ease of exposition we drop the superscript \( E,E \) in all the variables. When newspaper \( i \) chooses \( B_i^0 \), the cutoff points \( \hat{b}_i \) are still given as in (6). However, because readers have no ability to affect \( B_i^0 \) via UGC, they consider \( B_i^0 \) exogenous. Specifically,

\[
(B.38) \quad \hat{b}_i = \frac{B_i^0 + B_i}{2} + \frac{p_i - K_i (\phi + \chi)}{B_i - B_i^0} \frac{2 \phi^2}{(\phi+\chi)^2}.
\]
The expression for $b_{\text{indif}}$ remains as in (5) and the objectives of the firms are still given in (9) and (10). Optimizing with respect to $P_i$ and $K_i$ in Stage 3 and solving in terms of $B_i$ and $B_i^o$ yields for $P_i$ a solution identical to (11) and for $K_i$:

$$K_1 - c\delta = P_1 - c + \left[ \frac{c(1-\delta)}{2} + \frac{\phi^2}{(\phi+\chi)} (B_1 - B_1^o) \left( b_0 + \frac{B_1 + B_1^o}{2} \right) \right],$$

$$K_2 - c\delta = P_2 - c + \left[ \frac{c(1-\delta)}{2} + \frac{\phi^2}{(\phi+\chi)} (B_2^o - B_2) \left( b_0 - \frac{B_2 + B_2^o}{2} \right) \right].$$

Substituting the expressions for $P_i$ and $K_i$ back into the expression for $\hat{b}_i$ in (38), yields that segmentation is feasible at the symmetric equilibrium, specifically at $0 < \hat{b}_2 < b_0$ if:

$$\frac{\phi^2}{(\phi+\chi)} (B_2^o - B_2) \left( b_0 + \frac{B_2^o + B_2}{2} \right) < \frac{c(1-\delta)}{2} < \frac{\phi^2}{(\phi+\chi)} (B_2^o - B_2) \left( b_0 + \frac{B_2^o + B_2}{2} \right).$$

This implies from (39) that $K_i - c\delta > P_i - c$. In the second stage, each newspaper chooses $B_i$ and $B_i^o$. For Newspaper $I$, differentiating (9) with respect to $B_1$ and $B_1^o$ yields:

$$\frac{d\pi_1}{dB_1} = \frac{1}{2b_0} \left\{ \left[ \frac{\partial b_{\text{indif}}}{\partial B_1} + \frac{\partial b_{\text{indif}}}{\partial P_2} \frac{\partial P_2}{\partial B_1} \right] (P_1 - c) + \left[ (K_1 - c\delta) - (P_1 - c) \right] \frac{\partial \hat{b}_1}{\partial B_1} \right\},$$

$$\frac{d\pi_1}{dB_1^o} = \frac{1}{2b_0} \left\{ \left[ (K_1 - c\delta) - (P_1 - c) \right] \frac{\partial \hat{b}_1}{\partial B_1^o} \right\},$$

where $b_{\text{indif}}$ is given in (5) and $\hat{b}_1$ in (38). Evaluating (41) and (42) at the symmetric equilibrium where $-B_1 = B_2$, $-B_1^o = B_2^o$, $b_{\text{indif}} = 0$, $-\hat{b}_1 = \hat{b}_2$, while using the equilibrium expressions for $P_i$ and $K_i$ from (11) and (39) yields:

$$\frac{d\pi_1}{dB_1} = \frac{1}{2b_0} \left\{ \left[ \frac{4}{3} B_2 b_0 - 2b_0^2 \right] + \frac{1}{2} \left( b_0 - \frac{B_2^o + B_2}{2} \right) \left( \frac{B_2^o}{2} - \frac{3}{2} B_2 + b_0 \right) \right\} \frac{\phi^2}{(\phi+\chi)} + \left\{ \frac{c(1-\delta)}{2} - \frac{c^2(1-\delta)(\phi+\chi)}{8(B_2^o - B_2)^2 \phi^2} \right\},$$

