ON THE USE OF FIXED POINT THEORY TO DESIGN COUPLED CORE WALLS

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ABSTRACT

The use of Fixed Point Theory (FPT) to optimize the design of coupling beams in coupled core wall (CCW) systems is demonstrated. The basis for optimization is minimizing the transmissibility of horizontal ground motion by appropriately linking two coupled wall piers having different dynamic properties with beams having appropriate stiffness and damping characteristics. Using 21 example CCW structures illustrating a range of pier properties, it was shown that the resulting optimization of coupling stiffness is quite small and other design considerations will require stiffer, non-optimal coupling beams. Nonetheless, the potential to leverage the small amount of coupling available in a 'slab-coupled' series of wall piers in order to reduce transmissibility is suggested by the findings of this study.

INTRODUCTION

Hull and Harries (2008) identified Fixed Point Theory (FPT) as having potential applications to the performance-based design (PBD) of coupled core wall (CCW) systems. They identified the potential transition from CCW behavior under service lateral loads to a system of linked wall piers (LWP) under design seismic loads. Their work focused on the performance of the LWP system. Hull and Harries proposed a novel measure of performance: minimization of transmissibility of horizontal ground motion through the optimization of coupling beam stiffness resulting in the optimal engagement of two wall piers. Transmissibility is simply defined as the ratio of structural deflection to input horizontal ground motion. With the exception of very stiff structures, transmissibility is typically greater than unity. In a structure composed of multiple linked structural elements, transmissibility is affected by the ratio of dynamic properties of the coupled elements and the connection between these. By varying the relationship between dynamic properties of elements, transmissibility may be changed. Structures composed of dynamically identical components cannot be optimized using FPT; in such a case transmissibility is only a function of the sum of the element stiffnesses (Hull and Harries 2008).

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In this paper, the practical application of optimizing coupling beam stiffness between dynamically dissimilar wall piers using FPT will be investigated. The hypothesis being that the stiffness of the coupling beams for a given set of wall piers may be optimized to improve the CCW and subsequent LWP response to earthquake excitation. As shown in Figure 1, each wall pier is idealized as a single degree of freedom (SDOF) system having mass, stiffness and damping, m_i, k_i and c_i. The stiffness and damping (k_b and c_b, respectively) of the coupling continuum are represented by a spring and dashpot system and may be optimized so as to minimize lateral deflections X_1 and X_2 for a ground excitation U (Iwanami et al. 1996).

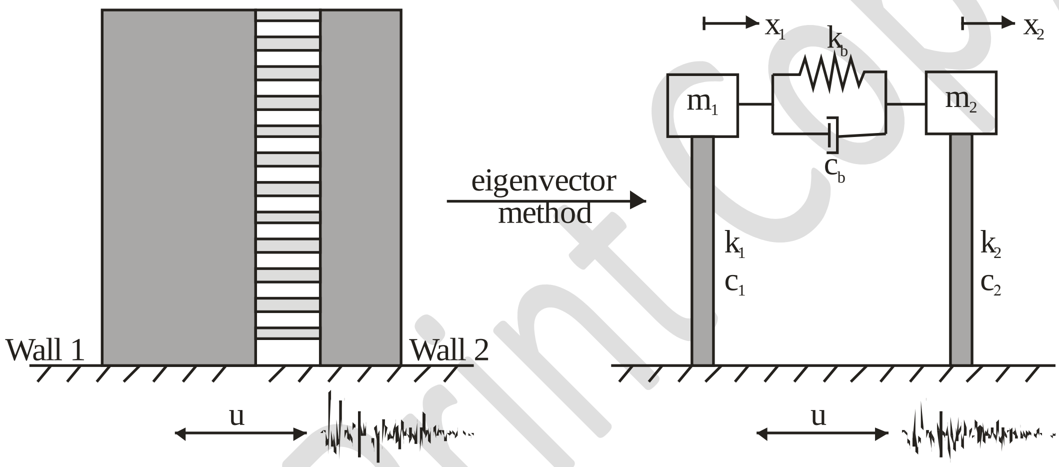
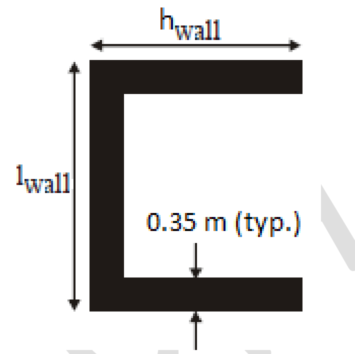


