Exploring the Relationship Between Teachers' Participation in Modified Lesson Study Cycles and Their Implementation of High-Level Tasks

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EXPLORING THE RELATIONSHIP BETWEEN TEACHERS' PARTICIPATION IN MODIFIED LESSON STUDY CYCLES AND THEIR IMPLEMENTATION OF HIGH-LEVEL TASKS

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This study explored the relationship between mathematics teachers' participation in professional development and subsequent changes in their instructional practices. This professional development aimed to help teachers to implement high-level tasks through the use of the *five practices*: anticipating, monitoring, selecting, and sequencing (Stein, Engle, Smith, & Hughes, 2008). Data were collected from teachers' participation with their school-based colleagues in modified lesson study cycles (MLSC). During these cycles, teachers took turns selecting, discussing, and reflecting on the implementation of high-level tasks (focus tasks). Specifically, prior to classroom instruction, teachers anticipated possible student solutions to the tasks and issues that might arise during instruction. Following classroom instruction, the teachers reflected on the lesson and how students actually engaged in the task. Audio recordings and meeting artifacts (e.g., teachers' anticipated solutions to the focus tasks) were collected. Data from the MLSCs were analyzed to determine teachers' level of participation and the key ideas that were shared in the MLSC meetings.

Four teachers' classroom instruction was also investigated. These teachers were observed teaching high-level tasks, including the focus tasks from the MLSCs. Data from these observations consisted of observation write-ups (detailed accounts of the lessons) and lesson artifacts (e.g., lesson plans, representations of displayed student work). These data were analyzed with regard to the level of cognitive demand of the task before and during the lessons and the teachers' use of the five practices.

Teachers' engagement in the professional development varied greatly. All of the teachers struggled to implement cognitively demanding tasks at a high level, and they used the five practices inconsistently and sporadically. Two possible explanations for the teachers' struggles are: (a) the teachers failed to consistently anticipate how students would engage in the task, and (b) the chaotic environment of the school negatively affected some teachers' participation in the professional development and their use of instructional practices. The results suggest that future professional development should focus on teachers' content-specific instruction, while also being conscious of and attending to the challenges they face in their particular teaching contexts.

Keywords: professional development, modified lesson study cycles, high-level tasks, five practices

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PREFACE

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1.0 CHAPTER 1: THE RESEARCH QUESTION

1.1 INTRODUCTION

For the past three decades the U.S. government has been concerned about the quality of education and level of student performance in our nation's schools as evidenced by reports such as *A Nation at Risk* (National Commission on Excellence in Education, 1983), federal laws like the No Child Left Behind Act (NCLB; Public Law No. 107-110, 115 Stat. 1425, 2002), and policy initiatives like the Race to the Top funding program (U.S. Department of Education, 2009). These concerns have been fueled by American students' poor performances on international comparisons such as the Trends in International Mathematics and Science Study (TIMSS) and Programme for International Student Assessment (PISA) when compared to their counterparts in high-achieving countries (Ginsburg, Cooke, Leinwand, Noell, & Pollock, 2005; Lemke et al., 2004; Mullis, Martin, Gonzalez, & Chrostowski, 2004; Stigler & Hiebert, 1999, 2004).

These concerns have led to calls for reforms of both mathematical teaching and the tasks used to educate students. Ginsburg et al. (2005) suggest that for American students to become more competitive with their peers from high-achieving countries, they need to improve their abilities to reason and use cognitively demanding skills on mathematically rigorous tasks as well as continue to hone their computational skills. Describing their vision for mathematics instruction, the National Council of Teachers of Mathematics (NCTM) advocates curriculum that is "mathematically rich, offering students opportunities to learn important mathematical concepts and procedures with understanding" (2000, p. 3). While the use of rigorous curriculum may be a necessary component to improve students' performance and learning, it is not the only aspect of mathematics education in need of reform. NCTM (1991, 2000) has also encouraged teachers to modify their instruction to facilitate students' engagement in such tasks by providing them with opportunities to struggle and grapple with complex mathematical problems and ideas as well as encourage student participation in discourse around these tasks. Critical aspects of classroom-discourse instruction suggested by NCTM (1991) include the need for teachers to pose questions, push students to justify their reasoning, and make decisions about what mathematical elements to highlight during the discussion and when to do so in order to assist students in progressing their mathematical understanding.

Unfortunately, in many American classrooms students are still exposed to few of the complex tasks that educational reformers have called for (Stigler & Hiebert, 2004). Additionally, many teachers who do provide their students with rigorous tasks are not able to maintain the level of rigor of the task throughout their instruction (National Center for Education Statistics, 2003). The purpose of the study described herein is to investigate the impact of professional development on teachers' ability to select and implement cognitively demanding mathematical tasks as well as to conduct whole-class discussion around these tasks. The hypothesis is that as teachers engage in exploration of their own practice and become aware of effective methods for using mathematically rich tasks in their classes, they will begin to use these same skills in teaching lessons that are not the focus of the professional development and thus improve their practice.

1.2 BACKGROUND

1.2.1 High-Level Tasks Have the Greatest Impact on Student Learning

Stein, Grover, and Henningsen (1996) defined a mathematical task as a "classroom activity, the purpose of which is to focus students' attention on a particular mathematical idea" (p. 460). Mathematical tasks play a key role in classroom instruction and student learning (Doyle, 1983, 1988; Hiebert & Wearne, 1993; Stein et al., 1996). Yet, mathematical tasks differ from one another in some key aspects. The level of cognitive demand required to solve a task is an important feature that can be used to distinguish various types of tasks. Stein et al. (1996) explained that tasks requiring high levels of cognitive demand are those that involve "the use of formulas, algorithms, or procedures with connections to concepts, understanding, or meaning" (p. 467) or that "include complex mathematical thinking and reasoning activities such as making and testing conjectures, framing problems, looking for patterns, and so on" (p. 466). They also explained that tasks requiring lower levels of cognitive demand call for memorization or "the use of formulas, algorithms, or procedures *without connection* to concepts, understanding, or meaning" (p. 466, emphasis in original). They used these notions of high and low levels of cognitive demand to define *high-level tasks* as those that require a high level of cognitive demand. Following this convention, I will hereafter refer to tasks requiring lower levels of cognitive demand as *low-level tasks*. The Task Analysis Guide (Stein & Smith, 1998) (Appendix A) provides more detailed characteristics of high-level and low-level tasks.

The difference in the level of cognitive demand of mathematical tasks has received much attention during the last two decades as research has shown it influences student learning. Hiebert and Wearne (1993) investigated six second-grade classrooms, two of which used an alternative instructional approach. An important aspect of this alternative approach was the use of high-level tasks that were designed to "encourage students to develop procedures for adding and subtracting multidigit numbers based on their understanding of place value" (p. 398) as opposed to following procedures introduced to them by the teacher or the textbook. The other four classes involved in the study used "the more conventional textbook program" (p. 398). Using assessments of place value and multidigit addition and subtraction of fractions, Hiebert and Wearne found that the students in the classes which who used the alternate approach to instruction had larger performance gains than their peers in the conventional classrooms. While there were many possible factors that contributed to the improved student learning gains in the two classes that employed the alternative instructional approach, the use of high-level tasks in these classes is significant.

Stein and Lane (1996) reported on student learning outcomes from four middle schools that had participated in the QUASAR project (cf. Silver & Stein, 1996). Teachers' participation in QUASAR provided them with opportunities to collaborate with teacher educators or other resources partners from local universities, colleges, or agencies. These collaborations centered on aiding teachers to identify and implement high-level tasks. To determine the impact of the teachers' work on instruction and learning, researchers observed their classes to determine the types of tasks used and the level of thinking at which students engaged as they worked on the tasks. Researchers also assessed students yearly or bi-yearly to determine what they had learned. Stein and Lane found that students who consistently had opportunities to engage in high-level tasks—and engaged in these tasks at a high level—had greater performance gains than those who typically engaged in low-level tasks.

Boaler and Staples (2008) investigated the impact of two contrasting approaches to teaching and learning. They found that students who experienced curriculum and instruction using a reform-based approach, which included the use of high-level tasks as a critical piece, "learned more, enjoyed mathematics more and progressed to high mathematical levels" (p. 609) than those students who learned in a more traditional setting that included the more frequent use of low-level tasks. The students in this study who experienced the reform-based approach had significantly lower baseline mathematics scores than those who experienced the more traditional educational experience. However, after one year those students in the reform-based approach had assessment scores equal to their peers in the traditional schools and after two years were achieving significantly higher on the assessments. Additionally, Boaler and Staples found that "achievement differences between students of different ethnic groups were reduced in all cases and were eliminated in most" (p. 610).

In a similar study, Boaler (1998) examined the impact that two differing teaching approaches had on student learning. She describes the first as a traditional, textbook, back-to-basics approach. Boaler calls the second approach "process-based" and explains that it is comprised solely of open-ended activities. The tasks described in the traditional approach closely resemble Stein and her colleagues' (1996; 2009) description of low-level tasks and those tasks used in the process-based approach fit their description of high-level tasks. Boaler found that students in the process-based approach had higher scores on an assessment aimed at assessing applied activities based in the context of their schools than students in the traditional approach. She also found no differences in performance between the two sets of students on a second assessment that used traditional, closed questions. Boaler explained that the two approaches impacted students' ability to think and reason mathematically very differently. She posited that the student in the traditional approach developed an *inert* (Whitehead, 1962) knowledge, which they struggled to use in contexts other than questions similar to those in the textbook from which they learned the knowledge. These students expressed that they were unable to interpret unfamiliar questions or apply the algorithms and procedures they had learned to these unfamiliar questions. In contrast, Boaler reported that while the students in the process-based approach did not have as great a repertoire of learned algorithms, they were more capable of applying what they had learned, or developing new, unlearned procedures, to new contexts. She also found that these students had developed an ability to reason about and use mathematics in new contexts.

1.2.2 The Best Student Learning Happens When High-Level Tasks Are Implemented

Stein and her colleagues (1996) introduced a conceptual framework for considering the relationship between tasks, instruction, and student learning. The Mathematical Tasks Framework (see Figure 1.1) shows the evolution of a task as it passes through three stages: first as it appears in the curriculum or instructional materials, second as the teacher sets up the task with his or her students, and third as the task is actually implemented by the teacher and the students. It also shows how this evolution affects student learning. Stein et al. (1996) found that although a task may begin as a high-level task as it is presented in the curriculum materials, it may devolve into a low-level task during either the set up or implementation stages of instruction.

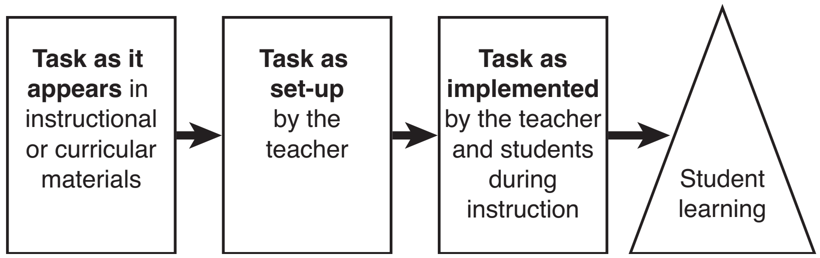

Figure 1.1: The Mathematical Tasks Framework (Stein & Smith, 1998)

While students' learning is positively impacted by increased opportunities to engage in high-level tasks, research has shown this positive affect to be even greater in situations where teachers are able to maintain the high level of the cognitive demand of the task throughout the set up and implementation stages of instruction. Stein and Lane (1996) found that student performance gains were greater for students in classrooms where tasks were set up and implemented at a high level than for students in classrooms where high-level tasks were selected but declined during the implementation stage.

1.2.3 High-Level Tasks are the Most Difficult for Teachers to Implement Well

Although the greatest student learning gains have been shown to occur when high-level tasks remain at a high level throughout all the stages of instruction presented in the Mathematical Tasks Framework, teachers tend to struggle to maintained high-level tasks at a high level throughout the process. Stein et al. (1996) examined the set up and implementation of over 140 mathematical tasks by teachers in the QUASAR study. They found that the majority of tasks that were set up as high-level by the teachers declined to low-level tasks during the implementation stage. The Trends in Mathematics and Science Study (TIMMS) video study investigated 100 eighth-grade mathematics classes each from seven countries (National Center for Education Statistics, 2003; Stigler & Hiebert, 2004). The study found that although 17% of the tasks selected by U.S. teachers were high-level tasks, none were implemented at a high level.

1.2.4 The Importance of Conducting Whole-Class Discussions Around High-Level Tasks

An important factor that affects the maintenance of high-level tasks is the manner in which teachers choose to conclude their lesson. In many cases, this portion of the lesson includes teachers conducting whole-class discussions about tasks (Otten, 2010; Stein, Engle, Smith, & Hughes, 2008). Classroom discussions are viewed as important for multiple reasons. Hatano and Inagaki (1991) found that students who participated in group discussions obtained knowledge that, in most cases, would not have been possible to acquire without participating in these discussions. Use of classroom discussions has also been shown to boost students' academic achievements for some sub-populations of students (Michaels, O'Connor, & Resnick, 2007).

However, the manner in which teachers conduct discussions around tasks has an impact on the type of thinking in which students engage. Kazemi and Stipek (2001) found that as teachers employed specific sociomathematical norms, they consistently pressed their students to engage in conceptual thinking. Stein et al. (2008) suggested that conducting class discussions in a manner that allows students to struggle with the critical mathematical concepts fundamental to the tasks being discussed supports student learning in three critical ways. First, this type of discussion allows students to learn mathematical discourse practices. Second, these discussions make students' mathematical thinking public, which in turn provides the opportunity for possible misconceptions to be addressed. Third, whole-class discussions encourage students to create their own mathematical ideas and allow them and others to challenge these ideas.

Whole-class discussions around high-level tasks in which teachers consistently press their students to think conceptually and allow them to struggle with fundamental mathematical concepts are aligned with the type of teaching and learning advocated by reform-oriented educational policy makers (National Council of Teachers of Mathematics, 1991, 2000; National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010). These discussions are critical to student learning because they "give students opportunities to share ideas and clarify understandings, develop convincing arguments regarding why and how things work, develop a language for expressing mathematical ideas, and learn to see things from other perspectives" (M. S. Smith, Hughes, Engle, & Stein, 2009, p. 549).

1.2.5 Effectiveness of Professional Development in Changing Teachers' Practice

One means of bringing about changes in teachers' instructional practices is professional development geared toward the desired changes. Studies on the impact of professional development have shown that focusing on specific instructional practices during professional development increases teachers' use of those practices in their instruction (Desimone, Porter, Garet, Yoon, & Birman, 2002; Garet, Porter, Desimone, Birman, & Yoon, 2001). There are multiple examples of professional development impacting teachers' practice, specifically with regard to their use of pedagogical moves aimed at improving classroom discourse, as well as their ability to select and maintain high-level tasks.

As part of a four-year collaboration with university-based mathematics teacher educators, eight secondary school mathematics teachers participated in professional development in the form of study group meetings aimed at assisting the teachers to better understand and adapt their classroom discourse practices to improve their instruction and their students' learning (cf.

Herbel-Eisenmann & Cirillo, 2009). One of the essential activities in which members of this collaboration participated was to read and discuss research literature on classroom discourse. These readings and discussions focused teachers' attention on various discourse practices and their potential impact on student learning. For example, they considered how the types of questions they asked and the patterns of questioning they used influenced the discourse and the math-talk communities they created in their classrooms. Investigating the impact of these readings and discussions on a specific discourse practice—"revoicing" (O'Connor & Michaels, 1993, 1996)—Herbel-Eisenmann and her colleagues (2009) found as the teachers participated in this collaboration over time, the number of connections they made between the educational theory about which they were reading and their own classroom practices increased. The researchers also reported that these connections grew increasingly detailed over time. Herbel-Eisenmann et al. also reported that their group discussions touched on important facets of revoicing that were not found in the existing literature such as the dilemmas teachers face when employing revoicing in their classrooms and the importance of the context of the conversation.

Nathan and Knuth (2003) studied the changes that one experienced middle school teacher made to her practice as a result of her participation in professional development focused on the analysis of classroom discourse. They found that over a two-year period this teacher was able to change her teaching so that it better aligned with the type of teaching advocated by the discussions and activities in the professional development.

Arbaugh and Brown (2005) investigated the impact of professional development that engaged a group of high school geometry teachers in examining their practice with regard to the types of instructional tasks that they used. The group read literature about the level of cognitive demand (LCD) of tasks, engaged in a task-sorting activity in which they determined the LCD of a set of mathematical tasks, and discussed the LCD of the tasks that they were using in their own teaching and how to modify low-level tasks so that they would have a higher LCD. The researchers compared the tasks that the participating teachers' used in their classrooms during the first week of the professional development to the tasks they used during the last week of the professional development (a span of approximately six months). They found that while this intervention did not produce statistically significant changes in the overall characteristics of the tasks teachers used, three of the six teachers from whom they collected data selected a much higher percentage of high-level tasks at the end of the professional development compared to the beginning.

Boston and Smith (2009) studied the impact of professional development geared at aiding teachers to select and then implement high-level, cognitively demanding tasks. They found that after participation in this professional development, teachers more frequently selected high-level tasks and were more frequently able to maintain these tasks at a high level throughout the implementation stage of instruction. During a follow up study, Boston and Smith (2011) found that two years after completing the professional development teachers continued to select high-level tasks and maintain these tasks at a high level during implementation at a statistically significant higher rate than before their participation in the intervention.

1.2.6 The Purpose of the Five Practices is to Help Teachers Implement High-Level Tasks Well

Stein and her colleagues (2008) suggest that teachers may struggle to maintain high-level tasks at a high level of cognitive demand during the implementation stage of instruction, particularly when conducting whole-class discussions, because of a lack of knowledge of how to prepare for and orchestrate these discussions. They point out that much of the focus around whole-class discussions has been around building norms of participation for the teachers and students (e.g., Cobb, Wood, & Yackel, 1993; Lampert, 1990). Yet teachers have not been given suggestions of specific instructional moves they could use to ensure that these discussions provide students with opportunities to constructively share their thinking and still allow the teacher to steer the conversation so that it will help students develop the critical mathematical understandings the conversation is meant to help them obtain.

Stein et al. (2008) proposed a set of instructional practices geared toward assisting teachers to conduct whole-class discussions around high-level, cognitively demanding tasks. These practices, *anticipating, monitoring, selecting, sequencing,* and *connecting*, (hereafter referred to as the *five practices*) are designed to help teachers be able to understand and productively use students' solutions to high-level tasks in such a way as to help progress student understanding (M. S. Smith et al., 2009; M. S. Smith & Stein, 2011; Stein, Engle, et al., 2008). The five practices is a set of practices designed to aid teachers by taking away the need to make on-the-fly decisions about students' solutions and assist teachers in planning how to deal with these choices in advance of teaching the lesson. As Stein et al. (2008) explained, "by expanding the time to make an instructional decision from seconds to minutes (or even hours) our model allows increasing numbers of teachers to feel—and actually be—better prepared for discussions" (p. 321).

1.3 THE STUDY

1.3.1 Purpose and Research Questions

Using qualitative research methods, this study examined teachers' participation in specific and purposeful professional development and how this participation influenced their instructional practices related to the topics of focus in the professional development. It sought to identify patterns in teachers' use of these practices and potential shifts in these patterns over time. The study explored the context, factors, and mechanisms that affected teachers' uptake (or lack thereof) of the ideas and suggestions discussed during the professional development. This study also investigated the impact of professional development on mathematics teachers' use of high-level tasks and the five practices. As part of the professional development, teachers participated in the following activities:

- Group members read and discussed existing research literature regarding the selection and use of high-level tasks, classroom discourse, and the five practices.
- Teachers identified tasks that they planned to use with their students, referred to as *focus tasks*, as well as the learning goals for these tasks.
- As a group, the teachers and university-based mathematics teacher educators discussed the focus tasks, the level of cognitive demand of these tasks, potential aspects of the tasks with which students may have struggled, and possible solution strategies students may have used while working on the task.
- Teachers refined the focus tasks as well as their learning goals and lesson plans for these tasks based on the discussion during the professional development. They then implemented the task with their students.

– Following their implementation of the focus tasks, teachers reflected with the other teachers and university-based mathematics teacher educators on their implementation of the task.

This study examined how the professional development described above influenced teachers' ability to select and implement high-level tasks and to use the five practices to conduct whole-class discussions around high-level tasks. Specifically this study addressed the following research questions:

I. To what extent do teachers participate in the professional development focused on selecting and implementing high-level tasks?

II. To what extent does teachers' use of high-level tasks change over the course of their participation in professional development focused on selecting and implementing high-level tasks?

III. To what extent does teachers' ability to maintain the cognitive demand of high-level tasks in the set up and implementation stages of instruction change over the course of their participation in professional development focused on selecting and implementing high-level tasks?

IV. To what extent does teachers' use of the practices of *anticipating*, *monitoring*, *selecting*, *sequencing*, and *connecting* in their instruction change over the course of their participation in professional development focused on selecting and implementing high-level tasks?

V. What relationship, if any, is there between teachers' use of the practices of *anticipating*, *monitoring*, *selecting*, *sequencing*, and *connecting* and their ability to maintain the level of cognitive demand of high-level tasks?

VI. To what extent do teachers incorporate the ideas and suggestions made about their focus tasks during the professional development meetings in their implementation of the focus task?

1.3.2 Significance

This study is valuable in that it investigated teachers' capacity to select high-level tasks and the impact of their use of the five practices on their ability to set up and maintain tasks at a high level. It also investigated the efficacy of professional development centered on supporting teachers' growth in these important areas. As teachers learn about the levels of cognitive demand of tasks, how differences in these levels impact student learning, and specific instructional moves they can use to conduct classroom discussion around these tasks, they will develop knowledge and skills that will aid them in maintaining the high level of cognitive demand of these tasks as students engage in them. This increased ability to select and maintain high-level tasks will then positively impact student learning.

By investigating the level of teachers' use of the five practices and the impact this has on their instruction, this study adds to the literature on the cognitive demands of mathematical tasks by adding to knowledge of the possible factors that aid teachers in maintaining high levels of cognitive demand throughout implementation of the task. The study also contributes to the existing knowledge base of each of the individual practices of anticipating, monitoring, selecting, sequencing, and connecting and how using these can assist teachers in their instruction.

This study contributes to existing literature regarding the teachers' selection and implementation of high-level tasks in that the context in which the study takes place combines important features of previous studies while including additional, potentially influential factors.

The QUASAR project focused on working with teachers at urban middle schools serving low-income student populations to improve student learning in an effort to disprove the belief that students in such schools perform poorly in mathematics because they lack the ability to learn mathematics well (Silver & Stein, 1996). In contrast, Boston and Smith (2009, 2011) studied teachers from both middle and high schools from suburban areas. The study herein combines these two features as it takes place in the context of a Grades 6-12 inner city, high-poverty school. However, this study differs from these earlier studies in that the teachers participating in the professional development were mandated by the school administration to do so. This is a significant difference, as in earlier studies teachers participated voluntarily (e.g., Arbaugh & Brown, 2005; Boston & Smith, 2009, 2011; Silver & Stein, 1996; Stein et al., 1996; Stein & Lane, 1996). A second notable distinction between this study and prior research is that the teachers in this study were required to use a mandated curriculum specified by the school district for which they worked. This was not the case in previous studies. Thus, the results of this study can be used as a comparison to the prior studies to investigate (a) the impact of allowing (or not allowing) teachers to choose to participate in professional development on their uptake of the ideas and suggestions provided during the professional development, and (b) the role a mandated curriculum may play in teachers' selection and implementation of high-level tasks.

The relationship between professional development and teachers' practice is also examined in this study. Kazemi and Hubbard (2008) posited that research on professional development should investigate influences on teacher learning both in professional development and in related instruction in teachers' classrooms. They advocated professional development that focuses on the participating teachers' own instruction and artifacts from this instruction, and that focuses on connections between the professional development and classroom settings. The professional development that is the focus of this study includes both of these aspects. This study provides evidence of the impact such professional development can have on teachers' instructional practices. Specifically, it investigated teachers' use of ideas and suggestions around focus tasks made in professional development meetings examining their implementation of those tasks. It also examined the transfer of such ideas and suggestions to teachers' instruction of tasks that are not discussed during professional development meetings. This investigation of teachers' use of key ideas and suggestions provided in professional development provides insight into the design and implementation of future teacher education and professional development.

While there have been recent anecdotal reports of teachers employing the five practices, there is a dearth of research with regard to teachers' uptake of the five practices in their instruction. This study explores teachers' use of the five practices by incorporating qualitative research methods as well as data collection and analysis tools designed specifically for this purpose. The findings of this study are a first step toward filling the void of knowledge about teachers' use of the five practices, and the data collection and analysis tools used herein can serve in future research in this area.

1.3.3 Limitations

This study has several limitations. This study investigated a small number of teachers. The teachers chosen for this study were chosen as a sample of convenience. They were participants of larger research study (cf. M. S. Smith, Cartier, Eskelson, & Tekkumru-Kisa, 2012; Stein, Russell, & Smith, 2011), of which this work is a part. As such, these teachers are not a representative sample and therefore any findings are not generalizable to all teachers. However, the findings in this study do not seek to establish generalizability, rather the intention is to explore individual teacher's instruction related to the professional development and possible changes in this instruction overtime. This study looked in depth at these teachers' participation in the professional development and their instruction and contextualized possible factors influencing their instruction as well as possible relationships between the professional development and their teaching practices.

The data collection used to document teachers' instruction is also a limitation of the study. A small proportion of teachers' instruction was captured as data and therefore may not be representative of their typical instruction. Further, the documentation of this instruction is dependent on the researcher's observational fieldnotes (Emerson, Fretz, & Shaw, 1995) as opposed to another method (e.g., video or audio recording) that could provide a more detailed account of the instruction.

This study only looked at the impact of the professional development meetings on teachers who selected the focus tasks for those specific meetings. It does not include any investigation of how the instruction of teachers who participated in the meetings but who did not select the focus tasks for those meetings, was impacted by the ideas and suggestions that were discussed. While that investigation could potentially provide important findings, it is beyond the scope of this study.

1.4 OVERVIEW

This dissertation is organized as follows. Chapter Two reviews the salient literature regarding three areas pertinent to the proposed study: (a) mathematical tasks, (b) classroom discourse, and (c) essential features of effective professional development. Chapter Three presents the methods of collecting, coding, and analyzing the data for the study. The results of the analyses described in Chapter Three are presented in Chapter Four. Chapter Five summarizes the results found in Chapter Four, and discusses the implications of these results and possible explanations for them as well as provides recommendations for further study.

2.0 CHAPTER 2: LITERATURE REVIEW

Improving teachers' instructional practices and the opportunities their instruction provides students to learn are the basis of much of the research in the field of education (e.g., Lester, 2007a, 2007b), as well as the emphasis of mathematics education reform (e.g., National Council of Teachers of Mathematics, 2000; National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010) and government policies such as the No Child Left Behind Act (NCLB; Public Law No. 107-110, 115 Stat. 1425, 2002). Two key aspects of mathematic instruction that affects student learning are the type of activities in which teachers choose to engage students and the type of discourse that occurs in classrooms. Enhancing teachers' ability to select and use quality tasks and to provide students with environments and opportunities to engage in mathematically rich conversations can positively influence student learning (National Council of Teachers of Mathematics, 1991, 2000; Stein & Lane, 1996). Teacher professional development is a common method used in an attempt to impact teacher practices (U.S. Department of Education, Office of Planning, Evaluation and Policy Development, Policy and Program Studies Service, 2007), yet the quality and type of professional development programs teachers engage in vary widely (Ball, Lubienski, & Mewborn, 2001; Guskey & Yoon, 2009).

The aim of this study is to investigate the impact of professional development designed to aid mathematics teachers in selecting quality mathematical tasks and implementing them at a high-level, particularly with regard to how teachers structure and conduct whole-class discussions around these tasks. In this chapter, salient information from existing educational literature and research regarding three topics that are germane to this study will be reviewed: tasks (particularly mathematical tasks), classroom discourse, and the essential features of effective teacher professional development. The first section of the chapter focuses on mathematical tasks and presents three frameworks that have been used in prior research to analyze teachers' selection and implementation of mathematical tasks as a method of examining teachers' instruction. The second section of the chapter reviews the importance of classroom discourse for student learning, the role teachers play in creating opportunities for classroom discourse, and the challenges teachers have using discourse in their classrooms. It also presents five instructional practices teachers can use to better facilitate classroom discussions around high-level mathematical tasks. The third section of the chapter examines research on teacher professional development, identifies six critical features of effective professional development, and explores these features in depth. The final section of the chapter ties these three topics together and presents a vision for how the study's intervention will attempt to impact teachers' instruction.

2.1 TASKS

2.1.1 Academic Tasks

The activities in which students engage and the manner in which they engage play a critical role on the impact students' educational experiences have on their learning (Doyle, 1983, 1988; Stein et al., 1996; Stein & Lane, 1996). Doyle (1983) proposed the concept of academic tasks as a useful focus of analysis when examining instruction and the impact it has on student learning. He defined *academic tasks* as consisting of three elements: (a) the product students are assigned to produce (examples in mathematics would include an answer on a test question, an equation that represents a given situation, and a conjecture based on observed phenomena), (b) the process by which students are to produce the desired product, and (c) the resources students use to produce the product.

Doyle (1983) suggested that tasks impact what and how students learn. He identified four types of academic tasks: (a) memory tasks, (b) procedural or routine tasks, (c) comprehension or understanding tasks, and (d) opinion tasks. *Memory tasks* are activities for which students reproduce previously learned information (e.g., repeating a given theorem such as the Pythagorean theorem[1]). *Procedural or routine* tasks are tasks in which students demonstrate their ability to correctly use a procedure to produce an answer (e.g., use the distance formula to find the distance between two points on the Cartesian plane). *Comprehension or understanding tasks* require students to apply previously learned concepts and procedures to similar, but unique contexts or to make inferences based on the previously learned material. An example of this would be making conjectures about an element in a sequence based on the first few elements of that sequence (e.g., predicting sum of the first 100 positive integers based on calculating the sum of the first two positive integers, then the first three, then the first four). *Opinion tasks* ask students to explain their preference of one choice over other viable options (e.g., choose one measure of central tendency over other possible measures to best calculate the data for a specific situation).

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[1] While the examples for each of the categories of tasks are given in mathematics, the

Doyle (1983) further posited that the type of tasks teachers provide students affects the learning experiences they are afforded. Task type determines what structural features of the content on which students focus. He pointed out that memory tasks focus on the surface structures of the task while comprehension tasks focus on the conceptual structure. In later work, Doyle (1988) suggested the cognitive level of academic tasks as a means for discussing the academic work in which students engage. He noted that different types of tasks have different levels of cognitive demand and that these levels can influence the opportunities student have to learn.

2.1.2 Mathematical Tasks

Building on Doyle's work, Stein, Grover, and Henningsen (1996) proposed that by narrowing in on mathematical tasks, as opposed to academic tasks in general, researchers might better study the relationship between teaching and learning in mathematics classrooms. They define *mathematical tasks* as a set of one or more activities that have the purpose of directing students' attention on a specific mathematical concept or idea. The National Council of Teachers of Mathematics (NCTM) (1991), in setting forth its vision of quality teaching, emphasized the importance of mathematical tasks. They suggested that tasks dictate the context in which students learn mathematics, determine the manner in which students learn (e.g., focusing in applying memorized procedures compared to exploring and making and testing conjectures about mathematical ideas), and convey messages about the type of mathematical thinking that is valued.

2.1.3 Mathematical Tasks as a Construct for the Analysis of Teaching

Stein et al. (1996) proposed using the mathematical tasks teachers choose to implement with their students and the manner in which these tasks are actually implemented as a construct for analyzing mathematics teachers' instruction and the opportunities it affords for student learning. Their work emanated from research done on the Quantitative Understanding: Amplifying Student Achievement and Reasoning (QUASAR) project. The QUASAR project was a multi-year intervention aimed at improving mathematics instruction in low-income urban middle schools (Silver & Stein, 1996). Schools were chosen to participate in the project based on applications submitted by collaborative teams at each school. These teams consisted of teachers at the school and university-based teacher educators near each site (Stein et al., 1996). The purpose of these collaborative teams was to assist teachers in selecting and implementing cognitively demanding tasks in their classrooms with the intent to provide students with improved learning opportunities. The QUASAR project focused on reform-based instruction as a key element of enhancing teachers' instruction. This emphasis, as well as the focus on the selection and use of mathematical tasks, resulted in remarkable changes in teacher practice as well as increased opportunities for student learning (Stein, Silver, & Smith, 1998).

The research on the QUASAR project produced three key analytical frameworks for investigating mathematics teachers' instruction and opportunities for students to learn from it: (a) the Task Analysis Guide, (b) the Mathematical Tasks Framework, and (c) the list of factors associated with decline and maintenance of the level of cognitive demand of mathematical tasks. The following sections will explore each of these frameworks in detail.

2.1.3.1 The Task Analysis Guide

Similar to Doyle's (1988) proposal, Stein and her colleagues (1996) explored the impact of mathematical tasks on student learning by categorizing tasks based on their level of cognitive demand. They proposed that tasks be divided in to four categories: (a) *memorization*, (b) *procedures without connections*, (c) *procedures with connection*, and (d) *"doing mathematics"*. Smith and Stein (1998) defined each of these categories as follows. *Memorization* tasks are those that require students to reproduce an answer by recalling a previously learned fact, rule, formula, or definition, or require students to memorize a fact, rule, formula, or definition. These tasks do not require a procedure to be solved and do not have a connection to underlying concepts or meaning of the piece of information being memorized. *Procedures without connections* tasks require students to use a procedure in order to be solved. However, these tasks are very straightforward and do not necessitate any connections between the procedure that is required and the underlying mathematical concepts. *Procedures with connections* tasks have a suggested method for students to use, however the purpose of the tasks is to highlight how the focal procedure connects to the fundamental mathematical concepts that form the basis of the task. *Doing mathematics* tasks engage students in the work of mathematics by requiring them to use complex and nonalgorithmic thinking. These tasks do not have a suggested path for students to follow and students must explore various solution strategies while grappling with the underlying mathematical ideas. The Task Analysis Guide (TAG) (Appendix A), produced by Smith and Stein (1998) provides a full description of the characteristics of each type of mathematical task. To further aid in the categorization of mathematical tasks, Stein et al. (1996) grouped the procedures with connections and doing mathematics tasks together in one category, termed *high-level* tasks, and memorization and the procedures without connections tasks together in another, named *low-level* tasks. The TAG describes high-level tasks as those tasks that require a higher-level of cognitive demand from students to solve them, while low-level tasks do not.

Determining how to categorize tasks based on their level of cognitive demand is not a trivial matter. Stein et al. (2000) explained this can be difficult because some tasks have superficial features that make them appear to be high or low level even though they are, in fact, the opposite. Just because a task has these features, if it overtly or implicitly implies the need to use a specific solution strategy without connection to the conceptual underpinnings of the mathematics, it is not a high-level task. Similarly, some tasks may appear on the surface to be low level, but in fact be high-level tasks.

Stein and her colleagues (2000) also explained that the level of cognitive demand of a task is dependent upon the students that engage in the task, the learning environment in the classroom, and the tools that are available to the students. The same task may require significant, deep cognitive thinking from early elementary school students but may be solved fairly easily through the use of previously learned algorithm by older students. For example, the task *"A fourth-grade class needs 5 leaves each day to feed its 2 caterpillars. How many leaves would the students need each day for 12 caterpillars?"* (M. S. Smith & Stein, 2011, p. 3) may require significant thinking and reasoning from fourth-grade students in a class in which they are required to justify their thinking to the teacher and the other students using words, diagrams, or other representations. However, for eighth-grade students who have previously learned the cross-multiplication method (i.e., for $\frac{a}{b} = \frac{c}{d}, \ ad = bc$) for solving for a missing value in a proportion and have access to a calculator this task would be routine and require little cognitive strain.

2.1.3.2 The Mathematical Tasks Framework

Stein et al. (1996) suggested that mathematical tasks may change as they move through three stages of instruction (a) as represented in curriculum materials, (b) as set up by the teacher in the classroom, and (c) as implemented by the students. They explained that in order to use tasks as a construct for examining teachers' instruction, it is useful and necessary to compare the same task at the various stages of instruction. Stein and Smith (1998) refined the framework presented in the work by Stein et al. (1996) and developed the Mathematical Tasks Framework (MTF) (see Figure 1.1).

The first stage consists of the task as it is presented in the curricular materials given to the students. *Task set up* is defined as "the task that is announced by the teacher" (Stein et al., 1996, p. 460). This includes the manner in which teacher presents the task. Teachers can do this by displaying the task as it is in the textbook, on a worksheet, as written on the whiteboard in the front of the room, presented using computer presentation software, or in some other form. Task set up also involves the verbal directions provided by the teacher regarding the task before students engage in it. It could include a description of what type of work is expected or specific instructions regarding the solution method student should use. Task set up would also incorporate teachers' references to students' prior knowledge or related experiences, as well as any instructions as to how to use provide tools or other resources. A number of teacher-related factors may cause the task to change from its initial form in the instructional materials to how the teacher sets it up. Stein et al. (1996) suggest that the teacher's goals for the task, their subject matter knowledge, or the knowledge of students may affect how he or she sets up the task, and may produce a change in the level of cognitive demand of the task from how it is presented in the curriculum materials. For example, consider the Land Sections Task (Appendix B). The task as presented in the textbook asks students to determine the amount of land of various landowners. Students must compare the sizes of various fractions and add fractions that may have different denominators, yet no algorithm for doing so is provided, nor has one been demonstrated to the students in prior tasks. For students in sixth grade this task could be considered a high-level cognitively demanding task with no set solution path. Sleep and Eskelson (2012) documented the use of this task by Maria, an experienced middle school teacher. Her goal for the task was to provide her students with practice for adding and subtracting fraction. She had previously taught them the conventional algorithms for these operations and she expected students to use them when working on the task. The task as Maria set it up was significantly different from how it was presented in the curriculum, as her students already knew a specific solution method for solving it and were expected to use this method as they engaged in the task. Thus, the cognitive level of the task changed from how it was presented in the curriculum materials to how the teacher set it up with her students because of her goals for using the task.

Task implementation is the manner in which students actually work on the task. Despite how a task is represented in curriculum materials and how the teacher sets it up, the students may implement it in a different manner. Stein et al. (1996) suggested four possible factors that may lead to alterations in a task between the set up and implementation stages of instruction. These are (a) classroom norms, (b) task conditions, (c) teacher instructional habits and dispositions, and (d) student instructional habits and dispositions. An example of a change in the level of cognitive demand of a task between the set up and implementation stages would be a teacher's use of an open-ended task such as asking student to find the quadrilateral with largest possible area given a fixed perimeter and asking students to provide a written explanation for why they believe the quadrilateral they identified had the largest area. The teacher may set up the task by briefly reviewing the ideas of area and perimeter and providing the students with manipulatives, graph paper, and other resources, but because of time constraints provide students with a very limited amount of time to explore the task. This may cause students to not engage in the task as intended but rather to either not attempt to solve it, randomly guess a few quadrilaterals and find which out of those few has the largest area, or engage in the task in a manner inconsistent with how the teacher set up the task.

2.1.3.3 Factors associated with maintenance and decline of the level of cognitive demand of mathematical tasks

Building on the first two frameworks (the TAG and MTF) and using data gathered as part of the QUASAR project, Stein et al. (1996) sought to identify specific factors that exist in lessons in which teachers set up a task at a high level and maintain it at a high level throughout the implementation stage (also see Henningsen & Stein, 1997). They also identified factors present when teachers set up tasks at a high level but the tasks are implemented at a low level. To do this they used a sample of over 100 lessons in which the main instructional task was determined to be set up as a high-level task. Trained researchers for the QUASAR project observed hundreds of classroom lessons taught by the teachers associated with the project. The researchers took extensive fieldnotes during the observations and created detailed summaries of the lesson afterward. These summaries were then coded using the classifications of tasks described in the TAG (i.e., memorization; use of formulas, algorithms, or procedures *without connection* to concepts; use of formulas, algorithms, or procedures *with connection* to concepts; and "doing mathematics"). These codes were applied to how the teacher set up the main instructional task of each lesson and how the students implemented the task. The researchers determined that a new category for task implementation, *unsystematic exploration*, should be added to the coding scheme, as it was present in multiple lessons. Based on previous literature, Stein and her colleagues created a list of possible factors present when students engage in tasks at a high level. They then examined each of the lessons from the data sample for which tasks were set up at a high level and remained at a high level through the implementation stage. They identified all of the factors from the list that were present in the lessons and counted the number of occurrences of each factor. The researchers repeated this process with the lessons that were set up at a high level but that declined to a lower level during implementation using a list of factors from prior research that appeared to be present when teachers had difficulty maintaining the high level of cognitively demanding tasks. Stein et al. (1996) found that seven factors were common in the lesson during which high-level tasks were maintained at a high level and that six factors were common among lessons during which high-level tasks declined to low-level tasks during implementation. The factors associated with the maintenance of high levels of the cognitive demands of tasks are (a) tasks build on students' prior knowledge, (b) scaffolding, (c) appropriate amount of time, (d) high-level performance modeled, (e) sustained pressure for explanation and meaning, (f) student self-monitoring, and (g) teacher draws conceptual connections. The factors related to the decline of high levels of the cognitive demands of tasks are (a) inappropriateness of the task, (b) classroom problems, (c) too much or too little time, (d) lack of accountability, (e) challenges become nonproblems, and (f) focus shifts to correct answer. Stein and Smith (1998) elaborated on these factors for both the maintenance and decline of the cognitive demand of high-level tasks and provided a fuller description of each one (please see Appendix C for their description of each factor).

These three frameworks (the TAG, the MTF, and the factors associated with the decline and maintenance of high-level cognitive demands) have been used together to investigate teachers' instruction and the impact it has on student learning (Arbaugh & Brown, 2005; Boston & Smith, 2009, 2011; Stein & Lane, 1996). Chapter Three, the methods section of this dissertation, will describe how these frameworks were used to analyze teachers' selection and implementation of mathematical tasks.

2.2 CLASSROOM DISCOURSE

Classroom discourse has been a fundamental element of the reform movement in mathematics education that has taken place over the last 20 years. NCTM (1991) published a set of standards for mathematics teachers that sets forth a vision of what mathematics teaching should entail. In this influential document, NCTM asserted that for the type of changes advocated therein to occur, teachers must become much more proficient at employing and conducting whole-class discussions around mathematical concepts. Not only must teachers provide opportunities for students to participate in classroom discussions, they must do so in a way that encourages students to investigate, reason about, and develop mathematical ideas. As part of the reform NCTM advocates, they suggested five major shifts in the environment of mathematics classrooms. These are shifts:

- Toward classrooms as mathematical communities - away from classrooms as simply a collection of individuals;
- toward logic and mathematical evidence as verification - away from the teacher as the sole authority for right answers;

- toward mathematical reasoning - away from merely memorizing procedures;
- toward conjecturing, inventing, and problem solving - away from an emphasis on mechanistic answer-finding; and
- toward connecting mathematics, its ideas, and its applications - away from treating mathematics as a body of isolated concepts and procedures. (p. 3)

Classroom discourse and whole-class discussion play a vital role in accomplishing each of these shifts in mathematics classrooms and the manner in which mathematics is taught. The movement away from individuals learning in isolation to learners participating in mathematical communities is critically dependent on the use of classroom discourse and discussions. As students have opportunities to share ideas with one another, they will begin to value one another's ideas and to work collaboratively to investigate mathematical problems. Classroom discussions can also play a fundamental role in aiding students and teachers in moving away from viewing teachers as the source of mathematics authority to a view of logic as the basis for determining the correctness of mathematical ideas. An example of this can be seen in Lampert's (1990) fifth-grade classroom. Lampert and her students were discussing what the last digit (the ones place) would be in 7 raised to the fifth power based on observations they had made of previous powers of 7. They did not want to ascertain the answer based on computation or with the use of a calculator, rather they were trying to determine the digit based on logic and reasoning. Describing this interaction, Lampert explained that she acted as a manager of the discussion and allowed the students to examine their own ideas and conjectures and those put forth by their classmates.

Classroom discussions also play a major role in aiding teachers in focusing mathematical learning and teaching on reasoning and away from a sole emphasis on procedural memorization and mastery as well as in allowing students to make and test conjectures. This can be seen in the case of Ball's third-grade classroom (Schoenfeld & Pateman, 2008). The class was discussing whether zero is an even or an odd number. As they discussed this, Sean suggested that six is both an even and an odd number. His classmate pushed against his claim and he was forced to reason using the mathematical definitions the class had developed for even and odd numbers to defend his conjecture. Classroom discourse and discussion can also play a vital role in assisting teachers to aid their students in making connections between various mathematical representations and concepts (Jansen, 2006). In describing the importance of classroom discourse and discussion NCTM (1991) explained that as students work collaboratively with one another to reason about and develop mathematical ideas, they engage in mathematical activities in which mathematicians and other users of mathematics in intellectual communities engage.

More recent reform documents have also emphasized the need for classroom discourse and discussions. In an updated version of their standards for mathematics teaching, NCTM (2000) noted that the standards they set forth in 1991 had, in many cases, been implemented in a superficial or incomplete manner; classroom discourse was singled out as an example of this. As such, the *Principles and Standards for School Mathematics* (PSSM) (NCTM, 2000) were designed to provide teachers, administrators, and others in the role of influencing how and what mathematics is taught to students as a guide in making these crucial decisions. PSSM presents six principles for teaching mathematics, five of which incorporate the use of classroom discussions in much of the same way as described in the 1991 document. The Common Core State Standards for Mathematics (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010) represents a consensus among mathematics educators, mathematicians, policy makers, and other stakeholders at state and national levels of education. The Common Core State Standards for Mathematics specifies which mathematical concepts should be taught in each grade level as well as the mathematical practices students should develop while learning these concepts. This document identifies eight such practices some of which necessitate classroom discussions. For example, the principle, *construct viable arguments and critique the reasoning of others* requires student to reason about and produce conjectures about mathematical ideas as well as to analyze the conjectures of others. In order to develop and master these skills, students will need to engage in mathematical discourse with one another as well as with their teachers.

2.2.1 Teachers' Role in Classroom Discourse

Teachers play a vital role in establishing and maintaining environments in which discourse is valued and students feel comfortable discussing mathematical ideas. NCTM (1991) proposed that teachers' roles in classroom discussions should consist of three elements. First, teachers are to stimulate student's thought about important mathematical concepts in ways that assist them to reason and ask questions about the concepts. This can be done by providing high-level, open-ended tasks as well as by asking thought-provoking questions. The second aspect of teachers' roles in classroom discourse is to move to the background and allow students to do the majority of the thinking and reasoning while still guiding the conversation toward the intended learning goals. The third element of conducting classroom discourse identified by NCTM is for teachers to monitor and be in control of who is participating in the discussion. Teachers must be aware of the level of understanding of the individual students and make decisions of who participates, how, and when based on this knowledge. In discussing the needed changes in teachers' practices and in curriculum material based on the newly proposed reforms in mathematics education, Lappan and Ferrini-Mundy (1993) noted that teachers play a crucial role in effecting quality classroom discussions. They also explained that teachers need to teach students how to engage in productive classroom discussions. Students will need to be taught how to listen and respond respectfully to other students and the teacher. They also need to know how to pose conjectures about mathematical concepts and understand that discussion of and disagreement with these conjectures does not imply personal judgment against individual students. Lappan and Ferrini-Mundy further suggested that teachers must also provide students with the tools necessary to investigate the mathematics under consideration. These tools may include physical manipulatives such as algebra tiles or base-ten blocks, technological tools like graphing calculators or computers, and strategies for investigating mathematical concepts (e.g., modeling or the use of visual or graphical representations).

In order to facilitate quality mathematical discussions, teachers must teach students to attend to nuances in mathematical discussions that are not present in discussions in other school subjects. Yackel and Cobb (1996) distinguished between sociomathematical norms and general social norms that exist for typical classrooms of all subjects. They defined sociomathematical norms as norms that are specifically related to the mathematical aspects of the activities in which students engage. Examples of sociomathematical norms are developed understandings of what makes ideas mathematically different or why one solution is mathematically more efficient than another solution. They pointed out that the teacher and her students are continually developing and refining these norms, and that sociomathematical norms are not a set of prescribe rules of engagement, rather they may differ from class to class. Yackel and Cobb also emphasized the need for teachers' active participation in classroom discourse. They explained that students, without the teacher's involvement and guidance, will not be able to effectively produce mathematical ideas, understandings, or conventions that are compatible with the existing field of mathematics. This exhortation that teachers must be active participants in classroom discussions combined with the recommendations given by NCTM would point to teaching reminiscent of the examples from Ball (Schoenfeld & Pateman, 2008) and Lampert (1990) described earlier.

Kazemi and Stipek (2001) provide an example of how Yackel and Cobb's notion of sociomathematical norms can impact classroom discussions. Their study was an effort to identify differences in the characteristics of discourse in classrooms with different levels of emphasis on conceptual thinking. The data for their study consisted of video recordings of four upper-elementary teachers' implementation of the same lesson. The researchers used qualitative data analysis techniques to analyzed transcripts of these lessons. This consisted of annotating the transcripts of the lessons and creating formal summaries of each lesson. These summaries were then examined for similarities and differences among the lessons. Researchers paid particular attention to both the teachers' and the students' roles in each lesson as well as how engaged students were in each lesson. The transcripts and the summaries were then used to identify the various social and sociomathematical norms as well as the level of press for conceptual thinking that was present in the teacher-student interactions in each lesson. Kazemi and Stipek observed a consistent difference between lessons that consisted of high-press teacher-student interactions and those with low-press interactions. The lessons with high-press interactions made use of four specific sociomathematical norms that were absent in the low-press lessons. Thus, the use of sociomathematical norms, at least in the case of the lessons in this study, aided teachers in conducting classroom discourse that maintained a high-press for conceptual thinking.

2.2.2 Teachers Struggle to Make Effective Use of Classroom Discourse

While the importance of classroom discourse has been well established, conducting quality classroom discussions is an extremely complex aspect of instruction that many teachers struggle to use effectively with their students.

In describing the importance of classroom discourse, Leinhardt and Steele (2005) pointed out that if done well this aspect of teaching can greatly empower student learning. However, they also noted that if classroom discourse is managed poorly by the teacher, it can result in student confusion and the possible development of misconceptions. To investigate the complexity of classroom discussions, Leinhardt and Steele studied a 10-lesson unit of instruction on functions and graphs taught by Magdalene Lampert to fifth-grade students. They acknowledged that Lampert was by no means an ordinary teacher. As such, the purpose of their study was to identify and examine the tools she, as an expert teacher, used to orchestrate classroom discourse. From previous research, Leinhardt and Steele identified three tools expert teachers use to organize the classroom activities and environment in which the discourse takes place. These tools are *routines*, *metatalk*, and *intellectual climate*. They defined *routines* as "small, socially shared, scripted pieces of behavior" (p. 91) and *metatalk* as discussion that aids student in connecting the activities in which the engage with the content they are learning. With regard to *intellectual climate,* they noted that classroom environment sends strong messages to students about what is and is not valued. This is true for the type of behavior and activities that are carried out in the classroom as well as for the type of discourse that occurs.

Leinhardt and Steele (2005) videotaped Lampert's instruction for the first five days of the study and Lampert videotaped herself for the last five days. The researchers also conducted brief interviews with Lampert before and after each of the first five lessons. The intent of these interviews was to gain insight into Lampert's instructional goals for the lessons and how well she felt she was able to accomplish these goals. Transcripts of the recorded lessons and interviews were used to identify and trace instructional moves that influenced classroom discourse throughout the lessons. Leinhardt and Steele's in-depth analysis of the Lampert's instruction demonstrates the complexity of conducting classroom discussions as wells as Lampert's mastery to do so. They explained that Lampert expertly facilitated the discourse around the mathematical ideas by "standing to the side of the dialogue" (p. 133). She did not lecture or simply provide mathematical ideas to student, yet she also did not allow students to engage in unsupervised and unrestricted mathematical explorations. Rather, she participated in the mathematical conversations in such a way that students were able to provide the discussed ideas, questions, and conjectures, but so that she was able to steer the dialogue to a predetermined mathematical destination.

In studying Lampert's instruction, Leinhardt and Steele found that she used the three tools they identified as important from previous research. They described Lampert's classroom as one in which there was a strong intellectual climate and engaging in deep discussions about fundamental mathematical concepts was ordinary. Students were encouraged to participate in these discussions and to serve as the main discussants. Leinhardt and Steele felt that Lampert's use of routines and metatalk were two important factors in establishing this environment. Lampert used routines to hold all members of the classroom community, including herself, accountable to each other and to the validity of the mathematics they were discussing. She used multiple forms of metatalk to highlight important aspects of discussion and to mark ideas or questions that the class might need to refer to in later discussions. In addition, Leinhardt and Steele also found that Lampert's skillful ability to use students' unexpected and unhelpful responses as well as occasions during which students did not answer questions positively impacted the discourse in her classroom. Leinhardt and Steele noted that Lampert's abilities to conduct whole-class discussions through the use of routines, metatalk, creating and intellectual climate, and directing the dialogue differ greatly from other teachers they have studied and are not skills typical teachers possess. Thus, while Lampert was able to negotiate the complexity of effectively facilitating classroom discussions to aid her students' learning, many teachers may struggle to do so as they have not yet developed the needed knowledge or skills.

Research shows that few teachers are able to use classroom discourse in a manner consistent with the ideas of instructional reform promoted by NCTM. In an effort to investigate the impact of mathematics educational reform on classroom teaching Spillane and Zeuli (1999) examined teachers' knowledge and their use of academic tasks and classroom discourse patterns. They used data gather as part of a larger research study examining state-level educational policy and policymaking and mathematics and science teaching. Data from the larger study consisted of survey data from the TIMSS Teacher Questionnaire for two different populations, third- and fourth-grade teachers and seventh- and eighth-grade teachers. The researchers analyzed teachers' responses to the questions regarding reform-based practice on the TIMSS questionnaire and identified a subset of 25 teachers who they then interviewed and observed. These 25 were selected because of their high self-reports of reform-based instructional practices. Of the 25 teachers, all reported being familiar with recent mathematical reforms and 21 stated that they were "familiar" or "very familiar" with NCTM standards. Eighteen of these 25 teachers were third- or fourth-grade teachers while the other seven were seventh- or eighth-grade teachers. The researchers observed each teacher's instruction twice, with the exception of one teacher who was only observed once.

The researchers examined the type of tasks and discourse patterns the teachers' used in their instruction. Three distinct patterns emerged from the data. The first pattern was the use of conceptually grounded tasks and conceptually centered discourse; four of the 25 teachers fell into this category. The tasks used by these teachers as well as the discourse practices they employed were aligned with the mathematics education reform with which they claimed to be familiar. The second pattern the researchers noticed was teachers' use of conceptually oriented tasks and procedure-bounded discourse. The instruction of 10 of the 25 teachers fit this pattern. While these teachers tended to select reform-based tasks, the discussions in their classrooms around these tasks were not focused on conceptual learning. The third pattern that the researchers identified was described as "peripheral changes, continuity at the substantive core" (p. 16). The remaining 11 teachers were placed in to this category. Although the majority of these teachers reported either being "fairly familiar" or "very familiar" with the NCTM standards, they used tasks and discourse that focused primarily on mathematical procedures and facts. Thus, although these 25 teachers reported being familiar with the NCTM standards for reform-based mathematics teaching, and a majority of them selected high-level tasks for their students to engage in, the vast majority (over 80%) did not engage students in productive discourse centered on conceptual learning. While this study in no way claimed to represent the general population of teachers in the United States, it does shed light on possible issues that may apply to many teachers throughout the country.

2.2.3 Challenges of Using Classroom Discourse Centered on High-Level Tasks

Many of the challenges that teachers face when implementing discourse in their classrooms are exacerbated when teachers conduct whole-class discussions around high-level cognitively demanding tasks. Silver and Smith (1996) described two of the challenges teachers in the QUASAR project faced as they attempted to build discourse communities and focus students' attention on high-level mathematical tasks. The first challenge teachers faced is best viewed as a tension between creating environments of mutual respect and trust and developing a culture of constructive criticism when establishing communities of discourse. Students must feel the classroom is a safe place to share ideas, conjectures, and questions without the risk of being attacked or degraded because of what they share. On the other hand, for students to grapple with and explore critical mathematical ideas, they must be able to critically question the concepts being discussed. Silver and Smith suggested that this tension can be negotiated by establishing norms for interacting with and responding to classmates' comments.

A second challenge teachers face when orchestrating classroom discussions identified by Silver and Smith (1996) is the balance between getting students to engage in the conversation and meaningfully steering the conversation toward learning goals based on specific mathematical concepts. Students at first may require encouragement to share ideas. However, as they participate in whole-class discussions and they become more willing and comfortable in doing so, teachers must ensure that they are not just sharing ideas solely for participation's sake. Getting students to share may be important, but unless it is done in a purposeful manner opportunities for students to build fundamental mathematical understanding may be lost in the sharing and the discussion itself. Silver and Smith (1996) explained that these challenges are, in part, a result of teachers' lack of experience participating in the type of discourse communities they are encouraged to build around high-level tasks as most pre-service and in-service teacher learning opportunities do not provide such experiences.

In other work describing some of the difficulties and successes of the QUASAR project, Silver (1996) noted that one challenge teachers in the project had to overcome while attempting to establish discourse communities was their students' lack of experience communicating about high-level mathematical tasks in the manner called for in these type of communities. This was aggravated by the fact that many students have had a traditional view of learning mathematics ingrained in them throughout their educational lives. These learning experiences often include working in isolation with little or no opportunities to work collaboratively or even to discuss their ideas and solutions.

Stein, Engle, Smith, and Hughes (2008) noted that perhaps the most difficult of challenges teachers face when conducting classroom discussion around high-level tasks is knowing how to effectively deal with the many differing solutions students may develop when working on such tasks. Teachers not only need to be able to determine the correctness of the different solutions, but they must also be able to discern student understanding and possible misconceptions from the various solutions. Perhaps most importantly, teachers must be able to guide the classroom discussion using students' ideas and solutions in a way that enhances students' understanding of the key mathematical concepts while moving toward accomplishing the learning goals they have set for the lesson. Achieving these outcomes while facilitating a classroom discussion is rife with challenges.

Stein and her colleagues (2008) explained that most of the research and practitioner literature regarding conducting classroom discourse has focused on the use of high-level tasks, establishing norms for discussions, and encouraging student interactions while exploring the task. They also pointed out that little attention has been given to active instructional moves teachers could employ to productively facilitate discussions toward their desired learning goals.

When teachers use open-ended high-level tasks with multiple solution paths, their discussions can easily fall into the pattern of what Ball (2001) referred to as “show and tell” discussions during which students take turns presenting their correct solutions strategies. Stein et al. (2008) suggested that these conversations are not productive in moving students’ understanding toward the desired learning goals as teachers using this method do not typically highlight important concepts or connect differing representations or solution methods to important ideas. In these cases, all solutions are viewed as equally good and there is no discussion of when one solution may be more useful than another solution. Thus, conducting “show and tell” discussions around high-level tasks does not ensure students’ mathematically understanding will increase.

A second challenge in conducting “show and tell” classroom discussions is the lack of student engagement in the discussions. These discussions turn into a cycle of one student presenting his or her solutions while the rest of the class listens. There is little, if any, opportunity for the other students in the class to question the presenting student’s ideas or press for further clarification. There is also no connection between the presenting student’s ideas and previously explained solutions. This lack of back-and-forth conversation can lead to students’ disengagement from the discussion. Stein et al. (2008) explained that this type of discussion provides motivation, mathematical or otherwise, for students to pay attention to and attempt to understand the methods shows by their classmates.

Conducting effective classroom discussion requires significant pedagogical knowledge and skill (e.g., Ball, 2001; Lampert, 1990; Leinhardt & Steele, 2005). These demands are greatly intensified by the ad hoc-ism of conducting discussions centered on students’ solutions, as teachers do not know beforehand how their students will attempt to solve the task or what questions will emerge during the discussion. Stein et al. (2008) acknowledged the issues this uncertainty presents and explained that expert teachers are able to cope with these issues by quickly assessing students' level of understanding of the mathematical concepts that are the focus of the discussion and then making decisions, on the fly, of how to guide the classroom discussion based on this information. However, they suggested that the ability to correctly and effectively make such rapid improvisational decisions requires content knowledge, pedagogical knowledge, and knowledge of how students learn mathematics that most teacher do not have. In particular, inexperienced teachers who are typically unable to anticipate the nuances and complexities that may occur during discussions around high-level tasks, have great difficulty in successful navigating these discussions.

2.2.4 Five Practices for Successfully Conducting Classroom Discourse Centered on High-Level Tasks

In an attempt to aid teachers in dealing with the complex nature of conducting classroom discussions centered on high-level tasks, Stein and her colleagues (2008) proposed a set of five instructional practices. These practices are specifically designed to eliminate as much of the improvisation of conducting classroom discussions as possible. They noted that the individual practices are not their novel ideas and that the practices have been suggested separately in previous literature. However, they suggested that by using these practices together as a set, teachers will have much more time to consider the instructional decisions they will face when facilitating class discussions. In this way, the need to make important choices on the fly will be greatly reduced and teachers will feel more confident in using discussions in their classrooms. These practices are:

1) Anticipating likely student responses to cognitively demanding mathematical tasks (*anticipating*);
2) monitoring students' responses to the tasks during the explore phase (*monitoring*);
3) selecting particular students to present their mathematical responses during the discuss-and-summarize phase (*selecting*);
4) purposefully sequencing the student responses that will be displayed (*sequencing*); and
5) helping the class make mathematical connections between different students' responses and between students' responses and the key ideas (*connecting*). (Stein, Engle, et al., 2008, p. 321)

Each of these practices will be discussed in depth in the following sections. One or more examples of each practice being used in instruction will also be provided.

2.2.4.1 Anticipating

While anticipating is a practice aimed at aiding teachers to conduct whole-class discussions, it does not occur during these discussions. In fact, teachers anticipate well in advance of engaging the students in the task around which the discussion will occur. Anticipating requires teachers to consider the multiple methods and solution strategies students might use when solving a task. This requires much more from teachers than solely begin familiar with the task; it requires teachers to engage in the task as students would, putting themselves in the students' position and using the type of reasoning and strategies their students would use. Teachers should anticipate both correct and incorrect solutions methods as well as the tools (e.g., representations, manipulatives) that student might use while working on the problem (M. S. Smith & Stein, 2011; Stein, Engle, et al., 2008). Stein et al. (2008) also suggested that in addition to using their own knowledge of students' mathematical understanding and skills, teachers can look to educational research on methods students use to work on and solve mathematical tasks (e.g., Fennema et al., 1996). Teachers can also consult curricula materials as some provide teacher resources that include possible methods students may use to work on tasks as well as misconceptions that may surface during classroom discussions.

Schoenfeld (1998) presented a comparison of a novice and an expert teacher that brings to light the importance of being able to anticipate how students will respond to a task. Mr. Nelson, a student teacher, was teaching a lesson on exponents with the goal that students would be able to reduce fractions of the form $\frac{x^a y^b}{x^c y^d}$ where $a \geq c$ and $b \geq d$. He had designed the task so that students would work on three problems (a) $\frac{m^6}{m^2}$, (b) $\frac{x^3 y^7}{x^2 y^6}$, and (c) $\frac{x^5}{x^5}$. He expected the students to be able solve problems (a) and (b) with relative ease and expected some confusion with (c). Mr. Nelson then planned to begin a classroom discussion by asking students for their response on part (a) and (b). He expected some variety in their answers and a few students who would not understand how to do them. He planned to use $\frac{x^5}{x^5}$ as an example to aid the students in working through their confusion. Mr. Nelson intended to rewrite $\frac{x^5}{x^5}$ as $\frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x \cdot x \cdot x}$, show how the each pair of x's (one in the numerator and on in the denominator) could be reduced to 1, and show that the fraction reduces to $x^0 = 1$. Once he had done this, he would note the case of $x = 0$ and which would result in the fraction being undefined.

However, upon engaging students in the task and following his plan with regard to problem (c), Mr. Nelson's students gave many unexpected answers to such as "x", "zero", and "zero over zero". Schoenfeld explains Mr. Nelson had not prepared for this confusion and thus had to improvise trying to explain to the students how problem (c) would reduce to 1 using an example involving numbers instead of variables. For the next several minutes he continued to try to help the students understand that $\frac{x^5}{x^5}$ would reduce to 1, however he was unsuccessful.

Contrast Mr. Nelson's experience with that of Mr. Minstrell also depicted by Schoenfeld (1998). Mr. Minstrell was an experienced physics teacher who had received the Presidential Award for Excellence in Science Teaching. He had designed the lesson that was the focus of the observation. The lesson focused on blood alcohol content and the lesson goal was to help the students to explore the advantages and disadvantages of using mean, median, and mode as measures of central tendency. Schoenfeld explained that while much of Mr. Minstrell's lesson went according to plan, he too encountered an unexpected student method for computing the average value. Schoenfeld pointed out that Mr. Minstrell was able to deal with this unexpected response in part based on his intimate knowledge of the task and of how students would engage in it. Mr. Minstrell used this unexpected response to further the class discussion by exploring it with the class and examining how it related to the standard definition of the mean. While Mr. Minstrell's content knowledge, extensive teaching experience, and pedagogical knowledge played a part in his ability to effectively use his students responses, particularly the unexpected method, his thorough knowledge of the task and his knowledge of how students would work on it were also vital to the success of his classroom discussion.

2.2.4.2 Monitoring

As with anticipating, monitoring is a task that occurs before the whole-class discussion begins; it takes place while students work on the task. Monitoring is the practice of observing students as they work on the task, examining the strategies they use to solve the task, and gaining insight into how they are thinking about and conceptualizing the fundamental concepts of the task (M. S. Smith & Stein, 2011; Stein, Engle, et al., 2008). The purpose of monitoring is to determine which solutions would be the most useful to discuss as a class with the intent of pushing students toward accomplish the learning goals of the lesson. Stein and her colleagues (2008) noted that monitoring typically occurs as the teacher circulates through the room while students work. This allows them to see how each student is working on the task, provides them with a sense of how the students (in general) are progressing on the task, and allows teachers to determine which of their anticipated solutions have been used by their students. It also provides teachers with opportunities to engage students in individual or small group conversations about their strategies to gain insight into their thinking and reasoning or to help steer them back on track if they are engaging in the task in a way that is completely off the mark and unfruitful. This does not suggest that teachers should correct all students that have an incorrect strategy or answer as incorrect methods can lead to fruitful discussions (Kazemi & Stipek, 2001). However, they may choose to do so if the student is working on the task in such a way that no productive learning can occur from it. Smith and Stein (2011) suggested that teachers use a monitoring tool as they monitor student's engagement in the task. This tool would be created before the instruction begins, most likely as the teacher anticipates possible student solutions. It lists the type of strategies the teacher expects to see the students use and includes a section in which the teacher can record which students actually use the each anticipated solution and a section to indicate a possible order in which the student solutions could be shared. The tool helps the teacher in that it provides a space for them to record how students are engaging in the task allowing them to focus on assessing and advancing student thinking while talking with them individually or in small groups instead of trying to remember which student used which method. It also allows the teacher, at a glance, to recognize patterns in how students are working on the task (e.g., the teacher can quickly see that over half of the class used an additive strategy while trying to solve a proportional reasoning problem). Figure 2.1 is an example of a possible monitoring tool for the task *"A fourth-grade class needs 5 leaves each day to feed its 2 caterpillars. How many leaves would the students need each day for 12 caterpillars?"* (M. S. Smith & Stein, 2011, p. 3).

