Commitment to "Forbidden Questions" in Quantum Phenomena Requires a Philosophical Stand

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The theory of quantum mechanics, as formulated by the Copenhagen school, has been controversial since its inception. Heisenberg’s uncertainty principle asserts that certain aspects of reality are not simultaneously defined, forbidding certain questions.

Recognition has recently been given to experimentalists who have asked these “forbidden questions”. Aephraim Steinberg at the University of Toronto conducted the double slit experiment using weak measurements to construct average trajectories of particles traveling through both slits. To an adherent of the Copenhagen view of reality, however, these average trajectories will constitute nothing more than a mathematical contrivance. Experiments like these will only prove fruitful if we are willing to reject quantum mechanics’ restrictive philosophical approach.

This paper will isolate the controversial physical postulate of quantum mechanics (the postulate of wave collapse) and the philosophical approach that gave rise to it. This approach reflects an instrumentalist philosophy which claims that science must only account for the results of measurements, and has nothing to say about their underlying causes. Such an approach has put an epistemic moratorium on discovering the causes underlying quantum phenomena.

Notable progress has been made by those who reject this moratorium. Steinberg et al. found the average particle trajectories by rejecting the idea that there is no underlying reality to our

measurements. Bell, more notably, was able to discover details of quantum entanglement by using his concept of “beables” to question the built-in epistemology of quantum mechanics.

Because quantum mechanics does not explicitly define wave collapse and prescribe what causes it and when it is supposed to happen, the theory cannot give explicit solutions to a certain class of experiments. This so called measurement problem is assuaged by Zurek’s theory of decoherence, which has had great success in predicting the results of recent experiments. Despite this, decoherence contains the same philosophical oversights as the original theory; it does not propose, or even address, the issue of the underlying causes for quantum phenomena. While most scientists try to steer clear of such philosophical controversies, underlying causes cannot be discovered without the conviction that it is the job of science to discover them.

TABLE OF CONTENTS

LIST OF FIGURES

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1. Introduction

The theory of quantum mechanics has been controversial since its inception. Contemporaries of the theory like Einstein and Schrödinger saw the apparent contradictions in the concept of superposition (as demonstrated by the famous "Schrödinger's cat" example). In the history of physics, controversial issues are typically resolved through further physical discovery. Such discoveries have been rare in the field of quantum phenomena, but recent work is popularizing the will to question long standing assumptions in the field. If this new work is to reach its full potential, physicists exploring its implications must consider philosophical issues that have been present in quantum theory from its start.

2. Asking "Forbidden Questions"

The top breakthrough Physicsworld.com named for 2011 involved the use of weak measurements to probe into the mysterious quantum world [26]. Weak measurements allow experimenters to make measurements on a quantum observable without appreciably damaging the state of complementary observables. Because of the uncertainty principle, this comes at the cost of inaccuracy in the weak measurement.

The award went to Aephraim Steinberg of the Center for Quantum Information and Quantum Control at the University of Toronto. Steinberg did weak measurements on a version of the double slit experiment to obtain measurements of both the position and momentum of photons. This was accomplished by weakly coupling each photon's transverse momentum to its polarization, then a precise, destructive measurement of position is made by a CCD. Using the imprecise momentum information together with the precise position information acquired at the

CCD, researchers were able to average over multiple trials to obtain the average trajectories of individual photons penetrating the two slits [27].

The assertion that these average trajectories have physical meaning flies directly in the face of quantum mechanics, which holds that there is no such thing as trajectory. The trajectories found are in fact consistent with trajectories predicted by the de Broglie/Bohm "pilot wave" formulation of quantum phenomena [10]. Does this mean we are on our way to a new, uncompromised understanding of the quantum world, an understanding unsullied by fundamental uncertainty? Not quite yet. An advocate of what has been become known as the "Copenhagen interpretation" would be unimpressed by these results, maintaining that these average trajectories are no more than a mathematical contrivance, that the trajectories are merely being inferred and not measured directly, and since there is no direct observation of trajectory, there is no proof that these results are anything more than a fantasy that matches a few data points. Steinberg admits "our work in no way modifies the well known predictions of quantum mechanics." However, he does say that his work will "push [physicists] to change how they think about things."

This paper will identify the philosophical approach contained in quantum mechanics and argue that some who have made progress, such as Steinberg, and most notably Bell [4], have done so by substituting this instrumentalist approach with one that is mindful of underlying causes.

3. What is Philosophy?

The branch of philosophy that mainly concerns us in this paper is epistemology. Epistemology is the field that asks what the proper method of acquiring knowledge is. An epistemic question salient to quantum phenomena is whether science is just a way of devising convenient descriptions of appearances, or if it is actually possible to identify causal processes that make things work the way they do. The answers a scientist gives to these questions, whether he has conscious knowledge of such philosophical issues or not, will determine what kinds of theories he can produce.

As an example of epistemology's physical importance, imagine a time when the theory of epicycles and heliocentric models were being considered (before Galileo observed the phases of Venus). One who holds the epistemic notion that theories can only be descriptions of appearances might say that the two theories are equally good, since they reproduce the exact same set of observations. One who holds that science is about identifying causes would prefer the heliocentric theory because it identifies the sun as the cause of planetary motions by placing it at common foci of every orbital ellipse [19]. One with this epistemic view might perhaps still be skeptical of Kepler's theory, but he would regard it as possibly correct, and would seek further confirmation by applying the causal connections the theory proposes to different situations. This astronomer would also reject the theory of epicycles *on philosophical grounds*, on the grounds that it is only meant as a model to fit observation and the assumption that the earth is at the center of the universe. Averroes, one such scholar who doubted epicycles on philosophical grounds before a heliocentric model was even proposed said "Ptolemy was unable to see Astronomy on its true foundations…We must, therefore, apply ourselves to a new investigation concerning that genuine astronomy whose foundations are principles of Physics."[19]. By "principles of physics," I take Averroes to mean "physical causes." When applied to quantum mechanics, it is exactly this kind of thinking that is needed when asking the kinds of "forbidden questions" Steinberg speaks of.

The next section will demonstrate the philosophical attitudes implicit in quantum mechanics. It should be noted that because I am talking about different theories pertaining to quantum phenomena, when I say quantum mechanics, I am talking about what is commonly called the "Copenhagen interpretation". I would contend that the very notion of a "physical interpretation" is improper. This concept implies that there is a mathematical formalism that describes appearances, and then after the fact we tell a story of what is physically happening. Although this is in fact the epistemic angle taken by many physicists, including the makers of quantum mechanics, I do not want to take that angle as the given. If it is the case that the purpose of science is to identify the causal processes behind our observations, then *differing "interpretations" constitute different theories*.

