THE IMPACT OF TEACHER POSITIONING ON STUDENTS' OPPORTUNITIES TO LEARN: A CASE STUDY OF AN ELEMENTARY MATHEMATICS CLASSROOM

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This case study aimed at investigating how teacher positioning influenced students' opportunities to learn in a 3rd grade mathematics classroom. This study focused on how norms were reflected in teacher positioning and how students were positioned in terms of their identities as mathematics learners.

Utilizing the socio-cultural view of learning (Lave & Wenger, 1991; Wenger, 1998) with positioning theory (Harré & van Langenhove, 1999; van Langenhove & Harré, 1999), the study analyzed classroom discourse during whole class discussions. It also examined interviews with the teacher. The analysis consisted of two phases. At the first phase, I investigated the transcripts of 8 lessons and 3 interviews and a newsletter written by the teacher to parents to identify what norms appeared and how the teacher communicated those norms. At the second phase, I analyzed the transcripts of classroom discussions to articulate what the interactional pattern looked like, how the teacher positioned students during discussions, and how students reacted to teacher positioning in terms of negotiating their identities as mathematics learners.

The analyses revealed that her teaching philosophy appeared as a hybrid discourse, which included different kinds of norms and multiple positions taken up by the teacher. They also showed that particular types of teacher positioning promoted or restricted students' participation in whole class discussions about problem-solving strategies. Finally, the results demonstrated that students engaged in social comparison to protect their current self-evaluations as mathematics learners or construct new positive self-evaluations as mathematics learners.

The findings suggested two conclusions. First, the teacher's positioning played a significant role in her students' participation in discussions about their strategies and these positions were aligned with her teaching philosophy Second, students' social comparisons appeared as discursive positions that mediated their identities as mathematics learners. The limitations of the study and implications for future mathematics education research were also provided.

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PREFACE

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1.0 INTRODUCTION

Fostering mathematical discussions has been viewed as one of the key features of successful mathematics teaching these days. The *Professional Standards* by the National Council of Teachers of Mathematics (1991) particularly emphasized the importance of classroom discourse as a standard for teaching mathematics. The discourse standard suggests that teachers orchestrate mathematical discussions to promote students' understanding, and that they pay attention to classroom culture to build the environment where students can have opportunities to learn equally. For example, teachers are expected to provide cognitively demanding tasks which requires students to justify their ideas, organize discussions well, and encourage students to participate in the discussions. Then, students are expected to not only respond to teachers but also actively engage in discussions at various points by presenting, clarifying, and justifying their strategies to build mathematical knowledge. From the discourse standard, orchestrating mathematical discussions is viewed as an important part of teachers' responsibilities.

A socio-cultural perspective of learning has taken discourse very seriously, since it is a cultural tool which people in a community or an activity system (e.g., school) should acquire and be skilled in using to be recognized as a full participant in their community (Lave & Wenger, 1991; Sfard, 1998; Wenger, 1998). In mathematics classrooms, for example, students participate in activities such as problem solving and discussions. If students are in another class such as science, they should have another type of discourse, that is, scientific discourse, to participate in scientific activities. Students' participation in intellectual activities depends on how much they acquire mathematical discourse and how well they are able to use it. Learning to use particular types of discourse (e.g., mathematical, scientific) plays a significant role when students learn.

Although mathematical discourse is essential for students' learning mathematics, it is necessary to keep in mind that there is no discourse that characterizes a whole mathematics classroom (Sfard, 2000). Such a universalistic view of discourse may underestimate the unique characteristics that each mathematics classroom has. All mathematics classrooms are not the same, since classes are taught by teachers who are from different backgrounds and their uniqueness is reflected in their classes. In addition, there are students with different backgrounds involved in those classes. In this sense, an investigation of classroom discourse requires us to capture and describe as many features and characteristics of classroom discourse as possible because those are what make each classroom unique as a community of mathematics learners.

1.1 PARTICIPATING IN A COMMUNITY OF PRACTICE

The idea of a classroom as a "community of learners" is a central concept of a socio-cultural view of learning (Lave & Wenger, 1991; Wenger, 1998). From this perspective, a mathematics classroom is viewed as a community where teachers and students are jointly working on various activities by using mathematical discourse and various tools such as algorithms, equations, graphs, and so on. Learning is, therefore, better characterized by participants' interactions rather than individuals' intellectual activities (Sfard, 1998).

If learning is discussed in terms participants' interactions, it is necessary to keep in mind that not all participants participate equally in interactions. For example, some students may be more vocal in discussions than others. Other students may have more mathematical knowledge and receive better evaluations from teachers than others when they are working together on the same problem. Teachers may prefer particular types of algorithms and favor students who present those algorithms in the discussions. This complexity seems to generate variation in students' participation in learning. This variation will be related to students' opportunities to learn in a classroom. We should identify what factor(s) would influence students' participation in classroom interactions, if we are to understand how students learn and why their participation varies.

1.2 OPPORTUNITIES TO LEARN IN CLASSROOMS

The principles suggested by the National Council of Teachers of Mathematics (2000) states the issue of equity in terms of excellence of mathematics education. According to the NCTM, equity refers to high expectations and support that all students should have in the course of learning. This means that all students should have expectations and support from teachers to maximize their learning.

Unfortunately, it seems very difficult for mathematics teachers to provide all students with high expectations and support for learning, because of the difference in teachers' understanding of students' ideas expressed in their talk, their support of struggling students, their understanding of mathematics, and so on. Although equity is one of the urgent issues in teaching and learning mathematics, it may be difficult for teachers to achieve in each classroom, since there many factors influence classroom learning.

One way which contributes to better understanding of equity is to approach to it in terms of opportunities to learn (OTL). OTL has been studied with different foci, such as standardized tests and the relationship between an individual and a learning environment (Haertel, Moss, Pullin, & Gee, 2008). Different foci generated different interpretations of the concept. For example, OTL has been viewed as “opportunities to learn what is tested” (Haertel et al., 2008, p. 1), while it has been conceptualized as “affordances for changing participation and practice” (Greeno & Gresalfi, 2008, p. 172). Another focus has been placed on the fair distribution of OTL (Esmonde, 2009). Since there are multiple types of conceptualization of OTL, it would be important to narrow the focus in this study.

Since this study took the socio-cultural perspective of learning, OTL was also conceptualized in the same vein. The socio-cultural view has emphasized participation in activities as an essential aspect of learning (Lave & Wenger, 1991; Wenger, 1998). Considering this emphasis, I interpreted OTL in this study as opportunities for students to participate in mathematical discussions in order to investigate classroom interactions.

1.3 STATEMENT OF THE PROBLEM

Whether or not students have a fair OTL has been an important educational issue in the United States (Haertel et al., 2008), equity has been a crucial principle to promote students’ learning in mathematics education (NCTM, 2000). The fact that equity has been the issue in education indicates that not all students have a fair OTL, and more importantly that there would be some factors that influence a fair distribution of OTL during learning.

Researchers who have taken a socio-cultural or situated cognition approach (e.g., Esmonde, 2009; Gee, 2008; Greeno & Gresalfi, 2008) have investigated OTL in relation to the environment in which students interact during learning. According to them, OTL is not constrained by whether students are equally exposed to the same amount of subject matter or whether they learn what is measured in standardized tests. Nevertheless, OTL is more social or even political because students are subject to the influence of the learning environment such as the dynamics of classroom interaction.

In order to understand students' OTL, it would be necessary to closely investigate what teachers actually do in classrooms and how students' participation in learning is affected by teachers. Such an investigation would require a micro-analysis of classroom interactions in which teachers and students discursively position themselves or are positioned in relation to subject matter and their actual or future identities as participants in a community of learners.

Looking at how participants (e.g., teachers and students) position themselves and are positioned within classroom interactions would provide useful information on their interactions in terms of discursive positions that they take up, what they do with their talk, and storylines which they develop during interactions, all of which are constitutive elements of conversations (Harré & van Langenhove, 1999; van Langenhove & Harré, 1999). Those elements would serve as an analytic lens to investigate how participants influence each other's positions in discourse. In mathematics classrooms, teachers and students use and participate in mathematical discussions and arguments, where teachers, for example, align one students' idea with another, interrupt students, and/or validate particular ideas for their teaching purposes. Such teacher positioning may affect students' OTL, that is, opportunities for students to participate in mathematical discussions and arguments which promote their learning. Therefore, a micro-analysis of classroom interactions is necessary for understanding the social factors influencing OTL for students.

It is also as important as investigating teachers' influence on students' OTL to examine how students respond to their teacher's positioning, because their responses would show what they are doing to negotiate their participation in learning. In a socio-cultural view, learning is conceptualized in terms of changes in participation in practices of a community such as a classroom (Greeno & Gresalfi, 2008). This idea indicates that, when students learn, their participation changes. In order to articulate changes in students' participation, it would be necessary to investigate not only how teacher positioning influences students' participation but also how students react to teacher positioning and why they do so. Looking at participation from both teachers' and students' perspectives would be able to provide better understanding of the subjective meanings of changes in participation.

1.4 PURPOSE OF THE STUDY

The purpose of this study was to investigate how teacher positioning influenced students' opportunities to learn (OTL) in an elementary mathematics classroom. Taking the socio-cultural view of learning, OTL was defined in this study as opportunities for students to participate in mathematical discussions and arguments in a classroom. As an analytical framework, I used positioning theory. Positioning theory is a useful tool for discourse analysis, because its idea that conversations consist of positions, speech acts, and storylines allows for micro-analyses of classroom discussions.

This is a case study of a 3rd grade mathematics classroom where a female teacher with over 20-year teaching experience taught 17 students (10 boys and 7 girls). Utilizing positioning theory (Harré & van Langenhove, 1999; van Langenhove & Harré, 1999), I investigated what classroom norms and culture looked like and how those were functioning as a context for influencing students' participation in whole class discussions.

Mathematics classrooms usually share social and socio-mathematical norms (Yackel & Cobb, 1996), which shape classroom culture and practices. According to Yackel and Cobb, social norms are general and applicable to any classroom, while socio-mathematical norms are defined as what counts as mathematically different, sophisticated, elegant, and efficient through interactions between teachers and students. They also argued that socio-mathematical norms influence not only a mathematical discussion itself but also students' participations in the discussion. The attributes of socio-mathematical norms indicate the possibility that sophisticated levels of students' problem-solving strategies might regulate students' contributions to mathematical discussions and position students according to the efficiency of their strategies. Therefore, it would be useful to consider socio-mathematical norms as a factor differentiating students' positions.

While this study investigated the influence of teacher positioning on students' participation in whole class discussions, the study also examined how students responded to teacher positioning and why they did so. I paid particular attention to students' social comparisons as an important outcome of classroom social processes. Social comparison is a psychological process in which individuals compare themselves with others to have positive (or biased) self-evaluations (Dijkstra, Kuyper, van der Werf, Buunk, & van der Zee, 2008). Since the socio-cultural perspective conceptualized learning as a process of becoming a different person through participating in practices of a community (Lave & Wenger, 1991; Wenger, 1998), it would be assumed that students might be engaging in social comparison as a way of negotiating their participation to be recognized as a particular type of learner in a classroom (e.g., advanced student, contributor).

In conclusion, this case study largely relied on a socio-cultural theory of learning and positioning theory as frameworks and examined the classroom culture/norms and teacher positioning in terms of their influence on students' OTL. Social comparison was also considered as an outcome of classroom social processes.

1.5 CONTRIBUTION TO THE FIELD

This case study would be able to contribute to educational research in several ways. First, the study would contribute to mathematics education research especially on equity. Socio-cultural researchers argued that equity is closely associated with students' OTL (e.g., Esmonde, 2009; Gee, 2008; Greeno & Gresalfi, 2008), and they agreed that OTL is differently distributed to students. As one of the factors influencing OTL, Greeno and Gresalfi pointed out the role of positioning and argued that "different OTL is afforded to each individual from moment to moment, depending on how he or she is positioned in the interaction" (p. 183). The use of positioning theory helps to provide useful information on not only how teachers position students but also how teacher positioning influences students' OTL.

The next contribution is related to research on teacher discourse in mathematics classrooms. Greeno and Gresalfi pointed out that the relationship between positioning and OTL would lead to greater consideration of teacher discourse. As the authority in a classroom, teachers position students in classroom interactions, for example, giving them positions such as contributors, struggling students, and role models. Those instances of positioning are identifiable in what teachers say. Therefore, it would be important to study teacher discourse with the consideration of its influence on students' OTL.

Using positioning theory, this study carefully investigated teacher discourse and identified the types of teacher positioning (e.g., interrupting, validating comparing). Teacher positioning located students differently; students' participation in classroom discussions were enhanced or restricted depending on what positions the students assumed. Those instances seem very useful for understanding not only teacher discourse but also its role and influence during classroom interactions.

Lastly, this study would also contribute to future research on the social construction of students' identities as learners. This study also investigated how students responded to teacher positioning in terms of social comparison. Social comparison is closely related to one's self-evaluation, and its motives are to maintain ones' positive self-evaluation (Tesser, Miller, & Moore, 1988). According to this assumption, students' social comparison observed in the mathematics classroom implies that students engage in social comparison to maintain their positive self-evaluations as mathematics learners or to be recognized as such.

Sfard & Prusak (2005) conceptualized learning as filling a gap between one's current identity (i.e., what I am) and his/her future identity (i.e., what I want to/should be). Studying students' social comparisons in mathematics classrooms would help us better understand students' identity work as they try to be recognized as a good mathematics learner.

1.6 ORGANIZATION OF THE PAPER

This paper consists of 5 chapters. The first chapter is an introduction of the paper and describes the research problem that I investigated in this study. The second chapter reviews the literature of (1) a socio-cultural view of learning, (2) OTL, and (3) social comparison. I also explain positioning theory, which was used as an analytical framework in this study. The third chapter describes the design and methods I used in this study. The fourth chapter presents the results of the study and discusses the findings in detail in terms of the influence of teacher positioning on students' participation in classroom discussions. The last chapter provides the limitations of this study and implications for future educational research.

2.0 THE REVIEW OF THE LITERATURE

This chapter provides the review of the literature relevant to this study. First, I explain the socio-cultural view of learning, which I used to conceptualize learning in this study. Second, I discuss the concept of opportunities to learn (OTL) with the specific emphasis on mathematics education literature. Third, I describe social comparison, which is an important outcome of social processes among children. Fourth, I explain positioning theory which serves as an analytical framework for this study in terms of its theoretical roots and characteristics. I introduce several educational studies which used positioning theory to demonstrate the theoretical and methodological potentiality of positioning. Lastly research questions are presented.

2.1 SOCIO-CULTURAL VIEW OF LEARNING

A socio-cultural view was influenced by Vygotsky and supports the idea that human thinking is social and dependent on historical, cultural, and situational factors (Kieran, Forman, & Sfard, 2001). In the following sections, I focus on some central concepts of a socio-cultural perspective such as participation and identity and explain those ideas in terms of learning. Also I review the studies which investigated mathematics education from a socio-cultural perspective of learning.

2.1.1 Learning as participation

Sfard (1998) described two types of conceptualizations of learning as "acquisition" and "participation." Especially in mathematics education research, the acquisition paradigm represents the traditional, didactic approach to learning and emphasizes the acquisition of knowledge and skills. According to van Oers (2001), traditional mathematics classrooms have focused on mastering arithmetical operations, applying abstract principles and structures to concrete situations, and solving problems through mathematical activities. In those classrooms, students are either recipients or constructors of knowledge, depending on how they are taught and/or how much they are intrinsically motivated to learn. They are expected to "acquire" as much knowledge as possible, and teachers' main role is to transmit knowledge and skills to students efficiently.

On the other hand, the "participation" paradigm has viewed learning as participation in intellectual activities of a classroom community. That is, participation in classroom activities enables students to learn and makes them recognizable as classroom members. The notion of legitimate peripheral participation (LPP) (Lave & Wenger, 1991) describes how novices or newcomers become experts or old-timers in the course of mastering knowledge and practices which had been historically, culturally shared in a community. In LPP framework, becoming full participants means that one has mastered the discourse and activities to (re-)build, maintain, and contribute to the community that he or she belongs to.

If we apply the concept of LPP to classroom settings, learning would entail changes of students' participation in intellectual practices. In other words, it would be considered a process through which students achieve full participation in a classroom. If students are novices, their participation is peripheral due to their limited capabilities. However, as they increase capabilities, their participation becomes a central contribution to classroom activities. Changes in participation indicate that students have better understanding of knowledge and have more responsibility as agents or actors in learning (Greeno & Gresalfi, 2008).

In the socio-cultural perspective, learning through participation is not separable from interactions which occur in a context of learning (Rex, Steadman, & Graciano, 2006). For example, when students learn, they interact with other participants such as teachers and peers, subject materials, learning tools, and other intellectual resources. From the socio-cultural perspective, it would be important to take into consideration how students are positioned or position themselves in relation to other participants and/or subject matter, because their positioning would shape their participation in learning. Therefore, investigating classroom learning is closely associated with examination of ways in which students are positioned or position themselves during interactions with teachers, peers, and other learning resources.

2.1.2 Social construction of identities

While a socio-cultural perspective conceptualizes learning in terms of participation, it also involves a process of constructing identities. As Wenger (1998) describes that learning is a "process of becoming" (p. 215), one becomes a different person, for example, from a novice to an expert, when learning occurs. This means that, when a person learns, his/her participation in practices changes, and that he/she becomes recognized differently as a learner.

However, as Sfard and Prusak (2005) argued, identity has not been well operationalized in those studies. Actually it is quite difficult to define identity in a socio-cultural framework, because it has been long studied by mainstream psychology, which has viewed identity as a fixed, stable, single entity located in one's mind. The socio-cultural operationalization requires a conceptual shift from individual to more relational, which expects us to view identity as more dynamic, unstable, and multiple. Identities are, then, considered as amenable to changes and negotiations in contexts.

Base on those socio-cultural assumptions, several researchers have provided definitions of identity. For example, Gee (2000) defined identity as being recognized as a particular type of person in a given context, and provided four types of identities (e.g., nature identity, institution identity, discourse identity, and affinity identity). From his socio-linguistic background, Gee paid particular attention to contexts in which identities appear and explained the ontological mechanism of each identity (e.g., institution identity authorized by the authority within institutions, discourse identity recognized in discourse of individuals). Holland, Lachicotte, Skinner, and Cain (1998) also introduced two types of identities (e.g., figurative identities and positional identities) based on their ethnographic studies. Their famous concept of "figured world" has been used by several researchers to describe mathematics classrooms (e.g., Boaler & Greeno, 2000; Esmonde & Langer-Osuna, 2013; Wagner & Herbel-Eisenmann, 2009). According to Holland et al., figurative identities come from the stories, artifacts, and practices which shape a community as a culturally specific world, and positional identities are associated with one's social positions within the community.

Some researchers have paid particular attention to discourse and its nature for defining identity. For example, Sfard and Prusak (2005) defined identity as a collection of reifying, significant, and endorsable stories about a person. The stories include the ones which told by both the person and others. From their definition, identities are considered as discursive constructs. Also, identity was defined as the social positioning of self and other (Bucholtz & Hall, 2005).

Those definitions have been applied to the studies on students' mathematical identities in relation to types of classroom activities, classroom norms, discursive positioning, and so on. In the next section, I review how mathematics education research has studied students' mathematics identities, that is, their identities as mathematics learners.

2.1.3 Identities as mathematics learners

The construction of learner identities has recently become a popular topic in mathematics education research, and many socio-cultural/situated cognition researchers have examined identities in terms of learning mathematics. According to Bishop (2012), one of the reasons why identities has been rigorously studied came from the NCTM's recognition the significant role of students' dispositions toward mathematics in their educational success. That is, the NCTM paid particular attention to the affective components of mathematics learning as a factor promoting students' mathematics learning. Bishop argued that such a disposition included students' ideas or beliefs about not only mathematics but also mathematics learners, and that it was well reflected in the NCTM's goal for creating confident, flexible learners of mathematics.

Socio-cultural/situated cognition researchers have studied identities with various foci (e.g., Bishop, 2012; Boaler & Greeno, 2000; Cobb, Gresalfi, & Hodge, 2009; Gresalfi, Martin, Hand, & Greeno, 2009; Sfard & Prusak, 2005; Wood, 2013; Yamakawa, Forman, & Ansell, 2009). For example, Boaler & Greeno (2000) examined high school students' mathematical identities in relation to mathematical activities that they had in the classrooms, viewing a classroom as a figured world of mathematical practices. They found that students from the classes with more dialogic interactions showed more positive identities toward mathematics than those who were from the classes with traditional, didactic interactions.

Cobb et al. (2009) proposed two types of identity, “normative” and “personal,” as an interpretive scheme for mathematics classrooms. According to them, normative identities have to do with the norms and responsibilities for being a good mathematics student in a particular classroom, while personal identities are associated with to what extent students conform to those norms and responsibilities. Comparing two classrooms (e.g., algebra classroom, design experiment classroom) which were different in how teachers taught a class, they found that students from the discussion-oriented classroom (design experiment classroom) had viewed normative identities as more positive than the students from the algebra classroom where the teacher was viewed as the authority.

Sfard and Prusak (2005), proposing their definition of identity, compared identities of two different groups of students (e.g., Israeli students, students who immigrated to Israel). They examined a set of discourse about students based on two types of identities, that is, their actual/current identity and designated/future identity. They argued that learning was to fill the gap between those two identities.

There is another group of researchers who paid particular attention to moment-to-moment interactions in a classroom to understand how students’ identities are socially constructed (Wood, 2013; Yamakawa et al., 2009). They have focused on discursive positioning which appears during classroom interactions and examined what discursive positions appear, how students are positioned or position themselves, and how they negotiate those positions in terms of constructing mathematical identities.

Although those studies about students’ mathematical identities have different foci, the studies have paid particular attention to interactions. Interactions involve the use of cultural tools, practices, and discourse that have shaped a community. It is through interactions that people learn to become its legitimate members. In mathematics classrooms, students learn through classroom interactions, which provide mathematical tools, practices, and discourse. They also learn to become an effective mathematical thinker/learner in a mathematical classroom. In terms of attaining a legitimate membership, interactions can be considered as a place for students' identity work.

2.2 SOCIAL COMPARISON AS AN INTERACTIONAL OUTCOME

Social comparison has been studied in terms of its influence on one's self-evaluation in mainstream psychology. The research has focused on how individuals choose comparison targets, for what purpose(s) they compare themselves with others, what individuals do when their self-evaluations are threatened, in various settings. Since social comparison studies have been working on one's self-evaluation, it is easily assumed that social comparison is associated with ones' identity. Recently mathematics education research has paid particular attention to students' identity as a crucial factor influencing their mathematics learning. Investigating social comparison in mathematics classrooms might be useful for better understanding of students' identity as learners.

However, social comparison has been studied in mainstream psychology, which strongly supports the view that identity is a fixed, stable, single entity located in individual mind. On the other hand, in mathematics education research, identity has been studied by socio-cultural researchers who support the view that identity is multiple, changeable, unstable and that it is socially constructed. In order to investigate social comparison in relation to constructing identities as mathematics learners, an alternative conceptualization of social comparison would be necessary.

A socio-cultural view of learning assumes that social interactions play a significant role in learning. It is through social interactions that people acquire skills, discourse and knowledge, learn to participate in practices of a community, and become recognized as legitimate full members of the community (Lave & Wenger, 1991; Sfard, 1998; Wenger, 1998). In other words, social interactions provide opportunities to construct new identities as learners.

However, social interactions are so complex that their dynamics might produce various interactional outcomes. Social comparison can be considered as one of those outcomes. Simply stated, social comparison is a psychological process through which people compare themselves with others. Social comparison researchers have agreed that, when people compare themselves with others, they usually have a desire for positive self-evaluation (Dijkstra et al., 2008; Tesser et al., 1988). If we think about classroom interactions from the social comparison point of view, we might be able to say that students have a desire for positive self-evaluation in the domains which are relevant to them (e.g., test scores, popularity) and compare themselves with others based on such a desire.

Sfard and Prusak (2005) conceptualized learning as filling a gap between actual identity (i.e., what a person is) and designated identity (i.e., what a person wants to/should be), defining identity as a collection of reified, significant, and endorsable stories about a person. For example, Student A may in the interview, “I am not good at division, but I want to be able to solve a long division problem by myself.” The sentences indicated his/her current identity (e.g., “I am not good at division”) and future identity (“I want to be able to solve a long division problem by myself”) as a mathematics learner. Since designated identities appear in future discourse indicating desires, constructing new identities would be associated with a desire for constructing positive self-evaluations. Social comparison might appear as an outcome caused by such a desire, and it might be considered as an instance which would support acts of constructing designated identities.

Sfard and Prusak's (2005) conceptualization of identity implies that learning is, to some degree, subject to desires or obligations to become better learners in terms of filling a gap between ones' actual identity and designated identity. We might be able to assume that social comparison is an outcome of classroom interactions through which students construct new identities as learners. Conceptualizing social comparison as an outcome of social interactions would be helpful for better understanding of how students' identities as mathematics learners are socially constructed. In the subsequent section, I explain social comparison with particular focus on developmental changes in social comparison and a desire to maintain positive self-evaluations. Those foci would be useful for understanding social comparison in classroom settings.

2.3 SOCIAL COMPARISON AMONG CHILDREN

Social comparison has been a frequent research topic in psychology since Festinger (1954) introduced social comparison theory. Many researchers investigated it in relation to academic performance/ability (e.g., Altermatt, Pomerantz, Ruble, Frey, & Greulich, 2002; Pomerantz, Ruble, Frey, & Greulich, 1995; Toyama, 2001). Dijkstra et al. (2008) reviewed the studies on social comparison particularly in classroom settings and provided a sufficient summary. Based on their meta-analysis on social comparison in classrooms, the following sections describe the characteristics of social comparison in educational settings.

2.3.1 Developmental changes

Social comparison is viewed as any process of relating one's characteristics/attributes to those of others. Festinger's (1954) classical theory of social comparison assumed that individuals tended to compare themselves with similar persons to meet their basic needs for accurate self-evaluation (i.e., similarity hypothesis), but many researchers are now supporting the view that individuals seek their biased/positive self-images through social comparison (Dijkstra et al., 2008).

The ability to compare with others is one of the developmental milestones and is necessary for young children to develop the sense of self. According to Dijkstra et al. (2008), even preschoolers are capable of comparing themselves with others to evaluate themselves. Their social comparison is usually more focused on knowing how well they can accomplish tasks rather than seeking for relative evaluations of performance.

Several studies (e.g., Frey & Ruble, 1985; Pomerantz, Ruble, Frey, & Greulich, 1995; Toyama, 2001) have shown that as children get older, their ways of engaging in social comparison change. For example, Frey and Ruble (1985) found that children used direct forms of social comparison (e.g., "I am the best," "Mine is better than yours") most frequently at the grade one and used them less frequently as they got older. Also they found that, at the grade one or two, children got more interested in performance evaluation and used social comparison information for self-evaluation.

According to Pomerantz et al. (1995), direct forms of social comparison were significant among young children, while indirect forms (e.g., progress inquiry such as "Where are you now on that page?") were used more frequently as they got older. In the same vein, Pomerantz et al. (1995) and Toyama (2001) discussed that, as children became older, they came to recognize the negative aspects of direct forms of social comparison and that they found indirect forms more useful for self-evaluation. Their findings suggested that as they grew, children became capable of making their social comparisons more socially acceptable.

Butler's (1992) study found that children under the age of 7 compared themselves with others to seek for the information about appropriate behavior or improving ability, while older children engaged in social comparison to gain positive evaluations/judgments or avoid negative evaluations/judgments. Butler described this difference in social comparison goals as mastery-oriented (or self-improvement) and performance-evaluation (or self-enhancement) respectively. This finding tells us that children's social comparison goals change around the age of 7 and that older children get more interested in showing their performances are superior to those of others.

2.3.2 Self-Evaluation Maintenance (SEM) model

The SEM assumes that people are motivated to maintain positive self-evaluations and that the relevance of performance domain/dimension and psychological closeness play a significant role in social comparisons (Beach & Tesser, 2000; Tesser et al., 1988). The relevance of performance domain refers to how important the domain is to determine one's self-evaluation, and psychological closeness to how psychologically close one feels the comparison target is. In the SEM model, one feels that his/her self-evaluation is threatened, when the other person outperforms in the domain highly relevant to the one (e.g., academics, sports), while the other's better performance does not hurt one's self-evaluation when the domain is not highly relevant. For example, when a student thinks that math is very important to him and his classmate does better than he in the math exam, he may feel that his self-evaluation is threatened. He may not feel that way, however, if math is not relevant to him and he thinks that baseball is more important to him. Moreover, when a person compares him-/herself with the outperforming other in the domain highly relevant to him/her, the closer the other is, the more his negative feeling is intensified.

Tesser and his colleagues argued that there are two psychological processes involved in social comparison from the SEM perspective. One is the comparison process and the other is reflection. According to them, the comparison process is activated when one "compares" oneself with the psychologically close other who outperforms in the domain highly relevant, and he or she is more likely to feel that his/her self-evaluation is threatened by the other's superiority. On the other hand, even though the psychologically close other outperforms, one's self-evaluation is not threatened if the performance domain is not relevant to him/her. Rather he or she is more likely to "reflect" the other's superiority and bask in it. In the SEM model, the relationship between self and other (or comparison target) plays a significant role in determining whether the person engages in comparison, which is likely to produce negative feelings such as envy and threat to self-evaluation, or reflection, which is more likely to generate positive feelings such as enhancement to self-evaluation.

2.3.3 Consequences of social comparison

Dijkstra et al. (2008) summarized consequences of social comparison in the following 3 domains: affection, cognition, and behavior. Concerning affective consequences, they stated that evaluative anxiety and stress tended to become significant when academic standards of the classroom were high. According to their summary, another affective consequence was that, in classrooms, students were more likely to engage in identification than contrast. As mentioned in the previous section, identification is more frequently observed when students engage in upward comparison. Identifying themselves with those who perform better, students seem to hope to receive better scores/grades like a comparison target. Dijkstra et al. mentioned that girls tended to respond empathetically, while boys did in an egocentric/hostile way. Also they stated that downward comparison generated positive feelings for those who were egocentric or wanted to be superior to others in tasks, but not for task-oriented people.

Cognitive consequences of social comparison are concerned with self-concepts. According to Dijkstra et al., social comparison influences students' academic self-concepts through either assimilation or contrast. That is, students tend to assimilate to (or identify with) others who can perform better as a sources of knowledge and motivations, while students' academic self-concepts are threatened when they compare (or contrast) themselves with those who can perform better. Dijkstra et al. also stated that students' academic self-concepts decrease when they are compared with those who perform better, while their academic self-concepts increase when they are compared with those who do not perform as well as they do. Another interesting result on cognitive consequences was that relative evaluation influences academic self-concepts more than absolute evaluation (Dijkstra et al., 2008).

Dijkstra et al. summarized that studies on social comparison in classrooms had focused on students' performance and that the studies found a positive influence on social comparison on academic performance. For example, group work (or working with someone) had more positive influence than working alone. Even when others were in the same room, positive influence was reported. In addition, Dijkstra et al. mentioned that upward comparison was more effective in classrooms (i.e., increase of academic performance) than downward comparison. For instance, when low-achieving students' best friends became more high-achieving, low-achieving students performed better academically, although they evaluated themselves less positively than low achievers with a similarly low-achieving best friend. However, Dijkstra et al. also reported that upward comparison had both positive and negative influences on students, depending on the standard of social comparison (i.e., particularistic or universalistic). That is, usually upward comparison had positive effects due to the assimilation to (or identification with) better-performing students, but when students felt that everybody in a classroom was better or performed better, they felt less competent. When students was (or felt) the lowest academically, their self-concepts also suffered.

2.4 OPPORTUNITIES TO LEARN (OTL)

OTL is closely associated with equity, which has been an important goal in mathematics education. The traditional view of OTL was based on the knowledge representation framework and assumed that students have equal OTL if they are exposed to the same information (Gee, 2008). Gee, however, pointed out that the traditional view of OTL did not pay attention to the complex issues about knowledge representations which may influence OTL (e.g., prior knowledge, effectiveness of representation, and difference between input and uptake). Rather he proposed the socio-cultural view of OTL and conceptualized OTL in terms of the relationship between learners and the learning environment.

According to Gee (2008), a socio-cultural perspective has assumed that students do not have the same OTL (Greeno & Gresalfi, 2008), for example, pointed out tasks and positioning as the factors influencing students' OTL. They argued that tasks that students work on determine how students participate in learning with particular attention to cognitive demands of tasks and agencies involved in performing tasks. According to them, cognitive demands are determined by students' prior knowledge and the content of task. That is, if students do not have enough prior knowledge, the task may be too difficult for them and they may not have opportunities to participate in productive discussions. Also, Greeno and Gresalfi discussed two kinds of agency: disciplinary agency, which students exercise in performing a mathematical procedure, and conceptual agency, which they need for formulating questions, developing and explaining strategies conceptually. They argued that those agencies were associated with students' OTL. For example, if students work on the tasks which only requires disciplinary agency, they may miss opportunities for conceptual thinking in learning.

In addition to cognitive demands and agency, Greeno and Gresalfi discussed the influence of positioning on OTL. In a classroom, students are not equally positioned for several reasons. Teachers may position some students in particular ways, for example, because those students know better than others and contribute to discussion with useful information. Or students may resist participating in a certain task or classroom discussions for other reasons. Those instances of positioning may influence some students' participation in activities and hinder their OTL in the classroom.

Esmonde (2009) also discussed OTL with particular attention to its fair distribution to students. Her focus on the fairness rather than equality brought in the issues of classroom ecology. According to her, classroom ecology influences students' learning, because it produces a particular type of interaction, positioning, and learning outcomes. For example, in a classroom where a particular ethnic group is dominant, the issues such as race and socio-economic status may be associated with students' learning outcomes or an achievement gap between racial groups. She stressed the importance of investigating the relationship between social categories (e.g., race, gender, socio-economic statues) and categories produced at school (e.g., ability groups, popularity, friendship) and how those two categories interact in terms of learning outcomes.

In conclusion, despite different foci, a socio-cultural approach agrees that students have different OTL in a classroom and that the interaction between students, tasks, and learning environments should be considered. OTL is not just the issue of the gap between what students know and what are assessed in tests. It is rather concerned with how students exercise their agencies in completing tasks, how they participate, and how they interact with people and environments for producing learning outcomes.

It would be useful to investigate how OTL plays a significant role in promoting students' learning of mathematics. Such an investigation would require a micro-analysis of moment-to-moment classroom interactions, and investigation of discursive positioning might be one of the approaches. Positioning consists of conversations and depicts how participants in a conversation are doing with their talk, from what position(s) they are talking, and what kind of storylines they are producing. A close look at positioning acts in a classroom would enable us to understand how students' participation is changing, restricted, and/or reinforced. Changes in students' participation can be considered as their learning trajectories and describes how their identities as learners change (Greeno & Gresalfi, 2008). Classroom discourse analysis would contribute to the understanding of not only classroom interactions but also students' participation in learning.

2.5 POSITIONING AS AN ALTERNATIVE APPROACH TO CLASSROOM DISCOURSE

In these days, the concept of positioning has appeared so frequently in educational research. Especially socio-cultural researchers seem to use this concept often, because they may think that the concept would help to explain a variety of students' participation in classroom discourse and how such participation is associated with the production of students' identities as learners.

Simply stated, positioning is an act in which people assign or are assigned particular positions within a conversation, that is, a joint action by conversation participants to locate each other in a particular storyline (Harré & van Langenhove, 1999; van Langenhove & Harré, 1999). Examining positioning means a close look at what people are doing with their talk (i.e., speech act), from what perspective they are speaking (i.e., position), and what story they are producing (i.e., storyline) within interactions. The application of positioning to investigations of classroom discourse would help to understand the complexity of interactions.

As an alternative approach to mainstream psychology, positioning theory was introduced Harré and his colleagues. In the subsequent section, I describe positioning theory and review how the theory was applied to educational studies.

2.5.1 Roots of positioning theory and its characteristics

Positioning theory finds its roots in discursive psychology which emerged through the paradigm shift called the "second cognitive revolution" which occurred as a reaction against the central-processing metaphor of cognitive psychology (Harré & Gillett, 1994; van Langenhove & Harré, 1999). A computer analogy typified by information-processing theory assumes that there is a mental structure (or schema) behind one's cognitive activities and, as a result, gave the impression that his or her cognition is universal, abstract, and individual (rather than social) in its nature. Those assumptions of cognitive psychology came to encounter a discursive turn in theory and method and were challenged by a new school of psychology called "discursive."

Quite literally, discursive psychology values a role of discourse in understanding psychological phenomena. As a result, discursive psychology assumes that (1) psychological phenomena, whether they are public or individual/private, are interpreted as properties and features of discourse, that (2) the public and/or individual uses of sign systems, which can be considered as a foundation of thinking, originates from interpersonal discursive processes, and that (3) the production of psychological phenomena in discourse depends on participants' skills, their social positions in a community, and unfolding storylines (Harré & Gillett, 1994). Again, those assumptions reject the notion of traditional psychology that there is a hidden mechanism inside human mind of controlling an inner mental state. Regarding the view of language, traditional psychology considers language as the representation (or expression) of human mind, while discursive psychology focuses on the individual use of sign system (e.g., language). In discursive psychology paradigm, attention is paid to what the individuals actually do with language, that is, discursive practices such as talking and writing.

The purpose of positioning theory is to explain how psychological phenomena are produced in discourse (Harré & van Langenhove, 1999). Therefore, if we want to know the process through which those phenomena (e.g., social comparison, construction of identity), we should take a close look at discursive processes such as how conversations develop, what discursive episodes emerge, and how they emerge. According to Harré and van Langenhove, there are three constitutive elements of conversations: positions, speech-acts, and storylines. In conversations, people (or participants in conversations) take up certain positions (e.g., storyteller, attentive listener, teacher etc.) or are positioned by others in conversations. As conversations develop, storylines are produced, changed, revised, maintained, and/or reproduced, because participants utter from a perspective accompanying a position taken up and positions usually include roles, responsibility, and moral obligations. This tells us that participants are constantly engage in positioning in conversations based on positions they have taken up (or are assigned by other participants) and talk from a certain point of view accompanying those positions. Positioning is defined as the "discursive process whereby people are located in conversations as observably and subjectively coherent participants in jointly produced storylines" (Davies & Harré, 1999, p. 37).

2.5.2 Similar concepts in positioning

It is important to clarify similar concepts for understanding better positioning theory. First, the concept of position is sometimes interpreted as something like role. According to van Langenhove and Harré (1999), positions are more dynamic forms of roles. Roles are static and stable but positions are dynamic and easily change in storylines. Due to the fluid nature of position, people may take up multiple positions and speak from multiple perspectives in a series of conversation. We might be able to view positions as "temporal" roles, which are subject to constant change and do not stay the same so long.

Another differentiation is necessary for using the word "storyline(s)." In the positioning literature, researchers interpreted storyline(s) differently. Wagner and Herbel-Eisenmann (2009) did not see storylines as a topical flow of stories but rather as "myths, or stories people live by." They also pointed out a conceptual similarity between their idea of storyline and "figured worlds" (Holland, Lachicotte, Skinner, & Cain, 1998), that is, a "socially and culturally constructed realm of interpretation in which particular characters and actors and recognized, significance is assigned to certain acts, and particular outcomes are valued over others" (p. 52). From their view, storylines were viewed as powerful discourse which would shape people's acts. While Wagner and Herbel-Eisenmann interpreted storylines in a broader sense, (Bomer & Laman, 2004) tried a narrower reading of storylines. They viewed storylines as relationships created between participants and what they say and write. More specifically, storylines are relational products which coordinate self, others and (oral and written) texts when people engage in positioning. Their way of interpreting storylines seems to pay more attention to the individual-collective nature of positioning suggested by Baker and Green (2011), since they did not end up simply identifying positions taken up by a particular person but rather looking at relational dimensions of positioning from multiple perspectives such as self, other, and their (oral and written) texts.

It is not my intention to decide which concept (position vs. role) and/or which interpretation of storyline is more useful. I believe both concepts and interpretations are necessary and help to understand positioning in different ways. For example, a broader reading of storylines is helpful for investigating a cultural context in which positioning occur, and a narrower reading is useful for exploring relationships among positioning categories (e.g. self positioning, text function etc.) at a micro level. The differentiation of the concepts and interpretations allows us to examine positioning events from different perspectives and has contributed to the better understanding of positioning itself.

2.5.3 Application of positioning theory to educational research

One of the advantages of using positioning is that it focuses on moment-to-moment interactions. By closely looking at how participants take up certain positions in particular storylines within discourse, positioning can describe the dynamics of interactions, for example, who has the power" and "who takes up what this/that position," "what one say from his/her position," and so on. Barnes (2004) investigated students' participation to high school mathematics classrooms to identity what positions would work effectively for collaborating learning. She identified students' positions (e.g., experts) which appeared within the classroom interactions. The results showed that it was desirable for students to be in a situation where they could move freely among the following positions, "Expert," "Critic," and "In Need of Help" and that students might lose an opportunity for collaboration if they stayed in one position. Barnes used positioning theory proposed by Harré and his colleagues and used it for identifying what positions students took up or were assigned in high school mathematics classrooms. In her studies, identifying students' positions helped her interpret not only how students collaborated but also in what situation their collaborative learning was effective.

