A NOVEL PWM SCHEME WITH TWO SWITCHING FREQUENCIES AND WIDER CARRIER TO IMPROVE THE THD IN VOLTAGE SOURCE CONVERTERS

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The integration of renewable energy resources presents a significant set of technical and infrastructure challenges to power grid operation and control. Besides, it has been proven that it is less harmful to the environment. Power electronics technology has the capability to mitigate the effects of the issues imposed on the power system by the rapid expansion of renewable energy development and increased penetration level at the utility scale. Although power electronics technology is considered a promising potential solution, power conversion efficiency in the rectifiers and inverters must be improved to optimize system performance.

A novel pulse width modulation technique is presented to improve the conversion efficiency. This method has two major concepts. First, widening the on-state of the converter’s operation will increase the output fundamental component. Inverse sinusoidal pulse width modulation is one of the methods that can achieve this concept. Second, the rate of change in the information at the peak of the sinusoidal reference is relatively small. The modulation of this period has less significance in comparison to the rest of the reference cycle. Hence, a frequency modulation technique is applied to fulfill this task.

The novel development and utilization of these two techniques in combination will prove that; in comparison to the sinusoidal pulse width modulation, the fundamental output will increase substantially. Also, this new technique extensively suppresses the harmonic distortion of the converter operation. Furthermore, these results are attained by using lower switching frequency to produce lower switching losses and improves converter operation efficiency.
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1.0 INTRODUCTION

1.1 MOTIVATION

Electric power infrastructure is being expanded and modernized on a global scale. As such, efficiency of power network operations must be taken into our consideration for future grid modernization effort. Improvements in energy efficiency overall, from the primary energy to the end-user, could save up to 20% in the primary energy alone [2]. Power electronics, as applied in current and future power system infrastructure, needs to be more efficient. For example, conventional pulse-width modulation (PWM) voltage-source converters lack good power conversion efficiency when their instantaneous AC output is lower than the DC input. In addition, the converter output contains significant levels of high-frequency distortion [3].

Therefore, researchers have realized the importance of power electronics efficiency and have expended more effort in improving the converter technology. Designing new topologies [4–8] is one approach to increasing power electronics systems, especially the converters. In [4] a proposed new topology that contains two more switches and diodes as compared to a single-phase full-bridge converter, can raise the efficiency up to as much as 97.9%. This topology is suitable for transformerless grid-connected photovoltaic (PV) systems. In [5] the authors used a current source converter that utilizes voltage-clamping and soft-switching techniques on a proton exchange membrane (PEM) fuel cell to increase the efficiency to 95%, and reduced the total harmonic distortion (THD) to 1.41% for an RL load. The proposed current source converter in [6] consists of a clamped circuit, coupled circuit, and full-bridge resonant converter. This topology applied both soft-switching and low-switching stress techniques to increase the efficiency up to 97%, and depressed the THD below 2%. The efficiency of the new topology proposed in [7] increased to 96.5%. This increase was realized
by using quasi time-sharing sinewave boost choppers with bypass diodes and a full-bridge inverter. The delta converter in [8] is connected in parallel with a conventional converter to compensate and improve the quality of the main converter output. This method reduced the output low-order harmonics and kept the THD below 20%.

Although these topologies improved the efficiency of the converters, they may not be practically applicable. In other words, it may be a challenge to upgrade or change the existing converters that are currently in operation using these techniques. Therefore, other researchers believe that the change in the PWM technique will improve converter efficiency in a more practical way. Increasing the fundamental component and reducing the THD are the focal points of several PWM techniques [9]. Researchers have created different PWM techniques [9–24] to accomplish these goals. In [10], the new PWM controls the switching in accordance with the line-to-line output instead of the phase output in three-phase systems. This technique improved the output by 15.47%, and reduced the switching operations to two-thirds of the conventional PWM method. The new modulation technique in [11] uses frequency modulation along with soft-switched PWM converter control. It includes the load current in the calculation of frequency, which results in efficiency improvement by 20%. Another frequency modulation technique [15, 16] improves the converter efficiency by utilizing a sinusoidal carrier. This carrier has a switching frequency that is a function of the slope of a trapezoidal reference wave. This technique reduced the switching to two-thirds of the conventional PWM and improved the THD of the single-phase inverter to 29%, if the switching frequency is 660 Hz. The radiated noise profile for this technique is preferred over the sinusoidal pulse width modulation (SPWM) technique [17]. The same previous frequency modulation technique is evaluated in [9, 12] except that the carrier wave is a triangular modulated with harmonic injected reference wave. The THD improvement in this technique is 47.3%. The heating losses on AC motor stators produced by using this technique is lower than the SPWM technique [13]. Also, the radiated acoustic noise spectrum is better than that of the SPWM [14]. Researchers [18] extended the improvement in the power conversion efficiency and suppressed the modulation during the reference wave peaks in the later frequency modulation technique. This results in noticeable advancement in THD, fundamental component, and the radiated noise. Another PWM technique [19–21] proposes
a solution to the overmodulation issue. It utilizes the DC input by replacing the triangular wave with an inverted-sinusoidal carrier. This technique improved the fundamental output, and THD. Researchers improved the modulation schemes in multi-level converter as well. For example, the modulation techniques in \[22–24\] applied variable frequency on a five-level cascaded converter with harmonic injected reference wave to increase the fundamental output and decrease the THD.

Power conversion efficiency has captured the attention of many researchers. This efficiency is affected by several elements such as fundamental component, THD, switching losses, complexity of the implementation, and more. The focus of this dissertation is to provide a new solution to improve the conversion efficiency while taking all of these elements into consideration.
1.2 OBJECTIVES

The mechanism of force-commutated PWM technique is that the switches in the converters turn on and off based on comparing a modulation signal $v_c$, which is the desired AC output, with a triangular carrier wave $v_\Delta$, which has the desired switching frequency. If the magnitude of $v_c$ is greater than $v_\Delta$, the upper switch in one leg of a voltage source converter will be in the on-state (ON) and the lower is in the off-state (OFF). Conversely, when the magnitude of $v_c$ is less than the $v_\Delta$, the upper switch in one leg is OFF and the lower is ON. When the modulation signal is sinusoidal the PWM technique will be referred to as sinusoidal PWM (SPWM). SPWM is the common modulation technique used in the industry today. Hence, it is the benchmark in all the comparisons being presented. The SPWM scheme reduces the filter’s size by pushing the energy at the lower order harmonics to the higher harmonics. However, this improvement in the filtration is associated with a reduction in the fundamental component and increases the switching losses considerably.

A new modulation technique is presented herein - a Variable Frequency Inverse Sinusoidal Pulse Width Modulation (VFSPWM) scheme - which improves the fundamental component, lowers the total harmonics distortion, and reduces the switching losses.

The VFSPWM technique replaces the triangle carrier $v_\Delta$ in the SPWM technique with an inverse sinusoidal variable frequency carrier. The new carrier wave $v_\Delta$ switches at higher frequency when the slope of the modulation signal $v_c$ is steep, i.e. $\frac{dv_c}{dt}$ is high. At the peak of $v_c$, the switching frequency of $v_\Delta$ is the lowest. The VFSPWM method is shown in Figure 1.1.

![Figure 1.1: Variable Frequency Inverse Sinusoidal Pulse Width Modulation](image)
VFSPWM increases the efficiency of the power conversions due to the better utilization of the DC source. Furthermore, the reduction in the total harmonics distortion results in providing smaller and more economical filter options. Since the switching at the peak of $v_c$ is low, switching losses are reduced in the VFSPWM scheme, resulting in a significant reduction in the power conversion cost.
1.3 SIGNIFICANCE

The controversy of the use of DC versus AC for electric power transmission systems started in the 19th century, when Thomas Edison supported a DC approach, and Nikola Tesla and George Westinghouse advocated for the AC system. The electric utility industry settled on AC systems at that time. Since then, power demand and power infrastructure have grown dramatically. According to the 2011 World Energy Outlook (WEO) by the International Energy Agency (IEA) [25], global energy demand will increase by 40% by 2035, while global installed power generation will be doubled. Therefore, the integration of other energy resources, i.e. renewable energy, has become inevitable [2]. Renewable energy resources such as wind, solar, hydro, biomass, ocean and geothermal are environmentally preferable and are becoming more competitive in the marketplace. However, the interconnection of renewable energy resources into the grid introduces technological and infrastructure challenges. To name a few, voltage instability, voltage regulation, reactive power consumption, and sub-synchronous resonance are common issues related to renewable energy integration. In addition, this increase in generation will require new transmission and distribution system expansion and designs of more efficient technologies to improve the utilization of the existing infrastructure [26]. Power electronics technology is a practical solution for many of these issues.

Power electronics applications are widely applied in power systems, from generators to end-user. Power electronics are used in different areas in the the power system, i.e. distributed generation, power electronic loads, power quality solutions, and for transmission and distribution system applications [27]. For example, a change in wind speed causes the output voltage, magnitude, phase, and frequency of the variable-speed of wind generators to fluctuate [28]. Therefore, variable-speed drives using power electronics technology is applied to connect the wind farm to the grid [29]. In the transmission system, AC transmission lines have to deal with the generated or absorbed reactive power, increasing the line power transfer capability, and avoiding overvoltage issues, for example. This can be achieved by applying compensation systems such as Flexible AC Transmission System (FACTS), i.e., SVC, STATCOM using VSC, or even switched shunt capacitors and inductors [30].
Cycloconverters are another power electronics solution that controls the frequency. These controllers are an AC/AC frequency converter with the ability to control voltage [31]. High Voltage DC (HVDC) is one of the preferred power electronics transmission technologies applied to transmit power in DC form. HVDC systems show good standing in different applications such as long distance transmission and offshore wind farm interconnections. Force-commutated converter type HVDC, which is conventionally noted by VSC-HVDC, has several features that meet the grid code requirements (steady-state and dynamic) for wind farms. One of these requirements, for example, is that the wind farms must support the grid during faults instead of shutting down. This code is known as low voltage ride through (LVRT), which is achievable through the application of VSC-HVDC. Another requirement is the reactive power capability that is also a feature of the VSC-HVDC technology. The black-start capability in the VSC-HVDC to start the wind turbines is another reason to apply it for the wind farms interconnections [30,32]. Battery storage systems with power electronics interfaces are a promising trend used for load leveling and mitigating fluctuations of unstable power outputs [28]. At the microgrid level, Electric Double Layer Capacitor (EDLC), a type of power electronics storage unit, has the ability to compensate voltage sags in uninterruptible power supply (UPS) systems for 20-60 seconds [28]. It is worth mentioning that power electronics technology is not limited to power system applications. For instance, electric and hybrid vehicles, which have low environmental pollution, are advancing rapidly in the past decade. This growth is partly due to continuous improvements in power electronics [33].

Overall, power electronics devices require advanced modulation techniques to ensure the proper operating states in power conversion. In order to be competitive in the power electronics industry, manufacturers have improved the converters performance, size, and cost, by using high-frequency modulation techniques [34]. This quest is advanced in this dissertation work, by providing a new high-frequency modulation technique, i.e., the VFSPWM.
Converters are called rectifiers when they convert the power from AC to DC, and called inverters given for DC to AC. There are two basic types of converter topologies: Current Source Converters (CSC) and Voltage Source Converters (VSC). The Current Source Converter (CSC) is the traditional converter that requires a large AC filter for harmonics elimination, in addition to the DC filters. It also needs reactive power supply to substitute for the consumed reactive power in the conversion in order to perform power factor correction. Converters usually consist of two three-phase converter bridges connected in series to form a 12-pulse unit, where each converter is a 6-pulse bridge. The switching valves are either transistors or thyristors. The switching between valves is known as commutation, and is the part of the power loss in the conversion. The switching loss results from inductive elements in the converter, e.g., the leakage inductance of the converter transformer makes the commutation not instantaneous. Or, put differently, it takes valve current a while to go to zero; hence, the product of the voltage and the current is not zero during the switching. In the traditional Current Source Converter (CSC) each valve is commutated only once per cycle. The meaning of this is that the commutation per valve occurs at the line frequency. Therefore, the converters that use this technology are called line-commutated converters (LCC). Although Current Source Converter (CSC) is the traditional converter, it is still extensively used because of its large power and voltage capabilities. [35–37]

Currents source converters have severely attracted the attention of the researchers in the last decades. For example, [37] proposed a new topology using GTO and PWM technique to improve the output voltage and current, mitigate the voltage spike caused by the change in the output current polarity. Furthermore, this topology provided excellent efficiency when the converter is connected to a motor as the only load. In addition, it enforce a
noticeable reduction in the noise level. [38] provided a form of Hybrid-HVDC that combines
the advantages of VSC and CSC in one transmission system. In the structure of the Hybrid-
HVDC, the sending side is a CSC due to its high reliability and reasonable cost. Since VSC-
HVDC has the ability to supply power into passive systems (no synchronous machines or with
very small short circuit capacity), it has been used on the receiving side to supply remote
isolated loads, for example. Additionally, it avoids increasing the short circuit capacity,
because the AC fault current is controllable. While A VSC technology is utilized on the
receiving side, where the converter is composed of an IGBT-based bridge, converter reactor
(to link the VSC to the AC system), DC capacitor (to boost the DC voltage up and to
reduce the effect of the impulse current when the bridge is off), and AC filters.

Voltage Source Converter (VSC) technology has some remarkable features such as its
requirement for more economic AC filter for high order harmonics elimination. Furthermore,
energy storage capacitors on the DC side control the power flow besides acting as
DC harmonic filters. The switching occurs at high frequency, few kHz, using one of the
pulse width modulation (PWM) techniques in order to get a smoother signal and elimi-
nate low-order frequency harmonics. This switching technique is called self-commutated or
forced-commutated because the converter forces the converter switches to turn ON and OFF
frequently in one cycle. This technology has resolved several issues. For example, both active
and reactive power can be controlled independently using one of the PWM techniques. In
other words, it does not require a reactive power source as it can operate in any quadrant-
i.e. rectifier/inverter operation with lagging/leading power factor. [30,35,39]

The performance evaluation of the VFSPWM is conducted on Inverters only. Inverters,
in general, are static power converters that mainly produce, from a DC source, a controllable
AC waveform output. The output waveforms of the force-commutated inverters are discrete
due to the forced switching feature. Hence, the transition in the inverters is extremely
quick at the expense of the wave quality (smoothness). Although, it is desired to produce a
sinusoidal waveform, the discrete output behaves like one [31]. The generated AC voltage is
composed of discrete values, high dv/dt, that constrains the load to be inductive to smooth
the output. If the load is capacitive, an inductive filter is required between the VSI AC
side and the load to suppress any generated current spikes. The switching frequency in
the VSI configuration can be high compared to the line frequency, for instance, 1 kHz or below. For high-voltage applications, each switch group consists of two or more switching devices (IGBT, GCT, or GCT) connected in series to withstand the high power and voltage rating [40]. Single-phase VSI can be configured as a half-bridge or full-bridge VSI. The former consists of two non-simultaneous switch groups (to avoid short circuiting the DC link voltage source and the undefined switch states), and two large capacitors to set a neutral point N and the low-order current harmonics injected by the operation of the inverter. On the other hand, a full-bridge VSI has a similar structure to the half-bridge except that it has an additional leg to set the neutral point to the load, Figure 2.1. Also, the output AC voltage could only reach up to the DC link voltage value, unlike the half-bridge where it could reach up to one half the DC link voltage value. The three-phase VSI, discussed in section 7, is the inverter suitable for medium- to high-power applications. There are three legs in this inverter with eight valid switching states (each leg cannot operate simultaneously, the top and the bottom switches of any leg cannot both be on or off simultaneously). In order to ensure the selection of the valid states only, a controller that uses different modulation techniques is implemented to send the gating signal to the switching devices. The most commonly used ones are carrier-based pulse width modulation. [31,37,40]
There are several types of PWM techniques. We found it convenient to follow this categorization: carrier-based PWM and programmed PWM. Although the programmed PWM technique shows better voltage output quality with lower switching losses, they require complicated control circuits and very advanced computers. Therefore, carrier-based PWM techniques are more viable [41]. The mechanism of the carrier-based PWM is that the switches turn on and off based on comparing a modulation signal \( v_c \), which is the desired AC output, with a triangular carrier wave \( v_\Delta \), which has the desired switching frequency. If the magnitude of \( v_c \) is greater than \( v_\Delta \), the upper switch in one leg will be on and the lower is off. Conversely, when the magnitude of \( v_c \) is less than the \( v_\Delta \), the upper switch in one leg is off and the lower is on. When the modulation signal is sinusoidal the PWM technique will be referred to as sinusoidal PWM (SPWM). This modulation technique is used in the single-phase full-bridge VSI can be bipolar, where the output voltage is positive or negative due to the utilization of only one modulation wave just like the half-bridge VSI as shown in Figure 2.2. Unipolar SPWM utilizes two \( 180^\circ \) out of phase modulation waves and one triangular carrier wave, as shown in Figure 2.3, to eliminate the odd multiples of the switching frequency and produces output voltage that has positive, negative, or zero value. The switching on each leg is controlled by one modulation wave to produce the AC output shown in Figure 2.3 [42, 43]. Unipolar SPWM can be applied by using an easier format that is composed of one modulation wave and unidirectional triangular carrier wave as shown in Figure 2.4 [44].

The case for the three-phase VSI is shown in Figure 2.5. This modulation technique is similar to the single-phase full-bridge VSI, except that it uses three \( 120^\circ \) out of phase modulation waves (\( v_a, v_b, \) and \( v_c \)) instead of just one like the single-phase half-bridge/full-bridge bipolar. The SPWM technique has two main factors to control the fundamental-frequency component in the inverter output voltage: 1) amplitude-modulation ratio, and 2) frequency-modulation ratio. Amplitude-modulation ratio, also known as modulation index \( (m_a) \), is the ratio of the peak value of \( v_c \) to the peak value of \( v_\Delta \) that controls the amplitude of the fundamental component of the AC output voltage. Usually this ratio is less than one; however, if \( m_a \) is greater than one, overmodulation will occur with higher fundamental AC output voltage (around 22% increase) but with more low-order harmonic components. If this ratio is too high, i.e., \( m_a > 3.24 \), it could lead to a square-wave modulation where the
Figure 2.2: Bipolar SPWM VSI Carrie, Modulating signal and Output

Figure 2.3: Unipolar SPWM VSI Carrie, Modulating signal and Output
each leg in the inverter is ON for one-half cycle of the AC output period. The frequency-modulation ratio \((m_f)\) is the ratio of the frequency of \(v_\Delta\) to the frequency of \(v_c\). The highest harmonics are having the orders equal to this ratio and its multiples. If \(m_f\) is an integer, which means the carrier wave is synchronized with the modulation wave, this scheme will be called synchronous PWM. Obviously, asynchronous PMW is another case where \(m_f\) is not an integer. Hence, subharmonics or noncharacteristic harmonics (not a multiple of the fundamental frequency) will be presented in the output AC voltage. To use one carrier wave only and keep the SPWM features in the three-phase VSI, \(m_f\) should be an odd multiple of 3. [31, 40]

A modified SPWM technique is presented in [45]. It has been observed that the data change rate at the peaks is low leading to the conclusion that the modulation in this period can be ignored. Therefore, the carrier wave has been modified to stop the switching for 60° centered at the peaks, Figure 2.6. This modulation technique improved the harmonics profile and lowered the switching losses. Besides, it increased the fundamental component [44].

Instead of using overmodulaiton technique to boost up the inverter output voltage, [46] proposed a technique known today as third harmonics injection that can be utilized and
Figure 2.5: Three-phase SPWM VSI Carrie, Modulating signal and Output

Figure 2.6: Modified Sinusoidal Pulse-Width Modulation
increase the output voltage up to \( \frac{2}{\sqrt{3}} \) \( \approx 15.5\% \) while maintaining the harmonics profile similar to that of the SPWM technique. As the name is self-explanatory, this technique adds a third harmonic component \( \frac{1}{6} \) the fundamental component, to the sinusoidal modulation wave to result in a modulation wave that is flattened on the top (the peak value of \( v_c \) does not exceed the peak value of \( v_\Delta \)). The third harmonic in the inverter output voltage (line-to-line) is canceled out by subtracting the line-neutral phases from each other [31, 40]. The single-phase harmonic injection pulse width modulation technique is shown in Figure 2.7.

Figure 2.7: Harmonics Injection Pulse-Width Modulation
3.0 RESEARCH PLAN

The research plan is to compare the presented VFSPWM technique to the other commonly used methods, e.g., SPWM, Sawtooth PWM, and Modified PWM. The evaluation this novel PWM performance and its improvement in the power conversion efficiency is conducted in several steps. In this chapter we cover the analytic procedure used to develop this new PWM technique. Then we will present a model to simulate the modulation technique. A brief introductory of the the comparison measurements are then presented. A validation procedure for the model is used to ensure the reliability of this model used for the evaluation. Finally, a description of the model for the other PWM techniques is presented.

3.1 MODULATION SCHEMES

As mentioned before, the goal of the modulation techniques is to enforce the inverter to generate a waveform that has a fundamental component equal to a desired sinusoidal wave. The first attempt to achieve this objective was a simple form known as square-wave modulation. The switching frequency in this modulation technique is equal to the line frequency, i.e., 60Hz or 50Hz. In other words, the inverter output has a positive value for one half-cycle and negative for the other half-cycle. This method produces excellent fundamental component but has uncontrollable magnitude. Therefore, a single-pulse-width modulation is used to control the inverter fundamental output. This modulation technique has one pulse per cycle as well. The width of this pulse can be increased or decreased, thus controlling the magnitude of inverter output. Unfortunately, these two techniques contains high distortion.
One method to generate an output with reduced distortion is known as multiple-pulse-width modulation. This technique substituted the single pulse output by several pulses that has a total area equal to the area of the single pulse. The width of these pulses is constant. This process is achieved by comparing a DC wave (reference) with a triangular wave (carrier). When the reference wave is greater than the carrier wave, the gating pulse in on the ON state, closing the inverter switch. The inverter output is a function of the ration between the amplitude of the reference wave to the carrier wave, known as modulation index ($m_a$). The output, however, still has distortion that mandates the use of expensive filters.

Another modulation technique that uses the same principle of the high-frequency switching as in the multiple-pulses-width modulation is the Sinusoidal PWM (SPWM). The width of the gating signal is modulated according to the instantaneous amplitude of a sinusoidal wave (reference). This method reduces the inverter output distortion significantly and the relationship between the reference frequency and the carrier frequency is known as frequency modulation ($m_f$). When $m_f$ is high, the harmonics distortion will be having the frequency of this $m_f$, thus reducing the filter size. However, the increase in the switching frequency leads to significant increase in the switching losses which affect the conversion efficiency. Besides, this technique may not reach the switching devices capacity and limit the inverter output to the $m_a = 1$.

The presented modulation technique utilizes the same concept of the pulse width of the square-wave technique. By way of explanation, it increases the width of the pulses while maintaining the same number of pulses per cycle to improve the utilization of the DC input. Hence, the fundamental component increases proportionally to the increase of the width of the pulse. In addition, this new modulation technique takes advantage of the high-frequency feature in the SPWM technique. The switching frequency is divided into high and low frequencies. The low-frequency period is 60° and starts when the data change in the reference wave is small. Therefore, the low-frequency should be centered at the peak of the reference wave. That is due to the fact that the use of high-frequency modulation in this period may cause more switching losses with unnoticeable increase in the inverter output accuracy. The high-frequency is placed in the beginning and ending for each cycle of a sine wave. This new modulation technique is shown in Figure 3.1.
3.2 PSCAD MODEL

The model is divided into three primary components: Voltage-source inverter model, carrier generators, and gate pulse generators. The inverter model represents a single-phase full-bridge voltage-source inverter circuit, shown in Figure 3.2. It includes a DC source that represent a DC bus or link and 2 shunt capacitors on the DC side to set a neutral point N. Each capacitor has the same theoretical voltage $V_{DC}/2$.

1. **The inverter model** has two legs, leg A and leg B, where leg A produces the positive half of the inverter and leg B produces the negative half to synthesize an output that has $+V_{DC}$, 0, and $-V_{DC}$ values. There are two switches per leg, upper and lower, where each switch consists of a Insulated-Gate Bipolar Transistors (IGBT) and a feedback diode. This diode is required to keep the inductive load current flowing and prevent the voltage spikes during the switching commutations. The load has been chosen arbitrary to be
inductive for two reasons: first, the model has to reflect real industrial projects where the load in the power grid is primarily inductive. Second, if the load is resistive, the change in the power factor will be unnoticeable. Besides, if the load is capacitive, extensive current spikes may be generated and inductive filters are likely to be required. The value of the inductance is equal to 26 mH.

2. **The carrier generator** is a model that creates a unidirectional variable-frequency inverse sinusoidal wave. This carrier produces the two switching frequencies, low and high. In Figure 3.3, \( m_{f_{\text{low}}} \) is equal to 6 and \( m_{f_{\text{high}}} \) is equal to 15. A frequency selector is connected to these two frequency sources to alternate between these two frequencies. The fundamental frequency is 60 Hz, which can be changed to any other frequency such as 50 Hz. In order to maintain the inverter output symmetry, the sinusoidal wave, that alternates between 6-15 times 60Hz, has been chosen to be a cosine function. This cosine signal is then run through an absolute function to change it into unidirectional wave. Then it flips vertically by multiplying the signal by -1. Finally, the unidirectional wave is shifted to the positive side by offsetting it by 1 to form the carrier shown in Figure 3.1.

