ESSAYS ON SOVEREIGN DEFAULT AND HOUSEHOLD PORTFOLIO CHOICE

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This dissertation analyzes portfolio choice problems in different contexts. In the first chapter, Nominal Exchange Rate Volatility, Default Risk and Reserve Accumulation, I investigate how nominal exchange rate volatility affects a sovereign's portfolio choice between how much debt to acquire and how much reserves to accumulate. First, I document a positive correlation between nominal exchange rate volatility and sovereign default risk and show that this relationship becomes stronger when more of the external debt is denominated in foreign currency. Then, I build a sovereign default model to rationalize these findings and to quantify the channels that contribute to the large reserve holdings among emerging countries.

In the second chapter, Household Portfolio Accounting, we document and analyze the substantial heterogeneity in household portfolio composition in the United States. We consider a standard life-cycle model with labor income risk and portfolio choice, augmented with a savings wedge that lowers the return on saving, and a risky wedge that lowers the relative return on risky assets. Using U.S. survey data (2004-2016), we compute the household-level wedges that rationalize the data. The chapter has two main contributions: first, it uses the wedges to guide plausible frictions that researchers should consider in their models. Second, it analyzes the extent to which household characteristics can account for the wedges.

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Preface

I would like to thank many people who helped me during my years of graduate study, they not only helped me gain the knowledge and skills, but also made me realize my enthusiasm and potential in becoming a good researcher. I am especially grateful to my main advisors Marla Ripoll and Sewon Hur for their continued support, guidance, and trust. I also greatly appreciate the expertise and key insight from my committee members Daniele Coen-Pirani and Christopher Telmer. Besides, I would like to thank my parents Xingu Yang and Lixia Dai, and my friend Pingfang Guo for their selfless dedication and warmful encouragement. With all your help, I am very proud of pursuing the PhD degree and working in the academic world.

1.0 Nominal Exchange Rate Volatility, Default Risk and Reserve Accumulation

1.1 Introduction

During the last thirty years, emerging countries have borrowed extensively from international markets in foreign currency. From 1990 to 2015, 90% of long-term public debt is denominated in a foreign currency among emerging countries. Large access to credit can facilitate investment and growth, but it may also add to vulnerabilities. Borrowing in foreign currency can make an indebted government vulnerable to volatile exchange rate fluctuations. Whenever the nominal exchange rate depreciates, external debt burden in terms of domestic currency increases, making it more difficult for the government to repay its debt, thereby increasing its default risk.

The underlying intuition is consistent with the recent crisis in Turkey. On August 10th 2018, Turkey's domestic currency Lira experienced a rapid depreciation of 18%, after US announced a doubling of the steel and aluminum tariffs on Turkish imports. The large depreciation was accompanied by a downgrade Turkey's sovereign debt ratings, which indicates a higher default risk. Specifically on Augest 16th 2018, European rating agency Scope Ratings downgraded Turkey's sovereign ratings to BB- from BB+. On Aug 17th, credit ratings agencies Standard & Poor's and Moody's downgraded Turkey's debt rating further into junk, down to B+ (S&P) and Ba3 (Moody's). Similar stories also happened in the 1997 Asian financial crisis, the 1995 Mexican peso crisis and other emerging country crises. Given the widespread vulnerability due to foreign currency debt and volatile exchange rate fluctuations, it is essential to have a precautionary measure to insure against the risk.

During the same period, there is a well-known fact that emerging countries have accumulated foreign reserves at a fast pace. The paper argues that increasing reserve accumulation is a natural response to the vulnerability resulting from foreign currency debt and volatile exchange rate fluctuations. Moreover, the paper aims to investigate how nominal exchange rate volatility affects a sovereign's default risk, the incentive to accumulate debt and reserves,

and whether it can explain the large reserve holdings observed in the data.

To address these questions, I start by constructing a quarterly panel of 15 emerging countries from 1991Q1-2015Q4 to study the empirical relation between nominal exchange rate volatility, foreign currency debt, and sovereign default risk measured by interest rate spread. The main novel finding is that higher nominal exchange rate volatility is significantly correlated with higher default risk, and the positive correlation becomes stronger when more of the external debt is denominated in foreign currency. The finding suggests that nominal exchange rate volatility and foreign currency debt is important in understanding default risk. The next step is to study this link in a sovereign default model, and assess its implications on reserve accumulation.

The model builds on the Eaton and Gersovitz (1981), and Arellano (2008) sovereign default model, where a government with income shocks borrows defaultable real debt to smooth consumption. The paper extends the classic model along three dimensions. First, it introduces two nominal assets denominated in foreign currency: One is defaultable long-term debt, the other is non-defaultable short-term reserves. With different maturity for debt and reserves, the government can borrow long-term debt and save in short-term reserves to help smooth consumption when future borrowing becomes costly. This channel is well studied in Bianchi et al. (2016): Because future value of long-term debt depends on the realizations of future states, but future value of reserves is independent of future states, issuing debt to accumulate reserves can effectively transfer resources from future good states to bad states.

Second, the paper considers a nominal exchange rate depreciation shock in addition to the standard income shock. The shock captures unexpected capital inflows and outflows that could affect a country's nominal exchange rate. It implies that the depreciation shock can change the domestic currency value of foreign currency debt and reserves. To allow the shock to have a real effect, the model needs to isolate the effect of a depreciation shock on domestic goods price. Following Aghion et al. (2004), the good price is assumed to be preset for one period before knowing the future realization of depreciation shock. The sticky price assumption can be thought of limited pass-thourgh from exchange rate to domestic price, as shown in Goldfajn and Werlang (2000). The assumption implies that purchasing power parity holds only ex ante for good prices, but it holds at any time for the asset prices. See

Dornbusch (1976) for the different pass-through between asset prices and goods prices.

Third, to capture the idea that central banks may use reserves in foreign exchange market to counter disruptive exchange rate movements, the model allows higher reserve holdings to exogenously reduce the exchange rate volatility. One can interpret higher reserves as a signal for higher capacity for the central bank to intervene in foreign exchange market, which discourages speculative attacks and reduces the exchange rate volatility.

With these key features, the model provides three channels to explain the reserve accumulation. First, reserves can help smooth consumption when future borrowing becomes costly, as in Bianchi et al. (2016). Second, reserves can provide a currency hedge against bad depreciation shock. Since both debt and reserves are denominated in foreign currency, whenever nominal exchange rate depreciates, debt burden in terms of domestic currency increases, but at the same time reserves in domestic currency also increases, which alleviates the effect of exchange rate depreciation on the default risk. Third, reserves can reduce the exchange rate volatility exogenously, and prevent a large exchange rate depreciation. When the government makes its portfolio choice decision between debt and reserves, it faces a tradeoff between these three benefits of reserves with the cost of keeping larger gross debt positions to accumulate reserves. To investigate the quantitative importance of these channels, the model is then calibrated and applied for quantitative experiments.

The model is calibrated to mimic salient features of a typical emerging country: Mexico. The calibrated model can generate more than half of reserve accumulation as in the data, which suggests that the model channels are quantitatively important in explaining the reserve accumulation. Then the model is applied for three experiments. First, we test the model to see whether it can explain the increasing reserve holdings. The paper finds that if the economy starts at a relative low exchange rate volatility regime, the increase in exchange rate volatility can make the government optimally increase the reserve holdings. However, if the economy starts at a relative high exchange rate volatility regime, further increase in exchange rate volatility will push the government away from international market in the sense of reducing both borrowing and reserves. Since emerging countries are in the stage of moving from limited financial integration to rapid integration in the past thirty years, the model prediction is consistent with the increasing reserve holdings.

Next, the paper investigates the quantitative relevance of using reserves to hedge against currency depreciation. The experiment recalibrates the model where reserves are denominated in domestic currency. The result shows that in this case, the optimal reserve holdings decrease 50% compared with the benchmark where reserves are in a foreign currency. The result highlights the foreign currency feature of reserves to understand the reserve accumulation. Last, the paper shows that using reserve to reduce the exchange rate volatility is quantitatively important. The experiment shuts down this channel by letting exchange rate volatility to be independent with reserve level. After recalibrating the model, the reserve holdings become only one third of benchmark level. The result points out the essential role of using reserves for foreign exchange market management.

Related Literature. The paper is mainly related to two different literatures. The first is the sovereign default literature. The main theoretical background is established by Eaton and Gersovitz (1981). The later quantitative exercises are developed by Arellano (2008). Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2013) extend the Arellano (2008) to incorporate different durations of long-term sovereign bond instead of only one period short-term bond. The model setup in this paper is based on the long-term bond sovereign default model. The novel difference is that the paper considers the foreign currency denominated nominal bond instead of commonly used real bond. By incorporating the foreign currency nominal bond, the paper contributes to the literature by pointing out that the exchange rate volatility could be an important channel that affects emerging countries' default risk and borrowing decisions. The paper also quantitatively investigates how different exchange rate volatilities will affect their borrowing decisions and the bond spread.

The paper is also closely linked to the reserve accumulation literature. This literature tries to explain why emerging countries hold large amount of foreign reserves. There are two popular views in this literature. The first is the mercantilism view, which suggests that reserve accumulation can promote export growth by preventing appreciation (Dooley et al. (2004)). The second is the precautionary view, which argues that reserves can act as a self-insurance against future sudden stop events (Alfaro and Kanczuk (2009), Bianchi et al. (2016), Hur and Kondo (2016) and Jeanne and Rancire (2011)). The sudden stop is known as the quick and sharp reversal of private capital flows in emerging market economies. It is

usually modeled as an exogenous event that happens with certain probability. For instance, in Bianchi et al. (2016), the sudden stop is modeled as an exogenous situation where countries are not able to borrow external debts. This paper also illustrates the precautionary motives for reserve accumulation, but in the presence of currency mismatch and volatile exchange rate fluctuations. These two features with defaultable debts can endogenously generate tight borrowing constraint, which provides a micro foundation for the exogenous sudden stop events. Moreover, the paper contributes to the literature by arguing that the exchange rate volatility is an essential factor that increases the precautionary demand for reserve holdings, and the paper proposes three channels through which large reserve holdings can be rationalized.

The rest of the paper proceeds as follows. Section 1.2 illustrates the empirical motivation, section 1.3 presents the model and section 1.4 shows the quantitative results. Section 1.5 concludes.

1.2 Empirical Motivation

This section aims to study the empirical link between foreign currency debt, nominal exchange rate volatility and default risk in emerging countries. The focus on emerging countries is due to their significant foreign currency borrowing, volatile exchange rate fluctuations and relatively frequent default events. These features make emerging countries a good fit to investigate my research questions.

Relevant information is collected from various sources of data. The nominal exchange rate and foreign reserves are from International Financial Statistics. The sovereign bond spread is from JPMorgan Emerging Market Bond Index (EMBI). The interest rate spread is the difference between the yield on a country's foreign currency bond and the yield on a comparable bond issued by United States. The external debt information is from World Development Indicators. Specifically, for the external long-term debt, we can distinguish between public and private debt. Moreover, among the external public debt, we can calculate how much debt is denominated in a foreign currency. However, for the external short-term

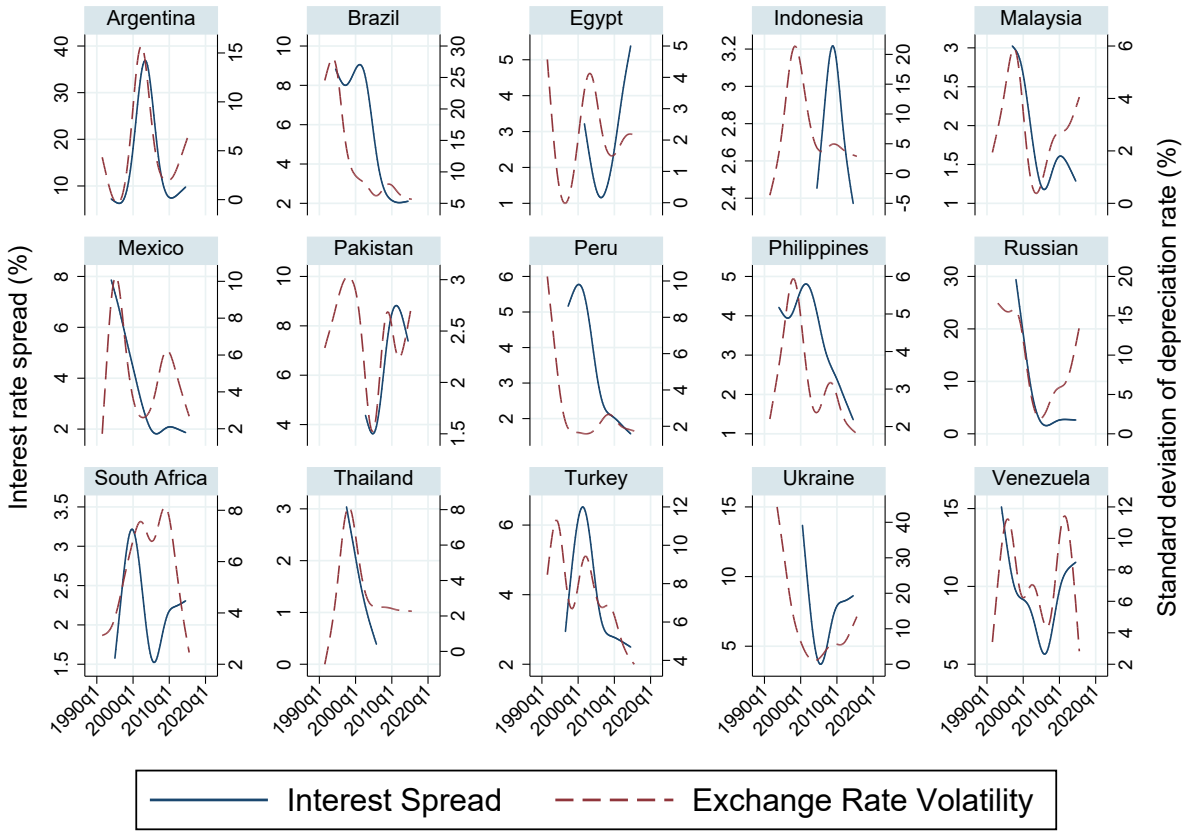
debt, we only know the sum of public and private debt, instead of the amount of each debt. The emerging countries in my sample include: Argentina, Brazil, Egypt, Indonesia, Malaysia, Mexico, Pakistan, Peru, Philippines, Russia, South Africa, Thailand, Turkey, Ukraine, and Venezuela. All the information allows me to construct a panel containing 15 countries from 1991Q1 to 2015Q4.