Figure 1 Idealized 2DOF system for application of fixed point theory (adapted from Hull and Harries 2008)

In this study, CCW prototype structures similar to those previously identified by Harries et al. (2004a) are used. These are 12 storey structures that have seven individual pier geometries labeled A through G, shown schematically in Table 1. The thickness of the wall piers is 0.35 m and the uniform storey height is 3.6 m. The other dimensions and resulting wall pier areas and moments of inertia are presented in Table 1. The coupling beam geometric information is not relevant at this point; indeed, this analysis is intended to lead to coupling beam stiffness requirements. The individual wall piers are paired into two-pier CCW systems, each pier matched with each other pier resulting in 28 unique analysis cases. Optimal coupling of identical wall piers based on transmissibility is meaningless (i.e.: Wall A coupled to Wall A); thus the number of unique analyses is 21. For example, case 16 (Wall D coupled to Wall E) is shown in Figure 2.

Table 1 Wall pier dimensions used in FPT analysis (Harries et al. 2004a)

Wall	Wall flange (h_{wall})	Wall web (l_{wall})	Gross wall area (A_g)	Gross wall inertia (I_g)	Wall geometry
	m	m	m^2	m^4	
A	7.00	9.00	7.80	40.20	
B	6.00	3.00	5.01	18.00	
C	4.00	3.00	3.60	5.83	
D	5.00	6.00	5.35	13.86	
E	3.00	6.00	3.96	3.32	
F	3.00	3.00	2.91	2.61	
G	4.00	9.00	5.70	8.51	

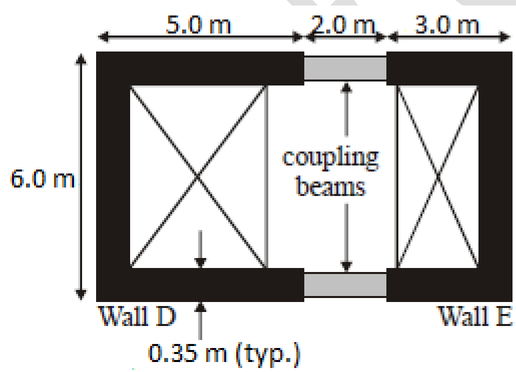


Figure 2 Example of prototype CCW Plan: Case 16: coupled Walls D and E

DERIVATION OF THE EQUIVALENT SDOF STRUCTURE

In order to model each MDOF wall pier as a SDOF system, it is represented by a massless beam-column member supporting a lumped mass at the top (Figure 1). Each beam-column is assigned geometric and material properties of the wall pier. The eigenvector method (Seto et al. 1987) is used to establish the equivalent SDOF mass. For each analysis case, the mass of the MDOF wall pier takes the form of a diagonal mass matrix, $\mathbf{M}_i$, with the diagonal values representing the portion of the storey mass assigned to each wall pier, i, based on its relative sectional area.

Each MDOF cantilever wall pier is assumed to have a fixed base and a single DOF at each floor level. The resulting stiffness matrix for each wall is therefore:

$$K_i = \begin{bmatrix} 2k_{iX} & -k_{iX} & 0 & \dots & \\ -k_{iX} & 2k_{iX} & -k_{iX} & 0 & \\ 0 & -k_{iX} & 2k_{iX} & & \\ \dots & 0 & & 2k_{iX} & -k_{iX} \\ & & & -k_{iX} & k_{iX} \end{bmatrix} \tag{1}$$

In which the lateral stiffness associated with each floor, x, of each wall, i, is $k_{ix} = 12EI_{ix}/h^3$.

