INVESTIGATING THE RELATIONSHIP BETWEEN PRE-SERVICE TEACHERS' ATTENTION TO STUDENT THINKING DURING LESSON PLANNING AND THE LEVEL OF COGNITIVE DEMAND AT WHICH TASKS ARE IMPLEMENTED

by

Scott Christopher Layden

Bachelor of Science, Indiana University of Pennsylvania, 2001

Masters of Science, Indiana University of Pennsylvania, 2005

Submitted to the Graduate Faculty of

the School of Education in partial fulfillment

of the requirements for the degree of

Doctor of Education

University of Pittsburgh

2015

UNIVERSITY OF PITTSBURGH

SCHOOL OF EDUCATION

This dissertation was presented

by

Scott C. Layden

It was defended on

October 29, 2015

and approved by

Melissa Boston, Associate Professor, Duquesne University

Charles Munter, Assistant Professor, Instruction and Learning

Margaret Smith, Professor, Instruction and Learning

Dissertation Advisor: Ellen Ansell, Associate Professor, Instruction and Learning

INVESTIGATING THE RELATIONSHIP BETWEEN PRE-SERVICE TEACHERS' ATTENTION TO STUDENT THINKING DURING LESSON PLANNING AND THE LEVEL OF COGNITIVE DEMAND AT WHICH TASKS ARE IMPLEMENTED

Scott C. Layden, EdD

University of Pittsburgh, 2015

This study investigated the relationship between attention to student thinking during lesson planning and the level of cognitive demand at which tasks are implemented for six pre-service teachers enrolled in a teacher education program that focuses on attention to student thinking during planning and instruction. Lesson plans were examined for attention to student thinking using two coding schemes, and samples of student work were examined to assess the level of cognitive demand at which tasks (associated with the enacted lesson plans) were implemented during instruction. Other planning related data sources were qualitatively drawn upon to support the extent to which pre-service teachers focused on student thinking with regard to planning.

One of the lesson planning coding schemes provides numerical scores indicating different degrees of attention to six elements of student thinking. The level of cognitive demand of task implementation for each lesson was able to be coded as high or low. In particular, the quantitative analysis suggested a trend that as overall attention to student thinking during lesson planning increases the odds of high level task implementation become greater compared to the odds of low level task implementation. Given a small sample size the quantitative results need to be considered within their limitations.

Qualitative analysis examining attention to student thinking during planning and task implementation supports the quantitative trend. In particular, the qualitative analysis suggests three findings. The first finding is that the two pre-service teachers who demonstrated the most attention to student thinking with regard to planning were the only pre-service teachers who implemented all of their tasks at a high level of cognitive demand. The second finding is that when receiving specific planning based support for a lesson as part of a university assignment, all the pre-service teachers were able to implement the task at high level of cognitive demand. The third finding is that a large majority of lessons using tasks accompanied by detailed planning support sources were implemented at high levels of cognitive demand.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

PREFACE

The journey to earning this degree was a long and winding road that lasted ten years. On a journey of such length many people are bound to have to assist you along the way. First and foremost, I would like to thank my wife, Jessica, who has sacrificed so much in order for me to achieve this dream. I would have never made it without you. You bore the responsibility of so many things yourself, and you are the only person I know that would be willing to do for someone else what you did for me for such a long time. What makes your sacrifices so meaningful is I know you have the creativity and talent that would allow you to be so much more successful than I could ever be, yet you set everything aside for the last 10 years to raise our family and help me. You were my rock, and the time you devoted to Anna, Ian, and Elle is reflected in how uniquely amazing they all are today. I promise to be the same for you as we move forward and go after one of your many dreams that you so willingly let remain just that during my studies. With your talent and ambition, I cannot even imagine how much you truly are able to achieve. I am so excited for the future, and as I look back I know I will never forget all that you did for me. For this, I am ever indebted to you, I love you, and I promise the rest is yours. I would also like to thank my children, Anna, Ian and Elle for being so understanding when daddy had to work on his “paper”. I love each one of you dearly and am so happy to be able to give you my full attention. I would also like to thank my in-laws for their ongoing help throughout the process, and for always helping get the kids where they needed to be. Thank you

to my parents for their exerted effort and different forms of help during the dissertation process, especially to my mom for making the long drive so many times on her own to come help.

I would also like to thank my committee for their focused attention and advice, especially my advisor, Dr. Ellen Ansell. I appreciate the thoughtful attention you gave every document I have ever sent you, and I also appreciate the times you pushed me forward as well as the times you “drew on the reins”. Dr. Peg Smith, thank you for your work on the Five Practices that my study drew heavily upon, and for helping me use and extend your rubric to enhance my study. Dr. Charles Munter, thank you for helping me with my methodology, and your continued support with my statistical analysis. Dr. Melissa Boston, thank you for your work with IQA toolkit which made it possible for me to conduct my study. Also, thank you for training me to use the toolkit and the openness you have demonstrated throughout this process. Additionally, I’d like to thank fellow graduate students (who all finished before me): Sam, Michelle for always keeping it real, and Steve for his continual support, advice and coding assistance. I would also like to thank Joe for his coding assistance. Also, thank you to Dr. Cathy Schloemer and Dr. John Uccelini for their advice and support.

Additionally, I’d like to thank the following professors who helped me along the way: Dr. Gaea Leinhardt, Dr. Linda Kucan, and Dr. Maureen Porter. At different times in my process you each provided me with exactly what I needed to keep going. Thank you to fellow teacher and student teacher, Traci Sexton and James Martin, who participated in a study. Thank you to Sarah for the editing help. If there is anyone I have missed, I thank you as well.

1.0 STATEMENT OF THE PROBLEM

1.1 INTRODUCTION

The main goal of teaching is to enhance and support student learning (Hiebert, Morris, Berk, & Jansen, 2007). The teaching of mathematics is a complex process that is not easily defined. According to Hiebert & Grouws (2007), teaching mathematics "consists of classroom interactions among teachers and students around content directed toward facilitating students' achievement of learning goals" (p. 372). The National Council of Teachers of Mathematics (NCTM) Standards (2000) called for instruction to build on students' prior knowledge, focus on problem solving, involve classroom discussion centered on the analysis of multiple solution methods, and press students to explain their work. The recently adopted Common Core State Standards for Mathematics (CCSSM, 2010) echo the NCTM Standards in their description of *Standards for Mathematical Practice* by recommending a similar set of "expertise that mathematics educators at all levels should seek to develop in their students" (CCSSM, 2010, p. 6).

Compared to previous standards documents, the CCSSM have received wide acceptance with regard to which mathematical ideas should be taught and the practices that students should engage in; however the CCSSM do not specify *how* the mathematics should be delivered (Munter, Stein, & Smith, 2015). Researchers agree that there is not one particular method of teaching recognized as being most effective in accomplishing all learning goals (Hiebert & Grouws, 2007); however, research does indicate that different instructional approaches are linked to differences in students' learning (Stigler & Hiebert, 1999; Hiebert, 2003).

One method of instruction that supports student learning of meaningful mathematics involves the teacher helping students grapple with key ideas as the students propose and justify their own claims and critique the claims of their classmates (Munter, et al., 2015). In order to do this, teachers must use cognitively demanding mathematical tasks, a critical dimension of effective classroom practice (Hiebert et al., 1997; Carpenter & Lehrer, 1999), to introduce new ideas and advance students' already existing knowledge. The implementation of cognitively demanding mathematical tasks supports students' engagement in meaningful mathematics and mathematical practices (NCTM, 2014). These tasks require students to go beyond memorization and the use of procedures by making mathematical connections and reasoning conceptually (Stein & Smith, 1998).

As students engage mathematically with such a task, the teacher circulates among the groups asking questions about individual students' specific thinking. The lesson then culminates with the teacher facilitating a whole-class discussion focusing on connections between different solution strategies and underlying mathematical ideas utilized by the students in the class. Through the discussion, the teacher guides students towards a particular mathematical goal (Senk & Thompson, 2003b).

While instruction involving cognitively demanding tasks is linked to increased student learning and achievement (e.g., Hiebert & Wearne, 1993; Boaler & Staples, 2008; Stein & Lane, 1996), such instruction is also difficult for many teachers to accomplish (e.g., Stein, Grover, & Henningsen, 1996; Stigler & Hiebert, 2004). Several avenues have been initiated to help teachers successfully incorporate cognitively demanding tasks into their practice to support their students' learning (e.g., Arbaugh & Brown, 2005; Boston & Smith, 2009; Smith, Bill, & Hughes, 2008).

For example, teachers participating in a researcher-led task analysis study group learned to critically examine different levels of cognitive demands of mathematical tasks, and even increased the use of high-level cognitive demand tasks in their own classrooms (Arbaugh & Brown, 2005). In another study, teachers who received support in selecting and implementing high-level tasks increased their ability to do so, and they significantly outperformed a control group of teachers, with regard to task selection and implementation, who did not receive support in the same areas (Boston & Smith, 2009). Another method used to help teachers implement high-level tasks is lesson planning that explicitly requires teachers to attend to student thinking around a mathematical task (Smith, Bill, & Hughes, 2008). Such lesson planning may be particularly useful for pre-service teachers, since novice teachers' plans and instruction often focus on actions of the teacher without consideration of student thinking (Leinhardt, 2003).

The purpose of this study is to investigate lesson planning and how it links to pre-service teachers' implementation of high-level tasks. In particular, the study seeks to examine the relationship between pre-service teachers' attention to students' mathematical thinking around cognitively demanding tasks during lesson planning and the cognitive demand at which students engage the same tasks when implemented during instruction. In this chapter, I argue first that the successful implementation of cognitively demanding tasks is important to student learning but yet difficult for teachers to accomplish. Secondly, I argue that research related to the link between planning and instruction suggests that lesson planning that focuses on student thinking can be a leverage for instructional change, particularly instruction involving cognitively demanding tasks. Finally, I argue that pre-service teachers are a meaningful population to investigate for this study.

1.2 MATHEMATICAL TASKS

A mathematical task "is defined as a classroom activity, the purpose of which is to focus students' attention on a particular mathematical idea" (Stein, Grover, & Henningsen, 1996, p. 460). Tasks vary in the level of cognitive demand required to solve them (Stein & Smith, 1998). Low-level tasks have limited possibilities for solution strategies and are focused on the correct answer. Such tasks involve memorization or the reproduction of a procedure provided by the teacher. Thus, they require little cognitive effort from students. In contrast, high-level tasks include problems that require students to draw connections to underlying concepts, and problems that require students to engage in complex thinking and reasoning (Stein & Smith, 1998; Stein et al., 1996). Such tasks often have multiple solution strategies, and they may present ambiguity. Students also need to put forth considerable cognitive effort in order to solve them.

The use of cognitively demanding tasks is linked to student learning and achievement as higher levels of student performance have been observed in classrooms where students have had the opportunity to engage with cognitively demanding tasks as compared to classrooms where students were not provided such opportunity (e.g., Stein & Lane, 1996; Hiebert & Wearne, 1993). One study examined the set up and implementation of 144 mathematical tasks across 4 middle schools (Stein et al., 1996). There were instances of tasks being set-up at a high-level and maintained as high-level during the implementation phase. Students in such classes demonstrated greater gains in performance compared to students in classes where tasks were set up as high level but declined to low-level during implementation (Stein & Lane, 1996). The majority of tasks that were high-level at the time of set-up followed the latter pattern; that is they declined to low-level during implementation (Stein et al., 1996). Thus, high-level tasks are important to student learning (Stein & Lane, 1996), but are difficult for many teachers to successfully implement (Stein et al., 1996).

One avenue intended to help teachers implement high-level tasks is the use of a detailed lesson planning tool that explicitly requires teachers to attend to student thinking around a task across the entire lesson (Smith, et al., 2008; Smith & Stein, 2011). The next section discusses the link between focusing on student thinking in planning and instruction, and how attention to student thinking in lesson planning can serve as leverage for instructional change.

1.3 FOCUS ON STUDENT THINKING IN LESSON PLANNING AND INSTRUCTION

The value of focusing on student thinking during planning and instruction has been supported by studies of Japanese Lesson Study (e.g., Lewis & Tsuchida, 1998; Yoshida, 1999; Kawanaka & Stigler, 1999) and expert-novice distinctions (e.g., Leinhardt, 1993; Schoenfeld, Minstrell, & van Zee, 2000; Zimmerlan & Nelson, 2000). Japanese Lesson Study is a form of professional development in which teachers focus on student thinking while planning a single lesson. The process involves teachers working together during the phases of planning, implementation and reflection. A comparison of Japanese and American teachers' planning and instruction showed that Japanese teachers focused primarily on student thinking during planning, while American teachers tended to focus more on teacher actions (Stigler, Fernandez, & Yoshida, 1996). During the instruction subsequent to the lesson plans, the students of the Japanese teachers were given more opportunities to think mathematically than their American counterparts.

Similar results can be found in expert teachers' planning and instruction (Leinhardt, 1993; Schoenfeld, Minstrell, & van Zee, 2000; Zimmerlan & Nelson, 2000; Livingston & Borko, 1990; Borko & Livingston, 1989). These studies found that *expert* teachers plan for and enact lessons based on the thinking of students in their classrooms. Expert teachers often do not have extensive written plans, but they do have rich and complex *agendas* (Leinhardt, 1993) or *lesson images* (Schoenfeld, 1998) which they can readily verbalize prior to teaching a particular lesson. Experts' plans provide evidence of careful consideration of student responses as well as clear goals for the lesson (e.g., Leinhardt, 1993; Schoenfeld et al., 2000).

Experts have specific plans for how to achieve their goals. During instruction subsequent to their plans, expert teachers make use of student thinking while working towards reaching the goal(s) of the lesson (Leinhardt, 1993; Schoenfeld, et al., 2000). Expert teachers discuss similar planning and instruction in analyses of their own classroom practices (e.g., Lampert, 2001; Ball, 1993; Schoenfeld, 1998). These teacher-researchers describe in detail how consideration of individual student's thinking in relation to the needs of the entire class motivates their planning and instructional decisions.

Novice teachers, on the other hand, make plans and subsequently implement lessons that often do not provide evidence of clear goals, and primarily focus on the actions of the teacher without consideration of student thinking (Leinhardt, 1993). For example, in a study involving pre-service teachers, Borko & Livingston (1989) found that, during their lesson planning, the pre-service teachers primarily focused on explaining strategies and content to students. While pre-service teachers' planned explanations and examples were mathematically sound, they experienced difficulty during interactive teaching when they had to provide explanations and examples on the spot (Borko & Livingston, 1989).

Though novice teachers can be faithful in implementing what they plan (e.g., Nelson & Zimmerman, 2000), the plans are generally not based on student thinking (Leinhardt, 1993). Difficulties tend to arise when students respond in ways that the pre-service teacher did not plan for (Borko & Livingston, 1989; Zimmerlan & Nelson, 2000). Basically, if there were not any students in the classroom, novice teachers would be able to implement their plans with a high rate of success.

Unfortunately, difficulties arise in teaching because student thinking is a mediating factor, and novice teachers have not yet developed knowledge structures possessed by expert teachers to successfully improvise during instruction (e.g., Livingston & Borko, 1990; Borko & Livingston, 1989). The complex lesson agendas of expert teachers account for a variety of student responses (Leinhardt, 1993), and novice teachers have not yet formulated such agendas. As a result, it is imperative that novice teachers explicitly consider much more information, in particular student thinking, than they generally do during planning so they can teach more like experts (Borko & Livingston, 1989).

1.3.1 Lesson Planning as Leverage for Instructional Change

In comparison to novice teachers, expert teachers plan more carefully for differences in student thinking and are better equipped to make use of student responses during instruction (e.g., Borko

& Livingston, 1989; Leinhardt, 1993). Novice teachers may be able to move in this direction if they would engage explicitly in practices that stimulate what expert teachers do tacitly. There is evidence that lesson planning can serve as a mechanism for pre-service teachers to build the capacity to attend to students' mathematical thinking during planning (Hughes, 2006). Before and immediately after a methodology course focused on attending to student thinking via lesson planning, pre-service teachers' attention to students' mathematical thinking during planning was examined. It was also then examined during the first semester of a full-year internship. Pre-service teachers' attention to student thinking during lesson planning did change over time. The pre-service teachers demonstrated a significant ability to attend to student thinking during planning in both the methods course and field experience (Hughes, 2006).

In the same study, the pre-service teachers were required to plan a limited number of lessons using a tool that explicitly focused their attention on student thinking around a high-level task. For these lesson plans, the pre-service teachers attended to student thinking at a higher level than when they were not required to use the tool. The study indicates that pre-service teachers have the ability to attend to student thinking during lesson planning, especially when explicitly directed to do so (Hughes, 2006). The study, however, does not provide any empirical evidence of how such planning impacts instruction.

There is a study currently being conducted that is investigating links between such planning and instruction under the umbrella of the Lesson Planning Project (Stein, Russell, & Smith, 2011; Smith, Cartier, Eskelson, Tekkumru-Kisa, 2012; Smith, Cartier, Eskelson, & Ross, 2013). According to Smith et al. (2012), "a core premise of the Lesson Planning Project was the belief that teachers' engagement in thoughtful, thorough lesson planning routines would lead to more rigorous instruction and improved student learning" (p. 118). These "thoughtful, thorough" routines focus on anticipation of student thinking. Anticipating student responses is the first practice in a specific set of 5 instructional practices designed to help teachers plan for and orchestrate a discussion around a cognitively demanding task (Smith & Stein, 2011).

These five practices (Smith & Stein, 2011) provide a "road map" to effectively plan for and implement a whole-class discussion which utilizes children's mathematical thinking. The idea is that by engaging in the five practices during planning teachers are able to extend "the time to make an instructional decision from seconds to minutes (or even hours)…increasing the number of teachers to feel- and actually be- better prepared for discussions" (Stein, Engle, Smith, & Hughes, 2008, p. 21). If engagement with the practices during planning helps ease the need to improvise during real-time instruction, then teachers are more equipped to maintain the cognitive demands of high-level tasks throughout the entire lesson.

The five practices are: (1) anticipating students' mathematical responses, (2) monitoring students' responses during the explore phase, (3) purposefully selecting student responses for public display, (4) purposefully sequencing student responses, and (5) connecting student responses during whole class discussion (Stein et. al, 2008). The successful implementation of these practices depends on the level of engagement with the preceding practice. Thus, anticipating student responses is a necessary prerequisite to each of the practices that follow (Smith & Stein, 2011).

One of the studies situated within the Lesson Planning Project investigated links between a teacher's collaborative anticipating efforts and her level of task implementation (Smith et al., 2013). The focus teacher participated in modified lesson study cycles where she and other group members anticipated responses to mathematical tasks she intended to use in upcoming lessons. In one lesson, the teacher had a specific learning goal and the high-level task was a "real-world problem". The teacher and group generated multiple anticipations focused on how students were *seeing* and *making sense* of the mathematics in the problem, and the task for that lesson was implemented at a high level of cognitive demand. In another lesson, the teacher had two general learning goals and the high-level task was considered "abstract mathematical". The majority of anticipations focused on what students were *doing* (rather than what they were thinking), and the task was implemented at a low-level of cognitive demand. In general with regard to planning, the authors argue that "the level of preparation in which the teacher engaged may contribute to her ability to maintain the level of demand of the task during instruction" (Smith et al., 2013, p. 40).

1.4 FOCUSING ON PRE-SERVICE TEACHERS' PLANNING AND INSTRUCTION

Instructional approaches that engage students in meaningful mathematics and mathematical practices support deep levels of learning. However, learning to teach mathematics in such a way can be a difficult matter (Brown & Borko, 1992). Pre-service teachers are learning to teach, and they need to be provided with opportunities to help them develop instructional approaches that foster student learning. Lesson planning that explicitly prompts pre-service teachers to focus on student thinking is one such specific opportunity.

As mentioned earlier, expert teachers tend not to provide extensive written plans (Schoenfeld, 1998). A small study within the Lesson Planning Project yielded similar results for in-service teachers. In particular, the majority of their anticipations were provided verbally and very few (if any) were provided in their written plans despite having access to a detailed planning tool prompting them to focus on students' thinking (Smith et al., 2013). In contrast, pre-service teachers enrolled in a particular teacher education program provided detailed lesson plans focused on student thinking especially when explicitly prompted by a detailed planning tool (Hughes, 2006); however, their engagement in such planning has not been linked to instruction. In general, pre-service teachers tend to struggle during live instruction (e.g. Livingston & Borko, 1990; Brendefur & Frykholm, 2000; Vacc & Bright, 1999).

Expert teachers and in-service teachers have the benefit of experience on their side, while pre-service teachers do not. Due to this lack of experience, it may be necessary for pre-service teachers to engage explicitly in detailed planning focused on student thinking. Through such planning, they may attempt to be prepared for different student responses and the general responsibilities of live teaching. Thus, pre-service teachers are a particular population that can potentially benefit from detailed planning that explicitly focuses attention on student thinking.

1.5 PURPOSE OF THE STUDY

The study proposed in this paper seeks to investigate the instruction of pre-service teachers after they have engaged in detailed lesson planning focused on student thinking. The form of lesson planning is intended to promote instruction that fosters student learning. Thus, this study seeks to investigate participants' engagement with an opportunity (i.e. lesson planning focused on student thinking) designed to promote the delivery of meaningful mathematics instruction for a population (i.e. pre-service teachers) who is learning to teach. More specifically, this study seeks to investigate the premise that “thoughtful” and “thorough” lesson planning will lead to improved instruction when pre-service teachers focus on student thinking.

In general, there is limited (if any) research regarding how pre-service teachers’ attention to student thinking during lesson planning is related to their implementation of lessons using high-level tasks. The purpose of this study is to examine the relationship between pre-service teachers’ lesson planning and instruction by addressing the following research question:

> What is the relationship between pre-service teachers’ attention to student thinking with regard to lesson planning around a mathematical task (perceived to be high level by the pre-service teacher), and the level of cognitive demand at which the task is implemented?

The study seeks to address this question by investigating the practices of pre-service teachers who are enrolled in a teacher education program which focuses on attending to students’ mathematical thinking during planning and instruction, and uses the five practices discussed earlier to frame much of their learning experience. The pre-service teachers are accustomed to using a detailed planning tool which focuses their attention on student thinking. The study will draw upon a rubric that uses the five practices as a framework to measure pre-service teachers’ attention to students’ mathematical thinking during lesson planning, and will seek to examine the relationship between their planning and the level of cognitive demand at which tasks are implemented. Through the use of interviews, the study seeks to examine whether pre-service teachers focus on student thinking when asked if their lesson went as planned.

1.6 SIGNIFICANCE OF THE STUDY

This study seeks to add to research linking planning and instruction. The study extends the investigation of pre-service teachers' attention to student thinking during lesson planning (Hughes, 2006) to instruction. More specifically, it adds to the work related to pre-service teachers' implementation of tasks (Mossgrove, 2006) by formally investigating how attention to student thinking in planning is linked to task implementation. Also, it investigates the core premise of the Lesson Planning Project (Smith et al., 2013) that "thoughtful, thorough" lesson planning leads to better instruction for a specific group of teachers (i.e. pre-service teachers).

Philipp (2008) argues that when pre-service teachers are provided support to engage with students' mathematical thinking, they begin to realize the importance of deeply learning the mathematics themselves. In turn, the pre-service teachers begin to view math through the lens of students' mathematical thinking and come to care about mathematics as teachers. This study may provide empirical evidence for a type of lesson planning that supports pre-service teachers' engagement with students' mathematical thinking which Philipp argues is so important.

Finally, the study involves secondary pre-service teachers. The vast majority of studies dealing with students' mathematical thinking are based at the primary level (e.g. Ball, 1993; Carpenter, Fennema, Peterson, Chiang, & Loef 1989; Fennema, Carpenter, Franke, M. L., Levi, Jacobs, V. B., Empson, S. B., 1996). Moreover, the nature of the study is not restricted to specific content at the secondary level. It could be the case that content is a factor in pre-service teachers' ability to attend to student thinking, but the study will observe different pre-service teachers who are teaching different content.

1.7 LIMITATIONS OF THE STUDY

The study being proposed has several limitations. First, the pre-service teachers are enrolled in a teacher education program which specifically uses the Five Practices as a framework to help focus their attention on student thinking as students work on high-level tasks. As a result, the pre-service teachers in the study are pre-disposed to particular elements of planning and instruction on which the study will focus the analysis. Thus, the results may not be generalizable to other teacher education programs. Furthermore, the study is analyzing lessons that were planned using a detailed electronic planning tool. Completion of these plans is a time-consuming process and it is not meant for everyday use in lesson planning, but this is not to say that student thinking is only attended to in detailed plans. More so, the results linked to such planning may not be generalizable in the relationship between the "normal" routines of daily planning and the subsequent instruction.

Additionally, the study is only offering a "snapshot" of the pre-service teachers' student teaching experience. The study is not examining whether pre-service teachers' attention to student thinking changes over time. For example, it is possible that their attention to student thinking during planning or level of task implementation may improve from the beginning of their full-year internship through the end based on their classroom experiences. Data will be collected during the middle to end of the field experience; therefore, this study will not offer any insight as to whether or not such change occurs or what may be responsible for it.

Another limitation is that this study will only be examining samples of student work as a proxy for classroom instruction (Boston, 2012). Without evidence from classroom observations, it is possible that the teacher lowered the cognitive demands through the level of support provided, and the student work would not indicate such actions. However, the diversity of student responses will provide some indication regarding the amount of scaffolding. Overall, student work can provide a picture of the quality of instruction, but consideration to each aspect must be given so that particular aspects of practice are not misrepresented (Boston, 2012).

1.8 ORGANIZATION OF THE DOCUMENT

The study proposed in this paper is anchored in three critical elements related to lesson planning, implementation of high-level tasks, and pre-service teachers. In the next chapter, literature pertaining to these three elements and how they are related will be reviewed. Chapter three will present the methodology of the study being proposed. Chapter four will report the results of the data analysis, and chapter five will summarize the results and present the implications of the findings.

2.0 LITERATURE REVIEW

"Good advance planning is the key to effective teaching" (Smith & Stein, 2011, p. 76). When a teacher develops "thoughtful and thorough lesson plans", she shifts her attention from her own actions to the thoughts and actions of the students in the classroom (Smith & Stein, 2011). Specifically, the teacher's attention shifts to the students' mathematical thinking, how they make sense of the content, and the advancement of their mathematical understanding. To help teachers plan in such a way, researchers suggest developing lessons using a framework that focuses on student thinking around cognitively demanding mathematical tasks and focuses on the important mathematical ideas in the lesson (Smith et al., 2008). By planning with such a framework, the teacher is prepared for much of what actually will happen during instruction (Smith & Stein, 2011).

Research suggests that pre-service teachers' lesson planning tends not to focus on student thinking. In turn, the instruction linked to such lesson planning also shows evidence of struggles to respond to students during instruction. The existing research regarding the link between pre-service teachers' planning and instruction raises an overarching question: *What is the nature of the relationship between pre-service teachers' planning and instruction when pre-service teachers engage in thoughtful and thorough lesson planning focused on student thinking?* It is possible that attention to student thinking in lesson planning would positively relate to instruction, but it is also possible that other challenges faced by pre-service teachers during live instruction “out-weigh” their planning. Currently, there is no particular study that addresses the above question; however, a review of the literature related to topics within the question informs the design of a new study. The overarching question has the following broad critical elements: lesson planning, implementation of high-level tasks, and pre-service teachers. In this chapter, literature pertaining to specific aspects of these three elements and how they are related will be reviewed.

In the first section, the broader body of research pertaining to lesson planning and its’ link to instruction will be reviewed, and a discussion of the Five Practices for Orchestrating Mathematics Discussions will be presented (Smith et al., 2008; Smith & Stein, 2011). The second section will focus on instruction. In particular, the section will discuss the research pertaining to the implementation of cognitively demanding mathematical tasks, which have been identified as a critical element of instructional practice (Hiebert et al., 1997; Carpenter & Lehrer, 1999). This section will also discuss the literature regarding lesson planning directly related to the implementation of cognitively demanding tasks, and the role the Five Practices play in such planning. The third section discusses the instructional practices of pre-service teachers with a particular focus on what influences their teaching. The chapter concludes with a summary of the findings which lead to the proposal of the study in this paper.

2.1 LESSON PLANNING AND INSTRUCTION

Since the mid 1900's lesson planning has been addressed in the education literature. Tyler (1949) described lesson planning as four processes: (1) identifying the right objectives, (2) selecting learning activities to meet those objectives, (3) meaningfully sequencing the activities, and (4) deciding how to appropriately evaluate the activities. An early review of planning literature concluded that the primary efforts of teachers' planning were focused on "structuring, organizing for, and managing limited classroom instructional time" (Clark & Peterson, 1986, p. 260). Such planning does not focus on what the students are actually thinking nor does it focus on interactions between the teacher and students. Planning in such a fashion often does not prepare teachers for the complexities of teaching. Research shows that difficulties can arise during instruction when students respond in unexpected ways (e.g., Borko & Livingston, 1989; Zimmerlan & Nelson, 2000).

An alternative method of planning focuses on student thinking and interactions between the teacher and students. Such planning involves teachers setting goals (e.g., Leinhardt, 1993; Schoenfeld et al., 2000), anticipating specific student responses (e.g., Zimmerlan & Nelson, 2000; Ball, 1993) and careful consideration of how to use those responses during instruction (e.g., Ball, 1993; Schoenfeld, 1998; Leinhardt, 1993). This type of planning is considered "thoughtful" and "thorough" as attention shifts from teacher actions to student thinking (Smith & Stein, 2011).

The next two sections discuss the literature pertaining to the two different "methods" of lesson planning just discussed. The first method is lesson planning that emphasizes procedures, structure and content. The second method is lesson planning that focuses on anticipating student thinking and interactions between the teacher and students.

2.1.1 Lesson Planning that Emphasizes Procedure, Structure, and Content

It wasn't until the 1970's that researchers began to investigate teacher planning in regards to individual lesson plans and their link to instruction. Clark & Yinger (1977) provide a literature review of a limited number of empirical studies done to that point pertaining to teacher planning. Of these five studies, one was more focused on curriculum planning than individual lesson planning (Taylor, 1970 as cited in Clark & Yinger, 1977), while others examined links between different aspects of planning and instruction (Zahorik, 1970; Peterson, Marx, & Clark, 1978; Morine, 1976; Yinger, 1977 as cited in Clark & Yinger, 1977).

About a decade later another literature review was produced providing a framework for the organization of teachers' thought processes. The proposed framework for teachers' thought processes consisted of three categories: "(a) teacher planning (preactive and postactive thoughts); (b) teachers interactive thoughts and decisions; and (c) teachers theories and beliefs" (Clark & Peterson, 1986, p. 257). Teacher planning was distinguished as its' own category because the researchers found a qualitative distinction across the literature between the type of thinking teachers do while planning compared to the type of thinking they do while actually teaching. However, they did not consistently find a distinction between the types of thinking teachers did before a lesson (pre-active thoughts) and the type thinking they did while reflecting on the lesson (postactive thoughts). For this reason, the researchers defined teacher planning to include "the thought processes that teachers engage in prior to classroom interaction but also includes the thought process or reflections that they engage in after classroom interaction that then guide their thinking and projections for future classroom interaction" (Clark & Peterson, 1986, p. 258).

Clark & Peterson's (1986) review concluded that the earlier studies on planning tended to focus on elementary teachers, and the primary efforts of teacher planning were focused on "structuring, organizing for, and managing limited classroom instructional time" (p. 260). Furthermore, eight different types of teacher planning were identified. These types included: weekly, daily, long range, short range, yearly, term planning, unit planning and lesson planning. The most commonly prescribed model used from 1970 to the date of the article was the linear model set forth by Tyler (1949). As mentioned earlier, this "model consists of a sequence of four steps: (a) specify objectives; (b) select learning activities; (c) organize learning activities; and (d) specify evaluation procedures" (Clark & Peterson, 1986, p. 44). While this was the most prescribed model, there was "reasonable agreement" in the literature that this model did not accurately describe the actual planning styles of experienced teachers. However, the value of the linear model may have been its use to train beginning teachers. The linear style model may have provided beginning teachers with a base format to develop plans compatible with their individual styles and classroom environments (Clark & Peterson, 1986).

Research has shown that when teachers engage in a linear model of lesson planning, they tend to make limited use of student thinking during instruction. For example, the first empirical study on classroom planning investigated the link between such structured linear lesson planning and instruction (Zahorik,1970 as cited in Clark & Peterson, 1986). Six of twelve teachers were provided a partial lesson plan with student objectives and an outline of future content to be covered. The remaining six teachers completed a task not related in any way to the lesson plan provided to the other teachers. These teachers were not aware they were going to be asked to teach. After one hour, all twelve teachers were asked to teach a lesson pertaining to credit cards. The actual lessons were recorded and analyzed in regard to how sensitive teachers were to students during the lesson. Clark & Yinger (1977) state that after examining the lessons "of the planners and non-planners, Zahorik noted that teachers who planned exhibited less honest or authentic use of the pupil's ideas during the lesson. He concluded from this that the typical planning model-goals, activities, and their organization and evaluation-result in insensitivity to pupils on the part of the teacher" (p. 281) In their review of the same study, Clark & Peterson (1986**)** point out that Zahorik did not investigate the extent to which the "planners" actually did plan for the lesson. A competing explanation is that since the "planners" were provided with instructional topics to be covered in the following two weeks, they were thus influenced to focus on the content. Meanwhile, the "non-planners" were "forced" to focus on students' thinking and experiences. In general, across the different studies that were reviewed, teachers demonstrated the ability to faithfully implement their lesson plans during instruction. Interestingly, even if a lesson was going poorly, teachers often still followed their plans (Clark & Yinger, 1977).

Research comparing the planning and teaching of expert and novice teachers found that the plans of novice teachers echo these results as they tended to primarily focus on teachers' actions as opposed to student thinking (e.g., Leinhardt, 1993; Schoenfeld et al., 2000; Zimmerlan & Nelson, 2000; Livingston & Borko, 1990). It was also found that when novice teachers engaged in such planning, they often experienced difficulty during instruction. For example, Livingston (1989) found that pre-service teachers primarily focused on developing strategies and explaining content to students during their lesson planning. It was found that while pre-service teachers' planned explanations and examples were mathematically sound, they experienced difficulty during interactive teaching when they had to provide explanations and examples on the spot (Borko & Livingston, 1989).

Another example of linear planning being linked to difficulties during instruction is provided by a case study involving a pre-service teacher (Zimmerlan & Nelson, 2000). The study investigated the lesson planning and subsequent instruction of Nelson (the pre-service teacher and one of the researchers) who was teaching the concept of subtracting exponents to an algebra class. The study found that Nelson's lesson planning included goals, specific problems and activities for students to work through, and the identification of problems with which he thought students would have difficulty. The lesson went smoothly when Nelson was able to follow his lesson plan; however, when students responded differently than he expected, he experienced trouble in providing appropriate explanations (Zimmerlan & Nelson, 2000).

In summary, teachers of various experience levels can implement plans that focus on content, structure and management of the lesson. However, difficulties can arise during instruction if students respond differently than expected. It seems that, focusing on the actions of the teacher, the planning for such lessons does not help the teacher prepare for unexpected student thinking. The next section discusses lesson planning that focuses on student thinking, and how such planning is linked to instruction.

2.1.2 Lesson Planning that Focuses on Student Thinking

While the lesson planning literature just discussed indicates an emphasis on procedures and classroom management for teachers of all experience levels, researchers did note a distinction between experienced teachers' planning practices and the planning practices of novice teachers.

Experienced teachers rarely identified lesson planning as an important part of the overall teaching process, yet it was “the one type of planning…addressed in all teacher education programs” (Clark & Peterson, 1986, p. 262). While experienced teachers did not consider planning to be an integral part of their practice, investigations indicated that experienced teachers’ written plans underestimated the amount of actual planning they performed mentally. Thus, one possible explanation for the distinction is simply what experienced teachers perceived to be planning (i.e. written plans) as opposed to how much they actually planned (Clark & Peterson, 1986).

Research comparing the planning and teaching of expert and novice teachers also supports this finding. Expert teachers’ written plans were often very limited; however they had “agendas” or “lesson images” that were not part of their written plans (Leinhardt, 1993; Schoenfeld, 1998). Through the use of pre-lesson interviews, teachers communicated to researchers their “agendas” or “lesson images” (i.e. plans) which were very rich and complex (Livingston & Borko, 1990; Leinhardt, 1993) as they indicated careful consideration of student thinking and responses. Expert teachers also indicated clear goals for the lesson along with specific plans on how to achieve those goals (e.g., Leinghardt, 1993; Schoenfeld et al., 2000). During instruction subsequent to their plans, expert teachers made use of student thinking while working towards reaching the goal(s) of the lesson (Leinhardt, 1993; Schoenfeld, Minstrell, & van Zee, 2000). Similar evidence relating planning and instruction is also present in the research on Japanese Lesson Study (e.g., Stigler, Fernandez, & Yoshida, 1996).

The remainder of this section will focus on the planning and instruction of expert teacher-researchers, as well as teachers who have participated in Japanese Lesson Study. The research related to expert teacher-researchers and Japanese Lesson Study indicates that when teachers plan with student thinking in mind, they are better equipped to respond to various forms of student thinking that arise during instruction and hence, to maintain the high-level of cognitive demands of task during implementation. Such findings during instruction are in contrast to the findings related to instruction associated with linear forms of lesson planning.

2.1.2.1 Planning and Instruction of Expert-Teacher Researchers

Expert teachers such as Magdalene Lampert, Deborah Ball, and Alan Schoenfeld discuss planning and instruction that focuses on student thinking in analyses of their own classroom practices and provide examples of such practice (e.g., Lampert, 2001; Ball, 1993; Schoenfeld, 1998). These researchers describe in detail how consideration of individual student's thinking in relation to the needs of the entire class motivates their planning and instructional decisions.

Ball (1993) for example goes into great detail regarding her decision on what task to present to students to teach negative numbers. In realizing that negative numbers had two important dimensional representations (*amount of the opposite of something* and *location relative to zero*), she wanted an example to capture them both. Upon considering various models (i.e. money (and debt), frog on a number line, game scoring), she decided to use a building model in which an elevator moves above ground and below ground. Ball (1993) explains that she chose the "building representation after weighing concerns for the essence of the content, coupled with what I knew to expect of 8-year olds' thinking…" (p. 380). She then explains the subsequent implementation of the task, and how student thinking with regard to the task influenced the decisions she made. As part of her planning she considered the use of other tasks such as money, so when students struggled to make sense of negative numbers with the elevator problem she had other alternatives to draw from.

Throughout the teaching episode, Ball (1993) carefully considers her students' responses to the task and she details her thoughts about how she should appropriately respond to student thinking with her original plan and goals in mind. She discusses specific questions she asked as well as a dilemma she faced in deciding whether to validate a students' incorrect thinking. Throughout the entire lesson, Ball explains how she balances the subject matter, goals of the lesson, and her students' thinking as she implements the lesson and orchestrates meaningful classroom discussion (Ball, 1993).

Magdalene Lampert is another teacher-researcher who exhibits planning and instructional practices similar to those of Deborah Ball. Lampert (2001) details her careful selection of a mathematical task and she anticipates several approaches students might use when solving the task. Both the selection of the task and anticipation of solutions are rooted in her knowledge of students' mathematical thinking. While the selection of the task and anticipation of approaches occur during planning, Lampert is thoughtfully attending to them so she is prepared to adjust her instruction accordingly. In addition to the selection of task and anticipating student responses, Lampert has also provided several analyses of her classroom discourse (e.g., Lampert, 1990; Lampert, 1992). These analyses show that Lampert's typical role in classroom discourse involves guiding the conversation by utilizing student responses to convey the mathematics she wants the students to learn as set forth in her "plan" for the lesson (e.g., Lampert, 1990; Lampert, 1992).

A description of Alan Schoenfeld's classroom practice (Schoenfeld, 1998) echoes that of Ball and Lampert as he anticipates solutions and crafts questions prior to the lesson. Schoenfeld's description adds to the planning and instruction conversation as he discusses how he selects and organizes students' solutions for whole class discussion. For example, in selecting student solutions to be shared with the class, sometimes the risk of confusing students may outweigh the potential gains of discussing an incorrect solution, and therefore the particular solution should be avoided.

Schoenfeld (1998) argues that even correct solutions should be avoided if they may lead students away from the mathematical goal of the lesson. Before the lesson begins, Schoenfeld (1998) has *planned* which solutions he wants presented to the class. For this reason, he will often insert his own solution strategies that are helpful in reaching the goal if such strategies were not provided by the class. After selecting the strategies he wants displayed, Schoenfeld (1998) purposefully sequences the strategies during public discussion to help achieve the goal of the lesson.

In general, all three teachers use their students' thinking in planning and instruction to determine particular moves and facilitate mathematical discussions around the goal of the lesson. Evident among Ball, Lampert, and Schoenfeld is how their plan for the lesson influences their instruction. While things may not always go exactly as expected, they still have an overarching plan which guides their decisions during implementation.

2.1.2.2 Japanese Lesson Study

Japanese Lesson Study is a form of professional development in which a team of teachers *collaboratively* plan, implement and reflect in detail on a single lesson focused on students' mathematical thinking related to the content. Lesson study in the United States has been receiving attention since Stigler and Hiebert's book *The Teaching Gap* was released in 1999 (Perry & Lewis, 2008). The book provided analysis of the Third International Math and Science Study (TIMMS) data in which they drew international comparisons of eighth grade mathematics classrooms from the U.S., Germany, and Japan (Stigler & Hiebert, 1999). They found that Japanese teachers possess and use knowledge of their students' thinking both during planning and instruction to help students gain a conceptual understanding of the mathematics being presented (Stigler & Hiebert, 1999). The aspects of planning engaged in by teachers participating in Japanese Lesson Study are similar to the aspects of planning attended to by the expert teachers just discussed. Also, research shows links between the planning and instruction of teachers who participate in Japanese Lesson Study.

With regard to lesson planning, participation in lesson study involves collaboratively attending to the following components: (1) selection and wording of the problem to begin the lesson, (2) materials needed by students to engage in the problem, (3) anticipation of student thinking, responses, and solutions, (4) questions the teacher will ask to assess/advance student thinking, (5) organization of the chalkboard, (6) time management, (7) handling of individual differences among students, and (8) closure of the lesson (Stigler & Hiebert, 1999). According to Lewis, Perry, and Hurd (2009), lesson study includes phases of investigation, planning, teaching, and reflection. During the investigation phase, the team of teachers must consider students' knowledge and student learning goals in relation to the particular content. The planning phase involves developing the lesson by anticipating student solutions to the selected task(s) and considering both the short term and long term goals of the lesson. The teacher must then teach the lesson while team members observe and collect data. Finally, the team reflects on the lesson as they share and discuss broad issues of redesign as well as specific issues including the understanding of particular students and the specific subject matter being taught (Lewis, Perry, & Hurd, 2009).

Even though lesson study is a collaborative practice engaged in by a team of teachers, there is much to learn from it due to the links to instruction associated with such planning. Within both mathematics and science education, research has linked lesson study to enhanced teachers' instruction and students' achievement (e.g., Lewis & Tsuchida, 1998; Yoshida, 1999; Kawanaka & Stigler, 1999). For example, in an empirical study contrasting Japanese and American teachers' lesson planning and instruction, researchers found that Japanese teachers focus primarily on student thinking in their written plans. During instruction based on those plans, the students of the Japanese teachers were given appropriate opportunities to think and learn mathematically (Stigler, Fernandez, & Yoshida, 1996). In particular, the Japanese teachers established an atmosphere in which they were able to elicit conceptual student thinking as well as validate it for the students. In contrast, American teachers' plans focused more on teachers' actions and subsequently less meaningful learning opportunities were made available to their students during instruction (Stigler, Fernandez, & Yoshida, 1996).

The focus on student thinking is a commonality among the planning of expert teachers and the planning that goes into Japanese Lesson Study. Empirical evidence indicates that teachers who focus on student thinking during planning are better prepared to deal with the complexities of student thinking during instruction. Thus, focusing on student thinking is a critical component of meaningful lesson planning. In the next section, this paper discusses a particular set of instructional practices that emphasize the importance of planning.

2.1.3 The Five Practices of Orchestrating Mathematics Discussions

Drawing upon a wide array of literature including that of expert teachers and Japanese Lesson Study, researchers have synthesized common planning and instructional practices of skillful teachers and integrated them into a “single package” (Stein, Engle, Smith, & Hughes, 2008). The *Five Practices for Orchestrating Productive Mathematics Discussions* provide a “road map” to effectively plan for and implement a whole-class discussion that utilizes children’s mathematical thinking (Smith & Stein, 2011).

> We think of the five practices as skillful improvisation. The practices that we have identified are meant to make student-centered instruction more manageable by moderating the degree of improvisation required by the teacher during a discussion. Instead of focusing on in-the-moment responses to student contributions, the practices emphasize the importance of planning. Through planning, teachers can anticipate likely student contributions, prepare responses that they might make to them, and make decisions about how to structure students’ presentations to further their mathematical agenda of the lesson. (p. 7)

The Five Practices are not intended to represent every practice teachers employ but rather ones that are commonly used and can be explicitly attended to in planning for a discussion. While the intention of the Five Practices is to help with mathematical discussions that focus on student thinking, engagement with the Five Practices helps teachers attend to student thinking throughout the three phases of a standards-based lesson (launch, explore, and summarize) using a high-level task. Whole class discussion takes place during the summarize phase; however successful implementation of the discussion requires attention to student thinking during practices that occur in the launch and explore phases as well (Stein & Smith, 2011).

The Five Practices are: (1) anticipating students' mathematical responses, (2) monitoring students' responses, (3) purposefully selecting student responses for public display, (4) purposefully sequencing student responses, and (5) connecting student responses (Stein & Smith, 2011). Prior to the Five Practices are two prerequisite practices: (0i) setting goals for instruction, (0ii) selecting an appropriate task. The following sections provide a complete description of each practice. The discussion of each practice will also include an example of a teacher's planning and/or instruction taken from the literature to illustrate what it means to attend to student thinking for the given practice.

2.1.3.1 Setting Goals for Instruction

According to Smith & Stein (2011), setting goals for instruction is "a critical starting point for planning and teaching a lesson" (p. 13). The mathematical goal for a lesson should include more than stating what students will do during a given lesson. An appropriate goal "clearly identifies what students are to know and understand about mathematics as a result of their engagement in a particular lesson" (Smith & Stein, 2011, p. 13). While setting a clear goal can be difficult, it should guide their decisions during later practices. For example, the goal should determine which solutions the teacher selects for class discussion and which questions she should ask about particular student solutions. Thus, the goal(s) of a lesson can lay the foundation for how teachers are able to attend to student thinking throughout the lesson.

Schoenfeld et al. (2000) provide an example of a teacher setting specific goals focused on students' mathematical thinking. One of the authors, Minstrell, who was also an experienced teacher, planned and implemented a physics lesson which was analyzed by himself and the other authors. In addition to discussing broader instructional goals which focused on students' mathematical thinking, Minstrell had content-specific goals for the lesson. The context of the lesson was measurements of Blood Alcohol Content. The following is one goal he identified:

- having the students (re-engage) with the issue of "which numbers count" – enumerating and elaborating the reasons one might or might not include specific data when calculating the "best number"

The goal states what students will do ((re-engage) with the use of "which numbers count" and what it means for students to do it (enumerate and elaborate reasons…). This particular lesson was one that Minstrell was "very familiar" with as he had taught it on several previous occasions. The other content goals he set for the lesson were also very specific. He identified specific things the students would do and say that would indicate whether or not a particular goal was being reached. Additionally, evidence from his teaching indicates that he had specific expectations (in relation to his goals) for both himself and the students which guided his actions throughout the implementation of the lesson. The evidence suggests that his goals served as a foundation for his engagement with students and their thinking (Schoenfeld et al., 2000). Such an example is evidence of how attending to a mathematical goal in planning is linked to move th teacher makes during instruction.

2.1.3.2 Selecting an Appropriate Task

As discussed in chapter one, mathematical tasks vary in the level of cognitive demand they place on students. Different opportunities to learn are provided to students based on the level of the task being implemented. Also, for successful implementation of the Five Practices, the task needs to be aligned with the mathematical goal of the lesson (Smith & Stein, 2011). Productive discussions become a real possibility when the teacher uses a high-level task. For example, the existence of multiple solution paths provides opportunities for differences in student thinking to emerge. The teacher can then use student responses to help the class as a whole think and reason about the mathematical ideas to be learned.

As previously discussed, in an analysis of her own teaching, Deborah Ball (1993) provides an example of attending to student thinking through the selection of an appropriate task. Ball (1993) writes about her decision on what task to present to students to teach negative numbers. In realizing that negative numbers had two important dimensional representations (amount of the opposite of something and location relative to zero), she wanted an example that captured them both. Furthermore, comparing magnitudes becomes difficult for students when dealing with negative numbers. "Simultaneously understanding that -5 is, in one sense, more than -1 and, in another sense, less than -1, is at the heart of understanding negative numbers" (Ball, 1993, p. 379). Ball (1993) ultimately decided on an elevator model and explains that she chose "the building representation after weighing concerns for the essence of content, coupled with what I knew to expect of 8-year olds' thinking…" (p. 380). She explains how elementary students tend to conceive negative numbers to be the same as zero. For example, "owing someone five dollars-i.e., -5 – seems the same as having no money" (Ball, 1993, p. 380). Clearly, the selection of an appropriate task in this case was based on Ball's subject matter knowledge of negative numbers and her knowledge of how students understand negative numbers. The task was specifically chosen to model negative numbers in a manner that would challenge the often occurring idea that negative numbers are equivalent to zero. Such an example illustrates how selecting an appropriate task based on student thinking is a critical part of the lesson planning process.

2.1.3.3 Anticipating Students' Responses

The previous example illustrates that anticipation of student thinking can be part of the task selection process. The practice of anticipating solutions to the task (once the task has been selected) involves considering all possible strategies students are likely to employ, how the teacher will respond to the strategies, and identifying the strategies that will help achieve the mathematical goal of the lesson (Stein & Smith, 2011). Furthermore, teachers should attempt to think about approaching the task in the way(s) of specific students with different levels of knowledge. Anticipation in light of student thinking prepares the teacher in advance to deal with various approaches and possible misconceptions (Stein et al., 2008). A practical benefit of anticipating student responses is that it frees up space in the mind of the teacher to have to think less spontaneously during instruction. When a teacher has anticipated a variety of solution paths and possible misconceptions, he/she has to do less thinking in the moment. If a student raises an idea the teacher has already thought about, the teacher is more mentally ready to deal with it. In examining and investigating her own practice, Lampert (2001) provides a detailed example of anticipating how students might approach a particular task. A problem similar to the one Lampert discusses is shown in Figure 1.

a.) _____ groups of 6 = 10 groups of 3
b.) 20 groups of 2 = _______ groups of 4
c.) _____ groups of 8 = ______ groups of 24

Figure 1 Modified Problem of the Day (Lampert, 2001)

Lampert discusses how different approaches have the potential to engage students in different mathematics for the specific problem presented in her book. The following solutions are similar to those discussed by Lampert but apply specifically to the numbers used in Figure 1. With regard to part a, one possibility is that the students attempt to first figure out the total for 10 groups of 3. Some students might realize that taking 10 times 3 to get 30 can do this. Another way is for a student to add three to itself 10 times to get the total. Yet another way is for students to start with ten 3's and pair them. This possible approach is shown in Figure 2.