Strategy	Who and What	Order
Unit rate Find the number of leaves eaten by one caterpillar (2.5) and multiply by 12 or add the amount for one 12 times		
Scale Factor Find that the number of caterpillars (12) is 6 times the original amount (2), so the number of leaves (30) must be 6 times the original amount (5)		
Scaling Up Increasing the number of leaves and caterpillars by continuing to add 5 to the leaves and 2 to the caterpillars, until you reach the desired number of caterpillars		
Additive Find that the number of caterpillars has increased by 10 (2 + 10 = 12), so the number of leaves must also increase by 10 (5 + 10 = 15)		
Other Scaling up by collecting sets of 2 leaves and 5 caterpillars		

Figure 2.1: *Example of a Monitoring Tool* (M. S. Smith & Stein, 2011, p. 9)

There are very few examples of monitoring provided in existing literature. However, one example of partial monitoring comes from the case of David Young who is teaching a patterns task (see Figure 2.2) with this sixth-grade class (M. S. Smith et al., 2005). He indicates the purpose of this task, as well as the unit the task is a part of, is to visualize and describe patterns, make conjectures about the patterns, and develop generalizations to fit the patterns. After setting up the task and asking the students to begin working on it, David describes how he monitors the students' work:

> I walked around visiting the pairs as they worked on the triangle train (Pattern 1). Again, students seemed to quickly see the pattern - add one more triangle - and count the sides to find the perimeter. I observed several pairs starting to build the tenth train and asked them to try to find another way. I suggested that they look at the four trains they had built and see if they could find any patterns that would help them predict the tenth train. In a few cases where the students were really stuck I suggested that they try to see if they could find a connection between the train number and the perimeter as a few students had done in the last pattern. (pp. 24-25)

In this example, David moved throughout the room monitoring and observing students' engagement in the task. He used the time to determine how individual students and small groups were thinking about the mathematics in the task. David also used strategic questions about the pattern to help advance students' thinking, and he was able to intervene in cases where students were struggling. However, there is no evidence that David used a tool to record how students were working on the task or that he used what he learned while observing student engage in the task to structure the discussion.

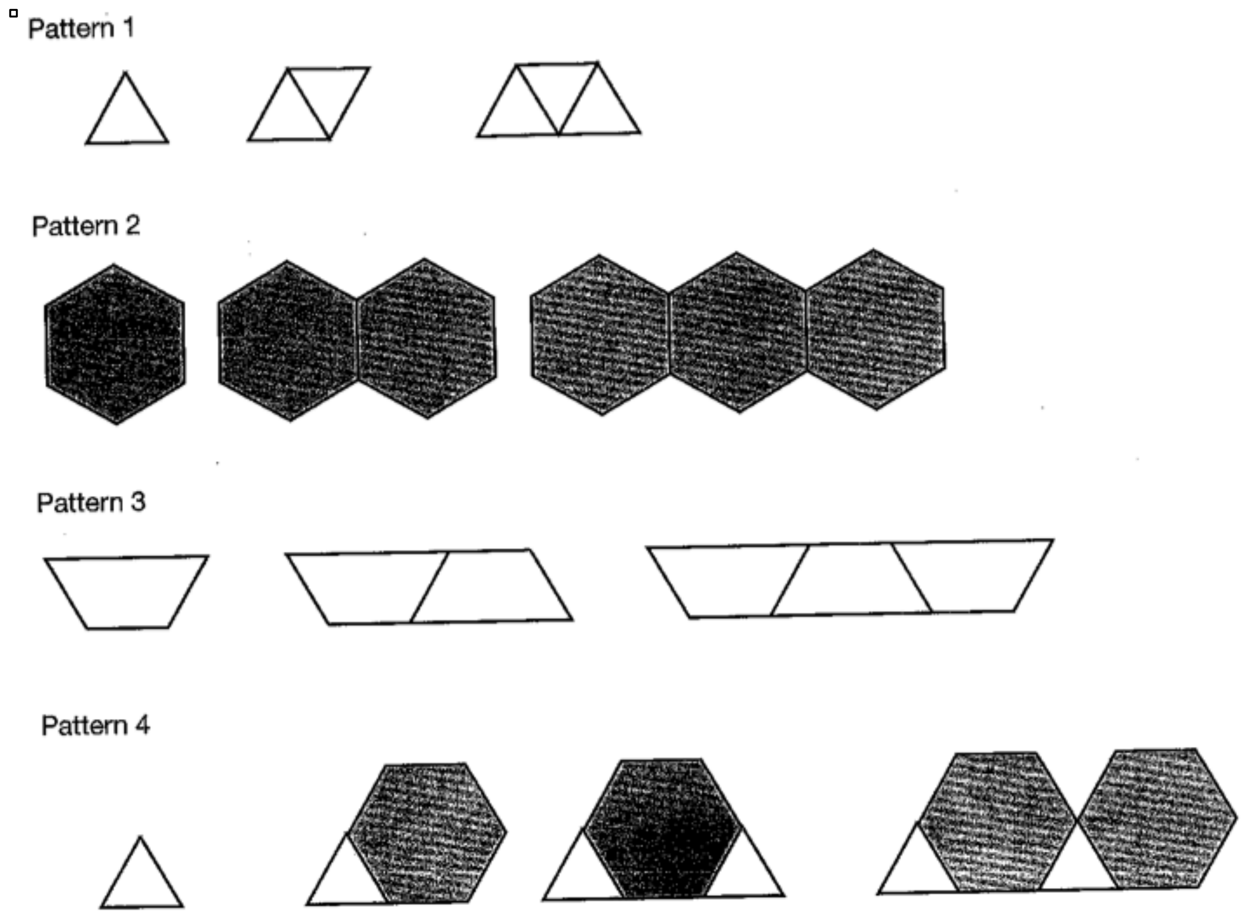


Figure 2.2: *The Pattern task used by David Young* (M. S. Smith & Stein, 2011, p. 9)

2.2.4.3 Selecting

Once a teacher has anticipated possible student responses and then monitored how her students engage in and attempt to solve the task, she will need to select which solutions to make public to the whole class. Stein et al. (2008) explained that the manner in which the teacher determines which solutions and ideas are highlighted and which are not plays a critical role in determining the effectiveness and success of the classroom discussion. One manner is to ask students to volunteer to share their solutions. This method of selecting students to share their ideas removes the control from the teacher and places it in the hands of the students, and typically results in the type of "show and tell" discussions described earlier. For example, while monitoring a teacher may observe that a few students have used a particular method she wants to highlight during the whole-class discussion. However, if she asks for volunteers, there is no guarantee that any of the students who developed the method she hopes to share will volunteer to share it.

If teachers are careful to select specific solutions with the intent of highlighting important mathematical concepts, they will greatly enhance the quality of the discussion. Stein and colleagues (M. S. Smith & Stein, 2011; Stein, Engle, et al., 2008) explained that there are multiple ways in which to do this. Typically, this is done as the teacher calls on specific individuals or groups to share their responses. They suggest that teachers can also ask for volunteers but then be particular about which "volunteers" are selected by purposefully choosing those students with the solutions she wishes to highlight. However, if a teacher decides to use this method, she must be careful to ensure that the risks of calling for volunteers describe above are mitigated. Smith and Stein (2011) suggest one way for the teacher to ensure students share the solutions she desires is to individually inform the student before a discussion begins that she would like him or her to share the solution during the whole-class discussion. In some cases, the teacher may have to introduce solutions to the class. Perhaps in her anticipation of student strategies, the teacher thought of a solution she felt was important for students to consider to help them in progressing toward accomplishing the learning goals of the lesson. Yet, this solution may not be used by any students in her class. In this case, the teacher can introduce student work with this solution from a different class or use a solution that she created herself prior to the lesson (Stein, Engle, et al., 2008).

Schoenfeld (1998) shared a depiction of his own teaching of an undergraduate course on mathematical problem solving during which he used the practice of selecting specific solutions to feature during whole-class discussions. For this course, Schoenfeld asked students to work on a task that asks if it is possible to inscribe a square within a given triangle using a compass and a straight edge. After the students had worked on the task for some time, Schoenfeld elicited multiple student solutions and recorded them on the board, but he did not discuss any of them in depth at this time. He then chose which of these solutions to take up and which to overlook. He also decided to introduce a solution that had not been produced by the students.

2.2.4.4 Sequencing

Class discussions can be further augmented as teachers purposefully determine the order in which the solutions are shared. Stein et al. (2008) posited that "by making purposeful choices about the order in which students' work is shared, teachers can maximize the chances that their mathematical goals for the discussion will be achieved" (p. 329). Stein et al. (2008) and Smith and Stein (2011) suggested multiple sequences of student solutions teachers may use during whole-class discussions. One possible sequence would be to ask a student to share the most common strategy first and then move to solutions that are more novel. This would "help validate the work that students did and make the beginning of the discussion accessible to as many students as possible" (Stein, Engle, et al., 2008, p. 329). Another option would be to have a common, but incorrect, strategy based on a misconception shared first. This would provide an opportunity for students to grapple with the mathematics of the task in a way to would aid them in developing a stronger conceptual understanding of the mathematics being discussed (National Council of Teachers of Mathematics, 2000, pp. 20–21). A third possible sequence would be to present more concrete strategies first and then move to more abstract strategies in an effort to make it easier for students to better grasp the abstract strategies (M. S. Smith & Stein, 2011). Teachers may also want to have students share related or contrasting strategies consecutively in an effort to aid the students in comparing them (Stein, Engle, et al., 2008). Stein et al. (2008) explained that by strategically sequencing the order in which solutions are shared, teachers maintain control of the classroom discussion and are able to build the discussion in a mathematically coherent manner.

An example of a teacher's use of sequencing can be seen in the continuation of the description of Schoenfeld's purposeful selecting of which solutions would be shared in his problem-solving class described in the *selecting* section above. Schoenfeld noted that after he gathered students' strategies and introduced one that they had not thought of, he was careful to discuss the strategies in a specific order. He said, "I did not take them simply in the order that they had been generated, but in an order that allowed various mathematical 'lessons' to emerge more naturally from the discussions" (1998, p. 68).

2.2.4.5 Connecting

Teachers can further assist students in developing conceptual understanding by providing opportunities during classroom discussions to form connections between mathematical ideas, representations, and strategies. Stein et al. (2008) explained, "rather than having mathematical discussions consist of separate presentations of different ways to solve a particular problem, the goal is to have student presentations build on each other to develop powerful mathematical ideas" (p. 330). Connecting students' strategies to one another and to the fundamental mathematical concepts being discussed helps students better understand the pros and cons of the various solutions. It also helps them see the same mathematical concept in different representations, which on the surface appear quite different, yet upon further investigation convey similar mathematical ideas.

Stein et al. (2008) suggested several methods teachers can use to form these connections. Teachers can hint at connections between solutions or representations as they direct the discussion from one to another. They can also ask students to compare and contrast two or more strategies or representations. By planning subsequent lessons based on students' solutions to tasks and the learning goals associated with the tasks, teachers can also create connections between lessons and the concepts discussed in each.

Hill and Charalambous (2012) compared two seventh-grade teachers' enactment of the same problem from *Connected Mathematics 2*. Their intention was to demonstrate the impact of teachers' *mathematical knowledge for teaching* (MKT) (cf. Ball, Thames, & Phelps, 2008) on the *mathematical quality of instruction* (MQI) (cf. Hill et al., 2008) of their lessons. One of the teachers, Mauricio, scored in the 93rd percentile of a representative sample of U.S. teachers on the MQI measurement while the other teacher, Wanda, scored in the 47th percentile. As Hill and Charalambous described Mauricio's lesson, they comment that he expertly connects various students' solutions methods while moving all students' understanding forward. One particular move Mauricio routinely uses is to ask students to draw connections between various shared solutions and then to revoice (O'Connor & Michaels, 1993) what these students say in a manner that emphasizes the important connections he wants the students to make. Wanda on the other hand, does not use the task as an opportunity to make connections between multiple solution strategies. She asks students to work on the task for approximately 15 minutes and then calls the class together for a whole-class discussion. However, this discussion is centered on one specific solution method, not elicited from students, but generated by the teacher. Wanda does not ask students to draw any connections between this method and what they have attempted to do nor does she make an effort to form any connections between this method and another strategy.

Because she did not make more that one solution publicly available, Wanda's students did not have an opportunity to discuss and compare various strategies. Thus, while there was some discussion about the task, there was little or no opportunity to engage in the type of dialogue suggested by NCTM around fundamental mathematical content. Rather, it is a conversation around one procedure for solving the task.

The set of five instructional practices suggested by Stein and her colleagues (M. S. Smith & Stein, 2011; Stein, Engle, et al., 2008) are intended to assist teachers as they conduct classroom discussions around high-level, cognitively demanding tasks. They do this by allowing teachers to consider and make instructional decisions (prior to discussions) about which solutions are vital to highlight in order to help progress student learning and how to effectively deal with the various strategies, both correct and incorrect, students may use while working on the task. They also provide teachers with tools and strategies for determining which solutions would be most beneficial to bring to the fore during classroom discussions and in what order they should be presented. Finally, the practices aid teachers in assisting students to form connections between the various solutions and representations that are presented during classroom discussions as well as connections with the underlying mathematical concepts that are at the root of the learning goals of the lesson. While it would be possible for a teacher to use one or more of the five practices in isolation, they are intended to be used as a set as they build on one another. For example, a teacher can observe how students engage in a task without first anticipating possible student solutions. However, it would be difficult to use a monitoring tool to record which students are using which strategies, as the teacher would have to make this tool while monitoring. Additionally, without anticipating or monitoring it is impossible for a teacher to effectively select or sequence the strategies to be shared in a whole-class discussion.

Classroom discourse centered on high-level mathematical tasks during which students are able to pose and critique ideas, question conjectures, and reason about fundamental mathematical concepts is a critical aspect of mathematics education reform. Teachers play a crucial role in conducting classroom discussion as they select the tasks that form the basis of the discussions, establish norms for participation, determine what will be discussed and by whom, supervise what is being discussed, and make course corrections in the discussion in order to steer students toward the intended learning goals. However, research has shown that many teachers struggle to facilitate discussions effectively. The five practices (anticipating, monitoring, selecting, sequencing, and connecting) proposed by Stein and her colleagues (M. S. Smith & Stein, 2011; Stein, Engle, et al., 2008) are active instructional moves teachers can use to enhance their ability to successfully orchestrate whole-class discussions.

2.3 PROFESSIONAL DEVELOPMENT

The vast majority of teachers in the United States engage in professional development each year (U.S. Department of Education, Office of Planning, Evaluation and Policy Development, Policy and Program Studies Service, 2007) with millions of dollars spent annually (Cohen & Ball, 1999) to provide it. The purpose of teacher professional development is to enhance teachers' knowledge and improve their instructional skills, with the end goal of increasing student learning. However, the types and quality of the professional development vary drastically, and differing professional development programs are often inconsistent and incoherent with one another (Ball et al., 2001). As such, researchers in the fields of education and teacher education have begun to investigate which types of professional development are more effective at achieving these goals and why. This research will provide much of the foundation for the study proposed in this dissertation.

2.3.1 Essential Characteristics of Effective Professional Development

Existing theory and research of professional development for mathematics teachers will be reviewed in this section. This review will focus on six salient characteristics of effective professional development. While the generated list is not meant to be viewed as exhaustive, the characteristics explicated herein are viewed to be critical for professional development to successfully bring about changes in teachers' practice. This list of essential elements has been distilled from several research studies (e.g., Desimone et al., 2002; Garet et al., 2001), published education research literature reviews (e.g., Hawley & Valli, 1999; M. Kennedy, 1998; Wilson & Berne, 1999), book chapters and articles proposing theories of teacher learning (e.g., Ball & Cohen, 1999) or explicating challenges of conducting professional development (e.g., Stein, Smith, & Silver, 1999), as well as from books written by expert professional development facilitators (e.g., Loucks-Horsley, Love, Stiles, Mundry, & Hewson, 2003; M. S. Smith, 2001). Each element will be presented in this section along with the theory or studies supporting it.

Desimone et al. (2002; also see Garet et al., 2001) studied the impact of various features of professional development that have been identified as potentially, positively impacting teachers' practice. They identified two types of features of professional development—structural features and core features. S*tructural features* are defined as "characteristics of the structure of a professional development activity" while *core features* are described as "characteristics of the substance of the activity" (p. 83). These terms will be used in this section to categorize the six essential elements of effective professional development identified in the literature that will be reviewed in this section. There are two structural features: (a) *teacher collaboration* and (b) *duration*; and four core features: (a) *active learning*, (b) *practice-based*, (c) *coherence*, and (d) *focus on subject matter content and pedagogical practices*. Each of these will be defined using existing literature and examples from pervious research will be given.

It is important to note that much of the justification used to support the selection of these six elements is based on research conducted by Desimone and her colleagues (Desimone et al., 2002; Garet, Birman, Porter, Herman, & Yoon, 1999; Garet et al., 2001; Guskey & Yoon, 2009). They employed the use of teacher surveys to gather information regarding teachers' participation in professional development and instruction as part of their large scale, longitudinal studies. The use of surveys to collect data regarding teacher professional development and instruction appears to be at odds with the findings of Spillane and Zeulli (1999) presented earlier. Desimone (2009) addressed concerns with regard to the use of surveys for this purpose. She reviewed past literature and showed that in several studies, teachers' self-reported data (via surveys) is highly consistent with observational data. Her review of the literature shows that the consistency of teachers' self-reported data and observational data is high when teachers are reporting about specific aspects of professional development and specified teaching behaviors. She suggested that while survey data can be consistent with observational or interview data, it is a complex process that must take into account compounding factors such as teacher beliefs and knowledge. Several studies cited in this chapter are based on data obtained via teacher surveys. These will be used to justify the selection of the six essential features of professional development as they have been vetted in peer reviews journals and have been accepted by the field of educational research. However, when possible, research that employs alternative forms of data collection (e.g., teacher observations or interviews) will also be included to strengthen the inclusion of the six proposed elements of professional development.

2.3.1.1 Structural features

2.3.1.1.1 Teacher collaboration. Teacher isolation as been an issue of concern of the profession since the 1970s (Davis, 1986; Griffin, 1995; Lortie, 1975; Sindberg & Lipscomb, 2005; Zielinski & Hoy, 1983). One manner in which professional development has tried to address this issue is by providing opportunities for teachers to learn and work collaboratively. Stein, Smith, and Silver (1999) acknowledged the changing landscape of mathematics education by pointing to the reforms called for by teacher groups (e.g., National Council of Teachers of Mathematics, 1989, 1991) and policy makers (e.g., National Board of Professional Teaching Standards, 1989). They hypothesized that as teachers adapt and relearn elements of their practice, providers of training and professional development for these teachers will need to do the same. As such, they described a "new paradigm" for teacher professional development that must to be transformative in nature in that it produces changes in teachers' beliefs, knowledge, and practice, as opposed to professional development that only adds new skills to teachers' current sets of practices. They proposed that this type of professional development will include certain vital characteristics, one of which is to work with "organizationally intact groups of teachers" (p. 240) (e.g., teachers from the same school, content area, or grade-band). Stein and her colleagues also suggested that it is critical for teachers to collaborate with "outside experts" (e.g., university-based teacher educators) with whom they have built trusting relationships.

In describing the consensus of what professional development for teachers should entail and identifying key design principles of this professional development, Hawley and Valli (1999) posited that teachers need to engage in collaborative problem solving. They suggest that this collaboration can take many forms but that by working with their colleagues, teachers are able to identify problems they are facing as well as possible remedies to these issues.

In one of the seminal pieces of literature on professional development for mathematics and science teachers Loucks-Horsley and her colleagues (2003) recommended various strategies to consider when designing professional development. They outlined three forms of collaborative structures and pointed out that these collaborative structures differ from other collaborative strategies in that the collaborative structures are the only processes used to group professionals together for learning whereas other collaborative strategies may have predetermined goals and/or procedures. They identified three types of collaborative structures: (a) partnerships with scientists and mathematicians in business, industry, and universities; (b) professional networks; and (c) study groups. They suggested that the choice of these and other strategies for professional development depend on the context, goals and purposes, and circumstances of the professional development.

Garet et al. (2001) conducted a study of the efficacy of various characteristics of professional development. They noted that prior to their study there had been little research on the impact different features of professional development. However, they used what research had been done as well as knowledge of professional development from "expert practitioners" to develop a list of features of quality professional development. They categorized the features as either structural features or core features using the same definitions provided at the beginning of this section take from their later study (Desimone et al. 2002). Among the structural features they identified was collective participation of teachers from the same school, department, or grade-level. Garet his and colleagues' (2001) study consisted of a national probability sample of over 1,000 mathematics and science teachers. They used data collected via a survey, which was given to evaluate the Eisenhower Professional Development Program. Teachers completing this survey described, in detail, the specific professional development activities in which they participated. The researchers used teachers' self-reporting of increased skills and knowledge in various areas (curriculum, instructional methods, approaches to assessment, use of technology, strategies for teaching diverse student populations, and deepening knowledge of mathematics) as well as changes in their instructional practices as outcome measures. Their analysis of the data found that collective participation in groups of teachers from the same school, subject, or grade positively impacted multiple core features of professional development (e.g., coherence and active learning opportunities), which then positively influenced teachers' knowledge, skills, and instruction.

Thus, organizing professional development activities in a way that encourages teacher collaboration, as opposed to teachers working in isolation, is a crucial feature of effective teacher professional development. This collaboration appears to be particularly fruitful when structuring professional development so that teachers are working with colleagues from their same schools, grades, and or departments.

2.3.1.1.2 Duration. The vast majority of stakeholders in education, from researchers and policy makers to administrators and teachers, regard "one-shot" workshops as ineffective, and "a waste of both time and money" (Guskey & Yoon, 2009, p. 496; M. Kennedy, 1998). While Guskey and Yoon as well as Kennedy argued that the structure of the professional development in the form of workshops may not be the reason some professional development is ineffective—rather, they feel it is due to the lack of attention to other key elements of professional development (e.g., the content of the professional development)— workshops are commonly used as an example of the ineffectiveness of professional development that is not sustained over long periods. Due to this, several researchers have identified the duration of professional development as a critical feature.

Shields, Marsh, and Adelman (1998) reported that in the early 1990s the National Science Foundation (NSF) funded Statewide Systemic Initiatives (SSI) in 25 states and the commonwealth of Puerto Rico in an attempt to encourage the growth of programs in these areas to promote reform based on the ideas disseminated by NCTM (1989, 1991). The states were free to implement the SSIs in whatever manner they saw fit as long as these programs were in alignment with the NCTM principles. Shields et al. (1998) studied the impact of the SSIs on teachers' instructional practices. They gathered data over a five-year period in the form of teacher interviews, classroom observations, teacher survey data collected for the SSIs' internal evaluations, and state reports to NSF. Using these data, they determined that the SSI program had a varied effect on teacher practice depending on the individual SSIs and the strategies they employed. Shields and colleagues' (1998) data suggests that the duration of the SSIs is related to the level of impact these programs had on teachers' instruction practices. They explained that the SSIs that targeted teachers early in the duration of the programs and that included high-quality professional development and long-term support had a higher likelihood of positively influencing teachers' classroom practices. Shields et al. (1998) did not provide specific information about what length of time is necessary for duration to have a positive effect. However, they did provide an example of effective professional development, which consists of a six-week summer program followed up with 10 days of additional professional development during the school year.

NSF also funded a program, called Local Systemic Change through Teacher Enhancement (LSC), in the mid 1990s aimed at improving science, mathematics, and technology instruction via teacher professional development. This professional development was provided at the school or district level and was intended for all teachers within the focus content (i.e., science, mathematics, or technology) at that level (Weiss, Montgomery, Ridgway, & Bond, 1998). LSC professional development activities tended to focus on pedagogical issues (e.g., specific instructional materials or pedagogical/classroom management strategies) and increasing teachers' content knowledge. Weiss and colleagues (1998) examined data collected over the first three years of the LSC program. These data were gathered via observations of professional development activities, classroom observations, teacher questionnaires, principal questionnaires, and teacher interviews. Participating teachers, via surveys, also rated the meetings favorably, and as the number of hours teachers spent in the professional development increase so did their opinion of it. This was common for both elementary as well as secondary mathematics teachers. Weiss et al. (1998) also found that the teachers who participated in 40 or more hours of the professional development were more likely to use the instructional practices that were the foci of the professional development in their subsequent teaching. However, they did not specify the length of time over which these 40 or more hours occurred (e.g., one year or over the length of the LSC program) as each of the LSC programs they studied were conducted over multiple years.

Supovitz and Turner (2000) also investigated the impact of LSC on teachers' instructional practices. Specifically, they studied whether science teachers' participation in professional development that utilizes curriculum based on science education (National Committee on Science Education Standards and Assessment, National Research Council, 1996) leads to an increased use of inquiry-based teaching practices. They used the data collected via teacher and principal surveys as part of program evaluations for LSC also used by Weiss et al. (1998). These data were collected from over 3,400 teachers and more than 650 principles from schools nationwide. Using hierarchical linear modeling, they determined that teachers who participated in 80 or more hours of the professional development associated with LSC used inquiry-based teaching practices significantly more often than did average science teachers. However, they also found that the teachers who participated in less professional development in LSC did not use these practices more frequently than average science teachers. Thus, in the case of LSC, the duration of the initiatives in the program (specifically, 80 or more hours of professional development) positively impacted teachers' use of instructional practiced discussed in those initiatives.

Garet, Porter, Desimone, Birman, and Yoon (2001) noted a dearth of research investigating the effects of various features of professional development for mathematics and science teachers on changes in teachers' knowledge and instructional practices. They sought to address this lack of information by analyzing survey data collected as part of the evaluation of the Eisenhower Professional Development Program. From prior research on professional development, they identified three structural features (*form* (e.g., type of activity), *duration*, and *collective participation*) and three core features (*content focus*, *active learning*, and *coherence*), which they used as the focus of their investigation. The researchers surveyed a nationally representative sample of teachers, obtaining responses from over 1,000 teachers. Teachers' responses to the survey were self-reports of their professional development experiences and their teaching practices. Garet and colleagues (2001) found that duration, measured as both the span of time over which activities occur and the number of contact hours spent, of professional development was positively correlated with both active learning and coherence of professional development. They concluded that professional development that is both sustained over a long period of time and that consists of a substantial number of hours is likely to be of higher quality.

However, they did not provide recommendations of the length of time over which a program should be conducted or the amount of contact hours teachers should receive as part of the program.

However, some educational researchers have argued that duration in and of itself is not a critical feature of professional development. For example, in Kennedy's (1998) review of literature on professional development for mathematics and science teachers, it is suggested that although most researchers focus on the form and length of professional development there are other features (e.g., the content of the professional development) that are of more importance.

Desimone and her colleagues (2002), conducted a follow up study to their prior work (Garet et al., 2001). They conducted a longitudinal study of professional development and teacher change focusing on the six features of professional development they had identified previously (i.e., *form*, *duration* (they used both time span and number contact hours to measure duration), *collective participation*, *content focus*, *active learning*, and *coherence*). The researchers selected one elementary school, one middle school, and one high school in 10 districts. These districts were chosen because they employed various types of professional development in addition to traditional workshops. This selection was purposeful, as Desimone et al. (2002) reasoned that if the professional development provided in these districts was found to be better than traditional workshops, then perhaps the teachers' instruction in these schools would be of better quality than that of the average teacher. They surveyed all the mathematics and science teachers in each of the 30 schools three times over the course of three years (for the elementary schools half of the teachers were given the mathematics survey while the other half was given the science survey). They used the teachers' instructional practices from the first year of the study as a baseline measure of instruction. They then used the professional development from year 2 of the study as a treatment, and measured its impact on teachers' practices in year 3 of the study. Desimone et al. (2002) used hierarchical linear modeling to analyze the data. Their analysis indicated that for this longitudinal data, measure of duration (time span and number of contact hours) did not significantly affect teachers' instructional practices either positively or negatively. This surprised the researchers as it seemed to contradict the findings of their previous study (Garet et al., 2001), in which duration positively impacted some aspects of teachers' instructional practices.

Thus, while there does not seem to be a consensus of the extent to which the duration of professional develop impacts teachers' practices, or of the number of hours that is considered to be sufficient for duration make an impact, duration of professional development does seem to have some positive influences on changes in teachers' instruction. It may not be a critical feature in isolation, but the findings from Garet et al. (2001) suggest that it does positively impact other features of professional development that are critical in changing teachers' practices. Intuitively, it seems that duration has the potential to positively impact teachers' practices. While duration is not the only feature that is needed for effective professional development, it likely is a necessary feature. As Smith (2001) stated, "although merely increasing the amount of time available for professional development is unlikely to make a difference, it is equally unlikely that, without being allotted more time, even the best professional development will be effective in accomplishing the ambitious reform agenda" (p. 48).

2.3.1.2 Core features

2.3.1.2.1 Active learning. Many experts of teacher professional development, including both facilitators as well as researchers, have pointed to active learning as a critical element of effective professional development. In their study designed to determine the essential features of professional development, Garet et al. (2001) identified active learning as one of the core features of professional development. They described active learning as activities during which teachers actively plan; discuss content, pedagogy, or other items germane to the work of teaching; engage in the work of teaching (e.g., collaboratively evaluate student work samples); observe other teachers; or develop new curriculum or teaching methods. Loucks-Horsley and her colleagues (2003) suggested 18 strategies for professional learning. Many of these strategies (e.g., action research, examining students work and thinking and scoring assessments, lesson study, immersion into problem solving in mathematics, demonstration lessons) fit the description of active learning given by Garet et al. (2001). In addressing the question of what matters in teacher learning, Darling-Hammond and Richardson (2009) suggested that designing professional development to include active learning activities is necessary for it to be effective. They suggested that these activities should include modeling instructional strategies as well as provide teachers with opportunities to practice and then reflect on these opportunities. Desimone (2009) also argued that active learning is one a critical feature of professional development. She suggested that a possible effective type of active learning is for teachers to observe other teachers and then provide constructive feedback, then be observed and receive reciprocal feedback.

Several studies give credence to the claim that active learning is a critical feature that determines the feature of professional development. Cohen and Hill (2001) examined the California teachers' learning opportunities aimed at assisting teachers in implementing reform-based policies and instruction in their classrooms. Part of their study specifically examined the professional development activities that were afforded to teachers. They noted that teachers' professional development experiences differed greatly and that a small portion of teachers experienced "unusual learning opportunities." During these unusual learning opportunities teachers became familiar with the content of specific units in new curriculum, implemented the lessons in this curriculum in their classrooms, and then shared their experiences using the curriculum with other teachers. While discussing their instruction, teachers would collaboratively work to address problematic issues they had faced, or would face, while teaching as well as plan for subsequent lessons. These teachers were also given opportunities to examine student work on assessments tied to the new curriculum they were using. As teachers discussed the student work, they would discuss possible mathematical misconceptions the students had as well as how to best address these misconceptions. However, the majority of the teachers in the study participated in what Cohen and Hill described as more conventional professional development. This professional development consisted of workshops lasting from a few hours to two days, many of which were provided by one of the teachers or administrators in attendance acting as a volunteer. Using survey data of teachers' self-reported instructional practices, Cohen and Hill found that the teachers who were provided with the unusual learning opportunities during professional development reported using the reform-based instructional practices that were the aim of the new California educational policies more than the teachers who participated in the more conventional professional development.

In their study described previously, Desimone et al. (2002) examined the impact professional development that included active learning activities had on teachers' classroom activities. The researchers used survey items related to each of the different aspects of active learning identified in their previous study (Garet et al., 2001) (i.e., observing teaching and being observed while teaching; planning classroom implementation; reviewing student work; and presenting, leading, and writing). Using the data, analysis, and outcome variables described earlier, the researchers found that the inclusion of active learning strategies in professional development positively impacts teachers' use of the instructional practices focused on during professional development.

Ingvarson, Meiers, and Beavis (2005) investigated the impact of various features of professional development programs for Australian teachers had on their knowledge, instruction and efficacy. They used data gathered for four individual studies over a two-year period of the Australian Government Quality Teacher Programme (AGQTP). The AGQTP was designed to help teacher improve their instructional skills and teaching knowledge. They used existing educational research to determine critical features of professional development on which to focus their investigation. One of these features was active learning. Survey data from over 3,200 teachers were used to evaluate the impact of these critical features on teachers' knowledge and instructional practices. Teachers were asked to describe the extent to which the AGQTP required them to engage in reflection on their teaching, specifically with regard to areas of their practice they needed to enhance. They were also asked the extent to which the AGQTP provided them with opportunities to try new teaching techniques. The survey questions also asked teachers to indicate the level of impact professional development activities had on their content knowledge and changes in their instructional practices. The researchers used a blockwise regression analysis to determine the impact of each feature on the outcome variables. Ingvarson et al. (2005) found that active learning in teacher professional development had a significant, positive impact on reported teacher knowledge in three of the four individual studies. They also found that active learning had a significant, positive impact on changes in teachers' instructional practices in all of the studies.

2.3.1.2.2 Practice-based. Another core feature of professional development that is related to active learning is the necessity for professional development to be practice-based. The notion of practice-based professional development has received great attention over the past decade. Ball and Cohen (1999) claimed that conventional teacher professional development consisting of "one-shot workshops" during which teachers are provided with overviews of the latest "best practices" or given tips of how to use the newest resources is inadequate at providing teachers with experience needed to successfully and positively change and improve their teaching practices. They proposed a dramatic new vision of teacher learning that aims to address the following questions:

a) What would teachers have to know, and know how to do, in order to offer instruction that would support much deeper and more complex learning for their students?
b) What sort of professional education would be most likely to help teachers to learn those things?
c) What do these ideas imply for the content, method, and structure of professional development.

Ball and Cohen (1999) addressed these questions and suggested that teachers need to learn their subject matter in very nuanced ways so that they understand not only how to perform computations using the proper algorithm (in the case of mathematics), but also how and why various representations of the same mathematical concept are connected, as well as possible misconceptions students may have and how these would be manifest in their work. This type of subject matter knowledge is very different than that used by experts in related, but different, fields (e.g., mathematicians, accountants, economists, computer programmers, engineers) or even of that of the general public (Ball & Bass, 2003; Ball, Hill, & Bass, 2005). They also proposed that teachers need to know about the students they are teaching, cultural differences between them and their students (as well as between students), how students learn, and pedagogical skills to teach students. Further, they claimed that it is not only sufficient for teachers to learn all of these things, but that they must do so within the context of their own teaching. Speaking of the complex work of learning to teach, they stated, “Teachers could not do such work unless they knew how to learn in the contexts of their work” (p. 10) and that in order to do this teachers must be provided with opportunities to analyze and reflect on their own work. Ball and Cohen claimed that teachers must be provided with these opportunities because one cannot learn to do the work of teaching outside of practice.

Two critical elements of the type of professional development advocated by Ball and Cohen (1999) are that it is focused on the essential practices in which teachers must engage while teaching, and that it includes opportunities for teachers to investigate their own practice. This does not, however, indicate that professional development must be done in teachers' classrooms during the school day. Rather, what Ball and Cohen referred to as “artifacts of practice” can be used to strategically capture what occurs during teachers' instruction for later analysis. Examples of artifacts of practice would be samples of student work, video recordings of teachers' lessons, curriculum materials, and teachers' lesson plans. The use of such artifacts is viewed as crucial to the success of the professional development. Ball and Cohen (1999) suggested that situating professional learning within actual artifacts of practice is essential as it is a means of grounding the discussions about teachers' practice in concrete items. They claimed that without this grounding, the conversations “become an exchange of buzzwords and slogans more than specific descriptions and analyses with concrete referents” (Ball & Cohen, 1999, p. 17).

The view that teacher professional development should be practice-based is shared by other educational researchers. In describing what they referred to as "a new paradigm for professional development," Stein, Smith, and Silver (1999) posited that professional development should be "embedded in or directly related to the work of teaching" (p. 239). They suggested that this is most effective when teachers' own practice is the focus of the intervention. They recommended that this be done by using various strategies such as: co-teaching, coaching, co-planning, joint reflection on instruction, and discussion around artifacts of practice.

Smith (2001) suggested that one possible design of practice-based professional development is the use of a three-stage cycle of investigation of teachers' practice. In the first stage, teachers, working either individually or in collaborative groups, select a learning goal and then determine an appropriate instructional tasks for students to engage in to achieve the learning goal. They then plan their instruction of the task. During the second stage of the cycle, they use the task while teaching their students. The final stage of the cycle consists of teachers' reflection on their instruction and their students' engagement in the task. This cycle is repeated with adjustments to future instruction made based on teachers' evaluation of their instruction. Smith suggested this design could be further enhanced by the inclusion of artifacts of practice. For instance, teachers could video record themselves teaching or use student work during the analysis stage of the cycle. Smith explained the purpose of practice-based instruction is to aid teachers in learning from their practice by becoming aware of particular elements of their instruction and apply lessons learned from their analysis of specific instructional experiences to their teaching in general.

Huffman, Thomas, and Lawrenz (2003) conducted a study that provides empirical evidence of the impact of practice-based professional development on teachers' instructional practices. They examined various types of professional development strategies identified by experts of professional development (Loucks-Horsley, Hewson, Love, & Stiles, 1998). These strategies were: (a) immersion strategies in which teachers engage in science or mathematical activities with scientists or mathematicians, (b) curriculum implementation that consisted of teachers using and then refining instructional materials, (c) curriculum development during which teachers help to create curriculum materials based on student need, (d) examining practice in which teachers discussed case studies or looked at "real classroom instruction", and (e) collaborative work that included study groups, coaching, mentoring, classroom observations, and feedback. These researchers collected data from over 90 science teachers and over 100 mathematics teachers. These teachers had voluntarily participated in the at least one of the types of professional development described above; some teachers had participated in multiple types of professional development throughout the course of the study. Researchers gathered survey data from the teachers regarding the type and duration of the professional development activities they had participated in during the designated period of time (the study does not indicate the length of this period of time). Self-reported survey data from teachers about their use of instructional practices were used as outcome measures. The researchers performed two regression analyses, one for the science teachers and one for the mathematics teachers. The researchers found that two of the five types of professional development (curriculum development and examining practice) were significantly related to teachers' use of standards-based instructional practices, the focus of all of the forms of professional development. This was true for both science and mathematics teachers in the study.

2.3.1.2.3 Coherence. In her book on designing effective professional development, Smith (2001) advocated coherence between professional development activities, all of which serve to achieve an over-arching goal. She explained that these activities must be well planned and coordinated and that the ad-hocism of much of traditional professional development will not produce desired changes in teachers' practices. Loucks-Horsley et al. (2003) also supported the use of a coherent professional development plan that incorporates connected activities. They proposed a framework for designing professional development (see Figure 2.3). This framework shows the need for interconnectedness in all phases of designing professional development. It also demonstrates that each professional development activity should be reviewed and the lessons learned from these activities should help shape future activities. By including multiple activities that are all connected in purpose, professional development provides multiple opportunities for teachers to develop the knowledge and skills that are the focus of the professional development program.

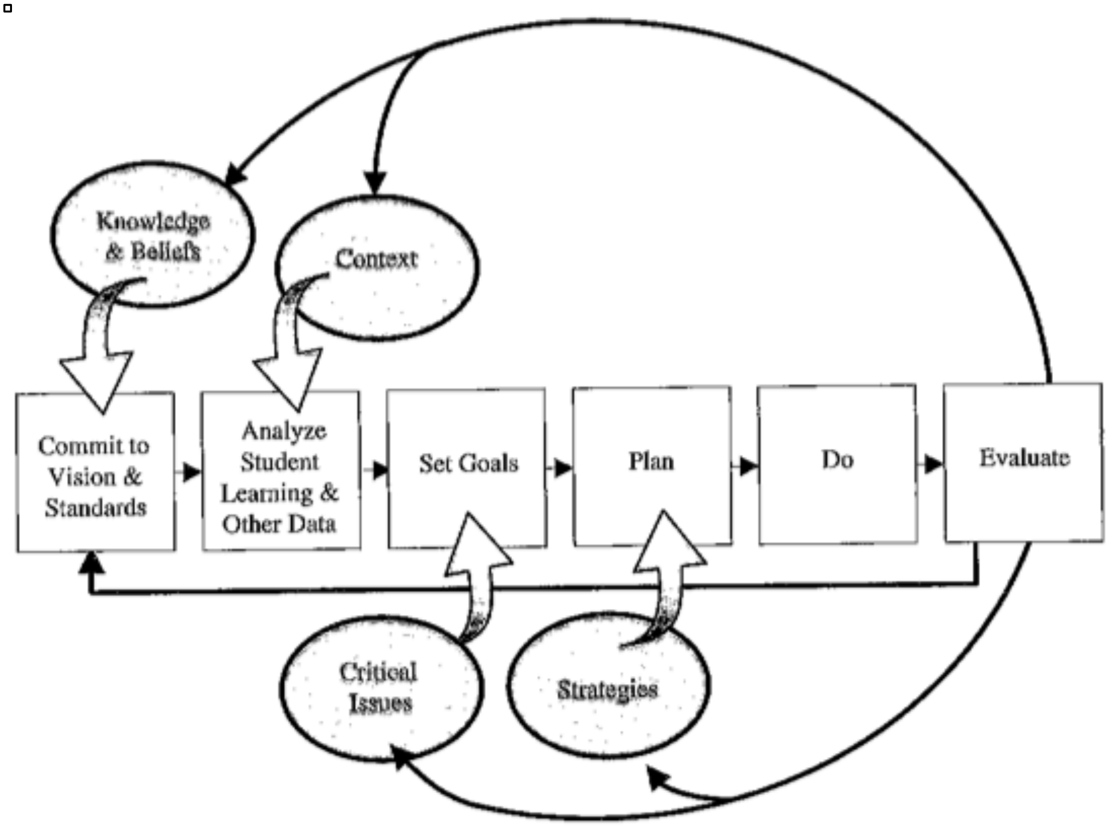


Figure 2.3: Design Framework for Professional Development in Science and Mathematics (Loucks-Horsley et al., 2003)

Garet et al. (2001) agreed with Smith (2001) as well as Loucks-Horsely and her colleagues (2003) about the need of coherence with in professional development activities and between these activities and other learning opportunities for teachers. They explained that professional development activities are more likely to positively impact teacher practices if they are part of a coherent program of teacher learning. They included this as a core feature of professional development in the study of the impact of professional development. Using teachers' self-reported data from surveys, they measured three aspects of the coherence of professional development: (a) connections with goals and other activities, (b) alignment with state and district standards and assessments, and (c) communication with others (i.e., continuing professional communication with other teachers). They then compared this data to outcome measures from teacher surveys on changes in their instructional practices. Garet and his colleagues found that teachers who participated in coherent professional development were significantly more likely to change their teaching practices to align with the focus of the professional development than were teachers who participated in professional development that is not coherent. In their three-year longitudinal follow up study (Desimone et al., 2002), these same researchers again measured the impact of coherence of professional development on teachers' practice. They found that when coherent professional development focused on a specific reform-based teaching strategy it significantly increased the likelihood of teachers using that strategy in their practice. For example, teachers who participated in coherent professional development that focused on alternate assessment practices reported using these practices significantly more than those who did not participate.

2.3.1.2.4 Focus on subject-matter content and pedagogical practices. Thus far, the critical features of effective professional development have focused on the characteristics of the program (e.g., collaboration, duration) or on the characteristics of the types of activities that are used within the professional development (e.g., active learning, practice-based). However, these are not the only vital components of professional development.

In her literature review of research on professional development for science and mathematics teachers, Kennedy (1998) found that most studies focused on the form and structure of professional development programs. Her review however, found that the most important difference between effective and ineffective professional development was the difference in the content of the programs (i.e., the substance of the professional development, not necessarily the subject matter). Kennedy restricted her investigation to studies in mathematics and science that used measurements of student learning as their outcome variable. She divided the research in her review into four categories according to their content:

(a) Teaching behaviors that could be employed generically in all school subjects,

(b) Teaching behaviors that could be employed generically in all school subjects but are suggested to apply to a specific school subject. Of these Kennedy says,

(c) Suggestions for teaching content and pedagogy of a specific school subject based on research of student learning, and

(d) Information about how students learn a specific school subject, but that does not include recommendations of how to teach the subject.

Kennedy found "that programs that focus on subject matter knowledge and on student learning of particular subject matter are likely to have larger positive effects on student learning than are programs that focus mainly on teaching behaviors" (p. 9).

While Kennedy's (1998) literature review was limited to studies with student learning as the outcome measurement, educational researchers have also theorized and found that focusing on content is a critical feature of professional development that positively impacts teachers' instructional practices. Wilson and Berne (1999) reviewed much of the research and teacher education theory regarding professional development prior to 1999. In their review, they synthesized many of the lists or recommendations of vital components of teacher professional development in to three categories, one of which is opportunities to talk about and do subject matter. The literature that Wilson and Berne reviewed did not provide evidence of the impact that talking about and working together on subject area content may have on teachers' instruction, rather it indicated that this appeared to be a critical feature that required further study. Loucks-Horsely and Matsumoto (1999) also conducted a review of research on professional development for mathematics and science teachers. They found that the best quality professional development should focus on aiding teachers to understand their subject matter, learners and learning, and effective instructional methods for teaching their subject to their students. Stein, Smith, and Silver (1999) suggested that for professional development to be effective in preparing teachers to align their instruction with educational reform efforts (e.g., National Council of Teachers of Mathematics, 1991, 2000), teachers need to engage in the subject matter themselves. Additionally, they asserted that professional development activities should scaffold teachers' learning of subject-matter content knowledge. In identifying features of high-quality professional development, Smith (2001) proposed that professional development should "be grounded in mathematics content" (p. 42) and "model and reflect the pedagogy of good instruction" (p. 43). This focus on pedagogical strategies based in the teachers' subject matter aids the teachers to both better understand what they are teaching as well as to learn quality instructional practices of how to teach it.

Loucks-Horsely et al. (2003) recommended multiple strategies for professional development, one of which is immersion experiences. In these experiences, teachers engage in doing mathematics or science with mathematicians or scientists. This strategy for professional development is based on the belief that teachers' learning is enhanced from these experiences just as their students' learning is increased as they engage in actually doing science or mathematics. It is also based on the assumptions that teachers need a deep understanding of the subject-matter content that they will be teaching and that teachers can deepen their content knowledge by engaging in these types of activities. Loucks-Horsely et al. also recommended that another strategy for effective professional development is to have teacher participate in “practice sessions.” This strategy is closely related to the need for professional development to be practice-based. It includes a focus on the pedagogical methods teachers would employ while teaching. This would be done via coaching, model lessons, or mentoring.

In addition to educational theory emphasizing the need to focus on content and pedagogy, empirical studies have also found this to be a fundamental feature of professional development that affects teacher’s practices. From prior research and hypotheses from educational researchers, Garet and his colleagues (2001) theorized that focusing on subject-matter content knowledge and pedagogical content knowledge (Shulman, 1987) were critical features of effective professional development. For their study, they gathered survey data from teachers regarding this topic. Specifically, teachers were asked to indicate the degree of emphasis (no emphasis, minor emphasis, major emphasis) the professional development activities they participated in gave to enhancing their content knowledge. The researchers then compared this to teachers’ self-reported changes in their instructional practices. They found that the professional development activities that had a major emphasis on teachers’ content knowledge were more likely have a positive impact on teachers’ knowledge and instructional skills. These researchers later replicated this finding in a three-year longitudinal study (Desimone et al., 2002).

An example from science education also provides evidence of the impact of professional development focused on subject matter content (in this case, science) on teachers’ instructional practices. T. M. Smith and his colleagues (2007) studied the impact of science teachers’ content knowledge on their ability to use inquiry-oriented teaching methods. To measure teachers’ content knowledge they collected data on teachers’ university degrees and majors as well as on the amount of professional development focused on science content in which they participated. The researchers used a national sample of over 1,000 eighth-grade science teachers’ responses on the teacher questionnaire from the 2000 NAEP Science Assessment. Smith et al. used teachers' self-reports of instructional activities to gather data on their classroom practices. The teachers in the study were also asked to indicate the amount of hours of professional development centered on science or science education they had participated in during the previous year. The researchers found that teachers' participation in between 30 and 40 hours of professional development centered on science content topics or pedagogy specific to science teaching was correlated to increases in teachers' us of reform-based practices. This was especially true for teachers who did not have a strong science background as part of their pre-service education. While Smith et al.'s work focused on science teachers, the lesson learned can also be applied to professional development for mathematics teachers as much of the theory and evidence regarding professional development deals with both mathematics and science teachers (e.g., Desimone et al., 2002; Garet et al., 2001; M. Kennedy, 1998; Loucks-Horsley et al., 2003).

2.3.2 Evidence of the Capacity of Professional Development to Affect Changes in Teachers' Practice

This section will provide two examples of mathematics teacher professional development that incorporates the critical structural features (teacher collaboration and duration) and core features (active learning, practice-based, coherence, and focus on subject-matter content and pedagogical practices) described above. These examples will also explain the impact that these professional development projects had on teachers' instructional practices.

2.3.2.1 Beyond implementation: Focusing on Challenge and Learning (BIFOCAL)

The Beyond Implementation: Focusing on Challenge And Learning project (BIFOCAL) was a multi-year practice-based professional development initiative with the aim of aiding middle school mathematics teachers to effectively implement innovative and challenging curriculum (Silver, Clark, Ghousseini, Charalambous, & Sealy, 2007; Silver, Ghousseini, Gosen, Charalambous, & Strawhun, 2005; Silver, Mills, Castro, & Ghousseini, 2006). Silver and colleagues (2005) explained that teachers typically receive professional development or other support when their school or district adopts new curriculum, particularly if this curriculum is viewed as challenging. However, this support usually tapers off once teachers become accustomed to using the curriculum. This may cause a situation in which teachers are familiar with a curriculum but have not gained sufficient skills in using it to its full potential in order to maximize student learning. This phenomenon is referred to as a *curriculum implementation plateau* (Silver et al., 2007, 2005, 2006). The BIFOCAL project was designed to aid teachers in overcoming this plateau. Twelve middle school teachers from five urban schools who had been using the *Connected Mathematics* curriculum (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2006a) for at least three years were invited to participated in the project. Principals from some of the schools also participated in the professional development. The teachers met in day-long professional development sessions once per month for eight months (Silver et al., 2005). During these sessions, teachers engaged in a two types professional development activities—case analysis and discussion and modified lesson study. Before reading or discussing a case, teachers would solve the mathematical task around which the case was structured. They then discussed the task, possible methods students might use to solve it, and the possible mathematical goals for which the task could be used to address. The cases that were examined were taken from materials specifically designed for professional development (Stein, Smith, Henningsen, & Silver, 2000), video samples of instruction from the TIMSS video study, or were produced using samples of student work from the *Connected Mathematics* curriculum (Silver et al., 2006). The modified lesson study design in which teachers engaged consisted of sequences of teachers selecting a task, collaboratively planning how to teach the task, one or more of teachers implementing the task with his or her students, and joint reflection on and analysis of the instruction of the task.

As teachers engaged in the professional development activities in BIFOCAL, the researchers found that they had multiple opportunities to learn about mathematics, make connections between related mathematical ideas, and ponder how the different methods in which these mathematical ideas could be presented to students would impact the student's opportunities to learn (Silver et al., 2007). Researchers also found that teachers' participation in this professional development led to some teachers using the instructional practices focused on during the sessions in their own teaching (Silver et al., 2005). For example, prior to the teachers' participation in BIFOCAL, they did not have much, if any, experience using multiple solutions in their instruction. Toward the beginning of the project, they spoke of the obstacles of using tasks with multiple solutions. However, through the duration of the professional development teachers discussed which of the many possible student solutions to select, how to sequence them, and the benefits of displaying and discussing erroneous solutions. The researchers noticed changes in teachers' ability to work with multiple solutions and noted a considerable improvement.

BIFOCAL incorporated all of the critical features of professional development discussed above. Both of the structural features (teacher collaboration and duration) were present in BIFOCAL. Teachers collaborated with one another in both the case analysis and modified lesson study portions of the program. BIFOCAL was implemented over eight months (approximately the length of one school year - October through May) and participated in many hours of professional development (eight day-long sessions - roughly five to six hours a day - would be approximately 40 to 48 hours of professional development). In addition to attending to the structural features of professional development, BIFOCAL also included each of the core features (active learning, practice-based, coherence, focus on subject-matter content and pedagogical practices). Teachers engaged in practice-based, active learning. They collaboratively designed lessons around specific tasks, used these in their classes with their students, and then jointly reviewed this instruction. They also collectively reviewed student work and discussed the understanding and possible misconceptions it conveyed. The activities in BIFOCAL were also coherent with one another as they all were focused on aiding teachers to be better able to implement the *Connected Mathematics* curriculum as well as tasks with multiple solutions in general. The professional development activities also focused on the concepts of mathematics as well as the pedagogical practices for implementing the tasks that were being discussed.

2.3.2.2 Enhancing Secondary Mathematics Teacher Preparation (ESP)

Boston and Smith (2009) studied the impact of professional development aimed at increasing teachers' knowledge of the cognitive demands of tasks as well as their ability to implement them in their classrooms. They theorized that by engaging teachers in activities designed to make them aware of the cognitive demands of tasks and the factors that impact the maintenance or decline of the cognitive demand of the task, teachers' ability to select and successfully use these tasks with their students would be enhanced. Their study was part of a larger project known as Enhancing Secondary Mathematics Teacher Preparation (ESP), which engaged teachers in professional development activities over a two-year period. As part of ESP, 18 secondary mathematics teachers participated in professional development during which they solved mathematical tasks, evaluated the levels of cognitive demands of mathematics tasks, and analyzed teachers' enactment of mathematical tasks. The professional development activities were centered on the Mathematics Tasks Framework (see Figure 1.1), the Task Analysis Guide (Appendix A), the factors influencing the maintenance or decline of the level of cognitive demand (Appendix C), and the Thinking Through a Lesson Protocol (M. S. Smith, Bill, & Hughes, 2008), a guide that helps teachers to anticipate how students will engage in high-level mathematical tasks in an effort to be more able to maintain the high level of cognitive demand of the task. During the professional development, teachers also read and discussed cases of anonymous teachers' implementation of high-level tasks (e.g., Stein et al., 2000). The teachers also were able to apply the fundamental ideas of the professional development to their own practice as they engaged in scaffolded field experiences (SFEs) (Borasi & Fonzi, 2002). These SFEs consisted of cycles of teachers selecting tasks from the school curriculum, analyzing the level of cognitive demand of the tasks, implementing them with their students, and then reflecting on their teaching of the task with the other group members.

Ten different secondary teachers were chosen to participate in the study as members of a contrast group; these teachers did not participate in the professional development activities. The teachers in ESP and those in the contrast group were comparable in regards to teaching experience, grade level, and secondary mathematics certification.

The 18 teachers in ESP provided the researchers with classroom artifacts (e.g., student work and instructional tasks). Eleven of these 18 teachers also agreed to allow researchers to observe their classroom instruction. Researchers gathered data from the ESP teachers in three five-day intervals dispersed over the school year (e.g., once in the fall, once in the winter, and again in the spring). For each data collection period, the ESP teachers submitted all instructional tasks they used over the five-day period of instruction for one of their courses. They also gave the researchers a class set of student work for three of the tasks that they used during the five-day period. The researchers observed each teacher implementing one of the tasks during the five-day period. These observations lasted for one class period. The ESP teachers were also given an assessment of their knowledge of the levels of cognitive demands of instructional tasks, as were the teachers in the contrast group. They were observed once during the school year (in the spring), and the researchers collected copies of the instructional tasks that were used during the lesson that was observed.

Boston and Smith (2009) used the Instructional Quality Assessment (IQA) Academic Rigor (AR) in Mathematics rubrics for *Potential of the Tasks* and *Overall Implementation* to analyze the data. The Potential of the Task rubric is used to determine the level of cognitive demand required for students to "produce the best possible response to the task" (p. 133). The Overall Implementation rubric is used to determine the actual level of cognitive demand used by the student to respond to the task. Boston & Smith performed multiple statistical analyses using the scores from the rubrics to then compare level of cognitive demand of the tasks teachers chose and their implementation of these tasks. This analysis revealed that the teachers in the ESP project improved significantly in their ability select high-level tasks and maintain these at a high level throughout their implementation from the first observation (fall) to the second and third observations (winter and spring). Upon completion of the professional development, ESP teachers also chose high-level tasks at a significantly higher rate than did the teachers in the contrast group and they were able to implement them at a higher level than the teachers in the contrast group.

The ESP project also employed the features of effective professional development. Teachers participated in the project over the course of approximately two school years, meeting together and working collaboratively to enhance their knowledge of the levels of cognitive demands of instructional tasks. They engaged in active learning and practice-based activities while gathering data from their own instruction to share and reflect on with group members. The professional development activities coherently built on one another to aid teachers in increasing their knowledge and instructional skills. The teachers also discussed pedagogy, specifically which instructional factors would aid them in maintaining the high levels of cognitive demand of tasks while employing them in their classes.

2.4 SUMMARY AND VISION FOR THE CURRENT STUDY'S INTERVENTION OF TEACHER PRACTICE

The tasks teachers select and the manner in which they implement them in their classrooms affect students' learning. An important feature that can be used to differentiate tasks is the level of cognitive demand that is required of students to solve the tasks. High-level tasks are those that require significant conceptual thinking from students to make connections between the solution strategies they are using to solve the task and the underlying mathematical concepts related to the task or that engage students in the process of "doing mathematics." The Task Analysis Guide (TAG) (Appendix A), created by Stein and Smith (1998), identifies the important features of high- and low-level tasks. Yet, tasks can change as they move through the different phases of instruction (Doyle & Carter, 1984; Stein et al., 1996). The Mathematical Tasks Framework (MTF) distinguishes between tasks in three stages of instruction: (a) tasks as presented in the curriculum, (b) tasks as set up by the teacher, and (c) tasks as implemented by the students. Stein et al. (1996) explained that the level of cognitive demand of tasks can change from one stage of instruction to another. They identified a set of factors common to instruction of tasks that are set up at a high level but that decline during the implementation stage to low-level tasks as well as a set of different factors typical of instruction when high-level tasks are maintained at a high-level throughout implementation. Together these three frameworks (the TAG, the MTF, and the factors associated with the decline and maintenance of the level of cognitive demand) can be used to analyze teachers' instruction.

Mathematics education reform (e.g., National Council of Teachers of Mathematics, 1991) has called for classroom discourse that allows students to engage in reasoning and justification; make, question, and defend conjectures; and form connections between various solution strategies, representations, and concepts related to high-level mathematical tasks. Teachers play a pivotal role in establishing a classroom environment in which such discourse is possible; they are also instrumental in conducting classroom discussions that develop students' mathematical understanding and move toward desired learning goals. However, teachers face many challenges when facilitating whole-class discussions around high-level tasks. Stein et al. (2008) put forth a set of five instructional practices teachers can use to carry out these discussions more effectively. These practices are: (a) anticipate possible student solution strategies; (b) monitor students work on the task; (c) strategically select student solutions that would be beneficial to share with the class; (d) purposefully sequence the order in which the selected solutions are shared; and (d) help students form connections across the shared strategies and between the strategies and the underlying mathematical concepts of the task.

Research on teacher professional development has shown that effective professional development programs share some common structural and core features. Structural features are related the format of the professional development while core features deal with the substance of the professional development. Two structural features of effective professional development are (a) opportunities for teacher collaboration, and (b) the duration of the professional development. Four core features of effective professional development are (a) active learning, (b) being practice-based, (c) coherence, and (d) a focus on subject-matter content and pedagogical practices.

This study proposes to examine professional development for mathematics teachers that contains the six features of effective professional development listed above. This professional development is focused on enhancing teachers' instructional practices by aiding them in selecting and implementing high-level tasks with their students. Much of the professional development will center on assisting teachers to conduct whole-class discussions around the high-level tasks they choose by using the five practices identified for this purpose by Stein and her colleagues (2008). As part of the analysis of the impact of this professional development, the TAG, the MTF, and the factors of decline and maintenance of the levels of cognitive demand of high-level tasks will be used to examine the tasks teachers select and their implementation of these in their classrooms. A more detailed explanation of the analysis methods for this study will be presented in Chapter Three.

While the proposed study contains many of the same features as the ESP and BIFOCAL projects, it differs from both in several key aspects. Teachers in the ESP and BIFOCAL projects participated on a voluntary basis. The vast majority of these teachers taught in suburban settings and for the most part they used the same reform-based curriculum materials. These teachers, while from similar settings, were not all from the same school or district. In contrast, the teachers participating in the proposed study are mandated by school and district administrators to participate in the professional development activities. They all work within the same school, yet they use a variety of curricula materials, some of which are traditional and others reform-based. Specific details regarding the participants and the context of the study are given in Chapter Three.

3.0 CHAPTER 3: METHODS

This study used qualitative data collection and analysis methods in an effort to better understand the influence professional development has on teachers' instructional practices. Specifically, it sought to examine the extent to which professional development centered on instructional practices which aid teachers in selecting high-level tasks and conducting whole-class discussions around those tasks impacted their ability to use said practices. The study drew on data obtained via observations and artifacts from the professional development meetings and teachers' classroom instruction.

3.1 CONTEXT OF THE STUDY

This study investigated the impact of professional development provided to mathematics teachers as part of a larger multiyear project, referred to hereafter as the Lesson Planning Project. The purpose of the Lesson Planning Project was to study the impact of an approach to schoolwide reform centered on improving teachers' instructional practices and instigating organizational change (M. S. Smith et al., 2012; Stein, Russell, Gomez, & Gomez, 2008; Stein et al., 2011). The Lesson Planning Project was housed in a collaborative effort between a university-based urban education center and a local, urban school district. This relationship can be seen in Figure 3.1. The Lesson Planning Project was a schoolwide intervention at Lincoln

Secondary School that consisted of Grades 6-12.[2] This intervention consisted of a multimember, university-based team which included researchers and teacher educators with expertise in organizational and school reform; instruction; and teacher education in mathematics, science, social studies, and English language arts (M. S. Smith et al., 2012). As can be seen in Figure 3.1, the Lesson Planning Project consisted of two classroom-based routines (in addition to organization-wide routines): one focused on lesson observations with feedback and the other on lesson planning. Subject-matter content areas further divided the lesson planning routine. The research conducted for this study examined the effects of professional development provided to the mathematics teachers as part of the lesson planning routine.

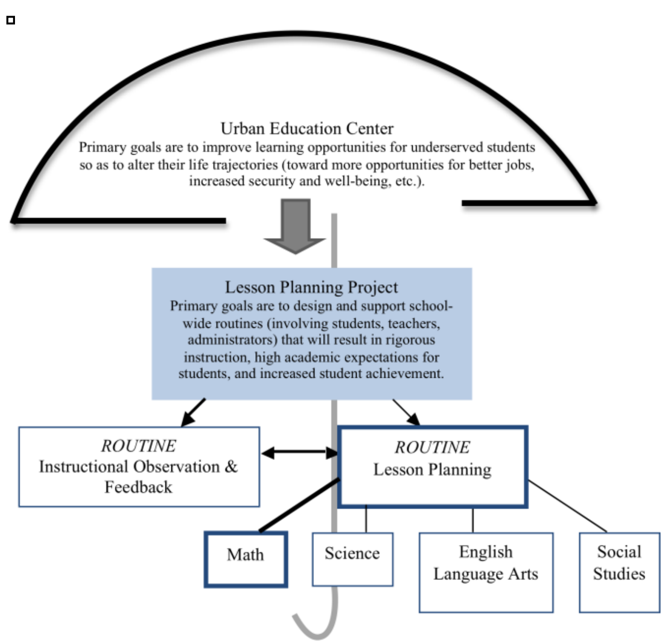


Figure 3.1: Overview of the Lesson Planning Project operating under the umbrella of the Urban Education Center (M. S. Smith et al., 2012)

[2] The names of the school, the teachers, and university-based professional development providers are pseudonyms.

As an additional element of the Lesson Planning Project's intervention at Lincoln, teachers were asked to use an electronic lesson-planning tool to create and submit their daily lesson plans. The purpose of the lesson-planning tool was to focus teachers' attention as they planned on aspects of student thinking that impact the manner in which they engage in high-level tasks (Stein et al., 2011). The lesson-planning tool aided teachers by prompting them to set specific learning goals, select tasks that appropriately help students achieve those learning goals, anticipate how students may engage in the selected tasks, and plan how to productively assist students in progressing in their work on the tasks (Stein et al., 2011). The lesson plans the teachers created using the electronic lesson-planning tool were frequently referred to during the professional development meetings by the teachers and university-based facilitators in an effort to ground discussions in the teachers' instructional practices.

Lincoln is a public school with a student body that is predominantly African American from low-income neighborhoods. There were approximately 650 students dispersed between Grades 6 to 12 at the time of the study (M. S. Smith et al., 2012). The school day at Lincoln was divided into eight periods, some 80 minutes long others only 40 minutes. Core academic subject classes (e.g., English language arts, mathematics, and science) were held during the 80-minute class periods.[3]

The mathematics classes in Grades 6-8 used both a traditional textbook from the Prentice Hall mathematics series (e.g., D. Kennedy, Charles, & Bragg, 2006) as well a reform-based textbook, *Connected Mathematics 2* (e.g. Lappan, Fey, Fitzgerald, Friel, & Phillips, 2006b). The 9th-12th grade mathematics classes used a variety of textbooks, which for the most part were

[3] The class schedule at Lincoln has subsequently changed so that it no longer includes 80-minute periods.

traditional in their approach to instruction. Most mathematics classes at Lincoln were directed by the district curriculum guides. These guides contained pacing schedules as well as supplemental tasks to be used in addition to the material in the textbooks for the 9th-12th grade courses. These additional tasks typically required higher levels of cognitive demand than those found in the traditional textbooks used by 9th-12th grade teachers. The curriculum guides also included information regarding the supplemental tasks such as when the tasks are to be used, the amount of time the tasks will require, the learning goals for each task, and teaching notes for each task. The teaching notes provided suggestions for setting up the task, questions to help assess students' understanding while working on the task, questions teachers can use to progress student thinking toward desired learning goals, and suggestions of questions to ask while discussing the task as a class. The curriculum guides also contained ideas for differentiating and scaffolding instruction of the supplemental tasks when needed. Upper-level math courses such as Elementary Functions and Calculus were not provided with these curriculum guides.

The Lesson Planning Project's university-based team of researchers and teacher educators worked with teachers at Lincoln during the two years before the 2011-2012 school year, the year the data for this study was collected.[4] The professional development at Lincoln during the 2011-2012 school year was based on the theory of change as presented by Smith et al. (2012). They explained that the professional development "was intended to improve teachers' knowledge, which in turn would influence the ways in which they planned for and enacted instruction" (p. 120). This theory of change is shown in Figure 3.2. While this theory of change remained intact throughout the course of the Lesson Planning Project, the design of the

[4] Two of the university-based team members had worked with teachers at Lincoln since the school opened in 2008.

professional development was modified each year because of the changing needs of the school personnel and the dynamic situation at the school (e.g., change in school administration, change in the district's teacher evaluation program, extremely high teacher turnover, and the addition of students who previously had attended schools with rival gangs). A detailed account of the Lesson Planning Project's work with the teachers at Lincoln during previous years will not be provided here, but can be found in Smith et al.'s (2012) description of the work.

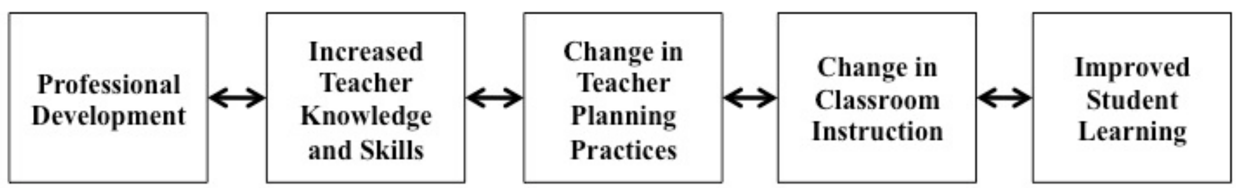


Figure 3.2: Framework for linking the effects of professional development on teachers and students (M. S. Smith et al., 2012)

3.1.1 Professional Development Design

The focus of the lesson planning routine of the Lesson Planning Project for the 2011-2012 school year was to work with teachers in grade-band teams (i.e., Grades 6-8 mathematics teachers in one team, Grades 9-10 science teachers in another) to collaboratively plan high-level tasks chosen by group members that they would then implement with their students. This was accomplished through weekly professional development meetings. These meetings were content specific (e.g., mathematics teachers attended professional development meetings separate from science teachers) and were facilitated by members of the university-based collaborative team. This section of this paper will only attend to the mathematics professional development, as that is the focus of this study. The mathematics professional development was conducted by two members of the university-based team: Dr. Travis (a professor and expert of mathematics education and teacher educator) and her graduate student assistant (the author of this study), who will hereafter be referred to as “the researcher.”

