

**USING THE TUTORIAL APPROACH TO IMPROVE PHYSICS LEARNING FROM
INTRODUCTORY TO GRADUATE LEVEL**

by

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In this thesis, I discuss the development and evaluation of tutorials ranging from introductory to graduate level. Tutorials were developed based upon research on student difficulties in learning relevant concepts and findings of cognitive research. Tutorials are a valuable resource when used either in-class or as a self-study tool. They strive to help students develop a robust knowledge structure of relevant topics and improve their problem solving skills. I discuss the development of a tutorial on the Lock-in amplifier (LIA) for use as both an on-ramp to ease the transition of students entering into the research lab and to improve student understanding of the operation of the LIA for those already making use of this device. The effectiveness of this tutorial was evaluated using think aloud interviews with graduate students possessing a wide range of experience with the LIA and the findings were uniformly positive. I also describe the development and evaluation of a Quantum Interactive Learning Tutorial (QuILT) that focuses on quantum key distribution using two protocols for secure key distribution. One protocol used in the first part of the QuILT is administered to students working collaboratively in class while the second protocol used in the second part of the QuILT was administered as homework. Evaluation of student understanding of the two protocols used in this QuILT shows that it was effective at improving student understanding both immediately after working on the QuILT and

two months later. Finally, I discuss the development and evaluation of four web-based tutorials focusing on quantitative problem solving intended to aid introductory students in the learning of effective problem-solving heuristics while helping them learn physics concepts. Findings suggest that while these tutorials are effective when administered in one-on-one think-aloud interviews, this effectiveness is greatly diminished when students are asked to use the tutorials as a self-study tool with no supervision. In addition, the development and evaluation of four sets of scaffolded prequizzes for introductory physics on the same topics as the tutorials is discussed. These prequizzes are designed to mimic the structure of the web-based tutorials and can be implemented in the classroom.

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1.0 INTRODUCTION

Physics education research (PER) is a field dedicated to understanding how students learn physics, determine what factors influence their learning and identifying what pedagogical strategies are effective in helping students transition from their initial knowledge state to the final knowledge state aligned with the goals of the instruction [1]. Much research conducted in PER is informed by results from cognitive science, a field in which researchers have explored (among others) many factors related to novice-expert behavior and developed theoretical frameworks to describe how the learning process takes place. Utilizing these basic building blocks, physics education researchers have focused on physics teaching and learning and developed curricular modifications as well as tools designed to accomplish the desired change in the student's knowledge state [2-9] and enhance student learning.

Tutorials are one such tool developed in an attempt to improve student understanding and help them build a coherent knowledge structure of physics concepts. Tutorials are primarily based on established theoretical frameworks of learning developed by cognitive scientists which emphasize providing appropriate scaffolding based upon students' prior knowledge to help students develop a robust knowledge structure. Tutorials can be effective in helping students at all levels learn physics, from introductory to graduate level and even beyond. This is because, regardless of level, when students are struggling to understand a novel topic, well-structured

tutorials that build on their current knowledge can provide appropriate guidance and support and help them extend and repair their knowledge structure. Additionally, research based tutorials can be used outside of class as a self-study tool. This allows students to work at their own pace as opposed to a “one size fits all” approach. Since having a good knowledge structure of physics is closely tied to students’ ability to solve problems (both quantitative and qualitative), research-based tutorials can also enhance students’ problem solving skills.

1.1 PROBLEM SOLVING

Many definitions for problem solving have been proposed and adopted in the literature [10-14], which are similar and have superficial differences between them. One such definition states that problem solving is a goal-directed behavior that often includes setting subgoals to enable the application of operators (actions that will transform one problem state into another problem state) [14]. Other definitions express that for an activity to be considered problem solving, the situation presented (e.g., starting conditions, end goal) must be novel to the problem solver [15]. This second more stringent definition implies that while a typical introductory physics problem for an introductory level student is a problem, it cannot be considered a problem for an expert with experience solving these types of problems [16]. This does not necessarily mean that no introductory physics problem would require a physics expert to engage in problem solving. If the problem posed to an expert is not intuitive (i.e., the expert does not automatically know what steps to take to solve the problem), he will have to engage in problem solving even in the context of introductory physics content [17]. Thus, for our purposes, problem solving is defined not only

by a goal-directed behavior to move from the initial problem state to the goal state, but by a process in which a strategy is devised to move between these two states rather than simply using “compiled” knowledge [18-20].

1.1.1 Problem Space

One common interpretation of problem solving comes from the notion of problem space [12]. Problem space is defined as the collection of all possible series of states leading from the initial problem state into other problem states. Not all of these series of actions will efficiently lead to the goal state and in fact many of them may lead to dead ends which will not allow the problem solver to achieve the goal state. Therefore, in this context, problem solving is equivalent to identifying a series of actions which will help the problem solver transition from the initial problem state to the goal state.

Moving between problem states within the problem space is achieved by the application of operators. Operators are defined as actions that change the problem state from any given initial state to a new modified problem state. Operators can be as simple as walking from one end of the room to the other in order to solve a simple problem involving being displaced from one’s goal location or as complex as applying physics concepts formally to solve a problem involving displacement making use of the set of conditions in the problem statement. In either case, problem solving means using operators available to the problem solver to find a path through the problem space to reach the goal state.

1.1.2 Methods for Identifying and Using Operators to Solve a Problem

To expand upon the available options that the problem solver has while navigating the problem space to solve a problem, it is vital to determine effective means of expanding the number of operators that the problem solver has available. The three common methods are discovery, instruction and examples [14]. Discovery is a method of operator acquisition which is defined as the user finding a new operator on his/her own to solve a problem. Discovery is an important method for the development and growth of any field since it can provide means of opening up the problem space to previously unknown operators. While this is a vital tool for expanding the number of available operators in a field, it is one of the least commonly used means of introducing new operators to a student.

The remaining two methods for gaining operators, instruction and example, are often used hand in hand to introduce students to new operators [14]. Instruction is generally characterized by explicitly telling the problem solver what actions to take to advance the problem state. One example involves simply exposing the problem solver to the operator in the context of a specific problem. The relative effectiveness of each of these two methods may be argued. In one study involving students performing a substitution and algebraic manipulations on a short equation, an example was shown to be a superior method for improving student performance [21]. An interesting finding of this study is that there is a strong cumulative effect from exposure to both instruction and example (i.e., the effect of using both instruction and example is larger than the simple addition of the effects of using instruction only and example only). This study illustrates that effective instruction should utilize both of these two methods to maximize student learning [21].

1.1.3 Navigating the Problem Space

Once the initial state and the goal state have been determined and the problem solver understands and can apply the operators required to move between states, there are still questions about how the operators are selected and what path is chosen to navigate the problem space. Though numerous methods for operator selection exist, here I will discuss three that are commonly used by students. These methods illustrate issues students might encounter while engaging in problem solving as well as one valuable method for operator selection which focuses on enabling blocked operators.

Backup avoidance is one means of operator selection that biases the problem solver against any action that would undo a previous operator [14]. While backup avoidance does make sense, in many cases it can serve as more of a hindrance than a help to students. For it to be helpful while navigating the problem space, one must be traveling down the proper path to the goal state while solving a problem. If any mistake has been made along the way, it will make it more difficult or even impossible to reach the goal state, and this bias will prevent the student from undoing the offending operator, making the problem solving process more difficult if not impossible.

Another commonly used method of operator selection is difference reduction, which involves finding and using the operator that appears to most quickly reduces the difference between the current problem state and the goal state while solving problems [14]. This is often useful for solving simple problems but it has several shortcomings. Chief among them is that difference reduction does not always lead to the goal state. While a given operator (or set of operators) may be superficially moving the problem state closer to the goal state, it may also be

leading to a dead end. This situation is of particular concern when difference reduction is coupled with backup avoidance, because the combination of these two methods of operator selection can result in the problem solvers getting stuck in suboptimal states.

Means-ends analysis is a third, more complex method of operator selection. Means-ends analysis begins by identifying major differences between the goal state and the current problem state and identifying an operator that will reduce this difference while moving the problem state to a new state [12]. If the operator proposed to move to this new state is blocked (cannot be applied), the problem solver temporarily changes the goal state to enable this operator (e.g., finding net force on an object to enable the blocked operator of using Newton's second law to determine the object's acceleration), effectively finding a sub-operator within the main operator. After this operator is enabled and applied, the problem solver repeats this process, now identifying major differences between this new state and the original goal state. This process goes on until a path of enabled operators exists between the initial problem state and the original goal state.

1.1.4 Taught Bias for Selecting Operators

Overexposure to a specified operator (or series of operators) can introduce bias in selecting operators to use, better known as a set effect [14]. One such set effect referred to as Einstellung effect, or mechanization of thought, is well demonstrated by examining students' ability to complete a series of water jug problems [22]. In each of these problems, the students were given three jugs labeled A, B, and C with various volumes as well as an infinite source of water for filling jugs. They are instructed to find a method for measuring a given volume of water only by

filling a jug completely from the infinite source, transferring water from one jug to another until the second jug is full or by emptying any jug. The students were given 10 such problems, all of which, with the exception of problem 8, could be solved by filling jug B, filling jug C from jug B and emptying jug C twice, and finally filling jug A from jug B leaving the specified amount in jug B. This series of actions is described by the equation $B-2C-A$ (e.g., “measure 100 fluid ounces by using jugs of 21, 127, and 3 fluid ounces”, where 21, 127, and 3 correspond to A, B, C, respectively, can be solved by the method described above). This series of operators is the simplest way to solve the first five problems, and can also be used to solve problems 6, 7, 9 and 10. However, problems 7 and 9 also have the simpler solution of $A+C$ and problems 6 and 10 also have the simpler solution of $A-C$. Additionally, problem 8, which cannot be solved by $B-2C-A$ can be solved by a simpler series of operators $A-C$. When students were given all 10 of these problems, roughly 80% of students used $B-2C-A$ on problems 6, 7, 9 and 10 rather than using the simpler option that existed for each of those problems, and 64% of students failed to solve problem 8 using any method.

If instead, students were only given the last 5 problems (numbers 6-10), their solutions to these problems changed dramatically. For this control group, less than 1% of students used $B-2C-A$ on problems 6, 7, 9 and 10 and only 5% of students were unable to solve problem 8. This exemplifies that the bias introduced by working through 5 problems that had $B-2C-A$ as the simplest solution significantly affected the way in which they solved these problems and in one case this bias prevented more than half of the students from finding any method to solve the problem (which, from an expert perspective, could be solved by using a simpler application of relevant operators, $A-C$, instead of $B-2C-A$). This implies that in any pedagogical approach, it is

important to consider that the way in which students are introduced to a method of solving problems may have a strong impact on how they will approach future problems.

1.2 RESOURCES DRAWN FROM COGNITIVE SCIENCE

As noted earlier, cognitive science has informed PER by providing theoretical frameworks which describe how learning takes place and providing guidelines for improving instruction. Additionally, the study of cognitive science can provide insight into the cognitive mechanisms underlying many of the difficulties that students have while engaging in physics problem solving [23,24].

1.2.1 Memory

Problem solving is a cognitive process and as such is done by the human information processing system. The human information process system or memory can be divided into two broad categories, long term memory (LTM) and short term memory (STM) also known as working memory [25,26]. LTM is where all of the accrued knowledge over the course of one's life is stored. One important aspect of the LTM is that it can store a functionally unlimited amount of information. STM, or working memory, is where active tasks are performed while engaging in problem solving. In particular, while problem solving, STM receives information through sensory buffers (e.g., eyes, nose, hands, ears, etc.) as well as retrieves prior knowledge from LTM, and uses working memory "slots" to process information. Unlike LTM, STM has a finite

capacity and it is estimated that the STM can only hold 7 ± 2 distinct pieces of information or chunks [15].

1.2.2 Cognitive Load Theory

The limited amount of working memory while solving a physics problem results in one of the major difficulties that beginning students have in solving problems. Students are likely to experience cognitive overload if more than the number of slots available in the working memory are required at one time while solving a problem [27]. Experts in physics do not have the same difficulty because they have larger knowledge chunks based upon their expertise and several pieces of knowledge can be invoked simultaneously. Cognitive load can be reduced by the use of an outside means for storing some of the necessary information (e.g. by writing down data, drawing a diagram, etc.) [28]. The cognitive load can increase if the person has to repeatedly draw information back into their working memory from the outside source. An added difficulty is that students may not always recognize the relevant pieces of information which must be processed at a given time, and, as a result, can keep in their working memory information that is not necessarily helpful in solving the problem. Instructional strategies developed by physics education researchers often focus on providing appropriate scaffolding support to keep cognitive load manageable for students[29-32]. These strategies often help students build a good knowledge structure so that they recognize what is relevant while solving problems and do not process extraneous information which is not useful (thus, also helping to decrease cognitive load).

1.2.3 Chunking

As noted, one way cognitive load can be reduced as one gains experience in a particular domain is via chunking relevant information. Chunking involves combining related pieces of information so that they can be processed in one slot in working memory. Chunking of knowledge is demonstrated with studies conducted on experts and novices in the game of chess [26,33]. In these studies, a chess board indicative of a good game of chess was shown to both chess experts and novices. The board was then disassembled and they were asked to reassemble the board to the best of their ability. Then, the process was repeated except this time the board was set up with a random arrangement of pieces. The results showed that the experts were able to reproduce many more pieces on the board than the novices when it was based on a good game of chess, but there was no difference between the abilities of experts and novices to recreate the randomized board. These results suggest that a chess expert is able to recognize the relative positions of the pieces in a good chess game by chunking the positions of the pieces based on their relative ability to attack or defend each other and other relations between them. On the other hand, for the random board, experts did not know how to chunk the information.

1.3 KNOWLEDGE STRUCTURE AND BEHAVIOR OF EXPERTS AND NOVICES

1.3.1 Importance of Knowledge Structure in Problem Solving

The structure of knowledge stored in the LTM is central to the ability to successfully chunk information [30,34]. One major feature of the knowledge structure of experts is that it is hierarchical [18,35,36]. An expert in physics organizes his knowledge from the top down in a pyramid shape starting with the most fundamental principles on the top and moving down to concepts that are more specific to the various contexts [35]. Furthermore, numerous links between various “nodes” (concepts, principles, facts, etc.) in this knowledge structure signify connections between related pieces of information (e.g., acceleration is the rate of change velocity with time). This knowledge structure in a knowledge domain influences the way experts retrieve knowledge when engaged in problem solving: they start by identifying the general principles and concepts that are applicable and design a plan (consistent with those principles and concepts) which is specific to the problem situation. In addition, the well-connected knowledge structure of experts allows them to identify multiple paths to follow in the solution process and to reassess their solution plan if necessary.

On the other hand, a typical knowledge structure of a novice consists of pieces which are only loosely connected and lack global consistency. Therefore, the way novices approach problem solving is by focusing on aspects of the problem which look familiar and identifying which formulas can be used (i.e., matching given information with equations without developing an overall plan or performing a qualitative analysis). It is important to note that these

descriptions of novice and expert are the extremes of a continuum and in a given course, students' "expertise" can show significant variation [37].

The discrepancy between the problem solving strategies of experts and novices is shown by a study in which advanced physics students and introductory physics students were asked to categorize many problems into groups based on similarity of solution [35]. While experts were more likely to group problems based on physics principles (Conservation of Energy, Newton's 2nd law, etc.), novices commonly grouped problems according to surface features (inclined plane, pulley problem, etc.).

1.3.2 Stages of Skill Acquisition

One difference between the novices and experts is the type of knowledge they possess in a particular domain [38]. Declarative knowledge is defined as knowledge that must be recalled explicitly and is based on memorized facts. Procedural knowledge in physics relates to knowledge of how to apply a particular principle or concept in an appropriate situation. Novices in physics (e.g., beginning students) have less robust knowledge structure and while they may possess relevant declarative knowledge, they may not necessarily know if they are applicable in a given situation. Even if they know that this declarative knowledge is applicable in a given situation, they may not be able to instantiate that knowledge appropriately in the given situation. In particular, they may struggle with procedural knowledge beyond plug and chug approaches and may have difficulty in applying appropriate knowledge to diverse situations. On the other hand, experts are significantly more facile at recognizing when a particular knowledge is applicable and using procedural knowledge appropriately to solve a problem.

The type of knowledge that is used in problem solving is tied to the three stages of skill acquisition of the problem solver [39,40]. Fitts and Posner refer to these three stages as the cognitive stage, the associative stage and the autonomous stage. These stages are not disjointed, with all students going through the cognitive stage followed by the associative stage etc., but rather students tend to develop skills along these three stages with overlap between them. The first stage of skill acquisition (the cognitive stage) is characterized by the collection and memorization of facts that are relevant to the problem solving process. This is the stage in which the student develops a base of declarative knowledge. Next, in the associative stage, the student develops his/her understanding of the knowledge (e.g., procedures for applying the declarative knowledge in various contexts). In this stage, students detect errors in their understanding and strive to eliminate them as they work through problems. Additionally, associations between various elements required to successfully complete a task are strengthened through the student's experience. Finally, as the student moves from using primarily declarative knowledge to using both declarative and procedural knowledge, they transition into the autonomous stage. This stage is associated with the task of problem solving becoming more automated. However, it is important to note that the type of problem solving students engage in during the associative stage can greatly influence the types of problem solving strategies which become more automated. For instance, in the example of repeated practice using B-2C-A to solve the three jug problems discussed earlier, students were biased to using this B-2C-A strategy for problems which had simpler solutions and were unable to recognize a simple solution in a problem in which the B-2C-A strategy could not be applied. This example shows that unless the problem solving practice students engage in is designed to help them make associations between different concepts and understand the underlying principles involved in solving problems, they can use problem solving

strategies which bypass a need for deep reasoning or robust understanding of the concepts and principals involved.

1.3.3 Behavior Indicative of Expert Problem Solving

One behavior that is common among experts is that they plan their problem solving strategy beginning with the initial state provided and move towards the goal state [39]. This expert like problem solving method is different from the working “backwards” strategy which is common among novices. Working backwards strategy starts with a focus on the operator that achieves the end goal. If this operator cannot immediately be enacted given the initial problem state, the problem solver will look for a way to enable this operator repeatedly until the end operator is enabled. Working backwards can cause issues for novices due to the excessive cognitive load caused by holding a series of operators leading from the original problem state to the goal state in their working memory simultaneously. Experts avoid this excessive cognitive load by using their experience and hierarchical knowledge structure and start modifying the initial problem state appropriately to move towards the goal state [39].

1.4 FRAMEWORKS FOR LEARNING FROM COGNITIVE PSYCHOLOGY

In order to help students make a transition from novice like behavior to expert like behavior, several cognitive frameworks have been proposed to scaffold student learning. These

frameworks have a strong influence on the development of instructional strategies and tools developed by physics education researchers including the tutorials discussed in this thesis.

1.4.1 Theory of Conceptual Change and Optimal Mismatch

Piaget proposed a framework emphasizing “optimal mismatch” to help students reconcile newly acquired knowledge with a preexisting knowledge structure [40]. In the case where the new knowledge is consistent with the preexisting knowledge structure, the student will build connections between the consistent components of their knowledge structure and the new knowledge, leading to what Piaget referred to as assimilation. On the other hand, when students are faced with new knowledge which is inconsistent with their preexisting knowledge structure, they must modify their preexisting knowledge structure in order to accommodate the new knowledge. Optimal mismatch is one effective method for enabling accommodation. It is based on eliciting cognitive dissonance in students by exposing them to inconsistencies between their prior conceptions and new observations. Once students realize that their conceptions are inconsistent with the observation, they are in a state of disequilibrium. This disequilibrium can create an optimal situation for students to repair, organize and extend their knowledge structure. To aid in this, students can be provided with appropriate tasks that help them understand why their prior conceptions are not applicable to the new case. Then, students can be provided guidance and support to resolve inconsistencies between prior conceptions and new observations so that they may build a robust knowledge structure. Posner et al. also emphasized the Piagetian “optimal mismatch” approach in their theory of conceptual change [41].

1.4.2 Preparation for Future Learning: Including Innovation and Efficiency in Instructional Design

In their theoretical framework, Schwartz, Bransford and Sears posited that to facilitate transfer of learning from one situation to another and prepare students for future learning, two orthogonal dimensions, efficiency and innovation, are important [42]. While the authors do not insist on a particular interpretation, in the context of instructional tasks (e.g., problem solving and learning activities), innovation can correspond to how creative and cognitively demanding a task is for students. If a task is too innovative compared to their current knowledge, students can experience cognitive overload and find it very difficult to complete the task. In this situation, they may get frustrated and give up. In the same context (of instructional tasks), the other dimension, efficiency (which can be thought of as orthogonal to innovation), can be interpreted to mean how similar a task is to other tasks that the student has experience performing and therefore is indicative of how easy a given task will be for a student. Continuous application of efficient tasks can lead to specialized and non-transferable expertise and students may become “routine experts” [43]. Additionally, if a task is only efficient, students will experience low cognitive engagement, and may become disengaged and gain very little from the problem solving process. Schwartz, Bransford and Sears argued that both of these elements are vital for enabling transfer of learning and preparing students for future learning from the problem solving process. Therefore, instructional design should include both elements of innovation and efficiency in order to keep students engaged and allow them to develop their knowledge structure while at the same time, decrease cognitive load and keep them from becoming frustrated.

1.4.3 Vygotsky's Notion of Zone of Proximal Development (ZPD)

Another framework for scaffolding student learning, proposed by Vygotsky, is associated with designing instruction consistent with a student's "zone of proximal development" (ZPD) [44]. For a particular student, this ZPD is a dynamical concept defined as the difference between what the student can accomplish on his/her own and what the student can accomplish with the aid of available teaching resources (teachers, peers, teaching interventions, etc.) In practice, application of this framework begins with understanding the initial knowledge state of the student and selecting a goal state that is within his/her initial ZPD. Educational interventions that are geared to stretch the student's knowledge state to this goal state can then be implemented. These interventions provide scaffolding that is rooted in the student's initial knowledge structure and is designed to promote the student's ability to work on this new level. In this way, the instructional intervention will lead to advancement of the student's knowledge structure and stretching of their ZPD. Continued application of this process can allow the student's knowledge state to be advanced to any state desired by deliberately and continuously building onto this dynamic ZPD.

1.4.4 Cognitive Apprenticeship Model

Another framework which informs the structure of the tutorials described in this dissertation is the cognitive apprenticeship model [45]. In this model, there are three important components: "modeling", "coaching and scaffolding", and "weaning or fading". Modeling refers to the educator demonstrating the skills required for optimal performance. Then, coaching and scaffolding refers to students practicing these skills on their own while receiving guidance and

support. The final aspect of this model of learning, fading or weaning, includes gradually reducing the support provided to students in order to help them develop self-reliance. These three stages of the cognitive apprenticeship model, modeling, coaching and scaffolding, and fading or weaning should be included in any instructional design to help students learn.

The cognitive apprenticeship model can be valuable in informing instruction designed to help students learn physics and develop effective problem solving strategies to solve physics problems. Coaching provides a good opportunity to help students practice the problem solving process in a way that is consistent with the process used by an expert. By exposing students to these good problem solving heuristics and providing helpful feedback as they solve problems, it is possible to guide them to adopt a more expert like approach to problem solving. Additionally, fading allows students the opportunity to build a robust knowledge structure as they increasingly rely on their own knowledge of how to solve problems [24].

1.5 KNOWLEDGE RETENTION AND RETRIEVAL

Each of the theoretical learning frameworks discussed earlier is designed to improve knowledge retention and retrieval at later times because in order for instruction to be effective, students must be able to retain the knowledge in LTM and retrieve it when the need to apply it in a particular situation arises. One common means of improving retention of knowledge is through practicing. One study examined the effects of practice on students who attempted to memorize a list of words with varying amounts of practice [14]. The study suggests that while memory decays as a

power law over time, the initial amount retained by students, and therefore the amount retained at all subsequent times was dependent on the practice students engaged in.

Beyond just the volume of practice, the type and quality of practice engaged in during problem solving is vital to improving knowledge retention and retrieval in appropriate situations. Practice is only useful in this regard when appropriate and useful skills are rehearsed during problem solving. Repetitive rote exercises are often not helpful and in some cases can be detrimental to learning [22]. Additionally, practice that does not adequately emphasize useful skills may fail to improve retention and retrieval of knowledge [22]. If the student does not see the merit of engaging in an intervention due to its lack of emphasis on skills that the student views as vital to their progress, they can become disengaged. One more possible pitfall of practice is that without guidance students may practice a given task incorrectly. By engaging in incorrect practice, students will develop a flawed knowledge structure and/or skills which will be harmful for developing a robust knowledge structure and learning effective problem solving skills.

Additionally, retention of knowledge can be impacted by the ways in which students were exposed to the knowledge [14]. In one study, students showed improved retention of a given list of words when they were taught these words by both a human and a computer over rather than exclusively by a human or exclusively by a computer. It is possible that students who encounter the same physics concepts in a variety of contexts (e.g., lecture, lab, recitation, online tutorials, etc.) can show improved retention and can use the concept appropriately when needed at subsequent times.

One important aspect of a learning intervention is ensuring that retrieval of knowledge is effectively enabled for students while solving problems. The failure of a student to invoke

relevant knowledge during problem solving when the relevant knowledge exists is most commonly attributed to the loss of access to the appropriate retrieval cues rather than to the loss of this knowledge from the student's memory. There are many strategies that are shown to enhance student's retrieval of knowledge [14]. Knowledge retrieval is enhanced if either the contexts or conditions under which the material is learned are the same as those (or are included in those) in which the student retrieves the knowledge. Also, in the case in which students have to generate the concept themselves rather than being told about it, knowledge retrieval for the generated concepts is enhanced later. Additionally, if the student properly stores the conditions under which the knowledge is applicable during the learning process, the student's ability to retrieve this knowledge will be improved. Since helping students build a robust knowledge structure and learn useful skills is fundamental to the research-based tutorials discussed in this thesis, they strive to enhance knowledge retrieval while solving problems.

1.6 RESEARCH-BASED TUTORIALS

This thesis focuses on developing and assessing research-based tutorials for introductory physics, upper-level undergraduate quantum mechanics and for physics graduate students who are involved in laboratory research. These tutorials are developed based upon the findings of cognitive research and take into account the common difficulties that students have in learning relevant concepts. In addition, many physics professors were consulted throughout the development of the tutorials. Having students work on research-based tutorials is one means of helping them engage in appropriate practice while ensuring that ineffective practice is avoided.

The tutorials discussed in this thesis build on students' prior knowledge and strive to help them build a robust knowledge structure of relevant concepts and improve students' problem solving and reasoning skills. They focus on providing optimal mismatch while staying in the ZPD of the students. They have elements of both innovation and efficiency built into them. The tutorial on the Lock-in Amplifier discussed in this thesis is envisioned mainly as a self-study tool. The introductory and quantum mechanics tutorials can be used as an effective supplement to traditional instruction similar to how the tutorials by University of Washington PER group are used in the recitations which supplement lectures [9]. However, they can also be used as self-study tools.

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2.0 IMPROVING STUDENTS' UNDERSTANDING OF LOCK-IN AMPLIFIERS

2.1 INTRODUCTION

Graduate students are one of the most under addressed groups in Physics Education Research. While they represent the culmination of many years of directed study within the field of physics, very few tools have been developed with the express intent of helping these students with their difficulties at the highest tiers of learning physics. This is especially true for students once they have moved beyond the classroom and into their roles as researchers. While these students have provided sufficient evidence of learning physics successfully by this stage in their academic career, they can still benefit from research-based approaches to learning physics. To provide an on-ramp and aid these students' entry into the lab setting, I developed a research based "OnRamp" tutorial focused on a versatile device, the lock-in amplifier (LIA).

The LIA is an instrument used extensively in lab research, especially in condensed matter physics [1-5]. However, many students who use this instrument for their research have only a limited understanding of the operation of the LIA. Often, graduate students use the LIA as a black box to find the amplitude of a signal at a given frequency without understanding the instrument's internal workings. Improper or inefficient use of the LIA, and misinterpretation of data obtained from them, are unfortunately quite common. Additionally, lack of understanding

can result in the student's inability to troubleshoot and modify the experimental setup when faced with anomalous results.

Computer and web-based learning tools are becoming increasingly common to aid in learning across many science and engineering fields [6-15]. These tools must be developed using a research-based approach to ensure that they will be effective and will suit both the level and the prior experience of the students they are intended to be used by [16-18]. I investigated the common difficulties that graduate students have with lock-in amplifiers in their experimental research by interviewing professors who oversee students who commonly make use of this device as well as by conducting think aloud interviews [19-21] with students who have experience with the LIA. Based on these difficulties, I developed the initial version of the OnRamp tutorial on the LIA to ease the transition into research of those who are just beginning to work in the lab setting, as well as to provide a firmer foundation for those who already use LIAs in their research. The LIA tutorial focuses on helping students build a robust understanding of the fundamental operation of a LIA, and helps students develop an intuition for many of the possible situations that they may encounter in their experiments. Ultimately, the goal of this tutorial is to help students integrate conceptual and quantitative understanding such that students will not use the LIA as a black box but will instead be able to interpret the LIA's output and troubleshoot as needed.

Many graduate students make use of the LIA in conjunction with an optical "chopper" or some type of modulator, e.g., an amplitude modulator in a laser. In this instance, many of the more complex behaviors of a LIA are bypassed, leading the student to believe (incorrectly) that they are qualified to use a LIA in other contexts. But even with a "chopping" experiment, the choices of chopper frequency and location can greatly affect the result. Specifically, the choice

of chopping frequency, and its proximity to noise sources (e.g., 50 Hz or 60 Hz “line” noise) can greatly affect the quality of the data obtained. Strategies for reducing noise also depend on where the noise enters into the signal stream [3]. Having a robust understanding of the LIA is therefore critical to making decisions regarding experimental design.

Our goal in developing this research-based tutorial was to develop tools that can instill an intuitive understanding of the basics of the LIA functions, so that students who use LIAs in their research understand more deeply how the input parameters affect the output. By merging conceptual and mathematical aspects of the instrument, the tutorial strives to help students learn the relationship between the input parameters and expected outputs so that they are able to troubleshoot unexpected outputs in their lab work.

In the following sections, I will begin with a description of an idealized LIA in which I will go into some details about the integration of the mathematical and conceptual aspects for a robust understanding of the device. This is followed by the details of the development of the tutorial including an investigation of student difficulties and iterative modifications of the preliminary version of the tutorial. I then discuss the structure of the tutorial including its evaluation tools and the associated simulations followed by a summary of student difficulties found via investigations in which faculty members and students were interviewed using think-aloud interviews. Finally, I discuss the results of the pretest and posttest to gauge the effectiveness of the tutorial.

2.2 THE IDEAL LOCK-IN AMPLIFIER

Throughout this chapter, as in the tutorial, I will treat the LIA as an idealized version of the instrument. Here, I assume that the signal of interest is centered on a frequency f_S which is present in the input signal. In our case, it will not necessarily be a pure frequency since the amplitude can change and amplitude modulation leads to sidebands that surround the central frequency. In the case in which there is no amplitude modulation introduced into the signal, I will treat this frequency as a pure frequency (a sinusoidal waveform consisting of a single frequency). Throughout this tutorial I make use of both idealized pure frequencies and input signals which are undergoing amplitude modulation to ensure that students will gain experience with both treatments of the LIA. To separate the signal of interest from unwanted noise, a reference signal is defined. The reference signal is selected to have a unit amplitude (for convenience). The (idealized) single-frequency input signal is first pre-amplified by a factor g , to give $V_I = gA_S \cos(2\pi f_S t + \varphi)$. This amplified signal is then multiplied (or “mixed”) by a reference signal $V_{RX} = \cos(2\pi f_R t)$ to form the “unfiltered” x-channel output of the LIA: $V_{MX}(t) = V_I(t)V_{RX}(t)$. Here, φ is the phase of the input signal of frequency f_S with respect to the reference signal, and A_S is the amplitude of the input signal with frequency f_S . For the creation of the unfiltered y-channel output of the LIA, the reference signal is shifted in phase by 90° before being multiplied by the amplified input signal. This results in the unfiltered y-channel: $V_{MY}(t) = V_I(t)V_{RY}(t)$, where $V_{RY} = -\sin(2\pi f_R t)$. The purpose of generating these two output signals based on two orthogonal reference signals is to ensure that the amplitude, A_S , can be measured regardless of the phase, φ , of the input signal (allowing an accurate measurement to be made of both the amplitude and phase of the frequency f_S).

The most common use of LIAs in the lab is to measure small signals in the presence of large background noise. This is accomplished by setting the reference frequency equal to the frequency which the experimentalist wants to analyze in the input signal (that is to say, $f_R = f_S$) though this is not the only case treated in this tutorial. For the case in which multiple frequencies are present in the input signal, each input signal was treated independently exactly as described above. The resultant unfiltered x-channel and y-channel outputs are determined by summing the outputs from each of these frequencies in the input signal.

To understand the effect of this multiplication, I rely on two trigonometric identities,

$$\cos(a) \cos(b) = \frac{1}{2} [\cos(a + b) + \cos(a - b)] \text{ and}$$

$$\cos(a) \sin(b) = \frac{1}{2} [\sin(a + b) - \sin(a - b)].$$

Application of these identities reveals:

$$V_{MX} = V_I V_{RX} = \frac{1}{2} g A_S [\cos(2\pi(f_S - f_R)t + \varphi) + \cos(2\pi(f_S + f_R)t + \varphi)] \text{ and}$$

$$V_{MY} = V_I V_{RY} = \frac{1}{2} g A_S [\sin(2\pi(f_S - f_R)t + \varphi) - \sin(2\pi(f_S + f_R)t + \varphi)].$$

In the most typical case experienced in a lab setting $f_S - f_R \ll f_R$ and $f_S + f_R \approx 2f_R$ and in the ideal case $f_S = f_R$, which results in $f_S - f_R = 0$ and $f_S + f_R = 2f_R$. The most typical case results in the unfiltered x-channel and y-channel outputs,

$$V_{MX} = V_I V_{RX} = \frac{1}{2} g A_S [\cos(\varphi) + \cos(2\pi(2f_R)t + \varphi)] \text{ and}$$

$$V_{MY} = V_I V_{RY} = \frac{1}{2} g A_S [\sin(\varphi) - \sin(2\pi(2f_R)t + \varphi)].$$

I note that in this ideal case, both V_{MX} and V_{MY} have one component with no frequency (a DC component) and a second component with a frequency of $2f_R$.