The measure of nominal exchange rate volatility for a country in a given period is the standard deviation of quarterly depreciation of nominal exchange rate over the period. The measure of default risk is the sovereign bond spread from EMBI, which is a common indicator for government default risk in the sovereign default literature. Figure 1 plots trends of interest spread and exchange rate volatility in each quarter for all countries in my sample. I use a three years rolling windows to compute both measures, where the time indicator represents the center of the window. The three years rolling windows are constructed to capture the link between short-run exchange rate volatility and spread.

Figure 1 shows that there is a strong association between exchange rate volatility and interest spread, especially for Argentina, Malaysia, Mexico, Pakistan, Russian, Thailand, Turkey, Ukraine and Venezuela. Among these countries, most of the spikes in exchange rate volatility and spread coincide with recent regional and global crises, e.g. 1997 Asian financial crisis and 2008 global financial crisis. It implies that in addition to country-specific factors, financial contagion and exogenous capital flows are also important in affecting the default risk. For other countries, the positive association does not hold perfectly, suggesting that other factors could also drive the change in spread. Overall, the figure is consistent with the idea that nominal exchange rate volatility is essential in determining the default risk.

The associations between exchange rate volatility and spread might be driven by other confounding factors. Next, I will use the regression analysis to control for time-invariant country fixed effect, common global time trend, and other potential confounding factors. The result is shown in Table 1. In the regression table, except for the nominal exchange rate volatility measure, all other variables indicate the corresponding average value in the three years rolling window. For the debt variables, short-term debt represents external short-term debt to GDP ratio, and all the remaining debt variables represent the corresponding long-term external debt to GDP ratio. Moreover, all the spread, exchange rate volatility and debt

Figure 1: Exchange Rate Volatility and Interest Spread



variables are measured in percentage or 0.01.

Specification (1) and (2) show a significant and positive relationship between exchange rate volatility and spread, which is consistent with Figure 1. Moreover, in specification (2), we can see that conditional on the level of total external long-term public debt, an additional unit of borrowing in foreign currency is significantly associated with a higher spread. It points out the importance of foreign currency debt in affecting the spread.

In specification (3), (4), (5), the interaction between foreign currency debt and exchange rate volatility is added into the regression. The positive and significant coefficients on the interaction term indicate that for country with a higher exposure in foreign currency debt, the

Table 1: Exchange Rate Volatility and Interest Spread

	(1)	(2)	(3)	(4)	(5)
	Spread	Spread	Spread	Spread	Spread
ExVol	0.602***	0.272***	-0.119**	-0.125***	-0.0848
	(0.076)	(0.033)	(0.052)	(0.047)	(0.062)
FC Public Debt		0.616***	0.446***	0.697***	1.038***
		(0.075)	(0.096)	(0.122)	(0.160)
Public Debt		-0.112**	-0.134***	-0.392***	-0.537***
		(0.046)	(0.041)	(0.071)	(0.092)
FC Public Debt × ExVol			0.0161***	0.0160***	0.00977***
			(0.003)	(0.002)	(0.003)
Private Debt				-0.0891***	-0.0874***
				(0.022)	(0.027)
Short-Term Debt				0.319***	0.174***
				(0.060)	(0.051)
N	1005	1001	1001	1001	811
adj. R^2	0.61	0.79	0.83	0.84	0.90
Countries	15	15	15	15	14
Other Controls	No	No	No	No	Yes

All regressions include country and year fixed effects.

Robust standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

association between exchange rate volatility and spread is stronger. This pattern is robust to adding other external debt: long-term private debt and short-term debt, and also robust to including other fundamental controls: real GDP, reserve-to-GDP ratio, current account-to-GDP, inflation, trade-to-GDP and capital account openness measure (Chinn-Ito index). Even though the coefficient on ExVol becomes negative after adding the interaction term, the total effect of ExVol depends on the coefficients on both ExVol and also the interaction term. We can see that when FC Public Debt reaches 10 percent, the overall effect of ExVol on Spread becomes positive. In my sample, only South Africa has on average FC Public Debt less than 10 percent. It implies that the regression coefficients on ExVol are still consistent with the idea that in general, a higher exchange rate volatility is associated with a higher spread.

The take-way from the above regression analysis is that foreign currency debt and exchange rate volatility are important in understanding the default risk. Next I will build a sovereign default model to study this link, and investigate its implications on reserve accumulation.

1.3 Model

To understand the pattern in the empirical section, I construct a sovereign default model with following three novel features. First, I introduce nominal foreign currency assets. Specifically, the sovereign can borrow long-term foreign currency bonds, and save in one-period foreign currency reserves. Second, I add an exogenous nominal exchange rate shock to the model in addition to the standard output shock, which stochastically moves the sovereign's domestic currency value of its foreign currency debt. Third, I introduce an exogenous link between reserves and exchange rate volatility, estimated from the data. Next, I will present the detailed description of the model.

1.3.1 Endowment

Time is discrete and indexed by $t = 0, 1, 2, ...$ The aggregate endowment is stochastic and follows an $AR(1)$ process:

$$\log y_t = (1-\rho)\mu + \rho \log y_{t-1} + \varepsilon_t, \ |\rho| < 1, \ \varepsilon_t \sim N(0, \sigma_\varepsilon^2).$$

1.3.2 Preferences

The preference of the representative agent is given by:

$$\mathbb{E}_t \sum_{j=t}^{\infty} \beta^{j-t} u(c_j),$$

where $\mathbb{E}$ is the expectation operator, β is the discount factor, and c is real consumption. The utility function is strictly increasing and concave.

I assume that the benevolent sovereign can borrow and lend from international financial markets, on behalf of its domestic hand-to-mouth agents to help them smooth consumption.

1.3.3 Nominal Exchange Rate Shock

To capture the idea that unexpected capital flows could disrupt the foreign exchange market, and change the real value of external foreign currency debt, the nominal exchange rate depreciation shock ξ is introduced. The depreciation shock ξ_t is defined as the net percentage depreciation of nominal exchange rate compared with last period nominal exchange rate. Mathematically we have

$$\xi_t = \frac{e_t}{e_{t-1}} - 1,$$

where e_t denotes the nominal exchange rate at time t, which indicates the domestic currency value of 1 unit of foreign currency. If exchange rate depreciates at time t, that is $e_t > e_{t-1}$ and $\xi_t > 0$, then 1 unit of foreign currency becomes more valuable in terms of domestic currency and vice versa. Therefore the exchange rate shock ξ_t can randomly change the

domestic value of foreign currency assets. The depreciation shock ξ_t is $AR(1)$ with time-varying volatility. The volatility of nominal exchange rate is assumed to be determined by the previous period's reserves-to-GDP ratio.

$$\xi_t = \rho_\xi \xi_{t-1} + \upsilon_t, |\rho_\xi| < 1, \upsilon_t \sim N(0, \sigma_t^2),$$

$$\sigma_t^2 = \exp(\alpha_1 + \alpha_2 \frac{a_{t-1}}{y_{t-1}}),$$

where $\frac{a_{t-1}}{y_{t-1}}$ denotes the ratio between last period's foreign reserves a_{t-1} and last period's income y_{t-1}. Based on my estimation which I describe in section 4, the parameter α_2 is negative. This implies that larger reserves can reduce the exchange rate volatility. It capture the idea that higher reserves indicate higher capacity for the central bank to intervene in foreign exchange market, which discourages speculative attacks and reduces the exchange rate volatility.

1.3.4 International Financial Markets

The government can issue long-term bonds from foreign risk-neutral lenders, but only in foreign currency. To model the long-term bonds in a tractable way, I follow Aguiar et al. (2016), Hatchondo and Martinez (2009), and Chatterjee and Eyigungor (2013). Specifically, each long-term bond will pay a coupon every period up to and including the period of maturity. Without loss of generality, the coupon payment is normalized to the risk-free nominal interest rate r per unit of face value as in Aguiar et al. (2016). The benefit is that with this normalization, a unit of risk-free long-term bond will have a price of one.

The long-term bond is assumed to mature randomly at probability $\delta \in [0, 1]$. That is, for an outstanding long-term bond of measure b, a δ fraction b will mature this period and the associated debt repayment is $(r + \delta)b$. The continuation face value of the long-term bond is $(1 - \delta)b$. If the sovereign changes its outstanding debt level at the end of this period to b'. Then the net issuance this period is $b' - (1 - \delta)b$. With this setup, the expected maturity of the long-term bond is $\frac{1}{\delta}$. The government also has access to a one-period risk-free foreign currency reserves, that pays $1 + r$ each period.

1.3.5 Budget Constraints

Following the international macroeconomics literature, the foreign price level p^* is set as the numeraire, and it is assumed to be constant for all periods. To allow the depreciation shock to have real a effect, I assume the domestic price is predetermined, that is, they are set a period in advance but can be adjusted fully at the end of the period. Under this setting, changes in nominal exchange rate will only affect the asset price in domestic currency, but the goods price in domestic currency remain the same. So there is a real effect from the depreciation shock. Given the model environment, we can formulate the budget constraints in both nominal and real term.

Denote $\tilde{b}_t \geqslant 0$ as the measure of outstanding foreign currency long-term bond, $\tilde{a}_t \geqslant 0$ as the measure of foreign currency reserves. Then without default, the nominal budget constraint is

$$p_t c_t = p_t y_t - (r+\delta)\tilde{b}_t e_t + q_t e_t(\tilde{b}_{t+1} - (1-\delta)\tilde{b}_t) + \tilde{a}_t e_t - \frac{\tilde{a}_{t+1} e_t}{1+r},$$

where q_t is the foreign currency nominal price of the bond issued by the sovereign, $(r+\delta)\tilde{b}_t$ is the foreign currency coupon payments and debt payments due, and all the terms are in nominal domestic currency.

Now define $b_{t+1} = \frac{\tilde{b}_{t+1} e_t}{p_t}, a_{t+1} = \frac{\tilde{a}_{t+1} e_t}{p_t}$ as the real value of foreign currency debt and reserves in terms of goods value at period t, when they are issued. The associated real budget constraint is

$$c_t = y_t - \frac{(r+\delta)\tilde{b}_t e_{t-1}}{p_{t-1}} \frac{p_{t-1} e_t}{p_t e_{t-1}} + \frac{q_t \tilde{b}_{t+1} e_t}{p_t} - \frac{q_t (1-\delta)\tilde{b}_t e_{t-1}}{p_{t-1}} \frac{p_{t-1} e_t}{p_t e_{t-1}} + \frac{\tilde{a}_t e_{t-1}}{p_{t-1}} \frac{p_{t-1} e_t}{p_t e_{t-1}} - \frac{\tilde{a}_{t+1} e_t}{(1+r) p_t}.$$

Since $\xi_t = \frac{e_t}{e_{t-1}} - 1$ and domestic price level is constant $p_t = p_{t-1}$, we have

$$c_t = y_t - (1+\xi_t)(r+\delta) b_t + q_t(b_{t+1} - (1+\xi_t)(1-\delta) b_t) + (1+\xi_t) a_t - \frac{a_{t+1}}{1+r},$$

where all the terms are in real value now, and we can see that nominal exchange rate depreciation $\xi_t > 0$ can result in real increase in foreign currency outstanding debt b_t and also foreign currency reserves a_t. Next, I will present the real budget constraint when the government defaults.