$$\frac{d\pi_1}{dB_1^o} = \frac{1}{2b_0} \left[ (K_1 - c\delta) - (P_1 - c) \right] \left[ \frac{(\phi+\chi)c(1-\delta)}{4\phi^2 (B_2^o - B_2)^2} + \frac{4B_2^o - B_2 - b_0}{2(B_2^o - B_2)} \right].$$
Assuming an interior equilibrium with \( B_1^o < B_1 < 0 \) implies that \( \frac{d\pi_1}{dB_1^o} = 0 \), and since 

\[ (K_1 - c\delta) > (P_1 - c) \],

(B.44) \[ \frac{c(1-\delta)}{2} = \left( b_0 + \frac{B_2}{2} - \frac{3}{2} B_2^o \right) (B_2^o - B_2) \frac{\phi^2}{(\phi + \chi)}. \]

Substituting (B.44) into (B.43) yields that:

(B.45) \[ \frac{d\pi_1}{dB_1} = \frac{1}{2b_0} \left[ 2B_2^o b_0 - \frac{3}{2} B_2 b_0 - 2(B_2^o)^2 + 2B_2 B_2^o - 2b_0^2 \right] \frac{\phi^2}{(\phi + \chi)}. \]

To ensure segmentation, the lower bound on \( \frac{c(1-\delta)}{2} \) from (B.40) should hold, which combined with (B.44) implies that \( B_2^o < b_0 \). Using the last inequality in (B.45) implies that \( \frac{d\pi_1}{dB_1} < 0 \) for all values of \( B_2 \) and \( B_2^o \). Hence, Newspaper 1 will choose the lowest bias consistent with \( B_1^o \leq B_1 < 0 \), implying that \( B_1^o = B_1 \) and no segmentation arises. A similar argument holds also for Newspaper 2.

**Locations Chosen Before Expansion Decision:**

If the timing of the decisions changed so that \( B_i \) were chosen first followed by the decision on whether to offer the online editions, there are circumstances under which the present equilibrium \( (B_{E,E}, B_{E,E}) \) would prevail. We now demonstrate these circumstances.

First note that because of the discrete nature of the expansion decision in Stage 2, when a given newspaper contemplates unilaterally deviating from \(-B_{E,E}\) or \(B_{E,E}\) in Stage 1, it does not anticipate that the competing newspaper or itself will change their expansion decisions, unless the deviation is sufficiently big. As we show in Proposition 4, \( \pi_{i,E,E} > \pi_{i,NE,E} \) and \( \pi_{i,E,E} > \pi_{i,E,NE} \) when both newspapers choose focal points in the neighborhood of \(-B_{E,E}\) and \(B_{E,E}\). In order to make a deviation worthwhile it has to change the choice of the competitor or itself from E to NE.
in the second stage. Such a unilateral deviation of Newspaper 1, \(-B^{E,E}\) to \(\hat{B}\) should yield that \(\hat{b}_1\) in (7) is equal to \(-b_0\) or \(\hat{b}_2\) in (8) is equal to \(b_0\). From (7) if \(\hat{b}_1 = -b_0\), it follows that:

\[
(B.46) \quad \sqrt{(\hat{B} - b_0)^2 - \frac{(4-\alpha)}{\alpha} \left( P_1 - K_1 \right) (\phi + \chi) \hat{b}^2} = \hat{B} + (3 - \alpha)b_0,
\]

and from (12) \((P_1 - K_1) = c(1 - \delta)\) when \(\hat{b}_1 = -b_0\).