Herbel-Eisenmann's (2009) study investigated teachers' language choice in the 8^{th} grade mathematics classrooms in terms of how teachers positioned textbooks. Using the idea of positioning, she focused on analyzing how teachers positioned textbooks rather than identifying social positions appearing during the classes. The results showed that there were several forms of positioning (e.g., privileging the textbook, aligning the teacher with the textbook, privileging the teacher, and aligning the teacher with the students). For example, when the textbook was read, teachers positioned the textbook as authoritative source to follow. Also, when the teacher made an additional comment on the textbook content, the teacher positioned herself as the authority that could add comments. Her study illustrated not only the dynamics of classroom interactions and teachers' language choices which would influence positioning textbooks and students. The result that teachers' utterances were involved positioning students and textbooks will help educators reflect their use of language in their own classrooms. Positioning will be a useful analytic tool for interpreting the complicated nature of classroom interactions.

Ritchie (2002) investigated the dynamics of interactions in an elementary school science classroom. He used positioning theory as an analytical framework and closely looked at how gender, status, and power relations were interwoven in the process of learning science. He used transcripts for classroom conversations for his analyses. The results revealed that students intentionally and strategically took up different positions (such as bossy students, victims, and good students) to the teacher and/or other students during the class. This study showed that classroom interactions were to large extent associated with power relations, which were made visible by acts of positioning.

The studies discussed above show that positioning made it possible to analyze the dynamics of moment-to-moment interactions. The micro-level analysis of discourse articulates not only what positions people took up but also how those positions functioned in the interactions. Positioning theory enabled researchers to investigate subject positions or identity, and this became possible since the theory viewed them as discursive products (or phenomena) emerging through interactions. The assumption that social/psychological phenomena emerge in discourse implies methodological potential and may help researchers investigate other issues from positioning perspectives if they view the issues as discursive products (or phenomena).

2.6 POSITIONING AND PSYCHOLOGICAL PHENOMENON

As Harré and his colleagues argued, positioning theory aims at understanding social/psychological phenomena emerging in discourse. Their argument allows for a new conceptualization of what has been investigated in the field of psychology by viewing it as social and/or psychological phenomena emerging in discourse. For example, it allows us to interpret something such as social comparison and the formation of identity as discursive products which are co-constructed by participants in particular discursive practices. The following sections describe how social comparison appears as a discursive product and how positioning helps to understand social comparison.

2.6.1 Modes of positioning

van Langenhove and Harré (1999) introduced different modes of positioning. Those modes include first-order/second-order positioning, moral/personal positioning, self/other positioning, and tacit/intentional positioning. According to van Langenhove and Harré's definition, first order positioning is the way by which a speaker positions him-/herself and others (or addressees) within a social space and moral order. That is, first order positioning is an initial act of positioning. First order positioning is usually categorized into either tacit or intentional. When one (Person A) tell or ask someone (Person B) to do something such as cleaning a room, Person A positions him-/herself and Person B in certain social space and moral order, for example, locating Person A as a parent who has the right to order Person B to clean a room and Person B as a child who has to follow his/her parent's order. If Person B does not question about cleaning a room or change the storyline produced (parent-child or discipline storyline), they are both located in first-order positions and accept their positions (e.g., parent, child) and moral responsibility (e.g., disciplining, obeying parents).

However, when first order positioning is challenged by Person B (e.g., Person B says, “Why should I clean a room?”), Person B engages in second-order positioning. This means that Person B’s first-order position is questioned and the storyline is also challenged. Person B may not be Person A’s child and not listen to Person A, or Person B may be playing a computer game and may not like to be disturbed by anyone. In a second-order positioning event, a storyline is usually subject to change.

In addition, van Langenhove and Harré (1999) differentiate modes of positioning based on moral orders in which people perform social actions. For example, when Person A (e.g., parent) says something to Person B (e.g., child) and Person B accepts it from their moral orders (e.g., disciplining, obeying parents) accompanied their first order positions, Person B engages in moral positioning. However, if Person B questions about what is told and does not accept first order positions for his/her personal reasons (e.g., “I don’t want to because I’m playing Super Mario now”), Person B engages in personal positioning.

In most cases, first-order positioning is tacit positioning. Since initial talk is usually uttered tacitly, it is not likely that people position themselves and the other intentionally in the initial talk. However, first-order positioning is sometimes done as intentional positioning, when a person’s intention is observed. For example, when a person makes fun of someone, tell a lie, and so on, he or she intentionally positions him-/herself and the other.

In sum, when people respond to first-order positioning regardless of the degree of ignitions, they have room for saying yes or no to their first-order positions and produced storylines for moral or personal reasons. A close look at modes of positioning shows that conversations consist of negotiations of positions and storylines, which make conversations dynamic.

2.6.2 Positioning and social comparison

Since social comparison is a directed to a comparison target, the comparison entails an alignment between a comparer and a comparison target. This alignment is a positioning act, and a comparer makes a different kind of alignment (e.g., upward, downward, lateral) with the target depending on his/her social comparison motive. In terms of producing alignments, we can approach to social comparison from the positioning perspective.

The following excerpt is from a 3rd grade mathematics classroom:

Teacher: OK, so you have one fourth, plus one fourth, plus one fourth.
In a way it's very similar, it looks different on your page, to this.
But, and, and it's a good way of thinking. She knew that four times
Four is 16, and she probably, you knew, even if you're not saying it.
That if you divided them into fourths, and you had four of them.

Student: Well, that's why I did so.

This short excerpt showed that the student accepted what the teacher previously uttered. In the positioning framework, the interpretation of the text above might be that the teacher took up positions as guide and evaluator to describe the strategy the student presented and made a comment on it, while the student was given a position as contributor to the class since her strategy provided a "good way of thinking" as the teacher acknowledged. In addition, the excerpt shows that positioning was interactive, since the teacher's utterance positioned the student (as contributor) as well as the teacher herself (as guide and evaluator).

In terms of social comparison motives, the student's utterance might be interpreted as an act of seeking for a positive self-evaluation, that is, the teacher made a positive comment on the student's strategy ("it's a good way of thinking") and the student aligned herself with the comment ("that's why I did so"). The students might be motivated by a performance-evaluation (or self-enhancement) goal to gain a positive judgment as a learner from the teacher.

When people talk, they utter from a certain position and their utterance is addressed to someone. This interactive nature of positioning helps us understand the relationship between the speaker and the addressee. For example, if Person A sees Person B as a rival in the performance domain which is important to Person A, Person A may feel that his/her self-evaluation was threatened and show envy to Person B. The storyline may be competitive in their conversation. On the other hand, if Person A admires Person B's outperformance in the same domain, Person A might view Person B as a role model to improve his/her ability by aligning him-/herself with Person B (upward comparison). Person A's utterances might be full of praises or his/her wishes to become Person B. The storyline appearing in this upward comparison will be different from the one which emerged in competitive relationships and look more favorable to both of them.

Although discursive approaches are not popular in social comparison research, positioning theory would provide a new possible way of analysis. Since positioning allows for a micro-level analysis of discourse (e.g., looking closely at constitutive elements of conversations such as positions and storylines), we would be able to understand the dynamics of human interactions and the social/psychological phenomena (e.g., social comparison) through emerging discursive practices.

Utilizing positioning theory with social comparison research, this study investigated how students engaged in social comparison through their discursive practices in their mathematics classroom. Social comparison is an important milestone in human development since it helps children understand themselves, which would contribute to the establishment of their identities.

Since mathematics education research (e.g., Bishop, 2012; Boaler & Greeno, 2000; Cobb et al., 2009; Sfard & Prusak, 2005; Wood, 2013; Yamakawa et al., 2009) has recently paid more attention to the role of students' identities in learning, the approach from social comparison to their identity construction in this study would provide a new perspective on mathematics learning. The next section describes the research questions in this study.

2.7 RESEARCH QUESTIONS

In response to the recommendations by the NCTM (1991, 2000), teachers are asked to create a learning community where students can actively participate in a wide variety of mathematical activities to improve their understanding of mathematical concepts. Therefore, teachers are expected to make their class more discussion-oriented. Also discourse is emphasized in the 2000 standards. Discussions have become more and more important in reform mathematics classrooms.

However, it is understandable that in discussion-oriented classrooms students may have some negative experiences accompanying social comparison. Active participation in discussion means that students are exposed to juxtaposition and/or comparison of different strategies for solving problems. Some of the ideas may be sophisticated or advanced in terms of problem solving, but others may not be. Students who elicit less sophisticated ideas may feel inferior or negative about themselves. Also, teachers may create a situation of social comparison. For example, what teachers say, even though they do not intend to compare students (i.e., tacit positioning), may trigger social comparison by referring to the contents or students who presented the contents previously and/or aligning one idea with another. In this way, social comparison would become an obstacle to reform effort.

In this study, I developed the following research questions in order to know how social comparison took place in the classroom discourse.

1. What is the nature of a teacher's instructional philosophy and how is the philosophy delivered to and reflected in the class discussions?
2. What do whole class discussions look like?
    - Are there any repeated patterns of interactions?
    - How does the teacher position students when they present their strategies and how does teacher positioning influence students' participation in discussions?
    - How do the students respond when the teacher positions themselves in terms of social comparison?

A teacher's instructional philosophy is useful for understanding the classroom culture, since it tends to have large influence on how she teaches students. If teachers follow the NCTM reform principles, for example, their classes would look different from those taught by teachers who do not conform to those principles. Understanding a teaching philosophy is, therefore, important for illuminating characteristics in a particular classroom. The second question, which is complemented by 3 sub-questions, aims at understanding the nature of classroom discussion. It would be useful for identifying whether a particular pattern of interaction took place during the discussion, since a pattern characterizes ways in which teachers and students interact for intellectual activities. If a pattern is observed, it would also important to examine how teachers and students positioned themselves and/or were positioned within such a pattern, since their acts of positioning determined not only teaching and learning in the classroom but also showed what else (or what other social phenomenon) was produced through interactions. The next chapter discusses details of the methods to answer the questions above.

3.0 METHODS

This chapter describes the methods and design to answer the following research questions:

1. What is the nature of a teacher's instructional philosophy and how is the philosophy delivered to and reflected in the classroom discussions?
2. What does whole class discussions look like?
    a. Are there any repeated patterns of interactions?
    b. How does the teacher position the students when they present their strategies and how does teacher positioning influence students' participation in discussions?
    c. How do the students respond when the teacher positions themselves in terms of social comparison?

The first section describes the research context such as a research setting, participants, and data collection. The second section explains the research design for this study. The last section is the overview of analysis and explains how I answered those questions.

3.1 RESEARCH CONTEXT

This study used the ethnographic data which were originally collected for different purposes (See Forman & Ansell, 2001; 2005). The data were collected from fall 1999 to winter 1999 in a private elementary school located in an urban neighborhood in a north-eastern state in the United States. The school was established more than 100 years ago and has a long history for educating young children. Most of the students were from upper-middle class families and their parents are highly educated. The class consisted of 7 girls and 10 boys who were all between 8 and 9 years old. Among 17 students, 10 students were European-American, 4 were African-American, and 3 were Asian-American. In the classroom, the students usually sat at 5 different tables as a group of 3 or 4 same-sex peers. However, they were allowed to move to another table or the floor to work with partners and/or alone depending on the assigned tasks.

Mrs. Porter[1] was in charge of teaching mathematics in this third grade class. She had a long teaching experience and taught at this school over 20 years. She did not follow a formal mathematics curriculum but rather taught using materials she developed and invented based on her experience and teaching journals. She highly encouraged students to be conscious about how they thought and used strategies which "made sense" to them whenever they solved problems and worked on other mathematical activities. Therefore, she valued processes of problem solving and paid close attention to students' thinking. She aimed at building a reform mathematics classroom since she wrote to parents of her students a newsletter which was entitled a "Reform Mathematics Teaching Philosophy: What I Believe about Teaching Mathematics." In the newsletter, Mrs. Porter emphasized the importance of students' sense-making experience during solving problems and their willingness to develop thinking and understanding.

[1] The names of the teacher and students in this study are all pseudonyms.

3.1.1 Data collection

The ethnographic data was previously collected for different research purposes (Forman & Ansell, 2001, 2005) during mathematics classes twice a week for four months (from September to December). The classes were observed, audio recorded, and transcribed. The field notes were also available and were used to support transcript production from audio tapes. Several formal and informal interviews with the teacher were also conducted, with the formal interviews being audio recoded, and transcribed. She also wrote to students' parents a newsletter which clearly addressed her teaching philosophy. This newsletter was also available.

The students' written work from each lesson were also collected. Since Mrs. Porter told students not to erase what they wrote in the process of problem-solving and even told them to use pens. Their written work helped to trace how they were trying to solve mathematics problems. In addition, the students were asked to describe the features of a good mathematical thinker.

3.1.2 Selection of class periods

Selecting class periods for analysis is crucial, since it directly affects results. I decided which classes to choose according to the following criteria.

The first criterion depended on the availability of transcripts. The dataset included transcripts of multiple classes, but not all the recorded classes were transcribed due to the audio quality. The original transcripts were made by one of the original ethnographers in this study, using the field notes and her memory to help her make sense of the audiotapes. For my analysis, therefore, I selected classes whose transcripts were available. Among 12 transcripts available, 4 were from September classes, 4 from October, 2 from November, and 2 from December.

The second one was related to the nature of the mathematical problem discussed in the class. For analysis, I selected classes where students did not work on questions with known answers (QWKA) or routine activities but rather more cognitively demanding problems. QWKAs do not promote discussions about problem-solving strategies because it is taken it for granted that such questions simply lead students to pre-defined answers. The following excerpt is an example of QWKA in an IRE sequence:

> T: They both have three digits. How do you spell digits?
> Ss: d-i-g-i-t-s.
> T: i-t-s. OK. They both have three digits.

On the other hand, a discussion about an invented strategy looks like in the transcripts as follows:

(Student is working on the subtraction problem, 1998-1789=209)

> S: Well, I put down the 1789.
> T: 1789.
> S: I plussed 11.
> T: And you added what?
> S: 11.
> T: 11. Why?
> S: So I could get to 1800.
> T: So you get to 1800. OK. You knew that 10 would get you to 1799 and one more would get you to 1800. OK.
> S: Then I added 98.
> T: So, then you got 1900 plus 98 equals 1998. And then?
> S: I added 11, a hundred, and 98.
> T: Then you added that up. And 1 and 8 is 9, and 1 and 9 is 10. And you got 209 also. That was faster, wasn't it? OK. Three different ways.

In the former excerpt, students' answers ("d-i-g-i-t-s") are subject to simple yes-no evaluations by teachers, which are commonly observed in IRE sequences. The IRE assigns fixed roles to both teachers and students and rarely locates them in a wide variety of positions. In the latter excerpt, the student was not just telling his answer but rather explaining his invented strategy with the teacher's prompts. In order to illuminate the dynamics of classroom discussions, I focused on the classes where students were allowed (or encouraged) to express their problem-solving strategies and their teacher placed the strategies on the discussion table.

Below are the classes I selected for analysis:

- Class on September 10, in which they discussed the problem, "You read for 15 minutes a day. How much time will you have spent reading one week?"
- Class on September 21, in which they discussed the problem, "The first post office in the U. S. was established in 1789. How long ago was that? How and tell the story of how you figured it out."
- Class on October 15, in which they discussed the problems, "A group of 252 ghosts were needed to haunt 9 cemeteries. How many ghosts went to each cemetery?" and "When Christopher Columbus began his journey, he had 51 men with him and 3 ships. There was one captain for each ship, and the rest were sailors. Each ship had the same number of sailors. How many sailors were on each ship?"
- Class on October 20, in which they engaged in the discussion of the "best" strategies for solving the problem of the weight of 18 brains when one brains weighed 3 lbs.
- Class on November 3, in which they discussed the problem, "The man who invented the sandwich was born on November 3 in 1798. Was that before or after the Jamestown Colony was begun in 1607? How much before or after?"
- Class on November 12, in which they discussed the problem, "We are going on a picnic. On one picnic there will be 18 people going and on the other picnic 16 people are going.

The problem is with the cafeteria people. They only made 12 sandwiches and in order to be fair, everyone has to share the sandwiches equally. Decide how to divide 12 sandwiches equally among 16 (and 18) people."

- Class of December 15, in which they discussed the problem, "(Twelve days of Christmas task, which is similar to the Hanukkah candle task) How many gifts did the true love send on the 12th day of Christmas? "
- Class on December 17, in which they discussed the problem, "The Eiffel Tower is 984 feet high. How many inches would that be?"

Since some of the problems listed above were solved with simple subtractions, they may look like QWKAs. However, I included them for analysis as long as two or more students were involved in discussions about how they thought and solved the problems.

It should be noted that October 20 class was different from the other classes in terms of the nature of the task. Usually Mrs. Porter introduced word problems and students were expected to work alone or with partners and present their solution strategies. However, the problem discussed on October 20 was provided with 5 solution strategies (See Appendix B) one week before, and students were expected to choose the strategy which they thought was the best and discuss why they chose it. The October 20 class was designed to promote students' reflexive and critical thinking about mathematical strategies (Forman & Ansell, 2005), while the other classes were instructed differently.

3.2 RESEARCH DESIGN: EMBEDDED, SINGLE-CASE DESIGN

This was the case study which relied on a secondary analysis of the ethnographic data previously collected in a 3rd grade classroom. The reasons for selecting a case study were derived from the arguments by Yin (2006) that the data collected in natural settings is relevant case studies which ask descriptive and explanatory questions, and that case studies are relevant for understanding a particular situation. This study used the data collected in a particular 3rd grade mathematics classroom in a natural setting and aimed at explaining how reform mathematics instruction influenced the classroom dynamics. The nature of the data and research questions was appropriate for case study methods, which helped to understand not only how the teacher implemented reform teaching but also how it influenced students' participation to the classroom discussions.

The research design of this study was a single-case design with embedded subunits. Mrs. Porter's classroom served as a single case and each student as a subunit embedded in the classroom. According to Yin (2006), there are 2 kinds of single-case studies: holistic and embedded case study. Yin argued that a holistic approach is relevant for investigating the whole nature or characteristic of the case (e.g., organization, program) and an embedded case study is relevant when researchers need to pay attention to subunits within the single case (e.g., employees of the organizations, services of the program). This study used the data collected only in Mrs. Porter's classroom (single-case) and aimed at examining the dynamics during classroom discussions created by the interactions between Mrs. Porter and her students (embedded subunits). Since the aim of the study required significant attention to the students as well as the classroom itself, I selected an embedded, single-case design for this study.

While embedded case studies enable researchers to examine a single case in depth with specific attention to subunits, they have pitfalls as well (Yin, 2006). According to Yin, a common pitfall is that researchers focus too much on subunits and fail to coordinate subunit-analyses with those in a larger unit. In order to avoid this pitfall, this study had 2 main research questions, one of which asked the nature of Mrs. Porter's reform mathematics classroom (i.e., "What was the nature of a teacher's instructional philosophy and how was the philosophy delivered to and reflected in the classroom?") and the other of which asked how students as subunits were positioned by Mrs. Porter in the classroom with the reform culture (e.g., "How did the classroom discussions look like?"), consolidated by a set of sub-questions (e.g., "Were there any repeated patterns of interactions?" "How did the teacher position the students when they presented their strategies?" "How did the students respond when the teacher positioned themselves?").

Some may be questioning the degree of generalizability due to the focus on particular cases instead of conducting highly controlled experiments. However, Yin (2006) argued for analytical generalization, not statistical one, in case studies and clearly stated that a case study was an empirical inquiry and comprehensive research strategy, which investigated contemporary phenomena in real-life contexts. According to Yin, case studies can be generalized to theoretical prepositions, which he defined analytical generalization, while statistical generalization to populations. That is, results of a case study may (or not) find similar phenomena supported by a particular theory, and they may become generalizable to the theory (or a rival theory).

This is an embedded, single-case study which aimed at examining the dynamics of classroom discussion in a reform-oriented elementary math class. In order to investigate the classroom dynamics, this study particularly focused on (1) the teacher's teaching philosophy and

(2) positioning acts of the teacher and the students. Segments from classroom discussions from the lessons selected served as sub-cases, and I examined those sub-cases from multiple data sources. In the next section, I present the overview of analysis and explain how I investigated those sub-cases.

3.3 OVERVIEW OF ANALYSES

In this section, I describe the overview of analysis I use in this study. The analysis was largely drawn from positioning theory. Positioning theory provides a useful framework for interpreting interactions at a micro level and I believe that the concept of positioning was a good analytical tool in this study.

The analyses consisted of two phases, which corresponded to the research questions. The first phase focused on identifying Mrs. Porter's teaching philosophy and examined how she had delivered her philosophy to the class and utilized it in her teaching. This phase of analysis aimed at understanding the classroom culture, including norms particular to her class, since it has influenced on how students (are expected to) act more or less. The second phase focused on investigating the dynamics of classroom discussion from positioning perspectives. This phase had three supplementary questions: the first one identified whether or there were any repeated patterns of the interactions, the second one focused on identifying whether or not a particular pattern of interaction existed, and the last one examined how the teacher and students positioned themselves or are positioned during a classroom discussion.

3.3.1 Phase One: Investigating teaching philosophy

The first phase of the analysis was conducted to investigate the nature of Mrs. Porter's teaching philosophy and how it was delivered to the classroom. At this phase, I analyzed 3 sets of transcripts of the interviews with Mrs. Porter and 8 sets of transcripts of whole class discussions.

In order to investigate how her teaching philosophy was delivered to the class, I used the idea of "lexical bundle" (Herbel-Eisenmann, Wagner, & Cortes, 2010). According to Herbel-Eisenmann et al., lexical bundles referred to "groups of three or more words that frequently recur, multi-word groupings, in a particular register" (p. 25) and were categorized into the following types: stance, discourse organizing, referential, and special conversational function. In this part of analysis, I focused on stance bundles, which describe personal attitudes in addition to personal feelings and value judgments/assessments. Herbel-Eisenmann et al. stated that attitudinal stance bundles had several subcategories (e.g., desire, obligation/directive, intention/prediction, ability). According to them, the obligation/directive category described a "commitment to something" or provides "instructions that one might follow" (p. 26) and almost always had 2nd person pronoun (i.e., you) as the subject or direct object rather than 1st pronoun "I." For example, the phrases such as "I want *you* to do," "*You* (don't) need to do," and "Do *you* have to" fell into this category. Since Mrs. Porter's teaching philosophy described what she wanted her students to do/become, the obligation/directive category was helpful for this part of analysis. The stance bundles functioned as linguistic markers to identify evidence that Mrs. Porter told her students about what they should do to be(come) a good mathematical thinker. The following excerpt is an example of attitudinal stance bundle (obligation/directive):

> T: ….. What ***I want you to do*** is show the whole story of how you're figuring this out. It's like writing a math story, in a way. ***You don't have to*** write a lot of words. But how exactly what you're thinking.

The two attitudinal bundles (obligation/directive) show what students should (not) do in Mrs. Porter's class, that is, designated identities as math learners, by indexing "you" as a the subject/direct object to conform to the identities. This analysis told us whether or not the students complied with the identities by taking risks in trying new strategies, being able to construct knowledge, and so on.

Especially for the analysis of interview data, I used the "I-poem" method (Gilligan, Spencer, Weinberg, & Bertsch, 2003). According to Gilligan et al., it aims at listening to participants' voices told by the first-person forms and understanding how they talk about themselves. The rule of I-poem method includes highlighting any "I" which is used with verbs and other important phrases and organizing like a stanza so that the I-phrases highlighted are juxtaposed in order of appearing in the texts.

Using the I-poem method, I selected from the interview transcripts the sentences and phrases which included the pronoun "I" to see not only how Mrs. Porter talked about herself but also how she thought about teaching and learning mathematics. Then I utilized Mrs. Porter's I-poems with positioning analysis to examine what kind of position she took up in her I-poems. This synthesis was useful since it provided better understanding of the complexity of Mrs. Porter's discourse about her teaching philosophy.

3.3.2 Phase Two: Investigating the nature of whole class discussions

This phase aimed at investigating the nature of whole class discussions in Mrs. Porter's classroom. Using positioning theory as an analytical framework, I analyzed the transcripts of classroom discussions from the perspectives of positions, speech-acts, and storylines in order to articulate how Mrs. Porter and the students engaged in interactive positioning. In other words, I investigated the nature of whole class discussions from their episodes of positioning.

The subsequent sections describe the unit and sub-unit of analysis and the supplementary questions to ask how classroom discussions looked like. Those questions aimed at identifying whether there were patterns of interactions and investigating how Mrs. Porter positioned the students and how they responded to her positioning.

3.3.2.1 Unit and sub-unit of analysis

Yin (2006) cited that a single case served as a unit of analysis, which means that Mrs. Porter's classroom is a unit of analysis of this study. This single case had a plenty of segments of verbal interactions between Mrs. Porter and student(s) and/or among students across classes, and the same students appeared multiple times in the interactions with Mrs. Porter within a particular class and/or across classes. In this sense, the sub-unit of analysis was defined as an interactional segment of verbal interaction in which a designated student expressed to Mrs. Porter and other students what (s)he did and/or thought about to answer a math problem during a whole class discussion. In each segment, a student talked about a newly developed strategy, or was not able to finish explaining what (s)he did to solve the problem. Also, when a student said something in the segment, Mrs. Porter verbally helped or did not help him/her present his/her idea. Another student may cut in during the presentation as well. The segments, therefore, usually consisted of multiple turn-takings between Mrs. Porter and individual students who volunteered to talk or were called on to talk.

In order to determine a sub-unit of analysis, it was necessary to decide when a designated student started presenting what (s)he did and when (s)he finishes his/her presentation. Focusing a sequence of turn-taking was useful for making such a decision. In the IRE/F framework, for example, teachers initiate interactions by asking a question, calling on a certain student to answer a question, and so on. Then, the student responds to the question, and the teacher evaluates or gives a feedback about the response. Attending to who initiates and ends an interaction was useful for determine a unit of analysis. Also, contextualization cues, which usually appear when a context changes, were another help for identifying a unit of analysis. The cues were a form of inviting students to a discussion (e.g., “Who wants to talk first?” “Did anyone solve the problem differently?”) or a word/phrase/sentence to imply a context shift (e.g., “OK.” “Let’s move on the next one.”). Since it was highly likely that Mrs. Porter encouraged as many students as possible to participate in a whole class discussion, there were multiple sub-units of analysis appearing in a class selected.

As Yin (2006) argued for the importance of the integration of embedded sub-units to a case for a detailed analysis, each interactional segment were grouped by students to summarize the analyses of the segments. The students, therefore, served as the first-level subunits, and the interactional segments as the second-level subunits. The following diagram (Figure 1) shows the structural description of (sub-)units of analysis of this study.

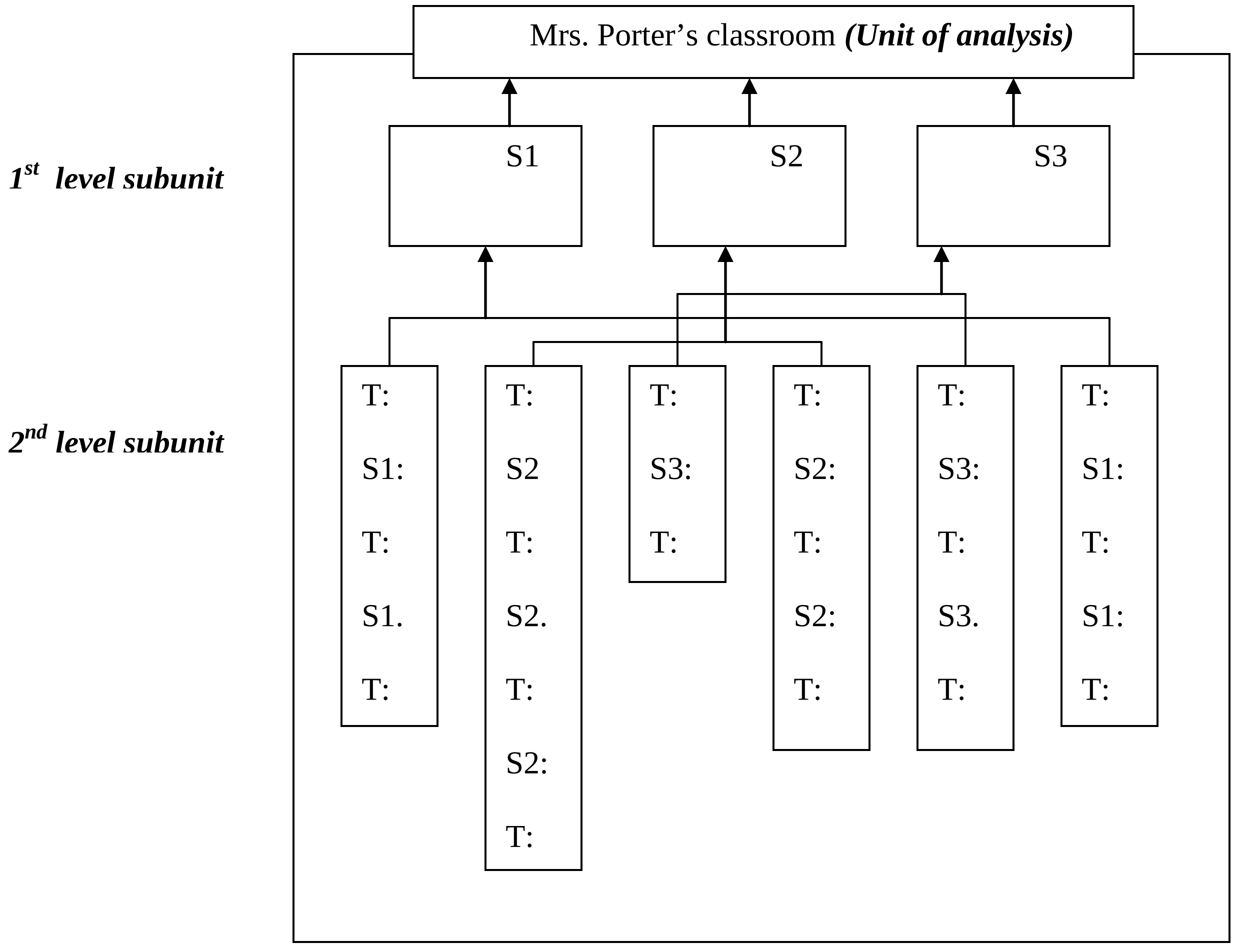


Figure 1. The structural description of (sub-)units of analysis

3.3.2.2 Investigating patterns of interactional segments

Investigating whether there were any repeated patterns of interactions in the segments was the next focus on the analysis. I used two concepts, which are (1) the IRE/F (Mehan, 1979; Wells, 1993) and (2) revoicing (O'Connor & Michaels, 1993, 1996), and examined whether the interactional segments fitted in the patterns of either IRE/F or revoicing.

The IRE/F is a triadic sequence commonly seen in classroom discourse, which is characterized by the pattern of initiation (by teacher), response (by student), and evaluation (by teacher) or feedback/follow-up (by teacher). Students' responses are subject to teacher evaluations or comments/feedback in the IRE/F sequence, and the questions initiated by teachers are not the one which promote a discussion but rather expects simple factual answers. The number of turns is quite fixed, and there are few variations of turn-taking in the sequence. In the IRE/F, teachers and students have fixed roles. Teachers are transmitters of knowledge and students are (passive) recipients of it. Here is an example of IRE/F:

T: How many days are there in a year? (Initiation)
S: 365. (Response)
T: Right. (Evaluation)

On the other hand, revoicing sequence does not fall into a simple triadic movement and has more turns. According to O'Connor and Michaels, revoicing is a discursive framework in which a student's position/idea is aligned with another by a teacher's repeating or recasting what the student says. Revoicing sequences look different from those of IRE/F due to the frequent appearance of "so," which serves as a warranted inference. O'Connor and Michaels stated that a warranted inference (i.e., so) shows that "the speaker is linking her utterance to that of the previous speaker and is making an inference that she believes to be warranted based on that previous utterance" (p. 323). The use of "so" indicates a connection between a current utterance and a previous one. The following excerpt is an example of revoicing:

1 T: Now, do 15 times 7 for us.
2 S: Well, 5 times 7 is 35. I left the 5 there and put the 3 on top of the 1.
3 T: OK.
4 S: Then I did 7 times 1 is 7.
5 T: Uh-huh.
6 S: Plus 3 equals 10.
7 T: OK. ***So***, you did it exactly the same way he did.

The roles of teachers and students are different in the revoicing modes. Teachers do not simply evaluate students' answers but rather facilitates students' responses, and students are given more opportunity to talk about his/her answers.

The sequence with revoicing looks different from the IRE/F by the length of sequence and nature of teacher's utterance. For example, the excerpt above has 4 turns by a teacher and does not fall into a simple IRE/F pattern. Among those 4 turns, the first one serves as initiation (I), the second and third ones as prompting (P), and the last one as evaluation/feedback (E/F). The example shows that the longer the pattern is, the more interactive the segment is, and the I-R-P-R-P-R-E/F pattern gives an opportunity to explore a problem-solving strategy to both the teacher and other students in the classroom. According to (Scott, Mortimer, & Aguiar, 2006), patterns of interaction articulate the level of teacher control over lessons. They argued that the most authoritative pattern was the IRE, which rarely allowed exploring or juxtaposing ideas, and that alternative patterns such as I-R-P-R-P-R-E and I-R-P-R-P-R- were more dialogic and helped students to talk about and elaborate their ideas. The frequent occurrences of alternative pattern would provide evidence of whether or not the class was interactive enough to let students talk about their strategies.

3.3.2.3 Analysis categories of positioning analysis

In positioning theory, conversations can be represented as the positioning triad, that is, the tri-polar structure (van Langenhove & Harré, 1999), which consists of positions, speech-acts, and storylines. This means that conversations have three constitutive elements, and that people take up a certain position and/or are assigned to a particular position within a storyline. Since positions are not fixed but rather change as a storyline develops, people may be assigned to and/or take up multiple positions in a conversation.

Position. A position is one of the three constitutive elements of conversation and a dynamic alternative to the concept of role (van Langenhove & Harré, 1999). Positions are not fixed or stable, but rather changeable and negotiable in a storyline. Therefore, people may not necessarily maintain the same position, but may have multiple positions for various reasons.

In this category, I looked at a position (or positions) that was (or were) assigned to the speaker in each of his/her utterance. Since people located themselves in relative to others in a conversation, we assumed that positions identified here would reflect the dual perspectives, i.e., speaker him-/herself and others. When examining what position(s) could be observed, I tried to identify the position(s) from the dual perspectives, that is, I asked, "When a certain position was assigned to a speaker, what position did the other person take up?"

In this study, I categorized all the possible positions of the teacher (Mrs. Porter) and her students into several categories such as advanced student, significant contributor, role model, participant, struggling student (for students' positions), initiator/facilitator, guide, and evaluator/feedback giver (for teacher's positions).

Identifying positions required attention to pronouns, especially plural pronominal form. A teacher, for example, tends to use "we" if (s)he wants to position him-/herself as one of the participants/learners in a class. In this case, the position might be different from the one when (s)he teaches in a authoritarian way. He or she, however, uses "we" as well by aligning him-/herself with some unnamed persons in order to add authority to the activities that (s)he wants the students to do (Pimm 1987 as cited in Rowland, 2000). The uses of "we" by teacher require consideration of with whom the teachers associate themselves.

In addition, one of the positioning genres Bomer and Laman (2004) mentioned might be of some help for identifying positions. The genre is called the "assignment of role ("you be x")."

Roles are slightly different from positions, according to van Langenhove and Harré (1999), but both of them share some similarities in terms of assigning.

Speech-act. Speech-act is another constitutive element of conversation (Harré & van Langenhove, 1999). According to Harré and van Langenhove, a speech-act means that people do something by saying something. Since speech-acts are closely related to positions and storylines, what a speaker is doing in his/her talk for various reasons is likely to affect the subsequent positioning. It may change a storyline, align the other with a different position, or affect the relationship between a speaker and the other. Speech-acts are a useful concept to interpret what a speaker is doing by his/her talks.

In this study, I categorized all the utterances (or turns taken by either Mrs. Porter, an individual student, or multiple students) into several groups. The categories included: explain strategies, respond to teacher, look for words, align oneself, challenge utterance (for students' speech-acts), initiate, clarify, revoice, prompt, compare, validate, negative comments, summarize, interrupt, and talk about norms (for teacher's speech-acts).

Storyline. As one of the constitutive elements of conversation, storylines are produced, changed, or challenged by people in a conversation, and people position themselves or are positioned by others in a storyline (Harré & van Langenhove, 1999). Bomer and Laman (2004) expanded this conventional idea of storyline and defined them as "what creates the relationship among self, other, and text" (p. 439)." To differentiate Bomer and Laman's interpretation of storyline from van Langenhove and Harré's definition, I called a storyline category "storylines-as-relationship" in this analysis.

In the storyline-as-relationship category, I closely looked at a speaker (= self), the addressee (= other), and 3rd person(s) mentioned in an utterance (= other), and then I examined what relationship was created among them through the utterance. Usually in one-on-one interactions between teachers and students, a simple teacher-student relationship (i.e., expert-novice relationship), is easy to observe. However, if either a teacher or a student mentions 3rd person(s) is involved in the interaction for some reason, the teacher-student relationship would be influenced by the positioning of the 3rd person(s) mentioned. Especially when a teacher compares ideas from two different students and one of them is more sophisticated that the other, it is likely that a storyline will become somewhat competitive. Therefore, it is necessary to pay attention to whether 3rd persons are mentioned in terms of the shift of relationship.

The nature of storyline-as-relationship was determined by second-level subunit of analysis (i.e., interactional segment between Mrs. Porter and a particular student) and categorized into 5 categories, including “authoritative,” “guiding,” and “collaborative,” which were modified based on the participant frameworks discussed in Tabak and Baumgartner’s (2004) study, and “self-protecting” and “competitive,” which especially focused on social comparison with the presences of 3rd persons as comparison targets.

Although I used Bomer and Laman’s notion of storyline as an analysis category, this does not mean that I would reject a broader interpretation of storyline as Wagner and Herbel-Eisenmann (2009) viewed it as a dominant discourse, or “myth.” I believe that a wide-reading of storyline is useful for understanding an interactional context of positioning.

3.3.2.4 Investigating teacher positioning

In whole class discussions, Mrs. Porter interacted with multiple students and positioned them in a certain way. Since Mrs. Porter strongly encouraged students to present how they solved problems (or what they tried to do when they could not finish problems), she positioned the students various ways while they were talking about their strategies. For example, she positioned some students as advanced when they used abstract strategies, while she negatively evaluated those who used advanced strategies without understanding what they were doing. To articulate Mrs. Porter's positioning, I picked up the following 3 from the speech-act categories mentioned in a previous section. Those categories were: interrupting, validating, and comparing. I chose them because it was those categories that would separate particular students from others.

Interrupting. Interruption usually occurs in a form of overlapping speech, and it is characterized by imbalance or asymmetrical communication between speakers, where a speaker, for example, frequently overlaps and the other often yields (Tannen, 1993). According to Tannen, in order to determine whether overlapping speech is interruption, it is important to pay attention to the context, speakers' habitual styles, and the interaction of their styles. Tannen mentioned that overlaps are more likely to be cooperative in casual conversations or when speakers are highly involved in conversations, and that overlapping speech is more likely to become interruption when speakers' habitual styles are different.

Considering Tannen's arguments about overlaps, I defined interruption in this study as Mrs. Porter's overlapping speech that prevented a student from finishing his/her utterance in their interaction during a whole class discussion. It would be useful as well to investigate the context where overlapping speech occurred in terms of classroom norms, because the norms usually determine what a class looks like and how teachers and students in the class should be. Mrs. Porter's interruption might be related to some of the norms.

Validating. If interruption takes place in a classroom, so does validation. By validation or validating, I meant that Mrs. Porter validated a particular student or strategy by making a positive comment. In order to identify the events of Mrs. Porter's validating students, I provided a coding scheme based on the norms which appear in the newsletter that Mrs. Porter wrote to parents. According to the norms which appeared in it, students were expected to take risks in developing new problem-solving strategies and become good at not only explaining their strategies but also making judgments about the efficiency of different strategies. In order for students to be able to explain their strategies, Mrs. Porter thought that they should be conscious about their thinking process, and that writing and discussion were necessary for successful math programs. It seemed that the ability to explain a strategy was crucial for students to be recognized as a "good" mathematical thinker in her class.

Since the norm which appeared in the newsletter showed what Mrs. Porter valued in a classroom, it was possible to speculate that she would validated the students who she thought conformed (or were trying to conform) to her teaching philosophy. In her class, students' ability to explain their strategies means that they were conscious about their thinking process. Therefore, I was able to assume that Mrs. Porter would validate those who were able to explain their strategies as "qualified" students. Based on this, the criteria for analyzing Mrs. Porter's validating strategies were as follows:

- Whether or not a designated student was able to explain his/her strategy in the segment of interaction with Mrs. Porter
- Whether or not Mrs. Porter made a positive comment on the student's explanation during the interaction with him/her

The interpretation of validation would be more relevant if it is utilized with that of interruption, and vice versa, since both of them would serve as contrastive examples to each other.

Comparing. Comparing was another event I investigated to identify Mrs. Porter's positioning pattern. A dictionary (The Oxford Dictionary of English 2nd ed.) defines "compare" as "estimate, measure, or note the similarity or dissimilarity between." In order to code comparing events, I approached by thinking, “What did Mrs. Porter do for the purpose of comparing strategies?” What she did might include juxtaposing multiple strategies from different students and aligning one student with another who expressed a similar/different strategy. Therefore, I analyzed comparing events from the following two perspectives: juxtaposition and alignment. The followings were criteria used for this part of analysis.

