Spare Capacity Allocation Using Partially Disjoint Paths for Dual Link Failure Protection

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***Abstract*— A shared backup path protection (SBPP) scheme can be used to protect dual link failures by pre-planning each traffic flow with mutually disjoint working and two backup paths while minimizing the network overbuild. However, many existing backbone networks are bi-connected without three fully disjoint paths between all node pairs. Hence in practice partially disjoint paths (PDP) have been used for backup paths instead of fully disjoint ones. This paper studies the minimum spare capacity allocation (SCA) problem using PDP within an optimization framework. This is an extension of the spare provision matrix (SPM) method for PDP. The integer linear programming (ILP) model is formulated and an approximation algorithm, Successive Survivable Routing (SSR), is extended and used in the numerical study.**

Keywords*— survivable network design, resilient traffic engineering, spare capacity allocation (SCA), shared backup path protection (SBPP), dual failure protection, partially disjoint path

I. INTRODUCTION

Communication networks are one of the critical national infrastructures upon which society depends, thus it is imperative that they be highly available and resilient to failures. In particular core optical backbone networks need high levels of fault tolerance. A variety of survivability techniques (e.g., multiple homing, self-healing rings, pre-planned backup routes, p-cycles, etc.) have been proposed for a range of network technologies (e.g., WDM, MPLS, etc.). The vast majority of the literature and implementations have focused on providing survivability for single link/node/SRLG failures. However, several recent studies have shown the need to address dual-link failures in real networks [8][9][10] [11][12] and [13]. For example, in [8] it was observed that in an operational IPTV network over a four month period 17% of link failures were dual failures. Recently, we studied the SCA problem for dual link failure protection [1]. It extends our previous work for single failures [4] and provides both optimization models and related heuristic solution algorithms for dual link failures.

In this paper we present the spare capacity allocation (SCA) optimization model for dual link failures using a modified shared backup path protection (SBPP) scheme with partially disjoint paths (PDP). The traditional shared backup path protection (SBPP) scheme preplans one working and two backup paths that are mutually disjoint to achieve dual link failure protection. Both backup paths reserve full capacity as their working path so that traffic flow does not have to be split upon failure. This method requires the network topology to be tri-connected or 3-connected. However, currently most backbone networks are bi-connected or two-connected. Adding physically diverse new links in the backbone network is always a slow and expensive process, sometimes even impossible. To quickly deploy survivable services that can be resilient to a set of protectable dual failure scenarios if not all of them, some flows need to use partially disjoint paths (PDP). Under this situation, the SCA problem using PDP has practical significance to minimize the total spare capacity or network overbuild for dual-link failure protection on bi-connected networks.

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II. RELATED WORK

A. Partial Disjoint Paths for Dual Failures

Many backbone networks nowadays are enforced to be bi-connected. This means that a flow might not have three mutually disjoint paths to protect any dual-link failure. This happens when the end nodes of a flow are separated by a set of cuts that contain only two links. These cuts are called the *cut-pairs* in [3]. Under this situation, the flow should be able to find the first two fully disjoint paths using the diverse routing algorithms in [4][6]. The two fully disjoint paths can be assigned as the working and primary backup paths, as blue and green curves in Figure 1.

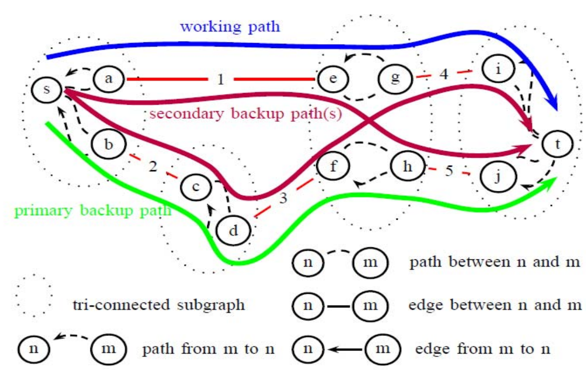


Figure 1: Partially disjoint secondary backup paths for a flow across four tri-connected subgraphs

Because of bi-connectivity, these two paths will pass through a set of cut-pairs. These cut-pairs can be aggregated into the *cut-groups* [2], which include several cut-pairs of this flow that share common links. For example in Figure 1, cut-pairs (1, 2) and (1, 3) can be merged into the cut-group (1, [2, 3]) because they share a common link 1. Dual link failures that contain the common edge 1 and either edge 2 or 3 have the same effect to interrupt the first two paths of the flow $s \rightarrow t$. After all cut-groups are identified for the flow, multiple secondary backup paths can be found to pass various halves of these cut-groups using the partial disjoint routing algorithm in [2]. The arrowed dashed curves or lines in Figure 1 indicate paths or links that have only reverse directions available to route these secondary paths. They enforce the diversity constrains and allow the trap avoidance in the max-flow based diverse routing algorithm. For partial disjoint paths, our goal is to recover the forward directions of some cut-pair links so the secondary backup paths can be found. For example, two secondary backup paths to improve protection from dual link failures are shown as red curves. These four paths can protect the dual link failures that happen in tri-connected subgraphs. However, they cannot protect against any dual failures that include the cut-pairs contained in the two cut-groups. With these four paths, the flow achieves the best survivability against dual-link failures.

B. Failure Dependent Path Protection for Single Failures

Previously, multiple partially disjoint secondary backup paths are used to protect dual link failures that straddle over the working and primary backup paths. Each of these partially disjoint secondary backup paths only protects a subset of dual link failures. This scenario is very similar to the failure dependent (FD) path protection for single failures in [5][6].

In FD path protection, multiple secondary backup paths q_r^k are associated with the flow failure matrix U, as shown in a cubicle diagram in Figure 2. These backup paths q_r^k are for flow r to protect failure scenario k. It can be represented in two alternate ways: either for each failure scenario k in Q^k, or for each flow r in Q_r. These matrices can be used to compute the spare provisioning matrix (SPM) using (1) as seen in [6]. This is slightly different from the normal way to compute SPM for failure independent (FID) SCA [1].