4. The Two Parts of Quantum Mechanics

Another note on my approach: when I refer to quantum mechanics, I am talking about the way it is currently formulated in many textbooks [14], [17], as opposed to the esoteric and often contradictory "interpretations" of Bohr or other specific contributors. I seek to treat quantum mechanics as one integrated set of ideas and identify its basic epistemic approach.

To help identify the basic epistemic approach of quantum mechanics I will briefly summarize quantum mechanics as consisting of two parts; the wave mechanics, and the postulate of wave collapse. The former is a correct theory based on observation; the later is a postulate with no physical support or precise definition. To draw a clear line between wave mechanics and the collapse postulate, I will start by summarizing wave mechanics, a theory that draws its observational support essentially from crystal diffraction of electrons and the photoelectric effect.

The photoelectric effect gave a solid indication that light has a particle nature. Photoelectrons are observed when light is incident on a piece of metal. If light is treated as energy spread out in a continuous field, a field of low energy would be able to free an electron from metal if only given enough time. In addition, one way to lessen this time lag, or to increase the speed of photoelectrons, would be to increase the intensity of the beam. No time lag for low frequencies is observed in such experiments, and an increase in intensity will only increase the number of photoelectrons, not their kinetic energy. Einstein's explanation of the photoelectric effect required that energy from light arrive in discrete packets of energy $h\upsilon$. This would explain why only light above a given frequency would produce photoelectrons, and why an increase in intensity (just an increase in the number of quanta, not the energy of any given quanta) will not change whether or not photoelectrons are produced. [14]

By comparing the extremum of action principle that applied to electrons in the Bohr atom to Fermat's principle that applied to minimization of time for traveling light, de Broglie realized that if these were merely two instances of the same principle, then $p = h/\lambda$ [9]. This was soon tested by Davisson and Germer by firing electrons into a crystal [11]. Interference patterns analogous to those seen in the Bragg diffraction of X-rays are observed, showing that somehow, matter in motion has some kind of wave-like property.

These two experiments demand that we accept matter and light have a particle and wavelike nature, or that they are particles that somehow propagate in a wavelike pattern. These waves must satisfy the following properties.[1]

(4.1) $\lambda = h/p$ and $\upsilon = E/h$ (Einstein and deBroglie)

---

[1] The following is an adaptation of the plausibility argument for the Schrodinger Equation found in Eisberg and Resnick's undergraduate quantum physics text [11].

(4.2) $E = \frac{p^2}{2m} + V$ (energy of a particle)

The potential energy V is based on physical phenomena that are already understood.

By substituting (4.1) into (4.2) we get

(4.3) $\frac{h^2}{2m\lambda^2} + V = h\upsilon.$

We then introduce the more convenient quantities

(4.4) $k = 2\pi/\lambda$, $\omega = 2\pi\upsilon$ and $\hbar = h/2\pi.$

We can now write (4.2) as

(4.5) $\frac{\hbar^2 k^2}{2m} + V = \hbar\omega.$

Wave functions can solve these equations so long as they are solutions to the right differential equation. Since any wave function, which must be square integrable, can be written as a fourier series.

(4.6) $\psi_{(x,t)} = \int\int \mathrm{a}_{\mathrm{k}\omega}\, \mathrm{e}^{i(kx-\omega t)}\, dk\, d\omega$

Summing over all frequencies ω and wave numbers k for particular values of $\mathrm{a}_{\mathrm{k}\omega}$. We can write ψ as a solution to a differential equation. The differential equation with ψ as its solution that satisfies (4.5) for each normal mode of (4.6) is the Schrödinger equation:

(4.7) $\frac{-\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi = i\hbar\frac{\partial\psi}{\partial t}.$

It is the two principles contained in (4.1) and these two principles alone that constitute the conceptual foundation of wave mechanics. While it has proven to be empirically correct in every case, the wave mechanics is nothing more than a well developed formalism to solve a differential equation which assumes these two principles hold in all cases. Identifying these two phenomena as the basic experimental foundation of the wave mechanics serves to indicate that the wave mechanics, while possessing enormous predictive power, identifies no explicit causal connections further than quantization of energy and the connection between momentum and the de Broglie wavelength.

The wave mechanics, found in the Schrödinger equation, are not the sum of the basic laws of quantum mechanics. There is in fact one extra rule (for which there is no explicit experimental support or description). When some aspect of a particle is measured (position, angular momentum, etc.), it is never measured as a wave, only as a particle. As the Hitachi lab's double slit experiment shows [33], the particles do arrive in a wavelike probability distribution, but they arrive, individually as particles (fig. 1).

