LEARNING BIOLOGY THROUGH CONNECTING MATHEMATICS TO SCIENTIFIC MECHANISMS: STUDENT OUTCOMES AND TEACHER SUPPORTS

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Integrating mathematics into science classrooms has been part of the conversation in science education for a long time. However, studies on student learning after incorporating mathematics in to the science classroom have shown mixed results. Understanding the mixed effects of including mathematics in science has been hindered by a historical focus on characteristics of integration tangential to student learning (e.g., shared elements, extent of integration). A new framework is presented emphasizing the epistemic role of mathematics in science. An epistemic role of mathematics missing from the current literature is identified: use of mathematics to represent scientific mechanisms, Mechanism Connected Mathematics (MCM). Building on prior theoretical work, it is proposed that having students develop mathematical equations that represent scientific mechanisms could elevate their conceptual understanding and quantitative problem solving. Following design and implementation of an MCM unit in inheritance, a large-scale quantitative analysis of pre and post implementation test results showed MCM students, compared to traditionally instructed students) had significantly greater gains in conceptual understanding of mathematically modeled scientific mechanisms, and their ability to solve complex quantitative problems. To gain insight into the mechanism behind the gain in

quantitative problem solving, a small-scale qualitative study was conducted of two contrasting groups: 1) within-MCM instruction: competent versus struggling problem solvers, and 2) within-competent problem solvers: MCM instructed versus traditionally instructed. Competent MCM students tended to connect their mathematical inscriptions to the scientific phenomenon and to switch between mathematical and scientifically productive approaches during problem solving in potentially productive ways. The other two groups did not. To address concerns about teacher capacity presenting barriers to scalability of MCM approaches, the types and amount of teacher support needed to achieve these types of student learning gains were investigated. In the context of providing teachers with access to educative materials, students achieved learning gains in both areas in the absence of face-to-face teacher professional development. However, maximal student learning gains required the investment of face-to-face professional development. This finding can govern distribution of scarce resources, but does not preclude implementation of MCM instruction even where resource availability does not allow for face-to-face professional development.

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1.0 INTRODUCTION

Two students are arguing over the meaning of the y-intercept on a graph in physics. Carlos is insisting that the y-intercept is 760 psi, and Juana is insisting that the y- intercept is the maximum pressure of the inflated balloon. To strengthen her argument, Juana points out that the teacher asked for the *conceptual* meaning of the y-intercept and 760 psi is just the quantitative value. Carlos remains unmoved by this reasoning, arguing that the y-intercept is the value where the line of best fit intersects with the vertical axis, and that value is 760 psi. Both students are using their understanding of mathematics in a science class, but they are generating different answers and demonstrating a different understanding of how mathematics is connected to the scientific concepts. Carlos' understanding is purely mathematical and based on a definition of a mathematical term. His quantitative values are unlinked to the scientific concepts that he is studying in science class. On the other hand, Juana shows an understanding of how the mathematics and the science are connected, such that the mathematical concept of a y-intercept (value when a best-fit line crosses the y-axis) is conceptually linked to a science concept (maximum pressure).

Studies of students solving problems involving mathematics in science class show that the dichotomy illustrated above is not uncommon and can be seen in biology (Stewart, 1983), chemistry (Taasoobshirazi & Glynn, 2009)), and physics applications (Bing & Redish, 2009). Furthermore, these studies suggest that students who fail to conceptually link the mathematics in science class with the scientific concepts show gaps in their understanding of the scientific concepts and struggle to solve quantitative problems. On the other hand, students who spontaneously connect their problem solving with the represented scientific phenomenon achieve insight in to their problem solving process, allowing them to succeed even with novel or complex problems (Taasoobshirazi & Glynn, 2009; Tuminaro & Redish, 2007).

Including mathematics in the science curriculum has produced mixed results. Some researchers have seen increased understanding of the underlying scientific phenomenon and improved problem solving while others have failed to show an effect (K. Becker & Park, 2011; Hurley, 2001). One reason for the discrepant results is a lack of clarity on what it means to integrate mathematics and science beyond simply characterizing how much of each discipline is taught or listing shared attributes. As the calls for mathematics integration into science class become stronger with the continuing push for integrated STEM (Science, Technology, Engineering and Mathematics) initiatives and with the advent of the Next Generation Science Standards in the US, there is a pressing need for a new framework that characterizes epistemologically different ways of integrating mathematics into science to aid in the design of curricula, and the evaluation and comparison of the effects of these curricula. After reviewing literature on integrating mathematics into science education (Chapter 2), I propose a new epistomological framework for classifying the forms of mathematics found within science education. I suggest that the current curricula and studies on quantitative problem solving in science fall in to one of three categories (Mathematics as Tool, Mathematics as Inscription, Grounded Mathematics) and identify a category that is missing, Mechanism Connected Mathematics (MCM). I define Mechanism Connected Mathematics as including mathematics in the scientific curriculum in such a way that students develop a mathematical model of a scientific phenomenon that connects the entities and the mechanism involved in the scientific phenomenon with the variables and functions of the mathematical model. For example, in the scientific phenomenon of inheritance, a variety of offspring types are produced from a set of parents because eggs and sperm (entities) join in such a way that any sperm can join with any egg and vice versa (scientific mechanism). A Mechanism Connected Mathematics equation of this phenomenon would be eggs x sperm = offspring outcomes, where the variables of eggs and sperm align with the entities in the phenomenon and the function of multiplication align with the multiple ways that eggs and sperm can join with one another.

Based on theories about the role of mathematics in scientific practice (Hestenes, 2010; Svoboda & Passmore, 2013), the structure of mathematical equations (Sherin, 2001) and the role of mathematics in science education (Hestenes, 2010; Redish & Kuo, 2015), I argue that instruction in an MCM curriculum will confer benefits to students. Specifically, compared to students exposed to a traditional curriculum, students who experience an MCM curriculum are predicted to 1) gain a better conceptual understanding of the scientific phenomenon that is being mathematically modeled and 2) be better able to solve quantitative problems in science, particularly novel or more complex problems.

To test out these hypotheses, in Chapter 3, I present a curriculum for a unit that incorporates MCM, and use quantitative analysis of pre and posttest multiple choice testing on over 1,000 students to assess the effect of this MCM unit versus a traditional unit on student conceptual understanding and quantitative problem solving. This analysis revealed that compared to traditionally instructed students, students who experienced an MCM unit showed a ten-fold gain in conceptual understanding of the modeled components, and a four-fold gain in ability to solve complex quantitative problems.

Chapter 4 discusses a theoretical rationale for how mechanism connected mathematics improves students' performance in solving complex quantitative problems. This theory is supported and further developed by qualitative analysis of student problem solving three groups of students contrasted in the following ways: 1) within-MCM instructed students: students who struggled with complex quantitative problems versus those who could competently solve those problems, 2) within-competent problem solvers: MCM instructed students versus traditionally instructed students. The results of this two contrasts suggest that exposure to an MCM unit allows competent students to make connections between their mathematical inscriptions and the underlying scientific phenomenon. They can then use this understanding of the scientific phenomenon to switch between mathematical and biologically oriented inscriptions, facilitating productive quantitative problem solving behaviors.

Regardless of the benefits that accrue to student learning with the inclusion of Mechanism Connected Mathematics, these benefits cannot be realized if teachers cannot enact the units because of inadequate preparation. Many teachers of science, particularly those in biology, have little background in mathematics (National Research Council, 2015) and most have only been exposed to traditional instruction where mathematical modeling (such as used in an MCM unit) has not been included in science (Watanabe & Huntley, 1998). Therefore, they have few resources to draw on when asked to implement science units with a mathematical emphasis, such as MCM instruction. To increase the likelihood of impacting student learning with science units that include mathematics, particularly those that require meaningful connections to be made between the mathematics and the science, professional development in mathematical modeling is needed. The fifth chapter investigates how much and what kinds of teacher supports are necessary to achieve gains in student learning and suggests that the answer to this question varies by content area (e.g. scientific concepts or quantitative problem solving).

2.0 LITERATURE REVIEW: MATHEMATICS INCLUSION IN SCIENCE EPISTEMIC (MISE) FRAMEWORK: A NEW FRAMEWORK FOR DESCRIBING INTEGRATION OF MATHEMATICS IN SCIENCE

The concept of integrating mathematics and science education is not a new one. In the first bibliography of integrated science and mathematics teaching and learning literature, covering the years 1901-1990, the first document on mathematics and science integration was published in 1905 (Berlin & Lee, 2005). Between then and 2001, over 800 papers have been published on integrating the two disciplines (Berlin & Lee, 2005). Most of these papers described curriculum and instruction. While research papers comprised only about twenty percent of the total publications and only half were empirical as opposed to theoretical, two metanalyses have revealed a small average benefit for mathematics and science learning (K. Becker & Park, 2011; Hurley, 2001). As a result, over the last two decades, integration of mathematics and science has been codified in policy initiatives over the last two decades (e.g. NGSS).

Despite the relatively long history of thought behind mathematics and science integration, there is still not a shared framework for how to define and characterize integration of mathematics and science in education. A review of the literature from 1901 through 2000 found a plethora of terms associated with integration including connections, cooperation, coordinated, correlated, cross-disciplinary, fused, interactions, interdependent, interdisciplinary, interrelated, linked, multidisciplinary, transdisciplinary, and unified (Berlin & Lee, 2005). Educational researchers have tried to impose structure on this chaos by generating methods of characterizing mathematics and science integration. Many of these earlier methods of classifying integration have focused on either the amount of integration between mathematics and science or the content that is being included.

In this literature review, I first review the reasons behind integrating mathematics with science and describe early methods of classification. I then argue that these previous classification schemes fail to capture a key aspect of mathematics use in the science classroom: the epistemic role of the mathematics. I argue that consideration of the epistemic role of mathematics in the science classroom has greater potential to explain student learning than a content or quantity based perspective. Combining ideas about the purpose of mathematics in science classrooms (Judson, 2013) and epistemological differences between the disciplines of mathematics and science (Lederman & Niess, 1998), I propose a new framework for classifying mathematics use in the science classroom: the Mathematics Inclusion in Science Epistemic (MISE) Framework. Through a review of the quantitative problem solving literature in science education, I show how this framework can be used to group previously published studies in to three categories (Mathematics as Tool, Mathematics as Inscription, and Grounded Mathematics). I also show how viewing studies on student quantitative problem solving through the lens of the MISE framework reveals a relationship between the function of mathematics in the science classroom, how mathematical expressions are used by students, and student learning outcomes. Finally, I identify an epistemic role that is largely absent from the quantitative problem solving literature in science education: Mechanism Connected Mathematics (using mathematics to focus attention on scientific mechanisms). While the MISE framework has potential for science education researchers in terms of explaining findings, I propose that it also has an important role to play for designers and instructors. Ultimately, I argue that better outcomes for student learning are possible if designers and instructors pay attention to the epistemic role of mathematics in the science classroom.

2.1 A NEW FRAMEWORK IS NEEDED TO CHARACTERIZE INTEGRATION IN MATHEMATICS AND SCIENCE EDUCATION

2.1.1 Historical rationales for integrating mathematics and science

Multiple rationales have been provided for integrating mathematics with science. It has been proposed that both disciplines a) Attempt to discover patterns and relationships, b) Are based on interdependent ways of knowing, c) Share similar processes (e.g. inquiry and problem solving), d) Benefit from connection to real-life situations, and e) Fundamentally require quantitative reasoning (Pang & Good, 2000). All of these rationales are based on commonalities between the two disciplines.

Some of these rationales have become encoded in policies advocating the integration of mathematics in to science. In 1993, Benchmarks for Science Literacy recommended that students study mathematics as part of their science classes:

> "For purposes of general scientific literacy, it is important for students (1) to understand in what sense mathematics is the study of patterns and relationships,…For the most part, learning mathematics in the abstract before seeking to use it has not proven to be effective. Thus, the curriculum should arrange instruction so that students encounter

any given mathematical pattern or relationship in many different contexts before, during, and after its introduction in mathematics itself."

The importance of connecting mathematics and science was echoed in 2000 by the National Council of Teachers in Mathematics (NCTM) when they called for linking mathematics to a context, particularly science, so that the processes and content of science can inspire mathematical problem solving (Berlin & Lee, 2005). Both of these policy statements, even though one is from science and the other is from mathematics emphasize the use of real world connections in science to help students learn mathematics (Rationale d), although there are subtle differences in perspective. The Benchmarks for Science Literacy Policy focuses on using real world contexts to overcome difficulties students have in using mathematics (presumably in science class), while the NCTM policy statement talks about the use of real world contexts to motivate students. The Benchmarks for Science Literacy policy statement also recognizes that mathematics and science share a common analytical framework for analyzing data: discovering patterns and relationships (Rationale a).

The most recent policy documents in science, the Next Generation Science Standards state, "Mathematics is a tool that is key to understanding science. As such, classroom instruction must include critical skills of mathematics" (p. 10, Appendix F, NGSS Lead States, 2013). The NGSS has taken great care to align its standards with those of the most recent policy document in mathematics, the Common Core State Standards in Mathematics, expressing the view that "Science is a quantitative discipline, so it is important for educators to ensure that students' science learning coheres well with their learning in mathematics" (p. 1, Appendix L, NGSS Lead States, 2013). The Common Core State Standards in Mathematics, in turn, recognizes science as one of the disciplines that can provide a real world context for mathematical problem solving and modeling (Common Core State Standards for Mathematics, 2012). Again, there is a subtle difference between the policy statements from science organizations and mathematics organizations, those from science organizations, as illustrated by the above quotes, treat mathematics as an inevitable, foundational part of the science curriculum, while those from mathematics organizations treat science as one of many real world contexts to which mathematics can be applied.

Despite a strong policy push (Berlin & Lee, 2005; Pang & Good, 2000) and a generally held view by educators that integration of mathematics and science is beneficial to student learning (Baxter, Beghetto, Ruzicka, & Livelybrooks, 2014; Berlin & White, 2010; M. M. Lee, Chauvot, Vowell, Culpepper, & Plankis, 2013), there have been comparatively few empirical studies. Of the literature published on integrating mathematics and science, only eleven percent of the articles from 1901-1989 and nineteen percent of the articles from 1990-2001 were research articles (Berlin & Lee, 2005). It has been reported that students who experience classes which integrate both mathematics and science instruction show increased motivation and a more positive attitude towards schooling (Stinson, Harkness, Meyer, & Stallworth, 2009). Moreover, two separate metaanalyses of studies published between 1935 and 1997 and between 1989 and 2009, showed small but positive effect sizes in both science and mathematics for most studies (K. Becker & Park, 2011; Hurley, 2001). These findings of overall positive effects for student learning and motivation have helped to maintain interest in finding productive ways to integrate mathematics and science. However, individual studies in these meta-analyses varied in their effects with some showing no or negative effects on learning in one or both disciplines (K. Becker & Park, 2011; Hurley, 2001) . Efforts to tease apart the reason behind these differences and thus identify effective models of integration have been stymied in part by the different methods that have been used to classify mathematics and science integration.

2.1.2 History of classifying mathematics and science integration

Prior schemes for categorizing the integration of mathematics and science in education have tended to focus on either the amount of integration between mathematics and science or the types of connections between the two disciplines. One of the earliest schemas developed by the Cambridge Conference in 1967 proposed characterizing integration between mathematics and science based on which discipline was primary and which was secondary. The Cambridge Conference defined five categories to describe interactions between mathematics and science: a) mathematics for sake of mathematics, b) mathematics for sake of science, c) mathematics and science, d) science for sake of mathematics, e) science for sake of science (Huntley, 1998). Lonning and DeFranco (1997) proposed a similar continuum model based on different criteria for what counts as equal treatment (category c). The Cambridge Conference proposed that the criterion for placing an interaction in the middle category is synergy where learning in each discipline is elevated by integration beyond what could be achieved independently. For Lonning and deFranco (1997), on the other hand, an interaction would be placed in the middle category if there were equal conceptual treatment of the two disciplines. Both of these frameworks assess the amount of "contact" between the two subjects to characterize integration. Hurley (2001), modified the amount of contact framework slightly to include the timing of instruction, generating five categories, ranging from little contact where science and mathematics concepts are planned and taught sequentially with one preceding the other, to total integration where science and mathematics are taught simultaneously in the same classroom in intended equality.

While the three frameworks described in the previous paragraph focus on the amount of integration, other frameworks focus on what is being integrated. A brief description of each of these frameworks will show how they share a focus on content of integration as opposed to extent of integration, even though the content might be parsed differently. Miller, Davison and Metheny (1995) suggest characterizing mathematics and science integration according to whether it is content specific, process based, methodology based, or thematic. In content specific integration, existing curriculum objects from mathematics and existing curriculum objectives from science are taught together (e.g. simple machines in physics and proportions in mathematics). Process integration is exemplified by a science class where both mathematics and science processes are needed to carry out an experiment. Methodological integration refers to situations where a similar instructional methodology is used for both science and math classes (e.g. concepts are investigated in both mathematics and science using inquiry and discovery). In thematic integration, schools use a common theme through which all the disciplines interact (e.g. examining the effects of an oil spill). Berlin and White (1994) propose a very similar framework with six categories of integration: 1) ways of learning, 2) ways of knowing, 3) process & thinking skills, 4) content knowledge, 5) attitudes & perceptions, 6) teaching strategies. Teaching strategies, process/thinking skills and content knowledge obviously align to the content specific, process based and methodology based categories of Miller et al (1995). As reviewed by Kurt and Phelivan (2013), ways of learning and ways of knowing seem to define a new category that deal with how knowledge is constructed in each of the disciplines.

2.1.3 Inclusion instead of integration

Both the content-focused and quantity-focused frameworks of integration seem to imply that the ultimate goal is complete integration of mathematics and science where instruction occurs simultaneously and focuses on the similarities between the two disciplines (in methodology, content, or process). Lederman and Niess (1998) in an introduction to a special issue on mathematics and science integration caution that there are fundamental differences between mathematics and science methodologically: “science seeks consistency with external world through empirical evidence, mathematics seeks consistency with its internal world through logical deduction” (p. 74, as summarized in Pang & Good, 2000). Moreover, there are some topics that are unique to mathematics or science. For example, multiplication of negative integers to yield a positive integer has no parallel in science (Koirala & Bowman, 2003). The process of scientific experimentation is not the same as the process of constructing a mathematical proof. Therefore, it seems important to maintain separate instruction of the two disciplines so that students can experience their unique content, methodology, and processes and come to understand the unique constraints and affordances of each discipline. For this reason, I am going to use the word “inclusion” in place of “integration” to indicate instruction in a primary discipline that includes the use of, or instruction in, a secondary discipline.

2.1.4 A new framework to categorize inclusion of mathematics in science

When applied to the scenario described in the first chapter where Carlos and Juana are arguing over the meaning of the y-intercept (a parallel concept in mathematics and science), the frameworks presented so far cannot capture why the students are disagreeing. For Juana, the graph is representing a phenomenon in the real world; for Carlos, this connection is not readily apparent. Returning to the distinction made by Lederman and Niess (1998), “science seeks consistency with the external world.” Thus, to honor the scientific epistemological perspective, when evaluating inclusion of mathematics into a science class, the connection of mathematics to real world phenomena should be considered to capture the epistemological alignment of the mathematics that has been inserted in to the science class. In other words, is the equation or graph simply an addition to the science class or has it taken on some of the attributes associated with science?

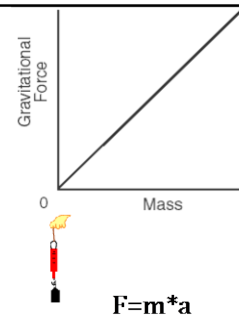
In his description of constructing the Mathematics Integrated into Science Classroom Observation Protocol (MISCOP) (2013), Eugene Judson brings up the idea that mathematics has purpose in the science classroom; as a tool to analyze data, as a method of communicating, to represent variables and their relationships, and to model scientific phenomenon. These uses are echoed and expanded on in the description of Practice 5: Use of Mathematics and Computational Thinking in NGSS (NGSS Lead States, 2013). These ideas of purpose of the mathematics in the science classroom and epistemological alignment capture two different dimensions. Purpose captures how the mathematics either is designed to be used or is being used in the classroom, while epistemological alignment captures the extent to which there is an attempt to have the mathematics, like science, align with a real world phenomenon. A scientific phenomenon is comprised of at least two parts: the entities within the phenomenon and the scientific mechanisms that explain how the entities interact to produce the observed phenomenon (Machamer, Darden, & Craver, 2000). Therefore, when considering epistemological alignment it is necessary to consider how well the mathematics aligns with both the entities and the mechanism of the phenomenon.

Derived from literature on curricular designs for incorporating mathematics into science, and studies of student quantitative problem solving, I use the two criteria suggested above (purpose and epistemological alignment) to propose a new framework, the Mathematics Inclusion in Science Epistemic (MISE) Framework, for categorizing inclusion of mathematics in science. Following a brief description of the framework, I provide a more detailed description of each category, using examples from the quantitative problem solving literature in science literature to group empirical studies and show how these groupings highlight the affordances and drawbacks of each category of inclusion. Finally, I propose a fourth category that is largely missing from the existing literature and argue that this last category offers unique affordances for student learning in science.

2.1.5 Summary of mathematics inclusion in science epistemic (MISE) framework

The four categories of the MISE Framework are 1) Mathematics as Tool, 2) Mathematics as Inscription, 3) Grounded Mathematics, and 4) Mechanism Connected Mathematics (Table 1). Mathematics as Tool is mathematics used to perform functions in science, generally to calculate a number. It can have multiple purposes that fall within that category (e.g. analyze data, make a numeric prediction). While some students, or teachers, or the curricular designers may recognize the connections to science entities or science mechanisms, they are not generally obvious to the novice, and don’t have to be present for the tool to be used for its intended purpose.

Table 1. Summary of mathematics inclusion in science epistemic (MISE) framework[1]

	Mathematics as Tool	Mathematics as Inscription	Grounded Mathematics	Mechanism Connected Mathematics
Example	F=m*a		 F=m*a	a=F/m
Purpose	Perform a calculation	Communicate	Connect entities in phenomenon with variables in the equation; displaying relationships within and among those entities	Develop mathematical inscriptions that embody the scientific mechanism in the phenomenon
Epistemological Alignment	None ⟶			High
Connection to Science Entities	Not necessarily present	May be present, but often ambiguous	Present	Present
Connection to Science Mechanisms	Not necessarily present	Not necessarily present	Not necessarily present	Present
Potential Affordances and Constraints				
Affordance Example	Fast	Express an idea or relationship	Entities within the scientific phenomenon are used to connect different representations enabling representation switching	Focuses attention on both the scientific mechanism and the entities within the phenomenon
Constraint Example	Difficulties with transfer	Meaning can be ambiguous or lost to novices	Mechanism underlying phenomenon may be obscured	Potential complexity of mathematical representation of mechanism

[1]While this will become more evident as each category is described in detail, each mathematical representation used as an example is a distinct entity with its own meaning. The expression F=m*a is not the same as the expression a=F/m because while multiplication and division are reciprocal functions, they have distinct grammatical meaning (Sherin, 2000).

Mathematics as Inscription encompasses the idea that mathematics is often the language through which scientific ideas are communicated so it is necessary to understand for students to understand the conventions. Examples of this type of use are graphing data or using units, because that's what scientists do when they communicate. There may be connections to the science entities and mechanisms, but these are not always obvious, particularly for the novice (Roth, Tobin, & Shaw, 1997).

Grounded Mathematics has two forms but both contain the idea that mathematical expressions are one way to represent scientific phenomena. The mathematical representations are grounded (anchored to) other scientific phenomenon by other representations (verbal, graphical, pictorial) Examples of this type of use are developing graphs from drawings of plant growth (Lehrer & Schauble, 2004) or developing mathematical expressions that are connected to pictorial and graphical representations of change in position of an object over time (Hestenes, 2010). The entities of the phenomenon and their relationships with one another are explicitly represented, but the mechanisms that produced those relationships are not obviously represented (at least for novices).

In the fourth category, which I believe is missing from the existing literature, Mechanism Connected Mathematics, the variables and mathematical processes in the mathematical representation (generally an equation) have connections to BOTH the entities and mechanisms in the scientific phenomenon. Moreover, the purpose of these mathematical representations is to make those connections. Once those connections are not present or are not developed, then the expression is no longer an example of Mechanism Connected Mathematics. An example of this type of mathematics is development of the equation, # of egg types * # of sperm types = # of offspring types, to express the number of possible outcomes of fertilization (Schuchardt &

Schunn, 2016). The variables of this equation are connected to the entities involved (sperm and eggs) and the process of multiplication is connected to the scientific mechanism that explains how interaction of the entities (any sperm can connect to any egg) produces the observed outcomes of fertilization (different offspring types).

In the MISE Framework, it is not possible to just look at an equation of graph and decide which category it belongs in, the context needs to be considered: what the mathematical expression is being used for, how the mathematics is being developed, and the intent. Thus, categorization may not be fixed and may shift as a unit moves from curriculum development to teacher enactment to student enactment. Moreover, this framework is not meant to imply that one category is a priori better than another. There are affordances and constraints to each usage (shown in Table 1). Mathematics as a tool is fast for example, but without the epistemic connections to science, students may not be able to apply to more complex problems independently.

In the following sections of the literature review, I will define each category in more detail, presenting the literature on quantitative problem solving that led to the development of that category and empirical findings about the effect of including mathematics in science in that way on student learning. I will then describe the demands that including mathematics in science in the ways described places on teachers and systems for teacher education. Finally, I will summarize some of the existing gaps in the literature on including mathematics in science.

2.2 THE CATEGORIES OF THE MISE FRAMEWORK

2.2.1 Mathematics as tool

The phrase "math as tool" of science appears frequently in the literature on integration of mathematics and science (Berlin & Lee, 2005). More recently this phrase has become reified in the Next Generation Science Standards, when mathematics is described as "a tool that is key to understanding science" (p.10, Appendix F, NGSS Lead States, 2013). A view of math as a tool to solve problems in a science context or to interpret data is commonly held among many college professors of teacher education programs (M. M. Lee et al., 2013; Watanabe & Huntley, 1998) and means that pre-service teachers are exposed to integrated settings where mathematics tends to be used mainly for calculations (Watanabe & Huntley, 1998). Thus, it is perhaps not surprising that many teachers interpret integration to mean mathematics used as a tool in science investigations (Chauvot & Lee, 2015). Descriptions of integrated curriculum by preservice or inservice teachers (even when codesigned as a collaboration between both math and science teachers) tends to be centered around using mathematics as a tool within science (e.g. to measure, to estimate distances, to convert units, to display data, to make predictions) (Frykholm & Glasson, 2005; Stinson et al., 2009).

Borrowing from the criteria for mathematics used as a tool in the MISCOP and from Miller, Davison, & Metheny (1993), The category, "Mathematics as Tool" in the MISE Framework contains mathematical operations used in science to carry out a function associated with science, but not to understand the shared underlying concepts, or processes/mechanisms. Functions associated with science could include collecting data (e.g. measuring daily growth of plant), analyzing data (e.g. calculating percent error), making a prediction (e.g. predicting whether ball rolled down ramp will reach target), solving for an unknown variable (e.g. given Force exerted on object of known mass, determining acceleration). When students are solving problems with a Mathematics as Tool approach, they are working algorithmically, manipulating symbols with little to no awareness of their connection to science processes or their meaning within the problem context. When a problem solver is working algorithmically, they know how to do things, but they don't know why. In the example in Table 1, students can use the equation F=m*a to predict a if they are given F and m, but they don't need to know or understand how and why the underlying entities of force and mass and acceleration are related in the physical world, or even what each of the variables mean beyond an equivalent word (e.g. F is the symbol for the word "Force").

While an algorithmic approach may allow students to arrive at answers quickly, research into student problem solving in a number of scientific disciplines has revealed shortcomings of this approach to problem solving (Chi, Feltovich, & Glaser, 1981; Gabel, Sherwood, & Enochs, 1984; Mason, Shell, & Crawley, 1997; Nakhleh, 1993; Salta & Tzougraki, 2011; Stewart, 1982, 1983; Taasoobshirazi & Glynn, 2009; Tuminaro & Redish, 2007). Namely, this approach is fragile to specific uses and will be misapplied in atypical/unfamiliar situations that require a change in procedure.

In biology, students are often taught to calculate the probability of inheriting particular gene combinations using a Punnett square, a tabular approach to listing combinations of genes in parents and predicting the possible combinations that will be present in the offspring. Many students can apply this approach with ease to simple cases, but struggle as the number of genes increases, even though the approach is logically the same (Stewart, 1983). They continue to carry out the setup exactly as they did in the simpler case (placing one letter in each of the input squares), even though based on the biology, when problem solving for cases with more than one gene, they should be placing combinations of letters in the input squares. This tendency to apply steps in exactly the same way regardless of whether the steps or order of steps makes sense is characteristic of an algorithmic/procedural approach that is not supported by understanding of the underlying concepts. Students who make these mistakes with the Punnett square often fail to connect the elements of the Punnett square to the objects of the biological process of inheritance and the generation of combinations to the mechanism of fertilization that produces those combinations. It has been suggested that this failure to make biological sense of the mathematical tool contributes to students' failure to transfer the process from simple to more complex problems (Stewart, 1983).

Prior work on problem solving in physics suggests that novices tend to approach problem solving algorithmically, using an approach where the equation (the algorithm) has primary importance (Chi et al., 1981; Mestre, Docktor, Strand, & Ross, 2011). On the other hand, for experts, the concepts tend to be the starting point for problem solving (Chi et al., 1981; Mestre et al., 2011). In many situations, the algorithmic approach tends to be successful in the class context because students can use contextual clues, such as the book section or what was recently covered in class, to select the appropriate equation as a starting point (Mestre et al., 2011). However, similar to what has been found in biology, the algorithmic approach begins to break down when problems become more complex or when students are presented with problems they have not seen before (Bing & Redish, 2008; Walsh, Howard, & Bowe, 2007).

The inability of students to make connections between a mathematical tool and the scientific phenomenon it encapsulates is a common problem that occurs across the sciences. For example, when student problem solving in chemistry was examined, more than half of all students failed to use a conceptual understanding of the problem in combination with an equation based approach and none of these students could successfully solve a conceptually related quantitative problem that had not been covered in class (Gabel et al., 1984). The researchers argued that student reliance on algorithms did not just prevent them from solving the problem but had become a substitute for understanding the concepts (Gabel et al., 1984). Students' reliance on algorithms as a substitute for understanding is supported by the finding that while in both 9th and 11th grade, students are more likely to correctly answer a chemistry problem that can be answered using an algorithmic as opposed to a conceptual approach, 11th graders are more likely than 9th graders to get the algorithmic problem correct and less likely to get the conceptual problems correct (Salta & Tzougraki, 2011).

This tendency to rely on an algorithmic approach to problem solving as a substitute for understanding of concepts can have greater impacts as students progress in their study of science. As students move into more complex problems in undergraduate studies, students using algorithmic problem solving strategies are less likely to answer problems correctly than students who use a conceptual approach (Taasoobshirazi & Glynn, 2009). Use of the algorithmic strategy is characterized by this student's explanation "I started out trying to solve for specific heat of the metal. Then I realized I had everything to solve the equation except for the heat lost by the metal, which is the same as the heat gained by the water. I needed this before I could solve for the specific heat of the metal. So I formed an equation, actually a few, to set things up so I could solve for the heat gained by the water"(Taasoobshirazi & Glynn, 2009). In contrast, students who used a conceptual approach decided initially on the value that they needed to solve for, constructing the needed equations and then worked systematically towards that goal. "I knew that ultimately I'd need to solve for specific heat of the metal. But in order to get the specific heat of the metal, I'd need the heat gained by the water. So I started to solve for the heat gained by the water first, and then getting that allowed me to solve for the specific heat of the metal" (Taasoobshirazi & Glynn, 2009). Both types of students end up solving for the correct variable, but the backwards-working approach is more unfocused, involves more equations, and is more prone to error (Taasoobshirazi & Glynn, 2009). These approaches were correlated with conceptual understanding the conceptual approach associated with greater conceptual understanding of chemistry problems than the algorithmic approach (Taasoobshirazi & Glynn, 2009).

2.2.2 Mathematics as inscription

Another way that mathematics is often included in science class is as an inscription: a conventional (scientifically accepted) visual way of communicating scientific ideas in mathematical form. Inscriptions could be any externalized written visual presentation of a scientific phenomenon that is used to communicate with others (Roth & McGinn, 1998). Teachers recognize this communicative aspect of mathematics in science declaring "Math is language in which scientists communicate with one another" (p. 63, M. M. Lee et al., 2013). The second most common perception of mathematics given by preservice science teachers (second only to mathematics as tool for science) is that mathematics is the language of science (Watanabe & Huntley, 1998). The idea of using mathematics to communicate precisely in science has become embodied in the language of NGSS which provides as examples of the use of mathematics, "use mathematical representations to describe and/or support scientific conclusions and design solutions" and "use mathematical, computational, and/or algorithmic representations of phenomena or design solutions to describe and/or support claims and/or explanations”(p. 10, Appendix F, NGSS Lead States, 2013). The use of mathematics in science in both of these examples is focused around communicating with others.

Thus, the second category of the MISE Framework for inclusion of mathematics in science is Mathematics as Inscription. Within the framework, the primary purpose of including mathematics as inscription in science instruction is for communication and for students to learn the conventions associated with that communication. This contrasts with the Mathematics as Tool category where the primary purpose is to calculate a quantity. An example of Mathematics as Inscription is contained in a qualitative study of a physics professor’s lecture that showed college students how to translate the motion of a ball down a ramp to graphs of the motion (Roth et al., 1997). No explicit justification was given for moving from the phenomenon to a data table to a graphical depiction, beyond the lecturer saying that “pictures are a lot nicer” (meaning graphs), nor was the graph presented as adding additional meaning. According to the researchers, no new conceptual information was added in this translation, but rather the lecture’s work was to translate a phenomenon into different mathematical inscriptions (data tables and graphs) (Roth et al., 1997). A more explicit representation of this form of inclusion of mathematics in science comes from an online tutorial program known as the Khan Academy on algebraically balancing chemical equations. The tutor, Sal Khan, explains why an equation cannot be balanced by adding 1.5 in front of an oxygen molecule by saying that “the convention is we don’t like having 1.5 oxygen molecules” (Khan Academy).

In the case of balancing the chemical equation, the symbols are treated as names for entities in the phenomenon (e.g. the molecule Aluminum). However, beyond that, the connection to the phenomenon is unclear. In terms of the graph development case, the professor talks aloud his calculation of “x” and “v”, but they are not explicitly labeled and the relationship to entities in the phenomenon such as displacement or position is unclear (Roth et al., 1997).

Other examples of the use of Mathematics as Inscription at lower grade levels would be teachers telling elementary school students to include units because that’s what we do in science, and a middle school student drawing a graph or citing an equation as a warrant for an argument but not asserting a connection to objects within a scientific phenomenon.

To contrast Mathematics as Inscription with Mathematics as Tool, examine the two equations for expressing the relationships between force, mass and acceleration shown in Table 1. For Mathematics as Inscription, the formula is written in the table as as $\Sigma F = m*a$ (with vector arrows above the “a” and “F”). While this is not most commonly the form shown to students, even to communicate Newton’s second law, I chose this form of the equation to illustrate how much meaning an inscription can have. First, the summation symbol means that Forces are added and that it is the net force that results in the acceleration. Second, the arrows above the “F” and “a” indicate that the acceleration will be in the same direction as the net force. Compare this to the Mathematics as Tool equation, $F=m*a$. There is no indication that multiple forces might be involved, nor that these might need to be added together to get a net force. Moreover, there is no indication of direction, either of the force or the acceleration.

In the context of science practice, inscriptions used for communication are socially constructed entities with complex, layered meanings (Redish & Kuo, 2015; Roth & McGinn, 1998). However, in the context of the traditional classroom, or in high school and college textbooks, these meanings are often glossed over or ignored completely (Aydin, Sinha, Izci, & Volkmann, 2014; Bowen & Roth, 2002; Roth et al., 1997). Thus, it is not possible to rely simply on the form of a mathematical expression to determine whether mathematics inclusion is

Mathematics as Inscription or Mathematics as Tool. The same equation, F=m*a, could be presented as a tool to solve for acceleration or it could be presented, and often is, as an alternative way to communicate Newton’s second law (in a form that is simplified for instructional purposes).

This stripping away of meaning presents difficulties for student understanding because often these layers of meaning are readily apparent to expert practitioners, but are not available to newcomers to the field (Redish & Kuo, 2015; Roth & McGinn, 1998; Roth et al., 1997). In the example of F=m*a, experts could infer the existence of the summation symbol and the vectors above F and a, but students would have no way of knowing they should be there without additional information. I am going to use two different examples to illustrate the ways in which meaning can be stripped away from two different mathematical inscriptions (graphs and equations) and the consequences this can have for student learning.

Graphs are a common inscription in science textbooks (Bowen & Roth, 2002). Examination of how one graph was changed as it was translated from a scientific publication to a high school ecology textbook revealed that multiple changes were made that made its meaning less clear, even to graduate students and professors of science (Bowen & Roth, 2002). For example, to combine three graphs into one, the scalar quantity and label for the vertical axis were changed from proportion of population and percent cover to “relative importance”. Not even the science experts could attribute a meaning to this relative importance variable. Data points were removed from the graphs and curves were smoothed. Both of these transformations removed any indication of data variability, a central part of both ecological processes and more general scientific inference about meaningful differences. The legend was approximately the same length for both the journal and textbook graph, but the journal graph referred readers back to the main text for additional information while the textbook graph did not. These changes were not unique to this graph, but were provided as a representative example of the differences between textbook and journal graphs.

Not represented by this example, but perhaps more worrisome was the difference in the length of captions and text supporting the graphs: graphs in high school or college textbooks had much less supporting text (either in captions or the main text) than graphs in journal publications (Bowen & Roth, 2002). Students have been shown to have difficulty reading and interpreting graphs (Leinhardt, Zaslavsky, & Stein, 1990; Shah & Hoeffner, 2002). Bowen and Roth (2002) propose that this difficulty for students stems from insufficient experience socially constructing the meaning of graphical inscriptions. Bowen and Roth base this claim, in part, on an examination of the abilities of 8th grade students to becoming increasingly sophisticated in their use of graphical inscriptions as they participated in an ecological unit where they acted as scientists, gathering and analyzing data, and using their findings to support their claims (Roth & Bowen, 1994). Others have also found that when graph construction and interpretation are used as part of social construction of meaning in scientific inquiry units, then students' steadily improve in their ability to both interpret and construct graphs (Wu & Krajcik, 2006).

