SECONDARY SPECTRUM MARKETS: FROM "NAKED" SPECTRUM TO VIRTUALIZED COMMODITIES

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ABSTRACT

SECONDARY SPECTRUM MARKETS: FROM "NAKED" SPECTRUM TO VIRTUALIZED COMMODITIES

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The creation of secondary spectrum markets emerged as a means to enable flexible spectrum-use mechanisms and abandon a rigid spectrum allocation and assignment approach, which resulted in severe inefficiencies in the use of this resource. At the core of the deployment of spectrum markets lie the definition of electromagnetic spectrum as a tradeable commodity, the reallocation of spectrum rights, the creation of incentives for resource owners to lease or transfer their spectrum holdings and the appropriate regulatory framework to support and enforce market transactions. It follows that the viability of spectrum markets depends on technical, economic and regulatory frameworks to render this approach a meaningful alternative for spectrum allocation and assignment.

In this research work, we explore the conditions associated with spectrum markets viability. For this purpose, we utilize Agent-based Modeling in order to study markets under different commodity definitions as well as network configurations. These configurations are gathered in three research stages, which start with the analysis of markets as stand-alone institutions where electromagnetic frequencies, without any associated infrastructure (i.e., "naked" spectrum), are traded. This allows us to explore the degree in which the limitations in spectrum fungibility impact the trading process and outcome.

In the second stage, we focus on refining the tradeable commodity in such a way that allows to circumvent the physical limitations of spectrum. To this end, we rely on technologies such as LTE-Advanced and virtualization in order to define a fungible, virtualized spectrum

commodity and explore the benefits that this provides for market deployment.

The final stage aims at extending the range of applicability of virtualized commodities and providing opportunities that could address current spectrum service and connectivity requirements. Hence, we explore markets as part of more complex network arrangements, where we rely on middleman theory, matching markets and simple auctions in order to enable resource trading. This requires the analysis of multiple factors that impact market design from the definition of tradeable commodities to the characterization of the role and objectives of market participants. These factors stem from relevant technical, economics and regulatory frameworks, which we explore to determine whether our spectrum markets proposal can be considered as a viable and applicable solution.

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PREFACE

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1.0 EXECUTIVE SUMMARY

The focus of this dissertation is to investigate the conditions and parameters involved in the design of secondary spectrum markets, while taking into account the complexity of electromagnetic spectrum as a commodity, and the regulatory framework needed to enable them. In this manner, our analysis explores three key axes for the study of spectrum markets: technology, economics and policy. To the best of our knowledge, there are no working, real-world examples of secondary spectrum markets. Hence, we have relied on Agent-based modeling to represent market settings of interest. We have focused on three particular market scenarios, which have been divided in three research stages.

Stage 1 focuses on analyzing the effects of the lack of spectrum fungibility on market viability. To this end, we build upon an existing market model [1], which considers three types of market participants: a) Spectrum License Requesters (SLRs), b) Band Manager (BM) and c) Spectrum License Holders (SLHs). The commodity available in the market is defined as spectrum bandwidth units in a specific frequency band. Additionally, we define alternate technology units (ATs), which are options that SLRs can find outside the market to fulfill their traffic requirements (i.e., wireline deployments, unlicensed spectrum, infrastructure enhancements, etc.) For spectrum fungibility considerations, we calculate a capacity fungibility score, which represents the ratio between the capacity obtained with an available frequency band, and that obtained with a preferred band. Then, we incorporate this measure to the market model. The objective is for the interactions among market participants to reflect their distinct valuation for preferred and available frequency bands.

The market transactions correspond to a Stackelberg auction, where SLRs post their bids and the BM assigns resources. Market viability is calculated by analyzing the results of our model simulations according to a set of predefined market viability criteria: 1) probability of demand being greater than supply; 2) probability of having empty bid lists; 3) percentage of resources assigned in the market; 4) average number of alternate technology units per SLR, and 5) average auction clearing price.

Results from this stage show that as spectrum fungibility decreases, we find fewer scenarios where markets are viable. Markets are viable when we do not face spectrum oversupply conditions, and this is further accompanied by higher auction clearing prices and a higher percentage of resources assigned in the market. It is important to note that viable scenarios are those with a larger number of market participants, which is not consistent with the current structure of the Telecommunications market.

The aforementioned findings point to the need of adding thickness to the market. In this way, we could develop a market setting that would not only attract a larger number of market participants (i.e., SLRs), but also provide with commodities that are suitable to a wider range of buyers. To this end, in **Stage 2**, we address this issue by adopting a technical definition of the commodity to trade in the market. Indeed, we no longer focus on trading "naked" spectrum; instead, we appeal to Virtualization concepts and the opportunities stemming from LTE-Advanced in order to define a more homogeneous and adaptive spectrum-related commodity. As such, this commodity stems from the definition of LTE Physical Resource Blocks (PRBs).

To fully exploit market thickness, we focus on defining a homogeneous commodity. For this purpose, our commodity to trade is derived from LTE-A bands in the range of 700 MHz. The choice of AT units is TV white space spectrum in the same frequency range. In this manner, we create a pseudo-fungibility environment, enabled by technology. Our objective is to create a marketplace where SLRs can express their spectrum needs in terms of capacity (in Mbps) instead of bandwidth units of a specific frequency. To enhance resource access, we consider that available spectrum commodities are *pooled*, and the resulting common pool of resources is administered by the Band Manager. Further, the BM is in charge of making the translation between PRBs and their resulting capacity, before assigning resources in the market.

The market structure in this stage is fairly similar to that of Stage 1. Hence, we work with the same market participants; however, we make the necessary adjustments to fit the new market commodity. We tested two different scenarios that focused on different access opportunities to the licensed spectrum auctions. Our results stem from comparing simulation results to the same viability criteria utilized for Stage 1. We find that when SLRs have the opportunity to cease utilization of unlicensed spectrum to participate in every market bidding round, all the tested scenarios are viable. Viability conditions include scenarios with only a few SLRs, thus pointing to the advantage of developing a more flexible, homogeneous commodity and marketplace.

Stages 1 and 2, while different in settings and commodity definitions, focus on the analysis of markets as standalone entities. In this manner, in our third stage, the objective is to extrapolate the definition of a flexible market commodity to a more complex network setting, where we could take into account further characteristics that influence market development.In **Stage 3**, we study a network model that utilizes markets for the assignment of spectrum, but also focuses on the interactions among participants and how these account toward the final market viability assessment. To provide a solid basis for the development of the model in this stage, we appealed to the Institutional Analysis and Development (IAD) framework, and focused on the interactions between an action situation and the existing agents, environment and the applicable governance mechanism.

The agents in this model are now 1) Service Providers (previously SLRs), 2) Resource Providers (previously SLHs), and 3) Virtual Network Builders. Service providers are new market entrants or existing providers who need to obtain resources in the market to fulfill the demand of their customers. Resource providers are incumbents who have excess spectrum resources and have the option of making them available, in a common pool, for subsequent trade. The VNBs are a new addition to the model, and their characteristics are the result of adapting middleman theory concepts. As middlemen, the VNBs' objective is to form partnerships with existing service providers, learn about their resource needs, and obtain an appropriate set of resources from the market.

To explore this stage, we have studied two different sets of interactions: a) VNB – SP interactions and b) VNB – RP interactions. The first set of interactions solve the VNB – SP partnership forming problem. We address this task by utilizing matching markets theory and thus basing possible partnerships on the compatibility between SPs(VNBs) choices and

VNBs(SPs) preferences. Choices refer to the values assigned by SPs and VNBs to parameters regarding their operations (e.g., demand, reputation) and preferences refer to what SPs and VNBs are looking for in members of the opposite set.

Once the partnership forming process is over, each VNB learns the demand of its customers and the price they are willing to pay for resources. In this way, it can obtain an appropriate set of resources from the pool by participating in a sealed-bid auction. At the end of the market transactions, each VNB assigns the obtained resources to its customers and receives a payment for its resource aggregation services. Note that a VNB will receive a payment only from customers that received resources. Similarly, RPs obtain their payment for the resources they assigned. After all payments are made, each entity (i.e., SPs, VNBs and RPs) have the opportunity to adjust their advertised fees and payments in order to remain competitive in the market and maximize their surplus.

Results from this stage show that RPs' participation in the market is always profitable and that SPs pay their true valuation for spectrum resources. VNBs' payment analysis shows that their remunerations are consistent with their reputation. VNBs' surplus analysis shows that they are capturing the risk in the market, and that holding a large number of SP partners may cause them to incur in negative surplus. We also find that VNBs' activities are crucial for easing congestion in scenarios with a large number of participants. Indeed, by aggregating the demand of their customers, VNBs provide a better alternative for managing the geographical demand, and converting it into manageable market demand. It is important to note that in our model, there is no exchange of information between SPs and RPs. This factor alleviates information sharing concerns which may discourage entities from participating in the market.

Overall, the model of Stage 3 constitutes an important alternative for bootstrapping the spectrum market, and provides a detailed definition of a network where virtualized commodities can be traded.

1.1 CONTRIBUTIONS

The main contribution of this work is that, to the best of my knowledge, this is the first study that takes deeply into account policy, economics and technical frameworks for the analysis of spectrum markets. This permits to draw market viability conclusions from different angles, such as the need for and advantages from the definition of a homogeneous, spectrum-related commodity, and the economic implications of adding a middleman in the market. In addition, our analysis provides us with opportunities for the applicability of different governance methods.

In what follows, we list additional contributions that stem from this research work.

- The limitations associated with the lack of spectrum fungibility have been taken into account when utilizing "naked" spectrum as a market commodity. This analysis has also been presented as a Conference paper [2] in the Telecommunications Policy Research Conference in 2013.
- We provide an analysis of the benefits stemming from defining a homogeneous market commodity. Spectrum homogeneity is further supported by an existing technical framework. The work comprising Stage 2 of this dissertation has been presented as a Conference paper [3] in the Telecommunications Policy Research Conference in 2014.
- A complex, yet adaptive, model has been developed for the analysis of Stage 3. This model can be modified to account for different market structures, spectrum commodity definitions and technical scenarios.
- We present a new application of matching markets within the spectrum trading context, as a mechanism that permits market participants to form partnerships. This partnership-forming process further helps to explore how parameters advertised by market participants and their preferences influence their performance and, ultimately, market results.
- We present market alternatives that aim at reducing the burden placed on potential spectrum buyers at the time of expressing their resource needs. To this end, we assign the resource seeking and aggregating task to a more specialized entity, which in our study is represented by the VNB. The specific rules behind the behavior of the VNB are drawn from middleman theory.

- We have taken steps towards defining spectrum as a Common Pool Resource. This has been key to adapt this concept, studied in different spectrum contexts in [4, 5], to a spectrum trading setting. In turn, this provides us with ample opportunity to more thoroughly adapt polycentric governance concepts to the model at hand.

2.0 INTRODUCTION

Secondary Spectrum Markets have been analyzed as an alternative for promoting efficient use of electromagnetic spectrum. This is evident if we take into account one of the markets' underlying objectives, which is to assign resources to users who value them most. If this is achieved, we can ultimately overcome spectrum scarcity and aid in the provision of flexible spectrum-use mechanisms.

Creating secondary spectrum markets requires defining electromagnetic spectrum as a tradeable commodity, which does not prove to be a simple task. Spectrum is known by its physical multidimensionality, which has been key in the advance of communication systems. However, that same multidimensionality makes it difficult to commodify spectrum. For instance, given that spectrum varies in frequency, time, space, and several other dimensions (up to seven as presented by Matheson [6]), we cannot treat it as a fungible commodity. In other words, we cannot expect all frequency bands to serve the same purposes, nor to be in equal demand and supply conditions.

Spectrum, although a very valuable resource, is not sufficient for providing telecommunications services. Indeed, these services are the result of complex communication systems which are defined by enabling technologies as well as applicable economic and policy frameworks. In consequence, when looking at spectrum in the market context, we need to focus, not only in its physical multidimensionality, but also on the technical, economic and policy frameworks surrounding the system for which it is an input.

The goal of our research is to study the creation of secondary spectrum markets that take into account the aforementioned characteristics, constraints and frameworks. For this purpose, we have divided our study into three main stages.

1. In the first stage, we focus on a market mechanism where "naked" spectrum is traded. This calls for a particular focus on spectrum fungibility and the limitations it poses in the trading process. In general terms, this stage focuses on market analysis combined with the physical constraints inherent to electromagnetic spectrum.
2. The second stage aims at finding a spectrum related commodity that could permit to increase market thickness and hence liquidity. Additionally, our objective is to approach market settings that are more likely to appear in real-world scenarios. For this purpose, in addition to performing a market analysis and taking into account the physical constraints of spectrum, we incorporate an adaptive technology that permits us to explore additional opportunities.
3. The focus of the third stage is to analyze markets as part of a complex system where technology, policy and economic concepts guide the opportunities as well as the boundaries of our design.

Throughout the three research stages our focus is to define the conditions that lead to market viability i.e., the conditions where markets prove to be a viable solution for efficient spectrum use. We focus on this particular analysis, given that it permits us to evaluate various factors that are not only inherent to spectrum as a tradeable commodity, but also shed light on the three axes we deem essential to explore: technology, policy and economics. By performing this multidimensional analysis, we can comprehensively evaluate the success of markets, not only as stand-alone resource allocation mechanisms, but also as entities that actively react to the network operation and, in turn, provide the network entities with important information about their performance.

To make the multidimensional analysis of the market possible, we find it suitable to frame our market design on the recommendations provided by Roth in [7], where he argues that for successful market design we need to address thickness, congestion, safety and simplicity. Thickness implies that the market attracts a sufficient number of participants who are willing to engage in negotiations. Congestion can be a result of market thickness; however, it can be overcome by providing enough time for participants to transact, or by making these transactions fast enough so that participants can consider enough possibilities until they arrive at the most satisfactory ones. Finally, safety and simplicity will encourage users to participate in the market instead of transactions outside of it and avoid adopting strategic behavior that could reduce welfare.

The feasibility of the aforementioned recommendations is tightly linked to the technical, economic and regulatory characteristics of the entire system. We shall remember that technology will determine what resources can be shared, how flexible they can be and thus how they can be transferred to other users. In turn, the underlying policy framework should allow for new technologies to be deployed and dictate the rules for the sharing process, including the participants' allowed behavior or in more general terms, how users are allowed to interact with each other. In consequence, the characteristics derived from technology and policy, together with the market design guidelines can provide us with a comprehensive framework to analyze the overall market and network viability. Figure 1 summarizes the links that exist between markets and the accompanying technical and regulatory frameworks.

In this work, we take into account these links in order to establish the characteristics of our agents, the workings of the market model and the details of the environment where the market transactions take place.

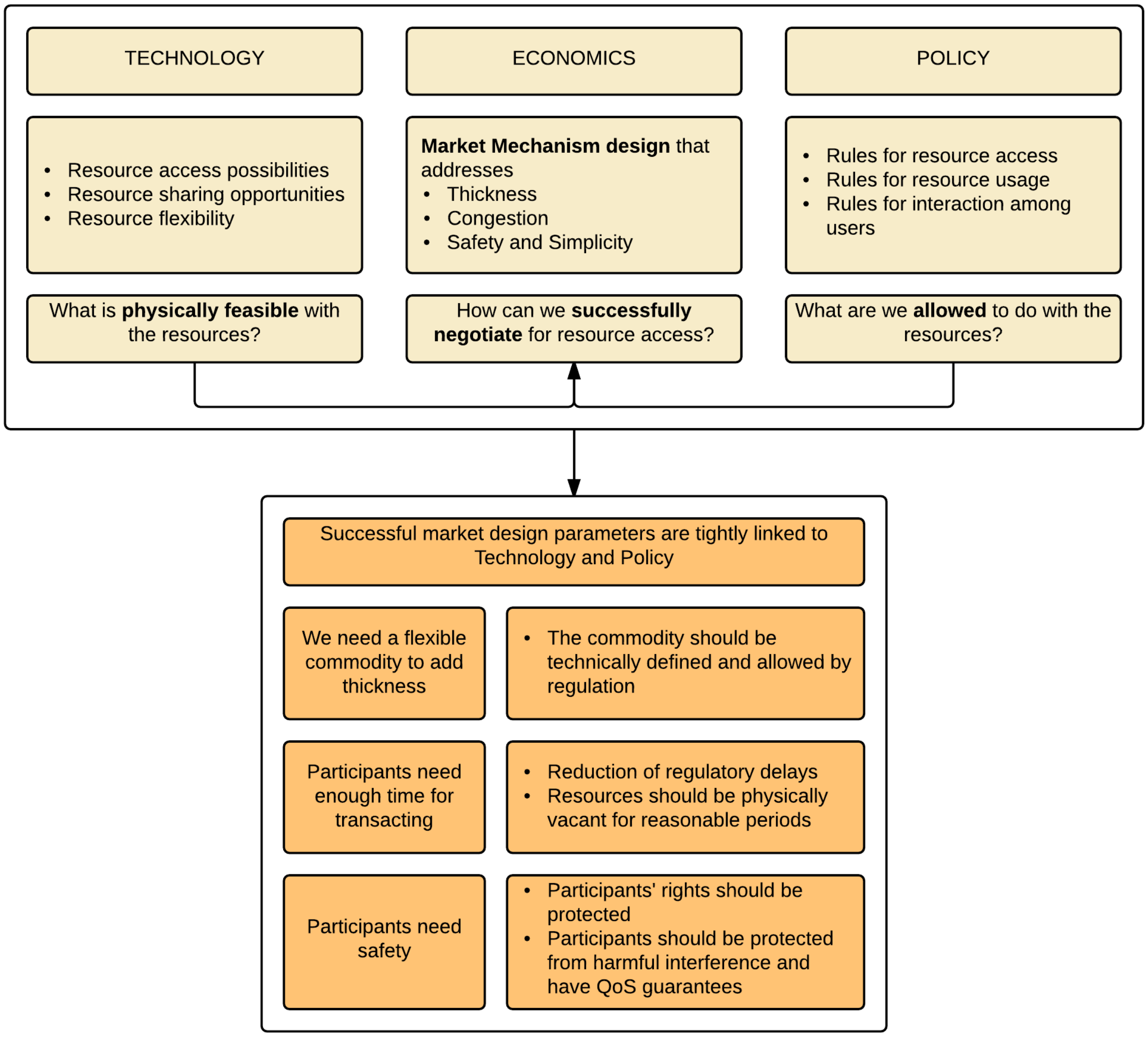


Figure 1: Comprehensive scenario for the analysis of secondary spectrum markets

3.0 BACKGROUND

Throughout the literature, secondary spectrum markets are analyzed from technical, regulatory and economics perspectives. Indeed, these aspects are key to defining the boundaries for spectrum markets design. The research path that we trace in this dissertation aims at reaching the point where technology, policy and economics converge, pointing us to feasible and perhaps successful solutions for the development of secondary spectrum markets. In sections 3.1, 3.2 and 3.3 , I elaborate on specific works that show the aforementioned approaches and how these account towards the definition of the core problem addressed in this dissertation.

In addition to conceptual influence stemming from existing literature, I find it important to provide background information on the mechanism that we have chosen towards modeling and analyzing the market scenarios of interest. In this manner, section 3.4 elaborates on Agent-based modeling and its suitability for this work.

3.1 TECHNICAL BACKGROUND

The physical characteristics of electromagnetic spectrum make it a multidimensional resource. Various authors have defined multiple levels and dimensions in which spectrum can vary [6, 8]. For instance, in [6], Matheson and Morris define seven of these dimensions: frequency, time, three dimensions of location (latitude, longitude and elevation) and two dimensions of arrival (azimuth and elevation angles). This multidimensionality implies that spectrum cannot be perfectly substituted by another frequency band, unless both frequencies share exactly the same characteristics in all possible dimensions.

Deploying markets in this context would imply that we would require a "one-to-one" match of demand and supply, or as expressed in [9], "[s]uccessful secondary market transfers require an alignment of the buyers' demands for spectrum of a particular dimension with the willingness of spectrum holders to supply spectrum in the same dimension". This means that multiple secondary markets would be required, one for each type of spectrum. Consequently, the information obtained from one market would not be indicative of the characteristics (e.g., market price, supply, demand) of the market for another frequency [10].

To address this issue, the authors in [11] have studied conditions under which spectrum is replaceable or fungible. Further, the authors developed *fungibility scores*, which are quantitative measures for spectrum fungibility. In this manner, market participants (in our context) would have a means to assess to what extent the resources available in the market fit their particular requirements and how to value them. Nevertheless, the assessment on spectrum fungibility does not relieve the lack of spectrum replaceability; and, in order to deploy secondary spectrum markets, it would be ideal to count on a flexible resource that could adapt to the needs of various types of market participants. For this purpose, we explore a relevant technical framework that could help in the definition of a flexible spectrum-related commodity, while taking into account the physical constraints inherent to electromagnetic spectrum.

3.1.1 Enabling Technology

Diverse technologies have been deployed with the objective of adding flexibility in the use of electromagnetic spectrum: spread spectrum mechanisms, multiple-access techniques, supercell and mini/micro cell deployments, cellular reuse, directional antennas for spectrum reuse, software-defined and cognitive radios, among others. These technologies have been key for enabling multiple spectrum sharing scenarios as they focus on allowing various users to access spectrum simultaneously and exploiting resources that would be otherwise underutilized.

In the spectrum markets context, we are interested in a technology that could permit us to create markets, with sufficient supply and demand, where users have multiple options from which to choose (i.e., thick markets). To this end, we focus on a technical mechanism that could allow us to alleviate some of the physical limitations of electromagnetic spectrum and the impact they have in the definition of spectrum as a commodity. We find promising opportunities stemming from wireless network and resource virtualization.

Virtualization has been widely studied in the Computer Science context, where it is defined as “any form of partitioning or combining a set of network resources, and presenting (abstracting) it to users such that each user, through its set of partitioned or combined resources has a unique, separate view of the network. Resources can be fundamental (nodes, links) or derived (topologies), and can be virtualized recursively. Node and link virtualization involve resource partition/combination/abstraction; and topology virtualization involves new address spaces” [12]. In the Wireless Network context, virtualization is currently under exhaustive study which makes it difficult to find a unified definition. Indeed, it is defined according to the area of application and the scope it covers. Nevertheless, focusing on the previously presented concept, and adapting it to the definition of wireless resources, we shall expect that the different components of the network will be partitioned, combined and abstracted, yielding multiple virtual instances. In turn, each of these virtual instances may be different from the other, depending on the partition or abstraction to which they belong. Consequently, with each virtual network, we would have the notion that we are dealing with a new network, different from the original [12].

One of the major advantages from virtualization is that each virtual instance could have the ability to operate without being aware of the underlying virtualization process. In this light, individual virtual networks could be running operator-specific protocols and architectures, which may differ from one co-existing virtual instance to another [13, 14]. For this to be possible, we require a strict level of isolation among virtual instances, which still remains a significant challenge for this technology.

On a more practical view, in the same manner as the technologies mentioned at the beginning of this section, Wireless Network Virtualization (WNV) also promises to provide spectrum access opportunities to a greater number of users by presenting increased alternatives for spectrum use, sharing and assignment. Additionally, it is expected that virtualization will allow operators to make changes in their current network (through expansion or shrinkage), as needed, without incurring in prohibitive costs [14].

3.1.1.1 Adding resource flexibility through Virtualization In our work, we are interested in analyzing the opportunities in terms of flexible-use of resources that can be derived from virtualization. Network virtualization in general provides a convenient mechanism for sharing resources among a wide set of users, while permitting integration with distinct virtualized substrates [15]. In fact, as pointed out in [13], a fully-virtualized and open infrastructure will allow to share infrastructure resources and it will also make it possible for multiple virtual instances (e.g., Mobile Virtual Network Operators (MVNOs)) to deploy different protocol stacks over the same radio resources. Additionally, virtualization promotes the decoupling of service providers' functions from those of the infrastructure providers. This would make it possible to decouple resources from services [16]. It is also important to note that virtualization can be paired with sophisticated underlying technologies (e.g., LTE-Advanced) in order to add granularity in the definition of the fundamental units of virtualization. In turn, these virtualized units can be allocated via multiple access, multiplexing and spectrum slicing techniques [13]. Along these lines, the authors in [13] present the different degrees of virtualization that can be achieved, and the corresponding levels of granularity depending on the aspects (i.e., scope and depth, underlying wireless technology, and virtualization of the client or infrastructure side of the network) and perspectives (i.e., flow-based, protocol-based or spectrum-based) of the virtualization process and their possible combinations.

From the description above, greater levels of flexibility can be reached from deeper levels of virtualization and its pairing with additional technologies. Indeed, in current literature, we find various efforts that merge virtualization with the creation of resource pools, with the objective of increasing efficiency in the utilization of resources and developing cloud-like environments for spectrum access [17–20]. The particular analogy with the cloud emphasizes the possibility of creating the illusion of an infinite amount of resources, which are available on demand, without the need to incur high upfront commitments and where users have the ability to invest on a short-term basis or as needed [20]. These characteristics open up a series of opportunities for the deployment of new, service-driven networks, where operators can obtain resources from the providers that best suit their requirements [20, 21].

Resource pooling has the added benefit of reducing scarcity. As mentioned in [22], as contributions of spectrum to the pool increase, lower is the probability that a given user, who has access to the pool, will experience spectrum shortages. Increased spectral efficiency in resource pooling settings results from the fact that they permit to overlay new radio systems on existing ones without requiring changes in the licensed system that is currently in place [18]. Note that higher benefit can be achieved in environments (i.e., geographical areas) where there is a large number of infrastructure providers (e.g., dense urban areas with overlapping cellular networks) [16].

Placing the definition of virtualized commodities within the spectrum markets context, we find that market mechanisms (i.e., auctions) are considered as effective approaches for resource assignment [23,24]. Indeed, as pointed out in [21], when combined with virtualization, we expect auctions to be performed on continuous goods, rather than discrete items; in other words, spectrum requesters may express their requirements, not in terms of a specific item, but instead in terms of their particular constraints and conditions. In consequence, spectrum requesters can utilize the auctioned resources towards the provision of more specialized services[1].

From a technical perspective, we consider that the flexibility opportunities provided by virtualization, especially when combined with underlying wireless technologies that further enhance this flexibility, can represent a significant advantage for developing a spectrum-related market commodity and framing secondary markets within a plausible, technical environment.

3.2 ECONOMICS BACKGROUND

3.2.1 Economics perspective on Secondary Spectrum Markets

When secondary markets are deployed, we can ensure that, with changes in demand and supply, spectrum will migrate to more efficient uses, which include parties outside of the initial resource allocation (i.e., the primary market) [9]. Note that the prices set through

[1]A more detailed description of auction mechanisms utilized in the spectrum context is presented in section 3.2 and chapter 4.

the market have the ability to capture information regarding demand and supply in such a way that outperforms the capabilities of a centralized entity [25]. Hence, we expect markets to reflect the actual interaction of buyers and sellers and thus more accurately portray the valuation of resources.

Defining spectrum markets requires us to analyze what are the costs associated with this activity. In [26] the author points out that the success of a secondary spectrum market depends on choosing a trading mechanism that minimizes the transaction costs and maximizes the traders' surplus[2]. Transaction costs may stem from diverse factors. For instance, these can be the result of laying out the ground for the trading activities; the time and efforts spent on negotiating in order to reach an agreement between the market participants; carrying out enforcement and administrative processes, among others [25, 27]. Additionally, it is important to consider that transaction costs are proportional to the number of participating entities [8].

An additional factor that is tightly linked to cost generation is the presence of externalities. Indeed, some economic activities generate incidental benefits (external economies) or harm (external diseconomies) to third parties for whom these benefits/harms are not intended. In this light, the total costs and/or benefits resulting from the economic activity do not match the costs incurred or the benefit obtained by the primary (intended) actors [8, 27, 28]. In this context, we refer to transaction costs as the costs that a market participant needs to incur in order to negotiate with other participants to alleviate the externalities (i.e., compensate others for the effects of unintended consequences) or to internalize these external costs.

As Coase states, "[o]nce the legal rights of the parties are established, negotiation is possible to modify the arrangements envisaged in the legal ruling, if the likelihood of being able to do so makes it worthwhile to incur in the costs involved in negotiation" [29]. In this manner, if it is possible to reach better outcomes through negotiation, the involved parties may start the negotiation process. Nevertheless, the costs of these negotiations should be lower than the benefit that can be obtained from the rights granted to the users.

[2] The Oxford Dictionary defines surplus as: "an excess of income or assets over expenditure or liabilities in a given period, typically a fiscal year".

Consequently, each of the parties will be willing to invest in further negotiations as long as this represents a positive revenue or explicit benefit.

In a majority of cases, market mechanisms in the form of auctions are utilized for the assignment of spectrum related resources. In fact, the first efforts for the deployment of spectrum markets took place in the 1990s with the adoption of auction mechanisms for the assignment of spectrum licenses [30]. Nowadays, there is vast work in terms of auction and mechanism design which allows to define different types of auctions for different types of resources.

Markets are not confined to auction design. In fact, Roth defines two extremes in which markets can fall: commodity and matching markets [31]. In commodity markets, there is no differentiation among resources and a participant's acquisition capabilities are given by whether they can afford resources or not. On the other hand, "a market involves matching whenever price isn't the only determinant of who gets what." [31]. The classical work on matching markets was developed around the college admissions and marriage stability problems [32]. In this work, a deferred acceptance algorithm is utilized to match students with colleges and women with men. Matching markets were further utilized for the "match" process for medical students who were entering their residency stage and for matching kidney donors with patients in need of a transplant [31, 33]. The study of matching markets has been expanded to a significant number of applications, which include monetary exchanges, and the ability for one-to-many and many-to many matches [34–36]. In [31], Roth asserts that auctions can be regarded as matching markets where sellers are matched with those buyers who most value what is being sold. Also, according to Roth, one of the benefits of auctions is their signaling capabilities. Consequently, in auctions "the high bid not only signals how goods should be allocated but also pays the seller of the goods" [31].

We make special emphasis on auctions in the context of matching markets because matching makes the markets more expressive or "personal". Indeed, matching markets are those where "prices don't do all the work, and in which you care whom you deal with". [31]. This is key for the type of spectrum market analysis that we aim at developing in this dissertation.

3.2.2 Market Design

In this section we present important factors that lead to successful market design. According to Roth [7],“[t]o work well, marketplaces have to provide thickness, i.e., they need to attract a large enough proportion of the potential participants in the market; they have to overcome the congestion that thickness can bring, by making it possible to consider enough alternative transactions to arrive at good ones; and they need to make it safe and sufficiently simple to participate in the market, as opposed to transacting outside of the market, or having to engage in costly and risky strategic behavior.”

For thickness to be addressed, participants should be ready to transact with one another. A well-known example of a thick market is the Amazon marketplace, where there are many participants who are ready to participate in many different types of transactions. Furthermore, this thickness results in more sellers being attracted by all the potential buyers and more buyers coming to this marketplace due to the increasing variety of sellers [31].

Overcoming congestion requires providing participants with enough time or with fast-enough transactions to consider sufficient alternative transactions in the market before arriving at those that are satisfactory [7]. Congestion becomes a salient problem in markets where the transactions are heterogeneous and the offers are particular to specific areas (i.e., cannot be made to the entire market). In congested markets, participants may react in ways that damage other market properties. For example, to avoid congestion participants may try to gain time by starting their transactions before others. This would lead to a series of thinner markets happening at various times, rather than one single, thick market [7].

There are two important factors that may deter bidder entry to an auction: risks and unmanageable complexity [7]. If it is too risky to participate in the market, “individual participants may try to manage their risk in ways that damage the market as a whole” (e.g., employers making exploding offers before applicants can assess the market, one party trying to prevent their trading counterparts from receiving other offers, etc.). In the second case, when markets are excessively complex, participants are not able to formulate the bids and assess their opportunities at each market stage, thus slowing the auction [7].

3.3 POLICY BACKGROUND

3.3.1 Allowing the creation of Secondary Spectrum Markets

The barriers to the use of spectrum inherited from legacy spectrum management and regulatory methods led to the artificial scarcity of this resource. In the particular case of the U.S., the initial regulatory approach adopted by the Federal Communications Commission (FCC) did not allow for significant modifications on the resource use prescribed by the spectrum license. In addition, regulation limited the license transferability, which translated in reduced opportunities for profitable license resale [28]. Hence, the inefficiency in resource utilization, and the resulting scarcity, encouraged regulators (e.g., the FCC[3]) to deploy policy mechanisms which target at providing opportunities for flexible spectrum use.

Flexible use mechanisms are comprised within two poles: exclusive-use and commons approaches. These two approaches aim at granting users sufficient autonomy to choose the uses and services to be provided with spectrum, the technology appropriate to the spectrum environment and the right to transfer, lease or subdivide spectrum rights [37]. Elaborating on the latter, we could take advantage of secondary markets in order to negotiate the transfer of resources (or rights to use them) from one user to another. In fact, in areas where scarcity is the rule and spectrum is subject to competing demands, market approaches would be especially suitable for assigning this resource to its highest valued uses (and users).

In a continuous effort to provide users with multiple flexibility approaches, regulators worked toward enhancing the opportunities for deploying secondary markets in spectrum. In the "Second Report and Order for Promoting Efficient Use of Spectrum through Elimination of Barriers to the Development of Secondary Markets" [38], the Commission modified the rules for leasing spectrum, which provided licensees with further opportunities to cooperatively share their resources through market mechanisms. The leasing arrangements could comprise any amount of spectrum within the geographical area assigned to the licensee and any period within the term of the license [37]. Two secondary market configurations were proposed: spectrum manager leasing and de facto transfer lease. The difference between

[3] In this document we refer to the Federal Communications Commission interchangeably as the FCC and the Commission.

these configurations lies "on the scope of the rights and responsibilities to be assumed by the lessee" [37]. This maintained a record of the accountability for the use of spectrum. In this manner, at the core of spectrum markets configurations, we have users negotiating for spectrum access or usage rights, and achieving mutually agreeable terms.

The Spectrum Policy Task Force (SPTF) was confident that secondary markets would present the opportunities for encouraging spectrum users to employ and develop novel technologies (e.g., opportunistic access technologies), find ways to reduce transaction costs and ultimately achieve efficient spectrum usage. As pointed out by the FCC, the purpose of secondary markets has not been to replace the existing spectrum allocation process; instead, it is considered that "...a robust and effective secondary market for spectrum usage rights could help alleviate spectrum shortages by making unused or underutilized spectrum held by existing licensees more readily available to other users and uses and help to promote the development of new spectrum efficient technologies".[4] Additionally, in his statement regarding secondary markets, Professor Cramton stated that "secondary markets are essential for the efficient and intensive use of spectrum. Secondary markets identify gains from trade that are unrealized by the primary market which in this case is the FCC spectrum auctions."[5] As expressed by Coase, in [29], when resources are assigned administratively, agencies do not possess all the information that is relevant for the business owners who will be making use of those resources. Consequently, the success of spectrum markets is derived from the immediate knowledge of the market participants in terms of what resources are more appropriate for their services and the valuation they have for them [28].

The FCC, in its Policy Statement from December 2000, presented five essential elements for a market system to operate effectively:

- Clearly defined economic rights
- Full information on prices and products available to all participants
- Mechanisms for bringing buyers and sellers together so that transactions take place with minimum administrative costs and delays

[4]Federal Communications Commission Policy Statement in the Matter of Principles for Promoting the Efficient Use of Spectrum by Encouraging the Development of Secondary Markets. p. 1.

[5]Professor Peter Cramton Statement at the Secondary Market Forum of the Federal Communications Commission. May 31, 2000.

- Easy entry and exit to/from the market by both, buyers and sellers.
- Effective competition, with many buyers and sellers.

According to Coase and Hazlett, the creation and enforcement of property rights was necessary and sufficient for economic development [29, 39]. Indeed, when trading spectrum, it is impossible to transfer or lease the actual frequency bands; instead, we trade rights over the available resources. Consequently, rights need to be properly defined [28, 29] and once these are established, negotiations are likely to take place and modify the arrangements dictated by the regulatory frameworks, as long as the benefits derived from the modification of rules outweigh the costs inherent to the actual negotiation processes [29].

In the following section we explore more deeply spectrum rights and the relevant enforcement and governance systems.

3.3.2 Spectrum Rights and Governance systems

"When property rights are well defined and transferable in the absence of transaction costs, all government allocations of property rights are equally efficient, because interested parties will bargain privately to correct any externalities." [40]

Spectrum rights are a key aspect that defines what can be done with spectrum resources. Even if incumbents obtain licenses directly from the FCC, they are subject to the regulatory framework applicable to those licenses in order to deploy certain types of services and to subdivide, transfer or lease their current assets. For instance, in [31], Roth presents an interesting analogy: "you may own the land on which your home is built, but local zoning laws may prevent you from selling food or opening a nightclub there." In the same manner, spectrum licensees are still subject to the rules deployed by the FCC in order to define resource usage boundaries.

The flexible-use policy framework presented in the previous subsection provides spectrum licensees with less strict boundaries for the use of the licenses that they have been granted [9]. However, as Coase suggested, sufficient property rights in spectrum should be created, so that after being sold to private owners, they could, in turn, freely buy, sell and lease their own resources [25, 29]. Cui et al. [41] have pointed out that in sharing environments, we are not dealing with the transfer of individual rights. Instead, *bundles of rights* are transferred among the sharing parties, which can define usage protocols and procedures across multiple dimensions.

In all cases, the boundaries of the transferred rights should be enforced in order to remain meaningful [10, 25]. In this light, we find conservative and flexible approaches, which limit or enhance the rights transferred to the different users. For instance, when there are federal constraints at stake, exclusion zones could be defined in order to limit the access of commercial spectrum users [42]. Nonetheless, every type of enforcement has its costs; consequently, the higher the value of the protected resources, the higher the cost that resource owners are willing to incur for enforcement purposes [42].

In the specific spectrum sharing arrangement that we study, we find it suitable to explore an alternative type of governance in order to define the rights that should be shared and how to enforce them. We define this governance process in the following section.

3.3.2.1 Polycentric Governance and Common-pool Resources According to [43], "[c]ommon-pool resources are systems that generate finite quantities of resource units so that one person's use does subtract from the quantity of resource units available to others. Most common-pool resources are sufficiently large that multiple actors can simultaneously use the resource system and efforts to exclude potential beneficiaries are costly."

In a spectrum trading environment, where it is likely to find multiple buyers and sellers opting for a common set of resources, we can expect electromagnetic spectrum to match this definition of common-pool resources. This is especially true if we take into account its high subtractability of use and the difficulty to exclude arbitrary users from accessing it[6] [44]. As a consequence, we might expect *collective-action problems* to occur. According to Ostrom [43], an important way to deal with these collective-action problems is to adopt a polycentric approach through the development of systems of governmental and non-governmental organizations working at multiple scales. Indeed, polycentric systems can be defined as "the organization of small-, medium-, and large-scale democratic units that each

[6] See [44] for a detailed definition of spectrum as a common pool resource. In this work, the authors point out that the subtractability and excludability characteristics of spectrum are mainly associated with the underlying technology, which provides different alternatives for spectrum use and access.

may exercise considerable independence to make and enforce rules within a circumscribed scope of authority for a specified geographical area" [45].

By applying polycentric governance concepts, we avoid the mistake of designing systems with one single point of failure. Indeed, we can take the polycentric approach as a means for different entities in the network to learn from local knowledge, obtain feedback from their own local policy changes and learn from the experience of other parallel units. In this way, we can create a system that is responsive to the environment threats at multiple scales, thus being able to compensate the failure of some units with the successful response of others [45]. It is important to note that there is a level of redundancy added in the network, which is actually an alternative for keeping systems running under the presence of external or internal malfunctions [45].

Ostrom has developed eight design principles for systems that operate under the conception of common-pool resources and polycentric governance [43, 46]:

1. Clear definition of group boundaries.
2. Match the rules that govern the use of common goods to local needs and conditions.
3. Ensure that those affected by the rules can participate in modifying them.
4. Make sure that the rule-making rights of the community members are respected by external authorities.
5. Develop a system to monitor members' behavior, which should be carried out by the community members.
6. Use graduated sanctions for rule violators.
7. Provide accessible and low-cost mechanisms for dispute resolution.
8. The responsibility for governance of the common resources should be built in nested tiers, from the lowest level up to the whole interconnected system.

In [47], Agrawal has synthesized the facilitating conditions identified by Ostrom and other authors, thus providing a more comprehensive approach. These facilitating conditions have been further adapted to a spectrum sharing approach in [44].

3.4 MODELING BACKGROUND

In general terms, models permit us to work with representations of the real-world. Indeed, a model is a "simplification of the real world and does not contain all of the details and inconsistencies that are present in the real world" [48, 49].

Given that secondary spectrum markets have not yet emerged in the real world, modeling appears as a suitable tool for their representation. Due to the nature of spectrum markets, it is necessary to utilize a modeling tool that permits to capture the interactions among market participants and their approach toward the available resources. In turn, individual interactions can be analyzed from a global perspective, thus permitting to assess the results obtained. For this purpose, we present agent-based modeling as an appropriate tool for modeling and analysis of spectrum markets.

3.4.1 Agent-based Modeling

According to Wilensky et al. [49], "[a]gent-based modeling is a form of computational modeling whereby a phenomenon is modeled in terms of agents and their interactions". Agent-based modeling (ABM) parts from the premise that most world phenomena can be modeled through agents, an environment and the corresponding agent-agent and agent-environment interactions.

Generally, agent-based models have been utilized in social and natural sciences to study phenomenons such as the spread of diseases, traffic patterns, social interactions and peer-influence, among others[7]. In the Economics domain, we find Agent-based Computational Economics (ACE), a branch of ABM in which agents "can be represented as interacting goal-directed entities, strategically aware of both competitive and cooperative possibilities with other agents" [50]. This is possible due to the autonomy that characterizes ABM agents. More recently, we have evidenced a widespread adoption of ACE for modeling electricity markets that adapt to the electricity industry restructuring process [51].

In general terms, ABM relies on the modeling of agents, their interactions and the en-

[7]These are just a few examples extracted from the model library of the ABM tool: NetLogo. Information on NetLogo can be found at https://ccl.northwestern.edu/netlogo/docs/

vironment where they exist. In what follows, we provide a brief definition of each of these entities and their role in an agent-based model.

3.4.1.1 Agents constitute the basic unit of ABM. They are mainly defined by their properties (i.e., characteristics or behavior) and their actions.

Agents can be mobile, stationary or connecting agents. The latter refer to agents that link two or more agents and can be utilized to represent relationships between the agents they connect [49]. The characteristics chosen for each of the agents depend on the role they play within the modeled environment. In this light, agents attend to different levels of granularity, which define their complexity. Indeed, an agent's granularity represents the "fundamental level of interaction" that is applicable to the phenomenon we are modeling [49].

A key factor that differentiates agent-based from other modeling approaches is the fact that ABM agents can be designed with relatively more autonomy [50]. As stated by S. Franklin [52], "[a]n autonomous agent is a system situated within and part of an environment that senses the environment and acts on it, over time, in pursuit of its own agenda and as to effect what it senses in the future." We can thus refer to agents as goal-oriented and adaptive entities. Agents are goal oriented in that they seek to maximize their payoff or utility, and they are adaptive in that they have the ability to learn which actions to take in order to maximize their payoffs and achieve their goals [53].

3.4.1.2 Environment The modeling environment refers to the general conditions, or the habitat, surrounding the model agents. Given that this is the "area" where agents exist and interact, the environment does influence the decisions of an agent. In turn, agents' decisions and actions also affect their environment [49].

3.4.1.3 Interactions Agent interactions may refer to their relationship with other agents or to self-interactions. In this way, an agent does not only have the capabilities to interact with others, but it is also able to update its behavior according to its own experience [49]. In the particular case of ACE, events are driven by the interaction of agents, after a set of initial conditions have been specified. In this manner, ACE relies on the outcome from agents' interactions in order to determine whether the system reaches an equilibrium state over time [50]

3.4.1.4 Model Analysis

Agent-based modeling attends to general modeling analysis approaches. Indeed, agent-based models are subject to sensitivity analyses, validation and replication techniques. These features permit us to study the impact of varying model parameters in the results obtained; how the model agents and environment resemble real-world scenarios; if the results correspond to scenarios that are likely to emerge in the real world and finally; whether the results obtained are actually due to the interaction of agents instead of possible mistakes or oversights in the execution of the model.

3.4.2 Agent-based Computational Economics (ACE)

As previously mentioned, the specific branch of ABM that deals with economics research is Agent-based Computational Economics, ACE. This modeling technique has appeared as a response to the limitations presented by traditional economic modeling methods. The latter make it difficult, or impossible, to model factors that are characteristic of economies in general, such as imperfect competition, strategic behavior, asymmetric information, multiple equilibria, among others [50, 53]. Hence, according to Tesfatsion [54], ACE permits the "modeling of economic systems as locally-constructive sequential games." In this way, ACE permits to model economic processes as "open-ended dynamic systems of interacting agents".

The author in [54] has defined a set of modeling principles that frame ACE. These principles suggest that users of these methods can explore how changes in initial conditions may affect the outcomes in dynamic systems. Note that ACE relies on agents' definition, scope, their adjustment to local conditions and does not regard the modeler as an active participant while the model is executed. In fact, modelers are deemed observers, analyzers and reporters of the model outcome.

The objectives behind ACE are classified within four axes [53, 54]:

- Empirical understanding: seeks causal explanations to global regularities and analyzes how these result from agents' interactions at a micro-scale.

- Normative design: is concerned with models that capture properties of a system designed with a particular objective in mind. As the model develops, the modeler can observe whether the outcomes are efficient, fair and orderly in spite of agents' behavior.
- Qualitative design and theory generation: create phase portraits or representations of possible state trajectories starting from all possible initial states. This permits to "find necessary conditions for global regularities to evolve" [53].
- Methodological advancement: improve existing tools and develop new ones that permit the advancement of ACE-based research. This includes the development of "programming, visualization and empirical validation tools" [54].

4.0 RELATED WORK

In this chapter we provide an overview of the research that has been done regarding the development of spectrum trading mechanisms. To better explain the different factors that play significant roles in spectrum trading, we have divided this chapter in three sections: trading mechanisms, trading environment, and trading beyond spectrum. We conclude this chapter by presenting a summary of certain constraints, challenges and benefits that seem to prevail across the literature.

4.1 SPECTRUM TRADING MECHANISMS

According to Cramton [30], auctions are considered transparent mechanisms for the assignment of spectrum licenses. By utilizing auctions, all parties are aware of the identity of the auction winners and why they obtained the resources. Furthermore, when auctions are properly designed, there is a salient tendency for resources to be assigned to the parties that value them most, in addition to the fact that regulatory entities may obtain their expected revenues from this process.

Auctions have been utilized in various spectrum and wireless resource sharing (and trading) scenarios. Generally, auctions are utilized for resource allocation and price discovery purposes and the type of auctions chosen depends on the complexity of the frameworks that are analyzed. Indeed, we find applications of uniform pricing auctions [55], reverse auctions [56]; combinatorial (and reverse combinatorial) auctions [57, 58]; sequential auctions [59], double auctions [60], among others. Additional combinations of auction types include the clock-proxy auction [61], which is an approximation mechanism for solving combinatorial auctions, and the quantized-bid proportional auction [62], which has been applied in a spectrum "micro-trading" environment. The utilization of different types of auctions responds to the important constraints that should be taken into account for auction design. The authors in [63] provide an overview of these constraints, which include the winner's curse, collusion, the complexity of solving the actual auction problem, among others.

Combinatorial auctions have been chosen by a large number of researchers given the opportunities that they present for resource assignment. Nevertheless, their complexity has led to modifications of this type of auctions, which make their solution manageable. These modifications include the pairing of combinatorial auctions with other types of auction mechanisms or the simplification of the process to choose the appropriate set of resources [57, 58, 61, 64]. For instance, as presented in [57], participants can utilize their local decision-making capabilities in order to choose the optimal set of resources before placing their bids.

Taking a step further in auction design, in [65], Forde et al. elaborate on the parameters that make current auctions inflexible, or what they refer to as auctions that manage spectrum into scarcity. Further, the authors propose a combinatorial clock auction mechanism where participants can post "expressive bids" that allow them to bid for what they really need instead of opting for a limited set of resources.

Broader applications of game theory are also found in the spectrum trading context. These methods aim at further modeling the behavior and strategies of the market participants. These can be used to improve bidding strategies and adapt them to the particular environment where trading takes place. [62, 66–68].

4.2 SPECTRUM TRADING ENVIRONMENT

We find a comprehensive and detailed approach on a novel mechanism for the utilization and assignment of virtualized wireless resources in [19]. In this work, the authors propose the creation of service-driven networks where resources belonging to existing providers or incumbents are pooled and offered to new service providers through an intermediate entity (e.g., broker or middleman). The authors envision the utilization of combinatorial auctions for the allocation of resources, which are aggregated according to the particular requirements of the services covered by each new provider.

In [62], the authors develop a spectrum "micro-trading"[1] approach as a means to enable spectrum trading on a micro-scale in "at least three-dimensions: micro-spatial, micro-temporal and micro-frequency scales". For this purpose, the authors consider auctions with short spectrum lease durations (i.e., 15 minutes). This permits to account for the mobile operators' fast changing demand throughout the day. For the actual development of this approach, the authors utilize a simulation study and present results which demonstrate the performance and viability of a spectrum micro-trading market which could be used to improve the spectrum utilization and performance of mobile operators. The following metrics are utilized for evaluating the viability of markets in the spectrum micro-trading scenario: liquidity, trading volume, spectrum price, profitability, blocking ratio, spectrum allocation efficiency, spectrum allocation delay, interference temperature, user experience and social welfare.[2]

The commodity traded in the aforementioned environment responds to the spectrum micro-trading pixelation model introduced in [69, 70]. This approach permits to define spectrum in terms of pixels, each of them having three dimensions: micro-space, micro-frequency and micro-temporal. Evidently, the minimum tradeable unit is one pixel. Given this generalized commodity definition, the authors do not consider it necessary to make an explicit differentiation on the underlying frequency band; however, pixels are differentiated in terms of the physical characteristics of the environment where they are defined. To test the model proposed in [69], the authors implemented a simulator based on multi-agent reinforcement learning and focus on the trading of TV white space (TVWS) spectrum.

El-Refaey et al., in [55], utilize the aforementioned spectrum commodity definition in order to develop an auction mechanism for the assignment of time-frequency units in a cloud-based network. Similarly, this trading environment contemplates the utilization of a mediator or broker, which is in charge of handling the auction stage. The computational

[1] Micro-trading is defined as "the possibility to trade spectrum resources on the micro-scale in one or more of the spatial, temporal, or frequency dimensions" [62].

[2] As pointed out by the authors, these evaluation metrics have been defined in the EC project QoSMOS [69].

complexity of this auction is reduced by performing a location checking process, in which the location of the available time-frequency units is contrasted to the location of those requesting these units. In this way, the set of resources offered to a given buyer is restricted to those that match its location.

In [71] and [72], the authors utilize resource pooling and trading in order to achieve more efficient utilization of optical network resources. Additionally, these mechanisms are applied to a virtualized network environment, where the physical network provider (carrier) is in charge of mapping virtual nodes to physical optical nodes and virtual links to physical optical paths in order to assign resources to the different users, or virtual optical network (VON) providers. In both works, the authors rely on a Stackelberg game for evaluating the proposed mechanism.

In [73], the authors point out important factors, from an economics and engineering perspective, that would motivate and constrain the development of spectrum markets. The model they propose is a two-tiered market, where "the upper tier consists of spectrum owners that trade spectrum assets analogous to land rights, and the lower tier consists of spot markets for limited-duration rentals of spectrum assets from owners at particular locations". In terms of the tradeable commodity, from a technical perspective, the authors propose the definition of an adaptive power mask that could vary according to time, space or frequency. From an economics perspective, the authors emphasize on the need for an appropriate definition of property rights, which should be clear and easily enforceable, transparent, flexible and it should also facilitate efficient allocations.

In [63], the authors focus on the creation of a spectrum broker as the central entity for the development of spectrum markets in TV white spaces (TVWS). The responsibilities of the broker include: "planing the possible broad uses of the available spectrum in the TVWS; packaging the spectrum for short-term disposal through trading mechanisms; serving the broker's customers with spectrum-leasing contracts; and acting as the port of call to handle interference caused by its customers to the primary DTV systems or between its customers themselves". To fulfill its duties, the broker acts under two different modes: merchant and auction, which depend on the level of resource supply and demand in the market. In this light, the authors consider an analysis of opportunity costs for spectrum trading and calculation of reserve prices that could accompany the broker's operational modes.[3]

In [74], the authors analyze the competition in secondary spectrum markets from the perspective of resource providers instead of the requesters. The particular environment where trading takes place is a *private commons* setting. To this end, the authors take a game theoretic approach where they identify the market equilibrium prices. Their analysis shows that providing secondary access represents an immediate revenue for the providers which can be contrasted to its opportunity cost due to the primary revenue that is lost. Additionally, the authors conclude that the market equilibrium prices in the studied setting point to a price war won by the resource providers with the lowest break-even prices.

Gao et al. [75] consider short-term secondary spectrum trading between one seller and multiple buyers. The authors study a hybrid spectrum market with guaranteed contracts, i.e., futures market, and spot transactions, i.e., spot market, where the goal is to maximize the expected profit of the spectrum seller under stochastic network information. Their results show that when information is symmetric, the optimal solution corresponds to a perfect price discrimination mechanism. When information is asymmetric, the authors utilize an "integrated contract and auction design –ContrAuction", which permits them to derive an optimal ContrAuction mechanism that maximizes the seller's profit with and without efficiency constraints.

Zhu et al. [76] present a market scenario with two different stages: one between primary users or spectrum holders and brokers and a second one between secondary service providers and secondary users. The authors call the first a primary market and refer to the latter as a secondary market. The authors focus on the selection of the appropriate secondary provider by the secondary users. "The objective of this service selection is to maximize the individual satisfaction (i.e., utility) jointly considering performance and cost". The selection process starts with the secondary users randomly choosing a service provider. Nevertheless, each secondary user is able to refine their selection based on the price and observed quality of service. The process is modeled through game theory, which permits to manage the lack of complete information available to the secondary users.

[3] This model has been tested in a practical setting in the city of Munich, Germany, where the authors have been able to test the feasibility of their proposal.

4.3 TRADING ENVIRONMENTS BEYOND SPECTRUM

It is important to point out additional applications of auctions and trading mechanisms, which are relevant to our area of study, even though they do not deal with spectrum-related resources. These applications are comprised by the development of *cloud* and *electricity* markets.

As the authors in [77–81] point out, the utilization of auctions in the Cloud business is rather novel. The cloud market model generally involves interested parties adopting a "fixed pay-as-you-go pricing plan wherein the consumer is charged the amount of time a VM instance was used at a fixed rate" [77]. Cloud providers have realized that the level of unassigned resources is significant, partly because of the conception that the cloud provides an infinite amount of resources, and partly because reports point out that only a small fraction of physical resources are indeed assigned [77, 82]. This has prompted big companies such as Amazon, with their EC2 system to adopt auctions for developing a spot market to sell their spare capacity [83].

In the Amazon case, the company establishes a spot price, which fluctuates according to changes in supply and demand for spot instances. Buyers advertise their bids for a Virtual Machine instance hour to Amazon Wireless Services (AWS). Subsequently, AWS determines the market-wide spot price and grants access to users with bids above this price. When the bid of a user falls below the current spot price, the user is given warning so that they can either re-adjust their bid or be aware that their service will terminate [77, 83].

The Amazon example has inspired researchers to investigate the development of auction-like mechanisms for cloud services. In the same manner as with the spectrum case, auctions attend to objectives such as bid truthfulness and expressiveness[4] and system characteristics such as providers' profit maximization or social-welfare. Additionally, authors have realized the need to develop an ontology that could help consumers better define and express their business needs, as well as helping providers diversify their service and resource offerings [78]. This effort aligns with the support system that Amazon has put in place in order to help users

[4] We refer to bid expressiveness as the possibility to auction for a wide-variety of resource sets, which adapt to particular user needs. This aligns with the definition presented in [65].

choose competitive bids, however, in [78], the authors aim at making support compatible with multiple systems, not only that of Amazon.

In summary, the overall objective of adding new market mechanisms to the Cloud is to adopt dynamic pricing methods that help optimize the profits of cloud providers and simplify the service choice process for the customers, thus allowing them to find the option that best suits their needs.

In the electricity domain, the U.S. industry has been undergoing substantial changes in structure and architecture. The goal has been to shift towards competitive markets, where prices are derived from supply and demand forces rather than the exercise of market power [51].

There is a significant line of research which focuses on the utilization of Agent-based modeling to design electricity markets. The main reason behind using ABM is that these tools allow for the modeling of restructured electricity systems “as commercial networks of strategically interacting traders and regulatory agencies learning to operate through time over realistically rendered transmission grids” [51].

An important survey which points to the different approaches that have been explored using ABM is presented in [53]. The authors emphasize on the vast methods adopted to simulate electricity markets. In this way, they focus on presenting shortcomings of salient proposals and the open issues that remain to be addressed by ACE researchers. For the evaluation of the proposed alternatives, the authors focus on four broad aspects: agent learning behavior, market dynamics and complexity, model calibration and validation, and model description and publication. The authors emphasize on the need of establishing unified guidelines for model evaluation as well as on the need to address trading strategies (e.g., bilateral trading) that would be more realistic and adaptive to the electricity field.

As an important step toward advancing the ABM-oriented research, Tesfatsion [51] offers a wide overview of current and past work on electricity markets. Simulation add-ons and tools have been developed that adjust to the specific requirements of electric markets. This is to point out that ABM has been indeed successful for modeling market mechanisms where the central commodities are complex physical and technical resources.

Regarding specific market strategies, electricity markets operate under an open access market model, where energy is priced (differently) at every time and location [84]. At the core of the electricity market model, we have forward auctions and real-time auctions. In the forward transactions, participants are able to plan ahead and lock in prices according to their needs. In turn, real-time markets permit to send appropriate price signals to manage congestion efficiently in the short-term [84]. Given that prices are particular to each period and location, this type of pricing is called *locational marginal pricing* in the real-time market. With high transparency, we do not only achieve efficient short run decisions but it also provides us with valuable market information for long-term planning and future market investments [84].

These types of markets are relevant to our study in that we are interested in the short-term allocation of resources, which change in price and availability depending on the time of day and the geographical location. An important difference between the electricity domain and the wireless communications environment is that "[e]lectricity markets have large 'lumpy' resources that are expensive to turn on and limited in the speed with which they can make adjustments. In contrast, wireless network elements are fast to respond and are efficiently controlled with marginal prices" [84]. In the Cloud domain, virtualization is in a far more advanced stage than in the case of wireless networks. This eases several constraints at the moment of defining the appropriate commodity to trade.

4.4 SUMMARY

At the core of the definition and design of spectrum trading approaches we find vast applications of game theory, especially represented by auctions. Multiple authors focus on this particular mechanism due to the transparency and efficiency in the resulting assignment of resources. Nevertheless, finding an appropriate auction design is not a trivial task. Each studied scenario calls for modifications, enhancements and combinations of auction mechanisms in order to find the solution that best fits each approach.

Finding the appropriate resource allocation mechanism is not the only concern when addressing the development of secondary markets for spectrum. Among the salient constraints, we find in the literature a common concern in terms of the definition of spectrum as a tradeable commodity and the duration of the lease obtained in the market. These factors influence the incentives that users have to participate in the market and the resulting efficiency of the market assignment of resources.

The authors in the aforementioned works have developed different methods to define the tradeable commodity in terms of the main axes in which spectrum varies: time, frequency and space. Along these lines, we find the definition of adaptive power spectral masks [67, 73]; bandwidth units defined in micro-space, micro-time and micro-frequency scales [62], also referred to as "pixels" [69].

Regarding lease duration, it is a general agreement that leases should be flexible and short-enough to capture the traffic variation and thus requirements of spectrum buyers. In this way, we observe markets with leases lasting periods as short as 15 minutes [62] and those which capture day-long durations (i.e., 8 hours) [63]. Nevertheless, this does not eliminate the need for lease scalability, which would permit to adapt to a broader range spectrum user's needs and services.

For developing spectrum markets that adapt to next-generation systems, we can borrow from experiences in the Cloud and electricity markets. Although differences remain among the tradeable resources, modeling techniques and advancements can serve to add a new perspective to the spectrum problem.

To conclude this literature review, it is important to point out the benefits of the development of secondary spectrum markets that seem to prevail in spite of the complexity of their design.

- Through secondary spectrum markets, spectrum ownership could be separated from the provision of wireless services, thus lowering entry barriers and facilitating the diversification of services [62, 73].
- Secondary markets lower the resource prices and open the market to small players, which may ultimately result in social benefits [63].
- By increasing the amount of available spectrum, spectrum markets would motivate the deployment of different types of radio systems, some of which may operate at lower spectral efficiencies [73].

- Secondary markets provide greater economic efficiency, given that trade would only occur when the buyer values the spectrum more than the current owner [73, 85].
- Market forces enhance the efficiency in resource management and use, even in constrained settings [86] by allocating spectrum fairly. In addition, it is more responsive to changes in spectrum demand over time [55].

5.0 MOTIVATION

There are multiple aspects that have motivated this work. On one hand, there is the curiosity to merge engineering concepts with more pragmatic ones such as economics and policy. On the other hand, there is the desire to provide more meaningful, comprehensive solutions to problems that for a long time have been considered from a one-dimensional point of view. In other words, I have been interested on the study of spectrum markets beyond the optimal resource allocation perspective, hence delving into additional factors that influence the final success of these markets.

If we take a look at markets in practice, there are markets that are necessary from a human and social perspective; markets that become a necessity as generations change and markets that simply never take off. There is a rationale behind the workings of these markets which stem from how the products are manufactured or defined, how these resources are valued and priced, and also from the regulation affecting those markets. In addition, markets depend on the feedback from the environment where they operate. This is why some products that may be very successful in Eastern countries may not be well received in the Western world.

These thoughts have prompted me to take into account considerations from three important contexts into my study of spectrum markets. These contexts, which I refer to as *research axes*, are: technology, economics and regulation. From the background information and the literature review presented in the previous chapters, we find that significant contributions have been made for an efficient use of spectrum resources from a technical, regulatory and economics perspective. Indeed, secondary spectrum markets are a good example of spectrum sharing mechanisms where these three axes have a significant impact. On their own, studies in these individual aspects permit us to assess the constraints we may face toward developing spectrum markets and ways in which we may overcome them. Nevertheless, when looking at the big picture, additional concerns emerge, which result from the combination of multiple factors. Indeed, in these emerging concerns we may find the reason why secondary markets for spectrum have not been adopted in practice.

In this manner, this research work aims at presenting an alternative to spectrum markets where factors stemming from technology, policy and economics are taken into account. I consider that such an approach is of critical importance, given that technical improvements can remain far from being realizable if there is not a policy configuration supporting them. In turn, technology may fail to be adopted if its economic sustainability is not verified. The converse is true as well, given that economic and regulatory efforts may fail to be adopted if these are not paired with feasible technical counterparts. In consequence, I expect the research methods and approaches presented in this work to be useful for assessing the conditions that lead to the creation of viable, and feasible, secondary markets for spectrum.

6.0 RESEARCH FRAMEWORK

The focus of this dissertation is to find the conditions that lead to viable outcomes in secondary spectrum markets. To date, we find works that adapt market mechanisms for the assignment of spectrum related resources; we find policies that guide the spectrum sharing process, and we also find technologies that enable users to share, lease or trade their resources. Evidently, each of this approaches correspond to larger areas of study, namely economics, policy and technology. These three topics, when independently addressed, provide us with significant insights on spectrum sharing opportunities. Nevertheless, we consider that a comprehensive analysis that combines the three axes can provide a more realistic approach towards defining whether a suggested market approach is adopted and how successful it can be.