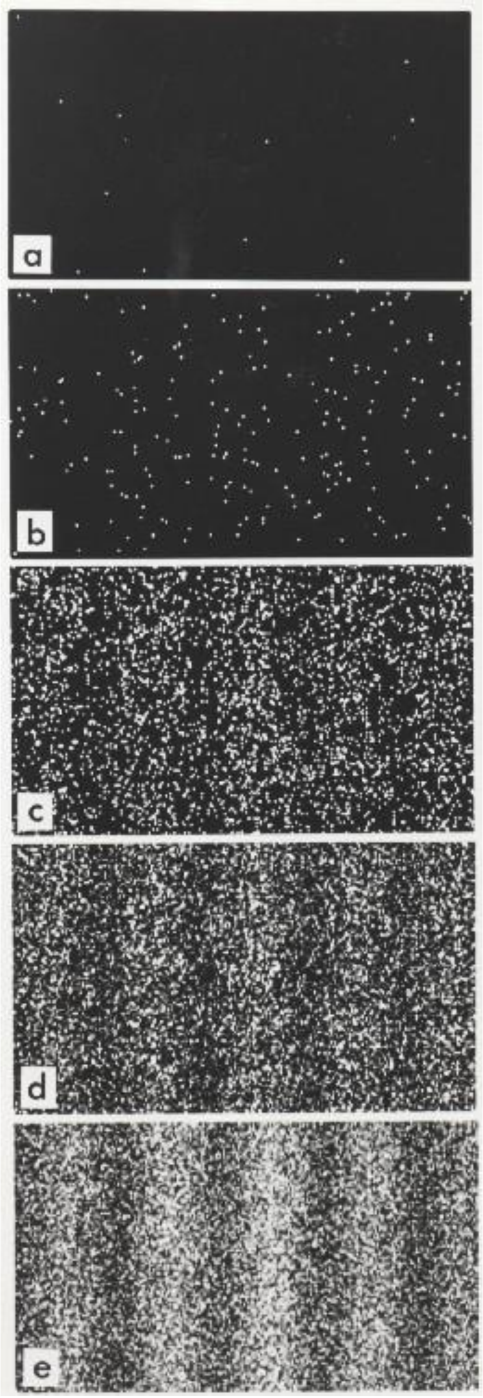


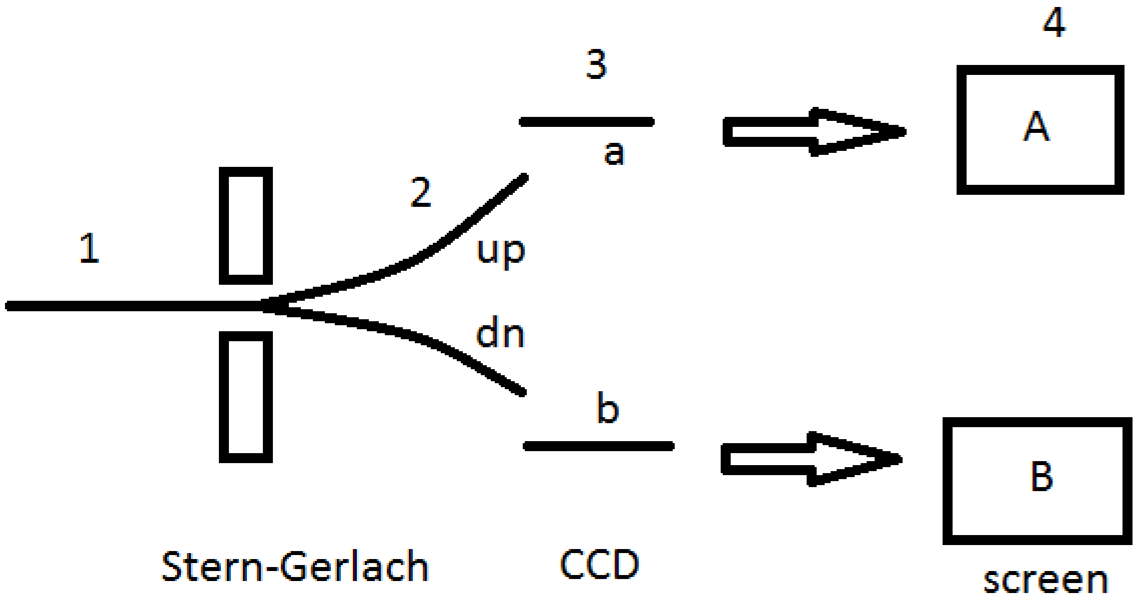
Figure 1: The slow evolution of the single electron double-slit experiment. It can be seen that although electrons arrive in an interference wave pattern, they arrive individually as particles. [33]

Quantum mechanics accounts for this by saying that when a wave state is measured, it "collapses" to an eigenstate of whatever observable was just measured. This is the problem of measurement, or “wave collapse”. No indication of what collapse physically entails is given and the process by which a given eigenvalue is selected during any particular measurement is apparently irreducibly random; having no physical cause whatever.

Just as Ptolemy invented epicycles to preserve a geocentric picture of the universe, quantum mechanics added on the postulate of wave collapse to bridge the chasm between wave mechanics and what is actually measured. Both theories postulate some kind of unprecedented physical process with no known cause. As a result, both theories account for known observations, but are uninterested, and have often closed discussion on, the subject of their underlying cause.

5. The Measurement problem

The philosophical flaws with the measurement postulate inevitably lead to physical problems. One of the most glaring problems with the postulate of wave collapse is that it is not at all clear when it is supposed to occur. Take the following example in Figure 2.



(1) $\psi = \frac{1}{\sqrt{2}}(|\uparrow> +|\downarrow>)$

(2) $\psi = \frac{1}{\sqrt{2}}(|\uparrow> |up> +|\downarrow> |dn>)$

(3) $\psi = \frac{1}{\sqrt{2}}(|\uparrow> |up> |a> +|\downarrow> |dn> |b>)$

(4) $\psi = \frac{1}{\sqrt{2}}(|\uparrow> |up> |a> |A> +|\downarrow> |dn> |b> |B>)$

Figure 2: Quantum mechanics contains no physical law describing which interaction; (2) the Stern-Gerlach machine, (3) the CCD or (4) the read-out, causes wave collapse.

Say that a spin ½ particle approaches a Stern-Gerlach apparatus in state (1). If the particle collapses to the up state it will be launched in one direction and if it collapses into the down state it will be launched in the opposite direction. But what if the wave function does not collapse? The particle will now not only be in a superposition of spin states, but also a superposition of velocity states (2). One would think that the wave function surely collapses when the particle is detected by a CCD, but there is nothing that prevents us from saying that the particle is in a superposition of crashing into two different CCDs at once (3). This superposition can apply in principle to the output on a computer screen (4), or further, to be truly perverse, you could describe the mind of the experimenter as being in a superposition having seen both A and B at once after he has looked at the screen.

Using an adaptation of a thought experiment by Bell [4], [7], I will demonstrate that the collapse postulate leaves holes in the predictive power of traditional quantum mechanics.

Imagine the following experimental setup in figure 3 below.

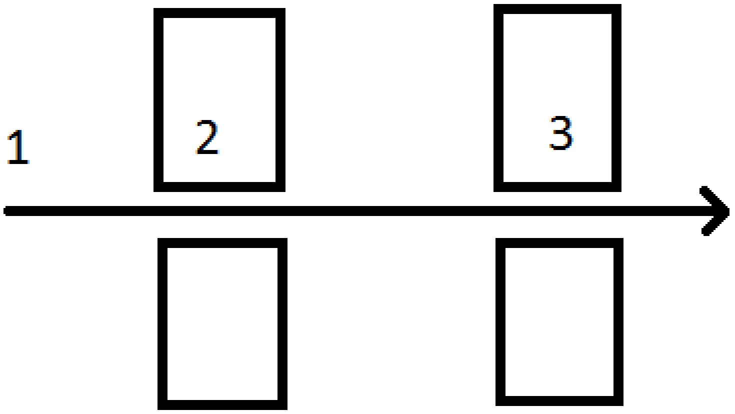


Figure 3: A spin 1/2 particle in a superposition of states undergoes an interaction (2) and a measurement (3) in an orthogonal basis. The final result depends on whether (2) counts as a measurement or not.