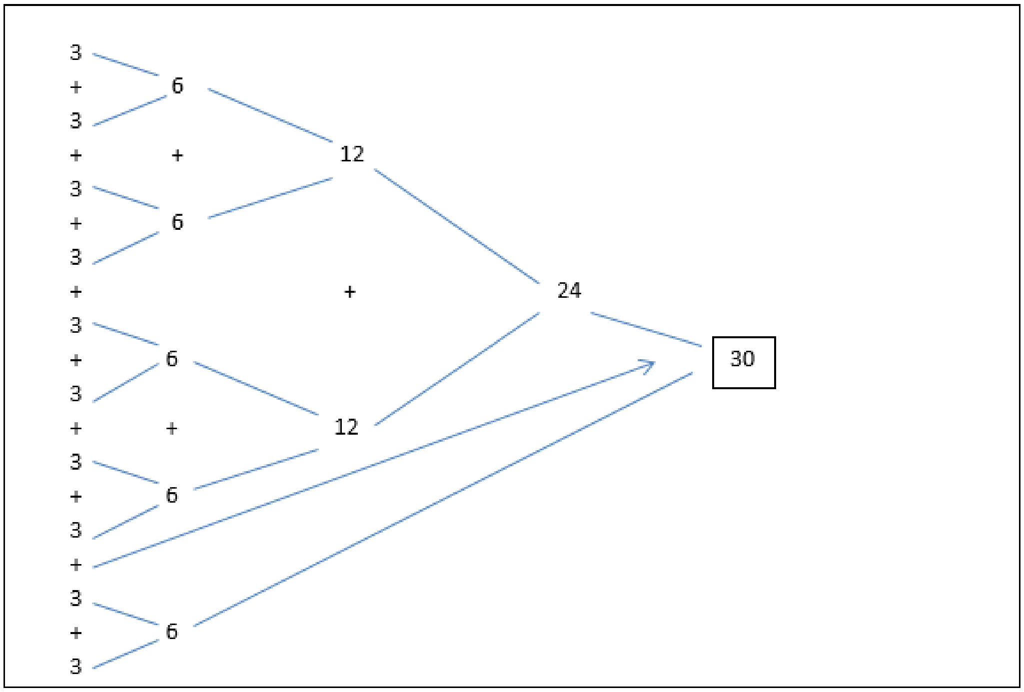


Figure 2 Modified Possible Approach to Problem of the Day, part a (Lampert, 2001)

In this particular approach the groups are being merged by pairing them starting with the threes, then pairing the larger groups, ultimately leading to thirty. The student could then look back and see that at one point 30 was equal to 5 sixes. Lampert then discusses how students who are not familiar with addition and multiplication relationships may approach it even differently.

An alternate approach she considers is students beginning by drawing 10 groups of 3 objects. This would also show that the right side of the equation is worth 30, and it would give a pictorial representation of the problem. An example of this possible approach is shown in Figure 3.

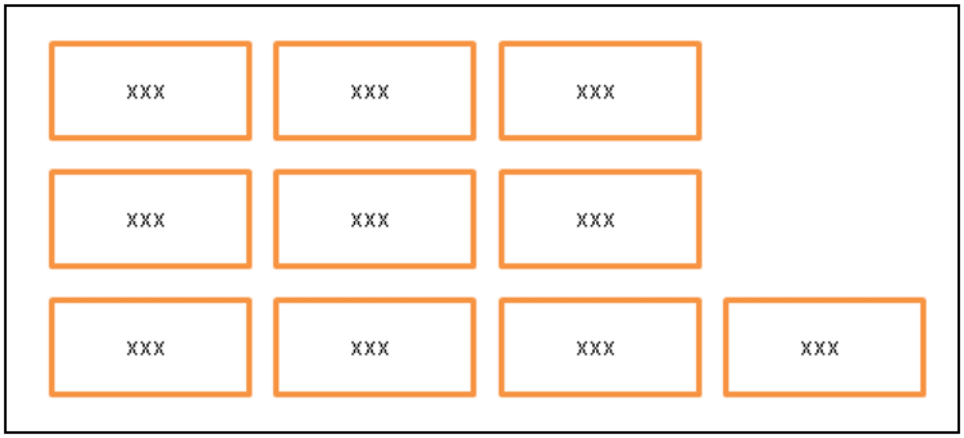


Figure 3 Modified Alternative Approach to Grouping Problem (Lampert, 2001)

The ability to anticipate student responses often is a result of a teacher's experience. An example from another study illustrates how a teacher uses an incorrect solution to highlight a common misconception for a particular task (Nathan & Knuth, 2003). The "painting problem" reads "Suppose Mrs. Jones, an experienced painter, can paint a wall in 3 hours, while rookie painter Mr. King paints the same wall in 7 hours. How long will it take them if they work together? Choose the most sensible answer from those given below. Explain your decision." (Nathan & Knuth, 2003, p. 187). The article also provides answer choices a thru h following the statement of the problem.

The teacher in the study acknowledged that he knew that many students approach the problem by doing "1/3 + 1/7 = 10/21", and they misinterpret the fraction 10/21 as time worked. When approximately six students made this same error, the teacher called one student who performed it to present the solution at the board to highlight a common conceptual error in how students interpret the fractions (Nathan & Knuth, 2003). While there is no evidence that the teacher anticipated the solution in writing within his lesson plan, there is an indication that he was mentally prepared for such a solution. As mentioned earlier, many experienced teachers do not write their plans, but have complex lesson images which include anticipated student responses (Schoenfeld, 1998).

2.1.3.4 Monitoring Students' Thinking

Through thoughtful anticipation of student responses during planning, teachers are better equipped to monitor student thinking while students are working on the problem's solution (Lampert, 2001 and Schoenfeld, 1998 as cited in Stein, Engle, Smith, & Hughes, 2008). Monitoring student responses involves the teacher listening to students' mathematical thinking and mentally noting, as well as physically recording different student approaches and strategies (Stein et al., 2008). During this time, teachers have opportunities to ask questions to probe student thinking. Furthermore, teachers ask questions that assess student thinking in relation to the mathematical goal of the lesson (Stein et al., 2008).

Hill and colleagues (2008) present case-studies in which they analyzed pre-service teachers' mathematical knowledge for teaching and the quality of instruction they were able to deliver. In describing the case of Noelle, the study provides a detailed example of a teacher monitoring student thinking. The students in Noelle's class are working on a lesson in which they have to generate different spatial arrangements for different numbers of cubes. After setting up the task, students were given time to explore different attributes of a cube. During this time "Noelle listens carefully and respectfully to students' ideas, asks for more clarifications, and lists the nominated attributes on the board" (Hill et al., 2008, p.470).

The following is one short episode of Noelle engaging with a student during this activity:

Shaun: It has four like edges, I mean eight.

Noelle: Okay, would you explain what the edges are? What you're calling "edges"?

Shaun: Like the sides or the point of a cube.

Noelle: Okay, so you're talking about a point or the actual side where the two parts come together?

Shaun: Like the point.

Noelle: The point. Okay. [Turning to the rest of the class: Would you agree that there are eight points? Look at your cube. He's talking about the points on the cube.] [She touches the vertices of the cube she holds in her hands.] Would you agree that there are eight points on there? (Hill et al., 2008, p. 471)

In the above episode, Noelle listens to what Shaun says, and she probes his thinking with further questions instead of assuming what he is thinking or correcting him. At the end of the episode, Noelle asks the entire class to verify the number of "points" on a cube. After receiving verification, she then responds appropriately to the whole class (not shown in the episode) by introducing the correct mathematical terms (vertex and vertices) for "point" and "points" respectively. This introduction of correct terms was only after Shaun brought forth the term "point".

During the episode, Noelle illustrates characteristics of other teachers who successfully monitor student thinking. She listens to her students' thinking, utilized the chalkboard to keep track of their thinking, asked appropriate questions to informally assess students, and responded with correct mathematical information at an appropriate time (Hill et al., 2008). If a teacher is able to attend to monitoring student thinking during planning (i.e. writing questions to assess students), the she may be more likely to successfully monitor students during instruction.

2.1.3.5 Purposefully Selecting and Sequencing Student Responses

Once teachers have monitored student thinking, they must decide how to facilitate the whole-class discussion most effectively. To make these decisions, the teacher must determine which student solutions will bring forth the key mathematical ideas. The teacher must also determine how to order the presentation of solutions so that ultimately the key ideas can be connected.

According to Smith & Stein (2011), "Selecting is the process of determining which ideas *(what)* and students *(who)* the teacher will focus on during the discussion" (p. 43). Selecting is an important practice because the teacher has control over the topics that will be discussed. Through selecting the teacher can ensure that the mathematics being discussed is in some way related to the mathematical goal of the lesson (Smith & Stein, 2011)

Also the authors define sequencing as "the process of determining the order in which the students will present their solutions" (Smith & Stein, 2011, p. 44). In order to sequence solutions effectively, a teacher must have knowledge of how various strategies are connected. The teacher must decide the best way to order them to build to the ultimate goal of the lesson. A common approach in sequencing is to start with the most frequently occurring strategy, but this is not necessarily always the case. For example, it may be beneficial to start with addressing a misconception (Smith & Stein, 2011).

The literature provides evidence of teachers selecting and sequencing responses (Nathan & Knuth, 2003; Fennema et al., 1993; Schoenfeld, 1998); however, many articles do not explicitly describe the selecting and sequencing process for an entire lesson. For this reason, the detailed example is taken directly from Smith & Stein (2011). In the case of Nick Bannister (Smith & Stein, 2011), Nick uses the Calling Plans task. The task has multiple solution methods, several of which Nick anticipated before beginning to teach the lesson. After setting up the task and monitoring student thinking, Nick considered the goals(s) of his lesson and carefully selected and sequenced the responses students generated. The problem reads "Long distance Company A charges a base rate of $5.00 per month plus 4 cents a minute that you're on the phone. Long distance Company B charges a base rate of only $2.00 per month, but they charge you 10 cents per minute used. How much per month would you have to talk on the phone before subscribing to Company A would save you money?" (Achieve, 2002, p. 149 as cited in Smith & Stein, 2011, p. 32).

As Nick had anticipated, groups solved the problem using tables, graphing or with an equation. The most commonly used strategy was using a table, so that is the strategy that Nick started with. While there were different table approaches available, Nick decided to use the representation of 10-minute intervals (instead of 20-minute intervals) because it showed the point of intersection. Nick hoped by showing this first it "would launch a discussion about what we do or do not know about the functions from the table and what else we might need to do to answer the question" (Smith & Stein, 2011, p. 48).

Following the presentation of the table approach, Nick selected a particular group to present their graphical approach. Rather than having the group explain their approach, Nick asked the class to attempt to explain what knowledge the group possessed that enabled them to create the graph. This particular sequencing provides an opportunity for students to connect the table approach with the graphical approach.

Finally, Nick had the group who produced the equation present last. Only one group had used an equation to solve the problem. Nick's overall sequencing went from most commonly used strategy (table approach) to the least commonly used strategy (equation); however, there was more to his decision-making than simply frequency. The sequencing progressed from more concrete approaches of a table and graphing to a more abstract approach of using an equation. Furthermore, the key features of the equation (slope and y-intercept) could be examined and analyzed in the contexts of the table and graph (Smith & Stein, 2011).

Selecting and sequencing can be more meaningful and less difficult if a teacher attends to the practices in lesson planning. The teacher can give himself more time to make meaningful decisions about what solutions and in which order they should be presented so that students have the best opportunity of reaching the mathematical goal of the lesson. For example, a specific prompt in a detailed electronic planning tool discussed later asks the following questions: *How will you orchestrate a class discussion about the task so that you can accomplish your learning goals? Specifically, what student responses do you plan to share during the discussion and in what order will they be discussed?*

2.1.3.6 Connecting Students' Responses

All of the previous practices lead up to the connecting of student responses during the summarize phase of the lesson. When connecting student responses, "the goal is to have student presentations build on each other to develop powerful mathematical ideas" (Stein et al., 2008, p. 330). Through appropriate selecting and sequencing of student responses teachers need to help the students draw mathematical connections.

Smith & Stein (2011) point out that this may be the most difficult of the Five Practices to implement as "it calls on the teacher to craft questions that will make the mathematics visible and understandable" (p. 49). During this practice questions must go beyond probing and focus on mathematical relationships and connections between ideas and representations. Furthermore, teachers need to ask questions that give consideration to both what the students know about the mathematics being taught and the mathematical goal of the lesson (Smith & Stein, 2011). Thus, connecting does not just refer to making links between mathematical relationships, but also making links between the mathematics of the lesson and student thinking.

An example of the practice of connecting can be found in an article in which Magdalene Lampert investigates her own teaching of a fifth-grade class (Lampert, 1992). In particular, Lampert analyzes the discourse she and the class engage in. In the beginning of the article, Lampert states that her "goal as a teacher is to have my students learn to do authentic mathematics" (p. 295). Lampert asked the class to analyze the relationship between the x and y values presented in a chart. The article analyzes the discussion surrounding students' engagement with the task. This particular example is a case of a teacher using a range of strategies and question types to help students make meaningful mathematical connections during whole class discussion. Throughout the episode, Lampert guides the discussion using multiple strategies such as revoicing (Forman & Ansell, 2002), making clear the social norms (Yackel & Cobb, 1996), and validating student contributions (Ball, 1993), as well as accessing and making use of student thinking through the act of questioning.

Engagement in the Five Practices is intended to help teachers plan for and implement a discussion around high-level tasks. Research suggests that implementing high-level tasks is difficult for many teachers. The next section discusses the research related to the implementation of high level tasks, and it describes the role the Five Practices play in planning lessons around them. The section also discusses research related to such lesson planning.

2.2 TASKS AND PLANNING LESSONS AROUND THEM

2.2.1 Academic Tasks and Mathematical Tasks

Different curricula across academic disciplines can be viewed as collections of academic tasks within a particular subject area (e.g., Doyle, 1979b, 1980b as cited in Doyle, 1983). From this perspective, the term task does not necessarily refer to a particular problem or set of problems students are working on, but rather are defined by the types of solutions students are expected to produce and the strategies they use to reach those solutions (Doyle, 1983). According to Doyle (1983), a "'task' focuses attention on three aspects of students' work: (a) the products students are to formulate…(b) the operations that are used to generate the product, such as memorizing a list of words or classifying examples of a concept; and (c) the "given" or resources available to students while they are generating a product…" (p. 161).

The nature of tasks and how students are expected to engage with the task influence students' opportunity to learn (Doyle, 1983). Academic tasks are differentiated in terms of the cognitive demands students are required to utilize to work on the task. The general types of academic tasks include: (1) memory tasks, (2) procedural or routine tasks, (3) comprehension or understanding tasks, and (4) opinion tasks (Doyle, 1983).

Memory tasks require students to reproduce information verbatim and thus often require limited cognitive demand. Procedural or routine tasks require a student to apply a previously learned algorithm or formula to a problem presented in a standard form the student is already accustomed to. Procedural tasks require more cognitive demand than memory tasks, but still can be completed with lack of understanding of the how the procedure is connected to mathematical concepts. Comprehension or understanding tasks may require students to apply prior knowledge to new situations, decide on the appropriate algorithm or formula to use in a given situation, or draw inferences and make predictions. Thus, understanding tasks require the highest level of cognitive demand if students are to appropriately engage with them. Opinion tasks require students to express what they think about a particular topic (Doyle, 1983).Opinion tasks do not necessarily require any level of cognitive demand since students are just expressing their preference (i.e. their favorite topic in the chapter).

Building on the work of academic tasks, researchers extended the concept of different task types to apply specifically to mathematics. According to Stein, Grover & Henningsen (1996), a mathematical task "is defined as a classroom activity, the purpose of which is to focus students' attention on a particular mathematical idea" (p. 460). Mathematical tasks (like academic tasks) vary in nature based on the levels of cognitive demand required of students when engaging with them. The Task Analysis Guide, shown in Figure 4 details the varying levels of cognitive demand inherent in the different types of mathematical tasks that a teacher can set up and implement (Smith & Stein, 1998).

Lower-Level Demands	*Higher-Level Demands*
Memorization Tasks • Involves either producing previously learned facts, formulae, or definitions OR committing facts, rules, formulae, or definitions to memory. • Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure. • Are not ambiguous – such tasks involve exact reproduction of previously seen material and what is to be reproduced is clearly and directly stated. • Have no connection to the concepts or meaning that underlie the facts, rules, formulae or definitions being learned or reproduced.	**Procedures with Connections Tasks** • Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas. • Suggest pathways to follow (explicitly or implicitly) that are broad, general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts • Usually are represented in multiple ways (e.g., visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations helps to develop meaning. • Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.
Procedures without Connections Tasks • Are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task. • Require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it. • Have no connection to the concepts or meaning that underlie the procedure being used. • Are focused on producing correct answers rather than developing mathematical understanding. • Require no explanations, or explanations that focus solely on describing the procedure that was used.	**Doing Mathematics Tasks** • Requires complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example). • Requires students to explore and to understand the nature of mathematical concepts, processes, or relationships. • Demands self-monitoring or self-regulation of one's own cognitive processes. • Requires students to access relevant knowledge and experiences and make appropriate use of them in working through the task. • Requires students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions. • Requires considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process required.

Figure 4 The Task Analysis Guide (Smith & Stein, 1998)

The cognitive demands required for low-level task types (memorization tasks and procedures without connections) are similar to those required for the memory tasks and procedural or routine tasks described in Doyle's work. The high-level task types (procedures with connections and doing mathematics) are similar to the comprehension or understanding tasks from Doyle's work.

2.2.1.1 The Implementation of Tasks in Mathematics Classrooms

In addition to similarities in task classifications between mathematical tasks and academic tasks, there are also some similarities when the tasks were implemented in classrooms. That is, teachers tend to struggle when trying to implement tasks with higher-level cognitive demands. Doyle (1988) examined two junior high mathematics classes with respect to the nature of tasks presented to students and their management within the classrooms. Consideration was taken to select teachers who demonstrated good classroom management and the use of higher-order tasks in their instruction. Through observational records and various classroom artifacts several findings regarding the use of classroom tasks and achievement were identified.

On a broad level, classroom work could be separated into two categories: familiar and novel (Doyle, 1988). Familiar work involves little ambiguity and little risk as students are accustomed to it and it requires little cognitive demand. Novel tasks in mathematics require the synthesis of multiple sources of information which may involve decision making or the combination of different algorithms. Novel tasks contain high degrees of both ambiguity and risk. The broad categories of familiar and novel work were found to be related to several other characteristics of the classrooms which included work flow, work production, and accountability.

In relation to work flow, Doyle (1988) found that during familiar work, classroom activity moved smoothly and classroom management was not an issue. Novel work, on the other hand, "stretched the limits of classroom management" (Doyle, 1988, p. 174). During lessons involving novel tasks, the work rate was slow and student errors and un-finished tasks occurred frequently. As a result some teachers avoided the struggles by not assigning novel work. Teachers who did assign novel work often provided explicit support reducing the cognitive demands of the task, lowered accountability, or continued assigning familiar tasks (Doyle, 1988).

In an attempt to analyze instruction specifically around mathematical tasks, Stein et al., (1996) conducted a study within the context of the QUASAR Project. The QUASAR Project was a national reform project "based on the premise that prior failures of poor and minority students were due to lack of opportunity to participate in meaningful and challenging learning experiences, rather than to a lack of ability or potential" (Stein, et al., 1996, p. 458). The study involved teachers across four middle schools who were working in collaboration with local universities to enhance classroom instruction.

The study focused on the task-set-up and task implementation phases within the Mathematical Tasks Framework (MTF), and used the Task Analysis Guide to assess the cognitive demands of the tasks during each phase of the MTF. A modified version of The Mathematical Tasks Framework (MTF) is shown in Figure 5.

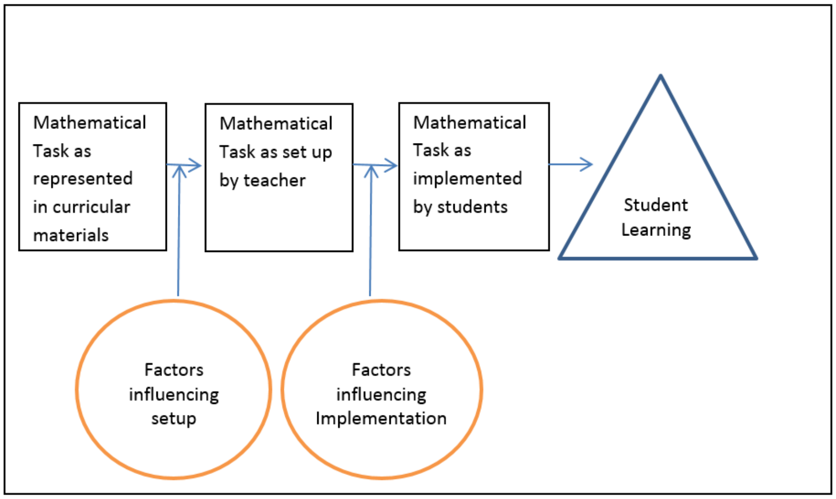


Figure 5 Modified Version of Mathematical Task Framework (Stein, Grover, & Henningsen, 1996)

The phases of classroom practice (represented by the rectangular boxes) include: the mathematical task exactly as it is presented in the curricular materials, the mathematical task as set up by the teacher before students begin working on it, and the way the task is implemented by the students during class. In this model, the type of work students engage in is in direct relation with student learning (represented by the triangular diagram) at the far right of the framework (Stein, et al., 1996).

The circular portions of the framework represent factors that can potentially influence the nature of the task from one phase to another. The teacher, for example, may alter the task as it was represented in the curricular materials in order to meet certain learning goals. The teacher's subject knowledge or knowledge of her students may influence the teacher to set up the task in a manner different from how it was originally presented. During the implementation phase, the task may change from how it was set up due to previously established classroom norms, the nature of the task, or teacher or student dispositions (Stein et al., 1996). The researchers utilized narrative summaries of classroom observations to focus on the main instructional tasks of each lesson they observed (Stein et al., 1996). The task that occupied the largest amount time during each lesson was the focus of analysis. In total there were 144 lessons observed which means there were 144 tasks analyzed during the set-up phase and implementation phase.

With regard to level of cognitive demand, 58 (40%) were set-up as Doing Mathematics, 49 (34%) were Procedures with Connections, 26 (18%) were Procedure without Connections, and 2 (1%) were Memorization. Non-mathematical and Other types of tasks comprised the remaining 7 (5%) tasks. During the implementation phase, tasks set up with lower level demands (i.e. memorization and procedures without connections), remained the same. However, tasks set up with higher level demands (i.e. procedures with connections and doing mathematics) tended to decline during implementation (Stein et al., 1996). It was very unlikely for a task to increase in cognitive demand from the time of set up to implementation, as only 2 of the 144 tasks analyzed were characterized in this way.

Specifically, 49 tasks were characterized as procedures with connections and 58 tasks were characterized as "Doing Mathematics" at time of set-up. During task implementation, 28 (57%) of the procedures with connections tasks declined either to procedures without connections, memorization, or no mathematical activity. Only 21 (43%) remained at procedures with connections during implementation. During task implementation, only 22 (38%) were maintained at the same cognitive level, and 8 (14%) tasks declined to procedures with connections. Interestingly, 10 (17%) of the tasks declined to no mathematical activity, and 15 (26%) were classified as other (Stein et al., 1996).

In summary, the cognitive demands of tasks tended to decline during implementation. The way in which the cognitive demands of tasks declined varied depending on the level of the task at the time of set up. More specifically, the tasks that were set-up at higher cognitive demand were more likely to decline during implementation.

Stein and colleagues identified classroom-based factors associated with both the decline and maintenance of cognitive demands from task set up to implementation. In total, 61 of the 144 tasks exhibited a decline in cognitive demands with several different factors being associated with the decline. Such factors include: challenges become non-problems, inappropriateness of task for students, focus shifts to correct answer, too much or too little time, lack of accountability, and classroom management. Within the analysis, it was possible for more than one factor to be associated with the decline of a given task (Stein et al., 1996).

The factor identified most frequently was challenges become non-problems. This is similar to Doyle’s finding of teachers tending to provide explicit support when students were working on novel tasks. Stein and colleagues often observed the teacher making instructional moves that reduced the cognitive demands of the problem. Either from being pressed by students or simply unable to watch students struggle, teachers often directed students to perform specific steps or explicitly told students how to do a given task. In such instances the teacher took away the students’ opportunity to meaningfully engage with a given task and lowered the cognitive demand by making it procedural in nature (Stein et al., 1996).

The second most prevalent factor associated with decline was inappropriateness of task for students. Tasks were classified as inappropriate based on motivational appeal, pre-requisite knowledge needed for successful engagement, and the potential for students to make progress on their own. The researchers noted that many students failed to engage at a deep cognitive level due to “lack of interest, motivation, or prior knowledge” (p. 480, Stein et al., 1996). Thus, for these reasons, such factors were considered inappropriate for a particular group of students.

Interestingly, classroom management problems were the least prevalent factor. This finding is somewhat in contrast with the findings of Doyle (1988) which suggested teachers' attempts to maintain control of the classroom were a predominant factor in lowering cognitive demands. The researchers acknowledge the common conception that skill practice keeps classrooms under control, but argue that engaging students in appropriate and meaningful tasks can also minimize disruptions.

The researchers further explored the maintenance and decline of Doing Mathematics tasks by identifying specific patterns (Henningsen & Stein, 1997). The patterns of decline included: (1) decline into using procedures without connections to concepts, meaning, and understanding, (2) decline into unsystematic exploration, and (3) decline into no mathematical activity. Respectively, the three most prevalent factors associated with tasks that declined into using procedures without connections to concepts, meaning, and understanding were challenges become non-problems, too much or too little time, and focus shifts to correct answer. In general, these factors are associated with a quick pace of instruction in which the teacher reduces the cognitive demands of the task during implementation by "taking over" and removing the challenging aspects of the problem. The same three factors were also most prevalent when tasks declined into unsystematic exploration. Although for this particular pattern, inappropriateness of the task was the most prevalent of the three. Thus, lending more support to the notion of a teacher choosing a task with motivational appeal, appropriate level of difficulty, and proper degree of explicitness. Unsystematic exploration refers to students attempting to explore the important mathematical ideas embedded within a task, but failing to sustain progress with different solution strategies or understanding (Henningsen & Stein, 1997). This particular pattern was created when researchers observed students trying to make meaningful attempts, but the nature of their engagement could not be characterized into one of the already existing patterns.

Tasks that declined into no mathematical activity were observed to be associated most frequently with inappropriateness of the task, classroom management problems, and too much or too little time (Henningsen & Stein, 1997). The researchers note that “classroom management problems appeared to play a large role when tasks declined into a complete lack of mathematical engagement on the part of the students” (Henningsen & Stein, 1997, p. 537). Such a finding indicates that teachers struggled to keep students focused and engaged with the task. Interestingly, for this particular pattern, too much time was the problem in regards to time management (Stein & Henningsen, 1997).

There were also several factors present with tasks whose cognitive demands were maintained from the time of set up to implementation. In total, 45 of the 144 tasks remained at a high level during implementation. Factors associated with the maintenance of high-level tasks included: tasks build on students’ prior knowledge, appropriate amount of time, high-level performance modeled, sustained pressure for explanation and meaning, scaffolding, student self-monitoring, and teacher draws conceptual connections.

The most prevalent factor, present in 82% of tasks remaining at a high level, was that the task builds on students’ prior knowledge. Thus, in a similar sense how tasks inappropriately designed around student knowledge are linked to decline, posing tasks of at the appropriate level is important to the maintenance of the cognitive demands during implementation. Appropriate amount of time and modeling of high-level performance were the next most frequent occurring factors as both were present in 71% of the tasks that remained at high-level during implementation. Thus, giving students the appropriate amount of time (not too much or too little) to sufficiently and thoughtfully explore a task was an important feature of instruction. Also, presentations by the teacher or other students in front of the whole class which illustrated multiple approaches, multiple representations, exploration and justification were an important factor (Stein et al., 1996).

Sustained press for justification "was evident through teacher questioning, comments, and feedback" (Stein et al., 1996, p. 481). Based on consistent instructional norms established by teachers, students realized there was an expectation to provide more than correct answers. Through the use of appropriate scaffolding, teachers were able to help students remain engaged with the task without reducing the cognitive demand. Such assistance was judged to be sufficient enough to keep the students on track so that they could continue meaningfully working on a given problem (Stein et al., 1996).

In total, there were 22 tasks that were set up as Doing Mathematics and remained as Doing Mathematics during implementation. Interestingly, there was a nearly uniform distribution of five factors associated with the maintenance of these high level tasks. The factors include: (1) task builds on students' prior knowledge, (2) scaffolding, (3) appropriate amount of time, (4) modeling of high-level performance, and (5) sustained press for explanation and meaning (Henningsen & Stein, 1997). Such a finding suggests that for tasks to be implemented at the level of Doing Mathematics a number of factors in conjunction with each other need to be present.

The overall findings of the literature related to mathematical tasks indicate that when certain factors are present during instruction, it is possible for tasks that are set up at a high-level to remain that way during implementation. However, the general tendency is for the cognitive demands of tasks to decline during implementation. When this happens different patterns of decline emerge and particular factors are associated with each of the patterns. Thus, there are instructional factors associated with both the maintenance and decline of high level tasks.

2.2.1.2 Supporting Teaching in Improving Ability to Implement Tasks

Teachers who have received professional development directly related to the selection and implementation of cognitively demanding tasks have shown evidence of outperforming teachers who have not received such support (Boston & Wolf, 2006; Boston & Smith, 2009). For example, teachers from a district that participated in professional development consistently provided high-level tasks, and student work provided evidence that the cognitive demands of the tasks were maintained during implementation. In contrast, teachers from a district that did not participate in the same professional development provided tasks with varying levels of cognitive demands, and student work showed limited engagement with the tasks (Boston & Wolf, 2006).

Another study examined teachers from a district that “had recently adopted a standards-based middle school mathematics curriculum and engaged teachers in a professional development initiative, both of which promoted the use of cognitively challenging instructional tasks and provided support for task implementation and for conducting whole class discussions” (Boston, 2012, p. 87). With regard to assignment collections, teachers demonstrated the ability to select cognitively demanding tasks, and student work indicated that high levels of cognitive demand were maintained during implementation (Boston, 2012).

In summary, studies have investigated how what teachers do during instruction impacts the maintenance or decline of cognitively demanding tasks (Stein et. al., 1996; Henningsen & Stein, 1997), and studies have investigated how professional development directly related to the selection and implementation cognitively demanding tasks impacts teachers’ practice (Boston &

Wolf, 2006; Boston & Smith, 2009; Boston, 2012). However, there is a limited number of studies that attempt to identify or explain the underlying supports that lead to the development of specific teacher instructional actions that influence the cognitive demands of tasks during implementation (Wilhelm, 2014).

The purpose of the study proposed within this paper is to investigate the ways in which attention to student thinking during lesson planning is an underlying support to the development of actions that influence the cognitive demands of tasks during implementation.

2.2.2 Planning Lessons Around High-Level Tasks

The research regarding the implementation of high-level mathematical tasks, indicates that the majority of teachers struggle to maintain their cognitive demands during instruction. As discussed earlier, a particular form of lesson planning has been identified as one way to help teachers successfully implement high-level tasks. According to Smith & Stein (2011), "good advance planning is the key to effective teaching" (p. 76). When a teacher develops "thoughtful and thorough lesson plans", she shifts her attention from her own actions to the thoughts and actions of the students in the classroom (Smith & Stein, 2011). Specifically, the teacher's attention shifts to the students' mathematical thinking, how they make sense of the content, and the advancement of their mathematical standing.

A case study involving two pre-service teachers (Paige and Keith) investigated how contextual factors related to the pre-service teachers' settings were linked to differences in planning and implementation of mathematical tasks (Mossgrove, 2006). One particular area of instructional practice was the examination of the level of cognitive demands of mathematical tasks within their lesson plans and during implementation. Additionally, the study examined different contextual settings associated with their different field placements in relation to their planning for and enactment of specific mathematics lessons.

With regard to the cognitive demands of mathematical tasks, it was found that Keith demonstrated more success than Paige in selecting tasks with a potential for a high-level of cognitive demand and maintaining that level during implementation. When examining contexts of Paige's and Keith's field placements, the analysis focuses primarily on differences found in curriculum and mentor (Mossgrove, 2006). The type of feedback Keith and Paige received from their mentors and the curriculums they used were identified as critical differences that may have contributed to their differences in planning for and enacting high level tasks. It is important to note that Mossgrove's study does not attempt to correlate Keith and Paige's level of planning to their instruction. Rather, the contextual influences of mentor and curriculum were linked to differences in the planning (i.e. selection of the task with high-potential) and enactment of the tasks (i.e. maintenance or decline of cognitive demands).

While Mossgrove (2006) did not formally investigate whether either pre-service teachers' planning had any link to their instruction, Smith & Stein (2011) analyze one of Paige's lesson plans and one of Keith's lesson plans in their book on the Five Practices. Smith & Stein do not attempt to describe any causal relationship between Paige or Keith's planning and any instructional practice, but rather use them as examples to illustrate different degrees of thoroughness and thoughtfulness along a lesson planning continuum. Paige's lesson plan is an example of "no planning" as it falls on the far left of the continuum. Keith's lesson plan is an example of moderate planning as it falls in the middle of the continuum. Smith & Stein (2011) then go on to explain how the use of a particular lesson planning framework can help teachers reach the far right of the continuum, and they ultimately provide a sample plan based on the framework as an illustration. The rationale behind such planning is that it leads to improved instruction and maintenance of high-level tasks (Smith et al., 2011). A modified lesson planning continuum is shown in Figure 6.

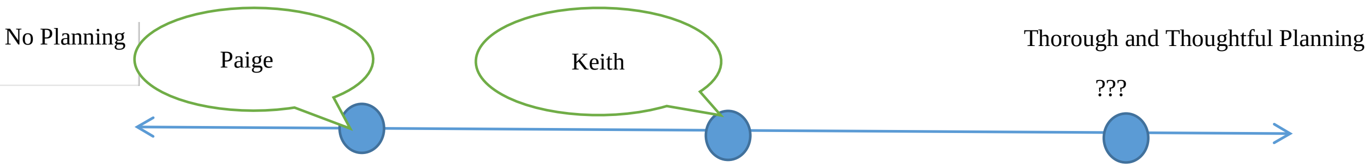


Figure 6 Modified Lesson Planning Continuum (Smith & Stein, 2011)

The next section begins with a discussion of a lesson planning framework known as the Thinking Through a Lesson Protocol (TTLP) (Smith, Bill, & Hughes, 2008), and its relationship to the Five Practices (Smith & Stein, 2011). Secondly, empirical evidence is presented indicating that the TTLP increases attention to student thinking during planning (Hughes, 2006). This is followed by a discussion of how the Five Practices have been used to measure attention to student thinking (Hughes, 2006). Lastly, the section will discuss the recent professional development work regarding the use of the electronic planning tool based on the TTLP (Smith et al., 2012; Smith et al., 2011; Smith et al., 2013).

2.2.2.1 Relationship Between TTLP and The Five Practices

The TTLP is a detailed planning process used prior to teaching that is intended to help teachers be more successful when implementing high-level tasks (Smith et al., 2008). In particular the TTLP "provides a framework for developing lessons that use students' mathematical thinking as the critical ingredient in developing their understanding of key disciplinary ideas" (Smith et al., 2008, p. 133). According to Smith & Stein (2011), the Five Practices are a "significant subset" of the TTLP. When teachers engage in the TTLP process they are actually attending to aspects of the Five Practices and the groundwork practices. Portions of the TTLP related to the Five Practices are shown in Figure 7. A full version of the TTLP is presented by Smith, Bill, & Hughes (2008).

Part 1: Selecting and Setting Up A Mathematical Task

What are your mathematical goals for the lesson (i.e. what do you want students to know and understand about mathematics as a result of the lesson)?

In what ways does the task build on students' previous knowledge, life experiences and culture? What definitions, concepts or ideas do students need to know to begin work on the task? What questions will you ask to help students access their prior knowledge and relevant life and cultural experience?

What are all the ways the task can be solved?

- Which of these methods do you think your students will use?
- What misconceptions might students have?
- What errors might students make?

Part 2: Supporting Students' Exploration of the Task

As students work independently or in small groups, what questions will you ask to:

- Help a group get started or make progress on the task?
- Focus students' thinking on the key mathematical ideas in the task?
- Assess students' understanding of key mathematical ideas, problem-solving strategies, or the representations?
- Advance students' understanding of the mathematical ideas?
- Encourage all students to share their thinking with others or to assess their understanding of their peers' ideas?

Part 3: Sharing and Discussing the Task

How will you orchestrate the class discussion so that you accomplish your mathematical goals?

- Which solution paths do you want to have shared during the class discussion? In what order will the solutions be presented? Why?
- In what ways will the order in which the solutions are presented help students' understanding of the mathematical ideas that are the focus of your lesson?
- What specific questions will you ask?

Figure 7 Portions of TTLP Related to the Five Practices

An examination of Part 1 of the TTLP reveals attention to the mathematical goal, mathematical task, and anticipation of student responses. Part 2 of the TTLP is intended to help teachers support students' exploration of the task. This includes the teacher designing questions he/she will ask to focus, assess, advance, and encourage student thinking. Part 3 of the TTLP is focused on helping the teacher orchestrate the whole class discussion. The first and second questions, respectively ask, teachers to consider both which solution paths will be shared (selecting), and the order in which they will be presented (sequencing). The remainder of Part 3 engages the teacher in thinking about developing students' understanding of the mathematical ideas present in the lesson. Specifically, teachers are asked to identify different questions including ones designed to help students make sense of the math, as well as questions designed to help students draw connections across the different strategies (connecting).

While the Five Practices make up a significant portion of the TTLP, the TTLP extends to other issues not addressed by the practices (Smith & Stein, 2011). Such issues include considering the needs of English Language Learners and struggling students, introducing the activity so *all* students have access to the problem, ensuring students will remain engaged with the task, determining what needs to be heard or seen to know *all* students understand the important mathematical ideas, and deciding how the lesson will continue during the next class period. Engagement in the TTLP is a worthwhile process, but it is also time-consuming and demanding. The intention of the TTLP is not for teachers to use on a daily basis (Smith et al., 2008). "Rather, teachers have used the TTLP periodically (and collaboratively) to prepare lessons so that, over time, a repertoire of carefully designed lessons grows" (Smith et al., 2008, p. 135). The goal of the TTLP is to change how teachers plan for and think about lessons. The rationale behind periodic engagement with the TTLP is to get teachers thinking about its' content on a regular basis without actually physically completing the process.

2.2.2.2 TTLP Increases Attention to Student Thinking During Planning

Hughes (2006) examined whether pre-service teachers' attention to students' mathematical thinking changed over time. In particular, the study measured pre-service teachers' attention to students' mathematical thinking before and immediately after a methodology course, and during the first semester of a full-year internship. The methodology course "emphasized students' mathematical thinking as a key element of planning" (Hughes, 2006, p. 63). In order to measure pre-service teachers' attention to student thinking during lesson planning, the study utilized a scoring rubric closely linked to the Five Practices. The scoring rubric is shown in Table 1.

Table 1 Scoring Matrix for Lesson Plans Element of Attending to Students' Thinking (Hughes, 2006)

Element of Attending to Students' Thinking	Score = 0	Score = 1	Score = 2	Score = 3
Mathematical Goal	A mathematical goal does not exist	Vaguely describes concepts OR focuses on skills students will exhibit OR focuses on things students will do to complete the task	Specifies concepts and what it means to "understand" the concept	N/A
Anticipating Students' Correct Thinking	Evidence of anticipating students' correct thinking does not exist	Vaguely describes correct strategies/thinking students may use when working on the problem	Specifically describes at least one correct strategy/approach students may use when working on the problem. However, the strategies/approaches are limited and do not represent an attempt to describe the many ways in which students may solve the problem(s).	Specifically describes correct strategies/thinking students may use when working on the problem AND there is an attempt to identifying the many possible solution strategies or representations students may use
Anticipating Students' Incorrect Thinking	Evidence of anticipating students' incorrect thinking does not exist	Vaguely describes incorrect ways in which they may think about the problem	Specifically describes at least one incorrect way in which students may think about the problem or specific question students may ask or difficulty students may encounter as they work on the problem, however the challenges and misconceptions are limited and do not represent an attempt to describe the many challenges or misconceptions that students may have	Specifically describes incorrect ways in which students may think about the problem or specific questions students may ask or difficulties students may encounter as they work on the problem AND there is an attempt to identifying the many challenges or misconceptions students may encounter with the given mathematical task
Questions to Assess and Advance Students' Thinking	Specific example questions do not exist	Provides a specific example question to ask students but the circumstances under which the question is appropriate are not given, are not based on students' mathematical thinking about the problem, or only one circumstance based on students' mathematical thinking is present	Provides a specific example question to ask students AND the circumstances under which the question is appropriate (circumstances based on students' mathematical thinking about the problem). There must be at least two different circumstances based on students' mathematical thinking with a corresponding specific question(s)	N/A
Discussion Building on Students' Thinking	Evidence of building on student thinking does not exist	Selects and/or sequences students' solutions to be discussed but does not provide any specific questions to ask related to the student work OR identifies a question to ask, but is vague about for which student solution the question is appropriate, OR simply asks students to explain or share his/her solution without specific questions that highlight mathematical ideas	Identifies specific questions that highlight the mathematics in a specific student solution	N/A
Discussion Making the Mathematics Salient	Evidence of thinking about making the mathematics of the lesson salient does not exist	Identifies questions that are vague or so few that a particular mathematical idea is not being well-developed OR expresses specific mathematical ideas that they wish to address in the discussion, but offer no specific questions to ask in order to achieve their mathematical intentions	Identifies a series of specific questions that develop mathematical ideas	N/A

The rubric is related to the Five Practices and groundwork practices as it includes dimensions related to the mathematical goal, anticipation of student responses, questions for assessing/advancing student thinking, selection and sequencing of student responses, and questions that could be used during the whole class discussion. While the rubric does not contain anything related to the level of the mathematical task, the study did compare scores of lesson plans that used high-level tasks with scores of plans using low-level tasks. The study found that teachers demonstrated higher levels of attention to students' mathematical thinking when planning a lesson with a high-level task versus a lesson with a low-level task (Hughes, 2006). This finding supports the notion that the selection of a high-level task is a critical groundwork practice prior to engagement in the Five Practices.

Furthermore, the study found a relationship between teachers' scores in anticipating students' correct thinking and their attention to orchestrating a discussion that builds on students' thinking. In particular, a large percentage of teachers who had high scores in the orchestrating a discussion category also had significantly high scores in anticipating students' correct thinking (Hughes, 2006). This finding also supports the embedded nature of the Five Practices in which a preceding practice such as anticipating plays a role in the later practice of orchestrating the discussion.

One other specific aspect of the study was that the pre-service teachers engaged in lesson planning using the TTLP at a point in the study. In general, the study found that participants' attention to students' mathematical thinking was higher when asked to explicitly attend to it through the TTLP versus lesson plans that were designed without the use of the TTLP (Hughes, 2006). This finding is not surprising considering the purpose of the TTLP is to focus teachers' attention to students' mathematical thinking while planning a lesson; however, it does support the notion that the use of the TTLP can move teachers (particularly pre-service teachers) towards more thorough and thoughtful lesson plans (Smith & Stein, 2011).

2.2.2.3 The Lesson Planning Project

Planning with the TTLP focuses heavily on anticipation of student responses. In order to help teachers enact such instruction, researchers have designed the Lesson Planning Project. According to Smith et al., (2012), "A core premise of the Lesson Planning Project was the belief that teachers' engagement in thoughtful, thorough lesson planning routines would lead to more rigorous instruction and improved student learning" (p. 118).

The Lesson Planning Project team developed an "internet-based electronic planning tool" that draws heavily from the TTLP. Based on prior research regarding the use of tools and lesson planning, the research based lesson planning tool was used as the primary catalyst for school wide change within the Lesson Planning Project (Stein et al., 2011). The project engaged teachers in co-planning groups with other teachers and a university-based partner. The co-planning groups focused on the identification, set-up and enactment of high-level tasks as well as setting learning goals and anticipation of student responses. Lastly, teachers and university-based partners co-observed instruction to help build a coherent understanding of dialogic instruction.

Within the Lesson Planning Project, the researchers conducted a study by focusing on two teachers (one science and one math). For the purposes of this paper, only the second research question in the study involving the math teacher will be discussed. The mathematics study addressed the following research question:

How does collaborative planning focused on anticipating student responses to challenging tasks impact the way in which the task is implemented in the classroom?

The subject was Cara Nance, a beginning teacher of elementary functions and calculus who was an “enthusiastic” member of the professional development group and “appeared to embrace the five practices model of instruction” (Smith et al., 2013, p. 9). Data sources came from two-week group meeting cycles referred to as modified lesson study cycles (MLSC). For each MLSC, a focal teacher selected a high-level focus task and posted a lesson around the task in the electronic planning tool. Before the first-week meeting, all members of the group (UBs included) solved the task as many ways as they could. “They also anticipated ways in which students would work on the task, what solution methods they would attempt to use, and possible misconceptions they might have while working on the task” (Smith et al 2013, p. 6). The group would discuss the task at the first meeting, and then the focal teacher would modify the lesson in the electronic planning tool based on the group’s feedback and anticipated solutions. The focal teacher would then implement the task and collect artifacts from the lesson (e.g., samples of student work, diagrams or representations produced by students). At the second meeting, the focal teacher would share the artifacts and reflect on the lesson. The group would listen and ask questions about how the task was implemented. Upon completion of one cycle, another would begin with a different teacher as the focal teacher,

For the study, Cara was the focal teacher of the MLSC (i.e. transcripts of group meetings, electronic lesson plan, artifacts of practice). Additionally, data was collected from six other classroom observations not associated with the MLSCs. The purpose of this was to see if the MLSC lessons were similar or different from other lessons taught throughout the year. The six other lessons were selected by Cara based on when she would be using a high-level task. Data from these lessons included the electronic lesson plan, the main instructional task, samples of student work, detailed write-ups by the UBs, and lesson goals posted during the actual lesson (Smith et al., 2013). In order to address the research question (stated above), the coding of the data involved analyzing the tasks, and anticipated student responses.

Tasks were coded based on both selection and enactment. The Task Analysis Guide (TAG) and Academic Rigor (AR1): Potential of the Task rubric from the Instructional Quality Assessment (IQA) toolkit were used to code the task as it appeared in the lesson plan. The AR2: Implementation the Task rubric from the IQA toolkit was used to code the lesson write-ups. The use of the rubrics will be discussed further in the next chapter, as they pertain to the study proposed within this paper.

Of the selected tasks, 6 out of 8 were coded as procedures with connections according to the TAG and received a score of 4 out of 4 on the AR1: Potential of the Task rubric. Such a score indicates the task has the potential of being Doing Mathematics or Procedures with Connections and explicitly prompts student for evidence of understanding. The coding indicates that these 6 tasks were considered to be high-level as they appeared on the lesson plan. The remaining 2 tasks were coded as procedures without connections and received a score of 2 on the AR1 rubric. Such coding indicated that these two tasks were considered to be low-level.

During implementation, none of the tasks coded as a “4” for potential remained as a “4”. Of the six tasks coded as a ”4”, four of them declined to a “2” during implementation. The remaining two tasks coded as a “4” for potential were coded as a “3” during implementation. The authors note that such a change in score “is not considered a decline. Rather, it indicates that students’ reasoning was not made explicit during the lesson…while the teacher did not proceduralize the task and tell students what to do or how, there was no explicit evidence of the students' reasoning and they were not pressed to provide explanations regarding why they did what they did" (p. 26 -27, Smith et al., 2013).

Anticipation of student responses was coded by looking at solutions produced by all teachers (including focal teacher) in the MLSC, as well as all solutions posted in the focal teacher's electronic lesson plan. Each anticipated solution was coded as one of four types based on it's focus: 1.) logistics, 2.) doing, 3.) seeing, 4.) making sense. Smith et al., (2013) describes each type of anticipation:

> logistics – focused on something other than learning related to the task (e.g., how students will be grouped);
>
> doing – focused on what students will actually do while engaging in the task but provided no insight into why they are doing it or how they are thinking about it (e.g., students will use a calculator, students will try to solve it algebraically);
>
> seeing – focused on what students will or will not see or recognize (e.g., students will not notice the asymptote at zero because of the way they drew the graph); and
>
> making sense – focused on making sense of what students noticed or making a connection, engaging in some action to establish meaning (e.g., students will see that there is a limit to growth and how this is represented in the equation and be able to use the equation to find the y-intercept). (p. 17)

According to these types, Cara recorded no anticipation in the electronic planning tool for any of the 8 lessons. This is despite the tool having a specific section where responses could be entered that Cara had been introduced to early in the project. The only records of anticipation of responses were those produced by the group for the two lessons discussed in the MLSC meetings. The authors indicate that Cara may have anticipated on her own, but there was no record of it. Of the 25 anticipated responses across the two lessons discussed in the MLSC meetings, Cara produced 13 of them. The remaining 12 were produced by the group members.

For the lesson associated with MLSC 1, there was a total of 12 anticipated responses. Several of these anticipations were coded as seeing or making sense. Evidence from the lesson write up indicated that 10 of them actually matched solutions produced by students during the lesson. The authors point out that “in many instances it appeared that the teacher was prepared to deal with what had been anticipated” (p. 37, Smith et al., 2013). In the MLSC 2, 9 of 13 anticipated responses actually occurred during the lesson. The anticipations for the MLSC 2 lesson primarily focused on what the students would *do* incorrectly, and to some degree, correctly. For incorrect solutions, the teacher led them to the correct solution, and correct solutions were simply acknowledged. Emphasis appeared to be on the correct answer as opposed to student thinking, which was reflective the MLSC 2 meeting.