The eigenvalues, ω_{in}, representing the natural frequencies, and the eigenvectors, φ_{in}, representing the solution to the undamped free vibration equation of each wall, $\mathbf{M}_i\ddot{x} + \mathbf{K}_i x = 0$, are calculated. The effective equivalent SDOF modal mass of each wall, M_{in}, corresponding to each mode, n, is (Chopra 2009):

$$m_{in} = \frac{\left(\sum_i^N m_i \varphi_{in}\right)^2}{\varphi_{in}^T M_i \varphi_{in}} \tag{2}$$

Where N is the number of degrees of freedom (storeys) in the MDOF structures, and m_i is the storey mass associated with each DOF. The equivalent SDOF stiffness of each wall, K_{in}, is defined as (Chopra 2006):

$$k_{in} = \omega_n^2 m_{in} \tag{3}$$

For the present study, only the fundamental natural frequency is considered; thus n =1 in all equations. Due to the assumed vertical uniformity of the wall piers, considering only the first mode results in a modal participation factor equal to greater than 0.90 in all cases (Eljadei 2012).

FIXED POINT THOERY

Using the SDOF systems derived in the previous section, FPT is used to determine optimal values of coupling stiffness, k_b, and damping, c_b, that result in the lowest transmissibility for the 2DOF system shown in Figure 1. The transmissibility is defined as the ratio of the structure top displacement (x_i) to the displacement induced by the ground motion (u). The complete derivation of the closed-form solutions for the 2DOF system using FPT is presented in Richardson (2003); only necessary equations are presented here. In this formulation, it is mathematically necessary to designate walls 1 and 2 such that the frequency ratio, $\gamma = \omega_2/\omega_1 > 1.0$. The equation of motion for each individual SDOF is (Iwanami et al. 1996):

$$m_1 \ddot{x}_1 = k_1(u - x_1) + k_b(x_2 - x_1) + c_b(\dot{x}_2 - \dot{x}_1) \tag{1a}$$

$$m_2 \ddot{x}_2 = k_2 (u - x_2) + k_b (x_1 - x_2) + c_b (\dot{x}_1 - \dot{x}_2) \tag{4b}$$

Where all parameters are shown in Figure 1. Considering that the system is subjected to harmonic motion, Iwanami et al. (1996) derived the displacement transmissibility, x_1/u and x_2/u, shown in Equations 5a and 5b for two SDOF piers connected at the top. The equations are functions of the properties of each SDOF pier as well as the connecting element properties.

$$\left|\frac{x_1}{u}\right| = \sqrt{\frac{\left[1-\left[\frac{\omega}{\omega_2}\right]^2+\left(\frac{\omega_1}{\omega_2}\right)^2\left(\frac{\eta}{\mu}\right)+\eta\right]^2+\left[2\xi\left(\frac{\omega}{\omega_2}\right)\left[1+\mu\left(\frac{\omega_2}{\omega_1}\right)^2\right]\right]^2}{\left[\left[1+\eta-\left(\frac{\omega}{\omega_1}\right)^2\right]\left[1-\left(\frac{\omega}{\omega_2}\right)^2\right]+\left(\frac{\eta}{\mu}\right)\left[\left(\frac{\omega_1}{\omega_2}\right)^2-\left(\frac{\omega}{\omega_2}\right)\right]\right]^2+\left[2\xi\left(\frac{\omega}{\omega_2}\right)\left[1+\mu\left(\frac{\omega_2}{\omega_1}\right)^2-(1-\mu)\left(\frac{\omega}{\omega_1}\right)^2\right]\right]^2}} \tag{5a}$$