Mathematical equations are also an inscription of science that require enculturation and they can be more abstract and difficult to interpret than graphs (Roth & McGinn, 1998). Part of the complexity stems from mathematical equations having both complex meaning and syntax (grammar) (Redish & Kuo, 2015; Sherin, 2001). But there are also additional complexities of mathematical equations as inscriptions in science. In mathematics, the work of an equation is to express abstract relationships; in science, the equations represent meaning about physical systems (Redish & Kuo, 2015). This difference reiterates the epistemological distinction made about mathematics and science disciplines by Lederman and Neiss (1998).

Students often ignore the meanings embedded in scientific equations (Hammer, 1994). They tend to talk about the meanings of the symbols in terms of their names, but do not connect it to the physical situation (Redish & Gupta, 2009). In part, this disconnect may occur because of the way mathematical inscriptions are presented to students, ready-made, often with some of the details left out to make them simpler to use (Redish, 2005; Redish & Kuo, 2015; Tang, Tan, & Yeo, 2011). Failure to see the meanings embedded in scientific equations can cause students to become stuck in their problem solving efforts (Tuminaro & Redish, 2007), accept incorrect answers even though they do not make sense (Hammer, 1994), fail to transfer from one situation to a related situation, or provide an incorrect answer because they are not filtering their problem solving process through their knowledge of physics (Redish & Kuo, 2015). Importantly for instruction, failing to see the physical meaning of scientific equations is not a permanent or inevitable outcome for students, and some students will come to recognize the connections between the symbols in the equation and the physical world (often through interactions with others) (Gupta & Elby, 2011; Tang et al., 2011). When these connections are made, students often make breakthroughs in their understanding of how to solve the problem they are working on (Bing & Redish, 2008; Gupta & Elby, 2011; Tang et al., 2011).

It is not just the symbols in an equation that carry meaning; the way the symbols are arranged and the operations that relate them also carry meaning (Sherin, 2001). For example, $d_t = d_1 + d_2$ has a "whole equals the sum of its parts" relationship, meaning that the total distance an object has traveled is comprised of the distance traveled for time 1 and time 2. Whereas d_1/d_t has a "part of whole" relationship meaning that d_1 is some proportion of the total distance traveled. It is worth noting that the meaning of the equation is not solely dependent on the syntax. For example, $x_f = x_0 + \Delta x$ is commonly interpreted as having a "base + change" relationship as is the expression $x_f = x_0 + vt$, even though these are both addition equations like $d_t = d_1 + d_2$. That is, both of the x_f equations say that the final position, $x_{f,}$, is equal to the initial position (x_0) plus the change in position. However, to recognize their equivalence, it is necessary to understand that change in position (Δx) is determined by the velocity of the object and the amount of time it has been traveling and therefore is equal to "vt".

Inscriptions that present the syntactical relationship between two entities in ways that do not recognize the underlying relationships between the two entities in the phenomenon may contribute to student confusion. For example, the mathematical expression in Table1 which is commonly written, F=ma, may cause students to think that force is the product of mass times acceleration and thus they speak in terms of "the force due to acceleration" (Freedman, 1996; Redish & Gupta, 2009) when acceleration is actually the result of a force acting on a mass (i.e., there is a confusion of cause vs. effect). However, when syntactic arrangement of a mathematical expression is consistent with the physical phenomenon, recognition of the particular type of syntactic expression (e.g. "parts of whole", "base + change") can have powerful affordances for problem solving (Kuo, Hull, Gupta, & Elby, 2012).

2.2.3 Grounded mathematics

In response to evidence from numerous studies showing that students are better able to solve quantitative problems in science when they make connections between mathematics and the scientific phenomenon it represents, some members of the education community have designed curricular interventions which act to "ground" the mathematics in the scientific phenomenon (e.g. Lehrer & Schauble, 2004; Levy & Wilensky, 2009b; Mestre et al., 2011; Roth & Bowen, 1994; Wells, Hestenes, & Swackhamer, 1995; Wu & Krajcik, 2006). Some curricula aim to help students recognize links to the scientific phenomenon as they engage in the problem solving process (Mestre et al., 2011). However, the primary purpose of mathematics within these curricula is still mathematics as tool for producing an answer. In contrast, the other curricula mentioned are designed to have students derive the mathematical inscription as a way to describe the scientific phenomenon. These curricula focus on connecting different inscriptions (graphical, equation, tabular, pictorial, written) to each other and to the physical phenomenon that is being studied. Moreover, the mathematical inscriptions are constructed by students (often in a social context), even if other inscriptions are provided to students. Thus, these curricula place a greater emphasis on alignment with the scientific phenomenon than the other categories (Table 1). The purpose of the inclusion of mathematics in these curricula is to develop a description of the phenomenon under study. This purpose contrasts to the purposes for Mathematics as Tool and Mathematics as Inscription that are calculating a quantity, and communication, respectively.

To recognize both the description development approach and the purposeful construction of conceptual links to the phenomenon, I place these curricula in a third category: Grounded Mathematics. An example of this category is shown in Table 1. Students use a spring scale to measure the amount of force as they change the mass of the objects suspended from it. They graph the force (F) and mass data and derive an equation from the best fit line, F= slope*mass where the slope is equal to the value for g, the acceleration due to the gravitational field. To illustrate some of the key features of grounded mathematics, I will discuss three examples of Grounded Mathematics that have been developed in three different fields of science for different grade levels.

Lehrer and Schauble studied elementary school students as they developed mathematical inscriptions to describe variations in plant height. Over time, students' mathematical inscriptions and their conversations about those inscriptions became increasingly more sophisticated until students were distinguishing between the median spread of the data set and outliers (Lehrer & Schauble, 2004). Their understanding of plant height variation in nature mirrored their increased understanding of the mathematical inscriptions, moving from a completely random phenomenon to one that could be predicted within a given range (Lehrer & Schauble, 2011). From the rich descriptions provided of student conversations, it is clear that the generation of mathematical inscriptions and students' understanding of what they represent is grounded both in their experiences of measuring plant growth over time and in their discussions with peers that occur while they are constructing and presenting their inscriptions. Thus, they show that even young children can engage in constructing mathematical inscriptions that are grounded in a phenomenon. Moreover, interviews with students after completing the unit reveal that they can apply their acquired understanding to other scientific phenomenon.

In a Connected Chemistry unit on gas laws using virtual tools, a central focus of the curriculum was on getting high school age students to connect physical phenomena with inscriptions of that phenomena (macro and micro) to allow students to develop a mathematical inscription that encapsulates the phenomenon (Levy & Wilensky, 2009a, 2009b). During the unit, students were both asked to choose a canonical form of a gas law equation to represent data plotted on a graph and to type out a functional relationship that represented that data. Students successfully chose a canonical form, but only fifty to sixty-eight percent (depending on the data being represented) could write out a functional relationship that accurately described data on a graph (Levy & Wilensky, 2009b). Pre and posttesting of the 933 high school students who were involved in the study showed that students increased their conceptual understanding, but did not show significant gains on solving quantitative problems (Levy & Wilensky, 2009b). Researchers posit that the failure to see increased problem-solving scores is because students did not have a chance to apply the gas laws that were developed. Alternatively, it is implied that interaction with the computer simulation was mainly individual or small group. The lack of whole class discussions with opportunities to communicate meaning to others through the mathematical inscription, and with opportunities for instructor-mediated facilitation may have left connections between the mathematical inscription and the physical phenomenon underdeveloped (N. Becker, Stanford, Towns, & Cole, 2015).

Both of the curricula described above offer proof of concept: students can develop mathematical inscriptions grounded in an understanding of the physical phenomenon. Moreover, both show an increase in conceptual understanding of the phenomenon at the end of these units. However, both describe stand-alone units, which may be inherently limited in achieving changes in students. Therefore, I also present a full curriculum example of this grounded mathematics approach, which is for high school physics students and is designed to encompass an entire year. This approach, known as Modeling Instruction™, is based on a theory developed by David Hestenes (Hestenes, 2010). Hestenes defines a scientific model as “a representation of structure in a physical system or process” (p. 18, Hestenes, 2010). The structure of a system is defined as the relations between the objects in the system, and Hestenes recognizes five structure types: 1) systemic, which specifies composition of system, links among parts, or links to external objects; 2) geometric, which specifies configuration and location of objects; 3) object structure, which specifies intrinsic properties or parts or roles of objects; 4) interaction structure, which specifies properties of links, usually causal interactions (e.g. forces, transport, information exchange); and 5) temporal structure, which specifies change over time.

In Modeling Instruction, students engage in a modeling cycle which consists of gathering and analyzing data about a system, constructing a model of the system, extracting information from the system in the form of a prediction or explanation, validating the model, and then deploying the model in a different context (Halloun, 2007; Hestenes, 2010). Constructing a model of the system consists of developing representations (what I have been referring to as inscriptions) of the system (e.g., diagrams (system schemas), graphs, written descriptions, equations).

In the phases of the modeling cycle, students share their representational tools with the class, and with the facilitation of the teacher, class discussion is centered on developing a complete coordinated, and consistent scientific model using these tools. Thus, in this curriculum, unlike the other two described, which also aim to have students develop a model of a system (Lehrer & Schauble, 2004; Levy & Wilensky, 2009a), mathematics has no particular primacy. In fact, as Hestenes describes it, it is the system schema, the picture, from which all else should flow (Hestenes, 2010). Mathematics becomes, then not the language of science, but part of the language of science.

Multiple classes, in multiple locations, in multiple grade levels have been shown to increase their conceptual understanding of physics after a year of instruction in this curriculum and to perform better than students taught using traditional methods of instruction (Dye, Cheatham, Rowell, Barlow, & Carlton, 2013; Malone, 2008; Wells et al., 1995). Students who completed a first year physics class using the Modeling Instruction™ approach had higher posttest scores than students at the same school who completed traditionally instructed first year physics class (Malone, 2008). Moreover, interviews with modeling instructed and traditionally instructed students as they solved physics problems showed that they made fewer mistakes and were more likely to catch their errors (Malone, 2008). This study on problem solving involved only a few students in each set of classes. As with the other two Grounded Mathematics curriculum, more research needs to be done both to generalize findings and to explore the affordances and constraints of this approach for conceptual understanding and problem solving in science.

2.2.4 Mechanism connected mathematics

All of the grounded mathematics approaches have a common goal: to describe a scientific phenomenon. That description is centered around the objects of the phenomenon and the relationships between those objects. Causal mechanisms that determine those relationships might be included but they do not need to be included and are not a primary emphasis for evaluating the mathematical inscription against the phenomenon. For example, consider again the F=m*a inscription shown in the Grounded Mathematics column in Table 1. Hestenes, borrowing from Sherin (2001), explains that this expression expresses that force is proportional to acceleration (Hestenes, 2010). However, what is not present in this form of the equation is the reason that acceleration and force are proportional with mass as the proportionality constant. If the equation is rewritten as a=F/m, two concepts become clearer: 1) acceleration of an object is due to the force acting on the object (Redish & Gupta, 2009), and 2) that the reason acceleration is proportional to the force and not equal to it, is that the force is distributed over (divided by) the mass.

In science, the reason why particular outcomes occur is known as the scientific mechanism. Philosophers of science disagree over the details of the definition of a scientific mechanism ((Bechtel & Abrahamsen, 2005; Glennan, 2005; Illari & Williamson, 2012; Machamer et al., 2000). However, the definitions all seem to have in common the idea that a scientific mechanism describes how entities within a phenomenon interact to produce outcomes associated with this phenomenon. Examples of outcomes associated with a phenomenon include distinct termination events such as production of offspring from the joining of eggs and sperm, and acceleration resulting from the action of force on an object, as well as those resulting from ongoing phenomenon such as production of carbon dioxide from the breakdown of glucose in cellular respiration.

Inclusion of mathematics in science with an emphasis on developing not just the relationships between objects but consistency between the syntax of the mathematical inscription and the mechanisms of the phenomenon is largely missing from the science education literature. However, I propose that this fourth category is needed, both to emphasize an approach that emphasizes the "why" of scientific phenomenon as much as a description of science, and as a research mechanism to see whether and how alignment of equation syntax with scientific mechanism affects student understanding of science. Theoretically, aligning equation syntax with scientific mechanism should enhance student problem solving (Redish & Kuo, 2015; Sherin, 2001). When students spontaneously do recognize the syntax underlying a physics equation, they are able to solve problems that stump students who are relying on a more algorithmic approach (Kuo et al., 2012). It is possible, but not yet empirically tested, that developing a connection between scientific mechanisms and mathematics processes, will also develop a deeper understanding of the underlying scientific processes as well. One such example of how such an alignment might help conceptual understanding is presented above, where rearranging the equation F=m*a to the more mechanistically aligned a=F/m, may clear up causal confusion (Redish & Gupta, 2009).

However, this example does not show how making students aware of the alignment between scientific mechanisms and algebraic expressions of those mechanisms might facilitate problem solving. After all, there does not seem to be much difference between dividing and multiplying two variables from a problem-solving standpoint. An example from chemistry might make it clearer how helping students recognize the mechanistic alignment between scientific phenomena and mathematical expression may help students solve problems. Consider the balanced equation for producing water (H_2O) from hydrogen (H_2) and oxygen (O_2), $2H_2 + O_2 \rightarrow 2H_2O$. From this equation, a student can calculate how many grams of water will be produced if given a certain mass of oxygen or hydrogen. Usually, students are taught to do this using an algorithmic approach (i.e. take the atomic weight of oxygen, multiply it by the subscript, divide the product into the mass provided to get the number of moles of oxygen and divide that number by the number in front of the product to get the number of moles of water produced. Then, multiply the number of moles of water produced by the molecular weight of water (obtained by adding up the atomic weights)). Needless to say, this is a long process with multiple steps and it is easy for students to get lost in the problem solving process and many (sixty-six percent) have difficulty answering such mass/mass conversion problems correctly (Lythcott, 1990). A chemical equation describes a scientific mechanism: separate hydrogen and oxygen molecules are combined (the combination process is indicated by the plus sign) to produce (as indicated by the arrow sign) a new molecule containing both oxygen and hydrogen. If curricula were developed that focused on helping students make connections between the variables and process symbols in the chemical equation and the entities and scientific mechanisms they are intended to represent, it may be possible to get students to engage with the sense-making aspect of equations. As a result, they might, for example, draw out the changes that are occurring and use that as a way to structure their problem solving process, thereby potentially achieving greater success with mass/mass conversion problems. Moreover, student engagement with this aspect of chemical equations could increase conceptual understanding of chemical equations, which has been found to be woefully lacking (Lythcott, 1990).

With both of the examples provided above, the effect of connecting scientific mechanisms to mathematical processes remains in the realm of speculation. In part, this may be because until the development of the MISE Framework presented here, this category of connection between mathematics and science has not been clearly specified. To provide a concrete example of how connections between mathematical processes and scientific mechanisms might be developed in a curriculum and the effect of this curriculum on student quantitative problem solving and conceptual understanding, I will provide a brief description of a unit in inheritance that was developed to emphasize the connection of mathematical processes with scientific mechanisms (Schuchardt & Schunn, 2016). As with Grounded Mathematics, students began the unit by exploring the scientific phenomenon of inheritance. They were presented with data about the genes carried by two different sets of parents and their offspring. Through a series of hands-on tasks involving the objects of inheritance (eggs, sperm, genes), students were asked to develop an account of the mechanisms that led to the patterns they observed, and they were provided with ways to represent these objects pictorially. Following a task designed to encourage students to recognize the limited predictive power of the pictorial representations, students were presented with a set of data that they were instructed to use to develop a mathematical inscription As they presented their mathematical inscriptions to the class, the teacher was instructed to hold students accountable for consistency with both the data and the objects and mechanisms of inheritance. After testing the proposed mathematical inscriptions against additional data, only one inscription could meet both the connection to objects and mechanism test and be generalizable to more complex cases: # of types of offspring = (# of egg types) * (# of sperm types).

In this unit, not only do the variables in this equation map on to the objects of inheritance (e.g., # of types of eggs maps onto the number of eggs containing different combinations of the genes of interest), but the mathematical process of multiplication does too. In these example equations, multiplication maps on to the mechanism of fertilization where any egg type can join with any sperm type and vice versa.

Within the unit, this conceptual mapping was also supported by the use of pictorial representations. While such connections between inscriptions of different types was important in this unit too, unlike with Grounded Mathematics, the primary goal of the pictorial representations here is to make explicit the connection of the mathematical inscription with the scientific mechanism, rather than the connection between the entities within the phenomenon. As a result, students who experienced this unit gained a stronger understanding of the processes of inheritance and were also better able to solve complex quantitative problems in inheritance than students who were instructed traditionally (Schuchardt & Schunn, 2016).

The examples of Mechanism Connected Mathematics provided above, one from physics, one from chemistry, and one from biology show how mathematical inscriptions can be used to emphasize the connection between mathematical processes and scientific processes. The description of the biological example also described one way that this connection can be developed, and showed that developing these connections can have positive effects on student problem solving and mechanistic understanding. However, across the sciences, the effect of this approach to inclusion of mathematics in science has received little empirical examination. This gap in the research is probably partly because until now there has not been a structure for distinguishing the different ways that mathematics has been, or could be, included in science. Thus, mathematical expressions that describe the entities of a phenomenon and their algebraic relationships (i.e. force is proportional to acceleration) and mathematical expressions that specifically connect the mathematical processes to the scientific mechanisms (i.e. division can represent distribution of force over a mass) have been treated as equivalent.

2.2.5 The MISE Framework in science education

I started this review by describing frameworks for categorizing integration of mathematics and science. These frameworks focused mainly on how much integration was occurring and what topic of the disciplines were being integrated (e.g. processes, content, instructional methods). From the standpoint of curriculum design, these frameworks can be useful. However, there is perhaps a shared implied assumption of these frameworks that complete integration of mathematics and science is desirable. Because of the different epistemological standpoints of mathematics and science, Niess and Lederman (1998) dispute that this is a desirable goal. Therefore, there is a need for these frameworks to be elaborated with descriptions of what is appropriate, or especially useful, for such integration.

The framework that I have described here contains four categories: Mathematics as Tool, Mathematics as Inscription, Grounded Mathematics, and Mechanism Connected Mathematics. These categories are defined by the purpose of including mathematics in a science context, and the connections of the variables and processes of the mathematics inscription to the entities and mechanisms of the scientific processes. There is no intended hierarchy of inclusion intended by this presentation. At times, it may be most appropriate to use mathematics as a tool, such as when a speedy calculation is needed (e.g. during data transformation). This use is likely particularly effective as students move through their scientific career and may have already built an understanding of the purposes or conventions of particular mathematical inscriptions. Conversely, the level of mathematics required for students to connect mathematical processes to scientific mechanisms may not be available to students at particular grade levels, and thus, this may not always be an appropriate goal. For example, some scientific processes of ecology discussed in middle school and early high school are grounded in advanced calculus.

As is clear from the descriptions of the categories, the boundaries between the four categories are not dependent on the form of the mathematical inscription, but depend on context. One advantage of the framework that is presented here is that the inclusion of purpose and connection to meaning implies that it should be possible to use the framework to describe how inclusion of mathematics shifts from curriculum design to enactment by teacher to use by student.

2.3 TEACHER EDUCATION AS A LIMITING FACTOR IN INCLUDING MATHEMATICS IN SCIENCE

Any type of inclusion of mathematics beyond using mathematics as a tool to calculate percentages or simple graphing is going to require training of both pre-service and in-service teachers (Furner & Kumar, 2007; Offer & Mireles; Sorgo, 2010). Prior to professional development, teachers tend to agree that integration of science and mathematics is important and a natural fit (Berlin & White, 2010; M. M. Lee et al., 2013; Offer & Mireles). However, most teachers initially have unsophisticated views of integration that are not well specified (Berlin & White, 2010; Koirala & Bowman, 2003; Stinson et al., 2009). They refer mostly to the notion of mathematics as the language of science and as a tool of science, offering examples such as “Mathematics and Science overlap in so many ways. To do science, you have to know Mathematics (p. 148, Offer & Mireles) and “Integrating math and science instruction means how the two are interwoven together in all facets and aspects of world around us. Math is language in which scientists communicate with each other” (p. 163, Lee et al., 2013). Most science teachers at all levels feel uncomfortable with their level of preparation in mathematics (Frykholm & Glasson, 2005; Furner & Kumar, 2007; Watanabe & Huntley, 1998). Studies on preservice teachers have revealed that this concern is warranted. Preservice teachers use unsophisticated representations and make errors when using mathematics to carry out experiments and analyze data (Lewis, Alacai, O'Brien, & Jiang; Lunsford, Melear, Roth, Perkins, & Hickock, 2007). These errors range from simple calculation mistakes to conceptual errors (Lewis et al., 2002).

From both a policy and a research standpoint, it is important to improve science teacher’s preparation in mathematics. From a research standpoint, in order to implement all the categories of inserting mathematics in science mentioned in the MISE Framework, teachers need to be well prepared not only in science, but also in mathematics. From the studies mentioned above, it is clear that teachers feel, and are, underprepared to include mathematics in their science instruction. With this gap in preparation, it is hard to see how it is possible for academic researchers and curriculum developers to study the impact on student learning of different categories of the MISE Framework. The gap in expectations and preparation are particularly apparent for the categories of Grounded Mathematics and Mechanism Connected Mathematics which require teachers to not only understand scientific concepts, but to also understand mathematical variables and their relationships, and the mathematical processes and syntax well enough to make the connections to science for themselves and their students.

Overcoming these difficulties by ignoring the role of mathematics in science and trying to teach science without mathematics is not an option. Biology is often taught in high school as though mathematics is not, and has not, been necessary to the development of biological findings (Steen, 2005). College professors have complained that secondary education institutions are doing their students a disservice by not including mathematics in biology because students are unprepared for the realities of college biology (Orton & Roper, 2005). While this is not a problem for high school physics where mathematics and science are often so intertwined that doing mathematical problems has become equated with doing physics (Redfors, Hansson, Hansson, & Juter, 2014), it is no longer becoming a valid policy option to ignore the role of mathematics in other scientific disciplines or other grade levels. Over the years, many organizations have put forth policy documents emphasizing the importance of including mathematics in science (American Association for the Advancement of Science, 1993; National Science Teachers Association, 2002). With the advent of the Next Generation Science Standards that emphasize the importance of students engaging in scientific practices to develop their conceptual understanding of all science disciplines, mathematics in science takes on a key role in all disciplines and at all grade levels (NGSS Lead States, 2013; Osborne, 2014). While the Next Generation Science Standards includes using mathematics as one of the eight key practices in science, other practices such as analyzing data and modeling often require an understanding of mathematics concepts and practices (NGSS Lead States, 2013).

It is unlikely, however, that all teachers and students are going to be equally prepared to meet the mathematical demands of revised science education (National Research Council, 2015). Mathematics performance of students with lower SES s lags behind those with higher SES (National Center for Education Statistics, 2011). Moreover, science teachers in lower SES districts tend to have less experience in science teaching and often less education in science than those in higher SES districts (Banilower et al., 2013). Thus, teachers from lower SES districts are likely to be less knowledgeable about how to include mathematics in meaningful ways in their science classrooms. The districts that have the greatest need for professional development often have the fewest resources to invest in supplemental education for their current teachers (Archibald, Coggshall, Croft, & Goe, 2011; O. Lee, Miller, & Januszyk, 2014). One solution is to have teachers come out of their teacher education programs ready to insert mathematics in science in all of the categories specified in the MISE Framework. Another solution is to engineer in-service teacher training so that it is as efficient as possible requiring the fewest resources; the least investment in space, teacher time, and money (Archibald et al., 2011).

There are only a few studies on the effect of preservice teacher education reforms on including mathematics in science. The intervention generally involves having preservice teachers engage in science activities that include mathematics (Lewis et al., 2002) or design science activities that include mathematics for current or future students (Frykholm & Glasson, 2005), or combined the two interventions (Berlin & White, 2010; Koirala & Bowman, 2003). Most of these studies are qualitative and don’t always contain pre/post comparisons. Preservice teachers recognize the value of connecting mathematics and science (Berlin & White, 2010; Koirala & Bowman, 2003), and the perceived value of integration does not change by the end of a six course sequence on mathematics and science integration (Berlin & White, 2010). However, preservice teachers views on how to connect mathematics and science did change, becoming more nuanced, commenting on specific connections rather than vague generalities (Berlin & White, 2010), although often mired in mathematics as tool examples (e.g., to work with unit conversions, or to use different mapping scales) (Frykholm & Glasson, 2005). Moreover, at the end of coursework that included interventions designed to promote integration, preservice teachers perceived the difficulty and tensions in connecting the two disciplines (Berlin & White, 2010; Frykholm & Glasson, 2005), expressing awareness of areas of difficulty (e.g. the different meanings in mathematics and science for "variable") (Frykholm & Glasson, 2005), and of the gaps in their own knowledge (Berlin & White, 2010). Only one study looked at how preservice science teachers mathematical content knowledge changed as a result of participating in a science methods class that used a project-based approach to science, where they are expected to gather and analyze data and present their results (Lewis et al., 2002). The number of inscriptions used increased over time and become more mathematized, shifting from verbal descriptions to data tables and graphs, and in a few cases mathematical equations (Lewis et al., 2002). This brief survey of research studies on preservice teacher education interventions on inserting mathematics in science. A field that seems to be in its infancy, there is obviously need for more studies with greater number of participants and a greater emphasis on pre/post analyses, whether qualitative or quantitative. In many cases, it is not clear what type of mathematics inclusion in science preservice teachers are being exposed to, nor is it clear what type they are including in their curriculum development. Use of the MISE Framework could help better capture both the design of the intervention and the shifts in epistemology that preservice teachers might be undergoing.

However, it is not possible to rely on changes to preservice teacher training alone to close the gap between expectations and existing science teachers' knowledge of mathematics. Policy documents expect teachers to implement the suggested practices in their classroom within the next few years (National Research Council, 2015), and researchers interested in implementing new curriculum or assessing the effect of policy changes are generally working with teachers who are already in the classroom. In general, metanalysis of professional development in science focused programs or math/science programs has shown that students of teachers who participated in these programs achieved statistically significant gains on science assessments (Scher & O'Reilly, 2009). However, the authors offer the following moderating comments about their own findings: 1) Many of the included studies did not always meet best practices for measuring student achievement (e.g., not verifying equivalence of pretest scores for comparison and intervention groups), 2) There was too little variability in mode of professional development delivery to assess these effects, and 3) There were too few studies which were math/science focused to compare the effects of this type of dual intervention to the science focused interventions. Neither of the two science/math focused studies were specifically about including mathematics in science instruction, but involved long-term, large scale initiatives to either improve science and math teachers' pedagogy (Breckenridge & Goldstein, 1998)or to expose teachers to research experiences (Dubner et al., 2001).

Empirical research studies on the effect of educating in-service teachers on mathematics and science integration during professional development are few in number, and most have been conducted within the last decade. There are three main programs that have been researched, two involve middle school teachers (M. M. Lee et al., 2013; Offer & Mireles) and one involves elementary school teachers (Baxter et al., 2014). All three approaches involved long-term (a year or more) interventions that had teachers experience activities where math and science were connected, educated teachers on different viewpoints of math/science integration, and promoted reflection on math/science integration both in their own and others' lessons. Based on self-report, elementary school teachers, after PD, felt that their knowledge of science, but not their knowledge of mathematics had increased (Baxter et al., 2014). Pre/post testing revealed that middle school teachers' understanding of mathematics concepts had increased as a result of the intervention (Chauvot & Lee, 2015; Offer & Mireles). Similar to the results with preservice teachers, inservice teachers valued integration of mathematics and science before the intervention (M. M. Lee et al., 2013), and as a result of the intervention became more likely to develop connected lessons (Baxter et al., 2014; Offer & Mireles) and more aware of impediments to integration, including their own knowledge (Offer & Mireles). Paralleling the studies with preservice teachers, it seems that studying the effect of educating teachers on the connections between mathematics and science is still in its infancy and thus, there is much research remaining to be done. Particularly notable is that none of these studies on professional development included high school teachers. It may be that high school teachers are becoming educated on ways to include mathematics in science through curricular interventions, such as the Grounded Mathematics ones mentioned earlier. The teachers in Modeling Instruction participate in a three to four week workshop on the curriculum for example (Hestenes, 2010). All of the PD interventions mentioned so far are resource intensive, in terms of teacher time, workshop leader time, space, and school district support. As such, the current designs are potentially limiting in terms of scalability and equity (Archibald et al., 2011; Spillane, Gomez, & Mesler, 2009). Moreover, none of these studies mentioned look at the effect of PD interventions on student learning. There is a need to examine how changing the ways in which teacher support is delivered and the amount of time invested in PD interacts with student learning. This information will allow administrators, teachers and researchers to make informed decisions about the tradeoffs between investments and outcomes (Archibald et al., 2011).

2.4 FUTURE DIRECTIONS

With the advent of NGSS and its emphasis on learning of concepts through scientific practice, the question isn't any more whether mathematics should go into the science curriculum, but how it should go in. Once students are expected to engage in scientific practices to acquire content knowledge, they will need to collect, analyze, and inscribe data, all of which requires engaging in mathematical practices. The questions become what form of inclusion of mathematics works best under what circumstances, when engaging in which practices and to master what content.

Much is known about the shortcomings of students using Mathematics as Tool and Mathematics as Inscription without connecting to science content. Not much research has been done about when it is appropriate to assume that students' problem solving ability will not suffer when mathematics is included in this way. Does it depend on the context of use, or the mathematical skills of the student or prior exposure to methods of inclusion that have already developed connections? However, even less research has been done on the overall effect of including Grounded Mathematics or Mechanism Connected Mathematics in science on student learning, let alone the contextual settings when this type of inclusion is appropriate.

There is much need for developing curriculum that includes mathematics in science in different ways and for researching the effect of curriculum on student learning and problem solving. Many of the studies that have been done on existing curriculum to date, need to be expanded to include both quantitative and qualitative methods so that both generalizability of outcomes can be assessed and the effect on student conceptual understanding and problem solving can be described. There is also a need to study how variations on teacher preparation programs will impact student learning when using innovative curriculum and whether this impact varies by topic area (i.e., science content, mathematics in science context, or mathematics not in a science context). Particularly important is which variations will allow for reduction of resource investment to allow scaling across districts with different resource levels, and thus true equity.

I present here three studies that further these research goals. The first study (Chapter 3) describes the effect on student learning of a new unit designed to have students develop mathematical equations which model the scientific phenomenon of inheritance, emphasizing the connections between mathematical processes and the mechanisms of inheritance. The second study (Chapter 4) is a qualitative analysis of how students exposed to inheritance mechanism connected mathematics solve quantitative problems in an inheritance context. The third study (Chapter 5) tests the effect on student learning of reducing investment of resources in the professional development of teachers implementing the inheritance unit containing mechanism connected mathematics.

3.0 MODELING SCIENTIFIC PROCESSES WITH MATHEMATICS EQUATIONS ENHANCES STUDENT QUALITATIVE CONCEPTUAL UNDERSTANDING AND QUANTITATIVE PROBLEM SOLVING

Amid calls for integrating science, technology, engineering and mathematics (iSTEM) in K-12 education, there is a pressing need to uncover productive methods of integration. Prior research has shown that increasing contextual linkages between science and mathematics is associated with student problem solving and conceptual understanding. However, few studies explicitly test the benefits of specific instructional mechanisms for fostering such linkages. We test the effect of students developing a modeled process mathematical equation of a scientific phenomenon. Links between mathematical variables and processes within the equation and fundamental entities and processes of the scientific phenomenon are embedded within the equation. These connections are made explicit as students participate in model development. Pre-post gains are tested in students from diverse high school classrooms studying inheritance. Students taught using this instructional approach are contrasted against students in matched classrooms implementing more traditional instruction (Study 1) or prior traditional instruction from the same teachers (Study 2). Students given modeled process instruction improved more in their ability to solve complex mathematical problems compared to traditionally instructed students. These modeled process students also show increased conceptual understanding of mathematically modeled processes. The observed effects are not due to differences in instructional time or teacher effects.

3.1 INTRODUCTION

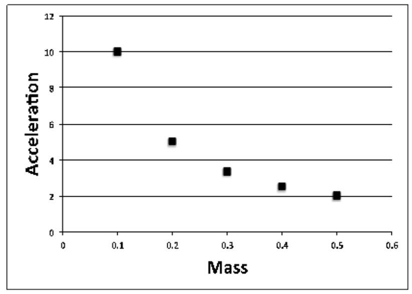
There have been many calls for integrating science, technology, engineering, and mathematics (STEM) instruction in K12 schools in order to enhance student learning (Honey, Pearson, & Schweingruber, 2014). Cited reasons include: 1) make mathematics and basic science appear more relevant to students to improve motivation during learning and thereby broaden participation in STEM (NGSS Lead States, 2013; PCAST, 2010); 2) produce STEM undergraduates who are better able to apply what they learn in mathematics to science, and in mathematics and science to engineering (Apedoe, Reynolds, Ellefson, & Schunn, 2008; Fortus, Dershimer, Krajcik, Marx, & Mamlok-Naaman, 2004; Litzinger, Lattuca, Hadgraft, & Newstetter, 2011); and 3) produce a general citizenry and workforce who are more technologically fluent through improved understanding of the scientific and engineering basis of modern technologies (PCAST, 2010; Peters-Burton, 2014). Unfortunately, given the generally siloed nature of instruction, particularly in high school, there are still questions about what constitutes productive models of STEM integration (Morrison, 2006; Peters-Burton, 2014). In this paper, we will present one instance of an integrated STEM (iSTEM) unit taught within a high school science class and examine its effect on quantitative problem solving and qualitative conceptual understanding.

The iSTEM unit integrates all four areas of STEM. An engineering design task motivates and deepens the learning, while technological advances in molecular biology allow students to visualize the normally invisible and indirectly measured objects of inheritance. The primary focus of the unit and our analysis in this paper, however, is the integration of mathematics with science. We assess whether a particular form of integration of mathematics with science, done via modeling of a process, enhances students' ability to solve problems in and improve their understanding of inheritance.

3.1.1 Forms of embodiment of mathematics in science education

Mathematics has long been a part of science education, particularly in chemistry and physics. There are different ways in which mathematics can be integrated into science education (Table 2). We review forms that are more typically present and then turn to alternative approaches that may be more productive for learning.

Table 2. Embodiment of mathematics in science education

	Calculated Procedure	Display of Data	Modeled Process
Example Representation	$\Delta x=1/2at^2+v_0t$ P1*P2 Pn = probability of getting genotype of gene n		$2H_2 + O_2 \rightarrow 2H_2O$ $\frac{W1 * W2}{(\text{\# egg types})(\text{\# sperm types})}$ Wn = ways of getting genotype combination of gene n
Example Operation	To produce a numerical answer	To display data and the relationships between variables	To express and test ideas about scientific processes
Connections to Science	Variables and mathematical processes do not have to be connected to entities and processes within phenomenon.	Variables are linked to entities within phenomenon.	Variables and mathematical processes correspond to entities and processes within phenomenon.

3.1.2 Mathematics as data presentation and calculated procedures

Two of the most common embodiments of mathematics in science education are as a summary of data and as a calculated procedure. As an example of data presentations, students might plot data on a graph from their experiment on mass and volume. As a common example of calculated procedures, students in physics are asked to memorize the equation for calculating the change in position of an accelerating object ($\Delta x=1/2at^2+v_0t$) and taught to plug in the values for acceleration (a), time (t), and initial velocity (v_0) to get the answer. In biology, calculated procedures are less common, but still exist. For example, students are taught how to use a Punnett square to calculate the probability of a set of parents generating an offspring with a specified gene combination (Appendix, Table A).

Both of these forms of mathematics are experienced by most students as relatively meaningless symbol manipulation (Stewart, 1983; Walsh et al., 2007). They are missing either data (calculated procedure) or operation (data representation) (Larkin & Simon, 1987). There has been increasing awareness of the shortcomings of the embodiment of mathematics as symbol manipulation. For example, when student problem solving strategies in chemistry were examined, more than half of the students failed to use reasoning about content together with their equation-based approach and none of these students could successfully solve a conceptual transfer problem (Gabel et al., 1984). The authors argued that student "reliance on algorithms is a substitute for understanding the concepts" (p. 232, Gabel et al., 1984). Other researchers have replicated this finding in chemistry and physics (Chi et al., 1981; Mason et al., 1997; Nakhleh & Mitchell, 1993; Salta & Tzougraki, 2011; Tuminaro & Redish, 2007; Walsh et al., 2007). Students tend to ignore connections to underlying concepts that could allow them to transfer their understanding to superficially different, but structurally similar problems (Chi et al., 1981). Simply exposing students to more problems does not increase conceptual understanding (Byun & Lee, 2014; Kim & Pak, 2002). It is becoming increasingly apparent that a more productive method for incorporating mathematics in science is needed – one that allows students to learn science concepts and transfer problem solving strategies to novel problems.

3.1.3 Mathematics as a modeled process

Rooted in the theory of scientific models and modeling (Buckley et al., 2004; Giere, 2004; Hestenes, 2010; Svoboda & Passmore, 2013), a rarely used third embodiment of mathematics in science education holds promise for improving student understanding: treating mathematics as a model of a scientific process. Specifically, including mathematics as a modeled process of a scientific phenomenon involves linking both the variables in the mathematical representation and the mathematical operations to entities and processes in the modeled phenomenon. Such a representation includes both data (mathematical variables connected to scientific entities) and operations on that data (mathematical operations paralleling scientific processes) (Larkin & Simon, 1987). In chemistry and biology, the ubiquitous chemical equation, although more often presented as a calculated procedure, can be an example of this modeled use. Consider the balanced equation for producing water (H_2O) from hydrogen (H_2) and oxygen (O_2), $2H_2 + O_2 \rightarrow 2H_2O$. From this equation, a student can calculate how much water will be produced if given a certain amount of oxygen or hydrogen (data). More interestingly, however, the equation describes an operation: separate hydrogen and oxygen molecules are combined (the combination process is indicated by the plus sign) to produce (as indicated by the arrow sign) a new molecule containing both oxygen and hydrogen. Student engagement with this aspect of the equation could increase conceptual understanding.

