A METHOD OF IMPROVING THE RESPONSE OF RESISTANCE-TYPE VOLTAGE DIVIDERS FOR HIGH-VOLTAGE IMPULSE MEASUREMENT

By

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FOREWORD

The author wishes to express appreciation to Mr. S. B. Griscom and Mr. A. R. Hileman who were co-workers with the author in this investigation. The advice and assistance of Dr. C. F. Wagner and Mr. J. W. Skooglund is also gratefully acknowledged.

I. INTRODUCTION

Most of the electrical equipment on transmission or distribution lines is exposed to the hazard of lightning. It is necessary that the manufacturer be able to test equipment in the laboratory to make certain that the field performance of the equipment will be satisfactory.

The lightning generator, or surge generator, has been developed to produce voltages in the laboratory which simulate lightning voltages on transmission and distribution lines. A surge generator can have output voltages as high as several million volts. Since voltages of this magnitude cannot be measured directly, it is necessary to have a voltage divider capable of reducing this voltage to a level suitable for measurement.

The resistance-type voltage divider is the most useful and versatile divider for laboratory testing.[1] It has long been recognized, however, that the response of the resistance divider falls off as divider length and resistance are increased.[2]

An investigation was undertaken by the Westinghouse Electric Corporation to determine the sources of error which have the most effect in distorting the response of high-voltage, high-resistance dividers. The investigation was extended to include methods of compensating the resistance divider to overcome the principal sources of error without decreasing the total resistance of the divider.

The results of that investigation are described in this thesis.

II. RESPONSE CHARACTERISTICS OF A RESISTANCE-TYPE VOLTAGE DIVIDER

A. Theory of the Resistance-Type Voltage Divider

The schematic diagram of an ideal resistance divider is shown in Figure 1. The upper part of the divider is composed of a high-voltage resistor, usually of 2000 to 20,000 ohms. At the lower end of the high-voltage resistor is a voltage-reduction resistor, or tap-off resistor, and a co-axial cable. The cable is terminated in its characteristic impedance to prevent reflections within the cable.

As a voltage is applied across the divider, the voltage applied to the cable is

$$E_R = E_S \frac{R_D}{R + R_D} \qquad (1)$$

where E_S is the applied voltage, R is the resistance of the high-voltage resistors, and R_D is the resistance of the voltage-reduction resistor and the cable in parallel. The voltage at the end of the terminated cable is applied to the plates of a cathode-ray oscilloscope.

Unfortunately, the circuit elements shown in Figure 1 are not all the elements effective in determining the response characteristics of a resistance divider. The more accurate representation in Figure 2 shows the presence of series inductance, l_s, stray capacitance to ground, c_g, and distributed series capacitance, c_s'. The resistance of the high-voltage resistors also varies with frequency. While these sources of error are kept as low as possible, they will still influence divider response if the applied wave has sufficient high-frequency content.

The voltage-reduction resistor and the resistor terminating the cable must have negligible inductance and capacitance. They must also have a resistance which is constant over wide ranges of voltage and frequency. Since satisfactory resistors of this type are commercially available, it will be assumed that these resistors introduce no errors.

It will be assumed that attenuation in the co-axial cable is negligible.

B. Resistance Divider Selected for Study

The divider selected for detailed study is a 12,000 ohm, 12-foot resistance divider. The maximum withstand voltage is in the order of 1600 kilovolts. A schematic diagram of the physical form of this divider is shown in Figure 3. The high-voltage unit is composed of 12- 1000 ohm, one-foot resistor units. Each resistor is composed of two coils of high-resistance wire wound on a one-inch Micarta tube. The coils are wound in opposite directions to minimize the series inductance. Three resistors are placed in each of four three-inch Micarta tubes. The tubes are filled with oil to increase the voltage-withstand ability of the divider.

A voltage in the order of 1000 volts is a typical voltage to apply to the oscilloscope plates. The maximum current through the divider is $\frac{1,600,000}{12,000} = 133$ amperes. To obtain a voltage of about 1000 volts across R_D, $R_D \doteq 7.5$ ohms. R_D might be as large as 25 ohms if a suitable potentiometer were placed between the oscilloscope and the terminated end of the coaxial cable. The voltage divider ratio $\frac{R + R_D}{R_D}$ will be in the order of 500 to 1500.

The divider described above was selected as being typical of high-voltage, high-resistance dividers. Its response characteristics will be given along with methods for correcting errors in response. These methods are applicable for divider of length and resistance different from that chosen above.

C. Circuit Constants of the 12,000 Ohm, 12-Foot Resistance Divider

The circuit constants of Figure 2 were determined for the 12,000 ohm, 12-foot resistance divider shown in Figure 3.

The resistor units were constructed to have a resistance of 1000 ohms per linear foot. The series inductance, l_s, was measured on a General Radio R. F. Bridge; the average inductance was found to be about five microhenries per foot.

Bockman and Hylten-Cavallius, for a resistance divider of similar construction, showed that the effect of the stray series capacitance, c'_s, is unimportant in determining the response of resistance dividers.[3]

It was found that the high-voltage resistor units do not maintain a constant resistance as frequency is varied. The measured resistance-frequency characteristic is shown in Figure 4.

It has long been recognized that one of the principal sources of error affecting the response of physically long, high-resistance dividers is the stray capacitance to ground. Tests were conducted at the Westinghouse Trafford High-Voltage Laboratory to determine the magnitude of this stray capacitance.

Stray capacitance, by its nature, is a difficult quantity to measure. In the case of the voltage divider, direct measurement of the stray capacitance would be very questionable because of the effect of the measuring leads. An indirect measurement was made to determine this stray capacitance.

The laboratory test set-up is shown in Figure 5. The output of a signal generator was impressed across the full length of the divider. The resistor units were tied together electrically, but the bottom of the lower unit was isolated from ground. The applied voltage, E_S, and the voltage to ground at the bottom of the divider, E_R, were measured with a vacuum-tube voltmeter. The measured ratio $\frac{E_S}{E_R}$ is plotted as a function of frequency in Figure 6.

If the stray capacitance to ground is assumed to be uniformly distributed along the length of the divider, the divider can be treated as a transmission line with uniformly distributed constants. It is shown in Appendix I that the distributed stray capacitance to ground can be calculated from the $\frac{E_S}{E_R}$ ratio at a given frequency. The capacitance to ground calculated at five megacycles was 4.67 micromicrofarads per foot. This value of capacitance was used to calculate the $\frac{E_S}{E_R}$ ratio at 1, 2, and 3.5 megacycles. The excellent correlation between the measured and the calculated values is shown in Figure 6.

If the concept of the ABCD network is applied, the ordinate of Figure 6 is the A constant. It is important to note that the A constant of a transmission line is a more sensitive indication of the effect of shunt loads to ground than is the B constant.[4] Since the B constant will prove to be the most important in determining the response of a resistance divider, complete confidence is felt in response calculations based on the assumption of uniformly distributed stray capacitance to ground.

Figure 6 describes a 15,000 ohm, 15-foot resistance divider. The data for Figure 6 was not taken for the 12,000 ohm, 12-foot divider. The longer divider presents an even more convincing argument for the validity of assuming the stray capacitance to be uniformly distributed. The longer physical length would allow for greater variation in capacitance from top to bottom.

It is felt that for purposes of this study sufficient accuracy will be obtained by assuming a uniformly distributed capacitance to ground of five micromicrofarads per foot.

D. Frequency Response of the 12,000 Ohm, 12-Foot Resistance Divider

One of the most illuminating indications of divider response is its frequency response characteristic. Knowledge of this characteristic gives considerable insight into the relative effects of inductance, resistance variation with frequency, and stray capacitance to ground. Figure 7 shows the calculated frequency response curves for a 12,000 ohm, 12-foot resistance divider. The effects of inductance, resistance variation with frequency, and stray capacitance to ground are considered separately (see Appendix II).

Curve A of Figure 7 shows the divider response considering only the presence of the measured five microhenries per foot. Curve B of Figure 7 shows the divider response considering only the resistance variation with frequency shown in Figure 4. Curve C of Figure 7 shows the effect on divider response of a uniformly distributed capacitance to ground of five micromicrofarads per foot.

Frequency Response Index, the ordinate of Figure 7, is defined as the ratio of voltage across the voltage-reduction resistor to the voltage that would exist if the divider were perfect in response.

Four principal assumptions were made in deriving the curves of Figure 7: (a) the divider is assumed to feed a resistive load, R_D; (b) R_D is assumed to be very small compared to the resistance of the high-voltage units; (c) the voltage across R_D, hence the current through R_D, is used as a measure of response; and (d) the inductance and the stray capacitance to ground are assumed to be uniformly distributed. Each of these assumptions has either been discussed earlier or is discussed in Appendix II.

Table I has been prepared from Figure 7 to show the frequencies at which inductance, stray capacitance to ground, and resistance variation with frequency become effective.

Table I

Source of Error	Frequency in Mc at 90% Response
Series Inductance	15.3
Resistance Variation with Frequency	4.2
Stray Capacitance to Ground	0.95

It is important to note from Table I and from Figure 7 that stray capacitance to ground exercises the greatest influence on the divider response. Much better response would be possible if the effect of stray capacitance could be overcome.

The frequency response of the 12,000 ohm, 12-foot resistance divider has been calculated considering all three sources of error listed in Table I (see Appendix II). This calculated response is shown as Curve A of Figure 8. Curve B of Figure 8 is a re-plot of Curve C of Figure 7 and includes only the presence of stray capacitance to ground. The very small difference between Curves A and B offers more graphic illustration of the predominance of the stray capacitance compared to the other sources of error.

Laboratory tests were undertaken to verify the calculated response curves for the 12,000 ohm, 12-foot divider. The laboratory test set-up is that shown in Figure 5 with S closed and R_D = 200 ohms. Voltages E_S and E_R were measured over wide ranges of frequency. The experimental frequency response characteristic is shown as Curve C of Figure 8. The very close correlation between the calculated and measured curves of Figure 8 gives confidence in the analytical determination of response characteristics.

