APPLYING MATH ONTO MECHANISM: INVESTIGATING THE RELATIONSHIP BETWEEN MECHANISTIC AND MATHEMATICAL UNDERSTANDING

by

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Physical manipulatives are commonly used to improve mathematical understanding. However, it is unclear when physical manipulatives lead to significant benefits. We investigated whether understanding the mechanism of a manipulative would affect mathematical use and understanding. Participants were asked to navigate a physical robot through a maze, and to create a strategy that could navigate differently sized robots through the same maze. Participants with a better understanding of the robot's mechanism were more likely to utilize complex mathematical strategies during the maze task than participants with lower mechanistic understanding. These participants with higher mechanistic understanding also showed greater understanding of the mathematical relationships within the robot. The study provides evidence for a relationship between mechanistic understanding and mathematical understanding, suggesting that mechanistic manipulatives, upon which mathematics can be applied, may be especially beneficial for fostering mathematical understanding.

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1.0 INTRODUCTION

As mathematical skills become increasingly important for success in today's world, an increasing amount of effort has been dedicated to finding effective strategies that promote mathematical understanding. One popular strategy is the use of physical manipulatives (e.g., Gravemeijer, 2002; Hiebert et al., 1997; NCTM, 2000), which are thought to facilitate understanding by grounding abstract mathematical concepts onto concrete experiences (Bruner, 1966).

Physical manipulatives provide several unique affordances, including sensorimotor interactions and experience with physical artifacts. However, it remains unclear when these affordances lead to learning increases, and which types of manipulatives provide the most benefit. Many studies have found positive learning effects from using physical manipulatives (e.g., Cass, Cates, & Smith, 2003; Martin & Schwartz, 2005), but many have found no benefit, or even adverse effects, when using physical manipulatives (e.g., McNeil, Uttal, Jarvin, & Sternberg, 2009) In their review of 23 studies, Suydam and Higgins (1977) found 11 studies where physical manipulatives had a positive effect, 2 studies where manipulatives had a negative effect, and 10 studies that showed no effect. Similarly, Sowell (1989) found a large range of results when comparing physical manipulative instruction to other instructional types, ranging from negative to positive effect sizes, though she concluded that mathematics achievement could be improved through manipulative use. On a more practical level, physical manipulatives come with a number of limitations, including physical space, set-up and clean-up time, and availability of manipulatives, which may impede the use of physical manipulatives in classrooms. A common workaround for these physical limitations is the introduction of virtual learning environments; however, virtual environments may lose unique physical learning benefits and decrease the effectiveness of mathematics education.

One way to alleviate this problem is to determine the cognitive mechanisms that underlie physical learning benefits, so that these mechanisms can be integrated into other educational media. Furthermore, understanding the cognitive mechanisms may provide insight into the types of learning situations for which manipulatives are most beneficial. The current study aimed to investigate one cognitive mechanism that may mediate physical manipulative benefits: more detailed or flexible mental representations for generating mathematical functions caused by higher mechanism understanding.

1.1 INCREASING MATHEMATICAL UNDERSTANDING VIA MECHANISTIC UNDERSTANDING

Mathematics involves the discovery and understanding of patterns. The ability to recognize and understand these mathematical regularities can lead to the discovery of mathematical relationships. Mechanisms serve a similar purpose. Defined by Machamer, Darden, and Craver (2000) as “entities and activities organized such that they are productive of regular changes from start or set-up to finish or termination conditions” (“activities” being producers of change, and “entities” being those that carry out activities), mechanisms describe the processes that lead to regular phenomena. Mechanisms can also be abstracted into “mechanism schemas” (Machamer et al., 2000), abridged mechanism descriptions that can be filled with specific descriptions of entities and activities depending on the situation. These schemas support the discovery of strategies and regularities: a person can formulate a schema based on their hypothesis of how entities and activities work within a mechanism, and then test whether their proposed mechanism leads to the hypothesized outcome. If the schema fails, then the person can revise their schema to more accurately predict the observed outcome. If the schema succeeds, then that schema can be used to explain regular outcomes across several related situations (e.g., through Peirce's theory of abduction; see Hartshorne & Weiss, 1935, Burks, 1958). In other words, mechanism schemas provide an understanding of how a system consistently works over many different circumstances.

Mathematics can then be applied to these mechanisms to describe the regularities that occur in a given phenomenon, simultaneously providing a perceptual basis for abstract mathematical patterns and emphasizing the consistency of such patterns. Students may be able to connect the regularities that are inherent in both mechanism and mathematics: mechanistic understanding allows students to recognize the relations that exist among a series of entities; they can then map and analogize those relations onto mathematical situations to understand how quantities relate to and change each other (see Gentner, 1983). For example, consider a phenomenon whose mechanism can be explained as, "Entity 1 is directly connected to Entity 2. When Entity 1 spins, this causes Entity 2 to spin." After examining this mechanism, the student may come to understand that Entity 1 leads to regular changes in Entity 2 (that is, if the number of spins of Entity 1 changes, the number of spins of Entity 2 also changes in a constant way).This could then be used as a basis for understanding proportional reasoning, which quantitatively describes a particular form of regular changes between two numbers. Furthermore, students may come to recognize that proportional reasoning can be applied in situations in which regular changes occur between entities, allowing them to recognize the applicability of mathematics across representationally similar situations. Thus, we predict that increasing students' understanding of a mechanism will a) lead to increases in mathematical understanding of the mechanism, and b) increase their likelihood of applying the appropriate math to situations involving that mechanism.

1.2 PHYSICAL MANIPULATIVES AND THE DISCOVERY OF MECHANISMS

Physical manipulatives are almost always manipulated with one's hands. Research has shown that people show heightened attention, slower visual search rates, greater visual memory, and enhanced cognitive control for objects in hand space than for objects away from this space (Abrams, Davoli, Du, Knapp, & Paull, 2008; Reed, Grubb, & Steele, 2006; Schendel & Robertson, 2004; Tseng & Bridgeman, 2011 Weidler & Abrams, 2012). In particular, the hands focus attention on objects' details. For example, Davoli, Brockmole, and Goujon (2012) asked people to visually search geometrical patterns while holding their hands near or far from the stimuli. When visual features and patterns were the same across images, there were no processing differences in relation to the hands; however, when images differed in their colors, then participants with their hands near the stimuli showed decreased performance. These processing differences are thought to be caused by a shift in the use of perception-based parvocellular pathways to the use of action-based magnocellular pathways for objects near the hands (Abrams & Weidler, 2013; Gozli, West, & Pratt, 2012).