3. **The gate pulse generator** provides a control signal to the inverter. It consists of two comparators and two logical inverters. The comparators in Figure 3.4 generate a pulse
when the modulated carrier is less than a generated sinusoidal wave (reference). And generate a zero when the carrier is over the reference wave. Each carrier generator is designated to one leg in the full-bridge inverter and one sinusoidal reference wave. The two sinusoidal reference are 180° out of phase. Since the upper and lower switches per leg should be in the opposite states, the output of the comparator gets the opposite value by using the logical inverter. The generated pulses then control the switches in the full-bridge inverter to produce an output that has an RMS value close to the reference wave. The gating signal is shown in Figure 3.1.

3.3 PERFORMANCE PARAMETERS

To evaluate and validate the new PWM technique and the model, a set of quality measurements needs to be defined. *Weighted Total Harmonic Distortion* (WTHD) and the
fundamental component \((V_1 \text{ or } I_1)\) are the two widely used parameters for comparisons. In order to understand the concepts of the WTHD and the fundamental components, we need to explain briefly the Fourier series analysis.

### 3.3.1 Fourier series analysis

Jean-Baptiste Joseph Fourier invented a method to describe periodic waves with trigonometric functions that have multiple frequency of the fundamental period \(T\). This method is known as *Fourier Series*. The periodic function is represented as

\[
f(t) = a_v + \sum_{n=1}^{\infty} a_n \cos n\omega_o t + b_n \sin n\omega_o t \tag{3.1}
\]

where \(a_v\), \(a_n\), and \(b_n\) are known as *Fourier coefficients*. And \(\omega_o\) is the *fundamental frequency* and its multiples are the *harmonic frequencies*. The Fourier coefficients can be calculated as follows:

\[
a_v = \frac{1}{T} \int_{t_0}^{t_0+T} f(t) \, dt \tag{3.2}
\]

\[
a_n = \frac{2}{T} \int_{t_0}^{t_0+T} f(t) \cos (n\omega_o t) \, dt \tag{3.3}
\]

\[
b_n = \frac{2}{T} \int_{t_0}^{t_0+T} f(t) \sin (n\omega_o t) \, dt \tag{3.4}
\]
Forming the signal symmetrically is beneficial in a sense that the calculation of the harmonics is convenient. For example, even-function symmetry has this form

\[ f(t) = f(-t) \]  \hspace{1cm} (3.5)

This form reduces the tedious calculation of the coefficients and expresses them as follows

\[ a_v = \frac{2}{T} \int_{0}^{T/2} f(t) \, dt \]  \hspace{1cm} (3.6)
\[ a_n = \frac{4}{T} \int_{0}^{T/2} f(t) \cos (n\omega_0 t) \, dt \]  \hspace{1cm} (3.7)
\[ b_n = 0, \text{ for all } n. \]  \hspace{1cm} (3.8)

The function is odd when it has this form

\[ f(t) = -f(-t) \]  \hspace{1cm} (3.9)

In this case, the coefficient calculation is simpler and is expressed as follows

\[ a_v = 0 \]  \hspace{1cm} (3.10)
\[ a_n = 0, \text{ for all } n. \]  \hspace{1cm} (3.11)
\[ b_n = \frac{4}{T} \int_{0}^{T/2} f(t) \sin (n\omega_0 t) \, dt \]  \hspace{1cm} (3.12)

When the wave is repeated in an opposite sign after one-half cycle, it will be called half-wave symmetry. It gains more simplification and has the following mathematical expression

\[ f(t) = -f(-t - T/2) \]  \hspace{1cm} (3.13)

This form can be even, odd or neither. The coefficients will have to following forms

\[ a_v = 0 \]  \hspace{1cm} (3.14)
\[ a_n = \frac{4}{T} \int_{0}^{T/2} f(t) \cos (n\omega_0 t) \, dt, \text{ for } k \text{ odd.} \]  \hspace{1cm} (3.15)
\[ a_n = 0, \text{ for } k \text{ even.} \]  \hspace{1cm} (3.16)
\[ b_n = \frac{4}{T} \int_{0}^{T/2} f(t) \sin (n\omega_0 t) \, dt \text{ for } k \text{ odd.} \]  \hspace{1cm} (3.17)
\[ b_n = 0, \text{ for } k \text{ even.} \]  \hspace{1cm} (3.18)
The calculation of the coefficient can have the simplest form if the function is quarter-wave symmetry. When a function is half-wave and has a symmetry at the positive or negative midpoint, this function will be known as quarter-wave symmetry. This symmetry can be even or odd. If the function is even, the coefficient will be calculated this way

\[ a_v = 0 \]  \hspace{1cm} (3.19)  \\
\[ a_n = \frac{8}{T} \int_{T/4}^{T/4} f(t) \cos(n\omega_0 t) \, dt, \text{ for } k \text{ odd.} \]  \hspace{1cm} (3.20)  \\
\[ a_n = 0, \text{ for } k \text{ even.} \]  \hspace{1cm} (3.21)  \\
\[ b_n = 0, \text{ for all } k. \]  \hspace{1cm} (3.22)

When the function is odd, the coefficients are similar to the ones in Equations (3.19)-(3.22) except they have the following modifications

\[ a_v = 0 \]  \hspace{1cm} (3.23)  \\
\[ a_n = 0, \text{ for all } k. \]  \hspace{1cm} (3.24)  \\
\[ b_n = \frac{8}{T} \int_{T/4}^{T/4} f(t) \sin(n\omega_0 t) \, dt, \text{ for } k \text{ odd.} \]  \hspace{1cm} (3.25)  \\
\[ b_n = 0, \text{ for } k \text{ even.} \]  \hspace{1cm} (3.26)

### 3.3.2 Fundamental component and WTHD

As mentioned previously in section (3.3.1), a periodic function can be expressed in this form

\[ f(t) = a_v + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t \]

\[ = a_v + a_1 \cos \omega_0 t + a_2 \cos 2\omega_0 t + a_3 \cos 3\omega_0 t + \ldots \]

\[ + b_1 \sin \omega_0 t + b_2 \sin 2\omega_0 t + b_3 \sin 3\omega_0 t + \ldots \]  \hspace{1cm} (3.27)

Since the inverter output has quarter-wave symmetry, the fundamental components \((V_1 \text{ or } I_1)\) can be simply expressed by calculating the first term in either equation (3.20) for the even function or equation (3.25) for the odd function.
The total harmonic distortion (THD) is the ratio between the fundamental component and the rest of the harmonics. In other word, this factor depicts how close the inverter output to desired reference wave. The THD for half-wave symmetric function can be calculated using the following expression

\[
THD = \frac{1}{V_1} \sqrt{\sum_{n=3,5,7,...}^{\infty} V_n^2}
\]  

Or

\[
THD = \sqrt{\left(\frac{V_{\text{rms}}}{V_{1,\text{rms}}}\right)^2 - 1}
\]  

The weighted total harmonic distortion (WTHD) is similar to the THD except that it assigns weights to the harmonics. The low-order harmonics get more weight than the high-order harmonics. This is beneficial in a sense that this parameters indicates who much the curve depends on the low-order harmonics. WTHD can be expressed as following

\[
WTHD = \frac{1}{V_1} \sqrt{\sum_{n=3,5,7,...}^{\infty} \left(\frac{V_n}{n}\right)^2}
\]  

These measurements can be calculated using the build-in functions in PSCAD. For example, the on-line Frequency Scanner (FFT) determines the harmonics coefficient, including the fundamental component, of the input signal. After specifying the base frequency, the number of phases and the number of harmonics needed, it calculates the harmonic magnitude, phase and the DC offset as well. Harmonic Distortion Calculator is another built-in function that can finds the WTHD. This can be seen in Figure 3.5. On the other hands, PSPICE using OrCAD calculates the WTHD of a signal by toggling the FFT function in the PSPICE Schematics.
3.4 MODEL VALIDATION

To ensure the validity of the PSCAD model, a mathematical representation has been conducted. Besides, PSPICE using OrCAD has been utilized to support the PSCAD model by matching the inverter output fundamental component and the WTHD. This validation examines two modulation methods, SPWM and VFSPWM. We added the SPWM to the validation process because this modulation technique has been studied extensively by several researchers. Therefore, it supported the validation of the VFSPWM. Besides, SPWM is the benchmark method.

3.4.1 Mathematical Model

In this section we examine the VFSPWM technique mathematically. As we mentioned in section 3.3, the fundamental components and the WTHD are calculated using the Fourier coefficients. The quarter-wave symmetry property in the inverter output makes equation (3.25) the focal point. Since the output of the inverter has several pulses with different width as shown previously in Figure 3.1, it is required to calculate the Fourier coefficients for each pulse. These coefficients are then added together to provide the harmonics coefficient of the inverter output. This coefficient can be expressed as follows
\[ b_n = \frac{2}{T} V_{dc} \left[ \int_0^{\alpha_1} \sin n\omega t \, d(\omega t) + \int_{\alpha_1}^{\alpha_2} \sin n\omega t \, d(\omega t) + \cdots \right] (3.31) \]

\[ + \int_{\alpha_m}^{\pi/2} \sin n\omega t \, d(\omega t) \]

\[ = \frac{2V_{dc}}{T} \left( \frac{1 - \cos n\alpha_1 + \cos n\alpha_2 + \cdots + \cos n\alpha_m}{n} \right) \] (3.32)

\[ = \frac{2V_{dc}}{nT} \left[ 1 + \sum_{k=1}^{m} \cos(n\alpha_k) \right] \] (3.33)

Since \( T = \frac{\pi}{2} \), then

\[ b_n = \frac{4V_{dc}}{n\pi} \left[ 1 + \sum_{k=1}^{m} (-1)^k \cos(n\alpha_k) \right] \text{ for } n=1,3,5,\ldots \] (3.34)

where \( \alpha_1 < \alpha_2 < \alpha_3 < \cdots < \frac{\pi}{2} \). And to find the RMS value, simply divide \( b_n \) by \( \sqrt{2} \). We used MATLAB to calculate the fundamental component and the WTHD for the SPWM and VFSPWM.

### 3.4.1.1 Sinusoidal Pulse Width Modulation

The analysis of the SPWM has been done already by different researchers. Therefore, we are built a MATLAB code that represents the sinusoidal pulse-width modulation technique mathematically. This MATLAB code has several parts. First, the input parameters are the arbitrary chosen initial switching angles. Then we set the other initial parameters such as the modulation index \( (m_a) \) and the modulation ratio \( (m_f) \). If the input parameters are in seconds, they need to be converted into radian unit.

```matlab
function s = spwm(x)
Ma=1;
Mf=24;
x=x*120*pi;
```

The next part is for finding the switching angles that form the chain of the gating pulses. Finding the angles is achieved by using a built-in MATLAB function called `fzeros`. This function finds the zeros of the the difference between two parts. The first part is the triangular

```matlab
```
carrier and the second part is the sinusoidal reference wave. The triangular wave can be
expressed by taking the trigonometrical inverse of the cosine function that has a frequency
equal to \( m_f \). This term is normalized by dividing it by \( \pi \). These zeros are the angles when
the reference wave and the triangular wave are equal. If the gating signal starts with the
OFF state, then we need to add another \texttt{fzeros} equation with an initial value equals to 0.
The number of angles here is 4.

\[
\begin{align*}
\mathbf{s} &= \left[ \texttt{fzero}( @(x)(\text{acos}(\cos(M_f x))/\pi)-M_a \sin(x),x(1),[0 \pi/2]) ; \\
& \texttt{fzero}( @(x)(\text{acos}(\cos(M_f x))/\pi)-M_a \sin(x),x(2),[0 \pi/2]) ; \\
& \texttt{fzero}( @(x)(\text{acos}(\cos(M_f x))/\pi)-M_a \sin(x),x(3),[0 \pi/2]) ; \\
& \texttt{fzero}( @(x)(\text{acos}(\cos(M_f x))/\pi)-M_a \sin(x),x(4),[0 \pi/2]) \right] ;
\end{align*}
\]

After we find the angles (\( \alpha_1 \ldots \alpha_k \)), we calculate \( b_1, b_3, \ldots, b_n \) using equation (3.34). This
example calculates the harmonics up to the 63rd order. Then we use the these Fourier
coefficient to calculate the WTHD using equation (3.28) or (3.29).

\[
\begin{align*}
H &= 0; \\
\text{for } n &= 3:2:63 \\
H(n) &= \text{abs}((400/(n*pi))*(1-\cos(n*s(1))+\cos(n*s(2))-\cos(n*s(3))+\cos(n*s(4)))); \\
\text{end}
\end{align*}
\]

\[
\begin{align*}
V_1 &= (400/(\pi))*(1-\cos(s(1))+\cos(s(2))-\cos(s(3))+\cos(s(4))) \\
\text{THD} &= 100*\text{sqrt}(\text{sum}(H.^2))/V_1
\end{align*}
\]

3.4.1.2 VFSPWM

- Output Voltage Calculation

To calculate the inverter RMS output voltage of the \( m \)th pulse, let us assuming each
pulse width is \( \delta \), Figure 3.6:
Figure 3.6: Inverter Output using VFSPWM Technique

\[ V_{LL,m} = \left[ \frac{2}{2\pi} \int_{\alpha_m}^{\alpha_m+\delta_m} V_{DC}^2 d(\omega t) \right]^{\frac{1}{2}} \]
\[ = \left[ \frac{V_{DC}^2}{\pi} \left( \alpha_m + \delta_m - \alpha_m \right) \right]^{\frac{1}{2}} \]
\[ = \left[ \frac{V_{DC}^2}{\pi} \delta_m \right]^{\frac{1}{2}} \]  
(3.35)

And for all the pulses, the inverter RMS output voltage will be:

\[ V_{LL} = \left[ \frac{V_{DC}^2}{\pi} \sum_{m=1}^{2p} \delta_m \right]^{\frac{1}{2}}, \; p = mf - 1 \]
\[ = V_{DC} \left[ \sum_{m=1}^{2p} \frac{\delta_m}{\pi} \right]^{\frac{1}{2}} \]  
(3.36)

The instantaneous output voltage should have this form:

\[ v_o = \sum_{n=1,3,5,...}^{\infty} B_n \sin (n\omega t) \]  
(3.37)

As shown in Figure 3.6, there is a negative pulse that has a width, \( \alpha_m \), equal to a positive pulse that is displaced by \( \pi \) and start at \( \delta_m \). With this structure, this wave has the even-symmetry property. Therefore, the Fourier Coefficient \( A_n \) is equal to 0 and \( B_n \) is calculated for odd harmonics only. We are calculating the coefficient \( B_n \) for each pair of pulses and we denoted it \( b_n \). Eventually we are adding these \( b_n \) coefficients to find the final \( B_n \) value. This coefficient can be derived as follows:
\[ b_n = \frac{2}{\pi} \left[ \int_{\alpha + \frac{\delta_m}{2}}^{\alpha + \delta_m} V_{DC} \sin(n\omega t) \, d(\omega t) - \int_{\pi + \alpha + \frac{\delta_m}{2}}^{\pi + \alpha + \delta_m} V_{DC} \sin(n\omega t) \, d(\omega t) \right] \]
\[ = \frac{2V_{DC}}{\pi} \left[ \frac{-\cos(n(\alpha_m + \delta_m))}{n} + \frac{\cos(n(\alpha_m + \delta_m))}{n} \right. \]
\[ + \left. \frac{\cos(n(\pi + \alpha_m + \frac{\delta_m}{2}))}{n} - \frac{\cos(n(\pi + \alpha_m))}{n} \right] \] (3.38)

Let us assume that:

\[ X_1 = n(\alpha_m + \frac{\delta_m}{2}) \quad \text{and} \quad Y_1 = n(\alpha_m + \delta_m) \]
\[ X_2 = n(\pi + \alpha_m + \frac{\delta_m}{2}) \quad \text{and} \quad Y_2 = n(\pi + \alpha_m) \]

We can say that:

\[ \frac{X_1 + Y_1}{2} = \frac{1}{2} \left[ n(\alpha_m + \frac{\delta_m}{2}) + n(\alpha_m + \delta_m) \right] \]
\[ = \frac{1}{2} \left[ n(2\alpha_m + 3\frac{\delta_m}{2}) \right] = n(\alpha_m + \frac{3}{4}\delta_m) \] (3.39)
\[ \frac{Y_1 - X_1}{2} = \frac{1}{2} \left[ n(\alpha_m + \delta_m) - n(\alpha_m + \frac{\delta_m}{2}) \right] = n\frac{\delta_m}{4} \] (3.40)
\[ \frac{X_2 + Y_2}{2} = \frac{1}{2} \left[ n(\pi + \alpha_m + \frac{\delta_m}{2}) + n(\pi + \alpha_m) \right] \]
\[ = \frac{1}{2} \left[ n(2\pi + 2\alpha_m + \frac{\delta_m}{2}) \right] = n(\pi + \alpha_m + \frac{\delta_m}{4}) \] (3.41)
\[ \frac{Y_2 - X_2}{2} = \frac{1}{2} \left[ n(\pi + \alpha_m) - n(\pi + \alpha_m + \frac{\delta_m}{2}) \right] = -n\frac{\delta_m}{4} \] (3.42)

Let us substitute equations (3.39)-(3.42) in this geometric function:

\[ \cos(X) - \cos(Y) = 2\sin\left( \frac{X + Y}{2} \right) \sin\left( \frac{Y - X}{2} \right) \]

We are getting the following expressions:

\[ \cos(X_1) - \cos(Y_1) = 2\sin\left( n(\alpha_m + \frac{3}{4}\delta_m) \right) \sin\left( n\frac{\delta_m}{4} \right) \] (3.43)
\[ \cos(X_2) - \cos(Y_2) = 2\sin\left( n(\pi + \alpha_m + \frac{\delta_m}{4}) \right) \sin\left( -n\frac{\delta_m}{4} \right) \]
\[ = -2\sin\left( n(\pi + \alpha_m + \frac{\delta_m}{4}) \right) \sin\left( n\frac{\delta_m}{4} \right) \] (3.44)

Because n is odd, n=1,3,5,...
Substituting equations (3.43) and (3.44) in (3.38) will result the following:

\[
b_n = \frac{4V_{DC}}{n\pi} \sin(n\frac{\delta_m}{4}) \left[ \sin\left(n\left(\alpha_m + \frac{3}{4} \delta_m\right)\right) - \sin\left(n\left(\pi + \alpha_m + \frac{\delta_m}{4}\right)\right) \right]
\]

(3.45)

The Fourier Coefficient \( B_n \) for all the pulses is:

\[
B_n = \sum_{m=1}^{2p} \frac{4V_{DC}}{n\pi} \sin(n\frac{\delta_m}{4}) \left[ \sin\left(n\left(\alpha_m + \frac{3}{4} \delta_m\right)\right) - \sin\left(n\left(\pi + \alpha_m + \frac{\delta_m}{4}\right)\right) \right]
\]

(3.46)

The final \( v_o \) expression can be found by substituting equation (3.46) in (3.37) to get:

\[
v_o = \sum_{n=1,3,5,...}^{\infty} \sum_{m=1}^{2p} \frac{4V_{DC}}{n\pi} \sin(n\frac{\delta_m}{4}) \left[ \sin\left(n\left(\alpha_m + \frac{3}{4} \delta_m\right)\right) - \sin\left(n\left(\pi + \alpha_m + \frac{\delta_m}{4}\right)\right) \right] \sin(n\omega t)
\]

(3.47)

• The Number of Angels

To find the number of the intersections between the carrier and the reference waves, the number of \( \alpha \)'s, each half-cycle is divided into three areas as shown in Figure 3.7. Each area has a period of 60°. In each area the number of \( \alpha \)'s can be found using the following expression:

\[
\text{number of } \alpha_{(\text{Area } i)} = \frac{2}{3} (m_f - 1) + X_{i-1}
\]

(3.48)

Where \( m_f \) is the modulation ration. \( X \) is the residue of rounding the value of \( \alpha \) from the previous area, \( X=0 \) in the first region. Let us take Figure 3.7 as an example where the modulation ratios are 8 and 2:

Number of \( \alpha \)'s in area 1:

\[
\frac{2}{3} (8 - 1) = 4.666 \quad \text{rounded to 5 } \alpha \text{'s}
\]

\( X_1 \) is equal to -0.333

Number of \( \alpha \)'s in area 2:

\[
\frac{2}{3} (2 - 1) + (-0.333) = 0.333 \quad \text{rounded to 0 } \alpha \text{'s}
\]

\( X_2 \) is equal to 0.333
The total number of α’s is 10. The calculation of α’s should follow these rules:

1. When $m_{f_{high}}$ is not triplen, the last α in area 1 must shift to area 2.
2. When $m_{f_{high}}$ is not triplen, the first α in area 3 must shift to area 2.

- Finding the Angles

It is better to analyze one pulse first and calculate the angles, α’s, in it and then generalize this calculation for the rest of the pulses in a cycle. Each pulse is divided into two areas: even and odd. The even α’s, $\alpha_2, \alpha_4, \alpha_6$ and more, are in the even region in each pulse. Similarly, $\alpha_1, \alpha_3, \alpha_5$ and more are in the odd region of each pulse. Figure 3.8a shows the carrier and the reference waves between $\alpha_7$ and $\alpha_8$ in Figure 3.7 that forms a single pulse. The calculation of the angles in the mth pulse follows this procedure:
Figure 3.8: VFSPWM Alpha’s Calculation
1. Even Area:

As shown in Figure 3.8b, the carrier function is

\[ A_c(1 - |\cos(m_f\omega t)|) \] (3.52)

The function of reference wave is

\[ A_r \sin(\omega t_m) \] (3.53)

Equate equation (3.52) to equation (3.53) to get

\[ A_c(1 - |\cos(m_f\omega t)|) = A_r \sin(\omega t_m) \] (3.54)

Where the time in the mth pulse is equal to:

\[ t_m = t_x + \frac{m T_s}{2} \] (3.55)

Equation (3.54) can be expressed as follows:

\[ A_c(1 - |\cos(m_f\omega t)|) = A_r \sin(\omega(t_x + \frac{m T_s}{2})) \]

\[ 1 - |\cos(m_f\omega t)| = m_a \sin(\omega(t_x + \frac{m T_s}{2})) \] (3.56)

Where m_a is the modulation index
2. Odd Area:

As shown in Figure 3.8c, the carrier function is

\[ A_c(1 - |\cos(m_f \omega t)|) \]  \hspace{1cm} (3.57)

The function of reference wave is

\[ A_r \sin(\omega t_m) \]  \hspace{1cm} (3.58)

Equate equation (3.57) to equation (3.58) to get

\[ A_c(1 - |\cos(m_f \omega t)|) = A_r \sin(\omega t_m) \]  \hspace{1cm} (3.59)

Where the time in the \( m_t h \) pulse is equal to:

\[ t_m = t_x + \frac{m T_s}{2} \]  \hspace{1cm} (3.60)

Equation (3.59) can be expressed as follows:

\[ A_c(1 - |\cos(m_f \omega t)|) = A_r \sin(\omega(t_x + \frac{m T_s}{2})) \]

\[ 1 - |\cos(m_f \omega t)| = m_a \sin(\omega(t_x + \frac{m T_s}{2})) \]  \hspace{1cm} (3.61)

Since the equations (3.56) and (3.61) are the same, then solving for \( t_x \) is easier. After finding \( t_x \), we substitute it in equation (3.60) to find the angles of the \( m_t h \) pulse in seconds. This unit of the angles can be changed into degrees by multiplying the founded \( t_m \) by \( 2\pi f \).

The VFSPWM MATLAB code has almost the same structure of the SPWM code. It has two modulation ratio, \( m_{f1} \) and \( m_{f2} \) that are equivalent to \( m_{fhigh} \) and \( m_{flow} \) respectively. In this code \( m_{fhigh} \) is equal to 15 and \( m_{flow} \) is equal to 6. The unit conversion for the initial angles was eliminated because the angles were already in radian.

```
function H = myfun(x)
Ma=0.1
Mf1=15; % High frequency
Mf2=6; % low frequency
```
The implementation of the function \texttt{fzero} has two parts as well, carrier and reference wave. Since the carrier has high-frequency and low-frequency, the set of \texttt{fzero} equations need to be divided into two groups. The first group is to calculate the notches for the period $[0, \frac{\pi}{3}]$. On the other hand, the second group finds the zeros for the period $[\frac{\pi}{3}, \frac{\pi}{2}]$.