E. Voltage Distribution along a Resistance Divider

A concept of the influence of stray capacitance to ground on the response of a resistance divider can be obtained by considering the voltage distribution along the divider at different frequencies. With a direct applied voltage the effect of the stray capacitance to ground is eliminated; the voltage distribution is linear because of the linearly distributed resistance. This linear voltage distribution is shown as Curve A of Figure 9. Curve B shows the calculated voltage distribution at five megacycles for the 12,000 ohm, 12-foot resistance divider. Curve B includes only the presence of resistance and of stray capacitance to ground (see Appendix III).

It is shown in Appendix III that the series current along the divider is proportional to the slope of the voltage distribution curve. It is stated in Section II(D) that the current from the lower end of the divider can be used as a measure of response. Figure 9 shows that the voltage distribution curves for d-c and for five megacycles differ in slope by a ratio of about three to one at the lower end of the divider. Figure 7 verifies that the response at five megacycles is only about 33 per cent of the response at very low frequencies.

It can be seen that frequency response errors caused by stray capacitance can be overcome by correcting the voltage distribution along the divider.

III. METHODS OF COMPENSATING THE RESISTANCE DIVIDER TO OVERCOME THE EFFECT OF STRAY CAPACITANCE TO GROUND

A. Decreasing the Total Length or Total Resistance of the Divider

Obviously, the effect of stray capacitance to ground can be decreased by decreasing the length of the divider. This is equivalent to lowering the magnitude of the total capacitance to ground. The required minimum length of the divider is usually fixed by the maximum voltage to be measured. For the 12-foot divider being considered, it would be impossible to reduce the length appreciably without causing the divider to flash over at the rated 1600 kilovolts.

One very effective method for minimizing the effect of stray capacitance is to lower the total series resistance of the divider. It is shown in Appendix II that the divider response for a given frequency and a given length is determined by the product RxC_g, where R is the total divider resistance and C_g is the total stray capacitance to ground. (The previous statement applies only for the case where stray capacitance to ground is the principal source of error.) It is possible to obtain better response characteristics by reducing the total series resistance, hence making the RxC_g product smaller.

Discretion must be used in attempting to correct divider response by decreasing the series resistance. The lower limit of divider resistance might be fixed by the ability of the test circuit to supply the divider current. To illustrate this with an extreme case, assume that the divider resistance has been decreased to 1000 ohms and that the surge generator has a total series capacitance of 0.02 microfarads. The discharge time constant of the test circuit would be 1000(0.02) = 20 microseconds. A normal output voltage wave would have a tail that reached one-half crest voltage in about 15 microseconds. This wave would be of little value for apparatus testing since the standard test wave has a tail reaching one-half crest voltage in 40 microseconds.[1]

Even if the loading on the measuring circuit is not a criterion, there is a limit to how far the divider resistance can be decreased to gain better response. Even if the effect of stray capacitance has been overcome, the series inductance is still present to cause errors. If the resistance is halved, for example, without a decrease in the series inductance, all of the frequency coordinates for Curve A of Figure 7 will be halved. If this decrease of resistance is carried to extremes without commensurate lowering of total series inductance, the response errors can become as serious as those originally caused by stray capacitance to ground.

Because decreasing the total divider resistance does cause greater loading on the measuring circuit, no further discussion will be made concerning this possible means of overcoming the effect of stray capacitance to ground. It should be emphasized, however, that, within limits, better response can be obtained by decreasing the series resistance of high-resistance dividers.

B. Use of Grading Shields or Use of Large Capacitors Shunting the Resistor Units

One method of overcoming the effect of stray capacitance to ground is the use of a grading shield at the high-voltage end of the divider. If the dimensions of the shield are properly chosen, the electrostatic gradient along the divider can be made almost uniform. The voltage distribution curve of the divider can be made to approach Curve A of Figure 9, even for very high frequencies. Figure 10 shows the good distribution that can be obtained by careful selection of the grading shield. (Figure 10 was extracted from an article by Bockman and Hylten-Cavallius.[3])

Another method of overcoming the effect of stray capacitance to ground is to parallel the resistor units with capacitors which are large enough to completely overwhelm the stray capacitance to ground. An example of this type of compensation is the divider used by Siemens-Schuckertwerke at the Nurnberg high-voltage laboratory.[5] This divider exceeds 25 feet in length and has a series resistance of about 7000 ohms. Across each of the six resistor units is placed a coupling capacitor of 0.0018 microfarads. This high value of shunting capacitance almost completely overcomes the effect of stray capacitance to ground.

Both of the methods of compensation just described have the disadvantage of introducing a large capacitance to ground viewed from the high-voltage end of the divider. This large capacitance could conceivably have two undesirable effects. The divider is connected directly across the test piece whose voltage is desired; hence the large capacitance to ground might make it impossible to obtain voltages with large high-frequency content. The divider, then, might make it impossible to obtain the steep voltage waves which it was designed to measure. The second disadvantage of using a divider with a large capacitance to ground is the ease with which this capacitance can be shocked into oscillation with inductance in the test-circuit leads.

It should be noted that the large capacitance to ground introduced by the two schemes described above affects the part of the wave having large high-frequency content. Decreasing the total divider resistance, discussed in the previous section, affects primarily the low-frequency tail of the wave.

Compensation by use of a properly proportioned grading shield or by use of large shunting capacitors is an effective method of equalizing the voltage gradient along the divider. The undesirable effects of the large capacitive loading should be investigated closely for each particular application.

C. Tapered-Capacitance Compensation

The reason for the non-linear voltage distribution along a resistance divider is that the charging current for the stray capacitance to ground must be supplied through the series resistance. This error can be corrected by shunting the series resistance with distributed capacitance which is just sufficient to carry the charging current for all the units below. If this compensation is performed correctly, the voltage distribution along the divider will be uniform.

To better understand the theory of tapered-capacitance compensation, it is convenient at first to ignore the presence of the series resistance. The distributed series capacitance, c_s, can be determined such that in the presence of uniformly distributed capacitance to ground, c_g, the voltage distribution along the capacitive chain will be uniform. Once this compensation has been performed, uniformly distributed series resistance can be introduced without causing a difference in voltage distribution.

Appendix IV shows a derivation of the minimum amount of distributed series capacitance that will provide uniform voltage distribution. This capacitance distribution is shown in Curve A of Figure 11 for a 12-foot capacitance chain. It is obvious that once compensation has been obtained by applying Curve A of Figure 11, the voltage distribution will not be upset if the distributed capacitance is paralleled by additional uniformly distributed capacitance. A typical series capacitance distribution is shown as Curve B of Figure 11, where the series capacitance exceeds the minimum shown in Curve A.

When the capacitive loading introduced by the divider is critical, the load should be kept as small as possible by correcting along Curve A of Figure 11. It is shown in Appendix IV that the minimum capacitive load viewed from the high-voltage end of the divider is $\frac{Lc_g}{2}$, where L is the total length of the divider.

If very low capacitive loading is not required, it might be desirable to distribute the series capacitance along a curve such as Curve B, Figure 11. If c_g is not exactly uniformly distributed, the additional series capacitance above the minimum would help minimize resulting errors in voltage distribution.

In a practical application, the compensated divider must supply current, I_R, to some load; the voltage across that load must be indicative of the voltage applied to the divider. It is of importance, then to determine the current from the bottom of the high-voltage resistor. It is shown in Appendix IV that the current from the bottom end of the capacitive chain is zero when correction has been obtained from Curve A, Figure 11. The compensated divider should feed a purely resistive load since the capacitive component of I_R is zero.

If the distribution curve lies above Curve A, for example along Curve B of Figure 11, there will be a capacitive current flow from the bottom of the capacitive chain. If this current is to be used as an indication of applied voltage, it must be fed into a capacitive load. The compensated divider must feed a load composed of a resistor and a capacitor in parallel. The relative magnitudes of the resistor and capacitor must be so chosen that there will be no interchange of resistive and capacitive currents between the two.

It is desired to apply the principles of tapered-capacitance compensation to a 12-foot divider with a uniformly distributed capacitance to ground of five micromicrofarads per foot. Figure 12 shows the theoretical minimum curve for the required variation of series capacitance. The total capacitive loading viewed from the high-voltage terminal of the divider is 30 micromicrofarads. The capacitive current fed from the bottom of the divider is zero.

It is of interest to compare the results of tapered-capacitance compensation with those obtained by adding uniformly distributed series capacitance of 500 micromicrofarads per foot. The loading of the capacitive chain has been increased to $\frac{500}{12} = 41.6$ micromicrofarads for an increase of almost 40 per cent. The voltage distribution along the capacitance chain is still found to be non-uniform, Figure 13 (see Appendix V). It is apparent, then, that tapered-capacitance compensation makes possible better voltage distribution with smaller loading than can be obtained with uniformly distributed series capacitance.

The principles of tapered-capacitance compensation will be applied to the 12,000 ohm, 12-foot resistance divider. The response characteristics of the compensated and the uncompensated dividers will be compared.

IV. A 12,000 OHM, 12-FOOT RESISTANCE DIVIDER WITH TAPERED-CAPACITANCE COMPENSATION

A. Electrical Characteristics

The theory of tapered-capacitance compensation has now been established. It remains to utilize this theory in designing a practical divider for high-voltage surge measurements.

The principal problem is to obtain the series capacitance distribution shown in Figure 12. It is impossible to obtain truly distributed capacitance by the use of lumped capacitors. Judgement must be used in deciding how closely the actual capacitance distribution curve must follow the theoretical curve. It was felt that adequate response characteristics could be obtained by using three lumped capacitors to represent the theoretical curve. Each capacitor would be connected across only one-fourth of the total divider. Figure 14 shows the theoretical distribution and the actual distribution selected for study. The inset of Figure 14 shows the manner in which the series capacitors would be applied to a 12,000 ohm, 12-foot resistance divider.

It will be noted that no capacitor is shown across the lower one-fourth of the divider. The capacitor required for this lower unit is about three micromicrofarads. No attempt was made to add a capacitor this small in magnitude.