The task from the lesson associated with MLSC 1 had a Potential score of “4” and was implemented as a “3”. Thus, the cognitive demands of the task were maintained during implementation. However, the task from the lesson associated with MLSC 2 had a potential of “4” and was implemented as a “2”. Thus, the cognitive demands of the task declined during implementation. One possible explanation are the types of anticipations and goals associated with the two different lessons. The lesson with MLSC 1 included multiple anticipations coded as “seeing” or “making sense” and a specific learning goal, whereas the majority of anticipations for the MLSC 2 lesson were coded as “doing” with general learning goals.

The authors argue that “*the level of preparation* in which the teacher engaged may contribute to her ability to maintain the level of demand of the task during instruction” (p. 40, Smith et al., 2013). As stated earlier, the majority of tasks selected as high-level declined during implementation. Overall, "the physical lesson plans that Cara Nance produced for her lessons were skeletal – less than two-pages in length – and provided virtually no details regarding what students were likely to do during a lesson, how she would respond to what students would do, or how the work produced by the students would be used to develop the mathematics understanding of the group" (Smith et al., 2013, p. 40-41).

Basically, the authors are suggesting that Cara's lack of detailed planning may be linked to the overall poor enactment of her lessons. The question remains, what would instruction look like if a teacher did engage in detailed lesson planning? Overall, the research from the lesson planning project has not yet yielded evidence of teachers actually engaging in thoughtful and thorough planning. Thus, there is no evidence linking such planning to instruction. As mentioned earlier, research does indicate that pre-service teachers enrolled in a teacher education program focused on attending to student thinking during lesson planning have the ability to thoughtfully plan lessons, especially when explicitly directed to so by the TTLP. Thus, this study seeks to investigate the core premise (thoughtful/thorough planning is linked to better instruction) of the Lesson Planning project by investigating a population (pre-service teachers) who have shown evidence of thoughtful/thorough planning.

2.3 PRE-SERVICE TEACHERS' INSTRUCTIONAL PRACTICES

There is evidence to support the hypothesis that pre-service teachers can engage in thoughtful and thorough planning (Hughes, 2006). Also, there is evidence from a small case study that pre-service teachers can experience different levels of success in implementing high-level tasks

(Mossgrove, 2006). Several studies indicate that pre-service teachers struggle during live instruction (e.g., Borko et al., 1992; Eisenhart et al., 1993; Livingston & Borko, 1990; Borko & Livingston, 1989; Zimmerlin & Nelson, 2000; Brendefur & Frykholm, 2000; Vacc & Bright, 1999; Karp, 2010). To date, there is no research investigating how thoughtful and thorough planning is linked to instruction (in general) or the implementation of high-level tasks for pre-service teachers. Since this study is investigating a link between a particular aspect of practice (i.e. planning) and instruction, the remainder of this section discusses links between other aspects of practice and pre-service teachers' instruction. These include teacher knowledge, beliefs about how math should be taught, mentoring, curriculum, and classroom management.

2.3.1 Pre-Service Teachers' Knowledge

In recent years, researchers have developed a framework to describe the various aspects and domains of knowledge required to effectively teach mathematics (Ball, Thames, & Phelps, 2008). While this paper acknowledges this work, and realizes the relevance of the Mathematical Knowledge for Teaching (MKT) Framework, it is beyond the scope of this paper to discuss the literature on pre-service teachers' knowledge within the context of MKT. That is because the study proposed in this paper does not seek to explicitly assess nor formally analyze pre-service teachers' knowledge as an influential factor. Rather it seeks to see if there is any evidence suggesting that pre-service teachers identify knowledge as playing a role in their ability to teach.

Several studies suggest that pre-service teachers' content knowledge (or lack thereof) plays a role in pre-service teachers' ability to deal with common misconceptions (e.g., Livingston & Borko, 1990), teach for conceptual understanding (e.g., Borko et al., 1992), provide mathematically connected explanations (e.g., Livingston & Borko, 1990), stay directed towards the mathematical goal (Borko & Livingston, 1989), and meaningfully respond to students during live instruction (e.g., Livingston & Borko, 1990; Borko & Livingston, 1989).

A particular study illustrates ways how knowledge deficits can be linked to instruction. Karp (2010) investigated the classroom teaching of 25 pre-service teachers enrolled in a course (prior to student teaching) that required them to teach algebra and geometry on Saturdays. The researcher did not examine actual classroom instruction, but drew on a variety of data sources such as reflective journals, online discussions, written lesson plans, and transcripts of problem solving interviews with individual students, and final letters from the pre-service teachers. Through comparative analysis of all data sources, common features in perceived difficulties were identified. It should be noted that this study utilized the MKT framework to analyze knowledge; however, the particular domains of MKT are not identified in this paper for the reasons set forth at the beginning of this section.

Pre-service teachers identified the varying levels of student knowledge and ability as a surprising factor. The pre-service teachers were surprised by what some students did know, while at other times they were surprise by students' lack of knowledge. Furthermore, the pre-service teachers expressed difficulty in understanding the way students understood certain concepts (Karp, 2010). In some specific instances there are links between knowledge deficits and instruction. For example, in one case a student provided a solution founded on two misconceptions, and the pre-service teacher responded incorrectly (Karp, 2010). Another problem was that the pre-service teachers had difficulty anticipating the number of solutions that could be produced for a single problem (Karp, 2010). Furthermore, they were not ready to respond to particular solution strategies, and they had difficulty trying to understand certain explanations and providing appropriate responses (Karp, 2010). Finally, the pre-service teachers often inappropriately used instructional materials (Karp, 2010).

2.3.2 Pre-Service Teachers' Beliefs of How Math Should Be Taught

It is often the case that pre-service teachers enter teacher education programs with views of teaching and mathematics based on their experiences as students (e.g., Brown & Borko, 1992; Thompson, 1992). Pre-service teachers often have *traditional* experiences as students in which the teacher is the authority, and the teacher transmits knowledge to the students (Cady, Meier, & Lubinski, 2006). As a result, mathematical knowledge is often viewed as absolute and certain. Such a view is in contrast with the engagement of students in mathematical reasoning, and teachers viewing themselves as a source of mathematical authority (Cady, Meier, & Lubinski, 2006). Research indicates pre-service teachers' beliefs about how math should be taught may or may not play a role in their ability to implement standards-based instruction in mathematics. Case studies involving Ms. Daniels within the Learning to Teach Mathematics Project in part focused on perceptions about teaching mathematics and their relationship to instruction (Borko et. al, 1992; Eisenhart et. al, 1993). Ms. Daniels exhibited beliefs that were aligned with the ideas of standards-based instruction, but she struggled to implement instruction that fostered conceptual knowledge. The researchers suggest that Ms. Daniels "good" belief about teaching mathematics "could not be supported by her knowledge of mathematics and mathematics pedagogy" (Borko et. al, 1992, p. 220).

While the case of Ms. Daniels does not support a link between "good" beliefs and "good" teaching, there is research that indicates differences in such perceptions can be linked to difference in instructional practice. Brendefur & Frykholm (2000) examined two secondary student teachers' perceptions about how math should be taught and instructional practices within the context of mathematical communication. The study used a frame involving four types of communication (uni-directional, contributive, reflective, and instructive) to examine the pre-service teachers' conceptions and practices in regards to communication as a method for student learning of mathematics. *Uni-directional* communication is lecture style, teacher directed with very limited opportunities for students to engage in conversation. *Contributive* communication involves teacher-student interaction but requires little thought. *Reflective* communication consists of students sharing ideas and strategies with peers, and the teacher and students utilize mathematical discussions to further explore concepts. *Instructive* communication occurs when the teacher uses what the students are thinking, doing, and saying to modify subsequent instruction (Brendefur & Frykholm, 2000)

The study also examined the communication practices of two secondary pre-service teachers (Becky and Brad) during their student teaching experiences. Despite similarities in age, content preparation, university coursework, courses taught during student teaching, style of cooperating teachers, and support received from university mentor, the two student teachers exhibited contrasting degrees of communication practices. By the end of the student teaching experience, Becky was able to comfortably and consistently implement reflective communication. Brad, on the other hand, implemented uni-directional communication throughout his entire student teaching experience.

The researchers discuss how qualitative differences in perceptions about practices related to the teaching of mathematics of the two pre-service teachers are potentially linked to the varying levels of instruction they provided. Brendefur & Frykholm (2000) "suggest that Becky had a disposition toward reflection that Brad did not possess, as well as broader vision and beliefs about the nature of mathematics" (p. 145). Becky expressed being uncomfortable acting as the sole mathematical authority in the classroom, and she believed students learned best by being actively involved in the mathematics at hand. Brad, on the other hand, expressed more traditional beliefs in which the teacher demonstrates the procedures students need to solve problems (Brendefur & Frykholm, 2000). Thus, Becky and Brad expressed views about their teaching that aligned with how researchers viewed their teaching.

Another research study also indicates that certain *initial* beliefs held by pre-service teachers play an important role as they develop as classroom teachers trying to implement standards-based instruction (Cady, Meier, & Lubinski, 2006). The study examined the beliefs and practices of K-8 teachers who participated in a project five years earlier as pre-service teachers. During the project, which was during their teacher education program, participants received support in implementing practices which were NCTM-CGI aligned. One aspect of the study examined teachers' locus of authority while they were still in the teacher-education program. An *internal locus of authority* indicates that a teacher believes he/she has the ability to "reflect on situations and make decisions based on one's own knowledge, experience and understanding" (Cady, Meier, & Lubinski, 2006, p. 297). An *external locus* indicates that a teacher believes knowledge is an absolute truth and correct answers are dependent on external sources such as a teacher or textbook.

The teachers who started the project with an internal-locus of authority were more likely to implement, continually refine and maintain *standards-based* practices than teachers who exhibited an external locus of authority (Cady, Meier, & Lubinski, 2006). The locus of authority was difficult to change, and it played an important role five years down the road in the teachers' ability to implement instruction focused on student thinking (Cady, Meier, & Lubinski, 2006).

Other beliefs, however, that were measured in the same study were more susceptible to change. The project was successful in changing the cognitive based beliefs the participants had when entering as pre-service teachers. In general, beliefs related to the teaching and learning of math and focus on student thinking continued to develop over the five-year span of the project (Cady, Meier, & Lubinski, 2006). Participants identified factors that influenced their change in their cognitively based beliefs to include "confidence through experience in their own classrooms, being in the project itself, having an increase in pedagogical content knowledge, the passing of time, and having supportive environments" (Cady, Meier, & Lubinski, 2006, p. 298).

With the changes in beliefs, the researchers also cited changes in the teachers' practices. Such changes included more confidence in discussing new ideas during a lesson, more confidence in evaluating student understanding, better classroom and time-management, better use of manipulatives and representations to develop concepts, better equipped to respond to student questions and incorrect solutions, and better use of terminology (Cady, Meier, & Lubinski, 2006).

2.3.3 Mentoring of Pre-Service Teachers

The term *mentor* often refers to the classroom teacher where the pre-service teacher is completing his/her field experience; however, it may also refer to a professional from the university who is regularly involved with the pre-service teacher (Little, 1990). Within this section, the term cooperating teacher will be used to refer to a mentor who is the classroom teacher during the field experience, and the term university supervisor will refer to a mentor associated with the pre-service teachers' university coursework. The term *mentoring* may refer to support received from either of the two sources.

There are studies in which researchers have identified links between differences in mentoring of pre-service teachers and their instructional practices (e.g., Mossgrove, 2006; Vacc & Bright, 1999). Mossgrove (2006) examined the selection and implementation of high level tasks, use of tools, questioning practices of two pre-service teachers, Paige and Keith. In general, the study found that Keith outperformed Paige in these areas. One influential factor that was different for Paige and Keith was the type of mentoring they received from their various mentors.

The type of mentoring Paige received was referred to as emotional support mentoring, whereas the type of mentoring Keith received was referred to as educative mentoring. Emotional support mentoring creates a safe-environment in which the primary goal is for the pre-service feels comfortable trying new ideas (Feiman-Nemsar, 2001). In educative mentoring, the pre-service teacher is pressed to see connections between theories of teaching and learning, and use instructional practices that are informed by these understandings (Feiman-Nemsar, 2001). In general, Paige's mentors provided emotional support by helping her deal with specific problems as they arose. Keith's mentors held him to high standards by consistently providing him specific observational feedback and continually identify specific events from lessons to identify areas of improvement (Mossgrove, 2006).

Vacc & Bright (1999) identify differences in mentoring as one possible factor for the different beliefs and instructional practices of two student teachers. Helen's cooperating teacher had extensive exposure to the principles of CGI, and Helen's beliefs (as measured by the CGI scale) continued to grow during her student teaching experience. Andrea's cooperating teacher had very limited exposure to the principles of CGI, and Andrea's beliefs (which increased during university coursework) leveled off during student teaching. Furthermore, while Helen still did struggle to incorporate CGI principles into her instruction, she still did so more frequently than Andrea (Vacc & Bright, 1999).

The analysis of the data in the case study with Ms. Daniels also indicates the cooperating teacher as an influential factor (Eisenhart et al., 1993). Specifically, the analysis suggests that Ms. Daniel's perceptions about her cooperating teacher's instructional focus influenced her instructional decisions. For example, the cooperating teacher intended for "Morning Math" to focus on procedural skills. In turn, Ms. Daniels "Morning Math" often consisted of short lessons covering a narrow spectrum within the curriculum (Eisenhart et al., 1993). The above findings (Mossgrove, 2006; Vacc & Bright, 1999; Eisenhart et al., 1993) stem from the observations of the researcher Other studies provide evidence that pre-service teachers themselves also perceive mentors as having an influence on their instructional practices (e.g., Frykholm, 1996; Brendefur & Frykholm, 2000).

One particular study suggests that cooperating teachers have the most significant influence on pre-service teachers' philosophies and practices regardless of whether or not cooperating teachers espouse the values taught in the teacher education program (Frykholm, 1996). In a study examining multiple aspects of 44 secondary teachers' implementation and perceptions about the NCTM standards, it was found that a significant majority of student teachers began to mirror the teaching styles of their cooperating teachers (Frykholm, 1996). Furthermore, analysis indicated that while doing so, student teachers were aware that their cooperating teachers did not involve the Standards as an integral part of their instruction. A common explanation provided by student teachers was that they had a limited number of models to look to as examples; thus, to handle the challenges faced by a pre-service teacher, they emulated the cooperating teachers' practices (Frykholm, 1996).

Another perceived pressure by student teachers is the necessity to keep pace with their cooperating teacher who is teaching another section of the same class (Brendefur & Frykholm, 2000). In the case study involving two student teachers, one of them (Brad) wanted to directly "tell" learners how to solve the mathematical problems posed in class. In addition to expressing fear about students explaining incorrect strategies, Brad also wanted to "tell" so he could keep pace with his cooperating teacher (Brendefur & Fykholm, 2000).

Not all research suggests that cooperating teachers have the most significant influence on student teacher's instructional practices (Brendefur & Frykholm, 2000). Becky's cooperating teacher espoused a uni-directional approach to classroom communication. Becky, however, seemed to not be affected by this style of instruction, and she remained open to reform minded approaches. Becky had support from her university supervisor, and she actively sought and openly received suggestions from him on how to move towards reflective communication. Brad also had a source of support in his university supervisor, but despite efforts to help him, Brad remained very uni-directional in his approach (Brendefur & Frykholm, 2000).

2.3.4 Pre-Service Teachers' Use of Curriculum

Research suggests that different curricula provide varying levels of support to teachers attempting to implement standards-based instructional practices (e.g., Lloyd & Frykholm, 2000; Senk & Thompson, 2003b). Mossgrove (2006) identified curriculum as a possible influence to the instructional practices of the two pre-service teachers (Paige and Keith) whom she investigated. As mentioned earlier, Keith was more successful at implementing high-level tasks than Paige.

While both pre-service teachers used a traditional textbook, "Keith had access to and experience with the reform oriented curriculum, CMP" (Mossgrove, 2006, p. 211). Paige, on the other hand, relied on the teacher's edition of her traditional text where she primarily utilized surface level support. Keith *viewed* and *utilized* the CMP materials as a source of substantial support to implement student-centered instruction. Keith modified the suggestions from the traditional textbook through the use of the CMP materials (Mossgrove, 2006).

2.3.5 Pre-Service Teachers' Classroom Management

In an extensive review of studies pertaining to pre-service and beginning teachers, Kagan (1992) found classroom management to be a common issue for novice teachers. Novice's expectations of the classroom are not in line with the complex reality that actually exists. Kagan (1992) writes "obsessed with class control, novices may also begin to plan instruction not designed to promote learning, but to discourage misbehavior" (p. 145). Classroom management and instruction become intertwined in a way that takes away from the type of learning that should be occurring in a classroom.

One of the student teachers (Becky) discussed earlier demonstrated concern about classroom management (Brendefur & Frykholm, 2000). Becky found that in trying to promote higher levels of communication, there were situations arising in which few students were responding meaningfully. As a result other students were starting to get off task. When this occurred early in her experience, analysis showed that Becky was reverting to uni-directional communication to avoid such situations. Fortunately in Becky's case she was able to work with her university supervisor to resolve such a dilemma (Brendefur & Frykholm, 2000).

2.4 CONCLUSION

In looking across the literature that has been reviewed, a synthesis of findings from different studies supports the development of a new study. The literature on mathematical tasks suggests that teachers have difficulty maintaining the cognitive demands of high level tasks due to a variety of classroom factors (e.g., Stein et al., 1996; Henningsen & Stein, 1997). Specific forms of lesson planning such as the TTLP and frameworks such as the Five Practices have been identified as ways in which teachers can meaningfully plan and prepare for instruction involving high-level tasks.

The notion that lesson planning in a particular manner is linked with improvements in instruction arises from literature pertaining to Japanese Lesson Study (e.g., Stigler, Fernandez, & Yoshida, 1996), teacher researchers analyses of their own practice (e.g., Ball, 1993; Lampert, 2000; Schoenfeld, 1998), and expert-novice distinctions (e.g., Borko & Livingston, 1990; Zimmerlan & Nelson, 2000; Leinhardt, 1993). Findings from the studies indicate that if teachers focus on certain things during planning (i.e. the mathematical goal, anticipating student responses, meaningfully selecting and sequencing student solutions for discussion), then they are more likely to do those things during instruction. The findings also indicate that these practices are generally found to occur with expert teachers, while novice teachers tend to struggle with such practices in both planning and instruction. The TTLP and Five Practices Framework have in part drawn from these findings to develop ways for non-expert teachers to explicitly focus on such aspects of practice during planning in order to help ease the difficulties of instruction.

The Lesson Planning Project is built on the idea that thorough and thoughtful planning will lead to improved instruction (Smith et al., 2012). To date, the analysis of data from this project provides little evidence of moving teachers in the direction of thorough and thoughtful planning practices even when directed to use an electronic planning tool based on the TTLP (Smith et al., 2013, Smith et al., 2011). It is noted that the current results from the Lesson Planning project stem from a single study with a small number of participants; therefore, the findings may not be generalizable. However, the empirical findings from the study do not provide any indication whether thoughtful and thorough planning leads to better instruction for in-service teachers, and they suggest that changing the practices of in-service teachers may be difficult.

While it may be difficult to get in-service teachers to engage in such planning practices, there is reason to believe that pre-service teachers (who are enrolled in a teacher education program that focuses on attention to student thinking) can produce thorough and thoughtful plans, especially when directed to do so with the TTLP. Pre-service teachers have demonstrated the ability to meaningfully attend to student thinking around a high-level task during lesson planning (Hughes, 2006); however, the study does not investigate how such planning is linked to the maintenance of cognitive demands of the task during implementation.

While many studies indicate that pre-service teachers struggle during live instruction (e.g., Zimmerlan & Nelson, 2000; Livingston & Borko, 1990; Vacc & Bright, 1999), there is evidence from a case study that a pre-service teacher had the ability to maintain the cognitive demands of high level mathematical tasks during instruction (Mossgrove, 2006). Again it is noted that this is one teacher from one study, but it indicates that it is not impossible for a pre-service teacher to experience success in implementing tasks at high level of cognitive demand. In short, the Hughes study indicates that pre-service teachers can attend to student thinking during lesson planning, and the Mossgrove study indicates that a pre-service teacher can implement tasks at a high level of cognitive demand. However, there is no evidence linking thoughtful and thorough planning on the behalf of pre-service teachers to their ability to implement tasks at a high level of cognitive demand.

From the synthesis of this research, the following question arises: *What is the relationship between thoughtful and thorough lesson planning by pre-service teachers and subsequent task implementation?* The evidence from Hughes' (2006) study provides promise that pre-service teachers will engage in thoughtful and thorough planning when prompted to do so. By investigating the practice of several pre-service teachers, it is likely there will be different degrees of thoughtfulness and thoroughness in their plans. The synthesis of literature led to several hypotheses.

Based on Hughes (2006) work it seems reasonable to expect that some pre-service teachers' plans will be placed on the far right of the lesson planning continuum described earlier. It also seems reasonable to expect that some will not be to the far right despite detailed planning with the electronic planning tool. In other words, by establishing a common detailed planning practice (i.e. the electronic planning tool) among several different participants, there is the likelihood of variation in degrees planning between participants, and it increases the likelihood that some participants will plan thoughtfully and thoroughly.

Variations in degrees of planning allows for an investigation of the relationship between pre-service teachers level of attention to student thinking during lesson planning and the level of cognitive demand at which the task was implemented during instruction. It is expected that better plans will lead to better instruction (i.e. maintenance of high-level tasks) for some pre-service teachers. However, due to different sets of classroom based factors associated with the decline of tasks and the research pertaining to pre-service teachers' instruction, it is possible that some pre-service teachers will produce thoughtful and thorough plans, but still struggle during instruction (i.e. decline of high-level tasks).

Anticipating student responses is one prominent practice in the type of planning that the pre-service teachers will engage in using the tool. Despite the literature indicating that anticipation is sparse for in-service teachers using the tool, it is likely that pre-service teachers will produce a variety of anticipated student responses. One reason is that pre-service teachers in Hughes (2006) study showed evidence of anticipating correct and incorrect solutions, and they received maximum scores within the anticipation dimensions of the attention to student thinking rubric. Furthermore, when pre-service teachers create a detailed lesson plan with the electronic planning tool, they are required also to create a monitoring tool. A monitoring tool typically consists of a chart which includes strategies anticipated by the teacher and space for the teacher to take notes when students employ those strategies.

Better anticipation should be linked to better instruction. Hughes (2006) study indicated that anticipating in lesson planning was linked to planning for the orchestration of a discussion that builds on students' thinking. In particular, a large percentage of teachers who had high scores in the orchestrating a discussion dimension of the rubric also had significantly high scores in anticipating students' correct thinking (Hughes, 2006). While this is only a link within planning, a theoretical argument could be made that task implementation would be higher when the teacher is better prepared for the discussion.

Also (as previously discussed), a study from the lesson planning project indicated that one lesson planned within a Modified Lesson Study Cycle (MLSC) had better anticipation than another lesson taught by the same focal teacher (Smith et al., 2013). When the lesson plan with the better anticipation was implemented, the task remained at a high level. When the other lesson plan was implemented, the task declined to low level. For both lessons, there was a reasonably high level of accuracy between anticipated solutions and what actually happened during the lesson; however it was the type of anticipating performed that may have accounted for differences in cognitive demand during task implementation.

Thus, more thoughtful anticipation of student responses should be linked to the maintenance of cognitive demands of high level tasks for some pre-service teachers. Because this study is dealing with pre-service teachers, it is possible that thoughtful anticipation can occur and the task will still decline during implementation. Such an instance occurred with a teacher in the lesson planning project who thoughtfully anticipated a misconception, but still struggled when trying to help students understand the mathematical ideas behind it (Stein et. al, 2011). It is also possible that pre-service teachers may struggle to produce anticipated solutions for some tasks. This may be due in part to lack of experience or lack of knowledge in relation to a particular task. It is very possible that it will be difficult for a pre-service teacher to accurately anticipate what students will do, if she never taught the lesson before.

In summary, the proposed study seeks to examine the relationship between thoughtful and thorough planning and task implementation. The rationale for choosing pre-service teachers is that as a group research suggests they will likely produce thoughtful and thorough plans when prompted to do so. The hypothesis of the lesson planning project is that thoughtful and thorough planning will lead to better instruction; however the research related to pre-service teachers' instructional practices suggests that there are several aspects of practice linked to their ability to deliver live instruction.

3.0 METHODOLOGY

The purpose of this study was to investigate the relationship between pre-service teachers' lesson planning and instruction. In particular, the study sought to examine whether, in what way, and if so, how pre-service teachers' attention to student thinking during lesson planning related to the level of cognitive demand at which students engaged with the main mathematical task during instruction. The participants' planning was examined via written lesson plans, and their instruction was examined via samples of student work produced during the enactment of those written plans. The focus of their thoughts about whether enacted lessons went as planned was investigated through the use of semi-structured interviews.

The study was both quantitative and descriptive in nature. That is, the study sought to quantitatively examine as well as describe the relationship between planning and instruction for pre-service teachers enrolled in a teacher education program that emphasizes attention to student thinking around cognitively demanding tasks. The pre-service teachers produced all lesson plans examined in the study using a detailed template that explicitly directed their attention to student thinking. The purpose of the study was not to compare this type of planning to any other type of planning, nor was it to compare the type of instruction associated with this type of planning to instruction associated with other forms of planning. Rather, it was to describe (quantitatively and qualitatively) the nature of the relationship that existed between such planning and the level of cognitive demand of task implementation for this specific set of pre-service teachers. Through the collection and analysis of artifact and interview data, the study sought to address the following research question:

> What is the relationship between pre-service teachers' attention to student thinking with regard to lesson planning around a mathematical task (perceived to be high level by the pre-service teacher), and the level of cognitive demand at which the task is implemented?

3.1 PARTICIPANTS

The study investigated the instructional practices of six pre-service teachers enrolled in a post-baccalaureate secondary mathematics Master of Arts in Teaching (MAT) program at a large urban university in the United States. Participants entered the teacher preparation program with an undergraduate degree in mathematics or the equivalent. The pre-service teachers were seeking a MAT degree and secondary (7-12) Instructional I mathematics certification. The study participants completed a full school year internship field experience (September through June) in a middle or high school classroom while attending courses full-time. Five of the participants were teaching in middle schools and one was at a high school.

David, Nick, Chris and Renee (four of the participants interned at a middle school) were placed across three different schools within a large city school district using the Connected Mathematics Project curriculum (CMP). CMP is a "problem-centered curriculum promoting an inquiry-based teaching-learning classroom environment" (https://connectedmath.msu.edu). Quinn (also middle school) was at a laboratory school associated with the university in which the pre-service teachers were enrolled. The middle level math program (MathScape) is inquiry based providing a foundation in computational skills and algebraic problem solving. Marian (the only participant teaching at a high school) was in an Algebra 2 classroom using the Discovering Advanced Algebra curriculum. The curriculum is designed to balance inquiry based teaching with both skill and procedural mastery while providing opportunity for students to draw connections to the real world.

3.1.1 Teacher Education Program

Pre-service teachers in the MAT program completed 36 credit hours which included several mathematics education courses as well as coursework related to adolescent learning, special education, and English language learners. The pre-service teachers attended courses throughout their teaching internship. During their methods courses the pre-service teachers made use of several tools and frameworks pertinent to planning and task implementation. These include: the Task Analysis Guide (TAG) (Smith & Stein, 1998), Mathematical Tasks Framework (MTF) (Stein et al., 1996), The Five Practices (Smith & Stein, 2011), and the Thinking Through a Lesson Protocol (TTLP) (Smith et al., 2008).

During the year of this study, the participants program was formed by the Mathematics Teaching Practices (NCTM, 2014). These practices include: *Establish mathematics goals to focus learning, Implement tasks that promote reasoning and problem solving, Use and connect mathematical representations, Facilitate meaningful mathematical discourse, Pose purposeful questions, Build procedural fluency from conceptual understanding, Support productive struggle in learning mathematics,* and *Elicit and use evidence of student thinking*. The eight practices “represent a core set of high-leverage practices and essential teaching skills necessary to promote deep learning of mathematics (p. 9, NCTM, 2014).

3.1.2 Internship

During their field experience internship, the pre-service teachers were at their placement school for 20 hours per week in the fall and 30 hours per week in the spring. The field experience took place during the normal school day, and the pre-service teachers attended courses during the evening. Each pre-service teacher interned in the classroom of a teacher who served as their mentor throughout their entire field experience. The field experience was a gradual immersion into the full responsibilities of teaching. It began with observations, and the completion of course-based assignments. As the year progressed, the pre-service teachers increased their responsibility until they were eventually teaching at least 80% of a full-schedule. Interns in the large city school district were part of an urban scholars program, and have an extra four hours per week at their internship site.

The university supervisor visited the classroom about every 2 weeks. The supervisor and pre-service teacher had a pre-lesson conversation about the lesson plan and a post-observation conference with each visitation. There were a total of three different supervisors for the interns involved in this study. All of the interns at the large city school district had the same university supervisor. The intern at the laboratory school and the intern at the high school each had a different supervisor.

3.1.2.1 Planning, Teaching, Reflecting (PTR) Assignment

During their internship, as part of a particular course requirement each pre-service teacher completed a planning, teaching and reflecting (PTR) assignment. The assignment consisted of two lesson cycles. The first cycle was completed by mid-October and the second cycle was completed by early December. Each cycle involved planning, teaching and reflecting for a different lesson. The course syllabus explained to the intern that the purpose of the "assignment is to provide you with an opportunity to experience the teaching cycle first hand by planning a lesson, teaching the lesson to students in one of the classes you teach, and reflecting on your teaching following the lesson" (p. 4, Teaching and Learning in Secondary Math 2, Course Syllabus).

During each lesson cycle, the pre-service teachers received feedback on their lessons plans from a course instructor before they taught the lesson. Assignment requirements included the use of a high level task. Note for Cycle 2 the task had to be *doing mathematics*. Secondly, interns were required to complete the lesson plan within an electronic planning tool that is explained in the next section. Other requirements included addressing the standards, attachment of a monitoring tool to the lesson plan, and providing a written reflection.

3.2 DATA SOURCES

The research question is focused on examining the nature of the relationship between two aspects of practice: planning and instruction. Thus, the study needed to draw upon appropriate data sources that represent each aspect of practice. In order to investigate planning, the study primarily examined written lesson plans produced by the pre-service teachers. The study also looked at task coversheets and post-lesson thoughts for evidence of attention to student thinking in relation to planning. In order to investigate the instruction associated with these lesson plans, the study examined samples of student work produced during lessons for which the detailed plans were written.

More specifically, the data sources are shown in Table 2. Each data source is described in detail in the following sections. The collection size represents the range of number data sources collected from each participant.

Table 2 Data Sources

Data Sources	Collection Size
Written Lesson Plans	3 to 4 per participant
Sets of 6 – Samples of Student Work	3 to 4 per participant
Task Coversheets	2 to 3 per participant
Clinical Interviews linked to lessons	2 to 3 per participant
Written Reflection from Assignment	1 per participant

Each pre-service teacher was asked to submit a lesson plan and student work from Cycle 2 of the planning, teaching reflection (PTR) assignment described in the previous section that was taught in December 2014. This lesson was accompanied by a reflection (instead of an interview) written after it was enacted. In addition to this lesson, each pre-service teacher was asked to submit task coversheets, written lesson plans, samples of student work and participate in a post lesson interview for three lessons enacted between the beginning of February and end of May 2015. It was not the intention of the study to examine if planning and/or implementation changed over time, but rather to examine relationships across a time frame consistent for all participants. The four month time period allowed adequate time for each participant to plan three detailed lessons in addition to the PTRs. Figure 8 shows the sequence of data collection for each lesson collected between February and May.

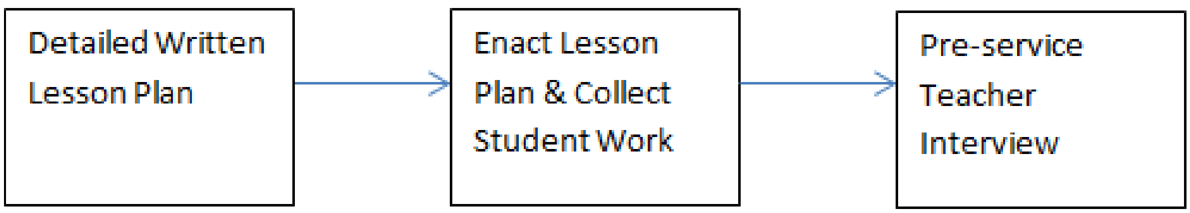


Figure 8 Sequence of Data Collection for Lessons Enacted between February and May

3.2.1 Written Lesson Plans

Pre-service teachers were expected to plan lessons taught during their field experience using an electronic planning tool. The lesson planning tool (LPT) supports writing lessons at two levels: a general daily plan and a detailed lesson plan. For daily planning, interns were expected to address 4 core elements at a general level: mathematical learning goals, describe the task, anticipate student responses, and plan how to assist students as they work on the task.

The detailed lesson plan contains the same core elements as the daily lesson plan, but, through specific prompts, requires the pre-service teacher to plan in greater detail, particularly with regard to student thinking. For the lesson plans collected as data sources, the pre-service teachers were expected to submit a detailed lesson plan around the task they selected to be high-level. *Perceived* means that the pre-service teacher thought that the task had the potential to engage students at a high-level of cognitive demand based on her knowledge of mathematical tasks and the TAG. The rationale behind the pre-service teacher's *perceived* task selection was to provide possible information related to the pre-service teacher's ability to select a task. For example, if a teacher consistently selected low-level tasks even though they perceive them to be high-level, then this may relate to planning and implementation in some way.

Pre-service teachers were required to periodically complete the detailed lesson plan template during their field experience. In particular, this happened when the university supervisor observed a lesson. An example of a completed detailed lesson plan template is in Appendix C. As part of the detailed lesson plan, pre-service teachers were expected to create a monitoring tool. A monitoring tool is typically a chart containing the solution strategies anticipated by the pre-service teacher with space allocated to take notes during the exploration phase of the lesson. The monitoring tool is intended to help the pre-service teacher keep track of solution strategies produced by students. It is also intended to help the pre-service teacher select and sequence student solutions for public display during the share and discuss phase of the lesson. Table 3 provides a generic example of a blank monitoring tool.

Table 3 A Blank Chart for Monitoring Students' Work

Solution Strategy	Student Name and Specifics	Presentation Order

The left column has space for the teacher to write down different strategies used by students. The pre-service teachers were required to fill in this column with anticipated strategies prior to teaching a lesson. In the corresponding space in the middle column, the pre-service teacher can record which groups perform each strategy and detail exactly what they did. The *order* column on the right allows for recording of the selected solutions and the sequence order in which they will be discussed. Monitoring tools created during lesson planning can vary in detail from one lesson to another and from one teacher to another. For example, anticipated solutions may be worked out completely in contrast to just naming the strategy. Pre-service teachers may have included a column in which they list questions they would ask to explore student thinking about a particular strategy. A sample completed monitoring tool is in Appendix D.

While the decisions of selecting and sequencing student responses for public display are ultimately done during instruction, the pre-service teacher is asked to attend to them during planning. For example, a specific prompt in the detailed planning tool asks the following questions: *How will you orchestrate a class discussion about the task so that you can accomplish your learning goals? Specifically, what student responses do you plan to share* *during the discussion and in what order will they be discussed?*

Finally, the planning tool asks the pre-service teacher to explain how she will know that students are reaching the learning goal(s), and identify what (and how) evidence will be collected that will allow the pre-service teacher to gain such information. In general, the detailed planning tool is based on the Thinking Through a Lesson Protocol (TTLP) (Smith et al., 2008) and is aligned with the Five Practices (Smith & Stein, 2011). Thus, engagement with the detailed lesson plan is intended to help teachers plan a lesson utilizing student thinking around a high-level task.

There were two reasons for expecting lesson plans to be created with the detailed planning tool. First, the study sought to examine the relationship between attention to student thinking in planning and implementation of high-level tasks. The detailed lesson plan is intended to help teachers incorporate high-level tasks into their practice. Also, the detailed planning template should have increased their attention to student thinking since it explicitly asks for such attention, and allowed for an examination between such planning and task implementation. Secondly, the engagement in the detailed planning tool provided uniformity in terms of what the pre-service teachers were prompted to include in their planning. The depth of plans may have varied from teacher to teacher (or even within a single teacher's plan from lesson to lesson); however the uniformity prompted the pre-service teachers to produce written plans in the same manner.

3.2.1.1 Procedure for Collection of Written Detailed Lesson Plans

This study examined a minimum of three written lesson plans per pre-service teacher that were expected to be written using the detailed LPT and enacted in the classroom. It was expected that each detailed plan focused on implementing what the pre-service teacher perceived to be a high-level mathematical task, and included in addition to the task, anticipated solution strategies, responses to all prompts within the template, and a monitoring tool. Anticipated solution strategies were required to be included as part of the monitoring tool.

The electronic planning tool is internet-based. Some pre-service teachers posted their lessons in the data base and others did not. Those who did not, submitted their plans electronically directly to the researcher. For the plans uploaded to the data base, the researcher had access to the website. Each participant was given a pseudonym by the researcher. Upon receipt of each lesson plan, the researcher removed any information relating to the participant's identity (name, school district, building, class, student name, cooperating teacher name), and labeled the lesson with the pseudonym. A random pseudonym common to the gender of each participant was used. Both a hard copy and an electronic copy were saved in a safe, locked (or password locked) location. Since each participant completed three or four lessons, each lesson was labeled by the pseudonym initials and lesson number (chronologically based on submission date). For example Quinn Brady (a male participant completing four lessons) had his plans labeled as Q.B. LP 1, Q.B LP 2, Q.B LP 3, Q.B LP 4.

3.2.1.2 Gathering of Additional Information About Written Detailed Lesson Plans

After the detailed lessons were collected, additional information was gathered regarding the lesson plan and the task it was planned around. More specifically, information about additional resources and/or experiences related to the lesson plan and task were gathered from a university instructor, university supervisors, and the interns themselves. For example, it was determined whether the interns engaged with the task the lesson planned around during their coursework.

Such engagement may have included launching the task, exploring the task as students themselves, and participating as students in a full-out modeled lesson around the task. Other information that was gathered included whether the lesson plan was already existing/fully fleshed from another resource base, and the type of feedback the intern received while planning. Please see Appendices J, K, and L for the checklists that were used to gather additional information from the university instructor, supervisors and interns respectively.

3.2.2 Samples of Student Work

In order to investigate the level of cognitive demand at which students engaged with the task, the study collected samples of student work produced during the main instructional task during lessons for which the detailed lesson plans were written. The Instructional Quality Assessment (IQA) toolkit was used to rate the potential of the task and the actual level at which students engaged the task. The development of the IQA toolkit and IQA Academic Rigor Mathematics Rubrics built on earlier work in language arts (Clare, 2000; Matsumura et. al., 2002), and extended to mathematics classrooms the validity of assignment collections as a measure of instructional quality (Boston & Wolf, 2006).

In particular that "four samples each (two medium quality and two high quality) and rated by two raters yield a generalizability co-efficient high enough (i.e. G>.80) to use assignments and student work as valid indicators of classroom practice" (p. 16, Boston & Wolf, 2006). What constitutes "medium quality" and "high quality" student work is at the discretion of the teacher. Boston & Wolf's (2006) generalizability study yielded a G coefficient of .91. Six pieces of student work also provides a valid indicator of classroom practice (Boston, 2012). The pre-service teachers in this study were asked to select six pieces of student work to create a sample from each lesson.

3.2.2.1 Procedure for Collection of Samples of Student Work

The pre-service teachers were expected to identify work produced by six different students on the main task. The 6 individual pieces together made up one sample of student work. The pre-service teachers were not rating the student work in the same way the researcher was; however, there were two criteria for pre-service teachers to select the sample of student work: 1.) select medium or high quality student work according to the expectations (high-medium-low) as described by the pre-service teacher on a coversheet. If 6 pieces of medium/high quality student work were not able to be identified, the pre-service teacher submitted 6 pieces of the highest quality available. 2.) If possible, the pieces of student work reflected the different types of work engaged in by students in the classroom. That is, if students produced medium/high quality work on the task in different ways, those ways were represented in the sample. This information was communicated to each pre-service teacher through a direction sheet given to them by the researcher. The direction sheet is in Appendix F.

To accompany the submission of student work, each pre-service teacher was expected to fill out a coversheet adapted from the coversheet designed for the IQA Toolkit (Boston, 2012). The coversheet is in Appendix A. On it, the pre-service teachers identify the task as *typical* or *especially challenging*, explain any directions given to the students regarding the main instructional task, identify the task's source, and list the participation structure of the activities (i.e. individual work, group work, think-pair-share) students actually engaged in while completing the task. The document also asks the pre-service teacher to describe expectations for high, medium and low quality work on the task. The cover sheet asks the teacher to explain if and if so how the performance criteria was conveyed to the students. Finally, it asks the pre-service teacher which quality of work expectation level (high, medium, or low) best describes the actual student work, and whether there was anything that influenced students' work on the task.

The participant was expected to remove any information from the student work that could indicate the identity of the student. The pre-service teacher scanned the student work (or photograph) into an electronic file and emailed them to the researcher. The participant also emailed the completed coversheet to accompany the student work. The researcher ensured that there is absolutely no information that may identify the participant or a student whose work has been submitted. Both a hard copy and an electronic copy were saved in safe, locked (or password locked) location. Since each participant is submitting coversheets and sets of student work, each set was labeled by the pseudonym initials and lesson number corresponding to the written lesson plan.

3.2.3 Interviews about Whether Enacted Lessons Went as Planned

The study also sought to examine whether pre-service teachers focused on student thinking when discussing the enactment of their lesson plan immediately after the lesson was taught. Semi-structured interviews with each participant were intended to provide a qualitative data source to gain insight into the focus of their post-lesson thoughts. The full protocol is in Appendix B. In this section, the questions within protocol are shown and the rationale is described. Figure 9 provides the interview protocol that was used following the enactment of each of the lessons enacted between February and May.

Supervisor Interview Questions:

1.) A.) Did your lesson go as planned? Why or why not? Feel free to provide specific examples and explanations. Also, please elaborate in your response and provide as much detail as possible.

(Pre-service teacher provides response without interruption from the supervisor)

Note to interviewer:

If participant suggests that lesson did go as planned and does not provide any explanation as to why, please probe by asking the following questions:

Can you explain what it means that the lesson went as planned?

Can you provide reasons or explanations as to why it went as planned or what helped it go as planned?

If participant suggests that lesson did not go as planned and does not provide any explanation as to why, please probe by asking the following questions:

Can you explain what it means that the lesson did not go as planned?

Can you provide reasons or explanations as to why it did not go as planned or what hindered it in going as planned?

B.) Is there anything else you would like to say about whether your lesson went as planned and why think so?

Figure 9 Interview Protocol Questions

The purpose of these questions was to identify what the pre-service teacher focused on when asked whether the lesson went as planned. One pre-service teacher may have responded with regard to teacher actions, while another may have responded with regard to student thinking. Or a pre-service teacher may have responded with regard to how they thought the task was implemented, while another may have responded with regard to covering all portions of the lesson plan. Based on the pre-service teacher's response, the interviewer probed for further explanation if needed, as described in the protocol, asking the pre-service teacher to explain why he or she thought the lesson unfolded in relation to the plan as it did.

3.2.3.1 Procedure for Collection of Interview Data

After each lesson enacted between February and May, each pre-service teacher participated in a semi-structured interview using the interview protocol. Each interview took place with either the pre-service teacher's university supervisor immediately following each targeted lesson or within two days with the researcher. Each interview followed the protocol and lasted approximately between 5 and 10 minutes. All interviews were audio-recorded. The audio recording was uploaded to a password secure Box account. The interview was identified by the pseudonym initials in the transcript and each transcript was labeled in correspondence with its enacted lesson.

3.2.4 Data Sources Submitted by Each Pre-Service Teacher

Each pre-service teacher planned and enacted a total of 3 or 4 lessons during the study. Table 4 indicates the data sources that were submitted for each lesson. For several of the lessons, pre-service teachers submitted all data sources; however there were some lessons for which pre-service teachers did not submit all data sources.

Table 4 Data Sources Submitted by Each Pre-Service Teacher

Pre-Service Teacher	Lesson #	**Date Mo./Yr**	**Lesson Plan**	**Student Work**	**Coversheet**	**Interview**	**Reflection**
David Upton	1 (PTR)	Dec. '14	X	X			X
	2	Feb. '15	X	X	X	X	
	3	May '15	X	X	X	X	
	4	May '15	X	X	X	X	
Quinn Brady	1 (PTR)	Dec. '14	X	X			X
	2	Feb. '15	X	X	X	X	
	3	Mar. '15	X	X	X	X	
	4	May '15	X	X	X	X	
Marian Turner	1 (PTR)	Dec. '14	X	X			X
	2	Apr. '15	X	X	X	X	
	3	May '15	X	X	X	X	
Renee Norris	1 (PTR)	Dec. '14	X	X			X
	2	Apr. '15	X	X		X	
	3	May '15	X	X		X	
Nick Newman	1 (PTR)	Dec. '14	X	X			X
	2	Feb. '15	X	X	X	X	
	3	May '15	X	X	X	X	
	4	May '15	X	X	X	X	
Chris Cain	1	Feb '15	X	X	X	X	
	2	May '15	X	X	X	X	
	3	May '15	X	X	X	X	
	PTR	Dec. '14		X			X

The lesson numbers represent the chronological order in which they were enacted as indicated by the date. For each pre-service teacher (except Chris), the PTR assignment is identified as Lesson 1 because it was enacted first. Chris's PTR assignment is not considered one of his lessons for the study, since his lesson plan was not available to be analyzed. Also, the PTR assignment did not include a coversheet or interview, but rather a written reflection. It also should be noted that one lesson submitted by Marian Turner was not included in the data set due to no student work being submitted for the main instructional task.

3.3 CODING

Coding for each of the three data sources – lesson plans, student work, and interviews is described below. Please note that task coversheets are not discussed because there was no coding scheme applied to them. Analysis of data involved both quantitative and qualitative methods.

3.3.1 Coding the Written Lesson Plans

Lesson planning data was coded for attention to student thinking and potential cognitive demand of the main instructional task. Two schemes were used to code teachers' attention to students' mathematical thinking in written plans: 1.) the study used a scoring rubric designed and used by Hughes (2006) based on the TTLP, containing elements related to mathematical goal, anticipating student thinking, questions to monitor student thinking, and orchestrating classroom discussion; and 2.) anticipating student thinking was also coded separately based on quality of types of anticipations (Smith et al., 2013).

Table 5 displays the Hughes (2006) scoring rubric. Similar to Hughes, the rubric is used to examine pre-service teachers' attention to individual elements of attention to student thinking, as well as provide a holistic measure of such attention. Each element could earn a total score of 2 or 3. The first element is the *mathematical goal*. Participants received a score of 0,1, or 2 based on the extent to which they a.) specify concepts, and b.) what it means for students to understand the concepts for the goal of the lesson. The second and third elements are *anticipating students' correct and incorrect thinking,* respectively. For each, participants received a score of 0, 1, 2, or 3 based on the extent to which they described strategies/approaches and misconceptions, and identified several solutions or representations that are either correct or incorrect, respectively. The fourth element is *questions to assess and advance students' mathematical thinking*. Participants received a score 0,1, or 2 based on the number, specificity, and circumstances under which certain questions should be asked. The fifth and sixth elements are *discussion building on students' thinking* and *discussion making the mathematics salient*. Participants received a score of 0, 1, or 2 in each element based on the extent to which they identify a series of questions to highlight mathematics in specific students' solutions and a series of questions to develop mathematical ideas, respectively. Each written lesson plan was scored across each of the six elements using the scoring criteria set forth for each. In addition to each element receiving a score, an overall score was assigned which is the sum of the element scores. The highest a teacher could score on an individual lesson was 14 points (Hughes, 2006). Appendix C contains a sample lesson plan, and Appendix D contains a sample monitoring tool that is part of the lesson plan. Appendix E illustrates how the overall lesson plan (including monitoring tool) is coded. Appendices C, D and E were used to train the second rater in coding lesson plans.

Table 5 Scoring Rubric Containing Elements of Attending to Student Thinking (Hughes, 2006)

Element of Attending to Students' Thinking	**Score = 0**	**Score = 1**	**Score = 2**	**Score = 3**
Mathematical Goal	A mathematical goal does not exist	Vaguely describes concepts OR focuses on skills students will exhibit OR focuses on things students will do to complete the task	Specifies concepts and what it means to "understand" the concept	N/A
Anticipating Students' Correct Thinking	Evidence of anticipating students' correct thinking does not exist	Vaguely describes correct strategies/thinking students may use when working on the problem	Specifically describes at least one correct strategy/approach students may use when working on the problem. However, the strategies/approaches are limited and do not represent an attempt to describe the many ways in which students may solve the problem(s).	Specifically describes correct strategies/thinking students may use when working on the problem AND there is an attempt to identifying the many possible solution strategies or representations students may use
nticipating Students' Incorrect Thinking	Evidence of anticipating students' incorrect thinking does not exist	Vaguely describes incorrect ways in which they may think about the problem	Specifically describes at least one incorrect way in which students may think about the problem or specific question students may ask or difficulty students may encounter as they work on the problem, however the challenges and misconceptions are limited and do not represent an attempt to describe the many challenges or misconceptions that students may have	Specifically describes incorrect ways in which students may think about the problem or specific questions students may ask or difficulties students may encounter as they work on the problem AND there is an attempt to identifying the many challenges or misconceptions students may encounter with the given mathematical task
Questions to Assess and Advance Students' Thinking	Specific example questions do not exist	Provides a specific example question to ask students but the circumstances under which the question is appropriate are not given, are not based on students' mathematical thinking about the problem, or only one circumstance based on students' mathematical thinking is present	Provides a specific example question to ask students AND the circumstances under which the question is appropriate (circumstances based on students' mathematical thinking about the problem). There must be at least two different circumstances based on students' mathematical thinking with a corresponding specific question(s)	N/A

Table 5 (continued)

Discussion Building on Students' Thinking	Evidence of building on student thinking does not exist	Selects and/or sequences students' solutions to be discussed but does not provide any specific questions to ask related to the student work OR identifies a question to ask, but is vague about for which student solution the question is appropriate, OR simply asks students to explain or share his/her solution without specific questions that highlight mathematical ideas	Identifies specific questions that highlight the mathematics in a specific student solution	N/A
Discussion Making the Mathematics Salient	Evidence of thinking about making the mathematics of the lesson salient does not exist	Identifies questions that are vague or so few that a particular mathematical idea is not being well-developed OR expresses specific mathematical ideas that they wish to address in the discussion, but offer no specific questions to ask in order to achieve their mathematical intentions	Identifies a series of specific questions that develop mathematical ideas	N/A

The instrument has face validity for such lesson plans in this particular study, because the detailed lesson planning tool utilized by the pre-service teachers is designed to help teachers plan for and enact a lesson around a high-level task. See Hughes (2006) for examples of how the rubric assesses various elements of attention to student thinking.

