INTERGENERATIONAL ASSOCIATIONS IN NUMERICAL APPROXIMATION AND MATHEMATICAL ABILITIES

by

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Although growing evidence suggests a link between children's math skills and their ability to estimate numerical quantities using the approximate number system (ANS), little is known about the sources underlying individual differences in ANS acuity and their relation with specific mathematical skills. To examine the role of intergenerational transmission of these abilities from parents to children, the current study assessed the ANS acuities and math abilities of 54 children (5-8 years old) and their parents, as well as parents' expectations about children's math skills. Children's ANS acuity positively correlated with their parents' ANS acuity, and children's math abilities were predicted by unique combinations of parents' ANS acuity and math ability depending on the specific math skill in question. These findings provide the first evidence of intergenerational transmission of an unlearned, non-verbal numerical competence and are an important step toward understanding the multifaceted parental influences on children's math abilities.

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INTRODUCTION

Educated children and adults have access to at least two different systems for representing and processing numerical information. The first, an *approximate* number system (ANS), allows for rapid but imprecise estimates about the number of items in a collection, and as such, provides the basis for rapid comparisons and approximate calculations in the absence of verbal counting (Barth, La Mont, Lipton, & Spelke, 2005; McCrink & Spelke, 2010). The ANS is present in infants and non-human animals (e.g., Agrillo, 2015; Beran, Perdue, & Evans, 2015; Izard, Sann, Spelke, & Streri, 2009; Xu & Spelke, 2000) and is therefore not tied to language or an understanding of symbols. The second, an *exact* number system, can be used to represent numerical information precisely through counting and number symbols, which is essential for school mathematics (Miller & Paredes, 1996). Unlike the ANS, the uniquely human ability to represent numbers symbolically and perform exact calculations is acquired over the course of development through formal and informal instruction (Baroody & Wilkins, 1999). The distinction between these two numerical systems provides a unique platform from which to study intergenerational transmission of an unlearned numerical competence and culturally transmitted mathematical abilities. Here, we examine specific intergenerational correlations between parent–child math ability and parent–child ANS acuity.

1.1 INTERGENERATIONAL TRANSMISSION OF COGNITIVE ABILITIES

Intergenerational transmission refers to the general process by which parents intentionally or unintentionally influence their offspring behaviorally or psychologically, and as such includes genetic and environmental factors. Several studies point to the existence of intergenerational transmission of cognitive abilities from parents to their offspring (Anger & Heineck, 2010; Björklund, Hederos Eriksson, & Jäntti, 2010; Black, Devereux, & Salvanes, 2009). For example, Anger and Heineck (2010) found children's general intelligence to be positively correlated with their parents' intelligence even when accounting for education, parental occupation, socio-economic variables, and marital status. However, far less is known about the influence of intergenerational transmission in specific academic domains.

Only a few previous research studies have examined the link between parents' and children's cognitive abilities in the domain of mathematics (Blevins-Knabe, Whiteside-Mansell, & Selig, 2007; Brown, Mcintosh, & Taylor, 2011; Crane, 1996; Duncan, Kalil, Mayer, Tepper, & Payne, 2005). One study with 5- to 9-year-old children, found children's math achievement to be significantly correlated with their mothers' general cognition, even after controlling for socio-economic status and the quality of the home learning environment (Crane, 1996). However, the maternal assessment included both language and mathematics making it difficult to examine the unique contribution of each. Evidence from a more recent, large-scale cohort study, found a positive association between parents' arithmetic scores assessed during childhood and their school-aged children's performance in mathematics (Brown et al., 2011). In the present paper, we focus not only on parents' math abilities, but also on the role of parents' ANS acuity for their children's emerging mathematical understanding.

1.2 THE DEVELOPMENT OF SCHOOL MATH ABILITIES

The ability to use an exact number system to represent numerical information symbolically and perform exact calculations depends on a complex integration of concepts acquired over years of instruction. According to the Common Core State Standards for Mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) by the end of kindergarten children are expected to count to 100, write numerals from 0 to 20, compare quantities, understand the relation between counting and cardinality, and develop a conceptual understanding of addition and subtraction (i.e., addition as "putting together"; subtraction as "taking from"). As children progress to first grade, they are expected to integrate these skills in order to relate counting to arithmetic (e.g., counting on three is the same as adding three), develop more refined strategies to solve arithmetic problems, and understand the reciprocal relation between addition and subtraction (e.g., use knowledge of 2 + 4 = 6 to know 6 – 2 = 4). By second grade, children are expected to increasingly use mental strategies for arithmetic and develop fluency within the number range from 0 to 100 for addition and subtraction.