Imagine a spin ½ at (1) particle starts out in state

(5.1) $$\psi_1 = \alpha|\uparrow> \ +\beta|\downarrow>$$

Where $\alpha^2 + \beta^2 = 1$ and α and β are real.

An interaction of some kind then happens at (2). If this interaction counts as a measurement, the particle will collapse into either the up or down state. If it does not, it will continue to be in a superposition of two spin states.

The final interaction (3) is a measurement in an orthogonal basis, whose eigenvectors are:

(5.2) $$|X \uparrow> \equiv \frac{1}{\sqrt{2}}(|\uparrow> + |\downarrow>)$$

(5.3) $$|X \downarrow> \equiv \frac{1}{\sqrt{2}}(|\uparrow> - |\downarrow>)$$

The outcome of this measurement is dependent upon whether the first interaction, (2) counted as a measurement. If it did, the state of the particle after (2) would be either

(5.4) $$\psi_2 = |\uparrow> = \frac{1}{\sqrt{2}}|X \uparrow> + \frac{1}{\sqrt{2}}|X \downarrow>$$ with probability α^2. Or

(5.5) $$\psi_2 = |\downarrow> = \frac{1}{\sqrt{2}}|X \uparrow> - \frac{1}{\sqrt{2}}|X \downarrow>$$ with probability β^2.

It follows that the probability of measuring $|X \uparrow>$ during the second measurement is

(5.6) $$\frac{1}{2}(\alpha^2 + \beta^2)$$

On the other hand, if (2) does not count as a measurement, the state is still

(5.7) $$\psi_2 = \alpha|\uparrow> + \beta|\downarrow>.$$

Which, in the orthogonal basis is

(5.8) $$\psi_2 = \frac{\alpha+\beta}{\sqrt{2}}|X \uparrow> + \frac{\alpha-\beta}{\sqrt{2}}|X \downarrow>.$$

In this case it follows that the probability of measuring $|X \uparrow>$ during the second measurement is

(5.9) $$\frac{1}{2}(\alpha^2 + 2\alpha\beta + \beta^2).$$

Without an explicit formulation of what kind of interaction at (2) constitutes a state collapsing "measurement" and what does not, quantum mechanics does not give an outcome for this simple experiment.

Because there is no physical description of what sets a "measurement" apart from a mere coupling interaction, we have no way of knowing how the system will evolve if we put some interacting device in the place of the first measurement. Say for example, in the place of the first interaction we place a Stern-Gerlach device which splits the beam of spin ½ particles in series with a device that recombines the beam. If we used such a device, quantum mechanics gives no explicit indication of what the final result would be. Experimenters have developed good intuitions about which kind of interactions will “cause collapse” and which will not, but an intuition is not a theory. Mankind had very good intuitions about gravity and inertia before Newton (so good that they could build structures like the Coliseum), but a real scientific theory of gravity and inertia requires explicit mathematical equations based on known cause and effect relationships. Quantum mechanics tells us that the cause and effect relationships of measurement are epistemically off limits, putting a vague postulate of wave collapse in their place.

To discover these cause and effect relationships, instead of merely predicting observations, a new formalism will ultimately have to be devised that rejects the philosophical underpinning that science deals only with observables. Only then will progress on this subject be possible.

6. Bell Asks “Forbidden Questions”

This is exactly the philosophy Bell applied to his work. By rejecting the philosophic standards implicit in quantum mechanics and using his own instead, Bell has allowed us to glimpse the boundary of a non-local world. It is only by following the same epistemic standards that Bell did that further progress will be possible.

The epistemic concept Bell applied is not a very new or complicated one, but in this context it bears a very peculiar name. Bell believed in beables. “Beable” is pronounced like the word “observable” only the “observ(e)” is replaced with “be”. The structure of this intentionally silly word contains its own meaning. A beable is a real attribute of a quantum system that is responsible for the observations that we make [31]. Although Bell did not trumpet this, “beable” just means “underlying cause”, the very thing that quantum mechanics’ epistemology ignores, denies, or remains agnostic about, as demonstrated above. We have grown used to the term “observable” because quantum mechanics forbids discussion of “beables”. The construction of the wording simultaneously defines Bell’s key philosophical concept and mocks the sort of terminology quantum mechanics has saddled us with.

Bell’s inequality uses a very similar setup to the EPR experiment. A source is made to emit two particles, each of them of spin one half. These particles are in the singlet state. The particles are launched in opposite directions, each toward a Stern-Gerlach measuring device. If these devices are both oriented in the same way, a measurement of the spin of one particle will indirectly reveal the spin of the other, since they must have opposite spins with respect to some chosen axis.

Bell devised a way to determine if this joint singlet state that describes the two particles has a local beable that is responsible for its observable properties [4], [5]. This beable would constitute some real attribute of the two particles that was endowed to them by their common source which makes them return the measurements that they do. Because this beable is not part of the quantum mechanics theory, it would constitute what has been come to be called a “local hidden variable”. It is important to reiterate that this entire line of investigation is only possible once one embraces the idea of beables, which is made possible only by rejecting the epistemology implicit in quantum mechanics that beables are not the business of science.

If the two Stern-Gerlach machines are allowed to rotate, so that they may measure the spin along whatever axis perpendicular to the particle’s path that they wish, the particles will no longer necessarily return opposite spins (since they are being measured from different angles).