- Whether or not Mrs. Porter juxtaposed multiple strategies from different students
- Whether or not Mrs. Porter aligned one student with another who expressed a similar/different strategy or others (e.g., adults in general)

In order to investigate comparing events, the concept of “revoicing” (O’Connor & Michaels, 1993; 1996) was also useful. Revoicing includes positioning structurally.

1 T: Now, do 15 times 7 for us.
2 S: Well, 5 times 7 is 35. I left the 5 there and put the 3 on top of the 1.
3 T: OK.
4 S: Then I did 7 times 1 is 7.
5 T: Uh-huh.
6 S: Plus 3 equals 10.
7 T: OK. ***So***, you did it exactly the same way he did.

The alignment observed in the excerpt above (Turn 7) is a form of revoicing and indicates that a student was located in the same position of the other student who did 15 times 7 in that way. That is, alignments are positioning acts attained by revoicing.

3.3.2.5 Investigating students’ reactions to teacher positioning

In whole-class discussions, a teacher positions students for certain instructional purposes. For example, a teacher might align one student with another who elicits a similar solution in order to illuminate similarities of ideas, or the teacher might reject a certain idea expressed by a student as “wrong” or “less accurate” for evaluation. Classroom discussions usually include such positioning acts by teachers. These kinds of teacher positioning are well-acknowledged instructional discourse in classrooms, and we would hardly question the effectiveness and consequences.

While some students may accept teacher positioning without a doubt, others may not for various reasons. For example, they may react to teacher positioning by challenges or silences in order to show that they are not willing to conform to teacher positioning just because they do not agree with the teacher. Or they may challenge teacher positioning in order to maintain their self-evaluation since they feel that what the teacher said threatens their social status in class. Like this, teacher positioning is able to show a lot more when we investigate it through adding students' points of view.

In order to analyze how the students react to Mrs. Porter's positioning, I investigated the students' acts of positioning from two perspectives: accepting and challenging (or rejecting). According to van Langenhove and Harré (1999), people engage in second-order positioning when they challenge an initial positioning. This mode of positioning, or second-order positioning, is a speaker's representation of rejecting the previous positioning and shows that the speaker thinks a storyline negotiable. The followings are the criteria second-order positioning:

1. Whether a student uses negative words (e.g., not, never, no etc.) to respond to the former positioning
2. Whether a student uses paradoxical conjunctions (e.g., but, however etc.) to respond to the former positioning

In the following excerpt, for example, a student challenges the teacher's positioning.

S: I did it in a different way. I did it, I wrote down, 15, 7 times down.
T: Yes, I know. 15 – 15- 15 – 15 – 15, 1, 2, 3, 4, 5, 15 and 15.
And you put pluses?
S: ***Not*** exactly like that.

However, if a teacher expects students to answer “no” when asking a question, the students’ saying “NO” is not second-order positioning, since they don’t think that a storyline should be negotiated. For example, “No” in the second line is not the marker of second-order positioning.

> T: Was he a bad mathematician?
> Ss: ***No***.
> T: No, he was a good one (= mathematician).

Another example of second-order positioning is as follows:

> T: …….. When you, if you have something, and you want to take something away from it, what you have, has to be bigger than what you’re taking away. Right? This is a lot bigger, 1900 is more than 1700.
> S: ***But*** then I switched it around the second time.

If students’ responses do not fall into the criteria above, I viewed all of them as evidence that students accepted or agreed with the teacher positioning. One of the reasons why I viewed this way came from the nature of elementary classroom. It is common in elementary school classrooms that teachers serve as the authority and have much more control and power than students. In such a situation, students can hardly challenge their teachers as long as they do not have a strong reason for doing so. Therefore, I rather categorized students’ responses to teacher positioning into either challenging or “not” challenging.

3.3.3 Identifying social comparison

The investigation of students’ responses to Mrs. Porter’s positioning was utilized with social comparison in order to articulate in what kinds of positioning context the students engaged in social comparisons.

In order to select the students' social comparisons episodes, I drew on Wood's (1996) summary of ways of responding to social comparison. According to Wood, people react to social comparison cognitively (e.g., self-evaluating, distorting/refuting the comparison), affectively (e.g., feeling jealous/proud), and behaviorally (e.g., imitating, conforming, joining the group). Based on this summary, I chose students' utterances which indexed social comparison. Then I examined social comparison goals/motives behind the statements. Social comparison theories point out 3 main motives for social comparisons, which are self-evaluation, self-improvement, and self-enhancement (Dijkstra et al., 2008). I closely looked at the statements in terms of those motives.

It was useful for the analysis to examine social comparison motives in positioning contexts, since positioning episode showed in what position people were located and those positions indicated the relationship between those people, including a power relationship and social status. Articulating the relationship between people also helped the analysis of social comparison motives, which usually entail the directions to social comparison targets such as lateral comparison for self-evaluation, upward comparison for self-improvement, and downward comparison for self-enhancement (Dijkstra et al., 2008).

3.3.4 Summary of analysis

The first phase of analysis focused on Mrs. Porter's reform-oriented teaching philosophy and examined what it was about and how it was delivered to the students in her class. This phase was important since it provided a large amount of information on the research context and then served as a foundation for subsequent analyses. In order to examine how Mrs. Porter communicated her teaching philosophy in the class, I used the concept of attitudinal stance bundles, which refers to recurring phrases when people commit to something or tell someone what to do. Since her teaching philosophy described what students were supposed to be, Mrs. Porter communicated it in a class by utilizing stance bundles such as “I want you to …,” “You need to …,” and so on. The stance bundles themselves included positioning structurally, and the bundles identified in this phase helped subsequent positioning analysis. The data sources of the phase one analysis were a newsletter to parents, where Mrs. Porter wrote about her teaching philosophy, the transcripts of interviews with her, and the transcripts of her math classes.

In addition, the “I-poems” method was used at the first phase to investigate Mrs. Porter’s thoughts about not only herself but also teaching mathematics. I used this method especially for the analysis of interview data and found that she took up many different positions throughout the interviews. The evidence that she positioned herself differently indicated the hybridity of her discourse about being a teacher and teaching mathematics.

The focus of the second phase was placed on positioning analysis to investigate the nature and characteristics of interactions between Mrs. Porter and her students. Subunits of analysis were interactional segments between Mrs. Porter and a designated student during a classroom discussion. Since Mrs. Porter usually encouraged and helped the students to explain their strategies for discussions, the interactions included multiple sets of time segments in which Mrs. Porter frequently revoiced the designated students’ utterances. Therefore, the discourse patterns in the segments did not look like a typical IRE/F sequence, but rather more complex modes such as I-R-P-R-P-R-P-R-E/F. Turn-taking appeared more frequently than in the IRE/F. In addition, Mrs. Porter seemed to allow another student to join the interaction if necessary. Also, it seemed that Mrs. Porter tried to hear about strategies from as many students as possible.

For analysis, there were the following analysis categories of positioning: position, storylines-as-relationships, and speech-acts. All the categories were from the tri-polar structure of conversation (Harré & van Langenhove, 1999). In order to investigate the nature of interactions, three supplementary questions were provided. The first question asked whether there were any specific interactional patterns in Mrs. Porter's interactions with the students, and the second one asked how Mrs. Porter positioned the students, and the third one asked how they responded to her positioning in terms of social comparison. Examining interactional patterns was important since it showed us Mrs. Porter's typical interactions with her students. In positioning analysis, I investigated the students' positioning acts in terms of second-order positioning (= challenging a previous positioning) and those of Mrs. Porter in terms of whether she interrupted, favored, or compared the students. In order to code second-order positioning, I looked at whether the students responded to Mrs. Porter's positioning with negative forms (e.g., no, never) or paradoxical conjunctions (e.g., but, however). Such students' responses were closely related to Mrs. Porter's positioning acts such as interrupting, validating and comparing, which might appear when she practiced her reform philosophy. I examined interrupting in terms of overlapping speech, validating in terms of students' ability to explain what they did for problem-solving, and comparing in terms of juxtaposing and/or aligning multiple strategies.

For interpretation of results of positioning analyses, social comparison was one of the focal points. In a reform classroom, as Mrs. Porter stated in the letter to the students' parents, discussions are highly valued. This implied that, once ideas and strategies were expressed in public, they were subject to comparison and/or juxtaposition. Mrs. Porter's teaching technique such as comparing (or contrasting) might or might not be associated with students' social comparison behaviors. How students reacted to Mrs. Porter's comparing act was crucial for interpretation. Also, identifying whether there were any students whose attendance levels were different due to a certain student's presence or absence provided some clues for examining social comparison.

Another focal point was on whether Mrs. Porter's interrupting or validating was related to social comparison among the students. Since validating and interrupting highly affect students' social statuses, such positioning acts would be related to the students' social comparison behavior due to status changes which might (or might not) be triggered by Mrs. Porter's acts of giving negative or positive comments.

Lastly, I focused on the methodological potentiality of positioning to analyze classroom discourse and its accompanying phenomena such as social comparison. Discursive approach to the analysis of social comparison, which I conducted here, seemed quite unique and would give a new direction to social comparison research.

3.3.5 Triangulation of coding

Triangulation is crucial for obtaining the reliability of data analysis. In this study, I triangulated the analysis by (1) drawing on multiple data sources to answer each of the research questions and (2) double-coding the data. The following table (Table 1) describes the summary of triangulation.

Table 1. Triangulation matrix

Research Questions	Data Source			
	Transcripts	Field notes	Interviews	Newsletter
<u>Teaching philosophy</u>: What norms did Mrs. Porter mention?	X		X	X
<u>Interactional patterns</u>: Are there any repeated interactional patterns in whole class discussions?	X*		X	
<u>Teacher positioning</u>: How did Mrs. Porter position students?	X*	X	X	
<u>Social comparison</u>: How did students respond to Mrs. Porter's positioning?	X*	X		

* These analyses were subject to double-coding.

Since there were two phases of analysis in this study, each phase had a different focus on analysis. The first phase investigated Mrs. Porter's teaching philosophy in terms of norms which appeared across the data. Since norms were easily identifiable through predetermined stance bundles (e.g., obligation/directive stance such as "You have/need to …" and "You cannot …," and the phrases with value/judgment adjectives such as "important" and "necessary"), the analysis was not double-coded. In addition, the I-poem method was integrated at this phrase. This method was heavily relied on collecting the expressions which started with the pronoun, I. The I-poem analysis was not double-coded since the I-phrases were easily identified throughout the data.

The second phrase examined positioning episodes in a classroom. Since positioning analysis required us to look at conversations from multiple perspectives (i.e., position, speech-act, and storyline), the interpretation was more complex than those of stance bundles and/or I-poems. Since this complexity has the possibility of generating erroneous interpretation of data, the positioning analysis was double-coded to assess the inter-rater reliability. In addition, this phase also analyzed social comparison statements. Since the statements needed examining from social comparison motives, the interpretation was as complex as that of positioning analysis. The social comparison analysis was also double-coded for the inter-rater reliability.

Among the transcripts from 7 classes, 3 were randomly selected and coded by another coder to calculate inter-rater agreements. Coders worked on the transcripts according to the coding scheme of positioning analysis developed for this study. It aimed at coding positions, storylines, and speech-acts in the classroom discussions (See Appendix C.1).

The agreements of positioning analysis were 89.4% for positions, 92.9% for speech-acts, and 90.9% for storylines (in December 15 transcripts); 96% for positions, 95% for speech-acts, and 82.35% for storylines (in November 12 transcripts); 87.7% for positions, 89.1% for speech-acts, and 80% for storylines (in September 21 transcripts).

There were 35 students' social comparison statements found in 8 transcripts of lessons. Since the number was relatively small (N=35), all the statements were categorized by a second coder according to the coding scheme (See Appendix C.2). The inter-rater agreement was 85.7%. The disagreements were resolved by re-examining and discussing different coding with coders.

3.3.6 Some issues around case studies

This case study is based on a secondary analysis of the ethnographic data collected for the previous studies (See Forman & Ansell, 2001, 2005). That might generate concerns in two ways. The first one is associated with the nature of case study method. There is a common stereotype that conclusions drawn from case studies tend to be subjective because the conclusions are based on the analysis of one particular case/sample and hard to be applied to larger contexts (Yin, 2009). This study investigated a particular 3rd grade classroom and the conclusions from the study may not be applicable to other classrooms. The second one is concerned with the nature of secondary data analysis. Since the data was already collected for a different research purpose, secondary data analyses have some weaknesses such as limited access to the actual context of data collection and the availability of data. Especially in this study, the transcripts of all the observed classes were not available.

Although this case study has multiple weaknesses, which are originated from the case study with a secondary data analysis, the study also has strengths. As Yin (2009) suggested, a case study provides deep, insightful understanding of a particular case which is investigated. The data is very rich since researchers collected as much data as possible in multiple ways (e.g., interviews, observations), and as a result, the data consists of plenty of information about the case and enables researchers to look at the issue or problem from multiple perspectives.

At the same time, however, the richness of data is not always an advantage of a case study, because it sometimes makes researchers feel as if they had gotten lost in the data or distracted by extraneous issues. In addition, some of the original data may have been ignored because it seemed irrelevant to the primary research questions. Considering those issues, a secondary data analysis would be a good way to investigate not only the same case through different researchers' eyes but also afford opportunities to study different research questions.

4.0 RESULTS

This chapter provides the findings from the following research questions:

1. What is the nature of Mrs. Porter's teaching philosophy and how is the philosophy delivered to and reflected in the classroom discussion?
2. What do whole class discussions look like?
    a. Are there any repeated patterns of interactions?
    b. How does Mrs. Porter position students when they present strategies?
    c. How do the students respond when Mrs. Porter positions them in terms of social comparison?

First, I talk about the results of Question 1, that is, the nature of Mrs. Porter's instructional philosophy and how she was delivering and reflecting it during whole class discussions. The teaching philosophy was examined in terms of attitudinal stance bundles, that is, a collection of phrases which represent obligations and/or directions, and I-poems.

Second, I discuss the findings from micro-analysis of whole class discussions. Utilizing positioning theory, the study examined the characteristics of interactional segments between the teacher and designated student(s) and identified whether there were any repeated patterns of interactions appearing. Then, I present the findings from the analysis of teacher positioning. The study examined teacher positioning in terms of interrupting, validating, and comparing, and identified the characteristics of those positioning events. In addition to teacher positioning, the study also investigated how students responded to teacher positioning. The focus was on whether the students agreed with or challenged teacher positioning.

4.1 THE NATURE OF MRS. PORTER'S TEACHING PHILOSOPHY

To articulate the nature of Mrs. Porter's teaching philosophy, I analyzed multiple data from different sources: transcripts of classroom discussions, the newsletter to parents, and transcripts of interviews with Mrs. Porter. The analysis focused on what kind of norms emerged in the data and how Mrs. Porter delivered those norms to students.

4.1.1 Emergent norms in Mrs. Porter's discourse

The results showed that there were several norms about mathematics teaching and learning emerging across the different data sources. The norms were categorized as social and socio-mathematical (Yackel & Cobb, 1996), teaching, and other (See Table 2). The social category included the norms for learning in general, such as learning from peers, respecting others, sense making, taking risks in problem solving, being conscious about the thinking process, trying hard, not giving up, and so on. Socio-mathematical norms refers to normative aspects which make mathematics sophisticated, efficient, and elegant (Yackel & Cobb, 1996), and the following norms fell into this category: efficiency/sophistication of strategy, multiple approaches to a problem, mathematical application, and so on. Those two kinds of norms were observed in all the data sources.

In addition to social and socio-mathematical norms, the data found that Mrs. Porter mentioned teaching norms in the newsletter and interviews. Those norms included: successful math program, developmental appropriateness of contents, opportunity to learn, guidance/guided teaching, consideration of individual differences of students, teaching differently (or practicing reform methods), building students' confidence in learning mathematics, improvisations, mathematical discussions. Those norms were what Mrs. Porter thought were important for teaching. The teaching norms were observed in both the newsletter and interviews. Also, Mrs. Porter mentioned another type of norm, which did not fall into any of social, socio-mathematical, or teaching, and appeared only in the interviews. For example, she said, "Mathematics should be fun," "People understand through language," and "Learning takes time."

The social and socio-mathematical norms that appeared in the classroom served as normative identities for the students as mathematics thinkers. Normative identities are a cluster of obligations with which students would have to comply in a classroom. These are composed of general classroom obligations and specific mathematical obligations (Cobb et al., 2009). In Mrs. Porter's classroom, social and socio-mathematics norms created normative identities and described what students should be like as mathematics learners. Normative identities can be considered a collection of her utterances about norms and also discursively constructed in those utterances.

The data revealed that Mrs. Porter talked about social and socio-mathematical norms to the students during the discussions and that, to parents, she mentioned teaching norms as well as social and socio-mathematical ones. This result was quite understandable, because the discussions and the newsletter had different addressees or audiences. In the classroom, her norms were restricted to what was important for learning and what counted as mathematically efficient and sophisticated, because those norms were what the students should follow when learning mathematics. On the other hand, the audience of the newsletter was parents. Parents are usually interested in knowing what teachers believe in and how they teach classes. Adding teaching norms explained Mrs. Porter's attitude towards teaching mathematics and positioned herself as a teacher with a clear vision of teaching.

The interviews were different from the other sources of data (e.g., classroom discussions and newsletter) in terms of the number of emergent norms. The data found more norms than the newsletter and classroom discussions and Mrs. Porter talked about both socio-mathematical and teaching norms more often in the interviews. This might have happened because the interviewer was a developmental psychologist.(Mann, 2011) argued that interviewers would influence the production of data and suggested that researchers pay attention to their role in interviews. Considering Mann's argument, we could assume that Mrs. Porter's talk might have been influenced by the presence of that interviewer and that she might have positioned herself differently in front of the interviewer to produce her talk.

In summary, Mrs. Porter was purposefully selecting norms to communicate according to the audiences or addressees. It was interesting to see the difference in emergent socio-mathematical norms between the classroom and the newsletter. Socio-mathematical norms define what counts as a good mathematical thinker in a classroom (Cobb et al., 2009), but she emphasized different socio-mathematical norms respectively. This indicates that normative identities in her classroom consisted of multiple obligations, and that she emphasized different aspects of normative identities depending on whom she communicated with. In addition, the evidence that the interviews identified the most norms in every category among all the data sources supports that Mrs. Porter purposefully selected norms depending on who was the audience.

Table 2. Emergent norms across different data sources

Norms	Classroom discussions	Newsletter to parents	Interviews with Mrs. Porter
Social	• Conscious about thinking process* • Learning from peers* • Respecting each other • Sense making* • Taking risks* • Trying hard or not giving up*	• Conscious about thinking process* • Constructing knowledge • Sense making* • Taking risks*	• Learning from mistakes • Learning from peers* • Sense making* • Social interaction • Taking risks* • Trying hard or not giving up*
Socio-mathematical	• Multiple approaches to a problem*	• Efficiency*	• Efficiency* • Mathematical application • Multiple approaches to a problem* • Sophistication
Teaching		• Developmental appropriateness* • Opportunity to learn • Successful math program	• Building students' confidence • Developmental appropriateness* • Discussion • Guidance or guided teaching • Improvisations • Individual differences • Teaching differently
Other			• Language skills for understanding • Learning takes time • Math as fun

* Norms which appeared in multiple data sources

4.1.2 Patterns of Mrs. Porter's language use to communicate norms

The next step of the analysis was to examine Mrs. Porter's language use in communicating norms. I analyzed the data (e.g., transcripts of classroom discussions and interviews, newsletter) in terms of patterns of Mrs. Porter's language use. In the analysis, I specifically focused on stance bundles (Herbel-Eisenmann et al., 2010). The subsequent sections present the analysis of her language use to communicate norms.

4.1.2.1 Classroom discussions

The data showed that there were repeated patterns of Mrs. Porter's language use to deliver norms to the students. I observed 81 phrases which she used to talk about the norms throughout the data and identified 5 particular patterns of language use among them (See Table 3).

One of the patterns was the use of adjectives which indicated values, such as "useful," "nice," "good/better/best," "fine," "right," "wrong," and so on. For example, Mrs. Porter said, "It's only a *good* way, if you understand why you're doing it," and "It's *important* that you see how other people do this," to the students during the class. Nearly one third (32.1%) of the norm statements (N=81) was the use of those adjectives.

Another pattern was the use of obligation/directive stances, such as "I (don't) want you to," "Do (Don't)," "You (don't) have/need to," and so on. Mrs. Porter used those expressions very frequently during the class and more than half of the norm statements (54.3%) were occupied by the use of obligation/directive stances. For example, Mrs. Porter said, "What *I want you to* do is show the whole story of how you're figuring this out," "You'*ve got to* do it a way that makes sense to you," "If you think it does not make sense, *explain* why it doesn't make sense," and so on.

Table 3. Frequency of the patterns of language use of Mrs. Porter to communicate norms in a classroom

Norm category	Norm	Adjective	Obligation/ directive	Anecdote/story emotion
Social	Conscious about thinking process*	2	24	2
	Learning from peers*	6	5	1
	Respecting each other	2	1	0
	Sense making*	10	6	3
	Taking risks*	0	0	2
	Trying hard/Not giving up*	3	4	3
Socio-mathematical	Multiple approaches to a problem*	3	4	0
(total)		26	44	11

* Norms which appeared in multiple data sources

Mrs. Porter sometimes used the words which explained her psychological/emotional states to communicate the norms. For example, she said, "I'*m* not so *worried about* the answer, as I *am about*, how are you going to start," "And again I'm going to say, I *love* the way this class accepts a challenge. I *love* the way you dig right in and you work at things," and so on. In addition, she used metaphorical statements as well. For example, she said, "It's *like* writing a math story, in a way," and "I want to tell you something about *good math students* and *good mathematicians*."

The results showed that she connected what she reinforced with the phrases that entailed value judgments to students (i.e., "Is it *important* to listen to others since I/we can learn from them?" "Do I *have to* write everything I did since Mrs. Porter *wants me to* explain how I figured it out?"). Especially the adjectives such as "important," "useful," and "good" sounded very clear about what is valued and seemed to help students understand the norms easily. The use of those adjectives sounded so direct that students would not have doubt about what to do in the classroom. It was interesting to find Mrs. Porter delivered her classroom norms in those ways.

The use of obligation/directive stances served similarly to the use of adjectives for purpose of communicating the norms, although the stances she used varied depending to the degree of obligations. The obligation/directive stances showed students very clearly what they were supposed to do in the classroom, especially when Mrs. Porter used (second person) imperative forms (e.g., "*Show* exactly what you're thinking," "Do what makes sense"). Her use of imperative forms generated strong statements and the students left no room for doubt about what to do in solving problems.

Among the obligation/directive stance, Mrs. Porter used the word "want" many times, for example, "I (don't) want you to," "I (don't) want to," and "I (don't') want." Out of 44 obligation/directive stances, 20 included the word "want." The use of "want" sounded less directive than that of imperative forms (e.g., "Use strategies that make sense"), but her frequent use of "want" played a significant role in telling students what to do in the classroom. Usually the use of "want" shows a speaker's desire, but the fact that Mrs. Porter was a teacher indicated that she took control over what to do. Her frequent use of "want," therefore, communicated that what Mrs. Porter wanted was what students were expected to do in her classroom.

In order to communicate norms, Mrs. Porter sometimes used the phrases which indexed emotional/psychological states, such as "worry about," "admire," "love" and "sad." For example, she said in the classroom, "I'*m* not so *worried about* the answer, as I *am about*, how are you going to start." This utterance showed that she wanted to know the students' thinking trajectories/processes of problem-solving and also implied that she did not want them to give an answer simply without understanding. In the same light, she said in a different lesson, "But I think, I am *so sad* that when I was little, people made me do it this way. Because I used to get all mixed up with multiplication and I used to think, I don't know how to do this. And when I had a list of problems and I remembered the rule, I could get the right answers. But I really didn't understand what I was doing." The word "sad" in this anecdotal utterance was so powerful as to make her students recognize how important sense-making was.

In addition, Mrs. Porter used anecdotes and stories to communicate norms. For example, she said one time, "It's like writing a math story, in a way," to tell the students how to articulate their thinking process, and the other time, "This is a class where we all teach each other," to show what her class should look like as a learning community. To describe a "good" mathematician, Mrs. Porter mentioned Einstein such as, "And I said, that I *admired Einstein* because he never - gave - up. He worked on one problem for years and years and years. And he'd think he had it figured out, and then he'd see that something was wrong. After all, it would take days, because some problems are very, very long." This utterance indicated that good mathematicians were like Einstein, and that it was important not to give up since good mathematicians did not do so. Also, the word "admire" implied that Mrs. Porter highly valued Einstein and that she wanted her students to see Einstein as a role model. The use of the metaphorical phrases contributed to communicating the classroom norms as the other patterns did so.

4.1.2.2 Newsletter to parents

I examined the newsletter in the same method that I used for analyzing the classroom transcripts. Regarding her language use, the data showed that Mrs. Porter used adjectives and obligation/directive bundles more than the other types such as anecdotes, story, and emotions. (See Table 4) [Also see Appendix A for the copy of newsletter]

Table 4. Frequency of the patterns of language use of Mrs. Porter to communicate norms in the newsletter

Norm category	Norm	Adjective	Obligation/ directive	Anecdote/story emotion
Social	Conscious about thinking process*	0	2	0
	Constructing knowledge	0	2	0
	Sense making*	2	0	0
	Taking risks*	0	1	0
Socio-mathematical	Efficiency*	0	1	0
Teaching	Developmental appropriateness*	1	0	2
	Opportunities to learn	0	1	1
	Successful math program	1	0	0
(total)		5	7	2

* Norms which appeared in multiple data sources

As Mrs. Porter used adjectives (e.g., *important*, *essential*) and obligation/directive stances (e.g., *must*, *should*) more often than the other types of phrases during the discussions, she used the same phrases in the newsletter as well. This showed that she again positioned herself as the authority in the newsletter too, because those phrases were related to values or judgments.

The newsletter had one anecdotal phrase which read, "*Experience has shown me* that students need opportunities to explore new concepts using materials and procedures that are appropriate to their developmental levels." This phrase seemed to provide supporting evidence that she positioned herself as the authority, since it sounded that she knew many things from her teaching experiences. In terms of anecdotes to communicate norms, she positioned herself as a puzzled student in the classroom, but she did not in the newsletter. She purposefully selected positions depending on how she presented herself to the audience.

In addition, the I-poems stated below gave Mrs. Porter another position.

- *I* try to develop assignments, tasks, etc. that promote understanding

- *I* encourage students to explore, conjecture, and think

The words such as “try” and “encourage” contributed to positioning Mrs. Porter as an enthusiastic teacher who was helping children learn. The I-poems sounded like her declaration about how to teach. The effective use of the first person pronoun helped to articulate her attitude as about teaching and seemed to present herself to parents in a desirable way.

4.1.2.3 Interviews

The same analytical method was used to examine the interview data. When Mrs. Porter talked about the norms in the interview, she also used adjectives (N=58), obligation/directive stances (N=48), emotional phrases (N=26), anecdotes (N=3), and metaphors (N=19). The results showed that there were many more norms that appeared in the interviews than in the newsletter and classroom discussions. (See Table 5) [Also see Appendix F for the transcripts for the interviews]

Table 5. Frequency of the norms which appeared in the interviews

Norm category	More than 10 times	5 – 9 times	Less than 5 times
Social	• Sense making* • Thinking process*	• Taking risks*	• Social interaction • Trying hard* • Reflection
Socio-mathematical		• Mental math • Multiple approaches to a problem • Place value	• Efficiency* • Multiplication tables • Regrouping • Sophistication
Teaching	• Guidance/guided teaching	• Discussion • Individual difference • Teaching differently	• Building students’ confidence • Direct teaching • Improvisations
Other		• Learning takes time	• Language skills for understanding • Math as fun

* Norms which appeared in multiple data sources

Table 5 showed that Mrs. Porter mentioned most frequently the norms such as guide/guided teaching, sense making, and thinking process among all the norms which appeared. This indicates that Mrs. Porter valued those norms the most.

The data showed that Mrs. Porter's language use in the interview was more complex than those in the classroom discussions and newsletter. Although I used the same categories (e.g., adjectives, obligation/directive, metaphor, anecdote, and emotion), the data required me to add further interpretations to categorize norms. For example, Mrs. Porter said, "So, I know then, that they're not *ready* to see that other step in thinking," and "And I find that (= children have difficulty working in a group) *typical*." Mrs. Porter used adjectives such as *ready* and *typical* to describe children's developmental states, but those were not related to values or judgments like *important* and *necessary*. I examined *ready* and *typical* in terms of what norms would make her use those adjectives in the sentences. Then I categorized them into the norm of developmental appropriateness. Similarly, I examined the sentence such as "But I was so fascinated with what they didn't understand," which appeared in the emotion category, and categorized it into the norm of thinking process (or cognitive skills).

Since it was relatively difficult to categorize the words/phrases according to the stance bundles used previously, I utilized the I-poem method to support my analysis. The collection of I-phrases, or I-poems, revealed that Mrs. Porter took up more positions in the interview than the newsletter and classroom discussions, and that the some I-phrases reflected certain types of norms (See Appendix D). For example, saying, "I'll try and find the reason why they've done it that way, and find a positive reason," Mrs. Porter positioned herself as a teacher researcher who was interested in children's cognition. Her interest was viewed as evidence that she paid attention to how children thought about and approached a problem and valued children thinking process as a norm. In another I-phrase such as “I mean my job is to teach and to help children learn” indicated that Mrs. Porter positioned herself as guide and thought it important to help children learn as a norm.

Other I-phrases did not clearly indicate norms but indexed her positioning. For example, the sentence such as “I had no confidence in my ability to, to do things mathematically” showed that Mrs. Porter took up a position of a novice teacher who was not comfortable in teaching mathematics. It was not clear which norms governed this utterance but it is possible to interpret from what position she said so. Similarly, the utterances such as “I wish I knew why kids feel that ultimately,” and “I hope I don’t convey that as a goal” did not indicate norms clearly. However, the I-phrases such as “I wish” and “I hope” indexed that Mrs. Porter might have certain dilemmas of teaching mathematics and discursively constructed a position of teacher with dilemmas (See Table 6).

Table 6. Examples of Mrs. Porter’s I-poems and positioning from interview data

Positions	I-poems
Puzzled math student	- I’d be always be wrong. - I just assumed I didn’t have much ability to work with numbers.
Authority	- I do believe that many adults believe that times tables are what multiplication is. - I don’t believe in forcing.
Novice math teacher	- I didn’t feel that I understand how math worked. - I had no confidence in my ability to, to do things mathematically.
Teacher with dilemmas	- I wish I knew why kids seem to feel that ultimately. - I hope I don’t convey that as a goal.

The synthesis of stance bundles and I-poems made it possible to not only examine norms more deeply but also to articulate Mrs. Porter's positioning in the interviews. As a result, I was able to find that Mrs. Porter drew on stance bundles to describe norms too, and that she talked about more norms in the interviews than the newsletter and classroom discussions. Especially, the I-poems allowed me to understand Mrs. Porter's positioning that was not identified by the analysis of stance bundles. It was useful to know through the I-poems constructed by the interview data that she shared her past experience of being a novice math teacher but also faced teaching dilemmas. The I-poems helped me interpret Mrs. Porter's positioning despite the complexity of the interview discourse.

4.1.3 Summary of the results

The results showed that there were patterns of Mrs. Porter's language use to deliver different kinds of norms to the class. The patterns included the use of adjectives, obligation/directive stances, metaphors, and phrases which indicated her psychological/emotional states including her anecdotes. The closer look at the patterns revealed that Mrs. Porter communicated the norms by associating it with adjectives so that they might sound like more general statements, such as "Trying is what is most important," and "The way that's best is the one that makes sense to you," and that she did so by giving more subjective statements which described what she wanted/valued/expected, such as, "I want you to think about what you decided would be logical to do," "I want the whole story," and "And again I'm going to say, I love the way this class accepts a challenge. I love the way you dig right in and you work at things" (See Appendix E).

The combination of more general and more subjective statements might play a significant role in positioning Mrs. Porter as an expert or old-timer. The authorship of such general statements seemed to position Mrs. Porter as the authority of the classroom (or learning community), and her frequent use of subjective statements with the word "want" prevailed as rules that the students should follow to get Mrs. Porter's recognition. In short, her strategic uses of both general (objective) and subjective statements solidified her authority position in the community and contributed to positioning the students as novices or new-comers who should learn the norms.

In terms of types of norms, Mrs. Porter communicated social and socio-mathematical norms to the students during classroom discussions, whereas she added her teaching norms in the newsletter and the interviews. The same pattern of language use was observed in both data sets. Especially in the newsletter, she maintained the authority position as she did in the classroom. In addition to that, she seemed to successfully position herself as an experienced teacher by including not only teaching norms and an anecdotal phrase (e.g., "Experience has shown me").

In the interviews, Mrs. Porter took up multiple positions. Some of the positions reflected certain norms, but others did not. Since the interview data were more complex than the other data, I utilized the I-poem method so that I could examine the utterances which did not fall in the stance bundle category that I used for analyzing the transcripts of classroom discussions and the newsletter. Although there were many I-phrases which did not clearly show the norms, some of them enabled me to articulate her positioning.

The analysis of the data from different sources led me to the idea that Mrs. Porter's teaching philosophy would be a hybrid discourse from multiple positions that she took up. The emergent norms could be viewed as pieces that consisted in those positions. In other words, as a position is a cluster of the rights and obligations embedded in it (van Langenhove & Harré, 1999), norms could be viewed as parts of the obligations that the position entailed. Mrs. Porter's teaching philosophy was constructed discursively from multiple voices from different positions that she took up, and she delivered it purposefully or differently depending on contexts, which could be determined by the nature of audience and addressees that she had.

4.2 THE NATURE OF WHOLE CLASS DISCUSSIONS

The subsequent sections discuss the nature of whole class discussions. First, I investigated whether there were any repeated patterns of interactions during the discussions. The investigation focused on the interactional sequences such as the IRE/F and revoicing in each subunits of analysis. Then I examined how Mrs. Porter positioned students and their strategies during the discussion. The analysis was based on looking at positions, storylines, and speech-acts in every utterance and articulating how those positioning elements made students recognizable as particular types of students in the class. Lastly, I looked at how students responded to Mrs. Porter's positioning and examined whether or not their responses were motivated by social comparisons.

4.2.1 The patterns of interaction in Mrs. Porter's class

It was very common in her classroom that she called on students to initiate an interaction, although volunteering by students sometimes started a new interaction. The interactions were usually between Mrs. Porter and individual students who were called on. I observed 154 segments of interaction throughout the data, and 34 out of 154 were the IRE/F. The others were not the IRE/F or similar forms, and they were longer and included Mrs. Porter's prompting and revoicing (See Table 7).

Table 7. Interactional patterns in Mrs. Porter's class

Type of interaction	Frequency	Nature of interaction
IRE/F	34 (22%)	Asking QWKAs - collecting mathematical facts/info - making sure which strategy students used
non IRE/F	120 (78%)	Prompting, revoicing

The results showed that the IRE/F sequence was not common in Mrs. Porter's classroom, but that she used it for several reasons. First, the IRE/F was observed when she asked questions with known answers (QWKAs). She sometimes asked those questions before (or during) the strategy discussions in order to collect mathematical facts/information necessary for solving problems (e.g., "We have 4×12, and you should all know that one. What is it?").

She also used the IRE/F to request from the students as many strategies/opinions as possible. For example, in the task presented to the class on October 20, Mrs. Porter gave 5 possible strategies to solve the problem to the students, who were expected to choose their favorite strategy. She asked them what they chose to make it available to the class. Mrs. Porter simply called on the students for participation, such as, "OK, Nathan," and then the students responded.

These two categories indicated that the IRE/F was not the main interactional sequence in Mrs. Porter's classroom but was used to promote whole class discussions. It might be helpful for students to know that there were multiple ways of solving problems, and her use of the IRE/F just to hear about what they did might reinforce her reform policy such as sense-making and risk-taking. Also asking QWKAs might help students establish cognitive links between what they actually did and what was being discussed by pointing out something important and/or refreshing the flow of the interactions.

The results showed that the interactional pattern with prompting and revoicing was the most common in Mrs. Porter's classroom during the whole class discussions that were in my sample. Since, as a sample, I selected 8 lessons where students were working on cognitively demanding problems, the results were not applied to all the lessons but only to the lessons that I chose for this analysis. Roughly 75% of all the interactions were not the IRE/F and consisted of prompting and revoicing. Mrs. Porter's prompting and revoicing were strategically used to help the students explain their strategies and articulate their understanding of not only the problem but also the strategy itself. The following excerpt showed the turn-taking pattern between Mrs. Porter and a female student (Ophrah). The class was discussing their strategies for solving two division problems: $12 \div 18$ and $12 \div 16$.

107	Ophrah:	First I drew a box, just box.
108	Mrs. Porter:	OK.
109	Ophrah:	I put, four times four is sixteen in it.
110	Mrs. Porter:	OK, ***so*** you're figuring out what equals sixteen so that it's a way to get started.
111	Ophrah:	And then after that I put eight times two is sixteen.
112	Mrs. Porter:	And you know that eight times two is sixteen. OK, then once you knew that?
113	Ophrah:	I drew twelve, (pause) in a straight line.
114	Mrs. Porter:	Do they have to be in a straight line?
115	Ophrah:	No.
116	Mrs. Porter:	OK, cause I, it'll be hard to find the room here. But sometimes they need to be in certain groupings and that's why I wanted to ask you that. OK, ***so*** now we've got our twelve boxes, and then?
117	Ophrah:	Put them into fourth.
118	Mrs. Porter:	Why did you put them into fourth? What gave you the clue?
119	Ophrah:	Because they were,
120	Mrs. Porter:	Now did you use anything from over here?

121	Ophrah:	No, I just wanted to see how I could get to sixteen.
122	Mrs. Porter:	OK (not audible)
123	Ophrah:	(Not audible) 18.
124	Mrs. Porter:	OK, you used that for the 18 one, all right. ***So*** why did you put them in fourths, is there a reason or did you decided just to try it?
125	Ophrah:	Just to try it.
126	Mrs. Porter:	OK, and then? So far you've done pretty much what Raj has done. So the what did you do?
127	Ophrah:	Then on the bottom, wait, then I circled four of them.
128	Mrs. Porter:	Like this? Yes.
129	Ophrah:	A little different.
130	Mrs. Porter:	All right, it doesn't matter, but you circled four. And what did you say to yourself?
131	Ophrah:	I said to myself, um, um, if there's sixteen, I'll be one fourth with them. And then when I finished I wrote each person would get three fourths.
132	Mrs. Porter:	OK, ***so*** you have one fourth, plus one fourth, plus one fourth. In a way it's very similar, it looks different on your page, to this. But, and, and, it's a good way of thinking. She knew that four times four is sixteen, and she probably, you knew, even if you're not saying it. That if you divided them into fourths, and you had four of them.
133	Ophrah:	Well, that's why I did it.
134	Mrs. Porter:	I thought so. And so that makes three fourth for each person.
135	Ophrah:	But I used (not audible)
136	Mrs. Porter:	OK, but you used it in a way there, because you knew that four fourths was sixteen. OK, a different way, Lindsay?

The interactional segment above is quite long and shows that Mrs. Porter used prompting and revoicing to help Ophrah explain how she did the problem. Turns 110, 116, 124 and 132 show Mrs. Porter used revoicing with warranted inferences "***so***" (O'Connor & Michaels, 1993, 1996) to reformulate what Ophrah said. The use of revoicing generated multiple turns and promoted a longer interaction. In addition, Mrs. Porter's strategic prompts showed clearly how Ophrah solved the problem. For example, Ophrah talked about the details of what she did, such as "First I drew a box, just box (Turn 107)," "I put, four times four is sixteen in it (Turn 109)," "I drew twelve, (pause) in a straight line (Turn 113)," and so on. The phrase such as "say to oneself," which appeared in Turns 130 and 131, was unusual since it seemed that Mrs. Porter encouraged

Ophrah verbalize her thinking process with this phrase. Mrs. Porter's strategic revoicing and prompting promoted interactions with the students and made those interactions longer. This may reflect her reform policy that aimed at reinforcing students' sense-making practices through articulating their thinking processes.

While Mrs. Porter encouraged the students to share and talk about their strategies, she seemed to shorten the interaction in some occasions. The following is another excerpt from the class which discussed the division problem.

277	Mrs. Porter:	OK, so each person got two thirds of each sandwich. And this "wich" is not the same kind of witch as a Halloween witch. OK, Miranda.
278	Miranda:	First I made
279	Mrs. Porter:	Wait a minute, is there anybody who hasn't told us a way yet that they would like to share?

The pattern of this interaction was the tri-structure like the IRE/F, but the last phase was different from typical IRE/Fs. It seemed that Mrs. Porter did not intend to give an evaluation or feedback but rather intentionally interrupted Miranda's utterance so that she could move on to another student who used a different strategy.

In another class, there was an interesting pattern observed. The class discussed the "12th Day of Christmas Problem" which contained consecutive numbers. They had worked on a similar problem ("Hanukkah Candles Problem") in the previous lesson.