Finally, V_{MX} and V_{MY} are each fed through a low-pass filter with a “time constant” $\tau = f_0^{-1}$ (where f_0 is the cutoff frequency). The “rolloff” can be set to one of four values typical

of a lab low pass filter (6 dB/octave, 12 dB/octave, 18 dB/octave, and 24 dB/octave). The values selected for both the time constant and the rolloff should be chosen carefully based upon the nature of the experiment. Terms in the unfiltered output of frequency $f \ll f_0$ are passed with unity gain, while signals with $f \gg f_0$ are attenuated as f^{-n} for $6n$ dB/octave filters (e.g., $\propto f^{-2}$ for 12 dB/octave filters). This filtering yields the two output signals V_{OutX} and V_{OutY} . In the idealized version of our most typical case ($f_R = f_S$), the time constant will be selected such that the low-pass filter will remove the second-harmonic ($2f_R$) term from both V_{MX} and V_{MY} resulting in a time-independent output signal. Removing the second-harmonic results in the output signals providing information about the magnitude and phase of the input signal: $V_{OutX} = \frac{1}{2}gA_S \cos \varphi$, and $V_{OutY} = \frac{1}{2}gA_S \sin \varphi$, or $A_S = \frac{2}{g}\sqrt{V_{OutX}^2 + V_{OutY}^2}$, and $\varphi = \tan^{-1}(V_{OutY}/V_{OutX})$.

While the LIA is most commonly used in the ideal case ($f_R = f_S$ with the second-harmonic attenuated) with no other coherent frequency in the input signal, this situation is not the only one that can commonly occur in a lab setting. Many times there is more than one frequency present in the input signal. This can be caused by coherent noise (i.e., the 60 Hz frequency introduced by electrical lines) and can result in the output signal failing to measure the targeted frequency's amplitude. Additionally, the reference signal frequency could be selected so that $f_R \neq f_S$. In this case, there are several possible output signals that could result depending upon the time constant selected. Even when $f_R = f_S$, the output signal will not always be that observed in the ideal case. If the time constant is incorrectly selected, the $2f_R$ signal can pass through the low-pass filter (either partially or fully) causing the output signal to have a time varying component. While these cases are not necessarily the goal of a typical lab use of the LIA, they are cases that may be experienced by an experimentalist (as I found in the interviews with faculty

members during the initial development of the tutorial) and are therefore covered throughout the tutorial because experimentalists should know what can cause an unexpected output signal to allow them to better troubleshoot.

2.3 METHODOLOGY

I individually interviewed five physics professors at the University of Pittsburgh (Pitt) who conduct research in condensed matter physics and commonly work with graduate students who use LIAs in their research. A typical interview time was 60-90 minutes, during which each professor was also asked to articulate what he expected his students to know about LIAs, what was the goal of the professor's experiment(s) utilizing LIAs and how LIAs are useful in the broader framework of research. Additionally, I had an opportunity to use an actual LIA in a research lab. Using the feedback from professors as a guide along with a cognitive task analysis of the underlying knowledge involved in the operation of a LIA, I developed a preliminary tutorial along with a pretest and posttest (to be given before and after the tutorial, respectively). Alongside the development of the initial version of the tutorial, I oversaw the development and refinement of a LIA simulation (see Fig. 2.1) which was built into the structure of the tutorial (development of the simulations code was done by Dr. Jeremy Levy and an undergraduate student Alexandre Gauthier) [22]. The simulation was built with the intention of allowing students to develop an understanding of the LIA by giving them the opportunity to experience the device first hand.

I then interviewed graduate students while they made use of the most up to date version of the tutorial using a think-aloud protocol to better understand their difficulties and to fine-tune the tutorial as well as its associated pretest, posttest and simulation. In these semi-structured interviews, students were asked to think aloud while they worked through the pretest, tutorial and the posttest. The interviewer tried not to disturb students' thought processes while they answered the questions except to encourage them to keep talking if they became quiet for a long time. Later, the interviewer asked students for clarification of points they had not made clear earlier in

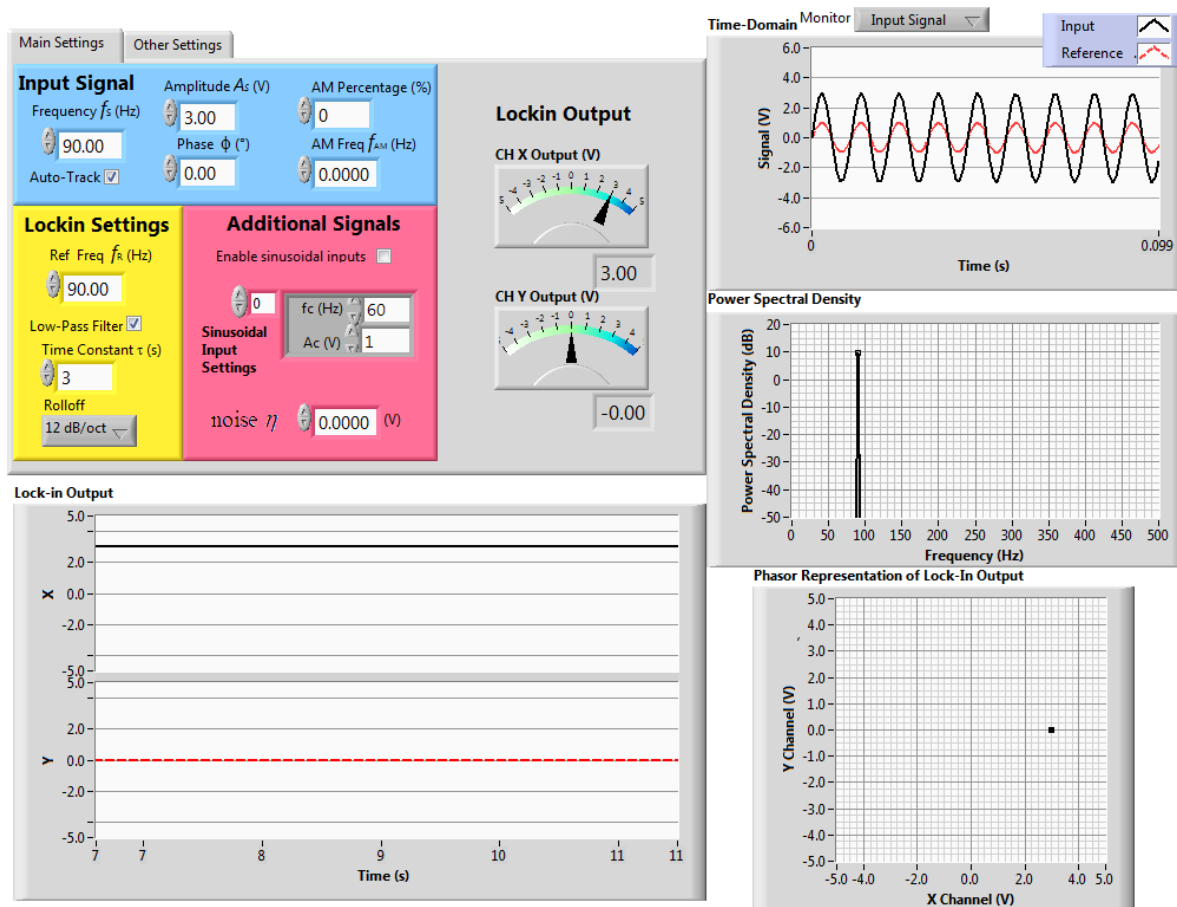


Figure 2.1. Screen capture of the simulation's interface displaying a typical set of input parameters and output signals.

order to understand their thought processes better. Some of these questions were planned out ahead of time while others were emergent queries based upon a particular student's responses to questions during an interview.

The tutorial (along with the pretest and posttest) was iteratively refined over 30 times,

Table 2.1. Summary of the grading rubrics used for the pretest and posttest questions discussed [23].

Rubric 1: Correct answer should have no frequency and zero DC offset present in the output signal.					
Non-zero frequency present?	Yes	Magnitude of either DC component (x,y) is non-zero?	Yes	0 points	
			No	5 points	
	No	Magnitude of either DC component (x,y) is non-zero?	Yes	0 points	
			No	10 points	
Rubric 2: Correct answer should have no frequency and non-zero DC offset present in the output signal.					
Non-zero frequency present?	Yes	0 points			
	No	Magnitudes of DC components (x,y) are?	Correct	10 points	
			Incorrect but non-zero	5 points	
			Zero	0 points	
Rubric 3: Correct answer should have a non-zero frequency and zero DC offset present in the output signal.					
Non-zero frequency present?	Yes	Magnitudes of both DC components (x,y) are zero?	Yes	5 points	+2.5 points for correct frequency +2.5 points for correct amplitude
			No	2.5 points	
	No	0 points			
Rubric 4: Correct answer should have a non-zero frequency and non-zero DC offset present in the output signal.					
Non-zero frequency present?	Yes	Magnitudes of both DC components (x,y) are correct?	Yes	5 points	+2.5 points for correct frequency +2.5 points for correct amplitude
			No	2.5 points	
	No	Magnitudes of both DC components (x,y) are correct?	Yes	2.5 points	
			No	0 points	
Rubric 5: Correct answer should have one frequency in the input signal.					
Frequency Correct?	Yes	Amplitude Correct?	Yes	10 points	Divide score by 2 if a second input signal is given that would result in a different output signal
			No	5 points	
	No	Amplitude Correct?	Yes	5 points	
			No	0 points	
Rubric 6: Correct answer should have one frequency in the input signal with a phase angle.					
Frequency Correct?	Yes	Amplitude Correct?	Yes	5 points	+5 points for correct phase angle Divide score by 2 if a second input signal is given that would result in a different output signal
			No	2.5 points	
	No	Amplitude Correct?	Yes	2.5 points	
			No	0 points	
Rubric 7: Correct answer should have two frequencies in the input signal.					
Frequency Correct?	Yes	Amplitude Correct?	Yes	5 points	Use the rubric for each of the two frequencies that should appear in the input signal and sum the points
			No	2.5 points	
	No	Amplitude Correct?	Yes	2.5 points	
			No	0 points	

based upon feedback from graduate students and professors. While professors worked through the different versions of the tutorial and associated simulations at their convenience and provided feedback afterward in one-on-one meetings, I used a think-aloud protocol when graduate students worked on any version of the tutorial. Based upon feedback from professors and graduate students, I refined the tutorial. As the tutorial and its supplementary material underwent this process of revision and fine-tuning, it was administered to 26 physics graduate students who had not been involved in the development phase of the tutorial. These students have a wide range of prior experience with the lock in amplifier. Students at the low end of this range have no firsthand experience with operating the LIA but have a basic grasp of what a LIA is and know how it is commonly used in the lab. The high end of this range is comprised of students who have extensive experience with the LIA and either concurrently used a LIA for their research or have made extensive use of one in the past. Overall, five students with no firsthand experience with the LIA and 21 students who had made use of the LIA in their research worked on the tutorial. These students were administered the pretest and posttest before and after the tutorial, respectively, in order to assess its effectiveness.

To adequately evaluate the effectiveness of this tutorial despite the modifications that were made to the pretest and posttest throughout the course of the development of these tools, a set of generalized grading rubrics were developed. To this end, four researchers jointly deliberated a series of seven rubrics that could be utilized in the scoring of all questions in all versions of the pretest and posttest. The agreement between the researchers was better than 90% on the rubrics presented in Table 2.1 for scoring performance on pretest and posttest questions.

2.4 THE TUTORIAL STRUCTURE

The interview begins with a pretest to assess student's initial understanding of the LIA before working on the tutorial. The questions on the pretest ask about what should happen in an ideal LIA as in the simulation that was developed by us alongside the tutorial. For each of the questions, the students are provided with a mockup of the simulation's interface with either the output omitted or certain components of the input signal removed and they are asked to provide the missing information. Students are provided supplementary information to aid them as they work through the pretest. The supplementary information provided contains a description of the simulation interface showing students how each of the controls modify the input signal, reference

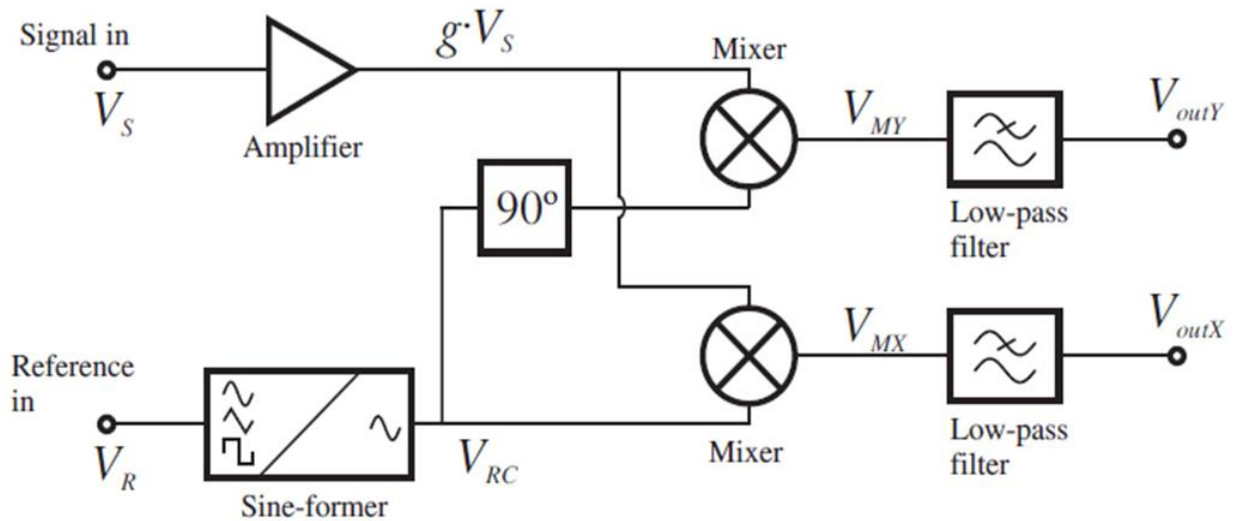


Figure 2.2. Diagram of the dual-channel LIA provided to students within the tutorial as well as with the pretest and posttest. This diagram was originally drafted for a lab manual for LIAs by the Norwegian University of Science and Technology [18].

signal and low-pass filter. In addition, students are provided with several graphs that diagram the dual channel LIA (Fig. 2.2) and describe the effect of the low pass filter (Fig. 2.3) and a description of the effect of amplitude modulation in this simulation. After students complete the pretest, they proceed to the tutorial. The tutorial begins with a brief comparative analysis of several other measurement devices (the voltmeter, oscilloscope and spectral analyzer) and the LIA. This serves to motivate the value of the LIA for making accurate measurements of the amplitude of a specific frequency within a given signal. This is followed by a guided approach to help students develop a good comprehension of the dual channel LIA. This section begins with a short narrated video followed by a series of slides that provide a detailed explanation of the diagram of the dual channel LIA (Fig. 2.2), including the basic function of the primary components. Then, students learn to integrate the conceptual and mathematical treatments of the

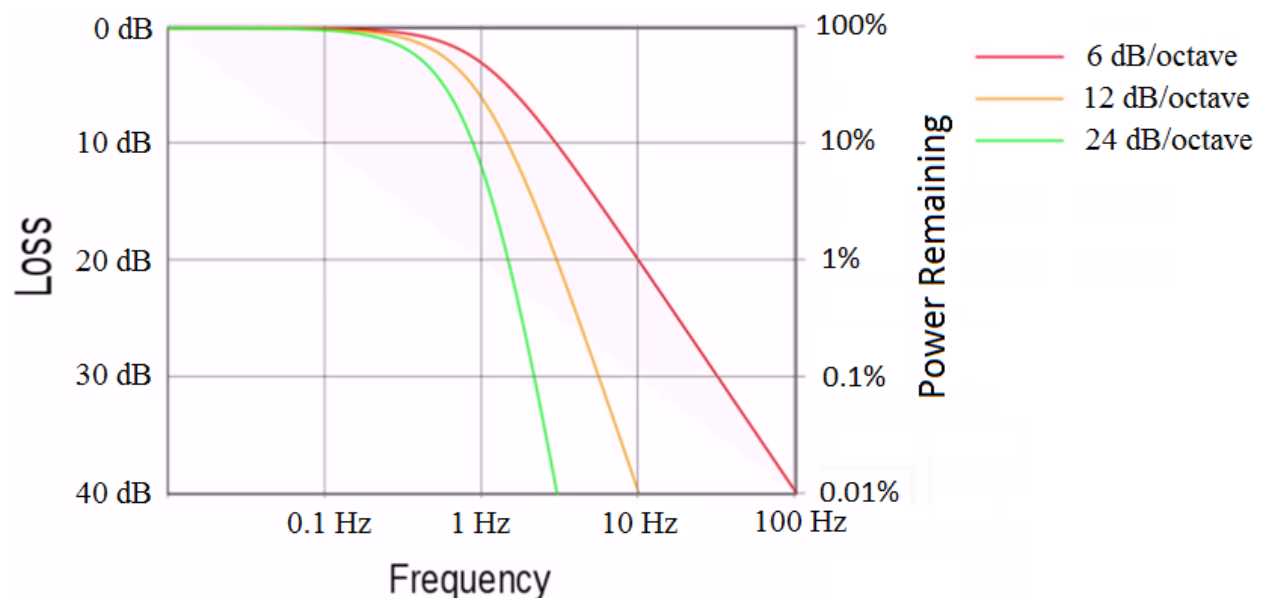


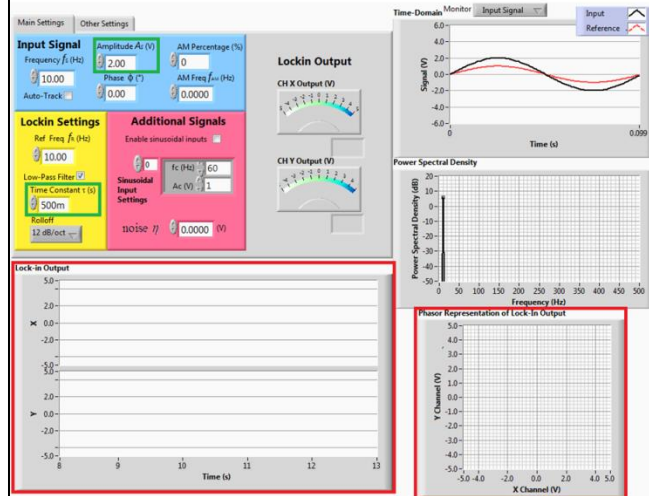
Figure 2.3. Graph of the amplitude loss of a signal with respect to the signal frequency after it passes through a low-pass filter with a time constant of 1 s (provided to students as part of the tutorial).

effect of the mixer in the LIA. The students are guided to find the mathematical expressions for V_{MX} and V_{MY} both in the general case (when $f_S \neq f_R$) and in the most typical case (when $f_S = f_R$) and make sense of these findings. Particular attention is paid to emphasizing which frequencies will be in the unfiltered output signals for each of these two cases to ensure that students internalize both the sum and difference frequency. This focus on integration of conceptual and mathematical treatments of the LIA allows students to accurately predict the output or input parameters of the LIA even in situations that they haven't experienced, after which it allows them to check their predictions throughout the tutorial. Many students, even those with extensive experience using the LIA, realized that they had not integrated the conceptual and mathematical treatments of the LIA's operation and had little understanding of the LIA outside of the most commonly experienced case. This is followed by the part that helps students learn about the role of the low pass filter. This part begins with a diagram of the amplitude loss with respect to frequency for the low-pass filter for 6 dB/octave, 12 dB/octave and 24 dB/octave rolloff as shown in Figure 2.3. While examining this diagram, students are asked to note two rules of thumb for the 12 dB/octave rolloff which is used throughout the remainder of the tutorial. Students are asked to assume that for all frequencies $f \leq 0.1 \times \tau^{-1}$ there is practically no attenuation of the signal as it passes through the filter while for all frequencies $f \geq 10 \times \tau^{-1}$ the amplitude is 99% attenuated. Students learn these rules of thumb alongside a more conceptual analysis of the low-pass filter and its relation to the time constant. The importance of this section became apparent throughout the course of student interviews. In particular, many students had a poor understanding of how the time constant affects the output signal before working on the tutorial. One student who used LIA frequently expressed this common difficulty with the time constant when he said "Ya, time constant is something that I don't always think about" Most

of the students interviewed either completely ignored the time constant or expressed that they did not know how to properly take it into account during the pretest.

After the basic treatment of the low-pass filter, students work on the rest of the tutorial and make use of the simulation to verify whether their predictions about what should happen in a particular situation are correct and to reconcile the differences between their predictions and observations. The simulation allows the students to manipulate all of the settings commonly found on a LIA as well as to modify the characteristics of a simulated input signal. The LIA settings that the student is able to manipulate include the reference frequency, the time constant and the rolloff of the low-pass filter. The simulation allows for a considerable degree of control over the input signal. First, the user can specify the frequency and amplitude of the primary input frequency. Additionally, the simulation allows several adjustments to be made to the input signal. One of these is the phase of the input signal with respect to the reference signal. A sinusoidal amplitude modulation can also be introduced to the input signal by specifying the frequency of the modulation and the percentage of the initial amplitude that the primary input frequencies amplitude will change by. A secondary signal can also be added to the input signal to simulate sources of coherent noise and the amplitude and frequency of this second signal can be specified. Finally, a white noise can be introduced to the input signal to demonstrate the effectiveness of the LIA at measuring the amplitude of a specific frequency despite the presence of white noise. The rolloff of the low pass filter can be set to 6 dB/octave, 12 dB/octave, 18 dB/octave or 24 dB/octave by use of a pull-down menu while all other characteristics of the LIA and input signal are able to be changed to any value using a text box. The output of the LIA simulation is provided to the user in three formats to ensure that students can internalize the

Simulation 2



- Sketch your predictions for what the output should be in the incomplete sections (outlined in red).
- Set the simulation's inputs to those shown in the "Main Settings" on the left.
- Check your prediction making use of the simulation.

[Explanation](#)

[Mathematical Explanation](#)



Simulation 2 Purpose

This simulation demonstrates a very similar case to that shown in simulation 1. This simulation serves to demonstrate one of the most common uses of the lock-in amplifier which is to measure the amplitude of a specific frequency in the input signal by setting $f_S = f_R$.



Figure 2.4. Example of a typical simulation question (left) and the associated "purpose" slide (right).

concepts involved in a representation that is most familiar and pedagogically effective for a student.

The simulation component is introduced to students by helping them understand the interface used in the simulation (in a similar manner to the pretest supplement). Additionally, students are walked through an example of how to predict the output signal of the LIA in a given situation. The students then work through a series of problems designed to be used with the simulation similar to the one in Figure 2.4. For each problem, students are asked to predict the output signal when provided with the input signal and LIA settings. After they have sketched their predictions, they input the parameters into the simulation and compare the output signal that the simulation provides with their prediction. For each prediction and simulation, both a mathematical and a conceptual explanation of the output signal based on the input parameters are provided (e.g., as shown in Figure 2.5), to aid students if they cannot reconcile their predicted

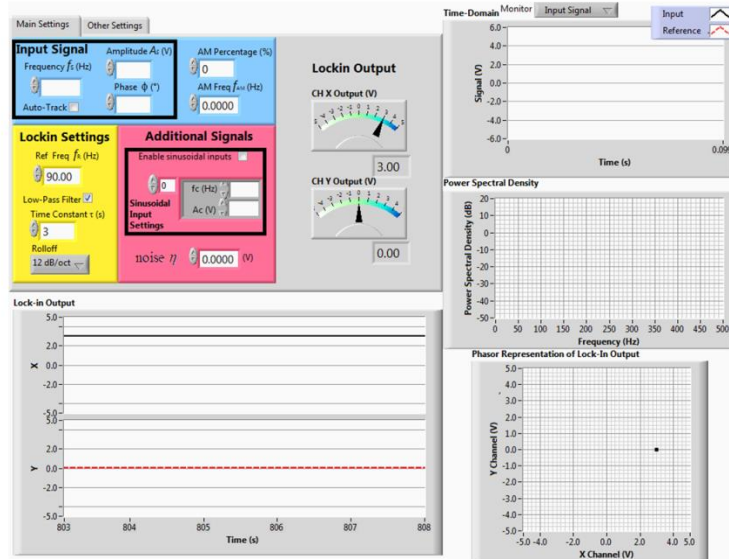
Explanation	Mathematical Explanation
<p>Since $f_s = f_R$, the multiplication of the input signal and reference signal yields both a constant and a 20 Hz component to the mixer output signal to be fed to the low-pass filter. Since $\tau = 0.5 \text{ sec}$, the cutoff frequency is 2 Hz and the 20 Hz signal is almost completely attenuated after passing through the low-pass filter. This yields the constant output signal of amplitude 2 V that you observe in the simulation.</p> <p>Return</p>	$V_{MX} = 2 \text{ V} [1 + \cos(2\pi(20 \text{ Hz})t)]$ $V_{MY} = -2 \text{ V} [\sin(2\pi(20 \text{ Hz})t)]$ <p><i>unattenuated frequency</i> $< 0.1 \times (0.5 \text{ s})^{-1} = 0.2 \text{ Hz}$</p> <p><i>attenuated frequency</i> $> 10 \times (0.5 \text{ s})^{-1} = 20 \text{ Hz}$</p> $V_{OutX} = 2 \text{ V}$ $V_{OutY} = 0$ <p>Return</p>

Figure 2.5. Example of an explanation provided to students for a simulation if they need guidance (Simulation 2 in this case). The “Explanation” (left) features a conceptual description of the problem’s solution while the “Mathematical Explanation” (right) features the mathematics required to solve the problem.

output with the output shown on the screen in the simulation. The conceptual explanations provided to scaffold their learning describe the reasons for a particular type of output for a given input without explicitly working out the mathematics associated with the mixer. This scaffolding allows students an opportunity to improve their conceptual understanding if the situation presented in the problem is challenging. The mathematical explanation provides the mathematical approach used to determine the low-pass filter signal, the low-pass filter rules of thumb and the filtered output signal in a given situation allowing students to check their conceptual understanding with the mathematical underpinnings and integrate qualitative and quantitative understanding. Student interviews suggest that indeed these mathematical explanations alongside conceptual explanations aid students in integrating conceptual and quantitative understanding of the LIA.

Following these questions in which students are provided input parameters and asked to predict the corresponding output, students work on a second set of questions that ask them to identify characteristics of either the input signal or other inputs in the LIA's settings by examining the provided output signal as shown in Figure 2.6. The students are asked these types of questions either in a free-response format where they have to predict the input signal or asked to answer a multiple choice question in which one of the choices may be correct for the input parameters corresponding to a particular output provided. To scaffold student learning, guidance and support are provided throughout. In particular, several hints in the form of guiding statements or questions are posed to help students for each of these questions as in Figure 2.7. One type of hint commonly provided to students as a first hint takes the form of a hypothetical discussion between two students who are attempting to solve the problem. The hint provides two

1) The lock-in amplifier's output is as shown below. Fill out the blank portions of the input parameters (outlined in black) that would make this output possible. (You need not fill out the Additional Signals section if only one input signal is necessary.)

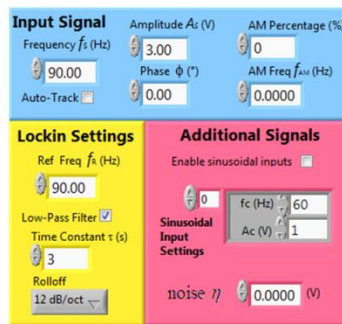


Hint 1

Hint 2

Answer

Answer



Since $f_S = f_R$, both a constant component and a $2f_S$ component are created by the mixer. The low pass filter with a time constant of 3 s heavily attenuates the $2f_S$ component. This yields the output shown.

Figure 2.6. An example of one of the tutorial questions which asks students to predict the input signal characteristics (left) and the answer to said question (right).

Hint 1

The following discussion takes place between two students attempting to work out this problem.

Student 1: The only frequency in the input signal that can create a DC component in the output is one that equals the reference frequency. If this condition isn't met, the output shown cannot occur.

Student 2: I don't agree. The signal could have a frequency close enough to the reference frequency such that it is not filtered out of the output. This case could also yield a DC output like that shown here.

With which student do you agree?

Hint 2

Which of these three sets of input parameters could result in the output shown?

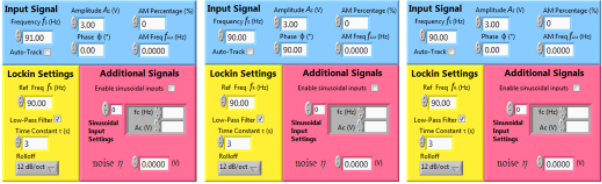


Figure 2.7. Examples of two types of hint commonly given to students as they answered the questions in the guided approach to learning used in the LIA tutorial.

viewpoints for students to consider on how to approach the problem, one of which is often correct and the other incorrect. A more heavily scaffolded second hint commonly provides three possible answers with some differences (dealing with common difficulties) for students to consider. This allows students to think about how each variation in the input signal results in a different output signal. A lightly scaffolded hint is often followed by a more heavily scaffolded hint for students who may need it. The initial scaffolding support is less direct (light scaffold) than the later support (heavy scaffold) to allow students to develop self-reliance (wean them off the scaffolding support) since I expect them to become less dependent upon the hints as they work through the tutorial [23]. In the guided approach to learning that the tutorial uses, after students answer each question to the best of their ability, the correct answers are provided along with an explanation for why each answer must be correct if the students wish to check their predictions or have difficulty with why their predictions do not match the observations in the simulations (e.g., see Figure 2.6). These guiding questions keep students actively engaged

throughout and are designed carefully based upon the common difficulties and a cognitive task analysis of how to help students build a coherent understanding of LIA. They give students an opportunity to practice many situations with different input parameters and understand how different input parameters impact the output signal of the LIA. The guided sequence is designed to provide students ample opportunity to build a solid conceptual understanding of the LIA by integrating conceptual and quantitative understanding. Interviewed students appreciated this guided approach to learning. For example, one interviewed student who initially needed more scaffolding from the hints provided but was later able to predict what should happen in a given situation without relying on the hint said “I think after doing more of these [tutorial problems] the intuition is getting better.”

The posttest is structured in the same manner as the pretest. The questions focus on similar concepts related to the LIA as in the pretest [see Appendix]. Students are again allowed access to the same supplementary material for the posttest as in the pretest. To ensure that any change in the score between the pretest and posttest was solely the result of the tutorial and not any difference in the specific questions asked in each test, the pretest and posttest were switched for roughly half of the students.

2.5 STUDENT DIFFICULTIES

Throughout student interviews, many difficulties with the LIA were identified when students made predictions about either the output or the input signal of the LIA in a given situation. As noted earlier, while five interviewed students had no firsthand experience with a LIA (they only

had a brief exposure to the LIA via discussions with their research advisor and/or group members who had used LIAs), others had used the device in the lab setting and several students had used it for several years as part of their research work. Interviews suggest that even those with extensive experience using the LIA often had shallow conceptual understanding of the LIA's operation and many of the interviewed students who had used it extensively for many years were only capable of predicting the input or output for the simplest of cases. The difficulties found in our investigation have roots in a lack of coherent understanding of the fundamentals of the LIA. For example, students often had little understanding of what the mixers in a LIA do. Even in the cases that are most common in the lab setting, interviewed students often demonstrated a superficial understanding of the LIA. The range and prevalence of these difficulties demonstrate that students are often using the LIA as a black box to make measurements at a targeted frequency without fully understanding how the device provides the output in a given situation. These students will, therefore, have difficulty in troubleshooting if anomalous outputs are observed as confirmed by some of the faculty members who were interviewed initially during the development of the tutorial. Below, I describe the common difficulties mainly observed among interviewed students before they worked on the tutorial or as they worked through the early parts of the tutorial (before they had gained much from the tutorial).

2.5.1 Difficulty in calculating and using the cutoff frequency:

Interviews suggest that one aspect of the LIA that is often overlooked by graduate students is the effect of the low pass filter on the output signal. For example, interviewed

students had great difficulty with the fact that the frequencies that will make it into the output signal can be estimated by making use of the time constant, τ . The time constant is inversely proportional to the cutoff frequency, which is the frequency at which the low-pass filter will cause half of the power in the signal to be lost. Frequencies higher than this cutoff frequency will experience increased signal attenuation.

Many students ignored the time constant throughout the pretest with some commenting that they didn't understand how to account for the time constant when predicting the output signal. During a think aloud interview, one student who had worked extensively with LIAs in the lab setting noted after working through the tutorial that "the most helpful part [of the tutorial] is [that] it explains the [low-pass] filter". The student went on to explain that he didn't know how to set the time constant and had little understanding of its purpose in the LIA despite having worked with the device in his research. Another student said "It is clear to me that when I use the lock-in, I always have some problems with having the right time constant. Somehow, if I don't have the right time constant, the [output] signal will be unstable." Other interviews also suggest that while students that are making use of the LIA understand that the time constant of the low-pass filter is important and its value must be carefully selected, they have little understanding of how to properly set the time constant to make a measurement. The confusion about the effect of the time constant (mainly in the pretest) resulted in students not being able to properly predict what frequencies will make it through the low-pass filter unattenuated and thus what frequencies will appear in the output signal.

2.5.2 Difficulty with the case in which $f_S = f_R$ and the $2f_R$ signal is attenuated:

Since the lock in amplifier is most commonly used to measure the amplitude of a targeted frequency present in an input signal, the case in which the signal and reference frequency are equal and the time constant is high enough to attenuate the $2f_S$ signal is the most familiar to most of those who have used the LIA in a lab setting. Despite this being the most common case, some students still had difficulty in predicting the output signal from a LIA under these conditions. Before working on the tutorial, these students often claimed that the LIA's output should have a frequency equal to that of the signal frequency and that the LIA is providing an output similar to that displayed by an oscilloscope measuring the input signal. This difficulty demonstrates a misunderstanding about the purpose of the LIA in these common situations. The most common purpose of the LIA is to measure the amplitude of the input signal at the targeted frequency (determined by the reference frequency) by multiplying the input signal by the reference frequency resulting in a DC output. Therefore, a non-zero frequency cannot be present in the output if $f_S = f_R$ and the $2f_S$ signal is attenuated.