If the particle pair is endowed with specific qualities (beables) that determine the probabilities of certain properties being measured about them, then the measured spins should follow logically consistent rules [16]. One of such rules is that the measurement by one of the Stern-Gerlach machines, A, will depend only on the angle the machine has been rotated, a, and the beable λ, endowed to the particles at the source. The same will be true for the other machine, B. The measurement each machine makes will not depend on the result of the other. Therefore

$$A(a,\lambda) = \pm 1 \quad and\, B(b,\lambda) = \pm 1. \tag{6.1}$$

The beable λ, which affects the probability measurements for the spins of the particles, satisfies the following integral:

$$\int \rho(\lambda) d\lambda = 1. \tag{6.2}$$

For an integral over all possible values of λ. Where ρ is the probability distribution of λ. Notice that even though we do not know ρ, or anything about λ, this equation still must hold. The expectation value for the product of A and B can be found using

$$\int A(a,\lambda)B(b,\lambda)\rho(\lambda)d\lambda =< A(a)B(b) >. \tag{6.3}$$

We can write down the following equation, introducing some other angle, c.

$$< A(a)B(b) > -< A(a)B(c) >= \int [A(a,\lambda)B(b,\lambda) - A(a,\lambda)B(c,\lambda)]\rho(\lambda)d\lambda. \tag{6.4}$$

Using the result of the traditional EPR experiment we can say that

$$A(\theta,\lambda) = -B(\theta,\lambda) \tag{6.5}$$

and rewrite the integral like so

$$< A(a)B(b) > -< A(a)B(c) >= -\int [A(a,\lambda)A(b,\lambda) - A(a,\lambda)A(c,\lambda)]\rho(\lambda)d\lambda. \tag{6.6}$$

We then use the fact that

$$A(a,\lambda)A(a,\lambda) = 1 \tag{6.7}$$

to rewrite the integral as

$$< A(a)B(b) > -< A(a)B(c) >= -\int [1 - A(b,\lambda)A(c,\lambda)]A(a,\lambda)A(b,\lambda)\rho(\lambda)d\lambda. \tag{6.8}$$

We take the absolute value of both sides:

(6.9) $$|< A(a)B(b) > - < A(a)B(c) >| = |\int [1 - A(b,\lambda)A(c,\lambda)]A(a,\lambda)A(b,\lambda)\rho(\lambda)d\lambda.|$$

We can then use the fact that

(6.10) $$A(a,\lambda)A(b,\lambda) = \pm 1$$

To drop $A(a,\lambda)A(b,\lambda)$ from the integral and write the following inequality:

(6.11) $$|< A(a)B(b) > - < A(a)B(c) >| \leq |\int [1 - A(b,\lambda)A(c,\lambda)]\rho(\lambda)d\lambda.|$$

This is true since the factor $A(a,\lambda)A(b,\lambda)$ inside the integral can only make the result of the integral smaller or the same, depending on the distribution of $\rho(\lambda)$. We can now use $A(\theta,\lambda) = -B(\theta,\lambda)$ and evaluate the integral to get a Bell inequality.

(6.12) $$|< A(a)B(b) > - < A(a)B(c) >| \leq (1 + < A(b)B(c) >)$$

Keep in mind that even though we have just done a bunch of math, this is just the result of enforcing common sense rules on a beable endowed to the particles at their source, and the spin measurements of two Stern-Gerlach machines, $A(a)$ and $B(b)$, whose outcomes are not dependent on one another's orientations.

Quantum mechanics predicts that

(6.13) $$< A(a)B(b) > = -\cos(a - b)$$

Plugging this prediction into the Bell inequality gives

(6.14) $$|\cos(a - c) - \cos(a - b)| \leq 1 - \cos(a - c)$$

Plugging in a=0°, b=45° and c=90° gives

(6.15) $$0.707 \leq 1 - 0.707$$

A false statement! [16]

Similar inequalities have been experimentally tested by Aspect et al., confirming the quantum mechanical prediction. The inequality is violated even when the two measurements are spacelike separated [3], [8], [18]. Bell was interested in identifying the underlying cause (hidden beable) of quantum phenomena. His equality shows that if such a beable exists, a measurement at one detector must influence the measurement at the other, even if the two measurements are spacelike separated, meaning the beable, if it exists, must be non-local.

It is important to note that since we have no theory identifying and describing this non-local beable, we are unable to control it. In the case of two entangled particles, we can measure a particle, causing it to collapse to a single state, but we are unable to control what state it collapses into, meaning we are unable to control what state its spacelike separated partner collapses into. As a result, if there is a faster than light interaction we are unable to signal with it. It is for this reason that some argue that there is no faster than light causal influence (or "non-local hidden variable"), since there can be no faster than light signaling. This argument does not stand, at least not on its own. Just because we don't have a theory that explains a physical phenomenon, and are therefore unable to manipulate it, does not mean it is fundamentally random.

Another problem non-local beables presents is a violation of temporal causality in relativistic reference frames. Bell, however realized a way around this. In another one of his papers, Bell showed how we can account for what we now call relativistic effects by throwing out special relativity in favor of Lorentz ether theory [2], [4], [6]. In order to do this, we must introduce a privileged reference frame (an ether). In this picture, time dilates and lengths contract for objects in motion to the reference frame. This picture would still reproduce the same time dilations and length contractions. Although Lorentz ether theory produces the same results as relativity, it was rejected in the early 1900s since there was no explicit evidence for an ether. The introduction of a privileged rest frame however, would allow causal influences to travel faster than the speed of light without causal influences traveling backward in time in certain reference frames. Under this model, light would not travel the same speed in all frames; it would just appear to do so due to contraction of measuring devices and time dilation caused by their motion relative to the ether. If this model were actually the case, then the appearance of a causal influence going back in time would just be an illusion caused by the assumption that light travels at the same speed in all reference frames.

Full elaboration of this point would deserve its own paper; I only wish to give a taste of the far reaching implications of Bell's work. By self consciously rejecting the philosophy implicit in quantum mechanics, that we should not be concerned about the underlying causes of what we observe, Bell was able to glimpse through a doorway to what very well could be a non-local world.

7. Decoherence: A Proposed Solution to the Measurement Problem

In section 5 I put forth a simple experiment in which quantum mechanics is unable to make a prediction because of the vague nature of the postulate of collapse. However, an outcome can be correctly predicted by understanding the nature of interaction (2) in figure 3 in terms of a unitary operator which acts on the state of the system and (if the interaction counts as a measurement) couples the evolution of the system to the evolution of the measurement apparatus.

Say that the first measuring device at (2) is represented by a spin ½ particle that starts in the $|\uparrow>$ state. This is just one of many ways one can represent a macroscopic system that couples to a quantum system to make a measurement.

If the interaction at (2) counts as a measurement, the unitary operation representing the interaction will affect both the system and the measurement apparatus, causing them to become entangled. One example of such an interaction would be a unitary operator that causes the measurement apparatus to take on the same spin state as the system.