The current study sought to establish a reliability of 85% or higher using two raters. Before coding the actual data, the researcher met with the second rater to introduce and explain the rubric. The second rater completed his doctoral degree at the same university and was already knowledgeable with the University's lesson planning format. As part of training, the researcher and rater coded a sample lesson plan together using the rubric. The researcher and rater independently coded a sample of 3 lesson plans (not from the actual data set). The two raters each coded 100% of the actual lesson planning data independently by using the rubric to assign scores for each of the six elements within each lesson plan. The researcher and second rater established 90% agreement from independent coding, and through discussion of discrepancies reached 100% agreement of the lesson plan data.

One limitation of the rubric is that a maximum score can be received for the anticipation dimensions by only identifying and describing possible correct and incorrect solutions, and it does not account for the quality of different types of anticipated solutions. For this reason, this study employed an additional coding method used by Smith et al, (2013) described below.

3.3.1.1 Coding the Type of Anticipated Student Responses

Drawing from Smith et al. (2013), anticipated responses produced by a pre-service teacher in collected lesson plans was coded as one of four types, increasing in quality of focus from one type to the next:

> logistics – focused on something other than learning related to the task (e.g., how students will be grouped);
>
> doing – focused on what students will actually do while engaging in the task but provided no insight into why they are doing it or how they are thinking about it (e.g., students will use a calculator, students will try to solve it algebraically);
>
> seeing – focused on what students will or will not see or recognize (e.g., students will not notice the asymptote at zero because of the way they drew the graph); and
>
> making sense – focused on making sense of what students noticed or making a connection, engaging in some action to establish meaning (e.g., students will see that there is a limit to growth and how this is represented in the equation and be able to use the equation to find the y-intercept) (Smith et al., 2013, p. 17).

In addition to anticipated student responses, this study extended this coding scheme not only to anticipation of what students would do during the lesson but also what students would learn as a result of the lesson (Personal Communication with Peg Smith, 9/11/2015). That is, the entire plan was coded for instances of logistics, doing, seeing, and making sense (not just anticipated solution strategies). For example, the mathematical goals were coded based on whether they focused on mathematics students would do, see or make sense of as a result of engaging in the lesson. Also, the planning around the discussion was coded based on whether the discussion was focusing on what students did or the mathematics the teacher wanted the students to see and/or make sense of during the summarize phase. Using two raters, the current study established 100% agreement. Specifically, the two raters independently coded 100% of the lessons for anticipations related to what students would do during the lesson and reached 87% agreement. Through discussion discrepancies were resolved and 100% agreement was reached. Also, the two raters independently coded 33% of the lessons for anticipations related to what students would learn as a result of the lesson and reached 92% agreement. Through discussion discrepancies were resolved and 100% agreement was reached. The researcher coded the remaining 67% of lessons himself for anticipations related to what students would learn as a result of the lesson.

3.3.1.2 Coding the Main Instructional Task in the Detailed Lesson Plan

To identify the main instructional task, the same criteria was used as Hughes (2006). The main instructional task was identified by meeting one or more of the following criteria:

> it was identified in the teachers' lesson plan as the main instructional task; it was described in the teachers' lesson plan as taking the largest amount of time in the lesson; it

was the central task for the lesson (i.e., not the warm-up problem or extension/homework problems) (p. 90, Hughes, 2006).

After identifying the main instructional task, the Potential of the Task (AR1) rubric from the Instructional Quality Assessment (IQA) Toolkit (Boston, 2012) was used to code the cognitive demand. The AR1 Rubric is shown in Table 6. The AR1 rubric allows for a numeric score to be given to the potential level of cognitive demand of the task, and allows for summarizing the data across lessons. The AR1 rubric also allows for distinctions to be made between the extent to which the task calls for reasoning to be explicit (i.e. difference between a 3 or a 4). If a task is separated into parts and there are discrepancies in the level of cognitive demand across parts, the overall score (from AR1) was based on the score of the part with the highest level of cognitive demand. For example consider a task with parts a and b: part a receives a 3 and part b receives a 4, then then the overall task is rated as a 4.

Table 6 AR1: Potential of the Task (Boston, 2012)

<table>
<tr><td colspan="2">Rubric AR1: Potential of the Task</td></tr>
<tr><td>4</td><td>The task has the potential to engage students in exploring and understanding the nature of mathematical concepts, procedures, and/or relationships, such as:

• Doing mathematics: using complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example); OR
• Procedures with connections: applying a broad general procedure that remains closely connected to mathematical concepts.

The task must explicitly prompt for evidence of students' reasoning and understanding.

For example, the task MAY require students to:
• solve a genuine, challenging problem for which students' reasoning is evident in their work on the task;
• develop an explanation for why formulas or procedures work;
• identify patterns and form and justify generalizations based on these patterns;
• make conjectures and support conclusions with mathematical evidence;
• make explicit connections between representations, strategies, or mathematical concepts and procedures.
• follow a prescribed procedure in order to explain/illustrate a mathematical concept, process, or relationship.</td></tr>
<tr><td>3</td><td>The task has the potential to engage students in complex thinking or in creating meaning for mathematical concepts, procedures, and/or relationships. However, the task does not warrant a "4" because:

• the task does not explicitly prompt for evidence of students' reasoning and understanding.
• students may be asked to engage in doing mathematics or procedures with connections, but the underlying mathematics in the task is not appropriate for the specific group of students (i.e., too easy <u>or</u> too hard to promote engagement with high-level cognitive demands);
• students may need to identify patterns but are not pressed for generalizations or justification;
• students may be asked to use multiple strategies or representations but the task does not explicitly prompt students to develop connections between them;
• students may be asked to make conjectures but are not asked to provide mathematical evidence or explanations to support conclusions</td></tr>
</table>

Table 6 (continued)

2	The potential of the task is limited to engaging students in using a procedure that is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task. **There is little ambiguity about what needs to be done and how to do it.** The task does not require students to make connections to the concepts or meaning underlying the procedure being used. **Focus of the task appears to be on producing correct answers rather than developing mathematical understanding (e.g., applying a specific problem solving strategy, practicing a computational algorithm).** **OR** There is evidence that the mathematical content of the task is at least 2 grade-levels below the grade of the students in the class
1	**The potential of the task is limited to engaging students in memorizing or reproducing facts, rules, formulae, or definitions. The task does not require students to make connections to the concepts or meaning that underlie the facts, rules, formulae, or definitions being memorized or reproduced.**
0	Students did not engage in a mathematical activity.
N/A	Reason:

The different dimensions of the AR1 were designed based on the different levels of cognitive demands required by students for different task types in the TAG described on page 47. The AR1 has been used in other studies for this purpose and has proven to be reliable (e.g., Boston, 2012; Switala, 2013). The researcher and second rater were trained in coding tasks for potential cognitive demand with the AR1 rubric. Each rater independently rated 100% of the tasks using the AR1 rubric. The independent coding resulted with the two raters having exact agreement on 19 of the 21 tasks, and for 2 of the tasks the two raters were off by only one scoring unit on the rubric. Through discussion, 100% agreement was reached. In the next section, coding samples of student work is discussed. The section concludes with an example taken from the study illustrating how tasks were coded for both potential and implementation.

3.3.2 Coding the Samples of Student Work

For each lesson, the samples (6 pieces of student work) from the main instructional task were coded for the level of cognitive demand evident in the students' written explanations. To assess the level of cognitive demand at which the task is implemented, the study used the AR2: Task Implementation Rubric from the IQA Toolkit (Boston, 2012). Table 7 presents the AR2 rubric. The AR2 rubric provides a holistic score for the implementation of the main instructional task by indicating the highest level of cognitive demand engaged by the majority of students in their written work (Boston, 2012).

Table 7 Rubric AR2: Implementation of the Task (Boston, 2012)

	Rubric AR2: Implementation of the Task
4	Student-work indicates use of complex and non-algorithmic thinking, problem solving, or exploring and understanding the nature of mathematical concepts, procedures, and/or relationships.*
3	Students engage in problem-solving or in creating meaning for mathematical procedures and concepts BUT the problems, concepts, or procedures do not require the extent of complex thinking as a "4"; OR The "potential of the task" on page 1 was rated as a 4 but Ss only moderately engage with the high-level demands of the task.*
2	Students engage with the task at a procedural level. Students apply a demonstrated or prescribed procedure. Students may be required to show or state the steps of their procedure, but are not required to explain or support their ideas. Students focus on correctly executing a procedure to obtain a correct answer.
1	Students engage with the task at a memorization level. Students are required to recall facts, formulas, or rules (e.g., students provide answers only). OR Students do not engage in mathematical activity.

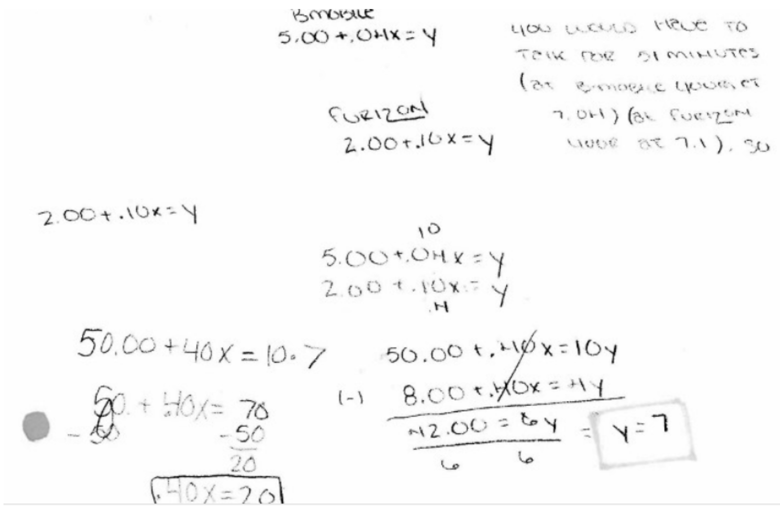
The collection of student work from each lesson was given a score of 1 thru 4 based on the corresponding specifications in the AR2 rubric. The collection was examined as a whole. For example, from a given lesson six pieces of student work are collected and scored as a whole using the AR2 rubric. The holistic score for the lesson is a 3, because the majority of pieces represented different strategies that engaged students in creating meaning connecting procedures and concepts (e.g., Boston & Wolf, 2006; Boston, 2012).

A score of 3 or higher indicates that the high-level cognitive demands of the task were maintained. This is true even if the task started with a potential score of 4. Smith et al. (2013) note that such a change in score (from 4 to 3) "is not considered a decline. Rather, it indicates that students' reasoning was not made explicit during the lesson…while the teacher did not proceduralize the task and tell students what to do or how, there was no explicit evidence of the students' reasoning and they were not pressed to provide explanations regarding why they did what they did" (Smith et al., 2013, p. 26-27). A score of 2 or lower indicates that the high-level cognitive demands of the task were not maintained. Using two raters, this study established 100% agreement. Specifically, the researcher and second rater received specific training using the AR2 rubric prior to each rater independently coding 100% of the tasks. The two raters had exact agreement on 18 of 21 student work samples, and for 3 of the tasks the two raters were off by only one scoring unit on the rubric. Through discussion, 100% agreement was reached. Please see the task and actual student work sample submitted by the intern David for his second lesson in Figure 10. An explanation of the coding of task potential and implementation follows the example.

Cell phone company B-Mobile charges a base rate of \$5.00 per month plus 4 cents a minute that you are on the phone. Furizon charges a base rate of only \$2.00 per month but they charge you 10 cents per minute used. How much time per month would you have to talk on the phone before subscribing to B-Mobile would save you money? (Institute For Learning, University of Pittsburgh, 2006)

Figure 10 Task and Student Work from David's Second Lesson

Figure 10 (continued)



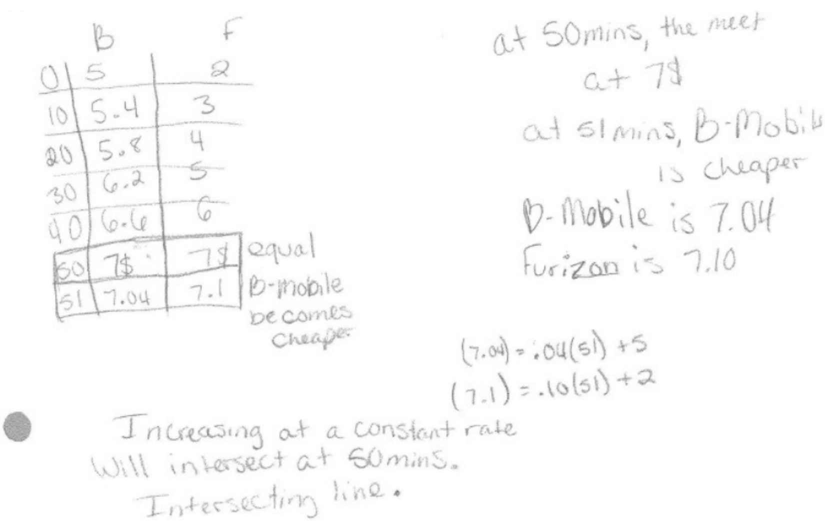


Figure 10 (continued)

B--Mobile = $5.00+.04x

Furizon = $2.00+.10x

5+.04x30=6.2
2+.10x30=5
6.2≠5

5+.04x3=5.12
2+.10x3=2.3
5.12≠2.3

5+.04x50=7
2+.10x50=7
7=7

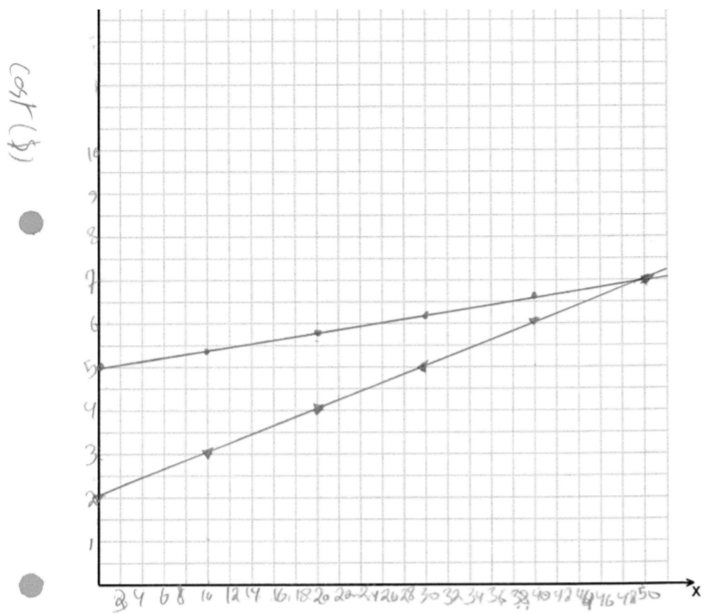


Figure 10 (continued)

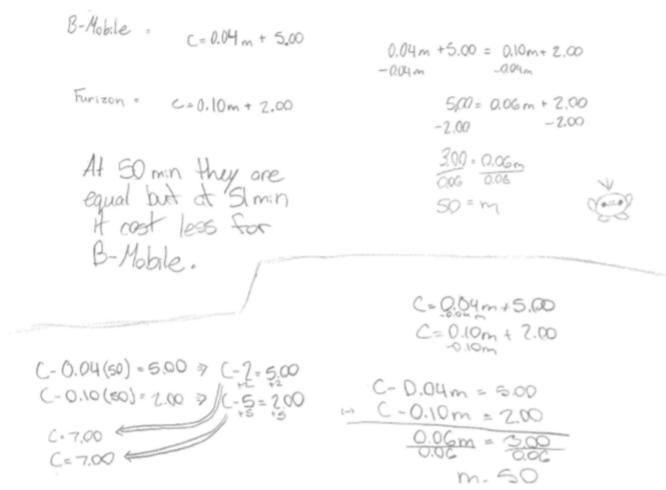
At first I put $5x + .04$ but then I noticed that was backwards So I made it $.04x + 5$. Then I put $.10x + 2$ also in the calculator and kept going down till they was equal and then the B-mobile was cheaper.

Minutes	B-mobile	Furizon
10	5.4	3
20	5.8	4
30	6.2	5
40	6.6	6
50	7	7
60	7.4	8

STUDENT 4
(CONT.)

$0.04m + 5 = .10m + 2$
$-.04 \quad\quad -.04$
$5 - 2 = .06 + 2$
$\frac{3}{.06} = \frac{0.06}{.06}$
$50 = m$

Figure 10 (continued)



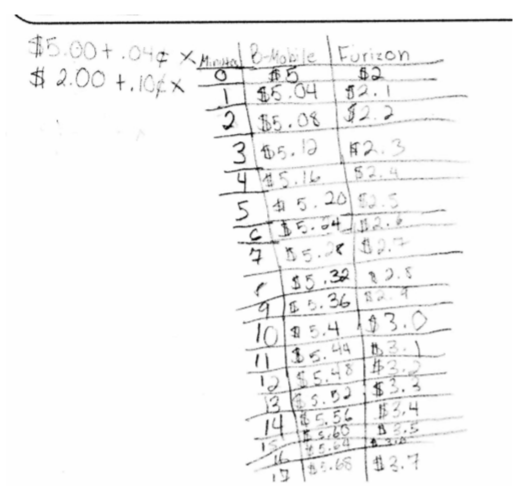
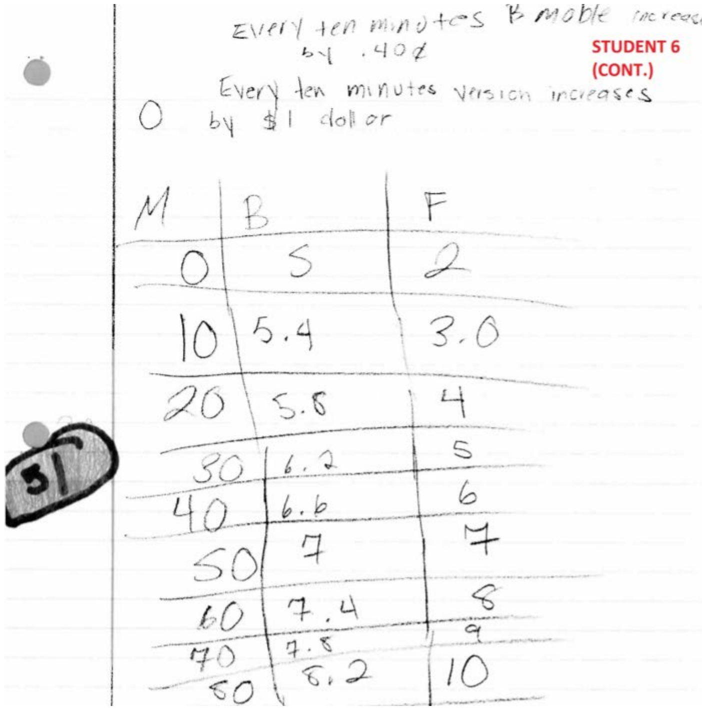


Figure 10 (continued)



The potential of this task received a score of 3 from the IQA AR1 Rubric because the task does not suggest a particular strategy or pathway, and students may engage in doing math or procedures with connections. The task did not warrant a 4 because it does not explicitly prompt students for evidence of reasoning or understanding.

The implementation of this task also received a score of 3 from the IQA AR2 Rubric because students engaged in problem solving and used multiple strategies to solve the problems. The students created meaning for mathematical procedures (i.e. tables, graphs, and systems of equations) within the real world context of the problem. The rating of student work did not warrant a 4 because students did not provide evidence of understanding how the different procedures were related, and the use of the procedures did not require the complex thinking necessary to receive a score of 4. Still, the task was considered to have high level potential, and the high level cognitive demands were maintained during implementation.

3.3.3 Coding the Interview Data

Following the transcription of the interview audio-tape, the transcripts were coded. The coding was performed in three general steps: 1) segmenting the data based on the context of the response; 2) identifying thematic responses that could be labeled as *student oriented* or *teacher oriented*. Each step with definitions of terms is described below. After the steps are described a table illustrating examples is provided.

A segment is defined as a complete thought expressed by the participant related to a specific topic (Guest, MacQueen, & Namey, 2012). The researcher performed segmentation by identifying a beginning and ending point for each segment (Guest et al., 2012). A complete thought may vary in length, and may include an exchange between participant and researcher. According to Guest et al., (2012), "the boundaries of a given segment should allow the thematic features of the segment to be clearly discerned when it is lifted from the larger context" (p. 52). Given the structure of the particular interview protocol, a segment will be defined by the nature of the response to a particular question.

The main question in the protocol asks: *Did your lesson go as planned? Why or why not?* Given the nature of the question, the participant was likely to respond whether the lesson went as planned and provide a response as to why. The response as to why is likely to be one of two natures: 1.) the participant provided an explanation as to what it means for the lesson to go (or not go) as planned. 2.) the participant provided reasons why the lesson did go (or didn't go) as planned. The protocol only has two follow up questions that are also aligned with these two possibilities. Thus, a segment will be labeled in one of four ways: 1.) *what it means for lesson to go as planned*; 2.) *what it means for lesson to not go as planned*; 3.) *reasons why lesson went as planned*; 4.) *reasons why lesson did not go as planned*.

Following segmentation, the second step began where each segment was labeled as *teacher oriented, student oriented* or *other*. Teacher oriented means the response was focused on something the teacher did during the lesson to explain what it meant whether it went as planned or provide a reason why. Student oriented means the response was focused on something the students did during the lesson to explain what it meant whether it went as planned or provide a reason why. Other means the response was not teacher or student oriented. Table 8 provides examples from the actual data of each segment type and associated orientation. Any response, or portion of response, did not receive any segment label if the participant is referring to a lesson that is different than the lesson the interview follows.

Table 8 Interview Segment Label, Brief Example Response, Orientation

Segment Label	Brief Example Response	Orientation
What it Means for Lesson to Go as Planned	Everything that I had planned to do from um my task in doing the detailed lesson plan, I kind of stuck to that	Teacher
	Students seemed to be picking up on the concept	Student
What it Means for Lesson to Not Go as Planned	Some groups didn't get as far (as planned)	Student
	Students didn't really work as hard as I would have hoped	Student
Reasons Why Lesson Went As Planned	They are very energetic and enjoy diving into mathematics…	Student
	I think something that helped it go as planned, I kind of like modeled for them an example	Teacher
Reasons Why Lesson Did Not Go as Planned	It took far longer than expected…behaviors, um students talking, uh students disrupting…	Student
	We were running out of time	Other

As indicated in Table 10, the segment label distinguishes whether the participant was talking about what it means or reasons why the lesson did (or did not) go as planned. The orientation distinguishes whether the segment was focused on the teacher, student or something else. At each of the two steps described above, the researcher and a second coder independently coded all of the transcripts. Initial agreement reached the desired 85% reliability level. Through discussion, the study established agreement of 100% using this coding scheme.

3.4 DATA ANALYSIS

Table 9 provides a summary of data sources, corresponding coding tools and outcomes from coding tool.

Table 9 Summary of Coding

Data Source		Coding Tool	Possible Outcome
Written Lesson Plans:	Overall Plan	Attention to student thinking rubric	0-14
	Main Task	AR1 Rubric	0-4
	Anticipation of what students will do or learn	Quality of anticipations rubric	Logistic, doing, seeing, making sense
Student Work Samples		AR2 Rubric	1-4
Interviews		Emergent coding	Teacher, student, or other orientation

All written lesson plans were scored with the attention to student thinking scoring rubric (Hughes, 2006), and all tasks and samples of student work were scored with the AR1: Task Potential and AR2: Task Implementation rubrics, respectively. For every lesson plan each of the six elements related to attending to students' mathematical thinking received a score. Also each lesson plan received an overall score for attention to students' mathematical thinking that is the sum of scores received for each of the six elements. Thus, lesson plan scores can range from 0 to 14. Each lesson plan was also analyzed for quality of anticipation related to what students would do during the lesson or learn as a result of the lesson (Smith et. al., 2013)

Each task as it appears in the lesson plan was coded on a scale from 0 to 4 using the AR1 Rubric from IQA Toolkit: high level (a score of 3 or 4) or as low-level (a score 0, 1 or 2). Each sample of six pieces of student work was coded on a scale of 1-4 using the AR2 Rubric from the IQA Toolkit to measure the level of cognitive demand at which the task is implemented. If a task started at 3 or 4 for potential and the sample of student work remains at 3 or 4 for implementation, then it was concluded that the level of cognitive demand was maintained at a high level during the lesson.

The research question was addressed by analyzing the relationship between attention to student thinking during planning and level of cognitive demand of task implementation both quantitatively and qualitatively. The quantitative relationship between total planning score and task implementation level was examined through logistic regression analysis. Also, logistic regression analysis was used to explore relationships between individual elements of attention to student thinking and task implementation. In addition to the written lesson plans, other data sources were drawn upon qualitatively to provide examples that illustrate the quantitative trend and provide possible explanations why certain lessons don't fit the quantitative trend.

3.4.1 Quantitative Analysis: Lesson Planning Scores vs. Implementation

The quantitative analysis focused on the relationship between the total lesson planning score (sum of all six elements each plan) and associated task implementation scores for the lessons of each pre-service teacher. The analysis also focused on the relationship between each individual element's score and the associated task implementation scores for the lessons of each pre-service teacher. All lesson planning scores, task potential scores, and task implementation scores that were entered into a STATA software spreadsheet. Table 12 in the next chapter indicates how the data was entered.

After the data was entered, the researcher performed logistic regression using STATA software to explore the relationship between the total lesson planning score as well as the score of each of the six individual elements of teachers' attention to student thinking during lesson planning (independent variables) and the level at which tasks are implemented (dependent variable). This type of analysis nested the data by pre-service teacher. For example, when running the regression on total lesson planning score versus implementation, the total lesson planning score results produced by the same pre-service teacher were considered one data point (as were the implementation results for that pre-service teacher). Nesting the data in this way took into consideration that planning and implementation scores were dependent upon the pre-service teacher producing them. A limitation to the analysis was the very small sample size (n=21) of the data set which were nested across six individuals. In consideration of this limitation, the purpose of the quantitative analysis was not to make causal claims about the relationship between planning scores and task implementation scores. Rather, the primary purpose is to detect if any relationship existed for this particular data set that would warrant the same analysis in future studies.

In order to run logistic regression, the implementation scores were dichotomized. A new variable "HighImp" (high implementation) was created. In the "HighImp" column, 1s were placed next to implementation scores of 3 or 4, and 0s were placed next to implementation scores of 1 or 2. Please recall, according to the AR2 rubric, an implementation score of 3 or 4 indicates the high level cognitive demands of the task were maintained during implementation. The analysis was run using total lesson plan score as a continuous independent variable and "HighImp" as the dependent variable. It was also run using each individual element as an *interval* score for the independent variable and "HighImp" as the dependent variable.

By dichotomizing the implementation scores the analysis was comparing total lesson planning scores for lessons implemented at a high level of cognitive demand to total lesson planning scores for lessons implemented at a low level of cognitive demand. For the individual elements, the analysis was run by identifying each element as an interval variable (as opposed to continuous). This was due to the small range of possible scores that could only be integer values for each element. For example, Mathematical Goal scores could only by 0, 1, or 2 and Anticipating Students' Thinking Scores could only be 0,1,2, or 3. The interpretation of logistic regression analysis involves odds ratios. Specific results and interpretations are discussed in the next chapter. As mentioned earlier, the quantitative analysis nests the data by pre-service teacher. That is, it was not treating the scores for each lesson as independent event but rather considering it in relation to the pre-service teacher who produced the lesson.

Prior research guides the results of the analysis by providing an indication of what can be considered overall high planning scores and overall high implementation for a single pre-service teacher. Hughes' (2006) study pertaining to attention to student thinking during lesson planning, and recent work involving the IQA toolkit (Wilhelm & Kim, 2015) help provide this guidance. Hughes (2006) provided a "snapshot" of a single pre-service teacher's attention to student thinking during lesson planning by calculating average lesson planning scores across lessons planned during his field experience, and also indicated that individual pre-service teacher's planning scores were consistently close when planning detailed lessons with the TTLP.

Also, Hughes (2006) suggests that a total score of 9 from the elements rubric is considered meaningful attention to student thinking. In relation to task implementation, recent work has found that for teachers who have had training in reform oriented teaching (i.e. Math Task Framework, using launch, explore, summarize, using the Five Practices) as little as three lessons is enough to achieve a reliable measure of instructional practice using the IQA Toolkit (Wilhelm & Kim, 2015). (For teachers who have not had such training, as many as 9 lessons is required to achieve a reliable measure using the IQA Toolkit due to variability.) Since the pre-service teachers in this study have had training in reform oriented teaching focusing on whole class discussion, a minimum of three lessons being collected is enough to gain a reasonably accurate picture of their ability for task implementation.

3.4.2 Descriptive Data to Support/Explanation of Quantitative Trend

Consideration of the lesson planning “snapshot” (Hughes, 2006) and the reliable measure of teacher task implementation (Wilhem & Kim, 2015) made it possible to situate pre-service teachers along a trend detected by the quantitative analysis. Illustrations and explanations of how pre-service teachers fit along the trend are discussed in the next chapter. In order to provide those illustrations and explanations the enacted lessons were also analyzed qualitatively. The qualitative analysis drew on the coding of the detailed written lesson plans, and other data sources including the tasks as they appear in lesson plans, samples of student work, and post-lesson interview transcripts. To provide an overall portrait of the attention to student thinking in written lesson plans, the coding for quality of anticipation (Smith et al., 2013) with regard to what students would do during the lesson and learn as a result of the lesson was added. Actual tasks in the lesson plans that were coded for potential cognitive demand with the IQA AR1 Task Potential rubric are provided, and examples of student work that were coded with the IQA AR2 Task Implementation Rubric to assess the cognitive demand at which students engaged the task are also presented. To provide contextual qualitative evidence of attention to student thinking with regard to planning around the task, interview transcripts, cover sheets and written reflections are also drawn upon. Recall, interview responses were coded based on the focus of orientation (i.e. on students’ thinking or actions, or on what the teacher did during the lesson, or something other than students or teacher). Task cover sheets and written reflections were not coded, but used to illustrate and/or support other findings. For example, task source information was taken from the coversheet, and the pre-service teachers expectations were also used when discussing task selection and implementation. Evidence from the written reflections was used to illustrate the pre-service teacher's thoughts about whether the lesson went as planned.

The first part of the qualitative analysis was examining the general planning structure utilized by each pre-service teacher. Secondly, each lesson was examined based on the chronological progression in which the data should have been generated: the mathematical goal, selected task and expectations for student work on the task, planning around the task, task implementation, and post-lesson thoughts. For example, entire written lesson plans (beginning with mathematical goals) were analyzed based on instances of quality to which they anticipated what students would do during the lesson or see and/or make sense of a result of the lesson. Also, based on the total score for attention to the elements of student thinking each plan was classified as having a low, moderate, or high attention to student thinking score. A high score was 9 or greater (Hughes, 2006). A moderate score was a 7 or 8, and a low score was 6 or less.

The selected tasks were analyzed in relation to the specific mathematics described in the expectations of high, medium and low quality work provided by the pre-service teacher. Specific strategies present in student work were analyzed in comparison to expectations and anticipations in the lesson plan. The coding of post-lesson thoughts analyzed links between orientations of focus (student or teacher) and task implementation.

3.5 SUMMARY

The study drew upon the lesson planning data and student work to describe any possible links between attention to student thinking during lesson planning and the cognitive demand evident in student work during task implementation. In addition to a quantitative relationship, it sought to qualitatively examine links between pre-service teachers' planning and task implementation to illustrate the relationship and possibly explain any inconsistencies. This was accomplished through the analysis of a variety of data sources using multiple coding schemes.

4.0 RESULTS

This chapter is organized by first presenting the results of quantitative analysis, and then presenting the results of the qualitative analysis to provide support and possible explanations in addressing the following research question:

> What is the relationship between pre-service teachers' attention to student thinking with regard to lesson planning around a mathematical task (perceived to be high level by the pre-service teacher), and the level of cognitive demand at which the mathematical task is implemented?

The quantitative results are representative of the collective group, and explore the relationship between attention to the elements of student thinking scores and task implementation scores. The results from each individual lesson plans, task potential and task implementation are shown in Table 10. The qualitative results draw on additional data sources generated from the point of view of the pre-service teachers, and are presented to describe and explain the relationship between attention to student thinking during lesson planning and task implementation indicated by the quantitative trend. Following the presentation of the qualitative results, patterns or trends that emerged from across the six pre-service teachers are described.

Table 10 All Scores for Lesson Plans, Task Potential and Task Implementation

	Lesson Plans							Task	
Teacher Initials (Pseudonym) and Lesson	Goal	Anticipate Student Thinking		Questions	Discussion		Total	Potential	Implementation
		Correct	Incorrect		Builds on Student thinking	Makes Math Salient			
(Max. Poss)	(2)	(3)	(3)	(2)	(2)	(2)	(14)	(4)	(4)
D.U. 1	2	3	3	2	2	2	14	4	3
D.U.2	2	3	3	2	2	2	14	3	3
D.U.3	1	3	3	1	0	0	8	4	4
D.U.4	1	3	2	2	2	2	12	3	3
Q.B.1	2	1	1	2	2	2	10	4	3
Q.B. 2	2	3	3	2	2	1	13	3	3
Q.B. 3	1	3	3	2	1	2	12	3	3
Q.B. 4	2	1	0	2	2	1	8	4	3
M.T.1	2	1	3	2	2	2	12	4	4
M.T 2	2	0	3	1	0	0	6	3	3
M.T. 3	2	1	1	0	0	0	4	3	2
R.N.1	1	1	2	2	1	0	7	3	3
R.N.2	1	3	2	2	2	2	12	4	2
R.N.3	1	3	0	2	0	0	6	4	2
N.N 1	1	1	3	2	1	0	8	3	3
N.N. 2	1	0	1	2	1	1	6	3	1
N.N. 3	1	0	1	2	1	1	6	3	3
N.N. 4	1	0	3	2	1	1	8	2	1
C.C.1	2	1	1	1	1	1	7	2	2

Table 10 (continued)

C.C.2	1	1	0	0	1	0	3	3	2
C.C.3	1	0	0	1	0	1	3	2	1

The data in a single row of Table 10 represents the written lesson plan scores, task potential score and task implementation score for one lesson enacted by a pre-service teacher. For example, the data in the row labeled C.C.3 are the scores from Chris Cain's third lesson. The order of lessons for each pre-service teacher is chronologically based on enactment date of the lesson. The order of the pre-service teachers is based on the average implementation score. For example, David is first because his average implementation score across four lessons is 3.25. Quinn is second with an average across four lessons equal to 3. Marian also has an average equal to 3; however that is based on three lessons.

4.1 QUANTITATIVE RELATIONSHIP BETWEEM ATTENTION TO ELEMENTS OF STUDENT THINKING IN PLANNING VS. COGNITIVE DEMAND OF TASK IMPLEMENTATION

As mentioned in the analysis section, there is a limitation to consider when looking at these results. That is, the study examined a small number of lessons (n=21) over which the relationship between attention to student thinking in lesson planning and task implementation is explored. With this consideration in mind, the following results are presented.

The first result is generated from running logistic regression using total lesson planning score as a continuous independent variable and "HighImp" as the dichotomized dependent variable. Recall, by dichotomizing the implementation scores, this analysis is comparing total attention to student thinking during planning scores for lessons implemented at a high level to total scores for lessons implemented at a low level. The results are presented in Table 11. "Coef" represents the value of the independent variable. "SE" represents the standard error and "OR" represents the odds ratio.

Table 11 High Implementation vs. Total Lesson Plan Score (Logistic Regression)

HighImp	Coef	SE	OR
Total LP Score	0.46^{**}	0.22	1.58

Note. N = 21 across 6 groups
$^{*}p < .1$. $^{**}p<.05$.

The p-value (p = 0.033) indicates that there is a significant relationship between total lesson planning score and high task implementation. The interpretation of the coefficient (0.46) from the logistic regression results in Table 13 requires the use of odds ratios. The odds ratio is calculated by $e^{0.4609475} \approx 1.58$. This suggests that as planning scores increase by a score of 1 the odds of high task implementation are 1.58 times larger than the odds of low task implementation. More specifically, it suggests that as attention to student thinking across the six elements of planning increases, the odds of students' implementing the task at a high level of cognitive demand are larger than them implementing the task at a low level of cognitive demand.

In addition to the total lesson plan score analysis, logistic regression was run with each individual element being the independent variable, and "HighImp" being the dependent variable.

Recall, the scores for each individual element were run as interval variables (as opposed to continuous) due to the small range of possible scores that could only be integer values for each element. The results of this analysis for the Mathematical Goal element are shown in Table 12.

Table 12 High Implementation vs. Mathematical Goal Element Score (Logistic Regression)

HighImp	Coef	SE	OR
Goal Score = 2	1.15	1.42	3.16

Note. N = 21 across 6 groups
$^{*}p < .1$. $^{**}p<.05$.

This particular analysis compares the implementation of lessons with the highest goal score received (i.e. score of 2) with the implementation of lessons with the lowest goal score received (i.e. score of 1, note: no lesson received a 0 for goal element which means every lesson contained a goal statement). The p-value (p = 0.418) suggests there is no significant difference with regard to implementation for lesson plans receiving a goal element score of 2 versus lesson plans receiving a goal element score of 1. Four of the remaining five elements also suggest there was no relation between different scores for each particular element and task implementation. These elements include Anticipating Students Correct Thinking, Questions that Assess/Advance Student Thinking, Discussion Builds on Student Thinking, and Discussion Makes Math Salient.

There were results for certain elements that do suggest a relationship or at least trend between lesson planning element scores and task implementation. In particular, the elements of Anticipating Students' Incorrect Thinking has a marginally significant p-value, and the results from the Discussion Builds on Student Thinking element suggested a trend between planning scores and task implementation. The results for the Anticipating Students' Incorrect Thinking element are shown in Table 13.

Table 13 "HighImp" vs. Anticipating Students' Incorrect Thinking (Logistic Regression)

HighImp	Coef	SE	OR
Anticipate Incorrect Score = 1	0.78	1.71	2.17
Anticipate Incorrect Score = 2	2.11	2.08	8.22
Anticipate Incorrect Score = 3	3.29^{*}	1.84	26.87

Note. N = 21 across 6 groups
$^{*}p < .1$. $^{**}p<.05$.

To put the element scores in context, a score of 3 for anticipating students' incorrect thinking meant that the lesson plan specifically described several incorrect ways students might approach the problem and identifies misconceptions the students might have. A score of 2 meant that the lesson plan specifically described one incorrect way but did not identify the many challenges or misconceptions students might have. A score of 1 meant the lesson plan vaguely described incorrect ways students might approach the problem. A score of 0 meant that the lesson plan did not provide any evidence of anticipating students' incorrect thinking. As the Anticipating Students' Incorrect Thinking scores increase (from 1 to 2 to 3), the p-values decrease and become closer to being significant. Also, the p-value (p = 0.074) for lesson plans with a score of 3 is marginally significant. The odd's ratio is $e^{3.290899} \approx 26.87$. This suggests that as planning scores for this particular dimension increase from 0 to 3, the odds of high task implementation are 26.87 times larger than the odds of low task implementation.

More specifically, the increasing trend in the coefficients suggests that as attention to this particular element of student thinking increases, the odds of high level task implementation become greater than the odds of low level task implementation for lesson plans that attended to this element compared to lessons that did not attend to it all. For example, the odds of high level task implementation for lessons with a score of 3 in anticipating students' incorrect thinking (when compared to lessons with a score of 0 for the element) are greater than the odds of high level task implementation for lessons with a score of 2 for the element (when compared to lessons with a score of 0 for the element).

A somewhat similar trend found with the Discussion Builds on Students' Thinking element. As the Discussion Builds on Student Thinking scores increased (from 1 to 2), the p-values decreased. As mentioned earlier, the p-values were non-significant; however, the p-values had a downward trend, and the coefficients are increasing. This suggests that as scores for this element increase, the odds of high level task implementation become greater than the odds of low level task implementation for lesson plans that attended to this element compared to lessons that did not attend to it all. As mentioned earlier, due to the small data set, it is difficult to draw any firm conclusions; however the trend towards significance as scores increase for these two elements suggests that future studies using such analysis could be warranted.

Table 14 summarizes task potential and implementation results for all 21 enacted lessons. As the table shows, 13 lessons started with high level tasks and remained as high level during implementation. Also, 5 lessons that started with high level tasks declined during implementation. The remaining 3 lessons started with low level tasks that remained low level during implementation.

Table 14 Task Potential and Implementation Summary of Results for All Lessons

	High Implementation	Low Implementation	Total
High Potential	13	5	18
Low Potential	0	3	3
Total	13	8	21

An additional logistic regression analysis was run to control for the potential of the task. More specifically, the analysis was run to gain insight into the extent to which the total score of planning (versus high level task selection) is responsible for high implementation scores. In order to control for potential, the variable “HighPot” was created to dichotomize potential scores. All tasks with potential of 3 or 4 received a 1, and all tasks with a potential of 1 or 2 received a 0. Table 15 shows the results of logistic regression with “HighImp” as the dependent variable and Total Lesson planning scores and “HighPot” as the independent variables.

Table 15 High Implementation vs. Total Lesson Plan Score Controlled for Potential

HighImp	Coef	SE	OR
Total LP Score	0.44*	0.24	1.55
"HighPot"	19.14	2873.46	205303514.4

Note. N = 18 across 6 groups
*p < .1. **p<.05.

The model of analysis that provides results in Table 15 excludes the implementation scores for tasks that were low potential to begin with when running Total LP score as the independent variable. More specifically, there were 3 lessons that had tasks with low cognitive demand potential, and all three were implemented at a low level of cognitive demand. The planning and implementation scores for these lessons were not included when the analysis was run to generate the results in Table 15. For these scores there is no variation to be explained by Total LP score because the low implementation is likely contributed to the low potential. Also, in relation to the coefficient for "HighPot", there is no variance between high potential scores and high implementation scores.

Basically, the results in Table 15 are generated from a model that is using Total LP to explain the relationship between planning and implementation for the 18 lessons that had tasks with high level cognitive demand potential. More specifically, it explains differences in planning scores between the 5 lessons in which the cognitive demands declined during implementation, and the 13 lessons in which the high-level cognitive demands were maintained. The p-value (p = 0.071) for Total LP Score in Table 15 is marginally significant. The odds ratio for Total Lesson planning score is approximately 1.55 which is similar to the approximate 1.58 when potential was not controlled for. Thus, even while controlling for potential, as attention to student thinking in lesson planning increases, the odds of students engaging the task at a high level of cognitive demand are larger than them engaging the task at a low level of cognitive demand.

As mentioned earlier, due to the very small sample size these results are not intended for causal claims. The purpose of the analysis was to examine relationships to determine if such analyses would be warranted in future studies with larger data sets. Logistic regression suggested a significant relationship between total lesson plan score for attention to student thinking and task implementation, and a marginally significant relationship between attention to anticipating students' incorrect thinking and task implementation. Logistic regression also indicated a trend in the results for the Discussion Builds on Student Thinking element suggesting that increased scores in that particular dimension may be linked to high level task implementation. These findings suggest that future studies with larger sample sizes could use these analyses to find more accurate and meaningful results.

Taking a "snapshot" look at the average total lesson planning score (Hughes, 2006) of each pre-service teacher in relation to their implementation "average" discussed in the analysis section provides context to the quantitative trend just discussed. Table 16 shows the total lesson planning average and information related to task the implementation. The researcher recognizes limitations to the "raw average" implementation statistic; however, it is provided since it follows the planning/implementation trend.

Table 16 Average Total Lesson Planning Score versus Implementation "Average"

Pre-service Teacher	Average Total Lesson Planning Score	Total Number of Lessons Implemented High or Low	
		Low	High
Chris	4.33	3	0
Nick	7	2	2
Renee	8.33	2	1
Marian	7.33	1	2
Quinn	10.75	0	4
David	12	0	4

Recall the quantitative analysis performed by the logistic regression nested the data by pre-service teacher. The results in Table 16 illustrate the basic idea of the total attention to student thinking score and implementation trend found by the analysis which was as planning scores increase the odds of high task implementation (versus low) also increase. The results in Table 16 are ordered from along a continuum of low planning – low implementation to high planning – high implementation.

Begin with Chris whose total planning average is 4.33 (indicating low attention across the six elements of student thinking), and he has 0 tasks implemented at a high level with a raw average of 1.67. Now consider, Nick, Renee and Marian whose lesson planning averages represent an increase compared to Chris's, and the number of lessons with tasks implemented at a high level also increase compared to Chris. Finally consider, Quinn and David whose lesson planning averages represent an increase compared to the other pre-service teachers, and the number of lessons with tasks implemented at a high level also increase in comparison. Also note is that Quinn and David were the only pre-service teachers with average planning scores greater than 9 which was suggested to represent meaningful attention to student thinking (Hughes, 2006). They are also the only two pre-service teachers who implemented tasks with high level cognitive demands across all of their lessons.

4.2 QUALITATIVE RESULTS SUPPORTING AND EXPLAINING TREND BETWEEN ATTENTION TO STUDENT THINKING DURING LESSON PLANNING AND TASK IMPLEMENTATION

The quantitative analysis just discussed suggested a significant positive relationship between total scores for attention to elements of student thinking during lesson planning and scores related to students' engagement with the task. Qualitative analysis expands on this result by providing specific lesson based evidence to support and explain the relationship between overall planning and task implementation for each pre-service teacher.

In this section, discussion of each pre-service teacher is presented in reverse order of the results indicated in Table 16. That is, the discussion begins with the high planners/high implementers (David and Quinn) and progresses to the discussion of the low planner/low implementer (Chris). The purpose of this progression is that some of the individual lesson results of the high planners/high implementers are similar to the results of individual lessons of other teachers. By discussing the results high planning/high implementation first certain findings can be drawn upon later and redundancy is avoided.

Both David and Quinn planned and enacted a total of four lessons for the study. For both David and Quinn, three of their four lessons are consistent with the quantitative trend just discussed as they all received high total planning scores and all were implemented at a high level of cognitive demand. Both David and Quinn also each have one lesson that has a total planning score of 8 (which is not consistent with the other high scores) but still the lesson was implemented at a high level of cognitive demand. Thus, for David and Quinn, the qualitative results of one lesson consistent with the quantitative findings are discussed to illustrate the trend, and the one lesson with moderate planning is discussed to provide possible explanations for the inconsistency. For the other pre-service teachers the selection of results presented is based on the idea that particular findings support or add to results that have already been discussed. The specific reasons for the selection of particular results are presented within the discussion of each pre-service teacher. While there are consistencies between pre-service teachers, the results of each pre-service teacher is separately presented in its own section.

The qualitative discussion of each pre-service teacher's results begins with a general overview of their lesson plans (i.e. common structure across plans and self-planned versus resource based plan), and an overview of the qualitative results for each lesson they planned and enacted as part of the study. Please recall the qualitative analysis of each lesson for each pre-service teacher involved examining the chronological sequence of planning and enactment. That is, the examination focused on goals, selected task and expectations for student work, planning around the task, task implementation, and post lesson thoughts. For several of the individual lessons that will be discussed, the same chronological progression is used to present the results.

For this study, an *unmodified* plan is defined as a lesson plan that was taken directly from a resource (i.e. website, textbook, or a university resource base), and submitted (by the pre-service teacher) as the lesson plan used for a given lesson for this study. A self-written lesson plan is defined as a lesson plan that was written by the pre-service teachers using what appeared to be a self-designed structure or within an already existing template. That is, the pre-service teacher did not submit the lesson plan exactly as it appeared from another resource. For example, a lesson designed within the EPT is considered self-written because the pre-service teacher wrote it within the EPT template. Also, a self-written plan may have drawn on other resources, but those resources were included within the general self-designed (or template) structure of the plan. The extent to which pre-service teachers drew on resources in all of their lessons is discussed later in this chapter after the results of each teacher are presented.

4.3 DAVID UPTON

For his written lesson plans, David drew heavily upon already existing resources. More specifically, David used three unmodified lesson plans, and he had one self-written plan. Two of the three unmodified plans are identical in structure since they were designed by the same university following the TTLP format. The third unmodified plan and self-written plan were not identical in structure, but still addressed the launch, explore and summarize phases of a typical standards based lesson. All of David's lesson plans addressed setting up the task, included anticipation of students' correct and incorrect thinking, and included a portion for the facilitation of classroom discussion.

All of David's lesson plans included more than one mathematical goal, and all the lesson plans were designed around tasks with high-level cognitive demand potential. A summary of results from David's lessons are provided in Table 17. Overall, the lesson plans indicate a high degree of attention to student thinking. During the enactment of all four lessons, David's students engaged the tasks in ways that maintained the high level cognitive demands. In general, David thought his lessons went as he planned, and he often focused on what students were doing during the lesson when explaining what it meant for the lesson to go as planned.

Table 17 Summary of Results from David Upton's Enacted Lessons

Lesson	Attention to Student Thinking During Planning			Implemented	Post-lesson Thoughts
	Goal(s)	Task Selected	Planning Around Task		
1 (PTR)	Making Sense	High	Unmodified Plan High Element Score Seeing & Making Sense	High	Met Goals Student Focus
2	See & Making Sense	High	Unmodified Plan High Element Score Seeing & Making Sense	High	As Planned Teacher Focus
3	Doing	High	Self-written Moderate Element Score Making Sense	High	As Planned Not as Planned Student Focus
4	Doing	High	Unmodified Plan High Element Score Seeing & Making Sense	High	As Planned Student Focus Teacher Focus

David's lesson planning and task implementation are relatively consistent across his four lessons; however, lesson 3 indicates an inconsistency in relation to attention to the elements of student thinking in the written lesson plan. Interestingly, lesson 3 is the one lesson for which David wrote the lesson plan himself. The remainder of this section seeks to gain further insight into the relationship that David exhibited between attention to student thinking during planning and task implementation. To accomplish this, discussions of one lesson for which David used an unmodified plan, and the self-written lesson are now presented.

4.3.1 David Upton's Lesson Using Unmodified Plan

For his second lesson, David selected a task commonly used in his teacher education program, and he used a lesson plan that was designed by the university that he attends. Interns engage with the Calling Plans Task in their university coursework, and participate in learning around it. The lesson plan accompanying the task is a model plan designed using the TTLP. This lesson is selected to serve as an example of the three resource based lessons David enacted during this study. All unmodified plans indicated a high degree of attention to the elements of student thinking and all the lessons associated with those plans had high level task implementation.