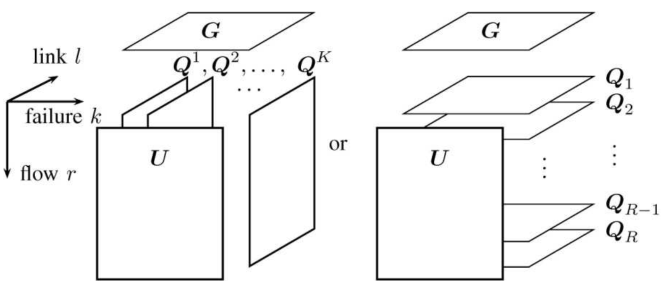


Figure 2: Cubical structure for failure dependent path restoration for single failures [5][6]

$$G_k = Q^{k^T} M U_k, 1 \leq k \leq K \qquad (1)$$

The k-th column vector $G_k = \{g_{lk}\}_{L\times 1}$ in G is determined by U's k-th column vector $U_k = \{u_{rk}\}_{R\times 1}$, M, and Q_k in (1). We use capital variable U_k here for a column vector to distinguish it from the row vector u_r. The same usage of the capital variables is for G_k representing a column vector for failure k.

The SCA model with partially disjoint paths for dual failure protection can use the above method similarly. We arrange all secondary backup paths aligned with their protected dual link failures and compute SPM accordingly.

C. SCA for Dual Link Failure Protection without PDP

The notation for dual link failure protection is in Table 1.

Table 1: Notation for Dual Link Failure

N, L, R, K	Numbers of nodes, links, flows, & failures
n, l, r, k	Indices of nodes, links, flows, and failures
i, j	Indices of links, $1 \leq i, j \leq L$
$B = \{b_{nl}\}_{N\times L}$	Node link incidence matrix
$D = \{d_{rn}\}_{R\times N}$	Flow node incidence matrix
$P = \{p_r\} = \{p_{rl}\}$	Working path link incidence matrix
$Q = \{q_r\} = \{q_{rl}\}$	Primary backup path link incidence matrix
$Z = \{z_r\} = \{z_{rl}\}$	Secondary backup path link incidence matrix, for failure independent case
$Z^k = \{z_r^k\}_{R\times 1} = \{z_{rl}^k\}_{R\times L}$	Secondary backup path to link incidence matrix for failure k, in failure independent case
$Z_r = \{z_r^k\}_{K\times 1} = \{z_{rl}^k\}_{K\times L}$	Secondary backup path to link incidence matrix for flow r, for failure independent case
$M = \text{Diag}(\{m_r\})$	Diagonal matrix of bandwidth m_r of flow r
$G_r^{[y]} = \{g_{lk}^{r[y]}\}_{L\times K}$	Contribution of flow r's y-th backup path to G, $y = 1$ or 2 for primary or secondary backup paths
$G = \{g_{lk}\}_{L\times K}$	Spare provision matrix, g_{lk} is spare capacity on link l for failure k
$U^{[y]} = \{u_{rk}^{[y]}\}_{R\times K}$	The incidence matrix for flow r's y-th backup path and failure k, $u_{rk}^y = 1$ *iff* failure k causes flow $r$to use its y-th backup path, y=1 or 2
$T^{[y]} = \{t_{rl}^{[y]}\}_{R\times L}$	Flow's tabu-link matrix for its y-th backup path, $t_{rl}^{[y]} = 1$ *iff* link l should not be used on flow r's y-th backup path, $y = 1$ or 2
S_1, S_2	Total spare capacity reserved for the primary or secondary backup paths
Y_r	Number of cut-groups for flow r
(ϕ_{ry}, ψ_{ry})	Cut groups of flow r, $1 \leq y \leq Y_r$, where $\phi_{r,y}$ and $\psi_{r,y}$ are binary vectors whose i-th element is 1 *iff* the i-th link belongs to the first (ϕ) or second (ψ) half of the cut-group, and $1 \leq i \leq L$
τ_{ry}	Binary vector whose k-th element indicates dual link failure k will be protected by the y-th secondary backup path of flow r, $0 \leq y \leq Y_r$
χ_{ry}	Binary vector whose k-th element indicates dual link failure k will not be protected due to either bi-connectivity or the maximum number of paths for flow r, $1 \leq y \leq Y_r$
ω_r^y	The y-th secondary backup path corresponding to the y-th cut-group for flow r, which bypasses the first half of the y-th cut-group and the second halves of the other cut-groups.

On tri-connected networks, the SCA problem for dual link failure uses three fully disjoint paths in [1]. First, we briefly discuss a flow r and its spare provision matrix G_r. The element g_{lk}^r gives the spare capacity required for flow r on link l when a dual link failure k happens. The total number of dual link failures is $K = \left\{ {L \atop 2} \right\}$. Each failure $k \in 1 \dots K$ contains a pair of failed links i, j and the index k is determined by $k = (i-1) \times L + (j - i)$ where $1 \le i < j \le L$. In dual link failure k, the working and backup paths of flow r will be interrupted and the spare capacity on their backup paths need bandwidth reservation as shown in the following two cases:

1. When a dual link failure k breaks the working path, but not the primary backup path, traffic will be protected by the primary backup path. The links on the primary backup path requires a bandwidth m_r.
   In formulation, $p_{ri} = 1$ *iff* link i is on the working path p_r and $q_{rj} = 1$ *iff* a link j is on the primary backup path q_r. Then $p_{ri}(1 - q_{rj}) = 1$ shows that link i is on the working path while link j is not on the primary backup path. Let $u_{rk}^{[1]} = p_{ri}(1 - q_{rj}) \oplus (1 - q_{ri})p_{rj}$, where $\oplus$ is the logical "OR" operation which gives $1 \oplus 1 = 1$. Then $u_{rk}^{[1]} = 1$ indicates failure k contains one link on the working path p_r but does not contain any link on the primary backup path q_r. For this failure case k, the spare capacity on another link l on the primary backup path should reserve bandwidth m_r. This is formulated in $g_{lk}^{r[1]} = m_r q_{rl} u_{rk}$. These equations can also be shown in a vector or matrix format in (2) and (3).

$$G_r^{[1]} = \left\{ g_{lk}^{r[1]} \right\} = m_r q_r^T u_r^{[1]} \tag{2}$$

$$u_r^{[1]} = \text{vec}(p_r^T \bar{q}_r \oplus \bar{q}_r^T \, p_r) \tag{3}$$

   In the above equations, vec(·) converts a matrix with index (i, j) into a row vector with index k and $\bar{q}_r = e - q_r$ where e is a unit row vector with size L. These equations are shown in the diagram in Figure 3.
   Notice that the length of the row vector $u_r^{[1]}$ is L^2 instead of number of failures $\binom{L}{2}$. This makes it easier to maintain matrix formulation and conversions between k and i, j. The actual failure size can be controlled by removing duplicate cases in the implementation. For this reason, we use $k = (i-1)L + j$ and $K = L^2$ below.
2. When the failure case k contains one link on the working path p_r and another link on the primary backup path q_r, traffic is rerouted to the secondary backup path z_r. Thus, the links on the secondary backup path need spare capacity to meet bandwidth demand m_r for failure case k. In this case, $u_{rk}^{[2]} = p_{ri} q_{rj} \oplus q_{ri} p_{rj}$. Then $u_{rk}^{[2]} = 1$ indicates failure case k breaks the working path p_r and the primary backup path q_r at the same time. Hence, the spare capacity on link l on the secondary backup path is m_r. This gives $g_{lk}^{r[2]} = m_r z_{rl} u_{rk}^{[2]}$. These equations can be rewritten in matrix format in (4) and (5).

$$G_r^{[2]} = \left\{ g_{lk}^{r[2]} \right\} = m_r z_r^T u_r^{[2]} \tag{4}$$

$$u_r^{[2]} = \text{vec}(p_r^T q_r \oplus q_r^T \, p_r) \tag{5}$$

With two cases above, the per-flow based spare provision matrix is given in (6). These equations are shown in the diagram in Figure 4.

The overall spare provision matrix is in (7).

$$G_r = G_r^{[1]} + G_r^{[2]} \tag{6}$$

$$G = \sum_{r=1}^{R} G_r \tag{7}$$

Using the spare provision relations above, the SCA model is formulated in (8)–(15), as given in [1].

$$\min_{Q,Z,G,s} \quad S = e^T s \tag{8}$$

$$s.t.: \quad s = \max G \tag{9}$$

$$G = Q^T M \, U^{[1]} + Z^T M \, U^{[2]} \tag{10}$$

$$P + Q \le 1 \tag{11}$$

$$P + Q + Z \le 1 \tag{12}$$

$$Q B^T = D \tag{13}$$

$$Z B^T = D \tag{14}$$

$$Q, Z: binary \tag{15}$$

The Integer Mathematical Programming problem above has

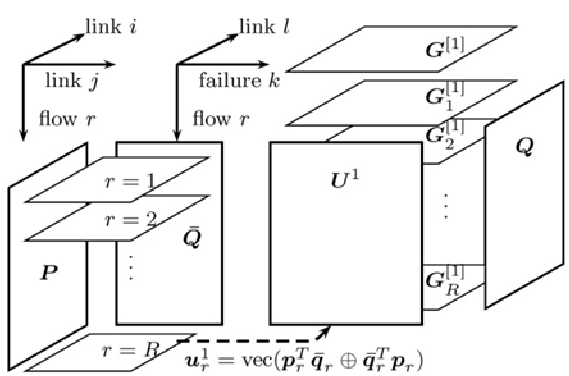


Figure 3: Spare provision matrix computation for primary backup paths in Q

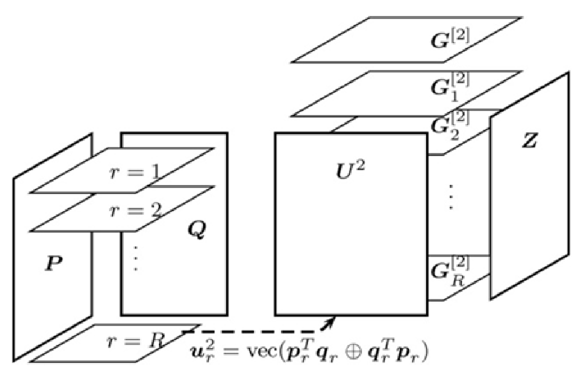


Figure 4: Spare provision matrix computation for secondary backup paths in Z

the objective function (8) to minimize the total spare capacity reserved on the network. The decision variables include the spare capacity s; the spare provisioning matrix G, the primary backup path matrix Q, and the secondary backup path matrix Z. In constraint (9), the spare capacity in a column vector s is derived from the maximum elements in each rows, across all failures, in the spare provision matrix G. In actual networks, this represents that the required spare capacity on a link is equivalent to the highest "watermark" among all possible dual-link failures when each individual dual-link failure uses the spare capacity or leaves a "watermark." In constraint (10), the spare provision matrix is derived from backup paths and their triggering failures. It is in a matrix format, equivalent to the aggregation of per-flow based computation in (7), (6), (2), and (4) earlier. Constraints (11) and (12) require the working and two backup paths to be mutually disjoint, i.e., these paths will use the same link at most once. Constraints (13) and (14) are the flow balance requirements, to guarantee backup paths in Q and Z to be valid routes. Constraint (15) requires backup path decision variables to be binary so that each backup path will not be bifurcated. Row vectors $u^{[y]}$ in $U^{[y]}$, $y = 1, 2$ are derived in (3) and (5) to indicate which failure case k could cause traffic detour to its primary or secondary backup path.