76	Mrs. Porter:	What was, what was similar to the Hanukkah candles? Kevin.
77	Kevin:	It was also using in the triangles.
78	Mrs. Porter:	Yes. But not only in what he was doing. What was similar about this twelfth day of Christmas problem and the, Hanukkah candles problem? Ophrah.
79	Ophrah:	The Hanukkah candles problem, you had to try to see how many candles you would need, and in this one you just had to see how many presents you get.
80	Mrs. Porter:	And, and was there something similar about the numbers?
81	Ophrah:	Yes.

82 Mrs. Porter:	What?	
83 Benjamin:	Uhm, they keep coming again and again.	
84 Mrs. Porter:	So they were.	
85 Student:	Like the Hanukkah candles keep melting.	
86 Mrs. Porter:	Yes. They kept coming and kept melting and here we had gifts and we keep bringing the same ones the next day. OK. They were both consecutive numbers problems.	

In this excerpt, four students were involved in the interaction. Mrs. Porter’s initial initiation was addressed to Kevin, who mentioned the strategy that his classmate had used to solve the previous problem, which looked like a tree diagram strategy (Turn 77). It seemed that his reply was not what Mrs. Porter expected, and she asked a different student the same question/reformulated question (Turns 78 and 80). After that, two students were involved in the interaction (Turns 83 and 85).

Since four students were involved in the interaction, this sequence resulted in multiple IRE/F sequences between Mrs. Porter and the students. Usually IRE/F sequences are used for QWKAs or closed questions and give an impression that the interaction is one-way. However, this example sounded more dialogic than typical IRE/Fs, because several students responded to the same question and Mrs. Porter guided them to the answer/conclusion she was looking for (“They were both consecutive numbers problems” in Turn 86).

In conclusion, Mrs. Porter used both IRE/Fs and non-IRE/Fs to promote discussions. The most common interactional pattern was the one with revoicing and prompting, which facilitated interactions between her and the students. It seemed that she took advantage of the IRE/Fs for several reasons such as collecting the mathematical facts necessary to solve problems and orchestrating different answers from the students. She, however, sometimes used the IRE/F to shorten the interaction and move on.

4.2.2 Mrs. Porter's positioning during whole class discussions

The following sections reported the finding from the analysis of Mrs. Porter's positioning during whole class discussions. Through the analysis, I closely looked at what positions appeared, what Mrs. Porter was doing with her utterances, and what the relationship between her and the students looked like. My specific foci were placed on her aligning multiple strategies/students with each other (comparing/juxtaposing), her making positive comments on strategies/students (validating), and her not giving students opportunities to talk (interrupting).

4.2.2.1 Episodes of validating students' strategies

In order to analyze Mrs. Porter's validations of students' strategies, I closely looked at her evaluations or comments on the strategies presented by the students and coded utterances as "validating" whenever Mrs. Porter made "positive" comments or evaluations of strategies.

There were 39 events of validating throughout the data. Most of the students (N=16) appeared in the events of validating, although some of them appeared more often the others. The common words/phrases included: "work," "good," "another way," "fast(er)," "efficient," "nicely," "quickly," "interesting," "(very) well," "make sense," "That's right/That's it," not a bad idea," "fine," and the alignment of students with adults such as parents. There was one student (Benjamin) whose strategy was not validated by Mrs. Porter in my sample. This might be because of my selection of classes rather than Mrs. Porter purposefully avoided validating his strategies. The small sample size restricted this analysis to some degree.

Although Mrs. Porter gave positive comments/evaluations to most of the students, several students were favored more often than others. Mrs. Porter praised two students most often (5 times), two students 4 times, two students 3 times, 3 students twice, and 7 students once (See Table 8).

In most cases, Mrs. Porter praised the students when they contributed to the discussion by presenting multiple problem solving strategies. She usually valued different kinds of strategies as long as the students used them with understanding. For example, in September 10 class, in which the class was working on the multiplication problem (15×5, 15×7), multiple strategies were discussed. One student wrote the multiplication table of 15, such as "15×1=15, 15×2=30, 15×3=45, ….., 5×5=75." Another student used a doubling strategy, such as "15+15=30, 30+30=60, 60+15=75." There was a student who used tally marks to solve 15×5, while there were two students who used an abstract strategy as below:

$$\begin{array}{r} 15 \\ \times\ 5 \\ \hline 75 \end{array}$$

When the students presented their strategies that worked well, Mrs. Porter gave positive evaluations, saying, "That's another way of doing it," "that's another thing that people might write down as a way to try it," "You see he/she did it differently," and so on. This kind of attitude toward strategies indicated that she did not intend to provide the student with the idea that one strategy was the "only" way of solving a problem. This seemed to be supported by the previous finding that she valued multiple approaches to a problem as one of the socio-mathematical norms.

Table 8. Frequency of Mrs. Porter's validating strategies

Frequency of validating	Student whose strategy was validated (N=16)
5 times	Pulak, Ophrah
4 times	Nathan, Raj
3 times	Michael, Lissa
Twice	Lyndsey, Bettina, Karl
Once	Ethan, Aisha, Bernard, Kevin, Joshua, Sabrina, Miranda

Mrs. Porter encouraged the students to try when they had difficulty solving problems. She gave positive feedback to the students who found the problem difficult, saying, "OK, that's a good way to start," "So far, you have a good strategy," and so on. Trying or not giving up was one of the social norms in her class, and she delivered it to the students by talking about Einstein, who she believed had not given up on solving difficult problems.

However, there were some interesting pieces of evidence that Mrs. Porter sent mixed messages to some of the students. She gave positive comments on the strategies but also said something negative or discouraging to a few students (e.g., Miranda and Bettina). The results showed that there were nine events of sending mixed messages, and five were addressed to Miranda and four to Bettina. For example, Mrs. Porter mentioned during the discussion that Bettina used a tally-mark strategy to solve 15×5. Using tally marks were viewed as a very inefficient and concrete strategy, which might be appropriate for younger children.

Table 9. Frequency of Mrs. Porter's sending mixed messages

Frequency of sending mixed message	Student who received mixed message
5 times	Miranda
4 times	Bettina

This is one of the examples of sending mixed messages. The class was working on the subtraction problem (1718-1607).

61	Bettina:	I counted backwards from 17, 1718, all the way to 1607.
62	Mrs. Porter:	OK. And she counted by ones. So she said 1717, 1716, 1715, and counted and counted and counted. And she finished it. And she got the right answer. It took a very – long – time. It is a strategy that works, but it takes a – very – long – time. And it's a logical thing to do. OK, who did it differently? Lyndsey.

Another example is below. The class was discussing the division problem (12 ÷ 16, 12 ÷ 18).

146	Mrs. Porter:	Miranda, did you do something different?
147	Miranda:	Yes, but it looks like Bettina's.
148	Mrs. Porter:	OK, but what were you thinking, that was different? Go ahead and tell us.
149	Miranda:	First I put squares then I put a half in each box (not audible). Then I put everybody's name.
150	Mrs. Porter:	Oh, that took a long time I'll bet. OK, you put everybody's name in one and you knew there was enough for everyone.
151	Miranda:	Then I counted, uhm, if I could, four squares in once box, and then I put everybody's name in one of them.
152	Mrs. Porter:	In one square.
153	Miranda:	In one square and then I thought (not audible).
154	Mrs. Porter:	OK, that's another way to do it. It took you a long time to write all those names, but it certainly worked well for you, good thinking.

Both examples above indicated that Mrs. Porter was sending mixed messages to the students. Although she made some positive comments about sense-making (e.g., "it's a logical thing to do," "it certainly worked well for you, good thinking"), she pointed out that the strategies were not efficient since they took a long time. The utterance, “that’s another way to do it,” in Turn 154 seemed less positive in this context because it was part of a mixed message, although the same phrase appeared frequently during validating interactions. The use of mixed messages by Mrs. Porter seemed to reflect conflicts in norms or teaching dilemmas. For example, some approaches to a problem and sense making might not correspond with mathematical efficiency or accuracy. Allowing students to take multiple approaches to a problem might generate concrete or less sophisticated strategies, although they were logical and made sense to the students. In the excerpts above, Bettina and Miranda received both positive and negative feedback at the same time from their teacher. Both of them preferred to use very concrete and inefficient strategies. Mrs. Porter seemed to accept their strategies since she appreciated multiple approaches to a problem and sense-making, but she evaluated them negatively as inefficient. Sending mixed feedback on strategies might contribute to ranking students since the strategies authored by the students who presented them evoked more negative evaluations of their strategies than those of other students. For example, as the former excerpt above showed, Bettina counted backwards by ones to solve 1718 – 1607. However, in the same class, Ophrah used subtraction and Lyndsey counted upwards from 1607 to 1618 by ones and then added 100 to 1618 to get to 1718. While Mrs. Porter gave mixed feedback on Bettina’s strategies (e.g., It is a strategy that works but it takes a very long time”), her feedback on Lyndsey’s and Ophrah’s strategies was relatively positive (e.g., “Right, she knew that when she got to 1618, that was a hundred years difference” to Lyndsey, “For some people it’s a good way of doing it. It’s especially good if there’s no regrouping” to Ophrah).

Table 10. Multiple approaches to subtraction problem 1718 - 1607

Name	Strategy used	Nature of Mrs. Porter's feedback
Bettina	Counting backwards by ones	Mixed (positive and negative)
Lyndsey	Counting and addition	Positive
Ophrah	Subtraction	Positive

Multiple approaches to a problem allowed the students to use their own strategies but also generated a sophistication gap between the strategies. Mrs. Porter's mixed feedback on certain strategies resulted in pointing out such a gap and implying that one strategy would be more (or less) efficient than the other. Since giving feedback is an act of positioning, regardless of whether feedback is positive, negative, or mixed, Mrs. Porter's feedback did position or rank Bettina, Lyndsey, and Ophrah. Giving both positive and negative feedback at the same time might not favor the students who got such feedback but rather locate them in less advantaged positions.

In conclusion, the results found that Mrs. Porter favored the students by making positive comments/evaluations on their strategies during the discussion. Through validating, Mrs. Porter positioned those students as "contributor" to discussions. While most of the validating events were associated with evaluations of strategies, some were related to alignments with adults. Pulak, who often used abstract algorithms, was aligned with adults in terms of his strategy use. Mrs. Porter's utterances such as "This is something you don't do in 3^{rd} grade," "That's the way a lot of your parents would do it" implied that Pulak knew better than the other 3^{rd} graders in the class, and as a result, positioned him as advanced in mathematics.

The results also showed that Mrs. Porter made positive comments to the students who made mistakes in solving problems. She used those mistakes as common errors that the students fell into, and reminded them of the errors. This indicated that Mrs. Porter positioned the student as another type of contributor or model, and that she emphasized learning from mistakes as one of the social norms.

Lastly, the findings showed that Mrs. Porter sometimes sent mixed messages to the students when they shared inefficient strategies. Interestingly, those comments were addressed to two female students, Bettina and Miranda. Her sending mixed feedback on strategy might reflect contradictions of norms (e.g., multiple approaches to a problem vs. efficiency, sense making vs. efficiency) and position the students differently or rank them according to how efficient the strategies were.

4.2.2.2 Episodes of interrupting students

The analysis of interrupting was initially determined by the presence of overlapping speech, but the findings were more complicated. The results revealed that overlapping speech appeared for various reasons. The data identified 18 events of Mrs. Porter’s overlapping speech. There were 9 students involved in the interactions with overlapping speech and interruptions. The student who appeared most often was Miranda, and she experienced Mrs. Porter’s overlapping speech or interruptions four times. Ophrah and Nathan experienced it three times, and the other students experienced twice or just once.

Although there were 18 instances of overlapping speech observed, the nature of each was slightly different. The results revealed that overlapping speech appeared not only for the purpose of interrupting but also for other purposes such as scaffolding, prompting, evaluation, and so on.

The following example of interrupting is from the class in which the class discussed the multiplication problem (15×5, 15×7).

182	Mrs. Porter:	OK, OK. Uhm, Miranda.
183	Miranda:	I did it, sort of like Ophrah's, but only, I did it by 15 plus 15 is 30, 30 plus 15 is 45, and 45 plus 15 and then like I carried the one and (not audible).
184	Mrs. Porter:	And did you get 75 also?
185	Miranda:	I'm not finished.
186	Mrs. Porter:	You didn't get finished. OK.
187	Miranda:	Then I got 15 plus 15 is 30, 30 plus 15 is 45 (Mrs. Porter interrupts)
188	Mrs. Porter:	OK, Now did somebody, we've got four 15s equals 60 minutes and that's one hour, right? So, how many minutes are leftover? Nathan?

Miranda started her explanation by saying that she used a similar strategy as Ophrah, who used a multiplications table of 5 to solve the same problem. Miranda's repeated addition strategy was already used by some of the students, but she said that she didn't get finished (Turn 185). Miranda, however, continued her explanation, and as a result, Mrs. Porter interrupted her and moved to the next topic, "How many minutes are leftover?"

Probably this happened since Miranda failed to understand her teacher's expectations after she admitted that she could not finish the problem. In Turn 186, Mrs. Porter confirmed that Miranda did not get finished, but Miranda just continued talking about the same thing. Mrs. Porter might have said in Turn 186, "OK," to signal the end of the interaction with Miranda and move on to a different task. It seemed that Miranda might not have understood the context change and maintained the on-going storyline of talking about her strategy.

Another example is below. Miranda is again involved in the following interaction. The class was working on the subtraction problem (1718-1607).

50	Mrs. Porter:	I want to know if he was born before or after.
51	Miranda:	After.
52	Mrs. Porter:	How do you know that?

53	Miranda:	Because, first I put …
54	Mrs. Porter:	I don’t want to know how you solved the problem. How do you know it was after and not before? That’s is the only thing I want to know.

Miranda was interrupted in Turn 53 by Mrs. Porter when she was responding to the question. Mrs. Porter’s utterance in Turn 54 sounded very directive. Especially the statement, “That’s the only thing I want to know,” implied that Mrs. Porter was not satisfied with what Miranda said or was trying to say, and that Miranda did not understand what Mrs. Porter wanted to hear from her. Interestingly, Miranda was interrupted again in a different class. When Mrs. Porter called on Miranda (“OK, Miranda”) and Miranda said, “First I made …,” Mrs. Porter interrupted her, saying, “Wait a minute, in there anybody who hasn’t told us a way yet that they would like to share?” Regardless the fact that Mrs. Porter called on Miranda, she did not give her an opportunity to talk.

Nathan was also interrupted by Mrs. Porter, but only once. In December 17 class, they were discussing the multiplication problem (984×12, or how many inches are there 984 feet?). The problem was too difficult for the students to solve, and only one student obtained the correct answer (by adding 984 twelve times). Usually Pulak was able to use abstract and efficient strategies and get the correct answer, but he was absent from that class. Since the class did not have Pulak as a role model to present the right answer, the whole discussion was not so organized. Although the students had their own strategies, the strategies were not so efficient. In the excerpt below, Nathan seemed to be trying hard but had difficulty getting to the right answer.

20	Nathan:	Then I, I, I knew, uhm, 7, 7 times 12.
21	Mrs. Porter:	Wait a minute, what did you decide to multiply?
22	Nathan:	12 times 7 equals 84.
23	Mrs. Porter:	Why, why did you decide to multiply12 times 7?
24	Nathan:	Cause I wanted to get 84.
25	Mrs. Porter:	Why?

26	Nathan:	Because that’s, those are the digits, the tens and the ones, so 84,
27	Mrs. Porter:	OK.
28	Nathan:	and I would know how many inches that was.
29	Mrs. Porter:	OK.
30	Nathan:	So then it would be easier for me.
31	Mrs. Porter:	I don’t know. I could understand. I could understand what you are saying. Nathan sees this and he’s saying, “What times 12 equals 84? Right?
32	Nathan:	Uh-huh.

Before the overlapping speech, Nathan was called on by Mrs. Porter to share his strategy. He said that he thought about division first but then changed his mind since he found out that dividing 984 made a smaller number. Upon his utterance, Mrs. Porter reformulated what he said and positively evaluated Nathan, saying, “That’s why we estimate. And that’s why we make predictions and guesses. To help you know that that can’t be right. So Nathan knew it couldn’t be right.”

Nathan’s utterance was, however, overlapped by that of Mrs. Porter (Turns 22 and 23). The interaction indicated that what Nathan did made no sense (Turn 31). He only looked at the last two digits of 984 and found the divisors of 84. He did not understand how to do the problem. Her overlapping speech might have been intended to redirect Nathan to an appropriate approach to the problem, but could be interpreted as her attempt to let him know that his strategy would not work. In terms of preventing him from keeping his old idea, this overlapping speech might be a case of negative validation.

The following example shows that overlapping speech appeared when Mrs. Porter prompted the students. The excerpt was from the class where the class was working on the problem with consecutive numbers. Sabrina was called on and shared what she did.

131	Sabrina:	Uhm, I added them. The first one I did, uhm, didn’t work. So I added 12 plus 11.
132	Mrs. Porter:	OK. So you started at the 12 and, and you did, then plus 11. And then did you do it this way?

133 Sabrina: Uhm. No, I didn’t, uhm, that way.
134 Mrs. Porter: 12 plus 11 equals
135 Sabrina: 23.
136 Mrs. Porter: And then? 23.
137 Sabrina: Plus 10.
138 Mrs. Porter: Plus 10 equals 33.
139 Sabrina: Equals 33.
140 Mrs. Porter: And 33 plus 9 equals
141 Sabrina: 33 plus 9, equals 42

Mrs. Porter prompted Sabrina so that she could explain her strategy to the class. In Turns 138 to 141, overlapping speech appeared. This indicated that Mrs. Porter knew how Sabrina did the problem (i.e., adding 12, 11, 10, 9, …, 1) and that both Sabrina and Mrs. Porter were jointly producing the storyline of Sabrina’s strategy. In this case, overlapping speech was not used for interrupting but for helping/prompting students. This type of overlap was interpreted as “cooperative overlap” (Tannen, 1993, 2005). Cooperative overlap indicated that, between two speakers, one speaker shows one’s interest in the other speaker’s talk, and that it is not viewed as interruption.

In the same way, overlapping speech appeared when Mrs. Porter revoiced what the students said.

111 Ophrah: And then after that, I put 8 times 2 is 16.
112 Mrs. Porter: And you know that 8 times 2 is 16. OK, then once you knew that?

Also, they appeared when Mrs. Porter evaluated what the students said.

2 Lyndsey: Doing 984 twelve times and adding it together.
3 Mrs. Porter: That’s it. Lyndsey decided that she could add 984 twelve times.

In conclusion, the results showed that overlapping speech appeared for various reasons, such as interrupting students, prompting/revoicing students, co-constructing a storyline, and evaluations. Except when Mrs. Porter interrupted students, her overlapping speech sounded “cooperative” (Tannen, 1993, 2005) as a way of showing her support for students and agreement with their ideas. This type of overlap was different from those used for interrupting. Miranda was interrupted by Mrs. Porter more often than the other students. Mrs. Porter's frequent interruptions might contribute to positioning Miranda as a student who was a struggling student.

4.2.2.3 Episodes of comparing or juxtaposing strategies

Juxtapositions and comparisons are similar in terms of multiple strategies being put on the discussion table. However, in the process of analyzing utterances which mentioned multiple strategies, I found it useful to differentiate comparisons from juxtapositions in terms of instructional purposes. In the data, juxtapositions occurred when Mrs. Porter showed the students different approaches to the problem, while comparisons appeared when she evaluated strategies by highlighting the difference in efficiency between strategies

The analysis found 52 instances where Mrs. Porter either compared or juxtaposed multiple strategies during classroom discussions. Most of the students were involved in either comparisons or juxtapositions. There were 30 juxtapositions, 20 comparisons, and 2 unclear events observed.

Juxtapositions. When Mrs. Porter was juxtaposing students' strategies, she frequently used the words/phrases such as "(just) like," "this way," "the same way," "do what he/she did," "is similar but look different," "another way," "a lot of people," and so on. Those words/phrases produced (opposite) alignments of strategies with one another and helped to show that there were different kinds of solutions for the problem. The following excerpt is the example of this alignment.

224 Mrs. Porter: OK. So she made it as a chart, what it's *like this*, it's *what Ophrah did*, but using a chart *instead*.

The utterance was made by Mrs. Porter after a student (Sabrina) talked about her strategy. Mrs. Porter aligned her strategy (i.e., making a chart) with Ophrah's and acknowledged that Sabrina provided another strategy for the problem. In another class, Mrs. Porter made similar statements such as, "In a way it's *very similar*, it *looks different* on your page," and "And it's really *the same thing*, just done in *another way*. This type of alignment helped not only provide multiple solution strategies but also to demonstrate the similarities among the strategies. In the excerpt, Mrs. Porter mentioned the similarity between Sabrina's strategy, which was making a chart representing the repeated addition of 15, and Ophrah's, which was using the multiplications table of 15, and the difference between the strategies (i.e., "instead") as well. Showing the association between/among strategies might contribute to the students' understanding of the problem.

Mrs. Porter used the phrases such as "some people," and "a lot of people" to generate alignments. The following excerpt is one of the examples.

5	Lyndsey:	Doing 984 twelve times and adding it together.
6	Mrs. Porter:	That's it. Lyndsey decided that she could add 984 twelve times. Does this make sense?
7	Students:	Yes.
8	Mrs. Porter:	Why does it make sense? It does make sense to me. *Some people* didn't think of doing it. Why does it make sense? Who can explain why it does make sense? Why could you get the answer by adding 984 twelve times? Raj.

Lyndsay stated her approach to the problem ("How many inches are there in 984 feet?") in Turn 5. While Mrs. Porter agreed with the approach, she said in Turn 8, "Some people didn't think of doing it." This juxtaposition produced the opposite alignment to Lyndsey, which indicated that there were different kinds of approaches to the problem. In a similar way, Mrs. Porter said in another class, "Nathan added up these numbers, but *some of you* didn't. *Some of you* added up 78, 66, 55, 45, and then 36." The phrase such as "some people" and "some of you" created juxtapositions so that Mrs. Porter could show the students multiple approaches to the problem.

The next example is an interesting case of juxtaposition which influenced the students' positions.

60 Mrs. Porter: I saw a lot of people doing that. And that's fine because counting by tens is something *a lot of you* are very comfortable doing.

Mrs. Porter uttered the statement above after Ophrah admitted that she messed up counting and did not finish the problem. The utterance indicated that Ophrah's strategy was popular among the students and that other students might have made similar counting errors as Ophrah did. The phrase such as "a lot of you" aligned Ophrah with those who used the same strategy, and this alignment served to maintain Ophrah's current position which could have been lowered by the errors that she made.

However, the same phrase was used differently in another class.

13 Mrs. Porter: I'll bet *a lot of people* know you couldn't right off. And do you know why I knew you couldn't do it? When you, if you have something, and you want to take something away from it, what you have, has to be bigger than what you're taking away. Right? This is a lot bigger, 1900 is more than 1700.

This statement was uttered after Miranda explained how she did the subtraction problem. (1998-1789=209). Miranda said that she subtracted 1998 from 1789, which revealed that she did not know how to do subtraction. In this utterance, Mrs. Porter differentiated Miranda from other students by aligning her opposite to those who knew how to do subtractions. Although Mrs. Porter used the similar phrase such as "a lot of you (people)," Ophrah and Miranda were positioned differently. Ophrah was positioned equally to other people despite the fact that she made an error, but Miranda was positioned as struggling due to that her lack of knowledge of subtraction. The phrase “a lot of people” seemed to justify Mrs. Porter’s implication that other students did better than Miranda.

In summary, juxtapositions were used to generate alignments. In terms of instructional purposes, Mrs. Porter juxtaposed the strategies to show the students that there were multiple ways of solving problems by articulating the differences in the strategies. Juxtaposing multiple strategies in the discussion helped students build a conceptual links between strategies and at the same time showed who authored the strategies. Making authorship available to the class may result in contrasting the strategies and positioning the students according to the degree of efficiency of strategies. If the strategy was sophisticated, the student who presented it might be recognized as advanced since people thought that the sophistication belonged to him/her. On the other hand, if the student presented the strategy which was not efficient, he or she might not be recognized as advanced since people thought that efficiency was not associated with him or her. The examples showed above would conclude that juxtaposition was serving as an instructional technique (i.e., articulating similarities and/or difference between/among strategies) and also served as one of the factors influencing students’ positions depending on how Mrs. Porter aligned their strategies.

Comparison. The data found that Mrs. Porter compared strategies by emphasizing their efficiencies. Mrs. Porter used comparatives such as “faster,” “more efficient,” “better,” “easier,” and “longer” for comparisons. The following utterances are the example of Mrs. Porter’s use of comparatives.

94 Mrs. Porter: It’s *easier* to count by tens.
And much *easier* to count by hundreds.

When the statement above was made, two students (Bettina and Lyndsey) had already presented their strategies. Both used a counting strategy, but Bettina did so by tens while Lyndsey by hundreds. Before Bettina and Lyndsey were called on by Mrs. Porter to explain their strategies, Ophrah had already presented hers, which was the same strategy that Bettina used. Ophrah, however, failed to count by tens and did not finish the problem. Mrs. Porter compared those two strategies and said that counting by 100s was a lot easier than counting by 10s, because she knew that another student (Ophrah) had already failed to count by 10s. This utterance implied that one strategy (counting by 100s) cause fewer errors than the other (counting by 10s).

Another example is below.

126 Mrs. Porter: You know that now. Right? Because adding 984 twelve
time is the same as adding twelve (pause) 984 times.
127 Aisha: Except that takes longer.
128 Mrs. Porter: Oh boy, one takes a lot *longer* than the other, doesn't it?
129 Students: Uh huh.
130 Male student (possibly Nathan): I, I could do it by one.
131 Mrs. Porter: So, so it's *more efficient* when you know that you have to
add this many times to say, "What if I turned it around?"
Because with adding, you can always turn numbers around.
It's the same answer. And with multiplication, you can
always turn numbers around.
You can't do it with division and subtraction.
But you can always do it with adding.
So when you know you've got all these times to add,
it's good to stop and say, "Why don't I turn that around?"
I could just add this number 12 times.
It's a big number, but it won't take me nearly as long.
So that's something to think about for the future.

In the excerpt above, Aisha presented her strategy, which required her to add twelve 984 times. At first Mrs. Porter aligned her strategy with another one (i.e., adding 984 twelve times) and mentioned the similarity between them, but she did not recommended her strategy because it took too much time. Mrs. Porter clearly said that Aisha's strategy was not as efficient as adding

984 twelve times and suggested that students adding bigger numbers fewer times than adding smaller ones a lot more times.

In addition to using comparatives, Mrs. Porter seemed to reinforce the difference in students' mathematical knowledge through comparisons. Mrs. Porter invited the students to talk about their strategies, which were very different in terms of levels of sophistication. Some students presented very concrete strategies, and others presented abstract ones. That was common in her classroom, because she promoted multiple approaches to a problem and it appeared as one of socio-mathematical norms in the classroom.

However, discussions of multiple strategies sometimes resulted in ranking students based on the sophistication levels of their mathematical knowledge. For example, Mrs. Porter made the following statement when Miranda failed to do 984 times 12.

332 Mrs. Porter: Because we forgot what's ones and what's tens
and what's hundreds. And that's important.
So *this doesn't work for everyone*.

Miranda was showing how she was doing 984 times 2, but her procedure revealed that she was doing so without understanding place value. The last sentence, "this doesn't work for everyone," implied that multiplying 984 with 12 was not an appropriate strategy for those who did not understand place value such as Miranda, and that those who knew little about place value should use other easier strategies. Her statement implied that she divided students into two groups, that is, a group of students who understood place value and were capable of doing 984 times 12 and a group of students who did not.

In another class, Mrs. Porter said as follows.

221 Mrs. Porter: And *some people* might find it very hard in 3rd grade,
especially early in 3rd grade to know what 3 times 18 is.
For *some kids who know 3 times 18*,
that would be an easy way to do it.

> But if you don’t know how to do 3 times 18,
> then you might need another strategy,
> which is adding it up, adding 3 each time.
> And that works.
> Or by doing 3 times 10 and then 3 times 8
> and adding them together.
> *It depends on what you already know and what makes sense to you*. Three times 18 is a wonderful strategy
> *if you know* how to do 3 times 18.
> But *if you don’t*, and some day you probably will be able
> to do that easily, then use a different strategy.

In this class (the October 20 problem), the students were told to pick up a strategy that they thought was the best from five different strategies for solving 3 times 18. Those strategies included using tally marks, writing a tree diagram for calculation, using multiplication tables of 3, combining two algorithms (3×8+3×10), and so on. Some students chose concrete strategies (tally marks, tree diagram) because they were easy to understand, and other chose an abstract strategy (3×8+3×10) because it was quick and did not take up much room for calculation. Mrs. Porter’s utterance above indicated that there were individual differences in mathematical understanding among the students, and she admitted that students’ prior knowledge may have influenced which strategy they chose. The phrases such as “some people,” “some kids who know 3 times 18,” and “if you (don’t) know how to do 3 times 18” seemed to categorize the students according to what they knew. Especially the statement, “It depends on what you already know and what makes sense to you,” implied that the difference in strategy choice reflected differences in mathematical knowledge.

Validating, which was referred to making positive comments on strategies or students, also seemed to produce a pecking order in the classroom by locating a particular student in a very high position. For example, Mrs. Porter said in one class (early in the school year), “This is something you don’t do in 3rd grade,” and “And again, I don’t expect 3rd graders to know how to do this." She made those comments after a student (Pulak) presented an abstract solution for a division problem. His strategy was more abstract and sophisticated that those used by the other students.

A similar example was also observed in students' utterances. Since Pulak frequently presented abstract strategies, there was a student who did not understand what Pulak did.

73	Nathan:	I don't understand what Pulak did.
74	Mrs. Porter:	That's OK. You'll learn more of it.

In another class, Nathan said something similar.

283	Nathan:	I don't understand what Pulak did.
284	Mrs. Porter	You're right, you probably don't.

These excerpts indicated that Pulak was more advanced than Nathan, and especially Mrs. Porter's alignment with Nathan's utterance ("You're right, you probably don't") consolidated Nathan's less-advanced position. The excerpts did not show that Mrs. Porter compared their strategies side by side, her responses to Nathan's utterances resulted in contrasting Nathan and Pulak in terms of understanding mathematics.

In conclusion, the results showed that Mrs. Porter compared multiple strategies to determine the efficiencies of strategies, while she juxtaposed the strategies to show the students different kinds of problem-solving strategies. The comparatives such as "more efficient," "faster," "better," and "easier" were observed as linguistic markers for comparisons. Those comparatives, more or less, implied that Mrs. Porter valued the quickness or efficiency of strategies, which were both reinforced in regular classrooms as socio-mathematical norms. The results also found the case of indirect comparison. Indirect comparisons did not entail side-by-side comparisons of multiple strategies but led to contrasting the differences between/among the strategies in terms of efficiency. Also, students' comments, such as "I don't understand what Pulak did," sometimes contributed to indirect comparisons because the comments reflected the difference in the level of understanding mathematical knowledge. This type of contrast assigned to each of the students particular positions such as "more advanced" and "less advanced," which could produce social ranks in the classroom.

4.2.3 Students' reactions to teacher positioning

The previous sections discussed how Mrs. Porter positioned the students in the classroom, focusing on the acts of validating, interrupting, and juxtaposing/comparing. Those positioning acts were to be discussed bilaterally, that is, to be analyzed from the perspective of the students, since positioning is interactional. In this section, I describe how the students reacted to Mrs. Porter's positioning in terms of social comparison.

4.2.3.1 Students' responses to teacher positioning

This section describes how students responded Mrs. Porter's positioning, especially validating, interrupting, and comparing/juxtaposing.

Responses to validating. Mrs. Porter made positive comments on the strategies or students during the whole class discussions where individual students were usually called on and given opportunities to talk about their strategies. The interactions with the students were not like the IRE/F sequences, but longer sequences with Mrs. Porter's revoicing and prompting. In most instances, Mrs. Porter made positive comments in the last phase of the interaction before initiating a new interaction with a different student.

Most of the students who received positive comments from Mrs. Porter did not show any special reactions (recorded in the verbal transcript) to her validating utterances. These students seemed to simply accept the comments without responding verbally, although their nonverbal responses might have indicated something different.

There was only one instance (in the verbal transcript) in which the students reacted to Mrs. Porter's validation. It happened in the middle of an interaction with Ophrah. The excerpt is as follows.

132	Mrs. Porter:	OK, so you have one fourth, plus one fourth. In a way it's very similar, it looks different on your page, to this. But, and, and, *it's a good way of thinking*. She knew that 4 times 4 is 16, and she probably, you knew, even if you're not saying it. That if you divided them into fourths, and had four of them.
133	Ophrah:	*Well, that's why I did it.*
134	Mrs. Porter:	I thought so. And so that makes three fourths, for each person.

Ophrah aligned herself with Mrs. Porter's positive evaluation ("it's a good way of thinking") saying, "Well, that's why I did it," and Mrs. Porter also aligned herself with Ophrah ("I thought so").

Those two types of reactions may reflect the students' desires to maintain or construct positive self-evaluations as math learners. The first type of reaction indicated that the students who had positive comments accepted the "good strategy" storylines produced by Mrs. Porter and located themselves as they were within the storylines. In those storylines, Mrs. Porter positioned them as contributors who presented good strategies for discussion. No reactions or utterances from them, probably, were their silent agreement to accept the "contributor" positioned assigned by Mrs. Porter. In addition, most of the positive comments were made in the last phases of interaction. Therefore, those comments seemed to serve as the official evaluations or feedback from Mrs. Porter, who had control over the class. Her positive comments might have sounded authoritative to the students and they might have accepted the comments tacitly.

The second type of reaction seemed to be a more active form than the first one. Ophrah aligned herself with Mrs. Porter's "good strategy" storyline without challenging it, which meant that she took up the "contributor" position that emerged in the storyline. Ophrah's reflexive (or self) positioning seemed to play a significant role in terms of constructing her self-evaluation as a math learner, because Mrs. Porter aligned herself with Ophrah. Her utterance, "I thought so," indicated that Mrs. Porter accepted Ophrah's positioning and that her confirmation consolidated Ophrah's "contributor" position.

In conclusion, Mrs. Porter's validations seemed to produce the storylines that positioned those who presented good strategies as contributors to the discussion. Usually validating took place in the last phase of the interaction and served as official evaluations from the teacher. Most of the students accepted the storylines and allowed themselves to be positioned as contributors. I was able to find only one case of validation that was followed by the student's self-positioning, in which Ophrah took up a contributor position by aligning herself with the teacher's positive comment.

Responses to interrupting. Compared to the instances of validating, I found fewer instances of interrupting. Overlapping speech was used not only for silencing students' voices but also for other purposes such as validating, prompting, scaffolding and co-constructing their mathematical story about their problem solving.

Miranda was the student who was interrupted by Mrs. Porter most frequently (N=5). Among three instances, she was silenced twice and received a mixed evaluation from Mrs. Porter once. Mrs. Porter silenced Miranda by initiating a new interaction with a different student or moving on to a different task.

182	Mrs. Porter:	OK. OK. Uhm, Miranda.
183	Miranda:	I did it, sort of like Ophrah’s, but only, I did it by 15 plus 15 is 30, 30 plus 15 is 45, and 45 plus 15 and then like I carried the one and (not audible).
184	Mrs. Porter:	And did you get 75 also?
185	Miranda:	I’m not finished.
186	Mrs. Porter:	You didn’t get finished.
187	Miranda:	Then I got 15 plus 15 is 30, 30 plus 15 is 45 (Mrs. Porter interrupts)
188	Mrs. Porter:	OK. Now did somebody, we’ve got four 15’s equal to 60 minutes and that’s one hour, right? So, how many minutes are leftover? Nathan?

After Miranda was interrupted in Turn 187, Mrs. Porter asked a new question for a new initiation. Mrs. Porter called on Nathan and was going to start a new interaction with him. Miranda’s talk was terminated intentionally by Mrs. Porter’s interruption because it seemed, as discussed before, that Miranda did not understand what she was expected to do. The utterance in Turn 184 indicated what Mrs. Porter expected from Miranda, in other words, she simply wanted to know whether Miranda had gotten the right answer or not. Miranda replied to it, saying, “I’m not finished.” In the next turn (186), Mrs. Porter repeated Miranda’s reply and reinforced that she did not complete the problem to signal that the interaction with Miranda was over. Miranda, however, kept talking about the same thing again, which showed that she was not aligning herself with the storyline of a new problem (“How many hours and minutes is 75 minutes?”) Mrs. Porter was going to produce. A new initiation terminated Miranda’s turn, and as a result, she was silenced. The same evidence was observed in a different class. Despite calling on Miranda, Mrs. Porter interrupted Miranda by initiating a new interaction, saying, “Wait a minute, is there anybody who hasn’t told us a way yet that they would like to share?” Again, Miranda was silenced and did not have a chance to talk further. Those instances suggested that Miranda did not challenge Mrs. Porter's silencing (in the verbal transcript).

Nathan, however, challenged Mrs. Porter's silencing by using overlapping speech. In December 17 class, the class discussed the multiplication problem (984×12, "How many inches are there in 984 feet?") but Nathan somehow thought he needed to use a division strategy.

22	Nathan:	12 times 7 equals to 84.
23	Mrs. Porter:	Why, why did you decide to multiply?
24	Nathan:	Cause I wanted to get 84.
25	Mrs. Porter:	Why?
26	Nathan:	Because that's those are the digits, the tens and the ones, so 84.
27	Mrs. Porter:	OK.
28	Nathan:	And I would know how many inches that was.
29	Mrs. Porter:	OK.
30	Nathan:	So then it would be easier for me.
31	Mrs. Porter:	*I don't know*. I could understand. I understand what you are saying. Nathan sees this and he's saying, "What times 12 equals 84?" Right?
32	Nathan:	Uh-huh.
33	Mrs. Porter:	And you're saying 12×7=84. Seven-times-twelve-equals-84. (It sounds like she's writing this on the board as she says the Words.)
34	Nathan:	So then
35	Mrs. Porter:	*But I'm not exactly sure why you wanted to, to do that*. I can see the digits up here. OK, go ahead, let's see what you decided to do.
36	Nathan:	There were 7 feet and 84 inches. So, so then I knew I had to, I had to, I had to figure out what, what 900, how to multi… multiply.
37	Mrs. Porter:	How to multiply something to get to 900?
38	Nathan:	**No, how to divide** (indiscernible talk) I could divide something.
39	Mrs. Porter:	You're saying, "What times 12 equals 900?"
40	Nathan:	Uh-huh.
41	Mrs. Porter:	OK. Now does, OK. We said, does this make sense? Do people understand this and somebody explains to us why this makes sense. Now does this make sense and why? Do you understand what he Was trying to do?
42	Students:	No.
43	Mrs. Porter:	Does that make sense?
44	Students:	No.
45	Mrs. Porter:	OK. If you think it does NOT make sense, explain why it DOESN'T make sense. Yes.
46	Students:	(silence)

Here Mrs. Porter was not convinced that Nathan's strategy was workable (Turns 31 and 35). The first instances of overlapping speech (Turns 22 and 23) showed that Mrs. Porter asked Nathan for clarification for why he wanted to 12 times 7 since the problem did not require division, and the overlap seemed to signal that Mrs. Porter questioned Nathan's strategy. The overlapping speech (in Turns 36 and 37) showed that Mrs. Porter helped Nathan complete the sentence, but the next instance of overlapping speech indicated Nathan's second-order positioning, that is, he challenged Mrs. Porter's utterance using an overlapping speech. However, Mrs. Porter was trying to maintain the storyline that Nathan's strategy was not appropriate.

The findings indicated that Mrs. Porter interrupted Miranda's and Nathan's utterances and that Miranda and Nathan reacted differently. Miranda did not react verbally to Mrs. Porter's interruption, while Nathan used second-order positioning. Although their responses were different, they were both given silenced positions by Mrs. Porter. In Miranda's cases, initiating a new interaction reinforced silencing and excluded Miranda from the ongoing storyline. In Nathan's case, Mrs. Porter was trying to silence him by initiating a new question (Turns 41 and 43) and positioning other students as evaluators of Nathan's strategy despite his second-order positioning.

Further analysis of overlapping speech: Mrs. Porter's negative comments. Since the data found only a few pieces of evidence that overlapping speech was used for interruption, I expanded the analysis to investigate what else produced silenced positions. The analysis looked closely at Mrs. Porter's negative comments on strategies, since I thought that negative comments might lower students' ranked positions and create silenced positions.

The data found 12 instances of Mrs. Porter's negative comments, although some of them were also categorized into either comparison or validation with mixed evaluations. Negative comments sometimes appeared when Mrs. Porter compared and evaluated two strategies and sent both positive and negative comments at the same time. Mrs. Porter gave negative comments when the following students talked about their strategies: Bernard, Bettina, Miranda, and Nathan. Miranda received the most negative comments (N=6), Nathan and Bettina three times, and Bernard twice.

Mrs. Porter made negative comments when the strategies were not efficient, too complicated to understand, or the students used strategies that they did not understand. In her interactions with Miranda, she said that subtraction was tricky for Miranda and that Miranda's strategy took a long time. Bettina were also given similar comments due to the ineffectiveness of her strategies. For example, Mrs. Porter said, "OK. You were counting backwards form 1998, and you were counting by ones. *That's why you're not finished yet*. Certainly we can do that, but it takes so long. It's easier to count by tens. And much easier to count by hundreds."

