An Framework of Efficient Hybrid Model and Optimal Control for Multihop Wireless Networks

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ABSTRACT

The performance of multihop wireless networks (MWN) is normally studied via simulation over a fixed time horizon using a steady-state type of statistical analysis procedure. However, due to the dynamic nature of network connectivity and nonstationary traffic, such an approach may be inappropriate as the network may spend most time in a transient/nonstationary state. Moreover, the majority of the simulators suffer from scalability issues. In this work, we presents a performance modeling framework for analyzing the time varying behavior of MWN. Our framework is a hybrid model of time varying connectivity matrix and nonstationary network queues. Network connectivity is captured using stochastic modeling of adjacency matrix by considering both wireless link quality and node mobility. Nonstationary network queues behavior are modeled using fluid flow based differential equations. In terms of the computational time, the hybrid fluid-based model is a more scalable tool than the standard simulator. Furthermore, an optimal control strategy is proposed on the basis of the hybrid model.

Categories and Subject Descriptors

C.4 [**Computer-Communication Networks**]: Performance of Systems—*Modeling techniques*

General Terms

Performance, Theory

Keywords

Nonstationary, fluid flow model, optimal control

1. INTRODUCTION

In multihop wireless networking research, while experimentations have taken off during the last years, we are still relying on simulations for examining the performance of various protocols on large scale networks. Nevertheless, many of the existing simulation tools are known to be lacking scalability. Another weakness of most simulation studies of multihop wireless networks are that steady state analysis are used even though transient or nonstationary periods will occur often and likely dominant the network behavior. Hence, the network control techniques designed and evaluated via steady state analysis may not make optimum use of network resources after a link failure or during nonstationary periods.

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In our preliminary study, we propose a novel dynamic performance modeling and control framework for multihop wireless networks by considering their unique characteristics (e.g. node mobility, wireless links quality, dynamic routing, and etc). The principal advantages of this framework are its generality in modeling different arrival and service queuing processes, its computational efficiency in analyzing large scale networks and its ease in formulating optimal control strategy under nonstationary conditions.

2. NETWORK CONNECTIVITY MODEL

Consider a multihop wireless network with M nodes, the network topology in terms of connectivity at time t is modeled by a $M \times M$ adjacency matrix denoted as $\mathbf{A}(t) = (a_{ij}(t))$. Here, $a_{ij}(t)$ represents the binary link connectivity between node i and j (i.e., $a_{ij}(t) = 1$ if link from node i to j exists, otherwise $a_{ij}(t) = 0$).The link availability is judged by whether receiver node is within the coverage of transmitter node. Taking into account node mobility, we can manipulate the elements of the adjacency matrix according to a stochastic model. The random waypoint mobility (RWM) model study [2] shows that the link connectivity can be modeled as a Markov process with connected/disconnected states, and the durations of both states follow exponential distributions [2]. This model can represent the stationary link statistics of RWM model without a long warm-up simulation period. Also, imperfect wireless link indicates that a transmitted packet could be destroyed by MAC layer collision with probability τ_{ij} or PHY layer propagation error with probability p_{ij}. Hence, the probabilistic link connectivity is represented by $a_{ij}(t) = (1 - \tau_{ij})(1 - p_{ij})$.

3. NONSTATIONARY QUEUE MODEL

In order to describe the time varying behavior of the queue at each network node, we adopt the concept of the Pointwise Stationary Fluid Flow Approximation (PSFFA) [3]. For a scenario of a single FIFO queue with nonstationary arrival process, we define $x(t)$ as the state variable representing the *ensemble* average packet number in the node at time t. Let $\dot{x}(t) = dx/dt$ be the rate of change of the state variable with respect to time. Following the flow conservation principle, we have $\dot{x}(t) = f_{in}(t) - f_{out}(t) = \lambda(t) - \mu C G(x(t))$. Here, $\lambda(t)$ represents the ensemble average arrival rate at time t. $1/\mu$ refers to the average packet length (bits), and C defines the server capacity (link bandwidth, bps). Thus μC denotes the average service rate (pkt/s). $G(x(t))$ represents the average link utilization at time t as a function of $x(t)$.