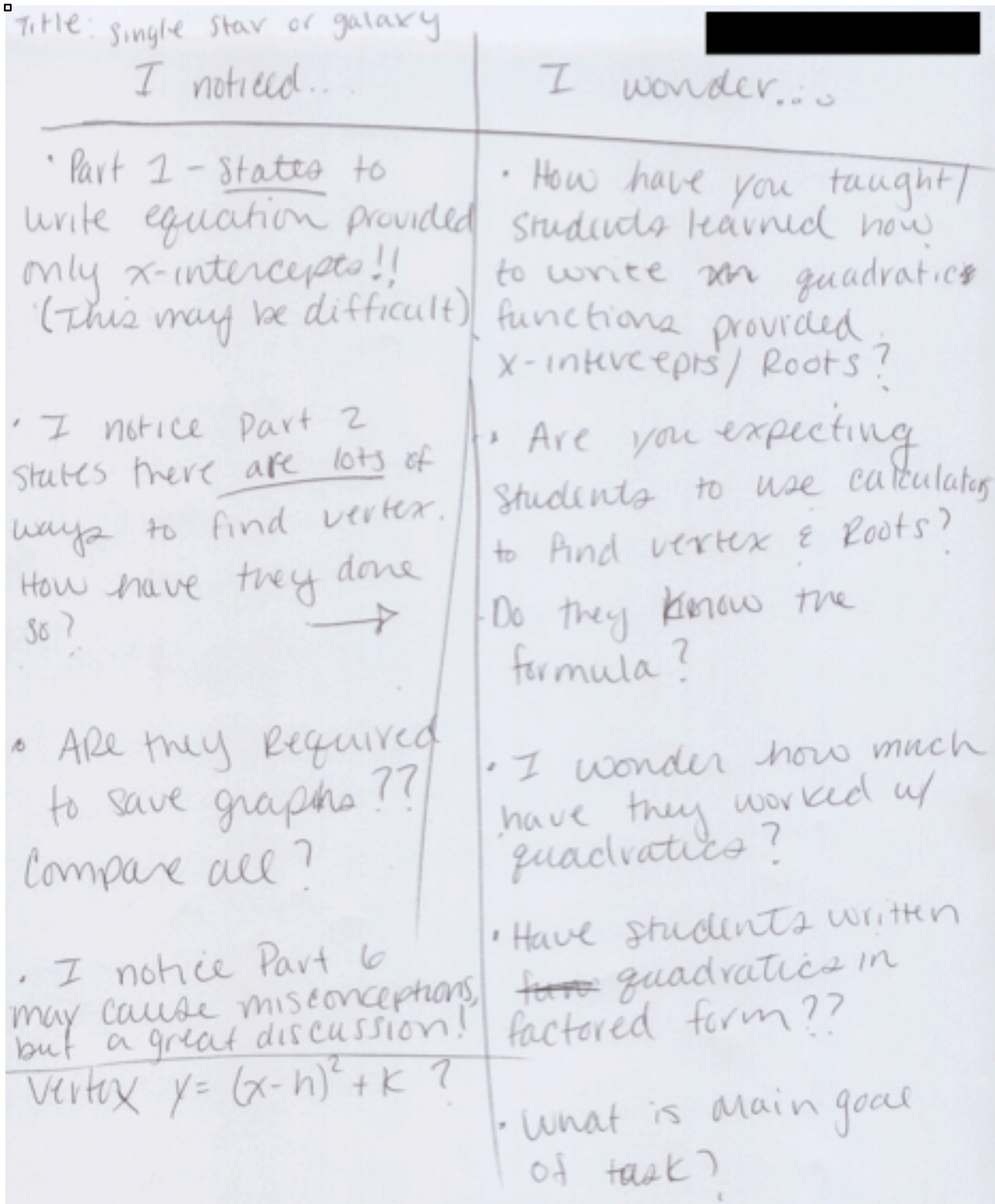
All mathematics teachers at Lincoln were required to participate in the weekly meetings. They were divided into three teams based on their grade band. Team A was comprised of the eleventh- and twelfth-grade teachers, Team B of the ninth- and tenth-grade teachers, and Team C of the sixth-, seventh-, and eighth-grade teachers. The members of each team changed during the first several weeks of the school year due to changes in class schedules, teachers leaving the school, and new hires during the school year. Once the teams stabilized, there were four members on each team. Table 3.1 shows the grades and courses taught by the teachers in each of the teams. Special education teachers assigned to assist the mathematics teachers with certain classes also frequently attended the professional development meetings.

Table 3.1: Professional development team member characteristics

	Teacher	Grades taught	Courses taught
Team A	Teacher 1	11th and 12th	– Advanced placement statistics – Probability and statistics
	Teacher 2	11th and 12th	– Elementary functions (pre-calculus) – Advanced placement calculus
	Teacher 3	11th and 12th	– Algebra II
	Teacher 4	11th and 12th	– Remedial algebra (this was a special course for students with learning disabilities)
Team B	Teacher 5	9th and 10th	– Algebra I – Remedial algebra – Geometry
	Teacher 6	9th and 10th	– Algebra I – Algebra II – Remedial algebra
	Teacher 7	9th and 10th	– Remedial algebra
	Teacher 8	9th and 10th	– Algebra I – Remedial algebra
Team C	Teacher 9	6th	– Sixth-grade mathematics
	Teacher 10	7th	– Seventh-grade mathematics
	Teacher 11	8th	– Eighth-grade mathematics
	Teacher 12	6th and 7th	– Sixth-grade mathematics – Seventh-grade mathematics

Each grade-band team met with at least one of the two university-based team members (Dr. Travis and the researcher) each week. The mathematics coach assigned to the school and the mathematics instructional team leader at the school would frequently attend these meetings. The initial professional development meetings (during September and November of 2011) were used to introduce the teachers to the electronic lesson-planning tool, the Task Analysis Guide (TAG) (see Appendix A), and the idea of differentiating mathematical tasks based on the level of cognitive demand required to solve them (Stein et al., 1996). The group members also learned about and discussed the five practices for orchestrating classroom discussions around high-level tasks known as the *five practices* (M. S. Smith & Stein, 2011; Stein, Engle, et al., 2008). For the fourth meeting of the year, each teacher was asked to select a high-level task based on the TAG, that he or she would be teaching during the following week to bring to their respective team meeting. During this meeting the teachers shared the tasks they brought with their team members and the team discussed how they would classify the task using the TAG. This activity set the foundation for the routine that would be followed during these team meetings throughout the remainder of the school year.

Once teachers became familiar with the notion of high-level tasks and the five practices, the professional development meetings settled into a series of two-week cycles referred to as *modified lesson study cycles*. For these cycles, one teacher on each team was assigned to select a *focus task*, a high-level task that would serve as the focus of the work in the professional development meetings during the two-week cycle. Prior to the first week of the cycle, the teacher would provide the other team members with the focus task and her learning goals for her instruction around the task. Before the first meeting of the modified lesson study cycle, all group members (i.e., the teachers and the university-based team members) were asked to engage in the focus task. They anticipated ways in which students would work on the task, what solution methods they would attempt to use, and possible misconceptions they might have while working on the task. The group members would record their work and possible solution strategies as well as produce *noticings and wonderings* (M. S. Smith, 2009) using a Noticings and Wonderings Recording Sheet (see Appendix D). The purpose of using the noticings and wonderings in these meetings was to respectfully provide instructional support and suggestions as well as to raise questions about how the teacher might implement the task. For example, consider the *Single Star or Galaxy?* task (see Appendix E). This was a focus task provide by one of the teachers in the 11th- and 12th-grade team. After working on the task, another teacher in this team produced the noticings and wonderings shown in Figure 3.3.



Title: Single Star or galaxy [redacted]

I noticed...	I wonder...
• Part 1 - states to write equation provided only x-intercepts!! (This may be difficult)	• How have you taught/ students learned how to write ~~an~~ quadratics functions provided x-intercepts/ Roots?
• I notice Part 2 states there are lots of ways to find vertex. How have they done so? →	• Are you expecting students to use calculators to find vertex & Roots? Do they know the formula?
• ARe they Required to save graphs?? Compare all?	• I wonder how much have they worked w/ quadratics?
• I notice Part 6 may cause misconceptions, but a great discussion!	• Have students written ~~form~~ quadratics in factored form??
Vertex $y = (x-h)^2 + k$?	• What is main goal of task?

Figure 3.3: Noticings and Wonderings for the Single Star or Galaxy Task

After the first meeting in the two-week cycle, the teacher who selected the task modified her lesson plan in the electronic lesson-planning tool (and possibly the task itself) based on the feedback provided during the meeting by the group members, the simulated student work on the task the group members produced, and the group members' noticings and wonderings. She would then implement the lesson in her classroom and collect artifacts (e.g., samples of student work, diagrams or representations produced by the students) from the lesson. During the second meeting of the two-week cycle, the teacher who had selected the focus task would share the artifacts she collected from the lesson and she would reflect on her teaching of the lesson, often attending to many of the noticings and wonderings that were discussed during the previous week's meeting. The other group members would listen to this reflection and would then ask her questions in the form of noticings and wonderings about the implementation of the task. Once this cycle was complete, another teacher in the group would select a focus task and the cycle would begin anew. This cycle was repeated throughout the remainder of the school year so that each teacher had at least two opportunities to select, plan, teach, and reflect on a focus lesson.

3.1.2 Connections to Features of Effective Professional Development

The professional development in the form of modified lesson study cycles contained the six features of effective professional development identified and discussed in detail in chapter two (teacher collaboration, duration, active learning, practice-based, coherence, and a focus on content and pedagogy). This section will provide evidence of each of these features of the professional development.

The professional development afforded teachers opportunities to collaborate with their colleagues, who not only were from the same school, but who taught related classes in the same grade-band level (e.g., all the teachers in Team B taught some form of algebra to ninth- and tenth-grade students). The professional development also allowed teachers to collaboratively attend to instructional issues they face while teaching. For example, the teachers identified specific misconceptions their students would face while working on the focus tasks and discussed methods for aiding students in overcoming these misconceptions. The duration of professional development can be measured both as the length of the time over which the professional development program spans as well as the total number of contact hours between the participants and the facilitators (Garet et al., 2001). The professional development that is the focus of this study attended to both these aspects of duration. The program spanned the length of the school year (i.e., September 2011 through May 2012) and the meetings were approximately 45 minutes in length each week.

The activities in the modified lesson study cycle employed during the professional development meetings engaged teachers in active participation as opposed to listening to a lecture or participating passively in some other form. The professional development was also practice-based as it centered on the work the participating teachers were doing in their own classrooms. This can be seen in the modified lesson study cycles when teachers would select tasks they would be using in *their* class and bringing artifacts from *their* implementation of the tasks. While each teacher was only able to focus on a task from his or her own teaching once every eight weeks, the teachers were still able to attend to issues faced by other teachers in their own school that were related to their teaching. The teachers also had an opportunity to discuss each other's practice and relate it to their own. Thus, while the professional development was not focused on each teacher's practice every week, all teachers were provided with opportunities to draw connections to their practice and were given frequent opportunities to focus directly on their own teaching.

The coherence of the professional development activities is seen in the cyclical nature of the program. As teachers repeated the professional development cycles, they continued to focus on content and pedagogical issues. Throughout the duration of the professional development, the work in which the teachers engaged was connected to the overarching goal of improving teacher instruction by attending to their ability to select and implement high-level tasks. By evaluating the teacher-selected tasks and focusing on specific issues that they would potentially face when using the task with their students, teachers were better prepared to implement that task in their classrooms.

The professional development also provided teachers with opportunities to engage in discussions of mathematical concepts and pedagogical practices. Frequently, as the group members discussed the various solutions they had anticipated students might use on the focus tasks, individuals would have questions about mathematical content. These questions would lead to in-depth conversations about the underlying mathematical ideas of the task. For example, one of the teachers in the ninth- and tenth-grade team selected the Swimming Practice task (see Figure 3.4) as a focus task. During the first meeting of the two-week cycle, as the group members discussed the task they discovered that three of the team members (two of the teachers and one of the university-based team members) had used three distinct methods for solving the task (see Figure 3.5). During the meeting, the team members had to grapple with mathematical concepts to make sense of the three solutions. Specifically they had to discuss what the variable x represented in each of the solutions as it represents the total number of practice days in solution

A, but it represents the additional days in the other two solutions. In addition to focusing on mathematical content, the group members also discussed the instructional practices teachers could use to aid their students when implementing the focus task.

Swimming Practice

Last month, Diana started training for the swim team. She swam 20 laps every day for 18 different practice days. This month Diana wants to swim 215 more laps that last month. She will swim 3 more laps per practice day and add more practice days to her monthly schedule.

(Remember to show all of your calculations in order to get your credit for this work!)

1. If Diana adds 5 practice days to her schedule this month, will she swim 215 more laps?
2. If she adds 10 practice days to her schedule this month, will Diana swim 215 more laps?
3. If Diana adds 6 practice days to her schedule this month, will she swim 125 more laps?
4. Use the guess/check/generalize method to write an equation for finding the number of additional practice days she needs to swim 215 more laps.
5. Now solve your equation. Explain your solution.

Figure 3.4: Swimming Practice Task

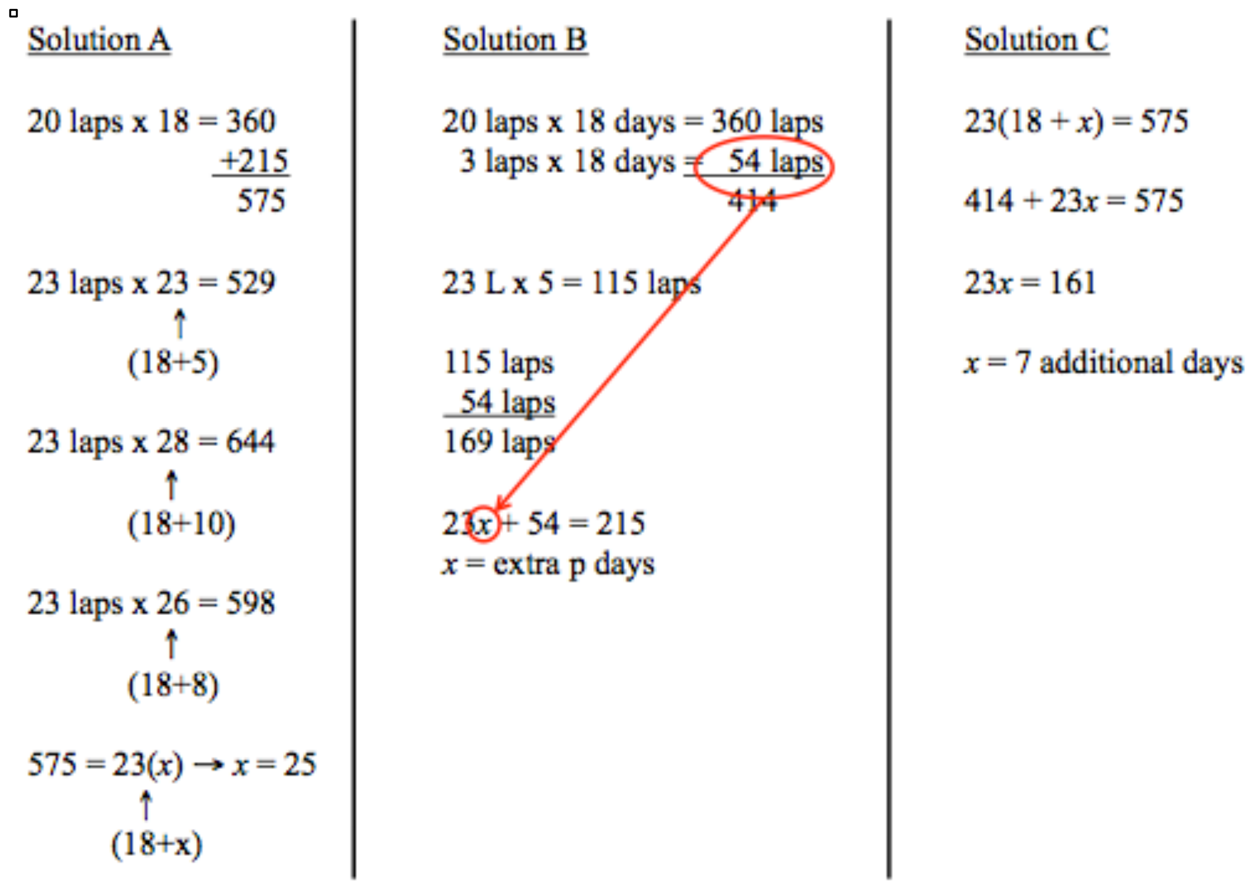


Figure 3.5: Solutions to the Swimming Practice Task

3.2 PARTICIPANTS

The purpose of this study was to evaluate the impact teachers' participation in the professional development as part of the Lesson Planning Project had on their selection of high-level tasks, their ability to maintain tasks at a high-level during the implementation stage of instruction, and their use of the five practices. While participation in the professional development was mandatory for all mathematics teachers at Lincoln, not all of the teachers were chosen as participants for this study. The nature of the study and the resources available limited the number of participants that could be included. This study employed qualitative data collection methods such as classroom observation and the collection of artifacts of practice. As each participant was to be observed multiple times and only one person was available to gather data, it was unreasonable to include all of the mathematics teachers at Lincoln in the study.

Of the 12 mathematics teachers at Lincoln, five were on instructional improvement plans based on poor teaching performance. The researcher, Dr. Travis, and the other university-based members of the Lesson Planning Project felt that it would be inappropriate to include these teachers in this study. Of the seven remaining teachers, two were part of Team A (the 11th- and 12th-grade team), three were on Team B (the 9th- and 10th-grade team), and two were on Team C (the 6th-, 7th-, and 8th-grade team). Due to a lack of resources, it was not feasible to track more than four teachers. The researcher and Dr. Travis were familiar with one of the two teachers on Team A and both of the teachers on Team C from prior working experiences. They felt that these teachers would be likely to cooperate in additional data collection. Thus, these teachers, along with the second eligible teacher on Team A, were invited to participate in this study. These four teachers agreed to participate in this study, and as such, they agreed to additional data collection (described in the subsequent section). Two of the teachers, Cara Nance (Teacher 2 in Table 3.1) and Nicole Nesmith (Teacher 3 in Table 3.1), were on Team A. The two remaining teachers, Gloria Xavier (Teacher 10 in Table 3.1) and Nathan Ingram (Teacher 9 in Table 3.1), were members of Team C. Demographics, grade, course, and teaching experience information for each teacher are presented in Table 3.2. It is important to note that as both Gloria and Nathan had taught at Lincoln prior to the study, they had participated in professional development with Dr. Travis and the researcher for the previous two years as part of the Lesson Planning Project.

Table 3.2: Characteristics of teacher participants

	Demographics				Grade and course information		Number of years teaching prior to the study	
	Teacher	Race	Gender	Age Range	Grade(s) taught	Courses taught	Overall	At Lincoln
Team A	Cara Nance	White	F	21-30	11 & 12	Elementary Functions, AP Calculus	.5*	.5*
	Nicole Nesmith	White	F	21-30	10, 11, & 12	Alg. II	.5*	0
Team C	Gloria Xavier	African American	F	41-50	7	Math-7	More than 10 years	2
	Nathan Ingram	White	M	31-40	6	Math-6	5	2

* Cara Nance and Nicole Nesmith had worked as long-term substitutes for half of the year before the study.

3.3 DATA SOURCES

Data were collected both from the professional development meetings as well as from observations of teachers' classroom instruction in order to address the research questions identified in Chapter One. These research questions are:

I. To what extent do teachers participate in the professional development focused on selecting and implementing high-level tasks?

II. To what extent does teachers' use of high-level tasks change over the course of their participation in professional development focused on selecting and implementing high-level tasks?

III. To what extent does teachers' ability to maintain the cognitive demand of high-level tasks in the set up and implementation stages of instruction change over the course of their participation in professional development focused on selecting and implementing high-level tasks?

IV. To what extent does teachers' use of the practices of *anticipating*, *monitoring*, *selecting*, *sequencing*, and *connecting* in their instruction change over the course of their participation in professional development focused on selecting and implementing high-level tasks?

V. What relationship, if any, is there between teachers' use of the practices of *anticipating*, *monitoring*, *selecting*, *sequencing*, and *connecting* and their ability to maintain the level of cognitive demand of high-level tasks?

VI. To what extent do teachers incorporate the ideas and suggestions made about their focus tasks during the professional development meetings in their implementation of the focus task?

Data from the professional development meetings included: (a) audio recordings and transcripts of weekly meetings; (b) meeting summaries (produced from listening to the audio recordings); and (c) meeting artifacts (e.g., tasks, lesson plans, samples of student work, diagrams produced by group members while discussing mathematical tasks or concepts). The data from classroom observations consists of: (a) detailed write-ups describing the classroom instruction; and (b) lesson artifacts such as the teacher's lesson plan, the task(s) used during the lesson, and pictures of displayed student work, lesson objectives, and lesson agendas. Table 3.3 shows which data will be used to address each of the research questions. These data types will be described in detail in this section.

Table 3.3: Data sources used to address each research question

				Research questions					
				I	II	III	IV	V	VI
Data sources	Professional development		Audio recordings						X
			Meeting transcripts						X
			Meeting summaries						X
		Meeting artifacts	– Focus tasks						X
			– Members' work on focus tasks	X					X
			– Members' noticings and wonderings	X					X
			– Pictures of diagrams used during the meetings						X
	Classroom observations		Write ups			X	X	X	X
		Lesson artifacts	– Lesson plans		X	X	X		X
			– Tasks		X	X			X
			– Pictures of displayed student work			X	X	X	X

3.3.1 Data Sources from Professional Development

Each professional development meeting was audio recorded. From these recordings, the researcher was able to produce summaries of the meetings. These summaries provided basic information about the meetings such as the date and time of each meeting, the teachers and university-based team members in attendance, and the purpose of the meeting. More importantly, they provided a synopsis of what occurred during the meetings. The summary of each meeting includes important events that occurred along with time markers corresponding to the audio recordings for convenience when referring to the audio recordings when investigating specific details of the meetings. See Appendix F for an example of a professional development meeting summary.

Transcripts of some of the professional development meetings were created from the audio recordings. These were used as additional professional development meeting data sources. Due to limited resources, not all of the professional development meetings were transcribed, however all of the meetings critical to the analyses of the research questions were transcribed. The process for determining which meetings will be transcribed will be described in the Coding section.

The meeting artifacts include scanned copies of the focus tasks (e.g., Figure 3.3 and Figure 3.5) selected by the teachers for each meeting, the teachers' work on these tasks (e.g., Figure 3.6), and the noticings and wonderings they produce while working on the task (e.g., Figure 3.4). Other artifacts shared in many of the professional development meetings include teachers' lesson plans, samples of student work or other items produced by students while working on the task (e.g., a poster of a histogram displaying the information provided in the task), and diagrams or representations of mathematical concepts produced by team members during discussions of the focus tasks.

3.3.2 Data Sources from Classroom Observations

Cara Nance and Gloria Xavier were observed eight times between October 2011 and May 2012. Nicole Nesmith and Nathan Ingram were each observed seven times during this same time frame. Teachers were observed teaching both lessons using the focus tasks they selected for discussion during the professional development meetings as well as lessons involving non-focus tasks. For the lessons with non-focus tasks, the teachers were asked to select lessons during which they would be implementing high-level tasks. The teachers selected the dates for the observations so that the lessons would occur naturally in the sequence of the content the students were learning. Ms. Nance, Ms. Nesmith, and Mr. Ingram each taught two lessons using focus-tasks, while Ms. Xavier only taught one. Table 3.4 shows when the classroom observations occurred and which of the observed lessons involved focus-tasks.

Table 3.4: Dates of classroom observations

Teacher	Obs. 1	Obs. 2	Obs. 3	Obs. 4	Obs. 5	Obs. 6	Obs. 7	Obs. 8
Cara Nance	10/25/11	11/14/11	01/09/12	02/28/12	03/19/12	04/18/12	05/08/12	05/30/12
Nicole Nesmith	10/25/11	12/01/11	01/24/12	02/27/12	04/16/12	05/03/12	05/29/12	--
Gloria Xavier	10/27/11	12/08/11	02/24/12	03/23/12	04/20/12	04/27/12	05/11/12	05/24/12
Nathan Ingram	10/27/11	12/13/11	01/05/12	03/01/12	03/12/12	04/30/12	05/22/12	--

Note: Shaded regions indicates lesson that used focus tasks from the professional development cycles. Non-shaded regions indicated lesson that used non-focus tasks.

During the classroom observations, the researcher would arrive prior to the beginning of the lesson to document the arrangement of the desks and note where the students sat as they entered the room. He would capture significant information posted on the chalkboard or the walls (e.g., lesson objective and agenda, previous work that was referred to during the lesson) either by replicating the information or diagrams in his fieldnotes or by taking pictures of the items. He documented what occurred during the lesson in the form of in-depth fieldnotes (Emerson et al., 1995). These notes focused on the instruction of the teacher and how the students responded to this instruction as well as how they implemented the tasks given them during the lesson. Particular attention was paid to instructional and classroom factors that might impact the maintenance or decline of the cognitive levels of the task (Stein et al., 1996) as well as the teachers' use, or lack thereof, of the five practices. After the meeting, these notes were used to produce classroom observation write-ups. These write-ups serve as the permanent record of the lesson.

The classroom observation write-ups were recorded in part A of the Classroom Observation Instrument (COI) (see Appendix G). The COI is a research tool that is used to both collect and analyze data from classroom instruction. Information about the origin of the COI is given in the Coding section of this chapter as it is primarily about the use of the COI as an analytical tool. However, part A of the COI, which is used to capture data from classroom instruction, will be described here. Part A of the COI consists of three sections: (a) the cover sheet; (b) Description of Lesson; and (c) Identification of Instructional Activities. The cover sheet contains fields for information about the lesson and classroom being observed such as the teacher's name, what subject/grade-level is being taught, when the lesson occurred, how many students are present, and a description of the physical layout of the room. This information is meant to help the researcher identify the lesson when performing subsequent analyses and to provide general background information about the lesson. The second section, Description of Lesson, is where the researcher records what occurs in the lesson. For this study, this section of the COI is where the classroom observation write-up of each observed lesson is contained. In the third section of the COI, Identification of Instructional Activities, the researcher, as the observer of the lesson, uses the description contained in the second section of the COI to identify all activities that occurred during the lesson and group them into one or more tasks. All activities during the lesson that focused students' attention on the same topic or mathematical concept are grouped together and considered one task. Appendix H is a completed example of part A of the

COI including the classroom observation write-up; it is from the observation of Cara Nance on May 8, 2012.

To ensure the classroom observation write-ups reliably captured the events of the lessons, Dr. Travis and the researcher performed reliability checks on three of the observation write-ups. They used the TAG and the list of factors associated with the decline and maintenance of high-level cognitive demands to determine the level of the tasks that were used and whether the cognitive demand of the task changed between the teacher's set up of the task and the students' implementation of it. They also used the Look Fors Sheet (see Appendix I; this will be discussed in detail in the Coding section) to assess the teachers' use of the five practices. Dr. Travis' coding of the write-ups was compared to that of the researcher, who had observed each lesson and thus was able to evaluate the lesson based on the actual instruction as opposed to a written summary. In one case, Dr. Travis had also attended with the researcher while observing a lesson. In this case, the write-up and subsequent coding of it were also compared against her recollection of the lesson. These reliability checks occurred during the data collection process and were used to make adjustments to this process. Based on recommendations from Dr. Travis throughout the data collection process, the researcher made adjustments to the level of detail captured in the observation write-ups to ensure sufficient information was captured so that they could be coded for level of cognitive demand and with the Look Fors Sheet.

3.4 CODING

This section will present the tools and processes that were used to code the data for the study.

3.4.1 Classroom Observation Analytical Tools

Two tools were used to perform the analysis of the data from the classroom observations: (a) the Classroom Observation Instrument and (b) the Look Fors Sheet. These tools will be described in this section.

3.4.1.1 Classroom Observation Instrument

As noted above, the Classroom Observation Instrument (COI) is used both to capture details of the classroom instruction as well as to analyze the collected data. The particular version of the COI used for this study is a modified version of the original COI developed and used as part of the QUASAR study (Stein et al., 1996). The original COI underwent multiple revisions and was pilot tested to ensure validity and reliability (Stein et al., 1996). The modified version of the COI used in this study (see Appendix G) was altered from the original in that it does not include some of the elements of the original, as they are not pertinent to this study. It also uses more standard language based on the subsequent work from the QUASAR project with regard to the cognitive demand of mathematical tasks. The modified version was validated as remaining true to the original version by one of the original designers of the COI.[5]

Part B of the COI, which consists of the section entitled Cognitive Demand of Instructional Activities, is comprised of a series of analytical questions based on the TAG, the MTF, and the list of factors of maintenance and decline of high-level, cognitively demanding tasks. The coder(s) (the researcher in all cases, and an additional secondary coder in some cases) answered the questions in part B of the COI about the main instructional task that occupied the most time during the lesson as identified in part A. The coder(s) categorized the task, as

[5] *Note*: Hereafter, COI will refer to the modified version of the Classroom Observation Instrument.

presented in written form (i.e., the textbook, the worksheet given to the students), based on the level of cognitive demand it requires of students. The task was coded as one of the four types of tasks listed on the TAG (*memorization*, *procedures without connections*, *procedures with connections*, and *doing mathematics*) or as one of two additional categories (*little or no academic thinking required* and *other*). The coder(s) then described how the teacher set up the task as well as the teacher's goals for the activity. The coder(s) also provided evidence from the description in the write-up of the lesson to support these claims. The teacher's set up of the task was then coded using the same six types of task as were used to code the task as presented in written form. The coder then determined how the task was implemented using the same codes and one additional code, *unsystematic and nonproductive exploration*. The final two questions in part B of the COI attend to the factors that lead to the decline or maintenance of the cognitive demand of the task. If the teacher set the task up at a low level (e.g., coded as *little or no academic thinking required*, *memorization*, *procedures without connections*, or *other*) these questions were not completed. If the task was determined to be at a high level during the set up but declined to a low level during the implementation stage, the coder(s) selected all the factors present during the lesson from the list of factors associated with the decline of cognitive demands as shown in Appendix C. Likewise, if the teacher set up that task at a high level and it remained at a high level throughout implementation, the coder(s) determined which factors were present during the lesson from the list of factors associated with the maintenance of high-level, cognitively demanding tasks (see Appendix C). For each question in part B of the COI, the coder(s) also provided justification for the selected answers.

3.4.1.2 Look Fors Sheet

The Look Fors Sheet (Appendix I) is a tool designed to be completed either during a classroom observation or when reviewing classroom instruction (e.g., via video or a detailed account of a lesson) as a means from assessing teachers' use of the five practices. For the purpose of this study, it was used to code the classroom observation write-ups contained in part A of the COI. Dr. Travis, the researcher, and another member of the Lesson Planning Project team co-designed the Look Fors Sheet. It has undergone several rounds of pilot testing and revisions, both in use during live classroom observations as well as being applied to write-ups of observations. After each use, the designers reevaluated the instrument and altered the design to improve its ability to accurately capture the desired aspects of instruction and assess teachers' application of the five practices.

The Look Fors Sheet consists of nine sections. The first section is a cover sheet similar to the cover sheet on the COI as it provides background information on the lesson. The coder(s) used this information to identify which lesson was being coded and where to obtain artifacts connected to the lesson. The remaining sections focus the coder's attention on the learning goals of the lesson; the cognitive demand of the task in written form, as set up by the teacher, and as implemented by the teacher and the students; or each of the five practices (anticipating, monitoring, selecting, sequencing, and connecting). In each section the coder(s) determined whether or not the teacher used specific instructional moves related to the five practices (e.g., for monitoring, "Teacher keeps a record of what students do and say as they work on the task"), identified how the teacher applies the five practices (e.g., for selecting, "Teacher appears to **randomly** select students to share their approaches to the task"), and provided evidence to justify the selected codes from the classroom observation write-up.

After completing the Look Fors Sheet for each observation, the researcher completed a Five Practices Summary Sheet (see Appendix J) based on the information on the Look Fors Sheet. He used the Five Practice Summary Sheet to indicate the extent to which the teacher used the five practices in the lesson. The researcher selected the level of use of each of the five practices using the following scale (*N = No use of the practice*, *L = Little use of the practice*, *P = Partial use of the practice*, and *H = High use of the practice*) based on an individual rubric for each of the five practices (contained in the Five Practices Summary Sheet).

3.4.2 Reliability Coding and Interrater Reliability

Reliability coding, or check-coding, is considered a suitable means for establishing the reliability of qualitative data coding (Miles & Huberman, 1994). The researcher coded the classroom observation write-ups using part B of the COI, and the Look Fors Sheet. To ensure the researchers' coding of the data was reliable, a secondary coder coded a subset of the data (8 of 30 lessons or 26.7%). Miles and Huberman (1994) suggested that intercoder reliability be measured using the following formula:

$reliability = \dfrac{number\ of\ agreements}{total\ number\ of\ agreements\ +\ disagreements}$. This formula was used to establish the level of interrater reliability between the primary coder (the researcher) and the secondary coder.

3.4.2.1 Refined coding rules

The researcher randomly selected three observations for training purposes. After training the secondary coder with regard to the COI and the Look Fors sheet, the researcher and the secondary coder individually coded these three lessons. After which they discussed their coding and refined their understanding of the codes. During this time, the researcher and the secondary coder established some specific coding rules related to the COI and the Look Fors sheet. The following two rules pertained to the coding of the COI:

- When coding the level of cognitive demand of the task as it appeared in the curriculum material, if any portion of the task contained a feature of a level of cognitive demand that was higher than the other portions of the task, the task would be coded at the higher level of cognitive demand. For example, if the majority of task included features consistent with a procedures without connections task, but it contained a question that required students to make a conjecture and justify the conjecture (features of a procedures with connections task), the task would be scored as procedures with connections.
- When coding the level of cognitive demand of the task as implemented by the teacher and the students, the level of cognitive demand would be determined by that of the majority of the students during the majority of the lesson. For example, if during an 80-minute lesson for which the main instructional task occupied the entire class period, the majority of the students were disruptive and refused to engage in the task for 60 of the 80 minutes but then during the last 20 minutes most of the students participated in a whole-class discussion and shared their reasoning about the task, the implementation of the task would be coded as little or no academic thinking, as the majority of the students did not participate in the task for the majority of time they spent working on it.

The researcher and secondary coder also set the following two coding rules with regard to the Look Fors Sheet:

- The term “redirects” in the code *“Teacher redirects students as they work on the task when needed”* under monitoring can refer to any of the following: (a) steering students down a new solution path, (b) aiding students to begin working on the task, and (c) getting students to engage in the task if they are currently off task.
- The term “student approaches” as used in multiple codes under selecting, sequencing, and connecting refers to instances in which student thinking is revealed. An explanation of student thinking (how or why the student did something), not just a statement of what they did must be provided. Thus, students stating their answers to specific questions would not be considered “student approaches” unless they also include a description of the mathematical thinking behind it. For example, a teacher asking a student to show her graph of a function would not be considered a “student approach” unless the student, or another class member, was pressed to provide insight into why she created the graph in the manner in which she did.

3.4.2.2 Interrater reliability

After coding the lesson observations as part of the training, the researcher and the secondary coder then separately coded eight additional observations as a means of establishing the reliability of the researcher’s coding of the data. Their interrater reliability was determined separately for the COI and the Looks Fors Sheet as well as combined.

3.4.2.2.1 Interrater reliability with regard to the level of cognitive demand of the tasks using the COI. The interrater reliability score for coding of the level of cognitive demand of the task calculated using the coding on the COI was 63%. This was the case after the training and establishment of coding rules. This is much lower than the recommended level of 80-90% (Miles & Huberman, 1994). One possible explanation for the low interrater reliability score is the secondary coder's lack of knowledge regarding the non-traditional mathematical curriculum used by the middle school teachers, *Connected Mathematics 2* (Lappan et al., 2006b). The secondary coder was chosen due to her familiarity with the larger research project of which this study was a part as well as her familiarity with the level of cognitive demand coding scheme and the Look Fors Sheet coding instrument used to analyze teachers' use of the five practices. However, she was not familiar with *Connected Mathematics 2*, or middle school mathematics in general, as she was a former secondary science teacher. Looking at the interrater reliability of the individual observations, this pattern becomes more apparent. The observations that the secondary coder coded were selected randomly. Of the eight observations she coded, five were of one of the middle school teachers (Gloria Xavier or Nathan Ingram). Of these five, never did the researcher and secondary coder agree on all three codes for the level of cognitive demand (as the task appears in the curriculum, as it is set up by the teacher, and as it is implemented during the lesson). They did not agree on any of three codes for one of the observations, they agree on one code for two of the observations, and they agreed on two of the three codes for two of the observations. In contrast, for each of the three observations of the high school teachers (Cara Nance and Nicole Nesmith), the researcher and the secondary coder agreed on all three of the codes for the level of cognitive demand. Because of the small number of codes for the level of cognitive demand for each observation (only three), disagreement on just one of the codes affected the overall reliability score greatly.

3.4.2.2.2 Interrater reliability with regard to the teachers' use of the five practices using the Look Fors Sheet and combined interrater reliability. The interrater reliability score for coding of The Look Fors Sheet for the eight observations was 78%. The overall interrater reliability score combining the coding of the level of cognitive demand of the task and The Look Fors Sheet for the eight observations was 75%.

3.4.3 Analysis of the Data from the Professional Development Meetings

The Professional Development Preparation Rubric is an analytic tool that was used to examine the data taken from the professional development meetings. This tool along with the process used to analyze this data will be addressed in this section.

3.4.3.1 Professional Development Preparation Rubric

The Professional Development Preparation Rubric (Appendix K) is a tool designed to measure teachers' preparation for professional development meetings. This tool does not measure their participation directly, rather it uses preparation as a proxy indicator for participation in the meetings based on the assumption that teachers' ability to participate is impacted by their level of preparation (e.g., well-prepared teachers will be able to participate in professional development at a higher level than will poorly-prepared teachers).

The tool measures teachers' preparation by examining the work on the focus tasks they created before the week 1 meetings of the modified lesson study cycles as well as the noticings and wonderings the teachers generated while producing this work. The work on the focus task the teachers create was scored as *no preparation*, *low preparation*, *medium preparation,* and *high preparation* based on the number and type of solution methods the teachers produce.

Teachers' noticings and wonderings were scored using the same categories, but based on the number and content of their noticings and wonderings. Thus, for the week 1 meeting in each modified lesson study cycle, teachers received two levels of preparation scores, one for the work on the focus task and one for their noticings and wonderings.

3.4.3.2 Coding the data from the professional development meetings

In order to examine the data from the professional development meetings, the two professional development meetings in each of the modified lesson study cycles for which one of the participating teachers selected and then reflected on a focus task were transcribed. The researcher coded the data from these transcripts to investigate the relationship between the professional development and changes in teachers' instructional practices. This amounts to 14 meetings in total (Cara Nance, Nicole Nesmith, and Nathan Ingram each chose focus tasks for two cycles, and Gloria selected a focus task for two cycles, but was only able to teach the task and complete the cycle with one of these tasks). The transcripts of these meetings were use to identify the key ideas and suggestions discussed during each meeting. A *key idea or suggestion* is defined as any idea or suggestion that is initiated by any group member and that is then taken up for discussion by other group members, one of which must be the teacher who selected the focus task. For example, suppose that during a meeting for which Gloria Xavier selected a task, Dr. Travis offers a suggestion (Suggestion A). Other group members briefly discuss Suggestion A, but Gloria never joins the conversation about it. In this case, Suggestion A would not be identified as a key suggestion. However, now suppose that during the same meeting Gloria wonders aloud about an idea (Idea B). Some of the other group members then respond to Gloria's wondering regarding Idea B. In this case, Idea B would be considered a key idea. Each of the professional development meetings in the modified lesson study cycles for which a participating teacher selected a focus task was examined for key ideas and suggestions.

Two important analysis rules were developed based on this definition of a key idea prior to coding the data. First, no ideas presented in the team members' noticings and wonderings material that were not discussed in the meeting were considered key ideas. Second, no ideas presented by any group members that were not followed up on by the teacher who had selected the focus task were identified as key ideas. A third coding rule was developed as the researcher began to examine the data. If an idea was shared by a team member and followed up on by the teacher who selected the focus task, but after the teacher then stated that he or she already planned on incorporating it in his or her instruction, the idea was not counted as a key idea. This rule was developed as these ideas do not seem to provide evidence of the impact of the professional development meetings on teachers' instruction. Teachers' subsequent instruction of the task as well as the transcripts and artifacts of the week 2 meeting of the modified lesson study cycle were searched for evidence of the teachers' uptake of the key ideas.

3.5 DATA ANALYSIS

Various qualitative analyses of the data from the classroom observations— which were coded using part B of the COI and the Look Fors Sheet— as well as the data from specific professional development meetings were used to address the research questions. This section will explain the analyses that were used to address each research question. Table 3.5 shows the data, coding tools or methods, and analyses used to answer each of the research questions.

To aid with the analysis of the data, the classroom observations were grouped into three time frames. Time frame 1 measured baseline data, time frame 2 measured intervention data, and time frame 3 measured maintenance data. This grouping was determined separately for each teacher (see Tables 3.6, 3.7, 3.8, and 3.9). Time frame 1 (the baseline measurement) includes all observed lessons before the teacher's first opportunity to select the focus task for his or her professional development meetings. In each case, this consists of the first two observations. Time frame 2 (the intervention measurement) contains the observations associated with the first and last modified lesson study cycles for which each teacher selected a focus task for the professional development meetings. It also includes all observations of both focus and non-focus tasks that occurred between these observations. Time frame 2 includes Observations 3, 4, and 5 for Cara Nance; Observations 3, 4, and 5 for Nicole Nesmith; Observations 3, 4, 5, 6, and 7 for Gloria Xavier; and Observation 3, 4, and 5 for Nathan Ingram.[6] Time frame 3 (the maintenance measurement) includes all observations after the final observation associated with a focus task from the professional development meetings. Time frame 3 consists of Observations 6, 7, and 8 for Cara Nance; Observations 6 and 7 for Nicole Nesmith; Observation 8 for Gloria Xavier; and Observations 6 and 7 for Nathan Ingram.

[6] Only one of Gloria Xavier's observations included the use of a focus task (Observation 7) due to scheduling issues. However, she was given the opportunity to select a focus task for the professional development meetings prior to Observation 3, thus Observations 3, 4, 5, 6, and 7 are all included in time frame 2 for her.

Table 3.5: Data, coding tools or methods, and analyses used to answer the research questions

<table>
<tr><th rowspan="2">Research question</th><th colspan="2">Data sources</th><th rowspan="2">Coding tools or methods</th><th rowspan="2">Analysis</th></tr>
<tr><th>Professional development</th><th>Classroom observations</th></tr>
<tr><td>I</td><td>– Meeting artifacts
○ Work on focus tasks
○ Noticings and wonderings</td><td></td><td>– Professional Development Preparation Rubric</td><td>– Rate teachers' work on the focus tasks and their noticings and wonderings from the week 1 meeting of each MLS cycle to determine their level of participation in the meetings</td></tr>
<tr><td>II</td><td></td><td>– Lesson plans
– Task</td><td>– Part B of COI</td><td>– Compare the level of cognitive demand of tasks selected by each teacher across time frames</td></tr>
<tr><td>III</td><td></td><td>– Classroom observation write-up (part A of COI)
– Lesson plans
– Task
– Additional lesson artifacts</td><td>– Part B of COI</td><td>– Compare the number of tasks set up at a high level and implemented at a low level by teacher across time frames
– Compare the number of tasks set up at a high level and implemented at a high level by teacher across time frames</td></tr>
<tr><td>IV</td><td></td><td>– Classroom observation write-up (part A of COI)
– Lesson plans
– Task
– Additional lesson artifacts</td><td>– Look Fors Sheet
– Five Practices Summary Sheet</td><td>– Compare the level of use of each of the five practices by each teacher across time frames</td></tr>
</table>

Table 3.5 (cont.): Data, coding tools or methods, and analyses used to answer each research question

Research question	Data sources: Professional development	Data sources: Classroom observations	Coding tools or methods	Analysis
V		– Classroom observation write-up (part A of COI) – Lesson plans – Task – Additional lesson artifacts	– Look Fors Sheet – Five Practices Summary Sheet	– For each teacher, compare the level of use of each of the five practices with the level of cognitive demand of tasks during implementation for each task set up at a high level – For each teacher, compute the total number of occurrences of each possible combination of level of use of a particular practice and the level of cognitive demand of the task during implementation (e.g., 4 tasks were implemented as procedures without connections during which the teacher made little use of monitoring)
VI	– Audio recordings – Meeting summaries – Meeting transcripts – Meeting artifacts	– Classroom observation write-up (part A of COI) – Lesson plans – Task – Additional lesson artifacts	– Identification of key ideas or suggestions	– For each teacher, examine the teacher's instruction of focus tasks for evidence of key ideas and suggestions identified in the professional development meetings

Table 3.6: Time frames of classroom observations for Cara Nance

Time frame 1		Time frame 2			Time frame 3		
Obs. 1 10/25/2011	Obs. 2 11/14/2011	Obs. 3 1/9/2012	Obs. 4 2/28/2012	Obs. 5 3/19/2012	Obs. 6 4/18/2012	Obs. 7 5/8/2012	Obs. 8 5/30/2012

Note: The grey regions indicate lessons that used focus tasks from the professional development cycles. The white regions indicate lessons that used non-focus tasks.

Table 3.7: Time frames of classroom observations for Nicole Nesmith

Time frame 1		Time frame 2			Time frame 3	
Obs. 1 10/25/2011	Obs. 2 12/1/2011	Obs. 3 1/24/2012	Obs. 4 2/27/2012	Obs. 5 04/16/12	Obs. 6 5/3/2012	Obs. 7 5/29/2012

Note: The grey regions indicate lessons that used focus tasks from the professional development cycles. The white regions indicate lessons that used non-focus tasks.

Table 3.8: Time frames of classroom observations for Gloria Xavier

Time frame 1		Time frame 2					Time frame 3
Obs. 1 10/27/2011	Obs. 2 12/8/2011	Obs. 3 2/24/2012	Obs. 4 3/23/2012	Obs. 5 4/20/2012	Obs. 6 4/27/2012	Obs. 7 5/11/2012	Obs. 8 5/24/2012

Note: The grey regions indicate lessons that used focus tasks from the professional development cycles. The white regions indicate lessons that used non-focus tasks.

Table 3.9: Time frames of classroom observations for Nathan Ingram

Time frame 1		Time frame 2			Time frame 3	
Obs. 1 10/27/2011	Obs. 2 12/13/2011	Obs. 3 1/5/2012	Obs. 4 3/1/2012	Obs. 5 3/12/2012	Obs. 6 4/30/2012	Obs. 7 5/22/2012

Note: The grey regions indicate lessons that used focus tasks from the professional development cycles. The white regions indicate lessons that used non-focus tasks.

3.5.1 Research Question I

To address Research Question I (To what extent do teachers participate in the professional development focused on selecting and implementing high-level tasks?) the researcher examined the data collected from the professional development meetings. Specifically, the researcher used the Professional Development Preparation Rubric to measure teachers' level of engagement in anticipating solutions to the focus task and in recording their noticings and wonderings prior to the week 1 meetings. A detailed analysis of the discourse of each meeting would aid in providing a more complex description of each of the teacher's level of participation in the professional development, however such an analysis is beyond the scope of this study. Rather, teachers' preparation for each modified lesson study cycle, as seen in their anticipated work on the focus task and the noticings and wonderings they recorded, was used as indicators of their level of engagement in the professional development activities. While this method of measuring teachers' participation does not capture the number or depth of teachers' contributions during the professional development activities, it does indicate the extent to which the teachers were *able* to participate based on their preparation for these meetings.

3.5.2 Research Question II

In order to answer Research Question II (To what extent does teachers' use of high-level tasks change over the course of their participation in professional development focused on selecting and implementing high-level tasks?), the researcher compared the level of cognitive demand of teacher-selected tasks during the classroom observations (as coded in part B of the COI) across the three time frames. This comparison between time frames was done within a single teacher's instruction. For example, the level of cognitive demand of the two tasks selected by Cara Nance during time frame 1 was compared to the level of cognitive demand of the three tasks in time frame 2 and the level of cognitive demand of the three tasks in time frame 3. This procedure was repeated for each teacher.

3.5.3 Research Question III

Research Question III (To what extent does teachers' ability to maintain the cognitive demand of high-level tasks in the set up and implementation stages of instruction change over the course of their participation in professional development focused on selecting and implementing high-level tasks?) was addressed in a similar manner as Research Question II. In order to investigate change over time, the researcher used the time frames in Tables 3.6, 3.7, 3.8, and 3.9 to group the observations. Teachers' ability to maintain the high level of cognitive demand of tasks from set up to implementation was examined within individual teachers. All observations that included tasks coded on part B of the COI as high level at the set up stage of instruction were investigated for this research question. This analysis determined the number of tasks each individual teacher set up at a high level and then implemented at a low level, as well as those tasks set up at a high level and maintained at a high level during implementation in each of the three time frames.

3.5.4 Research Question IV

Research Question IV (To what extent does teachers' use of the practices of *anticipating*, *monitoring*, *selecting*, *sequencing*, and *connecting* in their instruction change over the course of their participation in professional development focused on selecting and implementing high-level tasks?) was examined using the coded data from the Look Fors Sheet and the Five Practices

Summary Sheet. As with the research questions II and III, this research question investigates changes in teachers' practices over time. To study these changes the researcher used the time frames designated in Tables 3.6, 3.7, 3.8, and 3.9 to compare individual teacher's use of the of the five practices in each of these time frames. To do this, data from Five Practices Summary Sheets for each observation were examined for notable differences in each teacher's use of the five practices between the three time frames.

3.5.5 Research Question V

Data from the COI, the Look Fors Sheet and the Five Practices Summary Sheet were used to answer Research Question V (What relationship, if any, is there between teachers' use of the practices of *anticipating*, *monitoring*, *selecting*, *sequencing*, and *connecting* and their ability to maintain the level of cognitive demand of high-level tasks?). As this question deals with teachers' ability to maintain the cognitive demand of high-level tasks, only observations during which teachers were determined to have set up tasks at a high level as coded in part B of the COI were used. Once these observations were identified, data from part B of the COI and the Five Practices Summary Sheet for each observation were compiled into a matrix (see Table 3.10) comparing the teacher's use of the five practices and the level of cognitive demand of the task during implementation for that observation. These matrices were used to identify patterns in each teacher's use of the five practices and the level of cognitive demand of the task during implementation. A master matrix for each teacher was created using the information from the individual matrices to show the amount of observations that fall into each category (see Table 3.11).

Table 3.10: Matrix used to track an individual teacher's level of use of five practices as related to the level of cognitive demand of implementation of tasks set up at a high level during a single classroom observation

	Level of cognitive demand during implementation																							
	Low-level cognitive demands																High-level cognitive demands							
	Unsystematic and nonproductive exploration				Little or no academic thinking occurred				Memorization				Procedures without connections				Procedures with connections				Doing mathematics			
Level of use of practice	N	L	P	H	N	L	P	H	N	L	P	H	N	L	P	H	N	L	P	H	N	L	P	H
Five Practices																								
Anticipating																				X				
Monitoring																			X					
Selecting																			X					
Sequencing																			X					
Connecting																		X						

Note: N = no use, L = little use, P = partial use, and H = high use. The data displayed in this table is hypothetical and is meant only as an example of what this matrix may look like after the analysis occurs.

Table 3.11: Matrix used to track an individual teacher's level of use of five practices as related to the level of cognitive demand of implementation of tasks set up at a high level across all classroom observations

	Level of cognitive demand during implementation																							
	Low-level cognitive demands																High-level cognitive demands							
	Unsystematic and nonproductive exploration				Little or no academic thinking occurred				Memorization				Procedures without connections				Procedures with connections				Doing mathematics			
Level of use of practice	N	L	P	H	N	L	P	H	N	L	P	H	N	L	P	H	N	L	P	H	N	L	P	H
Five Practices																								
Anticipating	1													1					1	1			1	1
Monitoring	1															1		1	1				1	1
Selecting		1													1			1	1				1	1
Sequencing	1														1			1	1					2
Connecting	1													1				2					2	

Note: N = no use, L = low use, P = partial use, and H = high use. The data displayed in this table is hypothetical and is meant only as an example of what this matrix may look like after the analysis occurs.

3.5.6 Research Question VI

In order to answer Research Question VI (To what extent do teachers incorporate the ideas and suggestions made about their focus tasks during the professional development meetings in their implementation of the focus task?), the researcher focused on the professional development meetings that centered on the focus tasks chosen by the four teachers participating in this study. Specifically, the researcher used the key ideas and suggestions identified during the coding of the transcripts of the professional development meetings and the classroom observation write-ups found in part A of the COI.

To investigate Research Question VI, the researcher examined each individual teacher's uptake of the key ideas and suggestions made during the professional development meetings. The researcher used the list of identified key ideas and suggestions from the MLSC week 1 meetings for each teacher-selected focus task. He looked for evidence from the classroom observation write-ups and corresponding artifacts of the teacher's implementation of the focus task as well as the transcript and artifacts from the MLSC week 2 meetings during which the teacher reflected on his or her instruction. The researcher then recorded the key ideas and suggestions the teacher made use of during the lesson.

3.5.7 Exploring the Relationship Between Teachers' Participation in Professional Development and Their Instructional Practices

The results of the analyses used to address the six research questions are presented in the form of narrative cases. These cases highlight the teachers, including their preparation for the professional development meetings as seen as seen in the work on the focus tasks and the noticings and wonderings. They also include detailed descriptions of teachers' instruction and provide examples to accompany the results of the analyses. Finally, these narrative cases provide examples of teachers' uptake of key ideas from the MLSCs week 1 meetings in their instruction of the focus tasks. The purpose of these narrative cases is to provide a robust portrait of teachers' participation in the professional development and their instructional practices used to implement high-level tasks, as well as to explore the relationship between teachers' participation in the professional development and their use of the five practices.

3.6 SUMMARY

This dissertation study took place in the context of a larger research study that investigated schoolwide reform aimed at enhancing teachers' instructional practices and producing organizational change. The purpose of this dissertation study was to investigate the impact the professional development, provided as part of the larger project, had on mathematics teachers' use of specific instructional skills that serve as the focus of the professional development. Specifically, teachers' ability to select and implement high-level, cognitively demanding mathematical tasks and facilitate classroom discussions around those tasks was analyzed. The professional development focused on the five practices, instructional practices geared toward preparing teachers to be better able to conduct whole-class discussions.

Four of the teachers participating in the professional development were selected as a convenience sample to participate in this dissertation study. These teachers agreed to allow the researcher to observe their classes and collect classroom artifacts throughout the school year during which they participated in the professional development. The researcher used two analytic tools to examine the teacher's instruction. The Classroom Observation Instrument (COI) focuses the researchers' attention on the cognitive demands of the tasks used by the teacher at three stages of instruction: as presented in written from, as set up by the teacher, and as implemented by the teacher and his or her students. The Look Fors Sheet analyzes the teachers' use of the five practices during his or her instruction. In order to draw connections between the professional development and the teachers' instruction, audio recordings, transcripts, and artifacts from the professional development meetings were used to determine specific ideas and strategies that were the focus of the meetings. Detailed write-ups of the observations of teachers' instruction were then examined to conclude to what extent these ideas and strategies were manifest in their instruction. Chapter Four will present the results of these analyses.

4.0 CHAPTER 4: RESULTS

This chapter reports the results of the data analyses described in Chapter Three used to answer the six research questions of the study. It is organized into four sections that present the results of the analyses for each of the four teachers in this study in the form of narrative cases. These cases provide descriptive details of the teachers' participation in the MLSC meetings and their instruction as examples supporting the reported results. Section 4.1 is the narrative case of Cara Nance, section 4.2 relates to Nicole Nesmith, section 4.3 is of Gloria Xavier, and section 4.4 is the case of Nathan Ingram. The chapter concludes with section 4.5, which is a cross-case comparison of the four teachers.

Recall the six research question of this study are:

I. To what extent do teachers participate in the professional development focused on selecting and implementing high-level tasks?

II. To what extent does teachers' use of high-level tasks change over the course of their participation in professional development focused on selecting and implementing high-level tasks?

III. To what extent does teachers' ability to maintain the cognitive demand of high-level tasks in the set up and implementation stages of instruction change over the course of their participation in professional development focused on selecting and implementing high-level tasks?

IV. To what extent does teachers' use of the practices of *anticipating*, *monitoring*, *selecting*, *sequencing*, and *connecting* in their instruction change over the course of their participation in professional development focused on selecting and implementing high-level tasks?

V. What relationship, if any, is there between teachers' use of the practices of *anticipating*, *monitoring*, *selecting*, *sequencing*, and *connecting* and their ability to maintain the level of cognitive demand of high-level tasks?

VI. To what extent do teachers incorporate the ideas and suggestions made about their focus tasks during the professional development meetings in their implementation of the focus task?

4.1 CARA NANCE

This section presents the results of the six research questions as related to Cara Nance. It does so by describing Cara Nance's participation in the professional development meetings as well as exploring her instruction during the observed lessons. It also provides examples illustrating these findings.

4.1.1 Participation in the Professional Development Meetings (Research Question I)

Cara Nance was a member of the 11th- and 12th-grade professional development team. This team met 27 times throughout the school year. Ms. Nance attended each of these meetings and participated as a positive and active member. During a subset of these meetings (December 2011 - May 2012), the team engaged in nine two-week modified lesson study cycles (MLSC)—two of which were centered on focus tasks Ms. Nance selected. Ms. Nance produced work on the focus tasks for all but one of the nine cycles. The cycle for which she did not produce work was the first cycle for which she selected the focus task. She also created noticings and wonderings for seven of the nine cycles; the two cycles she did not create them for were the two for which she selected the focus task. Thus, she provided feedback on all of the tasks selected by her colleagues. Table 4.1 displays how Ms. Nance's work on the focus task and the noticings and wonderings she created for each cycle were scored using the Professional Development Preparation Rubric. The shaded rows designate the MLSC week 1 meetings for which Cara Nance selected the focus tasks.

Table 4.1: Cara Nance's level of preparation for the week 1 meetings of the professional development cycles

Cycle number	Meeting date	Level of preparation	
		Work on the focus task	Noticings and wonderings
1	Dec. 14, 2011	Medium	High
2	Jan. 4, 2012	No	No
3	Jan. 18, 2012	Low	High
4	Feb. 8, 2012	Low	High
5	Feb. 22, 2012	High	High
6	Mar. 7, 2012	Low	No
7	Apr. 11, 2012	Medium	High
8	Apr. 25, 2012	Low	Medium
9	May 9, 2012	Medium	Medium

Note: The shaded rows designate the professional development cycles for which Cara Nance selected the focus task.

As can be seen from Table 4.1, the scores of Ms. Nance's work on the focus tasks for the professional development meetings ranged from low to high. Specifically, the work for four of the eight meetings for which she produced work was coded as low, the work for three of the meetings was coded as medium, and the work for the remaining meeting was coded as high. Ms.

Nance consistently produced noticings and wonderings for the meetings that were scored as either medium or high. The noticings and wonderings for five of the seven meetings for which she created noticings and wonderings were scored as high, and the noticings and wonderings for the remaining two meetings were scored as medium.

4.1.1.1 Work on the focus tasks for the MLSC week 1 meetings

Ms. Nance's work on the *Single Star or Galaxy* task (Appendix E) that Nicole Nesmith selected as the focus task of the MLSC week 1 meeting on January 18 is typical of the work she produced that was scored as low-preparation. For this task, Ms. Nance only identified one possible method students might use when working on the task even though the task itself explicitly states that there are several possible ways for doing so. She identifies the common formula for finding the *x*-value of a parabola ($x = \frac{-b}{2a}$), but she does not consider alternative methods (e.g., using the line of symmetry of the parabola or creating a table). She also does not consider possible errors students may commit or misconceptions they may have when working on the task.

Cara Nance's work on the *Single Star or Galaxy* task is typical of her work on focus tasks that was scored as low level. This means that the work on these tasks included only one anticipated solution method. However, the *Single Star or Galaxy* task itself is not typical of the focus tasks for which Ms. Nance's work was scored low level. In the other three cases, the nature of the tasks may have played a role in her work being scored low level. An important element in scoring the level of preparation for teachers' work on the focus task using the Professional Development Preparation Rubric is that it uses the number and type of anticipated solutions (e.g., correct versus incorrect) as factors in differentiating between low-, medium-, and high-preparation. Examining the focus tasks for the other three meetings in which Cara Nance's work was scored as low (the meetings on February 8, March 7, and April 25) reveals that in each case the nature of the tasks did not allow for multiple solutions strategies (e.g., the task designates a specific procedure students are to use when working on it, or the topic of the task is such that only one strategy is logical to use).

The work Cara Nance created on three of the eight focus tasks for the MLSC was scored as medium level. This work was coded as such because she anticipated multiple, correct solution methods for the task. However, in none of these instances did she anticipate possible incorrect methods for solving the task. Her work on the *Lucky Day* task (Appendix L) is an example of this. For this task, Ms. Nance anticipated two possible equations for representing one of the two contexts presented in the task. To represent the total amount of money after a given amount of days using Option 1 (a linear relationship) she suggested that Option 1 could be represented as equation (a) $y = 1000n + 9000$ or as equation (b) $y = 10000 + 1000(n - 1)$. While algebraically these equations are equivalent, in the context of the problem they represent two distinct ways of thinking. Both of these solution paths were viable possibilities for how students might engage in the task, and both seemed likely to occur during the lesson.

4.1.1.2 Noticings and wonderings related to the focus tasks for the MLSC week 1 meetings

In order to be scored at a high level, a teacher's noticings and wonderings needed to include three "noticing and wondering pairs" (i.e., one noticing connected with one or more wondering), and the noticings and wonderings need to touch on two or more of the following areas: (a) specific elements of the task; (b) instructional or pedagogical issues related to the teaching of the task; (c) the learning goals associated with the task (either those identified by the teacher who selected the task or potential learning goals); or (d) mathematical content related to the task.

Table 4.2 shows the noticings and wonderings Ms. Nance produced related to the *Single Star or Galaxy?* task (Appendix E) selected for a MLSC by Nicole Nesmith. These noticings and wonderings touch on at least three of the areas listed in the rubric on the Professional Development Preparation Rubric used to rate the noticings and wonderings. Pair 1, pair 2, and pair 4 focus on specific problems of the tasks, while pair 5 lists the general vertex form of quadratic functions and brings up the question of what is the goal of the task. In both of the cases for which Ms. Nance's noticings and wonderings were coded as medium-level, the noticings and wonderings were of similar nature, however there were only two noticings and wonderings pairs provided.

Table 4.2: The noticings and wonderings Cara Nance produced related to the *Single Star or Galaxy* task for the modified lesson study cycle week 1 meeting on January 18

N&W Pair	Noticings	Wonderings
1	"Part 1 - States to write equation provided only *x*-intercepts!! (This may be difficult)"	"How have you taught/ students learned how to write quadratic functions provided *x*-intercepts/roots?"
2	"I noticed Part 2 states there are lots of ways to find the vertex."	"How have they done so?"
		"Are you expecting students to use calculators to find vertex & roots?
		"Do they know the formula?"
3	"Are they required to save graphs?? Compare all?"	"I wonder how much have they worked with quadratics?"
4	"I notice Part 6 may cause misconceptions, but a great discussion!"	"Have students written quadratics in factored form?"
5	"Vertex $y = (x - h)^2 + k$?"	"What is main goal of task?"

To summarize, Cara Nance attended every team meeting and participated actively in these meetings. Her level of preparation with regard to the work she created for the meetings varied from low to high, with the work for four of the eight meetings for which she produced it coded as medium or high. She also prepared noticings and wonderings that showed she thought meaningfully about the task prior to the meeting as the noticings and wonderings of five of the seven meetings for which she produced them coded as high level and the other two coded as medium level.

4.1.2 The Level of Cognitive Demand of Tasks Used During Observed Lessons (Research Question II)

Table 4.3 displays the level of cognitive demand of the tasks as selected (in written form), set up, and implemented by Cara Nance in the eight observed lessons. For six of the eight lessons, Ms. Nance selected high-level tasks, all of which were procedures with connections tasks. In the other two lessons, she selected procedures without connections tasks. Both of the tasks Ms. Nance chose during the baseline measurement (time frame 1) were high-level tasks, two of the three tasks she used during the intervention measurement (time frame 2) were high-level tasks, and two of the three tasks in the maintenance measurement (time frame 3) were also high level. Thus, there does not appear to be a pattern with regard to changes in the level of cognitive demand of the tasks Ms. Nance selected across the three time frames.

Table 4.3: The level of cognitive demand of the tasks as selected, set up, and implemented by Cara Nance during her lesson observations

	Observation number and date	Level of cognitive demand of the task as Selected (in written form)	Set up	Implemented
Time frame 1	Obs. 1 10/25/2011	**PWC**	**PWC**	**PWC**
	Obs. 2 11/14/2011	**PWC**	**PWC**	PWoC
Time frame 2	Obs. 3 1/9/2012	**PWC**	**PWC**	**PWC**
	Obs. 4 2/28/2012	PWoC	PWoC	PWoC
	Obs. 5 3/19/2012	**PWC**	**PWC**	PWoC
Time Frame 3	Obs. 6 4/18/2012	**PWC**	**PWC**	PWoC
	Obs. 7 5/8/2012	**PWC**	PWoC	PWoC
	Obs. 8 5/30/2012	PWoC	PWoC	PWoC

LN = Little or no academic thinking required by the task/ occurred during the lesson
Mem = Memorization task
PWoC = Procedures without connections task
PWC = Procedures with connections task
DM = Doing mathematics task
Unsys = Unsystematic and nonproductive exploration
O = Other

Note: High-level tasks (e.g., procedures with connections task and doing mathematics tasks) are bolded in the table. The shaded rows designate the lessons that involve focus tasks from the MLSCs.

The sections that follow provide descriptions of two of tasks as they appeared in written form, which Cara Nance selected for her observed lessons. These tasks were chosen because they contain many of the characteristics that are common of the other tasks selected by Ms. Nance.

4.1.2.1 A high-level task

Cara Nance's lesson on January 9 (Observation 3) was centered on the *Modeling with Logistic Functions* task. Ms. Nance selected this task for the first MLSC focused on her instruction. She modified the task after the MLSC week 1 meeting based on feedback and suggestions made during the MLSC week 1 meeting (some of these modifications are discussed with regard to Research Question VI in section 4.2.5.1 below). Appendix M contains the modified version of this task. It requires students to explore two real-world situations that can be modeled using logistic functions, the spread of a rumor and the spread of a cold virus, both among a limited population of people. Throughout the task, students are asked to discuss the mathematical elements of the functions that represent the situations and to make distinctions between these elements in the real word and as dictated by the function representing the situation. For example, after graphing the function representing the spread of the rumor students are asked to give the domain and range of the function. They are then asked whether these are the same as the domain and range of the problem situation. This pushes students to consider the fact that mathematically the graph of the function representing the spread of the rumor has horizontal asymptotes at $y = 0$ and $y = 800$, thus bounding the range of the function between 0 and 800, without including either. Yet, mathematically the domain is unrestricted. However, students must reason about what the domain and range represent in the context of the problem. In this case, the domain represents the number of days since the rumor was started. Thus, negative x-values are not logical, as you would not consider the spread of the rumor before the time it was started. Furthermore, mathematically the y-value representing the number of students cannot reach 800, yet students must consider within the context of the problem whether it is logical to believe that every member of the population but one would have heard the rumor regardless of the number of days since it began. In these questions and in others throughout the task, students are asked to reason about the mathematics and the context of the problem and then provide an explanation for their reasoning. Thus, this task was coded as high-level (procedures with connections) as it appeared in written form.