2.5.3 Difficulty with the case in which $f_S \neq f_R$, the $f_r - f_s$ signal is unattenuated and the $f_r + f_s$ signal is fully attenuated:

The difficulty in understanding the operation of the lock in amplifier in this situation affects their ability to make predictions about other more complex LIA setups. In the case in which $f_S \neq f_R$, students demonstrated several difficulties rooted in a lack of understanding of the basic operation of the LIA. One common difficulty among students when confronted with this

situation involved students claiming that the LIA would not show an output for any frequency in the input signal that does not equal the reference frequency. These students commonly claimed that both the x and y output of the LIA would be zero. It is worth noting that this difficulty was often due to students commonly disregarding the time constant provided in the problem statement and instead claiming that the LIA will attenuate all output resulting from frequencies that are not equal to the reference frequency. One student demonstrated this difficulty when he said “So there’s a frequency difference between them (the reference and signal frequency)... So I expect zero for this [output] although the frequencies are close”. It appears from this quote that the student is considering the effect of the reference and signal frequency being close to one another. Despite this consideration, the student selected zero as the output for both channels. Additionally, a few minutes later in the interview the student concluded “...the entire point of the lock-in is that it should pick out only the frequency... only the component of the same frequency as the reference”, suggesting that he is being misled about the operation of the LIA in general by his understanding of the device’s most common usage.

Another common difficulty among students is that many thought that the LIA will yield a DC output even under the condition $f_S \neq f_R$. These students provided an output that would have been correct if the signal and reference frequencies had been equal to each other rather than an output with a frequency of $|f_S - f_R|$ present in it with no attenuation from the low-pass filter. Interviews suggest that this difficulty is at least partly due to their lab experience with the LIA primarily consisting of cases in which $f_S = f_R$. Since some students only make use of the LIA as a measurement device under the condition $f_S = f_R$ and they have no experience with situations that cause the LIA’s output to be anything but a DC signal, a lack of conceptual understanding about the workings of the LIA can cause them to incorrectly conclude that the output signal from

a LIA must always be a DC signal. Interviews suggest that most students, even those with extensive experience with the LIA, used it as a black box rather than developing a conceptual understanding of the operation of this device before making use of it.

2.5.4 Difficulty with the case in which f_S and f_R are out of phase ($\varphi \neq 0$):

The most common difficulty that students had with non-zero phase involves cases in which $f_S \neq f_R$, the $f_r - f_s$ signal is unattenuated and the $f_r + f_s$ signal is fully attenuated. Students commonly combined relevant factors for solving this problem situation correctly with factors that are used to solve the case in which $f_S = f_R$ and $\varphi \neq 0$. Most commonly, this resulted in a time varying output with different amplitudes for the x-channel and y-channel outputs. Students would determine the amplitude of the x-channel to be $A_S \cos(\varphi)$ and the amplitude of the y-channel to be $A_S \sin(\varphi)$. Additionally, students showed weaker performance on problems in which $f_S = f_R$ and $\varphi \neq 0$.

2.5.5 Difficulty with the case in which $f_S = f_R$ and the $2f_S$ signal is unattenuated:

Another situation that caused considerable difficulty is the case in which $f_S = f_R$ and the $2f_S$ signal appears in the output. The most common student response involves a DC output without a $2f_S$ signal. The difficulty in answering this type of question stems from two sources that have already been mentioned earlier. First, students often make use of the LIA as a black box without a conceptual understanding for the most common situation that a LIA is used for (observing a DC output if a sufficiently high time constant is chosen). Second, students'

difficulty in taking into account the time constant and how it affects the attenuation of signals in the output often led them to claim that the low pass filter will remove all frequencies in the output signal.

In addition to these two common difficulties, several students had an even more fundamental difficulty with the operation of the LIA in this situation. Interviews suggest that many students lacked a coherent understanding of the internal operation of the LIA and how the mixing of the input signal and reference signal results in frequencies of $|f_S - f_R|$ and $|f_S + f_R|$ in the output signal. Most of the students interviewed mentioned the $|f_S - f_R|$ frequency which resulted in a DC output in this case but failed to even mention the existence of the $|f_S + f_R|$ frequency which would result in the $2f_S$ signal in this case. Additionally, in the pretest, students seldom made use of the relevant relations involving the mixing of the input and reference signals in the mixer to gain insight into what output they should expect for the questions posed. It was clear from interviews that most students (including those with several years of experience using the LIA) had little conceptual understanding of the LIA and have not taken the time to integrate conceptual and quantitative aspects of the LIA required to predict the output signal. One interviewed student said: “I never realized that I didn’t actually calculate these things [he did not realize that there is a mathematical procedure that can be used to make sense of the output voltage for a given input parameter set]”. These students (despite having made use of the LIA for varying periods of time) have often had no training (other than that which was strictly necessary to deal with the most common case).

2.5.6 Difficulty with amplitude modulation of the input signal:

Another difficulty during the think aloud interviews became apparent in cases in which an amplitude modulation to the primary input frequency is introduced. This amplitude modulation treated in this tutorial involves the amplitude of the input signal varying with time such that $A(t) = A_S + \frac{P_{AM}}{100} A_S \sin(2\pi f_{AM} t)$, where P_{AM} is the percentage of the initial amplitude that the input signal's amplitude is changed by over time and f_{AM} is the frequency of the amplitude modulation. The most common difficulty was claiming that the amplitude modulation would affect both the x-channel and y-channel output signal even if the amplitude only affects the output in one of the two channels. Interviews suggest that this difficulty is due a lack of conceptual understanding of the LIA and resulted from a lack of experience with amplitude modulation and confusion with other cases that introduce frequencies into the output signal that students had experience with. Also, for all other cases in the tutorial that result in non-zero frequencies in the output signal of the LIA, the frequency appears in both channels of the output with equal amplitude. These experiences impacted student expectations about the possible output signal of the LIA in the case with amplitude modulation of the input signal. Moreover, students also had difficulty determining how the low-pass filter affects the amplitude modulation. Despite the amplitude modulation generating a frequency that is passed through the low-pass filter in the same manner as any other frequency, several students either claimed that amplitude modulation should not be affected by the low-pass filter or completely ignored the effects of the low-pass filter on this signal.

2.5.7 Difficulty with multiple frequencies present in the input signal:

One aspect of the LIA that practically all students had difficulty with was the case in which multiple frequencies are present in the input signal. Many students reacted to this situation as though they had never considered such a possibility. Although the difficulty with this case was common to all of the students interviewed, their responses to this problem on the pre-test were varied when students were encouraged to try to predict what should happen in this situation based upon what they knew. Most commonly, students ignored one of the two frequencies and continued to treat the other frequency as though it was the only one in the input signal. For example, one student stated “So, since I don’t really know how to incorporate the changing (points to the additional signal section on the simulation interface) into it (the output signal), I’m just going to stick with the amplitude and phase (points to the input signal section on the simulation interface)”. Thus, when the student was faced with a situation that he had no prior experience with, he opted to ignore the source of his difficulty instead of finding a way to incorporate it into the output signal.

2.6 STUDENT PERFORMANCE ON PRETEST AND POSTTEST

The student difficulties discussed were primarily identified via student interactions with the pretest as well as the tutorial. As noted earlier, to evaluate the effectiveness of the tutorial in addressing these difficulties, a pretest and posttest were given to all students who worked on the tutorial while talking aloud. Table 2.2 shows the pretest and posttest data in which the average

Table 2.2. Summary of the average pretest and posttest scores separated into three groups (students with no firsthand experience using a LIA, students with prior firsthand experience using a LIA, and all students regardless of their prior experience).

	Number of Students	Average Pretest Score	Pretest Standard Deviation	Average Posttest Score	Posttest Standard Deviation
Students with firsthand experience	21	41.1%	22.3%	86.1%	12.1%
Students without firsthand experience	5	43.0%	25.2%	90.5%	6.5%
All Students	26	41.4%	22.9%	86.9%	11.3%

cumulative score increases from roughly 40% before working on the tutorial to over 85% after working on it. Also, Table 2.2 shows that students who had never used the LIA and were only exposed to the workings of the LIA via discussions with their research-advisor or other group members performed about the same on both the pre-test and post-test compared to those who had firsthand experience with a LIA sometimes extended over several years. The similar performance suggests that firsthand experience with the LIA does not equip students to be able

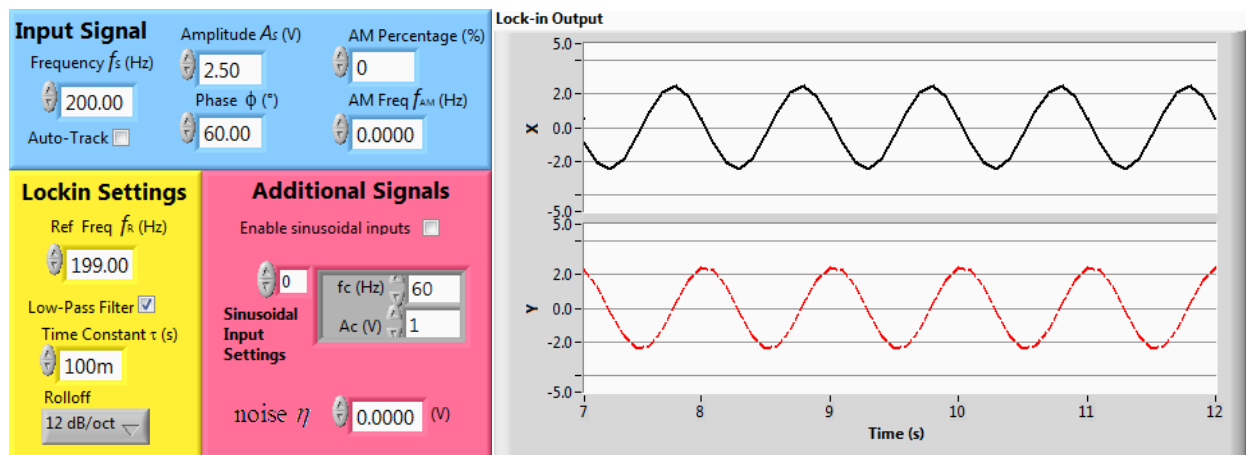


Figure 2.8. Screen capture of the input parameters (left) and the correct output signal (right) for a Rubric 3 pretest question.

to predict the output signal given the input signal or to determine the input signal given the output signal and students who have used the device extensively are no more prepared to troubleshoot issues with the LIA than those with no firsthand experience.

The pretest and posttest questions were consistent with the goals of the LIA tutorial. In particular, the LIA tutorial focuses on addressing student difficulties with six potential configurations of LIA settings and the pretest and posttest questions students answered reflected these situations. Below, each of these cases will be discussed along with an examination of the tutorial's effectiveness in addressing student difficulties related to each case. The pretest and posttest were both expanded throughout the time period in which student interviews were conducted to include additional questions focused on cases 1, 2, 3 and 6 (which demonstrated the situations most likely to occur in a lab setting). This resulted in multiple instances of these cases on each pretest and posttest in later versions of the pretest and posttest.

Table 2.3. Summary of the average score, standard deviation and number of instances of each of these cases present in both the pretest and posttest. Number of instances is the total number of problems demonstrating each case given to all students on either the pretest or posttest.

	Number of Instances in Pretest	Average Pretest Score	Pretest Standard Deviation	Number of Instances in Posttest	Average Posttest Score	Posttest Standard Deviation	p-value
Case 1	42	77.4%	39.6%	41	100%	0.0%	<0.001
Case 2	41	29.9%	37.5%	41	92.7%	19.3%	<0.001
Case 3	44	58.2%	40.8%	38	90.1%	22.5%	<0.001
Case 4	26	35.6%	35.4%	26	73.1%	30.2%	<0.001
Case 5	25	28.0%	34.2%	26	84.6%	32.6%	<0.001
Case 6	41	27.4%	37.4%	37	81.1%	32.6%	<0.001

2.6.1 (Case 1) $f_R = f_S$ with the $2f_R$ signal fully attenuated and $\varphi = 0$:

This case is the most commonly encountered lab situation in which students measure the amplitude of the frequency f_S in the input signal and generally correctly set the reference frequency equal to f_S and the time constant high enough to filter out the $2f_R$ signal. Despite this case being one that should be familiar for students who commonly use this device, some students had difficulty with these types of problems. Examining Table 2.3, I note that the average score across all pretest questions focusing on this case is roughly 75%. This less than perfect performance primarily results from some students claiming that the LIA would provide an output with frequency f_S in this case as discussed earlier. While student performance on questions involving this case is reasonable, their performance improves after they work on the tutorial with every student answering every question on the posttest related to this case correctly. Some of the students found these questions so easy, when encountered on the posttest, that they doubted whether questions related to the LIA could be this easy. For example, one of them commented “Did I solve it [the posttest problem demonstrating case 1] wrong? Why is it [the problem] so easy?” This improvement demonstrates the effectiveness of the tutorial in ensuring that all students understand how the LIA operates in this simple case.

2.6.2 (Case 2) $f_R \neq f_S$, the $f_R - f_S$ signal is unattenuated and the $f_R + f_S$ signal is fully attenuated:

This case is less commonly encountered in the lab setting. This situation could arise in a lab situation if the student meant to measure the amplitude of frequency f_S in the input signal by

setting $f_R = f_S$ and incorrectly sets f_R to a value near f_S . This situation also serves to illustrate the importance of integrating conceptual and quantitative understanding to understand the connection between the input and output in the LIA's operation. To correctly answer questions that relate to this case, students have to understand that since $f_R \neq f_S$, the $f_R - f_S$ frequency will not be zero as well as correctly understand the effect of the low-pass filter for the time constant and rolloff designated for the problem. Many of the students interviewed had no prior experience with this case and since they used the LIA as a black box, they did not know how to answer these questions correctly. For example, one student said "So since I don't really know how to incorporate the change [different values of f_R and f_S compared to the previous question] into it I'm going to stick to [predicting] amplitude and phase." Examining Table 2.3, I note a distinct lack of student understanding on the pretest with the average student score of roughly 30% across all pretest questions focusing on this case. This weak performance is caused by students lacking the in depth understanding of the LIA's operation required to treat this case. However, treatment of the LIA as a black box and extremely limited (if not nonexistent) exposure to cases outside of the most common case leaves students ill equipped to troubleshoot this situation if it were to arise by inadvertent setting of the parameters as noted above. After working through the tutorial student performance improved to an average of roughly 90% on these questions.

2.6.3 (Case 3) $f_R = f_S$ and the $2f_R$ signal is attenuated and $\varphi \neq 0$:

This case is very similar to case 1 (the most commonly faced lab situation) with the only difference being that the phase angle is no longer set to zero. This case could arise in the lab setting when the frequency of interest, f_S , is out of phase with the reference frequency. This case

also served to illustrate the importance of having both an x-channel and y-channel output. This case shows that the amplitude of the frequency, f_S , cannot, in general, be measured in either the x-channel or y-channel without the reference frequency being in phase with f_S (in which case only one channel is required). Student performance was not as good on these problems on the pretest when compared to case 1 with a score of roughly 60% on average. This lower performance is likely due to lack of conceptual understanding about the LIA and due to students having less experience with this case than with the case in which f_S is in phase with f_R (which is often either accomplished by cleverly selecting the source of the reference frequency or by adjusting the phase knob commonly present on LIAs). The average improved to roughly 90% after students worked on the tutorial.

2.6.4 (Case 4) $f_R = f_S$ with the $2f_R$ signal unattenuated and $\varphi = 0$:

This case can arise in the lab when students are measuring a frequency, f_S , that is very low and the time constant is incorrectly set too low resulting in the $2f_R$ signal being passed through the low-pass filter. This case is used to demonstrate the importance of correctly accounting for the effect of the low-pass filter and selecting a time constant that will properly limit the frequencies that can pass. Many students had practically no experience with this case before the tutorial and since they lacked conceptual understanding of the LIA to reason about new situations, it resulted in a very low average pretest score of roughly 35%. Since this case results in an output signal with both a DC offset (from the $f_R - f_S = 0$ component) and a non-zero frequency (from the $f_R + f_S = 2f_R$ component) and was therefore graded using rubric 4 from Table 2.1 students earned points for correctly identifying each of these two components.

Most of the points earned by students on the pretest, for this case, were the result of correctly identifying the DC offset with very few students correctly identifying the non-zero frequency. The tutorial was able to improve student understanding such that the average posttest score was roughly 75%. The relatively limited improvement in this case is partly due to the fact that this situation is different from all others in that in all other cases, the larger of the two frequencies generated by the multiplier is fully attenuated while passing through the low pass filter. This leads to the common difficulty of disregarding the effects of the $f_R + f_S$ term on the output signal.

2.6.5 (Case 5) Amplitude modulation present in the input signal:

The effects of a time varying input signal on the output signal of the LIA is another case students learn in the tutorial. Amplitude modulation in the form of a sinusoidal time dependent variance in the amplitude of the input signal is used in this tutorial to help students understand the effects of a particular time dependent amplitude on the output of the LIA. Additionally, this case demonstrates how the value of the time constant can affect the time required to take an accurate measurement. Since the output signal changes more slowly as the time constant increases, the time required for a stable output to be achieved in the most common case (case 1) will also increase as the time constant increases. It is useful for students to understand while making use of the LIA to make a large number of measurements to ensure that they select a time constant that is high enough to filter out any undesired time varying components but is low enough to allow quick measurements to be made (valuable when performing a large number of measurements). The average score on all pretest questions focusing on this case is roughly 30%.

This weak performance demonstrates students' limited understanding of the low-pass filter as well as how a change (of a specific type) in the input signal amplitude affects the output signal. Students showed a marked improvement in performance after completing the tutorial with the average score being roughly 85% on posttest questions related to this case.

2.6.6 (Case 6) Two frequencies present in the input signal:

The final case related to several pretest and posttest questions involves the presence of two frequencies in the input signal with the $|f_R - f_S|$ component of each signal passing through the low-pass filter virtually unattenuated while both $f_R + f_S$ signals are completely attenuated. This case demonstrates how the LIA processes multiple frequencies that are simultaneously present in the input signal to generate a single output signal. It also acts as an example of the potential lab scenario in which the experimentalist is attempting to measure the amplitude of one frequency (as in the most common case) but incorrectly sets the time constant too low, allowing part of the signal from a coherent noise source with a frequency close to the value of the reference frequency into the output. The average student score on the pretest for all problems that involve this case is 25%. This low average score illustrates students' inexperience with both cases involving two frequencies in the input signal and cases with frequencies in the input signal that do not equal the reference signal (similar to case 2). After students work on the tutorial, they obtained an average score of roughly 80% across all posttest questions that involve this case.

After examining the pretest and posttest results of the six cases covered holistically I note that the tutorial appears to be effective in improving student understanding in a variety of situations ranging from the most commonly experienced lab setting to a variety of other less commonly

faced cases that students may potentially encounter while making use of this device. In addition to improving student understanding of these potential cases, the tutorial provides scaffolding to improve students' general understanding of the operation of the LIA by integrating conceptual and quantitative understanding. I note that instead of grouping the pretest and posttest questions based on the potential experimental setups, the questions can also be grouped into two categories based upon what form the answer takes. In particular, the pretest and posttest questions either asked students to sketch the output signal that should result from a given set of input parameters as in Figure 2.9 or asked them to predict the characteristics of the input signal based on the LIA's settings and an output signal diagram provided as in Figure 2.10.

Questions in which students were asked to determine and sketch the output signal were included to examine students' ability to check the output signal to ensure that the input parameters are as they predicted in the lab. If the students are able to correctly answer problems of this type, they should be able to confirm that the output signal is what it should be, based on their assumed input parameters, when performing a measurement in the lab and if not, they can interpret it to imply that the input signal has characteristics that they did not take into account in their prediction of the output signal. This ability to predict the form of the output signal before making a measurement can ensure that students will be less likely to record anomalous outputs without making sense of them conceptually due to having an incorrect or incomplete understanding of how the LIA generates its output signal. In this investigation, students initially showed a limited ability to predict the output signal for the array of cases given in the pretest. Therefore, the average score was under 40% across all pretest questions that required that students sketch the output signal. This average improves to over 85% after students made use of the tutorial.

1) Sketch the output characteristics on the displays outlined in red, based on the given input settings.

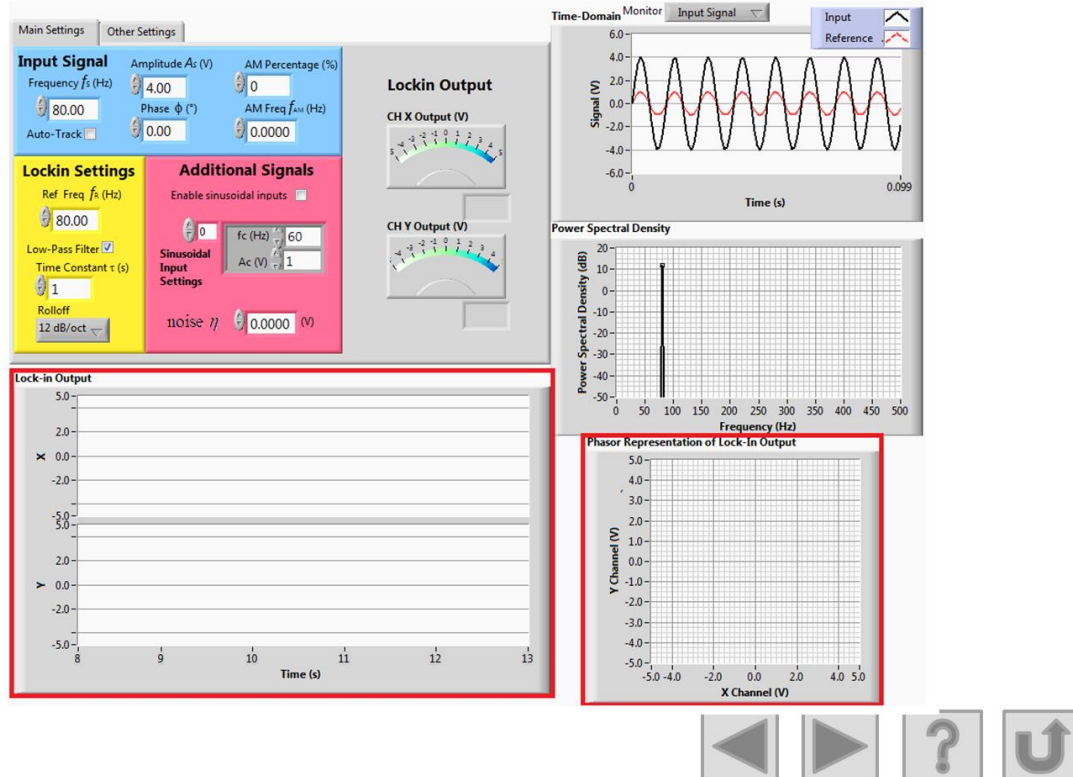


Figure 2.9. An example of a typical pretest or posttest problem in which the students are asked to sketch a prediction for the output signal for the given set of input parameters.

The remaining questions, which ask students to predict the characteristics of the input signal based on the output signal and the LIA's settings, measure students' ability to examine an output signal and determine what the input signal must be. This ability is valuable to students in the lab setting when they are faced with an output signal that differs from what they expected. Student performance before working on the tutorial is roughly 40% across all pretest questions that asked the student to predict the input settings and it improved to over 80% after working

7) The lock-in amplifier's output is as shown below. Fill out the blank portions of the input parameters (outlined in black) that would make this output possible. (You need not fill out the Additional Signals section if only one input signal is necessary.)

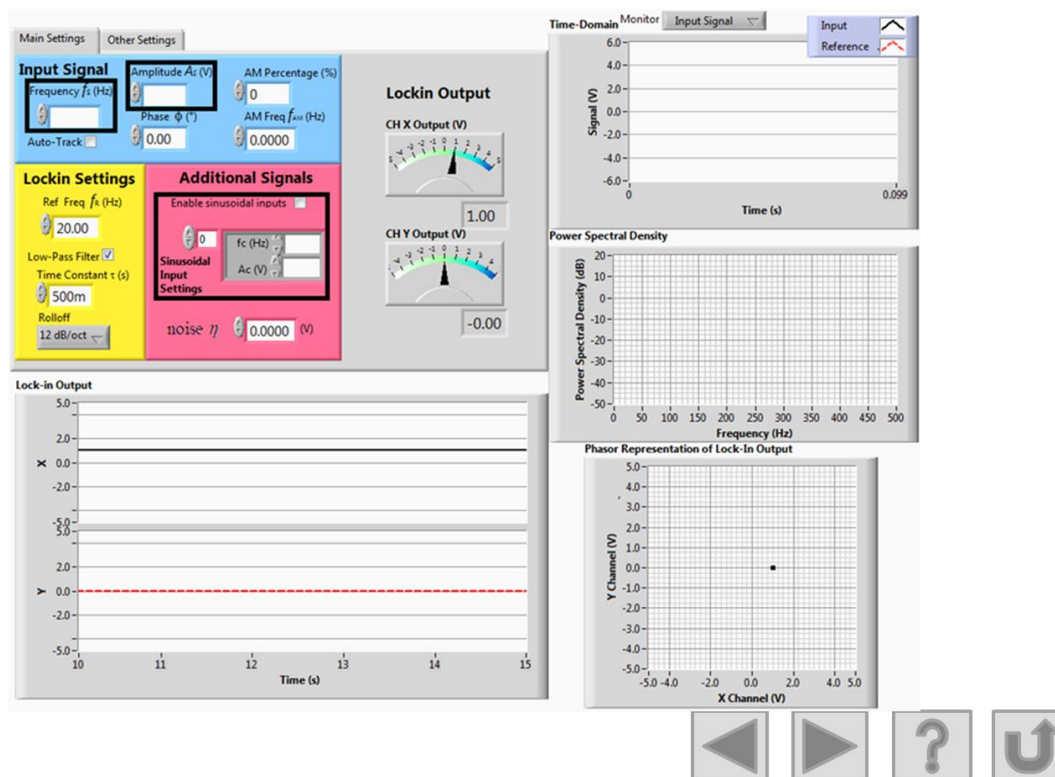


Figure 2.10. An example of a typical pretest or posttest problem in which the students are asked to sketch a prediction for the input parameters for the given output signal.

Table 2.4. Summary of the average score, standard deviation and number of instances in which students predicted characteristics of input or output of the LIA and the total averaged score on all questions of these two types on the pretest and posttest.

	Number of Instances in Pretest	Average Pretest Score	Pretest Standard Deviation	Number of Instances in Posttest	Average Posttest Score	Posttest Standard Deviation	p-value
Predicting Output	135	38.1%	42.6%	151	87.7%	27.7%	<0.001
Predicting Input	55	42.5%	42.1%	55	80.9%	30.5%	<0.001
Total score	190	39.4%	42.5%	206	85.9%	28.7%	<0.001

through the tutorial. Improved student performance on either of these two types of problems should correlate with increased ability to troubleshoot difficulties that may arise when making use of the LIA.

2.7 SUMMARY

I find that physics graduate students who use LIAs for their experimental research have many common difficulties with the basics of this instrument. Most students who used a LIA lacked conceptual understanding and used the device as a black box (with no understanding of how the LIA functions) to perform measurements. These difficulties made it challenging for students to correctly predict any but the most common cases encountered in the experiments involving LIAs with any degree of reliability. I have developed and evaluated a research-based tutorial that helps students learn about the LIA's operation and also helps them make connections between conceptual and quantitative aspects of the LIA's operation. This tutorial develops students' ability to predict output signals and input signals in a variety of cases all of which relate to situations they may encounter in the lab setting with the goal of enabling them to troubleshoot and check the validity of the measurements they make in the lab.

Examination of the average student scores on the pretests and posttests shows considerable improvement for all cases discussed. This improved ability to identify both typical (case 1) and less common (case 2-6) cases should improve students' ability to ensure that the output signals obtained are as expected and troubleshoot anomalous output signals or unexpected conditions present in the input signal when they arise. Additionally, the increase in scores across

a wide variety of students with varied levels of experience with the LIA suggests that the tutorial is useful at providing an OnRamp for students who are just being introduced into the lab setting and starting with the use of LIAs as well as providing an opportunity for those who have been using the LIA to improve their understanding of this versatile device.

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3.0 DEVELOPMENT AND EVALUATION OF AN INTERACTIVE LEARNING TUTORIAL ON QUANTUM KEY DISTRIBUTION

3.1 INTRODUCTION

Quantum mechanics is a particularly challenging subject for undergraduate students [1-14]. Based upon prior research studies that have identified difficulties [15-22], I have developed a set of research-based learning tools to help students develop a good grasp of quantum mechanics [23-36]. These learning tools include the Quantum Interactive Learning Tutorials (QuILTs) and concept tests [37-38] similar to those popularized by Mazur for introductory physics courses [39-40]. The QuILTs use an inquiry-based approach to learning [41], in which students are asked a series of guiding questions and strive to bridge the gap between quantitative and qualitative aspects of learning quantum mechanics and help students build a robust knowledge structure by guiding them to discern and learn the framework of quantum mechanics in a given context. They are developed based upon the findings of cognitive research and an investigation of students' difficulties in learning relevant concepts. The instructors can use the QuILTs either as in-class tutorials on which students can work in small groups or as homework supplements. The concept tests are integrated with lectures and when paired with peer instruction, they encourage students to take advantage of their peers' expertise and learn from each other. Here, I discuss the

development and assessment of a research-based QuILT on quantum key distribution (QKD) [42-46].

QKD is an interesting application of quantum mechanics useful for generating a shared secure random key for encrypting and decrypting information over a public channel [41-46]. A unique feature of secure QKD protocols is that two parties, e.g., a sender who is traditionally referred to as Alice and a receiver traditionally referred to as Bob, who generate a random shared key over a public channel, can detect the presence of an eavesdropper (traditionally referred to as Eve) who may be intercepting their communication during the shared key generation process to gain access to the key. Alice and Bob are connected via a quantum communication channel which allows quantum states to be transmitted during the random shared key generation process. When photon polarization states are used for shared key generation, the quantum communication channel used is generally an optical fiber or free space. In addition, during their shared key generation process, Alice and Bob also communicate certain information with each other via a classical channel such as the internet.

The ability to detect an eavesdropper in secure QKD protocols is due to the fact that physical observables are in general not well-defined in a given quantum state but measurement collapses the state and gives the observable a definite value. In particular, secure key generation exploits the fact that in general, measurement disturbs the state of a quantum mechanical system which can be in a superposition of linearly independent states and that a random unknown quantum state cannot be cloned [47]. This indeterminacy is unique only to quantum mechanics and can be used to determine if someone eavesdropped during the QKD process and if so, how much information was gained by the eavesdropper, Eve. In particular, Eve must measure the state of the system sent by Alice to Bob during the shared key generation process over the public

channel. But since quantum measurement in general changes the state of the system, she will not know, and will be forced to make an informed guess about, what replacement quantum state to send to Bob each time she intercepts the communication between Alice and Bob by measuring Alice's transmitted state. Even if Eve intercepts what Alice transmitted and uses clever strategies for sending replacement quantum states, since she must make a guess about what state to transmit to Bob at least in some cases, it will cause a discrepancy in the shared key that Alice and Bob generate in her presence. Also, if Bob is alerted by Alice that she has transmitted a state in the key generation process, and the eavesdropper does not send a replacement state to Bob to replace what Alice had transmitted, the discrepancy present in the shared key will be such that Alice and Bob will detect the presence of an eavesdropper. After the entire shared key is generated over the public channel, Alice and Bob can compare certain bits, e.g., every p^{th} bit, to ensure that they agree on the shared key generated (they discard the bits they compare so that it is not part of the shared key). If they find discrepancy in the bits they compare (beyond a threshold discrepancy due to decoherence in actual situations), they will abort the shared key generated because someone may be eavesdropping.

Thus, QKD relies on foundational issues in quantum mechanics and uses a simple two state system including the fact that a quantum state can be in a superposition of linearly independent states and measurement in general changes the state of the system and collapses the state into an eigenstate of an operator corresponding to the observable measured. QKD is already in use by the banking industry [46]. This real world application makes QKD a great vehicle for helping students learn about the fundamentals of quantum mechanics in an undergraduate course.

One protocol students learn via the research-based QKD QuILT involves generating a shared key over a public channel using single photons with non-orthogonal polarization states

(known as the B92 protocol [43]) and another protocol makes use of entangled states of two spin- $\frac{1}{2}$ particles (known as the BBM92 protocol [44]). The warm-up for the QuILT helps students learn foundational topics including the fact that in a two state system, a quantum state can be in a superposition of two linearly independent states but measurement of an observable will collapse the state and we obtain a definite value for the observable measured. In the first part of the QuILT after the warm-up, students learn about an insecure QKD protocol in which two orthogonal polarization states of single photons are used. Then they learn about a secure QKD protocol which uses two non-orthogonal polarization states of single photons via a guided inquiry-based approach to learning in which students are actively engaged in the learning process throughout. In the second part of the QKD QuILT, students are provided guidance and support in learning another secure QKD protocol which uses two entangled spin- $\frac{1}{2}$ particles. We deliberately selected one protocol to involve polarization states of single photons and another protocol to involve entangled states of two spin- $\frac{1}{2}$ particles because we wanted students to learn that both are effective two state systems for secure QKD protocols in which the presence of an eavesdropper can be detected.

In the next section, we review the B92 (for the last name of the original inventor of the protocol, Bennett, and the year it was published) [43] and BBM92 [44] protocols students learn via the QKD QuILT. This is followed by an investigation of student difficulties with relevant topics and the development and evaluation of the QuILT. We then discuss the findings from the pretest and posttest used to evaluate the effectiveness of the first secure QKD protocol and relevant foundational concepts related to quantum states and measurements that students learn. The pretest was administered after traditional instruction of the fundamentals of quantum mechanics required for QKD but before students worked on the QuILT. The posttest was

administered after the QuILT. We then discuss student understanding of the BBM92 protocol covered in the second part of the QuILT by administering a final exam question two months later on this topic.

3.2 THE QKD PROTOCOLS STUDENTS LEARN

A classical key distribution protocol cannot guarantee that the key shared over a public channel is secure. Such a classical protocol depends on the computational difficulty of mathematical functions [49]. On the other hand, as noted earlier, the secure QKD protocols are based upon the foundations of quantum mechanics including the fact that a quantum state can be in a superposition of linearly independent states, the measurement of an observable in general will change the quantum state and a random unknown state cannot be cloned. In particular, in order to securely generate a random shared key for encrypting and decrypting information over a public channel, QKD uses properties of quantum states and involves encoding information in quantum states which can be, e.g., in a superposition of linearly independent states of a single particle or entangled states of two particles. However, when measurement is performed in order to gain information, the state of the system collapses and one “bit” of information is obtained. These QKD protocols involve preparing and sending special quantum states and measuring an observable which yields one of two outcomes (either bit 0 or 1) in the prepared states with a certain probability. Though eavesdropping is possible for both secure quantum key generation protocols that students learn, comparing a small subset of the bits in the key at the end of the

shared key generation process will reveal if an eavesdropper was present and if so, to what extent the key was compromised.