The critical distinction between the use of mathematics in science as a modeled process versus either a summary of data or a calculated procedure is that the modeled process contains links to scientific entities (variables) and processes (operations), encouraging students to engage in meaning making (Hestenes, 2010; Sherin, 2001). Meanwhile, mathematics as summary of data and calculated procedure too often devolves into manipulation of symbols with little link to the underlying science. Therefore, even though the latter two embodiments of mathematics in science education are more common and still have a purpose in science education, it seems likely that when the goal is learning about the phenomenon, converting mathematics use in science education to modeled processes might help students learn scientific concepts as well as improve their problem solving abilities.

3.1.4 Exemplifying the embodiment of mathematics in science education

We argue that it is possible to transform the use of mathematics for a particular topic from data presentation or calculated procedure to modeled process, rather than simply assuming that specific science topics require calculation or data summary approaches. To illustrate, consider Newton's second law (conceptually: more effort is required to get a heavier object into motion than a lighter one). Students can investigate this phenomenon by using a string with weights to exert a constant force on a cart on a frictionless track (Figure 1).

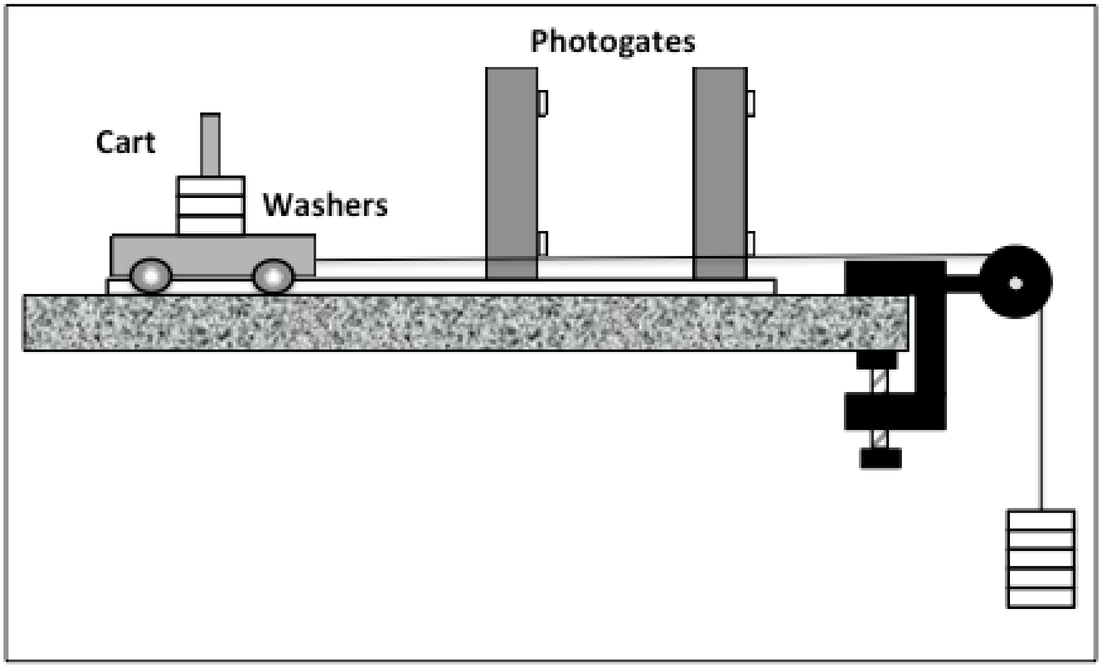


Figure 1. Measuring the effect of mass (determined by the number of washers) on acceleration of the cart.

Different amounts of mass can be added to the cart and sensors are placed on the track so that acceleration of the cart can be determined from the time required to travel the distance between the two sensors, using the calculated procedure equation: $a=2*\Delta x/t^2$. This is a calculated procedure for two reasons: 1) the equation is only being used to derive a quantity, rather than part of some sense-making process; and 2) the constant multiplier "2" and the operation time squared have no clear process meaning. For example, there is no entity that is time squared; instead the representation is shorthand for the more meaningful equation, $\Delta x/t/t$, which represents the change in velocity per unit time.

By changing the mass on the cart and calculating the resulting acceleration, students can produce a data table, as shown in Table 3.

Table 3. Example data table used in science instruction

Force (N)	Mass (Kg)	Acceleration (m/s^2)
1	0.1	10
1	0.2	5
1	0.3	3.3
1	0.4	2.5
1	0.5	2

The table reveals that acceleration decreases as the mass increases for a fixed force. This statement–captures the core phenomenon, but provides no hints about the underlying causal mechanism. By contrast, a student could present their understanding of this phenomenon with the following statement: as mass increases, the force is distributed over more mass, diluting the resulting acceleration. This idea could be represented mathematically by acceleration = Force/mass (i.e., a=F/m). This kind of equation is a modeled process. The symbolic form of the equation (Sherin, 2001) matches a conceptual understanding of the physical phenomenon. First, each variable represented in the equation has meaning in the phenomenon. Acceleration is the amount of time it takes for an object to go from the velocity at sensor 1 to the velocity at sensor 2. Mass is the amount of stuff on the cart. Force is the pull exerted by the string. Second, the mathematical operation (division) has meaning as well: the pull of the string is getting distributed over the amount of stuff of the cart. The equals sign describes the result of a physical process applied to inputs (the effects of a force applied to a mass), rather than simply noting a mathematical equivalency that is convenient for calculation (i.e., the force happens to be equal to the acceleration times the mass). Such connections of variables and operations in the equation to the entities and processes in the scientific phenomenon frame the equation in such a way that students may be more likely to engage in physical mapping between the mathematics and science (Bing & Redish, 2008). Participation in problem solving using this equation may therefore tend to occur more often through the more productive epistemic game of mapping meaning to mathematics as opposed to recursive plug and chug (Tuminaro & Redish, 2007).

Contrast this approach with the way the relationship between force, mass and acceleration is often presented in a textbook. The equation is rewritten as F=m*a and students are asked to memorize this equation as a way to calculate the force exerted by a given object. It is difficult to reason how or why mass should be multiplied by acceleration to give a greater force. It is possible to see that each particle within the object will contribute its own acceleration – but then why isn't the function addition rather than multiplication? It is also difficult to reason how acceleration causes a force? Because the equation is presented rather than derived by students and the variables and mathematical processes within the equation have little connection to the entities and processes within the physical phenomenon, this use of a mathematical equation has much more of the flavor of a calculated procedure. Continued presentation of physics as mathematical formulas to be memorized is one possible explanation for why students have a hard time transferring ideas in physics (Sherin, 2001; Tuminaro & Redish, 2007) and applying their understanding to engineering problems (Litzinger et al., 2011).

3.1.5 Mathematics linked to science concepts facilitates problem solving

Students who are able to solve more complex problems in physics and chemistry have not only an understanding of how to use the mathematics, but also an understanding of how that mathematics is linked to the concepts (Bing & Redish, 2008; Chi et al., 1981; Taasoobshirazi & Glynn, 2009; Walsh et al., 2007). Bing and Redish describe an attempt at problem solving by upper level physics majors where the students, stuck in computing the mathematics and failing to engage in connection of the mathematics to the system of interest, are unable to solve the problem, despite their obvious facility with mathematics. It is not until one student asks for the relationship between the equations and the physics particles that the group is able to progress (Bing & Redish, 2008). In chemistry, a student who successfully uses a conceptually-based strategy to solve a thermochemistry problem talks about the problem solving process in terms of the concept first, "…I need to find the heat gained by the water first" and then applied the mathematics, while an unsuccessful student expresses his algorithmic approach in this way, "I just came up with an equation to solve for the problem, but I think I plugged in the wrong values or something…" (p. 184, Taasoobshirazi &Glynn, 2009).

Several approaches elevating the contextual element of mathematics within a K-12 scientific curriculum (e.g. problem-based learning, qualitative explanations of problem solving, analogies, model development) have improved student conceptual understanding and/or problem solving. (Dori & Kaberman, 2012; Lehrer & Schauble, 2004; Litzinger et al., 2011; Novick, 1988; Savery, 2006; Wells et al., 1995). However, these studies did not test the effect of embedding understanding of scientific entities and processes within a modeled process mathematical equation, as opposed to simply embedding the equation in a scientific context). The current study focuses specifically on this equation as modeled process approach.

3.1.6 Mathematics in biology education

Almost all of the research that has been discussed so far has revolved around the use of mathematics in chemistry and physics, likely because mathematical representations of phenomena have been a part of physics and chemistry instruction for a longer time (Steen, 2005).

However, over the last two decades, rapid changes in biology understanding combined with advances in research technologies (i.e. new measurement tools and computer simulations) require that biology students, not just physics students, learn how to reason in the language of mathematical symbols (Bialek & Botstein, 2004). Further, the most recent scientific standards (Next Generation Science Standards) identify using mathematics as a core practice of science that all students should learn (NGSS Lead States, 2013). Since so many students take high school biology (Lyons, 2013), it is particularly important that mathematics becomes a greater part of the high school biology curriculum. Thus, there is a need both for biology curricula that incorporates mathematics as modeled processes into instruction, and research into the effect of this approach on student understanding. We seek to determine whether students become better problem solvers and better understand underlying biological processes.

3.1.7 Inheritance and mathematics as a modeled process

Inheritance presents a good opportunity for researching the effects of introducing mathematics as modeled process. Inheritance instruction has typically involved predicting the probability of getting a particular type of offspring from a set of parents. That is, mathematics has been at least a small part of high school instruction in heredity for a long time, and therefore we can test the effects of changing the approach to mathematics rather than simply adding (any form of) mathematics. Moreover, teachers report that inheritance is one of the hardest topics for students to understand (Stewart, 1982), so there is great need and opportunity to improve instruction on this topic.

Previous research suggests that when studying inheritance, students have difficulty understanding the underlying biological processes of inheritance (meiosis and fertilization) and how these processes affect the units of inheritance (alleles) that are counted in the mathematical procedures (Moll & Allen, 1987; Stewart, 1983; Chi-Yan Tsui & Treagust, 2010). Students can also struggle to connect the appearance of an organism with the underlying combination of alleles, particularly across generations (Chi-Yan Tsui & Treagust, 2010). One approach that has been pursued is to explicitly develop and connect the process of meiosis with inheritance either through the use of a computer simulation (Buckley et al., 2004) or through tracing the movement of alleles using drawings (Moll & Allen, 1987). The results of these interventions have been mixed. When college students are instructed in how to trace alleles through drawings of meiosis, over half continue to use an algorithmic method to solve genetics problems (Moll & Allen, 1987). Students who draw out meiosis are more successful at solving problems involving one gene than students who use an algorithmic approach, but not more successful at solving problems involving two genes (Moll & Allen, 1987). As the authors point out, drawing out meiosis is a relatively labored and detailed procedure as compared to the speed of the algorithmic approach (Moll & Allen, 1987). The computer-based intervention resulted in higher posttest scores than traditional instruction (Buckley et al., 2004), but studies on a different population of students using the same computer program suggested that genetic reasoning was only improved for the easier types of problems for most students and that a key variable was the mindfulness of student interaction with the different representations of inheritance (Chi-Yan Tsui & Treagust, 2003). The computer-based intervention also requires that students have sustained access to computers during class time, a resource that may not be available to most schools.

All of the approaches to modifying inheritance instruction have focused on enhancing student understanding of the biological processes of inheritance. None have suggested fundamentally altering the embodiment of mathematics within the inheritance curriculum.

Currently, similar to the worst examples of math-science integration in physics and chemistry, the textbook mathematical expression for predicting inheritance outcomes embodies mathematics as calculated procedure and is devoid of any meaningful connection to biological entities or processes. Instead, as is exemplified in a commonly used high school biology textbook("Bscs biology: A molecular approach blue version," 2001), students are exposed to a short didactic introduction to probability, which reminds students, “Probability is usually expressed as a fraction. The chance of the coin landing heads up is one out of two, or ½,” (p.351). There is no exploration of why probability is expressed as a fraction or how this fractional representation relates to the entities of inheritance. Students are then shown how to use an algorithmic procedure, the Punnett square, for determining how the parental genes for a single trait will recombine in the offspring (Appendix, Table B). When students progress to considering inheritance of two gene combinations, they are told that they multiply the probability for getting a particular genotype from each separate gene because "and" means multiply. At this point, there is no biological correlate to the probability for getting a particular genotype from each separate gene and there is no connection drawn to the biological process by which a combination is formed; that is both the constituent probabilities and the mathematical operator on them is not biologically motivated. Unsurprisingly, studies on how students solve inheritance problems show that they tend to use an algorithmic (calculated procedure) method whether they use a pictorial representation followed by counting (the Punnett square), or the mathematical probability method outlined above (Moll & Allen, 1987; Stewart, 1983). Students struggle to extend what they have learned from simple to more complex genetic probability problems and show little ability to connect the mathematics with the biology (Cavallo, 1996; Moll & Allen, 1987; Stewart, 1983).

We have developed a new inheritance unit that changes the way mathematics is embodied from calculated procedure to modeled processes (summarized in Figure 2). Following the modeling cycle (Halloun, 2007; Passmore, Stewart, & Cartier, 2009), students engage in scientific practice by analyzing modern technology-based data (e.g., PCR data) to develop a model of inheritance which includes modeled process mathematical representations (i.e., equations that capture data patterns but also reify the underlying genetic process; Figure 3). Prior research in the physical sciences presented above suggests that some form of embedding mathematics in a scientifically rich context could improve problem solving and understanding of scientific content. We theorize that by specifically changing the use of mathematics from (teacher-presented) calculated procedures to (student-developed) modeled processes that embed biological concepts within the mathematics, students will be better able to solve inheritance problems, and will also demonstrate better understanding of the mathematically modeled processes. We assess the benefits of instruction via modeled process equations on student learning. Specifically, we asked two questions:

1) Is conceptual understanding improved when students are taught mathematics in science as a modeled process rather than calculated procedure?
2) Is quantitative problem solving also improved when students are taught mathematics in science as a modeled process rather than calculated procedure?

The benefits on both conceptual understanding and quantitative problem solving are examined in terms of breadth and scope of benefits to help frame the extent of benefits and the likely mechanisms of change (e.g., general engagement effects of using an iSTEM unit vs. specific modeling of particular processes; general benefits on quantitative reasoning vs. specific benefits to more difficult transfer problems).

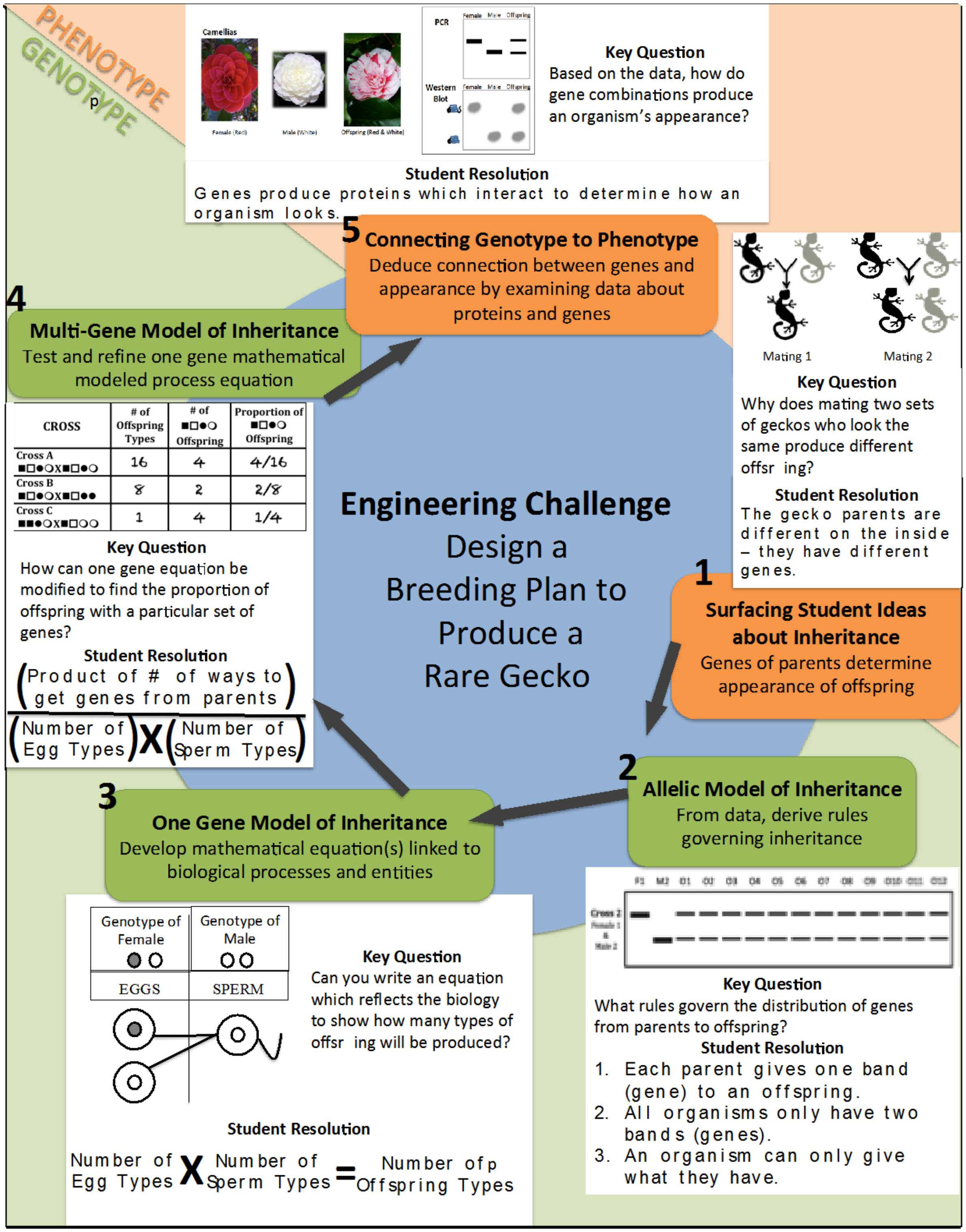


Figure 2. Unit overview of iSTEM inheritance unit.

The unit begins and ends with the engineering challenge that is revisited at the end of tasks 2, 3 and 4. The colored boxes show the product of each task. The numbers indicate their order in the unit. The white boxes show a phenomological representation provided to students, the key questions students engage with and the target student resolution.

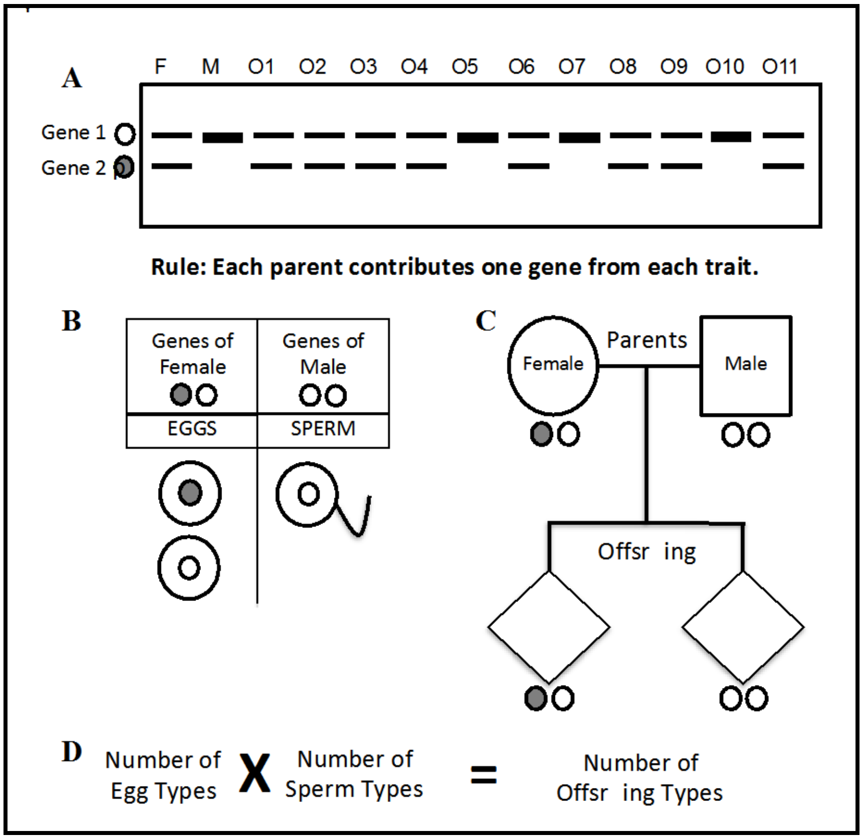


Figure 3. Multiple representations used in the iSTEM inheritance unit

A) PCR Diagram, B) Egg/Sperm Table, C) Prediction Pedigree and D) Initial Mathematical Modeled Process Equation. F = Female, M= Male, O= Offspring

3.1.8 The iSTEM unit and mathematical modeled processes of inheritance

The iSTEM inheritance unit begins and ends with an engineering challenge: design a breeding plan to develop a rare gecko so that a zoo can attract visitors (Figure 2). The initial exposure to the design challenge is designed to help students see genetics knowledge as useful in a real world context and therefore serve as a motivation to understand the phenomenon of inheritance. The unit is constructed as a modeling cycle (Halloun, 2007; Passmore et al., 2009) to develop increasingly complex conceptual models, interconnected ideas and representations that describe or explain a simplified version of the phenomenon which can be used to make predictions (Etkina, Warren, & Gentile, 2006). Each nascent model is developed through analysis of data, followed by argumentation with peers to resolve differences in interpretation and representation and reach a consensus (i.e. Task 2 in Figure 2). Revisiting the engineering challenge after the development of each model permits students to test the model's sufficiency. For example, the design challenge specifically asks for a rare gecko in order to push students beyond a simple one gene breeding design to more complex multigene models. This move to multigene modeling of outcomes makes the need to quantitatively predict outcomes more salient and thus serves as a motivation for mathematical representation becoming a key part of the inheritance models (Tasks 3 and 4 in Figure 2).

The development of this mathematical representation occurs in Tasks 3 and 4. However, the groundwork is laid in Task 2, when students are shown the physical entities of inheritance (the genes), which are revealed in parents and their offspring via a technological application (Polymerase Chain Reaction or PCR) (Task 2 in Figure 2). Students are asked to analyze the gene patterns they observe, and derive basic qualitative rules that summarize the way in which genes are transferred from one generation to the next. These rules both preview the predictability of inheritance patterns and encapsulate part of the biological processes that will later be represented mathematically (Figure 3A).

Using their newly generated rules, students are directed to work with manipulatives depicting biological entities of inheritance, such as sperm, eggs, and genes, to make predictions about the outcomes of breeding two parents. The manipulatives are designed to enable students to see the relationships between the entities of inheritance (genes, eggs, and sperm), the processes of inheritance (i.e., the packaging of genes and joining of egg and sperm), and the quantitative inputs (number of genes in parents) and outputs (number of offspring types). However, using manipulatives to make predictions is relatively time consuming. The unit asks students to recognize this constraint and introduces the affordances ~~to~~ of developing a mathematical model of the process to make predictions. The relationships between entities, processes, inputs and outcomes are maintained through the inclusion of pictorial representations (Figure 3B Egg/Sperm Table, Figure 3C Prediction Pedigree).

In their first attempt at modeling the process mathematically, students are directed to examine a data table showing the offspring outcomes for three different combinations of parents. They are then instructed to develop equations that fit the data available to them and map on to the biological processes and entities that they have represented pictorially. Only two possible equations fit these requirements: Number of Different Offspring Outcomes = (number of egg types) * (number of sperm types) or Number of Different Offspring Outcomes = (number of gene types for trait in female) * (number of gene types for trait in male) (Task 3, Figure 2). The pictorial representations shown in Figure 3 make connections to the symbolic form of the mathematical equation (Sherin, 2001). Specifically, they support connections of mathematical operations (multiplication as combination) to the biological entities and processes (the sperm can join with either egg to produce two new entities).

The engineering design challenge is designed to push students to consider multiple traits, which then encourages refinement of the mathematical model. As part of the application of the single gene model to multiple genes, students are expected to deduce that the equation, Number of Different Offspring Outcomes = (Number of sperm types)*(Number of egg types), is the only one which generalizes, because in the inheritance process, the genes for each trait are packaged independently into sperm and eggs before they are combined in an offspring. This process of testing and subsequent refinement of mathematical representations for inheritance is thus supposed to allow students to gain a deeper understanding of one of the fundamental processes of inheritance. The unit then asks students to recognize that the probability of a desired event is equal to the number of desired outcomes as a proportion of the total number of possible outcomes, to allow the development of the final equation shown in Figure 3, Task 4.

Table A (Appendix) compares the modeled process equation to the calculated procedure methods that are used in traditional inheritance instruction, which uses the Punnett square. In this traditional instruction, there is little connection provided to the underlying biology as no biology is needed to teach the approach or solve a given problem. The purpose is not to model an idea about how inheritance of genes occurs, but rather only to calculate the correct answer.

In contrast, the modeled process equation makes explicit connections between the biology and the mathematical process (Figure 3). For example, the variables in the equation are expressed as eggs and sperm, entities in the inheritance process. Egg types are multiplied by sperm types, because each egg could theoretically join with each sperm. Furthermore, the explicit purpose of the equation within the unit is to model ideas about how inheritance occurs and therefore multiple equations are initially developed and tested against additional data, allowing students to refine their ideas about the biological process of inheritance.

It is important to note that the context of the mathematical representation is a big determinant of whether it is a calculated procedure or a modeled process. The modeled process inheritance equation could be a calculated procedure if students were just shown the equation and taught a formulaic approach for plugging in the variables. The embodiment of mathematics in science and education is not simply about the structure and use of the mathematics, but rather about how it is taught to and taken up by students.

We present two studies that examine the effects on student learning (conceptual understanding and problem solving ability) of changing from traditional instruction to using an iSTEM unit. The first study involves comparison between teachers implementing the new or traditional instruction and the second study focuses on teachers implementing both the new unit and traditional instructional approaches. The iSTEM unit involves several types of instructional changes (e.g., inclusion of engineering challenges, and use of technologies like PCR) and thus the intervention is broadly labeled iSTEM. However, in this paper, we focus our analytic lens on changing the treatment of mathematics in inheritance instruction from a calculated procedure to a modeled process. This focus is achieved by examining in detail the nature of changes on student learning (e.g., broadly on all aspects of inheritance or more narrowly on aspects of inheritance most directly connected to the modeled processes).

3.2 STUDY 1 METHODS

3.2.1 Participants

All teachers were from public school districts in a Midwestern state, drawn from urban, suburban, and rural areas. A local educational agency sent out notices inviting teachers to an exposure meeting. Teachers who attended this meeting signed up to participate in professional development. A subset of the teachers who finished professional development volunteered to implement the iSTEM unit in their classrooms and participate in our study. These volunteers recruited additional teachers from their schools as implementers (Table B, Appendix). The implementing teachers helped to recruit other teachers within their school to serve as controls, using their usual instructional unit for inheritance (described below). Characteristics of the iSTEM and traditional samples are shown in Table 4. Generally, the teachers and students were well matched. Both groups taught honors and nonhonors classes for 9th and 10th grade first year biology students. Additional individual teacher and school characteristics (including standardized test scores) are shown in Table B in the appendix. Professional development was conducted by the research team and primarily focused on teachers experiencing the unit as learners, although some pedagogy was covered in the longer professional development sessions.

Table 4. Characteristics of participants within each group

	iSTEM Unit	**Traditional Instruction**
Participants	12 Teachers, 745 Students	6 Teachers, 321 Students
Percent of students eligible for free/reduced lunch program	38%	41%
Grade & Biology Level	9th and 10th grade, 1st year biology	9th and 10th grade, 1st year biology
Teacher Biology Education	80% masters or undergraduate degree in biology	80% masters or undergraduate degree in biology
Years Teaching Biology	80% more than 6 years	80% more than 6 years
Professional Development	Yes (4-25 hours)	None
Instructional Hours	820 minutes, (4 weeks, Planned)	890 minutes (4.5 weeks, Average)

Teachers who implemented the iSTEM unit received a curricular plan that included daily instructions for lessons and teachers were observed at least once. Teachers who engaged in traditional instruction kept a daily lesson journal consisting of a 2-3 sentence summary of the day's events for each class. Five out of six teachers submitted a journal. An analysis of these journals revealed that the traditional teachers were indeed engaging in inheritance instruction as usual:

- All five teachers showed or instructed students on how to set up Punnett squares to solve probability problems (e.g., "students were shown how to do single trait crosses using Punnett squares").
- The phrases used by all five teachers suggested that the inheritance laws were learned as a set of dictates handed down by Gregor Mendel (e.g., "We revisited the notes and added to them with Mendel's laws of segregation and independent assortment.").
- Four out of five teachers did not mention basic objects and processes of inheritance (including eggs, sperm, fertilization, and gamete formation) in their journals, let alone linking them with mathematical solutions.

3.2.2 Assessments

Pre and post tests were administered to students to examine the effects on student learning. To allow for a sufficiently broad set of questions for each knowledge subcategory but still use only one class period for the assessment, a matrix sampling protocol was used, drawing from a pool of 42 inheritance questions (genetics terminology, genetic processes, genetic probability) and 11 mathematical probability questions. The question categories were chosen *a priori* for the reasons outlined below. An exemplar question from each category is shown in Table 4.

Genetics terminology. Because terminology changes were not part of the intervention, genetics terminology questions provide convergent evidence that teaching ability and student ability were roughly equivalent across conditions.

Genetic processes. Genetics process questions assessed whether students qualitatively understood genetic processes, and were divided into two subtypes: processes that were mathematically modeled (packaging of genes into sperm and eggs and combining eggs and sperm to form offspring, Task 2 and 3 in Figure 2) and processes that were not modeled mathematically (how an organism's appearance is determined by its genes, Task 5 in Figure 2). Larger condition effects for the processes that were mathematically modeled would provide evidence in favor of the effects of modeling mathematical processes in particular.

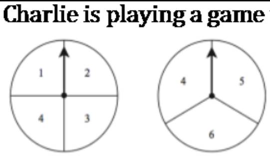
Genetic probability. Genetic probability questions required students to make probabilistic predictions in the context of inheritance. These questions were also subdivided into two categories: simple genetic probability questions asked about simple probability in a genetics context; and complex probability questions required students to apply compound probability to a genetics context, which is then necessarily more complex.

Mathematical probability. Because students' ability to make predictions in a genetics context might be influenced by their understanding of probability in a mathematics context, a category of questions assessing students' understanding of and skill with simple and compound probability in a mathematical context was included. Based on state standards, simple and compound probability had been covered by 9^{th} grade ("Comparison of mathematics michigan k-8 grade level content expectations (glce) to common core standards," 2010), but that did not mean their performance was universally high.

Because no single previously published assessment contained a sufficient number of questions in all of the categories, the pool was constructed by aggregating questions from previously published assessments (Adamson et al., 2003; Blinn, Rohde, & Templin, 2002; delMas, Garfield, Ooms, & Chance, 2007; Garfield, 2003; Nebraska Department of Education, 2010; "Project 2061: AAAS science assessment beta," 2013; Tobin & Capie, 1984; C.-Y. Tsui, 2002)

Based on two posttests of 26 and 27 questions, average KR-20 is 0.72 (average discrimination = 0.46; average difficulty = 0.50). For a subset of students (N= 365), there were no student identifiers on pretests, therefore it was not possible to match up student posttest score with student pretest score, even though a teacher average could be calculated. The deidentified pretest scores were calculated using multiple imputation. The coefficients for multiple imputation were based on Hierarchical Linear Modeling (HLM) (Raudenbusch & Byrk, 2002)) results involving the variables that best predicted student posttest scores: 1) membership in an honors biology class, and 2) participation in unit implementation as well as: 1) student posttest score, 2) the average of student pretest scores for each teacher, and 3) the difference between the teacher's average posttest scores and average pretest scores. When deidentified pretest scores were imputed using these variables, the observed average difference between the observed mean student pretest score for each teacher and the mean calculated from averaging the identified and imputed student pretest scores for each teacher was only .0023 (Range: -.03 to .05, SD=.02).

Table 5. Question categories on pre and post assessments

Categories		Number of Questions	Example Question
Genetic Processes	Mathematically Modeled	7	The genotypes of the sperm from one male and the genotypes of the eggs from one female are shown below. Male Sperm: FQ, fQ Female Eggs: FQ, fQ, Fq, fq Which answer lists all the possible genotypes that could be expected in the offspring: a. FFQQ, ffQQ b. FFQQ, FfQQ, FFQq, FfQq, ffQQ, ffQq c. FFQQ, FfQQ, FfQq, ffQq d. FFQQ, FFQq, FFqq, ffQQ, ffQq, ffqq, FfQQ, FfQq, Ffqq
	Unmodeled Mathematically	20	Flies can have red or brown eyes and straight or curly wings. Red eyes (R) is dominant to brown eyes (r) and straight wings (S) is dominant to curly wings (s). If you have a fly which has brown eyes and straight wings, what combination of genes could it have? a. Rrss c. rrss b. RrSs d. rrSs
Genetic Probabilities	Simple	3	In dogs, the gene allele (e) for drooping ears is recessive to E for erect ears. A male dog with genotype Ee was mated to a female dog with genotype ee and gave birth to a litter of 10 puppies. What is the expected proportion of drooping-eared puppies (ee) in the litter? a. ¼ c. 1 b. ½ d. Don't know
	Complex	7	If organisms of type BbSs and type bbSs are crossed, what is the probability that the offspring would be BbSs? a. 1/16 d. 1/2 b. 1/8 e. 9/16 c. 1/4
Genetics Terminology		5	The appearance resulting from a given gene combination is referred to as the: a. Genotype d. Allelotype b. Prototype e. Stereotype c. Phenotype
Mathematical Probabilities		11	Charlie is playing a game with two spinners.  All four sections on spinner A are the same size, and all three sections on spinner B are the same size. Charlie spins Spinner A and Spinner B one time and adds his results. What is the probability of getting a sum of six? a. 1/12 c. 1/4 b. 1/6 d. 1/2

3.2.3 Mathematics in biology survey questions

As part of a larger survey asking students about their attitudes towards the unit, students were asked two questions about the use of mathematics in the unit. Matrix sampling was used with the four survey versions distributed equally across all implementing teachers. Out of the approximately 630 students who took the survey, one quarter of them (157) answered a survey containing these two questions about mathematics use: 1) Did your group find math to be useful in solving the design challenge? YES or NO 2) If yes, list examples of the types of math you used. The examples the students provided were content coded by two independent raters into Biology Connected Mathematics versus Unconnected Mathematics with 91% agreement. Table 8 in the results section provides code definitions and example statements.

3.3 STUDY 1 RESULTS

3.3.1 Overall effects on student problem solving ability and understanding of science content

Generalizability of the effects of an intervention can either be assessed by examining consistency of patterns across teachers and students, as is typically done, or by examining consistency of patterns across test questions. Statistically significant results can derive from effects limited to one strong teacher or subgroups of students (e.g., only the more interested students or only students in honors sections) or to a few particular questions within a conceptual subgroup of questions. More persuasive results are ones that show consistent and significant effects across students and teachers and across questions. We use both analytic approaches, but with statistical methods adapted to each given the constraints of the matrix sampling approach (e.g., individual students can have topic means but not question means) and the nature of the contrast (e.g., students are nested within teachers, but questions are not nested within teachers).

For the analysis of cross-question generalizability, a percent correct score was calculated for each teacher for each question, pre and post. An ANCOVA was conducted examining the effects of instructional condition on posttest scores within each test category, using category pretest score as a covariate. All critical assumptions for ANCOVA were met, including independence of variables, homogeneity, normality, and homoscedasticity.

Both instruction conditions generally showed gains in understanding from pre to post (Figure 4, dark bars compared to light bars). However, students from iSTEM teachers showed significantly greater adjusted post-test scores in their ability to make quantitative predictions about genetics outcomes ($F(1, 146)=6.4$, $\eta^2=0.03$, $p=0.015$ Figure 4, Genetic Probability). Students who received instruction in the iSTEM unit had an average sixteen point gain on genetic probability questions, approximately two times the seven point gain showed by students who were taught using traditional curricula.

Given the iSTEM unit's focus on mathematical modeling, the increased gain in quantitative problem solving is perhaps not surprising. But, we also theorized that mathematically modeling scientific processes by explicitly connecting mathematical symbols and functions with scientific entities and processes would help students understand scientific processes better (i.e., influence non-quantitative questions). When compared to teachers who used traditional curricula to teach genetics, students of iSTEM teachers showed significantly greater adjusted post-test scores for understanding of inheritance processes ($F(1, 419)=23.1$,

η^2=0.045, p< 0.001, Figure 4, Genetic Processes). The average gain in understanding of inheritance processes for traditional teachers was eight points, while classes taught by iSTEM teachers had an average gain of twenty-one points, an almost three-fold improvement.