We now extend the model to the multi-traffic class case. With S different traffic classes, each arrives at node with the rate of $\lambda_1(t), \lambda_2(t), \ldots, \lambda_S(t)$, respectively. The aggregated arrival streams can be considered as one arrival process or $\lambda_T(t) = \sum_{l=1}^{S} \lambda_l(t)$. The total number of packet in the node is defined as $x_T(t) = \sum_{l=1}^{S} x_l(t)$. The fluid flow model now becomes $\dot{x}_T(t) = \lambda_T(t) - \mu C(G(x_T(t)))$. The model can be further developed for each class with $G(x_l(t), x_T(t))$ as the average utilization function of the link by class l traffic in terms of $(x_l(t), x_T(t))$, i.e. $\dot{x}_l(t) = \lambda_l(t) + C(G_l(x_l(t), x_T(t)))$ $\forall l = 1, 2, \ldots, S$. The utilization function $G_l(x_l(t), x_T(t))$ depends on the stochastic modeling of the queue such as traffic arrival and service processes. Based on queuing theory, the closed form $G_l(x_l(t), x_T(t))$ can be obtained for standard queues such as $M/M/1$, $M/D/1$ or $M/G/1$ [3]. For the general queues, the measurement data $(\rho, (x_l, x_T))$ can be used for curve fitting to find $G_l(x_l(t), x_T(t))$ in the form of polynomial. Thus, this fluid flow model is general in nature [3] and able to represent the traffic with variable bit rates.

4. HYBRID MODEL & EVALUATION

In an M-node network, an arbitrary node i is shown in Fig.1. At each node, there are $M-1$ possible packet classes based on different final destinations. We assume that packets are generated at node i destined for node j with mean rate $\gamma_i^j(t)$, and $x_i^j(t)$ is the average number of packets at node i buffer destined for node j. A routing variable $r_{ik}^j(t)$ denotes a zero/one indicator that equals to one if traffic from node i destined to node j is routed through node k according to the specific routing scheme (e.g., DSR, AODV, etc.). To model the whole network, the first term to the right of equal sign in (1) represents the class j traffic flow routed into node i from other nodes namely fin_i^j, the second term denotes the class j offered traffic entering the network at node i, and the last term characterizes class j traffic flow out of node i namely $fout_i^j$. This hybrid model (1) can be solved via numerical integration techniques (e.g., Runge Kutta).

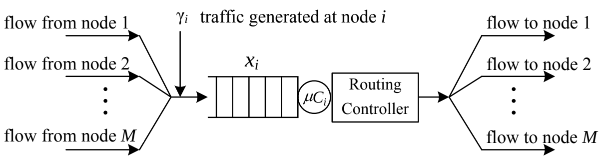

Figure 1: An arbitrary node i queueing model.

$$\dot{x}_i^j(t) = \sum_{l=1, l\neq i,j}^{M} \mu C_l(G_l^j(x_l^j(t), x_T(t)))(a_{ik}(t) r_{li}^j(t)) + \gamma_i^j(t) - \mu C_i(G_i^j(x_i^j(t), x_T(t))) \sum_{k=1, k\neq i}^{M} a_{ik}(t) r_{ik}^j(t) \quad \forall i, j \in [1, M] \quad (1)$$

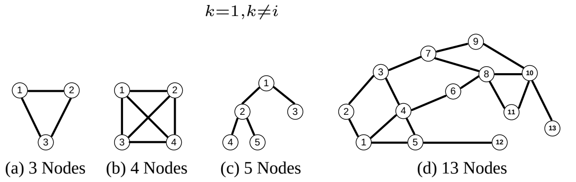

Figure 2: Topologies of sample networks.