We illustrate this point in this dissertation by analyzing markets at different stages. Indeed, each stage leads to the next with a specific question that drives its research framework and settings. In this manner, the broader questions that guide this research work are:

- What is the appropriate commodity to trade?
- What incentivizes users to participate in a secondary spectrum market?
- What makes a secondary spectrum market viable?

In an attempt to find answers to such broad questions as those previously presented, we have thought of three main axes that we can explore in order to address them. Indeed, those broad questions can be converted in the following:

- (Technology) What is physically feasible with the resources?
- (Economics) How can we successfully negotiate for resource access?
- (Policy) What are we allowed to do with the resources?

In turn, we can still work on the questions above in order to restrict our focus to more specific areas that can provide us with the information that we need for determining whether markets are viable. The following sections present the specific questions that frame our research work.

6.1 RESEARCH QUESTIONS

The questions included in this section correspond to a broad area of impact of the research we propose. In the upcoming sections, we will highlight the subset of questions that most adapt to the research covered in this dissertation and the corresponding hypotheses.

Q1. When are markets for "naked" spectrum viable?

- Q1.1 What is the impact of fungibility limitations on spectrum markets' viability?
- Q1.2 How does the valuation of resources change when the traded commodities are not perfect substitutes (i.e., perfectly fungible)?

Q2. What is the appropriate technical framework that will lead to successful spectrum trading?

- Q2.1 Can we apply virtualization concepts to define the resources shared in the network?
- Q2.2 What are the benefits from adding virtualization to the definition of spectrum related commodities?
  - Q2.2.1 Does virtualization provide flexibility to define the spectrum related commodities?
- Q2.3 How does the valuation of resources change when considering their virtualized counterparts?

Q3. What is the role of markets in a complex system?

- Q3.1 Where are negotiations likely to take place?
- Q3.2 How can markets provide feedback for the system?
- Q3.3 How does participants' behavior (e.g., risk averseness, competitiveness) influence the outcome on the negotiation process?

- Q3.4 Does the negotiation mechanism provide users with incentives to engage in this type of sharing?
- Q3.5 Is the VNB (i.e., a middleman) necessary for achieving system stability?
- Q3.6 How do costs influence the outcome of negotiations in this network?
  - [Q3.6.1] Influence of transaction costs
  - [Q3.6.2] Influence of opportunity costs
  - [Q3.6.3] Influence of agency costs

Q4. Define the policy framework for the network model we study

- Q4.1 How does this model fit within the Institutional Analysis and Development (IAD) framework?
- Q4.2 What types of rights are granted to users in the modeled network?
  - [Q4.2.1] How are rights defined?
  - [Q4.2.2] What rights are shared in the pool?
- Q4.3 Is polycentric governance a suitable policy framework for the modeled network?
- Q4.4 After polycentric governance methods, when is external regulatory intervention necessary (i.e., global regulators intervening instead of local regulators)?

Q5. Identify parameters for successful market design

- Q5.1 Assessing market thickness
  - [Q5.1.1] When does the number of participants (i.e., RPs and SPs) increase?
  - [Q5.1.2] When does the resource supply increase?
- Q5.2 Assessing market congestion
  - [Q5.2.1] How many participants obtained resources from the market?
  - [Q5.2.2] How many VNBs were able to serve their customers?
  - [Q5.2.3] How many resources were assigned from the pool?
- Q5.3 Assessing market safety
  - [Q5.3.1] Penalty history
    - [Q5.3.1.1] How high are the penalties assigned?
    - [Q5.3.1.2] How many users have been penalized?
    - [Q5.3.1.3] How many times have users been penalized?

6.2 RESEARCH SETTINGS

The research stages presented in this dissertation show three different avenues that can be explored in order to define market viability.

In stage 1 and 2, the analysis is centered in the performance of the market mechanism that is used to transfer resources from resource buyers to sellers.

The difference between these two stages lies mainly on the definition of the commodity to trade.

In stage 1, "naked" spectrum is the traded commodity. This calls for an analysis of the fungibility limitations among the existing and required electromagnetic frequencies. In general terms, the settings of stage 1 focus on market analysis and the physical constraints of electromagnetic spectrum.

In stage 2, we work toward defining a different market commodity, which can be more favorable from the market participants perspective and from a market design perspective. In fact, by exploring opportunities provided by LTE-A and Wireless Network Virtualization, we expect to find a means to design thicker markets while providing users with a more manageable commodity. In this light, in stage 2 we focus on market analysis, the physical constraints of spectrum and adapting an appropriate technical framework.

In stage 3 we take a bigger leap and our focus shifts from an entirely market-oriented perspective to the analysis of markets as part of more complex communications systems. We still consider wireless network virtualization for the definition of the traded commodities; however, we take into account additional economic and policy concepts that shed light on the interactions of users, the market design rules and the boundaries for resource use and allocation. In particular, we adapt matching markets design and concepts to create more expressive market transactions. Additionally, we explore the literature on *polycentric governance* to create a system that can be adaptive to local conditions.

For illustration purposes, the research settings have been categorized according to general parameters of interest. Each research stage addresses these parameters from multiple perspectives, relevant to the problem studied in each stage. The particular parameters, perspectives and corresponding stages are presented in Table 1.

Table 1: Research Settings

Parameter	Type	Stage
Network Type	Stand-alone markets	1,2
	Service-driven network	3
Market Commodity	“Naked” spectrum	1
	Virtualized commodities	2,3
Market Mechanism	Stackelberg Auctions	1,2
	Matching Markets	3
Resource Valuation	Based on fungibility level	1
	Based on capacity comparability	2
	Based on compatibility with service provided	3
Technical Settings	Technology-independent	1
	Wireless network Virtualization within LTE-A boundaries	2
	Virtualization with resource pooling	3
Policy Settings	Fixed rules provided by the regulator	1,2
	Polycentric Governance Framework	3

We have formulated a set of hypotheses that aim at evaluating factors relevant to the research questions from section 6.1 that will be addressed in this dissertation work. In each stage we aim at exploring factors that are associated with the set of hypotheses we have developed in order to shed light onto the research questions presented in section 6.1.

6.3 HYPOTHESES

We have formulated the following hypotheses as a means to evaluate criteria relevant to the research questions that were previously presented.

H1. Lower fungibility scores negatively impact the auction cutoff price in the market.

H2. The percentage of resources assigned attends to the demand and supply conditions rather than the actual fungibility level of resources.

H3. When the market commodities are homogeneous, through virtualization mechanisms, the availability of alternate technology units positively impacts the market demand.

H4. When market commodities are homogeneous, resource assignment is proportional to users' willingness to pay and supply conditions.

H5. The amount invested by the RPs depends on the uncertainty over the future price of resources in the system.

H6. The value of an RP's option to invest depends on the current and expected valuation of resources and the cost of investment.

H7. The investment level of the RPs is directly proportional to the amount of resources available in the market.

H8. SPs and VNBs utilize public and private information to formulate their preferences.

H9. Establishing matching preferences in terms of joint surplus increases the amount of *allowable* matches.

H10. The cost of penalties have an impact on a user's (good/bad) behavior.

H11. Historical data on resource prices help reduce uncertainty for investments in the system.

H12. System stability increases the amount of resources shared by RPs in the system (i.e., RPs can generate additional resources by investment in their current infrastructure or short selling resources.)

H13. The reputation of middlemen is directly proportional to the total number of matches of a VNB, and to the payment obtained from resource aggregation services.

H14. SPs that assign higher weights to the matching parameters perform better in terms of percentage of matched SPs, demand obtained from the market, and surplus, than those assigning lower weights. In the same way, VNBs that assign higher weights to the matching parameters perform better in terms fo percentage of matched VNBs, payment received, and surplus than those with lower weights.

The questions and hypotheses that will be explored in this dissertation work are summarized in Table (2). Note that some of the research questions are addressed through the framework that has been considered for the design of each stage, while other questions are explored through the experiments designed to test the aforementioned hypotheses.

Table 2: Correspondence among research stages, hypotheses and research questions

Proposed Research	**Research Questions**
Stage 1	Q1
H1 - H2	Q1.1, Q1.2
Stage 2 - 3	Q2, Q2.1
H3	Q2.2
H4	Q2.3
Stage 3	Q3, Q3.1
H4, H13	Q3.2
H7	Q3.3
H9	Q5.1
H14	Q5.2
Stage 3	Q4.1, Q4.2
Stage 1 - 3	Q5.1, Q5.2

7.0 SPECTRUM TRADING SCENARIOS

In this chapter, we provide a thorough description of the three spectrum trading scenarios that we study in this work. We refer to these scenarios as *Stages I, II and III.* The differences among these stages lie on the environment where markets are analyzed, the definition of the market participants, but most importantly on the definition of the commodity central to the market, or commodity to trade.

As a means to frame each of these stages within the broader scope of this research work, we also emphasize on the hypotheses that are relevant to each stage.

7.1 STAGE I

The focus of Stage I is to study the effect of the lack of spectrum fungibility on market viability. For this purpose we work with an existing spectrum trading model SPECTRAD, which was developed by Caicedo et al. in [1,87], and we follow the calculations of spectrum fungibility presented by Weiss et al. in [11]. Our work in this stage aims at adapting the measures of spectrum fungibility to SPECTRAD, so that the market interactions of the participants would reflect the level of comparability between preferred and available frequency bands.

7.1.1 General Description

7.1.1.1 Fungibility Measures The work presented in [11] aims at providing a quantitative measure for spectrum comparability and replaceability. These calculations take into account multiple dimensions in which spectrum can vary e.g., space, time, technology, regulation. This led the authors to present two metrics for spectrum fungibility: a probabilistic and a distance score. The probabilistic score represents the fraction of a given characteristic (e.g., coverage) obtained when utilizing an available frequency instead of the preferred one. The distance score represents the Euclidean distance between the results obtained with two different frequencies for the same metric (e.g., coverage). It follows that a probability score of 1 and a distance score of 0 correspond to ideal fungibility conditions. Equations (7.1) and (7.2) are utilized for the calculation of probabilistic and distance scores, respectively. Note that we refer to the preferred frequency parameters as $f_1(d_1)$ and to the available frequency parameters as $f_2(d_2)$.

$$\text{Probabilistic Score} = min\left(\frac{f_1}{f_2}, 1\right) \tag{7.1}$$

$$\text{Distance Score} = \frac{\max((d_1 - d_2), 0)}{d_1} \tag{7.2}$$

These scores could represent a useful means to determine the probability of success of a given transaction, when preferred frequencies are replaced by those that are available. Hence, we follow the definition of fungibility scores presented in [11] and calculate specific scores that would be applicable to a market scenario. Indeed, we focus on two important metrics: coverage and capacity. In this way we expect to determine the bandwidth needed with an available frequency to match the performance (i.e., coverage and capacity) of another, available frequency.

For calculating the coverage fungibility score, we utilize the link budget formula (7.3), where P_r is the received power, P_t is the transmitted power, G_t and G_r are the transmitter and receiver gains, respectively and L_p is the path loss. In order to capture various parameters relevant to the areas where the frequencies would operate, we rely on empirical propagation models for path loss calculations, such as Okumura – Hata and COST 231 Walfisch–Ikegami.

$$P_r = P_t(dBm) + G_t(dB) + G_r(dB) - L_p(dB) \tag{7.3}$$

If we work with a minimum allowed received power, we can determine the maximum distance at which this power can be achieved. This translates into the coverage attained with a particular frequency, which further provides us with the metric for calculating our probabilistic and distance scores. Along these lines, the coverage scores permit us to compare the maximum coverage obtained with the preferred frequency f_1 and an available frequency f_2.

A similar process can be utilized to calculate the capacity fungibility scores. In this case, we utilize the Shannon–Hartley Information Capacity theorem (7.4), which permits to determine the maximum rate achievable (C) in a given channel, with a particular bandwidth (B), and under the presence of noise.

$$C = B\log_2(1 + \mathrm{SNR}) \tag{7.4}$$

The signal to noise ratio (SNR) in this formula is defined as follows: the signal value corresponds to the power received at a specific distance (e.g., fixed distance from the transmitter or cell–edge); the noise power was estimated using (7.5) or its equivalent (7.6), where F is the noise figure of the receiver, k is the Boltzmann's constant, T is the reference temperature of 290 K and B is the considered bandwidth.

$$\begin{aligned} N &= FkTB && (7.5) \\ &= F(dB) + k(dBm/Hz/K) + T(dBm) + B(dBm) && (7.6) \end{aligned}$$

With the aforementioned formulas, we can determine the bandwidth that f_2 requires in order to achieve the same capacity reached with f_1. Additionally, if we consider both frequencies, f_1 and f_2, operating with the same bandwidth, we can obtain a measure of their comparability or replaceability. In this way, when adapted to a market scenario, the fungibility scores allow the market participants to place a cap on their valuation of an existing resource, when it is not exactly their preferred one.

7.1.1.2 Market Model As previously mentioned, we adapt the trading model presented in [87], SPECTRAD, to an imperfect fungibility scenario. Nevertheless, to maintain a basis for evaluating the results obtained with the new market setting, we make changes only where necessary and where the lack of spectrum fungibility applies. In what follows, we describe the general characteristics of the market type, the participants, the traded commodity and its valuation, and the market transactions.

- **Market Type:** We focus on an spectrum exchange with Band Manager (BM) functionality. In this scenario, the BM is in charge of auctioning and assigning its spectrum holdings. Note that these spectrum holdings correspond to the resources made available by current spectrum license holders (SLH). The BM we consider is in charge of granting authorizations to the spectrum buyers or spectrum license requesters (SLRs) to access the spectrum; however, it is not in charge of configuring the buyers' equipment for spectrum use.
- **Market Participants:** The main participants in this market scenario are the Band Manager and the Spectrum License Requesters. Note that we do not focus on the process in which the BM obtains the resources from the Spectrum License Holders. In this way, an SLH is a passive user and does not actively affect the operations of the market.

  The objective of the SLRs is to obtain resources in the market to fulfill their traffic demand. Evidently, depending on the service they provide, SLRs will have a specific preference for the frequency band they seek. As a consequence, the frequency available in the market (i.e., in the BM holdings) may not be the same as their preferred frequency.
- **Traded Commodity:** In the same way as the original SPECTRAD model, the basic trading units are spectrum bandwidth units (BBUs). In [87], these are defined as 200KHz bandwidth units of spectrum in the 1900 MHz band. Throughout our Stage I study, this frequency corresponds to the available frequency band.
- **Spectrum Valuation:** An SLR values the BM holdings in the measure that they compare to its preferred frequency. In this way, we assume that the maximum value that an SLR is willing to pay for an available frequency is limited by the degree in which it can replace the exact frequency the SLR needs. This means that we can utilize the calculated fungibility scores to limit the maximum amount that every SLR is willing to

pay.

It is important to note that an SLR can opt for alternate technologies (ATs) to fulfill their traffic requirements. ATs are also considered in the original SPECTRAD model and these represent technical alternatives (e.g., wireline deployments, unlicensed spectrum, infrastructure enhancements, among others) that could permit an SLR to fulfill its demand when spectrum is not available. This would be the case when there are no resources available in the market or when the bids of the SLR are not competitive enough. We assume that AT units provide the same performance as the BBUs of an SLR's *preferred* frequency. Thus, the maximum amount that an SLR is willing to pay for a BBU in the market corresponds to the amount that it would pay for an AT unit. Equation (7.7) expresses this relationship, where numBBUs and numATs represent the required number of BBUs and ATs respectively and LimitPricePerBBU and LimitPricePerAT correspond to the maximum price to pay for either BBUs or ATs.

$$(\text{numBBUs})(\text{LimitPricePerBBU}) = (\text{numATs})(\text{LimitPricePerAT}) \tag{7.7}$$

In turn, numBBUs and numATs can be defined through (7.8) and (7.9).

$$\text{numBBUs} = \frac{\text{trafficToServe}}{\text{capacityPerBBU}} \tag{7.8}$$

$$\text{numATs} = \frac{\text{trafficToServe}}{\text{capacityPerAT}} \tag{7.9}$$

It follows that the maximum price to pay can be defined as (7.10), which in turn can be expressed in terms of the corresponding fungibility score (7.11).

$$\text{LimitPricePerBBU} = \frac{\text{capacityPerBBU}}{\text{capacityPerAT}} \times \text{PricePerAT} \tag{7.10}$$

$$\text{LimitPricePerBBU} = \text{FungScore} \times \text{PricePerAT} \tag{7.11}$$

- **Market Transactions:** We utilize the Stackelberg auction model for the market transactions.[1] In general terms, each SLR will post a bid in each bidding round for the amount of resources it requires. Bids are sorted in descending order and at the end of each bidding round, the cutoff price of the auction is set as the last bid to receive resources (when the demand is greater than the supply), or as the reserve price or minimum cutoff price (when the supply is greater than the demand). The cutoff price is then announced and, subsequently, each SLR adjusts its price in order to remain competitive in the market. Evidently, their price adjustments are limited by (7.11). At the end of the auction, the SLRs whose bids were above the cut-off receive the corresponding number of BBUs. The SLRs that did not obtain enough resources from the auction have the option to utilize AT units to fulfill their demand.

7.1.2 Hypotheses

H1. Lower fungibility scores negatively impact the auction cutoff price in the market.

H2. The percentage of resources assigned attends to the demand and supply conditions rather than the actual fungibility level of resources.

7.2 STAGE II

In the second research stage, our goal is to define a new commodity to trade in the market. In consequence, the commodity that we consider for the new market model is no longer *naked* spectrum; instead, we look for a spectrum-related commodity that could permit to add thickness to the market. For this purpose, we are interested in a mechanism that allows us to circumvent some of the physical constraints inherent to spectrum, thus presenting the market participants with a more manageable method to evaluate the suitability of the available resources.

[1] This is consistent with the auction model utilized for the Spectrum Exchange with BM functionality portion of the original SPECTRAD model.

7.2.1 General Description

We take advantage of current resource utilization technologies in order to explore further scenarios where markets can be viable. We center our attention on two technical alternatives: virtualization and the LTE-Advanced standard. These technologies enable more efficient resource-use methods, which are appropriate for creating enhanced spectrum sharing scenarios. Our particular approach is to pair the definitions of virtualization with the concept of resource pooling. In this way, we envision a pool of spectrum-related resources at the center of the market, which can be accessed by the different market participants (or SLRs).

Note that one of our objectives is to provide the SLRs with the opportunity of expressing their requirements, not in terms of an specific frequency band and its bandwidth, but instead in terms of the capacity (in Mbps) that they require to serve their customers. For this approach to be successful, we require the process to be seamless to the SLRs, which means that they are not aware of the exact resources they are using, nor the specific physical characteristics. Virtualization comes into play in the creation of this seamless environment, or in other words, in the translation of physical electromagnetic spectrum into capacity as a commodity.

To perform this virtualization process, we appeal to the opportunities presented by LTE-Advanced. This is a mature technology which focuses on providing flexible spectrum allocation mechanisms in order to reach higher speeds and efficiency in the utilization of resources. In order to define our market commodity, we focus on the basic element for radio resource allocation of LTE, which is the Physical Resource Block (PRB). The PRBs are sets of resource elements defined in time and frequency, which are used for uplink and downlink transmissions. For transmission, the PRBs are aggregated in sub-frames and frames. A sub-frame is a 1 millisecond unit, which is formed by two PRBs and a frame corresponds to a 10 millisecond unit composed by twenty PRBs. In frequency, one PRB corresponds to 12 subcarriers of 15KHz each, totaling 180 KHz per PRB [88].

For resource allocation purposes, the LTE standard [89] dictates the number of resource blocks that can be assigned and the capacity that can be obtained. In Table 3, we include the parameters that are considered in the standard for downlink transmission. These data

Table 3: LTE Parameters for Downlink Transmission

Number of Resource Blocks	6	15	25	50	75	100
Number of Occupied Subcarriers	72	180	300	600	900	1200
Transmission Bandwidth [MHz]	1.4	3	5	10	15	20
Occupied Bandwidth [MHz]	1.1	2.7	4.5	9.0	13.5	18.0
Guardband [MHz]	0.32	0.3	0.5	1.0	1.5	2.0

permit us to establish a direct link between the number of PRBs and their associated capacity. Taking these factors into account, we find that the resource definition and aggregation properties of LTE are a significant addition to the virtualization process that we devise. Indeed, by means of the LTE standard, we can establish the translation between capacity and physical spectrum resources that we seek. As a result, we can define the commodity to trade in terms of the LTE resource allocation units.

7.2.1.1 Market Model Our focus in this second stage is to capture the improvement that can be reached when we define a more flexible, spectrum-related commodity. In this light, we maintained the general characteristics of the market model from the first stage intact, except for the modifications necessary to adapt the new market commodity.

New market commodity: The commodities in the market can be defined as virtualized resources that are aggregated in a *pool* and which now constitute the BM holdings. As previously explained, for the virtualization process, we rely on the mapping between PRBs and bandwidth offered by LTE and their further translation into capacity (in Mbps). To this end, we utilize expressions (7.4) and (7.5) to calculate the capacity that can be achieved with the bandwidth aggregated through LTE PRBs.[2] Given this commodity definition, we expect the SLRs to express their market demand in terms of the capacity they require to

[2]In expression (7.4), the signal value is calculated through the COST 231 Walfisch-Ikegami model, utilizing the LTE frequencies relevant to this experiment.

fulfill their customers' requirements. In this way, the BM would be in charge of performing the PRB – capacity mapping, so that the process becomes entirely transparent from the SLRs' perspective.

There are multiple LTE frequencies defined by the standard; however, in this stage we choose those frequencies that allow us to create a pseudo perfect fungibility environment.[3] In this way, the pooled resources correspond to PRBs from the following LTE bands: 13 (746 MHz - 756 MHz); 14 (758 MHz - 768 MHz) and 17 (734 MHz - 746 MHz). Following the details presented in Table 3, these three 10 MHz-bands provide us with a minimum of 6 PRBs and a maximum of 50 PRBs per band. Considering carrier aggregation properties, the pooled assets would range from 18 PRBs to 150 PRBs. To further comply with the LTE standard, we propose the leasing time for these commodities as the duration of an LTE frame i.e., 10 milliseconds.

It is important to remember that the setting of this research stage aims at creating a homogeneous commodity through virtualization. In this way, part of this effort is to find an AT unit which would also adapt to the homogeneous scenario. Hence, we define the alternate technology units (ATs) as unlicensed TV White Space (TVWS) spectrum in the 700 MHz band.

7.2.2 Hypotheses

H3. When the market commodities are homogeneous, through virtualization mechanisms, the availability of alternate technology units positively impacts the market demand.

H4. When market commodities are homogeneous, resource assignment is proportional to users' willingness to pay and supply conditions.

[3] This perfect fungibility scenario relies on our assumption that LTE-A capable devices should be able to tune to multiple LTE-A frequencies. In addition, to further enable fungibility, we have chosen frequencies that are rather similar in range.

7.3 STAGE III

In the third stage of this dissertation, we are interested in studying the performance of the overall system where markets are deployed. We find it valuable to place this market model within a broader framework that permits to take a more comprehensive view of the entire system. We do this by situating our study, model and tests within the Institutional Analysis and Development (IAD) framework.

Presenting the model in this manner allows us to place the problem within a broader context, which also points to possible future directions and applications.

In what follows, we introduce key elements of the IAD framework and how these apply to the specific context of our study.

7.3.1 Framework Overview

We follow guidelines and concepts presented in the Institutional Analysis and Development (IAD) framework literature. Figure 2 summarizes the general components of the network we analyze from the IAD perspective. This framework is of particular interest given that it takes into account the interplay of the multiple agents and entities in the system. This implies that the actions of each one of them impacts the system performance; but, at the same time, feedback is provided from current outcomes, which may influence subsequent operations.

7.3.2 Agents

We have three main types of agents in Stage III, which in turn constitute the participants in the market designed for this section of our study.

- **Resource Providers (RPs):** RPs are current spectrum license holders and/or infrastructure owners. After serving their customers' needs, these users have excess resources which they may share in the system. In this way, the RPs are making their resources available in the *pool* so that these could serve the traffic demand of new entrants to the system or providers who lack enough resources to fulfill their demand. We expect the RPs

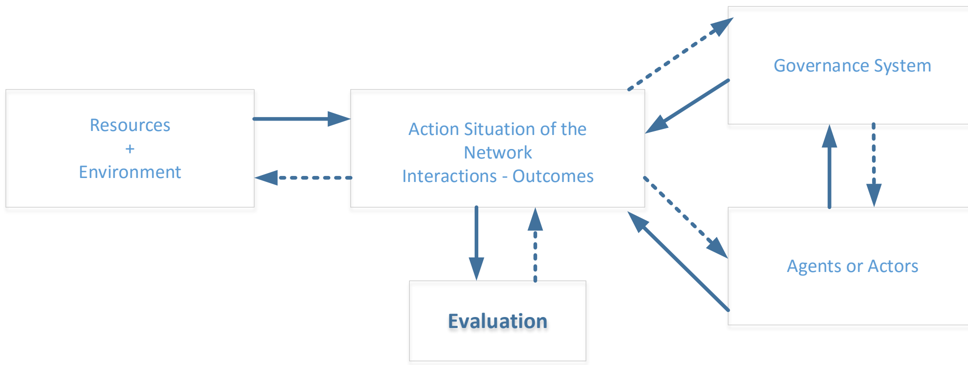


Figure 2: Components of the network from an IAD Framework perspective

to require a remuneration for the resources they are sharing. To this end, they should announce their reserve price (i.e., minimum accepted price) for the shared resources. Any price below the minimum would signify a loss to the RP.

- **Service Providers (SPs):** SPs are new market entrants or existing users who do not possess enough resources to fulfill the demand generated by their customers (i.e., end users). Each SP aims at providing a specific type of service which has particular requirements in terms of resource quality, amount and availability. For example, SPs may focus on providing voice, data or video services and their resource demand will attend to the particular characteristics of those services. However, we aim at simplifying this process by allowing the SPs to express their demand in general terms, leaving the complex task of finding the matching resources to an intermediate entity known as Virtual Network Builder.

- **Virtual Network Builders (VNBs):** This entity acts as a broker or middleman who is in charge of aggregating resources from the pool and assigning them to the SPs who are requesting them. Indeed, the VNBs deal with the complexity of analyzing the pooled resources and assembling them in a manner that satisfies the SPs' requirements.

  As a middleman, the VNB has several functions in general settings, which are illustrated in [90]. Indeed, the author classifies middlemen within six categories. In what follows we explain the categories that are relevant to the VNB.

– Bridge: Reduces the “physical, social, or temporal distance” between buyers and sellers. In this way, this type of middleman is able to find opportunities between two disconnected sets of participants.

– Insulator: Limits the information flow between buyers and sellers, or in this particular case between RPs and SPs.

– Certifier: Provides value for both, buyers and sellers, by screening available options, scouting for the requirements of buyers and endorsing its findings through their own reputation. In this manner, the middleman reduces the asymmetry of information between buyers and sellers.

– Enforcer: Makes sure that the buyers and sellers (i.e., RPs and SPs) are not failing to provide the service or utilize the resources as convened.

– Risk bearer: Reduces uncertainty for both negotiating parties. In fact, “[b]y building diversified portfolios, [these middlemen] are better able to weather volatility than their trading partners” [90].

In the particular case of resource access, a middleman can minimize the *intellectual* barriers posed by the knowledge required in order to successfully obtain resources from an auction.

In this light, a VNB fulfills middleman duties that correspond to the transactions and negotiations it performs with the RPs and SPs. The particular activities and parameters that are relevant to the VNBs are more thoroughly explained through their interactions in the system, which are defined in section 7.3.4.

7.3.3 Resources and Environment

The resources defined in the two previous stages shed light on the type of commodity that could be suitable for the new market analysis. Indeed, in the second stage we defined a perfectly fungible, virtualized commodity, which is bounded within a specific frequency range. In this section we assume the existence of virtualized commodities that can be translated into specific throughput requirements. In this way, this commodity can be defined as the commodities of stage II or as the result of more complex virtualization processes. The resource pooling approach still applies to the scenario we investigate in this stage.

For the pooled resources to be assigned to their new users, i.e., service providers, it is important that their characteristics are announced. For instance, for resource aggregation purposes, a VNB needs to know what are the main physical characteristics (e.g., frequency, time and bandwidth available), the technical characteristics (e.g., how the resource can be sliced for virtualization purposes and compatible technologies) and regulatory characteristics (e.g., maximum allowed power, leasing time). In this way, a VNB can choose more accurately the resources that are useful for its customers and this can also represent a better means to place an appropriate valuation for the available resources.

The environment where this system is deployed corresponds to a medium-sized, semi urban area, where we can find more than one resource provider and where there would be significant interest for various SPs to participate. Nevertheless, our system can be decomposed and applied to reduced areas within a city. For instance, we can have VNBs operating at *neighborhood-level*, thus permitting providers to share resources according to the needs of particular areas within a city. Additionally, a smaller scope reduces the uncertainty over resource availability and suitability.

7.3.4 Interactions

For the proposed service-driven network model to be viable, we require interactions among the different participants so that the available resources can be transferred from RPs to SPs with the intervention of the VNB. In this dissertation, we utilize markets for the resource negotiation and allocation process.

Placing markets within this system permits us to explore parameters such as the participants' behavior, the rationale behind the prices they pay and the profit they seek, their incentives to participate, among others. Overall, markets allow us to study the interactions among participants and to learn about the conditions that lead to viability or failure.

It is important to mention that these behaviors, characteristics and interactions are tightly linked to the environment where the markets are placed. In turn, the environment is framed within regulatory and technical parameters which delineate the feasible (and practi-

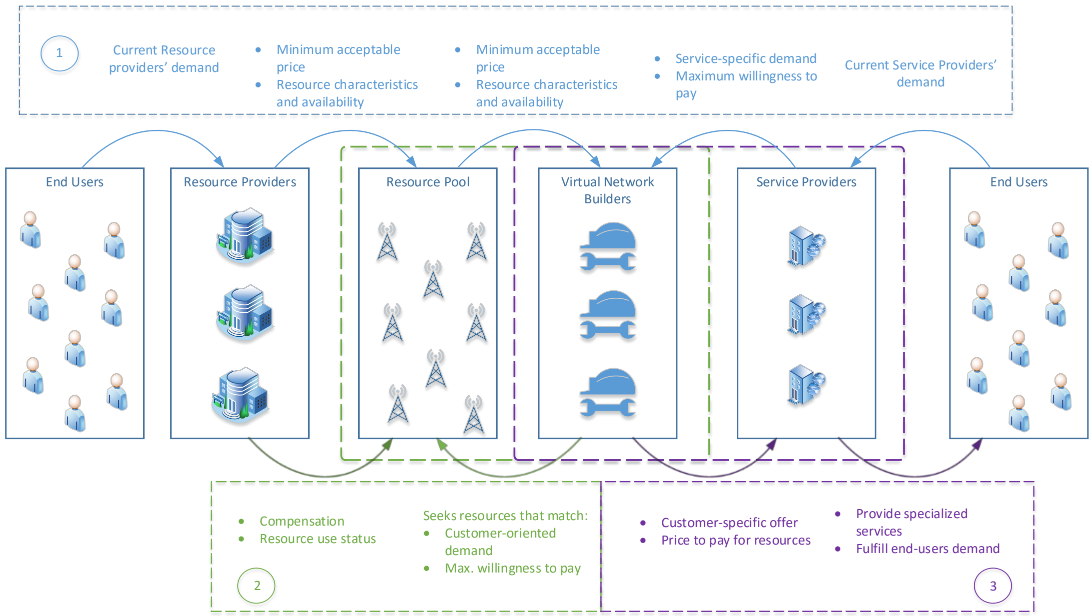


Figure 3: General Market Model

cal) boundaries we face. Exploring markets within these boundaries will allow us to provide insights not only on the market viability outcomes, but also in the general regulatory and technical approaches that surround this system. Ultimately, this will provide us with a comprehensive view on the feasibility of our proposal. This results in an "information cycle", where the market outcomes serve to adjust network and participants parameters and these adjustments influence the market results.

Figure 3 shows the different entities that are part of the service-driven network, the information they provide to each other and the interactions (i.e., negotiations) that take place among them.

Along these lines, there are two important instances where markets are analyzed in our virtualized network: VNB–SP negotiations and VNB–RP negotiations. The applicable negotiation mechanisms should adapt to the characteristics, needs and objectives of the participants in each specific scenario, therefore, the market mechanisms used to allocate resources in each of these instances need not be equal.

We have approached VNB–SP negotiations as a matching market that creates partnerships between VNBs and SPs. This permits us to capture the preferences and objectives that these two entities have when they participate in the market, the need for a middleman that eases the market transactions for SPs, and the SPs' valuation for this type of service. The specific approach we have taken towards developing this matching process is presented in section 8.3.1.

VNB–RP negotiations are regarded as a regular market process where supply and demand levels determine the price to pay for resources. Behind market demand we find the matching market between VNBs and SPs. In turn, the market supply is determined by the level of participation of RPs. In this way, behind the market setting of the VNB–RP interactions, there is also a set of RP characteristics that are important to take into account. In section 8.3.3, we present the specifics of the supply side of the market and the subsequent interactions that determine the final assignment of resources in the market.

7.3.5 Action Situation

Action situations are defined as "the social spaces where individuals interact, exchange goods and services, solve problems, dominate one another, or fight (among the many things that individuals do in action situations)" [91]. An action situation is additionally defined as the component where "individuals (acting on their own or as agents of organizations) observe information, select actions, engage in patterns of interaction, and realize outcomes from their interaction" [92]. In this section we focus on describing the action situation that corresponds to our analysis.

In [91], Ostrom points out that the structure of an action situation can be defined by the following set of variables.

1. The set of actors – Who and how many individuals withdraw resource units from this resource system?
2. The specific positions that will be filled by participants – What positions exist?
3. The set of allowable actions and their linkage to outcomes - Which types of harvesting technologies are used?

4. The potential outcomes that are linked to sequences of actions – What geographic region and what events in that region are affected by participants in these positions? What chain of events links actions to outcomes?
5. The level of control each participant has over choice – Do appropriators take the above actions on their own initiative, or do they confer with others?
6. The information available to participants about the structure of the action situation – How much information do appropriators have about the condition of the resource itself, about other appropriators' cost and benefit functions and about how their actions cumulate into joint outcomes?
7. The costs and benefits assigned to actions and outcomes – How costly are various actions to each type of appropriator, and what kinds of benefits can be achieved as a result of various group outcomes?

Consequently, in the network we analyze, we can briefly define the action situation as follows:

- **Actors:** We have three types of actors in this network: resource owners or providers (RPs), resource aggregators or virtual network builders (VNBs) and resource buyers or service providers (SPs).
- **Positions:** The positions of the different actors are derived from the set of rights that are assigned to each of them. The specific types of rights will be explained in section 7.3.6.
- **Set of allowable actions:** By means of wireless network virtualization, RPs make their resources available in the pool. Virtual Network Builders can aggregate resources from the pool and, in turn, SPs can access the resources offered by the VNB with which they are associated. It is thus implied that SPs cannot access directly the resources from the pool.
- **Potential Outcomes:** These are the result of the interactions that take place among the actors in the network. For instance, some of these outcomes will be a consequence of the matching process between VNBs and SPs and from the auction for the assignment of pooled resources. We mention some of them in what follows:

1. SPs form a partnership with the VNB they prefer.

2. VNBs are able to aggregate their preferred set of resources from the pool, via competitive bidding.

3. RPs receive a compensation for the resources that they shared in the pool.

4. SPs obtain the resources that they need for providing their specific service from their partner VNB.

5. VNBs obtain a compensation for their aggregation activities.

6. End users obtain the service they contracted from the SP.

7. Resources are more efficiently utilized in the area of study.

8. RPs obtain a compensation (profit) for resources that would otherwise remain unutilized.

9. SPs can obtain resources through more economical, and accessible methods.

- **Level of control over choice:** RPs should be allowed, by regulation, to share their resources in the pool. The partnership formed by SPs and VNBs depends on their preference over members of the opposite set, which is formed by individual weights and valuations of each entity.

- **Information available:** the information available in the system will depend on the part of the system we are analyzing.

  The RPs have information on the actual amount of resources that they have available for sharing. Additionally, they know what is the minimum payment they should receive in order for the sharing process to be profitable for them.

  The VNBs have information about the characteristics of the resources that are available in the pool, such as reserve price, resource type, among others. In this way, they can opt for their preferred (and suitable resources). In turn, their set of preferred resources will attend to the requirements of their SP customers. This means that they will also have information on the SPs' resource preferences.

  The SPs will have information about the price that the VNBs are charging for their resource aggregation services. They know what is their actual demand (from the number of end users they should serve) and the specific details of their business model and service to provide. The latter are key to establish their valuation for the resources offered by the VNB.

- **Costs and benefits:** The costs can be associated with participation in the matching process to form a partnership with the preferred VNB and SP. Additionally, we should take into account the resource aggregation costs and opportunity costs of sharing resources and participating in the system.

  The payoffs are defined in terms of the surplus of the partnership i.e., the surplus that can be gained aggregating the gain/loss of both members of the partnership instead of individual surplus.

7.3.6 Rules, Rights and Governance

The aforementioned characteristics define the initial action situation of the network that we study. It is important to note that the values and parameters associated to these working components of the action situation respond to an applicable set of rules. In [92], the author defines boundary, authority, aggregation, scope, information, and payoff rules.

An important factor that we need to take into account in any sharing process is the definition of the appropriate bundle of rights that is assigned to each member of the network. In [92], the author points out that "property rights determine which actors have been authorized to carry out which actions with respect to a specified good or service". Along these lines, the available property rights are defined in terms of which actors are allowed to access, withdraw, manage, exclude or alienate either resources, rights or other actors. The following positions are then defined, which account for specific bundles of the aforementioned rights:

- **Authorized Entrant**, who has only access rights.
- **Authorized User**, who has access and withdrawal rights.
- **Claimant**, who has access, withdrawal and management rights.
- **Proprietor**, who has all previously mentioned rights except alienation.
- **Owner**, whose bundle includes every right previously mentioned.

From these definitions, we can initially consider SPs as authorized users, VNBs as proprietors and RPs as owners.

Together with the definition of the appropriate rules, the property rights define the action situation that applies to the network we study. It is important to note that the aforementioned rules and rights should be defined through an appropriate governance process. Given the nature of the system we analyze, we find polycentric governance as a suitable mechanism for addressing the collective action problems that may arise. This area has been broadly explored by Elinor Ostrom and her fellow researchers at Indiana University Bloomington[4].

A key factor in our analysis of this system is feedback. Similar to what is presented in [91] the definition of the action situation receives inputs from the governance system, the actors and the resource system. At the same time, the interactions and outcomes that take place within the action situation, can generate feedback that will serve to update the behavior of the actors, the governance system, and resource provision and management mechanisms.

7.3.7 Hypotheses

H7. The investment level of the RPs is directly proportional to the amount of resources available in the market.

H9. Establishing matching preferences in terms of joint surplus increases the amount of *allowable* matches.

H13. The reputation of middlemen is directly proportional to the total number of matches of a VNB, and to the payment obtained from resource aggregation services.

H14. SPs that assign higher weights to the matching parameters perform better in terms of percentage of matched SPs, demand obtained from the market, and surplus, than those assigning lower weights. In the same way, VNBs that assign higher weights to the matching parameters perform better in terms fo percentage of matched VNBs, payment received, and surplus than those with lower weights.

[4]Please refer to `https://ostromworkshop.indiana.edu` for detailed information on the research work that has been done by the *Ostrom Workshop*.

8.0 MODEL IMPLEMENTATION AND EXPERIMENTS

In chapter 7, we provided a broad overview of the workings of each of the market stages we analyze in this study. In this chapter, we delve into the particulars of the modeling process of each stage by elaborating on the parameters utilized, working assumptions and scope. These modeling considerations will lead us to the results presented in chapter 9.

8.1 STAGE I

8.1.1 Fungibility Scores

In order to calculate the relevant fungibility scores, we have implemented MATLAB code. Our code utilizes Okumura–Hata, COST231–Hata and Walfish Ikegami empirical propagation models, depending on the applicable distance and frequency ranges. For subsequent adaptability of these scores to our market model, we have focused on the calculation of *capacity fungibility scores*. Hence, we calculate scores that compare the achievable capacity with two different frequencies: the available frequency and the preferred one. Table 5 presents the parameters utilized for the calculations of these scores.

8.1.2 Market Model

In this stage, we rely on the Agent-based model utilized for building the first version of a spectrum trading model, named SPECTRAD, which was introduced in [1,87]. SPECTRAD was developed using Java and REPAST Simphony, an agent-based modeling platform that works with the Eclipse Integrated Development Environment.[1] For determining market viability, we follow the criteria defined in [1, 87], which represent conditions associated with market liquidity. We briefly define these criteria in what follows.

- **Probability of empty bid list:** This results from comparing the number of market runs in which no SLRs post bids for the available resources to the total number of market runs. This situation results from resource prices being above the SLRs' willingness to pay, from available resources not being suitable to the SLRs' demand or from the SLRs relying entirely on AT units for their traffic. In this way, when the value of this probability is too high, it represents an adverse condition for market liquidity.
- **Probability of demand greater than supply:** This condition results from the situation where the resource requirements of the SLRs are larger than the amount of resources available in the BM holdings. This situation calls for a competitive bidding in order to obtain resources from the market; hence, the cutoff price reflects the interaction of the market participants (i.e., their bid adjustments) and, consequently, their willingness to pay. On the other hand, when the demand is not greater than the supply, every bidder obtains the requested resources and pays the minimum price established by the BM (i.e., the BM's reserve price). As a result, we would expect that in scenarios where the demand is greater than the supply, the resources are assigned to the users who value them most (or those who are willing to pay more to obtain them). Consequently, higher values of this probability positively contribute toward market viability as they represent a high interest of the participants to obtain resources from the market.
- **Average cutoff price:** As previously mentioned, the cutoff price in the auction will be determined by the existing level of demand and supply and, in consequence, by the interactions of the SLRs. Note that no cutoff price could be lower than the BM's reserve price; however, it could rise according to the level of competitiveness in the market and the willingness to pay of the SLRs interested in the resources. In this way, a positive market outcome shows cutoff prices that are well above the minimum established by the BM.

[1] See https://eclipse.org for full information on this IDE.

- **Average number of AT units per SLR:** If SLRs are actively obtaining their spectrum requirements from the market, we assume that they have a low incentive to obtain AT units outside of the market. In this light, when the SLRs keep their AT holdings low, we have a positive condition for market liquidity.
- **Percentage of assigned bandwidth units:** As a measure of efficiency, it would be desired for the majority of the BM holdings to be assigned once the bidding rounds end. In this way, a high percentage of assigned BBUs implies active participation of the SLRs in the market, which is positive for its viability.

Based on the aforementioned criteria, we assess overall market viability by defining pass/fail thresholds and scores for each criterion. To define the thresholds, we evaluate the data obtained from our model simulations and determine patterns and trends that are associated with market success. In this way, the thresholds are the resulting breaking points in the data. In absence of these breaking points, we utilize parameters that would correspond to ideal performance. The scores have been defined in [1] according to the market impact of each viability criterion. In this model, we follow these choices as we aim at utilizing the original SPECTRAD model as our basis for comparison.

An overall market score can be obtained by adding all the individual scores. Consequently, market scenarios will be considered viable when their score is greater than zero. Table 4 presents the scores applicable to this stage.

8.1.3 Experiments

Our experiments aim at capturing market viability conditions when different levels of spectrum supply, spectrum valuation and fungibility are considered.

- The level of spectrum supply is defined by R, in equation (8.1). R takes into account the ratio of resources available in the market (numBBUs) to the number of market participants (numSLRs). In this way, lower values of R (e.g., 5, 10) render spectrum undersupply conditions, whereas higher values of R (e.g., 20, 25) represent spectrum oversupply.

$$R = \frac{\text{numBBUs}}{\text{numSLRs}} \tag{8.1}$$

Table 4: Viability Criteria and Corresponding Market Scores

Viability Criteria	
Criteria	**Score Pass / Fail**
P1 - Bid List Empty	1 / -1
P2 - Demand Greater than Supply	1 / -1
P3 - Cutoff Price	0 / -1
P4 - Percentage of Assigned BBUs	1 / -1
P5 - Number of ATs per User	0 / -1

- SLRs are configured with different levels of resource valuation: low, medium and high. This will determine the maximum amount that they are willing to pay for an AT, and thus for a BBU in the market. We shall also remember that this value will be further limited by the applicable fungibility score. In order to capture a worst-case scenario, we consider the case where all users have a low valuation for the available resources.
- Finally, the fungibility score we calculate represents the physical difference between frequencies. However, for users to be able to utilize a frequency different than their preferred one, they may incur in additional costs associated with equipment compliance, quality of service, among others. For this reason, we consider the calculated fungibility score and two lower values, A and B in order to account for the additional costs:

$$\text{Calculated fungibility score} = \frac{\text{capacityPerBBU}}{\text{capacityPerAT}} \tag{8.2}$$

$$\text{Value A} \sim 0.8 \times \text{Calculated fungibility score} \tag{8.3}$$

$$\text{Value B} \sim 0.4 \times \text{Calculated fungibility score} \tag{8.4}$$

Table 6 summarizes the general market parameters to be considered in these experiments.

Table 5: Reference parameters for the calculation of fungibility scores in Stage I

Fungibility Score Parameters	
Parameter	**Reference Value**
Preferred Frequencies	700, 1000, 1500, 1700, 1900 and 2000 MHz
Available Frequency	1900 MHz
Reference Bandwidth	200 KHz
Distance from transmitter	1 Km
Transmitted Power	1mW
Minimum Required Received Power	-80 dBm
Base Station Height	50 m
Geographic Environment	Medium / Small city
Mobile Antenna Height	1 m
Noise Figure	0 dB
Width of road	20 m
Building separation	40 m
Building Height	15 m
Phi	90

Table 6: Market model parameters of Stage I

Market Model Parameters	
Market Type	Band Manager Exchange-based market
Number of Market Participants	numSLRs = {4, 5, 6, 10, 20, 50}
Distribution of Users' Spectrum Valuation	All users have low valuation for the *available* spectrum
Available Spectrum	Calculated using (8.1), where R = {5, 10, 15, 20, 25}
Fungibility level	FungScore = {Calculated score, 0.25, 0.15}
Mean traffic demand	4.0 Mbps
Traffic inter arrival time	Uniformly distributed between 10 and 25 simulation time units

Table 7: Viability Criteria and Market Scores for Stage II

Viability Criteria	
Criteria	**Score** **Pass / Fail**
P1 - Bid List Empty	1 / -1
P2 - Demand greater than Supply	1 / -1
P3 - Cutoff Price	0 / -1
P4 - Percentage of Assigned BBUs	1 / -1
P5 - Number of ATs per User	0 / -1

8.2 STAGE II

8.2.1 Market Model

In stage II, the model is very similar to that of stage I (see section 8.1). The modeling tool and simulation environment are still ABM and Repast Simphony, respectively. There are only certain variations in the model, which account for adapting the new virtualized commodity, as defined in 7.2.1.1 to the market.

As a result of our choice of market commodities and alternate technology units (ATs), we are now working on a pseudo-fungibility environment that can be found in real-world scenarios. As such, we deal with a homogeneous commodity, which does not require the fungibility considerations that were key for the previous stage.

For viability evaluation, we utilize the same criteria as the original model of SPECTRAD and the first stage of this work (section 8.1). The applicable scores for this stage are presented in Table 7.

8.2.2 Experiments

To test the success of this virtualized setting, we propose two simulation scenarios, which focus on the unlicensed spectrum usage period:

1. We follow the same AT units duration that was presented in the original version of SPECTRAD: a random period, uniformly distributed between 90 and 110 simulation time units.
2. We consider that the unlicensed spectrum usage time will be the same as the duration of the licensed spectrum lease (i.e., 10 milliseconds).

The objective of the second scenario is to account for possible degradation of service relative to the use of unlicensed spectrum and to permit the SLRs to enter the market once a new bidding round starts.

In each of these scenarios, we test the same resource undersupply and oversupply conditions that are defined through R in equation (8.1). Nevertheless, in this stage the amount of resources available (i.e., $numBBUs$) is restricted by the LTE standard parameters. In this way, for the values defined by (8.1) that do not match an LTE value, we choose the closest allowable amount of PRBs.

To take into account the SLRs' resource valuation we analyze scenarios where one-third of users belongs to each, high, medium and low, licensed spectrum valuation levels. Nevertheless, when comparing licensed vs. unlicensed spectrum, the SLRs should take into account the possible difference in resource quality. In this way, their valuation for the AT units or unlicensed TVWS spectrum will be inversely proportional to their licensed spectrum valuation. Table 8 summarizes the parameters that we consider appropriate for the simulations in this stage and their corresponding values.

8.3 STAGE III

The model we utilize for this section has been entirely developed for this dissertation. In this way, this section includes all the details regarding the implementation of this model and

Table 8: General Parameters for the model of Stage II

General Second Stage Model Parameters	
PRBs occupied bandwidth [MHz]	Based on the three available 10MHz LTE Bands: [1.08, 2.7, 4.5, 9] Using carrier aggregation, we can obtain up to 27 MHz.
Traffic capacity of a PRB [Mbps]	Calculated using (7.4) and the standard specified bandwidth. Min = 4.06 Mbps - Max = 15.5 Mbps
Traffic capacity of a TVWS bandwidth unit [Mbps]	1. 18 Mbps – Calculated for a bandwidth of 180 KHz with a 700 MHz band
PRBs lease time	10 simulation time units (represent the 10 millisecond duration of an LTE frame)
Duration of unlicensed spectrum usage	Case 1: uniformly distributed between 90 and 110 simulation time units Case 2: 10 simulation time units
Number of spectrum users	numSLRs = {4, 5, 6, 10, 20}
Mean traffic demand	4.0 Mbps
Mean traffic inter arrival time	Uniformly distributed between 10 and 25 simulation time units

its two interaction instances: VNB – SP and VNB – RP negotiations.

Although the scope of the market analysis in this stage is greater than that of the previous stages, the entities or agents participating in this model maintain similar characteristics in terms of their behavior and what they seek from the market. Indeed, Service Providers (SPs) correspond to the Spectrum License Requesters (SLRs) from Stages 1 and 2. Similarly, Resource Providers (RPs) correspond to Spectrum License Holders (SLHs) in Stages 1 and 2. Note that in Stage 3, RPs become active participants in the system and their actions will influence the final market outcome.

For the study of the network and market settings of Stage 3, we have also developed an Agent-based Model. Due to the nature of the data that we aim at generating and the level of detail we utilize for defining our agents, we deem more appropriate to utilize a tool with different computational capabilities, such as MATLAB.

As presented in this section, there is a significant number of parameters, and their corresponding levels, that come into play. To maintain the tractability of the problem we analyze, we have utilized uniform distributions for assigning SPs, VNBs and RPs to different risk profiles and valuation levels. The same reasoning supports the definition of the thresholds we utilize for differentiating between low/averse, medium/neutral, and high/taker valuation/risk levels, respectively. This approach has allowed us to focus our attention on how the model works, as a whole. Future work on this stage includes modifications to these distributions in order to determine their effect on our results.

This section has been divided into two main subsections: VNB–SP negotiations and VNB–RP negotiations. VNB–SP negotiations account for the matching and partnership forming process between these two entities. VNB–RP negotiations refer to the actual market transactions that lead to the assignment of spectrum-related resources.

8.3.1 VNB – SP negotiations

Matching markets are at the core of the VNB–SP negotiations. Matching markets are a vast research area, applied to various lines of investigation and which has been mainly led by the Nobel Laureate, Alvin E. Roth. Initial proposals on matching processes are represented by the work of Gale and Shapley, which present a solution based on marriage and college admission problems [32]. Since the appearance of this work in the 1960s, the research of matching markets has widely expanded, covering issues such as matching medical students with residency positions, students with public schools, and dealing with life-or-death situations such as matching organ donors with recipients.

Matching markets are of interest in this specific part of our work because we aim at building a more expressive system for matching resource buyers, sellers and intermediaries. In this way, we utilize matching markets to form partnerships between VNBs and SPs, mimicking our real-life interactions with middlemen. These interactions are generally based on the reputation of middlemen and how much we trust them. For partnerships to form, we rely on an underlying set of preferences we design according to our expectations (e.g., ease of interaction, middlemen expertise and accuracy) and our own known information (e.g., budget, willingness to pay, valuation of resources to obtain, risk perspective). In this setting, matching represents an economic construct rather than a technical one. Hence, this fulfills partnership forming purposes outside of technical resource or operation compatibility.

From the previous description, we can infer that for matches to form, we need to have a clear definition of preferences and be aware of the individual limitations. Consequently, in what follows, we present the specific details of how a matching mechanism has been implemented for the market model we devise.

Let $\mathcal{S} = \{s_1, s_2, ...s_n\}$ be the set of n participating SPs and $\mathcal{B} = \{b_1, b_2, ...b_m\}$ the set of m participating VNBs.

Each of the agents in $\mathcal{S}$ and $\mathcal{B}$ are assigned a risk profile, which guides the values they assign to their own parameters and the preferences they express with regards to the members of the other set. For this purpose, we assign to all $s \in \mathcal{S}$ and all $b \in \mathcal{B}$, a risk value defined as $rv = U(0, 2)$. In this way, the variable rv can take a uniformly distributed integer in the range [0,2]. Consequently, the risk profile of s_i and b_j is assigned as follows:

$$riskProfile(rv) = \begin{cases} \text{averse} & \text{if } rv = 0 \\ \text{neutral} & \text{if } rv = 1 \\ \text{taker} & \text{if } rv = 2 \end{cases} \tag{8.5}$$

8.3.1.1 Configuring real and advertised SPs' and VNBs' prices and fees In this subsection we explore how SPs and VNBs define the real and advertised values of the fees they are willing to pay and expecting to receive, respectively.

Service Providers: Each SP has a *real* and an *advertised* value that indicates its willingness to pay for the service of a VNB. The real valuation of an SP can be translated into a measure of how interested is an SP in transacting with a VNB. To capture this, we assign a level of valuation for each SP in the system, vl_i, which is a uniformly distributed integer in the range [0,2].

$$vl_i = U(0,2) \quad \forall i \in \mathcal{S}$$

In this way, the valuation level of s_i is finally assigned as follows:

$$valuationLevel(vl_i) = \begin{cases} \text{low} & \text{if } vl_i = 0 \\ \text{medium} & \text{if } vl_i = 1 \\ \text{high} & \text{if } vl_i = 2 \end{cases} \tag{8.6}$$

We assume that there is a minimum price (reserve price) advertised in the system for the revenue expected by a VNB and also a maximum, general, price that any VNB can charge. In this way, we work with a range of prices, p_i, chosen by s_i according to its valuation level vl_i. The ranges we have defined are included in Table 9.

These ranges represent the real price that each SP is willing to pay for VNB services. The limits utilized for price assignment are the following:

$$p_{min} = 25 \tag{8.7}$$

$$p_{max} = 100 \tag{8.8}$$

$$p_{med} = \frac{p_{max} + (2 \times p_{min})}{3} \tag{8.9}$$

$$p_h = \frac{(2 \times p_{max}) + p_{min}}{3} \tag{8.10}$$

Table 9: VNB price range according to valuation level

Valuation Level (vl_i)	**Price Assigned**
0	$p_i = U(p_{min}, p_{med})$
1	$p_i = U(p_{med}, p_h)$
2	$p_i = U(p_h, p_{max})$

The *advertised* price of each SP, on the other hand, depends on its risk profile. As such, these prices will include a level of shading, which is consistent with the risk a given SP is willing to take. Table 10 shows the three levels of price shading that have been defined.

Taking into account the price shading levels in Table 10, the advertised prices are defined in (8.11), and these apply for all $i \in \mathcal{S}$

$$advPrice_i = (1 - d_r) \times p_i \tag{8.11}$$

.

Note that d_r represents the price shading applicable to s_i, according to its own risk level.

To handle the creation of preference sets, the particular price advertised by s_i is mapped to a *price level*, as shown in Table 11.

Table 10: Percentage of price shading according to each SP's risk level

Risk Level	**Price Shading**
Averse	$d_r = d_A = U(0, 0.05)$
Neutral	$d_r = d_N = U(0.05, 0.10)$
Taker	$d_r = d_P = U(0.10, 0.15)$

Table 11: Price levels assigned according to range of prices advertised by s_i

Price Level	**Price Advertised**
0	$p_{med} > advPrice_i \geq p_{min}$
1	$p_h > advPrice_i \geq p_{med}$
2	$p_{max} > advPrice_i \geq p_h$

Virtual Network Builders: In the same manner as the SP, each VNB also defines two types of fees: *real* and *advertised*. The real fee it can charge depends on its quality or reputation. To bootstrap the market, the quality level of VNB b_j, ql_j, is randomly assigned (8.12); however, we expect to incorporate a reputation building mechanism based on the performance history of b_j.

$$ql_j = U(0, 2) \tag{8.12}$$

In this way, the final quality, q_j, of b_j is given by 8.13. It is expected that a higher quality VNB can charge higher fees for its services. The actual values assigned according to the quality level are included in Table 12. Note that these constitute the real fees a VNB requires as a payment.

$$q_j(ql_j) = \begin{cases} \text{low} & \text{if } ql_j = 0 \\ \text{medium} & \text{if } ql_j = 1 \\ \text{high} & \text{if } ql_j = 2 \end{cases} \tag{8.13}$$

The limits utilized for the fees assigned are defined as follows:

$$f_{min} = 25 \tag{8.14}$$

$$f_{max} = 100 \tag{8.15}$$

$$f_{med} = \frac{f_{max} + (2 \times f_{min}}{3}) \tag{8.16}$$

$$f_h = \frac{(2 \times f_{max}) + f_{min}}{3} \tag{8.17}$$

Table 12: Real fee of a VNB according to its quality (or reputation) level.

Quality Level	**Real fee of b_j**
0	$f_j = U(f_{min}, f_{med})$
1	$f_j = U(f_{med}, f_h)$
2	$f_j = U(f_h, f_{max})$

In a similar manner as the price shading performed by the SPs, the VNBs' advertised fees are set according to the risk level of each VNB. In this way, the advertised fees include a percentage increase on a VNBs real fee. The percentage increase is presented in Table 13.

Following the percentage of price shading shown in Table 13, the price advertised, $advFee_j$, by b_j is given by (8.18)

$$advFee_j = (1 + i_r) \times f_j \tag{8.18}$$

We also assign price levels to each VNB according to the range of the advertised price, $advFee_j$, as shown in Table 14.

Table 13: Percentage of fee increase according to each VNB's risk profile

Risk Level	**% of Fee Increase**
Averse	$i_r = i_A = U(0, 0.05)$
Neutral	$i_r = i_N = U(0.05, 0.10)$
Taker	$i_r = i_P = U(0.10, 0.15)$

Table 14: VNBs' fee level according to the range of the advertised fees

Price Level	Price Advertised
0	$f_{med} > advFee_j \geq f_{min}$
1	$f_h > advFee_j \geq f_{med}$
2	$f_{max} > advFee_j \geq f_h$

8.3.1.2 Service Providers' Demand Each end user in the system is randomly set as a customer of one of the existing Service Providers. For this purpose, each end user will be assigned a uniformly distributed random integer between 1 and the number of SPs. This number will correspond to the ID of the SP that will be serving this particular end user. In consequence, the traffic that each SP needs to serve will be the aggregate of the demand of its end users. This is defined in (8.19), where T_i is the total traffic of s_i, u_{ij} is the j-th user of s_i, and t_{ij} is u_{ij}'s individual traffic.

$$T_i = \sum_{j=1}^{p} t_{ij} \tag{8.19}$$

The actual traffic of each end user is defined as an exponentially distributed random number with mean tm. In current tests of the model, I use the value $tm = 4.0$Mbps

SPs' Demand Calculation: The demand is calculated by comparing the coverable traffic with the traffic to serve. The coverable traffic of s_i, Tc_i, is defined in (8.20), where rsc_i is the amount of resources already available to s_i and C is the capacity per resource.

$$Tc_i = rsc_i \times C \tag{8.20}$$

If the coverable traffic, Tc_i is greater than the traffic to serve, the demand of s_i is zero. Otherwise, the *resource* demand of s_i is given by (8.21) and the *throughput* demand of s_i is given by (8.22).

$$d_i = \frac{\lceil T_i - Tc_i \rceil}{C} \tag{8.21}$$

$$d_i = \lceil T_i - Tc_i \rceil \tag{8.22}$$

Table 15: SPs' demand levels according to the range of their demand value

Demand Level	**Demand Ranges**
0	$0 \leq d_i < d_{med}$
1	$d_{med} \leq d_i < d_h$
2	$d_h \leq d_i < d_{max}$

In order to form the preference vector, I have also classified the actual demand values into three demand levels. These levels are defined in (8.23 – 8.26). Note that the maximum total demand or d_{max} considers the case where an SP needs to serve all the end users in the area, i.e., o end users, and each end user has a traffic demand equal to the average t_m. Table 15 further illustrates how the demand of s_i is classified into multiple levels.

$$d_{min} = 0 \tag{8.23}$$

$$d_{max} = o \times t_m \tag{8.24}$$

$$d_{med} = \frac{d_{max} + (2 \times d_{min})}{3} \tag{8.25}$$

$$d_h = \frac{(2 \times d_{max}) + d_{min}}{3} \tag{8.26}$$

8.3.1.3 Choices and Preferences of VNBs and SPs

This model considers a set of choice and preference parameters for s_i and b_j, for all $i \in \mathcal{S}$ and $j \in \mathcal{B}$, respectively. It is important to clarify that *choices* refer to the set of parameters that are particular to the workings of each VNB or SP. These entities decide on what level/value to assign to each choice parameter according to their risk profile or their individual settings. Subsequently, these become parameters advertised (perhaps after being shaded) in the matching process. Conversely, *preferences* refer to the set of parameters that VNBs and SPs observe in the members of the opposing set. In other words, the preferences have to do with the individual analysis of a VNB(SP) regarding the choice parameters of a given SP(VNB). In what follows, I refer to how these choices have been assigned and how they account toward defining the final set of preferences of SPs and VNBs.

8.3.1.4 Choices of SPs The two following parameters are considered for the choices of the Service Providers:

- **Price:** defined according to the advertised price levels presented in Table 11.
- **Demand:** defined according to the demand levels included in Table 15.

These parameters take integer values between 0 and 2, which stand for low, medium and high levels, respectively. In order to manage these parameters, I have represented the value level associated with each parameter as a 1×3 vector where the kth element can take a value of 0 or 1, depending on whether the value corresponds to a low, medium or high level (8.27). These vectors are pv_i and dv_i for price value and demand value, respectively.

$$\text{level vector} = [l, m, h] \tag{8.27}$$

$$\text{low level} = level0 = [1, 0, 0] \tag{8.28}$$

$$\text{medium level} = level1 = [0, 1, 0] \tag{8.29}$$

$$\text{high level} = level2 = [0, 0, 1] \tag{8.30}$$

8.3.1.5 Choices of VNBs In a similar manner to the case of the SPs, the following choice parameters have been considered as relevant for the matching process between VNBs and SPs:

- **Quality:** As previously presented, the quality level or reputation is randomly assigned to each VNB in the initialization process.
- **Fees:** The fee level of each VNB is defined according to Table 14.

These parameters take integer values between 0 and 2, which stand for low, medium and high levels. These levels are also expressed as vectors, qv_j and fv_j for quality and fees, respectively (8.27).