4.3.1.1 Attention to Student Thinking During Planning

(a) Mathematical Goals

The lesson plan specifies one goal in particular that addresses what it means for students to understand solutions to systems of equations:

> The graphical solution to a system of linear equations is the point of intersection of the lines and represents the one coordinate pair that is a solution to both of the equations. (DU, LP2, p. 1)

The goal addresses mathematics related to a system of equations that students should be able to see and make sense of as a result of the lesson. For example, students should realize that the point of intersection on the graph is the solution to the system, and make sense of the solution by understanding what it represents in relation to both equations.

(b) Selecting a Mathematical Task

In order to achieve the learning goals, the lesson is designed around a phone calling plans situation that can be represented using systems of equations. David slightly modified the task from the resource by changing the names of the companies. The task is shown in Figure 11.

Cell phone company B-Mobile charges a base rate of \$5.00 per month plus 4 cents a minute that you are on the phone. Cell phone company Furizon charges a base rate of only \$2.00 per month but they charge you 10 cents per minute used. How much time per month would you have to talk on the phone before subscribing to B-Mobile would save you money? (Institute for Learning, University of Pittsburgh, 2006)

Figure 11 Calling Plans Task Used in David's Lesson

The task received a score of 3 according to the AR1 Potential rubric since it has the potential for students to engage in high-level cognitive demands as they draw connections using a variety of strategies. The task did not receive a score of 4 because it does not explicitly prompt students to explain their reasoning.

David's expectations are that "students are to discuss the task collaboratively in groups and understand that multiple approaches may be found. Students are expected to be able to present their ideas to the class and participate in explaining and discussing the methods to solve the task". David's considerations of high, medium and low quality work generally focus on what a single “student” should produce. For example, for high level work, David expects each student to solve the problem in multiple ways, be able to justify his/her work and share the methods with other students. Medium quality work is when a single student solving the problem in 1 or 2 ways and is able to explain work. Following the same trend, low quality work is partially described as student only solving the problem 1 way. David does not identify any particular solutions or mathematical concepts related to the task in distinguishing between different qualities of work.

(c) **Planning Around the Task**

Overall, the lesson plan aligns with the goals, and provides evidence of a high level of attention to student thinking as students engage and discuss the task. The lesson plan received a score of 14 out of a possible 14 points when rating the written lesson plan with the elements of attention to student thinking scoring rubric (Hughes, 2006). According to David, the written lesson plan was drawn from his university resources and he used it as is. That is, he did not alter any parts of the lesson plan.

The total score of 14 indicates that David’s lesson plan earned a maximum score in all six of the attention to student thinking elements. This makes sense because the lesson plan is designed to be a model lesson plan by university educators, and it was written following the TTLP model which addresses all of the dimensions.

The explore phase included several possible solutions and each possible solution included a worked out example, questions for the teacher to ask specifically related to the solution, possible misconceptions/errors, and questions to address those particular misconceptions. In total, there were 16 anticipated student solutions in the lesson plan. The majority of anticipations were coded as *doing*; however, there were also *seeing* and *making sense* anticipations. The fact that four anticipations go beyond the level of doing, indicate that the lesson provided information related to how students might be thinking about the specific mathematics they produce. For example, one anticipation that was coded as seeing focused on students' incorrect thinking. In relation to students using a table to solve the task, the lesson plan states "students do not realize that they are not just looking for instances of where the costs are the same, but where the costs are the same for the same number of minutes (e.g. not 40 min for Furizon and 25 min for B-Mobile)" (Calling Plans Lesson Plan, University of Pittsburgh, 2006).

In addition to the attention provided to possible student solutions, the remainder of the lesson plan also provides similar detail and level of attention to student thinking when addressing the other dimensions. The lesson plan includes "rationale and mathematical ideas" that correspond to the discussion of particular solutions that provide the teacher guidance on how to help students make sense of the mathematics in those solutions. For example, the lesson plan includes a sequence of solutions to be presented for public display. The first suggestion is having a group present the solution using a table with even time increments starting at zero. The following rationale is provided for this one particular solution and its order in the sequence:

Students will have an opportunity to discuss their procedure for calculating the cost for any number of minutes, which can be linked with the equation. The y-intercept can also be discussed as the cost for zero minutes for each plan, and can be linked to the graph. The table also provides an opportunity to see the point of intersection as the common coordinate pair for the two plans. The goal is to discuss mathematical ideas associated with cost-per-minute (rate of change/slope) and monthly fee (y-intercept) for each plan. Students should be able to say why one plan starts out as the less expensive plan but over time it is the most expensive plan. They should understand how this information is represented in a table, a graph, and an equation. (Calling Plans Lesson Plan, University of Pittsburgh, 2006)

Overall, the rationale is directed towards aiding the teacher in facilitating a discussion that helps the students make sense of the underlying mathematics. Not only does it highlight the mathematics of the particular solution, but also the connections to other solutions. Also, the context of the problem is always under consideration. Thus, it is intended not only at making sense of the math, but the math in relation to the particular context.

4.3.1.2 Task Implementation

Student work indicates that students worked on the problem in ways anticipated in the lesson plan. The sample indicates students engaged in a variety of strategies and the high level cognitive demands of the task were maintained. Figures 12 and 13 provide examples of student work on the task. Notice that one student solves the problems using equations and indicates an understanding of the solution in relation to the context of the problem. The other student solves the problem using a table and indicates a similar understanding within the problem context.

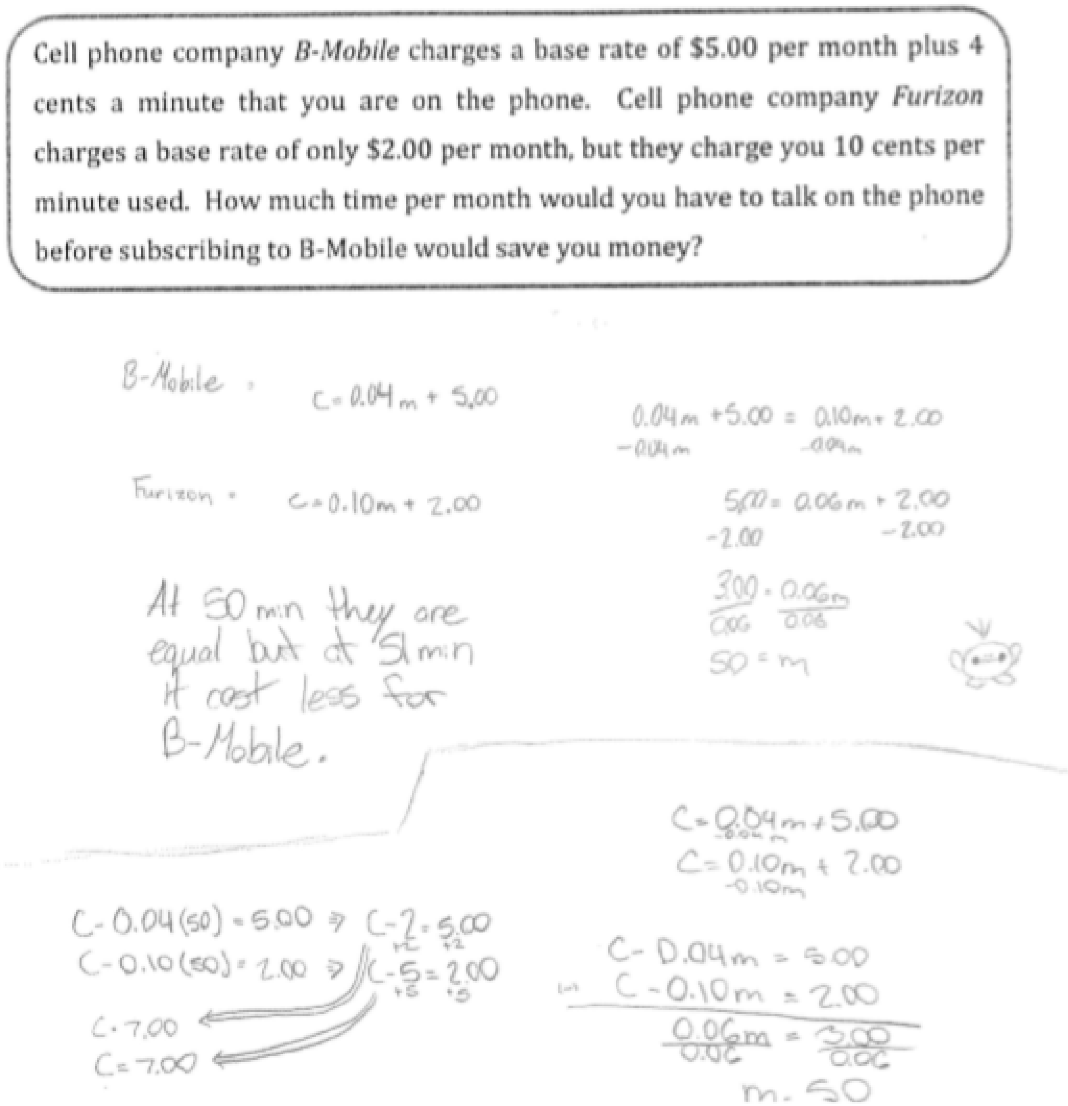
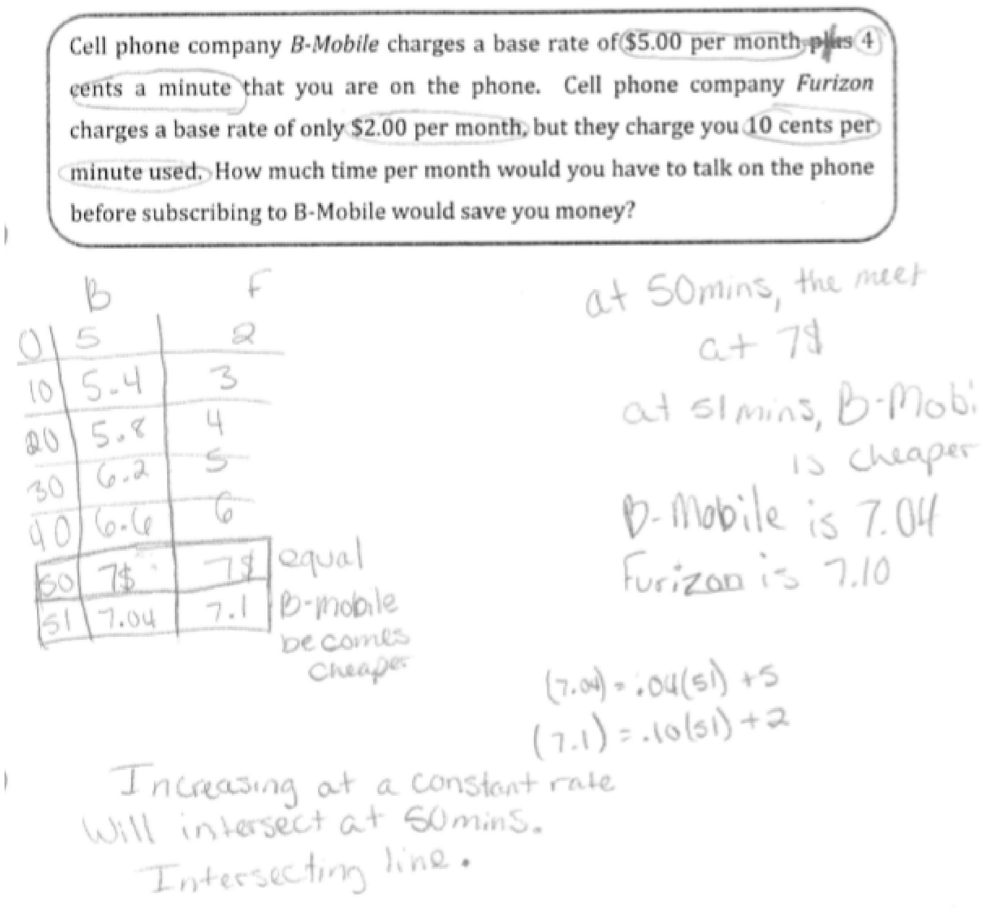


Figure 12 Example of Student Using Equation Approach



Cell phone company *B-Mobile* charges a base rate of $5.00 per month ~~plus~~ 4 cents a minute that you are on the phone. Cell phone company *Furizon* charges a base rate of only $2.00 per month, but they charge you 10 cents per minute used. How much time per month would you have to talk on the phone before subscribing to B-Mobile would save you money?

	B	F	
0	5	2	
10	5.4	3	
20	5.8	4	
30	6.2	5	
40	6.6	6	
50	7$	7$	equal
51	7.04	7.1	B-mobile becomes cheaper

at 50mins, the meet at 7$

at 51mins, B-Mobil is cheaper

B-Mobile is 7.04

Furizon is 7.10

(7.04) = .04(51) + 5

(7.1) = .10(51) + 2

Increasing at a constant rate

Will intersect at 50mins.

Intersecting line.

Figure 13 Example of Student Using Table Approach

In addition to the above examples, there was also a student who used a graphical approach. In general, the student work sample indicates students were able to use procedures they already knew (i.e. graphing, tables, solving equations) and connect them to this particular situation to find a solution. David considered the student engagement with the task to meet his expectation of medium quality work, because they were able to solve the problem in more than one way and demonstrate understanding of what the task was asking.

4.3.1.3 Thoughts about Whether the Lesson Went as Planned

When asked if the lesson went as planned, David only provides one brief response that is more *teacher oriented*, and he does not refer to any particular mathematics performed by the students or their thinking.

> DU: Um, I do think it went as planned um I planned my launch kind of like with a you tube video to have them um kind of get engaged in the task. I gave them um time to read it individually and eh I um had someone read it aloud, I had them circle words, so everything that I had planned to do from my um task in doing the detailed lesson plan um I kind of stuck to that. Um some of the timing was a little bit off but I do feel that progressed along in in terms of me monitoring them appropriately, me selecting responses to use, um for the discussion, and kind of closing out with an exit slip, so I feel like it went according to plan. (D.U., Post Lesson Interview Lesson 2)

In this response, David refers to many teacher actions (or things that he did). For example, David says "I planned my launch…I gave them um time to read it…I had them circle words…I do feel that progressed along in terms of me monitoring them appropriately, me selecting responses to use…" There is no attention to what students actually did during the lesson. The above comment is also an indication that David did not stick exactly to the resource based plan, as that plan did not include the use of you-tube video to launch the task. However, David is indicating the lesson went as planned because in his opinion he successfully "progressed" through the essential components of it.

4.3.1.4 Summary of David's Second Lesson

The results of David's second lesson indicate a mathematical goal focused on students seeing and making sense of mathematics related to systems of equations. Evidence of attention to student thinking was provided in the written lesson plan that was designed around task with high level cognitive demand potential. More specifically, the written plan addressed all elements of attention to student thinking at a maximum level, and it provided instances of what students should see and make sense of in relation to systems of equations as result of engaging in the lesson. David's expectations for student work on the task align with the multiple anticipated approaches set forth in the plan. Students engaged the task at a high level of cognitive demand, and David thought the lesson went as planned. However, he does not refer to student engagement with the task when explaining what it meant for the lesson to go as planned. Overall, for Lesson 2, David's lesson plan shows a high level of attention to student thinking, his expectations for student work aligned with the plan, and students engaged the task in ways that the high level cognitive demands were maintained.

4.3.2 David Upton's Self-written Lesson

For his third lesson, David used a self-written plan around several convincing and proving tasks. The purpose of presenting the results of this lesson is that the total planning score for attention to elements of student thinking is not consistent with the scores from David's other lesson plans. Despite the inconsistency in planning, the task was still implemented with high level cognitive demands. A qualitative look at the results provides a possible explanation there is inconsistency in planning yet task implementation remained consistent with his other lessons.

4.3.2.1 Attention to Student Thinking During Planning

(a) Mathematical Goal(s)

David's lesson plan includes two goals specifying concepts related to proof that students will learn, but does not elaborate on what it means for students to learn or understand those concepts:

> Students learn that one cannot provide a result by simply generating illustrative examples
>
> Students learn that results which are clearly true in limited domains are not necessarily true in wider domains (D.U., LP3, p. 1)

Neither goal states any specific mathematics students will see or make sense of during the lesson in order to "learn" the indicated result. For this reason, each of the above goals was coded as something the students would *do* during the lesson.

(b) Selecting a Mathematical Task

In order to achieve his learning goals, David selected a variety of convincing and proving tasks in which students had to prove if statements were always, sometimes or never true. Overall, the tasks have the potential to engage students in high-level cognitive demands. According to his coversheet, the task was taken from the MARS database. Figure 14 provides an example of one of the statements students had to prove.

Directions:

For the statement below, say whether it is always, sometimes, or never true. You must provide several examples or counter examples to support your decision. Try also to provide convincing reasons for your decision. You may even be able to provide a proof in some cases.

$3x^2 = (3x)^2$

Is this always, sometimes, or never true? (D.U. LP3)

Figure 14 One Convincing and Proving Task from David's Second Lesson

The convincing and proving tasks received a score of 4 according the AR1 Rubric due to their open-endedness and explicit prompting for students to explain their reasoning.

David's expectations are that students "provide several examples or counterexamples to each task and work toward convincing (in a mathematical way) that the statement was always, sometimes, or never true. Students were told that showing a few examples was not enough. Students were also expected to be able to present ideas during class discussion." When considering high, medium and low quality work, David does not identify specific solution strategies, but he does describe the components of proof he considers necessary for work to qualify at each quality level. For example, in his description of high quality work David writes:

> Student provides numerous examples and counterexamples and provides mathematically sound reasoning as to why the statement is always, sometimes, or never true. The student

works toward proving the statement is valid or invalid. The student is able to justify his/her own work (as well as others work) and provides convincing reasons for his/her position about the mathematical statement. (David Upton, Coversheet Lesson 2, p. 2)

David's descriptions of medium and low quality work are similar in nature with regard to what he expects them to provide in relation to providing a proof. Primary differences in the descriptions of medium and low quality work (compared to high quality work) include the amount of examples/counterexamples provided, level of engagement in providing a convincing argument, and ability to justify work. Even though, David is not referring to particular solutions, he is distinguishing quality of work based on students' mathematical performance in relation to proof. Given the multitude of different problems used in the lesson, it makes sense how David describes quality of work. That is, it would have been difficult to discuss specific mathematics of each problem since there was such a variety provided in the plan.

(c) Planning Around the Task

For this lesson, the plan provides evidence of a moderate level of attention to the elements of the student thinking. As shown in Table 18, the lesson plan received a score of 8 out of a possible 14 points when rating the written lesson plan with the elements of attention to student thinking scoring rubric (Hughes, 2006).

Table 18 Lesson Plan Dimensions Scores from David's Lesson 3

	Lesson Plans						
Teacher (Pseudonym)	Goal	Anticipate Student Thinking		Questions	Discussion		Total
		Correct	Incorrect		Build on Student Thinking	Math	
(Max. Poss)	(2)	(3)	(3)	(2)	(2)	(2)	(14)
David Lesson #2	1	3	3	1	0	0	8

As mentioned earlier, the mathematical goals only specified what students should understand, and not what it meant for them to understand it. Also, for the Questions to Assess/Advance Student Thinking dimension, David provides specific questions to ask while students explore the task, but the circumstances under which the questions are appropriate is not provided. With regard to Discussion dimensions, David writes that he plans to facilitate a discussion around student solutions and question them, but he does not indicate any particular solutions or any specific questions. For this reason, he received a score of 0 in both dimensions because he does not specifically attend to student thinking in planning for the discussion.

There were only two dimensions where David received maximum scores: Anticipating Students' Correct and Incorrect Thinking. The plan provides at least two correct solutions that specifically describe the math students might perform and how they will *make sense* of the mathematics. For example, consider the following task statement:

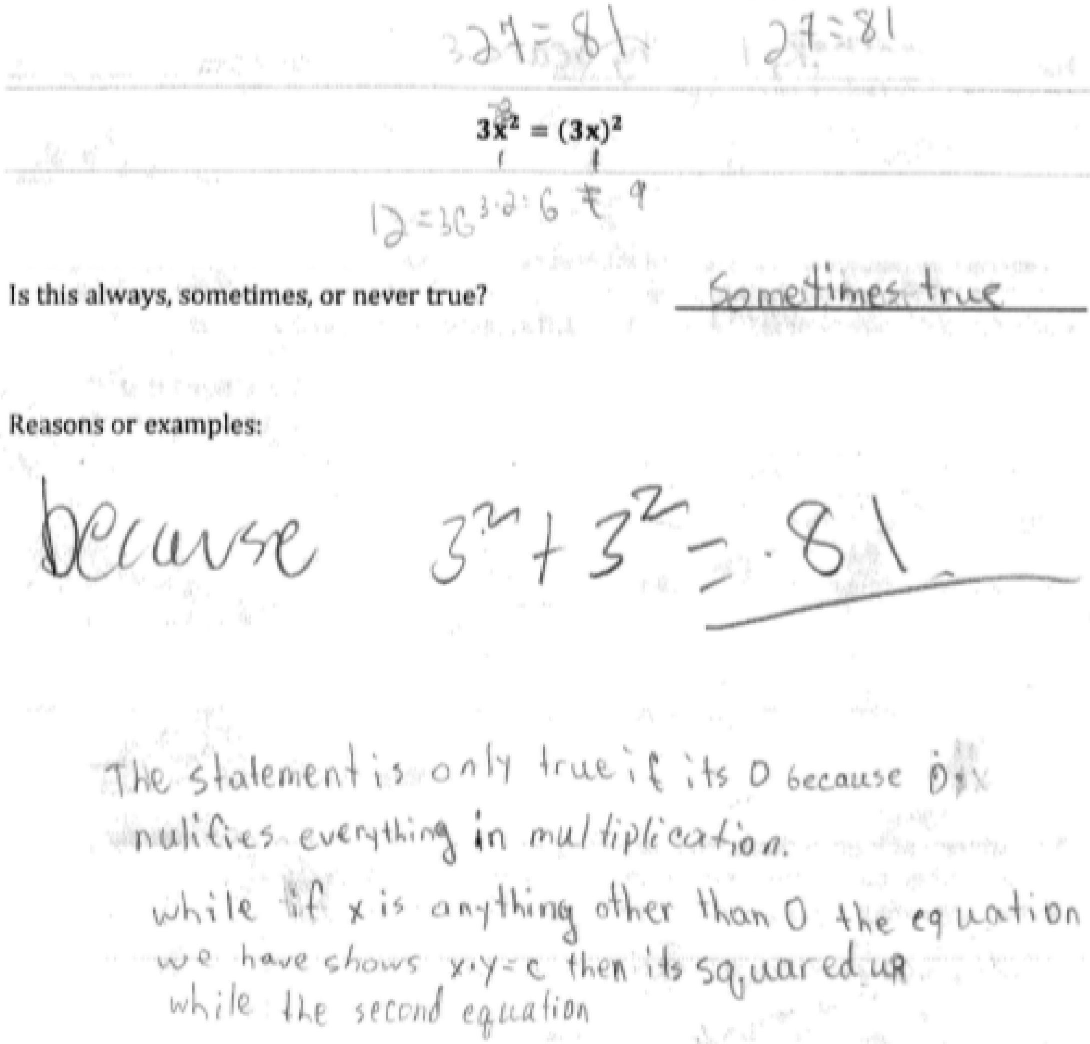
> Pentagons have fewer right angles than rectangles. Is this always, sometimes, or never true?

> Sample solution: Always true. Pentagons can have at most 3 right angles. Rectangles must have four. If one tries to draw a five-sided polygon with four or more right angles, then it either degenerates into a rectangle, or has three parallel side and then cannot be closed. (D.U., LP3, p. 13)

This particular sample solution was classified as *making sense* because the anticipation describes how students are making sense of the idea that a Pentagon can have at most 3 right angles. There are other anticipated strategies that also focus on making sense of the mathematics in the particular solution. In addition to correct solution strategies, the plan also identifies and describes common misconceptions the students might have when generating proofs. For this particular lesson plan, there are no instances of seeing, and all instances of making sense are from anticipated solution strategies.

4.3.2.2 Task Implementation

Student work indicates that students engaged the task in ways David expected. The sample of student work suggests students used a variety of strategies and made efforts to justify their claims. Thus, the high level cognitive demands of the task were maintained during implementation. Figures 15 and 16 provide examples of student work on one of the problems within the overall task. Notice how one student explores the problem with some specific numbers greater than 0 to provide a counterexample and then provides a written response explaining why 0 works as a solution. Notice the other student uses different numbers to provide counterexample and then shows how 0 works in the equation.



327 = 81   27 = 81

$3x^2 = (3x)^2$

$12 = 36 \quad 3\cdot 2 = 6 \neq 9$

Is this always, sometimes, or never true? Sometimes true

Reasons or examples:

because $3^2 + 3^2 = 81$

The statement is only true if its 0 because 0 x nulifies everything in multiplication.
while if x is anything other than 0 the equation we have shows x·y=c then its squared up
while the second equation

Figure 15 Student Work Sample 1 on Convincing and Proving Task

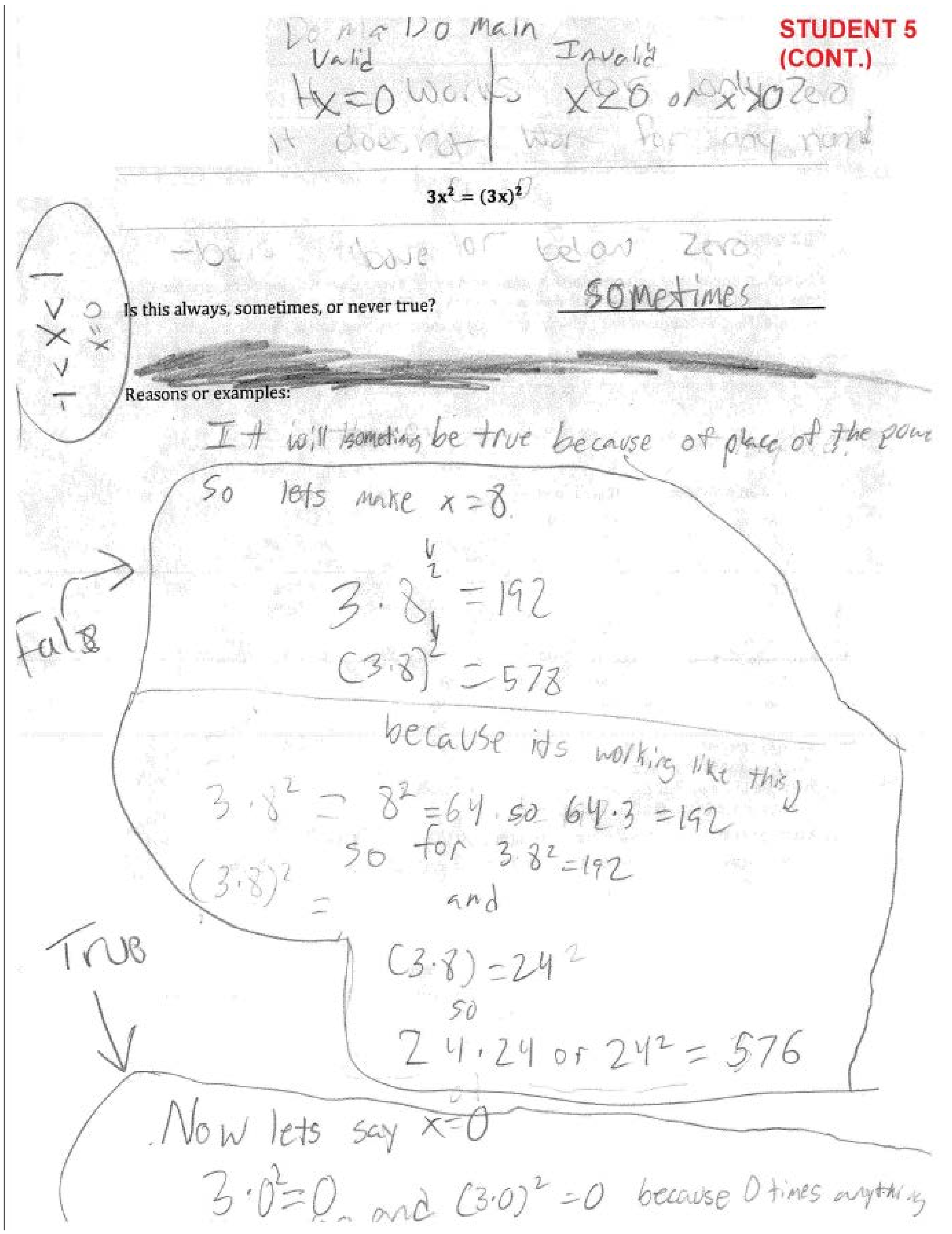


Figure 16 Student Work Sample 1 on Convincing and Proving Task

The examples provided indicate that students understood the concept that one counter example is enough to prove a statement false. Also, each student identified that 0 works which was an indication the statement was only sometimes true. David indicates that students engaged in what he considered medium quality work since they provided examples and counterexamples to make their argument, but generally did not provide complete proofs to any of the statements.

4.3.2.3 Thoughts about Whether Lesson Went as Planned

When asked if the lesson went as planned, David focused on how students approached the task. David thought part of his lessons went as planned and part didn't go as planned. In both instances, he referred to what students did when explaining what it meant as to whether it went as planned.

> DU: Ok, um I think the lesson um partly uh went the way I thought it would go so for the convincing and proving tasks um students did provide examples of um instances when um certain mathematical statements were always true, they were sometimes true, and when they were never true. Um some of it really didn't go as planned was um the students idea of like um giving a proof for it. So students didn't really work um as hard as I would have hoped um in in constructing a proof. Um I think the (inaudible) did that today as well um students really didn't um work to providing as explicitly mathematical reasoning they just provided examples instead of proving it to be mathematical fact.
>
> I: Ok, so what um when you said that…, when you said like it did go as planned um and you said that they in terms of the examples they provided the examples, can you can you elaborate more on that, what you meant by that.
>
> DU: Yeah, so um what I meant by that is I kind of planned for them to give like so for example um the mathematical statement was um the more digits a number has then the larger its value so some students they recognized that was sometimes true but they they

gave examples of why that was true. So they said that negative numbers um it doesn't necessarily work um whereas with positive integers it would work. (D.U., Post Lesson Interview, Lesson 2)

David talks both about proof and specific mathematics students performed when explaining what it means for the lesson to go as planned and not as planned. In contrast to his post lesson interview for Lesson 1, David is primarily focusing on student thinking in the interview. However, this is not surprising; David's responses align with his expectations and written lesson plan's attention to anticipated responses as he focuses on students' work on the task.

4.3.2.4 Summary of David's Lesson 3

The mathematical goals for David's third lesson indicate concepts students learn with regard to proof but not what it means to learn them. He selected problems with high level cognitive demand potential to help students meet the learning goals, and the primary area where attention was focused was how students engaged with the task. That is, David's expectations, written lesson plan, and thoughts on whether the lesson went as planned all primarily focus on students' approaches to the problems. Ultimately, the cognitive demands of the task were maintained by the students during implementation. Thus, for lesson 3, David provides a specific evidence of attending to student solution strategies, and the task was engaged by students at a high level of cognitive demand.

4.3.3 Summary of Planning and Implementation Relationship Exhibited by David

Overall, David represents a case of a pre-service teacher who selects tasks with high-level cognitive demand potential that are accompanied with detailed lessons plans that focus on student thinking about the task. The unmodified plans provide a high degree of evidence to the six elements of attending to student thinking, and they provide instances of seeing and/or making sense throughout the explore and summarize phases of the lesson plans. The lesson plan David wrote himself does not provide attention to all of the six elements of attending to student thinking, and the instances of making sense are only found in the anticipated solution strategies in the explore phase.

Despite differences in overall attention to student thinking in David's own planning versus the unmodified plans, the high-level cognitive demands of the tasks he selected were maintained during implementation of all lessons. One finding in the lesson for which David wrote the plan himself is that David's expectations, written plan, and thoughts about whether the lesson went as planned all focus on students' anticipated solution strategies. For the most part, this finding aligns with other evidence of attention to student thinking with regard to planning found in his other lessons.

4.4 QUINN BRADY

Three of Quinn's lesson plans were self-written, and one was unmodified. The three self-written plans were similar in structure. In particular, they all include the following headings: learning and performance goals, connection to previous and following lessons, materials, launch, supporting students exploration of the task (monitoring tool included), and sharing and discussing the task (monitoring tool included). One of Quinn's self-written lesson plans can be found in Appendix G.

The performance goal in each plan states what students will do or learn, and the learning goal specifies what it means for students to understand the concept. In the connection to the previous and following lesson section included in his lesson plans, Quinn writes about related concepts students learned prior to the lesson, how they will use that knowledge in the current lesson, and how the concepts relate to the following lesson. Under the materials heading, Quinn listed things both he and/or the students would need to engage in the lesson (i.e. whiteboard and markers, copy of task for each student, document camera).

In the launch section of each plan, he lists questions he plans to ask the students before they begin exploring the task. The questions either connected to previously learned content or were aimed at getting students interested in problem by relating it to something familiar to them. After the launch, the lesson plan transitioned to supporting students' exploration of the task. In this section of each plan, Quinn included a monitoring tool. Each monitoring tool had the same structure consisting of one column listing possible student approaches and another column with corresponding questions to ask for each approach. The last portion of each self-written lesson plan presented a table for sharing and discussing the task. Each monitoring was similar in structure, and contained two additional columns (connections between strategies and key points) compared to the monitoring tools in the explore phase. In the connection between strategies column, Quinn provides information regarding how the corresponding approach relates to other approaches listed in the table. The key points column provides information about how the corresponding solution relates to the underlying mathematics of the task.

All of Quinn's lesson plans included performance goals and corresponding learning goals, and all the lesson plans were designed around tasks with high-level cognitive demand potential. A summary of results from Quinn's lessons are provided in Table 19. Overall, the lesson plans indicate a high degree of attention to student thinking. During the enactment of all four lessons, Quinn's students engaged the tasks in ways that maintained the high level cognitive demands. In general, Quinn thought his lessons went as he planned, and he often focused on what students were doing during the lesson when explain what it means for the lesson to go as planned.

Table 19 Summary of Results from Quinn Brady's Enacted Lessons

Lesson	Attention to Student Thinking During Planning			Implemented	Post-lesson Thoughts
	Goal(s)	Task Selected	Planning Around Task		
1 (PTR)	Seeing	High	Self-Written High Element Score Seeing	High	Met Goals Student Focus
2	Seeing & Making Sense	High	Self-Written High Element Score Seeing & Making Sense	High	As Planned Student Focus
3	Doing	High	Unmodified Plan High Element Score Seeing	High	As Planned Student Focus
4	Seeing & Making Sense	High	Self-Written Moderate Element Score Seeing	High	As Planned Teacher Focus

Quinn's lesson planning and task implementation are relatively consistent across his four lessons; however, the results of lesson 4 indicate some inconsistencies in relation to attention elements of student thinking in the written lesson plan and his thoughts about whether the lesson went as planned. The remainder of this section seeks to gain further insight into the relationship that Quinn exhibited between attention to student thinking during planning and task implementation. To accomplish this, discussions of lesson 2 results and lesson 4 results are now presented. Lesson 2 serves as a representative example of one Quinn's lessons that is consistent with the majority of his lessons. Lesson 4 indicated inconsistencies in element planning score (moderate vs. high in all others) and his focus during post lesson thoughts about whether lesson went as planned (teacher vs. student oriented in all others). Lesson 4 is examined because despite these inconsistencies with regard to planning, the task was still implemented at a high level of cognitive demand.

4.4.1 Quinn Brady - Lesson 2

For his second lesson Quinn planned a around a Tiles task that is used in his teacher education program. The task is based on a lesson that appears in the book on the Five Practices (Smith & Stein, 2011). The structure of the lesson plan is similar to the three other self-planned lessons; however that is not the reason this lesson is discussed. Rather, this lesson is discussed because the overall attention to student thinking with regard to planning for this lesson is consistent with two of his other lessons (one self-planned and one resource-based plan).

4.4.1.1 Attention to Student Thinking With Regard to Planning

(a) Mathematical Goal(s)

Quinn's lesson included two performance goals with corresponding learning goals that focused on how students would see and make sense of generalizing a mathematical pattern. The performance goals state what the students will do and the learning goals state what it means for students to understand what they did:

Learning Goals

> Students will understand that generalizations about a pattern hold true for every part of that pattern.
>
> Students will recognize that the pattern can be broken into sections, and the sections can be represented by polynomials.

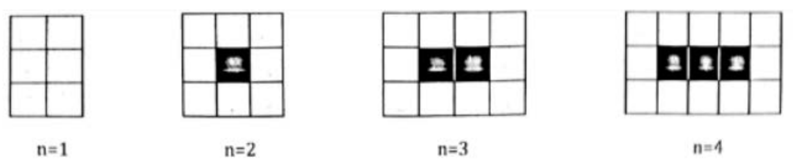
Performance Goals

> SWBAT write a generalization about the side length and area of a pattern
>
> SWBAT to utilize polynomial arithmetic to create a generalization about a pattern
>
> (Q.B., LP2, p. 1)

One of Quinn's learning goals indicates what it means for students to make sense of the broader principle of generalizing by understanding that "generalizations about a pattern hold true for every part of that pattern". The other learning goal indicates students will see "that the pattern can be broken into sections, and the sections be represented by polynomials". Thus, Quinn's learning goals go beyond what students will do mathematically, and focus on the way students will think about and understand pattern generalization as a result of the lesson.

(b) Selecting a Mathematical Task

In order to achieve his learning goals, the lesson was designed around a task with high-level cognitive demand potential. The task is a version of a similar task in Smith & Stein (2011). Quinn identifies specific mathematics aligned with his mathematical goals to explain the differences in what he considers high, medium, and low quality work on the task. The task is shown in Figure 17.



1. What patterns do you notice in the set of figures?

2. How many total square tiles (white and black) are going to be in figure number 6? How did you figure this out?

3. Determine an expression (*generalization*) for the total number of square tiles in any figure.

4. Determine an expression (*generalization*) for the number of black square tiles in any figure.

5. Determine an expression (*generalization*) for the number of white square tiles in any figure.

6. How many white tiles will figure number 23 have?

Figure 17 Main Instructional Task from Quinn's Lesson 1

The task received a score of 3 according to the AR1 Potential rubric since it has the potential to engage students in high-level cognitive demands. The task did not receive a score of 4 because it did not explicitly require students to explain their reasoning.

Quinn frequently refers to specific mathematics the students will (or will not) perform and understand when explaining what he considers high, medium and low quality work on the task. Thus, with regard to quality of work, Quinn's explanations provide evidence that he is attending to student thinking specifically related to the underlying mathematics of the task. For example, when explaining what he considers high quality work on the task, Quinn refers to specific mathematics the students will do and/or recognize. When explaining medium quality work, Quinn refers to specific mathematics the students will perform and also identifies what students will not be able to do and how they will be confused. Specifically, Quinn wrote:

> Medium quality work would be describing the pattern from one step to another but not being able to represent it as an expression. At this level they would recognize that the number of black and white tiles can be represented by an expression but they would be confused how to express it correctly. (Q.B., Coversheet Lesson 2, p. 2)

For low quality work, in addition to identifying mathematics the students will not perform, he also states that students "not being able to explain how the pattern changes from step to step is also low quality work." For each level of quality of work, Quinn's description identifies specific mathematics aligned with his goals that students will perform and what they will (or will not) recognize as a result of doing the mathematics.

(c) Planning Around the Task

The remainder of Quinn's lesson plan aligns with his learning goals, and provides evidence of a high level of attention to student thinking as students engage and discuss the task. Table 20 shows the scores Quinn received for each lesson planning dimension for Lesson 2. The lesson plan received a score of 13 out of a possible 14 points when rated according to the elements of attention to student thinking scoring rubric (Hughes, 2006).

Table 20 Lesson Plan Dimensions Scores from Quinn's Lesson 2

	Lesson Plans						
Teacher (Pseudonym)	Goal	Anticipate Student Thinking		Questions	Discussion		Total
		Correct	Incorrect		Build on Student Thinking	Math	
(Max. Poss)	(2)	(3)	(3)	(2)	(2)	(2)	(14)
Quinn Lesson #1	2	3	3	2	2	1	13

As seen in Table 20, a score of 13 indicates that Quinn's written lesson plan provided evidence of "maximum" attention to student thinking across all but one dimension (Discussion Makes Math Salient). For example, Quinn specifically describes more than one correct and incorrect solution strategy, and he provides specific example questions to assess and advance students' thinking that correspond to at least two different circumstances based on their thinking.

The quality of Quinn's focus throughout the lesson plan is on what students will do and see mathematically, and in some places how they will make sense of what they do and/or see. For example, in his monitoring tools, he identifies correct and incorrect strategies students will *do*, a specific pattern students might not *see*, and he elaborates how particular strategies can help students see or make sense of the mathematics of the lesson. Table 21 provides a portion of the monitoring tool from the explore phase in Quinn's plan.

Table 21 Portion of Quinn's Explore Phase Chart from Lesson 2

Possible Student Approaches	**Questions to Ask**
Students give numerical answer to the pattern	• What does "n" mean? • Do you remember what a generalization means? • How can you generalize your answers?
Students not seeing the pattern that they are multiplying the height (3) by the width (n+1)	• How many tiles high are the figures? Does this every change? • How many tiles wide is figure 2? Figure 3? Figure 4? How does this width relate to our step number? Can we make a generalization?

The first approach in Table 21 states something students will do (i.e. give numerical answer to pattern). The second approach states something students will not see while working on the pattern (i.e. multiplying the height (3) by the width (n +1)). These are examples of Quinn anticipating what students will do and see (or not see) while they work on the task themselves in the explore phase.

In the table he created for summarize phase of the lesson plan, Quinn elaborates on what students should learn from certain solutions presented during the discussion. Table 22 provides a portion of the table from the summarize phase in Quinn's plan.

Table 22 Portion of Quinn's Summarize Phase Table from Lesson 2

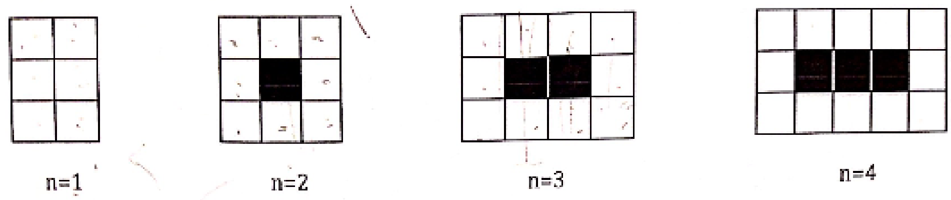
Solutions to Share	Questions about Strategy	Connection Between Strategies	Key Points
Students represent white tiles as 6 + 2(n-1)	• What does this simplify to? • Can you compare this to the first solution we presented? • Can you show me in the figure how you went about creating this expression? • Why are you adding the *2(n+1)*? What does this expression represent?	• Both strategies simplify to the same expression	In this method, students recognize that 3 white tiles will always remain on the left and right sides of the figures, totaling a constant of 6 white tiles. Each subsequent figure is adding two more white tiles and one additional black tile.
Students represent white tiles as 5+(2n -1)	• What does this simplify to? • Can you compare this to the previous two solutions we presented? • Can you show me on the figures how you created this expression? • Why are you adding *(2n+1)*? What does this expression represent?	• Simplifies to the same expression as the other two strategies	This method recognizes that 5 tiles remain as constants. The additional tiles added to the figure are represented by *2n+1*. I think it is unlikely students will choose this method, but I put it on here in the event that it does come up.

The information associated with the solution [Students represent white tiles as 5+(2n -1)] goes beyond doing to state something the students will notice. It is classified as *seeing* based on the information provided in the "Key Points" column (i.e. this method recognizes the 5 tiles remain as constants. The additional tiles added to the figure are represented by 2n-1). Also, in the "connections between strategies" column, the plan indicates the recognition of a connection between the simplified form of this expression and the simplified forms of the other expressions.

The same type of connection is listed for the 6 + 2(n-1) solution. Further information associated with this anticipation focuses on *making sense* of what the students notice. For example, under the “Key Points” column for this anticipation, Quinn writes: “In this method, students recognize that 3 white tiles will always remain on the left side and right sides of the figures, total a constant of 6 white tiles”. He then goes beyond this recognition to establish the meaning of 2(n-1) by stating “Each subsequent figure is adding two more white tiles and one additional black tile”. The degree of detail provided by Quinn in this lesson plan in particular indicates that he is going beyond what students are doing. He is attending also to what students are noticing and thinking about mathematically, and what they should notice or understand as a result of the entire lesson.

4.4.1.2 Task Implementation

Student work samples indicate that students worked on the problems in some of the ways Quinn anticipated in his lesson plan. Overall, the entire sample of student work indicates students engaged in a variety of strategies and the high level cognitive demands of the task were maintained. Also, when talking about whether the lesson went as planned, Quinn refers to how students engaged the task. Figures 18 and 19 provide examples of student work on the task. Notice the particular solution strategies each student provided to item #5.



1. What patterns do you notice in the set of figures?

You always add a in the middle

2. How many total square tiles (white and black) are going to be in figure number 6? How did you figure this out?

7 ← Always # +1

3 × 7 = 21

3

Always 3

3. Determine an expression (*generalization*) for the total number of square tiles in any figure.

$3(x+1)$ $3x+3$

4. Determine an expression (*generalization*) for the number of black square tiles in any figure.

$x-1$

5. Determine an expression (*generalization*) for the number of white square tiles in any figure.

$3(x+1) - (x-1)$ $-x+1$

$3x+3-x+1$

6. How many white tiles will figure number 23 have?

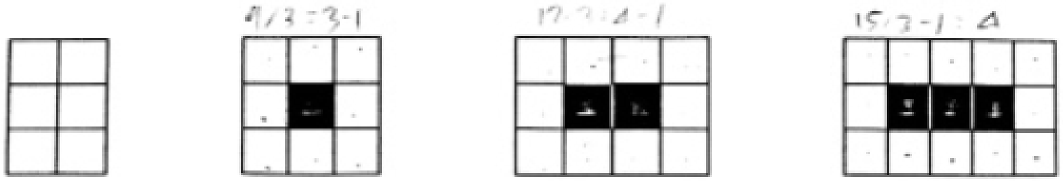
$72 - 22$

$$\begin{array}{r} 72 \\ -22 \\ \hline 50 \end{array}$$

$3x+4-x$

$2x+4$

Figure 18 Student Providing 3n + 3 – (n-1) Solution in Quinn's Plan



n=1 n=2 n=3 n=4

1. What patterns do you notice in the set of figures? adds one more black block
The black block always first one. The black blocks are horizontal to the figure number. The always one less than the middle row. The black blocks are always the always three, and height of the white blocks is always one more than the length is the figure #
2. How many total square tiles (white and black) are going to be in figure number 6? How did you figure this out? $6+3(6-1)=21$
the 6= the 2, 3 white blocks on the side
the 3(6-1)= the white blocks not part of the sides and the black blocks

3. Determine an expression (*generalization*) for the total number of square tiles in any figure.
$6+3(n-1)$
$3+3n$

4. Determine an expression (*generalization*) for the number of black square tiles in any figure.
$n-1$

5. Determine an expression (*generalization*) for the number of white square tiles in any figure.
$6+2(n-1)$
$4+2n$

6. How many white tiles will figure number 23 have?
$6+22\cdot 3=72$

Figure 19 Student Providing 6+2(n-1) Solution in Quinn's Plan

The two examples provided reflect different ways students worked on the task, and both approaches were anticipated in Quinn's plan. Quinn stated that the work student produced was medium to high quality according to the descriptions discussed earlier. That is, students coming up with particular solution strategies and recognizing what they represent in relation to the context of the problem.

4.4.1.3 Thoughts about Whether the Lesson Went as Planned

When asked if his lesson went as planned, Quinn said he thought that it did, and he said that students responding the way he expected is what it meant for the lesson to go as planned. For example, consider the following excerpt:

> QB: Um, I would say it did go as planned. Um, you know not per I didn't get perfect responses from the, from what I had expected but it was it was very close and overall I felt they were really hitting the main points and seeing how this pattern was growing over time.
>
> I: So what were you doing that made you feel like this?
>
> QB: So, um as I was calling up each individual student to present the problem um they were they were first of all their expressions were correct, um the expressions I was looking for, and more than that I was having other students repeat those, why their expressions were the way they were and I was hearing um the answers that I was looking for in that regard. (QB, Post Lesson Interview, Lesson 2)

While Quinn does not elaborate on specific solutions, he says that students' "expressions were correct" and they were "the expressions I was looking for". Later, when asked for a reason as to why he thought the lesson went as planned, Quinn primarily credited the students in the class, and their willingness and ability to "dive in" into mathematics and communicate mathematically. Quinn also acknowledges that his own knowledge of the task and the task structure may have helped the lesson go as planned, but he continually referred to the students' engagement with mathematics and the classroom discussions. While Quinn does not refer to specific solutions or specific instances of student thinking, his responses indicate that the lesson went as planned based on what students were doing/thinking as they engaged with the mathematical task selected to meet the learning goals.

4.4.1.4 Summary of Quinn's Second Lesson

The results from Quinn's second lesson indicate that Quinn attended to student thinking with regard to planning the lesson around a high-level task. The written lesson plan addressed the elements of attention to student thinking (Hughes, 2006), and it showed instances of what students would do, see and make sense of mathematically (Smith et al., 2013). Quinn's descriptions of different levels of student work quality align with his mathematical goals, and his explanation of why the lesson went as planned was based on the mathematics students produced during the class. During the enactment of the lesson, the high-level cognitive demand of the task was maintained. Thus, for lesson 2, Quinn showed evidence of attending to student thinking with regard to lesson planning, and the task was engaged by students at a high level of cognitive demand. In general, the results of lesson 2 are representative of the results of Quinn's Lessons 1 and 3.

4.4.2 Quinn Brady – Lesson 4

For Quinn's fourth lesson the task came directly from a resource given to Quinn by one of his professors. The anticipating he did within the plan was drawn directly from a resource based plan written around the task. Despite drawing on the resource, Quinn's scores for attention to anticipated solution strategies were drastically low in comparison to his other lessons according the elements of student thinking rubric (Hughes, 2006). These low scores are responsible for a moderate total element score which is inconsistent with his other lessons. Also, the focus of his post lesson thoughts about the lesson going as planned is teacher oriented while the other lessons have a student oriented focus. Consideration of the quality of anticipations rubric (Smith et al., 2013) helps possibly explain why despite inconsistency in planning scores in relation to his other lessons, the high-level cognitive demands of the task were still maintained during implementation.

4.4.2.1 Attention to Student Thinking With Regard to Planning

(a) Mathematical Goal(s)

Quinn's lesson plan included three performance goals with corresponding learning goals all related to how students should see and make sense of slope, linear equations and linear relationships:

Learning Goals

> Students understand that the graph of a linear relationship is a line that models the relationship between the variables in the context. The coordinates of the points on the line form the solution set for the associated linear equation.