Table 1: Computation Time Comparison

# Nodes	# Diff. Eq.	Simulation	Hybrid Model
3	6	134.7s	**0.24s**
4	12	1136.2s	**0.51s**
5	20	12592.4s	**0.8s**
13	156	384729.3s	**7.06s**

In Fig.2, the Poisson traffic arrivals and the exponential service times are configured for all nodes, and all links are switched between on/off periodically. The computation time of hybrid model conducted by Matlab is compared with nonstationary simulation [3] with 5000 independent runs by Opnet 14.5 in Table I; both are implemented on the PC with a Intel T7400 2.16 GHz duo-core processor and 2GB memory. All the results from hybrid model are within 98% confidence interval of the simulation results. Our hybrid model is shown to be an accurate and scalable performance evaluation tool.

5. OPTIMAL CONTROL FORMULATION

Based on hybrid model, an optimal control problem is formulated to determine the minimum-delay path for the virtual circuit on MWN. Here, the time varying average number of packets in the nodes along the path, which in a sense is a measure of the end-to-end delay, is minimized. We seek the optimal routing of the new nonstationary traffic (s, d) from source node s to destination node d by considering its impact on the existing background traffic b at a possible virtual circuit path $p_{sd}^{(r)}$. Let N_{sd} denote the number of possible paths from node s to d, and $p_{sd}^{(r)}$ represents the set of all nodes on r-th path except destination node d, $\forall r \in [1, N_{sd}]$. All the possible paths from node s to d can be determined from network topology prior by graph theoretic algorithms. Thus the effect of v-type traffic on the network delay is updated during the time interval $[t_0, t_f]$, which depends on the dynamics of network topology, as below:

$$\min_{\Gamma_s^d} \quad \int_{t_0}^{t_f} \sum_{r=1}^{N_{sd}} \sum_{i\in p_{sd}^{(r)}} (x_i^{d(r)} + \sum_{\substack{j=1 \\ j\neq i,d}}^{M} x_i^{j(r)}) d\tau \quad (2)$$

$$\text{s.t.} \quad \dot{x}_s^{d(r)} = fin_s^{d(r)} + \gamma_s^{d(r)} - fout_s^{d(r)} \quad (3)$$

$$\dot{x}_{i'}^{d(r)} = fin_{i'}^{d(r)} + \gamma_{i'}^{d(r)} - fout_{i'}^{d(r)} \quad i' \in p_s^{d(r)} - \{s\} \quad (4)$$

$$\dot{x}_i^{j(r)} = fin_i^{j(r)} + \gamma_i^{j(r)} - fout_i^{j(r)} \quad i \in p_s^{d(r)} \quad (5)$$

$$\Gamma_s^d \in U \quad (6)$$

The optimal control vector Γ_s^d is the vector of arrival rate of the traffic (s, d) to all the possible paths (i.e. $\Gamma_s^d = [\gamma_s^{d(1)}, \gamma_s^{d(2)}, \ldots, \gamma_s^{d(N_{sd})}]$). This routing problem is formulated by considering how to best distribute (route) a given external load $\gamma_{s\_v}^d$ among all the possible paths. However, only one path should be switched on and all of the external traffic be assigned to this path, which is constrained by (6). In order to force only one element of Γ_s^d greater than zero and equal to the offered load, (6) can be accomplished by $\gamma_s^{d(r)} \geq 0$, $\sum_{r=1}^{N_{sd}} \gamma_s^{d(r)} = \gamma_s^d$, $\gamma_s^{d(r)} \gamma_s^{d(q)} = 0$ $(r, q \in [1, N_{sd}], \forall r \neq q)$ all together. This optimization problem can be solved analytically by combining Hamilton-Jacobi arguments and mathematical programming techniques [1].

6. REFERENCES

[1] L. L. Frank, V. Draguna, and L. S. Vassilis. *Optimal Control*. John Wiley & Sons, 3rd ed. edition, 2012.

[2] S. K. Hwang and D. S. Kim. Markov model of link connectivity in mobile Ad Hoc networks. *Telecommunication System*, 34(1–2):51–58, 2007.

[3] W. Wang, D. Tipper, and S. Banerjee. A simple approximation for modeling nonstationary queues. In *Proc. of IEEE INFOCOM*, Mar. 1996.