When students use physical manipulatives, their attention may be focused toward the manipulatives and their details. Though it seems counterintuitive to focus on details when the goal is to learn abstract mathematical concepts, increased attention may make it more likely for students to recognize and discover the mechanisms involved in the physical manipulative (assuming such a mechanism exists in the manipulative). Students can then learn the relationships that exist between the parts of the mechanism and integrate these relationships into their mental representation of the system, leading to a more detailed or flexible representation (Behr, Lesh, Post, & Silber, 1983; Goldin & Schteingold, 2001) upon which abstract mathematical principles can be applied (see Figure 1).

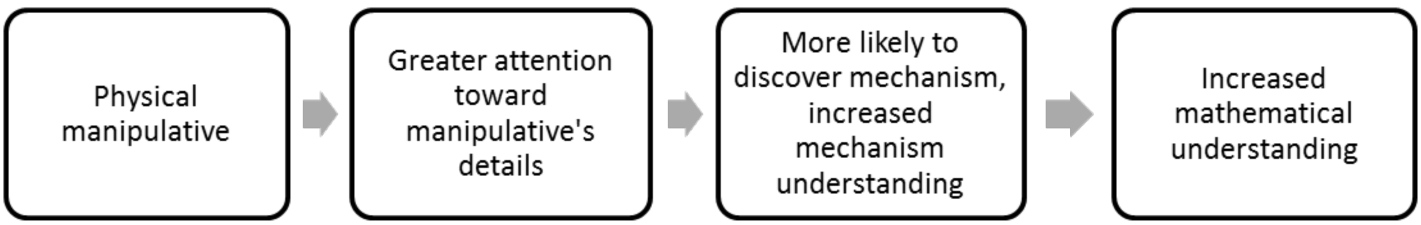

Figure 1. Proposed relationship between physical manipulative use and mathematical understanding.

In previous work, we found evidence that physical manipulatives lead to greater attention to the manipulative’s details. Students who worked with a physical robot during a task were more likely to include and accurately draw the robot’s details from memory than students who worked with a virtual robot during the task (Liu & Schunn, 2013). In the current study, we are investigating the latter half of Figure 1: whether students who understand the manipulative’s mechanism will show greater mathematical understanding and be more likely to propose accurate mathematical relationships.

1.3 EXPERIMENTAL OVERVIEW

To test our proposed relationship between mechanistic understanding and mathematical understanding, we used a robotics task. Robotics has been used to successfully improve mathematics performance (e.g., Nagchaudhuri, Singh, Kaur, & George, 2002; Petre & Price, 2004; Silk, 2011) and is a rich domain to integrate mathematics with other STEM domains. In addition, robotics can be especially resource-intensive: physical robots require much set-up time, take up much physical space, and can be expensive to buy for classroom use. In response, several virtual robotics environments have been created to address these physical concerns; thus, being able to integrate physical learning affordances into these virtual environments may be especially beneficial for this domain.

The current study used the widely used LEGO NXT robot as its physical manipulative. The robot's mechanism consists of three entities: the robot's program, motors, and wheels. The program (which used a C-based language called ROBOTC; www.robotc.net) consists of commands that tell the robot which direction to move and the number of times to rotate its motors (see Figure 2 for an example). When the program is run through the robots interface, the robot's motors rotate the number of times designated in the commands, which causes the robot's wheels to rotate, which causes the robot to move. Importantly, the parts of the robot that are involved in its mechanism (i.e., the motor rotations and wheel rotations) are also proportionally related (i.e., one motor rotation will equal the same number of wheel rotations, which will equal the same distance traveled by the robot).

```
task main()
{

forward(50); // Robot will move forward and rotate its motor 50 times

backward(20); // Robot will move backward and rotate its motor 20 times

turnRight(10); // Robot will turn right and rotate its motor 10 times

turnLeft(10); // Robot will turn left and rotate its motor 10 times

}
```

Figure 2. Examples of the available commands in the programming task.

We expected that participants who were taught the robot's mechanism (i.e., the causal connection between the robot's motors and wheels) would be more likely to include the mechanism in their mental representation of the robot than participants who were not taught the mechanism. Because the robot's mechanism is also involved in its proportional relationships, participants who understood this mechanism would be more likely to discover and understand the robot's quantitative relationships as well, and would be more likely to utilize math based on these quantitative relationships in tasks involving the robot. To test these hypotheses, we used a maze navigation task: participants were asked to navigate a robot through a maze, and to create a generalizable strategy that could navigate differently sized robots through the same maze. Although non-mathematical strategies (e.g., guessing and checking) could be used to navigate the maze, strategies needed to utilize proportional reasoning to fulfill the strategy portion of the task. Thus, we hypothesized that:

1) Participants with high mechanistic understanding would show greater understanding of the quantitative relationships that exist within the robot than participants with low mechanistic understanding.

2) Participants with high mechanistic understanding would use more frequent and more complex mathematics in their maze navigation strategies than participants with low mechanistic understanding.
3) The participants with greater understanding of the robot's quantitative relationships would be the same participants who use more frequent and more complex mathematizations during the maze navigation task.

Based on our model in Figure 1, all participants should show heightened attention toward the robot, because all participants used a physical robot. However, it is possible that participants with higher mechanistic understanding will attend more to the proportionally relevant parts of the robot than those with lower mechanistic understanding. It is also possible that participants need to visualize the robot and its mechanisms before they can mathematically utilize their mechanistic understanding. Thus, we explored participants' attention toward the robot and their spatial visualization ability as potential factors in the relationship between mechanistic understanding and mathematical use and understanding.

2.0 METHODS

2.1 PARTICIPANTS

Participants consisted of fifty undergraduate students recruited through the University of Pittsburgh Psychology department's subject pool and compensated with course credits. Twenty-four students were randomly assigned to the High Mechanistic condition, and 26 students were assigned to the Low Mechanistic condition. Students majoring in robotics-related or math-heavy majors (i.e., robotics, technology, engineering, mathematics, statistics, physics, chemistry) were not eligible to participate in the study.

