THREE BUSINESS ANALYTICS ESSAYS ON TRANSPORTATION MANAGEMENT

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Transportation service has a major impact on economic development and growth. Efficient management in transportation allows organizations to handle complicated situations. The studies in my dissertation focus on developing novel methodologies, strategies and decisions to help address three different practical demands from organizations by using business analytical tools. Although we present our methodologies in three particular business contexts, the frameworks of three essays can be easily generalized to other industries.

The first essay is to explore the strategy for global market expansion of a private air medical company. We assess the global medical aviation market and identify the most suitable regions for the company. We combine the Analytic Hierarchy Process (AHP) and the Grey Number theory (GN) to analyze the potential foreign market. We evaluate all countries and areas in the world and make our recommendations through our novel AHP-GN model.

The second essay targets a booming industry-the China express delivery industry. With rapid development of e-commerce in China, its express industry has experienced phenomenal growth in recent years. Investors are particularly eager to discover how to gain a better understanding of the market and compare between operating express delivery firms to understand their respective strengths and weaknesses. We investigate the top 12 express delivery companies in China and evaluate their independent business performance. The Analytical Network Process (ANP), a multi-criteria decision-making methodology, is used to develop an

evaluation framework. In addition to the ANP method, we employ the center-point triangular whitenization weight function to convert uncertain information into a unique value and rank the 12 express delivery companies.

The third essay studies the car-sharing industry. The three fundamental management issues in car-sharing industry are: 1). Branch Station Location Selection; 2). Station Size/Capacity; 3). Strategies for imbalance of vehicle distribution at each station. In this study, we develop novel approaches to address these questions. Our models require few inputs and offer quick analytic results. Application of the models to the Zipcar setting is used to illustrate our models and to derive managerial insights.

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1.0 INTRODUCTION

Transportation is an essential part of a society. It plays a significant role in the functioning of modern economy and has a major impact on the financial performance of businesses. Being able to manage transportation functions effectively is essential for customer satisfaction and firm profitability. Examples of activities in transportation management include logistics, materials handling, industrial packaging, outsourcing, planning, scheduling, and supply chain management. Research on transportation management has a long history. However, along with the development of advanced technologies, globalization and changes in laws and regulation, new business models have been developed to serve the public. These new transportation models may require different management approaches. As transportation costs account for a significant portion of a business' expenditure, firms more than ever before are looking for effective transportation management systems.

This dissertation aims to propose new management models for emerging transportation systems in the modern economy. In particular, it targets three new transportation businesses, i.e. the air medical transport service; e-commerce induced express delivery industry; and the car-sharing business in the US.

The first research is on air medical transport. Air medical transport has gained increasing popularity in last decade. However, the global market of this industry has not been fully explored either in theory or in practice. The first essay in chapter 2 assesses the medical aviation market and identifies the most suitable regions for a private air transport company to expand its business on a global scale. We combine the Analytic Hierarchy Process (AHP) and the Grey Number (GN) theorem to analyse the potential foreign market. Compared with conventional methods, the proposed model mitigates the adverse effects of uncertainty while providing a practical approach that considers management's subjective judgments. The integrated model is comprehensive and flexible for assessing the demand for air medical service in different parts of the world. In addition to the air jet service industry, the proposed GN-AHP model can be easily generalised to evaluate many other markets and industries.

The next research presented in Chapter 3 is on the Express Delivery Industry in China. With the rapid development of e-commerce, the express industry in China has experienced phenomenal growth in recent years. The demand for express delivery services is growing exponentially and has far exceeded public expectation. Many profit-motivated investors are seeking partnership opportunities with express delivery firms so as to effectively enter the industry. Investors are particularly interested in analyzing the market, and compare firms in order to understand their respective strengths and weaknesses. We investigate the top 12 express delivery companies in China and evaluate their business performance. The Analytical Network Process (ANP), a multi-criteria decision-making method, is used to develop an evaluation framework. In addition to the ANP method, we employ the center-point triangular whitenization weight function to convert uncertain information into a unique value, and rank the 12 express delivery companies. Compared with conventional methods, the proposed model can reduce the adverse effects of uncertainty and provide a practicable approach to reflect the sentiments of experts. It is comprehensive, flexible and easily applicable to other industries and markets.

The last research study presented in Chapter 4 focuses on the car-sharing industry in the US. Economic, environmental and social concerns have increased the popularity of car sharing programs. More firms have considered entering this market to satisfy growing demands. Car sharing firms in general face three practical problems: 1) Station location selection; 2) Station size/capacity decision; and 3) Rebalancing vehicles among stations. In chapter 4, we develop two novel models to address these questions. Our models require few inputs and offer quick analytic results. Application of the models to Zipcar is used to validate our models and to derive managerial insights.

These three essays re-investigate classical questions from the strategic level down to detailed operational levels. We propose three new model frameworks, which are different from the perspectives of the literature. As an overview of this dissertation, the subsequent three sub-sections in this chapter focus on introducing the backgrounds of these three industries.

Chapter 2 presents a comprehensive framework for market expansion evaluation and offers specific recommendations for the case company. Chapter 3 presents an extended and more sophisticated model to address uncertain judgments, and then the model is applied to evaluate the China Express Delivery industry for potential investors to prioritize potential business partners. Finally, Chapter 4 discusses car-sharing management issues through control theory and uses a Zipcar expansion scenario to validate the proposed model.

1.1 AIR MEDICAL JETS GLOBAL MARKETS

A private jet company that engages in air medical transport motivated this research. Air jet medical transport has become an essential part of modern healthcare. The studied firm has recently received a license to operate internationally, and as a result, the senior management of the firm must make strategic decisions regarding global expansion. For the sake of confidentiality, the company is referred to using the *fictitious name* Medical Aviation Service (MAS).

MAS is a premier air medical transport provider that offers critical care patient services and specialises in medical transport with fixed-wing aircraft, charter services, aircraft maintenance and aircraft management/sales. It endeavours to achieve high aviation safety, quality and performance standards by focusing on client needs and developing a strategic partnership with the urgent care centre of a world-renowned hospital system. MAS was founded in the early 2000s and has experienced tremendous growth since its inception. In 2013, MAS attained a special operating certificate from the Federal Aviation Administration (FAA) and is now authorised to fly almost anywhere in the world, rather than being constrained within the U.S. domestic market.

Accordingly, MAS is interested in exploring potential markets around the world and to determine from where they could launch their services on a global scale. Given that air medical transports are extremely expensive, to effectively help MAS identify markets and address management concerns, all countries around the world must be evaluated based on their economic data and healthcare conditions. We weigh potential markets and prioritise countries that have a high concentration of wealth combined with relatively poor healthcare environments. For example, markets with many extraordinarily wealthy people and low healthcare service quality are attractive candidates, even if their income distribution is uneven.

Our main contributions include introducing the GN-AHP model, which uses intervals representing uncertain comparison results and carries this interval information throughout the whole process to rank potential markets, to improve the extant methodology and providing a comprehensive, practical, and flexible approach for managers to assess the global market for the private air medical transport business. Our proposed model turns out to be very useful when vagueness and uncertainty are present in judgment and comparison.

1.2 CHINA EXPRESS DELIVERY INDUSTRY

Benefitting from the explosive development of e-commerce, China's express industry has experienced phenomenal growth in recent years. According to the China Express Delivery Report 2013-2016 (2013), in the first three quarters of 2013, private express delivery enterprises in China delivered more than 4.77 billion pieces of mail and parcels, a 69.2% growth from the previous year; the total revenues generated during the same time period were more than \$10 billion, an increase of 51.6% from the previous year. During the "Singles' Day" sales period in 2013, which is known as China's "Black Friday," a total of 323 million packages needed to be delivered in a short period of time; however, China's domestic express delivery capacity, which includes international carriers' China branches, can handle only 50% of that volume. Therefore, this market still has great potential for growth because the entire industry is projected to maintain more than 20% growth in both package volume and revenues over the next 4 years. Investors have recognized this upward market and many have either invested in or are considering entering the express industry. As noted in Deloitte's China Express Sector Development Report 2014, the market share by business volume gained by domestic private enterprise climbed from 35.4% in 2007 to 78.9% in 2013, and the market share by business operating income gained by private enterprises increased from 23.7% in 2007 to 67.5% in 2013. Faced with this golden opportunity, both Chinese and international investors are flocking to the area to gain a share in this promising market. However, the express sector is a capital-intensive industry, so a comprehensive understanding of China's express industry is a must-have for both domestic and international investors, especially those who wish to compare existing express carriers to identify their competitive advantages and disadvantages before they make final investment decisions.

This study was motivated by the needs of one such investor. In China, more than 5000 express business entities are registered in the State Post Bureau of China. Among these registered express firms, many domestic express firms are eager to attract internal or external funds to sustain their business; others need capital for business expansion. Faced with numerous choices, investors need to understand whether a firm is worth the investment and, if so, what part of the firm would benefit from investment.

In the study, we combine the analytic network process (ANP) (Saaty, 1996) and the center-point triangular whitenization weight function (CTWF) (Liu and Lin, 2006) to resolve the disparity in judgments when performing the assessment. The ANP method is a valuable performance evaluation technique and is widely accepted and implemented to improve the evaluation process's efficiency and thoroughness. The proposed CTWF-based ANP model is an integrated multi-criteria decision support framework that is well matched to address our problem. The integrated model also provides a clear and replicable approach to examine firms in different industries.

In addition to the improved methodology, we assess the performance of China's express delivery industry from an investor's perspective, as this has not been explicitly studied. The contributions of this research are therefore twofold:

a). From the perspective of managerial implementations: (i) Our research is the first examination of the performance evaluation on China's express delivery industry. We make use of all public data and carry out an extensive survey. Our results can help investors to better understand each individual company's competitive advantages and disadvantages, and the proposed model can be easily extended to other express companies beyond these 12 firms. (ii) The rankings clearly indicate the differences between the three types of enterprises, i.e., private, state-owned and international carriers. For each type of firm, investors should have different strategies to gauge potential investment opportunities. (iii) Our ranking of 12 major express companies provides valuable information to all potential firms aiming to compete in the Chinese market.

b). From a methodological perspective: (i) We borrow CTWF concept, one from grey number theory, to tackle uncertainty; in particular we used the center-point triangular whitenization function to convert interval information into one unique weight for the ANP model. (ii) The CTWF-based ANP model follows the ANP procedure while providing greater flexibility on subjective weights, with minor add-on computations. The proposed methodology is easy to implement for any similar situation in practice.

1.3 THE CAR-SHARING INDUSTRY

Car-sharing is a type of car rental whereby people lease automobiles for short-term use. Car-sharing companies charge by the hour or by the distance driven. The business model is attractive to consumers who make occasional use of the vehicles. The car-sharing companies make available certain cars in selected locations (stations) to provide service. A customer can make reservations online or by phone; then they can pick up the vehicle at the scheduled time and return the vehicle to either the original location or any other authorized station.

Car-sharing has emerged as an important alternative to public transportation, and its market has expanded tremendously because of many of its benefits (Barth and Shaheen, 2002). Shaheen and Cohen (2014) revealed that car-sharing membership has grown exponentially from 16,000 in 2002 to more than one million in a decade. They pointed out that shared vehicles used in the US have risen from 696 in 2002 to 19,115 in 2014. Navigant Consulting (2013) reported that global car-sharing revenue will expand from $1 billion in 2013 to $6.2 billion by 2020.

A car-sharing company charges rental fees and a monthly fee if available. The rental fee includes a reserved parking space, fuel, and insurance. Thus, it significantly reduces the financial burden of those who are already cash stretched, and it prevents them from paying for unused time and fees that are common with conventional rentals.

Besides adding financial value for consumers, most car-sharing programs are environmentally friendly; i.e., they reduce carbon emissions, air pollution, and traffic jams. According to a San Antonio government report (2011), each car-sharing vehicle can take 4.6~20 cars off the road. Car sharing saves drivers time and money with reserved spaces, avoiding the need to look for parking spots. Because of its environmental and social benefits, Enoch and Taylor (2006) urged governments to offer incentives for car-sharing companies. Several cities in Texas have partnered with the car-sharing company Car2Go, and they also motivate their government employees to use such services by providing free city street parking (Hu 2010).

Zipcar is a US-based car-sharing company and the largest car club service in the world. It provides an alternative to car rental and personal car ownership. Zipcar members only need to pay a fixed monthly fee and rental based on actual driving; gas and insurance are included in the hourly rate. It is a self-service, 24/7 global car-sharing business. Zipcar is attractive to customers, as the stations are often just minutes away from public transportation stops and/or close to high-density population areas; e.g., universities, hospitals, downtown, and airports. Members are able to view vehicle availability and reserve a self-service car via various channels, such as telephone, Internet, and online mobile devices. Members can easily pick up cars at these stations and return them with guaranteed parking spaces. Zipcar has attracted more than 850,000 members all over the world, with more than 10,000 vehicles served in North America (Shaheen and Cohen, 2014).

Our research is motivated by the need to make strategic and operational decisions for Zipcar. Besides vehicle purchasing and maintenance, important strategic decisions faced by Zipcar are: 1) Location decisions. For expansion, management of Zipcar needs to estimate the number of potential users in various locations and select areas with the highest likelihood of success; 2) Capacity/Size decision. Once the location is determined, Zipcar needs to decide how many vehicles are needed in the station and how to resolve the vehicle distribution imbalance situation; namely, how to balance the number of vehicles at different locations when demand is skewed. Supplying more vehicles or not satisfying customers' demand are two decisions that managers need to balance. The location and capacity issues in the car-sharing industry may look similar to those in traditional industry. However, because of the unique features of car sharing, commonly employed methodologies cannot be easily applied to address the car-sharing problems. For example, in addition to the coverage issue, Zipcar emphasizes providing customers with easy access to vehicles.

Car-sharing companies do not rely on traditional marketing channels; instead, they primarily depend on users' word of mouth. Therefore, it is best to expand its service near the existing service station. From this perspective, we find that selecting a new service station/region resembles predicting diffusion from the current station to regions not already served. This view of expansion differs from the traditional one and motivates us to look into the issue further.

1.4 CONCLUSIONS AND FUTURE WORK

The key conclusions and results from each essay are summarized to emphasize the advantages of using each new model to identify global markets for the air medical Jets industry, to evaluate leading companies of the Chinese express delivery industry and to better manage regular operations issues in the car-sharing industry. The future work of each model includes gaining more precise data to make better recommendations, and linking model parameters in the third essay to make conclusions more consistent.

2.0 FOREIGN MARKETS EXPANSION FOR THE AIR MEDICAL TRANSPORT BUSINESS

In this chapter, we are interested in determining which country/region could be the most valuable market for a private air medical transport company, again for the sake of confidentiality, the company is referred to using the *fictitious name* Medical Aviation Service (MAS). We assess the business opportunity for MAS from both the tangible and the intangible perspectives. We identify evaluation criteria (i.e., factors, attributes) and derive composite scores for each market. To determine the priority of all candidate markets, the importance of each criteria and sub-criteria must be determined. However, the management team at MAS was unsure about assigning the weights for each factor and was hesitant to provide comparison judgments for each pair of criteria. Thus, it was difficult for the MAS team to reach meaningful results.

To evaluate the global market in such an environment, we combine the AHP (Saaty, 1977, 1980, 1986) and grey number (GN) (Deng, 1990) to address the uncertainty of judgments in market evaluation. The AHP is an effective evaluation tool that is widely known for its thoroughness and performance improvement. However, the AHP requires that decision makers clearly know the weight of each factor relative to other factors, and thus it is difficult to select an appropriate market when multiple criteria, concerns, and measures are involved. The proposed GN-based AHP model is an integrated multi-criteria decision support framework that is well suited to help management select the most appropriate regions for business expansion when uncertainties are present. The GN-AHP model provides a systematic and replicable method for examining markets on a global scale.

The chapter is organised as follows. In Section 1, we review the literature; Section 2 lays the theoretical foundation for the proposed model; Section 3 presents the unified evaluation framework; in Section 4, we analyse a market's objective performance by applying the GN-AHP model, and we summarise the computational results, validate the model, and offer business implications. Subsequently, in Section 5 we investigate a market's subjective performance and make a final recommendation based on both the objective and the subjective rankings. We summarise and conclude this study in Section 6.

2.1 LITERATURE REVIEW

2.1.1 Analytic Hierarchy Process (AHP)

The AHP, one of the most widely used multi-criteria decision making (MCDM) methods, is comprised of three principles: decomposition, comparative judgment, and synthesis. In the AHP, a complicated decision problem is decomposed into a rational decision hierarchy based on related attributes or criteria. Decision makers are allowed to make subjective judgments, but they can also include objectively measured information. In our study, MAS must use pairwise comparisons to assess tangible and intangible factors regarding multiple markets based on knowledge, experience, and publicly available data to yield consistent and systematic judgments.

2.1.2 Grey Number (GN)

The grey system, proposed by Deng (1990), is a mathematical theory originated from the grey set concept. It can effectively tackle problems with incomplete discrete data and uncertainty. In the grey system, if information is known completely, it is called a white system. When information is totally unknown, it is called a black system. A system with partially known and partially unknown information is called a grey system. A grey system thus contains uncertain information presented by grey numbers and grey variables.

Liu and Lin (2006) formalize the grey number by defining $\otimes a = [\underline{a}, \overline{a}] = \{a \mid \underline{a} \le a \le \overline{a}; \underline{a}, \overline{a} \in R\}$, where $\otimes a$ includes two real numbers, with $\underline{a}$ being the lower limit of $\otimes a$ and $\overline{a}$ being the upper limit of $\otimes a$. The grey system is effective in dealing with insufficient and incomplete information and it has been applied to several disciplines (Pai et al. 2007; Kayacan et al. 2010; Bai and Sarkis, 2010; Zhao et al. 2012). Because the information in the real world is often incomplete and uncertain, extending from the crisp number to the grey number is necessary for many real world applications. In our study, we primarily use definitions from Liu and Lin (2006) to define grey numbers and grey matrices.

2.1.3 Integrating AHP and Grey System

Many researchers have studied multi-criteria decision-making (MCDM) problems by using different tactics (Ho et al., 2010), of which the AHP and the grey system are two important methods. Integrating the grey system with AHP seems to be a logical approach to tackle uncertain information under AHP framework. Research on such integration can be generally divided into two streams:

The first stream is to integrate AHP and Grey Relational Analysis (GRA), which is one of the primary research areas in grey system theory. GRA was derived from grey system theory

(Deng 1982, 1988). Given predetermined measurement rules, GRA can be used to effectively measure the correlations between the reference factors and compared factors, and then rank all alternatives in terms of the calculated correlations. Due to its ease of implementation, a lot of GRA applications have appeared in the literature (Birgun and Gungor, 2014; Samvedi et al. 2012; Yang and Chen, 2006; Zeng et al. 2007). Maniya and Bhatt (2011) proposed a modified GRA algorithm that need not normalize raw sequence data before performing GRA to derive rankings. Dou et al. (2014) propose using crisp value instead of traditional grey relational coefficient in GRA method, and then use this value in the ANP framework to make choices.

The second stream is to handle interval judgments by using grey number theory. Three approaches have been used to handle interval judgments:

i) Using fuzzy set and probability theory (Buckley 1985, Saaty and Vargas, 1987, Haines 1998);

ii) Using preference programming (Arbel 1989, Arbel and Vargas, 1993, Islam et al. 1997, Salo and Hamalainen, 1992,1995); and

iii) Using grey system theory.

Fuzzy AHP has been widely discussed by researchers and both the theories and applications in fuzzy AHP are well established. Lee (2002) compared the preference programming and the grey system and concluded that the grey system is easier to implement, and can solve problems more effectively than the preference programming model.

Once an interval pairwise comparison matrix is established, there are two ways to tackle the interval matrix:

1). Using whitenization functions to convert the grey interval to one single number: Chen et al. (2013) construct whitenization functions to convert interval information to one crisp number; then follow the AHP framework to rank alternatives. In whitenization, Li et al. (2008)

used arithmetic average to find several experts' lower bound of intervals to compare alternatives. The whitenization process is easy to implement, but it loses information during the process. The whitenization functions may differ significantly depending on their purposes and lead to different rankings.

2). Retaining interval information to find each alternative weights. Both Xu (1993) and Lee (2002) use maximal eigenvalue and eigenvector to find the weight interval. No literature has employed interval information throughout the whole decision process; our study fills in this gap.

In this study, we propose the GN-AHP model to process interval judgments while carrying all interval information, rather than taking one single number or only using the maximal eigenvalue & eigenvectors. Instead we retain all information to find each alternative's weight interval, and then make rankings based on the derived intervals. Our model provides a valuable approach to handle uncertain information and make decisions with group experts.

2.1.4 Literature on Private Jets Industry

Given that the private jet industry is a niche market, the few studies that have examined the field primarily focus on Jet operational scheduling and coverage problems. Ronen (2000), Keskinocak and Taylor (1998), and Espinoza et al. (2008) use 0-1 integer programming to plan air jet routing. Carnes et al. (2013) develop a mathematical model to optimize jet routes for a Canadian air ambulance, while Carnes (2010) discuss how to park air medical craft to achieve minimal daily transportation costs. Thus, the private jet market, especially the air medical transport market, has not been fully explored; we fill this void in the literature.

2.2 THE THEORETICAL FOUNDATION OF THE GN-AHP MODEL

2.2.1 Preliminary

The preliminaries necessary for constructing the GN-AHP model are as follows.

Definition 1. Grey characteristic value and vectors.

Assume $D(\otimes)=(\otimes_1,\otimes_2,\cdots,\otimes_n)^T$, such that

$$D(\otimes)W(\otimes)=\lambda(\otimes)W(\otimes) \tag{2.1}$$

Thus, $\lambda(\otimes)$ is a grey eigenvalue of $D(\otimes)$, and $W(\otimes)$ is an eigenvector of $D(\otimes)$ associated with $\lambda(\otimes)$. From (1), we can easily find:

Theorem 1. If $D(\otimes)W(\otimes)=\lambda(\otimes)W(\otimes)$, then

$$(\underline{d}_{ij})(\underline{w}_{ij})=\underline{\lambda}(\underline{w}_{ij}) \text{ and } (\overline{d}_{ij})(\overline{w}_{ij})=\overline{\lambda}(\overline{w}_{ij}),\ \ i,j=1,2,\cdots,n \tag{2.2}$$

Theorem 2. Suppose $A(\otimes)=[\underline{A},\overline{A}]$, $\underline{\lambda}$ and $\overline{\lambda}$ are the eigenvalues of $\underline{A}$ and $\overline{A}$, respectively. Then,

(i). $\lambda(\otimes)=[\underline{\lambda},\overline{\lambda}]$ is the eigenvalue of $A(\otimes)$

(ii). $X=[p\underline{x},q\overline{x}]$ is the eigenvector of A associated with λ, where $\underline{x},\overline{x}$ correspond to $\underline{\lambda},\overline{\lambda}$ and p,q are nonnegative real numbers satisfying $0<p\underline{x}\le q\overline{x}$.

Proof: See Appendix A.1 for the Proof of Theorem 2.

2.2.2 Building the Grey Number Judgment Matrix in the AHP

The mechanism to build the grey number judgment matrix in the AHP is as follows.

Definition 2. $D(\otimes)$ is an AHP grey number judgment matrix, if $D(\otimes)=[\otimes_{ij}]_{n\times n}$ and $i,j=1,2,\cdots,n$, such that (i) $\otimes_{ij}=[\underline{d}_{ij},\overline{d}_{ij}]$ and $\frac{1}{9}\le\underline{d}_{ij}\le\overline{d}_{ij}\le 9$ and (ii) $\otimes_{ij}=\dfrac{1}{\otimes_{ji}}$.

Given $D(\otimes)=[\otimes_{ij}]_{n\times n}$ is a grey number matrix and $w(\otimes)=(w_1(\otimes),w_2(\otimes),\cdots,w_n(\otimes))^T$ is its weight vector, then $\otimes_{ij}=\dfrac{w_i(\otimes)}{w_j(\otimes)}$ is the judgment interval for $i,j=1,2,\cdots,n$.

Definition 3. For $D(\otimes)=[\otimes_{ij}]_{n\times n}$, if $\otimes_{ij}=\dfrac{1}{\otimes_{ji}}$, $\otimes_{ij}\cdot\otimes_{jk}=\otimes_{ji}\cdot\otimes_{ik}$, then $D(\otimes)$ is a consistent grey number judgment matrix.

To construct the grey number judgment matrix and develop a formula to compute the grey number weight, we introduce the quasi-uniformity concept of a real number matrix in Appendix A.2. Owing to the quasi-uniformity properties of $(\underline{d}_{ij})_{n\times n}$ and $(\overline{d}_{ij})_{n\times n}$, we have

$$\frac{w_i(\otimes)}{w_j\otimes}=[\Omega\underline{d}_{11}\underline{d}_{ij},\Omega'\overline{d}_{11}\overline{d}_{ij}] \tag{2.3}$$

and

$$\Omega\underline{d}_{11}=\frac{1}{\Omega'\cdot\overline{d}_{11}},\quad (i,j=1,2,\cdots,n) \tag{2.4}$$

Therefore, we find

(i). when $\Omega\underline{d}_{11}=1$, $\dfrac{w(\otimes)_i}{w(\otimes)_j}=\otimes_{ij}$;

(ii). when $\Omega\underline{d}_{11}>1$, $\dfrac{w_i(\otimes)}{w_j(\otimes)}\subset\otimes_{ij}$. By letting $\otimes=[\Omega'\overline{d}_{11},\Omega\underline{d}_{11}]$, we have $\otimes_{ij}=\otimes\cdot\dfrac{w_i(\otimes)}{w_j(\otimes)}$;

(iii). when $\Omega\underline{d}_{11}<1$, $\dfrac{w_i(\otimes)}{w_j(\otimes)}\supset\otimes_{ij}$. By letting $\otimes=[\Omega\underline{d}_{11},\Omega'\overline{d}_{11}]$, we have $\dfrac{w_i(\otimes)}{w_j(\otimes)}=\otimes\cdot\otimes_{ij}$;

where $\Omega=p/q$ and $\Omega'=q/p$ (see Appendix A.2 for the values of p and q).

From the quasi-uniformity properties above, we obtain the properties of the consistent grey number judgment matrix as follows.