The smooth capacitance distribution curve of Figure 14 has now been synthesized by the addition of only three lumped capacitors. Because of the approximation involved, there will be interchange of currents between the resistance and the capacitance chains. It might be expected that the current out of the bottom of the divider would have both a resistive and a capacitive on the response characteristics of the compensated divider than for the uncompensated divider where no capacitors are available to supply the corona current.

It is felt that the response characteristics shown in Figure 15 are excellent. Better response characteristics could be obtained only at the expense of increased loading or considerably more a complicated physical structure. It was decided, therefore, that the resistance divider with tapered-capacitance compensation shown in Figure 14 satisfied the purposes of this investigation.

B. Physical Form

The physical form of the 12,000 ohm, 12-foot resistance divider with tapered-capacitance compensation is shown in Figure 16. The high-voltage resistors and the lower three oil-filled Micarta tubes are the same as those described in Section II(B). The top Micarta tube is four inches in diameter.

The series capacitors which shunt the high-voltage resistors are made up of series-parallel combinations of smaller capacitors. The small capacitors are cylindrical in shape, about one inch in diameter and one inch in length. The capacitors are of the ceramic type; the dielectric has a high permissible gradient and a high dielectric constant.

The resulting physical structure is scarcely less complicated than that of the uncompensated resistance divider.

V. SUMMARY

The response of a resistance divider is affected by the presence of series inductance, resistance variation with frequency, and stray capacitance to ground. The response of high-voltage, high-resistance dividers is affected primarily by the stray capacitance to ground.

The response of a resistance divider of given length can be improved by decreasing the total series resistance. If the series inductance cannot be decreased along with the resistance, caution must be exercised to prevent serious errors arising because of the effect of the series inductance. Extreme lowering of total resistance might also cause excessive loading on the measuring circuit.

The effect of stray capacitance to ground can be overcome by use of a suitable grading shield or by use of large capacitors shunting the individual resistor units. Both of these methods cause a large capacitive load on the test circuit. This capacitive load might cause serious difficulties when voltages with appreciable high-frequency content are being measured.

The most satisfactory method for overcoming the effect of stray capacitance to ground is the use of tapered-capacitance compensation. This method of compensation makes possible uniform voltage distribution down the divider, along with good response characteristics. These desirable aims are accomplished without imposing a large capacitive load on the measuring circuit.

The method of tapered-capacitance compensation was applied to a 12,000 ohm, 12-foot resistance divider. The response characteristics were found to be much better than those for the uncompensated divider. The method of taper-capacitance compensation has been proved to be a satisfactory method of improving the response characteristics of high-resistance, high-voltage dividers without materially increasing the loading on the measuring circuit.

APPENDIX I

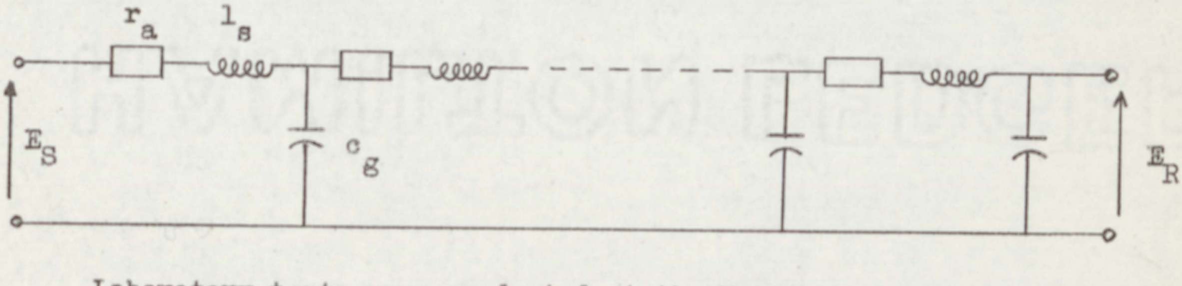
Determination of Distributed Capacitance to Ground for a 15-Foot Resistance Divider

Consider a 15-foot resistance divider with a distributed d-c resistance of 1000 ohms per foot. It is desired to find the distributed capacitance to ground.

An alternating voltage, E_S, is applied across the divider. If the lower end of the divider is isolated from ground, the voltage to ground at the lower end of the divider, E_R, can be calculated in terms of E_S and the circuit constants.[4]

$$\left|\frac{E_S}{E_R}\right| = \left|\cosh \sqrt{(R_a + j\omega L_s)\ (j\omega C_g)}\right| \qquad (1)$$

R_a is the total a-c resistance from Figure 4, ωL_s is the total inductive reactance, and ωC_g is the total shunt capacitive susceptance. These quantities per unit length are shown in the figure below.



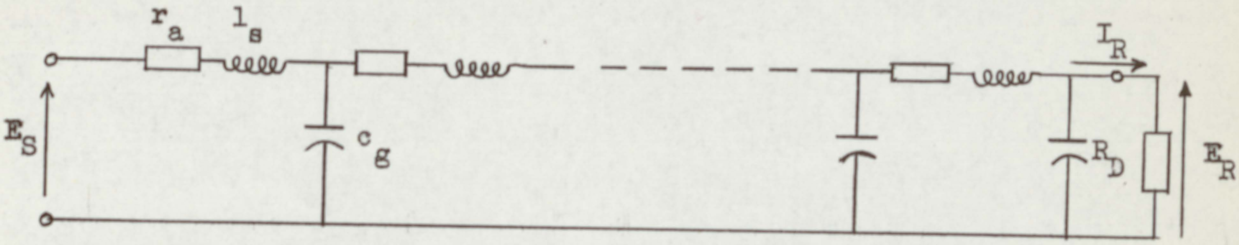
Laboratory tests were conducted at the Westinghouse Trafford High-Voltage Laboratory to determine the $\frac{E_S}{E_R}$ ratio of Equation (1). The output of a signal generator was applied as E_S. Voltages E_S and E_R were measured with a vacuum-tube voltmeter. The measured $\frac{E_S}{E_R}$ ratio at a given frequency caused all the quantities of Equation (1) to be known except C_g. At a frequency of five megacycles with $L_s = 75 \times 10^{-6}$ henries, Equation (1) was solved for C_g. C_g was found to be 70 micromicrofarads or c_g = 4.67 micromicrofarads per foot.

Figure 6 has been plotted to give the experimental $\frac{E_S}{E_R}$ ratios. By use of Equation (1) and the derived value of c_g = 4.67 micromicrofarads per foot, the $\frac{E_S}{E_R}$ ratio has been calculated at frequencies of 1, 2, and 3.5 megacycles. These results are also plotted in Figure 6.

APPENDIX II

Frequency Response of a Resistance Divider

Consider a divider network feeding a resistive load, R_D, as shown below. R_a is the total a-c resistance of the high-voltage resistors = Lr_a, L is the total length of the divider, L_s is the total series inductance of the high-voltage units = Ll_s, and C_g is the total stray capacitance to ground = Lc_g. The resistance, inductance, and capacitance are assumed to be uniformly distributed along the divider.



Let an alternating voltage, E_S with frequency f, be applied to the divider; let E_R, the voltage across R_D, be the indication of divider response. It is convenient to represent the network above by its equivalent ABCD network.[4] From the fundamental equations of the ABCD network,

$$E_S = AE_R + BI_R. \tag{1}$$

But $I_R = \frac{E_R}{R_D}$ hence,

$$E_S = E_R \left(A + \frac{B}{R_D}\right), \text{ or}$$

$$E_R = E_S \frac{R_D}{AR_D + B}. \tag{2}$$

If the divider were ideal, see Figure 1,

$$E'_R = E_S \frac{R_D}{R + R_D}, \tag{3}$$

where R is the total d-c resistance of the high-voltage units. The Frequency Response Index is

$$F.\,R. = \frac{E_R}{E'_R} = \frac{\frac{R_D}{R} + 1}{\frac{AR_D}{R} + \frac{B}{R}}\,. \tag{4}$$

It is shown in Section II(B) that $\frac{R_D}{R}$ is in the order of 0.001. This ratio will be assumed small enough to be neglected in Equation (4). Equation (4) then becomes

$$F.\,R. = \frac{R}{B} \tag{5}$$

Now let r_a be determined from Figure 4 for the nominal 1000 ohm resistors, l_s = five microhenries per foot, c_g = five micromicrofarads per foot, L = 12 feet, and R = 12,000 ohms.

If the inductance is considered as the only source of error, B = R + $j\omega L_s$. The Frequency Response Index becomes

$$F.\,R. = \frac{1}{1 + \frac{j\omega L_s}{R}}. \tag{6}$$

For the circuit constants given above, Equation (6) is plotted as a function of frequency in Figure 7.

If the only source of error is assumed to be the variation of resistance with frequency, then B = R_a. Equation (5) becomes

$$F.\,R. = \frac{R}{R_a}\,. \tag{7}$$

For the circuit constants given above, Equation (7) is plotted as a function of frequency in Figure 7.

If the only source of error is assumed to be uniformly distributed stray capacitance to ground, $B = \sqrt{\frac{R}{j\omega C_g}}\,\sinh\sqrt{j\omega RC_g}$.[4]

Equation (5) becomes

$$F.\ R. = \frac{\sqrt{j\omega RC_g}}{\sinh\sqrt{j\omega RC_g}} . \tag{8}$$

For the circuit constants given above, Equation (8) is plotted as a function of frequency in Figure 7.

If all three sources of error are considered to be acting together, the B constant becomes[4]

$$B = \sqrt{\frac{R_a + j\omega L_s}{j\omega C_g}}\ \sinh\sqrt{(R_a + j\omega L_s)\ (j\omega C_g)} . \tag{9}$$

The Frequency Response Index is found by inserting Equation (9) into Equation (5). The resulting response for the circuit constants given above is plotted as a function of frequency in Figure 8.

