Framing for Sense Making in Whole-Class Mathematics Discussions

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For nearly 30 years, education has been researching and implementing practices and teacher strategies to better support students to engage in more meaningful interactions with mathematics. In particular, moving away from teacher-centered approaches to mathematics learning and into more student-centered ones has been a focus. Whole-class discussions have been the subject of investigation in much of this research. Studying how teachers can better support students to discuss their perspectives on, work within, and pose questions about the mathematics of study has been one line of inquiry. Students need to have opportunities to make sense of the mathematics – to make decisions about what strategies to use, to engage with one another's ideas, and to productively struggle to reach new, more complete understandings. Similar sense making-driven efforts in science education research have used the sociological construct of framing (Goffman, 1974) to examine students' interactions with one another, the content, and their teachers (i.e., Hammer et al., 2005). My aim was to further understand a teacher's work of framing and how that may or may not influence students' interactions around the mathematics, particularly their engagement in sense making during whole-class discussions. To do so, I employed an explanatory case study design to study a 6th grade mathematics classroom. Based on a detailed micro-analysis of three videotaped lessons as well as teacher and student interviews, this study's logic of inquiry followed six different phases. The analysis revealed that the teacher's framing in this classroom mattered for the ways in which her students framed the activity and engaged in sense-making activity. Specifically, when the teacher framed the activity as a sense-making endeavor in which they were expected to be the authority, co-construct mathematical explanations, and to engage in productive struggle, the students did so. When the teacher engaged in less productive framings that were more teacher-centered, the students aligned with that framing as well. The implications of this study for teacher practice include the need to support teachers in setting and maintaining interactional expectations. In addition, supporting students to reach those expectations through particular framings is an added layer to consider beyond instructional practices.

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Preface

I became interested in framing as a way to describe classroom interactions after analyzing quite a bit of classroom video data. I was noticing differences across the teachers' classrooms that mathematics instructional practices alone could not describe. Berland & Hammer's (2012a) study making use of epistemological framing spoke directly to some differences that I was seeing. I went on to learn more about framing and wanted to apply it to classroom video data in a mathematics class. The result is what you will find herein.

First, I would like to thank some influential people who helped me throughout this endeavor. I would like to express my *deepest* appreciation to Zach, Ben and Camryn for their constant support and encouragement throughout this process. I would also like to thank a few colleagues along the way for their friendship and support: Dr. Christina Ashwin; Dr. Elaine Lucas-Evans, Cara Haines, and Dr. Hannah Sung. To Hannah in particular, the many hours we spent together talking about classroom interactions, and specifically Ms. Ellis's class, were important to this study and to my broader perspective. I would also like to thank Dr. Melissa Boston and Dante Orsini for their help in coding the videos with the IQA rubrics.

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1.0 Introduction

Over the last few decades, the National Council of Teachers of Mathematics (NCTM) has been supporting a shift in mathematics instructional practices toward more student-centered approaches and away from teacher-centered ones to foster deeper, more meaningful mathematics learning experiences for students. Munter, Stein and Smith (2015) termed the two instructional models dialogic and direct instruction, respectively. Munter et al. (2015) described the direct instruction model as the teacher providing students with example problems, as well as the ways in which they should be solved. Then, students can be invited to practice solving similar-type problems. They described the dialogic model as inviting students to think about new mathematical ideas without being shown or told how they should complete them. Not only do students think about those ideas based on their own perspectives on them and prior knowledge, they then share and discuss their solution strategies to reach new understandings about the mathematics they are studying. Their discussions are meant to not only share their approaches, but to build on them, defend them, connect them with other students' ideas and the mathematical goals more generally, and to critique them.

Not only is the dialogic model much more open-ended and less predictable in terms of how students might think about or approach a given problem, it also requires the teacher to coordinate those student-driven approaches to support students' collective sense making and progress toward reaching the mathematical learning goals. Because the teacher must coordinate so many moving parts and students need to learn how to interact in those culminating, whole-class discussions, it has been the focus of much of the research in mathematics education over the last several years (i.e., Forman, Larreamendy-Joerns, Stein & Brown, 1998; Michaels & O'Connor, 1993; Stein, Engle, Smith & Hughes., 2008).

Michaels, O'Connor and Resnick (2008) developed Accountable Talk® as a set of practices to help support students to engage in such discussions. In so doing, they pointed out the importance of three facets to Accountable Talk®, with emphasis on *accountability* – accountability to the community, to knowledge, and to reasoning. Part of their work focused on providing sentence-starters that would equip students with discursive tools to participate in discussions (i.e., "I agree with Sam because…"). Focusing on teachers' work in discussions, Stein et al. (2008) provided a set of five practices to support teachers in making whole-class discussions productive. By productive, they were referring to the ways in which teachers could make use of students' solution strategies and student thinking and connect them to one another's. Included in those practices is *anticipating* how students might approach a problem, as well as planning out the ways to *connect* students' approaches to one another's. Forman et al.(1998) considered both the teacher and students in discussions by specifically studying how teachers might support argumentation in whole-class discussions.

Equipped with these practices and tools for having more productive mathematics discussions, students and teachers still have trouble engaging in them in meaningful ways. Wood and Turner-Vorbeck (2001) discussed 'show and tell' as one kind of discussion in which students might participate, but merely by sharing their solution strategies with the class and not discussing them further. Selling (2016) similarly pointed out the difficulties that teachers and students have.

In addition to the difficulty students face in engaging in these discussions, there have been renewed calls to support student engagement in these sense-making practices by the *Common Core State Standards for Mathematics* (CCSSM; National Governors Association Center for Best

Practices (NGA) & Council of Chief State School Officers (CCSSO), 2010). The national move sent the message that mathematics is no longer a fixed set of rules and procedures to impart on students in school, rather a social endeavor that calls for students to make their thinking known, engage with others' ideas, and grapple with novel mathematics problems. This shift in thinking, and push for engagement in mathematical practices, can sometimes result in scripted, or over-proceduralized, enactments (Selling, 2016). Rather than engaging in sense making about the mathematics of study, students might conform to their teacher's expectations of them without engaging in meaningful interactions.

Researchers in science education have similarly pointed to this tension between sense-making activity and teacher-pleasing activity that can result from over-proceduralizing (i.e., Berland & Hammer, 2012a; 2012b). Berland and Hammer (2012a; 2012b) used 'pseudoargumentation' to refer to a category of classroom activity focused on argumentation. Such 'pseudo'-activity is done to satisfy some authority or when "students' attention is on doing what they expect the teacher will value rather than on the substance of the ideas at hand (Berland & Hammer, 2012b, p.88). Teachers might also engage in this kind of pseudo-activity by doing what Bloome, Puro and Theodorou (1989) termed 'procedural display.' Procedural display broadly entails two general ideas: (a) it is "a set of academic and interactional procedures that…count as the accomplishment of a lesson, and (b) the enactment of lesson is not necessarily related to the acquisition of intended academic or nonacademic content or skills" (p. 272). That is to say that even though students, and their teacher, might be engaged in practices, or other activities, that seem like they are meeting structural components of what should be occurring in dialogic classrooms, they might be doing so without regard for the underlying intentions or goals for engaging in such activities – sense making. Three other examples of this general idea of classroom experiences focused on teacher-appeasement rather than student sense making emerged in the literature. Jiménez-Aleixandre, Rodriguez, and Duschl (2000) followed Pope's (2003) work on 'doing school' and termed the opposite kind of activity 'doing science.' That so many different versions of "teacher-controlled activity," even in light of efforts for more student-centered activity exist suggests that it is a prevalent problem that should be addressed (Berland & Hammer, 2012a, p. 72).

Selling (2016) considered a different version of this idea in work related to access to engaging in the Standards for Mathematical Practice. She sought to determine what level of explicitness was appropriate to allow students to engage in the mathematical practices without stifling their thinking by proceduralizing them. In so doing, she articulated eight teacher moves that opened the door for students to engage in the practices in ways that supported them in making sense of the mathematics. Some of these teacher moves included explaining the rationale behind a given practice and expansively framing the practices for students. Expansive framing is done when the teacher sends messages that make the work at hand relevant in other contexts, to other people, or to other scenarios for students (Engle, 2006). Expansive framing, then, sends the message that what the students are doing in class is not just for the sake of 'doing the lesson,' but is relevant to other experiences and people outside of the classroom walls.

Selling (2016) was not the first to link framing with sense making, as will become evident in the literature review that follows. By building on work in science education, and bridging it with work in mathematics education that focuses on sense making, the study herein aims to learn how teachers might better support students to engage in sense making. That is, I will make use of one way that science education has discussed sense making – epistemological framing – to understand how a mathematics teacher might support student engagement in sense making through her framing. The study will answer the following research questions:

1) In what ways, if any, does the teacher frame whole-class discussions for sense-making activity (i.e., framing students as the authority or framing the activity as working together, among others)?
2) To what extent, or in what ways, do the teacher's students align with her framing?
    a. Which students, if any, respond in ways that align with a teacher's framing?
3) If students do align with the teacher's framing for sense-making activity, what mathematically are they making sense of?

To begin, I provide a literature review of the underlying intentions and theories behind supporting engagement in dialogic instruction. I define sense making in relation to the mathematics education research literature as well. I provide a brief review of the literature on framing, its origins, and how it has been used in science and mathematics education research. I go on to detail the six-phase data analysis plan in which I engaged to micro analyze three lessons of classroom videos from a sixth-grade mathematics class. Within that, I indicate how I defined and coded for framing in the dataset. I also provide information on how I supported my findings with data from other sources. In Chapter 4, then, I describe the findings that were a result of my microanalysis. Finally, in Chapter 5, I discuss the findings, their implications and limitations of this study.

2.0 Literature Review

In this literature review, I first establish what it means to know and do mathematics in dialogic classrooms, much like the one in which this study was conducted. In so doing, I emphasize the link between communication and learning in mathematics education research for the sake of sense making. I will then define some components of sense making in the classroom that have been identified and studied in mathematics education, specifically in relation to communication: students as the authority; co-constructing mathematical explanations; and productive struggle. Addressing the link between sense making and epistemological framing will follow, along with what framing is, and how it has been used in mathematics and science education research. Science education provides a valuable contribution within this study – epistemological framing. While framing has been addressed in mathematics education, epistemological framing has been used in one way. Science education research's sophisticated development of epistemological framing will be influential in the analyses proposed in this study.

2.1 Communication and Learning in Mathematics Education

Though the Common Core State Standards and the Standards for Mathematical Practice are in place on the national stage, a single pedagogy for addressing them is not (Munter et al., 2015). One of two frameworks for what it means to learn and do mathematics that Sfard and Cobb (2014) articulated was participationism. Participationism entails a similar pedagogy to dialogic instruction, described earlier by Munter et al. (2015). Participationism stands in contrast to acquisitionism in which mathematics is treated as a body of knowledge that others possess and can impart on those who do not yet possess it – similar to traditional instruction (Munter et al., 2015). Within participationism, learning mathematics refers to "a gradual transition from being able to play a partial role in the implementation of the given types of tasks to becoming capable of implementing them in their entirety and of one's own accord. Eventually, a person can perform on her own and in her unique way entire sequences of steps, which, so far, she would only execute in collaboration with others" (Sfard, 2008, p. 78). In engaging in mathematical activity with others, and playing progressively more sophisticated roles in that activity, the learner comes to know mathematics and is able to competently do it independently over time.

One particular instantiation of participationism "views discourse…as the thing that changes in the process of learning" (Sfard & Cobb, 2014, p. 558). Sfard and Cobb (2014, p. 553) go on to state that, within this view, "learning mathematics is equivalent to changes in patterns of participation in discourse." Therefore, "studying mathematics learning is synonymous with investigating processes of discourse development" (p. 558). While discourse can involve a single person, it is articulating one's own ideas and engaging with others' ideas that make the learning visible in the classroom settings. Webb, et al. (2014, p. 80), citing Forman and Cazden (1985), provided an explanation for one way in which this learning might occur: "formulating ideas to be shared and then communicating the ideas, students offering explanations may recognize their own misconceptions, or contradictions or incompleteness in their ideas more than they would when simply vocalizing aloud to oneself."

In related work addressing communication more broadly in mathematics learning, Lampert and Cobb (2003, p. 237) stated that "if schools are to involve learners doing mathematical work, classrooms will not be silent places where each learner is privately engaged with ideas. If students are to engage in mathematical argumentation and produce mathematical evidence, they will need to talk or write in ways that expose their reasoning to one another and to their teacher." Such reasoning, Schoenfeld (1992, p. 339) argued, was "an act of sense making, socially constructed and socially transmitted." Lampert and Cobb (2003, p. 239) went further to state that many studies "see talking and writing to be aspects of doing mathematics and regard the classroom as a community of learners, led by the teacher, in which learners are socialized to accept new norms of interaction and learn new meanings for mathematical words and symbols as they work together on problems."

To summarize, the participationism framework for mathematics learning encourages participation and communication as ways of coming to know and do mathematics – to learn it. Communication and learning cannot be separated from this view. While communication that is silent or written is possible, when studying classroom teaching and learning, the verbalized communication provides insight into what is taking place for both the teacher and students within the classroom community.

2.2 Sense Making in Mathematics Education

Within the participationism view, participating in mathematical practices collectively is a goal. Moreover, students interacting to make sense of the mathematics is central to their learning. It is not sufficient to communicate in mathematics classrooms and assume that learning is taking place; students must engage with others' ideas, as well as articulate their own (Webb et al., 2014). Beyond that, when students prepare themselves for peer critique, questioning, and review in constructing their classroom contributions, they are better off (Lampert and Cobb, 2003).

These researchers, among others, allude to what it means to engage in sense making activity in the classroom. It is not only sharing one's own thoughts, but also engaging in those of others. It is also insufficient to put ideas on the table; reaching shared understandings about these ideas is central to both learning from others and articulating one's own ideas sufficiently (Staples, 2007; Wood & Turner-Vorbeck, 2001). Within classrooms, such sense making activity has been articulated, and studied, in a variety of ways. Three components of sense making in classroom activity include: students as the authority; students co-constructing mathematical explanations; and productive struggle. These will be discussed further below. Additionally, the tasks in which students are engaging to make sense of the mathematics are particularly important. One curriculum, the Connected Mathematics Project 2 (CMP2; Lappan, Fey, Fitzgerald, Friel & Phillips, 2009), will be discussed in relation to sense making in the classroom.

2.2.1 Students as the Authority

For students to participate in the mathematics they are studying, and make sense of it, they need to have the opportunity to make their own thinking known in the form of conjectures, explanations, or solution strategies (Forman & Ansell, 2002). When students are authoring ideas that make sense to them and making them public, they are providing insight into what it is they know and understand. They are also, with the support of their teacher, inviting their peers and their teacher to think with them. When the teacher and students share authority, they are providing explanations for their thinking, critiquing one another, and reflecting on their reasoning (Engle & Conant, 2002; Smith, 2000).

Forman and Ford (2014), in their discussion of disciplinary engagement, specified two different kinds of authority that are relevant to sense making. The first is 'interactional' and refers to students sharing their ideas and providing explanations. The second is 'disciplinary' and refers to deciding what knowledge is within the discipline and whether or not a given explanation qualifies as new knowledge within the community (the classroom). Others have referred to similar distinctions with respect to authority in the classroom. Boaler (cf. 2003) called the latter discipline-specific authority. She suggested that tending to the authority of the discipline and its standards for accepting knowledge were critical to classroom interaction. Berland and Hammer (2012a), in their discussion involving 'doing the lesson,' called the two types of authority social and epistemic. According to them, for classrooms in which 'doing the lesson' is taking place, the teacher maintains the social and epistemic authority.

Authority encompasses two general ideas, then. The first is that students are authors within the class. They articulate their own understandings and conjectures about the problem or topic at hand. They also go a bit further, though, and decide what a sufficient explanation is, press one another when there is confusion, and decide what will qualify as new knowledge. Students might critique one another in respectful ways when they share authority in the class. They also will question one another, take responsibility for their own learning, or decide the plausibility of an explanation or argument (Hufferd-Ackles, Fuson, & M. Sherin, 2004; NCTM, 2014).

The discipline-specific interpretations of students as the authority relies partly on students also have social authority. Students talking directly to one another, asking questions of one another, and deciding who will contribute and when are all examples of students having social authority in the classroom setting.

2.2.2 Productive Struggle

Struggle is a necessary component of students' learning, particularly within the dialogic instructional model (Munter et al., 2015; Schoenfeld, 1985). Classrooms in which "instruction embraces a view of students' struggles as opportunities for delving more deeply into understanding the mathematical structure of problems and relationships among mathematical ideas, instead of simply seeking correct solutions support student learning" (NCTM, 2014, p. 48). Productive struggle refers to a particular kind of struggle that fosters learning with understanding instead of keeping students focused on the same idea that they do not understand (Warshauer, 2011). That is, supports need to be in place when students encounter uncertainty or struggle so that they can work through it to progress in their learning. A productive instance of such confusion is when students articulate just what it is that is confusing them instead of giving up on the problem or being told what the solution path or answer is (NCTM, 2014).

Hiebert and Grouws (2007) summarized the link between productive struggle, sense making, and understanding by stating that:

> "When students struggle (within reason), they must work more actively and effortfully to make sense of the situation, which, in turn, leads them to construct interpretations more connected to what they already know and/or to reexamine and restructure what they already know. This yields content and skills learned more deeply." (p. 389)

Their key features of productive struggle implicitly build on students as the authority. Students are not being told what is right or wrong or what to think next. Rather, students are working to come to new understandings based on their prior knowledge, thus coming to new knowledge.

2.2.2.1 Productive Struggle in Action

While productive struggle can occur in a variety of ways (i.e., within the mind, as one draws or writes ideas, or in working together), Warshauer (2015) identified struggle that is visible to teachers. Her work was meant to support teachers in assisting their students when they encountered struggle. She suggested there were four kinds of struggle that students encountered that were visible to the teacher: 1) getting started; 2) carrying out a process; 3) uncertainty in explaining and sense making; and 4) misconceptions/errors. In most instances of struggle, or uncertainty, students asked questions, gestured uncertainty, or did not write anything on their papers. Granberg built on the uncertainty aspect of productive struggle in her analysis of errors in relation to productive struggle. She stated that "making, discovering and correcting errors may generate effort that can engage students in productive struggle" (Granberg, 2016, p. 34). The extent to which struggle is productive, though, is dependent upon whether or not the student emerges from the struggle with a deeper understanding. These accounts of struggle align with the NCTM's (2014) descriptions of what students are doing when engaging in productive struggle: Students are asking questions of one another; articulating the source of their struggle; not giving up on a problem; and supporting classmates without giving away the answer or solution path in their description.

In related work that built from Warshauer's (2015) kinds of struggles, Sung (2018) explored power dynamics at play while small groups of 6^{th} grade students worked on mathematical tasks[1]. She went beyond categorizing types of struggle to situating the struggle within an individual

[1] The dataset on which Sung conducted her analyses is the same dataset on which this study was completed. Sung's analyses focused on small-group, 'Explore' time, while the proposal here will focus on whole-class discussions during the 'Summarize' phase of the lessons.

student, across multiple students, or in collective group work. She looked at student questioning, but also their non-verbal indicators of struggle, or uncertainty.

The distinction that Sung's (2018) work pointed out is important in considering productive struggle. Students might individually engage in productive struggle. They may be confused about a particular aspect of a solution strategy and ask a question about it. Then, they might emerge from that interaction with a deeper understanding of the strategy. Another version, though, might occur for the collective. That is, students might collectively be presented with a challenge, mistake, or question that prompts them to talk through it together to reach a new *collective* understanding. So, productive struggle might be individual or dispersed across a group of students.

Productive struggle is evident in a variety of ways, as have been discussed here, but the ways in which it is arguably most visible in classroom interactions is in students asking questions. That students are asking questions of one another suggests that they are attempting to make sense of the mathematics by engaging in another kind of sense making in classrooms: co-construction.

2.2.3 Co-constructing Mathematical Explanations

As students articulate their own ideas, ask questions of others about their explanations, and answer questions about their own explanations, they are laying the groundwork for the co-construction of mathematical explanations. Co-construction goes beyond articulating one's own ideas and answering questions aimed at reaching new understanding, co-construction involves engagement with others' ideas. Webb et al. (2014), citing Hatano (1993), provided the following articulation of co-construction:

> "Characterization of engagement that requires students to generate ideas and to attend to and engage with each other's ideas is co-construction, where students contribute different

pieces of information and build upon others' explanations to jointly create a complete idea or solution" (Hatano, 1993, p. 80).

Critiquing, building on, attempting to understand and connecting ideas across students or groups of students is what co-construction is about. It is engagement with others' ideas and reaching some new conclusion together that has built upon prior contributions.

Within co-constructing mathematical explanations, there are two versions that both fit the descriptions here. The first is that students form a more complete mathematical explanation or understanding of a given strategy, idea or concept. Students in this instance are focused on the same problem-solving approach or strategy and come to more articulate ways of describing it, thus supporting deeper understandings for all involved. The other version is one that Stein et al. (2008) articulated as one of their five practices. They did not call it co-constructing mathematical explanations, they referred to it as connecting student responses. They distinguished between students discussing individual strategies or approaches to solve problems and connecting those strategies to one another. They stated that within fostering whole-class mathematics discussions, "the goal is to have student presentations build on each other to develop powerful mathematical ideas." Moving beyond co-constructing explanations for individual approaches, they pointed out the need to build across individual approaches or strategies as another means of co-constructing mathematical explanations as related to a mathematical concept. Within their five practices framework, they highlight the teacher's role in supporting students to make those connections. Webb et al. (2014) similarly pointed out the importance in reaching new, shared meaning through co-constructing.

Sense making is not universally defined within the mathematics education literature, but these three selected components of sense making provide some indication of what one might see in a classroom in which it is occurring. Students are authoring ideas about the mathematics in which they are engaged, they are determining whether or not solutions are plausible with respect to the discipline, they are productively struggling with some new ideas and problems for which solution paths are not immediately evident, and they are engaging within and across one another's ideas to reach new understandings. There are additional indicators in classrooms that might suggest that sense making is likely to occur. One such indicator is in the kinds of tasks or problems students are solving (Henningsen & Stein, 1996). One curriculum in particular will be addressed next that was designed with reasoning and communication in mind to promote sense making.

2.2.4 Curriculum for Sense Making

The Connected Mathematics Project 2 (CMP2) is a National Science Foundation-funded curriculum that provides tasks that "allow students to make sense of them" ("Connected Mathematics Project," 2018). The CMP2 curriculum is typically associated with a more dialogic pedagogies and hence supports more student-driven approaches to solving novel problems. The curriculum is broken up into books that comprise units. One book is called "*Bits and Pieces* I" and focuses on the study of fractions, decimals, and percentages. Each book is comprised of tasks that are intended to be completed in a single class period or more.

2.2.4.1 Tasks

Tasks typically provide multiple entry points for students so that they can engage in the task in ways that are meaningful to them. The openness of the solution paths, then, encourage different ways of solving the problem about which students can discuss later in the lesson. In this sense, the tasks are designed to promote discourse among students and to allow teachers to assess student learning as they solve them. The tasks also support students as the authority in the class since they are the ones with the ideas about how to solve problems. Productive struggle comes in to play as students solve novel tasks in small groups.

Tasks within the CMP2 can be considered high cognitive demand ones (Stein, Grover, and Henningsen, 1996; Stein & Lane, 1996; Stein & Smith, 1998). Smith and Stein (1998) categorized kinds of tasks by the level of student-thinking involved in solving them. Tasks that ask students to memorize or to perform some procedure were considered to have a low cognitive demand. High cognitive demand tasks could still involve following a procedure, but the procedure in those tasks would be accompanied by some connection to their underlying concepts. Such tasks would still require some original thinking on students' parts. At the highest level, Smith and Stein (1998) termed the tasks "doing mathematics." For those tasks, there is not a procedure to be followed and the solution is not obvious for students. Such tasks would require the most original cognitive efforts from students. Stein et al. (1996) noted that tasks with higher levels of cognitive demand were more difficult to implement and that teachers often struggled to maintain that high-level of cognitive demand. That is, oftentimes teachers would take away the challenge associated with such problems by providing clues or procedures that helped students come to a solution without grappling with the ideas in meaningful ways.

2.2.4.2 Lesson Structure

Lessons within the CMP2 curriculum are designed to follow a 'Launch' – 'Explore' – 'Summarize' structure. During the 'Launch', the teacher introduces the task for the day that is typically embedded within some context (i.e., A school is holding a fundraiser with each grade setting a goal for raising money). The teacher addresses the prior knowledge that students will be building on, as well as how that information might support them in solving the given task. The teacher, though, does not provide a clear strategy or solution path for students to complete the task – that is left open to students (Stein et al., 1996). Once students have been introduced to the task, they are invited to 'Explore' the it individually, in small groups, or as a whole class. The teacher guide provides indicators for how each task should be completed (in small groups, individually, etc.). Typically, though, tasks are completed in small group settings. The 'Summarize' phase follows the 'Explore' phase. During the 'Summarize' phase, the teacher selects student groups to present their solution strategy or ideas about a given problem to the class. Students, then, "pose conjectures, question each other, offer alternatives, provide reasons, refine their strategies and conjectures, and make connections" (CMP, 2018).