These previous works could be implemented in the protection of mesh networks such as MPLS and OTN, actual implementation method can be seen in [14].

III. SCA MODEL WITH PARTIALLY DISJOINT PATHS

The SCA model using partially disjoint paths to protect dual link failures on bi-connected networks is introduced here. The partial disjoint routing algorithm in [2] is used first to identify the dual-link failures that a set of secondary backup paths could protect. The failure dependent path protection SCA formulation in [5][6] is followed to align these secondary backup paths to their corresponding dual-link failures. Next, the spare provisioning matrix is computed and used for spare capacity sharing under dual-link failures.

In Figure 5, the matrix relationship to compute the spare provision matrix from multiple secondary backup paths is shown. These secondary backup paths will protect different sub-sets of dual link failures, while still be able to share their spare capacity with other backup paths. Similar to Figure 2, these secondary backup paths can be represented as Z_r for flow r, or alternatively for specific failure k as Z^k. Both matrices contain the same path link incidence vector z_r^k for the

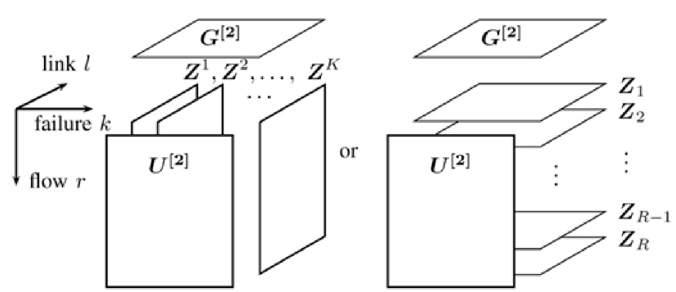


Figure 5: Matrix structure to compute spare provision matrix using multiple partially disjoint paths in Z^k or Z_r

secondary backup path as defined in Table 1.

Next we describe how to prepare the content in vector z_r^k and the failures protected by these partial disjoint paths.

A. Model Cut-Groups & related Secondary Backup Paths

Using the partial disjoint routing algorithm in [2], a flow on a bi-connected network could encounter a set of cut-groups whose enclosed cut-pairs can partition the source and destination nodes. The number of cut-groups is identical to or greater than the minimum number of secondary backup paths to cover all possible dual link failures.

We denote the number of cut-groups as Y_r and the cut-groups as (ϕ_{ry}, ψ_{ry}), $1 \le y \le Y_r$, where ϕ_{ry} and ψ_{ry} are binary vectors whose element i is 1 *iff* the i-th link belongs to the first (ϕ) or second (ψ) half of the cut-group, and $1 \le i \le L$. An example is given below, showing the use of binary vectors for two cut-groups (1, [2, 3]) and (4, 5) in Figure 1, where $r = (st)$, $y = 1, or\ 2$, and a comma is added between (st) and y for clarity.

$$\begin{aligned}
\phi_{(st),1} &= (10000\ \cdots) \\
\psi_{(st),1} &= (01100\ \cdots) \\
\phi_{(st),2} &= (00010\ \cdots) \\
\psi_{(st),2} &= (00001\ \cdots)
\end{aligned} \tag{16}$$

The corresponding two secondary backup paths for flow $r = (st)$ are represented in $\Omega_r = \omega_r^y$, where, the index y represents the cut-group whose first half links are used by this path, while its second half links are protected by this path.

$$\begin{aligned}
p_r &= (10010\ \cdots) \\
q_r &= (01101\ \cdots) \\
\omega_r^1 &= (10001\ \cdots) \\
\omega_r^2 &= (01110\ \cdots)
\end{aligned} \tag{17}$$

The original failure vector for secondary paths $u_r^{[2]}$ in (5) takes all dual links on the first two paths. Due to the existence of the cut-pairs in these Y_r cut-groups, multiple secondary backup paths end up protecting different subsets of the dual link failures. In the following, additional notation is introduced to model various dual link failures related to the cut-groups.

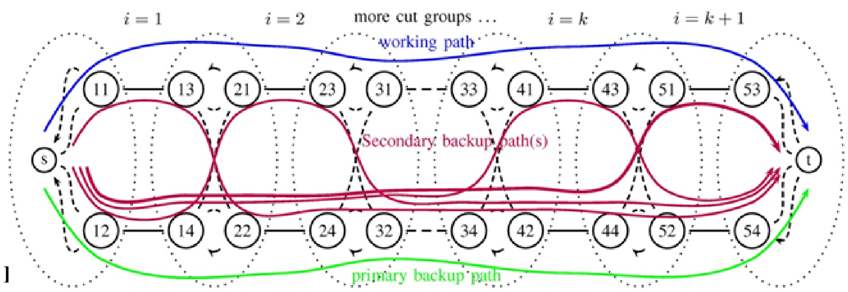
B. Dual Link Failures Protected by Partially Disjoint Secondary Backup Paths

Using the algorithm in [2], we construct multiple secondary paths to protect corresponding cut-groups so that the y-th secondary backup path always protects dual link failures that include the second half of the y-th cut-groups and the first halves of other cut-groups. This is accomplished by forcing this backup path passing the first half of the y-th cut-group and the second halves of any other cut-groups, as shown in Figure 6.