4.1.2.2 A low-level task

Cara Nance used the *Law of Cosines* task (Appendix N) during her lesson on May 30 (Observation 8). This task introduces students to the law of cosines by presenting a large triangle divided into two smaller triangles by the altitude of the triangle. A step-by-step process is then provided on the first two pages of the task resulting in the law of cosines. Practice problems are then presented that require students to follow the precise process that they just completed. It does not contain any questions that require students to conjecture, reason, or demonstrate understanding or connections between the formula developed during the beginning of the task and the underlying mathematical concepts. The final problem is set in a real-world context of a baseball game. However, it only requires students to manipulate the procedure learned in the beginning of the task, with no conceptual connections or cognitive challenge. As such, this task was rated as low level (procedures without connections) as it appeared in written form.

4.1.3 Implementation of High-Level Tasks and Use of the Five Practices During Instruction (Research Question III and Research Question IV)

4.1.3.1 Set up and implementation of high-level tasks

As shown in Table 4.3, the level of cognitive demand of each task as set up by Cara Nance matched the level cognitive demand of the task as it appeared in the curriculum materials for all but one of the tasks. In this case (May 8 - Observation 7), the level of cognitive demand decreased from procedures with connections to procedures without connections as Ms. Nance set up the task. Thus, five of the eight tasks were set up at a high level, all of which were set up as procedures with connections tasks. Of these five tasks, only two were implemented at a high level, both of which were coded as procedures with connections tasks during implementation. The remaining three tasks that were set up at a high level decreased to low-level tasks during implementation as the level of cognitive demand of all three was scored as procedures without connections. There appears to be no change in Ms. Nance's ability to maintain high-level tasks at a high-level during implementation between the three time frames, as the level of cognitive demand of tasks set up at a high level was maintained for only one task in each of the first two time frames and for none of the tasks in the third time frame.

Table 4.4 shows the factors associated with the maintenance and decline of high-level tasks during implementation present in Cara Nance's instruction. During the two lessons in which the level of cognitive demand was maintained, Ms. Nance pressed students for explanations and reasoning. She failed to do this during two of the three lessons in which the cognitive demand declined. Instead, she allowed the focus of the activities to shift to the correctness of the answers. During the other lesson in which the cognitive demand was not maintained (April 18 - Observation 6), Ms. Nance failed to provide the students with sufficient time to engage in the demanding aspects of the task. Specifically, the students spent the entire period working with manipulatives to create graphs, but they were not given the opportunity to express reasoning or connections to the underlying concepts. In this case, it is unclear as to whether students were provided with time the following day to engage in the cognitively challenging portion of the task as the researcher was unable to obtain this data.

Table 4.4: The factors associated with decline or maintenance of the cognitive demand of tasks set up by Cara Nance at a high level

Observation number and date	Level of cognitive demand as set up/implemented	Factors associated with decline present in the lesson	Factors associated with maintenance present in the lesson
Obs. 1 10/25/2011	H/H	n/a	– Teacher or capable students model high-level performance – Teacher presses for justifications, explanations, and meaning through questioning, comments, and feedback – Task builds on students' prior knowledge
Obs. 2 11/14/2011	H/L	– The teacher shifts the emphasis form meaning, concepts, or understanding to the correctness or completeness of the answer (there was a lack of press for reasoning)	n/a
Obs. 3 1/9/2012	H/H	n/a	– Teacher presses for justifications, explanations, and meaning through questioning, comments, and feedback
Obs. 5 3/19/2012	H/L	– Problematic aspects of the task become routinized – The teacher shifts the emphasis from meaning, concepts, or understanding to the correctness or completeness of the answer (there was a lack of press for reasoning)	n/a
Obs. 6 4/18/2012	H/L	– Not enough time is provided to wrestle with the demanding aspects of the task	n/a

On the surface it appears the factors related to the decline of the level of cognitive demand varied in the three lessons that involved tasks set up at a high level but implemented at a low level. However, another factor may have existed as a common underlying cause. Cara Nance's lessons often seemed to move at a fast, at times almost frenetic, pace. As such, there was often little press for reasoning, and in the case of the lesson on April 18 (Observation 6), no discussion of the task. During many of her lessons, it appeared that students' behavior was close to becoming a problem. In most cases, Ms. Nance was able to deal with the possible behavioral issues before they boiled over, but in at least one lesson, this was not the case. During the lesson on May 8 (Observation 7) multiple behavioral issues occurred. These included a student pounding on the classroom door (it was locked after the beginning of the class due to school policy) and screaming obscenities for about 5 minutes at the beginning of class; students teasing a student to the point that he bites a pencil so hard that it breaks, then that student yelling at his classmates and later, punching objects in the hall; and students running from one side of the room to the other after someone passed gas. Ms. Nance may have recognized the potential for classroom management and behavioral issues during her lessons and as such felt that it could be problematic to slow down and push for reasoning as part of in-depth discussions. This seems to be contrary to the factor for the decline in the lesson on April 18 as she gave the students too much time to work on the task and insufficient time for a whole-class discussion. However, this lesson involved a hands-on task in which students were using manipulatives and this was engaging for the students. Thus, minimizing some of the opportunities for behavior issues.

4.1.3.2 Use of the five practices

Table 4.5 displays Cara Nance's level of use of the five practices over the eight lessons she was observed teaching. There was no evidence of anticipating in Ms. Nance's lesson plans for any of the eight lessons. However, Ms. Nance's instruction did include the use of monitoring during every lesson. A deeper look into the scoring for monitoring on The Five Practices Summary Sheet reveals further information regarding Ms. Nance's use of monitoring during her lessons. On The Five Practices Summary Sheet, monitoring is scored as "partial use" if the teacher observes students as they work on the task and either (a) uses a monitoring tool to record students' responses, or (b) asks students assessing and advancing questions, but not both. To score "little use" for monitoring, the teacher only observes the students work, but does not use a monitoring tool or assessing and advancing questions. During every lesson for which she received a score of "partial use" for monitoring, Ms. Nance asked students assessing and advancing questions while observing their work on the task. Thus, in none of Ms. Nance's observed lessons did she use a monitoring tool. Ms. Nance's level of use of selecting, sequencing, and connecting varied by lesson from "no use" to "partial use" for selecting, from "no use" to "little use" for sequencing, and from "no use" to "high use" for connecting.

Table 4.5: The level of use of the five practices by Cara Nance during her lesson observations

	Observation number and date	Level of use of the five practices				
		Anticipating	Monitoring	Selecting	Sequencing	Connecting
Time frame 1	Obs. 1 10/25/2011	N	P	L	L	L
	Obs. 2 11/14/2011	N	P	N	N	N
Time frame 2	Obs. 3 1/9/2012	N	P	P	L	H
	Obs. 4 2/28/2012	N	L	N	N	N
	Obs. 5 3/19/2012	N	P	P	L	P
Time Frame 3	Obs. 6 4/18/2012	N	P	N	N	N
	Obs. 7 5/8/2012	N	L	N	N	N
	Obs. 8 5/30/2012	N	P	N	N	N

N = No use of the practice
L = Little use of the practice
P = Partial use of the practice
H= High use of the practice

Note: The shaded rows designate the lessons that involve focus tasks from the MLSCs.

Looking at Cara Nance's level of use of the five practices within each lesson, there are three lessons for which she uses four of the five practices (monitoring, selecting, sequencing, and connecting). During the other five lessons, Ms. Nance only used monitoring. In order to receive the score "no use" for selecting, sequencing, and connecting, one of two things must occur: either (a) there is no discussion of the task, or (b) there is a discussion of the task but no student approaches are made public during the discussion (as determined by the coding on The Look Fors Sheet). In two of the five lessons for which Ms. Nance's level of use of selecting, sequencing, and connecting was scored as "no use," there was no whole-class discussion of the task. During the remaining three lessons, there was a class discussion of the task, but no student approaches were made public. With regard to the lessons in which Ms. Nance did use the practices of selecting, sequencing, and connecting, two of the three lessons were lessons in which she implemented a focus task she selected for the MLSCs.

The data in Table 4.5 show that there is no distinct pattern of change in Cara Nance's use of the five practices from the baseline measurement (time frame 1) to the intervention measurement (time frame 2). Her lesson plans did not indicate that she had anticipated in any of the lessons, and her monitoring was scored as "little use" or "partial use" for each lesson. During time frame 1, Ms. Nance used the remaining three practices (i.e., selecting, sequencing, and connecting) in one lesson at "little use" and did not use them in the other lesson. During time frame 2, Ms. Nance used selecting, sequencing, and connecting in two of the three lessons. Her level of use for selecting during these lessons increased to "partial use," her level of use for sequencing remained the same, and her level of use for connecting increased to "partial use" for one of the lessons and "high use" for the other lesson. However, Ms. Nance did not use any practice other than monitoring during the maintenance measurement (time frame 3). During this time frame, her level of use for monitoring fluctuated between "little use" and "partial use" as it had done in the other time frames.

4.1.3.3 Descriptions of instruction

This section contains detailed descriptions of two of Ms. Nance's observed lessons. One of these lessons featured a task set up and maintained at a high level through implementation. The other is of a task that was set up at a high level but declined to a low level during the implementation. These descriptions also portray Ms. Nance's use of the five practices in her instruction.

4.1.3.3.1 Instruction of a task set up and implemented at a high level. This section describes Cara Nance's implementation of the *Modeling with Logistic Functions* task (described above) during her lesson on January 9 (Observation 3). To set up this task with her students, Cara Nance began the lesson by having the students work on a warm up problem similar to those in the task (see Appendix O). As the students entered the room at the start of class, this warm up problem was displayed on the front board using a document camera. It asked the students to use their graphing calculators to graph the function, describe the viewing window on the calculator, state the domain and range, and compare and contrast exponential and logistic functions. After allowing the students to work on the warm up for eight minutes, Ms. Nance asked a student to display his graph and domain and range of the function using the document camera. He showed his graph (Figure 4.2) and the domain (-∞,∞) and range (0,∞). Ms. Nance then asked the student what the viewing window on the calculator was. He was not sure and handed the calculator to her. She found the information on the calculator and wrote it on his paper. Ms. Nance then asked the class where the asymptotes of the function would be. A female student replied that they are at 0 and 900. Ms. Nance added these asymptotes to the graph (see Figure 4.3) and asked if the domain and range listed on the paper were correct. The same female student responded that the range was incorrect, explaining "because of the asymptote at 900." As a class, they briefly reviewed the domain and range of the basic exponential function $f(x) = e^x$ and the basic logistic function $f(x) = \frac{1}{1+e^{-x}}$. She had a student read the context of the rumor problem. Ms. Nance then told the class that they would need to change the viewing window of the calculators and she reminded the students that they would not need much room beneath the x-axis. She asked the class why this was the case. A student replied, "Because the graph won't go there."

Ms. Nance agreed saying, “Right, because we can’t really talk about a negative number of people.”

Figure 4.1: Recreation of the graph of the warm up problem produced and displayed to the class by a student

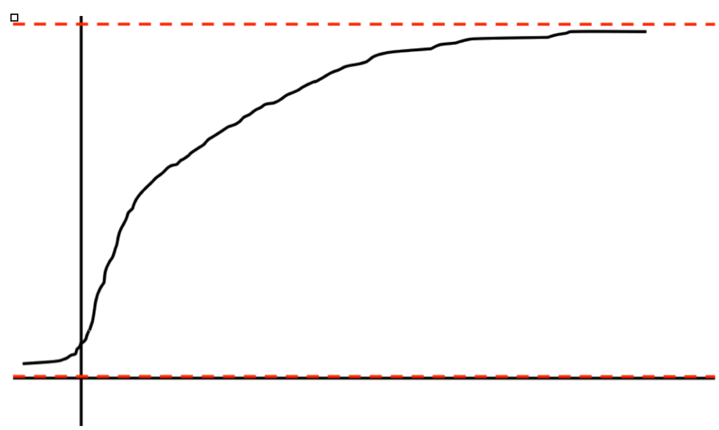


Figure 4.2: Recreation of the graph of the warm up problem produced and displayed to the class by a student with the asymptotes added by Ms. Nance

As the students worked on the task, Cara Nance constantly circulated through the room observing their work, redirecting their efforts when needed, and asking students questions to gauge their understanding and to help them make progress on the task. For example, as the students began to work on the task, some struggled to use their graphing calculators correctly to explore the graph of the function. Many students entered the equation of the function in their graphing calculators correctly, but the calculators were producing graphs that did not appear to match that of the function (recall that the function in this task was $S(t) = \frac{800}{(1+19e^{-0.8t})}$). The calculators displayed graphs similar to that in Figure 4.4, which suggests that the function has the shape of an exponential function. Seeing a pair of students who were struggling with this problem, Ms. Nance asked them, "What do you know about these functions that can help you know how to adjust your window?" Here she was referring to the fact that the students knew that logistic functions should level off due to a second vertical asymptote. They also knew that the value of this asymptote was related to the numerator in of the function. These were both items they had discussed while looking at the student's work on the warm up problem earlier in this lesson. By adjusting their window to show a greater range of *y*-values, the calculator displayed the graph in Figure 4.5. This graph shows the correct shape of the graph as the graph levels off near the *y*-value 800. After asking the students this question, Ms. Nance then left them to use what they knew about logistic functions specifically and graphing functions in general, to correctly adjust their calculators, and thus produce the correct graph. Ms. Nance's intervention then, did not take away from of the thinking needed to correctly engage in the task. Rather, it successfully pointed students in the right direction while leaving the cognitively challenging aspects of the task intact. Ms. Nance used this type of strategic questioning throughout the lesson as she observed students working on the task to redirect their thinking or to respond to their questions.

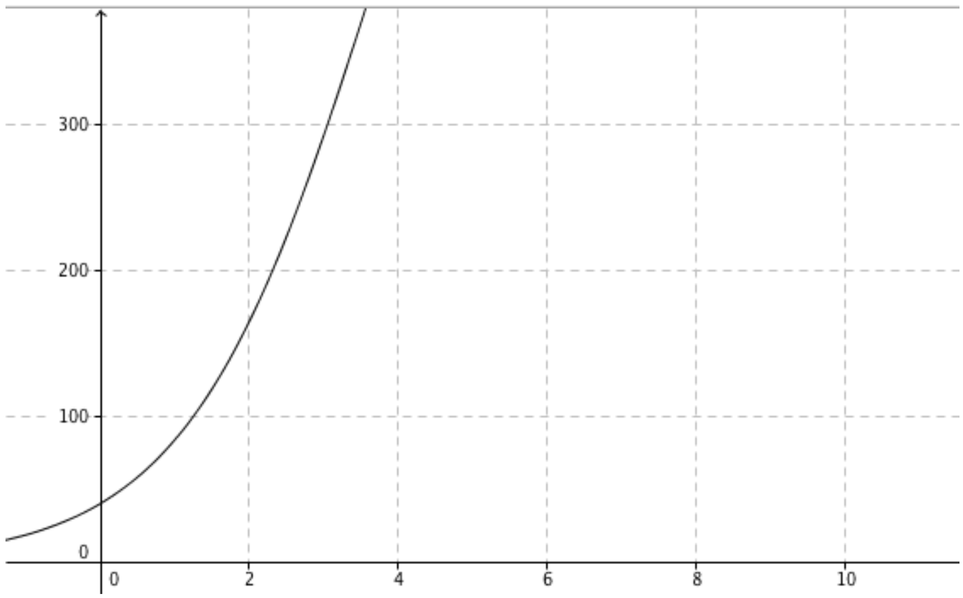


Figure 4.3: Recreation of the graph of the rumor function as displayed by many students' calculators

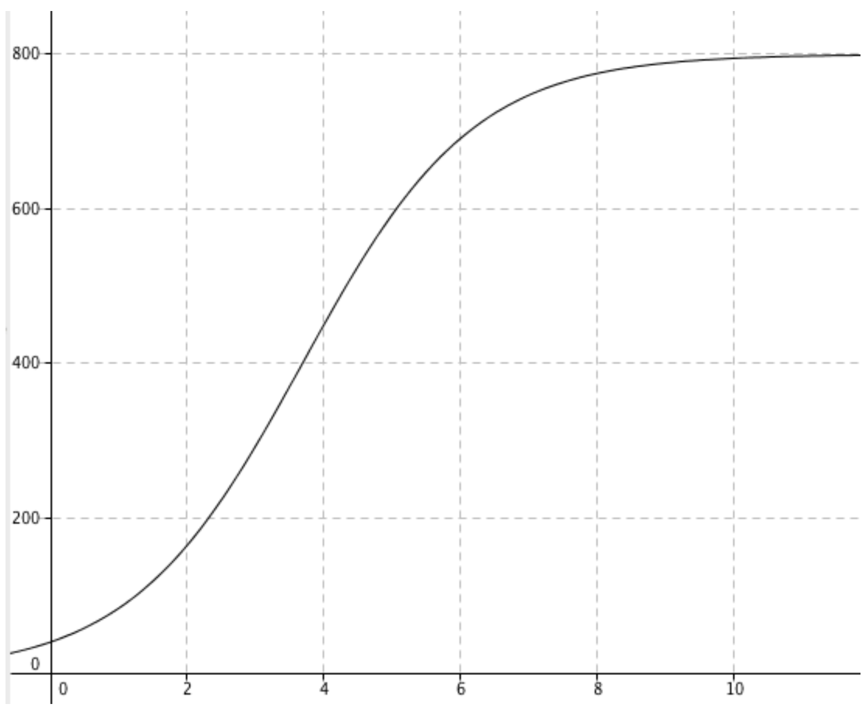


Figure 4.4: Recreation of the graph of the rumor function once the viewing window on the calculator was adjusted

After allowing students to work on the task for about 35 minutes, during which time she continued to circulate and talk with the students about how they were working on the task, Cara Nance hung a large piece of chart paper with the letters A through G written on it on the front board. As students continued to work, she approached individual students and asked them to write their answers to specific portions of the problem on chart paper (e.g., the first student Ms. Nance spoke to put his answer to part A on the paper, and the second to put her answer to part B on the paper, etc.). While there is no data that provides insight into how Ms. Nance selected these students, her selection appears to have been made purposefully as she individually asked these students to put their answers to the specific portions of the task on the chart paper as the rest of the class continued working.

After an additional 10 minutes during which the class continued to work on the task and the selected students placed their responses on the chart paper, Ms. Nance began a whole-class discussion about the task using these responses. Ms. Nance and the students discussed the students' responses to the task on the chart paper with Ms. Nance asking the class questions about the reasoning for the answers. For part A, a student had identified $y = 800$ as an asymptote of the graph. Ms. Nance asked the student that identified this asymptote why there was one at $y = 800$. The student responded that there were only 800 students in the school. Ms. Nance later asked the class what the x-intercept of the graph would be. One of the students told her that there was not one because of the asymptote at $y = 0$.

This pattern of discussing one response to each question on the task continued for most of the lesson with one important exception. When the discussion arrived at part F "According to our model, how many students heard [the rumor] at the end of day 3? Explain or show how you got your answer." Ms. Nance purposefully asked students about two methods they had used to find the answer. She first asked if any of the students had used algebra to solve this question. One student indicated that she had. This student described to the class how she did this, saying that she substituted 3 into the equation of the function and solved it. At Ms. Nance's request, she worked through the computation at the front of the room for the rest of the class to see. As the student did this, the students and the class noticed that her answer was slightly different from the answer written on the poster by another student. Ms. Nance asked the other students in the class how they had work on this portion of the task. Most students said that they had used the trace function on their graphing calculators to determine the value of the function when the *t*-value was 3. Ms. Nance told the students that the difference between the answers was most likely due to rounding issues. Ms. Nance's implementation of the task was coded as high level (procedures with connections).

Ms. Nance made use of four of the five practices during the lesson, failing to use only anticipating. Ms. Nance's use of monitoring was scored as "partial use" as she circulated the room and asked students assessing and advancing questions as they worked on the task. For example, after looking at a students' work Ms. Nance asked the students about domain and range of the context of the task. Ms. Nance asked the student what this domain would be and the student was unable to tell her. Ms. Nance then asked the student if domain deals with the *x* or *y*-values. The student said domain was the *x*-values. She then asked the student what the *x* values represent in the context of the problem, and the student replied that it was the number of days. Ms. Nance followed up by asking the student what the domain would be. The student said that it would be "negative infinity to infinity." Ms. Nance responded, "Is that for the function or the problem situation?" The student still was not sure, so Ms. Nance asked, "Can you have negative days?" She then let the student consider this on her own.

Ms. Nance's use of selecting was purposeful as evident in her selection of students to record their responses on the chart paper that formed the bases of their discussion as well as how she specifically chose to highlight two methods for working on part F during the discussion. Her level of use of selection was coded as "partial use." While she seemed to purposefully select student approaches to focus on during the discussion, it is less clear whether Ms. Nance sequenced the two approaches (solving for $t = 3$ algebraically versus using the graphing calculator as a tool) purposefully or if this was done randomly. Thus, her use of selecting was scored as "little use." Ms. Nance's use of connecting during the implementation of the task was coded as "high use." She drew connections between the two methods she had asked students to share during the discussion around part F. In addition, she emphasized in her comments and questioned the connection between the domain and range of the function and the graphical representation of the function. This was related to one of the goals Ms. Nance had identified for the lesson.

4.1.3.3.2 Instruction of a task set up at a high level but implemented at a low level. In her lesson on March 19 (Observation 5), Cara Nance used the *What is a Radian?* task. She selected this focus task for the second MLSC centered on her instruction. This task was coded as high level (procedures with connections), both as it appeared in written form and as set up by Ms. Nance. However, despite her use of monitoring, selecting, sequencing, and connecting during the lesson, the task was implemented at a low level.

Cara Nance modified the *What is a Radian?* task based on suggestions made during the MLSC week 1 meeting in preparation for her use of it (some of these modifications are discussed with regard to Research Question VI in section 4.2.5.2 below). Appendix P is the modified version of the task. This task asks students to investigate the relationship between the length of the radius of a circle and the measure of the angle formed by the arc of the same length as the radius (a radian). Students are asked to explore this task by using a piece of string the length of the radius of a circle to form the arc corresponding to the measure of one radian of the circle. They then use previously learned facts about circles (e.g., the length of the circumference C of a circle is related to the length of the radius r by the formula $C = 2\pi r$) to explore the relationship between degrees and radians as units of measurement of angles of a circle. While much of the task is procedural (e.g., it asks students to convert several angle measurements given in degrees to radians and vice versa), the task was considered to be a procedures with connections task because it requires students to provide an explanation as to the number of radians needed to complete a full rotation of a circle. It asks students to describe a procedure to convert degrees to radians and a procedure to convert radians to degrees.

To set up the task, Cara Nance reminded students of a lesson they had done during the previous week. She displayed a diagram of a compass using the document camera and asked a specific student, M2, to explain to the class how he had worked on this task using the diagram of the compass.[7] He explained that he broke up the compass into equal sized pieces. As he says this, Ms. Nance drew lines on the compass diagram to show the class what M2 was describing (see Figure 4.6). Ms. Nance then clarifies that doing this splits the compass into 16 pieces. M2 added that each pieces is 22.5° and said, “So just count the number of pieces and multiply by 22.5.” Ms. Nance then asked the students to work in pairs and she passed out a protractor, pieces of papers each with a circle with a different size radius, and a piece of string to each pair of students. She read the directions given on the first page of the task and explained that the piece

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[7] Students are referred to as M# (males) or F# (females) based on the observation write-ups as a way of preserving anonymity.

of string that they received was too long to be the radius of the circle on the piece of paper that they have, emphasizing, “You can’t do the task if the string is too long.” She then told the students that they needed to hold the string at the center of the circle and she would cut the string to the correct size. At this point, she began to move through the room cutting the string and the students started to work on the task.

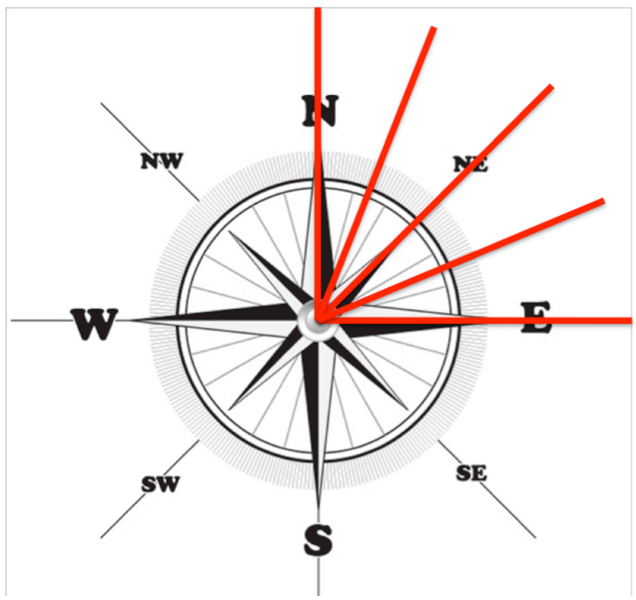


Figure 4.5: Recreation of the diagram Ms. Nance created during the set up of the *What is a Radian?* task

The implementation of the task, however, did not maintain the same level of cognitive demand, as it was coded as low level (procedures without connections). As students began to work, several had difficulty engaging in the task as directed. One student had answered question #2— which asks for the number of radians needed to complete a circle— with 6 (the actual answer was 2π as the length of the circumference of any circle can be found by multiplying the length of the radius by 2π). When Ms. Nance saw this student’s answer, she asked him how he found the answer and he said he guessed based on how far the one piece of string had gone around the circle. Ms. Nance did not press the student to reason any further about this question. Two other students had begun working on the task incorrectly as they had drawn a diagram similar to Figure 4.7 with a right angle in the circle as the measure of how far one radian would go around the circle (the actual measure of the angle equivalent to one radian is approximately 57°). Seeing this, Ms. Nance used the string to show the students that the angle needed to be much smaller than 90° and she then asked them to erase the 90° angle and to begin again. Another student explained to Ms. Nance that he used the string to mark several angles around the circle, but that the measures of the angles are all different (59°, 64°, and 61°). Yet, another student determined that 6.31 radii were needed to move around the circle. He found the measure of the arc length created by placing the string on the circumference, which represented one radius, on the circle to be approximately 57°. He then divided 360° by 57° to determine he would need about 6.31 radii to complete one rotation around the circle.

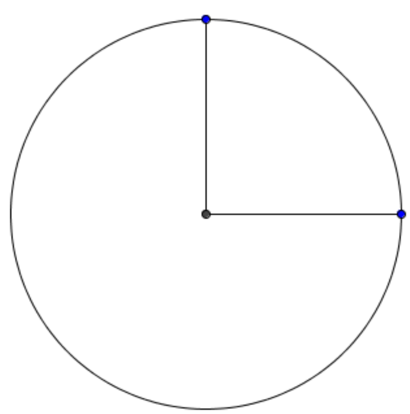


Figure 4.6: Recreation of a diagram drawn by two students in Ms. Nance's class

After about 13 minutes of letting the students work on the beginning of the task, Ms. Nance called for a whole-class discussion about what the students had done. She showed the work of the student who had divided 360° by 57° in order to find the number of radii needed to complete the circle. The student explained what he had done and why he divided 360° by 57°, but there was no press by Ms. Nance for a conceptual reason for doing so. Rather, Ms. Nance explained that because each of the angles of the radians created by stretching the string along the outside of the circle were the same they will all have a measure of 57°. At no point did they discuss that the length of the string stretched on the circle (the radian) is the same as the length of the radius of the circle. Nor did they discuss that although each pair of students had different sized circles, they all needed exactly the same number of radians to travel around the circle they were given and that the arc formed by the radian is the same number of degrees as the central angle.

Later as the students worked on the portion of the task for which they had to find the equivalent radian measures for specified degree measure, the focus was on the process, not the reasoning behind it. When any reasoning did occur, it was due to heavy leading by Ms. Nance. For example, two students were trying to determine the equivalent radian measure for 45° and they had divided the circle into eighths. Seeing this, Ms. Nance pointed to the piece of the circle between 0° and 45° and said, "So this is one eighth of the whole circle, right?" The students agreed that it was. Ms. Nance then asked, "So what's all the way around?" One of the students said that it was 2π. Ms. Nance responded, "So what's an eighth of 2π?" The two students were not sure how to answer this question. Ms. Nance told them that the numerator of the fraction would be 2π and the denominator would be 8 and she wrote $\frac{2\pi}{8}$. She then asked the students if they could reduce the fraction. As such, students were not explicitly pressed to reason about the task and, in fact, the reasoning that did occur was done by Ms. Nance. Furthermore, in many instances, students were told by Ms. Nance how to work through many of the cognitively challenging aspects of the task, thus reducing the cognitive demand of the task.

Cara Nance made use of four of the five practices during her instruction, omitting only anticipating. Her use of monitoring can be seen in the description above of how she circulated and observed students' work on the first page of the task. She moved through the room visiting many students. She asked some of them questions about how they were working on the task; in the case of the two female students who had drawn the 90° angle inside the circle, Ms. Nance redirected their efforts. Thus, her monitoring was rated as "partial use."

After allowing the students to work on the first page of the task for about 13 minutes, Ms. Nance and the students had a brief discussion about this portion of the task. She then asked them to continue work on the task, and they did so for about 25 minutes. This was followed by another whole-class discussion about the remainder of the task. During this discussion, Ms. Nance purposefully selected two groups' approaches to part 2 of the task (finding equivalent radian measures for angles with measures given in degrees). To begin this discussion, Ms. Nance asked a specific group (Group 1) for their work and she displayed it via the document camera (see Figure 4.8). While looking at their work, she told the class that Group 1's diagram made it appear as though the circle was cut into eight pieces or that the top of the circle was cut into four pieces. She said that the first mark was 1/4 of the top, which is π, and that this was equivalent to $\frac{\pi}{4}$. They then used this method to discus radian measures for the marks at 135°, 225°, and 315°.

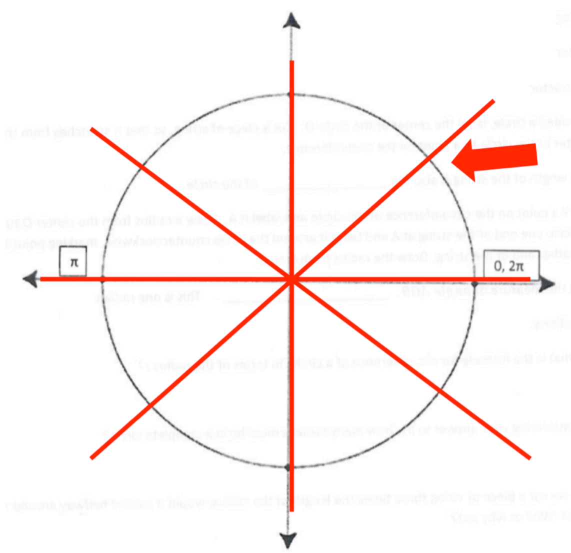


Figure 4.7: Recreation of the diagram of Group 1's approach to part 2 of the *What is a Radian?* task

After discussing Group 1's approach to the problem, Ms. Nance next asked another group (Group 2) if she could show the class their work (see Figure 4.9). She asked the members of Group 2 to explain what they had done to find the angle measures in radians. One of the group members said that she was confused and was not sure what Ms. Nance wanted her to say. Ms. Nance then explained to the class that this group said that the pieces between 0 radians and the first line above it in the diagram was one third of a half so it would be $\frac{\pi}{2}$ multiplied by 1/3. She then wrote $\frac{\pi}{2} * \frac{1}{3} = \frac{\pi}{6}$. This portion of the whole class discussion demonstrates Ms. Nance's thoughtful selection of student approaches she wished to share with the class. It appears as though her selecting of these approaches was purposeful, as she did not ask for volunteers but immediately asked these groups if she could share their work. Furthermore, when the members of Group 2 were not sure what Ms. Nance wanted them to explain to the class, Ms. Nance had a specific approach that she wished to share with the class. Her level of use of selecting was coded as "partial use" as she selected multiple correct approaches and seemed to do so purposefully.

However, there is no evidence that Ms. Nance's sequencing of the two approaches she selected was purposefully done, as there does not seem to be a mathematical or pedagogical reason for the order in which they were shared. Nor did the order seem to allow her to use one approach to build on the other toward the goals of the lesson. Thus, her level of sequencing was scored as "little use."

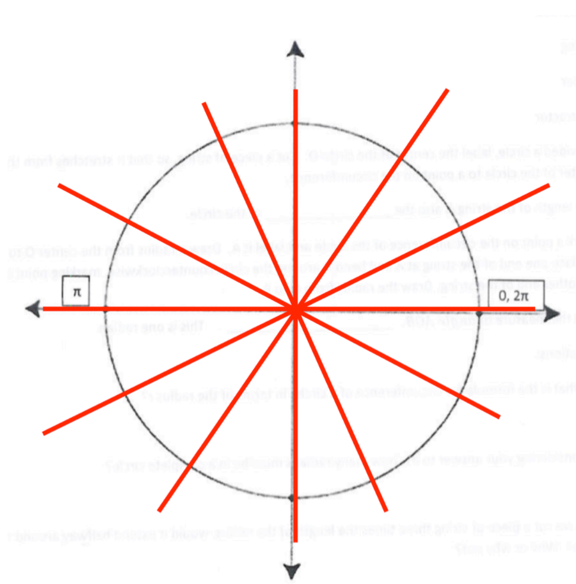


Figure 4.8: Recreation of the diagram of Group 2's approach to part 2 of the *What is a Radian?* task

Cara Nance also used the practice of connecting during the lesson. When discussing with the students part 3 of the task in which they were to describe a method for converting degrees to radians and vice versa, Ms. Nance referred to the Group 1's method (without implying that it was the method they shared). She formed connections between the approach of dividing the top half of the circle into a certain amount of pieces to find the amount of radians of an angle (e.g., she divided the top half of the circle (π radians) into fourths to find the radian measure for 45°, which is one fourth of the top half of the circle) and the challenge to convert radians to degrees. This also demonstrated a connection to one of the goals for the task that Ms. Nance identified in her lesson plan: "Students will discover the relationship between radian and degree measurements." As she began the discussion around part 3 of the task, Ms. Nance asked the students if they could work backwards going from radians to degrees. She showed part 3 on the document camera and pointed to the first angle measure given ($\frac{\pi}{4}$). She asked, "What is this?" (How many degrees?) A student replied that it was 45°. Ms. Nance asked the student why it was 45° and the student explained, "Because it is 180 divided by 4." Ms. Nance then asked about the next two radian angle measures, $\frac{\pi}{6}$ and $\frac{2\pi}{3}$, and the same student described how to find these angle measures in degrees. As the student talked, Ms. Nance wrote out the mathematical computation of what the student spoke (e.g., $\frac{2}{3} * 180 = \frac{360}{3} = 120°$). Ms. Nance then asked the students what rule they would use to convert radians to degrees. The students did not know how to respond to this. Ms. Nance asked, "What are you changing pi into?" The same student who had described how to convert $\frac{\pi}{4}$, $\frac{\pi}{6}$, and $\frac{2\pi}{3}$ replied that they were changing π into 180°. Ms. Nance asked the class, "And what could you multiply by?" Multiple students responded that they could multiply by 180°. Ms. Nance then referred to an example from a previous lesson of converting feet to meters. She pointed out that in addition to multiplying by the quantity "meters," they had to divide by the quantity "feet" (e.g., converting 15 feet to meters, multiply 15 by $\frac{1 \text{ meter}}{3 \text{ feet}}$ which is the same as multiplying be 1 meter and dividing by 3 feet), and she said, “So we also need to divide.” She then wrote “multiply by $\frac{180}{\pi}$” as the answer to question about converting radians to degrees on the bottom of page 4 of the task. Ms. Nance’s level of use of connecting was rated as “partial use” as she drew connections between the approaches that were shared and the goals of the lesson, but not between the approaches themselves.

4.1.3.4 Summary of implementation of high-level tasks and use of the five practices

Cara Nance struggled to maintain the high-level of cognitively demanding tasks throughout the implementation of the lessons. Only two of the five lesson in which Ms. Nance set up a task at a high level did she implement it as such. In the other three lessons, the level of cognitive demand decreased. Ms. Nance typically did not use the five practices as a set during her lessons. In every lesson, she used monitoring, but in only three of the eight did she use any of the other five practices. In these three lessons she used monitoring, selecting, sequencing, and connecting, but did so in varying degrees.

4.1.4 The Relationship Between Use of the Five Practices and the Ability to Maintain the Level of Cognitive Demand of High-Level Tasks (Research Question V)

Table 4.6 shows the relationship between Cara Nance’s use of the five practices and the level of cognitive demand of the tasks implemented in the five lessons in which she set up the task at a high level. The data suggest no discernable differences in Ms. Nance’s use of any of the five practices when comparing her successful and unsuccessful implementation of high-level tasks. In every lesson, Ms. Nance’s use of anticipation was scored as “no use.” In addition, in each observation Ms. Nance’s level of use of monitoring was coded as “partial use.” For the three tasks implemented at a low level, Ms. Nance scored “no use” during two of the observations and “partial use” during the third. In the two tasks coded as high level during implementation, Ms. Nance’s use of selecting was scored as “little use” once and “partial use” once. Of the three low-level tasks at implementation, Ms. Nance’s use of sequencing was coded as “no use” for two and “little use” for the third. During both of the observations for which the implementation of the task was coded as high level, Ms. Nance’s use of sequencing was also coded as “little use.” During the three observations of the tasks implemented at a low level, Ms. Nance use of connecting was scored as “no use” for two and “partial use” for one. Similarly, in the observation of the tasks implemented at a high level, her use of connecting was scored as “little use” for one task and as “high use” for the other. Thus, overall there does not appears to be any difference in Cara Nance’s level of use of the five practices between her lessons in which she maintains the cognitive demand of the task and those in which the demand decreases.

Table 4.6: Cara Nance's level of use of five practices as related to the level of cognitive demand of implementation of tasks set up at a high level across all classroom observations

	Level of cognitive demand during implementation																							
	Low-level cognitive demands																High-level cognitive demands							
	Unsystematic and nonproductive exploration				Little or no academic thinking occurred				Memorization				Procedures without connections				Procedures with connections				Doing mathematics			
Level of use of practice	N	L	P	H	N	L	P	H	N	L	P	H	N	L	P	H	N	L	P	H	N	L	P	H
Anticipating													3				2							
Monitoring															3				2					
Selecting													2		1			1	1					
Sequencing													2	1				2						
Connecting													2		1			1		1				

Note: N = no use, L = low use, P = partial use, and H = high use.

4.1.5 The Relationship Between the Modified Lesson Study Cycles and Implementation of the Focus Tasks (Research Question VI)

Cara Nance selected the focus task for two of the 11th- and 12th-grade team's MLSCs. Her uptake of the key ideas and suggestions given by her colleagues during the week 1 meeting of these cycles will be presented in this section.

4.1.5.1 Uptake of key ideas from the first MLSC

Cara Nance selected the task called *Modeling with Logistic Functions* (see Appendix Q) for the MLSC week 1 meeting on January 4. During this meeting five key ideas were discussed:

1. Express the domain and range in multiple notations - The team members suggested that it would be beneficial for Ms. Nance to have her students write the domain and range of the function in both inequality and interval notation.
2. Discuss limits - A team member felt that this task would be a good opportunity for Ms. Nance to discuss the mathematical concept of limits with her students.
3. Use calculators to graph the functions - The team discussed with Ms. Nance how accurate she wanted her students to be when graphing the function. She said that she wanted the graphs to be fairly accurate, and as such, the team members suggested she have the students use their calculators to help them graph the functions.
4. Bring up the idea of the function being a model - The team members felt that Ms. Nance should discuss the idea with her students that the mathematical function is

a model of a real-world situation and that what actually occurs in the real world may not be exactly what the model predicts.

5. Reword questions F and G to reflect the idea that the function is a model - Based on the key idea #4, the team members felt that Ms. Nance should reword these questions.

Cara Nance taught the *Modeling with Logistic Functions* task on January 9, five days after the week 1 meeting. Her instruction of the task and her subsequent discussion of this instruction during the MLSC week 2 meeting on January 11 provide evidence that she incorporated three of the five key ideas (key ideas #3, #4, and #5 listed above) from the MLSC week 1 meeting.

4.1.5.1.1 Key idea 1 - Expressing the domain and range in multiple notations. At no time during her lesson using the task did Cara Nance emphasize the need to write the domain and range in more than one notation. When she discussed the functions with the students and their work, which stated the domain and range of the functions, she only used one type of notation. During the week 2 meeting of the modified lesson study cycle during which Ms. Nance debriefed her teaching of the task, she provided samples of student work on the task. On each sample, the students all used the same type of notation to state the domain and range.

4.1.5.1.2 Key idea 2 - Discuss limits. Ms. Nance did not take up this key idea in her instruction. There was much discussion of asymptotes when the students created the graphs of the functions, but at no point did Ms. Nance introduce the concept of limits.

4.1.5.1.3 Key idea 3 - Use calculators to graph the functions. After the week 1 professional development meeting during which her task was discussed, but prior to her instruction of it, Ms. Nance modified the task. Part of these modifications was to change the instructions from “Sketch the graph” to “Plot the points of the graph of the function. Label!” Ms. Nance also created a “Calculator Tips” poster she displayed at the front of the room during the lesson that contained advice for how to properly use the graphing calculators to best graph the functions. In addition, during her set up of the task Ms. Nance explicitly told the students to use their graphing calculators to graph the functions on the task.

4.1.5.1.4 Key idea 4 - Bring up the idea of the function being a model. Ms. Nance posted the following as an objective of the lesson on the chalkboard at the front of the class: “I can model real world situations using logistic functions.” During the lesson, Ms. Nance explicitly referred to the functions as models representing the situation described in the task. She said, “So based on the model, not the situation, what’s the highest number of students?” Later in the lesson, Ms. Nance highlighted the fact that what occurs with regard to the situation in reality may differ that what is represented in the mathematical model. As described in the write-up of the observation:

> Ms. Nance ask[s] the students why the answer to part G is 799. One student says, “because of the asymptote.” Ms. Nance asks, “in reality could all 800 students hear it?” The students say that they could. Ms. Nance tells the class that “model” is the key word for this part of the problem. (120109.Ms.CN writeup)

Further evidence of her uptake of this idea can also be seen in her incorporation of the key idea 5.

4.1.5.1.5 Key idea 5 - Reword questions F and G to reflect the idea that the function is a model. As noted earlier, Ms. Nance modified the task after the week 1 professional development meeting but before using it in class. This included a rewording of questions E, F, and G so that they included the phrase “according to our model” or “based on our model.”

4.1.5.2 Uptake of key ideas from the second MLSC

The second MLSC for which Cara Nance selected the focus task was centered on the *What is a Radian?* task (see Appendix R). The MLSC week 1 meeting for this cycle was held on March 7. Seven key ideas emerged during this meeting:

1. Have a pre-drawn circle for the task - The task called for students to use a string with a given length to draw a circle with a radius the length of the string. The team members discussed the potential problems students would have doing this and that it would be better to remove these issues to help students focus on the mathematics of the task.
2. Reword question #1 on the task - The team member suggested rewording this question in order to help students make progress toward the mathematical goals of the task.
3. Add a question or directions prior to question #1 - The team members felt that a question or specific instructions directing students to use the string to determine the number of radii needed to go completely around the circumference of the circle would steer students toward the goal of the lesson.
4. Explicitly state the correspondence between rotations, degrees, and radians - Ms. Nance wonders how explicit she should be about the corresponding number of rotations around the circle, the number of degrees, and the number of radians after students work on part 1 of the task but prior to beginning part 2. The team members suggest that she be very clear about this.
5. Ensure students consider a full rotation around the circle is one whole unit - The team discussed that some students may view half a rotation around the circle as

the unit because it is represented as π radians. Some team member felt that this would cause confusion for the students and urged Ms. Nance to emphasize that the unit is a complete rotation (2π radians) around the circle.

6. Remove the warm up of the problem - Ms. Nance had identified a problem to use as a warm up that dealt with the distance the tip of a hand of a clock moves in a certain amount of time. The team member felt that this might not be beneficial to use as a warm up problem for this lesson.
7. Modify the degrees to radians conversion problems on the last page - The team members point out to Ms. Nance that these problems produce "very messy answers" (e.g., $\frac{31}{36}\pi$ radians). They suggest changing two of the problems so that they work out to "nicer" answers and leave one with a "messy" answer.

Cara Nance's lesson involving the *What is a Radian?* task occurred on March 19. Her instruction and the debriefing of it during the MLSC week 2 meeting on March 28 provided evidence that she incorporated five of the seven key ideas (key ideas #1, #2, #4, #6, #7) from the week 1 professional development meeting. She did not integrate key idea #3 or #5 into her instruction.

4.1.5.2.1 Key idea 1 - Have a pre-drawn circle for the task. Ms. Nance modified the task after the week 1 professional development meeting but prior to her instruction of it. The modified version reflected this key idea. She gave students an additional sheet of paper with a pre-drawn circle on it. The modified version of the task asks student to cut a piece of string the length of the radius of the pre-drawn circle and to use this piece of string to determine the number of radii needed to complete the circumference of the circle.

4.1.5.2.2 Key idea 2 - Reword question #1 on the task. The modified version of the task that Ms. Nance created after the week 1 meeting demonstrated her uptake of this key idea. The wording of this question on the original version of the task is: "What is the circumference of the circle in terms of radius r?" On the modified version of the task, this question reads: "What is the formula for the circumference of a circle, in terms of the radius r?"

4.1.5.2.3 Key idea 3 - Add a question or directions prior to question #1. The evidence from Ms. Nance's instruction and from the week 2 professional development meeting suggest that she did not incorporate this idea in her teaching of the task. The modified version of the task did not include an alteration of the instructions prior to question #1 and the original instructions did not ensure students would use the string to answer the questions in part 1 of the task. Further, during the lesson, many students did not use the string to determine the number of radii needed to complete a full rotation of the circle. Rather, they found the length of one arc length using the string and then used algebra to determine the needed number of radii (e.g., some students found the measure of the arc length created by placing the string that represented one radius on the circle to be approximately 57°. They then divided the total number of degrees in the circle, 360°, by 57° to determine that they would need approximately 6.32 radii to complete one rotation around the circle). When this occurred during the lesson, Ms. Nance did not redirect the students to place the string repeatedly on the circle. This was also evident during the week 2 meeting when Ms. Nance shared samples of student work that showed their use of algebra to work on part 1 of the task.

4.1.5.2.4 Key idea 4 - Explicitly state the correspondence between rotations, degrees, and radians. Evidence from her instruction suggests that Ms. Nance did take up this key idea. Question #1 of part 2 of the task was altered in the modified version of the task so that it explicitly stated this correspondence for both a half and full rotation of the circle. In the original version of the task, the question read: "If half the rotation (180°) is equal to π radians, what is the radian measure for 90°?" This question in the modified version of tasks reads: "If half a rotation (180°) is equal to π radians and the whole rotation around the circle is 2π radians (360°), what is the radian measure for 90°?" During the lesson, after the students had worked on part 1 of the task and they had discussed it, Ms. Nance explicitly told the students that the entire purpose of part 1 of the activity (what they had worked on up to that point) was for them to realize that 2π radians are needed to go completely around the circumference of a circle and π radians are needed to go half way around.

4.1.5.2.5 Key idea 5 - Ensure students consider a full rotation around the circle is one whole unit. There is evidence from the observation of Ms. Nance's lesson that she did not incorporate this idea into her instruction. During the lesson Ms. Nance spoke about the circle in such a way that a full rotation around the circle was the unit they were considering. This can be seen in this excerpt from the observation write-up:

> Ms. Nance is circulating and working with the students. She is working with M1 and M2. They have divided the circle into eight equal pieces. Ms. Nance points to the piece between 0° and 45° and says, "So this is one eighth of the whole circle, right?" The students agree that it is. Ms. Nance says, "So what's all the way around?" M1 says that it is 2π. Ms. Nance then says, "So what's an eighth of 2π?" The two students are not sure how to answer that. Ms. Nance says that the numerator of the fraction would be 2π and the denominator would be 8 and she writes $\frac{2\pi}{8}$. She then asks the students if they can reduce the fraction. M2 says, "We can take 2 out of each." Ms. Nance agrees and

M1 says, "Oh! It's pi fourths ($\frac{\pi}{4}$)." To which Ms. Nance responds, "Great!" (120319.Ms.CN writeup)

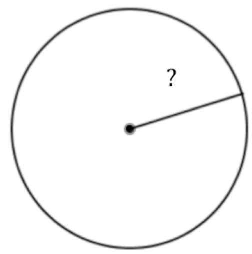
However, directly after this encounter, Ms. Nance talked with another group that spoke about the circle using the notion of half of the circle being the unit:

> Ms. Nance is working with M5 and F5 who are trying to determine who many radians are equivalent to 210°. Ms. Nance asks them how many pieces they have divided the circle into and M5 say 12. Ms. Nance points to one of the pieces of the circle and says, "Ok, so this is one twelfth of the whole circle or 2π. Or it is one sixth of half of the circle, or π." F5 says that it would then be 1.16π. Ms. Nance asks her why. F5 says, "Because 90° is half [SE: Here she is implying that 90° is half of π] and a third of a half is point one six (0.16). So this (she points to 210°) is one point one six pi (1.16π)." Ms. Nance responds, "But what is it as a fraction?" F5 replies, "What?! We have to do it as a fraction?!" Ms. Nance says that she wants them to write the answer as fractions "because it will help later on." (120319.Ms.CN writeup)

Ms. Nance specifically told this group to use fraction notation (as opposed to decimal notation), but she did not redirect them to speak about the full rotation as the whole unit.

4.1.5.2.6 Key idea 6 - Remove the warm up of the problem. Ms. Nance did integrate this key idea into her teaching of the task. She did not use the warm up problem about hands of the clock that she shared with her team during the week 1 meeting. Rather, she used an entirely different warm up shown in Figure 4.10.

Warm Up
1) Find the Radius of the circle if C=8ft.



2) Convert from MPH to ft/sec.
a) 60 MPH

b) 70 MPH Leave answer as a fraction.

Figure 4.9: Warm up used by Ms. Nance during her implementation of the lesson

4.1.5.2.7 Key idea 7 - Modify the degrees to radians conversion problems on the last page. As part of her modification of the task after the week 1 meeting, Ms. Nance altered the questions regarding the conversion from degrees to radians so that the answers to the questions produced "nicer" answers (i.e., the fractional part of the answers were more common compared to those of the questions on the original version of the task).

4.1.5.3 Summary of uptake of key ideas in instruction

Cara Nance's colleagues provided several suggestions related to her focus tasks during the MLSC week 1 meetings. These suggestions ranged from possible modifications to the focus task, to key concepts that should be discussed with the students, to possible methods for how students should be encouraged to engage in the task (e.g., encourage students to use a calculator to graph the function, push students to use an algebraic method for finding the vertex of the parabola). Cara Nance incorporated the majority of the suggestions her colleagues proposed (three of five related to the first MLSC and five of seven related to the second MLSC).

It is important to note the nature of the key ideas in the two MLSCs. During the first MLSC associated with her lesson on January 9 (Observation 3), three of the five key ideas centered on important ideas or concepts the team members felt Ms. Nance should highlight during the lesson (e.g., explicitly discuss the idea that the function is a model of the real-world situation and thus may not exactly represent what actually occurs), while only one idea dealt with a possible method students could use to work on the task. The fifth key idea during the first MLSC was a possible modification to the task. Conversely, during the second MLSC, which was connected to the lesson on March 19 (Observation 5), five of the seven key ideas discussed during the week 1 meeting related to possible modifications to the task, while only two corresponded to key ideas or concepts students needed to consider during the lesson. These differences between the type of key ideas discussed during the two MLSCs suggests that the conversation in the first MLSC was more centered on conceptual understanding needed by students, and that the focus of the second MLSC was more focused on changes that needed to be made to the task.

4.1.6 Summary of Cara Nance

Cara Nance was a dedicated member of the 11th- and 12th-grade professional development team. She attended every team meeting and came prepared for the MLSC week 1 meetings by anticipating possible student solutions methods to the focus tasks and recording insightful noticings and wonderings related to these tasks. The only MLSCs for which she did not produce these materials were for the MSLCs center on task she had selected. Cara Nance selected high-level tasks during the majority (6 of 8) of her observed lessons. However, she typically was unable to implement these tasks at a high level, doing so in only two lessons. The factors related to her in ability to do so varied, but in two of the cases, it was due to a lack of press for student reasoning and explanation. Cara Nance's use of the five practices was inconsistent. She never anticipated possible student responses in her lesson plans, but she engaged in monitoring in every lesson. She employed selecting, sequencing, and connecting to varying levels in three of the eight lessons, and always used these together (e.g., there were no lessons in which she used selecting but did not use sequencing or connecting). There are no noticeable differences with regard to her level of use of the five practices used during lessons with tasks implemented at a low level and lessons with tasks implemented at a high level. Cara Nance's colleagues provided her with several suggestions during the MLSC week 1 meetings. There is evidence that she incorporated the majority of these ideas, doing so either in preparation for her lesson (e.g., modifying the focus task based on group members' suggestions) or in her instruction of the tasks. Chapter Five will present possible explanations for her struggles using high-level tasks and her sporadic use of the five practices.

4.2 NICOLE NESMITH

This section address the six research questions with respect to Nicole Nesmith. It presents a narrative case that describes her participation in the professional development meetings and of her instruction.

4.2.1 Participation in the Professional Development Meetings (Research Question I)

Like Cara Nance, Nicole Nesmith was also a member of the 11th- and 12th-grade professional development team. Of the 27 meetings this team had during the school year, Ms. Nesmith attended 24. She was present for eight of the nine MLSC week 1 meetings and selected the focus tasks for two of these meetings. Ms. Nesmith produced work on the focus task and noticings and wonderings for the six cycles for which she did not select the focus task; she did not produce any materials for the two meetings centered on her focus tasks. Table 4.7 shows the scores from the Professional Development Preparation Rubric for Ms. Nesmith's work on the focus tasks and noticings and wonderings. The shaded cells denote the meetings for which she selected the focus tasks. Of the six meetings for which Ms. Nesmith produced work on the focus task, the scores of this work extended from low to high. Work for two of the meetings was scored as low, the work for three of the meetings was scored as medium, and the work for one meeting was scored as high. All of the noticings and wonderings that Ms. Nesmith produced were scored as high.

Table 4.7: Nicole Nesmith's level of preparation for the week 1 meetings of the professional development cycles

		Level of preparation	
Cycle number	Meeting date	Work on the focus task	Noticings and wonderings
1	Dec. 14, 2011	Medium	High
2	Jan. 4, 2012	Medium	High
3	Jan. 18, 2012	No	No
4	Feb. 8, 2012	Low	High
5	Feb. 22, 2012	Medium	High
6	Mar. 7, 2012	High	High
7	Apr. 11, 2012	No	No
8	Apr. 25, 2012	Low	High
9	May 9, 2012	*Ms. Nesmith did not attend this meeting.*	

Note: The shaded rows designate the professional development cycles for which Cara Nance selected the focus task.

4.2.1.1 Work on the focus tasks for the MLSC week 1 meetings

The codes for medium-preparation and high-preparation with regard to teachers' work on the focus tasks on the Professional Development Preparation Rubric are similar, differing only in that work coded as medium-preparation only includes correct answers while high-preparation work also includes incorrect answers. Thus, Ms. Nesmith's work was prepared at a quality level for all but one MLSC week 1 meeting. The work Nicole Nesmith created on the *What is a Radian?* task selected by Cara Nance (see Appendix L) for MLSC week 1 meeting on March 7 was scored as high preparation. It is typical that the work she produced that was coded as medium preparation or high preparation. Her work on this task was coded as high preparation because she anticipated multiple approaches to the task and included both correct and incorrect answers to the task.

The Professional Development Preparation Rubric scores work on a focus task as low preparation if it includes only one method for solving the task. As noted in section 4.1.1 discussing Cara Nance's work on the focus tasks, the nature of some of the tasks may have pushed the team members' level of preparation to a low level. The work for the two MLSC week 1 meetings that Ms. Nesmith created that was scored as low preparation (February 8 and April 25) corresponds with the meetings centered on these focus tasks. For example, the focus task used during the MLSC week 1 meeting on February 8 was a task called *Hear It and Read It: Ratios and Fractions*. This task includes a page of examples of how to write fractions and ratios in various forms (e.g., as fractions and using a colon). The problems in this task dictate exactly how students should work. For instance, problem one contains several situations that can be represented as ratios. After each, students are asked to write the ratio as a fraction and using a colon. Thus, it is not logical to solve the task using an alternate method.

4.2.1.2 Noticings and wondering related to the focus tasks for the MLSC week 1 meetings

Nicole Nesmith's noticings and wonderings were coded as high-preparation because they contained at least three noticings and wonderings pairs and touched on at least two of the areas designated in the Professional Development Preparation Rubric. Table 4.8 lists the noticing and wondering pairs Ms. Nesmith created for the MLSC week 1 meeting on January 4 with regard to the *Modeling Logistic Functions* task selected by Cara Nance (Appendix Q). All of the noticings and wonderings pairs she created touch on specific elements of the task, whether they are individual sections of the task or the representations students will produce when working on it. Pairs 3, 4, and 5 address instructional issues the teacher may face during the lesson, and pairs 4 and 5 point out possible trouble areas related to how students may try to make sense of the mathematics in the task.

Table 4.8: The noticings and wonderings Nicole Nesmith produced related to the *Modeling with Logistic Functions* task for the modified lesson study cycle week 1 meeting on January 4

N&W Pair	Noticings	Wonderings
1	"[I noticed] that you asked for domain and range of the logistic functions and problem situation."	"Will students need clarification that the logistic function is the 'generic' function?"
2	"[I noticed] that (b) and (e) correspond to one another."	"Should you put them right after one another?"
3	"They have to sketch graphs."	"Will students recall that exponential functions have an asymptote at $x = 0$?
		"Maybe hint to them to extend graph on calculator to see it."
		"Hint to them to use a range of points (ex. $x = 0, 5, 10, 15, 20$)."
4	"Part G (Part 1) and Part F (Part 2) deal with answers that happen in between days.	"Remind students to be aware of this."
5	"When I graphed it, my graph looked exponential."	"How are you going to target this misconception?"

In summary, Nicole Nesmith was an active participant in her team's MLSC meetings. She produced work and the noticings and wonderings on a consistent basis, only failing to do so when the meeting focused on tasks she selected. Additionally, the work and noticings and wonderings she created were thoughtful and showed that she prepared for the meetings in such a way as to be able to engage in discussion around the focus tasks, providing insights about the tasks and suggestions about how to implement them.

4.2.2 The Level of Cognitive Demand of Tasks Used During Observed Lessons (Research Question II)

Table 4.9 shows the level of cognitive demand of the tasks selected (in written form) set up, and implemented by Nicole Nesmith in her seven observed lessons. All but one of the tasks she used during these lessons were scored as procedures with connections tasks (high-level tasks). The task she used during the last observation was a low-level task coded as procedures without connections. As all but one of the tasks were coded the same, there was no change between the baseline measurement (time frame 1), the intervention measurement (time frame 2), and the maintenance measurement (time frame 3).

Table 4.9: The level of cognitive demand of the tasks as selected, set up, and implemented by Nicole Nesmith during her lesson observations

	Observation number and date	Level of cognitive demand of the task as: Selected (in written form)	Set up	Implemented
Time frame 1	Obs. 1 10/25/2011	**PWC**	**PWC**	LN
	Obs. 2 12/1/2011	**PWC**	**PWC**	LN
Time frame 2	Obs. 3 1/24/2012	**PWC**	**PWC**	LN
	Obs. 4 2/27/2012	**PWC**	**PWC**	LN
	Obs. 5 4/16/2012	**PWC**	**PWC**	**PWC**
Time Frame 3	Obs. 6 5/3/2012	**PWC**	**PWC**	LN
	Obs. 7 5/29/2012	PWoC	PWoC	PWoC

LN = Little or no academic thinking required by the task/ occurred during the lesson
Mem = Memorization task
PWoC = Procedures without connections task
PWC = Procedures with connections task
DM = Doing mathematics task
Unsys = Unsystematic and nonproductive exploration
O = Other

Note: High-level tasks (e.g., procedures with connections task and doing mathematics tasks) are bolded in the table. The shaded rows designate the lessons that involve focus tasks from the MLSCs.

Below, two of the tasks selected by Nicole Nesmith are described as they appeared in written form. The high-level task is representative of the other high-level tasks selected by Ms. Nesmith as they shared similar qualities with respect to the coding of the level of cognitive demand. The one low-level task selected by Ms. Nesmith is also described to show what distinguished it from the other tasks she used.

4.2.2.1 A high-level task

The lesson on January 24 (Observation 3) used the *Single Star or Galaxy* task (Appendix E). This task was the first task selected by Nicole Nesmith to be the focus of a MLSC. The *Single Star or Galaxy* task was coded as a high-level task (procedures with connections) due to the fact that, while it does not explicitly state how students are to work on the task, the steps to follow are implied based on the given information. For instance, in Question #1 students are asked to find the equation of a quadratic function given its two *x*-intercepts ($x = 2$ and $x = 6$). They are not told to use a specific procedure to do so, but based on their familiarity with the various forms of equations of quadratic functions and the information given it is fairly obvious that they should begin by setting up the equation $f(x) = (x-2)(x-6)$ and then find the product. Yet, the *Single Star or Galaxy* task was considered high level because it requires students to describe how various transformations of the original function are related to one another. They also must conjecture about the needed amount of known points necessary to determine a unique quadratic equation and then provide an explanation to support this conjecture.

4.2.2.2 A low-level task

The task Nicole Nesmith selected for her lesson on May 29 (Observation 7) was the *Radical Roller Coaster* task. This task allows students to work with radical equations in the context of designing roller coasters. To do this, it states two formulas. The first is for the velocity of the roller coaster at the bottom of a hill $v = \sqrt{64h}$ where v = velocity and h = the height of the hill. The second is for the velocity of the roller coaster after completing a loop $v = 8\sqrt{h - 2r}$ where v = velocity, h = the height of the hill, and r = the radius of the loop. The task then presents several practices problems in which students have to identify some of these variables from the description of the roller coaster and find the missing variable. However, there is no connection to the underlying mathematical concepts related to rational functions and students could engage in the task by simply plugging in the values of v, h, and/or r without considering how they are related. Thus, this task was scored as low level (procedures without connections). This task differed from the other tasks selected by Nicole Nesmith in that while all the tasks dictated a specific manner in which to engage in the mathematical problems, only this one failed to push students to reason, conjecture, provide explanations, or make mathematical meaning or connections to underlying concepts.

4.2.3 Implementation of High-Level Tasks and Use of the Five Practices During Instruction (Research Question III and Research Question IV)

4.2.3.1 Set up and implementation of high-level tasks

As seen in Table 4.9, the level of cognitive demand of the tasks set up by Nicole Nesmith always match the level of the task as it appeared in written form. Thus, six of the seven tasks used by Ms. Nesmith in her lessons were set up at a high level. However, only one of these tasks was implemented as such (April 16 - Observation 5). This task was coded as procedures with connections as it appeared in written form and as set up and implemented by Ms. Nesmith. In each of the five remaining lessons in which Ms. Nesmith was unable to maintain the high-level of the task throughout the lesson, her implementation of the task was coded as little or no academic thinking occurred during the lesson. As was the case with her selection of the tasks, it appears that there is no difference between the baseline, intervention, and maintenance measurements with regard to the level of cognitive demand of the task as set up and as implemented.

Table 4.10 displays the factors associated with the maintenance or decline of the level of cognitive demand during implementation in Ms. Nesmith's lessons. During each of the five lessons in which tasks were set up at a high level but implemented at a low level, Ms. Nesmith struggled to maintain control of the class. As a result of the poor classroom management, the majority of the students did not engage in the task for the majority of the lesson, rather they were off task and in many instances a distraction to other students. During the lesson on April 16 in which she was able to maintain the high level of cognitive demand, she did not have problems managing student behavior and the task built on students' prior knowledge of mathematical concepts during the lesson. The differences between Ms. Nesmith's implementation of the task at a high level on April 16 and her implementation of tasks at a low level during other lessons will be illustrated in the subsequent sections containing detailed descriptions of her instruction and her uptake of her group members' key ideas regarding her instruction of the focus task in the MLSC week 1 meetings.

Table 4.10: The factors associated with decline or maintenance of the cognitive demand of tasks set up by Nicole Nesmith at a high level

Observation number and date	Level of cognitive demand as set up/implemented	Factors associated with decline present in the lesson	Factors associated with maintenance present in the lesson
Obs. 1 10/25/2011	H/L	– Classroom-management problems prevent sustained engagement in high-level cognitive activities	n/a
Obs. 2 12/1/2011	H/L	– Classroom-management problems prevent sustained engagement in high-level cognitive activities	n/a
Obs. 3 1/24/2012	H/L	– Classroom-management problems prevent sustained engagement in high-level cognitive activities	n/a
Obs. 4 2/27/2012	H/L	– Classroom-management problems prevent sustained engagement in high-level cognitive activities	n/a
Obs. 5 4/16/2012	H/H	n/a	– Teacher presses for justification, explanations, and meaning through questioning, comments, and feedback. – Task builds on students' prior knowledge
Obs. 6 5/3/2012	H/L	– Classroom-management problems prevent sustained engagement in high-level cognitive activities	n/a

4.2.3.2 Use of the five practices

Nicole Nesmith's level of use of the five practices in the observed lessons is shown in Table 4.11. There was evidence of anticipating in Ms. Nesmith's lesson plans for only two of her seven lessons. Both of these lessons were the lesson during which she used a focus task she selected for the modified lesson study cycles. Ms. Nesmith made use of monitoring in every lesson. Her monitoring was scored at either a little-use or a partial-use level. During each lesson for which Ms. Nesmith's monitoring was scored as partial use, it was due to her use of assessing and advancing questions. She did not use a monitoring tool during any of the observed lessons. Ms. Nesmith did not use the practice of connecting during any of her lessons. She only used the practices of selecting and sequencing in two of the seven lessons, only one of which was a lesson for which she anticipated. Of the two lessons for which Ms. Nesmith made use of selecting, the level of use was coded as partial use for one lesson and little use for the other. During both instances in which Ms. Nesmith used the practice of sequencing, it was scored as little use.

Table 4.11: The level of use of the five practices by Nicole Nesmith during her lesson observations

	Observation number and date	Level of use of the five practices				
		Anticipating	Monitoring	Selecting	Sequencing	Connecting
Time frame 1	Obs. 1 10/25/2011	N	L	N	N	N
	Obs. 2 12/1/2011	N	P	N	N	N
Time frame 2	Obs. 3 1/24/2012	P	P	P	L	N
	Obs. 4 2/27/2012	N	L	N	N	N
	Obs. 5 4/16/2012	L	P	N	N	N
Time Frame 3	Obs. 6 5/3/2012	N	L	L	L	N
	Obs. 7 5/29/2012	N	L	N	N	N

N = No use of the practice
L = Little use of the practice
P = Partial use of the practice
H= High use of the practice

Note: The shaded rows designate the lessons that involve focus tasks from the MLSCs.

Looking at Nicole Nesmith's use of the five practices within each lesson, she made use of four of the five practices (anticipating, monitoring, selecting, and sequencing) in one of her seven observed lessons. During another, she used three of the practices (monitoring, selecting, and sequencing), and she used two of the five practices (anticipating and monitoring) during a third lesson. During the other four lessons, she only used monitoring. The lesson for which she used four of the practices involved a focus task that she selected for use in one of the professional development modified lesson study cycles.