Table 16: SP preference vectors according to risk level

Risk level of s_i	**Reputation/Quality Preference of s_i**	**Price Preference of s_i**
Averse	$qp_i = [0, 0, 1]$	$pp_i = [0, 0, 1]$
Neutral	$qp_i = [0, 1, 1]$	$pp_i = [0, 1, 1]$
Taker	$qp_i = [1, 1, 1]$	$pp_i = [1, 1, 1]$

8.3.1.6 Preferences of SPs SPs express their preferences regarding the following parameters (or choices) of VNBs:

- VNB reputation or quality[2]
- VNB advertised fee

These preferences are expressed as vectors, which represent the *preference* for a low, medium or high value for each of the aforementioned parameters. Further, these preferences are linked to the risk profile of each SP, which justifies the preference level. In this manner, the quality preference vector (qp_i) and the fee preference vector (pp_i), are represented as 1×3 vectors, where the kth element can take either value of 0 or 1. The kth element is equal to 1 if s_i prefers that value level for a particular parameter. This is further illustrated in Table 16.

8.3.1.7 Preferences of VNBs A VNB expresses its preferences regarding the following SP values:

- SPs' advertised price
- SPs' demand level

In the same manner as the SPs' case, the values to these parameters are assigned according to the risk level of each VNB. The vector corresponding to each preference is presented in

[2] We refer to reputation and quality interchangeably throughout the document, regarding this specific characteristic of the VNBs.

Table 17.

Table 17: VNBs' preference vectors according to their risk level

Risk level of b_j	**Price Preference of b_j**	**Demand Preference of b_j**
Averse	$pp_j = [1, 0, 0]$	$dp_j = [1, 0, 0]$
Neutral	$pp_j = [1, 1, 0]$	$dp_j = [1, 1, 0]$
Taker	$pp_j = [1, 1, 1]$	$dp_j = [1, 1, 1]$

8.3.1.8 Comparing Choices and Preferences In order to create the final preference vectors of each SP and VNB, I create a matrix for each of their preference parameters. In the case of the SPs, the ijth matrix element is the result of multiplying the preference vector of s_i times the transpose of the corresponding value vector of b_j. Q_s corresponds to the reputation (or quality) preference matrix (8.31) and R_s corresponds to the price preference matrix (8.32)

$$Q_s(i,j) = qp_i \times qv_j^T \tag{8.31}$$

$$R_s(i,j) = pp_i \times pv_j^T \tag{8.32}$$

For the VNBs, the ijth elements of the preference matrices take into account the preferences of each VNB and the values assigned to the corresponding parameter by the SP. D_b corresponds to the demand preference matrix (8.33), and R_b represents the price preference matrix (8.34).

$$D_b(i,j) = dv_i \times dp_j^T \tag{8.33}$$

$$R_b(i,j) = pv_i \times pp_j^T \tag{8.34}$$

Table 18: Weights assigned to SPs' and VNBs' preference parameters

Quality / Demand	**Price(VNB) / Price(SP)**
H	H
H	L
L	H
L	L

8.3.1.9 SPs' and VNBs' utility The next step is to define the utility of each SP and VNB. The idea behind defining this utility is for the SPs(VNBs) to find a subset of VNBs(SPs) that would be part of their final preference set. Once a preference set is defined, we can then proceed to apply the deferred acceptance algorithm for matching, which will be explained in section 8.3.2.

To this end, I propose to define weights that each SP and VNB can give to the different parameters that are being considered. These weights are defined as uniformly distributed random numbers within a specific range set by expressions (8.35) and (8.36). This definition has been arbitrarily chosen to avoid increased complexity stemming from different weight distributions.

$$W_h = U(0.6, 1) \tag{8.35}$$

$$W_l = U(0.1, 0.5) \tag{8.36}$$

Given that each SP and VNB takes into account two parameters for their preferences, we have a final set of four different combinations of weights and parameters, as shown in Table 18. The actual fashion in which an SP and a VNB choose the weight to assign is defined in the experiment design section 8.3.6.

The individual utility of SPs and VNBs, i.e., the utility of a matching between s_i and b_j, is given by (8.37) and (8.38), where w_q and w_p are the weights assigned by s_i to the quality and price factors, and w_d and w_p are the weights assigned by b_j to the demand and price factors.

$$U_s(i,j) = w_q \times Q_s(i,j) + w_p \times R_s(i,j) \tag{8.37}$$

$$U_b(i,j) = w_d \times D_b(i,j) + w_p \times R_b(i,j) \tag{8.38}$$

We also obtain a matrix A, where the ijth element corresponds to the joint utility derived from the partnership between s_i and b_j (8.39).

$$A(i,j) = U_s(i,j) + U_b(i,j) \tag{8.39}$$

The values in the individual and joint utility matrices are utilized for creating the final preference vectors of s_i and b_j. Given the aforementioned calculation of utilities and the value that weights can take, the maximum *individual* utility of s_i or b_j is 2, while the minimum *individual* utility is 0.2. In the case of *joint* utility, the maximum value is 4 and the minimum is 0.4.

These values are utilized to rank each member of the opposite group and choose those that will be part of the final preference vector. I assume that an SP and a VNB having the lowest score for their partnership should not be included in each other's preference vector. In this way, there should be a minimum allowed threshold (between 0.4 and 4) that represents an acceptable partnership. For our working model purposes, we have assumed this acceptable threshold to be the middle point in the joint utility range, namely 1.8[3].

8.3.2 Matching SPs and VNBs

The subset of feasible, or acceptable, partnerships corresponds to the preference vectors of s_i and b_j. As pointed out by Roth in [93], regarding the marriage problem posed by Gale and Shapley [32], "[p]references can be represented as rank order lists of the form $P(m_i) = w_3, w_2, ...m_i$, denoting that man m_i's first choice is w_3, his second choice $w_2[w_3 >_{m_i} w_2]$

[3] Other values for the acceptable threshold have been analyzed, and the corresponding results are included in section 10.2

and so on, until at some point he prefers to remain unmatched (i.e., matched to himself)." The same applies to the problem at hand. In this case, the preference vector of s_i and b_j will contain a subset of members of the opposite set with whom it is possible to form a partnership (8.40) (8.41). These subsets, or preference vectors are sorted in descending order of preference.

$$P(s_i) = b_k, b_l, b_m, ..., s_i \tag{8.40}$$

$$P(b_j) = s_o, s_p, s_q, ..., b_j \tag{8.41}$$

The matching between SPs and VNBs is implemented utilizing the *deferred acceptance algorithm* for the many-to-one matching case. This means that a VNB can form a partnership with n SPs, where n = VNBs quota or partnership size; while an SP can only form a partnership with one VNB. The value of n has been set to m i.e., the total number of SPs in the network[4].

In [93] Roth presents the description of the deferred acceptance algorithm as it would be applied to the Gale and Shapley's marriage problem. In what follows, we adapt this definition to fit the problem we explore.

- Step 1a. Each SP, s_i, proposes a partnership to the first choice in its preference vector (if it is not empty).
- Step 1b. Each VNB, b_j, rejects any unacceptable proposals and, in case there are multiple acceptable proposals, b_j holds the n most preferred ones and rejects all others.
- Step ka. Any SP rejected at step *k-1* makes a new proposal to the next VNB in the preference vector, which has not rejected it. (If no more acceptable proposals remain, the SP makes no further proposals).
- Step kb. Each VNB holds its most preferred, acceptable proposals to date and rejects the rest.
- The algorithm stops when no further proposals are made. Then we match each VNB to the SPs (if any) whose proposals it is currently holding.

[4]This value was assigned as a means to establish uniformity among the VNBs in the network and avoid forcing a specific market structure. Nevertheless, it can be adjusted to fit scenarios of interest

The outcome of this matching game is a matching $\mu : \mathcal{S} \cup \mathcal{B} \rightarrow \mathcal{S} \cup \mathcal{B}$ such that $b = \mu(s)$ if and only if $\mu(b) = s$. For all s and b, either $\mu(s)$ is in $\mathcal{B}$ or $\mu(s) = s$; and, either $\mu(b)$ is in $\mathcal{S}$ or $\mu(b) = b$. This means that the outcome matches SPs with VNBs, or to themselves, and if s is matched to b, then b is matched to s [93].

It is important to note that we consider the case in which the SPs *propose* a partnership first, which leads to an SP-optimal matching, μ_S [93]. For the actual implementation of this algorithm in the model, we have also followed important considerations presented in [94], [32], [35].

Once the matching process is over and we obtain the final matching μ, each VNB learns which are its customers and each SP will learn the ID of the VNB with whom it will be working. The following step consists of the matched SPs communicating their demand and resource price parameters to their VNB partners. As a result, the matched SPs' demand now constitutes the *market* demand, which should be obtained from the pool.

8.3.2.1 Market Demand As previously mentioned, the market demand consists of the throughput needed by the matched SPs. Let's refer to the set of SPs matched with b_j as $MS_j = \{ms_{1j}, ms_{2j}, ..., ms_{nj}\}$. In turn, we can refer to these SPs as VNB b_j's customers. In this manner, each VNB should gather information about the actual resource demand of each of its customers and the maximum price they are willing pay for these resources[5]. Demand includes the quantity (in Mbps) and the type of resources required. In this model I assume that SPs can be divided in two types, regarding the services they offer to their customers. In this way, resources of type 1 are those utilized to provide video streaming services and resources of type 2 are those required for low throughput, bursty traffic such as Internet of Things applications.

The price that SPs will pay for resources is consistent with s_i's valuation level, vl_i, and it is defined in Table 19. In this way, this valuation level is associated with the fee an SP is willing to pay for VNB services and the maximum amount to pay per resource unit.

The limits utilized for resource price assignment are shown in (8.42,8.43,8.44,8.45). These

[5]Note that there is a difference between the VNB fee and the resource price. The first is intended to cover the cost incurred by each VNB in obtaining the resources from the pool, while the latter corresponds to the valuation that each SP has for the spectrum resources.

Table 19: VNB price range according to valuation level

Valuation Level (vl_i)	**Resource Price Assigned**
0	$rscp_i = U(rscp_{min}, rscp_{med})$
1	$rscp_i = U(rscp_{med}, rscp_h)$
2	$rscp_i = U(rscp_h, rscp_{max})$

values represent monetary units.

$$rscp_{min} = 3 \tag{8.42}$$

$$rscp_{max} = 10 \tag{8.43}$$

$$rscp_{med} = \frac{p_{max} + (2 \times p_{min})}{3} \tag{8.44}$$

$$rscp_h = \frac{(2 \times p_{max}) + p_{min}}{3} \tag{8.45}$$

It follows that each VNB b_j creates a demand inventory (quantity and price) per resource type and it looks for the corresponding type of resources in the common pool.

8.3.3 VNB – RP negotiations

After the market demand is defined through the matching market, the VNBs need to find the appropriate set of resources from the pool. As mentioned in previous sections, the pool of resources is populated with the excess spectrum belonging to existing incumbents in the area, which we refer to as Resource Providers (RPs). The VNBs bring to this side of the market the resource demand and pricing information that they learned from their partnering SPs. Note that, for market transactions, the VNBs utilize the exact information provided by their SP partners (i.e., they do not seek an additional profit). Figure 4 depicts these interactions.

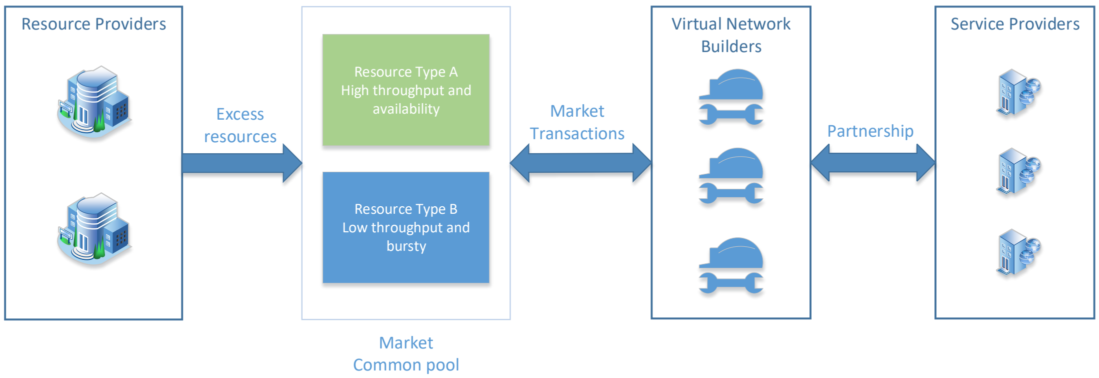


Figure 4: General overview of the transactions that lead to the VNB–RP negotiations

There are various aspects that drive the amount of resources shared by the RPs in the pool (i.e., the market supply), and their reserve price. In this section, we delve into these details and the approach we have taken towards modeling them. Note that the main characteristics of the VNBs have been already presented in subsection 8.3.1. For this reason, this section focuses on the description of the RPs and the subsequent market interaction.

We represent the set of RPs as $\mathcal{RP} = \{rp_1, rp_2, ..., rp_n\}$. In the same manner as the VNBs and SPs, an RP, rp_k has a risk profile that will dictate its choices regarding the amount of resources to offer and the corresponding reserve prices. For this purpose, I assign to every $rp \in \mathcal{RP}$ a risk value defined as $rv = U(0, 2)$. In consequence, rp_k receives a risk profile according to (8.5).

The RPs in this model, and the resources they possess, are differentiated in terms of the service they provide to their end users. For ease of representation in the model, I assume that RPs are divided into providers of resources for video streaming and IoT, or type 1 and type 2, respectively.

8.3.3.1 Managing the Resources of the RPs In the same manner as the SPs, RPs have a set of end users, whose demand they need to serve. Let $UR_k = \{ur_{1k}, ur_{2k}, ..., ur_{qk}\}$ be the set of q end users of resource provider rp_k. The resulting demand that rp_k needs to cover is given by (8.46), where TR_k represents the total traffic of rp_k, t_{jk} is the traffic of end user ur_{jk}, and ur_{jk} is the j-th user of rp_k

$$TR_k = \sum_{j=1}^{q} t_{jk} \tag{8.46}$$

Every RP has a given amount of existing resources or holdings, H_k, each providing a throughput x. In this way, the coverable traffic of rp_k is given by TC_k in (8.47).

$$TC_k = H_k \times x \tag{8.47}$$

In order to determine the amount of resources that rp_k will make available in the pool, it must compare the traffic coverable with its current holdings to TR_k. We refer to rp_k's resources available for pooling as offer, O_k, which is given by (8.48). As expected, the offer of each RP results from subtracting the traffic required, TR_k, from the coverable traffic, TC_k. Given that O_k corresponds to the entire amount of rp_k's resources that are available for pooling, I will refer to this value as the *real* offer of rp_k.

$$O_k = TC_k - TR_k \tag{8.48}$$

As pointed out at the beginning of this section, the risk profile of rp_k defines the final amount of resources that it will offer for pooling. The underlying assumption is that the more prone to risk rp_k is, the greater amount of resources it will pool. In other words, the risk profile of rp_k dictates whether this RP shares 100% of O_k or a smaller fraction of it. It follows that the final market offer (i.e., resources pooled by rp_k) is given by (8.49), where MO_k is the market offer, O_k is the real offer of rp_k and pm_k is the fraction of resources shared by rp_k. pm_k is a function of the risk profile of rp_k and it is defined in (8.50).

$$MO_k = pm_k \times O_k \tag{8.49}$$

Table 20: Market offer of each RP as a fraction of real offer

Risk Level	**Market Offer**
Averse	$MO_k = U(0.7, 0.8) \times O_k$
Neutral	$MO_k = U(0.8, 0.9) \times O_k$
Taker	$MO_k = U(0.9, 1) \times O_k$

$$pm_k(rv_k) = \begin{cases} U(0.9, 1) & \text{if } rv_k = 0 \\ U(0.8, 0.9) & \text{if } rv_k = 1 \\ U(0.7, 0.8) & \text{if } rv_k = 2 \end{cases} \tag{8.50}$$

A more complete translation of this assumption is presented in Table 20.

8.3.3.2 Resource Prices Each RP has a *real* reserve price, p_k, for the resources it offers. Due to their risk profile, RPs may shade their real price and advertise a different, higher, price in the market. The percentage increase of rp_k, i_k, is given by (8.51).

$$i_k(rv_k) = \begin{cases} i_k = U(0.1, 0.15) & \text{if } rv_k = 0 \\ i_k = U(0.05, 0.1) & \text{if } rv_k = 1 \\ i_k = U(0, 0.05) & \text{if } rv_k = 2 \end{cases} \tag{8.51}$$

In this way, the advertised price, pa_k of rp_k is given by (8.52).

$$pa_k = (1 + i_k) \times p_k \tag{8.52}$$

8.3.3.3 Market Supply The supply in the market model is given by the resources that have been pooled by the RPs. For this purpose, we work with a pool construct that contains the amount of available resources, the minimum price to ask for them, their type, and their owner. After this process, the pool is ready for the VNB to find the resources that match the type, quantity and price requirements of its customers. In other words, the system is ready for transferring resources from RPs to VNBs, and subsequently to SPs.

8.3.4 VNB - RP Market

The actual market transactions that permit VNBs to obtain resources from the pool are modeled via a simple market setting. The idea behind this market model is to match supply and demand and find a resulting market clearing price (i.e., cutoff price) through a sealed-bid auction.

To model this market, we first divide the supply and demand according to the type of resources. In this manner, we will have the following demand and supply sets: $D_t = \{d_{t1}, d_{t2}, ..., d_{tl}\}$, $O_t = \{o_{t1}, o_{t2}, ..., o_{tv}\}$ where the subindex t stands for type, which can be 1 or 2, and l and v are the total number of demand requests and supply offers, respectively.

Each member of the demand and supply sets is associated with either a bid price or an offer (reserve) price, respectively. We arrange the demand-bid pairs (i.e., bids (bp) associated with D_t) in descending order, and the supply-offer pairs (i.e., offers (op) associated with O_t) in ascending order. Then, we aggregate the demand values as the price decreases. Similarly, we aggregate the supply values as the price increases. This is presented in Tables 21 and 22

The clearing price results from the last point in which $o_{ti} \geq d_{ti}$ and $bp_{ti} \geq op_{ti}$. A graphical representation is included in figure 5. The bold line area shows the region where supply and demand intersect. The prices associated with this demand constitute the market clearing price (cutoff price).

After this market interaction, each VNB receives its set of resources with prices above the clearing price. We have modeled this market as a one-time interaction; hence, resources are assigned after a single bidding round. Additionally, we assume that the market transactions for both types of resources are independent from each other, thus they take place

Table 21: Market demand as a function of price. Note that the bid values are sorted in descending order

Demand	**Bids**
d_{t1}	bp_{t1}
$d_{t1} + d_{t2}$	bp_{t2}
...	...
$\sum_{i=1}^{l}(d_{ti})$	bp_{tl}

Table 22: Market offer as a function of price. Note that the offer values are sorted in ascending order

Supply	**Offer**
o_{t1}	op_{t1}
$o_{t1} + o_{t2}$	op_{t2}
...	...
$\sum_{i=1}^{v}(o_{ti})$	op_{tv}

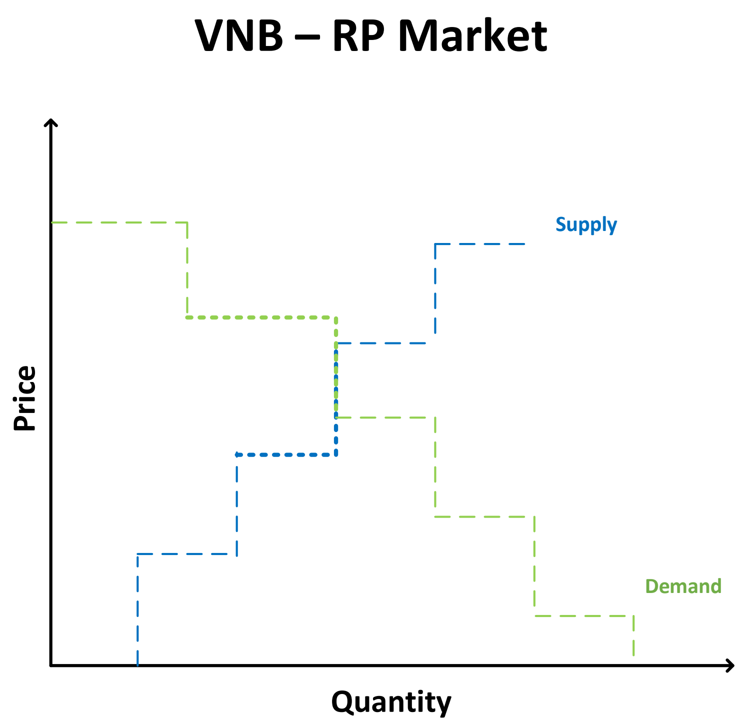


Figure 5: Scheme of the market between VNBs and RPs

simultaneously.

8.3.5 Resource Assignment and Price Adjustment

Once it has received the resources with the most competitive prices, a VNB assigns the obtained resources to the corresponding SPs. At the same time, the VNB reveals the cutoff price to its customers. SPs then calculate the total amount they need to pay for resources (8.53), and the corresponding surplus. As shown in (8.54), an SP's surplus results from the difference between the price it is willing topay for resources, $rscp_i$, and the cutoff price, multiplied by s_i's demand.

$$RscPayment_{sp} = cutoffprice \times d_i \tag{8.53}$$

$$RscSurplus_{sp} = (rscp_i - p_{cutoff}) \times d_i \tag{8.54}$$

The cutoff price information also allows SPs to adjust their advertised price $advPrice_i$ for subsequent market participation. Indeed, an SP has the option of increasing the advertised price if it did not obtain resources from the market, or reducing the advertised price in case its surplus is negative.

There is an additional payment that the SP needs to consider: VNB payment. The rule for this payment is as follows: an SP pays its VNB partner for its services only if the VNB obtained the SP's (partial or full) demand. Given that in our matching model we consider allowable matches based on the joint utility between SPs and VNBs, the final payment that b_j receives from s_i is the average fee advertised by said SP and VNB (8.55). This permits to adjust possible individual losses resulting from the joint utility model.

$$payment_{ij} = \frac{advFee_j + advFee_i}{2} \tag{8.55}$$

Both, SPs and VNBs, can calculate the surplus stemming from this payment. It results from the difference between the real fees they are willing to pay/accept (i.e., p_i and f_j) and the final payment they make/receive (i.e., $payment_{ij}$). Expressions (8.56) and (8.57) show these surplus calculations for SPs and VNBs, respectively. Learning whether this surplus is positive or negative allows both, SPs and VNBs, to adjust their advertised fees for subsequent participation in this market. Evidently, if the surplus is negative, SP(VNB) fees should be reduced(increased) and vice versa.

$$FeeSurplus_{s_i} = p_i - payment_{ij} \tag{8.56}$$

$$FeeSurplus_{b_j} = f_j - payment_{ij} \tag{8.57}$$

On the RP side, each RP that assigned resources will receive the corresponding remuneration. Evidently, this remuneration will correspond to the fraction of resources assigned, pmo_k. This payment is made by the SPs through the VNB and the total amount received by rp_k is given by (8.58). The surplus from this transaction corresponds to the difference between rp_k's real reserve price and the market cutoff price and it is given by (8.59).

$$payment_k = cutoffprice \times pmo_k \tag{8.58}$$

$$surplus_k = (p_k - p_{cutoff}) \times pmo_k \tag{8.59}$$

In this model we assume that RPs decide at each instance whether it is convenient for them to participate. In this light, the payment and surplus information serve each RP to make this decision. For instance, a negative surplus could be an indicator to either adjust the advertised prices for subsequent interactions or to leave the market.

It is important to note that throughout these interactions, only VNBs know the SPs' demand and their bids. Additionally, the VNBs do not communicate to the SPs the identity of the resource owners, and they do not necessarily need to know it. Indeed, the RPs could be even shielded from providing information regarding their identity to the intermediaries, as the only information VNBs need from the pool is the type of resources and their advertised price. In this light, the only information that is known by all participants is the final market clearing price. The rest of the information necessary for the different sets of transactions is either private (i.e, only known by the entity) or public within the transaction domain (i.e., known only by the transacting parties).

Table 23: Division of SPs and VNBs into experimental groups according to weights assigned

	Demand / Reputation	**Fees to Charge / Pay**
Group 1	L	L
Group 2	L	H
Group 3	H	L
Group 4	H	H

8.3.6 Experiments

In order to evaluate the model developed for Stage III, we have designed an experiment that aims at testing the influence of a set of factors in the market outcome. To this end, we divide the entire set of SPs and VNBs (i.e., the experiment subjects) in four experimental groups. Each experimental group assigns a different weight to the matching parameters (i.e., experiment factors) relevant to SPs and VNBs. More precisely, SPs consider two main factors for their matching: fee to pay to the VNB and VNBs' reputation. Similarly, VNBs consider the fee to receive from SPs and the level of SP demand. Given these four parameters, we consider that each VNB/SP can assign a low or high weight to the parameters relevant to them. As a result, we have four different combinations of values, which we translate into four experimental groups, as presented in Table 23.

In the model, low and high weights are defined as a uniformly distributed random numbers. The range for low weights is [0.1 - 0.5] and the range for high weights is [0.6 - 1.0].

SPs and VNBs are randomly assigned to each group at the beginning of the simulation and, at the preference forming stage, the weights appropriate to their group are utilized. Figure 6 shows an scheme of the experiment design and how it was applied to the model at hand.

RPs are not subjects of this experiment as they are not involved in the matching process. Nevertheless, the factors and weights considered in the experiment do influence the market transactions as these will dictate which SPs participate in the market; hence, the market

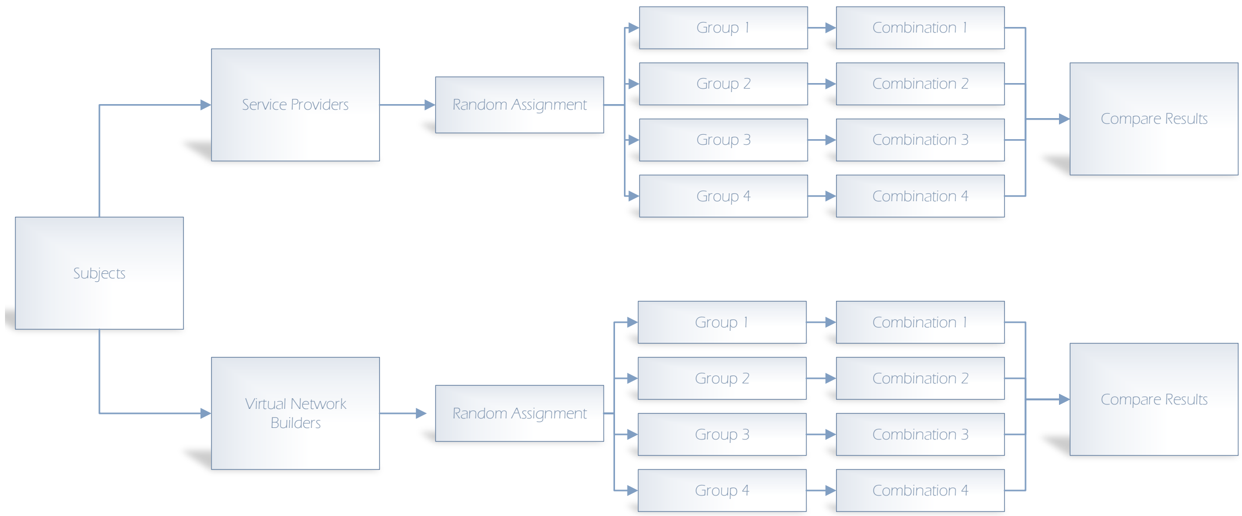


Figure 6: Experiment design for the third stage of market analysis

demand.

In addition to the workings of this experiment design, we have tested our model according to different configurations of the SP, VNB and RP market. These variations allow us to consider different levels of resource supply and demand, in addition to competition levels of VNBs. The factors that we have considered are included in Table 24.

In general terms, this approach permits us to evaluate how heterogeneous users interact with each other. In consequence, the results to obtain from this stage are divided into two sets: results that show how experimental groups perform in selected areas, and a general overview of the model performance taking into account all groups.

8.3.6.1 Overall Model Performance For the overall model performance, we focus on several parameters that we consider key for the success of a market, and which are congruent with the workings of our model. These parameters are the following:

- SP - VNB Matching performance

    Percentage of matched SPs

    Average number of SP customers per VNB

Table 24: Parameter Values for simulation of Stage III model

Parameter	**Values**
Number of VNBs	Test monopoly and oligopoly conditions $VNBs = \{1, 2, 3, 4\}$
Number of SPs	Test different levels of market demand, consistent with Stage I and II configurations $SPs = \{4, 5, 6, 10, 20, 50\}$
Number of RPs	Test different levels of market supply and RPs participation in the market $RPs = \{5, 10, 15\}$

- Percentage of satisfied demand
- Percentage of resources assigned by RPs
- Payment received by RPs and VNBs
- Market participants' surplus

  VNBs' surplus from payments received.

  SPs' surplus from VNB payment and resource payment.

  RPs' surplus from resource payment received.

8.3.6.2 Experimental Group Performance The objective in this set of results is to define whether there is one group that outperforms the rest. We measure this performance in terms of a specific set of factors:

- Percentage of matched SPs
- SPs' served demand
- SPs' surplus from VNB fees
- Percentage of matched VNBs
- VNBs' payment received

- VNBs' surplus from SP payment

These factors are associated with the matching process, where we expect the experiment to have a larger effect. We have also included surplus as an indicator of the performance of the overall system.

9.0 RESULTS

9.1 STAGE I RESULTS

In this section, we present the results from the experiments described in section 8.1.

9.1.1 Fungibility Scores

Our analysis starts by calculating the fungibility scores that will be applied to the market model. As detailed in 8.1, we focus on the calculation of capacity fungibility scores. In this light, we present the fungibility scores resulting form comparing the difference in capacity (in Mbps) that can be achieved with the available frequency, operating at varying bandwidths, and the preferred one, operating at a fixed bandwidth. Table 25 and Figure 7 show the resulting levels of fungibility when comparing 1900 MHz (i.e., the available frequency) to various *preferred* frequency bands.

In figure 7, we show the bandwidth required to match the performance of a preferred frequency with the available band. When we compare 1900 MHz to lower frequencies, such as 700 MHz, the maximum capacity score obtained is approximately 0.8 with the maximum bandwidth explored. However, as we explore higher frequencies, we can indeed reach the maximum fungibility score of 1. In this light, the closer the preferred-available frequencies are to one another, lower the bandwidth required to obtain similar performance.

Including a measure of bandwidth that further explains how and when to reach perfect fungibility conditions, allows users to better assess whether the available frequency adjusts to their needs, and thus place a better valuation on the available commodities.

Table 25: Resulting capacity fungibility scores at 1 Km distance from the transmitter.

Capacity Fungibility Score at 1 Km			
Preferred Frequency [MHz]	**Bandwidth of Available Freq. [MHz]**	**Distance Score**	**Probabilistic Score**
700	0.2	0.6552	0.3448
	1	0.3856	0.6144
	20	0.1968	0.8032
1000	0.2	0.5649	0.4351
	1	0.2246	0.7754
	11.22	0	1
1500	0.2	0.3085	0.6915
	0.483	0	1
1700	0.2	0.1686	0.8314
	0.3	0	1
1900	0.2	0	1
2000	0.2	0	1

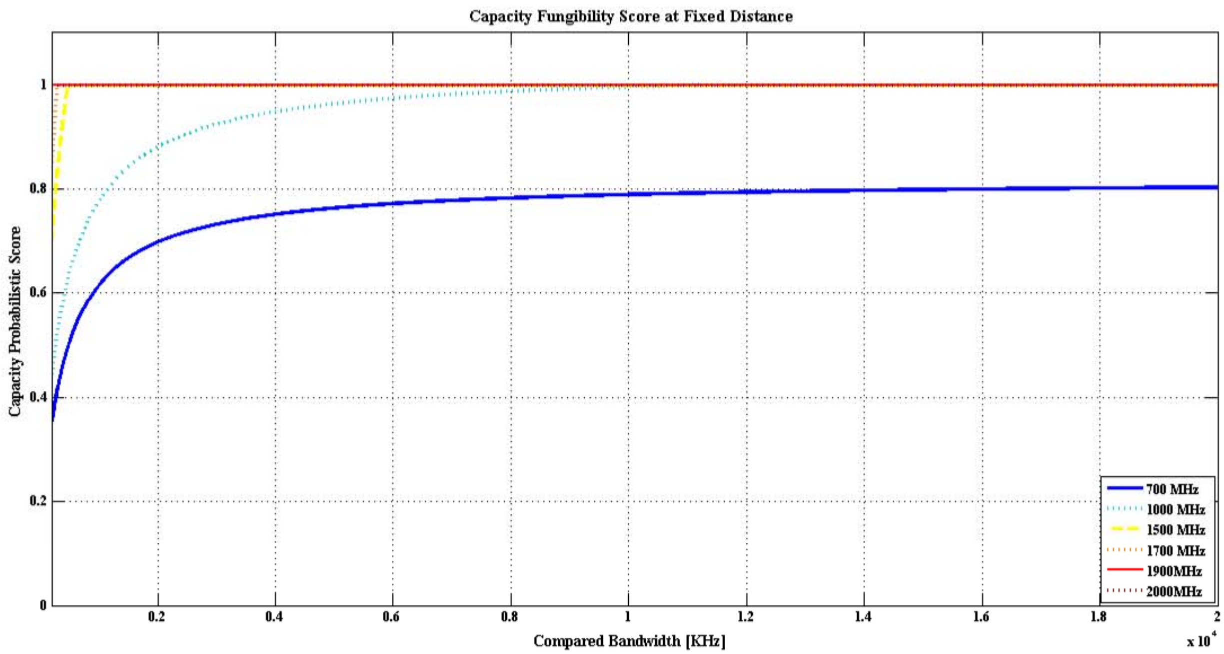


Figure 7: Capacity fungibility score calculated at 1Km from the transmitter.

9.1.2 Market Viability Results

We have adapted the fungibility scores to the Exchange-based Band Manager model of SPECTRAD. The focus of this analysis has been a *worst-case* scenario, where all market participants, i.e., SLRs, have a low valuation of the available resources. Indeed, the frequency available in the market is 1900 MHz and the SLRs' preferred band is 700 MHz. Considering that the available BBUs have a fixed bandwidth of 200 KHz, we utilize the probabilistic fungibility score of 0.3448, as presented in Table 25. Nevertheless, we include in our analysis additional, lower fungibility scores as a means to account for further costs that the SLRs may incur in order to adapt their systems to the available frequency band. In total, we consider four fungibility scenarios for our analysis:

- Perfect fungibility
- Calculated fungibility = 0.3448
- Fungibility Score = 0.25

Table 26: Viability Criteria and Market Scores

Viability Criteria			
Criteria	**Pass Value**	**Fail Value**	**Score Pass / Fail**
P1 - Bid List Empty	<1%	$\geq$ 1%	1 / -1
P2 - Demand Greater than Supply	$\geq$ 10%	<1%	1 / -1
P3 - Cutoff Price	N/A	<31	0 / -1
P4 - Percentage of Assigned BBUs	$\geq$ 62%	<62%	1 / -1
P5 - Number of ATs per User	N/A	$\geq$ 3	0 / -1

- Fungibility Score = 0.15

As included in Table 6, we analyze multiple scenarios of spectrum undersupply and oversupply, which attend to the various combinations of spectrum requesters, $numSLRs$, and available resources, $numBBUs$, given by the parameter R. The results included in this section stem from the analysis of one hundred simulation runs of each of these market scenarios, which were active for 5000 simulation time units. The first 3000 time units were considered as a warm up period and the data utilized for our results corresponds to the last 2000 time units.

Observing the aggregate data gathered from the multiple simulation runs, we could obtain the average values of each of the five viability criteria included in section 8.1. Nevertheless, in order to assess the overall viability of markets, we followed the process developed in [1, 87], and hence, we empirically defined pass/fail market viability thresholds that could permit us to define a quantitative measure of market viability. These thresholds stem from the observation of the average values of each viability criteria that are associated with positive outcomes. In fact, we observed the trends in the results and the breaking points where the majority of the criteria shift from positive to negative market characteristics. In this way, we defined the threshold values presented in Table 26. Each of the pass/fail values included in this table is associated with the pass/fail scores already presented in Table 4.

We contrast the simulation results that each individual market scenario presents for each viability criterion to the thresholds in Table 26 and determine the corresponding score. By adding all five scores, we obtain a final, market viability score. When the latter is greater than zero, we consider the conditions of the corresponding scenario as viable. In this way, we find the combination of fungibility level, number of SLRs and resource supply conditions that yield market liquidity. In the next subsection, we include the viability results for each fungibility level and the corresponding market scenarios.

9.1.3 Viability scores for different Fungibility levels

In this section we present the viability scores for each scenario that presented **positive** results in our analysis. For each fungibility case, we present the combination of R and $numSLRs$ associated with market viability and the number of positive outcomes that this represents.

9.1.3.1 Perfect Fungibility When fungibility is perfect, we assume a fungibility score of 1. This corresponds to the market scenario studied in [87] and we consider this case as a basis to evaluate the results that we obtain in imperfect fungibility cases. Table 27 presents the individual results for each viability criteria for all the markets with positive final scores. We find nine viable market outcomes corresponding to the following scenarios:

- $R = 5$ and $6 \leq numSLRs \leq 50$
- $R = 10$ and $5 \leq numSLRs \leq 50$

9.1.3.2 Calculated Capacity Fungibility Score In this fungibility case, as presented in Table 28, we find 8 positive market viability outcomes, which correspond to the following scenarios:

- $R = 5$ and $10 \leq numSLRs \leq 50$
- $R = 10$ and $5 \leq numSLRs \leq 50$

Compared to the perfect fungibility case, we lose one positive outcome in spectrum undersupply conditions.

Table 27: Market viability score results for the perfect fungibility scenario

Perfect Fungibility								
NumSLRs	**NumBBUs**	**R**	**P1**	**P2**	**P3**	**P4**	**P5**	**Score**
6	30	5	1	0	0	1	-1	1
10	50	5	1	1	0	1	-1	2
20	100	5	1	1	0	1	0	3
50	250	5	1	1	0	1	0	3
5	50	10	1	0	0	1	0	2
6	60	10	1	0	0	1	0	2
10	100	10	1	0	0	1	0	2
20	200	10	1	0	0	1	0	2
50	500	10	1	0	0	1	0	2

Table 28: Market Viability Scores obtained for capacity probabilistic fungibility score

Fungibility Score equal to Capacity Probabilistic Score								
NumSLRs	**NumBBUs**	**R**	**P1**	**P2**	**P3**	**P4**	**P5**	**Score**
10	50	5	1	0	0	1	-1	1
20	100	5	1	1	0	1	-1	2
50	250	5	1	1	0	1	-1	2
5	50	10	1	0	0	1	0	2
6	60	10	1	0	0	1	0	2
10	100	10	1	0	0	1	0	2
20	200	10	1	0	0	1	0	2
50	500	10	1	0	0	1	0	2

Table 29: Market Viability Score obtained when the fungibility score is 0.25

Fungibility Score equal to 0.25								
NumSLRs	**NumBBUs**	**R**	**P1**	**P2**	**P3**	**P4**	**P5**	**Score**
10	50	5	1	0	0	1	-1	1
20	100	5	1	1	0	1	-1	2
50	250	5	1	1	0	1	-1	2
6	60	10	1	0	-1	1	0	1
10	100	10	1	0	-1	1	0	1
20	200	10	1	0	0	1	0	2
50	500	10	1	0	0	1	0	2

9.1.3.3 Fungibility score equal to 0.25

As presented in Table 29, this lower fungibility scenario yields seven positive market outcomes. In fact, we find viability when:

- $R = 5$ and $10 \leq numSLRs \leq 50$
- $R = 10$ and $6 \leq numSLRs \leq 50$

9.1.3.4 Fungibility Score equal to 0.15

In the lowest fungibility level studied, we find yet an additional decrease in the number of viability outcomes. Table 30 includes the particular values, which show that viability is achieved when:

- $R = 5$ and $20 \leq numSLRs \leq 50$
- $R = 10$ and $6 \leq numSLRs \leq 50$

9.1.4 Summary of Results

Figure 8 presents a summary of the market viability scores resulting from our analysis. For ease of representation, we have included results for $R = 5, 10, 15$ only, as we have not found any viable market configurations under spectrum oversupply conditions (i.e., $R \geq 15$).

Table 30: Market Viability Scores obtained when the fungibility score is 0.15

Fungibility Score equal to 0.15								
NumSLRs	**NumBBUs**	**R**	**P1**	**P2**	**P3**	**P4**	**P5**	**Score**
20	100	5	1	1	-1	1	-1	1
50	250	5	1	1	-1	1	-1	1
6	60	10	1	0	-1	1	0	1
10	100	10	1	0	-1	1	0	1
20	200	10	1	0	-1	1	0	1
50	500	10	1	0	-1	1	0	1

As presented in the analysis of each fungibility case, we find that as the fungibility score decreases, so do the number of liquid markets. Additionally, the markets that remain liquid are those that require a larger number of market participants or SLRs. In this light, it is important to note that none of the spectrum fungibility levels analyzed presented viable outcomes when $numSLRs = 4$ or $R \geq 15$ (i.e., spectrum oversupply conditions).

Where market viability is present, we highlight the following characteristics:

- When there is no spectrum oversupply, the probability of demand being greater than the supply rises and hence a higher percentage of spectrum is assigned at prices determined by the market transactions.
- When demand is greater than the supply, the cutoff price lies above the BM's reserve price.
- When $R = 5$, the number of AT units per SLR tends to compare to the *fail* threshold
- When $R = 10$, the AT holdings of the SLRs fall within the *pass* threshold.

In cases where markets are not viable and there are spectrum oversupply conditions, we observe the following:

- A low percentage of BBUs is assigned in each bidding round and the cutoff price remains close to the BM's reserve price.

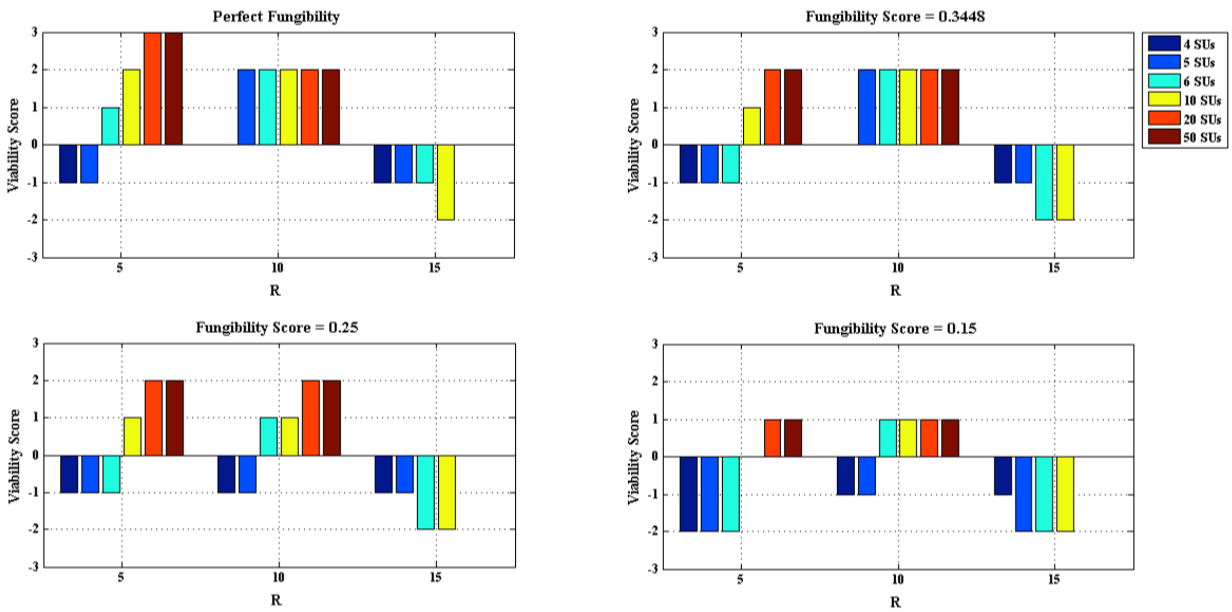


Figure 8: Market viability results contrasting perfect fungibility conditions to three additional fungibility levels.

- The probability of having an empty bid list is significantly low
- The SLRs have nearly null AT holdings
- Even if the two last characteristics are positive for market viability, these do not overcome the effects of the BBUs assignment percentage and the cutoff price. Hence, markets under these circumstances are still not viable.

In the overall market analysis, we found that the cutoff price falls as the fungibility level decreases, as presented in Figures 9, 10 and 11. We expect this to be the case as the SLRs' valuation of resources is proportional to the fungibility score of the available frequency. These results are relevant for hypothesis H1.

In terms of the percentage of resources assigned in the market, we still find that this is consistent with the demand and supply levels rather than the actual fungibility level. This can be observed in Figures 12, 13 and 14 where the overall percentage of assigned BBUs drops as the level of supply increases; however, it remains relatively constant despite changes

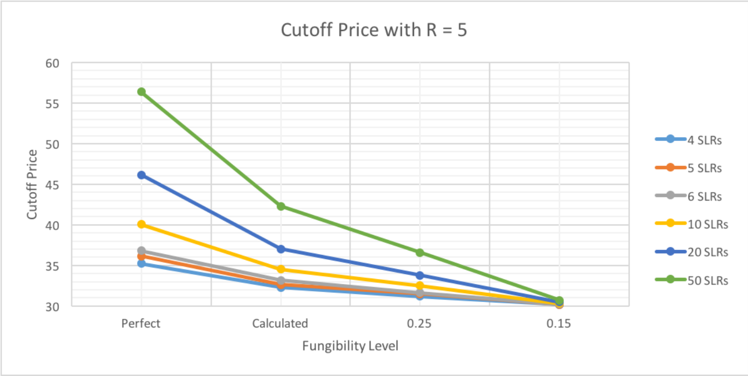


Figure 9: Cutoff Price with R = 5

Figure 10: Cutoff Price with R = 10

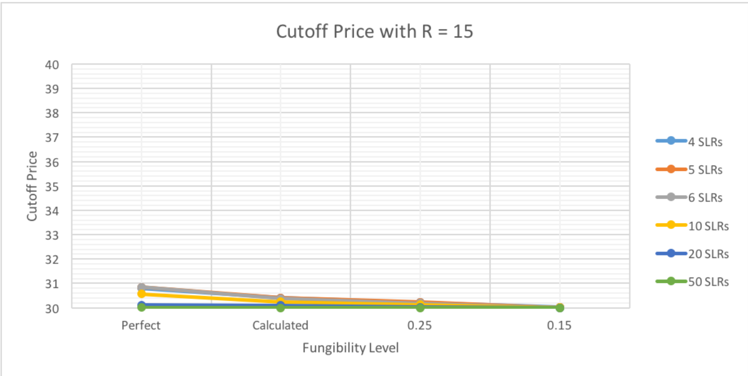


Figure 11: Cutoff Price with R = 15

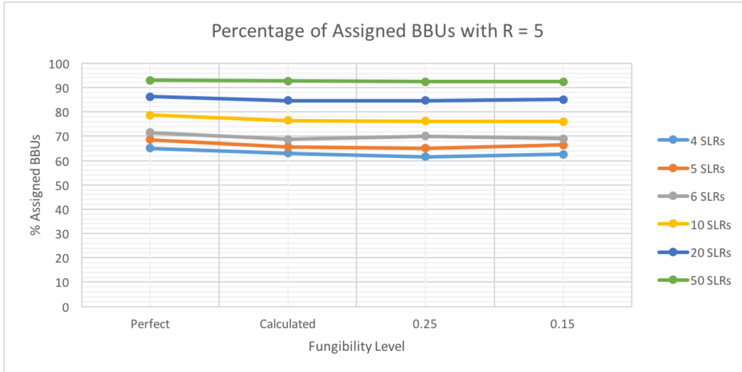


Figure 12: % BBUs Assigned, R= 5

in fungibility. These results are relevant for testing hypothesis H2.

Regarding the AT holdings of each SLR, as presented in Figures 15, 16 and 17, these are lower as the fungibility level decreases from perfect to the calculated fungibility score. As the fungibility score decreases even further, there is not a significant change in the average number of AT units per user. Indeed, the changes we observe are consistent with the spectrum demand and supply levels in the market. Indeed, as spectrum supply increases, the actual difference in AT holdings across fungibility levels starts to fade. For instance, in the case of 50 SLRs and $R = 15$, the AT holdings value remains at zero.

In light of these results, it is important to note that our study contemplates AT units that are perfectly fungible with the preferred frequency of the SLRs. In this way, the price to pay for these AT units is proportional to high resource valuation and, evidently, higher than the price to pay for less fungible spectrum.

It is important to note that even if we found different market configurations that lead to viable outcomes, these configurations do not resemble the current configuration of the telecommunications market. In fact, in current practical scenarios, we find a limited number of market participants (or incumbents) capturing the majority of the market, under spectrum scarcity conditions. This is an additional motivation for the incorporation of a different technical framework to our market model, which could permit us to find market liquidity in scenarios that adjust to the existing situation.

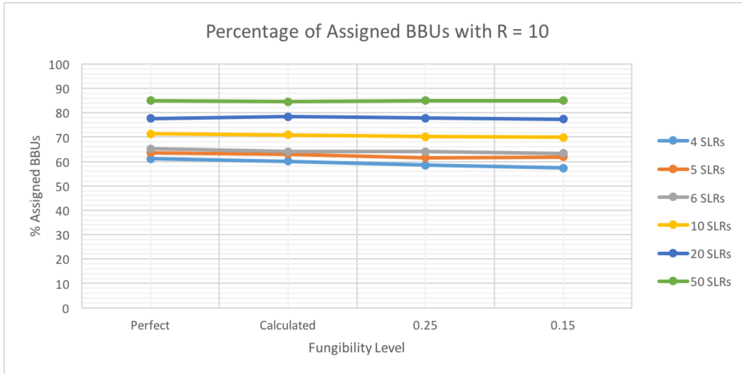


Figure 13: % BBUs Assigned, R = 10

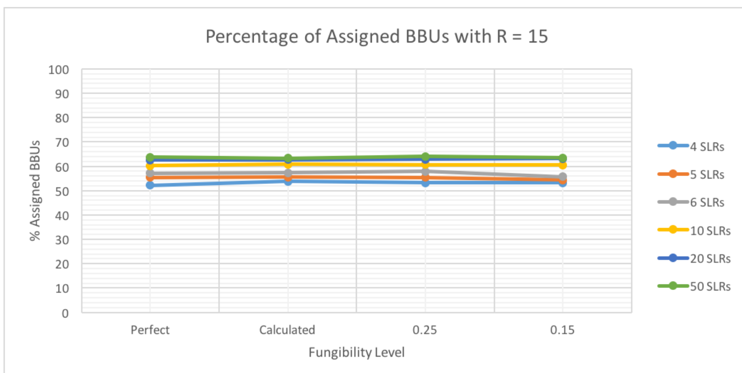


Figure 14: % BBUs Assigned,R = 15

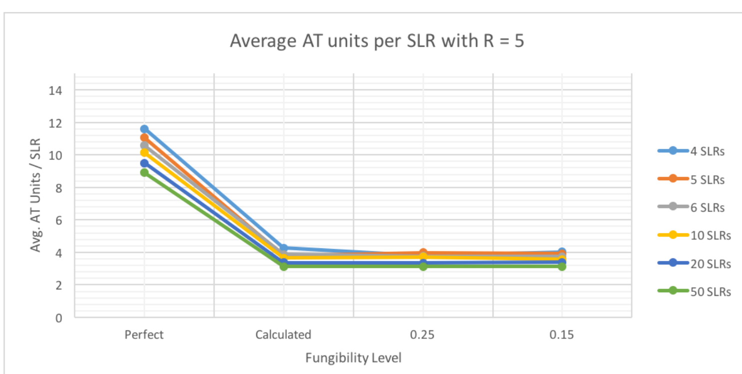


Figure 15: Avg.ATs per SLR, R= 5

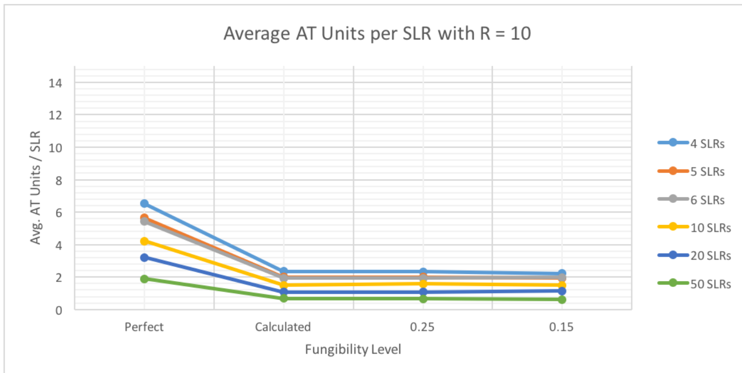


Figure 16: Avg.ATs per SLR, R = 10

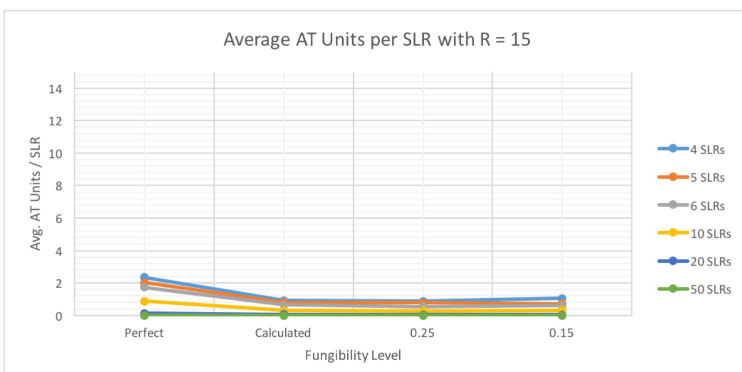


Figure 17: Avg.ATs per SLR, R = 15

9.2 STAGE II RESULTS

The research work presented in [3] focuses on defining a tradeable, spectrum-related commodity that could add thickness to the market while avoiding the significant constraints inherent to the physical characteristics of electromagnetic spectrum. In section 8.2 we report the results obtained in that work, which portray the experiments presented in section 8.2.

The results presented in this section correspond to the average values of one hundred simulation runs, for each combination of parameters applicable to our model. Note that since we have considered that the available LTE bands are three 10 MHz bands in the 700 MHz frequency range, we are able to obtain a spectrum pool with a minimum of 18 PRBs and a maximum of 150 PRBs. The actual simulation scenario is constructed following the same approach of [87]. In this way, we utilized the variable R in order to determine the number of PRBs available in the market. This availability is given by the expression in (9.1).

$$\text{number of PRBs} = \text{number of SLRs} \times R \tag{9.1}$$

For each simulation scenario we considered different combinations of users and available PRBs given by $R = \{5, 10, 15, 20, 25\}$.

It is important to note that for the availability of PRBs, we utilized values that could be aggregated through the addition of allowable quantities of PRBs from the three 700 MHz bands (see Table 3)[1]. In case the exact value, given by (9.1) was not consistent with the standard, we considered the closest (allowed and higher) amount of PRBs. For example, in the case of 4 SLRs and $R = 5$, we would form a pool of 20 PRBs in our simulation. We could not use a standard-compliant amount of PRBs from the three bands to obtain this value. Instead, we used a pool of 21 PRBs, which results from aggregating 15 PRBs from one band and 6 PRBs from another. In the same way, we have limited our simulated combinations to

[1]The current LTE standard contemplates the association of only certain bands for intra-band and inter-band carrier aggregation. However, for ease of implementation, we have assumed that the bands chosen for our model can be associated with each other for carrier aggregation purposes, even if this is not supported by the standard yet. As the LTE standard has been continuously evolving, we expect carrier aggregation to expand to additional bands. In this way, our assumption does not limit the applicability of our model.

Table 31: Viability Criteria and Market Scores

Viability Criteria			
Criteria	**Pass Value**	**Fail Value**	**Score Pass / Fail**
P1 - Bid List Empty	$= 0$	> 0	1 / -1
P2 - Demand greater than Supply	$\geq 10\%$	$< 1\%$	1 / -1
P3 - Cutoff Price	N/A	< 51	0 / -1
P4 - Percentage of Assigned BBUs	$\geq$ Average all scenarios	$<$ Average all scenarios	1 / -1
P5 - Number of ATs per User	N/A	≥ 4	0 / -1

those resulting in values ≤ 150; hence, our simulated scenarios for 10 SLRs end at $R = 15$, in the case of 20 SLRs, we consider only $R = 5$ and we have not tested the case of 50 SLRs.

In what follows, we present the results obtained for each scenario, regarding the five different viability criteria. In the same manner as in Stage 1, we utilize the values obtained across the simulation runs in order to define pass/fail thresholds that would allow us to provide an overall (quantitative) assessment of market viability. In this way, Table 31 shows the values that we utilized for the evaluation of our results.

9.2.1 Scenario 1

The first scenario we tested was a rather *conservative* approach, as we tried to remain as close as possible to the original SPECTRAD model, while still being able to incorporate the virtualization notions for the definition of the traded commodity. In this manner, the duration of the AT units' usage is the same as those from the original model: uniformly distributed between 90 and 110 simulation time units.

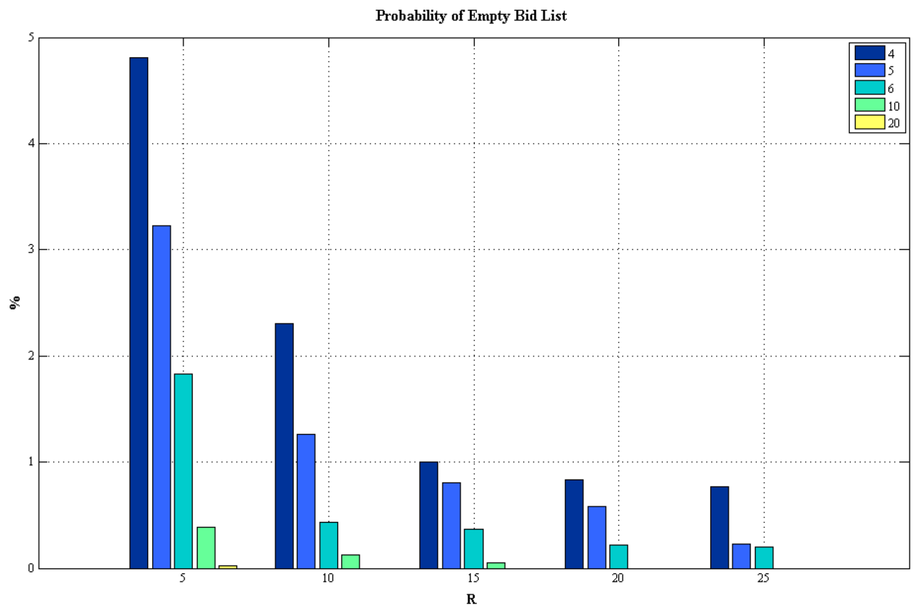


Figure 18: Probability of having an empty bid list in Scenario 1.

9.2.1.1 Probability of Empty Bid List When spectrum is scarce, it is rather difficult for an SLR to obtain spectrum from the market. Hence, SLRs begin to increase their AT holdings, which in turn reduces their bidding activity. In this way, as each SLR accumulates AT units, the probability of having an empty bid list is higher. In Figure 18, we can observe how this probability varies depending on the number of SLRs and the existing spectrum supply (given by R).

9.2.1.2 Probability of Demand being greater than Supply As spectrum availability grows in the market (higher values of R), the probability that the demand is greater than the supply decreases. Conversely, as the number of SLRs grows in the market, greater is the probability that the market demand will surpass the BM spectrum inventory. Figure 19 presents the results we obtained regarding this criterion. As it can be observed, this probability is at its highest when $R = 5$ and there are 20 SLRs in the market, which is a clear condition of spectrum undersupply. The lowest value of this probability is observed in

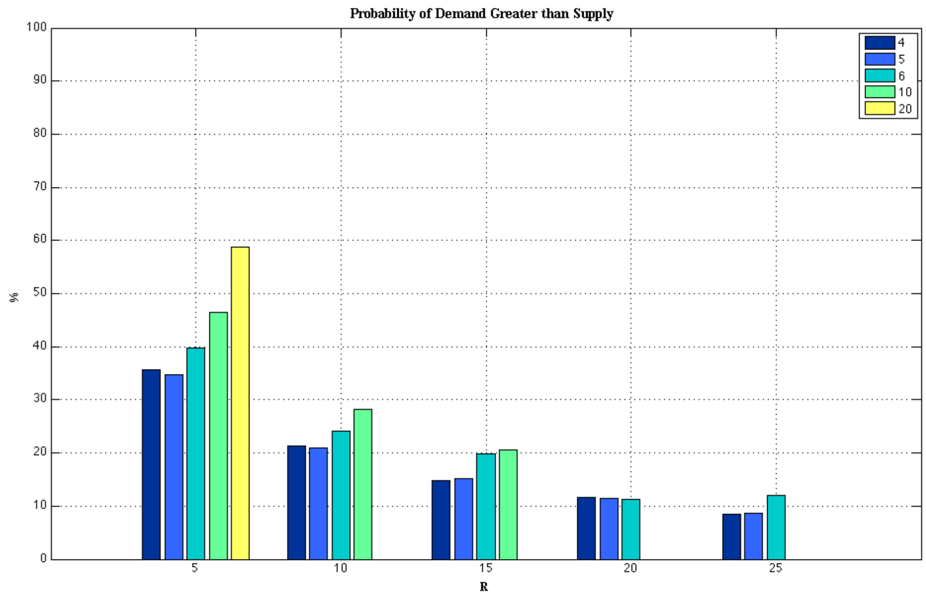


Figure 19: Probability of demand being greater than supply in Scenario 1.

spectrum oversupply conditions (i.e., R = 25 and numSLRs = 4).

9.2.1.3 Average cutoff price We utilized this criterion as a means to evaluate the valuation of the spectrum-related resources in the market. In spectrum undersupply conditions, we can expect SLRs to pay higher prices for spectrum than in spectrum oversupply conditions. This trend is corroborated in Figure 20, where we find an average cutoff price as high as 101.8 monetary units when R = 5 and numSLRs = 20, and as low as 57.6 monetary units, when R = 25 and numSLRs = 4.

9.2.1.4 Percentage of assigned spectrum When the spectrum supply increases, we find less efficiency in spectrum assignment. Regarding this criterion, the highest efficiency, 85.7%, is achieved when R = 5 and there are 20 SLRs participating in the market i.e., spectrum oversupply conditions. The worst case corresponds to the scenario given by R = 25 and numSLRs = 4, where only 43.4% of resources are assigned. When analyzing

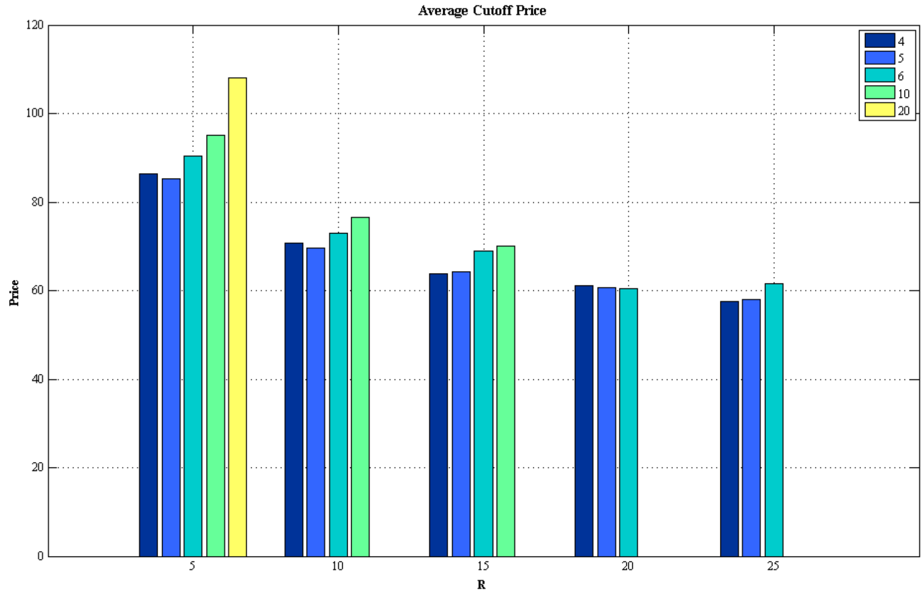


Figure 20: Average cutoff price in Scenario 1

the results from our simulation, the average percentage of assigned resources is 62%. In consequence, this is the value that we utilized for the market viability evaluation of this scenario (see Table 31).