In this example, we tried to find eight angles.

\begin{verbatim}
ang = [fzero(@(x)1-abs(cos(Mf1*(x)))-Ma*sin(x),x(1),[0 pi/3]);
fzero(@(x)1-abs(cos(Mf1*(x)))-Ma*sin(x),x(2),[0 pi/3]);
fzero(@(x)1-abs(cos(Mf1*(x)))-Ma*sin(x),x(3),[0 pi/3]);
fzero(@(x)1-abs(cos(Mf1*(x)))-Ma*sin(x),x(4),[0 pi/3]);
fzero(@(x)1-abs(cos(Mf1*(x)))-Ma*sin(x),x(5),[0 pi/3]);
fzero(@(x)1-abs(cos(Mf1*(x)))-Ma*sin(x),x(6),[0 pi/3]);
fzero(@(x)1-abs(cos(Mf1*(x)))-Ma*sin(x),x(7),[0 pi/3]);
fzero(@(x)1-abs(cos(Mf1*(x)))-Ma*sin(x),x(8),[0 pi/3]);
fzero(@(x)1-abs(cos(Mf2*(x)))-Ma*sin(x),x(9),[0 pi/3]);
fzero(@(x)1-abs(cos(Mf2*(x)))-Ma*sin(x),x(10),[0 pi/3]);
fzero(@(x)1-abs(cos(Mf2*(x)))-Ma*sin(x),x(11),[pi/3 pi/2]);
fzero(@(x)1-abs(cos(Mf2*(x)))-Ma*sin(x),x(12),[pi/3 pi/2])]
\end{verbatim}

Finally this part uses the calculated coefficient to find the WTHD which is based on 63 harmonics. The result of the validation is presented after we explain the PSPICE model. One needs to notice that the variables name are not similar to the SPWM code but they achieve the same purpose.

\begin{verbatim}
H=0;
for n=3:2:63
H(n)=abs((400/(n*pi))*(1-cos(n*ang(1))+cos(n*ang(2))-cos(n*ang(3))+cos(n*ang(4))-cos(n*ang(5))+cos(n*ang(6))-cos(n*ang(7))+
\end{verbatim}
V1=(400/(pi))* (1-cos(ang(1))+cos(ang(2))-cos(ang(3))+cos(ang(4))-
cos(ang(5))+cos(ang(6))-cos(ang(7))+cos(ang(8))-cos(ang(9))-
cos(ang(10))-cos(ang(11))+cos(ang(12)))

THD=100*sqrt(sum(H.^2))/V1

3.4.2 PSPICE Model

As we mentioned before this is a second validation step where at the end of this chapter we are presenting the results of the validation. Before we show the result of the validation, we are explaining a PSPICE model that is used for the validation. The process of the validation is following the same sequence as in the mathematical section. The first part explains the PSPICE model used to simulate the SPWM technique. The second part is the PSPICE model of the VFSPMW modulation technique.

3.4.2.1 Sinusoidal Pulse Width Modulation This model consists of two 180° out of phase AC sources that represent the two sinusoidal reference waves explained in the previous sections. Also, it includes the carrier source that generates a triangular wave. These sources are shown in Figure 3.9. These two reference waves are compared to the triangular carrier to generate the gating signal. The generator signal creates a train of pulses based on the instantaneous amplitude of the reference waves and the carrier. The output of these generates is one when the reference wave is greater than the carrier wave, and zero otherwise. To perform this comparison, several process should be taken: First, the gating generator subtracts the carrier wave from the reference to determine the crossing points. The generator then transforms this signal into a one-sided by using an absolute function. By adding these two signal, the negative side is eliminated. Then a limiter with a
high gain is used to regulate the one-sided signal and ensure the transition between 1 or 0 is rapid. Figure 3.10 shows the two PSPICE gate generators where each one is assigned with a reference wave. This mechanism can be depicted in the following expression

\[ gate_{1,2} = reference_{1,2} - carrier + |reference_{1,2} - carrier| \]  

(3.62)

The signal denoted "gate\(_1\)" in Figure ?? goes to the inverter upper switch of leg A and lower switch of leg B. Where the signal "gate\(_2\)" controls the inverter lower switch of leg A and upper switch of leg A. Figure 3.11 shows the model of the full-bride single-phase inverter. The blocks denoted "Sbreak" in the figure is acting as ideal switches. The blocking resistance for these ideal switches is adjusted to 1 \(M\Omega\) and the conduction resistance is set to 10 \(m\Omega\). The modification to these resistances was mandatory to mimic the IGBT switches in the PSCAD model. Hence, the switching time (slope) of the two models, PSCAD and PSPICE, should be similar. This step ensures the accuracy in the validation. The gating signal and the inverter output are shown in Figure 3.12. The upper plot is the triangular carrier with the two sinusoidal reference waves. The two next plots are the gating signals for leg A and leg B. The last plot is the inverter output that represents the desired wave (reference wave).
Figure 3.10: SPWM Gating Generator

Figure 3.11: Full-Bridge VSI Model
3.4.2.2 VFSPWM  This model has several common parts with the SPWM model such as the sinusoidal wave sources, the gating generator and the full-bridge voltage-source inverter. It is better to describe the carrier because it is the only part that is different from the SPWM PSPICE model. The carrier model shown in Figure 3.13 consists of a high-frequency carrier generator and a low-frequency carrier generator, $m_{f_{\text{high}}} = 5$ and $m_{f_{\text{low}}} = 3$ respectively. Each generator contains a sine wave source that is rectified to the positive side by using the absolute built-in function. This rectified wave is then flipped to the negative side by multiplying it by (-1). Then it is shifted to the positive side to form the inverse sinusoidal carrier. These two carrier signals are then multiplexed to create the one carrier that switches in high-frequency for the first and last 60° per half-cycle while switching at low-frequency for 60° in the middle of the half-cycle. This process is achieved by using two digital clocks. Each clock has a frequency of 120° and a duty cycle equals to 50%. They control two ideal switches, denoted "Sbreak" in the Figure 3.14, to alternate between the two carrier signals. The gating signal in Figure 3.10 is using this carrier to control the switches of the VSI in Figure 3.11. The new carrier, gating signal, and the inverter output are all shown in Figure 3.12.
3.15. The upper plot is the variable-frequency inverse sinusoidal carrier with the two sinusoidal reference waves. The two other plots are the gating signals of leg A and leg B. The last plot is the inverter output that represents the desired wave (reference wave).

![Figure 3.13: Two VFSPWM Carrier with Different Frequencies](image)

3.4.3 Validation Result

To ensure that the PSCAD model is accurate to assist in evaluating the VFSPWM algorithm, a validation stage must be conducted. In this section the validation is based on two measurements parameters: the spectrum and the WTHD of the inverter outputs. We are comparing the WTHD and the harmonic coefficients, up to 63 harmonics, for the PSCAD, PSPICE, and the mathematical model. The SPWM technique is validated first then we apply the same validation process to the VFSPWM technique.

![Figure 3.14: VFSPWM Carrier Generator](image)
3.4.3.1 Sinusoidal Pulse-Width Modulation  We start the comparison when \( m_f = 12 \). Figure 3.16 shows the Spectrum for the SPWM with \( m_a = 0.9 \) and \( m_f = 12 \). The spectrum for the three models is almost identical. Figure 3.17 shows the \( V_1 \) percent error of the PSPICE and the PSCAD models, besides, the \( V_1 \) percent error of the mathematical and PSCAD model to measure the differences among the three models versus \( m_a \). The highest error percent occurred at \( m_a = 1 \) and its value is smaller than 1.2\%. The spike at \( m_a = 1 \) is caused by the different behavior of the models when the peak of the reference wave is equal to the peak of the carrier wave. The WTHD percent error versus \( m_a \) is shown in Figure 3.19. The difference among the three models is really small. This difference is calculated in percent error, with PSCAD as reference, to expose the difference clearly, Figure 3.18. The percent error is low where the highest one is at \( m_a \) and is equal to 1\% because the MATLAB code does not include the blocking and conduction resistances in the calculation. The rest of the percent error are smaller than 0.5\%. This confirms the validity of the the PSCAD model.
Figure 3.16: SPWM Spectrum - Ma=0.9, Mf=12

Figure 3.17: SPWM Error

Figure 3.18: SPWM THD
3.4.3.2 VFSPWM  We arbitrary chose $m_{f_{\text{low}}} = 3$ and $m_{f_{\text{high}}} = 9$ for this validation process. Figure 3.20 shows the Spectrum for the VFSPWM when $m_a = 1$, for 63 harmonics. The three models are almost identical and the difference is significantly small. Figure 3.21 shows the percent error of the PSPICE and the PSCAD models, besides, the percent error for the mathematical and PSCAD model with the change of $m_a$. The percent error is smaller that 0.3% which confirms the validity of the PSCAD model. The next step is to compare the WTHD. The percentage error of the WTHD versus $m_a$ is shown in Figure 3.20. The three models are closely matched. The percentage error of the WTHD is depicted in Figure 3.22. As can be seen, all the values of the percentage error reside below 1.2%. After these two validation steps we can assure the validity of the PSCAD models.

Figure 3.19: SPWM THD Percentage Error

Figure 3.20: VFSPWM Spectrum - $m_a = 0.9$, $m_{f_{\text{low}}} = 3$ and $m_{f_{\text{high}}} = 9$
Figure 3.21: VFSPWM Percentage Error

Figure 3.22: VFSPWM THD

Figure 3.23: VFSPWM THD Percentage Error
I the previous sections we presented the analysis of the VFSPWM technique. Also we described the models we built to help in evaluate the performance of this new modulation algorithm. Furthermore, we emphasized these procedures by analyzing and building models of the SPWM because it is the benchmark of this evaluation. Before we show the results of the comparisons between the VFSPWM and SPWM technique, we prefer to add two more commonly used modulation technique to help in generalize the comment in the results to an acceptable extent. The two additional techniques are the sawtooth PWM and the modified PWM. In this section we built and described two PSCAD models that represent the sawtooth PWM and the modified SPWM. The difference between the SPWM model and these two is the carrier generator only. Therefore, we changed and explained the components of the carrier generator while used the other SPWM components.

3.5.1 Sawtooth Pulse-Width Modulation

The carrier generator in this model is similar to that of the SPWM model. In order to generate a sawtooth carrier, the carrier configurations, shown in Figure 3.24, need to be changed. If the duty cycle is 50% the carrier generator will create the SPWM triangular wave. This duty cycle needs to be changed into 100% to form the sawtooth carrier designated for the sawtooth PWM technique. In this Figure, the $m_f$ is equal to 12 (720 Hz) and the phase shift is zero. The carrier waveform along with the two reference waves are shown in Figure 3.25. This carrier is known as trailing edge modulation where the modulation is occurring during the sloppy curve. If the sloppy curve is left sided, then this technique is called leading edge modulation. In this section we are discussing the trailing edge due to the fact that the performance of the two modulation techniques is similar. The comparison between the this carrier and the reference wave is achieved by the same comparator used for the the SPWM. This comparator generates the gating signals that control the switches in the full-bridge voltage-source inverter. This technique performances exactly like the SPWM during the linear region, $0 \leq m_a \leq 1$. 
3.5.2 Modified Pulse-Width Modulation

The carrier generator for the modified PWM is complicated in comparison to that of the SPWM and sawtooth PWM. This carrier, depicted in Figure 3.26, consists of two frequency sources, triangular wave generator and a zero source. A two input selector is used to alternate between these two sources. This selector switches to channel A (the triangular wave) when the value of the selector control input is 1. On the other hand, it switches to channel B (zero frequency) when the selector control input is 0. A pulse generator is connected to the two input selector control to control the selector by creating pulses varies between 0’s and 1’s. This pulse controller switches for 120Hz (twice the fundamental frequency) with duty cycle equals to 2/3 the cycle. To ensure that the carrier and the reference wave are in phase, the phase angle of the carrier is set to 120°. This structure guarantees that the switching is happening during the first and last 60° per half-cycle with no switching at all between these two period. The carrier and the two reference waves are shown in Figure 3.27.
Figure 3.25: Sawtooth PWM

Figure 3.26: Modified PWM Carrier Generator
Figure 3.27: Modified PWM Carrier
4.0 SINGLE-PHASE VFSPWM

The comparison in this section shows the result of the fundamental component and the WTHD vs the modulation index. The SPWM technique is the reference (benchmark) algorithm due to the fact that this technology is the most commonly used in the industry. The modulation techniques are grouped into the same switching frequency and the same stress.

4.1 SIMILAR PULSES

To evaluate the new modulation technique, one way to do the comparison with the SPWM is by checking the fundamental component and the WTHD for the modulation techniques while the number of pulses in the inverter output voltage is the same; hence lower switching losses. In the VFSPWM there are two switching frequencies. The combination of these two frequencies must generate a carrier wave that has the same pulses per cycle for the SPWM. Figure 4.1 shows an example of the SPWM and the VFSPWM having the same number of pulses in half-cycle, 5 pulses. This can be achieved by equating the average of these two frequencies to the SPWM frequency. If we examine the VFSPWM carrier for one cycle, we will find that it can be divided into 6 areas. 4 areas switch at high frequency and 2 areas switch at low frequency. Therefore, equation (4.3) shows the calculation of the VFSPWM switching frequencies that would result in the same number of pulses per cycle as in the SPWM. An example of the VFSPWM switching frequencies that correspond to the SPWM technique with $m_f$ equal to 10 and 12 is shown in Table 4.1.
Figure 4.1: Same Pulses Test (SPWM_{12} and VFSPWM_{7-4})

\[ f_{SPWM} = \frac{4f_{VFSPWM_{H}} + 2f_{VFSPWM_{L}}}{6} \quad (4.1) \]

Since the VFSPWM technique uses the unidirectional form, the new \( f_{SPWM} \) should be one half \( f_{SPWM} \) to yield the following

\[ \frac{f_{SPWM}}{2} = \frac{4f_{VFSPWM_{H}} + 2f_{VFSPWM_{L}}}{6} \quad (4.2) \]

\[ f_{SPWM} = \frac{4f_{VFSPWM_{H}} + 2f_{VFSPWM_{L}}}{3} \quad (4.3) \]

Where \( f_{SPWM} \) is the SPWM switching frequency,
\( f_{VFSPWM_{H}} \) is the VFSPWM high switching frequency and
\( f_{VFSPWM_{L}} \) is the VFSPWM low switching frequency.
Table 4.1: Switching frequencies corresponding to the SPWM switching frequency

<table>
<thead>
<tr>
<th>$f_{SPWM}$</th>
<th>$f_{VFSPWM_H}$</th>
<th>$f_{VFSPWM_L}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>12</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>12</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>12</td>
<td>9</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 4.2: The Fundamental Component (V1) Comparison in the Similar Pulses Test
4.1.1 Voltage Fundamental Component

The comparison of the modulation techniques can be illustrated in Figure 4.2. In this comparison, $m_f$ of the SPWM technique has been chosen arbitrary to be equal to 12. The switching frequency of the MSPWM can be calculated as follows

$$f_{MPWM} = \frac{3}{2} f_{SPWM}$$  \hspace{1cm} (4.4)

As can be seen, the increase of $V_1$ is, generally, proportional to the increase in $m_a$. Starting at $m_a = 0.1$, $VFSPWM_{9-0}$ has the highest fundamental component all the way to $m_a = 1$. where $VFSPWM_{8-2}$ increases rapidly to be the second highest at $m_a = 0.4$ because the inverter switches are staying ON longer in the period of the low frequency. Unlike the $VFSPWM_{8-2}$, $VFSPWM_{7-4}$ has a smaller slope but higher than that of the MPWM at $m_a = 0.9$. The three VFSPWM curves gain the highest fundamental component for most of the $m_a$ values. This result leads to the conclusion that all the VFSPWM techniques are showing better utilization of the DC source over the SPWM, Sawtooth PWM and MPWM methods. Yet, they all lack for the linearity associated with the SPWM, Sawtooth PWM and MPWM techniques. The sudden increase in the MPWM at $m_a = 1.9$ caused by the change of the multiple gating pulses per cycle into one single pulse, this signal is known as square wave. This square wave usually causes significant increase in the output distortion.

4.1.2 Harmonics Distortion

As we mentioned before, The harmonic reduction is of great importance. The WTHD equation (3.30) stated before shows that an increase in the harmonics would cause a proportional increase in the WTHD as well. However, any increase in the fundamental component that is greater than the increase in the harmonics would decrease the WTHD. Therefore, we found it beneficial to present the square values of the harmonics to examine any increase isolated from the fundamental component. This harmonic examination is shown in Figure 4.3. SPWM and Sawtooth PWM have the lowest harmonics. However, these harmonics increase with the increase of $m_a$. This increase is sharper than the other modulations because the lower-order harmonics are proportionally increasing. Hence, their harmonics are
worse than MPWM and $VFSPWM_{9-0}$ at $m_a = 0.3$. Between $m_a = 0.4$ and $m_a = 0.5$ the highest harmonic in the SPWM technique is the 11th harmonic, $V_{11} = 36$ where the 13th harmonic is the highest in $VFSPWM_{8-2}$, $V_{13} = 28.6$. Therefore, $VFSPWM_{8-2}$ does better at this point. Unfortunately, $VFSPWM_{7-4}$ carried the highest harmonics through out the comparison because the 11th harmonic is the highest in addition to the 3rd, 5th, 7th and 9th. In the overmodulation region, all the VFSPWM techniques are about %33 higher that the other modulations. This is due to the fact that the 3rd harmonic has the highest weight and it is increasing by the increase of $m_a$ in the overmodulation region.

It is recommended to show the ratio between $V_1$ and the weighted harmonics. Therefore, we are using WTHD in equation (3.30). Figure 4.4 shows the WTHD of the modulation techniques. for low $m_a$, the WTHD for $VFSPWM_{9-0}$ and MPWM are indicating good performance because the $V_1$ is considerably high. Since $V_1$ and the harmonics for $VFSPWM_{8-2}$ are improving rapidly, the WTHD is decreasing sharply to make $VFSPWM_{8-2}$ the second best modulation technique among the others at $m_a = 0.6$. When $m_a \leq 1$ the three VFSPWM methods started with %45 to %80 better performance than the SPWM and Sawtooth SPWM. On the other hands, they tend to be closer to each other and when $m_a \geq 1$. 

Figure 4.3: The Weighted Voltage Harmonics in the Same Pulses Test
Figure 4.4: Voltage WTHD in the Same Pulses Test

Figure 4.5: Current WTHD in the Same Pulses Test
The ratio between the current ripple goes through the load and the fundamental current $I_1$ is shown in Figure 4.5. $VFSPWM_{9-0}$ method stated as the best technique to achieve the minimum WTHD at $m_a = 0.75$. The WTHD for $VFSPWM_{8-2}$ has improved quickly to be the second best method up to $m_a = 0.85$. At $m_a = 1$, SPWM, Sawtooth PWM, $VFSPWM_{9-0}$ and $VFSPWM_{8-2}$ have almost the same WTHD value.

4.2 SIMILAR STRESS

Previously we compared the modulation technique with the same number of pulses per cycle. In this section we examine the modulation techniques with the same stress, $\frac{dv}{dt}$. In other words, we chose the VFSPWM techniques that satisfies this expression $f_{\text{SPWM}} = f_{\text{VFSPWM}_{H}}$. That means the number of pulses in the first and last 60° per half-cycle is the same of the modulation techniques we are evaluating. For example, If $f_{\text{SPWM}} = 12$, then the same stress test will assign the following switching frequencies to the VFSPWM technique : (6-3) and (6-0) along with $MPWM_{m_f=12}$. Figure 4.6 show an example of the same stress test.

![Diagram](image)

Figure 4.6: Same Stress Test (SPWM 12 and VFSPWM 6-3)
Figure 4.7: V1 vs Ma, Mf=12 with the same stress

4.2.1 Voltage Fundamental Component

In Figure 4.7, it can be observed that VFSPWM\(_{6-0}\) has the highest fundamental component while the second highest is the MSPWM technique up to \(m_a\) is equal to 0.7 where the VFSPWM\(_{6-3}\) is becoming the second highest fundamental component. At this point VFSPWM\(_{6-0}\) and VFSPWM\(_{6-3}\) are behaving identically when \(m_a\) is greater than 0.7. The average of the increase in the fundamental component in the VFSPWM\(_{6-0}\) is about 80% of the SPWM technique. On the other hand, the average of the improvement in the fundamental of the VFSPWM\(_{6-3}\) is about 33.3% of the SPWM. Both VFSPWM\(_{6-0}\) and VFSPWM\(_{6-3}\) does not lose the linearity feature in the linear region, however, the slope of the VFSPWM\(_{6-0}\) technique is low which may be considered as a draw back as we will see in the STATCOM application in Chapter 8.
4.2.2 Harmonics Distortion

As can be seen in Figure 4.8, $VFSPWM_{6-3}$ has the highest weighted harmonics in this examination. At $m_a = 0.3$ the sharp increase in harmonics for the SPWM and Sawtooth PWM techniques made $VFSPWM_{6-0}$ and MPWM the preferred modulation methods. The harmonics that form the $VFSPWM_{6-3}$, SPWM and Sawtooth PWM are almost the same starting from $m_a = 0.7$. Although, the VFSPWM technique is not as good as the SPWM, Sawtooth PWM and MPWM in the overmodulation region, $m_a \geq 1$, this region is rarely manipulated in the industry.

The trade off between $V_1$ and the weighted harmonics is shown in Figure 4.9. Although, VFSPWM technique has higher harmonics, the WTHD shows that they are better than the SPWM and Sawtooth PWM when $m_a \leq 1$. For example, the WTHD at low $m_a$ for the VFSPWM techniques are lower by 25% to 50% of that for the SPWM and Sawtooth PWM techniques. At $m_a = 0.7$ most of the VFSPWM have the same weighted harmonics and improvement in the WTHD by 20% to 40%. In the overmodulation region, the WTHD for all the modulation techniques are the close to each other.
Figure 4.9: Voltage WTHD of the modulation techniques with the same stress
5.0 REAL MODEL

In section (3.4) we represented the model mathematically and built an ideal PSCPICE model for the validation. In this section we are examining the performance of the VFSPWM modulation techniques in the industry by simulate a PSCPICE model using real components instead of ideal components. These real components of the PSCPICE model are described first and then the performance of the model will be evaluated.

5.1 PSPICE MODEL

This model has the same two blocks of the ideal PSCPICE model we explained before, VSI and controller, except that these blocks contain different parts in the real model.

5.1.1 Voltage-Source Inverter

The single phase voltage-source inverter consists of a DC source that feeds through two legs, leg A and leg B. Leg A is designed to generate the positive part of the inverter. On the other hand, the negative part is generated from the switching devices in leg B. Each leg has an upper and lower switching devices. The switching device is composed of one of more IGBT devices in parallel with one or more feedback diodes. Figure 5.1 shows the inverter containing the DC source and the two legs including the upper and lower switching devices. The ”IXGH40N60” model is chosen to be the IGBT devices in the voltage-source inverter because it has a high switching capability, 300 ns to rise and 800 ns to fall. The datasheet of the ”IXGH40N60” IGBT is presented in the appendix.
Figure 5.1: The Real Model Single-phase Voltage-Source Inverter
5.1.2 Inverter Controller

Each converter requires a controller that generates a set of pulses as a gating signal to turn the switching devices ON and OFF in a special pattern that forms a desired output.

5.1.2.1 Reference Generators  This model contains reference generators to synthesize two sinusoidal waves. The two sinusoidal waves are 180° out of phase. The first one is dedicated for the positive side which is denoted as leg A. The second wave is for the negative pulses, leg B. Figure 5.2 shows the two reference generators where the first wave generator is labeled reference 1. This wave is shifted to −60° to simplify the synchronization with the carrier wave that is described in the next section. The second wave generator is labeled reference 2 and shifted to 120°.

![Figure 5.2: The Real Model Reference Waves Generator](image)

5.1.2.2 Carrier Generator  The carrier generator consists of two parts, carrier waves source and multiplexer. The carrier waves source provides a high frequency carrier and a low frequency carrier. Each carrier wave is generated from a sinusoidal source. Then it is rectified by using an absolute function. This rectified function is then flipped to the negative side by multiplying the wave by (-1). Finally, this wave is offset to the positive side by adding a constant value (1). This process can be seen in Figure 5.3. The two input multiplexer combine these two carriers to form the VFSPWM carrier where these two carriers are the input waves. This multiplexer consists of two switches. Each switch turns ON for 60° and
OFF for another 180°. Two digital clocks "DSTM" are controllers that generate switching pulses to operate the switches. The first controller is for high switching frequencies and the second one is for low switching frequencies. Figure 5.4 shows the multiplexer.

![Figure 5.3: The Real Model Carriers Generator](image)

![Figure 5.4: The Real Model Carriers Generator](image)

5.1.2.3 Comparator The comparator’s function is to generate a binary signal, ones and zeros, by comparing the value of the two inputs. The two inputs are the reference wave and the carrier. In Figure 5.5 the comparator is an operational amplifier "LM324" where it generates +15 volt DC when the positive input (reference) is greater than the negative input (carrier) and -15 Vdc otherwise. A limiter is connected to the OP Amp to set the boundaries between 0 and 1. The gain of the limiter is high enough (1K) to enforce the transition between 0 and 1 in a very short time. In this model there are two comparators, one for leg A and the other one for leg B. The datesheet is included in the appendix.
5.2 REAL MODEL COMPARISON RESULTS

One of the major differences between the ideal switching devices and the more realistic ones is that the device does not switch instantly. To turn the switch on, the internal capacitance is required to be fully charged to a threshold voltage. The time it takes to reach this charge level is known as the turn on delay $t_{d(on)}$. Then it takes some time for the voltage level to keep rising up from the threshold voltage into a full-gate voltage where the switching device operates in a linear manner. In the turn off process, the stored charges need turn off delay $t_{d(off)}$ to reach the pinch-off region. Then, the internal capacitance starts discharging to reach the threshold level. This discharge time is known as fall time $t_f$. In this section, we are comparing the ideal model fundamental component and WTHD with the real model. We are presenting two examples; the first one has the low switching frequency set at $m_{f_{low}} = 3$ and the high modulation ratio set at $m_{f_{high}} = 9$. For simplification, it is labeled like this $VFSPWM_{9-3}$. While the other example has a high switching frequency equal to $m_{f_{high}} = 15$ and the low frequency is $m_{f_{low}} = 6$, $VFSPWM_{15-6}$.
5.2.1 VFSPWM_{9-3}

5.2.1.1 Spectrum  Figure 5.6 shows the comparison of the voltage spectrum between the ideal model and the real model when \( m_a = 1 \). It can be noticed that the real fundamental value is slightly less than the ideal because of the delay time it takes to rise. This difference is not significant. Besides, we can conclude that applying this modulation technique will mimic the simulation to a good extend. To generalize this conclusion, it would be required to present the fundamental component versus a range of modulation index \( m_a \) which can be seen in Figure 5.7 in error percentage instead of real values to notice the difference. When the modulation index \( m_a \) is small, the width of the pulses is small. Having such narrow pulses will be clearly affected by the delay in the switching. For instance, at \( m_a = 0.1 \), the difference between the real and ideal fundamental component is 17\%. Fortunately, this difference can be neglected when the modulation index \( m_a \) reaches 0.5 where the percentage error is less than 5\%.