What factors influence individual differences in children's school math abilities? In the behavioral genetics literature, twin studies are used to provide a detailed understanding of the genetic and environmental effects underlying many of our human traits (see Polderman et al., 2015 for a meta-analysis). Specifically in the domain of mathematics, behavioral genetics studies of twins have identified a substantial genetic component to mathematical ability, as well as an influence of the shared environment (Hart, Petrill, Thompson, & Plomin, 2009; Kovas et al., 2007; Thompson, Detterman, & Plomin, 1991). In the behavioral psychology literature, a number of cognitive and social factors are known to influence individual differences in children's mathematical abilities. These include social factors such as income level (Griffin, Case, & Siegler, 1994; Jordan, Kaplan, Ramineni, & Locuniak, 2009), amount of number-relevant input from teachers and parents (Gunderson & Levine, 2011; Klibanoff, Levine, Huttenlocher, Vasilyeva, & Hedges, 2006; Levine, Suriyakham, Rowe, Huttenlocher, & Gunderson, 2010), and the home learning environment (Melhuish et al., 2008), as well as general cognitive factors, such as short-term memory, working memory, attention, language, and general intelligence (Geary, 2004; LeFevre, Fast, et al., 2010; Mabbott & Bisanz, 2008; McLean & Hitch, 1999; Passolunghi & Siegel, 2004; Wilson & Swanson, 2001). Finally, a growing number of studies highlight the importance of a number-specific competence, ANS acuity, for children's mathematical understanding.

1.3 THE LINK BETWEEN SCHOOL MATH ABILITIES AND APPROXIMATE NUMBER SYSTEM ACUITY

Numerical representations in the ANS are noisy with considerable overlap between neighboring representations of number. The degree of overlap increases with increasing number, meaning that smaller quantities are represented more precisely than larger quantities, and discriminability of any two ANS representations is a function of the ratio between them (e.g., Dehaene, 1992). There are developmental improvements in ANS acuity, such that in infancy there is much more noise associated with representing a particular number and these representations sharpen throughout childhood (Halberda & Feigenson, 2008; Libertus & Brannon, 2010; Lipton & Spelke, 2003; Piazza et al., 2010; Xu & Spelke, 2000). There are also substantial individual differences in ANS acuity starting in infancy (Libertus & Brannon, 2010; Starr, Libertus, &

Brannon, 2013). ANS acuity is typically assessed using non-symbolic number comparison tasks, in which participants are asked to select the more numerous of two briefly displayed sets of objects. Individuals with more precise approximate number representations are more accurate on these tasks and are able to discriminate between non-symbolic quantities with finer ratios.

Importantly, there is now a wealth of evidence to suggest that individual differences in ANS acuity are correlated with and predictive of school math ability (Bonny & Lourenco, 2013; Libertus, Feigenson, & Halberda, 2011, 2013; Libertus, Odic, & Halberda, 2012; Lourenco, Bonny, Fernandez, & Rao, 2012; Mazzocco, Feigenson, & Halberda, 2011). New research has started to examine potential mechanisms to explain the link between individual differences in ANS acuity and school math ability. In one explanation, as children learn the meaning of number words and Arabic digits, they map these symbolic representations to the corresponding representations in the ANS, and it is the strength of these mappings that is critical for math abilities (Brankaer, Ghesquière, & De Smedt, 2014; Pinheiro-Chagas et al., 2014). Another possible explanation is that since children are able to solve approximate addition and subtraction problems before they can solve symbolic addition and subtraction problems (Barth et al., 2005), a basic understanding of approximate arithmetic based on ANS representations serves as a foundation for understanding exact arithmetic (Park & Brannon, 2014; Pinheiro-Chagas et al., 2014).

Although very little is known about the sources underlying individual differences in ANS acuity, there is some recent evidence that environmental factors, such as education and socio-economic status (SES), are influential. Piazza and colleagues (2013) found that among the Mundurucu – an Amazonian tribe whose language does not have number words larger than 5 – their level of math education was significantly correlated with their ANS acuity. Furthermore, there is some evidence that children from higher SES backgrounds displayed better ANS acuity on number estimation tasks (Mejias & Schiltz, 2013) and approximate addition problems (McNeil, Fuhs, Keultjes, & Gibson, 2011). However, the possibility of intergenerational transmission of ANS acuity from parents to their children has not yet been considered.