In the B92 protocol Alice randomly sends a series of single photons with either a $+45^\circ$ or 0° polarization. Bob examines each photon by passing it through a polarizer with a polarization axis of either -45° or 90° with a photodetector set up behind the polarizer which clicks if the photon passes through his polarizer. In the two possible cases in which the photon's polarization is not orthogonal to the polarization axis of Bob's polarizer, there is a 50% chance for the photon to pass through and a 50% chance for the photon to be absorbed. In the other two possible cases, in which the photon's polarization is orthogonal to the polarization axis of Bob's polarizer, there is a 100% chance for the photon to be absorbed. Therefore, on average, 25% of the time the photon will pass and be detected. Since the photon must not be polarized orthogonal to the polarization axis of Bob's polarizer for it to pass through, Bob can determine the polarization of the photon when it passes through the polarizer and is detected by the photodetector behind the polarizer. If the photon is not detected, it could be polarized along either of the two possible directions (though it is more likely that it is polarized orthogonal to the polarization axis of Bob's polarizer) and Bob cannot be certain of the polarization of the photon. Every time the photon is detected Bob contacts Alice over public channels and informs her that he detected that photon and both Alice and Bob record the polarization of that photon as the next bit in the key. If we introduce an eavesdropper (Eve) who is using the same protocol as Bob to intercept photons sent by Alice and replaces them to the best of her ability, she (like Bob) will not be certain of the polarization of photon that she intercepted 75% of the time. Even if Eve makes an educated guess for all cases in which she is not certain and assumes that the photon is polarized orthogonal to the polarization axis of her polarizer which will be correct in $2/3$ of these cases, she will send

replacement photons with the incorrect polarization of photon 25% of the time resulting in a discrepancy between Alice and Bob's key which can be detected by Alice and Bob comparing a subset of the total key over public channels after it has finished being generated.

In the BBM92 protocol Alice generates a pair of entangled spin- $\frac{1}{2}$ particles. She examines one of these entangled particles in her lab and the other is sent along to Bob for him to analyze in his lab. Both Bob and Alice examine one of the entangled particles by passing it through a SGA randomly oriented along either the X or Z axis and measuring the particle's deflection on a screen. In 50% of cases both Alice and Bob have their SGAs oriented along the same axis as each other. In these cases Alice and Bob will each know the deflection that the other measured. In the remaining 50% of cases in which Alice and Bob's SGAs are oriented along orthogonal axes, neither Alice nor Bob will be certain of the measurement made by the other. After each measurement is made, Alice and Bob contact each other via a public channel and compare their SGA orientations. If their SGA is oriented along the same axis as the other person's, they will record their measurements as the next bit of the shared key. Now we introduce an eavesdropper to this case who uses the same protocol as Bob and generates replacement particles to send to Bob after intercepting the entangled particle sent by Alice to Bob. When Eve is eavesdropping, there are two cases in which a bit of the key will be recorded by Alice and Bob. In the case in which Eve's SGA is oriented along the same axis as Bob's and Alice's, Eve be able to reproduce with 100% certainty a spin- $\frac{1}{2}$ particle that when measured by Bob will be indistinguishable from the one he would have measured if Eve had not intercepted. In this case, Eve's presence will go undetected. On the other hand, in the case in which Eve's SGA is oriented along the orthogonal axis compared to Bob's and Alice's (Bob's and Alice's are aligned with each other so they will record the bit as part of their shared key), Eve will generate a

replacement particle that when measured by Bob will, 50% of the time, be the same as the measurement he would have made if Eve hadn't interfered and, the other 50% of the time, be the opposite of the measurement he would have made. Overall this leads to a 25% discrepancy between the bits recorded by Bob and those recorded by Alice which can be detected by Alice and Bob comparing a subset of the total key over public channels after it has finished being generated.

Before learning about the B92 protocol for secure key distribution involving two non-orthogonal polarization states of single photons, students learn about an *insecure* key distribution protocol using two orthogonal polarization states of single photons. In the insecure protocol, Eve can find out the polarization states of each photon that Alice sends to Bob while generating the shared key over a public channel with 100% certainty without her presence being detected by Alice and Bob. In particular, students learn that an eavesdropper in the insecure QKD protocol can generate a replacement photon with the same polarization as the one sent by Alice to Bob during the shared key generation process and transmit it to Bob without him realizing that the photon he intercepted did not come from Alice but came from Eve. The QuILT helps students learn that the ability for an eavesdropper to gain access to the key without her presence being detected makes this protocol insecure for generating a shared key over a public channel.

As noted, the B92 secure QKD protocol that students learn in the QuILT uses generation of a shared key securely over a public channel using two non-orthogonal polarization states of single photons [43]. Alice randomly sends single photons with one of the two non-orthogonal polarization states (e.g., with 0° and 45° polarization states) and Bob randomly intercepts the single photons with one of two non-orthogonal polarization states (e.g., randomly with 90° and -45° polarization states if Alice randomly transmits 0° and 45° polarized photons) together with a

100% efficient photo-detector behind his polarizers to detect the photons transmitted by Alice. After the measurement of polarization, the photon is polarized in the state it was measured, with all information about its initial polarization lost. A systematic comparison, e.g., of every p^{th} bit in the shared key, after a sufficiently long key is generated by each person will display at least a minimum threshold error if Eve was eavesdropping no matter how innovative her protocol for replacement of photon is. The QKD QuILT guides students through an ideal situation in the absence of decoherence. However, students learn that error can be introduced in the shared key generated by Alice and Bob due to decoherence as well (e.g., interaction of the photon with the surroundings which can change its polarization state or lead to scattering or absorption of the photon). They learn that the effect of decoherence can be neglected over small distances, e.g., QKD schemes, similar to the one they learn have been tested successfully and the highest bit rate system currently demonstrated exchanges secure keys at 1 Mbit/s (over 20 km of optical fiber) and 10 kbit/s (over 100 km of fiber) with applications in the banking industry [44].

The second secure QKD protocol students learn, the BBM92 protocol and developed by Bennett, Brassard and Mermin [44], makes use of an entangled state of two separate spin- $\frac{1}{2}$ particles (which are intertwined in such a way that they must be described by a combined quantum state). In particular, there is no basis in which an entangled state can be written as a product of the quantum states of each particle individually. Therefore, in an entangled state, performing a measurement on one particle affects the measurement of the other. As noted, the BBM92 protocol involves creating an entangled state of two spin- $\frac{1}{2}$ particles such that one of these particles travels towards Alice's detector and the other particle travels towards Bob's detector in the shared key generation process over a public channel. Alice and Bob randomly choose either the z basis or the x basis for their SGAs, and they communicate their chosen basis

with each other over the public channel and only record a measured “bit” as part of their key if they both choose the same basis (in which case they know with certainty what the other person will record). In this protocol, anyone intercepting either particle alters the entangled state. This alteration of state can reveal the presence of an eavesdropper and the amount of information that is gained by her. Students learn that the presence of an eavesdropper in this protocol can also be detected by comparing some shared “bits” at the end of the key generation process similar to the first protocol.

3.3 STUDENT DIFFICULTIES

During the development of the QKD QuILT, we conducted 15 individual semi-structured think-aloud interviews [48] with physics undergraduate students enrolled in quantum mechanics and physics graduate students to understand their difficulties with relevant concepts in order to effectively address them in the QuILT. During the semi-structured interviews, students were asked to verbalize their thought processes while they answered the questions about the QKD basics either as separate questions before the preliminary version of the QuILT was developed or as a part of the QuILT, which included various protocols for generating insecure or secure shared key over a public channel. Students were not interrupted during these think-aloud interviews unless they remained quiet for a while (if they became quiet they were asked to keep talking). After the students answered the questions to their satisfaction, we asked them for clarification of the issues they had not made clear earlier. In the interviews that were conducted before the development of the QuILT, students were asked general questions relevant for the QKD QuILT.

In interviews conducted with different versions of the QuILT, students were asked to work on different parts of the QuILT including the basics while thinking aloud. In addition to the interviews, students were also asked open-ended questions about issues relevant for QKD.

During the interviews throughout the development of the QuILT, some students claimed that the polarization states of a photon cannot be used as basis vectors for a two state system due to the fact that a photon can have any polarization state. They argued that since a polarizer can have any orientation and the orientation of the polarizer determines the polarization state of a photon incident on the polarizer, it did not make sense to think about polarization states of a photon as a two state system. These students were so fixated on their prior experiences with polarizers from introductory classes (which can be rotated to make its polarization axis whichever way one wants with respect to the polarization of incident light) that they had difficulty thinking of polarization states of a photon as vectors in a two-dimensional Hilbert space. Some students displayed a fundamental difficulty with what constitutes a basis vector in this context. For example, one interviewed student said “I’m not sure what a basis vector is [in this context]”.

It is interesting to note that most students who had difficulty accepting that two polarization states of a photon can be used as basis states for a two state system had no difficulty accepting that spin states of a spin- $\frac{1}{2}$ particle can be used as basis states for a two state system despite the fact that the two systems are analogous. Interviews suggest that this difference in their perception was often due to how a spin- $\frac{1}{2}$ system and polarization were first introduced to them and the kinds of mental models they had built about each of these systems. Generally, students are introduced to polarization in an introductory physics course in classical optics and they are introduced to spin- $\frac{1}{2}$ systems in a quantum mechanics course. Since students had

learned about the spin- $\frac{1}{2}$ system only in quantum mechanics, thinking of spin states of a spin- $\frac{1}{2}$ particle as vectors in a two dimensional Hilbert space did not create a similar conflict. This difficulty in reconciling what students knew from classical optics about light passing through a polarizer and polarization states of a photon as vectors in a two dimensional Hilbert space is somewhat similar to the difficulty that introductory students have reconciling their everyday notions about force and motion with the established laws of physics learned in introductory physics courses.

As noted, some students who had difficulty connecting the measurement of polarization in a laboratory with polarization states of a photon were relatively comfortable with reasoning about measurement of a particular component of spin and thinking of spin states as vectors in a two-dimensional Hilbert space. During interviews, some of them even mentioned that measurement of S_z will collapse a spin state which was initially in a superposition of eigenstates of \hat{S}_z to an eigenstate of \hat{S}_z . However, they had difficulty with similar reasoning about the measurement of polarization of a photon collapsing the polarization state which is initially in a superposition state (e.g., the simplest case, in a superposition of two orthogonal polarization states chosen to be parallel and perpendicular to the polarization axis of the polarizer) into an eigenstate of the polarization measured. Even after students were reminded that a spin state of a spin- $\frac{1}{2}$ particle can be in a superposition of eigenstates of \hat{S}_z and that eigenstates of \hat{S}_z can be used as basis states to write any state in this two dimensional Hilbert space, some interviewed students still had difficulty thinking of spin- $\frac{1}{2}$ and polarization states of photon as analogous and coming to terms with polarization states of a photon as vectors in a two dimensional Hilbert space. They had difficulty with the notion that we can associate a vector in the Hilbert space to correspond to any polarization state (which is an eigenstate of the corresponding polarization)

and that any two orthogonal states in that space can be used as basis states to represent any polarization state. Interviews suggest that the difference in how students were first introduced to spin and polarization was at least partly responsible for why they often reasoned about these analogous two-state systems in very different ways.

Some written responses and interviews suggest that some students incorrectly thought that whenever single photons with a given polarization were sent through a polarizer, they would be completely blocked (absorbed) by the polarizer only when the photon polarization was orthogonal to the polarization axis. They claimed that a photon with any other polarization would make the detector behind the polarizer click. Individual discussions suggest that this difficulty was often related to the difficulty of applying the measurement postulate of quantum mechanics to the single photon incident on a polarizer situation. In particular, some of these students thought that a single photon incident on a polarizer can partly pass through the polarizer and partly get absorbed (e.g., for polarized photons, they claimed that the cosine squared of the angle between the polarization axis of the polarizer and polarization of the photon yields the fraction of a photon that transmits as opposed to the probability of the photon being transmitted). They did not realize that a single photon will either get absorbed by the polarizer or it will pass through with a certain probability. Interviews suggest that this difficulty often had its origin in the fact that students were not considering single photons passing through a polarizer probabilistically and were interpreting the situation by mixing quantum mechanical and classical ideas. In particular, students often confused the situation of a single photon incident on a polarizer with that of a beam of light incident on a polarizer. They knew from classical optics that the intensity of light generally decreases after passing through a polarizer and is only completely absorbed if

the polarization of incident light is orthogonal to the polarization axis of the polarizer and extrapolated this, incorrectly, to the single photon case.

Examining holistically, the written responses and interviews suggest that some students felt that the polarizer will only block photons that are polarized perpendicular to the axis of the polarizer and will allow any other photons to pass through completely. Moreover, other students claimed that the polarizer will act in the opposite extreme, blocking all photons that are not polarized parallel to the axis of the polarizer. In the written responses, these two types of difficulties were less common but still serve to illustrate difficulties with the basics of how single photons interact with a polarizer, something that is important to help students understand the quantum mechanics principles at work in B92 QKD protocol.

A common difficulty before working on the B92 QKD protocol in the QuILT involves a lack of understanding of when both parties in the key generation process have successfully generated the next “bit” in the shared key in this protocol. Students with these difficulties did not realize that for each photon polarization that Alice sends, there is a non-zero probability of the photon being absorbed completely due to quantum measurement collapsing the state but only one of the two possible polarizations sent by Alice has a non-zero probability of being transmitted through each of Bob’s two polarizers that he randomly uses for his measurement. Often this difficulty resulted in students incorrectly assuming that it is possible for Bob to determine the polarization state of the photon Alice sent both if the photon is absorbed (not detected by the detector behind Bob’s polarizer) or transmitted (detected by the detector behind Bob’s polarizer). Students who had this difficulty often claimed that Bob will know the polarization of the photon if his detector does not click (as discussed earlier) due to their incorrect mental model that polarization of the photon must be perpendicular to the polarization

axis of the polarizer for the photon to be completely absorbed. They also claimed that if the detector clicks they will know the polarization because a photon will be partly transmitted and partly absorbed (resulting in the detector clicking) in all situations in which the photon polarization is neither parallel nor orthogonal to the polarization axis of the polarizer. As a result of this difficulty, some students claimed that for all cases except when the photon polarization is perpendicular to Bob's polarization axis in the given situation, Bob's detector behind his polarizer would click. Due to these two difficulties, they incorrectly claimed that whether the detector clicks (signifying a photon passed through Bob's polarizer) or not the next "bit" of the key can be generated by the two parties.

Another relatively common way in which students misinterpreted when the next "bit" of the key is generated is that they thought that Bob is only certain about the polarization of the photon when the photon Alice sent is polarized perpendicular to the polarization axis of his polarizer. These students incorrectly claimed that since photons with polarization perpendicular to the polarization axis of Bob's polarizer are always blocked, Bob must always know the polarization of the photon Alice sent when it is perpendicular to the polarization axis of the polarizer. In the case when photons are neither perpendicular nor parallel to the polarization axis of the polarizer, these students often claimed that since there is a non-zero chance for the photon to be either transmitted or absorbed, Bob cannot infer the polarization of the photon sent by Alice. Thus, these students incorrectly claimed that Bob can only generate the next bit of the key when there is only one possible outcome (when Bob's polarization axis is perpendicular to the polarization of the photon that Alice sends). These students did not realize that what is important is whether a given outcome (transmitted or absorbed) exists for both polarizations of photon that Alice could send (as is the case for the absorbed outcome) or only for one of the polarization's

(as is the case for the transmitted through the polarizer outcome in which case the photodetector behind the polarizer will click). Bob knows about the photon polarization only when his detector clicks because his detector may not click when photons are absorbed which can occur for either of the two polarizations of photon that Alice could send.

Students also had difficulty in distinguishing between a “qubit” and a “bit” and how a qubit can be in a superposition state but once a measurement of an observable is performed, we get one bit of information. In addition, some students had difficulty in determining which angle was relevant for determining if the photon will pass through or get absorbed by a polarizer. In particular, these students were unsure whether the relevant angles were those between the polarization of the photon and the polarization axis of the polarizer. For example, some students made use of the polarization of the photon with respect to the horizontal axis in determining the probability of transmission or absorption.

3.4 THE DEVELOPMENT OF THE QUILT

The QKD QuILT began with an analysis of student difficulties with related concepts. The QuILT strives to build connections between the formalism and conceptual aspects of quantum mechanics without compromising technical issues in a context which provides an exciting application of quantum mechanics. It was developed based on the difficulties found by written surveys and interviews. It helps students learn quantum mechanics using a simple two state system and was developed based upon the findings of cognitive research and physics education research. It builds on the prior knowledge of students found via investigation of difficulties and

uses a guided inquiry-based approach in which various concepts build on each other gradually. The development of the QuILT went through a cyclic interactive process which included the following stages:

- (1) Development of the preliminary version based on a theoretical task analysis of the underlying knowledge structure and research on student difficulties with relevant concepts.
- (2) Implementation and evaluation of the QuILT by administering it individually to students and getting feedback from faculty members who are experts in these topics.
- (3) Determining its impact on student learning and assessing what difficulties were not adequately addressed by the QuILT.
- (4) Refinements and modifications based on the feedback from the implementation and evaluation.

As noted, in addition to written free-response questions administered to students in various classes, individual interviews with 15 students were carried out using a think-aloud protocol [48] to better understand the rationale for their responses throughout the development of various versions of the QuILT and the development of the corresponding pre-test and post-test, given to students before and after they engaged in learning via the QuILT. The QuILT asks students to predict what should happen in a particular situation and after their prediction phase is complete, they are provided a figure and table from which they can infer what they should have predicted. Then, they are asked to reconcile the differences between their prediction and what they infer from the information provided (in the second BBM92 protocol for secure key generation using two entangled particles, students use a simulation to check their prediction and then reconcile the differences between their prediction and what the simulation shows). After

each individual interview with a particular version of the QuILT (along with the administration of the pre-test and post-test), modifications were made based upon the feedback obtained from students. For example, if students got stuck at a particular point and could not make progress from one question to the next with the hints already provided, suitable modifications were made to the QuILT. We also iterated all components of the QKD QuILT with three faculty members and made modifications based upon their feedback. When we found that the QuILT was working well in individual administration and the post-test performance was significantly improved compared to the pre-test performance, the first part that uses non-orthogonal polarization states of photon was administered in class and the second part that uses entangled particles was given to students as a homework after traditional instruction on relevant concepts including instruction on two state systems (e.g., spin- $\frac{1}{2}$, polarization states of photon) and addition of angular momentum (which was useful for understanding entangled states). On average, students spent around 1.5 hours on the entire QuILT (many of the QuILT questions are in multiple-choice format to ensure that it can eventually be turned into a web-based tutorial which students can use as a self-study tool and get appropriate feedback if they select an incorrect option for a question).

The QuILT uses an educational approach that includes both elements of innovation and efficiency deemed important for preparing students for future learning by Sears, Bransford and Schwartz [50]. Innovation and efficiency are both incorporated in a guided active-learning approach to learning via the QuILT in which students are challenged to think through carefully designed questions (innovation) and are provided sufficient guidance (efficiency) to make progress. The QuILT strives to provide enough scaffolding to allow students to build a good knowledge structure while remaining engaged in the learning process. The guided aspect of the QuILT is illustrated by one interviewed student noting that “[answers to] most of these

[questions] I can figure out if I just think about it.” For the BBM92 protocol, after working through a series of questions that asked about hypothetical situations regarding various SGA orientations for both Alice and Bob, a student said: “If they have different orientations [for their SGAs] then they won’t even talk on the phone and be like ‘Alice record this’ [i.e., they will not record the measurement because only when they have the same orientation do they record the bit]”. It appeared that by working through the QuILT, most students had developed a good grasp of the quantum mechanical concepts involved in the quantum key distribution protocols.

3.4.1 QKD QuILT PART I (Based on B92 Protocol)

The QuILT warm up first helps students learn the basics related to bits, qubits, polarization states of a photon, spin- $\frac{1}{2}$ systems, and effect of measurement on a two state system in a superposition of linearly independent states. Many of these basics are asked in a series of multiple choice questions in a guided approach to learning. The need for reinforcing these basics in a guided approach was evident from answers to written free-response questions and during individual interviews with students. For example, after going over the basics, one interviewed student noted “If I hadn’t known the stuff in the basics I would be really confused with the QKD part”. The following questions (not necessarily consecutive) in the basics section help students learn about a qubit:

Question 1:

The quantum-mechanical analogue of a classical bit is called a “qubit”. A qubit can be described as $|q\rangle = \alpha|0\rangle + \beta|1\rangle$ where $|0\rangle$ and $|1\rangle$ are two orthonormal states of a quantum

system and α and β are two complex numbers such that $|\alpha|^2 + |\beta|^2 = 1$. Let \hat{Q} be an operator corresponding to a physical observable such that $\hat{Q}|0\rangle = 0|0\rangle$ and $\hat{Q}|1\rangle = 1|1\rangle$.

Choose all of the following statements that are correct:

- (I) A bit cannot be in both $|0\rangle$ and $|1\rangle$ states. However, a qubit can be in a superposition of $|0\rangle$ and $|1\rangle$ states.
- (II) A measurement of observable Q in a state $|q\rangle$ can only produce one of two possible values.
- (III) When you perform a measurement of Q in a qubit, you obtain one bit of information.

- (a) (I) and (II) only
- (b) (II) and (III) only
- (c) (I) and (III) only
- (d) All of the above

Question 2:

Choose all of the following that can form a qubit (i.e., that can be used as basis states for a qubit):

- (I) the eigenstates of \hat{S}_z for a spin one-half particle
- (II) the two orthogonal polarization states of a photon
- (III) the ground state and the first excited state of a one dimensional infinite square well.

- (a) (I) only
- (b) (I) and (II) only

- (c) (I) and (III) only
- (d) (I), (II) and (III)

We find that discussing these kinds of questions with other students while working on the QuILT helps students grasp the basic concepts that are necessary to understand how quantum key distribution works and why a particular protocol is secure or insecure. The following question is also asked in the warm up (basics section) after a series of questions about specific polarization states of a photon:

Question:

If a single photon with a normalized polarization state $|S\rangle = \alpha|H\rangle + \beta|V\rangle$ is incident on a horizontal polarizer, which one of the following is true?

- (a) The photon passes through the polarizer with a probability $|\alpha|$.
- (b) The photon passes through the polarizer with a probability $|\beta|$.
- (c) The photon passes through the polarizer with a probability $|\alpha|^2$.
- (d) The photon passes through the polarizer with a probability $|\beta|^2$.

Once students work through the basics, they are first led through the *insecure* protocol using orthogonal polarization states of photons. In this protocol, students are led through a series of questions without the eavesdropper. Then, they are asked about Eve's interference. Students are guided to realize that Eve is capable of intercepting and replacing photons without being detected. This insecure protocol gets students used to the process by which a shared key can be generated but illustrates a flawed method that is prone to eavesdropping going undetected.

In the QuILT, students learn the following protocol for insecure shared key generation over a public channel:

Alice and Bob want to generate a shared binary “key” (for encoding and decoding secret information) over a public channel (where a third party can eavesdrop on what is being sent). Alice and Bob discuss the following protocol over a public channel over which a third party can eavesdrop:

- Alice will send to Bob single photons with either horizontal or vertical polarization randomly with equal probability.
- Bob will intercept the photons sent by Alice using a polarizer which is randomly oriented either horizontally or vertically with equal probability. Bob’s ideal polarizer has a 100% efficient photo-detector behind it. When a photon passes through his polarizer, the photodetector clicks.
- They agree to denote a horizontally polarized photon state as bit ‘1’ and the vertically polarized photon state as bit ‘0’.
- Every time Alice sends a photon, she will alert Bob over the public channel that she sent a photon without telling him the polarization of the photon.
- They also decide that every time Bob detects a photon in his polarization measurement, he will send an email (which is a public channel) to Alice saying “I got it” without writing to her whether he was using a horizontal or vertical polarizer.
- They decide that every time Bob says “I got it” both Alice and Bob will record that bit as part of the key they are generating for encoding and decoding information.

- A photon can only be intercepted by one person's polarizer/detector system. If Eve intercepts a photon sent by Alice with her polarizer/detector system, she sends to Bob a replacement photon.
- If Eve is eavesdropping and intercepting the photons sent by Alice with her polarizer, she will have to send to Bob a replacement photon, otherwise Bob will not receive a photon when Alice alerted him and he will know someone has been tampering with the system.
- Assume that when Eve is eavesdropping and intercepting the photons sent by Alice with her polarizer, she is using a polarizer with a horizontal or a vertical polarization axis with equal probability (same as Bob).
- Assume that the time it takes Eve to replace a photon is negligible so that Bob does not notice any time-lag if Eve intercepts Alice's photon and sends to Bob a replacement photon in its place.

They learn that the above protocol for shared key generation over a public channel is not secure due to the use of orthogonal polarization states and eavesdropping will go undetected. Next, students are guided through the B92 protocol which generates a secure key in which Alice, Bob and Eve each use two non-orthogonal polarization states of a photon as follows:

Alice and Bob discuss the following protocol over a public channel where a third party can eavesdrop:

- Alice will send single photons either with $+45^\circ$ polarization or 0° polarization (horizontal polarization) randomly with equal probability.

- Bob will use a polarizer with polarization axis either at -45° or $+90^\circ$ randomly with equal probability. The 100% efficient photodetector placed behind his polarizer detects every photon that passes through his polarizer.
- Over the phone they decide to label the $+45^\circ$ polarization state as bit “1” and the 0° polarization state as bit “0”.
- Every time Alice sends a photon, she alerts Bob on the phone that she is sending something but does not reveal its polarization.
- Every time Bob measures a photon in his photodetector (i.e., he gets a “click” in his photodetector), he sends Alice an email with “I got it” without saying “what” polarization he measured and both of them note down that “bit” as part of their shared key.

If Eve is eavesdropping, we will assume that she uses the following protocol:

- If Eve is eavesdropping and intercepting the photons sent by Alice, she does it with her polarizer with polarization axis either at -45° or 90° with equal probability, identical to Bob’s protocol.
- A photon can only be intercepted by one person’s polarizer/detector system because it gets absorbed. If Eve intercepts a photon sent by Alice with her polarizer/detector system, she sends to Bob a replacement photon.
- Eve sends to Bob a replacement photon even if the detector behind her polarizer does not click (and she does not know the polarization of the photon sent by Alice although she knows that Alice had alerted over a public channel that she was sending a photon); otherwise Bob will not receive a photon when Alice alerted him. If Eve did not send replacement photons to Bob for cases in which













her detector did not click, the probability of Bob's detector (placed behind his polarizer) not clicking will be higher than if Eve was not eavesdropping.

Therefore, Bob will know that someone was tampering with the system.

- Assume that the time it takes Eve to replace a photon is negligible so that Bob does not notice any time-lag if Eve intercepts Alice's photon and sends to Bob a replacement photon in its place.

Following the basics, students are guided through a series of questions in which there is no eavesdropper. These questions culminate in Table 3.1 that students complete which spans all possible combinations of Alice's polarization, Bob's polarization and whether or not Bob's detector clicks. In each situation, students are asked if a bit is recorded by Alice and Bob and if so which bit is generated (0 or 1). After completing this table in which students predict what should happen in each situation, they were provided with a figure that illustrates what should

Table 3.1. In the B92 protocol in the QuILT, students are asked to complete the empty boxes in the table below predicting what should happen in a given situation based upon their understanding of the QKD protocol involving non-orthogonal polarization states of a photon.

Alice's polarization:						
Bob's polarization:						
Bob's detector clicks:	N	N	Y	N	N	Y
Bit is recorded: (Y or N)						
Which bit they record: (0 or 1 or -)						

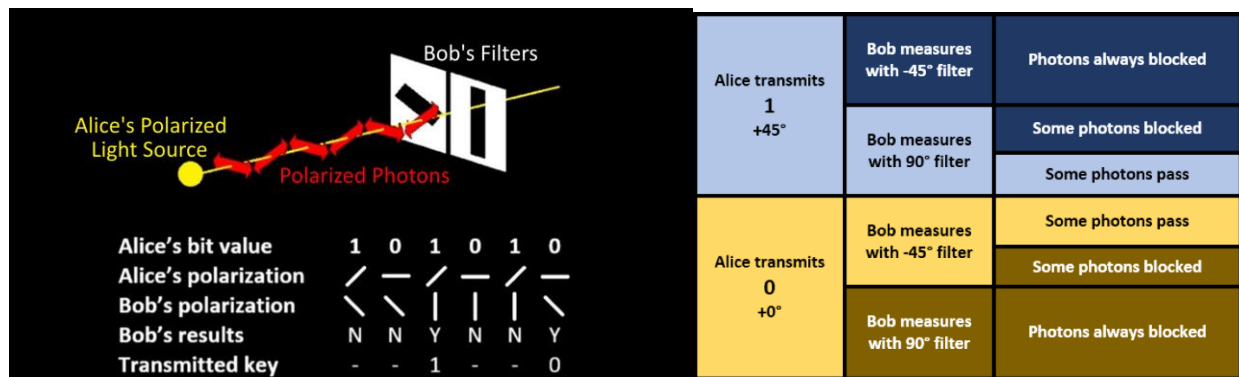


Figure 3.1. In the QuILT, students are first asked to make predictions about what should happen in a given situation and later asked to check whether their predictions are consistent with the information in this figure about the B92 QKD protocol involving non-orthogonal polarization states of single photons. Students are then asked to reconcile differences between their predictions and what happens in a given situation if their predictions are not consistent with this figure before they proceed.

happen in a given situation (Figure 3.1). Figure 3.1 displays the probabilities that a bit is recorded in each case. Students are then guided to address cases in which Eve has eavesdropped on the key generation process. The questions culminate with students calculating that Eve will introduce an error in 25% of the bits that Bob records (in the ideal case with no decoherence, eavesdropping is the only source of error). They learn that if Alice and Bob compare a small subset of the shared key after the entire key is generated, they will know that this error is due to eavesdropping and they will discard the key and attempt the key generation process at another time, preferably through other channels. The following is an example of a question students answer as the culmination of a series of questions regarding Eve's interference in the key generation process:

Question:

When Eve's detector does not click after she intercepts Alice's photon, she is not sure about the polarization of the photon sent by Alice. Eve will have to guess the polarization of the replacement photon she will send to Bob (in place of the one she intercepted). Suppose Eve is intercepting every photon sent by Alice and sending a replacement photon to Bob. Choose all of the following statements that are correct about the strategy for sending replacement photons to Bob when Eve is not sure what she intercepted from Alice:

- (I) If Eve's polarizer is at 90° when she intercepted the photon sent by Alice and the detector does not click, she has less chance of making an error if she sends a replacement photon with 0° polarization to Bob
- (II) If Eve's polarizer is at -45° when she intercepted the photon sent by Alice and the detector does not click, she has less chance of making an error if she sends a replacement photon with 45° polarization to Bob
- (III) If Eve's polarizer is at 90° and the detector does not click, she has the same chance of making an error regardless of whether she sends 0° or 45° polarization photon to Bob to replace it

- (a) (I) only
- (b) (II) only
- (c) (III) only
- (d) (I) and (II) only

3.4.2 QKD QuILT PART II (based on BBM92 Protocol):

The QKD QuILT Part II helps students learn the BBM92 protocol and involves entangled states of two spin- $\frac{1}{2}$ particles for secure key generation over a public channel. This part of the QuILT first helps students review the basics of spin- $\frac{1}{2}$ systems and entangled states. For example, in the following question from the QuILT, students convert the entangled state given in a basis (e.g., involving the eigenstates of S_z) to a basis involving the eigenstates of S_x and find that the particles remain entangled in both bases.

Question:

A) Suppose we have two entangled spin $\frac{1}{2}$ particles (particles A and B) described in the Z basis by the state $|\Psi_{AB}\rangle = 1/\sqrt{2}[|\uparrow_A\rangle_Z|\downarrow_B\rangle_Z - |\downarrow_A\rangle_Z|\uparrow_B\rangle_Z]$. If $|\uparrow_A\rangle_Z = 1/\sqrt{2}(|\uparrow_A\rangle_X + |\downarrow_A\rangle_X)$ and $|\downarrow_A\rangle_Z = 1/\sqrt{2}(|\uparrow_A\rangle_X - |\downarrow_A\rangle_X)$, write down the entangled state of the two particles in the X basis and simplify the expression.

B) How does this expression for the state in the X basis compare to the state in the Z basis?

In the QuILT, students consider the case of entangled spin $\frac{1}{2}$ particles (particles A and B) with all measurements referring to the measurements of spin performed using a Stern-Gerlach Apparatus (SGA) and a screen. The SGA consists of a region of non-uniform magnetic field along a given axis (either along the Z axis or X axis). Measurement of the spin state of the particle along the axis of the SGA can then be made by using a screen to detect how the particle was deflected (as shown in Figure 3.2). Measurement of the particle on the screen produces a

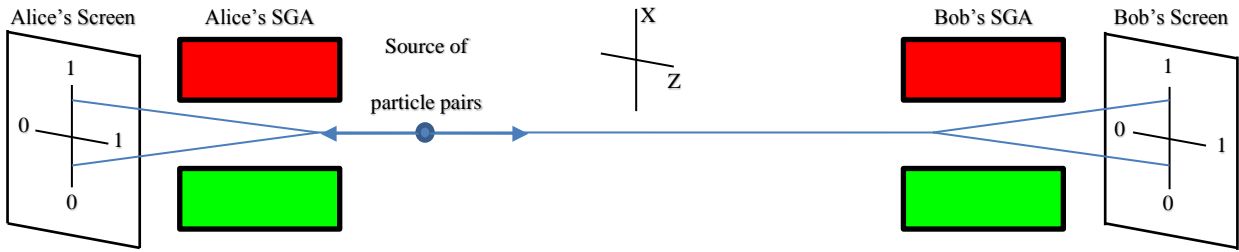


Figure 3.2. Schematic diagram of the setup for BBM92 protocol for QKD.

flash (the flash may be shifted up/down or left/right depending on the SGAs' magnetic field gradient and the particle's spin state into which the system collapses upon measurement). For spin- $\frac{1}{2}$ particles, the particles separate into two distinct streams, e.g., one deflected upwards (which we call measurement outcome 1), one deflected downwards (which we call measurement outcome 0) along this axis. Students are guided via questions such as the following two questions to help them make sense of the BBM92 QKD protocol:

Question 1:

A) Particle A's spin state is measured along the z axis. In what basis will the outcome of the measurement of particle B's spin state be deterministic?