Theorem 3. Let $D(\otimes)=[\otimes_{ij}]_{n\times n}$ be a consistent grey number matrix and $w^*(\otimes)=(w_1^*(\otimes),w_2^*(\otimes),\cdots,w_n^*(\otimes))^T$, $w^\#(\otimes)=(w_1^\#(\otimes),w_2^\#(\otimes),\cdots,w_n^\#(\otimes))^T$ be the eigenvector satisfying

$D(\otimes)w^{*}(\otimes)=tr(D(\otimes))w^{*}(\otimes)$ and $D(\otimes)w^{\#}(\otimes)=tr(D(\otimes))w^{\#}(\otimes)$. Then, $\frac{w_i^{*}(\otimes)}{w_j^{*}(\otimes)}=\frac{w_i^{\#}(\otimes)}{w_j^{\#}(\otimes)}$ if and only if there exists a nonnegative constant μ such that $w^{*}(\otimes)=\mu w^{\#}(\otimes)$ or $w^{\#}(\otimes)=\mu w^{*}(\otimes)$.

Proof: See Appendix A.3 for the Proof of Theorem 3.

Theorem 4. Suppose $D(\otimes)=[\otimes_{ij}]_{n\times n}$ is a consistent grey number matrix, $\underline{w}=(\underline{w}_1,\underline{w}_2,\cdots,\underline{w}_n)^T$ and $\overline{w}=(\overline{w}_1,\overline{w}_2,\cdots,\overline{w}_n)^T$ are nonnegative normalised eigenvectors of $(\underline{d}_{ij})_{n\times n}$ and $(\overline{d}_{ij})_{n\times n}$ associated with the maximal eigenvalue. Then,

$$w(\otimes)=[p\underline{w},q\overline{w}]=(w_1(\otimes),w_2(\otimes),\cdots,w_n(\otimes))^T \tag{2.5}$$

and a necessary and sufficient condition for $\otimes_{ij}=\frac{w_i(\otimes)}{w_j(\otimes)},(i,j=1,2,\cdots,n)$ is

$$\Omega=\sum_{j=1}^{n}\frac{1}{\sum_{i=1}^{n}\overline{d_{ij}}}=\frac{1}{\sum_{j=1}^{n}\frac{1}{\sum_{i=1}^{n}\underline{d}_{ij}}} \tag{2.6}$$

where $p=\sqrt{\sum_{j=1}^{n}\frac{1}{\sum_{i=1}^{n}\overline{d}_{ij}}}$ and $q=\sqrt{\sum_{j=1}^{n}\frac{1}{\sum_{i=1}^{n}\underline{d}_{ij}}}$.

$=$

Proof: See Appendix A.4 for the Proof of Theorem 4.

2.2.3 A Decision Support System for Implementing the GN-AHP Model

Based on the above discussion, we build the GN-AHP model following the sequence below.

(i). Construct the grey number judgment matrices.

(ii). Compute the maximal eigenvalue of the grey number matrix $(\underline{d}_{ij})_{n\times n}$ and $(\overline{d}_{ij})_{n\times n}$ and determine the associated nonnegative normalised eigenvector $\underline{w}=(\underline{w}_1,\underline{w}_2,\cdots,\underline{w}_n)^T$ and $\overline{w}=(\overline{w}_1,\overline{w}_2,\cdots,\overline{w}_n)^T$.

(iii). Determine parameters p and q :

$$p = \sqrt{\sum_{j=1}^{n} \frac{1}{\sum_{i=1}^{n} \overline{d}_{ij}}}, \quad q = \sqrt{\sum_{j=1}^{n} \frac{1}{\sum_{i=1}^{n} \underline{d}_{ij}}} \tag{2.7}$$

which are derived through the proof of Theorem 4.

(iv). Normalise the weight vector.

$$w(\otimes) = [\, p\underline{w}, q\overline{w}] = (w_1(\otimes), w_2(\otimes), \cdots, w_n(\otimes))^T \tag{2.8}$$

(v). Rank the preference order:

We first employ a lower limit judgement matrix on the AHP method, with which we can find a ranking of all alternatives with corresponding weights. It is the lower limit of the weight intervals for ranking, indicating the minimum weight of an alternative that could be possibly reached. Similarly, we can find the upper limits of weight intervals by applying an upper limit judgement matrix through the AHP process. Then we can combine the lower and upper limits of weight intervals, and obtain a complete weight interval for each alternative.

Once the upper and lower limits of weight intervals are completed, a key issue is how to rank alternatives by using such interval results, namely, how do we prioritize each alternative's interval that can characterize their potential rankings. To this end, we give the following three criteria: consider two alternatives: $M_1(x_1, y_1)$ and $M_2(x_2, y_2)$, x_i and y_i represent one alternative rating's lower and upper limits respectively.

(i) If $x_1 > y_2$, then M_1 is ranked higher than M_2;

(ii) If $x_1 > x_2$ and $y_1 > y_2$, then M_1 is ranked higher; finally

(iii) If $x_2 < x_1$, and $y_2 > y_1$, we create a new indicator-score density- to help us rank alternatives; namely let $\rho_1 = \frac{x_1 + y_1}{2(y_1 - x_1)}$, $\rho_2 = \frac{x_2 + y_2}{2(y_2 - x_2)}$ represent two alternatives M_1 and M_2's score densities (Interval average divided by interval distance). When $\rho_1 > \rho_2$, alternative M_1 is ranked higher than M_2; when $\rho_1 < \rho_2$, alternative M_1 is ranked lower than M_2.

The above three situations can completely portray a ranking relationship between the two grey weight intervals, and we can use these three criteria to determine the final ranking of the potential markets.

2.3 MODEL DEVELOPMENT FOR THE MAS EXPANSION DECISION

2.3.1 Problem description of MAS

Management at MAS strives to identify the best market for global expansion. To analyze various locations and offer sensible recommendations, we employ a sequential decision-making process in which prior choices affect subsequent valuations. More precisely, we break down the whole process into two parts: objective criteria evaluation and subjective criteria evaluation. We first use objective criteria to filter and evaluate all candidates and then apply subjective criteria to evaluate the candidates chosen from the objective-criteria based evaluation. We finally combine the two types of results and make the recommendation. By doing so, we significantly lower the complexity of the problem and reduce the computational burden.

To determine the weights of the objective and the subjective criteria in the AHP, one must build comparison matrices. In the conventional AHP, a single value is assigned to each pair of the compared elements (e.g., criteria, sub-criteria, or alternatives). However, the management of MAS is uneasy about assigning a specific value to each pair of comparisons, i.e., they have doubts about using a single crisp number to reflect their true sentiment. In particular, the executives participating in the project are uncertain about many pairwise comparisons, such as how much more important the wealth criterion is than the healthcare or the patient acuity criterion. We tackle such a dilemma in this chapter and address this issue through the proposed GN-AHP model to provide a robust recommendation.

2.3.2 The GN-AHP Model

The complexity of a decision problem grows with the uncertainty in judgments. Many traditional evaluation models cannot comprehensively and accurately reflect the environment and understand the knowledge and opinions of the experts. Different judgments could be made not only by different decision makers, but also even the same decision maker, who some of the time is unable to provide an exact view regarding a specific comparison. As mentioned earlier, this research contributes to the improvement of the methodology (i.e., through the GN-AHP model) and the practice (i.e., resolving issues encountered by the air jet medical service industry).

We use AHP as the main structure to integrate with the grey number to effectively incorporate uncertain information and comprehensively aggregate judgments throughout the evaluation process. The proposed GN-AHP model reduces the time needed to prioritize individual markets and identify the best region for global expansion. Figure 2.1 presents the proposed framework.

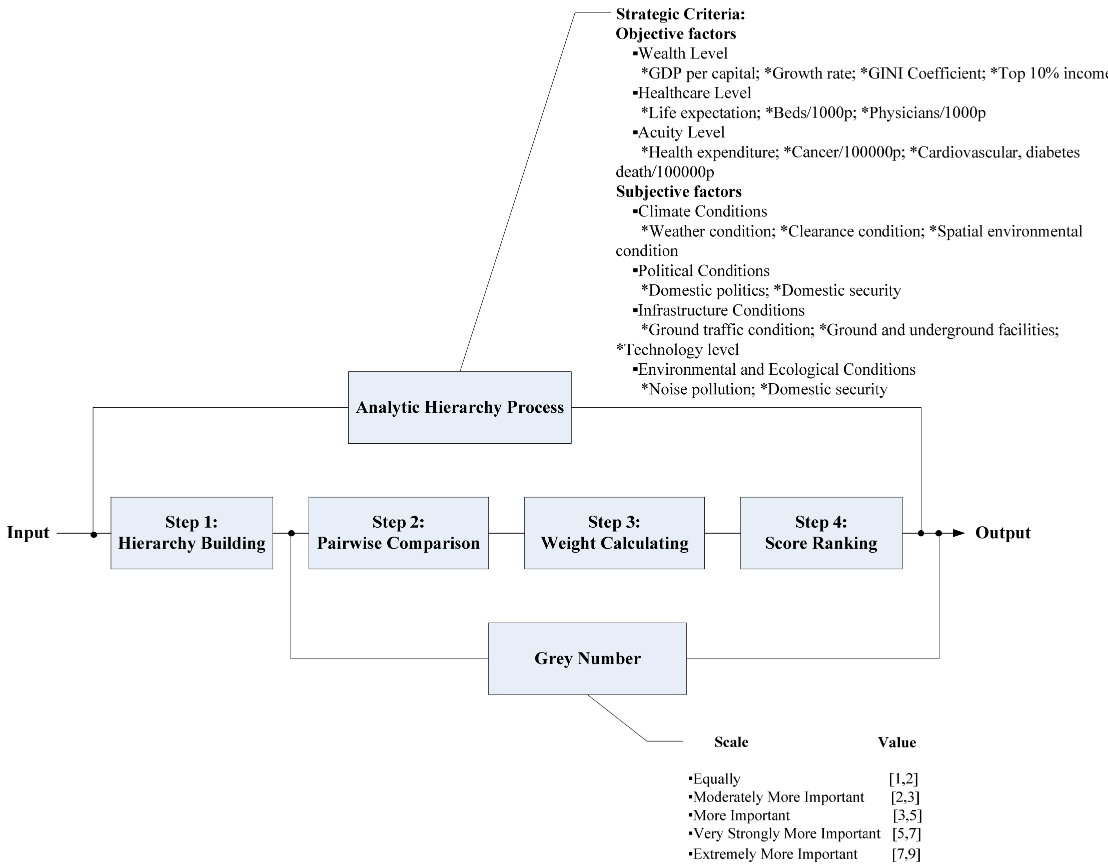


Figure 2.1. The proposed GN-AHP framework

2.4 APPLYING THE GN-AHP TO OBJECTIVE DATA

Based on objective criteria, we evaluate all candidates by using three factors: level of wealth, level of healthcare, and level of patient acuity. Wealth concentration is essential, as only people with high incomes can afford air medical transport services. On the other hand, the quality of healthcare providers is essential in differentiating potential markets. Nations with low accessibility to quality healthcare would be more likely to need help from MAS to transport (wealthy) patients to US hospitals, while those with advanced accessibilities in their regions have less need for MAS' transcontinental services. Alternatively, a market's healthcare expenses on cancer and cardiovascular cases are also important indicators. In general, higher healthcare expenses mean more sick patients, which suggests a greater need for air jet transportation services, as evidenced by the fact that the majority of MAS clients are cardiovascular and cancer patients.

2.4.1 Objective Criteria Selection

Under each evaluation criterion (wealth, healthcare quality and patient acuity level), we identify several sub-criteria to support the evaluation.

2.4.1.1 Wealth Level (WL)

As air jet service is designed mainly for affluent patients, it is necessary to confirm the affordability of the market. Traditionally, the GDP per capita is used to measure a region's living standards (European Parliament and The Guardian 14-09-2009). Because the GDP as a measure has been challenged, however, Dasgupta (2010) proposed using GDP growth over population growth as the second factor to measure a market's wealth.

We include four sub-criteria to measure the wealth of a market: (i) GDP per capita; (ii) GDP growth relative to population growth; (iii) GINI coefficient; (iv) income share held by the highest 10%. GINI measures the distribution of income or consumption expenditure deviating from a perfectly equal distribution among individual households within an economy, thus indicating income inequality within countries, with 0 representing no inequality and 100 signifying perfect inequality. The top 10% of the income population indicates the percentage of wealth belonging to the top 10% of the earners in a country. As this is the wealthiest group of people, it hints at the income concentration of a nation.

2.4.1.2 Healthcare Level (HL)

We define the measure of quality of healthcare by (i) life expectancy; (ii) number of beds per 1000 people; and (iii) number of physicians per 1000 people. The last two sub-factors are healthcare attainment scores as defined by the World Health Organization (WHO).

2.4.1.3 Patient Acuity (PA)

The acuity of patients' illnesses in a nation can be measured by (i) cardiovascular and diabetes deaths per 100,000 people; (ii) all types of cancer deaths per 100,000 people; and (iii) health care expenditures. Cardiovascular disease and cancer are the leading causes of death worldwide. Ninety percent of MAS' patients belong to these illness categories, and accordingly, regions with higher health expenditures possess greater potential to generate revenue for MAS.

2.4.2 Data Collection

The data set was constructed from both publicly available sources and from information collected through surveys and interviews with MAS. The data from countries and regions are retrieved from The World Bank (WB), the World Health Organization (WHO) and the World Fact book of the Central Intelligence Agency (WFCIA). For very small states, as the public database does not contain their information, we retrieved such information from the states' official websites. Taking Hong Kong, Macao and Taiwan as examples, all WB, WHO and WFCIA world organizations do not have information regarding cardiovascular and cancer diseases. Thus, we must search the government's websites to gather the information. For even smaller nations, because there is no government source that compiles the necessary information, we fill in the data using regional averages corresponding to the size of the countries. For example, in the WB database, because there is no information on "Health Expenditure" for some of the Caribbean islands such as Saint Lucia, we fill in this information using the average stats of its neighboring islands rather than all of Latin America.

With respect to data screening, the complete dataset we compiled is comprised of statistics for 226 countries and regions. It includes all lands on Earth except Antarctica and Greenland. However, given the high expenses related to air jet transport, it may not be effective to incorporate all nations and regions in our detailed evaluation. Thus, we create a simple ratio to screen out nations with little potential for MAS expansion. We seek to identify markets that demonstrate a great need for jet services and can afford the high costs associated with air jets. The ratio is derived by dividing the GDP per capita by the number of hospital beds per 1000 patients. The GDP per capita represents a nation's wealth, while the number of hospital beds corresponds to healthcare quality. The higher the ratio, the more attractive the market to MAS, as it either has higher wealth or lower healthcare quality. We use Canada's ratio as a threshold because its air jet industry is well developed in Canada. Nations whose ratios are higher than that of Canada's have great potential and are retained for further consideration, while those with lower ratios are removed. In all, there are 32 states with ratios that are greater than that of Canada (See Table 2.2 for details).

2.4.3 The GN-AHP Model for Attribute Rating

When expressing their preference in paired comparisons, decision makers often feel that the value of a paired comparison could be in a neighborhood, it is very difficult to single out a unique value. Under such circumstances, the grey number offers approximate ranges to reflect their true sentiments. The GN-AHP model not only addresses judgments with uncertainty, but it also computes an inconsistency ratio to ensure that only justifiable rankings are used. The details associated with applying the Figure 2.1 GN-AHP model to MAS are discussed herein.

Step 1: Build the hierarchy

Figure 2.2 shows the AHP hierarchy. At the top is the objective, i.e., to identify the best market for MAS expansion. The second level displays the three main criteria. A market with a high wealth score is more attractive to MAS. The lower the healthcare delivery quality, the greater the market needs for MAS. Finally, the physical illnesses of people living in the region, which suggest high healthcare costs and disease frequencies, may require increased demands on medical aviation services. Higher morbidity and mortality correspond to higher health expenditures, thus implying a more attractive market for MAS to enter.

Step 2: Make pairwise comparisons with grey numbers

After establishing the AHP structure, the decision makers need to pairwise compare the criteria and sub-criteria using grey numbers. Each element is compared in turn with every other element at the same level in the hierarchy. If one is equally as important as the other, then the ratio is defined as one or two, i.e., [1,2]; if one is moderately more important than the other, the ratio is [2,3]; if one is strongly more important, the ratio is [3,5]; if one is very strongly more important, the ration is [5,7]; and if one is extremely more important, the ratio is [7,9]. Responses in between these grey numbers are allowed. These judgments thus lead to matrices of comparisons.

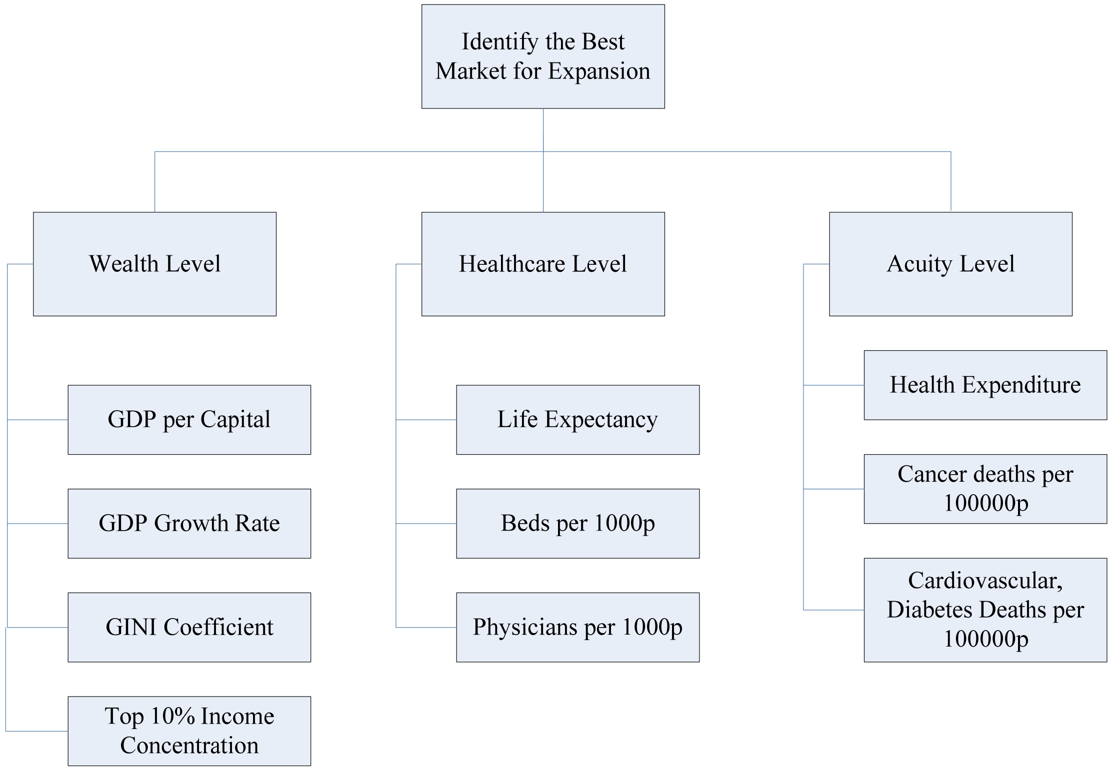


Figure 2.2. AHP hierarchy for identifying the expansion target markets

Table 2.1. Pairwise comparison of the main criteria in the GN-AHP Model

	Wealth Level	Healthcare Level	Acuity Level	Global Weight
Wealth Level	[1,1]	[3,5]	[5,7]	[0.60,0.63]
Healthcare Level	[1/5,1/3]	[1,1]	[3,5]	[0.25,0.30]
Acuity Level	[1/7,1/5]	[1/5,1/3]	[1,1]	[0.07,0.10]

The management of MAS provided all judgments in Table 2.1. From Table 2.1, it is clear that patient acuity is the weakest indicator for market attractiveness; healthcare level is three to five times more important than patient acuity, but it is only 1/7 to 1/5 as important as the wealth level of a nation. The diagonal of the matrix consists of [1, 1]s, as each criterion is equally important when compared with itself. The lower triangle of the matrix is the inverse of upper responses. This pairwise comparison process continues at each sub-criterion level, except at the bottom of the hierarchy.

Step 3: Translate the comparison matrices into weights

This step converts the pairwise comparisons into sets of weights. The GN-AHP uses eigenvector scaling to translate the judgments into numerical scores. The global weights of the three main criteria in the rightmost column in Table 2.1 are calculated using Eq. (8). For the symmetric judgment matrix of the AHP, if we choose the lower value (smaller value) of the grey number in Table 2.1, we can calculate the lower limit of the global weight; otherwise, we can find the upper value of the global weight. The lower and the upper limit judgment matrices could be as follows.

Lower judgment matrix: $\begin{pmatrix} 1 & 3 & 5 \\ 1/3 & 1 & 3 \\ 1/5 & 1/3 & 1 \end{pmatrix}$ Upper judgment matrix: $\begin{pmatrix} 1 & 5 & 7 \\ 1/5 & 1 & 5 \\ 1/7 & 1/5 & 1 \end{pmatrix}$

By following the Steps (II)-(IV) for implementing the GN-AHP Model in Section 3.3, we obtain the global weights. The first two levels in Fig. 3 show the global weights for each criterion and sub-criterion.

Step 4: Combine weights and judgments to establish composite scores

As there are 32 markets to be evaluated simultaneously, there would be too many pairwise comparisons to be made using the conventional AHP method. Therefore, we adopt the AHP rating method as it creates an ideal alternative, which is a conceptual alternative that receives the highest possible rating under each factor. The ratings of all candidates can be found by examining how far the alternatives are from the ideal. The last level in Figure 2.3 indicates the scores for each of the 32 candidate markets derived by the AHP rating methods. Names corresponding to the abbreviated country symbols are provided in Table 2.2, while details of the calculations are presented in Table 2.3 and 2.4.

Table 2.3 presents the global weight for each sub-criterion and the respective grey number score under each sub-criteria. The composite scores and idealized rating for each candidate are given in rows 1 and 2 in Table 2.4. For example, the composite score of market Bermuda is [11.96, 21.93]. As it is rated the highest among all 32 potential markets, it becomes the idealized rating of [100.00, 100.00]. Ratings of other markets are then normalized based on the Bermuda ratings. The idealized ratings of all 32 markets are sorted to determine their ranks.

As the ratings are in grey number form, there may exist interval crossover. We can compare the lower and upper values to determine the rank of the potential markets. By following section 3.3 step (V), we find the top ranked market is Bermuda.

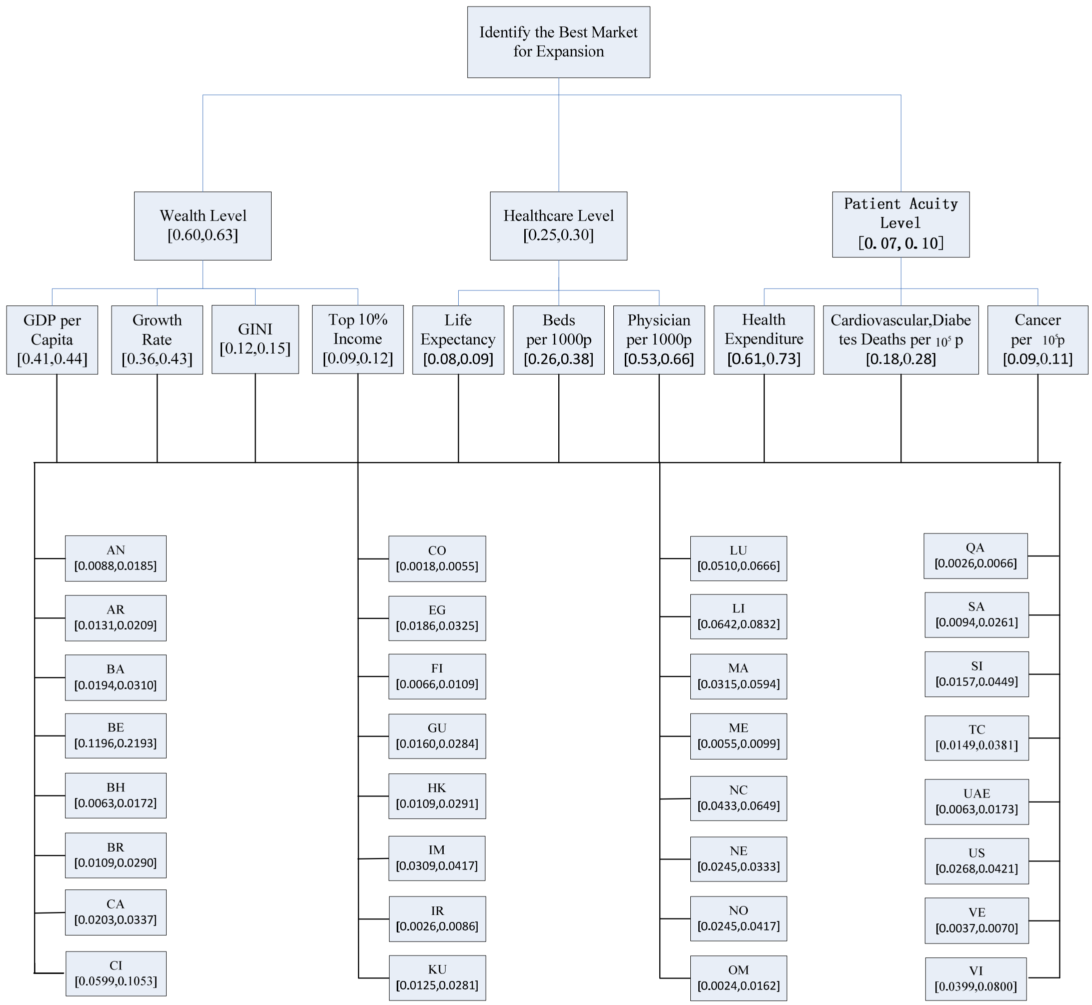


Figure 2.3. AHP hierarchy and corresponding weights in grey number

Table 2.2. Abbreviated country names

Table of Markets							
Full	**Abbreviation**	**Full**	**Abbreviation**	**Full**	**Abbreviation**	**Full**	**Abbreviation**
Andorra	AN	Cayman Islands	CI	Kuwait	KU	Oman	OM
Aruba	AR	Colombia	CO	Liechtenstein	LI	Qatar	QA
Bahamas	BA	Equatorial Guinea	EG	Luxembourg	LU	Saudi Arabia	SA
Bahrain	BH	Faroe Islands	FI	Macau	MA	Singapore	SI
Bermuda	BE	Guam	GU	Mexico	ME	Turks and Caicos Islands	TC
British-Virgin Islands	VI	Hong Kong	HK	Netherlands	NE	United Arab Emirates	UAE
Brunei	BR	Iran	IR	New Caledonia	NC	United States	US
Canada	CA	Isle of Man	IM	Norway	NO	Venezuela	VE

Table 2.3. Grey number weights of sub-criteria and potential markets (1/10,000)

