AN EXAMINATION OF THE POTENTIAL OF SECONDARY MATHEMATICS CURRICULUM MATERIALS TO SUPPORT TEACHER AND STUDENT LEARNING OF PROBABIILITY AND STATISTICS

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The Common Core State Standards for Mathematics (CCSSSM) suggest many changes to secondary mathematics education including an increased focus on conceptual understanding and the inclusion of content and processes that are beyond what is currently taught to most high school students. To facilitate these changes, students will need opportunities to engage in tasks that are cognitively demanding in order to develop this conceptual understanding and to engage in such tasks over a breadth of content areas including probability and statistics. However, teachers may have a difficult time facilitating a change from traditional mathematics instruction to instruction that centers around the use of high-level tasks and a focus on conceptual understanding and that include content from the areas of probability and statistics that may go beyond their expertise and experience. Therefore, curriculum materials that promote teacher learning, as well as student learning, may be a critical element in supporting teachers' enactment of the CCSSM. This study examines three secondary mathematics curriculum materials with the intention of determining both the opportunities they provide for students to engage in high-level tasks and the opportunities for teacher learning. Tasks in the written curriculum materials involving probability and statistics as defined by the CCSSM will be examined for evidence of these opportunities. The results of this examination suggest that one of the three secondary mathematics curriculum materials, Core-Plus Mathematics Project (CPMP), contains high-level tasks addressing many of the probability and statistics standards from the CCSSM. A second curriculum, Interactive Mathematics Program, also contains high-level tasks but has far fewer high-level tasks than CPMP. The third curriculum, Glencoe Mathematics (GM), addresses many of the probability and statistics standards from CCSSM but does so with low-level tasks. None of the three curricula provides ample opportunities for teacher learning in the areas of anticipating student thinking and providing transparency of the pedagogical decisions made by the authors when designing the materials.

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1.0 RESEARCH PROBLEM

This study examines secondary mathematics curriculum materials with the intention of determining both the opportunities for students to engage in high-level tasks and the opportunities for teacher learning. Tasks in the written curriculum materials involving probability and statistics as defined by the Common Core State Standards for Mathematics (CCSSM) will be examined for evidence of these opportunities. With that end in mind, this chapter will argue the following points in order to justify such a study:

1) CCSSM will necessitate change in mathematics education
2) Curriculum materials will play a vital role in the change that CCSSM hopes to facilitate
3) CCSSM may require student engagement in high-level tasks
4) Teacher learning may be necessary for high-level tasks to be implemented well
5) Curriculum materials are one potential source of teacher learning (educative curriculum materials)
6) Probability and statistics are important content areas where high-level tasks and educative curriculum materials may be especially useful

1.1 CCSSM WILL NECESSITATE CHANGE IN MATEHMATICS EDUCATION

The Common Core State Standards represent the first time in United States history that common standards will be used across most of the country. Forty-three states have adopted the CCSSM along with the District of Columbia, four territories, and the Department of Defense Education Activity (National Governors Association Center & Council of Chief State School Officers, 2012). The intention of CCSSM is to provide a more focused, coherent set of goals for what students are expected to learn than what currently exists among the states in the United States (National Governors Association Center for Best Practices, 2010). The CCSSM emphasizes conceptual understanding and additional content beyond what is currently taught in most high schools with the goal of college and career readiness for all students (NGACBP, 2010).

Another addition relative to previous standards in CCSSM specific to mathematics (CCSSM) is the inclusion of Standards for Mathematical Practice. According to the CCSSM, these Standards for Mathematical Practice incorporate important processes and proficiencies from the NCTM process standards and the strands of mathematical proficiency from the National Research Council (NGACBP, 2010). These Standards for Mathematical Practice describe to educators the *processes and proficiencies* that should be developed in their students. Including practices along with specified content sets the CCSSM apart from many of the state level standards that focus on content only.

The CCSSM were created because of the results of both national and international assessments of student performance. For example, poor results from the National Assessment of Education Progress (NAEP) led to a Commission on No Child Left Behind (NCLB) leading a call for a voluntary national curriculum and assessments that would match (Goertz, 2010). Both the original NAEP (2008) which is referred to as the Long-Term Trend assessment (LTT) and the New NAEP (2009) do not demonstrate a significant improvement for secondary students in mathematics (Kloosterman & Walcott, 2010). Internationally, 15 year olds from the United States do not compare favorably in mathematics, ranking 26th out of 34 countries according to the Program for International Student Assessment (PISA) data (OECD, 2013).

These results were not consistent with the results being reported from state level assessments. For example, a state might report that 70% of the students tested were proficient in 8th grade mathematics. However, NAEP data would suggest that only 30% of the students from that state were proficient in 8th grade mathematics. This discrepancy between individual state results and a national assessment revealed one of the potential problems with the United States educational system. There is disparity between each state's standards. One would think the release of the *Curriculum and Evaluation Standards for School Mathematics* by the National Council of Teachers of Mathematics (NCTM) in 1989 would have facilitated the states coming together to teach a common set of standards. While the NCTM provided what could have been a guiding document, many states created mathematical standards of their own that were often unfocused and lacked coherence (Geortz, 2010). Additionally, the state standards varied greatly and often did not match what the NCTM was proposing (Porter et al., 2009; Reyes, 2006).

The lack of consistent standards across states made it difficult for curriculum developers to produce quality curriculum materials that would be suitable across the country. One option would have been to create textbooks specific to each state's individual standards. However, this was far too expensive and therefore was only done for states with large populations such as California, New York, and Texas. For the rest of the United States, the alternative was that curriculum developers made textbooks that were large and incoherent in an attempt to satisfy the many different demands of these states. As a result, some have suggested that the textbooks are not adequate and will need to be improved if significant changes in mathematics education are to be made (Willoughby, 2010).

The *Curriculum and Evaluation Standards for School Mathematics* proposed significant changes in mathematics education. The NCTM further developed their recommendations with the release of the *Principles and Standards for School Mathematics* in 2000. This document refined the previous recommendations by specifying learning expectations for different grade bands beginning with Pre-K all the way through grade 12. These two documents intended to facilitate a major shift in mathematics education, to change dramatically the way many think about mathematics instruction, and ushered in what is known as the *Standards Era*.

The widespread adoption of the CCSSM, the specificity of the standards, and the potential use of assessments that will be aligned to them could accomplish what the NCTM started in 1989. Based on the disparity already mentioned between the state standards and NCTM *Standards*, one might conclude that the state standards likely differ greatly from the guidelines set forth by CCSSM. This suggests that states will need to make significant changes if the guidelines of CCSSM are to be met. Porter et al. (2011) refers to this change as, "An unprecedented shift away from disparate content guidelines across individual states" (p. 103). Porter et al. (2011) suggest that CCSSM is considerably different than what states currently have in their standards and assessments, is more focused than what states standards are in mathematics, and is different than what teachers currently report they are teaching.

CCSSM hopes to push the level of conceptual understanding for students beyond the current U.S. levels. However, accomplishing the goal of increasing conceptual understanding may be especially difficult because many current curriculum materials may not support the demands of CCSSM. Many of the textbooks used in the United States are conceptually weak which leads to mathematics instruction that is too mechanical (Ginsburg et al., 2005). This may be because it is much easier to write curriculum that caters to low-level thought (Willoughby 2010).

1.2 CURRICULUM MATERIALS WILL PLAY A VITAL ROLE

Many researchers agree that textbooks have a significant impact on what students learn (e.g., Schmidt, Houang, & Cogan, 2002; Stein, Remillard, & Smith, 2007; Valverde et al., 2002; Willoughby, 2010). Research has demonstrated that teachers have a strong dependence on textbooks and other resource materials (Remillard, 2005) likely due to their important role in supporting both teaching and learning (Fan & Zhu, 2007; Boaler, 2002). Because of the important role of textbooks and teacher dependence on them, textbooks are often a way to try to influence classroom practices and affect student achievement (Senk & Thompson, 2003). In many cases, the textbook is the curriculum (Hudson, Lahann, & Lee, 2010). Other researchers suggest that textbooks affect how teachers teach (Ball & Cohen, 1996; Reys, Reys, & Chavez, 2004). Begle (1973) suggested that changing textbooks may be the only way to affect student learning and that textbooks are so powerful that they may have more impact on student learning than the teacher does.

Curriculum materials will play an important role in the CCSSM era if CCSSM is to be effective. Shaughnessy (2007) suggests that national standards without curriculum materials to accompany them are not useful. Curriculum materials will likely need to adapt and evolve to meet the new recommendations set forth in CCSSM. It has been suggested that for a curriculum to be effective for the students it must first have an effect on the teachers (Remillard & Bryans, 2004). Therefore, curriculum materials should provide opportunities for teacher learning and support for teachers in order to maximize the effectiveness of their instruction.

According to Martin et al. (2001), when the NCTM proposed their new standards in 1989, no textbooks at the time were consistent with what NCTM was proposing, so new curriculum materials were required. NCTM was advocating for students to have opportunities to engage in problem solving, communication, reasoning, and extended connections to other concepts. As a result, the National Science Foundation (NSF) funded multiple textbook projects at the elementary, middle, and high school levels. These NSF funded textbooks are referred to as *Standards*-based textbooks. Even though *Standards*-based textbooks were designed to address the demands of the NCTM *Standards*, because they differed so greatly from the textbooks that had been traditionally used, they were often rejected. In some communities, they were the source of considerable controversy (Schoenfeld, 2004).

The CCSSM may be positioned to enact change at a larger scale than the NCTM *Standards* because there are assessments aligned to CCSSM that provide accountability for schools to implement CCSSM. Schools may need to examine every aspect of their mathematics programs including which textbooks they are using. If schools are using textbooks that are not consistent with the demands of CCSSM, schools will be in the market for new materials. Both traditional and *Standards*-based textbooks are making the claim to being aligned with the CCSSM through updated versions of older texts or the publication of new versions, but the legitimacy of those claims is still in question.

1.3 CCSSM MAY REQUIRE STUDENT ENGAGEMENT IN HIGH-LEVEL TASKS

The adoption of the CCSSM may force textbook publishers to reevaluate their products and improve them. Textbook publishers will need to incorporate tasks that require high-level cognitive demand in order to develop conceptual strength. Additionally, school districts will need to evaluate their current curriculum materials to determine if they will be able to meet the demands of these new standards and assessments. To meet the challenges of the CCSSM, schools may need to challenge their students with tasks that place a higher cognitive demand on them. One possible step toward meeting these challenges could be the adoption of new curriculum materials that would contain these cognitively demanding tasks.

In 1979, Doyle initially introduced the notion of task as a potential unit for analysis. Doyle (1983) suggested that tasks are important because the intellectual and physical products students are expected to create, the operations students are to use to create these products, and the resources available for students to use can all be traced back to the task. Doyle's (1983) theory of the importance of tasks is driven by the notion that the mathematical concepts students are to learn are embedded in the tasks provided by the teacher. If a task is designed to elicit high-level thinking, students will then have an opportunity to approach a concept with higher order thinking. If a task is designed to elicit low-level thinking, students will then only have the opportunity to approach the concept with a focus on low-level procedures. Thus, one can reach Doyle's (1983) conclusion that tasks are a vital part of mathematical learning.

Stein and Lane (1996) and Stein, Grover, and Henningsen (1996) advanced the notion of analyzing tasks specifically in mathematics education. Organizations such as NCTM and MAA have called for students to develop deeper understandings about mathematics as opposed to simple memorization or procedural knowledge. Stein and Lane (1996) suggest that tasks have a significant influence on the kinds of thinking students may engage in and therefore significantly influence learning outcomes. The work of Stein et al. (1996) led to the development of the Task Analysis Guide found in Appendix C (Stein et al., 2000). The Task Analysis Guide can be used to differentiate between mathematical tasks that have the potential for either low or high cognitive demand. Low cognitive demand tasks are those that involve either memorization or using procedures without connection to meaning. High cognitive demand tasks are those tasks that involve using procedure while also making connections or tasks defined as doing mathematics. More detail about the Task Analysis Guide will be presented in chapters 2 and 3.

Since textbooks are so widely used in secondary mathematics education and tasks tend to drive instruction, one could reasonably conclude that it would be important to look at the level of tasks found in textbooks. The level of cognitive demand of tasks may also be indicative of the potential for a task to engage students in the Standards for Mathematical Practice from the CCSSM. Low-level tasks (*memorization* and *procedures without connections*) have little ambiguity about what needs to be done or how to do it (Smith & Stein, 1998). Based on this characterization, many of the Standards for Mathematical Practice are already beyond *memorization* and *procedures without connections*. For example, making sense of problems, reasoning abstractly, constructing arguments, looking for structure and repeated reasoning are all components of the Standards for Mathematical Practice that all would require, at a minimum, that a task be somewhat ambiguous about what needs to be done or how to do it. Given the characterization of low-level tasks from Smith and Stein (1998) which indicates that little ambiguity exists, low-level tasks are unlikely to engage students in the Standards for Mathematical Practice. Therefore, while the level of cognitive demand will not reveal the extent to which students will actually engage in a specific practice, it is reasonable to assume that a high-level task is more likely to provide potential opportunities for students to engage in the Standards for Mathematical Practice.

1.4 TEACHER LEARNING MAY BE NECESSARY

Adapting to the demands of CCSSM, specifically the increased conceptual level associated with CCSSM, may be difficult for teachers. Most teachers were taught in a traditional manner and most teachers tend to teach in the same manner in which they were once instructed and find it difficult to change their routines (Putnam & Borko, 2000). However, providing students with opportunities to engage in tasks that require a high-level cognitive demand will require improvements in teaching practices (Boston & Smith, 2009). To improve teaching practices, teachers need opportunities to challenge long-held beliefs by thinking about the types of tasks students should engage in, what it means to know and understand mathematics, and how to help students as they engage in high-level thinking and reasoning (Boston & Smith, 2009). All of these suggestions are related to promoting teacher learning in addition to student learning.

Davis and Krajcik (2005) suggest that promoting teacher learning is no easy task and therefore may not successfully occur through one method. While it may be easy to add new ideas, teachers must use knowledge in real time in the classroom and need to make connections between the new and existing ideas. These ideas are what Shulman (1986) describes as pedagogical content knowledge (PCK). PCK is knowledge that teachers have that differs from experts in a field, content knowledge, and general pedagogy shared by all educators, pedagogical knowledge. PCK is knowledge of how content and pedagogy are combined into effective instructional practices for specified content. The need for PCK makes promoting teacher learning different from promoting student learning.

Ball and Cohen (1996) suggest that teachers must adapt curriculum to meet the needs of their own students. However, Ball and Cohen further suggest that curriculum materials often overlook the role of the teacher. The result is that the teachers make adaptations to the curriculum that create a gap between what the curriculum writers intended and what is actually enacted in the classroom. In some cases, the teacher may even disregard the curriculum altogether and create his or her own lesson. Curriculum materials that promote teacher learning could assist the teacher in adapting the curriculum to fit their local needs while helping them to avoid making changes that would be detrimental to the curriculum.

Stein and Kaufman (2010) suggest that curriculum materials that are designed to elicit more ambitious forms of student learning will be significantly more challenging for teacher learning because they are different from what teachers are used to. *Standards*-based curriculum materials differ greatly from what people in the United States would remember about their own educational experiences (Robinson, Robinson, & Maceli, 2000 cited in Senk & Thompson, 2003). These types of materials challenge currently held beliefs about what of mathematics education is important and how these important items would best be taught (Hudson, Lahann, & Lee, 2010; Senk & Thompson, 2003). The foreign nature of these curriculum materials is one reason that they should strive to be educative in nature (Remillard, 2005).

1.5 CURRICULUM MATERIALS ARE ONE POTENTIAL SOURCE FOR TEACHER LEARNING

Educative curriculum materials are materials that aim to promote teacher learning. The notion that curriculum materials could promote teacher learning has been suggested by several researchers (Ball & Cohen, 1996; Davis & Krajcik, 2005). Curriculum materials that are educative have the potential to provide learning and support for teachers while maximizing the effectiveness of their instruction. Educative curriculum materials have demonstrated the ability to facilitate changes in instruction (Ball & Cohen, 1996). Since, as has already been established in this chapter, teachers rely heavily on curriculum materials, including textbooks, educative curriculum materials may provide a means of influencing large numbers of teachers and thus large numbers of students (Stein & Kim, 2009).

Ball and Cohen (1996, p. 7) proposed that, "Materials could be designed to place teachers' learning central to efforts to improve education." Ball and Cohen assert that the need for curriculum materials to be educative is based on how individual teachers shape their instruction based on their own understandings about the curriculum materials they are using, beliefs about what is important, ideas about students, and a notion of what the role of the teacher should be. Curriculum materials then should attempt to address each of these areas. Ball and Cohen noted that curriculum developers often overlook the teacher, acting as if their materials can work on students without teachers. As a result, many curriculum materials with the potential to improve student learning have failed to improve student learning because they have not provided enough support to the teacher to implement the curriculum effectively.

According to Ball and Cohen (1996), educative curriculum could be valuable by pointing out the following areas where school districts miss opportunities by setting the wrong goals for change. School districts often see the adoption of a new curriculum as a way of changing instruction, but they miss the opportunity of a new curriculum to facilitate teacher learning. Additionally, school districts focus professional development on fidelity of implementation when they could focus on developing professionals by promoting increases in their *capacity to teach*. Building the *capacity to teach* could promote teachers adapting curriculum materials for their personal needs while still reaching the instructional goals of the curriculum. The focus would shift from fidelity of implementation to fidelity of student learning.

Educative curriculum materials are important because teacher learning is potentially not as simple as student learning. A number of areas that teachers may benefit from learning exist. Research has shown that not all teachers are equipped with enough knowledge to teach high school mathematics effectively. Specifically, they fail to see the connections between concepts that could maximize their effectiveness. Teachers may also benefit from understanding more about the goals, rationales, and approaches of the curriculum they are being asked to implement. Finally, teachers could benefit from an increased ability to anticipate what students are thinking. Students will develop their understandings by connecting new information to prior knowledge, so anticipating student thinking is an important part of effective instruction (Stein & Kim, 2009). Anticipating student thinking involves considering how students will interpret the problem, the strategies they may use to solve the problem, and how those strategies relate to what the teacher would like the students to learn (Stein & Kim, 2009). Educative curriculum materials have the potential to address each of these three areas.

To summarize the argument thus far, the CCSSM is positioned to usher in an era of mathematics education that will focus more on conceptual understanding and include content and processes that are beyond what is currently taught to most high school students. To facilitate these changes, students will need opportunities to engage in tasks that are cognitively demanding in order to develop this conceptual understanding and to engage in such tasks over a breadth of content areas. However, teachers may have a difficult time facilitating a change from traditional mathematics instruction to instruction that centers around the use of high-level tasks and a focus on conceptual understanding and that include content that may go beyond their expertise and experience. Therefore, curriculum materials that promote teacher learning, as well as student learning, may be a critical element in supporting teachers' enactment of the CCSSM in mathematics.

1.6 PROBABILITY AND STATISTICS ARE IMPORTANT CONTENT AREAS

Probability and statistics education has been identified as important for several reasons. These include the need to create productive citizens of all students, the emergence of probability and statistics in the workplace, and the importance of probability and statistics in many college level classes (Jones & Tarr, 2010).

To elaborate on the need of probability and statistics for productive citizens, Garfield and Ahlgren (1988) suggest that all citizens should have knowledge of probability and statistics as a part of basic literacy in mathematics because knowledge of probability and statistics could be valuable in interpreting data presented in the media, understanding games of chance such as the lottery, or other examples that appear in everyday life. Cobb and Moore (1997) argue that variability is omnipresent thus making probability and statistics important to study.

Even though people are surrounded by probability and statistics, their reasoning in these areas may be flawed. Researchers have shown that there are widespread, persistent misconceptions (to be discussed in Chapter 2) in these areas that need to be addressed (Garfield & Ahlgren, 1988). These misconceptions are similar at all age levels, exist among all levels of ability, and are difficult to change (Garfield & Ahlgren, 1988; Konold et al., 1993; Pratt, 2000).

These reasons are likely a major part of why the NCTM *Standards* (1989; 2000) included two widely ignored content areas, probability and statistics, among its ten content areas. The NCTM was advocating for the inclusion of probability and statistics as a vital part of mathematics education along the same lines as algebra or geometry (Shaughnessy, 2007). CCSSM has included probability and statistics as one of the six conceptual categories for high school mathematics and included probability and statistics as a domain in sixth through eighth grade. This once again puts probability and statistics on equal ground as the other conceptual categories such as algebra and geometry.

The Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report was written to provide recommendations for a comprehensive statistics education program that spanned K-12 education. The goal of the report was to promote *statistical literacy* among all high school graduates (Franklin et al., 2007).The GAISE Report argued that *statistical literacy* was needed for the following reasons:

1) Creating good citizens - Citizens are informed by polls which are based in statistics C
2) Making good personal choices - Data is presented to us about food quality, drug effectiveness, toy safety, investment choices, etc.
3) Developing better workers - Quality control practices and accountability systems allow for the identification of improvements in manufacturing and are based in statistics

Teaching probability and statistics will not be easy. Konold (1989) suggests that teaching probability and statistics is difficult because students possess strong, often incorrect, conceptions prior to any instruction. More dangerous is that even when these conceptions are inaccurate they can sometimes learn quantitative skills well enough to convince the teacher and themselves that they have an accurate understanding of probability and statistics concepts. Martin et al. (2001) suggests that including probability and statistics could be especially difficult for schools and could require school districts to make a significant commitment to developing both pedagogical and content knowledge for teachers in their district. In a review of literature, Jones, Langrall, and Mooney (2007) found that there was evidence of many issues dealing with teachers' content knowledge in probability. Jones and Tarr (2010) suggest that more efforts must be dedicated to the education of teachers in the areas of probability and statistics in order to improve student understanding of probability and statistics.

Probability and statistics bring an added level of complexity not typically associated with other mathematics content. Shaughnessy (2007) suggests that unlike other mathematics problems, statistics problems add the challenge of dealing with bias, contextual issues, and uncontrolled variation. Additionally, probability is an area where students have diverse levels of reasoning thus making it even more difficult to teach (Jones et al., 2007).

Some teachers eliminated probability and statistics concepts from instruction altogether due to lack of time and a fear that including it might take away from other parts of their curriculum (Gattuso & Pannone, 2002). This type of thinking may stem from the era before the NCTM *Standards* and CCSSM when probability and statistics were not a part of mainstream curriculum and thus were not seen as important as they are today. Failure to assess these areas has provided teachers no motivation to change their thinking. Jones and Tarr (2010) suggest that teachers might not provide students with opportunities to learn probability and statistics because they never had such an opportunity themselves. This is especially concerning since researchers have identified probability and statistics as a very important area for learning in high school.

The reasons mentioned here likely contributed to the poor performance by high school students in the areas of probability and statistics on national (NAEP) and international (PISA) assessments. In some cases, scores from United States students improved when compared to previous years (Data analysis, statistics and probability scores for 12^{th} grade were 150 in 2005 and 153 in 2009 according to NAEP), but despite improvement in these areas, the probability and statistics scores were still below proficient levels (only 26% of 12^{th} graders at or above proficient in 2009 in mathematics according to NAEP) or the averages of other countries (U.S. score of 481 was below the average score of 494 for all countries involved and lower than 29 other educational systems according to 2012 PISA). One could interpret these data as demonstrating that the NCTM did have a positive impact on probability and statistics since there was some improvement, but the United States was so far behind that the impact was not enough to bring students up to acceptable levels of performance.

CCSSM and the assessments aligned to them will likely include probability and statistics. Because probability and statistics are going to be assessed, school districts nationwide will be taking steps to ensure probability and statistics are taught in their classrooms. Given the difficulties with instruction, the lack of teacher knowledge, and the importance of probability and statistics, educative curriculum could be especially useful in this area. Since probability and statistics are often excluded from curricula, school district leadership will need to find a textbook that includes them. Since it has been established that textbooks drive curriculum, the inclusion of probability and statistics in a textbook could be the best way to ensure their inclusion in the curriculum. Finally, based on the literature cited here, it would seem that probability and statistics are areas with severely underdeveloped content knowledge for both teachers and students. As a result, the use of high-level tasks would represent an even greater challenge for teachers in probability and statistics than in other areas of mathematics.

1.7 PURPOSE AND RESEARCH QUESTIONS

The purpose of this study is to evaluate textbooks currently in use in secondary schools for teaching mathematics to determine the extent to which those textbooks have the potential to prepare students and teachers to meet the demands of the Common Core State Standards with regard to probability and statistics. Specifically, this study answers the following research questions:

1) To what extent do current secondary mathematics textbooks provide opportunities for students to engage in the probability and statistics content recommended by the Common Core State Standards?
2) What are the cognitive demands of the tasks that are aligned with the Common Core State Standards recommendations for mathematical content in probability and statistics?
3) To what extent does the teachers' guide provide support for enacting high-level tasks that address the Common Core State Standards recommendations related to probability and statistics?
    a) To what extent does the teachers' guide provide suggestions related to *anticipation* on high-level tasks that reflect content recommendations of the Common Core State Standards?

b) To what extent does the teachers' guide provide *transparency* on high-level tasks that reflect content recommendations of the Common Core State Standards?

This study defines *anticipation* and *transparency* in the same manner as Stein and Kim (2009). *Anticipation* involves, "Expectations about how students might interpret a problem, the array of strategies – both correct and incorrect – they might use to tackle it, and how those strategies and interpretations might relate to the mathematical concepts, procedures, and practices that the teacher would like her students to learn (Stein & Kim, p. 45, 2009)." *Transparency* involves talking about, "The mathematical and pedagogical ideas underlying these tasks – thereby making their agendas and perspectives accessible (Stein & Kim, p. 44, 2009)." Stein and Kim (2009) suggest that this goes beyond providing steps to follow, questions to pose to the students, and answers to give. Instead, they propose that *transparency* equips teachers with the necessary information to select and adapt tasks. Finally, Stein and Kim suggest *transparency* may include providing information about how the task is connected to other activities in the curriculum. In summary, *transparency* is about making the mathematical purpose of the task clear to the teacher.

Only those tasks coded at high-level cognitive demand were analyzed for *anticipation* and *transparency* because they are the only tasks that would require such support for the teacher (Stein & Kim, 2009). Because low-level tasks offer a restricted path, following previously learned algorithms or recalling facts, there is no need for the teacher to anticipate multiple strategies and interpretations or be transparent about underlying mathematical and pedagogical ideas. However, high-level tasks, specifically *doing mathematics* tasks, have an open-ended nature without a predictable pathway to follow. Therefore, guidance in the areas of *anticipation* and *transparency* would be very valuable.

1.8 SIGNIFICANCE

There is a variety of groups that could benefit from this study. The largest benefactor would likely be those schools or districts considering one of the curricula reviewed for adoption. Analyzing the cognitive demand of instructional tasks speaks to both the instructional design and the content emphasis of a textbook as suggested by Hudson, Lahann, and Lee (2010). Schools can then decide what type of textbook is appropriate for their school climate. Textbooks with high-level tasks will require a great deal of professional development, may cause a lot of conflict with the beliefs held by teachers, and will be difficult to implement (Hudson, Lahann, & Lee, 2010). School decision makers will have to decide if they have the time, resources, and staff to take on such a challenge. The analysis of the potential for teacher learning provides decision makers with an idea of how supportive the curriculum materials are of their own implementation. In addition, each textbook was analyzed to determine its alignment with the CCSSM in regards to probability and statistics. While most publishers are going to make the claim of alignment, the textbooks analyzed have had that claim tested in one specific content area.

In addition to providing specific information relating to probability and statistics, the analysis of tasks provided by this study could serve as a framework for further evaluation of curriculum materials. For example, if a district uses curriculum materials that have not been reviewed here, they could apply the same analysis on their own to determine how their curriculum materials would fit in with those that are reviewed in this study. This study brings together research on tasks that require high-level cognitive demand, research on educative curriculum materials, and applies them to the CCSSM in such a manner that could be applicable to any one of the content areas identified by the CCSSM. Therefore, anyone wishing to evaluate content areas other than probability and statistics as defined by the CCSSM could benefit from this study as well.

Finally, this study provides a foundation for understanding the potential of curriculum materials that could be used as an aide when observing teachers using these materials. If an observer could be educated in the same way as the teacher, the observer may be able to provide feedback to the teacher more effectively. For example, if someone observing had a better understanding of anticipated student responses, connections between topics, and transparency related to key ideas of a task, he or she might have a different perspective during observation. This understanding of the potential of the curriculum could also be beneficial when planning in-service activities that could work in cooperation with the curriculum materials to maximize teacher learning and instructional effectiveness.

1.9 LIMITATIONS

The primary limitation of this study is that it only focuses on the written curriculum. According to the math task framework, Appendix D, this study is only focusing on the tasks as they appear in the curricular/instructional materials. It does not take into account how the teachers will set up the task, how the task will be implemented, or what student learning will actually occur. This study is only focused on the potential each task has as it is written in the curriculum. Of course, the potential of each task is critical since if a task does not have the potential to do something, it most likely will not. That makes this study an important first step of many for researchers wishing to understand the impact of tasks on student learning. Another limitation is that the study focuses on probability and statistics only. There are six different conceptual categories in the

CCSSM for high school. Probability and statistics are only one of the six categories. If one or all of the other conceptual categories were to be analyzed, they very well may tell a different story about each curriculum. Additionally, this study is based on the assumption that high-level tasks will better address the Standards for Mathematical Practice than low-level tasks. While this assumption is reasonable, it does not tell the entire story. Not every high-level task will address all of the eight Standards for Mathematical Practice and not every low-level task fails to address all of the eight Standards for Mathematical Practice. An analysis of tasks with a focus on the extent to which each of the Standards for Mathematical Practice are addressed would provide greater detail about how each of these standards is being addressed in the curriculum materials.

Finally, this study is limited in that it only analyzes three sets of current curriculum materials. These three sets of materials provide a snapshot of the landscape of secondary mathematics education materials, but they may not paint the entire picture of what is available. Including more curricula from an even wider variety of publishers could reveal more about available curriculum materials.

1.10 SUMMARY

The following points were argued to justify a study that examined secondary mathematics curriculum materials with the intention of determining both the opportunities for students to engage in high-level tasks and the opportunities for teacher learning on tasks in the written curriculum materials involving probability and statistics as defined by the Common Core State Standards:

1) CCSSM will necessitate change in mathematics education through more focused, coherent goals that emphasize conceptual understanding and specific mathematical practices
2) Curriculum materials will play a vital role in the change that CCSSM hopes to facilitate
3) CCSSM may require student engagement in high-level tasks
4) Teacher learning may be necessary for high-level tasks to be implemented well
5) Curriculum materials are one potential source of teacher learning (educative curriculum materials)
6) Probability and statistics are important content areas where high-level tasks and educative curriculum materials may be especially useful

The next chapter reviews literature, which provides a research foundation for the points, argued here. This literature will provide a basis for why probability and statistics are important, difficult to teach, and an overview of the myriad of misconceptions in this content area. The literature will also provide information regarding the potential power of curriculum materials to educate not only students but teachers as well. The potential of curriculum to be educative in nature could be especially important in meeting the demands of CCSSM, which may require schools to provide students with opportunities to engage in tasks that require high-level cognitive demand. The literature will provide a background on the importance and implementation of high-level tasks in mathematics education. Finally, the probability and statistics standards of CCSSM will be analyzed in connection with both secondary mathematics education research and the Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report. All of the literature referenced will further build on the argument made in this chapter while providing the basis for the methodology of the study.

2.0 REVIEW OF LITERATURE

Chapter 2 is dedicated to reviewing salient literature related to this study. This chapter begins with an examination of research in probability and statistics. This examination includes why probability and statistics are important, what makes them difficult to teach, including common misconceptions, and connections between the CCSSM and research in probability and statistics. After reviewing research on probability and statistics, this chapter turns its focus to educative curriculum materials. Educative curriculum materials are materials that promote teacher learning. Most curricula are written with student learning in mind. However, researchers have recently suggested that it could be possible for teacher learning to be a consideration in the design of curriculum materials. Since this chapter will establish a number of reasons that probability and statistics education could be difficult to teach and learn promoting teacher learning will then potentially be a very important step in providing enough support to promote student learning. Finally, this chapter turns its attention to tasks. The importance of tasks was established by Doyle (1983) and has since been elaborated specifically in mathematics education. The latest research on tasks discusses the importance of tasks requiring high-level cognitive demand for students to complete. An important connection made by Stein and Kim (2009) is that if a task potentially requires high-level cognitive demand, it will also put a high-level demand on the teacher to implement well. Therefore, it may be even more important for curriculum materials to be educative in nature if those materials incorporate many high-level tasks.

2.1 PROBABILITY AND STATISTICS

The primary purpose of this section is to make the following argument: Probability and statistics are important topics but are difficult to teach due to many factors including that misconceptions are widespread across content and for students at all grade levels. Once this argument has been made, the chapter will move on to suggestions related to statistics education. Next research in probability and statistics is connected to the curricular suggestions of the CCSSM. Finally, a study analyzing the tasks found in textbooks relating to probability from a historical perspective is reviewed.

However, before moving on to the argument, it might be beneficial to define what probability and statistics education might entail. The GAISE Report suggests that instructional programs should enable all students to do the following (Franklin et al., p. 5, 2007):

1) Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them;
2) Select and use appropriate statistical methods to analyze data;
3) Develop and evaluate inferences and predictions that are based on data; and
4) Understand and apply basic concepts of probability

In order to accomplish this, the GAISE Report suggests that students will need to understand the nature of variability, the role of context, probability, and chance variability (Franklin et al., 2007).

After an extensive review of research in statistics education, Garfield et al. (2008) suggest the following ideas as being important to statistics education:

1) Data
2) Statistical models

3) Distribution

4) Center

5) Variability

6) Comparing groups

7) Sampling and sampling distributions

8) Statistical inference

9) Covariation

One can see many similarities between the GAISE Report (Franklin et al., 2007) suggestions and those made by Garfield et al. (2008). The sense of agreement between the two becomes even greater when both are investigated in detail. For example, Garfield specifically identifies center as an important idea for statistics education. Even though center has not been explicitly listed as an instructional goal for the GAISE Report, it spends considerable effort in developing a student understanding of center in service of addressing the goals that are explicitly listed. This agreement is of no surprise since both the GAISE Report and Garfield et al. are based on prior research.

2.1.1 Probability and statistics are important

Probability and statistics have been identified as an area of importance by many researchers (Casey, 2010; Garfield et al., 2008; Hawkins & Kapadia, 1984; Hirsch & O'Donnell, 2001; Jones et al., 1997; Jones et al., 1999; Konold et al., 1993). Some researchers approach the importance of probability and statistics by identifying everyday situations where the average person may interact with probability and statistics. For example, Garfield et al. (2008) suggests that advertising has become more persuasive through presenting data. Because of this Garfield et al. suggest that it would be important for someone to be able to evaluate the claims the advertisers are making and be able to make sound arguments themselves as the person in question makes decisions. Therefore, all citizens should be educated in statistics. Because of the importance of probability and statistics, statistics education is increasing at the elementary, middle school, secondary, and post-secondary levels (Casey, 2010; Garfield et al., 2008). Others consider the implications of probability and statistics in professional settings. Hirsch and O'Donnell (2001) suggest that probability is vital in all careers and most everyday decisions.

2.1.2 Probability and statistics are difficult to teach

There are many issues associated with probability and statistics education. One issue is that when compared to other areas of research, probability and statistics education is relatively new (Garfield et al., 2008). Educational research on probability and statistics has only existed for the past twenty years. Prior to the NCTM *Standards* (1989), probability and statistics were not considered part of most mathematics curricula in schools. Research on probability and statistics prior to the *Standards*, was primarily conducted by psychologists in an attempt to understand subjects' judgments in situations of uncertainty and the misconceptions that caused errors in judgment (Shaughnessy, 1992).

Another issue related to research on probability and statistics education is the lack of connection between research results and suggestions for instruction (Garfield et al., 2008). Garfield et al. elaborate by saying that research is too often conducted in labs using quantitative methods that don't transfer to classrooms. Often this occurs because researchers do not feel that qualitative methods would be relevant and the researchers are more comfortable outside of the classroom setting.

In addition to these research concerns, teachers often lack preparation specific to teaching probability and statistics (Bataner, Godino, & Roa, 2004). Casey (2010) suggests that one cause of this may be that few teachers have studied statistics, and the few that have studied statistics were taught with an emphasis on procedural knowledge. Most teachers are being asked to teach something they have never themselves experienced, reasoning with statistics (Casey, 2010; Pfannkuch, 2006).

Statistical reasoning involves, “Making interpretations based on sets of data, graphical representations, and statistical summaries (Garfield, 2002).” Garfield (2002) further suggests that statistical reasoning is a combination of ideas about data and chance, making inferences, and interpreting results. Even for those who are proficient in mathematical reasoning, there are three areas of difficulty associated with statistical reasoning. Statistical reasoning is difficult because it is contextual (Garfield, 2003), requires an aggregate view (McGatha, Cobb, & McClain, 1998), and can be counterintuitive (Batanero & Sanchez, 2005; Baterno, Henry, & Parzysz, 2005; Hawkins & Kapadia, 1984).

Contextual refers to the need to pay attention to contexts. In statistics, data alone is meaningless. The contexts of the data provide all of the meaning. Mathematical reasoning is abstract which means it attempts to remove the contexts and focuses on the underlying mathematical rule or idea. Because of their stance on contexts, statistical reasoning and mathematical reasoning are in direct conflict with one another (Garfield, 2003). An aggregate view is a view that considers all the data as a whole instead of focusing on individual data points (McGatha et al., 1998). Casey (2010) suggests that the inability to see data from an aggregate view causes difficult for secondary students and prevents them from understanding topics that are otherwise developmentally appropriate such as correlation coefficient. Reasoning related to probability can be counterintuitive which differs greatly from logical reasoning and causal reasoning (Batanero & Sanchez, 2005; Baterno, Henry, & Parzysz, 2005; Hawkins & Kapadia, 1984). For example, if drug A is better for right handed people and drug A is better for left handed people, one would reason that drug A is better for all people which is not necessarily true (Hawkins & Kapadia, 1984). Conversely, in mathematics counterintuitive results only occur at the highest levels while in probability they occur even at the elementary level (Baterno, Henry, & Parzysz, 2005). Therefore, not only are teachers inexperienced in this form of reasoning, but the experiences they have in mathematical reasoning can be contradictory to what they would be asked to teach in statistical reasoning. It is not unreasonable to conclude that teachers with degrees in mathematics may have difficulty teaching probability and statistics (Garfield et al., 2008).

Garfield et al. (2008) suggest that both preservice and inservice teachers demonstrate difficulty with understanding and teaching the core concepts of probability and statistics at all levels K through 12. Teacher knowledge in statistics needs to be developed and determining ways to develop such knowledge should be explored (Casey, 2010; Garfield et al., 2008).

Garfield et al. (2008) suggest that another problem is that teachers were taught statistics via a lecture format and then chose to teach statistics in the same manner. Garfield et al. further suggest that even though many efforts are made to lead teachers away from lecture-based formats of instruction, few teachers actually change their methods. One suggestion for why this takes place is because a lecture is much easier to prepare for than an activity. However, much as in other topics in mathematics, lecture oriented approaches fail to develop deep understandings and thus leave students with knowledge that quickly disappears (Garfield et al., 2008).

Garfield et al. (2008) suggest that studies have demonstrated that students have difficulties with even the most basic concepts in statistics. They conclude that promoting student learning will be very difficult. In addition, studies have also demonstrated that preservice teachers have limited or even incorrect notions related to the concept of sample even after they have taken a statistics course. Similarly, Garfield et al. also reference studies that have demonstrated participant failure to use relevant content when comparing groups of data even after they have taken a methods course. Additionally, studies of students who earned an A in a college statistics course showed that shortly after completion of the course, the students had limited understandings of mean, standard deviations, and the Central Limit Theorem (Garfield et al., 2008). When all of these factors are added together, it makes sense that confidence would be a serious issue for anyone being asked to teach statistics (Garfield et al., 2008).

Casey (2010) conducted a study of three mathematics teachers that were attempting to teach students to think and reason statistically as well as becoming statistically literate. The statistics content being taught was correlation coefficient. According to Casey, correlation coefficient is developmentally appropriate for secondary students but is difficult to understand because students fail to see data as aggregate and rely too much on personal beliefs about the data. Additionally, Casey suggests that students struggle the most with inverse associate or negative correlation. Casey observed during this study that for teachers to teach anything beyond basic calculations of correlation coefficients, the teachers needed to possess a conceptual understanding of correlation. This knowledge would include how to compute correlation, why correlation is computed in that manner, and what the implications of this computation are. In other words, teachers needed to know the meaning of correlation not just the computation. As a result, Casey suggests that teaching statistics requires three knowledge components: knowledge of meaning, knowledge of terminology, and knowledge of context.

Research by Konold (1995) demonstrated that formal instruction often fails to impact students. Konold used questionnaires and interviews to learn about the beliefs of college students in relation to their prior education in statistics and found that the beliefs held by the participants were unaffected by the classes they had taken. For example, Konold gave a questionnaire to 119 students asking about the accuracy of the weather forecast that claims a 70% chance of rain both before and after they participated in a variety of different statistics courses and workshops. The results showed only a 6% increase in the number of correct responses after instruction.

Finally Garfield et al. (2008) summarize other issues in statistics education by suggesting that statistics is challenging to both teachers and learners for the following reasons:

1) Concepts and rules are complex and often counterintuitive
2) Students struggle with the underlying mathematics
3) Contexts can be misleading
4) Often confused with mathematics where there is one right answer and problems are not as messy

Perhaps the greatest concern to educators in the areas of probability and statistics are the widespread misconceptions in these areas. While misconceptions are an issue in mathematics education, their role in probability and statistics education may be significantly stronger.

2.1.3 Misconceptions are widespread across content and among everyone

Most of what has been written in regards to probability and statistics focuses on the myriad of misconceptions associated with them. There are many misconceptions, primarily among statistically naïve thinkers, but even among those whom have been educated in probability and statistics. Therefore, one might suggest that probability and statistics is an area of utmost importance in education. However, these misconceptions could also make probability and statistics an area of extreme difficulty to teach.

Hawkins and Kapadia (1984) note that there are many historical examples of mathematicians themselves making errors when it comes to basic probability. One example given is that a number of mathematicians felt that when flipping two coins, the probabilities of both heads, both tails, and one of each were all equally likely (each being 1/3). One should realize that flipping one of each is twice as likely (1/2) as the other two (1/4 and 1/4). Similarly, Batanero, Henry, and Parzysz (2005) reference a famous mathematician, D'Alembert, who argued that the probability of getting at least one tail in the same two flips of a coin situation was 2/3 even though it should be ¾.

Hirsch and O'Donnell (2001) suggest that misconceptions related to probability and statistics are developed outside of the classroom through informal experiences. Students are exposed to complicated problems and develop heuristics to estimate the probabilities associated with these problems. Unfortunately, in many cases these heuristics are faulty. Even though the heuristics are faulty, they are deeply held and thus resist changing even with formal instruction (Batanero, Henry, & Parzysz, 2005; Hirsch & O'Donnell, 2001; Konold, 1995). Hirsch and O'Donnell suggest that students will passively go along with instruction but actually still hold on to their misconceptions. As a result, students can choose correct answers to problems without correct reasoning behind it.