It should be noted that Equation (5) is exactly the same as the equation that would be derived if $R_D = 0$ and I_R were used as the indication of response. Illustration of this is shown below.

$$I_R = \frac{E_S}{B}$$

$$F.\ R. = \frac{E_S}{B} \cdot \frac{R}{E_S} = \frac{R}{B} \tag{10}$$

APPENDIX III

Voltage Distribution along a Resistance Divider as Determined by Stray Capacitance to Ground

Consider a resistance divider with uniformly distributed series resistance, r, and uniformly distributed capacitance to ground, c_g. If an alternating voltage of 1.0 is applied to the divider, it can be shown that[6]

$$e = \cosh x\sqrt{j\omega c_g r} - \frac{\cosh L\sqrt{j\omega c_g r}}{\sinh L\sqrt{j\omega c_g r}} \sinh x\sqrt{j\omega c_g r}, \qquad (1)$$

where x is the distance along the divider measured from the top and e is the voltage at point x. Equation (1) is plotted in Figure 9 for L = 12 feet, c_g = five micromicrofarads per foot, r = 1000 ohms per foot, and $f = \frac{\omega}{2\pi}$ = five megacycles.

One of the differential equations describing the above network is

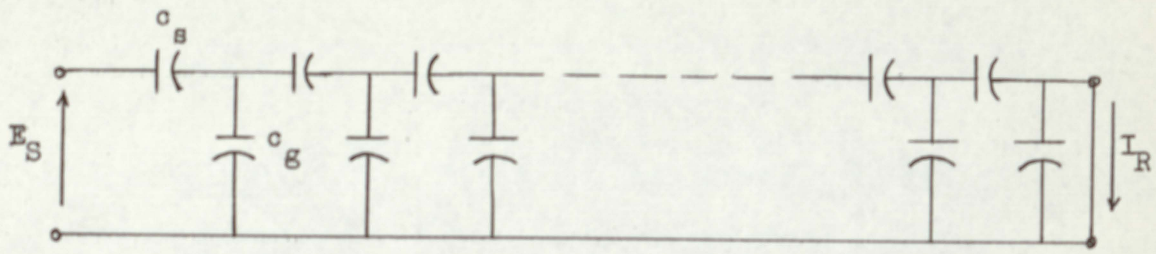
$$\frac{de}{dx} = - ri, \qquad (2)$$

where i is the series current at point x along the divider.[6]

APPENDIX IV

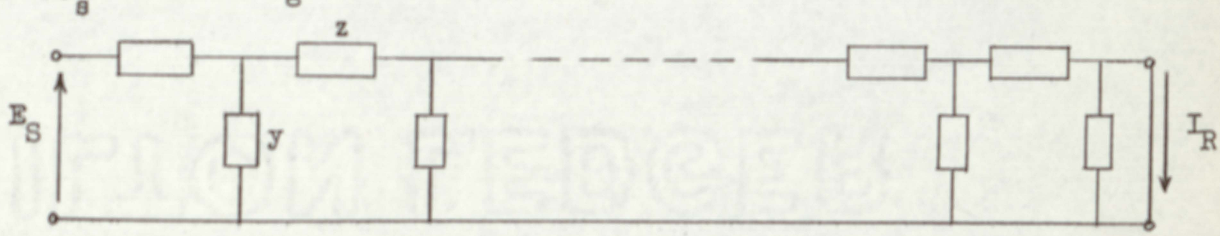
Method of Applying Tapered-Capacitance Compensation

Consider a capacitance chain having uniformly distributed capacitance to ground and distributed series capacitance. This chain is shown in the figure below.



where c_s is the distributed series capacitance times unit length, c_g is the uniformly distributed capacitance to ground per unit length, L is the total length, and x is the distance from the sending end of the chain. Let E_S be an applied alternating voltage and I_R be the current out of the shorted end of the chain.

It is convenient to alter the figure above to that shown below with $z = -j\frac{1}{\omega c_s}$ and $y = j\omega c_g$.



The differential equations relating voltage, e, and current, i, can now be written

$$\frac{di}{dx} = - ey, \tag{1}$$

$$\frac{de}{dx} = - iz. \tag{2}$$

Let it now be required to find the variation of z, hence c_s, required to cause uniform voltage distribution along the capacitance chain. This condition requires that

$$\frac{de}{dx} = - \frac{E_S}{L} , \tag{3}$$

or

$$e = E_S \left(1 - \frac{x}{L}\right). \tag{4}$$

Equations (1) and (4) can be used to find i:

$$\frac{di}{dx} = - yE_S \left(1 - \frac{x}{L}\right),$$

$$i = \frac{yE_S}{2L} (x^2 - 2Lx + K), \tag{5}$$

where K is an arbitrary constant of integration.

If Equations (3) and (5) are inserted into Equation (2),

$$- \frac{E_S}{L} = - \frac{2yE_S}{2L} (x^2 - 2Lx + K),$$

or

$$\frac{1}{z} = \frac{y}{2} (x^2 - 2Lx + K). \tag{6}$$

But since $\frac{1}{x} = j\omega c_s$ and $y = j\omega c_g$,

or

$$\frac{c_s}{c_g} = \frac{x^2 - 2Lx + K}{2} . \tag{7}$$

Equation (7) makes it possible to determine the required distribution of c_s for any known c_g and any previously selected K.

The significance of the factor K must be discussed further. The only condition that has been imposed on the solution of Equations (1) and (2) is that the voltage distribution along the chain be uniform. There is one other physical limitation which must be imposed; that is that $C_s \geq 0$. Equation (7) shows that $K \geq L^2$ satisfies this physical requirement. The meaning of the factor K can best be visualized by considering i at x = 0.

$$i_0 = \frac{yE_S}{2L} K. \tag{8}$$

The total capacitance loading viewed at x = 0 is

$$C_L = \frac{i_0}{E_S j\omega} = \frac{c_g}{2L} K \tag{9}$$

$$C_L \geq \frac{c_g L}{2} \tag{10}$$

Equation (10) shows that if $K = L^2$, the capacitive loading viewed from x = 0 is the minimum that can be obtained with a uniform voltage distribution. It is also apparent that uniform voltage distribution can be obtained with $K > L^2$. It is convenient to consider the factor K to be composed of two components, $L^2 + NL^2$. The quantity $N \frac{c_g L}{2}$ represents the amount by which the capacitive load exceeds the minimum value $\frac{c_g L}{2}$. The factor N represents this additional loading in multiples of sub-multiples of the minimum load. Equation (9) then becomes

$$C_L = \frac{c_g L}{2} (1 + N) \tag{11}$$

With $K = L^2 (1 + N)$, Equation (8) becomes

$$\frac{c_s}{c_g} = \frac{(x - L)^2}{2} + \frac{NL^2}{2} \tag{12}$$

The term $\frac{(x - L)^2}{2}$ gives the variation of c_s necessary to provide uniform voltage distribution along the capacitive chain. The second term shows merely that once uniform voltage distribution has been obtained, the distribution is not upset by the addition of uniformly distributed series capacitors.

It is important to consider the current I_R at $x = L$ through the shorted end of the chain. Substituting $x = L$ into Equation (5) gives

$$I_R = \frac{yE_S}{2L} (K - L^2) = \frac{yE_S NL}{2} . \qquad (13)$$

Equation (13) shows that when compensation has been performed to give the minimum circuit loading, there is no current at the lower end of the string. When additional uniformly distributed capacitors are added, there is capacitive current flow from the bottom of the capacitive chain.

APPENDIX V

Voltage Distribution along a Capacitive Chain Composed of Uniformly Distributed Series and Shunt Capacitors

Assume c_s = 500 micromicrofarads for one foot and c_g = five micromicrofarads per foot, with the quantities defined as in Appendix VII. It can be shown that with 1.0 unit of voltage applied and the lower end of the string grounded, it can be shown[6] that

$$e = \cosh x\sqrt{\frac{c_g}{c_s}} - \frac{\cosh L\sqrt{\frac{c_g}{c_s}}}{\sinh L\sqrt{\frac{c_g}{c_s}}} \sinh x\sqrt{\frac{c_g}{c_s}} \tag{1}$$

Equation (1) is plotted in Figure 13 for the vaÌes of capacitance given above and for a network 12 feet in length.

REFERENCES

1. Standards for Dielectric Testing, American Standards Association, Proposed Revision No. 4, (Committee Report).

2. The Measurement of High Surge Voltages, P. L. Bellaschi. AIEE Transactions, volume 52, 1933, pages 544-567.

3. Errors in Measuring Surge Voltages by Oscilloscope, Marius Bockman, Nils Hylten-Cavallius. Technical Achievements of ASEA Research (Vasteras, Sweden), March, 1946, pages 7-21.

4. Electrical Transmission and Distribution Reference Book (book). R. R. Donnelley and Sons, Chicago, Illinois, 1950.

5. The New High-Voltage Laboratory of Siemens-Schuckertwerke in Nurnberg (pamphlet), Richard Elsner. Siemens-Schuckertwerke, Germany, September, 1952.

6. Advanced Mathematics for Engineers (book), H. W. Reddick, F. H. Miller. John Wiley and Sons, Inc., New York, N. Y., 1947.