Substituting for \(P_1 - K_1\) in (B.46) and solving it for \(\hat{B}\) yields:

\[
(B.47) \quad -\hat{B} = \frac{b_0(2-\alpha)}{2} + \frac{T}{\alpha b_0}.
\]

To support the equilibrium we derive in the study, Lemma 1 asserts that:

\[
\frac{\alpha(\alpha+1)}{2} b_0^2 < T < \frac{(4-\alpha)(2(\alpha+2)-\sqrt{(4-\alpha)^2-3a(2-\alpha)}}{72 \alpha} b_0^2.
\]

Substituting the lower bound on \(T\) back into (B.47), yields that: \(-\hat{B} > \frac{3}{2} b_0\). Hence, in order to change its own expansion to NE Newspaper 1 will have to make its print edition more extreme. Next, we derive the condition necessary to make such a large deviation unprofitable. After deviating to \(\hat{B}\), by using (11) in (5) one can derive the new expression for the value of \(b_{indiff}\) as: \(b_{indiff} = \frac{B^{E,E} + \hat{B}}{6}\). Then from (9) and (11) the profits of Newspaper 1 after the deviation are:

\[
(B.48) \quad \hat{\pi}^{dev}_{1} = \frac{1}{2b_0} \left( \frac{B^{E,E} + \hat{B}}{6} + b_0 \right) \left[ \frac{\phi^2}{(\phi + \chi)} \left( B^{E,E} - \hat{B} \right) \left( \frac{B^{E,E} + \hat{B}}{3} + 2b_0 \right) \right],
\]

where \(\hat{B} = -\left( \frac{b_0(2-\alpha)}{2} + \frac{T}{\alpha b_0} \right)\) from (B.47).

From (B.35)

\[
\pi_{1}^{E,E} = \frac{\phi^2}{(\phi + \chi)} \left( 3b_0^2 - \alpha(b_0-B^{E,E})(-3a(b^{E,E})^3 + (8 + \alpha)b_0(b^{E,E})^2 + (2 + 3\alpha)b_0^2 b^{E,E} + (14 - \alpha)b_0^3) \right)
\]

and from (16) \(B^{E,E} = \frac{3}{4b_0} \left( \alpha(\hat{b}^{E,E})^2 + (2 - \alpha)b_0^2 \right)\).
Hence,

\[ b_{\text{Indif}} = \frac{b_{E,E} + \hat{B}}{6} = \frac{3\alpha(\hat{b}_{E,E})^2}{4b_0} + \frac{3(2-\alpha)b_0}{4} - \frac{(2-\alpha)b_0}{2} - \frac{T}{\alpha b_0}. \]

Substituting for \( b_{\text{Indif}} \) and \( B_{E,E} - \hat{B} \) into (B.48) yields:

\[ (B.49) \quad \hat{\pi}_{\text{dev}}^1 = \frac{\phi^2}{b_0(\phi + \chi)} \left[ \frac{3\alpha(\hat{b}_{E,E})^2}{4b_0} + \frac{(2-\alpha)b_0}{4} - \frac{T}{\alpha b_0} + b_0 \right]^2 \cdot \frac{3}{4} \alpha \left( \hat{b}_{E,E} \right)^2 + \frac{5(2-\alpha)b_0}{4}. \]

A comparison of (B.35) and (B.49), yields that there are values of \( \hat{b}_{E,E} \in \left[ b_0, \frac{b_0}{2} \right] \) under which \( \pi_{\text{dev}}^1 \hat{b}_{E,E} \), implying that the large deviation to \( \hat{B} \) is unprofitable for Newspaper 1.

Note that when \( B_i \) is chosen prior to the expansion decision of the newspapers, Newspaper 1 cannot induce \( \hat{b}_2 \) to be equal to \( b_0 \). According to (8), the value of \( \hat{b}_2 \) is only a function of \( B_2 \) and not \( B_1 \). Hence, for a fixed value of \( B_2 = B_{E,E} \), any unilateral deviation of Newspaper 1 will not change \( \hat{b}_2 \).