Hirsch and O'Donnell (2001) were able to generate evidence of students using faulty logic to generate correct answers in their research. Hirsch and O'Donnell gave students multiple choice and open-ended questions related to probability and probabilistic reasoning. The multiple choice items would ask students which of the following is either least or most likely. Then there would be a follow up multiple-choice item asking students to provide an explanation for their answer. The results of this study showed that many students provided correct answers to probability questions without providing the correct reasoning on the follow up question. This can be especially dangerous for statistics education. Students and teachers would in essence be seeing a false positive test. The positive being the correct answer but with false reasoning used to determine the correct answer. Teachers may then be compelled to believe that the students have mastered the concept due to the positive response without such mastery actually occurring. Since the reasoning associated with the concepts is still faulty, future learning may be impeded as well.

In some cases, instruction in probability and statistics has actually caused students to rely more on faulty heuristics. Research by Morsanyi, Primi, Chiesi & Handley (2009) demonstrated that psychology students relied on the equiprobability bias heuristic more at the end of their college educations than they did at the beginning. An example of how the use of this heuristic was assessed is in the following question from Morsanyi et al. (p. 213, 2009):

The two most common causes of learning difficulties among university students are dyslexia and dyscalculia. Out of 15 university students with learning difficulties, approximately nine are dyslexic, and six have dyscalculia. Joe is a student with a learning difficulty. Which of the following is most likely?

a) Joe is dyslexic
b) Joe has dyscalculia
c) Both are equally likely

In this example, the appropriate response is choice A. Based on the data provided, Joe is most likely dyslexic because nine of 15 university students with learning difficulties are dyslexic. However, students using the equiprobability heuristic respond with C because they falsely assume that two outcomes must be equally likely even though they have been provided data that demonstrates this assumption to be untrue.

Research on misconceptions was initially conducted by psychologists, not educators. As a result, reviews of literature on probability and statistics often trace research back to either Piaget and Inhelder from the 1950's or Tversky and Kahneman's work from the 1970's (Shaughnessy, 1992; Chernoff & Sriraman, 2010; Garfield, 2008). These psychologists developed many theoretical perspectives on probability and statistics and identified specific heuristics subjects in their studies used to make decisions under conditions of uncertainty that led to misconceptions of probability and statistics. Specifically, Tversky and Kahneman (1973) were able to establish the existence of the representativeness and availability heuristics, which have led them and other researchers to determine many other misconceptions that exist in the areas of probability and statistics. A table of identified misconceptions can be found in Appendix A.

Teachers may benefit from being aware of these common misconceptions. If teachers are able to anticipate potential misconceptions that students might have, they might be better able to deal with those misconceptions during instruction. Since these misconceptions occur in people at all levels of education, the teachers themselves might even have some of these misconceptions. If the teacher has a misconception, it is vital that the teacher has an opportunity to change his or her thinking. Therefore, tasks that address commonly held misconceptions could be beneficial to both the teacher and students. However, since these misconceptions are so strongly held and widespread, the curriculum materials containing such tasks will need to be educative in nature in order to support building teacher knowledge. Without such support, it is possible that the misconception will either never be addressed or even worse, the misconception could be reinforced if it is held by teachers and passed on to students.

2.2 GUIDELINES FOR ASSESSMENT AND INSTRUCTION IN STATISTICS EDUCATION REPORT

In an attempt to help educators deal with all of the previously mentioned issues, the Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report was written. The GAISE Report was written on the premise that, "Every high-school graduate should be able to use sound statistical reasoning to intelligently cope with the requirements of citizenship, employment, and family and to be prepared for a healthy, happy, productive life (Franklin et al., p. 1, 2007)." The GAISE Report references advances in technology, a society that is filled with data in the information age, and the NCTM as justification for statistics and probability being key components to mathematics curriculum beginning as early as pre-K and continuing all the way through 12th grade. However, incorporating probability and statistics is not as easy as just adding it to the existing curriculum.

The GAISE Report concurs with the research previously referenced that suggests that teachers have difficulty with teaching probability and statistics for many reasons (Franklin et al., 2007). As previously suggested, one of these reasons is that probability and statistics are new topics for many mathematics teachers. Because of this, teachers have not had the opportunity to develop their knowledge of the concepts and underlying practices that they will be teaching. This lack of knowledge leads to a vision of the curriculum that lacks cohesion. Another significant difference that was previously mentioned and is suggested by the GAISE Report is that mathematics and statistics differ greatly. The GAISE Report notes that mathematics is simply about numbers, but statistics and probability are numbers with context. Garfield et al. (2008) further explain that in mathematics contexts are discarded because they can be distracting, hence the need for abstraction. These fundamental differences cause students to react differently to each and therefore teachers need to be prepared differently depending on which one they are to teach.

One area where these differences are evident is that statistics focuses on variability. Franklin et al. (2007) define multiple types of variability in the GAISE Report. The basis of statistics is comparing *natural variability* to *induced variability*. *Natural variability* refers to the idea that measurements on individuals will vary. For example, if one were to measure the heights of different people, not everyone is the same height. *Induced variability* refers to experiments that are set up with the intention of creating variation. An example of this would be giving someone a drug as compared to giving them a placebo. In addition to these two main types of variability, Franklin et al. (2007) define two others, *measurement variability* and *sampling variability* that are important to statistics. *Measurement variability* refers to the idea that even repeating measurements on the same subjects can yield different results. For example, if a person blood pressure is measured more than once, it is possible that the measurements will differ. *Sampling variability* refers to the idea that two samples of the same population will likely yield different results.

Many other researchers join Franklin et al. (2007) when they propose that all students should have *statistical literacy* (Batanero & Sanchez, 2005; Ben-Zvi & Garfield, 2004; Garfield et al., 2008; Jones & Thornton, 2005). Garfield et al. (2008) define *statistical literacy* as understanding both the language and tools of statistics. They suggest that this includes an understanding of terms, symbols and representations of data, and the ability to interpret, evaluate, and communicate about data. They further suggest that this occurs through five knowledge bases: literacy, statistical, mathematical, context, and critical. Finally, they specifically identify three levels of *statistical literacy* as knowledge of terms, understanding terms in context, and critiquing claims.

In the GAISE Report, Franklin et al. (2007) suggest that *statistical literacy* should emphasize data collection design, exploring data, and interpreting results. This emphasis is evident in the GAISE Report's suggestions for statistical problem solving. Franklin et al. suggest the following four processes be included:

1) Formulating questions by clarifying the problem and determining what questions the data can answer
2) Collecting data by designing and then employing a plan to collect data appropriate for the question
3) Analyzing data with appropriate numerical and graphical methods
4) Interpreting results in relation to the original question

Franklin et al. (2007) suggest that variability plays an important role in the above process and that an increased role of variability is indicative of maturation in the process. Specifically they note the following:

1) To be a stats question, there must be variability
2) Acknowledge the variability and use randomness and other designs to minimize it
3) Use distributions (confidence intervals) to account for variability

4) Generalizations must incorporate additional variability (make data go from sample to population)

Based on all of these ideas, the GAISE Report was created. One purpose of the report was to generate a framework that would represent a clear, coherent vision of what statistics education might look like in the pre-K through 12 classrooms. The GAISE Statistics Framework consists of three developmental levels. These levels are often equated with grade levels, but the intention of them is to be based on levels of statistical literacy as opposed to age. Thus, an adult with no experience in statistics would begin at level A even though some might consider level A to be elementary level statistics. This is an important component of using the framework since it would be inappropriate to have high school students working in level C if they have not first experience levels A and B in elementary and middle school.

The distinction between the three levels is the role of variability (Franklin et al., 2007). In level A, variability within a group is considered. Level B considered variability between groups and covariability. Finally, at level C, students consider modeling aspects of data analysis. Franklin et al. provide examples related to word length to illustrate the differences between the levels. At level A, one might consider how the lengths of words on a single page differ. At level B, one might consider how the lengths of words from third grade books compare to lengths of words from fifth grade books and be able to describe the differences with statistical relationships such as every grade the words get two letters longer. At level C, one might consider a regression line predicting the lengths of words at each grade level book and determine if it predicts the lengths well. The framework for each level can be found in appendix B.

2.3 COMMON CORE STATE STANDARDS AND PROBABILITY AND STATISTICS

Before moving forward, it may be helpful to summarize this chapter so far. The chapter has established that probability and statistics are important topics and should be included in mathematics education. It has been established that probability and statistics will be difficult to teach because those with the responsibility to teach it are typically experts in mathematics, which uses a different type of reasoning than probability and statistics. Mathematical reasoning often involves abstraction, which requires eliminating contextual features of a problem. Probability and statistics reasoning is just the opposite because the contexts are vital to interpreting the data (Garfield, 2003). Probability and statistics are also difficult to teach because of the widespread misconceptions strongly held by many people that will likely be present in the students and even possibly the teachers.

2.3.1 Probability and statistics in curricula and standards

As information on probability and statistics has become available and more prevalent, studies have begun to determine how much probability and statistics exist in current curricula and standards documents. With the NCTM's push to make probability and statistics mainstream topics, one would expect that textbooks, state standards documents, and assessments would all contain a variety of probability and statistics topics.

Porter, Polikoff, and Smithson (2009) analyzed state standards and compared them to each other and NCTM Standards at the fourth and eighth grade levels. They found that they were significantly different. There was nationwide agreement on 13 topics in mathematics, which represented on average only 18.6% of each state's total curriculum and 21.4% of the NCTM's suggested content. Of the 13 topics, most indicated using low-level cognitive demand.

This is particularly disturbing since Porter, Polikoff, and Smithson (2009) suggest that having clear consistent standards is the first step in standards based reform. However, Porter et al. continue by suggesting that just having standards is not enough. In addition, assessments need to be created to match the expectations of the standards or the standards will be ineffective. Professional development and instructional materials can be aligned with standards and assessments to create a coherent system of education that will better promote student learning.

CCSSM intend to change all of this. The argument has already been made for how the CCSSM will facilitate such a change and therefore provides a fertile basis for this research. Therefore, the next step is to analyze the suggestions found in CCSSM. Since this study uses CCSSM as its guide to what areas of probability and statistics should be included in the curriculum that was analyzed, it makes sense to make connections between suggestions found in the literature and suggestions found in the CCSSM as part of a review of literature.

The GAISE Report was released in 2007. The authors of the GAISE Report were guided by the findings from decades of prior research. CCSSM was released in 2010, which means that the GAISE Report was able to influence what suggestions were made by CCSSM. This relationship between research influencing GAISE and GAISE influencing CCSSM is demonstrated in the tables that follow. There are three tables, each representing one of the domains from CCSSM in the area of probability and statistics.

1) Interpreting Categorical and Quantitative Data
2) Making Inferences and Justifying Conclusions
3) Conditional Probability and the Rules of Probability

Within each of these domains, there is a cluster of standards. For example, there are nine standards in the cluster associated with Interpreting Categorical and Quantitative Data. The rows of the tables are organized by specific standards from CCSSM. Any suggestions from the GAISE Report and suggestions found in research are summarized in the row with the CCSSM standard to which the suggestions correspond.

The similarities across the rows are not coincidental. The tables demonstrate that the GAISE Report influenced CCSSM and that both CCSSM and the GAISE Report were influenced by research. Therefore, even if CCSSM were not adopted by many states nationwide, it would still provide an appropriate basis for studying probability and statistics since it is based on prior research and the GAISE Report.

2.3.2 Interpreting Categorical and Quantitative Data (S-ID)

The S-ID standard focuses on interpreting data. Specifically, there is a focus on summarizing, representing, and interpreting data. Graphical representations of data are prevalent as well as uses of measures of center and spread. A key point of emphasis is that the focus is not just on drawing graphs and calculating measures of center and spread. The focus is on interpreting and understanding what the graphs represent and what the measures of center and spread mean. The keys to these understandings are interpreting each with contexts. In mathematics, contexts are often intentionally ignored in favor of abstracting mathematical concepts. In statistics, contexts are vital and cannot be ignored. Another key to interpreting data is an aggregate view. Students must develop the ability to look at data as a whole rather than focusing on individual data points. Table 2.1 provides links for each of the standards in this cluster with the recommendations in the GAISE Report and relevant research.

Table 2.1. Comparison of CCSSM S-ID cluster to GAISE report and research

CCSSM Standard	GAISE Report	Relevant Research
S-ID-1 Represent data with plots on the real number line (dot plots, histograms, and box plots)	At level A, students use dot plots and box plots to explore distributions and association. Students begin using histograms at level B for summarizing and comparing distributions as well as more sophisticated uses of dot plots and box plots. Potential confusion between bar graphs and histograms is noted as a misuse of statistics in level A.	Garfield et al. (2008) suggests that histograms and box plots cause confusion for students because the students think they are the same thing even though they are significantly different. Bakker, Biehler, and Konold (2004) suggest that boxplots present unique challenges for students because the median and quartiles are not easily understood and individual cases are not perceiveable.
S-ID-2 Use statistics appropriate to the shape of the data distribution to compar9e center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	Measures of center and spread are introduced at level A and increase in sophistication through levels B and C. For example, the mean evolves from an interpretation as “fair share” to “balance point” from level A to level B and then sample means are used for making statistical inferences. Measures of spread start with range at level A, progress to the Mean Absolute Deviation at level B, and then standard deviation and applications of measures of spread at level C.	Groth and Bergner (2006) suggest that students need to understand measures of center including which measure is most useful for a given problem. Ben-zvi (2004) suggests that spread is fundamental to statistical thinking. Reading and Reid (2006) suggest that variation (spread) affects all other areas of statistics. Reading (2004) suggests that center is overemphasized while variability is underemphasized or even ignored due to difficulty. Delmas and Liu (2005) suggest that students will have difficulty with variability and as a result cannot make inferences or understand distributions. Konold and Pollatsek (2002) suggest a signal (center) amongst the noise (variation) view of center and spread.

(table continues)

Table 2.1 (continued)

CCSSM Standard	GAISE Report	Relevant Research
S-ID-3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers)	Context is viewed very differently in statistics than it is in mathematics. In mathematics, we strip away contexts, but in statistics, context is what gives the numbers meaning. Students will interpret differences in data sets throughout all three levels of the framework with degrees of sophistication being developed throughout.	Chance (2002) suggests that data without context is useless. Casey (2010) suggests knowledge of context is important in teaching statistics. Garfield et al. (2008) suggests that making comparisons between groups allows students to develop an understanding of contexts and that boxplots may be useful for making such comparisons. Pfannkuch (2006) suggests that boxplots are difficult for both students and teachers because they are conceptually demanding, obscure information, condense data, and summarize data. Pfannkuch (2006) also suggests that justifying inferences is difficult and that traditional statistics instruction neglects making inferences with box plots.
S-ID-4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.	Students should develop an understanding of appropriate analysis as analysis that leads to inferential statements regarding population parameters that can be justified. Normal distributions should be introduced as a model for sampling distributions and students should be familiar with finding areas under the normal curve using appropriate technology.	Pfannkuch (2006) suggests that both students and teachers need to improve their abilities to communicate in the area of distribution. Garfield et al. (2008) suggests that normal distribution and fitting data to normal distribution are important topics and prerequisites to formal studying of sampling distributions.

(table continues)

Table 2.1 (continued)

CCSSM Standard	GAISE Report	Relevant Research
S-ID-5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.	Using a two-way frequency table to summarize categorical data for two categories is explicitly suggested as part of level B. Interpretations of data are part of this suggestion including recognizing associations and trends.	Understanding context is vital (Casey, 2010; Chance, 2002; Garfield et al., 2008). Batanero et al. (1996) studied conceptions of association in frequency tables and suggest that three misconceptions exist: dependence can only exist if the two cells containing disagreement between variables have a frequency of zero; inverse association is a form of independence; judgments are based on the cell that contains the maximum frequency and ignores the other cells.
S-ID-6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.	Representing two quantitative variables on a scatter plot and describing how they are related is incorporated at all three levels (A, B, and C) with varying levels of sophistication. These comparisons range from basic comparisons like as one gets larger the other gets larger at level A to estimating lines of best fit at level B and finally using least squares to calculate a line of best fit.	Garfield et al. (2008) suggest that processing, analyzing, and representing the data is one of four stages of data analysis. Hubbard (1997) suggests that students are presented open-ended questions in a standard form leading to memorization that teachers misinterpret as understanding (i.e., create a scatter plot, describe the relationship, find the correlation coefficient and say if it agrees with the suggested relationship, find the regression model and write the equation, plot regression model, state if the model does a good job predicting).
S-ID-7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	Interpretations of both slope and intercept are both explicitly discussed at level C. This discussion includes situations in which interpretations of intercept are unrealistic based on contexts.	Garfield et al. (2008) suggests that contexts are used to explain patterns or deviations from patterns when generating a model and that models are the foundation of statistical thinking yet are often neglected in statistics courses. Zieffler and Garfield (2009) suggest that student interpretation of rate of change is slow to develop and often is not seen as relating to covariation.

(table continues)

Table 2.1 (continued)

CCSSM Standard	GAISE Report	Relevant Research
S-ID-8 Compute (using technology) and interpret the correlation coefficient of a linear fit.	At level B, the calculated correlation coefficient is the Quadrant Count Ratio. This notion is built upon to develop the use of Pearson's correlation coefficient at level C.	Falk and Well (1997) suggest that correlation coefficient, specifically Pearson's *r*, is used in education, psychology, the social sciences, and is central to many statistical methods, but current instructional practices lead to an impoverished understanding of conception of correlation. Rumsey (2002) suggests time focused on calculating correlation coefficients can inhibit understanding.
S-ID-9 Distinguish between correlation and causation	Students begin distinguishing between correlation and causation at level B and then continue to develop the ability to distinguish between the two at level C. Specific suggestions are given for each how to facilitate students making this distinction at both levels.	There is a common misconception that correlation implies causation. (Chance, 2002; Delmas et al., 2007; Garfield, 2003)

Throughout Table 2.1, the influence of the GAISE Report and research on CCSSM can be seen. The GAISE Report and research suggested much of the same content found in CCSSM prior to CCSSM being released. In addition to these content suggestions, the GAISE Report and research also emphasize a focus on understanding, how each fits into the big picture of statistics, and cautions associated with each. Additionally, misconceptions and errors in emphasis during instruction are identified as important points to be made about how these suggestions should be taught. Both GAISE and research emphasize the importance of contexts in probability and statistics education. The authors of CCSSM incorporated all these content suggestions and the emphasis on contexts.

2.3.3 Making Inferences and Justifying Conclusions (S-IC)

The S-IC standard focuses on making inferences and conclusions about a population based on a sample of that population. The focus is not just on being able to make an inference or draw a conclusion, but to understand why one can make such an inference or draw such a conclusion. In addition, students are expected to understand the role of randomness in these inferences and conclusions. Additionally, students should be able to look at the inferences and conclusions of others and decide if they are appropriate.

Table 2.2. Comparison of CCSSM S-IC cluster to GAISE report and research

CCSSM Standard	GAISE Report	Relevant Research
S-IC-1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population	One of the four components identified is the *process* component. At level A, students do not make inferences. At level B, making inferences is considered reasonable by students. At level C, students are able to make inferences about the population.	Garfield et al. (2008) suggest making inferences based on samples is a central idea of statistics but students are reluctant to make inferences about a population regardless of the sample. They further suggest that students have multiple difficulties and multiple misconceptions in the area of sampling.
S-IC-2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*	Possible reasons for inconsistent models are provided. At level C, p-values are used to make judgments when a model is in question. *The specific example of determining if a coin is fair by using 5 tosses is explicitly discussed in the introduction.*	Garfield et al. (2008) suggest that students should understand how data are produced, how data are collected, where data comes from, the types of analysis, and the conclusions that can be made. They further suggest that students lack an understanding of the importance of sample size. Sample size is important to consider in the case of tossing a coin 5 times.
S-IC-3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.	*Collect data* is one of the four identified process components. At level A, the differences are not considered. At level B, differences begin to be considered with sample surveys and comparative experiments being used. At level C, students develop a full understanding of each type of statistical study and how randomization is important to each.	Garfield et al. (2008) suggest that students should understand the differences between random sampling and random assignment. Smith and Sugden (1988) suggest that surveys, experiments, and observational studies are important to the work of applied statistics and propose a framework for examining each.

(table continues)

Table 2.2 (continued)

CCSSM Standard	GAISE Report	Relevant Research
S-IC-4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.	At level C, an *appropriate analysis* is defined as one where justifiable inferential statements about population parameters can be made. Specifically, population mean is identified for numerical data and population proportion is identified for categorical data. Multiple explicit suggestions for estimating a population mean or proportion and a margin of error calculated based on the sampling distribution are provided.	Garfield et al. (2008) suggest making inferences based on samples is a central idea of statistics and should include how data are produced, how data are collected, where data comes from, the types of analysis, and the conclusions that can be made. Yilmaz (1996) suggests statistics education is important for many students not majoring in statistics yet has been ineffective. Yilmaz suggests a course design that includes studying population, sampling, drawing conclusions, and statements regarding error using appropriate technology.
S-IC-5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.	Level C provides specific suggestions regarding randomized experiments including using simulations to approximate a p-value and decide if the differences are significant.	No specific references to this particular standard were found in the research.
S-IC-6 Evaluate reports based on data.	An overall theme of the report is that data governs our lives. It suggests that students understand how statistics are commonly misused in reports so that students may be equipped to identify such things in the real world. Historical examples of these misuses are presented.	Garfield et al. (2008) suggest that advertising has become more persuasive through presenting data, so it would be important for someone to be able to evaluate the claims the advertisers are making.

The GAISE Report initially suggested many of the concepts later identified by CCSSM and provided suggestions for developing these concepts throughout varying levels of sophistication. For example, the GAISE Report suggests that students first consider making inferences about the population at level B and are unable to make such inferences until level C. Building on this idea, CCSSM suggests that students understand making inferences about the population as one of the standards found in under the domain of Making Inferences and Justifying Conclusions. Therefore, we can once again see that the suggestions found in CCSSM are built upon the suggestions of the previously released GAISE Report. Research also plays an influential role as many cautions that educators need to made aware of including areas where students have misconceptions, reluctance, or tend to lack understanding are addressed. For example, Garfield et al. (2008) suggests that students will be reluctant to make inferences based on a scholarly review of research. Understanding this suggestion from research could be why the GAISE Report does not address making inferences until its highest level of sophistication, level C, and suggests that students will not even consider making inferences until level B.

2.3.4 Conditional Probability and the Rules of Probability (S-CP)

The S-CP standard focuses on independence, conditional probability, and rules of probability. Multiple interpretations of independence are addressed both using and not using rules of probability. Rules of probability are addressed with suggested example problems and methods of interpreting results that may demonstrate appropriate understanding of each.

Table 2.3. Comparison of CCSSM S-CP cluster to GAISE report and research

CCSSM Standard	GAISE Report	Relevant Research
S-CP-1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").	While describing events as subsets of sample space using these characteristics is not explicitly addressed, the use of two-way frequency tables and suggestions regarding association require an understanding of unions, intersection, and complements.	Batanero, Henry, & Parzysz (2005) suggest that sample space and compound events are important concepts for probability instruction. Jones & Thornton (2005) suggest that middle school and high school age students struggle with sample space.
S-CP-2 Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.	Acknowledges the importance of students understanding independence, but defines independence in the context of random sampling providing independent observations as opposed to using the product of probabilities as a characterization.	Independence is an important concept (Batanero, Henry, & Parzysz, 2005; Batanero & Sanchez, 2005). However, Batanero, Godino, and Roa (2004) suggest that although independence can be expressed by this multiplicative rule, probability instruction is moving away from this characterization because it often leads to an incomplete understanding of independence. Hirsch & O'Donnell (2001) suggest that students may be able to demonstrate use of formal rules while still holding on to misconceptions.

(table continues)

Table 2.3 (continued)

CCSSM Standard	GAISE Report	Relevant Research
S-CP-3 Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.	While not stating the rule explicitly, the GAISE Report interprets independence in the same manner by suggesting that independence is the chance of one outcome not being effected by knowledge of another outcome (if a coin landed on heads on the second flip that doesn't change the probabilities associated with the fourth flip of that coin).	Independence is an important concept (Batanero, Henry, & Parzysz, 2005; Batanero & Sanchez, 2005). Batanero, Godino, and Roa (2004) make instructional suggestions for developing an understanding of conditional probability and independence that include playing a game with three cards. One card is red on both sides, one blue on both sides, and one that is red on one side and blue on the other. Cards are randomly drawn with replacement and only one side shown to students. Students are then asked to predict what color the other side is.
S-CP-4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*	Suggestions addressing this standard are introduced at level A and then further developed at levels B and C with the explicit use of a two-way table.	Rossman and Short (1995) suggest that an intuitive understanding of conditional probability can be developed using genuine data and two-way frequency tables. They present multiple examples and suggest that conditional probability provides opportunities for important and interesting examples to be included in statistics education. Batanero & Sanchez (2005) suggest that students will benefit from working with real data and have multiple misconceptions in conditional probability. Chance (2002) suggests that students will benefit from working through the entire statistical process as opposed to textbook problems that eliminate steps for them.

(table continues)

Table 2.3 (continued)

CCSSM Standard	GAISE Report	Relevant Research
S-CP-5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer*.	Acknowledges the importance of students understanding independence, but defines independence in the context of random sampling providing independent observations. *Specifically addresses an observational study involving smoking and lung cancer at level C.*	Rossman and Short (1995) suggest that the distinction between P(A/B) and P(B/A) is subtle yet crucial. They specifically reference an example where students are asked to interpret a two-way table of data and assess the statement "most Democratic senators are women" and "most women senators are Democrats" and refer to making such an interpretation as an essential skill.
S-CP-6 Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.	While this standard is not explicitly addressed, other suggestions are closely related and could be used in a manner consistent with the suggestion of CCSSM. For example, suggestions for discussing association as an interpretation of conditional probabilities readily lend themselves to this suggested understanding of conditional probability.	No specific references to this particular view of conditional probability were found in the research.
S-CP-7 Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.	Once again, while this standard is not explicitly addressed, using the suggestions regarding association could easily incorporate the addition rule and then interpreting the results of the addition rule in terms of the population based on the model.	Hansen, McCann, and Myers (1985) research demonstrates that students who learned from text that focused on conceptual learning as opposed to rote learning were able to apply six formulas including P(A or B) – P(A) + P(B) – P(A and B) more effectively due to their ability to categorize problems by underlying concepts as opposed to surface features which could be misleading.

Both the GAISE Report and research likely influenced the authors of CCSSM in the area of independence. Both make the assertion that an understanding of independence is vital to probability and statistics, which was then adopted by CCSSM. What is interesting is the stance each takes on how that understanding is developed. The primary influence of the research is cautioning against the use of formal rules because of how formal rules have the ability to mask misconceptions. Because of this suggestion from research, the GAISE Report suggests formal rules should be saved for advanced classes such as discrete mathematics or calculus and opts for a more informal approach to developing an understanding of independence in earlier classes. CCSSM incorporates multiple ways of understanding including both informal methods and the use of formal rules. For example, standard S-CP-3 seems to reflect the suggestions of the GAISE Report even though it relies on an application of a formal rule for conditional probability. Standard S-CP-2 seems to contradict the suggestions of both GAISE and research since it focuses on using a formal rule to define independence.

2.4 TEXTBOOK STUDIES

As argued in chapter 1, textbooks represent a way to influence classroom practices and affect student achievement (Senk & Thompson, 2003). In some cases, research has suggested that textbooks have more of an impact on student learning than the teacher (Begle, 1973). This is because textbooks impact what students have the opportunity to learn (Schmidt, Houang, & Cogan, 2002; Stein, Remillard, & Smith, 2007; Valverde et al., 2002; Willoughby, 2010) and the teachers are dependent on them (Remillard, 2005). For these reasons, analyzing textbooks has been an important method of research employed in both probability and mathematics education.

2.4.1 Analysis of probability in textbooks

Jones and Tarr (2007) set out to determine the nature of probability topics in middle school textbooks with a specific focus on the levels of cognitive demand. Jones and Tarr selected two textbooks from four different eras of mathematics education published over the last 50 years. Those four eras are New Math (1957 – 1972), Back to Basics (1973 – 1983), a focus on Problem Solving (1984 – 1993), and the National Council of Teachers of Mathematics Standards (1994 – 2004) era. The textbooks were selected based on their popularity, which was determined by the market share during a given era. Due to a lack of data, the popularity of textbooks during the New Math era was determined by a consensus of mathematics educators familiar with the curriculum during that era. In order to qualify for selection, textbooks must have been intended for average students in grades 6, 7, and 8. For example, algebra textbooks were not considered because they would have been intended for advanced students. Only student editions of the textbooks were analyzed because Jones and Tarr were only concerned with tasks students may have encountered.

In addition to examining popular textbooks, Jones and Tarr (2007) also analyzed what they referred to as alternative textbooks. Alternative textbooks were ones that were identified by the previously mentioned consensus of mathematics educators as being potentially innovative, influential, or being a departure from the current popular series. Table 2.4 is a list of the eras, popular textbooks, and alternative textbooks analyzed by Jones and Tarr.

Jones and Tarr (2007) used the task analysis guide (Appendix C) from Smith and Stein (1998) as the basis for their analysis. Table 2.5 shows the codes from Smith and Stein and the resulting description used by Jones and Tarr for their research in probability.

Table 2.4. Textbooks selected for analysis from different mathematical eras

Era	Popular (Publisher)	Alternative (Publisher)
New Math (1957 – 1972)	*Modern School Mathematics: Structure and Use 6* *Modern School Mathematics: Structure and Method 7 & 8* (Houghton Mifflin)	*Mathematics for the Elementary School, Grade 6* *Mathematics for Junior High School, Vols. I & II* (Yale University Press)
Back to Basics (1973 – 1983)	*Holt School Mathematics: Grades 6, 7, & 8* (Holt, Rinehart, & Winston)	*Real Math: Levels 6, 7, & 8* (Open Court)
Problem Solving (1984 – 1993)	*Mathematics Today: Levels 6, 7, & 8* (Harcourt Brace Jovanovich)	*Math 65: An Incremental Development* *Math 76: An Incremental Development* *Math 87: An Incremental Development* (Saxon Publishers)
Standards (1994 – 2004)	*Mathematics: Applications and Connections: Courses 1, 2, & 3* (Glencoe/McGraw-Hill)	*Connected Mathematics* (Dale Seymour)

Note. From "An examination of the levels of cognitive demand required by probability tasks in middle grades mathematics textbooks," by Jones & Tarr, 2007, *Statistics Education Research Journal, 6*(2), p. 12. Reprinted pending permission

Table 2.5. Comparison of codes from Smith and Stein (1998) to Jones and Tarr (2007)

Smith and Stein (1998)	Jones and Tarr (2007, p. 8)
Memorization	Simply memorize information
Procedures without Connections	Routinely perform algorithms without giving any attention to the meaning or development of the procedure
Procedures with Connections	Focus on the meaning of a procedure or algorithm
Doing Mathematics	Explore and analyze the mathematical features of a situation

Jones and Tarr (2007) found that most probability tasks across textbooks were at the level of procedures without connections. However, two textbooks contained both more high-level tasks and a higher percentage of high-level tasks than all others did. Those textbooks were the standards era alternative series (*Connected Mathematics*) and the Back to Basics era alternative series (*Real Math: Levels 6, 7, & 8*). The standards era alternative series was particularly impressive because a majority (59%) of its tasks required high-level cognitive demand. By applying the finding that tasks either stay at the same level or decline during implementation (Stein et al., 1996), Jones and Tarr suggest that most textbooks across each of the four eras analyzed would have only provided students with opportunities for engagement at lower levels of cognitive demand and thus severely limit their views and understandings of probability.

2.4.2 Textbook studies in mathematics education

Other studies where textbooks were analyzed also provide important insights for the proposed study. Thompson, Senk, and Johnson's (2012) analysis of high school mathematics textbooks for opportunities to learn reasoning and proof is of particular interest. Thompson et al. (2012) claimed that, "Textbook analysis is a first, but important, step in understand students' opportunities to learn reasoning and proof (p. 282)." Thompson et al. analyzed both the narratives and exercises of textbooks in order to determine what opportunities to engage in this process were available in U.S. secondary textbooks. Thompson et al. analyzed the narratives because they provide opportunities for teachers to introduce reasoning and proof to students. Thompson et al. analyzed the exercises because they provide opportunities for students to practice with reasoning and proof.

Thompson et al. (2012) analyzed a variety of textbooks for their study. They began with the Algebra I, Algebra II, and Precalculus textbooks from each of the large textbook publishing companies (Glencoe, Holt, and Prentice-Hall). These major companies were included in the study because they represent a majority of the textbooks being used by secondary schools. They also analyzed Interactive Mathematics Program textbooks because of their reputation for developing innovative curriculum materials. Finally, they analyzed textbooks from two different curriculum development projects, Core-Plus Mathematics (courses 1 – 4) and the University of

Chicago School Mathematics Project (Algebra I, Algebra II, Functions, Statistics, and Trigonometry, and Precalculus and Discrete Mathematics). This sample of textbooks allowed them to analyze both traditional and *Standards*-based textbooks.

Thompson et al. (2012) suggest that students have few opportunities to engage in proof and reasoning in both the narratives and exercises. Furthermore, many of the justifications found in the Algebra I textbooks that were analyzed were related to a specific case rather than a general case. Thompson et al. suggest that this focus on justifications with specific cases may contribute to the willingness many students have to confuse an argument based on a specific example as a proof.

Another key finding by Thompson et al. (2012) is the differences between the curriculum materials based on their pedagogical design. Thompson et al. found that Core-Plus Mathematics contained the largest percentage of proof and reasoning exercises with Interactive Mathematics Program and the University of Chicago School Mathematics Project also rating as above average in the percentage of proof and reasoning exercises. However, the style of the proof and reasoning opportunities were not the same. Core-Plus Mathematics and Interactive Mathematics Program provided students with more opportunities for making conjectures while the University of Chicago School Mathematics Project provided more opportunities for students to read proofs. Thompson et al. suggest that this is because Core-Plus Mathematics and Interactive Mathematics Program are both investigation based while the University of Chicago School Mathematics Project focuses more on the study of mathematical properties.

These results suggest that including textbooks in the current study that differ with respect to the underlying philosophy of teaching and learning may also lead to differences with respect to the level of cognitive demand the tasks require of students. In other words, investigation-based materials may require a different level of cognitive demand than materials that focus on mathematical properties. On a related note, teachers may require more support to implement one type of curriculum material than another may. Depending on the nature of the tasks found in each textbook, teachers may be required to learn as much or more than the students are required to learn in order for the curriculum to be implemented with fidelity.

2.5 EDUCATIVE CURRICULUM

Educative curriculum materials are curriculum materials that are written to educate the teachers and students as opposed to those curriculum materials that only have student learning in mind. The argument has just been made that probability and statistics are important topics but are difficult to teach due to many factors including that misconceptions are widespread across content and for students at all grade levels. Because of this argument, educating teachers along with students may be vital in probability and statistics education.

2.5.1 The birth of educative curriculum materials

Ball and Cohen (1996) are often credited with initiating the notion that curriculum materials could be written with the intention of educating the teacher along with the students. Ball and Cohen suggest that textbooks represent an important avenue to teacher education because they are a central fixture in teaching, intimately connected to teaching, well positioned to influence individual teachers, and already a part of the routine of schools. Ball and Cohen suggest the drawback of using curriculum materials to influence instruction is that the teachers and sometimes parents will reject the new textbooks.

Ball and Cohen (1996) suggest that curriculum materials often fail because they overlook the teacher and all the needs the teacher will have in order to implement the curriculum well. They suggest that since teachers shape instruction based on their understanding of the material, their personal beliefs about what is important, and their perception of the roles students and teachers should play in instruction, curriculum materials may be doomed to fail without strong curricular guidance. Unfortunately, Ball and Cohen also suggest that lacking this guidance is a common characteristic in our educational system.

One would assume that curriculum developers would prefer that their curriculum materials be implemented with fidelity. However, Ball and Cohen (1996) suggest that teachers often adapt curriculum materials to fit local needs that curriculum developers may not have been able to predict. In addition, Ball and Cohen suggest that the educational system we operate in often disparages textbooks and promotes the notion that the best teachers do not follow textbooks. Ball and Cohen suggest that there is a significant gap between teachers and textbook designers with little work being done to bridge this gap or study the relationship between the two.

The premise of this work by Ball and Cohen (1996) is that “Curriculum materials could contribute to professional practice if they were created with closer attention to processes of curriculum enactment” (p. 7). Ball and Cohen later assert that, “Materials could be designed to place teachers in the center of curriculum construction and make teachers’ learning central to efforts to improve education” (p. 7). Based on this belief, the notion of educative curriculum materials was born.

Ball and Cohen (1996) suggest that there are five intersecting domains teachers work across while enacting curriculum materials (p. 7):

1) Teachers are influenced by what they think about their students, what students bring to instruction, students' probable ideas about the content at hand, and the trajectories of their learning that content.
2) Teachers work with their own understanding of the material, which shapes their interpretations of what the central ideas are, how they hear, evaluate, and respond to students' ideas, and how they decide how to focus and frame the material for students
3) Teachers fashion the material for students, choose tasks or models, and navigate instructional resources such as textbooks in order to design instruction.
4) Teachers must keep their eye on the group, and on the ways of knowing, interacting, and working that seem possible. This requires attention to patterns and norms of discourse, the nature of tasks, and the roles played by the teacher and student.
5) Teachers are influenced by their views of the broader community and policy contexts in which they work, and by the expressed ideas of parents, administrators, and professional organizations.

Ball and Cohen (1996) suggest that curriculum materials could be designed to take into account the work that teachers must do in each of these five domains. They use knowledge of students as an example. Ball and Cohen suggest that while each individual student may differ some from the others, much of what students may think or do can be anticipated. Ball and Cohen continue by suggesting that teachers' guides could then offer examples of student work with comments on the meaning of each example to aide teachers in interpretation and anticipation of student thinking.

Ball and Cohen also suggest that teachers' guides could support teachers in learning content better. This could be done by providing alternative representations and the connections between them and the merits each would provide. Curriculum guides may be able to illuminate the possibilities of curriculum materials that may have gone unnoticed by teachers.

Ball and Cohen (1996) also suggest that curriculum developers could make their pedagogical judgments explicit to teachers. If teachers were made aware of pedagogical thinking that went into specific tasks, their decisions on adaptation or omission of a task may be impacted. In addition, teachers may be able to better present the materials if the pedagogy behind them were made explicit instead of being kept secret.

Ball and Cohen (1996) suggest that rather than approaching a new curriculum with the previously mentioned goal of fidelity of implementation, perhaps it would be more beneficial to think of new curriculum materials as an opportunity for professional development. Ball and Cohen acknowledge the difficulty in such a task. Curriculum materials would need to change the way they are designed to incorporate things such as examples anticipated student work. However, Ball and Cohen suggest that the results could be an increased capacity to teach.

2.5.2 Design heuristics for educative curriculum

Davis and Krajcik (2005) state that teacher learning is:

> Developing and integrating one's knowledge base about content, teaching, and learning;

> Becoming able to apply that knowledge in real time to make instructional decisions; participating in the discourse of teaching; and becoming enculturated into (and engaging in) a range of teacher practices. Teacher learning is situated in teachers' practice. (p. 3)

Davis and Krajcik's (2005) definition of teacher learning is multifaceted, complex, and has many components. Teacher learning requires subject matter knowledge, pedagogical knowledge, and pedagogical content knowledge as suggested by Shulman (1986). Davis and Krajcik (2005) further suggest that connections between ideas must be established as a part of teacher learning while new instructional approaches are being developed and teaching principles are addressed. Careful consideration must be given to possible student ideas that might arise.

Given all the needs and difficulties of teachers learning, what can educative curriculum materials do? The positive potential of educative curriculum materials was described by Ball and Cohen in 1996 and was advanced by Davis and Krajcik in 2005. Davis and Krajcik make five suggestions regarding educative curriculum materials. These five suggestions then lead Davis and Krajcik into developing nine design heuristics.

The first suggestion from Davis and Krajcik (2005) is based on Ball and Cohen (1996) suggesting that educative curriculum materials could help teachers to anticipate student thinking and help teachers consider what to do in reaction to this anticipated thinking during instruction. Davis and Krajcik suggest that curriculum materials could also explain why the students might be thinking that way. Additional support related to anticipating and dealing with student thinking could include knowledge of different instructional representations such as analogies, models, or diagrams.

The second suggestion by Davis and Krajcik (2005) is to promote teachers' learning of subject matter. Once again, this suggestion is based on Ball and Cohen (1996). The typical notion of subject matter knowledge should obviously be included here, but one could also consider the disciplinary practices associated with a subject area. This would lend itself to the notion of doing mathematics as a mathematician might instead of in the procedural world that mathematics education often becomes in traditional classrooms.

The third suggestion made by Davis and Krajcik (2005) is that educative curriculum could help teachers relate units during the year. Once again, this suggestion is based on Ball and Cohen (1996). Davis and Krajcik suggest that this could move beyond providing teachers with simple objectives. Instead, teachers could have lesson objectives presented in such a way that they promoted the teachers reflecting on the lesson and how it fit into the context of the bigger picture of the curriculum. This could promote a more coherent instructional program overall and foster some discussions between teachers as they consider the courses they teach in relation to the courses taught by their colleagues.

A fourth suggestion by Ball and Cohen (1996) that was expanded upon by Davis and Krajcik (2005) is that educative curriculum materials could make the curriculum developers' pedagogical judgments visible to the teachers using them. Davis and Krajcik suggest that by providing rationales to the teachers, teachers will be able to better integrate their knowledge bases and stronger connections will be made between theory and practice. This could improve the flexibility with which the knowledge could be applied and could promote autonomy by helping teachers make decisions about adapting curriculum materials to their own classrooms.

The fifth and final suggestion by Davis and Krajcik (2005) is that curriculum materials might promote a teacher's ability to use resources either provided in the curriculum or provided personally to adapt curriculum materials to fit local conditions while still achieving productive instructional goals. They refer to this ability in a teacher as *pedagogical design capacity*. The theory behind this idea is that teachers enact a curriculum with their students in the classroom. This enactment ideally may involve changes that are made to the curriculum materials but the essence of the original curriculum materials are still addressed. In other cases, the teacher may intentionally move away from the essence of the original materials, which could also be acceptable. However, teachers may move away from the essence of the original curriculum materials in such a way that is devastating to the intended learning of the materials. Given these possible scenarios, it could be important to arm teachers with an improved ability to make decisions regarding the enactment of curriculum materials in productive ways.