The 'Summarize' phase is, ideally, when all three of the sense making components discussed above are visible. Students act as the authority as they share their solution strategies, conjectures, and explanations for the problem. They also engage in productive struggle with one another as they ask clarifying questions to build more sound explanations and reasons for claims. Co-construction of mathematical explanations occurs, as well, as the teacher supports students to engage with one another's ideas and make connections across them. Students are also acting as the disciplinary authority as they determine what counts as a sufficient explanation or solution to the task. Ideally, students walk away from the task with some new, more robust understanding of the problem itself and the various ways to solve it.

Sense making can occur in a variety of ways in classrooms. The components addressed above were meant to illustrate some ways in which sense making might be visible in mathematics classrooms. There are other ways to study student sense making, too. For more than ten years, science education researchers have been developing and using epistemological framing to study student sense making.

2.3 Sense Making and Epistemological Framing

Epistemological framing is one way of discussing a shift in school learning from a place where students ‘do school’ to one where student sense making is the goal. Hammer et al. (2005) provided an illustrative example of such a shift involving one of their college-level physics students in a reform-oriented freshman physics course. After failing the course’s midterm, the student, Louis, met with the instructor to discuss “his approach to learning in the course” (p. 13). A sharp spike in his exam score on the retake prompted the researchers to approach Louis for an interview. He said that his instructor had two influential pieces of advice that he followed: 1) “When you study, try to explain it, try to explain it to a ten-year-old” and 2) “Think of an analogy” (Louis quoting his instructor, p. 13 & 14). Rather than attempting to memorize problems and formulas, Louis began to try to make sense of the physics he was learning. For example, he began to talk about voltage in terms of dump trucks instead of thinking of voltage as a formula to memorize. The researchers named the shift in Louis’s study habits, and success in the physics course (he finished with a high B, one of the highest grades in the class), a different epistemological framing. The framing that his professor cued called on *resources* from Louis’ own, outside of school experiences to understand the material as opposed to attempting to memorize the procedures or formulas he was learning.

Addressing Louis’ epistemological shift in terms of the resources on which he drew emerged from the *resources-based framework* that Hammer, et al. (2005) devised. They contrasted their framework with ideas about transfer. Generally, transfer implies that knowledge is imparted as intact units that are taken to a variety of settings and applied. These researchers went on to say that within the resources framework, learning is not acquisition, but a “state the learner enters or forms at the moment, involving the activation of multiple resources” (p. 5). Their acknowledgement of, and dissatisfaction with, learning as acquisition in science overlaps quite a bit with arguments Sfard and Cobb (2014) made in mathematics.

The resources approach, Redish (2004, p. 9) stated, was attempting to answer the question: "When a student responds to an instructional environment to build new knowledge, what existing resources are activated and how are they used." He hypothesized that students' "reasoning consist[ed] of weakly organized resources" and that those resources could be activated in particular ways (p. 23). Epistemologies came into play when considering appropriate resources to activate in a given situation. For example, Hammer et al. (2005) illustrated different epistemologies by talking about how a child knows what is for dinner on a given night. The child knows this answer because a parent told him/her. In that sense, the knowledge came from an authority figure, or was *knowledge as transmitted stuff*. A contrasting example they gave is how a child knows her mom got her a birthday gift. The child knew it was her birthday and saw her mom hiding something. They termed this "the activation of the resource *knowledge as fabricated stuff*" (p. 8). These varying ways of coming to decide what knowledge is and where it comes from stem from epistemologies that are fluid and draw on a variety of resources, depending on what is activated at a given time.

The resources approach, and epistemological framing more specifically, are helpful in thinking about the shift in mathematics education research and pedagogical practices that align with the participationism framework and dialogic instruction. The resources on which students are expected to draw have shifted from memorized procedures and definitions to their own prior knowledge, ideas, and their classmates. The authority, or main resource, is no longer expected to be the textbook or teacher, rather students themselves fill this position for one another and themselves (cf. Boaler, 2003). Berland and Hammer (2012a) put it another way in relation to ‘doing school’ when they stated the following:

> “In one framing, the students and teacher expect the teacher to be in charge not only of what ideas are correct but also, for example, of who is entitled to speak and when. In another framing, they might have quite different expectations regarding what is appropriate behavior” (p. 72)

Epistemological framing, then, is a lens for determining on which resources students are drawing, or perceive to be relevant, based on their explicit and implicit messages. What follows is a brief look at framing more generally and its origins. Framing and its uses in mathematics education follow, along with what framing in mathematics education has not yet considered. Finally, specific ways in which epistemological framing have been examined in interaction will conclude the section as a lead-in to the methods used in this study.

2.4 Framing and Its Origins

While epistemological framing has taken hold in science education research, it originated in anthropology with Bateson (1972). Studying monkeys at play, he noticed their ability to find common ground in understanding that they were, in fact, just playing and did not need to defend themselves. He suggested that metamessages were being sent to signal this understanding to one another so that the playful interaction could continue. Metamessages are those that provide additional information about how to interpret a given message. Tannen (1993, p. 3) summarized Bateson’s ideas in stating that “no communicative move, verbal or nonverbal, could be understood without reference to a metacommunicative message, or metamessage, about what is going on – that is, what frame of interpretation applies to the move."

Goffman, a sociologist, expanded the idea of frames to sociology and is most widely cited in science and mathematics education research in explicit discussions about framing. He suggested that frames were always in play and they accounted for individuals' answers to the question, "What is it that's going on here?" (Goffman, 1974, p. 8). He saw prior experiences as a major influential factor in their answer to the above question and stated that they affect what an "individual can be alive to at a particular moment" (p. 8). That is, an individual's sense of what is going on in an interaction will have implications for what he/she pays attention to in that interaction – whether it aligns with their expectations or not might dictate the extent to which they focus on particular features.

Just as Goffman is most widely cited for framing in science and mathematics education research, the methodological means for studying and examining frames are typically attributed to Tannen (1993) and Tannen and Wallat (1993). These linguists acknowledged Goffman's ideas about framing and believed that linguistics could be used to make determinations about individuals' framings. In work with a video-recorded medical exam, the researchers used evidence from the doctor's register, intonation, hesitation (and lack thereof), and pauses, among other linguistic features, to describe changes in framing depending on whom the doctor was addressing (the child, her mother, or medical students who would later view the recording).

Framing has been examined in a variety of ways and for a variety of purposes. Researchers have looked at different kinds of framing, including 'doing,' 'expansive,' 'epistemological,' and 'cognitive,' to name a few (Berland & Hammer, 2012a; Engle, 2006; van de Sande & Greeno, 2012). In addition to different kinds of framing that can be examined, there are also multiple facets to framing. Engle (2006) pointed out that the ‘who,’ ‘when,’ ‘where,’ ‘what,’ ‘how,’ and ‘why’ of a learning situation can all be framed and that framing can have an influence on learning. Similarly, Redish (2004) identified various components of framing that one might attend to in a learning environment: social; physical; skills; affect; and epistemological. Therefore, there are different kinds of framing and various aspects of framing that exist and can be examined. For example, the physical arrangement of the desks in a room have the potential to frame the learning environment in a particular way (Hammer et al., 2005). Desks arranged in rows and columns might indicate that mostly lectures and independent work will take place. On the contrary, tables with many chairs surrounding them in a room might indicate that there will be a lot of group work in that setting.

At times, individuals within an interaction could have framings that are misaligned. For example, during a lesson, a teacher might ask a question. Several students might raise their hands to answer and another student might just shout out an answer. In that instance, the ways in which the students with their hands raised were framing the interaction would not be aligned with the ways in which the person who shouted the answer were framing it. Subsequent teacher moves, such as acknowledging or ignoring the student who shouted out, could indicate his/her alignment with a particular framing.

2.5 Framing in Mathematics Education

While science education researchers put forth the idea of epistemological framing, van de Sande and Greeno (2012) made use of this work to a limited extent in mathematics education. In considering alignment of frames in problem solving to reach new, shared understandings, the researchers combined ideas from epistemological framing and cognitive framing to name a new kind of framing: conceptual framing. Their references to epistemological framing were in addressing pairs' or groups of students' understandings of the purpose in problem solving, for example to get an answer. They also discussed alignment with respect to ways of finding and justifying an answer – repeated addition in one instance. In addition to conceptual framing, these researchers named another kind of framing: positional. Positional framing takes into account the social organization within an interaction. Specifically, it "refers to the expectations that members of a group have for the pattern of interaction amongst themselves" (van de Sande & Greeno, 2012, p. 72). This framing extended to include "the establishment of who in the group is entitled, expected, or perhaps obligated, to initiate topics and questions, to question or challenge others' presentations, to indicate that a topic has been resolved, and so on" (p. 72).

In their analysis of framing, van de Sande and Greeno (2012) used narrative to justify their interpretation of framings, and their alignment, within interactions. That is, they talked about the interactions together to make determinations related to individuals' framings and the extent to which they aligned. These researchers also did not explicitly address sense making in relation to epistemological framing in mathematics education.

More work has been done in mathematics education research with respect to frame alignment. Forman et al. (1998) addressed alignment of frames in terms of argumentation. They argued that the teacher within a classroom can reframe the activity to orient the class in specific ways toward the practice of argumentation. The teacher's work of aligning frames with respect to both the mathematics being studied and the social roles within that work were both important to coming to understand what argumentation meant for the class participants.

Addressing frames and frame alignment in a different way, Heyd-Metzuyanim, Munter and Greeno (2018) analyzed a co-planning session between a researcher and a mathematics teacher.

Their analyses of both the planning session and the subsequent lesson revealed that the two individuals were framing the lesson in different ways throughout the interaction. In their case, the misalignment had implications for how the subsequent lesson played out. In their study, they referenced a shift from more direct pedagogical models to more dialogic ones. In the interaction they analyzed, the researcher was framing the co-planning session in relation to the dialogic model while the teacher had not quite made it there entirely and was still framing in ways that were related to the direct model.

Heyd-Metzuyanim et al. (2018) referred to the researcher's frame as an 'exploration frame,' while the teacher's frame was termed a 'doing' frame (p. 14). In contrast to sense-making activity, the 'doing frame' is more procedure-oriented, much like 'doing the lesson' that was addressed in the introduction. The overlap for related frames is in the source of the information, or in the expected source of the information. Rather than attempting to figure out the mathematics based on one's own prior knowledge or ability to think through the problem at hand (sense making), there is a rule or expected procedure to follow. Though Heyd-Metzuyanim et al.'s (2018) work did not make use of epistemological framing explicitly, the general ideas that they were applying to the co-planning session and the subsequent lesson mapped closely onto ideas from epistemological framing and 'doing the lesson' on which others in science education have built.

As was briefly introduced in the introduction, Selling (2016) engaged in some work with framing in school mathematics with an equity-oriented focus. In her attempt to address students' opportunities to engage in the mathematics practices, she made use of framing to talk about the level of explicitness that students might need. Being explicit, she argued, about what the mathematical practices are and how students should engage in them without over-proceduralizing them was a balance she was searching for in teacher practice. Within that endeavor, she used a different kind of framing from science education research, *expansive* framing (Engle, 2006). Expansive framing was one of eight instructional moves that teachers might use to be more explicit about the mathematics practices. Much like Heyd-Metzuyanim et al.'s contribution, Selling's was addressing the difficulty associated with engaging in dialogic teaching for the sake of sense making. In other words, she was trying to address the issue in classrooms with 'doing the lesson.'

Other mathematics education researchers have made use of framing, but in a variety of ways for different purposes. Bannister (2015); Brasel, Garner and Horn (2016); Horn and Kane (2015); and Louie (2016) have used framing in communities of practice to track, and make claims about, teacher learning. Hand and Gresalfi (2015) made use of framing to address identity formation, while Hand, Penuel and Gutierrez (2013) discussed framing in relation to more equitable educational opportunities.

In conclusion, within mathematics education research, Heyd-Metzuyanim et al. (2018) and Selling (2016) have made the closest links between sense making and framing. Their examinations, though, did not consider students' framings in the interactions. They also did not address the ways in which one person, namely a teacher, could influence the framing of a student or students over time. Some work within science education is helpful in thinking about students' epistemological framings and how their framings might be affected in interaction.

2.6 Framing in Science Education

Framing has been much more widely researched within science education. What those science educators have found is that frames can be manipulated, students respond to such framings, and teachers can support students to engage in sense making, at least at an individual level. They also considered teachers' framings and how to support changes in them. Most of their work has focused on two particular kinds of framing: *expansive* and *epistemological*.

Berland and Hammer (2012a) applied epistemological framing to examine the ways in which a teacher influenced students' engagement in scientific argumentation. As keeps coming up in this review, they were contrasting 'doing the lesson' framings with more discipline-oriented framings of argumentation that were intended for the purpose of convincing another person or group with support for claims. They provided detailed analyses of two different instructional sessions that were representations of what normally happened in the class. They contrasted the two sessions by terming one an idea-sharing session and the other an argumentative one. Within the idea-sharing session, the students were not engaging with one another's ideas and their contributions were to the teacher instead of to one another. The argumentative discussion, however, involved students talking directly to one another and critiquing one another's contributions – arguing with them. They attributed subtle differences in the two interactions to students' and the teacher's framing of it. That a student addressed another student's challenge to one of his claims indicated that the student who made the initial argument expected to defend it to more than just the teacher. In addition, the teacher's physical location during the argumentative discussion was different. He was seated behind the students so that they could address one another instead of looking to him. He also gave the student presenter a yardstick to identify him/her as the one in charge. Broadly what Berland and Hammer's (2012a) work did was point out specific aspects of the teacher's and students' interactions that were a difference in framing. They attributed differences in framing to subtle moves by the teacher.

Various other researchers have studied epistemological framing in different settings (i.e., Andrade, Delandshere & Danish, 2016; Haglund, Jeppsson, Hedberg & Schonborn, 2015; Louca, Elby, Hammer, Kagey, 2004). One way that Russ, Lee & B. Sherin (2012) applied epistemological framing was to cognitive clinical interviews. Interested in understanding how students interpreted such interactions, and thus the resultant conclusions researchers draw from the interviews, these researchers used epistemological framing as a way of interpreting students' behaviors in the interaction. Different from other researchers' works noted thus far with respect to framing, Russ et al. (2012) relied on clustered behaviors to make determinations about the ways in which students were framing the interviews. Though they did not call it such, they alluded to students' framings in the interviews as almost a test-taking scenario as opposed to a knowledge-seeking one in which the students would try to reason through the question to come to an answer. These distinctions seemed similar to 'doing the lesson' as opposed to other more productive framings of the interaction. They went beyond identifying the ways in which students were framing the interviews to study the ways in which the *interviewer* could influence students' framings. Ultimately, they found that particular talk moves were supportive of invoking certain desired frames and even maintaining desired frames once they were in place.

Their work further contributed to the idea that frames can be manipulated in interaction. Though the setting was not a classroom, it still demonstrated the ways in which interactions could cue particular framings for participants. They suggested that more work needs to be done to determine the ways in which desirable frames can be initiated and stabilized so that information gleaned in interactions can align more closely with its aims – sense making in the case of mathematics education.

Finally, a different take on epistemological framing was evident in Watkins, Coffey, Maskiewicz and Hammer's (2017) study of teachers' epistemological framings in summer professional development. What they were trying to understand was not only teachers' epistemological framings within certain summer professional development activities, but how their epistemological stances might change over time. While teachers' framings were not the target of their study, they did use the analytic lens to help reveal the ways in which some teachers demonstrated more productive epistemologies in relation to the act of engaging in 'doing science' (Jiménez-Aleixandre et al., 2000). In describing some more favorable interactions with the activities during the summer professional development, the researchers pointed out that the teachers were resolving scientific challenges in ways similar to what they would do if they were in a different, non-educational setting. They referred to such interactions as sense-making that might occur at home.

Their work addressed a different aspect related to a teacher's work of framing – their own epistemological stances toward the discipline. What they encountered with teachers in a summer professional development was that their epistemological framings were similar to that of 'doing the lesson.' They demonstrated an expectation that the professional development providers would be the authority over the scientific content and would share it with them. That is, that the professional development would be a teacher-centered (teacher here meaning professional development provider) endeavor as opposed a learner-centered one (with the learners in this case being classroom teachers engaged in professional development). Their work was similar to Heyd-Metzuyanim et al.'s (2018) study in that both found that the teachers' framings of the interaction in relation to the discipline had implications for how they framed the activity in which they were engaging.

2.7 Summary of the Literature Review

Sense making is the goal for students in mathematics classrooms. Such sense making activity is likely to occur, based on a variety of factors including curriculum choice, during the whole-class discussions. As students are providing explanations to one another about their own ideas and solution strategies, their classmates are asking questions of them to build their own understanding about the topic of study. Such co-construction of mathematical explanations requires students to shift from looking to the teacher or textbook as the authority to making use of one another and their own ideas to make sense of the mathematics. Epistemological framing is one way of examining the extent to which students are engaging in such sense making activity. Russ et al. (2012) and Berland & Hammer (2012a) provided findings that suggest that a teacher within an interaction can influence the ways in which students are framing the activity, and thus the extent to which they engage in sense-making activity. These studies were focused in science education, though, and Russ et al.'s (2012) work was situated within cognitive clinical interviews. This study set out to contribute to the literature with respect to a teacher's work of framing in a mathematics classroom, particularly during whole-class mathematics discussions.

The literature indicates that sense making activity done collectively and collaboratively in mathematics classrooms is a goal. Examining the ways in which a math teacher epistemologically frames the mathematical activity to support her students in sense making during whole-class discussions is a worthwhile endeavor. I plan to examine both the teacher's and her students' epistemological framings in the classroom interactions, as well as examine the influence of the teacher's explicit talk moves on the ways in which her students frame the whole-class discussions, particularly for sense making.

3.0 Methods

In this section, I describe my position as a researcher in this explanatory case study. I go on to detail the design of this secondary analysis, including the logic of inquiry that guided my analyses. I then discuss the data sources around which this secondary analysis was focused as well as their origin. That section is followed by information related to the specific classroom context from which the classroom videos were recorded, as well as a description of the teacher, Ms. Ellis[2]. Part of this included detailing her instructional approach, her curriculum, and the tasks she used in her lessons. The details of my analysis plan follow in phases, including the ways in which I reduced the sample from ten classroom videos to three that focused my microanalysis of framing. I also provide my codebook for framing and how I went about coding. I go on to detail how I made sense of those codes and attended to rigor. I start by stating my research questions in order to better focus this section:

1) In what ways, if any, does the teacher frame whole-class discussions for sense-making activity (i.e., framing students as the authority or framing the activity as working together, among others)?
2) To what extent, or in what ways, do the teacher's students align with her framing?
    a. Which students, if any, respond in ways that align with a teacher's framing?
3) If students do align with the teacher's framing for sense-making activity, what mathematically are they making sense of?

[2]This is a pseudonym that is consistent with original research on this teacher conducted by Sung, Wallace and Williams (Sung, 2018; Wallace & Sung, 2017; Wallace, Sung & Williams, 2014; Williams, Wallace & Sung, 2016).

3.1 Researcher Positionality

As a former mathematics teacher in middle and high school settings who has now been engaged in research in schools while pursuing a doctoral degree in mathematics education, I approached this study as a teacher first. While examining this data, I compared what I was seeing in this classroom video dataset to my own instructional practices, experiences with my students, and what I imagine the ideal mathematics classroom to be. I identified with Ms. Ellis as a white female teacher with similar educational training while I was a teacher. I also taught for about the same amount of time as Ms. Ellis when the video was recorded. My views on this data were also influenced by my most recent role as a researcher - what I have studied in other classroom settings on video, in-person, and through reading the literature. I approached this data, as well, as a former pre-service teaching supervisor – one who supported teachers as they engaged in classroom-based experiences to grow in their practice as effective mathematics teachers who supported students in taking a central role in the classroom.

With this dataset specifically, I had engaged in a previous study that partially focused on Ms. Ellis with Munter, Sung, Wallace (each with work(s) cited within this document). Within that study, we were trying to understand how our two perspectives on classroom teaching were similar and how they were different. To do so, we focused our analyses on teachers who we believed to be engaged in high-quality instructional practices.

In that study, Sung and I extensively reviewed classroom videos from various teachers, including Ms. Ellis, in order to reach some new understanding about one another's perspectives and our own. We spent quite a bit of time analyzing Ms. Ellis's classroom together. We did so in comparison to other teachers in our dataset. We found that we jointly valued some structural aspects of Ms. Ellis's classroom, but that we noticed different things beyond that with respect to the teacher's support for her students and what the students were actually engaged in during the lesson. Much like this study, Sung ended up completing her doctoral dissertation in Applied Developmental Psychology on the same data I am using – that collected from Ms. Ellis's classroom (see Sung, 2018).

Through the time I spent analyzing Ms. Ellis's class in that study, I had noticed some aspects of her instructional practice that stood out to me as high-quality in reference to the mathematics education literature. I detail some of those below when I justify the explanatory case study.

3.2 Design of the Study

In order to answer the above-stated research questions, I employed an explanatory case study design and followed a logic of inquiry that was informed by prior research on framing in mathematics and science education research. Below I justify both the explanatory case study design, as well as the choice of Ms. Ellis as the case. I then go on to explain my logic of inquiry and why it was a fitting choice for examining framing specifically.

3.2.1 Case Study Justification

I conducted an explanatory case study (Yin, 2003) in which Ms. Ellis's work of framing and her students' subsequent alignment (or not) of that framing were analyzed. My aim was to further understand a teacher's work of framing and how that may or may not influence students' interactions around the mathematics of study, particularly their engagement in sense making during whole-class discussions. As Yin (2003, p. 6) pointed out, explanatory case studies are useful when “operational links nee[d] to be traced over time, rather than mere frequencies or incidence.” The dataset used in this study allowed for such ‘tracing.’ By tracing Ms. Ellis’s contributions, as well as each of her students’, I was able to look at some possible implications of her framing over the span of a few lessons. I was also able to see deviations from the typical routines of interaction that provided further explanations of the inner workings of the classroom and the ways in which students’ engagement in sense making activity might have been attributed to the teacher’s framing work.

Based on my history with this dataset, I knew that this teacher was one who was engaging in mathematical practices considered productive within the mathematics education literature. Her students were taking ownership of their ideas, sharing them, and asking questions of one another and one another’s approaches to solving the problems – taking a dialogic approach (NCTM, 2014; Munter et al., 2016). She was using a curriculum that was associated with greater conceptual understanding than traditional curricula and more student communication around the mathematics they were studying (Ben-Chaim, Fey, Fitzgerald, Benedetto, & Miller, 1998; Cady & Hodges, 2015; Cai, 2014; Cai, Wang, Moyer, & Nie, 2011). She was also implementing the curriculum as intended in two ways. She followed the ‘Launch,’ ‘Explore,’ ‘Summarize’ format by allowing her students time to come up with their own approaches to the problem and talk about those in small groups. She followed that up with a whole-class discussion in which students shared and defended those strategies.

The other way in which Ms. Ellis implemented the curriculum as intended was in maintaining the cognitive demand of the tasks as they appeared in the text (Henningsen & Stein, 1996; Stein & Lane, 1996). That is, she did not implement the tasks in a way that reduced the tasks to procedures to be followed or formulas to apply. She supported students to originate solution strategies to the novel tasks from the textbook. The fact that Ms. Ellis was engaging in these practices that the mathematics education literature linked to sense making activity made it more likely that I would actually find sense making in this dataset. Because my study was looking at the ways in which a teacher's framing might support students to engage in sense making, I needed a dataset in which the classroom activity would likely yield productive interactions worthy of further examination.

A comparative case study would have been ideal for answering my research questions. Having another teacher who was engaging in productive mathematics practices, using a similar curriculum, and for whom there was extensive classroom video data available would have helped answer the research questions. However, another such teacher was not available in this dataset, or any other to which I had access. Another mathematics teacher within the broader dataset that Wallace, Sung, and Williams collected similarly had a culture of respect, but was not engaging in these kinds of mathematical practices. She was teaching with a direct approach, making her a poor comparison.

Ms. Ellis was a teacher who was engaged in productive mathematics instructional practices that suggested to me that I would be more likely to find sense-making activity taking place in her class. She was doing so around tasks that were more likely to encourage higher levels of cognitive demand from her students (Stein et al., 1996). That is, there was mathematics content around which I could expect students needed to make sense on their own. Not only was Ms. Ellis engaging in teaching that aligned with the participationism framework, she was doing so outside of the context of a research study focused on her mathematics teaching efforts. Therefore, Ms. Ellis was a case of a teacher, with some level of training in reform-oriented teaching practices (i.e., Accountable

Talk®), but who was not part of a mathematics education-specific research project. She was also a teacher for whom I could examine her framing over multiple lessons.

3.2.2 Logic of Inquiry

Green, Skukauskaite, Dixon and Cordova (2007) articulated in their interactional ethnography analytic framework a cyclic analysis that looks at the individual and the collective. Their stance was that, by going back and forth between the individual and the collective, "the group construction of the discourse and the individual discourse use and take up, construct a picture of where, when, and how knowledge was made available, what counts as knowledge, and how common knowledge and access to knowledge are socially constructed both in the moment and over time" (p. 120). The nature of interactional ethnography is well suited for examining framing as it helped to reveal what aspects of the classroom interactions were overlapping for the teacher and her students. It was particularly useful as I was looking at an individual's (Ms. Ellis's) framing and how that mapped onto the collective's (the students') interactions with each other around the mathematics during the lesson. In order to complete this work, I started with Ms. Ellis's explicit messages, and then traced what her students were doing after those. This involved looking back and forth at Ms. Ellis and what her students were doing to determine alignment in framing.