There are no apparent patterns in Nicole Nesmith's use of the five practices, and there is no pattern of change in her use of the five practices between the baseline measurement (time frame 1), intervention measurement (time frame 2), and maintenance measurement (time frame 3). Ms. Nesmith taught two lessons during the baseline measurement, and in both she only used monitoring. However, during the intervention measurement Ms. Nesmith taught one lesson during which she used four of the five practices, one lesson during which she only used one practice, and one lesson during which she used two of the practices. In the maintenance measurement, Ms. Nesmith taught two lessons, one of which included the use of three of the practices while the other included the use of only one practice. There were no noticeable differences in Ms. Nesmith's use of the five practices between the lesson in which she successfully implemented the task at a high level and those in which she did not.

4.2.3.3 Descriptions of instruction

Nicole Nesmith's instruction during two of her lessons is described in this section. The first lesson illustrated here is a lesson during which she set up a task at a high level, but was unable to maintain it as such during implementation. The second lesson described in this section is of instruction during which she was able to maintain the high level of cognitive demand throughout the lesson. These descriptions will also portray Ms. Nesmith's use of the five practices during these lessons.

4.2.3.3.1 Instruction of a task set up at a high level but implemented at a low level. This section focuses on Nicole Nesmith's lesson on January 24 (Observation 3) during which she implemented the *Single Star or Galaxy* task described above (also see Appendix E).

During the lesson, Nicole Nesmith set up the task by passing out colored pencils to the students and reading the information provided on the top of the first page of the task. However, as she is doing this, many of the students were out of control, insubordinate, and disruptive. For example, when Ms. Nesmith first asked a student to read the beginning paragraph of the task, the student refused. Ms. Nesmith asked her again and said that she (the student) would lose her participation points for the day if she did not comply. The student responded by yelling back, "I don't care!" and again refused to read the information. Ms. Nesmith then called on a second student to read the paragraph. He did so, but many of the students were talking with each other so loudly at this point that it is very difficult to hear him read. Ms. Nesmith pressed on with the set up of the task despite the poor student behavior. She had a student read the second paragraph and then discussed with the students the meaning of the word "unique." She explained to the class that she gave each group of students four different colored pencils and told them that they will draw four different graphs all on the same coordinate grid. She asked a student to give the points associated with the x-intercepts provided in the instructions and the student did so. She also talked with students about what would cause the graph to reflect of the x-axis as called for in Question #3. Lastly, Ms. Nesmith tried to direct the students' attention to Question #4, and she told them that the two factors they select for this question must be different from the two factors their partner chooses. She then asked them to begin working on the task. However, throughout the set up of the task the majority of the students were extremely loud and did not pay attention to what Ms. Nesmith was telling them. Rather, they were talking with or yelling at each other, texting, watching videos on their phones, or otherwise off task. While the majority of the students were not focused on Ms. Nesmith during the set up of the task, this set up was coded as procedures with connections as it did not lessen the need for students to reason about the task while working on it. For example, students were still required to make and defend a conjecture about the needed number of points to determine the equation of a unique quadratic function.

Some students worked on the task during the next several minutes of the lesson. During this time, Nicole Nesmith circulated the room trying to get the remainder of the students on task and helping those students who requested help. Many of the students engaged in the task when Ms. Nesmith worked directly with them. However, as soon as she would leave to help other students, the students she had been helping immediately became off task. Most of these students seemed content not to work on the task unless Ms. Nesmith was speaking with their small groups, while a few others constantly yelled across the room for Ms. Nesmith to return and continue helping them. Of the 21 students in the room, only about six remained on task for a significant portion of the lesson. The rest were talking, texting, putting their heads down on their desks to sleep, or disturbing their classmates.

After approximately 25 minutes, Ms. Nesmith called for the students' attention and began a whole-class discussion about the task. She displayed a blank copy of the task using a document camera. As she began the discussion, most of the students continued to talk loudly over her and did not pay attention. Additionally, a student who was not a member of the class entered from the hallway and began talking disruptively, distracting even more students. At this point Ms. Nesmith had to escort the student out of the classroom. When she returned, she led the students through a discussion of the questions on the task and shared some students' work on the task. However, the majority of the students continued to talk loudly and ignored what she was saying. During this time, she was again interrupted by another student who entered from the hallway and disrupted her students. The level of cognitive demand of the task as implemented during the lesson is determined by the cognitive effort of the majority of the students over the majority of the lesson. While Ms. Nesmith did not proceduralize the task for the students or shift the emphasis away from meaning making or reasoning, it was clear that the majority of the students were not engaged in the task for the majority of the lesson. Thus, the implementation of the task was coded as low level (little or no academic thinking occurred). This type of poor, disruptive student behavior was rampant during each of the lessons for which the implementation of the task was coded as little or no academic thinking occurred. The implementation of the task in this lesson was typical of these lessons in this regard.

Nicole Nesmith used anticipating, monitoring, selecting, and sequencing at varying levels during her instruction of the *Single Star or Galaxy* task. Her anticipating for this task was coded as partial use as she anticipated multiple, specific solution methods students could use while working on the task. It was not coded as high use as she did not anticipate any incorrect strategies. In her lesson plan, Ms. Nesmith anticipated two possible strategies that students might use to find the vertex of the parabola in Question #2, (a) a symbolic approach and (b) a graphical approach. However, she did not state specifically what each of these approaches entailed. Along with anticipating these approaches, Ms. Nesmith also included questions she identified as assessing and advancing questions for each of these approaches as well as for each of the other questions on the task. The questions Ms. Nesmith included for the two approaches she identified are:

For Part 2, if students take a symbolic approach:

1. Which form of the equation will be more helpful in giving the information you need to determine the vertex and why?
2. Can you relate this to how you might determine the vertex from the graph?

If students take a graphical approach:

1. Since you know the intercepts, can you sketch what this parabola might look like?
2. Where is the vertex on your sketch? If you made tick marks for a scale on your sketch, would that help you find the axis of symmetry?

Nicole Nesmith's use of monitoring during the lesson was coded as partial use. During the portion of the lesson in which students were asked to work on the task in small groups, Ms. Nesmith circulated through the room trying to get students to engage in the task and assisted those who were working on it. During this time, she observed the methods students used as she talked with them about the task, but she did not use a monitoring tool. Ms. Nesmith also used some of the assessing and advancing questions she identified while speaking with the students about the task. For example, soon after the students began working in small groups Ms. Nesmith noticed that several of them were struggling with Question #1, which asks them to write the equation of the parabola that has the given *x*-intercepts. Ms. Nesmith moved from one small group to another and asked the students what information was given in the task that would be useful for them to create the equation. Once students mentioned the *x*-intercepts, Ms. GX would point to three posters of the different forms of the equation of quadratic functions posted on the classroom wall (see Figure 4.11) and asked which form they could most easily write using the information they had. She would then talk with them about which form they would need the equation to be in to find the vertex easily.

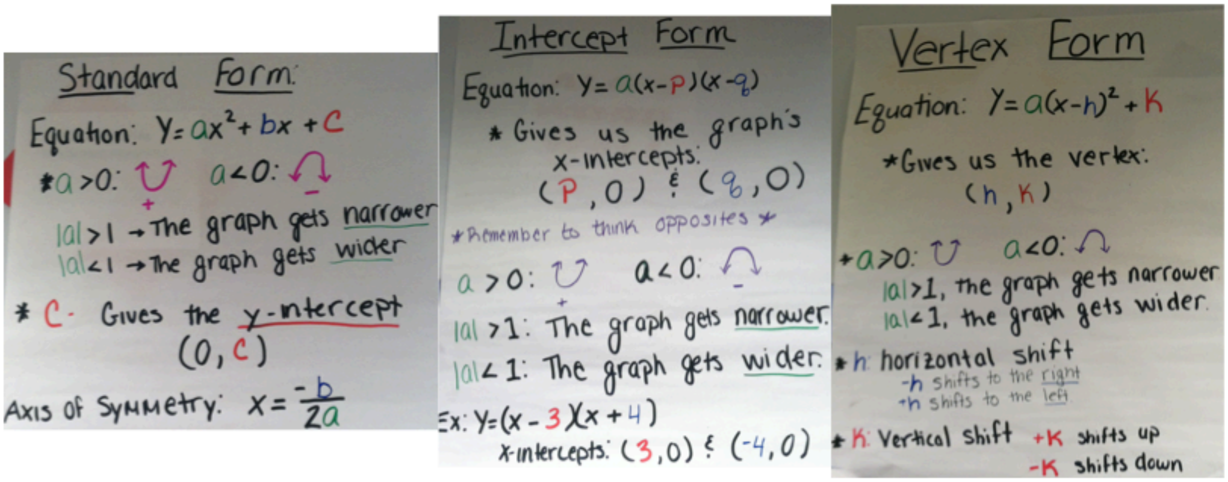


Figure 4.10: Posters of the different forms of the equation of quadratic functions created by Ms. Nesmith before the lesson

Nicole Nesmith also used the practice of selecting (at a partial-use level) and sequencing (at a little-use level) during the lesson. During the whole-class discussion when they arrived at Question #2, Ms. Nesmith asked a specific student to describe how he found the vertex of the parabola. This student said that he used his calculator. Ms. Nesmith displayed the graph of the function on the calculator using the document camera and explained how the student had used the trace function on the calculator to find the lowest point on the graph. After displaying this method for finding the vertex, Ms. Nesmith told the class that another group had used the axis of symmetry of the parabola to determine the x-value of the vertex. Additionally, at her request one of the group members explained that they found the y-value of the vertex substituting the x-value into the equation and then solving for y. This brief discussion showed that Ms. Nesmith purposefully selected three different methods for finding the coordinates of the vertex to be shared during the whole-class discussion around the task. However, there was no evidence that her sequencing of the three solution methods was purposeful, as she did not build on one to set up another. Ms. Nesmith did not use connecting during the lesson.

4.2.3.3.2 Instruction of a task set up and implemented at a high level. On April 16 (Observation 5), Nicole Nesmith taught the *Lucky Day* task—the focus task of the second MLSC. This lesson was unique among Ms. Nesmith's lessons in that it was the only lesson for which Ms. Nesmith was able to maintain the high level of cognitive demand throughout the lesson. During this lesson, Ms. Nesmith only used two of the five practices, anticipating and monitoring.

The *Lucky Day* task (Appendix L) allows students to compare linear and exponential functions in a context of money. Students are asked to contrast two plans of payment, one based on each type of function. The task forces student to use multiple representations of each type of function (e.g., symbolic notation, table, graph) and make connections between them. Further, it asks them to provide explanations and mathematical justifications for their answers. For example, Question #3 asks student to write an equation for the total amount of money they would receive after n days for each plan. They must do this using the information they gather in the table in Question #1. Question #3 also asks students to provide an explanation of what each part of this new equation represents in the context of the task. Thus, while the *Lucky Day* task gives a prescribed method for working on the task, it forces students to make conceptual meaning of what they are doing. Thus, it is considered a high-level task (procedures with connections).

To set up the task, Nicole Nesmith asked a few students to each read a paragraph on the first page of the task. She then asked them to individually consider which of the two plans they would choose. Almost immediately, students began to tell her which they would select and after some prompting, they explained why. Most students selected Option 1 (the linear relationship). A few of the reasons given for this were "Because you make more money" and "Pennies a day are dumb as (expletive)! You can't use pennies at the store." Ms. Nesmith asked the student that said Option 1 would provide more money why she thought this was the case. The student responded, "Because you start with ten thousand dollars." A few of the students said that they would select Option 2. When asked why, one student explained, "Because your money is doubling every day." When Ms. Nesmith pressed him on this by saying, "But you're starting with one penny," he told her, "It doesn't matter." Ms. Nesmith next showed the students the tables on the top of page 2 of the task and then briefly reviewed what they will need to do for Questions #2 and #3 of the task. She also stressed that Questions #4 and #5 deal with the domain and range of the function. They quickly review that domain is the input or possible *x*-values of the function and that range is the output or possible *y*-values of the function. She then told the students that they had 35 minutes to work on the task. During this set up Ms. Nesmith highlighted the important aspects of the task and provided clear instructions about what she wanted students to do while working on the task. The cognitive challenges of the task remained intact as students were still required to reason about and connect the various representations they used in the task and to provide explanations for the responses to the questions in the task. Thus, the task as set up by Ms. Nesmith was also coded as high level (procedures with connections).

It was common for Ms. Nesmith, as with many of the other mathematics teachers at the school, to have a student from the local university volunteer as a tutor during her lessons. Once the students began working on the task in small groups, this tutor would work with a small group on the task, at times remaining with one group for the entirety of the class and at other times moving from group to group. During this particular lesson, Ms. Nesmith had two of these tutors in her room to assist her. She also had the Instructional Team Leader (ITL) in the room assisting with the lesson. The ITL was a former mathematics teacher at the school who now served as a mathematics instruction expert to aid the mathematics teachers at the school with their teaching. Thus, there were four adults in the room to help the students engage in the task. Ms. Nesmith and the ITL circulated through the room observing the students as they worked on the task and talking with them about how they were attempting to solve the problems on it. The tutors each worked with one group for the duration of the lesson. As Ms. Nesmith worked with students in their small groups, she used strategic questioning to help them make progress on the task. For example, as she talked with a group of four female students about Question #3, Ms. Nesmith asked them how they thought Option 1 could be represented. Two of the students told Ms. Nesmith that it was a linear function and when Ms. Nesmith asked why, one of the students said that they would use the form $y = mx+b$. Ms. Nesmith agreed that this was the case and then asked what the independent and dependent variables represented. The student replied that the days was the independent variable and that the total amount of money was the dependent variable. Ms. Nesmith responded, "What does *m* [the slope of the linear equation] represent?" The student said that it was "the amount of increase each day." Ms. Nesmith then asked what the point "where the line crosses the axis at *x* equals zero" represented. The student told her that this was the *y*-intercept. Ms. Nesmith agreed and rephrased it as, "The starting point." She then said, "So that's usually at x equals zero, but we don't have a zero. Where do we start?" The student said that they start at day 1, and Ms. Nesmith replied, "Good, use that." This type of interaction is representative of how Ms. Nesmith worked with the students as they worked on the task in small groups, pressing them to reason about the task.

After allowing the students to work on the task in small groups for approximately 30 minutes, Ms. Nesmith called them back together for a discussion about their work on the first few pages of the task. The students provided answers to the questions and the class discussed the type of relationships they felt modeled each of the options. Students easily recognized Option 1 as a linear relationship and they were able to identify the starting amount as the *y*-intercept as well as the fact that it increased by $1,000 each day. With regard to Option 2, one student said that it was a quadratic function, but she was quickly corrected by another student who said that it was exponential. Ms. Nesmith referred the first student to a poster on the wall that they had created during a previous lesson that had the characteristics of exponential functions. After looking at the poster the student exclaimed, "Oh, it starts slow and then shoots up." The majority of the students engaged in the task at a high level, making connections between the various representations and demonstrating understanding of the underlying concepts through their responses to Ms. Nesmith's questions. Thus, the implementation was viewed as maintaining the high level of cognitive demand of the task at set up and was considered to be procedures with connections.

Nicole Nesmith only made use of two of the five practices (anticipating and monitoring) during her instruction of the *Lucky Day* task. Her level of use of anticipation was scored little use indicating that she only anticipated how students would respond to the task generally. In the lesson plan she stated that she wanted to focus on the domain and range of the problem situation as students had been struggling with this concept, thus indicating that students may struggle with it while working on the task. Ms. Nesmith's use of monitoring was coded as partial use because she moved through the room observing how students worked on the task and used assessing and advancing questions to aid them in making progress on the task. Her use of these questions was illustrated above in the description of her implementation of the task. Ms. Nesmith did not use of a monitoring tool to record students' approaches to the task. During the whole-class discussion, students provided answers to some of the questions on pages 2 and 3 of the task, but at no time did they share their methods for working on the task (e.g., provide an answer to a questions as well as a description of how and why they went about solving the task in the manner in which they did). Thus, while there was a discussion about the task and students shared reasoning about the mathematics involved, Ms. Nesmith was considered not to have used the practices of selecting, sequencing, or connecting.

4.2.3.4 Summary of implementation of high-level tasks and use of the five practices

Nicole Nesmith was not capable of regularly implementing tasks at a high level as she did so only during one lesson. In each lesson when she set up a task at a high level but failed to maintain it at this level throughout the lesson, it was due to her inability to effectively deal with student behavior issues. However, she did demonstrate that she was capable of implementing high-level tasks when provided with the additional support. This was evident in the lesson on April 16 (Observation 5) when she had two tutors and the Instructional Team Leader in the room with her to help her engage students in the task and reduce opportunities for students to disrupt the class. Ms. Nesmith's use of the five practices was sporadic, not following a perceived pattern.

4.2.4 The Relationship Between the Use of the Five Practices and the Ability to Maintain the Level of Cognitive Demand of High-Level Tasks (Research Question V)

The relationship between Nicole Nesmith's use of the five practices and the level of cognitive demand of the task she implemented in the six lessons in which she set the main instructional task up at a high level is shown in Table 4.12. The data suggest that there are no noticeable differences in her use of any of the five practices between the lesson in which she implements tasks at a high level and the lesson in which she implemented tasks at a low level. Ms.

Nesmith’s use of anticipating and selecting ranged from no use to partial use. Her use of monitoring fluctuated from little use to partial use, and her use of sequencing was either scored as no use or little use. She did not use connecting in any lesson regardless of the level of implementation.

Table 4.12: Nicole Nesmith's level of use of five practices as related to the level of cognitive demand of implementation of tasks set up at a high level across all classroom observations

	Level of cognitive demand during implementation																							
	Low-level cognitive demands																High-level cognitive demands							
	Unsystematic and nonproductive exploration				Little or no academic thinking occurred				Memorization				Procedures without connections				Procedures with connections				Doing mathematics			
Level of use of practice	N	L	P	H	N	L	P	H	N	L	P	H	N	L	P	H	N	L	P	H	N	L	P	H
Anticipating					4		1											1						
Monitoring						3	2												1					
Selecting					3	1	1										1							
Sequencing					3	2											1							
Connecting					5												1							

Note: N = no use, L = low use, P =

4.2.5 The Relationship Between the Modified Lesson Study Cycles and Implementation of the Focus Tasks (Research Question VI)

Two of the 11th- and 12th-grade team's MLSCs were centered on focus tasks selected by Nicole Nesmith. The key ideas and suggestions given by her colleagues during the MLSC week 1 meetings and her uptake of these key ideas and suggestions are described in this section.

4.2.5.1 Uptake of key ideas from the first MLSC

The task that Nicole Nesmith selected for the first MLSC was the *Single Star or Galaxy?* task (Appendix E). Her team discussed it in the MLSC week 1 meeting on January 18. Six key ideas were suggested during this meeting:

1. Push students to use both the algebraic formula to find the vertex as well as calculators and compare the methods - Ms. Nesmith believed that most students would use their calculators to find the vertex of the quadratic functions. The group members urged her to push students to also use the algebraic formula to find the vertex (i.e., the *x*-value of the vertex can be found using the formula $x = \frac{-b}{2a}$ if the function is given in the form $f(x) = ax^2 + bx + c$).
2. Graph the transformations of the function on the same grid - Ms. Nesmith had originally planned to have the students graph their various transformations of the function on different coordinate planes. However, the team members suggested that it would be beneficial for the students to graph the transformations on the same coordinate plane.

3. Students should make a regression model of the functions using the information provided in the task - One of the team members felt that Ms. Nesmith should have students use their calculators to create regression models of the functions in the task.
4. Graph without using calculators - One of the team members suggested that Ms. Nesmith should push students to graph the functions in the task without using their calculators.
5. Discuss what “unique” means in the whole-class discussion of the task at the end of the lesson - The group members felt that it would be very important for students to understand what “unique” means when working on the task, as this will affect how they are able to answer the last problem, which pushes them to make meaning of the mathematics they have done up to that point. They urged Ms. Nesmith to include a conversation about what “unique” means during the discussion after the students have worked on the task even though she already planned to discuss this during her set up of the task. They encouraged her to bring it into the conversation for a second time during the lesson.
6. Have students select different factors to use in problem #4 - The team members felt it would be beneficial to have the students select different factors than the group members with which they were working in order to better explore the mathematical concept that is the focus of the task.

Nicole Nesmith used the *Single Star or Galaxy* task during the lesson observation on January 24. She discussed her implementation of the task with her group members during the MLSC week 2 meeting on January 25. The data for the classroom observation and from the

MLSC week 2 meeting suggest that Ms. Nesmith took up two of the key ideas (key ideas #2 and #6) and did not incorporate three of the ideas (#3, #4, and #5). There is conflicting data with regard to her uptake of key idea #1.

4.2.5.1.1 Key idea 1 - Push students to use both the algebraic formula to find the vertex as well as calculators and compare the methods. During the lesson observation, only two methods for finding the vertex were discussed by Ms. Nesmith or the students, (a) using the graphing calculator, and (b) using the line of symmetry of the parabola. Neither of these incorporates the use of the algebraic formula for finding the *x*-value of the vertex of the function. However, during the week 2 professional development meeting, Ms. Nesmith explained that in a different section of the course that was not observed for this study, she had pushed one of her students to use the algebraic formula.[8] Thus, it is unclear to what extent Ms. Nesmith took up this key idea in her instruction of the task.

4.2.5.1.2 Key idea 2 - Graph the transformations of the function on the same grid. Ms. Nesmith's lesson plan for this task accessed the day of the lesson states, "Students will complete numbers 1-6 and graph the different parabolas on the same grid." Ms. Nesmith did not have a lesson plan for the task at the time of the week 1 meeting, so no comparison can be made between this lesson plan and a previous version. However, during the MLSC week 1 meeting Ms. Nesmith stated that she had planned to have the students graph all of the transformations on separate grids. During the lesson, Ms. Nesmith explicitly directed her students to graph all of the parabolas on the same coordinate plane.

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[8] Nicole Nesmith taught three sections of the same course. The observation relates to the implementation of this task only takes into account her use of the task in one of these three sections.

4.2.5.1.3 Key idea 3 - Students should make a regression model of the functions using the information provided in the task. Ms. Nesmith did not take up this idea in her implementation of the lesson. At no time during the lesson did Ms. Nesmith direct the students to create regression models using their calculators. Instead, the task directed the students to find the exact equation of the function and then to graph that function.

4.2.5.1.4 Key idea 4 - Graph without using calculators. There is no evidence that Ms. Nesmith incorporated this idea into her instruction. During the lesson, a few of the students graphed the functions without the use of their calculators, but at no time did Ms. Nesmith suggest that they do this. The majority of the students used their calculators to graph the functions during the lesson.

4.2.5.1.5 Key idea 5 - Discuss what "unique" means in the whole-class discussion of the task at the end of the lesson. Ms. Nesmith did not take up this key idea while teaching the task. During her set up of the task, Ms. Nesmith discussed with the students the meaning of the word "unique." However, this is never addressed again during the remainder of the lesson despite her colleagues' suggestion to again discuss it during the conversation at the end of the lesson. During the MLSC week 2 meeting, Ms. Nesmith confirms that this was the case for all classes in which she used this task.

4.2.5.1.6 Key idea 6 - Have students select different factors to use in problem #4. There is evidence that Ms. Nesmith integrated this key idea into her instruction of the task. As she set up the lesson, Ms. Nesmith explicitly directed the students to select a different factor than that of their partner (most of the students worked in groups). Further support for this occurred during the whole-class discussion at the end of class when Ms. Nesmith asked the members of one of small the groups to give the two factors that they used for problem #4 and the students identified two different factors.

4.2.5.2 Uptake of key ideas from the second MLSC

Nicole Nesmith selected the *Lucky Day* task (see Appendix L) as the focus task of the second MLSC centered on her instruction. This task was discussed during the MLSC week 1 meeting on April 11. During this meeting six key ideas were discussed:

1. Use this task to let students practice scaling the axes of the coordinate plane when graphing - Ms. Nesmith told the team that her students struggle to correctly create graphs and that their ability to correctly scale the axes of the coordinate plane is particularly weak. One group member suggested that this task would be a useful opportunity for students to practice this skill.
2. This task should not be used as an introduction to exponential functions - The team members did not feel that this task was an appropriate task to use as an introduction to exponential functions. Ms. Nesmith had planned to use it as such. They suggested that she have the students engage in other activities related to exponential functions before working on this task.
3. Adjust how the students talk about the situations in the problem by "adjusting the calendar" - One of the team members who had taught this task in previous years suggested that Ms. Nesmith let the students come up with their own equations for the situations in the task (which most likely will be incorrect) and then help them adjust how they use the calendar (e.g., if the question asks for the amount of money on day 30, give the answer for day 29). He felt that most students would be able to develop the equation $y = .01(2^x)$, which is incorrect, but that they

would have a difficult time creating the equation $y = .01(2^{x-1})$, which is correct. By having the students give the amount of money for the day before the one asked for, they could use the equation $y = .01(2^x)$.

4. Adjust the problem by changing the context of the problem - In order to address the potential problem of students struggle to develop the equation $y = .01(2^{x-1})$, it was suggested that Ms. Nesmith change the context of the task to fit the equation $y = .01(2^x)$.
5. Change the warm up to the "paper folding activity" - Some of the group members who are familiar with the *Connect Mathematics 2* curriculum used in the middle school grades suggested that Ms. Nesmith use a specific activity from this curriculum that they referred to as the "paper folding activity." This activity is the *Making Ballots* task in investigation 1.1 of the 8th-grade CMP book called *Growing, Growing, Growing* (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2006c). This task models the situation $y = 2^x$. The team members felt that working on this task before the *Luck Day* task might help students to think more productively about the situation in the *Luck Day* task that is modeled by $y = .01(2^{x-1})$.
6. Use an activity from the *Connected Mathematics* unit on exponential functions as a warm up - Members of the team suggested that Ms. Nesmith could use one or more activities from the *Connected Mathematics* unit on exponential functions as a way to help students find patterns in situations modeled by $y = 2^x$, $y = 3^x$, and $y = 4^x$.

Nicole Nesmith used the *Lucky Day* task during the observed lesson on April 16. She discussed her use of the task with her team during the MLSC week 2 meeting on April 18. Of the six key ideas suggested during the MLSC week 1 meeting, there is evidence that Ms. Nesmith incorporated three of the key ideas (#2, #5, #6) into her teaching of the task. There is also evidence that she did not take up two of the key ideas (#3 and #4). It is unclear whether she used key idea #1.

4.2.5.2.1 Key idea 1 - Use this task to let students practice scaling the axes of the coordinate plane when graphing. It is unclear whether or not Ms. Nesmith took up this key idea. Ms. Nesmith did not provide directions regarding the scaling of the axes during the lesson that was observed. However, on the samples of student work that Ms. Nesmith brought to the MLSC week 2 meeting, the graphs of two of the students have identical scales for the two options. This is unexpected because of the scale the students used. For option 2 in the task, the exponential option, both students used a scale on the *y*-axis that began at zero and increased by 0.45 for each line on the grid (see Figure 4.12). The choice of 0.45 was very peculiar as this not a natural section. It would seem that either these students were given this scale by the teacher as they copied the graph from a displayed solution, or that the calculators they used provided them with the same window as they graphed the points. No graphical solutions to the task were shared during the lesson that the researcher observed. However, it is unclear whether these student solutions were from the class period that was observed or from another period. Graphical solutions could have been shared during another class period.

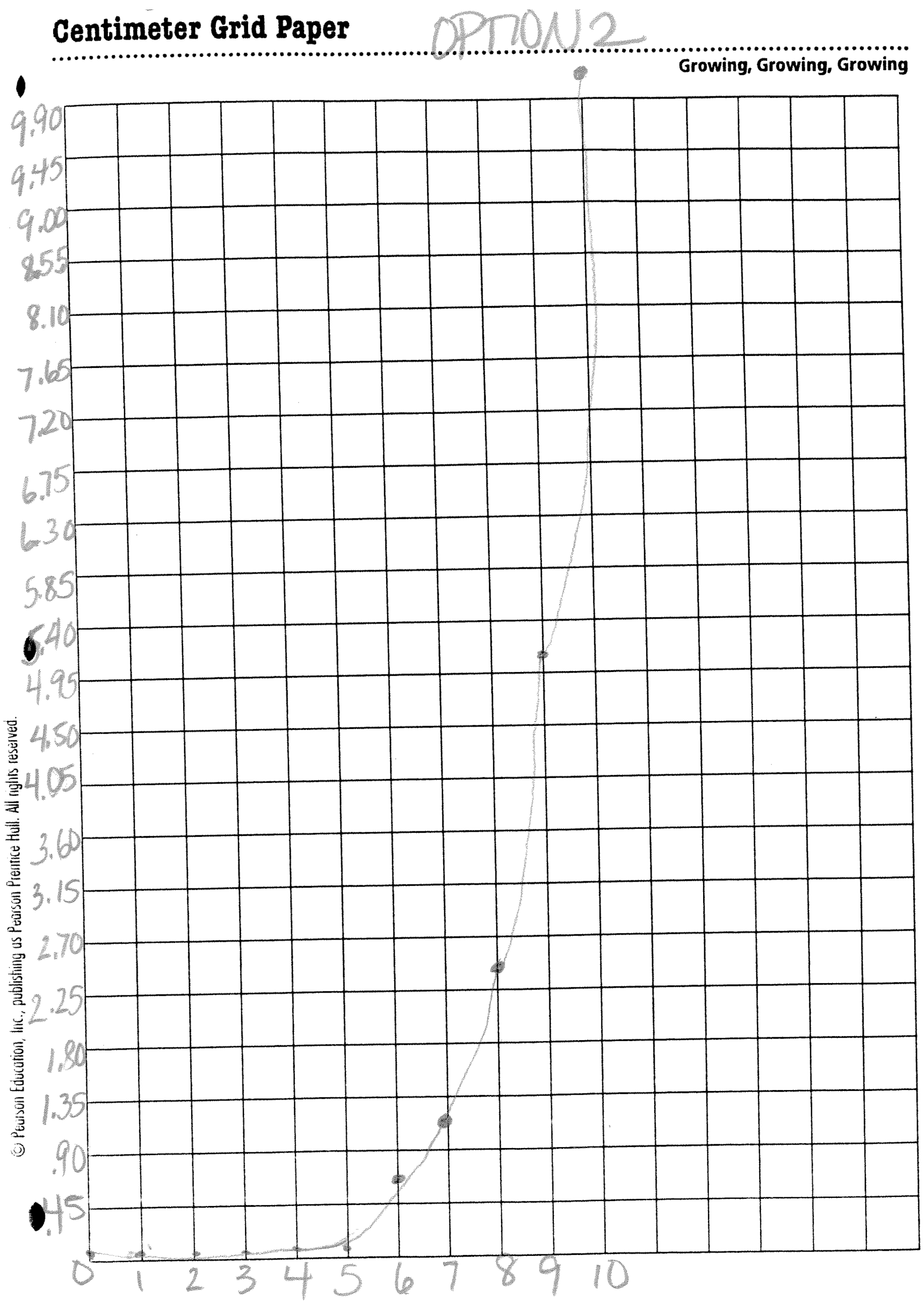

Figure 4.11: Graph created by student working on the *Luck Day* task in Ms. Nesmith's class

4.2.5.2.2 Key idea 2 - This task should not be used as an introduction to exponential functions. There was no evidence during the lesson to suggest that this was, or was not, the first task the students engaged in that dealt with exponential functions. However, during the MLSC week 2 meeting, Ms. Nesmith explained that she—along with the aid of the instructional team leader at the school—had given the students activities from the *Connected Mathematics 2* unit on exponential functions during some of the lessons leading up to the lesson using the *Lucky Day* task as a means of introducing the students to exponential functions. The instructional team leader participated in these preparation activities as she was familiar with *Connected Mathematics 2* and Ms. Nesmith was not.

4.2.5.2.3 Key idea 3 - Adjust how the students talk about the situations in the problem by "adjusting the calendar". Ms. Nesmith did not take up this suggestion. During the lesson she gave each group a calendar to use if they wanted, but she did not talk with the students about using the previous day's amount as a way to deal with the situation modeled by $y = .01(2^{x-1})$.

4.2.5.2.4 Key idea 4 - Adjust the problem by changing the context of the problem. Ms. Nesmith did not adjust the context of the task as she made no modifications to it, and thus she did not incorporate this key idea.

4.2.5.2.5 Key idea 5 - Change the warm up to the Paper Folding activity. Ms. Nesmith did not use the "paper folding activity" (the *Making Ballots* task in *Connected Mathematics 2*) as the warm up during this lesson. However, during the MLSC week 2 meeting, she explained that this was one of the tasks from *Connected Mathematics 2* that she used during the days leading up to the lesson as a way to prepare students for the *Lucky Day* task. Thus, she did incorporate this key idea into her teaching.

4.2.5.2.6 Key idea 6 - Use an activity from the Connected Mathematics 2 unit on exponential functions as a warm up. As explained with regard to Key ideas #2 and #6, Ms. Nesmith did not use a related activity from *Connected Mathematics 2* as a warm up. However, she did use multiple activities from *Connected Mathematics 2* in the days leading up to the *Lucky Day* task. Hence, she also took up this suggestion in her instruction.

4.2.5.3 Summary of uptake of key ideas in instruction

During the MLSC week 1 meetings centered on Nicole Nesmith's focus tasks, her colleagues suggested several key ideas. These ideas included suggestions regarding which solutions methods to encourage students to use, ideas to highlight during whole-class discussions, modifications to the focus tasks, and possible activities to use as warm ups or on days prior to the focus task to prepare students. There is evidence that Ms. Nesmith took up roughly half of these ideas in her instruction of the focus tasks.

4.2.6 Summary of Nicole Nesmith

Nicole Nesmith attended most of her professional development teams' meetings. She came prepared to the MLSC week 1 meetings having anticipated multiple solution strategies to the focus task and recorded her noticings and wonderings to share with the team members. She consistently selected high-level tasks to use in her classes, however she was not able to implement these tasks at a high level in all but one of her lessons. In each case, she struggled to maintain the high level of the tasks, not because she routinized the challenging aspects of the task, rather because she was unable to maintain proper student behavior during her lessons. Students would ignore her or directly disobey her requests. They would distract their classmates and make it extremely difficult for Ms. Nesmith to teach. During the one lesson in which Ms. Nesmith was able to maintain the high level of the task throughout the lesson, there were three other adults in the room aiding the students to successfully engage in the task. She had also worked with the Instructional Team Leader to use activities prior to this lesson in order to prepare students to work on the challenging task. Thus, in Nicole Nesmith we see a teacher that chose high-level tasks, and who, with additional support, was able to implement one such task at a high level.

4.3 GLORIA XAVIER

The six research questions are addressed in this section with regard to Gloria Xavier's participation in the professional development and her instruction. It includes descriptive examples to illustrate these findings.

4.3.1 Participation in the Professional Development Meetings (Research Question I)

Gloria Xavier was member of the 6th-, 7th-, and 8th-grade team. This team met 26 times during the 2011-2012 school year. Ms. Xavier attended 21 of these meetings. During the 2011-2012 school year, the school district which included Lincoln Secondary School implemented a new teacher evaluation program. As part of this effort, teachers from each school were asked to participate in training to prepare them as building representatives to help communicate the expectations and requirements of this program as well as to serve as liaisons for the program with the teachers at their respective schools. Ms. Xavier was one of the teachers participating in this training and as such, she frequently missed the professional development meetings to attend this training. It appears that at least five of Ms. Xavier's six absences were due to these trainings.

The 6th-, 7th-, and 8th-grade team took part in nine modified lesson study cycles. Ms. Xavier attended eight of the nine MLSC week 1 meetings, and she selected the focus task for two of these meetings. Table 4.13 shows the scores from the Professional Development Preparation Rubric for Ms. Xavier's work and noticings and wonderings. The shaded rows in the table indicate the meetings for which she selected the focus tasks. Ms. Xavier only created work on the focus task for one of the eight MLSC week 1 meetings she attended; this work was scored as low. She also only produced noticings and wonderings for one of the MLSC week 1 meetings that she attended, and because of the nature of this meeting the team members produced their noticings and wonderings during this meeting. Thus, Ms. Xavier did not create any noticings and wonderings in preparation for the MLSC week 1 meetings. Hence, Ms. Xavier was not prepared for the professional development meetings.

Table 4.13: Gloria Xavier's level of preparation for the week 1 meetings of the professional development cycles

		Level of preparation	
Cycle number	Meeting date	Work on the focus task	Noticings and wonderings
1	Dec. 14, 2011	No	Medium
2	Jan. 4, 2012	No	No
3	Jan. 18, 2012	*Ms. Xavier did not attend this meeting.*	
4	Feb. 8, 2012	No	No
5	Feb. 22, 2012	No	No
6	Mar. 7, 2012	Low	No
7	Apr. 11, 2012	No	*
8	Apr. 25, 2012	No	No
9	May 9, 2012	No	No

Note: The shaded rows designate the professional development cycles for which Cara Nance selected the focus task.
* The data regarding the teachers' noticings and wonderings for the meeting on April 11, 2012 was lost.

4.3.1.1 Work on the focus tasks for the MLSC week 1 meeting on March 7th

Gloria Xavier produced work on the focus task for the MLSC week 1 meeting on March 7. This meeting centered on the focus task selected by Nathan Ingram called *Using the Mean* (appendix S). This task requires students to consider a set of data displayed both as a table and in a stem plot. The students consider how adding additional data points to a set of data will affect the mean of the set. Ms. Xavier's work on this task was scored as low preparation. She answered each question on the task, but only wrote out the answer or explanation asked for. There was no evidence that Ms. Xavier considered additional manners of thinking about the task, and her work did not contain multiple approaches to the task.

4.3.1.2 Noticings and wonderings related to the focus task for the MLSC week 1 meeting on December 14th

Gloria Xavier produced noticings and wonderings during the MLSC week 1 meeting on December 14. This meeting was centered on a focus task selected by another member of the team called *Box and Whisker Plots* task. This task provides students with a table of data of various characteristics of peanut butter as well as the quality rating of many brands of peanut butter. It introduces students to box and whisker plots and asks students to calculate the minimum value of the data, the maximum value of the data, the median, the lower quartile, and the upper quartile of the data set. It also has students compare the box and whisker plots of two natural peanut butter brands versus regular peanut butter brands. Lastly, it introduces student to the interquartile range and the concept of outliers. Ms. Xavier produced two noticing and wondering pairs (see Table 4.14) that both touched on the instructional issues related to the task. As she produced multiple pairs, but both addressed the same area of focus, her noticings and wonderings were scored as medium preparation.

Table 4.14: The noticings and wonderings Gloria Xavier produced related to the *Box and Whisker Plots* task during the modified lesson study cycle week 1 meeting on December 14

N&W Pair	Noticings	Wonderings
1	"I noticed you use the MR (Mathematical Reflections) as a quiz."	"I'm wondering how are the grades looking from this and how do you prepare them to succeed at it."
2	"I noticed this lesson seems very high level."	"I'm wondering what foundation was laid to prepare them (reviewed in previous lessons and 6th and 7th grade."
		"I'm also wondering what scaffolds will be in place to catch students when they get stuck."
		"Also, is the vocabulary an issue for the students and how will they keep from getting confused?"

4.3.1.3 Possible explanation for poor preparation and participation in the professional development meetings

A possible explanation for Gloria Xavier's lack of preparation for the professional development meetings is that it may have been a result of a feud she and several of the middle school teachers were having with the administration at Lincoln Secondary School. These teachers, along with all the members of the 6th-, 7th-, and 8th-grade professional development team, were hesitant to engage in work outside of the professional development meetings. This hesitancy was based on concerns some teachers at the school had that the school administration was unjustly saddling the teachers with extra work outside of their given duties as teachers and outside of their contracted work time.

During the MLSC week 1 meeting on February 22, this became readily apparent. None of the teachers of the 6th-, 7th-, and 8th-grade team had anticipated possible solutions strategies for the focus task selected for this MLSC, nor had they created noticings and wonderings. There had been a trend in many of the previous MLSC week 1 meetings in which only one of the team members brought work on the focus task or noticings and wonderings to the meetings. At the beginning of the meeting when the mathematics coach learned that no one had prepared for the meeting, she paused the discussion about the focus task and told the teaches that it was difficult to have discussion about the task if no one had worked on it ahead of time. Ms. Xavier, forcefully yet respectfully, said that they had not done the work on the task because they were not given the time to do so. At the beginning of the school year, teachers were given professional development time built into the school day to work on assignments related to the professional development meetings. However, during the weeks before this meeting, this time had been filled with additional responsibilities (e.g., hall monitoring, additional meetings). Ms. Xavier said that the teachers had talked with the administration about these concerns but nothing had changed. The mathematics coach decided to leave the meeting and get the principal so that the teachers could express their concerns directly to him. Once the math coach left, Ms. Xavier explained to University-based team members (UBs) that she had no problem doing extra work and she pointed to the fact that she had done extra work outside of her contract time with one of them and other teachers once a week during the previous school year. However, she said that the administration at Lincoln was taking advantage of the teachers in what they were asking them to do, and the teachers union had warned the teachers at Lincoln not to take on extra work without compensation. Ms. Xavier and the other team members stated that they had no problem attending the meetings and working with the UBs, but the UBs unfortunately were "caught in the middle" of their battle with the administration regarding the extra duties they were being asked to fulfill.

To summarize Gloria Xavier's preparation for and participation in the MLSC meetings, she did very little in the form of preparation for the MLSC week 1 meetings having only produced work on a focus task for one of the eight meetings she attended; this work was rated as low preparation. Further, she failed to create noticings and wonderings in preparation for any of the meetings. Thus, although she attended the majority of the team meetings, her participation as measured via meeting preparation was minimal.

4.3.2 The Level of Cognitive Demand of Tasks (Research Question II)

The level of cognitive demand of the tasks as selected (in written form), set up, and implemented by Gloria Xavier are presented in Table 4.15. During six of the eight lessons, Ms. Xavier used high-level tasks, five of which were coded as procedures with connections tasks and one as a doing mathematics task. The tasks selected for the other two observations were low-level tasks; both were coded as procedures without connections tasks. The data suggest that there was an increase of the level of cognitive demand between the baseline measurement (time frame 1) and the intervention and maintenance measurements (time frames 2 and 3). The two tasks Ms. Xavier used during baseline measurement were both low-level tasks, whereas all of the tasks during the two latter time frames were high-level tasks.

Table 4.15: The level of cognitive demand of the tasks as selected, set up, and implemented by Gloria Xavier during her lesson observations

	Observation number and date	Level of cognitive demand of the task as Selected (in written form)	Set up	Implemented
Time frame 1	Obs. 1 10/27/2011	PWoC	PWoC	PWoC
	Obs. 2 12/8/2011	PWoC	PWoC	PWoC
Time frame 2	Obs. 3 2/24/2012	**PWC**	**PWC**	PWoC
	Obs. 4 3/23/2012	**PWC**	PWoC	PWoC
	Obs. 5 4/20/2012	**DM**	**PWC**	PWoC
	Obs. 6 4/27/2012	**PWC**	LN	LN
	Obs. 7 5/11/2012	**PWC**	**PWC**	PWoC
Time Frame 3	Obs. 8 5/24/2012	**PWC**	**PWC**	PWoC

LN = Little or no academic thinking required by the task/ occurred during the lesson
Mem = Memorization task
PWoC = Procedures without connections task
PWC = Procedures with connections task
DM = Doing mathematics task
Unsys = Unsystematic and nonproductive exploration
O = Other

Note: High-level tasks (e.g., procedures with connections task and doing mathematics tasks) are bolded in the table. The shaded rows designate the lessons that involve focus tasks from the MLSCs.

Following are descriptions of two of the tasks selected by Gloria Xavier with explanations for how they were scored with regard to their level of cognitive demand in written form. One rated as high level, the other as low level.

4.3.2.1 A high-level task

For her lesson on April 20 (Observation 5), Gloria Xavier selected the task called *Walking to Win*, from the *Connected Mathematics 2* curriculum (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2006d). The *Walking to Win* (Appendix T) task gives the walking rate of two brothers, Emile and Henri, and suggests a race in which the slower of the two brothers receives a head start. The students must explore the situation to determine the length of the race to ensure a specified outcome (e.g., the brother with the slower walking rate wins, but by a small margin). They also must describe the strategy they used to find the length of the race, and they must justify the length as well as the strategy they describe. This task was coded as high level (doing mathematics), as students must explore the two linear relationships that represent the walking rates of the brothers, the head start given to one of the brothers, and the conditions that would produce the desired outcome. No possible strategies for doing this are suggested or implied in the task. Thus, students must explore various solution paths.

4.3.2.2 A low-level task

Gloria Xavier used the *Drawing Wumps* (Appendix U) task from the *Connected Mathematics 2* curriculum (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2006e) during her lesson on October 27 (Observation 1). This task provides students with a very long list of coordinate pairs (points on a graph), all of which are in the first quadrant of the Cartesian plane. When connected, these points form a block figure that resembles a human. Students are then given "rules" [e.g., ($2x$, $2y$) - indicating that each x- and y-value should be multiplied by 2] to produce similar figures. These figures are transformations of the original shape (e.g., they have been magnified or stretched). Students are asked to find the points for four additional figures using these rules and then are asked to graph each figure. Lastly, students are asked to compare and contrast the figures: Question C1 - “Compare the characters to Mug. Which are the impostors?”; Question C2 - “What things are the same about Mug and the others?”; and Question C3 - “What things are different about the five characters?” While students are asked to make comparisons, these comparisons do not necessarily push them to make connections to mathematical ideas or reason mathematically. Thus, this task was scored as low level (procedures without connections).

4.3.3 Implementation of High-Level Tasks and Use of the Five Practices During Instruction (Research Question III and Research Question IV)

4.3.3.1 Set up and implementation of high-level tasks

As seen in Table 4.15 above, of the six tasks coded as high level in written form, Ms. Xavier set four of them up as high-level tasks. Both of the high-level tasks (as they appeared in written form) that were set up at a low level were considered procedures with connections tasks as found in the curriculum material. One declined to a procedures without connections task during set up, and the other declined to have little or no academic thinking required by the task once it was set up by Ms. Xavier. The level of cognitive demand of all four of the tasks set up as high-level tasks declined to a low level—in each instance the tasks were coded as being implemented as procedures without connections tasks.

There is no data with regard to Ms. Xavier’s ability to maintain the level of cognitive demand of tasks set up at a high level during the baseline measurement (time frame 1), as she did not set up any tasks at a high level during this time frame. There was no change in this ability during the intervention and maintenance measurements (time frames 2 and 3 respectively) because each task that was set up at a high level during these time frames declined to a low level during implementation.

Table 4.16 presents the factors of decline of high-level tasks that were present in Gloria Xavier's instruction. In three of the four lessons during which Ms. Xavier was unable to sustain the high level of the task throughout the lesson, it was due to a lack of press for reasoning. This in turn led to a shift in emphasis away from conceptual meaning making toward the correctness of the answers to the problems. In one lesson, this was compounded as there was insufficient time for the students to engage in meaning making during a whole-class discussion. The factors that led to the decline during the lesson on April 20 (Observation 5) are unclear. The students seemed to have the requisite prior knowledge and Ms. Xavier did not appear to shift the emphasis from conceptual understanding to correct answers. However, the students were unable to correctly reason about the task. Ms. Xavier's implementation of this task will be discussed in detail in a subsequent section of this chapter.

Table 4.16: The factors associated with the decline of the cognitive demand of the tasks set up at a high level by Gloria Xavier

Observation number and date	Level of cognitive demand as set up/implemented	Factors associated with decline present in the lesson
Obs. 3 2/24/2012	H/L	– The teacher shifts the emphasis form meaning, concepts, or understanding to the correctness or completeness of the answer (there was a lack of press for reasoning) – Not enough time is provided to wrestle with the demanding aspects of the task
Obs. 5 4/20/2012	H/L	– Unclear: The students seemed to have the required prior knowledge, but they were unable to correctly reason about the task.
Obs. 7 5/11/2012	H/L	– The teacher shifts the emphasis form meaning, concepts, or understanding to the correctness or completeness of the answer (there was a lack of press for reasoning)
Obs. 8 5/24/2012	H/L	– The teacher shifts the emphasis form meaning, concepts, or understanding to the correctness or completeness of the answer

4.3.3.2 Use of the five practices

Table 4.17 shows Gloria Xavier's level of use of the five practices during her eight observed lessons. For each practice, there were at least two observations during which Ms. Xavier's level of use of the practice was considered little use or partial use. There was evidence in the lesson plans of three of the observed lessons of anticipating, two at a partial-use level and one at a little-use level. Ms. Xavier used monitoring, selecting, and sequencing at a level of either little use or partial use in five of the eight lessons, however these did not always occur with in the same lesson. In all four of the observations for which Ms. Xavier's level of use of monitoring was scored as partial use, this level was achieved because she used assessing and advancing questions. She did not use a monitoring tool during any of the observations. Ms. Xavier only used connecting in two of the eight lessons, once at a partial-use level and once at a little-use level.

Table 4.17: The level of use of the five practices by Gloria Xavier during her lesson observations

	Observation number and date	Level of use of the five practices: Anticipating	Monitoring	Selecting	Sequencing	Connecting
Time frame 1	Obs. 1 10/27/2011	N	P	N	N	N
	Obs. 2 12/8/2011	N	N	L	L	N
Time frame 2	Obs. 3 2/24/2012	P	P	L	L	N
	Obs. 4 3/23/2012	N	P	P	L	P
	Obs. 5 4/20/2012	L	P	L	L	N
	Obs. 6 4/27/2012	P	N	N	N	N
	Obs. 7 5/11/2012	N	L	L	L	L
Time frame 3	Obs. 8 5/24/2012	N	N	N	N	N

N = No use of the practice
L = Little use of the practice
P = Partial use of the practice
H= High use of the practice

Note: The shaded rows designate the lessons that involve focus tasks from the MLSCs.

Looking at Gloria Xavier's use of the five practices within each lesson also shows a varied level of use of the practices. Her use of each of the practices fluctuated between no use and partial use, but never reached a level of high use. During her lesson on May 24 (Observation 8), she did not use any of the five practices. During two of the lessons (October 27 - Observation

1 and April 27 - Observation 6), Ms. Xavier only made use of one of the five practices while during the lesson on December 8 (Observation 2) she employed two of the five practices. In the remaining four lessons Ms. Xavier used four of the five practices, however she did not always use the same four practices during these observations.

The data suggest that Gloria Xavier's use of the five practices was sporadic, as there are no patterns of change in her use of the practices between the baseline measurement (time frame 1), intervention measurement (time frame 2), and maintenance measurement (time frame 3). During the baseline measurement, Ms. Xavier taught a lesson during which she only used one of the five practices (monitoring at a partial-use level) and another lesson during which she used two of the five practices (selecting and sequencing both at a little-use level). During the intervention measurement Ms. Xavier taught five lessons. In four of these five lessons, Ms. Xavier made use of four of the five practices. During the remaining lesson in this time frame, she only used one of the five practices. Ms. Xavier taught only one lesson during the maintenance measurement and in this lesson she did not use any of the five practices.

4.3.3.3 Descriptions of instruction

This section contains a detailed illustration of Gloria Xavier's instruction. Only one lesson is described as Ms. Xavier failed to implement a task at a high level during any of her lessons. This lesson was chosen because it was typical of Ms. Xavier's instruction in which high-level tasks were implemented at a low level.

4.3.3.3.1 Instruction of a task set up at a high level but implemented at a low level. This section focuses on Gloria Xavier's use of the *Walking to Win* task (Appendix T) described in section 4.3.2.1.

Gloria Xavier began her set up of this task by asking the students to look at the first page of the task. She explained the context of the task and wrote the walking rate of each of the brothers on the board. She asked the students which of the brothers is faster, and after they identified that Emile is faster Ms. Xavier asked the students for reasons why. A male student responded, "Emile's faster because he walks 2.5 meters in 1 second and Henri only walks 1 meter in 1 second." Ms. Xavier replied that she liked this answer and then said, "It sounds like you are already making a table in your head." They continued to converse about the task, discussing what the variables were in the task. Ms. Xavier asked the students to identify which of the variables was the independent variable (time) and which was the dependent (distance) variable and to explain why. Ms. Xavier told the class that they know that the independent variable was *x* and the dependent variable was *y* and she writes:

ind $\rightarrow x$

dep $\rightarrow y$

She then asked, "Which [time or distance] is *x* and which is *y*?" A female student said, "So distance is *y* and time is *x*." Ms. Xavier asked the student why, and the student said, "Because you can stop your distance but you can't stop your time." Ms. Xavier agreed with this and then made a table (see Table 4.18) and said this would be one way to solve the problem. She read problem 2.1 on the top of page 25. She then stopped and said that she was not going to complete the table she created, and she erased the numbers in the *x* column. Ms. Xavier then asked the class, "What have we worked with interchangeably?" A student replied that they have used equations, tables, and graphs in their past lessons. Ms. Xavier said that this was correct and asked, "So it [the problem] does not tell you how to figure this out, use whatever way you want." Ms. Xavier then directed the students to begin working on the task.

Table 4.18: Table created by Gloria Xavier during the set up of the *Walking to Win* task on April 20

Time *x*	Distance *y*
0	
1	
2	
3	
4	
5	

This set up of the task was coded as high level (procedures with connections). While Ms. Xavier attempted to direct the students toward using whatever method they thought would be productive, her prior suggestion of using a table implied that this was the proper way to work on the task. In fact, after the lesson Ms. Xavier commented to the researcher that she wished she had not put this table up as she felt it steered all the students to use tables on the problem instead of letting them work on it in whatever manner they wanted. It was also evident from the students' work on the task that this suggestion impacted their thinking on the task.

Although the task was set up at a high level, it was not implemented as such. The students all began creating tables to represent the two linear relationships and explore the task. However, in each case these tables were flawed either with erroneous data or not connected to the problem situation. For example, one student F6 created separate tables for the two brothers (see Tables 4.19 and 4.20). These tables correctly depicted the walking rate for each brother. However, the table representing Henri's distance (Table 4.20) did not take into account the head start he is given.

Table 4.19: Table created by F6 representing the distance traveled by Emile after *t* seconds

Emile	
Time	Distance
0	0
1	2.5
2	5
3	7.5
4	10
5	12.5

Table 4.20: Table created by F6 representing the distance traveled by Henri after *t* seconds

Henri	
Time	Distance
0	0
1	1
2	2
3	3
4	4
5	5

After Ms. Xavier pointed out to F6 that she has not accounted for the head start, F6 changed her table for Henri. However, the new table still did not properly represent the problem situation, as it was just an extension of the table she previously created for Henri with a gap between 0 and 45 seconds (see Table 4.21).

Table 4.21: Adjusted table created by F6 representing Henri's distance after *t* seconds

Henri	
Time	Distance
0	0
45	45
50	50
55	55
60	60
65	65

After allowing the students to work on the task individually or in small groups for 15 minutes, Gloria Xavier called for all the students' attention and began a whole-class discussion around what they had done up to this point. She asked one student to display the table that she had started. The student was hesitant because she had not completed it yet and she believed she might have made some mistakes. Ms. Xavier told this student that she wanted the other students to see how the table is laid out, however the student remained hesitant to share her work. Ms. Xavier did not push this student to show her table; rather she created a table using the same design as this student's table. This table consisted of three columns; one for time and the other two to display the distance each of the brothers had traveled. Ms. Xavier then noted that one of the students used an important idea when creating his table, and she asked this student to display his table. He did so and Ms. Xavier asked what this student had done. The other students noticed that the student had "skip counted" when selecting the values for time to place in the table (e.g., 0 seconds, 5 seconds, 10 seconds, etc.). They then discussed why this might be advantageous. Ms. Xavier led the students in completing the table she had created with both Emile's and Henri's distances displayed (see Table 4.22). They began increasing the *t*-values by 1 each time but quickly moved to skip counting by fives. They continued to complete the chart and identified at what time the distances of the two brothers would be equal. At this point, the class ended.

Table 4.22: Table created during the whole-class discussion during the lesson on April 20

Distance (m)		
Time (Sec)	Emile	Henri
0	0	45
1	2.5	46
2	5	47
3	7.5	48
4	10	49
5	12.5	50
10	25	55
15	37.5	60
20	50	65
25	62.5	70

Gloria Xavier's implementation of the *Walking to Win* task was coded as low level (procedures without connections). The students tried to use a table to represent the data, but their tables were flawed. They were able to verbalize that Henri had a head start and that Emil was moving faster, but they could not represent this correctly using the tables. They were also only able to correctly make a table and determine how the two linear relationships would interact after Ms. Xavier led them in creating it during the whole-class discussion at the end of the lesson. It is unclear what factors led to the decline of the task.

Gloria Xavier made use of four of the five practices during her instruction of the *Walking to Win* task. Her lesson plan suggests that students might use various methods for exploring the task, but it does not identify any of these methods specifically. This was viewed as anticipating how students would engage in the task generally, and thus her level of anticipation was scored as little use.

Ms. Xavier used monitoring at a level of partial use. This was due to the fact that she observed how students engaged in the task and used assessing and advancing questions to speaking with them about their work and their thinking on it as they worked in small groups. However, she did not use a monitoring tool. An example of her use of assessing and advancing questions occurred as she spoke to F6 about the two tables she originally created depicting the distance the two brothers had traveled after time *t* (Tables 4.26 and 4.27). Their conversation was captured in the field notes taken during the lesson. Here is the portion in which Ms. Xavier uses assessing and advancing questions:

Ms. GX: "Ok, so what does Henri have?"

F6: "A head start."

Ms. GX: "What does this means?"

F6: "There's a lot of distance between them."

Ms. GX: "When is there a lot of distance between the two boys?"

F6: "At the start."

Ms. GX: "So look at your tables, does that show a head start?"

Ms. Xavier purposefully selected students' work to display during the lesson. The first table she displayed did not include a unique strategy for solving working on the task. Rather, it was used to show the students a convenient way to display all of the data in one table. She also asked another student to show his table. This was done to display the idea of increasing the value of *t* by more than one for each row in the table. Combining these made public one method for working on the task. However, no other strategies (e.g., equations or graphs) were discussed. Thus, Ms. Xavier's use of selecting and sequencing was coded as little use, as only one approach was made public.

4.3.3.4 Summary of implementation of high-level tasks and use of the five practices

Although Gloria Xavier set up tasks in four of her lessons at a high level, she was unable to implement these as such during any of her lessons. She typically failed to do so as she failed to press students to reason conceptually about the tasks. Her use of the five practices was erratic as there were lessons in which she failed to use any while in other lessons she used four of the five. However, her use of the practices was usually scored as no use or little use as she seldom employed any practice at a partial-use level and never did so at a high level.

4.3.4 The Relationship Between Use of the Five Practices and the Ability to Maintain the Level of Cognitive Demand of High-Level Tasks (Research Question V)

Table 4.23 displays the relationship between Gloria Xavier's use of the five practices and the level of cognitive demand of the tasks implemented in her lessons during which she set up the tasks at a high level. Gloria Xavier set up the main instructional task at a high level in four of her eight observed lessons. In each case, the task was implemented as a low-level task (procedures without connections). As a result, no conclusions can be drawn from the data with regard to differences between her levels of use of the five practices between her instruction of high- and low-level tasks. However, the data does reveal that Ms. Xavier's level of use of the five practices was at no use or little use for the majority of the lessons in which the level of cognitive demand decreased during implementation.

Table 4.23: Gloria Xavier's level of use of five practices as related to the level of cognitive demand of implementation of tasks set up at a high level across all classroom observations

	Level of cognitive demand during implementation																							
	Low-level cognitive demands																High-level cognitive demands							
	Unsystematic and nonproductive exploration				Little or no academic thinking occurred				Memorization				Procedures without connections				Procedures with connections				Doing mathematics			
Level of use of practice	N	L	P	H	N	L	P	H	N	L	P	H	N	L	P	H	N	L	P	H	N	L	P	H
Anticipating													2	1	1									
Monitoring													1	1	2									
Selecting													1	3										
Sequencing													1	3										
Connecting													3	1										

Note: N = no use, L = low use, P = partial use, and H = high use.

4.3.5 The Relationship Between the Modified Lesson Study Cycles and Implementation of the Focus Tasks (Research Question VI)

Two of the 6th-, 7th-, and 8th-grade team's MLSC were centered on tasks selected by Gloria Xavier. However, due to scheduling difficulties and communication issues, she was only observed teaching the second of these two tasks. In addition, due to these same problems, she did not debrief her instruction of the first task she selected for the MLSC meetings during the second week of the cycle centered on it. Thus, this section only investigates Gloria Xavier's uptake of the key ideas made by her colleagues with regard to her focus task for during one MLSC.

4.3.5.1 Uptake of key ideas from the MLSC centered on focus task

Gloria Xavier selected the focus task *3.2 Exploring Equality* from the seventh-grade *Connected Mathematics* book *Moving Straight Ahead* (Lappan et al., 2006d) (see Appendix V). This was the focus task of the 6th-, 7th-, and 8th-grade team's MLSC week 1 meeting on April 25. During this meeting four key ideas were discussed:

1. Provide manipulatives for students to use while working on the task - The mathematics coach described how she used to teach this lesson when she taught seventh grade. She stressed the use of manipulatives to aid the students in exploring the task and suggested this to Ms. Xavier.
2. Use the Getting Ready portion of the task during the set up of the task - Another group member who also taught 7th grade and was ahead of Ms. Xavier's pacing of the curriculum had taught this task the day prior to the week 1 meeting. He said that he used the examples in the Getting Ready portion of the task to

introduce the concepts to the students and suggested that Ms. Xavier use this portion of the task as well.

3. Use a balance scale as an analogy of solving the problems in the task - The teacher who had taught the task the day prior to the meeting described how he used a balance scale as an analogy of how students needed to work on the problems in the tasks. Ms. Xavier liked this idea and said that she should use it during her lesson.
4. Model how to set up equations in the problem using the manipulatives - The group felt that if Ms. Xavier planned on providing manipulatives to the students to use as the work on the task that she should model how to use them during the set up of the task.

Gloria Xavier used the focus task in her observed lesson on May 11 and she discussed her instruction of the task during the MLSC week 2 meeting on May 16[9]. There is evidence that Gloria Xavier incorporated two of the four key ideas in her instruction of the task (key ideas #2 and #3). There is also conflicting evidence with regard to whether or not she took up key idea #1 while teaching the lesson. There is no evidence that suggests used key idea #4 during her lesson.

4.3.5.1.1 Key idea 1 - Provide manipulatives for students to use while working on the task. There is conflicting evidence concerning Ms. Xavier's use of this key idea. During the observed lesson of her instruction of the focus task, Ms. Xavier did not provide any manipulative for the students to use, rather she provided a lab sheet that had the diagrams of the coins and

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[9] Gloria Xavier had not used this task with her students prior to the MLSC week 2 meeting (on May 2) that was held one week after her team discussed the task with her. Typically, the team would have debriefed her instruction of the task during this meeting. However, as this was not possible, Ms. Xavier discussed it during a portion of the week 2 meeting of the following MLSC.

pouches shown on page 50 in the curriculum materials. However, during the discussion in the MLSC week 2 meeting, Ms. Xavier said that she forgot to distribute the manipulative during the class observed by the researchers, but that she did distribute counting chips that students used to represent the coins in the problem during a her instruction of the same lesson to another section of the course.[10]

4.3.5.1.2 Key idea 2 - Use the Getting Ready portion of the task during the set up of the task. Ms. Xavier did take up this idea. She used each example in the Getting Ready portion of the task while setting up the task with her students. Ms. Xavier worked through these with the students as she introduced the context and mathematical concepts related to the task.

4.3.5.1.3 Key idea 3 - Use a balance scale as an analogy of solving the problems in the task. Ms. Xavier also incorporated this key idea into her use of the task. In conjunction with key idea 2, Ms. Xavier used the analogy of a balance scale while working through the examples listed in the Getting Ready portion of the task. She drew diagrams on the board of a balance scale while working through the examples (see Figure 4.13).

[10] Gloria Xavier taught three sections of 7th-grade mathematics.

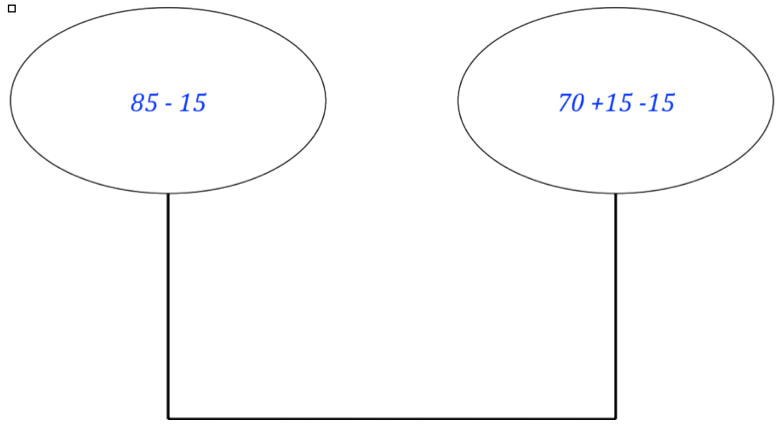


Figure 4.12: Recreation of a diagram of a balance scale used by Ms. Xavier during her set up of the focus task taught on May 11

4.3.5.1.4 Key idea 4 - Model how to set up equations in the problem using the manipulatives. There is no evidence that Ms. Xavier incorporated this idea into her teaching. As she did not have students use manipulatives during the observed lesson, she did not need to model how to use them. She made no mention during the MLSC week 2 meeting of modeling how to use the manipulatives for the students in her other classes in which she distributed them.

4.3.5.2 Summary of uptake of key ideas made during the MLSC meeting in instruction

The members of Gloria Xavier's professional development team made suggestions regarding her instruction of the focus task selected for the MLSC week 1 meeting on April 25. These included specific ideas of how to launch the task as well as the suggestion to provide manipulatives for the students to use and to model how to successfully use these manipulatives. Ms. Xavier used two of these suggestions during her observed lesson and suggested that she used a third while implementing the task with another section of students.

4.3.6 Summary of Gloria Xavier

Gloria Xavier was a member of the 6th-, 7th-, and 8th-grade professional development team. She attended the majority of the team meetings, however she failed to anticipate possible student solutions for the focus task and to create noticings and wonderings for all but one meeting. Thus, she arrived unprepared for the meetings. Ms. Xavier selected high-level tasks in six of her eight observed lessons. She set up four of these six tasks at a high level during the lessons, but she failed to implement any at a high level. During the lesson in which she failed to maintain this high level do cognitive demand throughout implementation, she typically failed to press students to reason conceptually about the task. Ms. Xavier's use of the five practices was sporadic, following no pattern with regard to which practice she used on a consistent basis. However, when she did employ these practices, they were typically used at a little-use level. Ms. Xavier received suggestions from her colleagues regarding her instruction of the focus tasks she selected. There is evidence that she incorporated some of these suggestions in her teaching.

4.4 NATHAN INGRAM

This section addresses the six research questions of the study as they relate to Nathan Ingram's participation the professional development meetings and his instruction. In so doing, it provides examples illustrating the findings in the form of a narrative case.

4.4.1 Participation in the Professional Development Meetings (Research Question II)

Nathan Ingram was also a member of the 6th-, 7th-, and 8th-grade team. He attended 16 of the 26 meetings. In addition to teaching 6th-grade mathematics, Nathan Ingram also served as the school's athletic director. His duties in this role pulled him away from the professional development meetings at times, particularly during the spring of 2012. For example, of his eight absences, six were the last six professional development meetings of the year (from mid-April through May). This responsibility may have been the cause of some or all of these absences.

The 6th-, 7th-, and 8th-grade team engaged in nine MLSCs. Mr. Ingram attended six of the nine MLSC week 1 meetings, and he selected the focus task for two of these meetings. Table 4.24 shows the scores from the Professional Development Preparation Rubric for Mr. Ingram's work and noticings and wonderings for these meetings. The shaded rows designate the meetings for which he selected the focus tasks. Mr. Ingram only created work on the focus task one of the six meetings he attended. This work was scored as low preparation using the Professional Development Preparation Rubric. Mr. Ingram also only produced noticings and wonderings for one meeting. However, as with Gloria Xavier, this was for the first cycle, in which the group members created their noticings and wondering during the meeting. The data regarding the group members' noticings and wonderings for the MLSC week 1 meeting on April 11 was lost, and it is unknown whether Mr. Ingram produced noticings and wonderings for this meeting. Thus, Mr. Ingram did not create noticings and wonderings prior to the MLSC week 1 meeting for any modified lesson study cycles. Hence, Mr. Ingram was not prepared for the team's meetings.