The recurrent negative comments to Miranda and Bettina might have affected their social ranks as mathematics learners. Since positioning itself is a transient action, ranks emerging within acts of positioning are also temporal. However, the recurrence of a certain ranking can stabilize those positions that temporally emerged (Wortham, 2006). Making negative comments on strategies temporally assigned the rank as "mathematically struggling students" to Miranda and Bettina. Nevertheless if this assignment frequently recurred, it would make them recognized in the classroom as mathematically struggling students. And if they did not challenge such negative positioning, the position as struggling student could be sustained and construct their identities as mathematics learners.

Nathan also received negative comments from Mrs. Porter several times. He, however, challenged Mrs. Porter's negative comment in the December 15 class where the students were working on the problem of consecutive numbers.

216	Mrs. Porter:	42, 40, 36, 30, 22, 12. Look at them. *I don't think Nathan noticed what he's got here*.
217	Nathan:	They're, they're going backwards.
218	Mrs. Porter:	Ya, ya.
219	Nathan:	*I knew that.*
220	Mrs. Porter:	OK. You did know that. The same numbers. Now why is that happening?
221	Nathan:	Oh, I know.
222	Mrs. Porter:	Why?
223	Nathan:	Cause, cause every time, there. It's just like going this way and this way. They're just crossing up and down.
224	Mrs. Porter:	They are. OK. Why, Aisha.
225	Nathan:	Because.
226	Mrs. Porter:	Does anybody know? Let somebody else tell us, thank you, Nathan. Kevin?

Mrs. Porter made a negative comment in Turn 216, which transiently positioned Nathan as one who was slow to understand, but he challenged this positioning, saying, "I knew that" (Turn 219) to reinforce the impression that he knew something. Although what he said ("They're, they're going backwards" was not anything Mrs. Porter wanted to hear, he said again, "I know" (Turn 221). Moreover, he cut in and tried to answer when Mrs. Porter called on Aisha to explain (Turn 225).

This pattern of interaction was interesting since it was different from the previous examples of Miranda and Bettina who had been positioned as struggling students. Here Nathan received a negative comment but challenged it, perhaps because he did not want to be positioned as a struggling student in this storyline. In the excerpt, he seemed to be negotiating his position so that he would be ranked in a higher position.

Bernard received negative comments from Mrs. Porter, but the position he received was different from “mathematically struggling students.” Mrs. Porter made those comments since Bernard’s strategies were difficult for other students to understand. For example, she said, “OK, let’s wait until we do the 18 with you. OK. Because *it will be harder for them to understand*,” and “He’d have another two thirds left over from each of these thirds. And then when he added them all up he had 18. OK, *that’s kind of complicated*.” Those negative comments on his strategies seemed to position him as a “mathematically advanced” student, who provided unique strategies that teachers did not usually think were “appropriate” for the rest of her third grade students.

Regarding the appropriateness of strategy used, Mrs. Porter seemed to have a belief that strategies should be matched with students’ level of cognitive development. In the October 20 class where the students were discussing their preferred strategy for calculating 18 times 3, Mrs. Porter mentioned her beliefs about developmental appropriateness.

112	Mrs. Porter:	If you know your multiplication tables well. It’s very quick and very efficient. If you don’t know your multiplication tables well, then it[2] wouldn’t be a quick and efficient strategy for you. Right? So sometimes, it depends on who’s doing it. Would it be a good strategy, Karl. You were saying 5^{th} graders were looking at this, they’d really understand it. What about the other kids in this school?
113	Karl:	Well, if they don’t know their multiplication tables, 2^{nd} graders might not like it.
114	Mrs. Porter:	They may not like it. Who, who in this school might have a little trouble understanding this Karl?
115	Karl:	Kindergarten.
116	Mrs. Porter:	OK, I think so.
117	Karl:	And first.
118	Mrs. Porter:	Do you think kindergarten and first might get Number Two[3]?
119	Several Students:	Ya. Uh-huh. Yes.

[2] Mrs. Porter was referring to the strategy which used several different algorithms to do 18×3, i.e., 10×3=30, 8×3=24, and 30+24=54.
[3] Number Two was the strategy that used 18 sets of tally marks grouped into 3s.

In the excerpt above, Mrs. Porter was trying help this student articulate his reasons why using a strategy involving the multiplication tables might not be appropriate for students younger than 3rd graders but might be appropriate for older students. Her utterance “it depends on who’s doing it” (Turn 112) seemed to reflect her belief about students’ strategy use and have some association with her similar statements in other classes such as, “I don’t expect 3rd graders to know how to do this” and “This is something you don’t do in 3rd grade,” which were addressed to an advanced student (Pulak).

In conclusion, I found only a few instances of interruptions of students using overlapping speech, which led me to conduct further analyses. These analyses revealed that not only overlapping speech but also negative comments might lower the students’ social ranks or give the students positions such as “struggling” students. Some students did not make any verbal response to Mrs. Porter’s interruptions and this response may have produced silenced positions. Two students (Miranda and Bettina), in particular, were positioned as struggling students more often than the other students. On the other hand, Nathan challenged Mrs. Porter’s interruption so that he could not be given a silenced position. He seemed to be negotiating his position, whereas Miranda and Bettina appeared to accept their silenced positions.

Further analysis was done to better understand the production of silenced positions. Silenced positions may also have been given when Mrs. Porter made negative comments on the strategies. Mrs. Porter usually did so when she found the strategies not efficient. In addition, those negative comments sometimes implied the students’ lack of understanding of certain mathematical skills or knowledge and seemed to contribute to giving silenced positions to the students whose strategies were negatively evaluated.

The analysis also found that Mrs. Porter made negative comments on the strategies which were unique and complicated. Bernard was the one who received such comments twice. Her comments did not appear to give him a silenced position but rather positioned him as an "advanced" student. The data suggested that Mrs. Porter might have believed that students' strategies should be matched with their level of cognitive development. This might be associated with the reasons why she made negative comments on the strategies that diverged from popular strategies used by average 3rd graders.

Response to juxtaposing and comparing. Mrs. Porter juxtaposed and compared several strategies in different phases of interaction, but she did so most frequently on the last phase (N=38). Most of those juxtapositions and comparisons appeared as (a part of) evaluations or feedback on the strategies previously discussed and Mrs. Porter started new interactions. Since new initiations sent a message that a new storyline would be produced, it seemed that the students usually accepted Mrs. Porter's evaluations and feedback and moved onto the next phase.

However, there were two instances where the students reacted to Mrs. Porter's comparisons. Interestingly, the same student (Pulak) was involved in both of these instances. One was observed when Pulak showed a sophisticated procedure of division during the whole class discussion. Mrs. Porter positioned his strategy as advanced, saying, "I don't expect 3rd graders to know how to do this." Mrs. Porter did not directly compare Pulak with anybody but her utterance did imply that Pulak's strategy was a lot more sophisticated than those used by the other students. This seemed to trigger Ophrah's affective reaction and she quietly said to the peers on her table, "He always does stuff like that." The other instance was observed when Pulak used an abstract strategy for doing multiplication. Mrs. Porter said, "This is something you don't do in 3rd grade," as her evaluation on Pulak's strategy. Nathan said that he did not understand what Pulak had done. Mrs. Porter, however, only said, "That's OK. You'll learn more of it," and she did not fully explain Pulak's strategy. Then Nathan challenged her utterance saying, "Can we learn it now?" Again, Mrs. Porter did not directly compare Pulak with anybody, but Nathan's utterance ("I don't understand what Pulak did") implied that Pulak was more advanced than Nathan. In addition, Mrs. Porter's utterance ("That's OK. You'll learn more of it") seemed to reinforce the differences in social ranks between them. Those two pieces of evidence showed that comparing multiple strategies might trigger the students' affective reactions and/or rank them according to their mathematical knowledge and skills.

4.2.4 Students' social comparisons

This section describes the patterns of their reactions to Mrs. Porter's positioning acts and discusses their ways of responding in terms of social comparison goals.

The data examined Mrs. Porter's positioning with the specific foci on validating, interrupting, and juxtaposing/comparing, and it revealed that some of those positioning acts were related to students' social comparisons. There were 35 instances of students' social comparisons identified in the data. (See Table 11) The students reacted to Mrs. Porter's positioning in particular ways in some circumstances. The data identified that Nathan engaged in social comparison most often (N=8), Ophrah five times, Miranda three times, Kevin twice, and Pulak, Sabrina, Ethan and Benjamin once.

Table 11. Summary of students' social comparison

Motives	Action	Frequency
Protecting self-evaluation	- Challenging teacher's comments (second order positioning) - Showing emotional reactions	6
Constructing positive self-evaluation	- Aligning oneself with teacher's positive comments - Aligning one's strategy with advanced one	28
unclear	- Positioning oneself as good math thinker	1
		N=35

4.2.4.1 Constructing positive self-evaluations as mathematics learners

Out of 35 instances of students' social comparisons, 28 of them were related to the social comparison goal of constructing positive self-evaluation as mathematics learners. To achieve this goal, they responded to Mrs. Porter's positioning in particular ways.

First, the students aligned themselves with Mrs. Porter's positive evaluations or feedback. Here is an example:

132	Mrs. Porter:	OK. So you have one fourth, plus one fourth. In a way it's similar. It looks different on your page to this. But, and, and, *it's a good way of thinking. She knew that 4 times4 is 16*, and she probably, you knew, even if you're not saying it. That if you divided them into fourth, and you had four of them.
133	Ophrah:	*Well, that's why I did it.*
134	Mrs. Porter:	I thought so. And so that makes three fourths for each person.

In the excerpt above, Mrs. Porter made a positive comment on Ophrah's strategy (Turn 132), and Ophrah aligned herself with the comment, saying, "Well, that's why I did it" (Turn 133). This alignment means that Ophrah actively took up a "contributor" position, which was usually given by Mrs. Porter's positive comments.

Second, the students aligned their strategies with better or more advanced ones. The example is below.

190	Nathan:	I can understand it (Number Five[4]), because it's 17 times. And each time they keep adding 3. Like
191	Mrs. Porter:	Right.
192	Nathan:	Three times three equals 9. And then they keep going on, but I think they could have been a little quicker.
193	Mrs. Porter:	OK. What could they have done to make it a little quicker?
194	Nathan:	They could have said, said, they could have done Number Four[5] instead.
195	Mrs. Porter:	OK.
196	Nathan:	Instead of like doing it that way, because it took so long.
197	Mrs. Porter:	So what do you think is better, thr-, four or five now?
198	Nathan:	Four.
199	Mrs. Porter:	So now that you're thinking about it and heard what other people say even though you chose, which two did you choose?
200	Nathan:	1[6] and 5.
201	Mrs. Porter:	1 and 5, now you would change your mind and choose 1 and 4?
202	Nathan:	Uh-huh.

The excerpt came from the class when the students decided the best strategy for doing 18×3. Before Nathan talked, the group which had Pulak, the most advanced student, said that he chose Number 4 strategy as the best one because he thought it was efficient. On the other hand, Nathan chose 1 and 5 (Turn 200) at the beginning but he changed his mind and picked up 1 and 4 (Turn 201). Turn 201 indicated that Nathan aligned himself with Pulak, who were usually positioned as advanced, in terms of strategy choice. His alignment with Pulak gave him the same transient position as Pulak and seemed to contributing to raising his social position in the classroom.

Third, the students took up a position of "good/knowledgeable math thinker/learner" and actively positioned themselves to construct positive self-evaluations. The example is below.

[4] Number Five was the strategy which used multiplication tables of 3 to solve 18×3.
[5] Number Four was the strategy which used multiple algorithms, 3×10=30, 3×8=24, 30+24=54, to solve 18×3.
[6] Number One was the strategy which used tree diagrams which represented repeated additions of 3s.

174	Mrs. Porter:	Don’t you when you tell time 5, 10, 15, how many are there on the clock? And then we go to 30, and then we go to 45, and then we go to 60. Four of them in an hour. You got a clock right there. You can even use the clock. Nobody did that with this problem. But you could have drawn a clock. And counted off the minutes too. So, four 15s equals 60 or 4 times 15 equals 60. And how many minutes are left then?
175	Male student:	*Can I say something? I drew a clock.*

In this excerpt, Mrs. Porter mentioned that using a clock might be useful to identify how many hours and minutes was 75 minutes, but that none of the students did so. A male student, however, challenged Mrs. Porter’s new initiation and claimed that he did use a clock. This showed that the student positioned himself as good math thinker, that is, someone who thought of a useful strategy and used it.

In addition to the example above, the data found that the students also positioned themselves as good/knowledgeable thinker/learner by presenting different strategies. Since discussing multiple approaches to the problem was a crucial part of Mrs. Porter’s instruction, the students seemed to share the idea that they would be able to contribute to the discussion if they presented a different strategy. From the data, I identified Mrs. Porter’s utterances, such as “Who did it differently?” and “Did anybody do it a different way?” and “This is a real different,” many times during the class.

28	Mrs. Porter:	So that’s a strategy quite a few people used. Some people wrote it across this way, but a lot of people. You used that strategy?
29	Kevin:	*Ya, but in a different way.* You see I did something like this but what I was doing was I put every number like in a plus way and then I like added it up and then I went to the [not audible].
30	Mrs. Porter:	You mean when you got to this part?
31	Kevin:	Well see I did.
32	Mrs. Porter:	You did the 13s.
33	Kevin:	I did it like this somehow and I did it like this.
34	Mrs. Porter:	Yes. But you did it across and he did it down.
35	Kevin:	Ya.
36	Mrs. Porter:	OK. Who did it a different way? Miranda didn’t you do it this way too?

In this example, Kevin emphasized the difference between his strategy and one of the strategies which were already discussed in the class (Turn 29), although the difference turned out to be very minor (Turn 34) and Mrs. Porter invited a new student for a new different strategy. It seemed that Kevin might have known that the difference was very small, since he immediately aligned himself with Mrs. Porter’s utterance, “You used that strategy?” saying, “Ya.” However, he quickly said, “but in a different way,” which indicated his challenge to the storyline that Mrs. Porter was going to produce, that is, the one that Kevin used the same strategy too. This challenge seemed to reinforce his attempt to position himself as a contributor or good math thinker.

Constructing positive self-evaluations could be matched with the self-improvement motive, which is usually identified by upward comparison (i.e., comparing oneself with someone who does better) (Dijkstra et al., 2008). Although comparison targets were not always actual (e.g., peers) but sometimes desired (e.g., imaginary self/person as good math thinker), all of the examples above supported upward comparisons in terms of the direction of comparisons. The next section discusses another comparison goal: protecting current self-evaluations as math learners.

4.2.4.2 Protecting current self-evaluation of mathematics learners

This section discusses the other social comparison goal, which is protecting current self-evaluations. The data identified six instances of this comparison goal and found several particular patterns of how the students protected their current self-evaluations in the classroom.

One of the patterns of this comparison goal was a second-order positioning, that is, challenging Mrs. Porter's negative comments/feedback on strategies. The example is as follows.

13	Mrs. Porter:	Do you know why you can't? I'll bet a lot of people knew you couldn't right off. And do you know why I knew you couldn't do it? When you if you have something, and you want to take something away from it, what you have, has to be bigger than what you're taking away. Right? This is a lot bigger, 1900 is more than 1700.
14	Miranda:	*But then I switched it around the second time.*

In the class where the excerpt above came from, the students were discussing how to do 1998-1789 but Mirada seemed unsure how to begin this problem (e.g., which number needs to be subtracted). Mrs. Porter's utterance indicated that when she compared Miranda with other students (who understood subtraction) she evaluated Miranda's approach negatively. The phrase such as "a lot of people (knew you couldn't right off)" seemed to make the statement sound legitimate and justify her positioning Miranda as a struggling student. As a result, the phrase distanced Miranda from the other students and positioned her as a struggling student.

However, Miranda challenged Mrs. Porter's utterance and told that she did it differently the second time. Miranda's utterance indicated that she tried to avoid being positioned as struggling and hoped to recover from this assignment. This seemed to reflect Miranda's motive for protecting her current self-evaluation as a math learner.

Second, the students reacted affectively to social comparisons. They seemed to show negative emotions such as envy. The data found this evidence occurring especially when Mrs. Porter positioned a particular student as more advanced than average students. As already mentioned before, Mrs. Porter validated Pulak's strategy by saying, "I don't expected 3rd graders to know how to do this," and then Ophrah reacted affectively and said, "He (=Pulak) always does stuff like that." Mrs. Porter's validation contrasted the gap in mathematical knowledge and skills between Pulak and the other students, and Ophrah seemed to have felt her self-evaluation threatened. As (Tesser et al., 1988) argued in their SEM theory, this kind of unpleasant comparison occurred when a comparer recognizes that the difference between him/her and his/her comparison target is significant and someone close (e.g., friend, family member) outperforms in the area that the comparer think important to him-/herself. Ophrah's emotional reaction seemed to support the SEM theory.

Third, the students distanced themselves from someone who had silenced positions (e.g., struggling student) when they were being aligned together. The example is below.

180	Mrs. Porter:	Think about it a little bit. Did you say, Bettina and Sabrina, that you also chose two different ones?
181	Sabrina:	She (=Bettina), she chose one, but I didn't want to choose that one. She chose it.

Mrs. Porter aligned Sabrina with Bettina in terms of strategy choice (the task from October 20). Sabrina, however, resisted the alignment and asserted that it was Bettina who chose those strategies, not her. Her resistance seemed to reflect her attempt to distance herself from Bettina, who was frequently positioned as struggling by Mrs. Porter. Sabrina's utterance indicated that she might be defending her current self-evaluation as a math learner by being aligned with Bettina, and that she might not want to be recognized as struggling in the classroom.

4.2.4.3 Summary of the results

In conclusion, the data revealed that the students engaged in social comparisons (1) to construct their positive self-evaluations as math learners and (2) to protect their current self-evaluations as math learners. The results found that the students engaged more often in the act of constructing their positive evaluations than the one of protecting their current self-evaluation, and that the act of self-protection took place especially when the students felt their self-evaluations threatened by Mrs. Porter's positioning. The result that the students were motivated more frequently by the goal of constructing positive self-evaluations suggested that the students were actively negotiating positions so that they would be recognized as good math thinkers/learners in the classroom. This seemed to support the SEM theory's assumption that people are motivated to maintain their positive self-evaluations. In the same light, their act of protecting their current self-evaluations or recovering their damaged self-evaluations also seemed to support this assumption.

Moreover, the results showed that there were particular students who engaged in social comparisons more often (e.g., Nathan, Ophrah) than the other students. The frequent involvement in social comparison seemed to suggest that Nathan and Ophrah might be more conscious about their self-evaluations as math learners than the other students and the relations between mathematics as a subject and themselves would be more significant than those of the other students. Social comparisons would be a crucial affective factor influencing how students locate themselves in mathematics.

It would be important to discuss students' social comparison from the perspective of reform mathematics. The data found that Mrs. Porter reinforced multiple approaches to a problem as one of the socio-mathematical norms in the classroom. She implemented the norm and tried to hear as many strategies as possible from the students. Multiple strategies were usually presented during whole class discussions and helped to show that the students could approach to the problem in various ways.

However, the data showed that reinforcing multiple approaches to a problem contrasted the students' strategies or their mathematical ability during the discussions (e.g., Pulak did 15×5, while Nathan said he did not understand Pulak's strategy and Bettina used a tally mark strategy). As the analysis of comparisons and juxtapositions revealed, Mrs. Porter contrasted multiple strategies and made comments in terms of efficiency. Since teachers' evaluations have significant influence on students' self-evaluation, Mrs. Porter' comparison episodes may promote contrasting of not only strategies but also students' mathematical ability and trigger social comparisons among students who receive the evaluations.

5.0 DISCUSSION AND CONCLUSIONS

This study aimed at investigating how teacher positioning influenced students' opportunities to learn in a 3rd grade mathematics classroom, utilizing the socio-cultural view of learning and positioning theory. The analysis had two phases. The first phase focused on Mrs. Porter's teaching philosophy in terms of the norms which appeared in the classroom and the positions that she took up. Investigating her teaching philosophy was necessary, since the philosophy created the environment which would influence classroom interactions and promote social comparison among students. The second phase focused on investigating the interactional patterns of classroom discussions and episodes of positioning during whole class discussions. First, I examined whether or not there were any repeated interactional patterns in classroom discussions. Second, I analyzed teacher positioning in terms of validating, interrupting, and comparing/juxtaposing and examined how students reacted to those instances of positioning. Lastly, I investigated students' social comparison statements in terms of teacher positioning, that is, how teacher positioning was related to students' social comparison.

This chapter provides discussion and conclusions of the results from the following perspectives: (1) teaching philosophy as a hybrid discourse, (2) the influence of teacher positioning on students' participation in learning, and (3) social comparison as discursive positioning to mediate students' identities as mathematics learners. The limitations of this study and implications for future mathematics education research are also provided.

5.1 TEACHING PHILOSOPHY AS A HYBRID DISCOURSE

The analysis of stance bundles was helpful for identifying the norms that Mrs. Porter mentioned. The results indicated that several norms appeared across the data, such as social, socio-mathematical, teaching, and other. In the classroom, Mrs. Porter delivered only social and socio-mathematical norms by using particular language patterns (e.g., obligation/directive stances, anecdotes/narratives, adjectives which require value judgments). In the newsletter to parents, she mentioned teaching norms as well as social and socio-mathematical norms with the same language patterns. In the interviews, however, Mrs. Porter mentioned the norms in a different way. The data found not only the same language patterns that appeared in the classroom and the newsletter but also other language patterns which were associated with the positions that Mrs. Porter took up, such as teacher-researcher who was interested in children's cognitive development, reform teacher who was motivated to teach differently, a guide who helped children learn, and a puzzled math student who was forced to use particular algorithms with little understanding. She frequently used words/phrases which described cognition, such as "realize," "understand," and "make clear (to me)," when she took up a position of teacher researcher. Those words/phrases indexed that, as a teacher researcher, she realized/understood something about children's cognitive development, which she utilized for her teaching. Also, when she took up a "guide" position, she often used the words/phrases such as "help," "lead," "empower," and "for them (= her students)," that signaled her intention to help the students learn. The positions she took up produced particular types of language use due to the rights and obligations that belonged to the positions.

In the positioning framework, Mrs. Porter's teaching philosophy can be viewed as something that was discursively constructed by the positions she took up and the norms which appeared. As the analysis of stance bundles showed, the positions she took up shaped her language use due to the rights and obligations which the positions had. For example, as a reform teacher, she said in the interview, “if they (= the students) don’t understand, then it’s something we *have to* work out. And *try to* make clearer and find other ways of looking at.” Through this example, we can tell that the position as a reform teacher allowed her to use the phrases such as “have to” and “try to,” which signal the responsibilities of reform teachers in a form of obligation/directive stance. Similarly, Mrs. Porter said, “It just depends, some children are ready and others *have to* really be nurtured and encouraged” and “you *cannot* let kids flounder,” taking up a guide position. Those obligation-directive bundles appeared in the interview as parts of the goals that Mrs. Porter thought were important for teaching and learning mathematics.

The “I poems” (Gilligan et al., 2003) that Mrs. Porter constructed in the interview showed that what she said reflected different voices (or positions) such as reform teacher, guide, teacher researcher, novice teacher, math student, and so on. Mrs. Porter took up more positions in the interview than in the whole classroom discussions, and the I-poems from the interview captured more deeply her thoughts about not only teaching mathematics but also children’s cognitive development and her experience of teaching mathematics. As the I-poems are a collection of what she said and/or was going to say, her teaching philosophy can be viewed as something constructed by the voices from different positions.

However, the voices from different positions are not always agreeable with each other because, according to the positioning theory, each position contains its unique rights and obligations. The data showed that Mrs. Porter positioned herself differently in the interviews, whole class discussions, and newsletter to parents. In the newsletter, for example, she restricted herself to a particular position (e.g., guide with knowledge and skills for teaching reform mathematics) and silenced other positions that appeared in the interview and during the classroom discussions (e.g., novice teacher, puzzled math student, teacher researcher). This implies that Mrs. Porter intentionally selected what positions were to be presented depending on to whom those positions were addressed. In other words, she was negotiating her different positions to produce a storyline that would be appropriate for both Mrs. Porter and the addressees.

As long as a teaching philosophy is constructed through the negotiation of multiple voices from different positions, we need to consider that a teaching philosophy might have been communicated differently depending on what positions appeared in a particular context. This claim is partially supported by the evidence that Mrs. Porter seemed to send to the students mixed messages about what should be valued in learning mathematics: efficiency versus sense-making. Such an inconsistency may create some confusion among students and become an obstacle to their conforming to classroom norms.

From the interpretive perspective, the way in which Mrs. Porter's teaching philosophy was constructed by multiple voices and delivered differently depending on contexts would lead us to the consideration of multi-layered discourse. (Harré, 2001) stated that "a remarkable feature of human social interaction, in contrast to that of other primates, is the overlaying of the first-order action, be it in a conversational medium or some other, with an interpretive gloss, a second-order discourse, an account" (pp. 699-700) and that "accounts address the question of the intelligibility and warrantability of actions" (p. 700). His statement suggests that we pay attention to differences of discursive actions which would appear with shifts of positions. The close look at accounts would be of great help for interpreting classroom interactions and provide greater understanding of those social activities.

5.2 INFLUENCE OF TEACHER POSITIOING ON STUDENTS' PARTICIPATION

The results showed that Mrs. Porter engaged in particular types of positioning such as validating and interrupting during classroom discussions. Those positioning acts were usually associated with Mrs. Porter's evaluations of students' problem-solving strategies. When validating occurred, she gave positive comments on the strategies. The students who presented those strategies were usually positioned as contributors to discussions, role models, or advanced students. On the other hand, Mrs. Porter gave silenced positions to particular students by interrupting their utterances or giving negative comments on their strategies.

It would be necessary to consider the possibility that teacher positioning might influence students' participation in learning. Giving positive comments on particular strategies means that the students who presented those strategies were also positioned positively as mathematics learners. Positions such as role models, advanced students, and/or contributors would provide opportunities to engage in discussions productively and construct knowledge. For example, Ophrah was often positioned as a contributor during classroom discussions even when she admitted that she messed up and could not complete a problem. Mrs. Porter did not give Ophrah negative comments but rather used her error as an example. As a result, not only Ophrah but also other students had opportunities to learn from mistakes.

On the other hand, another type of teacher positioning would limit students' participation in learning. One of the significant instances was that Mrs. Porter did not accept Nathan's request that he wanted to know the sophisticated algorithms presented by Pulak, who was usually positioned as an advanced student by Mrs. Porter. Rather Mrs. Porter said to Nathan (and other students) that the strategy was too difficult for 3rd graders to understand. Particularly this instance seems to be associated with the norm, "developmental appropriateness," which was observed as a teaching norm in the data. In the interview, Mrs. Porter admitted that she was a strong advocate for Constance Kamii's proposal that teaching algorithms to children just as procedures would be harmful for their mathematics learning (Kamii & Dominick, 1998). Kamii and Dominick's study was based on Piagetian developmental psychology, which is well known for the stage theory of development. Mrs. Porter seemed to be largely influenced by Kamii's argument about teaching algorithms and might have believed in developmental constraints (Metz, 1995) on children's learning. Probably Mrs. Porter thought that the strategy which Pulak presented was too difficult or not developmentally appropriate for average 3rd graders, when Nathan publicly showed his interest in Pulak's strategy. This type of positioning silenced Nathan's voice and hindered his opportunities to learn about a new strategy that he had not understood before.

Those instances show that validating and interrupting would influence students' OTL by either promoting or restricting their participation in mathematical discussions. This evidence seems to support Greeno and Gresalfi's (2008) argument that "different OTL is afforded to each individual from moment-to-moment, depending on how he or she is positioned in the interaction" (p. 183). Students do not have the same OTL just because they are exposed to the same information or subject matter. It is important to keep in mind that students' OTL is largely influenced by the ways they are positioned and that teachers also play a significant role in positioning students.

In addition to validating and interrupting, the results revealed that Mrs. Porter engaged in another type of positioning, that is, comparing or juxtaposing multiple strategies. This positioning is very popular in classrooms, because it is a common teaching technique. Comparing or juxtaposing multiple strategies helps students understand the conceptual links between/among strategies by generating alignments or contrasts. Mrs. Porter used this technique very often. She invited as many students as possible to explain how they solved the problem, saying, “Did anybody else do it differently?” Providing multiple strategies would be useful because it can be considered as one way of balanced instruction (Sfard, 2003). Sfard argued for the importance of meeting individual student’ needs in learning mathematics and proposed balanced instruction which does not teach a single strategy as a recommended solution but rather respects the multiplicity of students’ problem-solution ideas. From the perspective of balanced teaching, comparing or juxtaposing strategies seems useful for students’ learning.

However, it should be noted that such comparison and juxtaposition sometimes resulted in ranking students’ strategies according to sophistication. The data showed that this evidence became salient when Mrs. Porter compared two strategies and said that one was more efficient than the other. Such ranking might influence students’ social status in the classroom and position students who present sophisticated strategies as good math thinkers or advanced students. If they are positioned as such, then they might have more opportunities to get involved in productive mathematical arguments/discussions, because their ideas are valued and respected.

The study focused on the teacher’s validating, interrupting, and comparing/juxtaposing and examined how those instances of positioning influenced students’ participation in learning. The results found an important association of teacher positioning with students’ participation. The influence of teacher positioning on students’ participation is very complex and difficult to fully understand. However, we should pay special attention to this issue, because it is concerned with equity in terms of how students’ OTL is maximized.

5.3 SOCIAL COMPARISON AND STUDENTS' IDENTITIES

This study found that students engaged in social comparison frequently during whole class discussions. This finding suggests that social comparison is a common phenomenon among children in early elementary grades, and that learning is not purely cognitive work such as knowledge acquisition but rather more complex joint work by students and teachers. In this study, positioning theory articulated the dynamics of classroom interactions and revealed that the teacher and her students were negotiating various positions in the classroom.

This study identified two social comparison goals: constructing positive self-evaluations as mathematics learners and protecting current self-evaluations as mathematics learners. For example, students frequently aligned themselves with Mrs. Porter's positive comments despite the target of those comments. Also, some students distanced themselves from their classmates who were recognized as struggling math students. Other students challenged Mrs. Porter when she made negative comments on what they did.

Those two social comparison motives can be well explained by the self-evaluation maintenance (SEM) theory (Beach & Tesser, 2000; Tesser et al., 1988), which assumes that people are motivated to maintain positive self-evaluations. The SEM assumption can help us interpret that those two social comparison goals reflected the students' desire to be recognized as good math thinkers/learners. As a psychological mechanism of social comparison, the SEM model argues that people activate either reflection or comparison process depending on how close a person is to a comparison target and how relevant a comparison domain is. The reflection process usually occurs when the closeness and domain relevance are low, while the comparison process takes place when the closeness and domain relevance are high. In a situation of upward comparison (i.e., comparison with a superior target), a person can experience positive feelings through reflection and negative feelings though comparison. The difference between reflection and comparison is also helpful for interpreting Ophrah's case. In one class, Ophrah reacted affectively toward Pulak by saying, "He always does stuff like that," when Mrs. Porter gave very positive comments on Pulak's strategy. Pulak used a standard algorithm of division and Mrs. Porter mentioned that the algorithm was too difficult for 3rd graders to understand. From the SEM perspective, we would assume that Ophrah might have showed envy since she engaged in the comparison process. She might also have felt close to Pulak and mathematics might have been an important domain for her. In addition to the SEM, (Alicke & Zell, 2008) argued that a person feels envy toward an upward comparison target in the domain that is important to the person, and the person feels more envious as the closeness is higher. If we interpret Ophrah's reaction to Mrs. Porter's validation of Pulak as envy, it might be plausible to argue that Pulak was the target of Ophrah's upward comparison and math was an important domain for her.

It is worth noting that the interpretation of envy could be problematic. As the SEM explained, people usually engage in two psychological processes, reflection or comparison, when they experience social comparison. They feel negative emotions such as envy when they activate comparison, whereas they admire their comparison target when they activate reflection. That is, upward comparison would produce different emotions, depending which psychological process they use to process social comparison. Therefore, it would be important to pay particular attention to the context of social comparison for interpreting positive or negative emotional reactions to social comparison.

It was quite interesting that this study found that children as young as the age of eight engaged in social comparison in an ordinary classroom setting. They were not given any experiments such as peer nominations or social comparison tasks, but they were in regular math classes. This suggests that social comparison is quite common among children even in the early elementary grades and that they are engaging in social comparison on a daily basis. In this study, students used social comparisons for two goals, one for constructing positive self-evaluations and the other for protecting current self-evaluations. The desire for positive self-evaluation was quite significant among them, and it would be necessary to understand why they desired a positive self-evaluation in terms of developmental change in children's social cognition.

Butler (1992) differentiated children's social comparison goals and reported that children's social comparison goals change from mastery-oriented to performance-evaluation around the age of seven. This means that children over seven have needs for gaining positive evaluations or avoiding negative judgments and that they become conscious about showing that they are superior to others. In addition, Seidner, Stipek, and Feshback (1988) reported that feeling proud changed according to social comparison goals. Seidner et al. found that second graders started feeling proud by comparing their performances with those of others, and that children became less likely to feel proud of their mastery as they grew up. Their findings indicate that children need to feel superior to others to experience pride. This developmental change should be taken into consideration for understanding children's social comparisons.

Children's desire for constructing positive evaluations can be discussed in terms of their identities as mathematics learners. According to (Sfard & Prusak, 2005), learning is to fill the gap between one's actual identity and designated identity. In other words, when students learn, they are trying to become what they want to be. This study revealed that students engaged in social comparison based on their goals for constructing positive self-evaluations and protecting current self-evaluations as math learners. Their social comparisons appeared in the classroom as a form of aligning themselves with the teacher's positive comments regardless of the target of those comments, and a form of distancing themselves from struggling math students or challenging the teacher's negative comments. Those findings suggest that students made use of positioning to be recognized as good math thinkers/learners, and that social comparisons appeared when students need to negotiate multiple positions to construct a positive future identity as a learner. As not only a milestone of social cognition but also an outcome of social processes, social comparison seems to appear in students' everyday discourse and plays a crucial role in their learning of mathematics.

5.4 LIMITATIONS OF THE STUDY

This study has some limitations. One is concerned with the data. This study is a secondary analysis of previously collected data for examining different topics, and data collection was already completed. For this reason, analyses and interpretations will be restricted by necessity. The original dataset was comprised of transcripts of whole class discussions recorded on audiotapes. Thus, important nonverbal information was not recorded and could not be used to interpret the subtle and often contradictory cues available through people's gestures, facial expressions, body positions, etc. In addition, only a sample of those transcripts was analyzed in this study. As a result, the study's conclusions must be understood as based on a limited set of classroom interactions.

Another is concerned with the nature of the analysis. Since this study aims at examining the process of students' social comparison in terms of classroom interactions, analyses needed to be conducted on moment-to-moment bases. Such a micro-level analysis may fail to link to a macro-level of analysis of out-of-school contexts such as family relations, community, and social systems, although both types of analyses operate simultaneously. This will be one of the issues to be considered for future studies on social comparison in educational research.

Although this study aimed at investigating social comparison, it did not use the formal measures of social comparison. This is another limitation of the study. A future direction will have to focus on methodological considerations, which will aim at utilizing the formal measurements of social comparison as well as positioning analyses.

5.5 IMPLICATIONS FOR FUTURE RESEARCH

Based on the findings from this case study, this section provides implication for future educational research on the following issues: (1) equity, (2) discussion orchestration, and (3) social comparison.

5.5.1 Implications for future research on equity

As already mentioned, equity has been a critical issue in educational research. This case study conducted a micro-analysis of classroom discourse and investigated equity in terms of students' OTL. The results showed that students had different OTL and that teacher positioning and norms might influence students' participation in mathematical discussions.

What the results indicate is that multiple factors are involved in students' OTL such as teacher positioning, norms, and students' participation. That is, we should discuss OTL in terms of the relationship between learners and the learning environment, as Gee (2008) suggested. In order to do so, it would be necessary to investigate classroom discourse very carefully.

Classroom discourse is a very rich source of data that can provide plenty of information on how people (e.g., students, teachers) interact. It is considered as a place where social and psychological phenomena are discursively constructed (Harré & van Langenhove, 1999; van Langenhove & Harré, 1999) as well as a site for communication between teachers and students.

As one of ways to investigate classroom discourse carefully, positioning might be useful. Many researchers have already applied the concept of positioning to their studies (e.g. Anderson, 2009; Barnes, 2004; Bishop, 2012; Bomer & Laman, 2004; Herbel-Eisenmann, 2009; Ritchie, 2002; Wood, 2013; Yamakawa et al., 2009) and depicted the dynamics of classroom discursive interactions. Since students' OTL is subject to positioning (Greeno & Gresalfi, 2008), changes in positioning would enable us to understand how students' participation in classroom activities change. Such changes might be made by teacher positioning or students' self-positioning. A discursive approach such as the use of positioning would be able to contribute to investigating OTL.

Although a micro-analysis of classroom discourse is quite useful, it is necessary to consider larger contexts such as race, socio-economic status, and community for investigating equity. As Esmonde (2009) pointed out, equity is not fairly distributed to all students and an achievement gap exists among particular ethnic groups. She argued for the consideration of the fact students' social identities (e.g., race, socio-economic status, gender) influence their identities as learners. If a focus is placed only on classroom discourse, then investigations might miss an important link between social identities and learner identities.

In conclusion, research on equity is very important in terms of providing every student with OTL. The research would require micro-analyses of moment-to-moment interactions in classrooms, because the analyses depict how OTL is determined by different positioning.

Positioning theory would be helpful as an analytical framework for the research. In addition, the equity research would need to consider out-of-school settings in terms of the influence on the construction of students' identity.

5.5.2 Implications for research on discussion orchestration

Stein, Engle, Smith, & Hughes (2008) have proposed a new method for orchestrating productive discussions in mathematics classrooms. This proposal was made from struggles and dilemmas that teachers have been experiencing. In reform classrooms, teachers are expected to be a good facilitator of discussion. However, Stein et al. reported that many teachers had difficulty dealing with unpredictable questions and ideas that students presented during the discussions. Especially novice teachers are not used to improvisation since they have little knowledge of how to guide discussions. Stein et al. argued for the importance of teachers' roles during discussions and proposed five practices for making discussions productive, which are anticipating, monitoring, selecting, sequencing, and connecting. They call this proposal the 2^{nd} generation of reform practice.

From the reform mathematics point of view, Mrs. Porter was a teacher who rigorously engaged in reform practices. The reform practices of the 1^{st} generation suggested that teachers use cognitively demanding tasks that would promote students' participation in discussions, and emphasize creating social norms for learning (e.g., respecting different opinions, listening to others). Mrs. Porter usually encouraged students to present their strategies to show them that there were multiple approaches to the same problem. The interactional pattern in her classroom was not the typical IRE/F sequence but rather longer and more dialogic. In addition, she told the student about the importance of learning from mistakes and encouraged them to take risks to solve problems. Those findings suggest that Mrs. Porter was a 1st generation reform practitioner.

The study also revealed that Mrs. Porter's long teaching experience seemed to allow her to do some of five practices (Stein et al., 2008) proposed. For example, Mrs. Porter developed problems based on her teaching journals, which gave her ideas of how her past students worked on the problems. The journals were useful for her when she prepared for her whole class discussions and "anticipated" ideas that students would present in the discussions. The evidence that she often invited different strategies suggests that she practiced "anticipating." Her journal also helped her "monitor" students' strategies. Inviting multiple strategies helped students understand mathematical links between the strategies. By juxtaposing and comparing the strategies, Mrs. Porter explained the conceptual similarities and evaluated which one was efficient/quick. This indicated that Mrs. Porter was "connecting" different approaches to the problems.

While this five practice proposal is very useful for teachers in terms of providing practical methods for orchestrating discussions, this study showed that some types of teacher positioning (e.g., validating, interrupting, comparing) affected students' participation in discussions and restricted their access to particular strategies. Some types of teacher positioning could also rank the strategies according to their sophistication levels and influence students' social status in a classroom. Future research should include those issues.

5.5.3 Implications for future research on social comparison in classroom settings

This study has captured another type of dilemma that is related to reform efforts. Since NCTM proposals encourage students to participate in mathematical discussions, promoting their participation indicates that ideas and strategies are subject to comparison and evaluation. Although comparison and evaluation are made to promote their conceptual understanding and not separable from discussions, students' ideas could be compared with more efficient ones and positioned as less sophisticated. Recurrent negative positioning may affect students' self-evaluations as learners and may trigger unproductive social comparisons.

This study revealed that students reacted to a teacher's positioning and engaged in social positioning to construct or protect positive self-evaluations as mathematics learners. The results indicate that affective components were involved in learning mathematics. Since the NCTM recommended that students develop positive dispositions towards mathematics, identities have generated particular attention among researchers who have worked on affective aspects of mathematics education. In addition to identities, other affective issues such as social comparison should be investigated more rigorously in future research. Social comparison seems to be an important issue because it influences a person's self-evaluation in a domain which he/she thinks is relevant to him/her. Mathematics could be one of those domains of relevance.

Although traditional social comparison research has heavily relied on methods such as peer nominations and social comparison tasks/measurements, a discursive approach might be useful as an alternative way to investigate social comparison. This new approach enables us to conceptualize psychological issues as discursive products which would appear in social interaction. We can, therefore, investigate such issues considering the dynamics of social processes. Positioning theory allows for moment-to-moment analyses of interactions and could significantly contribute to future research with its methodological advantage.