$$\left|\frac{x_2}{u}\right| = \sqrt{\frac{\left[1-\left[\frac{\omega}{\omega_1}\right]^2+\left(\frac{\omega_1}{\omega_2}\right)^2\left(\frac{\eta}{\mu}\right)+\eta\right]^2+\left[2\xi\left(\frac{\omega}{\omega_2}\right)\left[1+\mu\left(\frac{\omega_2}{\omega_1}\right)^2\right]\right]^2}{\left[\left[1+\eta-\left(\frac{\omega}{\omega_1}\right)^2\right]\left[1-\left(\frac{\omega}{\omega_2}\right)^2\right]+\left(\frac{\eta}{\mu}\right)\left[\left(\frac{\omega_1}{\omega_2}\right)^2-\left(\frac{\omega}{\omega_2}\right)\right]\right]^2+\left[2\xi\left(\frac{\omega}{\omega_2}\right)\left[1+\mu\left(\frac{\omega_2}{\omega_1}\right)^2-(1-\mu)\left(\frac{\omega}{\omega_1}\right)^2\right]\right]^2}} \tag{5b}$$

Where ω = the forcing function frequency

ω_i = natural frequency of wall pier i: $\omega_i = \sqrt{k_i/m_i}$

η = fixed point stiffness ratio: $\eta = k_b/k_1$

μ = the mass ratio of the wall piers: $\mu = m_2/m_1$

ζ = damping ratio: $\zeta = c_b/\left(2\sqrt{m_2 k_2}\right)$

Figure 3 shows a schematic representation of the two transmissibility equations plotted by ranging the damping, ζ, from zero and infinity. Three curves result: one for each DOF (wall pier) when the damping is set to zero and a third curve for both walls when $\zeta = \infty$. Setting $\zeta = \infty$ in the latter case effectively constrains the two SDOF systems to behave as a single unit, and consequently the two walls have the same displacement and transmissibility (Hull and Harries 2008). The points P and Q in Figure 3 are the fixed points (FPT is also referred to as P-Q theory), corresponding to the maximum values of the transmissibility equations 5a and 5b. Hull and Harries (2008) showed that the optimal transmissibility of the system is achieved when the transmissibility values of P and Q are equal. The value of the fixed point stiffness ratio corresponding to this optimum case is obtained (Richardson 2003) as η = U/L; where: :

$$U = \frac{1}{4}\left(-\omega_1 + \omega_2\right)\left(\omega_1 + \omega_2\right)(-3\left(\mu+5\right)\left(\mu+1\right)^2 \omega_1{}^6 - 3\omega_2{}^2\left(7\mu+3\right)\left(\mu+1\right)^2 \omega_1{}^4$$

$$+\left[\left(3\mu^5 + 15\mu^2 + 33\mu^3 + 21\mu^4\right)\omega_2{}^4 - 3\sqrt{g} + \mu\sqrt{g}\right]\omega_1{}^2 \tag{6}$$

$$+\left(21\mu^5 + 51\mu^4 + 39\mu^3 + 9\mu^2\right)\omega_2{}^6 + \left(3\mu^2\sqrt{g} - \mu\sqrt{g}\right)\omega_2{}^2)\,\mu$$

$$L = (\omega_1{}^2\left(\mu+1\right)^2\left(\omega_2{}^2\mu + \omega_1{}^2\right)[\left(6\mu + \mu^2 + 5\right)\omega_1{}^4 + 13\mu^2 + 3 + \mu^3 + 15\mu)\,\omega_2{}^2\omega_1{}^2$$

$$+\left(7\mu^3 + +3\mu + 10\mu^2\right)\omega_2{}^4 + \sqrt{g})] \tag{7}$$

In which:

$$g = \left(\omega_2{}^2\mu + \omega_1{}^2\right)\left(\mu^3\omega_2{}^2 + 26\mu^2\omega_2{}^2 + 9\mu^2\omega_1{}^2 + 9\omega_2{}^2\mu^2 + 26\omega_1{}^2\mu + \omega_1{}^2\right)$$

$$\left(5\omega_1{}^2 + 3\omega_2{}^2 + \omega_1{}^2\mu + 7\omega_2{}^2\mu\right)^2 \tag{8}$$