In this method, the dual link failures protected by the y-th secondary backup paths can be represented in τ_{ry} as in (18).

$$\tau_{ry} = \mathrm{vec}\left(\hat{\phi}_{ry}^T \psi_{ry} \oplus \psi_{ry}^T \hat{\phi}_{ry}\right), \quad 1 \le y \le Y_r \tag{18}$$

where $\hat{\phi}_{ry} = \Phi_r - \phi_{ry}$, $\hat{\psi}_{ry} = \Psi_r - \psi_{ry}$, and $\Phi_r = \sum_{y=1}^{Y_r} \phi_{ry}$, $\Psi_r = \sum_{y=1}^{Y_r} \psi_{ry}$. For the first two partially disjoint



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is also Y_r. These paths are denoted as ω_r^y in previous section. We will use these backup paths and their protected dual link failures indicated in $\tau_{ry}, 0 \le y \le Y_r$ and ignore dual link failures in χ_{ry} that could not be protected at all.

secondary backup paths, they could also protect other dual link failures that straddle on the first two paths that do not belong to the cut-pairs defined above. They are represented using τ_{r0} in (19).

$$\begin{aligned}\tau_{r0} &= \mathrm{vec}(p_r^T q_r \oplus q_r^T p_r) - \mathrm{vec}(\Phi_r^T \Psi_r \oplus \Psi_r^T \Phi_r)\\ &= u_r^{[2]} - \mathrm{vec}(\Phi_r^T \Psi_r \oplus \Psi_r^T \Phi_r) \qquad (19)\end{aligned}$$

The dual link failures that cannot be protected by these secondary backup PDPs are the failures whose links belong to the first and second halves of the same cut-groups, modeled as χ_{ry} in (20).

$$\chi_{ry} = \mathrm{vec}\big(\phi_{ry}^T\, \psi_{ry} \oplus \psi_{ry}^T\, \phi_{ry}\big),\ \ 1 \le y \le Y_r \qquad (20)$$

It is easy to prove that the unprotected dual link failures in $\chi_{ry}, 1 \le y \le Y_r$, and the dual link failures protected by all secondary backup paths $\tau_{ry}, 0 \le y \le Y_r$ are identical to all dual link failures that could disconnect the first two paths $u_r^{[2]}$. In summary, the set of partially disjoint paths found here have covered the maximum possible set of dual link failures.

$$\begin{aligned}&\textstyle\sum_{y=1}^{Y_r} \chi_{ry} + \sum_{y=0}^{Y_r} \tau_{ry}\\ =\ &\textstyle\sum_{y=1}^{Y_r} \big(\chi_{ry} + \tau_{ry}\big) + \tau_{r0}\\ =\ &\textstyle\sum_{y=1}^{Y_r} \big[\mathrm{vec}\big(\phi_{ry}^T\, \psi_{ry} \oplus \psi_{ry}^T\, \phi_{ry}\big) +\\ &\quad \mathrm{vec} \phi ryT \psi ry \oplus \psi ryT\, \phi ry + \tau r0\\ =\ &\textstyle\sum_{y=1}^{Y_r} \big[\mathrm{vec}\big(\Phi_r^T \psi_{ry}\, \Psi_r \oplus \psi_{ry}^T\, \Phi_r\big)\big] + \tau_{r0}\\ =\ & vec(\Phi_r^T \Psi_r \oplus \Psi_r^T \Phi_r) + \tau_{r0}\\ =\ & u_r^{[2]} \qquad (21)\end{aligned}$$

C. Construction of Secondary Backup Path Matrix for Flow r and Failure k

In Figure 5, the secondary backup paths for flow r and failure k are denoted as a binary vector ω_r^k. It can be aggregated into Z^k, based on each failure k , $1 \le k \le K$; or based on each flow r into $Z_r, 1 \le r \le R$.

In the following, we use Z_r to represent all paths in ω_r^y for different dual link failures. It is not difficult to aggregate these partially disjoint secondary backup paths in ω_r^y using another similar approach with Z^k.

From the previous section, the number of cut-groups of a flow r is Y_r. Since we are using the partially disjoint routing algorithms in [2], the number of partially disjoint backup paths is also Y_r. These paths are denoted as ω_r^y in previous section. We will use these backup paths and their protected dual link failures indicated in $\tau_{ry}, 0 \le y \le Y_r$ and ignore dual link failures in χ_{ry} that could not be protected at all.

As we can see from (21), all dual link failures that disrupt the flow r as given in $u_r^{[2]}$ have been partitioned into the above two types of failures: (i) protectable failures indicated in τ_{ry} and (ii) unprotected failures in χ_{ry}. The unprotected failures are mainly because of the topology limitation of bi-connectivity, and/or the constraint enforced while performing the partially disjoint routing procedure, i.e. the maximum number of backup paths might be limited at a smaller number than the number of cut-groups. In the following, we assume the backup paths can be as many as possible so that we only consider unprotected failures due to topology limitations.

We organize all partially disjoint paths in ω_r^y into the per-flow based secondary backup path matrix Z_r as in (22), where τ_{r0}^1 and τ_{r0}^2 are two parts of τ_{r0} that indicates dual link failures to be protected by the first two secondary backup paths.

$$Z_r = \tau_{r0}^1\, \omega_r^1 + \tau_{r0}^2\, \omega_r^2 + \textstyle\sum_{y=1}^{Y_r} \tau_{ry}\, \omega_r^y\ ,1 \le r \le R \quad (22)$$

Thus far, we have described the content of $u_r^{[2]}$ and Z_r using the formulation of the cut-groups, their related dual-link failures in τ_{ry}, χ_{ry}, and the corresponding partially disjoint paths in ω_r^y.