APPENDIX A

MRS. PORTER'S NEWSLETTER TO PARENTS

Frances Porter
Third Grade

What I Believe About Teaching Mathematics

It is important that mathematical concepts, rules, and procedures make sense to students. Therefore, I try to develop assignments, tasks etc. that promote understanding.

I encourage students to explore, conjecture, and think as they estimate, compute, and develop strategies to solve problems and to construct personal knowledge.

As much as possible, the mathematics in my classroom is related to every day life. Skills are developed in the context of solving problems.

I agree with Marilyn Burns' statement, "Learning is an internal process that happens in individual ways and on individual time tables. . . [so it is important that3 the children push the curriculum, not the other way around." Therefore, problems need to be as open ended as possible allowing for interpretations at a variety of developmental levels.

We cannot give children knowledge and understanding. They must construct it for themselves. Experience has shown me that students need opportunities to explore new concepts using materials and procedures that are appropriate to their developmental levels. This exploration should take place BEFORE concepts are introduced formally, so instead of offering students a list of rules or procedures for solving a particular kind of problem or completing a task, students are encouraged to develop strategies on their own or in groups. It is when we discuss what has taken place that concepts are introduced and developed. Students look for patterns and connections that allow us to make generalizations, theories and predictions about the way numbers work.
Although this approach is sometimes frustrating at the beginning, and errors are often made in the process, it is important that students understand that mistakes and frustration are a part of

learning. Mistakes allow teacher and student to see where misconceptions lie, so that they can be addressed.

Writing and discussion are essential elements in a successful math program. In order to explain their approaches to solving problems, students must become conscious of their thinking processes. The emphasis shifts from getting the right answer to developing thinking and understanding.

We must believe that all children are able to learn and convey this message to students by continually commenting on the learning that is evident. We must resist the temptation to focus on errors and what children are unable to do. Instead, we must look carefully at what children demonstrate they are able to do and then use our experience and knowledge about the development of mathematical thinking to help them move forward.

Both traditional and alternative algorithms are discussed when appropriate, but one approach or algorithm is not valued over another. It is not expected that all children will use the same algorithm to arrive at an answer. Children are encouraged to develop and refine algorithms and procedures consistent with their thinking. All workable strategies and algorithms are accepted, but we continually discuss the relative efficiency of strategies and algorithms as we work.

Just as spelling, punctuation and grammar are the tools for writing, addition, subtraction, multiplication etc. are viewed as tools for doing math. Students are encouraged to continually "sharpen their tools" in order to do work more easily and efficiently.

Multiplication is an important element in the third grade program. During the year, students generally move from solving (what are usually considered) multiplication problems by using addition, to actually multiplying when appropriate. By the end of the third grade year, most students are able use multiplication comfortably as they work. Students can also be expected to be familiar with the terms multiple, factor, and product. They should have a sense of how multiplication relates to division and about how both multiplication and division relate to fractions.

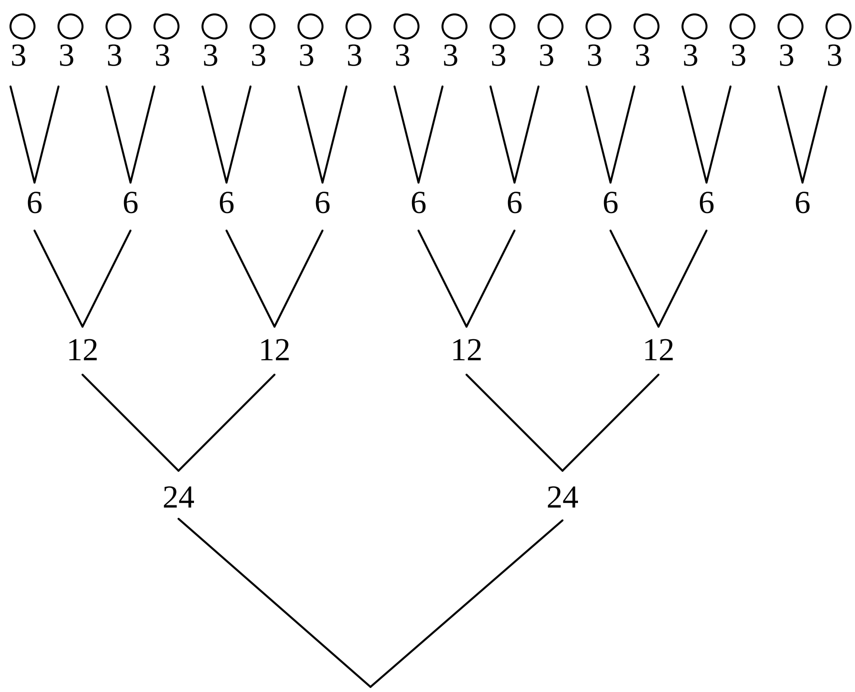
By the end of third grade, students should have become better at estimating and more willing to take risks. They should be able to explain the strategies they have chosen to solve problems and be able to make judgments about the efficiency of various strategies. They should understand the importance of discovering patterns and relationships and should have developed the habit of searching for them as they work. It is expected that students will be familiar with many mathematical terms such as, area, perimeter, circumference, and consecutive.

Students are also expected to have increased understanding of place value and large numbers, plane and solid geometry, some beginning algebraic notations, time and other measurements, money, probability, and statistics.

APPENDIX B

SOLUTION STRATEGIES DISCUSSED ON OCTOBER 20

Strategy 1



$$\begin{array}{r} 48 \\ +\quad 6 \\ \hline 54 \end{array}$$

Strategy 2:

3 | | |	3 | | |	3 | | |
3 | | |	3 | | |	3 | | |
3 | | |	3 | | |	3 | | |
3 | | |	3 | | |	3 | | |
3 | | |	3 | | |	3 | | |
3 | | |	3 | | |	3 | | |

Strategy 3:

18 × 2 (lbs each) = 36 lbs

36 + 18 (1 more lb each) = 54lbs

Strategy 4:

3 × 10 students = 30 lbs

3 × 8 students = <u>24 lbs</u>

54 lbs

Strategy 5:

3 lbs ×2 = 6	2 brains	3 × 10 = 30 lbs	10 brains
3 × 3 = 9 lbs	3 brains	3 × 11 = 33 lbs	11 brains
3 × 4 = 12 lbs	4 brains	3 × 12 = 36 lbs	12 brains
3 × 5 = 15 lbs	5 brains	3 × 13 = 39 lbs	13 brains
3 × 6 = 18 lbs	6 brains	3 × 14 = 42 lbs	14 brains
3 × 7 = 21 lbs	7 brains	3 × 15 = 45 lbs	15 brains
3 × 8 = 24 lbs	8 brains	3 × 16 = 48 lbs	16 brains
3 × 9 = 27 lbs	9 brains	3 × 17 = 51 lbs	17 brains
		3 × 18 = 54 lbs	18 brains

APPENDIX C

CODING SCHEMES

Appendix C describes the two coding schemes used in this study. The first one was used to analyze positioning events of Mrs. Porter and her students. It consisted of 3 sub-sections (position, storyline-as-relationship, and speech-act) with coding categories and definitions. The second one was used to examine students' social comparison statements. It included 3 codes based on social comparison goals.

C.1 CODING SCHEME FOR POSITIONING ANALYSIS

C.1.1 Position

1	Advanced student	Student who is capable of abstract problem solving strategies AND is aligned with people who have more knowledge of mathematics (e.g., adults, old siblings) in terms of strategy use
2	Significant contributor	Student whose problem solving strategy or idea is positively evaluated during discussions; strategies can be concrete or abstract
3	Role model	Student whose problem solving strategy or idea is treated as a good example by teacher

4	Participant	Student who talks, gives answers, or asks questions during discussions; Student may volunteer to talk or be called on by teacher
5	Struggling student	Student whose problem solving strategy is negatively evaluated by teacher OR who seems to have poor mathematical skills and knowledge
6	Initiator/facilitator	Teacher who initiates discussion/interactions by introducing problems, calling on somebody, and so on
7	Guide	Teacher who helps students talk during discussions by prompting, revoicing, and/or clarification
8	Evaluator/feedback-giver	Teacher who evaluates problem solving strategies, ideas, or previous utterances OR gives comments/suggestions on each of those
9	None of above	N/A

C.1.2 Speech-act

1	Explain	Students are talking about their strategy or what they did to solve problems
2	Respond	Students are responding to a previous utterance which is usually made by teacher
3	Look for words	Students are looking for words or thinking about what to say when they are asked questions by teacher
4	Align	Students are aligning themselves with other persons or previous utterances
5	Challenge	Students are challenging or rejecting previous utterances which were addressed to themselves
6	Initiate	Teacher is initiating a question OR interaction
7	Clarify	Teacher is asking a student for clarification of what he/she said
8	Revoice	Teacher is restating or reformulating students' utterances; Revoicing usually starts with "so," e.g., "So, you're saying ¾ is equal to 45 minutes."
9	Prompt	Teacher is helping students to talk with prompts such as providing words/cues, paraphrasing, and so on
10	Compare/juxtapose	Teacher is comparing or juxtaposing strategies, ideas, or people
11	Validate	Teacher is giving a positive evaluation/comment on strategies or ideas
12	Negative comment	Teacher is giving a negative evaluation/comment on strategies or ideas

13	Summarize	Teacher is summarizing strategies and/or ideas presented by students
14	Interrupt	Teacher is interrupting students' utterances by overlapping speech
15	Talk about norms	Teacher is talking about classroom norms (e.g., sense-making, learning from each other, not giving up etc.)
16	None of above	N/A

C.1.3 Storylines-as-relationship

1	Authority-recipient (authoritative)	Appears between a teacher and student with the IRE/F sequence; Teacher usually has control over interactions and students simply respond to the teacher
2	Guide-guided (guiding)	Appears between a teacher and students with sequences longer than the IRE/F; Teacher usually has/help students talk by prompting, revoicing, and/or clarifications
3	Authority-resistant (self-protecting)	Appears between a teacher and students especially when a teacher makes a comment on a student or his/her strategy and he/she challenges the comment
4	Team (collaborative)	Appears between a teacher and students especially when a teacher's control is very low AND a teacher works together with students on a problem/task
5	Rival (competitive)	Appears between/among students when one student feels competitive or shows envy to someone who outperforms
6	None of above	N/A

C.2 CODING SCHEME FOR SOCIAL COMPARISON STATEMENTS

	Social comparison motives	**Ways of reacting to social comparison**
1	Protecting a current self-evaluation as math learner	- Demeaning someone who is better-off
		- Showing envy to someone who is better-off
		- Distancing oneself from someone who is worse-off
		- Rejecting negative comments which lower/threaten one's self-evaluation
2	Constructing a new positive self-evaluation as math learner	- Aligning oneself with a person who has good math knowledge and skills
		- Conforming to positive evaluations or comments made by a teacher
		- Positioning oneself as advanced math student, good math thinker, or good/quick math learner
3	None of above	N/A

APPENDIX D

EXAMPLES OF NORMS WHICH EMERGED IN MRS. PORTER'S I-POEMS AND POSITIONING FROM INTERVIEW DATA

Norms	I-poems	Positions
Teaching differently	• I found a need to try things differently. • I've been trying to discover better and new ways ever since. • I began finding new ways of doing things.	Reform practitioner
Mental math (or thinking)	• I'm having kids do more mental math and taking them through the prediction part. • I have all year being much more careful to do that mentally with them and to talk about those strategies. • I am helping them focus on the things that I think they need to begin to focus on in order to make these mental calculations and decisions.*	Reform Practitioner
Sense making	• I had been able to communicate that, you, you have to use what makes sense to you.	Reform practitioner
Multiple approaches to a problem	• (that's why) I try to encourage them to develop their own strategies.* • I have students use a number of different strategies to solve a problem, often again in the past, some much more efficient than others.	Reform practitioner
Multiplications	• I'm always trying to give them problems that involve multiplication all year long. • I think that multiplications are all the things that the children are doing.	Reform practitioner
Thinking process	• I was trying to find out all the things that why they did it this way. • I'll try and find the reason why they've done it that way, and find a positive reason.	Teacher researcher

	• I became so fascinated with math, that I wanted to see what children could do.	
Taking risks	• I know that in our school, a lot of people do encourage children to take risks. • I think that a lot of children have been encouraged from the time they're very, very young, to take risks and to try things on their own.	Guide
Learning from mistakes	• I encourage them to leave them (=mistakes) there.	Guide
Efficiency	• I think that we are helping them find ways to become more efficient. • I want them to begin to understand that it is more efficient to multiply.**	Guide
Helping children learn	• I mean my job is to teach and to help children learn. • I look for ways to help them. • I think of it as guiding them to learning.	Guide
Building students' confidence	• I want them to feel that they can. • I'm trying to empower them. • I do want to feel that they can do it.	Guide
Opportunity to learn	• I want them to have all the experiences that I didn't have. • I present opportunities for it to be used. • I think it's really important to continue to expose them to things that may be more sophisticated.***	Guide

* Mrs. Porter uttered from two positions: reform practitioner and guide.
** The utterance reflected two norms: efficiency and multiplications.
*** The utterance reflected two norms: opportunity to learn and mathematical sophistication.

APPENDIX E

EXAMPLES OF MRS. PORTER'S LANUGAGE USE TO COMMUNICATE NORMS

1. Adjectives
   [e.g., good/better/best, right, fine, important, okay, the only, useful, nice]
   - *The way that's <u>best</u> is the one that makes sense to you.*
   - *Trying is what's most <u>important</u>.*
   - *Somebody did something in a very interesting way, that will be <u>useful</u> for you to know.*

2. Obligation/Directive
   [e.g., I (don't) want you (to), Do/Don't, You (don't) have to/need to/got to,
   You can(not), I can(not), You want to, You are going to, You'd better]
   - *<u>I want you to</u> show how you figured it out.*
   - *But if you see a way here that you think, boy that's a neat way to do it. Then that's what you <u>need to</u> write down, on your paper.*
   - *I <u>cannot</u> help you if you don't try.*

3. Anecdotes/stories/emotional phrases
   - *<u>It's like writing a math story</u> in a way.*
   - *This is <u>a class where we all teach each other</u>.*
   - *I want to tell you something about <u>good math students</u> and <u>good mathematicians.</u>*
   - *<u>When I went to school</u>, there was only one way. If we couldn't do it that way, then we were wrong even if the answer was right.*
   - *But I think, I am so sad that <u>when I was little</u>, people made me do it this way. Because <u>I used to</u> get all mixed up with multiplication and I <u>used to</u> think, I don't know how to do this. And when I had a list of problems and I remembered the rule, I could get the right answers. But I really didn't understand what I was doing.*
   - *I'<u>m</u> not so <u>worried about</u> the answer, as I <u>am about</u>, how are you going to start.*
   - *I <u>admired</u> Einstein because he never gave up.*
   - *And again I'm going to say, I <u>love</u> the way this class accepts a challenge. I <u>love</u> the way you dig right in and you work at things.*

APPENDIX F

TRANSCRIPTS OF INTERVEWS WITH MRS. PORTER

F.1.1 1st interview (Conducted by Dana in October 1999)

Transcript conventions:

- * underlined words indicate a stronger intonation of a particular word or phrase
- * [factual information stating time, interruptions, or tape count are enclosed in brackets]
- * (additional information from the researcher's perspective included within parentheses)
- * commas are used to indicate slight pauses in conversation
- * a longer pause is indicated by the word pause inside parentheses
- * slow - deliberate - speech - indicated - by - dashes - in-between - words
- * All names are pseudonyms

1	Frances Porter (FP)	Somewhere I have the pages I copied which I will look for before we finish. But I think I have the dates. Do you have all the dates that she sent me?
2	Dana:	You know what, I don't have them on me.
3	FP:	OK. Well, I think I have them. And it's possible that I don't actually have something for every one of those dates. They're in that pile some where. All right. Let's look at this.
4	Dana:	Today is October the 5th Tuesday afternoon, I'm with Frances Porter at Riverside Academy. It's 3:15 p.m. everything looks fine. I haven't used this since your last class.
5	FP:	Really.
6	Dana:	It seems to be going just fine. OK. Basically you just have to sign and date it and they need your social security number.
7		(Informal conversation took place.)
8	Dana:	I have several questions that I'm going to ask you.
9	FP:	Uh-huh.

10	Dana:	This is a piece of my dissertation research.
11	FP:	OK.
12	Dana:	I'm interested in your approach to teaching mathematics. And, after spending the last couple of years actually observing. I'm interested in how your students learn mathematics and how your instruction facilitates their mathematics learning. I'd like to start by explaining how I'm going to conduct the interview.
13	FP:	OK.
14	Dana:	I have a series of questions I'm going to ask. And if you feel that you've already answered them as we go on, uhm, through the interview, just let me know and we'll go on. And at the end of the question I'll ask you if there's anything else. And I'm doing that, uh, not necessarily that I want something else, but I just want to make sure if there's anything else you want to say.
15	FP:	Mm-huh.
16	Dana:	I want you to be able to do that.
17	FP:	Great.
18	Dana:	And I'm also, I'm not going to take notes because I have the tape player.
19	FP:	Right.
20	Dana:	But I may just jot some things down that I want you to clarify a little for me.
21	FP:	OK.
22	Dana:	And when I'm done I'm going to give you a copy of the questions I ask you. And if you do want a copy of the tape, cause I know you're giving a talk and maybe some of this will help you.
23	FP:	Yup, ya, maybe.
24	Dana:	With your presentation. I'll be happy to make a, a copy of the tape. I'm going to type up the transcript but I don't know if I can do that by October 11th.
25	FP:	No, don't worry. I probably got a feel for what I'm going to do. I don't know. It will be off the top of my head.
26	Dana:	(laugh) I'm sure it will be.
27	FP:	(laugh)
28	Dana:	OK. First question. OK, I know that your teaching style went through a major transformation about 10 years ago. Why did you see a need to teach differently?
29	FP:	I wouldn't say that my teaching underwent a major change 10 years ago. I would say that my teaching began to change about 10 years ago. And I didn't feel a need for a major change in teaching style. I always thought I did all right.
30	Dana:	Mm-huh.
31	FP:	But as I learned about various things and as I read and became intrigued by some of the things I read.
32	Dana:	Mm-huh.
33	FP:	I found a need to try things differently.

34	Dana:	Uh-huh.
35	FP:	So, from that point on things began to evolve. I also went back to school.
36	Dana:	Mm-huh.
37	FP:	I also need to get a different chair.
38	Dana:	OK.
39	FP:	Because one of my melodies at this moment is that I have a pinched nerve,
40	Dana:	Oh no!
41	FP:	in my leg and I might be able to, I may have to stand but I may also be able to sit here. I did get through a lecture last night sitting. So, I might, I might be able to sit. If not I'll stand.
42	Dana:	Oh.
43	FP:	Which I've been doing
44	Dana:	OK.
45	FP:	for 2 weeks. Uhm, when I went back to school, it became really clear to me, that no where in any of my education had anyone required me to think.
46	Dana:	Mm-huh.
47	FP:	And when I realized how much I learned when I was required not only to think but to explain my thinking. To begin to reflect, not only to do something, but to decide how I'd done it, and why I done it and then to take those thoughts, bring them to class and discuss them with others who may have done it differently or thought differently. I realized that not only was I learning more,
48	Dana:	Mm-huh.
49	FP:	But, the group of people that gathered together to discuss the ideas were creating knowledge and understandings that none of us could have created on our own. That my understanding was based on what I had done, could have been directly opposed or different from someone else's thoughts. But it was bringing together that group of people and talking about those differences, that we created an understanding that wasn't mine, or his, or hers, but was ours.
50	Dana:	Mm-huh. OK.
51	FP:	And that was a very exciting notion.
52	Dana:	Mm-huh.
53	FP:	And, it made clear to me the reason for school. Because I had also been reading books like Jane Healey's "Endangered Minds" in which she said that, uhm, schools used to be important because people had to come to a central place because that's the only place somebody would be available to give them information.
54	Dana:	Mm-huh.
55		[tape count: 90]
56	FP:	But nowadays we can get information anywhere. The Internet, we can get it from movies, from TV, we don't necessarily have to gather people together in one place called school.

57	Dana:	Mm-huh.
58	FP:	Schools are very expensive to run, you know. And if we're going to do things the way we've always done them and only transmit information, then we might better do it in a different way and save ourselves a lot of money.
59	Dana:	Mm-huh.
60	FP:	So, I could see the point, but then I realize that the reason for school is the social interaction.
61	Dana:	Mm-huh.
62	FP:	The sharing of ideas after [interruption by office intercom system] after we've had to reflect on what we've done, so I was <u>very</u> excited about that. It was about the same time that I went from teaching kindergarten to teaching first grade.
63	Dana:	Mm-huh.
64	FP:	And I wondered, if I could inspire first graders to also think about why they had done things and to talk about them and to do some of the same things. And I think it was at that time, that I began every year with what I would consider myself as little bit of a teacher researcher. I always wondered if I try this will something happen.
65	Dana:	Mm-huh.
66	FP:	What would happen if. And so as I say I didn't start teaching, I didn't undergo this big transformation. I began finding new ways of doing things and I think I've been trying to discover better and new ways ever since.
67	Dana:	Mm-huh. Can you give me an example? Like say, with the first graders, how you started to get them to reflect like you. You know, the same kind of thing that you learned.
68	FP:	Well I would do the same things that I do now and the same things that I even do with young adults that I'm working with. I say when they tell me the answer to something, "How do you know?"
69	Dana:	OK.
70	FP:	And a first grader would usually say, I just knew it. And I would say, no, you didn't just know it, what did you think to yourself first? And I would help them through or I would lead them through and I would reiterate. I can, my first thought is an example of something that happened that is not math related but is reading related. Every morning I used to write, a little bit of news on the board. Uhm, I would write it because it was the beginning of first grade and a very typical way. In a standard format, they could predict what a lot of it was going to be because it was the same every day. I'd write the date and then today is Monday and today you will go to gym, you will go to music. So, it was very, very much the same. And then I would ask volunteers to come up and read the individual lines with a pointer pointing to the words. And usually there'd be someone who could read a little. The second day of school, I remember, I can remember the little boy that did it. Uhm, somebody had read the sentence, you will go to gym. And the second

		sentence of a child who I knew couldn't read at all volunteered and came up and stumbled a little. You - will - go - to - and somebody, the other kids were kind of helping him along with music.
71	Dana:	Mm-huh.
72	FP:	And I said to him, how did you know this word was will? Because I knew he didn't know music. But I pointed to will and he said it with great authority, and he said because I saw it up here. It was in the sentence before and he had remembered what someone had read. So I said, Oh, you heard so-and-so read that sentence and you looked and saw this word was the same as that, so you knew this word had to sound the same as that word. So I put in his mouth, the words that he needed to explain how he knew. All he said was, I saw it there. But, I began to give children the words to tell me what I wanted to hear. I wasn't making something up.
73	Dana:	Mm-huh.
74	FP:	I was just helping him tell me in words, what he didn't have the words to tell me. So, I began to watch and then reiterate.
75	Dana:	OK.
76	FP:	And then before long, more were able to, you know, chime into that. And I would find whatever ways I could to help them learn to do that.
77	Dana:	OK. I think that's how I observe you in mathematics class.
78	FP:	Ya, I would do the same kinds of things.
79	Dana:	You ask the same questions.
80	FP:	I'd ask, I'm always asking, how do you know? How did you figure it out? Where did you begin? Why did you choose to start with number 20 or whatever it was?
81	Dana:	And then you help them with the words.
82	FP:	Right.
83	Dana:	Some times they'll say something and you'll like kind of take what they say.
84	FP:	Right. Because you had asked, or someone, I think it was you that had asked even some time during this last year. I notice that you tend to repeat everything that they say. Well I do. Uhm, and I try to sometimes clarify if it isn't clear. Sometimes I'm making sure I've understood what they're trying to tell me. And I'm also trying to make it clear to the other listeners. For me, I often have to hear something more than once. So, ya.
85	Dana:	OK. Uhm, is there anything else? Do you feel that pretty much doesit?
86	FP:	I think so.
87	Dana:	OK. All right. Uhm, all right. Are you comfortable?
88	FP:	I'm all right. I'll move around a little.
89	Dana:	OK. I've heard you say and also, you've stated in your written statement about mathematics teaching. That's what I dropped off yesterday. And I highlighted and I quote what you said, that mathematical concepts rules and procedures need to <u>make sense</u> to students. Can you give me an example?

90	FP:	Uhm, of it needing to make sense? Or I don't know what you want an example of.
91	Dana:	Of how it makes sense. The making sense part. What does that mean to you?
92	FP:	What it means to me is [interruption from office intercom system] OK. I guess what it means to me is that I know there are many children, even kindergartners who can say, 2 plus 2 is 4, or you know, 9 plus 9 is 18. But they don't really understand what that means. So that yes, I could teach children to follow any formula, and they could get the answers to problems right. But that doesn't mean they're going to <u>understand</u> what they're doing. So I think that, all of what we know about mathematics and what we, you know want to help them <u>get</u>, we want to make sense to them. And that is why I try to encourage them to develop their own strategies for solving problems because they're going to do things that make sense to them. And then when a child doesn't have anything they can do, perhaps when they listen to some of the others, some of what others do might help them make sense of what a question is.
93	Dana:	OK.
94	FP:	OK.
95	Dana:	Ya, that makes sense. But how do you know when it makes sense to them? What exactly do you do?
96	FP:	When, OK, if they can explain what they've done.
97	Dana:	Um-huh. OK.
98	FP:	OK. They can explain it in such a way that I know they are missing the point completely.
99	Dana:	OK.
100	FP:	So, in listening to them, I get an idea of whether something is clear or not.
101	Dana:	OK.
102	FP:	In fact, one of the lessons, the lesson that you did here. (FP points to the transcript from the 17th of December that I left with her the day before the interview.)
103	Dana:	Ya.
104	FP:	The whole point of this and the reason I probably went on and on and on and on with it as I did, was because I understood that they didn't understand some things that they thought they understood about multiplying 2-digit numbers.
105	Dana:	Mm-huh.
106	FP:	And, I was trying to find out all the things that why they did it this way. And it turned out that people had shown them, and uh, but they were doing something without understanding it. So they were in almost every case doing it incorrectly.
107	Dana:	Mm-huh.
108	FP:	And that was eye opening. And also upsetting and so I wanted to see where to go from here with it.
109	Dana:	So, once you get that information and you understand that they do

		understand or suppose they don't understand, then what do you do with that information as a teacher?
110	FP:	Well, if they do understand, then we can move on to something else. If they don't understand, then it's something we have to work out.
111	Dana:	OK.
112	FP:	And try to make clearer and find other ways of looking at it. Ya know whether it's using materials or coming from a different direction, just to help them get it.
113	Dana:	Do you get that from, like the whole-class discussion, or is it more individual work?
114	FP:	Uhm, I get it probably from lots of places.
115	Dana:	OK.
116	FP:	I get it from watching them as they work.
117	Dana:	Mm-huh.
118	FP:	I get it from looking at their faces when they read the task. And some seem so overwhelmed and others seem, like, oh this is a cinch. Uh, I get it when I see some straining to watch, to see what others are doing rather than relying on themselves.
119	Dana:	Mm-huh.
120	FP:	Uhm, so I watch, I listen, the whole-class discussion's important. Their individual papers are important. If they seem to have gotten it one time but then next time they're working in a different situation and can't apply it then I know they don't. So, it takes a while. You don't necessarily understand, or you can't necessarily access their understanding from one lesson.
121	Dana:	Ya, that makes sense. OK. Uhm, all right let's go on to the next question. OK. I understand there are many content areas that you emphasize in your mathematics instruction. I'm interested in how your students learn multiplication. Thinking back on last year, could you please describe the knowledge of your students, about the knowledge they had about multiplication when they entered the class?
122	FP:	Uhm, what I find, not only from last years kids, but almost every year.
123	Dana:	OK.
124	FP:	OK. Is that kids know the word multiplication. They know it means times tables. And some of them know some of their time's tables or most of them. Ah, and I do believe that many adults believe that times tables are what multiplication is. I think that they also have a sense that it's repeated addition.
125	Dana:	Mm-huh.
126	FP:	And I also think that they often believe that they are multiplying when they are not multiplying at the beginning of second grade. So, if I'm asking, I can't think of a problem, let's just think of one, cause I'm always trying to give them problems that involve multiplication all year long. And of course nothing is coming to my mind. Uhm, how many, uh, come on. Can you think of a multiplication?
127	Dana:	Uhm.

128		[Tape count: 250]
129	FP:	If one costs a certain amount of money, how many would 8 cost? OK.
130	Dana:	OK.
131	FP:	Uhm, yes we can figure that out. It would be 8 times 30 cents, if one costs 30 cents. And they will come up with an answer, usually a right answer. And they will write 8 times 30 equals, uhm, 8 times 30, 240. OK. And uhm, or $2.40 whatever. And then I'll say, so how did you do it? How did you know? And they will often say, well I added 30 eight times. And so, their understanding of multiplication, they know what it is, but they aren't using it to solve problems early in third grade.
132	Dana:	OK.
133	FP:	They are usually doing it by addition, but saying they're multiplying. And unless I ask, how did you know that, or how did you do it? I would assume, that they understood multiplication.
134	Dana:	OK.
135	FP:	So they know what it is, but they don't, they're not able yet, to use it. I also find that by the end of third grade, most students are using it.
136	Dana:	OK. So, can you tell me what do you want your students to really learn about multiplication during their third grade?
137	FP:	I guess I would like them to learn, it's interesting that I can just respond to you without thinking. (laughs)
138	Dana:	(laughs) I know.
139	FP:	I didn't know that I would be able to do that. What do I want them to learn about multiplication? I guess, I want them to begin to understand that it is more efficient to multiply then to add large columns of figures, (we both smile and chuckle) or long columns of figures. And I will design problems to help them see that. So that doing it in ways other than multiplying are very difficult. And uhm, because I can't put multiplication tables in their head. And because I can't force them, to learn them. And I don't believe in forcing. I suppose I could beat them in to it. I want them to, to feel it's important enough to do it. And, when they begin to see the advantages and how quickly a child who does understand that can do it, can do a problem, uhm, I think it is good incentive.
140	Dana:	OK. So you, you believe, you do believe in their learning multiplication facts?
141	FP:	Yes, but not in me drilling them.
142	Dana:	Not in drilling them. How do you believe they should learn them then?
143	FP:	I think they can drill themselves if they want to.
144	Dana:	OK.
145	FP:	I think they do have to probably memorize them. I don't know. Because I learned them by being drilled. But, uhm, I'm always giving them opportunities to use multiplication. And I will offer them, and I don't know if I did that last year while you were here, opportunities to challenge themselves. Would you like to try to do the four's times tables in one minute? Have I done that when you were here?

146	Dana:	I don't believe so.
147	FP:	We've done it already a couple of time this year. And most do not want to but some of them practicing love to do that and they realize a minute's a lot longer than they thought it was.
148	Dana:	(laughs)
149	FP:	And, so that too, creates some competition but it's not imposed competition. It is invited.
150	Dana:	OK.
151	FP:	And no one has to participate. We don't make a big deal about it. If you want to take the challenge it's available to you. If you don't, that's fine.
152	Dana:	Mm-huh. OK. (pause) OK. Children have different abilities and knowledge about multiplication. How do you find out about these differences?
153	FP:	Well I feel like maybe I've said some of that. But, they, by watching and listening, again. They, you know, there will be children mid-year toward the end of the year that are still adding things up. When they say they say they are multiplying they don't really understand, they understand the relationship. They understand that multiplication is repeated addition. But they still aren't using it. There are children that instinctively early, or maybe not even instinctively, they just understand numbers so well that they can multiply by 10 and a hundred, without blinking an eye. Because they know how numbers work. Uhm, and there are others that don't. So you just listen, and you can see with their work. When you've given one problem that can be solved in a lot of different ways it becomes very clear, the level of sophistication of individual students becomes very clear.
154	Dana:	Mm-huh. OK.
155	FP:	I think.
156	Dana:	OK. And then as you learn that they have different abilities, what do you do with that as a teacher?
157	FP:	Uhm, what do I do with it? I don't find it the problem that I think a lot of teachers do. I, I was in San Diego reading editorials, that were very upsetting, because they realize that in classrooms, they were talking about whether students should be grouped or not grouped according to ability. And I've never believed in ability grouping because I feel that students are labeled and then everyone, should have the same opportunities. And they said, yes but if you don't group students then a teacher would have to develop six different lesson plans. I don't believe that's true. I think if you develop open-ended questions, questions where there is an answer but it can be arrived at in a number of different ways. And even better, my questions are not open-ended. But when you say, how many ways can you find to do, something. There isn't one finite answer. You can be correct with a very simple and sophisticated answer and you can be correct with something very complex. And it allows children, of all kinds of abilities, to solve one problem and still, uhm what, be challenged.

158		[tape count: 353]
159	Dana:	Mm-huh.
160	FP:	And children will challenge themselves, if they are, if that kind of thinking is valued, I think.
161	Dana:	OK. Uhm, is there anything else that you feel is important to teaching, for children to learn multiplication?
162	FP:	I, I don't know. I think, I would never want to teach multiplication in isolation like this is, <u>this</u> is the solution to things. It's got to be taught as one of many <u>tools,</u> and used, you know, in lots of different ways.
163	Dana:	OK. All right. Now I'd like to go back to the last session that I observed.
164	FP:	OK.
165	Dana:	That's the transcript that I gave you. Uhm, and it was the 17th of December and the following assignment was written on the board, the Eiffel Tower is 984 feet high. How many inches would that be? Can I ask you, what did you think of the transcript?
166		[tape count: 366]
167	FP:	I didn't read it thoroughly,
168	Dana:	OK.
169	FP:	because I had a lecture last night and, I mean I only got it yesterday after school. Uhm, I remembered and I did not write as much as you might have thought because again it was at the end,
170	Dana:	OK.
171	FP:	But what I did write, was, (FP reads from her journal) the kids did the Eiffel Tower problem today, 984 feet high how many inches. The most efficient and quickest kids added 984 twelve times. Now again, this isn't end of year. This is end of, this is mid, not even mid-year yet. Bernard and Lyndsey compared differing answers and then revised. Because adding, even a number 12 times, allows for a lot of error. I'm beginning to notice more working together and sharing of information. But I want to make a note of kids' misconceptions about place value. And then I had written how I wish I had it on video tape. Now I'll have to listen to the audio tape again. And I hope it all comes clear on the audio tapes, I said. Karl, Nathan, and someone else were showing us what a parent had taught us about multiplying. And I wrote down their examples.
172	Dana:	Mm-huh.
173	FP:	It was <u>so</u> clear that kids did not understand why they were doing what they did. I said again and again that their parents meant well. I hope I did that. And that the way that they were shown <u>does</u> work. But that it was easy to get mixed up. And if they didn't understand it, they shouldn't use it. And then, the next day I sent to parents, the Newsletter. Did you get a copy of that?
174	Dana:	No, but I'd love to have a copy of it.
175	FP:	I did make a copy of it. It's on my desk some where.
176	Dana:	OK.
177	FP:	If I can't find it we'll make another one before you leave.

178	Dana:	All right.
179	FP:	And I wrote, students were involved in three explorations that revealed a lot about their mathematical understanding. And I wrote about the Hanukkah thing, and then I got to the Eiffel Tower. (FP's voice is starting to get more hoarse.) Listening to them explain their work, it is clear that most students were very confused by the traditional algorithm for multiplying two and three-digit numbers. A few students said they were taught this algorithm at home. And they chose to use it to solve the Eiffel Tower problem. In every case the answers were incorrect. And they were unable to explain the reasons for what they were doing. Their confusion points to the need for children to have a clear understanding of place value before such procedures are introduced.
180	Dana:	Mm-huh.
181	FP:	And, I think that the reason that I felt as I kind of glanced over this again, that I went on and on and seemed to beat certain parts of this to death with the kids was, that I was so <u>astounded</u> at their lack of understanding and I wanted to be <u>clear</u> about what they understood and what they didn't.
182	Dana:	Mm-huh.
183	FP:	And, unfortunately that was the end of the, before winter break. And then after winter break I immediately was ill. And didn't really get to go on with this. I mean, had they come to school the next day, which they did but it wasn't a real day. Uhm, had they come to school and had we really been able to pursue this, it would have been interesting to see where it would have gone. But, so that, the timing of all this was unfortunate.
184	Dana:	Can I ask, when ever I came in, you told me that just looking at the transcript brought things back and it was a little disturbing to you. Could you tell me why?
185	FP:	Ya, well I guess what was disturbing has to do with probably me, on a more personal note with my students. Uhm, I know there were points in here where you had written, that Michael was waving his hand frantically and I was calling on other people. I know I did that with Michael. Uh, he annoyed me. And I'm hoping it wasn't because of that, but it was ah, unsettling because I realized that, it was a purposeful avoiding but not only because he annoyed me, I don't think. I hope. Because he often waived, and waived, and waived and then had almost nothing to say. And <u>while</u> things were going on that were so intense, I really wanted to get to people like Bernard who I felt could really contribute some understanding,
186	Dana:	Mm-huh.
187	FP:	to the situation. But, if the kids do perceive that I'm avoiding certain children, that would be very upsetting to me and whether I did that a lot and it was noticeable, I don't know. But it's something to think about. That is upsetting as a teacher.
188	Dana:	Ya, OK. (pause) This goes back to a little bit of what you already talked

		about. But we talked about the several different strategies used to solve that Eiffel Tower problem. And the fact that, I noticed the students were fascinated with the standard algorithm. And I'm just wondering, why do you think they were so fascinated with it?
189	FP:	I wish I knew. I wish I knew why kids seem to feel that ultimately, this is the way they're suppose to solve problems. I hope I don't convey that as a goal. I don't believe it's a goal. Now I think their parents do and whether it comes from there, or not, I don't know. Whether I convey it, I don't know. Uhm, I, but they do, and they do every year.
190		[tape count: 475]
191	Dana:	So, you see that as well.
192	FP:	I do. Now, I know, I don't know if Pulak was there that day. Because he does understand the algorithm.
193	Dana:	Ya.
194	FP:	And I said that everyone that showed it to us didn't. So, it's possible that he wasn't there that day. And I do believe that the kids admired his math ability. So part of it could be that anything that Pulak was able to do was something that they wanted to be able to do. Whether I contributed to that, I don't know.
195	Dana:	Mm-huh.
196	FP:	I mean I was very impressed with his ability to understand things that children this age typically don't understand.
197	Dana:	Mm-huh.
198	FP:	And it wasn't just that he could do it. He knew exactly why and what he was doing. Whereas other children have been taught, and don't understand it at all.
199	Dana:	Ya, so, you think that it's possible that they admire him what he does because, he can do what they can't do or because you,
200	FP:	Because, ya, cause he always gets things right and he had this way that parents use to solve the problems. So, doesn't that make it something to be able to want to be able to do. And again, whether I and my attitudes about Pulak and the way I responded to what he did, or other ways, that I without knowing what I was doing conveyed that certain things about this would be desirable, I don't know. But they definitely, were intrigued and they always do want to learn it.
201	Dana:	All right. Uhm, well is there anything else you remember about that class that you believe was important for your students in understanding multiplication?
202		[tape count: 510]
203	FP:	There were probably a lot of things that went on. I do remember the part with the tens,
204	Dana:	Mm-huh.
205	FP:	where, uhm, 12 times 10 is 120, 20 times 12 is 240, and so by using that kind of strategy, I think is an important kind of thing when children understand those landmark numbers of 10 and 100, and how useful they can be. Because that's multiplication too, in a way. Uhm, so I think that

		it came home there with Raj and Ethan who were working together is what you have here.
206	Dana:	Mm-huh.
207	FP:	Uh, I don't know, I didn't read it thoroughly enough to be able to answer that question,
208	Dana:	OK.
209	FP:	for this particular lesson anyway.
210	Dana:	All right. We'll move on to the next question.
211	FP:	(FP was looking at the transcript.) Here it is. Well when they were saying just to add a zero because it's the magic of ten.
212	Dana:	Mm-huh.
213	FP:	OK. Those concepts come up again, and again, and again. It helps with the place value understanding again, I think. Because I always have that place value chart out and we refer to it and things like that. So there are places in here where you have, uhm, in the transcript where other things besides the traditional algorithm relate to multiplication and I think are important.
214	Dana:	Mm-huh. OK. Our next question, uhm, you often ask your students to write down their strategies and explanations in addition to solving the mathematics problem. Why do ask them to do this?
215	FP:	Well I think that gets back to the first questions you've asked me. How do I know what my kids understand? And their ability to share their thinking with others comes from being able to articulate it. And, uhm, discussion is one thing. Once their in third and even first graders can begin to write it down. It's much harder with first graders cause they don't even know how to write yet. But, uhm, I think that when they can put thoughts into words, they can really understand them and certainly others that write about mathematics support that view. Marilyn Burns say it all the time. When you write about what you have done, you have to revisit your thinking.
216	Dana:	Mm-huh.
217	FP:	And, you know think, and then again, being able to articulate it. (pause) Humans I think understand through language. Now, I have artists who would argue that point. I have argued that point a lot. They think there are other visual ways that people understand that don't require words. And I think we understand a lot without words. But I think our most complete understandings of things that are visual, or mathematical, or emotional, or anything else, come when we can articulate them through language. And I think it is the thing that separates us, makes us human, is our ability to communicate using a verbal language. Maybe I'll change that view at some point but at this point I still think it's important.
218	Dana:	As far as being a teacher, when you get that information, how does that help you?
219	FP:	It helps me to assess what they understand, and it helps me in my ability to, help children who don't understand. Because, well, it does a few

		things. When children can articulate what they know, and their misconceptions, they are showing me how to teach them.
220	Dana:	OK.
221	FP:	And, when they can, children can share ideas by discussing their strategies after they are able to articulate them. Then, children that wouldn't understand <u>my</u> explanation, might understand someone else's. And if they hadn't done this in the first place, I wouldn't even <u>know</u> about certain ways of thinking about things. Every year, every week usually, I learn some way of looking at something that <u>never</u> would have occurred to me. And, and so it allows me to be a better teacher. And it allows children to have more opportunities to understand because they're hearing it from lots of different points of view.
222	Dana:	Mm-huh. That makes sense. OK. My last question. OK. What do you think is the most important change that you've made in your teaching that has had the most positive impact on your students learning mathematics?
223	FP:	Well probably what I've, most of what I've said. Uhm, I used to think, way back when, that mathematics was this finite thing. I was afraid of it. I was never what I thought a strong math student. I thought it was something you had to do by following the rules. I think that my requiring students to think and basing, no, believing that the most important thing is <u>understanding</u> is the biggest change that's made the difference. And the kids even by the second week here this year have, you know will say, this is fun.
224	Dana:	Mm-huh.
225	FP:	Math isn't supposed to be fun. But they do think of it as fun, even when they're struggling and even when they're frustrated with it.
226	Dana:	Mm-huh. You mentioned <u>your</u> mathematics background and <u>your</u> mathematics learning. Is that, do you think that influences how you teach mathematics?
227	FP:	I think that it has made me see that I wasn't as bad a math student as I <u>thought</u> I was. I just didn't have the option of doing it any other way but one.
228	Dana:	OK.
229	FP:	So, that <u>understanding</u> has made it essential for me to offer my students options, because I, I realize that they <u>don't</u> all learn in the same way, and they need to. I mean some of them can't solve a money problem without getting coins and with others it would be a waste of their time to get the coins out.
230	Dana:	That's good. Thank you very much.
231	FP:	That's helped me too. Because my topic is what I've learned about (laughs) teaching mathematics and I guess, ya, what more can I say.
232	Dana:	These are the questions that I've asked and tried to ask and used to follow you. I really appreciate [end of tape].
233		[tape counter: 661]

F.1.2 2nd interview (Conducted by Dana in December 1999)

Transcript conventions:

- *italicized* words indicate a stronger intonation of a particular word or phrase
- [factual information stating time, interruptions, or tape count are enclosed in brackets]
- (additional information from the researcher's perspective included within parentheses)
- commas are used to indicate slight pauses in conversation
- a longer pause is indicated by the word pause inside parentheses
- slow - deliberate - speech - indicated - by - dashes - in-between - words
- underlined words indicate overlapping speech
- All names are pseudonyms

		[Side 1, tape A, count: 10]
1	Dana	Today is Tuesday, December 14th, 1999. I'm at Riverside Academy with Frances Porter. (pause)
2	Dana	OK, I'm going to conduct this interview the way same way I did the last time.
3	FP	OK.
4	Dana	*During the first interview, when we were talking about your mathematics background, and your mathematics learning, and how it influenced your teaching, and it made you see that, and I quote, 'I wasn't as bad a math student as I thought I was.' I was wondering about your comment. And I'd like to know if you can tell me more about that.*
5	FP	Uhm, I always when I was a child thought I wasn't any good at math, but I got, my grades were all right because I could learn the formulas, and I did what I was told, but I never really understood it. And I just assumed that some people understood things like this and some didn't and I was one of the ones that didn't. I, after teaching, and seeing the various ways that children solve things, so much of what they did made sense to me, and I really feel that if I had been encouraged to think in ways consistent *with* the way that I thought about things. I probably wouldn't have had that opinion of myself. I think I would have felt that I was able to do math, and that I wasn't bad at it. But there was that one approach, and that didn't suit the way I learned or thought about things. And I guess that's what I meant.
6	Dana	*OK. So it was your thinking, more of your ability, your mathematics ability, when you were thinking about?*
7	FP	Right. I just assumed I didn't have much ability to work with numbers, or to think of things in a logical way. Because I thought that the logical way meant the way that I was told I was suppose to do things.
8	Dana	Uh-huh.
9	FP	And my mind didn't work that way.