(7.1) $$\psi_2 = \alpha|\uparrow>|\uparrow> +\beta|\downarrow>|\downarrow>$$

At (3), the state of the system is measured while the state of the first measuring device is not. The entangled system as a whole, which includes the system and the first measuring apparatus, is in a pure state, while the state of the system and the state of the first measuring apparatus considered separately are both in a mixed state. To find the mixed state of just the system on its own we must consider the density matrix of the entire system:

(7.2) $$|\psi_2 >< \psi_2| = \alpha^2|\uparrow>|\uparrow><\uparrow|<\uparrow| + \alpha\beta|\uparrow>|\uparrow><\downarrow|<\downarrow| + \alpha\beta|\downarrow>|\downarrow><\uparrow|<\uparrow| + \beta^2|\downarrow>|\downarrow><\downarrow|<\downarrow|.$$

To find the density matrix of the quantum system alone, we take the trace of the density matrix for the combined system, tracing over the apparatus state. [30]

(7.3) $$Tr_{app}(|\psi_2 >< \psi_2|) =$$

$$|\uparrow\alpha^2 ><\uparrow|<\uparrow|\uparrow> +\alpha\beta|\uparrow><\downarrow|<\downarrow|\uparrow> + \alpha\beta|\downarrow><\uparrow|<\uparrow|\downarrow> + \beta^2|\downarrow><\downarrow|<\downarrow|\downarrow>$$

$$= \left|\psi_3 >< \psi_3\right| = \alpha^2|\uparrow><\uparrow| + \beta^2|\downarrow><\downarrow|$$

This is not a pure, but a mixed state; it represents classical probabilities, not a quantum superposition. Writing it in the orthogonal basis we get

(7.4) $$|\psi_3 >< \psi_3| = \frac{\alpha^2+\beta^2}{2}|X\uparrow>< X\uparrow| + \frac{\alpha^2-\beta^2}{2}|X\uparrow>< X\downarrow| + \frac{\alpha^2-\beta^2}{2}|X\downarrow>< X\uparrow| + \frac{\alpha^2+\beta^2}{2}|X\downarrow>< X\downarrow|$$

To get the probability of measuring the $|X\uparrow>$ state at (3), we use the projection operator $|X\uparrow>< X\uparrow|$ on the density matrix and find the trace of the resulting matrix.

(7.5) $$Tr(|X\uparrow>< X\uparrow\left|\psi_3 >< \psi_3\right|) = Tr(\frac{\alpha^2+\beta^2}{2}|X\uparrow>< X\uparrow| + \frac{\alpha^2-\beta^2}{2}|X\uparrow>< X\downarrow|) = \frac{\alpha^2+\beta^2}{2}$$

On the other hand, if the interaction does not count as a measurement, the unitary operator for the interaction will not evolve the system with the apparatus, giving us

(7.8) $$\psi_2 = (\alpha|\uparrow> +\beta|\downarrow>) \otimes |\downarrow>.$$

Where the first part of the tensor product is the state of the quantum system and the second part is the unaffected state of the measurement apparatus. This tensor product indicates that no coupling interaction has occurred between the measurement apparatus and the system. Taking the trace over the apparatus, we find that the system on its own is still in a pure state.

(7.9) $Tr_{app}(|\psi_2 >< \psi_2|) = Tr_{app}(\alpha^2| \uparrow >< \uparrow | + \alpha\beta| \uparrow >< \downarrow | + \alpha\beta| \downarrow >< \uparrow | + \beta^2| \downarrow >< \downarrow | \otimes | \downarrow >< \downarrow |) =$

$|\psi_3 >< \psi_3| = \alpha^2| \uparrow >< \uparrow | + \alpha\beta| \uparrow >< \downarrow | + \alpha\beta| \downarrow >< \uparrow | + \beta^2| \downarrow >< \downarrow |.$

Which is the pure state

(7.10) $$\psi_3 = \alpha|\uparrow > + \beta| \downarrow > = \frac{\alpha+\beta}{\sqrt{2}}|X \uparrow > + \frac{\alpha-\beta}{\sqrt{2}}|X \downarrow >.$$

Giving us a probability for measuring $|X \uparrow >$ that includes the interference term αβ.

(7.11) $$Tr(|X \uparrow >< X \uparrow |\psi_3 >< \psi_3|) = \frac{1}{2}(\alpha^2 + 2\alpha\beta + \beta^2)$$

It is also possible for (2) to be the sort of interaction which partially entangles the two systems, but not completely.

(7.12) $$\psi_2 = \delta[\alpha| \uparrow > | \uparrow > + \beta| \downarrow > | \downarrow >] + \gamma[(\alpha| \uparrow > + \beta| \downarrow >) \otimes | \downarrow >]$$

Where $\gamma^2+\delta^2=1$ and γ and δ are real. Where δ can be thought of as the extent of the entanglement achieved by the unitary operator at (2). The density matrix for the combined system is therefore:

(7.13) $$|\psi_2 >< \psi_2| =$$

$\delta^2[\alpha^2| \uparrow > | \uparrow >< \uparrow | < \uparrow | + \alpha\beta| \uparrow > | \uparrow >< \downarrow | < \downarrow | + \alpha\beta| \downarrow > | \downarrow >< \uparrow | < \uparrow | + \beta^2| \downarrow > | \downarrow >< \downarrow | < \downarrow |] +$

$\gamma^2[\alpha^2| \uparrow >< \uparrow | + \alpha\beta| \uparrow >< \downarrow | + \alpha\beta| \downarrow >< \uparrow | + \beta^2| \downarrow >< \downarrow | \otimes | \downarrow >< \downarrow |].$

We again carry out a measurement of the system by tracing over the state of the measuring apparatus.