It should be noted that given the specific LTE requirements for resource assignment, when there is spectrum undersupply we can observe that the resulting assignment is not completely smooth across the distribution of spectrum users. As the spectrum availability increases in the market, this irregularity disappears.

9.2.1.5 Average number of Unlicensed spectrum units per SLR It is expected that as spectrum supply increases, SLRs will have more opportunities to obtain licensed PRBs from the market, and hence they will seek less unlicensed spectrum units. This is portrayed in Figure 22, where we show the results we obtained in our simulations regarding the average number of unlicensed spectrum units per SLR.

Given the characteristics of our model, each SLR requires four unlicensed spectrum units,

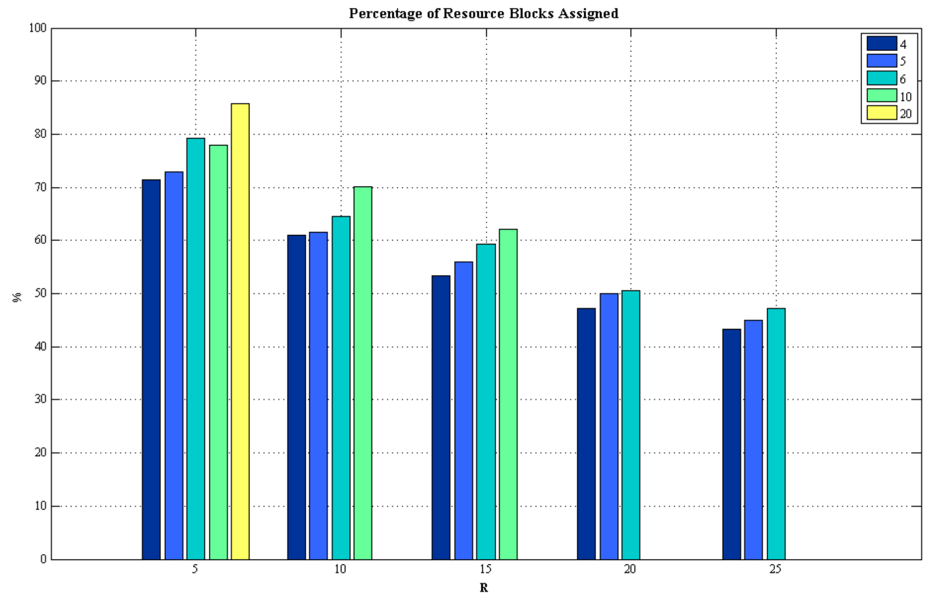


Figure 21: Percentage of assigned spectrum in Scenario 1

on average, to fulfill their traffic requirements. In this way, any value below this threshold is considered positive for market viability. Observing the results in Figure 22, we find find users with an average of ~ 1 unlicensed spectrum unit when there is spectrum oversupply, while SLRs accumulate more than 4 unlicensed spectrum units in undersupply scenarios.

9.2.1.6 Viability Score for Scenario 1 The values obtained for each viability criteria were evaluated using the pass/fail thresholds, and the corresponding scores, included in Table 31. In this way, Figure 23 includes the final market viability scores obtained for the market scenarios simulated in this stage.

It is important to point out that the goal of this scenario is to determine the feasibility (or viability) of markets designed for trading virtualized commodities. We also contrast this results with those obtained with the original version of SPECTRAD, in order to determine whether there are improvements regarding market liquidity conditions. In this way, contrasting our results with those of the original SPECTRAD model, we find one additional

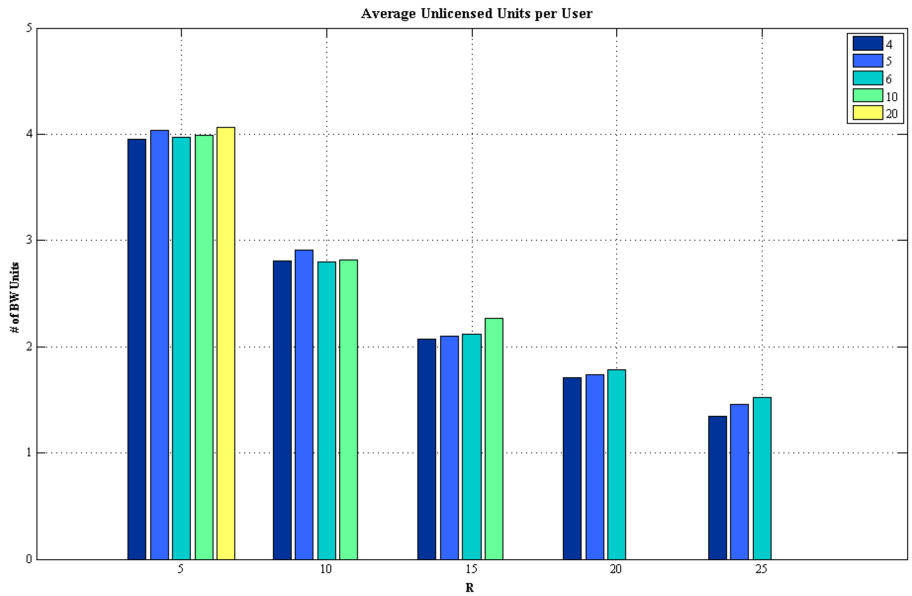


Figure 22: Average number of unlicensed spectrum units per SLR in Scenario 1

scenario where markets are viable, which is the case where R = 10 and there are 4 participating SLRs. The remaining viable scenarios coincide with the original model, and all of them belong to cases where $R = \{5, 10, 15\}$. We do not find viable markets in situations of spectrum oversupply i.e., $R = \{20, 25\}$.

9.2.2 Scenario 2

In this scenario we study the impact of the duration of the unlicensed spectrum usage in the final market viability outcome. Our objective is to map the duration of this alternate technology to that of the lease of a licensed PRB from the market. This permits us to account for possible degradation of service while using unlicensed spectrum, in addition to providing the SLRs with the opportunity to cease the utilization of unlicensed spectrum and participate in a new bidding round when it is available.

In the following sections, we present the results obtained for each viability criterion under these new model characteristics. We also evaluate how this impacts the overall market

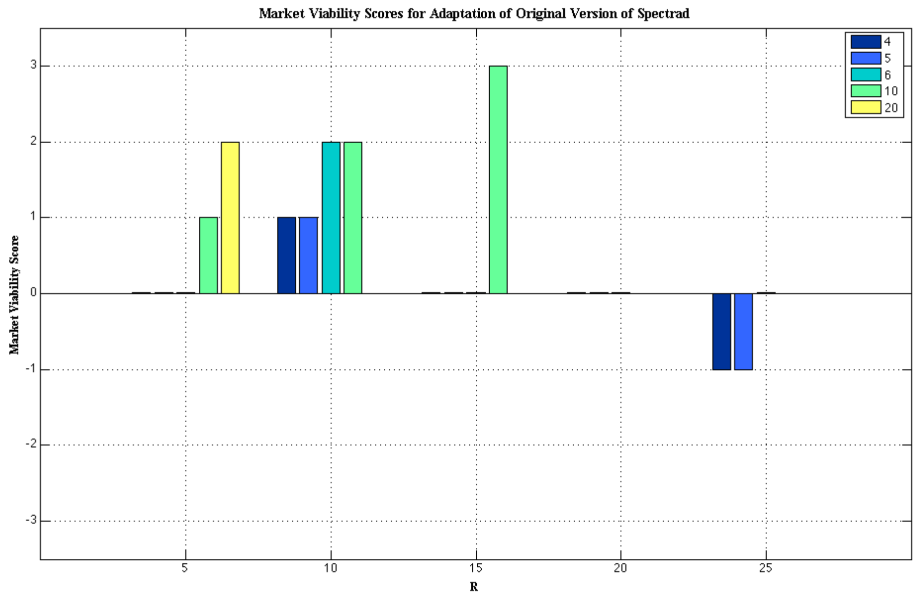


Figure 23: Market Viability Score for Scenario 1

viability score.

9.2.2.1 Probability of Empty Bid List In all the market configurations that we tested in this scenario, we obtained a zero probability of empty bid list. This results from the duration of unlicensed spectrum units usage, which allows the SLRs to bid for licensed spectrum in every bidding round.

9.2.2.2 Probability of Demand greater than Supply Given the shorter duration of unlicensed spectrum usage, SLRs are significantly more active in the market. This is reflected in Figure 24. In fact, when $R = 5$, we find 100% probability of demand being greater than supply. The worst case we find is that of $R = 25$ and $numSLRs = 4$, i.e., spectrum oversupply, where the analyzed probability drops to 18.3%. It is important to point out that even in the spectrum oversupply conditions, this probability does not reach levels as low as those of the original SPECTRAD model.

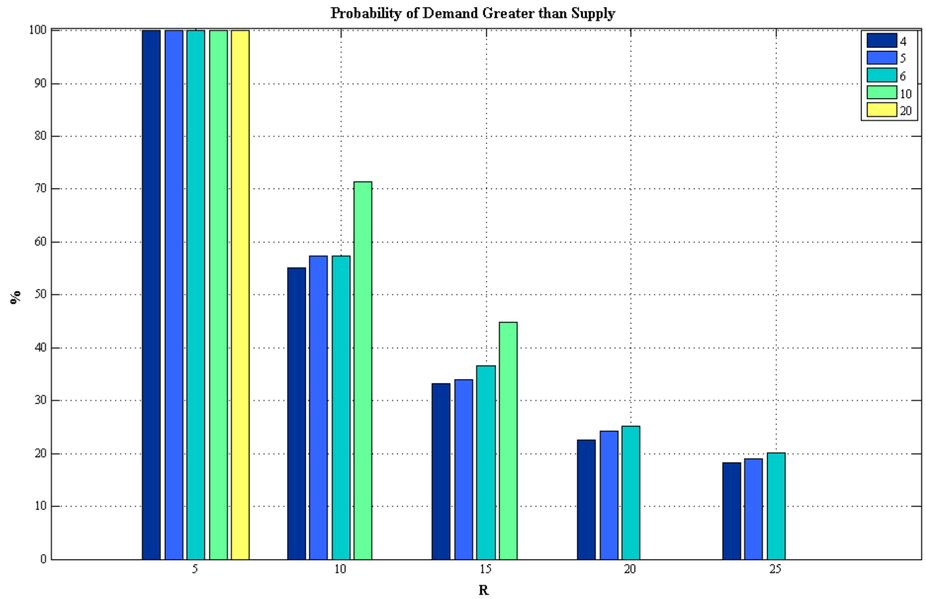


Figure 24: Probability of demand greater than supply results for Scenario 2

9.2.2.3 Average cutoff price The higher number of market participants also has an effect on the average auction cutoff price. In this scenario, the highest cutoff price rises to 161.8 monetary units in cases of spectrum scarcity ($R = 5$ and $numSLRs = 20$). The lowest cutoff price is obtained in spectrum oversupply conditions ($R = 25$ and $numSLRs = 4$), where it falls to 66.5 monetary units. Nevertheless, this lower value is still above the reserve price of the Band Manager (i.e., 50 monetary units). Figure 25 shows the results obtained for this criterion.

9.2.2.4 Percentage of assigned spectrum As evidenced in our previous simulation scenario, the resource assignment results are not entirely smooth under spectrum scarcity conditions. This is due to the constraints we face in order to follow the resource assignment rules of the LTE standard. We can observe in Figure 26 that the highest percentage of resources assigned corresponds to the case where $R = 5$ and $numSLRs = 6$, while the lowest efficiency appears when $R = 25$ and $numSLRs = 4$.

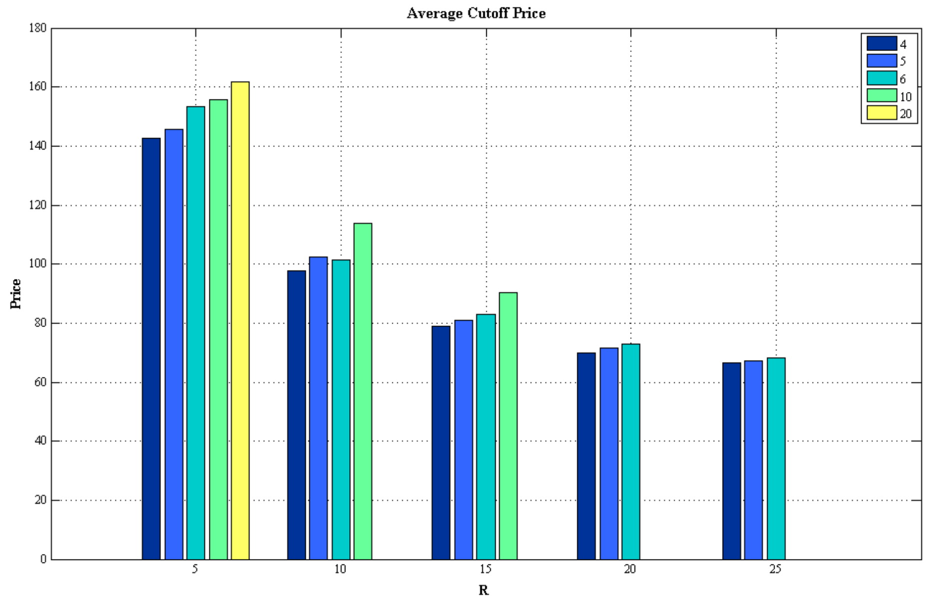


Figure 25: Average cutoff price for Scenario 2

When analyzing our aggregate simulation data, we find that the average percentage across all simulation runs is 76%. Consequently, this value becomes the threshold for evaluation of the market viability score in the second simulation scenario.

9.2.2.5 Average number of Unlicensed Spectrum units per SLR

Given that the major change in our simulation scenario affected the duration of the unlicensed spectrum usage, we expect to see a significant change in the results of the evaluation of this criterion. In fact, as shown in Figure 27, the maximum amount of unlicensed bandwidth units per user is approximately 2 in spectrum undersupply conditions. This value decreases until it reaches 0.37 when there is spectrum over supply. It should be noted that even the highest values we obtain are below the average number of bandwidth units that the SLRs need to satisfy their traffic requirements.

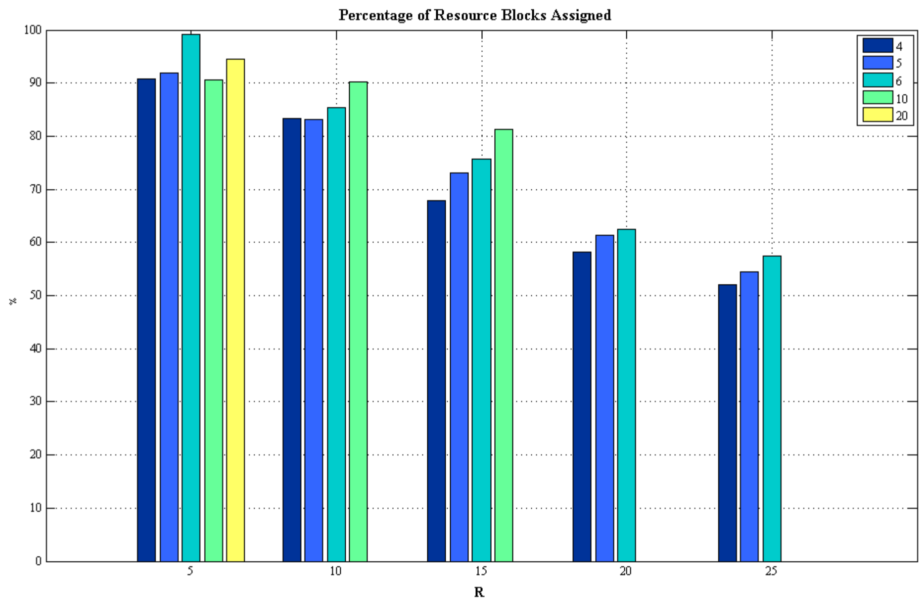


Figure 26: Percentage of assigned resource blocks in Scenario 2

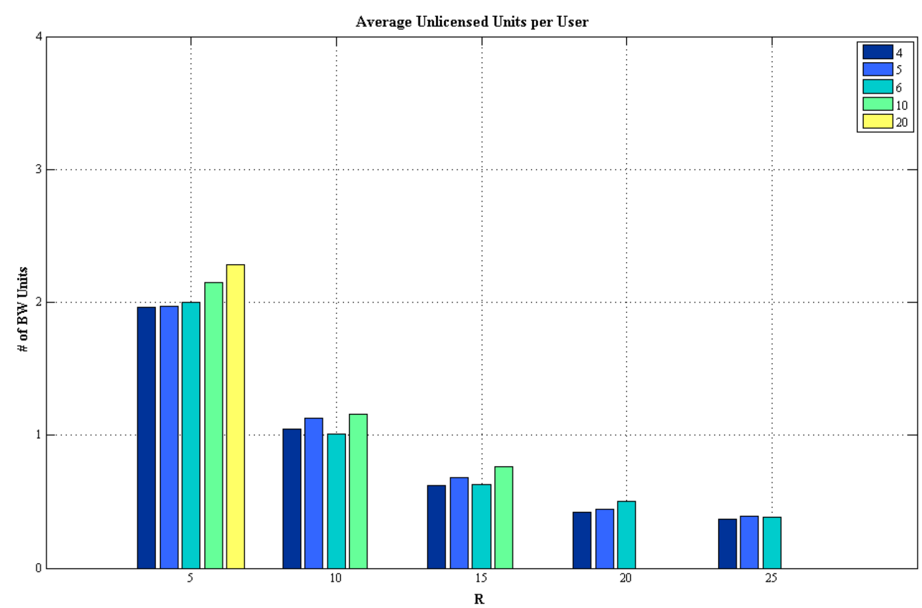


Figure 27: Percentage of assigned resource blocks in Scenario 2

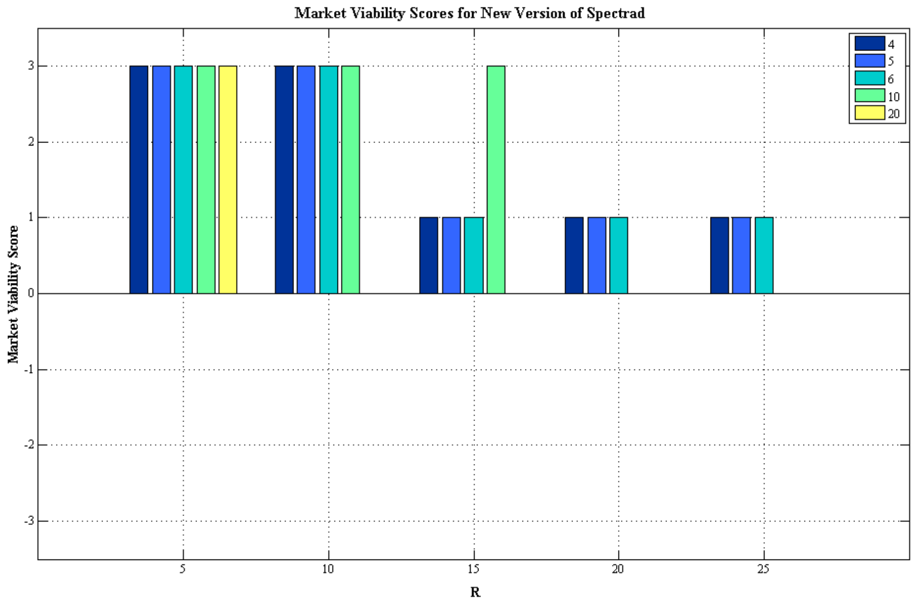


Figure 28: Market viability scores for Scenario 2

9.2.2.6 Viability Score for Scenario 2 The individual criteria showed significant improvements in the second simulated scenario. Figure 28 shows how these individual improvements impact the overall market viability score. As it can be observed, in this new scenario we find that each market configuration tested yields viable outcomes. Additionally, these favorable results can be evidenced even in situations where there are only a few market participants.

9.2.3 Summary of Results

The main objective of the second stage of this research work is to find whether a technical approach such as virtualization and the incorporation of a standard such as LTE-A would permit us to develop a new spectrum–related commodity that would add thickness to the market and hence improve the overall market liquidity outcome.

When comparing the results of our first scenario with those of the original SPECTRAD model, we could evidence only a slight improvement in terms of viable configurations. Nevertheless, we shall remember that in this new scenario our assumption of "perfect fungibility" is framed within an existing, and mature, technology: LTE-A. Further, if we focus on particular results such as those of the probability of demand being greater than supply, we find significant improvements which can be attributed to the implementation of the resource pool, and the carrier aggregation capabilities of our model.

The second simulation scenario presents individual improvements for the market viability criteria and an enhancement of the overall viability conditions. One of the main factors that contributes to this positive results is the null probability of having an empty bid list in all scenarios. This translates in the high level of SLR participation in the market, hence supporting our third hypothesis H3.

It is important to point out that when resources are homogeneous, the availability of alternative resources positively impacts the market outcome. This is particularly salient in the case of resource valuation (average cutoff price) and willingness to participate in the market (probability of empty bid lists and demand / supply conditions). These factors also support our hypotheses H3 and H4. Indeed, in the absence of resource compatibility issues, the resulting cutoff price is well above the minimum accepted by the BM. Additionally, having null empty bid list incidences points to higher levels of market demand for all bidding opportunities. Evidently, this homogeneous case is suitable for alternate technology units such as unlicensed spectrum, which can be easily accessed and do not represent high investment costs (e.g., acquisition of equipment, building infrastructure, among others).

The positive results we have obtained, especially in the second simulation scenario have shed light on the opportunities that we can derive from the incorporation of a flexible technical framework to the definition and design of spectrum markets. Indeed, these results lead to the third research stage of this dissertation, which aims at applying the notions of virtualization to a wider network and resource scope.

Figure 29 compares the viability scores obtained in the two scenarios pertaining to this stage (right-most figures) to the results of the original SPECTRAD model (left-most figure). The positive results shown in the right-most figure stem from positive market scores obtained throughout the viability criteria that we have considered. This includes the factors associated with our hypotheses H3 and H4.

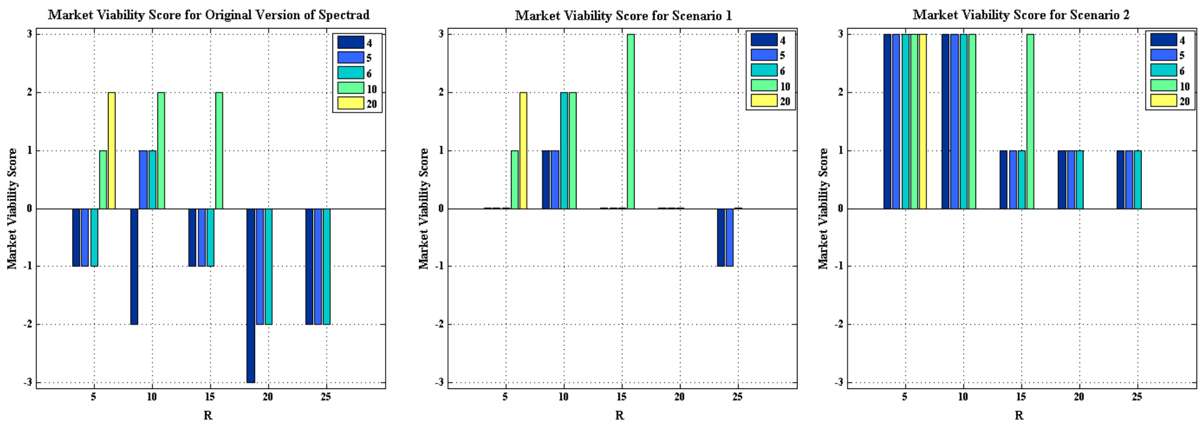


Figure 29: Comparison of the viability scores for different versions of the SPECTRAD market model

9.3 STAGE III RESULTS

In this section, we present the results from the experiment setting described in section 8.3.6. The complexity of the system modeled in this stage is significantly higher than that of the previous stages. We now deal with three different types of participants, whose characteristics and relevant parameters can vary, thus yielding interesting analysis settings. The results stemming from our simulation data are divided into: overall model performance assessment and experimental groups performance assessment. We expect these analyses to permit us to explore favorable conditions for market viability, as well as the issues that need to be addressed to achieve successful outcomes.

To determine an appropriate number of simulation runs for this stage, we assume we are dealing with a daily market. In this manner, we run the model for the equivalent of one year (360 simulation runs) and we replicate this process 10 times for consistency and error avoidance. To include a training period in our system, we divide our simulation runs into *monthly* interactions. In this way, we run the model 50 times and for our monthly data, we gather the results pertaining to the last 30 iterations.

9.3.1 Overall Model Performance

A global perspective on how the proposed model functions allows us to investigate details that are not always available, individually, from market mechanisms. Our overall model assessment is composed by the perspective of all participants, what they seek, and how they perform. For this purpose, in this subsection we focus on the results from the different parts of our model and study what are the outcomes perceived by SPs, VNBs and RPs. This can be captured through specific parameters from the VNB–SP matching process, the VNB–RPs negotiations, and the subsequent payments and price adjustment process.

9.3.1.1 SP - VNB Matching Performance To evaluate the SP–VNB matching market performance, we have obtained data regarding the percentage of matched SPs and the number of SPs with whom each VNB is matched. We have aggregated this data in terms of the average and these results are shown in what follows.

Given that RPs do not intervene directly in the matching process, the results do not change according to this factor. Consequently, we only include data from our scenarios with 10 RPs.

Percentage of Matched SPs: Figure 30 shows the results for the average percentage of matched SPs throughout our simulation (one year data). Each line corresponds to the relevant scenario regarding the total number of VNBs in the system. Indeed, we find that as the number of VNBs in the market increases, the percentage of matched SPs increases as well.

Average number of SP Customers per Virtual Network Builder: Figures 31, 32, 33 and 34 show the results for this parameter, considering the four different VNB market configurations. Reputation is a key factor for VNBs; hence, we differentiate the VNBs' performance according to their reputation. In each graph, reputation is referred to as "quality" and the numbers (0 - 2) are consistent with the reputation level (i.e., 0 corresponds to low,

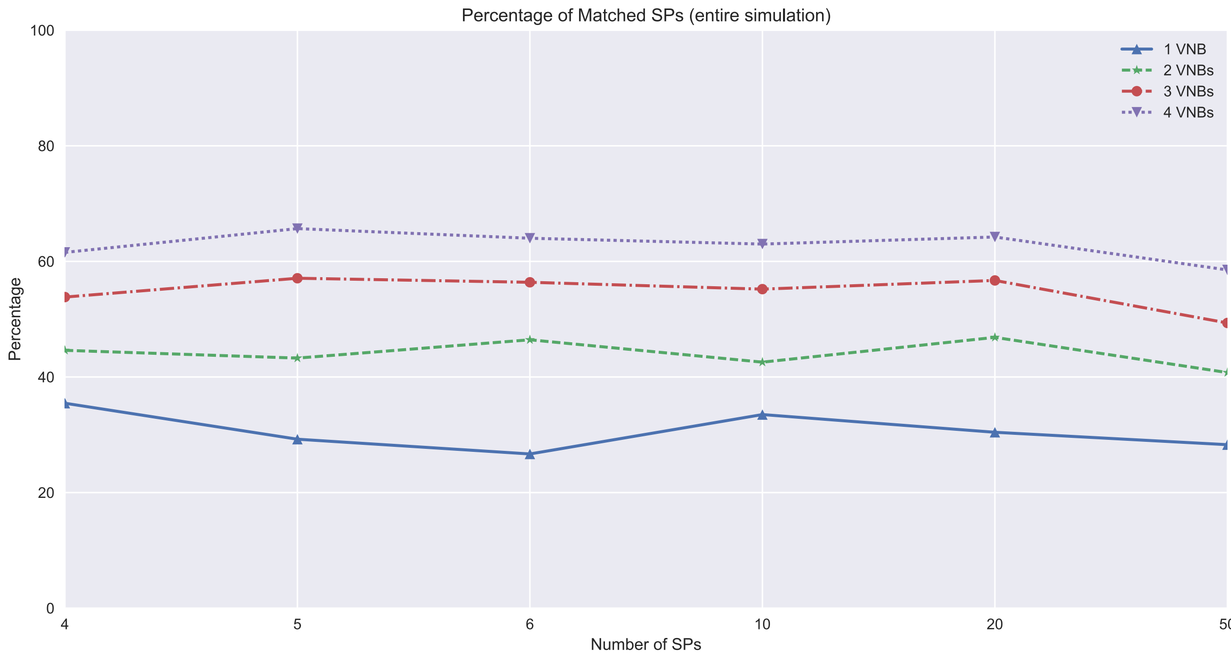


Figure 30: Percentage of Matched SPs with 10 RPs in the market

1 to medium and 2 corresponds to high).

The results show that VNBs with higher reputation are, on average, matched to a greater number of SPs. The difference between reputation levels becomes less evident as the number of VNBs increases. VNBs with higher reputation outperforming those with lower levels is somewhat expected, as this is an indicator of a VNB's trustworthiness and how well it performs in the system. Nevertheless, this preference may come at a higher cost to the SPs.

9.3.1.2 Percentage of demand satisfied in the market This is an important measure to explore, as it allows us to merge the results from both negotiation instances and assess the core requirement of this system: share spectrum resources and allow SPs to satisfy the demand of their customers. In this light, we have approached this parameter in terms of the average percentage of SPs that have been able to obtain resources in the market after they have been matched. Figures 35, 36 and 37 show the results for scenarios with 5, 10 and 15 RPs, respectively.

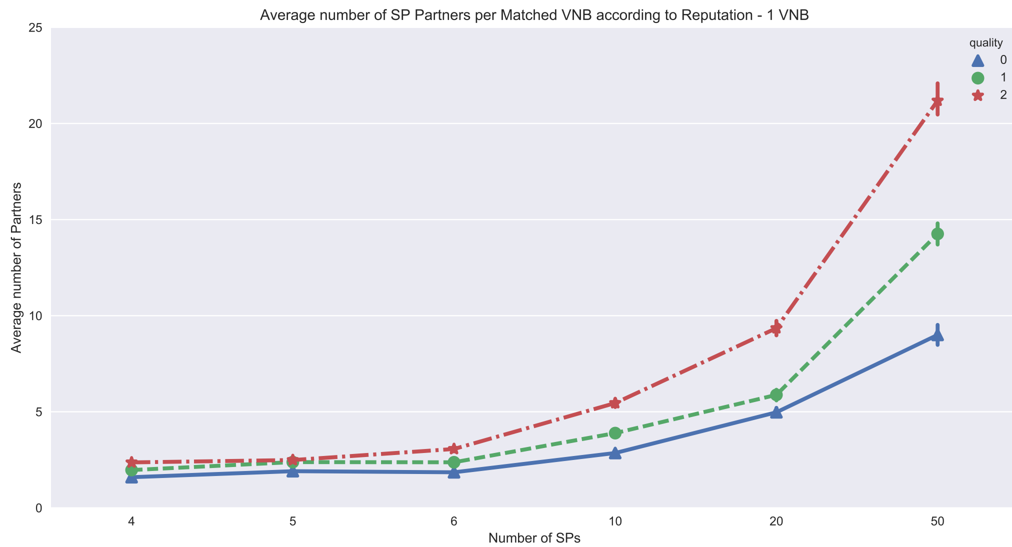


Figure 31: Average number of partners per VNB according to its reputation. Scenario with 1 VNB and 10 RPs

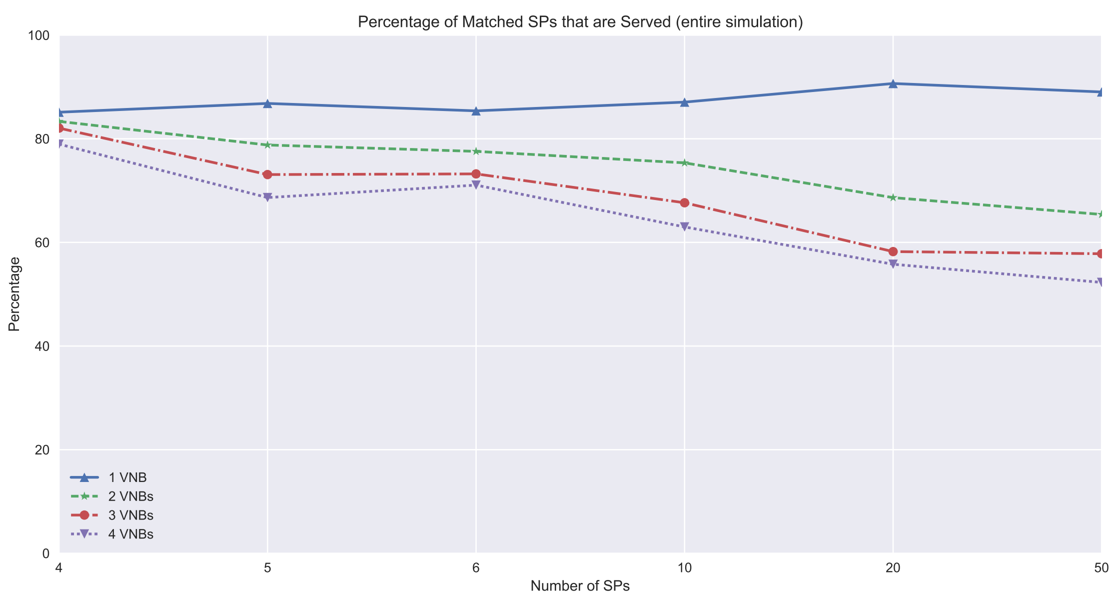


Figure 37: Percentage of SPs whose demand has been satisfied when there are 15 RPs in the system

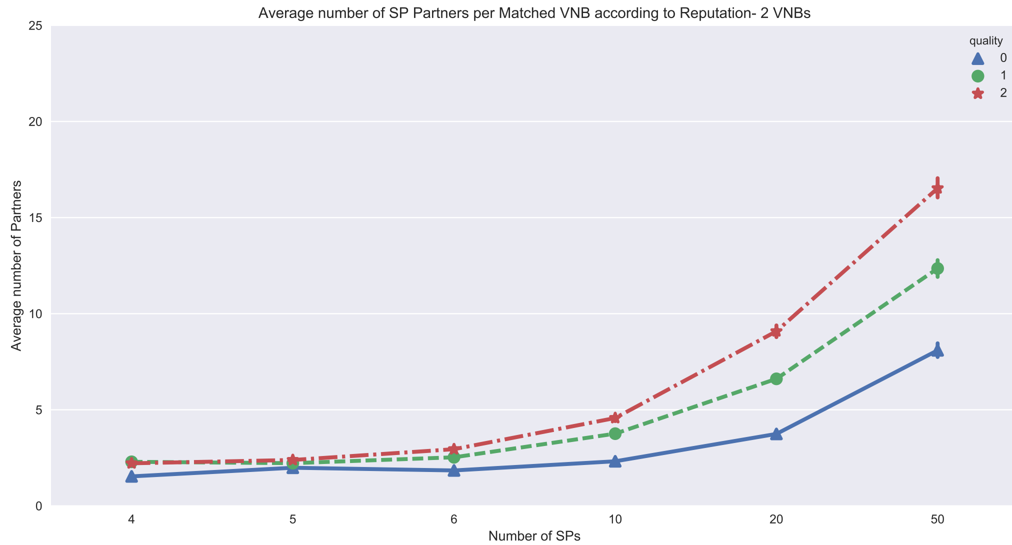


Figure 32: Average number of partners per VNB according to its reputation. Scenario with 2 VNBs and 10 RPs

There are several factors that are important to note in these figures. First, the difference between VNB market scenarios is not entirely clear when there are only 5 RPs in the system. This is especially the case when we are considering scenarios with less than 6 SPs (see figure 35). Scenarios with higher number of SPs and RPs show that the tendency is for VNB monopolies to perform better, although with approximately 20% difference, between the best and worst performances, at most. It is important to point out; however, that from the results presented in 9.3.1.1, in monopoly cases, VNBs were matched with less SPs than duopoly or oligopoly cases. This suggests that the positive performance of VNB monopolies is based on the fact that they initially had fewer SPs to serve. Evidently, a higher percentage of a small number of SPs may be comparable to a lower percentage of a greater number of SPs (as is the case of VNBs > 1). This reduces the performance breach among different numbers of VNBs, thus suggesting that their performance is rather consistent throughout our simulation scenarios.

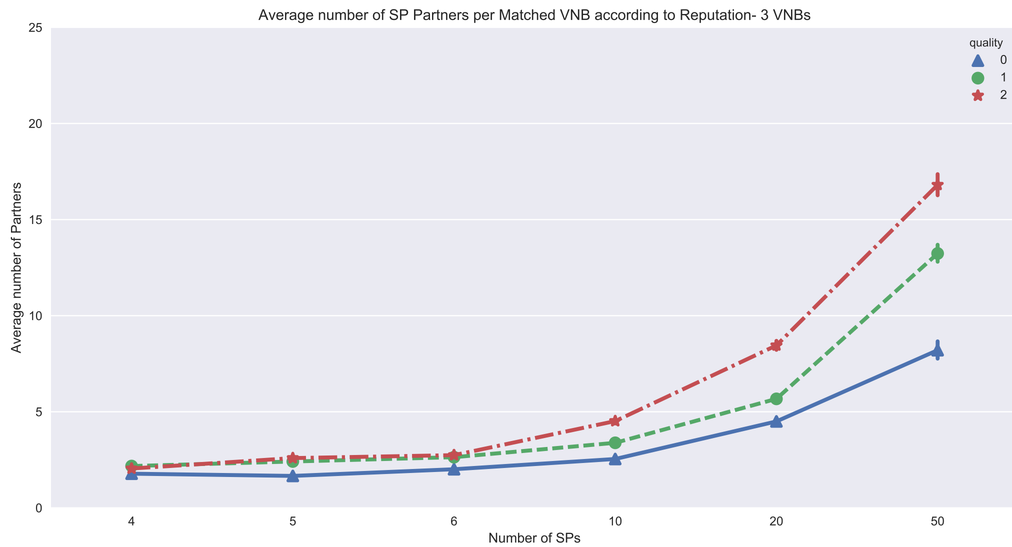


Figure 33: Average number of partners per VNB according to its reputation. Scenario with 3 VNBs and 10 RPs

9.3.1.3 Percentage of resources assigned by the Resource Providers In the figures to follow, we delve into the performance of RPs. For this purpose, we analyze the percentage of resources that they have successfully assigned in the market, on average. Figures 38, 39 and 40 show the scenarios with RPs = 5, 10 and 15, respectively.

In these figures we can observe that there is a significant performance improvement when we shift from 5 to 10 or 15 RPs. Indeed, scenarios with 10 and 15 RPs show only very slight differences. Note, however, that these figures show the percentage of resources assigned by each RP. Hence, the overall amount of resources assigned in the scenario with 15 RPs is higher.

Regarding the performance of the VNB market configuration, in the majority of cases, scenarios with a higher number of VNBs show a higher percentage of resources assigned. This trend is clearer in the case of 15 RPs (figure 40) where markets with 4 VNBs perform better for all SP configurations.

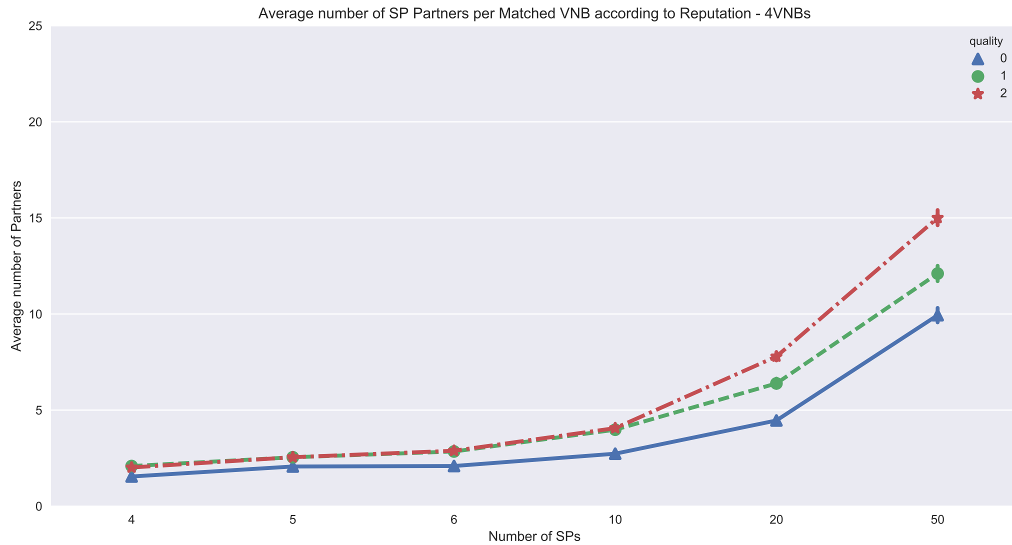


Figure 34: Average number of partners per VNB according to its reputation. Scenario with 4 VNBs and 10 RPs

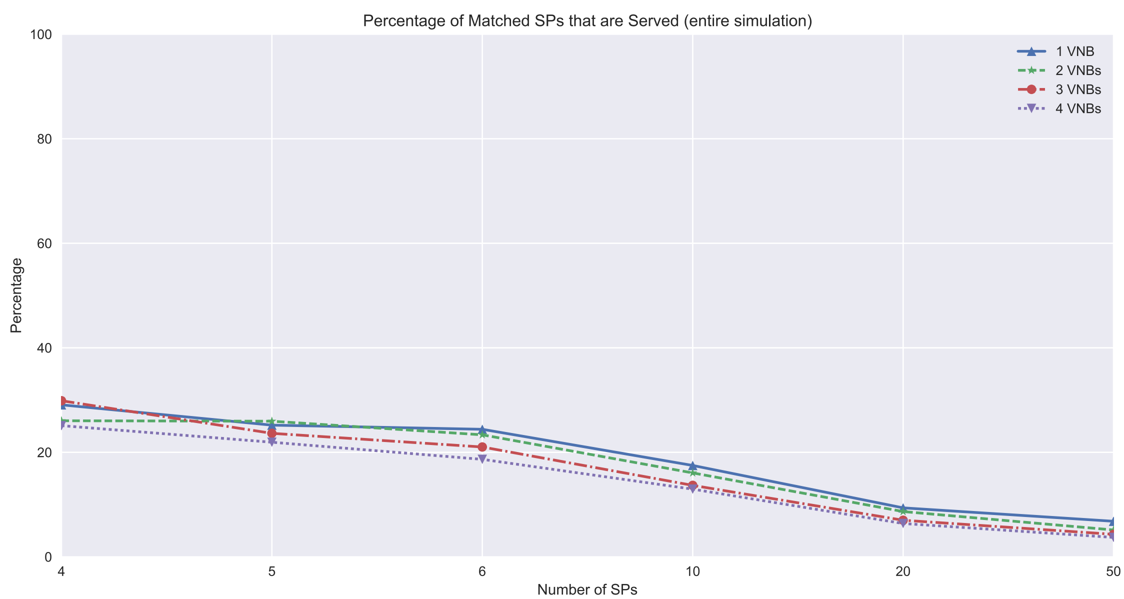


Figure 35: Percentage of SPs whose demand has been satisfied when there are 5 RPs in the system

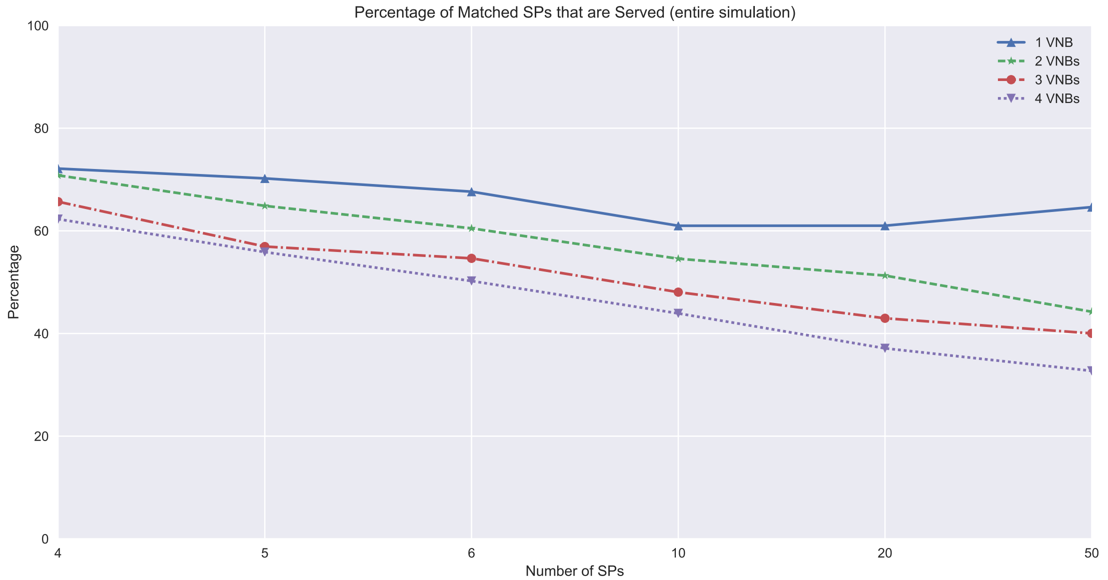


Figure 36: Percentage of SPs whose demand has been satisfied when there are 10 RPs in the system

Figure 38: Percentage of resources assigned by each Resource Provider (in average), when there are 5 RPs in the system.

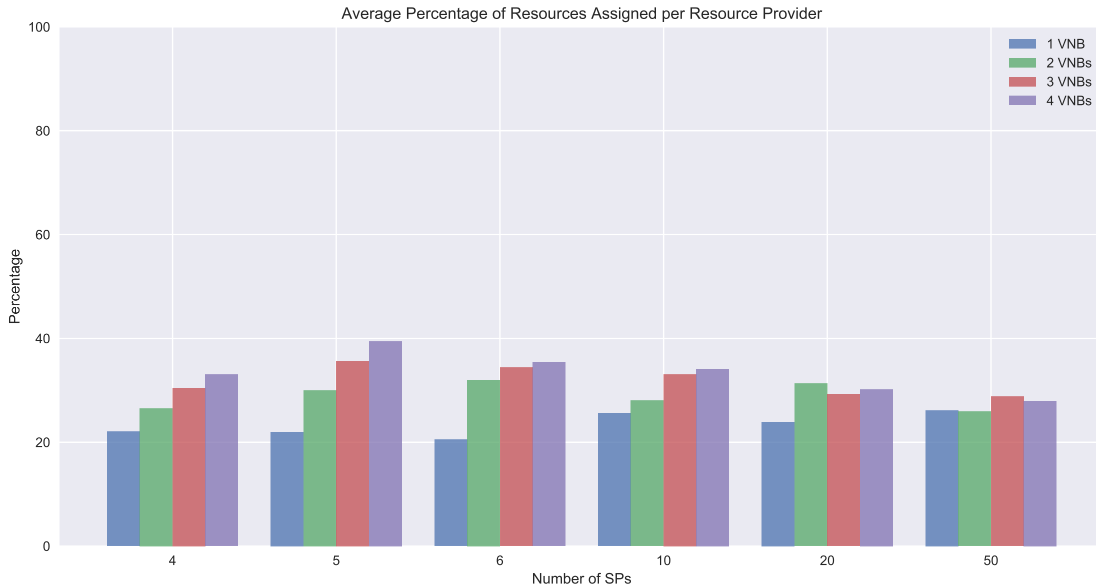


Figure 39: Percentage of resources assigned by each Resource Provider (in average), when there are 10 RPs in the system.

Figure 40: Percentage of resources assigned by each Resource Provider (in average), when there are 15 RPs in the system.

Figure 41: Average payment received by each RP when there are 5 RPs in the system.

9.3.1.4 Payments We now analyze the monetary remuneration received by the two profit-seeking entities in our market model: RPs and VNBs.

Payment Received by RPs: RPs receive a payment for the resources they assign in the market. Note that this payment is calculated with the market clearing price. Figures 41,42 and 43 show the results for scenarios with 5, 10 and 15 RPs, respectively.

The results shown in these figures are consistent with the amount of resources assigned in the system, shown in subsection 9.3.1.3. In consequence, scenarios with a larger number of RPs show higher revenue per RP especially when we shift from 5 to 10 or 15 RPs. It is important to note that the peak of performance in this analysis stems from the scenarios with 6 - 10 SPs. After this point, the revenue of RPs (as well as the amount of resources assigned) starts to decrease. This reduction can be associated with various factors: 1) market demand is the matched SPs' demand, not the entire SP population's demand , which extends to the amount of resources assigned and subsequent payment ; 2) given the market we have modeled for VNB–RP transactions, as there are more SPs in the system, more bids enter the market, hence increasing the opportunities for the market clearing price to be lower.

Figure 42: Average payment received by each RP when there are 10 RPs in the system.

Figure 43: Average payment received by each RP when there are 15 RPs in the system.

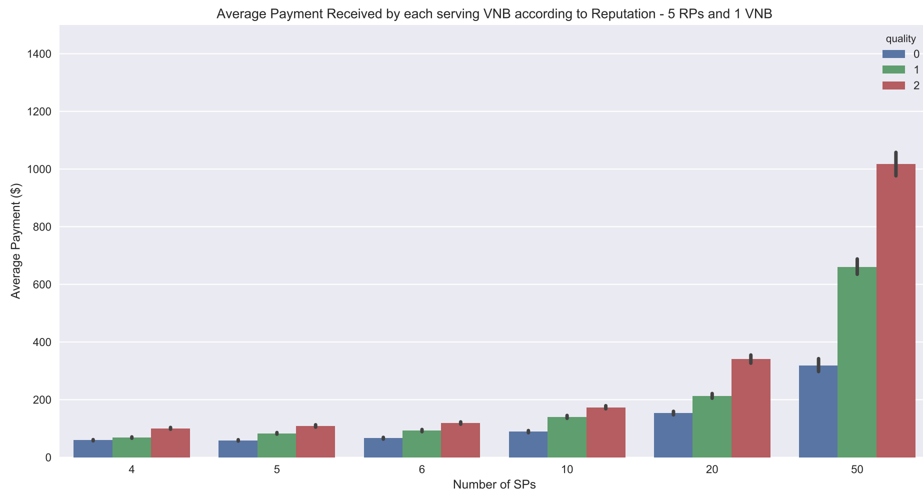


Figure 44: Average payment received by each VNB, according to its reputation. Scenario with 1 VNB and 10 RPs.

Payment Received by VNBs: VNBs receive a payment only when they have been able to obtain resources for their SP customers. For this purpose, we present payment data regarding the VNBs that were successful in the resource assignment process. In figures 44, 45, 46, and 47 we explore the payment received by each VNB while also differentiating these entities according to their reputation. The figures represent a scenario with 10 RPs. Other RP configurations are included in Appendix B.

In our model, VNBs with a higher reputation are allowed to charge higher fees for their services. This is reflected in our results, as in all cases, VNBs with a higher reputation (quality = 2) receive a higher remuneration.

9.3.1.5 Surplus This is an indicator of the profit each participant obtains in the market. Note that for all advertised prices, SPs, VNBs and RPs have the option of shading their real value. The surplus hence represents the difference between their real valuation for resources and services and the amount they received.

In the figures that follow, we present box plots showing the distribution of the surplus

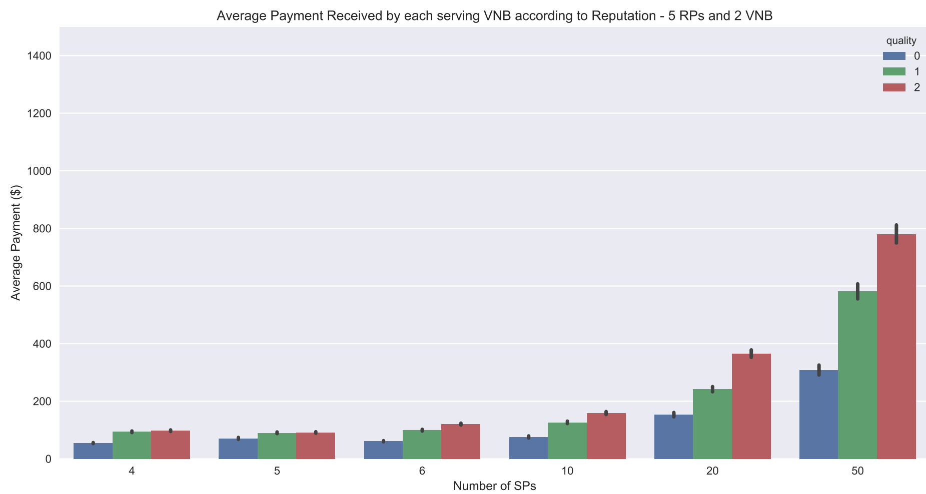


Figure 45: Average payment received by each VNB, according to its reputation. Scenario with 2 VNBs and 10 RPs.

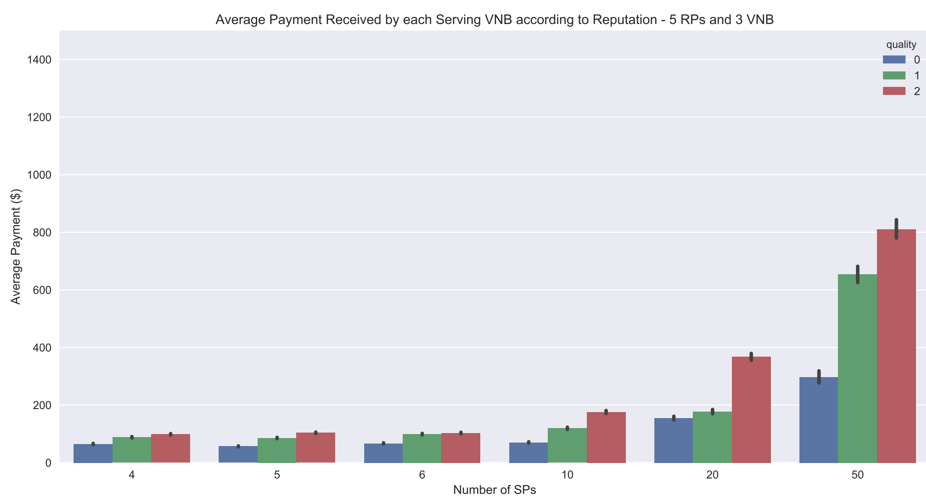


Figure 46: Average payment received by each VNB, according to its reputation. Scenario with 3 VNBs and 10 RPs.

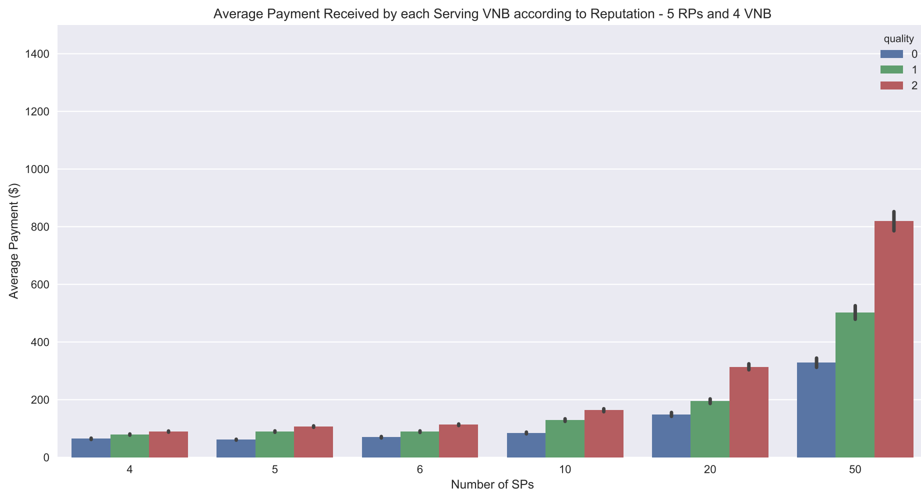


Figure 47: Average payment received by each VNB, according to its reputation. Scenario with 4 VNBs and 10 RPs.

values relevant to each entity.

RPs' surplus: After receiving a payment from the resources assigned, each RP can calculate the resulting surplus from its market negotiations. Figures 48, 49 and 50 show the surplus distribution for scenarios with varying RP and VNB configurations.

In the market setting we have modeled, the market clearing price should always be greater than or equal to the price advertised by the RPs. It follows that the minimum surplus perceived by each RP is zero, as shown in the figures above. Having a minimum surplus of zero indicates that even in the worst case scenario, RPs are not incurring in losses. This could result in a significant incentive for RPs to participate in the market.

The upper-limit value in the inter-quartile range increases with the number of RPs; however, in all scenarios it decreases as the number of SPs increases. This shows that the surplus distribution is consistent with the average payment, as it is lower for higher number of SPs.

SPs' surplus from resource payment: We now analyze the other side of resource payment. This results from the SPs' perspective and how the amount they pay for resources

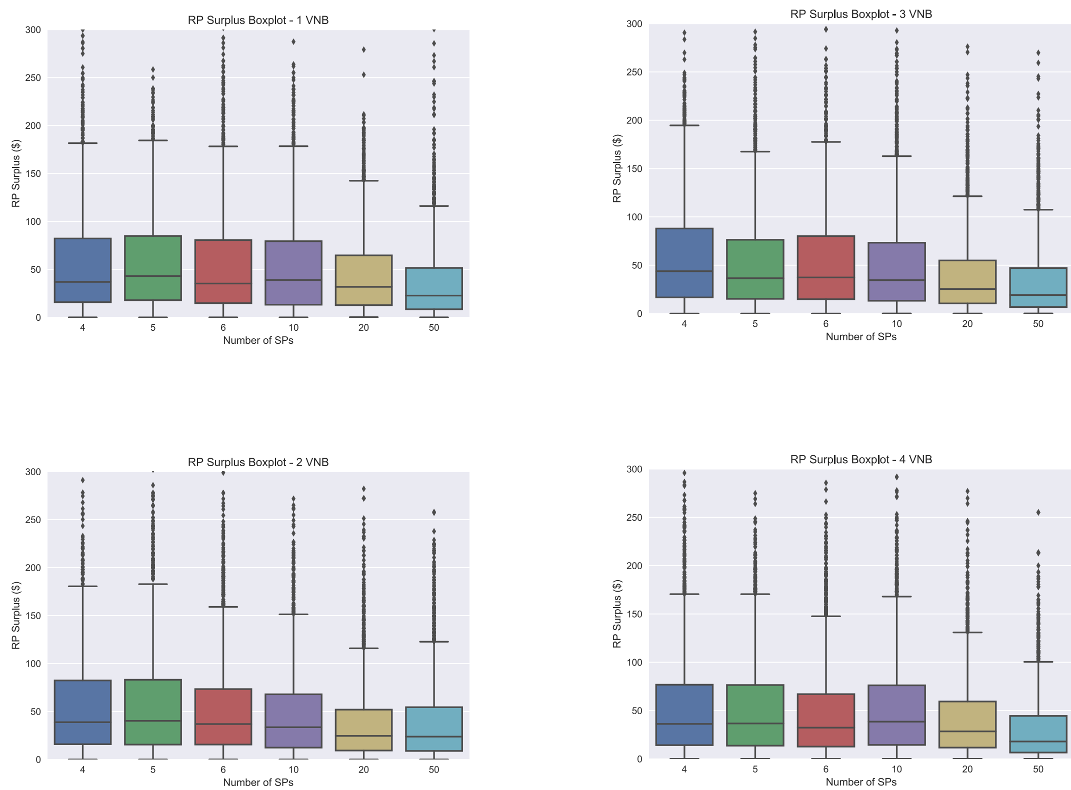


Figure 48: Box plot of the surplus perceived by each RP in a 5 RP scenario. This includes results form different VNB market configurations: top left: 1 VNB, top right: 2 VNBs, bottom left: 3 VNBs and bottom right: 4 VNBs.

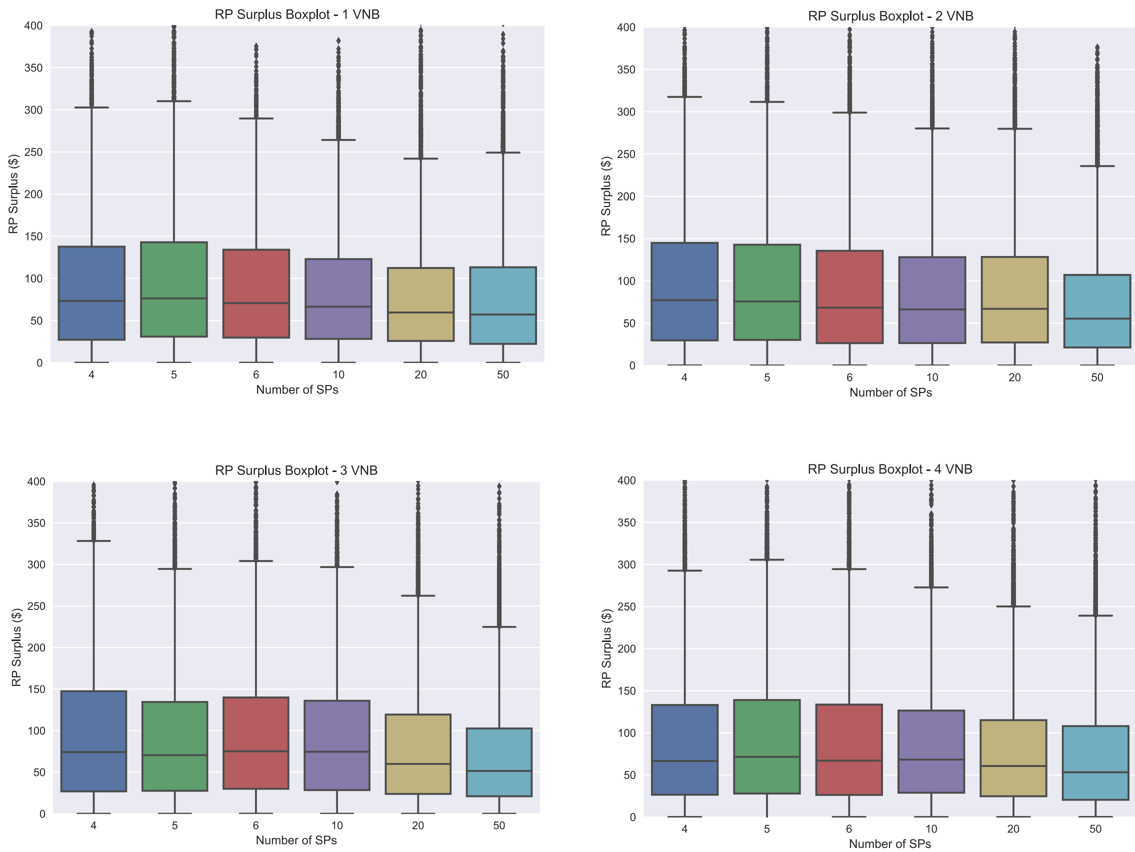


Figure 49: Box plot of the surplus perceived by each RP in a 10 RP scenario. This includes results form different VNB market configurations: top left: 1 VNB, top right: 2 VNBs, bottom left: 3 VNBs and bottom right: 4 VNBs.