D. Spare Provisioning Matrix for Partially Disjoint Secondary Backup Paths

For the y-th secondary backup path, its contribution to the spare provisioning matrix $G_{ry}^{[2]}$ can be obtained by (23) and (24).

$$G_{ry}^{[2]} = m_r {\omega_r^y}^T \tau_{ry},\ \ 3 \le y \le Y_r \qquad (23)$$

$$\begin{aligned}G_{r1}^{[2]} &= m_r {\omega_r^1}^T (\tau_{r1} + \tau_{r0}^1)\\ G_{r2}^{[2]} &= m_r {\omega_r^2}^T (\tau_{r2} + \tau_{r0}^2) \qquad (24)\end{aligned}$$

The spare provisioning matrix for the secondary backup paths is given in (25) and (26).

$$G_r^{[2]} = \textstyle\sum_{y=1}^{Y_r} G_{ry}^{[2]} \qquad (25)$$

$$G^{[2]} = \textstyle\sum_{r=0}^{R} G_r^{[2]} \qquad (26)$$

The ILP formulation of the SCA problem using partially disjoint paths composes of (8), (9), (11), (13), (18), (19) and (23)-(30). These equations are listed below again for clarity. The equations with their numbers followed by single quotation mark indicate that they are duplicates, i.e. (8') is a duplicate of (8). Note that the cut-groups related information in (ϕ_{ry}, ψ_{ry}) is pre-computed for each flow r according to the procedure in [2].

$$\min_{Q,\omega_r^y,G,s} \quad S = e^T s \qquad (8')$$

$$s.t.: \quad s = \max G \tag{9'}$$

$$P + Q \le 1 \tag{11'}$$

$$QB^T = D \tag{13'}$$

$$\tau_{ry} = \mathrm{vec}\left(\hat{\phi}_{ry}^T \psi_{ry} \oplus \psi_{ry}^T \hat{\phi}_{ry}\right), \ 1 \le y \le Y_r \tag{18'}$$

$$\tau_{r0} = \mathrm{vec}(p_r^T q_r \oplus q_r^T p_r) - \mathrm{vec}(\Phi_r^T \Psi_r \oplus \Psi_r^T \Phi_r) \tag{19'}$$

$$G_{ry}^{[2]} = m_r \omega_r^{y^T} \tau_{ry}, \ 3 \le y \le Y_r \tag{23'}$$

$$G_{r1}^{[2]} = m_r \omega_r^{1^T} (\tau_{r1} + \tau_{r0}^1)$$

$$G_{r2}^{[2]} = m_r \omega_r^{2^T} (\tau_{r2} + \tau_{r0}^2) \tag{24'}$$

$$G_r^{[2]} = \sum_{y=1}^{Y_r} G_{ry}^{[2]} \tag{25'}$$

$$G^{[2]} = \sum_{r=0}^{R} G_r^{[2]} \tag{26'}$$

$$G = Q^T M\, U^{[1]} + G^{[2]} \tag{27}$$

$$p_r + q_r + \omega_r^y - \phi_{ry} - \sum_{j=1; j \ne y}^{Y_r} \psi_{rj} \le 1, \quad 1 \le y \le Y_r, \ 1 \le r \le R \tag{28}$$

$$\omega_r^y B^T = D, 1 \le y \le Y_r, 1 \le r \le R \tag{29}$$

$$Q, \omega_r^y : binary, 1 \le y \le Y_r, 1 \le r \le R \tag{30}$$

Alternatively, we could use the construction of $u_r^{[2]}$ and Z_r in previous section to organize $G_r^{[2]}$ in (31). This approach has the same effect as the steps in (23)-(25) above but it keeps the matrix diagram as shown in Figure 5.

$$G_r^{[2]} = m_r z_r^k u_k^{[2]}, 1 \le k \le K, 1 \le r \le R \tag{31}$$

This ILP formulation can be solved using commercial software such as AMPL/CPLEX which enumerates all possible solutions in small cases. For large cases, heuristic algorithms are needed.

E. Extension of Successive Survivable Routing Algorithm

The Successive Survivable Routing (SSR) algorithm has been successfully used to find near optimal solutions in a short time for large scale SCA problems. SSR was originally developed for the basic single failure SCA problem in [1][4], and then extended to the failure dependent path restoration SCA problem in [5][6], to the two layer single failure SCA case in [6][7], and to the dual link failure case in [1]. This paper further extends the SSR algorithm for the SCA problem for dual link failures to use the partially disjoint paths. The original SSR flow chart in [1] is used here as the baseline and the related steps in the original flow chart are modified to handle differences for the partially disjoint paths.

The SSR algorithm finds the SCA solutions by routing the partially disjoint secondary backup paths iteratively. Here the working paths and primary backup paths are assumed given as described in [1]. Each backup path computation uses a shortest path algorithm. The link routing metric is the *incremental spare capacity* $v_r^y = \{v_{rl}^y\}$ for flow r's y-th secondary backup path. It is computed from the most recent spare provision matrix $G^{[2]}$ that is further based on previously routed backup paths. After all flows find their Y_r partially disjoint secondary backup paths, SSR continues to update existing backup paths whenever a new one could use less spare capacity. This process keeps reducing total spare capacity until it converges (i.e., no more backup path updates) or after reaching a preset iteration count. Different random orderings of the flows for routing backup paths are used to provide diversity and avoid local minima. The best solution is used as the final solution. The SSR scheme includes several steps for a given flow r and its y-th secondary backup path as shown in Figure 7.

Step 1 calculates the protectable failures in vector τ_{ry} for flow r's y-th cut-group based on the cut-group information stored in $\phi_{ry}$and ψ_{ry} found using the algorithm in [2], the working path in p_r, the primary backup path in q_r, and the source and destination nodes s_r, d_r. Then the protectable failures in τ_{ry} are computed in (18) and (19).

Step 2 first removes the current y-th secondary backup path of flow r if there is one, then updates the spare provision matrix $G^{[2]}$ using (24)-(26).