Green et al.'s (2007) interactional ethnography is complemented by another analytic approach described by Engle, Conant and Greeno (2007) called *progressive refinement of hypotheses.* Just as Green et al. (2007) provided an analytic framework for going back and forth between the individual and the collective to make determinations about what was going on in an interaction, Engle et al. (2007) described the ways in which researchers in the learning sciences go back and forth between theory and data. They initially draft hypotheses based on theory and then turn to the data to see whether or not there is support for them. In turn, researchers drafted new hypotheses, informed by both theory and data this time, and returned to the dataset to look for more evidence. This back and forth "can allow a single study to progress through multiple iterations of hypothesis generation and evaluation, making the resulting findings more robust than they might have been otherwise" (Engle et al., 2007, p. 240). Broadly, the theoretical underpinnings for my logic of inquiry rest within the work of Green et al. (2007), but are complemented by the procedural approach that Engle et al. (2007) provided.

Further, these complementary approaches were justified by existing work on framing in the literature specifically. Watkins et al., (2017) discussed the prior research on framing in science education research in which assignment of epistemological framing to a group of students as opposed to an individual was prevalent. Citing Conlin, Gupta and Hammer (2010), though, Watkins et al., (2017) pointed out that individuals influence the group's framing, thus both considerations are important. In this study, I hypothesized that an individual - the teacher - could influence her students' framings. At the same time, though, individual students' framings were where I turned to determine whether or not collective alignment to the teacher's framing was evident. Next, I turn to the data around which I employed this logic of inquiry and tested these hypotheses.

3.3 Data Sources

Wallace, Sung and Williams had previously collected this data and analyzed it as part of a broader, primary analysis on cultures of respect in classrooms. The following studies have been published on that larger dataset in the field of Applied Developmental Psychology: Sung (2018), Wallace and Sung (2016), Wallace, Sung and Williams (2014), and Williams, Wallace, and Sung (2016). Their studies ended up focusing on more than just cultures of respect; they examined autonomy-supportive practices, teachers' offerings of choice and power dynamics in group work.

Administration at the urban charter school system from which the data originated identified Ms. Ellis as one of six middle school teachers who demonstrated a culture of respect in her classroom. Sung, Wallace and Williams made use of student survey data to verify that students shared these perceptions of their teachers, including Ms. Ellis. They used survey instruments that measured the students' perceptions of their teacher's affective support, autonomy support, a self-report of student behavioral, emotional, and cognitive engagement, and teacher-student instructional interactions (Wallace, Sung, & Williams, 2014). Data collection for their primary analyses included classroom video for ten complete lessons in the fall of 2012, student surveys, audio-recorded student focus groups, two teacher interviews, and a teacher artifact binder.

I added to the data corpus by analyzing the instructional quality of Ms. Ellis's video-recorded lessons. I used the Instructional Quality Assessment (IQA) Toolkit to do so and detail that instrument more below (Boston, 2012).

3.3.1 Classroom Video

During October and November of 2012, the above-named researchers recorded ten classroom lessons of Ms. Ellis's using two wide-angle lensed-cameras in order to capture the entire class. The teacher wore a microphone, but individual students did not. Recording began as students were entering class for the last period of the day and ended when they were nearly all dismissed from the classroom. Most lessons included about five to seven minutes of cleanup, pack-up, and dismissal for the day during which Ms. Ellis made comments about student behavior and engagement in the lesson. Though not recorded on consecutive days, the ten lessons were from the same unit: Connected Mathematics Project 2's (CMP2's) '*Bits and Pieces I'* (2009) and included one day during which students were given a partner quiz (more detail on the curriculum is provided in the context of the study below). The partner quiz lesson took place after three lessons had been recorded, so on the fourth day of filming. Because there was no instruction on this day, I decided to focus my analysis on the nine instructional lessons that remained. While some tasks spanned multiple days, the entire 'Launch' and/or 'Summarize' phase of all of the lessons was not necessarily recorded. Because recording was done on non-consecutive days, some parts of the 'Launch' or 'Summarize' phase of the task completion were not recorded.

3.3.2 Teacher Interviews

The original research team conducted two interviews with Ms. Ellis – one in September before filming began and the other in April, after filming ended. Questions were related to how she provided choice to her students, how she tended to students' understanding and interest, and how she supported independent student thinking, among other things. These questions were not directly about the mathematics, but Ms. Ellis's answers did provide some insight into her general goals for her classroom instruction and some of the reasons behind the choices she made. For example, in response to a question regarding how Ms. Ellis thought about and supported independent thinking, she replied, "I [a hypothetical student] can bring a strategy to the table and know that it is okay to use because it works and this is what I believe and this is what I understand. And you can ask me questions about it, but I don't have to change that strategy if my, you know, peer wants to use a different one that is okay." Other statements like these that were relevant to my analyses were instrumental in triangulating my findings and interpretations after I analyzed the data.

The teacher interviews, in part, were completed around specific video clips from the dataset. That is, the researchers selected small portions of the recorded classroom interactions, showed them to Ms. Ellis and asked questions about them. Their questions were not directly about the mathematics instructional practices or the mathematics itself as that was not the intent of their study. However, as I stated above, some of the data around those clips was helpful in triangulating my own analyses – more on this process below.

3.3.3 Student Interviews

The original research team also conducted interviews with subsets of Ms. Ellis's students. These focus group interviews consisted of questions asked by the researchers, as well as video clips on which the students were asked to reflect or interpret. For example, at one point in the focus group interviews, the facilitator showed a video clip in which a student presenter was at the board presenting her strategy. The facilitator then asked the students, "What did you think? How do you think that student was feeling?" One student responded, "I liked how the class [respected] Wendy when she was up at the board, like they all looked at her and paid attention." The facilitator then asked why the students thought everyone was paying attention to her. One student said, "Because maybe some people didn't understand it and they wanted to understand it even more." In response to other questions, the students would build off of one another's contributions, providing a more detailed lens for how the students interpreted interactions in the class.

3.3.4 Ms. Ellis's Artifact Binder

The original research team asked Ms. Ellis to complete an artifact binder in which she provided pictures, handouts, newsletters, and descriptions of artifacts from her classroom that she thought would be helpful for understanding it. They did not stipulate what she included, but they did provide a form on which Ms. Ellis could describe the artifact, how it contributed to their understanding of her classroom, and reflect on whether or not that specific artifact was effective or not. For example, Ms. Ellis put encouraging notes in the binder that she passed out to individual students. In her reflection, she questioned how her students felt about those notes and suggested that she should ask them in the future. She also included a poster of the class's student-generated norms for group work in the binder (see Figure 1). On the bottom right corner of the poster that hung in her classroom it stated that they were "Created by 5/6 Students," meaning the students in those grades decided on the norms. As with the teacher interviews, it is important to again note that these data were not collected with discipline-specific aspects of Ms. Ellis's classroom instruction as the focus. Rather, these data were collected with a lens toward a culture of respect in the classroom. These data were, however, used to triangulate aspects of my data analysis. More detail on their uses appears after the data analysis plan.

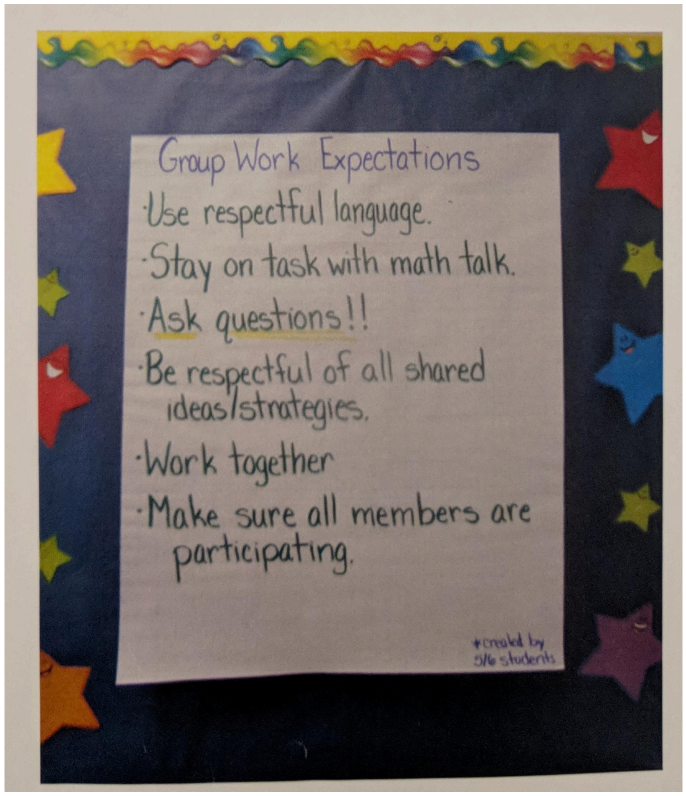


Figure 1 Copy of Student-Generated Group Work Exepctations Submitted by Ms. Ellis

3.3.5 Instructional Quality Assessment Toolkit Scores

Based on my history with the data and preconceptions based on that coming into this study, I wanted to verify that what I was seeing in Ms. Ellis's classroom was actually high-quality instruction. I also wanted a way to see variation across the nine lessons in a systematic way. To do so, I used the Instructional Quality Assessment (IQA) Mathematics Toolkit rubrics (Boston, 2012; Boston & Wolf, 2006; Matsumura, Garnier, Slater, & Boston, 2008) to assign five scores to each of the nine classroom lessons (the mathematical task, the implementation of the mathematical task, the quality of teacher's contributions; the quality of the students' contributions and the participation). Notably with respect to the design of this study, the IQA rubrics allowed me to consider the teacher's contributions (the individual) and those of the collective's (the students) in a systematic, unbiased way. That I had some unbiased measures of the quality of each of those contributions aligned with my logic of inquiry.

3.3.5.1 IQA Toolkit

The IQA toolkit (Boston, 2012; Boston & Wolf, 2006) is a set of nine classroom observation rubrics that broadly measure the potential cognitive demand of the mathematical tasks on which students are working and the quality of the discussions in which students engaged (see Appendix A for a list of the rubrics). The rubrics were designed to align with work on Accountable Talk®, making them particularly fitting to examine Ms. Ellis's teaching. Each rubric is divided into categories of scores ranging from zero to four (Michaels, O'Connor & Resnick, 2008). (See Boston (2012) for an example of two rubrics.) Generally, lessons with scores above two are considered high-quality and those scoring below two are lower in quality.

Two rubrics addressed the cognitive demand of the mathematics itself in which students had the potential to engage. Cognitive demand refers to Stein, Grover and Henningsen's (1996) Mathematical Tasks Framework in which they assigned levels of thinking that the task required to various mathematical tasks ranging from memorizing facts to engaging in novel mathematical thinking around tasks that did not have a clear and obvious solution path. The *Potential of the Task* rubric built from that framework but included at its highest level (a rating of four) a prompt from the task for students to explain their thinking or make their reasoning known. That rubric rated the task as it appeared on the paper or board – however it was presented to students. Rating the task as-written differed significantly from the other rubric – the *Implementation of the Task* rubric – in which the rating was dependent upon the teacher's actions- supports, questions, clues- around the task. To illustrate, a task might ask students to solve a novel problem with an unclear solution path, but the teacher might then tell students exactly how to solve the task. In that instance, the cognitive demand on the students was lowered because the teacher gave the students the procedure or path to the solution. Such a task might earn a high score on the *Potential of the Task* rubric, but a lower score on the *Implementation* rubric based on the supports the teacher provided to students. The distinction between the two rubrics that rate the task on which students are working is significant statistically. Childs, Dixon, Campbell-Sutherland, Bai & Boston (under review) did a factor analysis and found that the *Potential of the Task* rubric was a distinct factor from all of the other rubrics.

One of the IQA rubrics rated the levels of student participation based on what ratio of students participated in the whole-class discussion. The other six rubrics looked at the quality and kinds of classroom discourse. For example, the *Asking (Teacher Press)* rubric was meant to provide a rating of the extent to which the teacher pressed students for mathematical explanations. On the student side, the *Providing (Student Response)* rubric rated how often students gave explanations during the whole-class discussion.

I separated the rubrics into five categories to come up with five separate measures to compare all nine lessons: participation; task potential; task implementation; quality of students' contributions and quality of teacher's contributions. Table 1 shows which rubrics were used to come up with the ratings for each of the five categories. I generated a single score for each of the categories for all nine instructional lessons by averaging the included rubric scores within each category. From there, I decided which three lessons would comprise the sample for my microanalysis.

Table 1 Lesson Rating Categories by IQA Rubric

Lesson Category	Applicable IQA Rubric(s) from Boston & Wolf (2006)
Participation	• Participation
Task Potential	• Potential of the Task
Task Implementation	• Implementation of the Task
Quality of Students' Contributions	• Student Discussion Following the Task • Students' Linking • Providing (Student Responses)
Quality of Teacher's Contributions	• Teacher's Linking • Asking (Teacher Press) • Teacher Questioning

3.4 Classroom Context

The sixth-grade class in which this study took place was in an urban charter school in the Northeast United States in the fall of 2012. According to the school's website, they were committed to providing student-centered learning experiences to underserved populations. The charter system was made up of 80% students of color and 80% students qualifying for federal free and reduced lunches. This particular school was part of a broader charter school system that was being studied. Researchers Wallace, Sung and Williams spent time in the school system focusing on cultures of respect in classrooms. They studied teachers from multiple disciplines and grade levels as part of their original research. Further detail about this teacher, Ms. Ellis, and her students follows.

3.4.1 Participants

Ms. Ellis was a sixth-grade math teacher at the time of data collection with six years of teaching experience. Prior to becoming the sixth-grade mathematics teacher, Ms. Ellis also taught a self-contained fourth grade class and was an elementary math and science teacher. She obtained both a bachelor's degree in education and a Master of Education degree. She obtained her Master of Education degree while also being a full-time teacher. In addition, the particular class I studied was considered full-inclusion, meaning students receiving special education services were in her classroom with other students who were not.

Of the 24 students in Ms. Ellis's class, 69% of them were enrolled in the lunch subsidy program, while 15% were identified as having special needs. Additionally, 36% of the students identified as white with 64% identified as black. The students were in Ms. Ellis's room for mathematics only at the end of the day and were dismissed for the day from her class. For this reason, there was some downtime at the end of each lesson while students gathered their belongings and waited to be dismissed to their transportation home.

Ms. Pine[3], a resource teacher, supported Ms. Ellis during this particular period of the day. Ms. Pine served all students in the class, not just those who were receiving special education services. Though rare, in one of the lessons included in this microanalysis, Ms. Pine did take an instructional lead during a whole-class discussion. She did so while Ms. Ellis was supporting another student individually at a side whiteboard. Ms. Ellis was still listening to what was going on in the lesson and interjected from time to time to provide support or redirection. Typically,

[3] In Sung's (2018) study, she referenced Ms. Pine as the assistant teacher instead of using a pseudonym as I have done.

though, Ms. Pine did not take an instructional lead during any parts of the lesson and instead acted as a support for all students following Ms. Ellis's instructional lead.

It is important to note that Ms. Ellis engaged in Accountable Talk® professional development (Michaels et al., 2008). The entire school was part of the training, but the details of Ms. Ellis's involvement, including whether and/or how she was supported beyond the trainings, is not known. Accountable Talk® is a set of practices that supports teachers and students in being accountable "to the learning community, in which participants listen to and build their contributions in response to those of others;…accountab[le] to accepted standards of reasoning, talk that emphasizes logical connections and the drawing of reasonable conclusions;…and accountab[le] to knowledge" (Michaels et al., 2008, p. 283). According to their website, Accountable Talk® professional development "support[s] educators in implementing high-quality, discussion-based learning experiences in their classrooms, schools, and districts" (Accountable Talk®, 2015).

3.4.2 Curriculum

Ms. Ellis used Pearson's Connected Mathematics Project 2 (CMP2)[4] curriculum developed at The University of Michigan with National Science Foundation-funding by Lappan, Fey, Fitzgerald, Friel, and Phillips (2009). On their website, the curriculum was described as "a problem-centered curriculum promoting an inquiry-based teaching-learning classroom environment" ("Connected Mathematics Project," 2018). The curriculum as written is broken up

[4]There are various on-going and completed studies of the CMP curriculum series both at the University of Michigan and elsewhere. I reviewed these studies and did not find others that were directly linked to this one in terms of examining a particular teacher's framing of the activity when the activity was centered around CMP textbook materials.

into books by unit. The unit, or book in CMP2 terms, of study during the filmed portion of the class was called ‘*Bits and Pieces I’*. Its focus was rational numbers as represented in fractions, decimals, and percentages. ‘*Bits and Pieces I’* was comprised of four investigations for students to explore with between four and five tasks per investigation. Though Ms. Ellis often planned for tasks to be completed in a single, sixty-minute class period, many spilled over into other class periods. CMP2 printed a pacing guide for each of the tasks and suggested 45-minutes for each. The four investigations addressed estimating with fractions, comparing fractions, converting between fractions and decimals, and making use of percentages. More specifics on the tasks included in this analysis appear below, as well as in Appendix B.

Ms. Ellis followed the curriculum’s structure and adhered to the ‘Launch,’ ‘Explore,’ and ‘Summarize’ phases when students were engaged in a task. She did, however, modify the tasks at times. During the ‘Launch,’ Ms. Ellis introduced the mathematical task being investigated. She provided some information about the context of the problem for her students and sometimes provided some insight into what she expected of her students while they worked on the task. She also solicited some prior knowledge from her students. For example, in one lesson she asked the students what they knew about fractions larger than one. During the ‘Explore’ phase of the lesson, Ms. Ellis asked students to work in their table groups on the task. During that time, she walked around monitoring their work, asking them questions, reminding them to talk with their group members, and to stay on pace with group members. The ‘Summarize’ phase of the lesson was the time during which students shared their solution strategies and ideas about the problem with the entire class. This always took on a whole-class discussion format for the recorded lessons. Students usually occupied the typical teacher space at the front of the room during these times, but Ms. Ellis did interject, verbally and physically, at times during the discussion. For the most part, Ms. Ellis implemented the curriculum as intended.

While Ms. Ellis followed the curriculum as intended for the most part, her perceived and stated learning goals for each lesson or task were not known. I did access to the curriculum's goals as stated in the textbook, but I am less sure about how Ms. Ellis thought about those goals for her students specifically. Analyses using the Instructional Quality Assessment Toolkit (described in detail above) did reveal that Ms. Ellis maintained the cognitive demand of the tasks as written in all but one of her lessons (Henningsen & Stein, 1996; Stein & Lane, 1996). In Lesson 7, she reduced the cognitive demand of the task slightly by not requiring her students to explain their work.

3.4.2.1 Mathematical Tasks

As stated previously, the unit the students were studying while the classroom videos were collected was '*Bits and Pieces I*.' The specific tasks in the lessons on which I focused my microanalyses included: 'Folding Fraction Strips,' 'Fractions Between Fractions', and 'Naming Fractions Greater than 1' (Lappan et al., 2009). The tasks, along with their CMP2-stated goals appear in Appendix B. In Lesson 1, on 'Folding Fraction Strips,' the students were given various strips of paper with different colors and asked to fold the strips to create equal fractional parts of fractions ranging from halves to twelfths. As part of that, they had to state what their strategy for folding the strips was. The next lesson focused on 'Fractions Between Fractions,' Lesson 5, Ms. Ellis modified the task and asked her students to find *two* fractions in between each pair of fractions. The pairs of fractions had common and different denominators and were designed such that students could not complete the task merely by finding common-denominator fractions nor by merely using percentages to help find fractions. The most challenging pair in that task was to find two fractions that fit in between one tenth and one ninth. Finally, Lesson 6 focused on a contextualized problem in which students were working with fractions greater than one in the context of students cleaning litter from portions of a highway. In this task, students were asked to find how much of a highway a particular group of students needed to clear, where that portion would start or end, and how much highway was left to clean.

In the following section, I detail the analyses that I conducted on these data sources and in the final section of the chapter, I provide a summary of the ways in which each of these data sources contributed to answering and/or triangulating my research questions and answers.

3.5 Data Analysis

I turned to the above-described data sources to answer my three research questions. I completed my analysis in six phases, outlined below. I answered my research questions by first considering the teacher's work of framing, and second, looking at students' responses to said framing (how they responded, who responded, and what they were making sense of mathematically). In so doing, I followed my logic of inquiry by looking at the individual and then at the collective. Throughout analysis, I went back and forth between the two. Analyzing both the teacher's contributions and her students' helped me to explain a possible impact of a teacher's framing work.

3.5.1 Phases of Data Analysis

My data analysis occurred in six different phases. The first phase involved surveying the dataset and taking field notes. The second phase was to analyze the quality of instruction across the lessons using the IQA Toolkit. Doing so provided me information about the quality of Ms. Ellis's instruction in each lesson in order to reduce the data to three lessons for more detailed, microanalyses. I transcribed the 'Launch' and 'Summarize' phases of those three lessons completely. I began the third phase by identifying Ms. Ellis's explicit messages, and thus her framing, in those three lessons. The fourth phase continued my microanalysis by turning to look at the students' contributions. All teacher and student contributions were analyzed at the level of talk turns. I determined whether students' contributions aligned with Ms. Ellis's previous framing, as well as identified the significance of the students' mathematical contribution. In the fifth phase I looked back at the codes to gain a broader perspective on the data by dividing each lesson up into segments and looking at the codes within each. A segment was defined as either the 'Launch' or by student presenter at the board. That is, the 'Launch' was a segment, and then each student who went to the board to present their solution strategy during the 'Summarize' phase of the lesson constituted the segments. In the final phase, I attended to rigor by looking broadly at the dataset – re-viewing all ten of the lessons in their entirety, reading all of the other data sources, and checking in with a member of the original research team as a kind of member-checking. Table 2 provides a summary of these six phases of analysis.

Table 2 Phases of data analysis with brief descriptions.

Phase	Description
Phase 1: Survey the Data	• Watch all 10 lessons • Note 'Launch,' 'Explore,' and 'Summarize' phases • Take Field Notes
Phase 2: Instructional Quality Assessment and Data Reduction	• Rate all 9 instructional lessons using the IQA Toolkit • Select 3 high-quality lessons for microanalysis • Transcribe all of the 'Launch' and 'Summarize' phases for the 3 lessons
Phase 3: Teacher Framing	• Identify all explicit messages from Ms. Ellis • Code the kind of framing for each by turn-of-talk
Phase 4: Students' Contributions, Frame Alignment and Mathematics Analysis	For each student's contribution by turn-of-talk: • Note whether there is alignment to Ms. Ellis's framing • Note mathematical significance of student's contribution
Phase 5: Analyze Framing by Segment	• Separate lessons into segments ('Launch' or by student presenter) • Count the number of framings, alignments, and misalignments by segment
Phase 6: Attend to Rigor	• Re-view the entire data set • Read all other data collected – interviews; artifacts; focus groups • Discuss findings with at least part of the original research team

3.5.1.1 Phase 1: Survey the Data

To begin to deconstruct the dataset and engage in the logic of inquiry that I described above, I followed Erickson's (2006) recommendations by using a whole-to-part, inductive approach to my initial viewing of the video data. I watched each lesson completely, without stopping while also taking field notes about shifts in classroom activity and any noticings relative to Ms. Ellis's messages. For example, during the first pass, I noted the 'Launch,' 'Explore,' and 'Summarize' phases of each videotaped lesson, as well as took notes about Ms. Ellis's directions to her students. Some of these included her directions for how the students needed to work together as well as what they were expected to do while completing the mathematical task. In relation to my history with this dataset, it is important to note that I had not viewed the entire dataset – all ten lessons – prior to engaging in this dissertation analysis. I viewed, at most, two of Ms. Ellis's lessons with Hannah, but not the entire dataset.

3.5.1.2 Instructional Approach

Based on the ten videotaped lessons, I observed the following instructional routines in this classroom. During the sixty-minute block, Ms. Ellis had her students seated in groups of three to four students around tables. She intentionally grouped the students heterogeneously by ability (Sung, 2018). For the most part, Ms. Ellis implemented the curriculum as intended. For example, she supported students in exploring the mathematics in their small groups rather than collectively taking them through the tasks together at the board. She walked around asking questions of each group while the students grappled with the problems during the 'Explore' phase of the lesson. This could be evidence to suggest that she supported them to productively struggle (Hiebert & Grouws, 2007; Stein, Smith, Henningsen, & Silver, 2000; Warschauer, 2015).

She also engaged them in full-class conversations about their strategies during the 'Summarize' phase of the lesson. The 'Summarize' phase typically began with an individual or a group presenting his/her solution to the class at the front of the room while everyone else listened from their seats. Ms. Ellis then had that student presenter(s) stay at the interactive white board to field questions from the class. Often, students would raise their hands to ask questions of the student presenter(s). Ms. Ellis usually made it a point to have the presenter select students from whom to take questions and to answer those questions. Sometimes, if this was not taking place, Ms. Ellis would request, "Questions or comments?" She also participated as a student at times, asking questions herself of students when others did not have any. If Ms. Ellis did participate as a student and ask a question, she typically waited until all of the other students asked their questions first. One student in the focus group interviews stated that this was a typical practice of Ms. Ellis's.