Market	GDP/Capita [0.41,0.44]	GDP Growth rate [0.36,0.43]	GINI [0.12,0.15]	Top 10% Income [0.09,0.12]	Life Expectancy [0.08,0.09]	Beds/1000p [0.26,0.38]	Physicians per 1000p [0.53,0.66]	Health Expenditure [0.61,0.73]	CV, Diabetes Deaths/ 10^5 p [0.18,0.28]	Cancer deaths/ 10^5 p [0.09,0.11]
AN	[24,45]	[21, 43]	[7,12]	[6,10]	[2, 5]	[7, 20]	[15, 34]	[4, 13]	[1, 5]	[1, 2]
AR	[38, 51]	[35, 49]	[8,14]	[5,8]	[3, 5]	[11, 22]	[22, 39]	[6, 14]	[2, 5]	[1, 2]
BA	[54,70]	[52,66]	[12,24]	[10,20]	[5,8]	[16,33]	[32,57]	[9,21]	[3,8]	[1,3]
BH	[16,41]	[13,39]	[8,11]	[5,9]	[2,4]	[5,18]	[10,32]	[3,12]	[1,4]	[0,2]
BE	[375,609]	[355,579]	[33,52]	[27,38]	[30,55]	[98,231]	[199,402]	[55,148]	[16,57]	[8,22]
VI	[112,203]	[102,193]	[31,48]	[19,22]	[10,20]	[33,84]	[66,147]	[18,54]	[5,21]	[3,8]
BR	[31,74]	[26,64]	[9,17]	[6,13]	[3,7]	[9,31]	[18,53]	[5,20]	[1,8]	[1,3]
CA	[57,78]	[52,70]	[13,28]	[12,20]	[5,8]	[17,36]	[34,62]	[9,23]	[3,9]	[1,3]
CI	[188,287]	[178,277]	[18,29]	[12,21]	[15,26]	[49,111]	[99,193]	[28,71]	[8,27]	[4,11]
CO	[5,16]	[5,15]	[2,5]	[0,1]	[0,1]	[2,6]	[3,10]	[1,4]	[0,1]	[0,1]
EG	[55,85]	[53,80]	[9,17]	[5,8]	[5,8]	[15,34]	[31,60]	[9,22]	[3,8]	[1,3]
FI	[19,22]	[18,19]	[5,14]	[2,3]	[2,3]	[5,13]	[11,23]	[3,8]	[1,3]	[0,1]
GU	[45,65]	[43,62]	[11,23]	[7,16]	[4,7]	[13,30]	[27,52]	[7,19]	[2,7]	[1,3]
HK	[31,75]	[28,69]	[8,15]	[5,9]	[3,7]	[9,31]	[18,53]	[5,20]	[1,8]	[1,3]
IR	[8,22]	[7,18]	[2,7]	[1,3]	[1,2]	[2,9]	[4,16]	[1,6]	[0,2]	[0,1]
IM	[96,105]	[79,63]	[23,47]	[6,15]	[8,12]	[25,50]	[52,86]	[14,32]	[4,12]	[2,5]
KU	[38,76]	[30,50]	[11,32]	[3,6]	[3,7]	[10,30]	[21,51]	[6,19]	[2,7]	[1,3]
LI	[191,168]	[172,143]	[42,77]	[23,52]	[16,23]	[52,99]	[107,172]	[30,64]	[9,24]	[4,10]
LU	[168,176]	[110,131]	[38,66]	[12,17]	[13,19]	[42,79]	[85,138]	[24,51]	[7,19]	[3,8]
MA	[99,158]	[83,123]	[21,55]	[5,12]	[8,15]	[26,63]	[52,109]	[15,40]	[4,15]	[2,6]
ME	[16,26]	[12,20]	[6,9]	[2,3]	[1,2]	[5,10]	[9,18]	[3,7]	[1,3]	[0,1]
NE	[66,79]	[50,63]	[18,35]	[5,9]	[6,9]	[20,40]	[41,69]	[11,25]	[3,10]	[2,4]
NC	[113,165]	[111,131]	[32,58]	[13,24]	[11,16]	[35,68]	[72,119]	[20,44]	[6,17]	[3,7]
NO	[75,98]	[52,64]	[18,46]	[6,14]	[6,12]	[20,50]	[41,86]	[11,32]	[3,12]	[2,5]
OM	[21,42]	[18,31]	[6,16]	[3,5]	[2,4]	[6,17]	[12,30]	[3,11]	[1,4]	[0,2]
QA	[8,17]	[7,13]	[2,6]	[1,2]	[1,2]	[2,7]	[4,12]	[1,4]	[0,2]	[0,1]
SA	[29,71]	[25,52]	[6,24]	[2,5]	[2,6]	[8,27]	[16,48]	[4,18]	[1,7]	[1,3]
SI	[47,128]	[40,92]	[12,38]	[5,13]	[4,11]	[13,47]	[26,82]	[7,30]	[2,12]	[1,5]
TC	[46,104]	[35,75]	[14,36]	[3,7]	[4,9]	[12,40]	[25,70]	[7,26]	[2,10]	[1,4]
UAE	[19,46]	[17,34]	[4,16]	[2,3]	[2,4]	[5,18]	[10,32]	[3,12]	[1,4]	[0,2]
US	[82,112]	[70,85]	[18,38]	[6,10]	[7,11]	[22,44]	[45,77]	[12,28]	[4,11]	[2,4]
VE	[11,18]	[9,14]	[3,6]	[1,2]	[1,2]	[3,7]	[6,13]	[2,5]	[1,2]	[0,1]

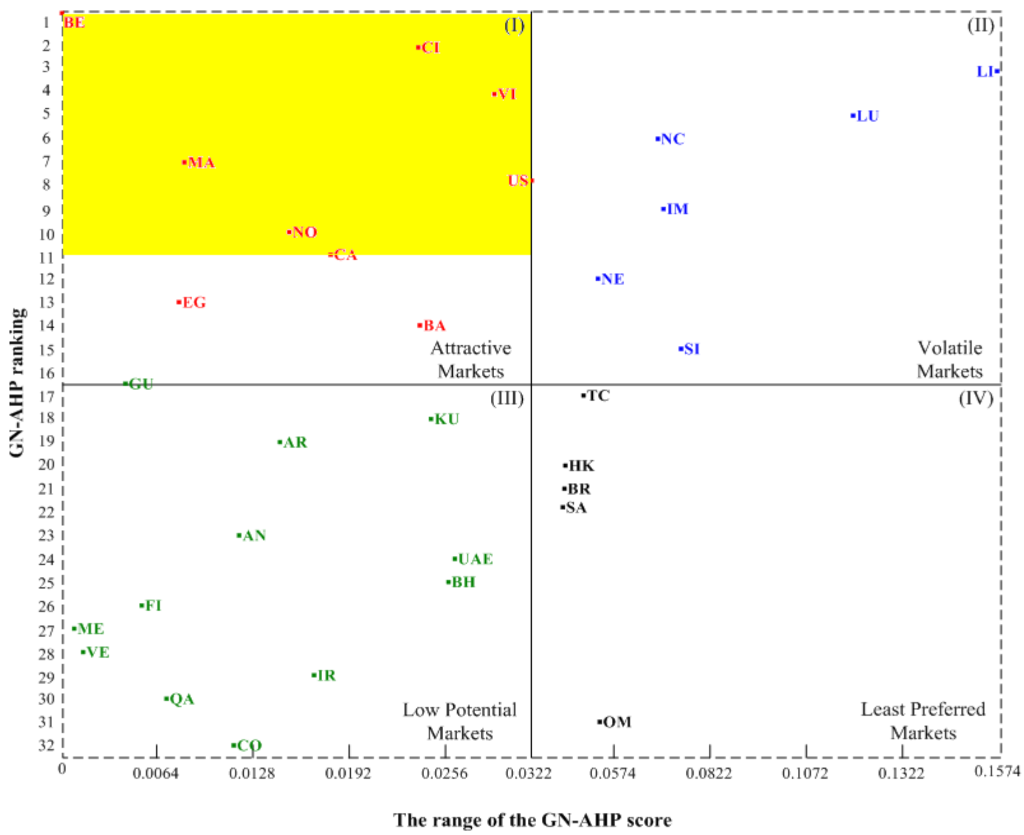
Table 2.4. Numerical ratings and ranks of potential markets (1/100)

Markets	Andorra	Aruba	Bahamas	Bahrain	Bermuda	Virgin Islands	Brunei	Canada
Composite score	[0.88,1.85]	[1.31,2.09]	[1.94,3.10]	[0.63,1.72]	[11.96,21.93]	[3.99,8.00]	[1.09,2.90]	[2.03,3.37]
Idealized rating	[7.35,8.43]	[9.53,10.95]	[14.13,16.22]	[5.26,7.84]	[100.00,100.00]	[33.36,36.47]	[9.11,13.22]	[15.36,16.97]
Rank	23	19	14	25	1	4	21	11
	Cayman Islands	Colombia	Equatorial Guinea	Faroe Islands	Guam	Hong Kong	Iran	Isle of Man
Composite score	[5.99,10.53]	[0.18,0.55]	[1.86,3.25]	[0.66,1.09]	[1.60,2.84]	[1.09,2.91]	[0.26,0.86]	[3.09,4.17]
Idealized rating	[48.01,50.08]	[1.50,2.50]	[14.81,15.55]	[4.97,5.51]	[12.95,13.37]	[9.11,13.26]	[2.17,3.92]	[19.01,25.83]
Rank	2	32	13	26	16	20	29	9
	Kuwait	Liechtenstein	Luxembourg	Macau	Mexico	Netherlands	New Caledonia	Norway
Composite score	[1.25,2.81]	[6.42,8.32]	[5.10,6.66]	[3.15,5.94]	[0.55,0.99]	[2.45,3.33]	[4.33,6.49]	[2.45,4.17]
Idealized rating	[10.45,12.81]	[37.93,53.67]	[30.36,42.64]	[26.33,27.08]	[4.51,4.59]	[15.18,20.48]	[29.59,36.20]	[19.01,20.48]
Rank	18	3	5	7	27	12	6	10
	Oman	Qatar	Saudi Arabia	Singapore	Turks and Caicos Islands	United Arab Emirates	United States	Venezuela
Composite score	[0.24,1.62]	[0.26,0.66]	[0.94,2.61]	[1.57,4.49]	[1.49,3.81]	[0.63,1.73]	[2.68,4.21]	[0.37,0.70]
Idealized rating	[2.01,7.38]	[2.17,3.01]	[7.85,11.90]	[13.12,20.47]	[12.45,17.37]	[5.26,7.88]	[19.19,22.41]	[3.09,3.19]
Rank	31	30	22	15	17	24	8	28

2.5 COMPUTATIONAL RESULTS

2.5.1 First Evaluation Profile

To explore potentials market, we project the GN-AHP results into a matrix of four quadrants. Figure 2.4 presents the four market quadrants according to the weighted ratings of the GN-AHP (higher ratings are placed at the top) and the grey interval range (smaller grey range is placed on the left). Note that the grey interval distance, d_i, of the potential market can be calculated using $d_i = \overline{x}_i - \underline{x}_i$, wherein $\overline{x}_i$ and $\underline{x}_i$ are the upper and lower limits of market $i's$ grey number. The horizontal line dividing the upper and lower quadrants is the median of the ranking based on the value of the GN-AHP rating, while the vertical line is the GN-AHP range of the US score, which is chosen because the US is the benchmark for further market expansion.



Note the quadrants are given in the upper right corners

Figure 2.4. Evaluation profile of the potential markets

The upper-left quadrant represents the markets with stable market performances and high ratings. More precisely, this area corresponds to countries having smaller grey interval distances and higher AHP ratings. Markets in this quadrant are preferred for the launching of new air jet medical services. We name this quadrant the "Attractive Markets". The upper-right quadrant reveals that the markets have high grey distances and high ratings. Larger grey distances imply higher uncertainty or risk involved in choosing this market. We refer to these markets as ‘‘Volatile Markets". Third, the markets in the lower-left quadrant have small grey interval distances and low ratings and are termed "Low Potential Markets". The last category belongs to those that have large grey interval distances and low ratings. Markets in this quadrant are the least preferred by MAS, and we name them the ‘‘Least Preferred Markets”.

2.5.2 Use of the Profile Matrix

We project all 32 potential markets into our profile matrix. The result is presented in Figure 2.4.

Attractive Markets: The first quadrant contains nine markets, i.e., BE, CI, VI, MA, US, NO, CA, EG and BA. We find that the grey interval range of these markets is smaller than that of the US. This indicates that the nine markets have high ratings and that the demands from these markets are stable. Therefore, they are highly desirable.

Volatile Markets: We find the LI, LU, NC, IM, NE and SI markets in quadrant II to have high ratings but also high volatility, which suggests the markets are not as stable as those in quadrant I.

Low Potential Markets: Twelve markets are located in quadrant III, including GU, AR and KU, etc. These markets exhibit a small grey interval distance (less than the US market), indicating that the evaluation results are robust but the ratings are relatively low, making the markets unattractive.

Least Preferred Markets: Markets of TC, HK, BR, SA and OM receive poor ratings and are considered high risk, and thus they belong to the least desired quadrant, i.e., quadrant IV. Fig. 4 shows that although the ratings of some markets, such as TC and HK, achieve an average rating, their grey interval range is higher than others, signaling that the results in this quadrant are highly unstable. For markets belonging to this quadrant, MAS management would exclude them from further consideration.

We map the GN-AHP market analysis results of all 32 candidates in Fig. 4 to the world map in Figure 2.5.

Note: 1st quadrant - house; 2nd quadrant - plane; 3rd quadrant - telescope; 4th quadrant – sun with cloud.

Figure 2.5. The geographic information of 32 candidate markets

Figure 2.5 reveals that several of the Caribbean island countries demonstrate great potential. First, most countries in the Caribbean area lack good healthcare services. Second, as many of these countries provide tax shelters for the wealthy to hide their money, they attract wealthy people; thereby this fact makes it an appealing market for MAS expansion.

Although North America has the most advanced healthcare service in the world, it is still highly ranked due to its high demand and wealth. The European market, on the other hand, is quite saturated. Accordingly, the stiff competition in this market could lead to high costs and lower than expected revenues. Therefore, it is not recommended.

South America's ability to pay for jet services is much less than that of other markets. Although many countries in the Middle East can afford the costs associated with jet services, our analysis finds the market attractiveness is low relative to other markets. Furthermore, South East

Asia is, at this time, not recommended based on the results of our analysis. However, due to its high economic development and population growth, it may become an important region for future expansion. Finally, Caledonia, Guam and Equatorial Guinea are geographically remote (distant from other continents), and thus it is financially implausible for MAS to conduct its business in those areas.

2.6 SUBJECTIVE CRITERIA ANALYSIS ON GN-AHP MODEL

2.6.1 Subjective Criteria Selection

After identifying the countries in quadrant I as the preferred markets for expansion, MAS looks for one exact location in which the company could try to expand its business. In the former discussion, we chose the quantifiable criteria and sub-criteria. In fact, management experience and opinions often play an important role in location selection. Based on the results of Section 2.5, the management of MAS further takes into account four subjective indicators essential when making a decision regarding location. These indicators are climate, politics, infrastructure, and environmental and ecological conditions. We also include sub-criteria to better detail each indicator.

Climate conditions include three factors: (i) weather conditions, (ii) clearance conditions and (iii) spatial environmental conditions. Weather conditions refer to the natural environment such as temperature, rainfall and wind power. Clearance conditions refer to the area around the airport, which sets the limitations on obstacles and buildings. Spatial environmental conditions refer to the range of spatial delineation as defined by flight needs. The International Civil

Aviation Organization (ICAO) has divided the airspace into various categories to ensure the safety and efficiency of aircraft operation and thus prevent collisions between aircraft and aircraft as well as aircraft and obstacles.

With respect to political conditions, we include two factors: (i) domestic politics and (ii) domestic security, both of which are related to the stability and development of the nation. A stable government is the cornerstone of the steady development of MAS' chosen market.

Infrastructure conditions are described by (i) ground traffic conditions; (ii) ground and underground facilities; and (iii) technology level. Ground traffic conditions measure ground facilities related to the airport. For example, roads used to access the airport, parking lots, public transportation, etc. Ground and underground facilities include power supply, water supply, gas supply, communications, roads, drainage and other public facilities. Technology level includes the availability of technical support.

Environmental and ecological conditions include two sub-factors: (i) noise pollution control and (ii) ecological escape. Noise pollution is the noise generated when planes take off and land. It warrants serious consideration to ensure that the noise is at the acceptable level and has a low impact on the surrounding buildings and people. Ecological escape measures the environmental disruption of the airport, such as bird habitats, which results in enormous economic damage and puts the lives of aircraft crew members and passengers at risk.

Figure 2.6 presents the AHP hierarchy of the subjective criteria. Figure 2.7 shows the global weight for each criterion, sub-criteria and the five attractive markets based on subjective factors.

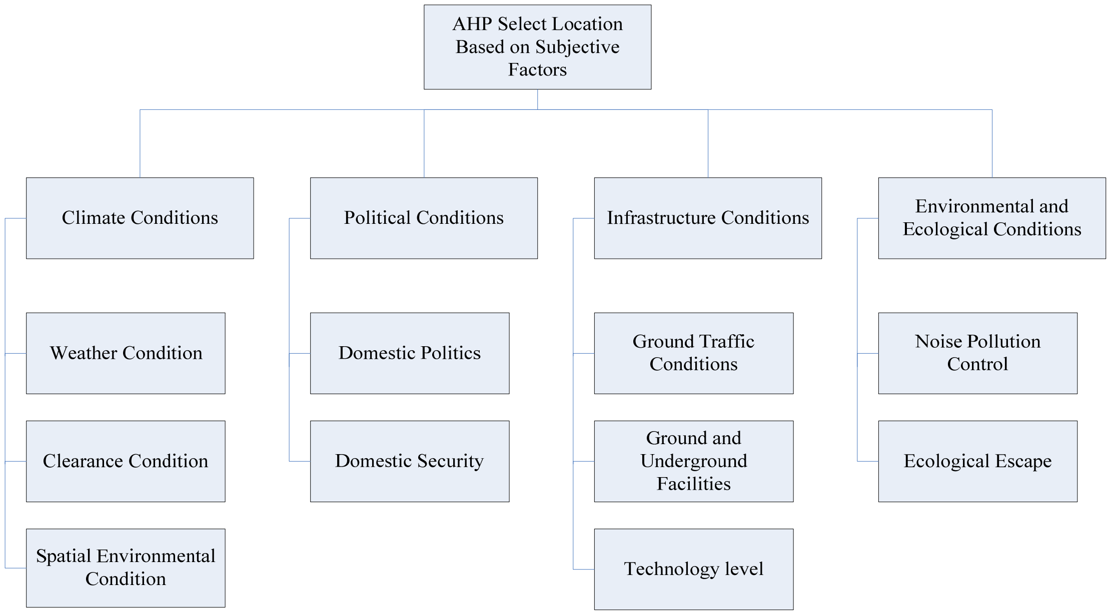


Figure 2.6. AHP hierarchy for selecting location based on subjective factors

2.6.2 Evaluation based on Subjective Criteria

From the objective criteria analysis in Section 2.5, we selected Bermuda, the Cayman Islands, the British-Virgin Islands, Macau and Norway (the shaded area of Quadrant I in Figure 2.4) for further analysis. The subjective evaluation results are presented in Figure 2.7.

We then combine the objective and subjective results to attain the composite performance. Table 2.5 summarizes the rating for each of the five markets and the corresponding ranking, from which we conclude that Bermuda is the best region for MAS to launch the next market, while the Cayman Islands and the British-Virgin Islands are runner-ups.

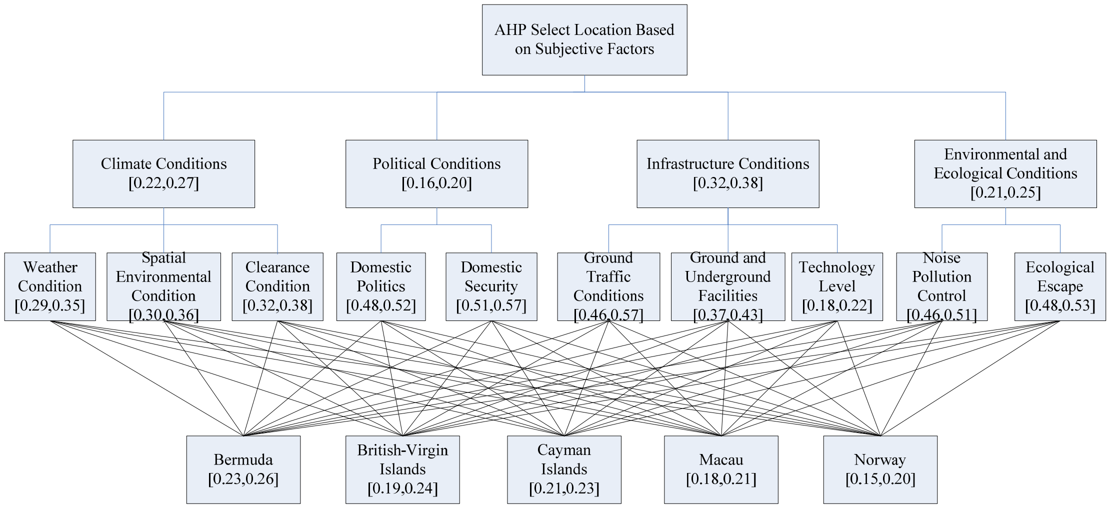


Figure 2.7. AHP hierarchy and the corresponding weight in grey number based on subjective factors

Table 2.5. Composite ratings/rankings for each of the five attractive markets

	Objective Factors (weight:0.6)	*Objective Rankings*	*Subjective Factors (weight:0.4)*	*Subjective Rankings*	*Overall Ratings*	*Overall Rankings*
Bermuda	[0.22,0.25]	1	[0.23,0.26]	1	[1.00,1.00]	1
British-Virgin Islands	[0.09,0.11]	3	[0.19,0.24]	3	[0.58,0.64]	3
Cayman Islands	[0.15,0.17]	2	[0.21,0.23]	2	[0.76,0.78]	2
Macau	[0.02,0.04]	5	[0.18,0.21]	4	[0.38,0.43]	5
Norway	[0.06,0.07]	4	[0.15,0.20]	5	[0.43,0.48]	4

2.7 CONCLUSIONS AND FUTURE WORK

We propose an international market evaluation model to help a private jet air medical transport company identify the potential market for global expansion. In this paper, we apply the grey number interval approach to tackle the uncertainty associated with pairwise judgments.

The results show that the interval data can effectively cope with the uncertainty in the process of evaluation. Based on the interval data, we use the AHP to derive the ratings and rank the 32 potential markets.

We recommend MAS to develop the Caribbean market first, i.e., Bermuda, the Cayman Islands, and the British-Virgin Islands. These three islands not only present great market potential, but also they are very close to MAS' US base. The Caribbean islands and Central America are crucial stepping stones as MAS seeks to enter the South American market. The European market may seem attractive as Norway is ranked fourth. However, as a US based firm, it may not be a smart move for MAS to enter that market as it would have to compete directly with strong European rivals.

When decision makers find it difficult to select a specific value in pairwise comparison, they are inclined to assign a range of values in judgment. In this study, we adopt a grey number to address such uncertainties. By using the grey interval number, the proposed GN-AHP method allows an approximate range of evaluation which provides valuable help in resolving decision makers' dilemma of choosing a single crisp number and improves the reliability of the assessment results in uncertain environments.

One limitation of this study is that we do not consider the interactions among the criteria. It is possible that some factors are correlated, and accordingly, the analytic network process (ANP) method may be more appropriate. By combining the ANP approach with the grey number, one can incorporate intricate relationships between evaluation elements. We leave this to future research. Furthermore, the data under study are collected on a country level. Yet, even within the same country, different areas may have different economic and healthcare conditions. Thus, if we could acquire the data at the state/province level, we may make even more accurate recommendations and provide additional options for MAS. From a methodological perspective, since our grey interval ranges are given in advance, one promising avenue of future research is to develop a method when comparison judgment ranges are kept as flexible rather than fixed in advance. How to handle overlaps of random intervals given by experts/reviewers could be a very interesting topic. Finally, in addition to developing the GN-AHP model, this study makes a contribution to the evaluation of the medical jet transport market. We anxiously look forward to seeing future extensions as discussed in this study.

3.0 PERFORMANCE EVALUATION FOR CHINA'S EXPRESS DELIVERY SERVICES: MULTI-CRITERIA INTERACTION WITH UNCERTAIN WEIGHTS

In this chapter, we study the China express delivery industry. We are particularly interested in gaining a better understanding of such a high growth industry.

To do so, we classify the whole China express delivery industry into three types of enterprises: (i) private enterprises, which mainly serve China's domestic express business; leading firms include SF Express, SHENTONG Express and YTO Express. (ii) State-owned express enterprises, which receive full or partial funding from Chinese government agencies; these include the Express Mail Service (EMS) operated by China Post; China Railway Express (CRE); and China Air Express (CAE). (iii) International express carriers, which include FEDEX, UPS, DHL, TNT, and so on. Our sample comprises, taken from all three groups, 12 leading express companies in China, including their subsidiaries that operate in provinces, municipalities and autonomous regions. Although only 12 express companies have been selected here for the sake of easy illustration, the framework we proposed can easily accommodate additional firms for comparison.

More specifically, we assess these 12 companies from five perspectives: operational strength, general company strength, cost efficiency, service quality and technology. We evaluate the criteria and the sub-criteria and then derive a composite score for each candidate. Specifically, because of high interdependency between criteria and sub-criteria, we use the analytic network process (ANP) (Saaty, 1996) to create the evaluation framework. Applying

ANP requires knowing each criterion's weight and pairwise-compared value relative to other criteria in order to determine the priority of alternatives. In practice, however, this type of information is not easy to determine explicitly as a single number. Our study therefore uses a survey conducted by Hefei University of Technology, which provided us with a range given by a group of experts rather than one consensus number on each weight and pair comparison value. For this reason, the ANP decision-making scheme is not sufficient to reflect the decision environment. If we simply take one point, say, the middle point, of any given range, it would mislead and inaccurately represent the experts' judgments. On the other hand, if we cover all expert opinions, the computations of ANP models could increase exponentially. Thus, we need a consistent approach to assign the weights for each criterion or sub-criterion to perform the evaluation while carrying affordable computations.

The remainder of the chapter is organized as follows. In Section 1, we review the literature; Section 2 lays the theoretical foundation for the proposed model; Section 3 describes governance in the express industry and proposes our model; in Section 4, we assess the 12 major Chinese express delivery companies and show the feasibility and practicability of the proposed model; Section 5 summarizes the computational results and offers managerial implications. Section 6 concludes.