Step 3 calculates the link metric v_r^y from $G^{[2]}$ and flow r's y-th secondary backup path contribution $G_{ry}^{[2]}$ in (23)-(24) as follows:

Given G, ω_r^y and $G_{ry}^{[2]}$ for current flow r, let $G^{-ry} = G - G_{ry}^{[2]}$ and $s^{-ry} = \max G^{-ry}$ be the spare provision matrix and the link spare capacity vector after current secondary backup path

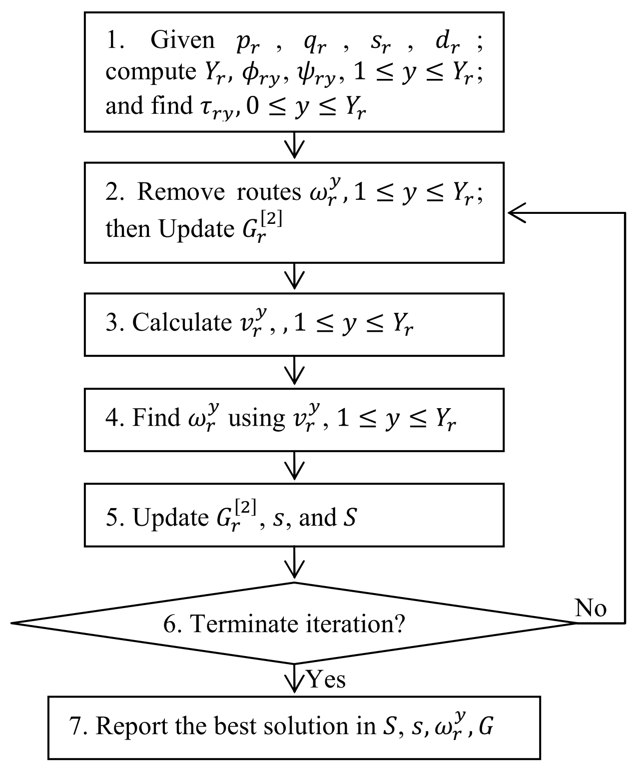


Figure 7: Flow chart for successive survivable routing (SSR) algorithm using partially disjoint secondary backup paths for dual link failure protection

ω_r^y is removed.

Let ω_r^{y*} denote an alternative backup path for flow r's y-th secondary backup path, and function $G_{ry}^{[2]*}(\omega_r^{y*}) = m_r \omega_r^{y*T}[\tau_{ry} + \tau_{r0}\,\delta(y == 1)]$, where $\delta(x) = 1$ if x is true or 0 otherwise. Then, this new path ω_r^{y*} produces a new spare capacity reservation vector in a function format of

$$s^*(\omega_{ry}^*) = \max[G^{-ry} + G_{ry}^{[2]*}(\omega_r^{y*})]. \qquad (32)$$

Let $\omega_r^{y*} = e - p_r - q_r + \phi_{ry} + \sum_{j=1;j\neq y}^{Y_r} \psi_{rj}$. This assumes the partially disjoint secondary backup path to use all other links not used by the first two paths, excluding those specified in the cut-groups as indicated in ϕ_{ry} and ψ_{rj}. Then, we can find the vector

$$v_r^y = \{v_{rl}^y\}_{L\times 1} = s^*(\omega_r^{y*}) - s^{-ry}. \qquad (33)$$

The element v_{rl}^y is the cost of the incremental spare capacity on link l if this link is used on the backup path.

Step 4 uses the shortest path algorithm with the link metric v_r^y to find a new or updated backup path $\omega_r^{y\prime}$. This path will be partially disjoint according to the constraint enforced in step 3. It also minimizes the total incremental spare capacity along the path due to the link costs used in step 3.

Step 5 replaces the original backup path ω_r^y with the new backup path $\omega_r^{y\prime}$ if the new one has the lower cost based on the link metrics in v_r^y, i.e., update to the new path only when $v_r^{yT}\omega_r^y > v_r^{yT}\omega_r^{y\prime}$.

Step 6, by default, returns to Step 2 for the next backup paths for all Y_r secondary backup paths corresponding to their cut-groups of flow r, and then for all flows $1 \leq r \leq R$. After the end of an iteration of all flows, the algorithm can stop and go to Step 7 if the termination condition is met. The termination condition can be: (a) there is no backup update for all flows in the recent iteration, or (b) when the maximum number of iterations is reached.

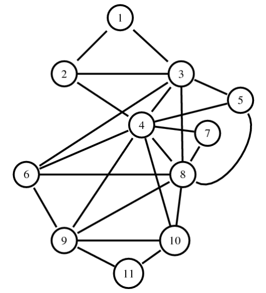
Step 7 reports the best results after terminating the iterations of the SSR algorithm.

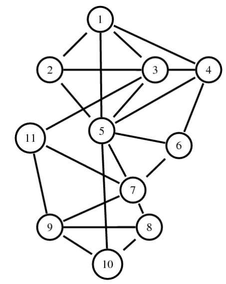
IV. NUMERICAL STUDY

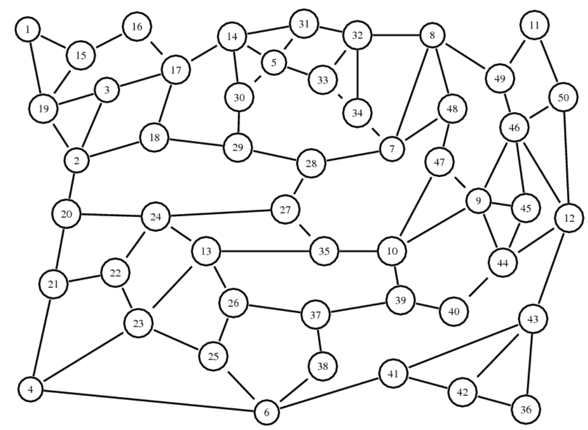
Four sample networks in Figure 9 are used in the numerical study. Two of them are bi-connected networks, and the other two are tri-connected and created by slightly modifying the previous two networks. For example, Net50x86 has four additional links than Net50x82 (i.e., links 1-16, 8-11, 36-40, and 38-41). The cut-pairs of two bi-connected networks are also listed in **Error! Not a valid bookmark self-reference.**.