When the government defaults, it does so on all current and future debt obligations. But we assume after default, the government can still keep it foreign reserves. A default triggers exclusion from borrowing in credit markets for a stochastic number of periods. The sovereign will regain access to credit markets with a constant probability $\theta \in [0, 1]$. In every period where the sovereign is excluded from credit markets, there is a real output loss of $\phi(y)$, which is increasing in the income.

$$\phi(y) = \max\{0, d_0 y + d_1 y^2\}$$

In case of default, the sovereign will still retain the control of their foreign reserves. Therefore the real budget constraint can be derived similarly

$$c_t = y_t - \phi(y) + (1 + \xi_t)a_t - \frac{a_{t+1}}{1 + r},$$

where the sovereign can only save in reserves to smooth consumption.

1.3.6 Timing

The timing within each period is as follows: First, the output shock and exchange rate shock are realized, where the variance of the exchange rate shock depends on the reserves at the beginning of the period. After observing these shocks, the sovereign chooses whether to default on its debt and then makes its borrowing and saving decisions.

1.3.7 Recursive Formulation

The recursive formulation of the sovereign's problem is described here. The sovereign cannot commit to repay the debt. So in each period, the sovereign will compare the value of repayment and the value of default to make its repayment decision.

Let V indicate the value function of a sovereign that is not in default. For any bond price function q, the function V satisfies the following functional equation:

$$V(b, a, y, \xi) = \max\{V^R(b, a, y, \xi), V^D(a, y, \xi)\}, \tag{1.1}$$

where the sovereign's value of repayment is given by

$$V^R(b,a,y,\xi) = \max_{a'\geqslant 0, b'\geqslant 0, c} \{u(c) + \beta \mathbb{E}_{(y',\xi')|(y,\xi)} V(b',a',y',\xi')\}, \tag{1.2}$$

subject to

$$c = y - (1+\xi)(r+\delta)b + q(b^{'} - (1+\xi)(1-\delta)b) + (1+\xi)a - \frac{a^{'}}{1+r}.$$

The value of defaulting is given by:

$$V^D(a,y,\xi) = \max_{a'\geqslant 0, c} u(c) + \beta \mathbb{E}_{(y',\xi')|(y,\xi)}[(1-\theta)V^D(a',y',\xi') + \theta V(0,a',y',\xi')], \tag{1.3}$$

subject to

$$c = y - \phi(y) + (1+\xi)a - \frac{a'}{1+r}.$$

The solution to the sovereign's problem gives the decision rules for default $\hat{d}(b,a,y,\xi)$, debt $\hat{b}(b,a,y,\xi)$, reserves in default $\hat{a}^D(a,y,\xi)$, reserves when not in default $\hat{a}^R(b,a,y,\xi)$, consumption in default $\hat{c}^D(a,y,\xi)$, and consumption when not in default $\hat{c}^R(b,a,y,\xi)$. The default rule $\hat{d}$ is equal to 1 if the sovereign defaults, and is equal to 0 otherwise.

1.3.8 Bond Prices

Sovereign bonds and reserves are priced in a competitive market inhabited by a large number of identical risk-neutral international investors. Investors discount future payoffs at the risk-free rate $1+r$. This implies that in equilibrium the bond-price function solves the following functional equation:

$$q(b',a',y,\xi)(1+r) = \mathbb{E}_{(y',\xi')|(y,\xi)}[(1-\hat{d}(b',a',y',\xi'))(r+\delta+(1-\delta)q(b'',a'',y',\xi'))], \tag{1.4}$$

where

$$\begin{aligned} b'' &= \hat{b}(b',a',y',\xi') \\ a'' &= \hat{a}^R(b',a',y',\xi'). \end{aligned}$$

This means that, in equilibrium, a risk-neutral investor is indifferent between selling a bond today, and keeping the bond to the next period. If the investor keeps the bond and the sovereign does not default in the next period, he will receive the coupon and the debt repayments $(r+\delta)$. In addition, his continuation value of bond worth $(1-\delta)$ times the price of a bond issued in the next period.

1.3.9 Recursive Equilibrium

I focus on Markov Perfect Equilibria. That is, in each period the sovereign's equilibrium default, borrowing, and saving decisions depend only on the payoff-relevant state variables. Then a Markov Perfect Equilibrium is defined by a set of value functions V, V^R and V^D, rules for default $\hat{d}$, borrowing $\hat{b}$, reserves $\{\hat{a}^R, \hat{a}^D\}$, and consumption $\{\hat{c}^R, \hat{c}^D\}$, and a bond price function q, such that: (i) given a bond price function q, the policy functions $\hat{d}, \hat{b}, \hat{a}^R, \hat{a}^D, \hat{c}^R, \hat{c}^D$, and the value functions V, V^R, V^D solve the Bellman equations (1), (2), and (3), and (ii) given the sovereign's policies, the bond price function q satisfies condition (4).

1.4 Quantitative Results

1.4.1 Calibration and Functional Forms

For the functional forms, the utility function displays a constant coefficient of relative risk aversion,

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}, \text{ with } \gamma \neq 1.$$

The endowment shock follows the standard $AR(1)$ process :

$$\log y_t = (1-\rho)\mu + \rho \log y_{t-1} + \varepsilon_t, \ |\rho| < 1, \ \varepsilon_t \sim N(0, \sigma_\varepsilon^2).$$

The depreciation shock follows $AR(1)$ process with time-varying volatility.

$$\xi_t = \rho_\xi \xi_{t-1} + \upsilon_t, |\rho_\xi| < 1, \upsilon_t \sim N(0, \sigma_t^2),$$

$$\sigma_t^2 = \exp(\alpha_1 + \alpha_2 \frac{a_{t-1}}{y_{t-1}}),$$

For the default cost function, I follow Chatterjee and Eyigungor (2013). That is $\phi(y) = \max\{0, d_0 y + d_1 y^2\}$, so that it is asymmetrically more costly to default in good times.

For the calibration, the paper targets Mexico as the benchmark economy, since Mexico is a common reference for studies on emerging economics. The data for Mexico is mainly taken

from International Financial Statistics and EMBI spread dataset from $1991Q1$ to $2015Q4$. Since the calibration uses quarterly data, a period in the model will correspond to a quarter in the data.

There are total fourteen parameters that need to be determined. The first three parameters discount factor β,default cost d_0, d_1 will be jointly calibrated to match three moments. These are the mean external debt-to-GDP ratio of 25%, standard deviation of the interest spread of 2.5% and the default probability of 2%.

Four parameter values will be set directly. The risk aversion γ will be set to 2 as in the sovereign default literature. The risk-free rate r will be set to 1% to match the three-month US treasury bill real return. The probability of reentry θ will be set to 0.083 to reflect that on average it takes three years for countries to regain access to credit markets after default. The debt duration δ will be set to 0.05 to match the 5 years average bond duration in Mexico as in Broner et al. (2013)

The remaining seven parameters $\mu, \rho, \sigma_\varepsilon^2, \rho_\epsilon, \sigma_\upsilon^2, \alpha_1, \alpha_2$ for output shock and nominal exchange rate shock will be estimated based on the HP-filtered Mexico real output and the HP-filtered nominal exchange rate data from $1991Q1$ to $2015Q4$. The benchmark calibration result is shown in Table 2.

The interest spread can be calculated as follows. Firstly the yield i is calculated, which is defined as the return an investor would earn if he holds the bond to maturity and no default is declared. This yield satisfies

$$i = \frac{r+\delta}{q} - (1-\delta)$$

The sovereign spread is then computed as the difference between the yield i and the risk-free rate r. The annualized spread is used here:

$$r_t^s = (\frac{1+i}{1+r})^4 - 1$$

Table 2: Parameter Values

Discount factor	β	0.981
Output cost of defaulting	d_0	-1.025
Output cost of defaulting	d_1	1.115
Risk aversion	γ	2
Risk-free rate	r	1%
Probability of reentry after default	θ	0.083
Debt duration	δ	0.05
Output autocorrelation coefficient	ρ	0.94
Depreciation autocorrelation coefficient	ρ_υ	0.21
Standard deviation of innovations for output	σ	1.5%
Standard deviation of innovations for depreciation	σ_υ	8.18%
Mean log output	μ	$-(1/2)\sigma_\varepsilon^2$
Variance for the depreciation shock	α_1	-4.77
Variance for the depreciation shock	α_2	-0.46

1.4.2 Simulation Results

Now, the paper will firstly show the model's fit. Then there will be quantitative exercises to quantify the channels of the model.

Table 3 shows that the model exactly matches the three targeted moments. The model can also generate a quantitatively reasonable interest rate spread. For the reserve holdings, the model can explain over half of the reserve holdings. It implies that the channels in the model are quantitatively important to understand the reserve accumulation. Meanwhile, there are also other mechanisms that could help us understand the reserve holdings.

Table 3: Simulation Statistics

	Model	Data
Mean Debt-to-GDP	25	25
$\sigma(r_s)$	2.5	2.5
Default probability	1.9	2
Mean r_s	2.5	3.4
Mean Reserve-to-GDP	4.4	7.6

1.4.3 The Effects of Exchange Rate Volatility

In this subsection, the paper wants to investigate how different exchange rate volatility regimes affect the government's incentives to borrow and accumulate reserves. Moreover, we want to test whether the model is able to replicate the following two empirical patterns.

First, if we interpret higher exchange rate volatility as a proxy for rapid financial integration and more volatile capital flows, then we want to know whether the rapid financial integration and more volatile capital flows can explain the well-known increasing reserve holdings. Second, in the empirical section, the paper documents a positive association between exchange rate volatility and interest rate spread. Here, we want to see whether the

model can generate similar pattern. Since the depreciation shock follows

$$\xi_t = \rho_\xi \xi_{t-1} + \upsilon_t, |\rho_\xi| < 1, \upsilon_t \sim N(0, \sigma_t^2),$$

$$\sigma_t^2 = \exp(\alpha_1 + \alpha_2 \frac{a_{t-1}}{y_{t-1}}).$$

We can see that part of the exchange rate volatility is exogenous and determined by α_1, and part of the volatility is endogenously determined by the reserve holdings $\alpha_2 \frac{a_{t-1}}{y_{t-1}}$. Here the experiment is to see how the increase in the exogenous part of the volatility α_1 affects the government's decisions. The result is shown in the Table 4. The label ExVol is the standard deviation of ξ_t if the economy starts with zero reserve holdings. Mathematically we have ExVol $= \frac{\exp \frac{1}{2}\alpha_1}{1-\rho_\xi}$.

Table 4: The Effects of Exchange Rate Volatility

ExVol	0.06	0.1(*Benchmark*)	0.12
Mean Debt-to-GDP	30	25	23
Mean Reserve-to-GDP	2.4	4.4	4.0
Mean r_s	1.4	2.5	2.8
Default Rate per 100 Years	1.2	1.9	2.0

Table 4 shows that if the economy moves from a low exchange rate volatility regime of 0.06 to the higher benchmark exchange rate volatility regime of 0.1, the government optimally reduces borrowing and increases reserve holdings. It suggests that when the exchange rate volatility is increasing from a relatively low level, increasing reserve accumulation is better than reducing external borrowing to limit the exposure to exchange rate fluctuations. So the change of volatility regime from 0.06 to 0.1 is consistent with the idea that rapid financial integration and more volatile capital flows can explain the increasing reserve accumulation.

However, this is not always true. If the exchange rate volatility is increasing from the benchmark of 0.1 to a higher level of 0.12, then the model predicts that using reserves to pay back debt is better than increasing reserve holdings. It suggests that in a relative high

volatility regime, rapid financial integration and more volatile capital flows may not explain the increasing reserve holdings.

For the relationship between exchange rate volatility and interest spread, the model predicts the same positive correlation as in the data, and it is true for both regime changes. It implies that high exchange rate volatility always increase the default risk even with precautionary measures by the governments.

1.4.4 Reserve Accumulation to Hedge the Depreciation

The model predicts three different channels for reserve accumulation: consumption smoothing when borrowing is costly, hedging against depreciation shocks, and reducing exchange rate volatility. Bianchi et al. (2016) show that different maturity of debt and reserves can make reserve a good hedge against rollover risk and help smooth consumption when future borrowing is costly. They also show that this channel is quantitatively important. Here, I want to investigate whether the two novel channels for reserve accumulation are also quantitatively important.