Once one solution strategy was presented and there were no more questions of it, another student from a different group would typically come to the board to present a different solution strategy for discussion. The students would either volunteer to present next or Ms. Ellis would select a student. Both Ms. Ellis in her interview and her students in the focus group interviews commented on this aspect of whole-class discussions – that multiple solution strategies were typically shared. There was evidence to suggest that Ms. Ellis, at least sometimes, supported students in drawing connections between the strategies. For example, after two students had provided opposing ideas, Ms. Ellis said, "And you're disagreeing, but I think you might have used a similar strategy to what Lyla just did. She said she looked at that extra above the goal as a piece to add on." Here, Ms. Ellis acknowledged that the two ideas were in opposition, but pointed out that the two were, on some level, viewing the problem in the same way. Though she did not always support such connections as will be further discussed later.

3.5.1.3 Phase 2: Instructional Quality Assessment and Data Reduction

With the initial phase of analysis behind me, I decided that I needed to tend to the quality of Ms. Ellis's instruction in order to select lessons to analyze that were conducive to sense-making activity. I also wanted to determine whether my anecdotal assessment of Ms. Ellis's instruction as high quality was consistent with the mathematics education literature and another coder. To do so, I used the Instructional Quality Assessment (IQA) Mathematics Toolkit (Boston, 2012; Boston & Wolf, 2006). I made use of the nine observation rubrics on all nine of the instructional lessons in order to determine which three lessons would make up an appropriate sample for more detailed, microanalyses of the classroom interactions as well as Ms. Ellis's framing.

A second coder coded eight lessons and we discussed our scores to reach consensus. We did not end up scoring Lesson 7 because there was not a whole-class discussion in it. Lesson 6's discussion and small-group worktime took place largely during Lesson 7 because Lesson 5 also ran long into Lesson 6. That left little time in Lesson 7 to actually focus on the task meant for that day. Therefore, the students only had time to get started on the task and work in small groups. We were both trained on the IQA Toolkit by Dr. Melissa Boston and have engaged in IQA coding on separate projects.

In summary, I used the single score from each of my five categories (task potential, task implementation, student contributions, teacher contributions, and participation) to compare the eight lessons. Because the IQA scores in all five sub-categories did not vary by much and because the 'Launch' of the tasks seemed significant in terms of examining a teacher's framing, some lessons were not considered. A brief overview of the rubrics and their link to my three components of sense making follows.

IQA and Sense Making

Since I was interested in the ways in which a teacher might support her students to engage in sense-making activity, possibly through framing, it was logical for me to target lessons in which sense making was most likely occurring. To better understand how the IQA would help me in terms of sampling for sense making, I mapped the three components of sense making onto the nine IQA rubrics. (See Table 3). Participation is in a category all its own and peripherally linked to sense making as one would hope that most students were contributing to a sense-making discussion. Unfortunately, it is not strongly linked to any one category of sense making (i.e., productive struggle, authority, co-construction). For that reason, it is left out of my mapping. Table 3 provides an analysis of the observable aspects of sense making in classroom interactions, along with how, and which IQA rubric measured each. The cognitive demand of the task seemed quite meaningful for productive struggle and for students as the authority. If a student were asked to merely memorize a fact or procedure, then the authority that he/she could exhibit would be limited. Relatedly, the amount of productive struggle in which students could engage would be limited. Therefore, the cognitive demand score for each lesson was most important in terms of determining which lessons were conducive to sense-making activity.

Table 3 Sense Making Mapped onto IQA Rubrics (IQA portions adapted from Boston & Wolf, 2006)

Sense Making Component	Observable Characteristics in Classroom Video	IQA Rubric for Assessing	What IQA Rubric Measures
Students as Authority	Students: Explaining Thinking/Solutions	Student Discussion Following Task	The extent to which students provide full, complete explanations for their strategies that connect to the underlying mathematics OR whether students discuss more than one solution strategy
	Students: Generating Solutions to Tasks Based on Prior Knowledge	Potential of the Task & Implementation of the Task	The potential of the mathematical task to encourage higher-order thinking and reasoning
	Students: Connect Explanations to Mathematical Ideas	Providing	The extent to which students provide evidence for their claims as appropriate to the discipline
	Teacher: Requesting Explanations	Asking	
Productive Struggle	Teacher: Asking for Explanations	Asking & Teacher Questioning	The extent to which the teacher presses students for conceptual explanations or to explain their reasoning when they do not provide them on their own
	Teacher: Provide Cognitively Demanding Tasks	Potential of the Task & Implementation of the Task	The potential of the mathematical task to encourage higher-order thinking and reasoning and the extent to which the teacher maintains the task's rigor when implemented

Co-constructing Mathematical Explanations	Students: Connect Others' Ideas	Students' Linking	The extent to which students relate their ideas to others and connect their contribution to others'
	Teacher: Connecting Students' Ideas to Others or Providing Space for Students to Connect Ideas	Teachers' Linking & Teacher Questioning	The extent to which the teacher connects or provides opportunities for students to show how ideas relate to one another

3.5.1.4 Phase 3: Teacher Framing

The literature review established that there are multiple facets to framing and various ways of evaluating one's framing in an interaction. For that reason, it is important to now turn to discuss the ways in which I viewed framing in this dataset particularly. I decided to narrow my focus to the teacher's explicit messages about what it was that was going on in a given lesson because I coded alone (Goffman, 1974). That is, while I consensus-scored each lesson for the IQA, I completed the rest of the phases on my own. Considering messages about what it was that was going on in a given lesson goes back to the sociological foundations of framing that were most widely cited in the science and mathematics education literature. I was interested in the teacher's work of framing and how that could have supported students to engage in whole-class discussions for the purpose of making sense of the mathematics – how the individual's actions mapped on to those of the collective's. My interest in how that framing affected students required that I turn my attention toward students' contributions that indicated their understanding of what was going on in the lesson to see if they aligned or not with the teacher's framing. Below I detail the ways in which I identified and coded the teacher and her students' framings.

3.5.2 Teacher Framing

For the initial research question in this study, my microanalysis focused on the teacher's work of framing via her explicit messages about what was going on in the class, lesson, task, etc. Other researchers have similarly looked at explicit messages to make claims about framing and how such messages influenced subsequent activity (i.e., Forman et al., 1998; Louca et al., 2004; Rosenberg, Hammer & Phelan, 2006; Russ et al., 2012). These studies have not all focused on mathematics or mathematics discussions, and were not all targeting classroom instruction.

The researchers named in the previous paragraph, at least in part, analyzed explicit messages that guided the learner(s) to understand what it was that was taking place in the interaction. I made use of the broader research base on framing in mathematics and science education by using previously identified frames to name the ways in which Ms. Ellis framed the activity and whole-class discussion for her students. It is possible that other kinds of framing were evident in the dataset, but I did not generate new codes as I did not see a need.

Table 4 provides a list of my codes for both the teacher and students' framings, along with a definition and example from my dataset. There were broad and sub-codes for epistemological[5] and social framings. The blue codes were all categorized as epistemological framing, while the red codes were all social framings.

Table 4 Framing codes, sources, and examples from within this dataset

Framing Code	Definition	Source(s)	Example from This Data
Bounded	Classroom events are referred to as being contained in the given class period, with the participants present in the classroom about the topic at hand, typically pre-determined.	Engle, 2006; Engle et al. 2012; Selling, 2016	"With today's lesson you're going to get nine different colors of paper that you will turn into fractions today."

[5] I distinguished code names throughout this text in this font.

	The students do not play a central role in the events and there are not references to other contexts.		
Expansive	Classroom events are referred to as spanning a broader time, place, audience, topic of study, or for various reasons. Audience members outside of those present can be referred to and the topic of study may be linked to other topics. Students do play a central role in classroom events.	Engle et al. 2012; Engle, 2006; Selling, 2016	"Continuing our work with fractions, we're going to kind of talk about fractions between fractions today."
Doing	Students and the teacher operate with the understanding that the teacher has the social and epistemic authority. Students and the teacher are focused on completing tasks related to finishing the lesson. The teacher is central to the contributions and decision-making.	Berland & Hammer, 2012a; Heyd-Metzuyanim et al., 2018; Jiménez-Aleixandre et al., 2000; Pope, 2003	"So if you take 90ths and you cut them in half, you're going to get 180ths, right?"
Epistemological	Participants interact with the understanding that students are the authority, the ones contributing ideas, deciding on their correctness, and explaining them. The source of knowledge is students and they decide what is new knowledge.[6]	Redish, 2004; Hammer et al. 2005; Watkins et al., 2017	"That is something you're going to have to find out. Um, if I say 'yes' or 'no' then I answer the question that we're trying to search for."
Epistemological: Explaining own strategy	A student defends or is invited to defend and/or explain, upon being questioned, his/her own way ideas or strategies. This code is different from a student originally sharing an idea when asked by the teacher; it is in response to a question.		"You tell me how, you're the one that did it on your paper."
Epistemological Student as originator	A student is asked to, or does, share his/her ideas or solution strategy. This typically occurs at the beginning of the discussion, after a different idea or strategy has been presented, or in answering the teacher's questions.		"You're going to take these nine strips of paper and you're going to fold them to show what would halves look like. How would you fold it for thirds? Think of a good strategy you want to use. You do not necessarily have to go in this order."
Epistemological: Understanding others' strategies	A student or the teacher asks a question in an attempt to understand another student's strategy; could be in relation to his/her own solution path or outside of it.		"Were you trying to turn fifths into twentieths where it says one out of five? Where did you get one out of five equals twenty?"
Social (positional)	An individual's expectations related to how to interact, who will speak, how conversation will be initiated, etc. This includes who will be questioned and who will do the questioning.	Hammer et al., 2005; van de Sande & Greeno, 2010	"Your group will catch you up, don't worry."

[6] Within the literature, epistemological framing is framing related to knowledge and/or knowing. Because the 'doing' frame accounted for one version of knowledge - a teacher-directed version, I established the codes so that epistemological framing referred to a more productive framing that was akin to sense-making activity. This interpretation aligned with descriptions provided in the cited sources in the table.

Social: Math is working together	The teacher or students encourage, or rely on, one another to think through the mathematics. This could be in the form of asking for help while at the board, while answering questions, or when the teacher indicates to students that they are expected to work with their group members.		"Does someone in Keri's group want to help her? I'm assuming you all did the same."
Social: Teacher space	A student, or students, is invited to or does occupy typical teacher space physically and/or discursively (e.g., student stands at the front of the room; student chooses who to call on; student selects next presenter)		"Lauren, when you're done marking that up you have a couple of questions to answer."
Affective	The teacher or students indicate how they are expected to feel about the interaction or activity or how they do feel about it.	Hammer et al., 2005; Redish, 2004	"You're going to have to discuss with your group what are your wholes?...This is not an easy task the five that you're coming across. You need to rely heavily on groupwork."

I coded by talk-turn, so some teacher and student contributions fit more than one of my code definitions. In those cases, I coded for both framings. An example of this arose in the following teacher contribution: "You need to make sure you are helping each other out. You're going to find that some of us are better at creating thirds than others. Some of us are better at creating fifths. Make sure you're working together and helping each other out." In this instance, I coded for both epistemological and social framing. The epistemological frame was apparent in that the teacher told her students not to expect to just know how to create all of the different fractional pieces by folding the paper – this was not a simple recall or apply-a-procedure kind of problem. This could have indicated to the students that they were not expected to know how to fold the paper for all fractional pieces, but that they could figure out how by working together. The social frame was cued by her telling her students to work together to help one another. Socially, they should have been talking to each other about the task for support. These two seemed to send related messages: math is something you do together to figure out some things you may not know and you should expect to support one another when doing math in this class.

Ms. Pine was a factor in this coding process. Though she did not typically take an instructional lead, Ms. Pine did lead some of the whole-class discussion times and she also contributed to the framing when Ms. Ellis took a lead. Because the research questions were geared toward the effect of teacher's framing, I felt it appropriate to account for Ms. Pine's framing in my coding. With that being said, I understand that the students had different relationships with Ms. Pine from what they had with Ms. Ellis, including their view of her power or authority in the classroom. At the same time, though, Ms. Ellis's framing accounted for an overwhelming number of the framing codes that I ended up coding. Ms. Pine framed the classroom activity far less. Additionally, after I coded, there was not a pattern of students aligning more or less with Ms. Pine over Ms. Ellis, nor vice versa.

The second research question was meant to address the implications, if any, of a teacher's work of framing. That is, examining Ms. Ellis's framing through explicit messages addressed how she was framing the whole-class discussion, and even the tasks, but it did not address whether there was evidence to suggest that such framing mattered for the students. I looked to students' contributions to determine how they aligned with Ms. Ellis's framing.

3.5.3 Students' Framing

To address the second research question, I coded students' contributions related to what it was that was going on in the class. I also coded utterances that indicated that students were aligned with a current framing that was in place. For example, a student utterance of "Yes" could have indicated that he/she aligned with a particular framing. In those instances, I coded the students' contribution as such. Just as with the teacher's framing, I coded at the level of talk-turn. For example, in Lesson 6 when talking about how much highway a group had cleaned as a mixed number when it was provided as an improper fraction of nine-fourths, Ms. Ellis stated, “You have to write that length as a mixed number. Show on the number line how you figured that out. Go ahead, Leona.” Leona then walked to the board and began writing on a number line on the interactive whiteboard as she said the following, “Ok, so we did it in- well, ok. So that would be one whole and then you go one-fourth, two-fourths, three-fourths, and four-fourths make the whole [writing the fractions on the number line]. And then the improper fraction would go down here [on a separate number line], so, um, five-fourths, six-fourths, seven-fourths, eight-fourths, then nine-fourths. Ok, so since this [4/4] is a whole, um, eight-fourths would be the second whole cause it’s like the numerator is double the denominator and then since they’re at nine-fourths, it would be one-fourth left -two and one-fourth.” Based on the teacher’s epistemological and social framing of the student as the originator of the solution and the student being invited to the teacher space to “show on the number line” at the front of the room, the student aligned with that framing. That Leona went to the front of the room where the teacher typically stood, indicated she aligned with the social framing of taking up teacher space. The explanation that followed, more than just an answer or marking up the number line, indicated that she also aligned epistemologically with the teacher in understanding that she was the originator of the solution idea and she should share that with the class in detail.

Just as I linked sense-making components to the IQA rubrics in Table 3, I similarly linked them to my codes for framing. I did so to see which portions of the class, based on the framing code, had the potential for students to engage in various aspects of sense making. For example, when the teacher framed students within the discussion as the originator of ideas, the potential for students to be the authority in those instances was probably higher than when the teacher invoked the doing frame. Similarly, when the teacher invited students to occupy typical teacher space through her social framing – spatially or discursively – that likely gave students an opportunity to be the authority as opposed to other times in which an alternate framing was invoked. Table 5 explicates these links between framing codes and sense-making activity.

Table 5 Sense-making components linked to framing codes

Sense-making Component	Applicable Framing Codes	Explanation
Students as authority	Epistemological (all three sub-codes); Social (both sub-codes)	Instances in which students are given the opportunity to explain their own solutions, ideas, or strategies to their peers either in front of the class or by raising their hands provide the potential for students to be the authority. If the students are given the opportunity to originate ideas, share them, and/or question others' ideas, then they have the chance to be the disciplinary and social authority in those interactions.
Co-constructing mathematical explanations	Expansive, Epistemological (student explaining own strategy and understanding others' strategies); Social (math is working together)	When students have the opportunity to question one another and respond to their peers, they have the chance to work with others' ideas to co-construct mathematical explanations. If the teacher expansively frames the participants in the class, students could have the opportunity to co-constructing mathematical explanations with other students or experts' ideas brought forth by the teacher.
Productive struggle	Epistemological (student explaining own strategy and understanding others' strategies); Social (math is working together); Affective	When students are explaining their own strategy, trying to understand another student's strategy and/or are working together, they have the opportunity to articulate what it is they do not know, reexamine what they thought they knew based on new input from peers, and support one another to reach new understandings. In addition, if the teacher affectively frames the activity for students by preparing them for some challenge or difficulty with the task, they could be prepared to engage in productive struggle.

3.5.4 Misalignment of Frames

While the previous paragraph provided an example of frame alignment, participants were not always aligned with one another. Part of looking into the effect of Ms. Ellis's framing on students' interactions involved some misalignment. Misalignment of frames was evident in tracing interactions over a period of time, sometimes a few talk turns. In the Lesson 1 discussion in which Ms. Ellis asked Nya to share her strategy for creating fractional parts there was evidence of a misalignment of frames. Because Ms. Ellis asked for the strategy as opposed to an answer is evidence that she framed the lesson epistemologically with the student as the originator of the ideas. Nya, then, went to the front of the room and responded to the question of her strategy by saying, "Twelfths." Here, Ms. Ellis epistemologically framed the contribution, but Nya framed it as doing by providing just a one-word answer with no explanation: "Twelfths." In that instance, Ms. Ellis went on to try to align Nya with the epistemological frame by asking again, "What was your strategy?" Then Ms. Ellis stated, "I just want to know, were there – did you have some type of strategy when you decided to make one of these?" Ms. Ellis attempted to support Nya in aligning with the epistemological frame until Nya eventually did align and provide an explanation for how she created twelfths. Examining misalignments helped answer the second research question in regard to students' alignment with Ms. Ellis's framing.

3.5.5 Orienting Students to the Task

In addition to looking for frame alignment and misalignment, it was important for me to consider how students were oriented to the mathematical task itself on which they worked and discussed. Ms. Ellis's explicit messages about how students should think about completing the task during the ‘Launch’ phase of the lesson provided some insight into how they subsequently interacted during the ‘Summarize’ phase. For example, when she began the discussion, Ms. Ellis did say in one lesson, “I’m expecting quite a debate when we get to the teachers” (meaning one particular bar measuring teachers’ progress toward a fundraising goal within the problem). She made this comment about one component of the problem that required students to estimate an amount and a fractional part of a goal based on pictures of shaded regions of fraction bars. The teachers’ fraction bar to which Ms. Ellis’s comment referred was the only one that exceeded the goal mark and required a fraction larger than one. This orienting statement may have let her students know that she was aware that there were different answers and she expected disagreements when they discussed that portion of the task. Her students’ debates around the answers to the teacher fraction bar could have indicated that her explicit message about what to expect supported such student interactions during the ‘Summarize’ phase of the lesson.

From the three lessons selected for my sample, I transcribed all of the dialogue during whole-class times, namely during the ‘Launch’ and ‘Summarize’ phases of the lesson. From those transcripts, I coded all of Ms. Ellis’s explicit messages about what it was that was going on in the lesson, task, discussion, or otherwise to make determinations about what kinds of framing she did for her students. Based on prior research, I coded using the codes described in Table 1, including bounded, expansive, doing, epistemological, social, and affective framing.

While I was open to naming new framings that emerged, I did not see a need. What I did decide to do during my analysis was to provide more detailed codes for some of the epistemological and social framings that I was coding. I divided the epistemological framings into: the student as the originator of ideas; understanding others’ strategies; and the student explaining his/her own strategy. I divided the social framings into: math is working together and student is in/invited to teacher space. Doing so allowed me to code at a greater level of detail for students' alignment to Ms. Ellis's framing. It was also helpful in seeing what kinds of interactions were taking place. For example, the epistemological code of understanding others' strategies did not occur very often. This code allowed me to see when students were asking questions that were aimed at understanding another student's way of explaining or solving the task. It did not, as it turned out, occur very often, which revealed something about the kinds of questions students did ask of one another during the discussion.

3.5.5.1 Phase 4: Students' Contributions, Frame Alignment, and Mathematics Analysis

Once I identified Ms. Ellis's work of framing in the three lessons, I began to analyze each student's contribution to note, as applicable: the alignment to Ms. Ellis's framing and the significance of the mathematical aspects of the contribution. I will further detail the logistics of these parallel analyses below.

Table 6 displays an excerpt of my analysis categories in table-format. The table includes a timestamp, a column for the teacher's contributions, a column for students' contributions, a column for the framing code, an alignment column and a column for the significance of the mathematical contribution. Because I linked sense-making to the codes (see Table 5), I did not include a separate column for the aspect of sense-making that was evident in the student's contribution. I also included a column for comments/notes about the contribution. The table used in my analysis sequentially accounted for all of the contributions from both the teacher and the students during the 'Launch' and 'Summarize' phases. Not all contributions were coded or had an indication in the mathematical significance column.

Table 6 Column Titles for Parallel Analyses

Teacher's Contribution
Student's Contribution
Framing Code
Timestamp
Mathematical Significance

Alignment

I looked at each student contribution to see how each one aligned, or did not, to Ms. Ellis's prior framing. Alignment to the most recent framing, or any previous framing in the lesson, was noted along with, at times, a brief description for my coding of such alignment. I also noted any evidence of misalignment in students' contributions. I noted when the teacher or students broke with the pattern of interaction leading up to that point as a misalignment of frames.

In determining alignment, I considered a student aligned to Ms. Ellis's framing if he/she aligned with her previous framing within the lesson. For example, sometimes students would be presenting and talk about their strategy using the pronoun "we" as opposed to "I." In those instances, I looked to see if the teacher had framed the mathematical task or activity in the 'Launch' socially as doing math together. If she had, then I considered that a student's alignment to the teacher's framing from the 'Launch'. Relatedly, if a student asked a peer a question in an attempt to understand his/her strategy, I considered that alignment epistemologically even if Ms. Ellis had not explicitly stated that the students needed to be trying to understand one another's strategies. That both were considered epistemological frames in my coding scheme was sufficient for me to term that alignment. In addition, if a student walked to the board when Ms. Ellis stated his/her name, I considered that social alignment to being invited to occupy teacher space. The student, by walking to the board without prompting, indicated that he/she understood the social framing discussions: students share their strategies at the board in the teacher's space. That is, I did not look for alignment with an immediately preceding framing by the teacher because that seemed too rigid a view of framing. Framings can last an extended period of time, so if I coded the teacher's framing in a particular way at all in that lesson for a given framing, I considered it alignment by the student.

One exception to this, which was evident in patterns in the data, was the doing frame. The doing frame was not invoked by the teacher very often during the 'Summarize' phase, and when it was, students always aligned immediately with it. For that reason, students were considered to align with the teacher's doing frame if Ms. Ellis had immediately prior indicated a doing frame verbally. Otherwise, as was in the case illustrated above with Nya during Lesson 1, the student was not considered to be aligned to the teacher's framing when invoking the doing frame.

Mathematics Analysis

In response to the third research question, I also completed an analysis of the mathematical significance of students' contributions, where applicable. I did so by initially marking all mathematics-specific contributions. That is, if there was mathematical content in a student's contribution or in the teacher's contribution, then I coded it as such. From there, I analyzed across all mathematics contributions to see the progression of the mathematics, or at least what it was that students were talking about. After that, I looked at each turn of talk to see what the student was saying and how that fit in with what was said previously. This process was informed by literature on rational numbers in the elementary/middle grades (i.e., Moss and Case, 1999; Hackenberg, 2007; Lamon, 2005). For example, in Lesson 5 when Keri was talking about how she found a fractional part in between one-fifth and two-fifths, she talked about dividing the fifths in half. In so doing, she was addressing the continuity of the number line. Her contribution was mathematically significantly different from her peers' contributions in that they had merely found other equivalent fractional parts with whole number numerators and denominators or found some percent that fell between the two percent-equivalents of the given fraction pairs. Her process, and the underlying conceptual meaning of her contribution, differed greatly from her peers'. I performed this mathematical analysis across all contributions with the thought, too, that if the teacher was framing for sense making, and students were taking up that framing, then knowing what the students were making sense of would be important.

3.5.5.2 Phase 5: Analyze Framing by Segment

In order for patterns to emerge and for me to make claims about what was going on with respect to Ms. Ellis's framing and her students' alignment, I needed a way to analyze the framing codes, alignment, and the mathematics contributions. To do so, I divided each of the three lessons up into segments. For each lesson, I considered the 'Launch' a segment. Then, once the 'Summarize' phase started, I segmented by student presenter. When a new student presenter was asked to go to the board to share his/her strategy, I considered that a new segment of the class. Segmenting could have been done in a lot of different ways, but doing so by student presenter allowed for consistency across lessons and also made sense in terms of the mathematics contributions. That is, segmenting in that way allowed for the framing analysis to be done around a single strategy with a particular underlying mathematical conception. Table 7 summarizes the number of segments per lesson. Because the 'Launch' counted as a segment in each lesson, you can see that Lesson 1 involved seven student presenters, Lesson 5 involved nine student presenters, and Lesson 6 involved five student presenters. This was not surprising considering that Lesson 5 took the most amount of time and was fully filmed; the 'Launch' and the 'Summarize' phases were both fully captured on video. Lesson 6 had the shortest amount of time on camera, and thus had the fewest number of presenters.

Table 7 Number of Segments by Lesson

Lesson	Number of Segments
Lesson 1	8
Lesson 5	10
Lesson 6	6

After segmenting, I also noted where each misalignment of frames occurred and for whom they occurred. I looked further to see whether or not there was evidence to suggest that Ms. Ellis attempted to support the student to align and, if so, whether or not the student showed evidence of subsequent alignment. In so doing, I found only three students who did not align with Ms. Ellis's framing. Each of those three students did align with Ms. Ellis at various other places in the dataset, suggesting that there was not systematic misalignment of frames for select students.