2.2 MATERIALS

Mechanism manipulation. Participants were shown two defective, physical robots and asked to predict whether the robot would be able to move forward in a straight line. On the first robot (Figure 3, left), the cord attaching the robot's brick (where the robot's programs and commands are stored) to the robot's motors was disconnected to emphasize the relationship between the robot's commands and wheel movements via motor rotations. The second robot (Figure 3, right) had two mismatched wheels (one large wheel and one small wheel) to emphasize the relationship between the robot's wheel size and movement distance. An experimenter ran each robot to test participants' predictions and show that the robots would not run properly. The experimenter also explained the cause of the robots' errors. After seeing both robots, participants were asked to describe the process through which a robot goes to move forward, starting from the moment a program is downloaded into the robot; if participants' explanations contained errors, the experimenter corrected them before moving on to the next task.

Mechanism Understanding questionnaire. As a manipulation check, the Mechanism Understanding questionnaire consisted of two open-ended questions about how the robot functioned: "Please explain the process that the robot goes through to move, starting from its motor rotating" and "Please draw a diagram of the process". The mechanism manipulation was considered successful if the participant articulated, in at least one of the two questions, that the robot's motor rotations caused the robot's wheels to rotate (i.e., they recognized the fundamental mechanism that powers the robot).

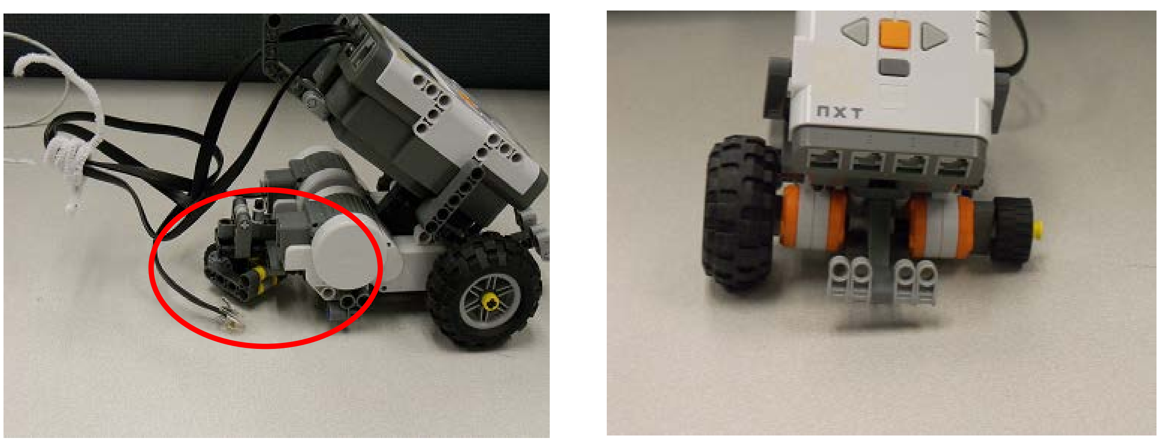

Figure 3. The two robots used in the Mechanism manipulation. Left, the USB cord connecting the robot's right motor to the robot's brick was disconnected. Right, the robot's two wheels were mismatched in size.

Maze task. Participants learned to program a robot in the programming language ROBOTC, using commands that told the robot which direction to move and the number of times to rotate its motor during each movement (e.g., forward(150), backward(200), turnLeft(20), turnRight(80)). Participants were given two objectives during this task: to navigate their robot through a maze (shown in Figure 4) and to create a strategy that could navigate another robot with different sized wheels through the same maze, without relying on guess-and-check methods. They were also provided with a tape measure to measure the maze or robot, though there was no requirement to use the tool. Participants were given an initial 30 minutes to complete the task. After creating their initial strategy (on average, after 26 minutes), participants were asked to explain whether they thought their strategy would generalize to other robots. Participants were then given 30 minutes to revise their initial strategy, with a recommendation to use a mathematical formula in their new strategy. They were also given access to a set of smaller robot wheels and were allowed to switch the smaller and larger wheels at will.

Two raters coded participants' initial and final strategies based on the type of mathematization used, with Kappa = 1.0 ($p < .001$). A more detailed description and an example of each strategy code are given in Table 1. The first two strategy types do not explicitly use mathematics, though prior research suggests that the Plausible Guesstimation strategy is a foundation upon which more sophisticated mathematical strategies can be built (Nouyvanisvong, 1999). The two latter mathematical strategies are both relevant to the task, but only the last one can fully solve the task.