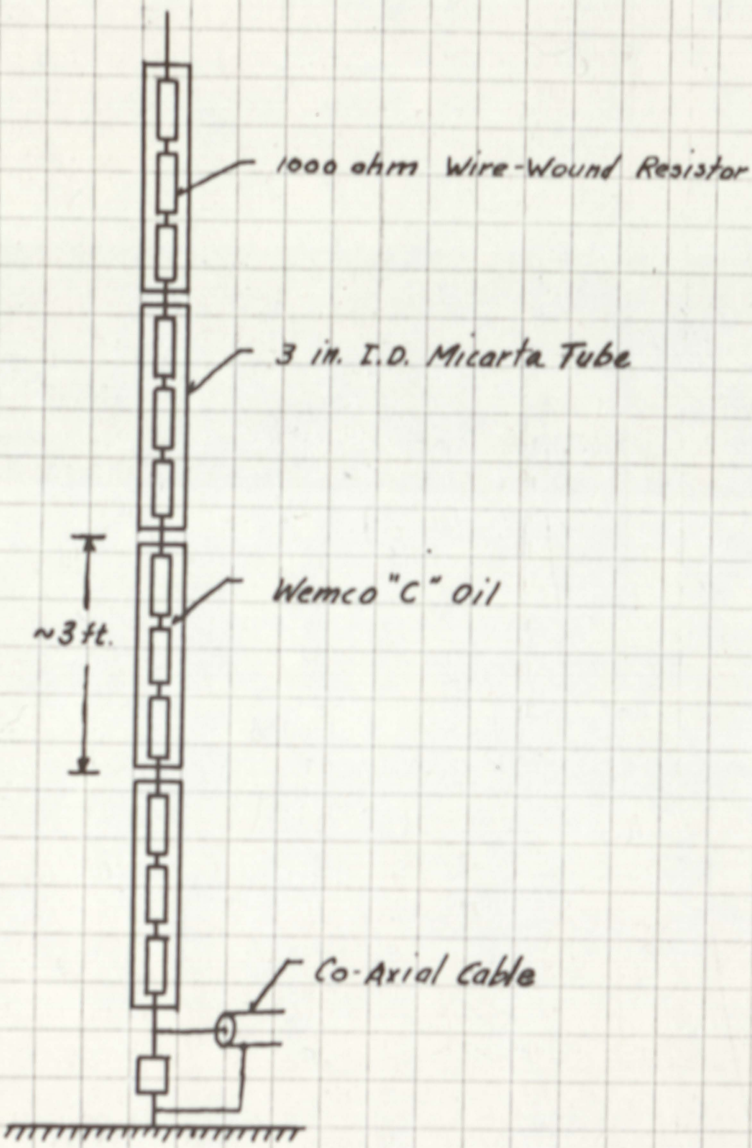


Figure 3. Physical form of a 12,000 ohm, 12-foot resistance divider.

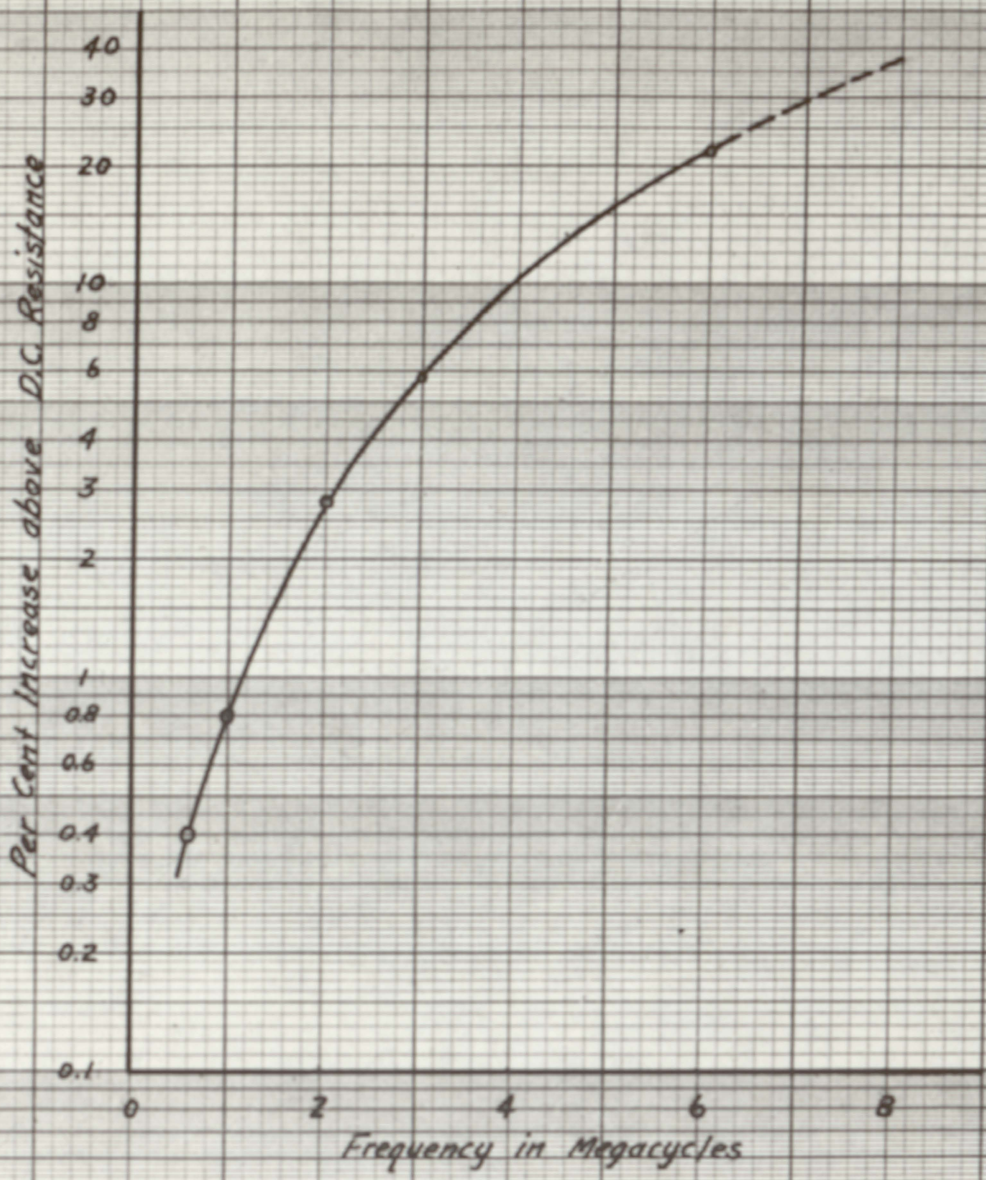


Figure 4. Variation of resistance with frequency for a 1000 ohm, 1-foot, high-voltage resistor. This variation applies for a 12,000 ohm, 12-foot resistance divider.

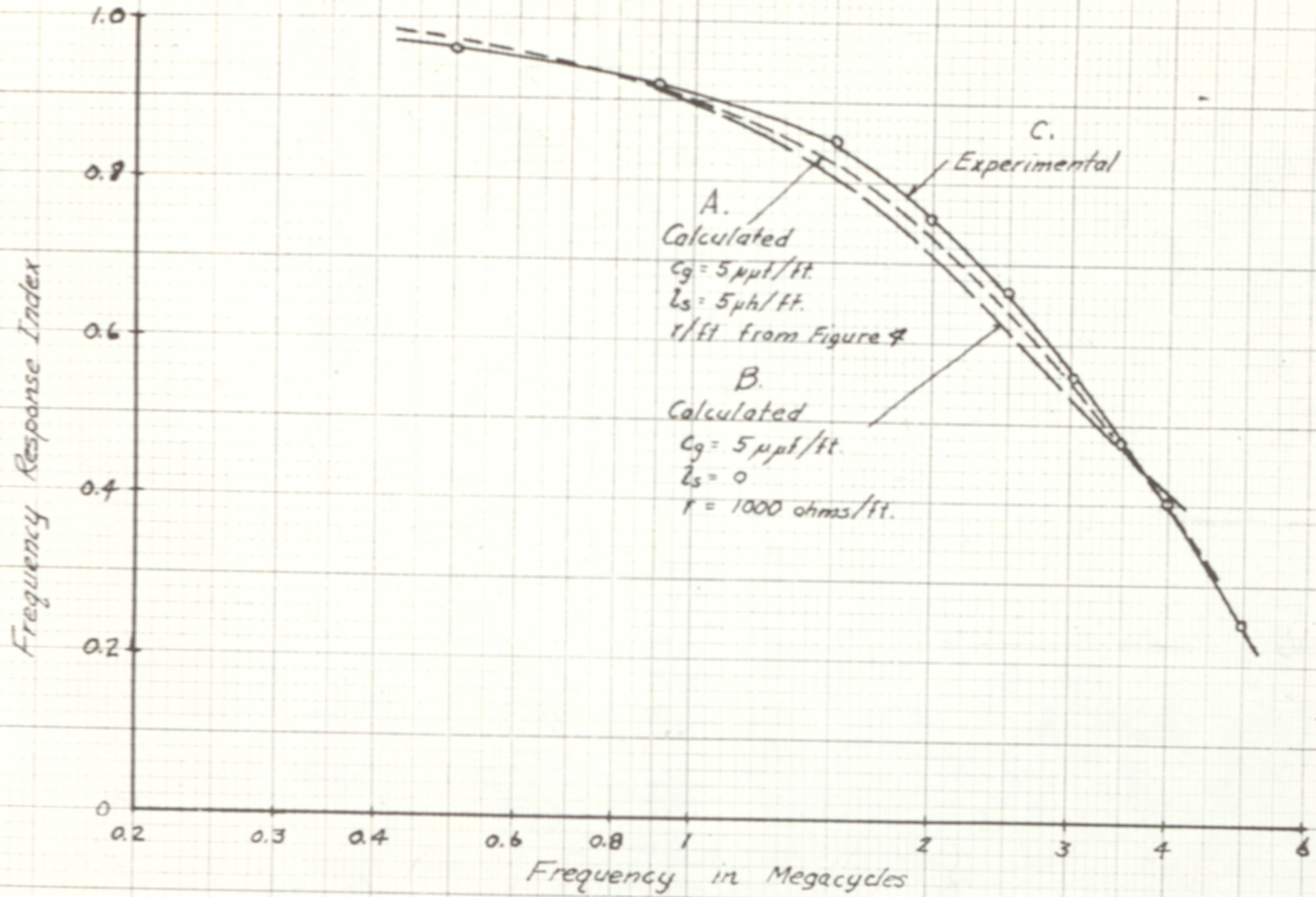


Figure 8. Measured and calculated frequency response of a 12,000 ohm, 12-foot resistance divider.