Analyzing the framings, alignments, and mathematical significance by segment allowed me to engage further in my progressive refinement of hypotheses as I had some hypotheses about what was going on in the class in terms of Ms. Ellis's framing. I then looked to my evidence for those hypotheses bearing out and, when they did not, I revised my hypotheses and went back to the data to see if there was evidence to support those. As an example, after my initial viewings of the data, I hypothesized that Ms. Ellis was framing the classroom activity for students to attempt to understand one another's strategies. When I looked at the evidence, though, the understanding others' strategies code did not emerge as much as I thought it would. I then revised my hypothesis to be that Ms. Ellis framed the classroom activity for students to co-construct mathematical explanations due to the amount of emphasis she placed on students' working together and questioning one another.

3.5.5.3 Phase 6 Attend to Rigor

Making determinations about framings is interpretive work. For this reason, it was important to be sure that the conclusions I drew were trustworthy and tended to rigor (Toma, 2006). In addition to relying on inter-rater reliability in double-coding the IQA Toolkit on all nine lessons, I engaged in other forms of enhancing credibility in my findings. Toma (2006) provided some guidelines for attending to rigor in qualitative research and specifically discussed case study research. He cited Miles and Huberman (1994) in describing the paramount aspect of qualitative research: being credible. One way to enhance credibility in qualitative case study research is by triangulating findings. Looking across multiple data sources to determine whether convergent evidence existed is one way that I enhanced my own credibility within this study.

Table 8 provides a list of the research questions and the applicable data sources that contributed to verifying my own findings. The classroom video data was the main evidence source for answering all three research questions. Outside of that, I used the IQA scores to objectively categorize the quality of instruction within each lesson. The IQA scores were helpful in triangulating the first and third research questions. Because the IQA has rubrics that categorize the teacher and students' contributions with respect to linking students' contributions to one another's, I had an additional lens with which to view one aspect of sense making that I was looking for: co-constructing mathematical explanations. In addition, the rubrics provided measures for the teacher requesting explanations and students providing explanations, another way of looking at students as the authority in the classroom. The teacher interviews were a helpful secondary lens for seeing

Ms. Ellis's classroom from her perspective. She provided insight into what her goals were for the interactions in her class. Her interviews were also helpful in the ways in which she did, or did not, address the mathematical substance of her lessons. Similarly, the student interviews provided a bit of the students' perspectives on the classroom interactions, making them helpful for interpreting the alignment and mathematical significance questions. The artifact binder was yet another contributor in understanding Ms. Ellis's perspective. Not only what she selected to put in the binder, but her commentary around each item were helpful to my own findings/interpretations.

Another resource within this study was one of the original researchers, Sung, who also completed her dissertation on this dataset. The amount of time that she spent analyzing the classroom videos, and collecting them, was very helpful when I discussed my own interpretations and findings with her. In that sense, I did a kind of member-checking. Finally, my own re-viewings of the entire video dataset were helpful in helping me understand and interpret my findings -more on this step in the next paragraph.

Table 8 Ways in which findings by research question were triangulated with other sources

Research Questions	Data Sources						
	Classroom Video	IQA Scores	Teacher Interviews	Student Interviews	Artifact Binder	Member-check with Original Team	Re-view Data
RQ1: Teacher framing for sense making	✓	✓	✓		✓		✓
RQ2: Student Alignment	✓			✓		✓	✓
RQ3: Making sense of what mathematically	✓	✓	✓	✓		✓	✓

In addition to triangulating, I searched for alternate ways of viewing the data to strengthen my arguments. Erickson (2006) suggested systematically looking back at the data sources to look for alternate explanations for what was going on in a given setting. Similarly, he advocated systematically looking for evidence to support claims in re-viewing the data. Re-watching all ten lessons in their entirety, with my conclusions in mind, is another way that I searched for alternate ways of viewing the data as well as those findings.

While I did re-view the three lessons of interest many times throughout my coding process, I did so again in their entirety once I had analyzed the framing codes and mathematical contributions. I also re-viewed all of the other lessons in the dataset sequentially in their entirety. Doing so with some preliminary conclusions in mind allowed me to corroborate some of those findings, find some counterexamples, and determine the typicality of the three lessons that I analyzed in further detail. While I re-viewed the lessons, I took field notes to keep track of some of my reactions in light of my preliminary findings, as well as to better account for the time that I did not micro-analyze.

I also looked to the other available data sources to see how they might better inform my preliminary conclusions, or not. I read all of the transcripts from Ms. Ellis's interviews and from the students' focus group interviews. I also looked back at everything in the artifact binder that Ms. Ellis provided to the research team. Again, doing so allowed me to see some corroborating evidence and some counterevidence for my claims. Generally, in this phase, I took a broader view of the data, this teacher, and her classroom to enhance the credibility of the conclusions I drew.

4.0 Findings

In this chapter, I begin by providing the IQA results that helped me choose which three lessons would be part of the sub sample that I micro-analyzed for framing. I go on to share the results for how the teacher and students in the class framed the activity. Then I provide greater detail about the teacher's framing, how that supported sense making and how her students aligned to that framing. I end by discussing the ways in which Ms. Ellis framed the activity that was *not* conducive to sense making. Throughout these findings, I integrated supporting evidence (and, at times, refuting) for my claims to attend to rigor as described in the triangulation matrix in Table 8. I did not analyze those other data sources, I merely looked at them in relation to my findings. So, I searched and interpreted the evidence in the other data sources through the lens of my findings.

As a reminder, the research questions were as follows:

1) In what ways, if any, does the teacher frame whole-class discussions for sense-making activity (i.e., framing students as the authority or framing the activity as working together, among others)?
2) To what extent, or in what ways, do the teacher's students align with her framing?
    a. Which students, if any, respond in ways that align with a teacher's framing?
3) If students do align with the teacher's framing for sense-making activity, what mathematically are they making sense of?

4.1 IQA Coding Results and Data Reduction

The results of Phase 2 of my data analysis – IQA and Data Reduction - appear in Table 9. A second coder coded the lessons as well and we discussed scores to reach consensus. Our inter-rater reliability was 80.5%. Table 9 provides the coding results separated into the five categories (the categories were explained in Table 1). Lessons highlighted in gray are the ones that ended up in the sub sample for microanalyses. Each category could have been scored with a high of '4' and a low of '0.' As stated in the methods section, the Task Potential is a distinct factor amongst the other rubrics, and thus was significant in my pursuit of finding sense making activity (Childs et al., under review).

Table 9 IQA Coding Results by Category

Lesson	Task Potential	Task Implementation	Student Contributions	Teacher Contributions	Participation
1	4	4	3.14	2.8	2
2	4	4	3.43	3.2	4
3	3	4	3.29	3	3
4	n/a	n/a	n/a	n/a	n/a
5	4	4	3.29	3.2	3
6	4	4	3	2.8	2
7	n/a	n/a	n/a	n/a	n/a
8	4	4	3.43	3.2	3
9	4	4	3.14	2.8	3
10	3	4	3.14	2.8	2

Because the IQA scores in all five sub-categories did not vary by much and because the 'Launch' of the tasks seemed significant in terms of examining a teacher's framing, some lessons were not considered for the sub sample. As a result, Lessons 1, 5 and 6 ended up being the ones that I micro-analyzed. Lessons 1, 5 and 6 all had Task Potential and Task Implementation scores of '4'. This indicated that the cognitive demand of the tasks on which students were working was higher than lessons in which those scores were lower (Smith & Stein, 1998; Stein et al., 1996). As for Student Contributions scores, Lesson 6 had the lowest score there. This was due to lower *Student Linking* and *Student Providing* scores on the IQA rubrics (scoring 2 and 3, respectively). The Teacher Contributions scores in those three lessons were all similar to one another and to the broader dataset. The only difference between Lessons 1 and 6 and Lesson 5 in terms of the Teacher Contributions scores was lower *Teacher Linking* scores on the IQA rubric (2 for Lesson 1 and 6 and 3 for Lesson 5). The 'Linking' rubrics within the IQA assess whether or not the teacher or her students drew explicit connections between strategies by comparing or critiquing more than one strategy.

Aside from IQA scores, there were other issues to consider in reducing the sample. For example, Lessons 2 and 8 had high scores in all areas, but the 'Launch' for those tasks was not recorded. Lesson 4 was a day in which the students took a partner quiz. Lessons 3 and 10 had lower *Task Potential* scores than the other lessons, thus decreasing the likelihood of sense-making activity. And, as for Lesson 9, most of the discussion was not filmed. It is important to note that these lessons were not filmed consecutively. For that reason, some of the 'Launches' were not included and the end of the discussions in Lesson 1 and Lesson 6 are not included in the video dataset. However, most of the discussion for those two lessons were recorded. This was disappointing, but a limitation of the dataset with which I had to work. I decided to privilege lessons in which the entire 'Launch' was filmed as that seemed to be the more important lesson phase in terms of framing than the lesson's conclusion. Therefore, I had to make concessions in terms of claims I could make about whether the lesson's goal was met mathematically. This left Lessons 1, 5 and 6 to include in my microanalysis sub sample. (See Table 9.)

4.2 Framing in the Classroom

In this section, I detail the ways in which Ms. Ellis and her students were framing the classroom activity in the three lessons of my sub sample. I provide the total numbers of each framing for both Ms. Ellis and her students. I also explain some specific examples for how I coded framing. I draw some comparisons in this section as well across Ms. Ellis's framing and her students' framings.

4.2.1 Ms. Ellis's Framing

Ms. Ellis framed the class in all of the ways that I predicted based on my review of the literature: **bounded**, **expansive**, **doing**, **epistemological**, **social** and **affective**. Table 10 provides the codes, an example of each code from the dataset, and a breakdown of the framings by lesson. It also contains a total column and row for grand totals of framings. In Table 10 you can also see that Ms. Ellis mostly framed the classroom activity for **epistemological** (180 times), **doing** (90 times) and **social** (70 times) frames across the three lessons. The epistemological codes and sub codes appear in blue while the social codes and sub codes are red. She framed the class in these ways the least: **expansively** (6), **bounded** (1), and **affective** (3).

Table 10 Teacher's Coded Framings by Lesson

Framing Code	Example	Lesson 1	Lesson 5	Lesson 6	Total Frequency (including sub codes)
Bounded	"With today's lesson you're going to get nine different colors of paper that you will turn into fractions today."	1	0	0	1
Expansive	"Continuing our work with fractions, we're going to kind of talk about fractions between fractions today."	1	4	1	6

Doing	"So if you take 90ths and you cut them in half, you're going to get 180ths, right?"	17	68	5	90
Epistemological	"That is something you're going to have to find out. Um, if I say 'yes' or 'no' then I answer the question that we're trying to search for."	0	24	1	25 (180)
Epistem: Explaining own strategy	"You tell me how, you're the one that did it on your paper."	22	49	10	81
Epistem: Student as originator	"You're going to take these nine strips of paper and you're going to fold them to show what would halves look like. How would you fold it for thirds? Think of a good strategy you want to use. You do not necessarily have to go in this order."	17	25	29	71
Epistem: Understanding others' strategies	"Were you trying to turn fifths into twentieths where it says one out of five? Where did you get one out of five equals twenty?"	1	1	1	3
Social (positional)	"Your group will catch you up, don't worry."	4	1	2	7 (70)
Social: Math is working together	"Does someone in Lauren's group want to help her? I'm assuming you all did the same."	3	6	5	14
Social: Teacher space	"Lauren, when you're done marking that up you have a couple of questions to answer."	14	22	13	49
Affective	"You're going to have to discuss with your group what are your wholes?...This is not an easy task the five that you're coming across. You need to rely heavily on groupwork."	1	0	2	3
Total		81	200	69	350

Ms. Pine, the resource teacher, did some of the framing work in these lessons. While she was present for all of the lessons, she only took an instructional lead in the Lesson 5 'Summarize' phase while Ms. Ellis supported a student at the side white board individually. Other than that instructional lead, Ms. Pine made comments from time to time during the 'Summarize' and the 'Launch.' Some of those comments were coded as framings if they fit the code definitions. Table

10 displays the combined framings from Ms. Pine and Ms. Ellis. Ms. Pine, however, framed far less than Ms. Ellis did. She did so 8 times in Lesson 1 and 29 times in Lesson 5. She did not do any work of framing in Lesson 6. Of the 37 total times Ms. Pine framed the classroom activity for students, 15 times were as doing. While Ms. Pine's framings are accounted for in the table, I mostly attributed framing to Ms. Ellis as she was the main teacher in the room. For that reason, Ms. Pine disappears in this results section unless she was the one framing in a particular instance that I have represented in transcript excerpts. I recognize that the students' personal relationships with Ms. Pine, as well as their perceived power differentials for her as the resource teacher could affect the ways in which they aligned with her framing. However, I did not find any patterns of misalignment when it came to Ms. Pine's framing versus Ms. Ellis's.

Looking at Table 10, it is clear to see that Lesson 5 was the longest lesson within this dataset as it contained the greatest number of teacher framings at more than double the two other lessons. Filming for that lesson began at the beginning of Lesson 5 and continued over half way into Lesson 6. Because the two lessons were consecutively filmed, I was able to capture the entire lesson. For Lesson 6, though, that left little time to discuss, meaning that lesson ran over into Lesson 7. Not only did Lesson 5 have the greatest number of framings, it also had a disproportionate number of doing frames coded totaling 68 - 75% of the total doing frames across the three lessons. There will be more detail about why this might have been in later sections of this chapter.

The least coded frames were bounded, expansive, and affective. Ms. Ellis only framed the students' work one time as being contained within a particular lesson. The limited number of bounded framings is a positive for Ms. Ellis's practice and for her students. Bounded framing is less productive for students as it constricts the work and thinking they are doing to that given time and space, making it less applicable in other settings (Engle, 2006). Ms. Ellis expansively framed the lessons a total of six times. Expansive framing is the opposite of bounded as it supports students to see themselves, their ideas, and their work as relevant beyond the given lesson, task, or group of people (Engle, 2006). Finally, Ms. Ellis framed the activity affectively three times across the lessons. When she did so, she referred to the difficulty that students might experience in solving a task.

During the 'Launch' phase of the lessons, three codes were more prevalent than the others. Ms. Ellis framed the activity as doing (32 times), socially as math is working together (5 times), and epistemologically as the originator of ideas (26 times). While these codes applied to the 'Launch' more often than any others during that phase, other codes were still evident during that time. This was not surprising as the 'Launch' was typically the time during which the teacher was soliciting students' prior knowledge and preparing them to successfully complete the day's task by establishing a shared context mathematically and situationally (Boaler & Brodie, 2004; Jackson et al., 2012). Ms. Ellis asked students for memorized facts during the 'Launch' often times, resulting in a lot of doing frame codes. She also asked less-specific questions that invited students' ideas, which were coded epistemologically as student as originator. For example, in Lesson 5 Ms. Ellis asked, "Why can we fit one third in between one fourth and one half?" Ms. Ellis also supported students to see the task they were about to engage with socially as math is working together via her social framing during the 'Launch.' Also in Lesson 5, Ms. Ellis socially framed the activity when she said, "The strategy you use will be dependent on what you feel you and your group members want to use." During the 'Summarize' phase, she still framed the activity as doing, but she engaged in more social and epistemological framings during that time.

4.2.2 Students' Framings

Ms. Ellis's students' framings followed some similar trends to her own. They epistemologically framed, or demonstrated alignment to **epistemological** frames, the most (184 times). (See Table 11.) The **doing** frame code was the second most prevalent for the students (66 times). Finally, the third most coded frame was the **social** framing (60 times). The students did not engage in any **bounded**, **expansive**, or **affective** framings during the three lessons that I analyzed. This result was not surprising as Ms. Ellis rarely framed the classroom activity in these ways.

Each kind of framing was coded at the turn-of-talk level. That is, grainsizes were consistent in how they were measured, but not necessarily in the length of utterance. For this reason, among others, each framing code was dependent upon the context around which it was said. And, just as was the case for Ms. Ellis, some turns-of-talk were coded with more than one framing. The results are summarized in Table 11 along with examples from this dataset for the students' frames. As framing is context-dependent, I contextualized some of Table 11's examples in the next paragraph.

Table 11 Students' Coded Framings by Lesson

Framing Code	Example (contextualized in the paragraph below the table)	Lesson 1	Lesson 5	Lesson 6	Total Frequency (including sub codes)
Bounded		0	0	0	0
Expansive		0	0	0	0
Doing	"Oh, so should it be fifteen hundredths?"	13	51	2	66
Epistemological	"Can we use our fraction strips?"	0	6	0	6 (184)
Epistem: Student Explaining Own Strategy	"Well, uh, we knew that, uh, 3/10 was, um 30% and then 3/12 we found out, um, if you drew a picture and we drew 3/12 that would be equal to ¼ and ¼ is 25%."	26	42	17	85
		8	32	39	79

Epistem: Student as originator	"We started out with one-fifth equals twenty percent. And two-fifths equals forty percent. So we knew we had to find in between 20% and 40%, so then we found, uh, out that three twelfths- Since we had to find two so then we had to find something that was either 20%, 25%, or 30%, so three twelfths was equal to 25%, so we knew that three twelfths was one and then we knew that three tenths was equal to 30%, so we used three twelfths and three tenths.				
Epistem: Understanding others' strategies	"Are you saying that you knew that ¼ is 25%, so you tried to find something that was equal to it, like 3/12?"	1	10	3	14
Social (positional)		0	0	0	0 (60)
Social: Math is working together	"Um, well, Makayla, can you help explain how we got 3/10?"	7	12	7	26
Social: Teacher space	"How would that start out with halves because the question was how could we use the halves strip to [make it]? You wouldn't start out with the halves."	8	16	10	34
Affective		0	0	0	0
Total		63	169	78	310

4.2.2.1 Students' Doing Framings

For most of students' framing that was **doing**, they demonstrated alignment to the teacher's **doing** frame by answering a simple teacher question with a single word or phrase. In addition, when students' contributions indicated that they were framing the interaction as a version of 'doing the lesson,' it was considered **doing** as well. 'Doing the lesson' comes from Jiménez-Aleixandre et al.'s (2000) comparison to 'doing science.' Broadly, 'doing the lesson' meant that the teacher was in control of the ideas, their correctness, and the topics of conversation. So, both students' indicated alignment to the teacher's framing and their work of framing the activity as such were considered doing. The excerpt below, Excerpt 1, provides an example of each.

Excerpt 1

Ms. Ellis: Since you were using percents, what's one whole as a percent?
Keri: One hundred.
Ms. Ellis: One hundred percent. So I think there might be some confusion with, um – You're saying one point five is a possible answer and Mary read that as one whole and five tenths, so if I'm over one whole, should I not be over 100%? And you're saying one whole and five tenths is 30%. Do you understand that?
Keri: Oh. So should it be fifteen hundredths?

The first two lines of the excerpt are an example of Keri aligning to Ms. Ellis's doing frame. Ms. Ellis asked a known-answer question to which the student replied, "One hundred" (Boaler & Brodie, 2004). Ms. Ellis went on to affirm the student's answer by repeating, "One hundred percent…" – note that she added the label of "percent" (Forman et al., 1998). Each of these three contributions was considered the doing frame because the teacher asked a question that did not require original thought on the student's part. She also likely knew that the student already knew the answer to the question. The student demonstrated evidence of alignment with that doing frame by giving the brief answer to the teacher's question.

The next example in Excerpt 1, and the one from the table above, is a bit different as it demonstrated an instance in which the student was framing the activity as doing (Line 7). By Keri stating her 'answer' as a question for Ms. Ellis to affirm, she demonstrated that she was framing the activity as doing - a version of school in which the teacher tells the student if her thinking is correct. Other answers to questions in this class were accompanied by an explanation from the student or even asking if a group member might help answer a question instead of relying on the teacher for the answer. Those kinds of explanations or contributions were considered epistemological because they did not demonstrate that the students saw the teacher as the authority.

4.2.2.2 Students’ Epistemological Framings

The general epistemological frame was rare for students. The example provided in the table was one in which the student’s contribution had something to do with knowledge, knowing and resources, but was not specific to any particular epistemological sub code. The example in Table 11 came from the ‘Launch’ of Lesson 5 in which students were finding fractions between fractions. The student asked, “Can we use our fraction strips?” The student asked about a resource that she wanted to use. Redish (2004), Hammer et al. (2005), and Berland & Hammer (2012a), among others, termed resource-related aspects of framing as a kind of epistemological framing. That is, because this student wanted to use the fraction strips as a resource to come to some new understanding, it was epistemological. All such general epistemological codes for students dealt with resources in this dataset.

Excerpt 2

Naomi: Yeah, ok, we started out with one-fifth equals twenty percent. And two-fifths equals forty percent. So we knew we had to find in between 20% and 40%, so then we found, uh, out that three twelfths- Since we had to find two so then we had to find something that was either 20%, 25%, or 30%, so three twelfths was equal to 25%, so we knew that three twelfths was one and then we knew that three tenths was equal to 30%, so we used three twelfths and three tenths.

[Mary’s hand is raised, among others.]

Naomi: Mary?

Mary: Well, how did you figure out what the percents for each of those fractions were? Or get fractions for each of those percents?

Naomi: Well, uh, we knew that, uh, three tenths was, um 30% and then three twelfths, we found out, um, if you drew a picture and we drew three twelfths that would be equal to one fourth and one fourth is 25%.
[Some other student questions in between]
Keri: Are you saying that you knew that one fourth is 25%, so you tried to find something that was equal to it, like three twelfths?
Naomi: Yeah.
Keri: Okay.

The sub codes for epistemological framing examples all came from the excerpt above, Excerpt 2. Again, the context is helpful for understanding the codes and how they were applied. Lines 1-5 were the example used for the student as originator code. This is the explanation that Naomi gave when she was invited to the board to share her strategy for finding two fractions in between one fifth and two fifths. She provided a thorough explanation as to how her group came up with the fractions of three twelfths and three tenths. The ideas came from her and her group, therefore they were original to the student.

Lines 8-9 and 14-15 were both considered understanding others' strategies. (See Excerpt 2.) In those lines, two separate students were asking Naomi for more information about how she found those two fractions. They were trying to understand her strategy better. Lines 10-12 then were considered student explaining own strategy. Because Naomi was responding to a peer's question, she was explaining her strategy further. The distinction between student as originator and explaining own strategy codes was that the originator code was used only for the initial response or strategy-sharing. The explaining own strategy code was *in response to* a student question about the initial explanation.

Coding the distinction between providing some explanation around which others could talk and supporting one's own explanation upon being asked a question was important in this dataset for illustrating what kinds of interactions were taking place. It was not just that students were welcome to share their ideas, it was that they were also accountable to defending those ideas to their teacher and classmates that suggested sense making was taking place. Students seemed to be the authority in sharing their original ideas, asking questions of others' ideas, and defending their ideas (Forman & Ford, 2014).

Relatedly, the understanding others' strategies code was a further distinction. This code was used when a student asked the student presenter how or why a certain aspect of his/her explanation was the way it was. This distinct code helped reveal further details about the kinds of interactions that students were having with one another. It revealed that they were attempting to engage in co-constructing mathematical explanations with one another and, in some cases, that they were engaging in productive struggle (Forman et al., 1998). Hiebert and Grouws (2007, p. 389) used the adjectives "actively and effortfully" to refer to how students struggled productively to move from what they knew to understanding what they did not yet fully grasp. The excerpts above, particularly Kari's in lines 14 and 15 in Excerpt 2, demonstrated a kind of productive struggle in which she articulated what it was that was confusing to her. Not all understanding others' strategies codes could be linked to productive struggle.

4.2.2.3 Students' Social Framings

The social framing code, or demonstrated alignment to the social framing code via utterances, was applied in a few ways. The example provided in Table 11 for a social framing code of math is working together was one such code – "Um, well, Makayla, can you help explain how we got 3/10?" If a student asked a group member for help or provided a group member help while at the board or trying to explain, I coded that as math is working together. Because asking for and offering help indicated that the student did not feel responsible for knowing all of the answers or providing all of the information. Similarly, that the students, in these moments of puzzlement, did not simply ask the teacher for help or give up was indicative of their understanding that co-constructing mathematical explanations was expected.

A slightly different version of the math is working together code emerged when students were providing explanations at the front of the room. When students would go to the front of the room and, as they were explaining, say "we decided" or "our strategy" was…, that was considered math is working together. Any variation of the collective pronoun used during an explanation that attributed an idea or strategy to the student's group instead of him or herself signaled framing of math is working together. Because these students saw the knowledge they were sharing as collectively produced by stating "we" instead of "I", that indicated that they were aligned to the social framing of working together with a group. An example is when a student stated, "We used either percents or we used the number line to break it up." Such statements indicated alignment with Ms. Ellis's emphasis on the group work and her explicit framing for collectively engaging in the mathematics during the 'Launch' phases.

Notably, students framed more often socially as math is working together (26 times) than Ms. Ellis did (14 times). (See Tables 10 and 11.) That is, her students demonstrated alignment to the idea that they were co-constructing mathematical explanations without much prompting from Ms. Ellis. One explanation for this could be that Ms. Ellis recognized that her students did not need to be reminded of that expectation of hers because they already demonstrated alignment to it sufficiently. Another plausible explanation is from the analysis that Sung (2018) completed during the 'Explore' phase of the lessons. While she did not examine the teacher's framing, she did point out that she noticed Ms. Ellis reminding students a lot more to explain their strategies to one another and to work collectively on the task instead of individually during that phase of the lessons (H. Sung, personal communication, February 18, 2019). I noticed this as well when I re-viewed the lessons in their entirety. The students' interviews and Ms. Ellis's interviews pointed to this stability as well. Both the students and Ms. Ellis referred to an activity from the beginning of the year in which the whole grade-level observed four students working together on some task. They called this activity the 'fish bowl activity.' The observers, then, noted what the group was doing well together and what they could improve upon. From that activity, they created the group work norms that appeared in Figure 1.