**Expansion of Readership Facilitated by Online Editions:** In this section we demonstrate that the profits of each newspaper may rise with the introduction of an online edition, if in the absence of offering such editions the market of readers is not fully covered. In Figure 7 we depict this possibility.
Less than full coverage implies that the expected utility of readers with very extreme political opinions is negative when newspapers offer only print editions, namely \( E[U_b] < 0 \) for \( b < \bar{b}_1 \) and \( > \bar{b}_2 \), and \( E[U_b] > 0 \) for \( \bar{b}_1 < b < \bar{b}_2 \). Readers located at \( \bar{b}_1 \) and \( \bar{b}_2 \) are just indifferent between buying the print edition of newspapers 1 and 2, respectively, and withdrawing from the market. Solving for \( \bar{b}_1 \) and \( \bar{b}_2 \) yields:

\[
\begin{align*}
\bar{b}_1 &= \frac{\phi B_1}{\phi + \chi} - \frac{\phi^2 B_1^2}{(\phi + \chi)^2} + \left[ \frac{\bar{u} - p_1}{\phi} - \frac{\phi B_1^2}{(\phi + \chi)^2} - \frac{\sigma^2 \chi}{(\phi + \chi)} \right], \\
\bar{b}_2 &= \frac{\phi B_2}{\phi + \chi} + \frac{\phi^2 B_2^2}{(\phi + \chi)^2} + \left[ \frac{\bar{u} - p_2}{\phi} - \frac{\phi B_2^2}{(\phi + \chi)^2} - \frac{\sigma^2 \chi}{(\phi + \chi)} \right].
\end{align*}
\]

Note that the expressions in the brackets included in the radicals of (B.50) are positive, since the expected utility of a reader located at \( b=0 \) is positive according to Figure 7. Hence, \( \bar{b}_1 < \frac{2\phi B_1}{\phi + \chi} \) and \( \bar{b}_2 > \frac{2\phi B_2}{\phi + \chi} \). The objectives of the two newspapers are:

\[
\begin{align*}
\max_{p_1, \tilde{b}_1} \pi_1 &= \frac{(b_{indf} - \bar{b}_1)(p_1 - c)}{2b_0}; \quad \max_{p_2, \tilde{b}_2} \pi_2 &= \frac{(\bar{b}_2 - b_{indf})(p_2 - c)}{2b_0}.
\end{align*}
\]
Optimizing with respect to $P_i$ yields at the symmetric equilibrium when $P_1 = P_2 = P$, $-B_1 = B_2 = B$, $b_{indif} = 0$, and $-\tilde{b}_1 = \tilde{b}_2 = \tilde{b}$ that:

$$\tag{B.52} p_{NENA}^{\text{less}} - c = \frac{4\phi^2 B \tilde{b} \left( \frac{\phi B}{\phi + \chi} \right)}{(\phi + \chi)^2 \left( \tilde{b} + \frac{\phi B}{\phi + \chi} \right)} \quad \text{and} \quad \pi_{\text{less}}^{\text{NENA}} = \frac{(p_{\text{less}}^{\text{NENA}} - c) \tilde{b}}{2b_0},$$

where the subscript $\text{less}$ in (B.52) indicates that the market is less than fully covered. It is possible to find an upper bound on the equilibrium profits in (B.52). Specifically,

$$\tag{B.53} \pi_{\text{less}}^{\text{NENA}} < 0.343 \phi \ b_0^2.$$

Now, assume that extending the product mix by introducing the online edition allows the newspapers to cover the entire market. Specifically, the expected utility of readers located at $-b_0$ and $b_0$ is strictly positive at the $\{E, E\}$ equilibrium. Hence, $E[U_{b_0}] > 0$ when reader $b_0$ is exposed to the bias $(B^o)^{E,E}$ and pays the fee $K^{E,E}$. From (B.35), it is possible to derive a lower bound on the expected profits of each newspaper for the region of $\alpha$ values that support segmentation, as specified in Lemma 1. Specifically,

$$\tag{B.54} \pi^{E,E} > \frac{2.6 \phi^2}{(\phi + \chi)} b_0^2.$$

A comparison of (B.53) with (B.54) implies that the expansion of the readership that is facilitated by the extension of the product mix will definitely increase the profits of each newspaper provided that $\chi < 6.58 \phi$, namely that readers are not overly concerned about inaccurate reporting. Note that this condition does not contradict the requirement for less than full coverage in the absence of segmentation. A necessary condition for the latter is that $\chi > 2\phi$.

Hence, there is a nonempty interval of values for the ratio $\frac{\chi}{\phi}$ that is consistent with the result that
introducing the online edition may increase the profits of the newspapers. This increase in profits is different from the result reported in Proposition 4, when the market was fully covered even in the absence of segmentation.