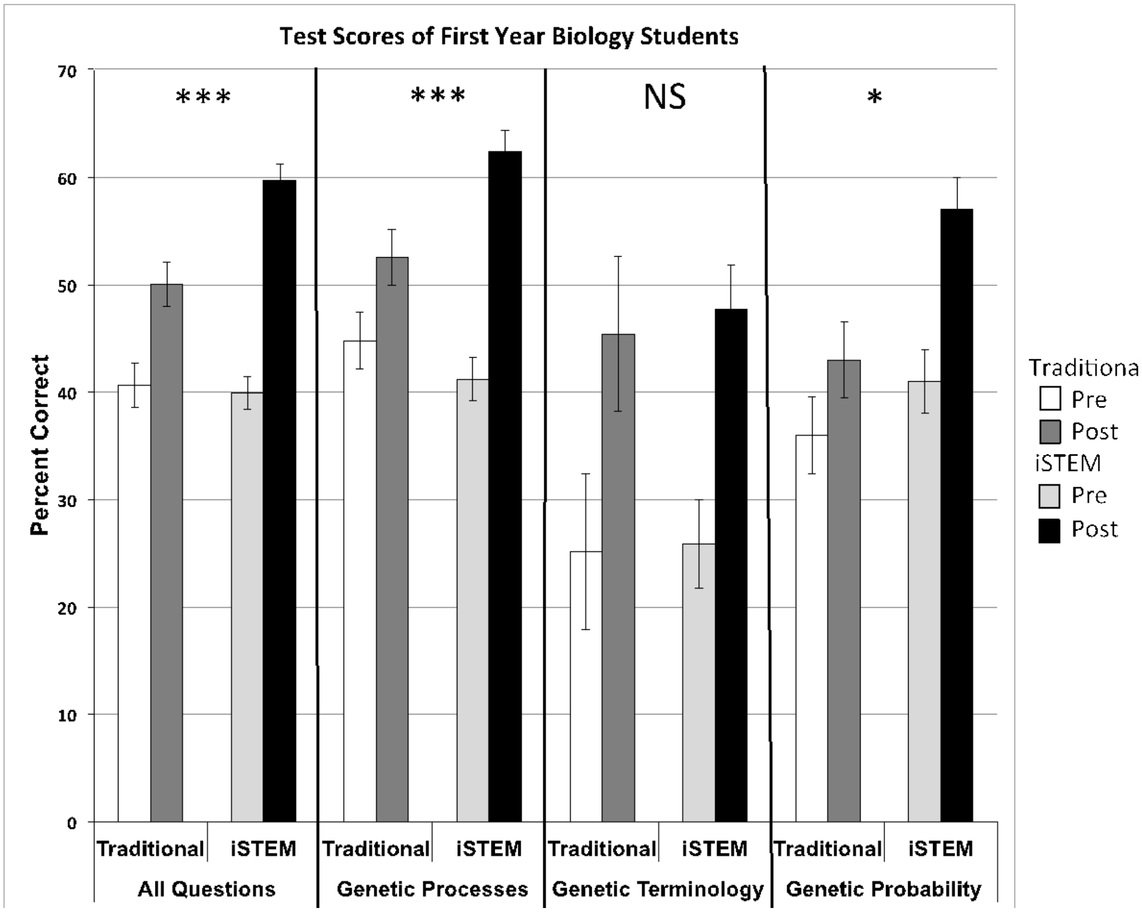


Figure 4. Pre-post teacher-level means (with SE bars) within each instructional condition for student problem solving and understanding of different forms of biology content knowledge

NS > 0.1, * $p < 0.05$, *** $p < 0.001$.

3.3.2 Specificity of problem solving benefits

Others have found that students taught using traditional instruction do not struggle with calculating simple genetic probability (Moll & Allen, 1987; Stewart, 1983), whereas they often do struggle to transfer this ability to more complex genetic probability problems (Moll & Allen, 1987; Stewart, 1983). Thus, it is likely that the problem solving benefits of the iSTEM instruction were only found in more complex probability problems. However, there are too few questions within subtypes to use the generalizability across question analytic approach. To approach this more fine-grained analytic question, we: 1) switch to a two-level Hierarchical Linear Model (733 students nested within 12 teachers) examining student means on simple and complex probability problem categories, and 2) include as an additional covariate a measure of ability to solve probability problems in general (i.e., with no biology content). Because of the sparse matrix sampling protocol for probability problems, individual students' pretest scores in mathematics with only a few questions each were not meaningful. Therefore, in this HLM analysis, we use a teacher mean score for mathematical probability, obtained from averaging all of the students' scores for the teacher. Five implementing teachers were dropped at this stage of the analysis because the version of the posttest that was administered to these classes had too few questions to generate reliable genetic probability scores for each student. In order to further reduce noise across the posttest variations used in the matrix sampling protocol, post-test scores were standardized within each test version. The variables included in the analysis were: 1) instructional condition, 2) mean pretest mathematical probability score of each teacher's students (Teacher Pretest Probability Score), 3) honors designation, and 4) each student's pretest score (Student Pretest Score). Condition and Honors variables were left uncentered; all other variables were grand mean centered. All key statistical assumptions of HLM were met (e.g., homoscedasticity, normality, independence, and linearity).

The HLM results confirm findings from prior research that most students can solve simple genetic probability problems. Traditionally-instructed students and iSTEM-instructed students were not significantly different (post-tests of 69% vs. 76% correct, HLM *b* =-0.02, *p* = 0.85, Honors and Teacher Pretest Probability Score as covariates). This null result for simple genetic probability problems held across all of the statistical models that were tested. By contrast, iSTEM-instructed students were significantly more able to calculate genetic probabilities for complex problems (52% vs. 34% correct, see Figure 5; HLM *b* = 0.27, *p* = 0.02, Honors and Teacher Pretest Probability Score as covariates). The condition effect on the difference between standardized mean question gains for simple vs. complex probability (by teacher) was statistically significant (F=5.4, *p=.03).*

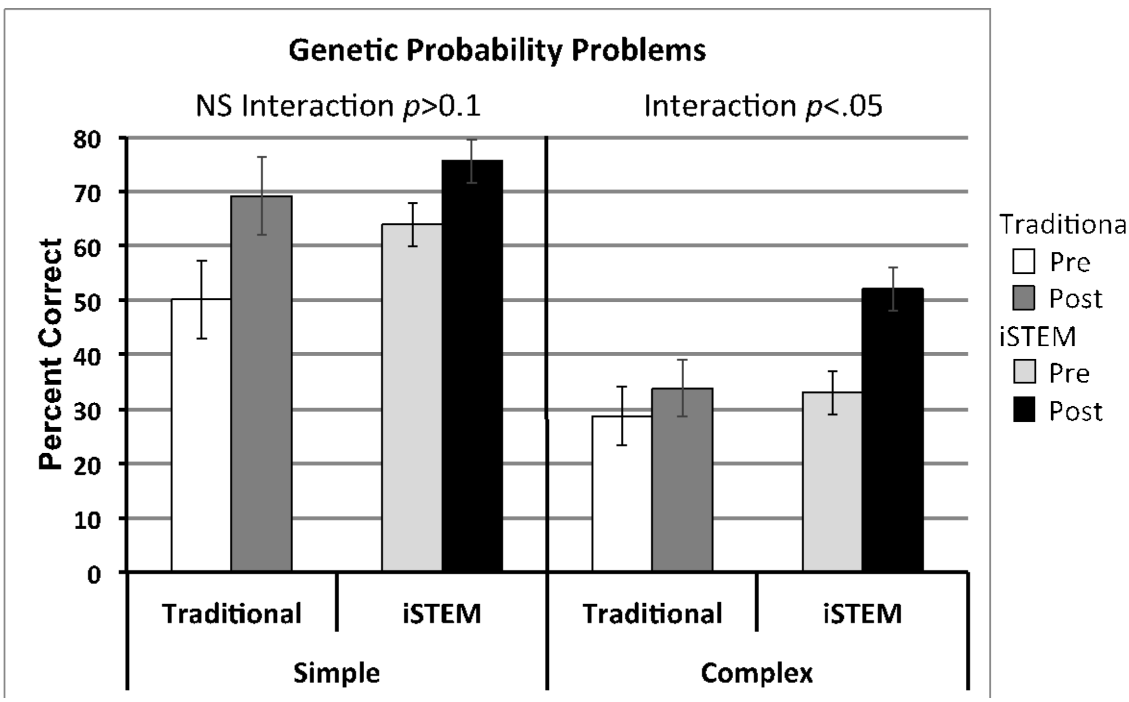


Figure 5. Mean pre and post-test scores (with SE bars) within each instructional condition on simple and complex genetic probability problems

To explore the robustness of these results across statistical assumptions and covariate choices, a number of different statistical models were tested (see Table 6). Models are arranged in order of best fit. The best fitting model, Model 1, includes the covariates of ability grouping

(Honors) and the classes' prior understanding of mathematical probability (Teacher Pretest Probability Score). It shows that implementation of the unit has an effect size of 0.27. This effect size of approximately 0.3 is maintained in the other models.

Mean pretest score for simple genetic probability problems is significantly greater for the iSTEM-instructed group as compared to the traditionally instructed group, which may better position them to learn complex genetic probability. Therefore, prior understanding of simple genetic probability (mean of student scores for each teacher due to the matrix sample approach) was added as a covariate. However, it was not found to be a significant predictor of complex genetic probability scores in any of the models. This finding is supported by the literature which has shown that students have difficulty transferring their understanding of simple genetic probability problems to more complex problems (Stewart, 1983).

Table 6. Regression coefficients and model fit statistics for HLM models for predicting complex genetic probability student posttest scores

	Model 0	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Teacher Level Variable Name	Fixed Effect Regression Coefficients (Standard Error)						
Instructional Condition		0.27* (.094)	.30* (0.104)		0.32* (0.137)	0.30+ (0.142)	
Honors		0.41* (0.139)	0.40* (0.155)	0.45* (0.199)		0.76*** (0.147)	0.43+ (0.214)
Teacher Pretest Probability Score		0.20* (0.066)	0.19* (0.072)	0.19+ (0.089)	0.34*** (0.063)		0.18+ (0.038)
Intercept		-0.17 (NS) (.098)	-0.19 (NS) (0.109)	-0.01 (NS) (0.119)	0.02 (NS) (0.106)	-0.36* (0.120)	-0.15 (NS) (0.123)
Student Level Variable Name	Fixed Effect Regression Coefficients (Standard Error)						
Student Pretest Score			0.08* (0.038)				0.08* (0.038)
Estimation of Variance Components							
Teacher Level	0.23	0.009	0.015	0.034	0.036	0.040	0.043
Student Level	0.87	0.87	0.86	0.87	0.87	0.87	0.86
df	11	8	8	9	9	9	9
Chi square	133.56	14.49	16.62	25.95	31.97	32.86	29.13
p-value	< 0.001	0.069	.034	0.002	< 0.001	<. 0.001	<. 0.001
Deviance	2007	1989	1992	1995	1995	1994	1999
Estimated Parameters	2	2	2	2	2	2	2

Teacher Pretest Probability Score is the mean student pretest score for each teacher. NS >.1, * *p*< .05, ** p <.01, *** p <.001

3.3.3 Benefits for qualitative understanding of genetic processes

The test items for qualitative understanding of genetic processes included both those processes that were modeled in the mathematical equations of the unit and those that were not. To distinguish between the effect of mathematical process modeling versus a general effect of the methods of iSTEM instruction (e.g., via improvements in overall student engagement or quality of classroom/group discussion), the effect of iSTEM instruction versus traditional methods was examined separately on modeled versus unmodeled genetic processes. If explicitly linking mathematical variables and processes with scientific entities and processes promotes student understanding of those processes, then there should be a differential effect of iSTEM instruction on modeled versus unmodeled processes. Again, given the more refined focus of analysis, significance testing was performed using a two-level HLM on student means across questions with 975 students nested in 17 teachers, and controlling for various other student or contextual factors.

Both traditional and iSTEM instruction showed improvement in student understanding of unmodeled processes (Figure 6). However, the HLM results show that the adjusted posttest scores for Traditionally-instructed students and iSTEM-instructed students were not significantly different (69% vs. 76%, b=0.10, p=0.55, Honors and Student Pretest Score as covariates). This null result for unmodeled process questions held across all of the statistical models that were tested.

By contrast, only iSTEM-instructed students showed a gain in their ability to answer questions about the mathematically modeled genetic processes (Figure 6). Moreover, HLM results show that the adjusted posttest scores for iSTEM-instructed students were significantly different from traditionally-instructed students (b=0.34, p = 0.025, Honors and Student Pretest Score as covariates). To explore the robustness of results across statistical assumptions and covariate choices, a number of different statistical models were tested (Table 7). Models are arranged in order of best fit.

Model 1, which is the simplest model that best explains both teacher and student level variance, shows that implementation of the unit has an effect size of 0.34. Across models, iSTEM instruction continues to be a significant predictor of modeled genetic process posttest scores, with an effect size of approximately 0.3 or greater across all models tested; removing the variable of iSTEM instruction from the model produces a worse fit (e.g., Models 5 and 6). Other explored covariates that did not have a consistent significant effect for either genetic process or genetic probability analyses included: teacher means of genetic process or genetic probability score, and school measures such as ACT and State Test scores, and school SES.

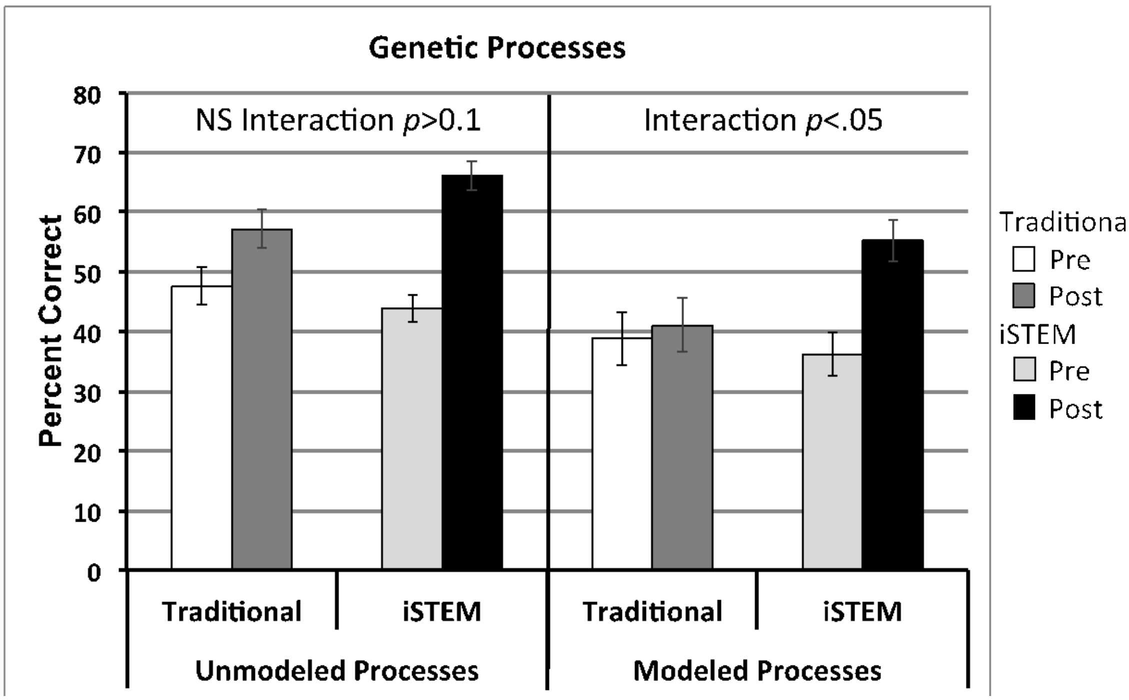


Figure 6. Mean (and SE bars) for pre and post-test scores within each condition for modeled and unmodeled genetic processes

Table 7. Regression coefficients and model fit statistics for HLM models for predicting modeled genetic process posttest scores

	Model 0	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Teacher Level Variable Name	Fixed Effect Regression Coefficients (Standard Error)						
Condition		0.34* (0.137)	0.31* (0.138)	0.38* (0.141)	0.40* (0.144)		
Honors		0.51** (0.145)	0.43* (0.160)	0.34 (NS) (0.229)		0.42 (NS) (0.267)	
Teacher Pretest Probability Score			0.07 (NS) (0.066)				
Teacher Pretest Score				0.31 (NS) (0.317)	0.68** (0.205)	0.10 (NS) (0.362)	0.54* (0.235)
Intercept		-0.31* (0.124)	-0.25* (0.095)	-.27+ (0.128)	-0.19 (NS) (0.117)	-0.03 (NS) (0.108)	-0.19 (NS) (0.117)
Student Level Variable Name	Fixed Effect Regression Coefficients (Standard Error)						
Student Pretest Score		0.21*** (0.037)	0.21*** (0.037)	0.20*** (0.037)	0.20*** (0.037)	0.20*** (0.037)	0.20*** (0.037)
Estimation of Variance Components							
Teacher Level	0.14	0.045	0.043	0.045	0.048	0.070	0.077
Student Level	0.91	0.882	0.883	0.882	0.883	0.883	0.884
df	16	14	13	13	14	14	15
Chi sauare	129.86	59.05	51.75	53.95	59.87	68.47	76.75
p-value	< 0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001
Deviance	2712	2673	2678	2675	2674	2677	2680
Estimated Parameters	2	2	2	2	2	2	2

Teacher Pretest Score is the mean student pretest score for each teacher. NS >.1, * *p*< .05, ** p <.01, *** p <.001

3.3.4 Student perception of mathematics in iSTEM unit

One hundred and forty-five students distributed across all teachers who implemented the iSTEM unit were asked if they thought mathematics was useful in designing a plan to breed a rare gecko and to give an example of how it was useful. Seventy-nine percent of students thought mathematics was useful in the design challenge and gave an example of its use. The examples these students provided were content coded into Biology Connected Mathematics versus

Unconnected Mathematics (see Table 8 for definitions and example statements). On average, forty percent of these examples involved a biological connection, and this rate was no lower than thirty percent for any teacher. Thus, we have evidence that many, although perhaps not all, of these students made connections between the mathematics they used and the biological phenomenon of inheritance.

Table 8. Codes for student examples of how mathematics was used to design a breeding plan for a rare gecko

Code	Definition	Example Statements
Unconnected Mathematics	Statements mention the calculations that would be performed (multiply, divide) or that math is used for calculating financial profit. No biological terms are used.	"We added and subtracted the cost of the geckos to the budget." "Probability,multiplication, fractions."
Biology Connected Mathematics	Statements about the use of mathematics make reference to biological entities or processes.	"We use egg type x sperm type to get the number of offspring." "We used a math equation to find out different possible ways that the genes could move (or combinations) when offspring was produced."

3.4 STUDY 2 METHODS

3.4.1 Participants

After receiving data on the effect of iSTEM instruction on student learning, two of the teachers in the traditional group volunteered to undertake twenty hours of professional development during the summer and used the iSTEM unit with their classes the subsequent year. In both years, the classes were nonhonors classes. Only students who took both the pretest and the posttest were included in the analysis (Year 1, Teacher 4, N= 55, Teacher 9, N = 45; Year 2, Teacher 4, N= 39, Teacher 9, N = 29).

3.4.2 Assessments

The assessments used were the same as described for Study 1, except that the genetics terminology category was eliminated in Study 2. Performance in each category or subcategory was calculated by obtaining a percent correct score for each question for each teacher and averaging.

3.5 STUDY 2 RESULTS

There were too few students total to conduct generalizability analyses across questions given the matrix sample approach, and therefore we focus on generalizability across students. Because there were only two teachers and the instructional contrast was within teacher, we conducted simple ANCOVAs (rather than HLMs) of the instructional condition effect on student posttest scores in each subcategory. Only those variables that were shown to be significant in the larger sample were used as covariates with this smaller sample (class mean of mathematics probability score, student composite pretest score). There were no Honors classes in the second study. Assumptions for ANCOVA were met (i.e., normality, homoscedasticity, and independence of variables).

Genetics process results were consistent with our between-teacher findings from Study 1 (Figure 7). Students showed gains for unmodeled processes with both traditional and iSTEM instruction (Traditional Gain = 17, SE=5, iSTEM Gain = 22, SE=5). However, only iSTEM instruction produced gains in student understanding of modeled processes (Traditional Gain = -4, SE= 6, iSTEM Gain = 22, SE= 6). With student composite pretest score as a covariate, adjusted student standardized posttest scores for iSTEM instruction were significantly different from traditional instruction for modeled (Traditional M = 34, iSTEM M = 56, $F(1, 165)=6.3$, $\eta^2=0.04$, $p=0.01$), but not unmodeled processes (Traditional M = 54, iSTEM M =62, $F(1, 165) = 2.20$, $\eta^2=0.01$, $p=0.14$).

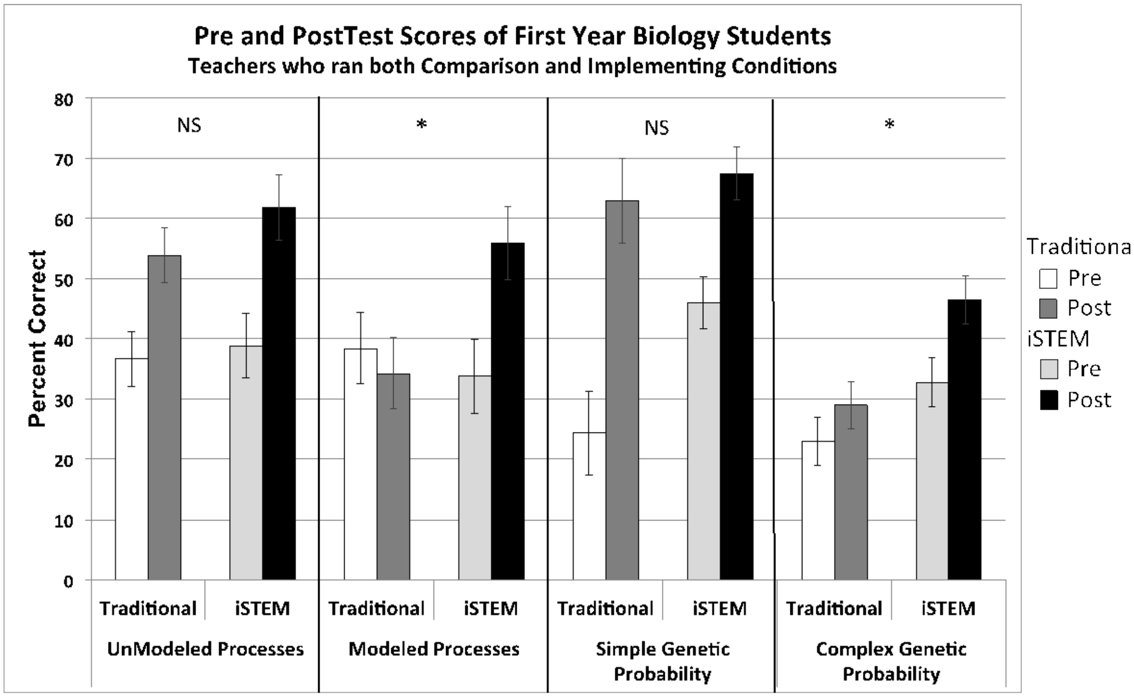


Figure 7. Pre-post means (and SE bars) within each instructional condition for student problem solving and understanding of different forms of biology content knowledge for traditional teachers who subsequently adopted iSTEM instruction

NS > 0.1, * p < 0.05

3.6 GENERAL DISCUSSION

We examined a curriculum that included many critical iSTEM practices that are typically absent from science instruction: It was organized around an engineering design problem, students had to develop explanations from data, and iteratively develop and elaborate various models. Most intensely, the curriculum focused on mathematical modeling of processes in biology. Using both between teacher comparison (Study 1) and within teacher comparisons (Study 2), students who were taught inheritance using this curriculum performed at higher levels on assessments than did traditionally instructed students. Differences were found on measures of solving quantitative inheritance problems (particularly more complex problems) and of answering qualitative questions about genetics process (particularly related to the processes that were modeled in the unit).

Because there was not random-assignment to condition, one might argue that prior differences in instructional ability (e.g., experience in teaching biology) or student characteristics (e.g., prior performance in mathematics and science) accounted for the results in Study 1. However, a number of factors argue against such possible confounds as the source of the performance differences: 1) Teachers were closely matched in a number of categories, including teaching experience and level of education; 2) Traditional teachers came from five of the same schools as teachers implementing the iSTEM condition; 3) school characteristics were not significant covariates in any of the HLM analyses; 4) The effect of iSTEM instruction on quantitative problem solving and qualitative understanding of genetics processes were robust even with the addition of prior ability covariates in the analytic models; and 5) Teachers that switched from traditional to iSTEM instruction showed an increase in student performance after the switch.

A different concern might relate to possible differences in time on task. Often inquiry-based instruction requires more time than does traditional instruction. However, from the teacher logs, the traditional-instruction teachers reported spending a mean of 890 minutes (approximately 22 days) on inheritance; in contrast, the iSTEM instruction only involved 820 minutes (approximately 20 days). By focusing on a major instructional target in the traditional curriculum, it was possible to engage students in many practices of science with the core science content without extending the length of instruction.

Thus, we have good evidence that instructional reform in high school science using the reform practices can significantly improve student understanding and problem solving ability. These improved instructional outcomes occurred in a range of instructional contexts and appeared on relatively traditional measures of student performance (i.e., multiple choice), similar to ones used for accountability purposes in many settings. Although rich instruction is likely to produce even stronger results on rich performance assessments, the results on simpler multiple choice assessments are practically important for influencing the reform movement in the United States and beyond.

3.6.1 Theoretical implications

This iSTEM intervention in inheritance was designed around mathematical modeling of genetic processes, based on the theory that asking students to develop a mathematical model of genetic processes and subsequently refine and use that model would cause them to connect mathematical variables and processes with scientific entities and processes, leading to a better understanding of the modeled scientific processes (Hestenes, 2010). In support of this theory, we demonstrate that a plurality of students who have been asked to develop a mathematical model of a biological phenomenon do indeed connect the use of mathematics with that biological phenomenon. Prior research in physics and chemistry also found that students who are able to link their mathematical equations with scientific concepts are better able to solve more complex problems (Bing & Redish, 2008; Taasoobshirazi & Glynn, 2009).

The current findings extend the prior research on quantitative problem solving in science by showing that deliberate instruction in modeled process mathematics can improve student problem solving as problems increase in complexity. That is, even with quantitative problem solving, there are benefits to linking equations to scientific concepts that are revealed on more complex problems. In inheritance, traditionally instructed students typically can solve simple genetic probability problems with ease, but struggle with more complex problems (Stewart, 1983). Stewart (1983) argued that students could not solve complex problems because they lacked an understanding of the underlying genetic processes and were using an algorithmic approach to solving the single gene problems that did not transfer well. The currently obtained results show a qualitative interaction between method of instruction and change in student scores for simple and complex genetic probability problems. Both traditionally instructed and iSTEM instructed students show a comparable and significant change pre to post in their ability to solve simple genetic probability problems, which if anything is slightly smaller for iSTEM compared to traditionally instructed students. However, traditionally instructed students show little to no change in their ability to solve complex genetic probability problems, while iSTEM instructed students show a significant increase pre to post instruction. The finding of a qualitative interaction between simple and complex genetic probability gains for students in the two conditions means that the difference in gains is significant. Indeed, the condition effect on the difference between standardized mean question gains for simple vs. complex probability (by teacher) was statistically significant ($F=5.4$, $p=0.03$). This interaction suggests a deeper explanation than that proposed by Stewart (1983): mathematical procedures that are directly connected to processes provide a method for students to generalize a learned procedure to more complex problems. In other words, it is not that understanding of scientific processes turns an algorithm into something that is generalizable; rather, we suggest that understanding must be connected to the mathematical procedures themselves to obtain generalizable performance. The mechanism of action is not fully resolved. Perhaps by framing the mathematical equation as rooted in and derived from the scientific phenomenon, students are more likely to engage in more productive problem solving procedures such as blended processing, by mapping meaning to the mathematical equation itself (Bing & Redish, 2008; Kuo et al., 2012; Tuminaro & Redish, 2007). Alternatively, the modeling cycle used to develop and modify the mathematical equation may foster better understanding of the connections between mathematics and the scientific phenomenon allowing for a "working forwards" approach to problem solving where students can represent and solve the problem in different ways and check their answers (Chi et al., 1981; Taasoobshirazi & Glynn, 2009)." We postulated that inclusion of modeled process mathematics would not only increase student quantitative problem solving ability, but also increase their understanding of the mathematically modeled scientific processes. Curriculum and instructional units that ask students to mathematically model scientific concepts have previously shown improved understanding of the modeled concepts (Lehrer & Schauble, 2004; Liang, Fulmer, Majerich, Clevenstine, & Howanski, 2012; Wells et al., 1995). Our study extends these findings in two ways. First, instead of embedding mathematical equations within a rich scientific context, this intervention specifically asks students to model scientific processes within the mathematical equation. Second, the study shows that within the same unit, processes that were modeled mathematically were better understood than those that were not modeled mathematically. This specificity of which qualitative understandings showed improvements suggests that benefits are unlikely to be due to a generalized effect of iSTEM instruction in general (e.g., increased student discussion, increased use of scientific practices such as analyzing data or developing an argument from evidence). However, we should note, that the gains from pre to post instruction for nonmodeled processes were directionally larger for the iSTEM instruction (an effect size of 0.1 SD). Given the sample size of the current studies, we cannot rule out that a larger sample size of teachers might also reveal a generalized, if perhaps smaller, effect of the iSTEM instruction.

The current studies did not directly address how the mathematical modeling of scientific processes increases student understanding of those processes. One possible explanation is that by embedding scientific processes within the mathematical model for solving quantitative problem solving, teachers and students are forced to spend more time on those scientific processes. Indeed, in the iSTEM unit, more time is spent on the modeled processes than in traditional instruction. Unlike in traditional instruction where teachers report only briefly presenting in PowerPoint or lecture format these key processes for understanding inheritance, in the iSTEM unit, students are forced to discuss these processes each time they engage in quantitative problem solving. Another possible explanation is that by asking students in the iSTEM unit to develop, and later refine, a mathematical model that is connected to entities and processes in the phenomenon of inheritance, students have to engage in deeper thinking about what entities and processes within the phenomenon are important and how they are linked to one another. Then, in the process of refining the model, students are asked to confront misconceptions about the processes. Thus, students work to construct and refine their understanding of the mathematically modeled processes. Other model-centric approaches could similarly have such benefits through deeper reflection. For example, Cartier and Stewart (2000) used a model-evaluation approach to provide students with opportunities to develop a better understanding of how knowledge claims are structured in genetics.

3.6.2 Practical concerns

Instructional approaches to science education that use mathematics raise questions about whether students' mathematics ability then serves as a barrier to accessing science (Maerten-Rivera, Meyers, Lee, & Penfield, 2010). Indeed, physics was historically placed last in the high school sequence because of concerns that the required mathematics was beyond the abilities of many 9^{th} graders (Sheppard & Robbins, 2005). We argue that some forms of mathematics are well within the reach of most 9^{th} graders and can serve a productive basis of science instruction, especially when treated in a modeling approach (i.e., not relying on previously memorized complex mathematical algorithms). The unit was effective in classrooms with relatively low prior ability in solving probability problems, and prior mathematical ability was not a strong predictor of performance, especially not qualitative understanding.

Further, the improved outcomes did not require large increases in instruction on mathematical techniques. Traditional instruction teachers reported spending on average 260 minutes on genetics probability instruction, as compared to approximately 270 minutes in the iSTEM unit. It was the nature of the quantitative instruction that was the larger difference. Traditional teachers report teaching only calculated procedures methods for problem solving versus the scientifically connected modeled process used in the iSTEM unit.

Others have designed instructional interventions that have increased student quantitative problem solving ability and/or understanding of the inheritance processes modeled mathematically in the iSTEM unit. One approach asked students to pictorially represent the processes (Moll & Allen, 1987). While students showed an increase in understanding of the pictorially represented processes, half of the students chose not to pursue the drawing method when engaging in quantitative problem solving. Moreover, those who used a calculated procedure method were more successful at solving complex problems. The authors speculated that this was because representing the processes pictorially became more cumbersome as problem complexity increased.

Two other groups have shown that students increase their understanding of genetics processes (Buckley et al., 2004; Chi-Yan Tsui & Treagust, 2003), and one has shown that students also increase their ability to solve genetics probability problems (Buckley et al., 2004), following instruction using a computer simulation that models genetics processes (described in Horwitz, 2010). However, many science classrooms do not have regular access to computers. Thus, the mathematical modeling of processes in the iSTEM inheritance unit described here provides a low-tech alternative, at least for the biology concepts that could be modeled with relatively simple mathematics. Other aspects of biology, involving more complex mathematics, might be best supported with computer simulation methods.

3.7 CONCLUSION

We have provided evidence that mathematical modeling of inheritance processes can increase students' ability to solve quantitative genetic probability problems and to answer qualitative questions about the modeled genetics processes. Thus, we have generalized prior findings (Bing & Redish, 2008; Taasoobshirazi & Glynn, 2009) which have suggested that making connections between a mathematical equation and the underlying scientific processes increases the ability to solve mathematical problems in a scientific context. Furthermore, we have provided support for a theoretical idea that modeling scientific processes and entities mathematically through explicit connections between mathematical variables and processes and the entities and processes within a scientific phenomenon increases understanding of the scientific phenomenon. While further research needs to be done into how including modeled process mathematics increases problem solving ability and student understanding of science, the unit on inheritance presented here provides a successful model of iSTEM instruction that integrates mathematics and biology in an engineering context.

4.0 MECHANISM CONNECTED MATHEMATICS IN SCIENCE EDUCATION: CHANGING STUDENTS' APPROACH TO PROBLEM SOLVING

Mathematics has been a part of scientific practice and science education for a long time. In scientific practice, mathematics serves several purposes, including as a tool, an inscription, and a model of the phenomenon (NRC, 2012). Traditional science education often ignores the use of mathematics as a model of a real world phenomenon that explicitly connects mathematical equations with science entities and mechanisms. With this narrow focus, science education not only excludes an important element of scientific practice, but also potentially shortchanges students in their understanding of both the science and the application of mathematics to the scientific context. However, few curricula have been developed that explicitly focus on mathematical modeling of scientific phenomenon to develop these connections in all students and even fewer research studies have been done that investigate how instruction that includes mathematical modeling affects students' problem solving. In this qualitative study of the strategies and representations used by high school biology students solving complex and unfamiliar inheritance problems, we examine the problem solving strategies of students who have been instructed in a unit that focuses on mathematical modeling of a scientific phenomenon and students who have been taught using a more traditional siloed mathematics-as-tool approach. In contrast to traditionally instructed students, students who have been exposed to mathematical modeling in science tend to connect their inscriptions with the scientific phenomenon and use multiple problem solving approaches.

4.1 INTRODUCTION

Mathematics has been incorporated into science instruction in many different ways, as a tool, an inscription, and a model of the phenomenon. Across the scientific disciplines, when students use mathematics simply as an algorithm to get the right answer and fail to make connections between their quantitative problem solving and the scientific phenomenon, students struggle to solve complex and unfamiliar problems (Kuo et al., 2012; Stewart, 1983; Taasoobshirazi & Glynn, 2009). If, instead, students spontaneously make connections between their mathematical problem solving process and the underlying scientific phenomenon during problem solving, they experience greater success (Kuo et al., 2012; Taasoobshirazi & Glynn, 2009; Tuminaro & Redish, 2007).

Instruction that grounds mathematical inscriptions in the scientific phenomenon, or explicitly develops connections between the phenomenon and the mathematical inscription benefits students' mathematical and conceptual understanding as well as their quantitative problem solving (Lehrer & Schauble, 2004; Malone, 2008; Schuchardt & Schunn, 2016; Wells et al., 1995). However, the mechanism behind these improved student outcomes remains relatively unexplored. To better understand how a curriculum that fosters connections between mathematical inscriptions and a scientific phenomenon can alter quantitative problem solving, we conducted a qualitative study of quantitative problem solving in three groups of students. To tease apart the effect of problem solving competence and the effect of instruction, two comparisons are made. Within students receiving mechanism-connected mathematics (MCM) instruction, one contrast is made between students who struggle with more complex and familiar problems and those who are successful with those problems. Within students who are successful at problem solving, a second contrast is made between students who received MCM instruction and those who received traditional instruction. We show that MCM instruction changes successful students approach to quantitative problem solving.

4.1.1 Science practice versus science education

The scientific practice of scientists is said to be a messy, open-ended endeavor which relies on a cyclical process of asking questions (usually to investigate relationships between entities), gathering data, analyzing the data, summarizing the data in a mathematical form, and testing predictions generated from the mathematical equation against the physical phenomenon in a never-ending cycle (Figure 8A) (as summarized in Hume, 2009). The mathematics may be an *ad hoc* expression of the patterns in the data or otherwise relate inputs to outputs (Smith, Haarer, & Confrey, 1997). More often, though, the mathematics serves as a first-principles model, a way of tying the scientific action to the mathematical form and explaining the scientist's ideas about the phenomenon (Smith et al., 1997)

Choosing which entities to include in the mathematical expression and how to functionally represent the relationships between them can foster conceptual development (Svoboda & Passmore, 2013). Scientists tend to place greater value on this aspect of mathematical equation development; when scientists work with mathematicians to develop equations representing physical phenomenon, they will often pressure the mathematicians to alter their equations so that they better represent the physical world (Smith et al., 1997; Svoboda & Passmore, 2013).

Once the mathematical expression is felt to accurately represent the data at hand, it is used by scientists to generate predictions about how the physical phenomenon might behave under a variety of conditions. A mismatch between prediction and data can lead to a modification of the mathematical expression, a modification of the understanding of the phenomenon, and/or a modification of the limits of the mathematical expression (Quale, 2011; Svoboda & Passmore, 2013). Eventually, the mathematical representation becomes an expression of general laws and procedures (Hume, 2009).