Table 4.24: Nathan Ingram's level of preparation for the week 1 meetings of the professional development cycles

Cycle number	Meeting date	Level of preparation: Work on the focus task	Level of preparation: Noticings and wonderings
1	Dec. 14, 2011	No	Medium
2	Jan. 4, 2012	No	No
3	Jan. 18, 2012	*	No
4	Feb. 8, 2012	*Mr. Ingram did not attend this meeting.*	
5	Feb. 22, 2012	No	No
6	Mar. 7, 2012	No	No
7	Apr. 11, 2012	Low	**
8	Apr. 25, 2012	*Mr. Ingram did not attend this meeting.*	
9	May 9, 2012	*Mr. Ingram did not attend this meeting.*	

Note: The shaded rows designate the professional development cycles for which Cara Nance selected the focus task.
* The member of the team who was assigned to select a focus task for the meeting was unable to do so due to illness. Thus, the team members were not asked to create work for this meeting.
** The data regarding the teachers' noticings and wonderings for the meeting on April 11, 2012 was lost.

4.4.1.1 Work on the focus tasks for the MLSC week 1 meeting on April 11

Nathan Ingram created work on the focus task for the MLSC week 1 meeting on April 11. The focus task for this meeting was a task called *Relating Fraction and Decimal Multiplication* that was selected by another member of the team. This task requires students to multiply numbers with decimals in the context of purchasing food. In part A, the task asks students to estimate the price of a given amount of food based on the information in the task; write the decimals in the problem as fractions with denominators of 10, 100, and 1000; and to then calculate the exact price of the food. In parts B and C, students are asked to explain when it is logical to use multiplication to solve problems such as those in part A. They are also asked to conjecture about the size of the product of two numbers, related to the size of one of the factors, when the other factor is greater than 1 as well as when the other factor is less than 1. For part A Mr. Ingram wrote out each product using the fraction notation asked for and found the exact cost. He did not include estimation. For parts B and C, he wrote one-sentence explanations for each question. Mr. Ingram's work was scored as low preparation as he only anticipated one manner in which students might respond to the task.

4.4.1.2 Noticings and wondering related to the focus task for the MLSC week 1 meeting on Dec. 14

Mr. Ingram produced noticings and wondering on the task called Box and Whisker Plots (described in section 4.3.1.2) during the professional development meeting on December 14. This task was described with regards to noticings and wonderings produced by Gloria Xavier during this meeting in section 4.3.1.2. Mr. Ingram created two noticing and wondering pairs related to this task (see Table 4.25). These addressed specific elements of the task and touched on specific mathematical concepts students would have to consider as they worked on the task. These noticings and wonderings were rated as medium preparation.

Table 4.25: The noticings and wonderings Nathan Ingram produced related to the *Box and Whisker Plots* task during the modified lesson study cycle week 1 meeting on December 14

N&W Pair	Noticings	Wonderings
1	"Notice that students will need to use data from beginning of investigation."	"Wonder how to address issues of 'real life' use of box and whisker plot.
2	"Notice that students will be using minimum, maximum, and median."	"Wonder if students will have difficult time with upper/lower quartile."
		"Wonder if students will understand what data to use and if they will order numbers correctly."
		"Wonder if they will be able to read box and whisker plots."

To summarize, Nathan Ingram did very little to prepare for the MLSC week 1 meetings as he only produced work outside of these meetings once. Even then, this work was rated as low preparation. He also never created noticings and wonderings in preparation for the meetings. Thus, his participation as measured via meeting preparation was minimal.

4.4.2 The Level of Cognitive Demand of Tasks (Research Question II)

Table 4.26 shows the level of cognitive demand of the tasks as selected (in written form), set up, and implemented by Nathan Ingram during his observed lessons. Each of the tasks was coded as high level. Five of the tasks were procedures with connections tasks and the other two were doing mathematics tasks. As all the tasks were coded as high-level tasks, there was no apparent change in the level of cognitive demand of the tasks Mr. Ingram selected across the baseline, intervention, and maintenance measurements (time frames 1, 2, and 3 respectively).

Table 4.26: The level of cognitive demand of the tasks as selected, set up, and implemented by Nathan Ingram during his lesson observations

	Observation number and date	Level of cognitive demand of the task as Selected (in written form)	Set up	Implemented
Time frame 1	Obs. 1 10/27/2011	**PWC**	**PWC**	PWoC
	Obs. 2 12/13/2011	**PWC**	**PWC**	PWoC
Time frame 2	Obs. 3 1/5/2012	**DM**	**DM**	PWoC
	Obs. 4 3/1/2012	**PWC**	PWoC	PWoC
	Obs. 5 3/12/2012	**PWC**	**PWC**	PWoC
Time Frame 3	Obs. 6 4/30/2012	**DM**	**DM**	PWoC
	Obs. 7 5/22/2012	**PWC**	PWoC	PWoC

LN = Little or no academic thinking required by the task/ occurred during the lesson
Mem = Memorization task
PWoC = Procedures without connections task
PWC = Procedures with connections task
DM = Doing mathematics task
Unsys = Unsystematic and nonproductive exploration
O = Other

Note: High-level tasks (e.g., procedures with connections task and doing mathematics tasks) are bolded in the table. The shaded rows designate the lessons that involve focus tasks from the MLSCs.

This section includes a description of two of the tasks Nathan Ingram used during his instruction. Both of these tasks were rated as high level, however one task was coded as doing mathematics and the other as procedures with connections. Each of the tasks is representative of either the doing mathematics or the procedures with connections tasks used by Mr. Ingram during his lessons.

4.4.2.1 A doing mathematics task

During his lesson on April, 30 (Observation 6) Nathan Ingram used a task called *Finding Measures of Parallelograms* (Appendix W). This task provides information to students about the height and base of parallelograms. However, no formula or procedure is described for finding the area or perimeter. Students are given six parallelograms placed on a grid and asked to find the area and perimeter of each. Further, students are asked to describe the strategies they used for finding the perimeter and area of the parallelograms. This task was coded as high level (doing mathematics) because students are not provided with any suggested procedures for how to work on the task. Additionally, it requires students to engage in significant cognitive thinking as they must consider how to calculate the length of the sides of some of the parallelograms which do not lie on the grid lines (e.g., parallelogram C) when finding their perimeters. Students must also reason about how to determine the area of these parallelograms as some of the squares of the grid that they occupy are not completely covered (e.g., parallelogram C).

4.4.2.2 A procedures with connections task

Nathan Ingram used the *Using the Mean* task (Appendix S) during his lesson on March 12 (Observation 5). This task presents students with a set of data of the number of movies watched individually by a group of students and asks them to find the mean of this data. It guides them through the process of finding the mean by first asking for the total number of students, then the total number of movies watched, and finally the mean. Students are provided with additional data points and are asked to determine how these new data impact the mean of the set. Finally, they are asked to describe what happens when an outlier is added to the set and when a cluster of data values near one end of the data set is added. They must also provide an explanation for their response. This task was coded as a procedures with connections task as it dictates the method students will use to solve the task and it includes prompts for students to express their reasoning.

4.4.3 Implementation of High-Level Tasks and Use of the Five Practices During Instruction (Research Question III and Research Question IV)

4.4.3.1 Set up and implementation of high-level tasks

Table 4.26 above indicates that in five of the seven lessons Nathan Ingram set up the task at the same level of cognitive demand that it had in written form. The two exceptions, March 1 (Observation 4) and May 22 (Observation 7), were lesson in which Mr. Ingram used a procedures with connections task but set it up as a procedures without connections task, thus changing the cognitive demand from high level to low level. The tasks in the remaining five lessons were all set up at a high level, three as procedures with connections tasks and two as doing mathematics tasks. Regardless of the categorization of the task (e.g., procedures without connections, doing mathematics) at set up, Mr. Ingram implemented the task in each lesson as low level (procedures without connections). As every task was implemented at a low level, there was no change in Mr. Ingram's ability to implement task at a high level over the baseline, intervention, and maintenance measurements (time frames 1, 2, and 3 respectively).

The factors associated with the decline of the level of cognitive demand of the tasks in Nathan Ingram's lessons are given in Table 4.27. During four of these five lessons, Mr. Ingram shifted the emphasis of the task away from meaning making and toward finding the correct answer. During the lesson on January 5 (Observation 3), the additional factor of the teacher, along with a few of the students, taking over the thinking of the lesson was also present. The lesson on April 30 (Observation 6) had the factor of the teacher not holding the students accountable for high-level processes. Specifically, during this lesson Mr. Ingram failed to acknowledge that some of the methods used by the students were mathematically erroneous, and he actually acknowledged these as feasible strategies during the whole-class discussion about the task.

Table 4.27: The factors associated with the decline of the cognitive demand of the tasks set up at a high level by Nathan Ingram

Observation number and date	Level of cognitive demand as set up/implemented	Factors associated with decline present in the lesson
Obs. 1 10/27/2011	H/L	– The teacher shifts the emphasis form meaning, concepts, or understanding to the correctness or completeness of the answer
Obs. 2 12/13/2011	H/L	– The teacher shifts the emphasis form meaning, concepts, or understanding to the correctness or completeness of the answer
Obs. 3 1/5/2012	H/L	– Problematic aspects of the task become routinized (specifically, Mr. Ingram and a few students "took over" the thinking and reasoning and told the other students how to do the problem) – The teacher shifts the emphasis form meaning, concepts, or understanding to the correctness or completeness of the answer
Obs. 5 3/12/2012	H/L	– The teacher shifts the emphasis form meaning, concepts, or understanding to the correctness or completeness of the answer (Mr. Ingram did not push the students to reason or make meaning)
Obs. 6 4/30/2012	H/L	– Students are not held accountable for high-level products or processes (students were allowed to use methods that were mathematically imprecise and incorrect)

4.4.3.2 Use of the five practices

Nathan Ingram's level of use of the five practices during his seven observed lessons is presented in Table 4.28. His use of the five practices oscillated between no use and partial use, but never elevated to high use. He employed anticipating and monitoring in five of the seven observed lessons. Mr. Ingram's level of use of monitoring was scored as partial use during one observation, and it was scored as no use or little use for the other observations. During the observation for which Mr. Ingram's monitoring was scored a partial use, Mr. Ingram used assessing and advancing questions. As was the case with the other three teachers, Mr. Ingram did not use a monitoring tool during any of his observations. Mr. Ingram used selecting and sequencing in two of the seven observations, and he did not use connecting in any of the observations.

Table 4.28: The level of use of the five practices by Nathan Ingram during his lesson observations

		Level of use of the five practices				
	Observation number and date	Anticipating	Monitoring	Selecting	Sequencing	Connecting
Time frame 1	Obs. 1 10/27/2011	L	L	N	N	N
	Obs. 2 12/13/2011	L	P	N	N	N
Time frame 2	Obs. 3 1/5/2012	N	N	L	L	N
	Obs. 4 3/1/2012	N	N	N	N	N
	Obs. 5 3/12/2012	L	L	N	N	N
Time Frame 3	Obs. 6 4/30/2012	P	L	L	L	N
	Obs. 7 5/22/2012	P	L	N	N	N

Nathan Ingram's use of the five practices varied across observations. During one observation (#4), he did not use any of the five practices. In five of the seven observed lessons (#1, #2, #3, #5, and #7) he used two of the practices. In four of these five instances, Mr. Ingram anticipated and monitored. However, during the other observation he used selecting and sequencing. During Observation 6, Mr. Ingram used four of the five practices, omitting only connecting.

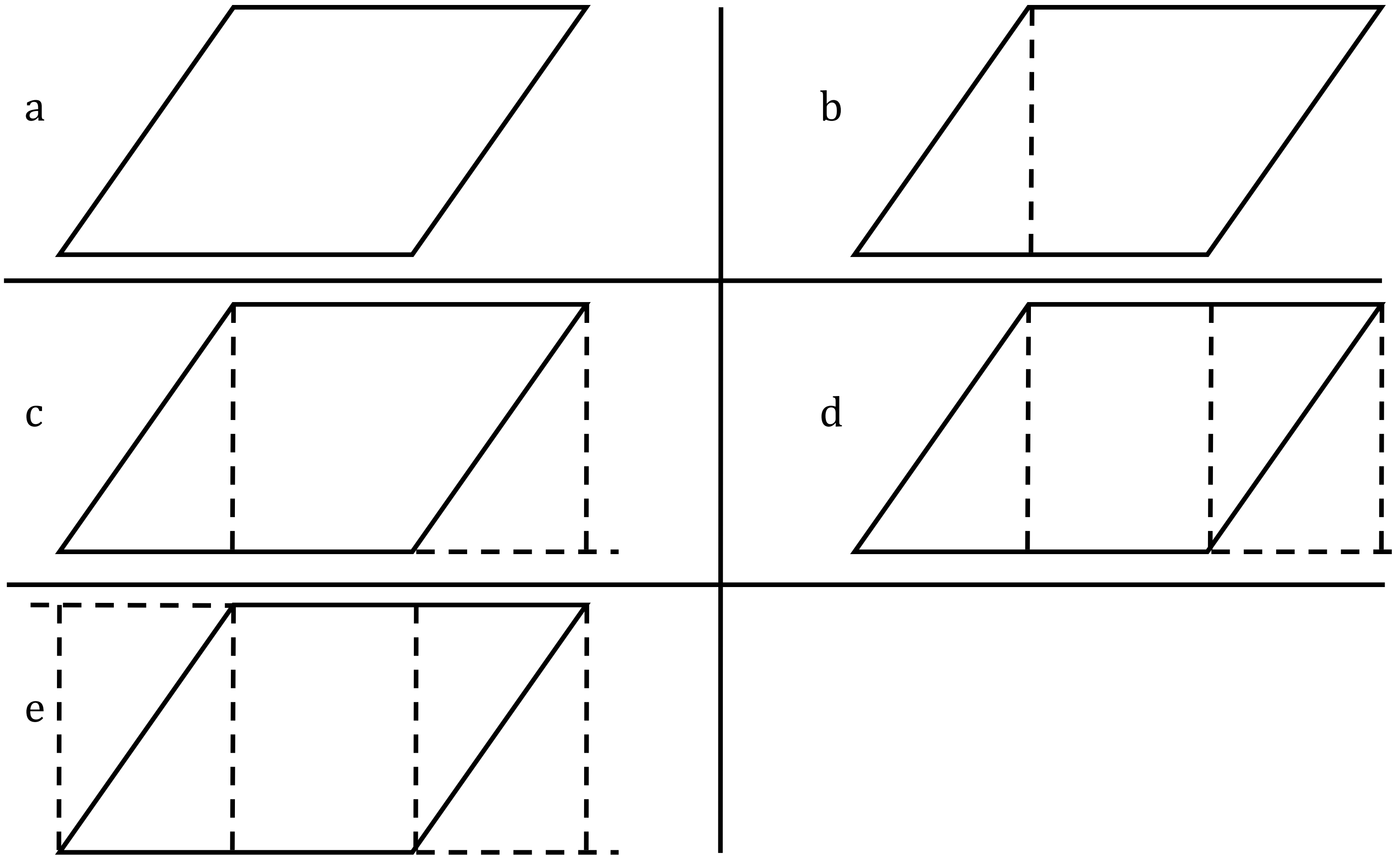
There were no patterns of change in Nathan Ingram's use of the five practices across the three measurements, baseline (time frame 1), intervention (time frame 2) and maintenance (time frame 3). During the two observations in the baseline measurement Mr. Ingram used anticipating (at little use during both observations) and monitoring (once at little use and once at partial use). During the intervention measurement Mr. Ingram used two of the five practices during two of the three observations and none during the third observation in this time frame. In one of the two observed lesson during the maintenance measurement Mr. Ingram used four of the five practices, excluding only connecting. In the other observed lesson, he used two of the five practices.

4.4.3.3 Description of instruction

During each of his lessons, Nathan Ingram implemented the task at a low level (procedures without connections in every case) regardless of the level of cognitive demand at which it was set up. Thus, a detailed description of only one task will be presented.

4.4.3.3.1 Instruction of a task set up at a high level but implemented at a low level. During his lesson on April 30 (Observation 6), Mr. Ingram used the *Finding measures of Parallelograms* task (described above). To set up this task, Nathan Ingram started by reviewing with the students how to find the height of any triangle or parallelogram including obtuse triangles for which the height is not contained within the shape. He showed students that the height of a parallelogram could be drawn on various locations of the shape. For example, Mr. Ingram drew a parallelogram on the chalkboard similar to the one in Figure 4.14-a. He asked where the height of the parallelogram is; F10 indicated it is a vertical segment drawn down from the top, left vertex.[11] Mr. Ingram drew this line in as seen in Figure 4.14-b. Another student, F1, said that the height is drawn from the top, right vertex. Mr. Ingram drew this dotted line on the diagram with the horizontal extension of the base as shown in Figure 4.14-c. A third student said that the line drawn from the top of the parallelogram to the bottom, right vertex is the height. Mr. Ingram drew this in the figure, see Figure 4.14-d. Mr. Ingram then said that the line drawn up from the bottom, left vertex was the height, and he added this with the horizontal extension of the top in the figure (see Figure 4.14-e). He then asked, "If we measure all of these, will they all be the same?" Multiple students said that they will.

[11] The students did not use the term vertex or speak as technical mathematically when referring to the diagram. They would say the height went "there" and then point or gesture to show Mr. Ingram where it would be drawn. This level of technicality was used to accurately describe the conversation.



Note: Mr. Ingram added the dotted lines as the students identified them. The letters labeling the lines have been added to assist the reader, but were not part of the diagram Mr. Ingram drew.

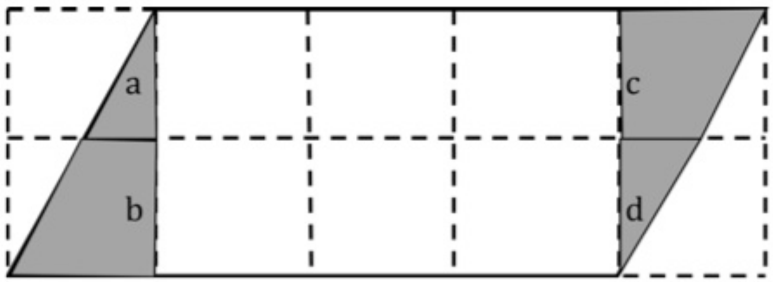
Figure 4.13: Recreation of the diagram drawn by Nathan Ingram during the set up of the *Finding Measures of Parallelograms* task used in the lesson on April 30

Continuing his set up of the task, Mr. Ingram asked the students to create a table on a blank piece of paper to record their answers, and he set up the table on the chalkboard for them to copy (see Table 4.29). He then passed out rulers and a worksheet that is a copy of the last page of the task— it shows the grid with parallelograms— to the students. He did not give any direction to the students to use the rulers other than saying that they are not required to use them, but could if they liked. He then read the problems in the task and emphasized that the students needed to describe the strategies they used when finding the area and perimeter of the parallelograms. With this, he told the students that they could work in pairs or individually and he directed them to begin working. This set up was coded as high level (doing mathematics). Mr. Ingram did not provide specific strategies for finding the perimeter or area of the parallelograms (e.g., possible procedures for finding the area: (1) find the product of the base and height, (2) divide the shape into rectangles and triangles, and then find the sum of the area of each, or (3) circumscribe the shape within a larger rectangle, find the area of that and subtract the area of any pieces in the larger rectangle that are not in the parallelogram). Further, he retained the requirement for students to provide descriptions of the strategies they used to find the perimeter and area of the parallelograms.

Table 4.29: A table created by Nathan Ingram during his set up of the *Finding Measures of Parallelograms* task used in the lesson on April 30

Design	Area (cm^2)	Perimeter (cm^2)
a		
b		
c		
d		
e		
f		

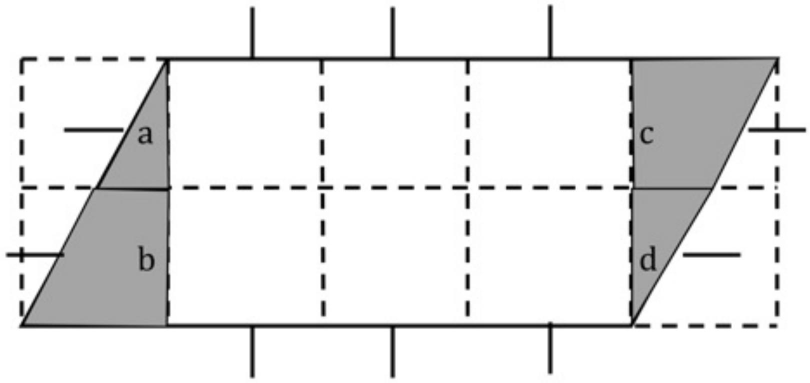
As the students worked on the task, Mr. Ingram circulated the room ensuring that they were on task. However he did not speak with the students about how they were working on the task, and thus it was not obvious how students were thinking about the task as they worked on it. In order to probe students understanding, the researcher conversed with a few of them individually after observing how they had worked on the task. He did not in any way try to steer them toward a certain strategy or give any sign of the correctness of the students answer. His conversation with the students revealed that many students did not demonstrate that they have solid conceptual understanding of perimeter or area as they work on the task. For example, the researcher asked M4 how he found the area of parallelogram A. M4 said that he "counted the squares." The researcher asked M4 how he counted the shaded squares in Figure 4.15. M4 pointed to one of the smaller partial squares (the triangle labeled a in Figure 4.15) and said that if you put it with "this square" (pointing to the triangle labeled d) that it makes one whole square. The researcher asked him if this was the same with the two larger partial squares (the shaded shapes labeled b and d), meaning if you put the two together it would make one whole square. M4 said that this was the case.



Note: The shaded squares are partial squares that were the focus of the conversation between the researcher and two of the students. These squares were neither shaded nor labeled on the students' tasks, and are labeled and shaded to assist the reader.

Figure 4.14: A diagram that represents parallelogram A in the *Finding Measures of Parallelograms* task

The researcher also spoke with F3 about her work on the task. She had found the area and the perimeter of all of the parallelograms. The researcher asked her how she found the perimeter on parallelogram A. She said that she counted each of the sides of the squares. He asked her about the sides of the parallelogram touching the shaded triangles in Figure 4.16. She said that each of these was also one, so she found a perimeter of 12 cm (she had placed tick marks by each of the sides of square she counted as 1 unit, see Figure 4.16). The researcher asked F3 about how she found the area of each parallelogram. She said that she "counts the blocks." He asked her how she counted the partial blocks (the partially shaded blocks in Figure 4.16, not referring to them as "partial blocks" but pointing to them). She told the researcher that the two smaller blocks (labeled a and d) go together to form a whole. Then she changed her mind and says that one small one (triangle a) and one larger one (triangle b) go together to form a whole.



Note: The shaded squares are partial squares that were the focus of the conversation between the researcher and two of the students. These squares were neither shaded nor labeled on the students' tasks, and are labeled and shaded to assist the reader.

Figure 4.15: A second diagram that represents parallelogram A in the *Finding Measures of Parallelograms* task

Some of the students used the rulers to find the perimeter or the parallelograms, while others counted all of the sides as having length 1 as F3 had. None of the students considered that the length of the sides passing diagonally through the squares of the grid would have a length longer than 1. One common method for finding the length of the sides of a parallelogram that do not lie on the horizontal or vertical lines of the grid would be to draw a vertical line from one of the vertices of the parallelogram forming a right triangle, then use the Pythagorean theorem to find the length of the diagonal side of the shape. While a few students drew vertical lines representing the heights of the parallelograms, none of them showed any work or made comments while working either in small groups or during the ensuing whole-class discussion to suggest that they had thought about finding the perimeter in this manner.

After 25 minutes of students working individually or in small groups, Mr. Ingram began a whole-class discussion. He began by asking if the students should all have the exact same answers or the perimeters of the parallelograms. A student said that they should not, and when

Mr. Ingram asked her why not, and she said, “Because we may have estimated.” Mr. Ingram agreed and said, “And if we all measure, there may be errors in how we measured, but the areas should be very close” [meaning that the student’s calculations for the area should not vary much]. He asked the class what the perimeter is for parallelogram A. The students gave varying answers: F4 said 13, another student said 12, and F9 said 12 6/16. Mr. Ingram thanks them for their answers and said that the actual answer is 12 1/2 (this is very close to the exact answer of $8+2\sqrt{5}$ which is approximately 12.47). They continued in this process of students giving the perimeter they found for each of the parallelograms and Mr. Ingram giving the answer he found, which was viewed as the correct answer. However, they did not discuss the methods students used to find the perimeter for any of the individual parallelograms, nor did they address any of the differences in the answers the students give with the answer Mr. Ingram identified as the correct answer. Only after going through all of the parallelograms did this occur. Mr. Ingram asked two students how they found the perimeters of the parallelograms. M4 said that he added up all four sides. Mr. Ingram said that this is a valid method. He then called on F5 to describe how she worked on this part of the task. F5 said that she used a ruler to find the lengths of all the sides and then added these lengths up. Mr. Ingram then steered the conversation toward the areas of the parallelograms, never addressing the potential issues of “counting the sides” of the parallelogram to find the perimeter.

Shifting the conversation, Mr. Ingram asked the students what they found for the area of parallelogram A. F4 said that the area was 9 and another student said that it was 10. Mr. Ingram says the area was not 10, but that it was 8; he then asked why it was 8. F1 said, “Because you don’t just count the small ones (meaning the colored partial squares shown in Figures X and Y), you got to add the two together to get 1.” Mr. Ingram agrees with this statement. He then asks what the area for B is and a student says that it is 8. Mr. Ingram agreed that this was correct. F5 asked why, and then before anyone can respond she said, "Oh, we just added the squares of the rectangle." They repeated the process of Mr. Ingram asking for the area of a shape, students calling out their answers, and then Mr. Ingram telling them the correct answer. The class finished with students working on practice problems finding the perimeter and area of additional parallelograms on grid paper.

The implementation of the task was coded as low level (procedures without connection). The students did not reason about the parallelograms in a conceptual way when finding the length of the sides that are not vertical or horizontal (e.g., they did not consider that they need to use the Pythagorean theorem when finding the perimeter). Rather, they either resorted to using the rulers and finding an estimate or they just "counted the sides" (i.e., each box that the side crosses through is one even if it crosses through at a diagonal). When finding the areas, the students resorted to "counting blocks" and combining the small partial blocks with the larger ones even if their sum is not equal to one. Mr. Ingram did not hold students accountable for accurate mathematical thinking and did not make them aware of possible misconceptions in their thinking during the lesson. His failure to hold students accountable and to discuss, either directly or by problematizing this thinking until the students determined it was erroneous, were important factors in the decrease of cognitive demand of the task.

Nathan Ingram used four of the five practices during his instruction of the *Finding Measures of Parallelograms* task at either a little-use or partial-use level. His lesson plan shows evidence of anticipation at a partial-use level. The lesson plan identifies two methods students might use when finding the area of the parallelograms, (a) divide the parallelogram into triangles along the diagonal of the parallelogram, or (b) cut the parallelogram down the height and rearranging the pieces to form a rectangle. Mr. Ingram monitored at a little-use level. He circulated the room observing students work on the task, but he did not use assessing and advancing questions, nor did he use a monitoring tool to record students' responses. Mr. Ingram's use of selecting and sequencing was also rated as little use during his instruction. Mr. Ingram had students discuss how they found the area and the perimeter of the parallelograms as described above. However, in the instances in which he asked the class members how they had worked on the task, some students raised their hands to answer, and he selected them to share in this manner. There is no evidence the he selected these students to share in a purposeful manner; rather, it appeared he selected those that raised their hands randomly. Mr. Ingram did not form any connections between the various methods students shared, nor between those methods and the conceptual mathematical ideas or learning goals of the lesson.

4.4.3.4 Summary of implementation of high-level tasks and use of the five practices

Although Nathan Ingram selected challenging tasks for his lessons and set the majority of them up at a high level, he was unable to implement any of them at a high level during his lessons. Mr. Ingram's use of the five practices was sporadic and did not show a pattern of change between the three time frame measurements.

4.4.4 The Relationship Between Use of the Five Practices and Ability to Maintain the Level of Cognitive Demand of High-Level Tasks (Research Question V)

Nathan Ingram set up five tasks at a high level during his observed lessons. Table 4.30 shows the relationship between his use of the five practices and his ability to implement these tasks at a high level. As none of the tasks were implemented at a high level, no comparison can be made between his use of the five practices and his implementation of tasks at a low and a high level.

However, as was the case with Gloria Xavier, it is evident that Mr. Ingram’s use of the five practices was minimal (e.g., little use or no use) during his lessons in which the level of cognitive demand decreased during implementation.

Table 4.30: Nathan Ingram's level of use of five practices as related to the level of cognitive demand of implementation of tasks set up at a high level across all classroom observations

	Level of cognitive demand during implementation																							
	Low-level cognitive demands																High-level cognitive demands							
	Unsystematic and nonproductive exploration				Little or no academic thinking occurred				Memorization				Procedures without connections				Procedures with connections				Doing mathematics			
Level of use of practice	N	L	P	H	N	L	P	H	N	L	P	H	N	L	P	H	N	L	P	H	N	L	P	H
Anticipating													1	3	1									
Monitoring													1	3	1									
Selecting													3	1										
Sequencing													3	1										
Connecting													5											

Note: N = no use, L = low use, P = partial use, and H = high use.

4.4.5 The Relationship Between the Modified Lesson Study Cycles and Implementation of the Focus Tasks (Research Question VI)

Two of the 6th-, 7th-, and 8th-grade team's MLSCs were centered on focus tasks selected by Mr. Ingram. His incorporation of the key ideas suggested by his colleagues during the week 1 meetings of these cycles is presented in this section.

4.4.5.1 Uptake of key ideas from the first MLSC

For the MLSC week 1 meeting on January 4, Nathan Ingram chose a task from the sixth-grade *Connected Mathematics* book *Bits and Pieces II* called the *Land Sections* task (Lappan et al., 2006b) (see Appendix B). During this meeting the team members suggested five key ideas:

1. Don't tell the students how to work on the task, allow them to develop a strategy and then share the strategy with the rest of the class - Mr. Ingram had expressed a frustration that he felt with regard to this task while teaching it in past years. He said that exploration portion of the task (part A) took the students an extremely long time and that many of the students became frustrated with it as they struggle to work on it productively. He said that in past years when teaching the lesson he had resorted to telling the students what strategies to use to complete part A. The other group members encouraged him not to do this as this took away from the cognitive challenge of the task. They suggested he let students work on the task and once many of them had developed various strategies for working on it, have the students share these strategies.
2. Use strategic questioning to help students, when working on part A, to compare smaller pieces of land to larger pieces - Mr. Ingram felt that many students would

be able to see that Lapp's land is one fourth of section 18. However, he did not feel that this was the best piece to work with when determining the relative size of the other pieces. The group members suggested that through strategic questioning Mr. Ingram could get the students to compare other, more useful pieces (e.g., Bouk and Kreb) to the larger pieces.

3. Provide pattern blocks that are the same sizes as Bouk's land for the students to use - Ms. Xavier had taught this task several times in previous years. She suggested that Mr. Ingram provide pattern blocks for the students to use because, even though they weren't designed for this purpose, one of the blocks is the same size as Bouk' land. She has had students use these in her classes to determine the sizes of other pieces of land.
4. Make the boundary line between section 18 and section 19 in the task more noticeable - The team members anticipated that because of how the diagram in the task is drawn that some students may consider the entire rectangle (sections 18 and 19) as the whole unit instead of thinking of each section as the unit amount. This thinking would impact the fractional amounts students find to represent the amount of land each person owns. The team member suggested that Mr. Ingram emphasize that there are two sections in the diagram by making the boundary line more noticeable.
5. Don't spend too much time working on part E of the task - Because Mr. Ingram was concerned about the length of the task and that part E addressed mathematical concepts that are not central to the goals of the lesson, the group members felt that he should not devote too much class time to this portion of the task.

Nathan Ingram used the *Land Sections* task during the lesson observed on January 5. He discussed his teaching of the task with the team members in the MLSC week 2 meeting on January 11. The data from the lesson observation and the week 2 meeting suggested that Mr. Ingram took up two of the key ideas (key idea #2 and #4) in his teaching. There is evidence of partial uptake of another key idea (key idea #5), and conflicting evidence of his incorporation of a fourth key idea (key #3). Mr. Ingram did not take up key idea #1.

4.4.5.1.1 Key idea 1 - Do not tell the students how to work on the task, allow them to develop a strategy and then share the strategy with the rest of the class. Mr. Ingram did not integrate this key idea into his teaching of the *Land Sections* task. During the observed lesson, Mr. Ingram did not allow students time to struggle while working on the task. As he was launching the task, M7 told Mr. Ingram that he had determined that Lapp's land is one fourth of Section 18. Mr. Ingram shared this information with the entire class and then divided the diagram of Section 18 displayed on the board into fourths for the students (see Figure 4.17). Directly after this, another student, M9, told Mr. Ingram and the class to split Gardella's land into fourths. This cannot easily be done. However, Mr. Ingram drew on the diagram and showed the students how by using Gardella's land and part of Stewart's land they could form sixteenths of the section (see Figure 4.18). Mr. Ingram continued a whole-class discussion about the task for the remainder of the class period without giving the students any time to work individually or in small groups on the tasks. He continued to use this same method of dividing the land into smaller and smaller pieces to determine the size of the pieces of land with his students, not allowing them to offer other strategies. During the week 2 professional development meeting Mr. Ingram shared with the team that for the first time he did not have to tell the students how to work on the task because one of them figured it out first. However, what he did not realize was that the remaining 23 students in the class had not had the opportunity to develop a strategy for working on the task.

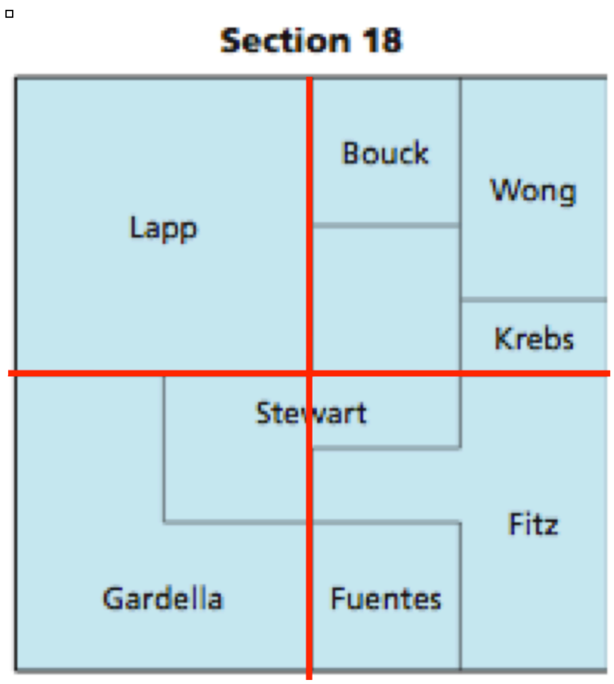


Figure 4.16: Recreation of a diagram drawn by Mr. Ingram during his set up of the focus task taught on January 4, 2012

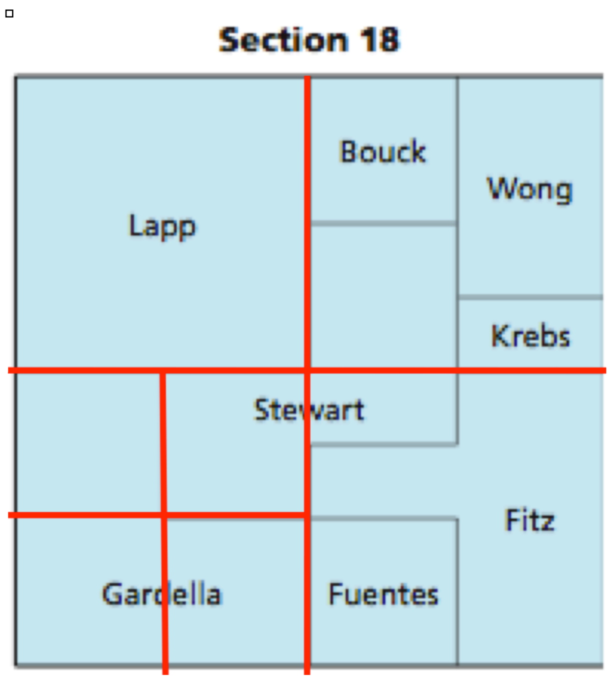


Figure 4.17: Recreation of a diagram drawn by Mr. Ingram during his set up of the focus task taught on January 4, 2012

4.4.5.1.2 Key idea 2 - Use strategic questioning to help students, when working on part A, to compare smaller pieces of land to larger pieces. There is evidence, while minimal, that Mr. Ingram did take up this key idea. During the discussion of how to divide the land so that they could use the smaller pieces to determine the sizes of the larger pieces of land, Mr. Ingram asked the students, "What family has the smallest amount of land?" Multiple students identified Krebs as the smallest piece of land. This then spurred a discussion of how use a rectangle the size of Krebs' land to determine the sizes of other pieces of land. This however is the only evidence of Mr. Ingram using strategic questioning during part A of the task.

4.4.5.1.3 Key idea 3 - Provide pattern blocks that are the same sizes as Bouk's land for the students to use. There is conflicting data regarding Mr. Ingram's incorporation of this key idea. During the lesson, Mr. Ingram did not provide pattern blocks to the students as tools to use while working on the task. However, during the week 2 professional development meeting, Mr. Ingram explained that he had the pattern blocks and other tools (e.g., extra copies of the diagram that students could cut up to help determine the size of the land) ready for the students, but that because his students suggested the strategies described with regard to key idea 1 he did not distribute them. Thus, it appears that Mr. Ingram planned to use this key idea, but during the lesson felt it was not necessary.

4.4.5.1.4 Key idea 4 - Make the boundary line between section 18 and section 19 in the task more noticeable. Mr. Ingram did use this key idea in his instruction of the *Land Sections* task. When he first presented the diagram to the students, Mr. Ingram drew the sections of land on the board as shown in Figure 4.19. This differed drastically from how the diagram was represented in the curriculum materials (see Appendix B). Later, as Mr. Ingram passed out a copy of the diagram in the curriculum materials for students to use, he directed the students to draw a thick, dark line between sections 18 and 19 and emphasized that the two sections were each considered as one, whole section.

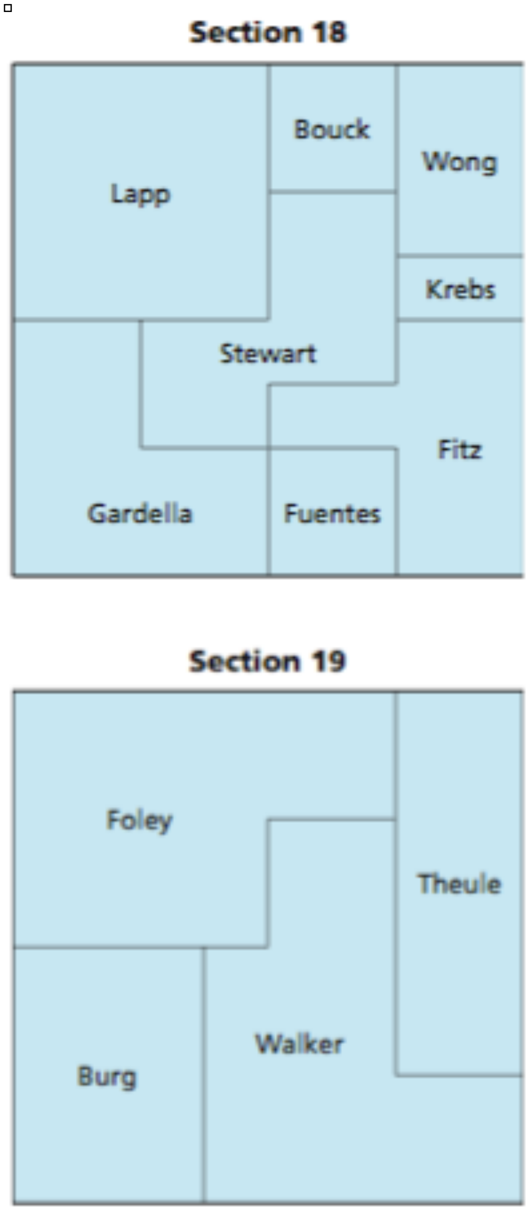


Figure 4.18: Recreation of a diagram drawn by Mr. Ingram during his set up of the focus task taught on January 4

4.4.5.1.5 Key idea 5 - Do not spend too much time working on part E of the task. There is partial evidence that Mr. Ingram took up this task in his instruction of the task. During the lesson observed on January 5 Mr. Ingram and the students did not work on or discuss part E of the task. However, during the week 2 professional development meeting, Mr. Ingram told the team members that they continued working on the task the following day. He said that during this subsequent lesson they discussed part E briefly, but did not spend much time on it.

4.4.5.2 Uptake of key ideas from the second MLSC

Mr. Ingram selected the task called *Using the Mean* from the sixth-grade *Connected Mathematics* book *Data About Us* (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2006f) (see Appendix S) for the second modified lesson study cycle centered around his teaching. He shared this task with his team during the week 1 professional development meeting on March 7. Four key ideas were suggested during this meting:

1. For part D of the task, conduct a whole-class discussion using a different data set than that provided in the task - The team members felt that it would be beneficial for the students to talk about a second set of data. They suggested the possibility of using data such as the number of letters in people's last names, height, or age.
2. Explicitly state that the students should include the data about Carlos given in part B of the task when working on part C - The group discussed how including or omitting the data point for Carlos given in part B impacts part C. They felt that it would be beneficial for the students to all work with the same data set and suggested that Mr. Ingram be explicit that they include this data when working on part C.
3. Have students make predictions about how the adding the additional data in each portion of the task will impact the mean - Team members suggested that Mr. Ingram should have the students make predictions about how the additional data impacted the mean, whether it increases or decreases. They also suggested that it would be good to have them predict how much the mean will change by, not

necessarily providing a numerical difference, but predicting it will change “a lot” or “a little.”

4. Use an exit slip to assess students’ understanding of how adding additional data impacts the mean - The team members provided various suggestions for what Mr. Ingram could ask on the exit slip, but they did not decide on a specific question. However, they did recommend that he use some sort of exit slip to assess student’s understanding of how the changes in the data impacted the mean.

Nathan Ingram was observed teaching the *Using the Mean* task on March 12, 2012. He debriefed his teaching of the task with his team members during the week 2 professional development meeting on March 28, 2012. There is evidence that Mr. Ingram incorporated one of the four key ideas in his instruction (key idea #4), and there is evidence of his partial uptake of two other key ideas (key ideas #1 and #3). Mr. Ingram did not take up key idea #2.

4.4.5.2.1 Key idea 1 - For part D of the task, conduct a whole-class discussion using a different data set than that provided in the task. Mr. Ingram partially took up this key idea in his instruction of the task. During the whole-class discussion Mr. Ingram provided a second data set consisting of test scores. He gave the students three test scores, 75, 79, and 83 (out of 100) and read question D1 and asked what would happen to the mean if they added a fourth test score of 11. After discussing this for a few moments, he asked what the impact on the mean would be if the fourth test score was 100 instead of 11. They then briefly discussed question D2, however they referred back to the original context of the task.

4.4.5.2.2 Key idea 2 - Explicitly state that the students should include the data about Carlos given in part B of the task when working on part C. Mr. Ingram did not use this key idea in his teaching of the task. During the set up of the task, Mr. Ingram read part C and told the students that they would need to make a new stem plot for this data. However, he did not give any directions with regard to whether or not students should use the data for Carlos given in part B when working on part C.

4.4.5.2.3 Key idea 3 - Have students make predictions about how adding the additional data in each portion of the task will impact the mean. Mr. Ingram partially incorporated this key idea in his teaching. As he set up the task and after Mr. Ingram read part B he asked the students what they thought would happen to the mean of the data. One student said that it would increase. Mr. Ingram asked her how much she thought it would increase and the student said she was not sure. Mr. Ingram responded by saying, "We'll see." He then moved on to describe part C. After reading part C, Mr. Ingram again asked for a prediction of how the additional data would impact the mean. A different student said that she thought the mean would again increase Mr. Ingram again said, "We'll see." These two students were the only ones who gave their predictions of what they thought would happen. Therefore, while Mr. Ingram did ask students to make predictions, only two of 22 students were able to do so.

4.4.5.2.4 Key idea 4 - Use an exit slip to assess students' understanding of how adding additional data impacts the mean. There is evidence that Mr. Ingram integrated this key idea into the lesson. As the class drew to an end, Mr. Ingram asked the students to answer the following prompt as an exit slip: "In your own words, describe how an outlier can effect the mean of a set of data." He also asked the students to provide an example if possible. This exit slip served the purpose of assessing students understanding of how additional data points impact the mean of a set of data as discussed during the week 1 professional development meeting.

4.4.5.3 Summary of uptake of key ideas in instruction

Nathan Ingram's colleagues provided several key ideas regarding the focus tasks he selected and his instruction of them. A few of these suggestions focused on general pedagogical techniques applied specially to the teaching of the focus task (e.g., provide proper manipulatives for the students to use, don't tell students what methods to use on part A of the task). Other suggestions dealt with specific aspects of the focus tasks (e.g., emphasize to the students that they need to include Carlos' data in the data set for part C). Of these key ideas, there is evidence that he incorporated a few completely, and others partially, during his instruction.

4.4.6 Summary of Nathan Ingram

Nathan Ingram was a member of the 6th-, 7th-, and 8th-grade team. He attended 16 of the team's 26 meetings. Mr. Ingram did not prepare for the MLSC meetings as he only produced work on a focus task for one of the nine MLSC week 1 meetings and he never created noticings and wonderings. The tasks he used in his lessons were high-level tasks. He typically set these tasks up at a high level, however he was never able to implement them as such. Typically, during his lessons the tasks were implemented at a low level because he would change the focus of the students' work on the task from conceptual meaning making to a focus on finding the correct answers. Mr. Ingram's use of the five practices was sporadic and did not reveal any noticeable patterns other than when he used the five practices, he did so at a no-use or little-use level. Mr. Ingram's colleagues provided key ideas and suggestions during the MLSC week 1 meetings centered on his focus tasks. Evidence from his instruction of these tasks and the MLSC week 2 meetings show that he incorporated some of these key ideas.

4.5 CROSS-CASE COMPARISON

Table 4.31 provides a summary of the findings for each of the six research questions for each of the teachers in this study. Looking across the four teachers, some patterns emerge. With regard to Research Question 1, it is apparent that the teachers' individual attendance and preparation for the professional development meetings varied greatly. Cara Nance and Nicole Nesmith attended nearly all of their team's meetings. They prepared for the meetings by thinking deeply about the designated focus tasks, creating anticipated student responses and recording their noticings and wonders about the task and the potential instruction of it. The work on the focus tasks and the noticings and wonderings were typically rated as either medium- or high-preparation. On the other hand, while Gloria Xavier and Nathan Ingram attended the majority of their team's meetings, they both missed several meetings. Further, neither was prepared for the meetings they did attend. They each only prepared work on the focus task for one of the team's meeting and then never created noticings and wonderings prior to the meetings. Thus, Ms. Xavier and Mr. Ingram typically arrived at the meeting have not prepared to discuss the task in a meaningful manner.

Addressing Research Questions 2 and 3, all of the teachers typically selected high-level tasks for their observed lessons, yet they all struggled to implement these tasks at a high level in their instruction. The factors associated with the decline of these high-level tasks in their lessons differed between the teachers. For Cara Nance, during lessons in which the task was implemented at a low level it was typically due to a lack of press for reasoning. Nicole Nesmith struggled with classroom management during each of her lessons in which she was unable to implement tasks at a high level. Many of Gloria Xavier's lessons also lacked a press for reasoning from the students. In Nathan Ingram's case, he typically shifted the students' focus when working on the task from conceptual understanding to the correctness of the answers or the completeness of the work.

Considering Research Question 4, all of the teachers used the five practices inconsistently and sporadically. The teachers' use of the five practices varied widely. For example, Nicole Nesmith never used connecting during her lessons, while the other teachers used it occasionally. However, when any of the teachers used any of the five practices, it was almost always at a level of low use or partial use. With regard to Research Question 5, there were no noticeable differences in the teachers' use of the five practices when comparing their instruction during lessons in which they implemented tasks at a high level and in lessons in which they implemented tasks at a low level.

Addressing Research Question 6, each of the teachers used several of the key ideas suggested by their teammates during the professional development meetings. Cara Nance used three of five during the first MLSC for which she selected the focus task and five out of seven for the second. Nicole Nesmith used two out of six ideas and three out of six of the key ideas in each of the two MLSCs centered on her focus tasks. Gloria Xavier only completed one MLSC. She used two of the four key ideas suggested by her teams during this MLSC. Nathan Ingram used three of the five key ideas suggested in the first MLSC and two of the four key ideas given during the second MLSC.

Table 4.31: Cross-case comparison of the four teachers with regard to the six research questions

	Cara Nance	Nicole Nesmith	Gloria Xavier	Nathan Ingram
Research Question 1	– Attended all 27 team meetings – Prepared for professional development meetings at a fairly high level	– Attended 24 of her team's 27 meetings – Prepared for professional development meetings at a high level	– Attended 21 of her team's 26 meetings – Did not prepare for the professional development meetings	– Attended 16 of his team's 26 meetings – Did not prepare for the professional development meetings
Research Question 2	– Typically selected high-level tasks (6 of 8 lessons)	– Typically selected high-level tasks (6 of 7 lessons)	– Typically selected high-level tasks (6 of 8 lessons)	– Selected high-level tasks for all seven lessons
Research Question 3	– Typically was not able to implemented tasks at a high level (2 of 6 lessons) – Factors associated with decline: • Lack of press for reasoning • Problematic aspects of the task become routinized • Not enough time is provided to wrestle with the demanding aspects of the task – Factors associated with maintenance: • Teacher presses for justifications, explanations, and meaning • Modeling of high-level performance • Task builds on students' prior knowledge	– Only implemented 1 of the 6 tasks at a high level – Factors associated with decline: • Classroom-management problems prevent sustained engagement in high-level cognitive activities – Factors associated with maintenance: • Teacher presses for justification, explanations, and meaning • Task builds on students' prior knowledge	– Did not implement any tasks at a high level – Factors associated with decline: • Lack of press for reasoning • The teacher shifts the emphasis form meaning, concepts, or understanding to the correctness or completeness of the answer • Not enough time is provided to wrestle with the demanding aspects of the task	– Did not implement any tasks at a high level – Factors associated with decline: • The teacher shifts the emphasis form meaning, concepts, or understanding to the correctness or completeness of the answer • Problematic aspects of the task become routinized • Students are not held accountable for high-level products or processes

Research Question 4	– Inconsistent use of the five practices • Never anticipated individually (as could be discerned in the lesson plans) • Always used monitoring, but at a low or partial level • Used selecting, sequencing, and connecting occasionally, and always together	– Sporadic and infrequent use of the five practices • Always used monitoring, but at a low or partial level • Never used connecting • Used anticipated, selecting, and sequencing each in 2 of the 7 lessons.	– Sporadic use of the five practices • Evidence of anticipating in 3 of 8 lesson plans • Frequency of monitoring, selecting, sequencing, and connecting varied widely • All practices used at a low or partial level	– Sporadic and infrequent use of the five practices • Evidence of anticipating in 5 of 7 lesson plans, but copied directly from the district curriculum • Used monitoring, selecting, sequencing, and connecting occasionally • All practices used at a low or partial level
Research Question 5	– No differences in use of the five practices between lessons with implementation of high-level tasks and lessons with low-level tasks	– No differences in use of the five practices between lessons with implementation of high-level tasks and lessons with low-level tasks	– No tasks were implemented at a high-level, thus no conclusions can be drawn regarding differences between her levels of use of the five practices in her instruction of high- and low-level tasks	– No tasks were implemented at a high-level, thus no conclusions can be drawn regarding differences between her levels of use of the five practices in her instruction of high- and low-level tasks
Research Question 6	– Incorporated the majority of her teammates' suggested *key ideas* regarding her focus tasks into her instruction of these tasks – Cycle 1 (3 out of 5), Cycle 2 (5 out of 7)	– Incorporated some of her teammates' suggested *key ideas* regarding her focus tasks into her instruction of these tasks – Cycle 1 (2 out of 6), Cycle 2 (3 out of 6)	– Incorporated 2 of 4 of her teammates' suggested *key ideas* regarding her focus task into her instruction of these tasks (Ms. Xavier only completed on MLSC centered on task she selected)	– At least partially incorporated some of teammates' suggested *key ideas* regarding his focus tasks into his instruction of these tasks – Cycle 1 (3 out of 5), Cycle 2 (2 out of 4)

Chapter Four addressed the six research questions using the analyses presented in Chapter Three. It did so in the form of a narrative case for each teacher. These cases provided illustrative descriptions of the teachers' participation in the professional development meetings, their implementation of high-level tasks, and their use of the five practices. Chapter Five explores possible explanations for the teachers' inability to implement high-level tasks at a high level, highlights the contribution this study provides to the field, and provides suggestions for future research based on this study.

5.0 CHAPTER 5: DISCUSSION

The purpose of this study was to examine teachers' participation in professional development focused on assisting them to select and implement high-level, cognitively demanding tasks, as well as to investigate the impact this professional development had on their practice. It investigated teachers' instruction by assessing their selection, setup, and implementation of high-level tasks, as well as their use of the five practices. The study also examined teachers' uptake of the specific suggestions regarding their instruction of focus tasks made during the professional development meetings.

The findings presented in Chapter Four suggest that across teachers, the level of engagement and participation in the professional development was drastically different. On one hand, Cara Nance attended every meeting and Nicole Nesmith missed very few. They both arrived at the team meetings prepared with anticipated solutions to the focus task and noticings and wonderings about the tasks. On the other hand, Gloria Xavier and Nathan Ingram both missed several of their team's meetings. When they did attend, they failed to bring possible solutions to the focus tasks to all but one of the meetings, and they never produced noticings and wonderings in preparation for the meetings. It might be expected that the varying levels of participation in the professional development would produce differences in teachers' ability to implement tasks at a high level during their lessons or in their use of the five practices. However, this was not the case as all the teachers struggled to implement tasks at a high level and inconsistently used the five practices in their lessons. Section 5.1 explores two possible explanations for the lack of impact the professional development had on teachers' instruction. Section 5.2 discusses the contributions this study makes to the body of literature related to the topics. Section 5.3 provides suggestions for future research based on the results in Chapter Four and the explanations presented in section 5.1.

5.1 EXPLANATIONS FOR THE PROFESSIONAL DEVELOPMENT'S LACK OF IMPACT ON TEACHERS' INSTRUCTION

5.1.1 Teachers' Lack of Anticipating

The fact that the teachers did not appear to anticipate the ways in which students might engage in and solve the tasks they were assigned may be a factor resulting in their inconsistent use of the five practices and their inability to implement high-level tasks. Stein (2013) explained that traditional lesson planning in the United States focuses on what teachers believe *should* happen during the lesson, typically not including contingency plans for how to deal with unexpected turns that may occur during the lesson. She proposed an alternative model of lesson planning based on considering several possible methods students may use to engage in the activities of the lesson. This approach to planning allows teachers to become familiar with the possible approaches students might use on the task, and it provides teachers with opportunity to prepare for how best to respond to these approaches in order to steer student learning toward the desired learning goals.

The five practices were designed with a purpose similar to that of the alternative approach to planning suggested by Stein (2013). When used as a set, they allow teachers to prepare for multiple student approaches and consider how to use these approaches in harmony to push student understanding toward the desired learning goals. Smith and Stein (2011) emphasized that the five practices are to be used as a set, and that each practice builds on the previous practice(s). For example, teachers cannot purposefully select student approaches to be shared during whole-class discussions if they have not first anticipated the possible approaches and then monitored to see what approaches were actually used by the students. Thus, the practices of monitoring, selecting, sequencing, and connecting are only viable when built on a solid foundation created through anticipating. If this foundation has not been created by thoughtful anticipation, then teachers are not able to use the remaining practices in more than a superficial manner.

The five practices are intended to assist teachers in conducting substantive, whole-class discussions around cognitively challenging tasks, thus supporting teachers in their ability to maintain the high-level of the tasks. However, the teachers in this study typically did not engage in anticipating, and when they did so it was in a superficial manner. Cara Nance did not anticipate any possible solution methods in her lesson plans for any of her lessons. Nicole Nesmith only anticipated student solutions for two of her seven lessons, neither being coded as high use. The lesson plans for three of Gloria Xavier's lessons and five of the plans for Nathan Ingram's lessons included anticipated student solution methods. However, none of these was coded as high use. Further, recall that Ms. Xavier and Mr. Ingram were provide with detailed curriculum guides from the school district. These guides included detailed plans for each of the individual lesson they taught. The researcher only had access to the district curriculum guides for one unit each of the 6th- and 7th-grade mathematics course, and neither of these units corresponded to the lessons taught by Ms. Xavier and Mr. Ingram that were observed for this study. However, the researcher did have access to all of the lesson plans submitted by the teachers during the 2011-2012 school year. Looking at the lesson plans submitted by Ms. Xavier and Mr. Ingram that do correspond to the curriculum guides that were available, it is apparent that these two teachers copied the plans in the curriculum guides directly into the lesson-planning tool. The lesson plans submitted by Ms. Xavier and Mr. Ingram were identical, both verbatim and in format, to those written in the curriculum guide. While it is impossible it compare the lesson plans submitted by these two teachers for the lessons examined in this study to the district curriculum guides, it should be noted that the plans for these lessons used the same wording, detail, format, and style as those lesson plans that were shown to be copied from the district curriculum. Hence, it is highly likely that the lesson plans that Ms. Xavier and Mr. Ingram created for the lesson observed for this study were also copied directly from the district curriculum guide. Thus, there is no firm evidence that Ms. Xavier and Mr. Ingram actually engaged in anticipating while preparing for the lessons, and in fact it is quite possible that they did not. For example, Ms. Xavier indicated that the lesson plans in the planning tool had no instructional value to her and were created solely to be compliant with the demand of the school administration, saying that lesson plans are good for "keeping the powers that be aware of what you're doing" (Russell & Stein, 2013). Thus, the lack of quality anticipating by the teachers did not provide a solid foundation upon which to build when employing monitoring, selecting, sequencing, and connecting, and when trying to implement tasks at a high level.

The notion that the lack of quality anticipating was a factor related to teachers' inability to maintain the level of cognitive demands of high-level tasks is partially supported by a deeper look at the MLSCs associated with Cara Nance's focus tasks and comparing these cycles to the 6th-, 7th-, and 8th-grade team's meetings. The findings in Chapter Four show that Cara Nance did not include any anticipated student solutions in her lesson plans. However, in a related study drawing on the same data sources, Smith, Cartier, Eskelson, and Ross (2013) found that Cara Nance and her colleagues on the 11th-12th grade team engaged in a great deal of anticipating during the MLSC week 1 meetings centered on her focus tasks. They also found that the majority of the anticipated items (e.g., possible student solution strategies, errors, or misconceptions) actually occurred during Ms. Nance's lessons involving these tasks and that Ms. Nance proactively addressed these things during the lessons. Thus, while Ms. Nance did not engage in anticipating on an individual basis (at least as can be determined from her lesson plans), the anticipating that occurred during the MLSC week 1 meetings impacted her instruction to a large degree. However, the outcome of the influence of the group anticipating during the MLSC week 1 meetings is not entirely clear. The group anticipating appeared to support Ms. Nance in implementing the focus task discussed during one of the MLSCs at a high level. However, it did not always guarantee a successful lesson, as she was unable to implement the other focus task discussed during a MLSC at a high level. This group anticipating may also not be an essential factor needed for the implementation of high-level tasks as Ms. Nance was able to implement a task at a high level that was not discussed during a MLSC.

Smith and her colleagues (M. S. Smith et al., 2013) suggested that the type of anticipating in which teachers engage is one of three important factors that supported Cara Nance in implementing the task in her lesson on January 9 at a high level. Smith et al. found that during the MLSC week 1 meeting during which the 11th- and 12th-grade team discussed the *Modeling with Logistic Functions* task, the team members anticipated not only how students might engage in the task, but what mathematical ideas students would and would not be able to discern while working on the task (e.g., students would not notice the asymptote at zero because of the way they drew the graph). In addition, they anticipated how the students would reason about these ideas (e.g., students would see that there is a limit to growth and how this is represented in the equation and be able to use the equation to find the *y*-intercept). Ms. Nance was able to implement the focus task discussed during this meeting at a high level. However, the team members' anticipating during the MLSC week 1 meeting centered on Cara Nance's focus lesson that she used in the lesson on March 19 focused almost exclusively on what students would do when working on the task (e.g., students will use a calculator, students will try to solve it algebraically), with little or no regard to what students might notice or reason about. Ms. Nance's implementation of this focus task was rated low level.

The 6th-, 7th-, and 8th-grade team's MLSC meetings did not include discussions around the focus tasks or anticipating of possible solution methods at the same depth as those of the 11th- and 12th-grade team. This was due to the lack of preparation of the team members for these meetings. During the majority of the MLSC week 1 meetings, only one team member produced any work on the focus task or brought noticings and wonderings. Thus, the team members looked at the focus task for the first time during the meetings. This resulted in discussions that only touched on surface-level aspects of the task, as the team members had not considered the mathematical complexities of these cognitively challenging tasks. Thus, neither Gloria Xavier nor Nathan Ingram benefited from the anticipating of their team members as Cara Nance did. Consequently, it appears as though the teachers' lack of anticipating individually and, in the case of Gloria Xavier and Nathan Ingram, the lack of quality group anticipating during the team meetings also prevented them from effectively using the remaining practices of the five practices as well as consistently implementing high-level tasks.

5.1.2 The Chaotic Nature of the School

The 2011-2012 school year at Lincoln Secondary School was very stressful for the teachers involved. As the school population had grown to include 11th grade the prior year and 12th grade at the beginning of the 2011-2012 school year, Lincoln's student body now included members of rival gangs who had previously attended separate schools. There had also been tremendous turnover of teachers from the previous year, and many of the newcomers had been placed on instructional improvement plans by school or district administrators. Many of the teachers new to Lincoln had not arrived at the school by choice; rather they had been displaced when the schools at which they had been working were closed due to declining enrollment in the district. Further, the district was beginning to implement a new teacher evaluation program based in part on teachers' instructional performance. In addition, many teachers at Lincoln (both in the middle and upper grades) noted extreme student behavior issues at the school such as fights, students roaming the halls during class time, and poor behavior in class. One teacher stated, "We had more fights this year than I have seen in my entire 12 years of teaching... We have lost good students because we have allowed people to beat people." (Russell & Stein, 2013). Russell and Stein (2013) also reported that teachers were frustrated with the lack of routines and consequences in place at Lincoln when dealing with student behavior problems. All of these issues, unrelated to instruction within a specific content area, impacted many teachers' outlook on the school year and their attitude about the professional development. At the end of the school year Gloria Xavier summed up the situation by stating:

> School was horrible this year. I just really had a rough year. So it [professional development and the electronic lesson-planning tool] was like an added piece on top of all of this roughness… We were in the midst of chaos, so any addition of work, any addition of anything just made the chaos spin. (Russell & Stein, 2013)

Clearly, the chaotic and stressful environment at Lincoln presented serious challenges for the teachers. Multiple teachers voiced their frustrations with their inability to cope with the problems they faced, and some expressed that they did not feel the professional development assisted them in successfully attending to these problems (Russell & Stein, 2013).

The chaotic environment at the school seemed to impact at least some teachers' participation in the professional development and their instructional practices. The quote from Gloria Xavier suggests that the stressful context in which the teachers were working negatively impacted her participation in the professional development. Evidence of the negative influence of the school context is also seen in the passive aggressive nature in which the middle school mathematics teachers fought the school administration's increase of teacher duties. As described in section 4.3.1.3, the middle school teachers felt that their time to work on professional development was being taken over with other duties and they did not feel they had time to do the assigned work for the professional development meetings (e.g., work on the focus task or create noticings and wonderings).

The chaotic school context seemed to impact Nicole Nesmith's and Cara Nance's instruction. Recall that in each lesson in which Nicole Nesmith was unable to maintain the high level of cognitive demand, it was due to an inability to maintain control of the classroom. In each case, student behavior became so disruptive that the majority of the class did not remain on task. During the one lesson a fight broke out, during other lessons multiple students who were not members of the class entered and disrupted her class. In general, she did not have the support she needed to properly manage her classroom. However, during the one lesson in which there were additional teachers and tutors in the room, student behavior was not an issue and Ms. Nesmith was able to implement the task at a high level. Cara Nance's lessons seemed to progress very quickly, at times not allowing for a press for student reasoning, and thus the tasks were often implemented at a low level. As described in section 4.1.3.1, Ms. Nance seemed to sense the potential for student behavior issues and this may have been one reason for the rush during her lessons. Section 4.1.3.1 also highlights a lesson in which Ms. Nance did lose control of her classroom. Thus, in the case of at least three of the four teachers who participated in this study, the undisciplined and chaotic school environment appeared to have some influence on either their participation in the professional development or their classroom instruction.

Research on education in contexts such as Lincoln (i.e., urban, high-poverty schools typically serving high proportions of African American and other minority students) suggests that in order to succeed when working in these environments teachers must develop knowledge and skills in many non-content areas. Milner (2013) argued that while content and pedagogical knowledge are necessary for teachers to be successful, these are insufficient, particularly for teachers working in contexts such as Lincoln. He suggested that one of the most critical skills that teachers in these environments need to develop is the capacity to learn about the students they are teaching. Haberman (1995) suggested that teachers must build positive relationships with their students in order to be successful. Positive teacher-student relationships have been found to increase students' motivation and effort (Wentzel, 1997) and decrease disruptive behavior (Battistich, Solomon, Watson, & Schaps, 1997). Building relationships requires teachers to be familiar with their students, both individually and as adolescents. Brighton (2007) posited that in order to form positive relationships with their students teachers must have an understanding of how students develop intellectually, socially, physically, emotionally, morally, and religiously.

The professional development investigated in this study, and the Lesson Planning Project of which it was a part, were not designed to assist teachers in dealing with the many issues they faced as a result of the chaotic nature of the school. The purpose of the professional development was to focus on aiding teachers in developing skills related to implementing high-level, cognitively demanding mathematical task. The university-based facilitators of the professional development were pedagogical content experts. They did not possess the expertise to provide teachers support in the non-content related areas of need. As such, they were unequipped to tackle the myriad of issues faced by the teachers at Lincoln. Thus, while the teachers engaged in the professional development focused on content-specific instruction on a weekly basis, it did not attend to many of their immediate needs. Furthermore, it does not appear that teachers were receiving sufficient support to address these issues from other means (e.g., additional unrelated professional development, assistance from school or district personnel). The lack of support for attending to the student behavior, school culture, and relationship-building issues and the disconnect between these issues and the content-specific professional development focused on instruction and pedagogy may have a created a barrier that prevented some teachers from fully engaging in and benefitting from the professional development.

5.2 LIMITATIONS OF THE STUDY

> Teaching work, like theater, unfolds in real time, in the presence of others. No teaching moment is quite like the next; even if the teacher were to repeat the same script, its reception would vary with every delivery. This phenomenon is a central challenge in teaching as well as in studying teaching. Because enactment flies by, and because each lesson or unit or encounter is not exactly like any other, teaching is difficult to describe even simply. (J. Lewis, 2008, p. 1)

In this quote, Lewis speaks to the difficulty of capturing and portraying teachers' instruction. The complexities of conducting research around teachers' participation in professional development and their teaching contribute to the various limitations of this study. These limitations are related to both the collection and analysis of the data and will be presented in this section.