**An Additional Dimension of Heterogeneity among Readers:**

In this section we extend our model by allowing for a second dimension of heterogeneity among readers with respect to their preference for the print versus the online editions unrelated to political opinions. This preference may be related, for instance, to the age of the reader, with younger readers usually preferring the online edition, and older readers being more comfortable with the traditional, print edition. Specifically, we assume that the expected utility of a young reader increases by \( \xi_{young} \) and that of an older reader decreases by \( \xi_{old} \) when choosing the online edition. Modifying (3) we obtain:

\[
E[U_b]_{online} = \left\{ \begin{array}{ll}
\bar{u} - \frac{\phi^2}{\phi + \chi} \left[ (B_l^O - b)^2 \right] - \frac{\chi \phi}{\phi + \chi} \left[ b^2 + \sigma_d^2 \right] - K_l + \xi_{young} & \text{if reader is young}, \\
\bar{u} - \frac{\phi^2}{\phi + \chi} \left[ (B_l^O - b)^2 \right] - \frac{\chi \phi}{\phi + \chi} \left[ b^2 + \sigma_d^2 \right] - K_l - \xi_{old} & \text{if reader is old}.
\end{array} \right.
\]

With the above modified expected utility, the threshold reader who is indifferent between the online and print editions of a given newspaper is different for the young and old populations. Specifically, \( \hat{b}_1^{old} < \hat{b}_1^{young} \) and \( \hat{b}_2^{old} > \hat{b}_2^{young} \). For Newspaper 2, for instance, adjusting (8) yields:

\[
\hat{b}_2^{young} = \frac{2 B_2 - (2-a) b_0}{(4-a)} + \frac{2 \sqrt{(B_2^{NE,E} + b_0)^2 - \frac{(4-a)}{a} (P_2 - K_2 + \xi_{young}) \frac{\phi + \chi}{\phi^2}}}{(4-a)},
\]

\[
\hat{b}_2^{old} = \frac{2 B_2 - (2-a) b_0}{(4-a)} + \frac{2 \sqrt{(B_2^{NE,E} + b_0)^2 - \frac{(4-a)}{a} (P_2 - K_2 - \xi_{old}) \frac{\phi + \chi}{\phi^2}}}{(4-a)}.
\]

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We assume that the populations of young and old readers are still each uniformly distributed on the interval \([-b_0, b_0]\) according to their political opinions, and that these distributions are independent of age. The proportions of young and old in the general population of readers are \((1-q)\) and \(q\), respectively. We derive Proposition WA.1 from this modified model.

**PROPOSITION WA.1:** When there exists an additional dimension of reader heterogeneity, unrelated to political opinions, each newspaper chooses to intensify the bias of its print edition and the expected size of the online segment declines. Moreover, when the variability in the population that is unrelated to political opinions increases (i.e., when \(q(1-q)\) and \((\hat{b}_{2\text{old}} - \hat{b}_{2\text{young}})\) are bigger), the polarization of the newspapers becomes more significant. Polarization remains, however, more moderate than in an environment with only print editions.

**Proof:** We drop the superscript \(E,E\) to simplify the notation, and write the objective of Newspaper 2 as:

\[
\pi_2 = \frac{q}{2b_0} \left\{ \left( \hat{b}_{2\text{old}} - b_{\text{indiff}} \right) (P_2 - c) + \left( b_0 - \hat{b}_{2\text{old}} \right) (K_2 - c\delta) \right\} + \frac{(1-q)}{2b_0} \left\{ \left( \hat{b}_{2\text{young}} - b_{\text{indiff}} \right) (P_2 - c) + \left( b_0 - \hat{b}_{2\text{young}} \right) (K_2 - c\delta) \right\}.
\]