3.1 LITERATURE REVIEW

3.1.1 Literature on Performance Evaluations

Performance evaluation is used to assess risk and analyze the operational status of a company. It identifies matters that may prevent a business from attaining its objectives, with the goal of determining how to assist the organization by providing tools to improve the effectiveness of current or planned operations (Murphy and Cleveland, 1995). Thus, performance evaluation plays a crucial role in a competitive business environment (Amado, et al, 2012).

The objectives, criteria and procedures adopted for performance evaluation have become a focus of study. For example, Morais and Camanho (2011) applied the data envelopment analysis (DEA) approach to evaluate the performance of cities in terms of how the public views their quality of life given the economic condition of the country. Amado et al. (2012) combined the Balanced Scorecard method with DEA to identify how to improve organizational performance in a multinational company.

Research on the performance evaluation process can be divided into two streams:

1) The first stream seeks to select judging criteria based on raters' preferences. In this stream, researchers usually select judging criteria in terms of the importance of each factor from their own perspectives. Studies such as those of Cleveland and Murphy (1992), Murphy and Cleveland (1995), Sarkis (2003), Yurdakul (2003), Chung et al. (2005), Lin et al. (2009), Abdi and Labib (2011), and Samvedi et al. (2012) made selections by following this stream.

2) The second stream studies performance appraisal and the organizational context. In this stream, researchers conduct performance evaluations based on certain industrial standards rather than using self-selected criteria (Tziner et al. 1998, Tziner 1999, Van Horenbeek and Pintelon 2014). In general, government agencies, authorities or other authoritative organizations industrial standards and require meeting specific criteria.

This study falls into the first category, i.e., we select important attributes primarily based on published studies of the express industry. Some attributes were chosen by reference to certain regions/countries' industry standards. Further, to best our knowledge, no performance evaluation research has been conducted on the China express delivery industry from an investor's perspective. This study aims to fill that gap.

3.1.2 Analytic Network Process (ANP)

The analytic network process (ANP) (Saaty, 1996), an extension of the analytic hierarchy process (AHP) (Saaty 1977, 1980, 1986), is a comprehensive decision-making technique that has the capability to include all of the relevant criteria in arriving at a decision. In the ANP, a complicated decision problem is decomposed into a rational decision hierarchy based on related attributes or criteria. The ANP can resolve complex multi-criteria decision problems when problems involve multi-criteria or hierarchy dependence relationships. Through the use of supermatrices, the ANP can effectively address problems whose elements interact and form a network structure. In our study, we use pairwise comparisons to assess tangible and intangible factors affecting express delivery companies based on knowledge, experience and available data to yield consistent and systematic judgments.

3.1.3 Decision-making on Third-Party Logistics and Performance Evaluation using the ANP

Third-party logistic problems have been extensively studied across various topics. Marasco (2008) and Aguezzoul (2014) provided comprehensive reviews of third-party logistics selection for the last two decades. Falsini et al. (2012) proposed a mathematic method that combined the AHP, the DEA and linear programing to evaluate logistics providers. Jharkharia and Shankar (2007) explored logistics providers by using the ANP method. Kayakutlu and Buyukozkan (2011) further investigated several factors that may have significant impacts on 3PL performance. In particular, they focused on both strategic and operational level factors, such as performance targets, planning activities, logistics operations and performance attributes.

3.1.4 Grey Number (GN) and Center-point Triangular Whitenization Weight Function (CTWF)

3.1.4.1 Grey Number (GN)

The grey system proposed by Deng (1982) is a mathematical theory originating from the grey set concept that can effectively tackle problems with incomplete discrete data and uncertainty. If information is known completely, it is called a white system. When information is totally unknown, it is a black system. A system with partially known and partially unknown information is called a grey system. Thus, a grey system contains uncertain information that is presented in grey numbers and grey variables.

Liu and Lin (2006) formalize the grey number by defining:

$$\otimes a = [\underline{a}, \overline{a}] = \{a \mid \underline{a} \le a \le \overline{a}; \underline{a}, \overline{a} \in R\}$$

where $\otimes a$ includes two real numbers, with $\underline{a}$ being the lower limit of $\otimes a$ and $\overline{a}$ being the upper limit of $\otimes a$. The grey system is effective in dealing with insufficient and incomplete information. And information in real-world is often incomplete and uncertain. Hence, extending from the exact information to the grey system is necessary for many real-world applications. Several studies had tried to apply the grey system into several disciplines (Pai et al. 2007; Kayacan et al. 2010; Bai and Sarkis 2010; Zhao et al. 2012).

3.1.4.2 Center-point Triangular Whitenization Weight Function

The center-point triangular whitenization function (CTWF) is a grey cluster evaluation method using grey information. The CTWF is applicable when the most likely value of each grey class is known but their boundaries are unclear. It can effectively whiten grey information and reduce the effect of incomplete discrete data and uncertainty over the decision results.

CTWF has received significant attention from researchers and has been employed in various practical scenarios. For instance, Liu et al. (2006) applied this model to the evaluation of regional industries, regional industrial parks and regional capabilities. Moreover, Yang et al. (2005) combined this model with the AHP method in their evaluations of investment of venture capital.

One of the recent trends in the AHP/ANP methodology is to incorporate uncertainty. The AHP/ANP are useful subjective methods, and different evaluators may have different opinions concerning each criterion. To include such uncertainty, extended ANP methodologies have become popular. Ayağ and Özdemir (2012) used a fuzzy ANP to evaluate machine tool attitudes given uncertain human inputs; Chen et al. (2010) also used a fuzzy ANP to study the operational performance of a university. Our study aims to make a methodological contribution in the spirit of this new trend, but instead of using fuzzy theory combined with the ANP, which has been debated in academic research, we integrate CTWF and ANP. In particular, we use the CTWF to handle uncertainty and then transfer output from the whitenization weight function to the ANP framework.

3.2 THE THEORETICAL FOUNDATION OF THE CTWF-ANP MODEL

In this section, we propose the CTWF-ANP model by combining the ANP and the center-point triangular whitenization weight function.

3.2.1 Building the CTWF

From the basic concept of grey numbers and the whitenization weight function (Liu and Lin, 2006), we can easily show the construction process of CTWF.

Assume that n objects have been clustered into s different grey classes according to m evaluation criteria. Let $x_{ij},\ i=1,2,\cdots,n,\ j=1,2,\cdots,m$ be the observed value of object i under criterion j. We must use $x_{ij},\ j=1,2,\cdots,m$ to assess object $i,\ i=1,2,\cdots,n$. To achieve this end, we need to go through the following steps.

(i) Let $\lambda_1,\lambda_2,\cdots,\lambda_s$ be the center points of individual intervals, which correspond to s grey classes, respectively.

(ii) Expand the grey classes in two different directions by adding a 0 and $(s+1)$ grey classes with their centers λ_0 and λ_{s+1} determined. Thus, the new sequences of centers are $\lambda_0,\lambda_1,\lambda_2,\cdots,\lambda_s,\lambda_{s+1}$. By respectively connecting the point $(\lambda_k,1)$ with the center $(\lambda_{k-1},0)$ and $(\lambda_{k+1},0)$ of the $(k-1)th$ and the $(k+1)th$ small grey classes, we obtain the triangular whitenization weight function $f_j^k(\cdot)$ for the kth grey class of the jth criterion, $j=1,2,\cdots,m,\ 1,2,\cdots,s$. See Figure 3.1 for details.

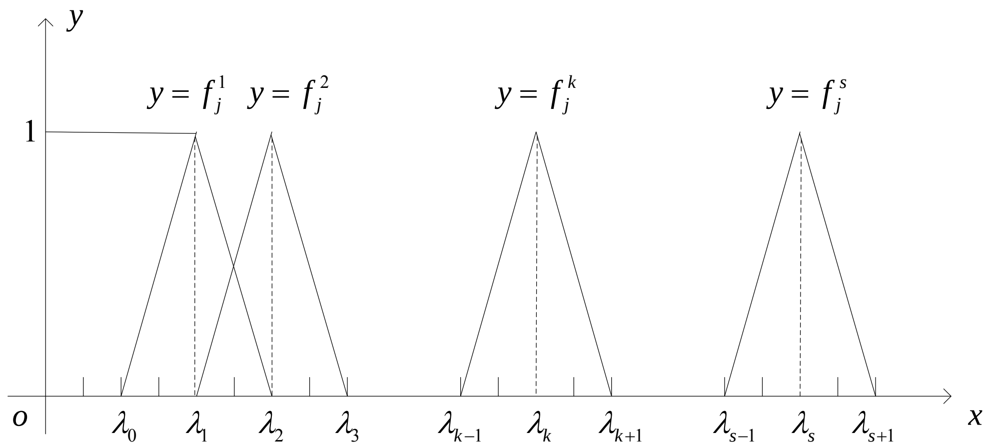


Figure 3.1. Center-point whitenization weight functions

For an observed value x under criterion j, we can employ the formula

$$f_j^k(x) = \begin{cases} 0, & x \notin [\lambda_{k-1}, \lambda_{k+1}] \\ \dfrac{x-\lambda_{k-1}}{\lambda_k - \lambda_{k-1}}, & x \in (\lambda_{k-1}, \lambda_k] \\ \dfrac{\lambda_{k+1}-x}{\lambda_{k+1}-\lambda_k}, & x \in (\lambda_k, \lambda_{k+1}] \end{cases}$$

to compute its degree of membership $f_j^k(x)$ in grey class k, $k=1,2,\cdots,s$.

(iii) Compute the comprehensive clustering coefficient σ_i^k for object i, $i=1,2,\cdots,n$, with respect to grey class k, $k=1,2,\cdots,s$

$$\sigma_i^k = \sum_{j=1}^{m} f_j^k(x_{ij}) \cdot \eta_j$$

where $f_j^k(x_{ij})$ is the whitenization weight function of the kth subclass of the jth criterion, and η_j the weight of criterion j in the comprehensive clustering.

(iv) From $\max_{1\le k\le s}\{\sigma_i^k\} = \sigma_i^{k^*}$, it is decided that object i belongs to grey class k^*. When there are several objects in grey class k^*, these objects can be ordered according to the magnitudes of their comprehensive clustering coefficients.

3.2.2 Modeling Steps of the CTWF-ANP Model

Based on the discussion above, we generalize the procedure of the CTWF-ANP model into five stages:

Stage 1: Construct the grey number judgment matrices.

Stage 2: Whiten each grey entry by using the center-point whitenization weight function $f_j^k(x_{ij})$.

Stage 3: Determine the whitening value $\max_{1\le k\le s}\{\sigma_i^k\} = \sigma_i^{k^*}$, $(i = 1, 2, \cdots, n)$ and construct the whitening judgment matrices.

Stage 4: Calculate the weight of each alternative by using the ANP principle.

Stage 5: Rank the preference order.

3.3 RANKINGS IN THE EXPRESS INDUSTRY AND THE PROPOSED MODEL

3.3.1 Express Company Rankings

Several consulting firms and researchers have touched upon China's express delivery sector; however, they either offer an overview of the Chinese express industry (Deloitte 2014, L.E.K. Consulting 2012, GF Securities 2012) or examine companies through a limited perspective (CRIIE 2014, Li 2010), such as consumer complaints or service responsiveness. Neither academic studies nor any third-party organization has done much work to evaluate Chinese express firms using multiple criteria. Our study aims to provide a comprehensive evaluation that includes as many fundamental judgments as possible. We attempt to rank express companies and lay out their competitive advantage in relation to one another for financiers who are interested in investing in the express delivery industry.

3.3.2 The CTWF-based ANP Model

The uncertainty in assessment often originates from various opinions of experts. To incorporate these valuable inputs without losing too much information, we propose a CTWF-based ANP model to resolve the ambiguity in judgments when conducting a performance evaluation of China's express delivery industry.

The proposed model offers a method to derive consensus when assessing firms based on the five basic standards mentioned above. Through the CTWF-ANP model, we can systematically and efficiently prioritize individual companies and detect the weakness/risks of each express delivery company for investors. Figure 3.2 shows the proposed unified framework.

The ANP is used as the main evaluation model, which is integrated with the grey numbers and the center-point triangular whitenization weight function to effectively incorporate uncertain information and comprehensively aggregate judgments throughout the evaluation process. The CTWF-ANP model is capable of incorporating uncertain information and requires only slightly more time to evaluate the firms than the conventional ANP method.

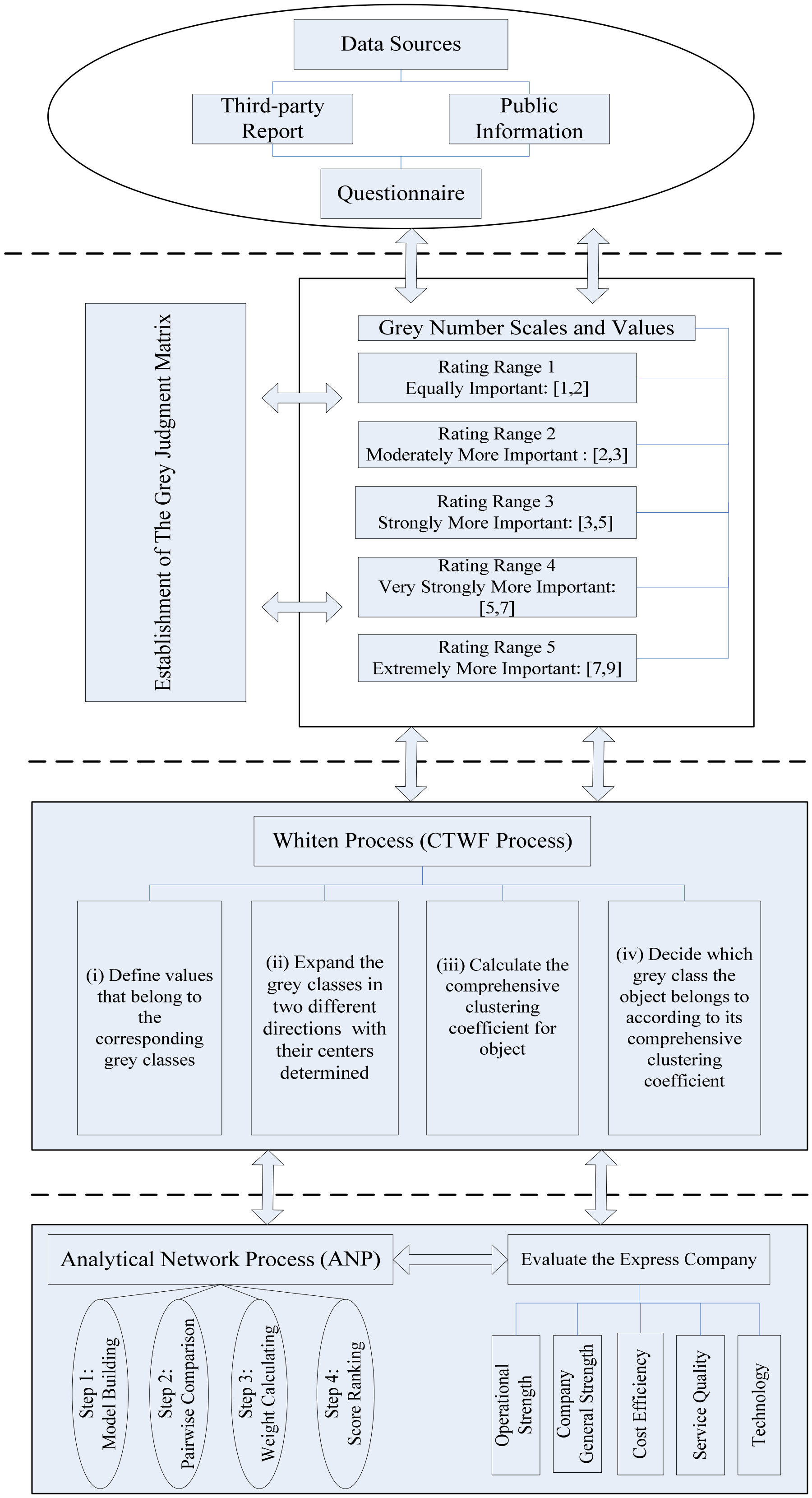


Figure 3.2. The proposed CTWF-ANP framework

3.4 APPLYING THE CTWF-ANP MODEL

3.4.1 Data Sources

We first collected data on China's express firms. We found our data from: (i) third-party reports; (e.g., China Express Delivery Industry Report, 2013-2016 from www.reportbuyer.com; China's Express Sector Development Report 2014 from Deloitte); (ii) publicly available sources; (e.g., China Express Management Institute (CEMI), a governing authority in China's express industry; China Research Institute of Industrial Economy (CRIIE), a special research organization that collects data from the development process in China's industrial areas; State Post Bureau of China (SPBC), the highest authority in China's mail/package delivery sector).

After data collection, through Hefei University of Technology, we invited several area experts from Beijing Jiaotong University, which is regarded as one of the best transportation research universities in China, to participate in a survey via questionnaire (see Appendix B for a sample table of the questionnaire). These experts answered questions based on their experiences and the data we collected. The details of how the experts accessed each sub-criterion will be discussed in 3.4.3.

3.4.2 The Evaluation Criteria

To make an evaluation, we selected five basic standards for comparison. These five basic standards are: (i) Operational Strength (OS); (ii) General Company Strength (GCS); (iii) Cost Efficiency (CE); (iv) Service Quality (SQ); and (v) Technology (TE). We define these five standards as follows:

(i) Operational Strength: a measure of an express company's normal business operations, such as business volume, operational mode and operational flexibility.

(ii) General Company Strength: a measure of an express company's financial health, brand reputation and internal management structure.

(iii) Cost Efficiency: this criterion measures a company's cost level and other business costs, such as partnership and damage incidence occurrence.

(iv) Service Quality: this measure focuses on customers' feedback and satisfaction with a given express company.

(v) Technology: this criterion measures the express companies' current technology levels in various areas, including advanced package tracking systems, real-time location for couriers/vehicles, and auto-classifying package systems. Furthermore, the company's future plan for technology is also important and includes general R&D expenses and employee training expenses.

Under each of the five control criteria, several sub-criteria are developed. Details can be found in Table 1.

3.4.3 The CTWF-ANP Model for Attribute Rating

As stated above, the CTWF-ANP model can effectively overcome the adverse effects caused by uncertainty. When the judgment has a variety of possibilities, decision makers are unable to choose a precise number to express the important degree in paired comparisons. Below following, we detail the steps of the CTWF-ANP model:

Step 1: Create the model

Figure. 3.3 shows the ANP network. The goal shown at the top is to rank selected express delivery companies. In the second level, it displays five control criteria. A company with worse operational strengths would receive lower evaluation scores. The better a company's general strengths, the higher the evaluation scores the company obtains. Further, lower cost efficiency and lower service quality lead to a lower evaluation score. Finally, a company with a lower technical level has a lower assessment score, as it may be not competitive.

Step 2: Make pairwise comparisons

The decision makers provide the pairwise comparison results in Table 3.1. Because there is no consensus given for each sub-criterion, the decision makers need to make a pairwise comparison in each strategic criterion and sub-criterion with grey numbers. Each element is compared to every other element at the same level in the hierarchy in turn. If one is equally as important as the other, the ratio is defined as one or two, i.e., [1,2]; if one is moderately more important than the other, the ratio is [2,3]. Similarly, for strongly greater importance, the ratio is [3,5]; for very strongly greater importance, the ratio is [5,7]; and for extremely greater importance, the ratio is [7,9]. Responses in between these grey numbers are allowed. These judgments thus lead to matrices of comparisons.

From the judgment results in Table 3.1, we found that General Company Strength is a weaker indicator. Operational Strength is between 3 and 4, which means it is more important than General Company Strength but only 1/3 to 1/2 as important as the Cost Efficiency of an express company. The diagonal of the matrix consists of [1,1] because each criterion is equally important when compared with itself.

Table 3.1. Pairwise comparison results of the main criteria in grey numbers

	Operational Strength	*Company General Strength*	*Cost Efficiency*	*Service Quality*	*Technology*	*Global weight*
Operational Strength	[1,1]	[3,4]	[1/3,1/2]	[1/4, 1/3]	[1,1]	[0.14,0.18]
Company General Strength	[1/4,1/3]	[1,1]	[1/5,1/4]	[1/5,1/4]	[1/4,1/3]	[0.05,0.06]
Cost Efficiency	[2,3]	[4,5]	[1,1]	[1,1]	[2,3]	[0.29,0.31]
Service Quality	[3,4]	[4,5]	[1,1]	[1,1]	[3,4]	[0.33,0.35]
Technology	[1,1]	[3,4]	[1/3,1/2]	[1/4,1/3]	[1,1]	[0.14,0.17]

The lower triangle of the matrix is the inverse of other responses. This pairwise comparison process continues at each level of the hierarchy with the exception of the bottom.

Step 3: Translate the comparison matrices into weights

Because of the uncertain decision environment, decision makers are uneasy about assigning a specific value to each pair of comparisons. Usually, they offer an interval to express their judgments at the expense of accuracy. Therefore, to improve the preciseness of evaluation, we need to whiten the pairwise comparison results of the control criteria before launching the ANP structure.

Table 3.2. Sub-Criterion descriptions and references

Control Criterion	Sub-Criterion	Sub-Criterion Descriptions	References
Operational Strength	Operational Coverage & Business Volume (OC & BV)	Measures an express company's network and capacity. Coverage consists of three types of business: international, cross-regions and intra-city. Business volume for each type indicates capacity and the main strength of a company.	Deloitte (2014), Falsini et al.(2012), Jharkharia&Shankar (2007),Kayakutlu&Buyukozkan (2011), Mitra et al.(2010)
	Operational Mode & Breadth (OM & B)	Infers what an express company has employed as its transport mode: air, ground and/or local. In addition to transportation, it evaluates any other value-added services offered, such as warehousing and freight forwarding.	Aguezzoul (2014), Deloitte (2014), Falsini et al.(2012), Kayakutlu& Buyukozkan (2011), Mitra et al.(2010)
	Operational Flexibility (OF)	This includes two main perspectives: supply chain flexibility and adoptability to changes of government policy. Good supply chain flexibility can offer a great buffer when a company is experiencing unpredicted events. The regulations in China's express industry are not well established; a company needs to prepare for future regulations changes from the government.	Aguezzoul (2014), Deloitte (2014), Falsini et al.(2012), Jharkharia&Shankar (2007)
General Company Strength	Financial Health (FH)	A good financial situation can boost a company's business and continuously support the company's development.	Aguezzoul (2014), Jharkharia&Shankar (2007), Kayakutlu&Buyukozkan (2011)
	Brand Reputation & Experiences (BR &E)	A good brand reputation in public and industry can help a company gain more business; more experience can also help a company better serve its customers.	Aguezzoul (2014), Jharkharia&Shankar (2007), Mitra et al.(2010)
	Management Structure (MS)	Includes two types of company structures: corporate and franchise. A corporate structure indicates a company that fully owns and operates all of its subsidiaries; a franchise structure gives more operational rights to its subsidiaries. A franchise structure can quickly extend a company's coverage, but may bring inconsistent services for its customers due to various subsidiaries' operational styles.	Falsini et al. (2012), Jharkharia&Shankar (2007), Kayakutlu&Buyukozkan (2011)
Cost Efficiency	Shipping Charge (SC)	Shipping rate is the core problem for any express company and directly reflects a company's cost level.	Aguezzoul (2014), Deloitte (2014), Falsini et al. (2012), Jharkharia&Shankar(2007), Mitra et al. (2010)
	Partnership (P)	Express delivery is part of a supply chain. Building and maintaining good relationships with downstream e-tailers and other partners is a key factor in success.	Aguezzoul (2014), Deloitte (2014), Jharkharia&Shankar(2007), Kayakutlu&Buyukozkan(2011)
	Damage Incidence Occurrence (DIO)	During shipping, losses and damages cannot be 100% avoided; therefore, expenses for damages and other losses play an important role in a company's costs.	Falsini et al.(2012), Jharkharia&Shankar (2007)

Service Quality	Integration of Services (IS)	Express business is beyond package delivery. A well-integrated express service can enhance customers' good experiences with a company. This includes employees' professionalism and the coordination of different express business processes.	Aguezzoul (2014)
	On-Time Delivery Reliability (OTDR)	On-time delivery and reliability are the most important indicators for customers when evaluating an express company. They include tracking capability, on-time rate and shipping speed.	Aguezzoul (2014), Falsini et al.(2012), Jharkharia&Shankar (2007), Mitra et al.(2010)
	Customer Relationships (CR)	Measures how a company handles end-customers' feedback, including complaints, claims and other service inquiries.	Aguezzoul (2014), Jharkharia&Shankar (2007), Kayakutlu&Buyukozkan(2011)
Technology	Adoption of Technology and Innovation (ATI)	Advanced technology and innovation can make deliveries more transparent to customers, enabling them to track their packages and increase confidence that a company can scale up to handle higher business volume and control its business.	Aguezzoul (2014), Deloitte (2014), Falsini et al.(2012), Jharkharia&Shankar (2007), Kayakutlu&Buyukozkan(2011)
	Investment in Technology & Development (IT & D)	This criterion indicates a company's plan for improving its future performance. It includes investment in information systems, equipment, warehouses, vehicles, materials handling and employee training.	Aguezzoul (2014), Falsini et al.(2012)

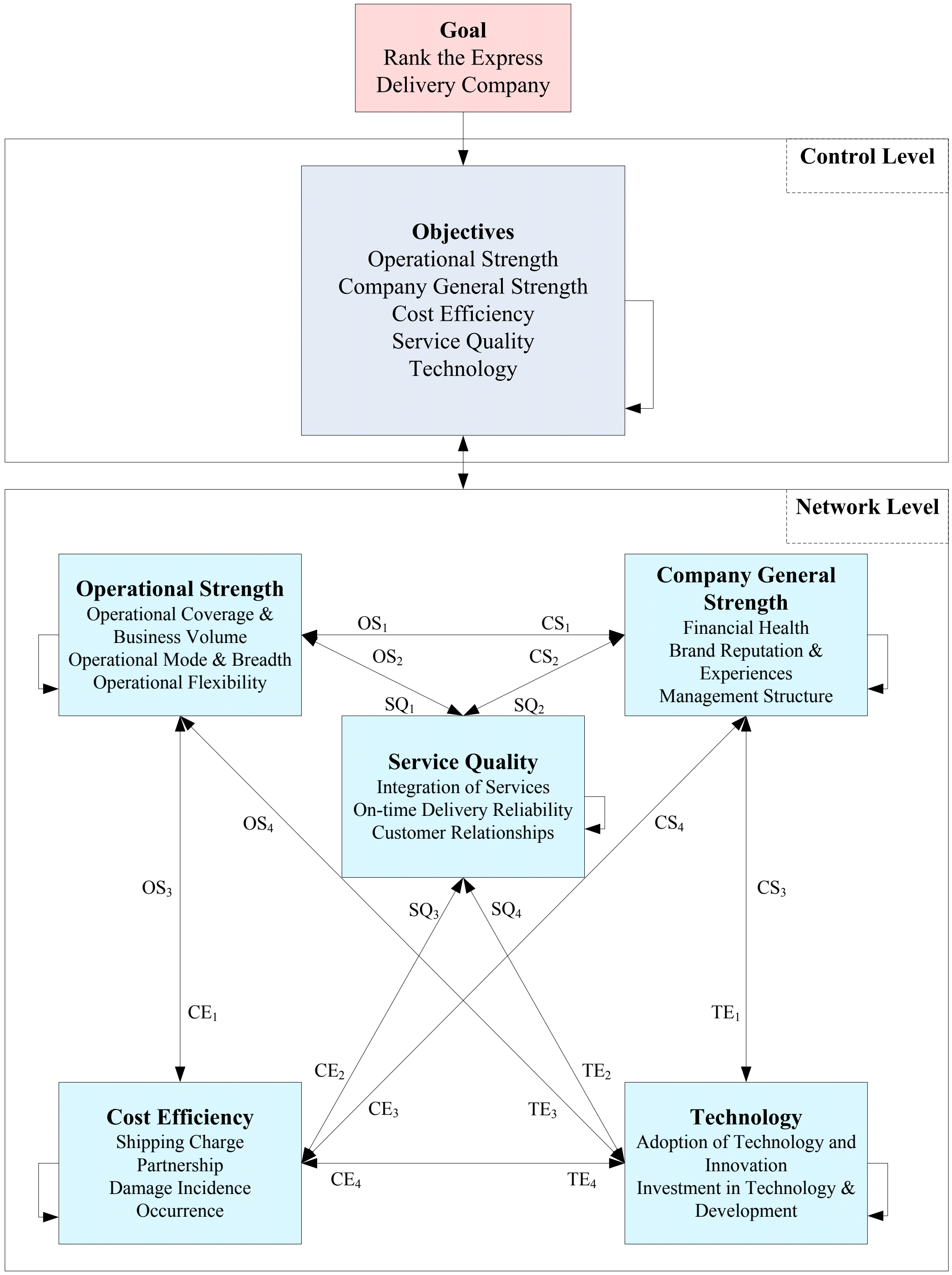


Figure 3.3. Generic ANP network structure for express delivery company evaluation

Next, we will show how the center-point triangular whitenization weight function works.