- (a) The x basis
- (b) The z basis
- (c) Either x or z basis, both yield deterministic outcomes
- (d) None of the above. In quantum mechanics, the outcomes are never deterministic.

B) Explain your reasoning for answer 1A)

Question 2:

Consider the following statements from two students:

Student 1: “The answer to question 2 A) must be option (b). The measurement of particle A along the z axis collapses the state $|\Psi_{AB}\rangle = 1/\sqrt{2}[|\uparrow_A\rangle_Z|\downarrow_B\rangle_Z - |\downarrow_A\rangle_Z|\uparrow_B\rangle_Z]$ into either $|\uparrow_A\rangle_Z|\downarrow_B\rangle_Z$ or $|\downarrow_A\rangle_Z|\uparrow_B\rangle_Z$. Therefore, the measurement of particle A’s spin state, as either $|\uparrow_A\rangle_Z$ or $|\downarrow_A\rangle_Z$, can be used to infer that measurement of particle B’s spin state will definitely be either $|\downarrow_B\rangle_Z$ or $|\uparrow_B\rangle_Z$, respectively.

Student 2: “No, The answer to question 2 A) must be option (d). A fundamental axiom of quantum mechanics is that the outcome of a measurement can never be predicted ahead of time.

Do you agree with “Student 1”, “Student 2” or neither? Explain your reasoning.

After working on the basics, the QuILT guides students through situations in which entangled spin- $1/2$ particles are generated in such a way that the spin state of one of them is measured by Alice and the spin state of the other is measured by Bob. Students are asked to predict the likelihood of Alice and Bob recording a “bit” in each case for the BBM92 protocol. They are guided to understand how the generation of a secure key with entangled spin- $1/2$ particles works with SGAs and screens on which the particles are measured. They are given a schematic diagram of the setup required to generate this key (Figure 3.2) as well as the following description that explains the protocol that Alice and Bob follow. This part of the QuILT does not include an eavesdropper. Students are given the following information before working on the QuILT and making sense of the BBM92 protocol.

Alice and Bob want to generate a binary “key” useful for encoding and decoding secret information over a public channel where a third party can eavesdrop on what is being

sent. Since Alice and Bob cannot meet in person, they discuss the following protocol for generating such a key over a public channel (e.g. internet). Their plan makes use of a source that produces two entangled spin- $\frac{1}{2}$ particles (ignore orbital angular momentum, assuming it is zero) in Alice's lab, two SGAs and two screens which act as measurement devices (the measurement of a particle at a particular point on the fluorescent screen is registered as a flash). The SGA consists of a region of non-uniform magnetic field along a given axis (either along the x axis or z axis). Measurement of the spin state of the particle along the axis of the SGA can then be made by using a screen to detect how the particle was deflected (e.g., as shown in Figure 3.2). Detection of the particle on the screen produces a flash (the flash may be shifted up/down or left/right depending on the SGA's magnetic field gradient and the particle's spin state into which the system collapses upon measurement). In general, spin- $\frac{1}{2}$ particles can separate into two distinct streams, e.g., one deflected upwards (which we call measurement outcome 1) and one deflected downwards (which we call measurement outcome 0) along the axis determined by the magnetic field gradient of the SGA.

- A source generates a pair of entangled spin- $\frac{1}{2}$ particles in Alice's lab which move in opposite directions towards Alice's and Bob's SGAs as shown in Figure 3.2.
- These two entangled particles A and B are described by the state $|\Psi_{AB}\rangle = \frac{1}{\sqrt{2}}(|\uparrow_A\rangle|\downarrow_B\rangle - |\downarrow_A\rangle|\uparrow_B\rangle)$ (where A is the particle that moves towards the measuring screen in Alice's lab and B is the particle that moves towards the measuring screen in Bob's lab).

- Particle A in the entangled state passes through a SGA in Alice's lab which she randomly orients along one of two orthogonal axes, denoted x and z , and she measures the deflection of the particle (up/down or left/right) and notes the orientation of her SGA (whether the direction of the magnetic field gradient is along the x or z axis).
- Alice and Bob have decided that upward deflection on Alice's screen in the z direction will be noted as a "1" and deflection to the right on Alice's screen in the x direction will be noted as a "1". The other two deflections in her lab will be noted as "0" in their shared binary key.
- Particle B in the entangled state passes over the public channel to Bob's lab and through his SGA which he randomly orients along one of the two orthogonal axes (x and z). Bob notes the deflection (up/down or left/right) and also notes the orientation of his SGA.
- After both Alice and Bob perform each measurement, they compare using a public channel the orientations of their SGAs (the direction of the magnetic field gradients) but not the observed deflection.
- If both Alice and Bob happened to use the same orientation for their SGAs for a particular measurement (had the same basis), Alice and Bob know with certainty what the other person obtained since their measurements are anti-correlated. In this case, Alice writes down her deflection measurement as the next bit of the shared binary key and Bob records the opposite of the deflection on his screen (to match Alice's bit) as the next bit of the shared binary key they generate (e.g. if Bob notes

upward deflection in the z direction and measures a 1 he records a 0 because that is what Alice must have recorded for her particle in an error-free measurement).

- If the comparison of Alice and Bob’s SGA orientations over the public channel show that they did not use the same orientation or “basis” (i.e., their SGAs had different magnetic field gradients), they discard that bit (do not include it in their shared key).
- They perform this procedure many times to generate a shared key of the length they want.

The QuILT asks students to predict when Bob is certain of Alice’s particle’s spin state after being informed of Alice’s SGA’s orientation over a public channel and why. After their predictions and explanation that requires understanding of foundational issues in quantum mechanics including measurement, students are asked to complete Table 3.2 with Bob’s

Table 3.2. In Part II of the QuILT, students are asked to complete the empty boxes in the table below predicting what should happen in a given situation based upon their understanding of the BBM92 QKD protocol involving entangled particles.

	Alice measures a 1				Alice measures a 0			
	Alice’s SGA is along the X axis		Alice’s SGA is along the Z axis		Alice’s SGA is along the X axis		Alice’s SGA is along the Z axis	
	Bob’s SGA is along the X axis	Bob’s SGA is along the Z axis	Bob’s SGA is along the X axis	Bob’s SGA is along the Z axis	Bob’s SGA is along the X axis	Bob’s SGA is along the Z axis	Bob’s SGA is along the X axis	Bob’s SGA is along the Z axis
Bob’s measurement (1, 0, or -)								

measurement for every possible combination of Bob's SGA orientation, Alice's SGA orientation and Alice's measurement. Students are guided to realize that Bob and Alice can only generate a bit of the shared random key when their SGAs are aligned along the same axis.

Next, an eavesdropper is included in this protocol (Figure 3.3) and they are provided with the following description about the assumptions made about Eve when performing her measurements during eavesdropping.

Now let us assume that a third party, Eve, is intercepting every particle in the entangled state sent to Bob with her own SGA which she randomly orients along the X or Z direction (similar to the strategy used by Alice and Bob). Then, Eve immediately generates a replacement particle (unlike the entangled state) to send to Bob. Let us assume that Eve can generate the replacement particle instantly and that she matches the particle that she intercepted (e.g., if Eve measures a 1 along the Z axis, she produces and sends a particle to Bob which is also in $|\uparrow_B\rangle_Z$ state).

Then, students are asked a series of guiding questions focusing on the cases in which Eve will mistakenly send to Bob a spin- $\frac{1}{2}$ particle with the incorrect spin state but Bob measures spin state along that axis so he will record that "bit" as part of the key. These questions ultimately lead students to realize that if Alice and Bob's SGAs are oriented along the same axis and Eve sends a replacement particle with the wrong spin state along that axis (due to her SGA being oriented perpendicular to Alice's and Bob's), they will record mismatched values for that bit in 25% of the cases. Therefore, a comparison by Alice and Bob over a public channel of a subset of bits in the shared key at the end of the key generation process will reveal this discrepancy. Finally,

students are directed to a web based simulation that models this protocol [34], and asked to check their predictions with the simulations and reconcile the differences between their predictions and simulation outcomes.

3.5 RESULTS FROM THE PRETEST AND POSTTEST FOR PART I

Students worked on Part I of the QKD QuILT (B92 protocol) in class in small groups in an upper-level undergraduate quantum mechanics course intended for physics majors. Before the QuILT, students learned about spin- $1/2$ systems, addition of angular momentum, polarization states of photons and a basic outline of quantum key distribution. Students were administered the pretest before working on the QuILT and they were given the full quiz score for trying their best on the pretest. The posttest was administered in the next class after all students had the opportunity to complete the QuILT at home if they did not finish it in class and it was counted as a quiz. The pretest and posttest are identical in structure with only the polarization angles changed between the pretest and posttest. The pretest and posttest each begin by briefly outlining a situation similar to the B92 protocol to ensure that the students are familiar with it. The pretest

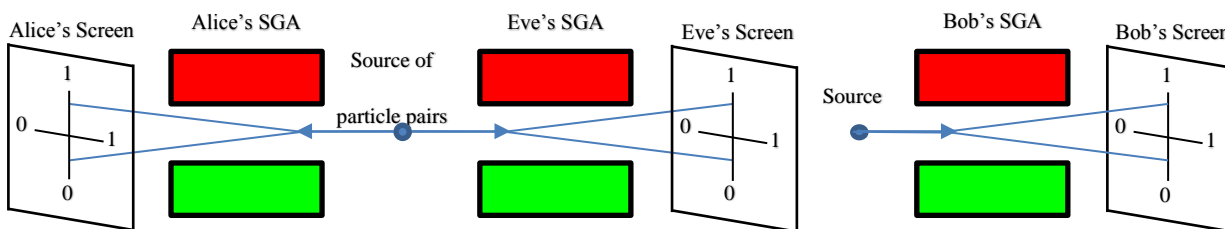


Figure 3.3. Schematic diagram of the setup for the BBM92 protocol with the eavesdropper included.

Table 3.3. Pretest and posttest results for the QKD QuILT broken down by question. The pretest (posttest) results are based on answers of 74 (76) students with an average score of 52.7% (90.9%).

Question #	% Correct on Pretest	% Correct on Posttest
1	51.4	93.4
2	63.5	92.1
3	41.9	90.8
4	63.5	98.7
5	52.7	98.7
6	48.6	78.9
7 Box 1	71.6	97.4
7 Box 2	45.9	96.1
7 Box 3	47.3	93.4
7 Box 4	79.7	98.7
8	13.5	61.8
Average	52.7	90.9

(and posttest) then asks a series of questions regarding a B92 protocol setup with Alice's polarization axes randomly switched between 70° or 0° (60° or 0° for the posttest) and the polarizer axes for Bob randomly switched between -20° or 90° (-30° or 90° for posttest).

Table 3.3 shows that student performance increases from roughly 53% on the pretest to roughly 91% on the posttest. Below, we discuss student performance on each question separately (see Table 3.3). The pretest is included in the appendix for reference.

3.5.1 Question 1

Bob uses the polarizer with a -20° polarization and his photodetector does not click.

Choose all of the following statements that can be inferred based upon the above protocol used by them:

- (I) Bob is 100% sure about the polarization of the photon sent by Alice.
- (II) Alice must have sent a photon with a 70° polarization.

(III) Alice must have sent a photon with a 0° polarization.

(a) (I) only

(b) (I) and (II) only

(c) (I) and (III) only

(d) None of the above

This question evaluates students' understanding of when a bit of the key cannot be generated. Since the detector does not click, Bob cannot infer anything about the polarization of the photon Alice sent and thus cannot use this photon to record the next bit. Examining the pretest data, we note that roughly 50% of students incorrectly claimed that a “bit” of the key can be generated even when no photon is detected. After working on the QuILT, practically no students had this difficulty. The major difficulty that was common in this question was discussed in the student difficulties section earlier. In particular, some students incorrectly claimed that the photon that Alice sent is polarized perpendicular to Bob's polarizer and Bob is 100% sure about the polarization of the photon sent by Alice.

3.5.2 Question 2

Bob uses a polarizer with a -20° polarization and his photodetector clicks. Choose all of the following statements that can be inferred based upon the above protocol used by Alice and Bob:

(I) Bob is 100% sure about the polarization of the photon sent by Alice.

(II) Alice must have sent photon with a 70° polarization.

(III) Alice must have sent photon with a 0° polarization.

(a) (III) only

(b) (I) and (II) only

(c) (I) and (III) only

(d) None of the above

This question supplements the first question. It evaluates student understanding of when a “bit” of the key can be generated. When the photodetector clicks, Bob knows that the photon that Alice sent cannot be polarized perpendicular to Bob’s polarizer and therefore he can determine the polarization of the photon he detected. The number of students who answered this question correctly on the pretest is likely to be somewhat inflated due to one of the common incorrect models that students used to describe QKD that leads students to select the correct answer due to incorrect reasoning. These students thought that the polarizer only blocked photons that were polarized perpendicular to its axis and that all photons that were not perpendicular to its axis were allowed to pass through (either fully or partially) causing the detector to click. Thus, they would answer that Bob knows the polarization of the photon that Alice sent and that the photon has the polarization which is not perpendicular to the axis of the polarizer that Bob used. Though these students made the correct selection from the options given, they incorrectly thought that this setup will always result in Bob’s detector clicking. Table 3.3 shows that about 40% of students failed to answer this question correctly on the pretest but practically no student had this difficulty after working on the QuILT.

3.5.3 Question 3

Suppose Alice transmits a photon with her polarizer set to 70° . Bob uses a 90° polarizer to intercept it. Write down the probability that the photon will pass through Bob's polarizer.

This question evaluates student understanding of how a polarizer interacts with single polarized photons. Two common student difficulties became apparent on the pretest. The first involved students using $\sin^2 \theta$ rather than $\cos^2 \theta$ which suggests a lack of understanding of the transmission probability through a polarizer for a polarized photon. The second difficulty was identifying the angle in the equation for transmission probability incorrectly. Many students selected one of the angles provided in the problem statement rather than taking the difference between photon polarization and the polarization axis of Bob's polarizer. These two difficulties and other less common mistakes resulted in more than half of the students failing to correctly answer this question on the pretest. After the QuILT, very few students made a mistake on this question.

3.5.4 Question 4

Alice transmits a photon with 0° polarization and Bob uses a -20° polarizer. Which one of the following statements is true?

- (a) The photon is blocked by Bob's polarizer with a 100% certainty.
- (b) The photon will pass through Bob's polarizer with approximately 88.3% likelihood.
- (c) The photon will pass through Bob's polarizer with approximately 11.7% likelihood.

- (d) The photon will pass through Bob's polarizer with a 100% certainty.

Question 4, is given to investigate students' understanding of the likelihood that Bob's polarizer would block the photon with a given polarization. Similarly to question 3, there are two major difficulties that students have with this question. One difficulty which was discussed earlier stems from using $\sin^2 \theta$ rather than $\cos^2 \theta$ to predict the probability of transmission. The second difficulty involves assuming that the photon is either always transmitted or always absorbed despite the fact that photon polarization and the polarization axis of Bob's polarizer are neither orthogonal nor parallel. These difficulties resulted in roughly 40% of students answering this question incorrectly in pretest. After working on the QuILT, practically none of the students answered this question incorrectly.

3.5.5 Question 5

Bob uses a 90° polarizer and the detector does not click. Can he infer the polarization state of the photon that Alice sent? If so, what is it?

- (a) Yes. Alice must have sent a photon with 0° polarization.
- (b) Yes. Alice must have sent a photon with 70° polarization.
- (c) No. Alice could have sent a photon with either polarization (0° or 70°).
- (d) None of the above.

Question 5 examines concepts similar to those examined in Question 1. The primary concept emphasized in this question involves understanding that the photon's polarization state,

and thus the next “bit” of information, can be determined only when the detector clicks. Two of the available options assert that the polarization of the photon that Alice sent can be determined and is one of the two possible polarization angles. A third option, “none of the above”, is logically unsound because the question statement gives every piece of information that Bob has in this situation meaning that he must be able to make some sort of determination, as he would during key generation. These three distracting options lead to roughly half of students selecting the incorrect answers in the pretest. The most common choice is based on the same common difficulty that students had with question 1. These students claimed that the photon is only blocked when the polarization of the photon is perpendicular to the axis of the polarizer and therefore they select the option that states that Bob can identify the polarization of the photon and that it is polarized perpendicular to the axis of his polarizer. After working on the QuILT, almost all students answer this question correctly.

3.5.6 Question 6

Choose all of the following statements that are correct based upon the protocol described:

- (I) Whenever Bob’s detector clicks, he can infer the polarization of the photon that Alice sent.
- (II) Whenever Bob’s detector does not click, he cannot infer the polarization of the photon that Alice sent.
- (III) If Alice sends a photon with 0° polarization and Bob uses a -20° polarizer, that photon will be partly absorbed and partly transmitted.

(a) (I) only

- (b) (I) and (II) only
- (c) (I) and (III) only
- (d) All of the above

This question evaluates student understanding of several important concepts relevant for the B92 protocol. It simultaneously tests whether students understand that if Bob's detector clicks, the polarization state of Alice's photon can be determined and thus the next bit of the key can be generated and if the detector does not click, the polarization state of the photon Alice sent cannot be inferred. This question also includes a good distractor related to a common difficulty students have before the QuILT, namely, that a single photon can be partly absorbed and partly transmitted (as discussed in the difficulty section earlier, this difficulty was often a result of over generalization of the fact that when a beam of light passes through a polarizer, the intensity of light generally decreases due to some light getting absorbed and some passing through). About half of the students selected the correct answer in the pretest. The two most common incorrect answers are based on two similar difficulties discussed earlier. Both difficulties involve Bob being able to determine the polarization of the photon that Alice sent regardless of whether the detector clicks or not. Students were often confused about what happens to photons that are polarized neither perpendicular nor parallel to the axis of the polarizer. They incorrectly claimed that the photon is either partially transmitted and partially absorbed resulting in the transmitted portion of the photon making the detector click or that they are completely transmitted allowing the detector to click. After the QuILT, roughly 80% of students correctly answered this question.

3.5.7 Question 7

Complete the third column of the following table by recording “the probability that a photon will pass through and hence Bob’s detector clicks”:

		Probability of detector clicking
Alice transmits 70°	Bob uses -20° polarizer	
	Bob uses 90° polarizer	
Alice transmits 0°	Bob uses -20° polarizer	
	Bob uses 90° polarizer	

Since this exercise only requires the students to take the difference between the two angles and use it with the equation for the probability that the photon will be transmitted ($\cos^2 \theta$) it is reasonable to expect some uniformity across these four questions. Upon examining the pretest data in Table 3.3, we observe that the percentage of students who correctly answered each sub-question ranges from roughly 50% to 80%. The difference in score for some questions is mostly due to the fact that some students who did not answer questions involving the photon transmission probability correctly knew that a photon that is polarized orthogonal to a polarizer cannot pass through it. After working on the QUILT, nearly all of the students answered these four questions correctly.

3.5.8 Question 8

Using the table above for the case described in the preceding question, calculate the percentage of measurements in which Bob is 100% sure about the polarization of the photon that Alice sent out of all of the experiments that Alice and Bob conduct.

The primary difficulty on this question was to properly account for each of the four probabilities in the table to determine the overall probability. In particular, several students ignored any probabilities from the table that were equal to zero and proceeded to average the remaining probabilities. Other students simply wrote down 50% due to there being a nonzero probability of generating a bit of the key 50% of the time. A slightly less common error that several students made involved multiplying the nonzero probabilities together to determine the average probability of generating a shared bit of the key. Slightly over 10% of students answered this question correctly on the pretest while roughly 60% of students determined this probability correctly on the posttest.

3.5.9 Overall Results for the Pretest and Posttest

The pretest and posttest responses suggest that many students have several alternative models and they have difficulty with various concepts including the polarization states of a single photon, quantum measurement and collapse of the single photon state upon measurement.

These alternative models are predominantly observed in student responses on the pretest. One of these alternative models is suggested by students claiming that a photon will only be blocked by a polarizer if its polarization angle is perpendicular to that of the photon polarization. This model results in students claiming that if the detector does not click, the polarization of the photon must be perpendicular to that of the polarization angle of the polarizer. Therefore, these students incorrectly claimed that a bit of the key can be generated every time a photon is sent to Bob. Additionally, use of this incorrect model will result in students answering questions about when Bob's detector will click correctly without correct reasoning.

A second model that is less commonly used by students includes the notion that only photons that are polarized parallel to the polarization angle of the polarizer are able to pass through and make the detector click. This model results in neither of the two possible polarizations of photon that Alice sends to Bob making Bob's detector click. We find that the students using this model generally had additional difficulties regarding when a bit of the shared key can be generated.

In addition, examination of students' responses (especially across the pretest questions) reveals inconsistencies in student reasoning. One set of inconsistencies is displayed by student responses to Questions 3, 4 and 7. Despite the fact that Questions 3 and 4 are nearly identical questions and are both contained within the answers to Question 7, it was common on the pretest to find an answer for one of these three questions that did not match the pattern being followed in the other two questions and in some cases the boxes for Questions 7 that were supposed to be similar to responses to Questions 3 and 4 had different student responses. Another common inconsistency is found when examining the responses to Questions 1 and 5 on the pretest. Despite these two questions being nearly identical to each other in subject matter, students often

selected inconsistent responses to these questions. On the pretest, even on other questions student reasoning was often inconsistent.

Interviews suggest that one cause of inconsistency across answers is the context of the questions. For example, when students are asked questions that require conceptual answers and require no equations or mathematical manipulations (e.g., Questions 1, 2, 5 etc.), they are more likely to answer them based upon their alternative model. If instead they are asked a question that requires a mathematical answer, e.g., the probability of a photon passing through a polarizer (e.g., Questions 3, 4, 7 etc.), students are likely to rely on equations even if the results disagree with their model (students may not even compare these answers with their model to check for consistency). Other subtle changes in the context also affect student responses to similar questions. When comparing Questions 1 and 5, the major difference is the change in the layout of the available answers but it was enough to cause students to answer these two questions inconsistently, especially on the pretest. Interviews suggest that some students on the pretest became overly focused on the details of each individual question. In the more extreme cases, this was taken to the point where students answered each question without thinking about its interrelation with other questions. This type of treatment was at least partly responsible for students answering Questions 3 and 4 differently than Question 7 (which requires the answers to Questions 3 and 4). Students became too focused on each individual question and did not check for consistency across questions.

Students' post-test responses are significantly more consistent across questions. Interviews suggest that this improvement in score is not only indicative of a better understanding of QKD but also suggests an improvement in student understanding of relevant basics. For example, interviews and written responses suggest that the QuILT improves student

understanding of polarization states of a photon and how to make connection between a polarizer in a physical space and polarization states of a photon in a Hilbert space. It also helps students think about single photons as having a probability of transmission or absorption rather than being partially transmitted and partially absorbed. The QuILT helps students understand that quantum measurement collapses the polarization state of a photon depending upon the polarizer used. As noted, before the QuILT, many students claimed that only a photon with a polarization orthogonal to the polarization axis of the polarizer will get completely absorbed and the detector does not click only for the case in which the photon polarization is orthogonal to the polarization axis because in all other cases the photon will partly get transmitted and partly get absorbed. Moreover, they incorrectly inferred that since the detector will not click only for the case in which the polarization axis of the polarizer and the polarization of the photon are orthogonal, Bob will know the polarization of the photon Alice sent only for the case in which the detector does not click. Overall, the QuILT was helpful in improving student understanding of topics emphasized.

3.6 RESULTS FROM FINAL EXAM EVALUATING EFFECTIVENESS FOR PART 2

The effectiveness of part II of the QuILT given to 18 students as graded homework was evaluated with a question at the end of the semester as part of the final exam. This question was posed as follows:

In homework, you learned a protocol for quantum key distribution (QKD) which involved using a source that produces two entangled spin- $\frac{1}{2}$ particles in Alice's lab, two Stern-Gerlach Apparati (which can be oriented either vertically or horizontally) and two screens which act as measurement devices in Alice and Bob's labs. Describe the protocol and explain why Eve's eavesdropping will necessarily introduce a minimum error between the shared key that Alice and Bob create using this QKD protocol.

This question probes student knowledge of the BBM92 protocol and was given to students two months after the students learned about this protocol via the QuILT. To score students' performance on this question, a rubric was developed. This rubric was used by one other researcher and I to independently grade the final exam problem and upon examining the results was found to have an inter-rater reliability of better than 90%. The rubric included six important elements of the BBM92 QKD protocol considered essential in students' responses. Three of these elements on which students were scored were from the QKD protocol without eavesdropping and three describe how eavesdropping would introduce error in the shared key generated. The six factors of the BBM92 protocol (weighed equally in scoring) that a student must describe to receive full credit are summarized as follows:

- The fact that Alice and Bob randomly orient their SGA's along two orthogonal axes.
- A measurement is only recorded as a "bit" if Alice and Bob have their SGAs oriented along the same axis (which they compare using a public channel).
- Each measurement recorded results in a "bit" in the shared key being generated.

- Error can be introduced by Eve in the shared key generated between Alice and Bob when Eve's SGA is oriented perpendicular to those of both Alice and Bob.
- When Eve's SGA is oriented perpendicular to Alice and Bob's SGAs (which are aligned), Eve cannot know the spin state of the spin- $\frac{1}{2}$ particle along the axis that Bob will be using.
- Eve's replacement particle in the case when her axis is perpendicular to Alice and Bob may create a discrepancy in the shared key being generated by Alice and Bob.

Table 3.4 breaks down student performance on this question into four broad categories. Examining the results in Table 3.4, we note that a third of the students were able to summarize all of the fundamentals of this secure key generation protocol correctly. One common difficulty that some students had in describing this protocol is keeping it separate from the B92 protocol. Roughly 20% of students mixed elements or terminology from the two protocols. For example, some students alluded to the click of the detector signifying a "bit" of the shared key being generated. Similarly, some students mentioned Bob calling Alice to say "Got it" rather than to

Table 3.4. Summary of the performance of the 18 students who answered the final exam question related to the BBM92 protocol covered in Part II of the QKD QuILT (BBM92 protocol). Students received an average score of 70.3% on this final exam question.

How the Student Answered the Final Exam Question	Percentage of Students	Average percentage Score of Students on Final Exam Question
Answered the Question Correctly	33	100
Some errors or omissions in Explaining the BBM92 Protocol	39	64
Incorporated Elements of B92 Protocol into BBM92 Protocol	22	54
Incorrect/only Superficial Elements of the Protocol	6	0

inform about the axis along which his SGA was aligned and to find out the alignment of her SGA. A few students who confused these two protocols mentioned the polarization of the particles being used for generation similar to what they had learned in the B92 protocol. An additional cohort of roughly 40% of students answered this question with some errors but the bulk of the protocol was sound. The main issue these students had was with explaining Eve's interference and how Eve's interference causes corruption in the shared key generated. Some students in this group also lost some points due to a lack of clarity regarding a certain step in the protocol (e.g., a student lost some points if he did not explicitly say that Alice and Bob's SGAs must be aligned for a measurement to be made even if everything else was correct). Only one student (5.6%) did not write anything intelligible about the method for generating a secure key using the BBM92 protocol, which is worth noting given that two months had passed between the implementation of the QuILT and the final exam. When the performance of students on this final exam question was graded using this rubric an average score of about 70% was obtained. While this score is not perfect, we note that a nearly perfect description of the BBM92 protocol was necessary to score 100% on this question and this exam was given roughly two months after the QKD QuILT.

3.7 SUMMARY

We developed and evaluated a research-based QuILT to help upper-level undergraduate students learn quantum mechanics in the context of quantum key distribution using B92 and

BBM92 protocols. The QuILT provides a guided approach to bridge the gap between the quantitative and conceptual aspects of quantum mechanics and strives to help students build a knowledge structure of foundational issues in quantum mechanics by helping students learn these two different protocols for secure key generation. It keeps students actively engaged in the learning process and evaluation shows that the QuILT improves students' understanding of concepts related to the QKD.

Performance on the QuILT part I pretest showed that students were able to answer only slightly over half of the questions about the B92 protocol after a typical classroom treatment of the quantum mechanics principles related to this protocol. After working through the QuILT, student understanding of the B92 protocol and the associated physics principles were improved such that the average score was roughly 90% on the posttest. Students' final exam performance on part II of the QuILT based upon the BBM92 protocol (which students had worked on as homework two months prior) suggests that students on average maintained a reasonable understanding of this QKD protocol even after two months. Finally, a survey given at the end of the semester showed that students found this QuILT to be very interesting and rated it highly.

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4.0 IMPROVING STUDENT UNDERSTANDING OF INTRODUCTORY PHYSICS VIA WEB-BASED TUTORIALS AND SCAFFOLDED PREQUIZZES

4.1 INTRODUCTION

With limited time available in the classroom, self-study tools provide a valuable opportunity to supplement classroom learning [1-13]. Effective implementation of self-study tools can provide students an opportunity to improve their problem solving and reasoning skills and knowledge structure of physics using an approach that allows each student to progress at a rate that is commensurate with their current knowledge. Effective self-study tools can help students learn to think like a physicist while engaging in problem solving [14-22], and expose them to physics principles in a way that scaffolds learning and emphasizes proper problem solving techniques [3-7, 23-25]. Research-based tutorials are one such self-study tool that can provide an effective approach to improving student learning of physics from the introductory material to advanced topics in physics [1,2,26-32]. Tutorials can be helpful to students with diverse backgrounds in physics, such as those in a typical introductory physics course, by challenging students with a guided approach to learning in which the guiding questions provide appropriate scaffolding and help students learn fundamental concepts and develop useful skills.

We developed several research based introductory physics tutorials each of which utilize on a quantitative physics problem involving physics principles commonly taught in introductory physics classes [33]. Each “tutorial problem” is divided into a series of sub-problems that deal with different stages of problem solving such as conceptual analysis of the problem, planning or decision making, implementation of the plan, and assessment and reflection on the problem solving process. By working through these sub-problems which are in a multiple-choice format with distractor choices based upon common student difficulties, students can learn expert like problem solving heuristics as they work on the tutorial problem while developing a more robust knowledge structure simultaneously.

Despite the ease with which students may access these web-based tutorials, there are difficulties with effectively implementing them as a self-study tool in a typical introductory level physics class. One issue is that students may not properly use these tutorials as they are instructed to do when not supervised. In particular students are asked to initially attempt to solve the problem before the tutorial, solve each sub-problem to the best of their ability and follow the appropriate link (for the multiple choice option that best fits their answer). For each sub-problem, if they answer incorrectly they are returned to the sub-problem to make another attempt at solving it after reading a short guiding statement and if they are correct they read a reinforcing statement and proceed to the next sub-problem. To partially address these issues, while still providing scaffolding similar to that provided in the tutorials, we also developed a series of prequizzes designed to be implemented in the classroom. These prequizzes were developed alongside the research-based tutorials on identical problems and designed to mirror the scaffolded approach to problem solving that makes up tutorials. In-class implementation of these prequizzes ensures that all students in a class follow the same scaffolded approach to problem

solving in a format that fits the time constraints of the classroom, though it lacks the feedback provided with each sub-problem in the tutorial.

Additionally, we examine the effectiveness of these tutorials when they are used by students in a closely monitored interview setting. The students were instructed to work on the tutorial problem, without scaffolding, to the best of their ability and then asked to work on the tutorial by solving each sub-problem. After working on the tutorial, students' knowledge of the associated physics principles was measured by evaluating their performance on an associated paired problem that is based on the same physics principle as the tutorial problem. Students carried out these tasks in a one on one interview with a physics education researcher and the students were prompted to think aloud while working on the tutorial problem and paired problem but otherwise not disturbed.

While this project began as an attempt to develop a suit of research-based tutorials and evaluate their effectiveness as a self-study tool, its scope has expanded to examining the relative effectiveness of scaffolded prequizzes. In particular, we investigate whether the implementation of a properly scaffolded prequiz is more or less effective than working on a scaffolded research-based tutorial as a self-study tool. Additionally, the scope of this study has been expanded to include a comparison of the relative effectiveness of these tutorials when students work on them in a one on one setting as a think aloud interview as compared to when students use them as a self-study tool with complete instruction but at their own discretion and under no supervision.

4.2 TUTORIAL DEVELOPMENT

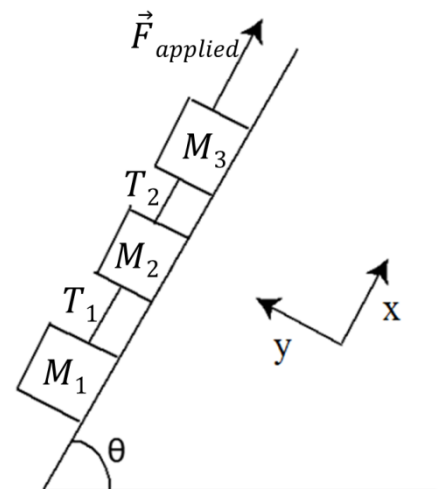
The tutorials discussed here were developed to improve student understanding of four physics principles which are typically covered in an introductory physics course (Projectile Motion, Application of Newton's Second Law, Conservation of Energy/Work-Energy Theorem, and Conservation of Angular Momentum).

All four tutorials were developed using the same protocol. First, a problem that requires use of one of the four physics principles was selected. Each selected tutorial problem was chosen to be somewhat more difficult than a typical homework problem associated with the chosen physics principle. This increased difficulty was chosen so that the problems could not be solved using a plug and chug approach and have enough depth to be able to help students learn an expert like problem solving approach. Then, a cognitive task analysis was performed by three graduate student researchers and one professor (all physics education researchers) to break down each tutorial problem into a series of sub-problems dealing with different stages of problem solving that must be answered to solve the tutorial problem. Each of these sub-problems was then posed as a multiple choice question. The incorrect options for each multiple choice question were chosen to emphasize common difficulties uncovered by having introductory physics students work on each sub-problem. If students selected an incorrect response, explanations for why those options are incorrect were provided and if students selected the correct response to a question, an explanation for why that option is correct was provided. These explanations were provided to reinforce student understanding of the reasoning behind the correct option and to aid students in repairing their knowledge structure when they selected an incorrect option. Using this approach, the first drafts of the tutorials were created. Each first draft was revised several times

based on interviews with introductory physics students and feedback from graduate students and several professors to ensure that they all were comfortable with the wording of the sub-problems and progression of the tutorial. During this revision process the tutorials were repeatedly implemented in one on one think aloud interviews with introductory level students and were shown to improve student performance on the paired problems developed in parallel with the tutorials (paired problems described below). The following are the tutorial problems provided for each of the tutorials.