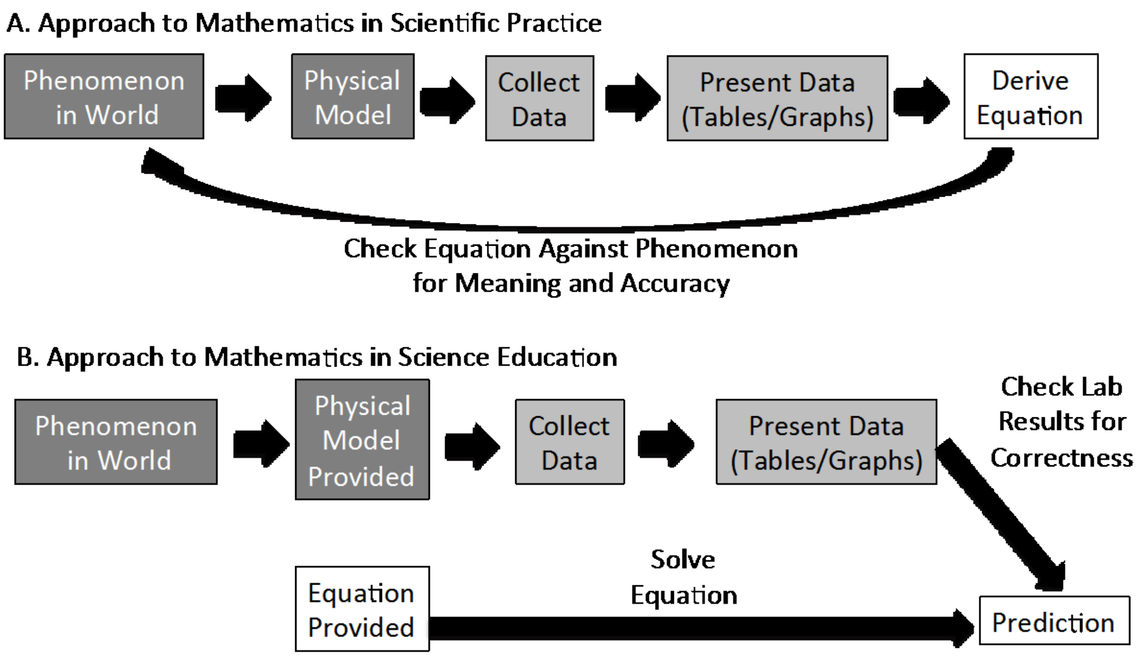


Figure 8. Approaches to mathematics in science education versus scientific practice

Unfortunately, it has been said that scientific inquiry as it is taught in K-12 classrooms often bears little resemblance to the cognitively challenging process of scientific inquiry in scientific practice (Figure 8B) (Hestenes, 2010; Hume, 2009; Quale, 2011). Instead, student laboratory work "focuses on recipe-style laboratory exercises…which involves closed problem solving and produces learning outcomes that are mainly content and skill-based" (Hume, 2009, p. 35). A common example of this occurs in chemistry when students are asked to complete a chemical reaction following a procedure, predict the amount of product based on a provided chemical equation, and then calculate the percent yield to determine how well they followed the procedure or whether they measured accurately. The mathematics becomes a known formula that is provided to the students who then apply it to see if the obtained data is "correct" (Hume, 2009; Smith et al., 1997; Svoboda & Passmore, 2013). While accuracy and fit to a known model is an important aspect of scientific practice, it is not the only purpose for mathematical expressions (Svoboda & Passmore, 2013).

4.1.2 Traditional use of mathematics in science education classes shortchanges students

By not challenging students to develop their own mathematical representations that mesh with both the phenomenon and the data, the traditional K-12 approach to scientific inquiry likely shortchanges students in several ways (Hestenes, 2010; Lehrer & Schauble, 2010, 2011; Schuchardt & Schunn, 2016; Svoboda & Passmore, 2013). Because students do not have the opportunity to consider which elements to include in the mathematical expression, they cannot consider which entities in the phenomenon are most important for their investigation or thinking (Svoboda & Passmore, 2013). In other words, "representational re-description of the world changes what students observe, and therefore, the questions they pursue" (p. 11, Lehrer & Schauble, 2010). Thus, by removing the task of creating (mathematical) representations, students' investigational space becomes narrowed to only observing and exploring within the space that has been provided to them. Deciding which scientific entities to include and how to relate them within an equation involves sense making of both the phenomenon itself and the relationship of the phenomenon with the mathematical expression (Hestenes, 2010; Lehrer & Schauble, 2000; Schuchardt & Schunn, 2016). It has been proposed that when students are provided with a mathematical expression without having the opportunity to derive it, students' understanding of the science becomes restricted in two ways: 1) their understanding of the phenomenon is decreased, and 2) their understanding of the mathematical expression is decreased (Hestenes, 2010; Lehrer & Schauble, 2000, 2010; Schuchardt & Schunn, 2016; Svoboda & Passmore, 2013).

This lack of connection between the phenomenon and the mathematical expression can manifest itself in multiple ways in student problem solving. Students will often apply the expression in a rote manner, manipulating symbols without understanding. The disconnect that can occur between symbols and the phenomenon being represented is apparent in Lehrer and Schauble's (2000) description of two middle school classes. In one class, where students were instructed to use triangles to depict an inclined plane, students drew equilateral and isosceles triangles, with no resemblance to an inclined plane. However, in a second class where students generated the idea of using a triangle as an inclined plane and then discussed the merits of this idea, including the connections between the features of an inclined plane and a triangle, students subsequently only drew the appropriate right triangles to represent an inclined plane. Furthermore, these latter students were able to illustrate the concept of steepness with their triangles (Lehrer & Schauble, 2000).

These types of difficulties with connecting mathematical representations to the real world (often associated with traditional instruction) persist beyond middle school (Gupta & Elby, 2011; Kuo et al., 2012). Students who are provided only with mathematical formulas and an explanation of how to use them in their physics class do not show increased performance on tests of conceptual understanding even after solving an average of 1,500 exercises and problems (Byun & Lee, 2014; Kim & Pak, 2002). Thus, simply increasing practice with symbols and equations does not appear to facilitate student sense-making of the phenomenon (Byun & Lee, 2014; Kim & Pak, 2002). Across the scientific disciplines, students who fail to make conceptual connections have difficulty solving quantitative problems and are unable to transfer their problem solving procedures to more complex or unfamiliar problems (Kuo et al., 2012; Stewart, 1983; Taasoobshirazi & Glynn, 2009). The lack of conceptual understanding is not simply a lack of understanding when and where to correctly use the formulas. Students who can correctly determine the probability of producing a particular offspring in genetics using a provided probability formula cannot explain what the variables in the equation represent in the real world or why, biologically, one outcome was obtained versus another (Stewart, 1982, 1983). While it has been suggested that teaching the "grammar" of equations may help students develop better understanding of the mathematical process (Redish & Kuo, 2015; Sherin, 2001), this approach fundamentally misses the potential role for the semantics of equations: connecting the mathematical inscription to the scientific phenomenon.

The few students who demonstrate that they understand how their mathematical problem solving is connected with the scientific phenomenon are more successful at problem solving, show greater flexibility in their approach, and are better able to solve complex problems (Kuo et al., 2012; Taasoobshirazi & Glynn, 2009; Tuminaro & Redish, 2007). Moreover, students who are allowed to develop their own mathematical representations show better understanding of how their representations are connected to the underlying scientific phenomenon (Roth & Bowen, 1994; Roth & McGinn, 1998). Combined, these findings about problem solving success and conceptual understanding suggest that one way to increase both conceptual understanding and student problem solving is to make science instruction more like scientific practice with respect to mathematical modeling of the scientific phenomenon. Compared to students exposed to traditional instruction, students who have experienced this type of instruction that more closely mimics scientific practice have been shown to have increased understanding of the science content as well as increased facility solving quantitative problems (Dye et al., 2013; Malone, 2008; Schuchardt & Schunn, 2016; Wells et al., 1995).

Within model-based curricula in physics and biology, students are asked to develop conceptual models of a scientific phenomenon through the iterative generation of representations of that phenomenon from interpretations of data gathered about the phenomenon. But there are important variations to modeling that have been explored in different curricula. One more common kind of focus during development of a mathematical inscription has been on developing a mathematical inscription based upon another inscription (often a table or graph) that has been generated from data (Hestenes, 2010). For example, developing the equation $F=m*a$ to inscribe a linear relationship between the Force and acceleration data on a graph. The student's main task is to find the equation that fits the table or graphed data. With this model fit focus, the connection of the mathematical inscription to the scientific phenomenon is implicit in that during equation development, students do not have to think about what is being modeled other than a bunch of dots on a graph or rows in table; the meaning of the variables is irrelevant for the model building process.

Another, less common, way of structuring the mathematical modeling task is to have students generate a mathematical inscription that fits the graph (or table) of data points *and* that also explicitly and transparently connects to the underlying mechanisms in the scientific phenomenon being modeled (Schuchardt & Schunn, 2016). Students should be able to explain how both the objects in the equation and the mathematical operations on the objects are connected to the entities and mechanisms in the scientific phenomenon[1]. For example, in $a=F/m$, to what do F, m, and a correspond in the underlying phenomenon, what does dividing F/m mean in the phenomenon, and why is the division operation plausible? To distinguish between the two types of equation development we have described, we will call this instructional approach Mechanism Connected Mathematics (MCM), and contrast it with the Inscriptional-Relational approach (IR) that emphasizes the relation of mathematics to other inscriptions. The more common form of the equation $F=m*a$ has fewer affordances for making connections with the phenomenon and simply serves to describe the relationship of the objects to one another on a graph, leading to some common misconceptions (Freedman, 1996). While not yet common, the MCM approach is better matched to authentic scientific practice and could be used in many instructional contexts. Relevant to theories of why any kind of mathematical modeling is useful for students, such a practice specifically pushes students towards conceptual understanding of mathematical inscriptions in science.

However, unlike in the scientific process, it is likely important for the educational process that the derived equation is written in scientifically meaningful terms instead of abstract variables and that the equation is not simply a convenient shortcut between inputs and outputs, but transparently represents the scientific mechanism. For example, an equation to express the

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[1] While there are some subtle differences in the definition of a scientific mechanism, we will be using the term in its broadest sense to mean the activities and operations that are carried out by the interaction of entities within a phenomenon to produce a change in state (Bechtel & Abrahamsen, 2005; Glennan, 2005; Machamer et al., 2000)

velocity of a car could be written as v=dx/dt or as “velocity=change in position/change in time”. While both equations expresses the relationship between inputs (position and time) and outputs (velocity), only the latter reminds students that velocity is conceptually how much the car has changed position over a given time interval. In this way, we postulate that both in the derivation and the subsequent use of these scientifically meaningful equations students repeatedly encounter the key scientific elements and the connections between them, resulting in improved conceptual understanding of these processes and elements.

4.1.3 Hypothesized advances of MCM instruction

We hypothesize therefore, that when MCM instructed students are solving quantitative problems they will be more likely than traditionally instructed students to make connections between their inscriptions and objects and/or mechanisms in the scientific phenomenon. In turn, this connection should help them solve quantitative problems, because students who make those connections spontaneously are more successful (Kuo et al., 2012; Taasoobshirazi & Glynn, 2009; Tuminaro & Redish, 2007).

Given its current rarity as an instructional approach, few have studied MCM instruction. In prior work on MCM instruction in high school biology, we have found that, relative to traditionally taught students, MCM-taught students showed two large performance benefits on post-tests (Schuchardt & Schunn, 2016). First, students showed much higher performance on questions addressing conceptual understanding of the phenomenon. This benefit was specific to topics in which students developed equations themselves, rather than other topics covered in the unit without derivation of equations, suggesting that deriving equations *per se* was important. Moreover, that the benefit was shown on conceptual questions without any quantitative element showed that understanding improved, rather than only facility with, or knowledge of, equations. Second, students showed much better performance on complex quantitative problems. However, the simple treatment vs. control performance pre-post gains did not provide insights into why (through what mechanism) the MCM instruction improved quantitative problem solving. Here we examine qualitatively the benefits conveyed by MCM instruction: why do students become better problem solvers?

4.1.4 MCM instruction in inheritance

To make more concrete the MCM instructional approach and how it differs from traditional use of mathematics in science, we examine the example of MCM instruction in biological inheritance. This description highlights three important features of the MCM instructional approach: 1) students derive equation; 2) students are asked to propose an equation that fits their understanding of the biological mechanism; and 3) the affordances of the particular mathematical inscriptions that are taught. This shift is from equations and inscriptions that are difficult to connect to biological mechanisms to equations and inscriptions that have more transparent relationships to underlying biological mechanisms. While the example of MCM instruction in inheritance serves as the context studied in this paper, the general approach also applies to physics, chemistry, and beyond; we return to other applications of MCM instruction in the general discussion.

Traditional Instruction in Inheritance. Predicting the probability of getting a particular type of offspring from a particular set of parents is a common (real world) quantitative problem solving task found in inheritance instruction. Furthermore, teachers report that inheritance is one of the hardest topics for students to understand (Stewart, 1982) and quantitative problem solving beyond the simplest cases is often quite weak (Schuchardt & Schunn, 2016; Stewart, 1983),

Unfortunately, the commonly-presented textbook mathematical expression for predicting inheritance outcomes is devoid of any meaningful connection to biological entities or processes. Instead, as is exemplified in BSCS (2001), a commonly used high school biology textbook, and as reflected in high school biology teacher journals of instruction (Schuchardt & Schunn, 2016), the method for teaching probability of offspring types and the biological mechanisms of inheritance (meiosis and reproduction) are taught separately (Figure 9B).

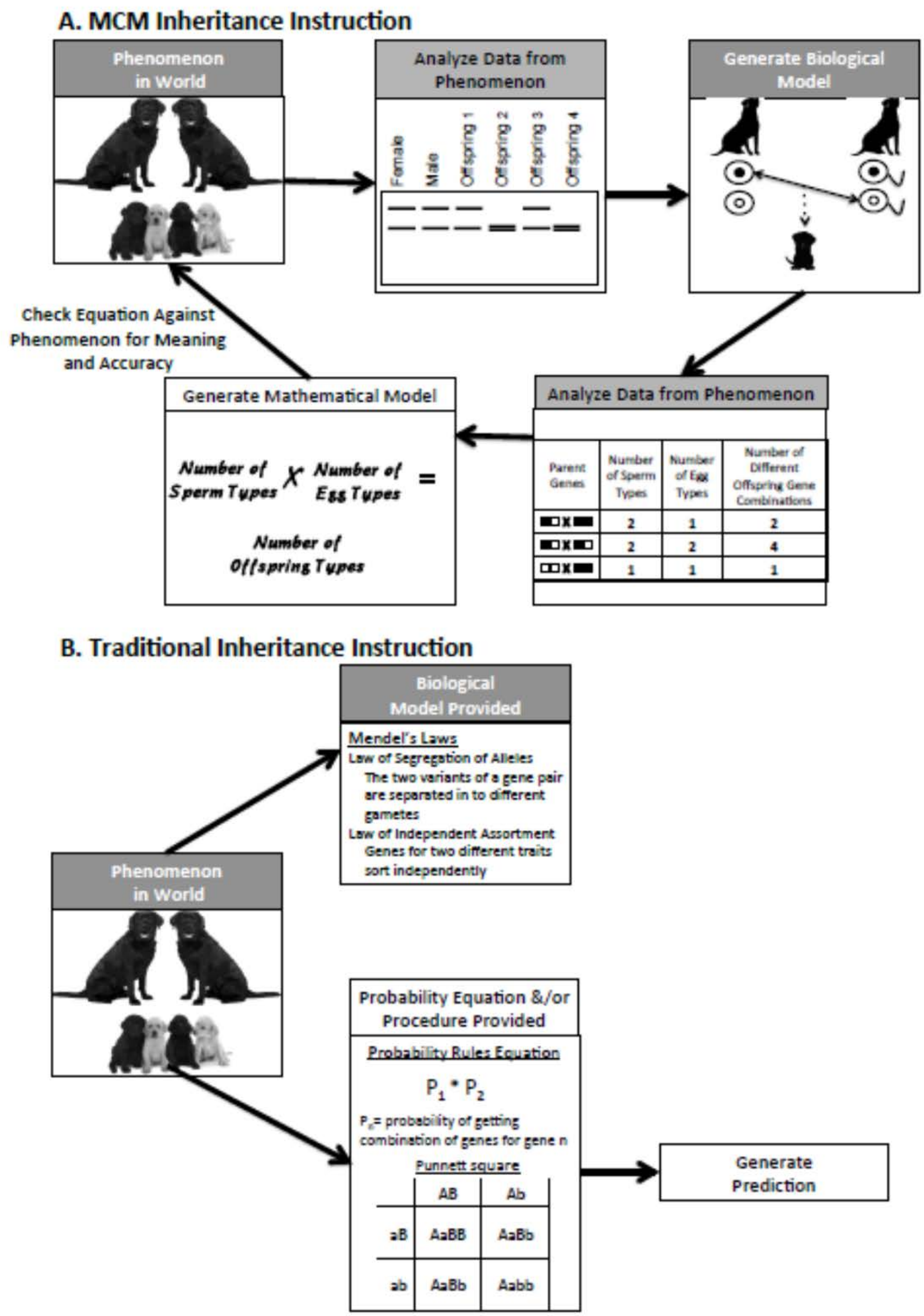


Figure 9. Two different approaches to mathematics in inheritance instruction

Here, the treatment of mathematics is not connected to the underlying biological concepts. For example, in a commonly used textbook, *BSCS Biology, a Molecular Approach* (2001), students are exposed to a short didactic introduction to probability, which reminds students, “Probability is usually expressed as a fraction. The chance of the coin landing heads up is one out of two, or ½,” (p.351). There is no exploration of why probability is expressed as a fraction or how this fractional representation relates to the entities of inheritance. Journals of instruction collected from teachers in traditional classrooms reveal that whether they are teaching students to use the mathematical probability method outlined above (Probability Rules equation) or a pictorial representation followed by counting (the Punnett square) (Figure 10), instruction in the method of calculating probability is separated from instruction in the mechanisms of inheritance.

Consistent with our claims on the importance of MCM approaches, students who have experienced this traditionally disconnected instruction in inheritance show deficits in their understanding of inheritance. They tend to use an algorithmic method and struggle to extend what they have learned to more complex quantitative problems and show little ability to connect the mathematics with the biology (Cavallo, 1996; Moll & Allen, 1987; Schuchardt & Schunn, 2016; Stewart, 1983).

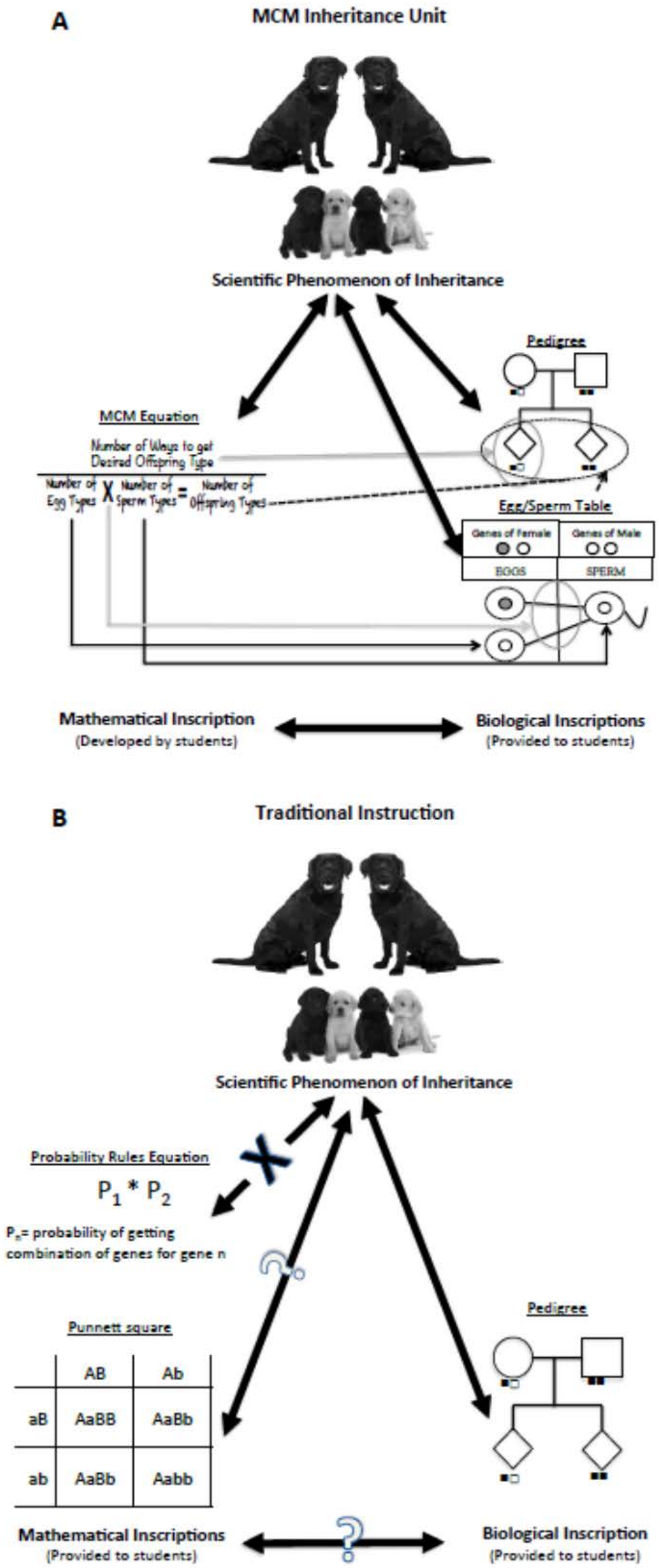


Figure 10. Comparison of inscriptions developed in MCM inheritance unit and Traditional instruction and development of connections to scientific phenomenon of inheritance

MCM Instruction in Inheritance. To examine the effects of having students derive and use mathematical equations that are meaningfully connected to a scientific phenomenon, we developed a MCM-based unit on inheritance. In an iterative cycle that mimics scientific practice, students develop a mathematical model (equation) to predict the probability that a set of parents will produce an offspring with a particular set of genes. Students begin by working with data displays and physical models of the phenomenon (paper cut-outs of eggs, sperm, and genes) to explore biological mechanisms (packaging of genes in to eggs and sperm, and joining of sperm and eggs to form offspring). From data tables showing the offspring outcomes of matings between parents with different sets of genes, students are asked to generate equations that fit the data. To reinforce the connection between the parts of the equation and the phenomenon, the variables within the equation are not reduced to letters, and during development of the equation, students must describe how the objects and processes within their equation are related to the entities and mechanisms in the phenomenon of inheritance. Subsequently, these connections are reinforced by providing students with inscriptions of the biological objects (Figure 10). After initial equation development, the unit asks students to test predictions made from the initial mathematical inscription against increasingly complex instances of inheritance, modifying both their knowledge of the biological mechanisms and their mathematical inscription when the predictions do not fit the data (Figure 9B).

Instructionally, this approach is different from the more traditional disconnected approach. Instead of studying the biological phenomenon and the mathematical method in parallel tracks, students move back and forth between exploring the biological mechanisms underlying data patterns and the mathematical inscriptions of those data patterns. The equation generated from this instructional approach (which we have labeled the MCM equation) also has different affordances and constraints when compared to the mathematical procedures (the Probability Rules Equation, and the Punnett square provided during more traditional instruction). The Probability Rules Equation is fast and does not involve many steps. However, connections between the variables and processes in the equation and the phenomenon of inheritance are obscure and they lack biological relevancy (Figure 10) (Stewart, 1982). The Punnett square is slower, involving more steps. Although it is designed to be connected to the phenomenon, many students do not connect the elements of the Punnett Square inscription with the elements in the phenomenon of inheritance and apply the method algorithmically (Figure 10) (Cavallo, 1996; Moll & Allen, 1987; Stewart, 1983). Moreover, the tedium involved in drawing out all the combinations causes many students to choose to use the less connected, but faster, Probability Rules equation (Moll & Allen, 1987). The MCM equation shares affordances with both the Probability Rules equation and the Punnett square. It is relatively fast, involving slightly more steps than the Probability Rules equation, but not necessitating drawing out all gene combinations like the Punnett square. The MCM equation is also designed to explicitly connect objects and processes in the equation to entities and mechanisms in the phenomenon of inheritance (Figure 10).

4.1.5 Why does MCM instruction improve quantitative problem solving performance?

It is relatively transparent why asking students to make conceptual connections would improve conceptual understanding: it is simply another opportunity to apply the conceptual knowledge. What is less clear is why MCM instruction improves quantitative problem solving. Is it the connections between biology and mathematics made during MCM instruction, or the relative affordances of the different mathematical inscriptions (MCM equation vs. Punnett square vs.

probability rules)? How do the connections between biology and mathematics work: are they providing a scaffolding to understanding and skill development which is lost by the end of the unit or do they allow for multiple problem solving pathways, making student problem solving less prone to errors or getting stuck? Alternatively, is the MCM equation simply less prone than the other mathematical inscriptions to errors perhaps because it is less involved than the Punnett square or easier to generalize to more involved or unfamiliar problems, because mapping is simpler?

Examining how MCM instruction enhances quantitative problem solving probes the theory behind the approach, allowing for improvement and appropriate use of the theory for designing instruction that incorporates mathematics in to science instruction. The goal of this paper is to describe quantitative problem solving for three groups of students: 1) students who have received MCM instruction but cannot solve complex quantitative problems (MCM Struggling), 2) students who have received MCM instruction and can solve complex quantitative problems (MCM Competent), and 3) a select group of students who have received traditional instruction and can solve complex and unfamiliar quantitative problems (Competent Traditional). To uncover effects due to problem solving success (rather than method of instruction), the MCM Struggling group will be compared to the MCM Competent group. Then the MCM Competent group will be compared to the Competent Traditional group, holding competence constant, and varying method of instruction. Student descriptions of their problem solving are examined to see whether and how they are making connections between the scientific phenomenon of inheritance and problem solving. Then, the types of inscriptions used and how they are used are described to gain insight in to students' problem solving processes and how connections between mathematical approaches and biological approaches are being used.

4.2 METHODS

4.2.1 Participants

Three experienced biology teachers implementing the MCM Unit in Inheritance for a second year agreed to participate in this study. At the end of the unit, teachers selected six students with a range of abilities who were willing to participate in the research interview. All eighteen students were in eleventh or twelfth grade. Twelve of them were first year biology students and six of them were second year biology students. Because of the structure of the school day, five of the 18 students did not have time to complete all interview tasks and were dropped from further analysis.

In order to see how problem solving strategies from the MCM instructional approach differed from typical problem solving strategies that student obtain from instruction with the Traditional approach, it was necessary to find an appropriate group of comparison students. This posed challenges because students from age and biology level matched classrooms taught using a Traditional approach generally exhibited a weak understanding of genetics mechanisms and a weak ability to solve genetics probability problems (Schuchardt & Schunn, 2016). If the students could not solve the problems, too few interviews would contain information about how students used traditional strategies. Therefore, a comparison group of seven students was taken from a context that was very likely to be high functioning in both their understanding of genetics mechanisms and their ability to solve genetics problems: twelfth graders from an academically selective school, enrolled in an Advanced Placement Biology class for their second year of biology. The teacher had a PhD in biology. The interview was conducted after the students had reviewed inheritance for the AP exam.

Eleventh and twelfth grade students were chosen for this study because in a previous study, it was noted that these older students often explained their reasoning process more thoroughly in writing. Thus, it was felt they might be more inclined to explain, or more aware of their reasoning process when problem solving, than younger students and therefore better able to articulate their problem solving process to an interviewer.

4.2.2 Instruments

Students were asked to solve two genetics probability problems for the interviewer. Students were told that they could explain their problem solving process either as they worked or afterwards. The interviewer asked follow-up questions to ascertain their understanding of how their problem solving process was linked to their understanding of the biological process of inheritance or probability. (The interview protocol is attached in Appendix C.) The interview for each problem consisted of two parts: I) Students explained their problem solving process to the interviewer either as they were solving the problem or immediately after they had finished; and II) Students answered follow-up questions from the interviewer probing for the types of connections that students were making between the mathematical inscription they were using and the phenomenon of inheritance. Interviews were video and audio recorded and student work was collected. Transcripts were made using the audio with annotations of student gestures and written inscriptions added from the video.

The two genetics probability problems (Figure 11) were similar to problems asked by James Stewart during interviews revealing that traditionally instructed students had difficulty with these types of problems (Stewart, 1982, 1983).

Problem 1

In guinea pigs, black coat color is dominant to white coat color and red eyes is dominant to brown eyes. If organisms of type BbRr and type bbRr are crossed, what proportion of their offspring will be bbRr?

Problem 2

Given a female with the genes: BbRrGg, what proportion of her eggs will contain genes “b” and “g”?

Figure 11. The two genetics probability problems used during student interviews

Both problems are examples of compound probability problems that are unlikely to be answered using memorized patterns. Problem 1 involves more steps than Problem 2 but is of a type that all students should have encountered during their biology classes. Problem 2 is one that should be unfamiliar to students because biology students are generally not asked to solve problems of this type. Complexity and lack of familiarity place two different stressors on student problem solving. By asking students to engage in multiple steps to achieve an answer, there is greater potential both for mistakes and for engaging in multiple strategies. Lack of familiarity moves students away from a potentially memorized approach to a particular type of problem. To highlight the distinguishing features of these two problems, Problem 1 will be labeled the Complex Problem and Problem 2 will be labeled the Unfamiliar Problem. During the interview, students were asked if they had seen these types of problem before and most confirmed that the first problem was familiar and the second was unfamiliar.

4.2.3 General coding protocol

The initial round of coding of the transcripts was done by the interviewer. The interviewer has a PhD in genetics, has taught high school genetics for a number of years, and was a developer of the MCM unit. Sixty percent of the transcripts from each instructional group (MCM and Traditional) were double coded by a second coder to verify coding reliability. The second coder has a degree in the life sciences and has taught high school genetics for several years. Both coders were blind to the instructional condition of the student. An average Cohen's kappa of .75 was obtained across all codes.

Assessing correctness. Student answers to the inheritance probability problems were assessed for correctness on a three-point scale. If the answer was correct, it received a two. If the answer was incorrect due to a nonconceptual error (e.g. counting or computational error), it received a one. If the answer was incorrect due to a conceptual error (either mathematical, or biological), it received a zero. Students who earned a two or a one on both problems, were categorized as Competent. Students who earned a zero on both problems were categorized as Struggling. Two students in each condition (MCM and MT) got one problem correct and one problem incorrect. They were considered transitional and dropped from further analyses.

Coding inscriptions. To describe the types and sequences of problem solving strategies that students were using, their written inscriptions were coded based on the structure of the inscription and students' talk about the inscription. Definitions, labels, and examples of some of the more common inscriptions are shown in Figure 12.

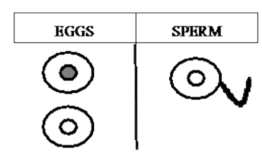
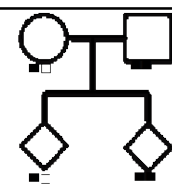
Inscription Label	Definition	Example
Punnett Square	Students have drawn a Punnett square on for a cross to determine proportion of offspring or # of types of offspring. Use whether accurate or inaccurate.	B b B BB Bb b Bb bb
Probability Rules Equation	Implicitly (no equation) or explicitly (equation), students are multiplying the proportion of offspring that could be expected from each single gene cross to get the proportion of offspring with a particular genotype.	"1/4*1/2*1/4"
MCM Equation	Students explicitly or implicitly use either the numerator or denominator or the combined equation that was developed in the MCM inheritance unit. w1*w2*w3….*wn/(Egg types)(sperm types), where w = ways to get offspring genotype for gene 1 from parental genotype	1*2=2 OR (eggs)*(sperm) =
Sperm/Egg Chart	Students have drawn a table (not Punnett square format) with the genotype combinations expected for sperm and eggs (whether accurate or inaccurate). Students must refer (either with words, writing or gestures) to either sperm or eggs in reference to this chart, either spontaneously or elicited.	
Pedigree	Students have drawn a tree indicating the connections between parents and offspring. Genotypes may or may not be indicated.	

Figure 12. Definition of inscription codes

The sequence of the inscriptions and the transitions between inscriptions were recorded. For additional qualitative information, transitions were also examined for evidence of why students were moving from one inscription to another.

4.2.4 Coding connections between mathematical inscriptions and phenomenon of inheritance

Student responses were examined for explicit connections between mathematical inscriptions (Punnett square, MCM Equation, and Probability Rules Equation) and the biological processes of inheritance. These connections could either be spontaneous if they occurred during students' description of the problem solving process or they could be elicited in response to the interviewer's question: "Are sperm and eggs represented here?" Students were said to have made a connection between the problem solving method and the biological process of inheritance if they could appropriately describe or indicate or label a drawn object or number in one of their mathematical inscriptions as egg and/or sperm (e.g., "And then for like the dad, he only has like two different options for sperm and those are those two" [Student point to the gene combinations written on the outside left of the Punnett square]). Student responses were coded as unconnected if they could not explicitly make such a connection (e.g., "Yeah, so like the eggs would just be like the mom's and the dad has the sperm" [Student points to mom and dad genotypes when talking about each]) or denied the existence of such a connection (e.g., "No. It wasn't relevant to the problem"). Connectedness and correctness were coded independently.

Representing student descriptions of problem solving. To identify patterns in problem solving, representations of student discourse and inscriptions were created. A visual representation of common student words was created for each group to facilitate extraction of patterns of mathematical and biological term use. To remove minor wording variations, students' problem solving descriptions and responses to the interviewer's prompts were modified in the following ways. Plural and singular words were combined (e.g., genes was substituted for gene), a single word was substituted for synonyms or words with closely aligned meaning (e.g., parent for mom and dad), numeric representations were substituted for number words (e.g., 1 for one). Mathematics and biology words were identified that occurred more than three times and occurred across two or more students. These lists of words and relative frequencies were entered into Wordle to produce a convenient visual representation of the frequency with which each group of students (MCM Competent, MCM Struggling, and Traditional Competent) were using mathematical and biological terms for each problem.

To identify patterns of inscription use in quantitative problem solving, the coded inscriptions for problem solving were arranged sequentially for each student. Within this representation, inscriptions were classified as biological if they depicted biological objects (Egg/Sperm Table and Pedigree) or whether students connected the inscription to biological objects (some Punnett square inscriptions). Inscriptions were classified as mathematical if they were an equation (Probability Rules Equation or MCM Equation) or if it was used as an algorithm that students did not connect to biological objects (some Punnett square inscriptions). Connections that students made between inscriptions were also coded.

4.3 RESULTS AND DISCUSSION

A summary of the primary differences in students' problem solving and talk about problem solving is provided in Figure 13. Students are grouped by instructional method and competence into three groups: 1) five MCM students (called MCM Struggling) who could not solve either quantitative problem; 2) the six MCM students who could solve both quantitative problems (called MCM Competent); and 3) five Traditional students (called Traditional Competent) who also solved both problems correctly. Pseudonyms are created based on these three groups (Struggling, Competent, and Traditional). First, we will discuss how the students talked about problem solving, focusing on the connections students made between their inscriptions and the associated scientific phenomenon. Second, we will turn our attention to how students used mathematical and biological inscriptions during problem solving.

Condition	Both Problems Correct	Student	Math-Science Connection	Multiple Inscriptions Problem 1
MCM Struggling	✖	Sailor	✖	✖
	✖	Scout	✖	✖
	✖	Sage	✖	✔
	✖	Schyler	✖	✖
	✖	Shaun	✖	✖
MCM Competent	✔	Camerin	✔	✔
	✔	Casey	✔	✔
	✔	Carter	✔	✔
	✔	Chris	✔	✔
	✔	Conner	✔	✔
	✔	Corey	✔	✔
Traditional Competent	✔	Tegan	✖	✖
	✔	Tristan	✖	✖
	✔	Terry	✖	✖
	✔	Tate	✖	✔
	✔	Taylor	✔	✖

Figure 13. Summary of characteristics of quantitative problem solving by condition

An “X” means “no” or not present, and a check mark means “yes” or present.

4.3.1 Relative frequencies of biology and math words in student problem solving descriptions

Student descriptions of their problem solving were analyzed for the types of words they were using. The results are displayed in both visual (Figure 14) and tabular (Table 9) form.

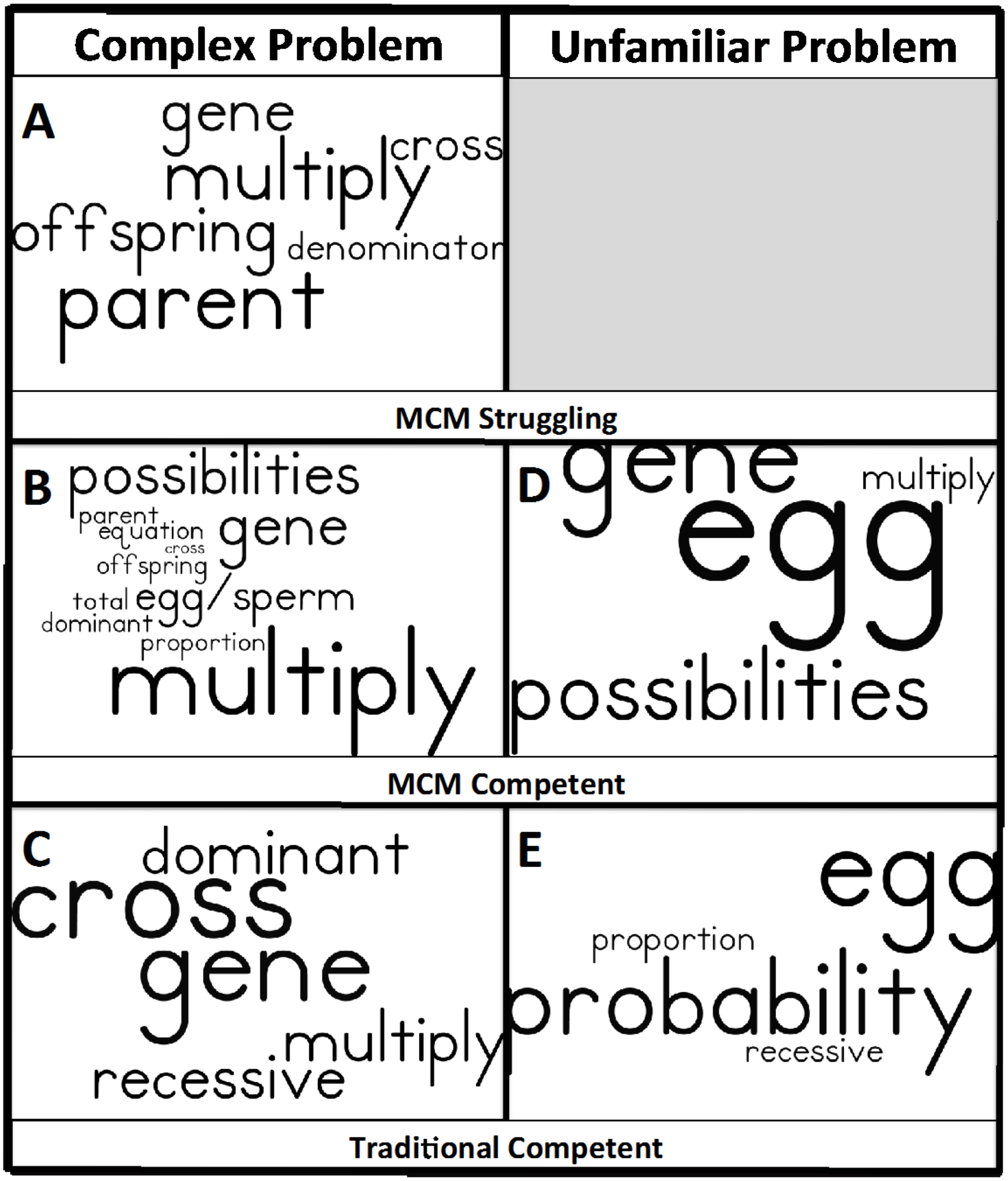


Figure 14. Relative frequencies of words associated with mathematics and biology in the three groups of students' spontaneous descriptions of their solutions for a complex problem (left) and an unfamiliar problem (right) The MCM Struggling group for problem 2 is not represented because only one biological/mathematical term (gene) was repeated multiple times.