Phase 1: For the required five grey classes, "extremely more important," "very strongly more important," "strongly more important," "moderately more important than the other," and "equally important as the other," we first determine the center points as follows: $\lambda_1=8$, $\lambda_2=6$, $\lambda_3=4$, $\lambda_4=\frac{5}{2}$, $\lambda_5=\frac{3}{2}$.

Phase 2: Expand the required grey classes by adding a class named "extremely more important-plus" and another class named "equal important than the other-minus." Assume that the centers of these additional classes are $\lambda_0=10$, $\lambda_6=\frac{1}{2}$ so that we obtain a new sequence of class centers: $\lambda_0=10$, $\lambda_1=8$, $\lambda_2=6$, $\lambda_3=4$, $\lambda_4=\frac{5}{2}$, $\lambda_5=\frac{3}{2}$, $\lambda_6=\frac{1}{2}$. By connecting point $(\lambda_k,1)$ and the center points $(\lambda_{k-1},0)$ and $(\lambda_{k+1},0)$ of the $(k-1)th$ grey class and the $(k+1)th$ grey class, we produce the triangular whitenization weight function $f_j^k(\cdot)$ for the kth grey class with respect to the jth criterion, $k=1,2,3,\cdots,5$, as follows:

$$f_j^1(x)=\begin{cases}0, & x\notin[6,10]\\ \frac{x-6}{2}, & x\in(6,8]\\ \frac{10-x}{2}, & x\in(8,10)\end{cases},\quad f_j^2(x)=\begin{cases}0, & x\notin[4,8]\\ \frac{x-4}{2}, & x\in(4,6]\\ \frac{8-x}{2}, & x\in(6,8)\end{cases},$$

$$f_j^3(x)=\begin{cases}0, & x\notin[\frac{5}{2},6]\\ \frac{x-5/2}{3/2}, & x\in(\frac{5}{2},4]\\ \frac{6-x}{2}, & x\in(4,6)\end{cases},\quad f_j^4(x)=\begin{cases}0, & x\notin[\frac{3}{2},4]\\ x-\frac{3}{2}, & x\in(\frac{3}{2},\frac{5}{2}]\\ \frac{4-x}{3/2}, & x\in(\frac{5}{2},4)\end{cases}$$

$$f_j^5(x)=\begin{cases}0, & x\notin[\frac{1}{2},\frac{5}{2}]\\ x-\frac{1}{2}, & x\in(\frac{1}{2},\frac{3}{2}]\\ \frac{5}{2}-x, & x\in(\frac{3}{2},\frac{5}{2})\end{cases}$$

Phase 3: Compute the comprehensive clustering coefficient σ_i^k for scale i, $i=1,2,3,\cdots,9$, $k=1,2,3,\cdots,5$. For details, see Table 3.

Table 3.3. Comprehensive clustering coefficient σ_i^k for control criterion

Grey class	OC&BV	OM&B	OF	OS
1	0	0	0	0
2	0	0	0.5	0.285
3	0	0.33	0.5	0.3807
4	0.5	0.67	0	0.2643
5	0.5	0	0	0.07
Grey class	**FH**	**BR&E**	**MS**	**CGS**
1	0	0.5	0	0.305
2	0	0.5	0	0.305
3	0	0	0	0
4	0	0	0.5	0.125
5	0.5	0	0.5	0.19
Grey class	**SC**	**P**	**DIO**	**CE**
1	0	0	0	0
2	0	1	0	0.58
3	0	0	0	0
4	0.5	0	0.5	0.21
5	0.5	0	0.5	0.21
Grey class	**IS**	**OTDR**	**CR**	**SQ**
1	0	0	0	0
2	0	0	0	0
3	0.33	0.33	1	0.6114
4	0.67	0.67	0	0.3886
5	0	0	0	0
Grey class	**ATI**	**IT&D**		**TE**
1	0.5	0		0.375
2	0.5	0		0.375
3	0	0.33		0.0825
4	0	0.67		0.1675
5	0	0		0

Phase 4: From $\max_{1\le k\le 5}\{\sigma_1^k\}=\sigma_1^3=0.3807$, $\max_{1\le k\le 5}\{\sigma_2^k\}=\sigma_2^1=\sigma_2^2=0.305$, $\max_{1\le k\le 5}\{\sigma_3^k\}=\sigma_3^2=0.58$, $\max_{1\le k\le 5}\{\sigma_4^k\}=\sigma_4^3=0.6114$, and $\max_{1\le k\le 5}\{\sigma_5^k\}=\sigma_5^1=\sigma_5^2=0.375$, we find that the composite scores of operational performance, general performance, service cost, service quality and express

technology are 0.3807, 0.305, 0.58, 0.6114, and 0.375, respectively. The corresponding weight of each criterion is 0.17, 0.14, 0.26, 0.27, and 0.17, which are the results of Table 3.3.

In particular, the center-point triangular whitenization weight functions use the points λ_k that most likely belong to the kth grey class as the center points so that these weight functions can be easily constructed using $\lambda_0, \lambda_1, \lambda_2, \cdots, \lambda_s, \lambda_{s+1}$. Conventionally, people tend to be more confident about the center points of grey classes than the grey classes themselves. Thus, the conclusions derived based on this cognitive assurance are more scientific and more reliable.

By following the above steps to implement the CTWF-ANP model in Section 3.2, we obtain the global weights. Tables in Appendix B. show the weights for each criterion and sub-criterion.

Step 4: Combine weights and judgments to establish composite scores

Evaluating all 12 express delivery companies simultaneously would result in too many pairwise comparisons for the conventional ANP method. Therefore, we adopt the ANP rating method because it creates an ideal alternative — a conceptual alternative that receives the highest possible rating under each factor. The rating of all candidates can be found by examining how far the alternatives are from the ideal. Tables in Appendix B indicate the weight for each of the 12 candidate express companies derived by the ANP rating methods.

Table. 3.4 gives the global score for each alternative, along with the subscore under each criterion. Based on the results, the priority of alternatives is obtained (the last Column in Table 3.4); SFEx is shown to have the highest score and is ranked as the top performer.

Table 3.4. Comprehensive ranking in sub-network and CTWF-ANP model (1/100)

	OS (W:0.17)	CGS (W:0.14)	CE (W:0.26)	SQ (W:0.27)	TE (W:0.17)	Global weight	Global ranking
EMS	5.71	3.93	8.23	9.80	6.51	7.4134	3
CREx	2.76	2.19	4.21	4.44	2.72	3.7976	7
CAEx	1.43	1.68	2.19	2.30	1.41	2.1128	11
SFEx	6.65	4.25	10.68	11.31	6.97	8.7409	1
ZJSEx	2.55	2.02	3.89	4.11	2.52	3.5122	9
STOEx	4.10	3.05	6.78	7.20	5.04	5.6876	4
YTOEx	5.72	4.02	9.26	9.82	6.53	7.7043	2
YUNDAEx	1.62	1.29	2.48	2.61	1.60	2.2343	10
TTKEx	1.61	1.27	2.46	2.60	1.59	2.0634	12
FedEx	2.32	4.75	3.54	7.69	6.51	5.1686	5
UPS	1.95	4.32	1.44	6.58	5.32	3.9917	6
DHL	1.49	4.11	1.11	6.36	5.11	3.7032	8

3.5 COMPUTATIONAL RESULTS

3.5.1 Overall Analysis

We use the overall priorities to evaluate the 12 express delivery companies. The CTWF-ANP results represented in Table 3.5 are divided into 8 columns. Column 1 shows the name of the express companies, and columns 2 through 6 represent the evaluation results of the express companies under the five control criteria (i.e., Criterion OS, CGS, CE, SQ and TE). The seventh column represents the synthesized priority of the express companies by incorporating the correlation between each control criterion. The final column is the ranking column; final ranking results of the 12 express companies are shown in this column.

Column 7 shows that the total score of company SFEx is higher than any of the other 11 express companies, indicating that SFEx is the best company for overall performance. Conversely, the total score of company TTKEx is the lowest, which means the overall performance of TTKEx does not rank very highly in terms of our evaluation system. The remaining 10 companies, YTOEx, EMS, STOEx, FedEx, UPS, CREx, DHL, ZJSEx, YUNDAEx and CAEx, occupy the second through eleventh positions, respectively. For details, see the ranking column in Table 3.5.

Table 3.5 represents the ranking results for the 12 express companies at the control criterion level. The general tendency of the ranking results show that private express companies are at the top of the rankings, with state-owned and international companies following; however, the ranking orders at each control criterion are not consistent with the

Table 3.5. Express Companies' ranking graphic for each control criterion and CTWF-ANP model

Company	Operational Performance	Company General Strength	Cost Efficiency	Service Quality	Express Technology	Total Score	Global Ranking
SFEx	6.65	4.25	10.68	11.31	6.97	8.74	1
YTOEx	5.72	4.02	9.26	9.82	6.63	7.71	2
EMS	5.71	3.93	8.23	9.8	6.51	7.41	3
STOEx	4.1	3.05	6.78	7.2	5.04	5.68	4
FedEx	2.32	4.75	3.54	7.69	6.51	5.17	5
UPS	1.95	4.32	1.44	6.58	5.32	3.99	6
CREx	2.76	2.19	4.21	4.44	2.72	3.79	7
DHL	1.49	4.11	1.11	6.36	5.11	3.71	8
ZJSEx	2.55	2.02	3.89	4.11	2.52	3.51	9
YUNDAEx	1.62	1.29	2.48	2.61	1.6	2.23	10
CAEx	1.43	1.68	2.19	2.3	1.41	2.11	11
TTKEx	1.61	1.27	2.46	2.6	1.59	2.06	12

global ranking orders, which shows that the developmental level of each express company is uneven. For example, at the company performance and express technology level, international carriers performed significantly better than the private and state-owned enterprises, while in the other control criteria, such as service costs, operational performance and international carriers, they underperformed. We explain this disorder through the following reasons: (i) each express company may have different priorities and strategies to develop its business, which leads to different assessment results under different criteria; (ii) a single criterion does not have enough weight to change the overall evaluation results; and (iii) the criterion with disorder receives a relatively low weight and therefore has a smaller impact on the final global rankings.

3.5.2 Classification Analysis

Outside investors are most interested in learning how to invest in China's express industry. In previous analysis, we viewed the 12 express companies individually while evaluating their business performance. These companies in essence can be grouped into three sectors: (i) private enterprises, namely SFEx, YTOEx, STOEx, ZJSEx, YUNDAEX and TTKEx; (ii) state-owned enterprises or EMS, CREx and CAEx; and iii) international carriers, here FedEx, UPS and DHL. By grouping all 12 firms, we draw Figure 3.4. In Figure 3.4, we calculate each group's average score under each criterion. We put the names of five criteria and total average score on the horizontal line, with the actual average numbers on the vertical line. Figure 3.4 clearly shows that the private sector enterprises stand out. Private enterprises outperform state-owned ones in all criteria; however, for some criteria, the difference between private and state-owned is not significant. The major issues for private enterprises are the following: (i) from a company performance perspective, private enterprises are eager to have more investors involved because the development of these firms still requires a significant amount of capital; on the other hand, the company's internal structures need to be reformed. Most of China's private express companies have adopted a franchise business model, which can rapidly expand a company's business coverage at the initial stage of development. When a firm grows to maturity stage, however, it needs to turn this business style into a corporate or self-running mode to consistently offer high-quality services to its clients. The firm should note that doing so requires significant capital. (ii) From an express technology perspective, private enterprises need to improve their technologies to better serve their customers.

All international carriers are in the middle range of the final ranking; however, for the group ranking, the international carrier group is not significantly superior. Figure 3.4 shows that the strengths of international carriers are their mature technology, excellent company overall performance due to having sufficient capital, established reputations and corporate structures, and their high-quality delivery services; on the other hand, these firms are dragged to the bottom by their high service costs and relatively poor operational performance. (i) For regulatory reasons, international carriers entered the Chinese market in the late 2000s, and since then they have not yet finished constructing their own delivery networks. For example, in the current Chinese express industry, more than 60% of total business volume comes from intra-city business. Surprisingly, international carriers have almost no business from this segment. They earn their business mostly from international deliveries.

(ii) Because international carriers usually have higher standards than their Chinese competitors, their services are more expensive, which impacts their clients. To reduce costs, carriers still need international giants to build up the delivery network, which would create economies of scale and cut costs.

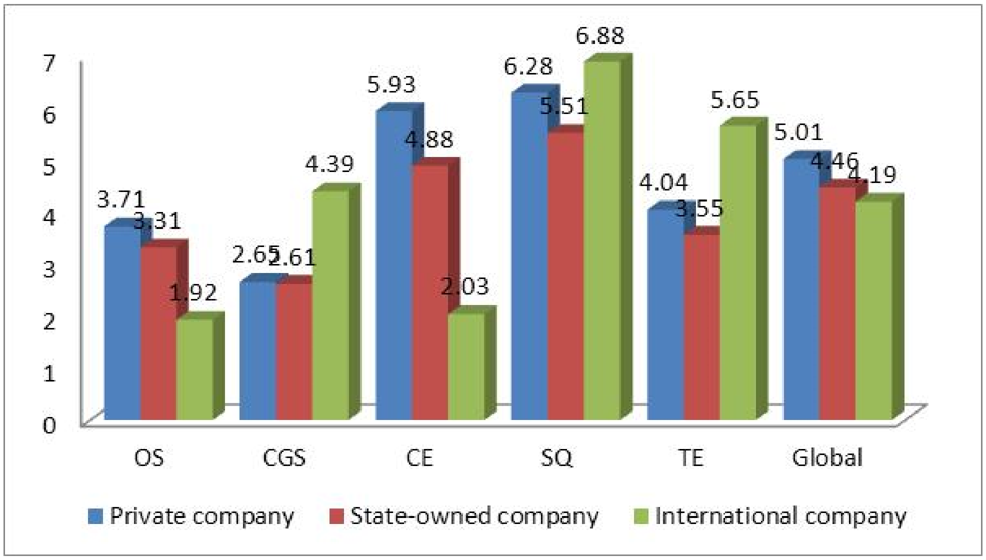


Figure 3.4. Comparison results for three different types of express companies

3.6 CONCLUSION AND FUTURE WORK

In this study, we propose a performance evaluation model to assess the business performance of express companies in China. We apply the center-point triangular whitenization function to tackle the uncertainty associated with pairwise judgments. The results show that the center-point triangular whitenization function can effectively cope with the uncertainty in evaluation. Based on the center-point triangular whitenization function, we use the ANP to derive the ratings and rank a sample of 12 express delivery companies operating in China.

Through the analysis of the evaluation results, for investors, we found that private enterprises are leading China's express delivery industry. In particular, private enterprises have competitive advantages in operational performance and service cost; however, they require investors' capital to reform their enterprises' internal structures and improve their technologies. State-owned enterprises, on the other hand, have excellent foundations in the delivery network and providing quality service; however, because of their own special structures, it is difficult for them to make changes. International carriers are ranked at the bottom in the final table, although they have sufficient capital, matured technologies and well-established corporate structures. Rather, because they have not built up their completed express delivery network in China, international carriers carry only marginal express business in China, which also leads to high service costs. We suggest that international carriers finish constructing a delivery network and dig their businesses into the intra-city level to improve their overall performance.

When decision makers find it difficult to select a specific value in pairwise comparison, they are inclined to assign a range of values in judgment. In this study, we adopt the center-point triangular whitenization function to address this uncertainty issue. By using the center-point triangular whitenization function, the proposed CTWF-ANP method introduces an effective means of evaluation, which provides valuable help in resolving decision makers' dilemma of choosing a single number; the model also improves the reliability of the assessment results in uncertain environments.

In future research, control criteria and their sub-criteria could be altered as the express delivery industry continues to develop. Under these circumstances, ranking results may be different. Thus, it could be very interesting to develop an even more comprehensive decision framework that uses a dynamic criteria system. We leave this to future research.

Finally, in addition to developing the CTWF-ANP model, this study makes a contribution to the performance evaluation of China's express delivery industry. We await future extensions as discussed in this study with great anticipation.

4.0 LOCATION DIFFUSION, RESOURCE ALLOCATION AND FLEET REBALANCING: A STUDY OF THE CAR-SHARING INDUSTRY

In this chapter, we are interested in developing novel approaches to address three management issues in the car-sharing industry. 1). Station Location Selection; 2). Station Capacity Design; 3). Strategy for vehicle replenishment when skew demand exists. Research has been active in studying location and fleet management problems. However, most of these studies have employed optimization models, which are powerful tools but may be difficult to generalize (Boyaci et al. 2015). Validating these models requires a significant amount of data. In practice, not all car-sharing companies have the data necessary for analysis. Therefore, a well-thought-out and easy-to-implement model with a limited data requirement will serve management better. Our research proposes a new approach to addressing location diffusion, resource allocation, and redistribution problems in the car-sharing industry.

The contributions of this research are threefold:

1). The car-sharing location decision in this research is market-driven and focuses on understanding demand potential rather than the conventional operations research (OR) approach, such as those in Jorge et al. (2014), Kek et al. (2009), and Smith et al. (2013).

2). Conventional models built on integer optimization require search algorithms to find optimal solutions for specific questions. Relative to the optimization approach, our model provides analytic results that can offer good solutions with significantly less data, time, and cost.

3). Methodologically, we make use of control theory (Wiener 1961; Astrom 1970) to model our problems and solve through differential equations. To the best of our knowledge, this is the first attempt to employ control theory to address car-sharing issues. We hope that such an approach can help management make better decisions and improve firm performance.

This chapter is organized as follows. In section 4.1, we review the literature on car-sharing business decisions from a methodological perspective. Section 4.2 models the market's diffusion, while section 4.3 deals with vehicle allocations and redistribution problems. In section 4.4, we use Zipcar to numerically examine our models and derive implications for managing the car-sharing service. Summary, conclusions, and future research directions are given in section 4.5.

4.1 LITERATURE REVIEW

4.1.1 Round-trip and Free-float Car-sharing Studies

Car-sharing contracts include round-trip, one-way, and free-float rentals. Round-trip rental requires users to return vehicles to the pick-up location, while one-way rental allows users to return vehicles to any location. Free-float rental, which is relatively new, does not have any fixed spots for users to pick up vehicles. Vehicles can be parked in a range of areas within an acceptable walking distance, and users can return the vehicles to any place in a particular area. Research on round-trip rental is not active due to lack of flexibility. Research on free-float rental is just emerging and is a promising research area. Free float has been getting more attention in recent times. Weikl and Bogenberger (2012) have developed decision support systems providing both user-based and operator-based relocation strategies. Finally, Wagner et al. (2015) used data analytics to identify factors critical for the successful expansions of free-flow business.

4.1.2 One-way Car Sharing

Researchers studying one-way rental have mainly focused on three issues: station location, station size, and number of vehicles to transfer between stations. One-way rental gives customers more flexibility, but it also causes vehicle imbalance among stations. Therefore, car-sharing companies need corresponding relocation policies to rebalance their vehicles. Barth and Todd (1999) pioneered such studies. To minimize the number of vehicle relocations, they recommend 18-24 vehicles per 100 trips. However, having so many vehicles in just a few stations is not only financially inefficient, but also implementationally impractical because of space limitations, especially in high population density areas. To improve the efficiency, researchers have tackled the one-way rental rebalancing issue from two directions: user-based and operator-based transfers. Details follow.

4.1.2.1 User-based Strategy

Barth et al. (2004) proposed that car-sharing companies adopt trip-split and trip-sharing strategies. The trip-split strategy suggests that companies incentivize customers to take separate vehicles when vehicles are needed at the destination station. Conversely, the trip-sharing strategy suggests that companies encourage multiple customers to share a vehicle when cars are limited in the origin station. They claim the strategy could lower relocation costs by 40%. Cepolina and Farina (2012) employed a pure user-based strategy. They advise customers to take public transportation to pick up vehicles in a different station as a rebalancing policy, where the transition time is viewed as customers' waiting time. The goal is to minimize both the car-sharing company's costs and customers' costs.

4.1.2.2 Operator-based Strategy

The literature mainly focuses on operator-based strategy. Adopting such a strategy requires the car-sharing company to address location, size, and rebalancing problems of multiple stations. These decisions involve various types of costs, e.g., parking, maintenance, operators, and the company-specific nature of the proposed models.

For single-period study, Kek et al. (2006) considered vehicle transfer costs and recommended a balancing technique to minimize the number of stations, staff, and relocations. Later, Kek et al. (2009) allowed one-way rental without scheduled return time. Their models focused on minimizing vehicle shortage time, parking space, and relocations.

Rickenburg et al. (2013) developed a rebalancing optimization model to minimize station setup, vehicle purchasing, and parking space costs. In addition to conventional costs, Jorge et al. (2014) also considered maintenance and depreciation costs. They developed a mathematical model and a simulation model to study real-time relocation policy in one-way rental. Smith et al. (2013) focused on rebalancing the operators by optimally routing vehicles and drivers. Furthermore, Boyaci et al. (2015) developed a multi-objective mixed-integer linear programming (MILP) model and used a branch-and-bound algorithm to jointly address all three car-sharing issues.

Alternatively, Correia and Antunes (2012) presented three trip selection models to address vehicle stock imbalance issues. Depending on the criterion chosen, the models can accept or reject requests from customers. Their policy rebalances vehicles at the end of each day.

In a multiple time period setting, Cheung and Powell (1996) considered multi-stage vehicle movement and built a dynamic programming model to minimize idle costs of vehicles. The model determines the number of vehicles to be relocated in each station (city) at each time period. Fan et al. (2008) factored in more inputs, e.g., fleet size, number of locations, and demand. They constructed a deterministic optimization model to adjust the number of vehicles on a daily basis. Fan and Xu (2013) considered uncertain demand and used stochastic programming in multi-stage periods to design a vehicle relocation strategy.

Since bicycle, or bike, sharing is similar to car sharing, we examine the literature on bike sharing. Lin and Yang (2011) studied a bike-sharing system and presented a non-linear integer model to look for the best location. The models consider station setup costs, customers' travel costs, and service quality (customer satisfaction) from the location and routing perspectives. A branch-and-bound algorithm was used to solve the model. They did not address bike number or how to rebalance the number of bikes in each station. Then, for the bike case, Shu et al. (2013) used periodical redistribution strategy to design a bike-sharing system in Singapore. Relocating bikes is easier than relocating vehicles, as moving multiple vehicles among stations is much more costly than transferring bikes. This makes effective car relocation even more crucial.

We summarize the literature's operator-based one-way car-sharing research as following table 4.1.

The table 4.1 shows that researchers have mainly tackled car-sharing issues using the optimization method, which often performs well when questions are specific, especially when input data are available. The solutions of optimization models can be very specific and present theoretically best solutions for the studied problem. However, the exact solution entails limitations. The model usually lacks flexibility, and it may be hard to generalize. Thus, researchers are continuously looking for other methods to address the managerial concerns since the conventional optimization models may become very complicated when the problem size becomes bigger. To obtain meaningful results, the modeler has to develop sophisticated searching algorithms through simulated settings, which is costly and time-consuming.

Table 4.1. Operator-based one-way rental literature summary

Paper	Methodology	Location	Station Size	Rebalance	Business Type
Cheung & Powell 1996	Dynamic Stochastic			X	Car
Fan et al. 2008	Multi-stage Stochastic Integer Linear			X	Car
Kek et al. 2009	Mixed-integer Linear			X	Car
Nair & Miller-Hooks 2010	Mixed-integer with Chance Constraint			X	Car
Lin & Yang, 2011	Non-linear Integer	X			Bike
Smith et al. 2013	Integer Linear			X	Car
Fan & Xu 2013	Multi-stage Stochastic Integer Linear			X	Car
Kek et al. 2006	Simulation Model	X		X	Car
Rickenburg et al. 2013	Mixed-integer Linear	X	X		Car
Shu et al. 2013	Linear	X		X	Bike
Jorge et al. 2014	Mixed-linear Integer		X	X	Car
Correia & Antunes 2012	Mixed-integer Linear	X	X	X	Car
Boyaci et al. 2015	Multi-objective MILP	X	X	X	Car
Chen et al. 2015	Differential Equation in Control Theory	X	X	X	Car

4.1.3 The Proposed Methods

Our models differ from traditional ones and are driven by market demand, which is crucial for service expansion. Conventional operational models emphasize cost reduction. We believe that cost is important but should not be a dominant factor, as demand potential should be considered first. Operational models should be applied to improve efficiency if and only if a company can identify the potential region for business expansion. Therefore, generalizable models need to be developed to relocate vehicles for multiple stations.