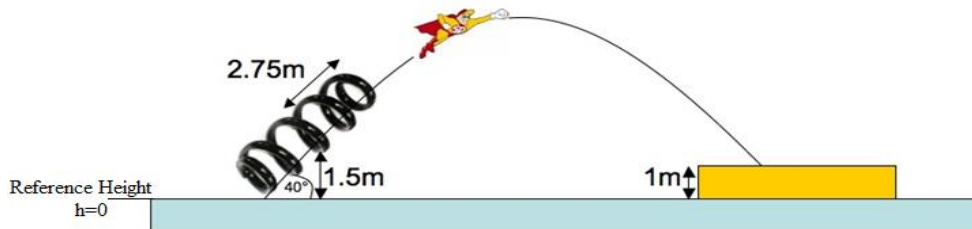
Projectile Motion Problem: A projectile is shot from ground level in a large, flat field with an initial speed v_o and at an angle θ_o above the horizontal. The maximum height that the projectile reaches is h_{max} and the horizontal range is R . Find the initial speed v_o and initial angle θ_o at which the projectile was launched in terms of h_{max} and R . The projectile lands at the same horizontal level from which it is launched. Throughout this problem we will consider the y axis to be vertical and the x axis as the horizontal.

Newton's Second Law Problem: Three blocks of masses $M_1 = 2$ kg, $M_2 = 4$ kg, and $M_3 = 6$ kg are connected by strings on a frictionless inclined plane with an angle of inclination $\theta = 60^\circ$, as shown in the figure below. A force of magnitude $F_{applied} = 120$ N is applied upward along the incline, causing the system of three masses to accelerate up the incline. Consider the strings to be massless and taut. What is the acceleration of



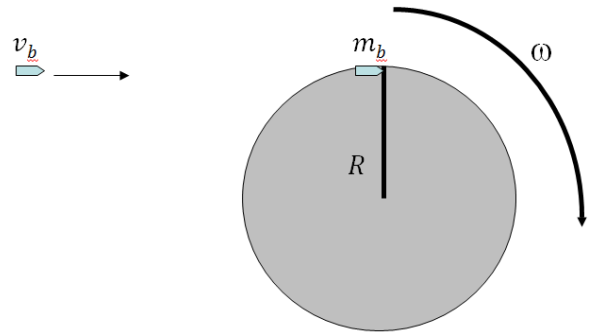
M_2 ? What are the tensions, T_1 and T_2 , in the two sections of string?

Conservation of Energy problem: You are the technical advisor for the David Letterman Show. Your task is to design a circus stunt in which Super Dave, who weighs 750 N, is shot out of a cannon that is oriented 40° above the horizontal. The “cannon” is actually a 1 m diameter tube that uses a stiff spring to launch Super Dave. The manual for the cannon states that the spring constant is 1800 N/m. The spring is compressed by a motor until its free end is level with the bottom of the cannon tube, which is 1.5 m above the ground. A small seat of negligible mass is attached to the free end of the spring for Dave to sit on. When the spring is released, it extends 2.75 m up the tube. The seat does not touch the sides of the 3.5 m long tube, so there is no friction. After a drum roll, the spring is released and Super Dave will fly through the air. You have an airbag 1 m thick for Super Dave to land on. The airbag will exert an average retarding force of 3000 N in all directions. Assume that this force is not enough to harm Super Dave. Note that this retarding force does not include the force due to gravity. You need to determine if the airbag is thick enough to stop Super Dave safely (with a good safety margin) – that is, he comes to a stop before he reaches ground level. Consider the seat and spring to have negligible mass. Ignore air resistance. Note that this is a real life problem and may have extraneous information. For this entire problem, we will consider the reference height, $h = 0$, to be ground level. Assume that Dave lands perpendicular to the airbag’s surface for a good safety margin.



Conservation of Angular Momentum Problem:

A large wooden wheel of radius R and moment of inertia I_w about its axis of symmetry is mounted on an axle so as to rotate freely. A bullet of mass m_b and speed v_b is shot and moves in a straight



line (neglect gravity) tangential to the wheel and strikes its edge, lodging in it at the rim. If the wheel was originally at rest, what would its angular speed be after the collision between the bullet and the wheel?

In the tutorial, the students are first provided with the problem and prompted to try their best to solve the problem without the aid of the tutorial. After the student has made an effort, he/she works on the tutorial and solve the sub-problems that relate to different stages of problem solving. For each sub-problem, the students select the answer that he/she thinks is correct from the four choices provided. If the student answers any sub-problem incorrectly, the explanation for why that answer is incorrect is provided and the student is returned to the sub-problem so that he/she can attempt to answer that sub-problem again. If the sub-problem is answered correctly, the student is provided with the justification for why that answer is correct to reinforce the reasoning behind the correct selection and the tutorial advances to the next sub-problem. By making use of this structure, students receive appropriate scaffolding on the sub-problems that they have difficulty with and less scaffolding on the sub-problems that they are able to solve without guidance and support.

Four paired problems which require use of the same principles as the tutorial problems were designed for the purpose of evaluating the effectiveness of the tutorials in improving

students' problem solving. After the initial design of the paired problems, they were iterated via interviews with introductory students and feedback from graduate students and professors. Each of these paired problems was designed to emphasize one of the physics principles used in each tutorial. These paired problems were also created to be similar to a typical weekly quiz in both length and difficulty. At the beginning of each of these paired problems, students were asked:

- Have you worked on the corresponding online tutorial?
- Was the tutorial effective at clarifying any issues you had with the problem covered in the tutorial?
- If the tutorial was ineffective explain what can be done to make it effective?
- How long did you spend on the tutorial?

Students were told that they will not be graded on their answers to these questions. This information can be helpful in determining the perceived effectiveness of the tutorial.

4.3 PREQUIZ DEVELOPMENT

In addition to the paired problems, three types of prequizzes focusing on different levels of scaffolding were developed corresponding to each of the four tutorials (Heavy, Light and No Scaffolding). While all three types of prequizzes in each set asked students to solve the same problem as in the associated tutorial, they differed in the level of scaffolding provided.

The Heavily Scaffolded prequiz was designed as a multiple choice quiz structured to be very similar to the tutorial that it was associated with. The Heavily Scaffolded prequiz asks the same questions as the tutorial's sub-problems and provides the same answer choices as those

available for the sub-problems in the tutorial. One difference between the tutorial and the Heavily Scaffolded prequiz is that if the answer that the student selects is incorrect, the tutorial provides immediate feedback explaining why it is incorrect and if the option is correct, the tutorial provides feedback to reinforce why that answer is correct. The Heavily Scaffolded prequiz, on the other hand, provided no feedback in response to what students selected as the correct response for each multiple-choice question dealing with different stages of problem solving.

In the Lightly Scaffolded prequiz the tutorial problem is broken up into four stages of problem solving. These explicit stages of problem solving are intended to guide students to solve the problem following an expert-like approach. In the “Analyze Qualitatively” step, students are asked to draw diagrams, list all known and unknown quantities, and to make predictions about the solution. In the “Plan/Make Decisions” step, the students are asked to divide the problem into sub-problems if necessary, determine what physics principles are applicable and useful to solve the sub-problems, choose the system or systems that is convenient for solving the problem, and write down equations that are necessary to solve the problem. The “Implementation” step asks students to solve the system of equations written down in the previous stage of problem solving for the desired quantities. Finally, the “Reflect and Assess” step asks students to check their results, e.g., by asking whether their solution makes sense, whether the solution has the correct dimensions, and whether the solution agrees with the student’s qualitative prediction.

The Unscaffolded prequiz asks students to solve the tutorial problem but provides no scaffolding. This prequiz provides a comparison to evaluate the effectiveness of the other two levels of scaffolding. For clarification, we note that all students regardless of the type of prequiz were asked to work on the corresponding tutorial as a self-study tool and the tutorial was posted

on their course website. The instructor had also mentioned to students that the tutorials will help them in their homework for that week and for the quiz next week.

After working on one of the three types of prequiz corresponding to a tutorial after instruction in relevant concepts, students handed in their prequiz problem and were given the paired problem corresponding to the tutorial problem as a second quiz in the same recitation. By examining the students' scores on the paired problems, we investigated the effect that the self-paced tutorial and various levels of scaffolding, provided by the prequizzes, had on students' performance.

4.4 RESEARCH METHODOLOGY FOR IMPLEMENTATION OF THE TUTORIAL AND PREQUIZZES

We implemented both of these interventions (tutorials and prequizzes) in two courses. Each of the four tutorials were posted on the course website as self-study tools after instruction in relevant concepts and three different types of prequizzes corresponding to each tutorial problem were randomly assigned to students in each recitation section. The first course was an algebra-based first semester physics course with roughly 385 students (split into two sections). The students come from varied backgrounds in math and science with a majority of them pursuing bioscience or neuroscience majors. The second course was a calculus-based first semester physics course with roughly 350 students (also split into two sections). The students in this course were almost entirely physical science, mathematics and engineering majors.

The tutorials were made available on the course website and could be used at a student's discretion after the associated physics principle was introduced in lecture but before students had the opportunity to do the associated homework problems. Students were made aware that no points would be awarded for completing the tutorial but announcements were made in class and posted on the course website informing students that the tutorials were available and may be helpful to them in their homework and quizzes.

One to two weeks after the associated physics principle was covered in class, each student was given one of the three prequizzes immediately followed by the corresponding paired problem for the tutorial. The amount of time given to each student for each quiz is given in Table 4.1. Examining this table, we note that the amount of time spent on the prequiz is kept the same to ensure that time on task remains consistent for all three types of prequiz corresponding to a given tutorial problem. Additionally, the type of prequiz that students in each recitation got was changed randomly for each implementation to avoid any cumulative effects of having students do a certain type of prequiz.

In the algebra-based course, some features of these interventions were not implemented for various reasons. For the algebra-based course, the Projectile Motion tutorial is not discussed here because students were given less time if they were given the Unscaffolded prequiz. This

Table 4.1. Time given for Prequiz and Paired Problem associated with each tutorial (in minutes).

	Prequiz	Paired Problem
Projectile Motion	15	15
Newton's Second Law	20	15
Conservation of Energy	10	20
Conservation of Angular Momentum	24	20

difference in time given makes it impossible to determine whether any positive effect for the Heavily scaffolded prequiz group is caused by the type of the prequiz or the different amount of time on task. Additionally, the algebra-based class was given only the Highly Scaffolded prequiz and Unscaffolded prequiz since the Light scaffolding prequiz was developed after implementation was completed in the algebra-based classes to provide a level of scaffolding between the Highly scaffolded and Unscaffolded prequizzes.

Rubrics were then developed by three graduate students and a professor for each of the paired problems and tutorial problems. The rubrics awards points for attempting each of the major steps from the problem (i.e. “Find Initial Angular Momentum” from Table 4.2) and points can be lost for making mistakes (i.e. “ $R_{Man} \neq R_{Wheel}$ ” from Table 4.2). The final score results from adding the points from all of the steps that they attempted and subtracting points from each of the mistakes they made in each step. The rubric in Table 4.2 is a good example of a typical rubric used to grade each of the paired problems and prequizzes and is standardized out of 10 points. Once the rubric was agreed upon, 10% of the quizzes and prequizzes were graded independently by three graduate students and a professor with that final version of the rubric. When the scores were compared, the inter-rater agreement was better than 90%. For the Highly Scaffolded prequiz, each sub-problem was graded based on its correctness and given equal weight in the overall score.

Apart from the large scale implementation of these tutorials in both algebra- and calculus-based courses as self-study tools, they were also implemented individually with students in one on one think aloud interviews. For each of these tutorials, students began by working on the tutorial problem without any scaffolding. The students were asked to work on the problem to

the best of their ability and to think aloud as they solved the problem. After students had attempted to solve the tutorial problem to the best of their ability, they were given the tutorial and asked to work on it by continuing to think aloud. Students were asked to solve each sub-problem in the tutorial that relate to different stages of problem solving. After they had an answer for a particular sub-problem, they were allowed to follow the link associated with the multiple choice questions that provided immediate feedback and support as needed. If they answered the sub-problem incorrectly, after viewing an explanation why the answer choice they selected is incorrect, they were returned to the same sub-problem. This process is continued until students complete the entire tutorial. Finally, students were given the paired problem that is based on the same physics principle as the tutorial and asked to solve the problem. Throughout

Table 4.2. Rubric for grading the Conservation of Angular Momentum paired problem which begin with a rotating merry-go-round and asks the student to find the angular velocity of merry go round when a man quickly sits on the edge of the merry go round.

Use Conservation of Angular Momentum	+1 point
Does not use conservation of angular momentum	-1 point
Find Initial Angular Momentum	+2 point
Misstates $L_o = I_{Wheel} \omega_o$	-1 point
Misstates $I_{Wheel} = 1/2 m_{Wheel} (R_{Wheel})^2$	-1 point
Find Final Angular Momentum	+5.5 point
Misstates $L_f = I_f \omega_f$	-1 point
Misstates $I_f = I_{Wheel} + I_{Man}$	-2 point
Misstates $I_{Wheel} = \frac{1}{2} m_{Wheel} (R_{Wheel})^2$	-1 point
Misstates $I_{Man} = m_{Man} (R_{Man})^2$	-1 point
$R_{Man} \neq R_{Wheel}$	-0.5 point
Find the Answer	+1.5 point
Incorrect ω_f	-0.5 point
Incorrect/No units for ω_f	-0.5 point
Incorrect conversion of ω_o to rad/s	-0.5 point

this process, the student was asked to talk aloud while a researcher recorded the observations. This process was repeated for each of the tutorials. Eight interviews were conducted with different students. In each of these interviews, between two and three tutorials were covered depending upon the pace of the student. These interviews focused on the tutorials pertaining to conservation of angular momentum, conservation of energy and Newton's second law. Due to several issues (discussed later) with the tutorial on Projectile motion, interviews were not carried out for this tutorial.

4.5 RESULTS FROM IN CLASS IMPLEMENTATION

Here, we will discuss the average scores for a recitation section on both the paired problems and the three types of prequizzes and how students are affected by both the tutorial and the scaffolding level in a particular type of prequiz. The prequiz acts as a “near” transfer problem since it is the same problem as the tutorial. The results of the prequiz can provide some insight into the effectiveness of the tutorials in improving student understanding of how to solve the tutorial problem. The paired problems can serve as “far” transfer problems since they are based on the same physics principles as the tutorial problem but are otherwise sufficiently different from the tutorial problem. Note that the Heavily Scaffolded prequiz is graded on a different scale than the other two prequizzes because it is a series of multiple choice questions rather than a free response question. This difference in grading scheme can result in the prequiz scores differing between the Highly Scaffolded prequiz and the other two prequizzes.

4.5.1 Results for Projectile Motion

Examining Table 4.3, we note that both Projectile Motion interventions had no statistically significant effect on the paired problem scores of students in the calculus-based course. This tutorial had no statistically significant effect on student performance on paired problem regardless of the scaffolding level of the prequiz students received. Therefore, we can conclude that this tutorial was ineffective in improving student problem solving of projectile motion problems. One interesting observation is a decrease in score of roughly 4.5% for those who were given the Lightly Scaffolded prequiz as compared to the comparison group which was given the Unscaffolded prequiz. While these results are not statistically significant, it is worth noting that

Table 4.3. Data on the Projectile Motion paired problem for the calculus-based class. The first set of data has the students separated based on either tutorial participation or prequiz scaffolding level. The second set of data has the students separated based on both tutorial participation and prequiz scaffolding level.

	Average Score	Standard Deviation	Number of Students		p-Value
Tutorial	6.27	3.13	158	Tutorial vs No Tutorial	0.621
No Tutorial	6.45	3.54	178		
Heavily Scaffolded	6.56	3.37	108	Heavy vs Light Scaffolding	0.255
Lightly Scaffolded	6.04	3.35	110	Light vs Unscaffolded	0.307
Unscaffolded	6.49	3.30	119	Heavy vs Unscaffolded	0.875

	Average Score		Standard Deviation		Number of Students		
	Tutorial	No Tutorial	Tutorial	No Tutorial	Tutorial	No Tutorial	p-Value
Heavily Scaffolded	7.03	6.88	3.36	3.37	52	56	0.817
Lightly Scaffolded	6.55	7.38	2.92	3.73	55	55	0.197
Unscaffolded	6.86	7.24	3.32	3.51	51	67	0.549

the Lightly Scaffolded prequiz group performed worse on the paired problem on projectile motion.

Upon examination of the student comments and review of student difficulties with this tutorial problem, we note that there are several issues with how the tutorial problem was structured. One difficulty that students had is that the Projectile Motion tutorial problem asked students to find the answers symbolically. This problem never gives students concrete numbers to plug into the equations at the end of the problem or sub-problems. This issue increased the difficulty of the problem for many students because they were only accustomed to solving problems with a numerical solution. Students also had difficulty due to the technically difficult algebra required to answer the tutorial problem in symbolic form. Because we asked that

Table 4.4. Data on the Projectile Motion prequiz problem for the calculus-based class. The first set of data has the students separated based on either tutorial participation or prequiz scaffolding level. The second set of data s has the students separated based on both tutorial participation and prequiz scaffolding level.

	Average Score	Standard Deviation	Number of Students			p-Value	
Tutorial	5.61	2.62	160	Tutorial vs No Tutorial		0.403	
No Tutorial	5.38	2.35	169				
Heavily Scaffolded	6.98	1.81	101	Heavy vs Light Scaffolding		<0.001	
Lightly Scaffolded	4.4	2.41	107	Light vs Unscaffolded		0.019	
Unscaffolded	5.16	2.44	122	Heavy vs Unscaffolded		<0.001	
	Average Score		Standard Deviation		Number of Students		
	Tutorial	No Tutorial	Tutorial	No Tutorial	Tutorial	No Tutorial	p-Value
Heavily Scaffolded	7.6	6.34	1.72	1.68	51	50	<0.001
Lightly Scaffolded	4.51	4.28	2.55	2.25	54	53	0.622
Unscaffolded	4.71	5.54	2.25	2.51	55	66	0.058

students find the magnitude and direction of the initial velocity in terms of the maximum height and range of the projectile rather than asking students to find the maximum height and range in terms of the initial velocity, roughly half of this tutorial involved difficult algebraic manipulations in symbolic form that students are not used to. While a good understanding of algebra is vital for success in an introductory physics class, it was not the intended purpose of this tutorial to reinforce facility in algebraic manipulations. Due to the complexity of the algebra required to complete this tutorial, students' attention was divided between physics principles and concepts relevant to solve the Projectile Motion problem and unnecessarily complicated algebraic manipulations. Additionally, since the complicated algebra increased both the length and the difficulty of the tutorial, it may have also increased the cognitive load on students as well as the likelihood that the student will remain engaged with the tutorial as intended.

For the calculus-based class, examining Table 4.4, we note that for the Lightly Scaffolded prequiz the results are statistically significantly lower than the comparison group. This reinforces the observation from Table 4.3 that the Light Scaffolding was not helpful for students. With the exception of the Heavily Scaffolded group, we note that the tutorial was ineffective even at improving scores on the prequiz which was based on the same question as the tutorial. This suggests that this tutorial was ineffective even for a near transfer problem.

4.5.2 Results for Newton's Second Law

Examining Table 4.5, we note that neither intervention caused a statistically significant improvement over the comparison group for the calculus-based students. As with the Projectile Motion data, we observed a decreased score on the paired problem for the Lightly Scaffolded

Table 4.5. Data on the Newton's Second Law paired problem for the calculus-based class. The first set of data has the students separated based on either tutorial participation or prequiz scaffolding level. The second set of data has the students separated based on both tutorial participation and prequiz scaffolding level.

	Average Score	Standard Deviation	Number of Students		p-Value		
Tutorial	7.75	2.71	135	Tutorial vs Non-Tutorial	0.142		
Non-Tutorial	7.28	2.89	197				
Heavily Scaffolded	7.78	2.44	147	Heavy vs Light Scaffolding	0.0376		
Lightly Scaffolded	7.00	3.25	104	Light vs Unscaffolded	0.295		
Unscaffolded	7.47	2.83	81	Heavy vs Unscaffolded	0.456		
	Average Score		Standard Deviation		Number of Students		
	Tutorial	Non-Tutorial	Tutorial	Non-Tutorial	Tutorial	Non-Tutorial	p-Value
Heavily Scaffolded	7.84	7.74	2.54	2.39	59	88	0.811
Lightly Scaffolded	7.63	6.43	2.96	3.44	50	54	0.0583
Unscaffolded	7.72	7.37	2.77	2.91	25	55	0.608

Table 4.6. Data on the Newton's Second Law paired problem for the algebra-based class. The first set of data separates the students based on either tutorial participation or prequiz scaffolding level. The second set of data separates the students based on both tutorial participation and prequiz scaffolding level.

	Average Score	Standard Deviation	Number of Students				p-Value
Tutorial	5.39	2.92	87	Tutorial vs Non-Tutorial			0.010
Non-Tutorial	4.46	2.91	274				
Heavily Scaffolded	5.16	2.84	191	Heavy Scaffolding vs No Scaffolding			<0.001
Unscaffolded	4.12	2.96	185				
	Average Score		Standard Deviation		Number of Students		
	Tutorial	Non-Tutorial	Tutorial	Non-Tutorial	Tutorial	Non-Tutorial	p-Value
Heavily Scaffolded	5.31	5.16	2.87	2.78	45	141	0.759
Unscaffolded	5.47	3.71	2.97	2.85	42	133	<0.001

group. Again, the difference in score between the Lightly Scaffolded and comparison (Un scaffolded) group is not statistically significant but the trend is worth noting.

Table 4.6 suggests that both interventions caused a statistically significant improvement in paired problem scores when compared with their respective comparison groups for students in the algebra-based course. Both of these interventions have a similar effect on the average paired problem score (an increase in score of roughly 10%). Despite these results being statistically significant, they are relatively small when considering the low scores of the comparison group and the small improvement in score with respect to how much room there is for improvement.

The limited effectiveness of these interventions may be due to students experiencing cognitive overload while engaging with the tutorial. This cognitive overload may be due to the

Table 4.7. Data on the Newton’s Second Law prequiz problem for the calculus-based class. The first set of data has the students separated based on either tutorial participation or prequiz scaffolding level. The second set of data has the students separated based on both tutorial participation and prequiz scaffolding level.

	Average Score	Standard Deviation	Number of Students				p-Value
Tutorial	8.37	2.15	135	Tutorial vs Non-Tutorial			0.187
Non-Tutorial	8.03	2.49	194				
Heavily Scaffolded	9.05	1.08	151	Heavy vs Light Scaffolding			<0.001
Lightly Scaffolded	7.03	3.03	106				Light vs Unscaffolded
Unscaffolded	7.79	2.6	75	Heavy vs Unscaffolded			<0.001
	Average Score		Standard Deviation		Number of Students		
	Tutorial	Non-Tutorial	Tutorial	Non-Tutorial	Tutorial	Non-Tutorial	p-Value
Heavily Scaffolded	9.08	9.03	1.18	1.02	59	90	0.790
Lightly Scaffolded	7.83	6.37	2.71	3.08	50	55	0.011
Unscaffolded	7.77	7.88	2.19	2.76	25	49	0.852

Newton's Second Law tutorial being long (requiring roughly an hour for an average student to complete it) and comprised of 17 sub-problems. The length of this tutorial also caused some students to ignore the instructions for how to properly use the tutorial. This is supported by several student comments that implied that the student read each slide of the tutorial rather than engaging with each sub-problem and attempting to work out the answer before getting support from the hints and feedback that were provided.

The results from Table 4.7 show that this tutorial, like the Projectile Motion tutorial, was ineffective at improving student scores on the near transfer prequiz problem. It also shows that the Lightly Scaffolded prequiz group had a lower score with respect to the comparison group. This decrease in score is not quite statistically significant but it is worth noting that this level of scaffolding is not effective at aiding students in the problem solving process.

Table 4.8 shows that the tutorial is effective at improving student performance on the prequiz problem for students in the algebra-based class. It also shows that the tutorial only causes a statistically significant improvement for students in the Unscaffolded group.

4.5.3 Results for Conservation of Energy

Examining the results from Tables 4.9 and 4.10 shows that neither of the Conservation of Energy interventions caused a statistically significant increase in student scores on the paired problem compared to the comparison group in either class. These data show that these two interventions are ineffective at improving students' knowledge and skills in a manner that their performance on a far transfer Conservation of Energy problem improves.

Table 4.8. Data on the Newton's Second Law prequiz problem for the algebra-based class. The first set of data separates the students based on either tutorial participation or prequiz scaffolding level. The second set of data separates the students based on both tutorial participation and prequiz scaffolding level.

	Average Score	Standard Deviation	Number of Students				p-Value
Tutorial	7.84	2.62	78	Tutorial vs Non-Tutorial			<0.001
Non-Tutorial	6.38	3.13	260				
Heavily Scaffolded	8.24	1.44	176	Heavy Scaffolding vs No Scaffolding			<0.001
Unscaffolded	5.14	3.49	176				
	Average Score		Standard Deviation		Number of Students		
	Tutorial	Non-Tutorial	Tutorial	Non-Tutorial	Tutorial	Non-Tutorial	p-Value
Heavily Scaffolded	8.62	8.14	1.39	1.44	40	131	0.498
Unscaffolded	7.05	4.6	3.25	3.36	38	129	<0.001

Table 4.9. Data on the Conservation of Energy paired problem for the calculus-based class. The first set of data has the students separated based on either tutorial participation or prequiz scaffolding level. The second set of data se has the students separated based on both tutorial participation and prequiz scaffolding level.

	Average Score	Standard Deviation	Number of Students				p-Value
Tutorial	8.18	2.75	185	Tutorial vs Non-Tutorial			0.385
Non-Tutorial	7.88	3.22	133				
Heavily Scaffolded	8.23	2.87	73	Heavy vs Light Scaffolding			0.529
Lightly Scaffolded	7.96	2.77	109				
Unscaffolded	8.03	3.14	135	Heavy vs Unscaffolded			0.643
	Average Score		Standard Deviation		Number of Students		
	Tutorial	Non-Tutorial	Tutorial	Non-Tutorial	Tutorial	Non-Tutorial	p-Value
Heavily Scaffolded	7.96	8.49	2.74	2.97	36	37	0.431
Lightly Scaffolded	8.35	7.27	2.52	3.06	70	39	0.0631
Unscaffolded	8.12	7.9	2.94	3.4	79	57	0.694

Table 4.10. Data on the Conservation of Energy paired problem for the algebra-based class. The first set of data has the students separated based on either tutorial participation or prequiz scaffolding level. The second set of data has the students separated based on both tutorial participation and prequiz scaffolding level.

	Average Score	Standard Deviation	Number of Students				p-Value
Tutorial	4.69	3.52	165	Tutorial vs Non-Tutorial			0.178
Non-Tutorial	4.15	3.82	172				
Heavily Scaffolded	4.77	3.54	179	Heavy Scaffolding vs No Scaffolding			0.064
Unscaffolded	4.05	3.84	184				
	Average Score		Standard Deviation		Number of Students		
	Tutorial	Non-Tutorial	Tutorial	Non-Tutorial	Tutorial	Non-Tutorial	p-Value
Heavily Scaffolded	5.05	4.46	3.12	3.77	75	95	0.266
Unscaffolded	4.39	3.77	3.79	3.84	90	77	0.297

Table 4.11. Data on the Conservation of Energy prequiz problem for the calculus-based class. The first set of data has the students separated based on either tutorial participation or prequiz scaffolding level. The second set of data has the students separated based on both tutorial participation and prequiz scaffolding level.

	Average Score	Standard Deviation	Number of Students		p-Value		
Tutorial	7.71	3.19	185	Tutorial vs Non-Tutorial	<0.001		
Non-Tutorial	4.65	3.63	129				
Heavily Scaffolded	8.52	1.42	72	Heavy vs Light Scaffolding	<0.001		
Lightly Scaffolded	6.15	3.89	115	Light vs Unscaffolded	0.299		
Unscaffolded	5.63	3.97	134	Heavy vs Unscaffolded	<0.001		
	Average Score		Standard Deviation		Number of Students		
	Tutorial	Non-Tutorial	Tutorial	Non-Tutorial	Tutorial	Non-Tutorial	p-Value
Heavily Scaffolded	9.1	7.99	1.41	1.21	35	37	0.007
Lightly Scaffolded	7.8	2.82	3.19	2.93	71	39	<0.001
Unscaffolded	7.04	3.5	3.54	3.62	79	53	<0.001

Table 4.12. Data on the Conservation of Energy prequiz problem for the algebra-based class. The first set of data has the students separated based on either tutorial participation or prequiz scaffolding level. The second set of data has the students separated based on both tutorial participation and prequiz scaffolding level.

	Average Score	Standard Deviation	Number of Students				p-Value
Tutorial	8.08	2.97	165	Tutorial vs Non-Tutorial			<0.001
Non-Tutorial	5.36	3.43	217				
Heavily Scaffolded	7.98	1.78	179	Heavy Scaffolding vs No Scaffolding			<0.001
Unscaffolded	5.16	4.2	230				
	Average Score		Standard Deviation		Number of Students		
	Tutorial	Non-Tutorial	Tutorial	Non-Tutorial	Tutorial	Non-Tutorial	p-Value
Heavily Scaffolded	9.09	7.15	1.35	1.62	75	95	<0.001
Unscaffolded	7.26	3.14	3.6	2.77	90	122	<0.001

This tutorial also has the same potential drawbacks as the Newton's Second Law tutorial. In particular, this tutorial problem is divided into 19 sub-problems making it longer than the Newton's Second Law tutorial. Additionally, the Conservation of Energy tutorial problem is a context rich problem. The addition of unnecessary information as well as the provided assumptions (no air resistance, assumption that the man hits the airbag with a velocity that is perpendicular to its surface, etc.) may distract students and may overload their working memory making it difficult for them to benefit from the tutorial fully.

Tables 4.11 and 4.12 show that the tutorial was effective in improving student scores on the near transfer problem in both the algebra and calculus-based classes. This tutorial improved the students' average score on this near transfer problem whether it was given as a Heavily Scaffolded, Lightly Scaffolded (for calculus-based students) or Unscaffolded prequiz. Also note

Table 4.13. Data on the Conservation of Angular Momentum paired problem for the calculus-based class. The first set of data has the students separated based on either tutorial participation or prequiz scaffolding level. The second set of data has the students separated based on both tutorial participation and prequiz scaffolding level.

	Average Score	Standard Deviation	Number of Students			p-Value	
Tutorial	6.91	2.62	184	Tutorial vs Non-Tutorial		<0.001	
Non-Tutorial	5.10	2.99	115				
Heavily Scaffolded	6.31	2.81	91	Heavy vs Light Scaffolding		0.971	
Lightly Scaffolded	6.32	2.94	98	Light vs Unscaffolded		0.549	
Unscaffolded	6.08	2.92	109	Heavy vs Unscaffolded		0.572	
	Average Score		Standard Deviation		Number of Students		
	Tutorial	Non-Tutorial	Tutorial	Non-Tutorial	Tutorial	Non-Tutorial	p-Value
Heavily Scaffolded	6.65	5.635	2.56	3.135	60	31	0.124
Lightly Scaffolded	7.435	4.775	2.47	2.835	57	41	<0.001
Unscaffolded	6.77	5.01	2.68	2.95	66	43	0.002

Table 4.14. Data on the Conservation of Angular Momentum paired problem for the algebra-based class. The first set of data has the students separated based on either tutorial participation or prequiz scaffolding level. The second set of data has the students separated based on both tutorial participation and prequiz scaffolding level.

	Average Score	Standard Deviation	Number of Students				p-Value
Tutorial	5.39	2.94	150	Tutorial vs Non-Tutorial			0.003
Non-Tutorial	4.40	3.19	186				
Heavily Scaffolded	5.10	3.14	180	Heavy Scaffolding vs No Scaffolding			0.025
Unscaffolded	4.35	3.08	175				
	Average Score		Standard Deviation		Number of Students		
	Tutorial	Non-Tutorial	Tutorial	Non-Tutorial	Tutorial	Non-Tutorial	p-Value
Heavily Scaffolded	7.51	4.85	2.64	3.16	90	79	<0.001
Unscaffolded	5.16	4.11	2.78	3.1	60	107	0.026

that the average score on the Lightly Scaffolded prequiz (for calculus-based students) is statistically indistinguishable from that of the Unscaffolded prequiz. While the scaffolding present in the Lightly Scaffolded prequiz does not result in a lower average score on the prequiz for Conservation of Energy, it does not cause a statistically significant improvement on the prequiz performance.

4.5.4 Results for Conservation of Angular Momentum

Tables 4.13 and 4.14 show that the Conservation of Angular Momentum tutorial produced a statistically significant improvement in average student score on the paired problem over the comparison (No Tutorial) group for both the calculus- and algebra-based students. This intervention was more effective in the calculus-based class where the tutorial yielded an almost 20% increase in score over the No Tutorial group. The algebra-based class received a roughly 10% increase in score. The relative increases in average score for these two classes is comparable.

We also note that in the calculus-based class, the Heavily Scaffolded prequiz and Lightly Scaffolded prequiz caused no statistically significant improvement on the average student grade for the paired problem when compared to the control group. In the algebra-based class, the Heavily Scaffolded group showed a statistically significant improvement on the average student grade for the paired problem. This improvement, though statistically significant, is a relatively small increase in score when the average score of the comparison group is less than 5

Despite the Conservation of Angular Momentum tutorial resulting in a small increase in paired problem score when compared to the comparison group's average score of roughly 5, this

tutorial was the most effective of the four evaluated. Examining this tutorial, several differences became apparent when compared with the other tutorials which are likely to account for the improved effectiveness. One difference is the relative length of the Conservation of Angular Momentum tutorial compared to the other tutorials. This tutorial included 7 sub-problems and could be completed in 15 to 20 minutes by an average student in either class. Additionally, this tutorial's sub-problems were predominately conceptual with almost no sub-problems focused on algebra. The decreased length of this tutorial may result in a decreased cognitive load on the student (compared to other tutorials) and the tutorial sub-problems focusing on the conceptual aspects of the tutorial problem emphasized important aspects of the problem solving process in a

Table 4.15. Data on the Conservation of Angular Momentum prequiz problem for the calculus-based class. The first set of data has the students separated based on either tutorial participation or prequiz scaffolding level. The second set of data has the students separated based on both tutorial participation and prequiz scaffolding level.