1.4 THE CURRENT STUDY

In the current study, we examine how individual differences in children's mathematical abilities and ANS acuity can be explained by different parental characteristics. To this end, we tested parent-child dyads on ANS acuity (a non-symbolic number comparison task) and a standardized test of math ability (Woodcock Johnson III Tests of Achievement; Woodcock, McGrew, & Mather, 2001). Children completed three subtests designed to measure their ability to perform mathematical computations in traditional written format (Calculation), to automatically recall basic number facts (Math Fluency), and to solve orally presented word problems with visual aids (Applied Problems). Due to time constraints during testing, parents only completed the Math Fluency subtest as a measure of mathematical ability. In order to assess a broader range of parental characteristics that may be important for explaining children's numerical or mathematical abilities, we included an additional measure of parental beliefs about the importance of children developing certain math skills (Math Benchmarks; Lefevre et al., 2009). Thus, we are able to examine specific intergenerational correlations between parent–child math ability and parent–child ANS acuity in the broader context of other parental characteristics. In this study we specifically ask the following questions:

1. Are parents' math abilities, ANS acuity or beliefs about math benchmarks correlated with their children's *mathematical abilities*? What are the unique contributions of each parental factor for predicting children's mathematical abilities?

2. Are parents' math abilities, ANS acuity or beliefs about math benchmarks correlated with their children's *ANS acuity*? What are the unique contributions of each parental factor for predicting children's ANS acuity?

2.0 METHOD

2.1 PARTICIPANTS

Fifty-seven parent-child dyads were recruited through flyers and local children's events to participate in a laboratory study. Children who were out of the age range (*n* = 1) or refused to complete the Approximate Number System Acuity task (see below, *n* = 2) were excluded, resulting in a final sample of 54 children (27 females and 27 males; mean age = 7.59 years, *SD* = 11.16 months, range = 5.61 years to 8.98 years) and their parents (46 mothers and 5 fathers; mean age = 38.60 years, *SD* = 5.13 years, range = 30 years to 59 years)[1]. Parent reports of race and ethnicity indicated that 45 children in our sample were identified as Caucasian, three as African-American, one as Asian and five as multiple races. Forty-six parents obtained a college degree or higher, three completed some college, one completed some high school, and one did

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[1] Originally, the sample included only 38 parent-child dyads. As suggested by a reviewer, we ran a power analysis with the original sample on the main association of interest between parents' and children's ANS acuity (*r* = .40). This analysis revealed that we would need a sample size of 53 to have more than 85% power to detect an effect of this size at an alpha level of .05. Thus, we collected data from additional participants to increase the sample size. The originally reported pattern of results and the interpretations of these findings remain the same. In addition, the final sample reported here included three sets of siblings (2 children each). When one child from each pair was excluded (all possible combinations of which were tested), the overall patterns of significance for the main findings did not change. Finally, some children in this sample did not complete the entire battery of standardized math assessments (see below), but missing data were not interpolated.

not report his or her education level. Prior to participation, all parents provided written consent for themselves and for their child, and children provided verbal assent. Parents were compensated with $8 for their participation and children received a small gift (e.g. book, stuffed animal, lunch bag).

2.2 MEASURES

2.2.1 Math Abilities

Parents' and children's math abilities were assessed using the appropriate subtests from the nationally normed Woodcock Johnson III Tests of Achievement (Woodcock et al., 2001).

2.2.1.1 Children

Children completed the Calculation, Math Fluency, and Applied Problems subtests. The *Calculation* subtest measured the ability to perform mathematical computations in traditional written format. The *Math Fluency* subtest measured automaticity with basic arithmetic facts and required children to solve simple addition, subtraction, and multiplication problems presented in traditional written format with a 3-minute time limit. All children started with the first item, and were told to work as quickly as possible without making mistakes. The *Applied Problems* subtest measured the ability to listen to a problem, recognize the procedure, and perform simple calculations. It required the child to analyze and solve orally presented word problems with visual aids. For this age range, the items included counting objects, identifying coins, telling time, performing single and double-digit arithmetic, performing calculations with money, and using simple fractions. Children gave verbal responses, but were allowed to use scratch paper if needed.