Students will understand that the slope between two points can be found by taking the difference of the *y* coordinates between two points and dividing by the difference of the corresponding *x* coordinate of the points: $(y_2 - y_1) / (x_2 - x_1)$.

Students will recognize that the y-intercept of a linear equation corresponds to an initial value and the slope corresponds to a constant change between the two variables.

Performance Goals

SWBAT find the slope of a line given two points.

SWBAT find solutions that satisfy equations.

SWBAT recognize how contextual situation can be modeled with a graph.

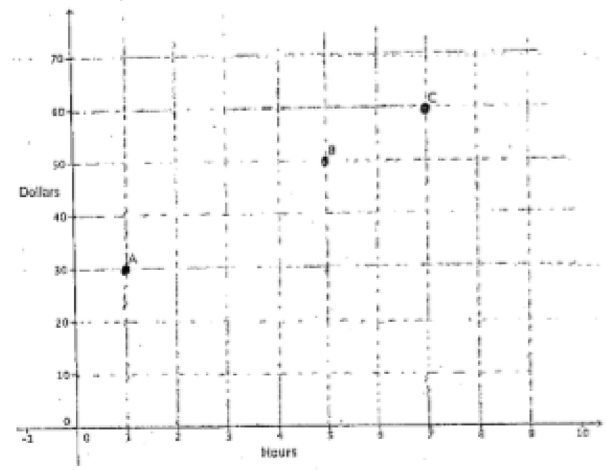
(Q.B., LP 4, p. 1)

Similar to his goals for Lesson 2, Quinn specifies mathematics students will see (i.e. the y-intercept of a linear equation corresponds to an initial value and the slope corresponds to a constant change between the two variables). He also indicates the students should make sense of the graph of a linear relationship (i.e. a line that models the relationship between the variables in the context. The coordinates of the points on the line form the solution set for the associated linear equation). Thus, Quinn's learning goals go beyond what students will do mathematically, and focus on the way students will think about components of linear equations and the broader idea of linear relationships.

(b) Selecting a Mathematical Task

In order to achieve his learning goals, Quinn's lesson was designed around a word problem involving a linear relationship that had high level cognitive demand potential. The task was designed by Institute for Learning, University of Pittsburgh. The task is shown in Figure 20.

Jose rents a surfboard for the day from a company that charges by the hour. The graph below shows the cost of renting a surfboard for different amounts of time.



1. What is the rental rate per hour for the surfboard? Explain your reasoning.
2. If the cost continues at this rate, calculate the cost of renting a surfboard for 12 hours. Show all work and explain your reasoning.
3. Calculate the number of hours Jose surfs if the rental cost is \$150.00. Show all work and explain your reasoning.

Figure 20 Jose's Surfboard (Institute for Learning, University of Pittsburgh)

The task received a score of 4 according to the AR1 Potential rubric since it has the potential to engage students in high-level cognitive demands, and it explicitly prompts students to explain their reasoning.

Quinn states that his expectations while students work on the task are that "students will be able to answer questions and explain their reasoning". His expectation for high quality work relates directly to one of his learning goals. More specifically, he describes high quality work as students using any solution method (table, function or graph) to recognize "an initial price that corresponds with the y-intercept and that the rate (slope) is increasing at $5 per hour". He also states that for work to be considered high quality, students must "show their work and explain their reasoning". Medium quality work is same as high quality work except "students show little work and do not explain their reasoning". Low quality work is when students do not recognize the mathematical concepts identified in his description of high quality work.

(c) Planning Around the Task

The remainder of Quinn's lesson plan aligns with his learning goals, and provides evidence of a moderate level of attention to student thinking as students engage and discuss the task. Table 23 shows the scores Quinn received for each lesson planning dimension for Lesson 4. Quinn received a score of 8 out of a possible 14 points when rating the written lesson plan with the elements of attention to student thinking scoring rubric (Hughes, 2006).

Table 23 Lesson Plan Dimensions Scores from Quinn's Lesson 4

	Lesson Plans						
Teacher (Pseudonym)	Goal	Anticipate Student Thinking		Questions	Discussion		Total
		Correct	Incorrect		Build on Student Thinking	Math	
(Max. Poss)	(2)	(3)	(3)	(2)	(2)	(2)	(14)
	2	1	0	2	2	1	8

As can be seen in Table 24, Quinn's lesson plan received maximum scores in three dimensions (Mathematical Goal, Questions that Assess/Advance Student Thinking, and Discussion Builds on Student Thinking) which indicates Quinn's lesson plan provided evidence of a high level of attention to student thinking for those particular elements. Quinn's lesson plan received very low scores for the anticipating student thinking dimensions. Quinn did not provide any incorrect solution strategies, and he did not provide worked out solutions for the correct strategies he included in the lesson plan.

The three correct strategies are provided in his monitoring tool, and include *extending the line*, *creating a table,* and *writing a function to model the situation*. As mentioned above, Quinn does not show what it means to extend the line, he does not provide a table, nor does he provide a specific function that will model the situation. According to the elements of student thinking rubric, these anticipations receive a low score. However, with regard to the quality of the anticipations, there is evidence that the anticipations are linked to mathematics students should see as a result of the discussion. Table 24 provides the Sharing and Discussing Task chart from Quinn's lesson plan.

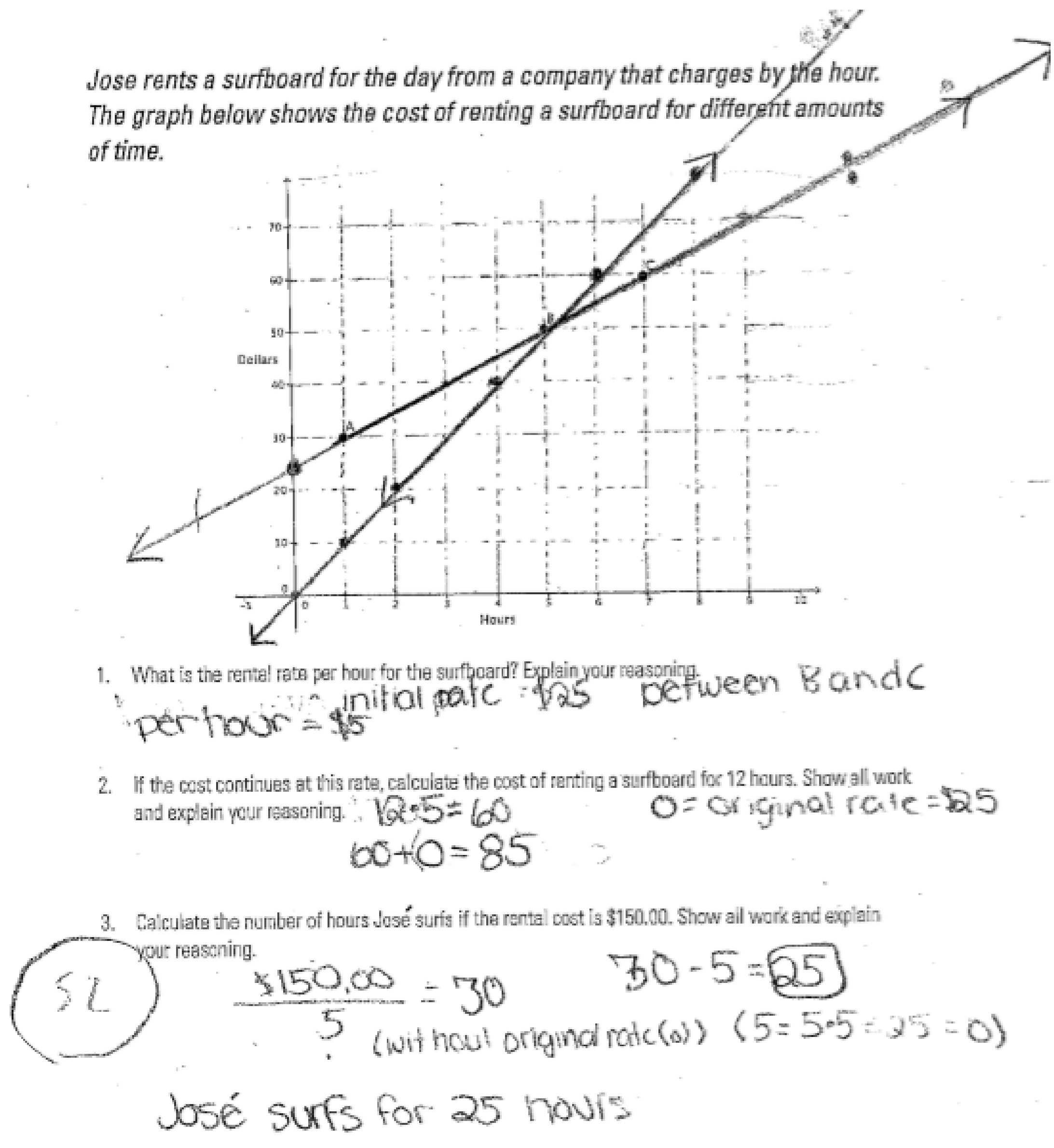
Table 24 Chart for Discussing Task From Quinn's Plan for Lesson 4

Solutions to Share	**Questions about Strategy**	**Connection Between Strategies**	**Key Points**
Extends the line	• How can we use this strategy to find the find the rate per hour of a surfboard? • How do we know what the cost of renting a board for 12 hours would be? • Is it possible to find the number of hours Jose surfs if the rental cost is $150?		This method will get us our answers, but it may be difficult to find how much the board costs for times greater than 10 hours due to the domain of the graph. We would have to extend the graph.
Table of the time in one column and the dollars in the other	• How can we find the rate using the table? • How can this method be used to find the cost for any random amount of time? • Are there any limitations to using this method?	The coordinates in the table lie on the line in the above strategy	We can see from the table how the cost goes up by 5 dollars per hour. We can also find the slope of a line using two of the pairs from the table.
Write a function to model the situation	• How did you find the rate? Where is it located in this function? • What do the variables in the function represent? How about the constants?	• If we plug a value for the hours from the table into the function, the corresponding value for dollars will result. • The slope of the line in the function is the difference between the cost for every increase in hour in the table. • The function is a symbolic model of the line.	The function models the cost for any hour. It allows the most efficient method to find the cost given the hour, or vice versa.

He lists the solutions again in the chart intended to help with the whole class discussion. Under the column titled “Key Points” he provides further information about the table solution by saying “we can see from the table how the cost goes up by 5 dollars per hour.” Quinn also lists several connections that should be recognized between strategies. For example, the plan indicates the connection should be made that “the slope of the line of the function is the difference between the cost for every increase in hour in the table”. So, despite not attending to student solutions by providing worked out correct responses or describing incorrect strategies, Quinn shows a high level of anticipation about what students should see as a result of the lesson. This particular lesson plan is an example why the quality of anticipations rubric (Smith et al., 2013) was added as another layer of coding, and why the use of the rubric was extended to the entire lesson plan. High element scores in the elements of student thinking rubric (Hughes, 2006) do not necessarily indicate quality.

4.4.2.2 Task Implementation

Student work samples indicate that students worked on the problem in a variety of different ways that maintained its’ potential high level cognitive demands. Figures 21 and 22 provide examples of student work on the task. Notice students engage with the graph and their calculations in different ways.



Jose rents a surfboard for the day from a company that charges by the hour. The graph below shows the cost of renting a surfboard for different amounts of time.

1. What is the rental rate per hour for the surfboard? Explain your reasoning.
initial rate = \$25 between B and C
per hour = \$5

2. If the cost continues at this rate, calculate the cost of renting a surfboard for 12 hours. Show all work and explain your reasoning.
12·5 = 60 O = original rate = \$25
60 + O = 85

3. Calculate the number of hours José surfs if the rental cost is \$150.00. Show all work and explain your reasoning.
SL
$\frac{\$150.00}{5} = 30$ 30 - 5 = $\boxed{25}$
(without original rate(o)) (5 = 5·5 = 25 = O)

José surfs for 25 hours

Figure 21 First Example of Student Work on Surfboard Task

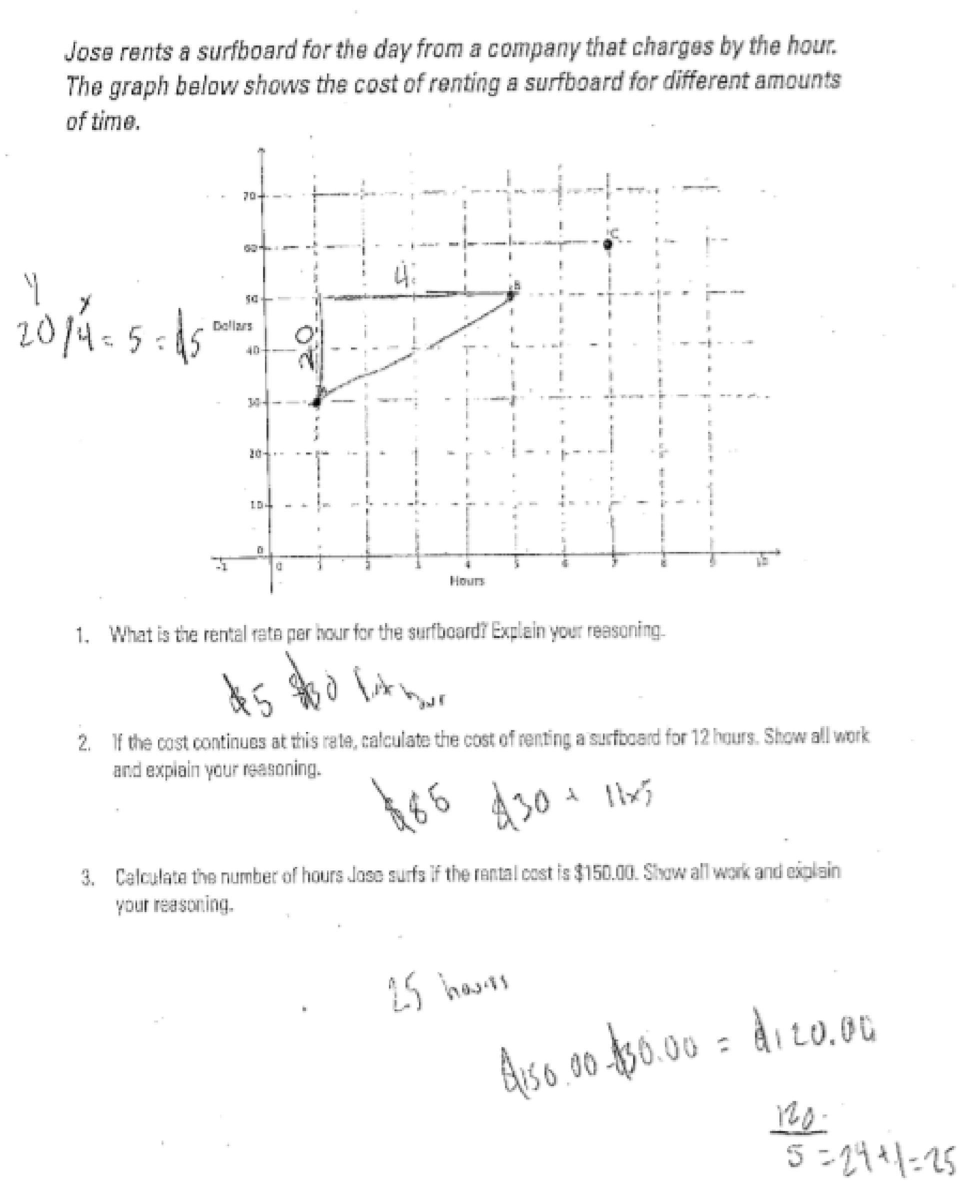


Figure 22 Second Example of Student Work on Surfboard Task

The student work sample received a score of 3 according to the AR2 Implementation Rubric due to the multiple strategies students used to make connections related to initial value and rate. The work sample did not receive a score of 4 because there was limited evidence of students explaining their reasoning. Recall, a score of 3 still indicates the high level cognitive demands of the task were maintained. Quinn's perspective on the student work aligns with the rating it received in the study. That is, Quinn considered student work to meet his expectations of medium and high quality work. He stated that nearly all students produced correct answers and met the learning goal related to recognizing initial value and rate, but "not every student was able to explain their reasoning".

4.4.2.3 Thoughts About Whether Lesson Went as Planned

When discussing whether the lesson went as planned, Quinn's very briefly and vaguely refers to students' work on the task by saying students "were able to engage in the task" and students "did respond the way I was hoping to". His responses are primarily teacher oriented reasons why the lesson went as planned. For example, in his initial response to whether the lesson went as planned, he says:

> Um but yeah overall they did respond the way that I was hoping to. Um the one area where I kind of had to push them along to get them to was when I was making the connection between slope and rate because no one really explicitly said that um once I pointed it out to them eh they got it. Like I , I , drew the line from one point to the next and talked about, well I didn't draw the line, I drew the change in y distance and change in x distance and I talked about eh what is this, where have you seen this, and someone said rise over run, okay well what is rise over run, it's the slope, ok great, so what can we say about the rate per hour and the slope. It's the same thing. Okay, yes great, so I kind of had to steer them to that and I was hoping that maybe some kids would see that on their own, that didn't happen um so that was the one area where I think, I don't want to

say didn't go as planned, but just needed more assistance from me I would say to get them there. (Q.B., Post Lesson Interview, Lesson 4)

In the above response, Quinn primarily identifies actions he performed to help the lesson go as planned. Near the end of the interview he was specifically prompted about "something the kids did" or something he did to help the lesson go as planned. In response to this question, he does credit students' prior knowledge, their' willingness to be vocal, the questions they asked, and the relatability of the task. However, the response is at a surface level, and he does address any specific student thinking.

4.4.2.4 Summary of Quinn's Fourth Lesson

For his fourth lesson, Quinn's learning goals are focused on students seeing and making sense of mathematics connected to linear equations and linear relationships. To help students meet these goals, he selected a task with high level cognitive demand potential. Quinn expects students to show work and explain their reasoning, and he identifies specific mathematics required for student work to be considered high quality. Quinn's written lesson plan did not provide as much attention to anticipating students' thinking according to the elements rubric, but other information associated with the anticipations focus on mathematics students should see during discussion about the solutions. Also, Quinn's post lesson interview provided primarily teacher oriented responses as to why the lesson went as planned. The cognitive demands of the task were maintained by the students during implementation.

4.4.3 Summary of Planning and Implementation Relationship Exhibited by Quinn

Quinn is a pre-service teacher who consistently set mathematical goals that focused on what students should see or be able to make sense of mathematically as a result of the lesson. He selected high-level tasks to achieve those goals, and thoughtfully planned lessons around them. Quinn used three self-written lesson plans and he found one online. Overall Quinn's plans provided evidence of attending to student thinking. In particular, Quinn provides evidence of attention to particular solution methods. In one of his lesson plans, Quinn did not provide any incorrect anticipated solutions nor did he work out the correct anticipations; however the plan attends to mathematics students should see during the discussion of specific solutions. He primarily focuses on students' engagement with the task when asked whether his lesson went as planned.

Overall, it is evident that Quinn thoughtfully considered his students' thinking and actions in relation to his planning. All of Quinn's tasks were implemented at a high-level of cognitive demand. Thus, Quinn is a case of a pre-service teacher who thoughtfully attends to student thinking with regard to lesson planning, and the high level cognitive demands of the tasks he selects are maintained during implementation.

4.5 MARIAN TURNER

Marian enacted a total of three self-written lessons for this study. Two of her lesson plans used the detailed template within the electronic planning tool, and these plans were not consistent in relation to how they addressed the template prompts. For example, she used the EPT for her lesson that was part of the university assignment. This particular plan addresses all questions within the template as well as including a monitoring tool and chart to help her orchestrate the classroom discussion. For the other lesson, the EPT plan indicates that Marian plans to have a whole class discussion, but it does not provide any response to a question regarding *how* she plans to orchestrate it. The lesson plan does not include a monitoring tool or chart to help orchestrate the discussion.

Marian's lesson that was planned without using the EPT was separated into three sections/subheadings: measurable student objectives, learning goals, and activity. The "activity" section addresses launching the task, students' exploration of the task, and a classroom discussion. The majority of the "activity" section focuses on students' exploration of the task in small groups. There is very limited information regarding the classroom discussion. This particular lesson plan can be seen in Appendix H.

All of Marian's lesson plans included measureable student objectives and corresponding mathematical goals, and all the lesson plans were designed around tasks with high-level cognitive demand potential. With regard to attention the elements of student thinking, two of Marian's lesson plans provide low to moderate degrees of evidence, and one lesson plan (from the PTR assignment) provides a high degree of evidence. The majority of instances of seeing or making sense throughout all her lesson plans are associated with the measureable objectives and learning goals. During the enactment of two of the lessons, students' engaged the tasks in ways that maintained the high level cognitive demands. For one lesson, the cognitive demands declined during implementation. In general, Marian thought each lesson had parts that went as planned and parts that did not. She focused on how students engaged the task to explain what it meant whether lesson went as planned, and she gave teacher oriented reasons to explain why parts of lessons went as planned.

Table 25 Summary of Results from Marian Turner's Enacted Lessons

Lesson	Attention to Student Thinking During Planning			Implemented	Post-lesson Thoughts
	Goal(s)	Task Selected	Planning Around Task		
1 (PTR)	Seeing & Making Sense	High	Self-written (EPT) High Element Score Making Sense	High	As planned Not as planned Teacher Focus Student Focus
2	Seeing	High	Self-written Low Element Score Doing	High	As planned Not as planned Teacher Focus Student Focus
3	Seeing & Making Sense	High	Self-written (EPT) Low Element Score Doing	Low	As Planned Not as planned Teacher Focus Student Focus

Lesson 1 and Lesson 3 in Table 25 are the lessons that Marian planned using the electronic planning tool. As indicated one lesson plan provided a high degree of evidence to attention to student thinking and was implemented at a high level. The other lesson plan provided a low degree of evidence of attention to student thinking and was implemented at a low level. These particular results correspond to the quantitative findings relating total planning score to implementation score. Perhaps the most interesting result of Marian's lessons, is Lesson 2. There was low attention to the elements of student thinking, and no instances of seeing or making sense beyond the objectives and goals; even so, the high level cognitive demands of the task were maintained. A closer look across the various data sources provides insight into the planning and implementation relationship exhibited for this particular lesson by Marian.

4.5.1 Marian Turner – Lesson 2

For her second lesson, Marian specified three measurable objectives and two learning goals that corresponded to the measureable objectives. One of the learning goals focused on mathematics students should recognize as a result of the lesson. Marian chose a high level task, and she provides low to moderate attention to student thinking when planning around the task. The students maintained the high level cognitive demands of the task during implementation. Marian refers to students' engagement with the task when explaining what it means whether the lesson went as planned, and she offers teacher oriented reasons to explain why parts of lesson went as planned.

4.5.1.1 Attention to Student Thinking with Regard to Planning

(a) Mathematical Goal(s)

Marian's lesson plan specifies three measureable student objectives focused on mathematics related to combinations and permutations that students will do and two corresponding learning goals. One of the learning goals indicates concepts related to permutations and combinations that students should recognize as result of the lesson:

Measureable student objectives:

> Students will be able to evaluate the number of permutations in a given scenario.
>
> Students will be able to evaluate the number of combinations in a given scenario.

Learning goals:

> Students will recognize that combinations are collections, whereas permutations are arrangements; they will understand the difference between the two. (M.T., LP 2, p. 1)

The measureable student objectives state what the students will do (i.e. evaluate number of permutations or combinations), and the learning goals specifies what students should see as a result of their doing (i.e. combinations are collections, permutations are arrangements). The learning goal also states students will understand the difference between what they see, but it dors not indicate how students will demonstrate that understanding.

(b) Selecting a Mathematical Task

In order to achieve the mathematical goals, the lesson is designed around an ice-cream parlor task in which different situations call for use of permutations or combinations. According to Marian, the task was self-created. A slightly modified task is shown in Figure 23.

5 flavors available:

- Chocolate
- Vanilla
- Strawberry
- Chocolate Chip Cookie Dough
- Mint Chocolate Chip

Customers can order their ice cream either in a cone or in a bowl.

1) A customer comes in and orders two scoops of ice cream – one chocolate and one vanilla. He tells you that he wants to eat the chocolate *first* and he wants his ice cream in a cone. How must you scoop the ice cream so that you can fill his order correctly?

2) What if the customer orders his ice cream in a bowl? How many ways can you scoop the ice cream so that you fill his order correctly?

3) Generalize your answers to #1 and #2 – what is the difference between how you scoop the ice cream when a customer orders a cone vs. a bowl?

4) The next customer comes in and cannot decide what flavors she wants. She tells you that she wants a cone, but she asks you to pick any two of the five flavors for her at random (they should be two different flavors). How many different ways can you scoop an order for her? Show your work.

5) Suppose the customer from #4 changes her mind and decides that she wants her ice cream in a bowl. How many ways can you scoop an order for her now? Show your work.

6) The scenarios in #2 and #5 are what we call *combinations.* Write your own definition of what you believe a combination is. How does this differ from a permutation?

Figure 23 Ice Cream Parlor Task from Marian's Second Lesson

The task received a score of 3 according to the AR1 Potential rubric. While the task prompts students to show work, it does not explicitly prompt them to explain reasoning.

As students engage the task, Marian expected students "to work productively within their groups. They were expected to finish the entire task and use prior knowledge to aid them as they worked through it. Their best work was expected. They were also expected to work as a group by sharing and discussing ideas." In her description of high quality work, Marian emphasizes specific mathematics and clarity of explanations:

> High quality work on this task included a clear written understanding of the difference between a combination and permutation. Specifically, in this scenario students had to see the difference between an order for a cone and a bowl. High quality work should show student work, and clear explanations of answers… (M.T, Coversheet Lesson 2, p. 2).

The primary difference between high, medium and low quality work is in the explanations provided by the students. For example, medium quality work is similar to high quality work but has "less clear or incomplete explanations…" and low quality work includes "little or no explanation…." Overall, the descriptions attend to student thinking by identifying specific mathematics and how students should convey it.

(c) Planning Around the Task

Marian's written lesson plan indicates a low level of attention to the elements of student thinking. Table 26 shows the scores Marian received for each lesson planning dimension for Lesson 2. The lesson plan received a score of 6 out of a possible 14 points when rating the written lesson plan with the elements of attention to student thinking scoring rubric (Hughes, 2006).

Table 26 Lesson Plan Dimensions Scores from Marian's Lesson 2

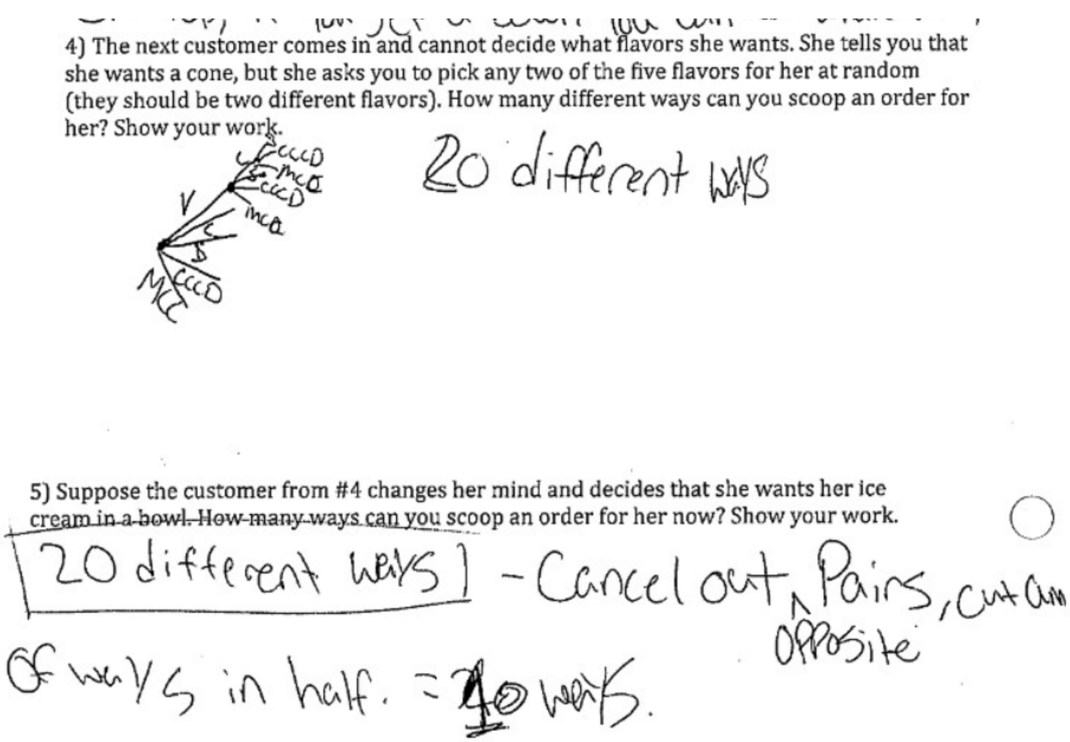
	Lesson Plans						
Teacher (Pseudonym)	Goal	Anticipate Student Thinking		Questions	Discussion		Total
		Correct	Incorrect		Build on Student Thinking	Math	
(Max. Poss)	(2)	(3)	(3)	(2)	(2)	(2)	(14)
Marian Lesson #2	2	0	3	1	0	0	6

As can be seen in Table 26, Marian earned a score of 0 in three dimensions which means the lesson plan did not provide any evidence of attention to student thinking for those elements. The lesson plan received a score of 1 for Questions to Assess/Advance student thinking because Marian did identify questions to ask, but the student thinking based circumstances under which to ask the questions was not specified. Her lesson plan earned a maximum score for Mathematical Goal and Anticipating Students' Incorrect Thinking.

Marian's anticipation of incorrect student thinking is related to the mathematical goals of her lesson plan. For example, problem 5 on the task is a combinations problem. Marian anticipates that students might solve it as a permutations problem and she identifies specific permutation strategies (tree diagrams and lists) that students might use incorrectly to approach the combinations problem. In total, the lesson plan includes four incorrect strategies which are all coded as *doing* (Smith et al., 2013) since they only identify what students will do incorrectly and not what they might notice as a result of that doing.

4.5.1.2 Task Implementation

Student work indicates that the high level cognitive demands of the task were maintained. Figures 24 and 25 show examples of student work on portions of the task. Notice the different strategies students use to find the possibilities.



4) The next customer comes in and cannot decide what flavors she wants. She tells you that she wants a cone, but she asks you to pick any two of the five flavors for her at random (they should be two different flavors). How many different ways can you scoop an order for her? Show your work.

20 different ways

5) Suppose the customer from #4 changes her mind and decides that she wants her ice cream in a ~~bowl. How many ways can you~~ scoop an order for her now? Show your work.

20 different ways - Cancel out opposite Pairs, cut am of ways in half. = 10 ways.

Figure 24 First Example of Student Work on #4 and #5 from Ice Cream Parlor Task

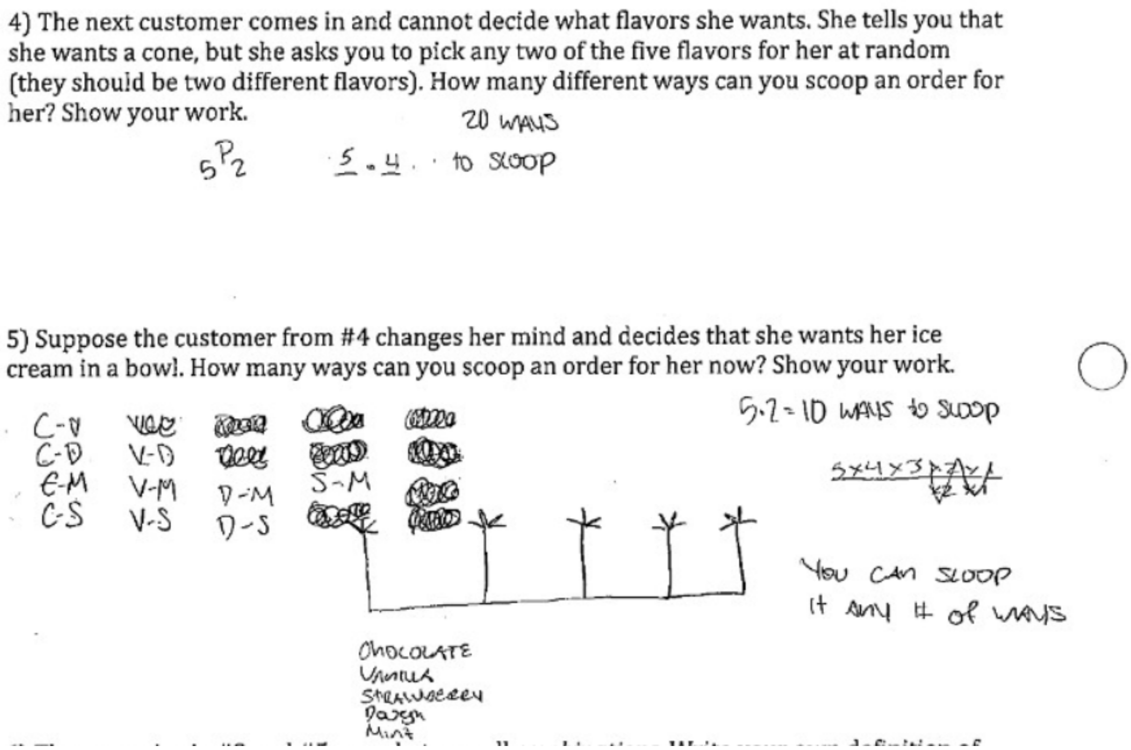


Figure 25 Second Example of Student Work on #4 and #5 from Ice Cream Parlor Task

In Figure 24, it is not exactly clear how the student came to answer using the tree diagram; however the explanation for item #5 indicates the student realized the situation was a combination and how to get the correct number of possibilities. The student work in Figure 25 indicates correct use of the permutation formula for item #4, and correct listing strategy for a combination situation. Marian describes the student work to meet her expectation of what she considered medium quality. She indicates that explanations were “not all complete or totally accurate”.

4.5.1.3 Thoughts About Whether Lesson Went as Planned

Marian thought "for the most part" her lesson went as planned, but she also talked about a portion that didn't. In both cases, her responses are student oriented. Marian focuses on students realizing (or not) the difference between permutations and combinations:

> So I think for the most part it did go as planned. There was one thing that I uh noticed the students were getting really caught up on because I did not anticipate. Um for question number 2 when it was talking about um filling the order the ice cream in the bowl. Um a lot of students were getting really confused as to like how you would scoop the ice cream into the bowl and it was the contextual part of the problem that I did not really, I just kind of assumed that they knew what I was talking about and that definitely hurt their entry into the task. So I think that was something I didn't plan for that um didn't go as planned. For the most part everything else did. Um they, I specifically said number 5 I knew that a lot of them would still be trying to think of permutations so a lot of them did do that so and it was something that I was able to address during the lesson. (M.T., Post Lesson Interview, Lesson 2)

When asked for the reason as to why it did or didn't go as planned, Marian provides a teacher oriented response, but it still focuses on things she did to help students realize whether to use a permutation or combination:

> Um number 4, when I asked them to um pick up a number of ways they could scoop the ice cream seemed more in order when they were actually permutations, that one definitely did go as planned. Um I knew that they would be trying to do the permutation notation cause that is what we just learned and I just wanted to make sure that I, I planned out questions to make sure that they knew why it was a permutation and I think because

we had done that recently and because we had really stressed the idea of permutations before so they were able to answer that question correctly. (M.T., Post Lesson Interview, Lesson 2)

Marian's post lesson thoughts focus both on students and herself, and provide explanations about whether and why the lesson went as planned. Regardless of her focus or type of explanation, Marian is referring to students' use of permutations or combinations relating back to the goals of the lesson.

4.5.1.4 Summary of Marian's Second Lesson

While Marian's written lesson plan received an overall low total score for attending to student thinking, her expectations, lesson plan (particularly goals and anticipation of incorrect thinking), and post lesson interview all align in their focus on students recognizing the difference between permutations and combinations. Thus, Marian provides evidence of attention to student thinking with regard to the mathematical concepts of permutation and combination. As mentioned earlier, Marian selected a high level task and the cognitive demands of the task were maintained by the students during implementation.

4.5.2 Summary of Planning and Implementation Relationship Exhibited by Marian

In her lesson 1, Marian used the detailed planning template and demonstrated a high degree of attention to the elements of student thinking, and the task was implemented at a high level of cognitive demand. In her third lesson, she used the detailed planning template but demonstrated a low degree of attention to student thinking, and the high level cognitive demands of the task declined during implementation. For her second lesson, Marian's written plan provided low attention to student thinking; however, her goals, expectations, written plan and post lesson thoughts all focused on students' use of permutations and combinations in appropriate situations.

4.6 RENEE NORRIS

The written plan for Renee's first lesson is in the same structure as TTLP format plans David used; however, Renee appears to have modified the content. For her second lesson plan, Renee used a TTLP formatted plan exactly as it appeared in the university resources. For her third lesson, Renee was implementing a scaffolded version of the same task that her second lesson was designed around. The lesson plan for her third lesson is hand-written, and it primarily includes worked out possible student solutions with corresponding questions to ask.

All of Renee's lesson plans include mathematical goals, and all of the lessons were planned around tasks with high-level cognitive demand potential. A summary of results from Renee's lessons are presented in Table 27. The lesson plan that was taken directly (without modification) from the university resource base indicates a high degree of attention to the elements of student thinking, while the other two plans indicate a moderate degree of attention. The first lesson (PTR assignment) was the only lesson in which the high level cognitive demands of the task were maintained during implementation. In general, Renee's thought about whether and why the lessons went as planned focus on teacher related actions.

Table 27 Summary of Results from Renee Norris's Enacted Lessons

Lesson	Attention to Student Thinking During Planning			Implemented	Post-lesson Thoughts
	Goal(s)	Task Selected	Planning Around Task		
1 (PTR)	Doing and Seeing	High	Slightly Modified Plan Moderate Element Score Doing	High	As Planned Student Focus Teacher Focus
2	Doing	High	Self-written Low Element Score Seeing	Low	As Planned Teacher Focus
3	Doing	High	Unmodified Plan High Element Score Seeing & Making Sense	Low	Not as planned Student Focus

Overall, Renee's planning and task implementation do not indicate any particular pattern across her three lessons. For example, lesson 1 was implemented at a high level of cognitive demand, had a moderate total planning score. In contrast, lesson 3 with the high planning score was implemented at a low level of cognitive demand. Interestingly, Renee's post-lesson thoughts about whether these two lessons went as planned align with the task implementation ratings. More specifically, for lesson 1 that she thought went as planned, the high level cognitive demands were maintained, and she focuses on student engagement after the lesson. For lesson 3 that she thought didn't go as planned, the cognitive demands declined, and she focuses on student engagement (or lack thereof) after the lesson.

One interesting finding is that Lesson 1 is the only lesson with a goal that goes beyond doing to state something students should see as a result of the lesson, and it is the only lesson for which the cognitive demands of the task were maintained. Perhaps even more interesting is that the summary of results for Renee's first lesson look very similar to the summary of results of Marian's second lesson. A closer look at Renee's first lesson reveals another similarity to

Marian's second lesson, which is alignment between the focus of the goals, anticipations in written plans, and thoughts as to why the lesson went as planned. The results of Renee's first lesson are presented to illustrate another example of a lesson where such alignment occurred and the task was engaged by students at a high level of cognitive demand.

4.6.1 Renee Norris – Lesson 1

For her first lesson, Renee's lesson plan included performance goals and a corresponding learning goal focused on what students should see mathematically as a result of the lesson. The lesson was designed around a high level task with a moderate degree of attention to student thinking. Students engaged the task at a high level and Renee focused on students' engagement when explaining why she thought it went as planned.

4.6.1.1 Attention to Student Thinking with Regard to Planning

(a) Mathematical Goal(s)

The three performance goals indicate mathematics related to exponential functions that students will do during the lesson, and the corresponding learning goal indicates what students will see mathematically:

Performance Goal:

> Identify the pattern of change between two variables that represent an exponential function in a situation, table, graph or equation.

Learning Goal:

Students will be able to make connections among the patterns of change in the table, graph, and equation, and formulate these patterns using exponential growth. (R.N, LP1, p. 2)

Neither the learning goal or performance goal indicates what it means for students to understand the patterns of change, but the learning goal does indicate students should be able to making the connections between the different representations. For this reason, the learning goal is considered to be focusing on what students will see mathematically (i.e. the connections between representations) as a result of engaging in the lesson.

(b) Selecting a Mathematical Task

In order to achieve the goals of the lesson, the lesson was designed around a real-life exponential situation task with high level cognitive demand potential. The task was drawn from a university resource base, but the context was altered to be more applicable to her students. The task is shown in Figure 26.

Ice Bucket Challenge

Throughout this past summer, having a bucket filled with ice, and cold water dumped on someone's head became pretty popular. Everyone, across the country was getting involved with this activity! But, why were they doing this?

In order to raise awareness for the ALS Association, people would challenge others to either donate money to the organization within 24 hours, or they would have to dump water on their heads. The creators of the Ice Bucket Challenge believed that if three new people were challenged every time, eventually, billions across the country would donate and know about the ALS Association.

At stage one of the process, a person challenged three others to take part in the Ice Bucket Challenge. At stage two, each of these three people challenged three others. How many people participated at stage five? How many people participated at stage 10? Describe a pattern, or a function that would model the Ice Bucket Challenge process at any stage.

Figure 26 Exponential Function Task from Renee's First Lesson

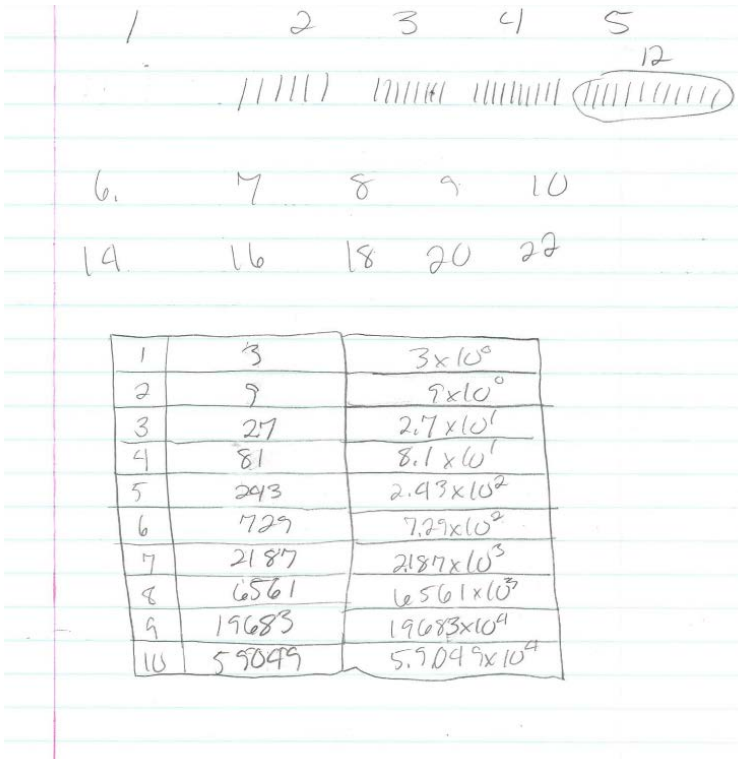
The task received a score of 3 according to the AR1 potential rubric. It does not suggest any specific pathway, and it has a variety of solution strategies that students can engage in to solve the problem. It does not explicitly prompt students to explain reasoning. As a reminder, Renee's expectations for student work on the task were not provided.

(c) Planning Around the Task

Overall, the lesson plan provided a moderate degree of attention to the elements of student thinking and primarily focused what students will do mathematically. The lesson plan lists several correct approaches students might take: pictorial representation, table/chart, graph, and equation. It also provides detail about one possible incorrect approach (i.e. students multiply number of people by stage number: 3x1 = 3, 3x2 = 6, 3x3 = 9, 3x4=12). The plan provides little information related to the classroom discussion around the solutions.

4.6.1.2 Task Implementation

Student work indicates that students' engaged the task in ways that the high-level cognitive demands were maintained during implementation. Examples of student work are provided in Figures 27 and 28. The two examples provided are instances of some of the different representations students used to solve the problem.



1	3	3×10^0
2	9	9×10^0
3	27	2.7×10^1
4	81	8.1×10^1
5	243	2.43×10^2
6	729	7.29×10^2
7	2187	2.187×10^3
8	6561	6561×10^3
9	19683	19683×10^4
10	59049	5.9049×10^4

Figure 27 Table Approach to Ice Bucket Task

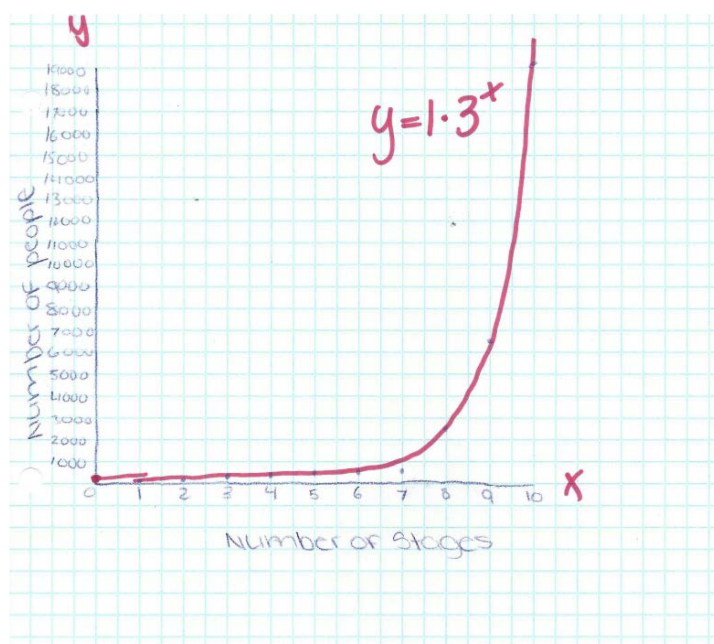


Figure 28 Graphical Representation of Ice Bucket Problem

In addition to the methods shown in Figure 27 and Figure 28 students engaged in a tree diagram, tally methods, and other similar methods to solve the problem. Overall, the student responses were very diverse in nature while leading them to the correct solution.

4.6.1.3 Thoughts About Whether the Lesson Went as Planned

Renee explains throughout her written reflection how overall she thought the lesson went as planned. In particular, Renee writes about the launch, explore and summarize phases of the lesson. With regard to the launch she writes, "In my opinion, the launch went very well. Students were engaged right from the start" (RN WR, p. 1). Later when referring to the explore phase she discusses how students used the different approaches she anticipated in her lesson plan. During the summarize phase she explains how she ordered the presentation of solutions differently than she planned, but by the end of the lesson the students "were able to determine where the growth factor was and to see first hand a model…" (RN WR, p. 4). Thus, with regard to meeting the mathematical, the lesson went as planned.

4.6.1.4 Summary of Renee's First Lesson

One of the mathematical goals of the lesson focused on students seeing connections between representations of exponential functions, and the lesson was designed around a task with high level cognitive demand potential. Renee's written lesson plan provided an overall low degree of attention to the elements of student thinking, and it primarily focused on what mathematics students would do during the lesson; however, students still engaged the task at a high level of cognitive demand. Renee's goal, the exploration phase of her written plan, and her thoughts about the lesson going as planned all focus on ways students engage the task and the mathematics they see as a result of their engagement. In particular, Renee’s goal and explanation of what it means for the lesson to go as planned focus on students making connections between the patterns of change in the representations of the exponential function.

The work produced by students reflects Renee’s goals and anticipations, and she refers to specific work to explain why she thought the lesson went as planned. Thus, the results of Renee’s first lesson are similar to the results of Marian’s second lesson. That is, with regard to planning there is alignment between goals, anticipations and focus when talking about the lesson going as planned, and the students engaged the task in ways that maintained the high level cognitive demands.

4.6.2 Summary of Planning and Implementation Relationship Exhibited by Renee

Overall, it is difficult to state any particular pattern or trend that crosses through Renee’s three lessons. For example, in lesson 1 has moderate planning and high implementation. Lesson 2 has low planning and low implementation, and Lesson 3 has high planning and low implementation. She did indicate that her third lesson did not go as planned based on student engagement, and the low level cognitive demand rating of student work corresponds with her thoughts. Renee’s first lesson was the only lesson in which students engaged the task with high level cognitive demands. Interestingly, the results from this lesson are similar to the results of Marian’s second lesson. More specifically, the overall plan shows low to moderate attention to the elements of student thinking, and primarily focuses on mathematics students are doing. There is alignment between her goals, anticipated strategies and thoughts about the lesson going as planned, and the high level cognitive demands of the task were maintained during implementation.

4.7 NICK NEWMAN

For all four of his lesson plans, Nick used the same self-written format that contained the following sections/headings: learning and performance goals, materials, setting up the task, and sharing and discussing the task. Under the setting up the task heading, each plan included a two column chart with “possible challenging aspects” as one column, “questions to ask” as the other column. Nick’s first lesson plan (PTR assignment) also included a list of possible solutions. The lesson plan for his first lesson was the only plan of the four to include any correct solution strategies. Nick’s first lesson plan is located in Appendix I. In general across his lesson plans, under the sharing and discussing the task section, Nick identified solutions he wanted to be shared during class discussion, and in some lesson plans listed questions he planned to ask.

Three of Nick’s lessons were designed around tasks with high level cognitive demand potential. A summary of results from Nick’s lessons are presented in Table 28. Nick’s lesson plans were not only similar in structure, but also in degree of attention to elements of student thinking. Overall, the majority of Nick’s lesson plan focus on what students are doing, but there are some instances of seeing found in his planning. During two of three lessons with high level task potential, the high level cognitive demands of the tasks were maintained during implementation. Nick’s thoughts about whether his lessons went as planned vary within and across lessons.

Table 28 Summary of Results from Nick Newman's Lessons

Lesson	Attention to Student Thinking During Planning			Implemented	Post-lesson Thoughts
	Goal(s)	Task Selected	Planning Around Task		
1 (PTR)	Doing	High	Self-Written Moderate Element Score Seeing	High	Met Goals Student Focus
2	Doing	High	Self-Written Low Element Score Seeing	Low	As planned Student Focus Teacher Focus
3	Doing	High	Self-Written Low Element Score Doing	High	As Planned Not as Planned Student Focus
4	Doing	Low	Self-Written Moderate Element Score Doing	Low	As Planned Not as Planned Student Focus Teacher Focus

The results from Nick's lessons do not suggest any particular relationship between planning and task implementation. The low implementation in the lesson four is likely attributed to the low level potential of the task to begin with. Nick's four lesson plans are nearly identical in format and degree to which they attend to student thinking. Of the three plans that used a task with high level potential, two were implemented in ways that maintained the high level cognitive demands of the tasks. An examination of Nick's Lesson 2 for which the high level cognitive demands of the task declined during implementation may provide some explanation as to why the implementation results were inconsistent with the other lessons for which he selected a high level task. Due to the similarity across lessons in attention to student thinking for Nick's goals and overall lesson plans, planning around the tasks will not be discussed for Lesson 2. Rather, the remainder of this section will focus on students' engagement with the task and Nick's thoughts about whether the lesson went as planned.