5.2.1 Limitations Regarding Data Collection

So much occurs during classroom instruction that it is impossible to capture it all. One must choose to focus on the teacher, a subset of students, all of the students, or everyone in the room. Each of these choices places varying restrictions on the amount of focus and detail that can be captured throughout the lesson. Ideally, researchers can use one or more video cameras to collect as much information as possible during instruction. However, even this tool presents limitations. Due to Institutional Review Board guidelines and restrictions from the district, which included Lincoln Secondary School, the use of video recorders was not an option in this study. Thus, the researcher had to depend on comprehensive fieldnotes to obtain as much detail regarding the classroom observations as possible. He paid particular attention to aspects of the lesson that were salient to the study (e.g., teachers' use of the five practices (or their lack of use); the level of cognitive demand of the task as set up and implemented during the lesson; possible factors related to the maintenance or decline of the level of cognitive demand of the tasks that were present during the lesson). However, it was impossible to capture all of the nuanced interactions that took place during the course of the 80-minute lessons and despite the researcher's best efforts, there were inevitably data that were not captured. The researcher used the detailed fieldnotes to produce the observation write-ups that were the primary data sources used to analyze teachers' instruction. However, the lack of video recordings of the lessons meant that the researcher and the secondary coder could not refer back to the actual instruction when analyzing the data. They only had access to the secondary depiction of the instruction in the form of the observation write-ups. This lack of video data and the dependence on these secondary descriptions of the lessons, may not have always provided the researcher and the secondary coder with a completely accurate account of what occurred during the lessons.

5.2.2 Limitations Regarding Data Analysis

Analyzing teachers' participation in professional development and their instruction, like data collection, also has certain limitations. Many of the limitations in this study were due to the fact that the work reported on in this study (e.g., analyzing teachers' use of the five practices, analyzing teachers' uptake of key ideas in professional development) had not been conducted in previous research in the manner in which it was undertaken herein. Thus, some of the analytical tools and processes had not been used prior to this study and as such, some unanticipated issues arose during data collection and analysis. For example, the Look Fors Sheet lacked precision in its recording of teachers' use of the five practices in some areas. With regard to anticipating, the Look Fors Sheet only takes into account the information in teachers' lesson plans. It does not account for thinking the teachers may do that is not recorded in their lesson plans (e.g., in the MLSC meetings or in individual discussions with other teachers). The Look Fors Sheet used "student approaches" as a method for determining a teacher's use of selecting, sequencing, and connecting. However, the definition of a student approach is not made clear. For this study, the researcher considered a student approach to be an explanation of student thinking (how or why the student did something), not just a statement of what they did or the answer that they found. Thus, in some of the observed lessons teachers asked students to share their responses during whole-class discussion, but because no explanation as to the thinking behind how they found the answers were discussed, these were not counted as student approaches. The Five Practices Summary Sheet also lacked precision in some areas, which may have produced an analysis of the teachers' instruction that was not completely accurate. For example, to obtain a score of low use with regard to monitoring, teachers only needed to observe their students as they worked on the task. Almost all of the mathematics teachers at Lincoln, including those not in this study, were in the practice of circulating the classroom as students worked on the task. In some cases, this may have been a form of classroom management to help ensure students remained on task. Therefore, using the fact that teachers circulated the classroom as students worked on the task most likely did not represent the true idea of monitoring as indicated in the five practices. As a result of these unanticipated complications, the analysis of the data using the Look Fors Sheet and the Five Practices Summary Sheet may have underrepresented teachers' use of some of the five practices. Changes to how the Look Fors Sheet accounts for teacher anticipation and a clearer definition of what constitutes a "student approach," as well as a revised coding rule for monitoring on the Five Practices Summary Sheet may produce data that are more accurate during data collection and results that are more reliable during data analysis.

The lack of high interrater reliability and the conservative nature of the tools that were used to analyze the data are also important limitations to this study. The interrater reliability with regard to the level of cognitive demand of the tasks and the teachers' use of the five practices was well below the recommended levels. This may have been partially due to the use of observations write-ups (as opposed to video recordings) or, as noted in Chapter Three, it may have been a function of the secondary coder's unfamiliarity with the mathematical content and curricula. The analytical tools, and the coding rules the researcher applied to them, were very conservative (e.g., what did and did not count as a student approach). This conservative scoring may have also erred on the side of providing lower estimates of teachers' use of the five practices and their implementation of high-level tasks than actually occurred.

5.2.3 Lack of a Measure of Teacher Knowledge

A limitation related to the data analysis in this study is the lack of a measure of teacher knowledge. This study did not take into account the possibility that teachers' knowledge, or lack thereof, may have played a role in their participation in the professional development meetings and their ability to implement tasks at a high-level in their lessons. Ball, Thames, and Phelps (2008) suggested that both subject matter knowledge as well as pedagogical content knowledge are needed for successful teaching. It is reasonable to assume that teachers must have knowledge of the mathematical concepts they are teaching in order to prepare for and teach high-level tasks. It is also likely that a firm understanding of these concepts would be important for teachers as they make decisions regarding the selecting, sequencing, and connecting of students' responses during whole-class discussions. This knowledge was not measured as part of this study, and therefore it is unknown how, if at all, teachers' level of knowledge impacted their ability to implement high-level tasks and their use the five practices. As the researcher and Dr. Travis (the other university-based facilitator of the professional development and member of the larger research project of which this study was a part) had worked with Mr. Ingram during the two years prior to this study, both had noted that he seemed to lack firm knowledge of many of the mathematical concepts he was asked to teach. Further, he commented on his unfamiliarity with several of these concepts during his three-year association with the researcher and Dr. Travis. Thus, it was apparent that Mr. Ingram had a lack of mathematical content knowledge related to the courses he taught. In at least one of his lessons, we saw a case in which his apparent lack of content knowledge was a factor in the decline of the level of cognitive demand of the task. In this lesson (described in Chapter Four), a student presented a flawed strategy for determining the perimeter of parallelograms; multiple students had used this strategy as they worked on the task. Mr. Ingram did not acknowledge that this strategy was erroneous, nor did he challenge the students' thinking about this strategy. Thus, in at least Mr. Ingram's case, lack of content knowledge may have impacted his implementation of high-level tasks and his use of the five practices. This also suggests that the teachers' level of content knowledge, as well as of their mathematical knowledge for teaching (Ball & Bass, 2003), may have hindered their implementation of high-level tasks and their use of the five practices. Mr. Ingram was only certified to teach elementary grades and Gloria Xavier was elementary certified with middle school certification added via testing. Hence, both had limited mathematical preparation. Although Cara Nance and Nicole Nesmith were certified to teach secondary grades (through 12th grade), this does not imply they have did not have issues related to content. Therefore, a knowledge assessment would be needed to determine the extent to which teachers' lack of knowledge could be a factor in their inability to implement tasks at a high level.

5.3 CONTRIBUTIONS OF THE STUDY REGARDING TEACHER EDUCATION AND RESEARCH OF TEACHERS' INSTRUCTIONAL PRACTICES

While there were several limitations to this study, it does provide many contributions to the existing knowledge of research of teachers' participation in professional development and of their instructional practices. This section will highlight these contributions.

5.3.1 Contributions to Professional Development

This study adds to the expanding body of literature regarding teacher professional development around the implementation of high-level mathematical tasks (e.g., Arbaugh & Brown, 2005; Boston & Smith, 2009, 2011; Silver et al., 2007, 2005, 2006; Silver & Stein, 1996; Stein et al., 2009). Most research studies that investigate teacher professional development, including those listed above, describe the activities in which teachers participated and the impact of the professional development on their subsequent instruction. However, few analyze teachers' participation in the professional development activities. The analyses in this study investigated teachers' participation in the professional development as well as their instruction. As such, this study provides useful information regarding what teachers actually do in preparation for and during such professional development. It also sheds light on possible factors that hindered this work from occurring as it was designed.

The information in this study around teachers' participation in the professional development is particularly insightful as it investigated professional development that was mandatory for all of the mathematics teachers at the school. This differs from many professional development programs that involve teachers who volunteer for, or are invited to participate in, the professional development (e.g., Boston & Smith, 2009; Silver et al., 2006), who apply for participation (Silver & Stein, 1996), or who seek funding to pay for participation in professional development (Arbaugh & Brown, 2005). As such, it highlights possible difficulties faced by the facilitators of the professional development in getting some teachers to engage in this process and possible causes. For example, consider Gloria Xavier's and Nathan Ingram's lack of preparation for the MLSC week 1 meetings. In the case of Ms. Xavier, this may have been due to disagreements with the school administration about the use of teachers' time. In the case of Mr. Ingram, it was possibly due to a disagreement in teaching philosophies with the university-based professional development facilitators.

The apparent lack of impact of the professional development on teachers' instructional practices (e.g., their inconsistent and low-level use of the five practices and their inability to implement cognitively demanding tasks at a high level) highlights the need for professional development to meet teachers where they are professionally and in their work environment. The teachers at Lincoln faced many challenges that hindered their ability to incorporate the focus of the professional development in their instruction. This was evident in Nicole Nesmith's inability to implement tasks at a high level during her lessons due to the student behavior issues she faced in her classroom. Creators and facilitators of professional development need to work with a broader set of experts to design a more integrated set of experiences for teachers that go beyond a focus on content in isolation. These experiences should take into account the challenges teachers face that, left unaddressed, may prevent them from utilizing or benefiting from the content-specific aspects of the professional development.

5.3.2 Insight into Teachers' Collaborative Exploration of Their Instruction

The study also adds to the knowledge base of teachers' collaborative exploration of their instruction (e.g., Arbaugh, 2003; Fernandez & Yoshida, 2004; Fernandez, 2005; C. Lewis, Perry, & Murata, 2006; C. Lewis, 2000; Sherin & Han, 2004). The findings show that the four teachers in this study participated in this collaboration in different ways. Cara Nance and Nicole Nesmith prepared for each meeting and used the materials they prepared to provide insightful suggestions to their colleagues regarding their instruction. On the other hand, while Gloria Xavier and Nathan Ingram also provided suggestions during their team's meetings, they did very little to prepare for the meetings. Thus, as there is no evidence that they had thought deeply about the focus task prior to the meetings, it is probable that the ideas they provided were not as thought out as if they had prepared. The study suggests possible reasons for Ms. Xavier's and Mr. Ingram's lake of preparation for these meetings.

5.3.3 Contributions to Research of Teachers' Instructional Practices

This study adds to the corpus of research on teachers' instructional practices. The findings of this study confirm those of Stein and her colleagues (1996) that teachers struggle to implement cognitively challenging tasks in a manner that maintains the challenge throughout the lesson. This study builds on the work by Stein and her colleagues as it suggests that even with support provided in the form of collaborative discussions with their peers about the use of high-level tasks teachers still tend to implement high-level tasks at a low level. This provides further evidence of the need for educational research regarding the conditions, support, knowledge, and abilities teachers need to be able to successfully use rigorous tasks with their students.

While there has been limited research on teachers' use of each of the five practices in isolation (e.g., Schoenfeld (1998) describes a novice teacher's and an experienced teacher's anticipations of possible student responses for two tasks and how the differences of their anticipations played out in their instruction of these tasks), there is a dearth of studies examining the use of the five practices as a set. This study suggests that high-level anticipating may be a crucial component of teachers' use of the other four practices as the failure to do so seems to have impeded teachers from consistently using monitoring, selecting, sequencing, and connecting. The lack of quality anticipating also seems to have been a factor in preventing teachers from implementing cognitively demanding tasks at a high level.

This study also provides new analytical tools for future studies of the five practices. The Look Fors Sheet was designed to focus researchers' attention on specific aspects of instruction that can be used as evidence of the teachers' use, or lack thereof, of each of the five practices. This tool along with the Five Practices Summary Sheet are instruments researchers can use to make evidence-based judgments regarding the level of use of each of the practices in teachers' instruction. While these tools lack some precision as they were used in this study, with some modifications, these tools could be useful resources for future investigation of teachers' instruction.

5.4 FUTURE RESEARCH

The outcomes of this study suggest the need for future research on professional development that (a) meets teachers where they are professionally and in their work environment; and (b) incorporates content-specific instructional theories and practices as well as addresses the non-content related issues. This notion of "meeting teachers where they are" does not imply facilitators should scuttle previously designed professional development programs at the first signs that teachers are struggling with issues outside of the intended design. Rather, it suggests that professional development must focus on teachers' content-specific instruction while also being conscious of and attending to the challenges they face in their particular teaching contexts. This suggests the need for experts in several areas of education (e.g., content-specific instruction, students' social and emotional development, behavior, motivation, etc.) to collaborate in designing and providing this type of professional development. Research around these types of collaborative efforts and their impact on teachers' instruction and student learning would greatly benefit the fields of teacher education and instruction and learning.

The results of this study also suggest that further research is needed with regard to professional development that incorporates evidence-based discussions that problematize aspects of teachers' instruction and includes an emphasis on reflection. While these were aspects of the original design of the professional development in the study, they were pushed aside as attention was shifted toward other aspects of the work. In the case of the 11th- and 12th-grade team's MLSCs, these aspects were drowned out by the amount of time taken up in the meetings on adaptations and modification to the task the teachers selected. The discussion centering on possible modifications to the focus tasks—as opposed to other goals of the professional development (e.g., what sequence of anticipated solution strategies would best build toward the learning goals of the lesson)—took up the majority of the time in these meetings, evidenced by the number of key ideas suggesting changes in the focus tasks selected by Cara Nance and Nicole Nesmith. With regard to the 6th-, 7th-, and 8th-grade team, the team members' lack of preparation made these types of in-depth discussions impossible. While teachers brought artifacts of practice (e.g., samples of student work) to the MLSC week 2 meetings, discussions around these artifacts did not problematize aspects of their instruction. Rather, the discussions typically fell into a pattern of the teacher who had selected the focus task recounting his or her instruction of it. Teachers were encouraged to reflect on their instruction formally in the tool, but due to complaints to school administrators regarding the amount of time the professional development took away from their other duties, these reflections were not stressed by the university-based professional development facilitator. Future research on professional development that includes evidence-based discussions problematizing teachers' instruction and that encourages teachers to reflection on their instruction would provide insight into the impact of these activities on teachers' uptake of the central ideas upon which the professional development is designed and changes in teachers' practices.

A final implication drawn from this study for the future development, facilitation, and research of professional development centered on the level of cognitive demand of instructional tasks and teachers' use of the five practices is the need to introduce the various items that are the focus of the professional development individually over an extended period of time. Smith and Stein (2011) suggested that determining clear learning goals and selecting appropriate (e.g., high-level) tasks is a critical foundation for conducting whole-class discussions and for teachers to use the five practices. The vital importance of anticipating with regard to teachers' use of the five practices and the ability to implement tasks at a high level is one of the findings from this study. This suggests that future professional development around these ideas should occur in the form of decomposition of practice. Grossman et al. (2009) explained that “decomposition of practice involves breaking down practice into its constituent parts for the purposes of teaching and learning” (p. 2058). A substantial portion of the time in the professional development meetings examined in this study was devoted to discussion of modifications of the tasks the teachers selected, the anticipated solutions to the tasks, or potential issues the teachers would face while teaching the tasks. As a result, there was very little if any focus on the four remaining practices. Professional development that uses the decomposition of practice approach would suggest that teachers should be introduced to the importance of learning goals, how to select high-level tasks, and each of the five practices one at a time. Teachers would also need to be provided with sufficient time and opportunities to put these into practice, reflect on their use of these practices, and refine their use of them. One such approach would be to introduce these practices to teachers as part of a multi-year professional development program. The first year would focus on selecting clear goals and appropriate tasks. The second year would build on these by focusing on anticipating student solutions to the tasks and using recoding tools to monitor how students engage in the task during the lesson. The third year would then incorporate selecting, sequencing, and connecting as built on the previous practices. Toward the end of the professional development program, teachers would be asked to incorporate each of these skills and practices together in their instruction. Professional development that incorporated the decomposition of practice would provide teachers with the opportunity to focus on the practices one at a time when they are initially introduced to the practices, as opposed to all at once. Thus, it would break up a very complex set of ideas, skills, and practices into more manageable sub-practices that teachers can master individually.

5.5 SUMMARY

This study investigated teachers' participation in professional development designed to aid teachers in selecting and implementing cognitively challenging tasks and the impact of this participation on their instruction of these tasks. The results provide insight into the challenges of engaging teachers in this type of work, as well as the struggles they have in using high-level tasks and the five practices. The discussion section explores two possible factors that may have impeded the teachers in successfully using the five practices and implementing cognitively challenging tasks at a high level. Finally, avenues for future research taking into account the results of this study and the suggested factors were presented.

APPENDIX A

THE TASK ANALYSIS GUIDE

The Task Analysis Guide (M. S. Smith & Stein, 1998).

Low-Level Cognitive Demands	High-Level Cognitive Demands
Memorization Tasks • Involve either producing previously learned facts, rules, formulae, or definitions or committing facts, rules, formulae, or definitions to memory. • Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure. • Are not ambiguous—such tasks involve exact reproduction of previously seen material and what is to be reproduced is clearly and directly stated. • Have no connection to the concepts or meaning that underlay the facts, rules, formulae, or definitions being learned or reproduced. *Procedures Without Connections Tasks* • Are algorithmic. Use of the procedure is either specifically called for or its use is	*Procedures With Connections Tasks* • Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas. • Suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts. • Usually are represented in multiple ways (e.g., visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations helps to develop meaning. • Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.

evident based on prior instruction, experience, or placement of the task. • Require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it. • Have no connection to the concepts or meaning that underlie the procedure being used. • Are focused on producing correct answers rather than developing mathematical understanding. • Require no explanations or explanations that focus solely on describing the procedure that was used.	*Doing Mathematics Tasks* • Require complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example). • Require students to explore and to understand the nature of mathematical concepts, processes, or relationships. • Demand self-monitoring or self-regulation of one's own cognitive processes. • Require students to access relevant knowledge in working through the task. • Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions. • Require considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process required.

APPENDIX B

THE *LAND SECTIONS* TASK

The *Land Sections* task (Lappan et al., 2006b), the focus task selected by Nathan Ingram for the MLSC week 1 meeting on January 4[12].

---

[12] From *CONNECTED MATHEMATICS BITS AND PIECES II* Copyright © 2006 by Michigan State University, G. Lappan, J. Fey, W. Fitzgerald, S. Friel, and E. Phillips. Used by permission of Pearson Education, Inc. All Rights Reserved.

Investigation 2

Adding and Subtracting Fractions

Knowing how to combine and separate quantities is helpful in understanding the world around you. The mathematical names for combining and separating quantities are *adding* and *subtracting*.

For example, if you own two acres of land and you buy another half-acre lot, you will have $2 + \frac{1}{2}$, or $2\frac{1}{2}$, acres of land. The number sentence that shows this relationship is:

$$2 + \frac{1}{2} = 2\frac{1}{2}$$

The *sum* refers to the $2\frac{1}{2}$ acres of land you own.

If you then sell $\frac{3}{4}$ of an acre of your land, you will own $2\frac{1}{2} - \frac{3}{4}$ acres of land. The number sentence that shows this relationship is:

$$2\frac{1}{2} - \frac{3}{4} = 1\frac{3}{4}$$

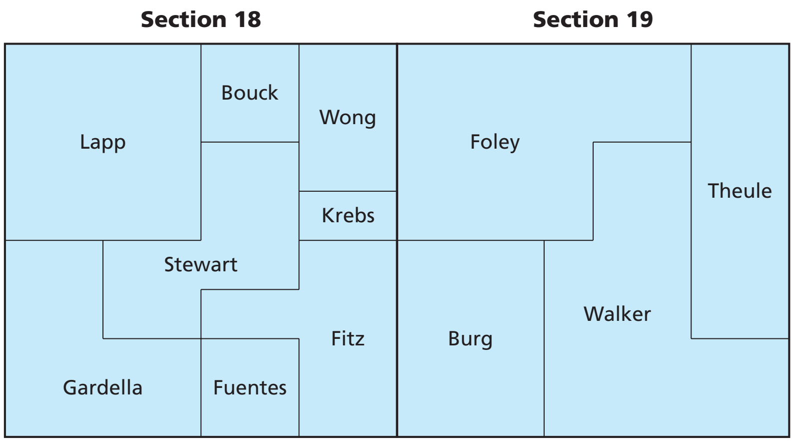
The *difference* refers to the $1\frac{3}{4}$ acres of land you will own.

The problems in this investigation require you to add and subtract fractions. As you work, use what you have learned in earlier units and investigations about fractions and finding equivalent fractions. Practice writing number sentences to communicate your strategies for solving the problem.

2.1 Land Sections

When Tupelo Township was founded, the land was divided into sections that could be farmed. Each *section* is a square that is 1 mile long on each side. In other words, each section is 1 square mile of land. There are 640 acres of land in a square-mile section.

The diagram below shows two sections of land that are *adjacent*, or side by side. Each section is divided among several owners. The diagram shows the part of a section each person owns.



Problem 2.1 Writing Addition and Subtraction Sentences

A. What fraction of a section does each person own? Explain.

B. Suppose Fuentes buys Theule's land. What fraction of a section will Fuentes own? Write a number sentence to show your solution.

C. **1.** Find a group of owners whose combined land is equal to $1\frac{1}{2}$ sections of land. Write a number sentence to show your solution.

2. Find another group of owners whose combined land is equal to $1\frac{1}{2}$ sections of land.

D. **1.** Bouck and Lapp claim that when their land is combined, the total equals Foley's land. Write a number sentence to show whether this is true.

2. Find two other people whose combined land equals another person's land. Write a number sentence to show your answer.

3. Find three people whose combined land equals another person's land. Write a number sentence to show your answer.

E. How many acres of land does each person own? Explain your reasoning.

F. Lapp and Wong went on a land-buying spree and together bought all the lots of Section 18 that they did not already own. First, Lapp bought the land from Gardella, Fuentes, and Fitz. Then Wong bought the rest.

1. When the buying was completed, what fraction of Section 18 did Lapp own?
2. What fraction of Section 18 did Wong own?
3. Who owned more land? How much more land did he or she own?

ACE **Homework starts on page 24.**

APPENDIX C

FACTORS ASSOCIATED WITH THE DECLINE AND MAINTENANCE OF HIGH-LEVEL COGNITIVE DEMANDS

(Stein & Smith, 1998)

Factors Associated with the Decline of High-Level Cognitive Demands	**Factors Associated with the Maintenance of High-Level Cognitive Demands**
1. Problematic aspects of the task become routinized (e.g., students press the teacher to reduce the complexity of the task by specifying explicit procedures or steps to perform; the teacher "takes over" the thinking and reasoning and tells students how to do the problem). 2. The teacher shifts the emphasis form meaning, concepts, or understanding to the correctness or completeness of the answer. 3. Not enough time is provided to wrestle with the demanding aspects of the tasks, or too much time is allowed and students drift into off-task behavior. 4. Classroom-management problems prevent sustained engagement in high-level cognitive activities. 5. Task is inappropriate for a given group of	1. Scaffolding of student thinking and reasoning is provided. 2. Students are given the means to monitor their own progress. 3. Teacher or capable students model high-level performance. 4. Teacher presses for justifications, explanations, and meaning through questioning, comments, and feedback. 5. Tasks build on students' prior knowledge. 6. Teacher draws frequent conceptual connections. 7. Sufficient time is allowed for explanation - not too little, not too much.

students (e.g., students do not engage in high-level cognitive activities because of lack of interest, motivation, or prior knowledge needed to perform; task expectations are not clear enough to put students in the right cognitive space). 6. Students are not held accountable for high-level products or processes (e.g., although asked to explain their thinking, unclear or incorrect student explanations are accepted; students are given the impression that their work will not "count" toward a grade).	

APPENDIX D

NOTICINGS AND WONDERINGS RECORDING SHEET

____

Title of Task

I noticed…	**I am wondering…**

APPENDIX E

THE *SINGLE STAR OR GALAXY?* TASK

The *Single Star or Galaxy?* task, the focus task selected by Nicole Nesmith for the MLSC week 1 meeting on January 18.

Single Star or Galaxy?

If the x-intercepts of a quadratic function are known, is there one equation with those particular intercepts or is there more than one?

By the end of this investigation, you will be ready to answer the question: How many points (ordered pairs) are necessary to determine a unique quadratic equation?

Since writing an equation for a parabola with special features is what you will be asked to do in most phases of the problem, think about whether a graph and/or table would be helpful representations to reason about the situation.

1) Write an equation for a parabola with x-intercepts at $x = 2$ and $x = 6$.

2) Determine the coordinates of the vertex for the parabola in part l). *Come on!* There are lots of ways you can do this.

3) Now, we are going to get flippy. Modify your equation from part 1) so that the graph is reflected across the x-axis. Where are the x-intercepts? Where is the vertex?

4) You are coming into the home stretch on this problem. Modify your equation from part 1) to apply a vertical stretch with a factor of 2. Where are the x-intercepts? Where is the vertex? Select another factor and predict how the graph will change, then test your conjecture.

5) Single Star or Galaxy? How many quadratic equations do you think there are with x-intercepts at $x = 2$ and $x = 6$? How are they related to one another?

6) How many points (ordered pairs) are necessary to determine a unique quadratic equation? Explain.

APPENDIX F

PROFESSIONAL DEVELOPMENT MEETING SUMMARY

Professional Meeting Summary from 11th and 12th grade team's meeting (Team A) on January 18. The task referred to in this summary is the *Single Star or Galaxy?* Task (Appendix E).

MATH PROFESSIONAL DEVELOPMENT MEETING WRITE-UP

Date: Jan. 18, 2011
Scheduled Time: 8:00 - 8:40
Actual Time: 8:05 - 8:42
Teachers present: Mr. EoLu, Ms. NN, Ms. CN, Mr. ET, Ms. ZD, Ms. NP
University Team Members Present: Sam Eskelson
Observer: Sam Eskelson
Official Purpose of the Meeting:

<u>Descriptive Account of Meeting</u>

00:00 Ms. NN talks about the task that she is using in her class and her goals for the task.

04:30 The group members share their noticings and wonderings about the task. Mr. EoLu asks about how Ms. NN expects students to find the vertex. Mr. BN asks how students are expected to write the equation of the function given only the *x*-intercepts. Ms. NN talks about these questions. They also talk about how Ms. NN expects student to use the calculators on the task.

08:09 Sam wonders whether or not if it would be useful to have the students graph the transformations of the functions or if this would lower the cognitive demand of the task. Ms. NN and the group discuss this. They also talk about how they think students may respond to question #6.

13:00 They continue the discussion they are having about how students will respond to question #6. They also discuss whether or not you really need three points to find a unique quadratic function.

15:27 Mr. EoLu talks about how students should be using a calculator and not a table at this point in their mathematical education. He talks about why. Sam pushes back about why the students need to use a calculator versus another method of graphing the functions. The group talks about the students' struggles to use graphing calculators.

20:35 Ms. NN briefly mentions a task that she made as a possible task to replace the one they are talking about or as a follow up to it (see 11 & 12 Grade Team artifact 5 from Jan. 11, 2012).

21:11 They go back to talking about problem #6 on the task, how students may respond to it and how Ms. NN could lead a discussion about it. They talk about the importance of having the student graph all the functions on the same grid to see how they are related.

26:06 They talk about problem #5 on the task and what type of thought and language Ms. NN is hoping the students use on this problem.

29:04 The group talks about when Ms. NN plans to teach this and how this is affected by the CBA tests scheduling. Ms. NN and Ms. CN express some frustrations about the testing schedules and their students' lack of progress.

31:39 Sam explains what the group will be doing during the following few weeks. The teachers talk about how to modify the computer file provided by the district that the task is on. They talk about the title of the task.

36:47 End

<u>University Facilitator's Reflections</u>
None

<u>Inferential Summary</u>
[Sam: All the teachers except Mr. ET participate well in the discussion of the task. Ms. NN has some very good ideas about how to teach it and the group gives her some nice ideas about how to enact it with her students. Ms. NN and Ms. CN express frustration over the testing schedule and the students' lack of progress.]

APPENDIX G

CLASSROOM OBSERVATION INSTRUMENT (COI)

PART A

COVER SHEET

Author's Name:
Date of Observation:
Teacher's Name (*pseudonym*):
Grade:
Subject:
Class/Block:
****
Scheduled Time Period for this Lesson: From ___ To ____.

Time Instruction Actually Began:
The first address (written or oral) given by the teacher to the class which relates to the current lesson, learning activities or homework
Time Instruction Actually Ended:
When the teacher stops interacting with the class about the current lesson, learning activities or homework
Total Length of Instructional Time:
Time between the beginning of actual instruction and the end of actual instruction

****
Number of students in the class during lesson:
Number of adults in the class during lesson:
Students' gender:

****
Briefly describe the physical features of the classroom (e.g., what is on the walls, how seats are arranged, temperature, etc.).

Provide a sketch of the room noting the physical layout. Mark where students are sitting (include gender, and if possible names of students). If the room arrangement changes during the lesson, include a description and sketch of all new arrangements.

DESCRIPTION OF LESSON

Write an account of the lesson that describes the chronological unfolding of instructional events, including a description of the content, teacher talk and actions, what students did and talked about. Also, include any unusual aspects of the setting [e.g., this was the day after school was cancelled because of a water main break] as well as a "learning context" (i.e., where today's lesson is situated with respect to lessons coming before and after it).

IDENTIFICATION OF INSTRUCTIONAL ACTIVITIES

1. Divide the lesson into the classroom activities with which students were engaged. Briefly identify and list these activities in chronological order. Include a descriptive label, identify how students were grouped (individually, small group, whole group), and indicate the number of minutes devoted to each activity.

2. Identify and append the resource materials that served as the basis of each of the activities.

3. Combine the activities that focused students' attention on the same topic or big idea as a *task* and list the tasks in order according to the amount of time each consumed. *For example, if the teacher opened the lesson with a whole-class discussion of the concept of ratio, then gave students a small group project that asked students to construct ratios to represent a variety of situations, and then conducted a whole-class share and summary of the work produced in the small groups, the three activities should be combined into one larger coherent activity (a task): Conceptualizing and using ratios*.

   It is the observer's prerogative to decide how to combine smaller activities into a larger task (if warranted) AND to give a label to that task (i.e., conceptualizing and using ratios).

   Select the 2 tasks that occupied the most amount of class time. (If the entire class was devoted to one task, please note.)

PART B

COGNITIVE DEAMAND OF INSTRUCTIONAL ACTIVITIES

Using the tasks identified in item 3 of part A, **answer questions 4 through 10 about *both* tasks.**

CURRICULUM MATERIALS

4. Identify the primary kind of cognitive demand of the activity *as it appeared in the resource materials*.
   i. Little or no academic thinking required
   ii. Memorization
   iii. Use of *procedures without connections* to meaning, concepts or understanding
   iv. Use of *procedures with connections* to meaning, concepts or understanding
   v. *"Doing Mathematics"* - engaging in the thinking practices of the discipline
   vi. Other

   Justify your selection.

SET UP

5. Describe the activity as presented either orally or in writing by the teacher to the class. *What were students told to do, with what resources, and what was the expected outcome of their work? If any of these were unclear, designate so.*

6. From the teachers' perspective, what appeared to be the goal of the activity *(i.e., what were students supposed to be learning to be able to do or to understand? What were the targets of learning: new words, formulas, learning to use a new kind of organizer, understanding a big idea like acceleration?)*.

   Justify your conclusion citing evidence from the lesson plan, the lesson materials or your observation.

7. As *set up by the teacher*, identify the primary kind of cognitive demand that this activity placed on students.

   i. Little or no academic thinking required
   ii. Memorization
   iii. Use of *procedures without connections* to meaning, concepts or understanding
   iv. Use of *procedures with connections* to meaning, concepts or understanding
   v. *"Doing Mathematics"* - engaging in the thinking practices of the discipline
   vi. Other

   Justify your selection citing evidence from the materials and your observations.

IMPLEMENTATION

8. Identify the manner in which *for the majority of the students* and the teacher *actually engaged with the activity for the majority of the time*.

   i. Little or no academic thinking occurred
   ii. Memorization
   iii. Use of *procedures without connections* to meaning, concepts or understanding
   iv. Use of *procedures with connections* to meaning, concepts or understanding
   v. *"Doing Mathematics"* - engaging in the thinking practices of the discipline
   vi. Unsystematic and nonproductive exploration
   vii. Other

   Justify your selection; if possible cite evidence from the materials and your observations.

FACTORS RELATING TO DECLINE OR MAINTANENCE OF LEVELS OF COGNITIVE DEAMND

If the **task was set up by the teacher as a *high-level* task complete questions 9 and 10**. If the **task was set up by the teacher as a *low-level* task DO NOT complete question 9 and 10**.

9. If the level of cognitive demand of the task <u>*declined*</u> from *High-Level* (*Procedures with Connections* or *Doing Mathematics*) during setup (question #7) to *Low-Level* (one of the other codes) during implementation (question #8), why? If the level of cognitive demand did not decline from high-level to low-level, please indicate such and skip this question. <u>*(select as many as apply)*</u>

    i. Problematic aspects of the task become routinized (e.g., students press the teacher to reduce the complexity of the task by specifying explicit procedures or steps to perform; the teacher "takes over" the thinking and reasoning and tells students how to do the problem).
    ii. The teacher shifts the emphasis from meaning, concepts, or understanding to the correctness or completeness of the answer.
    iii. Not enough time is provided to wrestle with the demanding aspects of the task or too much time is allowed and students drift into off-task behavior.
    iv. Classroom management problems prevent sustained engagement in high-level cognitive activities.
    v. Inappropriateness of tasks for a given group of students (e.g., students do not engage in high-level cognitive activities due to lack of interest, motivation or prior knowledge needed to perform; task expectations not clear enough to put students in the right cognitive space).
    vi. Students are not held accountable for high-level products or processes (e.g., although asked to explain their thinking, unclear or incorrect student explanations are accepted; students are given the impression that their work will not "count" toward a grade).
    vii. Other (explain)

    Justify your selection; if possible cite evidence from the materials and your observations.

10. If the level of cognitive demand of the task <u>*was maintained*</u> at a *High-Level* (*Procedures with Connections* or *Doing Mathematics*) from setup (question #7) through implementation (question #8), in what ways were students assisted to perform a high level? If the level of cognitive demand was not maintained at a high-level, please indicate such and skip this question. <u>*(select as many as apply)*</u>

    i. Scaffolding of students' thinking and reasoning.
    ii. Students are provided with means of monitoring their own progress.
    iii. Teacher or capable students model high-level performance.
    iv. Sustained press for justifications, explanations, and/or meaning through teacher questioning, comments, and/or feedback.
    v. Tasks build on students' prior knowledge.
    vi. Teacher draws frequent conceptual connections.
    vii. Sufficient time to explore (not too little, not too much).

viii. Other (explain)

Justify your selection; if possible cite evidence from the materials and your observations.

APPENDIX H

COMPLETED CLASSROOM OBSERVATION INSTRUMENT

Completed Part A of the Classroom Observation Instrument for the observation of Cara Nance on May 8.

PART A
COVER SHEET

Author's Name: Sam Eskelson
Date of Observation: May 8, 2012
Teacher's Name (*pseudonym*): Ms. CN
Grade: Mixed High School
Subject: Elementary Function
Class/Block: Block D
****
Scheduled Time Period for this Lesson: From ___ To ____. 1:25 - 2:45

Time Instruction Actually Began: 1:25
The first address (written or oral) given by the teacher to the class which relates to the current lesson, learning activities or homework
Time Instruction Actually Ended: 2:45
When the teacher stops interacting with the class about the current lesson, learning activities or homework
Total Length of Instructional Time: Approx. 80 min.
Time between the beginning of actual instruction and the end of actual instruction

****
Number of students in the class during lesson: 11
Number of adults in the class during lesson: 3 (Ms. CN, the photography teacher Ms. CN shares the room with, Sam Eskelson)

Students' gender: 5 Female, 6 Male

****

Briefly describe the physical features of the classroom (e.g., what is on the walls, how seats are arranged, temperature, etc.).

- The desks are arranged as shown in Figure 1. As Two students enter, they move two of the desks to the back left corner of the room so that the desks now are arranged as shown in Figure 2. Ms. CN allows them to keep there desks in this arrangement. There are windows in the back of the room.
- The class objectives are posted on the board (see artifact 2).
- The temperature was normal (i.e., not too hot or too cold).

Provide a sketch of the room noting the physical layout. Mark where students are sitting (include gender, and if possible names of students). If the room arrangement changes during the lesson, include a description and sketch of all new arrangements.

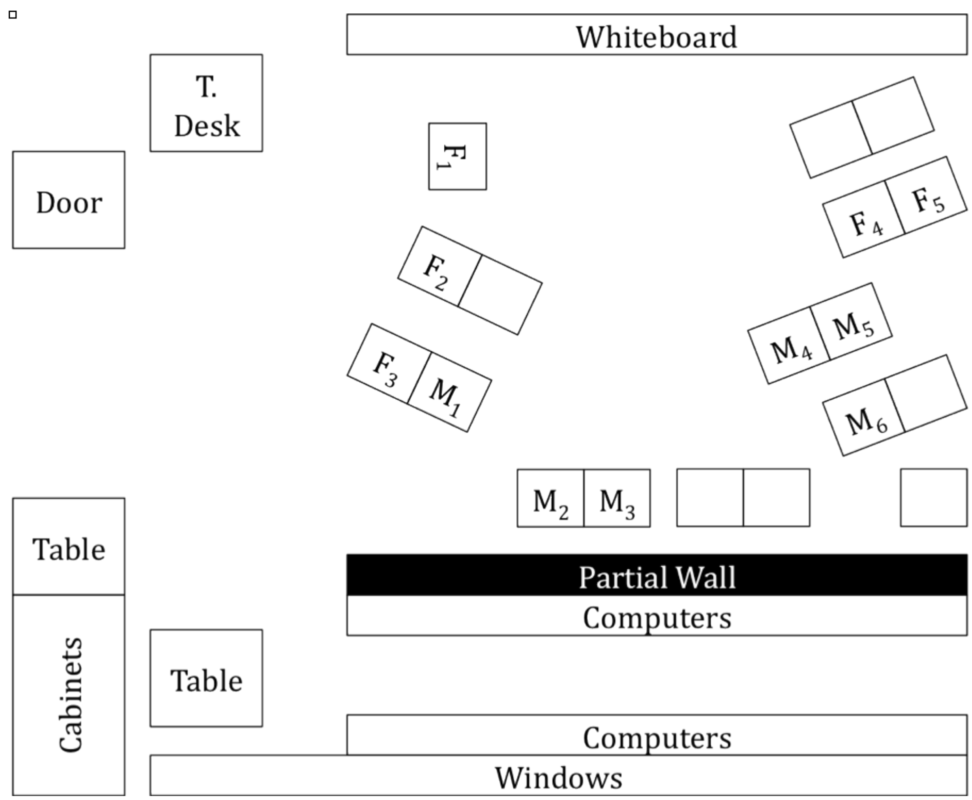


Figure 1

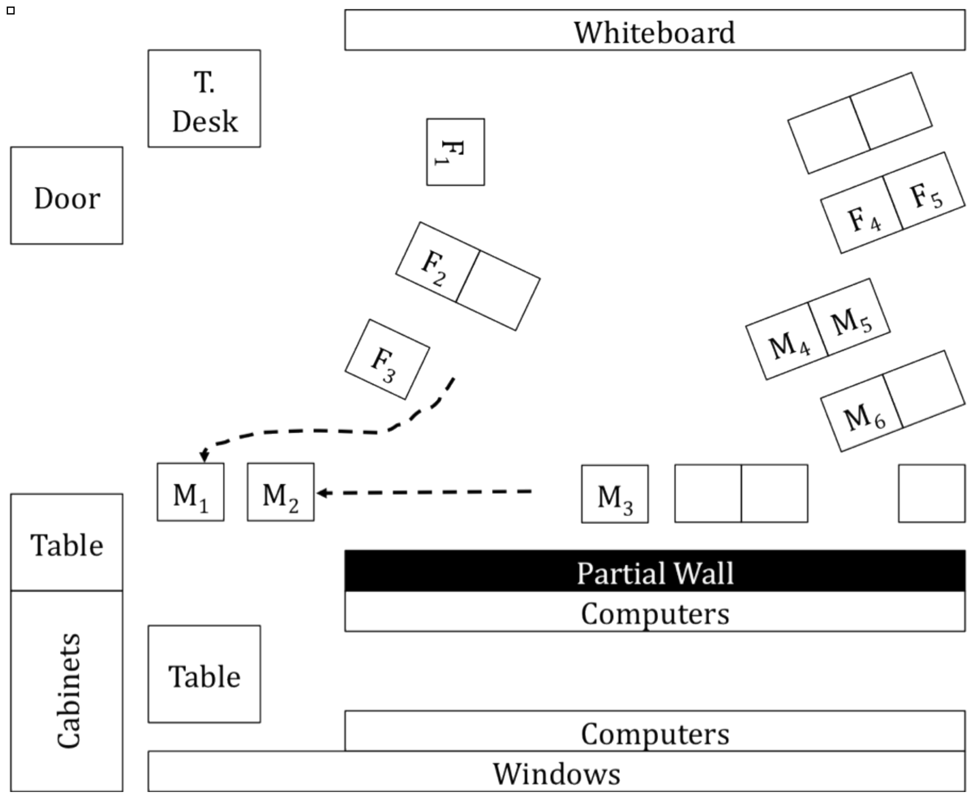
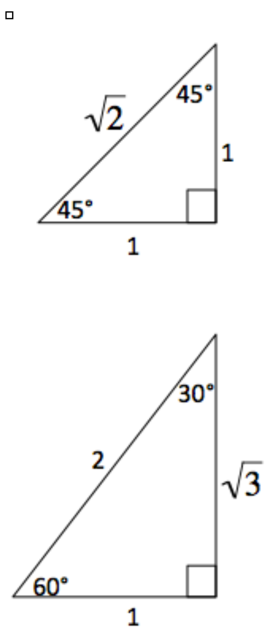

Whiteboard
T. Desk
Door
F_1
F_2
F_3
F_4
F_5
M_4
M_5
M_6
M_1
M_2
M_3
Table
Cabinets
Partial Wall
Computers
Table
Computers
Windows

Figure 2

DESCRIPTION OF LESSON

Write an account of the lesson that describes the chronological unfolding of instructional events, including a description of the content, teacher talk and actions, what students did and talked about. Also, include any unusual aspects of the setting [e.g., this was the day after school was cancelled because of a water main break] as well as a "learning context" (i.e., where today's lesson is situated with respect to lessons coming before and after it).

Pre class Ms. CN puts the warm up problem (see Figure 3) on the board before the students enter the room.



Warm up: Evaluate $y = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

Figure 3

1:25 The bell rings and there are 11 students in the room. They sit and enter. M_1 and M_2 move the desks as shown in Figure 2. Most of the students are looking at the warm up and trying to figure out what to do. Ms. CN looks the door [SE: This was a "hall sweep" period at the school during which teachers were asked to lock their doors once the bell rings and all student who are tardy are disciplined by the administration.] Ms. CN asks the students to write down what the equation in the warm up problem *"means"* if they do not know how to evaluate it.

1:30 A female student comes to the door and begins to pound on it and yell through the door to Ms. CN Ms. CN won't let her in because of the hall sweep and asks her to leave. The student pounds on the door and yells obscenities through it at Ms. CN because she won't let her in. Ms. CN asks the students in the room to ignore the student outside the door and not to look at her through the hole in the blinds covering the window in the door. Ms. CN begins teaching. The student continues to pound on the door and yell at Ms. CN for approximately 1 to 2 minutes.

Ms. CN asks F_5 to *"give the sentence of what the equation means."* F_5 says, *"y is the angle whose sine is negative square root of three over two."* Ms. CN asks M_6 what he drew on his paper [SE: She had seen it when she as he was working on the warm up]. M_6 says that he drew the *x* and *y* axes on his paper. Ms. CN draws these on the board and labels four points on them (see Figure 4).

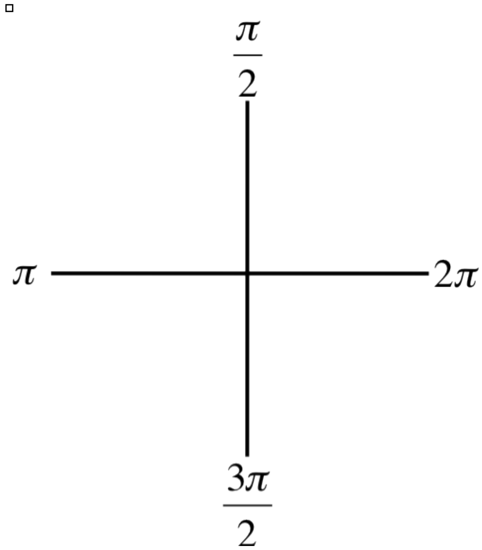


Figure 4

Ms. CN asks the class what the ratio for sine is. M_3 says it is *"opposite over r."* Ms. CN asks what the opposite is and M_6 says that it is the *y*-value. Ms. CN agrees and says that *r* (the radius) is the hypotenuse. She writes $-\frac{\sqrt{3}}{2} = \frac{opp}{hyp} = \frac{y}{r}$. Ms. CN then adds three triangles to the axes she drew earlier, making two of them a different color than the first (see Figure 5). She asks the students what special right triangle the black triangle is and M_6 says that it is *"a thirty, sixty, ninety triangle."* Ms. CN says, *"good, so this angle* (pointing to the angle in the black triangle formed by the *x*-axis and the hypotenuse) *is sixty degrees."* Ms. CN explains that the blue triangles are congruent to the black one. Together the determine that the hypotenuses of each of the blue triangles would also have a value of 2 and that the legs of the two blue triangles that extend from the *x*-axis would have a value of $-\sqrt{3}$. She labels each of these and then asks what the measure of the angle identified as α is (see Figure 6) [SE: Ms. CN did not label angle α or angle β in her drawing on the board, she just pointed to the angle, however, I labeled them to make it easier for the reader to understand what Ms. CN was asking about]. M_4 says that the angle is 240°. Ms. CN then asks what the measure of angle β is and M_4 says that it is 300°. Ms. CN asks how he found it and M_4 says that he subtracted from 360°.

Ms. CN points out to the students that the ratio of $\frac{y}{r}$ for the blue triangle in quadrant III would be $\frac{-\sqrt{3}}{2}$ and that they determined this to be 240°. She then asks what this would be in radians and a student says that it would be $\frac{4\pi}{3}$. Ms. CN writes $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) = 240° = \frac{4\pi}{3}$. They then repeat this process with the blue triangle in quadrant IV and determine the radians to be $\frac{5\pi}{3}$. Ms. CN writes $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) = 300° = \frac{5\pi}{3}$.

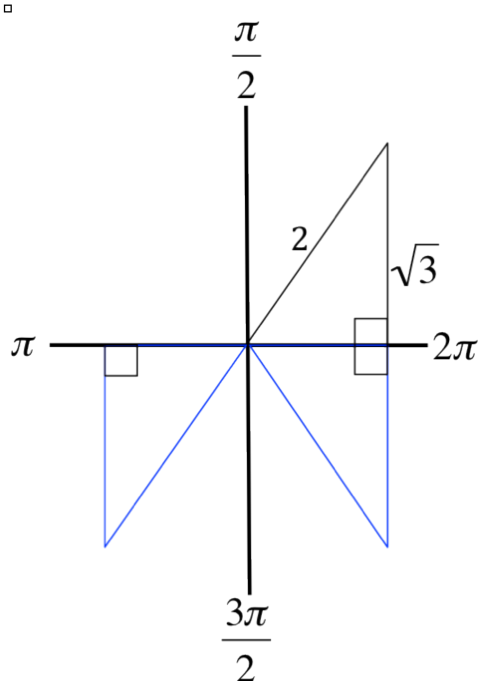


Figure 5

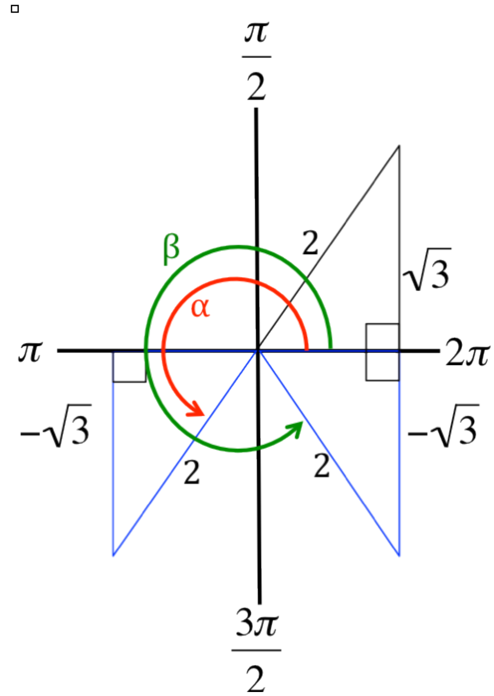


Figure 6

1:35 Ms. CN asks, *"what if we went around the circle one time and then kept going and stopped here* (pointing to the hypotenuse of the blue triangle in quadrant III)*?"* She then says that the triangle formed would be the same so the value of the inverse sine function of the new angle would be the same. She then extends what she wrote before $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) = 240° = \frac{4\pi}{3} + 2\pi n$ and $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) = 300° = \frac{5\pi}{3} + 2\pi n$, and she asks, how many answers there will be to the question. M_6 says that there will be an unlimited amount of answer. Ms. CN agrees and says this is because they can go around the circle as many times as the want to.

1:38 Ms. CN passes out the task (see artifact 3) and some grid paper and asks the students to look at it and do part 1. She tells them that they can work in pairs if they want to. She says that part 1 is similar to what they worked on the pervious day. A few students begin to work on the task, but most do not; they just sit at stare at the paper.

1:40 Ms. CN asks M_1 what the first step to finding the inverse of the equation function is. He says, *"switch the x and the y."* Ms. CN asks how to rewrite *y = sinx*. M_1 tells her that it would be *x = siny* Ms. CN then asks F_5 how to rewrite *y = cosx* and F_5 tells her it would be *x =cosy*. Ms. CN reminds the students that they cannot separate *"sin"* or *"cos"* from *"y."*

F_1 begins making quiet grunting and moaning noises under her breath. She does this off and on for the next five minutes.

Ms. CN writes $y = \sin^{-1}(x)$ and says that this is the same as $x = \sin(y)$. She asks M_5 to tell her a sentence of what this means. M5 says, *"y is the angle whose sine is x."* Ms. CN writes $y = \cos^{-1}(x)$ and says that this is the same as $x = \cos(y)$ and asks what this means. F_2 say, *"y is the angle whose cosine is x."* Ms. CN asks which students think that they understand the meaning of inverse sine and inverse cosine. Four students raise their hands.

1:45 F_1 is still making noises, both Ms. CN and several students ask her to stop. She glares at them but she stops. Ms. CN writes $y = \cos^{-1}\left(\frac{-\sqrt{2}}{2}\right)$ and asks F_3 for a sentence of what this means. F_3 whose head has been down the entire time, lefts her head and says, *"y is the angle whose cosine is negative square root of two over two."* She then puts her head back down. Ms. CN what for another ratio that is equivalent to $\frac{-\sqrt{2}}{2}$. A student says $\frac{-1}{\sqrt{2}}$. Ms. CN agrees that this is equivalent to $\frac{-\sqrt{2}}{2}$ and asks why when it is written it is written as $\frac{-\sqrt{2}}{2}$. F_2 says, *"because you can't have a root in the bottom."* Ms. CN agrees and says, *"right, but we can rewrite the negative square root of two over two as negative one over the square root of two to help us if we want."*

Ms. CN draws Figure 7 on the board [SE: again the angles labeled α and β were not in Ms. CN drawing; I added them for ease of explaining what occurred] and asks what type of triangle the two triangles are. Multiple students say that they are 45, 45, 90 triangles. Ms. CN asks what the measure of the angle labeled β is. M_3 says that they angle is 135°. Ms. CN asks what the measure of the angle labeled α is. M_3 says that they angle is 225°. Ms. CN says, *"oh he* [SE: M_3] *is being smart by using his unit circle* [SE: A handout from his notes that the had created in a previous class]. *"*

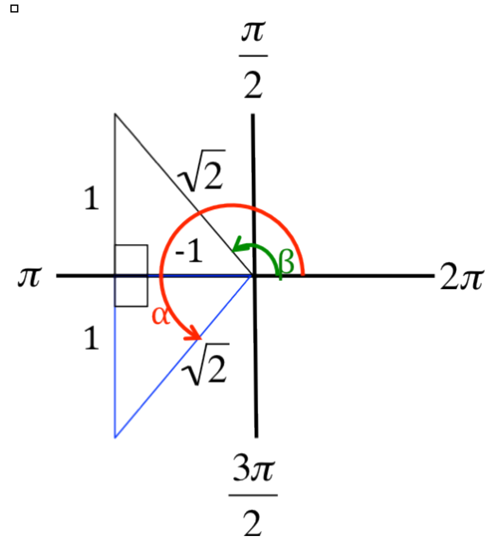


Figure 7

10:50 Ms. CN asks how they could show that there are an infinite amount of solutions for this question. M_3 says that 135° equals $\frac{3\pi}{4}$ and 225° equals $\frac{5\pi}{4}$. Ms. CN repeats her question of how they could show that there are an infinite number of solutions. F_3 says that they need to add, *"the thingy."* Ms. CN writes $\frac{3\pi}{4}+2\pi n$ and $\frac{5\pi}{4}+2\pi n$.

Ms. CN asks M_3 to read part 2 of the task. He does and then Ms. CN says that all of the students know something about functions and their inverses and she asks them to please write something down.

One minute later Ms. CN asks the students what know about functions and their inverses. F_5 says *"the domain and range."* [SE: there is a chance that she said more, but this is all I heard.] Ms. CN writes on the board, *"the domain and range interchange"*. Ms. CN asks F_3 what she and F_3 says, *"x and y reflect across the x-axis"*. Ms. CN says, *"they reflect, or are symmetric, over what?"* M_3 says *"the origin"* and Ms. CN says *"over the line y equals x"* and writes this on the board.

1:55 M_5 passes gas and M_4, F_4, and F_5 all leave their seats and go stand by the door while distracting the whole class with their loud talk about M_5's gas. Ms. CN waits for them for about one minute, then moves on, the students stand by the door for about 2 minutes before returning to their seats.

M_3 asks, *"don't most functions go left to right and inverses go up and down?"* Ms. CN asks, *"in order for a function to be a function, what must it do?"* M_3, F_3, and M_4 all say that it must pass the vertical line test. Ms. CN records this on the board. Ms. CN asks the students to look in their folders for some past work they did graphing the sine and cosine functions. Ms. CN shows F_3's work to the class so that the other students recognize what they are looking for. They spend the next minute or two looking for their work.

2:00 Ms. CN asks F_2 to read part 3 of the task, she does. Ms. CN creates two tables on the board (see Table 1 and Table 2). She says that the table for $y = sinx$ is in their past work in their folders. She then says that they will have a competition and whoever completes both tables on their paper and can put them on the board first will get a bonus point [SE: I'm not sure what the points are used for (grades, prizes, etc.)].

Table 1

x	$y = sinx$
-2π	
$-\frac{3}{2}\pi$	
$-\pi$	
$-\frac{1}{2}\pi$	
0	
$\frac{1}{2}\pi$	
π	
$\frac{3}{2}\pi$	
2π	

Table 2

x	$y = \sin^{-1} x$

The students begin to complete the tables for part 3 of the task and a student asks Ms. CN, *"how do you do this?"* Ms. CN says to the student, *"what do you know about inverses of functions?"* To which the student replies, *"oh, just switch them!"*

The students continue to work on the task. Ms. CN circulates throughout the room making sure that the students are doing it correctly. Ms. CN looks at F_2's work and tells her that she is copying the values for the cosine function not the sine function.

F_5 says that she is done and has Ms. CN check her work on her paper. Ms. CN says that she is correct and asks her to put the tables on the board. Ms. CN tells the class that F_5 won the race. F_5 goes to the board and puts her tables on the board (see Table 3 and Table 4). As F_5 put her tables up the other students complete there tables. M_3 goes up to the board to see what F_5 is doing [SE: he also told Ms. CN after class that he went up there to help F_5]. Some of the other students in the front of the room say things under their breath, teasing him because he didn't win the race. M_3 has his pencil in his mouth and gets so mad that he bits his pencil until it breaks in half inside his mouth. The students who were teasing him laugh and M_3 returns to his desk. He is visibly very anger.

Table 3

x	$y = sinx$
-2π	0
$-\frac{3}{2}\pi$	1
$-\pi$	0
$-\frac{1}{2}\pi$	-1
0	0
$\frac{1}{2}\pi$	1
π	0
$\frac{3}{2}\pi$	-1
2π	0

Table 4

x	$y = \sin^{-1} x$
0	-2π
1	$-\frac{3}{2}\pi$
0	$-\pi$
-1	$-\frac{1}{2}\pi$
0	0
1	$\frac{1}{2}\pi$
0	π
-1	$\frac{3}{2}\pi$
0	2π

2:00 Ms. CN asks the class what F_5 did to find the tables. F_3 says that F_5 *"switched the x and y"*. Ms. CN ask CN ask the class what the domain and range of $y = \sin^{-1} x$ is. A student tells her that it's switched from the $y = sinx$ function, *"so the domain is between negative one and one and the range goes from negative infinity to infinity."* Ms. CN writes $D:[-1,1]$ and $R:(-\infty,\infty)$.

The students in the front are making snide remarks about M_3 and teasing him about always needing to win as well as breaking the pencil in his mouth. M_3 gets really angry and begins yelling at the students. Ms. CN asks M_3 to step out of the room for a moment. She tells him, *"I'm not kicking you out, I just want you to cool down a bit."* He leaves the room and you can hear him punching and/or kicking the ways and other things outside in the hall. Ms. CN asks the photography teacher who is sitting in the very back of the room to go out in the hall with her to talk to M_3. They are out in the hall for approximately one minute. While they are out there the other students talk about how crazy they think M_3 is and laugh about how dumb they thought it was that he bit the pencil in half.

2:10 Ms. CN returns and talks to the whole class about how they need to quit teasing M_3 and they need to leave him alone. She talks to them for about two minutes about how they need to respect each other and not treat each other the way there were treating M_3.

Ms. CN asks the students to graph $y = \sin^{-1} x$ using the table they had just created. She tells the students that they'll need to label the *y*-axis using $\frac{\pi}{2}$ as their unit.

Ms. CN goes out into the hall again [SE: I found out later that she had M_3 work for the rest of the period in a small, empty room next to hers. She was giving him instructions for the rest of the assignment.]. While she is gone, the rest of the students talk about M_3 again. Ms. CN returns after about 1 minute.

2:15 Ms. CN circulates the room and makes sure the students are working on the task correctly. Multiple students including F_2 and M_6 have graphed the sine function similar to Figure 8, but with less detail. She tells them to *"flip it"* so that the *x*-axis becomes the *y*-axis and then to label the graph. She offers another bonus point for the first person that can correctly do this.

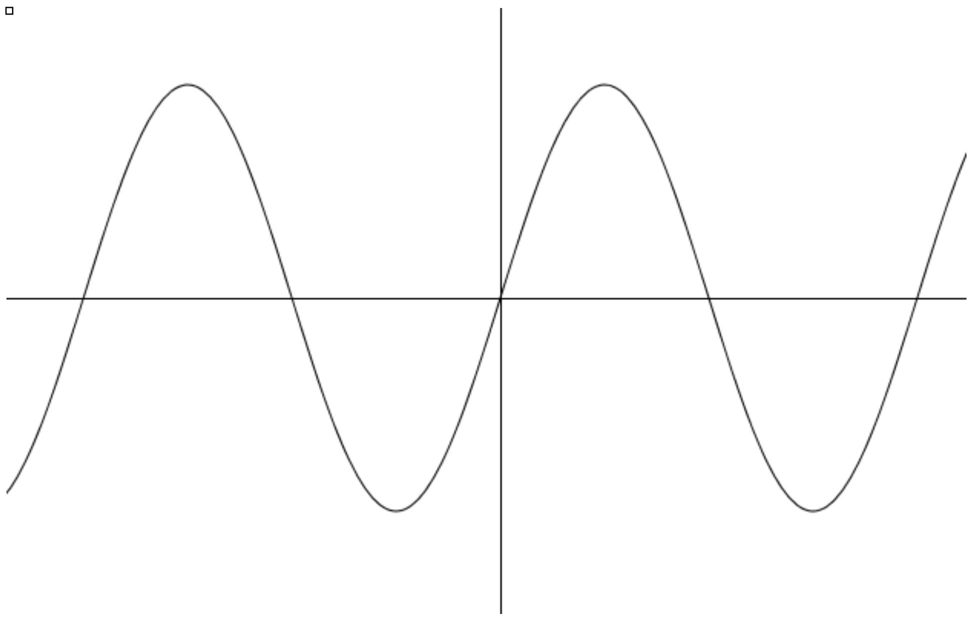


Figure 8

F_5 is the first student to finish. Ms. CN shows F_5's graph using the document camera, it look similar to the top graph in artifact 6 (also see Figure 9). Ms. CN asks why this would not be a function. F_2 says, *"because it doesn't pass the vertical line test."*

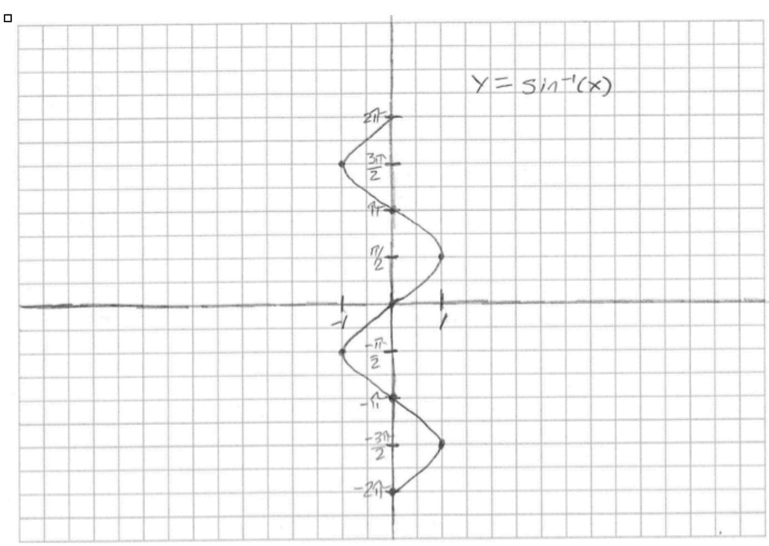


Figure 9

Ms. CN asks the students to turn the page on the task and work on part 5 to create the tables for $y = cos\ x$ and $y = \cos^{-1} x$. She says that again it will be a race and whoever creates the tables correctly first will get a bonus point. Ms. CN circulates and helps the students to complete the tables in part 5. F_2 says that she is done with the tables and Ms. CN goes and looks and tells her that she has done sine instead of cosine. F_2 erases her table and begins working again. F_5 again finished first. Ms. CN shows her table using the document camera (see Table 5 and Table 6).

Table 5

x	$y = cosx$
-2π	1
$-\frac{3}{2}\pi$	0
$-\pi$	-1
$-\frac{1}{2}\pi$	0
0	1
$\frac{1}{2}\pi$	0
π	-1
$\frac{3}{2}\pi$	0
2π	1

Table 6

x	$y = \cos^{-1} x$
1	-2π
0	$-\frac{3}{2}\pi$
-1	$-\pi$
0	$-\frac{1}{2}\pi$
1	0
0	$\frac{1}{2}\pi$
-1	π
0	$\frac{3}{2}\pi$
1	2π

Ms. CN asks what the domain and range of the cosine function is. M_1 says that it is the same as the sine function. Ms. CN agrees and then gives the domain and range while writing it on the board, $D:(-\infty,\infty)$ and $R:[-1,1]$. Ms. CN then asks what the domain and range are for $y = \cos^{-1} x$ and a student says, *"switch them"*. Ms. CN does this and writes $D:[-1,1]$ and $R:(-\infty,\infty)$.

M_5 passes gas again and M4, F4, and F5 all go crazy. They jump out of their seat, one of them sprays perfume to try to cover up the smell and then they run over by the door to get away from the smell. They are there for 1 or 2 minutes before returning to their seats.

2:25 Ms. CN asks the students to graph $y = \cos^{-1} x$. About half of the students work on this, the other half are talking or, in M_6's case, braiding his hair. Ms. CN moves through the room and looks at how the students are working.

Ms. CN shows the class M_2's graph, similar to that shown in Figure 10 (also see artifact 6). She asks, *"does this pass the vertical line test?"* M_2 says that it does not. Ms. CN asks, *"if a function isn't a function, what do we have to do?"* None of the students answer. Ms. CN draws a graph similar to Figure 11 and asks, *"what did we do to make* *this a function?"* F_4 says, *"get rid of the bottom"*. Ms. CN erases the bottom of the graph.

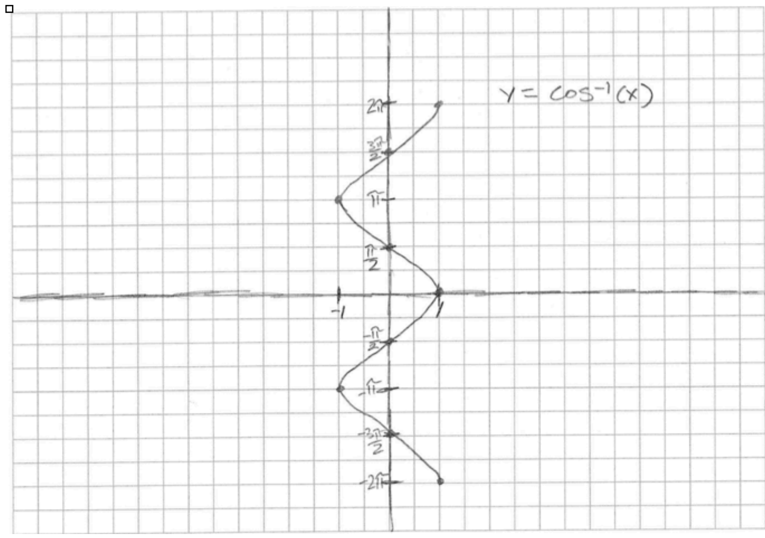


Figure 10

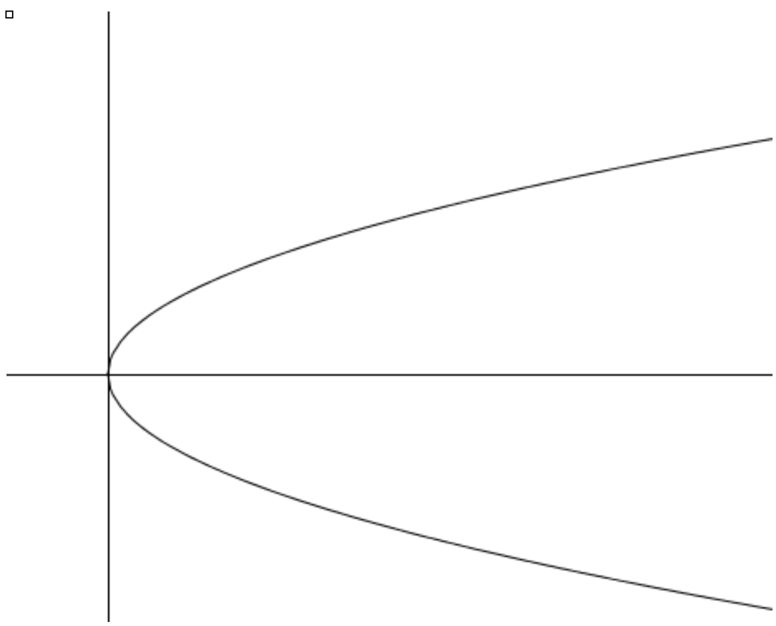


Figure 11

Ms. CN projects the inverse sine graph (Figure 9) and asks, *"what values could we get rid of?"* M_4 points to the section of the graph above $y = \frac{\pi}{2}$ and the part below $y = -\frac{\pi}{2}$ and says they can get rid of those sections. Ms. CN agrees with this answer and asks what the *"restricted range"* would be. M_4 says it would be between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$.