These five suggestions led to the creation of Davis and Krajcik's (2005) nine design heuristics. The heuristics are listed in Table 2.6. These heuristics are based in science, but the authors speculate that they are widely applicable to other fields, which could include mathematics. This would seem to be a reasonable suggestions since the challenges faced by teachers of science would seem to be similar to the challenge faced by teachers of mathematics. The need to anticipate student thinking or make connections across topics does not change just because the content does. Each of the nine heuristics includes what the curriculum materials should provide the teacher, how the materials could assist the teacher in understanding rationales behind decisions that were made by the developer, and how teachers could infuse their own ideas into instruction

Table 2.6. Educative curriculum design heuristics (Davis & Krajcik, 2005)

Design Heuristic	Description
Supporting Teachers in Engaging Students with Topic-Specific Scientific Phenomena	Materials should provide tasks for students to engage in, rationales for the teacher explaining why the tasks are appropriate, and suggestions for implementing the tasks well including potential difficulties and proper sequencing
Supporting Teachers in Using	Materials should provide instructional representations

Scientific Instructional Representations	such as models or diagrams, rationales for the teacher explaining why the representations are appropriate, and suggestions for using the representation well including what features are the most salient and support in adapting the representations
Supporting Teacher in Anticipating, Understanding, and Dealing with Students' Ideas About Science	Materials should identify likely student ideas and provide suggestions to help the teacher in dealing with those ideas
Supporting Teachers in Engaging Students in Questions	Materials should provide questions for teachers to use to frame the unit, guide class discussion, and engage students in asking and answering their own questions while providing rationales for why the provided questions are appropriate
Supporting Teachers in Engaging Students With Collecting and Analyzing Data	Materials should provide suggestions for approaches to help students collect, compile, and use evidence across multiple topics and provide the teachers with rationales for why using evidence is important
Supporting Teachers in Engaging Students in Designing Investigations	Materials should support teachers in helping students design their own investigations including ideas for appropriate designs and suggestions for improving inappropriate designs
Supporting Teachers in Engaging Students in Making Explanations Based on Evidence	Materials should provide suggestions for helping students make evidence based explanations including rationales for why engaging students in making evidence based explanations is important
Supporting Teachers in Promoting Scientific Communication	Materials should provide suggestions for helping students communicate productively including rationales for why engaging students in productive communication is important
Supporting Teachers in the Development of Subject Matter Knowledge	Materials should support teachers in developing knowledge of the content beyond the students level including possible student conceptions and misconceptions and relationships to real-world phenomena

While not the first to suggest the potential for curriculum materials to be educative, Davis and Krajcik (2005) are one of the most influential. Davis and Krajcik took the notions suggested by the likes of Ball and Cohen in 1996 and developed design heuristics that could help readers understand the potential for curriculum materials to promote teacher learning and thus be educative. They posed the question, "How can K-12 curriculum materials be designed to support teacher learning, and what might teacher learning with educative curriculum materials look like?" (Davis & Krajcik, p. 4, 2005). To answer this question, the design heuristics were intended to act as a guide to curriculum developers and a basis for discussion on how specific features of a curriculum might promote teacher learning.

Davis and Krajcik (2005) acknowledge the difficulties inherent in promoting teacher learning. They suggest that teacher learning includes developing and integrating a teacher's knowledge base regarding the content they are teaching, the pedagogy of teaching, and the teacher's own learning. Then the knowledge must be applied in real time during instruction all while trying to provide meaningful content to assist students to meet instructional goals in the context of authentic activities. Further complicating matters is the diverse nature of classrooms where all students are expected to succeed. Davis and Krajik suggest that all of this learning is situated in practice. This practice may include planning and modifying lessons, assessments, collaboration with colleagues, and communicating with parents.

To aide readers in understanding the complexity of teacher learning, Davis and Krajcik (2005) provide a comparison between student learning and teacher learning. Students are given a structured environment in school where they are provided a set of learning experiences intended to increase subject matter knowledge. Teachers are not placed in a structured learning environment and thus have to control their own learning. Teachers must also develop subject matter knowledge much like the students, but teachers must also develop pedagogical knowledge and pedagogical content knowledge as suggested by Shulman (1986). Since teachers are to apply their knowledge while making real time decisions in the classroom, teachers must acquire a much more flexible knowledge than students must. Because of these factors, one might suggest as Davis and Krajcik have that promoting teacher learning is different from promoting student learning.

Davis and Krajcik (2005) acknowledge that they have not empirically tested their design heuristics and thus do not refer to them as principles or standards. The term heuristic was specifically selected to suggest that their research is intended to provide useful suggestions that take research one-step closer to such principles or standards but that may require multiple iterations and revisions before such a goal may be obtained.

Davis and Krajcik (2005) acknowledge some limitations of educative curriculum materials. The educative nature of the curriculum may not be important if the content of the base curriculum is not of high quality. This means that a curriculum that is educative but filled with low-level tasks is not a good curriculum. A second limitation may be the teacher. Personal characteristics of the teachers using the curriculum are likely to have a significant impact on the effectiveness of the curriculum. The prior knowledge, beliefs held by the teacher, and the teacher's attitude toward improving his or her own instruction will all be possible factors in determining the effectiveness of how educative a curriculum can be. Finally, educative curriculum is not enough to facilitate change on its own. Multiple avenues of professional development should be used for maximum effectiveness.

If teachers can be educated through the curriculum materials, then instructional effectiveness could be maximized and curriculum could be implemented with fidelity. The next question one might ask is whether a specific curriculum is worth implementing well. Research in mathematics education suggests that the most worthwhile curriculum uses tasks that require high-level cognitive demand for students to complete (Boaler & Staples, 2008; Stein & Lane, 1996). It would not be a stretch to think that the research on high-level tasks in mathematics could apply to probability and statistics education as well.

2.5.3 Educative curriculum and CCSSM

Porter et al. (2011) suggest that CCSSM is considerably different than what states currently call for and what teachers are currently teaching. This suggests that for CCSSM to be implemented effectively, change must occur. For this change to occur, it may be necessary to have an impact on teacher knowledge. Even if a teacher does not need to be impacted to promote the changes by CCSSM, improving teacher knowledge can still be beneficial to instruction.

However, many approaches to improving teacher knowledge or even teaching in general are ineffective. Putnam & Borko (2000) suggest that learning experiences aimed at teachers that take place outside of the classroom do not have a meaningful impact because they are too removed from the day-to-day work of teaching. As a result, teacher educators are challenged with finding a way to facilitate learning experiences that actually relate to the work that teachers do. One way to facilitate learning experiences related to the work teachers do, may be with educative curriculum materials. Since teachers use curriculum materials as part of their day-to-day teaching duties, it seems logical that curriculum materials could represent a possible avenue for improvement in instruction that could have a meaningful impact since it is part of the day-to-day work of teaching.

2.5.4 Educative curriculum in mathematics education

The study most closely related to the current study was conducted by Stein and Kim (2009). Stein and Kim set out to analyze both the demands and opportunities for teacher learning of two *Standards*-based elementary school mathematics curricula. The rational for comparing two *Standards*-based curricula was that if *Standards*-based curricula were assumed desirable, what features make *Standards*-based materials different and therefore able to impact changes in instruction differently. District leaders could then consider the needs of their individual district, and decide which of these two desirable curricula would be better suited for their district. The two elementary school, *Standards*-based mathematics curricula Stein and Kim analyzed were *Everyday Mathematics* and *Investigation in Number, Data, and Space*.

Stein and Kim (2009) defined a few terms that are useful in the proposed study as well. Stein and Kim define base curriculum materials to mean, "That portion of the materials that is directly pitched to students and their learning (p. 10, 2009)." Stein and Kim define teacher materials as, "The parts intended to guide teachers as they use the materials (p. 10, 2009)." Much like the analysis by Stein and Kim, the proposed study focuses on both the base curriculum materials and the teacher materials.

The curricula chosen by Stein and Kim (2009) were carefully selected due to some specific features noted by the authors. Both curricula are designed to place an emphasis on the strategies used by students with special attention being paid to multiple representations as opposed to just correctness of solutions. *Everyday Mathematics* is a spiral curriculum in that students are exposed to concepts repeatedly but with increasing depth as they revisit the concepts throughout elementary school. *Investigations* is a module based curriculum where conceptual themes are developed into separate booklets and the order and pacing of the curriculum are less important than mastery of individual modules.

Stein and Kim (2009) randomly selected lessons from each curricula to analyze for their study. The main instructional task of each lesson was coded according to the math task framework, which is based on research by Stein et al. (1996). Each task was coded as either *memorization*, *procedures without connections*, *procedures with connection*, or *doing mathematics*. Next, the teacher materials were examined for evidence of *transparency* and *anticipation* of student thinking.

As expected, most of the tasks found in both curricula were high-level tasks meaning they were *procedures with connections* or *doing mathematics*. The *Everyday Mathematics* curricula had mostly *procedures with connections* (79%) while the *Investigations* curricula had mostly *doing mathematics* (89%). Although both types of tasks are challenging to implement well, the *doing mathematics* tasks are significantly more challenging for teachers because there is no specified pathway for students to follow in approaching these tasks. Therefore, teachers are charged with understanding both the right and wrong approaches student may use while completing *doing mathematics* tasks which makes a significant demand on teacher knowledge. *Procedures with connections* tasks tend to have a limited number of pathways for student thinking that makes them much more predictable than *doing mathematics* tasks.

Based on these differences, Stein and Kim (2009) coded *doing mathematics* tasks as placing high-level demand on teacher learning while *procedures with connections* tasks placed low-level demand on teacher learning. This is not to suggest that *procedures with connections* tasks are easy to implement. In fact, research would suggest otherwise (Stein et al., 1996; Henningsen & Stein, 1997). However, the challenges associated with *procedures with* *connections* tasks are not as demanding as those associated with *doing mathematics*. These results lead Stein and Kim to conclude that the *Investigations* curricula would place a higher demand on teacher learning than *Everyday Mathematics* due to the high number of *doing mathematics* tasks teachers would be asked to implement.

Based on the work of Ball and Cohen (1996) and then Davis and Krajcik (2005), Stein and Kim (2009) identified the potential for teacher learning as information in the teacher materials that provide teachers with the curriculum developers' rationales for including a particular task in the curriculum and information that will assist teachers in anticipating student thinking. Stein and Kim (2009) note that the notion of making curriculum developers' rationales visible to the teacher is referred to as being *transparent*. Stein and Kim reference Davis and Krajcik (2005) in suggesting that transparency could lead to teachers seeing connections between suggested activities rather than having teachers feel like they are completing a list of unconnected concepts. Stein and Kim suggest that many teachers' manuals fail to include rationales, assumptions or agendas that underscore the actions requested of the teachers and therefore limit the teacher's ability to intelligently select and adapt tasks. Stein and Kim (2009) elaborate on anticipating student responses by suggesting that curricula could provide teachers with discussion of typical student responses to tasks along with examples of student work. This suggestion stems from research suggesting that effective teacher preparation involves active envisioning of how students might approach a task both correctly and incorrectly.

Stein and Kim (2009) found that *Investigations* provide more opportunities for teacher learning than did *Everyday Mathematics*. *Investigations* was judged transparent for 80% of the tasks analyzed where *Everyday Mathematics* was only transparent for 21% of the tasks. Similarly, *Everyday Mathematics* only included examples of student work and thinking 30% of the time compared to *Investigations* where 91% of the tasks included some form of student responses, work, examples of potential difficulties, and/or explanations of how students may interpret the task. These results led Stein and Kim to classify *Investigations* as having a high number of opportunities for teacher learning while *Everyday Mathematics* had a low number of opportunities for teacher learning.

To summarize the findings of Stein and Kim (2009), *Investigations* places a higher demand on teacher learning than *Everyday Mathematics* to be implemented well, but also provides more support for teacher learning. A school leader must then consider the needs of his or her staff when deciding which of these two curricula he or she might choose. A staff that has a high rate of turnover with a high number of at risk students may not benefit as much from the same curricula as a staff with a low rate of turnover and a low number of at risk students.

2.6 HIGH-LEVEL TASKS

Stein and Kim (2009) were able to take two areas of research and combine them into one study. This chapter has already discussed educative curriculum materials, which is one area, represented in the Stein and Kim study. The other area is high-level tasks. Research on high-level tasks began with Doyle in 1983. Doyle’s work established the importance of tasks in education. As part of this important work on tasks, Doyle was also the first to classify tasks. However, Doyle’s work was not focused on any specific content area. Researchers in mathematics education (Stein, Grover, & Henningsen, 1996) then picked up where Doyle left off and refined his work to apply more specifically to mathematics education.

2.6.1 Establishing the importance of tasks

In 1983, Doyle explored the nature of academic work in both elementary and secondary schools. Doyle also hoped to discover what adaptations to academic work might improve student achievement. Doyle's approach to this analysis was to view curriculum as a collection of tasks. Doyle (1983) felt that, "tasks form the basic treatment unit in classrooms (p. 162)" and defined the focus of a task as follows (p. 161):

(a) The products students are to formulate, such as an original essay or answers to a set of test questions

(b) The operations that are to be used to generate the product, such as memorizing a list of words or classifying examples of a concept

(c) The givens or resources available to students while they are generating a product, such as a model of a finished essay supplied by the teacher or a fellow student

Doyle further clarified tasks as being defined by the answers students produce and the paths that the students use to obtain those answers.

Doyle (1983) distinguished the types of tasks by acknowledging that tasks influence learners because they direct the attention of learners to specific aspects of the curriculum and specific ways of processing the information. Doyle noted that this could be particularly important if the task directs the learner to process information in such a way that is based in meaning as compared to processing information based simply in surface features. Doyle also acknowledged that the resources provided with the task had a significant impact on the cognitive demand of the task. The cognitive demand of a task could be significantly lowered depending on the additional resources offered to the students.

Doyle (1983) categorized tasks four ways (p. 162):

1) *Memory tasks* in which students are expected to recognize or reproduce information previously encountered
2) *Procedureal or routine tasks* in which students are expected to apply a standardized and predictable formula or algorithm go generate answers
3) *Comprehension or understanding tasks* in which students are expected to (a) recognize transformed or paraphrased versions of information previously encountered, (b) apply procedures to new problems or decide from among several procedures those which are applicable to a particular problem, or (c) draw inferences from previously encountered information or procedures
4) *Opinion tasks* in which students are expected to state a preference for something

Doyle (1983) makes an important assertion that could be applied to explain the arguments made by those who support either traditional or *Standards*-based approaches to teaching mathematics. Dolye suggests that the completing one type of task can interfere with the goals of another type of task. Doyle specifically cites an example that learning an algorithm does not enable one to understand why it works or when to use it much like a supporter of *Standards*-based instruction would. Doyle further supports this view by noting that his analysis does not support the notion that drill and practice are required for acquisition of understanding. However, Doyle also suggests that understanding why an algorithm works and when to use it does not always lead to being able to use it correctly much like a supporter of traditional instruction might argue.

2.6.2 The relationship between cognitive demands of tasks as set up and implemented

Stein, Grover, and Henningsen (1996) advanced the notion of the importance of high-level tasks and brought task analysis to the forefront of research in mathematics education. Their research analyzed the characteristics, levels of cognitive demand, and fidelity of implementation of the level of cognitive demand of 144 tasks in classrooms in a reform-oriented mathematics project. The focus of the research was the relationship between when the teacher set the tasks up and how the tasks were actually implemented. Their goal was to examine the instructional tasks used and determine what causes high-level tasks either to be maintained at a high-level or to decline to a low-level.

Stein et al. (1996) have identified three phases that tasks must pass through as part of the math task framework. First, the task appears in the curriculum materials or instructional materials. Second, the teacher sets up the task. Third, the students implement them. Each of these phases can influence student learning. This relationship is illustrated in Appendix D.

The notion of engaging students in high-level mathematical tasks was inspired by national publications from the National Council of Teachers of Mathematics, the Mathematical Association of America, and the National Research Council suggesting students develop deep understandings of mathematics (Stein et al., 1996). The notion is that students should strive to "do mathematics" just as a mathematician might. Stein et al. define this as "framing and solving problems, looking for patterns, making conjectures, examining constraints, making inferences from data, abstracting, inventing, explaining, justifying, challenging, and so on (p. 456)." Stein et al. and their colleagues suggest that for students to be able to "do mathematics" students must be given the opportunity to engage in tasks that require high-level cognitive demand. Unfortunately, most mathematics classrooms follow an all too common problem of the teacher presenting a problem with a prescribed algorithm and then assigning a set of similar problems for students to practice individually. This type of instruction leads to either memorization or practicing procedures without understanding why the procedure works or when to use it.

Stein, Grover, and Henningsen (1996) conducted their research as part of the QUASAR Project. QUASAR (Quantitative Understanding: Amplifying Student Achievement and Reasoning) was a reform oriented project at the University of Pittsburgh aimed at studying the development and implementation of mathematics instructional programs in economically disadvantaged middle schools. QUASAR was a school level reform where teachers received professional development in an attempt to improve instructional opportunities for students who typically are not given an opportunity to participate in meaningful and challenging learning environments.

Stein, Grover, and Henningsen (1996) based the idea of analyzing mathematical tasks on Doyle's (1983) assertion of the importance of academic tasks. The authors note that a mathematical task is not a new task unless the underlying mathematical idea changes. Therefore, a lesson may be made up of multiples problems but if they were all focused on a single mathematical concept, they would be classified as one task.

Stein, Grover, and Henningsen (1996) categorized their codes into four categories: task description, task set up, task implementation, and factors of decline or maintenance. These codes included the duration of each task, the percentage of class time used for the task, the resources the task was based on, the mathematical topic that was the focus of the task, the context of the task, and if the set up was a collaborative effort among students. Codes specific to the set up and implementation of the task included the cognitive demands of the tasks, number of solution strategies, number and types of representations, and the requirements for communication, reasoning, or justification from students.

Of particular interest to the current study are the codes for the level of cognitive demand. Low-level tasks were coded as either memorizations or procedures without connection. The high-level tasks were coded as either procedures with connections or doing mathematics. The authors made a judgment call when faced with tasks that included multiple types of cognitive activity. Their decision was to code the task based on the task during set up and what a majority of the students were doing during implementation.

2.6.3 High-level tasks and student learning

The Math Task Framework, Appendix D, suggests that tasks pass through three phases prior to student learning. The first phase is the task as it appears in the curricular materials. The second phase is the task as set up by the teacher. The third phase is the task as implemented by the students. Stein and Lane (1996) investigated the link between tasks as set up by the teacher and student learning at four middle schools as part of the QUASAR Project.

Stein and Lane (1996) noted three possibilities for tasks as they moved from being set up by the teacher to being implemented by the students. The first is that high-level tasks were maintained throughout implementation and thus were implemented at high-level cognitive demand. The second was that tasks that were set up by the teacher to demand high-level cognitive demand were implemented at low-level cognitive demand. The third was that tasks that were set up by the teacher at low-level cognitive demand stayed at low-levels throughout implementation.

Stein and Lane (1996) suggest that the highest gains in student learning were from classrooms where the instruction was focused on high-level tasks. Conversely, classroom where the instruction focused on tasks that were set up and implemented with low-level cognitive demand demonstrated the lowest gains in student learning. Tasks that were set up with high-level cognitive demand even outperformed tasks that were set up with low-level cognitive demand when they were not implemented with fidelity. In other words, even when both implemented at low-levels, tasks set up for high-level cognitive demand still outperformed those set up for low-level cognitive demand.

A second study aimed at studying the use of high-level tasks and their impact on student achievement was conducted by Boaler and Staples (2008). Boaler and Staples analyzed student achievement and attitudes over a period of five years in three different schools. One of the schools, Railside, offered all students the same curriculum that the teachers had designed collaboratively using *Standards*-based resources such as IMP. In addition to designing their own curriculum, the teachers also developed their own method of enacting the curriculum that emphasized students working in groups on high-level tasks. Students were not grouped by ability level. Instead, every student at Railside was enrolled in the same Algebra course when entering the high school. The other two schools offered both traditional courses and IMP in classes that were grouped by ability level. Most students in these schools enrolled in the traditional courses.

An assessment based in middle school mathematics administered to first year students at the beginning of the study demonstrated that students at Railside achieved at significantly lower levels than students at the other two schools. This outcome was not unexpected since Railside is situated in an urban, low-income setting. The other schools were in a suburban setting. At the end of the first year, an algebra assessment indicated that the Railside students were still performing at a significantly lower level, but that they had closed the distance between the schools. At the end of year two, an assessment containing both algebra and geometry (which all students had received instruction in) demonstrated that the Railside students had not only surpassed the students from the other schools, but they scored significantly higher than those students did on the assessment. Additionally, students at Railside ended up taking more advanced mathematics classes their senior year than students at the other two schools.

In summary, students from a disadvantaged school with a significantly lower level of initial achievement were able to surpass their peers in only two years of instruction because of being given the opportunity to engage in high-level tasks. This study demonstrates the important relationship between high-level tasks and student learning. This study also demonstrates that instruction focusing on engaging students in high-level tasks can be more effective than traditional mathematics instruction.

2.7 SUMMARIZING CHAPTER 2

Probability and statistics are important topics because all people interact with them in a variety of ways. Additionally, probability and statistics are growing in importance in most professional careers. Because of their importance, probability and statistics education are becoming more prevalent at all levels of schooling. This educational importance is acknowledged at the secondary level by CCSSM since probability and statistics represents one of the six conceptual categories for high school mathematics. The probability and statistics standards found in CCSSM are built on suggestions by both the GAISE Report and scholarly research thus making these suggestions an appropriate basis for research in probability and statistics.

CCSSM represents a change from what mathematics instruction currently takes place in many classrooms across the United States. In order to meet the demands of CCSSM, students may need opportunities to engage in high-level tasks. Additionally, instruction focusing student engagement with high-level tasks will be the most effective way to promote student learning. However, high-level tasks are difficult to implement well and therefore teachers will need additional support to implement a curriculum designed to provide students with opportunities to engage in high-level tasks. Additionally, probability and statistics may be difficult for teachers implement well since most of these teachers are experts in mathematics and not probability and statistics. Finally, probability and statistics tasks are difficult to implement well because misconceptions are widespread, strongly held, and occur at all levels.

The combination of probability and statistics being exceptionally difficult to teach and high-level tasks being more difficult to implement well may suggest that teachers will need more support to meet the probability and statistics standards of CCSSM than any other conceptual category. One way to provide additional support to many of these teachers is through the curriculum materials they will be using. Curriculum materials that promote teacher learning in addition to student learning, known as educative curriculum materials, may be beneficial in aiding teachers in implementing the probability and statistics standards of CCSSM.

Based on this summary, it is appropriate to examine tasks found in secondary mathematics textbooks that correspond to the probability and statistics standards of CCSSM. One way to analyze these tasks would be to determine the potential of the task to engage students in cognitively demanding work. The higher the level of cognitive demand, the more potential the task will have in meeting the expectations of CCSSM. Once the level of cognitive demand has been established, it may be important to note the potential for teacher learning. Tasks of high demand will require more opportunities for teacher learning than those of low demand. When all of this data has been collected and analyzed, a clear picture of the potential a curriculum has to meet the expectations of CCSSM in probability and statistics will be available.

3.0 METHODOLOGY

Chapter 3 will focus on the methodology of the study. This chapter begins with a review of the purpose of the study and the research questions intended to address that purpose. The chapter will then discuss the textbooks that were included in the study and how the textbooks were selected. Next, the specific methodology for this study will be discussed. Finally, the chapter will connect the methodology of the study to the purpose of the study by discussing how the data that is collected will relate to the research questions.

3.1 PURPOSE AND RESEARCH QUESTIONS

The purpose of the study is to determine the extent to which secondary mathematics textbooks have the potential to prepare students and teachers to meet the demands of the content recommendations in the domain of probability and statistics as specified in the Common Core State Standards for Mathematics. Specifically, this study answers the following research questions:

1) To what extent do current secondary mathematics textbooks provide opportunities for students to engage in the probability and statistics content recommended by the Common Core State Standards?

2) What are the cognitive demands of the tasks that are aligned with the Common Core State Standards recommendations for mathematical content in probability and statistics?
3) To what extent does the teachers' guide provide support for enacting high-level tasks that address the Common Core State Standards recommendations related to probability and statistics?
    a) To what extent does the teachers' guide provide suggestions related to *anticipation* on high-level tasks that reflect content recommendations of the Common Core State Standards?
    b) To what extent does the teachers' guide provide *transparency* on high-level tasks that reflect content recommendations of the Common Core State Standards?

3.2 TEXTBOOK SELECTION

All of the research questions focus on the analysis of items and tasks as they appear in the written curriculum. Therefore, the curriculum selection is a vital part of the methodology of this study. Analyzing textbooks has proven to be a valuable avenue for research in the past with many examples of significant contribution being available in mathematics education alone (Jones & Tarr, 2007; Ross, 2011; Stein & Kim, 2009; Stylianides, 2009; Thompson, Senk, & Johnson 2012). Three secondary mathematics textbooks series (Core-Plus Mathematics, Glencoe Mathematics, and Interactive Mathematics Program) and the teachers' guides that accompany them were analyzed. Each of the identified textbooks series is described, including how it was selected, in the sections that follow.

The goal of the study was to analyze at least one traditional and one *Standards*-based curriculum. The traditional curriculum would be selected based on widespread use. Selection of the *Standards*-based curriculum began with an examination of the five curricula that were funded by the National Science Foundation (Core-Plus Mathematics Project, Interactive Mathematics Program, Math Connections, Mathematics: Modeling Our World, and SIMMS Integrated Mathematics). Of these five, the curricula selected to represent *Standards*-based materials would be that which had been suggested as being the most promising. For example, Martin et al. (2001) references mathematics programs identified by the U.S. Department of Education's Mathematics and Science Expert Panel as being exemplary. These exemplary programs include two of the National Science Foundation funded materials listed above, Core-Plus Mathematics Project and Interactive Mathematics Program. Additionally, researchers have examined the performance of students on multiple measures of achievement when using Core-Plus Mathematics Project or Interactive Mathematics Program as compared to traditional mathematics curriculum (Chavez et al., 2015; Grouws et al., 2013; Senk & Thompson, 2003; Tarr et al., 2013). In each case, the two *Standards*-based curricula have performed as well or better than their traditional counterparts have.

3.2.1 Glencoe Mathematics (GM)

The GM series is included because it represents a widely used textbook series (Ross, 2011). The GM series represents a traditional approach to mathematics education. The traditional approach means that the student editions include example problems with worked out solutions and explanations provided to guide students through the steps of the solutions. Then there are exercises at the end of each section often corresponding directly to one of these worked out examples. Additionally, the GM textbooks are organized by mathematical content meaning there is a book dedicated solely to Algebra I, another book specifically focused on Geometry, etc. The GM textbooks are published by McGraw-Hill and hold both the greatest collective market share and each textbook holds the greatest individual market share relative to other textbooks of type according to Ross (2011).[1] The GM textbook series is made up of four textbooks:

Algebra I (Carter et al., 2014)

Geometry (Carter et al., 2014)

Algebra 2 (Carter et al., 2014)

Advanced Mathematical Concepts: Precalculus with Application (Precalculus) (Holliday, Cuevas, McClure, Carter, & Marks, 2014)

The worked out example problems, the exercises at the end of each section, and the narratives were included in the analysis. A small sample from the GM Algebra textbook has been provided to exemplify each. Figure 3.1 is a worked out example from lesson 0-13 of the GM Algebra textbook. This example contains two items. The term items refers to the individual parts of a task. In Figure 3.1, the first item asks students to make a histogram of the frequency. The second item in Figure 3.1 asks students to make a histogram of the cumulative frequency. Figure 3.2 is one of the corresponding exercises from lesson 0-13 of the GM Algebra textbook. Once again, the exercise contains two items. Exactly like the worked out example, the first item asks students to graph the frequency, and the second item asks students to graph the cumulative frequency. Figure 3.3 is the part of the narrative of lesson 0-13 that is located prior to the worked out example and is indicative of the entire narrative for lesson 0-13.

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[1] GM held the largest market share in 2011 and no data is currently available regarding the 2014 edition.

Example 2 Make a Histogram and a Cumulative Frequency Histogram

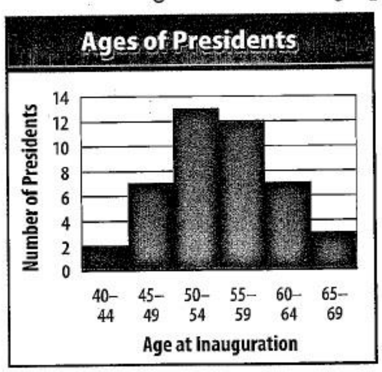
Make histograms of the frequency and the cumulative frequency.

Age at Inauguration	40–44	45–49	50–54	55–59	60–64	65–69
U.S. Presidents	2	7	13	12	7	3

Find the cumulative frequency for each interval.

Age	< 45	< 50	< 55	< 60	< 65	< 70
Presidents	2	2 + 7 = 9	9 + 13 = 22	22 + 12 = 34	34 + 7 = 41	41 + 3 = 44

Make each histogram like a bar graph but with no space between the bars.



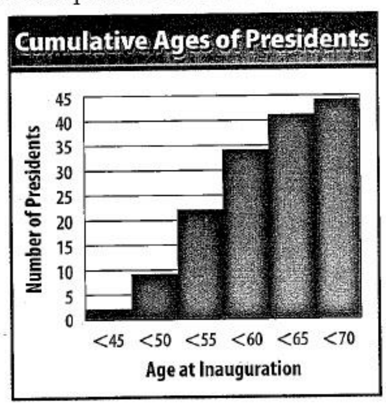


Figure 3.1. Example aligned to S-ID-1 from GM *Algebra 1* (Carter et al., p. 41, 2010)

2. PLAYS The frequency table at the right shows the ages of people attending a high school play.

a. Make a histogram to display the data.

b. Make a cumulative frequency histogram showing the number of people attending who were less than 20, 40, 60, or 80 years old.

Age	Tally	Frequency
0–19	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 II	47
20–39	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 III	43
40–59	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 I	31
60–79	𝍸 III	8

Figure 3.2. Exercise related to Figure 3.1 from GM *Algebra 1* (Carter et al., p. 45, 2010)

The **cumulative frequency** for each event is the sum of its frequency and the frequencies of all preceding events. A **histogram** is a type of bar graph used to display numerical data that have been organized into equal intervals.

Figure 3.3.Narrative found prior to Figures 3.1 from GM *Algebra 1* (Carter et al., p. 41, 2010)

3.2.2 Core-Plus Mathematics Project (CPMP)

The CPMP curriculum materials were funded by the National Science Foundation (NSF) and represent a *Standards*-based approach to secondary mathematics education. The CPMP curriculum materials have been identified as being an exemplary mathematics program by the U.S. Department of Education's Mathematics and Science Expert Panel (Martin et al., 2001). Additionally, research by Martin et al. (2001) demonstrated that five NSF funded curricula, including CPMP, were aligned with the NCTM Standards. Due to the existence of evidence suggesting alignment with NCTM, there was optimism that CPMP would also align well with CCSSM. Rather than have textbooks identified by content area, the CPMP textbook series organizes textbooks by years. There are four years of textbooks intended to be implemented in grades 9 through 12.

Core-Plus Mathematics, Course 1 (Hirsch, C. R., Fey, J. T., Hart, E. W., Schoen, H. L., & Watkins, A. E., 2015)

Core-Plus Mathematics, Course 2 (Hirsch, C. R., Fey, J. T., Hart, E. W., Schoen, H. L., & Watkins, A. E., 2015)

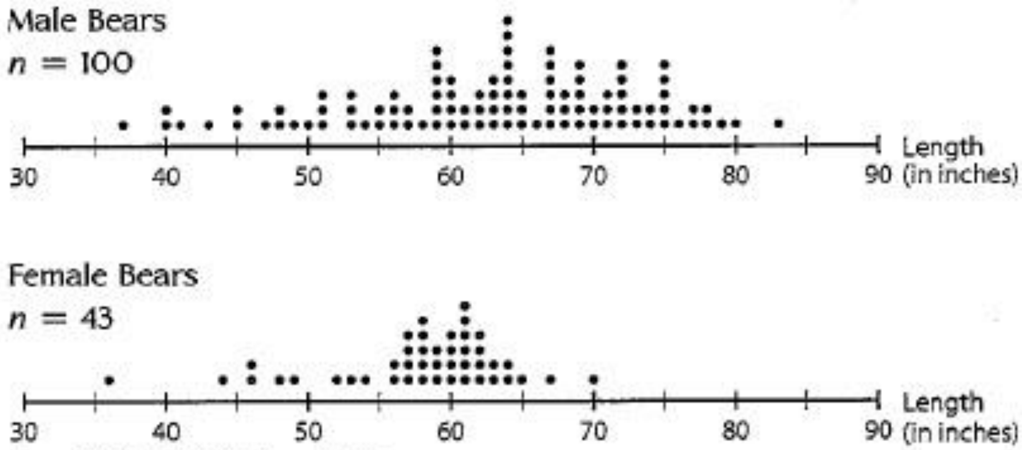
Core-Plus Mathematics, Course 3 (Hirsch, C. R., Fey, J. T., Hart, E. W., Schoen, H. L., & Watkins, A. E., 2015)

Core-Plus Mathematics, Course 4 (Hirsch, C. R., Fey, J. T., Hart, E. W., Schoen, H. L., & Watkins, A. E., 2015)

The CPMP curriculum materials are divided into *years* (1, 2, 3, and 4) which are then subdivided into *units*, and then *lessons*. Each *lesson* contains at least two *investigations* and an *on your own* section. Each *investigation* typically contained multiple items for instruction, summarizing, and checking for understanding. Each individual item was coded to paint a picture of the quantity of individual items as opposed to grouping all of the related items together as a single main instructional task and potentially masking the levels of cognitive demand of some parts of the curricula. In addition to coding items from the *investigations*, any items from the *on your own* section that correspond to the probability and statistics standards from CCSSM and the narrative parts of the text were also coded.

The following figures are from the first year of the CPMP textbook and exemplify what a typical *investigation* looks like. Figure 3.4 is from Year 1, Unit 2, Lesson 1, Investigation 1 of the CPMP curricula. It contains five items (note a-i and a-ii make up two of the five). Figure 3.5 is from Year 1, Unit 2, Lesson 1, Investigation 1. It contains four items. Figure 3.6 is the items at the end of Year 1, Unit 2, Lesson 1, Investigation 1 that are intended to summarize the investigation and be used for students to check their understanding. Finally, Figure 3.7 is the narrative part of Year 1, Unit 2, Lesson 1, Investigation 1.

1 As part of an effort to study the wild black bear population in Minnesota, Department of Natural Resources staff anesthetized and then measured the lengths of 143 black bears. (The length of a bear is measured from the tip of its nose to the tip of its tail.) The following **dot plots** (or *number line plots*) show the distributions of the lengths of the male and the female bears.



Source: Minitab Statistical Software Data Set

a. Compare the shapes of the two distributions. When asked to *compare*, you should discuss the similarities and differences between the two distributions, not just describe each one separately.

i. Are the shapes of the two distributions fundamentally alike or fundamentally different?

ii. How would you describe the shapes?

b. Are there any lengths that fall outside the overall pattern of either distribution?

c. Compare the centers of the two distributions.

d. Compare the spreads of the two distributions.

Figure 3.4. Items aligned to S-ID-1 from CPMP *Course 1* (Hirsch et al., p. 76, 2015)

2. When describing a distribution, it is important to include information about its shape, its *center*, and its *spread*.

a. *Describing shape.* Some distributions are **approximately normal** or *mound-shaped*, where the distribution has one peak and tapers off on both sides. Normal distributions are **symmetric**—the two halves look like mirror images of each other. Some distributions have a **tail** stretching towards the larger values. These distributions are called **skewed to the right** or **skewed toward the larger values.** Distributions that have a tail stretching toward the smaller values are called **skewed to the left** or **skewed toward the smaller values.**

A description of shape should include whether there are two or more clusters separated by gaps and whether there are outliers. **Outliers** are unusually large or small values that fall outside the overall pattern.

- How would you use the ideas of skewness and outliers to describe the shape of the distribution of lengths of female black bears in Problem 1?

b. *Describing center.* The measure of center that you are most familiar with is the *mean* (or average).

- How could you estimate the mean length of the female black bears?

c. *Describing spread.* You may also already know one measure of spread, the **range**, which is the difference between the maximum value and the minimum value:

range = *maximum value* − *minimum value*

- What is the range of lengths of the female black bears?

d. Use these ideas of shape, center, and spread to describe the distribution of lengths of the male black bears.

Figure 3.5. Items aligned to S-ID-1 from CPMP *Course 1* (Hirsch et al., p. 76-77, 2015)

Summarize the Mathematics

In this investigation, you explored how dot plots and histograms can help you see the shape of a distribution and to estimate its center and spread.

a. What is important to include in any description of a distribution?

b. Describe some important shapes of distributions and, for each, give a data set that would likely have that shape.

c. Under what circumstances is it best to make a histogram rather than a dot plot? A relative frequency histogram rather than a histogram?

Be prepared to share your ideas and reasoning with the class.

Check Your Understanding

Consider the amount of fat in the fast-food sandwiches listed in the table on page 82.

a. Make a dot plot of these data.

b. Make a histogram and then a relative frequency histogram of these data.

c. Write a short description of the distribution so that a person who had not seen the distribution could draw an approximately correct sketch of it.

Figure 3.6. Items aligned to S-ID-1 from CPMP *Course 1* (Hirsch et al., p. 83, 2015)

Investigation 1 Shapes of Distributions

Every day, people are bombarded by data on television, on the Internet, in newspapers, and in magazines. For example, states release report cards for schools and statistics on crime and unemployment, and sports writers report batting averages and shooting percentages. Making sense of data is important in everyday life and in most professions today. Often a first step to understanding data is to analyze a plot of the data. As you work on the problems in this investigation, look for answers to this question:

How can you produce and interpret plots of data and use those plots to compare distributions?

Figure 3.7. Narrative aligned to S-ID-1 from CPMP *Course 1* (Hirsch et al., p. 76, 2015)

3.2.3 Interactive Mathematics Program (IMP)

Much like the CPMP curriculum materials, the IMP curriculum materials were funded by the NSF and represent a *Standards*-based approach to secondary mathematics education. The IMP curriculum materials were also identified as being an exemplary mathematics program by the U.S. Department of Education's Mathematics and Science Expert Panel (Martin et al., 2001). IMP was also one of the five NSF funded curricula that demonstrated alignment with the NCTM Standards according to Martin et al. (2001). Once again, due to the existence of evidence suggesting alignment with NCTM, there was optimism that IMP would also align well with CCSSM. In the same fashion as CPMP, the IMP textbook series organizes textbooks by years. There are four years of textbooks intended to be implemented in grades 9 through 12.

Interactive Mathematics Program Year 1 (Fendel, D., Resek, D, Alper, L., & Fraser, S., 2009)

Interactive Mathematics Program Year 2 (Fendel, D., Resek, D, Alper, L., & Fraser, S., 2009)

Interactive Mathematics Program Year 3 (Fendel, D., Resek, D, Alper, L., & Fraser, S., 2009)

Interactive Mathematics Program Year 4 (Fendel, D., Resek, D, Alper, L., & Fraser, S., 2009)

The IMP curriculum materials were also divided into *years* (1, 2, 3, and 4) which were then subdivided into categories that are referred to as units although they were not explicitly called units by the curriculum materials. Each unit also had its own subcategories that resembled the *lessons* from CPMP. Each lesson then contains *activities, group activities,* and *problems of the week*. All three and any narrative sections were coded in the IMP curricula. Figure 3.8 is a group activity from the IMP curriculum called *What Are the Chances?*. This group activity contains ten items. Figure 3.9 is an activity from the IMP curriculum called *Rollin', Rollin', Rollin'*. This activity contains three items. Figure 3.10 is a problem of the week called *A Sticky Gum Problem*. Figure 3.11 is the narrative found at the beginning of the unit containing the group activity, activity, and problem of the week that are shown.

Group Activity

What Are the Chances?

Part I: Finding Probabilities

Sometimes the only way to find the probability of something is to use **observed probability**—a model based on using your own experience or making an educated guess. In other cases, you can use **theoretical probability.**

Items A through I pose questions about probability. In each case,

- Decide on the probability, using a theoretical model if you can.
- Describe how you decided on the probability. State whether your answer was based on a theoretical model or on observed results, or whether it was just a pure guess.

A. You pull one gumball out of a bag that contains three red gumballs, two blue gumballs, and four black gumballs. What is the probability that the gumball you picked is blue?

B. What is the probability of snow falling in Florida at least once next July?

C. You arrive at an intersection with a traffic signal. What is the probability that the light is red?

continued

D. You flip a coin twice. What is the probability of getting one head and one tail?

E. You roll a standard die. What is the probability of getting a prime number?

F. Your teacher selects two students at **random** from your class to run an errand. What is the probability that you are one of the two students?

G. You randomly point to a student in your mathematics class. What is the probability that this student is wearing sneakers?

H. You roll two dice. What is the probability of getting doubles?

I. You roll a pair of dice until you get doubles. What is the probability that you get doubles in three or fewer rolls?

Part II: Probabilities on the Number Line

Make a number line like the one below. For each item in Part I, indicate its probability by putting the letter in the proper place on the number line.

Figure 3.8. Items aligned to S-ID-1 from IMP *Year 1* (Fendel et al., p. 92-93, 2009)

Activity

Rollin', Rollin', Rollin'

Roll a pair of dice 50 times. With each roll, find the sum of the dice. Keep a record of your sums in an organized way.

1. Draw a **graph** of the data you gathered.
2. Write a paragraph about your results. You should summarize your observations about the data and discuss why the results came out the way they did.
3. What new thoughts does this experiment give you about how to play the counters game?

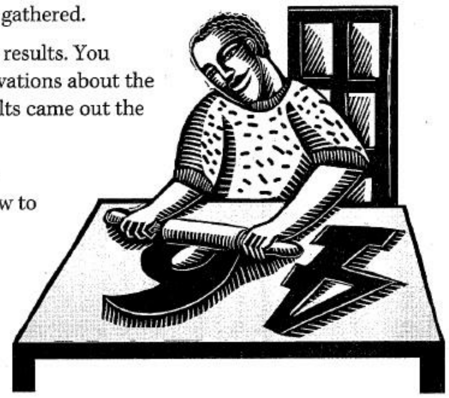


Figure 3.9. Items aligned to S-ID-1 from IMP *Year 1* (Fendel et al., p. 104, 2009)

PROBLEM OF THE WEEK 3

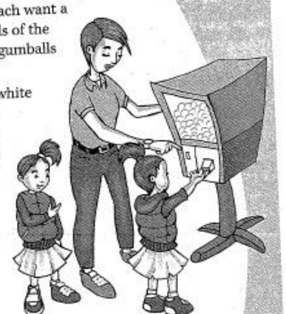
A Sticky Gum Problem

This Problem of the Week, or POW, starts with some specific situations. It then asks you to generalize what you've learned from the situations.

1. Ms. Hernandez comes across a gumball machine one day when she is out with her twins. Of course, the twins each want a gumball. They also insist on having gumballs of the same color. They don't care what color the gumballs are, as long as they're both the same.

   Ms. Hernandez can see that there are only white gumballs and red gumballs in the machine. The gumballs cost a penny each, and there is no way to tell which color will come out next. Ms. Hernandez decides to keep putting in pennies until she gets two gumballs of the same color.

   Why is three cents the most she might have to spend?



2. The next day, Ms. Hernandez and her twins pass another gumball machine. This one has three colors of gumballs: red, white, and blue.

   What is the most Ms. Hernandez might have to spend at this machine to get matching gumballs for her twins?
3. Mr. Hodges and his triplets pass the three-color gumball machine. Of course, his children insist that they all get the same color gumball. What is the most Mr. Hodges might have to spend?