The subtests were administered in the order they would be administered on full-length Woodcock Johnson III Tests of Achievement (1. Calculation, 2. Math Fluency, 3. Applied Problems). The raw scores from each of the three subtests were converted into standardized scores with an expected mean of 100 and SD of 15. Two Calculation scores, one Math Fluency score and two Applied Problems scores could not be calculated: normative data on the Calculation test was unavailable for the ages of two participants, one child refused to complete Math Fluency and Applied Problems and one child did not finish Applied Problems due to time constraints. Reports of internal consistency for all three subtests are high (Calculation: $r = .86$; Math Fluency: $r = .98$, Applied Problems: $r = .93$).

2.2.1.2 Parents

Parents completed the Math Fluency subtest of the Woodcock Johnson III Tests of Achievement (Woodcock et al., 2001). They were also allotted 3 minutes to complete the problems and their raw scores were converted into a standardized score. Four parents were unable to complete the Math Fluency measure due to time constraints during testing.

2.2.2 Approximate Number System Acuity

To measure ANS acuity, parents and children completed established versions of non-symbolic number comparison tasks, similar to those used by Halberda, Mazzocco, and Feigenson (2008) All stimuli were extracted from the freely available Psychological Assessment of Numerical Ability (Panamath; www.panamath.org) and used in a custom-made MATLAB script for the purposes of this study. ANS acuity was assessed as the proportion of correct responses (i.e., accuracy) on the non-symbolic number comparison task. Accuracy has been found to be a more reliable index of ANS acuity compared to other commonly used measures, such as Weber fraction (i.e., an index of the imprecision of participants' ANS representations), or ratio effects in accuracy or RT (Inglis & Gilmore, 2014). To measure parents' and children's ANS acuity, we calculated the percentage of correct trials on the non-symbolic number comparison task. All trials in which the participant's response time was above or below 2.5 (for parents) or 3 (for children) standard deviations from their average trial response time were discarded as outliers[2].

2.2.2.1 Children

Children were presented with arrays of yellow and blue dots that appeared simultaneously on the screen (i.e., yellow dots on the left half of the screen and blue dots on the right half of the screen) and were instructed to report whether there were more yellow or blue dots. They were encouraged to respond both quickly and accurately by providing a verbal response (i.e., "yellow" for more yellow dots or "blue" for more blue dots). The experimenter, who was unable to see the screen, immediately pressed a corresponding key on the keyboard to record their response. Each trial consisted of a 500 ms fixation cross, followed by a stimulus image for 1,500 ms, followed by a blank screen until the child made a response. Children were able to respond either during the image or during the blank screen. The next trial began immediately after the experimenter recorded the child's response.

[2] Different cut-offs were used for parents and children because they completed different numbers of trials.

Children were given six practice trials with experimenter feedback and encouragement to ensure that the child understood the task. The child then completed 72 test trials without feedback. The difficulty of the trial depended on the ratio between the numbers of yellow and blue dots. There were 18 trials for each of four ratio categories: 2.0 (e.g., 10 yellow dots and 20 blue dots), 1.5, 1.17, 1.14. The side of the larger quantity (i.e., the correct response) was counterbalanced across trials. To ensure that participants used numerical information instead of other perceptual cues to determine the correct response, three trial types were included: Congruent (i.e., the array with the larger number had the larger cumulative area), Incongruent (i.e., the array with the smaller number had the larger cumulative area but both arrays had equal cumulative perimeter), and Neutral (i.e., the arrays had equal cumulative area). Each dot array contained between 12 and 36 dots, and dot size varied within single arrays (average dot diameter = 36 pixels; allowed variation = 20%). The Spearman–Brown corrected split-half reliability for children's accuracy scores was 0.74, similar to what has been reported with adults in a previous study (Libertus, Odic & Halberda, 2012).

2.2.2.2 Parents

Parents completed a similar non-symbolic number comparison task that differed from the children's task only in terms of the response method, the number of trials, and the numerical ratios presented. Parents indicated their response by pressing one of two keys on the keyboard. After four practice images, parents were administered 150 test trials. There were 30 trials for each of five ratios: 1.33, 1.25, 1.2, 1.14, and 1.11. One parent did not complete the ANS measure due to inability to complete testing in the laboratory. The Spearman–Brown corrected split-half reliability for parents' accuracy scores was 0.73.