The optimum damping ratio, c_b, of the connecting element is taken as the average of the damping values associated with points P and Q (Hull and Harries 2008). Hull and Harries present the interaction of stiffness and damping properties and demonstrate that near the P and Q points, the optimization is relatively insensitive to the selection of damping, particularly in the range typical of engineered structures. Indeed, ‘near optimal’ solutions may exist for a relatively wide range of properties (Hull and Harries 2008). In this research only the optimum stiffness of the coupling beams is considered in addressing the objectives of the study.

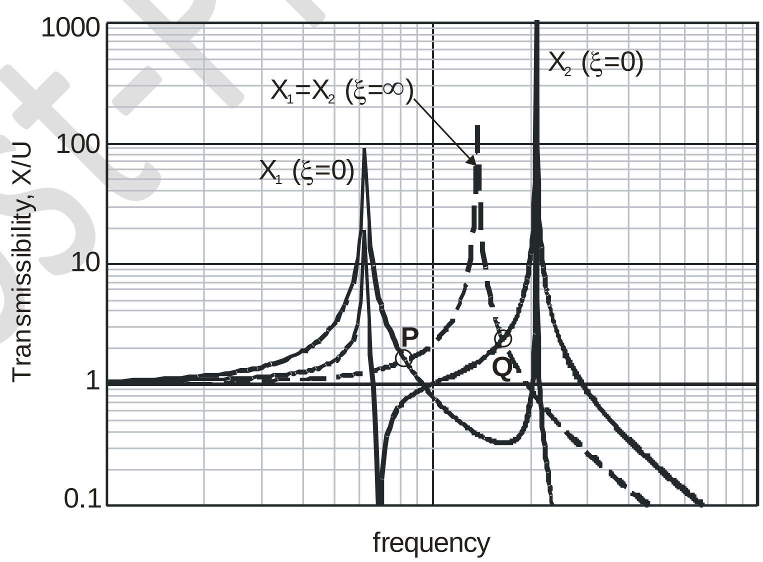


Figure 3 Schematic representation of transmissibility (Hull 2006)

PARAMETRIC ANALYSIS OF PROTOTYPE STRUCTURES

Twenty one prototype structures representing all combinations of different wall piers provided in Table 1 are used to explore the use of the FPT in optimizing CCW behavior. In these analyses, cracked concrete section properties are considered. In the calculation of the stiffness matrix (Eq. 1), the hinge region of the twelve-storey wall piers is assumed to form in the first two storeys, where a flexural stiffness of $0.35EI_g$ was used; $0.7EI_g$ was used for the upper ten storeys. The modulus of elasticity of concrete was assumed to be E = 28500 MPa. The total storey mass is assumed to be 10000 kN which is assigned to each wall pier based on its relative sectional area. The results, including the calculated optimal stiffness and damping ratios for the connecting elements, of the FPT analyses are summarized in Table 2 for the 21 cases.

When the natural frequency ratio, γ, approaches 1.0, the calculated fixed point stiffness ratio approaches zero. This represents the trivial case where two identical SDOF systems will have continued identical dynamic behavior (and thus equal transmissibility) regardless of the level of coupling and/or damping provided. Additionally, in the closed-form solution, when the product of the mass and frequency ratios, $\mu\gamma$, falls below 1.0, the optimization process yields negative stiffness values (cases 15 and 19 in Table 2). Although mathematically correct, such results are not physically meaningful — indicating a negative stiffness is required for optimization. In essence, coupling the wall piers in this case results in increased transmissibility compared to a system of uncoupled walls (Hull and Harries 2008).