Since both debt and reserves are in foreign currency, although exchange rate depreciation increases the real debt burden, it also increase the real value of reserves. So foreign currency reserves can be a good hedge against depreciation shock. To isolate the effect of currency hedge, I recalibrate a model where the reserves are denominated in home currency, to see whether there is large decrease in reserve holdings compared with the benchmark. The result is shown in Table 5.

The result shows that after recalibrating the model with domestic currency reserves, the reserve holdings decrease 50%, which implies that the foreign currency feature of the reserve is important to explain the large reserve holdings. On the other hand, even if the reserve is in home currency, the model still predicts 2.1% of reserve holdings. This suggests that it is important to have an asset that has the same return in all states of the world, so that the government will find it optimal to issue defaultable debt to purchase this assets, to transfer resources from future good states to future bad states.

Table 5: The Effects of Currency Hedge

Currency of Reserves	Foreign Currency	Domestic Currency
Mean Debt-to-GDP	25	25
$\sigma(r_s)$	2.5	2.4
Default probability	1.9	2
Mean r_s	2.5	2.6
Mean Reserve-to-GDP	4.4	2.1

1.4.5 Reserve Accumulation for Less Exchange Rate Volatility

Lastly, to quantitatively evaluate the role of using reserves to reduce the exchange rate volatility, I shut down this channel by setting $\alpha_2 = 0$. Then I recalibrate the new model to the same moments to see the quantify of reserve accumulation. The result is shown in Table 6.

Table 6: Role of Reserves to Reduce ExVol

	Benchmark	$\alpha_2 = 0$
Mean Debt-to-GDP	25	25
$\sigma(r_s)$	2.5	2.6
Default probability	1.9	2
Mean r_s	2.5	2.6
Mean Reserve-to-GDP	4.4	1.3

After shutting down the channel, we see a large drop in the reserve accumulation. It suggests that in an environment where the government is only allowed to borrow in foreign currency, the volatile exchange rate becomes a major risk for the government. The gov-

ernment is willing to triple the reserves from 1.3% to 4.4% to capture the benefits of using reserve to reduce the exchange rate volatility. The results also point out the important role of reserves for foreign exchange market management.

1.5 Conclusion

The paper investigates how nominal exchange rate volatility affects a government's default risk and its incentive to accumulate reserves. To understand the rationale, I construct a sovereign default model, where the government facing volatile exchange rate fluctuations can borrow and save in foreign currency.

In this model, whenever nominal exchange rate depreciates, debt burden in terms of domestic currency increases, leading to higher default risk. Based on the model structure, the paper proposes three channels through which reserve accumulation can insure against the volatile exchange rate movements. Specifically, it can help smooth consumption when borrowing becomes costly, hedge against depreciation in the exchange rate, and stabilize the fluctuations in exchange rate. Quantitatively, the paper shows that the calibrated model can generate more than half of the reserve holdings. It can also replicate the increasing reserve holdings and the positive link between exchange rate volatility and interest rate spread. Moreover, all the channels of reserve accumulation are shown to be quantitatively important.

From the paper, we learn that borrowing in foreign currency can add to vulnerability for a government if it faces volatile exchange rate fluctuations. Facing the vulnerability, if the government does not hold enough reserves, once a country-specific shock or financial contagion hits, it becomes much difficult for the government to repay its debt, and thereby increasing the default risk. In this sense, the model has an intuitive prediction that accumulating both debt and reserves are optimal to insure against the vulnerability.

However, the prediction is invalid if the nominal exchange rate volatility is relatively high. In this case, the additional reserve accumulation will increase the gross debt position, and the cost of higher gross debt and reserve level may outweigh the insurance benefits of

reserve accumulation. Therefore, the predicted optimal choice is to reduce debt and reserve to become closer to the financial autarky. This prediction hinges on the importance to regulate and limit massive amount of speculative capital flows.

2.0 Household Portfolio Accounting

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2.1 Introduction

Housing and stock markets can provide a great opportunity for households to invest and accumulate wealth. However, more than 20 percent of households do not own any risky assets, defined as stocks, real estate, and non-corporate business, according to the Survey of Consumer Finances (SCF, 2016). A large literature has focused on this extensive margin, for example, Attanasio and Paiella (2011), Chambers et al. (2009), Guvenen (2009), Khorunzhina (2013), Luttmer (1999), and Paiella (2007). Less is understood about the heterogeneity in the portfolio composition. For example, 10 percent of risky asset owners invest less than 40 percent of their wealth in risky assets, while 15 percent invest more than 200 percent of their wealth in risky assets (SCF 2016). In this project, we focus on accounting for differences in household portfolios, including this intensive margin.

We develop a benchmark life-cycle model with labor income risk and portfolio choice (Cocco et al. (2005)), augmented with a savings wedge that lowers the return on saving and a risky wedge that lowers the relative return on risky assets. In the first part of the paper, we show that various models are equivalent to our benchmark model with wedges. In the second part of the paper, we develop a methodology to estimate these wedges from the data. We apply this method to the Survey of Consumer Finances (SCF 2004–2016), the Panel Survey of Income Dynamics (PSID, 2005–2015) and the Survey of Income and Program Participation (SIPP, 2004–2012) and compute household-level wedges that rationalize the data, in the spirit of Chari et al. (2007). These wedges are a novel and useful diagnostic that is informative about how far household portfolios are from the optimum, controlling for

household characteristics.

This paper has two main objectives. First, we use the wedges to guide plausible frictions that researchers should consider. Second, we analyze the extent to which household characteristics can account for the wedges. For example, we find that risky wedges are smaller for college graduates, self-employed households, and home owners. White households have larger risky wedges but smaller savings wedges. Moreover, we find that risky wedges and savings wedges exhibit a strong negative correlation. One interpretation could be that households are heterogeneous in preferences and that more risk-averse households are less patient. Alternatively, we show that the negative correlation of risky and savings wedges can be generated with binding borrowing constraints. The next step in this research agenda is to quantitatively assess which frictions and model ingredients are the most significant in accounting for the differences in household portfolio choices in the U.S.

Our paper builds on the wedge accounting method developed by Chari et al. (2007), and is related to works by Hsieh and Klenow (2009) and Restuccia and Rogerson (2008) who measure firm-level wedges to measure resource misallocation. See Hopenhayn (2014) for an excellent review on the literature on firm-level wedges and misallocation. To our knowledge, our paper is the only research on measuring household-level wedges, with the exception of Berger et al. (2019) who focus on imperfect risk-sharing and aggregate fluctuations.

We build on a large literature that has used life-cycle models to understand portfolio choice. See, for example, Angeletos et al. (2001), Cocco et al. (2005), Davis et al. (2006), Gomes and Michaelides (2005), Hu (2005), Kaplan and Violante (2014), and Storesletten et al. (2007) for some recent works. Our paper is also related to works that attempt to account for differences in household portfolios (Bianchi 2018, Calvet et al. 2007, Cole et al. 2014, Fagereng et al. 2016, von Gaudecker 2015, Grinblatt et al. 2011, Guiso et al. 1996, Heaton and Lucas 2000, and Van Rooij et al. 2011) and in wealth (Carroll 2000, Castaneda et al. 2003, De Nardi 2004, Gabaix et al. 2016, Krusell and Smith 1998). Campbell (2006) and De Nardi and Fella (2017) provide excellent reviews of these literatures.

The rest of the paper is structured as follows. Section 2.2 develops the benchmark model with wedges and show that many models with frictions are equivalent to the benchmark model with wedges. Section 2.3 then describes the process of measuring wedges at the

household level and section 2.4 provides an analysis of the measured wedges. Finally, section 2.5 concludes by discussing directions for future research.

2.2 Model

In this section, we develop a benchmark model of portfolio with wedges and show that many models with frictions are equivalent to our model with wedges. We use the benchmark model with wedges later to account for household portfolios observed in the U.S. in the next section.

2.2.1 Benchmark Model with Wedges

Consider a life-cycle model with portfolio choice, uninsurable labor income risk, and mortality risk, similar to the Cocco et al. (2005). At age j, the household experiences one of finitely many shocks, denoted by s_j, and $s^j = (s_1, ..., s_j)$ denotes the history of events up through age j. The probability of any particular history s^j is given by $\pi_j(s^j)$. The benchmark model has four exogenous stochastic variables, all of which are functions of the underlying history s^j: the safe wedge $1 + \tau_{sj}(s^j)$, the risky wedge $1 + \tau_{xj}(s^j)$, labor income $y_j(s^j)$, and risky return $R_{xj}(s^j)$.

Households choose a sequence of consumption and safe and risky assets to maximize expected lifetime utility,

$$\begin{aligned}
\max \quad & \sum_{j=1}^{J} \sum_{s^j} \beta^{j-1} \pi_j\left(s^j\right) \left(\prod_{a=1}^{j-2} \psi_a\right) \left[\psi_{j-1} u\left(c_j\left(s^j\right)\right) + \omega(1-\psi_{j-1}) u\left(d_j\left(s^j\right)\right)\right] \qquad (2.1) \\
\text{s.t.} \quad & c_j\left(s^j\right) + \left[b_{j+1}\left(s^j\right) + x_{j+1}\left(s^j\right)\left(1+\tau_{xj}\left(s^j\right)\right)\right]\left(1+\tau_{sj}\left(s^j\right)\right) \\
& = Rb_j\left(s^{j-1}\right) + R_{xj}\left(s^j\right) x_j\left(s^{j-1}\right) + y_j\left(s^j\right) + T_j\left(s^j\right) \\
& d_j\left(s^j\right) = Rb_j\left(s^{j-1}\right) + R_{xj}\left(s^j\right) x_j\left(s^{j-1}\right) + y_j\left(s^j\right),
\end{aligned}$$

where $c_j(s^j)$ denotes consumption, $b_j(s^{j-1})$ is safe assets, $x_j(s^{j-1})$ is risky assets, R is the safe return, β is the discount factor, ψ_j is the survival probability from age j to $j+1$, $T_j(s^j)$ is lump-sum transfers, and $d_j(s^j)$ is the wealth the household bequeaths to its descendants

at death. To ease notation, we set the bequest motive ω to zero for the remainder of this section. The optimal allocation is summarized by

$$c_j\left(s^j\right)+b_{j+1}\left(s^j\right)+x_{j+1}\left(s^j\right)=Rb_j\left(s^{j-1}\right)+R_{xj}\left(s^j\right)x_j\left(s^{j-1}\right)+y_j\left(s^j\right), \tag{2.2}$$

$$1+\tau_{sj}\left(s^j\right)=\beta\sum_{s^{j+1}}\pi_j\left(s^{j+1}|s^j\right)\frac{u'\left(c_{j+1}\left(s^{j+1}\right)\right)}{u'\left(c_j\left(s^j\right)\right)}R, \tag{2.3}$$

$$1+\tau_{xj}\left(s^j\right)=\frac{\sum\limits_{s^{j+1}}\pi_j\left(s^{j+1}|s^j\right)u'\left(c_{j+1}\left(s^{j+1}\right)\right)R_{x,j+1}\left(s^{j+1}\right)}{\sum\limits_{s^{j+1}}\pi_j\left(s^{j+1}|s^j\right)u'\left(c_{j+1}\left(s^{j+1}\right)\right)R}, \tag{2.4}$$

where $\pi_j\left(s^{j+1}|s^j\right)$ denotes the conditional probability $\pi_j\left(s^{j+1}\right)/\pi_j\left(s^j\right)$.

It is straightforward to show that the benchmark model with no wedges but heterogeneity in discount factors are equivalent to the benchmark model with heterogeneous savings wedges. If risky returns follow $R_{xj}(s^j)=R+\mu+\eta(s^j)$ where $\mathbb{E}[\eta(s^j)]=0$, one can also show that benchmark model with no wedges but heterogeneity in excess returns, μ, are equivalent to the benchmark model with heterogeneous risky wedges. In the next subsections, we illustrate how other models map into the benchmark model with wedges.

2.2.2 Collateral Constraints

Consider a model with a borrowing constraint, in which households solve

$$\begin{aligned}
\max\quad & \sum_{j=1}^{J}\sum_{s^j}\beta^{j-1}\pi_j\left(s^j\right)\left(\prod_{a=1}^{j-1}\psi_a\right)u\left(c_j\left(s^j\right)\right)\\
\text{s.t.}\quad & c_j\left(s^j\right)+b_{j+1}\left(s^j\right)+x_{j+1}\left(s^j\right)\\
& =Rb_j\left(s^{j-1}\right)+R_{xj}\left(s^j\right)x_j\left(s^{j-1}\right)+y_j\left(s^j\right)\\
& -b_{j+1}\left(s^j\right)\leq\kappa x_{j+1}\left(s^j\right)
\end{aligned}$$

where $\kappa\geq 0$ is the fraction of risky assets that can be collateralized.