Table 9. Frequency of biology and math words in students' spontaneous explanations

Number in parentheses is the number of students who spoke the word. Total number of students: MCM Struggling (5); MCM Competent (6); Traditional (5).

Frequency of biology and math words in students' spontaneous explanations.

	Complex Problem			Unfamiliar Problem		
Biology Words	MCM Struggling	MCM Competent	Traditional	MCM Struggling	MCM Competent	Traditional
dominant		3(2)	4(3)			
recessive			4(2)			2(2)
gene	5(3)	7(2)	7(2)	3(2)	9(3)	
crossed/cross	4(3)	2(2)	8(2)			
offspring	6(5)	3(3)				
parent	7(2)	3(2)				
egg/sperm		5(3)			14(5)	7(5)
Math Words						
multiply	6(3)	11(5)	4(3)		3(2)	
probability						6(3)
denominator	3(2)					
equation		3(2)				
total		3(2)				
proportion		3(3)				2(2)
Possibilities		7(4)			7(3)	

Complex problem. In all groups, when talking about their problem solving process for the complex problem, students made frequent reference to the mathematical procedure of multiplying and to the biological entity, the gene. However, compared to the MCM Competent students, MCM Struggling students spoke only of a component (denominator) of the end product, while MCM Competent students also spoke of the mathematical goal of their process (finding possibilities or a proportion). Furthermore, while MCM Struggling and MCM Competent students both referred to the biological inputs (parents) and outputs (offspring) of the process of inheritance, only MCM Competent students referenced the mediating entities (eggs and sperm). This reference to the mediating entities was not solely due to competence at problem solving because Traditional Competent students also fail to reference eggs and sperm. Moreover, Traditional Competent students' biological words (recessive and dominance) were about describing the relationship of genes to one another (a relationship that is irrelevant to the question being asked) rather than describing the inheritance of genes from parents to offspring through eggs and sperm. Traditional Competent students, like MCM Struggling students, also did not refer to the mathematical goal of the question.

Unfamiliar problem. When students are asked to describe their problem solving process for the unfamiliar problem, the differences highlighted above become more apparent. The only biological and mathematical word used by more than two MCM Struggling students was the word "gene". MCM Struggling students made frequent reference to their uncertainty about how to tackle the unfamiliar problem, using phrases such as "I guess", "that's all I know" and "honestly I don't know this one". One student looked at the problem, thought about it, and then would not tackle it. Another student seemed to be writing down almost random numbers, as she expressed it, "I just wrote." Three MCM struggling students did complete the task using a problem solving process and representations, but two students, by their own admission, just did what they had done before and one jumped from one method to another without connecting them. MCM Struggling students had such difficulty explaining the rationale behind what they were doing that patterns in problem solving are hard to ascertain. Therefore, in the rest of the paper, for the unfamiliar problem, only MCM Competent and Traditional Competent problem solving processes will be compared.

As can be seen in Figure 14, Traditional Competent students, when describing their solutions for the unfamiliar problem, maintained their use of the descriptive word "recessive". However, other than the word "egg", which is provided by the problem and is part of their solution, they did not use any other biological word. In the unfamiliar problem, which is presumably less subject to memorized problem solving routines, several Traditional Competent students indicated that they were focusing on the mathematical conceptual target (calculating a probability). In contrast, MCM Competent students frequently mentioned the word gene – the biological entity that is getting packaged in to the eggs. Again, they were referring to not just the biological target of the problem, but the intermediaries that produce the result. Interestingly, the mathematical conceptual target (proportion) is no longer apparent, although the procedural word, "multiply" and one of the mathematical intermediaries (possibilities) for calculating that target are still used.

4.3.2 MCM competent students make connections between inscriptions and the scientific phenomenon

To determine whether eggs and sperm were really a meaningful biological concept for MCM Competent students or simply labels that they attached to a problem solving procedure, student responses were further coded for connections made between eggs and sperm and their inscriptions. As a reminder, if students did not make specific connections between egg and sperm and their inscriptions during their problem solving process, the interviewer asked "Are eggs and sperm represented anywhere?" Only connections that were specific to a particular object were coded as connected. For example, pointing to a gene combination in an egg/sperm table or Punnett square and saying that was an egg was coded as connected. Stating that the mom produced eggs and the dad produced sperm, but not being able to point to a specific object (drawing or number) in an inscription was coded as unconnected. Similarly, flatly denying the involvement of eggs and sperm in the problem solving process was also coded as unconnected. In general, only MCM Competent students make specific connections between inscriptions they use during problem solving and the entities involved in the scientific phenomenon (see column labeled Math-Science Connection in Figure 13).

Compared to the other two conditions, MCM Competent students in general also used more inscriptions during problem solving (Figure 13). Next we examined whether those inscriptions are mathematical or biological in nature and whether students made connections between those inscriptions.

4.3.3 MCM competent students use both biological and mathematical inscriptions during problem solving

Because descriptions of a problem solving process could be affected by students' vocabulary that has been provided to them through prior instruction or by their ability to explain themselves, we looked at the steps students took during problem solving as expressed in their inscriptions (including type and order) and their comments on those inscriptions. Keeping instruction constant, and varying on problem solving success, MCM Struggling students will be compared with MCM Competent students. Then, keeping problem solving success constant, and varying on instructional method, MCM Competent Students will be compared with Traditional Competent students.

Figure 15 shows the order (from left to right) of inscriptions used during problem solving. White indicates a mathematically oriented inscription (MCM equation, Probability Rules equation or a Punnett square that students have not connected to eggs and sperm). Dark grey indicates a biologically oriented inscription (Pedigree, a Punnett square that students have connected to eggs and sperm, and an Egg/Sperm listing (usually in table format)). Stippled shading indicates that it is unlikely that the inscription was biologically or mathematically connected for that student (see the description for Sage below). A heavy black bar indicates when the student stated an answer.

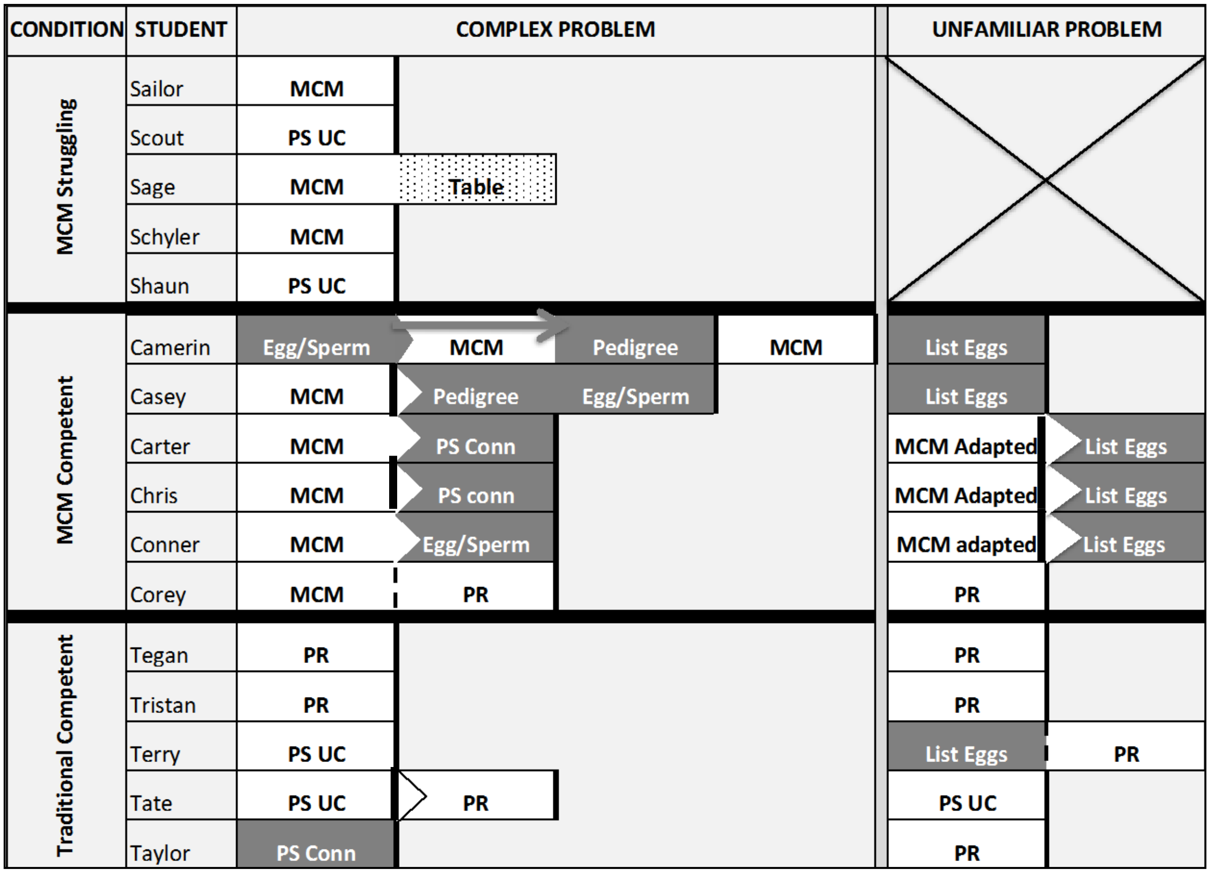


Figure 15. Order of inscriptions (left to right) during problem solving by student for each problem

MCM, Mechanism Connected Mathematics equation; PR, Probability Rules equation; PS UC, Punnett square that students did not connect to eggs and sperm; PS Conn, Punnett square that students connected to eggs and sperm; Egg/Sperm, listing of eggs and sperm. Dark grey indicates a biologically oriented inscription; white indicates a mathematically oriented inscription.

4.3.4 Inscription use while solving a complex problem

In general, MCM struggling students (top of Figure 15) tended to use only one inscription while solving the Complex Problem. The one exception was Sage, whose use of inscriptions was limited in other ways. She set up an MCM equation, wrote the number generated from the equation as the denominator and then, after rereading the question, said “I dunno, I do it like a table,” whereupon she drew a two column table with single letters from the genotype on each side. When the interviewer queried what the table meant to her, she stated, “I don’t know, it’s just like a way like how I’ll do it. I’ll, like, cross. It’s just like a better visual ‘n how I do it.” Later, when asked if eggs and sperm were represented on her paper, she gestures to where the column labels would be and said, “This would be like the egg and this would be the sperm I guess. I don’t know. It’s just how the table works I guess.” However, even with further questioning she does not indicate that one of the letters she wrote in her table represents an egg or sperm. Because it is doubtful that for this student, the objects in the table have an explicit connection to the egg and sperm entities in the biological phenomenon of inheritance, the inscription was labeled as a Table and shaded light grey to indicate it is not biologically or mathematically connected. Despite this one MCM Struggling student’s attempt to algorithmically use an inscription she had seen used in class in combination with an MCM equation, all other MCM Struggling students used only one inscription.

MCM Competent students (as shown in Figure 15), on the other hand, commonly used a table-like structure, called an Egg and Sperm Table, and they referred to the objects they listed as eggs and sperm. MCM Competent students also differed from MCM Struggling students by using more than one inscription in the problem solving process. In all but one case, at least one of the inscriptions they used was mathematically oriented (MCM equation) while another was biologically oriented. Moreover, information from one inscription was commonly used in another inscription, as indicated by the arrows connecting the inscriptions in Figure 15.

For example, Camerin set up a table listing eggs and sperm, then under the table she wrote the MCM equation with the number for each factor being the same as the number of objects in the column it was immediately under and the multiplication sign located under the column divider (Figure 16). She later explained that she got the denominator for the proportion in the following way "I got the four egg types and the two sperm types and multiplied it together to get eight different combinations". In this case, information from the egg/sperm table (a biological inscription) assisted with the mathematical inscription. Later, she used this same biological inscription (egg/sperm table to set up another biological inscription (a pedigree). She drew lines between the eggs and sperm as she wrote the offspring gene combinations in the pedigree. When asked what the lines meant, Camerin stated "It's pairing the egg and the sperm to produce the offspring."

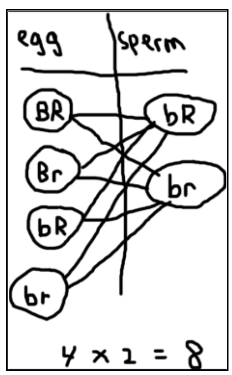


Figure 16. A tracing of Camrin's Egg/Sperm and MCM equation inscriptions

At other times, the information flow was between a mathematical inscription and a biological inscription. For example, after using the MCM equation to calculate that the number of possible offspring combinations would be eight, Casey immediately drew eight lines for her offspring in a pedigree inscription and then started to write in the genotype combinations. Carter makes a more explicit connection between the algebra in the MCM equation and the gene combinations listed in the biologically connected Punnett square when she says, “And then for like the dad, he only has like two different options for sperm [pointing to the algebraic notation 1*2 written under the genotype she has labeled as the dad’s] and those are those two [pointing to the two gene combinations for the dad’s sperm in the biological inscription]”. However, there was not a flow of information across inscriptions in all cases (indicated by the dashed line between the inscriptions in Figure 15). For example, Corey states that the information from his MCM inscription is not needed in his problem solving process with the Probability Rules equation. It is interesting that Corey was the only MCM Competent student to use a Probability Rules equation and not to use a biological inscription in his problem solving process raising questions that will be discussed later about affordances of different inscriptions for making connections.

From these comparisons of MCM Struggling and MCM Competent instructed students, it appears that Struggling students tended to only use one (mathematical) inscription during problem solving while Competent students tended to use multiple inscriptions, both mathematical and biological, and that Competent students made connections between the inscriptions. However, from only these two student groups, it is not clear whether the tendency to use multiple connected inscriptions is a feature of competence independent of instructional method or of an instructional approach that emphasizes building a mathematical model of the phenomenon of inheritance that is connected to both the entities and the mechanism of inheritance. Therefore, we compared two groups of students both of which can solve both problems of inheritance, but one has received MCM instruction and one has received more traditional instruction with siloed treatment of the biological and mathematical concepts.

The Traditional Competent students generally used only one inscription. One exception involved Tate, who for the complex problem used a single gene Punnett square to calculate the probability of a single event and then used this fraction as one of the factors he multiplied in his Probability Rules equation to calculate the probability of two independent events occurring together. One other Traditional Competent student (Terry) used a biologically connected Punnett square to solve the complex problem and no mathematical inscriptions. The relative lack of biological inscriptions and multiple inscriptions in the Traditional Competent group suggest that using multiple inscriptions, and biologically connected inscriptions to solve a problem is not a necessary hallmark of competence.

However, the complex problem, while requiring more steps than the unfamiliar problem, was more familiar to students. It may be that when solving that type of problem, students are simply applying the problem solving methods that they have been shown during instruction. We therefore examined Traditional and MCM Competent students problem solving on the unfamiliar problem for which they were unlikely to have received instruction. (As a reminder, students confirmed that the unfamiliar problem is not one that they have encountered before.)

4.3.5 Inscription use while solving an unfamiliar problem

The right half of Figure 15 shows a summary of the inscriptions (and the order) used by Traditional Competent and MCM Competent students when solving the unfamiliar problem. (MCM Struggling students are not included in discussions of this problem because their problem solving process lacked discernible patterns and they struggled to explain the rationale behind their process.) The label “MCM Adapted” in Figure 15 indicates that students adapted the MCM equation, using it to calculate the number of possible egg types from the number of alleles for each gene, instead of the number of possible offspring types. All but one MCM Competent student also uses a biological inscription whereas only one Traditional Competent student (Terry) uses a biological inscription. Moreover, Terry did not actually use the information from the biological inscription to solve the problem and it is questionable as to whether she was listing eggs in order to solve the problem. Right before Terry started listing eggs, she asked "Is this kind of what you were asking me, like?" This query probably referred back to a question the interviewer asked in Problem 1, "Are there egg and sperm involved in this anywhere?" Terry spent several minutes trying to figure this out and did not reach an answer that satisfied her. After listing only seven of the eight possibilities, she stops and explains verbally that "you have a fifty percent chance of getting a b, a big B, little b. a big R, little r or a big G, little g. Independently, like each is fifty percent, because it would like split like into the eggs. But they're sorting independently too. So, to get the proportion of her eggs… So…the total, I don't know. So I guess you have like a .25 percent chance of getting a little b and a little g together, I think." The interviewer asks her to write that down and she writes 0.25 and then above writes .5 x .5. She then goes on to say "I guess you could make all the eggs and then count them, but that takes like." After the interviewer asks "How many eggs do you think you would have to make?" Terry replies "Ummm, I don't know, a lot." When the interviewer asks Terry to explain where she got the .5 from, she talks about there being two of each gene so you would get fifty percent of each. Terry does not mention the listing of the eggs at all. This separation between the two inscriptions is demarcated in Figure 15 with a dashed line. Thus, it seems that most MCM Competent, but not Traditional Competent students, used a biological inscription to solve the unfamiliar problem. When a Traditional Competent student did have a biological inscription, it seemed to be addressing a previous query of the interviewer's instead of being used as a problem solving strategy.

A comparison of the three MCM Competent students who used both a biological and a mathematical inscription is revealing. All three students recognize a connection between these two types of inscription and use the information from the mathematical inscription in their biological inscription. Chris summarizes it for all three students when he says, "…so you get two genes from this, two genes from the r's and two genes from the g's. So you put that all together and you get 8. You get eight possibilities. I did my best, I could be wrong, but I did my best to put them all and these are all different possibilities of what can come out to the egg."

4.3.6 Connections between biological and mathematical inscriptions are used by MCM competent students in multiple ways

Having discovered that students were transitioning between biological and mathematical inscriptions, we examined their talk during and their descriptions of the transitions, to describe some of the reasons for switching.

Methodological affordances. Five MCM Competent students on at least one of the problems calculated the denominator of the proportion using the MCM equation and then switched to a biologically based method (such as a Connected Punnett square or an Egg/Sperm Table) to calculate the numerator. These students explained that they were switching either because the methods had different affordances, either personal (expressed as a preference for one method), or informational. Three MCM Competent students said for at least one transition they switched from one transcription to another because they preferred one method over another, especially for calculating one part of the proportion. Camerin, starts with an "Egg and Sperm

Table because it’s the easiest for me to understand”, moved to an MCM equation to calculate the denominator, and then to determine the numerator for the proportion ended with a “pedigree because those are also easier to understand than the actual equation.” She later said that she used a pedigree because, “just, for me, personally, pictures are more easier. They’re easier to understand than an equation.” Camerin later shows she can calculate the entire proportion (both numerator and denominator) mathematically for problem 1. In these instances, the students may or may not have recognized the different types of information provided by the different methods. However, four MCM Competent students who switched between biological and mathematical inscriptions showed a recognition that one method has affordances that the other method does not. For example, for problem 1, Conner explains, “It [the Egg/Sperm Table] is going to show me like what are the eight total outcomes. Like this [the MCM equation] just tells me there are going to be eight, this [the Egg/Sperm Table] shows me what they are, what they’re actually going to look like.” These students did not for that transition necessarily say they liked one method over another, rather that a particular method provided a particular piece of information.

Process checking. The recognition that mathematical and biological methods could provide different information was often (four out of five times), although not always, associated with process checking. During process checking, students are observed to use the information from one inscription to provide feedback on how they are progressing in problem solving when using another inscription. Three students used process checking in five different transitions across both the unfamiliar and the complex problems. As Chris indicated with his quote above, knowing how many possibilities there are makes it possible to determine if you have listed them all. In at least one situation, this knowledge of what the outcome should be helped a student overcome being stuck. Conner was struggling to solve the Unfamiliar Problem. He said, “It’s telling me here that I want to get, there's going to be eight different types for the female, but I don't know how to get those." The interviewer encourages him to explain his process and after a few utterances he stops and begins to work on the problem again,

> "Okay, I did everything with a capital B [All in these two lines said low voice, then drops to whisper] I did everything with [Silence 21 secs] [still low voice – to self] now, got to figure out that last one [Silence 19 secs] [Still low voice] Actually, I think I have my answer right here. It says how many eggs will contain big B and little g, err, little b and little g, I'm sorry. Uhh, so it's gotta be four under… 2 out of 8, ¼?"
>
> The interviewer asks him what made him keep trying and Conner said,
>
> "I just kind of looked at my work and thought I could add a bit more on and get eight, I don't know…"
>
> Interviewer: "So, what made you stop at eight? What made you decide to stop there?"
>
> Conner: "Well, the math says eight, but that's not the only reason, but, uhh, just going through, I really couldn't think of any more different types of combinations there could be."

Several key phrases suggests that Conner is process checking his listing of egg types against the answer he got from his math equation and that is helping him problem solve. For example, he mentions having to figure out the last one. The only way he could know there is a last one is if he is measuring how many eggs he has listed against how many he should have listed. Additionally, he says that when he looked at his work, when he was stuck, he thought he could add on a bit more and get eight. Finally, in the last quote, he shows that the mathematical and biological methods work in concert. He decides to stop not only because the math says there should be eight egg types, but because he can't think of any more egg types to add. In other instances of process checking, as exemplified by Chris' quote above, it is clear that students are assessing their progress using information from another inscription (usually the mathematical one), but this is not necessarily because they are stuck.

Answer checking. A third reason given for switching inscriptions was answer checking. While this is closely related to process checking, in answer checking a student completely solved a problem using one approach, stating an answer, and then resolved the problem using a different approach. Camerin, one of the students who switched transcriptions based on preference, later decides to check her answer. After calculating an answer as described above, she says, "I'm going to say 2 out of 8, but I'm going to check over it. [pause 10 seconds] Guess I can use the equation to check over it because it's got 2 (unintelligible word)." When asked why she used the math equation to check over her work, Camerin replied, "It was really the only other way that I could figure that I could know for sure that it was right. I didn't really know any other way to check it" indicating that she understood that both the biologically and mathematically based problem solving (as indicated by inscription use) should give the same answer. This differs from process checking because a student has given a final answer to the problem, whereas in process checking, students are checking their progress on the way to an answer. Camerin was the only student who explicitly stated that she was checking her answer. Interestingly, she calculated the denominator the same way in both problem solving paths and chose different paths for the numerator: the first one was biological, and the second one was mathematical.

4.4 GENERAL DISCUSSION

The results we have presented here suggest that not only does quantitative problem solving differ for competent and struggling students that reveals a useful role of making biological connections for successful quantitative problem solving, but that it also differs between groups of competent students who have been exposed to different instructional methods. We begin with a summary of the characteristics of each group's problem solving approach considering the kinds of resources students draw upon and the methods by which they select solution strategies.

Even though the Struggling students were exposed to a method of instruction that was designed to emphasize connection between the scientific phenomenon and the mathematical problem solving approach, this connection was not reflected in their problem solving efforts. The causes of this problem are taken up in the discussion. Here we focus on what it reveals about the mechanisms of MCM instruction. This group of students tended to use a single inscription during problem solving. While some students used an MCM equation and others used other inscriptions, they could not connect these inscriptions to the scientific phenomenon. Their descriptions of their problem solving attempts were characterized by words that described the beginning and end products of the scientific phenomenon and mathematical procedures, but did not indicate an understanding of either the scientific or mathematical concepts. Their disconnected, procedural approach to problem solving becomes particularly apparent when they face an unfamiliar problem. They either fail to develop a strategy or make unproductive attempts, doing what they did in the last problem or trying multiple disconnected strategies.

In contrast, the problem solving approaches of MCM Competent students reflected the connectedness between the scientific phenomenon and the mathematical approach designed in to the instruction. When solving familiar and unfamiliar problems, MCM Competent students used multiple inscriptions, typically including both biological and mathematical inscriptions. Moreover, all MCM Competent students used the MCM equation. MCM Competent students' descriptions of their problem solving were characterized by words indicating an understanding of the biological concept as well as the mathematical concept. They showed that they understood how their problem solving inscriptions are related to objects in the scientific phenomenon. Moreover, students recognized the connections between the biological and mathematical inscriptions. MCM Competent students switch back and forth between their inscriptions during problem solving, and they did so for multiple reasons, including personal preference, recognition of affordances of different inscriptions, process checking, and answer checking. This ability to switch is likely to lead to fewer mistakes and this ability provides an explanation of the prior finding that MCM instructed students outperform traditionally instructed students on solving complex or unfamiliar multiple choice questions.

The competent traditionally-instructed problem solvers took a substantially different approach to problem solving than did the MCM Competent students. Traditional Competent students' descriptions of problem solving were characterized by words indicating an understanding of the mathematical concept but not the biological concept. When solving a familiar problem, the traditionally instructed group tended to use only one inscription and that was mathematical in nature. Moreover, they fail to recognize a connection between the objects in their inscriptions and objects in the scientific phenomenon. On an unfamiliar problem, the focus on solutions and descriptive words that are mathematically oriented becomes even more apparent. Thus, although successful, these students had a fragile understanding and limited problem solving repertoire.

4.4.1 Leveraging mechanistic connections could facilitate conceptual understanding

The concept of joining eggs and sperm to produce offspring is one that was frequently expressed by MCM Competent instructed students, suggesting that they were cognizant of the underlying scientific mechanism behind formation of offspring during inheritance. An understanding of this mechanism could be seen to benefit problem solving when MCM Competent students drew out joining of eggs and sperm to determine what types of offspring were produced, cross checking with their prior calculation of how many types of offspring were produced. The determination of "what types" from the mechanistic understanding (facilitated by the biological inscriptions) generally determined the numerator of the probability expression and the determination of "how many types" from the MCM equation generally determined the denominator. However, even when Competent Traditionally instructed students vaguely recognized that eggs and sperm were involved, all but one student did not talk about eggs and sperm joining, but how genes were packaged into eggs and sperm (i.e., only part of the process, and not the critical last step for inheritance). Similarly, these students did not draw out joining of eggs and sperm. The contrast between these two groups of students suggests that leveraging mechanistic connections between inscriptions and the phenomenon as well as object connections may be important in facilitating conceptual understanding of the phenomenon represented by the quantitative problems. This in turn, may benefit quantitative problem solving by providing an alternative route to mathematical computation.

Taken together, these results suggest that social construction of linkages between objects and processes in the inscription to entities and mechanisms in the underlying scientific phenomenon allows students to synergistically engage in quantitative and qualitative problem solving pathways. Such synergistic behavior allows students to engage in productive problem solving behaviors such as checking answers and progress, and taking advantage of affordances offered by different pathways.

4.4.2 Key features of the MCM method contributing to productive quantitative problem solving

We postulated in the introduction that both the instructional methodology and the affordances of the MCM inscription might play a role in fostering productive problem solving behavior. We consider the following three features of the MCM unit: 1) Building a biology-connected inscription; 2) The affordances of the included inscriptions; and 3) Generating connections between inscriptions.

Building a biology-connected inscription. In the context of science practice, inscriptions are socially-constructed entities with complex layered meanings that are agreed upon by members of the community (Redish & Kuo, 2015; Roth & McGinn, 1998). Bowen and Roth showed that eighth grade students became increasingly sophisticated in their use of graphical inscriptions after participating in a unit that had students construct graphs through classroom discussion rather than only using them in a prescribed manner (Roth & Bowen, 1994). Others have also found that students' show improvement in their ability to interpret and construct graphs when graph construction and interpretation are used as part of a practice of socially constructing meaning while conducting investigations (Wu & Krajcik, 2006). A number of researchers have postulated that part of the difficulty students have connecting mathematical equations to the underlying phenomenon is because mathematical inscriptions are presented to students, ready made, instead of undergoing social construction of meaning (Redish, 2005; Redish & Kuo, 2015; Tang et al., 2011). The MCM unit is designed so that students socially construct the meaning of their algebraic equation and how it is connected to the scientific phenomenon it is modeling. However, it is unlikely that social construction alone could allow students to make meaningful connections between the inscription and the scientific phenomenon. An inscription must be able to be connected to the phenomenon. Too often, in inscriptions provided to students in science class, details of the scientific process have been left out of the inscription, thus making it hard to connect to the science (Redish & Kuo, 2015).

Affordances of problem solving inscriptions. Three different problem solving inscriptions for inheritance that are available to students (MCM equation, Probability Rules equation, and Punnett square) are postulated to vary in their affordances for easily connecting to the scientific phenomenon (see Figure 3). During the course of this study, these affordances in connecting to scientific phenomena were evident in Competent students' problem solving processes. The Probability Rules equation is most difficult to connect. One student who solved both problems correctly could easily connect eggs and sperm to the MCM equation, but, despite trying repeatedly, could not do so for the Probability Rules equation. In part, this difficulty likely occurs because the numerical objects within the Probability Rules equation are not designed to represent real world objects, but instead stand for a mathematical construct.

By contrast, the Punnett square is designed to represent the scientific phenomenon and also be a method for determining what types of offspring are produced from joining eggs and sperm. However, often during presentation to students some of the details are left out and so some students work out the connection and others do not (Moll & Allen, 1987; Stewart, 1983). Students who used the MCM equation along with the Punnett square made connections between the Punnett square and the scientific phenomenon, while with one exception, those who did not, failed to connect the Punnett square to the scientific phenomenon. In one case, this lack of connection caused one student (Scout) to apply the Punnett square algorithmically and inappropriately to the unfamiliar problem. Interestingly, one Competent Traditional student (Tess) could be seen trying to work out the connection when asked about it. She initially failed, and later succeeded. The behavior of this student underlines the issue with the Punnett square: it is not that connections cannot be made, but rather that students are often not exposed to those connections during traditional instruction. In addition, there is another problem with Punnett square regarding its limited usefulness for problem solving. In particular, as problem complexity increases, the complexity of this inscription also increases. As a result, increased complexity will often cause students to make greater errors during problem solving or switch to another method (Moll & Allen, 1987; Stewart, 1983). This problem of scaling is not true with either algebraic inscription.

Relating multiple inscriptions. The association between the MCM equation and sense-making with the Punnett square suggests that one other affordance of the MCM unit is multiple inscriptions that are related to one another. While the MCM equation was the only one designed to be built by students and designed to connect both scientific entities and mechanisms with the objects and processes in a mathematical equation, students were provided with two other inscriptions during the MCM unit. These inscriptions were drawings that showed the relationship between biological objects. During MCM Competent student problem solving, switching between mathematical and biological inscriptions was common and pivoted around objects that were represented in both sets of inscriptions. For example, Camerin wrote an MCM equation inscription under her egg/sperm drawings so that the numbers for egg and sperm types were aligned with their respective columns and then went on to determine the possible outcomes. Another student, Chris interrupted his listing of egg types to calculate how many possible eggs there could be using a modified MCM equation and then went back to listing eggs until he got to that number. To paraphrase Conner, the MCM equation tells how many there will be, the biological inscriptions show what they will be. Both in students' spontaneous descriptions and in their responses to the question of whether sperm and egg are represented, students showed that they understood that these shared objects had a connection to entities in the scientific phenomenon. It was clear that some students were also making mechanistic connections as well. Camerin wrote the multiplication symbol under the column divider between eggs and sperm in the Egg and Sperm Table and later drew lines across the column divider that as she explained represented joining of sperm and egg. Another student (Carter) referred to the Punnett square as another way of representing the multiplication of eggs and sperm in the MCM equation, as well as a way of showing joining eggs and sperm to produce offspring.

4.4.3 Applying MCM instruction beyond biology

The current work has shown that facilitating sense-making between quantitative problem solving and the scientific phenomenon changes student problem solving in ways that are likely to be beneficial. The method of modeling the scientific phenomenon through mechanism connected mathematics can be applied to scientific disciplines beyond biology. We noted in the introduction section how conceptualizing the relationship between force and acceleration as summed forces distributed over an object could facilitate student understanding. For example, instead of the traditional $F=m*a$ that leads to the oddly reversed impression that acceleration on an object causes a force (if students think about the meaning of the variables at all), students could derive an MCM equation that reads Acceleration = (Sum of Forces on Object)/Mass of the

Object. This more accurately reflects the mechanism behind the amount of acceleration that an object experiences: the sum of all the forces acting on the object distributed over its mass.

Similar strategies could also be used in chemistry. To illustrate, we propose a way to apply the MCM approach to more effectively teaching the complex and commonly memorized equation that describe the many factors contributing to Pressure, $PV=nRT$. In this form, the equation lacks meaning for many students and simply becomes a memorized algorithm. For example, the overall amount of pressure times volume (or nRT) does not correspond to any conceptual object or quantity, and rather is just a convenient calculation. However, if students were given the opportunity to derive an equation for pressure from experiences that led to an understanding of the equation and its terms, a richer conceptual understanding could be developed. Initially, or at lower grades, students could be asked to explore the definition of pressure as pressure = force/unit area. Then, the mechanisms behind changes in pressure could be explored and added to the equation. More particles will cause greater pressure, as will an increase in temperature, which causes each particle to move faster, so Pressure = number of particles * Temperature. Students could experiment with volume and pressure and come to understand that with the same number of particles at the same temperature, the amount of space available per particle determines the pressure because there will be more collisions. Initially, the R could be presented as a constant necessary to relate the terms. So at this intermediate stage, students would develop an MCM equation that Pressure=[(# of particles)*(Temperature)*(Relational Constant)]/Volume of Container. Later students could develop an understanding that R is a reflection of the average energy in the particles that is proportional to the average Force that will be exerted on the walls by the particles. In the end, by working with Mechanism Connected Mathematics, advanced students will develop an expert understanding of the mechanisms of pressure that relates both chemistry and physics, while high school age students will be able to understand the mechanisms governing changes in pressure.

4.5 CONCLUSIONS

Not all of the students exposed to MCM instruction were able to make connections between their inscriptions and the underlying scientific phenomenon, and these same students were not successful at problem solving. In addition, they did not show an awareness that eggs and sperm join to produce offspring even when they expressed that eggs and sperm were produced by the parents and that a pair of genes had to separate for this to occur. They also did not use multiple inscriptions and switch between them. As an observational study, we cannot isolate with certainty whether one of these differences is particularly important or even whether some other difference may have also been important. Since the MCM Struggling instructed students and the MCM Competent instructed students came from the same set of teachers, instructional differences are unlikely to account for the difference. The question becomes why did the MCM Struggling students fail to develop connected problem solving behavior. While it may be that the Struggling students lacked a key piece of biological understanding that enabled them to be successful, this does not fully explain why only one Struggling student attempted multiple inscriptions. Engle has shown that individual student engagement is also a key factor in students being able to transfer their knowledge in to new situations (Engle, 2006). One possible explanation is that the Competent, but not the Struggling students actively engaged in the social construction of the MCM equation and its connections to the scientific phenomenon and other inscriptions. In other words, MCM Struggling students were essentially provided the equation by their peers.

It is critical to point out that most students in traditional instruction are not successful with the types of problems that students were asked to solve in this study (Schuchardt & Schunn, 2016). We chose an extreme group of students to raise the likelihood of finding students that would be successful despite receiving traditional instruction. This raises the question of what prevents many students who receive traditional instruction from being successful at this type of problem solving. Although the current data did not directly examine this point, the current findings together with other literature on quantitative problem solving (Kuo et al., 2012; Taasoobshirazi & Glynn, 2009) suggest that they suffer from the fragility of using memorized algorithmic problem solving processes.

Many explanations have been put forth to explain how conceptual connections support increased facility with problem solving including the types of knowledge structures, the problem solving approach used by students, and the framing of the activity (Chi et al., 1981; Taasoobshirazi & Glynn, 2009; Tuminaro & Redish, 2007). Failure to see the meanings embedded in scientific equations can cause students to become stuck in their problem solving efforts (Tuminaro & Redish, 2007), accept incorrect answers even though they do not make sense (Hammer, 1994), fail to transfer from one situation to a related situation, or provide an incorrect answer because they are not filtering their problem solving process through their knowledge of physics (Redish & Kuo, 2015). The current study has extended these explanations to emphasize the importance of particular kinds of connections and to show how these connections influence the details of problem solving.

The results presented here go beyond simply extending to biology the prior finding in physics that fostering connections between mathematical problem solving methods and the scientific phenomenon can facilitate problem solving. Our results suggest that students instructed in a mechanism connected mathematics unit have a fundamentally different approach, suggesting a different concept of problem solving than traditionally instructed students. MCM Competent students generally appear to be solving a problem with a series of connected steps as opposed to Traditional Competent students who generally appear to be searching for a successful algorithm. Several aspects of MCM Competent problem solving suggest that they have a different concept of problem solving than MCM Traditional students. First, many MCM Competent students check for errors during the process of problem solving, while none of the Traditional Competent students exhibited this behavior. This suggests an approach based on figuring out an answer rather than assuming that once a mathematical approach is applied it will give an answer that is correct. Even more telling, when MCM Competent students checked for errors they generally switched inscriptions, usually pivoting around the biological entities that are part of the phenomenon of inheritance rather than rather than just relying on simple associations between paired representation. Such behavior is perhaps a reflection of their instruction that incorporated checking the derivation of a mathematical approach against the scientific phenomenon (Figure 9). This behavior also closely matches the ways in which some have described mathematics use in scientific practice (Figure 8) (Hume, 2009).

The MCM approach has the potential to be applied across the sciences to help students develop a conceptual understanding that supports problem solving unfamiliar and complex problems. It may be that some quantitative relationships are not easily transformed into fully conceptually approachable forms, especially in terms of conceptually justifying particular mathematical operators (e.g., why the operation is multiplication rather than addition in the torque rule). However, it is likely that even there having students consider the mechanistic plausibility of different equations will likely be helpful to conceptual understanding and more robust problem solving.