	Average Score	Standard Deviation	Number of Students		p-Value		
Tutorial	8.75	2.77	185	Tutorial vs Non-Tutorial	<0.001		
Non-Tutorial	4.34	4.05	115				
Heavily Scaffolded	8.9	2.08	92	Heavy vs Light Scaffolding	<0.001		
Lightly Scaffolded	6.08	4.34	99	Light vs Unscaffolded	0.627		
Unscaffolded	6.37	4.26	110	Heavy vs Unscaffolded	<0.001		
	Average Score		Standard Deviation		Number of Students		p-Value
	Tutorial	Non-Tutorial	Tutorial	Non-Tutorial	Tutorial	Non-Tutorial	
Heavily Scaffolded	9.58	7.56	1.49	2.38	61	31	<0.001
Lightly Scaffolded	8.5	2.72	2.99	3.66	57	41	<0.001
Unscaffolded	8.2	3.44	3.27	4.04	67	43	<0.001

manner that is more easily generalized for use in other problems that focus on the same physics principles.

Similar to the Conservation of Energy tutorial, Tables 4.15 and 4.16 show that this tutorial was effective at improving student scores on the near transfer problem in both the algebra- and calculus-based classes. This tutorial improved students' average score on this near transfer problem whether it was given as a Heavily Scaffolded, Lightly Scaffolded or Unscaffolded prequiz. Also note that the average score on the Lightly Scaffolded prequiz is lower than that of the Unscaffolded prequiz. While this result is not statistically significant, it does reinforce the trend that the scaffolding present in the Lightly Scaffolded prequiz does not aid students in the problem solving process when implemented only on the quiz.

Table 4.16. Data on the Conservation of Angular Momentum prequiz problem for the algebra-based class. The first set of data has the students separated based on either tutorial participation or prequiz scaffolding level. The second set of data has the students separated based on both tutorial participation and prequiz scaffolding level.

	Average Score	Standard Deviation	Number of Students				p-Value
Tutorial	8.12	3.2	140	Tutorial vs Non-Tutorial			<0.001
Non-Tutorial	5.5	3.68	180				
Heavily Scaffolded	8.59	1.96	173	Heavy Scaffolding vs No Scaffolding			<0.001
Unscaffolded	4.54	3.94	173				
	Average Score		Standard Deviation		Number of Students		
	Tutorial	Non-Tutorial	Tutorial	Non-Tutorial	Tutorial	Non-Tutorial	p-Value
Heavily Scaffolded	9.49	7.81	1.33	2.05	84	75	<0.001
Unscaffolded	6.05	3.86	3.97	3.7	56	105	<0.001

4.5.5 Results for Calculus-Based Class

Upon examining the results for the calculus-based course, we observe several patterns in the data. The most unexpected of these trends is that the student score on both the paired problems and prequiz for students who were given the Lightly Scaffolded prequiz tend to be either lower than or statistically indistinguishable from those given the Unscaffolded prequiz. Contrary to the initial hypothesis, that light scaffolding should help students use a more expert-like approach to problem solving, these data show that the Lightly Scaffolded prequiz results in no statistically significant improvement over the control group and in some cases results in a decrease in average student scores.

Another observation from the calculus-based course is that students did not perform statistically significantly better on the paired problem if they were given the Heavily Scaffolded prequiz when compared with those who were given the Unscaffolded prequiz. This trend shows that exposure to scaffolded sub-problems is not enough to improve student performance on these problems significantly without guidance, support and appropriate feedback as needed.

Additionally, we note that the Conservation of Angular Momentum tutorial caused a statistically significant increase in paired problem scores. A statistically significant improvement in student performance on the Conservation of Energy and Conservation of Angular Momentum prequizzes show that these tutorials may have caused enough of an improvement in student knowledge to aid them on solving a near transfer problem.

4.5.6 Results for Algebra-Based Class

For the tutorials related to Newton's Second Law and Conservation of Angular Momentum, students who made use of the tutorials scored statistically significantly higher on the paired problems. Additionally, students who made use of the Heavily Scaffolded prequiz scored higher on the paired problems for these two physics principles.

The tutorial and prequiz that focused on Conservation of Energy were less successful. Neither students who used the tutorial nor those who were given the Heavily Scaffolded prequiz showed statistically significant improvements in paired problem scores over their respective comparison groups. Though these interventions were ineffective in improving student performance on the far transfer paired problem, students did show improvement on the near transfer prequiz problem after engaging with either the Heavily Scaffolded prequiz or tutorial associated with Conservation of Energy.

Table 4.17. Student responses to the question “Was the tutorial effective at clarifying any issues you had with the problem covered in the tutorial?”

	Yes	No	No Response
Projectile Motion (Calculus)	85	73	12
Newton's Second (Calculus)	135	11	5
Newton's Second (Algebra)	76	7	4
Conservation of Energy (Algebra)	168	17	13
Conservation of Energy (Calculus)	139	19	7
Conservation of Angular Momentum (Algebra)	169	16	2
Conservation of Angular Momentum (Calculus)	121	22	7

4.5.7 Percieved Effectiveness

On the top of each of the paired problem quizzes, students were asked if the tutorial was effective at clarifying any issues that they had with the tutorial problem. Examining Table 4.17 we note that for all but one case the response was overwhelmingly positive regarding the tutorial's effectiveness. Therefore, of those students who made use of the tutorials, most felt that they were effective at clarifying any difficulties that they had with the problems. The exception to this trend is the Projectile Motion tutorial which received an even distribution of positive and negative responses. This less positive response is likely due to the complicated algebraic manipulations that make up the second half of the tutorial and the fact that this was a purely symbolic problem.

In addition to this yes or no question, the top of the paired problems also asked students to explain what can be done to make the tutorial effective, if the students thought it was ineffective. The comments were divided into several categories by researchers and tabulated in Table 4.17. Comments that were strictly positive or negative regarding the tutorial's effectiveness are placed in one of the first two columns. Comments that were unclear to the researchers or that were neither positive nor negative are accounted for in the third column. If the student suggested a change to the tutorial to make it more effective (which was overwhelmingly dominated by students asking for these tutorials to be made shorter), it is accounted for in the fourth column and if no comment was provided it is accounted for in the final column. Though this comment section was intended for suggestions and for students to mention issues that they had with the tutorial, it is surprising that almost as many students gave positive comments as negative comments for most of the tutorials. With the exception of the Projectile Motion tutorial,

most students rated the tutorials as effective at clarifying any issues they had with the tutorial problem and this is supported by the high proportion of positive comments on each of these paired problems.

4.6 RESULTS FROM INDIVIDUAL INTERVIEWS

By examining students in a one on one setting as they worked on these tutorials, we may be able to understand students' thought processes that lead to trends observed in the large scale implementation of these tutorials and the subsequent evaluation tools. Although several interviews were conducted with students individually during the development of the tutorials and the tutorials were found to be effective in individual administration in a think aloud interview format [34-36], we conducted 7 additional interviews with students who had been exposed to a typical classroom treatment of Newton's Second Law, conservation of energy/work energy

Table 4.18. Student comments in response to the question "If the tutorial was ineffective, explain what can be done to make it effective?"

	Positive	Negative	Neutral/ Confused	Change Suggested	No Comment
Projectile Motion (Calculus)	10	29	21	31	79
Newton's Second (Calculus)	10	10	5	6	124
Newton's Second (Algebra)	4	5	4	2	72
Conservation of Energy (Algebra)	5	8	7	21	157
Conservation of Energy (Calculus)	5	4	6	5	150
Conservation of Angular Momentum (Algebra)	4	7	10	7	159
Conservation of Angular Momentum (Calculus)	8	4	10	3	125

theorem, and conservation of angular momentum after the in-class administration of the tutorial to further understand how students worked on these tools individually. These students were exposed to this material in either a calculus-based or algebra-based introductory physics class. For each of the three tutorials examined in one on one interviews after the in-class administration, students began by working on the tutorial problem to the best of their ability without the scaffolding provided by the tutorial. Students then worked through the tutorial as outlined earlier. Finally, students worked on the paired problem used to evaluate the effectiveness of each tutorial. Throughout this process, students were asked to talk aloud so that researchers could understand their thought processes related to solving each of these problems. An interviewer was present to provide materials and instructions. The interviewer remained silent while the students worked unless they became quiet, in which case the interviewer prompted students to continue talking. After each student had completed all material related to one tutorial, the researcher asked clarifying questions for any issues that the students may not have made clear on their own.

4.6.1 Newton's Second Law

In the Newton's second law tutorial problem, three blocks, each of which is tied to the next, are placed upon an inclined plane. The topmost of these three blocks has a given force applied to it upwards along the plane. The student is asked to determine the acceleration of the middle block and the tension in each of the two ropes attaching the blocks together. The most common difficulty that students had in solving this problem was due to students using an incorrect form of Newton's second law to solve the problem. In particular, students used equations which either

included forces that were not acting on whichever block they were treating or failed to take into account either a force acting on said block or its acceleration (assumed it was a static equilibrium problem). These difficulties are often the result of difficulty with Newton's second law.

As the students worked on the tutorial, one common difficulty is best illustrated when examining one student's reasoning as he worked through the sub-problem shown in Figure 4.1. The student stated "So the force is applied in this direction [indicates the direction of applied force]. Ummm... and then the... mg is pointed straight downward and F ... the normal force opposes that so the answer will be C". By selecting "C" the student was choosing one of the incorrect options that included the normal force pointing opposite the direction of the gravitational force. This selection led the student to a hint which provides feedback about the

3. Which one of the following free body diagrams is correct for the system of three blocks?

Where $M = M_1 + M_2 + M_3$
 Normal Force is \vec{F}_N

(The lengths of the arrows do not represent the magnitudes of the forces here.)

A:

B:

C:

D:




Figure 4.1. Sub-problem 3 from the Newton's second law tutorial.

normal force always acting perpendicular to the surface upon which the object rests and that it is opposite to the direction of gravitational force only in certain cases. This instruction is enough for students to learn about this issue and carry on with the problem.

Another sub-problem that gave many students difficulty asks them to break down the gravitational force into two orthogonal components (one along the surface of the plane and one perpendicular to the plane's surface). All of the students interviewed correctly determined the axes along which $mg\sin\theta$ and $mg\cos\theta$ should be used though several students had issues with whether each of these components should be positive or negative along their respective axes. For each of the two multiple choice answers available for these cases, the scaffolding and guidance provided was enough for all interviewed students.

The next difficulty that several students had with this tutorial was demonstrated in the sub-problem shown in Figure 4.2. Since mass m_1 is connected to only one rope, the equation obtained from applying Newton's second law to the axis aligned along the incline will include one unknown, while that generated for mass m_2 , which is attached to two ropes would contain two unknowns. Thus, the use of mass m_1 is easier to use to find the tension T_1 . One student said "I think you can choose either. They have the same component, it's just that one has T_1 going one way, in the other T_1 is the other way." This illustrates a common difficulty students have when determining how they should choose the simplest system in this type of problem solving process. These students showed difficulty identifying factors that would make it easier to apply Newton's second law to solve for the unknowns. If students select an answer that states that both are equally easy, the feedback helps them think about the fact that only one of these two masses

10. Free-body diagram(s) for which object(s) would provide equations that would be easier to solve for tension T_1 ?

- A : M_1 only
- B : M_2 only
- C : Either M_1 or M_2 since both are equally easy to solve for tension T_1
- D : Neither M_1 or M_2 can be used to solve for tension T_1 .



Figure 4.2. Sub-problem 10 from the Newton's second law tutorial.

has just one unknown force acting along the inclined plane, which will make it easier to apply Newton's second law to that mass to solve for the unknown.

Students also had difficulty identifying the free body diagram for mass m_2 . The most common difficulty that students had in identifying the correct free body diagram was determining the direction of the tension forces, T_1 and T_2 . Upon a quick inspection of the available free body diagrams in the multiple-choice sub-problem, several students selected the free body diagram that was correct with the exception of the directions of the tension forces switched with one another. This incorrect selection was most commonly due to students not evaluating the available answers carefully. After students were provided feedback, they were able to realize their mistake and choose the correct answer.

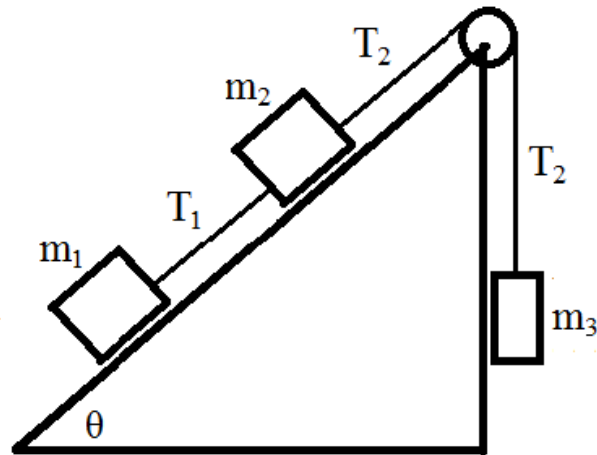


Figure 4.3. The diagram provided for the paired problem on Newton's Second Law.

The paired problem associated with this tutorial is:

Figure 4.3 shows two blocks on a frictionless plane with an angle of inclination of θ and a third block hanging as shown. The blocks are connected via massless cords. The cord that connects m_2 and m_3 goes around a frictionless, massless pulley. Find T_1 , T_2 , and the magnitude of the acceleration of the blocks in terms of m_1 , m_2 , m_3 , and angle of inclination θ . (Assume that the masses are such that m_3 moves down and m_1 and m_2 move up the inclined plane.)

Students generally performed quite well on this problem after making use of the tutorial, resulting in an average score of 89 %.

4.6.2 Conservation of Energy

The tutorial problem for conservation of energy (as shown earlier) is considered by all students interviewed to be very difficult when attempted without the scaffolding provided in the tutorial. In this problem, a man is fired from a spring loaded cannon and undergoes projectile motion ending with him impacting with an airbag which provides a retarding force of a constant magnitude (3000 N) against the direction of motion. This problem is given to students in a context rich format (a real world problem). This added complexity caused great difficulty for students when they first read the problem and searched for information relevant for solving the problem. Additionally, due to the length of the problem statement, many students have a hard time determining where to begin this problem without scaffolding. For example, one interviewed student stated “Ya, this paragraph [problem statement] is too long. You get lost in it.” Another student stated “I’m not even really sure where to start with this.” In addition to student difficulties with the length of the problem statement and the fact that the problem is written in a context rich format, several students were confused about which physics principles should be used to solve this problem. One interviewed student stated while working on this problem “... We can find the initial velocity that he is launched out of because we can set these equal to each other [spring potential energy and kinetic energy]. And after we set the initial velocity, we have to use... um. we have to use the kinematics equations which I do not remember.” The reason the student thought that kinematics would be a good approach to solving this problem is partly due to the projectile motion that the man follows after being fired out of the cannon. However, the student was unable to recall how to use kinematics equations to solve the problem.

The first sub-problem in the tutorial also demonstrates the difficulty students had with determining which physics principles should be used to solve this problem. The first sub-problem asks students to identify which physics principle or principles are “most suited” to solving the tutorial problem and offers several physics principles (or groups of several physics principles) as possible answers. The most common difficulty observed on this sub-problem, much like the tutorial problem without scaffolding, is students claiming that kinematics should be used to solve this problem.

Another sub-problem which uncovers another student difficulty is shown in figure 4.3. Students demonstrated several difficulties while answering this sub-problem. The most common among them is that students were overly eager to select the first statement. This is likely caused by students recognizing that this statement is true without determining that it is not a statement of conservation of total mechanical energy. Additionally, several students had difficulty identifying that the second statement is consistent with their definition of conservation of total mechanical energy. One such student claimed that “It’s not number two because the change in potential energy plus the change in kinetic energy equals the final [energy which is] not always zero.” Students have difficulty identifying the second statement as a statement of conservation of total mechanical energy because it is in a different form than the form they most commonly see (the form provided in the third statement).

One of the sub-problems asks if we can use the principles of conservation of total mechanical energy to solve the problem from when the man impacts the airbag until he comes to a stop. Though this problem did not elicit a strong alternative conception amongst the interviewed students, it is illustrative of why repetition of material as well as explicit statement of each step can be valuable in learning tools like these tutorials. While most of the students were able to

answer the question correctly, one interviewed student had difficulty. He stated “I’ll go on a whim and say typically yes you can [use conservation of total mechanical energy]” after spending several minutes unsure about whether or not conservation of total mechanical energy is applicable while the airbag applies a retarding force to the man. After selecting the incorrect options and obtaining feedback explaining why conservation of mechanical energy can’t be used in this situation, the student exclaimed “That’s incorrect because there’s a retarding force which is non-conservative. Therefore, we cannot use the principle of conservation [of total mechanical energy].” before going back to the sub-problem and selecting the correct answer. This sub-problem is valuable to reinforce correct ideas and repair students’ knowledge structure as appropriate.

2. Choose all of the following which are statements of Conservation of Total Mechanical Energy, E .

- i. Energy is not created or destroyed, but can change forms.
- ii. $\Delta U + \Delta K = 0$ where $\Delta U = U_f - U_i$ and U is the potential energy
- iii. $E_f = E_i$ or $(U_f + K_f = U_i + K_i)$

A: i only

B: ii only

C: i and iii only

D: ii and iii only



Figure 4.4. Sub-problem 2 from the Conservation of energy tutorial.

Another sub-problem asks students to find the total work done by the net force on the human projectile as he sinks into the airbag. The question is asked assuming that the man impacts the airbag at a normal incidence (traveling directly downwards into the airbag) and this assumption is justified to the student. The most common issue that students had with this sub-problem is finding the sign of the work done on the man by the net force. The incorrect answer selected most often states that positive work is done by the retarding force and negative work is done by the gravitational force. One student showed this confusion when he stated “Well there has to be an mg . The mg is pushing it down and the retarding force is putting it up... I mean pushing it up. I’m just not sure which one gets the negative sign... I guess the airbag would get the positive.” The scaffolding provided after the student selected the incorrect option was enough to help the student understand why he was not correct.

Another sub-problem asks students to use the work-energy theorem to write an equation that can be used to determine how far the man will sink into the airbag before he comes to a complete stop. This question provides an opportunity to examine another difficulty that makes this tutorial problem challenging for students. Students often have a difficult time identifying the velocity at each of the relevant points in time throughout the problem. This difficulty is primarily the result of the problem consisting of two sub-sections, one using conservation of energy and the other using the work energy theorem. Because the final velocity from the first part of this problem is the initial velocity for the second part of this problem, students are often confused about which velocity to use when solving this sub-problem. This difficulty is exemplified by one student who said “That [the results of the work up until this sub-problem] would make the initial velocity zero and that whole thing [the initial energy] will be zero and that doesn’t make sense.”

We note that this student has difficulty understanding an equation that does not agree with his conceptual understanding of this problem. Reviewing this student's interview up until this sub-problem, we observe that he has a good understanding of what is happening conceptually, but when asked to use the work energy theorem based on his conceptual understanding of the problem, this student as well as several others that were interviewed had a difficult time identifying the correct initial and final velocities.

The paired problem associated with this tutorial is:

In Figure 4.5, a horizontal spring with spring constant $k_1 = 800 \text{ N/m}$ is compressed 20 cm from its equilibrium position by a 4 kg block. Then the block is released. What would be the maximum compression of a spring ($k_2 = 500 \text{ N/m}$) on the 30° inclined plane when the 4 kg block presses against it? Assume that the track is frictionless, and the distance from A to B is 50 cm, where B is the edge of the uncompressed spring on the inclined plane.

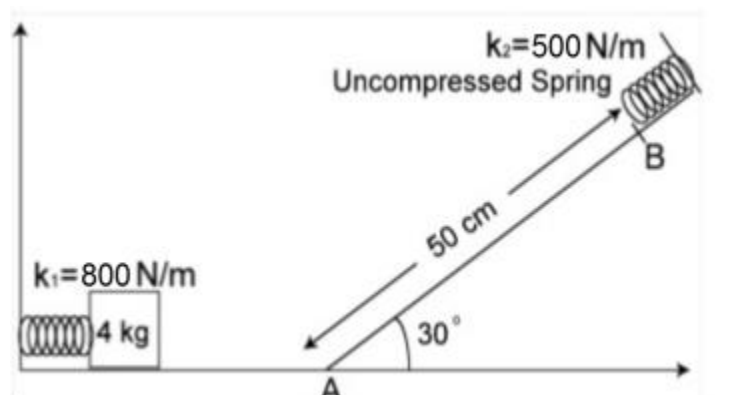


Figure 4.5. The diagram provided for the paired problem on Conservation of Energy.

Because the tutorial problem in this case is more complex than the paired problem, the tutorial provided ample opportunity for students to get practice with conservation of total mechanical energy. Most students in the interview group had little to no difficulty with the paired problem with an average score of 90.7% among interview students.

4.6.3 Conservation of Angular Momentum

The tutorial problem for conservation of angular momentum begins with a bullet impacting a wheel (uniform disk), which was initially at rest, and lodging in its edge. There were several common difficulties that students had when working on this tutorial problem without the aid of the tutorial's scaffolding. Many students were unable to determine the moment of inertia of the system both before and after the collision. Some students attempted to use the equation for the moment of inertia of a disk to find all moments of inertia. Additionally, several students had trouble determining how to find the angular momentum of the bullet initially. Since the bullet is moving in a straight line before the impact, students would often claim that the bullet had no initial angular momentum about the axle of the wheel.

In the tutorial sub-problems, students are asked if the system of bullet and wheel has an angular momentum about a point on the axle of the wheel before the bullet impacts the wheel. This sub-problem illustrates an issue that some students have about the bullet's initial angular momentum. As noted, students often claimed that the bullet initially has no angular momentum about the axle of the wheel. For example, one student stated the following in response to this question "I don't think so because it [the bullet] is not rotating". This student showed difficulty

conceptualizing an object having angular momentum if it is not moving in a circular path or rotating.

This difficulty is also addressed in the next sub-problem which builds on the prior sub-problem by asking students to find the magnitude of the angular momentum of the system of wheel and bullet about a point on the axle just before the collision. One student selected zero as the initial angular momentum saying “So, I guess it would be zero because the bullet hasn’t hit it [the wheel] yet.” The student’s difficulty with seeing the bullet and wheel as a single system with an angular momentum initially possessed entirely by the bullet shows that he has difficulty identifying the angular velocity of an object moving in a straight line. Additionally, this quote illustrates a difficulty that this student has with conservation of angular momentum. The student stated that the wheel has no angular momentum initially and that the wheel will spin after the bullet is lodged in it. Despite this prediction, the student claimed that the initial angular momentum of the system is zero. By claiming that both of these statements are true, the student is using inconsistent reasoning in applying conservation of angular momentum.

This tutorial also asks students to find the combined moment of inertia of the bullet and wheel about the axis of symmetry of the wheel after the collision, assuming that the bullet is a point mass (of mass m_b). Both $\frac{1}{2}m_b R^2$ and $m_b R^2$ were commonly selected incorrect answers for this sub-problem. The reason for this is demonstrated by the following response “I think I just remember this as an equation that we use” as he chose $\frac{1}{2}m_b R^2$. This student like several others made his selection based solely on his memory of an equation that he had used for moment of inertia rather than by considering each element present in the final system separately and summing their individual moments of inertia. These types of responses suggest that some

students think of physics problem solving as predominantly based on memorizing equations rather than on deliberate applications of physics principles and concepts.

The paired quiz problem for conservation of angular momentum includes two questions. The first paired problem associated with this tutorial is:

Suppose that a merry-go-round, which can be approximated as a disk, has no one on it, but it is rotating about a central vertical axis at 0.2 revolutions per second. If a 100kg man quickly sits down on the edge of it, what will be its new speed? (A disk has a moment of inertia $I = (1/2)mR^2$, mass of merry-go-round = 200kg, radius of merry-go-round = 6m)

The most common difficulty that students had with this question, much like the tutorial problem, was difficulty finding the final moment of inertia after the collision. Other than this difficulty, most students were able to solve the problem.

The second paired problem associated with this tutorial is:

A 20kg boy stands on a small stationary (at rest) merry-go-round near the edge of the merry-go-round. The total moment of inertia of the system of merry-go-round with the boy on it about the center is 120kg m^2 . The boy at the edge of the merry-go-round (radius of 2m) jumps off the merry-go-round in a tangential direction with a linear speed of 1.5m/s. What is the angular speed of the merry-go-round after the boy leaves it?

There are two difficulties that students commonly have with this problem. The first is best demonstrated by the following exchange between a student and the interviewer

Student: "I'm a little confused if, like, the angular velocity is like zero."

Interviewer: "You're confused as to whether it is [zero] or not?"

Student: “I think it is [zero] because it’s at rest but then that means that the final [angular momentum] has to be zero because of conservation of angular momentum. So whatever it’s like the momentum that the boy has and the wheel has, has to be zero... but the wheel is moving later. I don’t know.”

This exchange shows that this student has oversimplified conservation of angular momentum to mean that if the initial angular velocity of a system is zero, then the angular velocity of each of its components must always be zero. The second issue that students had with this question manifests in students’ attempts to find the moment of inertia of the boy and the merry-go-round. The problem statement provided the moment of inertia of the system of the merry-go-round and boy, the radius of the merry-go-round and the mass of the boy. Students must find the moment of inertia of the merry-go-round by calculating the moment of inertia of the boy and subtracting it from the moment of inertia of the system (which was given for this problem). Several students had difficulty in either calculating the moment of inertia of the boy or in subtracting this moment of inertia from the moment of inertia of the system to find the merry-go-round’s moment of inertia. Due to these difficulties, the average score on this paired problem quiz was 77.9% among interview students.

4.6.4 Comparative Effectiveness

The purpose of performing the individual interviews after the in-class implementation was not only to examine how the final version of the tutorial helps students learn physics but also

Table 4.19. Comparison of the average paired problem scores for students in the interview group as opposed to those in the two in-class implementation groups (self-study groups).

Physics Principle	Interview Group	Calculus-based class in-class Implementation	Algebra-based class in- class Implementation
Newton's Second Law	89.0%	77.5%	53.9%
Conservation of Energy	90.7%	81.8%	46.9%
Conservation of Angular Momentum	77.9%	69.1%	53.9%

to gauge the effectiveness of these interventions when administered in a controlled environment where a researcher can verify that the tutorials are being used as intended. Comparing the performance of students in the interview group to those in the in-class implementation group who self-reported (Table 4.19) that they had made use of the tutorial we find a considerably higher score in the interview group for all three tutorials tested.

One possible reason for these improved scores among the interview group, as compared to either the entire algebra-based or calculus-based class is ineffective approaches to using the tutorial and inaccurate self-reporting of tutorial participation. Students were made aware (by way of e-mails, announcements on the course web-page and a description of this project given to them verbally during their regularly scheduled class time) that working on the self-paced tutorials posted on the course website is voluntary in that working on the tutorials would not affect their grades. However, they were told that working on the tutorial may improve their homework and quiz performance. Upon examining student comments and other data gathered with their answers to the paired problems, it appears that some students who claimed to make use of the tutorial did not use it effectively. Many students commented that they “skimmed” or “looked over” the tutorial but such activities may not be helpful. Moreover, many students’ self-

reported time spent working on this tutorial seemed inconsistent with the time taken by students in interviews. For many students in the self-study group, the time they spent working on the tutorial according to self-report is often considerably lower than the time spent by the quickest students in one on one interviews. Other students reported that working on the tutorial took them a significantly longer time compared to interviewed students. This is particularly common for the conservation of angular momentum tutorial, which is the shortest. This tutorial took most interviewed students 15-30 minutes but some students in the self-study group reported spending up to 1.5 hours on this seven question tutorial. These students, both at the high and low end, are likely to have either not worked on the tutorial at all or may have engaged with it in a manner not conducive to learning.

As noted, it is likely that many of the students who self-reported as having engaged with the tutorials did not follow the protocol provided and therefore their performance did not reflect significant learning. Those who reported having “skimmed” through the tutorials most likely did not engage with each of the individual sub-problems as the protocol had suggested they do. Additionally, these students may not have attempted to solve the tutorial problem on their own without the scaffolding provided by the tutorial. The act of struggling with the tutorial problem before starting to work on each of the sub-problems in the tutorial can aid student learning. Additionally, struggling with the tutorial problem before engaging with the tutorial may increase students’ motivation to engage appropriately with the tutorial as the protocol outlined. Another issue that may have affected student learning from the tutorial is engagement with the tutorial. For some students, it is possible that engagement is high as students begin the tutorial but drops off as they work on each sub-problem, as instructed. It is possible that then these students may make a transition to using ineffective strategies for learning from the tutorial rather than

engaging with each sub-problem appropriately. This type of difficulty is likely to occur for a longer tutorial.

4.7 CONCLUSIONS

The Conservation of Angular Momentum tutorial was the only tutorial that caused a statistically significant improvement in student scores on the paired problem for both in-class implementations in algebra-based and calculus-based physics classes. We should reflect on what characteristics might have made it more effective than the other three tutorials as a self-study tool so that these characteristics can be used in the development of other self-study tools. One major difference is the length of this tutorial. The Conservation of Angular Momentum tutorial is quite short, consisting of seven sub-problems, while the other three tutorials have an average length of roughly 18 sub-problems. This was partially due to the tutorial problem being succinct compared to the other problems while still requiring the application of conservation of angular momentum. This shortened tutorial could be more effective in keeping the student engaged as a self-study tool while the longer tutorials may cause students to lose focus or experience cognitive overload.

One interesting pattern that was observed is that the Lightly Scaffolded prequiz resulted in, at best, no statistically significant improvement in student problem solving ability over the Unscaffolded prequiz and quite often resulted in decreases of the average score of students for the prequiz and associated paired problem. It appears that the light scaffolding used in the quiz acted as a distraction rather than an aid. The students in this intervention may have become

overly fixated on answering each of the scaffolding questions rather than on using the scaffolding questions to help them work towards a solution for the tutorial problem. Another possible reason for the light scaffolding's ineffectiveness may be that the large number of additional instruction might have caused cognitive overload.

While the three tutorials examined in one on one think aloud interviews were effective for students, only one of them was effective outside of these interviews for both calculus-based and algebra-based classes. The fact that students had to follow the correct protocol and start by solving the problem without any help on their own, followed by working on the tutorial as intended may have contributed to the success of the tutorials in one-on-one interviews. Moreover, the act of thinking aloud and being provided with a piece of paper may have increased the effectiveness of the tutorial for some in the one on one interviews. On the other hand, the lack of effectiveness when students used the tutorials as a self-study tool is likely due to students engaging with the tutorial in ways other than those outlined for them even when they used it as a self-study tool. Despite the encouragement, it is difficult if not impossible to ensure that students take an effective approach to working on these tutorials as a self-study tool outside of a controlled environment. Without students first attempting the tutorial problem and then engaging with the tutorial by deliberately thinking about and attempting to answer each sub-problem before looking at the scaffolding and feedback, the tutorial's effectiveness will be diminished. Each step in this process is designed to help students recognize and resolve any difficulties that they have while strengthening their knowledge of physics principles as they work on the tutorial problem. Students who refuse to use this deliberate approach outlined for them when engaging with these self-study tools are unlikely to benefit much from them.

The ineffectiveness of the tutorials when used as a self-study tool when they prove to be effective in individual one on one administration is analogous to the difficulty in making online tools effective for a diverse group of students. For example, studies typically find a high attrition rate in Massive Open Online Courses (MOOCs) [37-39]. These MOOCs commonly result in very few (roughly 5%) of the students who initially sign up completing the course. This may partly be due to students lacking the self-regulation skills, motivation or discipline to manage their time effectively to learn the material. This claim is further supported by the majority of those who complete the MOOC already possessing bachelor degrees, and thus having appropriate skills to take advantage of the self-study tools. A lack of self-regulation, motivation, discipline and time-management skills while engaging in learning outside of a classroom setting may turn out to be the biggest impediment in implementing these carefully designed research-based tutorials as self-study tools.

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5.0 FUTURE CONSIDERATIONS

In this thesis, we discussed the development and evaluation of research-based tutorials that range from introductory to graduate level physics. They were developed based upon findings of cognitive research and physics education research. Each of the tutorials developed and evaluations discussed provides an opportunity for further research. In the next few sections, I will briefly outline some possible avenues for continued research with each of these tutorials discussed.

5.1 INTRODUCTORY TUTORIALS

For the four tutorials developed and evaluated for introductory physics, further research opportunities are emergent from the results discussed in this thesis. These showed limited effectiveness when students were asked to use them as a self-study tool as compared to a markedly higher effectiveness when administered in one-on-one, think aloud, interviews. One possible approach to investigating the reason for this discrepancy would be to implement the tutorials in a different form which does not allow the students to rush through the tutorial and forces them to systematically answer each question in the order in which they are structured (e.g.

working through the tutorials in recitation with students working together in small groups). It would also be useful to have a form in which we can monitor the time students spent on each component of the tutorial. In particular, if the tutorials were administered to students in a way that would monitor the time that students spent on each sub-problem before selecting an answer, it would be possible to determine if students attempted to solve each sub-problem appropriately before selecting an answer. Additionally, it would be valuable to ensure that students are attempting to solve the problem on their own before working on the tutorial and answering each sub-problem as they were instructed rather than merely reading each slide as though the tutorial was a set of lecture slides (e.g. give the tutorial problem as a quiz in recitation before giving the tutorial). The ability to control for the way students interacted with the tutorials would allow for a more accurate measurement of the effectiveness of the tutorials as self-study tools.

Moreover, as discussed earlier, the lightly scaffolded prequiz resulted in no statistically significant improvement over the unscaffolded prequiz. This result is somewhat surprising because the scaffolding present in the prequiz is based on the scaffolding shown to be effective in other interventions in physics education research (though this was shown to be effective when implemented constantly over the course of a semester). One way to further examine the potential for this type of scaffolding to be effective is to either add additional hints to the lightly scaffolded prequiz to better guide students in the effective use of the scaffolding or explain to students the value of this scaffolding prior to them taking the prequiz. Moreover, one could ask students to work on homework assignments throughout the semester by using the same scaffolding to allow them continued practice with this type of scaffolding. Each of these modifications may aid students in effectively using the lightly scaffolded prequiz.