2.2.3 Math Benchmarks

Parents' academic expectations for their children were measured with the mathematics portion of the academic benchmarks section of the home numeracy questionnaire developed by LeFevre and colleagues (2009). The measure asked parents to rate how important they believed it was for their child to reach each of four math-related benchmarks prior to entering kindergarten (e.g. "Count to 10"; "Simple sums") on a 5-point scale. The ratings were averaged to derive a total score. The Spearman–Brown corrected split-half reliability for the math benchmark scale was .92.

2.3 PROCEDURE

Children completed the ANS task and math ability measures during a single 1-hour testing session in the laboratory, with the ANS task administered first to reduce between-subject variability due to test order. During the child's testing session, the parent completed the questionnaires (demographics and math benchmarks) followed by the ANS task and measure of math ability.

3.0 RESULTS

Descriptive statistics for all parent and child measures are displayed in Table 1. Since children's ANS acuity improves with age (Halberda & Feigenson, 2008), we calculated age-normalized residuals for children's ANS acuity and used the residuals in all further analyses. For consistency, we also used age-normalized residuals for parents' ANS acuity in all further analyses. Note that parents' and children's math ability measures were already age-normalized because we used a standardized test.

Table 1. Descriptive statistics for all parent and child measures

Measures	n	Mean	SD
Children			
Math Ability			
Calculation	52	111.48	17.33
Math Fluency	53	102.51	18.09
Applied Problems	52	112.27	165.84
ANS Acuity	54	.79	.9
Parents			
Math Ability			
Math Fluency	50	107.88	9.75
ANS Acuity	53	.81	.06
Math Benchmarks	54	2.98	.93

Note. Math ability scores are standardized with an expected mean of 100 and SD of 15. Math benchmark ratings are based on a 5-point scale (0-4) with larger numbers indicating higher expectations.

3.1 CHILDREN'S ANS ACUITY AND MATHEMATICAL ABILITIES

We first examined correlations among children's ANS acuity and children's math abilities, assessed using standardized scores from the Calculation, Math Fluency, and Applied Problems sections of the Woodcock Johnson III. As expected, children's math scores were all highly correlated with one another. Replicating previous studies (Bonny & Lourenco, 2013; Libertus, Feigenson, & Halberda, 2011, 2013), children's ANS acuity was also associated with all three of their math ability scores (Table 2).

Table 2. Pearson's correlations between parent and child measures

	1	2	3	4	5	6
1. Child ANS	-					
2. Child Calculation	.28*	-				
3. Child Math Fluency	.45**	.69**	-			
4. Child Applied Problems	.39**	.71**	.65**	-		
5. Parent ANS	.39**	.31*	.19	.52**	-	
6. Parent Math Fluency	.26†	.51**	.44**	.36*	.20	-
7. Math Benchmarks	.13	.32*	.33*	.16	.13	.21

Note. $^{\dagger}p<.10$. $*p<.05$. $**p<.01$.

3.2 PARENTAL INFLUENCES ON CHILDREN'S MATHEMATICAL ABILITIES

In order to examine parental influences on children's math abilities, we first examined correlations between each of the children's math ability scores with parents' math ability, ANS acuity, and beliefs about math benchmarks (Table 2). Children's Calculation scores were significantly correlated with parents' ANS acuity, Math Fluency scores, and math benchmark ratings. Children's Math Fluency scores were significantly correlated with parents' Math Fluency scores and math benchmark ratings. Children's Applied Problems scores were significantly correlated with parents' ANS acuity and Math Fluency scores.

To examine the unique contribution of each of the parent measures for predicting children's math scores, we performed three linear regression analyses predicting children's Calculation, Math Fluency, and Applied Problems scores respectively. Parents' Math Fluency scores, ANS acuity, and math benchmark ratings were all entered as possible predictors. As can be seen in Table 3, all three models captured a significant amount of variance in children's math scores. Table 3 also reports squared partial correlation coefficients that represent the proportion of variance in children's math scores, not associated with the remaining variables, that is explained uniquely by the listed factor. While parents' Math Fluency scores were the only unique significant predictor of children's Calculation scores and children's Math Fluency scores, parents' Math Fluency scores and parents' ANS acuity were both unique significant predictors of children's Applied Problems scores.