After you have answered these questions, create some examples of your own. You may want to begin with more examples about the Hernandez twins, using different numbers of colors. Or you may want to create examples using the three-color gumball machine and larger sets of children.

POW 3

As you create and solve examples of your own, look for a way to organize the information. Also look for patterns. Your goal is to find a formula that will guarantee each child a gumball of the same color. If someone tells you the number of gumball colors and the number of children, your formula will tell you the maximum amount of money the parent might need to spend.

○ *Write-up*

Begin your write-up for this POW with a discussion of Questions 1 through 3. Explain your answers to each question, and describe the process you used to solve them.

Then discuss the problems you made up and their solutions. Explain how you organized your information and the patterns you found.

Finally, state any general ideas you were able to formulate. Include conjectures you may have about the general problem, even if you can't prove them. For each general statement, explain why you think it's true. Provide examples to illustrate each statement. Describe the process by which you arrived at that generalization.

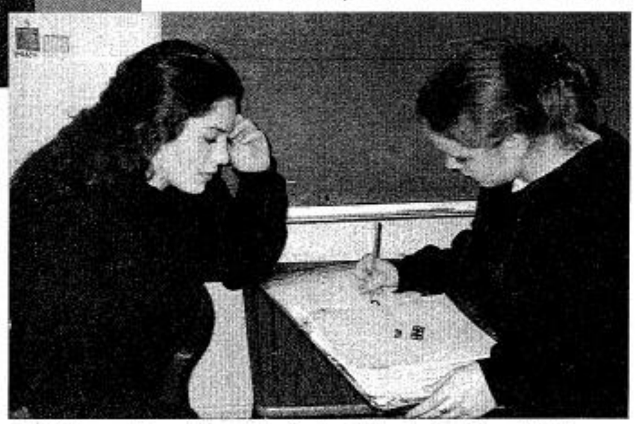
Figure 3.10. Items aligned to S-ID-1 from IMP *Year 1* (Fendel et al., p. 83-84, 2009)

Chance and Strategy

The theory of **probability** was developed in the seventeenth century, primarily to answer questions posed by gamblers. Mathematicians of the time saw that certain strategies could help increase the chances of success in many types of situations, even in those involving luck. Today, probability is used in many areas, from scientific research to helping people make good business decisions.

You'll enter the world of probability by exploring a dice game called Pig. In the first few activities of this unit, you will begin your search for the best **strategy** for the game by experimenting with different options.

Flipping coins is another way to investigate ideas about probability. In *The Gambler's Fallacy*, you'll use coin flips to arrive at a conclusion that may surprise you. Then you'll move on to the formal definition of *probability*. As you'll see, people study probability through both experimentation and theoretical analysis.



Leah Allen and Crystal Kovarik begin the unit by playing Pig and thinking about strategies they might use to play the game.

Figure 3.11. Narrative aligned to S-ID-1 from IMP *Year 1* (Fendel et al., p. 81, 2009)

3.3 METHODOLOGY

Stein and Kim's (2009) work in analyzing tasks found in the written curriculum of elementary mathematics textbooks provides the foundation of the methodology used for this study. Stein and Kim analyzed the demands and opportunities for teacher learning of two widely used elementary mathematics programs, *Everyday Mathematics* and *Investigations in Number, Data, and Space*. These textbooks both represent *Standards*-based curricula. Stein and Kim (2009) analyzed both the textbooks intended for the students, referred to as base materials, and the materials intended for the teachers, referred to as teacher materials, found in the teacher's edition of the textbook in close proximity to the lesson for the student. Stein and Kim did this because survey research in the districts where they did their analysis demonstrated that a majority of teachers did not consult materials in books that are separate from those intended for daily use. The base materials were analyzed to determine the level of cognitive demand of the textbooks using the Task Analysis Guide (Smith & Stein, 1998). The teacher materials were analyzed to determine what opportunities for teacher learning were available. Specifically, the teacher materials were analyzed for *transparency*, which refers to the curriculum writers being explicit about the mathematical purpose of the task, and *anticipation*, which refers to helping teachers to anticipate student responses.

This study also analyzed both base materials and teacher materials to determine the level of cognitive demand based on the Task Analysis Guide and the opportunities for teacher learning in the areas of *transparency* and *anticipation*. However, multiple grain sizes of analysis were used. Stein and Kim (2009) analyzed what they referred to as the main instructional task of each lesson. To ensure a complete picture of each curriculum, both a smaller grain size and more widespread analysis than looking only at the main instructional task was used for this study. The analysis began by following the Thompson, Senk, and Johnson (2012) methodology in identifying smaller pieces than tasks to be coded. These individual pieces are referred to as items. Thompson, Senk, and Johnson coded the lesson's narratives and all exercises within the lesson including review exercises. Anything in the textbook that represents an opportunity to learn was considered in their analysis. Once these data were collected, the individual items were then grouped together to form the main instructional tasks as defined by Stein and Kim (2009) and examined in a manner consistent with their methodology as well. By analyzing the textbooks using both the item view and task view, a clearer picture of the textbook was available than by simply looking at one or the other.

A task is defined as, “A classroom activity, the purpose of which is to focus students’ attention on a particular mathematical idea (Stein et al., p. 460, 1996).” Grouping items into tasks required looking at the mathematical idea behind each item. It is important to note that none of the narratives, review exercises, or parts of a curriculum that are not intended for instructional use such as the Problems of the Week in the IMP curriculum were coded at the level of task. For the IMP textbooks, the tasks were the activities or group activities since they are intended to be the instructional component of the curriculum and are organized to contain multiple items all focused on developing some common mathematical idea. For the CPMP textbooks, the investigations are divided into exercises for the student to work though. Each exercise would represent multiple items focused on the same mathematical idea, so each exercises often represented a task. If consecutive exercises represented the same mathematical idea, then they were group together and coded as a single task. The GM textbooks typically have multiple exercises grouped under the same set of directions. For example, the directions might say for problems 15 – 23 find the mean of the set of data. Since all of the problems from 15 – 23 involve finding the mean, they would be considered one instructional task. Because the GM textbooks follow the traditional pattern of providing an example and then providing exercises that correspond to that example, exercises were often grouped together with others that relate to the same example. This grouping of exercises constituted a task because they related to the same previous example and thus the same mathematical idea. For example, Figure 3.1 and Figure 3.2 would be consider a single instructional task because the exercise in Figure 3.2 was exactly like the example in Figure 3.1.

3.3.1 Identifying the items to be analyzed

Each of the curricula selected has an online resource that aligns the curriculum materials to the CCSSM. Only parts of the textbook identified in this online resource as being aligned to one of the standards for probability and statistics in the CCSSM were analyzed. However, there were cases where a subset of the sections identified by the online resource did not align to the CCSSM in the areas of probability and statistics. This often occurred because the item was included for review purposes. Only those items identified by the online resource that were verified as actually aligning with CCSSM in probability and statistics were analyzed.

A spreadsheet was created containing entries for every item where a curricula claims alignment with CCSSM. In the case of the CPMP textbooks, either page numbers or a page number with the specific items in alignment with the specified standard were identified. The GM curricula choose to identify alignment with a standard by providing a chapter and section number. For example, 12-3 would represent chapter 12, section 3. The IMP curriculum identified activities by titles that align to a specified standard. Table 3.1 is an example of that spreadsheet with a focus on standard S-ID-1.

Table 3.1. Sections aligned to S-ID-1

Standard	Textbook	Section
S-ID-1	CPMP-1	67
S-ID-1	CPMP-1	73-101
S-ID-1	CPMP-1	106
S-ID-1	CPMP-1	108-142
S-ID-1	CPMP-1	144-147
S-ID-1	CPMP-1	231 #29
S-ID-1	CPMP-1	454 #31
S-ID-1	CPMP-1	554-556
S-ID-1	CPMP-1	558
S-ID-1	CPMP-1	560-562
S-ID-1	CPMP-1	564
S-ID-1	CPMP-1	571-575
S-ID-1	CPMP-1	587
S-ID-1	GM-A1	0-13
S-ID-1	GM-A1	12-3
S-ID-1	GM-A1	12-4
S-ID-1	IMP-1	What Are the Chances?
S-ID-1	IMP-1	Rollin', Rollin', Rollin'
S-ID-1	IMP-1	Waiting for a Double

Once the online identification of sections of the textbooks was established, the next step was to identify the items within those sections that were in alignment with the given standard (in this case S-ID-1). Since the goal of this research was to gain a clear understanding of the opportunities that might exist in the areas of probability and statistics in a given set of curriculum materials, all parts of those curriculum materials were considered. Consistent with the methodology of Thompson, Senk, and Johnson (2012) both the narrative of the lesson and the exercises in the lesson were analyzed. This included those exercises intended for review. Thompson, Senk, and Johnson believed the narrative provided opportunities for teachers to introduce reasoning and proof (the focus of their analysis) while the exercises provide the students opportunities to engage in practice with reasoning and proof. In this case, an item may be introduced by the narrative or engaged in during the exercises and thus both require analysis.

Rather than look at all of the sections indicated as being in alignment with S-ID-1, it may be more beneficial to discuss the methodology with a more focused approach. Therefore, from this point forward, a subset of the sections will be used to continue this discussion of methodology. Table 3.2 is a subset of the same example spreadsheet from Table 3.1 with the “Item” column completed. This column is used to identify the parts of the identified sections that are aligned to the specified standard (in this case S-ID-1) from CCSSM. Often, multiple items were found in alignment with the specified standard in any one identified section. It is important to note that in some cases, in some cases, no alignment was found between the item and the identified standard from CCSSM. For example, “Waiting for a Double” in the IMP curriculum does not actually align with the standard S-ID-1 because the data are never represented with a plot on the real number line (see last row in Table 3.2).

Table 3.2. Items aligned to S-ID-1 from sections in Table 3.1

Standard	Textbook	Section	Item
S-ID-1	CPMP-1	73-101: U2-L1-I1	Narrative
S-ID-1	CPMP-1	73-101: U2-L1-I1	1a-i
S-ID-1	CPMP-1	73-101: U2-L1-I1	1a-ii
S-ID-1	CPMP-1	73-101: U2-L1-I1	1b
S-ID-1	CPMP-1	73-101: U2-L1-I1	1c
S-ID-1	CPMP-1	73-101: U2-L1-I1	1d
S-ID-1	CPMP-1	73-101: U2-L1-I1	2a
S-ID-1	CPMP-1	73-101: U2-L1-I1	2b
S-ID-1	CPMP-1	73-101: U2-L1-I1	2c
S-ID-1	CPMP-1	73-101: U2-L1-I1	2d
S-ID-1	CPMP-1	73-101: U2-L1-I1	3a
S-ID-1	CPMP-1	73-101: U2-L1-I1	3b
S-ID-1	CPMP-1	73-101: U2-L1-I1	3c
S-ID-1	CPMP-1	73-101: U2-L1-I1	3d
S-ID-1	CPMP-1	73-101: U2-L1-I1	4a
S-ID-1	CPMP-1	73-101: U2-L1-I1	4b-i
S-ID-1	CPMP-1	73-101: U2-L1-I1	4b-ii
S-ID-1	CPMP-1	73-101: U2-L1-I1	4c-i
S-ID-1	CPMP-1	73-101: U2-L1-I1	4c-ii
S-ID-1	CPMP-1	73-101: U2-L1-I1	4d
S-ID-1	CPMP-1	73-101: U2-L1-I1	5a
S-ID-1	CPMP-1	73-101: U2-L1-I1	5b
S-ID-1	CPMP-1	73-101: U2-L1-I1	6a
S-ID-1	CPMP-1	73-101: U2-L1-I1	6b
S-ID-1	CPMP-1	73-101: U2-L1-I1	7a
S-ID-1	CPMP-1	73-101: U2-L1-I1	7b
S-ID-1	CPMP-1	73-101: U2-L1-I1	7c
S-ID-1	CPMP-1	73-101: U2-L1-I1	7d
S-ID-1	CPMP-1	73-101: U2-L1-I1	8a
S-ID-1	CPMP-1	73-101: U2-L1-I1	8b
S-ID-1	CPMP-1	73-101: U2-L1-I1	8c
S-ID-1	CPMP-1	73-101: U2-L1-I1	8d
S-ID-1	CPMP-1	73-101: U2-L1-I1	9a
S-ID-1	CPMP-1	73-101: U2-L1-I1	9b
S-ID-1	CPMP-1	73-101: U2-L1-I1	9c
S-ID-1	CPMP-1	73-101: U2-L1-I1	9d
S-ID-1	CPMP-1	73-101: U2-L1-I1	SM1
S-ID-1	CPMP-1	73-101: U2-L1-I1	SM2
S-ID-1	CPMP-1	73-101: U2-L1-I1	SM3
S-ID-1	CPMP-1	73-101: U2-L1-I1	CYUa
S-ID-1	CPMP-1	73-101: U2-L1-I1	CYUb
S-ID-1	CPMP-1	73-101: U2-L1-I1	CYUc

(table continues)

Table 3.2 (continued)

Standard	Textbook	Section	Item
S-ID-1	GM-A1	0-13	Narrative
S-ID-1	GM-A1	0-13	Example 2
S-ID-1	GM-A1	0-13	Example 4
S-ID-1	GM-A1	0-13	Example 6
S-ID-1	GM-A1	0-13	Example 7a
S-ID-1	GM-A1	0-13	Example 7b
S-ID-1	GM-A1	0-13	Example 7c
S-ID-1	GM-A1	0-13	Example 8a
S-ID-1	GM-A1	0-13	Example 8b
S-ID-1	GM-A1	0-13	Exercise 2a
S-ID-1	GM-A1	0-13	Exercise 2b
S-ID-1	GM-A1	0-13	Exercise 3
S-ID-1	GM-A1	0-13	Exercise 4
S-ID-1	GM-A1	0-13	Exercise 5
S-ID-1	GM-A1	0-13	Exercise 7
S-ID-1	GM-A1	0-13	Exercise 8a
S-ID-1	GM-A1	0-13	Exercise 8b
S-ID-1	GM-A1	0-13	Exercise 8c
S-ID-1	GM-A1	0-13	Exercise 9a
S-ID-1	GM-A1	0-13	Exercise 9b
S-ID-1	GM-A1	0-13	Exercise 9c
S-ID-1	GM-A1	0-13	Exercise 10
S-ID-1	GM-A1	0-13	Exercise 11
S-ID-1	GM-A1	0-13	Exercise 12
S-ID-1	GM-A1	0-13	Exercise 13a
S-ID-1	GM-A1	0-13	Exercise 13b
S-ID-1	GM-A1	0-13	Exercise 13c
S-ID-1	GM-A1	0-13	Exercise 13d
S-ID-1	GM-A1	0-13	Exercise 14a
S-ID-1	GM-A1	0-13	Exercise 14b
S-ID-1	GM-A1	0-13	Exercise 14c
S-ID-1	GM-A1	0-13	Exercise 14d
S-ID-1	IMP-1	What Are the Chances?	Part I – A
S-ID-1	IMP-1	What Are the Chances?	Part I – B
S-ID-1	IMP-1	What Are the Chances?	Part I – C
S-ID-1	IMP-1	What Are the Chances?	Part I – D
S-ID-1	IMP-1	What Are the Chances?	Part I – E
S-ID-1	IMP-1	What Are the Chances?	Part I – F
S-ID-1	IMP-1	What Are the Chances?	Part I – G
S-ID-1	IMP-1	What Are the Chances?	Part I – H
S-ID-1	IMP-1	What Are the Chances?	Part I – I
S-ID-1	IMP-1	What Are the Chances?	Part II
S-ID-1	IMP-1	Rollin', Rollin', Rollin'	1

(table continues)

Table 3.2 (continued)

Standard	Textbook	Section	Item
S-ID-1	IMP-1	Rollin', Rollin', Rollin'	2
S-ID-1	IMP-1	Rollin', Rollin', Rollin'	3
S-ID-1	IMP-1	Waiting for a Double	No alignment

Within a single page or section of a textbook, there were often many items to be analyzed. Once again, in an effort to focus this discussion of methodology, a subset of the items will be used to continue this discussion. Each textbook has a different approach to instruction and thus an item in one textbook may look different than an item does in another.

Data was collected on how many items in a given textbook and in a given series are related to probability and statistics. Which textbook the items are found in may also be important since many states require only 3 years of mathematics. Any textbooks beyond the first three of each curricula (GM – Advanced Mathematical Concepts, CPMP – Year 4, IMP – Year 4) are more likely to be omitted for students completing only the minimum state requirements for graduation. Each of these more likely to be omitted textbooks was still analyzed for this research because they still represent opportunities for engagement. The results and discussion of this study include two analyses. The first analysis considers each curricula in its entirety. The second analysis considers only the first three years of each curricula.

3.3.2 Identify level of cognitive demand

Items were coded using the Task Analysis Guide as found in Smith and Stein (1998) (see Appendix C). The Task Analysis Guide specifies the characteristics of tasks in each of four categories: *memorization*, *procedures without connections*, *procedures with connections*, and *doing mathematics*. Each item was coded based on the highest potential level of cognitive demand it could achieve.

Once each item was assigned a code, both the frequency of each code and the percentage of items receiving each code are reported. These data are reported by standard, by textbook, and by overall curriculum. This was done to establish an overall rating of the cognitive demand in each manner described. In other words, the data rates the cognitive demand for each standard, each textbook, and each series overall.

Once again, to focus the discussion, a subset of the items presented earlier will be used to facilitate the discussion from this point. The items from Figure 3.2 from the GM textbook, Figure 3.4 and Figure 3.5 from the CPMP textbook, and Figure 3.8 and Figure 3.9 from the IMP textbook will be analyzed further. The items are shown in Table 3.3 with codes for cognitive demand. The CPMP examples highlight the rationale behind the methodology of coding each individual item. In both cases, if the analysis were limited to only coding the main instructional task, both would have received a code of *doing mathematics*. However, upon further inspection, in the first task from CPMP (Figure 3.4), three items are *doing mathematics*, one item is *procedures with connections* and one item is *procedures without connections*. In the second task from CPMP (Figure 3.5), one item is *doing mathematics*, two items are *procedures with connections* and one item is *procedures without connections*. By analyzing each individual item as opposed to only the task as a whole, a clearer picture of the curriculum may be available.

In some cases, there is not as much of a distinction. For example, all ten items of the IMP task *What Are the Chances?* are examples of *procedures with connections* because students are given suggested pathways that have connections to underlying conceptual ideas and multiple representations. Another example from IMP is that all three items of *Rollin', Rollin', Rollin'* are examples of *doing mathematics* because students are presented with open ended questions and must make a number of choices along the way as well as analyzing their own findings in a paragraph. The items from the GM textbook are coded as *procedures without connections*. Students are expected to use a learned procedure on the exercise with little connection to underlying concepts. A complete list of codes for the previously referenced items with the level of cognitive demand section completed is shown in Table 3.3 and with reference to the figures presented earlier.

Table 3.3. Items aligned to S-ID-1 from Table 3.2 with level of cognitive demand codes

Standard	Textbook	Section	Item	Cognitive Demand
S-ID-1	CPMP-1	73-101: U2-L1-I1	1a-i (Figure 3.4)	PWC
S-ID-1	CPMP-1	73-101: U2-L1-I1	1a-ii (Figure 3.4)	DM
S-ID-1	CPMP-1	73-101: U2-L1-I1	1b (Figure 3.4)	PNC
S-ID-1	CPMP-1	73-101: U2-L1-I1	1c (Figure 3.4)	DM
S-ID-1	CPMP-1	73-101: U2-L1-I1	1d (Figure 3.4)	DM
S-ID-1	CPMP-1	73-101: U2-L1-I1	2a (Figure 3.5)	PWC
S-ID-1	CPMP-1	73-101: U2-L1-I1	2b (Figure 3.5)	DM
S-ID-1	CPMP-1	73-101: U2-L1-I1	2c (Figure 3.5)	PNC
S-ID-1	CPMP-1	73-101: U2-L1-I1	2d (Figure 3.5)	PWC
S-ID-1	GM-A1	0-13	Exercise 2a (Figure 3.2)	PNC
S-ID-1	GM-A1	0-13	Exercise 2b (Figure 3.2)	PNC
S-ID-1	IMP-1	What Are the Chances?	I-A (Figure 3.8)	PWC
S-ID-1	IMP-1	What Are the Chances?	I-B (Figure 3.8)	PWC
S-ID-1	IMP-1	What Are the Chances?	I-C (Figure 3.8)	PWC
S-ID-1	IMP-1	What Are the Chances?	I-D (Figure 3.8)	PWC
S-ID-1	IMP-1	What Are the Chances?	I-E (Figure 3.8)	PWC
S-ID-1	IMP-1	What Are the Chances?	I-F (Figure 3.8)	PWC
S-ID-1	IMP-1	What Are the Chances?	I-G (Figure 3.8)	PWC
S-ID-1	IMP-1	What Are the Chances?	I-H (Figure 3.8)	PWC
S-ID-1	IMP-1	What Are the Chances?	I-I (Figure 3.8)	PWC
S-ID-1	IMP-1	What Are the Chances?	II (Figure 3.8)	PWC
S-ID-1	IMP-1	Rollin’, Rollin’, Rollin’	1 (Figure 3.9)	DM
S-ID-1	IMP-1	Rollin’, Rollin’, Rollin’	2 (Figure 3.9)	DM
S-ID-1	IMP-1	Rollin’, Rollin’, Rollin’	3 (Figure 3.9)	DM

Defining a task in these smaller units also allows for a more fair comparison between the curricula. If Figure 3.2 from the GM textbook is one task, Figure 3.4 or Figure 3.5 from the CPMP textbook is one task, and Figure 3.8 or Figure 3.9 from the IMP textbook is one task, the size of a task is dramatically different in each curricula. Coding Figure 3.4 and Figure 3.5 from CPMP as five items and four items respectively and Figure 3.8 and Figure 3.9 from IMP as three items and ten items respectively brings them much closer to the grain size of Figure 3.2 in the GM textbook, which is only two items.

3.3.3 Identify educative opportunities for teachers

The final step in the analysis was to examine the opportunities for teacher learning on the main instructional tasks, which were designated to be highly cognitively demanding in the curriculum materials. It is important to emphasize that this analysis is occurring at the level of main instructional task and not the item level. Items were grouped together consistent with the methodology from Stein et al. (1996), so items aimed at a particular mathematical idea were grouped together to form one main instructional task. The cognitive demand of these instructional tasks was also determined by using the highest code on any item within that task. While one might consider the average code or most frequent code to be more appropriate for an instructional task, these two alternative designations were not appropriate for the research questions posed in this study. It may be possible that lower cognitive demand items are included in a task in service of the higher demand item. Based on this possibility, the lower demand items are not the focus of the task. Additionally, the goal of this study was to examine the potential of the curriculum. If part of a task has the potential to be high-level, it would be inappropriate to suggest that the task is not high-level. Table 3.4 represents the same items from Table 3.3 collapsed into the level of task instead of item. Only the main instructional tasks of the textbooks that are designed to elicit high-level cognitive demand were coded for *anticipation* and *transparency*.

Table 3.4. Items aligned to S-ID-1from Table 3.3 grouped to form tasks

Standard	Textbook	Section	Task	Cognitive Demand
S-ID-1	CPMP-1	73-101: U2-L1-I1	1 and 2 (Figure 3.4 and 3.5)	DM
S-ID-1	GM-A1	0-13	Exercise 2 (Figure 3.2)	PNC
S-ID-1	IMP-1	What Are the Chances?	Entire Section (Figure 3.8)	PWC
S-ID-1	IMP-1	Rollin', Rollin', Rollin'	Entire Section (Figure 3.9)	DM

Figure 3.12 is part of a task where students are asked to relate a table or a graph to the Law of Large Numbers. The teacher's edition of the textbook provides an opportunity for anticipation as shown in Figure 3.13. Figure 3.13 anticipates two concepts that may cause conflict with students when it comes to the law of large numbers. Students may understand that the proportion of heads tends to get closer to the theoretical value of 0.5, but they may find difficulty in recognizing that the difference between actual value of heads and the expected value of heads typically increases.

f. Explain how your completed graph and table illustrate the Law of Large Numbers.

Figure 3.12. Task supported via anticipation in CPMP *Course 1* (Hirsch et al., p. 556, 2015)

f. The Law of Large Numbers says that as the number of trials increases, the estimated probability tends to get closer to the theoretical probability. The graph illustrates this because as the number of flips increases, the proportion of heads tends to get closer to 0.5.

POSSIBLE MISCONCEPTION Although the proportion of heads is converging to 0.5, the frequency of heads is diverging from the expected frequency. In the table in Part e, the expected number of heads in 10 flips is 5. The actual number is 4, for a difference of 1. After 50 flips, the proportion of heads is closer to 0.5 than for 10 flips, but the number of heads, 22, is 3 away from the expected number of heads, 25. Notice that while 3 is greater in magnitude than 1, it represents a smaller percentage of 50 than 1 does of 10.

This is an important idea for students to learn—that as you are flipping a coin, for example, the percentage of heads tends to get closer and closer to 50% as the number of flips increases, while the number of heads tends to get further and further from half the number of tosses. If students do not understand this, they will believe that the coin must balance out the numbers of heads and tails in the future by changing the probability that it will be a head. This idea comes up again in Reflections Task 17.

Figure 3.13. Support via anticipation in CPMP *Course 1* (Hirsch et al., p. 556T, 2015)

Figure 3.14 is a task from CPMP where the teacher's edition provides an opportunity for transparency. Figure 3.15 is the part of the teacher's edition that corresponds to student edition task shown in Figure 3.14. These figures demonstrate what typical opportunities for transparency look like in CPMP textbooks. As shown in Figure 3.15, the underlying focus of the task, use an informal understanding of conditional probability, is made explicit to the teacher. Additionally, the teacher is provided with an explanation of how the various methods of completing the task could be emphasized depending on the prior experience of the students in the class. This allows the teacher to adapt the task as needed without losing the conceptual understanding that the task intends to develop.

5 Suppose again the names of six boys and four girls are written on individual slips of paper and placed in a hat. This time you draw two names *without* replacement. That is, you draw one name, you do *not* return the slip of paper to the hat, then you draw a second name.

a. Find the probability that the first name drawn is a girl's name and the second name is a boy's name.

b. Explain why the answer to Part a is *not* $\frac{4}{10} \times \frac{6}{10}$.

c. Show how you can find the probability in Part a using the Multiplication Principle of Counting and the definition of probability given at the beginning of this investigation.

d. To find the probability in Part a, you can also use the **General Multiplication Rule** for any two events:

If A and B are events, then $P(A \text{ and } B) = P(A) \times P(B \mid A)$.

The notation $P(B \mid A)$ is read "probability of B given A." This means you find the probability of B assuming that you know A happened. Show how to use the General Multiplication Rule to find:

P(*girl's name on first draw* and *boy's name on second draw*).

Figure 3.14. Task supported via transparency in CPMP *Course 4* (Hirsch et al., p. 579, 2015)

5 **a.** Students may use various methods.

- Using the General Multiplication Rule (which students may use only implicitly until they get to Part d):
  The probability is $\frac{4}{10} \times \frac{6}{9} = \frac{24}{90} = 0.2\overline{6}$.
- Using the Multiplication Principle of Counting:
  P(*girl's name on first draw* and *boy's name on second draw*) =
  $\frac{\textit{number of outcomes corresponding to the event}}{\textit{total number of outcomes}} = \frac{4 \times 6}{10 \times 9} = 0.2\overline{6}$.
  (See the Mathematics Note in Problem 4 Part a.)
- Using permutations and the Multiplication Principle of Counting:
  $\frac{\textit{number of outcomes corresponding to the event}}{\textit{total number of outcomes}} = \frac{4 \times 6}{P(10, 2)} = 0.2\overline{6}$.

b. Informal response: The second factor is $\frac{6}{9}$ not $\frac{6}{10}$ because there are 6 boys but only 9 slips of paper left in the hat.

Formal response: You cannot use the Multiplication Rule for independent events because the events are not independent. The events in this analysis are "girl's name on first draw" and "boy's name on second draw." These events are not independent because the first slip of paper is not returned to the hat before the second slip is drawn; therefore, the result of the first draw changes the probability for the second draw.

c. Using the definition of probability given at the beginning of this investigation, you can compute the probability by counting outcomes. An outcome in this situation is a possible result of drawing two slips of paper when the first is drawn without replacement. That is, an outcome is a *sequence* of two names drawn. Each outcome is equally likely since each slip of paper is just as likely to be drawn as any other. Since the first slip of paper is not replaced before drawing the second, the total number of possible outcomes is $10 \times 9 = 90$. (Students may see this as $P(10, 2)$.) The number of outcomes corresponding to the event of "girl's name on first draw and boy's name on second draw" is $4 \times 6 = 24$. Thus, *P*(*girl's name on first draw* and *boy's name on second draw*) =
$\frac{\textit{number of outcomes corresponding to the event}}{\textit{total number of outcomes}} = \frac{24}{90} = 0.2\overline{6}$.

d. *P*(*girl's name on first draw* and *boy's name on second draw*) = *P*(*girl's name on first draw*) × *P*(*boy's name on second draw* | *girl's name on first draw*) =
$\frac{4}{10} \times \frac{6}{9}$.

INSTRUCTIONAL NOTE

Problem 5 references conditional probability. The context allows students to understand and apply conditional probability informally without formal development. Thus, depending on your students' experience with conditional probability, you may emphasize a formal approach to this idea or simply use an informal approach restricted to the specific contexts in this investigation.

Figure 3.15. Support via transparency in CPMP *Course 4* (Hirsch et al., p. 579T, 2015)

3.3.4 Reliability measures

To determine the reliability of the coding assignments, a stratified random sample of items were coded independently by the primary researcher and a secondary researcher. Stratified random sample refers to randomly selecting items from each of the textbook series individually as opposed to randomly selecting sections from all of the textbooks as a whole regardless of which textbook they were found in. One hundred forty seven items were selected for reliability coding. Using a stratified random sample as opposed to a random sample ensured representation by each textbook series. The CPMP and GM textbook series have many more items than the IMP series, so there was a concern that a random sample may have excluded IMP completely.

Once the items were selected, those items found near the selected items were also coded by the second coder. This allowed the second coder the opportunity to make judgments about not only the codes that should be assigned to items, but also what items should be group together to form tasks. This format of selection also ensures that the second coder reviewed both items and tasks from each of the individual textbooks. Training sessions were completed prior to coding the actual items used for this study to ensure reliability between the primary and secondary coders. These training sessions involved coding items and discussing discrepancies until the coders were able to provide consistent codes on randomly selected items reliably.

Cohen's κ was run to determine if there was agreement between the two coders' judgement on item alignment to the standards for probability and statistics of CCSSM. There was a moderate agreement between the two coders' judgments, $\kappa = .561$, $p < .0005$. Cohen's κ was also run to determine if there was agreement between the two coders' judgement on the level of cognitive demand of the items identified as being in alignment to the standards for probability and statistics of CCSSM. There was good agreement between the two coders' judgments, κ = .618, $p < .0005$. Finally, Cohen's κ was run to determine if there was agreement between the two coders' judgement on the presence of anticipation and transparency on high-level tasks. There was good agreement between the two coders' judgments, $\kappa = .615$, $p = .006$. In all cases, since $p < .01$, the kappa (κ) coefficients are statistically significantly different from zero.

3.4 HOW THE DATA RELATES TO THE RESEARCH QUESTIONS

The first research question was, "To what extent do current secondary mathematics textbooks provide opportunities for students to engage in the probability and statistics content recommended by the Common Core State Standards?" The following data are reported to answer this question:

1) Number of items identified as being in alignment with probability and statistics as defined by CCSSM in individual textbooks, curriculum materials overall, and the first three year of the series by individual standard (i.e. CPMP has 181 items in Year 1, 23 items in Year 2, 107 items in Year 3, 2 items in Year 4, 313 items overall, and 311 items in the first three years relating to standard S-ID-1). The total number of items for all probability and statistics standards in each textbook will also be reported
2) Number of tasks identified as being in alignment with probability and statistics as defined by CCSSM in individual textbooks, curriculum materials overall, and the first three year of the series by individual standard (i.e. CPMP has 11 tasks in Year 1, 1 task in Year 2, 7 tasks in Year 3, 0 tasks in Year 4, 19 tasks overall, and 19 tasks in the first three years

relating to standard S-ID-1). The total number of tasks for all probability and statistics standards in each textbook will also be reported

These data quantify the extent to which the textbooks provide opportunities for students to engage in the probability and statistics content recommended by the CCSSM. Additionally, the data quantifies the shortcomings of the curricula with respect to the probability and statistics recommendations of CCSSM. For example, a curriculum with zero items and tasks aligned to a specified standard could be identified as being deficient in relation to that specific standard.

As previously mentioned, most states require only three years of mathematics in high school. Any textbooks beyond the first three in a series may be less likely to be used for all students. If a textbook series saves all of the probability and statistics items and tasks for a fourth year or more advanced textbook, the opportunities for engagement in those items and tasks may not be taken advantage of for all students. To address this concern, the findings for just the first three books of each curricula are also reported.

The second research question is, "What are the cognitive demands of the tasks that are aligned with the Common Core State Standards recommendations for mathematical content in probability and statistics?" The following data are reported to answer this question:

1) The number of items and tasks in each textbook and each curricula receiving each of the codes for cognitive demand reported by standard and as an overall count (*memorization, procedures without connections, procedures with connection, doing mathematics*)
2) The percentage of items and tasks in each textbook and each curricula receiving each of the codes for cognitive demand reported by standard and as an overall count.

These data speak to the nature of the items and tasks in each textbook and each curricula overall. By reporting the number of items with each code, the number of opportunities for students to engage in high-level tasks in the area of probability and statistics is revealed. By reporting the number of tasks with each code, the number of opportunities for instruction with high-level tasks in the area of probability and statistics is revealed. Additionally, since high-level tasks are more difficult to implement with fidelity, especially *doing mathematics* tasks, this provides a report of the number of tasks that are likely to be challenging for the teacher to implement well. These data speak to the amount of support a district and the teacher materials would need to provide the teacher to promote proper use of these curriculum materials.

Reporting the percentage of items and tasks in each textbook and curricula receiving each code allows for some sense of the overall design of the textbook and curricula. Textbooks with a high percentage of *doing mathematics* items and tasks will demand much more from both students and teachers than those with higher percentages of *memorization* or *procedures without connections* codes. As previously argued in Chapter 1 (Section 1.3), a curriculum with high-level items and tasks will be more likely to promote students engagement in the Standards for Mathematical Practice. Additionally, reporting the percentage of items and tasks in each textbook and curricula receiving each of the codes for cognitive demand will also give an impression of how much support a district and the teacher materials would need to provide the teacher to promote implementation with fidelity. Finally, by reporting percentages for each individual textbook some interesting patterns emerge that can be used to reveal an inferred philosophy regarding how students learn of each curriculum.

The third research question is, “To what extent does the teachers’ guide provide support for enacting high-level tasks that address the Common Core State Standards recommendations related to probability and statistics?” The following data are reported to answer this question:

1) The number of high-level tasks in each textbook and in each curricula receiving each of the codes for teacher learning (*anticipation* and *transparency*).
2) The percentage of high-level tasks in each textbook and in each curricula receiving each of the codes for teacher learning.

These data speak to the extent that each textbook and each curricula overall support the teacher in enacting the probability and statistics recommendations of CCSSM. By reporting the number of high-level tasks with each code, the number of opportunities for teacher learning is revealed. Reporting the percentage of high-level tasks in each textbook and curricula receiving each code allows for some sense of the overall design of the textbook and curricula. Textbooks with a high percentage of high-level tasks receiving codes for *anticipation* and *transparency* provide more support for teachers than those with a low percentage of high-level tasks receiving those codes.

While more support will be needed to implement the curricula well, understanding what contributions the curriculum materials make to promoting teacher learning will help school districts decide what other types of support will be needed to promote proper use of these curriculum materials. With all of this data in hand, one could decide which curricula meets the needs of a given school district. Curricula with high-level tasks and little teacher support may be difficult to implement well without significant spending on other sources of teacher support. Curricula with low-level tasks may not need teacher support to be implemented with fidelity, but it may not suit the needs of a district looking to promote higher order thinking in preparation for CCSSM. In an area like probability and statistics where mathematics teachers are less likely to be comfortable with content and have a deep understanding of the concepts, understanding the demands the tasks place on both students and teachers and the support offered to teachers on tasks that will be highly demanding may be critical to a school district's success.

4.0 RESULTS

This chapter reports the results of the analysis described in Chapter 3. The research questions that guided this study, and the analyses conducted to answer them, are as follows:

1) To what extent do current secondary mathematics textbooks provide opportunities for students to engage in the probability and statistics content recommended by the Common Core State Standards?

   -determine the number of items identified as being in alignment with probability and statistics as defined by CCSSM in individual textbooks

   -determine the number of tasks identified as being in alignment with probability and statistics as defined by CCSSM in individual textbooks

   -determine the number of items identified as being in alignment with probability and statistics as defined by CCSSM in the entire curriculum

   -determine the number of tasks identified as being in alignment with probability and statistics as defined by CCSSM in the entire curriculum

   -determine the number of items identified as being in alignment with probability and statistics as defined by CCSSM in the first three textbooks of the curriculum

   -determine the number of tasks identified as being in alignment with probability and statistics as defined by CCSSM in the first three textbooks of the curriculum

2) What are the cognitive demands of the tasks that are aligned with the Common Core State Standards recommendations for mathematical content in probability and statistics?

   -the number of items receiving each of the codes for cognitive demand reported by standard and for all standards in the entire curriculum

   -the number of tasks receiving each of the codes for cognitive demand reported by standard and for all standards in the entire curriculum

   -the number of items receiving each of the codes for cognitive demand reported by textbook and in all textbooks for all standards

   -the number of items receiving each of the codes for cognitive demand reported by textbook and in all textbooks for all standards

3) To what extent does the teachers' guide provide support for enacting high-level tasks that address the Common Core State Standards recommendations related to probability and statistics?

   a) To what extent does the teachers' guide provide suggestions related to *anticipation* on high-level tasks that reflect content recommendations of the Common Core State Standards?

   b) To what extent does the teachers' guide provide *transparency* on high-level tasks that reflect content recommendations of the Common Core State Standards?

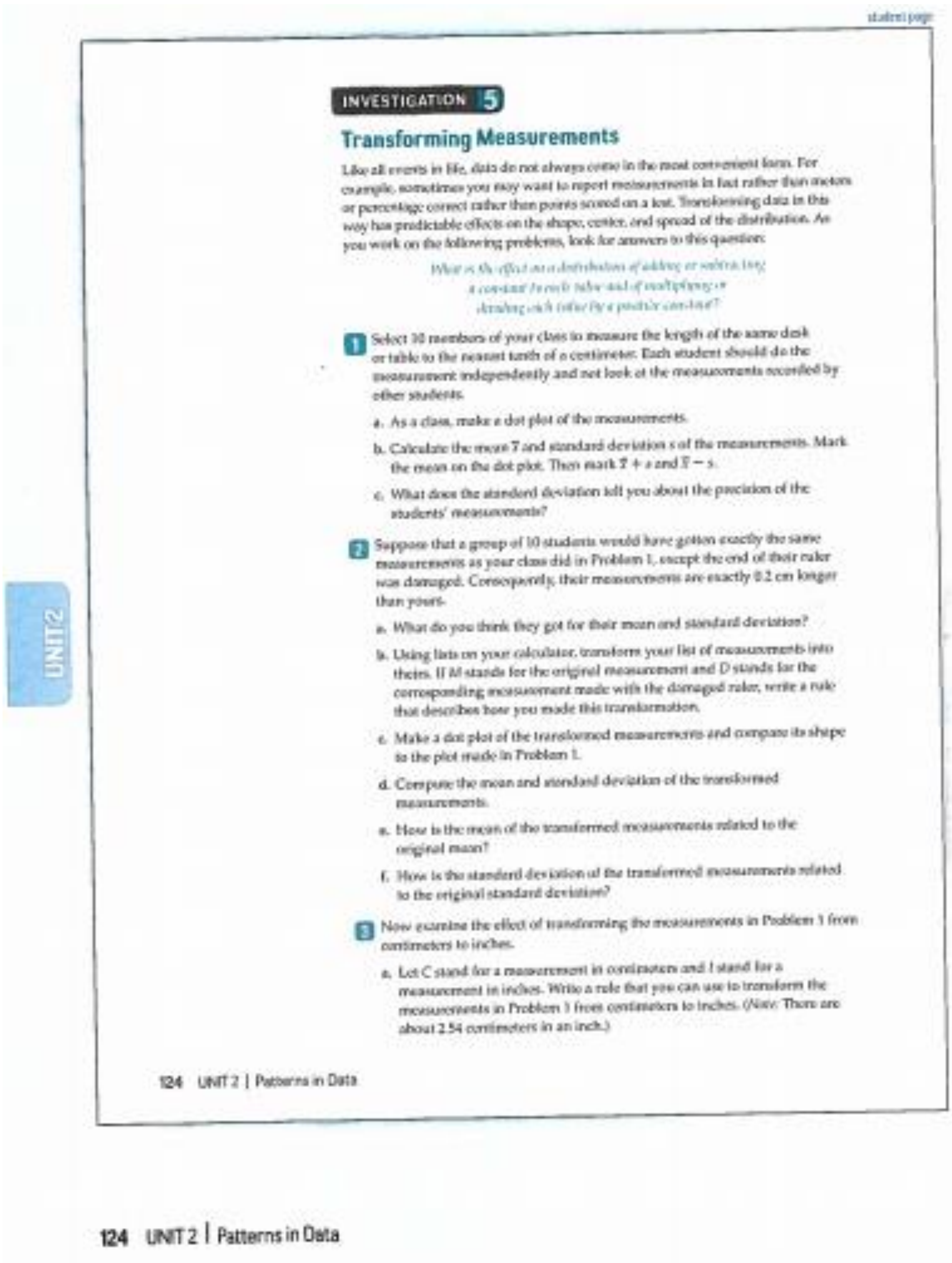
      -number of high-level tasks coded for teacher learning organized by textbook

      -number of high-level tasks coded for teacher learning for the entire series

4.1 DESCRIPTION OF TASKS AND ITEMS

The results are initially organized by textbook series, followed by a comparison between series. The term "task" refers to the main instructional task as defined by Stein and Kim (2009). A task can consist of many components or activities intended to focus students on a particular idea. As described in Chapter 3, the term "item" refers to each individual component of a task but also includes narratives, review problems, extra practice, or any other opportunity that students might have to engage in content that appears in the textbook (Thompson, Senk, & Johnson, 2012). Recall, this distinction was made to facilitate a more widespread analysis than looking only at the main instructional task. Additionally, this distinction will allow for more comparable grain sizes since the items in each curriculum are of similar size while the tasks are not. Figures 4.1 through 4.5 are pages from each of the different textbook series that exemplify the difference between items and tasks.