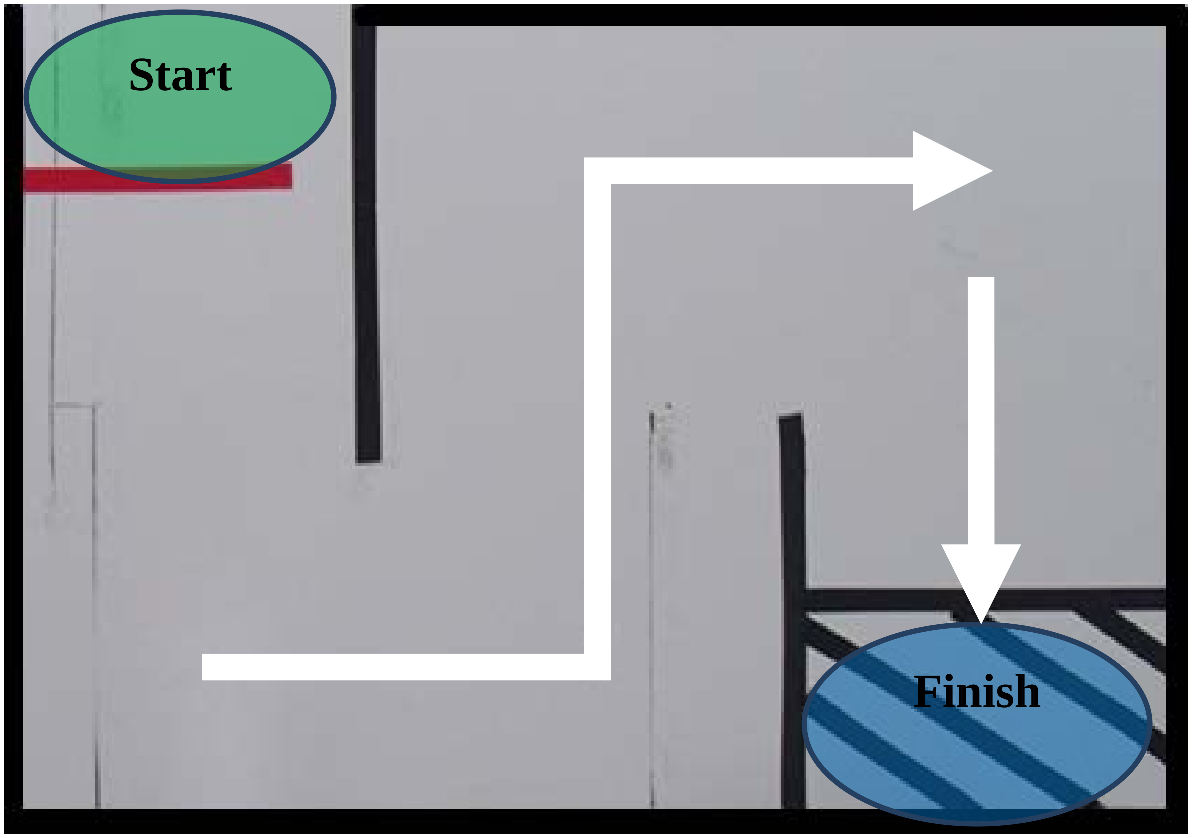

Figure 4. The maze through which participants navigated their robots.

Table 1.

Coding used for initial and final solution strategies in the maze task.

Code	Description	Example
Guessing	Participant created a guess-and-check strategy with no clear basis for guessed numbers, or gave exact commands they used as a strategy	1) forward(130), then turn left 2) turnLeft(160), then turn left 3) Go straight direction, forward(100) 4) turnLeft(28), 28 is still too large to turn, 100 is too long 5) Go straight like the first step, but the length is a little shorter, forward(100) 6) turnRight(24) 7) forward(100), go straight
Plausible guesstimation	Participant created a guess-and-check strategy, but guessed numbers were estimated using some situational basis	"Guess + test was my main strategy. After I learned that it took the robot 150 (approx.) motor rotations to go one straight stretch of the maze + 30 (approx.) motor rotations to make a turn in the maze, I just entered in the numbers in the computer until finally the robot got through the maze."
Specific proportional	Participant created a strategy utilizing proportional reasoning, but values were specific to their robot	"1. It is 0.1 inch per motor-rotation. 2. It needs 35 motor rotations for a left or right turn. 3. Measure the distance for each straight trait which is divided by 0.1 to get the number of motor-rotations for each straight trait."
General proportional	Participant created a strategy utilizing proportional reasoning that could be generalized to other robots	"First, start off with a given value for motor rotations (call this R1) and measure the distance the robot travelled for that number of rotations (D1). Second, measure the distance you would like the robot to travel to reach its intended destination (D2). Calculate the number of rotations it will take the robot to travel this distance (D2) using the formula R1/R2 = D1/D2 and solve for D2."

Mathematical Relationship Understanding questionnaire. The Mathematical Relationship Understanding questionnaire consisted of eight open-ended questions about how the robot's motor rotations, wheel rotations, and distances are quantitatively related. Questions asked directly about the mathematical relationships between components (e.g., "Are the number of wheel rotations related to the distance that the robot moves forward?"). Responses to each question were scored on the number of accurate mathematical relationships included in the answer, such that a higher score signified greater quantitative understanding (with a maximum score of 16 points). The Cronbach's alpha (α, a commonly used metric of instrument reliability; Cronbach, 1951) for this questionnaire was 0.64, suggesting potentially low internal consistency.

Robot drawing task. To determine the features of the robot to which participants attended during the task, participants were asked to draw the robot they had programmed from memory, and to include the important parts of the robot in their drawing. To assess and control for drawing ability differences across participants, a control drawing task was also given with the same instructions, except that participants could look at the robot they worked with as a reference while they drew. Both memory and control drawings were coded for the number of accurately drawn wheels and the number of motors included in the drawings (proportionally-relevant features), and whether or not the drawing included a detailed depiction of the robot's screen (a proportionally-irrelevant feature).

Paper Folding test. The Paper Folding test (Ekstrom et al., 1976) measures spatial visualization ability. A series of pictures depicts one to three folds made in a piece of paper, and the final picture shows where a hole is punched in the paper. Participants selected which of five options illustrated the reopened piece of paper. The test consisted of two parts with 10 questions each ($\alpha = 0.84$), with three minutes allotted for each part.

Santa Barbara Sense of Direction scale. The Santa Barbara Sense of Direction (SBSOD) scale is a standardized self-report scale of environmental spatial ability that has been shown to highly correlate with spatial knowledge tests that involve environment orientation and updating of location in space after self-locomotion (Hegarty et al., 2002). The scale consists of 15 statements ($\alpha = 0.84$) about one's spatial and navigational abilities, preferences, and abilities.

Participants rated their agreement with each statement on a scale of 1 (Strongly agree) to 7 (Strongly disagree).

Motivation questionnaire. As a control variable, the motivation questionnaire included nine questions about the participant's level of motivation during the maze task, building upon theories and measures of engagement (the Intrinsic Motivation Inventory; e.g., Ryan, 1982) and achievement goals (Elliot & Church, 1997). Three questions involved the participants' level of engagement (e.g., "I enjoyed the robotics tasks very much"; $\alpha = 0.91$), three questions involved the participants' level of performance-approach goals (e.g., "It is important to me to do well compared to others who do this experiment"; $\alpha = 0.86$), and three questions involved participants' level of mastery-approach goals (e.g., "I desire to completely master the tasks presented in this study"; $\alpha = 0.80$). Participants were asked to rate their level of agreement with each statement on a scale of 1 (Strongly disagree) to 7 (Strongly agree).

2.3 PROCEDURE

Participants first completed the Paper Folding test and the Santa Barbara Sense of Direction (SBSOD) scale. Next, an experimenter gave a brief, verbal introduction to the LEGO NXT robot that did not explain the mechanism of the robot. Participants in the High Mechanistic condition received the mechanism manipulation.