5.0 HOW MUCH TEACHER PROFESSIONAL DEVELOPMENT IS NEEDED WITH EDUCATIVE CURRICULUM MATERIALS? IT DEPENDS ON THE CONTENT DOMAIN

A large challenge facing wide-scale use of the Next Generation Science Standards (NGSS) is professional development of the existing teaching workforce (Reiser, 2013). It is an open question as to the amount and kinds of teacher support necessary to achieve student learning gains when implementing NGSS-aligned curriculum (Wilson, 2013), and how this varies across content domains within a discipline. Educative curriculum materials may support teacher learning on some content and thereby reduce the need for additional teacher professional development. In the context of an NGSS-aligned high school unit in genetics with extensively-developed educative curriculum materials, student results on assessments of science content administered pre and post unit implementation were examined across three conditions of teacher professional development. One condition (No PD) had no face-to-face professional development. The other two conditions varied by time spent on face-to-face PD: approximately 8 hours in the Reduced PD condition vs. 23 hours in the Extended PD condition. Students of participating teachers in all three PD conditions showed approximately equal gains in the domain of conceptual science content, suggesting the additional PD was not needed. However, learning in the domain of quantitative problem solving was lower in the Reduced PD and No PD conditions compared to the Extended PD condition. Combined, these findings suggest that the amount of required face-to face PD support that is necessary with educative curriculum materials may vary from none to over 20 hours depending upon the content domain.

5.1 INTRODUCTION

The publication of the National Research Council's Framework for Science Education, and subsequently, the Next Generation Science Standards, presents both a challenge and an opportunity for those involved in science education (Arkansas NGSS Review Committee, 2014; Bybee, 2014). These two documents call for multiple shifts in K-12 teaching and learning, such as from memorizing facts to building ideas, from isolated islands of knowledge to interconnected networks, and from learning about science to learning key concepts in science by engaging in scientific practices (Krajcik, Codere, Dahsah, Bayer, & Mun, 2014; Osborne, 2014). There are a number of efforts to develop new curricula aligned with the Next Generation Science Standards in multiple disciplines and for different grade levels (Roseman, Fortus, Krajcik, & Reiser, 2015). Some of these curricula have provided evidence of improvements in student learning over traditional methods of instruction (Schuchardt & Schunn, 2016; Plummer & Maynard, 2014).

It is clear that the large shifts in teaching and learning approaches embodied in these curricula are going to need to be supported by teacher professional development (Bybee, 2014; Doppelt et al., 2009; Reiser, 2013). Further, since science teachers have a variety of teaching and educational backgrounds and teach multiple grade levels, it is unlikely that a "one size fits all" professional development approach will be effective (National Research Council, 2015). Moreover, practicing teachers are better equipped to handle some of the changes associated with NGSS as compared to others (Arkansas NGSS Review Committee, 2014). Accordingly, the

National Research Council's Committee on Implementing NGSS has advised administrators to make sure that "professional development opportunities are structured to make effective use of teacher time and meet the teachers' needs. In general, this approach will require offering a menu of options and giving teachers some choices about how best to meet their professional development needs" (p. 49 (National Research Council, 2015). However, there continue to be open questions about the forms of professional development that are best suited to particular needed changes in teacher content knowledge and teaching practices (Wilson, 2013). Thus, as these reform curricula go to scale, there is a need for research on what kinds of teacher supports are going to be needed to support student learning through NGSS-aligned curricula. Here, we draw attention to different amounts of support required by different content domains. Borrowing from Gardner, we are using the term content domain to refer to the different "objects" (i.e., the content topics) which are studied as part of the content in a science discipline such as biology (Gardner, 1972). We focus on an old content divide in secondary science instruction—"conceptual" science vs. quantitative problem solving—that is brought in to greater relief with the NGSS call for increased integration of science practices and mathematics into science content instruction. We present here a study of the effect of variations in professional development supporting implementation of a NGSS-aligned curriculum, examining effects on student learning in those two different content domains. In so doing, we contribute to a growing literature base that provides evidence regarding the areas of student learning that will require more support in various forms.

5.1.1 Implementing NGSS: Critical areas needing support for teachers

While some changes in science instruction associated with NGSS will primarily affect curriculum designers (for example, the emphasis on vertical integration as students progress through science) or science education researchers (i.e. development of aligned assessments), other changes are particularly pertinent to teachers' ability to implement NGSS-aligned units in their classroom. These teacher-specific changes involve content knowledge and pedagogical knowledge (including pedagogical content knowledge) (Bismarck, Arias, Davis, & Palinscar, 2014; Wilson, 2013).

NGSS emphasizes depth of learning over breadth. This means that some teachers might find that they need additional conceptual science content support. For example, a review of NGSS by key decision makers in Arkansas suggested that many teachers at all grade levels are going to require a deeper understanding of content in order to help their students reach the depth of content understanding envisioned by NGSS (Arkansas NGSS Review Committee, 2014; National Research Council, 2015; Wilson, 2013).

Teachers are also going to need support to shift their practices so that they can help students engage in this in-depth content learning in a coherent way through the integrated use of science practices (Doppelt et al., 2009; Reiser, 2013). Within the NGSS classroom, students are seen as learning science not by being the passive recipients of facts or by mindlessly following scientific procedures, but instead by actively engaging in scientific practices (Bybee, 2014). Through practices such as asking questions, analyzing data, constructing explanation and engaging in argument from evidence, students can generate and refine models of scientific phenomenon that can be communicated to others (NRC, 2012). To assist their students in engaging in these practices, many teachers will need support on how to change their classrooms from teacher-centered explanation of facts to student-centered generation of concepts. This shift will have to include guidance on how to prompt students to generate productive questions and explanations, and how to guide data analysis and argumentation (Reiser, 2013).

Additionally, NGSS places an emphasis on interdisciplinary connections, between the focal discipline of science and other disciplines such as engineering, technology, English language arts, and mathematics (NRC, 2012). While engineering is intimately associated with all eight of the practices, technology and mathematics is most explicitly connected to Practice 3, Using Mathematics and Computational Thinking, and English language arts is more pronounced in Practice 8, Obtaining, Evaluating and Communicating Information (NGSS Lead States, 2013). However, connections to the Common Core State Standards in literacy and mathematics can be found across all of the practices (NGSS Lead States, 2013). This paper will focus on the intersection between science and the use of mathematical thinking. While some uses of mathematics are not very different from what is currently being done in science classrooms (e.g. applying ratios, percentages or unit conversions to measurement problems, or applying functions to represent and solve scientific problems), others are not as common, such as creating, testing, and revising mathematical functions to model data and phenomenon. This mathematical modeling of scientific phenomenon will require a greater conceptual understanding by teachers and students of both the mathematics and the science than a mathematics-as-tool, deliver-and-drill approach to solving quantitative science problems (Furner & Kumar, 2007; Offer & Mireles).

NGSS recommends that students develop core science content through application of scientific practices. When mathematical modeling is used as a practice to develop this core science content, there are two domains of student content learning that might be affected by teacher preparation: the core conceptual science content that is being developed through mathematical modeling; and the quantitative problem solving that is represented by the mathematical model.

The rationale behind having students engage in scientific practices to develop conceptual understanding of science is to improve student learning in science (NGSS Lead States, 2013). However, the types and amount of teacher support needed to achieve student learning gains in NGSS-aligned curriculum is not yet known. In this paper, we examine the effect of varying teacher support in implementing an NGSS-aligned curriculum centered on mathematical modeling on two domains of student content learning: 1) conceptual science content and 2) quantitative science problem solving.

5.1.2 Supporting teachers as they implement NGSS

In order to make these shifts in content and practice, in-service teachers are going to need access to various forms of effective professional development (Bybee, 2014; National Research Council, 2015; Reiser, 2013). Research and reviews on effective professional development have generally agreed that effective professional development (a) involves active learning, (b) has collective participation, (c) is embedded in subject matter, (d) is coherent, and (e) is of sufficient duration (Desimone, 2009; Garet, Porter, Desimone, Birman, & Yoon, 2001; Reiser, 2013; Wilson, 2013). The meaning of coherence has expanded over time to include both consistent with teachers' knowledge and beliefs, as well as coherence with school, state, and national policies and standards (Desimone, 2009; Reiser, 2013). There is some uncertainty over what constitutes sufficient duration (Desimone, 2009), although a review of nine studies indicated that professional developments with fewer than fourteen hours of contact time did not show changes in student achievement (Yoon, Duncan, Lee, Scarloss, & Shapley, 2007). However, there was little information about what additional supports, such as educative curricula materials or access to professional networks, were provided to teachers as contact time decreased. Furthermore, these studies were with elementary school teachers in non-science subjects. In this study, we will examine the effect of reducing contact time in the context of educative curricula materials that can be used in high-school science classrooms.

5.1.3 Taking professional development to scale

When talking about taking educational reform curricula to scale, the issue of time spent in professional development is critical, both in terms of demands on teacher time (National Research Council, 2015), and cost to the funding agency (Spillane et al., 2009). Professional development seeks to elevate human and social resources towards long-term gains, but concomitantly it requires an investment of physical, financial, and human resources in the form of time, money, space, and workshop leaders. Unfortunately, many educational systems are not equipped to provide large amounts of those resources (Spillane et al., 2009). To make NGSS more accessible in multiple contexts, and thus, truly equitable, solutions need to be found to the problem of how to provide professional development that elevates student learning while minimizing the investment of resources by local agencies. One potential solution to this problem is to change curricular materials so that they are more educative for teachers (Davis & Krajcik, 2005), helping teachers learn rather than just implement.

5.1.4 Professional development with educative curricular materials

The theory behind educative curricula materials is that materials that support teacher learning as well as student learning can promote educational reform by helping teachers teach meaningful content while ensuring that all students are successful (Davis & Krajcik, 2005). As applied to implementing NGSS, this means that curriculum materials should help teachers understand the scientific content at a deeper level, how to integrate the scientific practices, and the rationale for doing so (Davis et al., 2014).

Davis and Krajcik (2005) suggest teacher educative curricula materials can help teachers learn content, likely student responses to instructional activities, the relationships between units, the designers' rationale behind activities, and understanding of new pedagogies. Research suggests that teachers who use educative curricula materials do show changes in their science instruction, including using a greater number and more varied strategies to support learning and changes to teacher Pedagogical Content Knowledge (Cervetti, Kulikowich, & Bravo, 2015; Schneider, 2013). These studies conducted in science have not yet explored the effects on student learning, and in general, studies that explore the effects of educative curricula materials on student learning are not as common.

It has been suggested that educative curricular materials might be more effective if used in conjunction with other forms of support (Davis & Krajcik, 2005; Stein & Kaufman, 2010), such as professional development workshops. In the current study, we look at the effect of adding different amounts of face-to-face professional development, all in the context of teachers provided with educative curricula materials. Knowing the effect of variations in teacher supports will help policy makers, curriculum designers, and administrators make decisions about investments in these additional supports as NGSS curricula moves to scale.

In our study, we chose specifically to look at student learning rather than teacher outcomes. One reason for the focus on student learning is that the effect of professional development on student learning is under-investigated (Dede, Ketelhut, Whitehouse, Breit, & McCloskey, 2009; Doppelt et al., 2009; Luft & Hewson, 2014; Yoon et al., 2007). In general, very few studies on professional development in science have looked at student learning gains (Scher & O'Reilly, 2009; Wilson, 2013; Yoon et al., 2007), and some of these studies confound PD effects with intervention effects (e.g., Marek & Methven, 1991; Radford, 1998). Further, few of the studies involved secondary instruction, where the conceptual content demands and quantitative problem solving goals of instruction are high. Additional distinctions of kinds of student learning gains are rarely examined, potentially hiding important variation across areas needing differential support. For example, some researchers measure general science content knowledge (e.g., Diamond, Maerten-Rivera, Rohrer, & Lee, 2014; Doppelt et al., 2009; Radford, 1998) and others measure overall knowledge about specific units (e.g., Heller, Daehler, Wong, Shinohara, & Miratrix, 2012). Educative curriculum materials may be sufficient for content more closely tied to the units and thus less in need of additional professional development support.

Another reason for our focus on student learning is that NGSS is primarily focused on improving student science learning (NGSS Lead States, 2013). For example, in a document published by NGSS justifying the need for science standards, three of the four reasons cite the lagging achievement of US students in mathematics and science, and the need to ensure science and technological literacy of all students (Next Generation Science Standards, 2016). In clarifying the vision of the NGSS, the Guide to Implementing the NGSS states that the expectation of implementing the NGSS is that “more students and a more diverse group of students will want to continue their education in these areas to become scientists or engineers and, as citizens, will more deeply understand the processes and core ideas of science and engineering" (p. 10). While the guide acknowledges that science educators will need to change, it frames this change in the context of achieving the ambitious targets for student learning in science. Given this context, student learning gains become not just a crucial measure of the success of an NGSS curriculum, but also a relevant part of understanding the effects of teacher professional development.

We acknowledge that the effect of variants in teacher PD on student learning is not direct and has the potential to be affected by many intervening steps; research that studies these intervening steps is also important. However, research that only studies these intervening steps and never considers student learning is incomplete, and a research effort that studies how PD changes teachers, documents the changes in teaching, and examines the effects on students would necessitate a very large investment of resources (Luft & Hewson, 2014). Before such large scale studies are conducted, it has been recommended that initial studies are needed to understand how variations in PD affect different aspects of student learning (Luft & Hewson, 2014). Thus, the purpose of this study is to examine the student effects aspect of the larger research question of effects of teacher PD. Further, from a practical perspective, knowing the effects on student learning is likely to be a particularly salient question, especially research that helps district and school decision makers make sensible investments in teacher PD based on the context (e.g., for different disciplinary content domains).

5.1.5 Study context—Scaling strong learner outcomes with more feasible resources

This study is part of a larger research project that created an NGSS-aligned unit for biology and investigated the effects on student learning. Across the project, an NGSS-aligned curriculum and the associated teacher educative curriculum materials were iteratively developed, implemented, improved, and retested. After multiple years of pilot testing and major revisions, the version of the curriculum unit and teacher educative materials studied here was tested with varying supports across two years. During the first year of the study, student learning in conceptual science content and quantitative problem solving was compared to a comparison group that had not implemented the curriculum. Large student gains relative to the comparison group were observed in both categories of student science content learning (Schuchardt & Schunn, 2016). However, implementing teachers were supported with an average of twenty-three hours of face-to-face professional development in addition to the use of the educative curricular materials. This amount of professional development is likely not sustainable as the curriculum moves to scale. Therefore, in the second year of the study, we tested the effect on student science content learning of varying the professional development both in terms of time invested and the supports provided to teachers. The rest of this section describes the unit and the prior student learning outcomes obtained with extensive teacher PD.

The NGSS-aligned unit serving as the context of this study is a complex and multifaceted investigation of genetics/biological inheritance. As the focus of this study is on professional development in a unit consistent with NGSS, the unit description will focus only on the key features that distinguish this novel approach from the traditional approach to instruction in inheritance (which was verified with journals kept by the traditional instruction teachers). This NGSS-aligned inheritance unit was situated within the context of an engineering design problem and asked students and teachers to develop mathematical representations of genetic processes in an iterative cycle.

As written, the NGSS-aligned unit contained both differences in practices and differences in scientific content compared to how genetics has traditionally been taught (Table 10) . The unit included most of the key NGSS-aligned practices, and most intensely: (a) engineer solutions to a problem; (b) analyze data; (c) develop and refine models; and (d) argue from evidence. In addition, as specified in NGSS, these practices were tightly integrated with and used to develop the content that students were expected to learn, as opposed to highly scripted processes that are disconnected from content learning.

Table 10. Comparing instruction in science content and practice in Traditional Instruction and the NGSS-aligned curriculum

	Traditional	**NGSS Aligned**
Inheritance Laws	Memorized facts given by teacher (e.g. Law of independent assortment)	Developed as students make sense of and mathematically model data on inheritance outcomes
Probability Calculations	Memorized equations for combining probabilistic events (e.g. “AND” implies multiply)	Developed as students use data on inheritance outcomes and knowledge about mechanisms of inheritance to model the scientific phenomenon and build an understanding of mathematical relationships between event space and outcome space

The scientific content covered in the unit was also subtly but importantly different from the content contained in traditional instruction, given the NGSS-based emphases on greater depth, connections to mathematics, and development of content through engagement in scientific practices. First, instead of learning inheritance laws as rote facts, the NGSS-aligned unit asks students to develop inheritance laws through class discussion as they make sense of and mathematically model data on inheritance outcomes. Second, the analysis and modeling activities are designed so that students are asked to develop an understanding of probability in inheritance as the proportion of the total outcome space occupied by the event space (e.g., how many of the desired offspring types as a proportion of all possible offspring from the given parents). This contrasts with the focus in traditional inheritance instruction on memorizing the laws for combining probabilistic events (e.g., “AND” always implies multiply).

As noted above, students implementing this curriculum were found to show large gains in both conceptual understanding of inheritance and ability to solve complex quantitative inheritance problems (Author, 2016). But the improvements that were obtained in student learning relative to traditional instruction occurred in the context of a large investment in resources for extensive face-to-face professional development. Because of concerns of equity and practicality when taking a reform curriculum, such as this, to scale (National Research Council, 2015; Spillane et al., 2009), we wanted to test the effect of reducing, and even eliminating, face-to-face professional development on student learning, while continuing to provide educative curricula materials. By comparing the effects of differential teacher support on student learning in two content domains: conceptual science content and quantitative problem solving, we hoped to gain some insights on how much teacher support is necessary to realize gains in student learning. We hypothesized that aspects of change more foreign to biology instruction (e.g., making heavier use of another discipline like mathematics) will require greater support to obtain improved student learning outcomes, either because teachers are more willing to implement the changes that reside within a knowledge comfort zone or because teachers are better able to effectively implement reform when their supporting knowledge and skills are more robust.

5.1.6 Research questions

The overarching research question is: How much teacher support is necessary to achieve robust student learning gains following instruction in an NGSS-aligned unit in different science content areas (i.e, conceptual science content and quantitative problem solving)? We contrasted: 1) the full amount of PD that was iteratively developed in the early phases of the project to match the likely full amount many teachers could fit into their calendars in a given year (i.e., 20-25 hours); 2) a greatly reduced amount that district officials typically assign to address PD needs (i.e., one full day); and 3) the amount of teacher PD that typically happens in the US without special allocation of resources (i.e., none).

5.2 METHODS

5.2.1 Participants

Over the course of two years, twenty-four teachers were recruited from primarily urban and suburban school districts surrounding two metropolitan areas located in midwestern states. All teachers were compensated for their participation in the study. Recruitment was done through a flyer distributed via regional instructional support organizations soliciting teachers to attend a two-hour information session on implementing a unit in biology aligned with NGSS. During the first year of the study, after attending an information session that provided an overview of the unit, six teachers participated in all 23 hours of face-to-face professional development sessions and therefore are included in the Extended PD condition. During the second year of the study, the Extended PD was not an option because an investment of twenty-three hours in PD had been determined to be unfeasible for most school districts. Therefore, after the information session, the twelve teachers who volunteered to implement participated in one of two conditions: no face-to-face professional development (No PD, four teachers) or 8 hours of face-to-face professional development (Reduced PD, eight teachers). Three of the teachers in this Reduced PD condition did not have the option to participate in the No PD condition because they were participating as part of a continuing education program for their regional educational organization. The remaining nine teachers were asked which condition they would prefer to be in when they signed up; next, changes were made in condition assignment (with teacher consent) to insure a balance among these nine teachers by student and teacher characteristics across the Reduced PD and No PD conditions. In all conditions, teachers were provided with the same educative curriculum materials.

All of the classes included in this study were first year high school biology classes taken by 9th and 10th grade students, the most typical years for implementing high school biology in the US. Overall, all three groups were well matched based on teacher experience, teacher education, and school characteristics. Almost all of the teachers had either a masters or undergraduate degree in biology (Table 11), and most had been teaching for eleven or more years. The student characteristics in Table 11 illustrate both the diversity of contexts studied and strongly overlapping distributions across conditions—in the US, whether students qualify for free or reduced cost lunch is used as the primary indicator of socio-economic status. Statistical analyses of the student data include student and school characteristics as control variables.

Table 11. Teacher and student characteristics and professional development content of each professional development condition

Characteristic	Extended	Reduced	NoPD
Teachers			
Number of Teachers	6	8	4
Number of Teachers Reporting Educational Information	6	7	4
Number of Teachers with Undergraduate or Masters Degree in Biology	6	6	3
Number of Teachers with Masters Degree in Biology	1	1	0
Number of Teachers with Masters Degree in Education	1	4	2
Students			
Number of Students	265	377	162
Mean Percent of Students Qualifying for Free and Reduced Lunch(SD)	32 (23)	55 (23)	61 (26)
Mean Conceptual Science Content Pretest Score(SD)	36 (4)	32 (7)	35 (5)
Mean Quantitative Problem Solving Pretest Score(SD)	38 (11)	32 (9)	29 (7)
Professional Development			
Mean Total Number of Hours of PD	23	8	0
Mean Number of Hours of PD on Content	10	5	0
Mean Number of Hours of PD on Pedagogy	10	3	0
Mean Number of Hours of PD on PCK	3	0	0

5.2.2 Professional development conditions

In the Extended PD condition, all teachers received extended face-to-face professional development consisting of a weeklong summer workshop and two follow-up sessions during implementation of the unit. In the Reduced PD, teachers had fewer face-to-face professional development hours than the Extended PD condition.

The face-to-face professional development in both years of the study had the critical features of high quality professional development identified in the research literature: (a) active learning, (b) collective participation, (c) embedded in subject matter, and (d) coherent with NGSS (Desimone, 2009; Garet et al., 2001; Reiser, 2013; Wilson, 2013). The fifth characteristic, of sufficient duration, is what is being tested between years one and two: what counts as sufficient duration for which content areas (science content versus quantitative problem solving) in the context of educational curricular materials.

During both years, teachers in the PD workshops engaged in the NGSS-aligned unit as learners, participating in both small-group and whole-group discussions to develop their conceptual understanding of the material covered in the unit. These sessions were conducted in a way that was coherent with NGSS practices (NRC, 2012) with the workshop leaders acting as teachers and modeling the pedagogical practices that teachers would be expected to enact with their students. Two workshop leaders conducted the PD in the second year, and they were two of the four workshop leaders who conducted the PD in the first year. In both years, leaders had expertise in the biological sciences and pedagogy, participated in the design of the NGSS-aligned unit, and had multiple years of prior experience leading extended PD workshops around reform instruction in science.

Based on logs of teacher attendance at each event, face-to-face professional development during the first year of the study (Extended condition) averaged twenty-three hours across teachers. This time was divided between activities focused on pedagogy and content (see Table 11). Pedagogy was defined as any activity that focused on how the unit was designed to support student learning (i.e. the role of the engineering challenge, the relationship of the unit to NGSS and state standards, the role of multiple representations) or discussing teaching practices (i.e. facilitating student discourse, reflection on teaching, examining aligned and nonaligned enactments of unit instruction). Professional development activities that focused on such pedagogical aspects took approximately ten hours. Teachers also learned about the content of the unit by experiencing the unit as learners with the workshop leaders in the role of teachers. These content learning activities took about ten hours and were approximately evenly divided between a focus on learning conceptual science content (the genetic processes of meiosis and fertilization) and quantitative problem solving (solving genetic probability problems). In the NGSS-aligned inheritance unit, these two science content domains, conceptual science content (meiosis and fertilization) and quantitative problem solving (genetic probability) are taught simultaneously and synergistically rather than in separate silos. While particular activities might have more of a focus on one aspect of this knowledge, it is not possible to tease apart the exact amount of time spent on each science content area given the fluidity and integrated nature of discussions. An additional three hours was spent on developing pedagogical content knowledge (PCK, Shulman, 1987) around quantitative problem solving where teachers analyzed commonly seen student errors when solving genetic probability problems and developed pedagogical strategies to facilitate student learning.

As noted earlier, prior investigations of learner outcomes from the first year of the study indicated that twenty-three hours of face-to-face professional development combined with educative curricular materials was sufficient to produce large gains in student learning (Schuchardt & Schunn, 2016). We seek here to determine whether eight hours of teacher PD (a much more scalable quantity of PD) or relying on only the educative curriculum materials is also sufficient to generate comparable learning gains in their students in different content areas.

In order to sensibly shorten the face-to-face professional development during the second year of the study, it was necessary to make changes to the amount of time spent on both developing content knowledge and pedagogy (Table 11), with the greatest percent reduction on pedagogy (from ten hours to three hours on how elements of the unit were designed to support student learning) and less of a percent reduction on content learning (from ten hours to five hours, carried out in the same way as in the first year of the study). Time spent on developing pedagogical content knowledge in quantitative problem solving was eliminated (Table 11).

5.2.3 Curricular materials provided to teachers

All teachers were provided with student worksheets and manipulatives needed to implement the NGSS-aligned unit. Student worksheets and manipulatives were the same for all three implementing groups, as was the instructional scope and sequence.

Teacher support materials divided the overall unit into fourteen separate sections called tasks. The support materials provided an overview of the unit, content and scientific practice goals, and situated each task relative to the material that immediately preceded and followed.

The materials were designed to be educative, providing information about student ideas and teacher pedagogical practices. (An example of the materials is shown in Figure 17.) To support this claim about the educative nature of the materials, a representative set of materials from six of the fourteen tasks were analyzed for educative properties: two tasks dealt primarily with conceptual science content (meiosis and fertilization), two with quantitative problem solving, and two with both conceptual science content and quantitative problem solving. The criteria for educative quality were those used by Beyer, Delgado, Davis, and Krajcik (2009) in their analysis of teacher supports for high school biology curriculum. First, each task was analyzed holistically to determine which teacher knowledge domain and category could be represented (as defined by Beyer et al.; see below for definitions and examples of each). Then the teacher support materials were analyzed for each task to determine if the educative criteria for those knowledge domains were present (e.g., helping teachers use approaches for collecting and analyzing data, helping teachers use representations of scientific phenomenon with students). To increase independence and validity of results, the coding was completed by a person (the first author) who was familiar with the unit, but had not developed the teacher support materials.

Explain why collection/analysis approaches are necessary

NGSS Science Practice 4: Analyzing and Interpreting Data

"A major practice of scientists is to organize and interpret data through tabulating, graphing, or statistical analysis. Such analysis can bring out the meaning of data—and their relevance—so that they may be used as evidence." (NGSS, 2013)

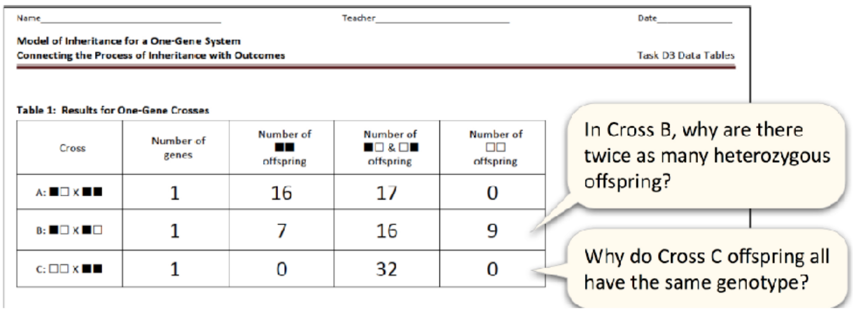
Students will analyze and interpret data by

- comparing simulation results across groups,
- pooling their results and comparing pooled data with their own results, and by
- using ratios to describe and draw conclusions about relationships between data groups.

Help teachers use approaches for analyzing data with their students

Purpose: To help students make sense of data and use it to check their predictions.

- Teacher hands out Task D3 Data Tables, and tells students the results in Table 1 come from actual gecko crosses.
- Students examine the data in Table 1.
- Teacher uses questions to help students make sense of the breeding results. (See examples in the image below.)


Name__________ Teacher__________ Date__________

Model of Inheritance for a One-Gene System
Connecting the Process of Inheritance with Outcomes
Task D3 Data Tables

Table 1: Results for One-Gene Crosses

Cross	Number of genes	Number of ■■ offspring	Number of ■□ & □■ offspring	Number of □□ offspring
A: ■□ x ■■	1	16	17	0
B: ■□ x ■□	1	7	16	9
C: □□ x ■■	1	0	32	0

Help teachers use particular communication approaches and representations with their students

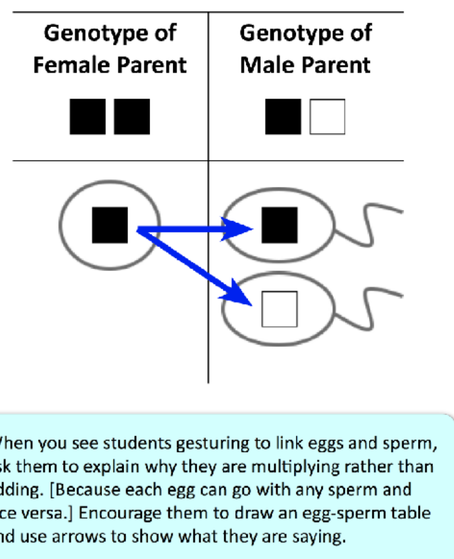


Figure 17. Excerpts from the educative curricula materials, illustrating some of the supports provided

The curricular materials addressed all three teacher knowledge domains from the Beyer et al. criteria: Pedagogical Content Knowledge (PCK) for Science Topics, PCK for Scientific Inquiry, and Teacher's Subject Matter Knowledge. Further, they provided support in eight out of the nine specific categories, including: (a) engaging students with topic-specific scientific phenomena, (b) using scientific instructional representation, (c) anticipating and dealing with students' ideas about science, (d) engaging students in questions, (e) engaging students with collecting and analyzing data, (f) helping students make explanations based on evidence, (g) promoting scientific communication, and (h) development of subject matter knowledge (Beyer et al., 2009). The ninth category, supporting teachers in engaging students in designing science investigations, was not relevant as students did not design science investigations in this unit; instead they analyzed provided data and engineered solutions. The type of support provided included both providing implementation guidance and rationales (Beyer et al., 2009).

The educative curricular materials were located online and provided a forum where teachers could ask questions, post student work, and share ideas. Access data showed that teachers logged in regularly during implementation to access the provided materials (a mean of 38 times, ranging from 16 to 80 times across teachers), but there was little variation across groups. Only a few of the teachers engaged in online discussions or posted revisions, and those who did, did so infrequently.

5.2.4 Student assessment

The focus of this investigation is the effect of different levels of PD on student content learning gains in: (a) Conceptual Science Content, and (b) Quantitative Problem Solving. Questions that assess conceptual science content cover the processes involved with transmission of genes between parents and offspring (meiosis and fertilization) (Figure 18). Questions that assess quantitative problem solving ask whether students can determine the probability of a particular outcome in a genetic context (for example, the probability that an offspring will contain a specific set of genes). Our prior results revealed that students showed gains from the NGSS-aligned unit in the quantitative problem-solving domain for questions involving complex genetic probability (two or more genes), but not simple genetic probability (one gene). Since the intent here is to see whether these gains are maintained when the amount of teacher support is reduced, only genetic probability questions involving two or more genes were included in this analysis.

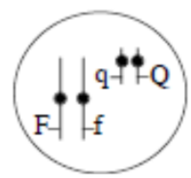
<table>
<tr><th colspan="2">Categories</th><th>Number of Questions</th><th>Example Question</th></tr>
<tr><td colspan="2" rowspan="2">Conceptual Science Content</td><td rowspan="2">7</td><td>The genotypes of the sperm from one male and the genotypes of the eggs from one female are shown below.
Male Sperm: FQ, fQ
Female Eggs: FQ, fQ, Fq, fq
Which answer lists all the possible genotypes that could be expected in the offspring:
a. FFQQ, ffQQ
b. FFQQ, FfQQ, FFQq, FfQq, ffQQ, ffQq
c. FFQQ, FfQQ, FfQq, ffQq
d. FFQQ, FFQq, FFqq, ffQQ, ffQq, ffqq, FfQQ, FfQq, Ffqq</td></tr>
<tr><td>In the germline cell below there are two pairs of chromosomes on which are shown the locations of two different genes. F and f represent two different alleles (versions or variants) of one gene, and Q and q represent two different alleles of another gene. If this cell divides <u>normally</u> to produce sperm, what are the possible sperm genotypes?

a. F, f, Q, q
b. Ff, Ff, Qq, Qq
c. FQ, fq, Fq, fQ
d. Ff, Qq, FQ, fq, Fq, fQ</td></tr>
<tr><td rowspan="3">Quantitative Problem Solving</td><td>Simple</td><td>3</td><td>In dogs, the gene allele (e) for drooping ears is recessive to E for erect ears. A male dog with genotype Ee was mated to a female dog with genotype ee and gave birth to a litter of 10 puppies. What is the expected proportion of drooping-eared puppies (ee) in the litter?
a. ¼ c. 1
b. ½ d. Don't know</td></tr>
<tr><td rowspan="2">Complex</td><td rowspan="2">7</td><td>If organisms of type BbSs and type bbSs are crossed, what is the probability that the offspring would be BbSs?
a. 1/16 d. 1/2
b. 1/8 e. 9/16
c. 1/4</td></tr>
<tr><td>Given a female with the genes: AaBbCc, what proportion of her eggs will contain genes "a" AND "b"?
a. 1/4 c. 1/2
b. 3/8 d. 2/3</td></tr>
</table>

Figure 18. Examples of questions on pre and post assessments

Because previously published assessments did not contain a sufficient number of questions in each category, the pool of questions was constructed by aggregating questions from these tests (Adamson et al., 2003; Blinn et al., 2002; delMas et al., 2007; Garfield, 2003; Nebraska Department of Education, 2010; "Project 2061: AAAS science assessment beta," 2013; Tobin & Capie, 1984; C.-Y. Tsui, 2002). The posttests had a mean KR-20 of .72 (mean discrimination = .46; mean difficulty = .50).

Teachers administered the assessments before and after instruction in inheritance following a matrix sampling protocol to allow for broad coverage of the conceptual content but not consume two full class periods for testing. In other words, there were multiple pretest versions and multiple posttest versions that teachers distributed randomly within each of their classes. From this method of testing, analyses focus on composite scores across students for each question rather than individual student scores aggregating across questions. Details of each specific statistical analysis procedure are presented within each relevant Results section.

5.3 RESULTS AND DISCUSSION

5.3.1 Equivalence of pretest scores across the three professional development conditions

The means and standard deviation of pretest scores by teacher for each content domain are shown in Table 11 and reflect the wide range of student backgrounds. Within each of the content domains, pretest scores were examined for statistically significant differences across the implementing conditions using a one way between-subjects ANCOVA conducted on the pretest question means for each teacher’s students, with professional development condition as the independent variable and free and reduced lunch (FRL) as the covariate. All required statistical assumptions were met (e.g. normality, homogeneity of variance, outliers, and independence).

While a marker of socioeconomic status, percent of FRL students did not show significantly different means across conditions, the standard deviations for each condition was quite large. Therefore, to err on the conservative side, FRL was kept as a covariate on the tests of differences in learning gains across conditions.

For Conceptual Science Content and Quantitative Problem Solving, pretest means adjusted for FRL were not significantly different across professional development conditions, $F(2,14) = .11.$, $p=.90$; $.F(2,14) = .59$, $p=.57$. This analysis shows that students in the three groups (Extended PD, Reduced PD, and No PD) had similar content knowledge in inheritance prior to instruction.

As noted earlier, because pre and post-tests involved matrix sampling, the data is analyzed for generalizability across questions rather than across students: a mean for each question was calculated for each teacher pre and post instruction. A change score for each question was calculated by subtracting the pre instruction mean from the post instruction mean. These question change scores were then aggregated within each content domain to provide a mean change score for each teacher. To remove the nuisance variance associated with initial differences in pretest scores across teachers, statistical analyses were conducted on these mean change scores. [2]

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[2] A similar pattern is found if the analysis is conducted on the difference of mean posttest and pretest scores by content area for each teacher.

Then, to examine consistency of change scores across teachers within each professional development condition, one-way between-subjects ANCOVAs (with FRL as covariate) were conducted on the teacher change scores for each content domain. All assumptions were met, (e.g. normality, homogeneity of variance, outliers, and independence). Two planned contrasts were performed for each content domain: (a) Reduced PD against Extended PD to determine whether reducing the number of face-to-face PD hours for teachers affected student learning, and (b) Reduced PD against No PD to determine whether educative curricular materials alone were sufficient to achieve student learning gains. Statistical test results and effect sizes (Cohen's *d*) are presented in the relevant figures.

5.3.2 Effect of reducing face-to-face PD on conceptual science content

In the Reduced face-to-face PD condition, where teachers had fewer overall hours of face-to-face PD compared to teachers in the Extended PD condition, the effect size was small and not statistically significant ($F(1,14)=0.07$, $p=.8$, 95% CI [-21, 16], Figure 19A). This result is useful for supporting the claims of equivalence of students and teachers across these two conditions.

To determine whether having any face-to-face PD support for teachers was necessary for student learning gains to occur in the NGSS-aligned unit, student performance in the Reduced PD group was compared to that in the No PD group. The lack of PD had a moderate effect on the mean performance of the NoP D group, but the mean was not significantly different from the Reduced PD group ($F(1,14)=0.6$, $p=.4$, 95% CI [-26, 11], Figure 19A). Moreover, the No PD group showed gains in learning pre to post, whereas students who receive traditional instruction had shown no learning gains on this measure (Figure 19A, Schuchardt & Schunn, 2016). Combined, these results suggest that educative curricular materials alone were sufficient to produce most of the student learning gains from using an NGSS-aligned curriculum in core science content, but does not entirely rule out small additional beneficial effects of having some face-to-face PD.