Let $c_j^*\left(s^j\right)$ and $\mu_j^*\left(s^j\right)\beta^{j-1}\pi_j\left(s^j\right)\left(\prod_{a=1}^{j-1}\psi_a\right)$ represent the optimal allocation for consumption and the Lagrange multiplier on the collateral constraint, respectively. The allocation in the problem with borrowing constraints is equivalent to the benchmark model with

wedges if and only if

$$1+\tau_{sj}\left(s^{j}\right)=\frac{u'\left(c_{j}^{*}\left(s^{j}\right)\right)-\mu_{j}^{*}\left(s^{j}\right)}{u'\left(c_{j}^{*}\left(s^{j}\right)\right)}, \tag{2.5}$$

$$1+\tau_{xj}\left(s^{j}\right)=\frac{u'\left(c_{j}^{*}\left(s^{j}\right)\right)-\kappa\mu_{j}^{*}\left(s^{j}\right)}{u'\left(c_{j}{}^{*}\left(s^{j}\right)\right)-\mu_{j}^{*}\left(s^{j}\right)}. \tag{2.6}$$

Notice that $\tau_{sj}\left(s^{j}\right)\leq 0$ with strict inequality when the collateral constraint is binding. Furthermore, $\tau_{xj}\left(s^{j}\right)>0$ if and only if the collateral constraint is binding and $\kappa<1$, which is the empirically relevant range, as discussed in Hur (2018).

2.2.3 Financial Knowledge

Consider a model with investment in financial knowledge, in which households solve

$$\begin{aligned}
\max \quad & \sum_{j=1}^{J}\sum_{s^{j}}\beta^{j-1}\pi_{j}\left(s^{j}\right)\left(\prod_{a=1}^{j-1}\psi_{a}\right)u\left(c_{j}\left(s^{j}\right)\right)\\
\text{s.t.} \quad & c_{j}\left(s^{j}\right)+b_{j+1}\left(s^{j}\right)+q\left(f_{j+1}\left(s^{j}\right)\right)x_{j+1}\left(s^{j}\right)+f_{j+1}\left(s^{j}\right)\\
& =Rb_{j}\left(s^{j-1}\right)+R_{xj}\left(s^{j}\right)x_{j}\left(s^{j-1}\right)+y_{j}\left(s^{j}\right)\\
& f_{j+1}\left(s^{j}\right)\geq 0
\end{aligned}$$

where $f_{j+1}\left(s^{j}\right)\geq 0$ is financial knowledge, which we—without loss of generality—assume is in units of the consumption good and changes the risky return by decreasing q. We assume that $q(0)=1$ and that q is strictly decreasing and convex.

Let $f_{j+1}^{*}\left(s^{j}\right)$ represent the optimal allocation for financial knowledge. The allocation in the problem with financial knowledge is equivalent to the benchmark model with wedges if and only if

$$\tau_{sj}\left(s^{j}\right)=0, \tag{2.7}$$

$$1+\tau_{xj}\left(s^{j}\right)=q\left(f_{j+1}^{*}\left(s^{j}\right)\right), \tag{2.8}$$

$$T_{j}\left(s^{j}\right)=-f_{j+1}^{*}\left(s^{j}\right). \tag{2.9}$$

Notice that the risky wedge is decreasing in financial knowledge.

2.2.4 Housing

Consider a model with housing, in which households solve

$$\begin{aligned}
\max \quad & \sum_{j=1}^{J} \sum_{s^j} \beta^{j-1} \pi_j \left(s^j\right) \left(\prod_{a=1}^{j-1} \psi_a\right) \left[u\left(c_j\left(s^j\right)\right) + v\left(h_{j+1}\left(s^j\right)\right)\right] \\
\text{s.t.} \quad & c_j\left(s^j\right) + b_{j+1}\left(s^j\right) + x_{j+1}\left(s^j\right) \\
& = R b_j\left(s^{j-1}\right) + R_{xj}\left(s^j\right) x_j\left(s^{j-1}\right) + y_j\left(s^j\right) \\
& h_{j+1}\left(s^j\right) \leq \zeta x_{j+1}\left(s^j\right)
\end{aligned}$$

where $h_{j+1}\left(s^j\right)$ and ζ are the consumption of housing services and the service flow rate of housing provided by the risky asset, respectively.

Let $c_j^*\left(s^j\right)$ and $x_{j+1}^*\left(s^j\right)$ represent the optimal allocations for consumption and risky assets, respectively. The allocation in the problem with housing is equivalent to the benchmark model with wedges if and only if

$$\tau_{sj}\left(s^j\right) = 0, \tag{2.10}$$

$$\tau_{xj}\left(s^j\right) = -\zeta \frac{v'\left(\zeta x_{j+1}^*\left(s^j\right)\right)}{u'\left(c_j^*\left(s^j\right)\right)}, \tag{2.11}$$

$$T_j\left(s^j\right) = x_{j+1}^*\left(s^j\right) \tau_{xj}\left(s^j\right). \tag{2.12}$$

Notice that the housing service role of risky assets, measured by ζ, reduces the risky wedge.

2.2.5 Interpretation of Wedges

We have demonstrated that risky wedges are related to the risky return, collateral constraints, financial knowledge, and service flows provided by risky assets. In general, we interpret differences in risky wedges as potentially being generated by differences in willingness to take risk (risk aversion), ability to take risk (constraints), actual or perceived differences in the risky return, which in turn could represent differences in financial knowledge, et cetera.

Differences in savings wedges can potentially be generated by differences in the safe return R and patience β, which in turn could represent unmodeled changes in family size and composition, differences in mortality risk, and any other factors that generate variation in the marginal utility of consumption (see, for example, Glover et al. 2011).

2.3 Wedge Estimation

We now describe the process for estimating the wedges. In this section, we assume that the stochastic processes are such that the household's relevant states are a set of exogenous shocks, s_j, which follow a Markov process, and an endogenous state: wealth w_j. We follow Cocco et al. (2005) for many functional forms, stochastic processes, and parameter values.

2.3.1 Labor Income Process

Households can live up to J periods, with exogenous retirement after age j^*. Working age ($j \leq j^*$) labor income y is given by

$$y_j(z, v, \varepsilon) = e^{f_j(z)+v+\varepsilon} \tag{2.13}$$

where f is a deterministic function of age j and other household characteristics z (education and race), $\varepsilon \sim N(0, \sigma_\varepsilon^2)$ is a transitory shock, and v is a persistent shock, which follows

$$v' = v + u \tag{2.14}$$

where $u \sim N(0, \sigma_u^2)$. For computational tractability, we assume that retirees ($j > j^*$) receive a constant fraction λ of permanent labor income in the last working age j^*.

2.3.2 Wedges

Similar to the labor income process, the wedges are assumed to have deterministic and stochastic components. Specifically, wedges follow

$$\tau_{mj}(z, \varepsilon_m) = g_{mj}(z) + \varepsilon_m \text{ for } m = x, s \tag{2.15}$$

where g_m is a deterministic function of age j and other household characteristics z and $\varepsilon_m \sim N(0, \sigma_{\varepsilon_m}^2)$ is an idiosyncratic shock that is independent across agents and over time.

2.3.3 Recursive Formulation

Let $w = Rb + R_x x + y$ denote household cash-on-hand. Furthermore,let $s \equiv \{z, v, \varepsilon, \varepsilon_x, \varepsilon_s\}$. The household's problem can be written as:

$$
\begin{aligned}
V_j(w,s) = \max_{b',x'} \;\; & u(c) + \beta \mathbb{E}_{R'_x, s'} \left[\psi_j V_{j+1}(w', s') + \omega \left(1 - \psi_j\right) u(w') \right] \\
\text{s.t.} \;\; & c + (x'(1+\tau_{xj}(s)) + b')(1+\tau_{sj}(s)) = w + T_j(w,s) \\
& w' = b'R + x'R'_x + y_{j+1}(s').
\end{aligned} \tag{2.16}
$$

Note that the state space can be simplified even further, given the assumption that the persistent shock is a random walk, as described in Cocco et al. (2005). See Appendix A for details. We impose the condition that

$$
T(w,s) = x^*_{j+1}(w,s)((1+\tau_{xj}(s))(1+\tau_{sj}(s)) - 1) + b^*_{j+1}(w,s)\tau_{sj}(s)
$$

where $x^*_{j+1}(w,s)$ and $b^*_{j+1}(w,s)$ denote the policy functions for risky and safe assets, respectively. Notice that households take contemporaneous transfers as given, but internalize the fact that investment decisions today affect future transfers. The solution to (2.16) can be characterized by:

$$
c^*_j(w,s) + x^*_{j+1}(w,s) + b^*_{j+1}(w,s) = w \tag{2.17}
$$

$$
1 + \tau_{sj}(s) = \frac{\beta \mathbb{E}_{R'_x, s'} \left[\frac{\partial \left(\psi_j V_{j+1}(w', s') + \omega \left(1 - \psi_j\right) u(w') \right)}{\partial w'} R \right]}{u'(c^*_j(w,s))} \tag{2.18}
$$

$$
1 + \tau_{xj}(s) = \frac{\mathbb{E}_{R'_x, s'} \left[\frac{\partial \left(\psi_j V_{j+1}(w', s') + \omega \left(1 - \psi_j\right) u(w') \right)}{\partial w'} R'_x \right]}{\mathbb{E}_{R'_x, s'} \left[\frac{\partial \left(\psi_j V_{j+1}(w', s') + \omega \left(1 - \psi_j\right) u(w') \right)}{\partial w'} R \right]} \tag{2.19}
$$

where $c^*_j(w,s)$ is the policy functions for consumption.

2.3.4 Algorithm for Solving Wedges

Here, we discuss the algorithm for solving the household-level wedges that rationalize the data. The last-period value function is given by $v_J(w, s) = u(w)$. For each household i, we observe age j, household characteristics z_i^{data}, labor income y_i^{data}, risky assets x_i^{data}, safe assets b_i^{data}, and in the case of the PSID, we also observe consumption c_i^{data}. We face the problem that we are unable to distinguish between transitory and persistent shocks in the data. We proceed by assuming that the transitory shock is zero, $\varepsilon = 0$, so that

$$v_i = \begin{cases} \log y_i^{data} - f_j(z_i^{data}) & \text{if } j \leq j^* \\ \log \dfrac{y_i^{data}}{\lambda} - f_{j^*}(z_i^{data}) & \text{otherwise} \end{cases} \tag{2.20}$$

where $f_j(z)$ is obtained by regressing $\log(y_i)$ on age and college dummies, their interaction, and race and year dummies. For a given guess of the wedge functions $g_{sj}(z)$ and $g_{xj}(z)$, by backward induction for $j < J$, we can obtain the risky wedges using equation (2.19), and in the case of the PSID, the saving wedges using (2.18). Once we have obtained the wedges, we regress the wedges on age j and household characteristics z to estimate g. We iterate these steps until g converges. See appendix B for details. Note that when we do not have consumption data as is the case with the SCF and SIPP, we can solve for the risky wedges and $g_{xj}(z)$ given any functional form for $g_{sj}(z)$. In particular, we use the $g_{sj}(z)$ estimated from the PSID. Since our focus is on the intensive margin, we only study households with positive risky assets.

2.3.5 Parameters and Functional Forms

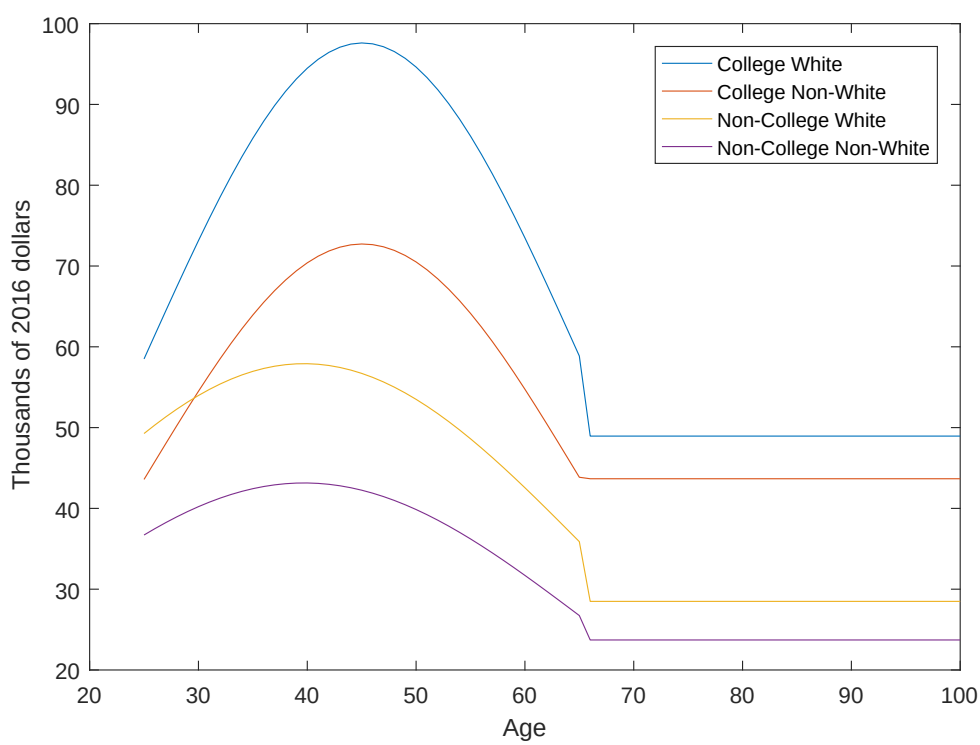
We use standard parameters and functional forms from the literature. Preferences are summarized by

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma} \tag{2.21}$$

where γ is the coefficient of relative risk aversion. We set the risk aversion to 2, which is a standard value in the macro literature. We set the bequest motive parameter, ω, to 1, one of the values considered in Cocco et al. (2005).