Finally, the teacher space code was applied in two different instances. When students voluntarily walked to the front of the room, a typical teacher space, to share their strategy or idea, which was teacher space. Even if the student did not ask permission or the teacher did not immediately prior ask the student to come to the front, the act of walking into that space without prompting indicated alignment to the social framing. Similarly, after a student was finished presenting and called on classmates whose hands were raised to ask questions I coded as teacher space. In these instances, the students socially aligned to the teacher's framing that they were in charge of the class's interactions at those moments when they were at the front of the room.

When a student would ask questions of one another that addressed some missing piece of an explanation, it was also considered teacher space. This version of the code applied when a student was asking a question related to some missing piece of the explanation that linked back to the directions of the task. From the Table 11 example, the student said, "How would that start out with halves because the question was how could we use the halves strip to [make it]? You wouldn't start out with the halves." The student's question pointed out that the presenter's previously-provided response to the question did not start with halves as the task stated it should. That was an example in which the student asked a teacher-like question. That is, she noticed that the students did not follow the task and pointed it out to them. Because the student presenters had not tended to the requirement of the question in their explanation, another student embodied the role of the teacher and pointed out that flaw to them. This code was only applied if the student's question was teacher-like and did not fit the epistemological sub codes of explaining own strategy or understanding others' strategies.

4.2.3 Framing in this Classroom Summary

This section explained the ways in which Ms. Ellis and her students were framing the classroom activity in the three lessons. It also provided some insight into the ways in which coding was context-dependent, especially for the students' contributions. Differences between Ms. Ellis's framing and her students' framings were also highlighted. Two differences included that the epistemological framing of understanding others' strategies and the social framing of math is working together were more evident in students' framings than in Ms. Ellis's. Another detail about the students' framing was that they epistemologically framed student as the originator (39 times) most in Lesson 6, the shortest lesson in the sub sample. This was due to the fact that Ms. Ellis asked a lot of questions in the 'Launch' about students' prior knowledge that was not necessarily memorized facts. Memorized facts or recall-type questions would have been considered doing. She was asking them what they knew about improper fractions. In answering, the students provided examples that were original to their own understanding. That is, they did not recite definitions, rather they talked about what they knew in their own terms. Sometimes, to gain a deeper understanding of their related knowledge, Ms. Ellis asked them to provide examples of what they were talking about. While I alluded to sense making in some parts of this section, I more explicitly link Ms. Ellis's framing to sense making in the next section. I also more directly address the three research questions around which this study was designed.

4.3 Framing for Sense Making

In response to the first research question, Ms. Ellis was framing the classroom activity for sense making by epistemologically, socially, expansively and affectively framing. These framings were linked to the three components I used to define sense making in a detailed way in Table 5, but I will further elaborate on these links-in-action in this section. Table 12 below is a simplified version of Table 5, without the explanations for the links to sense making. Doing and bounded framings were seen as less productive, not for sense making activity kinds of frames and thus do not appear in the table. The simplified table is meant to be a quick reference on how I linked sense making and the framing codes.

Table 12 Sense-making Components Mapped onto Framing Codes

Framing Code	Students As Authority	Co-Constructing Mathematical Explanations	Productive Struggle
Epistemological	✓		✓
Originator of Ideas	✓		
Explaining Own Strategy	✓	✓	✓
Understanding Others' Strategies	✓	✓	✓
Social		✓	
Teacher Space	✓		
Math Together	✓	✓	✓
Expansive		✓	
Affective			✓

Ms. Ellis framed the activity for sense making in a variety of ways, and did so consistently across the lessons (as evidenced in Table 10). As for student alignment, research question 2, her students, for the most part, aligned with such framings and engaged in sense-making activity. They aligned with her framing in 304/310 total framings of theirs. They demonstrated alignment by being the authority in the class (both the epistemic and social), engaging in co-constructing mathematical explanations, and, to a lesser extent, engaging in some productive struggle (Forman & Ford, 2014; Berland & Hammer, 2012a). That they engaged in sense-making activity was evidence that Ms. Ellis's framing did matter for her students. However, what the students were making sense of mathematically was even more helpful to understanding the impact Ms. Ellis's framing had on her students' engagement in sense making activity. As noted previously, her students engaged in more framing epistemologically as **understanding others' strategies** than Ms. Ellis did. Findings related to this bore out when I looked at what students were actually making sense of. Their sense making was limited to more thoroughly understanding individual strategies instead of drawing connections across them – a response to the third research question. In this section, I further explain these findings and corroborate them with evidence from other sources.

4.3.1 Ms. Ellis Framing Students as the Authority

All of the **epistemological** and **social** framing codes were mapped onto supporting students as the authority in the classroom and Ms. Ellis framed in those ways a grand total of 250/350 times across the three lessons. Therefore, Ms. Ellis framed students as the authority most often. Not only did Ms. Ellis frame students as the authority, she did so both with respect to one another and with respect to the discipline (Forman & Ford, 2014). Socially, she invited students to typical teacher spaces (**teacher space** code). Disciplinarily, Ms. Ellis framed her students as the originators of the ideas in the class – ones who were expected to defend and explain those ideas and approaches. She also framed her students as having the authority to question one another and to support one another in coming to more complete understandings (**explaining own strategy**; **understanding others' strategies**; and **math is working together** codes). The epistemological framing sub-code of **student as originator** was the second most prevalent of all of the epistemological framing codes. In this sense, she framed her students' ideas as the central element of the classroom activity. Her more prevalent epistemological framing of **explaining own strategy** further supported students' as the drivers of the discussion.

Ms. Ellis's framing of students as the authority was not only evident in the framing codes, it was also something that came through via other data sources. In relation to her students having the authority in the classroom, Ms. Ellis stated in one of her interviews that "the students take over" and that "it's all about student dialogue and what the students are getting from the lesson." In these comments, Ms. Ellis was reinforcing something gleaned from my coding of her framing - that her students really were taking over the authority in the class and their ideas were what she saw as central to the lesson.

This was further illustrated in an interview in which she referenced students who sometimes were not as confident in mathematics. She talked about encouraging those students and pointing out to them, "See you did do it. You thought of that on your own." That Ms. Ellis recognized that some students felt less confident and took it upon herself to remind them of their authority and ability in her class suggested that she really did view her students as capable and as being able to come to new understandings on their own. This belief may have supported her in framing the students as the authority. Without believing in their abilities, she may not have seen them as capable of being the authority in her class.

Ms. Ellis's comments above in relation to supporting students' confidence also implicitly highlighted, too, that she took a particular epistemological stance toward the mathematics in her class. Ms. Ellis did not, in her interviews, talk about giving students the strategies to solve problems or giving them answers, she referenced students coming up with their own strategies, justifying them, and attempting to understand others. At one point she said she "believes in the conceptual understanding and the 'why' behind the math."

In emphasizing students' own approaches, Ms. Ellis stated her expectation of students in question form: "Can you investigate and come up with a strategy you feel is efficient and one that works all the time, but yet you understand it?" Ms. Ellis also repeatedly mentioned her expectations for students solving problems with a variety of strategies. She was not so much concerned with what a student's strategy was, but more concerned that the student could present a strategy to his/her group and "know that it is okay to use because it works and 'this is what I believe and this is what I understand. And you can ask questions about it, but I don't have to change that strategy if my peer wants to use a different one.'" Ms. Ellis expected the students to both demonstrate the interactional and disciplinary authority over their idea and to communicate it in a convincing way. Her interview data also revealed her own epistemological shift in mathematics specifically and suggested that she wanted to support her students to see mathematics as a place to make sense of ideas for oneself.

4.3.1.1 Students' Alignment to Ms. Ellis's Framing as the Authority

Ms. Ellis's students aligned to her framing them as the authority, both socially and epistemologically, most of the time. Their interactions indicated alignment to her framing in that they gave explanations for their strategies and approaches, they questioned one another, they defended their approaches to one another, and they entered typical teacher spaces (Tabak & Baumgartner, 2004). There were times during which Ms. Ellis's students did not wait for her to call them to the board to share their strategy, they just went as soon as she said their name. Similarly, her students often had their hands raised to ask questions of the student presenter before Ms. Ellis explicitly asked for "Questions or comments." Her students' eagerness to ask questions suggested that they readily aligned with her framing them as the authority. She did not have to control all of the activity by constantly directing her students to come to the board or to ask questions of one another. This could mean that her consistent framing of students as the authority was meaningful for their understanding of how they were to interact both socially and with respect to the mathematics. That is, her students typically shared explanations for their strategies without being prompted. At times, of course, Ms. Ellis and peers did need to support one another in coming to more complete explanations, but they usually at least stated some explanation for their approaches initially.

These findings were corroborated by the IQA scores, the focus group interviews, and in my re-viewing of the data. As for the IQA scores, they accounted for the extent to which students volunteered explanations without being prompted (the *Providing* and *Student Discussion Following the Task* rubrics). Each of the three lessons scored the highest on these rubrics, except for Lesson 5. Within Lesson 5, the *Student Discussion Following the Task* score was slightly lower at a level of '3' as opposed to '4.' While '3' is still considered high quality, there was room for improvement. This lower score was due to extensive prompting from Ms. Ellis toward the end of the lesson, which was also corroborated by very high numbers of doing frames within that lesson. Nonetheless, the IQA scores did support the finding that students took on the role of the authority in Ms. Ellis's class.

In re-viewing the entire dataset, I noticed similar student interactions across all of the lessons. What I noticed with respect to students readily sharing their ideas and strategies and occupying typical teacher spaces was consistent across the entire dataset. That is, I did not see any lessons in which the interactions were significantly different during the 'Launch' or the 'Summarize' phases of the lessons.

Not only did Ms. Ellis's students' in-class interactions indicate alignment to being framed as the authority, they also alluded to this idea in their interviews. At one point in the focus group when students were shown a clip of one of their peers at the board presenting, the researchers asked them what their reaction to the clip was. The students responded by saying that the peers in the class were respectful of the student presenter, they looked at her, and they paid attention to her. When pressed for why students might look at her and pay attention to the student presenter, they responded that "maybe some people didn't understand it and they wanted to understand it even more." Her students spoke in terms of respect and paying attention to their peer. This could mean that they agreed that the student presenter was the authority both socially at the front of the room and in terms of the mathematics - they could listen to their peer to better understand something.

In terms of sharing their own ideas, at one point the students stated that "she wants to know what we're thinking." This statement suggested that the students understood Ms. Ellis's expectation to provide explanations around their thinking. Another student said that if he did not explain his reasoning, "she would probably be wondering 'how did I know that.'" Both of these quotes indicated that the students knew they were to share their original thoughts and that they were to explain their strategies – two codes that appeared frequently in the dataset.

Another different take on students as the authority that came up in Ms. Ellis's interviews and does support the idea that Ms. Ellis framed her students as the authority was related to confidence. The students said, "I think that she knew that, but I don't think that we knew that in ourselves" when talking about solving a task for themselves. Another student said, "She wants us to figure out the answers by ourselves and with our groups." Finally, another student said, "I think what she's saying is like that she believes us – believes in us more than we believe in ourselves during math class, like she knows we can do it by ourselves." It is probably unreasonable to expect students to talk about these ideas in terms of authority, but it is implicit in what they were saying. Their quotes demonstrated that they felt that Ms. Ellis wanted them to figure out the strategies and the solutions on their own without her having to tell them because she knew they could. That they said this in their focus group interviews suggested that they recognized and accepted that role as the authority. They also attributed that to Ms. Ellis. They did not say that they had always been good at math or that they had always had great experiences in math class, rather they said that she had the confidence in them even if they did not have it in themselves. That her students attributed that confidence building to her suggested that she did something different from what their other math teachers, and other teachers, did. It is possible that her continual framing of them as the authority contributed to this belief in themselves.

One last piece of evidence came from the artifact binder and is intertwined in Ms. Ellis's framing her students as the authority and instilling confidence in them. It is a more socio-emotional piece of evidence, but still speaks to the ways in which Ms. Ellis conveyed her belief in her students. The poster in Figure 2 was displayed in Ms. Ellis's room for her students to see. The top line explicitly told her students that she believed in them. She also stated that they were listened to, important, and would succeed. This source may speak more to the culture of respect that Ms. Ellis established, but it is also relevant to framing them as the authority. Not only did she talk about them as being competent in her interviews, tell them that via her framing, and reinforce it with encouragement, she displayed it to further remind them. The complete picture that these varying evidence-sources provided reveal different aspects of Ms. Ellis framing her students as the authority, and the ways in which her students aligned with her framing.

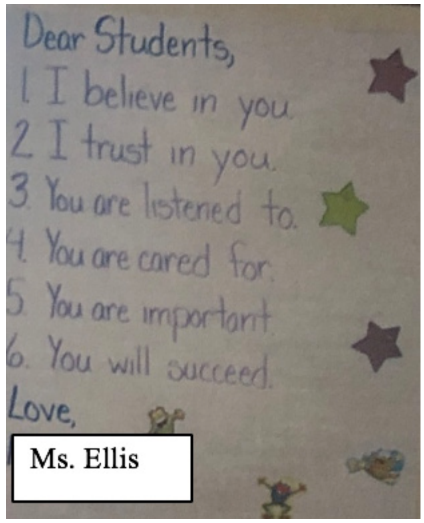


Figure 2 Poster of a Letter Ms. Ellis Wrote to Students

4.3.1.2 Student Misalignment to Ms. Ellis's Framing as the Authority

One student, Leona, misaligned with Ms. Ellis's framing her as the authority twice at the beginning of the Lesson 1 'Summarize' phase. Her misalignment accounted for one third of the total misalignments across the three lessons. This was the only student to demonstrate misalignment during the 'Summarize' phase. Ms. Ellis, along with Ms. Pine, tried to reinforce the epistemological framing of Leona as the authority. Excerpt 3 provides the dialog around the misalignment.

Excerpt 3

1 **Ms. Ellis**: Leona, what's a strategy you used?

2 **Leona**: Twelfths.

3 **Ms. Ellis**: Ok, so you used your strips to help you make your twelfths?

4 **Leona**: Yeah.

5 **Ms. Pine**: What was your strategy?

6 **Ms. Ellis**: Ok, what was your strategy?

7 **Leona**: What?

8 **Ms. Ellis**: I just want to know, were there- Did you have some type of strategy when you decided
9 to make one of these?
10 **Leona**: Oh, yeah, well, with help from Ms. Pine.
11 **Ms. Ellis**: That's ok.
12 **Leona**: Me and Ms. Pine, she said that twelve has a half point so it was six, so uh, you can fold it
13 in half.

Excerpt 3 above illustrates the ways in which Leona misaligned with Ms. Ellis's framing her as the originator of ideas and explaining her own strategy (Lines 1, 3, 6 and 8). Ms. Ellis asked Leona what strategy she used to fold the fraction strips. Leona was framing the activity as doing in that she was not explaining her strategy and she attributed the thinking to Ms. Pine (Lines 2, 10 and 12-13). It also shows how Ms. Ellis and Ms. Pine tried initially to reinforce the epistemological framing (Lines 5, 6 and 8-9). Lines 12 and 13, while coded as doing because Leona was attributing the thinking to Ms. Pine, were also the beginnings of her alignment to Ms. Ellis's epistemological framing. In those lines, she demonstrated that she was understanding that she needed to provide more of an explanation than simply trying to rely on Ms. Pine to give that explanation for her. That is, Leona eventually aligned with Ms. Ellis's framing of her as the authority- the one who needed to explain and defend her approach, even if she came to that approach with support from Ms. Pine.

Overall, Ms. Ellis's students aligned with her framing of them as the authority across all of the lessons. Though Leona misaligned with Ms. Ellis's framing in this interaction, she aligned with her framing in other interactions across the lessons. Therefore, it was not that Leona systematically misaligned with Ms. Ellis's framing. In addition, Leona did align with Ms. Ellis's social framing of teacher space by walking to the front of the room to share her strategy.

4.3.2 Ms. Ellis's Framing for Co-constructing Mathematical Explanations

As for framing to support students in co-constructing mathematical explanations, the epistemological framing sub codes of student explaining his/her own strategy and student understanding others' strategies codes were applicable. That both were only used after student ideas were already initially shared meant that they were about supporting students to respond to others' questions or ideas – thus supporting them in the co-construction of more thorough and complete mathematical explanations. Forman et al. (1998) similarly noted how a student was able to build an argument by integrating aspects of others' arguments. The social framing sub code of math is working together also had the potential to support students in this way. That the teacher framed the mathematical activity as a group activity requiring group support and justification to one's group indicated that co-constructing the mathematical explanations together was an expectation.

Finally, expansive framing could help students view their work as co-constructing mathematical explanations if the teacher provided some ideas from students in previous years or in other settings. Ms. Ellis's expansive framing was in referring to students' explanations and strategies from prior years. These framing codes that supported students to co-construct mathematical explanations occurred a total of 104 times within the three lessons, accounting for a little less than 30% of the framing codes. Some of these codes overlap with the codes counted as framing for authority. I did not distinguish which sense making component was evident in each framing as those lines were not clear. The total numbers of framings for co-construction and students as authority, though, indicated the framings that potentially supported student engagement in sense making.

The least evident of Ms. Ellis's epistemological framings was understanding others' strategies. For this code in particular, one might think that she did not support students in co-constructing mathematical explanations. To the contrary, though, she supported students in explaining their own strategies regularly across the three lessons. In fact, that was the most prevalent sub-code of epistemological framing in which Ms. Ellis engaged. Because this framing only applied when a student was responding to, or was asked to respond to, a peer's or the teacher's question in relation to his/her initial explanation of a strategy, she did frame for co-constructing mathematical explanations. Excerpt 4 below illustrates this.

Excerpt 4

Brad: I did the snake in [shows thirds at the front of the room]- And, like this and tried my best. I did like this and I tried my best with this one. And then I just folded it and there was already creases, so I had that. Like that and then I did it again.

Ms. Ellis: Questions for Brad?

[No students raise their hands.]

Ms. Ellis: I do, Brad, I have a question. You lost me after you created thirds. So, can you go back to the beginning? You created thirds and where did you go from there?

Brad: I, um, I folded the first thirds.

Ms. Ellis: Into what?

Ms. Ellis supported a student, Brad, to provide a more complete explanation than what he originally provided (Lines 6-7, 9). He was describing how he folded a fraction strip into ninths. Brad started out with an inarticulate explanation for how he folded his fraction strip (lines 1-3). The students referred to the fraction strips as "the snake" throughout Lesson 1. Ms. Ellis supported him in being more articulate and coming up with a more complete mathematical explanation (lines 6, 7 and 9) by pressing him for more detail. Ms. Ellis's contributions in those lines were coded as explaining own strategy because she was prompting the student, after his original explanation, to go into more detail and explain how he created ninths.

The epistemological explaining own strategy code was also applied when Ms. Ellis invited students to question their peers. It is noteworthy that Ms. Ellis first asked the class if they had questions for Brad in this interaction. She typically asked for such questions from peers before she asked her own questions. In this sense, she was further supporting her students as the authority and the ones deciding what would be discussed in relation to Brad's strategy. She was also supporting the students to co-construct mathematical explanations together and not just seeing that as her role. She was careful about the fact that her own question or comment might influence what her students said. Her students noted this at one point in their focus-group interview as well. One student stated, "We all get to say our own opinion because she waits 'til we're done asking questions and we're done saying comments to ask and say her comments – cause it's like she doesn't want to take what we have to say so we can't, like, say our own opinion and what we think is going on."

So, while Ms. Ellis did not frequently epistemologically frame the activity as understanding others' strategies, she did still frame for co-constructing mathematical explanations via the epistemological code of explaining own strategy. These are two different kinds of co-constructing mathematical explanations. The first is what Brad's excerpt above exemplified. The student was supported in co-constructing a more complete mathematical explanation of *his own* strategy by the teacher. In this sense, he was asked to be more articulate about his own thinking. While not represented in this excerpt, the code still applied when students were asking a student presenter about his/her strategy. Students asking students these kinds of questions fostered better mathematical communication on both sides of the interaction. The student asking the question had to be articulate about what it is that he/she did not fully understand from the original strategy sharing. The student presenter, then, had to either provide more detail or think about another way to say what he/she already said in presenting the strategy. This kind of mathematical communication about a student's strategy is mutually beneficial to those involved, and arguably to the broader classroom community.

This idea of building on other strategies and drawing connections among multiple strategies was what the IQA *Linking* rubrics addressed. So, my interpretation of this framing analysis for co-constructing mathematical explanations was connected to the lower *Linking* scores within these lessons on both the part of the teacher and her students. What the IQA *Asking* rubric scores did affirm was what was evidenced in the excerpt above. Ms. Ellis's request for Brad to provide more detail in his explanation would be counted toward her IQA score on the *Asking* rubric. Ms. Ellis scored the highest score of '4' on this rubric for all of the lessons in the sub sample. Further, she scored '4's' on that rubric in all of her lessons except Lesson 9. For that lesson, she still earned a '3.' This practice of hers, then, was consistent within this dataset.

In addition to the IQA scores as corroborating evidence of Ms. Ellis's lack of connecting students' strategies, the interviews pointed to the same issue. On three different occasions across Ms. Ellis's interviews, she talked about group work: setting up expectations around it very clearly in the beginning of the year and teaching students to respectfully disagree. What she did not emphasize as much was the mathematics within that and supporting one another to reach new mathematical understandings. She talked about group work more from a standpoint of helping one another out and sharing and proving strategies to one another. For communication, these are productive ways to talk support students, but they leave the mathematics and the mathematical goals of the lessons out of consideration.

4.3.3 Students' Alignment to Ms. Ellis's Framing for Co-Constructing Mathematical Explanations

Again, Ms. Ellis's students demonstrated alignment to her framings for co-construction, just as they did with her framing them as the authority. There was no evidence of misalignment to her framing for co-constructing mathematical explanations. Similar interactions to those described in the authority alignment section indicated alignment to her framing for co-constructing mathematical explanations: her students asked questions of one another; relied on each other for support in explaining their strategies at the board; and defended their strategies and approaches.

From Tables 10 and 11, you can see that students framed the activity as understanding others' strategies more often (14 times) than Ms. Ellis did (3 times). While I did not analyze Ms. Ellis's alignment to students' framings, it does seem as though her students were attempting to connect their strategies to one another's in some way. At least they were trying to understand each other's strategies, which would be the first step to connecting them and working toward achieving a broader goal. This could suggest that Ms. Ellis's lack of framing the classroom activity in this way contributed to the students' lack of progress toward the goal in the lessons, as noted above.

4.3.4 Ms. Ellis's Framing for Productive Struggle

Productive struggle was linked to the fewest framing codes in Table 12 (and Table 5) and was the least evident component of sense-making in this dataset. The teacher could have framed the mathematics activity for productive struggle in instances in which her framing was considered students explaining own strategy, understanding others' strategies, math is working together, and affectively. These framings were evident a total of 101 times across the three lessons, accounting for about 29% of Ms. Ellis's framings. While these framings had the potential to support students to engage in productive struggle, the extent to which they actually did engage in productive struggle would require further analyses. However, there is some evidence that aligns with the literature on productive struggle that the students did productively struggle somewhat.

In terms of framing, though, Ms. Ellis **epistemologically** and **socially framed** the activity for students to ask questions of one another and defend their approaches, which could each be linked to productive struggle. By asking questions of one another, the students were articulating what they did not know and supporting one another by explaining their strategies or approaches further (NCTM, 2014; Warshauer, 2015). Not only did Ms. Ellis invite students to productively struggle by asking questions of one another, she also did so in her affective framing. Ms. Ellis affectively framed the classroom activity for her students three times across the three lessons. Each time, she did so in a way that prepared students for the difficulty they should expect to encounter when solving the task. In addition to the example provided in Table 10, in Lesson 1 Ms. Ellis stated, "We'll probably get caught up on some hard ones." She was referring to difficulty they might encounter in folding their fraction strips for particular values. This affective framing about the challenge and difficulty the students might face with the tasks could have primed them for engaging in productive struggle. Her framing could have helped the students see that the tasks were not ones in which they were expected to memorize and apply information. They were going to have to figure some things out that were not immediately apparent.

4.3.4.1 Student Alignment to Ms. Ellis's Framing for Productive Struggle

As stated previously, the evidence of students' engagement in productive struggle was less apparent in this dataset. In terms of framing, Ms. Ellis's students did align with **explaining own** strategy; understanding others' strategies and math is working together, for the most part. Barring the misalignments that I discussed in the previous section, Ms. Ellis's students epistemologically aligned with Ms. Ellis's framing of the mathematical activity by coming up with their own strategies, explaining them, defending them, and questioning one another's strategies. They also aligned by not giving up or asking Ms. Ellis for support, instead they turned to one another to ask for help. These could all be versions of productive struggle, but I do not feel that I can make grand claims that they were engaged in productive struggle without some further evidence related to what the students understood after the 'Summarize' phase of the lessons.