Figure 4.1 is page 124 of the CPMP curriculum book 1A. This page is representative of the typical instructional portion of CPMP. As shown in Table 4.1, this page contains eleven individual items. The narrative at the top of the page poses a question for students to consider. This narrative is not considered part of an instructional task, but is included in the item analysis because it contains a question for consideration. The other ten items on this page comprise a single instructional task for this lesson. Item 1a is aligned with S-ID-1. Items 2b and 3a are not aligned with any of the probability and statistics standards of CCSSM. The other seven items on this page (1b, 1c, 2a, 2c, 2d, 2e, and 2f) are all aligned with S-ID-2. Because most of what the textbook refers to as number 1 (which includes three items 1a, 1b, and 1c) is aligned to S-ID-2, number 1 would be considered a task aligned to S-ID-2. Similarly, what the textbook refers to as problem 2 (which includes 2a, 2b, 2c, 2d, 2e, and 2f) is aligned with S-ID-2, number 2 would be considered a task aligned with S-ID-2. Since these two collections of items appear consecutively and have a majority of items aligned to the same standard, they would then be combined to form a single instructional task. Therefore, Figure 4.1, which is page 124 of CPMP book 1A, contains one task made up of nine individual items. Items 2b and 3a are not included in the analysis since they are not aligned with CCSSM content.



student page

INVESTIGATION 5

Transforming Measurements

Like all events in life, data do not always come in the most convenient form. For example, sometimes you may want to report measurements in feet rather than meters or percentage correct rather than points scored on a test. Transforming data in this way has predictable effects on the shape, center, and spread of the distribution. As you work on the following problems, look for answers to this question:

What is the effect on a distribution of adding or subtracting a constant to each value and of multiplying or dividing each value by a positive constant?

1 Select 10 members of your class to measure the length of the same desk or table to the nearest tenth of a centimeter. Each student should do the measurement independently and not look at the measurements recorded by other students.

a. As a class, make a dot plot of the measurements.

b. Calculate the mean $\bar{x}$ and standard deviation s of the measurements. Mark the mean on the dot plot. Then mark $\bar{x} + s$ and $\bar{x} - s$.

c. What does the standard deviation tell you about the precision of the students' measurements?

2 Suppose that a group of 10 students would have gotten exactly the same measurements as your class did in Problem 1, except the end of their ruler was damaged. Consequently, their measurements are exactly 0.2 cm longer than yours.

a. What do you think they got for their mean and standard deviation?

b. Using lists on your calculator, transform your list of measurements into theirs. If M stands for the original measurement and D stands for the corresponding measurement made with the damaged ruler, write a rule that describes how you made this transformation.

c. Make a dot plot of the transformed measurements and compare its shape to the plot made in Problem 1.

d. Compute the mean and standard deviation of the transformed measurements.

e. How is the mean of the transformed measurements related to the original mean?

f. How is the standard deviation of the transformed measurements related to the original standard deviation?

3 Now examine the effect of transforming the measurements in Problem 1 from centimeters to inches.

a. Let C stand for a measurement in centimeters and I stand for a measurement in inches. Write a rule that you can use to transform the measurements in Problem 1 from centimeters to inches. (*Note:* There are about 2.54 centimeters in an inch.)

124 UNIT 2 | Patterns in Data

UNIT 2

124 UNIT 2 | Patterns in Data

Figure 4.1. Instructional items in CPMP *Course 1* (Hirsch et al., p. 124, 2015)

Table 4.1. Data associated with items from Figure 4.1

Series	Textbook	Chapter	Lesson	Section	Standard	Cognitive Demand
CPMP	1A	2	2	Inv 5 – Narr	S-ID-1	DM
CPMP	1A	2	2	Inv 5 – 1a	S-ID-1	DM
CPMP	1A	2	2	Inv 5 – 1b	S-ID-2	PwC
CPMP	1A	2	2	Inv 5 – 1c	S-ID-2	DM
CPMP	1A	2	2	Inv 5 – 2a	S-ID-2	DM
CPMP	1A	2	2	Inv 5 – 2b	None	
CPMP	1A	2	2	Inv 5 – 2c	S-ID-2	DM
CPMP	1A	2	2	Inv 5 – 2d	S-ID-2	PnC
CPMP	1A	2	2	Inv 5 – 2e	S-ID-2	PnC
CPMP	1A	2	2	Inv 5 – 2f	S-ID-2	PnC
CPMP	1A	2	2	Inv 5 – 3a	None	

Figure 4.2 on page 129, is also taken from, CPMP 1A. CPMP is organized by chapters, which are divided into lessons. Each lesson contains multiple investigations. Figure 4.1 was from the Investigation 5 of Chapter 2 Lesson 2. At the end of each lesson there are problems referred to as “On Your Own”. These problems are intended as extensions, reviews, and connection making problems that students can work on after instruction as opposed to being part of the investigations, which make up the instructional portion of the textbook. Since these problems are not intended to be the focus of instruction, they are not considered a task. However, each portion of them can be considered an item. Therefore, Figure 4.2 contains six items that were analyzed but no instructional tasks

ON YOUR OWN

APPLICATIONS

1 The table below gives the percentiles of recent SAT mathematics scores for national college-bound seniors. The highest possible score is 800 and the lowest possible score is 200. Only scores that are multiples of 50 are shown in the table, but all multiples of 10 from 200 to 800 are possible.

College-Bound Seniors

SAT Math Score	Percentile	SAT Math Score	Percentile
750	97	450	29
700	93	400	15
650	86	350	7
600	75	300	2
550	62	250	1
500	46	200	0

Source: The College Board, 2011

a. What percentage of seniors get a score of 650 or lower on the mathematics section of the SAT?

b. What percentage get a score higher than 450?

c. Estimate the score a senior would have to get to be in the top half of the students who take this test.

d. Estimate the 25th and 75th percentiles. Use these quartiles in a sentence that describes the distribution.

2 In a physical fitness test, the median time it took a large group of students to run a mile was 10.2 minutes. The distribution of running times had first and third quartiles of 7.1 minutes and 13.7 minutes. When results were reported to the students, faster runners (shorter times) were assigned higher percentiles.

a. Sheila was told that she was at the 25th percentile. How long did it take Sheila to run the mile?

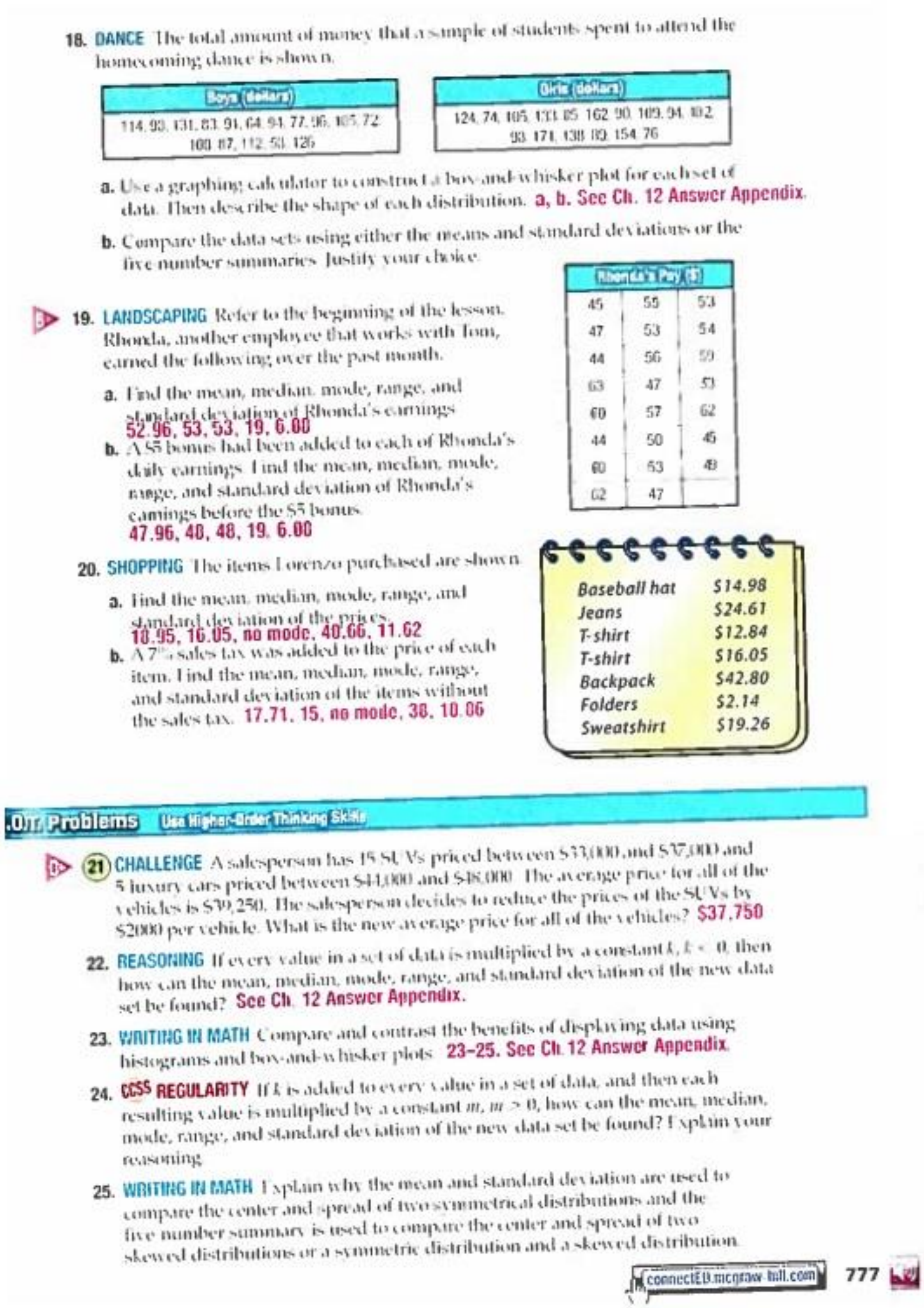
b. Mark was told that his time was at the 16th percentile. Write a sentence that tells Mark what this means.

Lesson 2 | Measuring Variability **129**

Figure 4.2. Items from On Your Own section in CPMP *Course 1* (Hirsch et al., p. 129, 2015)

Figure 4.3 and Figure 4.4 are the last two pages from section 12-4 of the GM Algebra I textbook. At the bottom of Figure 4.3 there are problems referred to as Higher Order Thinking Problems. Higher Order Thinking Problems are intended for enrichment and do not always relate to the example problems from the lesson in which they are found. For example, section 12-4 has example problems aligned to both S-ID-2 and S-ID-3. However, item 23 is aligned with S-ID-1. Because they are intended for enrichment as opposed to instruction, the Higher Order Thinking Problems are not considered instructional tasks. The items in the GM textbook that are intended for instruction have examples for the students and instructional notes for teacher. In the case of the Higher Order Thinking Problems, there are no examples for the students and the Teacher's Edition of the textbook provides the answer to the problem but no instructional notes. Therefore, even if the items address the same standard as the examples (section 12.4 items 21, 22, 24, and 25), they are not considered a task if they are found in the Higher Order Thinking Problems. Those items found prior to the Higher Order Thinking designation (18, 19, and 20) were considered a task since they appear in the main body of the exercises following the example problems.

Figure 4.4 contains three groups of items referred to as Standardized Test Practice, Spiral Review, and Skills Review. Item 30 is aligned to S-ID-1, which, once again, was not the focus of section 12-4. The other 21 items on the page are not aligned to any of the probability and statistics standards of CCSSM, so they clearly are not in alignment with the examples from the section in which they are found, 12-4, which is aligned with S-ID-2 and S-ID-3. These problems are designed as review problems, so they are not considered a task. However, since they are available for students, they were a part of the item analysis.



18. DANCE The total amount of money that a sample of students spent to attend the homecoming dance is shown.

Boys (dollars)
114, 93, 131, 83, 91, 64, 94, 77, 96, 105, 72, 100, 87, 112, 53, 126

Girls (dollars)
124, 74, 105, 133, 85, 162, 90, 109, 94, 102, 93, 171, 138, 89, 154, 76

a. Use a graphing calculator to construct a box-and-whisker plot for each set of data. Then describe the shape of each distribution. **a, b. See Ch. 12 Answer Appendix.**

b. Compare the data sets using either the means and standard deviations or the five number summaries. Justify your choice.

19. LANDSCAPING Refer to the beginning of the lesson. Rhonda, another employee that works with Tom, earned the following over the past month.

Rhonda's Pay ($)		
45	55	53
47	53	54
44	56	59
63	47	51
60	57	62
44	50	45
60	53	48
62	47	

a. Find the mean, median, mode, range, and standard deviation of Rhonda's earnings. **52.96, 53, 53, 19, 6.00**

b. A $5 bonus had been added to each of Rhonda's daily earnings. Find the mean, median, mode, range, and standard deviation of Rhonda's earnings before the $5 bonus. **47.96, 48, 48, 19, 6.00**

20. SHOPPING The items Lorenzo purchased are shown.

Baseball hat	*$14.98*
Jeans	*$24.61*
T-shirt	*$12.84*
T-shirt	*$16.05*
Backpack	*$42.80*
Folders	*$2.14*
Sweatshirt	*$19.26*

a. Find the mean, median, mode, range, and standard deviation of the prices. **18.95, 16.05, no mode, 40.66, 11.62**

b. A 7% sales tax was added to the price of each item. Find the mean, median, mode, range, and standard deviation of the items without the sales tax. **17.71, 15, no mode, 38, 10.06**

H.O.T. Problems Use Higher-Order Thinking Skills

21. CHALLENGE A salesperson has 15 SUVs priced between $33,000 and $37,000 and 5 luxury cars priced between $44,000 and $48,000. The average price for all of the vehicles is $39,250. The salesperson decides to reduce the prices of the SUVs by $2000 per vehicle. What is the new average price for all of the vehicles? **$37,750**

22. REASONING If every value in a set of data is multiplied by a constant k, $k < 0$, then how can the mean, median, mode, range, and standard deviation of the new data set be found? **See Ch. 12 Answer Appendix.**

23. WRITING IN MATH Compare and contrast the benefits of displaying data using histograms and box-and-whisker plots. **23–25. See Ch. 12 Answer Appendix.**

24. CCSS REGULARITY If k is added to every value in a set of data, and then each resulting value is multiplied by a constant m, $m > 0$, how can the mean, median, mode, range, and standard deviation of the new data set be found? Explain your reasoning.

25. WRITING IN MATH Explain why the mean and standard deviation are used to compare the center and spread of two symmetrical distributions and the five number summary is used to compare the center and spread of two skewed distributions or a symmetric distribution and a skewed distribution.

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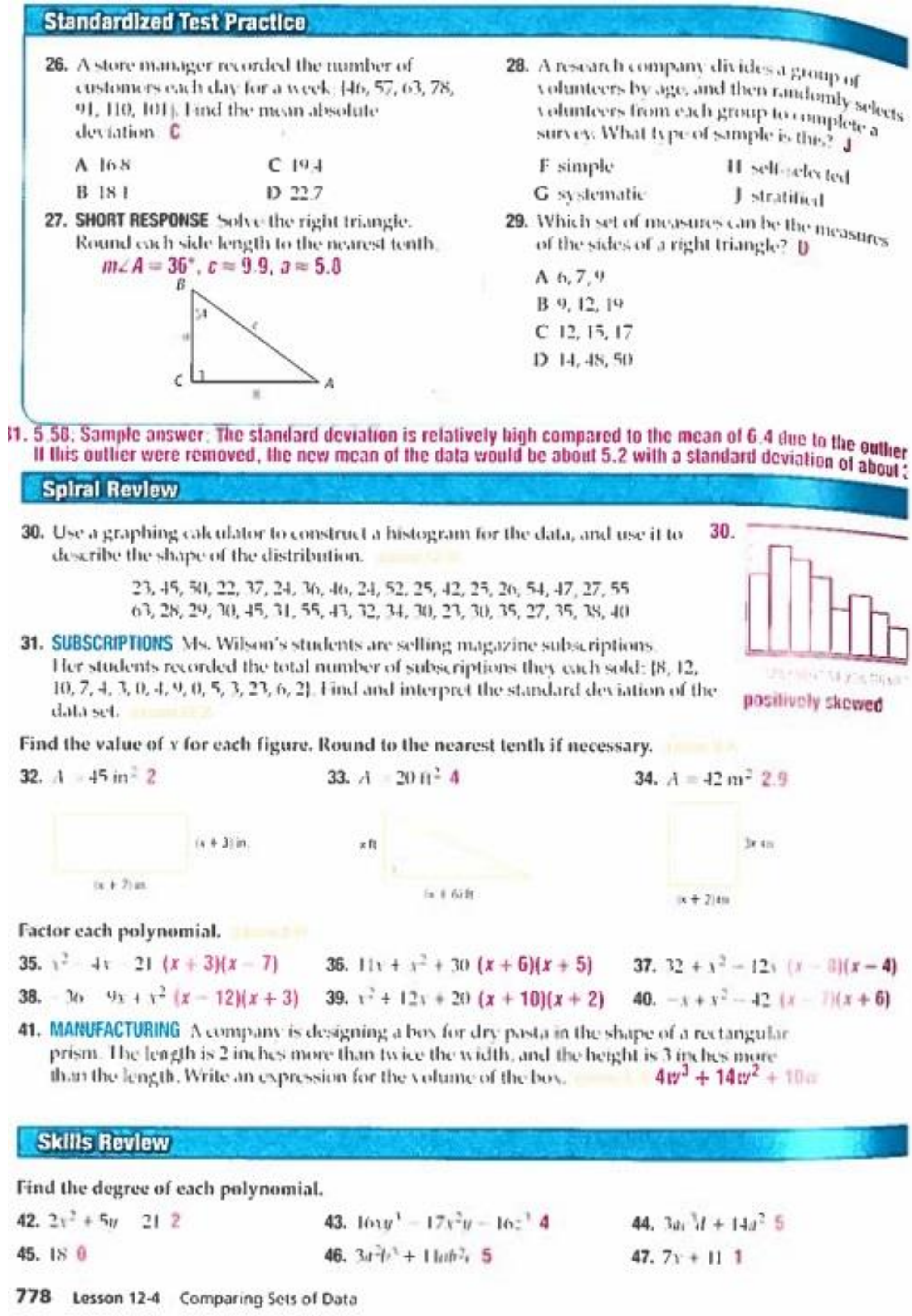
Figure 4.3. End of section items in GM *Algebra 1* (Carter et al., p. 777, 2010)



Standardized Test Practice

26. A store manager recorded the number of customers each day for a week: {46, 57, 63, 78, 91, 110, 101}. Find the mean absolute deviation. **C**

A 16.8 C 19.4
B 18.1 D 22.7

27. SHORT RESPONSE Solve the right triangle. Round each side length to the nearest tenth. $m\angle A = 36°$, $c \approx 9.9$, $a \approx 5.8$

28. A research company divides a group of volunteers by age, and then randomly selects volunteers from each group to complete a survey. What type of sample is this? **J**

F simple H self-selected
G systematic J stratified

29. Which set of measures can be the measures of the sides of a right triangle? **D**

A 6, 7, 9
B 9, 12, 19
C 12, 15, 17
D 14, 48, 50

21. 5.58; Sample answer: The standard deviation is relatively high compared to the mean of 6.4 due to the outlier. If this outlier were removed, the new mean of the data would be about 5.2 with a standard deviation of about :

Spiral Review

30. Use a graphing calculator to construct a histogram for the data, and use it to describe the shape of the distribution.

23, 45, 50, 22, 37, 24, 36, 46, 24, 52, 25, 42, 25, 26, 54, 47, 27, 55
63, 28, 29, 30, 45, 31, 55, 43, 32, 34, 30, 23, 30, 35, 27, 35, 38, 40

30. positively skewed

31. SUBSCRIPTIONS Ms. Wilson's students are selling magazine subscriptions. Her students recorded the total number of subscriptions they each sold: {8, 12, 10, 7, 4, 3, 0, 4, 9, 0, 5, 3, 23, 6, 2}. Find and interpret the standard deviation of the data set.

Find the value of x for each figure. Round to the nearest tenth if necessary.

32. $A = 45$ in^2 2
33. $A = 20$ ft^2 4
34. $A = 42$ m^2 2.9

Factor each polynomial.

35. $x^2 - 4x - 21$ $(x + 3)(x - 7)$
36. $11x + x^2 + 30$ $(x + 6)(x + 5)$
37. $32 + x^2 - 12x$ $(x - 8)(x - 4)$
38. $-36 - 9x + x^2$ $(x - 12)(x + 3)$
39. $x^2 + 12x + 20$ $(x + 10)(x + 2)$
40. $-x + x^2 - 42$ $(x - 7)(x + 6)$

41. MANUFACTURING A company is designing a box for dry pasta in the shape of a rectangular prism. The length is 2 inches more than twice the width, and the height is 3 inches more than the length. Write an expression for the volume of the box. $4w^3 + 14w^2 + 10w$

Skills Review

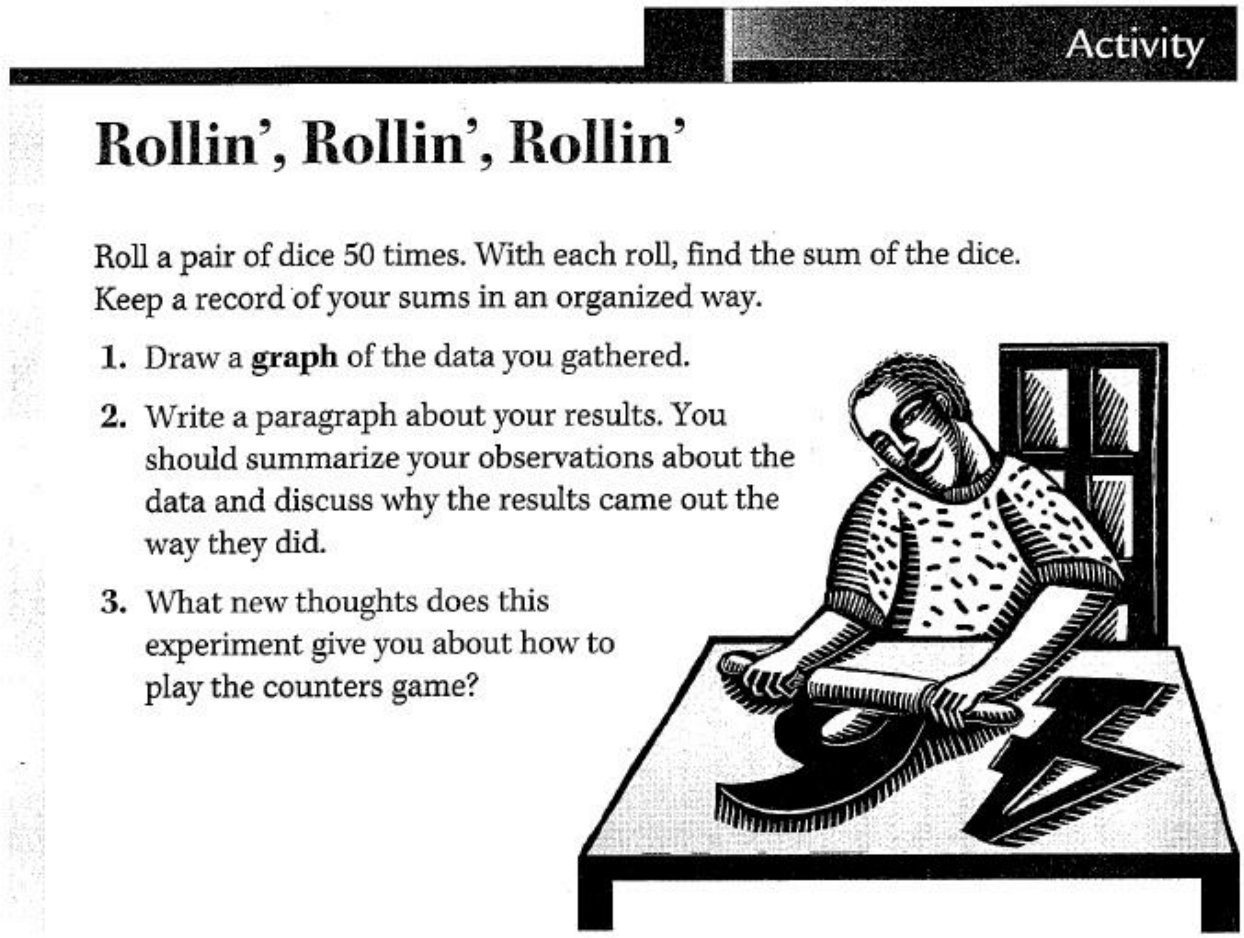
Find the degree of each polynomial.

42. $2x^2 + 5y - 21$ 2
43. $16xy^3 - 17x^2y - 16z^3$ 4
44. $3a^3d + 14a^2$ 5
45. 18 0
46. $3a^2b^3 + 11ab^2$ 5
47. $7x + 11$ 1

778 Lesson 12-4 Comparing Sets of Data

Figure 4.4. End of section items in GM *Algebra 1* (Carter et al., p. 778, 2010)

Figure 4.5 is from the IMP curriculum and contains three items that together form a single task. The first two items both align to S-ID-1. The third item does not align with any of the probability and statistics standards of CCSSM. Since most of the items are aligned to S-ID-1, the three items would be combined into one instructional task that is intended to address S-ID-1.



Activity

Rollin', Rollin', Rollin'

Roll a pair of dice 50 times. With each roll, find the sum of the dice. Keep a record of your sums in an organized way.

1. Draw a **graph** of the data you gathered.
2. Write a paragraph about your results. You should summarize your observations about the data and discuss why the results came out the way they did.
3. What new thoughts does this experiment give you about how to play the counters game?

Figure 4.5. Rollin', Rollin', Rollin' from IMP *Year 1* (Fendel et al., p. 104, 2009)

As shown in Figure 4.2, Figure 4.3, and Figure 4.4, the CPMP and GM textbooks provide problems at the end of each section for students to work on independently for the purpose of enrichment or review. These independent practice problems are not part of any instructional task. The tendency to provide many problems at the end of a section that are not part of the instructional portion of the text causes the CPMP and GM textbooks to have a high number of items when compared to the number of tasks. IMP does not provide these practice problems in the textbook. The lack of practice problems causes IMP to have fewer items per task than the other two curricula.

4.2 ONLINE STANDARD IDENTIFICATION LEADING TO ITEMS THAT DIDN'T CORRESPOND

In total 5283 items were analyzed from the three textbook series. Of the 5283 items, 3743 corresponded to the probability and statistics recommendations of CCSSM. There were 1540 items that did not correspond to the probability and statistic recommendations of CCSSM. These items were often the result of how the online resources referred to alignment with CCSSM.

The Core-Plus Mathematics Project online resource referred to pages in the textbook. For example, the Core-Plus Mathematics Project online resource suggested that items for S-ID-1 were on pages 108 to 142. That required the researcher to examine every task on those 35 pages. While many of the items did align with S-ID-1, not all of the items did. In many cases, there were problems that did not align with any of the probability and statistics recommendations of CCSSM so many items received a code of no correspondence. Figure 4.1, from page 124, and Figure 4.2, from page 129, both include tasks that fall in the range of pages identified by the online resource as containing S-ID-1 items. However, Table 4.1 shows that only two of eleven items in Figure 4.1 actually were aligned to S-ID-1. Two others were not aligned to any probability and statistics standard at all. A greater disparity between the online resource and the actual text is evident in Figure 4.2. None of the six items on this page is aligned with the probability and statistics standards of CCSSM let alone S-ID-1. The online resource identifies pages 108 to 142 because these pages are the investigations for Chapter 2 Lesson 2 of the textbook. Since S-ID-1 is one of the main focal points of this lesson, the online resource identified the entire lesson as being aligned to S-ID-1. However, there are parts of this lesson that are not actually aligned. Generalizing alignment in this manner caused the researcher to review many items from CPMP that did not actually align to any of the probability and statistics standards in CCSSM.

Similarly, the Glencoe Mathematics curriculum referred to sections of the book that contained many items. For example, S-ID-1 was found in the Glencoe Algebra I book in chapter 12 section 4. There are more than 50 items in chapter 12 section 4 that had to be analyzed based on this suggestion. However, not all of them actually corresponded to S-ID-1. Recall that Figure 4.3 and Figure 4.4 were both from chapter 12 section 4. As previously mentioned, one of the items in Figure 4.3 did not align to any probability and statistics standards of CCSSM. More dramatically, 21 of 22 items in Figure 4.4 did not align to any probability and statistics standards of CCSSM. Much like with the CPMP curriculum, the GM curriculum generalized sections of the textbook that addressed a specific standard even though that section contains problems at the end that often do not align. This caused the researcher to review many items from GM that did not actually align to any of the probability and statistics standards of CCSSM.

Because of the design of IMP, there were very few items identified as being in alignment that were not. IMP did not contain review problems or exercises for independent practice. Occasionally an item or items within a task would not align to a probability and statistic standards of CCSSM, but this was a rare occurrence. Item 3 in Figure 4.5 is an example of one such occurrence.

The 1540 items that did not correspond to the probability and statistics recommendations of CCSSM were removed from the analysis. The remaining 3743 items were grouped into a mere 193 tasks. It is expected that there would be many more items than tasks. However, there are more than 19 times as many items than there are tasks. This ratio does not suggest that typical tasks contain 19 individual items. This is more the result of many practice and review problems provided by the CPMP and GM curriculums that are not part of any instructional task as previously discussed.

4.3 GLENCOE MATHEMATICS

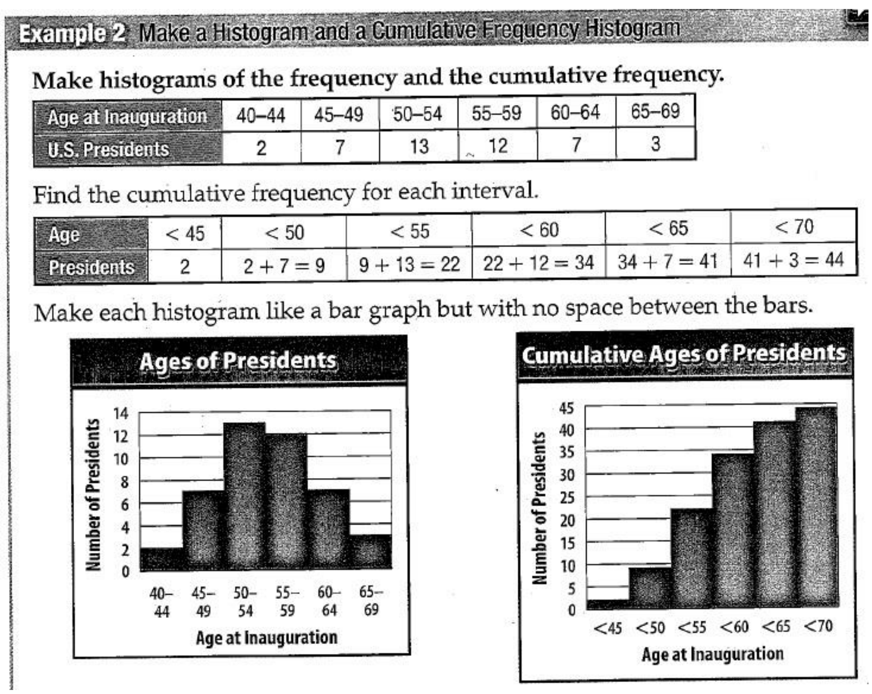
Glencoe Mathematics (GM) is a traditional approach high school mathematics textbook series organized by content (Algebra I, Algebra II, Geometry, and Precalculus) that is widely used based on market share data in Ross (2011). The traditional approach means that the student editions include example problems with worked out solutions and explanations provided to guide students through the steps of the solutions. Then there are exercises at the end of each section often corresponding directly to one of these worked out examples. Each example and its corresponding exercises are coded individually as items and then combined to form a task.

Figure 4.6 is a worked out example from lesson 0-13 of the GM Algebra textbook. This example contains two items. The first item asks students to make a histogram of the frequency. The second item asks students to make a histogram of the cumulative frequency. Figure 4.7 is one of the corresponding exercises from lesson 0-13 of the GM Algebra textbook. Once again, the exercise contains two items. Exactly like the worked out example, the first item asks students to graph the frequency, and the second item asks students to graph the cumulative frequency.

These two figures represent four items as shown in Table 4.2. The four items would be combined to form a single instructional task since they address the same standard in the same manner.

Table 4.2. Data associated with Figure 4.6 and Figure 4.7

Standard	Textbook	Section	Item	Cognitive Demand
S-ID-1	GM-A1	0-13	Example 2 Histogram (Figure 4.6)	PNC
S-ID-1	GM-A1	0-13	Example 2 Cummulative (Figure 4.6)	PNC
S-ID-1	GM-A1	0-13	Exercise 2a (Figure 4.7)	PNC
S-ID-1	GM-A1	0-13	Exercise 2b (Figure 4.7)	PNC


Example 2 Make a Histogram and a Cumulative Frequency Histogram

Make histograms of the frequency and the cumulative frequency.

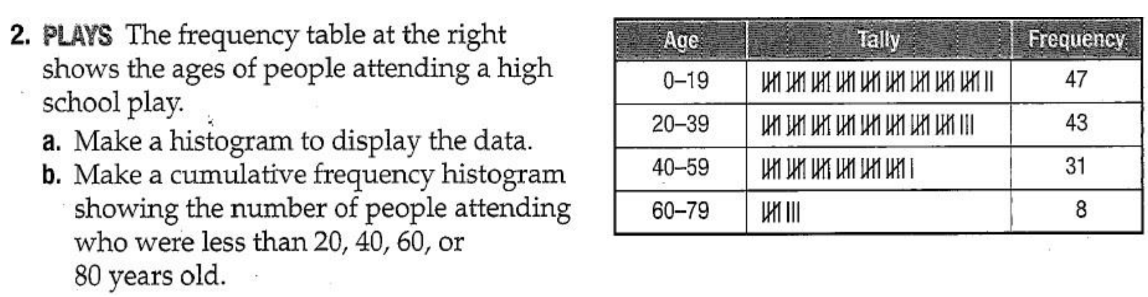
Age at Inauguration	40–44	45–49	50–54	55–59	60–64	65–69
U.S. Presidents	2	7	13	12	7	3

Find the cumulative frequency for each interval.

Age	< 45	< 50	< 55	< 60	< 65	< 70
Presidents	2	2 + 7 = 9	9 + 13 = 22	22 + 12 = 34	34 + 7 = 41	41 + 3 = 44

Make each histogram like a bar graph but with no space between the bars.

Figure 4.6. Example aligned to S-ID-1 from GM *Algebra 1* (Carter et al., p. 41, 2010)


2. PLAYS The frequency table at the right shows the ages of people attending a high school play.

a. Make a histogram to display the data.

b. Make a cumulative frequency histogram showing the number of people attending who were less than 20, 40, 60, or 80 years old.

Age	Tally	Frequency
0–19	卌 卌 卌 卌 卌 卌 卌 卌 卌 II	47
20–39	卌 卌 卌 卌 卌 卌 卌 卌 III	43
40–59	卌 卌 卌 卌 卌 卌 I	31
60–79	卌 III	8

Figure 4.7. Exercise related to Figure 4.6 from GM *Algebra 1* (Carter et al., p. 45, 2010)

4.3.1 Question 1

GM contains 1545 items that corresponded to the probability and statistics recommendations of CCSSM. Of the 1545 items, 822 (53%) of them correspond to only four of the standards (S-ID-1 has 161 items, S-ID-2 has 138 items, S-ID-4 has 257 items, and S-ID-6a has 266 items). That means the remaining 723 items are spread out over 20 remaining standards. The Algebra I, Algebra II, and Precalculus textbooks each have more than 400 items in them. However, the Geometry textbook only contains 146. Some standards have most of the items corresponding to them in the Precaluclus textbook. For example, S-IC-2 has 63 of 65 items in the Precalculus textbook. However, there are no standards that are solely addressed in Precalculus, so even though many opportunities would be lost by a student not enrolling in Precalculus as part of the GM program, nothing would be eliminated.

As shown in Table 4.3, the GM curriculum provides opportunities for students to engage in at least one item for every probability and statistics content suggestion of CCSSM. However, the number of opportunities varies greatly from one standard to another (S-CP-5 has only one item while S-ID-6a has 266 items). Of particular concern are the standards highlighted in Table 4.3, S-CP-5, S-CP-6, S-IC-5, and S-ID-5, which all had less than ten total items in the entire GM curriculum.

Table 4.3. Number of items in GM textbooks aligned to CCSSM probability and statistics

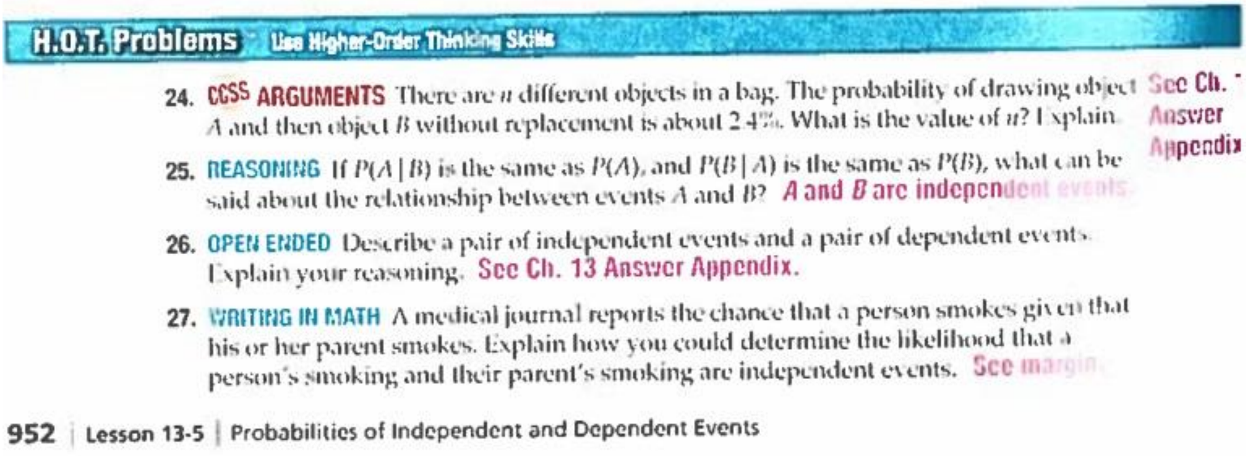
Standard	Algebra I	Algebra II	Geometry	Precalculus	Total	First 3
S-ID-1	60	55	33	13	161 (10%)	148
S-ID-2	70	29	0	39	138 (9%)	99
S-ID-3	14	12	0	1	27 (2%)	26
S-ID-4	4	76	0	177	257 (16%)	80
S-ID-5	6	0	2	0	8 (< 1%)	8
S-ID-6a	59	37	0	170	266 (17%)	96
S-ID-6b	5	4	0	8	17 (1%)	9
S-ID-6c	35	24	0	27	86 (5%)	59
S-ID-7	4	23	0	9	36 (2%)	27
S-ID-8	23	32	0	26	81 (5%)	55
S-ID-9	5	16	0	0	21 (1%)	21
S-IC-1	0	14	0	17	31 (2%)	14
S-IC-2	0	2	0	63	65 (4%)	2
S-IC-3	21	49	0	0	70 (4%)	70
S-IC-4	1	27	0	37	65 (4%)	28
S-IC-5	0	2	0	2	4 (< 1%)	2
S-IC-6	9	2	0	0	11 (1%)	11
S-CP-1	24	0	27	0	51 (3%)	51
S-CP-2	16	0	15	0	31 (2%)	31
S-CP-3	25	1	30	0	56 (4%)	56
S-CP-4	9	0	8	0	17 (1%)	17
S-CP-5	0	0	1	0	1 (< 1%)	1
S-CP-6	0	0	2	0	2 (< 1%)	2
S-CP-7	53	0	28	1	82 (5%)	81
Total	524 (33%)	405 (26%)	146 (9%)	590 (37%)	1584	994 (63%)

As shown in Table 4.4, the GM curriculum contains 59 total tasks that correspond to the probability and statistics standards of CCSSM. Of particular interest are the five standards from CCSSM that lack an instructional task (S-ID-5, S-ID-9, S-CP-4, S-CP-5, and S-CP-6). While all of the standards had at least one item associated with them, not all were part of an instructional task. This could mean they were part of an enrichment section or a special part of the homework exercises, but they were not included in the examples and main body of the homework exercises.

Table 4.4. Number of tasks in GM textbooks aligned to CCSSM probability and statistics

Standard	Algebra I	Algebra II	Geometry	Precalculus	Total	First 3
S-ID-1	3	1	0	4	8 (14%)	4
S-ID-2	4	1	0	1	6 (10%)	5
S-ID-3	1	1	0	0	2 (3%)	2
S-ID-4	0	1	0	2	3 (5%)	1
S-ID-5	0	0	0	0	0 (0%)	0
S-ID-6a	2	1	0	3	6 (10%)	3
S-ID-6b	1	0	0	1	2 (3%)	1
S-ID-6c	1	1	0	2	4 (7%)	2
S-ID-7	0	1	0	1	2 (3%)	1
S-ID-8	1	1	0	2	4 (7%)	2
S-ID-9	0	0	0	0	0 (0%)	0
S-IC-1	0	1	0	1	2 (3%)	1
S-IC-2	0	0	0	3	3 (5%)	0
S-IC-3	1	2	0	0	3 (5%)	3
S-IC-4	0	1	0	2	3 (5%)	1
S-IC-5	0	0	0	1	1 (2%)	0
S-IC-6	1	0	0	0	1 (2%)	1
S-CP-1	1	0	1	0	2 (3%)	2
S-CP-2	1	0	1	0	2 (3%)	2
S-CP-3	1	0	1	0	2 (3%)	2
S-CP-4	0	0	0	0	0 (0%)	0
S-CP-5	0	0	0	0	0 (0%)	0
S-CP-6	0	0	0	0	0 (0%)	0
S-CP-7	2	0	1	0	3 (5%)	3
Total	20 (34%)	12 (20%)	4 (7%)	23 (39%)	59	36 (61%)

Figure 4.8 shows the Higher Order Thinking Problems section at the end of chapter 13 section 5 of the GM Geometry textbook. Problem 27 is the only item in the entire GM textbook series that aligns with S-CP-5 from CCSSM. There is no example in this section related to this problem. Instead of being part of an instructional task, this problem is provided as enrichment at the end of a section with instructional tasks dedicated to S-CP-2 and S-CP-3.



H.O.T. Problems Use Higher-Order Thinking Skills

24. CCSS ARGUMENTS There are n different objects in a bag. The probability of drawing object A and then object B without replacement is about 2.4%. What is the value of n? Explain. See Ch. 13 Answer Appendix

25. REASONING If $P(A \mid B)$ is the same as $P(A)$, and $P(B \mid A)$ is the same as $P(B)$, what can be said about the relationship between events A and B? A and B are independent events.

26. OPEN ENDED Describe a pair of independent events and a pair of dependent events. Explain your reasoning. See Ch. 13 Answer Appendix.

27. WRITING IN MATH A medical journal reports the chance that a person smokes given that his or her parent smokes. Explain how you could determine the likelihood that a person's smoking and their parent's smoking are independent events. See margin.

952 | Lesson 13-5 | Probabilities of Independent and Dependent Events

Figure 4.8. Higher Order Thinking Problems from GM *Geometry* (Carter et al., p. 777, 2010)

4.3.2 Question 2

When examining the number of items receiving each code for cognitive demand by standard, the most glaring result shown in Table 4.5 is that the GM curriculum is dominated by procedures without connections items. Most of the individual standards have more items coded as procedures without connections than the other three possible codes combined. The few individual standards that do not have mostly procedures without connections items have only sixteen total items dedicated to them combined. Procedures without connections tasks represent more than 81% (1293 of a total 1584) of the items in the curriculum overall.