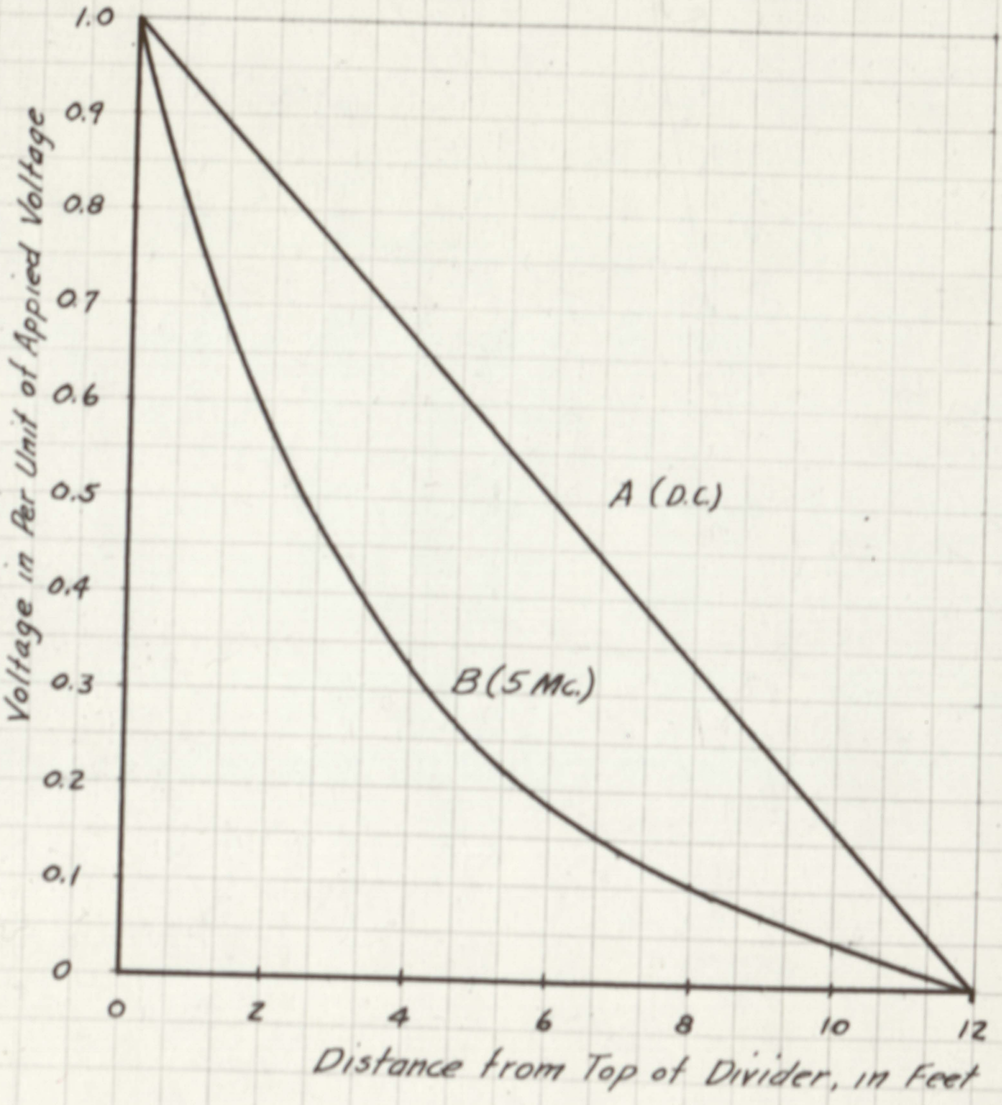


Figure 9_. Voltage distribution along a 12,000 ohm, 12-foot resistance divider at d.c. and 5 mc.

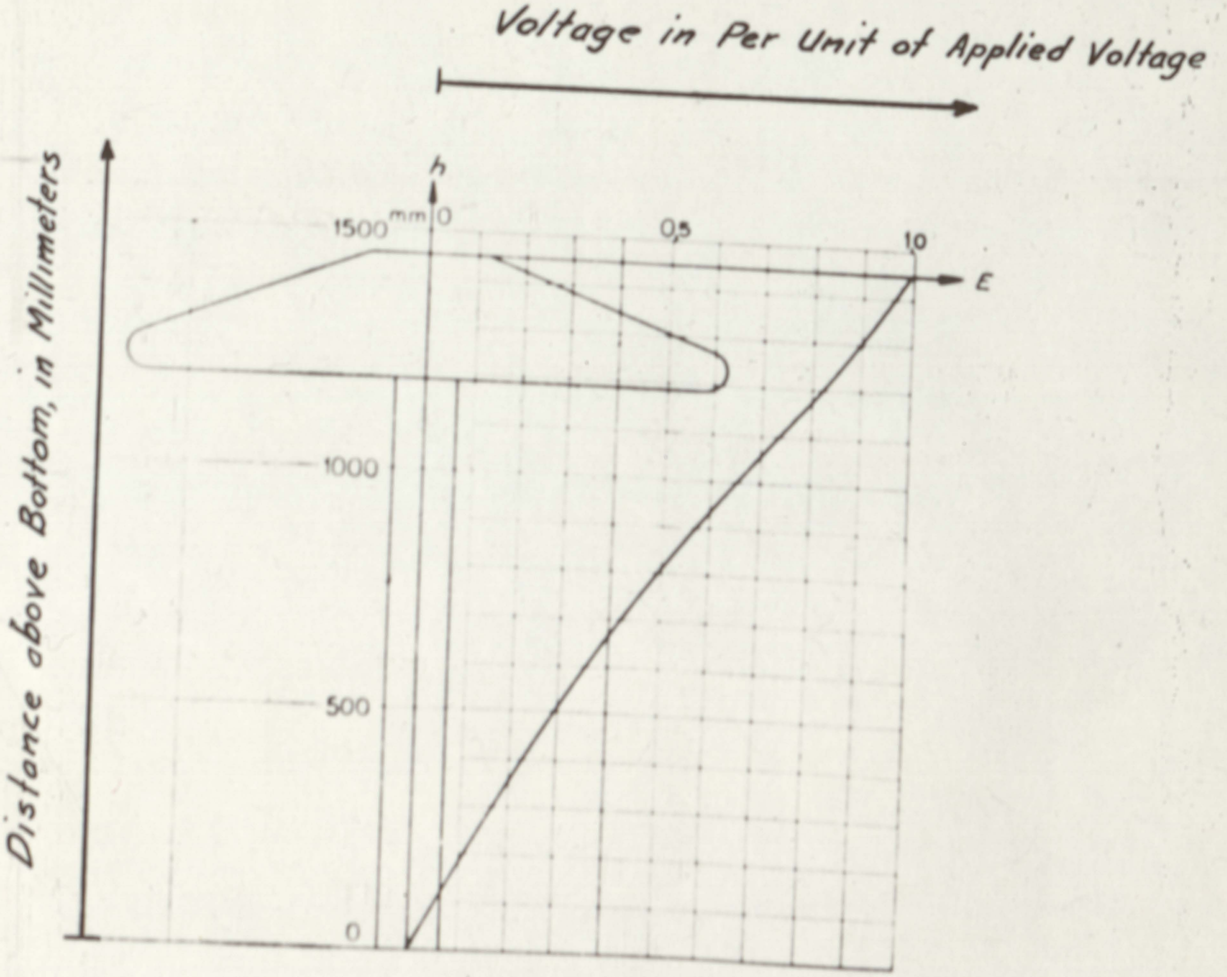


Figure 10. Electrostatic voltage distribution obtained by proper application of a grading shield.[3]

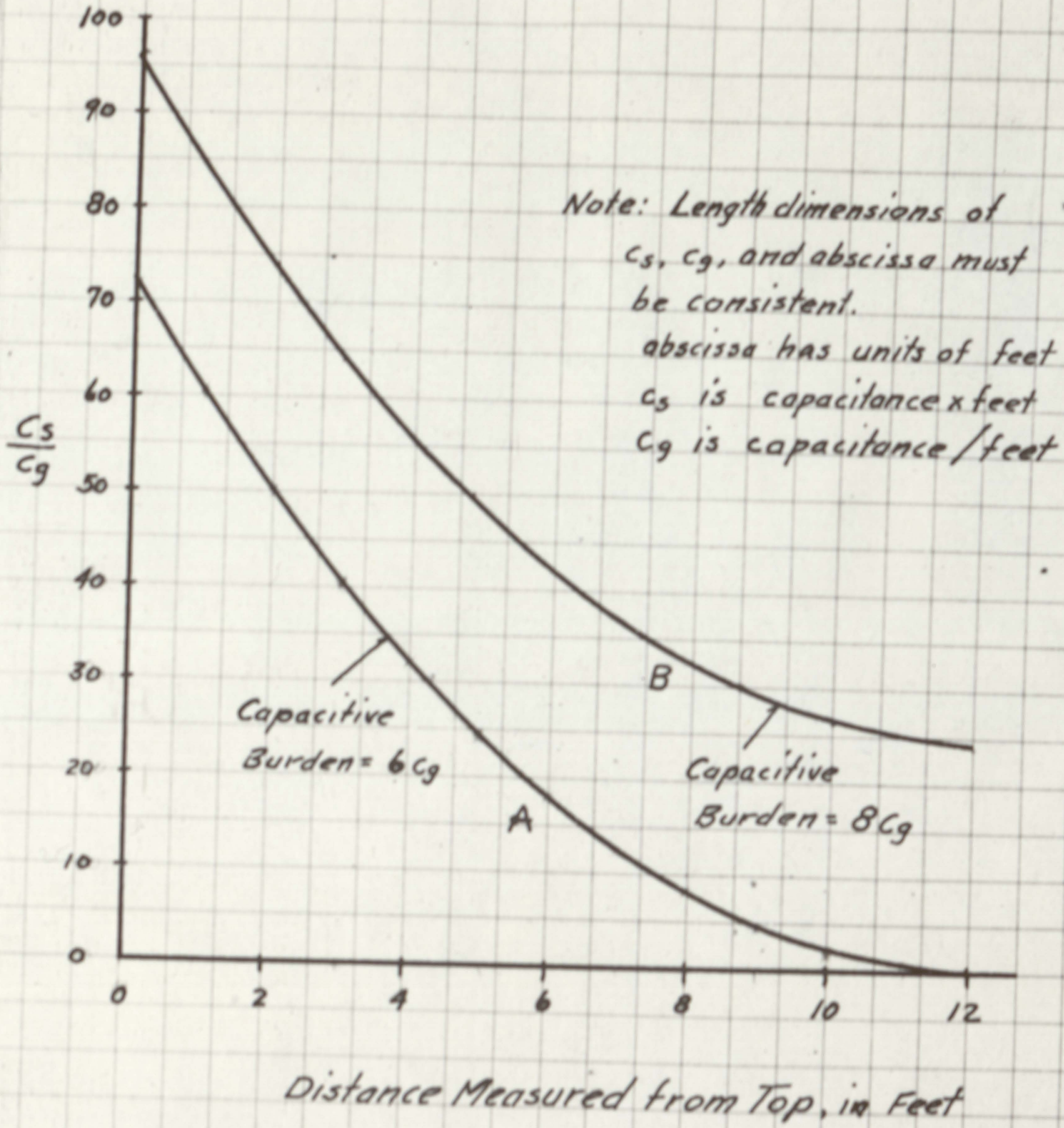


Figure 11. Required series capacitance distribution for uniform voltage distibution along capacitance chain of Appendix IV. Total divider length = 12 feet.