There were a few comments in the student focus group interviews that alluded to productive struggle. The students had watched a clip in which Ms. Ellis was supporting an individual student at his seat. The student reacted by saying that "She's supporting him because she's, like, not giving up on him and not letting him give up." The student's interpretation was not that she was dumbing anything down for that student, rather it was that she was supporting him and helping him persevere through his confusion. This is a kind of productive struggle – not giving up. Another student said that she felt that Ms. Ellis "does want us to work hard." While 'work hard' could have a few different interpretations, one could imagine one of them involving productive struggle. Again, just as was the case with the students as the authority, the students cannot be expected to talk in terms of productive struggle, but some of what they did say could still support such an interpretation.

4.3.5 Students' Mathematics Contributions

Once I established that students were actually engaged in sense making activity, I turned to their mathematical contributions to see what it was that they were making sense of. I have already stated several times that the students were more concerned with one another's individual strategies than they were with connecting their strategies. To their credit, though, they did make attempts to understand one another's strategies at times. Those attempts, however, did not result in many substantive mathematical connections. In this section, I will provide more content-specific information about the substance of students' contributions. I organized this section by lesson because each lesson took on a different focus and concerned a different task. The tasks and accompanying curriculum-stated goals are in Appendix B.

4.3.5.1 Lesson 1: Creating Fractional Parts

In Lesson 1, Ms. Ellis's students were completing the CMP2 *Bits and Pieces I* task titled 'Folding Fraction Strips' (Lappan, 2009, p. 7). In the task, her students were asked to fold strips of paper to create halves, thirds, fourths, fifths, sixths, eighths, ninths, tenths, and twelfths. Kieren (1994) noted that folding strips of paper was a better approach to thinking about fractional parts than cutting a pie. These students' use of the linear representation was, then, more productive in developing their sense for rational numbers. Ms. Ellis's students oftentimes took the approach to creating their fractional parts that involved splitting (Confrey, 1994; Moss & Case, 1999). Confrey (1994) suggested that children intuitively understand splitting before they understand other concepts. She also suggested that fractional representations that are close to what students naturally tend to do – split – are better for supporting more conceptual understandings with fractions. So, these students' approaches aligned with what is stated in the literature.

In terms of what the students were making sense of during the 'Summarize' phase of Lesson 1, it was more about the factors of each of the denominators than it was about creating equal parts. They did use the verb "split" to explain their strategies. For example, Leona created twelfths by first splitting her strip in half. Then she knew there needed to be six equal parts on each side of her halfway mark. She then split her halves of the fraction strip in half again, creating fourths (though she did not articulate the fourths). Finally, she divided the fourths into thirds. Max, on the other hand, started to create twelfths by first folding his strip into thirds and then splitting the thirds in half to make sixths. Finally, he split his sixths in half to make twelfths. He said, "Because six times two is twelve." The other student presenters during the Lesson 1 'Summarize' discussion similarly either started to create their fractional parts by dividing their strip in half or into thirds and then halves. Myers, Confrey, Nguyen and Mojica (2009) noted that students typically begin with splitting in half and struggle with splitting into thirds. Eventually, though, they do develop a better sense for splitting into thirds.

While what Max did was the opposite of what Leona did, Ms. Ellis did not point this out. Neither did her students. Doing so would have supported the students in making sense of two related goals of the lesson – to explore the part-to-whole relationships and to investigate the different partitioning strategies. Additionally, the students did not discuss fifths, but it is possible that they did so in the following lesson as it was not recorded.

What was evident in the classroom video of students' presentations was that they were not supported in connecting their strategies to one another's. They merely individually presented each strategy and then moved on to the next one – a version of "show and tell" (Ball, 2001; Wood & Turner-Vorbeck, 2001). There were five different student presentations recorded as part of the 'Summarize' phase of Lesson 1. Notably, there was only one peer-to-peer question asked in Lesson 1 and it was coded as **understanding others' strategies**. The question was less about the strategy specifically and more about how the student created the fractional pieces by starting with halves. Therefore, students were actually not making sense of one another's strategies. All of the explaining own strategy codes were coded when the student presenter(s) was responding to Ms. Ellis's questions.

Not once did Ms. Ellis support students in talking about their different choices of strategies - that is their choice of what to divide the fraction strip into first, halves or thirds. She also did not support them to think about the part-to-whole aspect of the fractional pieces they were creating (Kieren, 1994). If she did so in the remainder of the discussion that was not filmed, then she still missed the opportunity to support students in building up to that understanding based on what they had done individually within their groups. Therefore, there was evidence that individual student presenters (and, at times, a supporting group member) were making sense of their own individual strategies only. There was no evidence to suggest that the students were making sense of one another's strategies by asking questions about them and they certainly did not show evidence of connecting their strategies to one another's or to the lessons' goals.

4.3.5.2 Lesson 5: Fractions in Between Fractions

The mathematics of Lesson 5 were more closely linked to multiple representations of rational numbers, namely fractions and percentages. In Lesson 5, students were completing the CMP2 *Bits and Pieces I* task titled, 'Fractions Between Fractions' (Lappan et al., 2009, p. 25). In that task, students were finding two fractions in between each given pair of fractions. Notably, the textbook instructed students to find *a* fraction between each pair, but Ms. Ellis changed the task to ask students to find *two* fractions. This was particularly significant in part d of the task. In so doing, she demonstrated that she did not blindly follow the curriculum, but she made adaptations as she saw fit. The pairs of fractions were as follows:

$$\text{a.} \frac{3}{10} \text{ and } \frac{7}{10}; \quad \text{b.} \frac{1}{5} \text{ and } \frac{2}{5}; \quad \text{c.} \frac{1}{8} \text{ and } \frac{1}{4}; \quad \text{and d.} \frac{1}{10} \text{ and } \frac{1}{9}.$$

Just as in Lesson 1, the student presenters in this lesson referred to "splitting" their fractions. Splitting in this problem scenario was more abstract in that the students were doing so symbolically rather than physically with strips of paper. They also used equivalent fractions, and relied on percentages to help them find fractional equivalents in between the two fractions. Namely, they often used benchmark percentages such as one fourth, one fifth, etc. These students' reliance on percentages was different from what Ms. Ellis experienced with her earlier class in the day, as evidenced in the content logs.

Moss and Case (1999) pointed out that, around the fifth or sixth grade, students become comfortable with the whole numbers through 100. The students' level of comfort with those whole numbers might be why they chose to work with percentages instead of the fractions, though this did not explain why her earlier class did not choose to do the same. While percentages are a kind of rational number, when they are written as percentages, students could likely reason with them in a way that was more familiar to them – as whole numbers because the denominator was disguised and was the same for each of their percentages. Lamon (2005) discussed student strategies when they operated with percentages. She demonstrated the ways in which they went from known fractions or percentages to unknown ones. Ms. Ellis's students' demonstrated doing this by relying on benchmark percentages and fractions and then thinking through how they could find the unknown fractional equivalent they were looking for.

The student presenters in Lesson 5, Molly, Wendy, Naomi, Mary, Lyla and Tiana, all used some form of splitting, equivalent fractions, or benchmark percentages or fractions to find their two fractions in between fractions. Their strategies were all effective in helping them find fractions in between the given fractions. In part (a) of the task, they either thought about all of the tenths in between three tenths and seven tenths or they thought about 30% and 70%. From the 30% and

70%, they then found known percentages like 66% and 50% and converted them back to known benchmark fractions. As for part (b), some students split the fifths into tenths and twentieths or they considered the percentage equivalents (20% and 40%). From the percentages, the students then used 25% and 30% and converted those percentages back into fractions. One student shared her strategy for part (c). She repeatedly split the fourths until she had created 32nds. She found 6/32 and then reduced that to 3/16 to find her second fraction. For part (d), Lyla and Tiana both used equivalent fractions. Lyla stated that she was using the least common multiple of ninety, while Tiana talked about multiplying by a version of one, namely two over two, to find her equivalent fraction.

The exception, as will be further discussed in the next section, was with Makayla and Keri's strategy. What they chose to do in their group was to use splitting, but they did so with percentages. So, they were using a hybrid of the intuitive practice of splitting with percentages (which, when used as whole numbers by ignoring the implicit denominator of 100, are also very familiar to students). In finding fractions between one fifth and two fifths, they looked at the percent equivalent to one fifth, 20% and divided it in half. They then added that resultant 10% to 20% to get 30%. From 30%, they realized the fraction was three tenths. So, while they stated 30% as a fraction in between one fifth and two fifths, they came to it in a much different way from what their peers did.

The other fraction they came up with, though, was one and five tenths fifths $\left(\frac{1.5}{5}\right)$. This fraction also had a percentage equivalence of 30%. The way they talked about coming to their decimal-fraction-hybrid was by splitting a fifth in half. The "splitting" they referred to, though, only applied to the numerator and not to the denominator. That is, their version of splitting was not consistent with the splitting that others discussed. It was, however, consistent with what they did above when they divided the 20% by two. In addition, they did fail to treat the fraction as a number. Instead, they treated it as two numbers – numerator and denominator. What made them successful in this case, though, was converting the one fifth to a percent. What they did was consider the continuity of the number line by seeing that there was some piece in between each of the fifths. They decided to only divide the numerator and not the denominator. What they did not explicitly state in that approach was that after they halved the one in the one fifth, they had to add that half back on to the fifth. So, they were likely looking at the numerators, one and two, and thinking that a decimal would fit in between there. Then, they tacked on the denominator at the end. Their stated reasoning, though, relied on the one fifth equivalent of 20%. Again, they were dealing with the number that seemed whole and familiar – 20. Later descriptions reveal where Keri particularly got her explanation wrong. But the mathematics of their approach was different from others' approaches.

As for sense making, the students in this lesson were asking one another questions about how they got their equivalent fractions or how they knew the fractional equivalent to certain percentages. What they were not explicitly making sense of was their chosen partitions and how they could find infinitely many partitions to come up with infinitely many fractions in between any pair of fractions. Makayla and Keri were discussing this idea, but in a way that was more aligned with considering the continuity of the number line. While Ms. Ellis made a concluding statement about the limitations of relying on percentages to find fractions between fractions, she did not hit on the fact that they could just keep finding smaller and smaller parts in between any two fractions.

In addition to not making sense of the broader mathematical goals, the students were not connecting their strategies to one another's. Two of the **understanding others' strategies** codes emerged when students questioned Keri's explanation of her strategy that was different from the others. Other such codes applied when students asked one another how they knew particular equivalent fractions or why they chose to partition into particular numbers. While these questions of one another's strategies show something about how the student was connecting what he/she did to what the student presenter had done, they did not materialize to explicit connections. Additionally, again, Ms. Ellis did not support such connections. If she had, student sense making of one another's strategies and of the mathematical goals of the day could have been more evident.

4.3.5.3 Lesson 6: Measuring Distance with Fractions Greater Than One

Lesson 6, as stated elsewhere, was the shortest lesson because the Lesson 5 'Summarize' phase ran so far into the Lesson 6. Nonetheless, in Lesson 6, students were completing the CMP2 *Bits and Pieces I* task titled, "Naming Fractions Greater Than One" (Lappan et al., 2009) The part of the 'Summarize' phase that was recorded was only focused on the first two parts of the problem. That is, they worked on the first two parts in their small groups and then discussed those before moving on to the remaining parts of the problem. In this lesson the problem was contextualized as a highway cleanup activity in which groups of students at a school each volunteered to clean ten miles of highway. The students then cleaned particular sections of highway. One group cleaned a section nine fourths miles long. Another group started at mile 2 and cleaned five thirds miles. The full task is in Appendix B. The lesson's goals were related to writing improper fractions as mixed numbers and seeing fractions as a way to measure lengths between whole numbers.

The mathematics in this task, while still focused on rational numbers, was a bit different in terms of students' approaches. They were looking for mixed number equivalents for given improper fractions. They were also linking those representations to measuring distance. Students' strategies in this lesson were all very similar to one another. The extent to which they differed was in whether they chose to represent their fractional pieces bigger than one on a single number line or on multiple number lines. The distinction in choosing a single number line over two separate number lines reveals information about how students were thinking about improper fractions. Using separate number lines, as was evident most of the time in this class, demonstrated that the students were not yet thinking about improper fractions as a single number (Hackenberg, 2007). Instead, they were seeing nine fourths as two separate wholes with one fourth remaining. The nine fourths section, then, needed to be recorded on three number lines. Their approaches also differed in how they chose to label their number lines. Whether they chose to start labeling at mile zero or mile two was the extent of the differences in this sense.

While the students' approaches seemed very similar, there was sense making work to be done. Ms. Ellis still could have supported students in thinking about the difference between nine fourths, two and one fourth, and nine separate fourths. She could have supported this further by drawing on the context and addressing measurement. Further, Ms. Ellis's students did not see the approaches as much different as evidenced in their questions. They were focused around why they chose to divide their number line into thirds when the given improper fraction was five thirds. The richness of the mathematics in this shortened piece of the task was limited. That was partly due to the fact that, at least in what was recorded, the students only had the chance to 'Summarize' parts A and B of the task. There was not a lot to discuss beyond representing the fractions greater than one on the number line and doing so with mixed numbers.

Still, the students and Ms. Ellis did not show evidence of connecting these strategies or relating them to one's own strategy. So, sense-making again was limited to thinking about how individual groups approached the problem. While the students were still the authority in the class and engaged in co-constructing mathematical explanations, it was not mathematically very sophisticated.

4.3.6 Framing for Sense Making Summary

Overall, Ms. Ellis did frame the classroom activity for students for sense making and her students aligned with such framings. She framed the activity a total of 180 times in productive epistemological ways and 70 times in productive social ways. (See Table 10.) Similarly, her students framed the activity epistemologically 184 times and 60 times socially. While the extent to which students engaged in sense making activity was less clear, particularly for productive struggle, they did take on the role of the authority in the classroom. They did so epistemically and socially. They regularly shared their ideas in front of the class, asked questions of one another, and defended their own approaches. They also engaged in co-constructing mathematical explanations, though their co-construction was fairly limited in scope to individual strategies as opposed to more broadly connecting across strategies. The exception to this limitation was evident in students' understanding others' strategies framings, which were more commonly coded for them than they were for Ms. Ellis (Fourteen times for students and only three for Ms. Ellis). That epistemological frames were the most common ones coded in this dataset suggested that Ms. Ellis was actively supporting her students to engage with one another and the task in ways that supported them in making sense of the mathematics with one another as opposed to being told what to do. These findings were corroborated throughout this section from various other data sources. The member-checking that was possible through the interviews, as well as with one of the original researchers, Sung, was quite helpful in confirming my interpretations and findings. In the next section, I go on to talk about less productive framings – the doing frame.

4.4 The Doing Frame – Framing Not for Sense Making

The teacher did not always frame for sense making. The most prominent example of this was when she invoked the doing frame – accounting for approximately 25% of her framing codes. The doing frame was coded most during the 'Launch' phase of the lesson with the exception of Lesson 1. In Lesson 1, the doing frame was coded three times during the 'Launch' and five times during the first student presenter's, Leona's, explanation. It is important to note that Ms. Pine invoked the doing frame four times during Leona's presentation. Other than that exception, Ms. Ellis's contributions were coded as doing mostly during the 'Launch'.

Another example of a typical time during which Ms. Ellis framed the lesson as doing was when a student demonstrated confusion at the board. This was not necessarily mathematical confusion, but confusion about transferring one's work to the board for the class to see and for the presenter to explain. For example, during one lesson a student had copied a number to the board incorrectly and then became lost in her own work. Both teachers, and some students, supported the student by telling her which number to change and where she had gone wrong. These kinds of confusing moments should be expected and do not suggest anything about Ms. Ellis's instructional practices. They also, likely, did not have an impact on students' engagement in sense-making activity since they occurred when students were merely copying their work onto the board.

In terms of when the doing frame was coded most, it happened in Lesson 5. Again, Lesson 5 was the longest lesson recorded, so this should not be surprising. At the same time, though, in comparison to the other two lessons in this dataset, Ms. Ellis disproportionately invoked the doing frame. She used it four times as much as she did in the first lesson and more than thirteen times as much as she did in Lesson 6.

Though the doing frame was used most in Lesson 5, the way in which Ms. Ellis invoked it, and the apparent reason for doing so, during one of the student's presentations revealed something about teacher practice. The interaction occurred when the third student presenter of the discussion, Keri, was sharing her group's solution to finding two fractions in between the fractions one fifth and two fifths. Ultimately, the two fractions that Keri's group came up with were three tenths (3/10) and one-and-five-tenths fifths (1.5/5). While both fractions were, in fact, in between one fifth and two fifths, where Keri went wrong was in her labeling of the work she had done and in her explanation. While Keri was presenting her group's solution, Ms. Pine was leading the discussion and Ms. Ellis was at the side white board giving one-on-one assistance to another student. Because the previous part of the problem asked students to find two fractions in between three tenths and seven tenths, Ms. Pine led into this student presentation by saying, "This one is a little bit more tricky." What she meant by this was that the fractions one fifth and two fifths were separated by only a fifth, so the students had to find two fractions that did not have a denominator of five to fit in between the two given fractions. Even the next closest common denominator that could be used to find equivalent fractions, ten, would have only yielded one possible fraction in between the two fractions. Ms. Ellis, though, had changed the question as it was stated in the textbook to require students to find *two* such fractions. The excerpt below begins to illustrate the interaction.

Excerpt 5

Keri: And then, uh, we know one fifth as a percent is, um, twenty, so- and we know half of twenty is ten and so we did twenty plus ten and that equals thirty. So we knew that one fifth, wait- one point five was equal to 30% and 30% is less than two fifths since we know two fifths is 40%. So one of our numbers on the line was one point five and then for our second one, we got three tenths.

In the excerpt above, Keri provided her initial explanation at the front of the room (lines 1-4). Her explanation was coded epistemologically as originator of ideas and socially as math is working together and teacher space. Epistemologically, she was sharing her group's strategy and socially, she attributed her work to the group by using the pronoun "we." She also voluntarily walked to the front of the board when Ms. Pine called on her, indicating her alignment with the teacher space frame. Not represented in the excerpt, Ms. Pine asked Keri how she got three tenths. Keri went on to ask Makayla for help in explaining where the three tenths came from. Makayla supported Keri by explaining their strategy. For Makayla, her contribution was coded as both explaining own strategy and math is working together. What Makayla went on to explain was in line with the other strategies that students in the class used. They relied on known benchmark fractions and their equivalent percentages ($\frac{1}{5} = 20\%$; $\frac{2}{5} = 40\%$). Therefore, 30% is between 20% and 40% and $30\% = \frac{3}{10}$. That is, instead of relying on equivalent fractions, the students converted their fractions to percentages. Excerpt 5 above provides Keri's initial explanation. The interaction continues in Excerpt 6.

Excerpt 6

Ms. Pine: Thank you. So does anyone have any questions or comments about the first, uh, way, thing Ms. Keri did up there because I think Makayla – or you could comment on what Makayla just said. But, uh, Mary?

Mary: Um, where did she get one and five tenths from?

Ms. Pine: Your one point five you have circled. You're saying one point five is what you would put on the line? So it was 30%? So Mary is asking where you got it.

After Makayla supported Keri with her explanation, Ms. Pine went on to ask for other questions or comments. In this instance, she did something that Ms. Ellis did not usually do and she highlighted "about the first, uh, way" (line 1 in Excerpt 6). In this sense, Ms. Pine was pointing to the error and seemingly trying to get students to ask a question about that part. To her credit, Ms. Ellis would typically just ask for any other questions or comments without pointing to the mistake. After Mary asked a question about that mistake (line 4 in Excerpt 6), Ms. Pine elaborated on Mary's question (lines 5-6). Mary's contribution was coded as understanding others' strategies because she was trying to understand how she came to the one and five tenths.

In this interaction (Excerpt 6), Ms. Pine called on Mary. Typically when Ms. Ellis was leading the discussion, she relied on the student presenter, Keri in this case, to call on peers from whom to take questions. Notably, then, Mary directed her question to Ms. Pine as opposed to Keri. Ms. Pine then took away Keri's authority. This, again, was atypical and a further implication of framing. Ms. Ellis's typical practice of framing student presenters as the social authority likely supported students in talking directly to one another as opposed to making their questions filter through the teacher. That Mary asked her question to Ms. Pine instead of to Keri is evidence to suggest that such framing matters. Elsewhere in the transcript, Mary, as well as most other students, asked questions directly to the student presenters. The interaction continues in Excerpt 7.

Excerpt 7

Keri: So, ok, like, ok, one fifth is equal to twenty and then we did divide in half-
Ms. Pine: Twenty what?
Keri: Oh, 20%.
Ms. Pine: Ok.
Keri: So then we did divide by two because we were trying to get half of one fifth and that's half a percent. And then we added them back together and that equals thirty. And then we know one- and one whole and five tenths is equal to 30%.
[Wendy shared how her group used tenths and twentieths to find their two fractions.

In Excerpt 7, Keri attempted to explain how her group divided the fifth in half to find a fraction in between one fifth and two fifths (Excerpt 7, lines 1, 3 and 5-7). What she introduced, though not entirely accurately, was the idea of fractions with decimal numerators. Her group used a different approach to finding a fractional equivalent to 30% than what Makayla previously explained to the class. Mathematically what the group was addressing was the continuity of the number line – a conceptual point that can be difficult for students to grasp as it differs greatly from the integer number representations on the number line in which the number line is not continuous, but discrete.

Excerpt 8

Ms. Pine: Ok, but I still have a question about this one-and-five tenths equaling 30%.
Ms. Ellis: I do too, I have had my hand up.
Ms. Pine: Does anybody have a question about that? Cole?
Cole: I don't have a question about that, but-
Ms. Ellis: I do, my hand has been up.
Ms. Pine: Oh, you did something else? Ok?
Cole: Yeah, if one and five-tenths is 30%, how did you not get thirty one hundredths (30/100)?
Ms. Ellis: Let me ask my question real quick, Keri, just because my hand has been up. I know when I'm talking about one whole, what's one whole as a percent?
Keri: Um-
Ms. Ellis: Since you were using percents, what's one whole as a percent?
Keri: One hundred.
Ms. Ellis: One hundred percent. So I think there might be some confusion with, um – you're saying one point five is a possible answer and Mary read that as one whole and five tenths, so if I'm over one whole, should I not be over 100%? And you're saying one whole and five tenths is 30%. Do you understand that?
Keri: Oh. So should it be fifteen – fifteen hundredths, like-?
Ms. Ellis: Well, I think that depends. We now have to take the fifths that you have and figure out can we make them hundredths

In the excerpt above is where Ms. Ellis violated the typical epistemological framing of the discussion up to that point, and of most of the other discussions in the dataset. In Excerpt 8, Ms. Pine initially redirected students' conversations back to the 1.5=30% (Excerpt 8, lines 1 & 3). This was after Wendy raised her hand and shared that her group used tenths and twentieths to come up with their two fractions. Notably, Wendy just talked directly passed the issue with Keri's explanation and privileged her own solution strategy. It is possible that she thought Keri was trying to use equivalent fractions, as Wendy's group did. Nonetheless, Wendy's insertion was off-track with what the others were discussing at that point.

Line 2 in this excerpt is where Ms. Ellis began to interject, including interrupting Cole in lines 5 and 8. Within this dataset, Ms. Ellis did not typically interrupt her students and she did not typically take it upon herself to correct students' thinking until after no more students in the class had questions to ask of the student presenter. This was corroborated in the sense-making framing section by student interview data and by my re-viewings of the videos. That is to say that many times when a student presenter had made a mistake, the students helped the presenter see and correct the mistake without needing Ms. Ellis to interject. Cole (in lines 4 & 7) was attempting to do so. It is important to note that Cole's question was relevant and could have been helpful for pointing out fractional equivalences. Ms. Ellis, though, completely ignored his question and privileged her own. Ms. Ellis framed the classroom activity at that point as doing by turning it into a conversation between her and Keri (Excerpt 8, line 8 to the end). In the extended interaction, though not fully represented here, two separate students were asking Keri questions about how she came to the answer of one point five equals 30%. This time, though, Ms. Ellis did not let that play out. Instead, she invoked the **doing** frame and began funneling Keri's thinking into a direction that she was not actually going in her explanation

Given Makayla's support for Keri in explaining how the group came up with three tenths at the beginning of Keri's presentation, it is reasonable to wonder why Ms. Ellis did not ask one of Keri's group members to support her in her explanation. After all, Keri had already exhibited that her understanding of the strategy was limited when she didn't know where the three tenths came from. Additionally, Ms. Ellis and her students typically did turn to group members for support when they got stuck in their explanations (as evidenced in the beginning of Keri's presentation when Makayla supported her). If Ms. Ellis had stuck to that same practice and even turned to one of Keri's group members herself to ask if they could support her, they could have quickly resolved the confusion and prevented a **doing** framing. At the end of Excerpt 8, Ms. Ellis went on to instruct students to pack up because class had ended. She then had a private conversation with Keri off to the side while students were packing up their belongings to go home. (See Excerpt 9.)

Excerpt 9

Ms. Ellis: You come up here. You're saying when you put equals 20 what does 20 mean?

Keri: 20%.

Ms. Ellis: Ok, well you should have specified that like that. Why did you take 20% and divide it by two?

Keri: Because, we did it because we were trying to like find out what-

Ms. Ellis: Half of this is?

Keri: Yeah.