Table 3. Linear regression models predicting children's mathematics scores (Calculation, Math Fluency, Applied Problems) and ANS acuity from parent measures

	Children's Scores			
	Calculation	Math Fluency	Applied Problems	ANS Acuity
	$R^2 = .35$ $p < .001$	$R^2 = .26$ $p = .004$	$R^2 = .34$ $p < .001$	$R^2 = .18$ $p = .022$
Parent Predictors	pr^2	pr^2	pr^2	pr^2
Math Fluency	.21**	.15**	.13*	.03
ANS Acuity	.05	.01	.24**	.12*
Math Benchmarks	.06	$.07^{\dagger}$	<.01	<.01

Note. $^{\dagger}p<.10$. $*p<.05$, $**p<.01$

3.3 PARENTAL INFLUENCES ON CHILDREN'S ANS ACUITY

We then tested whether parents' math abilities, ANS acuity, and beliefs about math benchmarks were correlated with children's ANS acuity. Parents' ANS acuity was the only measure significantly associated with children's ANS acuity (Table 2). As can be seen in Figure 1, parents' performance on the ANS task was positively related to their children's performance on the ANS task even when children's and parents' age was controlled.

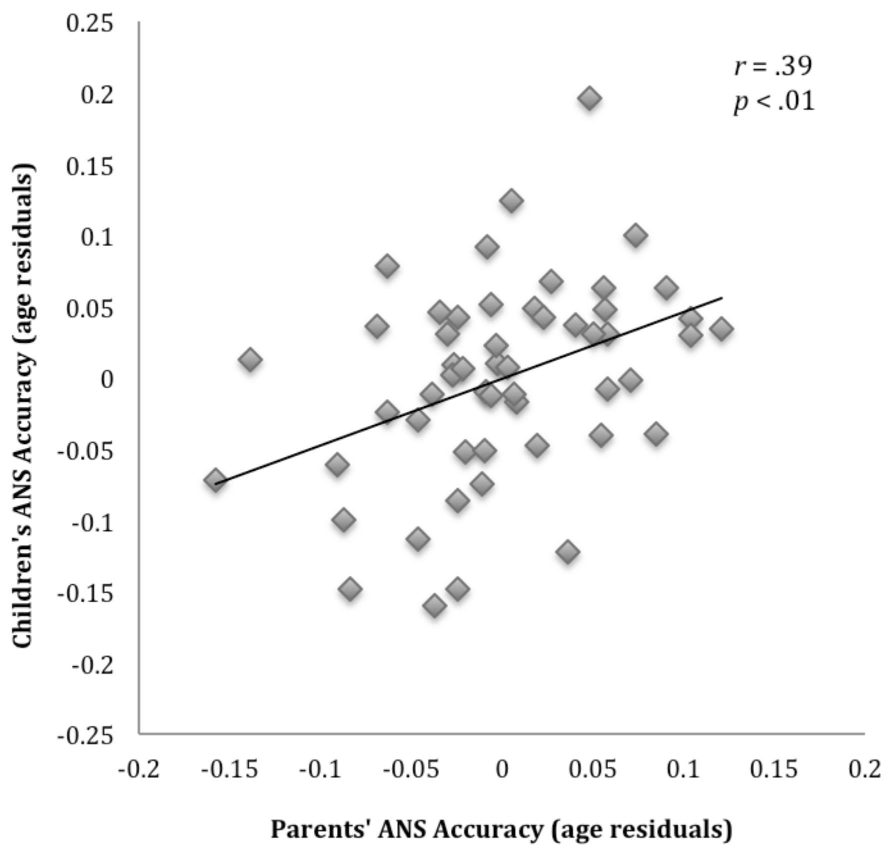

Figure 1. Correlation between parents' and children's ANS acuity as indexed by the proportion of correct responses on a non-symbolic number comparison task

Paralleling the analyses for children's mathematical abilities, children's ANS acuity was regressed on parents' Math Fluency scores, ANS acuity, and beliefs about math benchmarks

(Table 3). The model predicting children's ANS acuity from the parent measures was significant overall and parents' ANS acuity was the only unique significant predictor of children's ANS acuity.

4.0 DISCUSSION

The present study yielded two main findings. First, parents' math ability predicted their children's math ability across a number of math measures, including the ability to perform mathematical computations in traditional written format, to automatically recall basic number facts, and to analyze orally presented word problems with visual aids. Second, parents' ANS acuity correlated with their children's ANS acuity.