10	Dana	OK.
11	FP	So I always dreaded math.
12	Dana	*OK. So, uh, so dreading it, you didn’t like the subject of math as a student?*
13	FP	I didn’t, and it *frightened* me almost as a teacher, because I felt that if I’m not good at this how can I teach it well.
14	Dana	Oh, ya.
15	FP	But, I was encouraged by Ann Burgender, who was working here, to just relax and let the children show me. And when I found that I could do that, it, it did work. And I, and you know, I’ve learned a lot about math.
16	Dana	Uh-huh.
17	FP	From the children, and from trusting myself as well as them.
18	Dana	Uh-huh. Uhm, in the first interview, you stated that some point in your past, you were actually afraid of mathematics. That’s the words you used. I was wondering, why do you think you were afraid of it?
19	FP	I think, basically, from what I said, I didn’t feel that I understood how math worked.
20	Dana	Uh-huh.
21	FP	And that, and so that I had no *confidence* in my ability to, to do things mathematically.
22	Dana	OK.
23	FP	And so I was afraid to try because I figured I’d always be wrong, unless somebody showed me the way.
24	Dana	Uh-huh.
25	FP	And now, *even now*, you know, after all these years of learning more things, I’m much more intrigued now by problems and willing to tackle them, and I think even in a group situation I wouldn’t be afraid to, you know, try, knowing that if I didn’t get it, it wouldn’t be the end of the world.
26	Dana	Uh-huh.
27	FP	And that it would be OK. (laughs) *I’ve* become more of a risk taker myself. I’m going to open the window a little bit. It’s hot.
28	Dana	Ya, I’m warm in here.
29	FP	This room is always this way. You don’t wear jackets and sweaters.
30		[Tape 1, Side A, Count: 51]
31	Dana	Ya, I can feel that.
32	FP	OK.
33	Dana	*OK. Uhm, let’s see, as a teacher, how, so, you kind of stated this about your beliefs have changed, as far as what mathematics is. Then, how, how do you want your students to view mathematics?*
34	FP	Well I want them to be comfortable,
35	Dana	Uh-huh.
36	FP	with mathematics, I want them to feel empowered. I do want to feel that they can, do it,
37	Dana	Uh-huh.

38	FP	Uhm, that it’s, again, it’s not something to be afraid of.
39	Dana	Uh-huh.
40	FP	I want them (she laughs) to have all the experiences that I *didn’t* have, when I was growing up. Uh, I want them see math as a *real* and necessary part of people’s lives.
41	Dana	Uh-huh.
42	FP	Uhm, that helps them accomplish certain things in the world. Uhm, you know, shopping, whatever, making decisions about lots of different things, and I think that, what, that gearing or designing the lessons and things that I *give* them around everyday kinds of things.
43	Dana	Uh-huh.
44	FP	Might help accomplish that.
45	Dana	*Uh, how, how do you actually empower them, to make them feel that they can do mathematics? You know, suppose they’re afraid, like you were afraid.*
46	FP	Well, I think that, as we discuss things, when I accept all of what they do.
47	Dana	Uh-huh.
48	FP	When they share, and how they approach something, and I find and search for the *positive* parts of whatever it is they’re telling me.
49	Dana	Uh-huh.
50	FP	That even if they don’t have right answers, I can say, but look, this is very logical what you did up to here, it’s only this little part, perhaps, that, you know, you had a problem with. So what you’re doing is fine. And then we might look for ways to become more efficient or something like that. But, I keep accepting them,
51	Dana	Uh-huh.
52	FP	Trusting them, uhm, and in trusting them, I’m hoping that I’m develop confidence and letting them know *I’m* confident that they *can* do it.
53	Dana	Uh-huh.
54	FP	I think in all of those ways, but *mostly* in anything they tell me, that they’ve done, I try to find, I, I don’t say, no that’s not *right*. I’ll try and find the reason why they’ve done it that way, and find a positive reason.
55	Dana	Uh-huh.
56	FP	To *acknowledge* them, and to accept what they’ve done.
57	Dana	*OK. Uhm, so through those different ways, you do, do you believe you can change your students’ beliefs about mathematics, through this support and building confidence?*
58	FP	I think that I’ve *seen* it in a lot of children. I still have a couple of girls this year, one girl I’m thinking of in *particular*, whose still *very* sure she can’t, do things. Uh, she can do a lot more I think than she *thinks* she can, if she’d just relax. I also have a little boy who, if it doesn’t come to him immediately, almost, gets a mental block and can’t move *forward*. So, I still have a ways to *go*, but I’ve seen a couple of other children this year, who were *sure* they couldn’t do *math*, and they’re *just* starting to blossom. And, you can just see their confidence growing, and usually in

		the last few years I've seen children's confidence, you know, and their ability to do math grow.
59	Dana	Uh-huh.
60	FP	That's typical, and I do think that this approach, (pause)
61	Dana	Facilitates that,
62	FP	Right.
63	Dana	*OK. Uhm, you mentioned in e-mail to me last year that you noticed after listening to several of the tapes of your mathematics sessions, that there was a difference in how you spoke to your students early in the year in comparison to the end of the year. Could you comment on that difference, between how you spoke, the two different times?*
64	FP	Right. I, I remember sitting in, (she laughs softly) California last year, listening to the tapes and thinking that, and what I *remember* kind of is that, at the *beginning* of the year, I even think I speak more slowly. I, I *try* to be *very* specific, not with you will do it this way. But, making my expectations clear, asking them to explain themselves, explaining what I *mean* by that. Uhm, it takes that kind of talk at the beginning of the year to, to let kids know what you're looking for. Uhm, trying to encourage them to get started, giving them some ideas of how they might begin. But even after, you know, five or six weeks, I can simply say, here's the problem and they can go about it. And I, when they *begin* to have some confidence, when they're *beginning* to take some risks, I don't have to be as specific. Uhm, I think it's that kind of thing.
65	Dana	Uh-huh.
66		[Tape 1, Side A, count: 115]
67	FP	I *felt* at the beginning, when I was listening to those earlier tapes, that I went *over* things, and *over* things, and said them *again*, and *again*, and *again*. But I think they *need* that then.
68	Dana	Uh-huh.
69	FP	But later they don't.
70	Dana	Ya, and then you don't have to be so explicit, they already understand what your expectations are,
71	FP	Right.
72	Dana	and they can do that. OK. Uhm, *OK. In the same e-mail that you stated, that you worked hard to convince your students to be able to take risks, and you talked a little about this, and not to worry about making errors. Why do think being able to take risks is so important in learning mathematics?*
73	FP	I think being able to take risks is important in learning *anything*. You, unless, you want to produce students who are only proficient at reading directions and following them, they have to be risk takers. Because when they're presented then with a new problem that doesn't come accompanied with a set of *instructions*, they won't know how to begin.
74	Dana	Uh-huh.
75	FP	And to be able to look at a situation and dive in, and try things is the *only way* that we're going to produce thinkers, and learners, and people

that begin to *feel* empowered. People who begin to feel, hey, I can face this, I can do this. And it's hard for them and it's frustrating if they haven't done a lot of that *before*. But I think it's an essential part of learning just as errors are an essential part of learning. And that we learn the *most*, when we, *don't* do it well, and then we begin to see what the needs are to begin to do it better.

76 Dana Uh-huh. *Do you notice, uhm, when your children come in to the third grade from previous years, as far as being able to take risks. Is that something that's easy for them, do you find in general?*

77 FP Some. It just depends. I know that in our school, a lot of people do encourage children to take risks. Some people more than others. And, it will depend on their home situation too. Some *families*, I mean, having the right answer and being absolutely correct, and always neat and tidy is essential in some families. And you have to over come that, because I don't believe that learning *is* neat and tidy.

78 Dana Uh-huh.

79 FP Uh, I don't think, that there is a start here, and then take two steps forward. It's sometimes all over the place, and then we try to bring it all together to make some *sense* out of it. So, I think that a *lot* of children have been encouraged from the time they're very, very young, to take risks and to try things on their own. And others haven't so,

80 Dana Uh-huh.

81 FP It just depends, some children are ready and others have to really be nurtured and encouraged.

82 Dana Uh-huh. OK. Uhm, *also, you already mentioned a little about this, but, the focus of children being able to make errors, and not to be so hung up on worrying about having the correct answer. How do you feel that's so important in their learning?*

83 FP Because, I guess, again, I think that no one learns without errors along the way. And I tell children things like, the first time you got on a bike, you probably didn't ride a way. And the first time you tried tying your *shoes*, you probably couldn't do it. There was nothing wrong with that. We expect that to be the case. That it often takes lots of practice to get better, and better, and better, and finally be able to do something.

84 Dana Uh-huh.

85 FP And that it's *OK*. It's, it's *normal*. It would be, you know, very unusual to be able to look at a *new* problem, and know immediately how to do that.

86 Dana Uh-huh.

87 FP So, you've got to *try* some things, and maybe, maybe you, we will make mistakes. And I encourage them to leave them there. And then go on so they can look back and see what they did before, and what they've done afterward that might be better or maybe isn't and to be able to *think* about it.

88 Dana So you do see them doing that. I remember your doing that in class, and asking them to write in different color pens to kind of distinguish that.

So you do actually see them going back and following.

89 FP Uh-huh.

90 Dana OK.

91 FP I don't know if you want me to *go* here right now. But *this year* what I'm *noticing* and it is fascinating to me, and I guess I'm pleased about it. When I have students use a number of different strategies to solve a problem, often again in the past, some much more efficient than others. And then I say go back and write what you saw *someone else* do. Or if you were doing it again, would you do it this way, would you do it another way, and why? I would say out of my seventeen students, fifteen of them, almost always say, I would do it my way, *again*, because it makes sense to me.

92 Dana Huh.

93 FP And I had yesterday, someone write, my, it may not be the best way,

94 Dana Uh-huh.

95 FP But it's the best way for me because I understand it.

96 Dana Uh-huh.

97 [Tape 1, side A: count 184]

98 FP And *I* find that very rewarding, because I *do* say to children, often, that what they do needs to make sense. And that these are other ways, try them if they make *sense* to you, but *don't* try them if they don't.

99 Dana Uh-huh.

100 FP And they're *confident* then, that their way is all right as long as it makes sense.

101 Dana Uh-huh.

102 FP And while I would *hope* that some of the very inefficient ways, would gave way to something else that makes sense,

103 Dana Uh-huh.

104 FP Uhm, I, I, there's a confidence in making a statement like that, that I'm happy to see.

105 Dana Uh-huh. *I'm curious about the efficiency though. Because I do hear you say that throughout your teaching, to be more efficient.*

106 FP And we do *talk* about which strategies are more efficient. Yesterday we *did* a problem, that we probably did while you were here, I don't remember. It was Sir Frances Drake with 106 men on five ships and how many men were on each ship.

107 Dana Ya. (nods head in agreement) Uh-huh.

108 FP And uhm, interestingly, no one made 106 tallies or men and then grouped them.

109 Dana Uh-huh.

110 FP One person made five ships and started putting men on them. Uhm, most of them chose numbers that they thought would be close and started adding them up.

111 Dana Uh-huh.

112 FP Five times, uhm, and we talked about, and a couple of people, did some multiplication. They knew that 5 times 20 was a hundred, and started

		there. And we talked about how quickly some people had done it, and yet still, this way I would do again because it makes sense now. So, *I* know then, that they're not ready to *see* that other *step* in thinking.
113	Dana	Uh-huh.
114	FP	And I do think of it as a step because it's a more sophisticated way of looking. And that we *know* that as they get older, they understand math more completely, using the multiplication that they're learning about will become, more natural to them. But if they're not ready to that, yet then, they have to do what makes sense to them. Because when they try to use something they *don't* understand, that's when they get really mixed up.
115	Dana	*OK. So you, support that making sense, and then eventually after they do something repeatedly, they start to see a more efficient way, it's like a building process.*
116	FP	I think.
117	Dana	They have to slowly feel confident,
118	FP	And we keep showing it to them, just like when we have a very tiny child. We *don't* wait to talk to babies in sentences until they first understand words.
119	Dana	Uh-huh.
120	FP	We know they're not necessarily going to understand all the words we're saying. But we're setting up a *model* for them as we do it. And we're confident that little by little, they'll begin to understand eventually, the whole picture, and be able to talk in sentences, too. And we may *emphasize* certain words in our sentences, but we don't wait with these children, who don't understand the more sophisticated ways, to talk about them. We keep showing them, as *models*, but I think the *mistake* that a lot of us make, is *expecting* children to do it. I think they do it, when they're ready. We *encourage* them, uh, we try not to hold it up and say this is *the* way. But we do point our its efficiency. But, and they certainly *want* to be more efficient, (she laughs) they don't want to be struggling.
121	Dana	Uh-huh.
122		[Tape 1, Side A, Count: 234]
123	FP	And yet, they really can't use it well until they understand it. So, we keep presenting it, and little by little, one day, they'll go, ah yes! I see now.
124	Dana	Uh-huh. *OK So, you just discussed in the first interview, that there are multiple ways for students to do multiplication, and you talked about repeated addition, recalling the times tables, and using standard algorithms. We were wondering, how you think about multiplication. That is, what you expect to see in the work of a student who you thought understood multiplication?*
125	FP	When I read this question,
126	Dana	Uh-huh.
127	FP	I thought, this is going to be the hardest one to answer. And I don't

		know if I can answer this question.
128	Dana	OK.
129	FP	Uhm, I think that multiplication are all the things that the children are *doing*.
130	Dana	Uh-huh.
131	FP	Uhm, they tend to understand, no matter how they do it, that multiplication is repeated addition.
132	Dana	Uh-huh.
133	FP	Uh, at this point in this classroom, most of the kids are still adding,
134	Dana	Uh-huh.
135	FP	Really, there is *no one* that uses a standard algorithm in terms of two-digit numbers, ah, no one, that is even close. I'll say, did you write it on your paper, this way or this way, and nobody, no one at home has shown them and in a way that's good.
136	Dana	Uh-huh.
137	FP	Their sense of place value, uhm, has not been good and I feel that we're making really good progress there. And I can if you want to after, talk a little bit about specific things I'm doing this year that I think are contributing to that. Cause I'm, I'm learning.
138	Dana	OK.
139	FP	A little bit at a time. Uhm, I think that when I, a child explains their strategies, and says, I knew it because 5 times 6 is 30. And I, you know, they're using multiplication.
140	Dana	Uh-huh.
141	FP	Uhm, *even* when they start out with a problem, like the boat problem I just talked about, and say well I *knew* that 5 times 20 was a hundred, and then, so I knew I needed, uh, 60 more. And I knew that 5 times, uhm, 12 was 60, so I have 20 plus 12 on each ship. Well that child has a good sense of *multiplication,*
142	Dana	Uh-huh.
143	FP	and is *using it* to solve the problem.
144	Dana	Uh-huh.
145	FP	So, I think that just listening to them explain why and how they're *doing* things gives me a sense of how they're understanding it and how they're using it.
146	Dana	Uh-huh. Uhm, *what is the necessary background knowledge for the successful learning of multiplication?*
147	FP	(pause) I don't, background knowledge? I don't know. I mean, I don't think a child has to know their times tables to be successful in multiplying it just makes it easier.
148	Dana	Uh-huh.
149	FP	Uhm, I mean they can figure it out as though go. But to really be able to do, you know, to *rely* on multiplication, you do have to know the tables, because otherwise you're back to figuring it all out, which is using another process. I, I don't know, what do you mean, I'm not, let's talk about what we mean by background knowledge.

150	Dana	I would think, their knowing the times tables, you know, the multiplication, the arithmetic facts. Like knowing times tables would be some background knowledge, if they have that.
151	FP	And they, they are asked, you know, for homework especially, to work on different tables. And some children know them quite well, most do not.
152	Dana	Uh-huh.
153	FP	Uhm, they can solve problems without knowing, but it's certainly a child who knows, can use it.
154	Dana	Uh-huh.
155	FP	Ya, knowing those kinds of things is *helpful*. And I keep *telling* them that, but they have to want to. They have to realize the value themselves, and then they can learn them.
155	Dana	Uh-huh. *And then, what do you do to teach multiplication? Like what, what kinds of activities?*
156	FP	I just present problems that, I don't teach it formally, I teach it informally. I give them problems to solve,
157	Dana	Uh-huh.
158	FP	that, would, could use multiplication. Oh, if candy bars are 39 cents, how many, how much would 6 of them cost? And they usually add up the candy bars, you know. But then we talk about then, multiplying, and rounding. I don't teach any, *I don't think I* teach any skill in isolation. You know,
159	Dana	Uh-huh.
160		[Tape 1, Side A, Count: 305]
161	FP	So, you know the candy bars, what did I say, 35 cents? I don't know what I said,
162	Dana	Uh-huh. Ya.
163	FP	We talked about, wouldn't if be more *logical*, or wouldn't it be more efficient to round it to 40, and multiply by 6? And you know 4 times 6, and we use the magic of ten, and some children do that.
164	Dana	Uh-huh.
165	FP	And so we, we talk about it, I present *opportunities* for it to be used. And when they add, we look at how you can *also* arrive at that answer. And maybe more quickly,
166	Dana	Uh-huh.
167	FP	by using multiplication. So, that's one way I would, you know, *teach* it. Although, again, it's very informal kind of teaching.
168	Dana	You pulled that out of one of your student's strategies, like say if, they gave that example, you always ask did anyone else do something differently? So you would pull that out, and then give that back to the class.
169	FP	But, if *nobody* ever did it,
170	Dana	Ya.
171	FP	If they just weren't moving along, or if they were all doing almost the same kind of thing and nobody was looking at the *kinds* of things that

my experience tells me kids usually do. Then I would say, that I have *seen* people do, and I would show them that. Because, I would think probably that there would some students in the class that would really be intrigued.

172 Dana Uh-huh.

173 FP Or would be ready to pick up on that. Uhm, there are, every year, that I've seen in years past that nobody *does*. And so I will, I will offer that information, because it's typical after you've taught a number of years. And its happened every year, you figure, it's typical at this age. For at least a few children to see a certain thing. And you figure that some of your children, knowing your class, would be ready,

174 Dana Uh-huh.

175 FP to see that. So I do both. I take it, *mostly* I like to base it on what the children are doing.

176 Dana Uh-huh.

177 FP But if they're not moving along, and we're kind of bogged down in the same place, then I will also, throw in other things.

178 Dana *OK. OK, you mentioned that you were surprised that many students did not understand place value by mid December last year. What would you expect to see in the work of a student who you thought understood place value?*

179 FP I would expect them to be able to see, see immediately, if I gave, how long ago was 1547 to know that it was more than a hundred years ago.

180 Dana Uh-huh.

181 FP Uhm, early in the year, most children might start counting, most children, *some* children would begin to count by ones. They wouldn't even *think* of the fact that it might be, you know, almost an impossible *task*, because you want to be thinking about hundreds here, and tens. And because of that, there was *so much* of that this year, and last year I looked for some ways to address place value, and I uh. I don't know that they worked all that well. This year I tried a number of other things. This year I, started out by making digit cards, zero to nine, and giving them to kids, and having them arrange themselves as various numbers and then read the numbers and talk about who was in this place and who was in that place. We did some of that and they enjoyed that. Or given these digits, make yourself into the largest number you can, or the smallest possible number, given these digits.

182 Dana Uh-huh.

183 [Tape 1, Side A, Count: 360]

184 FP So they were doing those kinds of things. But even better, instead of giving those problems of how long ago someone, uhm, someone was born, or whatever. I always ask problems for them to work out. We've been doing *a lot* of those as discussion things, *real quick* mental math.

185 Dana Uh-huh.

186 FP Which I should have been doing before, I realize. So I'm doing, in a way its a little bit more of direct teaching of it, because, you know, I'll

		say, was it more than a hundred years ago? How do you know it's more than a hundred years ago? Oh, well because the number in the hundred's place,
187	Dana	Uh-huh.
188	FP	is, uh, you know, so many less than nine. You know,
189	Dana	Uh-huh.
190	FP	Or what about the tens? So, we're talking about those numbers, and so now I can put up most dates, especially with the year 1999, there just so easy. And, uhm, they can mentally with a minute or a minute and a half, figure out how long ago.
191	Dana	Uh-huh.
192	FP	Or I will give them, ask them to estimate, about how far, or about how many, using, you know, the place value things. But I want them mental thinking. Uhm, and I write horizontally, you know like, 49 cents and a dollar twenty-eight. *About* how much would this be,
193	Dana	Uh-huh.
194	FP	mentally, quickly. So, I'm asking them to round and I'm asking them to think about place value. And I'm finding a *great* improvement, in their ability to do these kinds of things.
195	Dana	Oh, that's good!
196	FP	And I should have been doing, you know, but you, it takes a while to figure it out.
197	Dana	Ya.
198	FP	But it has been very successful and I feel it's a more *direct* teaching.
199	Dana	Uh-huh.
200	FP	And I, and I've thought about why all this has taking me so long to come to. Because, and I suppose it's kind of selfish. But, I mean my job is to teach, and to help children learn, but *my passion* (she laughs) for the last few years has been *to find out* what they know.
201	Dana	Uh-huh.
202	FP	And what they *don't*. So, I didn't want to tell them much of anything, because I wanted to *find out*. I want, I was the researcher in the classroom,
203	Dana	(laughs)
204	FP	finding out without me telling them, and without anybody helping them and giving them clues. And I need to do both.
205	Dana	Uh-huh.
206	FP	I need to do both more. And I'm doing more teaching this year, not because I've lost my *interest* in the other. But I've realized that maybe that's kind of *selfish* on my part and I'm not *giving* enough.
207	Dana	Uh-huh.
208	FP	I was getting a lot more than I was giving. And some of them, you know, when I think of some of my strugglers in the past, uh, probably needed more from me. But I was so fascinated with what they didn't understand.
209	Dana	Uh-huh.

210	FP	That uhm, you know, it was part of my learning.
211	Dana	Uh-huh. So your comparison, I think back when I observed you, I don't remember seeing a lot of this direct teaching. So, how would that be different, then with what I observed?
212	FP	It's a lot of, you know, I'm not telling them what to do, but I'm,
213	Dana	Uh-huh.
214	FP	I'm facilitating more, we're doing more mental math.
215	Dana	Uh-huh.
216	FP	And we're doing it as a class, and we're discussing it as we go,
217	Dana	Uh-huh.
218	FP	And, I'm doing it *because*, I understand that looking at the hundreds, and I will *say*, look in the hundred's place. Whereas before, I would let them just do it. Now, I'm, when I *see* that they're having all this problem with place value, and that they, you know, getting an answer of 45 when it's 500 and something.
219	Dana	Uh-huh.
220	FP	Means they don't understand *place value*, then I'm now going *direct* them,
221	Dana	Uh-huh.
222	FP	to look at the places. And so, that's where it's more direct. I am helping them *focus* on the things that I think they need to begin to focus on in order to make these mental calculations and decisions.
223	Dana	Uh-huh.
224	FP	And instead of letting them flounder and find it for themselves, which they would eventually.
225	Dana	Uh-huh.
226	FP	But, there's no need to do that, you know, it's just my own fascination with their floundering, or something. (laughs)
227	Dana	(laughs) *So the mental math, is that just a group discussion.*
228	FP	Ya, ten minutes, five minutes, if we have a little bit of extra time. You know, sometimes it's combining amounts of money, sometimes it's dates, depending what's the event of the day, you know.
229	Dana	But they're not writing things down, it's just,
230	FP	No, it's just mental. I'll put a number on the board.
231	Dana	OK.
232		[Tape 1, Side A, Count: 438]
233	FP	And I'll say, or how many pages do we have left to read? Or,
234	Dana	OK.
235	FP	you know, there's so many pages in the book we're on, we're on page something, you know, we're on page something. And we're only taking a minute or two,
236	Dana	Uh-huh.
237	FP	or sometimes I'll do 4 or 5 different ones, if we have a few minutes. But I think all of that has helped them, begin to focus more on place value. And they're getting better with the zero as the placeholder, and, uhm, reading the larger numbers and things like that. We haven't gone much

		beyond thousand, we haven't gone much into the, we've *read* big numbers beyond ten thousand and a hundred thousand. But our mental math has focused more in the hundreds, and in the thousands.
238	Dana	Uh-huh.
239	FP	But we'll probably, as the year goes on, move on to some of the others as well.
240	Dana	OK. *So that answered the one question, I wondered what you did to teach place value,*
241	FP	Uh-huh.
242	Dana	*so those are the kinds of things you do. OK. You mentioned in your first interview that you learned a great deal when you had the opportunity to discuss your ideas in a small group during your graduate training. Do you think this type of learning situation works for young children as well as it did for you?*
243	FP	Ya, I read that, uhm, I don't, I don't do a lot of small group interaction and discussions. Sometimes, *late* in the third grade year, a group of children that have read the same book, I will sit on the side and listen to them discuss it.
244	Dana	Uh-huh.
245	FP	Maybe it's the age, and I have read in a book, by an author who I admire a lot, who teaches in New York, that third graders aren't quite ready to handle all of that on their own,
246	Dana	Uh-huh.
247	FP	I think *they* learn by discussion, but I think I need to lead the discussion, at this age.
248	Dana	Uh-huh, OK.
249	FP	I think that maybe fifth graders could. I think that, I mean, my learning came there,
250	Dana	Uh-huh.
251	FP	and, it was a very sophisticated kind of learning. Because we would read, and then we would give our *reflections* on the reading, so we would think about the why's and we'd have a whole *theory* about what we've read when we came in, and then the theories would differ, and then we would discuss the whole issue. We'd be so *convinced* of, you know, that we'd come to these *realizations*, and then find out there were all these other *interpretations*, and so,
252	Dana	Uh-huh.
253	FP	we would then construct all of our *understandings*, by sharing all of that, well that's pretty sophisticated.
254	Dana	Uh-huh.
255	FP	I don't think children could *that* kind of thing without leadership, but I *do* think that it is the *discussion* in the classroom,
256	Dana	But that's whole-class, right?
257	FP	That's whole-class, with some leadership, it could be in a small group, too, *possibly* if they were, they're, they wouldn't, they can discuss certain little things, but I don't think it would be enough to build a lot of

		new understandings. I, uhm, I will sit at the side and kind of listen after I get a group *started*, but, we don't do it as much with math.
258	Dana	Uh-huh.
259	FP	But I listen to them, you know, when they solve *problems* together,
260	Dana	Uh-huh.
261	FP	some of that kind of thing, happens. But I have lead class discussions, where I *know* that by children listening to other children, respond to a set question, and then we just go on with various *responses*, children that had *one* opinion of say what a pattern meant, in first grade, because I remember this one, very well, and I have it on audio tape. As one child who thought a pattern meant black white black white black *white*, and that there was only *one* pattern, up in front of us on the *wall*. But then another child said, oh, but *I* see, and then he pointed out another kind of a pattern, and as soon as that second child pointed out something different, it opened the minds of the others to look for others.
262	Dana	Uh-huh.
263	FP	If I had said, yes that's right and ended it after the first one, everyone's perception of what a pattern was would have been *that*.
264	Dana	Uh-huh.
265	FP	But it was like an invitation, as soon as everyone saw something *else*, for others to look for more. And they found all kinds of patterns, so that the child who had the original perception of what a pattern was, cause I said, do you see anything else, and he said no, there are no other patterns. But at the end he that pattern meant *more* than it did,
266	Dana	Uh-huh.
267	FP	when he first was questioned. So, *that* kind of thing because of discussion happens with very young children as *well*, but I think they need to be lead to it.
268	Dana	Uh-huh.
269	FP	The teacher doesn't always have to know where it's *going*, and the teacher doesn't have to have an end in *mind*, in fact, that's the most exciting discussion, where, the, the meaning, the understanding, the knowledge, grows, just because of what's *said*.
270	Dana	Uh-huh.
271	FP	And, and the teacher is finding out and discovering along with the students, that's very exciting, it happens occasionally.
272	Dana	Uh-huh.
273	FP	But I think it, it all, maybe I'm wrong, I don't know, but I have a, *I don't* understand how to help children do it individually in small groups.
274	Dana	Uh-huh. Ya, I remember last year, I think in your journal, the last day that I observed, you said something that you're noticing children starting to work together, so it wasn't until that time that they were even working together.
275	FP	Right, they work, on something, supposedly together, but they really wouldn't really be *interacting*.
276	Dana	Uh-huh.

277	FP	They would be doing their own thing with somebody else.
278	Dana	Right. Ya, I remember seeing the children, and it was difficult to get them to actually interact,
279	FP	Uh-huh.
280	Dana	where they came to an answer together.
281	FP	And they do, you know, these children in the school are encouraged to work cooperatively from kindergarten on.
282	Dana	Uh-huh.
283	FP	And they do, I mean they can cooperate, and they can share, but they don’t help one and other *build* understandings very often.
284	Dana	Uh-huh.
285	FP	And, it may be just be a developmental thing. It would be very interesting to pursue, that topic, and find out when it is, that you know, or *how* it is. And how we facilitate that kind of thing.
286	Dana	Sometimes I wonder if it’s a little more competitive maybe. That they’re learning something together, but at the same time they’re encouraged to have other ways to do things.
287	FP	Uh-huh.
288	Dana	And to show them that they’re expressing that other way. And in a way it’s like you’re teaching one thing but saying another thing. You need to express your own way of doing something but you also need to work together. And they still come across wanting to express something new.
289	FP	Ya, ya, uh-huh.
290	Dana	I don’t know if that makes any sense.
291	FP	I don’t know either.
292	Dana	But just from observing them. I remember seeing that because it would be hard, to be in those little groups, where you really did work together, cause everyone worked individually.
293	FP	Uh-huh. And I find that typical. And I find that pairs work better than larger groups at this age.
294	Dana	Uh-huh.
295	FP	And I know that four’s are very common, even in kindergarten they do four’s but it’s a more directed activity, I think, uhm, in fifth grade, fourth some times too, I don’t know as much about our fourth grade as the fifth. They have discussion groups of fours all the time.
296	Dana	Uh-huh.
297	FP	And that works very well, with them. So, you know, again I don’t know, maybe it’s just that I don’t facilitate that kind of thing, well that’s possible to. I need to see more, I haven’t spent much time watching teachers facilitate. I learn a lot, not necessarily by doing what I see other people do, but watching what they do and seeing what works well and what *doesn’t*, and then go back and say, OK, then I would do it this way, and try it. It takes months of reading and one viewing of about a half an hour (we both laugh). I learn so much by watching other people work, and I feel like, of course I’m not going on with this, but I’d be ready if I were going to continue to watch some other things that I’m

		not quite sure about.
298	Dana	Uh-huh. So, it’s, things are constantly changing,
299	FP	Uh-huh.
300	Dana	as you said, like, just how you changed this year and how your teaching,
301	FP	And my kids are different.
302	Dana	Ya.
302	FP	They’re very much less sophisticated math thinkers, do I say that every year? No, I had Bernard last year who was a very sophisticated math thinker and I had one other one last year. I have one child whose not very good at expressing at how he does things, but he’s a very sophisticated math thinker, too. But I don’t think he could touch, some of the others that I’ve had.
303	Dana	Uh-huh.
304	FP	I have one girl this year that’s a good, pretty good thinker too [interruption by intercom speaker].
305	Dana	Uh-huh. Well you had Bernard and Pulak, Pulak last year was very,
306	FP	Yes, Pulak was yes, in an adult way, he was. He’s still a very exceptional child.
307	Dana	Uh-huh.
308	FP	He’s just amazing, in every area, not just math.
309	Dana	He seemed like a very nice young boy.
310	FP	Ya.
311	Dana	All right Elaine, that’s it.
312	FP	OK
313	Dana	Yes that’s it. I appreciate your letting me come back and ask more questions.
314	FP	I’m sorry, I knew, I knew, I just couldn’t sit down and get it done. Cause every time I’d try, and you know, and here you had to come out on this ugly day.
315	Dana	Oh, it doesn’t matter, no.
316	FP	I said even on the phone, you know, just voice to voice I’d get it done really fast.
317	Dana	You can think about it, ya, but it’s hard.
318	FP	But it’s really hard to just sit down, because, then I think, what does she mean here? You know, and then you can just tell me and it’s easier.
319	Dana	No, I don’t mind at all, I was just thinking about your time.
320	FP	No, that’s all right.
321		[End of tape: Tape 1, Side A, Count: 648]

F.1.3 3rd interview (Conducted by Jane in March 2000)

Transcript conventions:

Symbol	**Meaning**
…	Ellipses indicate talk omitted from the data segment.
[ ]	Square brackets between lines or bracketing two lines of talk indicate the beginning ([ ) and end ( ]) of overlapping talk.
(0.4)	Numbers in parentheses represent silence measured to the nearest tenth of a second.
(.)	A dot enclosed in parentheses indicates a short, untimed silence (sometimes called a micropause), generally less than two- or three-tenths of a second.
end of line = = start of line	Equal signs are latching symbols. When attached to the end of one line and the beginning of another, they indicate that the later talk was “latched onto” the earlier talk with no hesitation, perhaps without even waiting the normal conversational rhythm or “beat.”
<stressed word>	word(s) enclosed in ‘< >’ symbols shows vocal stress or emphasis.
STOP	All-uppercase letters represent noticeable loudness.
Oh: no::	Colons indicate an elongated syllable; the more colons, the more the syllable or sound is stretched.
Wait a mi-	A hyphen shows a sudden cutoff of speech.
This is a (rehash)	Parentheses around words indicate transcriber doubt about what those words are, as in the case of softly spoken or overlapped talk.
This is a ( )	Empty parentheses indicate that some talk was not audible or interpretable at all.
((coughing))	Double parentheses enclose transcriber comments.
When? 'ats all right. Well, I don't know	Punctuation marks are generally used to indicate pitch level rather than sentence type. The apostrophe (') indicates missing speech sounds and normal contractions. The period indicates a drop in pitch; the question mark shows rising pitch (not necessarily a question); and the comma represents a flat pitch or a slight rising-then-falling pitch. When used, the exclamation point (!) shows "lively" or animated speech.
.hh	The h preceded by a period represents an audible inbreath. Longer sounds are transcribed using a longer string: .hhhhh
hh	The h without a leading period represents audible exhaling, sometimes associated with laughter; and laughter itself is transcribed using "heh" or "hah" or something similar. When laugh tokens are embedded in a word, they are often represented by an h in parentheses. (e.g., st(h)upid)
pt	The letters pt by themselves represent a lip smack, which occasionally occurs just as a speaker begins to talk.
Didjuh ever her uv 'im	Modified spelling is used to suggest something of the pronunciation.

[Side 1, tape A, count: 02]

1	Jane	Ok, this is uhm, March 7th ((2000)). And I’m talking to, this is Jane, I’m talking to Frances. And uhm, so, I dropped off some transcripts and some questions. And I understand, uhm, the transcripts are lengthy.
2	FP	Um-huh. I went through them.
3	Jane	Uh, so, I, that I just sort of like to about some of the, uh, topics
4	FP	Um-huh.
5	Jane	that I, uh, mention here. And I also, uhm, I want to say, ask you something ahead of time that’s not on that list. Uhm, first of all I want to say in person what I think I said, I mean, uh maybe not. How much I appreciate how open and honest you’ve <been> about your history and how and your feelings and how that relates to your teaching. And it really does, uh, help me a lot to understand, uh, what’s going on and what drives you and the <kids>.
6	FP	Um-hum.
7	Jane	And how you create this sort of, uh, motivational climate, uh, for learning. And that’s an important piece that you just don’t see much in the literature that <I> read at least.
8	FP	No, I don’t think you do either.
9	Jane	Uh, because you know that most people who write up this stuff talk only about the cognitive stuff, the <thinking> stuff. But I think you, the emotions and the attitudes and the motivations are <equally> important. And <mostly> we don’t, you know, know where those <come> from. And they come from things like personal history.
10	FP	And that’s why I think that cognitive things and <reflection> are so important, both for teachers and for kids.
11	Jane	Ya. So, uh, besides <thanking> you for being so open I, I wanted to also, uh, say that I would like to <use> some of that material in some of the things I write, but I feel, you know, it’s so personal.
12	FP	That’s all right.
13	Jane	And I want to be able to, you know, feel comfortable showing that to you so that I don’t feel like I’m, uhm, publishing anything or even presenting anything that’s, that <you> would find, uh, too, you know, personal and intrusive.
14	FP	I can’t imagine that I would. I mean I <didn’t> know much about math, I thought I was terrible at it. That’s not something that troubles me: now: or that I’d be upset about having someone know. I think I’ve come a long way. Shows ((laughs)) that I’ve come farther than people who were ok with it early on.
15	Jane	Ya, but, if I, of course your experience with <math> is not unusual for a lot of primary school
16	FP	[Right.
17	Jane	teachers,] many of whom are (.) women. And had, uh, similar experiences.
18	FP	((coughs))
19	Jane	And so, uh, that’s an important <part> of the story I think because it

does, uhm, address a <large> group of people who are responsible for getting kids started, in the, in the <field>.