Table 2: Network Cut-pairs and Results

Net	Cut-pairs for bi-connected	S_1	S
NJLATA	1-2, 1-3; 4-7, 7-8; 9-11, 10-11	184	278
Net11x22[1][2]	tri-connected	170	282
Net50x82[1]	1-15, 1-19; 6-38, 37-38; 6-41, 12-43; 11-49, 11-50; 15-16, 16-17; 36-42, 36-43; 39-40, 40-44	9706	15582
Net50x86[1]	tri-connected	9048	14428


(a) NJLATA, bi-connected
(*N, L, R*) = (11, 23, 110)


(b) Net11x22, tri-connected
(*N, L, R*) = (11, 22, 110)


(c) Net50x82, bi-connected, (50, 82, 2450)

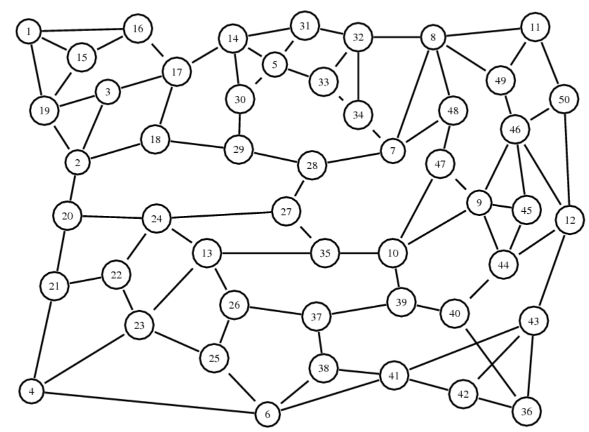

(d) Net50x86, tri-connected, (50, 86, 2450)

Figure 9: Networks for numerical study

This table also provides the total spare capacity values for all primary backup paths S_1 and the total spare capacity for both primary and secondary backup paths S.

The total spare capacity values in the first two networks are very close. This indicates that the shared spare capacity used by the bi-connected networks with partially disjoint paths has not been significantly increased compared to these in similar tri-connected networks. Analogous results are available between the last two networks as well.

Table 3 lists all the partially disjointed paths selected in the NJLATA network. Among these paths, the first two paths are always fully disjoint with each other. Note, that the primary backup paths might not be the shortest disjoint paths because the path is selected in a fashion that tries to minimize its spare capacity requirements by sharing capacity with other backup paths. For example, the paths for flow 1 → 4 and 8 → 11. The last two paths are partially disjoint against the first two paths. These last two paths are trying to share spare capacity among them. This can be observed by the large number of links duplicated among the two paths. This also indicates that the proposed SSR algorithm tried to achieve spare capacity sharing among these partially disjoint paths of the same flow as well.

Table 3: Partially disjoint paths in NJLATA network

Path 1	Path 2	Path 3	Path 4
1 2	1 3 2	1 2	1 3 4 2
1 3	1 2 3	1 3	1 2 4 3
1 2 4	1 3 4	1 2 3 5 4	1 3 5 4
1 3 5	1 2 3 4 5	1 3 8 5	1 2 4 3 8 5
1 3 6	1 2 3 4 9 6	1 3 8 6	1 2 4 8 6
1 2 4 7	1 3 8 7	1 2 3 5 8 7	1 3 5 4 7
1 3 8	1 2 3 4 8	1 3 5 8	1 2 4 5 8
1 2 4 9	1 3 8 9	1 2 3 6 9	1 3 6 9
1 2 4 10	1 3 6 9 10	1 2 3 5 8 10	1 3 5 8 10
1 2 4 9 11	1 3 8 10 11	1 2 3 5 4 6 9 10 11	1 3 5 4 6 9 11
2 4 7	2 3 8 7	2 1 3 5 4 7	2 1 3 5 8 7
2 4 10 11	2 3 4 9 11	2 1 3 6 9 10 11	2 1 3 6 9 11
3 8 7	3 4 7	3 5 8 7	3 2 4 7
3 8 10 11	3 6 9 11	3 4 10 11	3 4 9 11
4 7	4 8 7	4 7	4 10 8 7
4 10 11	4 6 9 11	4 8 9 10 11	4 8 9 11
5 8 7	5 4 7	5 3 8 7	5 3 4 7
5 8 9 11	5 4 10 11	5 3 6 9 11	5 3 8 10 11
6 8 7	6 4 7	6 3 8 7	6 3 4 7
6 9 11	6 3 4 10 11	6 4 9 11	6 8 10 11
7 8	7 4 8	7 8	7 4 9 8
7 4 9	7 8 9	7 4 10 9	7 8 6 9
7 4 10	7 8 10	7 4 9 11 10	7 8 9 11 10
7 4 9 11	7 8 3 4 6 9 10 11	7 4 10 11	7 8 10 9 11
8 10 11	8 9 11	8 6 9 10 11	8 6 9 11
9 11	9 10 11	9 11	9 4 10 11
10 11	10 9 11	10 11	10 4 9 11

V. CONCLUSIONS

This paper addresses the minimization of spare capacity for dual link failure protection on bi-connected networks. Partially disjoint secondary backup paths are used to protect all possible dual link failures. These backup paths are constructed to try and share their spare capacity whenever possible, to minimize the total spare capacity allocation. The heuristic algorithm, successive survivable routing, is extended to find these backup paths quickly. The numerical results illustrate the proposed SCA with PDP formulation and SSR algorithm on two bi-connected networks and show that the required redundancies are comparable to similar tri-connected networks.

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