![Figure 5.6: The Real Model Spectrum VFSPWM 9-3](image)

5.2.1.2 WTHD  We mentioned before that WTHD is one of the important measurement tools. Therefore, it is crucial to compare the WTHD between the real and ideal models. Figure 5.8 shows how the differences between the ideal and real WTHD is decreasing when the modulation index \( m_a \) is increasing. The calculated difference can be seen in Figure 5.9. It can be noticed how the percentage error is decreased to a decent level when the modulation index \( m_a \) is closing 0.5.
Figure 5.7: V1 Error Percentage of The Real Model VFSPWM (9-3)

Figure 5.8: WTHD of The Real Model for VFSPWM (9-3)

Figure 5.9: Percentage Error of The Real Model WTHD, VFSPWM (9-3)
5.2.2 VFSPWM

The second example is to present the comparison of the VFSPWM with two different frequencies, $m_{f_{\text{high}}} = 15$ and $m_{f_{\text{low}}} = 6$. The purpose of this example is to emphasize on the results of the previous example, $m_{f_{\text{high}}} = 15$ and $m_{f_{\text{low}}} = 6$.

5.2.2.1 Spectrum In Figure 5.10, the voltage spectrum up to the 63rd harmonics shows that the PSCAD model is almost similar to the real PSPICE model. This result is expected because the pulse in the VFSPWM is wider than the other common modulation techniques making the delay in the switching small in comparison to the width of the pulses. Since the difference between the spectrum of the real and ideal model is small, Figure 5.11 shows the change in the real model by percentage. When $m_a = 0.1$ The error is higher than the previous example, $m_{f_{\text{high}}} = 9$ and $m_{f_{\text{low}}} = 3$. That is due to the fact that the switching frequency is higher, hence the pulses are narrower and the switching delay has more effects on the width of the pulses.

![Figure 5.10: The Real Model Spectrum VFSPWM 15-6](image)

5.2.2.2 WTHD The WTHD, when the modulation index $m_a$ is low, has a noticeable difference between the real and ideal models. That can be shown in Figure 5.12. Fortunately, this difference is considered insignificant in the region that is frequently used which is defined between $m_a = 0.6$ and $m_a = 1$. Figure 5.13 depicts that in this range the percentage error is $\leq \%2$. We can conclude that the ideal model is acceptable to evaluate the VFSPWM technique.
Figure 5.11: V1 Error Percentage of The Real Model VFSPWM (15-6)

Figure 5.12: The Real Model WTHD for VFSPWM (15-6)

Figure 5.13: Percentage Error of The Real Model WTHD, VFSPWM (15-6)
6.0 HARMONICS ANALYSIS

Inverters are developed and deployed to fix several issues in the power system such as integrating renewable energy, increasing efficiently the capacity of the transmission system, and more. However, inverters and rectifiers are known as harmonic sources. They generate harmonics to a level that may threaten the quality of the power system and harm some of the main equipment like transformers. In this section mathematical equations are presented to calculate the harmonics for the SPWM and VFSPWM. Then, these harmonics are shown graphically to visualize the comparison between the modulation techniques. The comparison is divided into two sub-sections, similar pulses test and similar stress test. The difference between these two tests is explained in details in chapter 4.

6.1 HARMONICS CALCULATION

In this section we are presenting the mathematical equations that are derived previously in section 3.4.1. The harmonics that are analyzed in this section are $5^{th}$, $7^{th}$, $11^{th}$, $13^{th}$ and $17^{th}$. These harmonics are expressed mathematically as follows:

The fifth harmonic is calculated as follows:

\[ V_{5th} = b_5 \sin (1885t) \]  \hspace{1cm} (6.1)

where

\[ b_5 = \frac{4V_{dc}}{5\pi} \left[ 1 + \sum_{k=1}^{m} (-1)^k \cos(5\alpha_k) \right] \]
The seventh harmonic is calculated as follows:

\[ V_{7th} = b_7 \sin (2639t) \]  

where

\[ b_7 = \frac{4V_{dc}}{7\pi} \left[ 1 + \sum_{k=1}^{m} (-1)^k \cos(7\alpha_k) \right] \]  

The eleventh harmonic is calculated as follows:

\[ V_{11th} = b_{11} \sin (4147t) \]  

where

\[ b_{11} = \frac{4V_{dc}}{11\pi} \left[ 1 + \sum_{k=1}^{m} (-1)^k \cos(11\alpha_k) \right] \]  

The fifth harmonic is calculated as follows:

\[ V_{13th} = b_{13} \sin (4901t) \]  

where

\[ b_{13} = \frac{4V_{dc}}{13\pi} \left[ 1 + \sum_{k=1}^{m} (-1)^k \cos(13\alpha_k) \right] \]  

The fifth harmonic is calculated as follows:

\[ V_{17th} = b_{17} \sin (6409t) \]  

where

\[ b_{17} = \frac{4V_{dc}}{17\pi} \left[ 1 + \sum_{k=1}^{m} (-1)^k \cos(17\alpha_k) \right] \]  

where \( \alpha_1 < \alpha_2 < \alpha_3 < \cdots < \frac{\pi}{2} \). These angels are calculated using the MATLAB code in section 3.4.1.1 for the SPWM technique and from section 3.4.1.2 for the VFSPWM technique.
6.2 COMPARISON

The comparison between the different modulation techniques should follow an acceptable methodology. The challenging this comparison is that the SPWM, the benchmark technique, is operating at one switching frequency while the VFSPWM works on two combined switching frequencies. Therefore, the comparison is divided into two cases. The first one compares the techniques that have the same number of pulses per cycle, we call it similar pulses test. The second one compares the modulation techniques that have the share the same switching stress, noted similar stress test.

6.2.1 Similar Pulses

Since the SPWM technique is the reference, it is preferred to analyze it first and then analyze the other VFSPWM techniques. The modulation ratio $m_f$ that has been chosen arbitrary is 12. The corresponding VFSPWM techniques to this ratio are: (7-4), (8-2) and (9-0).

6.2.1.1 SPWM Figure 6.1a shows the fundamental component and the harmonics of the SPWM. While Figure 6.1b shows the harmonics in percentage of the fundamental. The 11th and the 13th harmonics are conserving the highest energy among the other harmonics because they are the sideband of the modulation ratio, $m_f = 12$. These harmonics start as high as the fundamental components then increase slowly until the modulation index, $m_a$, is 0.6 where they reach the maximum value which is 75% of the fundamental component. When the modulation index $m_a$ is greater that 0.6, both the 11th the 13th harmonics are decreasing and the the 7th starts substituting the losing energy in the the two harmonics. At $m_a = 0.9$, the 5th harmonic is slightly increasing.

6.2.1.2 VFSPWM$_{7-4}$ In the VFSPWM techniques, the carrier is composed of two modulation ratio, $M_{high}$ and $M_{low}$. The harmonics usually gain high values when there order is close to the modulation ratio. When the modulation ratios are 7 and 4, it is expected to observe an increase in the 4th, 7th harmonics and there multiples and there sidebands. In
Figure 6.2a, the energy of the harmonics is shared between several orders instead of one, the 13th, as in the SPWM. For low modulation index \( m_a \), all the harmonics are greater than 15\%, Figure 6.2b. Fortunately, in the operating region, \( m_a \geq 0.6 \), the highest is the 11th harmonic having \%40 of the fundamental.

6.2.1.3 VFSPWM\(_{8-2}\) This configuration of the modulation ratio \( m_f \) behaves in the same manner as in the previous technique in terms of the energy distribution. For instance, all the harmonics are sharing the energy generated from the inverter when the modulation index \( m_a \) is low. Figure 6.3a shows the amount of energy for each harmonic. In Figure 6.3b it is clear that the harmonics energy is distributed between the 13th and the 17th where both of them are less than 30\% of the fundamental. This is considered as a good result because it indicates that the size of the filter should be smaller and more economic.

6.2.1.4 VFSPWM\(_{9-0}\) This technique has an advantage over the other VFSPWM technique that it operates at one modulation ratio \( m_f \) only, which is 9 in this case. That means
Figure 6.2: VFSPWM 7-4 Harmonics Analysis

Figure 6.3: VFSPWM 8-2 Harmonics Analysis
the utilization of this technique should generate the harmonics in fewer orders in comparison to the other VFSPWM techniques. In Figure 6.4a, the 17\textsuperscript{th} harmonic reserve most of the harmonics energy between \( m_a = 0.1 \) and \( m_a = 1 \). Its value is considerably low in comparison to the SPWM technique. In Figure 6.4b, the maximum value of the 17\textsuperscript{th} harmonic is 20\% of the fundamental that occurs at \( m_a = 0.4 \). Therefore, this technique should be preferred because it reduces the harmonics values and shifts the harmonic to a higher order that will lead to smaller and more economic filters.

Figure 6.5 categorizes the harmonics according to their order and shows the results in peak values and percentage to the fundamental component. If we focus on the operation region, \( 0.6 \leq m_a \leq 1 \), it is evident that the highest harmonics occurs at \( m_a = 0.6 \) for the SPWM. These harmonics are the 11\textsuperscript{th} and The 13\textsuperscript{th}.

### 6.2.2 Similar Stress

The VFSPWM switching frequencies that maintain the same stress as in the SPWM with a modulation ration \( m_f \) equal to 12 is (6-0), (6,3) and (6-6). In this section we are analyzing the 5\textsuperscript{th}, 7\textsuperscript{th}, 11\textsuperscript{th}, 13\textsuperscript{th} and 17\textsuperscript{th} harmonics and then comparing them to the SPWM.
(a) 5\textsuperscript{th} Harmonic (peak value)

(b) 5\textsuperscript{th} Harmonic (% of the Fund)

(c) 7\textsuperscript{th} Harmonic (peak value)

(d) 7\textsuperscript{th} Harmonic (% of the Fund)

(e) 11\textsuperscript{th} Harmonic (peak value)

(f) 11\textsuperscript{th} Harmonic (% of the Fund)
Figure 6.5: Harmonics Comparison

(g) 13\textsuperscript{th} Harmonic (peak value)  
(h) 13\textsuperscript{th} Harmonic (% of the Fund)

(i) 17\textsuperscript{th} Harmonic (peak value)  
(j) 17\textsuperscript{th} Harmonic (% of the Fund)
6.2.2.1 VFSPWM_{6-0}  
This modulation technique has a feature that it has one switching frequency only. In other words, its harmonics will be a function of one modulation ratio, \( m_f = 6 \). Therefore, the harmonics that reserve most of the energy, besides the fundamental, are the 11\(^{th}\) and the 13\(^{th}\). The fundamental component of this modulation technique starts with high amplitude because the inverter conducts in the middle period of each half cycle. The harmonics amplitude are shown in Figure 6.6a. The maximum summation of these two harmonics is 40\% when the modulation index is about 0.4. In the operation range, \( 0.6 \leq m_a \leq 1 \), the other harmonics are at the lowest values where they are less than 10\%, Figure 6.6b.

6.2.2.2 VFSPWM_{6-3}  
The fundamental componenet of the VFSPWM_{6-3} has a steep slop but its low modulation ratio is small enough to put some energy in the 5\(^{th}\) and 7\(^{th}\) harmonics. It may require larger filters because the order of the harmonics is closer to the fundamental. Besides, the 11\(^{th}\) and 13\(^{th}\) harmonics get their energy from the high modulation ratio, \( m_f = 6 \), Figure 6.7a. Still, all the harmonics are less than 20\% in the operation region.
6.2.2.3 Sinusoidal Inverse Pulse Width Modulation

It is worth presenting the modulation technique that the VFSPWM is developed from. It switches at one frequency like the SPWM technique. Figure 6.8a shows how this technique behaves like the SPWM except that its fundamental is higher. The 11th and the 13th are gaining the highest energy, after the fundamental, because of the switching frequency. These two harmonics reserve very high energy that may demand high filters. They conserve about 80% of the fundamental during the operation region, specially when $m_a = 0.6$. Figure 6.8b depicts the energy distribution among the harmonics in percentage of the fundamental.

Figure 6.9 shows the comparison between these modulation techniques and the SPWM. It can be noticed that the 5th and the 7th harmonics are the lowest for the SPWM and the highest, total of 33% at $m_a = 0.6$, when using VFSPWM 6–3. On the other hand, 11th and the 13th harmonics are very high with the SPWM, total of 125% at $m_a = 0.6$. Where VFSPWM 6–0 and VFSPWM 6–3 are the lowest. the 17th harmonic is considered low using any of these techniques.
Figure 6.8: VFSPWM 6-6 Harmonics Analysis
(a) $5^{th}$ Harmonic (peak value)

(b) $5^{th}$ Harmonic (% of the Fund)

(c) $7^{th}$ Harmonic (peak value)

(d) $7^{th}$ Harmonic (% of the Fund)

(e) $11^{th}$ Harmonic (peak value)

(f) $11^{th}$ Harmonic (% of the Fund)
Figure 6.9: Harmonics Comparison

(g) 13th Harmonic (peak value)  
(h) 13th Harmonic (% of the Fund)  

(i) 17th Harmonic (peak value)  
(j) 17th Harmonic (% of the Fund)
In the previous chapters, the VFSPWM techniques have been tested and analyzed for one phase. We are analyzing the performance of this new modulation technique for three-phase. This chapter starts by presenting the three-phase PSCAD model. Then the components of the three-phase VFSPWM controller are explained. Then, the fundamental component and the THD of the three-phase VFSPWM is analyzed and compared to the SPWM technique. The three-phase VFSPWM waves along with the gating signal are shown in Figure 7.1. Figure 7.2 shows the output of the three-phase inverter using the VFSPWM technique.

![Figure 7.1: Three-Phase VFSPWM Waves and Gating Signals](image-url)
The function of the inverter is to convert the power from DC into AC. This process can be accomplished by using power electronics switches that turn on and off to form a signal that has a desired RMS value. This signal is basically a chain of square waves that should behave similar to the sinusoidal wave on the power equipment. The structure of the three-phase inverter is that it consists of three parallel half-bridge inverters, legs, with one common DC source. Each leg has an upper switch and lower one. When the upper switch is closed ”ON” while the lower is ”OFF”, the leg will be conducting and supplying positive voltage. For simplicity, it is common to refer to the whole leg as positive. On the other hands, when the lower switch is conduction and the upper one is blocking the leg is supplying negative voltage. Figure 7.3 shows the three-phase inverter. The DC capacitance connected in parallel with the DC source is to balance the DC voltage between the upper and lower switches. This is the inverter structure that can be controlled by most of the modulation controllers such as SPWM, Sawtooth modualtion, VFSPWM and more.

7.1 THREE-PHASE INVERTER
7.2 THREE-PHASE VFSPWM

In order to get a desired AC signal out of the inverter, a controller has to operate the inverter in a certain pattern. This controller defines the patterns based on two main inputs. The first one is the reference signal and the second one is the carrier. The Three-phase VFSPWM controller is similar to the single phase. It consists of three main components: reference generator, carrier generator and comparators.

7.2.1 Reference generator

This generator is producing two three-phase ideal sinusoidal signals that each should mimic the grid voltage wave (amplitude, frequency and phase). One signal is dedicated for the upper switch and shown in Figure 7.4a. The other one for the lower and shifted by 180° from the first one, Figure 7.4b. The controller’s job is to have the inverter RMS output equal to this sinusoidal signal’s RMS value. The amplitude of this sinusoidal signal is scaled from zero through 1. Figure 7.4 shows the three-phase reference generator.
Figure 7.4: Three-Phase Reference Generators
7.2.2 Carrier generator

The carrier generator is the component that provides a signal that replaces the triangular carrier in the SPWM. The purpose of this carrier is to determine the width of the pulses the inverter is producing. When the frequency of this carrier is high, the pulses will be narrow. Hence, the harmonics that the inverter generates are having a frequency, and its multiples, of the carrier frequency. That means the filters should be smaller and this is one of the main features in these PWM techniques.

Figure 7.5 shows the carrier generator. This generator consists of an alternator that determines the frequency of the a sinusoidal wave. This alternator switches between two frequencies to keep each one on for 60°. One of them is high and the other one is low. In this example the high is 6 times the fundamental frequency, 60 Hz, and the low is 3 times the fundamental frequency. The sinusoidal signal with the two frequencies is then rectified to have positive sign only. It is then subtracted by 1 and called "carrier A" for phase A. Two more signals are generated like carrier A where they have the same amplitude and frequencies. The first one is shifted by 120° and called "carrier B" for phase B and the second one is shifted by 240° and called "carrier C" for phase C.

Figure 7.5: Three-Phase Carrier Generator
7.2.3 Comparators

The reference signals and the carrier signals are compared to generate gating signals for the inverter switches. This process is accomplished by the comparator. For the upper switches, the comparator generates a pulse when the value of the reference greater than the carrier. The comparator generates a pulse when the reference is smaller than the carrier for the lower switches. Figure 7.6 shows the comparators of the three phases where each phase has two comparators, one for the upper leg and the other one for the lower.

![Three-Phase Comparators](image)

Figure 7.6: Three-Phase Comparators

7.3 THREE-PHASE COMPARISON RESULTS

This comparison is divided into two parts: same pulses and same stress. The two parameters that we are using for the comparison are the fundamental component and the WTHD.

7.3.1 Similar Pulses

Figure 7.7 shows the fundamental component of the SPWM and the VFSPWM configurations, (7-4), (8-2) and (9-0). The three VFSPWM techniques are greater than the SPWM by an average of 40%. VFSPWM_{9-0} maintains the highest fundamental component in the
linear region, $0 \leq m_a \leq 1$. Besides, the WTHD is lower for the three VFSPWM techniques, Figure 7.8. This indicates that the filters are smaller and cheaper.

![Graph showing comparison of V1 (L) vs Ma for different techniques]

Figure 7.7: Three-Phase Fundamental Component Comparison (Same Pulses)

7.3.2 Similar Stress

The VFSPWM techniques in the similar stress test are more linear than the case of the similar pulses in the linear region. Figure 7.9 shows the comparison of the fundamental component. VFSPWM$_{6-0}$ has the highest $V_1$ while VFSPWM$_{6-3}$ is almost identical to the original technology that this technique is developed from, simply we called it VFSPWM$_{6-6}$. SPWM has the lowest $V_1$ among them. In terms of WTHD, VFSPWM$_{6-0}$ has the lowest distortion. The VFSPWM$_{6-3}$ is out performing the VFSPWM$_{6-6}$ in the WTHD by an average of 32%. Figure 7.10 shows the WTHD comparison.
Figure 7.8: Three-Phase WTHD Comparison (Same Pulses)
Figure 7.9: Three-Phase Fundamental Component Comparison (Same Stress)
Figure 7.10: Three-Phase WTHD Comparison (Same Stress)
8.0 VFSPWM APPLICATION

In the previous chapters, the analysis of the VFSPWM modulation techniques confirm several major advancements. These techniques increase the fundamental component, reduce the switching losses and decrease the harmonics. In this chapter, we are presenting the benefit of using the VFSPWM technique in Flexible AC Transmission Systems (FACTS) application and compare it to the SPWM technique. First, A brief introduction on FACTS system is presented. Then, STATCOM, one of FACTS controllers, system is discussed in more details. Finally, a STATCOM model is built as a backbone of the comparison between the VFSPWM and SPWM technique. In this comparison we are using the unipolar technique, hence we are presenting the VFSPWM techniques of the same stress test only.

8.1 FACTS TECHNOLOGY BACKGROUND

The architecture of the power system was built to transfer the power from the generation to the load (distribution system) through the transmission system. This power system is divided into three categories: Radial systems, inter-connected areas and complex network. The use of the transmission system must maintain the adequacy and the security of the system. Unfortunately, This transmission system has steady-state limits such as angular stability, thermal and voltage limits. Besides, the transmission system is constrained by several stability conditions influenced by transient, dynamic voltage or sub-synchronous resonance phenomena. These issues can be solved by properly manipulating the system parameters. One of the common methods to control these parameters is by supplying and absorbing reactive power. For example, when the system under heavy load the voltage may fall below the allowed
limit. To solve this issue, it is required to supply reactive power to correct the load power factor. This can be achieved by applying the classical method, mechanically switched shunt capacitors (MSC), or engaging FACTS devices. FACTS controllers are used to increase the limits of the existing transmission system and mitigate the risk of the stability issues. It is preferred to use FACTS system over MSC because it has a rapid controllability of the voltage, current, frequency and phase of the power system. The FACTS controller that serve in this case is either Static Var Compensator (SVC) or Static Synchronous Compensator (STATCOM). STATCOM is the technology that we are using to evaluate the performance of the VFSPWM technique. [1,47–49]

8.2 STATCOM SYSTEM

STATCOM systems consist of a DC source behind a VSC devices that is connected to the grid through a coupling transformer. Figure 8.1 shows the single line diagram of the STATCOM system. The reactive power exchange between the converter and the grid is based on the voltage difference across the coupling transformer that has (X) reactance. In other words, when the voltage at the converter, \( E_s \) is greater than the grid voltage, \( E_t \), the reactive current \( I_q \) flows from the converter to the grid and the converter is acting as a capacitor and supplies reactive power. On the other hand, when the voltage at the converter, \( E_s \) is less than the grid voltage, \( E_t \), the reactive current \( I_q \) flows from the grid to the converter to act as an inductance and absorbs reactive power. These two voltages are in phase. The grid voltage \( E_t \) increases when it absorbs reactive power and decreases when supplies reactive power. The amount of the reactive power exchanged between the converter and the grid can be calculated using equation (8.1). The exchange in the reactive power is shown in Figure 8.2.

\[
Q = \frac{E_t^2 - E_t E_s}{X} \quad (8.1)
\]

The converter voltage \( E_s \) is controlled by changing the modulation index, \( m_a \) for the inverter. Initially, the value of the \( m_a \) is chosen to be equal to 0.8, the nominal value. Figure
Figure 8.1: STATCOM Single-Line Diagram

Figure 8.2: STATCOM Reactive Power Exchange [1]
8.3 shows the control of the converter voltage by changing $m_a$. The operation region is $0.6 \leq m_a \leq 1$. When $m_a$ is equal to 0.8, there is no reactive power exchange between the converter and the grid because the inverter output voltage $E_s$ is equal to the grid voltage $E_t$. When $m_a \leq 0.8$ the grid is supplying reactive power and when $m_a \geq 0.8$ the grid is absorbing reactive power from the converter. [1,47]

![Figure 8.3: STATCOM VSC Output Control](image)

8.3 SYSTEM ANALYSIS

In this section we analyze a network that is simplified to the Thevenin circuit. The PSCAD model of this system is shown in Figure 8.4. The AC Grid is represented by the short-circuit capacity (SSC) and system impedance that is described by the X/R ratio. These parameters can identify the strength of the system. For example, when the X/R ratio is small, the AC system may be called a ”weak system”. Also, when the SSC is low with respect to the actual system capacity, the AC system is weak as well.
The purpose of this section is to compare the VFSPWM techniques to the SPWM in the STATCOM application. The major advantage of the VFSPWM is the increase in the fundamental component of the inverter output voltage. In other words, when using the VFSPWM technique on the same system that uses the SPWM technique, the increase in the fundamental component allow using a smaller DC source and the inverter may generate the same nominal voltage. Table 8.1 shows a comparison between the ratings of the DC source. The reduction in the sizing of the DC source is ranging from 5% to 23.4%. This reduction introduces a potential savings in the STATCOM projects. The rest of this section presents the design of the filters for each technique. The AC system has the following specifications:

- **Power system network**
  - Voltage: 400kV

- **Transformer**
  - Voltage levels: 400/3.2 kV
  - Rating: 190 MVA
  - Impedance: \( Z_T = 0.38 + j11.995\% \)
  - Connection: Solidly-grounded Y / Resistance-grounded Y \( (R_g = 476.3\Omega) \)
Table 8.1: STATCOM DC Source Rating Comparison