We estimate the labor income process on disposable labor income from the PSID, similar to Krueger et al. (2016) and Hur (2018). First, we compute disposable labor income on the PSID 1970–1997 core sample of households (ages 23–65), by adding transfers to labor income and subtracting taxes estimated through TAXSIM version 9. Second, we obtain residuals from a regression with age, white, college, and year fixed effects and the interaction of age and college fixed effects. The deterministic age-profiles, $f_j(z)$, for white college and non-college graduates and non-white college and non-college graduates are also generated from this regression. Figure 2 plots these profiles. Finally, we estimate, using the generalized method of moments (GMM), the random walk process specified in (2.13). The estimation yields permanent shock variance of $\sigma_u^2 = 0.0120$, and a transitory shock variance of $\sigma_\varepsilon^2 = 0.1093$. For retired households, we estimate the replacement rate, λ, for each demographic group, based on the average log difference between real disposable labor income and predicted income.

Figure 2: Disposable Labor Income Profile



Following Cocco et al. (2005), we assume that the risky asset shock follows $R_x = R+\mu+\eta$ where $\mu = 0.04$ represents the average excess return and $\eta \sim N(0, \sigma_\eta)$ is normally distributed with $\sigma_\eta = 0.157$. Also following Cocco et al. (2005), the discount factor is set to 0.96 and the risk-free rate is set to 2 percent. The parameter values are summarized in Table 7.

Table 7: Parameter Values

parameters	values	target/source
last working age j^*	40	age 65
discount factor β	0.96	Cocco et al. (2005)
risk aversion γ	2	standard macro value
var(transitory shocks) σ_ε^2	0.1093	authors' estimation
var(permanent shocks) σ_u^2	0.0120	authors' estimation
replacement rate λ	0.79–1.00	authors' estimation
risk-free rate $(R-1)$	0.02	Cocco et al. (2005)
excess risky return μ	0.04	Cocco et al. (2005)
st. dev. of risky return σ_η	0.157	Cocco et al. (2005)
bequest motive intensity ω	1	TBD

2.4 Wedge Analysis

In this section, we present the estimated wedges. Risky and safe wedges have been winsorized at the 1 percent and the 10 percent level, respectively.

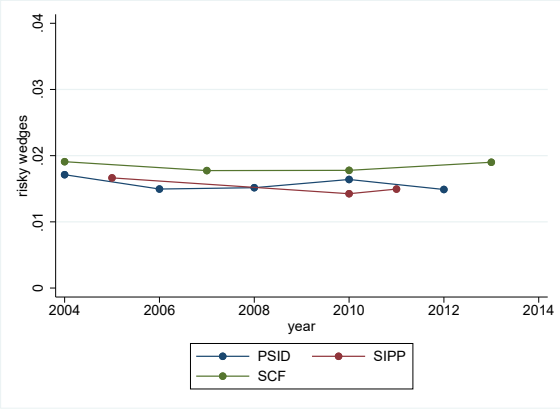
2.4.1 Wedge Patterns

Figure 3 plots the the time series of both wedges. Panel (a) demonstrates that the risky wedges are sizable and are consistent across the three data sets, while panel (b) shows that

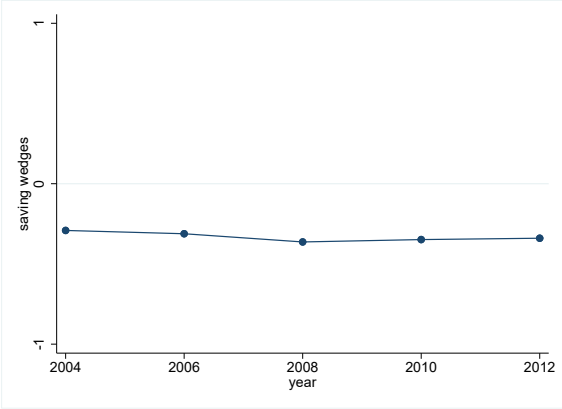
the savings wedge is negative and stable over time. For the remaining analysis, we report wedges estimated from the PSID.

Figure 3: Wedges over Time

(a) Risky Wedges



(b) Savings Wedges (PSID)



We find that risky wedges vary significantly across households, as shown in Figure 4 panel (a). In addition, more than 90 percent of households have a positive wedge, implying that these households are under-investing in risky assets. Panel (b) indicates an even larger variance for savings wedges.

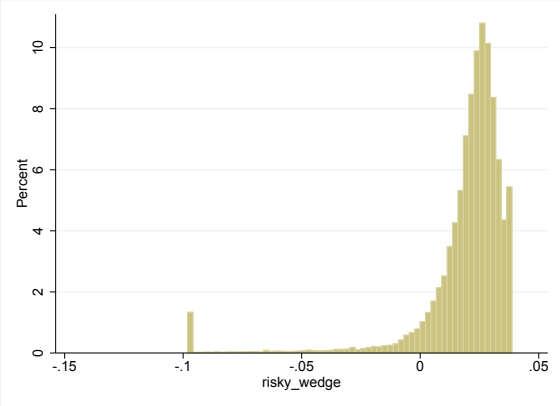
Figure 5 shows a bin scatter plot for risky and savings wedges. We find a strong negative correlation between risky and savings wedges. This is noteworthy since we documented that collateral constraints can generate wedges with positive risky wedges and negative savings wedges in section 2.2.2.

Figure 6 plots average wedges over the life-cycle. We find that risky wedges are relatively constant across age and that savings wedges are hump-shaped with a discontinuity around retirement. One factor that decreases the savings wedges in retirement is the fact that, in the model, retired households' only source of uncertainty is in their risky asset returns. Another factor is that retired households have different consumption bundles than working age households (for example, see Aguiar and Hurst 2005 and Aguiar and Hurst 2013).

Figures 7–11 plot the wedges by various household characteristics. We find that risky and savings wedges decline with education, suggesting that informational frictions may be

Figure 4: Wedge Distributions

(a) Risky Wedges



(b) Savings Wedges

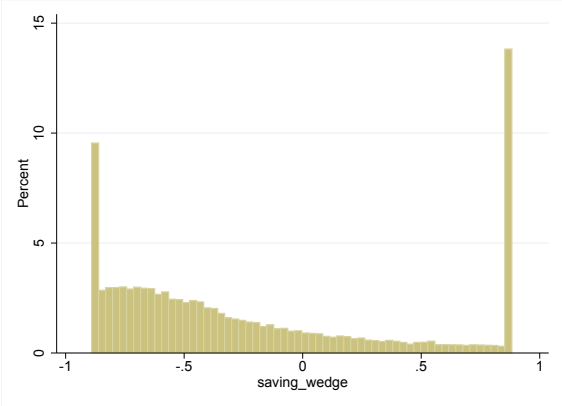


Figure 5: Risky and Saving Wedges

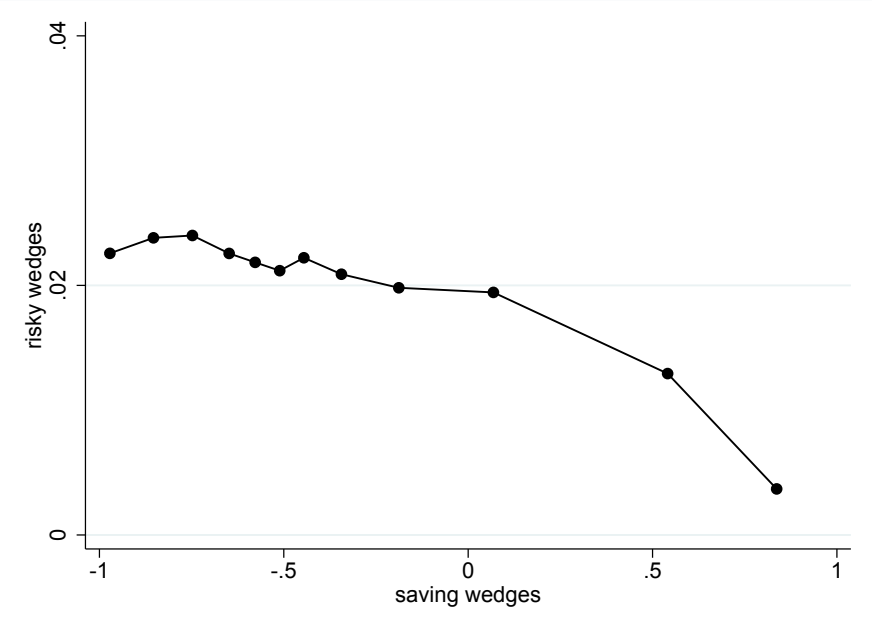
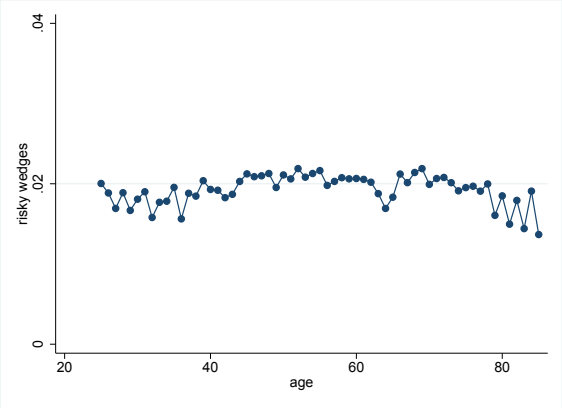
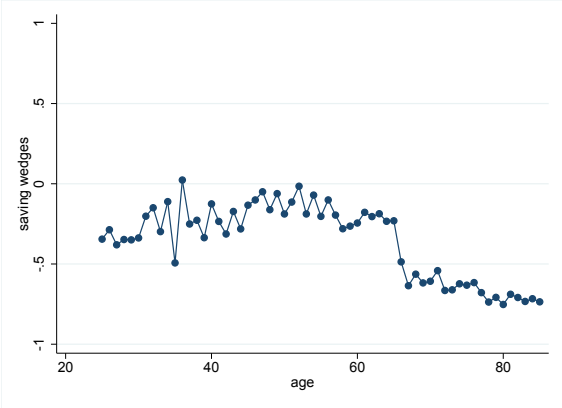


Figure 6: Wedges over the Life-Cycle

(a) Risky Wedges



(b) Savings Wedges

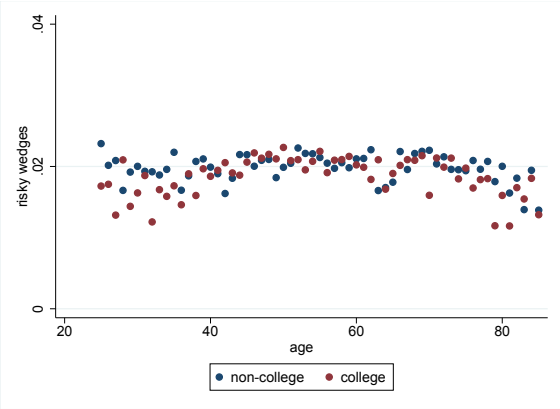


important. Surprisingly, risky wedges are higher for male heads. This implies that, controlling for household characteristics, including income and wealth, males do not take more risky asset positions, an implication that is at odds with the claim that women are more risk-averse than men (Jianakoplos and Bernasek 1998). There is a gender difference in the savings wedges, which could be the result of differences in, for example, family size and composition.

Home ownership and self-employment decrease risky wedges significantly, suggesting that these are important channels that need to be considered. We also find that white households have savings wedges but slightly higher risky wedges, a finding that is different Gittleman and Wolff (2004) who document that, controlling for income, there is no difference in savings rates or capital returns. Moreover, to the extent that the higher risky wedge and lower savings wedge could the result of a tighter collateral constraint (see section 2.2.2), our finding is in contrast to most of the literature on discriminatory lending practices, reviewed by Ladd (1998).