A similar expression can be derived for the objective of Newspaper 1. Using an approach similar to that leading to the first order condition (B.16), yields that the optimization with respect to \(B_2\) in the second stage can be expressed as:

\[
\left. \frac{dV_2}{d\hat{B}_2} \right|_{\text{symmetry}} = -\frac{\phi^2}{2b_0(\phi+\chi)} \left[ \alpha \left( b_0^2 - (E[\hat{b}_2])^2 \right) + \frac{4B_2b_0}{3} - 2b_0^2 \right] + \frac{\phi^2}{2b_0(\phi+\chi)} \left[ \frac{\alpha(b_0-E[\hat{b}_2])q(1-q)(\hat{b}_{2\text{old}} - \hat{b}_{2\text{young}})^2(\alpha-\alpha)}{(2-\alpha)b_0 - 2b_2 + (2-\alpha)(q\hat{b}_{2\text{young}} + (1-q)\hat{b}_{2\text{old}})} \right] = 0,
\]
where \( E[\hat{b}_2] = q\hat{b}_2^{old} + (1-q)\hat{b}_2^{young} \). The first term of (B.55) coincides with the first order condition (B.16) that was derived when politics was the only differentiating attribute among readers, with the only difference being that \( E[\hat{b}_2] \) replaces \( \hat{b}_2^{E,E} \) in (B.16). The second term is positive, and measures the extent of heterogeneity between the young and old populations. This second term is bigger when the variance due to the different ages in the general population is bigger (the product \( q(1-q) \)) and when the difference \( (\hat{b}_2^{old} - \hat{b}_2^{young}) \) is more significant (as implied by the different values of \( \xi^{young} \) and \( \xi^{old} \)). Evaluating (B.55) at the point when \( E[\hat{b}_2] = \hat{b}_2^{E,E} \) implies that \( \frac{dV_2}{dB_2} > 0 \), hence Newspaper 2 has to increase \( B_2 \) beyond \( B_2^{E,E} \) in order to satisfy the first order condition (B.55). Hence, bias intensifies, and since \( B_2 \) and \( E[\hat{b}_2] \) move in the same direction, \( E[\hat{b}_2] > \hat{b}_2^{E,E} \).

In addition note that

\[
- \frac{\phi^2\alpha(b_0 - E[\hat{b}_2])}{2b_0(\phi + \chi)} \left( b_0 + E[\hat{b}_2] \right) - \frac{q(1-q)(\hat{b}_2^{old} - \hat{b}_2^{young})^2}{(2-\alpha)b_0 - 2B_2(4-\alpha)(qb_2^{young} + (1-q)\hat{b}_2^{old})} < 0
\]

because the term inside the brackets is positive and bigger than \( b_0 \left[ \frac{3}{2} - \frac{1}{2(4-3\alpha)} \right] \), given that

\( E[\hat{b}_2] > \frac{b_0}{2}, (\hat{b}_2^{old} - \hat{b}_2^{young}) < \frac{b_0}{2}, \) and \( B_2 > b_0 \). As a result, to satisfy (B.55) \( \frac{4b_2b_0}{3} - 2b_0^2 < 0 \), implying that at the equilibrium \( B_2 \) is still smaller than \( \frac{3}{2}b_0 \). In addition, the expected equilibrium profits are still smaller when both editions are offered given that the newspapers would not choose segmentation according to politics if they had full control over the attributes of both editions.
The result reported in Proposition WA.1 is consistent with that reported in Proposition 5. According to Proposition 5, if newspapers could fully control the bias of their online editions, they would choose it to be identical to the bias of their print editions. Once some of this control is transferred to readers via UGC, the political segmentation of readers leads to intensified bias of the online version and reduced bias of the print version. However, if there is additional heterogeneity among readers that is unrelated to politics, newspapers can move closer to the outcome they would choose if they had full control over the characteristics of the online editions. Specifically, while the bias of the print version $B^{E,E}$ remains smaller than $\frac{3}{2} b_0$, it moves closer to this value. As well, the segment of consumers who choose the more biased online editions shrinks. Moreover, as the variability in the population that is unrelated to political opinions increases, the equilibrium moves closer to that described in Proposition 5, when newspapers have the exclusive right to choose the online bias.
BIBLIOGRAPHY