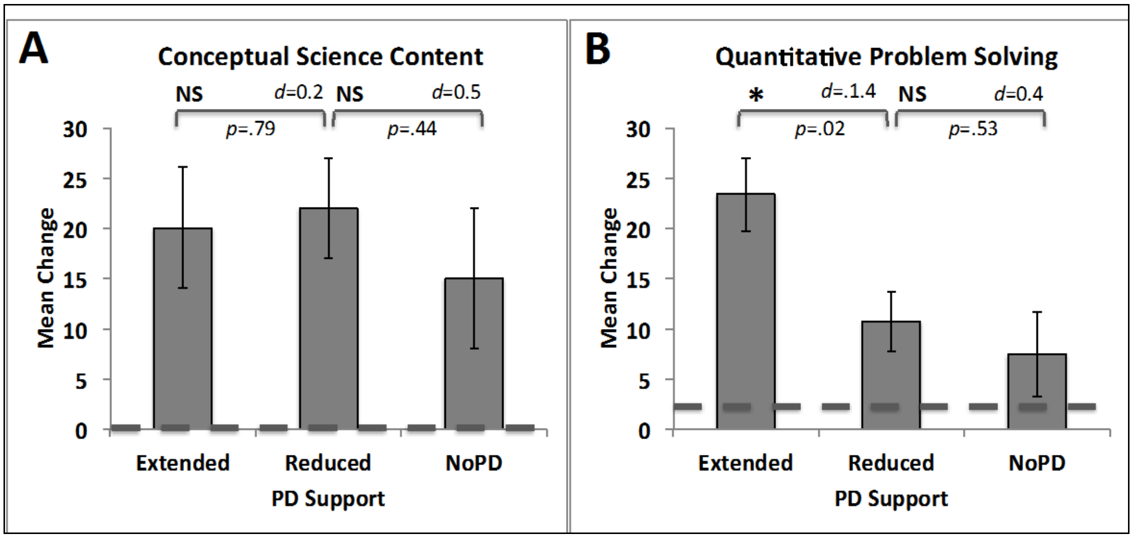


Figure 19. Mean pre-post change in each content domain

(A: Conceptual Science Content, B: Quantitative Problem Solving) as a function of teacher PD condition, with SE bars. Statistical significance test and effect size details are presented for each planned contrast. The dotted lines show the mean pre-post change in each content area for a group of comparison teachers using a traditional curriculum (Schuchardt & Schunn, 2016).

5.3.3 Effect of reducing face-to-face PD on quantitative problem solving

The time spent on training teachers on quantitative problem solving was also reduced between the Extended PD and Reduced PD conditions. This reduction in PD support had a large effect on pre-post gains, resulting in a significant decrease in teacher means of student change in quantitative problem solving (F(1,14)=6.5, p=.02, 95% CI [2, 23], Figure 19B). Comparison of the Reduced PD group with the No PD group showed that removing all face-to-face instructional support had only a small effect and did not result in a significant further drop on learning of this content domain (F(1,14)=0.4, *p*=.5, 95% CI [-14, 8], Figure 19B).

5.3.4 Other variables do not predict student content learning gains

An indicator of general teacher/student learning ability was assessed across conditions to determine whether there were significant variations and whether any variation might be responsible for the observed student learning gains in science content. Simple quantitative problem solving (single gene probability) showed no significant difference in learning gains between the students of implementing and comparison teachers. Moreover, the effect size of implementation was -.02. Thus, it is possible to use the learning gains in simple quantitative problem solving as an indicator of general student/teacher learning ability that is not biased towards either teaching method. The difference across PD conditions between the mean learning gains in simple quantitative problem solving was not significant for either planned contrast (Extended PD and Reduced PD, F(1,14)=.2, *p*=.6, 95% CI [-10, 16]; No PD and Reduced PD, F(1,14)=3.6, *p*=.08, 95% CI [-25, 2]). However, because the variation within each condition was large, gains in simple quantitative problem solving was added as a covariate. The addition of simple quantitative problem solving to FRL as a covariate in the planned contrasts had no effect on the finding of significant differences in learning gains for quantitative problem solving between the Extended PD and Reduced PD conditions (F(1,13)=7.4, *p*=.02, 95% CI [3,24]), and no effect on the finding of lack of significant differences in learning gains for the other contrasts in both quantitative problem solving (No PD and Reduced PD, F(1,13)=1.3, *p*=.3, 95% CI [-19, 6]) and conceptual science content (Extended PD and Reduced PD, F(1,13)=0.04, *p*=.8, 95% CI [-21, 17]; No PD and Reduced PD, F(1,13)=0.8, *p*=.4, 95% CI [-31, 13]). Moreover, across all conditions, there was no significant correlations between learning gains in simple quantitative problem solving and either complex quantitative problem solving (r=.01, *p*=.96) or conceptual science content (r=-.01, *p*=.97). These results suggest that general student/teacher learning ability is unlikely to be responsible for the observed differences in student learning gains by science content domain and PD condition.

Did teachers vary in the extent to which they needed support? Teacher education level is often cited as a variable that can influence student learning (Darling-Hammond, 2000). While there was not enough variance in teacher education level to test the effect on student learning of science content within conditions, it was possible to test the effect of having a master's degree across all conditions. Across all PD conditions, there was found to be no effect of teachers having earned a master's degree (in education or biology) on either student learning of quantitative problem solving, with FRL as a covariate (Masters: N=10, M=13; Undergraduate only: N=7, M=15; F(2,1) = 0.33, *p*=.58) or conceptual science content (Masters: N=10, M=19; Undergraduate only: N=7, M=21; F(2,1) = 0.05, *p*=.83). Thus, at least in the context of these educative curriculum materials, gains in student learning was not driven by education level, and any small differences in education level across PD conditions is unlikely to have been the cause of condition differences in student learning.

5.4 GENERAL DISCUSSION

The results presented here suggest that when teachers are provided with educative curricula materials to facilitate implementation of an NGSS-aligned unit, different levels of face-to-face PD support may be required to facilitate student learning for different content domains. For core conceptual science content (i.e., the most familiar to teachers), student learning gains can be achieved in the absence of additional Face-to-Face PD (i.e., just with educative curriculum materials), although some Face-to-Face PD is helpful. By contrast, for quantitative problem solving, which involves mathematical application in a science context and thus is more novel to teachers, student learning gains are greatly reduced when teacher Face-to-Face PD in this content domain is reduced.

At a minimum, these results provide support for the idea that when designing and assessing PD, there is a need to move beyond asking does PD have an effect or specifying fixed, general guidelines regarding the amount of PD to asking more subtle questions that explore what kind of content requires what kinds and amounts of teacher support (Borko, 2004; Dede et al., 2009). Different content may require different amounts of teacher support because teacher content knowledge is not monolithic (National Research Council, 2015). For example, in the context of this NGSS-aligned unit, biology teachers are well-prepared to teach about the processes of inheritance (Lyons, 2013). The unit requires no new understanding of this biology content, but rather involves a different way of helping their students learn this content. On the other hand, biology teachers are generally not as well versed in mathematics (Sorgo, 2010). Understanding and calculating probability in a new way in the science context would therefore likely require more support. The different content domain student effects observed here provide a different lens on the effect of teacher supports on student learning in the context of an NGSS-aligned unit, each of which is discussed below.

As implementation of NGSS-aligned curriculum moves beyond field trials of units to implementation at scale, it is encouraging that significant student learning gains were observed in the area of core science content when teachers were supported by educative curricula materials alone. These gains with minimal support are particularly impressive considering students in traditional instruction showed no significant gains in their understanding of these core processes that are at the heart of inheritance (i.e., this content is quite difficult for students). Note that the students in the no PD group were not specially prepared to master this material: 1) Their pretest scores were not significantly different than the other groups; 2) Their socioeconomic context as indicated by the percent of students that qualify for free and reduced lunch was approximately equal, if not slightly at the lower range, of the other implementing groups; and 3) The teachers for this group were also not better prepared (i.e., similar rates of masters or undergraduates degrees in biology).

It is important to emphasize that similar gains without PD would be unlikely if the curriculum materials were not educative, based on prior research findings (Ball & Cohen, 1996; Cervetti et al., 2015; Davis & Krajcik, 2005; Doppelt et al., 2009; Driel, Meirink, Veen, & Zwart, 2012; Heller et al., 2012). The educative curricula materials provided to implementing teachers here were of high quality, meeting almost all of the criteria put forth by Beyer et al. (2009). Of course, it remains an open question whether there were also other critical features within this particular NGSS-aligned unit that led to robust student gains. For example, the unit was designed so that teachers and students were asked to revisit the conceptual science content (the genetic processes of meiosis and fertilization) in multiple contexts (Schuchardt & Schunn, 2016).

All implementing conditions showed some student learning gains in quantitative problem solving. However, when time spent on this content domain in face-to-face PD was decreased, student learning gains were significantly lower, but still comparable to traditionally instructed students (Schuchardt & Schunn, 2016). One significant difference between the Extended PD and

Reduced PD conditions is that in the Extended PD condition, teachers spent three hours developing their pedagogical content knowledge using the new mathematical strategy by analyzing common student errors. For complex biological problem solving, extended professional development appears to be important for obtaining strong student learning outcomes even in the context of educative curriculum materials, possibly because both the content and pedagogical content knowledge were novel to teachers.

Pragmatically, relying solely on the teacher educative curricula materials did not significantly reduce quantitative problem solving scores beyond that of the Reduced PD group. This suggests that in terms of scalability, for mathematics in a science context, once the decision has been made to reduce PD in this content domain to two hours, little additional harm may be done by eliminating PD if teachers are provided with educative curricula materials. Consideration of threshold effects is important in optimally deploying school district resources (Archibald et al., 2011); although it may be counter-intuitive to some administrators, here we have an example in which providing a small amount of teacher professional development in this difficult content domain had no benefit over providing only access to educative materials. For those creating and offering teacher professional development it is also important to understand how much support is needed to be worthwhile.

5.4.1 Possible mechanisms

In both Reduced PD and No PD groups, the exact mechanisms that resulted in the drop in student learning of quantitative problem solving compared to the Extended PD condition require further investigation. It may be that the lack of support in the Reduced PD and No PD conditions made teachers feel unequipped to teach a new mathematical approach to a familiar scientific problem and thus they reverted to traditional methods (Collopy, 2003). Alternatively, it may be the lack of extended practice with the new approach in these two conditions meant they did not see the advantages of the new approach and thus could not transmit that to their students (Stein & Kaufman, 2010). Finally, it may be the unfamiliarity with common student errors in the Reduced PD and No PD conditions meant that teachers were ill equipped to help their students once they stumbled (Hill & Charalambous, 2012). What these results do show is that, unlike for conceptual science content, which may be generally more familiar to teachers, student learning in the context of quantitative problem solving can benefit from providing additional support for their teachers.

In this study that arose out of a larger study on learning gains in science from an NGSS curriculum, we separate out the effects of PD on two different content domains for student learning. The chain of causality from teacher PD to student learning outcomes is complex, likely including various changes in teacher content knowledge and teacher in class practices during unit enactment, and then changes in student enactment of activities. However, it is precisely because of this complexity and because of the NGSS emphasis on student learning gains that this paper focused on directly measuring student-learning gains as a consequence of different PD interventions. The goal was to determine how much face-to-face PD (from amounts of PD that are possible and typical in the US) is needed to achieve student learning gains in science content in the context of teacher educative curricula materials and whether that amount differs for conceptual science content (genetic processes) versus quantitative problem solving (cross-disciplinary science content). Thus we address an oft-cited but inadequately supported claim that teachers will need differential support for implementing different aspects of NGSS curricula (National Research Council, 2015; Wilson, 2013).

In general, the exact mechanisms for these content specific effects of face-to-face PD support in the context of implementation of an NGSS-aligned unit when teachers are provided with educative curricula materials is unclear based on this study. However, the results of this study suggest that when making decisions about scalability of NGSS and investment of resources into professional development, it is necessary to consider the desired student outcomes in specific content domains within a unit. Such considerations allow for efficient deployment of PD resources, which is critical in scaling units to all the contexts in need of new rigorous science curriculum materials (Archibald et al., 2011; Bybee, 2014; Reiser, 2013). Furthermore, in terms of research, the results presented here suggest productive avenues concerning the interaction of teacher support requirements for specific content domains, how they can be met, and student learning outcomes within specific content domains. As such, it seems necessary to reiterate the call for more small scale studies on professional development that include research on student learning outcomes (Luft & Hewson, 2014) to guide investment on the necessary and larger-scale studies that focus on connections between student outcomes and teachers' instructional practices, cognitions, and beliefs (Desimone, 2009; Driel et al., 2012; Luft & Hewson, 2014; Yoon et al., 2007).

6.0 CONCLUSION

6.1 SUMMARY OF FINDINGS

Previous attempts to characterize studies on curricula that include mathematics in science instruction have focused on the extent of integration between the two disciplines (e.g., Davison et al., 1995; Huntley, 1998; Hurley, 2001). In the literature review, I point out that this approach is problematic because it relies on superficial characteristics (e.g., time spent and sequence), ignoring the function that mathematics has within the science classroom (Judson, 2013). Because the functional role that mathematics is playing is more likely to be associated with changes in student understanding of quantitative problem solving and conceptual understanding, I propose a new epistemic classification scheme. Based on a review of the quantitative problem solving literature in science education, I suggest that to-date there are three main ways that mathematics has been included in science instruction; as a tool for calculating a quantity (Mathematics as Tool), as an inscription for expressing an idea or relationship (Mathematics as Inscription), and as one of a connected set of representations that is grounded in a scientific phenomenon (Grounded Mathematics). I conclude the literature review by arguing that there is a fourth function for mathematics in science instruction that has largely been ignored, an expression of the mechanistic underpinnings of the scientific phenomenon (Mechanism Connected Mathematics).

At the conclusion of Chapter 2 and the introductions of Chapter 3 and 4, I define Mechanism Connected Mathematics (MCM) and develop a theory of the instructional benefits of having students develop Mechanism Connected Mathematical expressions that model a scientific phenomenon within the context of a modeling cycle of data analysis, model development, model evaluation, and model refinement. To summarize, a Mechanism Connected Mathematical expression is defined as a mathematical expression where the variables and functions within the mathematical expression represent and mirror objects and scientific mechanisms within the scientific phenomenon. [Scientific mechanisms are defined as the interactions between objects within the phenomenon that produce the outcomes associated with the phenomenon (Machamer et al., 2000).] Building on prior theories about the significance of mathematical modeling in scientific practice (Hestenes, 2010; Svoboda & Passmore, 2013), I argue that by having to select both the variables in the mathematical expression and the way in which the variables are connected to each other through a mechanistically relevant function, students are forced to 1) decide which objects within the phenomenon are important mechanistically, and 2) how they are connected mechanistically. Thus, student attention is focused on mechanistically important aspects of the phenomenon initially during equation development and then during subsequent use of the equation, potentially increasing student conceptual understanding of the phenomenon. I further argue that maintaining students' ability to connect a mathematical expression for calculating a quantity with the represented scientific phenomenon has the potential to allow students to switch between scientifically and mathematically based approaches during quantitative problem solving. Based on prior studies of students who spontaneously connect their problem solving to the scientific phenomenon (Taasoobshirazi & Glynn, 2009; Tuminaro & Redish, 2007), I propose that having this ability to fluidly switch problem solving approaches will result in increased ability to solve quantitative problems, particularly for more complex or unfamiliar problems where algorithmic methods break down (Bing & Redish, 2009; Stewart, 1983; Taasoobshirazi & Glynn, 2009).

In Chapters 3 and 4, I describe results that provide support for the role that Mechanism Connected Mathematics in science education can play in enhancing conceptual understanding of a phenomenon and quantitative problem solving within that phenomenon. A large scale quantitative analysis of pre and post tests presented in Chapter 3 shows that including Mechanism Connected Mathematics in a unit of inheritance, results in a 1.5 fold increase in student conceptual understanding of the mathematically modeled mechanisms as compared to traditionally taught units that included Mathematics as a Tool. The inclusion of Mechanism Connected Mathematics as development of a model of the scientific phenomenon was associated with other designed changes in instruction that would not generally be present in a traditional classroom. These changes included more opportunities for data analysis, greater connections with other representations of the phenomenon (including drawings), and more opportunities for student discourse. It could be argued that these were the changes that led to the increase in student understanding of the mathematically modeled concepts. However, an aspect of inheritance that was not mathematically modeled showed no difference in pre/post change in conceptual understanding compared to traditional instruction, despite the MCM unit design for this aspect having the same instructional affordances (other than mathematical modeling) as the MCM modeled component of inheritance.

Statistical analysis of quantitative problem solving pre and post instruction also showed benefits for quantitative problem solving for students exposed to the MCM unit as compared to students exposed to traditional instruction. These benefits were only evident for complex quantitative problems, not for simple problems. As shown by Stewart (1983), traditionally instructed students don't struggle with simple inheritance problems which are amenable to algorithmic approaches, but do struggle with more complex or unfamiliar quantitative inheritance problems where the algorithmic approaches break down. Stewart (1983) speculated that students' lack of understanding of the connections between their algorithmic problem solving approaches and the underlying scientific phenomenon was behind their lack of success with the complex or unfamiliar problems. However, he did not provide evidence for this speculation.

Qualitative analyses of student problem solving presented in Chapter 4 explored reasons behind MCM instructed students' success on complex or unfamiliar quantitative problems. I theorized that connections between quantitative problem solving and the scientific phenomenon of inheritance developed through Mechanism Connected Mathematical Modeling would be apparent during successful problem solving. These connections would allow successful students to switch between approaches to problem solving centered around an understanding of the biological mechanism and approaches centered around a mathematical approach (use of an equation). In Chapter 4, I show that this is what is seen. MCM instructed students (Competent MCM) who can solve both a complex and an unfamiliar problem use more biologically connected words when talking about their problem solving as compared to MCM instructed students who can't solve either of those problems (Struggling MCM). Furthermore, Competent MCM students but not Struggling MCM students tended to use more than one inscription when solving problems, and generally one of the inscriptions was biologically oriented while the other was mathematically oriented. These inscriptions were not just linked through associated objects, students seemed to have an understanding of the mechanistic links between the inscriptions, aligning (verbally and/or pictorially) the function in the MCM equation with the interaction between objects of the mechanism in the biological inscription. Switching between inscriptions occurred for reasons that would seem to facilitate success, such as checking problem solving progress, making use of affordances of different approaches, and checking the final answer. The problem solving features of Competent MCM students summarized here (multiple inscriptions, inscription switching, connection between mathematical expression and mechanism of scientific phenomenon) appear to be features of MCM instruction not problem solving success. Traditionally instructed students who successfully solved both the complex and unfamiliar problem tended to be unable to connect their mathematical inscription with the underlying scientific phenomenon and used only one inscription.

As a group, the studies presented in Chapter 3 and 4 provide support for the theory I presented that having students develop a mathematical model of a scientific phenomenon where the functions and variables in the equation are connected to the mechanism and entities in the scientific phenomenon benefits conceptual understanding and quantitative problem solving. Moreover, these benefits were shown to accrue in the way that was predicted by the theory: i.e. that fostering mechanistic connections between the phenomenon and the mathematical expression would allow students to switch problem solving approaches with associated potential benefits for quantitative problem solving. However, there are important limitations both to the studies and to the MCM approach that need to be addressed. Because one of the approach limitations is addressed in Chapter 5, I will discuss limitations to the approach first and then discuss limitations associated with all three studies. As I discuss the potential impact of the limitations to the findings presented here, I will discuss how they open up avenues for future research.

6.2 LIMITATIONS TO THE MCM APPROACH

Limitations to the MCM approach can be grouped in to two categories: generalizability and capacity. By generalizability I mean the extent to which the MCM approach can be applied to other scientific phenomenon, and by capacity I mean the capacity of students to learn and teachers to teach using the MCM approach.

6.2.1 Generalizability to other scientific phenomena

The specific application of the MCM approach illustrated here was facilitated both by the choice of the scientific phenomenon and the choice of the mathematics used to model the phenomenon. The mechanism behind the transmission of genes from parents to offspring can be reduced to the interaction of two objects (eggs and sperm). Moreover, this interaction is a one-time event that is in itself not complex conceptually (eggs and sperm join so that any egg can join with any sperm and vice versa). Furthermore, the mathematics chosen involved relatively simple functions (multiplication and division) and concepts (proportions) that are within the capacity of many ninth grade students. (In support of this claim, the Struggling MCM students in the qualitative study of problem solving did not fail to solve problems because of an inability to multiply or divide, or set up the proportion.) There are other scientific phenomena across disciplines that meet these criteria and I have mentioned several of them previously. They include acceleration due to the effect of force on an object, and the joining of molecules in a chemical reaction. However, there are other phenomena that are more mechanistically complex. For example, those that have several layers of mechanistic explanations or interacting objects such as the behavior of a gas in an enclosed container or those that occur over time such as population growth or evolution. There are still other phenomena where the most complete mathematical representation of the mechanism may be too complex for most high school students, such as diffusion of a liquid through a solid. Questions then arise as to whether and how these phenomena can be modeled through an MCM equation. If adaptations are made, such as considering only certain aspects of a system at particular ages and building on that over time, what is the effect on conceptual understanding? Can computational modeling be used with phenomena that involve time or several decision points to build on an initial MCM model? What effect does this dual approach have on students' conceptual understanding, as well as their understanding of computational modeling? In some situations, are the complexities of the phenomenon or the mathematics so great, that students may be better served by another approach such as Grounded Mathematics?

6.2.2 Teacher capacity

Teachers in science, particularly in biology, do not necessarily have a strong background in mathematics (National Research Council, 2015; Watanabe & Huntley, 1998). Thus, they may struggle to implement the MCM approach in their classrooms in such a way that students can accrue benefits to conceptual understanding and quantitative problem solving. The amount of professional development that was provided in the initial studies presented in Chapters 3 and 4 could tax the resources of many school districts (Spillane et al., 2009). Chapter 5 presented an initial investigation in to the amount of support that teachers would need for students to show the gains to conceptual understanding and quantitative problem solving that were seen in these initial studies. This study showed that when teachers were provided with educative curricula materials (ECM), no further professional development was needed for students to show gains in conceptual understanding that are not significantly different from that shown when face-to-face professional development is added. However, it should be noted that the gains achieved while not significantly less are still less than is shown with face-to-face professional development, suggesting that there may be some benefits to the face-to-face interactions. However, reducing the time spent in face-to-face professional development (but not eliminating it) significantly decreases the gains to student quantitative problem solving. I speculate that teachers may need more support in this area because they are less well prepared in mathematics. However, this study did not examine individual differences and thus, while reporting on a phenomenon that occurs generally across teacher backgrounds in multiple contexts, the effect of individual teacher differences on the amount of support required to maximize student learning gains remains a question for future research.

6.2.3 Student capacity

I have already suggested that students might have to have a certain facility with mathematics to be able to engage in the MCM approach. The mathematics was carefully chosen within this example to be within the grasp of most ninth graders, and students in the qualitative study did not struggle because of application of the mathematical functions involved in the MCM equation. Moreover, teacher pretest probability score was included as a covariate in analysis of student quantitative problem solving ability and was not found to be a significant covariate for conceptual understanding. However, one drawback of the matrix sampling, which allowed for a breadth and depth of questioning pre and post implementation, is that it was not possible to assess whether individual mathematical competence affected students' conceptual understanding or quantitative problem solving in any of the studies. It remains an open question the extent to which student capacity in mathematics should affect design of an MCM curriculum and the benefits that may accrue from that curriculum. Because of this, it is also not clear what modifications and/or support for students could be used to maximize learning through an MCM approach.

6.3 LIMITATIONS OF THE STUDIES

Limitations of the studies can be divided in to two categories: Study Context and Study Design.

6.3.1 Study context

All of these studies took place in the context of a unit where the MCM equation was developed through a modeling cycle that included other features that are not typically found in traditional instruction. As discussed earlier, these features included multiple connected representations of the phenomenon, construction of the phenomenon from data analysis of contrasting cases, and coconstruction of representations with peers with associated increased opportunities for student discourse. It was found that conceptual gains were only associated with the mathematically modeled aspects of the phenomenon and not other nonmathematically modeled aspects that had the same instructional affordances, suggesting that mathematical modeling was an important feature of the gains. Moreover, students who were exposed to the MCM unit and were successful at problem solving showed problem solving behaviors that were consistent with the theoretical predictions of the benefits of the MCM equation. However, from the studies presented here, the effect of the MCM equation cannot be teased apart from how it was presented in the unit. While

I strongly suspect based on prior research showing the benefits of these additional instructional affordances (Chi & Wylie, 2014; Schwartz & Martin, 2004; Wells et al., 1995) that simple presentation of the MCM equation to students would not have the same effect, these studies do not address this question. Thus, the effects on student learning need to be interpreted as the effect of an MCM modeling cycle approach that includes the above-mentioned instructional affordances rather than an MCM equation in isolation.

6.3.2 Study design

The caveat about study context brings up an important feature of study design. The studies were not designed to test the MCM modeling cycle approach against other mathematical modeling approaches. Therefore, these studies cannot address how a designed instructional approach that includes MCM mathematical modeling compares to a Grounded Mathematics approach. Grounded Mathematics approaches (e.g., Lehrer & Schauble, 2004; Wells et al., 1995) include many of the instructional affordances of modeling (including student coconstruction of representations from data analysis, and multiple connected representations) but does not require that students represent in the mathematical expression the entities and mechanism underlying the phenomenon. Such a comparison would potentially reveal interesting constraints and affordances of the two model-based approaches. I suspect that these parameters will reflect interactions between teacher capacity, student capacity and the complexity of the scientific phenomenon. However, it should be noted that Grounded Mathematics is not a common approach in biology. All of our comparison teachers were not selected based on their approach to instruction in inheritance, instead their journals revealed that they engaged in traditional instruction of inheritance with mathematical approach to instruction presented to students and separated from a presentation about the scientific phenomenon. Thus, a comparison between the MCM approach as designed in the unit and the traditional approach serves a purpose in terms of revealing alternative and better approaches to business as usual and thus helping to change instruction towards more productive methods for student learning.

A second limitation embedded in the design of all three studies is that the intent was to assess the effect of the MCM unit on student learning in general. There was not an intention to assess the effect of individual differences in student capacity, teacher capacity, or instructional enactment. Efforts were made to control for differences. For example, the effect of the unit was assessed across multiple teachers in multiple contexts and differences in student capacity, teacher capacity, and school or class context were either controlled for (e.g. student pretest scores, honors designation, composite pretest math score across a teachers' classes) or assessed for an effect (SES, school test scores, teacher educational background, teacher experience). Quantitative differences in conceptual understanding and quantitative problem solving were even shown to exist when two teachers switched from traditional instruction in year one to implementing the MCM unit in year two after professional development during the intervening summer. Moreover, sporadic observations of teachers from all three studies during MCM unit implementation revealed a range of implementation fidelity. While these observations were not consistent enough across all teachers for implementation fidelity to be entered as a covariate, I am confident that the sample encompassed a range of implementation fidelity. Furthermore, within the design of the large-scale quantitative analysis, students were embedded within teachers in an HLM analysis controlling for outlier effects of individual teachers. However, examining for effects across a range of individual differences is not the same as examining for the effect of those differences on outcomes. Follow-up studies that focus on the effect on learning of individual differences could reveal interesting interactions. These potential interactions might either suggest areas that need further support (e.g. teachers with low mathematical background decreasing implementation fidelity during classes focused on quantitative problem solving) or reveal additional benefits to the MCM approach embodied in this unit (e.g. a greater change score for students with low pretest scores).

6.4 CONCLUSION

In this body of work, a new framework is provided for examining how mathematics is included in science education, which suggests that the epistemic role of the mathematics is important when thinking about educational outcomes. It is noted that one role of mathematics, focusing attention on mechanisms behind scientific phenomena has been largely overlooked in the literature on integrating mathematics in to science education. I propose an approach to address this oversight, Mechanism Connected Mathematics, and provide a theoretical basis for how an MCM approach could elevate student conceptual understanding and quantitative problem solving. This approach and theory were tested following design and implementation of an MCM unit in inheritance. Exposure to an MCM unit of instruction was shown across diverse contexts to increase students' conceptual understanding of mathematically modeled parts of the phenomenon and quantitative problem solving of complex, but not simple problems. The theoretical basis for the MCM approach was given credence through qualitative analysis of student problem solving. This analysis showed that students exposed to an MCM unit who competently solved complex and unfamiliar quantitative problems, but not those who were unsuccessful, showed problem solving behaviors that were predicted by the presented theoretical basis. The Competent MCM students recognized the mechanistic connections between mathematical inscriptions and the biological phenomenon and switched between biological and mathematical inscriptions during problem solving in potentially productive ways. These behaviors were not a function of competence, because Competent Traditional students generally did not make those connections or show switching between inscriptions.

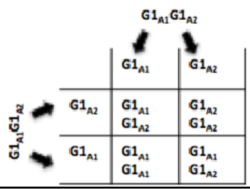
In addition, this body of work addresses a potentially important concern behind improving student learning through broader implementation of an MCM approach, lack of teacher preparation in mathematics (particularly in biology), and potential gaps in institutional capacity for professional development. In the context of educative curricula materials, no face-to-face professional development is needed to produce gains in student conceptual understanding or quantitative problem solving. However, the results suggest that some face-to-face professional development (1 day) is recommended to maximize student gains for conceptual understanding and more extensive face-to-face professional development (1 week) was needed to achieve maximal gains for quantitative problem solving.

Having provided evidence for a general effect of the MCM unit on student learning and for the proposed theoretical basis behind increased performance in quantitative problem solving, this work opens up avenues for additional exploration. These future studies could include, as suggested above, the effect of individual differences in student and teacher capacity and instructional context on student learning with an MCM unit, both on outcomes and throughout a unit. Such investigations will not only provide information about individual differences in outcomes, but can also provide evidence to support theoretical claims (e.g. for the effect of MCM on students' conceptual understanding). Another direction that these future studies could take is investigating the generalizability of this approach to other scientific phenomena, particularly the affordances and constraints either compared to different modeling methods and/or with respect to the characteristics of the phenomenon.

APPENDIX A

THREE WAYS MATHEMATICS CAN BE USED IN INHERITANCE INSTRUCTION

Table 12. Three ways mathematics can be used in inheritance instruction

EMBODIMENT OF MATHEMATICS IN INHERITANCE INSTRUCTION			
NAME	**Punnett Square**	**Probability Rules Method**	**Modeled Process**
Type of Embodiment	Calculated Procedure	Calculated Procedure	Modeled Process
Summary	Depiction of the rule that each parent will give one gene from each trait to offspring combined with an algorithm.	Treat the problem as purely a mathematical probability problem.	Apply a modeled process equation that makes explicit connections between the biology and the mathematical process.
Representation		P1*P2 Pn = probability of getting genotype of gene n	$\frac{W1 * W2}{(\text{\# egg types})(\text{\# sperm types})}$ Wn = ways of getting genotype combination of gene n
Worked Example	Calculate the probability of producing an offspring with the genes ■□ from breeding a male with ■□ genes and a female with ■■ genes.		
Step 1	Separate the 2 genes in the parents. <table><tr><td></td><td></td><td colspan="2">Male</td></tr><tr><td></td><td></td><td>■</td><td></td></tr><tr><td rowspan="2">Femal e</td><td>■</td><td></td><td></td></tr><tr><td>■</td><td></td><td></td></tr></table>	The male contains two types of alleles so the probability of passing on one of them is ½.	Determine the number of types of eggs and the number of types of sperm that can be produced (each parent can only contribute 1 gene for each trait): 2 sperm types and 1 egg type.
Step 2	Combine the genes from the parents in the inner squares. <table><tr><td></td><td></td><td colspan="2">Male</td></tr><tr><td></td><td></td><td>■</td><td></td></tr><tr><td rowspan="2">Femal e</td><td>■</td><td>■■</td><td>■□</td></tr><tr><td>■</td><td>■■</td><td>■□</td></tr></table>	The female contains one type of allele so the probability of passing on one of them is 1.	Because each egg type can join with each sperm type, multiply the number of egg types times the number of sperm types to obtain 2 possible offspring types.
Step 3	Count how many of the inner squares contain the gene combination of interest: ■ =2	Apply the appropriate probability rule: If both events are required then multiply the probability of the two events together.	Refer to the rule that each parent can only contribute one gene for each trait to determine the number of ways that each offspring genotype can be obtained.
Step 4	Count the number of total cells: =4	½*1=1/2	For ■□ offspring, the female can only contribute ■, the male must contribute: 1 way to get ■□ offspring.
Step 5	Place the number found in step 3 over the number found in step 4 = 2/4 and reduce the fraction = ½.		The probability of a desired event equals the number of desired outcomes as a proportion of the total number of possible outcomes. Place the answer from step 3 over the answer from step 4 = ½.

APPENDIX B

TEACHER AND SCHOOL CHARACTERISTICS

Asterisk means the teacher taught using both the iSTEM unit and traditional instruction. Teachers 4 and 9 used traditional instruction in Year 1 and then received professional development and used the iSTEM unit in Year 2. Only the data from their traditionally instructed classes is considered in Study 1. NA means that teacher's data was not available. Undesignated means that the school did not designate honors and regular biology classes. These classes were considered as nonhonors classes.

Table 13. Teacher and school characteristics

Instruction Condition	School ID	Teacher ID	Biology Level	Hours of Professional Development	Exposure Workshop Attendee/ Recruited	Number of Students	Years of Teaching Biology	Masters Degree	Biology Masters Degree	Biology Undergraduate Degree	School ACT	School ACT Science	School State Science	School State Reading	School State Math	Percent Minority Enrollment	Percent Free & Reduced Lunch
Traditional	1	1	Honors	0	Recruited	24	11+	Yes	No	Yes	19.2	19.2	23	51	22	68	57
	2	3	Regular	5	Recruited	42	0-1	No	No	No	21.6	21.2	35	66	42	26	16
	3	4*	Regular	0	Recruited	55	11+	Yes	Yes	No	17.5	18.2	11	39	13	26	68
	4	9*	Undesignated	0	Recruited	70	11+	Yes	No	Yes	19.4	19.9	16	47	37	15	43
	4	6	Undesignated	0	Recruited	71	NA	NA	NA	NA	19.4	19.9	16	47	37	15	43
	6	13	Honors	0	Recruited	59	11+	Yes	No	Yes	21.5	21.5	38	64	38	14	19
iSTEM	1	2	Honors	25	Exp Wkshp	19	11+	No	No	Yes	19.2	19.2	23	51	22	68	57
	2	3	Honors	5	Recruited	88	0-1	No	No	No	21.6	21.2	35	66	42	26	16
	3	5	Regular	22.5	Exp Wkshp	133	11+	No	No	Yes	17.5	18.2	11	39	13	26	68
	4	7	Undesignated	25	Exp Wkshp	49	6-10	No	No	Yes	19.4	19.9	16	47	37	15	43
	5	10	Honors	25	Exp Wkshp	15	11+	Yes	Yes	Yes	18.1	18.2	17	41	12	47	55
	6	11	Regular	25	Exp Wkshp	82	2-5	No	No	Yes	21.5	21.5	38	64	38	14	19
	6	12	Regular	25	Exp Wkshp	119	11+	Yes	No	Yes	21.5	21.5	38	64	38	14	19
	7	14	Undesignated	10	Recruited	39	11+	No	No	Yes	23.3	23.1	57	63	74	33	8
	7	15	Undesignated	25	Exp Wkshp	46	6-10	No	No	Yes	23.3	23.1	57	63	74	33	8
	7	16	Undesignated	25	Exp Wkshp	17	11+	No	No	Yes	23.3	23.1	57	63	74	33	8
	7	17	Undesignated	10	Recruited	24	NA	NA	NA	NA	23.3	23.1	57	63	74	33	8
	7	18	Undesignated	10	Recruited	114	NA	NA	NA	NA	23.3	23.1	57	63	74	33	8

APPENDIX C

INTERVIEW PROTOCOL

Data Gathered:

1) Students will solve 2 genetics probability problems.
    a. the 2 gene offspring problem.
    b. the 3 gene gamete problem.
2) Their written work will be preserved.
3) Audio recording will be done of interview.
4) Video/audio of students' paper/solutions (not of students) will be done to preserve a record of changes and order of problem solving.

Protocol

Introduction

Interviewer: I am gathering information about how different students solve genetics problems in order to help us make the unit better for other students. It is okay if you get an incorrect answer or aren't sure how to solve a problem. I am interested in what steps you might take to try and get an answer to the question.

I am going to ask you to solve two problems You can either explain what you are doing as you are working or after you are finished. I am going to record what you are saying so I can listen to it later. I am also going to video what you write on the paper. That is what the video camera is for. It is not recording your face – would you like to take a look to see what it is capturing. I will also be taking notes, just in case the audio or video does not work.

Problem 1

Student is given problem 1, the two gene offspring problem and asked to take a look at it.

Interviewer: Do you have any questions about what I am asking you to do?

Interviewer: Okay, can you show me how you would try to solve this problem?

If needed, interviewer prompts student to explain problem solving process: "Tell me what you did."

If needed, interviewer asks for clarification.

Interviewer: Have you seen this problem before? Where?

Problem 2

Student is given problem 2 (3 gene gamete problem) and asked to take a look at it.

Interviewer: Have you seen this problem before? Where?

Interviewer: Do you have any questions about what I am asking you to do?

Interviewer: Okay, can you show me how you would try to solve this problem?

If needed, interviewer prompts student to explain problem solving process: "Tell me what you did."

If needed, interviewer asks for clarification.

Interviewer: Have you seen this problem before? Where?

Interviewer: Okay, can you show me how you would try to solve this problem?

As needed, interviewer prompts student to explain what they are writing on the paper.

Post Problem Solving

If answers to these questions did not come up in the explanations, ask:

1) If eggs and sperm were not mentioned during the explanation:
    a. How are offspring produced from these crosses?
        i. Follow-up if egg and sperm are not mentioned: Do egg and sperm play a role?
        ii. Follow-up if relevant: Can you indicate where egg/sperm/fertilization are/occurred during your problem solving process?
2) If students have a punnett square or drew egg and sperm, Either:
    a. If students have one symbol from each gene in the egg and sperm/boxes: Why do you have one symbol from each gene in the egg and sperm/boxes?
    b. Can you explain to me why you put the symbols in the egg and sperm/boxes in the way you did?
3) If students have not already answered this:
    a. Draw out: If a female parent was AaBb, could they produce an Aa or a Bb egg? Why or why not?
4) If students have not already answered this:
    a. In what ways are problems 1 and 2 the same? How do they differ?

Questions to be used:

Problem 1

In guinea pigs, black coat color is dominant to white coat color and red eyes is dominant to brown eyes. If organisms of type BbRr and type bbRr are crossed, what proportion of their offspring will be bbRr?

Problem 2

Given a female with the genes: BbRrGg, what proportion of her eggs will contain genes "b" AND "g"?

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