Table 4.5. Cognitive demand of items in GM textbooks sorted by standard

Standard	Memorization	Procedures without Connections	Procedures with Connections	Doing Mathematics	TOTAL
S-ID-1	3	133	20	5	161 (10%)
S-ID-2	0	106	31	1	138 (9%)
S-ID-3	0	24	3	0	27 (2%)
S-ID-4	5	228	22	2	257 (16%)
S-ID-5	0	7	1	0	8 (1%)
S-ID-6a	1	178	61	26	266 (17%)
S-ID-6b	0	15	2	0	17 (1%)
S-ID-6c	0	77	7	2	86 (5%)
S-ID-7	0	30	4	2	36 (2%)
S-ID-8	1	61	17	2	81 (5%)
S-ID-9	0	15	5	1	21 (1%)
S-IC-1	3	20	4	4	31 (2%)
S-IC-2	0	53	12	0	65 (4%)
S-IC-3	2	64	4	0	70 (4%)
S-IC-4	1	57	6	1	65 (4%)
S-IC-5	1	1	2	0	4 (< 1%)
S-IC-6	0	2	1	8	11 (1%)
S-CP-1	0	45	4	2	51 (3%)
S-CP-2	0	31	0	0	31 (2%)
S-CP-3	1	52	3	0	56 (4%)
S-CP-4	0	12	3	2	17 (1%)
S-CP-5	0	0	1	0	1 (< 1%)
S-CP-6	0	2	0	0	2 (< 1%)
S-CP-7	0	80	2	0	82 (5%)
Total	18 (1%)	1293 (82%)	215 (14%)	58 (4%)	1584

When examining the level of cognitive demand of items by textbook, it is clear that procedures without connections dominate each textbook as well. As shown in Table 4.6, Algebra I contains 368 procedures without connections items out of 433 items. Algebra II has 329 items that are procedures without connections and 373 total items. There are 113 total items in the Geometry textbook, and 102 of them are at the level of procedures without connections. Finally, in the Precalculus textbook 460 out of 626 items are procedures without connection. Additionally, over half of the high-level items in the GM curriculum are found in the Precalculus text (157 of 268). During the first three years of the curriculum, students will have the opportunity to engage in 808 low-level items and only 111 high-level items.

Table 4.6. Cognitive demand of items in GM textbooks sorted by textbook

Textbook	Memorization	Procedures without Connections	Procedures with Connections	Doing Mathematics	TOTAL
Algebra I	0	368	53	12	433 (28%)
Algebra II	8	329	29	7	373 (24%)
Geometry	1	102	7	3	113 (7%)
Precalculus	9	460	123	34	626 (41%)
Total	18 (1%)	1259 (81%)	212 (14%)	56 (4%)	1545
First 3	9	799	89	22	919 (59%)

When organized by instructional tasks as opposed to items, the results once again contain mostly procedures without connections codes as shown in Table 4.7. Approximately 66% of the tasks were coded at the procedures without connections level. Tasks may have multiple components, and codes were given at the highest level of any individual component, so if any part of a task was at a high-level, the entire task was credited for being high-level. In other words, even coded generously, most of the tasks are low-level tasks.

Table 4.7. Cognitive demand of tasks in GM textbooks sorted by standard

Standard	Memorization	Procedures without Connections	Procedures with Connections	Doing Mathematics	TOTAL
S-ID-1	0	6	2	0	8 (14%)
S-ID-2	0	3	3	0	6 (10%)
S-ID-3	0	2	0	0	2 (3%)
S-ID-4	0	2	1	0	3 (5%)
S-ID-5	0	0	0	0	0 (0%)
S-ID-6a	0	1	2	3	6 (10%)
S-ID-6b	0	1	1	0	2 (3%)
S-ID-6c	0	2	1	1	4 (7%)
S-ID-7	0	1	0	1	2 (3%)
S-ID-8	0	3	1	0	4 (7%)
S-ID-9	0	0	0	0	0 (0%)
S-IC-1	0	2	0	0	2 (3%)
S-IC-2	0	0	3	0	3 (5%)
S-IC-3	0	3	0	0	3 (5%)
S-IC-4	0	3	0	0	3 (5%)
S-IC-5	0	0	1	0	1 (2%)
S-IC-6	0	1	0	0	1 (2%)
S-CP-1	0	2	0	0	2 (3%)
S-CP-2	0	2	0	0	2 (3%)
S-CP-3	0	2	0	0	2 (3%)
S-CP-4	0	0	0	0	0 (0%)
S-CP-5	0	0	0	0	0 (0%)
S-CP-6	0	0	0	0	0 (0%)
S-CP-7	0	3	0	0	3 (5%)
Total	0 (0%)	39 (66%)	15 (25%)	5 (8%)	59

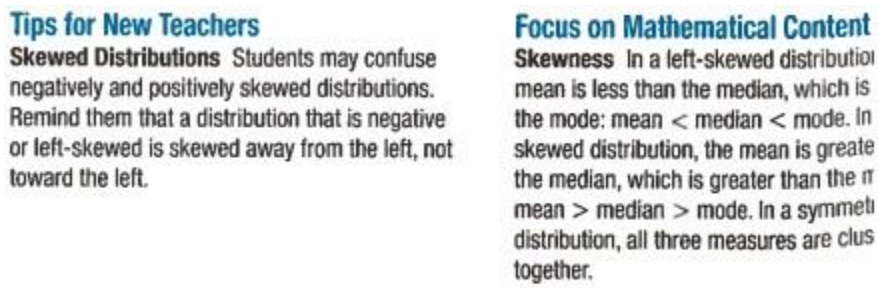
As shown in Table 4.8, the level of cognitive demand of tasks once again demonstrates the dominance of procedures without connections in the GM series with one exception. The Precalculus textbook actually has more high-level tasks (16) than low-level tasks (7). However, the rest of the textbooks have at least 85% of their tasks at the level of procedures without connections. When Precalculus is considered as part of the analysis, there are 39 low-level tasks and 20 high-level tasks, which represent 66% and 34% of the tasks respectively. When Precalculus is removed from the analysis, there are still 32 low-level tasks but only 4 high-level tasks representing 89% and 11% of the tasks respectively. The curriculum overall is limited in the number of opportunities for students to engage in high-level tasks related to probability and statistics. This limitation is magnified when the fourth textbook is not part of the curriculum.

Table 4.8. Cognitive demand of tasks in GM textbooks sorted by textbook

Textbook	Memorization	Procedures without Connections	Procedures with Connections	Doing Mathematics	TOTAL
Algebra I	0	17	3	0	20 (34%)
Algebra II	0	11	1	0	12 (20%)
Geometry	0	4	0	0	4 (7%)
Precalculus	0	7	11	5	23 (39%)
Total	0 (0%)	39 (66%)	15 (25%)	5 (8%)	59
First 3	0	32	4	0	36 (61%)

4.3.3 Question 3

Only one high-level task in the GM series, Advanced Mathematical Concepts: Precalculus with Applications section 11-1 examples 1, 2, and 3, provide support related to either anticipation or transparency. Specifically, this task anticipated students having a misconception about what skewed data looks like graphically, as shown in Figure 4.9. Other than this one task, the teachers' guide did not provide support for enacting high-level tasks in probability and statistics.



Tips for New Teachers

Skewed Distributions Students may confuse negatively and positively skewed distributions. Remind them that a distribution that is negative or left-skewed is skewed away from the left, not toward the left.

Focus on Mathematical Content

Skewness In a left-skewed distributio mean is less than the median, which is the mode: mean < median < mode. In skewed distribution, the mean is greate the median, which is greater than the m mean > median > mode. In a symmeti distribution, all three measures are clus together.

Figure 4.9. Teacher support via anticipation in GM precalculus (Holliday, p. 655-657, 2014)

4.4 INTERACTIVE MATHEMATICS PROGRAM

Interactive Mathematics Program (IMP) curriculum materials were funded by the NSF and represent a *Standards*-based approach to secondary mathematics education. Instead of being organized by content as the GM textbooks were, IMP represents an integrated approach organized by years. There are four years of textbooks intended to be implemented in grades 9 through 12.

4.4.1 Question 1

The highlighted entries in Table 4.9 are the probability and statistics content suggestions of CCSSM that the IMP curriculum does not provide opportunities in which students can engage. While the online resources for teachers did suggest online lessons that could be used to address these standards, S-ID-6b, S-ID-8, S-CP-3, S-CP-5, and S-CP-7 were not addressed in the student textbook and thus were not included in this study. No standard was addressed by more than 28 items and very few were addressed in more than one year of the textbook series. Of the 191 total items shown in Table 4.9, 189 are found in the first two years of the textbook series. If the four years of this textbook series are used during grades nine through twelve, then 99% of the opportunities found in the series will be available in ninth and tenth grade. That also means that probability and statistics will go unaddressed during the junior and senior years of students who attend schools using this textbook series.

Table 4.9. Number of items in IMP textbooks aligned to CCSSM probability and statistics

Standard	Year 1	Year 2	Year 3	Year 4	Total	First 3
S-ID-1	21	0	0	0	21 (11%)	21
S-ID-2	14	0	0	0	14 (7%)	14
S-ID-3	14	0	0	0	14 (7%)	14
S-ID-4	10	0	0	0	10 (5%)	10
S-ID-5	0	20	0	0	20 (10%)	20
S-ID-6a	18	0	2	0	20 (10%)	20
S-ID-6b	0	0	0	0	0 (0%)	0
S-ID-6c	3	0	0	0	3 (2%)	3
S-ID-7	8	0	0	0	8 (4%)	8
S-ID-8	0	0	0	0	0 (0%)	0
S-ID-9	0	1	0	0	1 (< 1%)	1
S-IC-1	0	6	0	0	6 (3%)	6
S-IC-2	1	7	0	0	8 (4%)	8
S-IC-3	0	1	0	0	1 (< 1%)	1
S-IC-4	0	2	0	0	2 (1%)	2
S-IC-5	1	13	0	0	14 (7%)	14
S-IC-6	1	5	0	0	6 (3%)	6
S-CP-1	28	0	0	0	28 (15%)	28
S-CP-2	13	0	0	0	13 (7%)	13
S-CP-3	0	0	0	0	0 (0%)	0
S-CP-4	0	1	0	0	1 (< 1%)	1
S-CP-5	0	0	0	0	0 (0%)	0
S-CP-6	1	0	0	0	1 (< 1%)	1
S-CP-7	0	0	0	0	0 (0%)	0
Total	133 (70%)	56 (29%)	2 (1%)	0 (0%)	191	191 (100%)

An examination of the results of the task analysis, as shown in Table 4.10, confirms the conclusions reached from the item analysis. There are five or less tasks associated with each standard. Most of the standards have tasks only in one year of the textbook series.

Table 4.10. Number of tasks in IMP textbooks aligned to CCSSM probability and statistics

Standard	Year 1	Year 2	Year 3	Year 4	Total	First 3
S-ID-1	2	0	0	0	2 (5%)	2
S-ID-2	2	0	0	0	2 (5%)	2
S-ID-3	2	0	0	0	2 (5%)	2
S-ID-4	2	0	0	0	2 (5%)	2
S-ID-5	0	3	0	0	3 (7%)	3
S-ID-6a	4	0	1	0	5 (12%)	5
S-ID-6b	0	0	0	0	0 (0%)	0
S-ID-6c	1	0	0	0	1 (2%)	1
S-ID-7	1	0	0	0	1 (2%)	1
S-ID-8	0	0	0	0	0 (0%)	0
S-ID-9	0	1	0	0	1 (2%)	1
S-IC-1	0	3	0	0	3 (7%)	3
S-IC-2	1	3	0	0	4 (10%)	4
S-IC-3	0	1	0	0	1 (2%)	1
S-IC-4	0	1	0	0	1 (2%)	1
S-IC-5	0	4	0	0	4 (10%)	4
S-IC-6	1	2	0	0	3 (7%)	3
S-CP-1	3	0	0	0	3 (7%)	3
S-CP-2	3	0	0	0	3 (7%)	3
S-CP-3	0	0	0	0	0 (0%)	0
S-CP-4	0	0	0	0	0 (0%)	0
S-CP-5	0	0	0	0	0 (0%)	0
S-CP-6	1	0	0	0	1 (2%)	1
S-CP-7	0	0	0	0	0 (0%)	0
Total	24 (57%)	17 (40%)	1 (2%)	0 (0%)	42	42 (100%)

4.4.2 Question 2

As shown in Table 4.11, the IMP curriculum has more high-level items than low-level items. With 31% of items being at the level of procedures with connections and 40% doing mathematics, the IMP curriculum has opportunities for students to engage in high-level items 71% of the time.

Table 4.11. Cognitive demand of items in IMP textbooks sorted by standard

Standard	Memorization	Procedures without Connections	Procedures with Connections	Doing Mathematics	Total
S-ID-1	0	15	3	3	21 (11%)
S-ID-2	0	3	7	4	14 (7%)
S-ID-3	0	3	3	8	14 (7%)
S-ID-4	1	7	2	0	10 (5%)
S-ID-5	0	13	4	3	20 (10%)
S-ID-6a	0	5	4	11	20 (10%)
S-ID-6b	0	0	0	0	0 (0%)
S-ID-6c	0	0	3	0	3 (2%)
S-ID-7	0	5	2	1	8 (4%)
S-ID-8	0	0	0	0	0 (0%)
S-ID-9	0	0	0	1	1 (1%)
S-IC-1	0	0	1	5	6 (3%)
S-IC-2	0	0	0	8	8 (4%)
S-IC-3	0	0	0	1	1 (1%)
S-IC-4	0	0	0	2	2 (1%)
S-IC-5	0	0	9	5	14 (7%)
S-IC-6	0	0	0	6	6 (3%)
S-CP-1	0	1	15	12	28 (15%)
S-CP-2	0	3	5	5	13 (7%)
S-CP-3	0	0	0	0	0 (0%)
S-CP-4	0	0	1	0	1 (1%)
S-CP-5	0	0	0	0	0 (0%)
S-CP-6	0	0	0	1	1 (1%)
S-CP-7	0	0	0	0	0 (0%)
Total	1 (1%)	55 (29%)	59 (31%)	76 (40%)	191

When organized by textbooks, all but two of the probability and statistics items in the entire IMP curriculum are found in the first two years of the textbook series as shown in Table 4.12. There is also a slight difference in the level of cognitive demand of the items when sorted by years. In year one, there is balance between procedures without connections, procedures with connections, and doing mathematics items with all three being in the forties. However, in year two, the numbers tend to lean more toward doing mathematics as there are more doing mathematics items than the other three possible codes combined.

Table 4.12. Cognitive demand of items in IMP textbooks sorted by textbook

Textbook	Memorization	Procedures without Connections	Procedures with Connections	Doing Mathematics	TOTAL
Year 1	1	40	45	47	133 (70%)
Year 2	0	13	14	29	56 (29%)
Year 3	0	2	0	0	2 (1%)
Year 4	0	0	0	0	0 (0%)
Total	1 (1%)	55 (29%)	59 (31%)	76 (40%)	191
First 3	1	55	59	76	191 (100%)

When instructional tasks were examined by standard, a majority were coded as doing mathematics, as shown in Table 4.13. More than 85% of the tasks were considered to have high cognitive demand as opposed to 70% of the items in Table 4.11. The greater disparity between high-level and low-level codes in tasks as compared to items may indicate that many of the tasks contain lower level items within them. Because they appear with high-level items, the low-level item codes do not show up in the codes for instructional tasks.

Table 4.13. Cognitive demand of tasks in IMP textbooks sorted by standard

Standard	Memorization	Procedures without Connections	Procedures with Connections	Doing Mathematics	Total
S-ID-1	0	1	0	1	2 (5%)
S-ID-2	0	0	1	1	2 (5%)
S-ID-3	0	0	0	2	2 (5%)
S-ID-4	0	1	1	0	2 (5%)
S-ID-5	0	3	0	0	3 (7%)
S-ID-6a	0	1	1	3	5 (12%)
S-ID-6b	0	0	0	0	0 (0%)
S-ID-6c	0	0	1	0	1 (2%)
S-ID-7	0	0	0	1	1 (2%)
S-ID-8	0	0	0	0	0 (0%)
S-ID-9	0	0	0	1	1 (2%)
S-IC-1	0	0	0	3	3 (7%)
S-IC-2	0	0	0	4	4 (10%)
S-IC-3	0	0	0	1	1 (2%)
S-IC-4	0	0	0	1	1 (2%)
S-IC-5	0	0	1	3	4 (10%)
S-IC-6	0	0	0	3	3 (7%)
S-CP-1	0	0	0	3	3 (7%)
S-CP-2	0	0	2	1	3 (7%)
S-CP-3	0	0	0	0	0 (0%)
S-CP-4	0	0	0	0	0 (0%)
S-CP-5	0	0	0	0	0 (0%)
S-CP-6	0	0	0	1	1 (2%)
S-CP-7	0	0	0	0	0 (0%)
Total	0 (0%)	6 (14%)	7 (17%)	29 (69%)	42

The idea of low-level items appearing in tasks with high-level items, in this case doing mathematics items, is exemplified by Figure 4.10 and Figure 4.11. These figures are from an IMP group activity called Making Friends with Standard Deviation. This group activity contains eleven total items as shown in Table 4.12.

Problem 1 in Making Friends with Standard Deviation begins with item 1a that does not align with the probability and statistics recommendations of CCSSM. Item 1b is a procedures without connections item aligned to S-ID-3. Item 1c is also aligned to S-ID-3, but increases in level of cognitive demand to procedures with connection. The fourth and final item for problem 1 is a doing mathematics item also aligned to S-ID-3. The bullet points with the fourth item were not coded as separate items because they provide guidance for students in addressing the initial portion of the item, “Explain why your pattern should occur” rather than represent independent items.

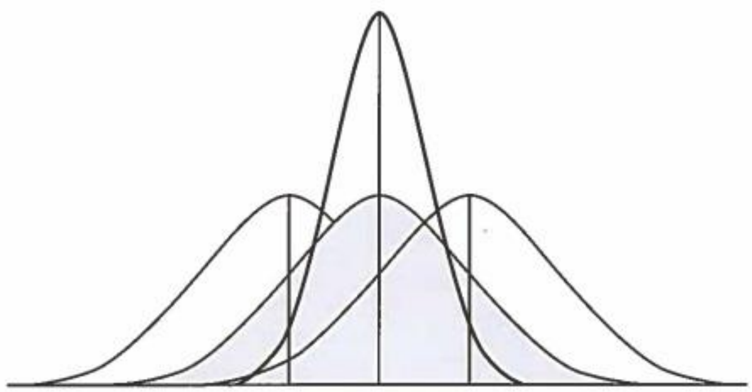
Items coded as procedures without connections, procedures with connections, and doing mathematics are also present in problem 2. All four items in problem 2 are aligned to S-ID-3. The first two are procedures without connections. The next item is at the level of procedures with connections. The fourth and final item is at the level of doing mathematics. The third problem in this group activity does not align with any probability and statistics recommendation of CCSSM.

In summary, the items in this group activity that are aligned with the probability and statistics recommendations of CCSSM are all aligned to S-ID-3. Since they are all aligned to the same item, this group activity is considered one instructional task. This one task contains three procedures without connections items, two procedures with connections items, and two doing mathematics items. The level of cognitive demand for the task is doing mathematics because that is the highest potential of any single item in the task. Coding the task at the level of doing mathematics masks the five codes were not at the level of doing mathematics.

Group Activity

Making Friends with Standard Deviation

You will be working with the concept of standard deviation to decide which variables actually have an effect on the period of a pendulum. It will be helpful for you to become familiar with what standard deviation means.



1. First explore what happens to the mean and the standard deviation of a set of data when you add the same number to each member in the set.
   a. As a group, make up a set of five numbers that are all different. Find the mean and the standard deviation of your set.
   b. Now choose a nonzero number and add it to each member of your set. Find the mean and the standard deviation of your new set.
   c. Repeat part b, using a different nonzero number. Add this number to each member of your original set of data, and find the mean and standard deviation of the new set. Keep repeating this process until you see patterns, and then describe those patterns.
   d. Explain why your pattern should occur.
      - Explain why the mean changes as it does when you add the same thing to each member of the set.
      - Explain why the standard deviation changes as it does when you add the same thing to each member of the set.

continued

The Pit and the Pendulum: Statistics and the Pendulum **331**

Figure 4.10. Items from a group activity in IMP *Year 1* (Fendel et al., p. 331, 2009)

2. Now explore what happens to the mean and the standard deviation of a set of data when you multiply each member in the set by the same number.

   a. Begin with the same set of data as in Question 1a. Choose a nonzero number other than 1. Multiply each member of your set by that number and find the mean and the standard deviation of the new set.

   b. Choose another nonzero number other than 1, and repeat what you did in part a.

   c. Keep choosing new nonzero numbers to use as multipliers for each member in your set. Find the mean and the standard deviation of each new set until you see patterns. Describe those patterns.

   d. Explain why your patterns occur.

3. Make up a set of data that satisfies each of the given conditions as closely as you can.

   a. Mean, 6; standard deviation, 1

   b. Mean, 10; standard deviation, 1

   c. Mean, 7; standard deviation, 2

332 INTERACTIVE MATHEMATICS PROGRAM

Figure 4.11. Items from a group activity in IMP *Year 1* (Fendel et al., p. 332, 2009)

Table 4.14. Data from items in Figure 4.10 and Figure 4.11

Series	Book	Chapter	Activity	Problem	Standard	Cognitive Demand
IMP	1	The Pit and the Pendulum	Making Friends with Standard Deviation	1a	None	
IMP	1	The Pit and the Pendulum	Making Friends with Standard Deviation	1b	S-ID-3	PnC
IMP	1	The Pit and the Pendulum	Making Friends with Standard Deviation	1c	S-ID-3	PwC
IMP	1	The Pit and the Pendulum	Making Friends with Standard Deviation	1d	S-ID-3	DM
IMP	1	The Pit and the Pendulum	Making Friends with Standard Deviation	2a	S-ID-3	PnC
IMP	1	The Pit and the Pendulum	Making Friends with Standard Deviation	2b	S-ID-3	PnC
IMP	1	The Pit and the Pendulum	Making Friends with Standard Deviation	2c	S-ID-3	PwC
IMP	1	The Pit and the Pendulum	Making Friends with Standard Deviation	2d	S-ID-3	DM
IMP	1	The Pit and the Pendulum	Making Friends with Standard Deviation	3a	None	
IMP	1	The Pit and the Pendulum	Making Friends with Standard Deviation	3b	None	
IMP	1	The Pit and the Pendulum	Making Friends with Standard Deviation	3c	None	

When looking at the tasks organized by textbook, both the first and second year have most tasks at the level of doing mathematics, as shown in Table 4.15. This resembles the results when looking at cognitive demand by standard.

Table 4.15. Cognitive demand of tasks in IMP textbooks sorted by textbook

Textbook	Memorization	Procedures without Connections	Procedures with Connections	Doing Mathematics	TOTAL
Year 1	0	2	6	14	22 (58%)
Year 2	0	3	1	11	15 (39%)
Year 3	0	1	0	0	1 (3%)
Year 4	0	0	0	0	0 (0%)
Total	0 (0%)	6 (16%)	7 (18%)	25 (66%)	38
First 3	0	6	7	25	38 (100%)

4.4.3 Question 3

While less than one third (31%) of the high-level tasks provide opportunities for teacher learning, Year 1 has eight of the ten total opportunities, as shown in Table 4.16. In Year 1, 40% of the high-level tasks contain opportunities to learn in the teachers' guide. Year 2 decreases to a mere 17%. Additionally, there are no opportunities for teacher learning through transparency. All ten tasks that contain opportunities for teacher learning do so through anticipation only. Figure 4.12 is part of the online teacher's guide of IMP. Specifically this part of the teacher's guide relates to Figure 4.11, Making Friends with Standard Deviation. The note in the teacher's guide refers to different ways students may come to understanding that adding a value to each term with change the mean but not the standard deviation.

Discussing and Debriefing the Activity

Focus the discussion on parts c and d of Questions 1 and 2.

Students' explanations of the patterns they observe in Question 1d may take several forms. For example, they may picture the data points on the number line, so that adding the same thing to each data point just moves the points along and hence also moves the mean. Or they may see the change in the mean algebraically (although it's unlikely they will have a full algebraic explanation involving the distributive law).

Students may attribute the lack of change in the standard deviation to the fact that the spread doesn't change when the set of data points is moved along. Or they may recognize that when all the data are changed the same way, the mean also changes, so the spread from the mean remains the same.

The explanations for Question 2d will be similar.

Fathom Dynamic Data™ software can be used to provide a visual demonstration of the effects on a simple data set of adding or multiplying by a constant.

Figure 4.12. Teacher's guide notes for the task in Figure 4.11 which contain anticipation

Table 4.16. Teacher support on high-level probability and statistics tasks in IMP

Textbook	Anticipation	Transparency	Total
Year 1	8/20	0/20	8/20
Year 2	2/12	0/12	2/12
Year 3	0/0	0/0	0/0
Year 4	0/0	0/0	0/0
Total	10/32	0/32	10/32
First 3	10/32	0/32	10/32

4.5 CORE-PLUS MATHEMATICS PROJECT

The Core-Plus Mathematics Proejct (CPMP) curriculum materials were also funded by the NSF and represent a *Standards*-based approach to secondary mathematics education much like IMP. CPMP also represents an integrated approach organized by years like the IMP materials. There are four years of textbooks intended to be implemented in grades 9 through 12.

4.5.1 Question 1

As shown in Table 4.17, the CPMP curriculum provides opportunities for students to engage in all of the probability and statistics content suggestion of CCSSM in the student text except S-CP-5. Some standards were given substantial attention, such as S-ID-6a with 444 items, while others were minimally addressed, like S-CP-6 with three items. No standard was found only in year four, which means students only completing three years of mathematics would not be missing any of the standards not already omitted. In fact, year four only contains 121 of the total 2018 items, which is only 6% of the total for this curriculum. Students can receive 94% of the opportunities this curriculum has to offer in the first three years of its textbooks.

Table 4.17. Number of items in CPMP textbooks aligned to CCSSM probability and statistics

Standard	Year 1	Year 2	Year 3	Year 4	Total	First 3
S-ID-1	181	23	107	2	313 (16%)	311
S-ID-2	169	8	43	0	220 (11%)	220
S-ID-3	36	4	8	0	48 (2%)	48
S-ID-4	0	0	117	0	117 (6%)	117
S-ID-5	0	37	0	0	37 (2%)	37
S-ID-6a	320	90	34	53	497 (25%)	444
S-ID-6b	0	39	1	24	64 (3%)	40
S-ID-6c	36	56	0	15	107 (5%)	92
S-ID-7	36	13	5	0	54 (3%)	54
S-ID-8	0	135	3	0	138 (7%)	138
S-ID-9	1	25	2	0	28 (1%)	28
S-IC-1	0	0	27	0	27 (1%)	27
S-IC-2	0	19	0	0	19 (1%)	19
S-IC-3	0	0	52	0	52 (3%)	52
S-IC-4	0	0	4	0	4 (< 1%)	4
S-IC-5	0	0	46	0	46 (2%)	46
S-IC-6	0	18	4	0	22 (1%)	22
S-CP-1	7	28	0	7	42 (2%)	35
S-CP-2	0	10	14	1	25 (1%)	24
S-CP-3	0	29	0	13	42 (2%)	29
S-CP-4	0	44	0	6	50 (2%)	44
S-CP-5	0	0	0	0	0 (0%)	0
S-CP-6	0	3	0	0	3 (< 1%)	3
S-CP-7	39	10	14	0	63 (3%)	63
Total	825 (41%)	591 (29%)	481 (24%)	121 (6%)	2018	1897 (94%)

As shown in Table 4.18, all but two standards (S-IC-4 and S-CP-5) have at least one instructional task associated with them. However, a majority of the tasks (54 of 96) are associated with three standards (S-ID-1 has 19, S-ID-2 has 12, and S-ID-6a has 23). This leaves 45 tasks to be spread among the 19 remaining standards. Similar to the item analysis, there are no standards only addressed in the fourth year of the curriculum. Only four of the total 96 tasks are found in the fourth year of the curriculum. That means students only completing three years of mathematics in this curriculum would still have the opportunity to engage with nearly 96% of the instructional tasks.

Table 4.18. Number of tasks in CPMP textbooks aligned to CCSSM probability and statistics

Standard	Year 1	Year 2	Year 3	Year 4	Total	First 3
S-ID-1	11	1	7	0	19 (20%)	19
S-ID-2	7	0	5	0	12 (13%)	12
S-ID-3	1	0	0	0	1 (1%)	1
S-ID-4	0	0	7	0	7 (7%)	7
S-ID-5	0	2	0	0	2 (2%)	2
S-ID-6a	16	5	1	1	23 (24%)	22
S-ID-6b	0	1	0	1	2 (2%)	1
S-ID-6c	1	2	0	0	3 (3%)	3
S-ID-7	2	0	0	0	2 (2%)	2
S-ID-8	0	5	0	0	5 (5%)	5
S-ID-9	0	1	0	0	1 (1%)	1
S-IC-1	0	0	1	0	1 (1%)	1
S-IC-2	0	1	0	0	1 (1%)	1
S-IC-3	0	0	2	0	2 (2%)	2
S-IC-4	0	0	0	0	0 (0%)	0
S-IC-5	0	0	1	0	1 (1%)	1
S-IC-6	0	1	0	0	1 (1%)	1
S-CP-1	0	1	0	0	1 (1%)	1
S-CP-2	0	1	1	0	2 (2%)	2
S-CP-3	0	2	0	1	3 (3%)	2
S-CP-4	0	3	0	1	4 (4%)	3
S-CP-5	0	0	0	0	0 (0%)	0
S-CP-6	0	1	0	0	1 (1%)	1
S-CP-7	1	1	0	0	2 (2%)	2
Total	39 (41%)	28 (29%)	25 (26%)	4 (4%)	96	92 (96%)

4.5.2 Question 2

The CPMP curriculum has more high-level items than low-level items much like IMP, as shown in Table 4.19. Of the 2018 total items, 75% of them were coded at a high-level. Of the high-level items, 56% were procedures with connections and 44% were doing mathematics.

Table 4.19. Cognitive demand of items in CPMP sorted by standard

Standard	Memorization	Procedures without Connections	Procedures with Connections	Doing Mathematics	Total
S-ID-1	1	79	162	71	313 (16%)
S-ID-2	0	42	98	78	218 (11%)
S-ID-3	0	3	26	19	48 (2%)
S-ID-4	2	14	79	22	117 (6%)
S-ID-5	0	29	6	2	37 (2%)
S-ID-6a	3	162	191	142	498 (25%)
S-ID-6b	0	19	22	23	64 (3%)
S-ID-6c	2	33	40	32	107 (5%)
S-ID-7	0	6	40	8	54 (3%)
S-ID-8	0	14	47	77	138 (7%)
S-ID-9	1	3	8	16	28 (1%)
S-IC-1	0	0	4	23	27 (1%)
S-IC-2	0	4	5	10	19 (1%)
S-IC-3	0	1	19	32	52 (3%)
S-IC-4	0	3	1	0	4 (< 1%)
S-IC-5	1	2	16	27	46 (2%)
S-IC-6	0	0	3	19	22 (1%)
S-CP-1	0	17	16	9	42 (2%)
S-CP-2	0	9	10	6	25 (1%)
S-CP-3	0	1	19	23	43 (2%)
S-CP-4	0	18	14	18	50 (2%)
S-CP-5	0	0	0	0	0 (0%)
S-CP-6	0	0	0	3	3 (< 1%)
S-CP-7	1	35	20	7	63 (3%)
Total	11 (1%)	494 (24%)	846 (42%)	667 (33%)	2018

Year one of the series has 565 high-level items out of a total of 825 items with is 68%, as shown in Table 4.20. Year two represents an increase in the percentage of high-level items as 450 of 582 items are high-level, which is 77% of the total items. Year 3 shows a continuation of the pattern as the percentage of high-level items increases to 82%, with 394 of the 481 total items being coded as high-level. Finally, 95 of 119 items (80%) were coded as high-level.

Table 4.20. Cognitive demand of items in CPMP textbooks sorted by textbook

Textbook	Memorization	Procedures without Connections	Procedures with Connections	Doing Mathematics	Total
Year 1	1	259	385	180	825 (41%)
Year 2	3	129	185	265	582 (29%)
Year 3	7	80	228	166	481 (24%)
Year 4	0	24	46	49	119 (6%)
Total	11 (1%)	492 (25%)	844 (42%)	660 (33%)	2007
First 3	11	468	798	611	1888 (94%)

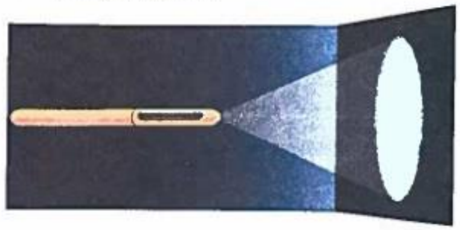
Somewhat different from the item analysis are the codes for tasks as shown in Table 4.21. A majority of the tasks were coded at the level of doing mathematics. This is a direct result of the coding scheme where the highest coded item within any task determined the level of the task. The coding scheme was designed to determine the highest potential of a task, not what the majority of the task was.

Table 4.21. Cognitive demand of tasks in CPMP sorted by standard

Standard	Memorization	Procedures without Connections	Procedures with Connections	Doing Mathematics	Total
S-ID-1	0	2	7	10	19 (20%)
S-ID-2	0	0	4	8	12 (13%)
S-ID-3	0	0	0	1	1 (1%)
S-ID-4	0	1	4	2	7 (7%)
S-ID-5	0	0	1	1	2 (2%)
S-ID-6a	0	2	11	10	23 (24%)
S-ID-6b	0	0	1	1	2 (2%)
S-ID-6c	0	0	0	3	3 (3%)
S-ID-7	0	0	2	0	2 (2%)
S-ID-8	0	0	0	5	5 (5%)
S-ID-9	0	0	0	1	1 (1%)
S-IC-1	0	0	0	1	1 (1%)
S-IC-2	0	0	0	1	1 (1%)
S-IC-3	0	0	0	2	2 (2%)
S-IC-4	0	0	0	0	0 (0%)
S-IC-5	0	0	0	1	1 (1%)
S-IC-6	0	0	0	1	1 (1%)
S-CP-1	0	0	0	1	1 (1%)
S-CP-2	0	0	0	2	2 (2%)
S-CP-3	0	0	1	2	3 (3%)
S-CP-4	0	0	2	2	4 (4%)
S-CP-5	0	0	0	0	0 (0%)
S-CP-6	0	0	0	1	1 (1%)
S-CP-7	0	0	0	2	2 (2%)
Total	0 (0%)	5 (5%)	33 (34%)	58 (60%)	96

For example, Figure 4.13 shows a task that contains nine items. Of these nine items, five of them were in alignment with the probability and statistics recommendations of CCSSM. Specifically, items 2bi, 2bii, 2biii, 2e, and 2fii all aligned with standard S-ID-6a as shown in Table 4.22. Of these five items, four were coded at the level of procedures without connections (2bi, 2bii, 2biii, and 2e) and one of them was coded as doing mathematics (2fii). Even though there are more codes for procedures without connections, the highest potential of the task is doing mathematics. Therefore, the task was coded as doing mathematics.

2 You could test your ideas about the (*distance, intensity*) relationship by collecting data from an experiment. But you can also get good ideas by mathematical reasoning alone. Consider what would happen if you were to enter a dark room and shine a small flashlight directly at a flat surface like a wall. The flashlight will create a circle of light on the wall.



a. Complete entries in the following table that contains measurements of light circle diameter for one flashlight that has been held at several distances from a wall. Distance and diameter measurements are in feet. Express the area in terms of π.

Light Circle Measurements

Distance from Light Source, x	1	2	3	4	5	6
Diameter of Light Circle, d	2	4	6	8	10	12
Radius of Light Circle, r						
Area of Light Circle, A						

b. Write rules that show:

i. diameter of light circle as a function of distance from the light source.

ii. radius of light circle as a function of distance from the light source.

iii. area of light circle as a function of distance from the light source.

c. Describe the relationships of the geometric variables diameter, radius, and area by completing sentences like this: "The variable ______ is proportional to ______, with constant of proportionality ______."

d. Light energy is measured in a unit called *lumens*. The intensity of light is measured in lumens per unit of area. As the light circle of a flashlight or lamp increases in size, the intensity of light decreases.

To explore how that decrease in light intensity is related to distance from source to target, suppose that the flashlight that gave (*distance, diameter*) values in Part a produces 160 lumens of light energy. Use the area data from Part a to complete this table relating light intensity I to distance x.

Light Intensity Measurements

Distance from Light, x	1	2	3	4	5
Area of Light Circle, A	π	4π			
Light Intensity, I	$\frac{160}{\pi}$	$\frac{160}{4\pi}$			

e. Write a rule that shows light intensity I as a function of distance x from source to receiving surface.

f. Study the graph of the light intensity function in Part e.

i. Which of the graph shapes in Problem 1 seems to best model the pattern of change in light intensity as distance from source to receiver increases?

ii. Explain in words what the pattern of change shown by the light intensity function and its graph tells about the effective range of a

Figure 4.13. Doing mathematics task in CPMP containing items below doing mathematics

Table 4.22. Cognitive demand of items in Figure 4.13

Series	Textbook	Unit	Lesson	Problem	Standard	Cognitive Demand
CPMP	2A	1	1	Inv 2 – 2a	None	
CPMP	2A	1	1	Inv 2 – 2bi	S-ID-6a	PnC
CPMP	2A	1	1	Inv 2 – 2bii	S-ID-6a	PnC
CPMP	2A	1	1	Inv 2 – 2biii	S-ID-6a	PnC
CPMP	2A	1	1	Inv 2 – 2c	None	
CPMP	2A	1	1	Inv 2 – 2d	None	
CPMP	2A	1	1	Inv 2 – 2e	S-ID-6a	PnC
CPMP	2A	1	1	Inv 2 – 2fi	None	
CPMP	2A	1	1	Inv 2 – 2fii	S-ID-6a	DM

As shown in Table 4.23, examining the tasks found in each textbook reveals a design that emphasizes high-level tasks. The type of high-level tasks differs from year one to the other years. Year 1 represents a balance between the two high-level task types with eighteen tasks considered procedures with connections and eighteen tasks considered doing mathematics. However, the other years represent a shift to an emphasis on doing mathematics tasks. Finally, it may be worth noting again that a student only completing three of the four years of the curriculum would not miss a substantial number of opportunities in comparison from the fourth year when compared to the previous three.

Table 4.23. Cognitive demand of tasks in CPMP textbooks sorted by textbook

Textbook	Memorization	Procedures without Connections	Procedures with Connections	Doing Mathematics	Total
Year 1	0	3	18	18	39 (41%)
Year 2	0	0	6	22	28 (29%)
Year 3	0	2	8	15	25 (26%)
Year 4	0	0	1	3	4 (4%)
Total	0 (0%)	5 (5%)	33 (34%)	58 (60%)	96
First 3	0	5	32	55	92 (96%)

4.5.3 Question 3

As shown in Table 4.24, there is minimal support for teachers enacting the curriculum. Of the 91 high-level tasks, only 13 of them (14%) provide opportunities for teacher learning. There appears to be a greater emphasis on anticipating student thinking than there is on providing transparency since 11 of the 13 opportunities for teacher learning are related to anticipation.

Figure 4.14 is part of a task where students are asked to relate a table or a graph to the Law of Large Numbers. The teacher's edition of the textbook provides an opportunity for anticipation as shown in Figure 4.15. Figure 4.15 anticipates two concepts that may cause conflict with students when it comes to the law of large numbers. Students may understand that the proportion of heads tends to get closer to the theoretical value of 0.5, but they may find difficulty in recognizing that the difference between actual value of heads and the expected value of heads typically increases.

f. Explain how your completed graph and table illustrate the Law of Large Numbers.

Figure 4.14. Task supported via anticipation in CPMP *Course 1* (Hirsch et al., p. 556, 2015)

f. The Law of Large Numbers says that as the number of trials increases, the estimated probability tends to get closer to the theoretical probability. The graph illustrates this because as the number of flips increases, the proportion of heads tends to get closer to 0.5.

POSSIBLE MISCONCEPTION Although the proportion of heads is converging to 0.5, the frequency of heads is diverging from the expected frequency. In the table in Part e, the expected number of heads in 10 flips is 5. The actual number is 4, for a difference of 1. After 50 flips, the proportion of heads is closer to 0.5 than for 10 flips, but the number of heads, 22, is 3 away from the expected number of heads, 25. Notice that while 3 is greater in magnitude than 1, it represents a smaller percentage of 50 than 1 does of 10.

This is an important idea for students to learn—that as you are flipping a coin, for example, the percentage of heads tends to get closer and closer to 50% as the number of flips increases, while the number of heads tends to get further and further from half the number of tosses. If students do not understand this, they will believe that the coin must balance out the numbers of heads and tails in the future by changing the probability that it will be a head. This idea comes up again in Reflections Task 17.

Figure 4.15. Support via anticipation in CPMP *Course 1* (Hirsch et al., p. 556T, 2015)

Figure 4.16 is a task from CPMP where the teacher's edition provides an opportunity for transparency. Figure 4.17 is the part of the teacher's edition that corresponds to student edition task shown in Figure 4.16. These figures demonstrate what typical opportunities for transparency look like in CPMP textbooks. As shown in Figure 4.17, the underlying focus of the task, use an informal understanding of conditional probability, is made explicit to the teacher. Additionally, the teacher is provided with an explanation of how the various methods of completing the task could be emphasized depending on the prior experience of the students in the class. This allows the teacher to adapt the task as needed without losing the conceptual understanding that the task intends to develop.

5 Suppose again the names of six boys and four girls are written on individual slips of paper and placed in a hat. This time you draw two names *without* replacement. That is, you draw one name, you do *not* return the slip of paper to the hat, then you draw a second name.

a. Find the probability that the first name drawn is a girl's name and the second name is a boy's name.

b. Explain why the answer to Part a is *not* $\frac{4}{10} \times \frac{6}{10}$.

c. Show how you can find the probability in Part a using the Multiplication Principle of Counting and the definition of probability given at the beginning of this investigation.

d. To find the probability in Part a, you can also use the **General Multiplication Rule** for any two events:

If A and B are events, then $P(A \text{ and } B) = P(A) \times P(B \mid A)$.

The notation $P(B \mid A)$ is read "probability of B given A." This means you find the probability of B assuming that you know A happened. Show how to use the General Multiplication Rule to find:

P(*girl's name on first draw* and *boy's name on second draw*).

Figure 4.16. Task supported via transparency in CPMP *Course 4* (Hirsch et al., p. 579, 2015)

5 **a.** Students may use various methods.