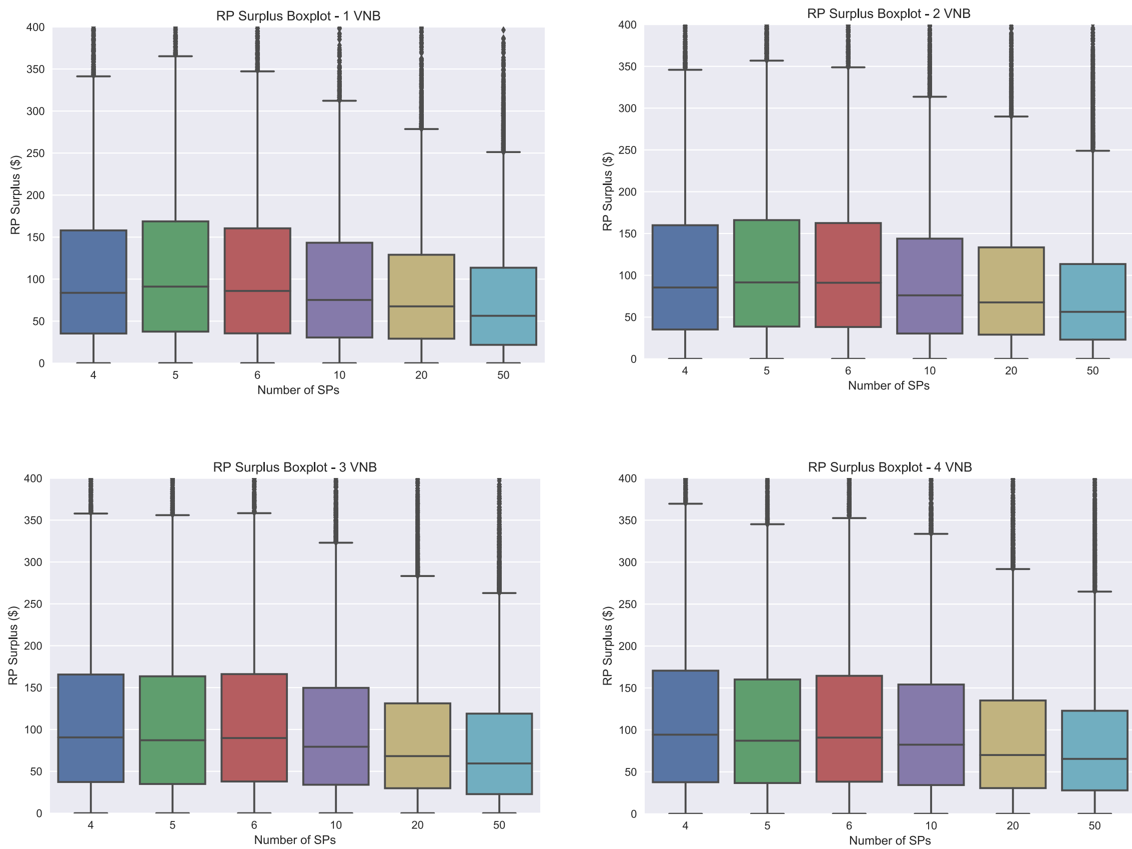


Figure 50: Box plot of the surplus perceived by each RP in a 15 RP scenario. This includes results form different VNB market configurations: top left: 1 VNB, top right: 2 VNBs, bottom left: 3 VNBs and bottom right: 4 VNBs.

compares to their real resource valuation.

Figures 51, 52 and 53 show the resulting surplus for scenarios with the available RP and VNB configurations.

The figures above show that in the majority of cases the SPs' surplus is zero. Only in the scenario with 15 RPs, we observe the interquartile range showing values above zero for large numbers of SPs (e.g., 10, 20 and 50). These results suggest that in the majority of cases, the SPs' payments match their real resource valuation.

SPs' surplus from VNB payment: SPs receiving resources need to make an additional payment. Indeed, they need to pay the convened fee to their VNB partners. In figures 54, 55 and 56, we show the distribution of the SPs' surplus stemming from the final fee they pay to the VNBs in scenarios with 5, 10 and 15 RPs, respectively.

The results presented in these figures show that there is a fraction of negative surplus values, which indicates that SPs incur in losses from their transactions with VNBs. Comparing all results, we observe that as the number of RPs increases, the surplus distribution becomes more stable. Particularly, for 10 and 15 RP scenarios, the 75% of surplus values (for 7 out of 8 cases) are positive. This indicates that in its majority, the SPs match with VNBs and the payment arrangement results in a profit for the SPs.

VNBs' surplus: This value stems from the payment received from the SPs. It should be emphasized that a VNB receives a payment only when it assigns resources. Hence, the figures we include in this section reflect the surplus of the VNBs who assigned resources to their customers.

The inter-quartile range of the box plots indicates that the surplus values in the analyzed scenarios follow a normal distribution. We find very low variability in the surplus values in scenarios with 5 RPs, and scenarios with 10 and 15 RPs where the number of SPs is lower than 10. In the case of 5 RPs, the surplus distribution is consistent, irrespective of the number of SPs considered. In the case of 10 and 15 RPs, the distribution is consistent for scenarios with less than 10 SPs. In the latter cases, as the number of SPs increases, there is a higher variability in the surplus values, which represents a higher probability of negative surplus. Nevertheless, results also show that the median surplus value, in the majority of cases, is zero.

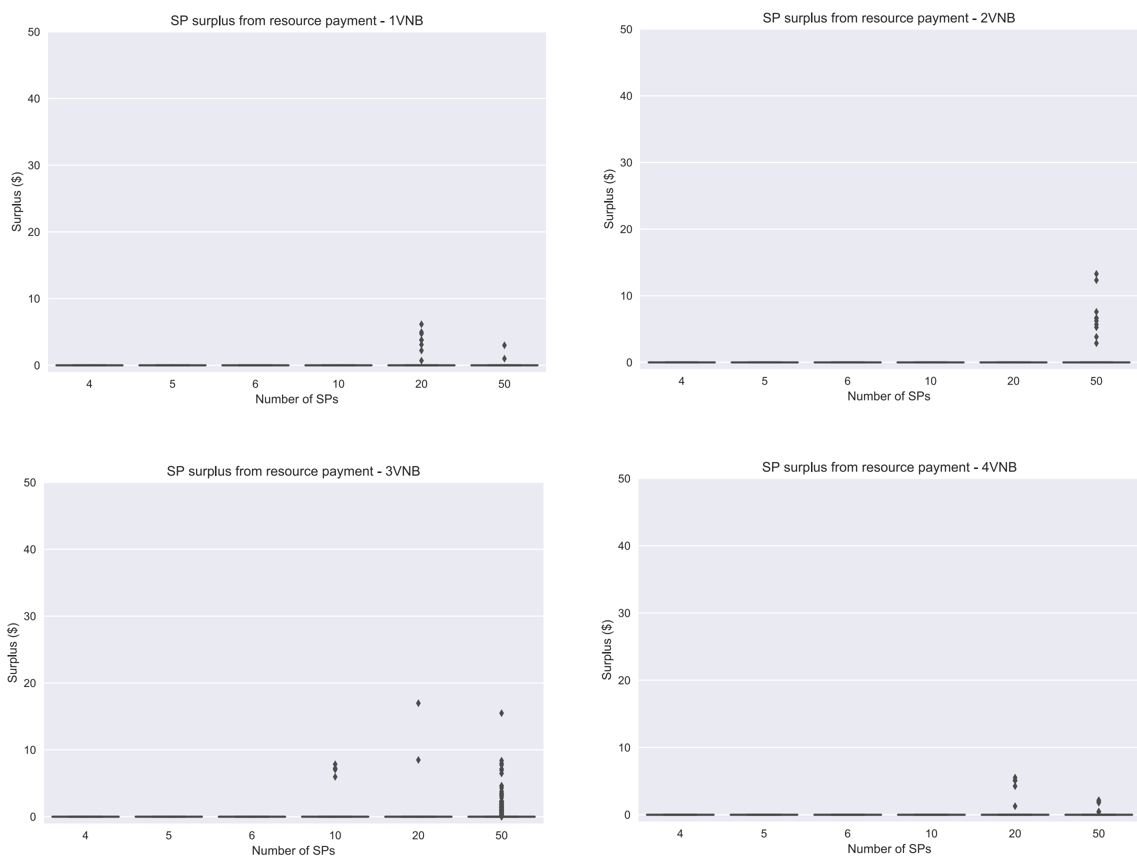


Figure 51: SPs' surplus from resource payment in a 5 RP scenario. This includes results form different VNB market configurations: top left: 1 VNB, top right: 2 VNBs, bottom left: 3 VNBs and bottom right: 4 VNBs.

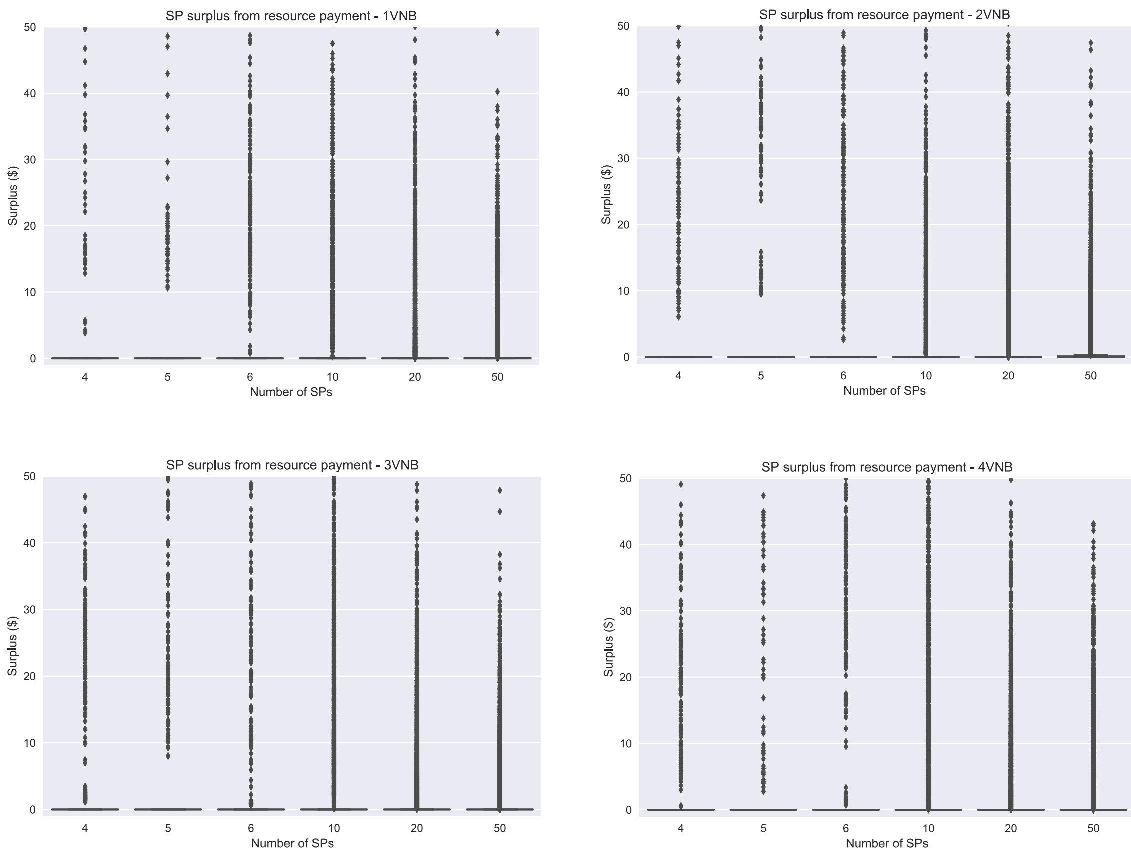


Figure 52: SPs' surplus from resource payment in a 10 RP scenario. This includes results form different VNB market configurations: top left: 1 VNB, top right: 2 VNBs, bottom left: 3 VNBs and bottom right: 4 VNBs.

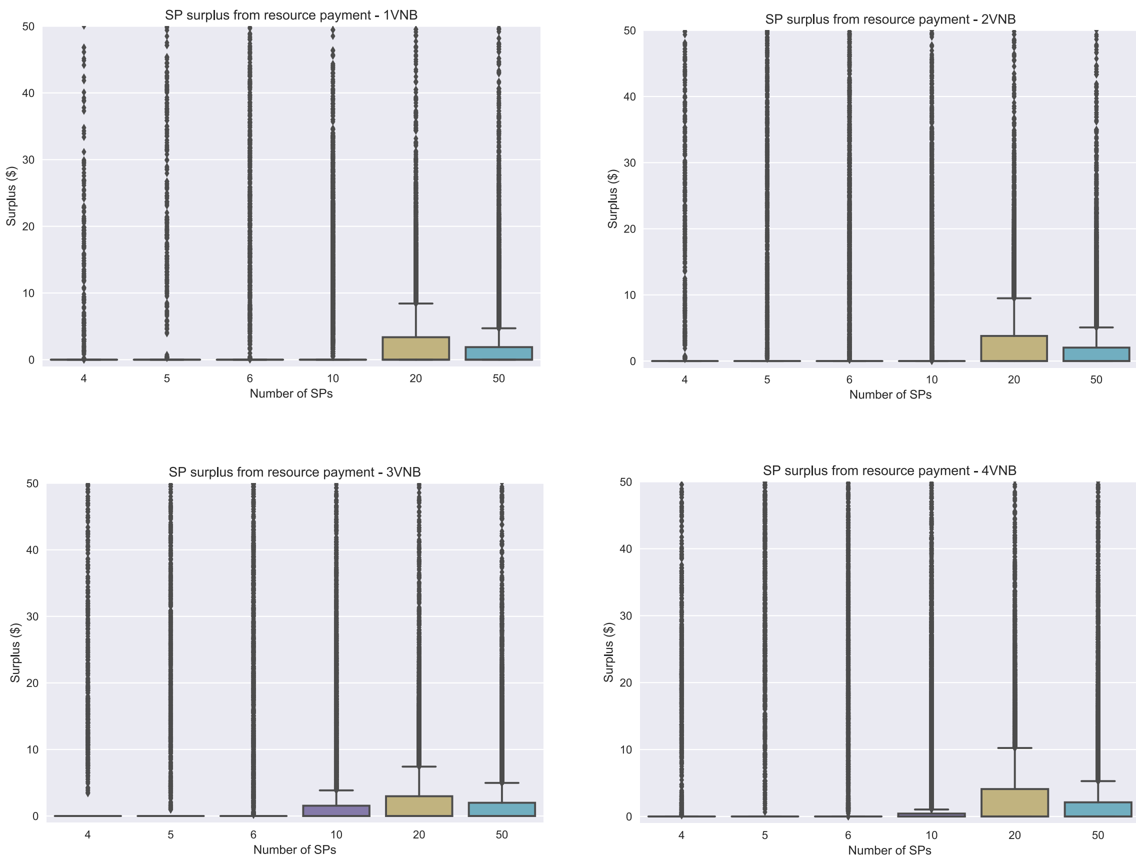


Figure 53: SPs' surplus from resource payment in a 15 RP scenario. This includes results form different VNB market configurations: top left: 1 VNB, top right: 2 VNBs, bottom left: 3 VNBs and bottom right: 4 VNBs.

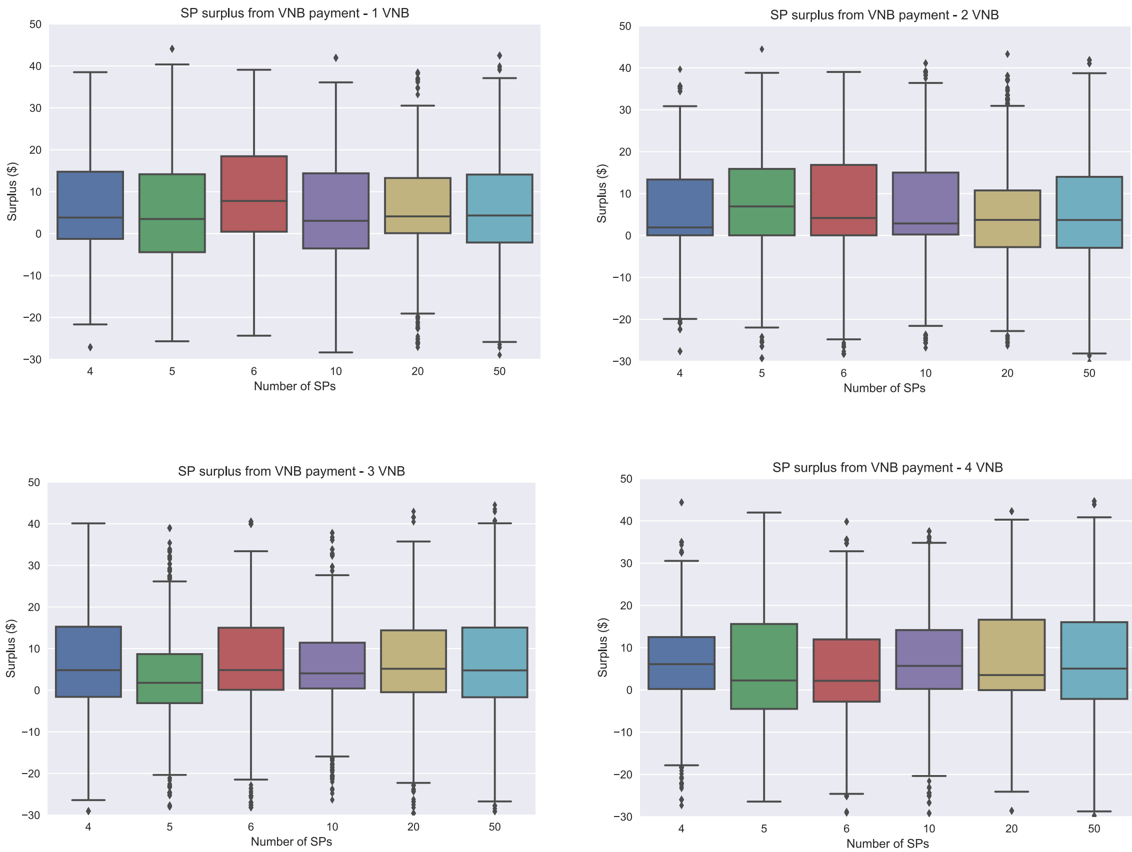


Figure 54: SPs' surplus from VNB payment in a 5 RP scenario. This includes results form different VNB market configurations: top left: 1 VNB, top right: 2 VNBs, bottom left: 3 VNBs and bottom right: 4 VNBs.

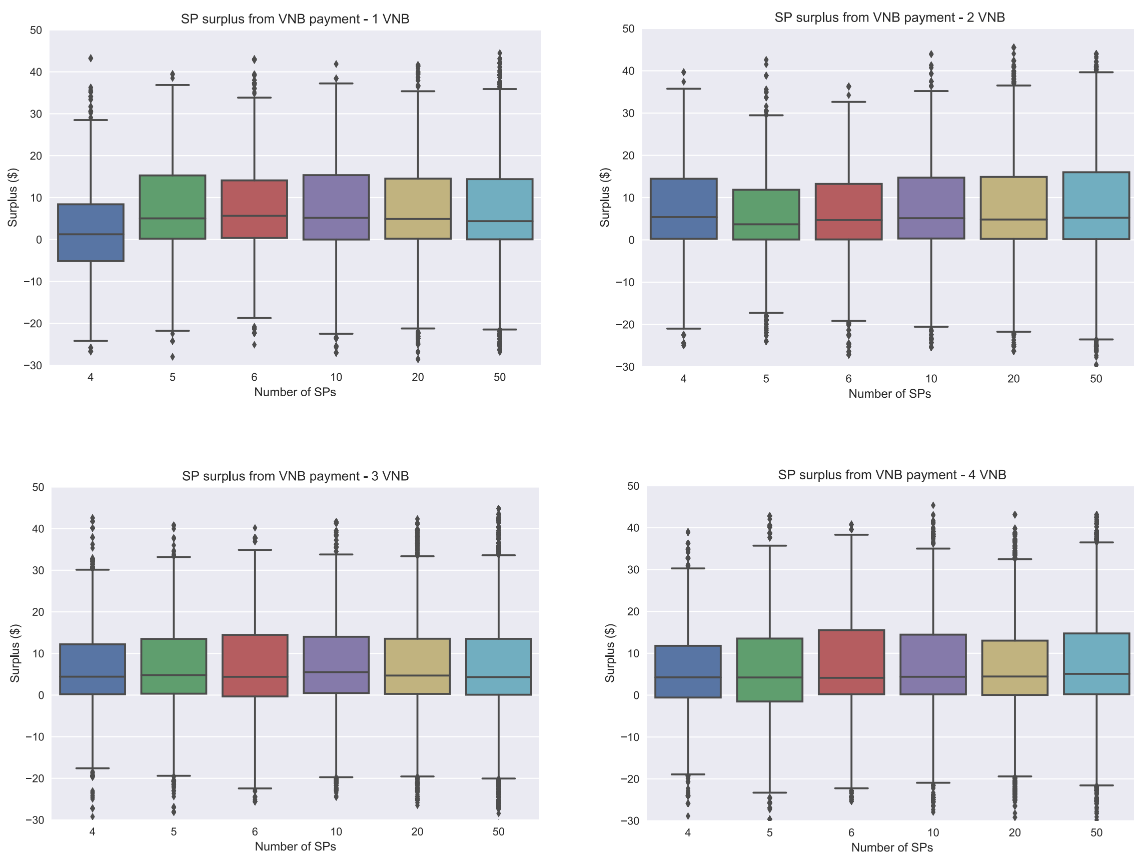


Figure 55: SPs' surplus from VNB payment in a 10 RP scenario. This includes results form different VNB market configurations: top left: 1 VNB, top right: 2 VNBs, bottom left: 3 VNBs and bottom right: 4 VNBs.

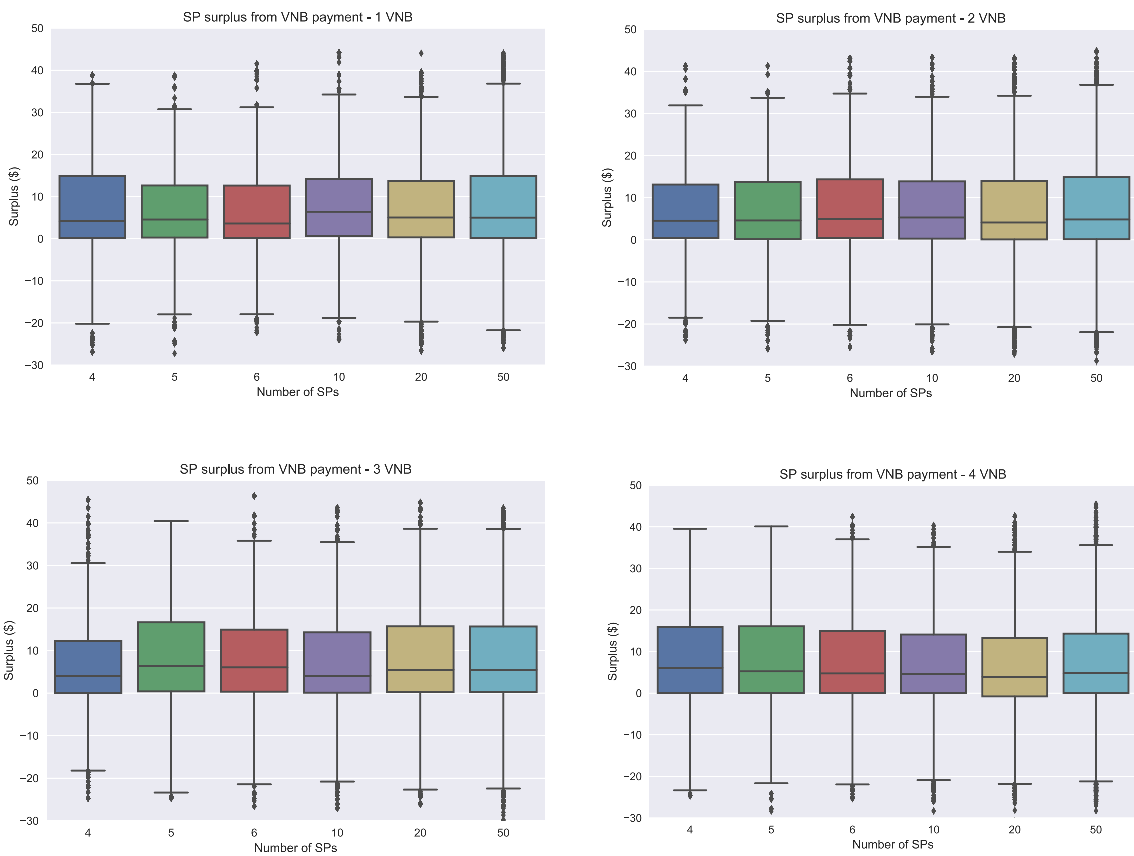


Figure 56: SPs' surplus from VNB payment in a 15 RP scenario. This includes results form different VNB market configurations: top left: 1 VNB, top right: 2 VNBs, bottom left: 3 VNBs and bottom right: 4 VNBs.

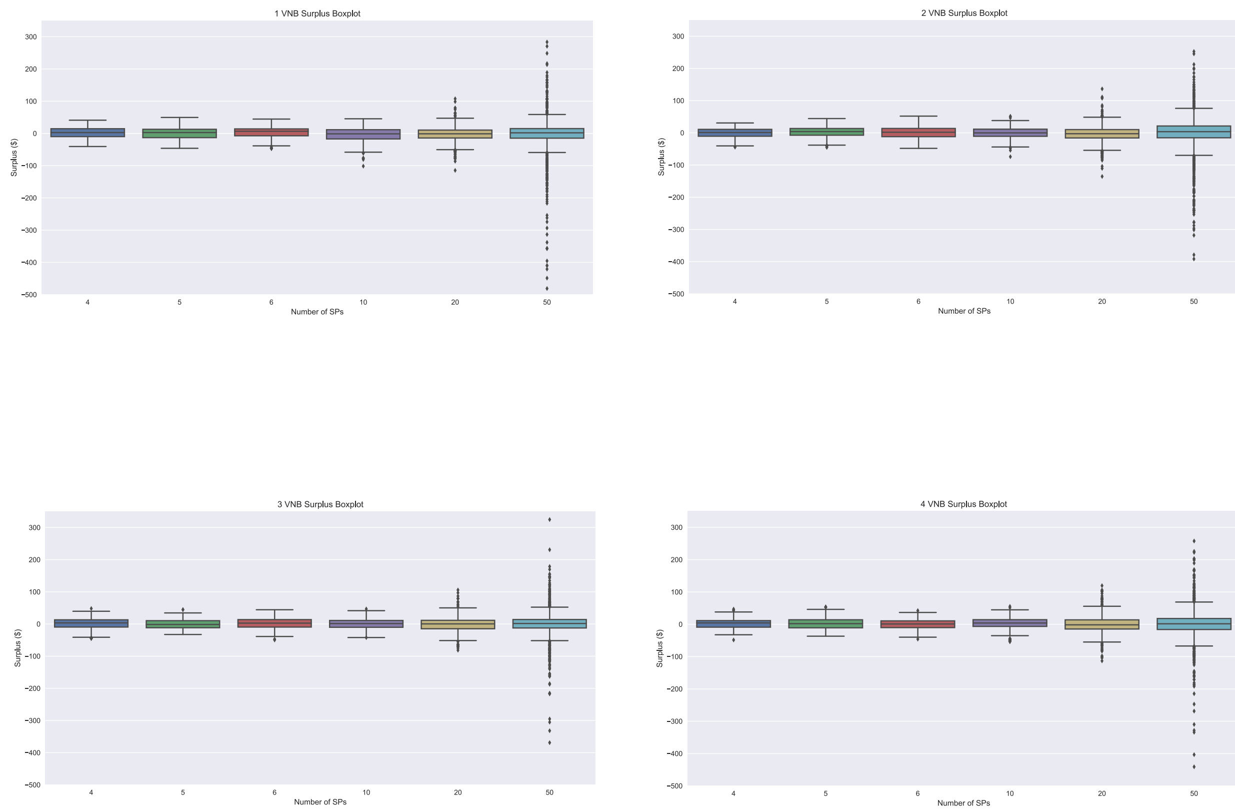


Figure 57: VNBs' surplus in a scenario with 5 RPs. This includes results form different VNB market configurations: top left: 1 VNB, top right: 2 VNBs, bottom left: 3 VNBs and bottom right: 4 VNBs.

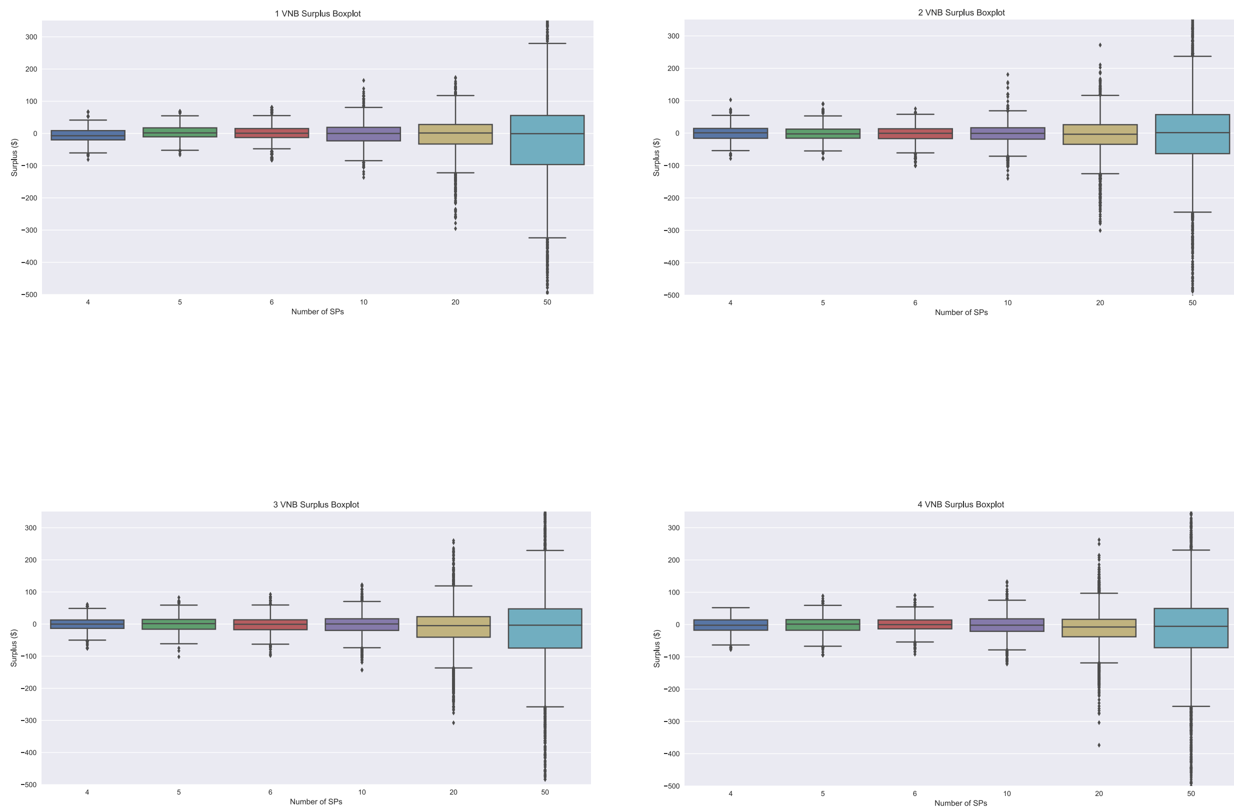


Figure 58: VNBs' surplus in a scenario with 10 RPs. This includes results form different VNB market configurations: top left: 1 VNB, top right: 2 VNBs, bottom left: 3 VNBs and bottom right: 4 VNBs.

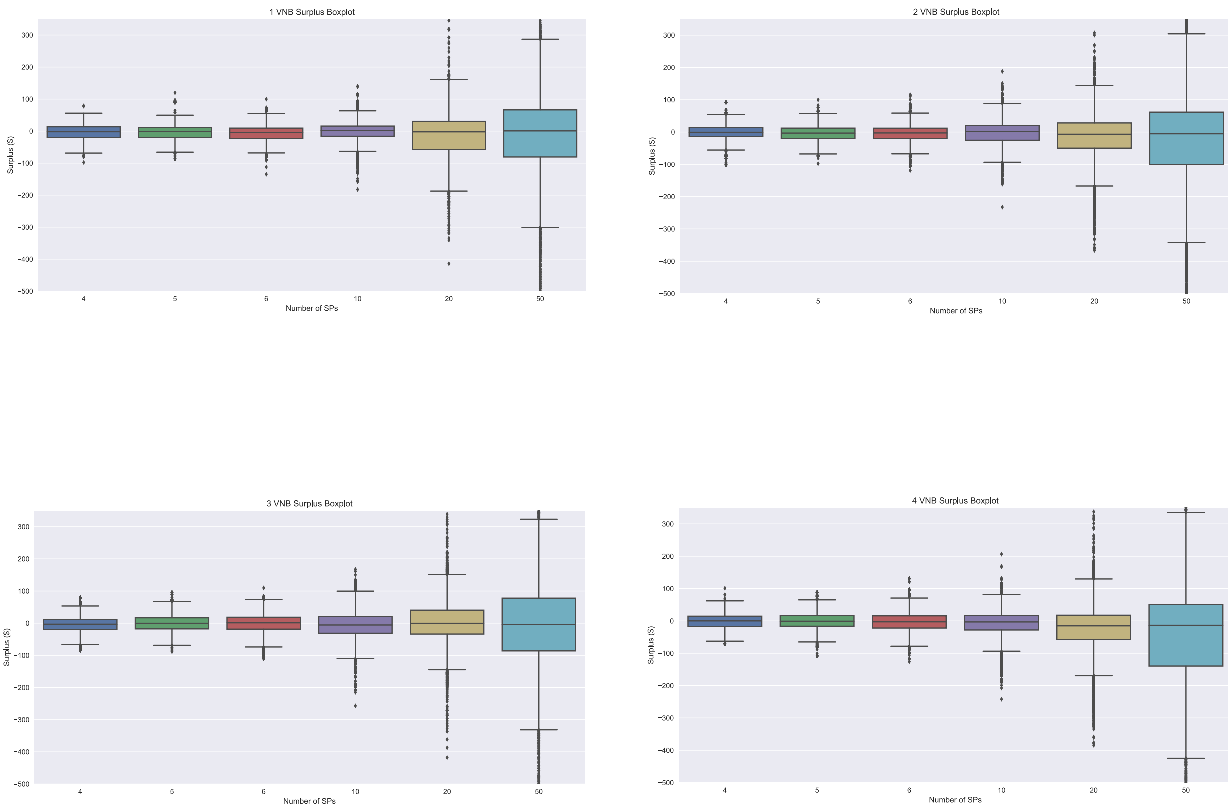


Figure 59: VNBs' surplus in a scenario with 15 RPs. This includes results form different VNB market configurations: top left: 1 VNB, top right: 2 VNBs, bottom left: 3 VNBs and bottom right: 4 VNBs.

9.3.2 Experimental Group Performance

In this subsection, we explore stage 3 results from an experimental group perspective. As such, we show which experimental group outperforms the rest according to a set of factors relevant to SPs and VNBs. To this end, we have processed simulation results relevant to the factors included in section 8.3.6, calculated the average values throughout all the scenarios analyzed, and chosen the group with the *best* performance. In what follows, we present the aggregate results showing the groups performing best for each factor.

In terms of the parameters we have chosen, best performance for SPs means:

- Highest percentage of matched SPs
- Highest percentage of served demand (i.e., percentage of matched SPs that received resources)
- Highest surplus from fees paid to VNBs

Similarly, best performance for VNBs means:

- Highest percentage of matched VNBs
- Highest payment received by VNBs
- Highest surplus from SP payment.

In figures 60, 61 and 62, we can observe that a higher percentage of SPs belonging to experimental group 4 are matched in all scenarios (i.e., all VNB, SP and RP combinations).

Regarding the percentage of SPs' demand satisfied, in figures 63, 64 and 65 we show that matched SPs belonging to experimental group 4 obtain the best results in the system.

Figures 66, 67 and 68 show the results for the group performance in terms of surplus. Group 4 members no longer outperform the rest. In fact, group 1 performs better in scenarios with 5 and 10 RPs, while group 2 performs better in the 15 RP case.

On the VNB side, we explore the group yielding the highest percentage of matched VNBs. Figures 69, 70 and 71 show that in the majority of scenarios, group 4 outperforms the rest. Exceptions are a few instances where group 2 performs better than the rest.

In figures 72, 73 and 74, we present the results regarding the payment received by VNBs in scenarios with different RP configurations. It can be observed that in cases with 10 and

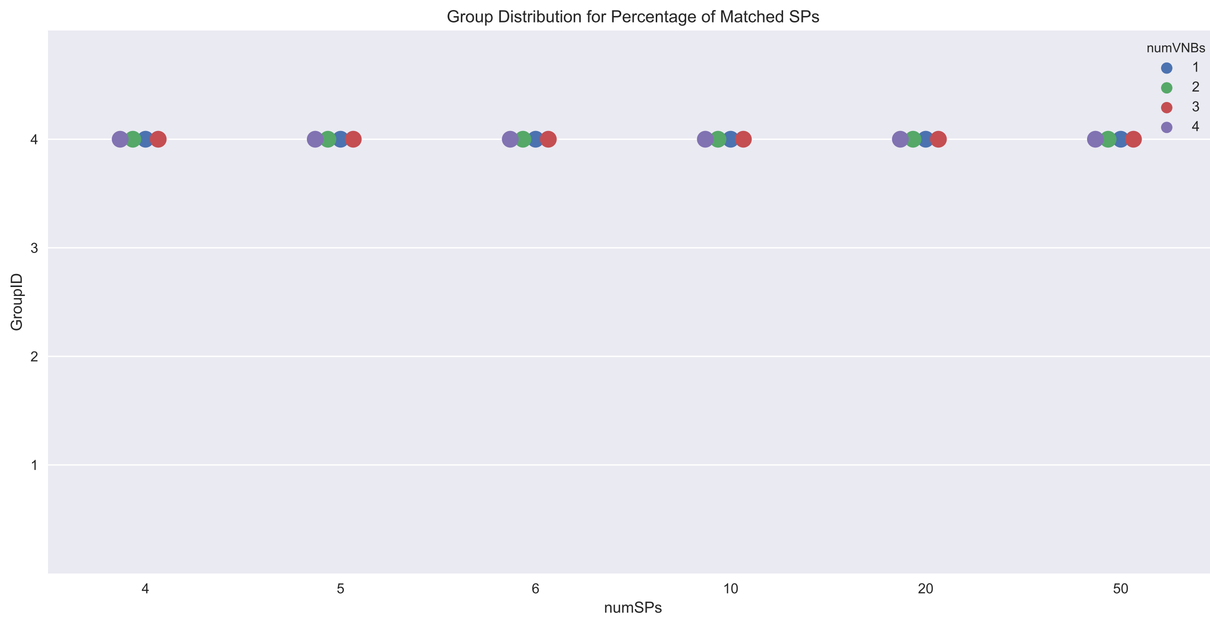


Figure 60: Groups with highest percentage of matched SPs in scenarios with 5 RPs.

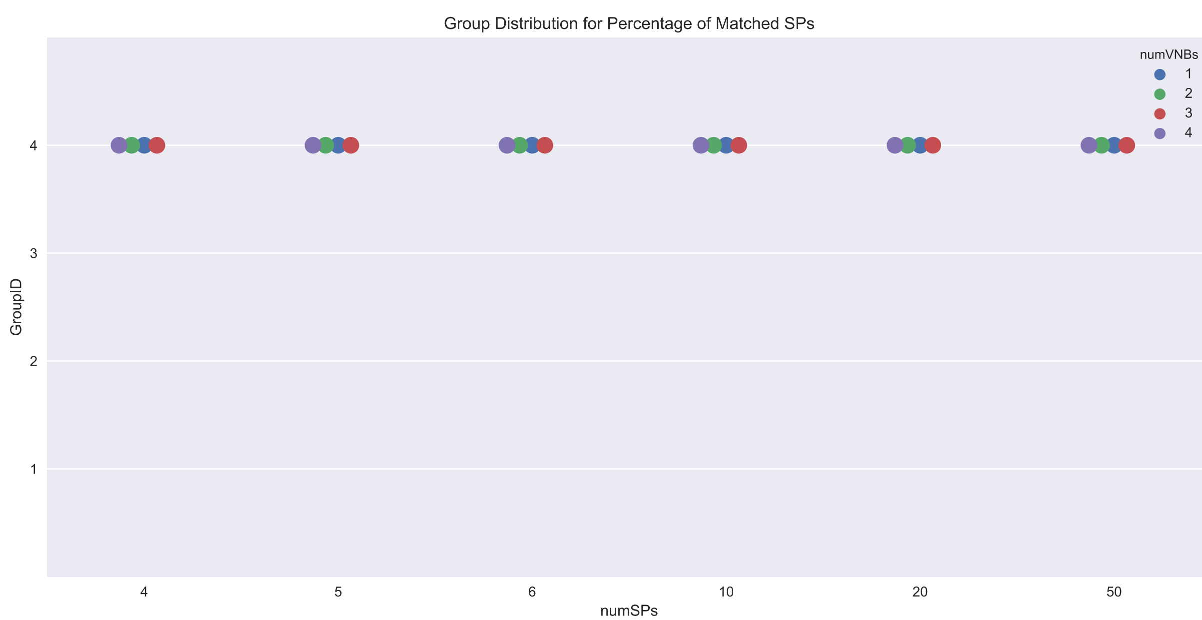


Figure 61: Groups with highest percentage of matched SPs in scenarios with 10 RPs.

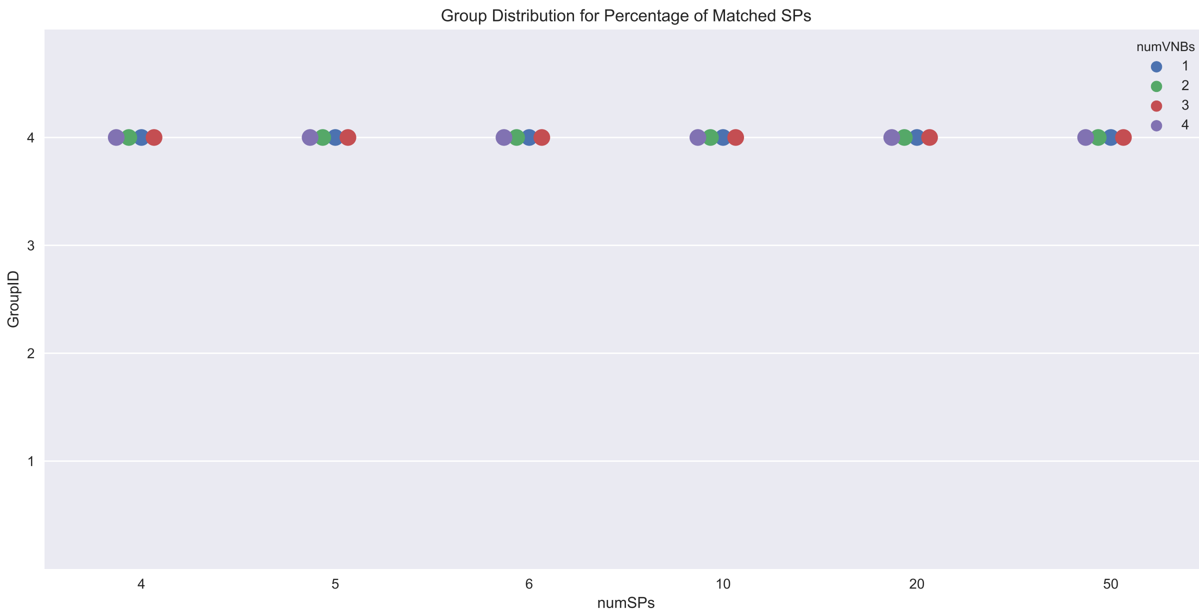


Figure 62: Groups with highest percentage of matched SPs in scenarios with 15 RPs.

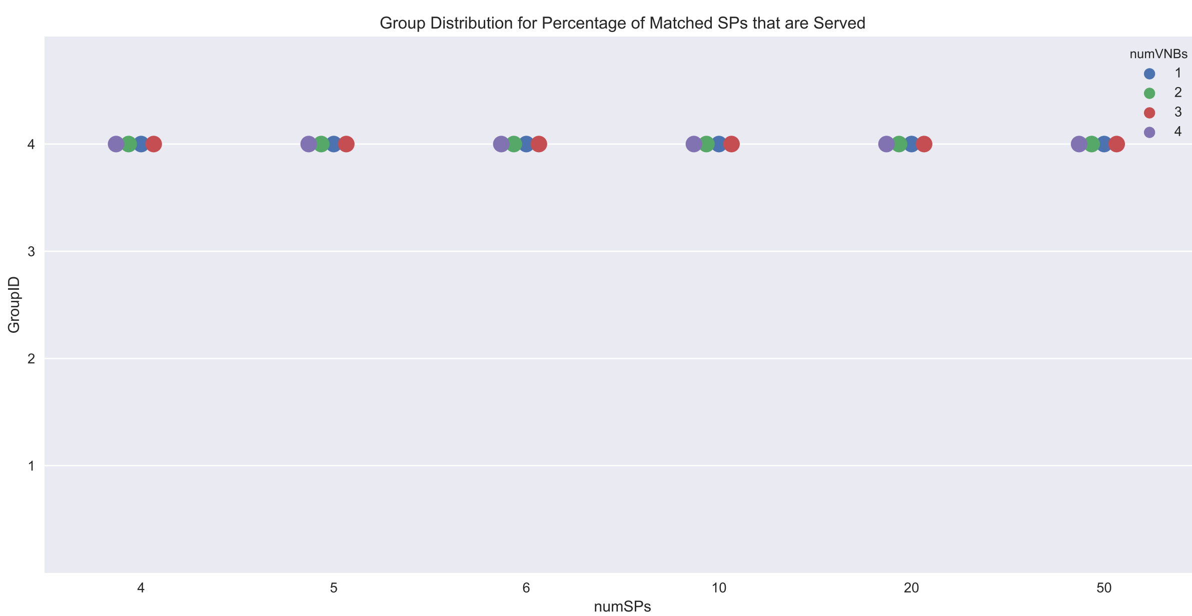


Figure 63: Groups with highest percentage of matched SPs receiving resources from the market. Scenario with 5 RPs

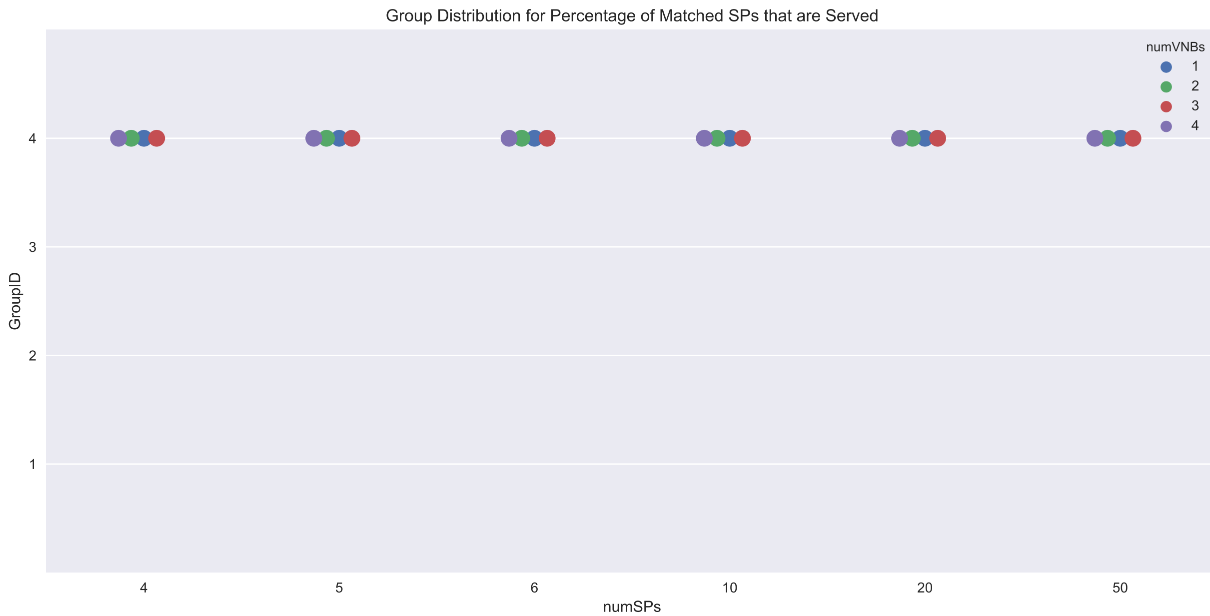


Figure 64: Groups with highest percentage of matched SPs receiving resources from the market. Scenario with 10 RPs

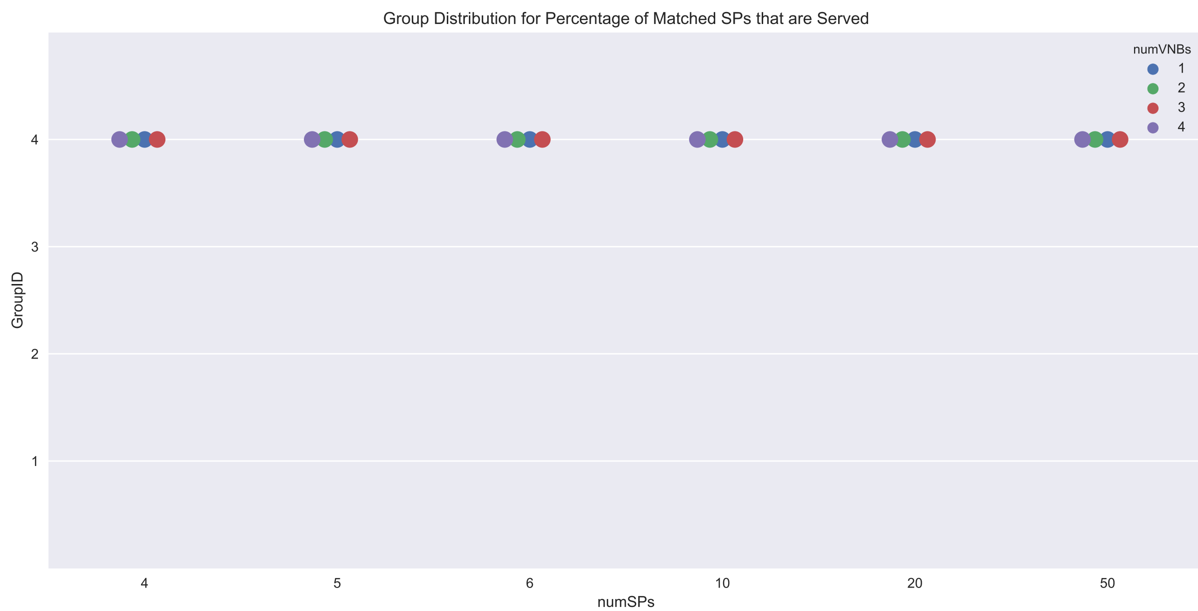


Figure 65: Groups with highest percentage of matched SPs receiving resources from the market. Scenario with 15 RPs

15 RPs, in the majority of scenarios, group 4 performs best. Nevertheless, it is hard to establish a solid conclusion in the case of 5 RPs, although there is a greater number of instances where group 3 outperforms the rest. Note that VNBs' payment depends on their ability to obtain resources in the market. In cases where supply is low (RPs = 5), this task is more complicated, hence a higher variability on the results.

Regarding VNBs' surplus, we find a similar outcome as that of the SPs. Figures 75, 76 and 77 show the variability in these results. In the scenario with 5 RPs, group 3 has the best performance; in the scenario with 10 RPs, group 4 performs best, and in the 15 RP scenario group 1 obtains the best performance. Nevertheless, the difference among groups is not significant.

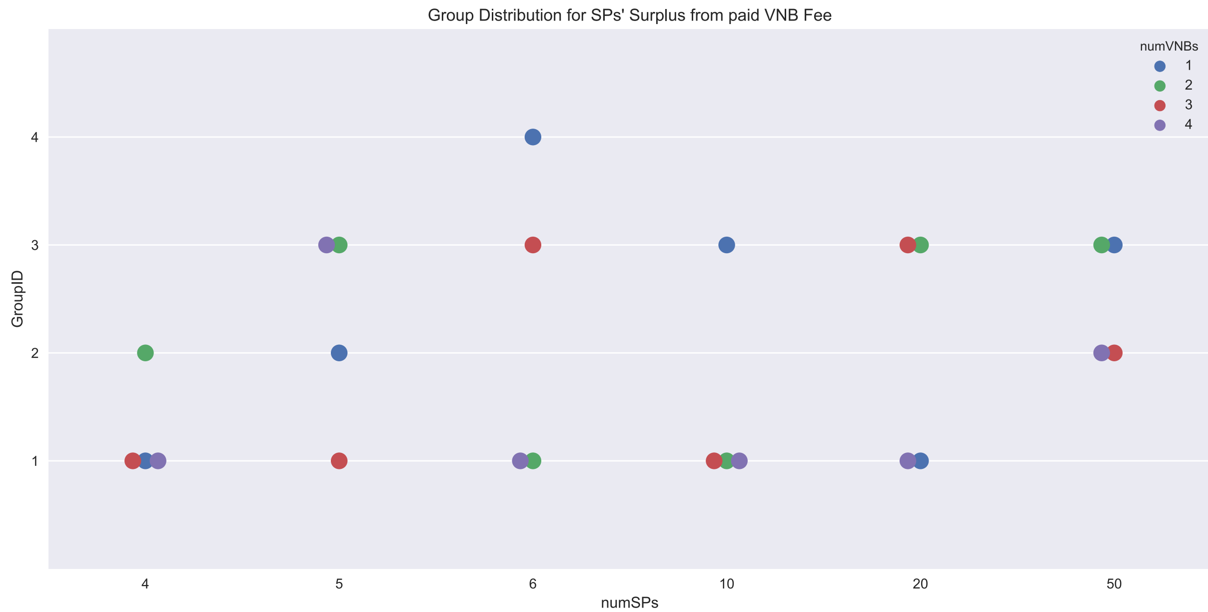


Figure 66: Groups where SPs obtain the highest surplus from VNB payment. Scenario with 5 RPs

9.3.3 Summary of Results

Through the analysis of the simulation results in this section, we have been able to draw important conclusions regarding the proposed market structure and its viability. The different combinations of market participants that we have tested, have allowed us to explore what market configurations become stable and which provide us with positive results. We highlight some important conclusions that may point to general spectrum market recommendations as well as future improvements of the current model.

Considering different RP configurations (i.e., 5, 10 and 15 RP scenarios) allows us to test different levels of RP investments. This has an evident impact on the amount of supply in the market, which is supported by figures 78, 79 and 80. Further, these results are relevant for hypothesis H7. Note that the VNB market configuration does not have a significant impact on the market supply. Indeed, this value remains rather constant in spite of changes in the number of VNBs.

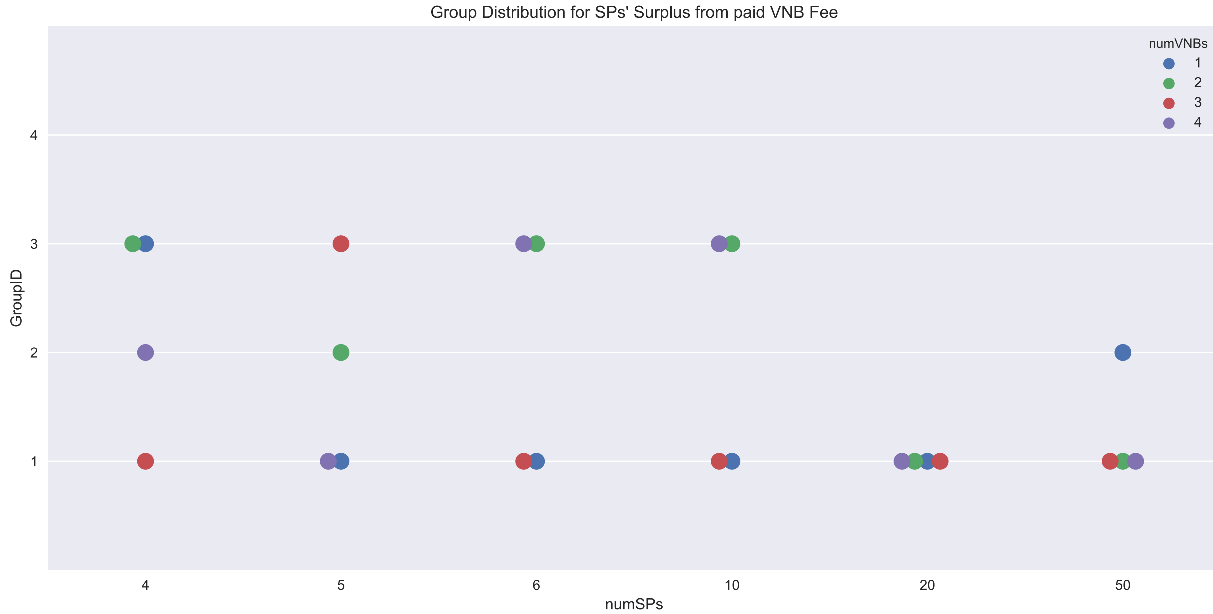


Figure 67: Groups where SPs obtain the highest surplus from VNB payment. Scenario with 10 RPs

In general, we find that market scenarios with only 5 RPs show less stable results. This is particularly the case when we explore them in combination with a small number of SPs participating in the market. As the number of RPs and SPs increases, we find repeating trends in the number of matched SPs and VNBs, the payments made and received, the amount of resources assigned, to mention a few factors. A small number of RPs in combination with a small number of SPs may be suggestive of initial market scenarios, as such, we can infer that external incentives may be necessary for these situations to work. For example, these incentives may target RPs and prompt them to share their resources in the market.