(7.14) $$Tr_{app}(|\psi_2 >< \psi_2|) =$$

$\delta^2[\alpha^2| \uparrow >< \uparrow | < \uparrow | \uparrow > + \alpha\beta| \uparrow >< \downarrow | < \downarrow | \uparrow > + \alpha\beta| \downarrow >< \uparrow | < \uparrow | \downarrow > + \beta^2| \downarrow >< \downarrow | < \downarrow | \downarrow >] +$

$\gamma^2[\alpha^2| \uparrow >< \uparrow | + \alpha\beta| \uparrow >< \downarrow | + \alpha\beta| \downarrow >< \uparrow | + \beta^2| \downarrow >< \downarrow |] =$

$|\psi_3 >< \psi_3| =$

$\delta^2[\alpha^2| \uparrow >< \uparrow | + \beta^2| \downarrow >< \downarrow |] + \gamma^2[\alpha^2| \uparrow >< \uparrow | + \alpha\beta| \uparrow >< \downarrow | + \alpha\beta| \downarrow >< \uparrow | + \beta^2| \downarrow >< \downarrow |] =$

$$\delta^2[\frac{\alpha^2+\beta^2}{2}|X\uparrow><X\uparrow| + \frac{\alpha^2-\beta^2}{2}|X\uparrow><X\downarrow| + \frac{\alpha^2-\beta^2}{2}|X\downarrow><X\uparrow| + \frac{\alpha^2+\beta^2}{2}|X\downarrow><X\downarrow|] +$$

$$\gamma^2[\frac{\alpha^2+2\alpha\beta+\beta^2}{2}|X\uparrow><X\uparrow| + \frac{\alpha^2-\beta^2}{2}|X\uparrow><X\downarrow| + \frac{\alpha^2-\beta^2}{2}|X\downarrow><X\uparrow| + \frac{\alpha^2-2\alpha\beta+\beta^2}{2}|X\downarrow><X\downarrow|].$$

By simplifying and using $\gamma^2+\delta^2=1$, we find that this density matrix is

(7.15) $|\psi_3><\psi_3| = \frac{\alpha^2+2\alpha\beta\gamma^2+\beta^2}{2}|X\uparrow><X\uparrow| + \frac{\alpha^2-\beta^2}{2}|X\uparrow><X\downarrow| + \frac{\alpha^2-\beta^2}{2}|X\downarrow><X\uparrow| + \frac{\alpha^2-2\alpha\beta\gamma^2+\beta^2}{2}|X\downarrow><X\downarrow|$

To get the probability of measuring the $|X\uparrow>$ state at (3), we use the projection operator $|X\uparrow><X\uparrow|$ on the density matrix and find the trace of the resulting matrix.

(7.16) $Tr(|X\uparrow><X\uparrow|\psi_3><\psi_3|) = Tr(\frac{\alpha^2+2\alpha\beta\gamma^2+\beta^2}{2}|X\uparrow><X\uparrow| + \frac{\alpha^2-\beta^2}{2}|X\uparrow><X\downarrow|) = \frac{\alpha^2+2\alpha\beta\gamma^2+\beta^2}{2}$

The extent of the entanglement, γ^2, will be proportional to the interference term in this simple example: $\alpha\beta\gamma^2$. Interaction processes with a small γ do not significantly reduce the off diagonal terms in the system's density matrix; such interactions are perfect for conducting the "weak measurements" mentioned in section 2 [30].

This process can help us understand an increasingly popular way of assuaging the measurement problem that was proposed by Zurek known as the process of decoherence [34]. Decoherence eliminates the problems associated with the measurement postulate by eliminating the postulate and replacing it with a more thorough description of the interactions between a quantum system and its environment in the same way that I explained the interactions between a quantum system and a measurement apparatus in the preceding example.

A simple way to understand this is to model is the following. Instead of treating a measurement as a "wave collapsing" process, it can be treated as an entangling interaction with another system, as described above. This second is system connected, usually through a series of intermediaries, to some larger macroscopic system that can be discerned with human senses (like a needle on a dial, or pixels on a computer screen). Because the second system is entangled with the state of the quantum system, the state of the second system on its own appears to be in a mixed state (since its state is found by tracing the combined density matrix over the state of the quantum system that was measured). This is completely analogous to how the state of (2) is traced over during the final measurement, putting the quantum system being measured in a mixed state.

Replacing the vague postulate of wave collapse with a more thorough application of the Schrödinger equation has allowed experimentalists to understand complex quantum systems and their interactions with macroscopic systems.

8. Recent Experimental Work

Early discussions on how to understand the quantum world have utilized Gendanken experiments, experiments that were possible in principle, but not possible with the instrumentation available. Within the last decade, many of these experiments are starting to become a reality, allowing researchers to probe the border between the micro and macroscopic worlds, and to test the predictions of decoherence theory.

Interference of Large Objects

Recent experiments have probed the edge of the quantum world by interfering particles of increasingly higher masses. By interfering objects of greater mass and geometric complexity, experimenters are able to probe the kinds of decoherence effects responsible for the difference between quantum and classical behavior.

Recent experiments have achieved interference with C_{60} fullerene molecules [23], [1]. A beam of these molecules is produced by effusion. Fullerenes are sublimated from a source inside of an enclosed chamber. This chamber has one small opening on the opposite end from the sublimation source. This opening has a diameter considerably less than the mean free path of the sublimated fullerenes. This ensures that the fullerenes are ejected from the chamber in a beam, since they will not suffer collisions with other fullerenes as they exit the chamber.

Far field interferometers are ill suited for interference experiments using large molecules because far field experiments require collimation angles much smaller than the diffraction angle of the incoming particles. Because large objects will tend to have smaller de Broglie waves, taking a very narrow portion of this diffraction angle will generate very little interference. The near-field Talbot-Lau interferometer is thus used instead [23].

A Talbot-Lau interferometer is a series of three periodic gratings with identical spacing. Experimenters used a silicon nitride membrane with a period of 100nm [12]. The particle wave diffracts through the first grating, creating a complex interference pattern down range of the grating. This pattern repeats periodically every Talbot length: $L_T = \frac{2d^2}{\lambda_{dB}}$. Where d is the spacing between the slits in the gratings. The second grating is placed a downrange of the first grating at a distance comparable to the Talbot Length. The fringe visibility is then read out by detecting collisions on a third grating.

Attempts to observe the interference of fullerenes have been thwarted by Van der Waals forces between the fullerene molecules and the second grating, causing decoherence of the fullerene wave. This problem can be eliminated by using a Kapitza-Dirac-Lambot-Tau interferometer [29], which substitutes the second grating with a standing light wave which performs the same function without causing a Van der Waals force. The standing light wave transfers momentum to the moving fullerenes only where the probability density wave of the photon interferes constructively with itself. The standing wave is specifically constructed to produce Kapitza-Dirac scattering.

The velocity of the fullerene beam must also be slowed to increase the de Broglie wavelength. This is a tradeoff, however, since the longer the molecule is in flight, the longer other forces, such as gravity, have to divert its trajectory.

Once all these specific influences have been eliminated or accounted for, experiments with large molecules allow us to understand environmental decoherence. When a particle interacts with it environment, it becomes entangled with one of the environment's degrees of freedom, leaving the fullerenes in a mixed state instead like in (7.3). Once all other sources of decoherence are eliminated, the degree of the fullerene's interaction with a given aspect of the environment can be measured.