<table>
<thead>
<tr>
<th></th>
<th>V_{DC} (kV)</th>
<th>DC Source Reduced (% of SPWM)</th>
<th>Ractive Power (MVar)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPWM</td>
<td>6.53</td>
<td>-</td>
<td>120-120</td>
</tr>
<tr>
<td>VFSPWM_{12-0}</td>
<td>5</td>
<td>23.4</td>
<td>23-17</td>
</tr>
<tr>
<td>VFSPWM_{12-3}</td>
<td>6.2</td>
<td>5</td>
<td>147-135</td>
</tr>
<tr>
<td>VFSPWM_{12-6}</td>
<td>5.6</td>
<td>14.2</td>
<td>71-71</td>
</tr>
<tr>
<td>VFSPWM_{12-9}</td>
<td>5.7</td>
<td>12.7</td>
<td>80-80</td>
</tr>
</tbody>
</table>

8.3.1 SPWM System Analysis

8.3.1.1 System Impedance The calculation of the system should be in per unit, therefore base values are the first thing that is calculated. The base values at the low-voltage bus are calculated as follows:

\[ S_{base} = 100 \text{ MVA} \]
\[ V_{baseL} = 3.2 \text{ kV} \]
\[ I_{baseL} = \frac{100 \text{ MVA}}{(\sqrt{3} \times 3.2 \text{ kV})} \]
\[ = 18.042 \text{ kA} \]
\[ Z_{baseL} = \frac{(3.2 \text{ kV})^2}{100 \text{ MVA}} \]
\[ = 0.1024 \Omega \]

The base values at the high-voltage bus

\[ S_{base} = 100 \text{ MVA} \]
\[ V_{baseH} = 400 \text{ kV} \]
\[
I_{\text{base}H} = \frac{100 \text{ MVA}}{(\sqrt{3} \times 400 \text{kV})} = 144.33 \text{ A}
\]
\[
Z_{\text{base}H} = \frac{(400 \text{kV})^2}{100 \text{ MVA}} = 1600 \text{ Ω}
\]

- 400 kV equivalent system

\[
SCC = 10000 \text{ MVA}
\]
\[
V = 400 \text{kV}
\]
\[
Z_{\text{sys}} = \frac{(400 \text{kV})^2}{10000 \text{ MVA}} \tan^{-1}(20)
\]
\[
= 16 \angle 87.14^\circ \Omega
\]
\[
I_{3\phi SC} = \frac{(400 \text{kV})}{\sqrt{3} \times 16 \angle 87.14^\circ}
\]
\[
= 14.43 \angle -87.14 \text{kA}
\]
\[
= 100 \angle -87.14^\circ \text{ pu}
\]

using \( I_{1\phi SC} = 0.9 I_{3\phi SC} \),

\[
I_{1\phi SC} = I_{3\phi SC}
\]
\[
= 12.99 \angle -87.14 \text{kA}
\]
\[
= 90 \angle -87.14^\circ \text{ pu}
\]
\[
Z^+ = \frac{1}{I_{3\phi SC}}
\]
\[
= 0.01 \angle -87.14^\circ \text{ pu}
\]
\[
= 0.0005 + j0.00999 \text{ pu}
\]
\[
Z^0 = \frac{3}{I_{1\phi SC}} - 2Z^+
\]
\[
= 0.013333 \angle 87.14^\circ \text{pu}
\]
\[
= 0.000666 + j0.013316 \text{ pu}
\]
\[
Z_{\text{sys}}(h) = \begin{cases} 0.000666 + j0.013316 \times h \text{ pu}, & h = 3n = 3, 6, 9, 12, \ldots \\ 0.0005 + j0.00999 \times h \text{ pu} \end{cases}
\]
• Transformer

\[ R_g = \frac{476.3}{0.1024} = 46.5137 \text{ pu} \]

\[ Z_T^+ = Z_T^- = \frac{S_{\text{new}}}{S_{\text{old}}} \ast Z_T \]

\[ = \frac{100}{190} (0.0038 + j11.995) \]

\[ = 0.002 + j0.063132 \text{ pu} \]

\[ Z_T^0 = Z_T^+ + \frac{3 + R_g}{Z_{\text{baseL}}} \]

\[ = (0.002 + j0.063132) + (3 \ast 46.5137) \]

\[ = 139.5411 + j0.063132 \text{ pu} \]

\[ Z_T(h) = \begin{cases} 139.5411 + j0.063132 \ast h \text{ pu, } h = 3n = 3, 6, 9, 12, \ldots \\ 0.002 + j0.063132 \ast h \text{ pu} \end{cases} \tag{8.3} \]

• Filters

This has four-branch filters where

\[ Q_{10} = 16 \text{ MVar} \]
\[ Q_{14} = 16 \text{ MVar} \]
\[ Q_{23} = 16 \text{ MVar} \]
\[ Q_{25} = 16 \text{ MVar} \]

Therefore, the total is \( Q_f = 64 \text{ MVar} \).
1. Branch 1: $h_n = 10$

\[
Q_{C_{10}} = \frac{h_n^2 - 1}{h_n^2} = 16\left(\frac{10^2 - 1}{10^2}\right) = 15.84 \text{ MVar}
\]

\[
X_{C_{10}} = \frac{V_{LL}^2}{3 \cdot Q_{C_{10}}} = \frac{3.2kV^2}{3 \cdot 15.84 \text{ MVar}} = 0.2155 \Omega
\]

\[
C_{10} = \frac{1}{2 \pi f X_{C_{10}}} = \frac{1}{2 \pi \cdot 60 \cdot 0.2155} = 12.31 \text{ mF}
\]

\[
X_{L_{10}} = \frac{X_{C_{10}}}{h_n^2} = \frac{0.2155}{10^2} = 0.002155 \Omega
\]

\[
L_{10} = \frac{X_{L_{10}}}{2 \pi f} = \frac{0.002155}{2 \pi \cdot 60} = 5.7162 \mu H
\]

\[
X_n = \sqrt{X_{C_{10}} \cdot X_{L_{10}}} = \sqrt{0.2155 \cdot 0.002155} = 0.02155 \Omega
\]

\[
R_{10} = \frac{X_n}{Q}, \text{ where } Q = 60\]
\[
= \frac{0.02155}{60} = 0.0003592 \Omega
\]

\[
Z_{F_1} = 0.0003592 + j(h \cdot 0.002155 - \frac{0.2155}{h}) \quad (8.4)
\]

99
2. Branch 2: \( h_n = 14 \)

\[
Q_{C14} = \frac{Q_{14} h_n^2 - 1}{h_n^2} = 16\left(\frac{14^2 - 1}{14^2}\right) = 15.91837 \text{ MVar}
\]

\[
X_{C14} = \frac{V_{LL}^2}{3 \ast Q_{C14}} = \frac{3.2kV^2}{3 \ast 15.91837MV_ar} = 0.214427\Omega
\]

\[
C_{14} = \frac{1}{2 \ast \pi \ast f \ast X_{C14}} = \frac{1}{2 \ast \pi \ast 60 \ast 0.214427} = 12.37\text{mF}
\]

\[
X_{L14} = \frac{X_{C14}}{h_n^2} = \frac{0.214427}{14^2} = 0.001094\Omega
\]

\[
L_{14} = \frac{X_{L14}}{2 \ast \pi \ast f} = \frac{0.001094}{2 \ast \pi \ast 60} = 2.902 \mu H
\]

\[
X_n = \sqrt{X_{C14} \ast X_{L14}} = \sqrt{0.214427 \ast 0.001094} = 0.015316\Omega
\]

\[
R_{14} = \frac{X_n}{Q}, \text{ where } Q = 60 = \frac{0.015316}{60} = 0.0002553\Omega
\]

\[
Z_{F2} = 0.0002553 + j(h \ast 0.001094 - \frac{0.214427}{h}) \tag{8.5}
\]
3. Branch 3: $h_n = 23$

\[
Q_{C_{23}} = Q_{23} \frac{h_n^2 - 1}{h_n^2} \\
= 16 \left( \frac{23^2 - 1}{23^2} \right) \\
= 15.9698 \text{ MVar}
\]

\[
X_{C_{23}} = \frac{V_{LL}^2}{3 \times Q_{C_{23}}} \\
= \frac{3.2kV^2}{3 \times 15.9698 \text{ MVar}} \\
= 0.213737 \Omega
\]

\[
C_{23} = \frac{1}{2 \times \pi \times f \times X_{C_{23}}} \\
= \frac{1}{2 \times \pi \times 60 \times 0.213737} \\
= 12.41 \text{ mF}
\]

\[
X_{L_{23}} = \frac{X_{C_{23}}}{h_n^2} \\
= \frac{0.213737}{23^2} \\
= 0.000404 \Omega
\]

\[
L_{23} = \frac{X_{L_{23}}}{2 \times \pi \times f} \\
= \frac{0.000404}{2 \times \pi \times 60} \\
= 1.07175 \mu H
\]

\[
X_n = \sqrt{X_{C_{23}} \times X_{L_{23}}} \\
= \sqrt{0.213737 \times 0.000404} \\
= 0.00929 \Omega
\]

\[
R_{23} = \frac{X_n}{Q}, \text{ where } Q = 60 \\
= \frac{0.00929}{60} \\
= 0.00015488 \Omega
\]

\[
Z_{F_3} = 0.00015488 + j(h \times 0.000404 - \frac{0.213737}{h}) \tag{8.6}
\]
4. Branch 4: \( h_n = 25 \)

\[
\begin{align*}
Q_{C25} &= Q_{25} \frac{h_n^2 - 1}{h_n^2} \\
&= 16 \left( \frac{25^2 - 1}{25^2} \right) \\
&= 15.9744 \text{ MVar} \\
X_{C25} &= \frac{V_{LL}^2}{3 \ast Q_{C25}} \\
&= \frac{3.2kV^2}{3 \ast 15.9744MV\text{ar}} \\
&= 0.213675\Omega \\
C_{25} &= \frac{1}{2 \ast \pi \ast f \ast X_{C25}} \\
&= \frac{1}{2 \ast \pi \ast 60 \ast 0.213675} \\
&= 12.414\text{mF} \\
X_{L25} &= \frac{X_{C25}}{h_n^2} \\
&= \frac{0.213675}{25^2} \\
&= 0.000342\Omega \\
L_{25} &= \frac{X_{L25}}{2 \ast \pi \ast f} \\
&= \frac{0.000342}{2 \ast \pi \ast 60} \\
&= 0.90687\mu\text{H} \\
X_n &= \sqrt{X_{C25} \ast X_{L25}} \\
&= \sqrt{0.213675 \ast 0.000342} \\
&= 0.008547\Omega \\
R_{25} &= \frac{X_n}{Q}, \text{ where } Q = 60 \\
&= \frac{0.008547}{60} \\
&= 0.00014245\Omega \\
Z_{F4} &= 0.00014245 + j(h \ast 0.000342 \ast \frac{0.213675}{h}) \quad (8.7)
\end{align*}
\]
The filter impedance is calculated as follows:

\[ Z_F = \frac{1}{Z_{\text{baseL}} \left( \frac{1}{Z_{F_1}} + \frac{1}{Z_{F_2}} + \frac{1}{Z_{F_3}} + \frac{1}{Z_{F_4}} \right)} \]

Figure 8.5a shows the impedance of the filter. It can be noticed that the filter is acting like a short circuit when the frequency is: 600 Hz, 840 Hz, 1380 Hz and 1500 Hz. The phase of the filter is depicted in Figure 8.5b. The phase is alternating between 90° and -90° at these resonance frequency. The filters supply the system with reactive power at the line frequency, capacitive. Figure 8.5c shows the filter resistance versus the reactance. This figure shows the four points where the filter is almost like a short circuit, very small resistance and zero reactance.

8.3.1.2 Impedance Scan  The impedance when we look from the 3.2 kV bus can be calculated as follows:

\[
Z_{3.2}(h) = Z_{\text{sys}}(h) + Z_T(h) \\
= (0.005 + j0.009999 \times h) + (0.002 + j0.063135 \times h) \\
= 0.0025 + j0.073155 \times h \text{ pu} \quad (8.8)
\]

Where the impedance of the system when looking for the converter end is as follows:

\[
Z_{eq}(h) = \frac{1}{\frac{1}{Z_F(h)} + \frac{1}{Z_{3.2}(h)}} \\ (8.9)
\]
Figure 8.5: Filter Impedance and Impedance Locus using SPWM
Figure 8.6: System Impedance using SPWM
8.3.1.3 Harmonic Injection  The voltage source harmonics, $V_{3.2}(h)$, is calculated using the equations in section 3.4.1.1. The filter current is calculated using equation (8.8):

$$I_F(h) = \frac{V_{3.2}(h)}{Z_F(h)} \quad (8.10)$$

From equation (8.8), the current that flows through the transformer to the system has the following relationship:

$$I_T(h) = I_{sys}(h) = \frac{V_{3.2}(h)}{Z_{3.2}(h)} \quad (8.11)$$

Finally the voltage at the 400 kV bus can be calculated as follows:

$$V_{400}(h) = Z_{sys}(h) * I_{sys}(h) \quad (8.12)$$

The impedance from the 3.2 kV bus, $Z_{3.2}$, and from the converter, $Z_{eq}$, are plotted in Figure 8.6. The system resonance is at 130 Hz with a value equal to 0.566 Ω. The rest of the resonance are caused by the filter and placed at the harmonic orders they are filtering, 10th, 14th, 23rd, and 25th. Table 8.2 shows the calculated harmonics. These values comply with IEEE 519 harmonic distortion limits:

$$V_h \leq 3\% \text{ and } THD_v \leq 5\% \text{ for } V \leq 69kV$$
$$V_h \leq 1\% \text{ and } THD_v \leq 1.5\% \text{ for } V \geq 161kV \quad [50]$$

The transformer is safe because the K-factor is 1.0615 which less than 4. The equation used to calculate the K-factor is:

$$K = \frac{\sum_{h=1} I_h^2}{1 + THD_i^2}$$

And the derating factor is

$$D = \frac{1.15}{1 + 1.15K}$$

By using the standard transformer, it can not be loaded more than 99.20%.
Table 8.2: SPWM Harmonic Distortion Results

<table>
<thead>
<tr>
<th>THD. %</th>
<th>3.2 kV bus</th>
<th>Converter Current</th>
<th>Current to Transformer</th>
<th>Current to Filter</th>
<th>400kV bus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fundamental</td>
<td>PU</td>
<td>1.08</td>
<td>1.33E-03</td>
<td>0.00173</td>
<td>0.02123</td>
</tr>
<tr>
<td>h=2</td>
<td>% of Fund.</td>
<td>0.1252</td>
<td>4.868</td>
<td>0.03386</td>
<td>3.024</td>
</tr>
<tr>
<td>3</td>
<td>0.1463</td>
<td>5.689</td>
<td>0.03958</td>
<td>3.535</td>
<td>0.01999</td>
</tr>
<tr>
<td>4</td>
<td>0.1616</td>
<td>6.282</td>
<td>0.04370</td>
<td>3.903</td>
<td>0.02208</td>
</tr>
<tr>
<td>5</td>
<td>0.3533</td>
<td>13.734</td>
<td>0.08554</td>
<td>8.533</td>
<td>0.04827</td>
</tr>
<tr>
<td>6</td>
<td>0.1200</td>
<td>4.666</td>
<td>0.03246</td>
<td>2.899</td>
<td>0.01640</td>
</tr>
<tr>
<td>7</td>
<td>0.2595</td>
<td>10.089</td>
<td>0.07019</td>
<td>6.268</td>
<td>0.03546</td>
</tr>
<tr>
<td>8</td>
<td>0.2114</td>
<td>8.217</td>
<td>0.05716</td>
<td>5.105</td>
<td>0.02888</td>
</tr>
<tr>
<td>9</td>
<td>0.0628</td>
<td>2.441</td>
<td>0.01698</td>
<td>1.516</td>
<td>0.00858</td>
</tr>
<tr>
<td>10</td>
<td>0.6635</td>
<td>25.792</td>
<td>0.17943</td>
<td>16.025</td>
<td>0.09064</td>
</tr>
<tr>
<td>11</td>
<td>0.5525</td>
<td>21.477</td>
<td>0.14941</td>
<td>13.344</td>
<td>0.07548</td>
</tr>
<tr>
<td>12</td>
<td>5.4791</td>
<td>213.002</td>
<td>1.48177</td>
<td>132.336</td>
<td>0.74856</td>
</tr>
<tr>
<td>13</td>
<td>0.2681</td>
<td>10.422</td>
<td>0.07250</td>
<td>6.475</td>
<td>0.03663</td>
</tr>
<tr>
<td>14</td>
<td>0.2828</td>
<td>10.992</td>
<td>0.07647</td>
<td>6.829</td>
<td>0.03863</td>
</tr>
<tr>
<td>15</td>
<td>0.3016</td>
<td>11.725</td>
<td>0.08156</td>
<td>7.284</td>
<td>0.04120</td>
</tr>
<tr>
<td>16</td>
<td>0.2716</td>
<td>10.558</td>
<td>0.07345</td>
<td>5.560</td>
<td>0.03711</td>
</tr>
<tr>
<td>17</td>
<td>0.1364</td>
<td>5.304</td>
<td>0.03690</td>
<td>3.295</td>
<td>0.01864</td>
</tr>
<tr>
<td>18</td>
<td>0.1378</td>
<td>5.357</td>
<td>0.03727</td>
<td>3.329</td>
<td>0.01883</td>
</tr>
<tr>
<td>19</td>
<td>0.5524</td>
<td>21.475</td>
<td>0.14939</td>
<td>13.342</td>
<td>0.07547</td>
</tr>
<tr>
<td>20</td>
<td>0.2152</td>
<td>8.364</td>
<td>0.05819</td>
<td>5.197</td>
<td>0.02939</td>
</tr>
<tr>
<td>21</td>
<td>1.2273</td>
<td>47.713</td>
<td>0.33192</td>
<td>29.644</td>
<td>0.16768</td>
</tr>
<tr>
<td>22</td>
<td>0.5482</td>
<td>21.312</td>
<td>0.14826</td>
<td>13.241</td>
<td>0.07490</td>
</tr>
<tr>
<td>23</td>
<td>0.8704</td>
<td>33.838</td>
<td>0.23540</td>
<td>21.023</td>
<td>0.11892</td>
</tr>
<tr>
<td>24</td>
<td>0.0432</td>
<td>1.678</td>
<td>0.01167</td>
<td>1.042</td>
<td>0.00590</td>
</tr>
<tr>
<td>25</td>
<td>0.8820</td>
<td>34.288</td>
<td>0.23853</td>
<td>21.303</td>
<td>0.12050</td>
</tr>
<tr>
<td>26</td>
<td>0.7493</td>
<td>29.131</td>
<td>0.20265</td>
<td>18.099</td>
<td>0.10238</td>
</tr>
<tr>
<td>27</td>
<td>0.2028</td>
<td>7.884</td>
<td>0.05485</td>
<td>4.898</td>
<td>0.02771</td>
</tr>
<tr>
<td>28</td>
<td>0.5638</td>
<td>21.918</td>
<td>0.15248</td>
<td>13.618</td>
<td>0.07703</td>
</tr>
<tr>
<td>29</td>
<td>0.1246</td>
<td>4.842</td>
<td>0.03369</td>
<td>3.009</td>
<td>0.01702</td>
</tr>
<tr>
<td>30</td>
<td>1.2102</td>
<td>47.048</td>
<td>0.32729</td>
<td>29.230</td>
<td>0.16534</td>
</tr>
<tr>
<td>31</td>
<td>0.3219</td>
<td>12.514</td>
<td>0.08706</td>
<td>7.775</td>
<td>0.04398</td>
</tr>
</tbody>
</table>

K-factor= 1.061505223
D= 0.992041427
8.3.2 VFSPWM\textsubscript{12–9} System Analysis

In this section we are calculating the harmonics at several points in the STATCOM system using the VFSPWM\textsubscript{12–9}. The calculation is brief because it follows the same procedure that presented in section 8.3.1.

8.3.2.1 System Impedance  The system impedance is the same. We are presenting the values here for convenience.

\begin{align*}
S_{\text{base}} &= 100\, MVA \\
V_{\text{base}L} &= 3.2\, kV \\
I_{\text{base}L} &= 18.042\, kA \\
Z_{\text{base}L} &= 0.1024\, \Omega
\end{align*}

The base values at the high-voltage bus

\begin{align*}
S_{\text{base}} &= 100\, MVA \\
V_{\text{base}H} &= 400\, kV \\
I_{\text{base}H} &= 144.33\, A \\
Z_{\text{base}H} &= 1600\, \Omega
\end{align*}
• 400 kV equivalent system

\[
SCC = 10000 \text{ MVA}
\]

\[
V = 400 \text{kV}
\]

\[
Z_{sys} = 16 \angle 87.14^\circ \Omega
\]

\[
I_{3\phi SC} = 100 \angle -87.14^\circ \text{ pu}
\]

using \(I_{1\phi SC} = 0.9I_{3\phi SC}\),

\[
I_{1\phi SC} = 90 \angle -87.14^\circ \text{ pu}
\]

\[
Z^+ = 0.0005 + j0.00999 \text{ pu}
\]

\[
Z^0 = 0.000666 + j0.013316 \text{ pu}
\]

\[
Z_{sys}(h) = \begin{cases} 
0.000666 + j0.013316 \times h \text{ pu}, & h = 3n = 3, 6, 9, 12, \ldots \\
0.0005 + j0.00999 \times h \text{ pu} 
\end{cases}
\]

(8.13)

• Transformer

\[
R_g = 46.5137 \text{ pu}
\]

\[
Z_T^+ = Z_T^- = 0.002 + j0.063132 \text{ pu}
\]

\[
Z_T^0 = 139.5411 + j0.063132 \text{ pu}
\]

\[
Z_T(h) = \begin{cases} 
139.5411 + j0.063132 \times h \text{ pu}, & h = 3n = 3, 6, 9, 12, \ldots \\
0.002 + j0.063132 \times h \text{ pu} 
\end{cases}
\]

(8.14)

• Filters

This has five-branch filters where \(Q_{11} = 30 \text{ MVar}, Q_{13} = 25 \text{ MVar}, Q_{18} = 10, Q_{24} = 20 \text{ MVar}\) and \(Q_{30} = 10 \text{ MVar}\). Therefore, the total is \(Q_f = 95 \text{ MVar}\).
1. Branch 1: \( h_n = 11 \)

\[
Q_{C_{11}} = 29.752 \text{ MVar} \\
X_{C_{11}} = 0.11473 \ \Omega \\
C_{11} = 23.121 \ \text{mF} \\
X_{L_{11}} = 0.000948 \ \Omega \\
L_{11} = 2.515 \ \mu H \\
X_n = 0.01043 \ \Omega \\
R_{11} = 0.00017383 \ \Omega \\
Z_{F_1} = 0.00017383 + j(h \ast 0.000948 - \frac{0.11473}{h}) \quad (8.15)
\]

2. Branch 2: \( h_n = 13 \)

\[
Q_{C_{13}} = 24.85207 \text{ MVar} \\
X_{C_{13}} = 0.137346 \ \Omega \\
C_{13} = 19.31315 \ \text{mF} \\
X_{L_{13}} = 0.000813 \ \Omega \\
L_{13} = 2.1557 \ \mu H \\
X_n = 0.010565 \ \Omega \\
R_{13} = 0.000176085 \ \Omega \\
Z_{F_2} = 0.000176085 + j(h \ast 0.000813 - \frac{0.137346}{h}) \quad (8.16)
\]
3. Branch 3: $h_n = 18$

$$Q_{C_{18}} = 9.9654 \text{ MVar}$$
$$X_{C_{18}} = 0.34252 \Omega$$
$$C_{18} = 7.744 \text{ mF}$$
$$X_{L_{18}} = 0.001185 \Omega$$
$$L_{18} = 3.1438 \mu H$$
$$X_n = 0.020148 \Omega$$
$$R_{18} = 0.0003358 \Omega$$
$$Z_{F_3} = 0.0003358 + j(h \ast 0.001185 - \frac{0.34252}{h}) \tag{8.17}$$

4. Branch 4: $h_n = 24$

$$Q_{C_{24}} = 19.96528 \text{ MVar}$$
$$X_{C_{24}} = 0.170963 \Omega$$
$$C_{24} = 15.515505 \text{ mF}$$
$$X_{L_{24}} = 0.000297 \Omega$$
$$L_{24} = 0.787318 \mu H$$
$$X_n = 0.007123 \Omega$$
$$R_{24} = 0.000118725 \Omega$$
$$Z_{F_4} = 0.000118725 + j(h \ast 0.000297 - \frac{0.170963}{h}) \tag{8.18}$$
5. Branch 5: \( h_n = 30 \)

\[
\begin{align*}
Q_{C_{30}} & = 9.9888 \text{ MVar} \\
X_{C_{30}} & = 0.341713 \text{ } \Omega \\
C_{30} & = 7.7626 \text{ mF} \\
X_{L_{30}} & = 0.00038 \text{ } \Omega \\
L_{30} & = 1.00714 \mu H \\
X_n & = 0.01139 \text{ } \Omega \\
R_{30} & = 0.000189891 \text{ } \Omega \\
Z_{F_5} & = 0.000189891 + j(h \ast 0.00038 - \frac{0.341713}{h}) \\
\end{align*}
\]

The filter impedance is calculated as follows:

\[
Z_F = \frac{1}{Z_{\text{base}L} \left( \frac{1}{Z_{F_1}} + \frac{1}{Z_{F_2}} + \frac{1}{Z_{F_3}} + \frac{1}{Z_{F_4}} + \frac{1}{Z_{F_5}} \right)}
\]

Figure 8.7a shows the impedance of the filter. The filter short circuits the voltages at 660 Hz, 780 Hz, 1080 Hz, 1440 and 1800 Hz. The phase of the filter is depicted in Figure 8.7b. The phase is alternating between 90° and -90° at these resonance frequency. The filters supply the system with reactive power at the line frequency, capacitive. Figure 8.7c shows the filter resistance versus the reactance. This figure shows the four points where the filter is almost like a short circuit, very small resistance and zero reactance.