5.2 QUANTUM KEY DISTRIBUTION QuILT

Investigating the effectiveness of the two part QKD QuILT we found that the two secure protocols in the two parts were effective in improving student understanding of QKD after working on the QuILT. One possible extension of this research is to investigate the effectiveness of each of these two protocols in the QuILT on its own. One may develop an additional pretest and posttest for the QKD QuILT Part II as well as a final exam question to evaluate the long term understanding of the B92 protocol in Part I of the QuILT. By evaluating the effectiveness of each of these two parts of the QuILT both in the short term and long term, we could draw conclusions about the relative effectiveness of an in class tutorial that students worked on in groups as compared to an out of class tutorial that students worked on as part of the homework with the aid of a computer simulation. It is even possible that if only one of the protocols is taught to the students, their retention of the protocol would improve because most of the difficulty discussed in this thesis related to retention of the BBM92 protocol was due to the mixing of certain components of the B92 and the BBM92 protocols.

Moreover, the development of a simulation similar to that used for the QKD QuILT Part II and evaluating its effectiveness is another possible future extension of this work. With this simulation, one could also implement each of these two tutorials as a self-study tool with the simulation used to reconcile the difference between the predictions and the observations and aid students when they are faced with difficulties while working through the tutorial. This would also allow further comparison of the effectiveness of these two parts of the tutorials in improving student understanding of relevant concepts. This would also allow us to evaluate if one of these two QKD protocols is more easily accessible to students due to the difference in the quantum

mechanics principles utilized in the protocols (non-orthogonal polarization states of single photons or entangled states of two spin- $\frac{1}{2}$ particles).

5.3 LOCK-IN AMPLIFIER TUTORIAL

The LIA tutorial was shown to be effective in improving student understanding when given to students in one-on-one interviews. One possible continuation involves a more in depth evaluation of the effectiveness of this tutorial with students who have no experience with LIA compared to those who have some experience with this device before using the tutorial. This would require performing more interviews with students who have no experience with the LIA, limited experience with the LIAs and extensive experience with the LIA.

Additionally, the effectiveness of this tutorial as a self-study tool could be evaluated. This investigation could be performed by administering students the pretest and posttest in an interview or classroom environment and allowing them to make use of the tutorial on their own time. It would be of value to compare the effectiveness of the LIA tutorial when used as a self-study tool with its effectiveness when used in a think-aloud interview. Due to the significantly higher effectiveness of the introductory tutorials when used in an interview setting, in which the interviewer can ensure that the students are correctly following the instructions provided in the tutorial for how to use it, it would be valuable to investigate whether a similar trend is observed with students using ineffective approaches to learning from the LIA tutorial when used as a self-study tool. Fortunately, initial reports of the effectiveness of the LIA tutorial when used as a self-study tool by students with no prior experience with the LIA have thus far been promising. It is

possible that the effectiveness of LIA as a self-study tool is partly due to the fact that advanced students engage with self-study tools in more meaningful ways than students in the introductory physics courses who generally have significantly less knowledge of physics and also have lower motivation to learn physics..

Another further extension of this work could be to investigate the long term effectiveness of this tutorial for students who make use of the LIA in the lab. This could be done by following up with each of the students who have made use of the LIA tutorial in a think aloud interview. One effective way to evaluate this would be with a survey asking students about their experiences using the LIA in a lab setting since using the tutorial. In this way, we could evaluate the aspects of the tutorial that were most successful at improving student understanding and troubleshooting related to the LIA in their lab work, which is the intended goal of this tutorial. Additionally, we could investigate in more depth the responses of students who had varying levels of experience using the LIA before working on the tutorial. This would allow us to see if different aspects of the tutorial appealed preferentially to students who are new to the LIA and those with some prior experience using the LIA.

5.4 OTHER CONSIDERATIONS

Another possible extension of this research as alluded to earlier comes from the observation that students on average gain less from using the introductory tutorials as a self-study tool, as opposed to when they use them in a one-on-one interview in which they must follow the protocol for using the tutorial as intended. It would be of interest to compare the relative effectiveness of a

tutorial for upper-level undergraduate and graduate students when administered in several different ways (in-class, in one-on-one interview, as a self-study tool). This investigation would allow us the opportunity to observe if this limited effectiveness of the tutorials as a self-study tool is only an issue in introductory courses or if it is also typical for more advanced undergraduate students and graduate students to not benefit from it as much when used as a self-study tool partly because they do not follow the guidelines for using the tutorial effectively in self-study and partly because of other distractions and obstructions. On the other hand, if tutorials are reasonably effective as self-study tools for more advanced students then this finding may prompt a further investigation related to the reason for the discrepancy between the effectiveness of self-study tutorials for introductory and advanced physics. One likely cause for the difference in performance among advanced students as compared to introductory students when using the tutorial as a self-study tool could be their self-monitoring skills and motivation to learn physics in addition to the existing knowledge of physics.

EXAMPLE OF THE PRETEST/POSTTEST USED TO EVALUATE THE EFFECTIVENESS OF THE LOCK-IN AMPLIFIER TUTORIAL

1) Sketch the output characteristics on the displays outlined in red, based on the given input settings.

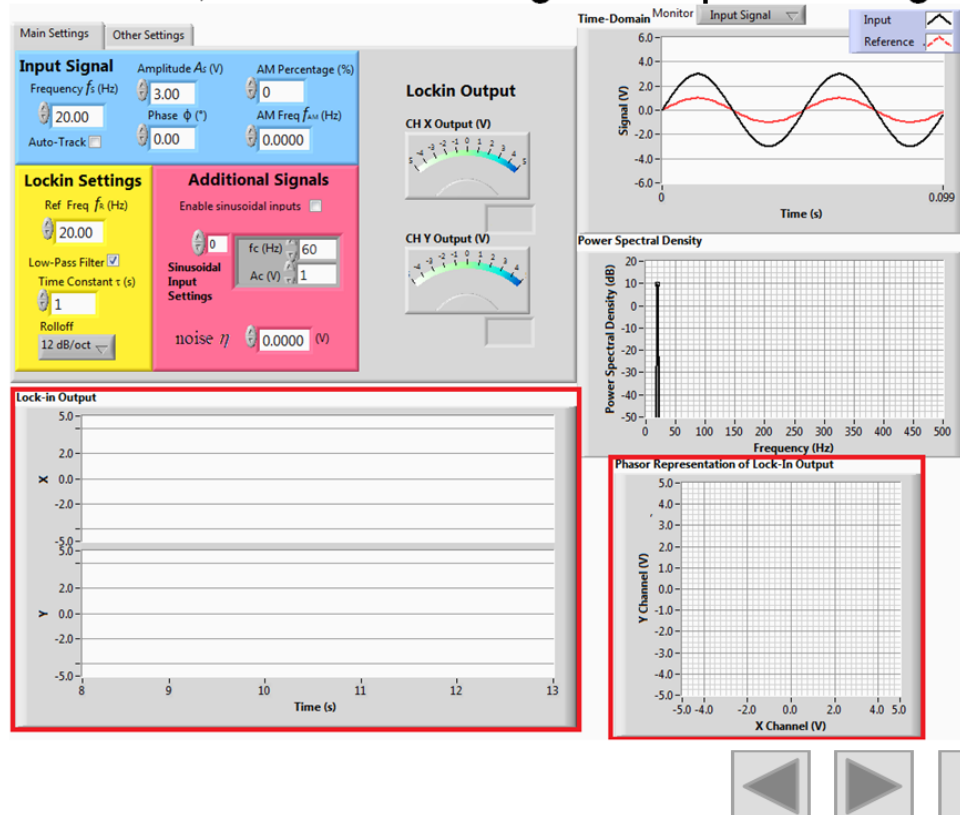


Figure 6.1. An example of case 1 in which students must predict the output signal.

2) Sketch the output characteristics on the displays outlined in red, based on the given input settings.

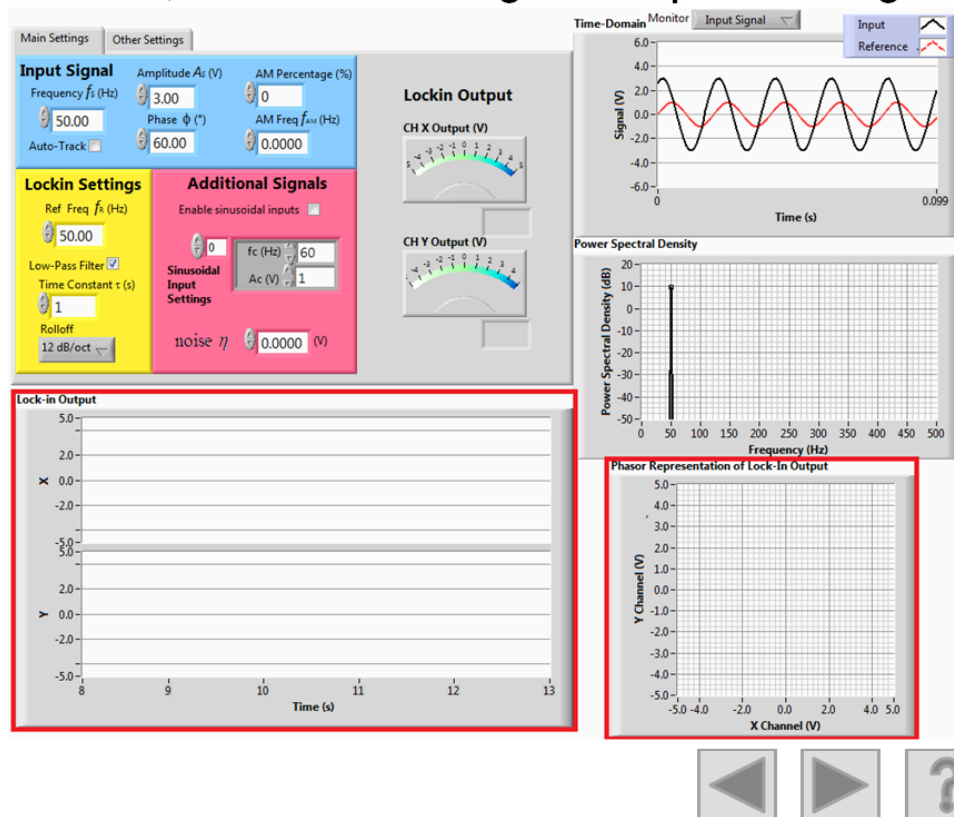


Figure 6.2. An example of case 3 in which students must predict the output signal.

3) Sketch the output characteristics on the displays outlined in red, based on the given input settings.

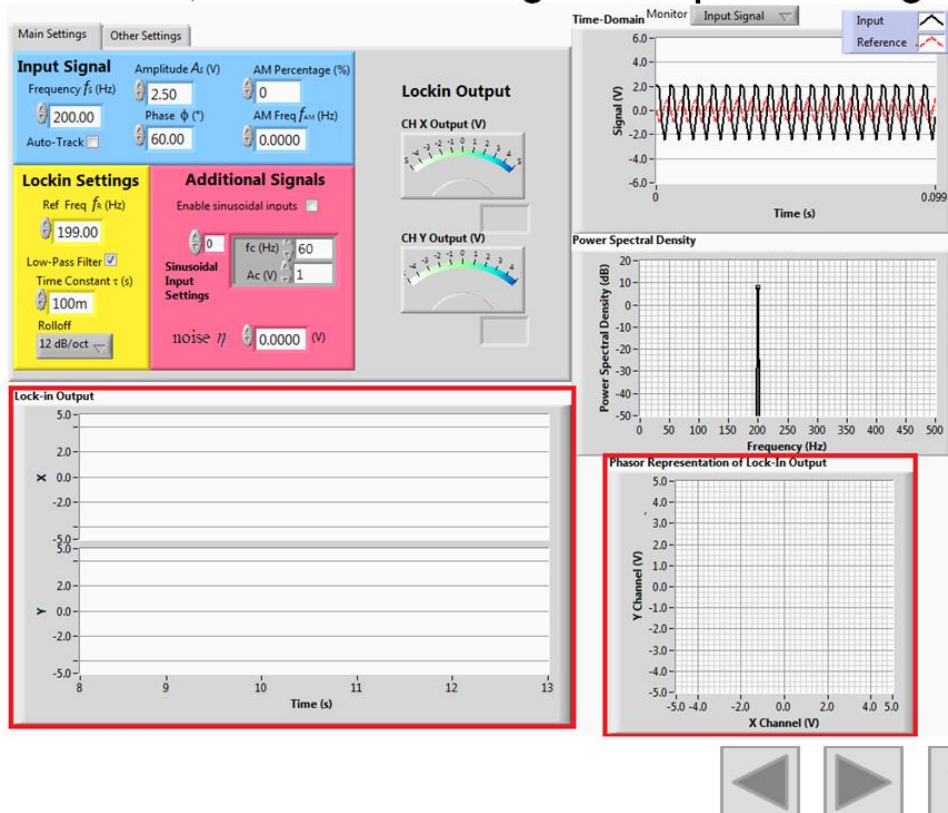


Figure 6.3. An example of case 2 in which students must predict the output signal.

4) Sketch the output characteristics on the displays outlined in red, based on the given input settings (note: an additional sinusoidal input signal is present).

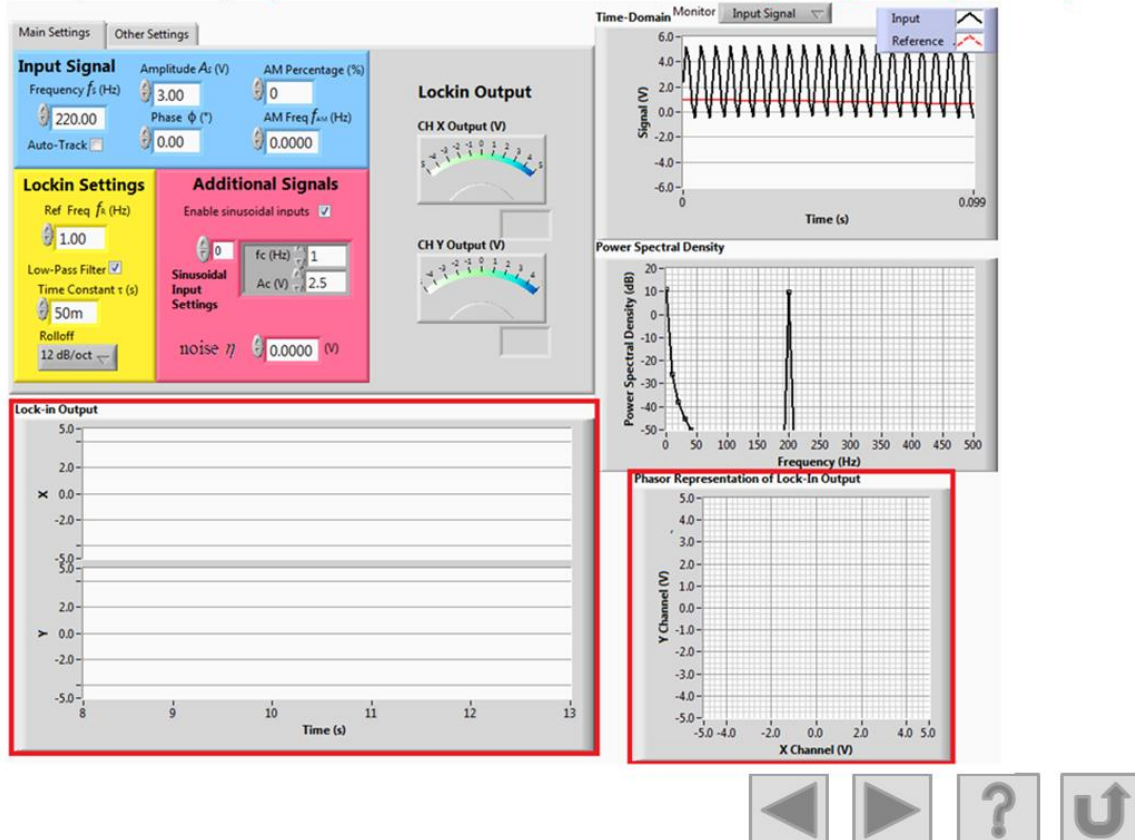


Figure 6.4. An example of case 4 in which students must predict the output signal.

5) Sketch the output characteristics on the displays outlined in red, based on the given input settings.

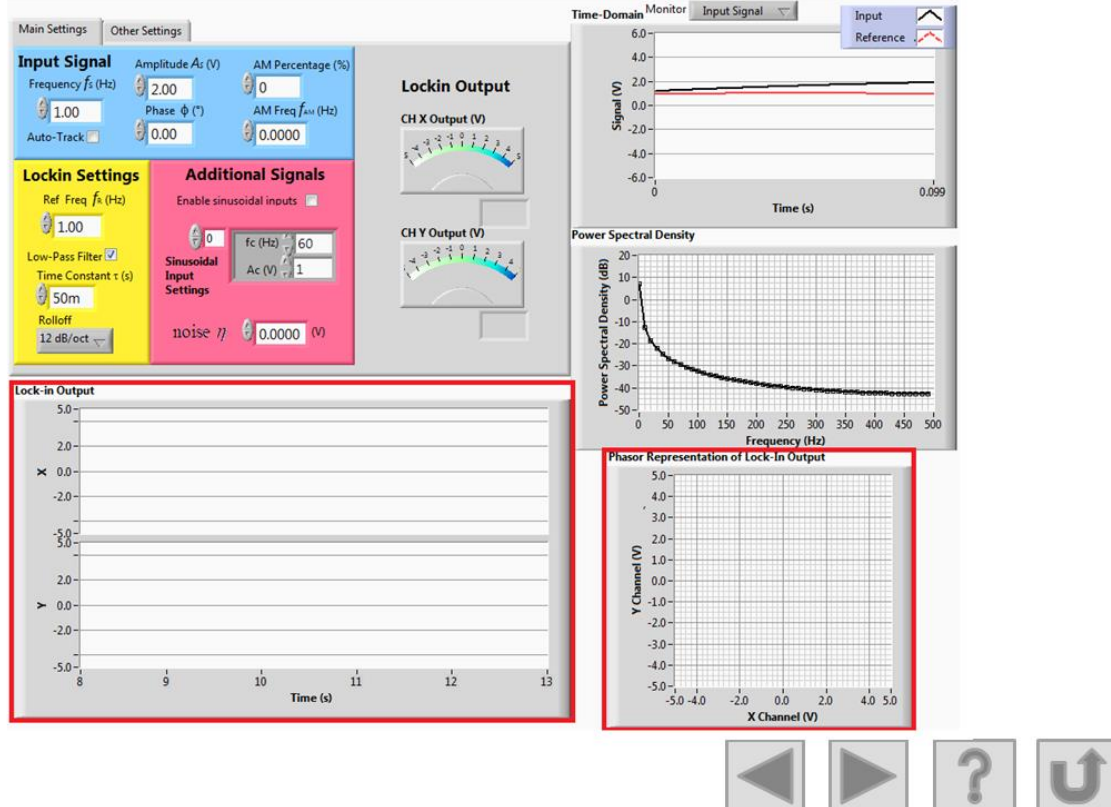


Figure 6.5. An example of case 4 in which students must predict the output signal.

6) The lock-in amplifier's output is as shown below. Fill out the blank portions of the input parameters (outlined in black) that would make this output possible. (You need not fill out the Additional Signals section if only one input signal is necessary.)

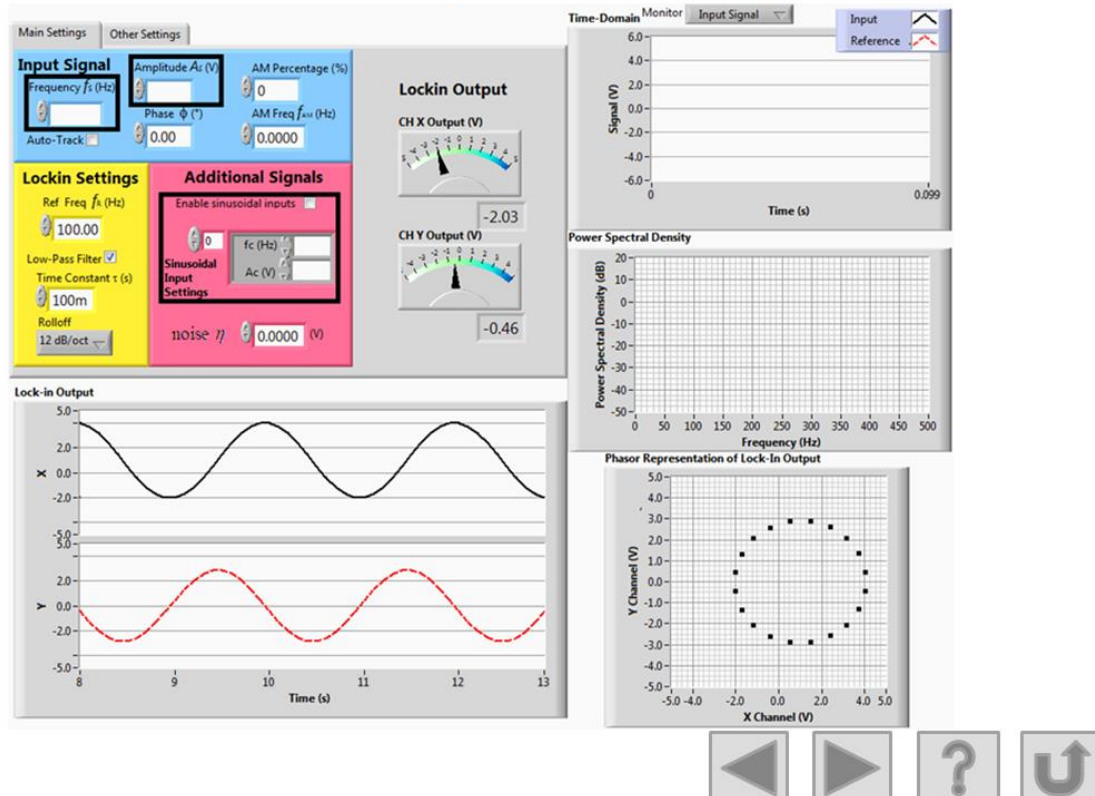


Figure 6.6. An example of case 6 in which students must predict the input parameters.

7) The lock-in amplifier's output is as shown below. Fill out the blank portions of the input parameters (outlined in black) that would make this output possible. (You need not fill out the Additional Signals section if only one input signal is necessary.)

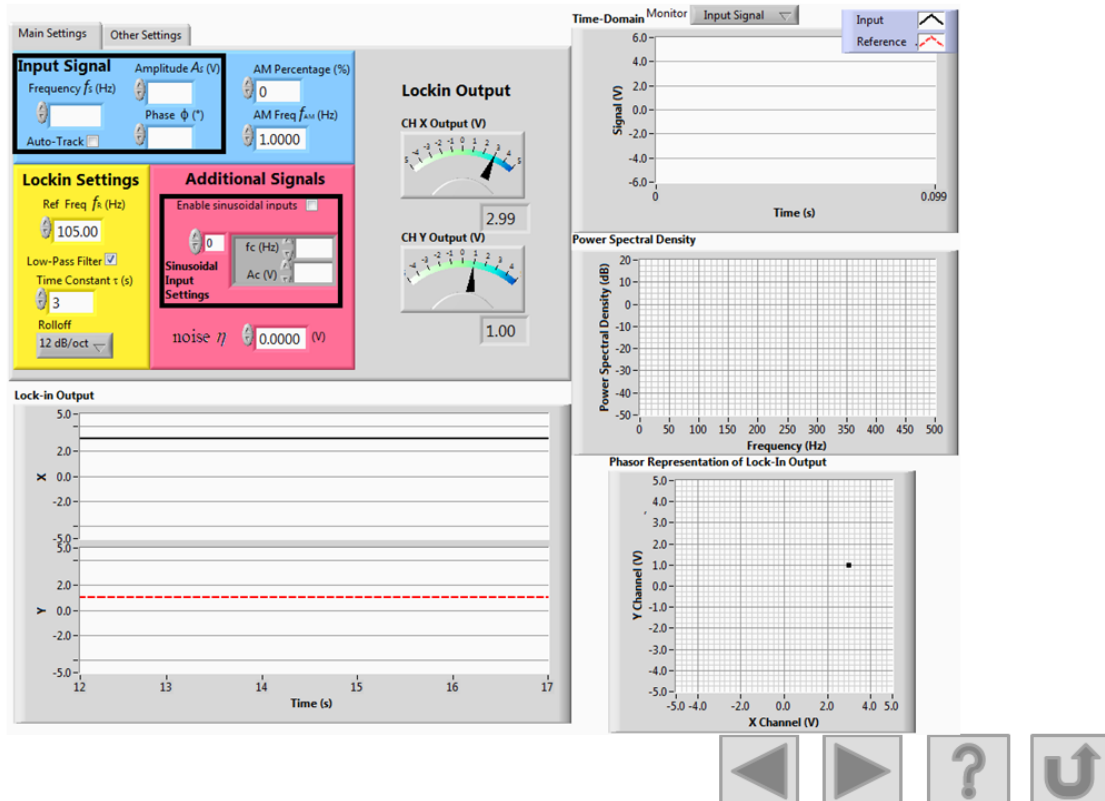


Figure 6.8. An example of case 3 in which students must predict the input parameters.

8) Sketch the output characteristics on the displays outlined in red, based on the given input settings (note: amplitude modulation is present).

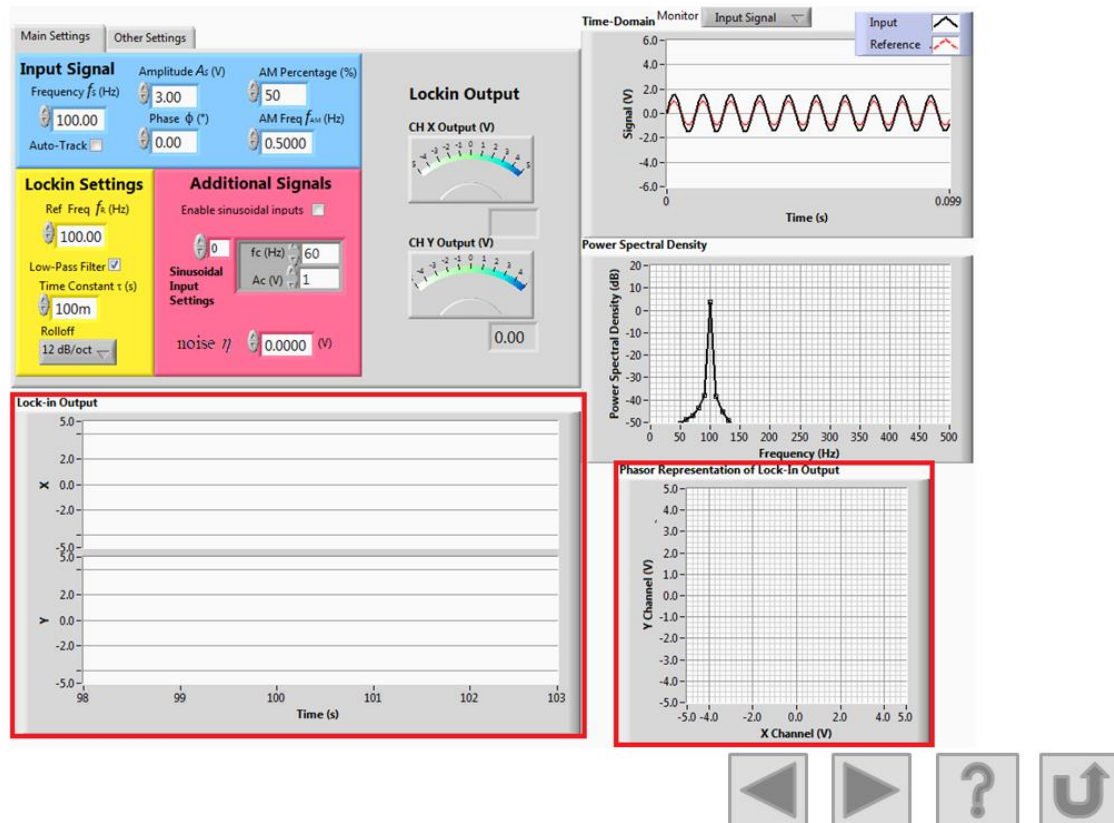


Figure 6.9. An example of case 5 in which students must predict the output signal.

9) The lock-in amplifier's output is as shown below. Fill out the blank portions of the input parameters (outlined in black) that would make this output possible. (You need not fill out the Additional Signals section if only one input signal is necessary.)

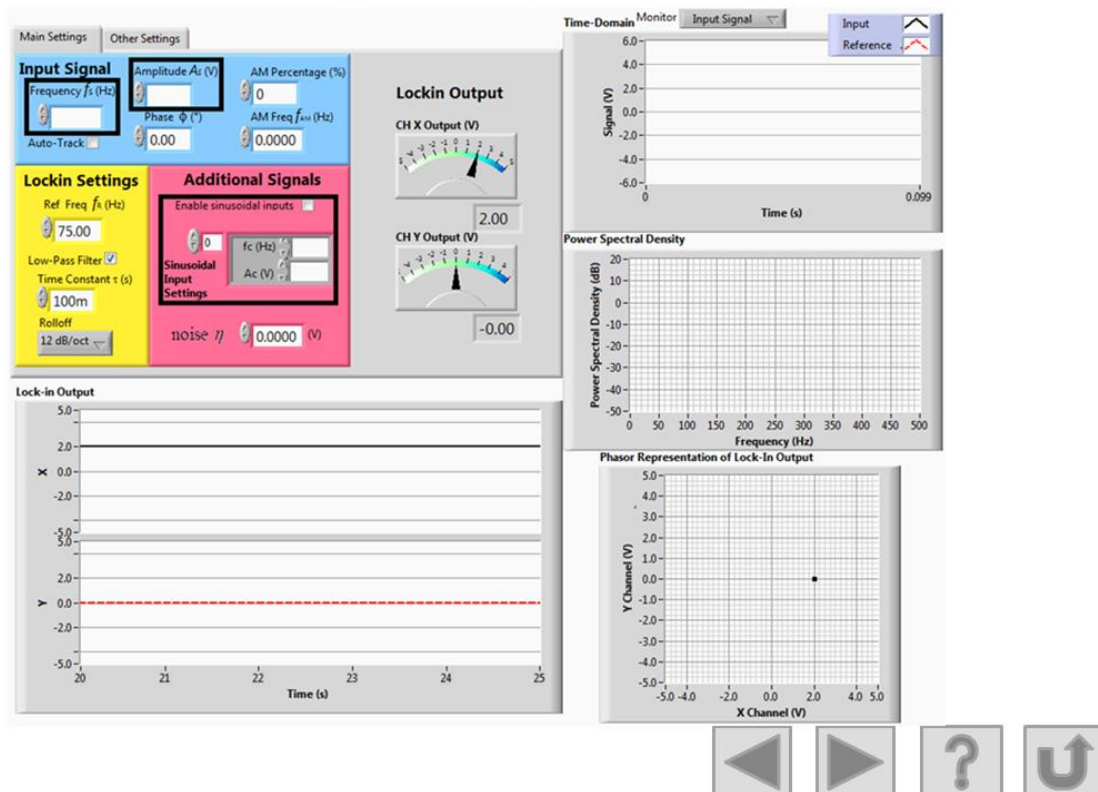


Figure 6.10. An example of case 1 in which students must predict the output signal.

10) The lock-in amplifier's output is as shown below. Fill out the blank portions of the input parameters (outlined in black) that would make this output possible. (You need not fill out the Additional Signals section if only one input signal is necessary.)

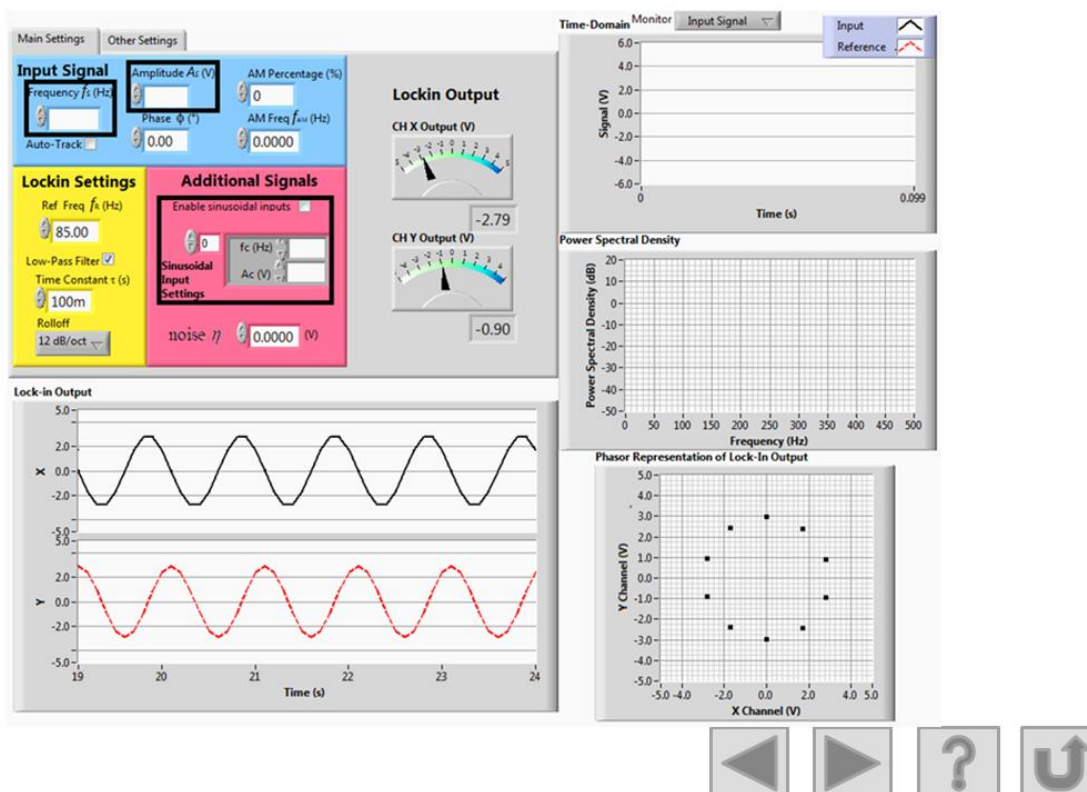


Figure 6.11. An example of case 2 in which students must predict the input parameters.