All participants then began the maze task, which was introduced as a programming task and included basic programming instructions. Participants were asked to navigate their LEGO NXT robot through a maze and to create a strategy that other students could use to navigate their own robots through the same maze; importantly, it was emphasized that other students' robots may have different sized wheels than the robot the current participant was using, so the strategy needed to work for robots with any sized wheels. Participants were given 30 minutes to work through the task. Afterwards, participants were asked whether they thought their strategy would generalize to other robots. After writing their answer, they were given an additional 30 minutes to revise their initial strategy while working with a robot with smaller-sized wheels.

After the maze task, participants were given the Robot Drawing and Control Drawing tasks. They then filled out the Motivation, Mechanism Understanding, and Mathematical Relationship Understanding questionnaires.

3.0 RESULTS

3.1 MANIPULATION CHECK

Because the purpose of the study was to investigate the effects of mechanistic understanding, we needed to ensure that our manipulation caused the High Mechanistic condition to have higher mechanistic understanding than the Low Mechanistic condition. If the mechanism manipulation was successful, then more individuals in the High Mechanistic condition would articulate the motor-wheel relationship (in their responses to the Mechanistic Understanding Questionnaire) than the Low Mechanistic condition. However, a Chi-square test of independence testing the relationship between condition and participant's score on the Mechanistic Understanding Questionnaire was not significant [X^2 (1, N = 50) = 1.53, p = .22]: 70.8% of individuals in the High Mechanistic condition recognized the motor-wheel relationship, compared to 53.8% of individuals in the Low Mechanistic condition.

We chose to redefine High vs. Low Mechanism in terms of whether participants correctly identified the robot's motor-wheel relationship, so that the two groups would differ in their level of mechanistic understanding as originally intended. Using this new definition, 31 participants were categorized as being in the High Mechanistic group, and 19 participants were categorized as being in the Low Mechanistic group. These group definitions were used for the remainder of the analyses.

3.2 MAZE TASK

We hypothesized that the High Mechanistic group would utilize mathematics during the maze task more frequently and with more complexity than the Low Mechanistic group. Analyses were computed separately on participants' initial strategies (completed before an experimenter recommended the use of a math formula in their strategy), and on participants' final strategy (completed after the participant was given the chance to revise their initial strategy). Tables 2 and 3 show the percentage of participants who created each type of initial and final strategy (i.e., Guessing, Plausible Guesstimation, Specific Proportional, and General Proportional), respectively, at each level of mechanistic understanding.

Table 2

Percentage of Participants Using Each Type of Initial Strategy by Level of Mechanistic Understanding

	Guessing	Plausible Guesstimation	Specific Proportional	General Proportional
High Mechanistic	16%	35%	26%	23%
Low Mechanistic	42%	37%	21%	0%

Table 3

Percentage of Participants Using Each Type of Final Strategy by Level of Mechanistic Understanding

	Guessing	Plausible Guesstimation	Specific Proportional	General Proportional
High Mechanistic	0%	23%	29%	48%
Low Mechanistic	26%	26%	42%	6%

We examined the frequency of mathematical strategies by comparing the level of mechanistic understanding with the use of mathematical strategies (i.e., strategies coded as either Specific Proportional or General Proportional). For initial strategies, a Chi-square test of independence was marginally significant [X^2 (1, N = 50) = 3.74, p = .053]: 49% of participants in the High Mechanistic group used a mathematical strategy, while only 21% of participants in the

Low Mechanistic group used a mathematical strategy. Meanwhile, for final strategies, 77% of High Mechanistic participants used a mathematical strategy, as compared to 48% of participants, and the Chi-square was significant [X^2 (1, N = 50) = 4.74, p = .029]. Thus, it appears that individuals who had a higher mechanistic understanding were somewhat more likely to use mathematical strategies, prior to any prompting to use mathematics. After an experimenter recommended the use of a mathematical strategy to all participants, individuals with higher mechanistic understanding were more able to generate mathematical strategies than individuals with lower mechanistic understanding.

To examine the two groups' strategy complexity, we computed two Mann-Whitney U tests to more finely compare the mathematizations used in participants' initial and final strategies (with Guessing being the least complex strategy possible = 0, and General Proportional being the most complex strategy possible = 3). The tests showed that complexity in initial strategies were slightly higher for the High Mechanistic group (mean rank = 29.4) than the Low Mechanistic group (mean rank = 19.1) [U = 173.5, p = .012, r = .36]. For final strategies, the High Mechanistic group (mean rank = 30.7) were much more likely to create complex strategies than the Low Mechanistic group (mean rank = 17.0) [U = 133.0, p = .001, r = .48]. Overall, participants with higher mechanistic understanding were more likely to create strategies that were more mathematically complex (and consequently more accurate), while participants with lower mechanistic understanding were more likely to rely on simple guessing or estimation strategies.

3.3 MATHEMATICAL RELATIONSHIP UNDERSTANDING

In addition to participants' strategies, we looked at whether the High Mechanistic group had higher scores on the Mathematical Relationship Understanding Questionnaire compared to the Low Mechanistic group. An independent-samples t-test confirmed that the High Mechanistic group (M = 5.52, SD = 2.42) had significantly higher scores on the questionnaire than the Low Mechanistic group (M = 3.32, SD = 2.36) [$t(48) = -3.15$, $p = .003$, $d = .92$], showing greater understanding of the quantitative relationships that exist within the robot (see Figure 5).

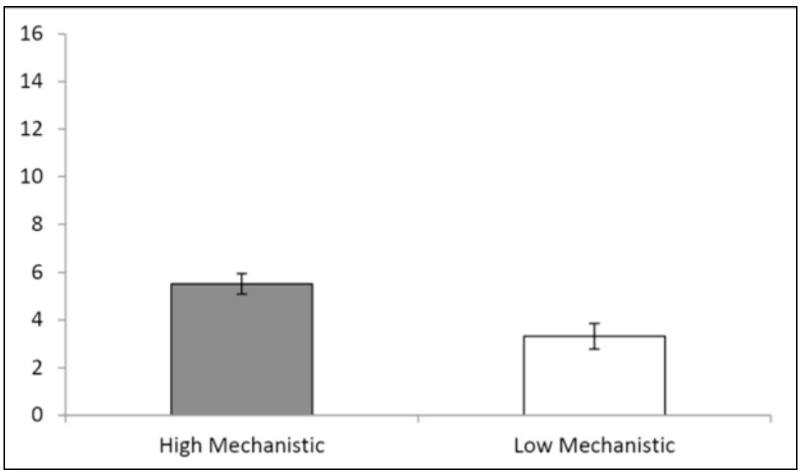

Figure 5. Average score on the Mathematical Relationship Understanding Questionnaire was significantly higher in the High Mechanistic group than the Low Mechanistic group.