4.7.1 Nick Newman – Lesson 2

For Nick's second lesson he planned a lesson around a high level task, and he co-taught the lesson with Quinn (the pre-service teacher in this study). According the AR2 rubric, the high level cognitive demands of the task were not maintained during implementation; however, both Nick and Quinn frequently talk about the maintenance of high level of cognitive demands evident during discussion.

4.7.1.1 Mathematics Goal(s)

Nick's lesson indicated one learning goal and four performance goals:

Learning Goal:

> Students will learn now to find the solution of a system of two linear equations by looking for the point of intersection of the graphs for the individual equations. (NN, LP2, p. 1)

Each of the performance were related to mathematics students would do during the lesson, and the learning goal was also coded as doing. The terminology used in the learning goal did not warrant it being coded as seeing. For example, the goal does not state that students will see or realize the point of intersection represents the solution to the system. Rather it says they will find the solution by looking for the point of intersection. This could mean students were directly told the point of intersection represents the solution, and they are simply using the graph to find the answer.

4.7.1.2 Task Implementation

In general, students provided solutions for the task, but they did not provide any evidence as to how they came up with those solutions. An example of student work is provided in Figure 29.

At a school band concert, Christian and Faith sell memberships for the band's booster club. **An adult membership costs $10**, and a **student membership costs $5. At the end of the evening, the students had sold 50 memberships for a total of $400**. The club president asked,

"How many of the new members are adults and how many are students?"

You can answer the question by writing and solving equations that represent the question and the given information.

A. Let *a* represent the number of $10 adult memberships and *s* represent the number of $5 student memberships.

1. What equation relates *a* and *s* to the $400 income total? $10A + 5s = 400$
   Explain what each term of the equation represents.

   A → adult
   S → student

2. Find three solutions for your equation from part (1). Write them in the form (a,s).

   Solution 1: (10, 60).
   Solution 2: (20, 40).
   Solution 3: (30, 20).

3. What equation relates *a* and *s* to the total of 50 new members? $a + s = 50$

4. Find three solutions for your equation from part (3). Write them in the form (a,s).

   Solution 1: (5, 45).
   Solution 2: (10, 40).
   Solution 3: (15, 35).

Figure 29 Example of Student Work from Nick's Second Lesson (Task from CMP3, Grade 8 Curriculum)

The example shown in Figure 29 is reflective of all the other student work in the sample. Each student provide equations to parts 1 and 3, and also provide solutions to parts 2 and 4. However, no students showed any evidence of how they came up with the equations or corresponding solutions. For example, the solutions could be found by trial and error, graphing, or solving the original equation for one variable then substituting a value and solving. The numerous solution strategies and potential connections that could be made between them is why the task is high level potential; however, the limited evidence of student thinking when providing their solutions does not indicate those high level cognitive demands.

4.7.1.3 Thoughts About Whether Lesson Went as Planned

Interestingly, when interviewed following the lesson, Nick says the lesson went as planned and he specifically refers maintaining the high level cognitive demands of the task when explaining what it means to go as planned. Also of note, this lesson was co-taught with Quinn Brady, who also participated in the interview. During the interview, Quinn agrees with Nick's assertion that the high level cogntive demands of the task were maintained.

It is clear from their thoughts about the lesson that the discussion portion of the lesson played a major role in their perception of the high level cognitive demands being maintained during implementation. To provide an accurate portrayal of their thoughts, a large portion of the interview is provided:

NN: Okay, so this is Nicholas speaking and I do think it went as planned. Um we had students who completed the whole worksheet together and um we were able to get to the talking points that we wanted to get to using their work and um we were able to keep it a high level task. Do you want to talk about anything else before?

QB: Um no. This is Quinn, um I would agree with what Nick said um as far as it being high level uh in addition to, to the students recognizing you know, what the terms and variables were supposed to, to mean and I think that the one, one reason they were able to recognize that was because of uh there were a lot of us here so we were able to ask a lot of questions to kinda dive into their thinking but also um another instance where I kinda felt like, okay they're getting this was when we were talking about the point of intersection on the graph and they recognized that that intersection corresponded to the same points that that worked in both equations. Um and they also recognized that there's only going to be one point that worked for both equations or line depending on how you're looking at it.

NN: And this is Nick speaking again and I just wanted to say another thing that I thought was really cool about keeping it high level was we had anticipated um a few different things happening throughout the task and they actually did happen so one of which was um on numbers 2 and 4 um talking about the order of saying thirty adults and twenty students or twenty students and thirty adults uh the students did have that happen and also um having number 5 occur where they so no, no pairs can satisfy both because they didn't come up with, of the three solutions they picked for each they didn't match so we anticipated that some students might have that happen and it was it was interesting and um I think it maintained high level when we were discussing those things that happened so I think the students were really able to jump in on really thinking about what it what it meant for everybody to have these different answers and it was okay and why it was okay. I think that that went really well in the discussion as well.

I: So what do you what did you do, um to make sure the lesson went as planned as well as it did?

NN: So uh this is Nick speaking again, one thing is as I was going around when students were working um whenever they asked a few questions some people said, “does it matter what order I put the adults and the students?” Um so people said, “I don’t have any pairs but so and so does which one of us is correct?” Instead of trying to say who’s correct and who’s not I’d try to say, “well write down the information you came up with and we’re going to use that to talk about in the discussion,” so I think having them actually put that down uh gave us um materials to uh to pull from whenever we were discussing it in the in the class discussion.

QB: Yeah, this is Quinn. Um also I think that clarifying what the, the variables um were in the two equations also helped the students to see what we were trying to get at like I know there was uh a while there, there were a few students who thought that a meant the money and S meant the money or the money for the adults and S meant the money from the students and um it was like, “no no no, let’s think about this. You said that it equals fifty so is that fifty dollars?” and then they’re like, “no that’s fifty people total.” “Okay so then if there’s no students, then how many adults are there going to be?” “Well fifty.” “Okay, so then what is a?” and then it was like, “oh okay, I get it.” So kinda, like I think you said, Nicholas said this that just from, from clarifying with them um during our um monitoring stage um of like what are these variables exactly and how do they fit into this equation I think that helped them to see a little bit better what they were trying to ultimately do.

NN: And this is Nick speaking again, I think that our choice of warm up was really important especially after um yesterday they were having a bit of trouble um going from standard form to slope intercept form and it was when the first time when we really clarified that they're both, they're both linear equations and it was really important for that warm up to get them to, to discuss when its best to use a slope intercept form or when its best to use a standard form and I think using that and jumping into us discussing the scenario having the uh important information bolded and actually really talking about what they were giving us um helped us to, helped the students to be able to use that information without us giving them too much. So in part a and part b we really didn't give them much whenever we launched I don't think we really we didn't write anything down but by focusing on what the scenario was and by giving that warmup they were able to really jump into the questions without much help. (NN, Post Lesson Interview, Lesson 2)

Basically, in every response, Nick or Quinn refer to talking points, questions they asked, or some discussion around the task to explain why the lesson went as planned. The purpose of pointing this out is that this particular lesson could be a case where student work underrepresented student engagement with the task (Boston, 2012). There are a couple different reasons why this is plausible in this situation. One reason is that Nick's two other lessons using tasks with high level cogntive demand potential were implemented at a high level of cognitive demand. In this lesson, the student work does not provide evidence against high level implementation, it just does not provide enough evidence to support it. From Nick's interview it is apparent that there was a great deal of discussion around the students' work on the task. Also, the co-teacher's comments about the cogntive demands being maintained are consistent with

Nick's comments. The co-teacher is Quinn Brady whose four lessons for this study were all implemented in ways that maintained the high level cogntive demands of the tasks he selected.

4.7.2 Summary of Planning and Implementation Relationship Exhibited by Nick

Nick set goals that were similar in attention to student thinking for all of his lessons. He selected tasks with high level cognitive demand potential for three out of four lessons, and his lesson plans were very similar in format and attention to student thinking across all lessons. The low level implementation found in lesson 4 is likely attributed to the low level task potential. Of the three lessons using tasks with high cognitive demand potential, two remained at high level during implementation. The overall consistency in his planning makes it difficult to attribute any link between planning and implementation; however, his post lesson thoughts offer a possible explanation as to why the implementation in Lesson 2 was inconsistent with the implementation of his other two lessons using high level tasks. That is, it is possible that the student work provided for Lesson 2 underrepresented the cognitive demands at which students engaged the task. If this is the case, Nick would represent a pre-service teacher whose students were able to maintain high level cognitive demands when the lesson was designed around a task with high level potential.

4.8 CHRIS CAIN

For all three of his self-written lesson plans, Chris used the detailed template within the electronic planning tool. Chris responded to each prompt within the electronic planning tool, and he attached a PowerPoint slideshow to each plan. Chris used the slides during class to guide the lesson. Since the slideshows were attached within the template, they were coded as part of the entire plan. None of Chris's lesson plans included a monitoring tool.

All of Chris's lesson plans included at least one mathematical goal, and only one lesson was designed around a task with high level cognitive demand potential. A summary of results from Chris's lessons is provided in Table 29. Overall, Chris provides a low degree of attention to the elements of student thinking, and they do contain some instances of seeing and making sense. During all three lessons, the tasks were implemented by the students at low-levels of cognitive demand. For all three of his lessons, Chris thought parts of the lesson went as planned and parts did not go as planned. Note: Chris's lessons shown in the table do not include a PTR assignment since his lesson plan was not available.

Table 29 Summary of Results from Chris Cain's Enacted Lessons

Lesson	Attention to Student Thinking During Planning			Implemented	Post-lesson Thoughts
	Goal(s)	Task Selected	Planning Around Task		
1	Seeing	Low	Self-written (EPT) Moderate Element Score Seeing	Low	Not as planned Student Focus
2	Doing	High	Self-written (EPT) Low Element Score Seeing & Making Sense	Low	As Planned Not as Planned Student Focus
3	Doing & Seeing	Low	Self-written (EPT) Low Element Score Doing	Low	As Planned Not as Planned Other Focus Student Focus Teacher Focus

Two of Chris's lessons started as low level tasks and remained at low level during implementation. As mentioned earlier, the low implementation of these lessons is most likely contributed to the low potential. The one lesson that began at high level declined to low level during implementation. Overall, Chris's planning shows low attention to student thinking. Due to the fact that Chris had only one lesson using a task with high level cognitive demand potential, it is difficult to identify any type of relationship between planning and implementation.

Overall, Chris represents an entirely different case than the other pre-service teachers simply by the fact that two of his three tasks had low-level cognitive demand potential. Across the other five pre-service teachers, there were only two other tasks with low-level potential. For this reason, a discussion of one or more of Chris's lessons does not provide any insight into the planning and implementation relationship. Rather, a look at some of the other ways Chris's lessons are different than the other five pre-service teachers seems more meaningful.

4.8.1 Chris's Lesson Plans Compared to Other Pre-Service Teachers' Plans

The primary difference between Chris's plans and the other pre-service teachers' plans is that none of his plans contained a monitoring tool or a chart designed to help with the explore phase of the lesson. Of the 18 other lesson plans submitted to the study, 15 of them contained some type of monitoring tool or chart for the explore phase. Each of the five other pre-service teachers had a chart or tool in at least one plan, and three of them included one tool in every lesson plan. Thus, Chris's lack of such a tool or chart is inconsistent in comparison to the vast majority of lesson plans submitted by other pre-service teachers in the study.

4.8.2 Chris's Thoughts about Lesson Going as Planned Compared to Others' Thoughts

Another difference between Chris and the other pre-service teachers arise in his explanations of what it means whether a lesson goes as planned. For the other pre-service teachers, the vast majority of all other explanations or reasons as to whether and why the lesson went as planned are either student oriented and teacher oriented. Chris, on the other hand, at least once in each lesson focuses on something other than the students or himself. For example, the following excerpts are taken from the post lesson thoughts for each lesson:

> CC: Okay, it did not go um completely as planned. The parts that we got through, we probably got through like three quarters of the lesson and all the main things, except we did not get to discuss the concepts at the end, which is unfortunate (C.C, Post-lesson interview for Lesson 1).

CC: Okay, so um the first part of class essentially went as planned. I was a little worried that it would take too long and I wouldn't be able to go into any detail with the more high level task during the second part um but I picture the first part being a little more procedure oriented (C.C, Post-lesson interview for Lesson 2).

CC: Okay. So um the lesson today, uh did not go exactly as planned uh because of time restraint and didn't expect certain aspects of those lessons to take as long as they did… Uh we did not have any time to talk about quadratics in (inaudible) form but our, the, the part of the lesson that was about quadratics in standard form did go as planned if a little bit (C.C., Post-lesson interview for Lesson 3).

CC: Okay, um because we included technology in this lesson, um things took a little bit longer. Uh usually if you hand them the paper, they're able to look at it and do it right away but whenever technology is involved, um there are computer issues, there it takes a while to boot up and even though it keeps the students ***(keeps them pacified***) in a sense or it keeps a lot of paper problems from happening. Um sometimes they get distracted while they're on their computer and they might be doing something for another class that they shouldn't be doing so the whole process takes a little bit longer to do, like it would in a different format so we only got through essentially half of the lesson that I wanted to get through today (C.C, Post-lesson interview for Lesson 3).

In each of the above responses, Chris refers either to time or not getting through parts of the lesson. In the final excerpt Chris refers to technology, and he provides it as a reason why time becomes an issue. Chris's focus on time restraints and not getting through parts of the lessons is different than the types of things the other pre-service teachers talked about when explaining whether their lessons went as planned. For example the other pre-service teachers focused on students' engagement with the task or instructional moves they made during the lesson.

4.8.3 Summary of Planning and Implementation Relationship Exhibited by Chris

Overall, Chris is a case of a pre-service teacher who demonstrated low attention to student thinking with regard to planning and low implementation. Even though overall planning was low and implementation was low in all lessons, identifying a relationship across lessons is difficult. The low planning and low implementation for two of the lessons is likely linked to the fact that tasks selected for those lessons had low level cognitive demand potential. Some insight is gained when comparing Chris to the other pre-service teachers. For example, none of Chris's lesson plans included a monitoring tool or chart for the explore phase. Meanwhile, the vast majority of all other lesson plans submitted to the study contain such a tool or chart. Also, Chris's thoughts about the lesson going as planned focus on time as an issue. Time constraint was not an issue of concern for the other participants.

4.9 PLANNING AND IMPLEMENTATION TRENDS ACROSS PRE-SERVICE

Looking across the results of six pre-service teachers some different trends emerge regarding the relationship between attention to student thinking during lesson planning and students' engagement with the task during implementation. In particular these trends include: 1.) Two pre-service teachers implemented all of their tasks at a high level of cognitive demand. Overall, these two pre-service teachers provided more evidence of attention to student thinking during planning than the other pre-service teachers. 2.) All pre-service teachers implemented their lessons that were part of a university assignment with high level cognitive demands. That is, when receiving specific planning based support from an instructor and university supervisor, every intern was able to maintain the cognitive demands of a selected high level task during implementation. 3.) Several of the tasks utilized during study are accompanied with detailed planning support resources. In the majority of lessons where such tasks were selected, the pre-service teachers were able to maintain the high level cognitive demands of these tasks during implementation.

4.9.1 Highest Implementers Showed Most Attention to Student Thinking During Planning

The lens one analysis suggested a significant positive relationship between attention to elements of student thinking total score and high level task implementation. More specifically, as total planning score increases the odds of high level task implementation also increase. David and Quinn's scores contribute to this result since overall they received the highest planning scores and implementation scores among the pre-service teachers. In addition to this quantitative trend, David's and Quinn's lesson plans also provide more instances of focusing on helping students see and/or make sense of the mathematics than the other pre-service teachers. Table 29 indicates the total instances of the different quality types of anticipations found in each lesson plan.

Table 29 Number of Instances by Each Quality Type of Anticipation in Each Lesson Plan

Pre-service teacher and lesson	Quality of Anticipations			
	Logistics	Doing	Seeing	Making Sense
David Upton Lesson 1	2	3	6	9
David Upton Lesson 2	5	12	3	2
David Upton Lesson 3	3	4	1	2
David Upton Lesson 4	1	3	2	3
Quinn Brady Lesson 1	0	7	5	0
Quinn Brady Lesson 2	0	4	5	2
Quinn Brady Lesson 3	4	15	4	0
Quinn Brady Lesson 4	0	4	7	1
Marian Turner Lesson 1	5	10	1	2
Marian Turner Lesson 2	1	8	1	0
Marian Turner Lesson 3	3	6	1	1
Renee Norris Lesson 1	2	7	1	0
Renee Norris Lesson 2	0	6	3	0
Renee Norris Lesson 3	2	6	5	1
Nick Newman Lesson 1	3	8	2	0
Nick Newman Lesson 2	3	7	3	0
Nick Newman Lesson 3	3	7	0	0
Nick Newman Lesson 4	3	7	0	0
Chris Cain Lesson 1	3	2	1	0
Chris Cain Lesson 2	2	6	5	1
Chris Cain Lesson 3	2	2	2	0

When considering the different quality levels of anticipation, there is no particular frequency established to indicate a particular degree of attention to student thinking. For example, there is nothing to support a claim stating there needs to be three instances of making sense or five instance of seeing in a lesson plan for it to show evidence of attention to student thinking. Rather, the value of these types of anticipations seems to be in the frequency of different qualities within a given lesson and comparison of such numbers across the pre-service teachers. Table 30 indicates the average number of instances of each anticipation type. Raw averages per lesson were calculated for each quality type of anticipation since each pre-service teacher did not teach the same number of lessons.

Table 30 Average Number of Instances of Each Quality Type Across Each Teacher

Pre-service teacher	Average Number of Quality of Anticipations			
	Logistics	Doing	Seeing	Making Sense
David Upton	2.75	5.5	3	4
Quinn Brady	1	7.5	5.25	.75
Marian Turner	3	8	1	1
Renee Norris	1.33	6.33	3	.75
Nick Newman	3	7.25	1.25	0
Chris Cain	2.33	3.33	2.67	.33

When looking at average instances per lesson, David has the greatest amount of making sense and is tied for second most in seeing instances. Quinn has the greatest average of seeing instances per lesson plan. Overall, these findings support and enhance the positive planning and implementation trend found by the lens one analysis. More specifically, they provide evidence that the pre-service teachers who were able to implement all tasks at a high level of cognitive demand, also provided more quality of focus across their entire plans when anticipating what students will do and learn as a result of the lesson.

4.9.2 Consistent High Implementation For Lesson that Part of University Assignment

One of the lessons submitted as a data point by all but 1 student was from the planning, teaching reflecting (PTR) university assignment. Even though this study does not have the lesson plan from this 1 student, the student work associated with the lesson was submitted. These particular lessons are all labeled as Lesson 1 in the previous discussion for each the five pre-service teachers who also submitted a lesson plan. This lesson was not included for the discussion of Chris in the previous section since he did not submit a lesson plan; however, it is included here because Chris's student work was submitted. Table 31 provides the task potential score and task implementation score for each lesson enacted as part of the PTR assignment. All the pre-service teachers received lesson planning feedback from the course instructor; however some tasks were also accompanied by additional support resources.

Table 31 Potential and Implementation Score for Each University Based on Assignment Lesson

Pre-service teacher	Pot.	Imp.	**Source**	**Description of Support**
David	4	3	Institute for Learning (IFL) (University of Pittsburgh)	• Engaged by intern during university coursework • Embedded in a case study • Fully fleshed out lesson plan • Received feedback from instructor and cooperating teacher
Quinn	4	3		• Received feedback from instructor
Marian	4	4	IFL	• Fully fleshed out lesson plan • Received feedback from instructor and cooperating teacher

Table 31 (continued)

Renee	3	3	University Resource Base	• Engaged by Intern during Coursework • Changed real-world context of original problem per advice from professor • Task was accompanied by a two-page narrative • Received Feedback from Instructor
Nick	3	3	Adapted from CMP3, Grade 8	• Received Feedback from Instructor
Chris	3	3	Adapted from NCTM	• Received Feedback from Instructor

While there may not necessarily be a link between the lesson planning evidence and task implementation scores for each of these lessons, there is the common ground of each pre-service teacher receiving lesson planning support and feedback. In particular, the university assignment requires the lesson to be designed using the TTLP, and the lesson plan must be submitted to the university instructor before enacting the lesson. Thus, each pre-service teacher focused to some extent on student thinking and received feedback related to student thinking during planning. The consistent findings suggest that when the pre-service teachers receive support focused on attention to student thinking, they are able to plan lessons in which students are able to engage the task with high level cognitive demands.

The cases of Chris and Renee make the finding particularly interesting since this was their only task with high level implementation. This suggests that even pre-service teachers who struggled to select and/or implement tasks at a high level of cognitive demand were able to do so when receiving focused support. Interestingly, this idea of detailed planning support stretches beyond the university based assignments to other lessons enacted during the study. This paper defines detailed planning support as support received from an instructor as part of a university assignment (i.e. PTR assignment or research seminar) or planning resources available (i.e. embedded in case study, fully fleshed out lesson plan).

4.9.3 High Rate of Implementation for Non-PTR Tasks with Detailed Planning Support

Several of the lessons enacted during the study that were not part of the PTR assignment were also accompanied by some certain types of planning support. The majority of lessons designed around such tasks were implemented with high level cognitive demands. Table 32 provides a list of these tasks with their potential and implementation scores, as well as the task source and brief descriptors of the support surrounding the task.

Table 32 Tasks and Associated Planning Support

Pre-service teacher (Lesson #)	Pot.	Imp.	Task Title	Source	Description of Support
David (2)	3	3	Calling Plans	Institute for Learning (University of Pittsburgh)	• Engaged by intern during university coursework • Embedded in a case study • Fully fleshed out lesson plan • Communicated with another intern who also used task • Received feedback from cooperating teacher
David (3)	4	4	Convincing and Proving	MARS	• Task was implemented as part of research seminar assignment in which he received feedback from instructor and classmates • Possible Anticipated Responses to Proofs Provided by Resource • Received feedback from cooperating teacher
David (4)	3	3	Flicks	Mathalicious	• Fully fleshed out lesson plan • Received feedback from cooperating teacher

Table 32 (continued)

Quinn (2)	3	3	Tiles	Modified from Smith & Stein (2011)	• Engaged by intern during coursework • Embedded in a case study
Quinn (3)	3	3	Pythagorean Theorem: Square Areas	MARS	• Fully fleshed out lesson plan • Communicated with another intern who also used task
Quinn (4)	4	3	Jose's Surfboard	Institute for Learning (University of Pittsburgh)	• Engaged with launching of task during university coursework • Fully fleshed out lesson plan
Marian (2)	3	3	Ice Cream Parlor Combinations	Self-created	• Planning feedback provided by University Supervisor in EPT
Marian (3)	3	2	Would you rather? Arithmetic vs. Geometric	Modified from Discovering Advanced Algebra 2nd Edition (Kendall & Hunt)	• Received feedback from cooperating teacher
Renee (2)	4	2	S-Pattern	Slight adaptation Institute for Learning (University of Pittsburgh)	• Engaged by intern during university coursework • Embedded in a case study • Watched a video of teacher using the task that focused on supporting productive struggle
Renee (3)	4	2	S-Pattern with Scaffolding	Slight adaptation Institute for Learning (University of Pittsburgh)	• Engaged by intern during university coursework • Embedded in a case study • Watched a video of teacher using the task that focused on supporting productive struggle

Table 32 (continued)

Nick (2)	3	1	It's In the System	CMP3, Grade 8 Curriculum	
Nick (3)	3	3	Frog's, Fleas, and Painted Cubes	CMP3, Grade 8 Curriculum	
Nick (4)	2	1	Words and Equations	MARS	
Chris (1)	2	2	Point Testers	Unknown	
Chris (2)	3	2	Solving Quadratics Without Factoring	Unknown	
Chris (3)	2	1	Optimization in Standard Form	Unknown	

In total, there were 16 lessons (in addition to the PTR assignment) represented in Table 38 that were enacted during the study. Eight of these lessons were implemented at a high level of cognitive demand, and eight were implemented at a low level. Interestingly, 6 out of the 8 high implemented tasks were accompanied by some type of detailed planning support. All 6 were implemented by David or Quinn.

David for example used the Calling Plans task which he engaged in as student during his university course work. The task was also embedded in a case study discussing its planning and implementation (Smith & Stein, 2011) that David had exposure to. Furthermore, the task was accompanied by a fully fleshed out detailed lesson plan created by the Institute For Learning (IFL) (University of Pittsburgh) that serves as a model TTLP lesson plan. David's other tasks (Convincing and Proving and Flicks) were also accompanied by different forms of support. For example, the Flicks lesson plan was provided in fully fleshed out form by the Mathalicious website. While the plan was not written to be a model TTLP lesson plan, it still addressed in detail the main components included in the TTLP.

Quinn also used tasks that were accompanied by detailed planning support. For example, the Tiles task was a modified version of a task embedded in a case study discussing its planning and implementation (Smith & Stein, 2011). The Pythagorean Theorem task was accompanied by a fully fleshed out lesson plan provided by MARS, and according to Quinn he got ideas from another intern on how to improve the lesson. Lastly, with regard to Jose's Surfboard, Quinn engaged with launching the task and had access to a fully fleshed out model TTLP lesson plan provided by the IFL. The anticipations in Quinn's "self-written" plan are identical to those in the IFL plan.

In contrast to these findings, only 2 of the 8 low-level implemented tasks represented in Table 38 were accompanied by detailed planning support, and they were similar tasks (S-Pattern and S-Pattern with scaffolding) enacted by the same pre-service teacher (Renee). The researcher is not aware of any detailed planning support for the remaining 6 tasks implemented at a low level. To put all of these findings in perspective, consider Table 33 which includes Chris's PTR task results.

Table 33 Task Results from Entire Study Including Chris's PTR 2

	High Implementation	Low Implementation	Total
High Potential	14	5	19
Low Potential	0	3	3
Total	14	8	22

In the entire study, there were 14 tasks implemented at a high level of cognitive demand, and they are shown in Table 34. To the researcher's knowledge, 12 of these tasks were accompanied by some type of detailed planning support.

Table 34 Tasks Implemented at High Level of Cognitive Demand

Pre-service teacher (Lesson #)	Task Title	Detailed Planning Support YES or NO
David (1)	Hexagon Trains	YES
David (2)	Calling Plans	YES
David (3)	Convincing and Proving	YES
David (4)	Flicks	YES
Quinn (1)	Lunch Combinations	YES
Quinn (2)	Tiles	YES
Quinn (3)	Pythagorean Theorem: Square Areas	YES
Quinn (4)	Jose’s Surfboard	YES
Marian (1)	Amazing Amanda	YES
Marian (2)	Ice Cream Parlor Combinations	NO

Table 34 (continued)

Renee (1)	Ice Bucket Challenge	YES
Nick (1)	Sneaky Andrew	YES
Nick (3)	Frogs, Fleas, and Painted Cubes	NO
Chris (PTR)	Batman	YES

Alternatively, there were 8 tasks implemented at a low level of cognitive demand, and they are shown in Table 35. Five of these began with high level potential and declined during implementation. Of these 5, only 2 were accompanied by some type of detailed planning support.

Table 35 Tasks Implemented at a Low Level of Cognitive Demand

Pre-service teacher (Lesson #)	**Task Title**	**Detailed Planning Support** **YES or NO**
Marian (3)	Would you rather?	NO
Renee (2)	S-Pattern	YES
Renee (3)	S-Pattern with scaffolding	YES
Nick (2)	It's in the system	NO
Nick (4)	Words and Equations	NO

Table 35 (continued)

Chris (1)	Point Testers	NO
Chris (2)	Solving Quadratics Without Factoring	NO
Chris (3)	Optimization in Standard Form	NO

In general the findings indicate that when pre-service teachers draw upon tasks accompanied by detailed planning support, then the tasks have high level cognitive demand potential and the high level cognitive demands are successfully maintained during implementation.

4.10 SUMMARY OF RESULTS

Overall, the results do address the research question:

> What is the relationship between pre-service teachers' attention to student thinking with regard to lesson planning around a mathematical task (perceived to be high level by the pre-service teacher), and the level of cognitive demand at which the mathematical task is implemented?

The quantitative analysis between attention to the elements of student thinking total score and level of task implementation suggest a possible significant positive relationship between planning and implementation, and it warrants further use in studies with larger sample sizes. The results from the cases of pre-service teachers further support these findings. That is, the two pre-service teachers with the highest planning scores also demonstrated more evidence of focusing on seeing and/or making sense when anticipating what students would do during the lesson or learn as a result of it. It so happens also that these two pre-service teachers were the only ones who implemented all of their tasks examined by this study with high level cognitive demands.

Two other trends were found related to the implementation of tasks accompanied by detailed planning support. One trend was the high level implementation (for all pre-service teachers) of the lesson that was enacted as part of a university assignment. A similar trend was the high success rate of high level implementation for the tasks that were accompanied by some type of detailed planning support. In general, all of the findings point towards answering the research question. That is, high level task implementation occurred at a high success rate when attention to student thinking was evidenced or supported during planning.

5.0 DISCUSSION

5.1 IMPORTANCE OF THE STUDY

Classrooms where students engage in mathematical tasks with high level cognitive demands have been linked to higher levels of student performance compared to classrooms where students do not have such opportunities (Stein & Lane, 1996; Hiebert & Wearne, 1993). More specifically, students in classes where the high level cognitive demands of tasks are maintained during implementation show greater gains in performance compared to students in classes where the cognitive demands decline (Stein & Lane, 1996). While the maintenance of the cognitive demands of high level tasks is important, teachers do not demonstrate a high success rate of such maintenance during implementation (Stein et al., 1996). As a result, several avenues have been developed to help teachers maintain the cognitive demands of high level tasks. One of these avenues is detailed lesson planning that focuses on student thinking around mathematical tasks across an entire lesson (Smith et al., 2008; Smith & Stein, 2011).

Instruction that focuses on student thinking is linked to greater student achievement (Stigler & Hiebert, 1999), and teachers who focus on student thinking during planning provide evidence of such focus during instruction (eg. Stigler, Fernandez, & Yoshida, 1996; Schoenfeld, et al., 2000; Ball, 1993). Thus, thoughtful lesson planning should "lead to more rigorous instruction and improved student learning" (p. 118, Smith et al., 2012). Expert teachers attend to student thinking during planning and instruction (Leinhardt, 1993; Schoenfeld et. al., 2000; Lampert, 2001). Novice teachers, on the other hand, tend to focus more on teacher actions (Leinhardt, 1993), and they must explicitly consider student thinking during planning so they can teach more like experts (Borko & Livingston, 1989).

Pre-service teachers are a particular set of novices who are learning to teach. They need to be provided opportunities to help them develop instructional approaches that foster student learning, because engaging students in meaningful mathematics and mathematical practices that deepen learning can be a difficult matter (Brown & Borko, 1992). While delivering meaningful instruction may be difficult for pre-service teachers, it still is possible. A case-study indicates that a pre-service teacher was able to consistently implement tasks with high-level cognitive demands (Mossgrove, 2006). In another study, pre-service teachers who were enrolled in a teacher education course focused on attention to student thinking were able to improve their attention to student thinking during lesson planning over time (Hughes, 2006). The same pre-service teachers showed increased attention to student thinking when explicitly engaging in planning using the TTLP.

Pre-service teachers are a meaningful population to study is because they have demonstrated the ability to engage in planning that attends to student thinking (Hughes, 2006); however such planning has not been explicitly linked to instruction. Mossgrove (2006) discussed differences in the planning practices of the two pre-service teachers in the study, but did not formally investigate attention to student thinking during planning. The purpose of this study is to investigate the premise of the lesson planning project that thoughtful planning leads to better instruction for a specific population of pre-service teachers. More specifically it seeks to investigate the relationship between pre-service teachers' attention to student thinking with regard to lesson planning and the level of cognitive demand at which students engaged mathematical tasks.

The investigation of this relationship brought forth different findings that include the following: 1.) Quantitative Analysis suggests that with further research a positive significant relationship between total score for attending to elements of student thinking during planning and maintenance of high level cognitive demands may be identified. 2.) Two pre-service teachers implemented all of their tasks at a high level of cognitive demand. Overall, these two pre-service teachers provided more evidence of attention to student thinking during planning than the other pre-service teachers. 3.) All pre-service teachers implemented their lessons that were part of a university assignment with high level cognitive demands. That is, when receiving specific planning based support from an instructor and university supervisor, every intern was able to maintain the cognitive demands of a selected high level task during implementation. 4.) Several of the tasks utilized during the study were accompanied by detailed planning support resources. In the majority of lessons where such tasks were selected, the pre-service teachers were able to maintain the high level cognitive demands of these tasks during implementation.

In general these findings align with the existing literature regarding the link between planning and instruction. The findings build upon the research related to pre-service teachers' ability to implement meaningful instruction. They also build upon the literature related to mathematical tasks by providing evidence of task implementation after engagement in explicit detailed planning related to student thinking around the task.

5.2 EXPLANATIONS AND IMPLICATIONS OF RESULTS

The lessons enacted during this study represent the pre-service teachers' "best efforts" at instruction. Thus, the above findings should be considered in this context. The term "best effort" indicates that certain conditions surrounded the enacted lessons examined by this study. These conditions include: pre-service teachers were aware of an expectation that all lessons should be planned around high level tasks using the detailed planning tool, the majority of lessons were enacted when a university supervisor was observing the classroom, and one lesson per pre-service teacher (with the exception of one participant) was part of a university assignment focused on planning and teaching. With these conditions in mind, each finding is discussed in relation to the existing literature.

5.2.1 The Quantitative Relationship Between Lesson Planning and Task Implementation

The small number of lessons enacted in this study (21) makes it difficult to draw any firm conclusions about a quantitative relationship between lesson planning and task implementation; however the results suggest that future studies with a larger sample size warrant similar analysis. In particular, this study found a significant positive relationship between total score for attending to elements of student thinking during planning and maintenance of high level cognitive demands for the limited number of enacted lessons.

Such a finding is linked to previous research relating planning and instruction. For example, Japanese teachers possess and use knowledge of their students' thinking both during planning and instruction to help students gain a conceptual understanding of the mathematics being presented (Stigler & Hiebert, 1999). Within both mathematics and science education, Japanese lesson study has been linked to enhanced teachers' instruction and students' achievement (e.g., Lewis & Tsuchida, 1998; Yoshida, 1999; Kawanaka & Stigler, 1999). In particular, Japanese teachers focused primarily on student thinking in their written plans. During instruction based on those plans, the students of the Japanese teachers were given appropriate opportunities to think and learn mathematically (Stigler, Fernandez, & Yoshida, 1996).

For this study, the total planning score is an indication of the degree to which written plans focused on student thinking, and the task implementation score is an indication of the degree to which students engaged with the mathematics during the lesson. Thus, the positive significant relationship aligns with the planning and instruction/student achievement relationship commonly found in Japanese Lesson Study.

One particular element of student thinking was also marginally significantly related to task implementation. The findings related to Anticipating Students' Incorrect Thinking suggested that as attention to student thinking for this element increased the odds of high level task implementation became greater. For example, the odds of high level task implementation for enacted lessons whose plans received a score of 3 for this element compared to those with a score of 0 were marginally significant. Also, these odds were greater than the odds for enacted lessons whose plans received a score of 2 for the element compared to those with a score of 0.

This particular finding fits with the concept of the Five Practices. In particular, one could argue that anticipating student thinking is the foundational practice. That is, the Five Practices rely on an embedded nature in which each practice builds upon the previous practice (Smith & Stein, 2011). As anticipating student thinking is the first practice, the remaining practices are built around the anticipation of students' responses.

5.2.2 Highest Implementers Showed Most Attention to Student Thinking During Planning

The findings related to David and Quinn illustrate the quantitative trend just discussed, and they represent specific examples of the positive relationship between planning and instruction. From a qualitative standpoint, David's and Quinn's results provide further support to the relationship through their overall focus on seeing and/or making sense of anticipating what students would do during the lesson or learn as a result of the lesson. In particular, their results fit in with the premise of the Lesson Planning Project that "teachers' engagement in thoughtful, thorough lesson planning routines would lead to more rigorous instruction and improved student learning" (p. 118, Smith et al., 2012). More specifically, in the Lesson Planning Project "thoughtful" and "thorough" refer to attention to the goals, tasks, and anticipated student responses during planning. David's and Quinn's lesson plans provide evidence of overall attention to student thinking via attention to these practices. Their average total scores for attention to the elements of student thinking (David =12 and Quinnn = 10.75) which represent different aspects of the Five Practices were the highest averages among the pre-service teachers. Also, their written lesson plans provided the most instances of seeing and/or doing among the pre-service teachers. A closer look at where the majority of instances occur reveals attention to different parts of the five practices. For example, several of Quinn's instances of *seeing* are found in the summarize phase of his lesson plan where he is focusing on connecting student responses (the fifth practice) that he used for public display. Similarly, several of David's instances of making sense are also in the summarize phase surrounding the discussion of students' solution strategies.

Even David and Quinn's post-lesson thoughts showed the greatest evidence of attention to student thinking with regard to planning. Both of their written reflections for the university assignment lesson described students' engagement with the task to explain why the lesson goals were accomplished. Also, for two of their other enacted lessons each of them focused on students' engagement with the task to explain what it meant for the lesson to go as planned. More specifically, they talked about how students responded to the task in ways they anticipated. Thus, even in their post lesson thoughts they were explaining how the first of the Five Practices helped their lesson go as planned. Each had one lesson where their post lesson thoughts focused on teacher actions, but in both cases the teacher actions described were in the context of what they did for the students to help the lesson go as planned.

It should be noted that David and Quinn's lesson planning was quite different. The majority of David's plans were drawn directly from resources, and the plans provided the detailed attention to student thinking. Quinn wrote the majority of his plans himself which means he provided the attention to student thinking. The difference in planning practice but similarity in implementation suggests that perhaps it does not matter who does the thinking to plan the lesson as long as the thinking in the plan is consulted. David's expectations and post-lesson thoughts indicate that he did consider his own students' thinking in relation to the lesson plans he was using. Other findings in the study are consistent with David's results. That is, when pre-service teachers used tasks accompanied by detailed planning support, they were able to implement high level cognitive demands with a high rate of success.

5.2.3 Implementation of Lessons with Tasks Accompanied by Planning Support

Similar to David lessons, several of the other lessons enacted for this study were designed around tasks that were accompanied by planning support. For example, each pre-service teacher enacted a lesson that was part of a planning, teaching, reflecting assignment, and each of those lessons were implemented at a high level of cognitive demand. Also, several of the other lessons enacted for this study were designed around tasks accompanied by detailed lesson plans that attended to student thinking. A large majority of these lessons were enacted at a high level of cognitive demand.

Literature has indicated that pre-service teachers often struggle during instruction. The finding in this study related to the implementation of tasks that were part of university assignment lessons indicates that when receiving appropriate support, pre-service teachers are able to implement meaningful instruction. In particular, this finding lends support to what the focus of teacher education should be. Hughes (2006) study showed that when pre-service teachers receive support in focusing on student thinking via lesson planning in a teacher education course, they were able to improve their attention to student thinking during planning over time. The particular finding in this study indicates that when pre-service teachers receive support in focusing on student thinking during planning, they are able to implement tasks at a high level of cognitive demand.

In this study, there were 14 tasks implemented with high level cognitive demands. Of these 14, 12 were accompanied by some type of planning support. Several of these were accompanied by detailed lesson plans. In general, the high success rate suggests that tasks accompanied by detailed planning support have an increased likelihood of being implemented with high cognitive demands.

The pre-service teacher David illustrates this through his majority use of lesson plans that were already designed, and some of the other teachers also used such plans for some of their enacted lessons. This finding suggests that perhaps more planning resources should be developed for teachers to consult. It is acknowledged that the TTLP is a time consuming process and it is not intended for everyday use (Smith et al., 2008). Thus, it is likely that when pre-service teachers are not being observed or putting forth “best effort” then their plans do not contain such detail. The findings from this study suggest that it may not be necessary for the pre-service teachers to spend the time creating the plans themselves. Rather, plans that are designed by mathematics educators could be successfully/effectively utilized by pre-service teachers.

5.2.4 Interesting Findings

The study also resulted in some findings that were not necessarily patterns or trends but perhaps should be given a further look in future research. Such findings include: 1.) enacted lessons with a particular focal alignment across planning data sources were implemented at a high level cognitive demand despite not necessarily having high planning scores, 2.) the structure of the lesson plans may play a role in degree of attention to certain elements of student thinking, and 3.) perhaps there are certain barriers/supports identified by pre-service teachers that are related to task implementation.

There were certain enacted lessons that did not show an overall high degree of attention to student thinking, yet the tasks were implemented with high cognitive demands. In two lessons there was a common focus across all data sources. Marian and Renee each enacted a lesson with low to moderate degree of attention to the elements of student thinking and relatively few instances focusing on how students would see or make sense of the mathematics. In each of these enacted lessons, all of their data sources showed evidence that aligned with the mathematical goal(s) of the lesson. More specifically, their expectations from the coversheet, the anticipated student responses, and their post lesson thoughts all focused on students' engagement with the task in relation to the mathematical goals. Since this occurred more than once, this finding could be suggesting that focusing on certain elements of attention to student thinking is more important than others. For example, these particular lessons suggest focusing on mathematical goals and anticipated students strategies aligned with those goals may be more important than focusing on other elements such as the discussion components. This finding also aligns with the earlier argument that anticipating student thinking is the foundational component of the Five Practices. Further research is required to determine whether planning attention to elements related to practices following anticipation is necessary, or if focusing on anticipated responses in relation to the goals yields the same results.

Another finding is that the structure of lesson plans may lend itself to greater attention to certain elements of student thinking. For example, lesson plans with charts during the explore phase with two columns (one for solutions and one for questions to ask with solution) all received maximum scores of 2 in the Questions to Assess/Advance Student Thinking element. Also, lesson plans that were written using the detailed planning template tended to have goals with more instances of seeing and making sense. This result is likely a function of the prompts within the detailed template that ask the planner about objectives and goals, and explicitly ask "how" the planner will know the objective or goal is met. Also, the resource based plans designed using the TTLP utilized charts throughout the launch, explore and summarize phases of the lessons. These plans overall received the highest total planning scores. In general these findings suggest that structure may be linked to attention to student thinking in planning, and it points to further research connecting such structure to implementation.

Another interesting finding relates to the pre-service teachers' post lesson thoughts about why certain lessons went as planned. In general, the majority of lessons were thought to have gone as planned, and in cases where the lesson didn't go as planned they explained why but often did not provide reasons. Thus, the post lesson thoughts did not seem to bring up anything particular related to barriers that would hinder a lesson from going as planned. With regard to lesson going as planned, many pre-service teachers referred to students' engagement with the task to explain what that means, and in some lessons provided teacher oriented reasons why the lesson went as planned. For two lessons that were implemented with a high cognitive demand, pre-service teachers (Renee and David) cited the launch as the main reasons why. In particular, each commented how the task was something the students personally related to and that helped spark their engagement. This finding points toward further research involving the implementation of tasks that relate to students interest. These tasks were not just real world problems, they were situations the students personally engaged with (i.e. Ice Bucket Challenge and Renting Movies).

5.3 CONCLUSIONS AND FURTHER RESEARCH

The findings of this study are important for different reasons. First, the results indicate that attention to student thinking in lesson planning is positively related to the level of cognitive demands at which students engage mathematical tasks. Perhaps the reason this is most important is that the study's subjects are pre-service teachers, a group who research indicates often tend to struggle during instruction. Also, high level task implementation is often difficult regardless of teacher experience. It would be interesting to follow these current pre-service teachers into their classrooms once they have jobs to study their implementation when "best effort" expectations are not in place. In what ways would their attention to student thinking during planning relate to task implementation when they are teaching in their own classrooms?

The fact that pre-service teachers showed attention to student thinking and implemented tasks with a high success rate suggests that similar results are certainly possible for teachers of different experience levels. Further similar research between lesson planning and task implementation for teachers of all experience levels would help further explain this relationship. The Lesson Planning Project (Smith et al., 2012) has investigated links between planning and instruction for in-service teachers but found limited evidence from planning. Perhaps, different methods to get at in-service teachers planning practices (i.e. lesson planning interviews) are required since written plans often underestimate experienced teachers actual plans (Schoenfeld, 1998).

Secondly, the link between task implementation and lessons that were accompanied by planning support suggests both the effectiveness of the particular teacher education program and the benefit of using tasks accompanied by detailed lesson plans. The pre-service teachers in this study were all part of the same teacher education program, and they all completed the same process to enact the planning, teaching, reflecting lesson. The 100% success rate of high level task implementation suggests the instruction/feedback they received was helpful to the task implementation. The finding can help inform other teacher education programs about practices that have evidence of translating to positive results in instruction.

The high success rate of implementation for tasks accompanied by detailed plans calls for research around other tasks with such plans already built around them. Further research could explain whether it important for individuals to engage in “thoughtful” and “thorough” planning or if drawing upon such lessons is sufficient. Results from such a study could push for the development of a large resource base or even a curriculum that consists of such detailed plans.

In general, the study calls for more research around the Five Practices. The pre-service teachers in this study are exposed to elements of attention to student thinking and enacting lessons with the Five Practices in mind since they frame much of their teacher education program. In particular a future study should focus on how attention to the Five Practices during planning plays out in classroom discussion since the primary goal of the practices is to orchestrate a discussion. Also, it would be interesting to see the relationship between planning and implementation for teachers who have not been exposed to learning about the Five Practices. Further research is needed to see what role the Five Practices play in the everyday planning and instruction of teachers with different experience levels.

APPENDIX A

INSTRUCTIONAL QUALITY ASSESSMENT

Cover Sheet for Collection of Tasks and Student Work[1]

Please answer all questions as specifically as possible. We're especially interested in question #4 to help us understand the assignment and student work.

Please check: This task is **typical** ☐ or **especially challenging** ☐.

1. Attach the task and any instructions that were given to students. If this is not possible, use the space below to state the task and describe the instructions.

____

____

2. If this task was drawn from a published source, please provide the following information:
   a. title: ______________________________
   b. volume : ______________________________
   c. publisher ______________________________

3. Please define the participation structure. (i.e. How are students organized: individual, pairs, small groups?)

____

____

[1] This cover sheet was adapted from Clare, L. (2000). Using Teachers' Assignments as an Indicator of Classroom Practice. Los Angeles: UCLA Center for the Study of Evaluation. Do not use or cite without permission from Lindsay Clare Matsumura (lclare@pitt.edu) or Melissa Boston (bostonm@duq.edu).

4.) What expectations do you have of students as they work on the task?

Expectations: ____

Were students aware of these expectations? If so, how did you make them aware?

____

____

On the this page, please explain what you consider high, medium and low quality work on the task.

High: ____

____

Medium: ____

____

Low: ____

____

5. Based on above explanations, how would you classify and describe the quality of work (high, medium, or low) that the students engaged in when working on the main instructional task? Why did you classify it this way?

____

____

____

6.) Is there anything that you think influenced students in producing such a quality of work you just described?

APPENDIX B

PLANNING AND INSTRUCTION INTERVIEW PROTOCOL

Planning and Instruction Interview Protocol

Interview will be conducted by university supervisor immediately after the lesson is taught.

Time when interview was started:________________

"This is (your name) interview (teacher's name) on (date) and this is the Planning and Instruction interview."

"Thank you for participating in this interview."

Supervisor Interview Questions:

1.) "Did your lesson go as planned? Why or why not? Feel free to provide specific examples and explanations. Also, please elaborate in your response and provide as much detail as possible."

(Pre-service teacher provides response without interruption from the supervisor)

<u>Note to interviewer</u>:

If participant suggests that lesson did go as planned and does not provide any explanation as to why, please probe by asking the following questions:

"Can you explain what it means that the lesson went as planned? "

"Can you provide reasons or explanations as to why it went as planned or what helped it go as planned?"

If participant suggests that lesson did not go as planned and does not provide any explanation as to why, please probe by asking the following questions:

"Can you explain what it means that the lesson did not go as planned?"

"Can you provide reasons or explanations as to why it did not go as planned or what hindered it in going as planned?"

2.) "Is there anything else you would like to say about whether your lesson went as planned and why think so?"

APPENDIX C

“ROB’S” LESSON PLAN – USED TO TRAIN SECOND CODER

The day will start off with the warm-up. The warm-up has students determine whether specific points are solutions to different equations. The warm-up also has students write an equation to represent a scenario involving a base rate and an hourly rate. The point of the warm-up is to give the students the introductory knowledge they will need for the day's main task, which involves finding the intersection point of two different equations representing phone plans. Next will come the launch in which we will discuss different phone plans. We will discuss things like what type of cell phones the students have, what type of plans they have, how often they talk on the phone, and what type of cell phone plan they would prefer. The point of the launch is to get the students engaged and involved with the day's lesson/conversation. Next we will go into a think/pair/share. The think/pair/share involves a monthly cell phone plan that includes a base rate and a per minute rate. The students are asked how much they would owe for a month if they talked on the phone 100 minutes that month. The point of this think/pair/share is to give students more introductory knowledge they will need for the day's main task. We will then go into another think/pair/share which includes the day's main activity. The activity says "There are two different phone companies you can choose to select. Phone company A charges a base rate of $5 per month, plus 4 cents for each minute that you are on the phone. Phone company B charges a base rate of only $2 per month, but charges you 10 cents for every minute you are on the phone." Students are asked a series of questions about the phone plans, involving one question asking "How much time would you have to talk on the phone before subscribing to company A would save you money? Explain why your answer makes sense." This question is the main question because it involves the intersection point of the phone plans. For this question, I plan on allowing different students to present their work who answered the question using an equation, a graph, and a chart. FInally, students will answer the closing question which asks students how to tell if a point is an intersection point and also to determine whether a point is the intersection point of two equations.

0

1

Attachments

Lesson_3.14.docx

What are your measurable objectives for this lesson? What will students be able *to do* at the end of the lesson?

Students will be able to find the intersection point of two systems of linear equations

WHEN?

This will be explored during the day's main task. Students will see that at 50 minutes the two phone plans have the same price, and since the same combination of minutes and price satisfies both equations, that point is the intersection point.

HOW?

Students will explain for question 3 that any time after 50 minutes, plan A would be the best because the graphs intersect at 50 minutes and $7. Students will also answer both closing questions, explaining how to know when a point is the intersection point of two equations and determining if a point is the intersection point of two different equations.