20	FP	And I think that if we <think> about it, it helps us not, to give our own the kids we're <teaching> the same negative kinds of experiences. You know, I don't <want> them to feel the way I did, so I look for ways to help them. And, uhm, you know, just having the right answer and having to do it someone else's <way> which I never <understood> was painful. And so I feel, probably I bend over too much the other way, that's all right. I want them to feel that they can. And that whatever they <try> is going to be ok.
21	Jane	Ya, uh, actually that, that's a nice, I think introduction to, uh, not the first question but, uhm, the question about girls and boys. Uhm, that I guess is question 2.
22	FP	That's all right. I thought about it. But, part of the problem with answering that question is that I taught boys at Riverside for so many years and it's only [been
23	Jane	Ya, I know.]
24	FP	the last, <three>?
25	Jane	Ya. Ya I know, you had girls, ya.
26	FP	Ya, my first girls are in the <sixth> grade now. So I've taught very few girls at this level.
27	Jane	Ya.
28	FP	I've taught girls long ago. But, uhm, and even in classes that are <co-ed> we have quite a number fewer girls.
29	Jane	Ya.
30	FP	than boys.
31	Jane	Ya.
32	FP	So I don't think it's <fair> to generalize.
33	Jane	Ya.
34	FP	Because I don't have enough to go on. In my first class, I had very strong girls. And in that <year> we also traded children, uhm, for what we called project time which was mostly math based. And, uhm, they would do it in both rooms but we mixed the classes up for the experience <because> there were so few girls we wanted them to have experience, not <only> with their own classmates, were there might only be three other <girls>. So we would put, we put <all> the girls together. Where you with us then?
35	Jane	No.
36	FP	Ok, we'd put all the girls together in one group. And then the other group was coed and we <changed> that coed group three times during the year. So that the boys had
37	Jane	((laughs))
38	FP	experience in single sex groups and in coed groups. <All> of the groups were, all the girls were in coed groups but there were quite a number of girls in the one. I think all together there were only 6 or 7 <girls>.
39	Jane	Ya.

40	FP	Out of all of the 30s, 34, 36 children.
41	Jane	Ya.
42	FP	So, uhm, the girls were always in a coed group because we had all the girls <together>. But the <boys> we wanted to be in single sex <some> of the time but only some of the time. So we,
43	Jane	Ok.
44	FP	There were 3 different arrangements.
45	Jane	Ya.
46	FP	And, uh, in that <year> both in <my> class and the other there were <several> <very> strong girls in math.
47	Jane	Uh-huh.
48	FP	And there weren't that many girls.
49	Jane	Uh-huh.
50	FP	Uh, both years that <you> came into my classroom, I think there weren't as many girls that were strong. Lyndsey last year, you know I was thinking as I was reading these over that they're already hard to remember <vividly> because I'm so
51	Jane	Ya. ( )
52	FP	into the ones I have now. Lyndsey was very strong, and uhm, I've had a, a few. But mostly the boys were the really strong math students. This year I have 2 girls out of my <7> girls that are quite strong. One is very <quiet> but she does volunteer information and her, uhm, comments indicate that she really understands. She thinks things through well. She develops her <own> strategies. The <other> one is a very competitive child. She's got, uhm, 2 or 3 older brothers plus a young, an older sister and some younger siblings. And she, she, her <brothers>, I had 2 of her brothers. Both of whom, well the younger of the 2 brothers was a <very> strong math student. And she tends to be like him. From day one even thought she's <new> to River Side this year, she was coming up with very <quick> responses and very well thought out things. Uh, so she's probably one of my top, although I have 2 or 3 boys. But then again I had 10 boys, that and, uh, that are quite good. <One> boy is very much like the girl that I have. They are probably my 2 top thinkers. The others are very thorough and they get things done well but they're not as <creative> in their math thinking. So, I guess I can only answer it that way, cause I've had so few, I: don't: think: I would, I would <argue> already even though I don't have anything to back it up. That there's nothing, uh, inherent in that ability. That if there's a difference it's got to be (.) based on, on the way society acts towards males and females. We <expect> boys to be able to do the math science kinds of things, but I <think> that because our school was <so> well prepared for coeducation, I always feel it was, we were <over> prepared. All the research. I think we've all been very careful to make sure that, uhm, you know, that girls are treated in appropriate ways. ( ) that probably has an effect.
53	Jane	Ya, I did ask Rose at one point, about a sort of similar issue and she, she

said she thought based on the entry-level exams and the kind of students who <apply> that, uhm, more of the top girls might go to a place like Hillside Academy. So that it, it might be that we’ve in fact got different groups of students ( ) boys and girls so, you know, that’s another more complicated factor. And of course, like you said, the numbers are very small.

54 FP Um-hum.

55 Jane Within the school as whole and certainly in any one classroom.

56 FP And in any year, when I had all males as well, there are some boys that are <awful>

57 Jane Ya.

58 FP and some that are very strong. Ya, I think it’s children in general rather than a gender thing. The little girls that’s so strong in the classroom came from ( ) interestingly. ((laughs)).

59 Jane Hm. So <somebody> thought

60 FP Um-hum.

61 Jane that may be a coed experience. ( )

62 FP They brought 3 girls in.

63 Jane Ok, uhm. I’m, I was very curious when I read the transcript from your interview in December with <Dana>, when you talked about, uhm, direct instruction. And you said that you felt that it was ( ) that kind of that it was in some ways selfish to <only> uhm, (.) focus on the, on the children’s strategies that there was times when you wanted to there were times when you wanted to do, that you thought it was maybe important to do other things. And I’m, what I’m wondering if you could just elaborate some more on that.

64 FP I don’t think it was, yes it was focusing on their strategies. I became so fascinated with math, that I wanted to see what children could <do>. All on their own.

65 Jane Ya, ok.

66 FP And I don’t think I gave them, when they were floundering, I don’t, I let them flounder, which I still do to some extent. But I then didn’t give them what they <needed>. Because I wanted to see how it would work itself <out>. And that, it was a selfish thing to do, I think. Because my JOB is to help them become better. Which I think they did but I could have done more. And I <don’t> view what I’m doing now as more DIRECT teaching but more guidance. And I wrote about it this year, in November. And I wrote this, I realize that there’s been a change in my math teaching this year. And I think it is a change for the better. In the past I’ve been so focused on wanting to find out how kids think, that I don’t think I helped them enough. This year I think I’m providing a better <balance> and I’m finding ways to provide <practice> with certain strategies and concepts. I’m having kids do more mental math and taking them through the prediction part. For example, today, Veteran’s Day, the Armistice was signed in 1918. Was that more than 100 years ago? How do you know? The number in the hundreds place is

9 etc. Where there 10 year, more than 10 years ago? How many tens are there? So, that while I always did those <problems> of how long ago, and it was clear to me that some children would start counting by <ones> when something was 3 or 400 years apart. I didn't help them. I would talk about it not being very efficient but I didn't help them find <strategies> that would result in them being more efficient with it. So I have all <year> being much more careful to do that mentally with them and to talk about those strategies. So you look at a number and you start to think to yourself before you start figuring out. Look at that number. Is it more than 200 years ago? How do you <know>? And, and as children started to talk about how they knew, others started to key in. Most of the class now <easily> does those things that others were having trouble without paper mentally. And some aren't so good with the, you know, if it's something 82, quickly coming up to 2000 but they're getting much better at that too. Because I know that 2 and 8 is 10 and I, you know, whatever. Uhm, so they're getting much better and maybe 2 or 3 aren't paying much attention now but the <others> some were VERY weak at the beginning, are, are doing that. So,

67 Jane ( ) What's the pay off for them to be able to do that better? I mean in terms of ( )

68 FP In that, that it's, well, let's see. First of all, they are more empowered mathematically. They who felt, I can't, now feel, I can. They are learning: that: you: can and probably <should> estimate first. Make some <predictions> about what probably should happen before you focus in on a little detail. You know, in a way it's like drawing. You know, you look at the whole <picture> you try to get a sense of something before you start drawing the toe. And uhm, I think that we are helping them find ways to become more <efficient>. (.) That did happen before after the fact when we would discuss how they had <done> it. By then some of them were so convinced they had done badly that maybe they weren't listening as well. And we still <do> that. I still talk and discuss <thoroughly> what strategies children have used to solve <problems>. Because I think it is an important thing to <do>. And I certainly learned a lot about their thinking and I <know> that they do continue to learn from one and other. But some do turn out, tune: out as well especially if they think they've done very badly. Where they, those are the ones you really want to listen. So <doing> it in advance, you are helping them get started. And without saying do: it: this: way:. You are offering ways, that they might, and it isn't always only one way, you know, we try lots of different ways. Or how did you, usually I'll put it up there and, you know, I may say it was it more than a hundred years ago. But then we still talk about how people are looking at this and how they are <beginning>. So I guess that, and I don't think of that as direct teaching, I think of it as guiding <them> to learning. Because direct teaching would be more like, this is how I want you to do the <problem>,

69	Jane	Ya.
70	FP	You know, or teaching an algorithm.
71	Jane	Some people use the term guided discovery ((Tape 1, Side A, count: 221)).
72	FP	Ya, and I guess that would be it, that’s very nice term.
73	Jane	Is that a nice term?
74	FP	A very nice term.
75	Jane	((laughs)) Ok.
76	FP	And I, I guess all I did after that was say, it seems like such a logical thing to do why didn’t I do more of this <before>? I mean it, I don’t know why it didn’t occur to me.
77	Jane	I think other people have had the same experience. Uh, that when, you know, when you start, uh, focusing on students’ <strategies> you, uhm, are reluctant to go back to something that’s <more> directive. But still sort of resonates as direct teaching until you reflect on it like you <did> and say, what a minute, uhm, isn’t, you know, there too much floundering or too much sort of aimless wandering. And, uhm, it really has to, I have to give them some kind of <channeling>.
78	FP	Right.
79	Jane	So that they can, uhm, continue to investigate but not <everywhere>.
80	FP	Yes.
81	Jane	Because [I know
82	FP	That’s right.]
83	Jane	( ) So you’re thinking more, sort of, uh, down, sort of <looking> down the road, [further
84	FP	Um-hum. Um-hum.]
85	Jane	for them, or [something
86	FP	Ya, um-hum.]
87	Jane	Ya, and then, sort of saying, well, <these> ways you’re going may not, I mean to <yourself>, these ways they’re going, may not work out so well, so I maybe I need to <push> in a couple of directions, and, and <channel> them more.
88	FP	And you need both. Because <until> you have the opportunity to find out HOW they’re thinking. You don’t know [how
89	Jane	Ya.]
90	FP	to help direct, or help, or coach.
91	Jane	Ya.
92	FP	Because, and THAT’S what I think the first part <is>. When you get involved in the strategies and the finding <out>. You don’t want <anyone> to tell them anything because you’re SO intent on finding out how they’re thinking. How are they doing this? How do kids do these things?
93	Jane	Ya.
94	FP	Because until I can understand that
95	Jane	Ya.
96	FP	I can’t know how they need to be helped.

97	Jane	Ya.
98	FP	And I guess that, that’s part of it. I think the same thing happened with <whole language>.
99	Jane	Ya. ((laughs))
100	FP	Completely hands off. Although I don’t think I <ever> was hands off with language.
101	Jane	Right.
102	FP	THAT all made sense to me from the beginning what my role was. But I think I was so much more comfortable with language, you know.
103	Jane	Ya.
104	FP	And, and.
105	Jane	Right.
106	FP	But, uhm, you know, people absolutely hands off, well.
107	Jane	Right.
108	FP	You can’t let kids flounder there <either>. You need to guide without being too ( )
109	Jane	Ya, yes. So it is happening a lot ( ) . I think that’s another part of the, <important> part of the <message>. Uh, that, people who’ve had <experience> with this different way of teaching for <quite> a WHILE, have come to realizations.
110	FP	Um-hum. ((Tape 1, Side A, count: 260))
111	Jane	That people NEW to the field are probably going to also <have> to come to, uh, but it would be helpful, for people who are <working> with them, to sort of, again, with the <teachers> do the sort of same thing,
112	FP	Right.
113	Jane	you did, with the, with your students and sort of say, maybe they’re floundering here? ((laughs)) Maybe they need
114	FP	Yes.
115	Jane	some more guided discovery.
116	FP	And, uh, presently in our teacher preparation classes we need to bring these issues <up>. I think they’re going to have to find new teachers. They’re going to have to find out some, ( ) but bringing the issues <up> and you know. Well, well maybe speed up the <process> a little wee bit. ( ) I’m going to have an opportunity, if all goes well, on the 20th, to go in and talk to one of Renee’s classes, I’m going to take her class that night, because she’s going to be out of <town> and she wants me to just <talk> about what I’ve learned,
117	Jane	Hmm.
118	FP	as a teacher. She’s doing the psychology, you now, Piaget, Vygotsky, all these other
119	Jane	Ya.
120	FP	THINGS. And, uh, difference, that’s the ( ) time, ((strange noise, sounds like a timer going off))
121	Jane	Ya, I know.
122	FP	Uhm, so more, so, you know she, that week she decided to devote to their lots of different ways to get to the same end. Uh, so she, she knows

		that I would be in, I would be interested in <doing> it. And, she's going to be away so I'm going to go in and talk about some of the things that I've learned. Now that the, ( ) examples of writing, I've done presentations on writing and on math, so, you know, just take a look at ( ). So, what kinds of things are you interested in finding out?
123	Jane	Ya, well I think that's uhm, needs to be. Well, we've had this conversation <before>, more and more in the ( ) courses as well as, you know more.
124	FP	And how.
125	Jane	Integrated with instruments.
126	FP	And HOW do you begin to go in, how do you go in to a room, with children you've <never> seen before and find out what they know? You know, uh, giving them a test, I don't think does it at all, because you get a very limited bit of <information>. But there are <lots> of good ways to do it depending on their age and everything, so I can share some of that too. Ya, it <seems> like everybody would know that but it isn't true.
127	Jane	No. ((laughs))
128	FP	((laughs)) It just seems so natural.
129	Jane	Ya, but, uh, ya you've been <immersed> in it.
130	FP	Ya.
131	Jane	That's what it seems like.
132	FP	Ya, there was an art teacher and art teachers never find out the rules you have to figure it out as you go along. I never expected there to be certain things you're suppose to do. I expected to go in and find out what I needed to do. And that helped I think.
133	Jane	Um-hm.
134	FP	That's a good lesson too. Teacher ( )
135	Jane	Uhm, (.) Well I think we've really addressed uh, questions 1:, 2:, and: 3. Unless you have other things?
136	FP	Uh-uh.
137	Jane	Uh, I would be interested in, uh, you're responses to the transcripts.
138	FP	Ok, but I need to know what you wanted to know.
139	Jane	Ya. No, I actually have a.
140	FP	And I <really> have almost nothing written. I have <absolutely> nothing written on one of them. So it wasn't that important to me. And I wasn't necessarily always doing a good job. I <filled> all of this for the half year, but I had <nothing> for, uhm, one of the dates. Let me see what I have for November 30^{th} was one of your times, wasn't it?
141	Jane	November 12^{th}. ((Tape 1, side A, count: 323))
142	FP	November 12^{th}. (.) I think that's the one I have nothing and I apologize. Except language stuff. (.) November 6^{th}. November 9^{th} through 13^{th}. (.) Tuesday the 10^{th}. Tuesday's the 10^{th} so Thursday would have been the 12^{th}. Thursday and Friday ((laughs)) I have nothing.
143	Jane	Ok.
144	FP	I have nothing. Just words. (.) Things ( ) made it ( ) Right. So there is

		nothing for that, but what was the other date?
145	Jane	October 20th.
146	FP	October 20th. I might have <something> for October 20th. October the 9th, ((sounds of shuffling paper muffles her words)) October 23rd, 19th, October 20th. (.) It’s difficult writing:, writing journals ( ) some mental math today. How many minutes in a half an hour? Quarter of an hour? I did that. But that’s not.
147	Jane	Ya. We discussed this problem that day, you discussed that problem.
148	FP	I don’t have anything down.
149	Jane	Ok.
150	FP	I’m sorry.
151	Jane	All right. Well which of the two transcripts did you write on? You said you wrote a few things. Or no that, we were talking about your journal.
152	FP	Ya, no. Ya, I thought I had something in my ( ) but it wasn’t anything. I, I looked at them this morning or last night and I thought there’s nothing in my journal.
153	Jane	Ok.
154	FP	And I can, I’ll answer whatever questions. I did read through both the transcripts.
155	Jane	Wow. [That’s a, that’s a ( )
155	FP	Not, not,] I read through them superficially, cause I was there. ( )
156	Jane	( )
157	FP	You know what I mean ((laughs)) And I, I didn’t need, [you know
158	Jane	Ya. ((laughs))]
159	FP	to, to, great thought in this. It <certainly> AGAIN: points out to me when I read through these. How long:: I draw certain things out and wonder if that’s in the best interest of children, you know. And go on, I go on and on and on and on with the discussion ( ). It’s fascinating to me. And probably for the ones who are really involved, but for those that are kind of on the edge (.) I <wonder> how useful that is. So that’s, ( ) having something like this makes me much more aware of it. And I don’t know the <answer> but it’s certainly something to consider.
160	Jane	Ya. (.) Well I know when I’m observing, I, I do try to pay attention to whether students seem to be fading,
161	FP	Uh-hum.
162	Jane	in serious ways, maybe they’re just, you know, very polite students. But I pick up a <lot> of that. I think sometimes as things got, uh, edged towards lunch and they would get antsy ( )
163	FP	And then there are those that are right up close, really involved.
164	Jane	Ya. Ya. Right.
165	FP	And then there are those that aren’t paying that much attention.
166	Jane	Right.
167	FP	But it’s always interesting. With the ( ) the comments that you or Dana noticing things where <I> really didn’t necessarily <notice> Nathan singing a song. Them talking about these <other> things while they were <working>.

168	Jane	Ya.
169	FP	Which is not a problem but it’s so interesting that there’s this whole other perspective.
170	Jane	Ya, I know.
171	FP	That I never pick up on.
172	Jane	Ya. Ya, there is a whole other <world> going on
173	FP	Yes, absolutely.
174	Jane	a microworld.
175	FP	Uh-hum.
176	Jane	Uhm, that always surprised us too. Uh, and NOW that we’re looking (.) more carefully at the whole-class discussions, the <field> notes are a surprise to us too. Cause I sort of forgot, and then of course I wasn’t always <there>. And so I look at other people’s field notes and that’s why, the, the uh, the data that we <have> is really a treasure. I feel like I’m going, I’m, I’m <prospecting> for buried treasure,
177	FP	((laughs))
178	Jane	something here. Because you know I, I find you know your journal sometimes and I find field notes that I hadn’t <remembered> or someone else took, and then we’ve got the transcript and then we got the student <work>. And there are all these little ( ) pieces and all these connections I hadn’t been able to make before. Uh, and it’s a really interesting process to, to have this kind of data. I haven’t co-, collected some of this kind of data but not in a classroom this way.
179	FP	Uh-hum.
180	Jane	Uhm, and certainly not at the beginning of the year. So, uh, you know, there’s just stuff that I hadn’t expected. Uhm, I, I’m been, I was fascinated by the November 12th problem, which is dividing sandwiches. Dividing,
181	FP	We did it twice this year too. We did the same problem
182	Jane	Twice, why did you do it twice?
183	FP	I don’t even realize that I did it twice. So I look back in my this year’s <journal> and I realized I had given the same problem
184	Jane	((coughs))
185	FP	basically the same problem, the sandwiches, the same numbers of things earlier in the year. Ahgh, ((laughs)) and they had just as much trouble the second time as the first. But,
186	Jane	What, so that, the, the one question that I, that we came up with for this one was. What, uh, were you trying to teach with this, task?
187	FP	Well, it’s
188	Jane	I mean, I’m asking this <naïve> question but I really wanted to hear it from you.
189	FP	I don’t always start out (.) with a purpose that I’m sure of. Uhm, I, I know that we had probably been talking a little about fractions, and a little about division. And, to get a sense of how kids can actually DEAL with those, while they might not <know> they’re dealing with them. You give a problem like <that>. There are others of course, and, and I

		<guess> the reason I gave this year's because I thought it had worked well, as a learning experience for me, and, and about how children think. So I may going to <use> the multiplication that they've been practicing. Uhm, I think this one, the transcript, had 16 or 18,
190	Jane	Ya.
191	FP	which was an interesting thing. I, I've only done 18, I think, this year. Uhm, so that the 3 times 6 is 18, for some of my children this year, they did this problem in 5 minutes. Probably both times. Because, immediately, they knew 3 times 6 is 18, and knew what they had to do. Uhm, others still: don't:, they still don't uhm, apply, <any> of the skills that they're learning to solving problems like that. So I guess part of it is, are they beginning to apply skills that we're working on to problem solving. And this kind of a problem, offers the opportunity for multiplication, division, and <fractions>. Probably addition and other things too. And, and, if they <usually> solve them in lots of different ways, which they did. And <some> children that seemed so good at things are completely stymie by something like this. If it can't be divided once and come out even. If there's anything left over they can't figure that they can divide that again in another way or something. So part, I guess a <lot> of it is seeing where they are with things and what comes next then.
192	Jane	Uh-huh.
193	FP	I guess almost all of what I <do> is designed, where are we? And what, what don't they understand yet? What kinds of things will I need to go by, from a lesson like this?
194	Jane	Uhm, one of the things, that interested me about this lesson ((clears her throat)) is that uhm, some of the kids seem to want to deal with it is a math problem without fractions.
195	FP	Um-hum.
196	Jane	And, you know, put the ideas of sandwiches sort of aside. And other kids were very much <emotional> ( ) the sandwiches. And, uhm, there's some confusing parts to it.
197	FP	Like?
198	Jane	Uhm, well, there's the confusion over pieces, and fractional parts. And in fact, in one place, I didn't notice this initially but Ellen did, one of the students, and it was Lyndsey, got very confused about parts and wholes. Which, you know, is an important issue in fractions, trying to keep parts and wholes straight.
199	FP	Lyndsey's here, on page 11.
200	Jane	Ya, page 11. Uhm, she started <out> by cutting the, you know, the sandwiches in half.
201	FP	Um-huh, um-huh. So she gave everybody half.
202	Jane	Right.
203	FP	So, she had 24 minus 18. Ok.
204	Jane	Right.
205	FP	And she had 6 left.

206	Jane	So she then-
207	FP	There were <6> pieces left.
208	Jane	Right, right. So, you, she said 6, you say 6 <pieces>,
209	FP	Um-hm.
210	Jane	She says 6 pieces,
211	FP	I should have probably said 6 <halves>, it should have been.
212	Jane	Right. And <then> she says 6 <sandwiches>. So this is a shift,
213	FP	Um-hm.
214	Jane	from pieces,
215	FP	Um-hm.
216	Jane	of sandwiches to <whole> sandwiches.
217	FP	And I went right along with <her>.
218	Jane	Ya:.
219	FP	Um-hm.
220	Jane	And so then she was, you know, dividing, those, what she thought of as <whole> sandwiches into thirds.
221	FP	Um-hm.
222	Jane	When in fact, if they were half sandwiches, they would have been-
223	FP	But I bet she knew they were half sandwiches. So how did she get, let's see what she got at the end. (.) I don't know, I don't know if you have the papers ( ).
224	Jane	Ya, we do.
225	FP	Ok, did she have-
226	Jane	I don't, I don't recall. Uh, and I, you know, would a, I, I, definitely-
227	FP	But I <said>, so you gave each person one <half> of a sandwich, and a third, the: sandwich:, so maybe that isn't.
228	Jane	Well you, oh you maybe, ya, you [( ) maybe
229	FP	( )] Just with hers.
230	Jane	Uhm, and <I> was kind of interested because, the uh <visual> representation which <many> of them were,
231	FP	Um-hm.
232	Jane	of the sandwich, uhm, is <easy> to get confused, because the, a half of a sandwich still looks like, or quarter of a sandwich, still <looks> like a sandwich except smaller. Uhm, you gave another problem involving, ah, pies.
233	FP	Uh-hm.
234	Jane	And a slice of a <pie> doesn't look like a whole pie. So, you know, in <some> context, some applications of fractions, the whole has a distinctive shape,
235	FP	Um-hm.
236	Jane	from a part, and in others, the whole is unfortunately confusingly <similar> to the parts.
237	FP	None of that ever occurred to me.
238	Jane	Uhm, ya, I was just-
239	FP	I wonder if, no but then I'd be directing. (.) <Suggesting> that they, have, if I had pieces of paper cut into squares that were sandwiches, that

		they could cut them if they wished with the scissors. That that would be more useful.
240	Jane	Well, ( ) they would be a different size.
241	FP	Right.
242	Jane	Certainly.
243	FP	Right.
244	Jane	And that would be ( )
245	FP	And they would have that
246	Jane	Ya.
247	FP	in front of them. Well, its-
248	Jane	Another thing that was interesting about this lesson is that some of the students who were stumped, and they were mostly boys, <refused> to draw sandwiches. Uhm, and,
249	FP	So, ya, they refused the concrete thing.
250	Jane	Ya. Right, and, and actually in Dana's fieldnotes, she talked about going over to one of the boy tables and <suggesting> that, and they, would have nothing to with it.
251	FP	Um-hm.
252	Jane	And yet other kids found that whole <idea> that you could make the concrete and work with the concrete, and then move,
253	FP	Um-hm.
254	Jane	between the concrete, and the, and the fractional parts. Uhm, they're useful. And in <this> case perhaps, the opposite, another, uh, thing happened, were using the concrete <parts> lead her to make the <inappropriate> translations, into fractional parts. [( )
255	FP	I'm not sure] she <did>, she may have <verbalized> it incorrectly. So, <that's> what I'm not sure of. She may have said sandwiches, meaning halves, of sandwiches, I don't know.
256	Jane	Ya, ya.
257	FP	And that's, we'd have to see it.
258	Jane	And then ( ) pieces
259	FP	Right.
260	Jane	is a difficult
261	FP	Right.
262	Jane	Cause pieces is not precise.
263	FP	That's right.
264	Jane	It works well in actually, in picnics, but it doesn't work well in terms of
265	FP	Right.
266	Jane	In terms of [fractional parts.
267	FP	And when we] can't see what we were drawing and what we were doing
268	Jane	Ya, doing
269	FP	then it's very hard to interpret.
270	Jane	Ya.
271	FP	Uhm,
272	Jane	And some of them, I think felt, and I saw this in the transcript other places, that if you <divided> it, you know a sandwich into <any>

		portion, it was a half. And so <you> intervened and said, no but if they're more than <2> of them in the whole then we call something else, something like that.
273	FP	And it's fairly early in the year too.
274	Jane	Ya.
275	FP	Actually.
276	Jane	Oh ya.
277	FP	Oh ya. There was a group once, and we were dividing, uh, a number of chocolate chip <cookies> among children. And they absolutely couldn't do it. And I finally said, to the whole class, well a few things. I want you to put everything away. And I cut out circles. And I gave them cookies, like I did when I taught first grade,
278	Jane	Ya, ya.
279	FP	I gave them the same problem. And it was something as simple as: uhm, 6 cookies for 4 <children>. I mean that simple.
280	Jane	Ya.
281	FP	And, asked them, to <imagine> that they were real cookies. And I wanted them,
282	Jane	Ya.
283	FP	to <share> them. And they need to that at third grade, <exactly> as my first graders had <chosen> to do that. Like I had cookies cut for them too. And the <explanations> of my <first> graders were probably as sophisticated, if not more sophisticated than my <third> grade. But they, you sometimes see that they simply
284	Jane	Ya.
285	FP	have to have the food, ((laughs)) to do it with. They even drew chocolate chips on the cookies, in third grade, too. It was <important> for them to have real cookies, some how.
286	Jane	But <one> of the students in, uhm, this class, uh, uh it was one of the boys, I forgotten exactly who. Uh, actually corrected <you> when you referred to
287	FP	Ya.
288	Jane	fractional parts of a sandwich as a <piece>, he said half or,
289	FP	Oh, good.
290	Jane	quarter. So, you know, and he was one of the boys who resisted drawing them.
291	FP	Um-hm.
292	Jane	Uh, so, you know, uh, this ends up, for me, an interesting transfer,
293	FP	Um-hm.
294	Jane	because we've got this, uh, ( ) application problem, and, and I <agree>. But you've got some kids who are really wedded to the concrete objects and to the story that they're familiar with.
295	FP	Yes.
296	Jane	Uhm, and <others> who are absolutely resisting <that> and seeing it as a, as a <fraction> problem, or division problem.
297	FP	And not <always> see that. I mean that is (.) I guess,

298	Jane	This just makes it clearer.
299	FP	Absolutely. I <guess> it's always in the back of my mind. There are <always> children who see that and <those> that only think <mathematically> (.) will not, will have problems occasionally forgetting what the <thing> is we're talking about, so they will divide balloons the way they do <cookies> and ( ) not do that ((laughs))
300	Jane	((laughs))
301	FP	So we talked about those kinds of things.
302	Jane	Ya adult mathematicians are like that too. ((laughs))
302	FP	But uhm, <that's> why I think it is important to relate that you don't give the problem (.) of how can you divide this number by this. Or how can you take (.) uhm, 12, and divide it into 18 parts and not have cookies or sandwiches or <something>. Because some children need the something. And others don't. And the ones that don't can get rid of the concrete, in their minds. The ones who need it, it's there for them. And the, I think it's <important> to relate it to things they understand, picnic and sandwiches, or cookies, or balloons, or party hats. Whatever it, happens to be.
303	Jane	(.) Ya. Uhm, the, have you done anything like this year?
304	FP	No. I have it my [( )
305	Jane	( )]
306	FP	folder, that could be done. It was interesting. Uh, I think we've talked, we may have actually, talked a bit about (.) ((sighs)) seems to me at some point when we solved some sort of problem in LOTS of different ways. I then added a few other ways that I had seen children in the past solve it.
307	Jane	Um-hm.
308	FP	And, and we talked about efficiency or (.) how do we decide which is the best one.
309	Jane	Ya.
310	FP	I also am having children, when I think of it, write more about how would you do it if you were going to do it again (.) and why? And,
311	Jane	On a regular basis? [( )
312	FP	I've tried,] I've tried before, but I don't always get there. Uhm, and I'm <real> intrigued by some, you know MOST of my children this year, would do it the same way, again. And one them wrote it in a way that's so right for children. It may not be the, it may not be the best way, but for ME, it's good because I understand it or something. It makes sense to me.
313	Jane	Ya.
314	FP	And, and that's, I was so pleased that I had been able to communicate that, you, you <have> to use what makes sense to <you>. Were as some kids strive to use, regrouping when they can't regroup, and they know that they should probably do a problem like this and they get it <completely> wrong. Their answers are <so ridiculous> and ( ) the question, that you can see that mathematical sense has nothing to do

with it. I don't see that so much this year. They're much: more: (.) uhm, likely to choose what makes <sense> to them, rather than what adults might consider more sophisticated. And be comfortable with that. And I think it's really important to continue to <expose> them to things that may be more sophisticated. But to remind them that you have to <use> what makes sense. Looking for ways to become more efficient. And I have one student, I'm trying to think what problem we did, just the other, yesterday or the day before. (.) It was a big number. OH, yesterday was the day when the first Oreo cookies were baked. And today's ovens which are as large as a football field produced 2000 Oreo cookies a minute. And I said how many Oreo cookies would be produced between 8:30 and 10 o'clock? Because they're not very good with time and I figure, well at least they have to get that (   ) first. And some of my, a couple of my very (.) good generally math thinkers were adding: things: up:. Others were so quick. A round number like 2000, uhm, 90 minutes, or most of them started with 30 minutes. Uhm, 30 times 2000 is 30 times 2 ((end of tape 1 side A, count: 738))

315 Jane ((Tape 1, side B, count: 2)) Uhm, one of the things that I was struck by in the October 20th discussion but I actually saw it from the very beginning of the year, is when you're talking about more sophisticated strategies, you obviously don't use that word, you talk about ways of doing it that your parents use. And October 20th it actually was highlighted because the conversation was, in terms of figuring out what's best, came around to best for whom.

316 FP Right. And it came around to it quickly.

317 Jane Ya.

318 FP It came around through them too though, didn't it?

319 Jane To some degree. I think in part, we did this. You planned this activity to be somewhat similar to the activity we did in the previous year.

320 FP I think it was <exactly> the same activity the previous year.

321 Jane Ok. And the kids <then> I think were the ones started ( )

322 FP So, I was anticipating it.

323 Jane You had that in the back of your head, and so you actually, it sort of came from the kids, you know, but I think it's one of the places where you were leading them.

324 FP Uh-hum.

325 Jane Guiding them. Then the discussion I think became very interesting, about the definition of what's best depends upon, who. And so it started out with discussion of people younger than you, people in your grade, people older than you,

326 FP Uh-huh.

327 Jane like your siblings, your parents. And, one of the things that interests me is how that connects for kids, what they're doing in that classroom at this moment with other situations, other groups of people, and the sort of push to get more efficient is not an arbitrary thing but a kind of thing in terms of this is what this group of older people who you know, respect,

		do.
328	FP	Ya, I know. And I wonder if that’s so good. I understand what you’re saying. And I think I’ve done much less with parents this year because we had Tarun last year who always used the traditional algorithms for things because he <understood> them. And it <bothered> me that everyone thought he was the best. Because he could do that. And I felt that, and maybe it’s <exactly> what you’re saying now. That lead children to believe that if <I> am equating what Tarun was doing with what adults do than that had to be something very <desirable> to attain. And yet truly I believe that you don’t ever have to solve problems using that algorithm, to be good at math thinking. And yet I was communicating the idea that you did. I think to those kids last year. I <don’t> think I’ve done that this year.
329	Jane	I actually don’t think that that’s a damaging message.
330	FP	I don’t think that that 2-digit by 2-digit multiplication algorithm is a very sensible one at all. I <don’t> think it’s a good algorithm.
331	Jane	But I think you gave a <multitude> of messages.
332	FP	But I think that because Tarun could understand it and it’s something adults <knew> and I communicated that. It made it sound as though that is the desirable way to go. Whereas, I don’t think if you’re multiplying, without pencil and paper, most people would do it that way, in their heads. You know, they would be <doing> 3 times 2000 or 30 times 2000.
333	Jane	Ya, well.
334	FP	And, and that’s fine.
335	Jane	Ya, for mental math.
336	FP	For pencil and paper math, why this, that destroys place value. Because until that sense of place value is, is REALLY strong, and until kids are estimating and <realizing> that the answers they get that are incorrect are so completely ridiculous, you can’t even. I even hesitate, to show it to them. In fact I, I know that since we’re doing square roots for this month’s calendar and the person that I left with the children for the rest of next week, where they’ll be getting into 13 times 13 to figure it out. I said please do not show them the traditional algorithm, because it will confuse those. Let them work out ways. Let them work together, whatever. And then to discuss those. Because they’re getting pretty good at figuring out ( ).
337	Jane	Have you had any conversations about this issue with teachers of the fourth graders?
338	FP	No. I don’t necessary agree. Well there’s one new fourth grade teacher, who I, I, who’s so quiet and because she’s new is nervous about expressing her ideas. I think she’d be fine. The fifth grade teacher and the other fourth grade teacher aren’t secure in what they do and go to the book. And so I can’t. But my friend Ann who has known math and been a strong math person ever since she was a child, uhm, she feels real strongly, about that. And it’s taken me a long time to understand and we

can see how place value is destroyed. I mean its become very clear to me. So she feels very strongly. She's REALLY strong on don't <ever> teach children a subtraction with regrouping. There is NO need for it. If they come to it themselves that's fine. But they <never> need it and it can be so destructive. You don't teach it to them if they're doing something like that and making errors because they stumble across it, you can help them. But, and so is Constance Kamii who's one of the researchers who has written articles in how <damaging> traditional algorithms can be for children. And the more I <see> what children do, and listen to what they say about what they do, the more I agree with that. So, I don't want to communicate to children that it is a better way. I really don't. And I think, I was aware last year that my kids that it was better and it was very disturbing. I think I've not communicated that this year. And I'm <pleased> that they feel that it's a good way for me because I understand it. And unless their way is so ridiculous ((laughs)) and they're just saying that because they're stubborn I want to do it my way. And I don't see that either.

339	Jane	Another thing that I was interested in on October 20th, is the, well first of all the students seemed really fascinated by the discussion, and continuing beyond the how did you solve it, is that the most efficient, you know.
340	FP	Uh-hum.
341	Jane	To the discussion of who might solve it one way versus another way and thinking about not just the age of the person, but what they knew.
342	FP	Uh-hum.
343	Jane	So if you are a first grader who knew your three times tables then it didn't matter that you were younger. It was the <knowledge> that you had that would lead you to choose one kind of strategy vs. another. And one of the students mentioned in response to some other students saying that I think the strategy with the tally marks was best, for younger children. He said, well I have a younger brother and he doesn't understand tally marks.
344	FP	Uh-hum.
345	Jane	So really thinking that it's not cut and dry. That thinking about other people's thinking and so that was interesting to me. And then also they really got into a kind of debate in the class.
346	FP	Yes they did.
347	Jane	With each other about this issue and strategies and what's best, and for whom. And you may have been less directive than you normally are in these discussions. And they were addressing more comments to each other.
348	FP	And that's important at times too.
349	Jane	Ya. So that was a really interesting whole class discussion. So I thought that the task seemed to generate a lot of involvement from the student and interesting thinking processes.
350	FP	Ya, I agree.

351	Jane	That’s a really all I wanted to talk about.
352	FP	OK.
353	Jane	I certainly appreciate your taking the time to come out now.
354	FP	It’s always interesting. It really is. ((tape recorder was shut off and turned back on)).
355	FP	It was only after the fact that I <realized> that I had done that. Now, looking back, this year, I am purposefully not giving that message.
356	Jane	That message about algorithms.
357	FP	Right. And one thing better than another. And I, you know, you have to, you have to just come to that realization because you can’t think about everything.
358	Jane	But you know these kids are.
359	FP	It isn’t a bad thing.
360	Jane	No.
361	FP	It was just a message that I was giving that I really didn’t intend to give. It was useful, especially for children that felt so <far> from being able to achieve something like that, you know. Where I’m trying to empower them I was probably helping them, I wasn’t helping them to feel empowered, I was making them feel <less> capable. I don’t know. I’m sure I’m over blowing it all.
362	Jane	Ya, well.
363	FP	But it’s still there and it’s something that I became aware of after the fact, as we all do every year with everything we do.
364	Jane	Yes, ya, ya.
365	FP	Well I sure won’t do that next year.
366	Jane	Well you know, people have always written about the liabilities of recording. Obviously there are obviously all the advantages of keeping a journal, making tapes, listening to them, reflecting. But there’s also a down side that you tend to see things more negatively when it’s all out in concrete. The subtleties of human interaction are not all distillable into a verbal transcript.
367	FP	Right.
368	Jane	So that we often mitigate our messages in a nonverbal ways that don’t come out in a verbal ( ).
369	FP	On paper,
370	Jane	Ya, I know,
371	FP	What I’m <saying> I’m not speaking in <sentences>.
372	Jane	No one speaks in sentences. You would be truly bizarre if you spoke in sentences ((laughs)).
373	FP	But it looks so weird. And I know that listening to it wasn’t as bad as seeing.
374	Jane	But linguists have studied that and they talked about the fact that complexity in oral language is communicated very differently than complexity in written language.
375	FP	Absolutely.
376	Jane	In written language it’s largely in the words we use. And in oral

language it's in the grammar, that we have these multiple embedded sentences. Then in writing,

377 FP The <grammar> is what I don't like in here,

378 Jane But no one speaks grammatically.

379 FP But also it's in tone of voice and that's in facial expressions.

380 Jane But that's not in the transcript.

381 FP I know.

382 Jane That's why video tapes are helpful. And you can actually put that in somewhere. But linguists who've looked at lot of this kind of data always say that social interaction looks much more conflictful, when you start writing it down. And even when you annotate it with nonverbal. And I saw that early on in the data I was collecting including the data that had <me> in it.

383 FP Uh-hum. Uh-hum.

384 Jane And then I really that everyone has to come to that realization when they do this stuff. That you think, oh my God, you know, I'm wondering why we didn't start fighting.

385 FP ((laughs)).

386 Jane You know, because it seemed like a power play all over the place. But it actually is not the, ( ) it does change the phenomena.

387 FP Whatever the negative is, the positives of doing this and what you learn about yourself and what you learn about what you could be doing, I think it far out weigh. The advantages far out weigh the disadvantages. And reflecting and,

388 Jane That's why you're ideal for this kind of thing. I <really> appreciate it. ((laughs))

389 FP Someone once said to me, you'd be so brave to use a tape recorder in your classroom. They weren't even thinking that no one else would be listening to it but me. But that doesn't make sense to me. You have to be terribly sensitive and have a tremendous ego, I think for that to be the case. I want to say that this year, the children because we just did reports and they did their reports, I asked them to develop a list of criteria, for various things, and one is what we talked about last year. But what makes a good math thinker, and one of the things they put down was, take risks. I didn't guide them in any way.

390 Jane Ya, ya.

391 FP And the last one they put down was, a good math thinker never gives up. ((laughs)) And I looked at, I kept saying Albert Einstein. He wasn't, it wasn't that he was that much more brilliant, he <never> gave up. I said, just think, because we talked about him a little bit. And about he'd do something and he'd think he got it and then somebody would should him or he'd realize and he, so what did he do, he tried again. You know, and years and years and years, so I was so pleased with the both of those comments that came out. They do what makes sense. They put that in there. And a bunch of other things.

392 Jane There are many places in which you are quite, last year, nobody has told

		me,
393	FP	Absolutely, of course not. ((end of tape))

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