### 8.3.2.2 Impedance Scan

The impedance when we look from the 3.2 kV bus can be calculated as follows:

\[
Z_{3.2}(h) = Z_{sys}(h) + Z_T(h)
\]

\[
\begin{align*}
&= (0.005 + j0.009999 \ast h) + (0.002 + j0.063135 \ast h) \\
&= 0.0025 + j0.073155 \ast h \text{ pu} \\
\end{align*}
\]

Where the impedance of the system when looking for the converter end is as follows:

\[
Z_{eq}(h) = \frac{1}{\frac{1}{Z_F(h)} + \frac{1}{Z_{3.2}(h)}}
\]
Figure 8.7: Filter Impedance and Impedance Locus using VFSPWM\textsubscript{12–9}
Figure 8.8: System Impedance using VFSPWM$\text{}_{12-9}$
8.3.2.3 Harmonic Injection The voltage source harmonics, $V_{3.2}(h)$, is calculated using the equations in section 3.4.1.2. The filter current is calculated using equation (8.20):

$$I_F(h) = \frac{V_{3.2}(h)}{Z_F(h)} \quad (8.22)$$

From equation (8.20), the current that flows through the transformer to the system has the following relationship:

$$I_T(h) = I_{sys}(h) = \frac{V_{3.2}(h)}{Z_{3.2}(h)} \quad (8.23)$$

Finally the voltage at the 400 kV bus can be calculated as follows:

$$V_{400}(h) = Z_{sys}(h) * I_{sys}(h) \quad (8.24)$$

The impedance from the 3.2 kV bus, $Z_{3.2}$, and from the converter, $Z_{eq}$, are plotted in Figure 8.8. The system resonance is at 130 Hz with a value equal to 0.566 Ω. The rest of the resonance are caused by the filter and placed at the harmonic orders they are filtering, 11th, 13th, 18th, 24th and 30th. Table 8.3 shows the calculated harmonics. These values comply with IEEE 519 harmonic distortion limits:

- $V_h \leq 3\%$ and $THD_v \leq 5\%$ for $V \leq 69kV$
- $V_h \leq 1\%$ and $THD_v \leq 1.5\%$ for $V \geq 161kV$ [50]

The transformer is safe because the K-factor is 1.458 which less than 4. The equation used to calculate the K-factor is:

$$K = \frac{\sum_{h=1}^{n} (\frac{hI_h}{I_1})^2}{1 + THD_i^2}$$

And the derating factor is

$$D = \frac{1.15}{1 + 1.15K}$$

By using the standard transformer, it can not be loaded more than 94.38%.
## Table 8.3: VFSPWM\textsubscript{(12–9)} Harmonic Distortion Results

<table>
<thead>
<tr>
<th>THD, %</th>
<th>3.2 kV bus</th>
<th>Converter Current</th>
<th>Current to Transformer</th>
<th>Current to Filter</th>
<th>400kV bus</th>
</tr>
</thead>
<tbody>
<tr>
<td>h=2</td>
<td>0.7567</td>
<td>107.910</td>
<td>0.20465</td>
<td>55.72</td>
<td>0.10339</td>
</tr>
<tr>
<td>3</td>
<td>1.0255</td>
<td>146.239</td>
<td>0.27735</td>
<td>75.51</td>
<td>0.14011</td>
</tr>
<tr>
<td>4</td>
<td>1.1672</td>
<td>166.445</td>
<td>0.31567</td>
<td>85.95</td>
<td>0.15947</td>
</tr>
<tr>
<td>5</td>
<td>3.5535</td>
<td>506.722</td>
<td>0.96101</td>
<td>261.65</td>
<td>0.48549</td>
</tr>
<tr>
<td>6</td>
<td>1.1184</td>
<td>159.476</td>
<td>0.30245</td>
<td>82.35</td>
<td>0.15279</td>
</tr>
<tr>
<td>7</td>
<td>1.1184</td>
<td>159.476</td>
<td>0.30245</td>
<td>82.35</td>
<td>0.15279</td>
</tr>
<tr>
<td>8</td>
<td>1.2830</td>
<td>182.947</td>
<td>0.34696</td>
<td>94.47</td>
<td>0.17528</td>
</tr>
<tr>
<td>9</td>
<td>1.0995</td>
<td>156.782</td>
<td>0.29734</td>
<td>80.96</td>
<td>0.15021</td>
</tr>
<tr>
<td>10</td>
<td>0.9104</td>
<td>129.814</td>
<td>0.24620</td>
<td>67.03</td>
<td>0.12437</td>
</tr>
<tr>
<td>11</td>
<td>5.2549</td>
<td>749.339</td>
<td>1.42114</td>
<td>386.93</td>
<td>0.71793</td>
</tr>
<tr>
<td>12</td>
<td>0.5902</td>
<td>84.160</td>
<td>0.15961</td>
<td>43.46</td>
<td>0.08603</td>
</tr>
<tr>
<td>13</td>
<td>3.6463</td>
<td>519.951</td>
<td>0.98610</td>
<td>268.48</td>
<td>0.49816</td>
</tr>
<tr>
<td>14</td>
<td>1.0177</td>
<td>145.127</td>
<td>0.27524</td>
<td>74.94</td>
<td>0.13905</td>
</tr>
<tr>
<td>15</td>
<td>0.8777</td>
<td>125.152</td>
<td>0.23735</td>
<td>64.62</td>
<td>0.11991</td>
</tr>
<tr>
<td>16</td>
<td>0.5256</td>
<td>74.945</td>
<td>0.14213</td>
<td>38.70</td>
<td>0.07180</td>
</tr>
<tr>
<td>17</td>
<td>3.9623</td>
<td>565.011</td>
<td>1.07156</td>
<td>291.75</td>
<td>0.54133</td>
</tr>
<tr>
<td>18</td>
<td>0.4485</td>
<td>63.962</td>
<td>0.12130</td>
<td>33.03</td>
<td>0.06128</td>
</tr>
<tr>
<td>19</td>
<td>1.5967</td>
<td>227.682</td>
<td>0.43180</td>
<td>117.57</td>
<td>0.21814</td>
</tr>
<tr>
<td>20</td>
<td>0.3341</td>
<td>47.637</td>
<td>0.09035</td>
<td>24.60</td>
<td>0.04564</td>
</tr>
<tr>
<td>21</td>
<td>2.8082</td>
<td>400.441</td>
<td>0.75945</td>
<td>206.77</td>
<td>0.38366</td>
</tr>
<tr>
<td>22</td>
<td>1.5354</td>
<td>218.937</td>
<td>0.41522</td>
<td>113.05</td>
<td>0.20976</td>
</tr>
<tr>
<td>23</td>
<td>1.5582</td>
<td>222.195</td>
<td>0.42140</td>
<td>114.73</td>
<td>0.21288</td>
</tr>
<tr>
<td>24</td>
<td>1.1247</td>
<td>160.379</td>
<td>0.30416</td>
<td>82.81</td>
<td>0.15366</td>
</tr>
<tr>
<td>25</td>
<td>3.1947</td>
<td>455.558</td>
<td>0.86398</td>
<td>235.23</td>
<td>0.43647</td>
</tr>
<tr>
<td>26</td>
<td>0.3786</td>
<td>53.990</td>
<td>0.10239</td>
<td>27.88</td>
<td>0.05173</td>
</tr>
<tr>
<td>27</td>
<td>0.2519</td>
<td>35.913</td>
<td>0.06811</td>
<td>18.54</td>
<td>0.03441</td>
</tr>
<tr>
<td>28</td>
<td>0.7113</td>
<td>101.434</td>
<td>0.19237</td>
<td>52.38</td>
<td>0.09718</td>
</tr>
<tr>
<td>29</td>
<td>0.8036</td>
<td>114.586</td>
<td>0.21731</td>
<td>59.17</td>
<td>0.10978</td>
</tr>
<tr>
<td>30</td>
<td>0.4711</td>
<td>67.180</td>
<td>0.12741</td>
<td>34.69</td>
<td>0.06436</td>
</tr>
<tr>
<td>31</td>
<td>2.7439</td>
<td>391.272</td>
<td>0.74206</td>
<td>202.04</td>
<td>0.37487</td>
</tr>
</tbody>
</table>

**K-factor=** 1.458680612

**D=** 0.943549414
8.3.3 VFSPWM$_{12-6}$ System Analysis

In this section we are calculating the harmonics at several points in the STATCOM system using the VFSPWM$_{12-6}$. The calculation is brief because it follows the same procedure that presented in section 8.3.1.

8.3.3.1 System Impedance  The system impedance is the same. We are presenting the values here for convenience.

\[ S_{\text{base}} = 100 \text{MVA} \]
\[ V_{\text{baseL}} = 3.2kV \]
\[ I_{\text{baseL}} = 18.042kA \]
\[ Z_{\text{baseL}} = 0.1024\Omega \]

The base values at the high-voltage bus

\[ S_{\text{base}} = 100 \text{MVA} \]
\[ V_{\text{baseH}} = 400kV \]
\[ I_{\text{baseH}} = 144.33A \]
\[ Z_{\text{baseH}} = 1600\Omega \]
• 400 kV equivalent system

\[
SCC = 10000 \text{ MVA} \\
V = 400 \text{kV} \\
Z_{sys} = 16 \angle 87.14^\circ \Omega \\
I_{3\phi SC} = 100 \angle -87.14^\circ \text{ pu}
\]

using \( I_{1\phi SC} = 0.9 I_{3\phi SC} \),

\[
I_{1\phi SC} = 90 \angle -87.14^\circ \text{ pu} \\
Z^+ = 0.0005 + j0.00999 \text{ pu} \\
Z^0 = 0.000666 + j0.013316 \text{ pu} \\
Z_{sys}(h) = \begin{cases} 
0.000666 + j0.013316 \times h \text{ pu}, & h = 3n = 3, 6, 9, 12, \ldots \\
0.0005 + j0.00999 \times h \text{ pu} 
\end{cases}
\] (8.25)

• Transformer

\[
R_g = 46.5137 \text{ pu} \\
Z^+_T = Z^-_T = 0.002 + j0.063132 \text{ pu} \\
Z^0_T = 139.5411 + j0.063132 \text{ pu} \\
Z_T(h) = \begin{cases} 
139.5411 + j0.063132 \times h \text{ pu}, & h = 3n = 3, 6, 9, 12, \ldots \\
0.002 + j0.063132 \times h \text{ pu} 
\end{cases}
\] (8.26)

• Filters

This has three-branch filters where \( Q_{10} = 30 \text{ MVar}, Q_{14} = 25 \text{ MVar}, Q_{21} = 20 \). Therefore, the total is \( Q_f = 75 \text{ MVar} \).
1. Branch 1: $h_n = 10$

\[
Q_{C_{10}} = 29.7 \text{ MVar} \\
X_{C_{10}} = 0.115 \ \Omega \\
C_{10} = 23.05 \ \text{mF} \\
X_{L_{10}} = 0.001149 \ \Omega \\
L_{10} = 3.048 \ \mu H \\
X_n = 0.01149 \ \Omega \\
R_{10} = 0.0001915 \ \Omega \\
Z_{F_1} = 0.0001915 + j(h * 0.001149 - \frac{0.115}{h}) \quad (8.27)
\]

2. Branch 2: $h_n = 14$

\[
Q_{C_{14}} = 24.87 \text{ MVar} \\
X_{C_{14}} = 0.1372 \ \Omega \\
C_{14} = 19.33 \ \text{mF} \\
X_{L_{14}} = 0.0007 \ \Omega \\
L_{14} = 1.857 \ \mu H \\
X_n = 0.0098 \ \Omega \\
R_{14} = 0.0001634 \ \Omega \\
Z_{F_2} = 0.0001634 + j(h * 0.0007 - \frac{0.1372}{h}) \quad (8.28)
\]
3. Branch 3: \( h_n = 21 \) (second-order damped filter)

\[
Q_{C_{21}} = 19.95 \text{ MVar} \\
X_{C_{21}} = 0.17105 \Omega \\
C_{21} = 15.51 \text{ mF} \\
X_{L_{21}} = 0.00038788 \Omega \\
L_{21} = 1.0289 \mu H \\
X_n = 0.008145 \Omega \\
R_{21} = Q \times X_n, \text{ where } Q=5 \\
= 5 \times 0.008145 \\
= 0.04073 \Omega \\
Z_{F_3} = \frac{jR_{21}hX_{L_{21}}}{R_{21} + jhX_{L_{21}}} - j\frac{X_{C_{21}}}{h} \\
= \frac{R_{21}(hX_{L_{21}})^2}{R_{21}^2 + (hX_{L_{21}})^2} + j\left(\frac{R_{21}^2 hX_{L_{21}}}{R_{21}^2 + (hX_{L_{21}})^2} - \frac{X_{C_{21}}}{h}\right) \\
= \frac{6.128 \times 10^{-9}h^2}{0.001659 + 1.5 \times 10^{-7}h^2} + j\left(\frac{6.4347 \times 10^{-6}}{0.001659 + 1.5 \times 10^{-7}h^2} - \frac{0.17105}{h}\right) (8.29)
\]

The filter impedance is calculated as follows:

\[
Z_F = \frac{1}{Z_{\text{base}L} \left(\frac{1}{Z_{F_1}} + \frac{1}{Z_{F_2}} + \frac{1}{Z_{F_3}}\right)}
\]

Figure 8.9a shows the impedance of the filter. The filter short circuits the voltages at 600 Hz, 840 Hz, and at 1260 Hz and higher. The phase of the filter is depicted in Figure 8.9b. The phase is alternating between 90° and -90° at these resonance frequency. The filters supply the system with reactive power at the line frequency, capacitive. Figure 8.9c shows the filter resistance versus the reactance. This figure shows the four points where the filter is almost like a short circuit, very small resistance and zero reactance.
Figure 8.9: Filter Impedance and Impedance Locus using VFSPWM$_{12-6}$
8.3.3.2 Impedance Scan The impedance when we look from the 3.2 kV bus can be calculated as follows:

\[
Z_{3.2}(h) = Z_{sys}(h) + Z_T(h)
\]

\[
= (0.005 + j0.009999 * h) + (0.002 + j0.063135 * h)
\]

\[
= 0.0025 + j0.073155 * h \text{ pu} \tag{8.30}
\]

Where the impedance of the system when looking for the converter end is as follows:

\[
Z_{eq}(h) = \frac{1}{\frac{1}{Z_F(h)} + \frac{1}{Z_{3.2}(h)}} \tag{8.31}
\]

![System Impedance using VFSPWM](image)

Figure 8.10: System Impedance using VFSPWM$_{12-6}$
8.3.3.3 Harmonic Injection  The voltage source harmonics, \(V_{3.2}(h)\), is calculated using the equations in section 3.4.1.2. The filter current is calculated using equation (8.30):

\[
I_F(h) = \frac{V_{3.2}(h)}{Z_F(h)} \tag{8.32}
\]

From equation (8.30), the current that flows through the transformer to the system has the following relationship:

\[
I_T(h) = I_{sys}(h) = \frac{V_{3.2}(h)}{Z_{3.2}(h)} \tag{8.33}
\]

Finally the voltage at the 400 kV bus can be calculated as follows:

\[
V_{400}(h) = Z_{sys}(h) * I_{sys}(h) \tag{8.34}
\]

The impedance from the 3.2 kV bus, \(Z_{3.2}\), and from the converter, \(Z_{eq}\), are plotted in Figure 8.10. The system resonance is at 130 Hz with a value equal to 0.566 Ω. The rest of the resonance are caused by the filter and placed at the harmonic orders they are filtering, 10\(^{th}\), 14\(^{th}\), 21\(^{th}\). Table 8.4 shows the calculated harmonics. These values comply with IEEE 519 harmonic distortion limits:

\[
V_h \leq 3\% \text{ and } THD_v \leq 5\% \text{ for } V \leq 69kV
\]
\[
V_h \leq 1\% \text{ and } THD_v \leq 1.5\% \text{ for } V \geq 161kV \ [50]
\]

The transformer is safe because the K-factor is 1.5277 which less than 4. The equation used to calculate the K-factor is:

\[
K = \frac{\sum_{h=1} \left( \frac{hI_h}{I_1} \right)^2}{1 + THD_i^2}
\]

And the derating factor is

\[
D = \frac{1.15}{1+1.15K}
\]

By using the standard transformer, it can not be loaded more than 93.56%.
Table 8.4: VFSPWM\(_{(12-6)}\) Harmonic Distortion Results

<table>
<thead>
<tr>
<th>THD, %</th>
<th>3.2 kV bus</th>
<th>Converter Current</th>
<th>Current to Transformer</th>
<th>Current to Filter</th>
<th>400kV bus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fundamental</td>
<td>1.11</td>
<td>1.23E-03</td>
<td>0.0019</td>
<td>0.0245</td>
<td>1</td>
</tr>
<tr>
<td>h=2</td>
<td>2.4570</td>
<td>49.724</td>
<td>0.66447</td>
<td>25.31</td>
<td>0.33568</td>
</tr>
<tr>
<td>3 % of Fund.</td>
<td>1.5794</td>
<td>31.964</td>
<td>0.42714</td>
<td>16.27</td>
<td>0.21578</td>
</tr>
<tr>
<td>4</td>
<td>3.6794</td>
<td>74.463</td>
<td>0.99506</td>
<td>37.90</td>
<td>0.50269</td>
</tr>
<tr>
<td>5</td>
<td>1.9454</td>
<td>39.371</td>
<td>0.52612</td>
<td>20.04</td>
<td>0.26579</td>
</tr>
<tr>
<td>6</td>
<td>0.9737</td>
<td>19.706</td>
<td>0.26334</td>
<td>10.03</td>
<td>0.13303</td>
</tr>
<tr>
<td>7</td>
<td>0.5665</td>
<td>11.464</td>
<td>0.15320</td>
<td>5.83</td>
<td>0.07739</td>
</tr>
<tr>
<td>8</td>
<td>3.1734</td>
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<td>0.43376</td>
<td>16.52</td>
<td>0.21913</td>
</tr>
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<td>0.35846</td>
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<td>0.21707</td>
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<td>15.04</td>
<td>0.19953</td>
</tr>
<tr>
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<tr>
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<td>0.20918</td>
<td>7.97</td>
<td>0.10567</td>
</tr>
</tbody>
</table>

K-factor= 1.527760527
D= 0.93559519
8.3.4 VFSPWM$_{12-3}$ System Analysis

In this section we are calculating the harmonics at several points in the STATCOM system using the the VFSPWM$_{12-3}$. The calculation is brief because it follows the same procedure that presented in section 8.3.1.

8.3.4.1 System Impedance  The system impedance is the same. We are presenting the values here for convenience.

\[ S_{\text{base}} = 100 \text{ MVA} \]
\[ V_{\text{base}L} = 3.2 \text{kV} \]
\[ I_{\text{base}L} = 18.042 \text{kA} \]
\[ Z_{\text{base}L} = 0.1024 \Omega \]

The base values at the high-voltage bus

\[ S_{\text{base}} = 100 \text{ MVA} \]
\[ V_{\text{base}H} = 400 \text{kV} \]
\[ I_{\text{base}H} = 144.33 \text{A} \]
\[ Z_{\text{base}H} = 1600 \Omega \]
• 400 kV equivalent system

\[
SCC = 10000 \text{ MVA} \\
V = 400 \text{kV} \\
Z_{sys} = 16\angle87.14^\circ \Omega \\
I_{3\phi SC} = 100\angle-87.14^\circ \text{ pu}
\]

using \( I_{1\phi SC} = 0.9I_{3\phi SC} \),

\[
I_{1\phi SC} = 90\angle-87.14^\circ \text{ pu} \\
Z^+ = 0.0005 + j0.00999 \text{ pu} \\
Z^0 = 0.000666 + j0.013316 \text{ pu} \\
Z_{sys}(h) = \begin{cases} 
0.000666 + j0.013316 \times h \text{ pu}, & h = 3n = 3, 6, 9, 12, \ldots \\
0.0005 + j0.00999 \times h \text{ pu} 
\end{cases} \tag{8.35}
\]

• Transformer

\[
R_g = 46.5137 \text{ pu} \\
Z^+_T = Z^-_T = 0.002 + j0.063132 \text{ pu} \\
Z^0_T = 139.5411 + j0.063132 \text{ pu} \\
Z_T(h) = \begin{cases} 
139.5411 + j0.063132 \times h \text{ pu}, & h = 3n = 3, 6, 9, 12, \ldots \\
0.002 + j0.063132 \times h \text{ pu} 
\end{cases} \tag{8.36}
\]

• Filters

This has seven-branch filters where \( Q_5 = Q_7 = Q_{11} = 15 \text{ MVar} \), \( Q_{13} = 20 \text{ MVar} \), and \( Q_{17} = Q_{23} = Q_{25} = 10 \text{ MVar} \). Therefore, the total is \( Q_f = 95 \text{ MVar} \).
1. Branch 1: $h_n = 5$

\[
\begin{align*}
Q_{C_5} & = 14.4 \text{ MVar} \\
X_{C_5} & = 0.237037037 \text{ Ω} \\
C_5 & = 11.190591 \text{ mF} \\
X_{L_5} & = 0.009481481 \text{ Ω} \\
L_5 & = 25.1504 \mu H \\
X_n & = 0.047407407 \text{ Ω} \\
R_5 & = 0.000790123 \text{ Ω} \\
Z_{F_1} & = 0.000790123 + j(h \ast 0.000790123 - \frac{0.237037037}{h}) \quad (8.37)
\end{align*}
\]

2. Branch 2: $h_n = 7$

\[
\begin{align*}
Q_{C_7} & = 14.69387755 \text{ MVar} \\
X_{C_7} & = 0.232296296 \text{ Ω} \\
C_7 & = 11.418971 \text{ mF} \\
X_{L_7} & = 0.004740741 \text{ Ω} \\
L_7 & = 12.5752 \mu H \\
X_n & = 0.033185185 \text{ Ω} \\
R_7 & = 0.000553086 \text{ Ω} \\
Z_{F_2} & = 0.000553086 + j(h \ast 0.004740741 - \frac{0.232296296}{h}) \quad (8.38)
\end{align*}
\]
3. Branch 3: \( h_n = 11 \) (second-order damped filter)

\[
\begin{align*}
Q_{C_{11}} &= 14.87603306 \text{ MVar} \\
X_{C_{11}} &= 0.229451852 \text{ } \Omega \\
C_{11} &= 11.560528 \text{ mF} \\
X_{L_{11}} &= 0.001896296 \text{ } \Omega \\
L_{11} &= 5.03009 \mu H \\
X_n &= 0.020859259 \text{ } \Omega \\
R_{11} &= 0.000347654 \text{ } \Omega \\
Z_{F_3} &= 0.000347654 + j(h \ast 0.001896296 - \frac{0.229451852}{h}) \\
\end{align*}
\] (8.39)

4. Branch 4: \( h_n = 13 \)

\[
\begin{align*}
Q_{C_{13}} &= 19.8816568 \text{ MVar} \\
X_{C_{13}} &= 0.17168254 \text{ } \Omega \\
C_{13} &= 15.450521 \text{ mF} \\
X_{L_{13}} &= 0.001015873 \text{ } \Omega \\
L_{13} &= 2.69469 \mu H \\
X_n &= 0.013206349 \text{ } \Omega \\
R_{13} &= 0.000220106 \text{ } \Omega \\
Z_{F_4} &= 0.000220106 + j(h \ast 0.001015873 - \frac{0.17168254}{h}) \\
\end{align*}
\] (8.40)
5. Branch 5: $h_n = 17$

\[
\begin{align*}
Q_{C_{17}} &= 9.965397924 \text{ MVar} \\
X_{C_{17}} &= 0.342518519 \Omega \\
C_{17} &= 7.74435 \text{ mF} \\
X_{L_{17}} &= 0.001185185 \Omega \\
L_{17} &= 3.1438 \mu\text{H} \\
X_n &= 0.020148148 \Omega \\
R_{17} &= 0.000335802 \Omega \\
Z_{F_5} &= 0.000335802 + j(h \ast 0.001185185 - \frac{0.342518519}{h}) \tag{8.41}
\end{align*}
\]

6. Branch 6: $h_n = 23$

\[
\begin{align*}
Q_{C_{23}} &= 9.981096408 \text{ MVar} \\
X_{C_{23}} &= 0.341979798 \Omega \\
C_{23} &= 7.756554 \text{ mF} \\
X_{L_{23}} &= 0.000646465 \Omega \\
L_{23} &= 1.7148E \mu\text{H} \\
X_n &= 0.014868687 \Omega \\
R_{23} &= 0.000247811 \Omega \\
Z_{F_6} &= 0.000247811 + j(h \ast 0.000646465 - \frac{0.341979798}{h}) \tag{8.42}
\end{align*}
\]
7. Branch 7: \( h_n = 25 \)

\[
\begin{align*}
Q_{C_{25}} &= 9.984 \text{ MVar} \\
X_{C_{25}} &= 0.341880342 \ \Omega \\
C_{25} &= 7.75881 \text{ mF} \\
X_{L_{25}} &= 0.000547009 \ \Omega \\
L_{25} &= 1.45099 \ \mu H \\
X_n &= 0.01367521 \ \Omega \\
R_{25} &= 0.00022792 \ \Omega \\
Z_{F_7} &= 0.00022792 + j(h \times 0.000547009 - \frac{0.341880342}{h})
\end{align*}
\] (8.43)

The filter impedance is calculated as follows:

\[
Z_F = \frac{1}{Z_{\text{baseL}} \left( \frac{1}{Z_{F_1}} + \frac{1}{Z_{F_2}} + \frac{1}{Z_{F_3}} + \frac{1}{Z_{F_4}} + \frac{1}{Z_{F_5}} + \frac{1}{Z_{F_6}} + \frac{1}{Z_{F_7}} \right)}
\]

Figure 8.11a shows the impedance of the filter. The filter short circuits the voltages at 300 Hz, 420 Hz, 660 Hz, 780 Hz, 1020 Hz, 1380 Hz and 1500 Hz. The phase of the filter is depicted in Figure 8.11b. The phase is alternating between 90° and -90° at these resonance frequency. The filters supply the system with reactive power at the line frequency, capacitive. Figure 8.11c shows the filter resistance versus the reactance. This figure shows the four points where the filter is almost like a short circuit, very small resistance and zero reactance.