4.2 IDENTIFY THE BEST LOCATION FOR EXPANSION

We address the Zipcar management issues in two phases. Phase 1 identifies the best location from the perspective of market diffusion, while Phase 2 determines how to effectively deploy and redistribute vehicles. We focus on Phase 1 in this section and discuss Phase 2 in section 4.

In Phase 1, we develop a discrete-time difference equation model for location selection. Prior to the setup of a new station, the population of the potential region is assumed to be composed of non-members. After the opening of the station, people in the region gradually recognize its presence. The population surrounding the station can thus be divided into two categories: Potential Members (non-member) and Members. People may switch their roles between these two categories. In order to estimate the number of people who would become members, we borrow the concept of control theory (Astrom 1970) to develop models. By comparing the regenerative rate from the proposed models for various locations, we are able to determine the best location for expansion.

Control theory has been widely applied to the epidemiology research (Hethcote 2000; Inanba and Nishiura 2008; Nishiura 2010). By adapting their diffusion view, we incorporate population dynamics (transitioning in and out of car-sharing membership) and demographic information into our model for business expansion decision making. Because the **P**otential Members may evolve into **M**embers, and subsequently become non-members (**P**otential Members), we name the discrete-time difference equation model *P-M-P*.

To address the impact of population mobility on market expansion, we develop three *P-M-P* models. We start with a base model and then build a *P-M-P* model on homogeneous customers and fixed population size. Thereafter, we introduce a more complicated *P-M-P* location model to tackle heterogeneous user groups and variable population size.

4.2.1 Notations for the Proposed Model

t : The discrete-time index, t = 0, 1, 2, ··· T

j: The index of user groups, j = 0, 1, 2, ··· J

Parameters without subscript j indicate that it does not have subgroups, i.e., it has a homogeneous population.

$F_j(t)$: Number of members at time t for user group j

$P_j(t)$: Number of potential members (non-members) at time t for user group j

$N_j(t)$: The population of a region at time t for user group j, where $N_j(t)=F_j(t)+P_j(t)$

$P_j(t)$: Number of potential members (Non-member of Car-sharing) in user group j at time t

$F_j(t)$: Number of Car-sharing members in user group j at time t

The following parameters are based on per-unit time, which may be a year, month, week, day, or hour. All parameters are positive. We define "Contact" as an effective communication to persuade an individual to become a member regardless of his or her background.

β_j: The average contact number by a Car-sharing member and converted to members for user group j

γ_j: The dropout rate of user group j. $0 \le \gamma \le 1$

η_j: The turnover rate of user group j. $0 \le \eta \le 1$

λ_j : The variation coefficient of user group j, which is close to 1.

4.2.2 Model Establishment

Case 1: Homogeneous Population (No Subgroups) with Same Composition

In this case, we assume the population size is constant. As members and non-members may transition from one group to the other, we approximate the size of each group by the following equations:

$$\begin{cases} P(t+1) = P(t) - \beta F(t)\dfrac{P(t)}{N} + \gamma F(t) \\ F(t+1) = F(t) + \beta F(t)\dfrac{P(t)}{N} - \gamma F(t) \end{cases} \tag{1}$$

where $\beta P(t)\dfrac{F(t)}{N}$ is the total number of people who become members after being contacted by members. From the notations and definitions above, we have $P(0)+F(0)=N$ and

$$\begin{cases} P(t) >= 0 \\ F(t) >= 0 \\ P(t)+F(t) = N \end{cases} \tag{2}$$

Eq. (2) implies that there exists a non-negative and unique solution for Eq. (1). Let the initial market be P(0) = N, and because of word of mouth, the market will grow. Then F(t+1) - F(t) > 0, and

$$\beta P(t)\frac{F(t)}{N} - \gamma F(t) > 0 \xRightarrow{P(0)\approx N} \beta N\frac{F(t)}{N} - \gamma F(t) \Rightarrow \beta F(t) - \gamma F(t) > 0 \Rightarrow \beta - \gamma > 0. \quad (3)$$

Let
$$D_0 = \frac{\beta}{\gamma}$$

as the regenerative rate, i.e., the number of non-members who become members after being contacted by a member. When $D_o \leq 1$, it implies that the dropout rate is higher than the entrance rate. If such a trend persists, members will diminish and eventually reduce to zero. On the other hand, $D_o > 1$ implies that the entrance rate is greater than the dropout rate. In the long run, everyone in the population will become members. Particularly, if $\gamma = 0$ *and* $\beta > 0$, then the region is recommended to grow business; if $\gamma = 0$ *and* $\beta = 0$, it is recommended to retain current strategies without making any further adjustments to operations.

Case 2: Homogeneous Population (No Subgroups) with Different Composition

Case 1 assumes the population composition is constant. We now allow the population composition to change, due to new arrivals and departures. For example, every year students graduate from the college and leave the campus. In the meantime, new students arrive and become potential members of the car-sharing program. The arrival and departure rates are approximately equal. Let η be the rate and termed the "turnover rate." We assume the new arrivals are non-members, e.g., the newly arrived freshmen had not used a car-sharing service before. Thus, Model (1) is rewritten as:

$$\begin{cases} P(t+1) = P(t) - \beta F(t)\frac{P(t)}{N} + \gamma F(t) + \eta N - \eta P(t) \\ F(t+1) = F(t) + \beta F(t)\frac{P(t)}{N} - \gamma F(t) - \eta F(t) \end{cases} \tag{4}$$

Given P(0) = N, to increase membership (i.e., $\Delta F(t) > 0$), we must have

$$\beta P(t)\frac{F(t)}{N} - \gamma F(t) - \eta F(t) > 0 \xrightarrow{P(0)\approx N} \beta N\frac{F(t)}{N} - \gamma F(t) - \eta F(t) \Rightarrow \beta - \gamma - \eta > 0.$$

We can then define the regenerative rate as $D_0 = \frac{\beta}{\gamma+\eta}$. This is used by management to quickly determine whether a region is suitable for starting a car-sharing service.

Case 3: Heterogeneous Population (containing Subgroups) with Different Composition

Only certain individuals in the US need a car-sharing service, as many people already own cars. Most car-sharing companies prefer to open locations around college campuses. Since students need vehicles occasionally, a car-sharing service fills this void and proves to be successful. Here, we divide the population into "user groups," as different user groups have different needs with respect to shared vehicles. The groups may be classified by age, occupation, income, etc. A statistical classification model can be applied to organize the population of a region. With different user groups, the model can depict the different market segments for a location problem.

We divide the population into $m+1$ user groups and use Fig. 1 to depict the flows of each group at time *t*.

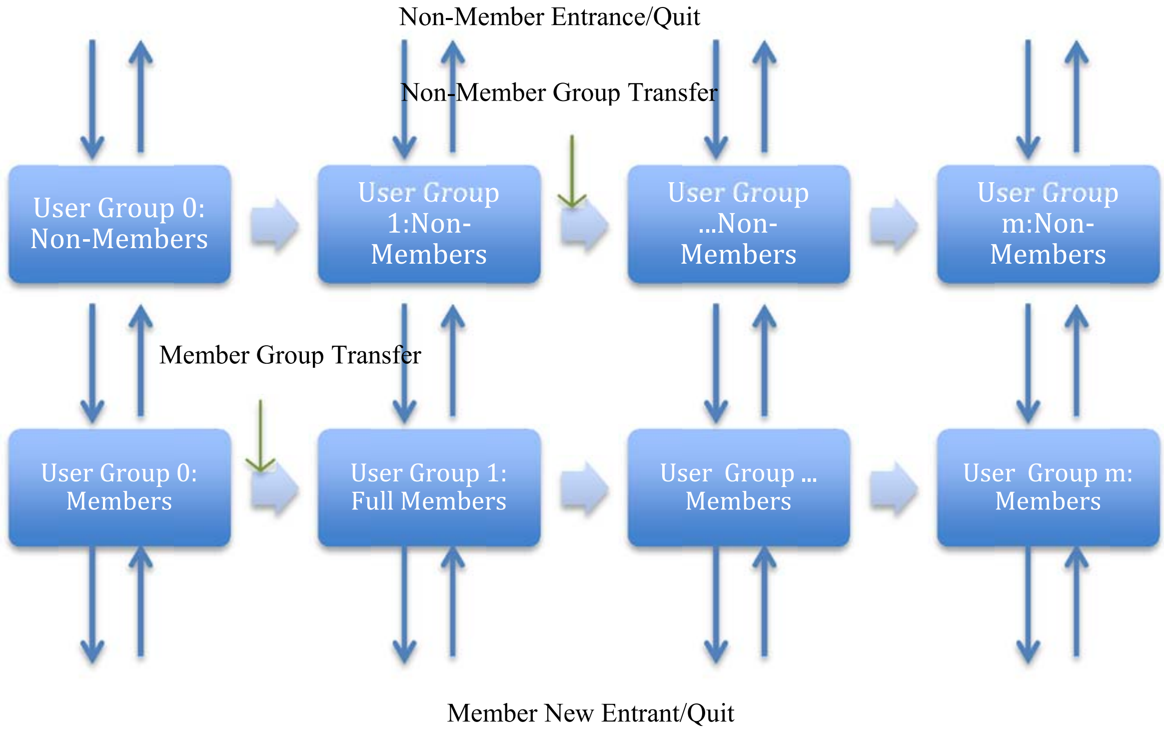


Figure 4.1. Individuals transition through various groups under Case 3

From Figure 4.1, we can estimate the potential members in each group:

$$\begin{cases} P_{j+1}(t+1) = P_{j+1}(t) - \beta_{j+1}F_{j+1}(t)\dfrac{P_{j+1}(t)}{N_j} + \gamma_{j+1}P_{j+1}(t) + \mu_{j+1,0}P_{j+1}(t) - m_{j+1,0}P_{j+1}(t) - v_{j+1,0}P_{j+1}(t) + v_{j,0}P_j(t), \\ \qquad j = 0,1,2,\cdots,m-2 \\ F_{j+1}(t+1) = F_{j+1}(t) + \beta_{j+1}F_{j+1}(t)\dfrac{P_{j+1}(t)}{N_j} - \gamma_{j+1}P_{j+1}(t) + \mu_{j+1,1}F_{j+1}(t) - m_{j+1,1}F_{j+1}(t) - v_{j+1,1}F_{j+1}(t) + v_{j,1}F_j(t), \\ \qquad j = 0,1,2,\cdots,m-2 \end{cases} \tag{5}$$

where

$\upsilon_{j,0}$: transfer rate from potential member user group j to potential member user group j+1;

$\upsilon_{j,1}$: transfer rate from member user group j to member user group j+1;

$\mu_{j,0}$: new member arrival rate of non-member user group j for non-members;

$\mu_{j,1}$: new member arrival rate of member user group j for members;

$m_{j,0}$: non-member leaving rate of non-member user group j;

$m_{j,1}$: member leaving rate of member user group j;

Other notations can be found in Section 3.1.

In addition to Eq. (5), we have group 0 and group m as follows:

$$P_0(0) = N_0, F_0(0) = 0$$

$$\begin{cases} P_0(t+1) = P_0(t) - \beta_0 F_0(t)\dfrac{P_0(t)}{N_j} + \gamma_0 F_0(t) + \mu_{0,0} P_0(t) - m_{0,0} P_0(t) - v_{0,0} P_0(t) \\ F_0(t+1) = F_0(t) + \beta_0 F_0(t)\dfrac{P_0(t)}{N_0} - \gamma_0 F_0(t) + \mu_{0,1} F_0(t) - m_{0,1} F_0(t) - v_{0,1} F_0(t) \end{cases}$$

$$\begin{cases} P_m(t+1) = P_m(t) - \beta_m F_m(t)\dfrac{P_m(t)}{N_m} + \gamma_m F_m(t) + \mu_{m,0} P_0(t) - m_{m,0} P_m(t) + v_{m-1,0} P_{m-1}(t) \\ F_m(t+1) = F_m(t) + \beta_m F_m(t)\dfrac{P_m(t)}{N_m} - \gamma_m F_m(t) + \mu_{m,1} F_m(t) - m_{m,1} F_m(t) + v_{m-1,1} F_{m-1}(t) \end{cases}$$

Then Eq. (5) can be simplified as:

$$\begin{cases} P_{j+1}(t+1) = q_{j+1} P_{j+1}(t) - \beta_{j+1} F_{j+1}(t)\dfrac{P_{j+1}(t)}{N_j} + \gamma_{j+1} F_{j+1}(t), & j = 0,1,2,\cdots,m-1 \\ F_{j+1}(t+1) = p_{j+1} F_{j+1}(t) + \beta_{j+1} F_{j+1}(t)\dfrac{P_{j+1}(t)}{N_j} - \gamma_{j+1} F_{j+1}(t), & j = 0,1,2,\cdots,m-1 \end{cases}$$

where $p_{j+1} = 1 + \mu_{j+1,1} - m_{j+1,1} - v_{j+1,1} + v_{j,1}$; $q_{j+1} = 1 + \mu_{j+1,0} - m_{j+1,0} - v_{j+1,0} + v_{j,0}$

Since a member in group j may contact people outside of the group, we need to take in all of the new members whom he or she influenced and assign them to the corresponding group. Thus, we have

$$\begin{cases} P_{j+1}(t+1) = q_{j+1} P_{j+1}(t) - \lambda_{j+1}\left(\sum_{k=0}^{m} \beta_k F_k(t)\dfrac{P_{j+1}(t)}{N(t)}\right) + \gamma_{j+1} F_{j+1}(t), & j = 0,1,2,\cdots,m-1. \\ F_{j+1}(t+1) = p_{j+1} F_{j+1}(t) + \lambda_{j+1}\left(\sum_{k=0}^{m} \beta_k F_k(t)\dfrac{P_{j+1}(t)}{N(t)}\right) - \gamma_{j+1} F_{j+1}(t), & j = 0,1,2,\cdots,m-1 \end{cases} \tag{6}$$

where λ_j is a variation coefficient, accounting for the uncertainty of new members estimated. Substituting $N_j - F_j$ into $P_j(t)$ in Eq. (6), we have

$$\begin{cases} F_0(0)=0 \\ F_{j+1}(t)=(p_{j+1}-\gamma_{j+1})F_{j+1}(t)+\lambda_{j+1}/N(t)\sum_{k=1}^{m}\beta_k F_k(t)(1-F_{j+1}(t)), \qquad j=0,1,\cdots,m-1 \\ F_j(0)=F_{j0}, \quad 0\le F_{j0}\le N_j, \quad j=0,1,\cdots,m \end{cases} \tag{7}$$

For ease of discussion, we denote

$$F(t)=\begin{bmatrix} F_1(t) \\ F_2(t) \\ F_3(t) \\ \vdots \\ F_m(t) \end{bmatrix}, \quad \Theta=\begin{bmatrix} \beta_1 & \beta_2 & \beta_3 & \cdots & \beta_{m-1} & \beta_m \\ \beta_1 & \beta_2 & \beta_3 & \cdots & \beta_{m-1} & \beta_m \\ \beta_1 & \beta_2 & \beta_3 & \cdots & \beta_{m-1} & \beta_m \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ \beta_1 & \beta_2 & \beta_3 & \cdots & \beta_{m-1} & \beta_m \end{bmatrix}$$

$$T=(T_{i,j})_{m\times m}, \quad T_{i,i}=\lambda_{i-1}, \quad i=1,2,\cdots,m,$$

$$B=(B_{i,j})_{m\times n}, \; B_{i+1,i}=p_i-\gamma_i, \; i=1,2,\cdots,m-1, \; G(F(t))=(G_{i,j})_{m\times m}, \quad G_{i+1,i}=\lambda_i F(t)/N_i, \quad i=1,2,\cdots,m-1.$$

Eq. (7) becomes

$$F(t+1)=BF(t)+T\Theta F(t)-G(F(t))\Theta F(t), \; t=0,1,2,\cdots \tag{8}$$

Define the time-independent solution for Eq. (8) as F; we then have

$$F=BF+T\Theta F-G(F)\Theta F \; . \tag{9}$$

Let $x=\sum_{k=1}^{m}\beta_k F_k$, then $0\le x\le \sum_{k=1}^{m}\beta_k N_k$.

In addition, we define

$$\lambda=\begin{bmatrix} \lambda_0 \\ \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_{m-1} \end{bmatrix}, \quad Q=\begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 0 \\ \lambda_1/N & 0 & 0 & \cdots & 0 & 0 \\ 0 & \lambda_2/N & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \lambda_{m-1}/N & 0 \end{bmatrix}.$$

Then, Eq. (9) can be expressed as

$$F = BF + xE\lambda - xQF$$

where $E \in I_{m\times m}$. As matrix $E - B + xQ$ is invertible, we have

$$F = x(E - B + xQ)^{-1}\lambda\ .$$

In the following, we present the derivation process to identify the regenerative rate given multiple user groups.

Let $\beta = (\beta_1, \beta_2, \cdots, \beta_m)^T$, then

$$\begin{aligned} f(x) = \beta_1\lambda_0 + \beta_2(\lambda_1 + \lambda_0 q_1(x)) + \beta_3(\lambda_2 + \lambda_1 q_2(x) + \lambda_0 q_1(x) q_2(x)) + \cdots \\ + \beta_m(\lambda_{m-1} + \lambda_{m-2} q_{m-1}(x) + \lambda_{m-3} q_{m-2}(x) q_{m-1}(x) + \cdots \\ + \lambda_1 q_2(x)) q_3(x) \cdots q_{m-1}(x) + \lambda_0 q_1(x) q_2(x) \cdots q_{m-1}(x)) \end{aligned} \tag{10}$$

Here, $q_j = q_j(x) = p_j - \gamma_j - x\lambda_j / N_j$, $j = 1, 2, \cdots, m-1$.

To simplify Eq. (10), we define $\kappa_0 = \beta^T[(E-B)^{-1}Q]^j(E-B)^{-1}\lambda, j = 0,1,\dots,m-1.$ Eq. (10) can thus be expressed as

$$\kappa_0 - 1 = \kappa_1 x - \kappa_2 x^2 + \kappa_3 x^3 + \cdots + (-1)^{m-1}\kappa_{m-2}x^{m-2} + (-1)^m \kappa_{m-1} x^{m-1}$$

And the regenerative rate becomes

$$D_0 = \kappa_0 = \beta^T(E-B)^{-1}\lambda \tag{11}$$

When $D_o \le 1$, it implies that the dropout rate is higher than the entrance rate. If such a trend persists, members will diminish and eventually reduce to zero. On the other hand, $D_o > 1$ implies that the entrance rate is greater than the dropout rate. In the long run, everyone in the population will become members.

4.2.3 Properties of the Regenerative Rate

Case 1: Homogeneous Population (No Subgroups) with Same Composition

The properties of the regenerative rate D_0 are described below.

Theorem 1.

(i) If $D_0 \le 1$, solution to Eq. (1) approaches the diffusion-free equilibrium $\lim_{t\to\infty} F(t) = 0,\ \lim_{t\to\infty} P(t) = N$.

(ii) If $D_0 > 1$, solution to Eq. (1) approaches the unique positive diffusion equilibrium

$$\lim_{t\to\infty} F(t) = \tilde{F} > 0, \qquad \lim_{t\to\infty} P(t) = \tilde{P} > 0 .$$

Proof. Please see Appendix.

Combining $D_0 = \frac{\beta}{\gamma}$ and Theorem 1, we can rank all candidates with $D_0 > 1$ for possible market expansion.

Case 2: Homogeneous Population (No Subgroups) with Different Composition

Similar to Case 1, we can prioritize candidates with $D_0 = \frac{\beta}{\gamma + \mu} > 1$ to identify markets for expansion.

Case 3: Heterogeneous Population (containing Subgroups) with Different Composition

Theorem 2.

(i) If $D_0 \le 1$, then Eq. (5) approximates the diffusion-free equilibrium;

(ii) If $D_0 > 1$, then Eq. (5) approximates the upper boundary N_j .

Proof. See Appendix.

4.3 DEPLOYMENT AND REDISTRIBUTION MODELS

After selecting the location for business expansion in Phase 1, car-sharing firms need to determine the number of vehicles necessary to satisfy customer needs in order to provide timely service. A good decision is crucial for business success, as it affects service quality and operating costs. To ensure efficient operations, management needs to develop strategies to balance vehicles between stations. Obviously, the number of vehicles required in a station depends on customer need. To meet demand, vehicles have to be redistributed so that no station is over-inventoried or starved. A car station's net demand = [Actual Demand – Returning Vehicles]. We use net demand rather than actual demand to ensure that the returning vehicles are taken into account in the relocation decision. The car-sharing company provides 24/7 services; therefore, a continuous-time differential equation model is fitting for identifying the best operating decisions.

4.3.1 Model Setting and Notation

The following notations and assumptions are necessary for model development.

4.3.1.1 Model Setting:

When demand in a car-sharing station is greater than current available vehicles, managers need to transfer vehicles to the shortage station. Like the traditional inventory model, we assume that at time 0 the station's inventory is at its maximal level. When net demand exceeds on-hand inventory, the company needs to replenish the station in order to meet demand to reach its maximal inventory level again; otherwise no replenishment is needed. We name the time when the station reaches its maximal inventory level as the receiving point. The time interval between two consecutive receiving points is called a "Cycle." We assume that a car-sharing company has a preference as to how long it should check the vehicle inventory in a station. We use Figure 4.2 to describe our settings.

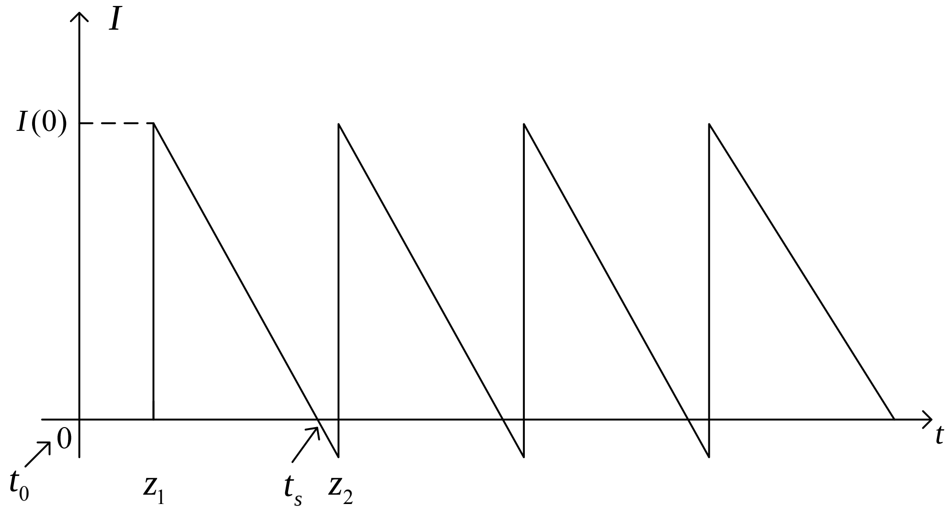


Figure 4.2. Relationships between different time points in replenishment

4.3.1.2 Model Notations:

t_s: Zero Inventory Time. The time when the vehicle inventory level drops to zero from its maximal level. This time is within a cycle.

z: The Replenishment Time. The time the company spends relocating vehicles. It is an independent and identically distributed (i.i.d.) random variable in $[z_1, z_2]$. z_1 and z_2 are two consecutive vehicles' receiving times, i.e., inventory reaches its maximal level at both z_1 and z_2.

r: Average demand per unit of time, $r \geq 0$.

$D(t)$: Net demand for a particular station at time t, which is the actual demand minus the returning number of vehicles, and can be expressed as:

$$D(t) = \begin{cases} r + \alpha I(t), & I(t) \geq 0 \\ r, & I(t) < 0 \end{cases}$$

$I(t)$: On-hand inventory level at time t.

α: Impact of inventory shortage from previous cycle, $0 \leq \alpha \leq 1$. It is the impact of lost customers (due to vehicle shortage). The more sales that are lost in the previous cycle, the more vehicles that are needed at time t; thus, larger α is needed to avoid further loss. It is equivalent to the proportion of lost demand.

DS(t): Deferred supply rate. When net demand is greater than on-hand inventory (i.e., D(t)>I(t)) and customers are willing to wait, the firm needs to request vehicles from other station(s) to satisfy customers' needs. This number of deferred supplies is depicted by *DS(t)*.

τ: Coefficient of supply shortage, $0 \le \tau \le 1$. It is the impact of shortage on deferred supply rate.

$S(t)$: The number of vehicles needed but unavailable (shortage) at time *t* when inventory drops to zero.

T: The length of the cycle.

The relationship between shortage and deferred supply rate can be described as follows. At time *t*, when the station does not have enough on-hand vehicles to fulfill net demand, customers may or may not be willing to wait. The deferred supply is associated with current shortage. As the shortage increases, the customer's willingness-to-wait time decreases. Hence, we can assume that the deferred supply rate (deferred supply at a unit time) is linearly associated with the supply shortage at time *t*. Thus,

$$DS(t) = r - \tau S(t), \qquad t_s \le t \le T$$

4.3.2 Model Establishment

We now establish the relationship between the company's set replenishment time (z) and the time at which the station's inventory reaches zero (time t_s). When *z* is less than t_s, there is no shortage in the station.

As the change in inventory is affected by the demand, we have

$$\Delta I(t)=\begin{cases}-r-\alpha I(t), & 0\le t\le t_s\\ -r, & t_s\le t\le T\end{cases}$$

By converting the above function into a continuous format, we can find the first order of inventory level $I(t)$ with respect to t :

$$\frac{dI(t)}{dt}=\begin{cases}-r-\alpha I(t), & 0\le t\le t_s\\ -r, & t_s\le t\le T\end{cases} \tag{12}$$

As $I(t_s)=0$, we have

$$I(t)=\begin{cases}\dfrac{r}{\alpha}(e^{\alpha(t_s-t)}-1), & 0\le t\le t_s\\ r(t_s-t), & t_s\le t\le T\end{cases} \tag{13}$$

$$I(0)=\frac{r}{\alpha}(e^{\alpha t_s}-1) \tag{14}$$

Then

(i) The expected level of inventory in a rebalance cycle is

$$\begin{aligned}\overline{I}&=\int_{z_1}^{t_s}\int_0^{z}\frac{r}{\alpha}(e^{\alpha(t_s-t)}-1)f(z)dtdz+\int_{t_s}^{z_2}\int_0^{t_s}\frac{r}{\alpha}(e^{\alpha(t_s-t)}-1)f(z)dtdz\\ &=\int_{z_1}^{t_s}[\frac{r(e^{\alpha t_s}-e^{\alpha(t_s-z)})}{\alpha^2}-\frac{rz}{\alpha}]f(z)dz+\int_{t_s}^{z_2}[\frac{r(e^{\alpha t_s}-1)}{\alpha^2}-\frac{rt_s}{\alpha}]f(z)dz\end{aligned} \tag{15}$$

Eq. (15) represents the average on-hand inventory level per cycle, where the first term represents the expected inventory without shortage, while the second term represents the expected inventory with shortage.