2:30 Ms. CN ask the students to draw the restricted range on their graphs of the inverse sine function and she highlights the portion of F_4's graph which she is showing with the document camera (see Figure 12). Ms. CN asks what the domain of the restricted graph is and M_4 says that it is -1 to 1. Ms. CN records this as $D:[-1,1]$. Ms. CN asks what the range is and M_4 says it is from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$. Ms. CN records this as $R:[-\frac{\pi}{2},\frac{\pi}{2}]$. Ms. CN asks if there still is an infinite number of angles they could use when answering a question like the one on the warm up. F_5 says that there is only one answer in the identified domain and range.

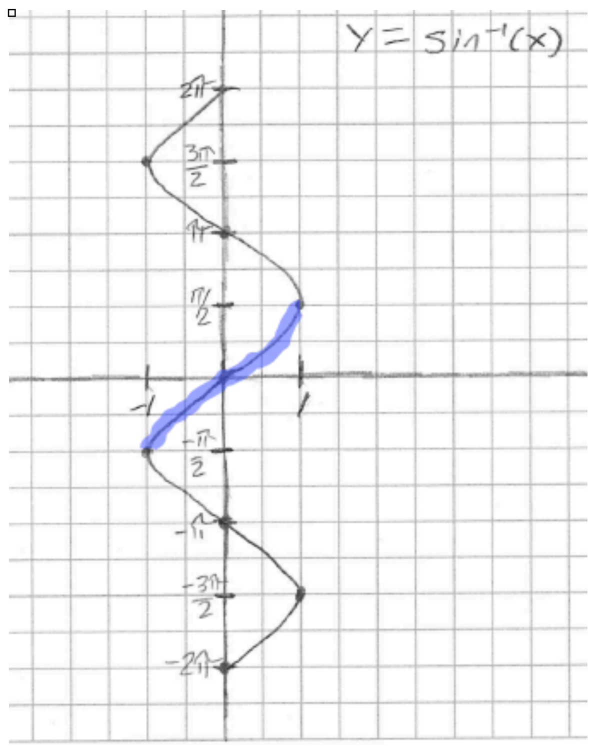


Figure 12

2:35 Ms. CN shows M_2's graph of the inverse cosine function (similar to Figure 10) and asks how to restrict the range so that it would be a function. M_4 says to restrict it between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$. He then corrects himself and says that it should be between $-\pi$ and 0. Ms. CN says, *"ok, how about the positive version of that."* M_4 realizes what she means and says that the range would be from 0 to π. Ms. CN agrees and highlights this on M_2's graph (see Figure 13). Ms. CN asks what the domain would be and F_2 says why they are only looking at the highlighted part. Ms. CN says, *"because it's restricted."* F_1 says the domain is -1 to 1 and Ms. CN writes this as $D:[-1,1]$. Ms. CN asks M_6 what the range would be and he says it is from π to 0. Ms. CN says that when you give the domain or the range you should give them starting with the smaller number and then giving the larger number. M_6 repeats the range, this time from 0 to π. Ms. CN writes it as $R:[0,\pi]$.

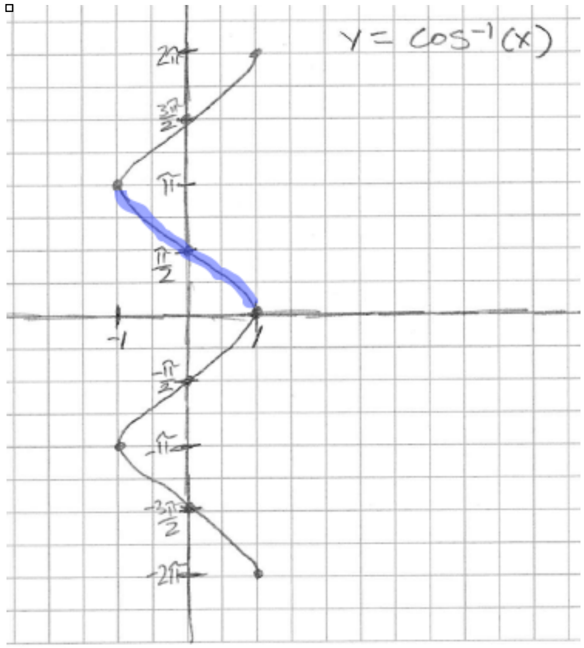


Figure 13

2:40 Ms. CN shows the students the last page of the task using the document camera and draws a sketch of the restricted sine function (see the top graph of artifact 4). Ms. CN asks for the domain and range and the students give them to her. Ms. CN writes them next to the graph. Ms. CN asks the students whether the sine function deals with the *x*-value or the *y*-value. The students tell her it is the *y*-value. Ms. CN draws two axes and labels quadrant I as positive and quadrant II ask negative (see Figure 14 and the second figure in artifact 4).

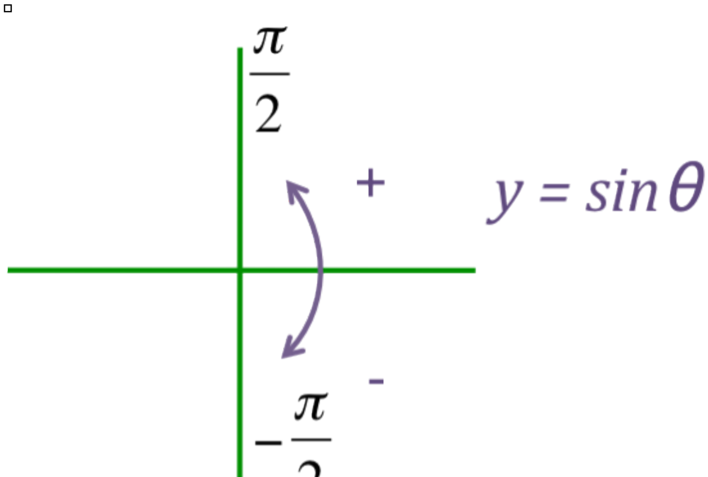


Figure 14

They repeat this process for the restricted inverse cosine function. The students give Ms. CN the domain and range and she records them as well as draws a sketch of the function (the third figure from the top of artifact 4). She then draws two more axes and identifies where the cosine has a positive and negative value (see Figure 15 and the bottom figure of artifact 4).

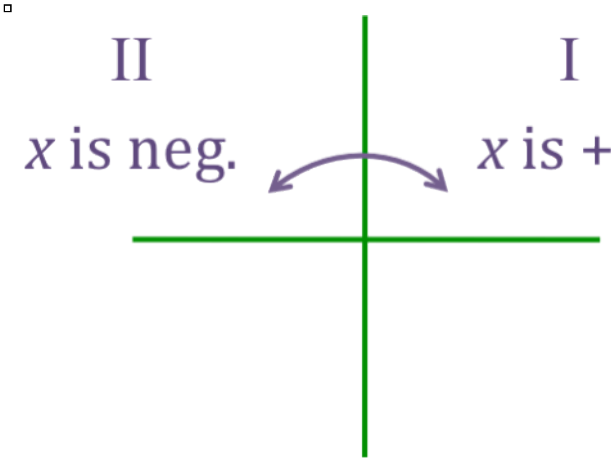


Figure 15

2:42 Ms. CN asks the student to get a blank piece of paper for their exit slip. She writes the problem for it on the board:

"Evaluate $\sin^{-1}(1)$ *and* $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$*."*

Several students begin putting their things away in preparation for the bell to ring. Ms. CN tells them not to and that they need to do the exit slip. She then tells the students that because the function is restricted they don't need to draw the triangles like they did for the warm up, rather they can use their unit circle. About seven students work on the exit slip. Ms. CN works with M_4, M_5, and M_6 showing them how to use the unit circle to work on the task. She tells them, *"inverse sine has to be in either quadrant one or quadrant two, what angle has a value of one in those quadrants?"* M_5 hesitantly says, *"pi halves (* $\frac{\pi}{2}$ *)?"* Ms. CN says, *"great, now do the other one."*

2:45 The bell rings and class ends.

IDENTIFICATION OF INSTRUCTIONAL ACTIVITIES

1. Divide the lesson into the classroom activities with which students were engaged. Briefly identify and list these activities in chronological order. Include a descriptive label, identify how students were grouped (individually, small group, whole group), and indicate the number of minutes devoted to each activity.

Activity	**Student Grouping**	**Amount of time**
Warm Up	– Individual	5 min
Warm Up Discussion	– Whole group	8 min
Work on Graphing Inverse Sine and Cosine Functions worksheet	– Whole group – Small group	67 min

2. Identify and append the resource materials that served as the basis of each of the activities.

   See artifacts from the lesson.

3. Combine the activities that focused students' attention on the same topic or big idea as a <u>*task*</u> and list the tasks in order according to the amount of time each consumed. *For example, if the teacher opened the lesson with a whole-class discussion of the concept of ratio, then gave students a small group project that asked students to construct ratios to represent a variety of situations, and then conducted a whole-class share and summary of the work produced in the small groups, the three activities should be combined into one larger coherent activity (a task): Conceptualizing and using ratios*.

   It is the observer's prerogative to decide how to combine smaller activities into a larger task (if warranted) AND to give a label to that task (i.e., conceptualizing and using ratios).

Task	**Activities comprising task**	**Amount of time**
Evaluating Inverse Sine Functions	– Warm Up – Warm Up Discussion	13 min
Graphing the sine and cosine	– Work on Graphing Inverse Sine and Cosine Functions worksheet	67 min

Select the 2 tasks that occupied the most amount of class time. (If the entire class was devoted to one task, please note.)

APPENDIX I

LOOK FORS SHEET

COVER SHEET

Observer's Name:
Date of Observation:
Teacher's Name (*pseudonym*):
Grade:
Subject:
Class/Block:
****
Scheduled Time Period for this Lesson: From ___ To ____.

Time Instruction Actually Began:
The first address (written or oral) given by the teacher to the class which relates to the current lesson, learning activities or homework
Time Instruction Actually Ended:
When the teacher stops interacting with the class about the current lesson, learning activities or homework
Total Length of Instructional Time:
Time between the beginning of actual instruction and the end of actual instruction

****
Number of students in the class during lesson:
Number of adults in the class during lesson:
Students' gender:

****
Please indicate which of the following artifacts have been collected as data. Please identify where these artifacts can be found (e.g., in the electronic planning tool for Alg. I on May 26, 2012, or appended to the observation tool). If these artifacts are not in the electronic planning tool, please append them to this observation tool.

- Lesson plan Obtained: Y / N Location: ___________________________
- Task used during the lesson Obtained: Y / N Location: ___________________________
- Pictures of student solutions Obtained: Y / N Location: ___________________________
- Other (Explain): ________________________ Location: ___________________________

  ____

  ____

Teacher: **Date of Observation:** **Block:**

LEARNING GOAL

In the Lesson Plan, as Posted in the Room, or as Described Verbally by the Teacher

Identify the learning goal for the lesson. How do you know this is the learning goal? Where did the evidence come form (i.e., what is the source of the goal - the lesson plan, verbally stated by the teacher, other)?

- Learning goal:

- Evidence:

- Source:

Select one

___ Identifies what students will **do** during the lesson

___ Indicates what students will learn at a **general level**

___ Identifies the mathematical understanding that students will develop at a level of specificity that can provide a clear instructional target which in turn can guide the selection of activities and the use of 5 Practices.

___ Missing

EVIDENCE/EXAMPLES

MAIN INSTRUCTIONAL TASK

Identify what the main instructional task is (e.g., CMP *Bits & Pieces II* Investigation 2.1) and where it can be found.

AS PRESENTED IN THE CURRICULUM MATERIALS (e.g., in the text book or worksheet)
Select one
___ No mathematics
___ Memorization
___ Procedures **without** connections to concepts, meaning or understanding
___ Procedures **with** connection to concepts, meaning, or understanding
___ Doing mathematics
___ Missing

EVIDENCE/EXAMPLES

During the Lesson

AS SET UP
Select one
___ No mathematics
___ Memorization
___ Procedures **without** connections to concepts, meaning or understanding
___ Procedures **with** connection to concepts, meaning, or understanding
___ Doing mathematics
___ Missing

AS ENACTED
Select one
___ No mathematics
___ Memorization
___ Procedures **without** connections to concepts, meaning or understanding
___ Procedures **with** connection to concepts, meaning, or understanding
___ Doing mathematics
___ Missing

EVIDENCE/EXAMPLES

SOURCES

In several of the remaining areas of the *Look Fors Sheet* you will be asked to identify the source when marking an option. This is to help identify where the information used to choose the codes related to it can be found. Please identify the source of this information. Sources could include, but are not limited to:

- The electronic lesson-planning tool
- The professional development meetings (if this is the source please identify whether the information comes from the artifacts from the meeting or the audio recorded discussions during the meeting)
- Comments by the teachers before, during, or after the lessons,

ANTICIPATION

In the Lesson Plan

Select as many as apply

__ Anticipates how students will respond in general (*e.g., "students will have a hard time with this task or activity"*)

__ Anticipates specific strategies that students might use to approach the task (*e.g., the teacher has worked out several possible solution strategies student might use work on the task.*)

__ Missing

Source:

Select as many as apply

Anticipated responses include:

- ___ correct responses
- ___ incorrect responses
- ___ multiple forms of representations (*e.g., diagram, symbols, graph, table*)
- ___ multiple strategies used to approach the task
- ___ common misconceptions

__ Anticipations include interpretations of what the responses signify about student understandings (*e.g., When comparing two or more proportions students find the difference between the two quantities in each proportion to determine which has "more" of a specific quantity. This may indicate that students are thinking about proportional relationships additively and not multiplicatively.*)

__ Plans how to respond to anticipations of what students will do (e.g., *If student have the misconception described above, ask them to consider "extreme" proportions like 1/1000.*)

__ Missing

Source:

EVIDENCE/EXAMPLES

MONITORING

In the Lesson Plan

Select one

___ Creation of monitoring tool with descriptions of anticipated responses
___ Missing

EVIDENCE/EXAMPLES

During the Lesson

Select as many as apply

___ No time is provided for the students to work on the task in small groups or independently
___ Time is provided for students to work on the task in small groups or independently, but the teacher does not observe students as they work on the task
___ Teacher observes students as they work on the task
___ Teacher keeps a record of what students do and say as they work on the task
___ Teacher asks students assessing and advancing questions as they work on the task
___ Teacher redirects students as they work on the task when needed

EVIDENCE/EXAMPLES

SELECTING

During the Lesson

How many student approaches to the task are made public?

Select one

*(Distinguish between options for "one" and "multiple" by determining whether multiple, **distinct** approaches are shared, not by the number of students who share the approaches.)*

___ No approaches to the task are discussed and/or displayed

___ One student approach to the task is discussed and/or displayed (*please use this choice if multiple student solutions are discussed but all use the same approach to the task*)

___ Multiple, distinct student approaches to the task are discussed and/or displayed

What types of students' approaches to the task are made public? How many of each type are made public?

Select all that apply

___ No student approaches to the task are shared

___ Correct approaches to the task are discussed and/or displayed

# of correct approaches ____

___ Incorrect approaches to the task are discussed and/or displayed

# of incorrect approaches ____

How are students' approaches to the task made public during whole-class discussion?

Select as many as apply

___ No student approaches to the task are made public

___ Students' approaches to the task are displayed

___ Students' approaches to the task are discussed

How do students' approaches appear to be selected?

Select as many as apply. Please indicate how often this occurs (once, a few times, several times, every time).

___ No student approaches to the task are made public

___ Teacher appears to **randomly** select students to share their approaches to the task.

How often does this occur? _______________

___ Teacher appears to **purposefully** select students to share their approaches to the task.

How often does this occur? _______________

___ Students volunteer to share their approaches to the task.

How often does this occur? _______________

How do you know if the teacher's selection of approaches is random or purposeful? (e.g., Teacher explicitly state that she chose a student's solution for a specific reason; After the lesson the teacher tells you she chose a student's solution for a specific reason; Judgment of the observer; Other)

Source:

SELECTING (Cont.)

During the Lesson

What is the appropriateness of the student approaches to the task that are made public?

Select one

___ No student approaches to the task are made public

___ Students' approaches to the task that are made public **are** aligned with the learning goals of the lesson

___ Students' approaches to the task that are made public **are not** aligned with the learning goals of the lesson

EVIDENCE/EXAMPLES

SEQUENCING

In the Lesson Plan

Select one

___ Lesson plan includes one or more suggested sequences of anticipated solutions to the task
___ Missing

EVIDENCE/EXAMPLES

During the Lesson

Select one

___ No student approaches to the task are made public
___ Only one approach to each part of the task is made public (*e.g., student 1 shares an answer for problem #32, student 8 shares an answer for problem #33, etc.*)
___ Teachers' sequencing of shared (discussed and/or displayed) student approaches to the task is **random** (*i.e., the shared approaches do not seem to be ordered in any meaningful way*).
___ Teachers' sequencing of shared (discussed and/or displayed) student approaches to the task is **purposeful** (*i.e., the shared approaches build on one another in some meaningful way*).

How do you know if the teacher's sequencing of approaches is random or purposeful? (e.g., Teacher explicitly state that she had a student share her solution first for a specific reason; After the lesson the teacher tells you she had a student share her solution first for a specific reason; Judgment of the observer; Other)

Source:

EVIDENCE/EXAMPLES

CONNECTING

In the Lesson Plan

Select as many as apply

___ Lesson plan includes one or more suggestions for connecting anticipated student approaches to the task to each other
___ Lesson plan includes one or more suggestions for connecting anticipated student approaches to the task to the key mathematical ideas
___ Missing

EVIDENCE/EXAMPLES

During the Lesson

Select as many as apply

___ No student approaches to the task are made public
___ Only one approach to each part of the task is made public (*e.g., student 1 shares an answer for problem #32, student 8 shares an answer for problem #33, etc.*)
___ Connections are made among various students' approaches to the task
___ Connections are made between students' approaches to the task and the learning goal(s) that is/are driving the lesson
___ Students are asked to compare/contrast various students' approaches to the task

EVIDENCE/EXAMPLES

LEARNING GOAL

During the Lesson

Select as many as apply

The learning goal(s) appear to inform:

___ Teachers' selection of activities

___ Teachers' selection of students' approaches to the task that are made public

___ Teachers' sequencing of students' approaches to the task that are made public

___ Teachers' efforts to form connections between various students' approaches to the task and/or between students' approaches to the task and the key mathematical ideas

EVIDENCE/EXAMPLES

APPENDIX J

THE FIVE PRACTICES SUMMARY SHEET

Five Practices Summary Sheet

Lesson Identification
Observer's Name:
Date of Observation:
Teacher's Name (*pseudonym*):
Grade:
Subject:
Class/Block:

Level of Use
Please rate the teacher's level of use of each of the five practices using the following scale. Please use the coding rules provided below to make your selection based on the information provided by the observer on the Look Fors Sheet.

Practice	**Level of Use**
Anticipating	
Monitoring	
Selection	
Sequencing	
Connecting	

N = No use of the practice
L = Little use of the practice
P = Partial use of the practice
H = High use of the practice

Coding Rules for Determining the Level of Use of the Five Practices

Anticipating

Level of use	Coding rules
N (No use)	– There is no evidence of anticipation in the lesson plan
L (Little use)	– The teacher only anticipates how students will respond generally – Only one response is anticipated
P (Partial use)	– The teacher anticipates specific strategies students might use on the task – At least two strategies are anticipated by the teacher
H (High use)	– The teacher anticipates specific strategies students might use on the task – At least two strategies are anticipated by the teacher – The teacher anticipates both correct and incorrect strategies – The teacher plans how to respond to anticipated solutions

Monitoring

Level of use	Coding rules
N (No use)	– No monitoring takes place during the lesson
L (Little use)	– The teacher observes the students as they work on the task, but does not recording anything on a monitoring tool and does not ask assessing and advancing questions
P (Partial use)	– The teacher observes the students as they work on the task – The teacher either a) uses a monitoring tool to record how students work on the task **OR** b) ask students assessing and advancing questions as they work, **but not both**
H (High use)	– The teacher observes the students work on the task – The teacher a) uses a monitoring tool to record how students work on the task **AND** b) ask students assessing and advancing questions as they work

Selecting

Level of use	Coding rules
N (No use)	– There is no discussion of the task **OR** – No student approaches are made public during the discussion of the task
L (Little use)	– Only one student approach is made public during the discussion **OR** – Multiple students approaches are made public, but students volunteer to share their approaches, **OR** – Multiple student approaches are made public, and the teacher appears to select students to share their approaches randomly

P (Partial use)	– Multiple student approaches are made public – Teacher appears to select students to share their approaches purposefully – Only correct approaches are shared
H (High use)	– Multiple student approaches are made public – Teacher appears to select students to share their approaches purposefully – Both correct and incorrect approaches are shared

Sequencing

Level of use	**Coding rules**
N (No use)	– There is no discussion of the task **OR** – No student approaches are made public during the discussion of the task
L (Little use)	– Only one student approach is made public during the discussion **OR** – Multiple student approaches are made public, and the teacher's sequence of shared approaches appears to be random
P (Partial use)	– There is no suggested sequence of anticipated solutions in the lesson plan – Multiple student approaches are made public, and the teacher's sequence of shared approaches appears to be purposeful
H (High use)	– There is a suggested sequence of anticipated solutions in the lesson plan – Multiple student approaches are made public, and the teacher's sequence of shared approaches appears to be purposeful

Connecting

Level of use	**Coding rules**
N (No use)	– There is no discussion of the task **OR** – No student approaches are made public during the discussion of the task **OR** – One or more student approaches are made public during the discussion but no connections are made between two or more approaches or between and approach and the goal(s) of the lesson
L (Little use)	– Only one student approach is made public during the discussion, and connections are made between the approach and the learning goal(s) that drive the lesson
P (Partial use)	– Multiple approaches are made public during the discussion and connections are made either a) between two or more approaches **OR** b) between one or more approaches and the learning goal(s) of the lesson **but not both**
H (High use)	– Multiple approaches are made public during the discussion and connections are made both a) between two or more approaches **AND** b) between one or more approaches and the learning goal(s) of the lesson

APPENDIX K

PROFESSIONAL DEVELOPMENT PREPARATION RUBRIC

Professional Development Preparation Rubric

Identification Information
Teacher's Name (*pseudonym*):
Meeting Date:
Grade-Band Team:

Level of Preparation
Please use the rubrics below to assess the teacher's level of preparation for the professional development meeting based on the data from the gathered work on the focus tasks and on the Noticings and Wonderings.

Item	**Level of Preparation**	**Notes**
Work on focus task		
Noticings & wonderings		

N = No preparation
L = Low preparation
M = Medium preparation
H = High preparation

Rubric for Determining the Level of Preparation from the Work on the Focus Tasks

Level of use	Coding rules
N (No preparation)	– The teacher does not provide any work on the focus task
L (Low preparation)	– The teacher anticipates only one solution method for the task
M (Medium preparation)	– The teacher anticipates multiple solution methods for the task – All the anticipated solutions use correct methods
H (High preparation)	– The teacher anticipates multiple solution methods for the task – The teacher anticipates both correct and incorrect solutions methods

Rubric for Determining the Level of Preparation from the Noticings and Wonderings
Evaluate the teachers' level of Preparation based on the noticings and wonderings they create using two categories, *quantity* and *quality*. Quantity is based on the number of *noticing and wondering pairs* they create[13]. Quality is based on the focus of the noticing and wondering pairs. Pairs that focus on the following areas will be given a higher score than those that focus on other areas:

- Specific elements of the task
- Instructional or pedagogical issues related to the teaching of the task
- The learning goals (either those identified by the teacher who selected the task or potential learning goals) associated with the task
- Mathematical content related to the task

Level of use	Coding rules	
	Quantity of noticings and wonderings	**Quality of noticings and wonderings**
N (No preparation)	The teacher does not provide any noticings and wonderings	n/a
L (Low preparation)	The teacher provide **ONE OR MORE** noticings and wonderings	The noticings and wonders **DO NOT ATTEND TO ANY** of the specified areas
M (Medium preparation)	The teacher provides at **ONE OR TWO** noticings and wonderings **OR** The teacher provide **THREE OR**	The noticings and wonderings attend to **ONE OR MORE** of the specified areas The noticings and wonderings attend

[13] A noticing and wondering pair consists of one noticing connected with one or more wonderings. For example, *"Questions #3 and #4 have students consider transformation of the graph."* (noticing) *"Would it be helpful to the students to graph these transformations when working on the problem?"* (wondering).

	MORE noticings and wonderings	to **ONLY ONE** of the specified areas
H (High preparation)	The teacher provide **THREE OR MORE** noticings and wonderings	The noticings and wonderings attend to **TWO OR MORE** of the specified areas.

APPENDIX L

THE *LUCKY DAY* TASK

The *Lucky Day* task, the focus task selected by Nicole Nesmith for the MLSC week 1 meeting on April 11.

Lucky Day

Today is your lucky day…you hit the jackpot!! You won the Publisher's Mystery Sweepstakes. The Publisher's wish to add to the intrigue by having you decide how you would like to get paid. You have to decide between the following two options.

Option 1: You can receive $10,000 on the first day of the month, and an additional $1000 a day for the remainder of the month (assume there are 31 days in the month).

Option 2: You can receive a penny on the first day of the month, two pennies on the second day, four pennies on the 3^{rd} day, eight pennies on the fourth day, and so on, for one month (assume there are 31 days in the month).

Make a prediction. Before you do a mathematical analysis, which plan do you think will yield more prize money at the end of the month? Turn and talk to your partner. What representation do you think will be helpful in making a prediction?

1. Make a table and examine the data. Indicate how much you would receive on a particular day and, in addition, the ***total amount*** you would have received by that day in the month.

DAY	AMT RECEIVED THAT DAY (dollars)	TOTAL AMOUNT RECEIVED (dollars)
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

DAY	AMT RECEIVED THAT DAY (dollars)	TOTAL AMOUNT RECEIVED (dollars)
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

2. Explain what you notice about how the TOTAL amount is changing in each option with respect to the day. Write at least three ideas for each option that you think may help you to write an equation to represent the option.

3. Write a function that will indicate the ***total amount*** received for each option on the n^{th} day of the month. Explain what each part of your equation represents.

   OPTION 1 OPTION 2

4. What is the domain and range for the functions you wrote in part 4? Explain the domain and range in the context of the Sweepstakes.

5. Graph the functions on your graphing calculator. What do you notice? Which option will you choose and why?

6. On what day of the month does Option 2 surpass Option 1? How do you know? What does this mean in terms of the growth rate of each function?

7. Since Option 2 is more lucrative, let's continue to work with that payment plan. Assuming you would again receive one penny on the first day, describe how the ***total amount*** received would change if the amount you receive each day triples.

8. What function would be used to represent the situation when the amount triples each day? Explain what each part of your function represents.

9. What is the domain and range for the function you wrote in part 9? Explain the domain and range in the context of the Sweepstakes.

CHALLENGE

10. Here's another variation on Option 2. Suppose you received 5 pennies on the first day, and at the end of each consecutive day the amount you received doubled. How much would you receive on day 2? Day 5? Write a function that will indicate the total amount received on the n^{th} day of the month.

Similarities and Differences

(Day 2) Lucky Day Task

$Y_1 = 0.01(2^x - 1)$; $Y_2 = 0.01(3^x - 1)$; $Y_3 = 1000x + 9000$

Similarities	Differences

APPENDIX M

THE MODIFIED *MODELING WITH LOGISTICS FUNCTIONS* TASK

The modified version of the *Modeling with Logistic Functions* task used by Cara Nance during her lesson on January 9. The original task (Appendix Q) was discussed during the MLSC week 1 meeting on January 4.

Modeling with Logistic Functions

Name_______________________ Date_____________ Block________

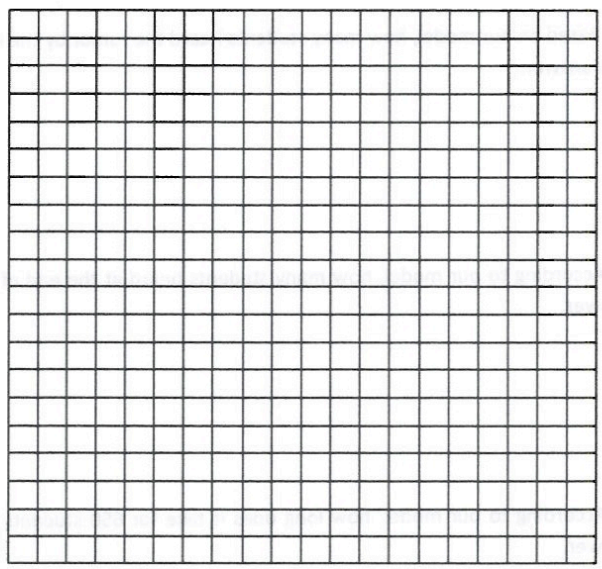
Part 1: Spreading a Rumor

University Prep High School has 800 students. Derrick, Ashley, and Mike start a rumor, which spreads logistically so that

$$S(t) = \frac{800}{1 + 19e^{-0.8t}}$$

models the number of students who have heard the rumor by the end of t days, where t = 0 is the day the rumor begins.

Plot the points to graph the Function. Label!



a). Identify and write equation(s) of any asymptotes of the function. Label on graph.

Explain why the asymptote(s) are relevant to the problem situation.

b). List any intercepts and explain how the intercept(s) relates to the problem situation.

Modeling with Logistic Functions

c). State the domain of the **logistic function.**

What is the domain of the *problem situation*?

d). State the range of the **logistic function.**

What is the range of the *problem situation*?

e). Based on our model, how many students heard the rumor by the end of day 0? Explain or show how you got this answer.

f). According to our model, how many students heard at the end of day 3? Explain or show how you got your answer.

f). According to our model, how long does it take for 650 students to hear the rumor? Show or explain your answer.

g). Based on the model, is there a maximum number of students who will hear the rumor?

If yes, what is the maximum?

If no, why not? Explain your reasoning.

Modeling with Logistic Functions

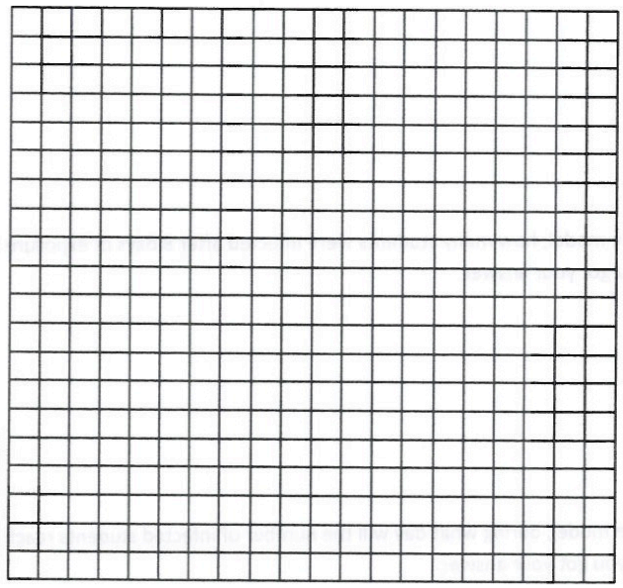
Part 2: Spread of a Cold Virus

A cold virus can spread through University Prep rather quickly. The function

$$s = \frac{600}{1 + 59e^{-0.2t}}$$

represents the total number of students infected (s) by a viral strain after t days, when t ≥0.

Plot points to graph the Function and Label.



a). Write the equation(s) of any asymptotes of the function. Lab el on graph.

Explain why asymptotes are relevant to the problem situation.

b). List any intercepts.

c). State the domain of the logistic function.

Is this also the domain of the problem situation? Why or why not?

d). State the range of the function.

Is this also the range of the problem situation? Why or Why not?

e). Based on the model, how many students were initially infected by the virus? Show or explain how you got your answer.

f). Based on the model, how many students were infected after 8 days of exposure to the cold virus? Show or explain how you got your answer.

g). Based on the model, during what day will the number of infected students reach 200 for the first time? Show or explain how you got your answer.

h). Based on the model, is there a maximum number of students who will be infected? If yes, what is the maximum? Explain your reasoning.

If no, why not? Explain your reasoning.

APPENDIX N

THE *LAW OF COSINES* TASK

The *Law of Cosines* task used by Cara Nance during her lesson on May 30.[14]

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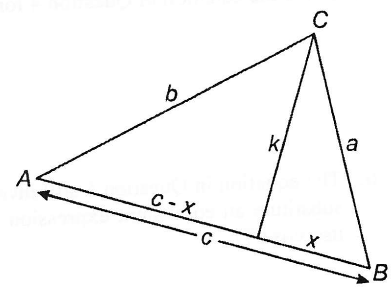
[14] The first two pages of this task were taken from a website sponsored by the National Council of Teachers of Mathematics (NCTM, n.d.). These pages are reprinted with permission from National Council of Teachers of Mathematics.

The Law of Cosines

NAME ____________________

The law of sines can be used to determine the measures of missing angles and sides of triangles when the measures of two angles and a side (AAS or ASA) or the measures of two sides and a non-included angle (SSA) are known. However, the law of sines cannot be used to determine the measures of missing angles and sides of triangles when the measures of two sides and an included angle (SAS) or the measures of three sides (SSS) are known. Since the law of sines can only be used in certain situations, we need to develop another method to address the other possible cases. This new method is called the **Law of Cosines.**

To develop the **law of cosines**, begin with ΔABC. From vertex C, altitude k is drawn and separates side c into segments x and $c - x$. (Why can the segments be represented in this way?)



1. The altitude separates ΔABC into two right triangles. Use the Pythagorean theorem to write two equations, one relating k, b, and $c - x$, and another relating a, k, and x.

2. Notice that both equations contain k^2. (Why?) Solve each equation for k^2.

3. Since both of the equations in Question 2 are equal to k^2, they can be set equal to each other. (Why is this true?) Set the equations equal to each other to form a new equation.

4. Notice that the equation in Question 3 involves x. However, x is not a side of ΔABC. As a result, we will attempt to rewrite the equation in Question 3 so that it does not include x. Begin by expanding the quantity $(c-x)^2$.

5. Solve the equation in Question 4 for b^2.

6. The equation in Question 5 still involves x. To eliminate x from the equation, we will attempt to substitute an equivalent expression for x. Write an equation involving both $\cos B$ and x. (Why use $\cos B$?)

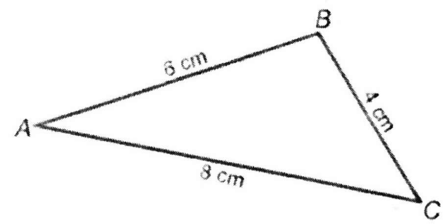
7. Solve the equation from Question 6 for x. (Why solve for x?)

8. Substitute the equivalent expression for x into the equation from Question 5. The resulting equation contains only sides and angles of ΔABC. This equation is called the **Law of Cosines.**

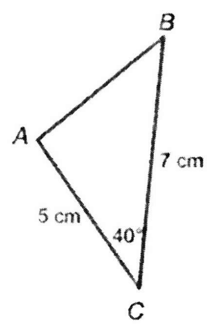
9. Using a similar method, two other forms of this law could be developed for a^2 and c^2. Based on your work for Questions 1–8, write the two other forms of the law of cosines for ΔABC.

The Law of Cosines:

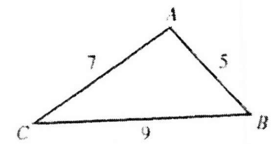
Example 1: Find all angles



Example 2: Solve the triangle. Find all unknown sides and angles.

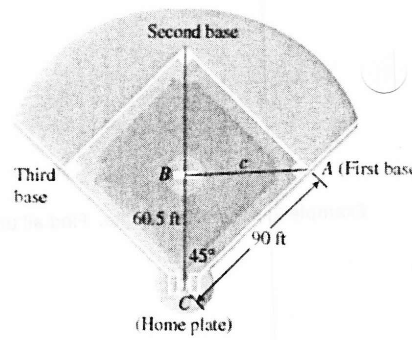


Example 3: Solve ΔABC if $a = 9$, $b = 7$, and $c = 5$.



Example 4: **Measuring a Baseball Diamond**

The bases on a baseball diamond are 90 feet apart, and the front edge of the pitcher's rubber is 60.5 feet from the back corner of home plate. Find the distance from the center of the front edge of the pitcher's rubber to the far corner of first base.



APPENDIX O

WARM UP PROBLEM

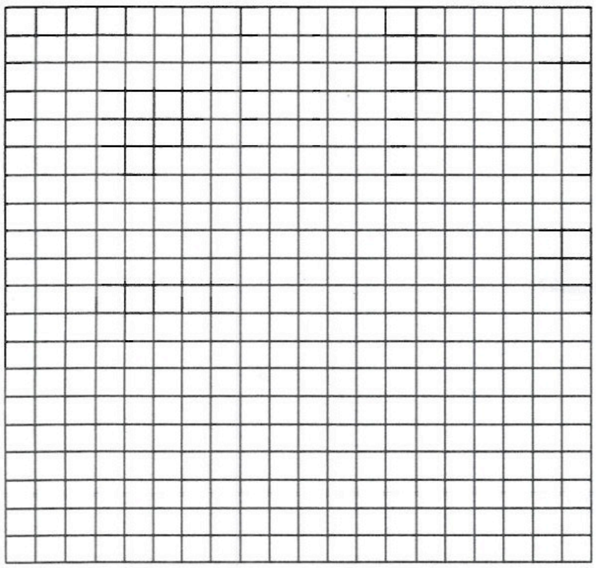
The warm up problem used by Cara Nance during her lesson on January 9 in conjunction with the modified version of the *Modeling with Logistic Functions* task (Appendix M).

Monday, January 09, 2012

Warm-up:

1. Graph the function and identify your viewing window.

$$f(x) = \frac{900}{1+14e^{-0.3x}}$$



Domain:

Range:

2. Compare and Contrast characteristics of exponential functions and logistic functions.

APPENDIX P

THE MODIFIED *WHAT IS A RADIAN?* TASK

The modified version of the *What is a Radian?* task used by Cara Nance during her lesson on March 19. The original task (Appendix R) was discussed during the MLSC week 1 meeting on March 7.

Name______________________ Date________________ Block_____

What is a Radian?

Exploration

Materials:

String

Paper

Protractor

Provided a circle, label the center of the circle O. Cut a piece of string, so that it stretches from the center of the circle to a point on the circumference.

The length of the string is also the ________________ of the circle.

Mark a point on the circumference of the circle and label it A. Draw a radius from the center O to point A. Place one end of the string at A and bend it around the circle counterclockwise, marking point B on the other end of the string. Draw the radius from O to B.

Find the measure of $angle\ AOB$. ____________________ . This is one radian.

Questions:

1. What is the formula for circumference of a circle, in terms of the radius r?

2. Considering your answer to #1, how many radians must be in a complete circle?

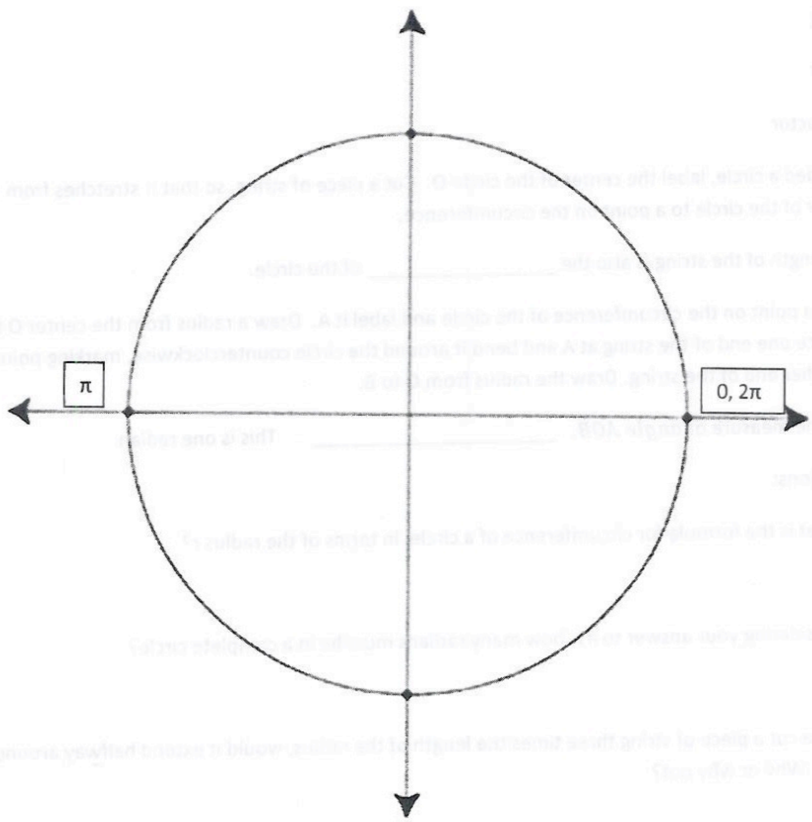
3. If we cut a piece of string three times the length of the radius, would it extend halfway around the circle? Why or why not?

4. How many degrees are in a straight angle?

How many radians are in a straight angle?

Part 2: Radian Measure of Special Angles.

Given the circle with $0, \pi, and\ 2\pi$ radians explore the radian measure of special angles. Label Each Radian and degree on the circle. Use a different colored pencil to label parts 1, 2, and 3. Round fractions to simpliest form.



1. If a half rotation (180°) is equal to π radians and whole rotation around the circle is 2π radians (360°), what is the radian measure of 90°? ____________ What is the radian measure for 270°?____________

0° = ____________

90° = ____________

180° = ____________

270° = ____________

360° = ____________

2. Given the relationship of 180° and π radians, use the circle provided and label 45°, 135°, 225°, and 315° using radian measure.

45° = ____________

135° = ____________

225° = ____________

315° = ____________

3. Using the same procedure as in #2, use the circle below and label 30°, 60°, 120°, 150°, 210°, 240°, 300°, and 330° using radian measure.

30° = ____________

60° = ____________

120° = ____________

150° = ____________

210° = ____________

240° = ____________

300° = ____________

330° = ____________

Part 3: Converting between degrees and radians.

Given your knowledge of the radian measure of special angles, and we know $\pi \; radians = 180°$, let's explore the connections between radian and degree measure. Based on your previous exploration, give the degree measure that is equal to each of the following radian measures.

$\frac{\pi}{4} =$

$\frac{\pi}{6} =$

$\frac{2\pi}{3} =$

$\frac{11\pi}{6} =$

Based on the knowledge you now have of radian and degree measure, explain to a student who missed this lab experience how to convert from radian to degree measure and then from degree to radian measure.

To convert from radian to degree measure you should

____

____

____

To convert from degree to radian measure you should

____

____

____

____

Based on your previous two answers, is there a mathematical formula that supports the procedures you described? If so, what are they?

____

____

____

Radian and Degree Measure Beyond The Special Angles

Now that you have made connections between radian and degree measures, and you have discovered how to convert between radians and degrees, you are ready to explore simple conversions for measures that are not special anlges.

Using the formula from the previous exercise, convert each radian measure into degrees and each degree measure into radians.

$\frac{7\pi}{6} =$ $\quad\quad$ $140° =$

$\frac{7\pi}{12} =$ $\quad\quad$ $260° =$

$\frac{3\pi}{18} =$ $\quad\quad$ $200° =$

$\frac{13\pi}{20} =$ $\quad\quad$ $108° =$

APPENDIX Q

THE *MODELING WITH LOGISTIC FUNCTIONS* TASK

The original version of the *Modeling with Logistic Functions* task, the focus task selected by Cara Nance for the MLSC week 1 meeting on January 4.

Modeling with Logistic Functions

Name________________________ Date_____________ Block________

Part 1:

Spread of a Cold Virus

A cold virus can spread through a school rather quickly. The function

$$s = \frac{600}{1 + 59e^{-0.2t}}$$

represents the total number of students infected (s) by a viral strain after t days, when t ≥0.

Note: Because a person cannot be partially infected, round down to nearest integer.

Sketch the graph

a). Write the equation(s) of any asymptotes of the function.

b). List any intercepts.

c). State the domain of the logistic function.

Is this also the domain of the problem situation? Why or why not?

d). State the range of the function.

Is this also the range of the problem situation? Why or Why not?

e). Based on the function, how many students were initially infected by the virus? Show or explain how you got your answer.

f). How many students were infected after 8 days of exposure to the cold virus? Show or explain how you got your answer.

g). During what day will the number of infected students reach 200 for the first time? Show or explain how you got your answer.

h). Based on the function, is there a maximum number of students who will be infected? If yes, what is the maximum? Explain your reasoning.

If no, why not? Explain your reasoning.

Modeling with Logistic Functions

Part 2:

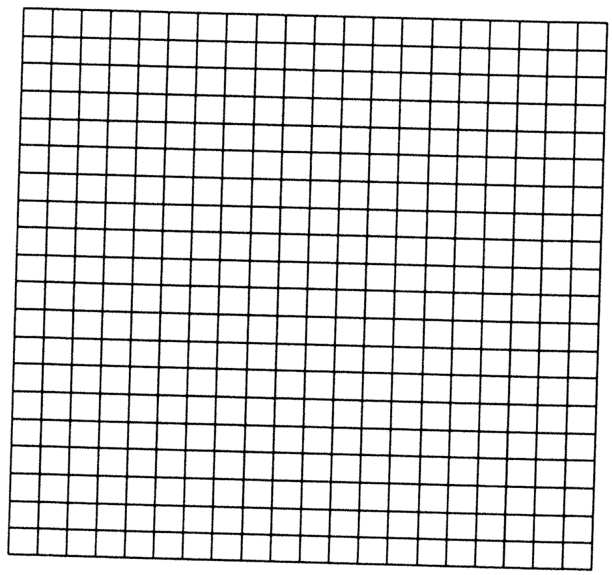
Spreading a Rumor

University Prep High School has 789 students. Derrick, Ashley, and Mike start a rumor, which spreads logistically so that

$$S(t) = \frac{800}{1 + 19e^{-0.8t}}$$

models the number of students who have heard the rumor by the end of t days, where t = 0 is the day the rumor begins.

Sketch the graph



a). Identify and write equation(s) of any asymptotes of the function.

Explain why the asymptote(s) are relevant to the problem situation.

b). List any intercepts and explain how the intercept(s) relates to the problem situation.

Modeling with Logistic Functions

c). State the domain of the logistic function.

What is the domain of the problem situation?

d). State the range of the logistic function.

What is the range of the problem situation?

e). How many students heard the rumor by the end of day 0? Explain or show how you got this answer.

f). How long does it take for 650 students to hear the rumor? Show or explain your answer.

g). Based on the function, is there a maximum number of students who will hear the rumor?

If yes, what is the maximum?

If no, why not? Explain your reasoning.

APPENDIX R

THE *WHAT IS A RADIAN?* TASK

The original version of the *What is a Radian?* task, the focus task selected by Cara Nance for the MLSC week 1 meeting on March 7.

Name______________________ Date______________ Block_____

What is a Radian?

Exploration

Materials:

String

Paper

Protractor

On a piece of paper draw a point in the center. Cut a piece a piece of string holding one end at the center point and tying the other end around a pencil. Carefully draw a circle. Label the center of the circle O.

The length of the string is also the ______________ of the circle.

Mark a point on the circumference of the circle and label it A. Draw a radius from the center O to point A. Place one end of the string at A and bend it around the circle counterclockwise, marking point B on the other end of the string. Draw the radius from O to B.

Find the measure of $angle\ AOB$. ____________________ . This is one radian.

Questions:

1. What is the circumference of the circle, in terms of the radius r?

2. Considering your answer to #1, how many radians must be in a complete circle?

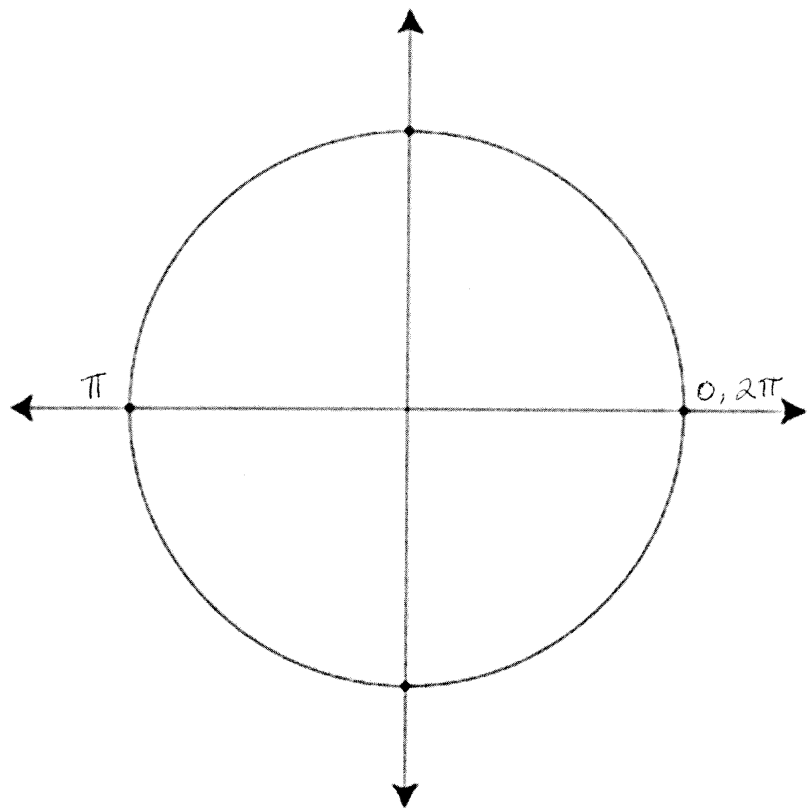
3. If we cut a piece of string three times the length of the radius, would it extend hallway around the circle? Why or why not? Try it if you need to!

4. How many degrees are in a straight angle?

How many radians are in a straight angle?

Part 2: Radian Measure of Special Angles.

Given the circle with diameter, $0, \pi, and\ 2\pi$ radians explore the radian measure of special angles. Label Each Radian and degree on the circle. Use a different color for part 1, 2, and 3.



1. If a half rotation (180°) is equal to π radians, what is the radian measure for 90°? _______
   What is the radian measure for 270°?___________

0° = __________

90° = __________

180° = __________

270° = __________

360° = __________

2. Given the relationship of 180° and π radians, use the circle provided and label 45°, 135°, 225°, and 315° using radian measure.

45° = __________

135° = __________

225° = __________

315° = __________

3. Using the same procedure as in #2, use the circle below and label 30°, 60°, 120°, 150°, 210°, 240°, 300°, and 330° using radian measure.

30° = __________

60° = __________

120° = __________

150° = __________

210° = __________

240° = __________

300° = __________

330° = __________

Part 3: Converting between degrees and radians.

Given your knowledge of the radian measure of special angles, and we know $\pi\ radians = 180°$, let's explore the connections between radian and degree measure. Based on your previous exploration, give the degree measure that is equal to each of the following radian measures.

$\frac{\pi}{4} =$ $\frac{\pi}{6} =$

$\frac{2\pi}{3} =$ $\frac{11\pi}{4} =$

Based on the knowledge you now have of radian and degree measure, explain to a student who missed this lab experience how to convert from radian to degree measure and then from degree to radian measure.

To convert from radian to degree measure you should

____

____

____

To convert from degree to radian measure you should

____

____

____

____

Based on your previous two answers, is there a mathematical formula that supports the procedures you described? If so, what are they?

____

____

____

Radian and Degree Measure Beyond The Special Angles

Now that you have made connections between radian and degree measures, and you have discovered how to convert between radians and degrees, you are ready to explore simple conversions for measures that are not special anlges.

Using the formula from the previous exercise, convert each radian measure into degrees and each degree measure into radians.

$\frac{5\pi}{2} =$ $115° =$

$\frac{7\pi}{2} =$ $155° =$

$\frac{3\pi}{18} =$ $310° =$

RADIAN APPLICATIONS

1. Your analog clock shows the passage of 45 minutes.

   A. Find the radian angle through which the minute hand moves in that time.

   1A. ___________

   B. Find the radian angle through which the hour hand moves in that time.

   1B. __________

APPENDIX S

THE *USING THE MEAN* TASK

The *Using the Mean* task used by Nathan Ingram during his lesson on March 12[15].

[15] From CONNECTED MATHEMATICS DATA ABOUT US Copyright © 2006 by Michigan State University, G. Lappan, J. Fey, W. Fitzgerald, S. Friel, and E. Phillips. Used by permission of Pearson Education, Inc. All Rights Reserved.

3.3 Using the Mean

A group of middle-school students answered the question: How many movies did you watch last month? The table and stem plot show their data.

Movies Watched

Student	Number
Joel	15
Tonya	16
Rachel	5
Swanson	18
Jerome	3
Leah	6
Beth	7
Mickey	6
Bhavana	3
Josh	11

Movies Watched

0	3 3 5 6 6 7
1	1 5 6 8
2	

Key: 1 | 5 means 15 movies

You have found the mean using cubes to represent the data. You may know the following procedure to find the mean: The **mean** of a set of data is the sum of the values divided by the number of values in the set.

Problem 3.3 Using the Mean

A. Use the movie data to find each number.

1. the total number of students

2. the total number of movies watched

3. the mean number of movies watched

B. A new value is added for Carlos, who was home last month with a broken leg. He watched 31 movies.

1. How does the new value change the distribution on the stem plot?

2. Is this new value an outlier? Explain.

3. What is the mean of the data now?

4. Compare the mean from Question A to the new mean. What do you notice? Explain.

C. Data for eight more students are added:

Tommy	5	Robbie	4
Alexandra	5	Ana	4
Trevor	5	Alicia	2
Kirsten	4	Brian	2

1. How do these values change the distribution on the stem plot?
2. Are any of these new data values outliers? Explain.
3. What is the mean of the data now?
4. Compare the means you found in Questions A and B with this new mean. What do you notice? Explain.

D. 1. What happens to the mean of a data set when you add one or more data values that are outliers? Explain.
2. What happens to the mean of a data set when you add data values that cluster near one end of the original data set? Explain.
3. Explain why you think these changes might occur.

ACE **Homework starts on page 56.**

APPENDIX T

THE *WALKING TO WIN* TASK

The *Walking to Win* task used by Gloria Xavier during her lesson on April 20[16].

[16] From CONNECTED MATHEMATICS MOVING STRAIGHT AHEAD

Investigation 2

Exploring Linear Functions With Graphs and Tables

In the last investigation, you examined relationships that were linear functions. For example, the *distance* a person walks at a constant rate is a function of the amount of *time* a person walks. The *amount of money* a person collects from a walkathon sponsor who pays a fixed amount per *kilometer* is a function of the distance walked. You used tables, graphs, and equations to answer questions about these relationships.

In this investigation, you will continue to solve problems involving linear functions.

2.1 Walking to Win

In Ms. Chang's class, Emile found out that his walking rate is 2.5 meters per second. When he gets home from school, he times his little brother Henri as Henri walks 100 meters. He figured out that Henri's walking rate is 1 meter per second.

Problem 2.1 Finding the Point of Intersection

Henri challenges Emile to a walking race. Because Emile's walking rate is faster, Emile gives Henri a 45-meter head start. Emile knows his brother would enjoy winning the race, but he does not want to make the race so short that it is obvious his brother will win.

A. How long should the race be so that Henri will win in a close race?

B. Describe your strategy for finding your answer to Question A. Give evidence to support your answer.

ACE Homework starts on page 31.

APPENDIX U

THE *DRAWING WUMPS* TASK

The *Drawing Wumps* task used by Gloria Xavier during her lesson on October 27[17].

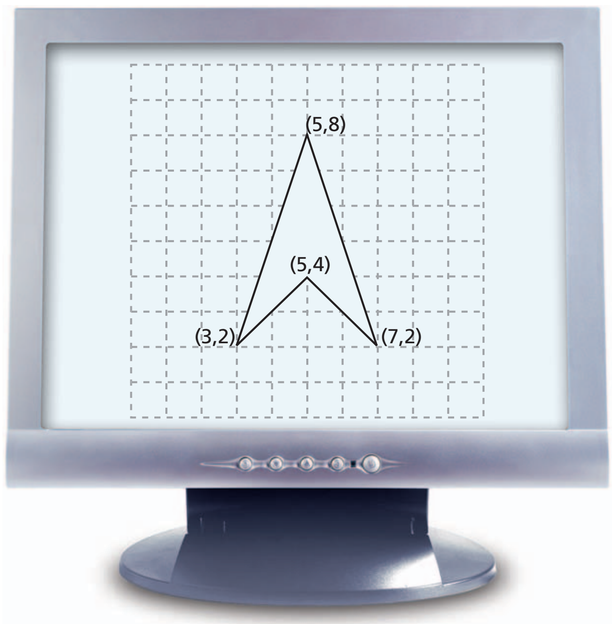
[17] From CONNECTED MATHEMATICS STRETCHING AND SHRINKING

Investigation 2

Similar Figures

Zack and Marta want to design a computer game that involves several animated characters. Marta asks her uncle Carlos, a programmer for a video game company, about computer animation.

Carlos explains that the computer screen can be thought of as a grid made up of thousands of tiny points, called pixels. To animate a figure, you need to enter the coordinates of key points on the figure. The computer uses these key points to draw the figure in different positions.



Sometimes the figures in a computer game need to change size. A computer can make a figure larger or smaller if you give it a rule for finding key points on the new figure, using key points from the original figure.

2.1 Drawing Wumps

Zack and Marta's computer game involves a family called the Wumps. The members of the Wump family are various sizes, but they all have the same shape. That is, they are similar. Mug Wump is the game's main character. By enlarging or reducing Mug, a player can transform him into other Wump family members.

Zack and Marta experiment with enlarging and reducing figures on a coordinate grid. First, Zack draws Mug Wump on graph paper. Then, he labels the key points from A to X and lists the coordinates for each point. Marta writes the rules that will transform Mug into different sizes.

Problem 2.1 Making Similar Figures

Marta tries several rules for transforming Mug into different sizes. At first glance, all the new characters look like Mug. However, some of the characters are quite different from Mug.

A. To draw Mug on a coordinate graph, refer to the "Mug Wump" column in the table on the next page. For parts (1)–(3) of the figure, plot the points in order. Connect them as you go along. For part (4), plot the two points, but do not connect them. When you are finished, describe Mug's shape.

B. In the table, look at the columns for Zug, Lug, Bug, and Glug.

1. For each character, use the given rule to find the coordinates of the points. For example, the rule for Zug is $(2x, 2y)$. This means that you multiply each of Mug's coordinates by 2. Point A on Mug is (0, 1), so the corresponding point on Zug is (0, 2). Point B on Mug is (2, 1), so the corresponding point B on Zug is (4, 2).
2. Draw Zug, Lug, Bug, and Glug on separate coordinate graphs. Plot and connect the points for each figure, just as you did to draw Mug.

C. 1. Compare the characters to Mug. Which are the impostors?

2. What things are the same about Mug and the others?
3. What things are different about the five characters?

ACE **Homework starts on page 28.**

active math online

For: Mug Wumps, Reptiles, and Sierpinski Triangles Activity
Visit: PHSchool.com
Web Code: and-2201

Coordinates of Game Characters

	Mug Wump	Zug	Lug	Bug	Glug
Rule	(x, y)	$(2x, 2y)$	$(3x, y)$	$(3x, 3y)$	$(x, 3y)$
Point	**Part 1**				
A	(0, 1)	(0, 2)			
B	(2, 1)	(4, 2)			
C	(2, 0)				
D	(3, 0)				
E	(3, 1)				
F	(5, 1)				
G	(5, 0)				
H	(6, 0)				
I	(6, 1)				
J	(8, 1)				
K	(6, 7)				
L	(2, 7)				
M	(0, 1)				
	Part 2 (Start Over)				
N	(2, 2)				
O	(6, 2)				
P	(6, 3)				
Q	(2, 3)				
R	(2, 2)				
	Part 3 (Start Over)				
S	(3, 4)				
T	(4, 5)				
U	(5, 4)				
V	(3, 4)				
	Part 4 (Start Over)				
W	(2, 5) (make a dot)				
X	(6, 5) (make a dot)				

APPENDIX V

THE *EXPLORING EQUALITY* TASK

The *Exploring Equality* task used by Gloria Xavier during her lesson on May 11[18].

3.2 Exploring Equality

An equation states that two quantities are equal. In the equation $A = 5 + 0.5d$, A and $5 + 0.5d$ are the two quantities. Both represent the amount of money that Alana collects from each sponsor. Since each quantity represents numbers, you can use the properties of numbers to solve equations with one unknown variable.

Before we begin to solve linear equations, we need to look more closely at equality.

What does it mean for two quantities to be equal?

Let's look first at numerical statements.

Getting Ready for Problem 3.2

The equation $85 = 70 + 15$ states that the quantities 85 and $70 + 15$ are equal.

What do you have to do to maintain equality if you

- subtract 15 from the left-hand side of the equation?
- add 10 to the right-hand side of the original equation?
- divide the left-hand side of the original equation by 5?
- multiply the right-hand side of the original equation by 4?

Try your methods on another example of equality. Summarize what you know about maintaining equality between two quantities.

In the Kingdom of Montarek, the ambassadors carry diplomatic pouches. The contents of the pouches are unknown except by the ambassadors. Ambassador Milton wants to send one-dollar gold coins to another country.

$1 gold coin

diplomatic pouch

His daughter, Sarah, is a mathematician. She helps him devise a plan based on *equality* to keep track of the number of one-dollar gold coins in each pouch.

In each situation:

- Each pouch contains the same number of one-dollar gold coins.
- The number of gold coins on both sides of the equality sign is the same, but some coins are hidden in the pouches.

Try to find the number of gold coins in each pouch.

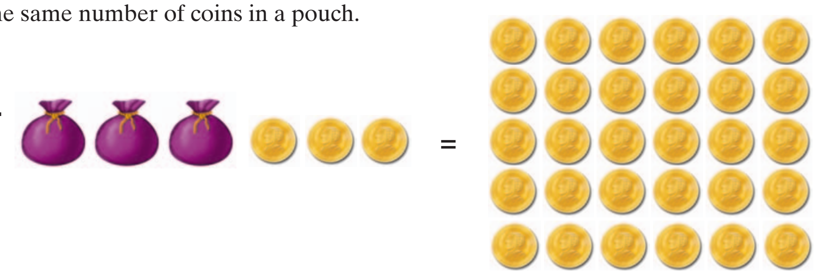
Problem 3.2 Exploring Equality

A. Sarah draws the following picture. Each pouch contains the same number of $1 gold coins.

How many gold coins are in each pouch? Explain your reasoning.

B. For each situation, find the number of gold coins in the pouch. Write down your steps so that someone else could follow your steps to find the same number of coins in a pouch.

1.



2.

3.

4.

5.

C. Describe how you can check your answer. That is, how do you know you found the correct number of gold coins in each pouch?

D. Describe how you maintained equality at each step of your solutions in Questions A and B.

ACE **Homework starts on page 57.**

APPENDIX W

THE *FINDING MEASURES OF PARALLELOGRAMS* TASK

The *Finding Measures of Parallelograms* task used by Nathan Ingram during his lesson on April 30[19].

[19] From CONNECTED MATHEMATICS COVERING AND SURROUNDING

Investigation 4

Measuring Parallelograms

In this unit, you have developed ways to find the area and perimeter of rectangles and of triangles. In this investigation you will develop ways to find the area and perimeter of parallelograms.

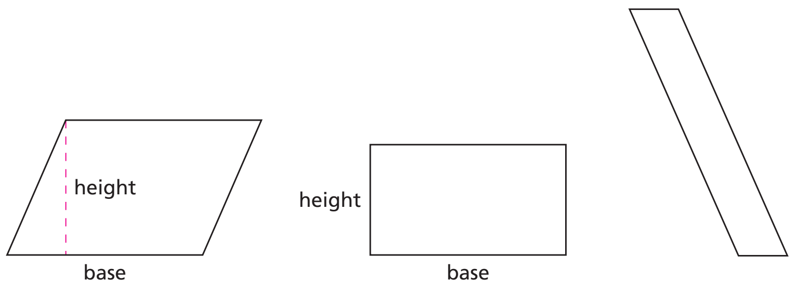
When you work with rectangles, you use measurements like length and width. For triangles, you use the side lengths, the base, and the height. Like triangles, parallelograms are often described by measures of side length, base, and height.

4.1 Finding Measures of Parallelograms

As you work with parallelograms, remember what you know about triangles and look for ways to relate these two figures.

Getting Ready for Problem 4.1

Here are three parallelograms with the base and height of two parallelograms marked. What do you think the *base* and the *height* of a parallelogram mean? How do you mark and measure the base and height of the third figure?



Problem 4.1 Finding Measures of Parallelograms

Six parallelograms labeled A–F are drawn on the centimeter grid on the next page.

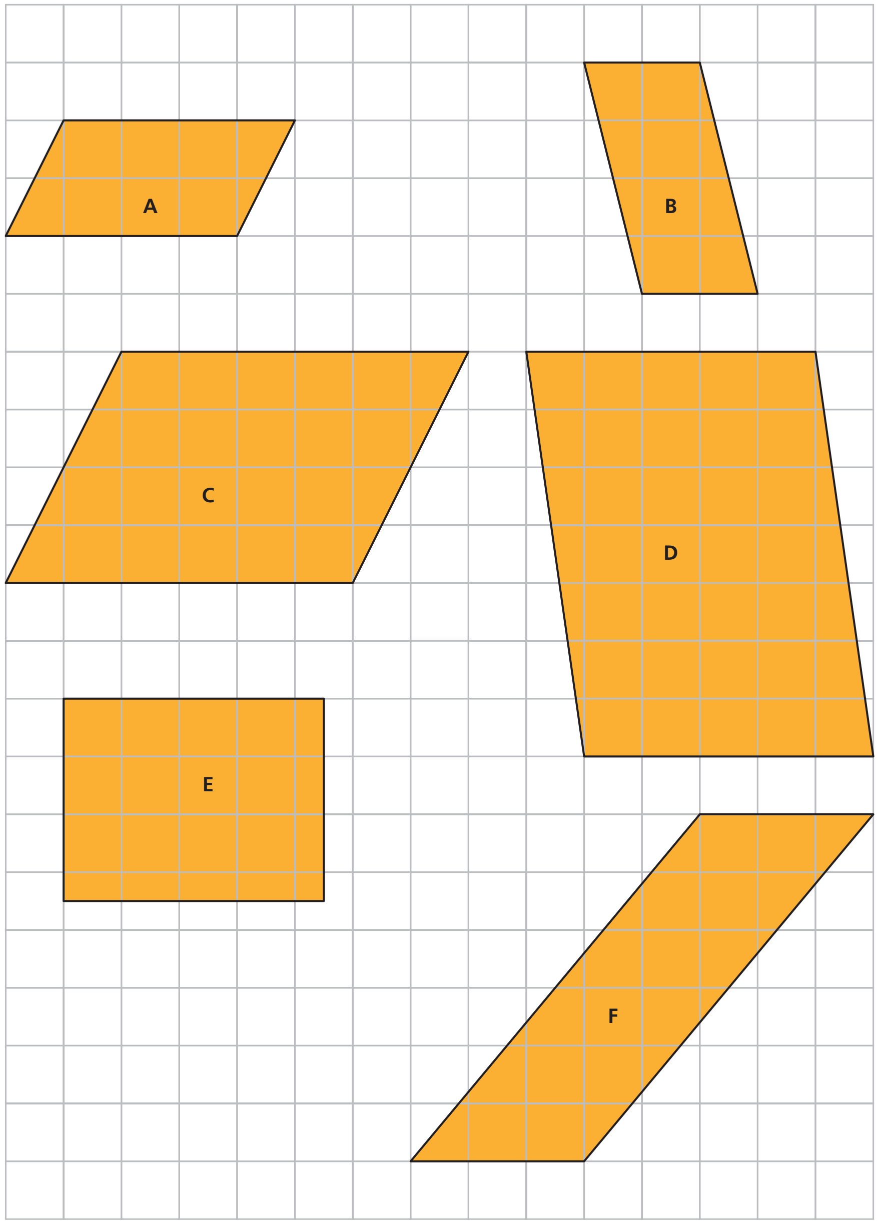
A. **1.** Find the perimeter of each parallelogram.

2. Describe a strategy for finding the perimeter of a parallelogram.

B. **1.** Find the area of each parallelogram.

2. Describe the strategies you used to find the areas.

ACE **Homework starts on page 60.**


A
B
C
D
E
F

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