0

What are your goals for the lesson? What is it that you want students to *know and understand* as a result of this lesson?

Students will understand that if a point satisfies two equations, it is the intersection point of those two equations.

0

In what ways does the task build on students' previous knowledge, life experiences, and culture?

This task builds on students previous knowledge as they have learned how to determine if a point satisfies an equation and is on the equations graph. This task builds upon students' life experiences and culture as it deals with cell phones, cell phone plans, and basketball. These are all relevant, real world applications for my students.

0

0

In what ways are your students ready to begin productive work on this task? In what ways are they not ready?

My students are ready to begin productive work on this task as they know how to create equations that represent word problems, they know how to graph equations, they know how to pick an x-value and find the resulting y-value in an equation, and they know how to determine whether or not a particular point satisfies an equation. My students are not ready to begin work on this as it will force them to struggle and really think. These are very open ended ideas and thoughts, and my students will struggle with that because they quickly and easily give up if they cannot find the correct answer immediately.

0

0

What are your expectations for students as they work on and complete this task?

Students are expected to actively participate and be engaged with the day's activity for the entire lesson. Students are expected to act maturely and politely the entire period. Students will need calculators, pencils, lines paper, graph papers, and the day's handouts. Students will work independently, with their table groups (about 3 per group) and with the class as a whole. Students will record their work on the day's handout, the lined paper, and the graph paper. Students will record their answers to the day's closing questions on their handout. All student work will be left in the class folder that remains in the classroom so I can assess student's effort and understanding for the day's lesson. Students will report on their work to their groups as well as to the class as a whole. I will walk around the room and monitor the students during the entire lesson.

0

0

How will you introduce students to the activity?

Students will first be introduced to the activity through the warm-up. The warm-up is meant to get students thinking about whether a point is a solution to an equation or not, and how to create an equation for a word problem. Next, students will be introduced to the activity through talk about cell phones, and a problem asking them to find the cost of a cell phone plan. This is meant to get students engaged with the lesson. Finally, students will use the previous example to help them complete the intersecting cell phone plans activity.

0

0

How will you provide access to all students while maintaining the rigor of the task? Note modifications for special education and ESL students.

The rules and expectations for the entire period will be covered and explicitly stated for all students. All students will have access to this because all students should be able to complete and understand the warm-up, if not on their own definitely after the class discussion. All students will be required to write what they think the cost of the first phone plan will be during the "think" portion with no help from me or classmates. This will help maintain the rigor. After this, all students will attempt to find the cost using their table groups for help. Finally, all students should be able to use what they wrote in order to help them participate in the class discussion. Next, students will partake in a think/pair/share about the two different phone plans and determine when the phone plans intersect. Students will be required to do this first on their own, which will help maintain the rigor. During the entire lesson, I will be monitoring and helping guide students in the right direction by using meaningful questions. I do not believe there are any students in my class who are not fluent in English. However, if I do have students like this, words that may be confusing, such as "down payment and per minute rate" will be explained by students in the class who understand these terms. Modifications for special education students will include extra time to work on the task, as most students with special needs are in the "block period" which does the same work as the single periods but has 2 periods to do so. Other modifications include preferential seating and the ability to "redo" the activity for full class participation points if they do not do well the first time.

0

0

Are you planning a whole class discussion of the task?

Yes

0

0

How will you orchestrate a class discussion about the task so that you can accomplish your learning goals? Specifically, what student responses do you plan to share during the discussion and in what order will they be discussed?

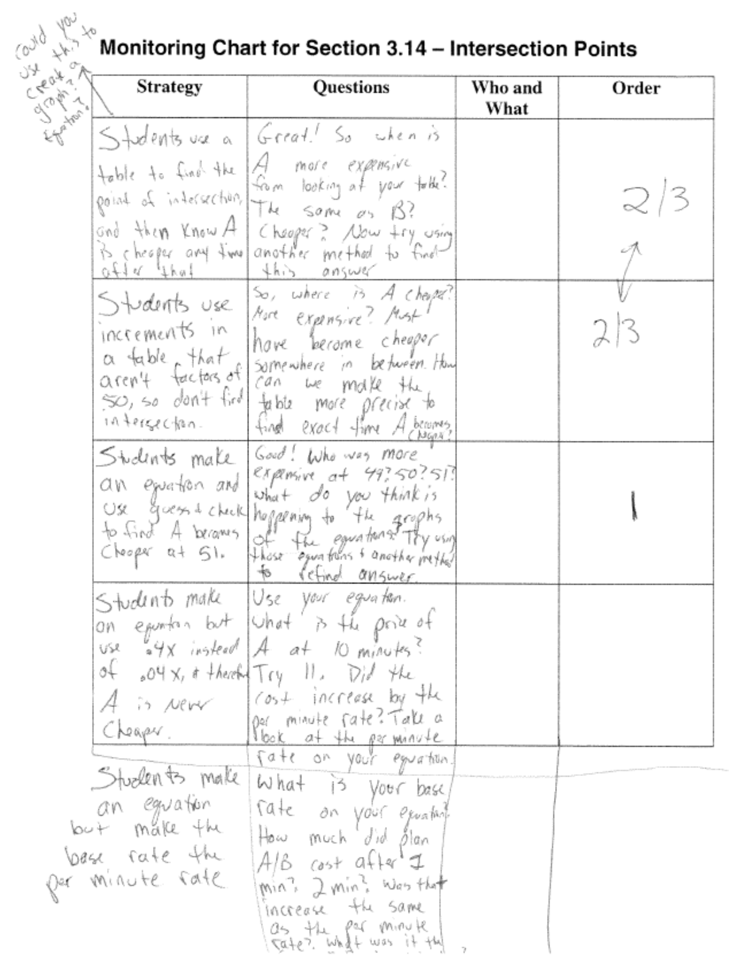
I will use a monitoring chart (see attached) to orchestrate a class discussion. I will have specific students present their answers for questions 1 and 2, and then multiple students will present their answers for number 3 about how they determined when subscribing to phone plan A would save money. Students will present in the order in which I have indicated on the monitoring chart. Then, as a class, we will discuss the student answers that are presented.

What will you see or hear that lets you know that students in the class have achieved the learning goals? Be specific, what and how will you collect evidence that all students have learned the desired content? [NOTE: Here you should be able to describe how the measurable Objectives of your lesson connect to / provide evidence of the Learning Goals that framed the lesson.]

While monitoring, I will see that students have successfully determined the different prices of the phone plans, and when phone plan A would be cheaper. I will also know how comfortable the students feel with this information based upon the "flow" of the class conversations. Also, I will check all student work because all student work is left in the folders. This will allow me to gauge how well the students understood the day's lesson and how well they accomplished the learning goals. Finally, I will see that students were successfully able to answer the closing questions.

APPENDIX D

"ROB'S" MONITORING TOOL – USED TO TRAIN SECOND CODER

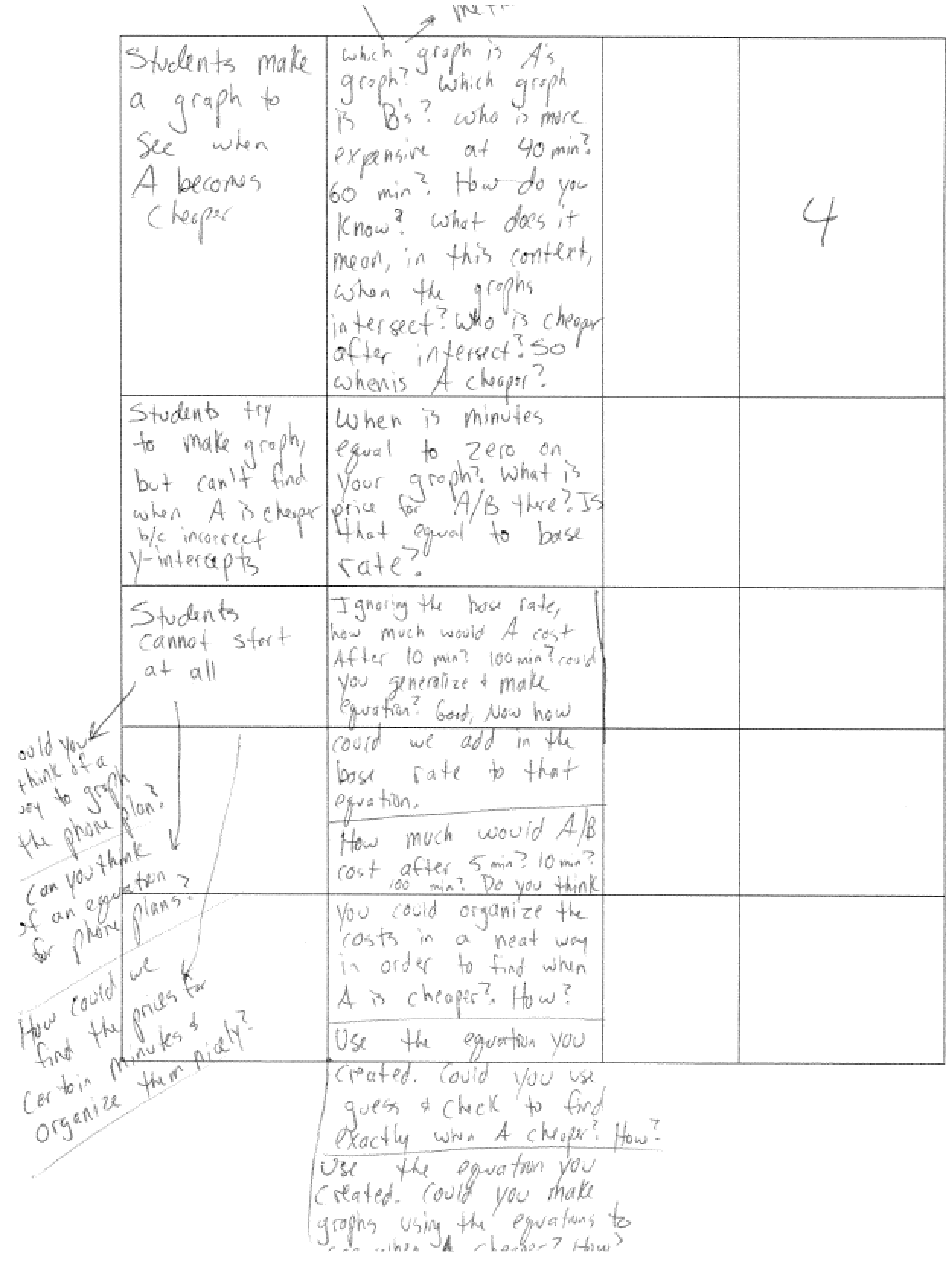


Could you use this to create a graph? Equation?

Monitoring Chart for Section 3.14 – Intersection Points

Strategy	Questions	Who and What	Order
Students use a table to find the point of intersection, and then know A is cheaper any time after that	Great! So when is A more expensive from looking at your table? The same as B? Cheaper? Now try using another method to find this answer		2/3
Students use increments in a table, that aren't factors of 50, so don't find intersection.	So, where is A cheaper? More expensive? Must have become cheaper somewhere in between. How can we make the table more precise to find exact time A becomes cheaper?		2/3
Students make an equation and use guess & check to find A becomes cheaper at 51.	Good! Who was more expensive at 49? 50? 51? What do you think is happening to the graphs of the equations? Try using those equations & another method to refind answer.		1
Students make an equation but use .4x instead of .04x, & therefore A is never cheaper.	Use your equation. What is the price of A at 10 minutes? Try 11. Did the cost increase by the per minute rate? Take a look at the per minute rate on your equation.		
Students make an equation but make the base rate the per minute rate	What is your base rate on your equation? How much did plan A/B cost after 1 min? 2 min? Was that increase the same as the per minute rate? What was it the		

Figure 30 Monitoring Chart



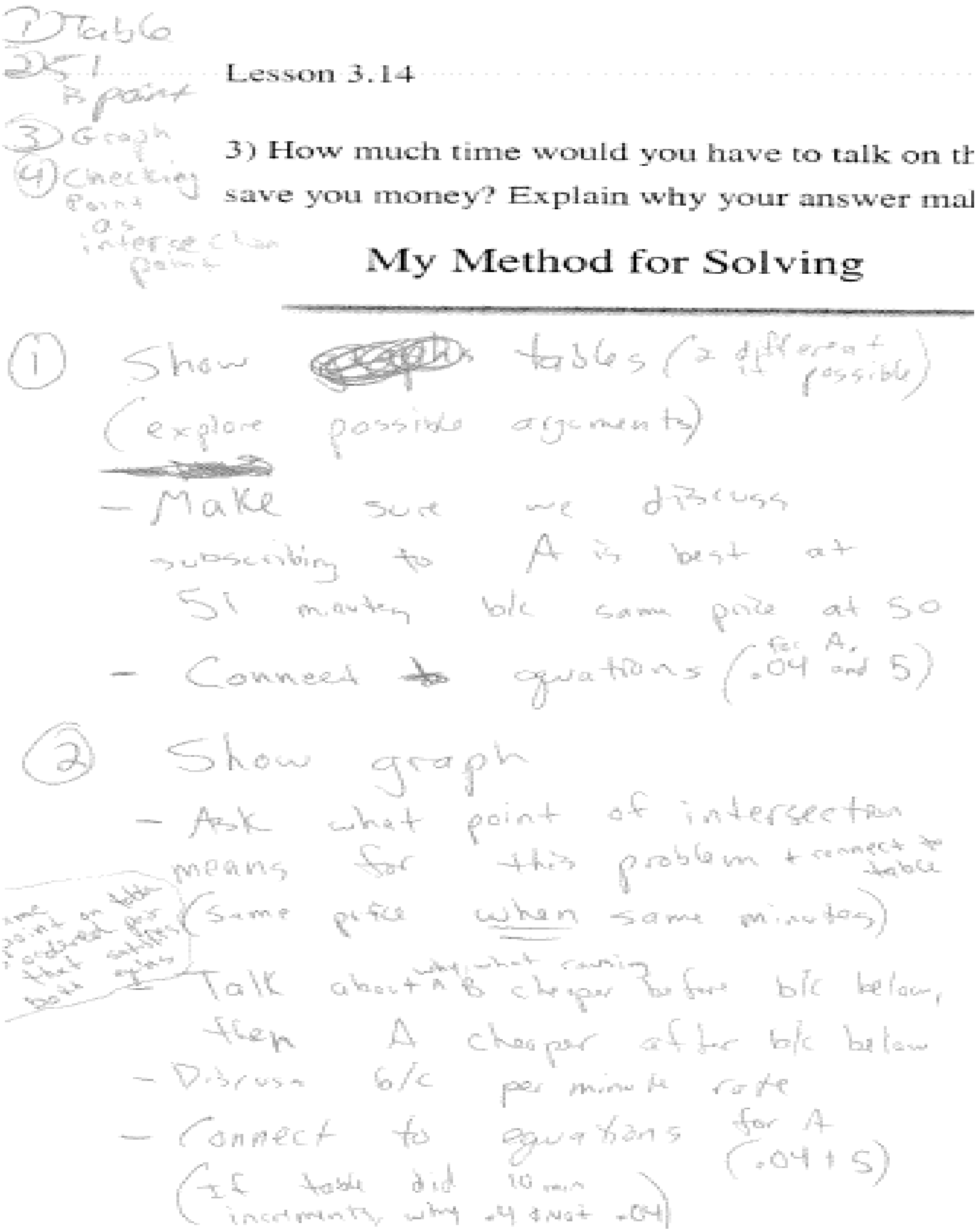
Students make a graph to see when A becomes cheaper	which graph is A's graph? Which graph is B's? who is more expensive at 40 min? 60 min? How do you know? what does it mean, in this context, when the graphs intersect? Who is cheaper after intersect? So when is A cheaper?		4
Students try to make graph, but can't find when A is cheaper b/c incorrect y-intercepts	When is minutes equal to zero on your graph? What is price for A/B there? Is that equal to base rate?		
Students cannot start at all	Ignoring the base rate, how much would A cost after 10 min? 100 min? could you generalize & make equation? Good, Now how could we add in the base rate to that equation.		
	How much would A/B cost after 5 min? 10 min? 100 min? Do you think you could organize the costs in a neat way in order to find when A is cheaper? How?		
	Use the equation you created. Could you use guess & check to find exactly when A cheaper? How?		
	Use the equation you created. Could you make graphs using the equations to see when A cheaper? How?		

Could you think of a way to graph the phone plan?

Can you think of an equation for phone plans?

How could we find the prices for certain minutes & organize them nicely?

Figure 31 Monitoring Chart



1) Table
2) 51
B point
3) Graph
4) Checking point as intersection point

Lesson 3.14

3) How much time would you have to talk on th
save you money? Explain why your answer mal

My Method for Solving

1) Show ~~graphs~~ tables (2 different if possible)
(explore possible arguments)
- Make sure we discuss subscribing to A is best at 51 minutes b/c same price at 50
- Connect ~~to~~ equations (for A, .04 and 5)

2) Show graph
- Ask what point of intersection means for this problem + connect to table
(same price <u>when</u> same minutes)

same point on table ordered pair that satisfies both eqns

- Talk about why what causing A B cheaper before b/c below, then A cheaper after b/c below
- Discuss b/c per minute rate
- Connect to equations for A (.04 + 5)
(If table did 10 min increments, why .4 & not .04)

Figure 32 Method of Solving

APPENDIX E

CODING ROB'S LESSON PLAN – USED TO TRAIN SECOND CODER

(i) Mathematical Goal.

Rob scored a maximum score of 2 for his mathematical goal because his goal(s) stated what students were to know and understand and what it meant for them to do so. Rob's goal(s) were stated as follows:

- Students will be able to find the intersection point of two systems of linear equations. Students will see that at 50 minutes the two phone plans have the same price, and the same combination of minutes and price satisfies both equations, thus that is the intersection point. Students will understand that if a point satisfies two equations, it is the intersection point of those two equations.

Rob states what students will know and understand (i.e. how to find an intersection point) and what it means to understand it (it satisfies both equations). He also makes specific reference to the problem students are working on.

(ii and iii) Anticipating Students' Correct and Incorrect Thinking.

Rob scored a maximum score of 3 in each element of anticipation. Rob's lesson plan contained an a variety of both correct and incorrect about how students might approach the problem. With regard to anticipating correct solutions, Rob specifically describes correct strategies/thinking students may use AND he attempts to identify as many strategies or representations as possible. Rob anticipated that students would make a table, write an equation and use guess and check, or create a graph. Not only did Rob list these strategies, he also shows evidence of actually solving the problem in each of these ways (See Appendix D).

With regard to anticipating Incorrect Solutions, Rob specifically describes incorrect ways in which students may think about the problem AND attempts to identify several challenges or misconceptions the students may encounter. For example, Rob anticipated that when students create the table it could be possible that some students would not increment by amounts that were factors of 50 and thus 50 would not appear in the table. If this were the case students would have difficulty finding the exact intersection point. With regard to using an equation, Rob anticipated that students might write the 4 cents per minute rate as .4x instead of .04x. Under this circumstance, Rob realized that plan A would not be cheaper and if students did this, they would not be able to solve the problem correctly. He also anticipated that some students may switch the base rate and per minute rate when writing the equation. With regard to the graph, Rob anticipated that students may make a mistake on the y-intercepts and thus not be able to find the intersection points. Rob also anticipated that some students may not be able to start the problem at all. Overall with regard to anticipating student thinking, Rob provided evidence that he considered a variety of both correct and incorrect student responses. Within the actual monitoring tool he had strategies listed, and he had attachments to the tool with each correct strategy completely worked out. His incorrect strategies were very specifically described.

(iv) Questions that Assess and Advance Students Thinking.

Rob also scored a maximum score of 2 in the element of questions that assess and advance students thinking. Within his monitoring tool (Appendix D), Rob listed several questions next to each correct and incorrect solution that he planned to ask as an individual student or small group of students worked on the specific approach listed. By having the questions associated with the specific solutions, Rob was able to clearly indicate the circumstances under which he was going to ask each question. A maximum score is received by providing a specific example question AND the circumstances under which asking it is appropriate. Also there must be at least two different circumstances based on students' thinking.

A specific rich example is provided when Rob anticipated the incorrect strategies students might use when attempting to solve the problem with the equation. Rob anticipated two circumstances (i.e. students write .4x instead of .04x and students switch the base rate and per minute rate). For each circumstance Rob lists a series of questions. To address the per minute rate issue, Rob asks "What is the price after 10 minutes? What is the price after 11 minutes? Did the cost increase by the per minute rate?" He then goes on to ask "What is your base rate for your equation? How much did plan A/B cost after 1 minute? 2 min? Was that increase the same as the per minute rate?" He also lists one more question that was illegible.

Overall, Rob provided evidence of questions that were associated with each different solution he anticipated. As already mentioned, all questions were listed in the monitoring tool under a column heading titled "questions". Each set of questions was listed in the block next to the solution strategy it as associated with. The formatting of the monitoring tool made it very clear the circumstances under which questions were intended to be asked.

(v) Orchestrating a Whole Class Discussion: Build on Student Thinking.

Rob earned a score of 1 on a scale of 0,1, or 2 for this element. Rob might have earned a maximum score of 2 but it was unclear if certain questions he listed were intended for the discussion or for the exploration phase. Rob did select and sequence students' solutions to be discussed, but it appears from the monitoring tool that questions associated with the solutions were intended to be asked during the exploration phase. Thus, a coding decision was made and is explained in the next paragraph. Such decisions will be consistent across the data set for the two coders. Rob says in his plan that he is going to use his monitoring tool during discussion, but he does not specifically indicate that he is going to ask certain questions during the whole class discussion in the planning text. According to his monitoring tool, Rob did order specific solutions for students to share publicly (1. Students make an equation and guess and check, 2. Students use an incorrect table, 3. Students use a correct table, 4. Students use a correct graph). Due to the fact that he selected and sequenced specific solutions, but it is not clear if the questions are intended for whole class discussion, Rob was given a score of 1 for this element based on the description provided in the rubric.

(vi) Orchestrating a Whole Class Discussion: Make Salient Mathematics of the Lesson.

Rob earned a maximum score of 2 for this element because he identified a series of specific questions that he planned to ask intended to develop mathematical ideas. In an attachment to his monitoring tool (last page of Appendix D), Rob lists that he plans to ask what the point of intersection means for this problem and how the graph connects to the table. He also indicates that he plans to ask why/what is causing plan B to be cheaper than plan A at first and why A becomes cheaper after. On this page, it is evident that Rob is planning to ask these questions during whole class discussion. Through the questions he intends to ask, it is clear that Rob is attempting to develop mathematical ideas of how the graph, table and equation are related.

APPENDIX F

DIRECTIONS FOR COLLECTING STUDENT WORK

Directions for Collection of Student Work

- Collect student work on main task and select work produced by six different students
    - Student work should be of medium/high quality (if available) as described by you on cover sheet
    - If six pieces of medium/high quality are not available, then select highest quality available
    - Student work should reflect different types of work engaged in by students (if students produced work in different ways, then that should be represented in sample)

- Remove student names or any other identifying information
- Complete cover sheet
- Email student work (scan or take picture) and completed cover sheet to Scott

Contact Information:

Scott Layden

scl15@pitt.edu

Cell:724 541 3834

APPENDIX G

QUINN BRADY'S LESSON PLAN FOR LESSON 4

Lesson Plan for Days 12 of Instructional Sequence

Grade 7, Pre-Algebra
Activity: Jose's Surfboard

Learning Goals

- Students understand that the graph of a linear relationship is a line that models the relationship between the variables in the context. The coordinates of the points on the line form the solution set for the associated linear equation.
- Students will understand that the slope between two points can be found by taking the difference of the *y* coordinates between two points and dividing by the difference of the corresponding *x* coordinate of the points: $(y_2 - y_1) / (x_2 - x_1)$.
- Students will recognize that the y-intercept of a linear equation corresponds to an initial value and the slope corresponds to a constant change between the two variables.

Performance Goals

- SWBAT find the slope of a line given two points.
- SWBAT find solutions that satisfy equations.
- SWBAT recognize how contextual situation can be modeled with a graph.

Connection to Previous and Following Lessons

Prior to this lesson, students found the slope of a line given two points of that line. Now they are using that knowledge and applying it to a real world scenario. They are also going to relate how certain solutions satisfy an equation, and those solutions are any point that belongs on the line. In the next lesson, *Joe's on the Beach Ice Cream*, students are given an explanation of a linear relationship instead of points, and they represent the line graphically and symbolically from this explanation.

Materials

- Whiteboard & Markers
- Task for each student
- ELMO document camera

Launch (5 minutes)

- Ask students if they remember the situation earlier in the year that we explored involving roller skate rentals. Have a student relay what that problem asked us to do.
    - o *The problem told us that roller skate rentals were $3 initially and then an extra $2 for every hour the skates were rented.*
- Ask how much money we had to pay up front, only one time: *The $3 initial cost*.
- Ask what the $2 stood for: *How much we had to pay for each hour of renting the skates. So 3 hours of skating was $3 plus $6, or $9.*
- Explain that we are going to look at a similar situation, but instead of receiving that cost information at the start, we are going to be given a graph that shows the cost of renting a surfboard for 3 different amounts of time. From that graph, we have to answer some questions about the rental rate of a surfboard.
- Pass out the task to each student. Allow them 10 minutes to work on their own and 10 minutes to work with their partner at their table.

Table 36 Supporting Students' Exploration of the Task (20 min)

Possible Student Approaches	Questions to Ask
Can't get started	• What can you tell me about this point (pointing to [1, 30])? What does it mean in the context of this problem? • How is the cost changing from 0 to 1 hours? 1 to 2 hours? • Do you remember how to find the change between two points?
Students create a table that shows the time in one column and the cost in the other column	• How did you determine the values in your table? How many values can you add to your table? • What patterns do you see in the table? How can you use these patterns to predict the cost of renting a surfboard for 12 hours?
Extends the line	• Can you explain your strategy? • How do you know extending the line is a strategy that works? • How can you use the line to find the cost of a 12-hour rental? • How can you use the line to find the number of hours you can rent for $150?
Write a function to model the situation	• What do the terms in your function mean with respect to the context? • How did you find the slope of your function? • How can you use the function to make predictions? • <u>Extension</u>: What if the initial cost was $10 and you had to pay $10 per hour to rent the surfboard? How would this compare to Jose's rates? Who would you rather rent from?

Table 37 Sharing and Discussing the Task (15 min)

Solutions to Share	Questions about Strategy	Connection Between Strategies	Key Points
Extends the line	• How can we use this strategy to find the find the rate per hour of a surfboard? • How do we know what the cost of renting a board for 12 hours would be? • Is it possible to find the number of hours Jose surfs if the rental cost is $150?		This method will get us our answers, but it may be difficult to find how much the board costs for times greater than 10 hours due to the domain of the graph. We would have to extend the graph.
Table of the time in one column and the dollars in the other	• How can we find the rate using the table? • How can this method be used to find the cost for any random amount of time? • Are there any limitations to using this method?	The coordinates in the table lie on the line in the above strategy	We can see from the table how the cost goes up by 5 dollars per hour. We can also find the slope of a line using two of the pairs from the table.
Write a function to model the situation	• How did you find the rate? Where is it located in this function? • What do the variables in the function represent? How about the constants?	• If we plug a value for the hours from the table into the function, the corresponding value for dollars will result. • The slope of the line in the function is the difference between the cost for every increase in hour in the table. • The function is a symbolic model of the line.	The function models the cost for any hour. It allows the most efficient method to find the cost given the hour, or vice versa.

APPENDIX H

MARIAN TURNER'S LESSON PLAN FOR LESSON 2

Lesson 10.6 – Combinations and Probability

Measurable Student Objectives:

- Students will be able to evaluate the number of permutations in a given scenario.
- Students will be able to evaluate the number of combinations in a given scenario.
- Students will learn the notation for combinations.

Learning Goals:

- Students will understand that there are fewer possibilities if order doesn't matter than there are if order is important.
- Students will recognize that combinations are collections, whereas permutations are arrangements; they will understand the difference between the two.

Activity:

- Recap what permutations are
- What if order does *not* matter? What if we have *collections* instead of *arrangements?* Will the number of total possibilities increase or decrease? (brief class discussion and predictions) *This did not go exactly as planned – I did not end up having a brief class discussion on this, as I was concerned about time (we had a shortened class period).*
- Get into groups of 4 – work on Ice Cream Parlor Task
    - #1: Make sure students understand that the chocolate needs to be scooped last in order that it can be eaten first

- #1: “How many ways can the ice cream be scooped?”
- #2: Make sure students understand the context – the order of the scooping does *not* affect how the customer eats his ice cream
- #3: Look for students who are using their prior knowledge of permutations – make sure they are using the word correctly (*arrangement*, not a number)
- #3: “What makes the scooping for a cone a permutation?” Why is scooping for a bowl not a permutation?”
  - Order matters vs. order doesn’t matter (specific vs. doesn’t matter)
  - Clarify, as needed, that there is no replacement in either scenario
- #4: “How many choices do you have for the first scoop? The second scoop?”
  - Look out for students who are using replacement
  - Remind students of prior knowledge – draw out slots, number of choices go in the slots, multiply number of choices together
  - Encourage students to use correct notation: ${}_nP_r$
- #5: “Is the number of ways to scoop going to increase or decrease? Why?”
- #5: Look out for tree diagrams, lists, still trying to use permutation strategy
  - Tree diagrams – encourage students to focus on one part of the tree diagram and extrapolate information to come up with the total number of possibilities
  - Lists – encourage students to come up with systematic ways of listing
- #5: “How many ways of scooping from #4 are no longer considered “different” ways of scooping now that order does not matter?”
- #6: Encourage students to move beyond “order mattering” and into more precise, specific ways of writing the definition
  - “Permutations are arrangements. What do we mean by arrangements?”
  - “Combinations are not arrangements. What might be a better word?”

- Class discussion (time permitting)
  - Definition
  - Notation

Start discussion of formula for ${}_nC_r$ – use lists to show connections

APPENDIX I

NICK NEWMAN LESSON PLAN FOR LESSON 1

Lesson Plan – It's in the System – 1.3 – Booster Club Memberships

Learning Goals: Students will learn how to find the solution of a system of two linear equations by looking for the point of intersection of the graphs for the individual equations.

Performance Goals:

a. Analyze and solve linear equations and pairs of simultaneous linear equations.
b. A-CED.A Create equations that describe numbers or relationships.
c. A-REI-C Solve systems of equations
d. A-REI.D Represent and solve equations and inequalities graphically.

Task (Attached)

Materials:

- ELMO with projector
- Copies of the task for each student
- Calculator for each student

Setting Up the Task:

- Warm-Up (Attached)

- Have students complete the Warm-Up for 5 minutes.
- Discuss the Warm-Up with students, ask such questions as:
  - Do some of these letters in our linear forms represent variables? Constants? Which?
  - When the values of one variable depend on those of another, what linear equation form is common to use?
  - When the values of the two variables combine to produce a fixed third quantity, what linear equation form do we use?
- Tell students we will be answering those questions in our new book.

- Introduce the task
  - Read the context of the problem aloud. Have students make guesses at what they think the numbers of adult and student memberships might be. Record their guesses on small pieces of paper and collect them. The person who had the closest guess wins a prize.
  - Let students know they will be finding two equations in Part A as instructed. They will then be finding 3 solutions to each by guessing one of the variables values.
  - Reinforce that the goal of this problem is to find the common solutions to two linear equations.
  - Let students know that they will then be graphing the two linear equations in Part B and answering various questions based on the graph and their findings.
- Tell students they will work independently for 7 minutes; they will then work in their groups of three to four for 15 minutes; and they will then participate in a whole group discussion where they share their findings.
- Tell students they may use any of the resources available to solve the task.
- Tell students they must be prepared to not only share their findings with the class but also to justify their reasoning for their findings.

Possible Challenging Aspects	**Questions to Ask**
Can't get started	• What information is the task giving you? Circle what is important.
Difficulty writing equations	• Does one variable depend on another? • Do the two variables combine to produce a third fixed quantity? • What are our variables? Our constants?
Difficulty finding solutions	• How did we find solutions in previous problems? • How does guessing one variable help us to find the corresponding value of the second?

Finish Immediately and can answer all questions asked about their approach	• Lead students to think deeply about what question D is asking. • (Offer students extensions for labeling, and adding more detail to solutions.)
Partners do not agree on a solution	• Could you take turns explaining your process to one another and determine who has made a mistake and why? • How do your solutions differ? • How are your solutions the same?

Sharing and Discussing the Task

- Groups share their findings using the ELMO.
- Ensure students know how to interpret the point of intersection from Question B graphically, algebraically, and contextually.
    - Graphically: This point is where the two lines meet/intersect.
    - Algebraically: This point is the coordinate (a,s) that satisfies both equations.
    - Contextually: The intersection point (30, 20) means that only the combination of 30 adult and 20 student memberships would lead to the reported $400 income with exactly 50 memberships sold.
- Check for understanding by asking students details about why points on either of the lines are not solutions to the system.

APPENDIX J

CHECKLIST USED TO GATHER ADDITIONAL LESSON PLAN INFORMATION FROM UNIVERSITY INSTRUCTOR

Table 38 Checklist for University Instructor

Table for (Intern Pseudonym)				
Task Name	**Question**	**Yes**	**No**	**Comments**
	Did interns engage with task during coursework?			
	Was task embedded in a case study that was made available to intern?			
	Was task accompanied by a two-page narrative that was made available to intern?			
	Was task accompanied by a fully fleshed out detailed lesson plan that was made available to intern?			
	Did you provide feedback on the lesson plan before the lesson was taught?			
	Did you provide any additional resources (not mentioned above) for this task?			

APPENDIX K

CHECKLIST USED TO GATHER ADDITIONAL LESSON PLAN INFORMATION FROM UNIVERSITY SUPERVISOR

Table 39 Checklist for University Supervisors

Table for (Intern Pseudonym)				
Task Name	**Question**	**Yes**	**No**	**Details**
	Did you provide any additional resources related to this task to the intern prior to the lesson being taught?			
	Did you provide any feedback as the intern was planning the lesson around this task?			
	Are you aware of any other experience/support the intern had with the task prior to teaching the lesson?			

APPENDIX L

CHECKLIST USED TO GATHER ADDITIONAL LESSON PLAN INFORMATION FROM INTERNS

Table 40 Checklist for Interns

Table for (Intern Pseudonym)				
Task Name	**Question**	**Yes**	**No**	**Details**
	Did you engage with this task during coursework?			
	Did you ever teach a lesson using this task before using it for this particular lesson?			*This does not mean a lesson taught the same day in a different period.
	Did you receive planning feedback for this lesson from a university professor?			
	Did you receive planning feedback for this lesson from your cooperating teacher?			
	Did you receive planning feedback for this lesson from your university supervisor?			
	Did you use any additional resources to help plan the lesson around this task?			

Table 40 (continued)

	Have you had any other experiences with the task not mentioned above?			

BIBLIOGRAPHY

Arbaugh, F., & Brown, C.A. (2005). Analyzing mathematical tasks: A catalyst for change? *Journal of Mathematics Teacher Education*, 8, 499-536.

Ball, D.L. (1993). With an eye on the mathematical horizon: Dilemmas of teaching elementary school mathematics. *The Elementary School Journal*, 93(4), 373-397.

Ball, D.L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59, 389-407.

Boaler, J., & Staples, M. (2008). Creating mathematical futures through an equitable teaching approach: The case of Railside School. *The Teachers College Record, 110*(3), 608-645.

Borko, H., & Putnam, R. (1996). Learning to teach. In D. C. B. R. C. Calfee (Ed.), *Handbook of Educational Psychology* (pp. 673-708). New York: Simon & Schuster Macmillan.

Borko, H., & Livingston, C. (1989). Cognition and improvisation: Differences in mathematics instruction by expert and novice teachers. *American Education Research Journal, 26*(4), 473- 498.

Borko, H., Eisenhart, M., Brown, C.A., Underhill, R.G., Jones, D., & Agard, P.C. (1992). Learning to teach hard mathematics: Do novice teachers and their instructors give up too easily? Journal for Research in Mathematics Education, 23, 194-222.

Boston, M.D., & Smith, M.S. (2009). Transforming secondary mathematics teaching: Increasing the cognitive demands of instructional tasks used in teachers' classrooms. *Journal for Research in Mathematics Education, 40*(2), 119-156.

Boston, M.D., & Wolf, M.K. (2006). Assessing academic rigor in mathematics instruction: The development of Instructional Quality Assessment Toolkit (CSE Tech. Rep. No. 672). Los Angeles: University of California, National Center for Research on Evaluation, Standards, and Student Testing (CRESST).

Boston, M.D. (2012). Assessing Instructional Quality in Mathematics. *The Elementary School Journal*, 113(1), 76-104.

Brendefur, J. & Fykholm, J. (2000). Promoting mathematical communication in the classroom: Two preservice teachers' conceptions and practices. *Journal of Mathematics Teacher Education*, 3(2), 125-153.

Brown, C.A. & Borko, H. (1992). Becoming a mathematics teacher. In D.A. Grouws (Ed), Handbook of Research on Mathematics Teaching and Learning (p. 209-221). NY: MacMillian.

Cady, J., Meier, S.L., & Lubinski, C.A. (2006). Developing Mathematics Teachers: The Transition from Preservice to Experienced Teacher. *Journal of Educational Research*, 99(5), 295-305.

Carpenter, T.P., Fennema, E., Peterson, P.L., Chiang, C., & Loef, M. (1989). Using knowledge of children's thinking in classroom teaching: An experimental study. *American Educational Research Journal*, 26(4), 499-531.

Carpenter, T. P., & Lehrer, R. (1999). Teaching and learning mathematics with understanding. In E. Fennema & T. A. Romberg (Eds.), *Mathematics classrooms that promote understanding* (pp. 19-32). Mahwah, NJ: Lawrence Erlbaum Associates.

Clare, L. (2000). Using Teachers' Assignments as an Indicator of Classroom Practice. (CSE Tech. Rep. 532). Los Angeles: University of California, National Center for Research on Evaluation, Standards, and Student Testing (CRESST).

Clark, C., & Peterson, P. (1986). Teachers' thought processes. In M. C. Wittrock (Ed.), *Handbook of research on teaching* (3rd ed., pp. 255-296). New York: Macmillan.

Clark, C., & Yinger, R. J. (1977). Research on teacher thinking. *Curriculum Inquiry, 7*(4), 279-304.

Common Core State Standards Initiative. (2010). *Common Core State Standards for Mathematics*. Retrieved from http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf

Doyle, W. (1979b). The tasks of teaching and learning in classrooms (R & D Rep. No. 4103). Austin: Research and Development Center for Teacher Education, The University of Texas at Austin.

Doyle, W. (1983). Academic Work. *Review of Educational Research*, 53(2), 159-199.

Doyle, W. (1988). Work in Mathematics Classes: The Context of Students' Thinking During Instruction. *Educational Psychologist*, 23(2), 167-180.

Eisenhart, M., Borko, H., Underhill, R., Brown, C., Jones, D., & Agard, P. (1993). Conceptual Knowledge Falls through the Cracks: Complexities of Learning to Teach Mathematics for Understanding. Journal for Research in Mathematics Education, 24(1), 8-40.

Fennema, E., Carpenter, T. P., Franke, M. L., Levi, L., Jacobs, V. B., & Empson, S. B. (1996). A longitudinal study of learning to use children's thinking in mathematics instruction. *Journal for Research in Mathematics Education, 27*(4), 403-434.

Forman, E.A., & Ansell, E. (2002). Orchestrating the multiple voices and inscriptions of a mathematics classroom. The Journal of the Learning Sciences, 11(2&3), 257

Franke, M. L., Kazemi, E., & Battey, D. (2007). Mathematics teaching and classroom practice. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 257-315). Charlotte, NC: Information Age.

Frykholm, J.A. (1996). Pre-service teachers in mathematics: Struggling with the *Standards*. Teaching & Teacher Education, 12, 665-681.

Garrison, 2014.

Guest, G., MacQueen, K. M., & Namey, E. E. (2012). *Applied thematic analysis*. Los Angeles: SAGE Publications, Inc.

Henningsen, M. & Stein, M.K. (1997). Mathematical Tasks and Student Cognition: Classroom-Based Factors That Support and Inhibit High-Level Mathematical Thinking and Reasoning. Journal for Research in Mathematics Education, 28(5), 524-549.

Hiebert, J., Carpenter, T., Fennema, E., Fuson, K., Wearne, D., Murray, H., Olivier, A., & Human, P. (1997). *Making sense: Teaching and learning mathematics with understanding.* Portsmouth, NH: Heinemann.

Hiebert, J. (2003). What research says about the NCTM standards. In J. Kilpatrick, W. G. Martin & D. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 5-23). Reston, VA: NCTM.

Hiebert, J. & & Wearne, D. (1993). Instructional tasks, classroom discourse, and students' learning in second-grade artimetic. *American Educational Research Journal*, 30, 393-425.

Hiebert, J., Morris, K.A., Berk, D., & Jansen, A. (2007). Preparing Teachers to Learn from Teaching. *Journal of Teacher Education*, 58(1), 47-61.

Hiebert, J., & Grouws, D. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp.371–404). Charlotte, NC: Information Age.

Hill, H.C., Blunk, M.L., Charalambos, C.Y., Lewis, J.M., Phelps, G.C., Sleep, L., & Ball, D.L. (2008). Mathematical Knowledge for Teaching and the Mathematical Quality of Instruction: An Exploratory Study. Cognition and Instruction, 26, 430-511.

Hughes, E.K. (2006). Lesson planning as a vehicle for developing pre-service secondary teachers' capacity to focus on students' mathematical thinking. *Unpublished Doctoral Disseration*, University of Pittsburgh, Pittsburgh.

Junker, B., Matsumura, L. C., Crosson, A., Wolf, M. K., Levison, A., Weisberg, Y., & Resnick, L. (2004, April). *Overview of the Instructional Quality Assessment*. Paper presented at the annual meeting of the American Educational Research Association, San Diego, CA.

Kagan, D. (1992). Professional growth among pre-service and beginning teachers. *Review of Educational Research, 62*(2), 129-169.

Karp, A. (2010). Analyzing and attempting to overcome prospective teachers' difficulties during problem-solving instruction. *Journal of Mathematics Teacher Education*, 13, 121-139.

Kawanaka, T., & Stigler, J. W. (1999). Teachers' use of questions in eighth-grade mathematics classrooms in Germany, Japan, and the United States. *Mathematical Thinking and Learning, 1*(4), 255-278.

Lampert, M. (2001). *Teaching problems and the problems of teaching*. New Haven, CT: Yale University Press.

Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal*, 27(1), 29-63.

Lampert, M. (1992). Practices and problems in teaching authentic mathematics. In Oser, F.K, Dick, A., Patry, J.L, (Eds), *Effective and Responsible Teaching: The New Synthesis*, (pp 295-314), San Francisco, CA: Jossey-Bass Publishers.

Leinhardt, G. (1993). On teaching. In R. Glaser (Ed.), *Advances in instructional psychology* (Vol. 4, pp. 1-54). Hillsdale, NJ: Lawrence Erlbaum Associates.

Lewis, C., & Tsuchida, I. (1998). A lesson is like a swiftly flowing river: Research lessons and the improvement of Japanese education. *American Educator, 22*(4), 14-17 & 50-52.

Lewis, C.C., Perry, R.R., Hurd, J. (2009). Improving mathematics instruction through lesson study: A theoretical model and North American case. *Journal of Mathematics Teacher Education*, 12, 285 – 304.

Little, J.W. (1990). The mentor phenomenon and the social organization of teaching. Review of Research in Education, 16, 297-351.

Livingston, C., & Borko, H. (1990). High school mathematics review lessons: Expert-novice distinctions. *Journal for Research in Mathematics Education, 21*(5), 372-387.

Lloyd, G.M., & Frykholm, J.A. (2000). On the development of “book smarts” in mathematics: Prospective elementary teachers' experiences with innovative curriculum materials. Issues in the Undergraduate Mathematics Preparation of School Teachers: The Journal*, 2*.

Matsumura, L., C., Garnier, H., Slater, S.C., & Boston, M.D. (2008). Toward measuring instructional interactions “at-scale.” *Educational Assessment*, 13, 267-300.

Morine, G. (1976). A study of teacher planning. *BTES technical report* 76-3-I. San Francisco: Far West Laboratory for Educational Research and Development.

Mossgrove, J.L. (2006). Examining the nature of instructional practices of secondary pre-service mathematics teachers. *Unpublished Doctoral Disseration*. University of Pittsburgh.

Munter, C., Stein, M.K., Smith, M.S. (in press). Dialogic and Direct Instruction: Two Distinct Models of Mathematics Instruction and the Debate(S) Surrounding Them.

Nathan, M.J., & Knuth, E.J. (2003). A study of whole classroom mathematical discourse and teacher change. Cognition & Instruction, 21(2), 175-207.

National Governors Association Center for Best Practices, Council of Chief State Schoo Officers (2010). *Common Core State Standards for Mathematics*. Washington, DC:Author.

NCTM. (2000). *Principles and Standards for School Mathematics*. Reston, VA: NCTM.

Perry, R.R. & Lewis, C.C. (2008). What is successful adaptation of lesson study in the U.S.? Journal of Educational Change 10(4), 365-391.

Peterson, P.L., Marx, R.W., & Clark, C.M. (1978). Teacher planning, teacher behavior, and student achievement. *American Educational Research Journal*, 15, 417-432.

Philipp, R.A. (2008). Motivating prospective elementary school teachers to learn mathematics by focusing upon children’s mathematical thinking. Issues in Teacher Education, 17(2), 7-26.

Schoenfeld, A., Minstrell, J., & van Zee, E. (2000). The detailed analysis of an established teachers' non-traditional lesson. *Journal of Mathematical Behavior, 18*(3), 281-325.

Schoenfeld, A., Minstrell, J., & van Zee, E. (2000). The detailed analysis of an established teachers' non-traditional lesson. *Journal of Mathematical Behavior, 18*(3), 281-325.

Schoenfeld, A. (1998). Toward a theory of teaching-in-context. Issues in Education, 4(1), 1-95.

Senk, S.L., & Thonpson, D.R. (2003b). School mathematics curricula: Recommendations and issues. In S.L. Senk and D.R. Thompson (Eds). Standards-based school mathematics curricula: What are they? What do students learn? (p. 3-30). Mahwah, NJ: Lawrence Erlbaum Associates.

Smith, M.S. & Stein, M.K. (2011). *Five practices for orchestrating productive mathematics discussions*. Reston, VA: NCTM.

Smith, M.S., Stein, M.K., Silver, E.A., Hillen, A., & Heffernan, C. (2001, April). Toward a practice based-curriculum for teaching: Integrating narrative cases and other artifacts of practice within a course for teachers of mathematics. *Symposium presented at the annual meeting of the American Educational Research Association*, Seattle, WA.

Smith, M.S., Bill, V., Hughes, E.K. (2008). Thinking through a lesson: Successfully implementing high-level tasks. *Mathematics Teaching in the Middle School*, 14(3), 132 – 138.

Smith, M. S., Cartier, J. L., Eskelson, S. L., & Ross, D. (2013, April). *Planning and teaching: An investigation of two teachers' participation in collaborative lesson planning activities and the impact of these activities on their instruction*. Paper presented at the annual meeting of the American Educational Research Association, San Francisco, CA.

Smith, M. S., Cartier, J. L., Eskelson, S. L., & Tekkumru-Kisa, M. (2012). Building a school university collaboration: A search for common ground. In J. M. Bay-Williams & W. R. Speer (Eds.), *Professional collaborations in mathematics teaching and learning: Seeking success for all. The seventy-fourth yearbook of the National Council of Teachers of Mathematics* (pp. 117–132). Reston, VA: National Council of Teachers of Mathematics.

Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Education Research Journal, 33*, 455-488.

Stein, M.K., Engle, R.A., Smith, M.S., & Hughes, E.K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning*, 10, 313-340.

Smith, M.S and Stein, M.K. (1998). Mathematical tasks as a framework for reflection: From research to practice. *Mathematics Teaching in the Middle School*, 3(4), 268-75.

Stein, M.K., & Lane, S. (1996). Instructional tasks and the development of student capacity to think and reason: An analysis of the relationship between teaching and learning in a reform mathematics project. *Educational Research and Evaluation, 2*(1), 50-80.

Stein, M.K., Russell, J., Smith, M.S. (2011). The Role of Tools in Bridging Research and Practice in an Instructional Improvement Effort. In Tate, W.F., King, K.D., Anderson, C.R. (Eds.), *Disrupting Tradition: Research Practice Pathways in Mathematics Education*, p. 33-44, Reston, VA: National Council of Teachers of Mathematics.

Stigler, J.W. & Hiebert, J. (1999). The Teaching Gap. New York: The Free Press.

Stigler, J.W. & Hiebert, J. (2004). Improving mathematics teaching. *Educational Leadership, 61*(5), 12-17.

Stigler, J. W., Fernandez, C., & Yoshida, M. (1996). Cultures of mathematics instruction in Japanese and American elementary classrooms. In T. P. Rohlen & G. K. LeTendre (Eds.), *and learning in Japan* (pp. 213–247). Cambridge: Cambridge University Press.

Thompson, A. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. Grouws(Ed.), Handbook of research on mathematics teaching and learning (pp. 127–146). New York: Macmillan.

Tyler, Ralph W. (1949). Basic Principles of Curriculum and Instruction. Chicago: The University of Chicago Press.

Vacc, N.N., & Bright, G. W. (1999). Elementary preservice teachers' changing beliefs and instructional use of children's mathematical thinking. *Journal for Research in Mathematics Education*, 30, 89-110.

Wilhelm, A.G. (2014). Mathematics teachers' enactment of cognitively demanding tasks: Investigating links to teachers' knowledge and conceptions. *Journal for Research in Mathematics Education,* 45(5), p. 636-674.

Wilhelm, A.G. & Kim, S. (2015). Generalizing from Observations of Mathematics Teachers Instructional Practice Using the Instructional Quality Assessment. *Journal for Research in Mathematics Education*, 46(3), 270-279.

Yackel, E. & Cobb, P. (1996). Sociomathematical Norms, Arugmentation, and Autonomy in Mathematics. *Journal for Research in Mathematics Education*, 27(4), 458-477.

Yoshida, M. (1999). *Lesson study: A case study of a Japanese approach to improving instruction through school-based teacher development.* Unpublished dissertation, University of Chicago.

Zahorik, J. A. (1970). The effect of planning on teaching. *Elementary School Journal*, 71(3): 143-51.

Zimmerlin, D., & Nelson, M. (2000). The detailed analysis of a beginning teacher carrying out a traditional lesson. *Journal of Mathematical Behavior, 18*(3), 263-27.