(ii) Expected deferred supply and expected sales loss in each cycle

Assume there is a linear relationship between deferred supply rate and shortage. As $(t)=r-\tau S(t)$, the change in shortage during unit time reflects the deferred rate. We thus have:

$$\begin{cases}\dfrac{dS(t)}{dt}=r-\tau S(t), & t_s\le t\le T\\ S(t_s)=0.\end{cases} \tag{16}$$

and

$$S(t)=\frac{r}{\tau}(1-e^{\tau(t_s-t)}),\qquad t_s\le t\le T \tag{17}$$

The expected deferred supply in each cycle is

$$\bar{B}=\int_{t_s}^{z_2}S(z)f(z)dz=\int_{t_s}^{z_2}\frac{r}{\tau}(1-e^{\tau(t_s-z)})f(z)dz \tag{18}$$

and since the total expected shortage in one cycle is the sum of expected deferred supply and expected sales loss, the expected demand loss of each cycle is

$$\bar{L}=\int_{t_s}^{z_2}r(z-t_s)f(z)dz-\int_{t_s}^{z_2}S(z)f(z)dz=\int_{t_s}^{z_2}[r(z-t_s)-\frac{r}{\tau}(1-e^{\tau(t_s-z)})]f(z)dz \tag{19}$$

The first term in Eq. (19) is the expected demand during shortage time; the second term represents the expected deferred supply when no vehicle is available at time t.

(iii) Expected replenishment per cycle

Since the goal of the replenishment is to reach its maximal capacity, the expected transfer in one cycle is the sum of the expected sales and the expected deferred supply, i.e.,

$$\begin{aligned}\bar{Q}&=\int_{z_1}^{t_s}[I(0)-I(z)]f(z)dz+\int_{t_s}^{z_2}[I(0)+S(z)]f(z)dz\\&=\int_{z_1}^{t_s}\frac{r(e^{\alpha t_s}-e^{\alpha(t_s-z)})}{\alpha}f(z)dz+\int_{t_s}^{z_2}[\frac{r(e^{\alpha t_s}-1)}{\alpha}+\frac{r}{\tau}(1-e^{\tau(t_s-z)})]f(z)dz\end{aligned} \tag{20}$$

4.3.3 Solution Methodology

After modeling station capacity and vehicle relocation problems in Eqs. (12)-(20), we now discuss the solution approach. Since t_s is the duration of depleting the initial inventory level, we have:

$$D(t)=\bar{Q} \tag{21}$$

Therefore

$$r+\alpha I(t)=\bar{Q} \tag{22}$$

Substituting $I(t)$ and $\bar{Q}$ into Eq. (22), we have

$$re^{\alpha t_s}=\frac{r}{\alpha}\int_{z_1}^{t_s}(e^{\alpha t_s}-e^{\alpha(t_s-z)})f(z)dz+\frac{r}{\alpha}\int_{t_s}^{z_2}(e^{\alpha t_s}-1)f(z)dz+\frac{r}{\tau}\int_{t_s}^{z_2}(1-e^{\tau(t_s-z)})f(z)dz \tag{23}$$

and

$$e^{\alpha t_s} = \int_{z_1}^{t_s} \frac{e^{\alpha t_s} - e^{\alpha(t_s - z)}}{\alpha} f(z)\,dz + \int_{t_s}^{z_2} [\frac{e^{\alpha t_s} - 1}{\alpha} + \frac{1}{\tau}(1 - e^{\tau(t_s - z)})] f(z)\,dz \,. \tag{24}$$

To simplify Eq. (27), we denote $\Phi = \int_{z_1}^{t_s} \frac{e^{\alpha t_s} - e^{\alpha(t_s - z)}}{\alpha} f(z)dz$, $\Psi = \int_{t_s}^{z_2} [\frac{e^{\alpha t_s} - 1}{\alpha} + \frac{1}{\tau}(1 - e^{\tau(t_s - z)})] f(z)\,dz$.

Thus,

$$t_s = \ln(\Phi + \Psi)^{\alpha} \ . \tag{25}$$

Theorem 3 (Existence and Uniqueness). Given $Z = [z_1, z_2]$ is a cycle between two consecutive vehicle received points, and $\forall t \in [z_1, z_2]$ is an i.i.d. random variable, there exists a unique time point $t_s \in [z_1, z_2]$ such that $I(t_s) = 0$.

Proof: See Appendix.

Substituting t_s into Eq. (14) and Eq. (17), we can effectively solve the problem of product deployment and redistribution.

4.4 MODEL APPLICATION TO ZIPCAR

To illustrate the proposed models, we collect a set of publicly available population data. We focus on the Zipcar business surrounding the University of Pittsburgh campus. Zipcar operates in nearly 30 major metro cities, on more than 350 college campuses, and 30 airports. Since 2004, Zipcar has focused on expanding service around the university area, as it recognizes that 18-32-year-olds, who account for 25% of the US population, are its most important customers, holding the key to Zipcar's future success. We focus on (i) selecting the best location to grow Zipcar's business and (ii) managing its resources (i.e., determining the number of vehicles in a station and a replenishment strategy).

4.4.1 Zipcar Location Selection Surrounding the University of Pittsburgh

We use the population in the University of Pittsburgh area as an example to verify the "*P-M-P*" model. In the following, we divide our discussion into steady and growth population scenarios.

Case 1: Homogeneous Population (No Subgroups) with Same Composition

We conduct surveys to obtain the necessary information. The University of Pittsburgh (Pitt) has 24,980 undergraduate and 10,034 graduate students at its main campus. Around Pitt's campus, the majority of people are students, and most of them are undergraduates; therefore, it is reasonable to assume that the population on the campus is quite homogeneous. We use the Pitt campus as an example to illustrate the scenarios in Cases 1 and 2.

Our survey is conducted on the south campus. We collect information from 550 students. Among them, 14 are Zipcar members; 4 are new members who have just joined recently (2014 Fall semester). Only one student terminated his membership. Thus, $\beta = {}^{4}/_{(14-4+1)} = .364$, indicating that, on average, .364 non-members will join Zipcar after being effectively "contacted" by a Zipcar member. The dropout rate is $\gamma = {}^{1}/_{(14-4+1)} = .091$. By applying this data in Eq. (3), we find the regenerative rate is:

$$D_0 = \frac{\beta}{\gamma} = 4$$

Since $D_0 > 1$, this shows that the number of full members will continue to grow and the selected location can be regarded as the best region in which to expand the Zipcar business.

Case 2: Homogeneous Population (No Subgroups) with Different Composition

From the survey taken in Case 1, it is reasonable to assume college students' turnover rate is about $\eta = .25$, as Pitt is a 4-year college, and each year seniors leave and freshmen arrive on campus. Thus, from Eq. (4) we find the regenerative rate is:

$$D_0 = \frac{\beta}{\gamma + \eta} = \frac{.364}{.091 + .25} \approx 1.07$$

Since $D_0 > 1$, we find that, after taking into account the large turnover rate, Pitt's south campus remains a promising area for expansion.

Case 3: Heterogeneous Population (containing Subgroups) with Different Composition

To validate the Case 3 model, we survey people in Squirrel Hill North. It is a residential area neighboring Pitt. As it is close to the shopping center, its residents include undergraduates, graduate students, and professionals. To account for the demographic differences, we include in the survey questions regarding occupation, income, age, and tendency to keep Zipcar membership after graduating or changing jobs. Based on the data collected, we apply cluster analysis to organize the population into groups. The population in the Squirrel Hill North area roughly corresponds to three user groups: undergraduate, graduate, and young professionals. We use three equations to depict the membership of the three groups.

(A) The Undergraduate group:

$$\begin{aligned} P_0(t+1) &= q_0 P_0(t) - \lambda_0 \textstyle\sum_{k=0}^{2} \beta_k F_k(t) \frac{P_0(t)}{N(t)}) + \gamma_0 F_0(t) \\ F_0(t+1) &= p_0 F_0(t) + \lambda_0 (\sum_{k=0}^{2} \beta_k F_k(t) \frac{P_0(t)}{N(t)}) - \gamma_0 F_0(t) \end{aligned} \tag{26}$$

(B) The Graduate student group:

$$\begin{aligned} P_1(t+1) &= q_1 P_1(t) - \lambda_1 \textstyle\sum_{k=0}^{2} \beta_k F_k(t) \frac{P_1(t)}{N(t)}) + \gamma_1 F_1(t) \\ F_1(t+1) &= p_1 F_1(t) + \lambda_1 (\sum_{k=0}^{2} \beta_k F_k(t) \frac{P_1(t)}{N(t)}) - \gamma_1 F_1(t) \end{aligned} \tag{27}$$

(C) The Young Professional group:

$$P_2(t+1) = q_2 P_2(t) - \lambda_2 \sum_{k=0}^{2} \beta_k F_k(t) \frac{P_2(t)}{N(t)}) + \gamma_2 F_2(t)$$
$$F_2(t+1) = p_2 F_1(t) + \lambda_2 (\sum_{k=0}^{2} \beta_k F_k(t) \frac{P_2(t)}{N(t)}) - \gamma_2 F_2(t) \quad (28)$$

Our survey has resulted in 546 effective answers from 580 questionnaires. We summarize the intermediate results in Table 4.2.

Table 4.2. Survey results of dynamic population in each of the user groups

Interval	$\mu_{j,1}$	$m_{j,1}$	$v_{j,1}$	$\gamma_{j,1}$	$\beta_j(t+1)$
Undergraduate Group (j=0)	0.2	0.2	.02	0.1	83/546 =.152
Graduate Group (j=1)	0.1	0.3	.001	0.05	30/546 =.0549
Professionals/Residents Group (j=2)	0.1	0.5	0	0.2	10/546 =.0183

By assuming λ=1, calculating $p_1 - \gamma_1 = .949, p_2 - \gamma_2 = .381$ and applying Eq. (11), i.e. $D_0 = \kappa_0 = \beta^T (E - B)^{-1} \lambda$, we find the regenerative rate of Squirrel Hill North is D_0 = (.152+.0549*.949+.0183*.381)+(.0549+.0183*.401)+.183=.292 > 1. As D_0 >1, we conclude that the membership function $F(t)$ has greater than 1 diffusion equilibrium. Thus, the number of memberships will grow, and the region is a good candidate for opening a new station.

4.4.2 Car Deployment and Rebalance at Pitt

In this section, we present numerical examples to validate the models developed in Section 4.2.

Eq. (25), i.e., $t_s = \ln(\Phi + \Psi)^{\alpha}$, requires the estimate of α,i.e., the impact of lost demand on the next cycle. If a customer cannot find a vehicle to rent and is not willing to wait, the sale is lost. To avoid loss in the next cycle, the company may need to add more vehicles to satisfy demand, and α is an index to help estimate the net demand. To estimate the value of α, we conducted a 10-day survey on the Pitt campus. Each day, we randomly selected 30 Zipcar members and asked:

If a Zipcar is not available for you to rent at the time of your need, how long are you willing to wait for the next available Zipcar?

The options for respondents' willingness-to-wait time start at 0 hours (no wait, will walk away immediately), with an increment of 0.5 hours, until it reaches 4.5 hours. Survey results are summarized in Table 4.3.

Table 4.3. Zipcar customer waiting time summary

T(Hrs) / Day	0	0.5	1	.5	2	2.5	3	3.5	4	4.5
1	1	5	8	5	5	4	2	0	0	0
2	1	2	10	6	2	2	2	3	2	0
3	3	4	6	8	2	3	2	1	0	1
4	2	3	8	8	6	3	0	0	0	0
5	1	5	7	4	2	3	2	3	2	1
6	0	3	9	6	5	1	2	3	1	0
7	3	3	8	0	5	0	1	0	0	0
8	0	6	5	7	5	5	2	0	0	0
9	1	7	8	7	2	3	0	2	0	0
10	1	3	9	6	6	3	1	1	0	0

Each column represents the maximal waiting time, from 0 to 4.5 hrs. The numbers in the table represent frequency, indicating how many respondents are willing to wait up to each time limit. For example, on Day 3, three people are not willing to wait, four people are willing to wait up to 0.5 hours, and so on. In order to find lost demand from Table 2, we need to select a cutoff (threshold time). We define the lost demand as the number of customers who will not make reservations if their willingness-to-wait time is less than the threshold time. Once the lost demand (requests not satisfied) is determined, we can then find α by dividing lost demand by sample size. Thus, if the cutoff is one hour, then α on Day 2 is (1+2)/30 = 0.10.

The average Zipcar customer waiting time is crucial for determining the threshold time. We use data from a Michigan Tech University report (2008) to estimate average waiting time. A total of 1,047 hours had been reserved in 251 transactions under a two-vehicle and two-month (October and November) setting. Thus, the average rental duration is 1047/251=4.17 hrs per transaction and 251/61=4.11 transactions per day. Assume a customer's interarrival time is exponentially distributed with a mean of 5.83 hours (=24hr/4.11). By applying the M/M/2 model from queuing theory, we find that the average waiting time for a Zipcar customer is

$$\text{average waiting time} \approx 0.6116 \text{ hours}$$

Thus, on average, customers need to wait 0.6116*60 = 37 minutes for an available Zipcar. Assume the company needs one hour to relocate vehicles. It is reasonable to assign the threshold waiting time in Table 2 to be 0.5 hours.

Based on the first columns of Table 2 and total sample size of 30 respondents per day, we can find α to be

$$\alpha = \frac{\text{sum of the first two columns}}{30*10 \text{ days}} = .18.$$

Based on the data collected, we also find: $\tau = .65, Z = [1, 3.5]$.

Substituting $\alpha = .18$, $\tau = .65$, and $Z = [1, 3.5]$ (hour) into Eq. (24), we have

$$e^{0.18t_s} = \int_1^{t_s} \frac{e^{0.18t_s} - e^{0.18(t_s - z)}}{0.18} f(z)dz + \int_{t_s}^{3.5} [\frac{e^{0.18t_s} - 1}{0.18} + \frac{1}{0.65}(1 - e^{0.65(t_s - z)})] f(z)dz \quad . \tag{29}$$

By solving Eq. (29), we obtain $t_s = 2.39$. Assume z is uniformly distributed. Substituting t_s into Eq. (14), we can then obtain the initial inventory level, which is also the expected replenishment inventory level in each cycle:

$$I(0) = \frac{1}{0.18}(e^{0.18\times 2.39} - 1) = [2.98] = 3 \ .$$

Based on the survey data, we find that Zipcar should check its station every 2.39 hours for a relocation decision. If replenishment is called for, it is expected to "refill" enough vehicles to ensure the number of "inventory" reaches its maximal inventory level (i.e., 3 vehicles in this case).

4.5 CONCLUSIONS AND FUTURE WORK

Car sharing has grown rapidly over the past two decades. The increase in car-sharing services not only helps consumers financially, but also creates tremendous benefits to society and the environment. As a fast-growing industry, companies constantly seek business expansion opportunities and respond to growing demand. In this research, we proposed innovative models to help car-sharing management make better decisions in identifying new station locations and determining the capacity and vehicle rebalancing need. We view the car-sharing station expansion decision as a marketing issue and use control theory to develop a location assessment model. We also borrow the concept of inventory management to quickly estimate the capacity necessary to serve the station, as well as relocate vehicles to meet customer demand.

Our models can be easily generalized and implemented. The models offer valuable recommendations to managers and require only limited inputs, which reduce the time and costs necessary for the car-sharing business.

One limitation of this research is that we did not consider the availability of parking spots. We assume that transferred vehicles will be guaranteed parking spaces, which may not be the case in practice. Also, our P-M-P model only makes use of one-time data to make a decision, so it may be necessary to repeat our P-M-P model multiple times in order to identify the mean and variances in order to confidently make final recommendations.

Our research can be extended in two directions. One is to study vehicle relocation between regions by allowing free float. This will be more costly and complex and require new strategies. Although the car-sharing industry is very promising, competition is intense in large cities. It would be interesting and important to examine how decisions vary in a competitive environment.

APPENDIX A

AIR MEDICAL JETS MARKET EXPANSION

A.1 PROOF OF THEOREM 2

For eigenvalue $\lambda(\otimes)$ *in Theorem 2 (i).*

From the definition of grey matrix, we have $\overline{A} \geq \underline{A} > 0$. Based on conclusion (i) and (iii) in the Perron-Frobonius (P.F.) theorem, we have $\overline{\lambda} \geq \underline{\lambda} > 0$. Similarly, conclusion (ii) in the P.F. theorem indicates the nonnegative eigenvectors $\underline{x}$ and $\overline{x}$ correspond to $\underline{\lambda}$ and $\overline{\lambda}$, respectively. If p and q satisfy $0 < p\underline{x} \leq q\overline{x}$, then $x = [p\underline{x}, q\overline{x}]$ and

$$A(\otimes)x = [\underline{A}, \overline{A}][p\underline{x}, q\overline{x}] = [p\underline{A}\underline{x}, q\overline{A}\overline{x}] = [p\underline{\lambda}\underline{x}, q\overline{\lambda}\overline{x}] = [\underline{\lambda}, \overline{\lambda}][p\underline{x}, q\overline{x}] = \lambda(\otimes)x$$

For eigenvector X *in Theorem 2(ii).*

If there exists an eigenvector $y(\otimes) = [\underline{y}, \overline{y}]$ of matrix A associated with $\lambda(\otimes) = [\underline{\lambda}, \overline{\lambda}]$, owing to theorem 1, we have $\underline{A}\underline{y} = \underline{\lambda}\underline{y}$ and $\overline{A}\overline{y} = \overline{\lambda}\overline{y}$; $\underline{\lambda}$ and $\overline{\lambda}$ are the maximal eigenvalues of $\underline{A}$ and $\overline{A}$. From conclusion (iv) in the P.F. theorem, we find there exist some p, q that satisfy $\underline{y} = p\underline{x}$ and $\overline{y} = q\overline{x}$. i.e., $y = [p\underline{x}, q\overline{x}]$. Therefore, $x = [p\underline{x}, q\overline{x}]$ is the eigenvector associated with all eigenvalues.

This completes the proof.

A.2 QUASI-UNIFORMITY

Definition A1. Let $B = [b_{ij}]_{n\times n}$ be a nonnegative real number matrix and $i, j, k = 1, 2, \cdots, n$ such that

$$b_{ij} \cdot b_{jk} = b_{ji} \cdot b_{ik} \tag{A1}$$

Then, B is a quasi-uniformity nonnegative real number matrix, and Eq. (A1) is the quasi-uniformity condition of B.

Theorem A1. A necessary and sufficient condition for $B = [b_{ij}]_{n\times n}$ is that there exist nonnegative vectors α, β, such that $B = \alpha\beta^T$

Proof: If there exist nonnegative vectors α, β such that $B = \alpha\beta^T$, then we find that B is a quasi-uniformity matrix. Conversely, if $B = [b_{ij}]_{n\times n}$ is a quasi-uniformity matrix,

$$\alpha = (\alpha_{11}, \alpha_{21}, \cdots, \alpha_{n1})^T, \quad \beta = (\frac{\alpha_{11}}{\alpha_{11}}, \frac{\alpha_{22}}{\alpha_{21}}, \cdots, \frac{\alpha_{nn}}{\alpha_{n1}})^T$$

and *k=1* in Eq. (A1), then we can easily show that $B = \alpha\beta^T$.

This completes the proof.

Theorem A1 yields the following corollary.

CorollaryA1. If $B = [b_{ij}]_{n\times n}$ is a quasi-uniformity matrix, then $\lambda = tr(B) \underset{=}{\Delta} \sum_{i=1}^{n} b_{ii}$ is the maximal eigenvalue. The rest of the eigenvalues are 0, and any column vector of B is the eigenvector associated λ.

Corollary A2. If $w = (w_1, w_2, \cdots, w_n)^T$ is an eigenvector of $B = [b_{ij}]_{n\times n}$ associated eigenvalue $\lambda = tr(B)$, then, for every $i, j = 1, 2, \cdots, n$, we have $b_{ij} = b_{jj} \dfrac{w_i}{w_j}$. Namely,

$$B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} (\frac{1}{b_1}, \frac{1}{b_2}, \cdots, \frac{1}{b_n}) \begin{bmatrix} b_{11} & & & \\ & b_{22} & & \\ & & \cdots & \\ & & & b_{nn} \end{bmatrix}$$

From Definition 5 and A1, we obtain the following theorem.

Theorem A2. Suppose $D(\otimes) = [\otimes_{ij}]_{n\times n}$ is a consistent grey number matrix, then $(\underline{d}_{ij})_{n\times n}$ and $(\bar{d}_{ij})_{n\times n}$ are consistent real number matrices.

Theorem A3. A necessary and sufficient condition for $D(\otimes)=[\otimes_{ij}]_{n\times n}$, a consistent grey number matrix, is that there exists grey number $\otimes_i$ $(i=1,2,\dots,n)$, such that

$$\otimes_{ij}=\frac{\otimes_i}{\otimes_j},\ \ (i,j=1,2,\cdots,n)$$

Proof: The sufficient condition is proven by using the results in Saaty (1980). Here, we only prove the necessary condition.

When $D(\otimes)$ is a consistent grey number matrix, from theorem A2, we know $(\underline{d}_{ij})_{n\times n}$ and $(\overline{d}_{ij})_{n\times n}$ are consistent real number matrices, i.e.,

$$\underline{d}_{ij}\underline{d}_{jk}=\underline{d}_{ji}\underline{d}_{ik},\ \ \overline{d}_{ij}\overline{d}_{jk}=\overline{d}_{ji}\overline{d}_{ik}$$

From Definition 5, we have

$$\underline{d}_{ij}=\frac{1}{\overline{d}_{ji}},\ \ \overline{d}_{ij}=\frac{1}{\underline{d}_{ji}}$$

Therefore,

$$\underline{d}_{ii}=\frac{1}{\overline{d}_{ii}}\le\overline{d}_{ii}=\frac{1}{\underline{d}_{ii}}$$

Namely, $\underline{d}_{ii}\le 1$. Let $\otimes_i=[\underline{d}_{ii},\overline{d}_{i1},\overline{d}_{i1}],\ \ (i=1,2,\cdots,n)$, we then have

$$\frac{\otimes_i}{\otimes_j}=[\frac{\underline{d}_{ii}\overline{d}_{i1}}{\overline{d}_{j1}},\ \ \frac{\overline{d}_{j1}}{\underline{d}_{ii}\overline{d}_{j1}}]=[\frac{\underline{d}_{ii}\underline{d}_{1j}}{\underline{d}_{1i}},\ \ \frac{\overline{d}_{jj}\overline{d}_{i1}}{\overline{d}_{j1}}]=[\underline{d}_{ij},\ \ \overline{d}_{ij}]$$

That is, $\otimes_{ij}=\frac{\otimes_i}{\otimes_j},\ \ (i,j=1,2,\cdots,n)$. This completes the proof.

Theorem A4. Suppose $D(\otimes)=[\otimes_{ij}]_{n\times n}$ is a consistent grey number matrix. Then,

$\lambda(\otimes)=tr[D(\otimes)]\underset{=}{\Delta}\sum_{i=1}^{n}\otimes_{ii}$ is the unique eigenvalue of $D(\otimes)$, and every column vector of $D(\otimes)$ is the eigenvector associated $\lambda(\otimes)$.

Proof: For every column vector $\otimes=(\otimes_{1j},\otimes_{2j},\cdots,\otimes_{nj})^T$, we have $\otimes_{ij}\cdot\otimes_{jk}=\otimes_{ji}\cdot\otimes_{ik}$. Thus, we find

$$\sum_{k=1}^{n}\otimes_{ik}\otimes_k=\sum_{k=1}^{n}\otimes_{ik}\otimes_{kj}=(\sum_{k=1}^{n}\otimes_{kk})\otimes_{ij}=\lambda(\otimes)\otimes_i$$

That is, $D(\otimes)\cdot\otimes=\lambda(\otimes)\cdot\otimes$.

If $\xi(\otimes)=[\underline{\xi},\overline{\xi}]$ is the eigenvalue of $D(\otimes)$, then according to Theorem 1, $\underline{\xi}$ and $\overline{\xi}$ are the respective non-zero eigenvalues of $(\underline{d}_{ij})_{n\times n}$ and $(\overline{d}_{ij})_{n\times n}$. Because $(\underline{d}_{ij})_{n\times n}$ and $(\overline{d}_{ij})_{n\times n}$ are a quasi-uniformity matrix, $(\underline{d}_{ij})_{n\times n}$ and $(\overline{d}_{ij})_{n\times n}$ have only non-zero eigenvalues. From Theorem 2, we have $\overline{\lambda}\geq\underline{\lambda}>0$. Thus, $\underline{\xi}=\underline{\lambda}$ and $\overline{\xi}=\overline{\lambda}$. That is, $\xi(\otimes)=\lambda(\otimes)$. Therefore, $\lambda(\otimes)$ is the unique eigenvalue of $D(\otimes)$. This completes the proof.

Corollary A3. Suppose $D(\otimes)=[\otimes_{ij}]_{n\times n}$ is a consistent grey number matrix. Then $w(\otimes)=[pw^*(\otimes),qw^{\#}(\otimes)]$ is the eigenvector of $D(\otimes)$ associated with eigenvalue $\lambda(\otimes)=tr[D(\otimes)]$, where $w^*(\otimes)$ and $w^{\#}(\otimes)$ are column vectors of $(\underline{d}_{ij})_{n\times n}$ and, respectively, p and q are nonnegative real numbers satisfying $0<pw^*(\otimes)\leq qw^{\#}$.

Corollary A4. Suppose $D(\otimes)=[\otimes_{ij}]_{n\times n}$ is a consistent grey number matrix, $w^*(\otimes)$ and $w^{\#}(\otimes)$ are eigenvectors of $D(\otimes)$ associated with eigenvalue $\lambda(\otimes)=tr[D(\otimes)]$. There then exists a grey number $\otimes$ such that $w^*(\otimes)=\otimes\cdot w^{\#}(\otimes)$ or $w^{\#}(\otimes)=\otimes\cdot w^*(\otimes)$.