### 8.3.4.2 Impedance Scan

The impedance when we look from the 3.2 kV bus can be calculated as follows:

\[
Z_{3.2}(h) = Z_{\text{sys}}(h) + Z_T(h) \\
= (0.005 + j0.009999 \times h) + (0.002 + j0.063135 \times h) \\
= 0.0025 + j0.073155 \times h \ \text{pu}
\] (8.44)

Where the impedance of the system when looking for the converter end is as follows:

\[
Z_{eq}(h) = \frac{1}{\frac{1}{Z_F(h)} + \frac{1}{Z_{3.2}(h)}}
\] (8.45)
Figure 8.11: Filter Impedance and Impedance Locus using VFSPWM$_{12-3}$
Figure 8.12: System Impedance using VFSPWM\textsubscript{12–3}
8.3.4.3 Harmonic Injection  The voltage source harmonics, $V_{3.2}(h)$, is calculated using the equations in section 3.4.1.2. The filter current is calculated using equation (8.44):

$$I_F(h) = \frac{V_{3.2}(h)}{Z_F(h)}$$ (8.46)

From equation (8.44), the current that flows through the transformer to the system has the following relationship:

$$I_T(h) = I_{sys}(h) = \frac{V_{3.2}(h)}{Z_{3.2}(h)}$$ (8.47)

Finally the voltage at the 400 kV bus can be calculated as follows:

$$V_{400}(h) = Z_{sys}(h) \cdot I_{sys}(h)$$ (8.48)

The impedance from the 3.2 kV bus, $Z_{3.2}$, and from the converter, $Z_{eq}$, are plotted in Figure 8.12. The system resonance is at 130 Hz with a value equal to 0.566 Ω. The rest of the resonance are caused by the filter and placed at the harmonic orders they are filtering, 5th, 7th, 11th, 13th, 17th, 23th and 25th. Table 8.5 shows the calculated harmonics. These values comply with IEEE 519 harmonic distortion limits:

$$V_h \leq 3\% \text{ and } THD_v \leq 5\% \text{ for } V \leq 69kV$$

$$V_h \leq 1\% \text{ and } THD_v \leq 1.5\% \text{ for } V \geq 161kV \ [50]$$

The transformer is safe because the K-factor is 1.5277 which less than 4. The equation used to calculate the K-factor is:

$$K = \frac{\sum_{h=1}^{\infty} \left( \frac{hI_h}{I_1} \right)^2}{1 + THD_i^2}$$

And the derating factor is

$$D = \frac{1.15}{1 + 1.15K}$$

By using the standard transformer, it can not be loaded more than 93.56%.
Table 8.5: VFSPWM\textsubscript{(12–3)} Harmonic Distortion Results

<table>
<thead>
<tr>
<th>THD. %</th>
<th>3.2 kV bus</th>
<th>Converter Current</th>
<th>Current to Transformer</th>
<th>Current to Filter</th>
<th>400kV bus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fundamental PU</td>
<td>0.1326</td>
<td>0.745</td>
<td>0.01860</td>
<td>1.01</td>
<td>0.01810</td>
</tr>
<tr>
<td>h=2</td>
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<td>0.15160</td>
<td>8.26</td>
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<tr>
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<tr>
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<td>0.22740</td>
<td>12.40</td>
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</tbody>
</table>

K-factor= 1.527760527
D= 0.93559519
8.3.5 VFSPWM\textsubscript{12−0} System Analysis

In this section we are calculating the harmonics at several points in the STATCOM system using the the VFSPWM\textsubscript{12−0}. The calculation is brief because it follows the same procedure that presented in section 8.3.1.

8.3.5.1 System Impedance  The system impedance is the same. We are presenting the values here for convenience.

\begin{align*}
S_{\text{base}} &= 100\, MVA \\
V_{\text{base}L} &= 3.2kV \\
I_{\text{base}L} &= 18.042kA \\
Z_{\text{base}L} &= 0.1024\Omega
\end{align*}

The base values at the high-voltage bus

\begin{align*}
S_{\text{base}} &= 100\, MVA \\
V_{\text{base}H} &= 400kV \\
I_{\text{base}H} &= 144.33A \\
Z_{\text{base}H} &= 1600\Omega
\end{align*}
• 400 kV equivalent system

\[ SCC = 10000 \text{ MVA} \]
\[ V = 400 \text{kV} \]
\[ Z_{sys} = 16 \angle 87.14^\circ \Omega \]
\[ I_{3\phi SC} = 100 \angle -87.14^\circ \text{ pu} \]

using \( I_{1\phi SC} = 0.9 I_{3\phi SC} \),

\[ I_{1\phi SC} = 90 \angle -87.14^\circ \text{ pu} \]
\[ Z^+ = 0.0005 + j0.00999 \text{ pu} \]
\[ Z^0 = 0.000666 + j0.013316 \text{ pu} \]
\[ Z_{sys}(h) = \begin{cases} 0.000666 + j0.013316 \ast h \text{ pu}, & h = 3n = 3, 6, 9, 12, \ldots \\ 0.0005 + j0.00999 \ast h \text{ pu} \end{cases} \]  

(8.49)

• Transformer

\[ R_g = 46.5137 \text{ pu} \]
\[ Z_T^+ = Z_T^- = 0.002 + j0.063132 \text{ pu} \]
\[ Z_T^0 = 139.5411 + j0.063132 \text{ pu} \]
\[ Z_T(h) = \begin{cases} 139.5411 + j0.063132 \ast h \text{ pu}, & h = 3n = 3, 6, 9, 12, \ldots \\ 0.002 + j0.063132 \ast h \text{ pu} \end{cases} \]  

(8.50)

• Filters

This has seven-branch filters where \( Q_{10} = Q_{14} = 20 \text{ MVar} \) and \( Q_{23} = Q_{25} = 15 \text{ MVar} \). Therefore, the total is \( Q_f = 70 \text{ MVar} \).
1. Branch 1: $h_n = 10$

$$Q_{C_{10}} = 19.8 \text{ MVar}$$

$$X_{C_{10}} = 0.172390572 \Omega$$

$$C_{10} = 15.387063 \text{ mF}$$

$$X_{L_{10}} = 0.001723906 \Omega$$

$$L_{10} = 4.57281 \mu H$$

$$X_n = 0.017239057 \Omega$$

$$R_{10} = 0.000287318 \Omega$$

$$Z_{F_1} = 0.000287318 + j(h * 0.001723906 - \frac{0.172390572}{h}) \quad (8.51)$$

2. Branch 2: $h_n = 14$

$$Q_{C_{14}} = 19.89795918 \text{ MVar}$$

$$X_{C_{14}} = 0.17154188 \Omega$$

$$C_{14} = 15.46319 \text{ mF}$$

$$X_{L_{14}} = 0.000875214 \Omega$$

$$L_{14} = 2.32158 \mu H$$

$$X_n = 0.012252991 \Omega$$

$$R_{14} = 0.000204217 \Omega$$

$$Z_{F_2} = 0.000204217 + j(h * 0.000875214 - \frac{0.17154188}{h}) \quad (8.52)$$
3. Branch 3: $h_n = 23$ (second-order damped filter)

\[
Q_{C_{23}} = 14.97164461 \text{ MVar} \\
X_{C_{23}} = 0.227986532 \text{ } \Omega \\
C_{23} = 11.63483 \text{ } \text{mF} \\
X_{L_{23}} = 0.000430976 \text{ } \Omega \\
L_{23} = 1.1432 \mu H \\
X_n = 0.009912458 \text{ } \Omega \\
R_{23} = 0.000165208 \text{ } \Omega \\
Z_{F_3} = 0.000165208 + j(h \ast 0.000430976 - \frac{0.227986532}{h}) \\
\] (8.53)

4. Branch 4: $h_n = 25$

\[
Q_{C_{25}} = 14.976 \text{ MVar} \\
X_{C_{25}} = 0.227920228 \text{ } \Omega \\
C_{25} = 11.638215 \text{ mF} \\
X_{L_{25}} = 0.000364672 \text{ } \Omega \\
L_{25} = 0.967324 \mu H \\
X_n = 0.009116809 \text{ } \Omega \\
R_{25} = 0.000151947 \text{ } \Omega \\
Z_{F_4} = 0.000151947 + j(h \ast 0.000364672 - \frac{0.227920228}{h}) \\
\] (8.54)

The filter impedance is calculated as follows:

\[
Z_F = \frac{1}{Z_{base L} \left( \frac{1}{Z_{F_1}} + \frac{1}{Z_{F_2}} + \frac{1}{Z_{F_3}} + \frac{1}{Z_{F_4}} \right)}
\]

Figure 8.13a shows the impedance of the filter. The filter short circuits the voltages at 600 Hz, 840 Hz, 1380 Hz and 1500 Hz. The phase of the filter is depicted in Figure 8.13b. The phase is alternating between 90° and -90° at these resonance frequency. The filters supply the system with reactive power at the line frequency, capacitive. Figure 8.13c shows the filter resistance versus the reactance. This figure shows the four points where the filter is almost like a short circuit, very small resistance and zero reactance.
Figure 8.13: Filter Impedance and Impedance Locus using VFSPWM$^{12-0}$
8.3.5.2 Impedance Scan  The impedance when we look from the 3.2 kV bus can be calculated as follows:

\[
Z_{3.2}(h) = Z_{sys}(h) + Z_T(h)
\]

\[
= (0.005 + j0.009999 * h) + (0.002 + j0.063135 * h)
\]

\[
= 0.0025 + j0.073155 * h \text{ pu} \quad (8.55)
\]

Where the impedance of the system when looking for the converter end is as follows:

\[
Z_{eq}(h) = \frac{1}{\frac{1}{Z_F(h)} + \frac{1}{Z_{3.2}(h)}} \quad (8.56)
\]

![Figure 8.14: System Impedance using VFSPWM](image)
8.3.5.3 Harmonic Injection  The voltage source harmonics, $V_{3.2}(h)$, is calculated using the equations in section 3.4.1.2. The filter current is calculated using equation (8.55):

$$I_F(h) = \frac{V_{3.2}(h)}{Z_F(h)}$$  \hspace{1cm} (8.57)

From equation (8.55), the current that flows through the transformer to the system has the following relationship:

$$I_T(h) = I_{sys}(h) = \frac{V_{3.2}(h)}{Z_{3.2}(h)}$$  \hspace{1cm} (8.58)

Finally the voltage at the 400 kV bus can be calculated as follows:

$$V_{400}(h) = Z_{sys}(h) * I_{sys}(h)$$  \hspace{1cm} (8.59)

The impedance from the 3.2 kV bus, $Z_{3.2}$, and from the converter, $Z_{eq}$, are plotted in Figure 8.14. The system resonance is at 130 Hz with a value equal to 0.566 Ω. The rest of the resonance are caused by the filter and placed at the harmonic orders they are filtering, 10th, 14th, 23th and 25th. Table 8.6 shows the calculated harmonics. These values comply with IEEE 519 harmonic distortion limits:

- $V_h \leq 3\%$ and $THD_v \leq 5\%$ for $V \leq 69kV$
- $V_h \leq 1\%$ and $THD_v \leq 1.5\%$ for $V \geq 161kV$ [50]

The transformer is safe because the K-factor is 2.25 which less than 4. The equation used to calculate the K-factor is:

$$K = \frac{\sum_{h=1}^{\infty} \left( \frac{hI_h}{I_1} \right)^2}{1 + \text{THD}_I^2}$$

And the derating factor is

$$D = \frac{1.15}{1 + 1.15K}$$

By using the standard transformer, it can not be loaded more than 85.96%.
Table 8.6: VFSPWM\(_{(12-0)}\) Harmonic Distortion Results

<table>
<thead>
<tr>
<th>THD. %</th>
<th>3.2 kV bus</th>
<th>Current Transformer</th>
<th>Current to Filter</th>
<th>400kV bus</th>
</tr>
</thead>
<tbody>
<tr>
<td>h=2</td>
<td>2.2790</td>
<td>103.9</td>
<td>0.71350</td>
<td>0.31140</td>
</tr>
<tr>
<td>3 % of Fund.</td>
<td>0.5749</td>
<td>26.2</td>
<td>0.18000</td>
<td>0.07850</td>
</tr>
<tr>
<td>4</td>
<td>3.7184</td>
<td>169.5</td>
<td>1.16420</td>
<td>0.50800</td>
</tr>
<tr>
<td>5</td>
<td>4.3504</td>
<td>198.3</td>
<td>1.36210</td>
<td>0.59440</td>
</tr>
<tr>
<td>6</td>
<td>0.6081</td>
<td>27.7</td>
<td>0.19040</td>
<td>0.08310</td>
</tr>
<tr>
<td>7</td>
<td>2.6559</td>
<td>121.1</td>
<td>0.83150</td>
<td>0.36280</td>
</tr>
<tr>
<td>8</td>
<td>0.4140</td>
<td>18.9</td>
<td>0.12960</td>
<td>0.05560</td>
</tr>
<tr>
<td>9</td>
<td>0.9429</td>
<td>43</td>
<td>0.29520</td>
<td>0.12880</td>
</tr>
<tr>
<td>10</td>
<td>3.2040</td>
<td>146</td>
<td>1.00320</td>
<td>0.43770</td>
</tr>
<tr>
<td>11</td>
<td>2.6909</td>
<td>122.7</td>
<td>0.84250</td>
<td>0.36760</td>
</tr>
<tr>
<td>12</td>
<td>4.3514</td>
<td>198.3</td>
<td>1.36240</td>
<td>0.59450</td>
</tr>
<tr>
<td>13</td>
<td>1.1596</td>
<td>52.9</td>
<td>0.36310</td>
<td>0.15840</td>
</tr>
<tr>
<td>14</td>
<td>3.8332</td>
<td>174.7</td>
<td>1.20010</td>
<td>0.52370</td>
</tr>
<tr>
<td>15</td>
<td>0.6927</td>
<td>31.6</td>
<td>0.21690</td>
<td>0.09460</td>
</tr>
<tr>
<td>16</td>
<td>0.4822</td>
<td>22</td>
<td>0.15100</td>
<td>0.06590</td>
</tr>
<tr>
<td>17</td>
<td>1.5352</td>
<td>70</td>
<td>0.48070</td>
<td>0.20970</td>
</tr>
<tr>
<td>18</td>
<td>0.5077</td>
<td>23.1</td>
<td>0.15900</td>
<td>0.06940</td>
</tr>
<tr>
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<td>67.7</td>
<td>0.46510</td>
<td>0.20300</td>
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<td>0.34720</td>
<td>0.15150</td>
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<tr>
<td>21</td>
<td>1.1143</td>
<td>50.8</td>
<td>0.34890</td>
<td>0.15220</td>
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<tr>
<td>22</td>
<td>0.7759</td>
<td>35.4</td>
<td>0.24290</td>
<td>0.10600</td>
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<tr>
<td>23</td>
<td>2.0195</td>
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<td>0.63230</td>
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<td>24</td>
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<td>25</td>
<td>1.6958</td>
<td>77.3</td>
<td>0.53090</td>
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<tr>
<td>26</td>
<td>0.5707</td>
<td>26</td>
<td>0.17870</td>
<td>0.07800</td>
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<tr>
<td>27</td>
<td>0.3252</td>
<td>14.8</td>
<td>0.10180</td>
<td>0.04440</td>
</tr>
<tr>
<td>28</td>
<td>0.4501</td>
<td>20.5</td>
<td>0.14090</td>
<td>0.06150</td>
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<tr>
<td>29</td>
<td>1.3474</td>
<td>61.4</td>
<td>0.42190</td>
<td>0.18410</td>
</tr>
<tr>
<td>30</td>
<td>0.5282</td>
<td>24.1</td>
<td>0.16540</td>
<td>0.07220</td>
</tr>
<tr>
<td>31</td>
<td>1.6020</td>
<td>73</td>
<td>0.50160</td>
<td>0.21890</td>
</tr>
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</table>

K-factor= 2.252  
D= 0.859620272
9.0 CONCLUSION AND FUTURE RESEARCH

9.1 CONCLUSION

The purpose of this research is to improve the efficiency of power electronics conversion systems. Power system equipment today is capable of operating at higher power ratings. Unfortunately, poor power conversion resulting in lower efficiency is one of the obstacles that prevents full utilization of existing infrastructure. Researchers realized that improvements in the power electronics technology efficiency can be accomplished in two ways. Several researchers preferred to modernize power electronics circuitry. These modifications raised the efficiency to an acceptable level. However, this solution may be economically unattractive. Other researchers provided new modulation controllers that lower the losses of the power electronic conversion. It is the latter approach that has been emphasized throughout this body of work. This dissertation shows a novel carrier based pulse width modulation (PWM) technique called the Variable Frequency Inverse Sinusoidal Pulse Width Modulation, VFSPWM, that combines several concepts to improve power converter performance.

A single- and three-phase PSCAD model has been built to simulate the VFSPWM operation. The carrier signal switches at two different frequencies. Each half-cycle is divided, evenly, into three regions. The carrier in the first and last regions switches at high frequency, while it switches at low frequency in the second area. The carrier has a shape of an inverse sinusoidal wave. This structure widens the pulses of the controller, hence, allow the switches to conduct longer. When the pulses are wider, the total area of the pulses is greater resulting in higher fundamental component.

A mathematical model has been represented to validate the PSCAD model. The fundamental component and the total harmonic distortions (THD) are calculated to validate the
PSCAD model. To increase confidence in the validation, a PSPICE model has been built to compare the converter fundamental component and THD to the PSCAD model. The results of the two validation methods guaranteed the reliability of the PSCAD model.

The PSCAD model is based upon deal operation of the converter switching devices. The output of the converter would be more realistic if the model added several switching characteristics such as switching rise/fall time and switching delay. Therefore, a PSPICE model that replaced ideal switching devices with switches that perform similarly to the real physical components was built. Also, the ideal comparators of the controllers were replaced by real operational amplifiers models. The comparison between the ideal and "real" model has showed that the converters in the ideal model performs similarly to the real model when the modulation index is greater than 0.4.

9.2 CONTRIBUTION

The analysis of the single- and three-phase versions of the VFSPWM techniques has been presented. Sinusoidal Pulse Width Modulation (SPWM) is the benchmark in this dissertation due to its widely used applications in power electronics circuits. Since VFSPWM operates at two combined switching frequencies, the performance comparison to the SPWM is presented in two methods. The first method is when the VFSPWM and the SPWM have the same number of pulses per cycle. In this dissertation, this method is denoted similar pulses test. The second method is to compare the VFSPWM technique to the SPWM while the switching devices are under the same switching stress. Therefore, this method is called similar stress test in the dissertation work.

VFSPWM shows a significant increase in the converter fundamental voltage and current in comparison to the SPWM technique. This increase in the fundamental component means better utilization of the DC source.

During the switching of the power electronics devices in the converter, the voltage across the switching devices and the current that flows through the device are not simultaneously zero. The multiplication of these two quantities is considered a loss, known as switching
power loss. VFSPWM reduces the total switching power loss because the number of pulses per half-cycle is reduced in comparison to the SPWM.

Improvement in the THD is observed as well. The analysis of the low-order harmonics ($5^{th}$, $7^{th}$, $11^{th}$, $13^{th}$ and $17^{th}$) has been conducted. When the switching rate is high, the harmonic components are shifted to high orders. The switching frequencies in the VFSPWM technique are greater than or equal to the SPWM switching frequencies. Therefore, the filters are more economic for the VFSPWM.

The performance of the VFSPWM technique was examined for a STATCOM application. The increase in the fundamental component results in substituting a DC source in the STATCOM application with a smaller and more affordable one. For example, the STATCOM application showed that the DC source using the VFSPWM technique is reduced by upto one-forth of that applying SPWM.

### 9.3 LIMITATIONS

The analysis of the VFSPWM technique neglected the protection circuits, known as snubber circuits. These circuits protect the converter switching devices from the overvoltage phenomena caused by voltage transient, and current spikes. It was better to analyze the model with omitted snubber circuits to reduce the sophistication of the evaluations. In addition, this approach helped to present clear, theoretical results and eliminate external unwanted factors.

The DC source applied in the analysis is assumed to be ideal. Usually the sources contain small ripple on top of the nominal DC value and have series impedances.

### 9.4 FUTURE RESEARCH

The VFSPWM technique can be used to other converter topologies. For example, it can be applied on multi-level converters such as the neutral-clamping point (NPC) technology. In
the industrial applications, most of the converters are multi-level resulting in an output with less harmonic content and sometimes require no filters.

Each SPWM switching frequency has several corresponding switching frequencies in the VFSPWM technique. Finding the VFSPWM optimal switching frequency is a great potential research area. For instance, linear programming technique can be applied to calculate the optimal switching combination based on several variables such as switching losses, fundamental component, WTHD, and more.

A STATCOM controller is the application used to evaluate the VFSPWM performance algorithm in this dissertation work. It is recommended to analyze network stability under dynamic contingencies. In addition, it is of great benefit to examine the VFSPWM technique when applied to different transmission and distribution systems. For example, High-Voltage AC (HVDC) technology is a viable application that can be used to evaluate the quality of the transmitted power. Also, discharge time of different industrial batteries can be studied when applying the VFSPWM technique.