4.1 PARENTAL INFLUENCES ON CHILDREN'S MATHEMATICAL ABILITIES

In the present study, parents' math ability correlated with children's math ability across a number of different math measures, a finding that extends previous research suggesting a link between parents' and children's math abilities (Blevins-Knabe et al., 2007; Brown et al., 2011; Duncan et al., 2005). Moreover, these relations remained robust even when controlling for parents' ANS acuity and math expectations for their children suggesting that this intergenerational transmission is not merely a reflection of associations between basic numerical skills and parental beliefs about the importance of math. Even though we find our measure of parental math ability to robustly predict multiple aspects of children's mathematical skills, future studies using more extensive measures of parental math ability, beyond the ability to automatically recall basic number facts as assessed here, as well as other measures of more general cognitive abilities are needed to clarify the specific relations to the various aspects of children's math ability.

It is important to note that our study design does not allow us to draw conclusions about the influence of living with one's parent over and above simply receiving 50% of their genes. Given the substantial genetic component of mathematical ability seen across twins (e.g., Kovas, Haworth, Dale, & Plomin, 2007), a children-of-twins study design involving adult twin pairs and their offspring (McAdams et al., 2014) could be valuable for determining the relative contributions of genetic transmission compared with environmental transmission of math abilities from parents to their children. Furthermore, given the positive associations between the frequency of parents' engagement in math-related home learning activities and children's math ability (Anders et al., 2012; Kleemans, Peeters, Segers, & Verhoeven, 2012; Lefevre et al., 2009; LeFevre, Polyzoi, Skwarchuk, Fast, & Sowinski, 2010; Ramani & Siegler, 2008; Skwarchuk, 2009), future studies using direct observational methods are needed to examine if and how intergenerational transmission of mathematical abilities might be mediated by parent-child interactions.

Parents' ANS acuity did not correlate with children's ability to automatically recall basic number facts, but did correlate with their ability to perform mathematical computations in traditional written format and to solve applied math problems. However, after controlling for the influence of the other parental variables, parents' ANS acuity only predicted a specific aspect of children's math ability, namely children's ability to solve applied math problems. One possible explanation is that parents with greater ANS acuity may encourage their children to think about mathematical applications more frequently and naturally talk more about numbers in everyday contexts. The amount of parent number talk during everyday activities in the home between 14 and 30 months of age has been found to be positively correlated with children's understanding of the cardinal meaning of number words at age 46 months, even when controlling for SES and overall amount of parental talk (Levine et al., 2010). In addition, in a sample of parents of 5- to 6-year-old children, Elliott, Braham and Libertus (under review) found that parents' ANS acuity was correlated with parents' number talk, especially for larger numbers. Thus, it is possible that parents with greater ANS acuity and math ability knowingly or unknowingly create different home learning environments and more opportunities to learn about math in everyday contexts that in turn shape differences in children's abilities to solve applied math problems.

Finally, we found that parental expectations about what math skills children should attain at a certain age correlated with children's abilities to perform mathematical computations in traditional written format and automatically recall basic number facts; however, parental expectations no longer predicted these math skills after controlling for parental math ability and ANS acuity. These findings are in line with previous work by LeFevre and colleagues (2009) who failed to find any links between parental benchmark expectations and children's math abilities in their data. In contrast, DeFlorio and Beliakoff (2014) assessed parental expectations about 23 different mathematical skills of 5-year-old children and found the accuracy of those beliefs to predict children's mathematical ability over and above the child's age and socio-economic status. Given these inconsistencies, it is possible that this relation may depend on the quantity and content of the items, on the specific math skill in question, or on the age of the children, warranting further careful examination.

4.2 PARENTAL INFLUENCES ON CHILDREN'S ANS ACUITY

This study is the first to examine the interrelations between parents' and children's numerical abilities. Parents with greater ANS acuity tend to have children who have greater ANS acuity themselves. It is important to note that parents' ANS acuity was the only parent measure that significantly correlated with children's ANS acuity. Furthermore, parents' ANS acuity significantly predicted children's ANS acuity when controlling for parents' math abilities and math expectations for their children. These findings may suggest a specific intergenerational relation of an unlearned numerical competence that is distinct from culturally transmitted mathematical abilities.