- Using the General Multiplication Rule (which students may use only implicitly until they get to Part d):
  The probability is $\frac{4}{10} \times \frac{6}{9} = \frac{24}{90} = 0.2\overline{6}$.
- Using the Multiplication Principle of Counting:
  P(*girl's name on first draw* and *boy's name on second draw*) =
  $\frac{\textit{number of outcomes corresponding to the event}}{\textit{total number of outcomes}} = \frac{4 \times 6}{10 \times 9} = 0.2\overline{6}$.
  (See the Mathematics Note in Problem 4 Part a.)
- Using permutations and the Multiplication Principle of Counting:
  $\frac{\textit{number of outcomes corresponding to the event}}{\textit{total number of outcomes}} = \frac{4 \times 6}{P(10, 2)} = 0.2\overline{6}$.

b. Informal response: The second factor is $\frac{6}{9}$ not $\frac{6}{10}$ because there are 6 boys but only 9 slips of paper left in the hat.

Formal response: You cannot use the Multiplication Rule for independent events because the events are not independent. The events in this analysis are "girl's name on first draw" and "boy's name on second draw." These events are not independent because the first slip of paper is not returned to the hat before the second slip is drawn; therefore, the result of the first draw changes the probability for the second draw.

c. Using the definition of probability given at the beginning of this investigation, you can compute the probability by counting outcomes. An outcome in this situation is a possible result of drawing two slips of paper when the first is drawn without replacement. That is, an outcome is a *sequence* of two names drawn. Each outcome is equally likely since each slip of paper is just as likely to be drawn as any other. Since the first slip of paper is not replaced before drawing the second, the total number of possible outcomes is $10 \times 9 = 90$. (Students may see this as *P*(10, 2).) The number of outcomes corresponding to the event of "girl's name on first draw and boy's name on second draw" is $4 \times 6 = 24$. Thus, *P*(*girl's name on first draw* and *boy's name on second draw*) = $\frac{\textit{number of outcomes corresponding to the event}}{\textit{total number of outcomes}} = \frac{24}{90} = 0.2\overline{6}$.

d. *P*(*girl's name on first draw* and *boy's name on second draw*) = *P*(*girl's name on first draw*) × *P*(*boy's name on second draw* | *girl's name on first draw*) = $\frac{4}{10} \times \frac{6}{9}$.

INSTRUCTIONAL NOTE

Problem 5 references conditional probability. The context allows students to understand and apply conditional probability informally without formal development. Thus, depending on your students' experience with conditional probability, you may emphasize a formal approach to this idea or simply use an informal approach restricted to the specific contexts in this investigation.

Figure 4.17. Support via transparency in CPMP *Course 4* (Hirsch et al., p. 579T, 2015)

Table 4.24. Teacher support on high-level probability and statistics tasks in CPMP

Textbook	Anticipation	Transparency	Total
Year 1	5/36	1/36	6/36
Year 2	5/28	0/28	5/28
Year 3	0/23	0/23	0/23
Year 4	1/4	¼	2/4
Total	11/91	2/91	13/91

4.6 COMPARISONS BETWEEN CURRICULUM MATERIALS

Looking at each set of curriculum materials individually has many benefits, but these results should also be examined in comparison with one another. This examination will once again progress through each of the research questions using the established relevant results to compare the three sets of curriculum materials.

4.6.1 Question 1

The CPMP curriculum provides the most opportunities for students to engage in all of the probability and statistics content suggestion of CCSSM based on the number of items in the student text as shown in Table 4.25. In many cases where CPMP lacks items for a specific standard, the other two curricula do as well. For example, S-CP-5 is not addressed by either CPMP or IMP and GM has only one item associated with S-CP-5. However, GM is lacking in S-ID-5 where the other two are not. IMP has four standards completely unaddressed (S-ID-6b, S-ID-8, S-CP-3, and S-CP-7) that the other two curricula address in some manner.

When looking at only the first three years of each curriculum, it is interesting to note that even though IMP has a much lower number of items than GM (191 compared to 994) as shown in Table 4.25, the IMP curriculum has more tasks than does the GM curriculum (42 compared to 36) as shown in Table 4.26. This is the result of the GM textbook providing enrichment and review problems at the end of each section where the IMP curriculum does not as discussed previously in this chapter. Refer to Figure 4.3 and Figure 4.4 from GM and Figure 4.5 from IMP from the beginning of this chapter for visual representation of the differences in the two curricula.

Table 4.25. Items aligned with CCSSM probability and statistics in each curriculum

Standard	Glencoe Mathematics Total	Glencoe Mathematics First 3	Interactive Mathematics Program Total	Interactive Mathematics Program First 3	Core-Plus Mathematics Project Total	Core-Plus Mathematics Project First 3
S-ID-1	161	148	21	21	313	311
S-ID-2	138	99	14	14	220	220
S-ID-3	27	26	14	14	48	48
S-ID-4	257	80	10	10	117	117
S-ID-5	8	8	20	20	37	37
S-ID-6a	266	96	20	20	497	444
S-ID-6b	17	9	0	0	64	40
S-ID-6c	86	59	3	3	107	92
S-ID-7	36	27	8	8	54	54
S-ID-8	81	55	0	0	138	138
S-ID-9	21	21	1	1	28	28
S-IC-1	31	14	6	6	27	27
S-IC-2	65	2	8	8	19	19
S-IC-3	70	70	1	1	52	52
S-IC-4	65	28	2	2	4	4
S-IC-5	4	2	14	14	46	46
S-IC-6	11	11	6	6	22	22
S-CP-1	51	51	28	28	42	35
S-CP-2	31	31	13	13	25	24
S-CP-3	56	56	0	0	42	29
S-CP-4	17	17	1	1	50	44
S-CP-5	1	1	0	0	0	0
S-CP-6	2	2	1	1	3	3
S-CP-7	82	81	0	0	63	63
Total	1584	994	191	191	2018	1897

As shown in Table 4.26, when examining the textbooks by task, the CPMP curriculum provides the most opportunities for students to engage in all of the probability and statistics content suggestion of CCSSM. When CPMP lacks tasks for a specific standard, the other two curricula do as well. The only exception is in standard S-IC-4 where the GM textbooks have three tasks and the IMP textbooks have one task. However, there are multiple examples of the other two textbook series not having a task for a specified textbook series but CPMP having at least one. There is even one standard, S-CP-4, where both GM and IMP do not have a task corresponding to the standard, but CPMP has four.

Table 4.26. Tasks aligned with CCSSM probability and statistics in each curriculum

Standard	Glencoe Mathematics Total	Glencoe Mathematics First 3	Interactive Mathematics Program Total	Interactive Mathematics Program First 3	Core-Plus Mathematics Project Total	Core-Plus Mathematics Project First 3
S-ID-1	8	4	2	2	19	19
S-ID-2	6	5	2	2	12	12
S-ID-3	2	2	2	2	1	1
S-ID-4	3	1	2	2	7	7
S-ID-5	0	0	3	3	2	2
S-ID-6a	6	3	5	5	23	22
S-ID-6b	2	1	0	0	2	1
S-ID-6c	4	2	1	1	3	3
S-ID-7	2	1	1	1	2	2
S-ID-8	4	2	0	0	5	5
S-ID-9	0	0	1	1	1	1
S-IC-1	2	1	3	3	1	1
S-IC-2	3	0	4	4	1	1
S-IC-3	3	3	1	1	2	2
S-IC-4	3	1	1	1	0	0
S-IC-5	1	0	4	4	1	1
S-IC-6	1	1	3	3	1	1
S-CP-1	2	2	3	3	1	1
S-CP-2	2	2	3	3	2	2
S-CP-3	2	2	0	0	3	2
S-CP-4	0	0	0	0	4	3
S-CP-5	0	0	0	0	0	0
S-CP-6	0	0	1	1	1	1
S-CP-7	3	3	0	0	2	2
Total	59	36	42	42	96	92

There are 24 individual standards related to probability and statistics in CCSSM. As shown in Figure 4.18, The GM series addressed all 24 standards with at least one item. This number is accurate when both the entire curriculum and only the first three years of the curriculum are considered. However, only 19 of the 24 probability and statistics standards are addressed by tasks over all four years of the GM curriculum and even less, 17, are addressed by the first three years of the curriculum. Both the entire CPMP series and the first three years of CPMP address 23 of the 24 probability and statistics standards with at least one item. The CPMP series addresses 22 of the 24 with a task both in the entire series and in the first three years. Finally, IMP addresses 19 of 24 probability and statistics standards with at least one item while addressing 18 of 24 with at least one task in the entire series. These numbers stay the same when only the first three years of the curriculum are considered.

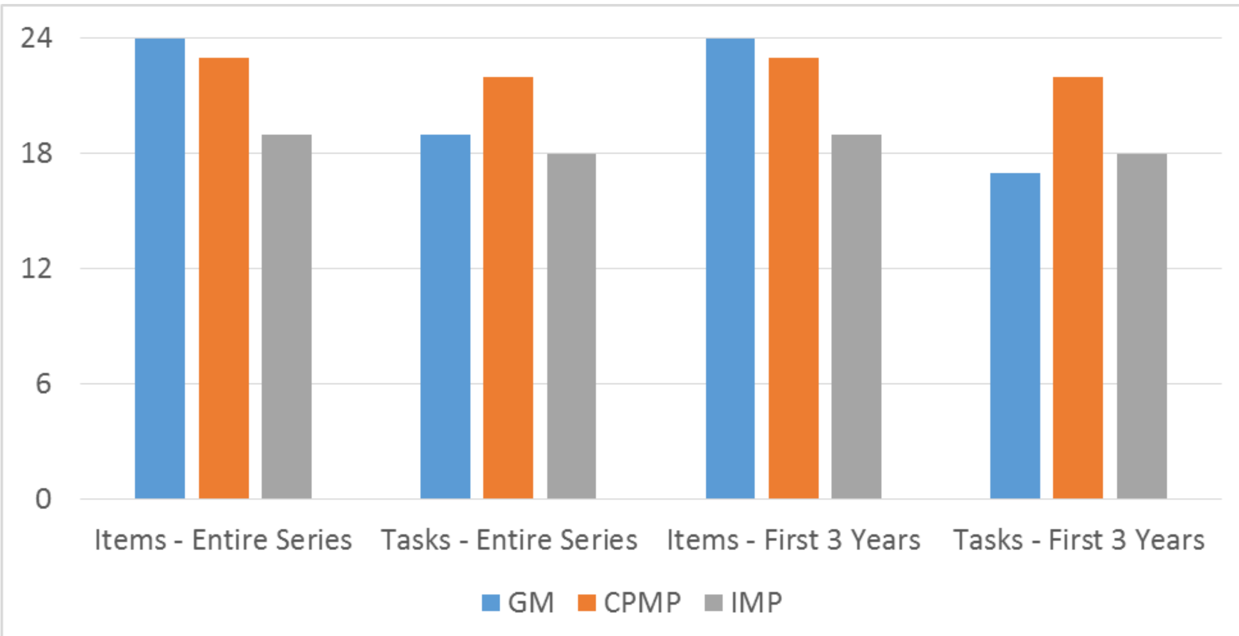


Figure 4.18. Number of CCSSM probability and statistics standards in each curriculum

Figure 4.19 shows the total number of items found in each curriculum that address any probability and statistics standard from CCSSM. The GM series has 1584 items in the entire series and 994 items in the first three year of the curriculum addressing probability and statistics standards from CCSSM. The CPMP series has 2018 items addressing probability and statistics standards from CCSSM in the entire curriculum and 1897 items in the first three years. Finally, IMP has 191 items in both the entire series and in the first three years since IMP has no probability and statistics content in alignment with CCSSM in the fourth year of the curriculum.

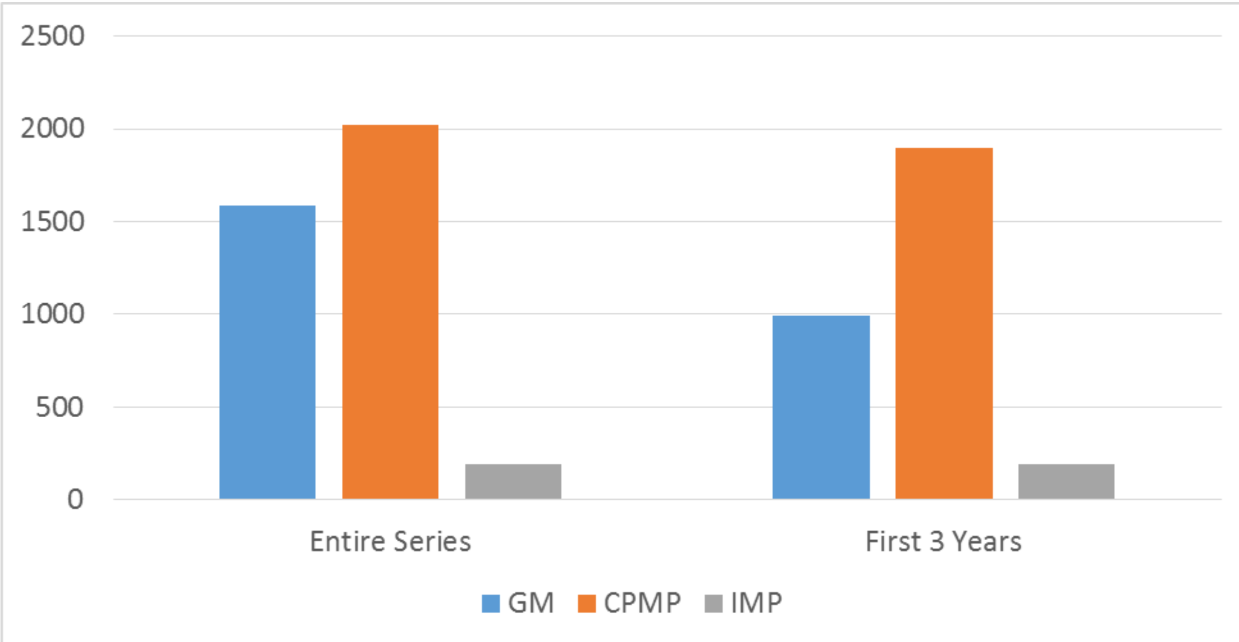


Figure 4.19. Number of items addressing CCSSM probability and statistics in each series

Figure 4.20 shows the total number of tasks found in each curriculum that address any probability and statistics standard from CCSSM. The GM series has 59 tasks in the entire series and 36 tasks in the first three year of the curriculum addressing probability and statistics standards from CCSSM. The CPMP series has 96 tasks addressing probability and statistics standards from CCSSM in the entire curriculum and 92 tasks in the first three years. Finally, IMP has 42 tasks in both the entire series and in the first three years.

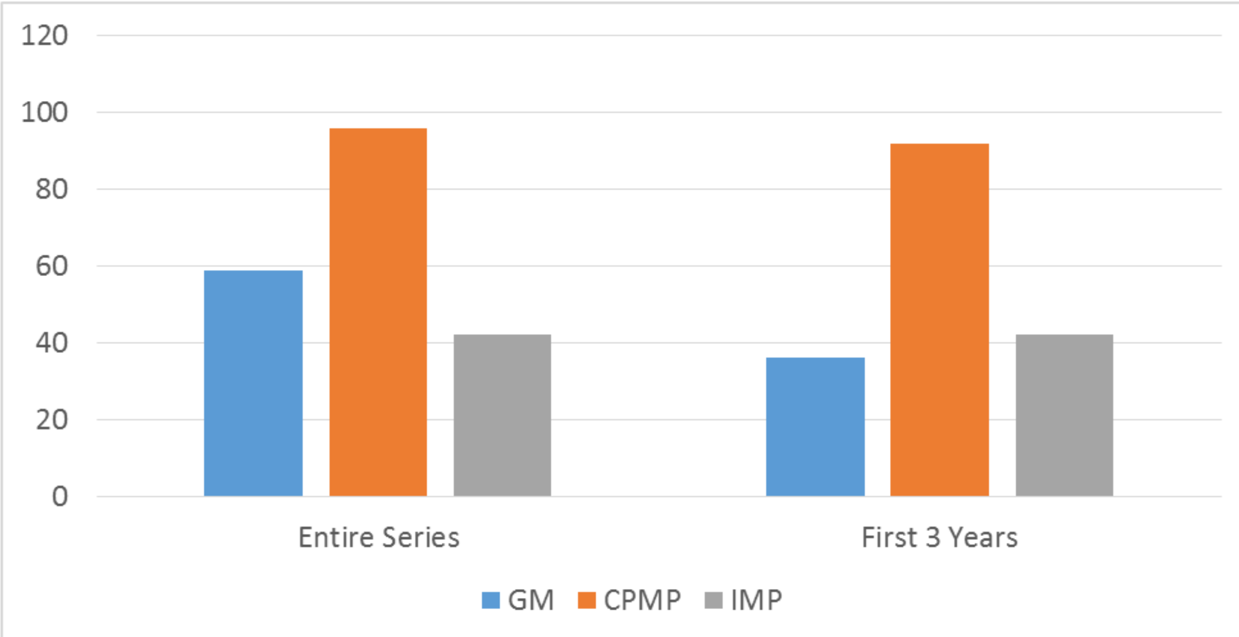


Figure 4.20. Number of tasks addressing CCSSM probability and statistics in each series

4.6.2 Question 2

As shown in Table 4.27, the GM curriculum is dominated by low-level items. Nearly 83% of the items in the GM curriculum are low-level. Contrarily, the IMP curriculum and CPMP curriculum have mostly high-level tasks with 71% of the IMP items and 75% of the CPMP items being high-level. In order to contrast the IMP and CPMP curriculums, a closer examination of the high-level tasks is necessary. In the IMP curriculum, 56% of the high-level tasks are doing mathematics. On the other hand, 56% of the high-level tasks in CPMP are procedures with connection.

Table 4.27. Cognitive demand of probability and statistics items in each curriculum

Textbook Series	Memorization	Procedures without Connections	Procedures with Connections	Doing Mathematics	Total
Glencoe Mathematics Total	18	1259	212	56	1545
Glencoe Mathematics First 3	9	799	89	22	919
Interactive Mathematics Program Total	1	55	59	76	191
Interactive Mathematics Program First 3	1	55	59	76	191
Core-Plus Mathematics Project Total	11	492	844	660	2007
Core-Plus Mathematics Project First 3	11	468	798	611	1888

Figure 4.21 is a graphical representation of the same data found in Table 4.27. Figure 4.21 clearly demonstrates the previously discussed tendency toward procedures without connections items in the GM series. The CPMP series has some low-level items, but there are more of each type of high-level item (procedures with connections and doing mathematics) than there are low-level items combined. The number of items in IMP increases with the cognitive demand. In other words, the higher the level of cognitive demand, the more items there are.

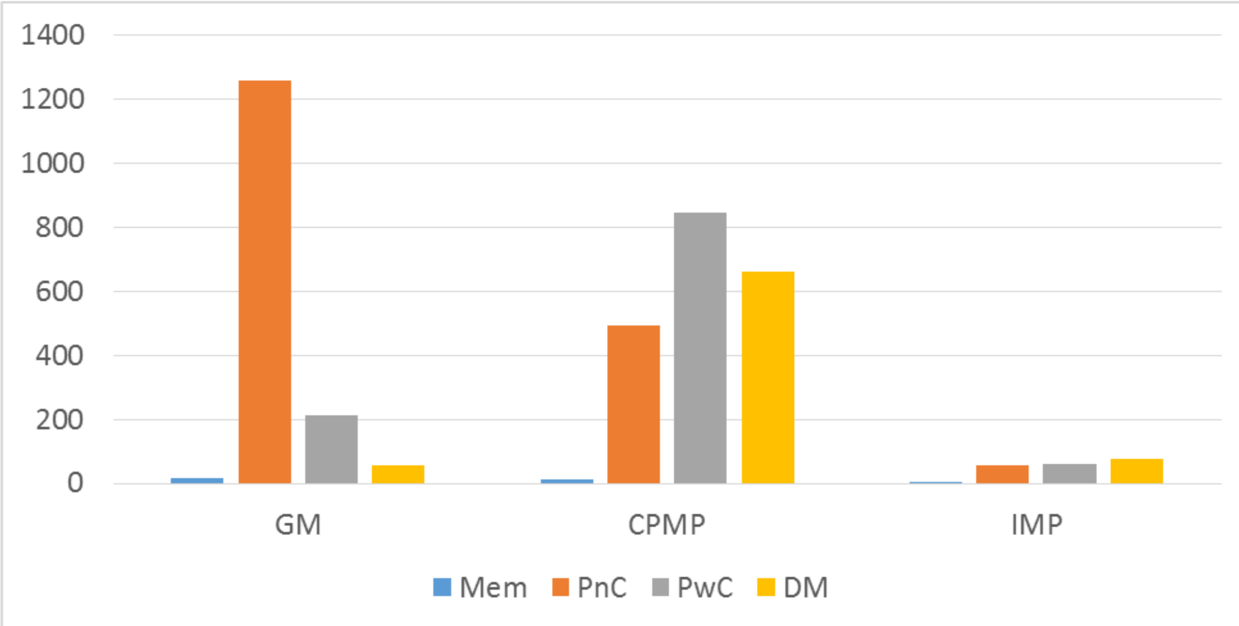


Figure 4.21. Cognitive demand of probability and statistics items in each curriculum

Much like the item analysis, the task analysis shown in Table 4.28 makes the traditional versus *Standards*-based designs visible with results. Only 34% of the tasks in the GM curriculum are high-level. However, both the IMP and CPMP curriculums have more than 60% of their tasks coded as being high-level.

Table 4.28. Cognitive demand of probability and statistics tasks in each curriculum

Textbook Series	Memorization	Procedures without Connections	Procedures with Connections	Doing Mathematics	TOTAL
Glencoe Mathematics Total	0	39	15	5	59
Glencoe Mathematics First 3	0	32	4	0	36
Interactive Mathematics Program Total	0	6	7	25	38
Interactive Mathematics Program First 3	0	6	7	25	38
Core-Plus Mathematics Project Total	0	5	33	58	96
Core-Plus Mathematics Project First 3	0	5	32	55	92

Once again, Figure 4.22 represents the data from Table 4.28 graphically. As shown in Figure 4.21, The GM series contains mostly procedures without connections tasks, has a few procedures with connections tasks, and even less doing mathematics tasks. The other two textbook series, CPMP and IMP, both have mostly doing mathematics tasks with some procedures with connections and procedures without connections tasks as well.

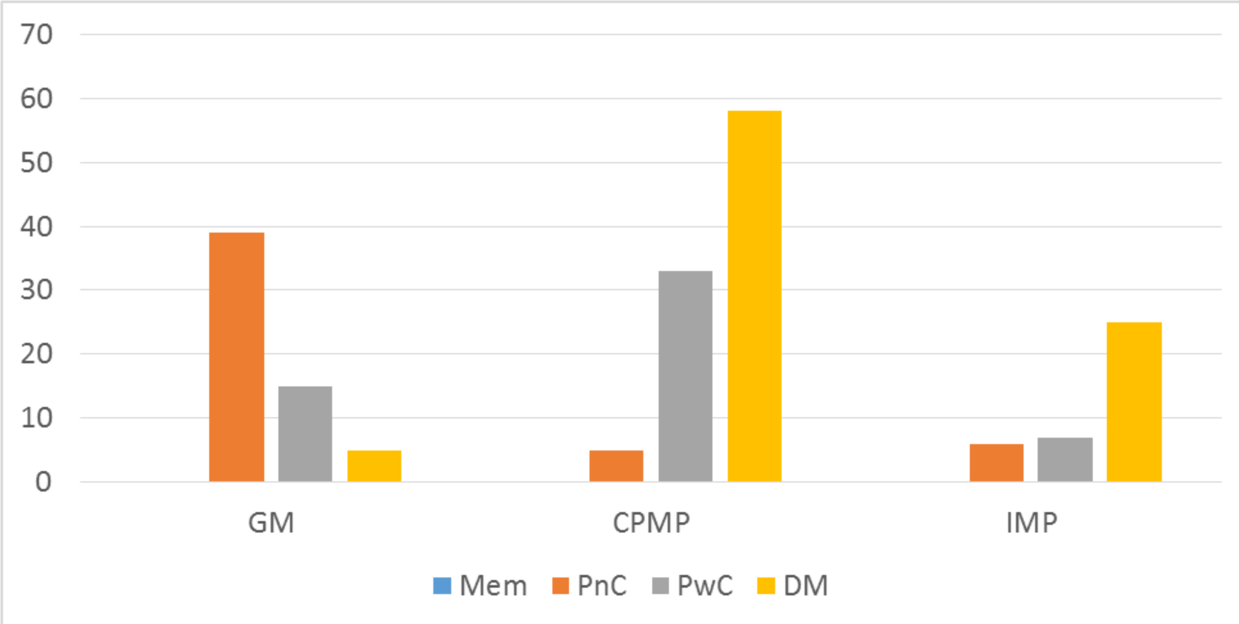


Figure 4.22. Cognitive demand of probability and statistics tasks in each curriculum

4.6.3 Question 3

As shown in Table 4.29, the IMP curriculum materials clearly provide the most support for teacher learning per high-level task. The GM curriculum does not provide more than one opportunity for teacher learning even though there are twenty tasks that are highly cognitively demanding for the students and thus demanding for the teacher to implement well. The ten opportunities for teacher learning in the IMP curriculum are close to the thirteen in the CPMP curriculum. However, ten opportunities out of 32 tasks means 31% of the high-level tasks in the IMP curriculum has opportunities for teacher learning. Thirteen opportunities out of 91 tasks means only 14% of the high-level tasks in the CPMP curriculum provide opportunities for teacher learning.

Table 4.29. Teacher support on high-level probability and statistics tasks in each curriculum

Textbook Series	Anticipation	Transparency	Total
Glencoe Mathematics Total	1/20	0/20	1/20
Glencoe Mathematics First 3	0/4	0/4	0/4
Interactive Mathematics Program Total	10/32	0/32	10/32
Interactive Mathematics Program First 3	10/32	0/32	10/32
Core-Plus Mathematics Project Total	11/91	2/91	13/91
Core-Plus Mathematics Project First 3	10/87	1/87	11/87

As shown in Table 4.23, none of the curricula provides teacher support in the form of opportunities for teacher learning through anticipation or transparency on most of the high-level tasks found in them. The CPMP curriculum has the most opportunities, but CPMP also has the highest number of high-level tasks. The IMP curriculum has nearly the same number of opportunities, 10 compared to 13, but only has 32 high-level tasks that would benefit from such opportunities compared to 91 high-level tasks in CPMP. The GM curriculum has both the fewest number of opportunities and the fewest number of high-level tasks among the three curricula.

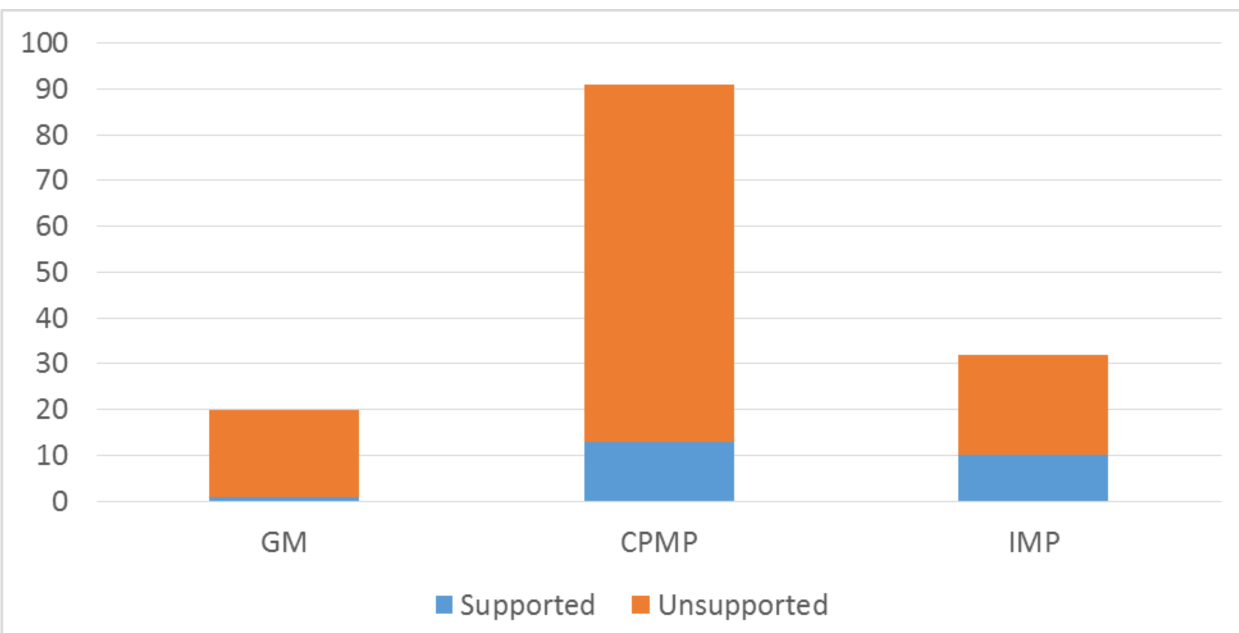


Figure 4.23. Supported and unsupported high-level tasks in each curriculum

4.7 CHAPTER 4 SUMMARY

The key results of the study can be summarized by the following:

1) CPMP has the most items (2018) and tasks (96) that address the probability and statistics standards found in CCSSM.
2) GM addresses the highest number of the 24 probability and statistics standards found in CCSSM via items (24/24) while CPMP addresses the highest number via tasks (22/24).
3) The majority of items found in the CPMP and IMP textbooks were of a high-level cognitive demand (75% high-level in CPMP and 71% high-level in IMP) while only 17% of the items in GM were high-level.
4) The majority of the tasks found in the CPMP and IMP textbooks were of a high-level cognitive demand (95% high-level in CPMP and 86% high-level in IMP) while only 34% of the tasks in IMP were high-level.

Less than one third of the high-level tasks in each of the three curricula provided opportunities for teacher learning (14% in CPMP, 31% in IMP, and 5% in GM)

5.0 DISCUSSION

This chapter contains a discussion of the results presented in chapter 4 and how the results provide insights regarding which secondary mathematics curriculum materials have the potential to support teacher and students learning of probability and statistics. Specifically, the purpose of this study was to analyze current secondary mathematics textbooks to determine the extent to which those textbooks have the potential to prepare students and teachers to meet the demand of the CCSS related to statistics and probability. Rather than repeating the results presented in chapter 4, here the results are used to consider which textbook would be the optimal choice for teaching probability and statistics. The extent to which this potential exists can be examined in multiple ways. The following questions will frame the discussion that follows:

1) Which textbook series provides the most comprehensive coverage of the CCSSM probability and statistics standards for content?
2) Which textbook series provides the most comprehensive coverage of the Standards for Mathematical Practice by providing high-level opportunities for students to engage in?
3) Which textbook series provides the most support for the teachers enacting the probability and statistics content?

The chapter begins by defining comprehensive coverage. Next, there is a discussion of each textbook series' inferred philosophy of how students learn. Then there is a discussion of the three framing questions. This will be followed in turn by discussion of: the limitations of the results;

the implications of the study; and the potential contributions of the study. Finally, concluding remarks including suggestions for future research are made.

5.1 DEFINING COMPREHENSIVE COVERAGE

It is important to notice the idea of comprehensive coverage showing up twice in these framing questions. Coverage is typically associated with content. However, the manner in which the content is addressed is as important as the content itself. If the demands of CCSSM are to be met, the content must be addressed in such a manner as to elicit the kind of thinking that would be required for one to engage in the Standards for Mathematical Practice that accompany the content standards of CCSSM. It has previously been suggested that high-level tasks will be required to elicit this type of thinking. Therefore, the concept of coverage must involve both content and cognitive demand.

Figures 5.1 and 5.2 have been provided to make this argument clear. The tasks in both Figure 5.1 and 5.2 could be considered as covering the content of CCSSM Standard S-ID-1. However, the manner in which that coverage occurs is very different. The task in Figure 5.1 instructs students to construct two specific graphs, a histogram and a cumulative frequency histogram with specified values, from a data set that is provided by the textbook. While this task does involve representing data with plots on a real number line, the task is of low cognitive demand since they involve following a specified procedure without any connections being made. The task in Figure 5.2 also asks students to create a graph. However, students generate the data themselves and are not instructed on the type of graph that should be drawn. Additionally, students are asked to write a paragraph discussing their observations and summarizing the results. Finally, students are asked to make connections between this task and a task they have previously completed. This task would be at the level of doing mathematics.

2. PLAYS The frequency table at the right shows the ages of people attending a high school play.

a. Make a histogram to display the data.

b. Make a cumulative frequency histogram showing the number of people attending who were less than 20, 40, 60, or 80 years old.

Age	Tally	Frequency
0–19	卌 卌 卌 卌 卌 卌 卌 卌 卌 II	47
20–39	卌 卌 卌 卌 卌 卌 卌 卌 III	43
40–59	卌 卌 卌 卌 卌 卌 I	31
60–79	卌 III	8

Figure 5.1. Procedures without connections items in GM *Algebra 1* (Carter et al., p. 45, 2010)

Activity

Rollin', Rollin', Rollin'

Roll a pair of dice 50 times. With each roll, find the sum of the dice. Keep a record of your sums in an organized way.

1. Draw a **graph** of the data you gathered.
2. Write a paragraph about your results. You should summarize your observations about the data and discuss why the results came out the way they did.
3. What new thoughts does this experiment give you about how to play the counters game?

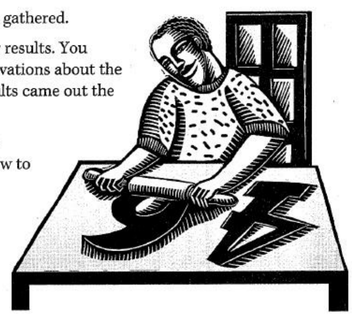


Figure 5.2. Doing mathematics items in IMP *Year 1* (Fendel et al., p. 104, 2009)

Given these two examples of tasks covering the same content in different ways, now consider them in the context of the Standards for Mathematical Practice from CCSSM (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010):

1) Make sense of problems and persevere in solving them
2) Reason abstractly and quantiatatively
3) Construct viable arguments and critique the reasoning of others

4) Model with mathematics
5) Use appropriate tools strategically
6) Attend to precision
7) Look for and make use of structure
8) Look for and express regularity in repeated reasoning

The doing mathematics task, Figure 5.2, provides students with an opportunity to engage in multiple Standards for Mathematical Practice. Because students must generate their own data set and are not given a specific type of graph to create, students are required to make sense of the problem and persevere in solving it, have the opportunity to look for and make use of structure, and may look for an express regularity in repeated reasoning. Requiring students to summarize their observations and discuss the results encourages the students to reason abstractly and quantitatively as well as look for and make use of structure. Finally, having students reflect on a previous task, the task shown in Figure 5.2 promotes students constructing viable arguments and critiquing the reasoning of others by having them reflect on a previously constructed argument and consider it in light of the new task.

The procedures without connections task, Figure 5.1, do not provide students with opportunity to engage in the Standards for Mathematical Practice because students are given the data set and provided specific instructions on what to do. By not allowing students to generate their own data or make a decision about what type of graph to create, students are not being engaged in the Standards for Mathematical Practice. Additionally, once the graphs are created, students do not do anything with them. There is no opportunity to use the graph or data for any high-level thinking.

5.2 INFERRED PHILOSOPHY OF HOW STUDENTS LEARN

Each textbook series has its own philosophy regarding how students learn that can be inferred from the examination of the materials. Glencoe Mathematics (GM) is a traditional textbook series where students are presented a few examples and then provided with many items to practice that mirror the examples and review previous lessons from other sections, chapters, or even textbooks. The GM philosophy appears to be one where students look at specific, detailed algorithms presented in the textbook or by the teacher who follows the steps found in the textbook and then uses the algorithm repeatedly on similar problems. Once the procedures has been observed and then mimicked, students are then provided opportunities to apply the procedure to more challenging items and review previously learned procedures. Thus, the inferred philosophy is that students learn best when provided an algorithm that they can repeat until it is locked into their memories. To ensure algorithms are not forgotten due to lack of use, items requiring their use may show up in subsequent sections for further repetition.

Interactive Mathematics Program (IMP) is a *Standards*-based curriculum where students are given activities to work through that are intended to develop student understanding of mathematics while students complete them. Rather than being provided algorithms, the students are encouraged to work through problems independently or in small groups. The teacher’s role is to provide support as opposed to direct instruction. The philosophy is that the students will develop meaning as they work through each of the activities. The IMP curriculum does not provide items for students to practice or review what they have learned. The philosophy instead is that students will have developed their own meaning and understanding. Since students have developed understandings on their own, there is no need for practice and review.

Core-Plus Mathematics Project (CPMP) is a blend of the two approaches. CPMP provides students with activities to work through, that they term investigations, which are intended to develop student understanding much like IMP. However, CPMP also provides exercises for student practice and review like GM. Supporters of the CPMP philosophy would assert that deep, meaningful understandings are developed through the investigations with the review and practice problems available to allow for repetition if needed. The potential impact of these inferred philosophies regarding how students learn will be part of the discussion throughout this chapter.

5.3 COMPREHENSIVE COVERAGE OF CONTENT STANDARDS

When considering the coverage of content standards, multiple grain sizes of analysis are possible. At the most detailed level of analysis, one could consider individual items in relationship to individual standards as was presented in chapter 4 (see Table 4.25). However, both the examination of individual standards and looking at items may be too detailed to capture the big picture of each curriculum. Rather than being bogged down by the detail of individual standards and items, a big picture approach using clusters of standards and instructional tasks will be used for the discussion of content coverage.

The 24 probability and statistics standards from CCSSM are grouped into three clusters by CCSSM. These clusters are Interpreting Categorical and Quantitative Data (S-ID), Making Inferences and Justifying Conclusions (S-IC), and Conditional Probability and the Rules of Probability (S-CP). These clusters represent the big ideas of statistics that students should learn while in high school. Looking at clusters instead of individual standards is justifiable because the absence of an individual standard may not be a glaring omission based on research in probability and statistics. In chapter 2, an extensive review of literature established a strong overall relationship between scholarly research and CCSSM overall. Additionally, the GAISE Report was also examined to determine the strength of the relationship between the GAISE Report and CCSSM, which was once again strong. However, when one considers individual standards, these two relationships were did not always exist. For example, S-CP-6 from CCSSM was not explicitly addressed in the GAISE report and no specific reference from the review of literature was found in relationship to this standard (see Table 2.3 for more). Given the lack of agreement between the GAISE Report, scholarly research, and CCSSM on an individual level, it may be reasonable to examine a larger grain size for a textbook analysis of coverage.

The notion of using instructional tasks instead of individual items is justified for multiple reasons. First, items that do not appear as part of the instructional section of the textbook are less likely to be engaged in by students than those that appear as part of an instructional task. While it was important to consider all parts of the textbook in the analysis, such as review and enrichment problems, students are going to be given opportunities to engage initially through instruction. Secondly, the item analysis yielded similar results to the task analysis. Looking at tasks instead of items will not have a great impact on the outcome. Those textbooks that primarily used high-level items were the same that primarily used high-level tasks. Similarly, those that used low-level items also had low-level tasks. While the number of items will be referenced occasionally to further develop the discussion of each curricula's inferred philosophy regarding how students learn, the focus of the discussion of comprehensive coverage will be on tasks.

Based on these arguments, the following will be examined to determine comprehensive coverage of content:

1) How many tasks related to each cluster of the CCSSM probability and statistics standards are addressed by each textbook series?
2) How many items and tasks related to the CCSSM probability and statistics standards are present in each textbook series?
3) Where do the opportunities for learning probability and statistics appear in the 4-year high school curriculum?

5.3.1 Coverage of clusters of standards

As shown in Table 5.1 and Figure 5.3, all three textbook series cover each of the content clusters from the probability and statistics standards of CCSSM. Recall that the three clusters are Interpreting Categorical and Quantitative Data (S-ID), Making Inferences and Justifying Conclusions (S-IC), and Conditional Probability and the Rules of Probability (S-CP). It is interesting to note that while IMP has the least number of S-ID and S-CP tasks, but it has the most S-IC tasks of the three curricula. Also of interest is the balance of coverage within each curriculum. In the GM curriculum, 37 of 59 tasks or 63% of the tasks are associated with the S-ID cluster. The CPMP has 80%, 77 of 96, of the tasks associated with the S-ID cluster. IMP is the only curriculum that does not invest a majority of its instructional tasks in probability and statistics to the S-ID standard by having 45% of its tasks in the S-ID cluster. This approach is more balanced than the GM and CPMP approaches. Finally, it is interesting to note that with exception to the high number of tasks in the S-ID cluster, the three curricula are relatively similar in the number of tasks in alignment with the probability and statistics standards of CCSSM.

Table 5.1. Clusters of tasks aligned with CCSSM probability and statistics in each series

Cluster	GM Tasks	IMP Tasks	CPMP Tasks
S-ID	37	19	77
S-IC	13	16	6
S-CP	9	7	13

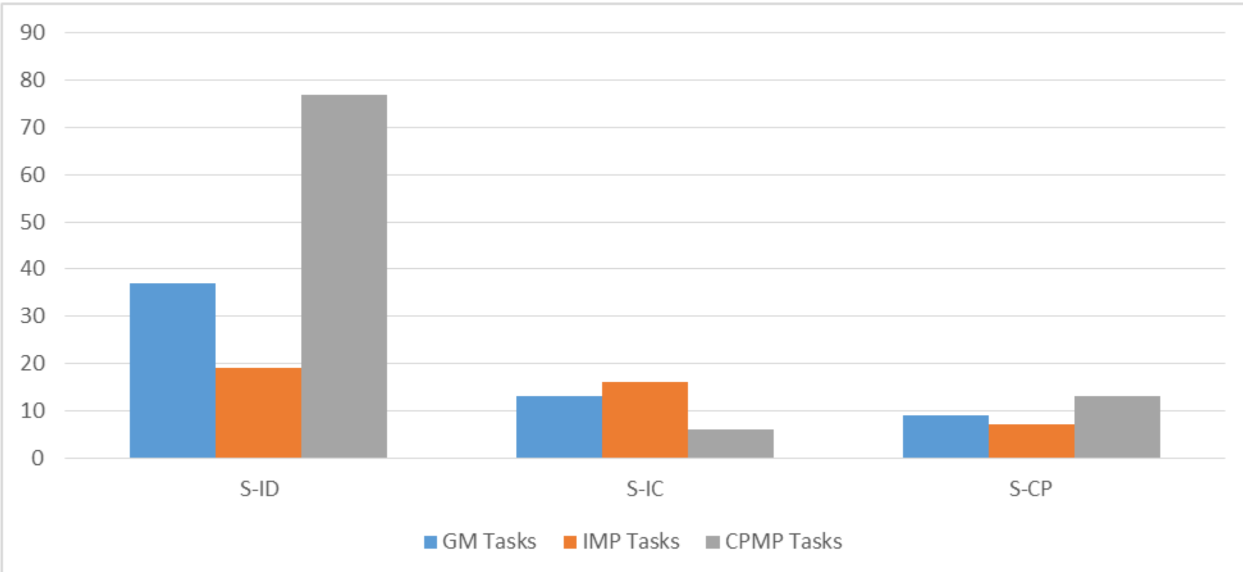


Figure 5.3. Clusters of tasks aligned with CCSSM probability and statistics in each series

The overall emphasis of the S-ID cluster over the S-CP cluster is not surprising given a similar emphasis in the assessments created by the Partnership for Assessment of Readiness for College and Careers (PARCC). PARCC is a collaborative effort representing multiple states and the District of Columbia to create assessments aligned with CCSS in both mathematics and English language arts. PARCC has organized CCSSM in both by traditional content (Algebra I, Geometry, and Algebra II) and in an integrated sequence (Mathematics Course 1, 2, and 3) with the idea of an end of courses assessment for each course. In each of these content structures, the standards from CCSSM are identified in order of importance as *major content*, *supporting content*, or *additional content*. As shown in Table 5.2, standards from the S-ID cluster are at least at the level of supporting standards and in some cases are considered major content in two of the three courses whether organized by content or integrated. S-CP is deemphasized as it appears in only one of the three courses and is considered additional content, which is the lowest level of importance in the PARCC framework.