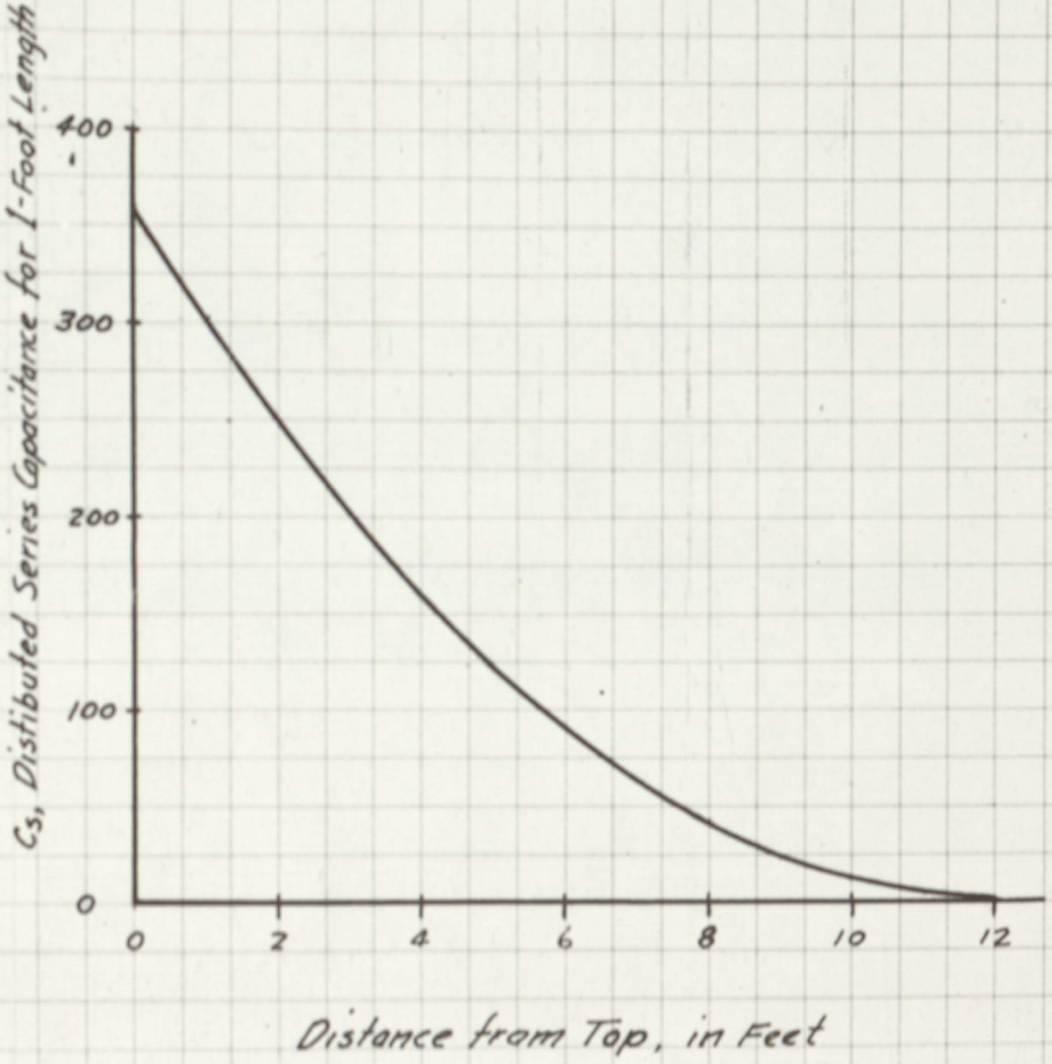


Figure 12. Series capacitance distribution required for uniform voltage distribution along a 12-foot divider. Distributed capacitance to ground = 5 μμf/foot.

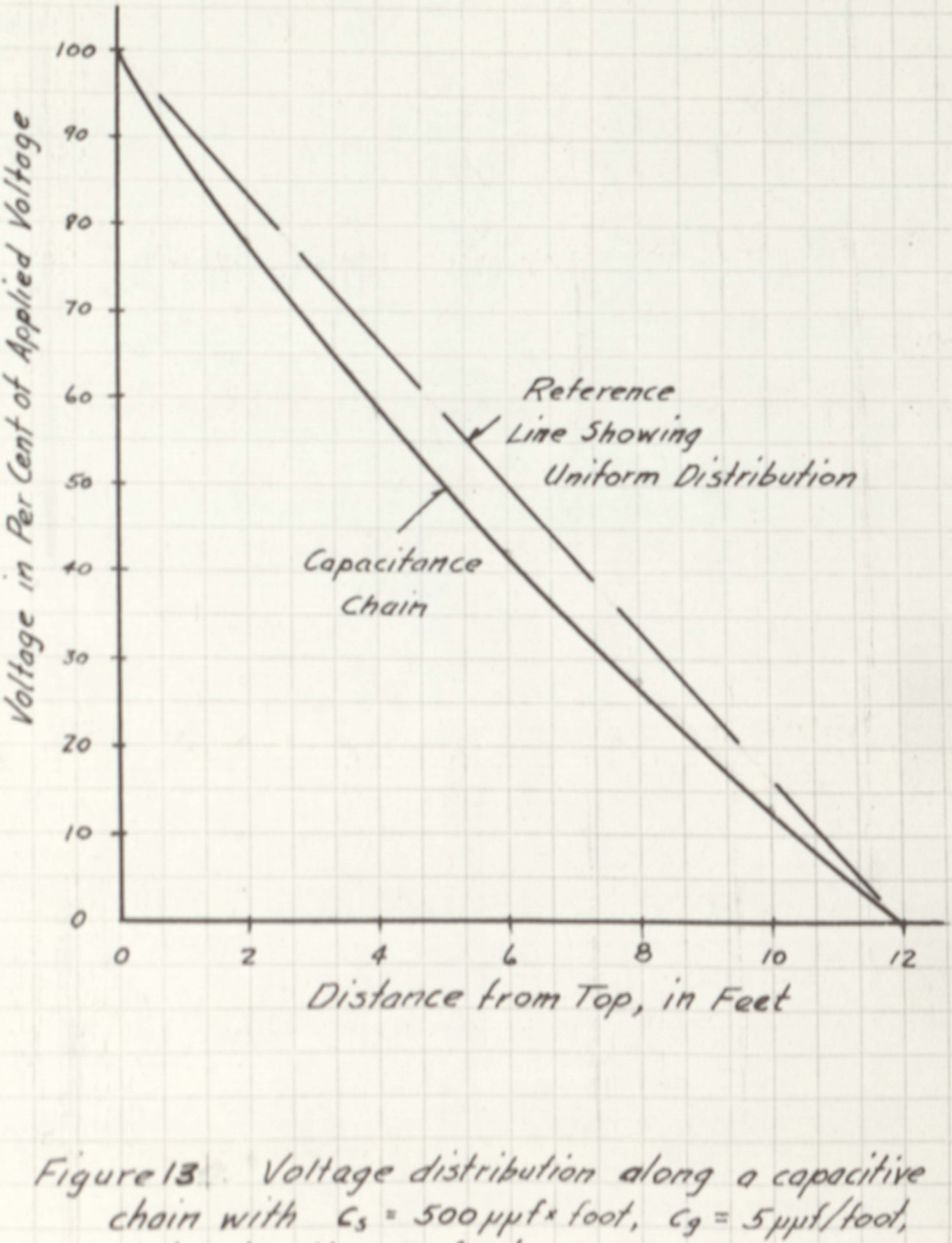


Figure 13. Voltage distribution along a capacitive chain with c_s = 500 μμf × foot, c_g = 5 μμf/foot, total length = 12 feet.