Further cross-sectional and longitudinal studies as well as more extensive twin studies are needed to examine whether the association between parents' and children's ANS acuity can be explained by shared genes or environment. A recent large-scale genetic investigation found modest, but significant, heritability of ANS acuity at the age of 16 (Tosto et al., 2014). However, it is unknown if ANS acuity is heritable at younger ages. Thus, further studies are needed to investigate the relative contributions of genetic and environmental factors on individual differences in ANS acuity, especially across development.

Although the ANS is unlearned, there is evidence that ANS acuity can be improved through education (Piazza et al., 2013) and training (Obersteiner, Reiss, & Ufer, 2013; Park & Brannon, 2013, 2014). Training first graders on approximate number estimations led to subsequent improvements in performance on a non-symbolic number comparison task (Obersteiner et al., 2013). Given the malleability of the ANS, these findings highlight the importance of examining how transmission of ANS acuity could operate through parent-child interactions. It is possible that parents with more precise ANS representations engage in more activities or discussions in the home (e.g., estimation, numerical comparison, discussion of equivalence) that lead to a strengthening of their children's ANS acuity.

4.3 CHILDREN'S ANS ACUITY MATHEMATICAL ABILITIES

In addition to associations between parental characteristics and children's numerical and math abilities, we also found that children's ANS acuity correlated with their mathematical skills across all of the math measures assessed here. As reviewed above, the ANS may be foundational for school math abilities through the mapping of number symbols to their corresponding magnitude representation or through a conceptual understanding of symbolic arithmetic, and these mechanisms may play a role concurrently or at different points in development. In our sample of 5- to 8-year-old children, the ANS may play a role in the understanding and execution of arithmetic. A basic understanding of arithmetic based on ANS representations may translate into improvements in exact arithmetic performance (Park & Brannon, 2014). Children with greater numerical estimation abilities may also be able to verify and reject outcomes of arithmetic operations with greater ease which is especially useful on arithmetic assessments such as the Calculation and Applied Problems subtests of the Woodcock Johnson (Woodcock et al., 2001). Furthermore, numerical estimation abilities may help children rapidly switch between solving addition and subtraction problems as is required on the Math Fluency subtest of the Woodcock Johnson (Woodcock et al., 2001).

4.4 PARENTS' ANS ACUITY AND MATHEMATICAL ABILITIES

Although not a primary interest of the current study, we acknowledge the non-significant correlation between parents' ANS acuity and their own math ability. Prior research studies do suggest that ANS acuity and math abilities are linked in adulthood (Dewind & Brannon, 2012; Halberda, Ly, Wilmer, Naiman, & Germine, 2012; Libertus et al., 2012; Lourenco et al., 2012). In the current study, these findings may be due to our assessment of adults' math abilities, which only narrowly assesses arithmetic fluency. Alternatively, our results may be due to a lack of power. Our sample size is smaller than that in previous papers examining ANS acuity and math ability in adulthood, but our effect sizes are comparable (Libertus et al., 2012) suggesting that we might have found a significant effect with a larger sample size.

4.5 LIMITATIONS AND CONCLUSIONS

While this work is the first to closely examine the interrelations between parents' and children's numerical and mathematical abilities, there are important limitations that should be addressed in future studies. First, our sample primarily included highly educated parents who had obtained a college degree or higher. Future research using samples with more variability in parental education and socio-economic status should investigate if these findings generalize to more diverse populations. Second, because the parents in this sample mainly consisted of mothers, it is unknown whether our findings generalize to fathers. In order to gain a complete understanding of the intergenerational transmission of these abilities, the cognitive abilities of both parents should be taken into account. Furthermore, it is possible that intergenerational transmission is stronger with the primary caregiver or within gender compared to between genders, as previous findings have demonstrate that father-son and mother-daughter associations in general intelligence are stronger than father-daughter and mother-son associations (Anger & Heineck, 2010). Future work is needed to elucidate whether daughters and sons might also show different relations to their gender-matched and gender-mismatched parents with respect to ANS acuity and math abilities.

In sum, our findings provide the first evidence of intergenerational transmission of an unlearned, non-verbal numerical competence (i.e., the approximate number system, ANS) from parents to their children. Importantly, only parents' own ANS acuity was a significant predictor of children's ANS acuity, whereas children's school math abilities were predicted by unique combinations of parents' ANS acuity and math ability. These findings are an important step towards a nuanced understanding of what shapes children's numerical and mathematical abilities and the intergenerational transmission of an unlearned numerical competence and culturally transmitted mathematical abilities.

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