Regarding the matching process, the method we propose considers the joint utility derived from an sp_i–vnb_j match. Hence, each SP's and VNB's preference vector is built in terms of this utility. For this to be possible, we require the sum of the utilities of sp_i and vnb_j to be greater than the threshold that we have set for allowable matches. In this way, we have two different possibilities for $vnb_j(sp_i)$ to enter sp_i's(vnb_j's) preference vector: 1)

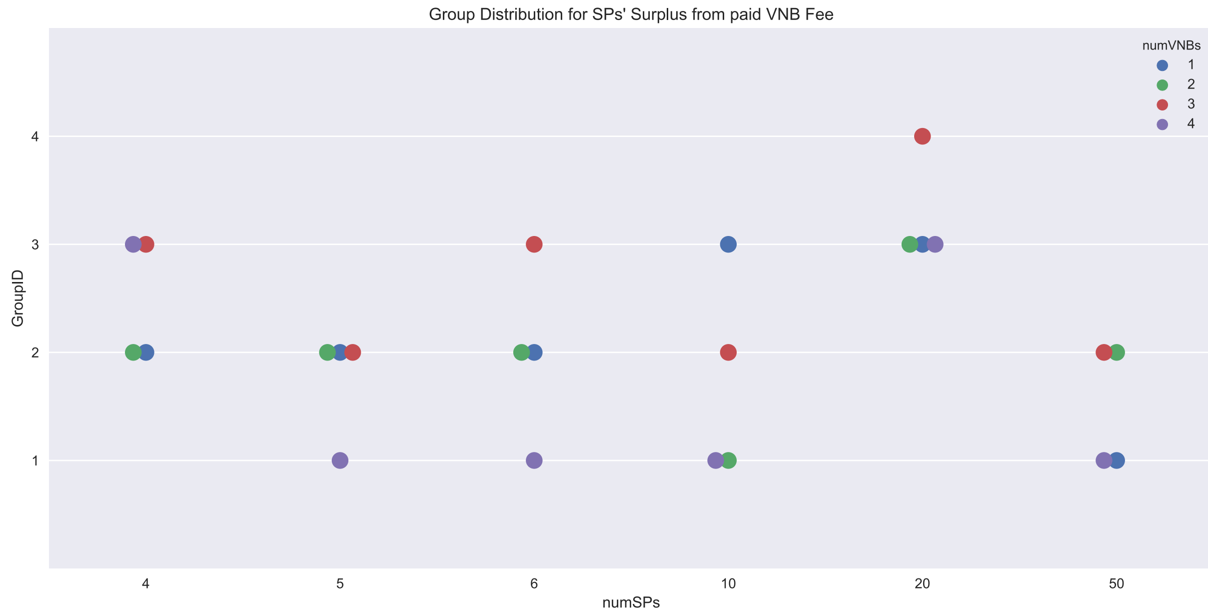


Figure 68: Groups where SPs obtain the highest surplus from VNB payment. Scenario with 15 RPs

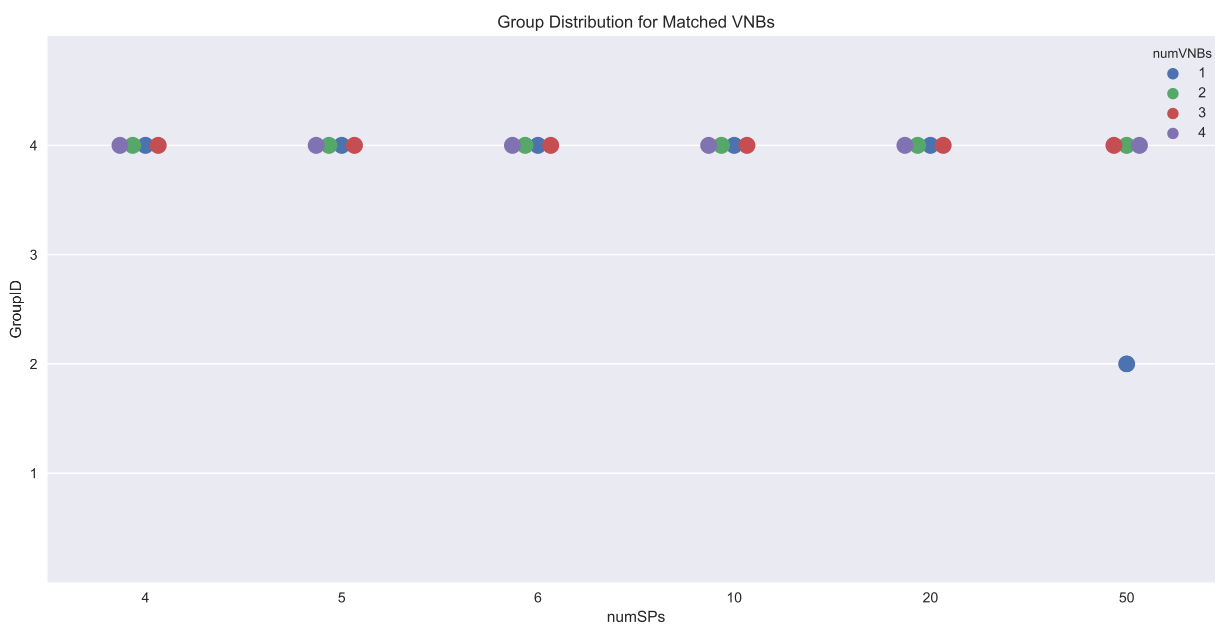


Figure 69: Groups with a higher percentage of matched VNBs. Scenario with 5 RPs

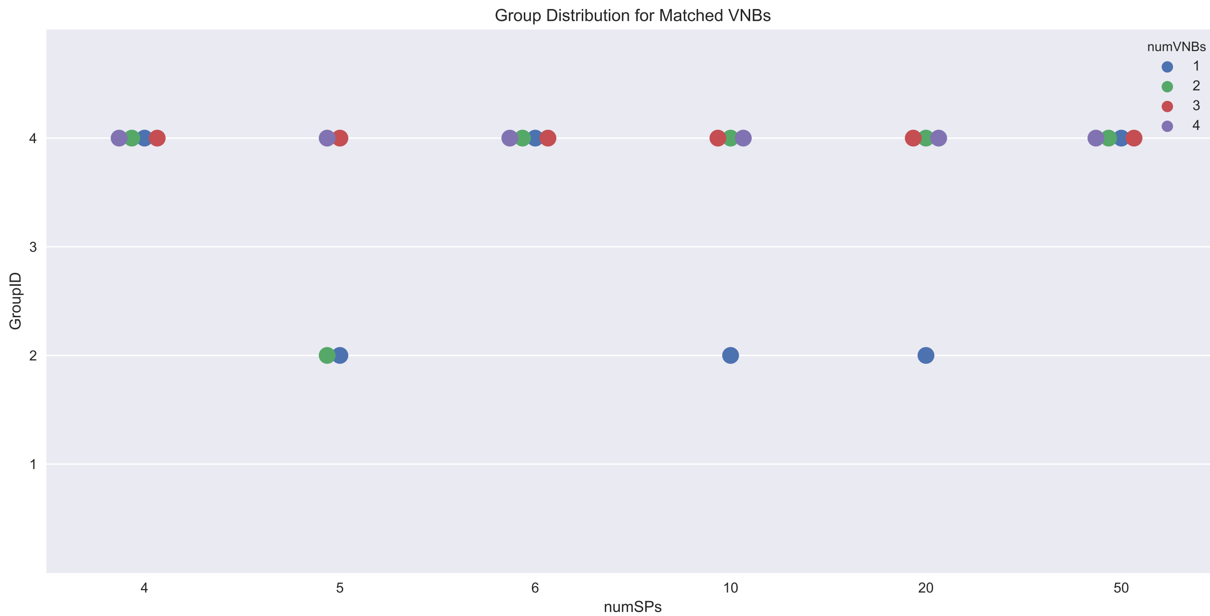


Figure 70: Groups with a higher percentage of matched VNBs. Scenario with 10 RPs

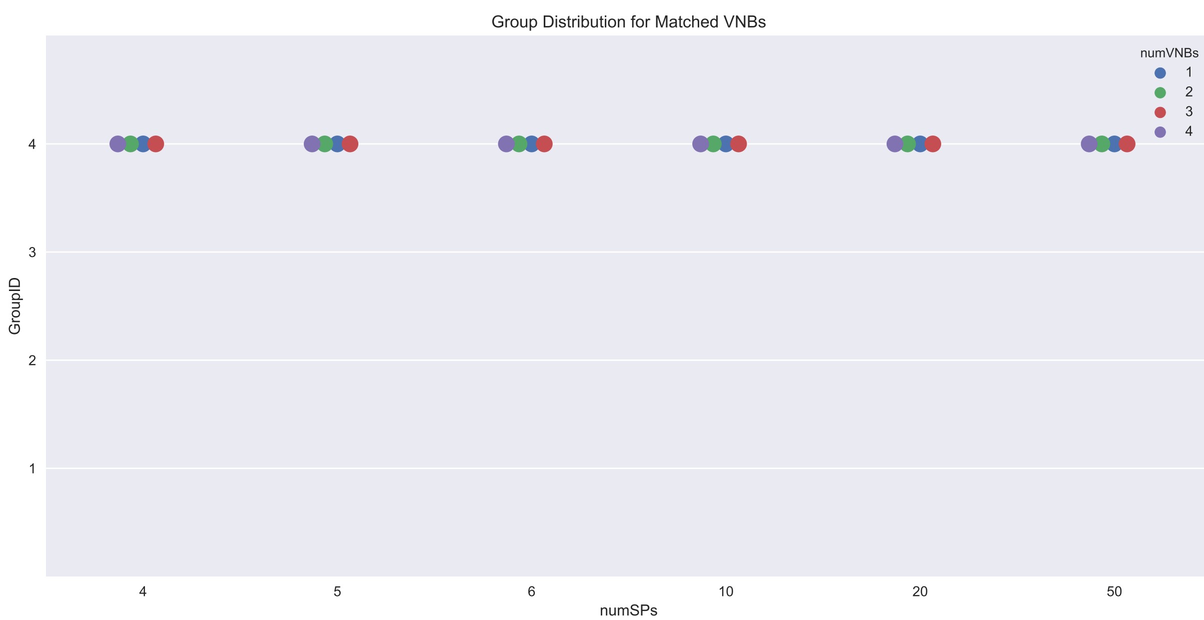


Figure 71: Groups with a higher percentage of matched VNBs. Scenario with 15 RPs

Figure 72: Groups where VNBs receive a higher payment. Scenario with 5 RPs

Figure 73: Groups where VNBs receive a higher payment. Scenario with 10 RPs

Figure 74: Groups where VNBs receive a higher payment. Scenario with 15 RPs

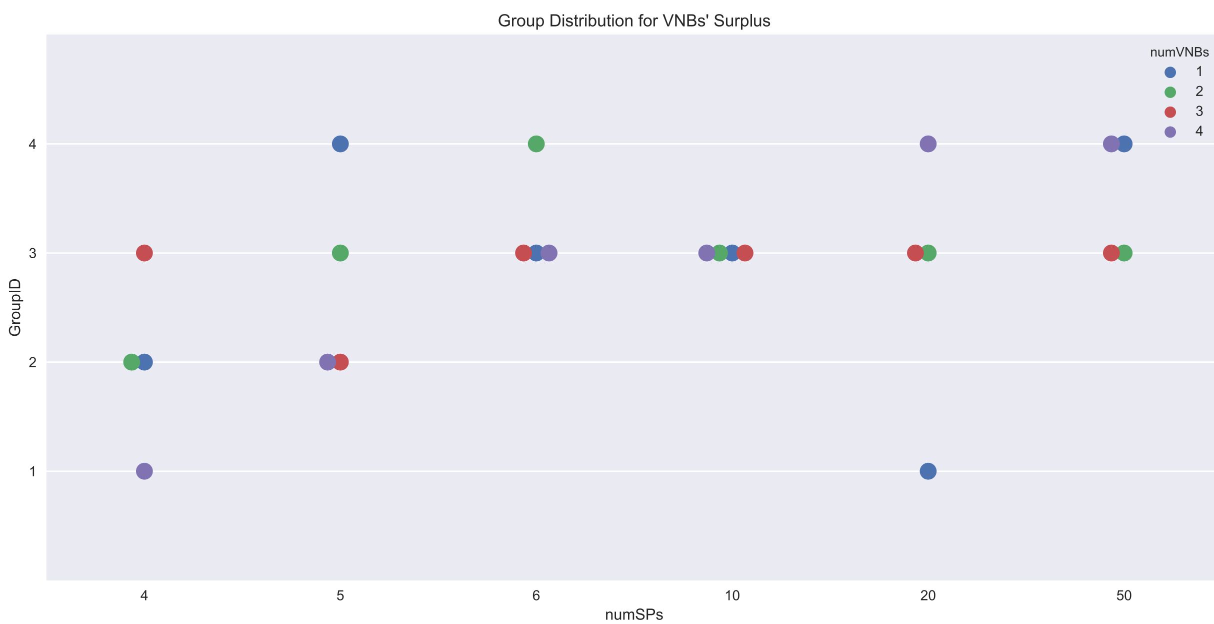


Figure 75: Groups with the highest average VNB surplus. Scenario with 5 RPs

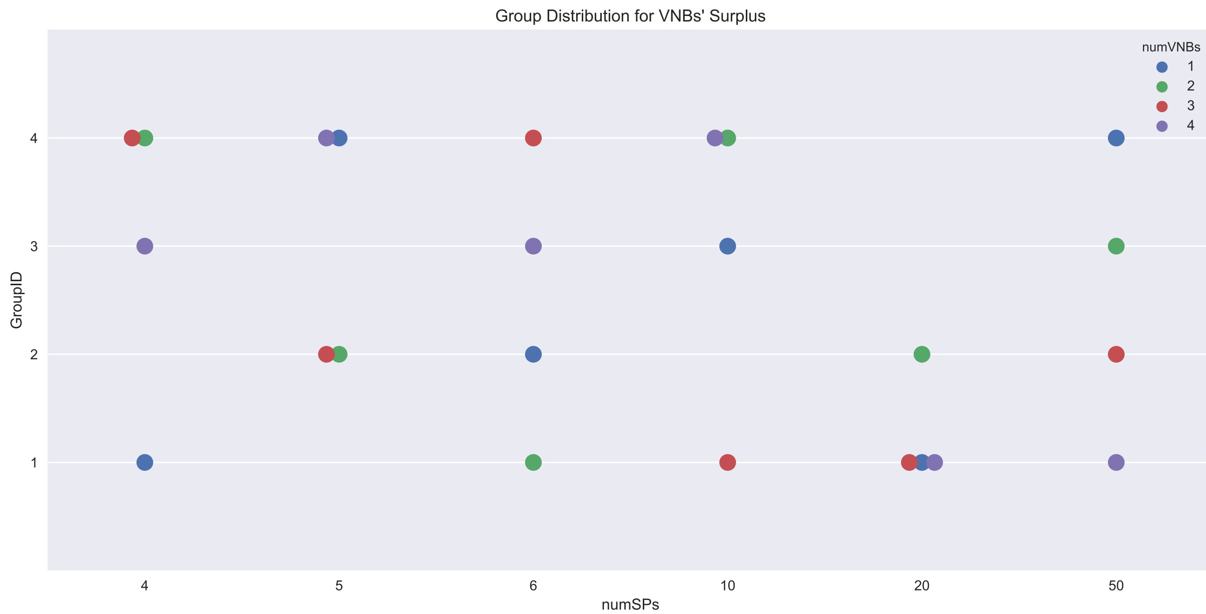


Figure 76: Groups with the highest average VNB surplus. Scenario with 10 RPs

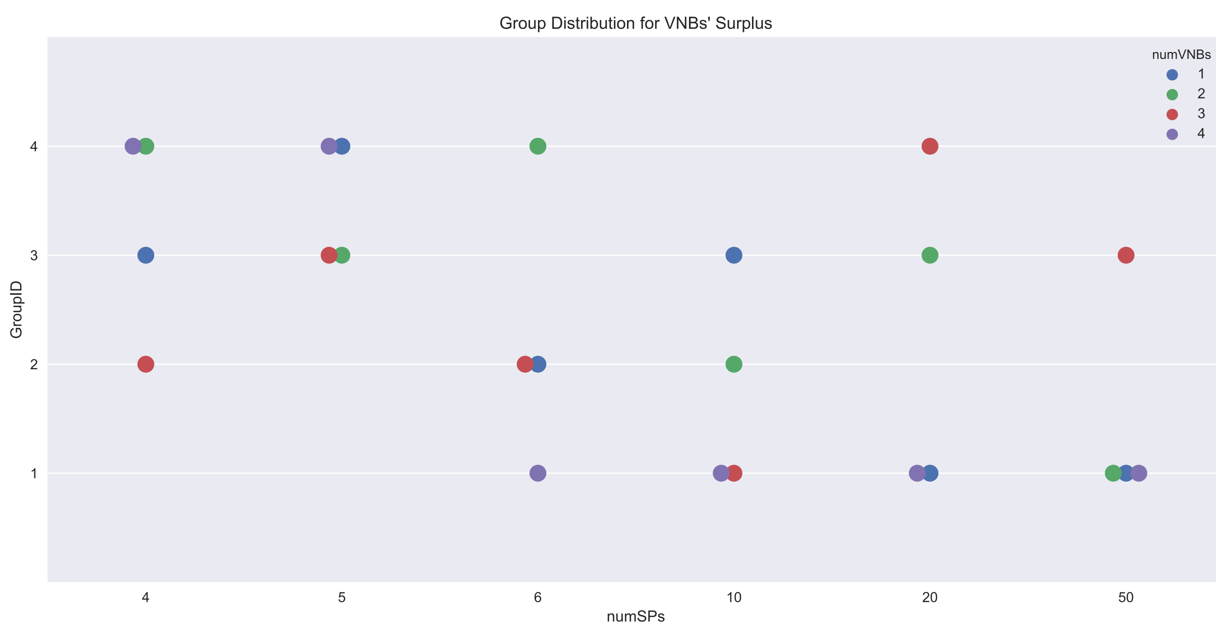


Figure 77: Groups with the highest average VNB surplus. Scenario with 15 RPs

Figure 78: Average supply per resource provider in a scenario with 5 RPs.

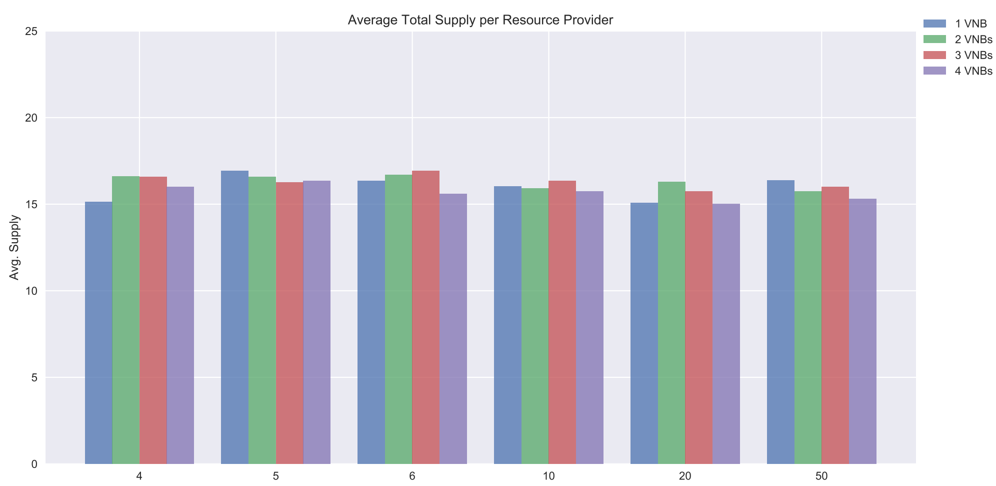


Figure 79: Average supply per resource provider in a scenario with 10 RPs.

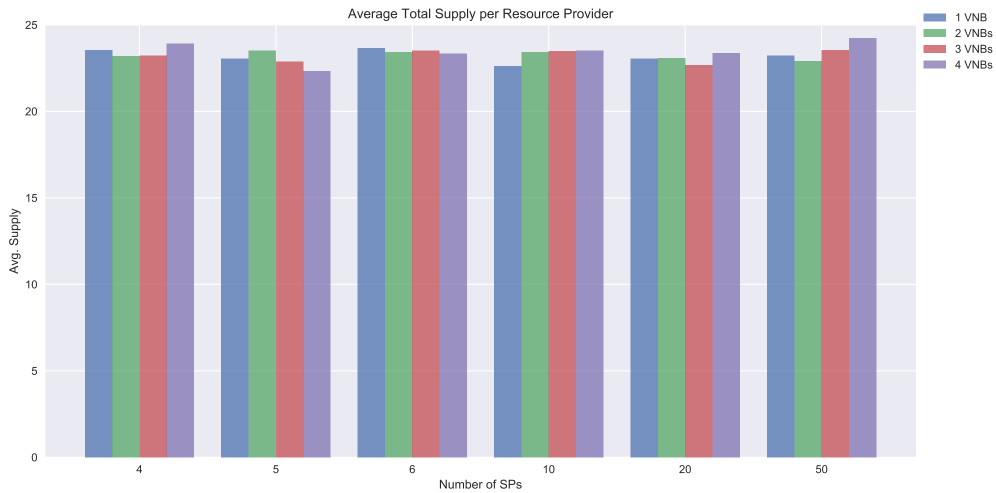


Figure 80: Average supply per resource provider in a scenario with 15 RPs.

vnb_j's utility is greater than the threshold minus sp_i's utility or 2) sp_i's utility is greater than the threshold minus vnb_j's utility. If we were to create preference vectors based on SPs' and VNBs' individual utilities, only one condition would lead to an allowable match: sp_i's utility *and* vnb_j's utility are both higher than the preset threshold. Evidently, the joint utility approach gives us additional flexibility for the definition of allowable matches, thus increasing their amount. This supports hypothesis H9. Defining matches in terms of joint utility has additional implications, which are later discussed.

The surplus results for SPs and RPs are very positive in general, which shows that their market participation is profitable. The situation of the VNBs is different. We showed that, in general, 50% of VNBs will have a profit. The losses associated with the other half stem from the pricing configuration and what leads to a positive surplus. As a reminder, the surplus stems from comparing the price VNBs expect to be paid, or their advertised fee, with the fee they receive if they assign resources. Given that our matching process is currently based on the joint surplus between SPs and VNBs, there may a disparity between prices. We have adjusted the fee that SPs pay to be the average between SPs' and VNBs' advertised fees. This improved our results; however, this does not completely turn all VNBs' profit into positive values.

Delving deeper into the factors leading to these circumstances, we found that VNBs with a higher reputation incur in higher losses than those with a lower reputation. This is directly associated with the prices these entities charge. Higher reputation VNBs are in average matched with a larger number of SPs (see figure 31) and hence, receive a higher payment for their service. This supports our hypothesis H13; nevertheless, high reputation VNBs also experience a larger disparity between the price they expect and the payment they receive. In consequence, VNBs with higher reputation incur in higher losses. What we need in the system to reduce VNBs' losses is for them to advertise lower fees and for SPs to be willing to pay higher prices for the VNB services. We could expect this to happen when the system we propose becomes common place and SPs highly value their participation in it.

It is also important to note that even if scenarios with a higher number of SPs result in VNBs' obtaining a higher overall payment, VNBs' positive surplus is more evident in scenarios with a lower number of SPs. This stems from the fact that small disparities in individual payments received by the VNBs are more evident as the number of customers increases (e.g., a small loss incurred in 50 individual payments is higher than that incurred with only 4 individual payments). In this light, the number of SP customers does influence the losses incurred by VNBs, hence suggesting that it is key for VNBs to define an appropriate set of partners with whom to interact.

From the surplus analysis, we find that VNBs are capturing the risk in the system, while SPs' and RPs' risk is minimized. This is a monetary risk; however, a deeper cost analysis may direct us in more specific types of risk and methods to handle it that may advance the system we propose. For instance, an alternative is for the VNB to charge an additional risk premium in addition to its service fee. This would resemble an insurance type of arrangement, which could account for the losses incurred by the VNBs. An additional approach is for the VNB to operate as a non-profit entity. We believe that VNBs would seek a payment to cover the cost associated with their duties; nevertheless, these costs could be covered with non-monetary payments. One example includes the potential profits stemming from data and information access.

Regarding the amount of resources assigned in the market, we can interpret this factor as a proxy for the success rate of VNBs. In the case of 5 RPs, given that supply levels are low, the performance of VNBs is affected. As supply increases, the effectiveness of VNBs increases as well. We find that the performance of different VNB market structures is fairly stable. As previously mentioned, it appears as VNB monopolies perform better than oligopolies, however, this is only the case because the number of matched SPs in a VNB monopoly is lower. Hence, when we take this factor into account, the actual difference in performance is slim. From a market perspective and future implications this may have, we would prefer there to be competition among VNBs, which could drive their pricing scheme towards market indicators rather than monopolistic practices.

From our experimental group analysis, we find that factors that more clearly depend on the matching process show that group 4 performs best for the factors explored. In this way, we find that when both characteristics (reputation and price for SPs and demand and price for VNBs) are given a higher weight, group 4 performance in the matching process is better. Nevertheless, when analyzing parameters that involve factors outside the matching scope (i.e., resource prices and RPs availability), there is not a definite trend on which group performs best. This provides us with information for testing hypothesis H14. It should be noted; however, that the matching process is what transforms geographical demand into market demand, hence group 4 does have an important impact on the overall process.

10.0 ANALYSIS

In the previous chapter we exposed the results obtained in the different market stages. In this chapter, we take a step further to present what these results imply in the general spectrum markets context, what they signify in terms of our research hypotheses and questions and how they account towards formulating recommendations and guidelines for future markets work.

10.1 HYPOTHESES TESTING AND STATISTICAL VALIDATION

In this section, we present the factors that we have explored in order to test the hypotheses relevant to the different stages of this work. In order to reject the null hypothesis, we have performed a paired or unpaired t-test with the available data. The results that we present in what follows correspond to a 95% confidence interval on the difference between the factors compared for each hypotheses. Note that this test has been applied for the factors that rely on aggregate measures.

In table 32 we show our analysis for hypothesis H1. We have compared the auction cutoff price under perfect and low fungibility conditions for different levels of resource supply. Our results show that for a given set of users, the cutoff price is higher under perfect fungibility conditions. This allows us to reject the null hypothesis (i.e., lower fungibility levels do not affect the auction cutoff price).

Table 33 shows the t-test for hypothesis H2. For testing this hypothesis, we compare the number of bandwidth units assigned under perfect and low fungibility conditions, taking also into account undersupply and oversupply scenarios.

Table 32: T-test results and analysis for hypothesis H1

Relevant Factors		T-test Result	Conclusion
Average cutoff price under perfect fungibility with R = 5 and SUs = 20	Average cutoff price under lowest fungibility with R = 5 and SUs = 20	15.67681 ± 0.04376	The average cutoff price is higher under perfect fungibility conditions, when R = 5 and SUs=20
Average cutoff price under perfect fungibility with R=10 and SUs=20	Average cutoff price under lowest fungibility with R=10 and SUs=20	4.18864 ± 0.01804	The average cutoff price is higher under perfect fungibility conditions, when R = 10 and SUs=20
Average cutoff price under perfect fungibility with R=15 and SUs=20	Average cutoff price under lowest fungibility R=15 and SUs=20	0.11331 ± 0.004602	The average cutoff price is slightly higher under perfect fungibility conditions, when R = 15 and SUs=20

The results show that in scenarios where the supply is low or normal, more BBUs are assigned when resources are perfectly fungible. In oversupply conditions, more BBUs are assigned in lower fungibility scenarios. However, it is important to note that the difference between perfect and low fungibility cases is small.

A larger difference between factors results when comparing supply conditions under a fixed fungibility setting. In these cases, we find that the assignment of spectrum units attends to supply and demand conditions, i.e., more BBUs are assigned when supply is larger (e.g., R = 15) and when demand increases (e.g., SUs = 20).

With these results, we can conclude that the resource assignment process responds in a larger degree to demand and supply conditions, rather than fungibility levels.

Table 33: T-test results and analysis for hypothesis H2

Relevant Factors		**T-test Result**	**Conclusion**
BBUs assigned under perfect fungibility with R = 5 and SUs=20	BBUs assigned under lowest fungibility with R = 5 and SUs=20	1.2119 ± 0.06919	More BBUs are assigned under perfect fungibility than lower fungibility conditions when R = 5 and SUs = 20
BBUs assigned under perfect fungibility with R = 10 and SUs=20	BBUs assigned under lowest fungibility with R = 10 and SUs = 20	0.829985 ± 0.1240	More BBUs are assigned under perfect fungibility than lower fungibility conditions when R = 10 and SUs=20

Table 33: (continued)

Relevant Factors		T-test Result	Conclusion
BBUs assigned under perfect fungibility with R = 15 and SUs = 20	BBUs assigned under lowest fungibility with R = 15 and SUs = 20	-1.62911 ± 0.239154	More BBUs are assigned under lowest fungibility than perfect fungibility conditions when R = 15 and SUs = 20
BBUs assigned under perfect fungibility with R = 15 and SUs=20	BBUs assigned under perfect fungibility with R = 5 and SUs=20	101.59935 ± 0.1945	In perfect fungibility conditions for 20SUs, more BBUs are assigned with R = 15 than with R = 5 (i.e., spectrum oversupply)
BBUs assigned under lowest fungibility with R = 15 and SUs=20	BBUs assigned under lowest fungibility with R = 5 and SUs = 20	104.44035 ± 0.1554	In the lowest fungibility scenario for 20SUs, more BBUs are assigned with R = 15 than with R = 5 (i.e., spectrum oversupply)
BBUs assigned under lowest fungibility with R = 15 and SUs=20	BBUs assigned under lowest fungibility R = 15, SUs = 4	157.63888 ± 0.15764	In the lowest fungibility scenario and with R = 15, more BBUs are assigned with 20SUs than with 4 SUs (i.e., higher spectrum demand)

Table 33: (continued)

Relevant Factors		T-test Result	Conclusion
BBUs assigned under perfect fungibility with R = 15 and SUs=20	BBUs assigned under perfect fungibility with R = 15 and SUs = 4	156.62044 ± 0.19483	In the perfect fungibility scenario and with R = 15, more BBUs are assigned with 20SUs than with 4SUs (i.e., higher spectrum demand)
BBUs assigned under lowest fungibility with R = 5 and SUs = 20	BBUs assigned under lowest fungibility with R = 5 and SUs = 4	72.57979±0.06402	In lowest fungibility conditions and with R = 5, more BBUs are assigned with 20SUs than with 4SUs (i.e, higher spectrum demand)
BBUs assigned under perfect fungibility with R = 5 and SUs=20	BBUs assigned under perfect fungibility with R = 5 and SUs = 4	73.29465±0.05326	In perfect fungibility conditions for R = 5, more BBUs are assigned with 20SUs than with 4 SUs (i.e., higher spectrum demand)

For hypothesis H4 we have compared the average number of BBUs and AT units assigned under different levels of demand and supply. In the scenario we analyze, AT units correspond to TVWS spectrum in a similar frequency band as BBUs. Our t-test shows that BBUs oversupply results in more BBUs assigned, and BBUs undersupply results in more AT units assigned, irrespective of the demand level. There is correspondence between supply and the price of resources. Indeed, in spectrum oversupply conditions, the price of resources tends to be low, whereas in undersupply conditions, the BBU price is significantly higher.

It should be noted that in our model, we consider users with different levels of valuation for BBUs in the market. Additionally, users' valuation for BBUs is inversely proportional to that of AT units. In this manner, our results also suggest that in cases where the price of spectrum is too high, users with a lower (available) spectrum valuation opt for their higher priced AT units. This shows that resource assignment, whether it is BBUs or AT units, depends on BBUs availability and users' resource valuation.

The specific t-test for hypothesis H4 is included in Table 34.

The t-test for hypothesis H7 is presented in Table 35. In this case, we compare the level of resource supply in the market in scenarios with different levels of RPs' participation. Our results show that a higher number of RPs in the market represents a larger market supply. In our model, RPs decide on what percentage of their available resources to make available in the market. Irrespective of the actual percentage, our results of this test show that higher RP participation is linked to a higher amount of resources available in the market, which allows us to reject the null hypothesis (i.e., the investment level of RPs is not proportional to the amount of resources available in the market.)

For hypothesis H13 we explore the effect of VNBs' reputation on the number of SP customers that each VNB has and the payment VNBs receive. Our results show that both parameters are larger for VNBs with a higher reputation. These results, which are included in Table 36 permit us to reject the null hypothesis (i.e., the reputation of middlemen is not proportional to the total number of matches of a VNB, and to the payment obtained from resource aggregation services).

In hypothesis H14 we explore results relevant to the groups defined in our experiment for the third market stage. In the case of VNBs, we find that the average payment received by Group 4 VNBs is higher than the payment received by Group 1 VNBs. However, in the case of surplus, Group 1 VNBs perform better than Group 4 VNBs. In the case of SPs, their average surplus stemming from resource and VNB payments is slightly higher for the members of group 4. These results are included in Table 37.

In the particular case of surplus, there are additional factors that influence its calculation,

Table 34: T-test results and analysis for hypothesis H4

Relevant Factors		T-test Result	Conclusion
Average BBUs assigned with R=15 and SUs=4 (lower cutoff price)	Average BBUs assigned with R=5 and SUs=4 (higher cutoff price)	23.00453 ± 0.04868	In a 4SUs scenario, more BBUs are assigned (in average) with R = 15 than with R = 5 (i.e., lower cutoff price conditions)
Average AT units assigned with R = 15 and SUs = 4 (lower cutoff price)	Average AT units assigned with R = 5 and SUs = 4 (higher cutoff price)	-1.30349 ± 0.0074316	In a 4SUs scenario, more AT units are assigned (in average) with R = 5 than with R = 15 (i.e., higher cutoff price conditions)
Average BBUs assigned with R = 15 and SUs = 10 (lower cutoff price)	Average BBUs assigned with R=5 and SUs=10 (higher cutoff price)	76.6528 ± 0.10531	In a 10SUs scenario, more BBUs are assigned (in average) with R = 15 than with R = 5 (i.e., lower cutoff price conditions)
Average AT units assigned with R=15 and SUs=10 (lower cutoff price)	Average AT units assigned with R = 5 and SUs = 10 (higher cutoff price)	-1.3926 ± 0.00653	In a 10SUs scenario, more AT units are assigned (in average) with R=5 than with R = 15 (i.e., higher cutoff price conditions)

Table 35: T-test results and analysis for hypothesis H7

Relevant Factors		T-test Result	Conclusion
Average supply with 15 RPs	Average supply with 5 RPs	20.76535±0.18857	The average supply with 15 RPs is greater than the supply with 5 RPs
Average supply with 10 RPs	Average supply with 5 RPs	11.83847±0.19433	The average supply with 10 RPs is greater than the supply with 5 RPs

Table 36: T-test results and analysis for hypothesis H13

Relevant Factors		T-test Result	Conclusion
Average number of customers of VNBs with high reputation	Average number of customers of VNBs with low reputation	8.02057 ± 0.35153	The average number of customers of high reputation VNBs is greater than that of low reputation VNBs
Average payment received by high reputation VNBs	Average payment received by low reputation VNBs	299.95348 ± 19.0905	The average payment received by high reputation VNBs is greater than the payment received by low reputation VNBs

which are not related to the weights assigned to matching parameters. This justifies the fact that surplus-related outcomes do not allow us to completely reject the null hypothesis. Nevertheless, for all other parameters, including those that do not correspond to aggregate measures, group 4 members outperform those of group 1.

10.2 SENSITIVITY ANALYSIS

The models defined for the different stages account for vast possibilities in terms of parameter combinations. The combination that we have deemed essential has been to vary the number of market participants and resource availability. This permitted us to explore how supply and demand variations influence the market results. The results stemming from this analysis have already been explored through our hypotheses and presented in our results section.

To account for further parameters that may influence the outcome of our model, we performed an additional sensitivity analysis, regarding the threshold utilized for establishing acceptable matches. As presented in section 8.3.1.9, the joint utility threshold, which defines acceptable matches was set as the middle point in the utility range: $[0.4 - 4]$, or 1.8. In this section, we explore the results obtained when considering two additional thresholds given by (10.1) and (10.2).

$$\text{Lower threshold} = 0.25 \times \text{utility range} = 0.9 \tag{10.1}$$

$$\text{Higher threshold} = 0.75 \times \text{utility range} = 2.7 \tag{10.2}$$

10.2.1 SP - VNB Matching Performance

In this subsection we explore the results obtained regarding the percentage of matched SPs and the average number of customers per VNB.

Table 37: T-test results and analysis for hypothesis H14

Relevant Factors		T-test Result	Conclusion
Average payment received by Group 4 VNBs - 15 RPs	Average payment received by Group 1 VNBs - 15 RPs	537.3233 ± 23.5892	The average payment received by Group 4 VNBs is greater than the payment received by Group 1 VNBs
Average Group 4 VNBs' Surplus - 15 RPs	Average Group 1VNBs' Surplus - 15 RPs	-14.3487 ± 4.7907	The average surplus of Group 1 VNBs is greater than the surplus of Group 4 VNBs
Average Group 4 SPs' surplus (from VNB payment)	Average Group 1 SPs' surplus (from VNB payment)	0.66187 ± 0.08282	The average surplus (from VNB payment) of Group 4 SPs is greater than the surplus of Group 1 SPs
Average Group 4 SPs' surplus (from resource payment)	Average Group 1 SPs' surplus (from resource payment)	0.79646 ± 0.0035	The average surplus (from resource payment) of Group 4 SPs is greater than the surplus of Group 1 SPs

10.2.1.1 Percentage of matched SPs: Lower thresholds imply a larger range of allowable matches. In other words, the matching process is less strict and permits matches between VNBs and SPs whose utilities are fairly low. In this manner, as it is presented in figure 81, the number of SPs that are matched is significantly higher when we utilize a lower threshold.

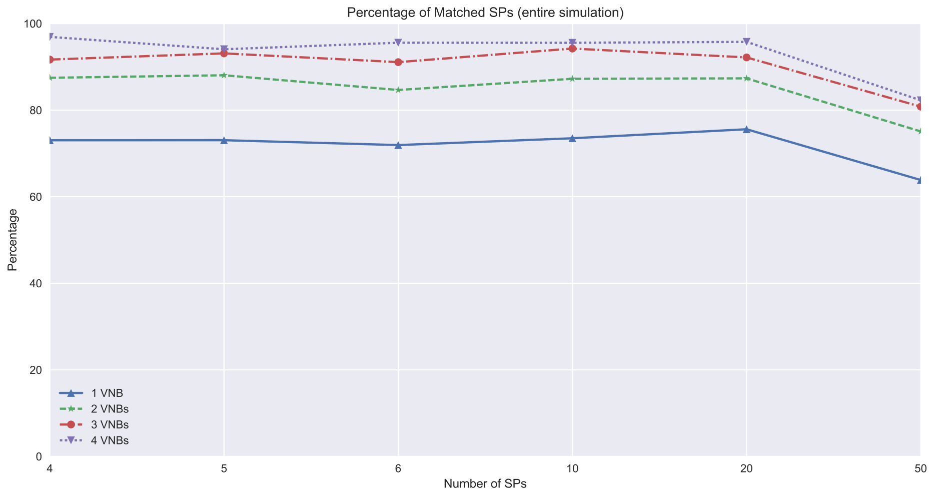
10.2.1.2 Average number of customers per VNB: Since there is a larger number of matched SPs in scenarios with lower thresholds, it follows that, on average, each VNB forms partnerships with a larger number of SPs. As expected, when we consider a higher utility threshold, we can observe lower number of partners per VNB. These results can be observed in figures 82, 83, 84 and 85, where we consider scenarios with different VNB configurations.

10.2.2 Percentage of demand satisfied in the market

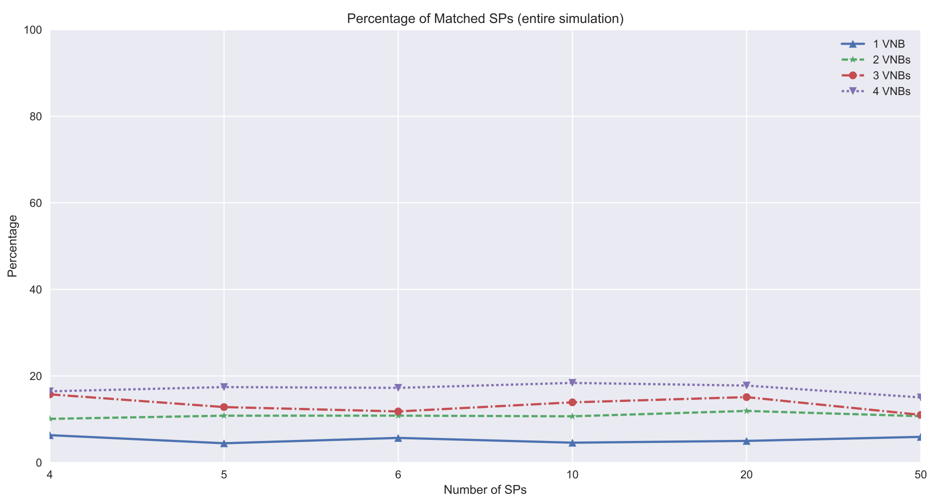
In this section, we present the percentage of matched SPs that have obtained resources from the market when considering different RP configurations (i.e., 5, 10 and 15 RPs), and the aforementioned lower and higher thresholds. As shown in figures 86, 87 and 88, lower thresholds signify lower levels of demand satisfaction. Note that lower thresholds result in higher market demand, hence, if we keep the supply levels constant, an increase in demand implies that a larger number of SPs may not obtain resources in the market. In other words, lower thresholds generate undersupply conditions. Additionally, it is important to point out that the percentage shown for each threshold is calculated over the entire set of matched SPs for each threshold level. In this way, the percentage of demand satisfied in the market for the lower threshold is calculated over a larger SP population than that of the higher threshold.

10.2.3 Percentage of resources assigned by RPs

The results presented in figures 89, 90 and 91, show how the percentage of resources assigned by the RPs varies according to the utility threshold. As expected, in scenarios with lower demand (i.e., higher thresholds), RPs assign a lower percentage of their resources.

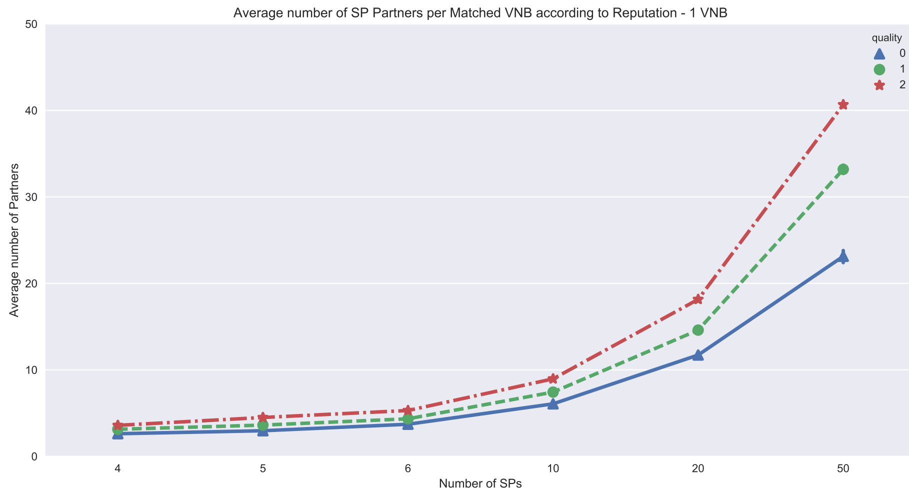


(a) Lower threshold results

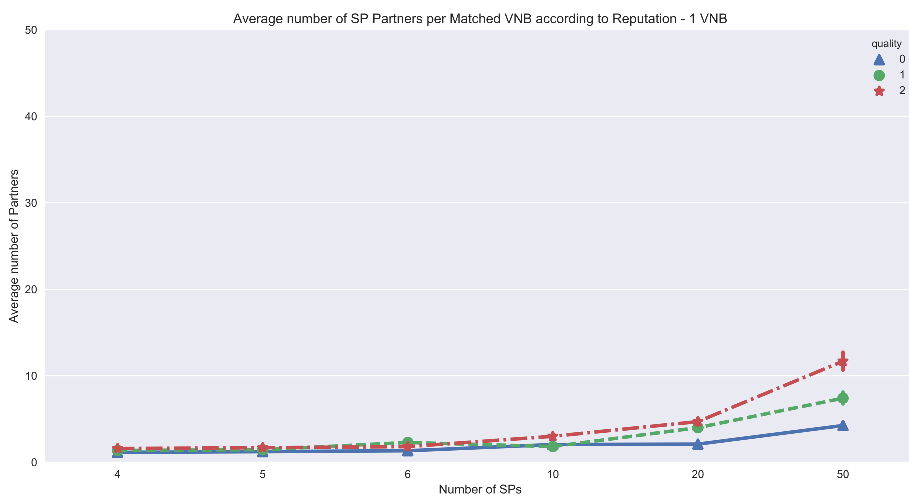


(b) Higher threshold results

Figure 81: Percentage of matched SPs in the entire simulation with varying number of VNBs and 10 RPs

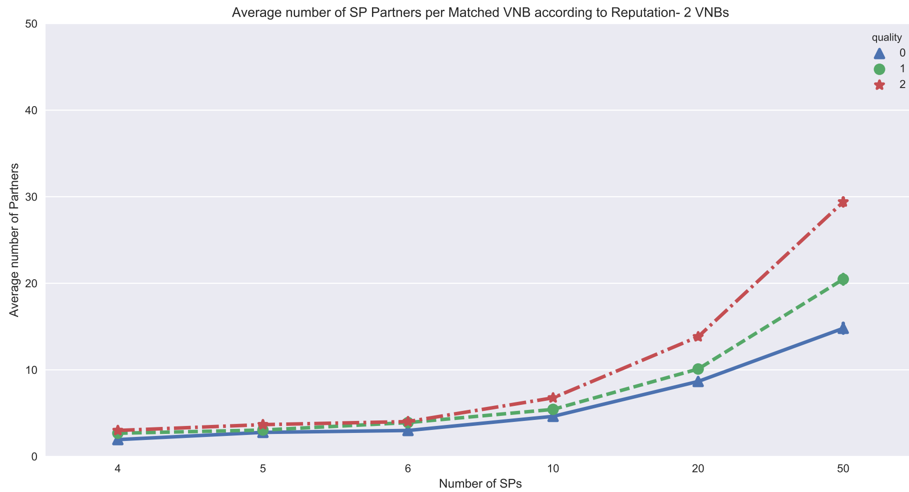


(a) Lower threshold results

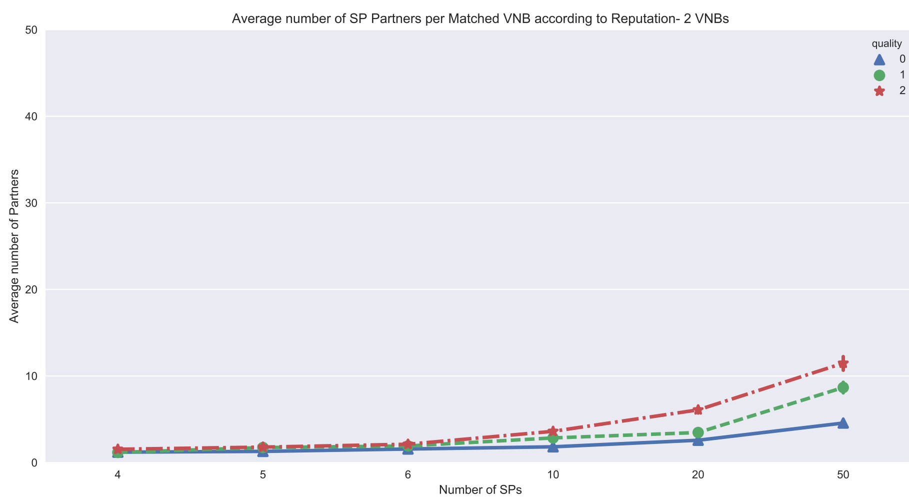


(b) Higher threshold results

Figure 82: Average number of SP customers per VNB in a scenario with 1 VNB and 10 RPs

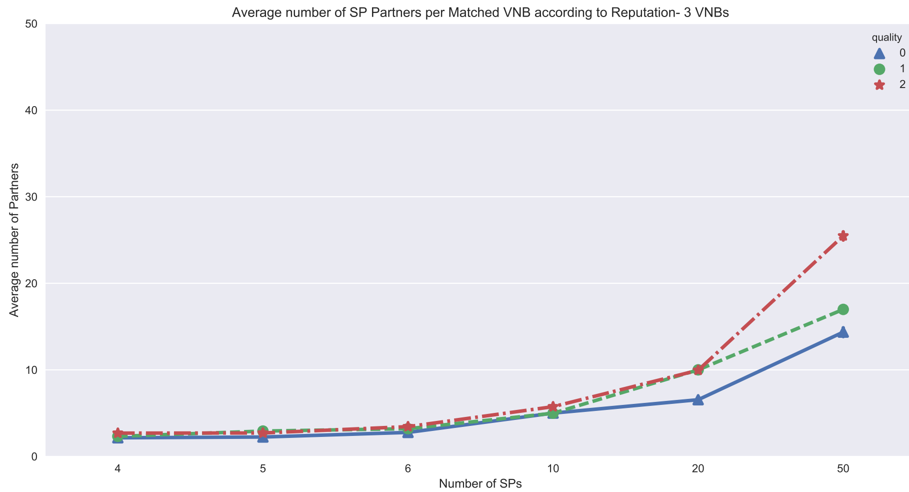


(a) Results with lower threshold

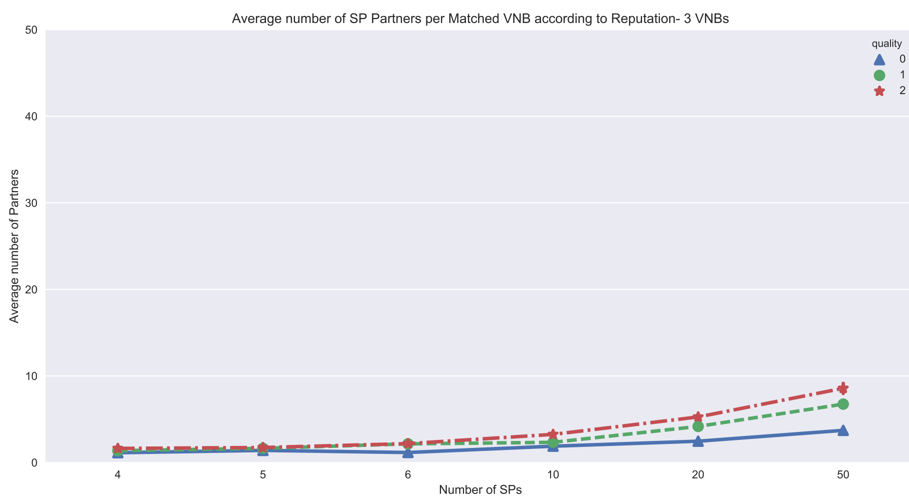


(b) Results with higher threshold

Figure 83: Average number of SP customers per VNB in a scenario with 2 VNBs and 10 RPs

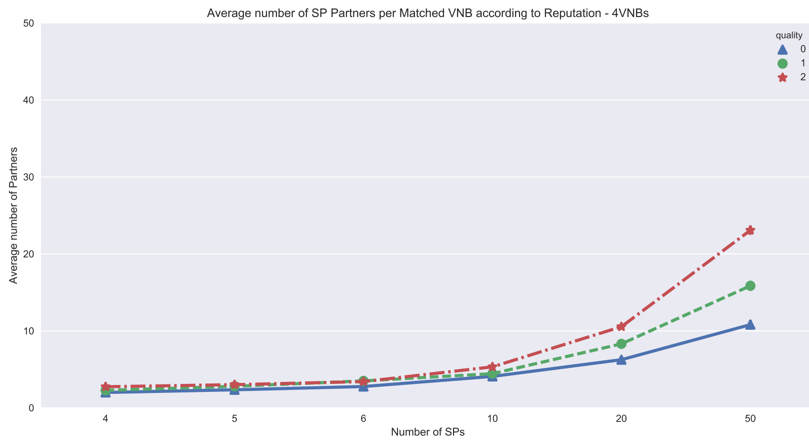


(a) Lower threshold results

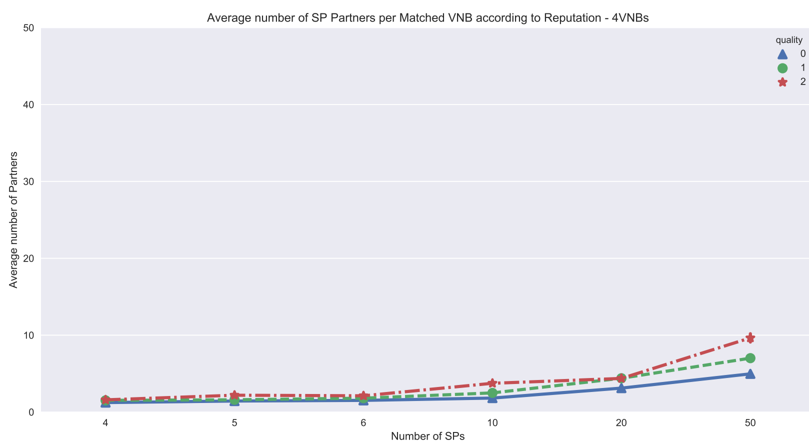


(b) Higher threshold results

Figure 84: Average number of SP customers per VNB in a scenario with 3 VNBs and 10 RPs.

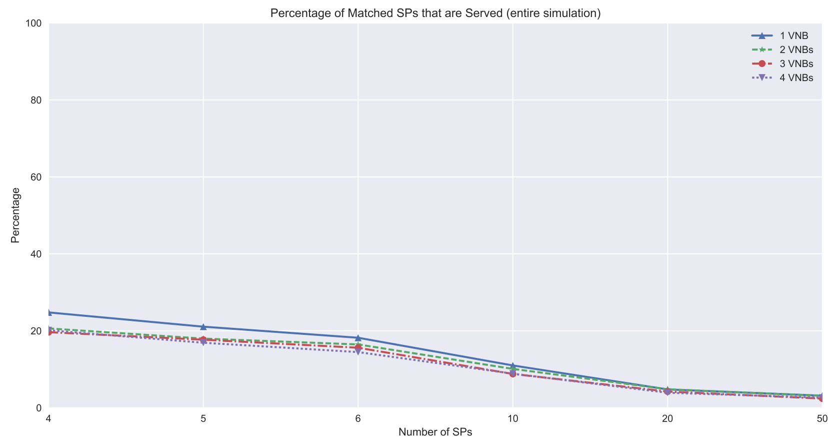


(a) Lower threshold results

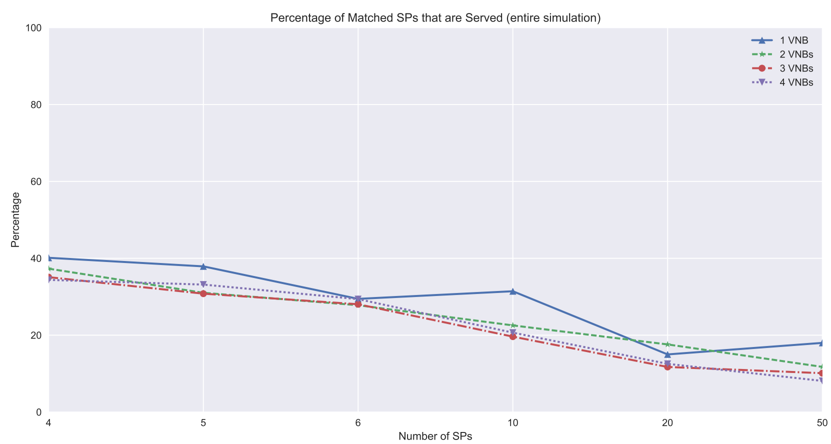


(b) Higher threshold results

Figure 85: Average number of SP customers per VNB in a scenario with 4 VNBs and 10 RPs



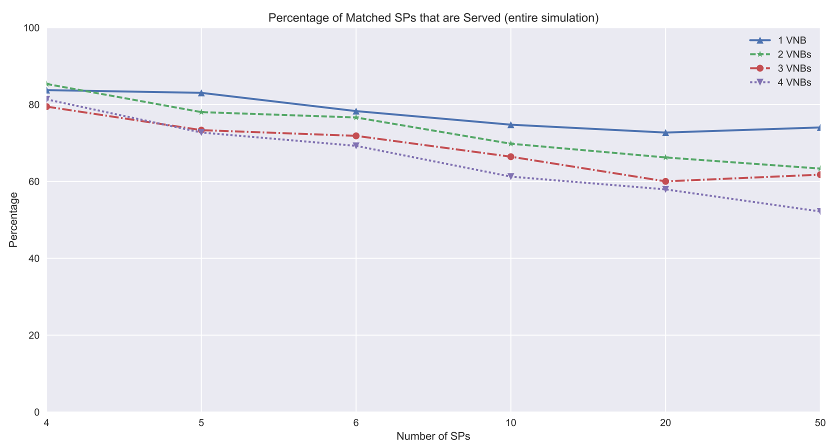
(a) Lower threshold



(b) Higher threshold

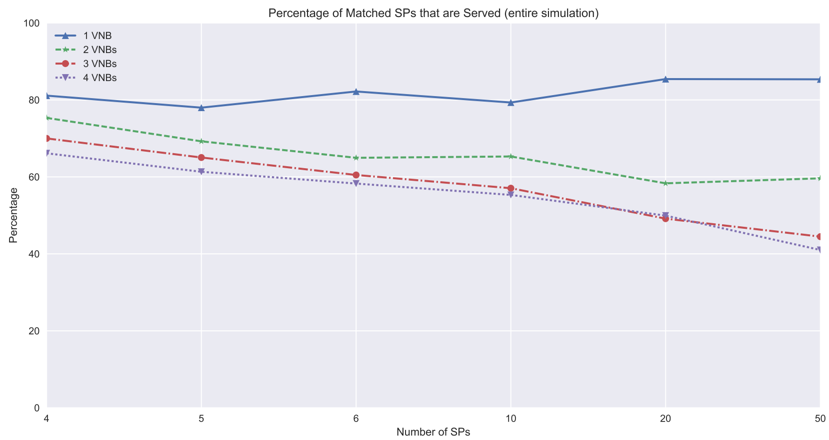
Figure 86: Percentage of matched SPs that obtained resources from the market. Scenario with 5 RPs

(a) Lower threshold

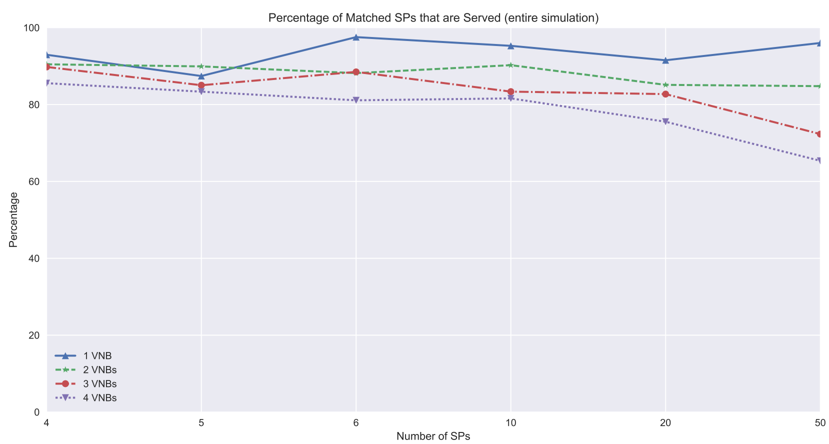


(b) Higher threshold

Figure 87: Percentage of matched SPs that obtained resources from the market. Scenario with 10 RPs



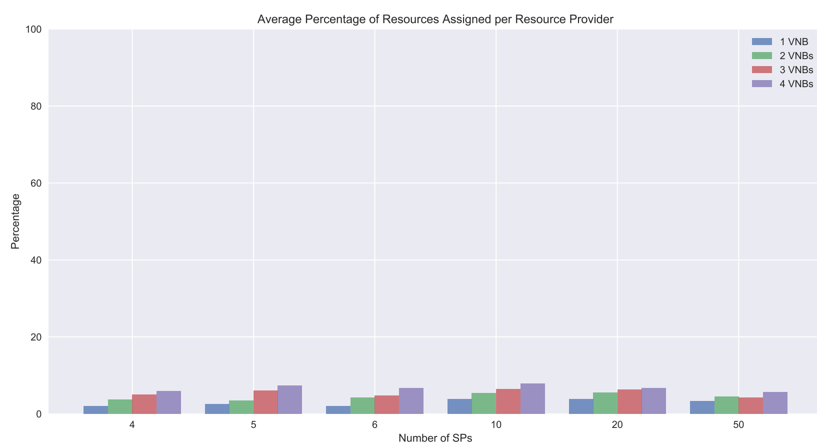
(a) Lower threshold



(b) Higher threshold

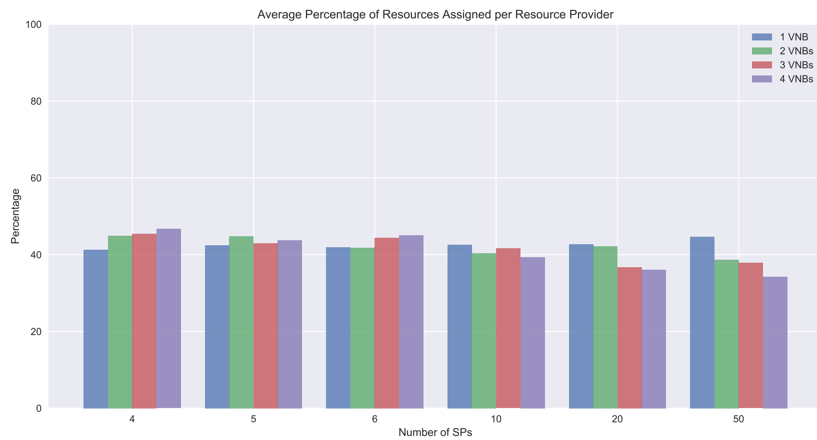
Figure 88: Percentage of matched SPs that obtained resources from the market. Scenario with 15 RPs

(a) Lower threshold



(b) Higher threshold

Figure 89: Percentage of resources assigned by each RP. Scenario with 5 RPs



(a) Lower threshold

(b) Higher threshold

Figure 90: Percentage of resources assigned by each RP. Scenario with 10 RPs

(a) Lower threshold

(b) Higher threshold

Figure 91: Percentage of resources assigned by each RP. Scenario with 15 RPs

10.2.4 Payments

In this section we explore how the payments received by RPs and VNBs change with the threshold variation.

10.2.4.1 Payments received by RPs The payments received by the RPs are consistent with the market demand. In this way, in scenarios with a larger number of matched SPs (i.e., lower utility threshold), RPs' opportunities to assign resources increase, and hence the payments they receive. Figures 92, 93 and 94 show the average payment received by the resource providers in scenarios with 5, 10 and 15 RPs, respectively.

10.2.4.2 Payments received by VNBs From a VNB perspective, more partners represent higher revenue opportunities. In figures 95, 96, 97 and 98, we show that under lower threshold configurations (i.e., more partners per VNB), VNBs' aggregate payments increase. Note that these figures reflect the average payment received by each VNB that has assigned resources to its customers in a scenario with 10 RPs.

10.2.5 Surplus

In this section we analyze the variations on the surplus perceived by each entity from the payments it makes or receives.

10.2.5.1 RPs' Surplus To illustrate how our sensitivity analysis impacts the RPs' surplus, in figures 99, 100, 101 and 102, we show the results obtained for a scenario with 10 RPs and each of the VNB market configurations that we have considered. As it can be observed, there is not a significant difference between these results and those presented in section 9.3. Indeed, the RPs' surplus remains positive for all the cases we have analyzed.

10.2.5.2 VNBs' Surplus In figures 103, 104, 105 and 106, we present how varying the utility thresholds impact the VNBs' surplus. In the same manner as presented in section 9.3,

(a) Lower threshold

(b) Higher threshold

Figure 92: Average payment received by the resource providers. Scenario with 5 RPs

(a) Lower threshold

(b) Higher threshold

Figure 93: Average payment received by the resource providers. Scenario with 10 RPs

(a) Lower threshold

(b) Higher threshold

Figure 94: Average payment received by the resource providers. Scenario with 15 RPs

(a) Lower threshold results

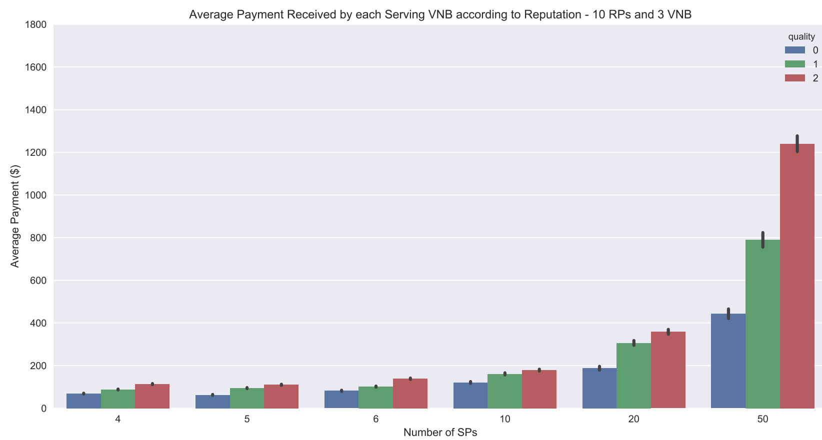
(b) Higher threshold results

Figure 95: Average payment received by VNBs in a scenario with 1 VNB and 10 RPs

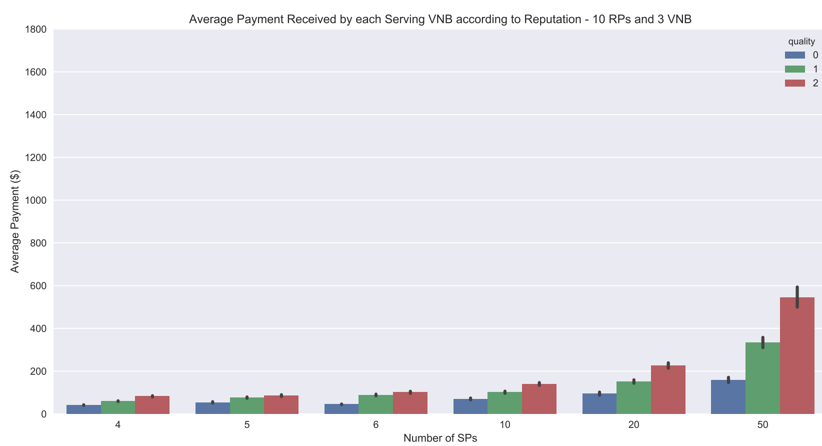
(a) Results with lower threshold

(b) Results with higher threshold

Figure 96: Average payment received by VNBs in a scenario with 2 VNBs and 10 RPs

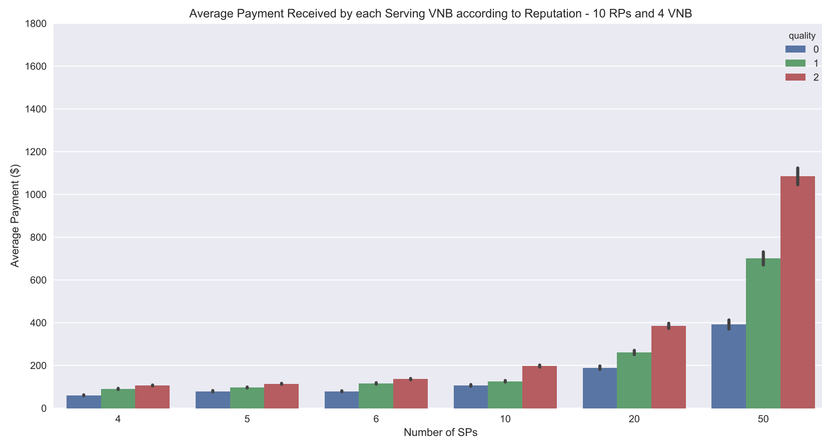


(a) Lower threshold results

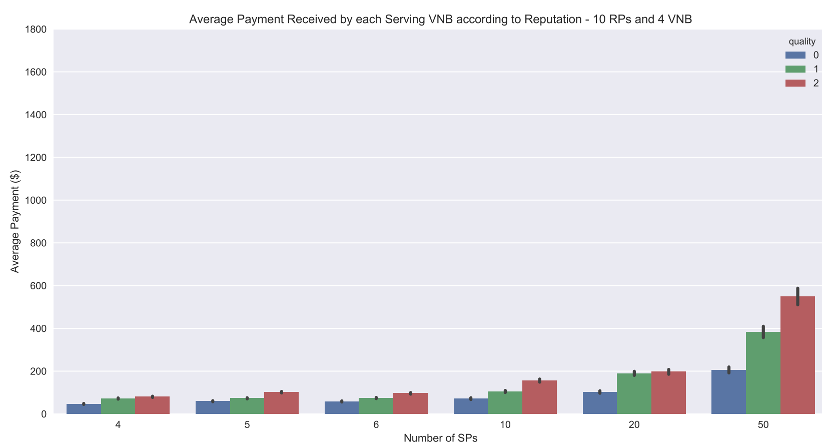


(b) Higher threshold results

Figure 97: Average payment received by VNBs in a scenario with 3 VNBs and 10 RPs.