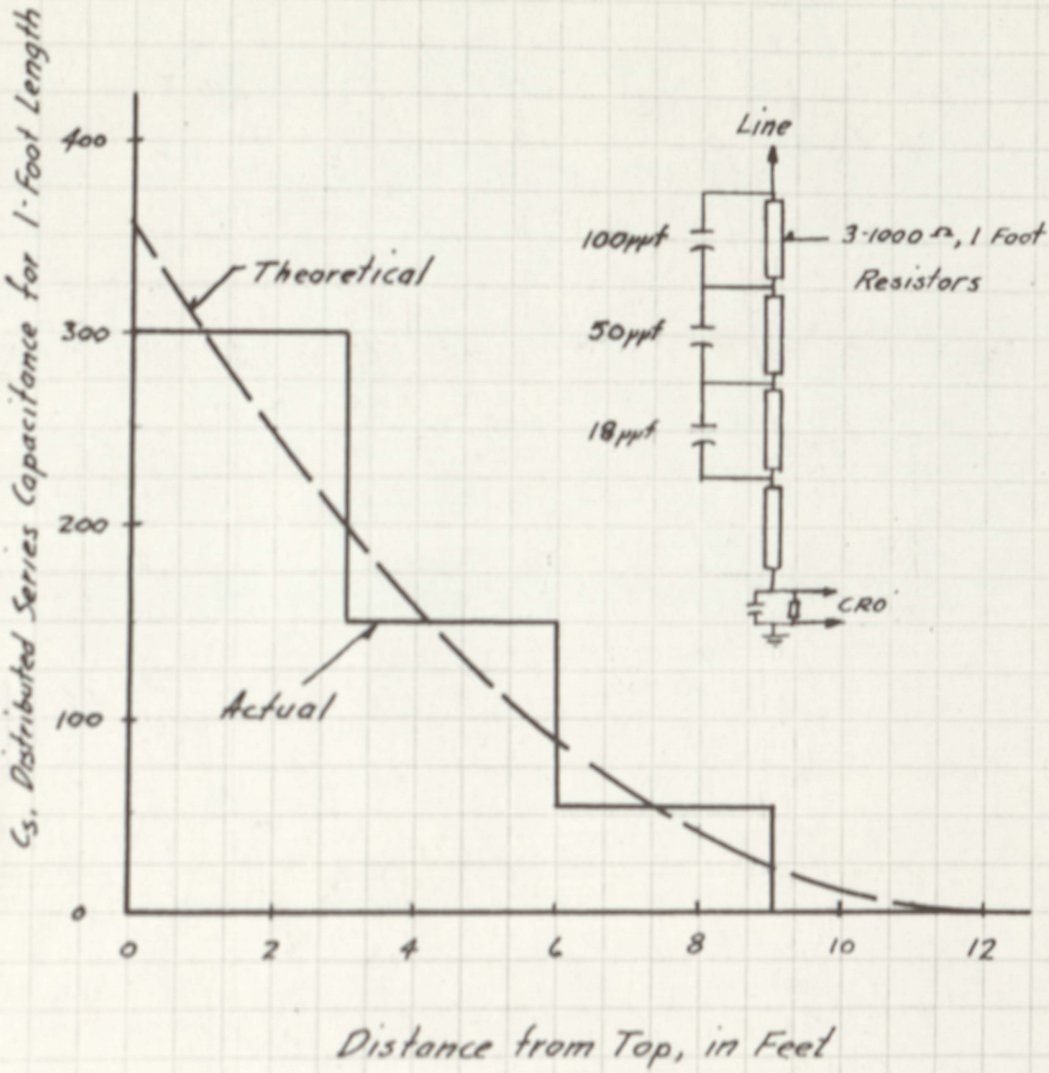


Figure 14. Comparison of actual and theoretical distribution of series capacitance for correcting voltage distribution of a 12,000 ohm, 12-foot divider.

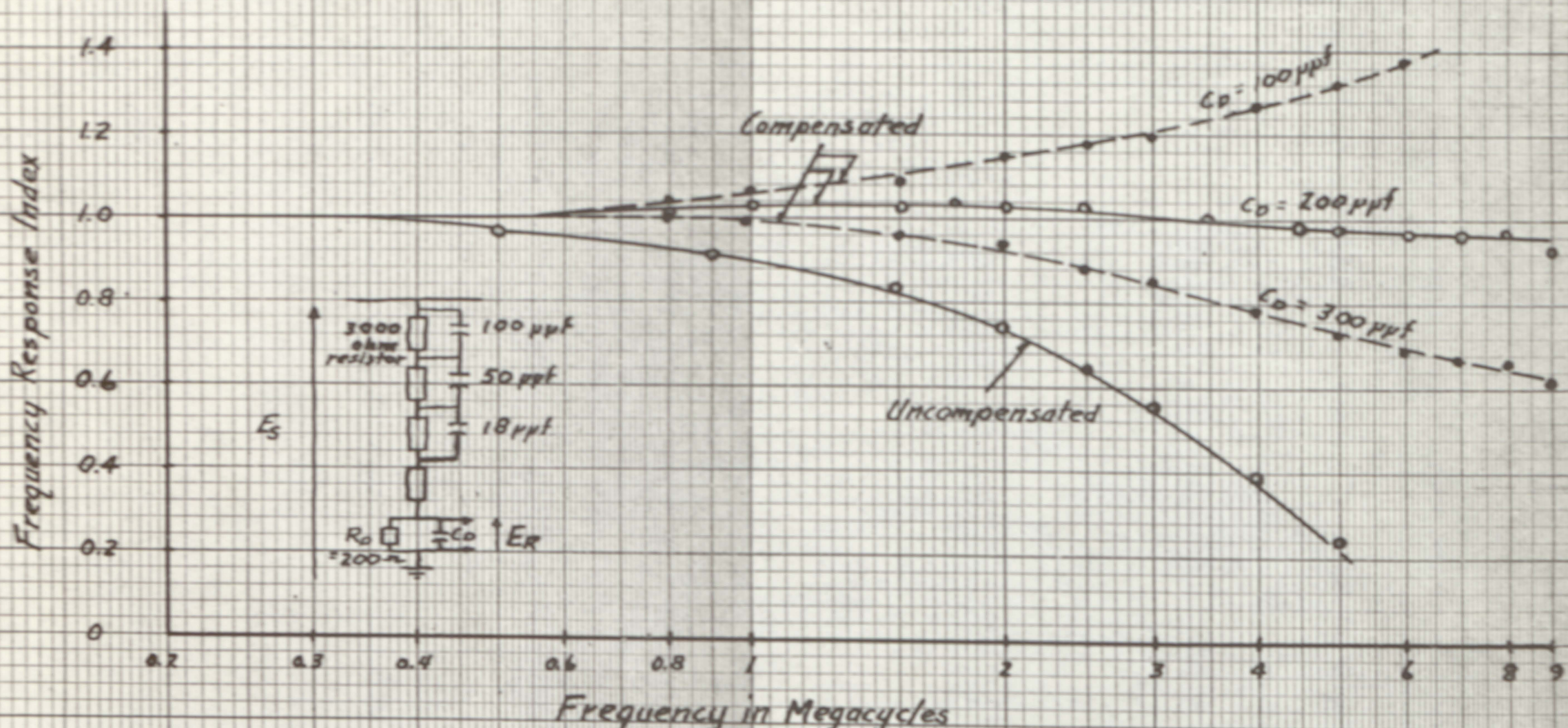


Figure 15. Experimental frequency response for a 12,000 ohm, 12-foot resistance divider with and without tapered-capacitance compensation.

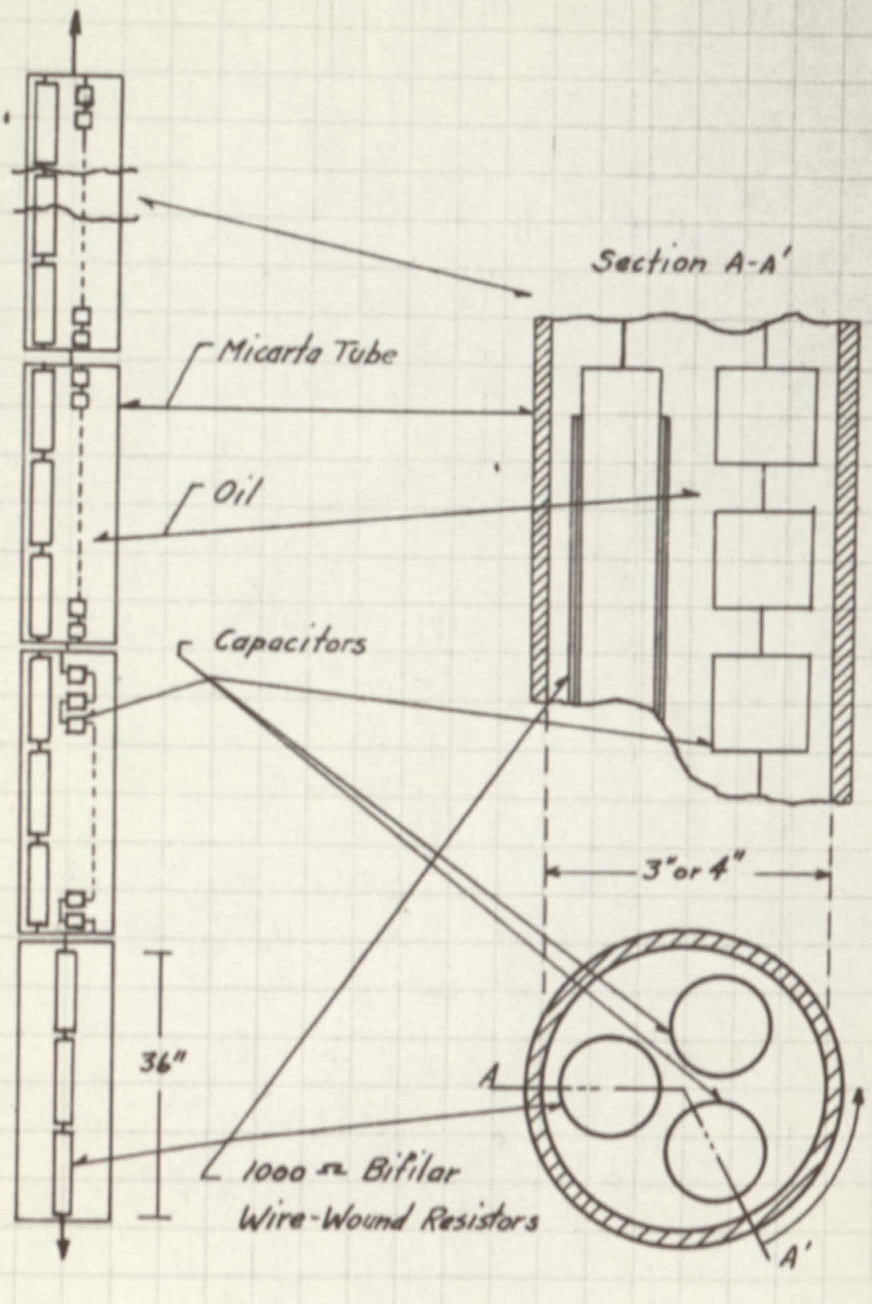


Figure 16. Schematic showing physical form of a 12,000 ohm, 12-foot resistance divider with tapered capacitance compensation.