DISTRIBUTION OF 2DOF OPTIMUM STIFFNESS TO MDOF SYSTEM

Having calculated the optimal stiffness, the next step is to determine the geometric dimensions of the coupling beams for these 12-storey prototype structures. The optimal coupling stiffness, k_b, is distributed to all coupling beams of the CCW system. This distribution of the total stiffness among the coupling beams should be proportional to the shear demand in the coupling beams associated with lateral loading, but as preliminary trial, a constant distribution is used. The beam stiffness and dimensions can then be determined:

$$k_{bi} = \frac{k_b}{N} = \frac{2\alpha AE}{L_b} = \frac{2\alpha h_b w_b E}{L_b} \tag{9}$$

Where h_b, w_b, and l_b are the coupling beam depth, width, and length respectively. N = 12 is the number of storeys and the factor 2 accounts for the two beams per storey (Figure 2). The factor $\alpha = 0.1$ is the reduction factor for axial stiffness in tension for reinforced concrete coupling beams (Kabeyasawa et al. 1983). The width and length of the coupling beams are assumed for all combinations to be equal to 0.35 m and 2.0 m, respectively (Figure 2). Thus, the required depth, h_b, of a single coupling beam can be determined. These calculated values are shown in Table 2. In most cases, the depth of beam, h_b, required to generate the coupling stiffness required, k_{bi}, is less than the thickness of a typical concrete slab.

Table 2 Optimization results of connecting elements using FPT

Case	Properties of Wall 1				Properties of Wall 2				Ratios		Properties of connecting elements			
	Wall	m_1	k_1	ω_1	Wall	m_2	k_2	ω_1	μ	γ	c_b	η	k_b	h
		Kg	kN/m	rad	-	Kg	kN/m	rad	m_2/m_1	ω_2/ω_1	-	k_b/k_1	kN/m	m
		x 10^5	x 10^5			x 10^5	x 10^5						x 10^4	
1	B	43.05	118.79	52.5	A	67.13	265.61	62.9	1.56	1.20	0.046	0.059	70.63	0.058
2	C	34.88	38.38	33.2	A	75.30	265.61	59.4	2.16	1.79	0.084	0.632	242.26	0.201
3	D	44.80	91.50	45.2	A	65.38	265.61	63.7	1.46	1.41	0.083	0.149	136.16	0.113
4	E	37.07	21.89	24.3	A	73.12	265.61	60.3	1.97	2.48	0.115	1.518	332.74	0.277
5	F	29.92	17.22	24.0	A	80.27	265.61	57.5	2.68	2.40	0.082	1.760	302.09	0.253
6	G	46.55	56.19	34.8	A	63.63	265.61	64.6	1.37	1.86	0.132	0.430	242.26	0.201
7	C	46.12	38.38	28.9	B	64.07	118.79	43.1	1.39	1.49	0.097	0.181	69.47	0.058
8	D	56.92	91.50	40.1	B	53.27	118.79	47.2	0.93	1.18	0.059	0.007	6.13	0.006
9	E	48.60	21.89	21.2	B	61.59	118.79	44.0	1.27	2.07	0.154	0.556	122.01	0.101
10	F	40.43	17.22	20.6	B	69.61	118.79	41.3	1.72	2.01	0.115	0.731	125.51	0.104
11	G	58.67	56.19	30.9	B	51.52	118.79	48.0	0.88	1.55	0.141	0.072	40.57	0.034
12	C	44.37	38.38	29.4	D	65.82	91.50	37.3	1.48	1.27	0.060	0.082	31.52	0.027
13	E	57.65	21.89	19.5	C	52.54	38.38	27.0	0.91	1.39	0.109	0.037	8.17	0.006
14	F	49.18	17.22	18.7	C	61.00	38.38	25.1	1.24	1.34	0.083	0.082	14.01	0.012
15	G	67.42	56.19	28.8	C	42.61	38.38	30.0	0.63	1.04	0.017	-0.005	-2.77	-
16	E	46.85	21.89	21.6	D	63.34	91.50	38.0	1.35	1.76	0.125	0.345	75.60	0.064
17	F	38.82	17.22	21.0	D	71.36	91.50	35.8	1.84	1.70	0.093	0.454	77.93	0.064
18	G	56.77	56.19	31.4	D	53.27	91.50	41.4	0.94	1.32	0.093	0.028	15.62	0.012
19	E	63.48	21.89	18.6	F	46.70	17.22	19.2	0.73	1.03	0.013	-0.003	-0.58	-
20	E	45.10	21.89	22.0	G	65.09	56.19	29.4	1.44	1.33	0.073	0.107	23.50	0.018
21	F	37.21	17.22	21.5	G	72.97	56.19	27.8	1.96	1.29	0.051	0.143	24.52	0.021