The comparison in this dissertation work is based on main measurements, i.e., fundamental component and WTHD. It is recommended to analyze the VFSPWM techniques using different factors such as power factor, distortion factor or lowest order harmonic factor.
APPENDIX

DATASHEETS
Low $V_{CE(sat)}$ IGBT

IXGH/IXGM 40 N60

600 V

75 A

2.5 V

High speed IGBT

IXGH/IXGM 40 N60A

600 V

75 A

3.0 V

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Test Conditions</th>
<th>Maximum Ratings</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{CES}$</td>
<td>$T_J = 25^\circ C$ to $150^\circ C$</td>
<td>600 V</td>
</tr>
<tr>
<td>$V_{CGR}$</td>
<td>$T_J = 25^\circ C$ to $150^\circ C$; $R_{GE} = 1 , \Omega$</td>
<td>600 V</td>
</tr>
<tr>
<td>$I_{C25}$</td>
<td>$T_c = 25^\circ C$, limited by leads</td>
<td>75 A</td>
</tr>
<tr>
<td>$I_{C90}$</td>
<td>$T_c = 90^\circ C$</td>
<td>40 A</td>
</tr>
<tr>
<td>$I_{CM}$</td>
<td>$T_c = 25^\circ C$, 1 ms</td>
<td>150 A</td>
</tr>
<tr>
<td>SSOA (RBSOA)</td>
<td>$V_{GE} = 15 V$, $T_J = 125^\circ C$, $R_G = 22 , \Omega$</td>
<td>$I_{CM} = 80 A$ @ $0.8 V_{CES}$</td>
</tr>
<tr>
<td>$P_C$</td>
<td>$T_c = 25^\circ C$</td>
<td>250 W</td>
</tr>
<tr>
<td>$T_J$</td>
<td>$-55 ... +150^\circ C$</td>
<td></td>
</tr>
<tr>
<td>$T_{JM}$</td>
<td>150 °C</td>
<td></td>
</tr>
<tr>
<td>$T_{stg}$</td>
<td>$-55 ... +150^\circ C$</td>
<td></td>
</tr>
<tr>
<td>$M_d$</td>
<td>Mounting torque (M3)</td>
<td>1.13/10 Nm/lb.in.</td>
</tr>
<tr>
<td>Weight</td>
<td>TO-204 = 18 g, TO-247 = 6 g</td>
<td></td>
</tr>
<tr>
<td>Maximum lead temperature for soldering</td>
<td>300 °C</td>
<td></td>
</tr>
</tbody>
</table>

1.6 mm (0.062 in.) from case for 10 s

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Test Conditions</th>
<th>Characteristic Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$BV_{CES}$</td>
<td>$I_c = 250 \mu A$, $V_{GE} = 0 V$</td>
<td>$V$</td>
</tr>
<tr>
<td>$I_{CES}$</td>
<td>$V_{GE} = 0.8 \cdot V_{CES}$</td>
<td>$200 \mu A$</td>
</tr>
<tr>
<td>$I_{GES}$</td>
<td>$V_{GE} = 0 V$, $V_{GE} = \pm 20 V$</td>
<td>$\pm 100 nA$</td>
</tr>
<tr>
<td>$V_{CE(sat)}$</td>
<td>$I_c = I_{C90}$, $V_{GE} = 15 V$</td>
<td>$40N60$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$V_{CES}$</th>
<th>$I_{C25}$</th>
<th>$V_{CE(sat)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>600 V</td>
<td>75 A</td>
<td>2.5 V</td>
</tr>
<tr>
<td>600 V</td>
<td>75 A</td>
<td>3.0 V</td>
</tr>
</tbody>
</table>

Features
- International standard packages
- 2nd generation HDMOS™ process
- Low $V_{CE(sat)}$ - for low on-state conduction losses
- High current handling capability
- MOS Gate turn-on - drive simplicity
- Voltage rating guaranteed at high temperature ($125^\circ C$)

Applications
- AC motor speed control
- DC servo and robot drives
- DC choppers
- Uninterruptible power supplies (UPS)
- Switch-mode and resonant-mode power supplies

Advantages
- Easy to mount with 1 screw (TO-247) (isolated mounting screw hole)
- High power density
**Symbol** | **Test Conditions** | **Characteristic Values** (\(T_j = 25^\circ C\), unless otherwise specified) | **min.** | **typ.** | **max.**
---|---|---|---|---|---
\(I_{gs}\) | \(I_{CS} ; V_{CE} = 10 V \)
Pulse test, \(t \leq 300 \mu s\), duty cycle \(\leq 2\%\) | 25 | 35 | S

\(C_{res}\) | \(V_{CE} = 25 V, V_{GE} = 0 V, f = 1 MHz\) | 4500 | pF | 300 | pF | 60 | pF

\(Q_g\) | \(I_{CS} ; V_{GE} = 15 V, V_{CE} = 0.5 V_{CES}\) | 200 | 250 | nC | 45 | 80 | nC | 88 | 120 | nC

\(t_{d(on)}\) | \(I_{CS} ; V_{GE} = 15 V, L = 100 \mu H\)
Switching times may increase for \(V_{CE} (Clamp) > 0.8 \cdot V_{CES}\) | 100 | ns | 200 | ns | 600 | ns | 200 | ns

\(t_{d(off)}\) | \(R_g = R_{off} = 22 \Omega\)
Remarks: Switching times may increase for \(V_{CE} (Clamp) > 0.8 \cdot V_{CES}\) | 4 | mJ | 600 | 2000 | ns | 300 | 800 | ns | 12 | mJ | 6 | mJ

\(t_{f}\) | \(R_g = R_{off} = 22 \Omega\)
Remarks: Switching times may increase for \(V_{CE} (Clamp) > 0.8 \cdot V_{CES}\) | 0.5 \(K/W\)

\(R_{thJC}\) | \(0.25 \ K/W\)

\(R_{thCK}\) | \(0.25 \ K/W\)

**TO-247 AD Outline**

**TO-204AE Outline**

IXYS reserves the right to change limits, test conditions, and dimensions.

IXYS MOSFETS and IGBTs are covered by one or more of the following U.S. patents: 4,835,592 4,881,106 5,017,508 5,049,961 5,187,117 5,486,715 4,850,072 4,931,844 5,034,796 5,063,307 5,237,481 5,381,025
Fig. 1 Saturation Characteristics

Fig. 2 Output Characteristics

Fig. 3 Collector-Emitter Voltage vs. Gate-Emitter Voltage

Fig. 4 Temperature Dependence of Output Saturation Voltage

Fig. 5 Input Admittance

Fig. 6 Temperature Dependence of Breakdown and Threshold Voltage
IXYS MOSFETS and IGBTs are covered by one or more of the following U.S. patents: 4,835,592 4,881,106 5,017,508 5,049,961 5,187,117 5,486,715 4,850,072 4,931,844 5,034,796 5,063,307 5,237,481 5,381,025

IXYS reserves the right to change limits, test conditions, and dimensions.
LOW POWER QUAD OPERATIONAL AMPLIFIERS

- WIDE GAIN BANDWIDTH : 1.3MHz
- INPUT COMMON-MODE VOLTAGE RANGE INCLUDES GROUND
- LARGE VOLTAGE GAIN : 100dB
- VERY LOW SUPPLY CURRENT/AMPLI : 375µA
- LOW INPUT BIAS CURRENT : 20nA
- LOW INPUT OFFSET VOLTAGE : 5mV max.
  (for more accurate applications, use the equivalent parts LM124A-LM224A-LM324A which feature 3mV max)
- LOW INPUT OFFSET CURRENT : 2nA
- WIDE POWER SUPPLY RANGE :
  SINGLE SUPPLY : +3V TO +30V
  DUAL SUPPLIES : ±1.5V TO ±15V

DESCRIPTION
These circuits consist of four independent, high gain, internally frequency compensated operational amplifiers. They operate from a single power supply over a wide range of voltages. Operation from split power supplies is also possible and the low power supply current drain is independent of the magnitude of the power supply voltage.

PIN CONNECTIONS (top view)

ORDER CODES

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<tr>
<th>Part Number</th>
<th>Temperature Range</th>
<th>Package</th>
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</thead>
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<td>-55°C, +125°C</td>
<td>N • D • P •</td>
</tr>
<tr>
<td>LM224</td>
<td>-40°C, +105°C</td>
<td>N • D • P •</td>
</tr>
<tr>
<td>LM324</td>
<td>0°C, +70°C</td>
<td>N • D • P •</td>
</tr>
</tbody>
</table>

Example : LM224N

June 1999
ABSOLUTE MAXIMUM RATINGS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>LM124</th>
<th>LM224</th>
<th>LM324</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vcc</td>
<td>Supply Voltage</td>
<td>±16 or 32</td>
<td>V</td>
<td>V</td>
<td></td>
</tr>
<tr>
<td>Vi</td>
<td>Input Voltage</td>
<td>-0.3 to +32</td>
<td>V</td>
<td>V</td>
<td></td>
</tr>
<tr>
<td>Vdi</td>
<td>Differential Input Voltage (*)</td>
<td>+32</td>
<td>+32</td>
<td>+32</td>
<td>V</td>
</tr>
<tr>
<td>Ptot</td>
<td>Power Dissipation</td>
<td>500 mW</td>
<td>500 mW</td>
<td>500 mW</td>
<td>mW</td>
</tr>
<tr>
<td></td>
<td>N Suffix</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>D Suffix</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Output Short-circuit Duration - (note 1)</td>
<td>Infinite</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>In</td>
<td>Input Current – (note 6)</td>
<td>50 mA</td>
<td>50 mA</td>
<td>50 mA</td>
<td>mA</td>
</tr>
<tr>
<td>Top</td>
<td>Operating Free Air Temperature Range</td>
<td>-55 to +125</td>
<td>-40 to +105</td>
<td>0 to +70</td>
<td>°C</td>
</tr>
<tr>
<td>Tstg</td>
<td>Storage Temperature Range</td>
<td>-65 to +150</td>
<td>-65 to +150</td>
<td>-65 to +150</td>
<td>°C</td>
</tr>
</tbody>
</table>

SCHEMATIC DIAGRAM (1/4 LM124)
### ELECTRICAL CHARACTERISTICS

$V_{CC^+} = +5V$, $V_{CC^-} = $ Ground, $V_O = 1.4V$, $T_{amb} = +25^\circ C$ (unless otherwise specified)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>LM124 - LM224 - LM324</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{io}$</td>
<td>Input Offset Voltage (note 3)</td>
<td></td>
<td>mV</td>
</tr>
<tr>
<td>$T_{amb} = +25^\circ C$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{min.} \leq T_{amb} \leq T_{max.}$</td>
<td>LM324</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$I_{io}$</td>
<td>Input Offset Current</td>
<td></td>
<td>nA</td>
</tr>
<tr>
<td>$T_{amb} = +25^\circ C$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{min.} \leq T_{amb} \leq T_{max.}$</td>
<td></td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>$I_{ib}$</td>
<td>Input Bias Current (note 2)</td>
<td></td>
<td>nA</td>
</tr>
<tr>
<td>$T_{amb} = +25^\circ C$</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$T_{min.} \leq T_{amb} \leq T_{max.}$</td>
<td></td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>$A_{va}$</td>
<td>Large Signal Voltage Gain</td>
<td></td>
<td>V/mV</td>
</tr>
<tr>
<td>$(V_{CC^+} = +15V, R_L = 2k\Omega, V_O = 1.4V$ to $11.4V)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{amb} = +25^\circ C$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{min.} \leq T_{amb} \leq T_{max.}$</td>
<td></td>
<td>50</td>
<td></td>
</tr>
<tr>
<td></td>
<td>LM324</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>25</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>SVR</td>
<td>Supply Voltage Rejection Ratio $(R_S \leq 10k\Omega)$</td>
<td></td>
<td>dB</td>
</tr>
<tr>
<td>$(V_{CC^+} = 5V$ to $30V)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{amb} = +25^\circ C$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{min.} \leq T_{amb} \leq T_{max.}$</td>
<td></td>
<td>65</td>
<td></td>
</tr>
<tr>
<td></td>
<td>LM324</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>65</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>110</td>
<td></td>
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<tr>
<td>$I_{cc}$</td>
<td>Supply Current, all Amp, no load</td>
<td></td>
<td>mA</td>
</tr>
<tr>
<td>$T_{amb} = +25^\circ C$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(V_{CC} = +5V$)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{min.} \leq T_{amb} \leq T_{max.}$</td>
<td></td>
<td>0.7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$V_{CC} = +30V$</td>
<td></td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>$V_{CC} = +5V$</td>
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<td>0.8</td>
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<tr>
<td></td>
<td>$V_{CC} = +30V$</td>
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<td>1.5</td>
</tr>
<tr>
<td>$V_{icm}$</td>
<td>Input Common Mode Voltage Range</td>
<td></td>
<td>V</td>
</tr>
<tr>
<td>$(V_{CC} = +30V)$ - (note 4)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{amb} = +25^\circ C$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{min.} \leq T_{amb} \leq T_{max.}$</td>
<td></td>
<td>0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$V_{CC}$ -1.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$V_{CC}$ -2</td>
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<td></td>
</tr>
<tr>
<td>CMR</td>
<td>Common-mode Rejection Ratio $(R_S \leq 10k\Omega)$</td>
<td></td>
<td>dB</td>
</tr>
<tr>
<td>$(V_{CC} = +30V)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{amb} = +25^\circ C$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{min.} \leq T_{amb} \leq T_{max.}$</td>
<td></td>
<td>70</td>
<td></td>
</tr>
<tr>
<td></td>
<td>LM324</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>60</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>80</td>
<td></td>
</tr>
<tr>
<td>$I_{source}$</td>
<td>Output Current Source $(V_d = +1V)$</td>
<td></td>
<td>mA</td>
</tr>
<tr>
<td>$(V_{CC} = +15V, V_o = +2V)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{amb} = +25^\circ C$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{min.} \leq T_{amb} \leq T_{max.}$</td>
<td></td>
<td>20</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$V_{CC} = +15V, V_o = +2V$</td>
<td></td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>$V_{CC} = +15V, V_o = +0.2V$</td>
<td></td>
<td>70</td>
</tr>
<tr>
<td>$I_{sink}$</td>
<td>Output Sink Current $(V_d = -1V)$</td>
<td></td>
<td>mA</td>
</tr>
<tr>
<td>$(V_{CC} = +15V, V_o = +2V)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{amb} = +25^\circ C$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{min.} \leq T_{amb} \leq T_{max.}$</td>
<td></td>
<td>10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$V_{CC} = +15V, V_o = +2V$</td>
<td></td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>$V_{CC} = +15V, V_o = +0.2V$</td>
<td></td>
<td>12</td>
</tr>
</tbody>
</table>
### ELECTRICAL CHARACTERISTICS (continued)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>LM124 - LM224 - LM324</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>V_HL</td>
<td>High Level Output Voltage</td>
<td>V_C = +30V</td>
<td>V</td>
</tr>
<tr>
<td></td>
<td>(V_C = +30V)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>T_{amb} = +25°C</td>
<td>26</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>R_L = 2kΩ</td>
<td>26</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>T_{min} ≤ T_{amb} ≤ T_{max}</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>R_L = 10kΩ</td>
<td>27</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td>T_{min} ≤ T_{amb} ≤ T_{max}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>V_LL</td>
<td>Low Level Output Voltage (R_L = 10kΩ)</td>
<td>V_C = +5V, R_L = 2kΩ</td>
<td>mV</td>
</tr>
<tr>
<td></td>
<td>T_{amb} = +25°C</td>
<td>5</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>T_{min} ≤ T_{amb} ≤ T_{max}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S_R</td>
<td>Slew Rate</td>
<td>V_C = 15V, V_I = 0.5 to 3V, R_L = 2kΩ, C_L = 100pF, unity gain)</td>
<td>V/μs</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.4</td>
<td></td>
</tr>
<tr>
<td>G_BP</td>
<td>Gain Bandwidth Product</td>
<td>V_C = 30V, f = 100kHz,</td>
<td>MHz</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>R_L = 2kΩ, C_L = 100pF</td>
<td>1.3</td>
<td></td>
</tr>
<tr>
<td>T_HD</td>
<td>Total Harmonic Distortion</td>
<td></td>
<td>%</td>
</tr>
<tr>
<td></td>
<td>f = 1kHz,</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>R_L = 2kΩ, V_O = 2V_{pp}</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>C_L = 100pF, V_C = 30V</td>
<td>0.015</td>
<td></td>
</tr>
<tr>
<td>e_r</td>
<td>Equivalent Input Noise Voltage</td>
<td>f = 1kHz, R_S = 100Ω, V_C = 30V</td>
<td>nV/√Hz</td>
</tr>
<tr>
<td></td>
<td></td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>D_{V0}</td>
<td>Input Offset Voltage Drift</td>
<td>7</td>
<td>30</td>
</tr>
<tr>
<td>D_{I0}</td>
<td>Input Offset Current Drift</td>
<td>10</td>
<td>200</td>
</tr>
<tr>
<td>V_{O1}/V_{O2}</td>
<td>Channel Separation (note 5)</td>
<td>1kHz ≤ f ≤ 20kHz</td>
<td>dB</td>
</tr>
<tr>
<td></td>
<td></td>
<td>120</td>
<td></td>
</tr>
</tbody>
</table>

**Notes:**

1. Short-circuits from the output to V_C can cause excessive heating if V_C > 15V. The maximum output current is approximately 40mA independent of the magnitude of V_C. Destructive dissipation can result from simultaneous short-circuit on all amplifiers.

2. The direction of the input current is out of the IC. This current is essentially constant, independent of the state of the output so no loading change exists on the input lines.

3. V_o = 1.4V, R_S = 0Ω, 5V < V_C < 30V, 0 < V_o < V_C - 1.5V

4. The input common-mode voltage of either input signal voltage should not be allowed to go negative by more than 0.3V. The upper end of the common-mode voltage range is V_C - 1.5V, but either or both inputs can go to +32V without damage.

5. Due to the proximity of external components insure that coupling is not originating via stray capacitance between these external parts. This typically can be detected as this type of capacitance increases at higher frequencies.

6. This input current only exists when the voltage at any of the input leads is driven negative. It is due to the collector-base junction of the input PNP transistor becoming forward biased and thereby acting as input diodes clamps. In addition to this diode action, there is also NPN parasitic action on the IC chip. this transistor action can cause the output voltages of the Op-amps to go to the V_C voltage level (or to ground for a large overdrive) for the time duration than an input is driven negative. This is not destructive and normal output will set up again for input voltage higher than -0.3V.
TYPICAL SINGLE-SUPPLY APPLICATIONS

AC COUPLER INVERTING AMPLIFIER

AC COUPLER NON-INVERTING AMPLIFIER

$A_v = \frac{R_2}{R_1}$

(as shown $A_v = 11$)
**TYPICAL SINGLE-SUPPLY APPLICATIONS**

**NON-INVERTING DC GAIN**

\[
A_V = 1 + \frac{R_2}{R_1} \quad \text{(As shown } A_V = 101) \quad A_V
\]

**DC SUMMING AMPLIFIER**

\[
e_0 = e_1 + e_2 - e_3 - e_4 \quad \text{where } (e_1 + e_2) \geq (e_3 + e_4) \quad \text{to keep } e_o \geq 0 \text{V}
\]

**HIGH INPUT Z ADJUSTABLE GAIN DC INSTRUMENTATION AMPLIFIER**

\[
e_o = \frac{1 + 2R_1}{R_2} (e_2 - e_1) \quad \text{As shown } e_o = 101 (e_2 - e_1).
\]

**LOW DRIFT PEAK DETECTOR**

* Polycarbonate or polyethylene
USING SYMMETRICAL AMPLIFIERS TO REDUCE INPUT CURRENT (GENERAL CONCEPT)

Aux. amplifier for input current compensation
TYPICAL SINGLE-SUPPLY APPLICATIONS

ACTIVER BANDPASS FILTER

\[ e_0 = (e_2 - e_1) \]

\[ \frac{R_1}{R_2} = \frac{R_4}{R_3} \]

(CMRR depends on this resistor ratio match)

HIGH INPUT Z, DC DIFFERENTIAL AMPLIFIER

VOLTAGE GAIN AND PHASE vs FREQUENCY

\[ V_{CC} = 30V \]
\[ RL = 2K \]
\[ CL = 100pF \]
\[ T_{amb} = 25^\circ C \]
LARGE VOLTAGE GAIN : 100dB
VERY LOW SUPPLY CURRENT/AMPLI : 375µA
LOW INPUT BIAS CURRENT : 20nA
LOW INPUT OFFSET VOLTAGE : 2mV

Applies to : LM124-LM224-LM324

** Standard Linear Ics Macromodels, 1993.
** CONNECTIONS :
* 1 INVERTING INPUT
* 2 NON-INVERTING INPUT
* 3 OUTPUT
* 4 POSITIVE POWER SUPPLY
* 5 NEGATIVE POWER SUPPLY

SUBCKT LM124 1 3 2 4 5 (analog)

.MODEL MDTH D IS=1E-8 KF=3.104131E-15
CJO=10F
* INPUT STAGE
CIP 2 5 1.000000E-12
CIN 1 5 1.000000E-12
EIP 10 5 2 5 1
EIN 16 15 1
RIP 10 11 2.600000E+01
RIN 15 16 2.600000E+01
RIS 11 15 2.003862E+02
DIP 11 12 MDTH 400E-12
DIN 15 14 MDTH 400E-12
VOFP 12 13 DC 0
VOFN 13 14 DC 0
IPOL 13 5 1.000000E-05
CPS 11 15 3.783376E-09
DINN 17 13 MDTH 400E-12
VIN 17 5 0.000000e+00
DINR 15 18 MDTH 400E-12
VIP 4 18 2.000000E+00
FCP 4 5 VOFP 3.400000E+01
FCN 5 4 VOFN 3.400000E+01
FIBP 2 5 VOFN 2.000000E-03
FIBN 5 1 VOFP 2.000000E-03
* AMPLIFYING STAGE
FIP 5 19 VOFP 3.600000E+02
FIN 5 19 VOFN 3.600000E+02
RG1 19 5 3.652997E+06
RG2 19 4 3.652997E+06
CC 19 5 6.000000E-09
DOPM 19 22 MDTH 400E-12
DONM 21 19 MDTH 400E-12
HOPM 22 28 VOUT 7.500000E+03
VPM 28 4 1.500000E+02
HONM 21 27 VOUT 7.500000E+03
VINM 5 27 1.500000E+02
EOUT 26 23 19 5 1
VOUT 23 5 0
ROUT 26 3 20
COUT 3 5 1.000000E-12
DOP 19 25 MDTH 400E-12
VOP 4 25 2.242230E+00
DON 24 19 MDTH 400E-12
VON 24 5 7.922301E-01

ENDS

ELECTRICAL CHARACTERISTICS

\[ V_{CC^+} = +5V, \ V_{CC^-} = 0V, \ T_{amb} = 25^\circ C \] (unless otherwise specified)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Conditions</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_o )</td>
<td>( V_{CC} = +5V )</td>
<td>0</td>
<td>mV</td>
</tr>
<tr>
<td>( A_{vd} )</td>
<td>( R_L = 2k\Omega )</td>
<td>100</td>
<td>V/mV</td>
</tr>
<tr>
<td>( I_{CC} )</td>
<td>No load, per operator</td>
<td>350</td>
<td>µA</td>
</tr>
<tr>
<td>( V_{com} )</td>
<td>-15 to +13.5</td>
<td>V</td>
<td></td>
</tr>
<tr>
<td>( V_{OH} )</td>
<td>( R_L = 2k\Omega ), ( V_{CC^+} = 15V )</td>
<td>+13.5</td>
<td>V</td>
</tr>
<tr>
<td>( V_{OL} )</td>
<td>( R_L = 10k\Omega )</td>
<td>5</td>
<td>mV</td>
</tr>
<tr>
<td>( I_{DS} )</td>
<td>( V_o = +2V ), ( V_{CC} = +15V )</td>
<td>+40</td>
<td>mA</td>
</tr>
<tr>
<td>GBP</td>
<td>( R_L = 2k\Omega ), ( C_L = 100pF )</td>
<td>1.3</td>
<td>MHz</td>
</tr>
<tr>
<td>SR</td>
<td>( R_L = 2k\Omega ), ( C_L = 100pF )</td>
<td>0.4</td>
<td>V/µs</td>
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</table>
PACKAGE MECHANICAL DATA
14 PINS - PLASTIC DIP

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<thead>
<tr>
<th>Dimensions</th>
<th>Millimeters</th>
<th>Inches</th>
</tr>
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<tbody>
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<td>0.51</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>1.39</td>
<td>1.65</td>
</tr>
<tr>
<td>b</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>b1</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
<td>20</td>
</tr>
<tr>
<td>E</td>
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</tr>
<tr>
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<td>2.54</td>
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</tr>
<tr>
<td>e3</td>
<td>15.24</td>
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</tr>
<tr>
<td>F</td>
<td></td>
<td>7.1</td>
</tr>
<tr>
<td>i</td>
<td></td>
<td>5.1</td>
</tr>
<tr>
<td>L</td>
<td>3.3</td>
<td></td>
</tr>
<tr>
<td>Z</td>
<td>1.27</td>
<td>2.54</td>
</tr>
</tbody>
</table>
# PACKAGE MECHANICAL DATA

14 PINS - PLASTIC MICROPACKAGE (SO)

<table>
<thead>
<tr>
<th>Dimensions</th>
<th>Millimeters</th>
<th>Inches</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.75</td>
<td>0.069</td>
</tr>
<tr>
<td>a1</td>
<td>0.1</td>
<td>0.004</td>
</tr>
<tr>
<td>a2</td>
<td>0.2</td>
<td>0.008</td>
</tr>
<tr>
<td>b</td>
<td>1.6</td>
<td>0.063</td>
</tr>
<tr>
<td>b1</td>
<td>0.46</td>
<td>0.018</td>
</tr>
<tr>
<td>C</td>
<td>0.5</td>
<td>0.020</td>
</tr>
<tr>
<td>c1</td>
<td>45° (typ.)</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>8.55</td>
<td>0.334</td>
</tr>
<tr>
<td>E</td>
<td>5.8</td>
<td>0.244</td>
</tr>
<tr>
<td>e</td>
<td>1.27</td>
<td>0.050</td>
</tr>
<tr>
<td>e3</td>
<td>7.62</td>
<td>0.300</td>
</tr>
<tr>
<td>F</td>
<td>3.8</td>
<td>0.157</td>
</tr>
<tr>
<td>G</td>
<td>4.6</td>
<td>0.208</td>
</tr>
<tr>
<td>L</td>
<td>0.5</td>
<td>0.050</td>
</tr>
<tr>
<td>M</td>
<td>0.68</td>
<td>0.027</td>
</tr>
<tr>
<td>S</td>
<td>8° (max.)</td>
<td></td>
</tr>
</tbody>
</table>
**PACKAGE MECHANICAL DATA**
14 PINS - THIN SHRINK SMALL OUTLINE PACKAGE

<table>
<thead>
<tr>
<th>Dim.</th>
<th>Millimeters</th>
<th>Inches</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.20</td>
<td>0.05</td>
</tr>
<tr>
<td>A1</td>
<td>0.05</td>
<td>0.15</td>
</tr>
<tr>
<td>A2</td>
<td>0.80</td>
<td>1.00</td>
</tr>
<tr>
<td>b</td>
<td>0.19</td>
<td>0.30</td>
</tr>
<tr>
<td>c</td>
<td>0.09</td>
<td>0.20</td>
</tr>
<tr>
<td>D</td>
<td>4.90</td>
<td>5.00</td>
</tr>
<tr>
<td>E</td>
<td>6.40</td>
<td></td>
</tr>
<tr>
<td>E1</td>
<td>4.30</td>
<td>4.40</td>
</tr>
<tr>
<td>e</td>
<td>0.65</td>
<td></td>
</tr>
<tr>
<td>k</td>
<td>0°</td>
<td>8°</td>
</tr>
<tr>
<td>i</td>
<td>0.50</td>
<td>0.60</td>
</tr>
</tbody>
</table>

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