Participants' scores on the questionnaire also positively correlated with the complexity of their final maze strategy [$r(50) = .375$, $p = .007$], indicating that participants who created more mathematically complex strategies in the maze task were those who possessed greater understanding of the quantitative relationships within the robot.

3.4 ROBOT DRAWING TASKS

The robot's mechanism consists of three primary parts: the robot's screen, motors, and wheels. However, only the motors and wheels are also involved in the robot's mathematical relationships; it is possible that attention and understanding of these particular parts of the robot contribute to mathematical understanding, rather than general mechanistic understanding. We explored whether the High Mechanistic group were more likely to include proportionally-relevant features of the robot in their drawings (i.e., the motors and wheels) than the Low Mechanistic group, and conversely, whether the Low Mechanistic group were more likely to include proportionally-irrelevant features of the robot (e.g., the robot's screen) in their drawings. Separate ANCOVAs were conducted on the number of accurately-drawn wheels and the number of motors included in the drawings, controlling for the number of wheels and number of motors in participants' control drawings, respectively. A Chi-square test of independence was also run to test the relationship between group and the likelihood of detailing the robot's screen in the drawing. Contrary to our hypotheses, there were no differences among group for the number of wheels included [$F(1, 47) = .055$, $p = .82$, $\eta_p^2 = .001$], the number of motors included [$F(1, 47) = .30$, $p = .59$, $\eta_p^2 = .006$] (Figure 6), or the likelihood of including screen details in the drawing[$X^2(1, N=50) = .42$, $p = .52$]. In addition, no correlations existed between Mathematical Relationship Understanding Questionnaire score and the number of wheels [$r(50) = .075$, $p = .61$], the number of motors [$r(50) = .15$, $p = .29$], or whether the screen was included [$r(50) = .17$, $p = .24$] in the drawing. Thus, both level of mechanistic understanding and level of mathematical understanding appeared to be unrelated to level of attention to basic features of the robot.

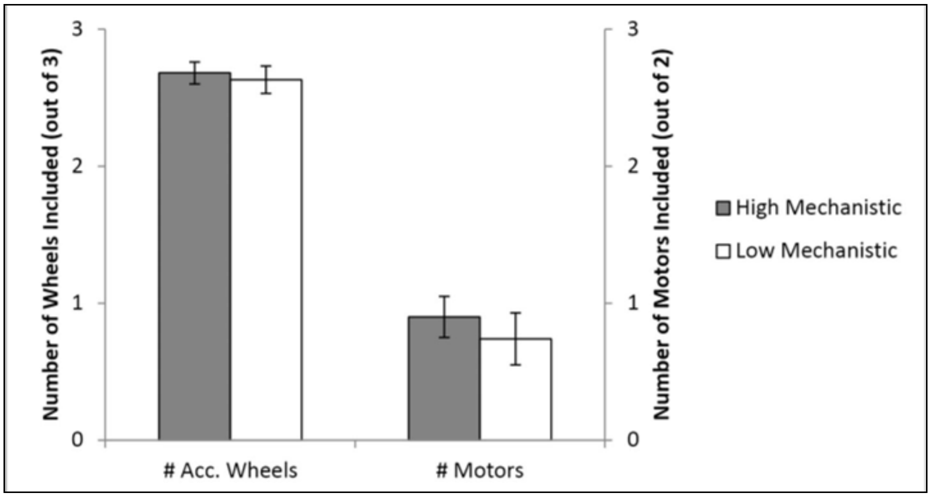

Figure 6. High Mechanistic group and Low Mechanistic group did not differ in the average number of wheels or motors included in their robot drawings.

3.5 INDIVIDUAL DIFFERENCES

To explore whether higher mechanism understanding was confounded with students' ability to visualize and simulate the robot's movements, we examined correlations of spatial visualization ability with maze strategy complexity and understanding of the mathematical relationships within the robot. However, there were no significant correlations between participants' scores on the Paper Folding Test and their initial strategy complexity [$r(50) = .23$, $p = .10$] or their final strategy complexity [$r(50) = .23$, $p = .11$]. Similarly, there was no correlation between Paper Folding Test score and Mathematical Relationship Understanding Questionnaire score [$r(50) = .21$, $p = .14$]. The control spatial measure, the SBSOD, was also uncorrelated with initial maze strategy complexity [$r(50) = .16$, $p = .26$], final maze strategy complexity [$r(50) = .11$, $p = .45$], and Mathematical Relationship Understanding Questionnaire scores [$r(50) = .21$, $p = .14$]. Thus, spatial visualization ability was not a confound in the performance differences between group and does not appear to play a role in participants' ability to effectively mathematize their understanding of the robot's mechanism.