DISCUSSION OF USE OF FPT OPTIMIZATION

Based on the fixed point theory (FPT) approach presented, it is seen that the performance objective of minimizing the transmissibility of horizontal ground motion through the optimization of coupling beam stiffness results in very small levels of required coupling stiffness. The 'required' coupling beam dimensions are generally smaller than the depth of the concrete slab, let alone a practically dimensioned coupling beam.

Such low levels of coupling stiffness are structurally impractical using either concrete or steel coupling beams and would result in unacceptably low values for the degree of coupling (doc). The premise of the FPT optimization is to permit the structure to degrade from a CCW to a LWP structure, essentially allowing the doc to fall to zero under the effects of significant ground motion (Eljadei 2012). Nonetheless, the coupling elements in a typical CCW geometry also participate in the gravity load resistance and must maintain sufficient residual capacity to do so. The calculated beam dimensions in this case were generally inadequate to provide the required residual capacity. The effect of providing coupling stiffness based on practical coupling beam designs is to move the dynamic system away from the optimum case for minimizing transmissibility. That is to say, other design considerations – primarily the target doc (El-Tawil et al. 2009) will control the design of these coupling beams.

FPT applications in structural engineering are generally most applicable to problems having large frequency ratios ($\gamma = \omega_2/\omega_1$) such as when isolating vibrating equipment from a structure. In practice, the frequency ratio of practical CCW systems (considering structural layout and efficient resistance of lateral load) will rarely exceed $\gamma = 2.0$. This relatively low ratio makes optimization impractical or trivial with respect to the global structural performance.

Considerably more research is necessary to identify a design space in which FPT is useful to the structural designer. As guidance for future study, the following applications are suggested:

1. The anticipated seismic performance of shear wall structures (those resisting lateral forces only through the summation of wall moments) may be enhanced by considering the beneficial effect of the small degree of coupling resulting from the presence of the floor diaphragm. While the diaphragm is not assumed to develop coupling frame action, it does act as a link between piers, affecting some interaction between individual piers and therefore also affecting the transmissibility of ground motion. Such an approach is not likely necessary in initial design but may serve the objectives of the seismic

assessment of existing structures. The beneficial effects of ‘slab coupling’ may mitigate the need for seismic strengthening in some cases.

2. ‘Mega-coupled’ wall structures are those having coupling elements at only a few discreet locations rather than at each floor. Such systems are analogous to ‘outrigger’ structures which are relatively common in modern high-rise design. The performance of such structures, which is dominated by few structural degrees of freedom, may benefit from the CCW to LWP design approach and therefore from the FPT optimization approach.

CONCLUSION

Based on the results presented, the primary objectives of this study of investigating the evolution process of a CCW structure to a collection of LWP structures does not appear to be enhanced through the FPT optimization of transmissibility between dynamically different wall piers. Other practical design considerations including the core having a practical floor plan and the need to develop a doc > 50% for an efficient CCW system (El-Tawil et al. 2009) appear to control the design of coupling beams. The use of the FPT approach to optimizing transmissibility is best suited to systems having large frequency ratios ($\gamma = \omega_2/\omega_1$); Practical CCW systems will rarely exceed $\gamma = 2.0$.

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