By varying the temperature of the ejected fullerenes, experimenters are able to probe decoherence effects caused by emission of thermal radiation from the fullerene molecules. At temperatures exceeding 1000K fullerenes radiate in a continuous spectrum similar to a blackbody and start to kick off C_2 molecules and thermal electrons. The fullerenes are allowed to emit some photons in the IR spectrum between the first and third gratings. It was found that the interference visibility decreases with increasing molecular temperature in agreement with the predictions of decoherence theory. The decoherence of these emissions can be understood by tracing the fullerene density matrix over the state of the emitted photon (which is not measured) The result is decoherence caused by thermal emission [1].

Collisional Decoherence

This same experiment has also been used to test decoherence due to collisions. To test the predictive power of the theory of decoherence, a gas was made to fill the space between the second and third gratings to determine the affect collisions will have on the interference pattern of the fullerene wave. The use of a Talbot-Lau interferometer ensured that particles would still be detected after collisions. A far-field interferometer only works for angles much smaller than the scattering angle of the fullerene molecules [24].

Collisions with gas particles in this chamber were found to reduce the visibility of the interference pattern in proportion to the pressure of the gas introduced. The diffracting fullerene molecules interact with the gas, thus entangling the momentum state of the fullerene with the momentum state of the gas particle. The measurement at the third grate is a trace of the combined fullerene/gas density matrix with a trace over the state of the gas particle. The reason we trace over the gas particle in this final measurement is because the position of the fullerene is being recorded while the position of the scattered gas particle is not, thus putting each fullerene that interacts with a gas molecule in at least a semi mixed state—an analogous situation to thermal emission. The experimental results were in complete agreement with the predictions made by decoherence theory.

Test of the Leggett Inequality:

Recent work has confirmed the violation of equalities proposed by Leggett [18], a result which purports to rule out a class of non-local hidden variables. Leggett derived an equality similar to the Bell inequality, but which does not rely on the assumption that the angle chosen by one detector has no influence on the result obtained by the second detector and vice versa. These inequalities are violated by the results of quantum mechanics, allegedly ruling out a large class of non-local hidden variable theories.

To confirm the result of quantum mechanics, a typical entangled photon setup was made, emitting correlated particles using parametric down conversion. Just as in the EPR-B experiment, both the detectors are allowed to be set to random angles with respect to one another. Both detectors are allowed to rotate in the plane that lies perpendicular to the direction of propagation for both photons. However, there is only a select group of angles for which quantum mechanics violates both the Leggett inequalities and the local CHSH inequalities simultaneously. Because of this, each detector must be able to rotate toward the direction of the photon's propagation. The results were in agreement with quantum mechanics, violating the inequality. [18]

Nanomechanical Resonators

Leaps of progress have also recently been made in constructing mesoscopic mechanical resonators. Recent experiments have been able to create resonators of as many as 10^{20} atoms and cool them to roughly 30 energy quanta above the ground state [1]. One current goal is to couple these large resonator states to typical quantum states such as electron spin, Cooper pairs and photons in optical cavities. Using Kerr-type interactions, trapped photon states can be coupled to mechanical resonator states through the collision transfer of photon momentum**.** By entangling a superposition photon state to the mechanical resonator, experimenters are hoping to produce a "cat state" for the mechanical resonator. "Cat state" is in reference to Schrödinger's cat; it means that the resonator is in a superposition of multiple non-overlapping amplitude states. These sorts of superposition states will not be achieved until certain challenges have been overcome. First, the coupling rate must be much faster than the decoherence rate that the trapped photon and the resonator have with their environments. The thermal decoherence rate of the mechanical resonator can be reduced by cooling the environment with a dilution refrigerator, since the rate of decoherence is proportional to temperature, and by maximizing the mechanical quality factor of the resonator, since the rate of decoherence is inversely proportional to it. The additional application of laser cooling to the mechanical resonator allows experimenters to eliminate thermally mixed states, allowing the resonator to settle into a pure superposition state. Once this is achieved, measuring the mechanical superposition state will be a challenge on its own since the resonator decoheres at a rate that is proportional to the square of the separation of states, making cat states especially hard to confirm.

9. Conclusion

Bell's primary concern with quantum mechanics is that the definition of measurement was vague. Although the vagueness of the measurement problem is solved by decoherence, and the predictions of decoherence have had impressive predictive power so far, the basic physical forbidden question remains unanswered: what is the underlying cause of quantum phenomena? The philosophical approach that prevents quantum mechanics from answering this is present in decoherence theory too; it implies that there are no underlying causes of quantum phenomena.

Decoherence eliminates the measurement problem by showing how quantum uncertainty turns into classical uncertainty during a measurement interaction. To recapitulate how this works: the system and the measurement apparatus are entangled, together forming a pure state; therefore the measurement apparatus and the system, each considered on its own, is in a classical probability state which accounts for the random results of any single measurement as well as the evolution of the system into the single measured eigenstate. Through this process, measurement interactions convert quantum probabilities into classical probabilities. There are two differences between classical probability distributions and quantum probability waves. First, a classical probability is the result of averaging over some unknown causal factor, while a quantum probability is supposed to represent a fundamental randomness in the nature of reality. In addition, a quantum probability wave, being a wave, is capable of producing interference as seen in (5.9); a classical probability distribution is not. Because the classical probabilities seen in one half of an entangled system do not show interference effects, it might be that they are classical probabilities in the sense that they are the result of our ignorance about some underlying causal factor. Such an approach comes from the philosophical perspective that science should primarily seek to discover causes of physical events instead of just devising systems to predict these events.

Aristotle classified freefall as a kind of “natural motion”. Instead of searching for an underlying cause for this motion, he took freefall as a primary fact, requiring no further explanation, and categorized it as one of many primary facts he called “natural motion”. An epistemology that is satisfied with coherence theory alone commits this same error; it assumes that the probability distribution found in the mixed state of half of an entangled system is a primary fact, requiring no causal explanation for the random results.

Commitment to forbidden questions really means a philosophical conviction that physical processes have causes and that the job of science is to find them. As successful as decoherence theory has been thus far, no new physical theory on its own will allow scientists to find the causal processes responsible for quantum phenomena if they don’t possess the conviction that they exist and are discoverable. A fundamental understanding of quantum phenomena will only be possible if scientists conscientiously arm themselves with that conviction.

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