Table 5.2. CCSSM content emphasis from PARCC Assessment Framework (PARCC, 2014)

Course	S-ID Cluster	S-IC Cluster	S-CP Cluster
Algebra I	3 Major 5 Supporting 3 Additional	0 Major 0 Supporting 0 Additional	0 Major 0 Supporting 0 Additional
Geometry	0 Major 0 Supporting 0 Additional	0 Major 0 Supporting 0 Additional	0 Major 0 Supporting 0 Additional
Algebra II	0 Major 3 Supporting 1 Additional	4 Major 2 Supporting 0 Additional	0 Major 0 Supporting 7 Additional
Mathematics 1	3 Major 5 Supporting 3 Additional	0 Major 0 Supporting 0 Additional	0 Major 0 Supporting 0 Additional
Mathematics 2	0 Major 0 Supporting 0 Additional	0 Major 3 Supporting 0 Additional	0 Major 0 Supporting 7 Additional
Mathematics 3	0 Major 4 Supporting 0 Additional	0 Major 2 Supporting 4 Additional	0 Major 0 Supporting 0 Additional

5.3.2 Number of tasks in the textbook series

A second method to addressing coverage of probability and statistics content is to examine the number of tasks each textbook series contains that are in alignment with the CCSSM probability and statistics standards. As shown in Table 5.3 and Figure 5.4, CPMP has 96 total tasks, the GM series has 59 tasks, and the IMP series has 42 tasks.

Table 5.3. Tasks aligned with CCSSM probability and statistics in each series

Textbook Series	Number of Tasks
GM	59
IMP	42
CPMP	96

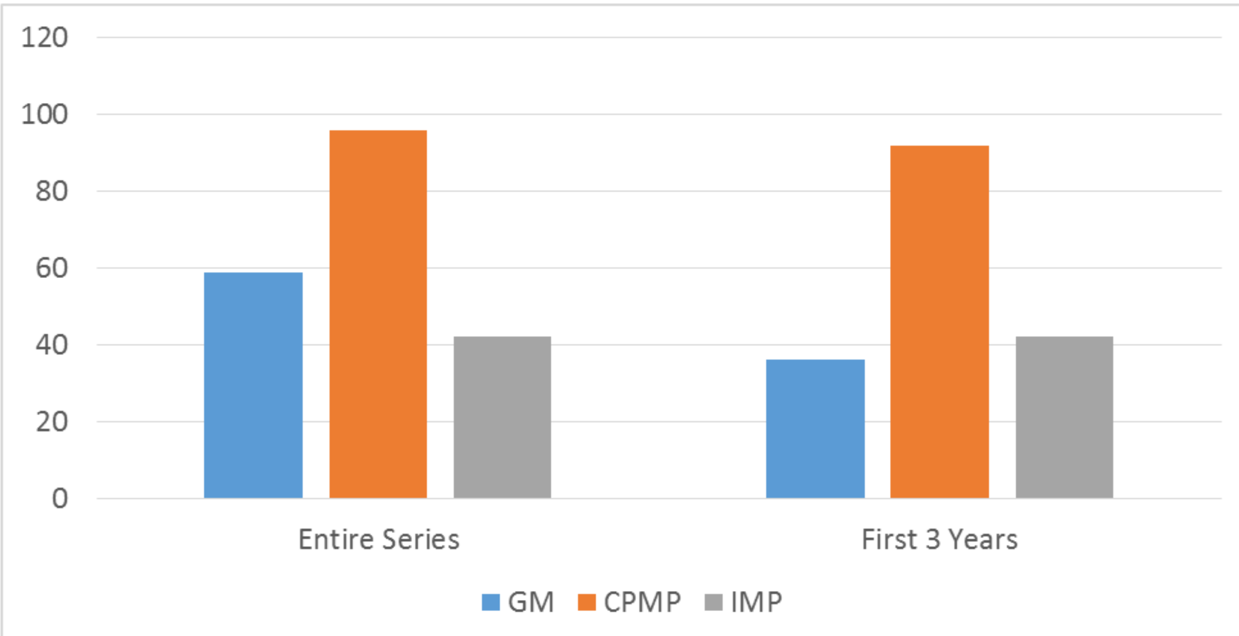


Figure 5.4. Tasks aligned with CCSSM probability and statistics in each series

5.3.3 Where the opportunities appear in the curriculum

A third consideration in addressing the question of which textbook series has the most comprehensive coverage is the location of the opportunities appear in the curriculum. Two points of discussion came from the results. One is the differences that emerge when comparing only the first three years of the textbook series as opposed to considering the entire series. The second is the integration of probability and statistics across the textbooks each curriculum.

Comparing the first three textbooks of each series

It is important to compare only the first three years of the textbook series because most states, 30 including the District of Columbia, only require three years of high school mathematics to graduate. Based on the three course minimum requirement, many students will not receive all four years of instruction in any of these curricula. Therefore, it may be important to consider what would be required in most or all states as opposed to the potential of the curriculum in its entirety.

When considering only the first three textbook of any series, the data for the number of tasks may tell a different story than when all four years are considered, as was the case in Table 5.3. There are differences for two of the curricula in this study, CPMP and GM. As shown in Table 5.4, the CPMP curriculum has 92 tasks in the first three books of the series. Recall that in Table 5.3, the CPMP curriculum had 96 tasks. This means a student only completing the first three years of the CPMP curriculum will have the opportunity to engage in 96% of the instructional tasks, as shown in Figure 5.4, in the curriculum. The GM curriculum loses a much higher percentage of opportunities when the Precalculus book is not considered in the data. Table 5.4 shows that the GM curriculum has 36 tasks in the first three years of the curriculum. When compared to the data in Table 5.3, which shows 59 tasks, there are 23 tasks, as shown in Figure 5.4, lost by not including the fourth year of the curriculum. This means that a student only completing the first three years of the curriculum would only receive 61% of the opportunities from tasks available in the curriculum.

Table 5.4. Probability and statistics tasks in the first three years of each series

Textbook	Number of Tasks
GM First Three	36
IMP First Three	42
CPMP First Three	92

There is no difference in the total number of tasks found in the IMP curriculum as shown in Tables 5.3 and 5.4 and Figure 5.4. This means that a student only completing the first three years of the IMP curriculum will not miss any of the opportunities to engage in probability and statistics content found in IMP.

The CPMP and IMP curricula show little and no change respectively when only the first three years of each series is examined. However, the GM curriculum does change when only considering the first three years of the curriculum. Of particular interest is the number of tasks in the GM curriculum when compared to IMP. When all four years are considered, the GM curriculum has more tasks associated with probability and statistics (59 compared to 42). That comparison looks differently when only the first three years are compared as the GM curriculum has six less tasks in the first three years (36 compared to 42).

Integration of Probability and Statistics into the curriculum

A second point of discussion that emerges from the data is how each textbook series integrates the CCSSM probability and statistics standards into the curriculum. One of the characteristics of a traditional approach, as seen in the GM series, is the dedication of textbooks to specified content. The GM series has books specially dedicated to Algebra I, Algebra II, Geometry, and Precalculus. The *Standards*-based approach is typically characterized by a more integrated curriculum where each textbook contains a variety of topics blended together in order to facilitate students making connection between the topics more easily. Both IMP and CPMP follow this integrated approach as indicated by the labels of their textbooks as years (Year 1, Year 2, Year 3, and Year 4) rather than content.

As shown in Table 5.5, each of the three textbook series follows a similar pattern of having the most tasks in the first textbook of the series with the number of tasks in each textbook after being less than the one that preceded it. The only exception to this pattern is the GM Precalculus book. The GM series starts with 20 tasks in Algebra I, decreases to 12 tasks in Algebra II, decreases again to 4 tasks in Geometry, but then increases to 23 tasks (39% of the tasks in the entire GM series) in the Precalculus textbook.

Table 5.5. Tasks aligned with CCSSM probability and statistics in each textbook

Textbook	Number of Tasks
GM Algebra I	20
GM Algebra II	12
GM Geometry	4
GM Precalculus	23
IMP Year 1	24
IMP Year 2	17
IMP Year 3	1
IMP Year 4	0
CPMP Year 1	39
CPMP Year 2	28
CPMP Year 3	25
CPMP Year 4	4

The data in Table 5.5 leads to a discussion of the integration of probability and statistics in each curriculum. Because there are tasks throughout each of the four textbooks in the GM textbook series, it would appear that probability and statistics have been integrated throughout the curriculum. However, since there are no tasks in the IMP Year 4 textbook and only in the IMP Year 3 textbook, these data suggest that probability and statistics has not been integrated throughout the IMP curriculum. This finding is of particular interest since, as previously discussed, the GM series follows a traditional design while the IMP curriculum follows a *Standards*-based approach. Drilling a little deeper into the data, 97% of the tasks in the IMP textbook series are in the first two books. Therefore, a student enrolling in four years of high school mathematics instruction based on IMP (assuming Year 1 as a freshman, Year 2 as a sophomore, Year 3 as a junior, and Year 4 as a senior), would stop engaging in probability and statistics content after completing his or her sophomore year. The CPMP series is more balanced than IMP is, but not as much as GM. The CPMP series has 70% of the probability and statistics tasks in the first two year of the curriculum as opposed to GM, which has only 54% of the tasks in the first two year. In summary, these data suggest that the traditional textbook series with content-based textbooks, GM, has integrated probability and statistics throughout all four textbooks better than the two textbook series, IMP and CPMP, which are typically characterized as being integrated approaches.

5.4 COMPREHENSIVE COVERAGE OF THE STANDARDS FOR MATHEMATICAL PRACTICE

The second point of discussion related to comprehensive coverage is associated with providing high-level opportunities (i.e. tasks) for students to engage in that will foster the development of the Standards for Mathematical Practice. It has been previously argued that CCSSM emphasizes conceptual understanding beyond what is currently taught in most high schools (NGACBP, 2010). Additionally, it has already been argued in Chapter 2 that one could reasonably assume that the Standards for Mathematical Practice will necessitate student engagement in high-level tasks. These two points necessitate an examination of the level of cognitive demand of the items and tasks found in alignment with the CCSSM probability and statistics standards.

As shown in Table 5.6 and Figure 5.6, only 33% of the instructional tasks in the GM curriculum are high-level. The IMP series has 84% of its tasks at a high-level. The CPMP series has 94% of its tasks at a high-level.

Table 5.6. Cognitive demand of probability and statistics tasks in each series

Textbook	Memorization	Procedures without Connections	Procedures with Connections	Doing Mathematics	TOTAL
GM Series	0 (0%)	39 (66%)	15 (25%)	5 (8%)	59
IMP Series	0 (0%)	6 (16%)	7 (18%)	25 (66%)	38
CPMP Series	0 (0%)	5 (5%)	33 (34%)	58 (60%)	96

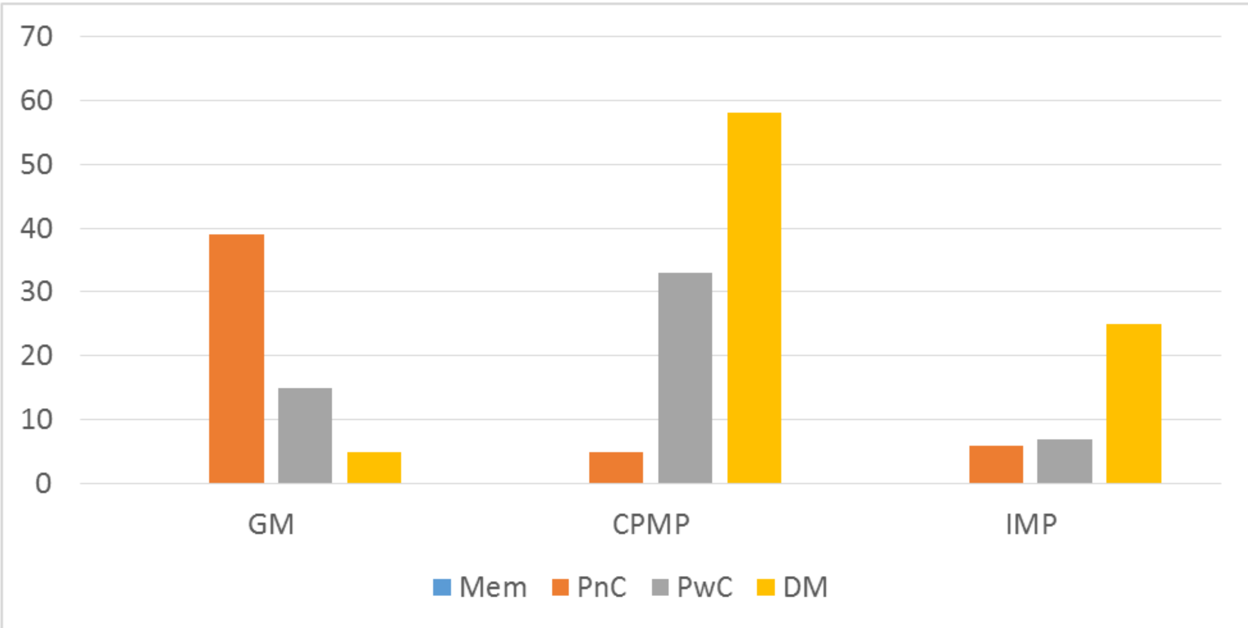


Figure 5.5. Cognitive demand of probability and statistics tasks in each series

There is a substantial difference in the level of cognitive demand of the opportunities found in the traditional curriculum, GM, when compared to the two *Standards*-based curricula, IMP and CPMP. Most of the items and tasks in the GM curriculum will likely not prepare students for the increased emphasis on conceptual understanding and the Standards for

Mathematical Practice in CCSS. However, both the IMP and CPMP series have the potential to engage students at a level that will allow them to meet these demands. Even if the high-level tasks from IMP and CPMP are not implemented with fidelity, they may still have increased opportunities to learn. Stein and Lane (1996) concluded that students who were in classrooms where high-level tasks were used but the cognitive demand not maintained during instruction still learned more than students who only had opportunities to work on low-level tasks. This conclusion suggests that even if IMP and CPMP are not implemented well, students with the opportunity to engage in the tasks in these textbooks will learn more than those students engaged in the GM curriculum.

As shown in Table 5.6 and Figure 5.6, the IMP and CPMP textbook series both have a higher level of cognitive demand than the GM curriculum. The lack of cognitive demand in the GM curriculum is more apparent when the GM Precalculus book is not considered. The GM Precalculus book contains 157 of the 268 high-level items and 16 of the 20 high-level tasks in the curriculum. Once again, if the fourth year of the curriculum is not required, students not given the opportunity to engage in the Precalculus textbook will not have the opportunity to engage in 59% of the high-level items and 80% of the high-level tasks in the curriculum.

5.5 SUPPORT FOR TEACHERS

The third question framing the discussion is, “Which textbook series provides the most support for the teachers enacting the probability and statistics content?” Support for teachers was examined in two ways. The first was an examination of *anticipation*. The second was an examination of *transparency*. Only high-level tasks were examined for indications of providing support for the teacher because low-level tasks do not require support to be implemented with fidelity.

As shown in Table 5.7 and Figure 5.7, less than one third of the high-level tasks found in each textbook series provide support for the teacher. The GM curriculum has the fewest occurrences of support (1), which represent 5% of the high-level tasks found in it. The CPMP has only 13 opportunities even though there are 91 high-level tasks in the curriculum. There is only support provided for the teacher on 14% of the high-level tasks found in the curriculum. The IMP curriculum provides the highest percentage of support with 31% of the high-level tasks having *anticipation* (none contained *transparency*).

Table 5.7. Teacher support on high-level probability and statistics tasks in each series

Textbook Series	Anticipation	Transparency	Total
GM	1/20	0/20	1/20
IMP	10/32	0/32	10/32
CPMP	11/91	2/91	13/91

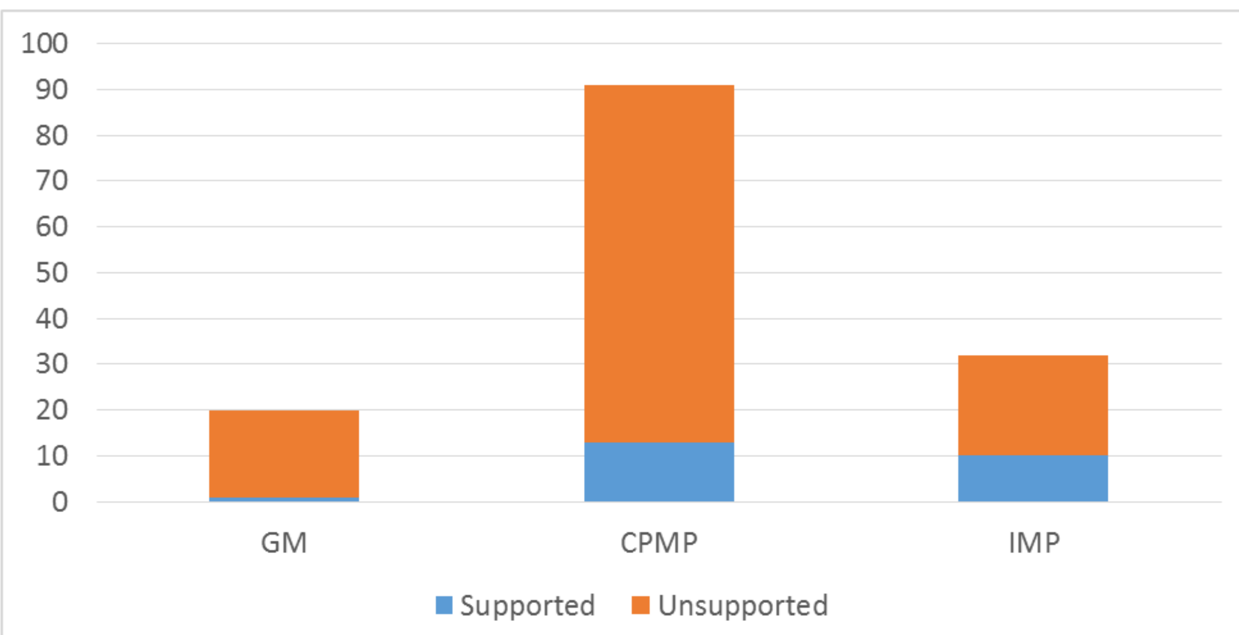


Figure 5.6. Supported and Unsupported high-level tasks in each curriculum

The IMP curriculum not only provides the highest percentage of support for teacher of the three textbook series, but it also provides the support for the teacher in a manner that may best promote the development of the program by providing the most support in the first year. Eight of the ten indication of support for the teacher occur in the first year of the IMP curriculum. Despite having nearly three times as many high-level tasks, the CPMP curriculum has only three more indications of support for the teacher than IMP. Finally, the GM curriculum does not provide any support for the teacher other than one instance, but there is not really a need for support since the curriculum will be much easier to implement at its highest potential, which is low-level.

Another point of discussion is the overall lack of transparency provided by any of the textbooks. There are only indication of support for the teacher through *transparency* (both in the CPMP series) in all three curricula combined. *Transparency* allows the teacher to select and adapt tasks by providing him or her with the mathematical purpose of the task (Stein & Kim, 2009). Teachers need to adapt the curriculum to meet the needs of their students (Ball & Cohen, 1996). However, without support for the teachers via *transparency*, the abilities of the teachers to make these necessary adaptations while maintaining fidelity of implementation may not exist. Given the argument by Stein and Kim (2009) that *doing mathematics* tasks are even more difficult to enact well than *procedures with connection*, the lack of *transparency* may be even more detrimental to tasks receiving the *doing mathematics* code.

5.6 CHOOSING A CURRICULUM FOR TEACHING PROBABILITY AND STATISTICS

There are multiple points to consider when choosing a curriculum for teaching probability and statistics. While many start with the notion that more is better, quantity should not be the only consideration. The discussion may begin with a count of how many standards are addressed or how many opportunities there are, but the quality of those opportunities must be considered. When considering quality, the level of cognitive demand becomes the focus of the discussion. However, highly demanding tasks are difficult for teacher to implement well. Therefore, any curriculum worth teaching will be more difficult to teach. Support for the teacher will be necessary if the curriculum is to be implemented with fidelity. Ball and Cohen (1996) suggest that curriculum materials often overlook the teacher, which leads to the enacted curriculum not matching the intentions of the written curriculum. It may not matter how many high-level opportunities a textbook provides if that same textbook does not take steps to ensure the teacher enacts them with fidelity.

School districts choosing a curriculum would be faced with a difficult decision given these curricular options. Both IMP and CPMP have the potential to provide students with opportunities to engage in high-level tasks and items related to probability and statistics. Both of these curricula will require substantial work by the teachers to implement well. While CPMP has more learning opportunities for students overall, IMP provides more support for the teacher. However, one must consider whether either curriculum provides enough support to be implemented with fidelity.

Part of the decision will likely involve the school district’s philosophy regarding how students learn. The IMP curriculum materials are clearly a *Standards*-based curriculum. The design focuses on high-level, instructional tasks that the students work through in order to develop a deep, conceptual understanding of mathematical content. The CPMP blends a similar *Standards*-based approach with some of the traditional opportunities for student practice problems and review exercises that many educators and students are accustomed to using. How the school district's philosophy matches with the philosophy of each curriculum will be an important component of the decision making process.

5.7 LIMITATIONS OF THE STUDY

One limitation of this study is that it only focuses on the CCSSM probability and statistics standards. There are six different conceptual categories in the CCSSM for high school. Probability and statistics represent only one of the six categories. It is possible that analyzing one or all of the other conceptual categories could tell a different story about each curriculum. This limitation would be true of a focus on any of the conceptual categories of CCSSM (for example if the study focused on functions), but is especially true of probability and statistics because these two areas have historically been widely ignored (Shaughnessy, 2007) and are still not up to the level of national document suggestions (Jones, Langrall, & Mooney, 2007). However, the philosophy of how students learn found in each curriculum is unlikely to be different for the other conceptual categories. Since the philosophy is the same, the other conceptual categories likely received similar treatment with regard to level of cognitive demand and teacher support when compared to the findings regarding probability and statistics. Therefore, it is likely that these findings do provide insight into the curriculum more broadly. The findings regarding content coverage may be different for the other conceptual categories and would require further investigation.

Another limitation is that this study only analyzes three sets of current curriculum materials. These three sets of materials provide a snapshot of the landscape of secondary mathematics education materials, but they may not paint the entire picture of what is available. Since only three curricula were included, it is not possible to conclude that any of the three is the best available in any of the dimensions analyzed. Including more curricula from an even wider variety of publishers could reveal more about available curriculum materials.

5.8 IMPLICATIONS AND POTENTIAL CONTRIBUTIONS OF THE STUDY

There is a variety of groups that could benefit from this study. The largest benefactor would likely be those schools or districts considering one of the curricula reviewed for adoption. Analyzing the cognitive demand of instructional tasks speaks to both the instructional design and the content emphasis of a textbook as suggested by Hudson, Lahann, and Lee (2010). Schools can then decide what type of textbook is appropriate for their school's philosophy regarding how students learn. Textbooks with high-level tasks will require a great deal of professional development, may cause a lot of conflict with the beliefs held by teachers, and will be difficult to implement (Hudson, Lahann, & Lee, 2010). School decision makers will have to decide if they have the time, resources, and staff to take on such a challenge. The analysis of the support provided for teachers will provide decision makers with an idea of how supportive the curriculum materials are of their own implementation. In addition, each textbook was analyzed to determine its alignment with the CCSSM in regards to probability and statistics. While most publishers are going to make the claim of alignment, the textbooks analyzed had that claim tested in one specific content area.

In addition to providing specific information related to probability and statistics, the analysis of tasks provided by this study could serve as a framework for further evaluation of curriculum materials. For example, if a district uses curriculum materials that have not been reviewed here, they could apply the same analysis on their own to determine how their curriculum materials would fit in with those that are reviewed in this study. This study brings together research on tasks that require high-level cognitive demand, research on educative curriculum materials, and applies them to the CCSSM in such a manner that could be applicable to any one of the content areas identified by the CCSSM. Therefore, anyone wishing to evaluate content areas other than probability and statistics as defined by the CCSSM could benefit from this study as well.

Finally, teachers who create their own curriculum could benefit from this study. In today's online world, many resources are available to teachers via the internet. However, not all of the resources are good ones. Teachers who design their own curriculum could benefit from this research because it provides information on what they might look for as they search through online resources. This research will help teachers to understand better the impact that the level of cognitive demand has on student learning. This research also provides teachers with an understanding of the importance of anticipation and transparency.

5.9 CONCLUDING REMARKS AND SUGGESTIONS FOR FUTURE RESEARCH

Despite each of the three curricula claiming to be aligned with CCSSM, the nature of that alignment varies considerably. Each presented a different approach (traditional, *Standards*-based, and blended) to addressing the CCSSM probability and statistics standards. While this study accomplished the goal of providing insight regarding the potential of these secondary curriculum materials to promote student and teacher learning in the areas of probability and statistics, the effectiveness of a curriculum is not based solely on potential. In order to determine which curriculum is actually the most effective, much more research needs to be done.

Establishing the CPMP and IMP curricula as having many more high-level items and tasks than low-level is merely one-step in the right direction. As has been argued previously, teachers will need to improve their teaching practices if students are to engage in high-level tasks (Boston & Smith, 2009; Stein & Kaufmann, 2010) because high-level tasks are more difficult to implement with fidelity. One-step in facilitating these improved practices would to be providing opportunities for teacher learning through the curriculum (Ball & Cohen, 1996; Davis & Krajcik, 2005). Will teachers take advantage of these opportunities? A logical next step would be to examine teachers as they set up high-level tasks to determine how, if at all, the teachers use the teacher materials. While it is important for a curriculum to provide opportunities for teacher support through anticipation and transparency, it is only useful if teacher take advantage of those opportunities. Understanding how teachers make use of the teacher materials could provide insight for textbook writers and publishers into how they can provide teachers with support they will be willing to use.

Previous research has already suggested that both IMP and CPMP perform at least equal to and in many cases better than their traditional counterparts on multiple forms of assessment.

For example, Senk and Thompson (2003) suggest that students in schools using NSF funded curriculum, which includes IMP and CPMP, perform as well as those students in schools using traditional curriculum, which characterizes GM, on assessments of procedural knowledge. However, on problem solving based assessments, the NSF curricula outperform their traditional counterparts. This is to suggest that the NSF curricula do no harm to procedural learning while improving problem solving ability.

A series of studies focusing on CPMP in comparison to traditional textbooks also suggests that CPMP performs as well or better than its traditional counterparts performs (Chavez et al., 2015; Grouws et al., 2013; Tarr et al., 2013). These studies focused primarily on the benefits of an integrated curriculum as opposed to one that separates textbooks by content but also incorporated other fields of research to their methodology. Grouws et al. (2013) compared CPMP Year 1 to Algebra I textbooks from traditional textbook series since both represented the first textbook in their respective series. Students with the opportunity to engage in CPMP Year 1 as opposed to traditional Algebra I textbooks scored significantly higher on all three measurement tools use in the study: a common objectives test, a problem solving and reasoning test, and a standardized achievement test. Additionally, Grouws et al. suggest that the number of opportunities to learn student were provided and teacher experience were also significant factors in predicting success on the three assessment tools. Tarr et al. (2013) compared CPMP Course 2 to Geometry since both are commonly used as the second textbook in their respective series. The same three types of measurement tools were used. The CPMP Course 2 students outperformed the Geometry students on the standardized achievement test. The two groups performed similarly on the other two assessments: common objectives test and problem solving and reasoning test. Once again, student opportunity to learn was a significant predictor of results. Finally, Chavez et al. (2015) compared third year CPMP to traditional Algebra II courses as a comparison of the third textbook in each series. Again, a common objectives test and standardized achievement test were used. The CPMP Year 3 students scored higher on the common objectives test. Both scored roughly the same on the standardized achievement test. Interestingly, opportunity to learn was not a substantial factor in the Chavez et al. (2015) study. Instead, Chavez et al. suggest that teacher beliefs and orientation about reform-oriented practices were significant factors.

These three more detailed studies (Chavez et al., 2015; Grouws et al., 2013; Tarr et al., 2013) confirm the previous suggestions of Senk and Thompson (2003). The CPMP curriculum performs as well or better than its traditional counterpart on multiple measures of student performance does. In other words, CPMP will do no harm overall while providing improvements in many areas.

The current study has addressed some of the suggestions made by these recent studies. Chavez et al. (2015) and Grouws et al. (2013) suggest that future research should incorporate an examination of more than one integrated curriculum. The current study has taken a step in this direction by analyzing both CPMP and IMP. Grouws et al. (2013) suggest that research specifically dedicated to examining how the opportunities provided for students to learn impact achievement is needed. Specifically, Grouws et al. suggest that a more detailed examination of the opportunities and more frequent classroom visits are necessary. The detailed examination of the opportunities provided by the three curricula in the current study provides the first step in addressing this suggestion by Grouws et al.

Next steps from the current study are similar to those suggested by Chavez et al. (2015), Grouws et al. (2013), and Tarr et al. (2013). All three of these studies suggest that more information is needed to determine which characteristics of a curriculum are important and under what circumstances they are effective. Specifically, Tarr et al. (2013) suggests researchers need to examine teacher enactment and student achievement to gauge curricular effectiveness. Chavez et al. (2015) suggests that future research could use a variety of implementation measures along multiple student outcome measures. Armed with the details regarding the potential of the three curricula analyzed in the current study, researchers could take these steps as suggested. Those researchers wishing to incorporate additional integrated curricula into their study, as suggested by Grouws et al. (2013), could start by repeating the methodology outlined by the current study in preparation for the next steps previously suggested.

APPENDIX A

PROBABILITY AND STATISTICS MISCONCEPTIONS IDENTIFIED IN RESEARCH

Table A.1. Probability and statistics misconceptions identified in research

Misconception	Citation	Description	Example
Availablity	Tversky and Kahneman (1973)	People rely on recall in place of statistics and therefore underestimate things that are difficult to recall.	The letters K, L, N, R, and V are more likely to be the 3rd letter in a word than the 1st but it is easier to think of examples where they are first so subjects think 1st is more likely.
Repesentativeness	Kahneman and Tversky (1973)	People favor samples that look like population characteristics instead of using statistics.	When flipping a coin 10 times, the result HTTHTHHTHT is considered more likely than HHHHTTHHHH but they are equally likely.
Base Rate Fallacy	Bar-Hillel (1980)	People tend to ignore base rates in favor of other information.	The taxi problem: the percent of green cabs in a city is ignored in favor of witness testimony even though both should be considered.
Conjunction Fallacy	Tversky and Kahneman (1983)	People will choose conjunctions as more likely than the individual outcomes.	It considered more likely someone is a lawyer who plays golf than someone is a lawyer even though being a lawyer must be more likely because it would include being a lawyer who plays golf and those who don’t.

(table continues)

Table A.1 (continued)

Misconception	Citation	Description	Example
Outcome Approach	Konold (1989)	Probability is seen as the ability to predict what will happen on the next individual trial.	If the weather forecast is calls for 70% chance of rain and it does not rain, then the forecast is considered wrong even though it allowed for a 30% chance of no rain.
Equiprobability	Tempelaar, Gijselaers, and van der Loeff (2006)	Random events are always equally likely.	When rolling a die the probability of rolling a 6 and not rolling a 6 are both considered 50% even though it should be 1/6 and 5/6 respectively.
Simpson's Paradox	Hawkins and Kapadia (1984)	If $a/b > c/d$ and $e/f > g/h$, then $(a + e)/(b + f) < (c + g)/(d + h)$.	If drug A is better for right-handed people and drug A is better for left handed people it is assumed drug A is better for all people but it is not necessarily true.
Birthday Paradox	Hawkins and Kapadia (1984)	People assume that nobody will have the same birthday even in a crowded room.	In a room of 30 people, it is actually very likely that two have the same birthday but it will be assumed unlikely.
Combinatorial Naivety	Hawkins and Kapadia (1984)	There are more combinations of small groups then there are large groups because they are easier to think of.	Given 10 people, it is assumed that there are more committees of 3 than there are committees of 7 even though they are the same
Gambler's Fallacy	Hawkins and Kapadia (1984)	The absence of a random outcome makes it more likely	In roulette if red has not come up in a while it's due to be next even though the probability is independent of prior outcomes.
Positive Recency Effect	Hawkins and Kapadia (1984)	A repeated outcome becomes more likely	In roulette if red has come up a lot it is more likely to do so again even though the probability is independent of prior outcomes.
Correlation is Transitive	Casey (2010)	If A and B have a positive correlation and B and C have a positive correlation, then A and C must also have a positive correlation	If hours sleeping and test scores are positively correlated, test scores and hours studying are positively correlated, then it will be assumed that hours sleeping and hours studying are positively correlated even though there may be no or even a negative correlation.

(table continues)

Table A.1 (continued)

Misconception	Citation	Description	Example
Law of Small Numbers	Tempelaar, Gijselaers, and van der Loeff (2006)	Small samples are judged to have the same characteristics as large samples	A sample of 12 can have all the same tests applied to it as a sample of 30.
Existence Correlation	Casey (2010)	Correlation is judged on existence instead of intensity	A correlation of .49 means no correlation exists but .51 means a positive correlation exists even though these are roughly the same intensity of positive correlation.

APPENDIX B

GAISE REPORT BY FRANKLIN ET AL. (P. 14-15, 2007)

Process Component	Level A	Level B	Level C
I. Formulate Question	***Beginning awareness of the statistics question distinction*** Teachers pose questions of interest Questions restricted to the classroom	***Increased awareness of the statistics question distinction*** Students begin to pose their own questions of interest Questions not restricted to the classroom	***Students can make the statistics question distinction*** Students pose their own questions of interest Questions seek generalization
II. Collect Data	Do not yet *design for differences* Census of classroom Simple experiment	Beginning awareness of *design for differences* Sample surveys; begin to use random selection Comparative experiment; begin to use random allocation	Students make *design for differences* Sampling designs with random selection Experimental designs with randomization
III. Analyze Data	*Use* particular properties of *distributions* in the context of a specific example Display variability within a group Compare individual to individual Compare individual to group Beginning awareness of group to group Observe association between two variables	Learn to *use* particular properties of *distributions* as tools of analysis Quantify variability within a group Compare group to group in displays Acknowledge sampling error Some quantification of association; simple models for association	Understand and *use distributions* in analysis as a global concept Measure variability within a group; measure variability between groups Compare group to group using displays and measures of variability Describe and quantify sampling error Quantification of association; fitting of models for association
IV. Interpret Results	Students do not look *beyond the data* No generalization beyond the classroom Note difference between two individuals with different conditions Observe association in displays	Students acknowledge that *looking beyond the data* is feasible Acknowledge that a sample may or may not be representative of the larger population Note the difference between two groups with different conditions Aware of distinction between observational study and experiment Note differences in strength of association Basic interepretation of models for association Aware of the distinction between association and cause and effect	Students are able to *look beyond the data* in some contexts Generalize from sample to population Aware of the effect of randomization on the results of experiments Understand the difference between observational studies and experiments Interpret measures of strength of association Interpret models of association Distinguish between conclusions from association studies and experiments
Nature of Variability	Measurement variability Natural variability Induced variability	Sampling variability	Chance variability
Focus on Variability	Variability within a group	Variability within a group and variability between groups Covariability	Variability in model fitting

Figure A.1. Process levels from the GAISE Report (Franklin et al., p. 14-15, 2007)

B.1 RECOMMENDATIONS AT LEVEL A BY FRANKLIN ET AL. (P. 23-24, 2007)

I. Formulate the Question

→ Teachers help pose questions (questions in contexts of interest to the student).

→ Students distinguish between statistical solution and fixed answer.

II. Collect Data to Answer the Question

→ Students conduct a census of the classroom.

→ Students understand individual-to-individual natural variability.

→ Students conduct simple experiments with nonrandom assignment of treatments.

→ Students understand induced variability attributable to an experimental condition.

III. Analyze the Data

→ Students compare individual to individual.

→ Students compare individual to a group.

→ Students become aware of group to group comparison.

→ Students understand the idea of a distribution.

→ Students describe a distribution.

→ Students observe association between two variables.

→ Students use tools for exploring distributions and association, including:

- Bar Graph
- Dotplot
- Stem and Leaf Plot
- Scatterplot
- Tables (using counts)
- Mean, Median, Mode, Range
- Modal Category

IV. Interpret Results

→ Students infer to the classroom.

→ Students acknowledge that results may be different in another class or group.

→ Students recognize the limitation of scope of inference to the classroom.

Figure A.2. GAISE Report recommendations for level A (Franklin et al., p. 23-24, 2007)

B.2 RECOMMENDATIONS AT LEVEL B BY FRANKLIN ET AL. (P.37, 2007)

I. Formulate Questions

→ Students begin to pose their own questions.
→ Students address questions involving a group larger than their classroom and begin to recognize the distinction among a population, a census, and a sample.

II. Collect Data

→ Students conduct censuses of two or more classrooms.
→ Students design and conduct nonrandom sample surveys and begin to use random selection.
→ Students design and conduct comparative experiments and begin to use random assignment.

III. Analyze Data

→ Students expand their understanding of a data distribution.
→ Students quantify variability within a group.
→ Students compare two or more distributions using graphical displays and numerical summaries.
→ Students use more sophisticated tools for summarizing and comparing distributions, including:
- Histograms
- The IQR (Interquartile Range) and MAD (Mean Absolute Deviation)
- Five-Number Summaries and boxplots

→ Students acknowledge sampling error.
→ Students quantify the strength of association between two variables, develop simple models for association between two numerical variables, and use expanded tools for exploring association, including:
- Contingency tables for two categorical variables
- Time series plots
- The QCR (Quadrant Count Ratio) as a measure of strength of association
- Simple lines for modeling association between two numerical variables

IV. Interpret Results

→ Students describe differences between two or more groups with respect to center, spread, and shape.
→ Students acknowledge that a sample may not be representative of a larger population.
→ Students understand basic interpretations of measures of association.
→ Students begin to distinguish between an observational study and a designed experiment.
→ Students begin to distinguish between "association" and "cause and effect."
→ Students recognize sampling variability in summary statistics, such as the sample mean and the sample proportion.

Figure A.3. GAISE Report recommendations for level B (Franklin et al., p. 37, 2007)

B.3 RECOMMENDATIONS AT LEVEL C BY FRANKLIN ET AL. (P. 61-62, 2007)

I. Formulate Questions

→ Students should be able to formulate questions and determine how data can be collected and analyzed to provide an answer.

II. Collect Data

→ Students should understand what constitutes good practice in conducting a sample survey.

→ Students should understand what constitutes good practice in conducting an experiment.

→ Students should understand what constitutes good practice in conducting an observational study.

→ Students should be able to design and implement a data collection plan for statistical studies, including observational studies, sample surveys, and simple comparative experiments.

III. Analyze Data

→ Students should be able to identify appropriate ways to summarize numerical or categorical data using tables, graphical displays, and numerical summary statistics.

→ Students should understand how sampling distributions (developed through simulation) are used to describe the sample-to-sample variability of sample statistics.

→ Students should be able to recognize association between two categorical variables.

→ Students should be able to recognize when the relationship between two numerical variables is reasonably linear, know that Pearson's correlation coefficient is a measure of the strength of the linear relationship between two numerical variables, and understand the least squares criterion in line fitting.

IV. Interpret Results

→ Students should understand the meaning of statistical significance and the difference between statistical significance and practical significance.

→ Students should understand the role of p-values in determining statistical significance.

→ Students should be able to interpret the margin of error associated with an estimate of a population characteristic.

Figure A.4. GAISE Report recommendations for level C (Franklin et al., p. 61-62, 2007)

APPENDIX C

TASK ANALYSIS GUIDE FROM SMITH AND STEIN (1998)

Levels of Demands

Lower-level demands (Memorization):

- Involve either reproducing previously learned facts, rules, formulas, or definitions or committing facts, rules, formulas or definitions to memory.
- Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure.
- Are not ambiguous. Such tasks involve the exact reproduction of previously seen material, and what is to be reproduced is clearly and directly stated.
- Have no connection to the concepts or meaning that underlie the facts, rules, formulas, or definitions being learned or reproduced.

Lower-level demands (Procedures without Connections):

- Are algorithmic. Use of the procedure either is specifically called for or is evident from prior instruction, experience, or placement of the task.
- Require limited cognitive demand for successful completion. Little ambiguity exists about what needs to be done and how to do it.
- Have no connection to the concepts or meaning that underlie the procedure being used.
- Are focused on producing correct answers instead of on developing mathematical understanding.
- Require no explanations or explanations that focus solely on describing the procedure that was used.

Higher-level demands (Procedures with Connections):

- Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas.
- Suggest explicitly or implicitly pathways to follow that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts.
- Usually are represented in multiple ways, such as visual diagrams, manipulatives, symbols, and problem situations. Making connections among multiple representations helps develop meaning.
- Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with conceptual ideas that underlie the procedures to complete the task successfully and that develop understanding.

Higher-level demands (Doing Mathematics):

- Require complex and nonalgorithmic thinking—a predictable, well-rehearsed approach or pathway is not explicitly suggested by the task, task instructions, or a worked-out example.
- Require students to explore and understand the nature of mathematical concepts, processes, or relationships.
- Demand self-monitoring or self-regulation of one's own cognitive processes.
- Require students to access relevant knowledge and experiences and make appropriate use of them in working through the task.
- Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions.
- Require considerable cognitive effort and may involve some level of anxiety for the student because of the unpredictable nature of the solution process required.

Figure A.5. Task Analysis Guide from Smith and Stein (1998)

APPENDIX D

D.1 MATH TASK FRAMEWORK FROM STEIN AND SMITH (P. 270, 1998)

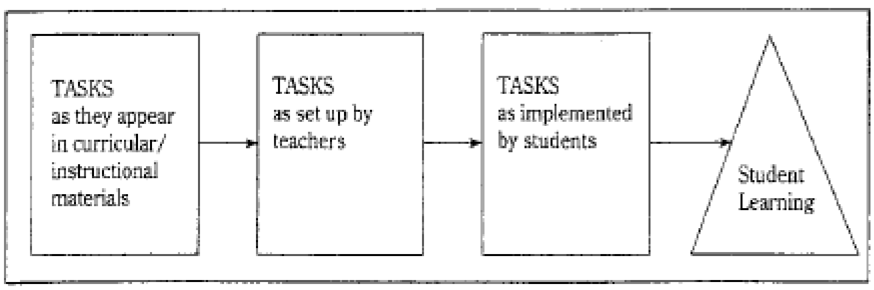


Figure A.6. Math Task Framework from Stein and Smith (p. 270, 1998)

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