Figure 7: Wedges and Education

(a) Risky Wedges



(b) Savings Wedges

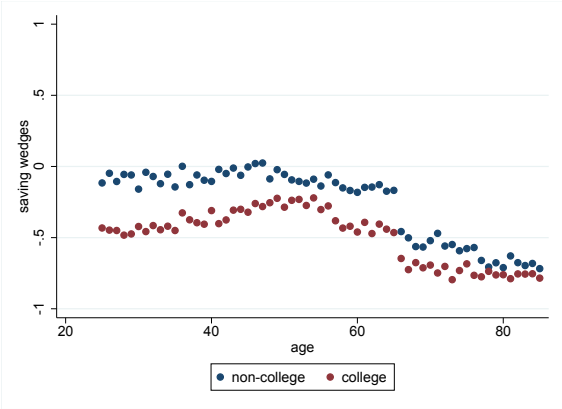
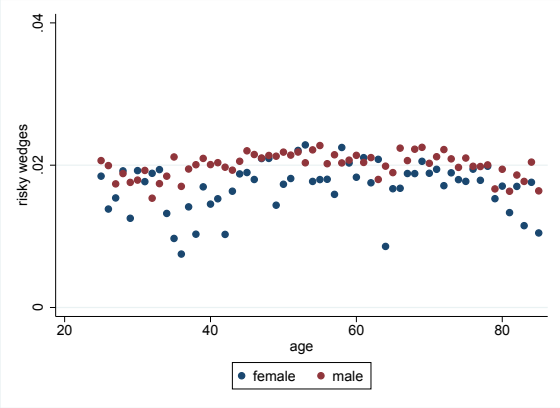


Figure 8: Wedges and Gender

(a) Risky Wedges



(b) Savings Wedges

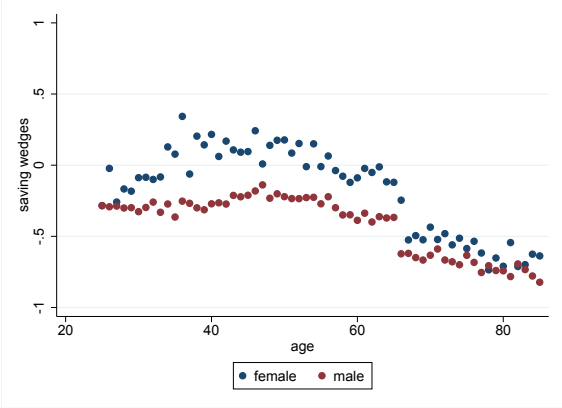
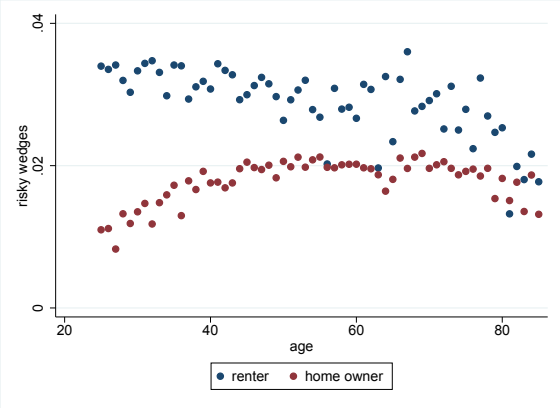


Figure 9: Wedges and Home Ownership

(a) Risky Wedges



(b) Savings Wedges

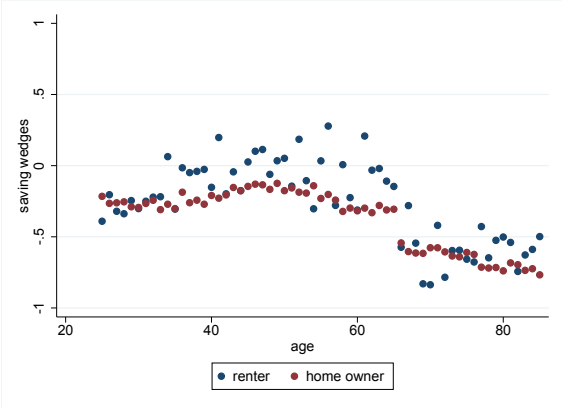
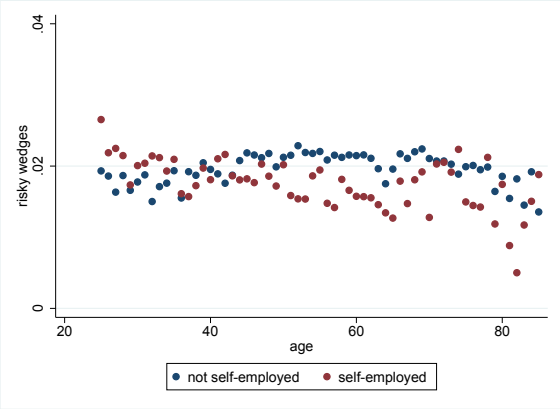


Figure 10: Wedges and Self-Employment

(a) Risky Wedges



(b) Savings Wedges

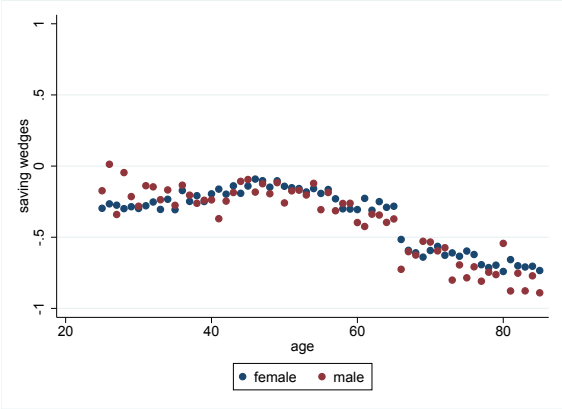
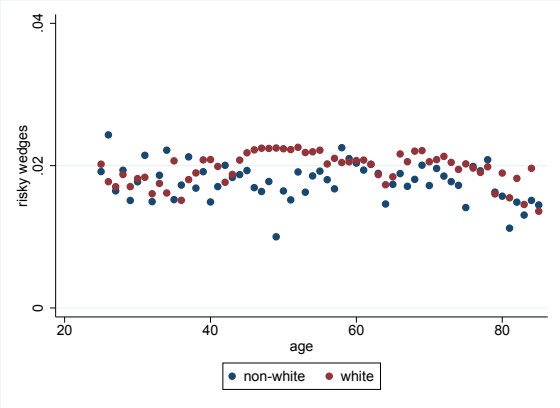


Figure 11: Wedges and Race

(a) Risky Wedges



(b) Savings Wedges

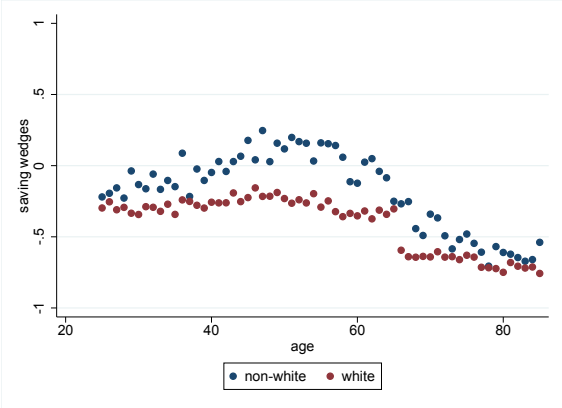
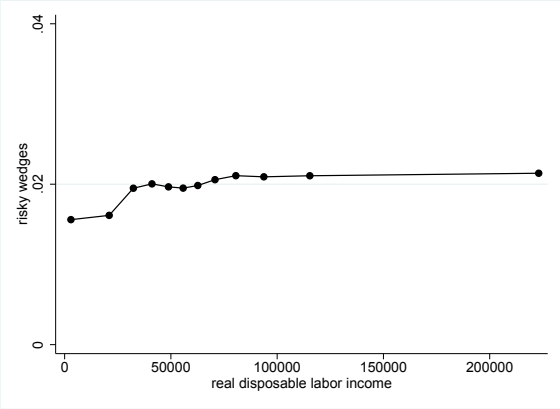


Figure 12: Wedges and Income

(a) Risky Wedges



(b) Savings Wedges

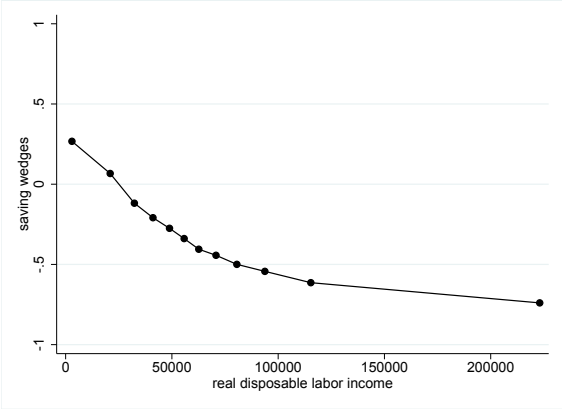
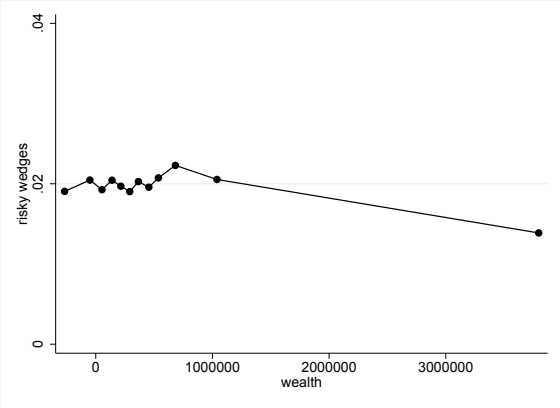
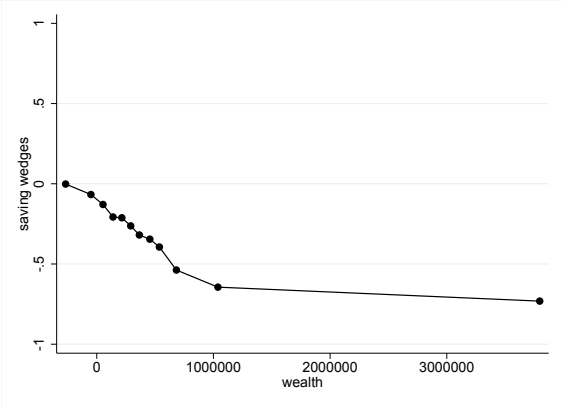


Figure 13: Wedges and Wealth

(a) Risky Wedges



(b) Savings Wedges



Finally, we find that risky wedges are increasing in disposable labor income and that savings wedges are declining in wealth (Figures 12 and 13). As we show below, however, some of the documented relationships, such as that between risky wedges and income are not robust in a more systematic analysis.

2.4.2 Wedge Regression

To analyze the wedges in a more systematic way, we regress the wedges on various household characteristics. In columns (1) and (4) of Table 8, we regress the risky and savings wedges, respectively, on the natural logarithms of disposable labor income and wealth. In the same table, columns (2) and (5) include household controls and columns (3) and (6) include household fixed effects.

A few observations are worth highlighting. First, notice that risky wedges increase with income and wealth, whereas savings wedges decline with income and wealth. Second, risky wedges are higher was for males, white, and retired households and are lower for college graduates, home owners, self-employed households, and stock owners. The effect for retired and self-employed households and stock owners are robust to including household fixed effects. Finally, savings wedges are higher for self-employed households and are lower for male, white, and retired households, college graduates, home owners, and stock owners. The effect for retired households and stock owners are robust to household fixed effects. Most of these relationships confirm the patterns documented in Figures 6–13.