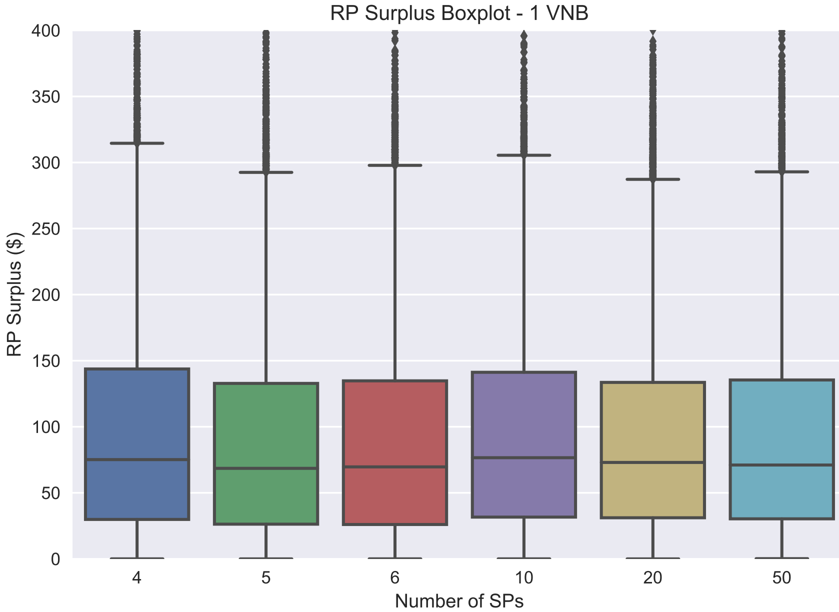


(a) Lower threshold results

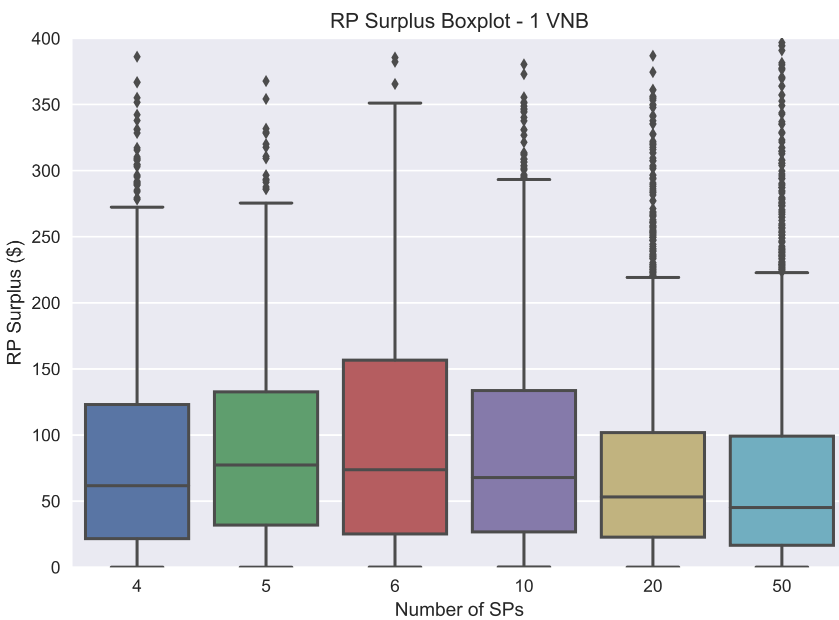


(b) Higher threshold results

Figure 98: Average payment received by VNBs in a scenario with 4 VNBs and 10 RPs

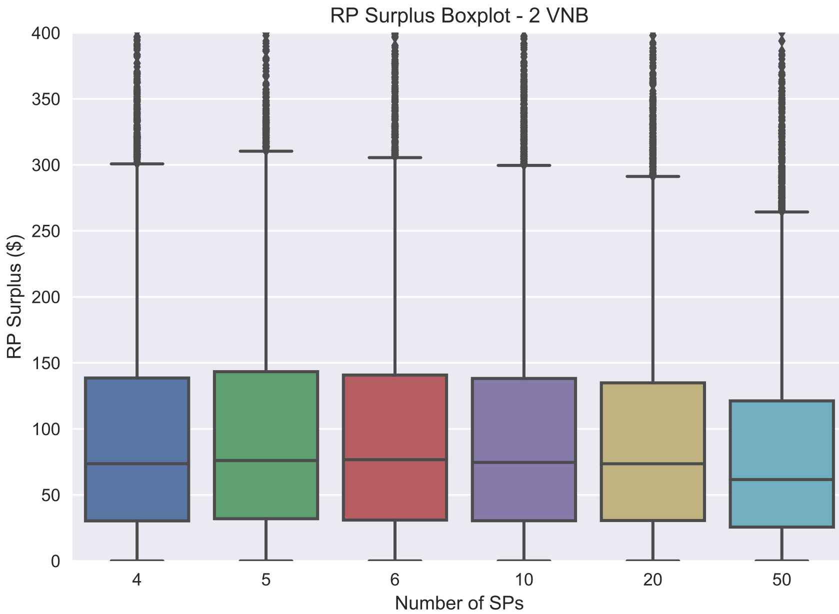


(a) Lower threshold results

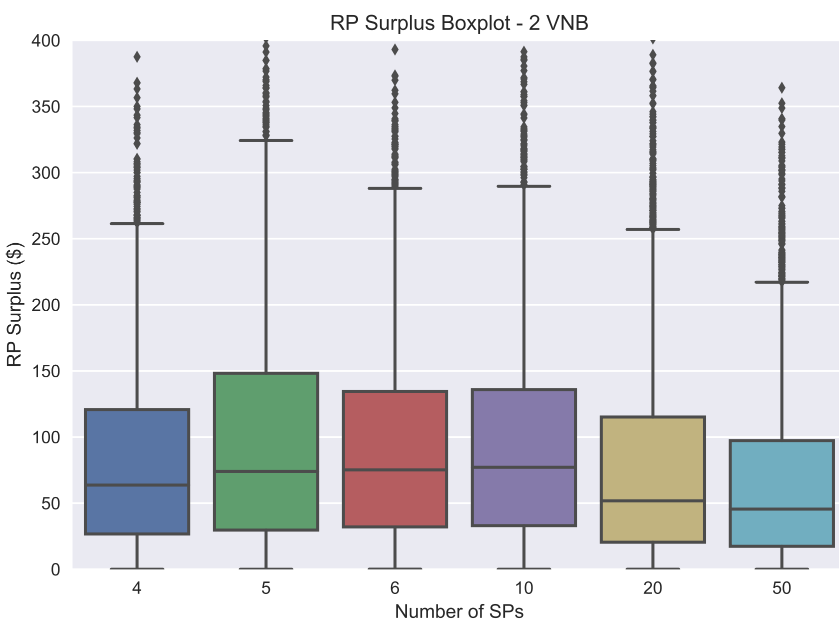


(b) Higher threshold results

Figure 99: RPs' surplus in a scenario with 1 VNB and 10 RPs

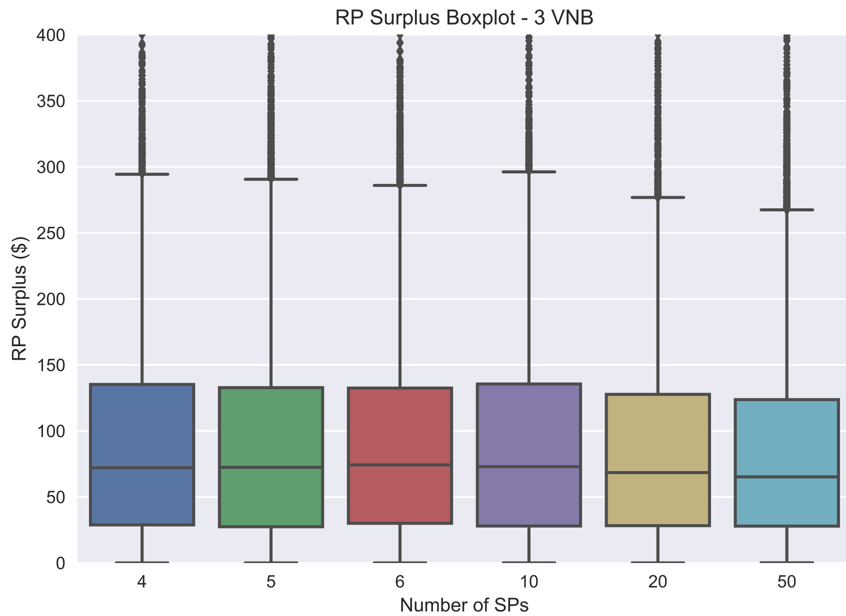


(a) Results with lower threshold

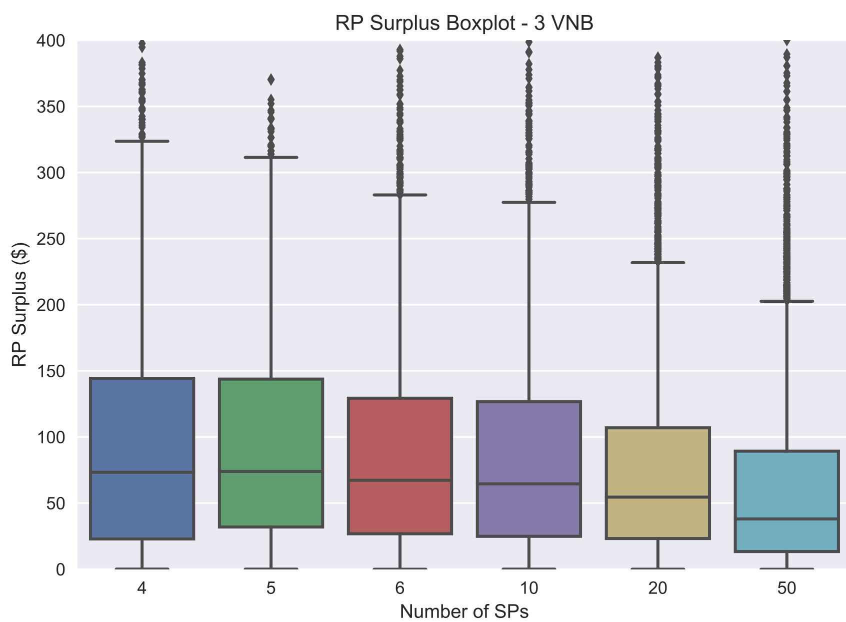


(b) Results with higher threshold

Figure 100: RPs' surplus in a scenario with 2 VNBs and 10 RPs

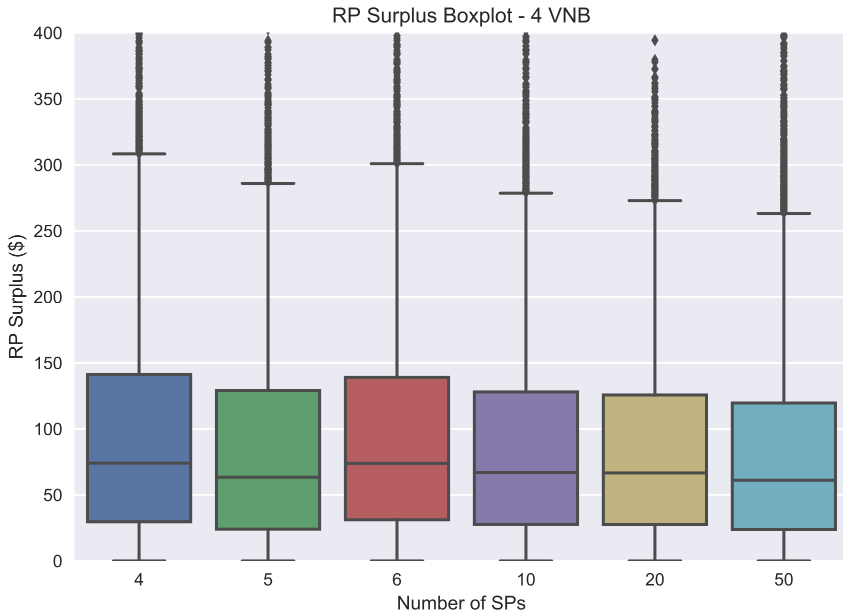


(a) Lower threshold results

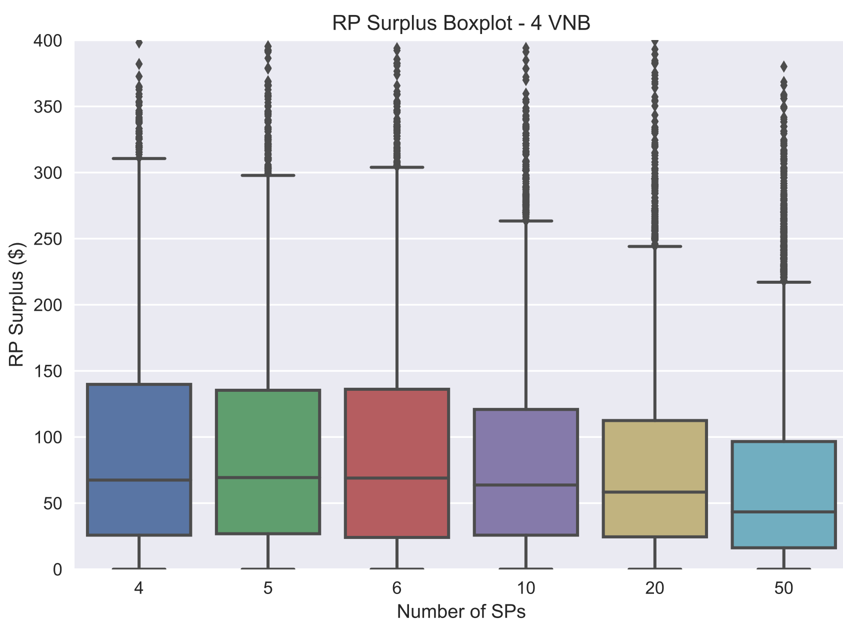


(b) Higher threshold results

Figure 101: RPs' surplus in a scenario with 3 VNBs and 10 RPs.



(a) Lower threshold results



(b) Higher threshold results

Figure 102: RPs' surplus in a scenario with 4 VNBs and 10 RPs

a larger number of SP customers implies greater gains, but given our joint utility configuration, this may also increase losses. This is corroborated with the results presented herein. Indeed, higher thresholds, i.e., less SP customers or partners, positively impacts the surplus distribution. These results suggests that, in terms of surplus, it is more profitable for VNBs to place more stringent conditions for their partner selection process.

10.2.5.3 SPs' Surplus There are two instances where SPs should calculate their surplus 1) from the amount they pay for resources, and 2) from the fee they pay for the VNB services.

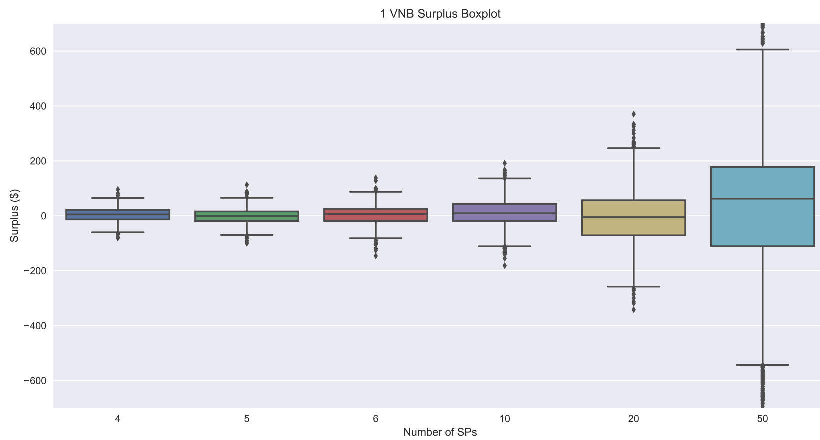
Figures 107, 108, 109 and 110 show the surplus distribution that stems from the SPs' resource payment. These figures represent a scenario with 10 RPs and different VNB configurations. As it can be observed, SPs generally do not make a profit from their resource payment, nor do they incur in losses. This is consistent with the results presented in section 9.3.

In figures 111, 112, 113 and 114, we present the surplus resulting from the VNB fee payment of those SPs who did obtain resources in the market. As it can be observed, the distribution in the two threshold levels are fairly similar between them, and they are also similar to the results presented in section 9.3.

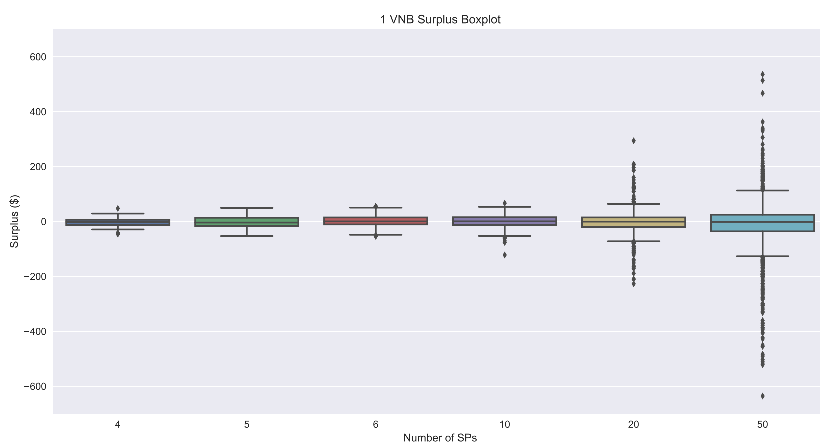
10.3 THINKING OUTSIDE THE BOX

The work presented in this dissertation has been designed with the objective of determining what we need in order to deploy successful spectrum markets. Nevertheless, it is also important to explore other axes of applicability of the model we propose, especially targeting areas of current technical interest.

At this moment, efforts are focused on the implementation of 5G technologies and the Internet of Things, which has found as one of its broader-impact applications the development of Smart Cities. In these technical settings, we may significantly benefit from sharing resources in smaller areas, as the frequencies of interest seem to be in the GHz range. In this manner, we may find 5G or IoT infrastructure owned by multiple parties, located in close

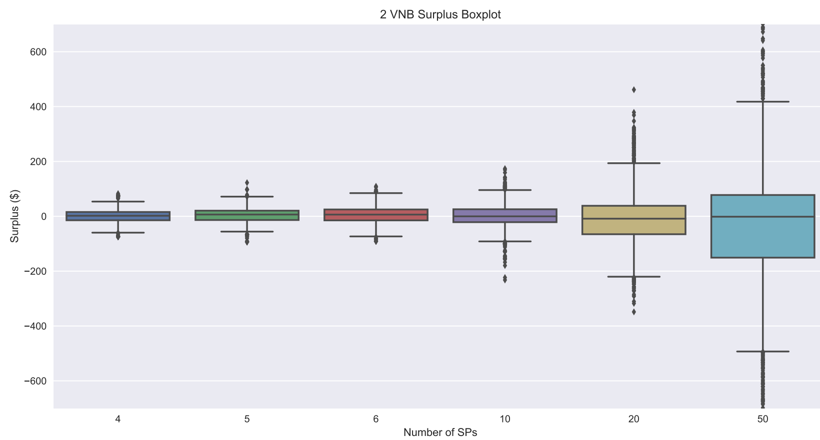


(a) Lower threshold results

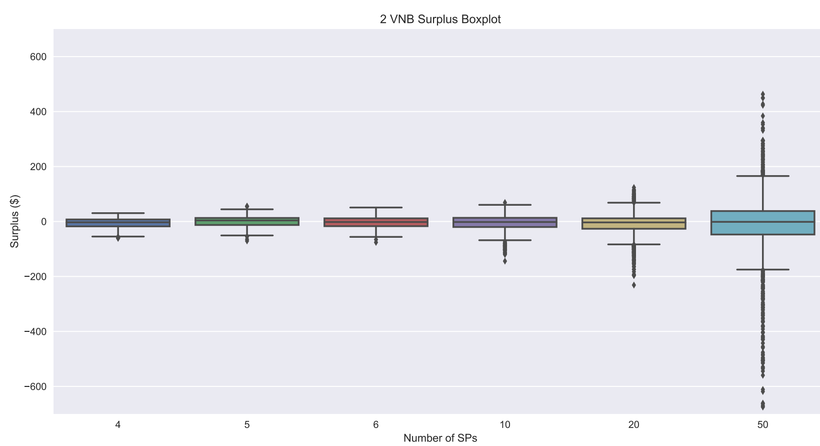


(b) Higher threshold results

Figure 103: RPs' surplus in a scenario with 1 VNB and 10 RPs

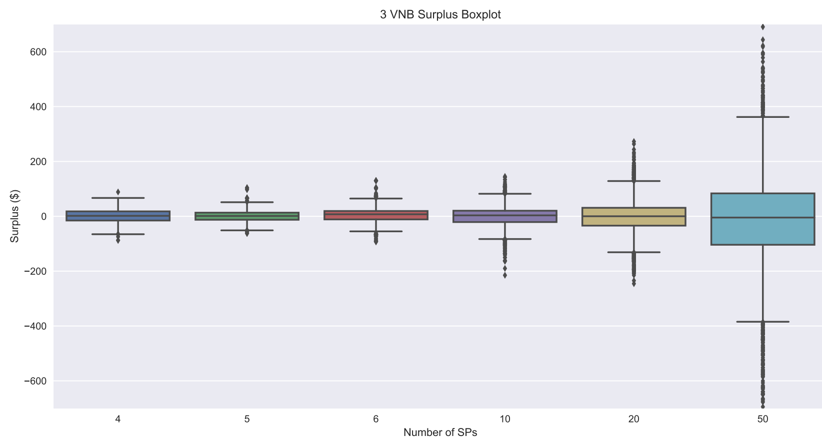


(a) Results with lower threshold

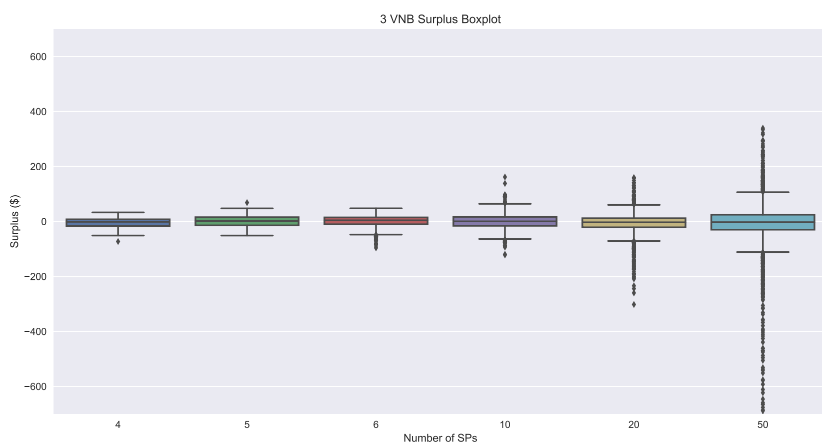


(b) Results with higher threshold

Figure 104: RPs' surplus in a scenario with 2 VNBs and 10 RPs

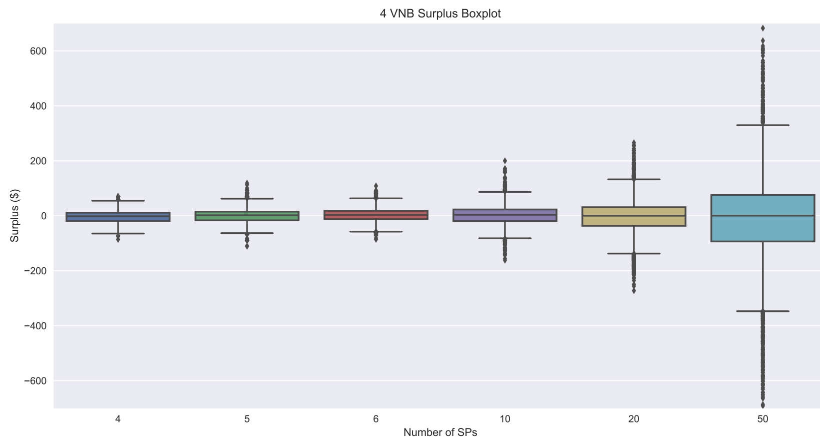


(a) Lower threshold results

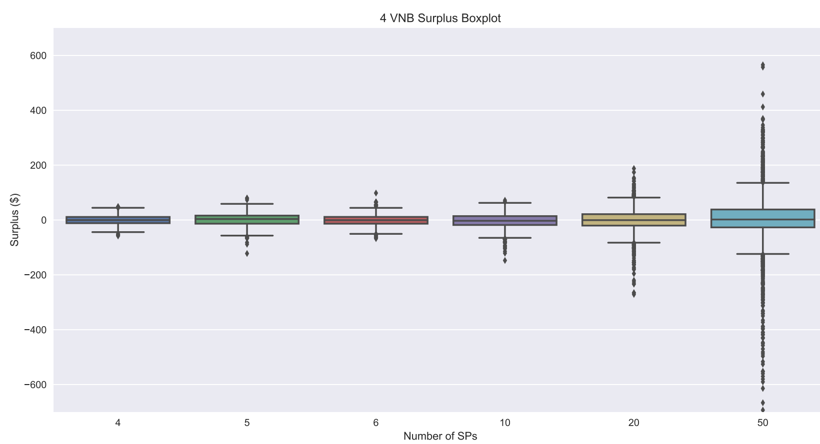


(b) Higher threshold results

Figure 105: RPs' surplus in a scenario with 3 VNBs and 10 RPs.

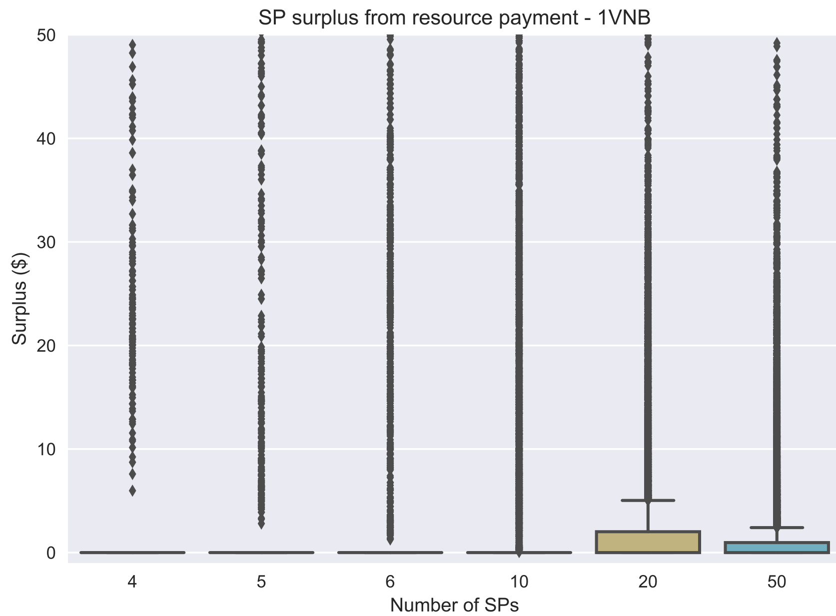


(a) Lower threshold results

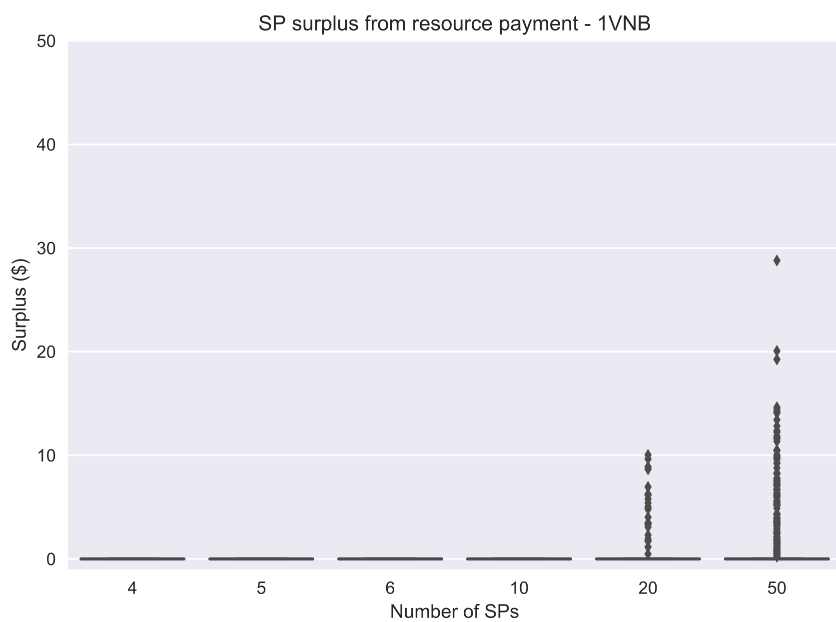


(b) Higher threshold results

Figure 106: RPs' surplus in a scenario with 4 VNBs and 10 RPs

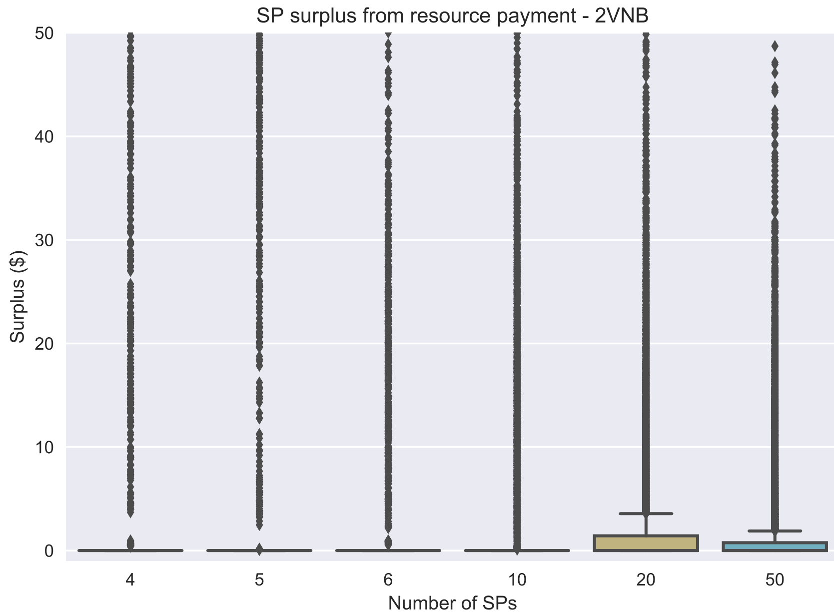


(a) Lower threshold results

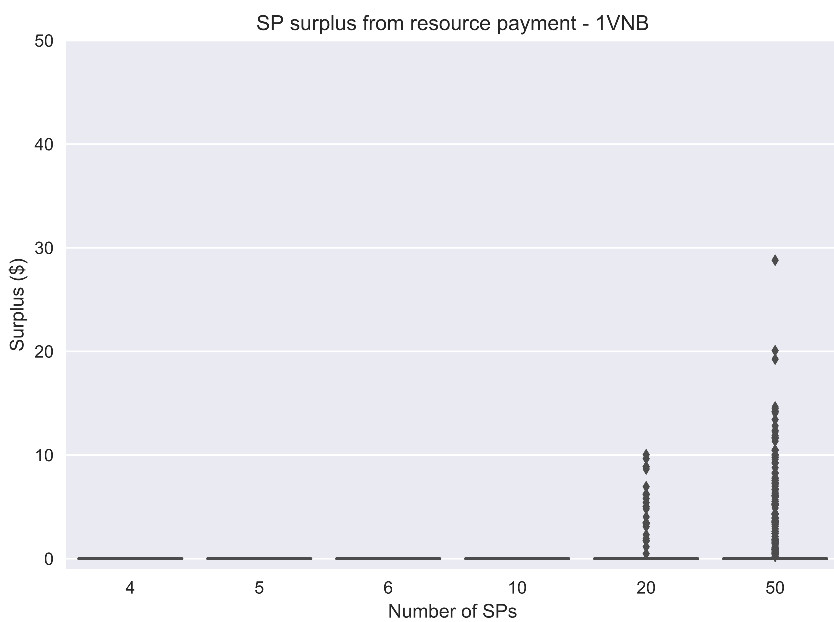


(b) Higher threshold results

Figure 107: SPs' surplus, obtained from their resource payment, in a scenario with 1 VNB and 10 RPs

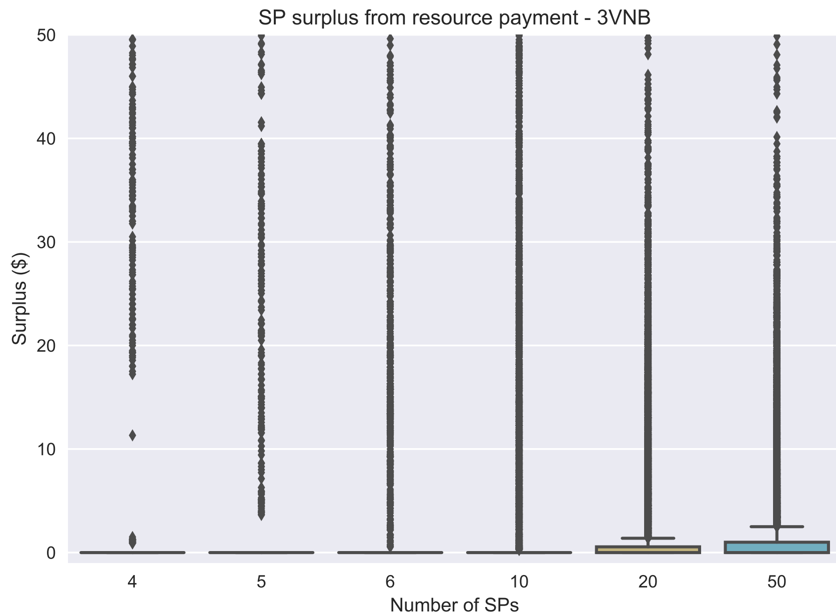


(a) Results with lower threshold

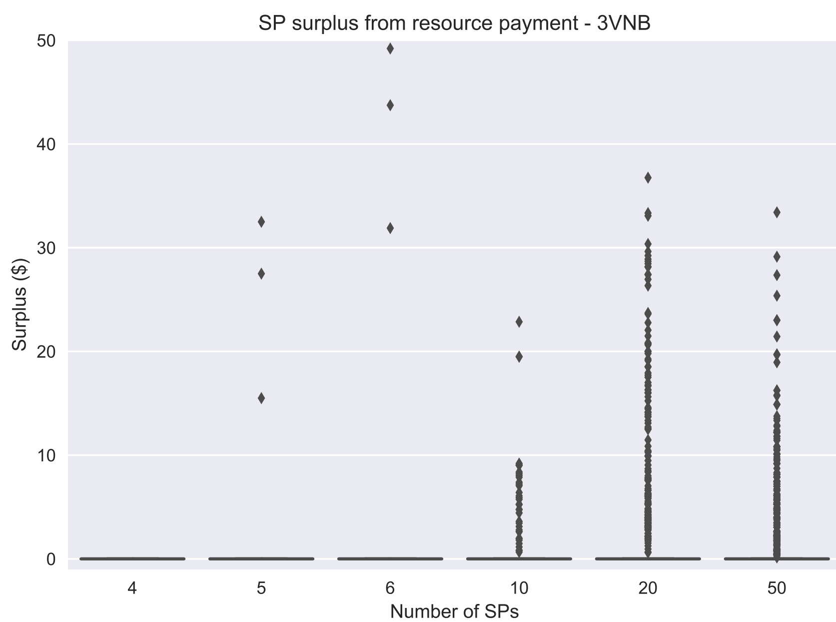


(b) Results with higher threshold

Figure 108: SPs' surplus, obtained from their resource payment, in a scenario with 2 VNBs and 10 RPs

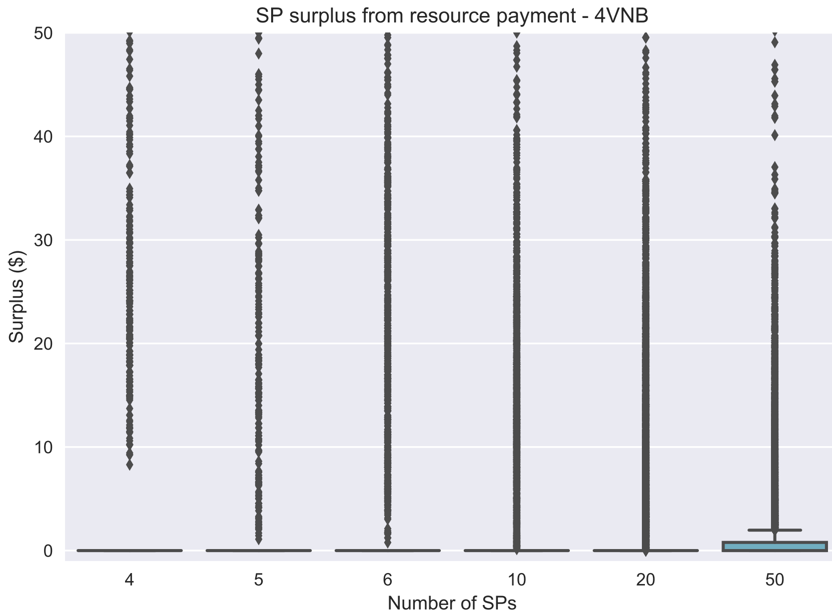


(a) Lower threshold results

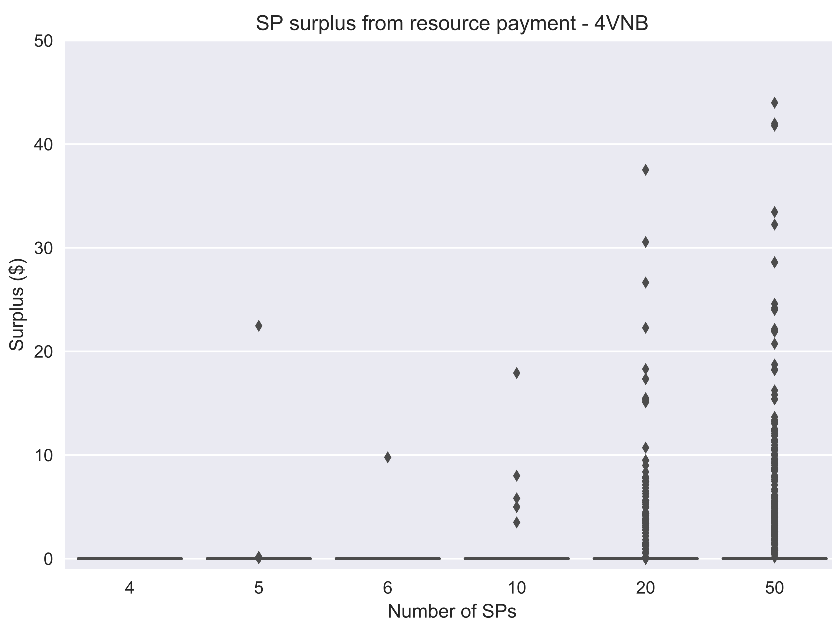


(b) Higher threshold results

Figure 109: SPs' surplus, obtained from their resource payment, in a scenario with 3 VNBs and 10 RPs.

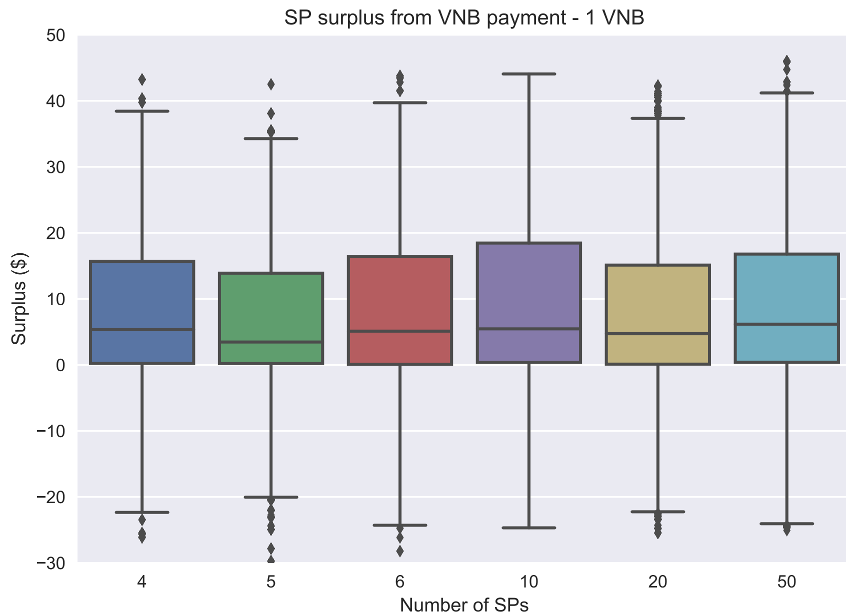


(a) Lower threshold results

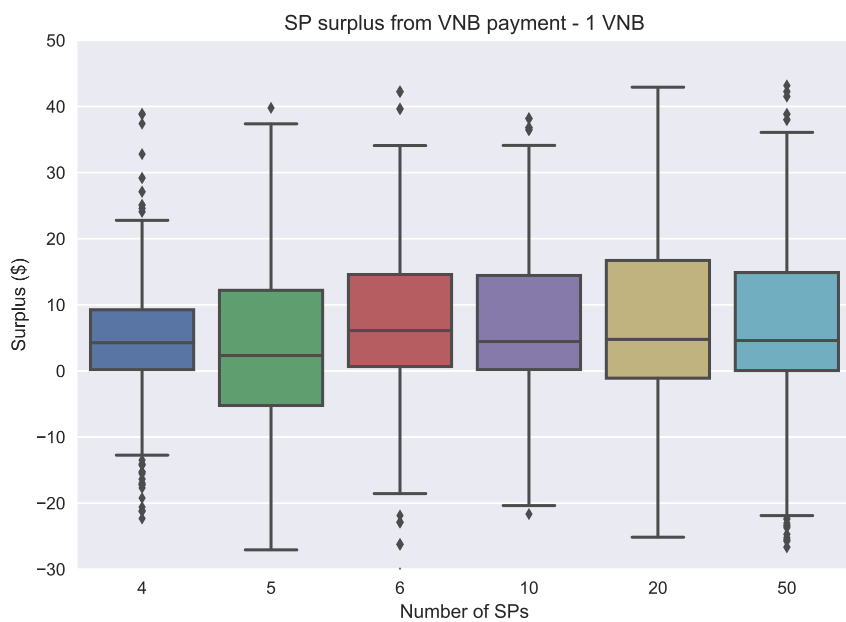


(b) Higher threshold results

Figure 110: SPs' surplus, obtained from their resource payment, in a scenario with 4 VNBs and 10 RPs



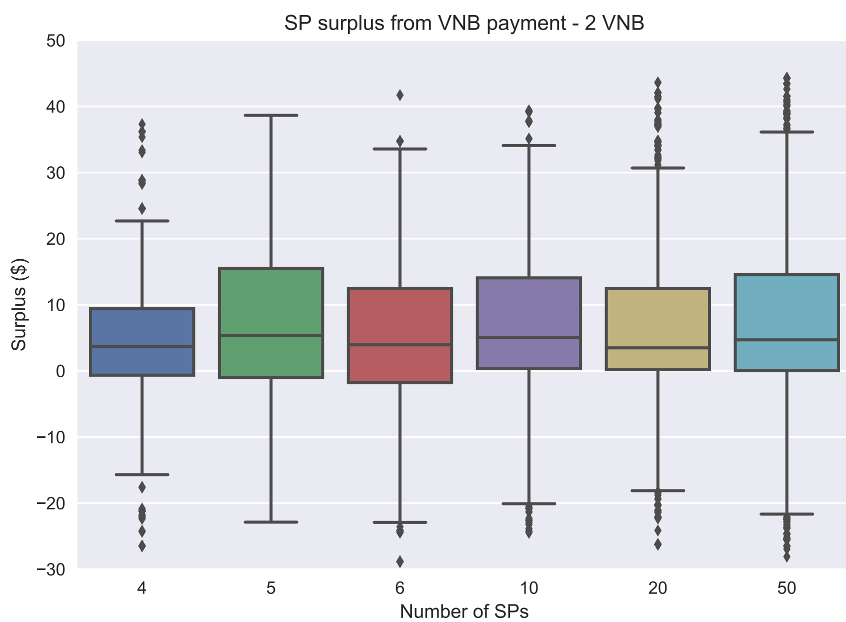
(a) Lower threshold results



(b) Higher threshold results

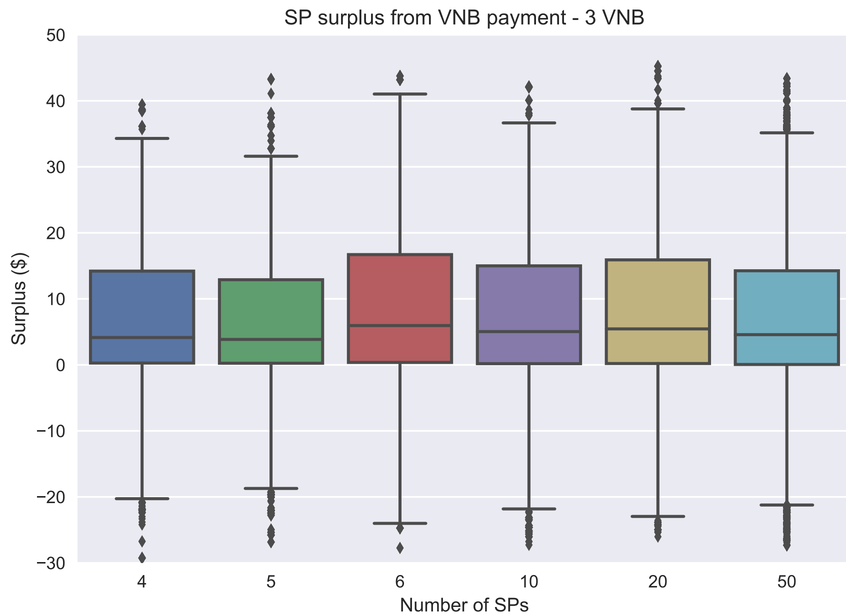
Figure 111: SPs' surplus, obtained from their VNB fee payment, in a scenario with 1 VNB and 10 RPs

(a) Results with lower threshold

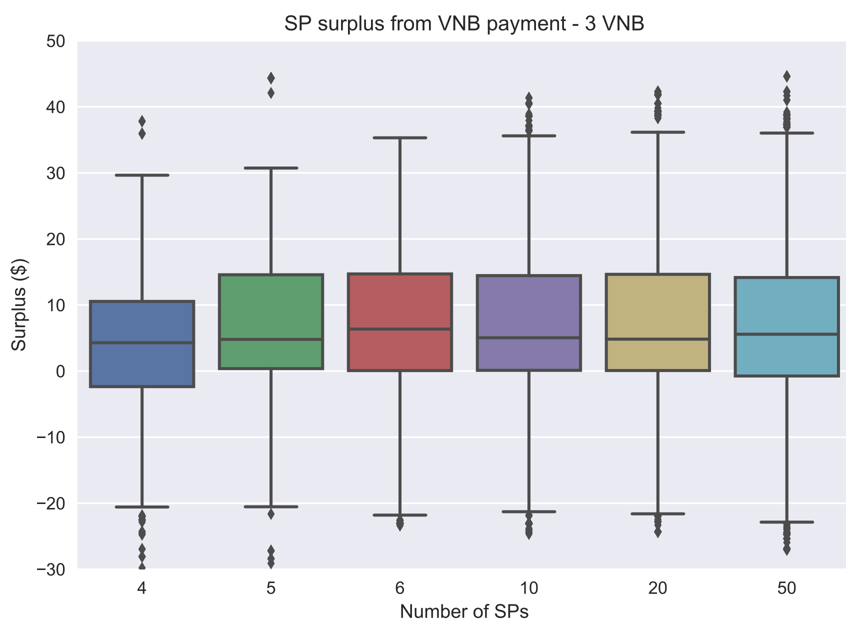


(b) Results with higher threshold

Figure 112: SPs' surplus, obtained from their VNB fee payment, in a scenario with 2 VNBs and 10 RPs

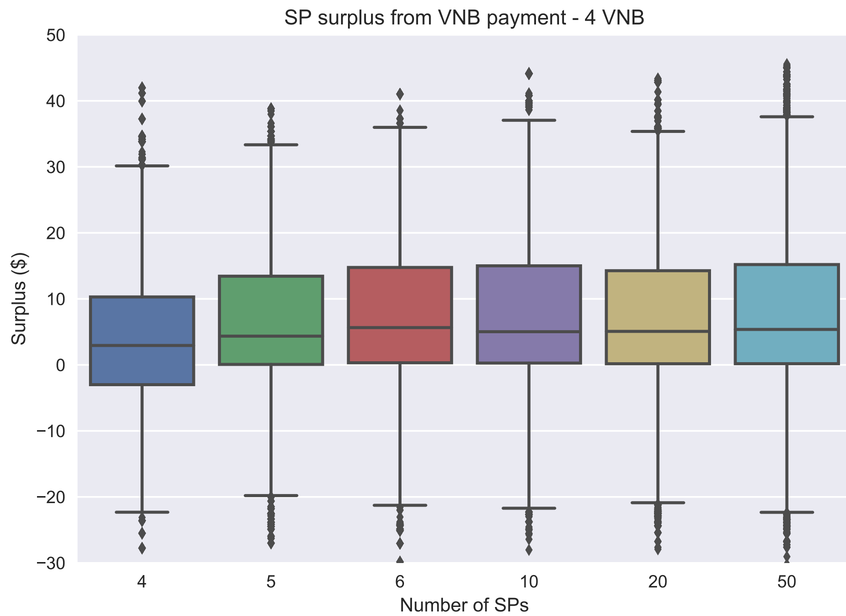


(a) Lower threshold results

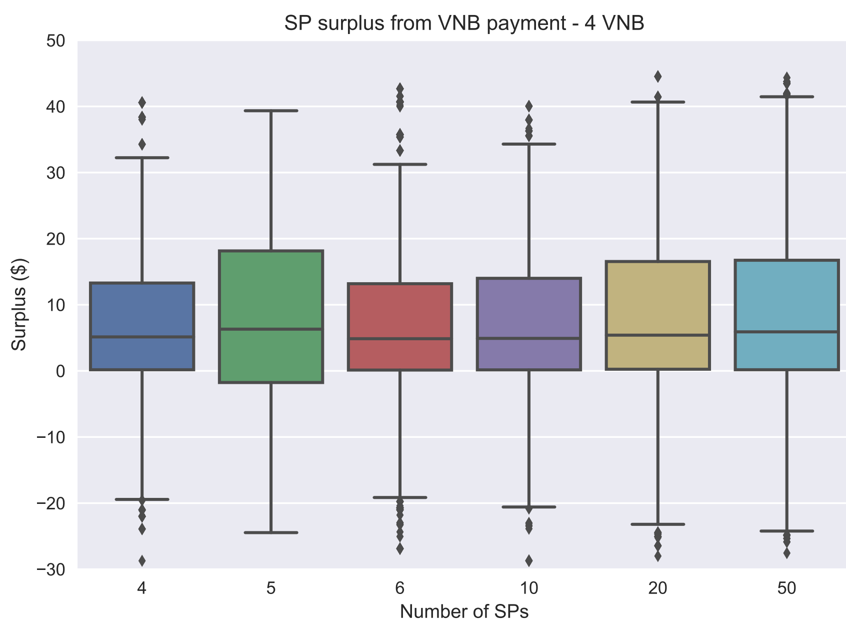


(b) Higher threshold results

Figure 113: SPs' surplus, obtained from their VNB fee payment, in a scenario with 3 VNBs and 10 RPs.



(a) Lower threshold results



(b) Higher threshold results

Figure 114: SPs' surplus, obtained from their VNB fee payment, in a scenario with 4 VNBs and 10 RPs

proximity. Along these lines, the VNBs' functions may go well beyond the distribution of unused capacity of large Resource Providers. Indeed, we could now refer as RPs to all entities that have deployed local networks. VNBs can fulfill resource aggregating functions, which may allow for more efficient uses of resources and the creation of larger scale networks, that result from the combination of small- and micro-cell configurations. Looking at the Smart Cities example, we can think of further applications[1] that may rely on a VNB for on-demand access to resources, as needed, in order to fulfill the demand specific to each area.

Exploring the cloud markets literature, we found that an important obstacle in resource access is defining an entity's appropriate needs and matching them with the available market offers. In the case of IoT and smart cities, and new service providers in general, a VNB could be also the entity that solves this problem. In this manner, a VNB could extend resource access to parties that could not do so individually.

From an enforcement perspective, thinking specifically of the Spectrum Access System (SAS), VNBs could make it easier for these entities to populate the databases. In fact, VNBs can keep track of areas where resources are being utilized, and the entities utilizing them. A report from the VNB to the SAS would permit the access system to maintain its databases up-to-date, while minimizing the amount of queries required in order to gather all the information.

Given the path that wireless communications services seem to be taking: reducing the scope and area of networks, our VNB market configuration may prove useful. Indeed, it could permit to bridge the gaps that slow-down the development of much needed spectrum (and resource) sharing schemes.

[1] The Smart Dublin project, which aims at converting Dublin, Ireland in a smart city, presents in the form of *challenges*, different aspects that need to be addressed. Some of these challenges include flooding alerts and monitoring (see `https://connectcentre.ie/news/connect-offers-smart-solution-flooding/`), indoor and outdoor wayfinding solutions, among others. For a full description of these challenges, see: `http://smartdublin.ie/challenges/`

11.0 CONCLUSIONS

The work that we have presented aims at providing a comprehensive view on the different aspects that influence the development of viable spectrum markets. We begin by exploring markets as stand-alone institutions where new entrants can access the spectrum holdings of a Band Manager via auction mechanisms. With the evolution of our study, we addressed technical characteristics of spectrum, which led us to find a new commodity to trade in the market. This new commodity relies on technical features and flexibility opportunities provided by resource virtualization. In the final stage of this dissertation, we adopted a new market perspective, by studying new market entities, their characteristics, the rules they follow, and their behavior. This was possible by placing our model within a broader framework which has also permitted us to set the stage for future analysis.

Throughout this three-stage study, we have been able to define what are the effects of the lack of spectrum fungibility in the market; to explore what are the consequences of defining more flexible tradeable commodities, and how a resource sharing environment that relies on market mechanisms may be deployed. In each stage results section, we have included the values and interpretation of the parameters of interest. Nevertheless, in what follows, we point out how our analysis addresses the successful market design guidelines defined by Alvin Roth in [7]: *thickness, congestion and safety.*

- Thickness: One of the main objectives in this work has been to define a commodity that can attract more participants to the marketplace. We have found that trading *naked* spectrum poses important limitations which impact resource valuation, and hence market success. By adopting a more technical definition of spectrum, we find that it is possible to offer resources that may be compatible with a wider range of services and devices, thus becoming more appealing to a wider set of buyers. Additionally, in our third stage, we include an additional entity, the virtual network builder, which is in charge of bridging additional differences between buyers and sellers. In absence of a VNB, these differences may ultimately restrict the amount of transactions that take place in the market.

- Congestion: Just as the VNBs bridge differences between SPs and RPs, the operations of these middlemen also reduce the amount of transactions that would be required if buyers and sellers were left to transact on their own. In other words, the final market transactions are performed with a reduced number of entities (VNBs), which represent the available buyers (i.e., SPs). In this manner, an additional function of the VNB is to reduce the congestion resulting from an increased number of market participants and possible transactions.
- Safety: Limiting the flow of information between transacting parties can be a source of safety for market participants. In this manner, buyers and/or sellers may not know what the business model of other participants is, hence avoiding restrictions on resource use. Additionally, the different market participants can base their future interactions on previous market outcomes. This allows them to place more competitive bids, avoid losses and choose the most suitable VNB for their needs.

Our study relies on the development of models that fit our scenarios of interest. For this purpose, we have utilized agent-based modeling (ABM) and its agent-based computational economics (ACE) branch to design and test models that best adapt to each of the stages described in this dissertation. Given the difference of scope between the first two stages and the third, the respective models show different levels of complexity. Nevertheless, the added complexity of our third stage model does not impact its stability, scalability and flexibility. In this manner, we are presenting a market analysis tool that can be adapted to fit further aspects of interest, i.e., upcoming technologies, more complex auction mechanisms and governance schemes.

Combining our modeling tools and the analysis that can emerge from them, we expect this work to elucidate how we can incorporate pragmatic market approaches into the academic study of spectrum markets. Indeed, we consider that our approach on matching market participants with brokers permits us to develop a more expressive resource access mechanism that can shed light on the incentives required and shortcomings to address, as we move towards adopting markets as feasible spectrum sharing alternatives.

12.0 FUTURE WORK

Forthcoming studies and analyses in spectrum markets are mainly related to our third stage model. As mentioned throughout this work, the model developed for Stage III constitutes a mechanism to bootstrap the market in more complex settings. In this manner, future work associated with this approach includes the development of a more complex market mechanism between RPs and VNBs (e.g., auction model or matching market) that would allow our agents to place more expressive bids. In turn, this would permit us to create a more refined service-driven network.

As a means to mimic market approaches that are successful in industries like electricity, we plan to implement additional transaction capabilities for the virtual network builders. In our future view of the model, VNBs will be capable of managing futures transactions as well as spot markets. This would permit us to capture different degrees of risk in the market transactions, thus making our VNBs' risk profiles richer and more pragmatic.

An important aspect of VNBs as middlemen is their reputation. In this light, we are interested in exploring different reputation building mechanisms that could adapt to our settings. Currently, there are myriad approaches that have been implemented for rating real-world middlemen (e.g., Amazon, OpenTable, Airbnb) that can shed light on features that would be applicable to our model.

Regarding the matching process, we aim at exploring different factors that may guide the preferences of SPs and VNBs. This would also lead us to study scenarios where SPs may be interested in matching with more than one VNB. On the resource provider side, RPs currently do not play a role in VNB–SP matching. Nevertheless, it is important to address how feedback from past transactions may incorporate parameters relevant to RPs into the matching process.

To date, our market model has been developed in such a manner that it allows for a deep analysis of governance mechanisms and the role these may play as the market evolves. In this way, our future work involves embedding specific polycentric governance considerations and applicable enforcement mechanisms in our market model. This would permit us to emphasize on the safety aspect of our market approach.

Finally, we aim at placing our market model within technical scenarios of interest. For instance, we aim at adapting our model to the spectrum requirements and market setup for 5G technologies and the Internet of Things. Throughout the different extensions of the work presented herein, our focus on studying factors from multiple disciplines remains, as we consider it important to advance spectrum sharing research from different perspectives and levels of abstraction.

APPENDIX A

EXPERIMENT RESULTS FROM STAGE III

A.1 SPS' GROUP PERFORMANCE

In this section we include an additional representation of SPs' group performance. In each of these graphs, we include the percentage of tested scenarios in which each group led to the best results for each criterion. The results presented in what follows correspond to scenarios with 10 RPs, given that the number of RPs does not influence the matching-related parameters.

A.1.1 Demand

As presented in section 9.3, SPs belonging to group 4 outperformed the rest of the groups in terms of demand. This is also illustrated in figures 115, 116, 117 and 118. Indeed, we observe that in all the scenarios we tested, members of group 4 had a higher percentage of the geographical demand converted into market demand.

The results regarding other criteria, such as the percentage of matched SPs and the percentage of SPs that obtained resources show an identical distribution as that of the demand graphs. In other words, for all these criteria, group 4 SPs outperform the members of other groups in all the tested scenarios. The parameter for which we find distinct results is that of the SPs' surplus stemming from the fees paid to the VNBs. We include these distributions in what follows.