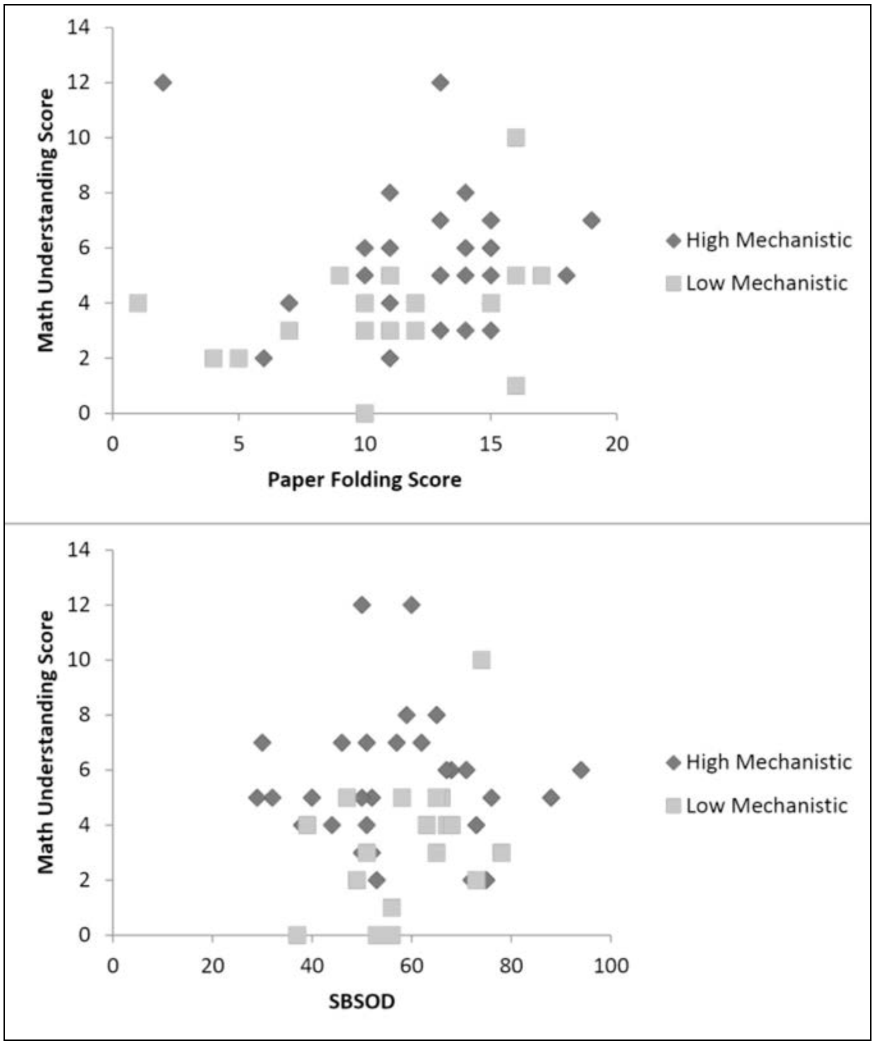

Figure 7. Neither the High Mechanistic group (dark gray diamonds) nor the Low Mechanistic group (light gray squares) showed correlations between math understanding score and spatial measure scores.

We also tested whether the two groups differed in their levels of motivation, and whether differing motivation could explain the differences seen in maze strategy complexity or Mathematical Relationship Understanding Questionnaire scores. An independent samples t-test revealed that the High Mechanistic group reported significantly higher engagement during the robotics task (*M* = 5.69, *SD* = 1.04) than the Low Mechanistic group (*M* = 4.58, *SD* = 1.70) [*t*(26) = -2.56, *p* = .016, *d* = .79]. There were no differences in reported levels of mastery goals [*t*(48) = -.97, *p* = .34, *d* = .28] or performance goals [*t*(47) = -.30, *p* = .76, *d* = .08] (Figure 7). However, the level of engagement did not explain the relationship found between mechanistic understanding and strategy complexity or mathematical understanding: after controlling for engagement, the main effect of mechanistic understanding level was still significant for initial maze strategy [$F(1, 47) = 5.41$, $p = .024$, $\eta_p^2 = .10$], final maze strategy [$F(1, 47) = 11.58$, $p = .001$, $\eta_p^2 = .20$], and Mathematical Relationship Understanding Questionnaire score [$F(1, 47) = 6.41$, $p = .015$, $\eta_p^2 = .12$]. To determine whether the creation of more successful maze strategies or better understanding of the robot's mathematical relationships may be driving increased engagement in the robotics tasks, we also conducted three ANCOVAs on engagement, using initial maze strategy complexity, final maze strategy complexity, and Mathematical Relationship Understanding Questionnaire scores as covariates, respectively. In all three tests, the main effect of level of mechanistic understanding remained significant [initial: $F(1, 47) = 6.0$, p = .018, $\eta_p^2 = .11$; final: F(1, 47) = 4.70, p = .035, $\eta_p^2 = .09$; math understanding: F(1, 47) = 4.84, p = .033, $\eta_p^2 = .09$], while main effects of initial strategy, final strategy, and math understanding was not. Therefore, level of mechanistic understanding appears to lead to differences in engagement, but engagement itself is not directly related to task performance or greater mathematical understanding of the robot.

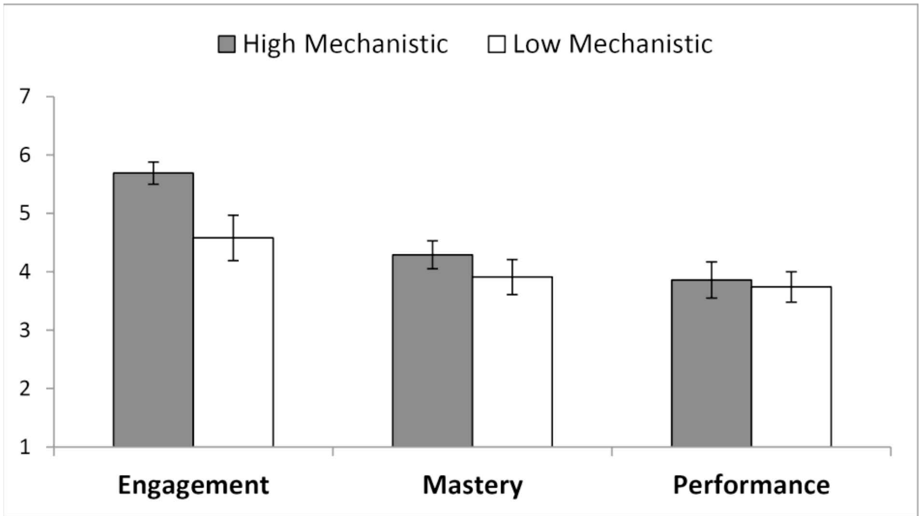

Figure 8. High Mechanistic group reported greater levels of engagement than the Low Mechanistic group, but equal levels of mastery and performance goals.

4.0 GENERAL DISCUSSION

The current study investigated the relationship between mechanistic understanding and mathematical understanding. Specifically, we examined whether understanding the mechanism of a physical robot would be associated with increased understanding of the mathematical relationships within the robot, and with higher frequency and complexity of mathematics used in a robotics task. The results showed that participants who understood the robot's mechanism also showed greater understanding of the robot's quantitative relationships. Furthermore, these participants were more likely to use math when navigating the robot through a maze, and were able to use more complex mathematizations for the task. We also found that higher mechanistic understanding was associated with greater engagement in the robotics task, which was not explained by higher mathematical understanding or better performance on the task, suggesting that mechanistic understanding *per* se may play a motivational role as well.