Theorem A5. Suppose $D(\otimes)=[\otimes_{ij}]_{n\times n}$ is a consistent grey number matrix and $w(\otimes)=(w_1(\otimes),w_2(\otimes),\cdots,w_n(\otimes))^T$ is the eigenvector of $D(\otimes)$ associated with eigenvalue $\lambda(\otimes)=tr[D(\otimes)]$. The following properties are then equivalent:

(i) $\dfrac{w(\otimes)_i}{w(\otimes)_j}=\otimes_{ij}$

(ii) $\dfrac{w_i(\otimes)}{w_j(\otimes)}\subset\otimes_{ij},\ \exists\otimes,\ st.\ \otimes_{ij}=\otimes\cdot\dfrac{w_i(\otimes)}{w_j(\otimes)}$

(iii) $\dfrac{w_i(\otimes)}{w_j(\otimes)}\supset\otimes_{ij},\ \exists\otimes,\ st.\ \dfrac{w_i(\otimes)}{w_j(\otimes)}=\otimes\cdot\otimes_{ij}$

Proof: From $D(\otimes)w(\otimes)=tr(D(\otimes))w(\otimes)$, we assume

$$w^*(\otimes)=(\underline{d}_{11},\underline{d}_{21},\cdots,\underline{d}_{n1})^T,\quad w^{\#}(\otimes)=(\overline{d}_{11},\overline{d}_{21},\cdots,\overline{d}_{n1})^T$$

From corollary A3, we find there exists nonnegative real numbers p,q such that

$$w(\otimes)=[pw^*(\otimes),\ qw^{\#}(\otimes)]$$

Namely,

$$\otimes_j = [p\underline{d}_{j1},\ \ q\overline{d}_{ji}],\ \ (j = 1, 2, \cdots, n)$$

Owing to the quasi-uniformity properties of $(\underline{d}_{ij})_{n\times n}$ and $(\overline{d}_{ij})_{n\times n}$, we have

$$\frac{w_i(\otimes)}{w_j \otimes} = [\Omega\underline{d}_{11}\underline{d}_{ij},\ \ \Omega'\overline{d}_{11}\overline{d}_{ij}]$$

Moreover,

$$\Omega\underline{d}_{11} = \frac{1}{\Omega' \cdot \overline{d}_{11}},\ \ (i, j = 1, 2, \cdots, n)$$

Therefore, we reach the following conclusion.

i) when $\Omega\underline{d}_{11} = 1$, $\dfrac{w(\otimes)_i}{w(\otimes)_j} = \otimes_{ij}$

ii) when $\Omega\underline{d}_{11} > 1$, $\dfrac{w_i(\otimes)}{w_j(\otimes)} \subset \otimes_{ij}$. Let $\otimes = [\Omega'\overline{d}_{11},\ \ \Omega\underline{d}_{11}]$, then $\otimes_{ij} = \otimes \cdot \dfrac{w_i(\otimes)}{w_j(\otimes)}$

iii) when $\Omega\underline{d}_{11} < 1$, $\dfrac{w_i(\otimes)}{w_j(\otimes)} \supset \otimes_{ij}$. Let $\otimes = [\Omega\underline{d}_{11},\ \ \Omega'\overline{d}_{11}]$, then $\dfrac{w_i(\otimes)}{w_j(\otimes)} = \otimes \cdot \otimes_{ij}$

where $\Omega = p/q$ and $\Omega' = q/p$ (similarly hereafter).

This completes the proof.

A.3 PROOF OF THEOREM 3

From Corollary A4 (see Appendix A.2), we know there exists a grey number μ such that $w^*(\otimes) = \otimes w^\#(\otimes)$ or $w^\#(\otimes) = \otimes w^*(\otimes)$. If $w^*(\otimes) = \otimes w^\#(\otimes)$, then $w_i^*(\otimes) = \otimes w_i^\#(\otimes), (i = 1, 2, \cdots, n)$. Therefore,

$$\frac{w_i^*(\otimes)}{w_j^*(\otimes)} = \frac{w_i^\#(\otimes)}{w_j^\#(\otimes)} = \frac{\otimes \cdot w_i^\#(\otimes)}{\otimes \cdot w_j^\#(\otimes)} = \frac{\otimes}{\otimes} \cdot \frac{w_i^\#(\otimes)}{w_j^\#(\otimes)}$$

Thus, grey number $\otimes$ is a constant. Let $\mu = \otimes$, then $w^*(\otimes) = \mu w^\#(\otimes)$. Similarly, we can prove $w^\#(\otimes) = \mu w^*(\otimes)$.

This completes the proof.

A.4 PROOF OF THEOREM 4

From the consistency property of $D(\otimes)$, we have

$$\underline{w}_i = \frac{1}{\sum_{i=1}^{n} \underline{d}_{ij}} \underline{d}_{ij}, \quad \overline{w}_i = \frac{1}{\sum_{i=1}^{n} \overline{d}_{ij}} \overline{d}_{ij}$$

From Theorem A3, we know there exists a grey number $\otimes_i (i = i, 2, \cdots, n)$ such that

$$\otimes_{ij} = \frac{\otimes_i}{\otimes_j}, (i, j = 1, 2, \cdots, n)$$

Namely,

$$\underline{d}_{ij} = \frac{\underline{d}_i}{\overline{d}_j}, \quad \overline{d}_{ij} = \frac{\overline{d}_i}{\underline{d}_j}$$

Thus,

$$\underline{w}_i = \frac{1}{\sum_{i=1}^{n} \underline{d}_{ij}} \underline{d}_{ij}, \quad \overline{w}_i = \frac{1}{\sum_{i=1}^{n} \overline{d}_{ij}} \overline{d}_{ij}$$

Therefore,

$$w_i = [\frac{p}{\sum_{i=1}^{n} \underline{d}_{ij}} \underline{d}_{ij}, \quad \frac{q}{\sum_{i=1}^{n} \overline{d}_{ij}} \overline{d}_{ij}] \text{ and } w_i = [\Omega \cdot \frac{\sum_{i=1}^{n} \overline{d}_{ij}}{\sum_{i=1}^{n} \underline{d}_{ij}} \frac{\underline{d}_i}{\overline{d}_j}, \quad \Omega' \cdot \frac{\sum_{i=1}^{n} \underline{d}_{ij}}{\sum_{i=1}^{n} \overline{d}_{ij}} \frac{\overline{d}_i}{\underline{d}_j}].$$

That is,

$$\Omega = \sum_{j=1}^{n} \quad = \frac{1}{\sum_{i=1}^{n} \overline{d}_{ij}} = \frac{1}{\sum_{j=1}^{n} \frac{1}{\sum_{i=1}^{n} \underline{d}_{ij}}}$$

Due to the symmetry of the weight vector, when $p = \sqrt{\sum_{j=1}^{n} \frac{1}{\sum_{i=1}^{n} \overline{d}_{ij}}}$ and $q = \sqrt{\sum_{j=1}^{n} \frac{1}{\sum_{i=1}^{n} \underline{d}_{ij}}}$, $\Omega = p/q$.

This completes the proof.

APPENDIX B

PERFORMANCE EVALUATION ON CHINA EXPRESS DELIVERY INDSUTRY

B.1 QUESTIONNAIRE

An executive interview had been conducted by Hefei University of Technology; the table below is the sample question of this survey.

Sample: Control criterion: Company General Strength (CGS).

Sub-criterion: Financial Health (FH); Brand Reputation & Experience (BR & E); Management Structure (MS).

Q1: How would you evaluate A company's Company General Strength to B company?						
	Equally	Moderately Better	Better	Very Strong	Extremely Better	
	1-2	2-3	3-5	5-7	7-9	Reverse
Equivalence						

Q2: How would you evaluate A company's Financial Health condition to B company?						
	Equally	Moderately Better	Better	Very Strong	Extremely Better	
	1-2	2-3	3-5	5-7	7-9	Reverse
Equivalence						

Q3: How would you evaluate A company's Reputation in industry and public relative to B company?						
	Equally	Moderately Better	Better	Very Strong	Extremely Better	
	1-2	2-3	3-5	5-7	7-9	Reverse
Equivalence						

Q4: How would you evaluate A company's Management Structure to B company?						
	Equally	Moderately Better	Better	Very Strong	Extremely Better	
	1-2	2-3	3-5	5-7	7-9	Reverse
Equivalence						

B.2 ANP CALCULATION TABLES 1-5

Table 1. Computing results of control system Operational Performance (1/100)

Computing results		Alternatives												Criterion: OP		
		EMS	CREx	CAEx	SFEx	ZJSEx	STOEx	YTOEx	YUNDAEx	TTKEx	FedEx	UPS	DHL	BS	DT	AGP
Alternatives	EMS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.48	5.13	10.08
	CREx	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.02	2.10	4.13
	CAEx	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.53	1.09	2.15
	SFEx	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.82	5.84	11.47
	ZJSEx	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.94	1.95	3.82
	STOEx	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.88	3.89	7.65
	YTOEx	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.48	5.13	10.08
	YUNDAEx	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.60	1.24	2.43
	TTKEx	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.60	1.24	2.43
	FedEx	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.85	4.75	5.46
	UPS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.35	3.68	4.39
	DHL	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.18	2.41	2.78
Computing results		Alternatives												Criterion: OP		
		EMS	CREx	CAEx	SFEx	ZJSEx	STOEx	YTOEx	YUNDAEx	TTKEx	FedEx	UPS	DHL	BS	DT	AGP
Criterion: OP	BS	8.54	3.5	1.82	4.62	1.54	3.08	4.08	0.98	0.98	8.54	3.5	1.82	0.00	0.00	0.00
	DT	17.69	7.25	3.77	9.57	3.19	6.38	8.41	2.03	2.03	17.69	7.25	3.77	0.00	0.00	0.00
	AGP	34.77	14.25	7.41	18.81	6.27	12.54	16.53	3.99	3.99	34.77	14.25	7.41	0.00	0.00	0.00
Computing results		Alternatives												Criterion: OP		
		EMS	CREx	CAEx	SFEx	ZJSEx	STOEx	YTOEx	YUNDAEx	TTKEx	FedEx	UPS	DHL	BS	DT	AGP
Weight		5.71	2.76	1.43	6.65	2.55	4.10	5.72	1.62	1.61	2.32	1.95	1.49	5.33	11.02	21.66

Table 2. Computing results of control system Company Performance (1/100)

Computing results		Alternatives												Criterion: CP		
		EMS	CREx	CAEx	SFEx	ZJSEx	STOEx	YTOEx	YUNDAEx	TTKEx	FedEx	UPS	DHL	FC	BR	CIS
Alternatives	EMS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.30	10.79	4.42
	CREx	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.94	4.42	1.81
	CAEx	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.49	4.07	0.94
	SFEx	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.62	12.28	5.03
	ZJSEx	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.87	4.09	1.68
	STOEx	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.74	8.18	3.36
	YTOEx	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.30	10.79	4.42
	YUNDAEx	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.56	2.60	1.07
	TTKEx	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.56	2.60	1.07
	FedEx	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.79	6.59	4.62
	UPS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.33	5.44	3.24
	DHL	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.17	4.36	3.04
Computing results		Alternatives												Criterion: CP		
		EMS	CREx	CAEx	SFEx	ZJSEx	STOEx	YTOEx	YUNDAEx	TTKEx	FedEx	UPS	DHL	FC	BR	CIS
Criterion: CP	FC	7.93	3.25	1.69	4.25	1.43	2.86	3.77	0.91	0.91	7.93	3.25	1.69	0.00	0.00	0.00
	BR	37.21	15.25	7.93	20.13	6.71	13.42	17.69	4.27	4.27	37.21	15.25	7.93	0.00	0.00	0.00
	CIS	15.25	6.25	3.25	8.25	2.75	5.50	7.25	1.75	1.75	15.25	6.25	3.25	0.00	0.00	0.00
Computing results		Alternatives												Criterion: CP		
		EMS	CREx	CAEx	SFEx	ZJSEx	STOEx	YTOEx	YUNDAEx	TTKEx	FedEx	UPS	DHL	FC	BR	CIS
Weight		3.93	2.19	1.68	4.25	2.02	3.05	4.02	1.29	1.27	4.75	4.32	4.11	3.97	18.61	7.63

Table 3. Computing results of control system Service Cost (1/100)

Computing results		Alternatives												Criterion: SC		
		EMS	**CREx**	**CAEx**	**SFEx**	**ZJSEx**	**STOEx**	**YTOEx**	**YUNDAEx**	**TTKEx**	**FedEx**	**UPS**	**DHL**	**P**	**OE**	**CC**
Alternatives	**EMS**	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.65	10.26	4.78
	CREx	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.09	4.21	1.96
	CAEx	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.57	2.19	1.02
	SFEx	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	3.02	11.68	5.44
	ZJSEx	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.01	3.89	1.81
	STOEx	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.01	7.78	3.62
	YTOEx	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.65	10.26	4.78
	YUNDAEx	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.64	2.48	1.15
	TTKEx	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.64	2.48	1.15
	FedEx	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.92	3.54	1.65
	UPS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.38	1.45	0.68
	DHL	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.20	0.76	0.35
Computing results		**Alternatives**												**Criterion: SC**		
		EMS	**CREx**	**CAEx**	**SFEx**	**ZJSEx**	**STOEx**	**YTOEx**	**YUNDAEx**	**TTKEx**	**FedEx**	**UPS**	**DHL**	**P**	**OE**	**CC**
Criterion: SC	**P**	9.15	3.75	1.95	4.95	1.65	3.3	4.35	1.05	1.05	9.15	3.75	1.95	0.00	0.00	0.00
	OE	35.38	14.5	7.54	19.14	6.38	12.76	16.82	4.06	4.06	35.38	14.5	7.54	0.00	0.00	0.00
	CC	16.47	6.75	3.51	8.91	2.97	5.94	7.83	1.89	1.89	16.47	6.75	3.51	0.00	0.00	0.00
Computing results		**Alternatives**												**Criterion: SC**		
		EMS	**CREx**	**CAEx**	**SFEx**	**ZJSEx**	**STOEx**	**YTOEx**	**YUNDAEx**	**TTKEx**	**FedEx**	**UPS**	**DHL**	**P**	**OE**	**CC**
Weight		8.23	4.21	2.19	10.68	3.89	6.78	8.26	2.48	2.46	3.54	1.44	1.11	8.7	33.61	15.66

Table 4. Computing results of control system Service Quality (1/100)

Computing results		Alternatives												Criterion: SQ		
		EMS	CREx	CAEx	SFEx	ZJSEx	STOEx	YTOEx	YUNDAEx	TTKEx	FedEx	UPS	DHL	AGP	OD	DC
Alternatives	EMS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	4.60	5.66	7.43
	CREx	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.89	2.32	3.05
	CAEx	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.98	1.21	1.58
	SFEx	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5.24	6.44	8.45
	ZJSEx	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.75	2.15	2.82
	STOEx	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	3.49	4.29	5.64
	YTOEx	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	4.60	5.66	7.43
	YUNDAEx	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.11	1.37	1.79
	TTKEx	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.11	1.37	1.79
	FedEx	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.59	1.95	2.56
	UPS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.65	0.80	1.05
	DHL	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.34	0.42	0.55
Computing results		Alternatives												Criterion: SQ		
		EMS	CREx	CAEx	SFEx	ZJSEx	STOEx	YTOEx	YUNDAEx	TTKEx	FedEx	UPS	DHL	AGP	OD	DC
Criterion: SQ	AGP	15.86	6.5	3.38	8.58	2.86	5.72	7.54	1.82	1.82	15.86	6.5	3.38	0.00	0.00	0.00
	OD	19.52	8	4.16	10.56	3.52	7.04	9.28	2.24	2.24	19.52	8	4.16	0.00	0.00	0.00
	DC	25.62	10.5	5.46	13.86	4.62	9.24	12.18	2.94	2.94	25.62	10.5	5.46	0.00	0.00	0.00
Computing results		Alternatives												Criterion: SQ		
		EMS	CREx	CAEx	SFEx	ZJSEx	STOEx	YTOEx	YUNDAEx	TTKEx	FedEx	UPS	DHL	AGP	OD	DC
Weight		9.80	4.44	2.30	11.31	4.11	7.20	9.82	2.61	2.60	7.69	6.58	6.36	15.90	19.56	25.68

Table 5. Computing results of control system Express Technology (1/100)

Computing results		Alternatives												Criterion: ET	
		EMS	CREx	CAEx	SFEx	ZJSEx	STOEx	YTOEx	YUNDAEx	TTKEx	FedEx	UPS	DHL	CIT	ES
Alternatives	EMS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	13.27	4.42
	CREx	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5.44	1.81
	CAEx	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.83	0.94
	SFEx	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	15.10	5.03
	ZJSEx	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5.03	1.68
	STOEx	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	10.07	3.36
	YTOEx	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	13.27	4.42
	YUNDAEx	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	3.20	1.07
	TTKEx	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	3.20	1.07
	FedEx	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	16.48	1.53
	UPS	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	14.82	0.63
	DHL	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	13.97	0.33
Computing results		Alternatives												Criterion: ET	
		EMS	ES	CAEx	SFEx	ZJSEx	STOEx	YTOEx	YUNDAEx	TTKEx	FedEx	UPS	DHL	CIT	ES
Criterion: ET	CIT	45.75	18.75	9.75	24.75	8.25	16.5	21.75	5.25	5.25	45.75	18.75	9.75	0.00	0.00
	ES	15.25	6.25	3.25	8.25	2.75	5.5	7.25	1.75	1.75	15.25	6.25	3.25	0.00	0.00
Computing results		Alternatives												Criterion: ET	
		EMS	CREx	CAEx	SFEx	ZJSEx	STOEx	YTOEx	YUNDAEx	TTKEx	FedEx	UPS	DHL	CIT	ES
Weight		6.51	2.72	1.41	6.97	2.52	5.04	6.53	1.60	1.59	6.51	5.32	5.11	28.13	9.38

APPENDIX C

LOCATION DIFFUSION, RESOURCE ALLOCATION AND FLEET REBALANCING: A STUDY OF THE CAR-SHARING INDUSTRY

C.1 PROOF OF THEOREM 1

Proof:

Denote the right side of $F(t+1)$ in Eq. (1) by $h(F)$.

$$h(F) = F + \frac{\beta}{N} P \cdot F - \gamma F$$

Note that

$$h'(F) = 1 + \frac{\beta}{N} P - \gamma$$

Given $F \in [0, N]$, since $0 < \gamma < 1$, then $h'(F) > 0$.

For case (i), because $D_0 \le 1$, $h(0) = 0$ and $0 < h'(0) < 1$ are known, either $h'(F) < 1$ or $h(F) < F$ holds, $\{F(t)\}$ would be a strictly decreasing sequence that will be bounded by zero as a lower limit. And also because there must exist a fixed point of h on $[0, N]$ for any given $\{F(t)\}$, and the only fixed point of h in $[0, N]$ is 0; hence, $\lim_{t \to \infty} F(t) = 0$.

For case (ii), because $D_0 > 1$, there exists a unique $\tilde{F} > 0$ such that $h(\tilde{F}) = \tilde{F}$, $h(F) > F$ as of $F \in (0, \tilde{F})$ and $h(F) < F$ as of $F \in (\tilde{F}, N)$. In this case, because of $h(0) = 0$, $h(N) < N$ and $0 < h'(0) < 1$, there must exist at least one fixed point such that $\tilde{F} > 0$, $h(\tilde{F}) = \tilde{F}$.

Let $\tilde{F}$ be the smallest positive fixed point, then $h(F) > F$ as of $F \in (0, \tilde{F})$ and $h'(\tilde{F}) < 1$. Given $h'(F) < h'(\tilde{F}) < 1$ in $F \in (\tilde{F}, N]$, if integrating of the last inequality over the interval $[\tilde{F}, F]$, we would have $h(F) < F$ for $F > \tilde{F}$. Thus, h has a unique positive fixed point $\tilde{F}$.

Particularly, when $\beta = N\gamma$ and $h(F) = F + \beta$, $\{F(t)\}$ would be a strictly increasing sequence associated with the increasing of β and $\sup_{\beta \to N - F(t)} F(t) \to N$.

The proof of $\tilde{P}$ is similar to $\tilde{F}$.

This completes the proof.

C.2 PROOF OF THEOREM 2

Proof:

Consider the linear part of Eq. (8)

$$F(t+1) = BF(t) + T\Theta F(t), \quad t = 0, 1, 2, \cdots \tag{c1}$$

Since

$$B + T\Theta = B + \begin{bmatrix} \lambda_0\beta_1 & \lambda_0\beta_2 & \lambda_0\beta_3 & \cdots & \lambda_0\beta_{m-1} & \lambda_0\beta_m \\ \lambda_1\beta_1 & \lambda_1\beta_2 & \lambda_1\beta_3 & \cdots & \lambda_1\beta_{m-1} & \lambda_1\beta_m \\ \lambda_2\beta_1 & \lambda_2\beta_2 & \lambda_2\beta_3 & \cdots & \lambda_2\beta_{m-1} & \lambda_2\beta_m \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ \lambda_{m-1}\beta_1 & \lambda_{m-1}\beta_2 & \lambda_{m-1}\beta_3 & \cdots & \lambda_{m-1}\beta_{m-1} & \lambda_{m-1}\beta_m \end{bmatrix}$$ is a non-negative matrix. By the Frebenious theorem (Horn & Johnson, 2012) of non-negative matrix, there would be only one positive eigenvalue ρ_0 and a positive eigenvector v associated with ρ_0; the module of other eigenvalues of matrix $B + T\Theta$ is not greater than ρ_0, that is $|\rho| \leq \rho_0$. Thus, the stability of Eq. (c1) is completely determined by ρ_0.

ρ_0 and v satisfy the following equation:

$$(B + T\Theta)v = \rho_0 E v$$

Since matrix $E - B$ is invertible, if $\rho_0 = 1$ holds, then the following equation must hold as a necessary and sufficient condition:

$$(E - B)^{-1} T\Theta\, v = v$$

That is

$$(E-B)^{-1}T\Theta=\begin{bmatrix} \beta_1\varpi_1 & \beta_2\varpi_1 & \beta_3\varpi_1 & \cdots & \beta_m\varpi_1 \\ \beta_1\varpi_2 & \beta_2\varpi_2 & \beta_3\varpi_2 & \cdots & \beta_m\varpi_2 \\ \beta_1\varpi_3 & \beta_2\varpi_3 & \beta_3\varpi_3 & \cdots & \beta_m\varpi_3 \\ \vdots & \vdots & \vdots & & \vdots \\ \beta_1\varpi_m & \beta_2\varpi_m & \beta_3\varpi_m & \cdots & \beta_m\varpi_m \end{bmatrix}$$

where

$$\varpi_1=\lambda_0$$

$$\varpi_2=\lambda_1+\lambda_0 q_1(0)$$

$$\varpi_3=\lambda_2+\lambda_1 q_2(0)+\lambda_0 q_1(0)q_2(0)$$

$$\cdots \qquad \cdots \qquad \cdots$$

$$\varpi_m=\lambda_{m+1}+\lambda_{m-2}q_{m-1}(0)+\lambda_{m-3}q_{m-2}(0)q_{m-1}(0)+\cdots+\lambda_1 q_2(0)q_3(0)\cdots q_{m-1}(0)+\lambda_0 q_1(0)q_2(0)\cdots q_{m-1}(0)$$

The characteristic equation of matrix $(E-B)^{-1}T\Theta$ is

$$\rho^{m-1}(\beta_1\varpi_1+\beta_2\varpi_2+\cdots+\rho_m\varpi_m-\rho)=0 \qquad \text{(c2)}$$

By Eq. (c2), we can obtain the positive eigenvalue

$$\rho_0=\beta_1\varpi_1+\beta_2\varpi_2+\cdots+\rho_m\varpi_m=(\beta_1,\beta_2,\cdots,\beta_m)(E-B)^{-1}\lambda=D_0$$

Here, D_0 is the Regenerative rate of Eq. (4) and Eq. (8).

Since

$$G(F(t))\Theta F(t)\geq 0$$

and by Eq. (8), we have

$$F(t+1)\leq BF(t)+T\Theta F(t) \qquad \text{(c3)}$$

By using the Comparison Principle (Zheng, 1994), it is clear to see :

(i) When $D_0<1$, membership function $F(t)$ has diffusion-free equilibrium 0; that is, the solution of Eq. (4) is stable, $\lim_{t\to\infty} F(t)=0$.

(ii) When $D_0>1$, membership function $F(t)$ has no diffusion-free equilibrium; that is, the solution of Eq. (4) is unstable, but has bounded solution $F(t)$.

From Eq. (4), we have

$$P_j(t)+F_j(t)=N_j, \quad j=0,1,\cdots,m, \quad t=0,1,2,\cdots.$$

That is

$$F_j(t)\leq N_j$$

Let $N_j,\quad j=0,1,\cdots,m$ be the upper boundary, then, we have

$$\sup_{t\to\infty} F_j(t) \to N_j,\quad j=0,1,\cdots,m,\quad t=0,1,2,\cdots.$$

This completes the proof.

C.3 PROOF OF THEOREM 3

Proof:

We first prove the existence of t_s.

Because t_s is the total time of the highest inventory level dropped to zero, then $I(t_s)=0,\ (z_1 \le t_s \le z_2)$. And t_s will fall into one of following circumstances.

(i) $t_s = z_1, I(z_1) = D$.

(ii) $t_s = z_2, I(z_2) = \bar{Q}$.

From the assumptions, we know the replenishment policy is

$$D(t) = \bar{Q} \tag{c4}$$

That is

$$I(z_1) = I(z_2) \tag{c5}$$

From Eq. (12) and Eq. (15), we know the station inventory function $I(t),\ t\in[z_1,z_2]$ is continuous on $[z_1,z_2]$ and differentiable over (z_1,z_2). According to Rolle's Mean Value Theorem (Stewart, 2009), when $I(z_1)=I(z_2)$ holds, there exists at least one point ξ in (z_1,z_2) where $I'(\xi)=0$. Let $t_s=\xi$, then, the existence of t_s is proved.

Next, we prove the uniqueness of t_s.

Assuming the business of each rental and replenishment cycle has similar pattern, thus we can focus on investigating only one cycle. Based on the assumptions, we know that as time point of replenishment, t_s will appear only once in one cycle. Once t_s arrives, the station inventory will gradually increase to the expected inventory due to replenishment; till station reaches its maximal inventory level, a new business cycle will start. Thus the uniqueness of t_s is proved.

This completes the proof.

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