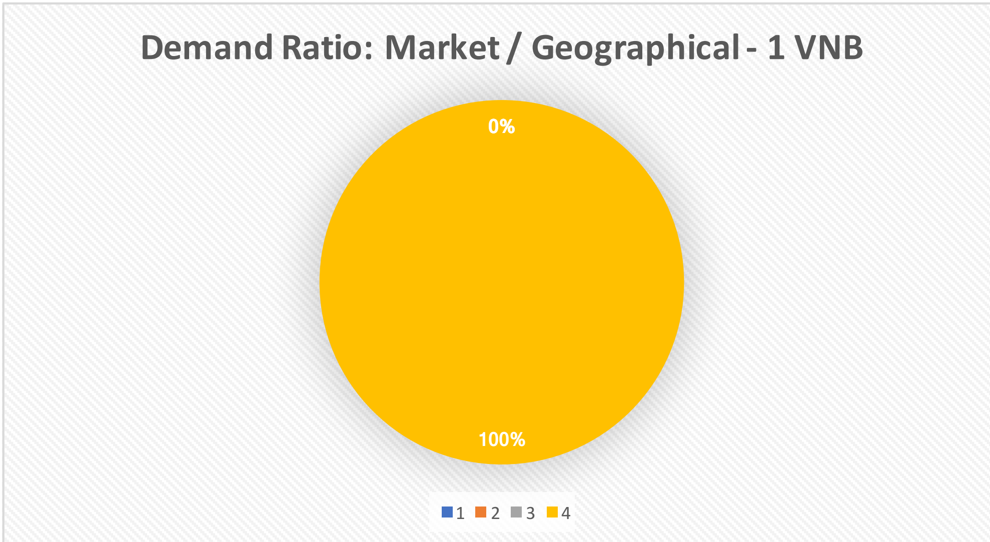


Figure 115: Group distribution of geographical demand that became market demand. Scenario with 1 VNB

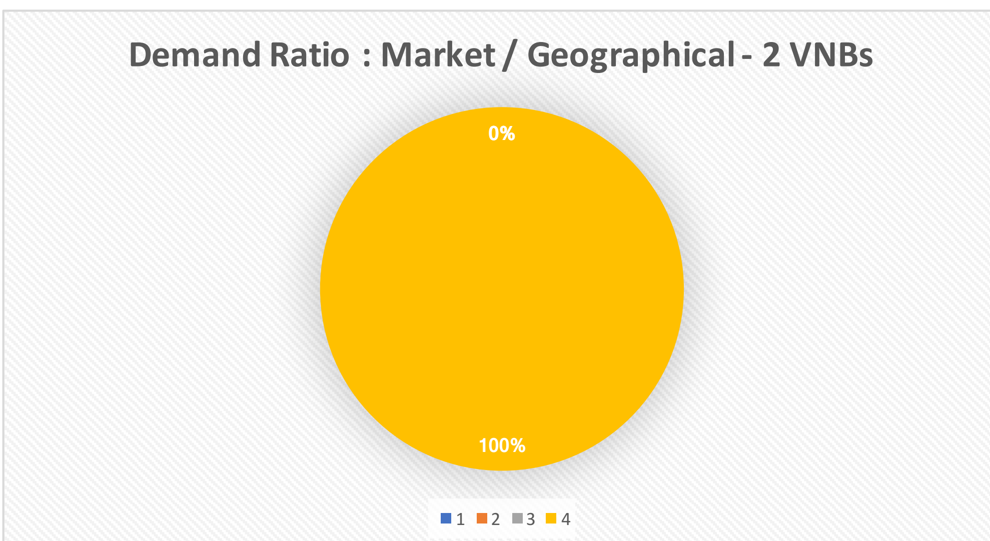


Figure 116: Group distribution of geographical demand that became market demand. Scenario with 2 VNBs

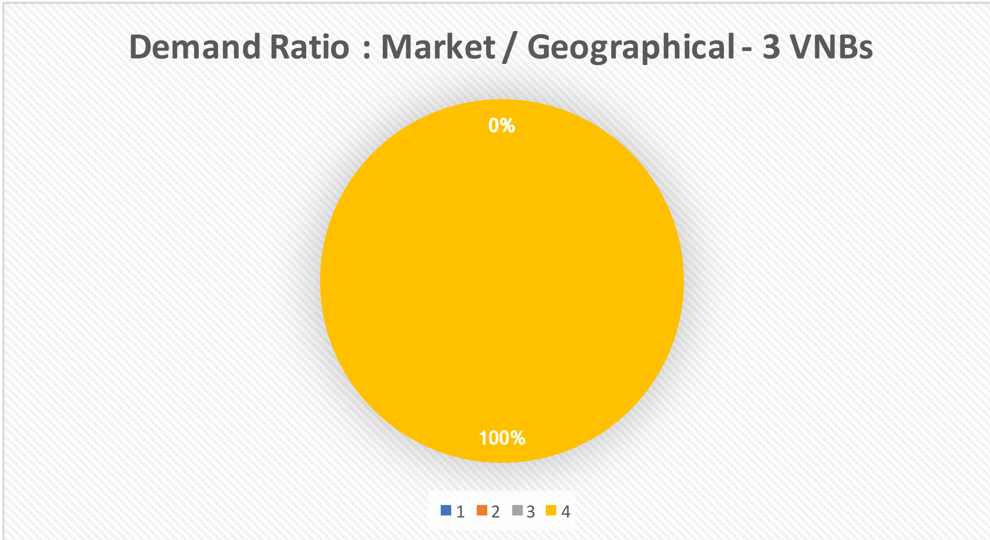


Figure 117: Group distribution of geographical demand that became market demand. Scenario with 3 VNBs

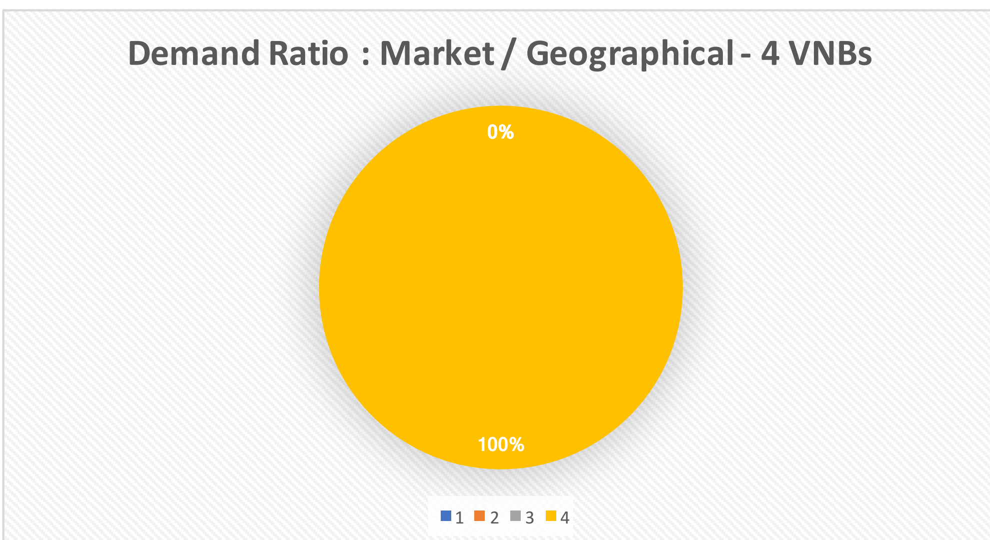


Figure 118: Group distribution of geographical demand that became market demand. Scenario with 4 VNBs

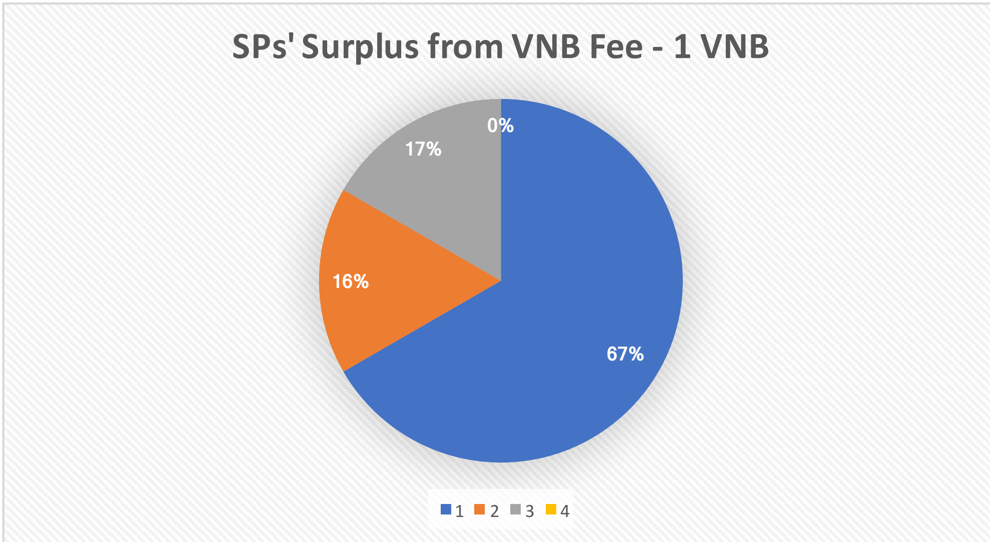


Figure 119: Group distribution of SPs' surplus from VNB fees Scenario with 1 VNB

A.1.2 SPs' surplus from VNB fees

A.2 VNBS' GROUP PERFORMANCE

In this section we present the percentage of the tested scenarios in which a given VNB group outperformed the rest regarding the parameters specified in each subsection. These graphs support the results presented in section 9.3, which show that contrary to the case of SPs, group 4 VNBs do not outperform the rest in every scenario.

A.2.1 Customers per VNB

A.2.2 Percentage of Matched VNBs

A.2.3 Payment Received by VNBs

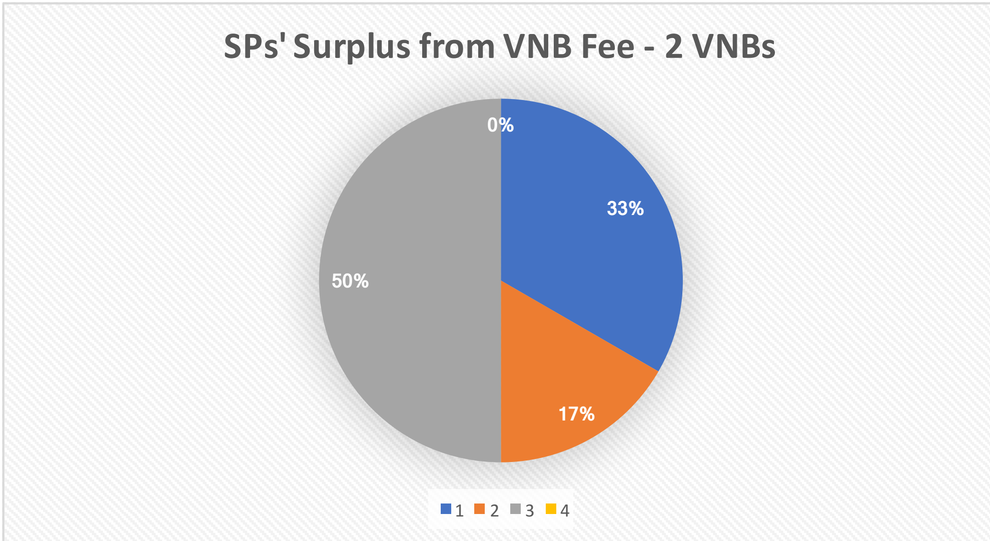


Figure 120: Group distribution of SPs' surplus from VNB fees Scenario with 2 VNBs

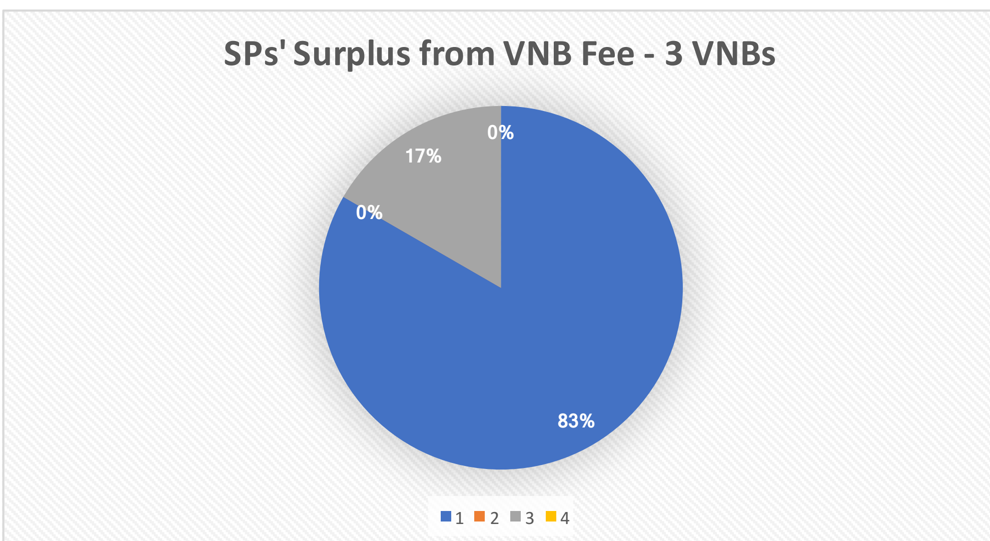


Figure 121: Group distribution of SPs' surplus from VNB fees Scenario with 3 VNBs

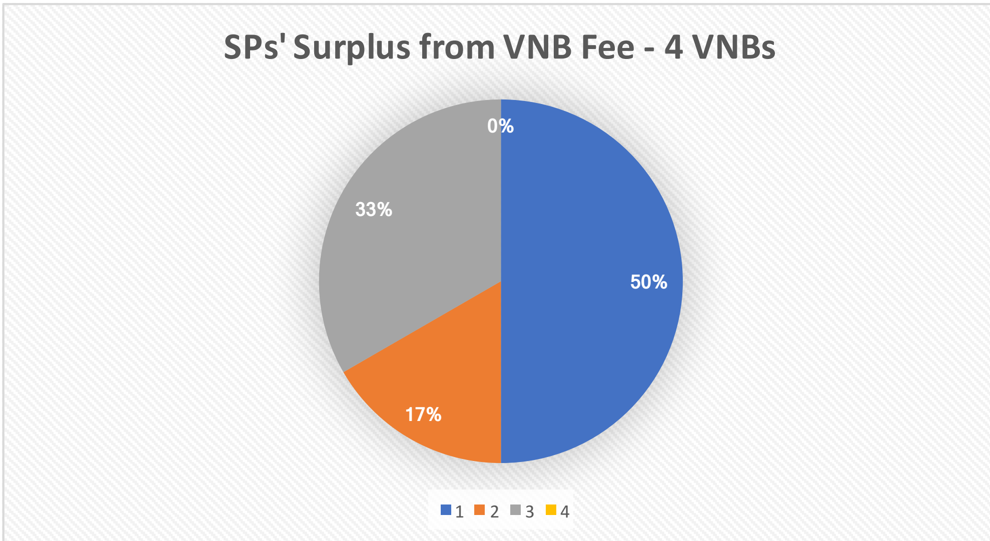


Figure 122: Group distribution of SPs' surplus from VNB fees Scenario with 4 VNBs

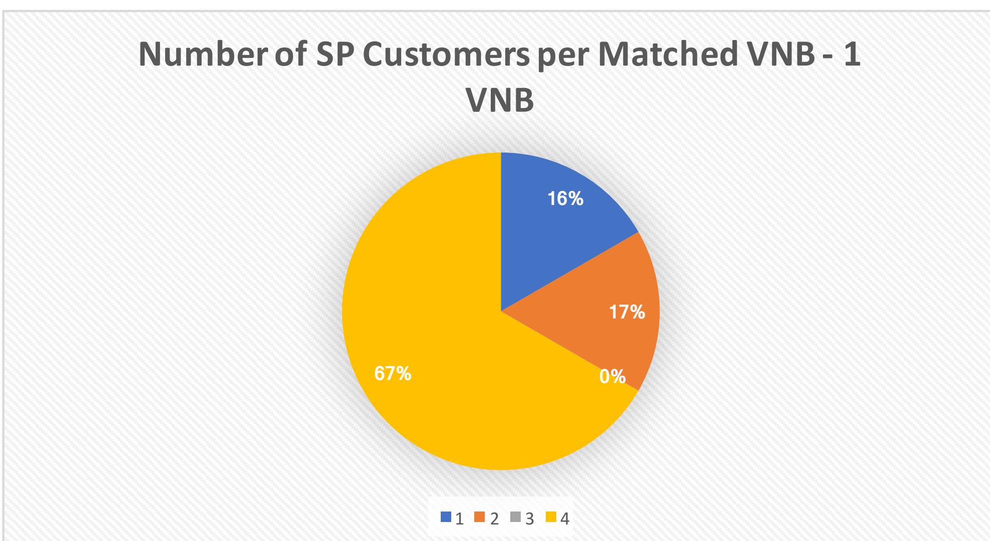


Figure 123: Group performance distribution regarding the average number of partners per VNB. Scenario with 1 VNB

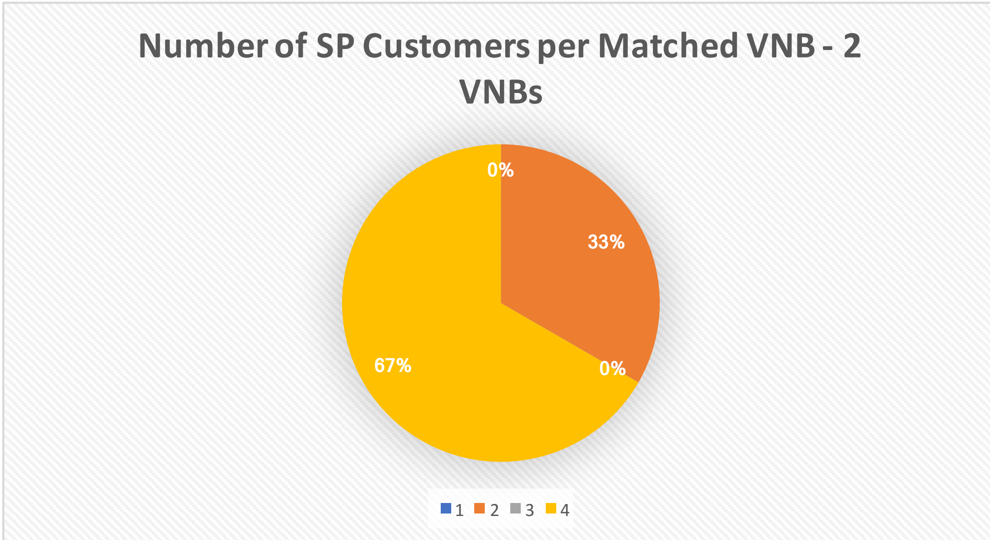


Figure 124: Group performance distribution regarding the average number of partners per VNB. Scenario with 2 VNBs

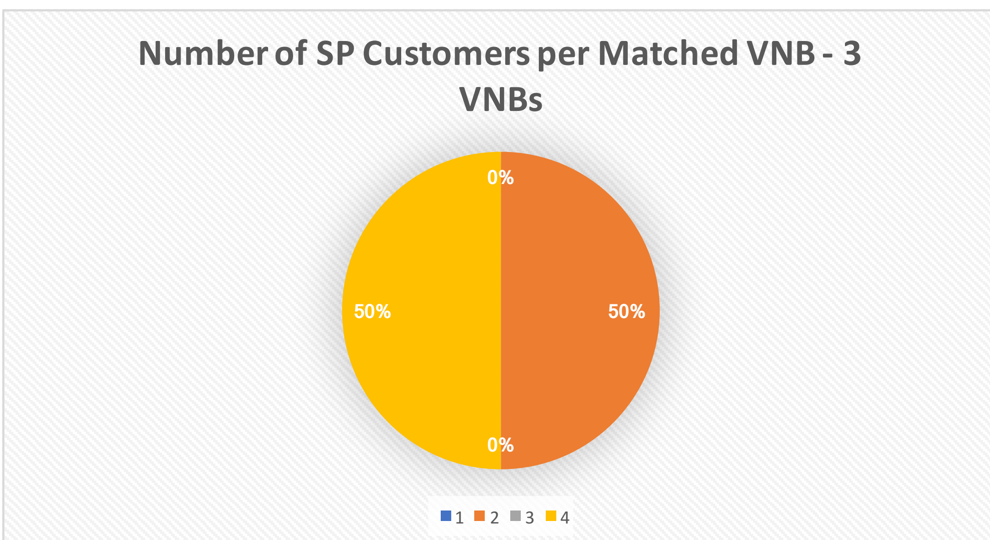


Figure 125: Group performance distribution regarding the average number of partners per VNB. Scenario with 3 VNBs

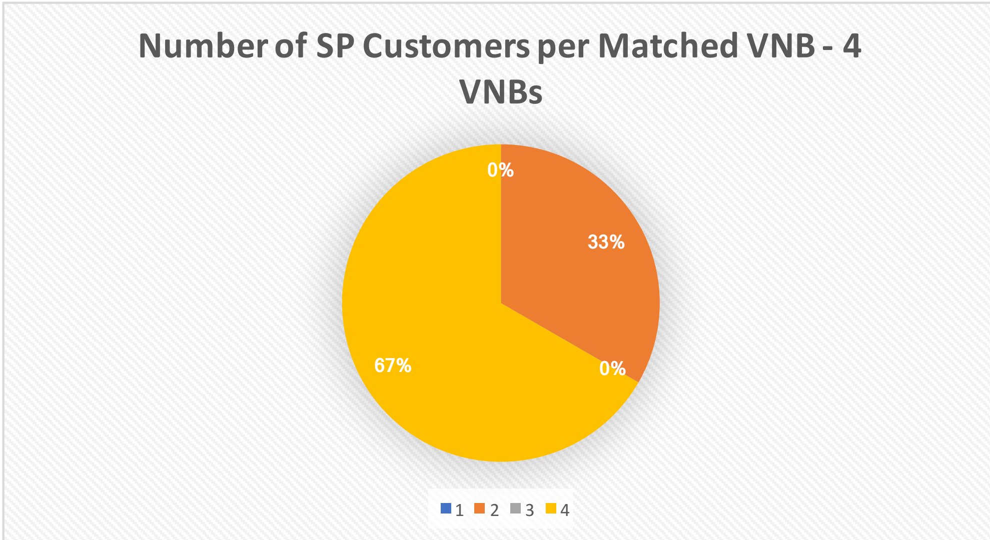


Figure 126: Group performance distribution regarding the average number of partners per VNB. Scenario with 4 VNBs

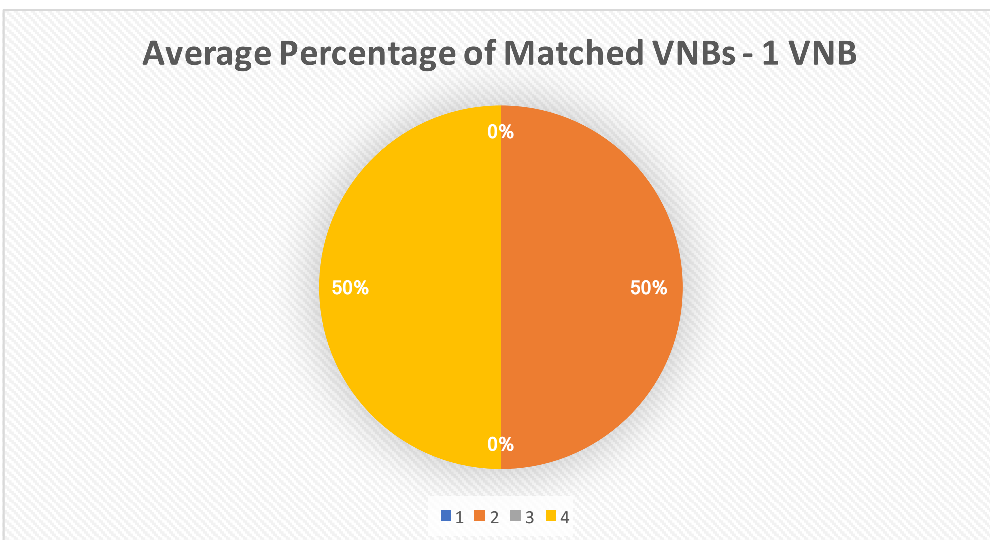


Figure 127: Group performance distribution regarding the percentage of matched VNBs (on average). Scenario with 1 VNB

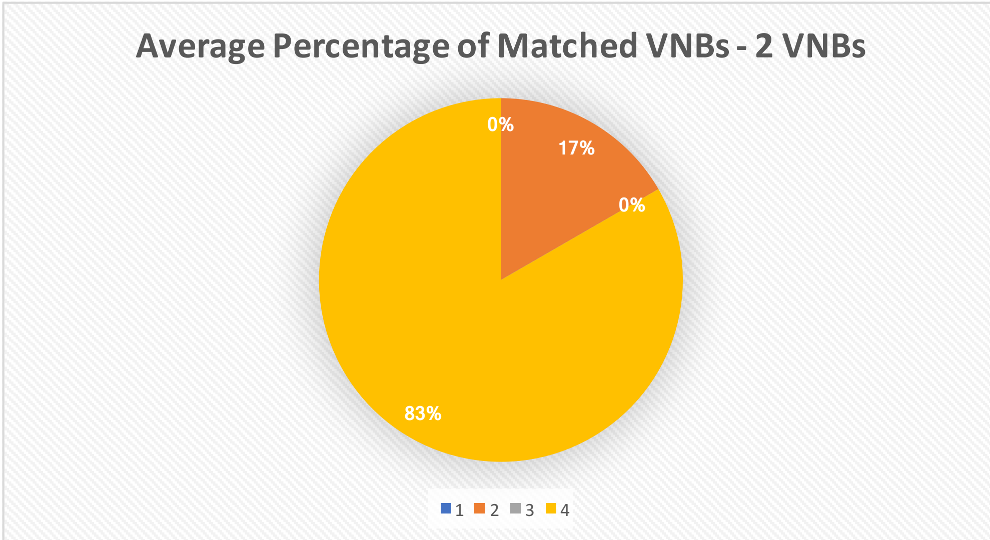


Figure 128: Group performance distribution regarding the percentage of matched VNBs (on average). Scenario with 2 VNBs

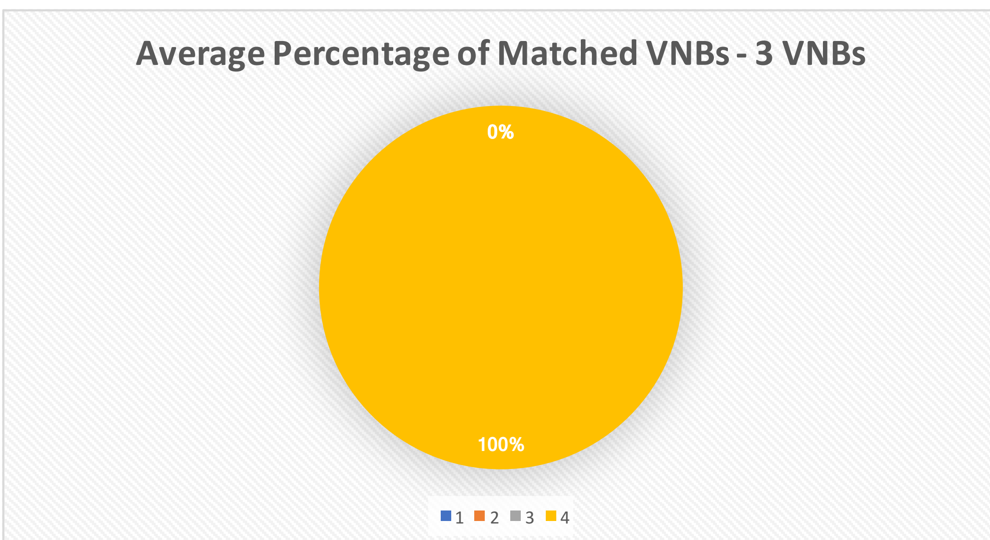


Figure 129: Group performance distribution regarding the percentage of matched VNBs (on average). Scenario with 3 VNBs

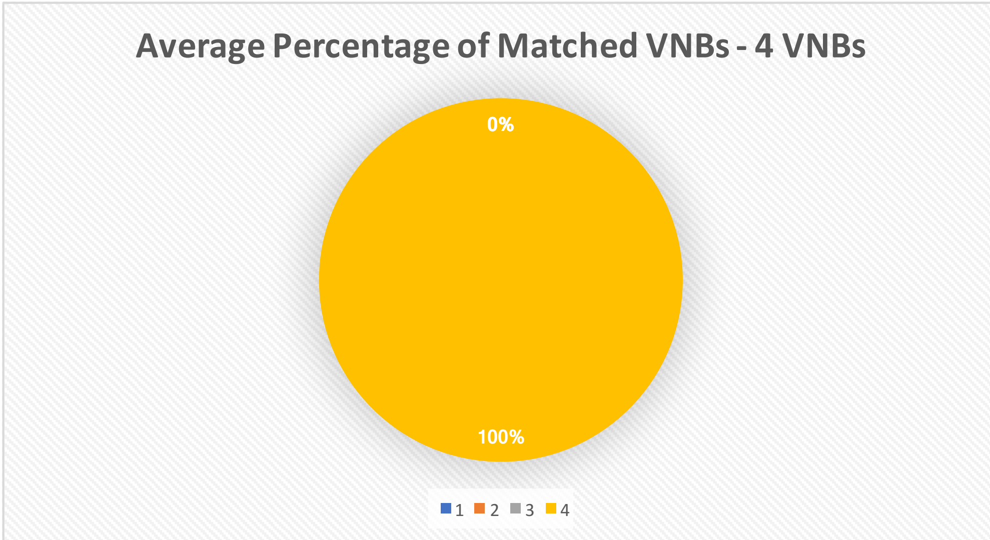


Figure 130: Group performance distribution regarding the percentage of matched VNBs (on average). Scenario with 4 VNBs

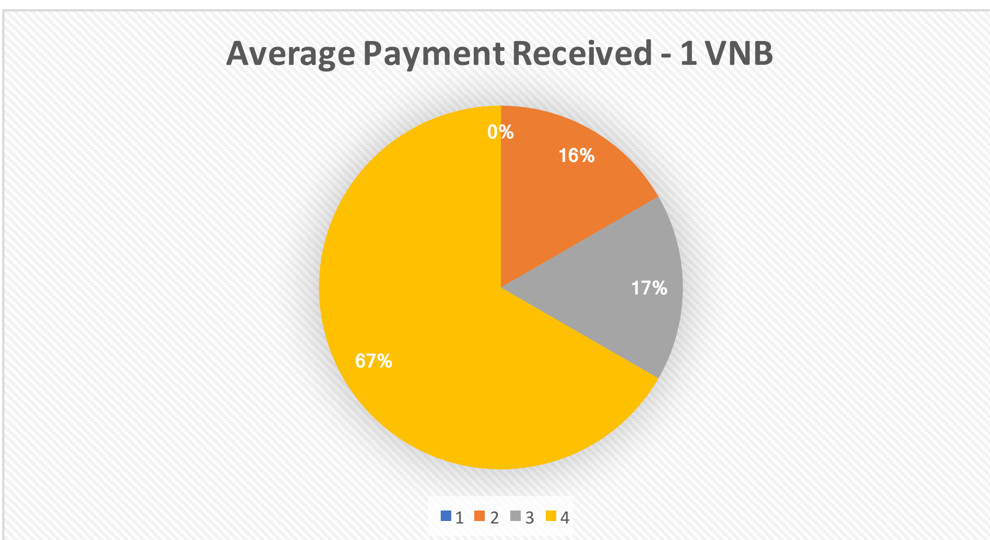


Figure 131: Group performance distribution regarding the average payment received by VNBs. Scenario with 1 VNB

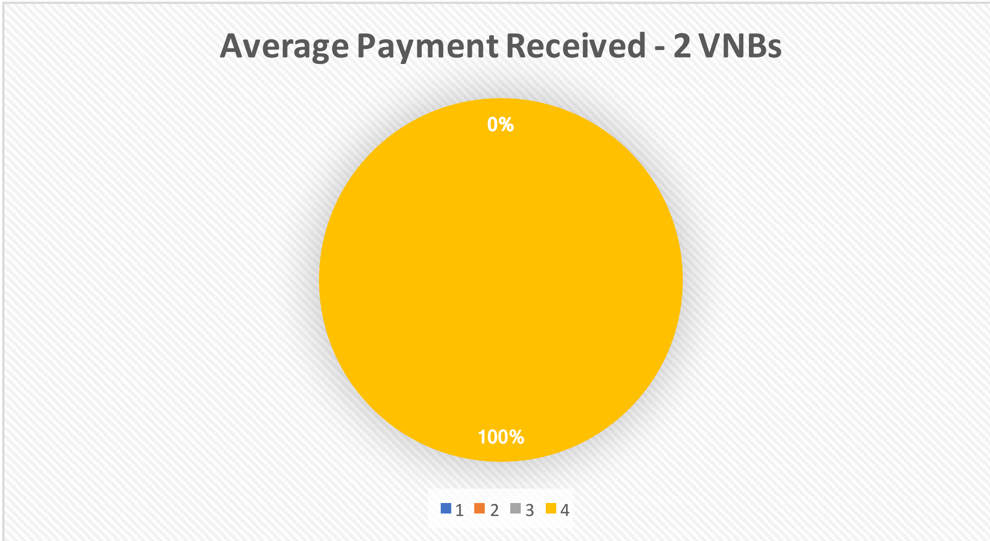


Figure 132: Group performance distribution regarding the average payment received by VNBs. Scenario with 2 VNBs

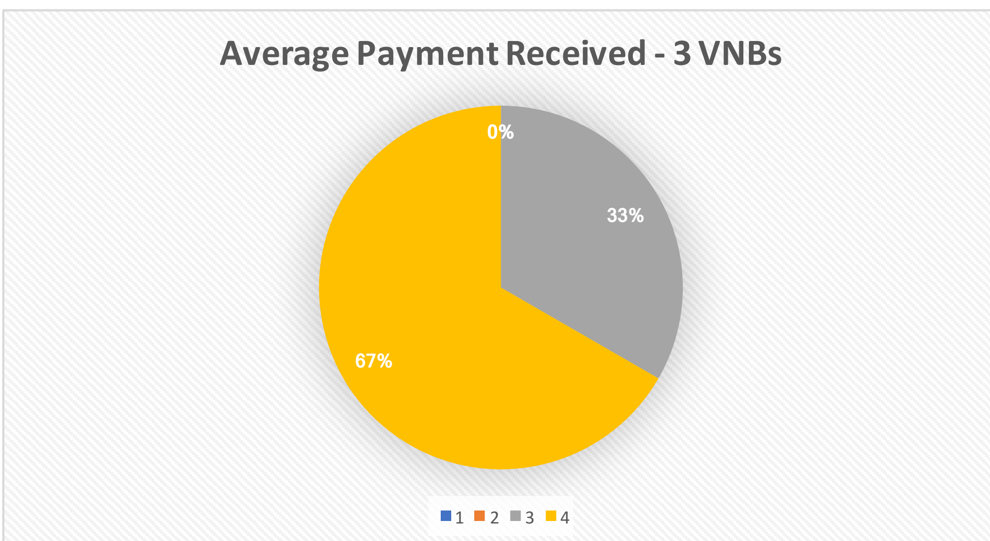


Figure 133: Group performance distribution regarding the average payment received by VNBs. Scenario with 3 VNBs

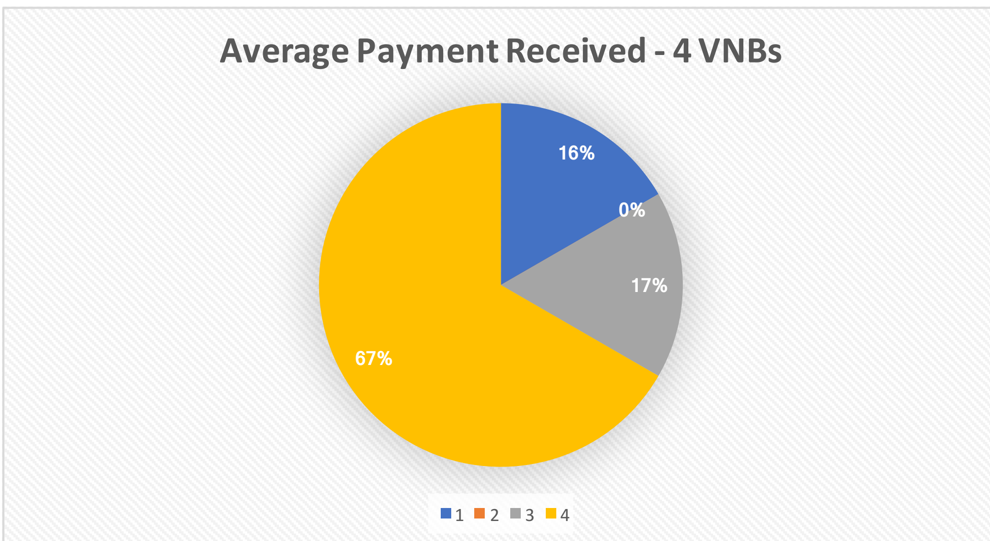


Figure 134: Group performance distribution regarding the average payment received by VNBs. Scenario with 4 VNBs

APPENDIX B

ADDITIONAL RESULTS FROM STAGE III

B.1 PAYMENTS RECEIVED BY VNBS

In this section we include results for the average payment received by VNBs in scenarios with 5 and 10 RPs.

B.1.1 Scenario with 5 RPs

B.1.2 Scenario with 15 RPs

B.2 VNBS' SURPLUS ACCORDING TO REPUTATION

In what follows, we include figures that illustrate the surplus distribution of VNBs according to their reputation. We present scenarios with 5, 10 and 15 RPs.

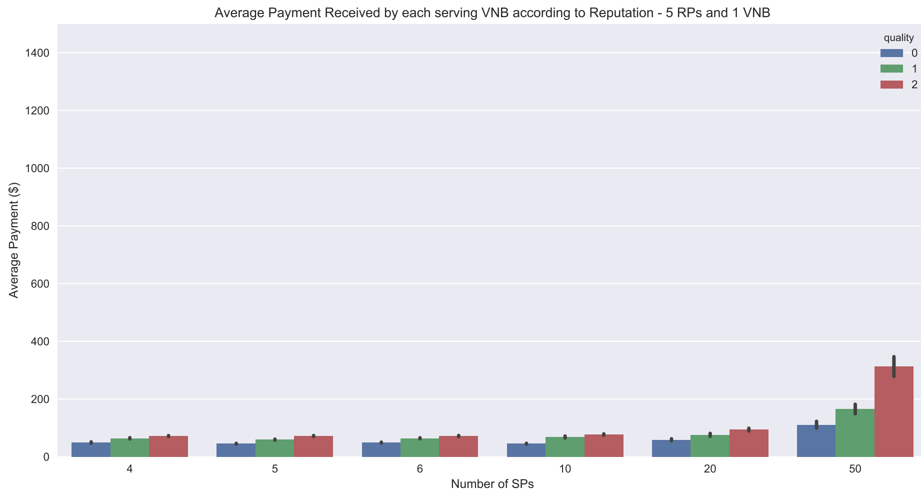


Figure 135: Average payment received by each VNB, according to its reputation. Scenario with 1 VNB and 5 RPs.

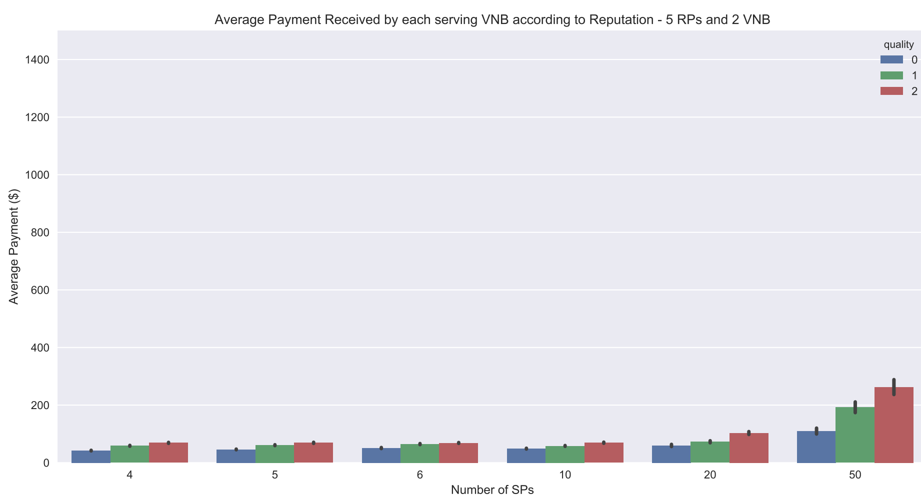


Figure 136: Average payment received by each VNB, according to its reputation. Scenario with 2 VNBs and 5 RPs.

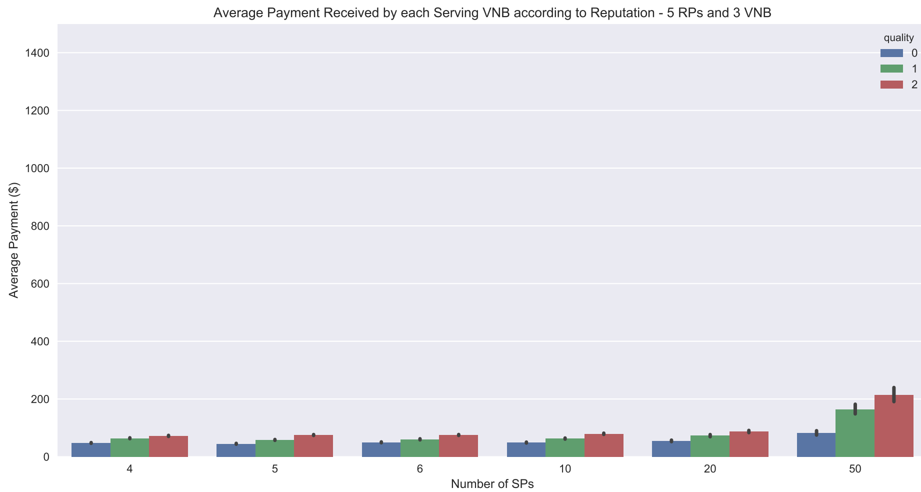


Figure 137: Average payment received by each VNB, according to its reputation. Scenario with 3 VNBs and 5 RPs.

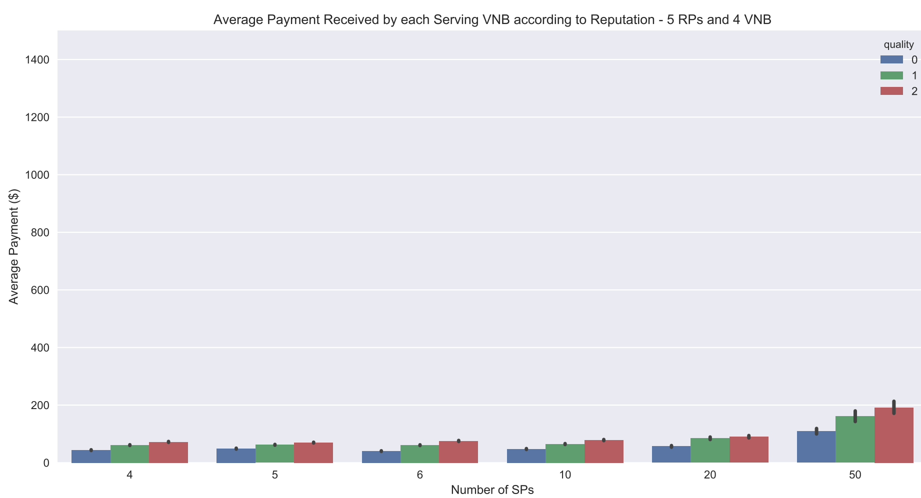


Figure 138: Average payment received by each VNB, according to its reputation. Scenario with 4 VNBs and 5 RPs.

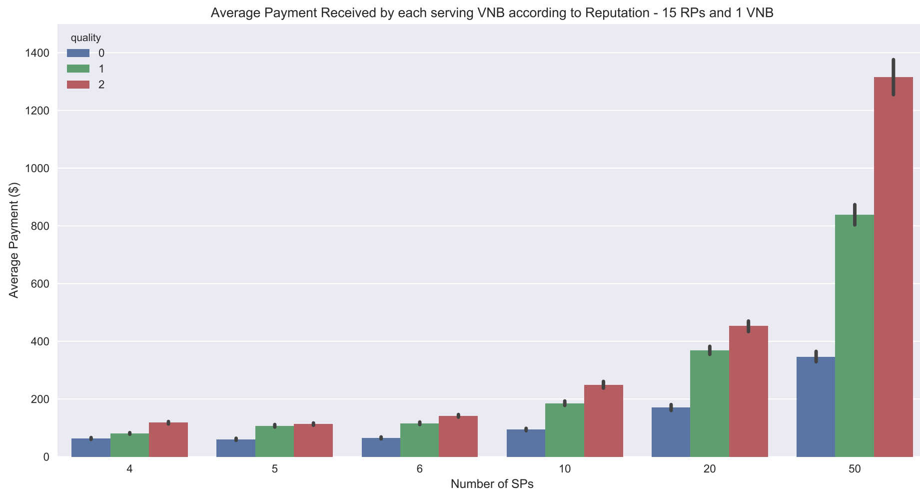


Figure 139: Average payment received by each VNB, according to its reputation. Scenario with 1 VNB and 15 RPs.

Figure 140: Average payment received by each VNB, according to its reputation. Scenario with 2 VNBs and 15 RPs.

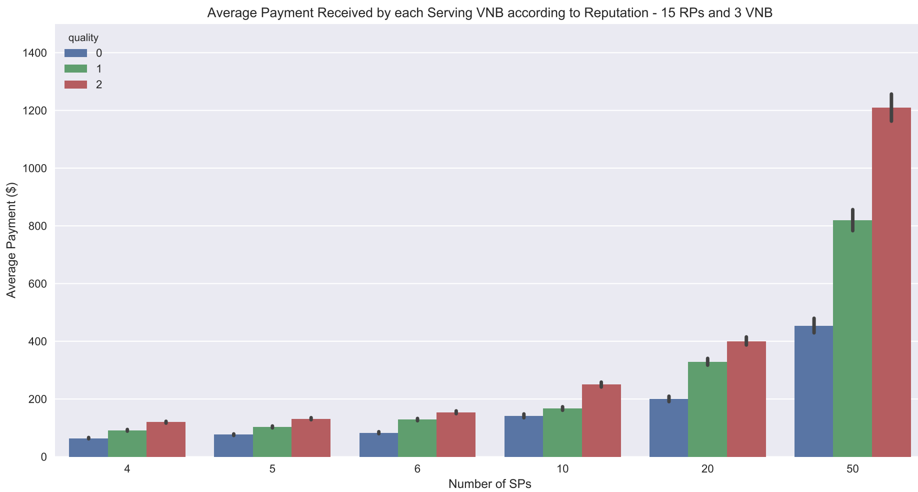


Figure 141: Average payment received by each VNB, according to its reputation. Scenario with 3 VNBs and 15 RPs.

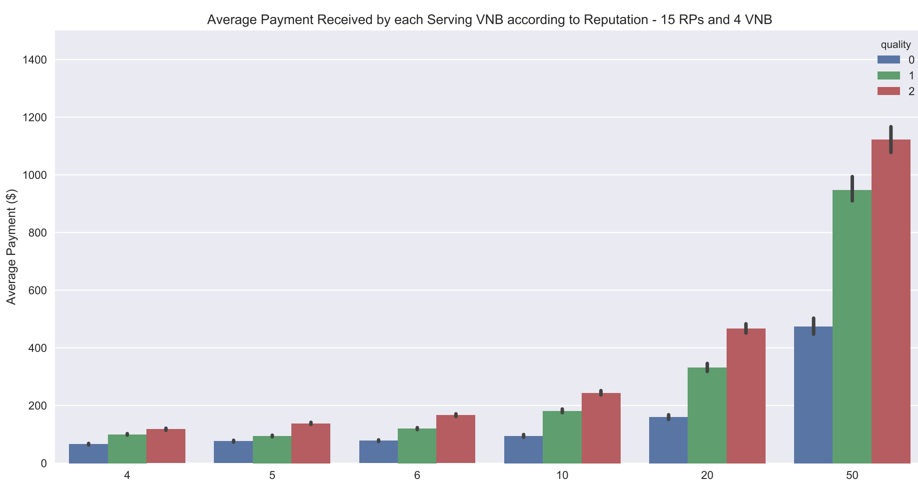


Figure 142: Average payment received by each VNB, according to its reputation. Scenario with 4 VNBs and 15 RPs.

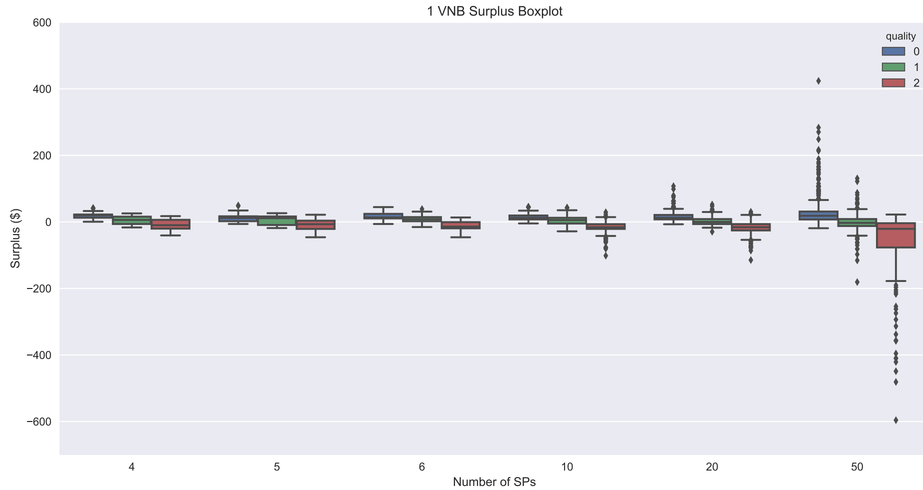


Figure 143: VNBs surplus distribution according to their reputation. Scenario with 1 VNB

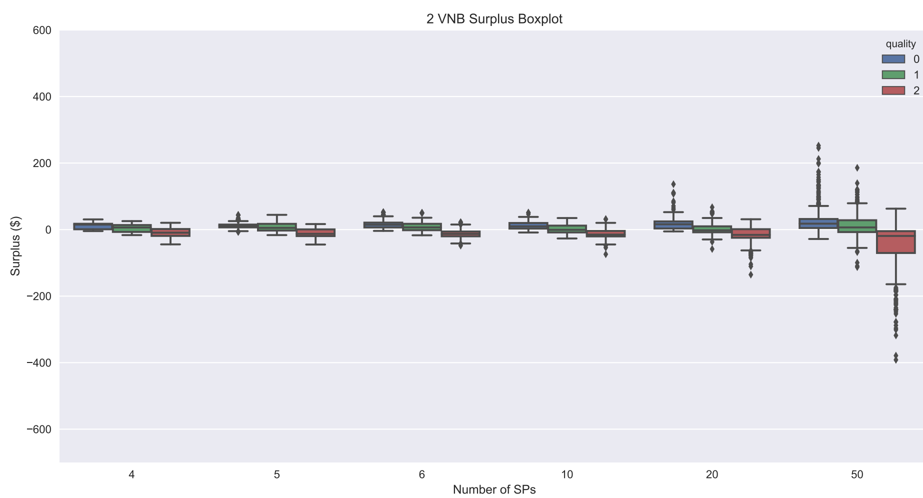


Figure 144: VNBs surplus distribution according to their reputation. Scenario with 2 VNBs

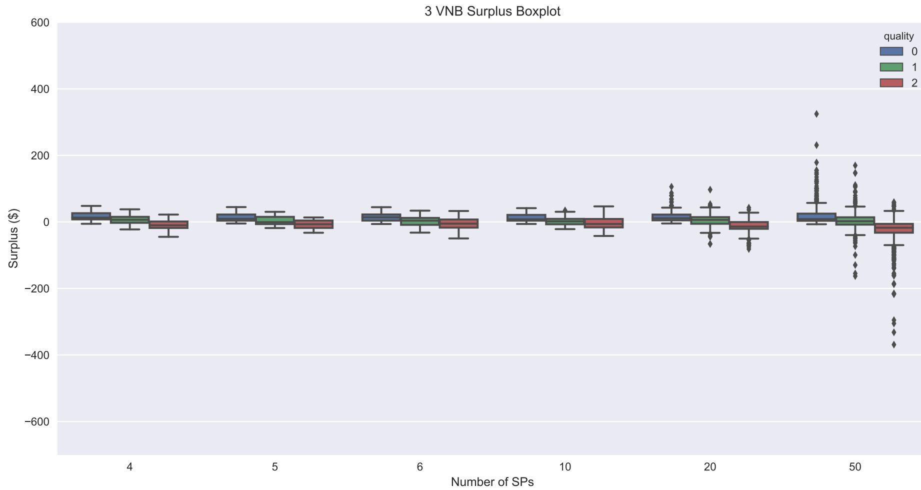


Figure 145: VNBs surplus distribution according to their reputation. Scenario with 3 VNBs

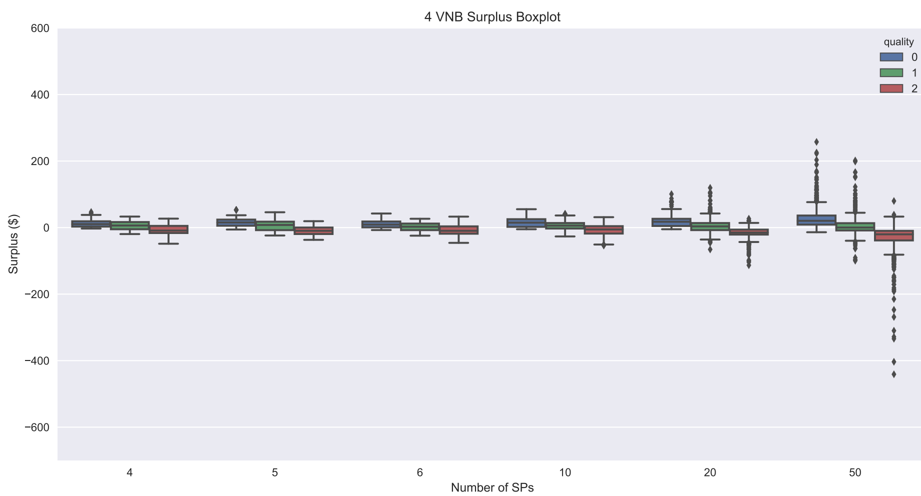


Figure 146: VNBs surplus distribution according to their reputation. Scenario with 4 VNBs

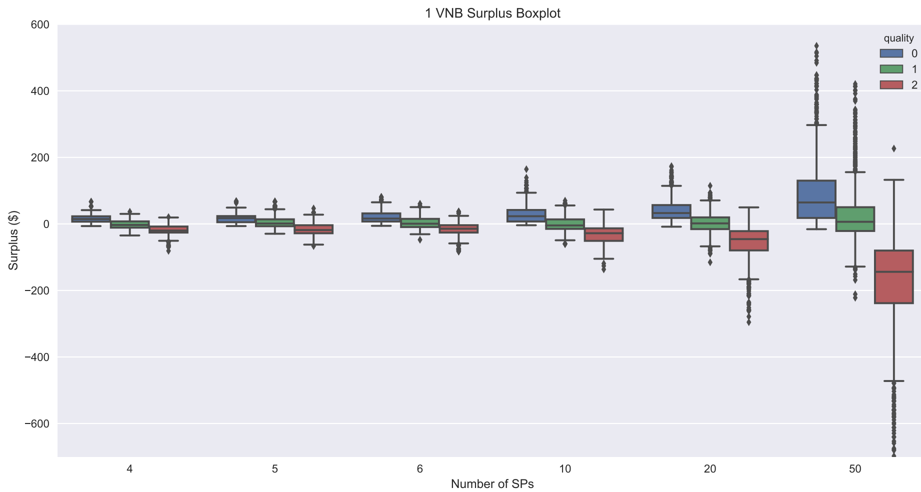


Figure 147: VNBs surplus distribution according to their reputation. Scenario with 1 VNB

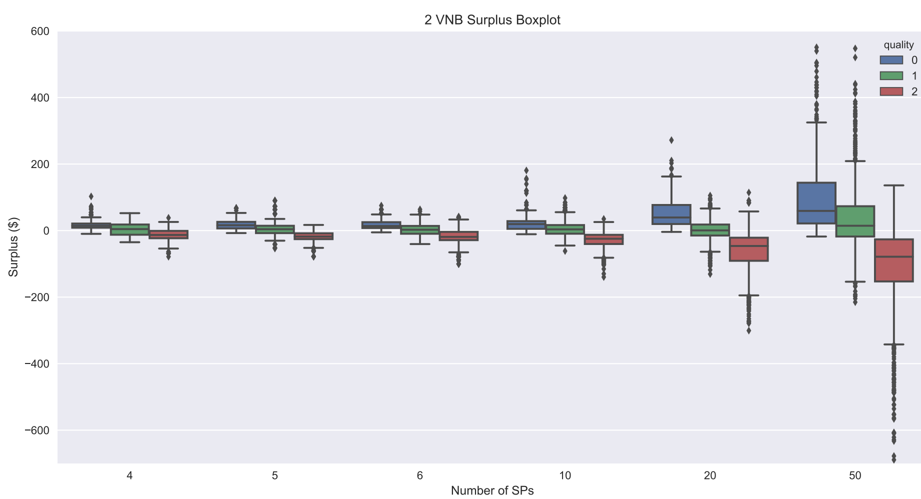


Figure 148: VNBs surplus distribution according to their reputation. Scenario with 2 VNBs

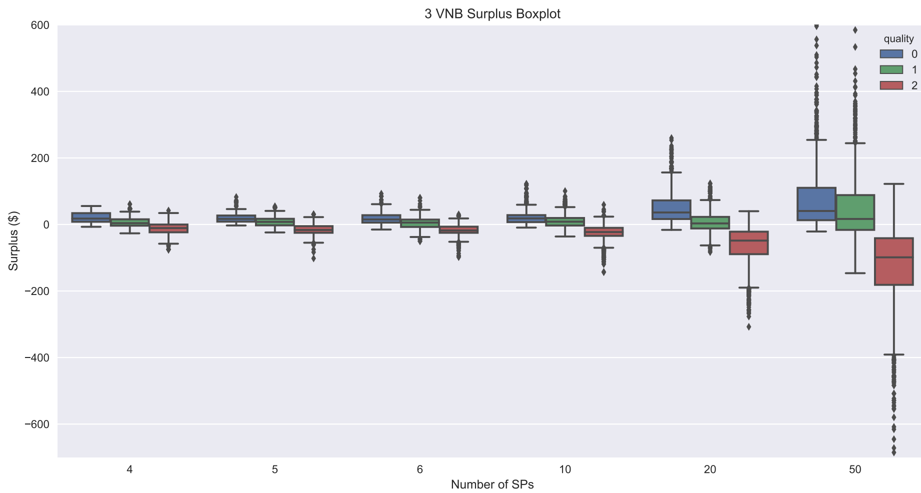


Figure 149: VNBs surplus distribution according to their reputation. Scenario with 3 VNBs

B.3 SCENARIO WITH 5 RPS

B.4 SCENARIO WITH 10 RPS

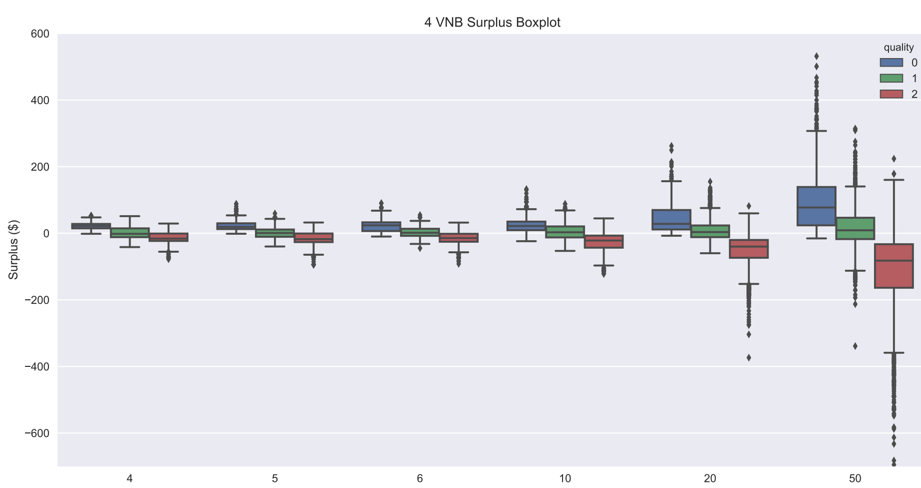


Figure 150: VNBs surplus distribution according to their reputation. Scenario with 4 VNBs

B.5 SCENARIO WITH 15 RPS

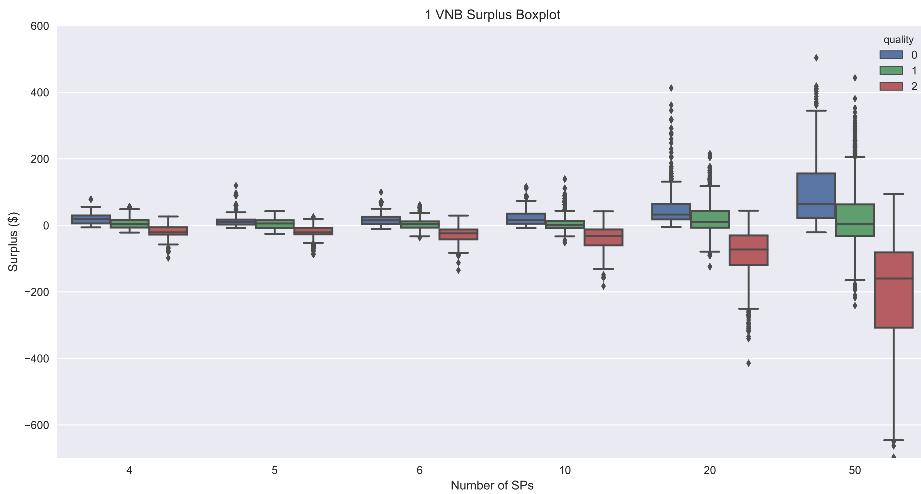


Figure 151: VNBs surplus distribution according to their reputation. Scenario with 1 VNB

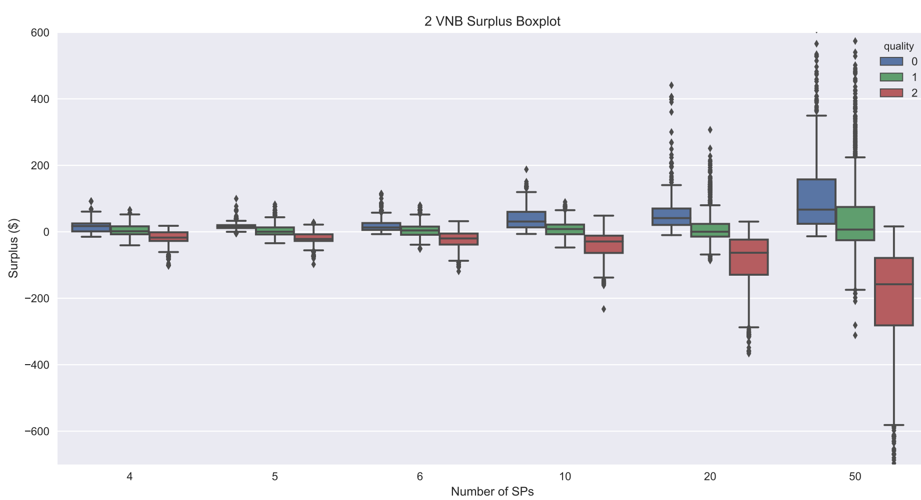


Figure 152: VNBs surplus distribution according to their reputation. Scenario with 2 VNBs

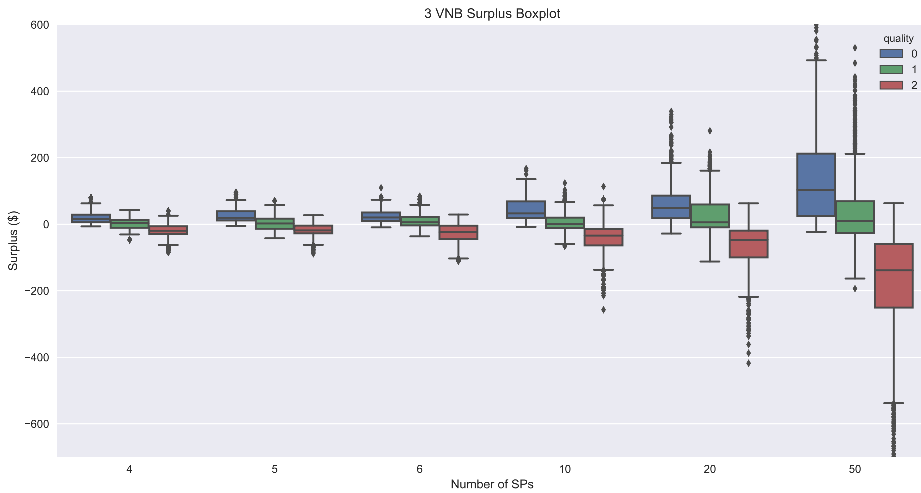


Figure 153: VNBs surplus distribution according to their reputation. Scenario with 3 VNBs

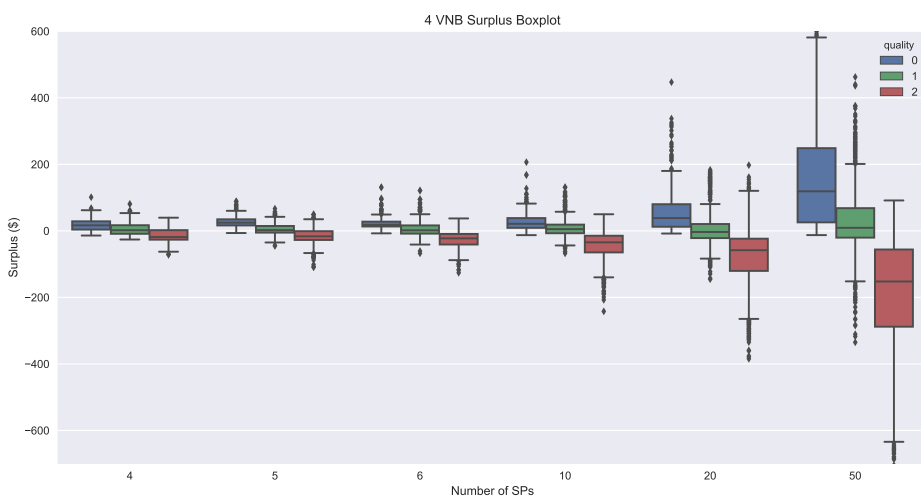


Figure 154: VNBs surplus distribution according to their reputation. Scenario with 4 VNBs

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