Table 8: Wedge Regressions

	risky wedge (percent)			saving wedge (percent)		
	(1)	(2)	(3)	(4)	(5)	(6)
	no controls	controls	household FE	no controls	controls	household FE
ln(income)	0.22***	0.34***	0.71***	-21.72***	-30.34***	-33.35***
	(0.01)	(0.02)	(0.02)	(0.38)	(0.40)	(0.61)
ln(wealth)	0.05***	0.09***	0.15***	-3.97***	-1.84***	-0.69***
	(0.00)	(0.00)	(0.01)	(0.12)	(0.11)	(0.15)
male		0.21***			-12.97***	
		(0.03)			(0.82)	
college		-0.30***			-2.20	
		(0.02)			(0.61)	
white		0.17***			-9.46***	
		(0.03)			(0.74)	
retired		0.29***	0.40***		-27.33***	-24.76***
		(0.05)	(0.05)		(1.32)	(1.40)
home owner		-1.63***	-1.81***		-8.60***	1.00
		(0.04)	(0.06)		(1.04)	(1.59)
self-employed		-0.51***	-0.05		2.75***	-0.06
		(0.03)	(0.05)		(0.83)	(1.12)
stock owner		-0.30***	-0.32***		-10.16***	-3.63***
		(0.02)	(0.03)		(0.61)	(0.74)
other controls	no	yes	yes	no	yes	yes
adj. R^2	0.02	0.10	0.47	0.16	0.40	0.62
N	24544	24521	24521	24544	24521	24521

Standard errors in parentheses. Other controls include age and age squared, and fixed effects for household size.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

2.5 Conclusion

From the wedge analysis, we find that many household characteristics are important to understand the heterogeneity in household portfolio choice. For instance, retired people, home-owners and stock owners have much different risky and saving wedges compared with others. This implies that standard portfolio choice choice models miss certain important ingredients to match the investment behaviors of the household.

The next steps in this research agenda is to quantitatively assess the significance of model frictions and ingredients in accounting for household portfolios in the U.S. Specifically, we plan to augment the benchmark model with various frictions (borrowing constraints, adjustment costs, etc.) and ingredients (housing, preference shocks, etc.) to evaluate which elements reduce both the average size and variance of the wedges.

Appendix A

Reducing the State Variables

The problem we need to solve is

$$V_t(w_t, e^{v_t}, \varepsilon_t, \varepsilon_t^x, \varepsilon_t^s) = \max_{x_{t+1}, b_{t+1}} u(c_t) + \beta\psi_t E_{\eta_{t+1}, u_{t+1}, \varepsilon_{t+1}} V_{t+1}(w_{t+1}, e^{v_{t+1}}, \varepsilon_{t+1}, \varepsilon_{t+1}^x, \varepsilon_{t+1}^s)$$

$$s.t.\ c_t + x_{t+1}(1+\tau_x(\cdot))(1+\tau_s(\cdot)) + b_{t+1}(1+\tau_s(\cdot)) = w_t + y_t(v_t, \varepsilon_t) + T_t$$

$$\begin{aligned} T_t &= x_{t+1}^*[(1+\tau_x(\cdot))(1+\tau_s(\cdot)) - 1] + b_{t+1}^*\tau_s(\cdot) \\ w_{t+1} &= x_{t+1}R_x + b_{t+1}R \end{aligned}$$

Where x_{t+1}^*, b_{t+1}^* denotes the optimal risky and safe investment. The problem can be rewritten as

$$V_t(w_t, e^{v_t}, \varepsilon_t, \varepsilon_t^x, \varepsilon_t^s) = \max_{s_t, \alpha_t} u(c_t) + \beta\psi_t E_{\eta_{t+1}, u_{t+1}, \varepsilon_{t+1}} V_{t+1}(w_{t+1}, e^{v_{t+1}}, \varepsilon_{t+1}, \varepsilon_{t+1}^x, \varepsilon_{t+1}^s)$$

$$s.t.\ c_t = [1 - s_t(1+\tau_s(\cdot))(1+\alpha_t\tau_x(\cdot))][w_t + y_t(v_t, \varepsilon_t)] + T_t$$

$$w_{t+1} = s_t(w_t + y_t(v_t, \varepsilon_t))(\alpha_t R_x + (1-\alpha_t)R)$$

where s_t^*, α_t^* denotes the optimal saving rate and risky share. Also we have

$$\begin{aligned} y_t(v_t, \varepsilon_t) &= e^{f(t, z_t) + v_t + \varepsilon_t} \\ v_t &= v_{t-1} + u_t \\ s_t &= \frac{x_{t+1} + b_{t+1}}{w_t + y_t(v_t, \varepsilon_t)} \\ \alpha_t &= \frac{x_{t+1}}{x_{t+1} + b_{t+1}} \end{aligned}$$

We can simplify the problem by normalizing the variables by the permanent income e^{v_t}

$$\begin{aligned} \tilde{w}_t &= \frac{w_t}{e^{v_t}}, \tilde{x}_{t+1} = \frac{x_{t+1}}{e^{v_t}}, \tilde{b}_{t+1} = \frac{b_{t+1}}{e^{v_t}} \\ \tilde{c}_t &= \frac{c_t}{e^{v_t}}, \tilde{a}_t = \frac{a_t}{e^{v_t}}, \tilde{T}_t = \frac{T_t}{e^{v_t}} \end{aligned}$$

Where a_t is the cash-on-hand

$$\begin{aligned} a_t &= w_t + y_t(v_t, \varepsilon_t) \\ \tilde{a}_t &= \tilde{w}_t + y_t(0, \varepsilon_t) \end{aligned}$$

We can show that the new problem $W_t(\tilde{a}_t, \varepsilon_t^x, \varepsilon_t^s)$ gives us the same decision rules s_t, α_t as in the original problem $V_t(w_t, e^{v_t}, \varepsilon_t, \varepsilon_t^x, \varepsilon_t^s)$. After the transformation, we reduce the number of state variables to one.

$$W_t(\tilde{a}_t, \varepsilon_t^x, \varepsilon_t^s) = \max_{s_t, \alpha_t} u(\tilde{c}_t) + \beta \psi_t E_{\eta_{t+1}, u_{t+1}, \varepsilon_{t+1}} (e^{u_{t+1}})^{1-\sigma} W_{t+1}(\tilde{a}_{t+1}, \varepsilon_{t+1}^x, \varepsilon_{t+1}^s)$$

$$s.t.\ \tilde{c}_t = [1 - s_t(1 + \tau_s(\cdot))(1 + \alpha_t \tau_x(\cdot))]\tilde{a}_t + \tilde{T}_t$$

$$\tilde{a}_{t+1} = s_t \tilde{a}_t(\alpha_t R_x + (1 - \alpha_t)R)/e^{u_{t+1}} + y_{t+1}(0, \varepsilon_{t+1})$$

Appendix B

Computational Algorithm

We solve for the value function and the optimal policy functions using a discrete state space approach. The algorithm entails an inner loop to calculate the value functions and individual wedges given the deterministic wedge profiles, and an outer loop to update the deterministic wedge profiles. Convergence occurs when the the deterministic wedge profiles is consistent with the fitted age profiles on individual wedges.

B.1 Discretize State Space

We discretize the state $\tilde{a}_t$ using N_a points, ε_t^x and ε_t^s both using N_x points. For the decision rules, we use N_s points for the saving rate, and N_{alpha} points for thhe risky share.

B.2 Discretize Income Process and Risky Return

Using the Tauchen method, we obtain the transition matrices and the grids for the permanent income shock u_t, idiosyncratic income shock ε_t, and stock return R_x.

B.3 Initial Guess for the Wedge Profiles

To solve for the value function and individual wedges, we need to know the wedge profiles for both risky and saving wedges. We set the first guess to zero, which corresponds to the

case with no wedges.

$$
\begin{aligned}
g_x(j,z) &= g_s(j,z) = 0 \\
std(\varepsilon_x) &= std(\varepsilon_s) = 0
\end{aligned}
$$

B.4 Value Function Iteration and Wedge Profile Calculation

1. Given wedge profiles $g_x(j,z), g_s(j,z)$, find optimal value function and decision rules by backward induction

   (a) No saving in the last period, $W_T(\tilde{a}_T, \varepsilon_T^x, \varepsilon_T^s)$ can be solved easily.

   (b) Need to guess for the individual transfer $\tilde{T}_t$ to solve the problem $W_t(\tilde{a}_t, \varepsilon_t^x, \varepsilon_t^s)$.

   (c) At each state $(\tilde{a}_t, \varepsilon_t^x\ \varepsilon_t^s)$, given the $W_{t+1}(\tilde{a}_{t+1}, \varepsilon_{t+1}^x, \varepsilon_{t+1}^s)$ and $\tilde{T}_t$, we use the grid search to solve for the optimal decision rules $s_t(\cdot), \alpha_t(\cdot)$. Then update the $\tilde{T}_t^{new}$ as

$$
\tilde{T}_t^{new} = (1-m_T)\tilde{T}_t + m_T(\alpha_t(\cdot)\tau_s(\cdot) + (1-\alpha_t(\cdot))((1+\tau_s(\cdot))(1+\tau_x(\cdot)) - 1))s_t(\cdot)\tilde{a}_t
$$

   where m_T controls the speed of updating.

   (d) Repeat step(c) until transfer convergence, i.e.

$$
\frac{\tilde{T}_t^{new} - \tilde{T}_t}{\tilde{a}_t} < \epsilon
$$

   Let the initial guess $\tilde{T}_t = 0$

2. After solving the value function $W_t(\tilde{a}_t, \varepsilon_t^x, \varepsilon_t^s)$, we can solve for the individual wedges as described in the main paper. One thing to notice is the mapping for the derivative calculation. We are interested in $\frac{\partial V_{t+1}(w_{t+1}, e^{v_{t+1}}, \varepsilon_{t+1}, \varepsilon_{t+1}^x, \varepsilon_{t+1}^s)}{\partial w_{t+1}}$

$$
\begin{aligned}
& \frac{\partial V_{t+1}(w_{t+1}, e^{v_{t+1}}, \varepsilon_{t+1}, \varepsilon_{t+1}^x, \varepsilon_{t+1}^s)}{\partial w_{t+1}} \\
= & \lim_{\Delta \to 0} \frac{V_{t+1}(w_{t+1} + \Delta, e^{v_{t+1}}, \varepsilon_{t+1}, \varepsilon_{t+1}^x, \varepsilon_{t+1}^s) - V_{t+1}(w_{t+1}, e^{v_{t+1}}, \varepsilon_{t+1}, \varepsilon_{t+1}^x, \varepsilon_{t+1}^s)}{\Delta} \\
= & \lim_{\Delta \to 0} \frac{V_{t+1}(\frac{w_{t+1}+\Delta}{e^{v_{t+1}}}, 1, \varepsilon_{t+1}, \varepsilon_{t+1}^x, \varepsilon_{t+1}^s) - V_{t+1}(\frac{w_{t+1}}{e^{v_{t+1}}}, 1, \varepsilon_{t+1}, \varepsilon_{t+1}^x, \varepsilon_{t+1}^s)}{\frac{\Delta}{e^{v_{t+1}}}} (e^{v_{t+1}})^{-\sigma} \\
= & \lim_{\frac{\Delta}{e^{v_{t+1}}} \to 0} \frac{V_{t+1}(\tilde{w}_{t+1} + \frac{\Delta}{e^{v_{t+1}}}, 1, \varepsilon_{t+1}, \varepsilon_{t+1}^x, \varepsilon_{t+1}^s) - V_{t+1}(\tilde{w}_{t+1}, 1, \varepsilon_{t+1}, \varepsilon_{t+1}^x, \varepsilon_{t+1}^s)}{\frac{\Delta}{e^{v_{t+1}}}} (e^{v_{t+1}})^{-\sigma} \\
= & \lim_{\frac{\Delta}{e^{v_{t+1}}} \to 0} \frac{W_{t+1}(\tilde{a}_{t+1} + \frac{\Delta}{e^{v_{t+1}}}, \varepsilon_{t+1}^x, \varepsilon_{t+1}^s) - W_{t+1}(\tilde{a}_{t+1}, \varepsilon_{t+1}^x, \varepsilon_{t+1}^s)}{\frac{\Delta}{e^{v_{t+1}}}} (e^{v_{t+1}})^{-\sigma} \\
= & \frac{\partial W_{t+1}(\tilde{a}_{t+1}, \varepsilon_{t+1}^x, \varepsilon_{t+1}^s)}{\partial \tilde{a}_{t+1}} (e^{v_{t+1}})^{-\sigma}
\end{aligned}
$$

3. Next, the individual wedges are regressed on a cubic function of age, and its iteration with education and race respectively to update the wedge profiles.

$$
\begin{aligned}
\tau_i^x &= \beta_x x_i + \varepsilon_i^x \\
\tau_i^s &= \beta_s x_i + \varepsilon_i^s
\end{aligned}
$$

where x_i denotes individual age cubic function, and its iteration with education and race. Then we update the wedge profiles according to

$$
\begin{aligned}
\beta_x^{new} &= m_x \beta_x + (1 - m_x)\beta_x^{old} \\
\beta_s^{new} &= m_s \beta_s + (1 - m_s)\beta_s^{old}
\end{aligned}
$$

The new standard deviation is fully updated based on $std(\varepsilon_i^x)$ and $std(\varepsilon_i^s)$.

4. Repeat step 1 until convergence, i.e.

$$
||\beta_x^{new} - \beta_x|| + ||\beta_s^{new} - \beta_s|| < \epsilon
$$

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