In regard to attention to details, we found no differences between participants with high mechanistic understanding and those with low mechanistic understanding: all participants were equally likely to attend to the proportionally relevant and irrelevant parts of the robot. Given that all participants used physical robots during the robotics task, all participants should also have the same heightened attention and processing for the robot's details, due to having the object in hand space; indeed, both high and low mechanistic participants generally drew the robot accurately (with high mean scores), suggesting that all participants highly attended to and remembered the features of the robot. Thus, attention to detail alone is not enough to discover mechanisms and mathematical relationships.

We also found that spatial visualization ability did not correlate with mechanistic understanding, mathematical use, or mathematical understanding. This finding appears to contradict previous findings that spatial ability correlates with accuracy on mechanistic reasoning problems (Hegarty & Sims, 1994; Hegarty & Steinhoff, 1997). However, research by Schwartz and Black (1996) suggests that people initially use mental simulations and mechanistic reasoning until a suitable rule is discovered, at which point people shift toward rule-based reasoning instead. Hegarty (1992) also found that people use other strategies concurrently with mental simulation. In the current study's robotics task, it is possible that participants' mechanistic understanding initially helped them to discover the constant relationship between the robot's motor rotations and distance movements. Once that relationship was found, participants may have stopped relying on mechanistic reasoning and shifted to other non-mechanistic strategies, such as rule-based reasoning, allowing them to avoid simulations of the motor-wheel relationship. Also, because participants would not have to rely as heavily on visualization of the mechanism, spatial ability may have played less of a role in the current study, explaining the lack of correlation between spatial ability and our mechanism and mathematical understanding measures.

4.1 INTERACTIONS BETWEEN MECHANISTIC AND MATHEMATICAL UNDERSTANDING

How does mechanism understanding benefit mathematical understanding? Mechanisms provide a perceptual ground for abstract mathematical concepts, a common affordance cited by physical manipulatives. In addition, the patterns and regularities that underlie many mathematical principles are also emphasized. By pointing out the regularities in mathematical principles, it may be easier for students to understand when mathematical principles can be generalized across settings. Furthermore, mechanisms may increase the likelihood that students use math in a learning situation: mechanism regularities may help students to see that mathematical principles, which are also regular, can be applied to the situation, while mechanism schemas, which include a hypothesized process and outcome, may encourage students to use math to test the proposed result of their mechanism (Machamer, Darden, & Craver, 2000). The current study provided evidence that mechanism influenced students' ability to see the applicability of math in the robotics situation, as high mechanistic participants were more likely to use math in their initial maze strategies, before an experimenter recommended for them to do so.

Mathematical studies often focus on the direction of physical experiences to math (e.g., Ahl, Moore, & Dixon, 1992; Bassock & Olseth, 1995), using what Schwartz and Moore (1998) call the "EQM" frame; that is, given an empirical situation, people determine which qualitative schema fits that situation best, and then determine which mathematical procedure to use based on the schema. Alternatively, the relationship between physical experiences and math may proceed in the opposite direction: mathematical understanding may lead to increased mechanism understanding. Indeed, previous research has suggested that mathematics can be used to make sense of physical experiences (e.g., Martin & Schwartz, 2005; Schwartz, Martin, & Pfaffman, 2005; Sherin, 1996).

Although the current study posited that increased mechanistic understanding would lead to greater mathematical understanding, it was not possible to conclusively test the direction of this relationship due to our unsuccessful mechanism manipulation. Several plausible third variable confounds were ruled out. However, participants may have used their mechanistic understanding to generate mathematical strategies and inform their mathematical understanding of the robot (i.e., mechanism to math direction); or, they may have first discovered the mathematical patterns between their inputted motor rotations and the robot's traveled distance and used that to conceptualize the robot's mechanism (i.e., math to mechanism direction); or, there may have been a constant conversation between mechanism understanding and mathematical understanding, where discoveries about mechanism and/or mathematical patterns were used to inform and revise their understanding of the other (i.e., a reciprocal mechanism and math relationship). This directionality question could be answered with future studies investigating the steps through which students proceed as they generate their mathematical strategies. Such data would also provide additional information about whether there are any differences between students who begin with mechanism or mathematical understanding in creating their strategies.

4.2 EDUCATIONAL IMPLICATIONS

Educationally, there has been a push toward virtual environments; not only does it avoid physical limitations associated with physical manipulatives, but it also engages students in technology, which is increasingly necessary as technology advances. Research focusing on virtual manipulatives have found that they also have unique affordances that physical manipulatives do not have, including reduced set-up and clean-up time, quicker feedback to students, increased student motivation, and increased exploration of strategies (e.g., Reimer & Moyer, 2005; Steen, Brooks, & Lyon, 2006; Suh, Moyer, & Heo, 2005; Yuan, Lee, & Wang, 2010). However, virtual environments may also lose benefits that physical environments provide. The current study suggests that mechanism understanding, which may be indirectly caused by physicality and more readily discovered in physical contexts, should also be emphasized in virtual contexts (for example, by showing a virtual simulation of the mechanism involved in the task at hand). Future studies examining whether presentation of virtual mechanisms lead to the same benefits as physical mechanisms would help to dissociate benefits caused by physical interactions from benefits caused by mechanistic understanding.

It is important to note that the current study does not discount the existence of physically unique benefits, such as those posited by embodied cognition. The current study investigated one physical affordance that was not tied directly to physicality; rather, we proposed that physically interacting with an object leads to increased attention toward the object, which leads to a higher likelihood of discovering the object's underlying mechanism. In cases in which the physical benefit is not directly caused by the physicality of the situation, investigating the direct cause of the benefit is beneficial, as they can then be integrated into other mediums to avoid physical limitations. In cases where physicality is the direct cause of learning benefits, then a combination of physical and virtual manipulatives may be ideal (e.g., Olympiou & Zacharia, 2011).

In sum, the current study shows that mechanistic understanding is associated with greater mathematical understanding. Teaching mathematics in the context of mechanisms, using mechanistic manipulatives that can be connected with mathematical principles, may provide several mathematical benefits, including increased use and complexity of mathematical strategies. Grounding mathematical concepts in concrete mechanisms and taking advantage of the regularities in both mathematical and mechanical systems allows students to see the applicability of mathematics to concrete situations, ultimately leading to a better understanding of both mechanism and mathematics, and the connections between the two.

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