Ms. Ellis: Then you would also take fifths – then why didn't you just take your fifths and split them in half? You're missing the zero which would kind of – if you're using percents it works that way. We all need to figure out where this came from. Do you understand why it's not 30%?

11 **Keri**: It came from one fifth out of five. [Writes 1.5/5 on the board.]

12 **Ms. Ellis**: Does one fifth mean one point five? Does two over one mean two point one?

13 **Keri**: No.

14 **Ms. Ellis**: Do you see what I'm saying?

15 **Keri**: Oh, ok.

Above is Ms. Ellis's private conversation with Keri (Excerpt 9). She continued funneling Keri's thinking (NCTM, 2014). The "20" to which Ms. Ellis was pointing was where Keri wrote that they divided twenty in half to get "half a percent." While "half a percent" is what Keri called it in her original explanation, what she meant was half of 20%, or 10%. Keri made a further mistake in her conversation with Ms. Ellis in line 11 when she said the 30% came from "one fifth out of five." What Keri meant to say was one and five tenths out of five, which was the fraction that her group used as their second one in between one fifth and two fifths. Keri actually wrote this on the board, even though she did not say it. Ms. Ellis interpreted Keri's statement to mean that she saw one fifth as the same as one point five (line 12). In line 14, Ms. Ellis asked if Keri saw what she was saying. Keri did not respond in the affirmative, instead, she said, "Oh, ok" (line 15). Her response indicated that she was not following Ms. Ellis's thinking.

The above interaction, Excerpt 9, demonstrated that Ms. Ellis was not listening to what Keri was explaining and she was genuinely confused. Specifically at line 11 when Keri said, "It came from one fifth out of five," Ms. Ellis completely missed that. True, Keri did not correctly say one-and-five tenths out of five, she was alluding to the denominator of five. That denominator of five was crucial to this misunderstanding. In this extended interaction, then, what harm was done by invoking the doing frame instead of listening to Keri or asking one of Keri's group members if they could support her explanation? First, Keri showed evidence of further confusion in the directly preceding excerpt when she said, "Oh, ok." She had shown similar evidence in Excerpt 8 when she said, “Oh. So should it be fifteen – fifteen hundredths?” What Ms. Ellis did in invoking the doing frame and funneling Keri’s thinking was confuse her further. Keri had some sense for what her group had done and she knew it dealt with one point five, dividing 20% by two, and 30%.

Not only did Ms. Ellis further confuse Keri, she encroached upon Keri’s potential for productive struggle, and the whole class’s opportunity to productively struggle with her. Warshauer (2015) noted that students can be engaging in productive struggle when they exhibit uncertainty in explaining something, as Keri was. That struggle, though, did not have the chance to become productive for Keri once Ms. Ellis took over her thinking. In addition, Cole’s question could have led to Keri asking Makayla for help. It may have also led to another student pointing out that there is more than one fractional equivalence for 30%, in fact infinitely many. The students in the class were very set on using known benchmark fractions and their equivalent percentages by that point in the discussion. Wendy’s contribution in which she stated that her group used tenths and twentieths was further evidence of the use of such benchmarks. What Keri’s group did was address a different mathematical point – the continuity of the number line. That there are infinitely many fractions, some containing decimals that the students could have found in between one fifth and two fifths, addressed the heart of the lesson. Ms. Ellis’s interjecting and taking over Keri’s thinking, though, may have influenced her students to see that approach as confusing or not as good as their own.

On the following day, again with the support of Keri’s group member Makayla, Ms. Ellis finally saw that the group’s fraction in between one fifth and two fifths was actually correct – 1.5/5. But Keri tried telling her that on the previous day privately, though not completely accurately.

During two other students' presentations in Lesson 5, Ms. Ellis and Ms. Pine invoked the **doing** frame quite a bit. Those students were presenting their ideas on finding a fraction between one tenth and one ninth. During Lyla's presentation, she became confused when trying to explain how her group found another fraction with a decimal numerator. What Lyla's explanation alluded to was nine-point-five ninetieths, though she did not articulate it very well in her explanation. Lyla became confused as she tried to come up with the equivalent fractions with denominators of ninety. Ms. Ellis and Ms. Pine invoked the **doing** frame quite a bit with her to support her through some writing mistakes at the board. Lyla seemingly became flustered and the teachers supported her explanation by asking several simple questions that required a one-word answer. For example, Lyla was saying point two ninetieths as an answer when discussing dividing by two when she actually meant zero-point-five ninetieths. The teachers' contributions were coded as the **doing** frame twenty-four times during Lyla's time at the board. Lyla aligned with their framing at that time as well.

Tiana presented next, answering Ms. Ellis's question of, "What would you do if I said, 'no decimals in your numerator'?" Tiana had the idea of using a common denominator of 180 to find equivalent fractions that fit between one tenth and one ninth. She struggled some through the explanation. For that reason, Ms. Ellis and Ms. Pine both invoked the **doing** frame again to support her in coming to an answer. Their contributions were coded fourteen times as **doing** and Tiana demonstrated alignment throughout.

Ms. Ellis's students' difficulty with these explanations demonstrated the limitation of the percent-equivalent strategy that students shared prior to Keri's presentation. Even the equivalent fraction strategy was not always helpful. These interactions further point to the importance of connecting students' individual strategies to the broader mathematical goals and to one another's.

If Ms. Ellis had been framing for collective co-construction of the mathematical topics of the day, her students might not have exhibited as much difficulty in their explanations with the pairs of fractions that were closer in size. In relation to the day's mathematical goal, Ms. Ellis made one concluding statement to the class in Excerpt 10.

Excerpt 10

> "Percents can't always take you the whole way because you don't know. Is it a good strategy? Yes, I think to a certain point though because a lot of you wouldn't have known what to do for 1/9 and 1/10 as a percent, yes. Sarah's strategy of cutting apart the number line and using equivalent fractions works for this one [one tenth and one ninth] more than percents does. Um, we're going to start [the next lesson]- Any questions?"

Mathematically, her concluding statement privileged the equivalent fraction strategy over relying on percentages to find values that fit between two given fractions. What her statement did not do, however, was emphasize the fact that the students could find as many equivalent fractions as they wanted in between any two fractions. Her "Launch" in Lesson 5 was alluding to that. She said, "Fractions work differently than whole numbers. Um, for example, if you talk about the numbers one and two, pretend this is my number line. Can you find any other whole numbers in between them?" After some student discussion, she went on to say, "No matter what fractions we're given – are we always going to be able to find another fraction between them?" With these questions, Ms. Ellis planted a seed for one of the lesson's goals – that smaller partitions can always be used to find another fraction in between two given fractions. More broadly, this goal was hinting at the continuity of the number line. What she did not do throughout the lesson, and only very briefly touched on in her concluding statement, was to support students in relating their individual strategies to that goal. Because they weren't supported throughout the lesson, they had to rely on her quite a bit to think through what that meant, specifically evidenced in Tiana and Lyla's presentations.

4.5 Rigor

I attended to rigor throughout this section by triangulating with multiple data sources – as outlined in Table 8. Those data sources included consulting with those who were studied (via interviews and the artifact binder), an original researcher (via conversation), and by consensus-coding the lessons using the IQA Toolkit. I read through Ms. Ellis's interview transcripts from before and after filming, students' focus group transcripts, Ms. Ellis's binder of classroom artifacts with her thoughts around them, and the original researchers' content logs by lesson. I also re-viewed the data to look for conflicting and supporting interpretations of what I found via my analysis (Erickson, 2006). I spent so much time with Lessons 1, 5 and 6 that it was important to revisit the broader perspective that I had on this classroom at one time by watching all of the videos. I did so with my summarized findings in mind so that I could see if there were alternate ways of viewing what I thought the data was showing.

While I did not systematically analyze these other data sources, I reviewed them with my findings in mind. Doing so may have affected the ways in which I interpreted the information they provided. With that being said, I tried to include the direct quotations here, and elsewhere in the discussion, to be as transparent as possible about what information was in the other data sources.

Additionally, as framing is interpretive. I grounded my claims about students' framings in their explicit verbal statements. I also transcribed all of the whole-class dialogue during the three lessons in my reduced sample. This increased the rigor in making determinations about Ms. Ellis's framing in that I was less likely to miss explicit messages. Relatedly, by simultaneously analyzing the mathematical component of the interactions based on prior research in mathematics education, I decreased the degree to which my own inferences affected the analysis.

5.0 Discussion & Conclusions

Supporting students to engage meaningfully with mathematics, and in all of their studies, is a topic that gets a lot of attention in education. Much emphasis has been placed on the need for better thinkers in today's economy. The CCSSM is one effort that highlights that need by including the Standards for Mathematical Practice. Through educational experiences, particularly in mathematics, students can be invited to engage in thinking for themselves and making sense of problems on their own. Researchers in science education have pointed out, not only the need for a shift to more meaningful classroom experiences, but a way to talk about engaging in authentic scientific practice instead of merely "doing" school science (Jiménez-Aleixandre et al., 2000). They talk about it in terms of framing with emphasis on epistemological framing that supports students in sharing the epistemic and social authority with their teachers (Berland & Hammer, 2012a; Hammer et al., 2005; Redish, 2004). When students are able to be an authority over the knowledge and social routines , they are better supported in becoming creative and independent thinkers.

Informed by this prior research, I set out to answer the following research questions:

1) In what ways, if any, does the teacher frame whole-class discussions for sense-making activity (i.e., framing students as the authority or framing the activity as working together, among others)?
2) To what extent, or in what ways, do the teacher's students align with her framing?
    a. Which students, if any, respond in ways that align with the teacher's framing?
3) If students do align with the teacher's framing for sense-making activity, what mathematically are they making sense of?

This study contributed to the literature on framing in classrooms, as well as understanding whole class discussions by examining the framing within a middle school mathematics classroom. The analysis revealed that students were engaging in more than just "doing the lesson," they were making sense of the mathematics of study (by being the authority, co-constructing mathematical explanations, and engaging in some productive struggle) (Jiménez-Aleixandre et al., 2000). Not only were they engaging in sense-making activity, that activity could be attributed to the teacher's work of framing the classroom activity as such (Berland & Hammer, 2012a). In addition, less productive, more "doing the lesson"-like framings were linked to the teacher's framing as well. The findings suggest that the teacher's framing mattered for the ways in which her students engaged in mathematics. They also suggested that a more nuanced view of the work of teaching that extends beyond considering instructional practices broadly can reveal meaningful details about the ways in which teachers can support their students to productively engage with the content and one another.

5.1 Discussion of Findings

5.1.1 Ms. Ellis's Framing Mattered

This study revealed that teacher framing mattered for students. Ms. Ellis's students aligned with her framing more than 98% of the time. All students who verbally contributed across the dataset demonstrated alignment to Ms. Ellis's framing at some point in the lessons. Ms. Ellis repeatedly framed the lesson in particular ways. Specifically, she framed the lesson most often epistemologically, socially, and as "doing" These three framings were mapped onto the previously-defined components of sense-making activity: students as the authority; students co-constructing mathematical explanations; and productive struggle. (See Table 5.) In addition, when students failed to align with Ms. Ellis's framing, she put forth effort to reinforce her epistemological framing to support their alignment to her expectations.

In the beginning of the Lesson 1 'Summarize' phase of the lesson, for example, Leona demonstrated that she was framing the discussion as **doing** instead of **epistemologically** and **socially** as a time to share her group's strategy for folding the fraction strip into twelfths. (See Excerpt 3.) It is possible that she did so because she was the first student to present on the first day of video recording available for this secondary analysis. Either way that Ms. Ellis and Ms. Pine both reinforced the epistemological framing and insisted that she explain her thinking/strategy could have also sent messages to future student presenters about what was expected of them when they volunteered.

Students demonstrated alignment to Ms. Ellis's framing throughout the rest of the Lesson 1 discussion. The implications from this instance of student misalignment are that Ms. Ellis consistently did the work of framing the activity epistemologically and she did not consent to Leona's misalignment. Further, the fact that Ms. Ellis's students aligned with her framing throughout the rest of the Lesson 1 discussion suggested that her students saw, through Leona's interaction, that the discussion was not going to be "doing the lesson." Instead, it was expected that students would share their strategies and defend them. Thus, Ms. Ellis's reinforcement of the epistemological framing for Leona could have supported the rest of the class to align with the teacher. In other instances, when other students in Ms. Ellis's class attempted to frame the activity as **doing** during the 'Launch,' Ms. Ellis again reinforced the epistemological frame. Other students could have recognized that switching to the **doing** frame and asking for clues or relying heavily on the teacher's explanations was not going to work. Therefore, maybe they were discouraged from framing the lesson as "doing." In this way, Ms. Ellis was able to maintain the cognitive demand of the tasks that her students were working with (Henningsen & Stein, 1996; Stein et al., 1996; Stein et al., 2000).

An example from when Ms. Pine, the assistant teacher, framed the classroom activity as "doing" helps to further illustrate the effect of framing in the classroom. In Excerpt 6, when Keri was presenting at the front of the room, Ms. Pine shifted the social framing. Instead of letting Keri choose who would ask her the next question, Ms. Pine selected Mary to ask the question about Keri's strategy. Because Ms. Pine reframed herself to be the social authority in that moment, she was framing the discussion at that point as doing. In response, Mary went on to direct her question to Ms. Pine instead of to Keri, the student presenter at the front of the room. Typically when Ms. Ellis was leading the discussions, she left that social authority up to the student presenter. In those instances, the student asking a question asked it directly to the student presenter. This subtle shift in Ms. Pine's social framing of the classroom activity affected the subsequent interaction. What it ultimately meant in that interaction was that Ms. Pine went on to rephrase Mary's question for Keri. The talk, then, went through the teacher instead of being student-to-student.

More generally, each time the teacher (either Ms. Pine or Ms. Ellis) framed the classroom activity as doing, students immediately aligned to that framing. Granted, students demonstrated immediate alignment most of the time, even for the epistemological and social framings. After all, there were only six misaligned frames across the sample. Still, the students consistently immediately aligned with the doing frame. This suggested that students' default framing was of doing. It also further highlighted the power a teacher's framing can have over the ways in which students interact in the classroom – particularly with respect to sense making.

While Ms. Ellis rarely expansively framed the classroom activity (only 6 times), the students expansively framed the skills they were learning in her class – specifically in relation to their future careers during the focus group interviews. They did so on three different occasions in the interviews. In reading through those interviews to triangulate my microanalysis findings, I found that the students talked about Ms. Ellis asking a student to get back on task. They said she did it "so the group [could] get back to work so they get a better job or use it in their everyday lives." Another student referenced the group work expectations and norms and stated that, "It's teaching us to learn how to work with different people…because when you get older you're gonna work with a lot of different people." These statements, among others, suggested that Ms. Ellis must have expansively framed these social skills at some point for her students (even though this was not observed in the three videotaped lessons). The fact that they saw their activity in her classroom, at least socially, as being relevant to their future lives is meaningful. Their framing in the focus group interviews indicated that they saw value in their class's social arrangement beyond completing mathematical tasks and helping one another. The context, for them, was expanded to their future selves, their future careers, and their interpersonal skills (Engle, 2006). It might also be related to what Wallace, Sung and Williams (2014) examined with respect to fostering cultures of respect in this classroom.

Prior research has addressed the significance of expansive framing. Engle (2006) and Engle et al. (2012) talked about how expansively framing students' in-class learning and activities could support them to take their learning and use it in other settings. More recently within mathematics education, Selling (2016) named expansive framing as one of eight instructional practices that could support students to engage meaningfully with mathematics. What Ms. Ellis's students showed in their interview data was that expansively framed skills were relevant to them. That is, they remembered this aspect of their work even when they were not asked specifically about it. This suggested, in concert with the more general finding that teacher framing mattered, that expansive framing had an impact on students and should be used in instruction more often.

5.1.2 Ms. Ellis's Framing for Sense Making

Ms. Ellis framed the classroom activity for sense-making activity a great deal throughout the lessons. (See Table 12 for the links between framing codes and sense-making activity.) Most evident was her framing of students as the authority. Second most often, she framed the activity for co-constructing mathematical explanations. However, productive struggle was the least frequent framing used in these three lessons. Ms. Ellis's students also demonstrated alignment to such framing for sense making by being the authority in the room, co-constructing mathematical explanations, and, to a lesser extent, engaging in productive struggle.

Her students demonstrated that they took on the role of the authority in the classroom in more ways than one. They were the social and epistemic authority in the room, suggesting that what they were engaged in was more than "doing the lesson" (Berland & Hammer, 2012a). Her students moved into typical teacher spaces without being prompted to do so (both discursively and spatially). They also questioned one another's strategies, reasoning, and choices. Ms. Ellis rarely needed to tell the student presenters when there was a flaw in their explanations or when they were not quite complete. Her students did this for each other. Further, Ms. Ellis's students framed the classroom activity as understanding others' strategies more than she did. Because that framing code was about a student trying to better understand another student's solution strategy, its presence in this dataset (14 times) suggested that students were interested in one another's strategies. That is, the fact that Ms. Ellis did not frame the activity in that way very much (3 times), but her students still did it suggested that her students were not asking those questions to please their teacher. Instead, they seemed to be asking them out of curiosity and not out of expectation. Granted, the number of codes is not huge, so the claims I can make are limited. However, they do signal some significance. The relation between the understanding others' strategies codes for Ms. Ellis and for her students further supported the claim that they were not engaged in "doing the lesson" (Berland & Hammer, 2012a).

Relatedly, though, Ms. Ellis's lack of framing as understanding others' strategies could be one explanation for the students' limited sense making with respect to co-constructing mathematical explanations and engaging in productive struggle. While Ms. Ellis did frame the classroom activity for a specific version of co-constructing mathematical explanations and for productive struggle, mathematically, there was limited evidence of students making progress toward the learning goals. They were not making sense of one another's strategies in relation to their own nor were they doing so in relation to the broader lessons' goals.

As related to co-constructing mathematical explanations, the students recognized that putting ideas together was beneficial to everyone in the group or class. Just as was the case in Ms. Ellis's interviews, they did not talk about co-constructing the mathematical explanations. They focused more on coming to new understandings about strategies and not in relation to some broader, overarching goal or idea. The students talked about putting ideas together, possibly disagreeing, but ultimately that the communication would support all participants in coming to an agreement on some new idea. One student also mentioned the benefit of multiple perspectives. That student said, "If you're working independently, you could, like, you could understand something but it could be wrong, so you would want other people to help you." This student pointed out that even if a student believes in his/her own strategy, their peers might help him/her see a flaw in it. Each of these quotes support Ms. Ellis's emphasis on group work, on co-constructing ideas and explanations, and the benefits of doing so.

Just as Ms. Ellis highlighted her focus on establishing group work norms in the beginning of the year, her students did as well. They referenced an activity in which they observed four students engaging in group work, took notes on the strengths and weaknesses within it and then established group work norms based on that activity. Her students were involved in creating those norms and the activity stuck with them from the beginning of the year.

That activity may have supported students in viewing their own role as being responsible for helping their peers. Socially, this seemed to have implications. In their focus group interviews, one student stated, "I think that people more like accept you if you don't understand something because they'll try to help you, too. Like, if somebody raises their hand and says, 'I don't get this', other people will try to tell them what to do." Another student referenced a stigma attached to asking for help. That student suggested that asking for help did not cause peers in the class to think a person was dumb or not paying attention. It is important to note that the student stated, "I think that not in our classroom." That she emphasized 'our classroom' suggested that the entire school may not have these same norms and social acceptance of students helping one another and revealing their own misunderstandings. This is another example in which students could be making such a comment based on the work Ms. Ellis did in establishing a culture of respect – the work on which Wallace, Sung and Williams focused. Nonetheless, Ms. Ellis framed the activity such that students were supporting one another, which could also be a contributing factor to this stance

Ms. Ellis was framing for co-constructing mathematical explanations and productive struggle in the sense that she encouraged her students to ask questions of one another and to defend their approaches. Her students, in turn, did formulate more complete mathematical explanations to explain their own solution strategies. Ms. Ellis's students clearly engaged in co-constructing mathematical explanations around particular solution strategies. They asked questions about each other's approaches and formed more complete explanations about them. However, the students did not show much evidence of co-constructing broader mathematical explanations for the underlying mathematical concepts within their approaches. That is, they did not show much evidence of connecting their approaches or of connecting their work to the mathematical goals of the lessons. In terms of productive struggle, they were able to articulate what it was that they did not quite follow in a student presenter's explanation – a defining feature of productive struggle (NCTM, 2014). Her students also asked their group members for help when they were confused, including when they were at the board presenting. Instead of admitting defeat and retreating to their seats, they relied on their classmates to support them.

While her students were engaging in those sense making components in that sense, they were not doing so in relation to one another. They were not drawing connections between strategies, considering the affordances or constraints of a particular strategy, or linking their individual strategies to broader mathematical ideas or concepts. Had Ms. Ellis framed the classroom activity in these ways more consistently, it is possible that the sense making components would have been more evident. It is also possible that there would have been evidence of the students making mathematical progress throughout the lessons toward the goals.

While the entire 'Summarize' phase of Lessons 1 and 6 were not recorded, Ms. Ellis still missed opportunities to support students in making progress toward those lesson goals. For example, in Lesson 6 when students made choices about representing the improper fractions on one or more number lines, Ms. Ellis could have pointed out that they were considering the structure of improper fractions in different ways. Some students viewed the fractions as a single number, while other students thought about the improper fractions in terms of mixed numbers. Those students representing their improper fractions on separate number lines demonstrated that they were treating the improper fraction as a whole number and a fraction – two separate parts (Hackenberg, 2007). Ms. Ellis could have asked questions related to the students' choice in representation. She also could have asked students who used opposing representations to explain one another's (Webb et al., 2014). Students, then, might have not only engaged in co-constructing mathematical explanations across strategies, but they also could have collectively engaged in productive struggle.

While fewer framings of Ms. Ellis's were linked to productive struggle, it did seem to be an implicit expectation of hers. At one point in her interview, Ms. Ellis said, "See, you did do it. You thought of that on your own." She alluded to some productive struggle in which a student engaged. Her statement suggested that both a student felt that he/she could not complete a task and that she did not give in to providing clues or a procedure for the student to follow. That she talked about congratulating a student on doing something on his/her own is possibly indicative of some kind of productive struggle. And when she said, "Can you investigate and come up with a strategy you feel is efficient and one that works all the time, but yet you understand it?" she was also referring to students doing some work to figure something out. Not only did she allude to students figuring something out, she pointed out that they also would understand whatever it was. Again, this could be a kind of productive struggle, depending on the context, of course.

In terms of the other data sources, Sung's (2018) study and my conversations with her, suggested that the students were displaying what she termed 'uncertainty' during the 'Explore' phase of the lesson when they were working together. Hannah felt that Ms. Ellis's students were productively struggling in their small groups to come to new understandings about the group's strategy. In my re-viewings, I noticed this as well. The students asked a lot more questions of one another during that time. While this could be indicative of productive struggle, it raised some concerns as well. If the 'Explore' phase of the lesson was, in fact, used by students to understand their own strategies, then they were prepared to think more broadly during the 'Summarize' phase of the lesson about how their strategies connected to others' and to the broader mathematical goals. After all, their framing of understanding others' strategies was evidence that they were at a place where they were trying to make sense of what others did. Ms. Ellis, however, did not capitalize on that, nor did she support that framing very often – at least not in what was recorded.

So, Ms. Ellis's classroom was structured for productive struggle in various ways. During the 'Explore' phase, students could grapple with ideas and come to some understanding about the problem and an approach to it. Then, during the 'Summarize' phase, they could have the opportunity to collectively struggle in connecting with they had learned during the 'Explore' phase to the broader goal. Nevertheless, that was not what happened. Ms. Ellis also showed a disposition toward productive struggle in selecting high cognitive demand tasks, maintaining that cognitive demand and encouraging students to ask questions of one another. She even affectively framed for productive struggle by warning students of difficulty she expected they would encounter with particular problems. In her interviews, she demonstrated further evidence of placing value on productive struggle. That is to say, Ms. Ellis structured her classroom and the activity within it in ways that could promote productive struggle. The extent to which that struggle was evident, though, was limited.

Again, Ms. Ellis could have actively promoted productive struggle more by asking students to compare their approaches to one another's or to describe how their own strategy related to another student's (Webb et al., 2014). She also could have asked questions that targeted the mathematical goals. For example, in Lesson 5 when she finally realized Keri's answer of 1.5/5 was correct, Ms. Ellis could have asked questions related to it and the continuity of the number line. She could have asked, "What about 1.1/5?" Instead, she decided to ask students to consider a fraction that did not involve a decimal. In addition, given that her students in that lesson preferred to use percentages to find fractions in between fractions, she could have asked questions related to non-benchmark fractions. For example, "Would 31% or thirty-one hundredths be between one fifth and two fifths?" These questions are examples of ways in which Ms. Ellis could have asked her students to relate what their strategies were to what the lesson's goals were. Doing so would have required them to think outside of their preferred solution strategies and think more broadly about the mathematics within it. In turn, that extended thinking could have promoted collective productive struggle.

Regardless of how Ms. Ellis's students entered her classroom at the beginning of the school year, she continued to frame her students as the authority and to emphasize complete explanations and understanding. She reinforced epistemological framings to support her students in being the authority within the mathematics. Their alignment to her framing by providing explanations, defending their approaches, and questioning others' approaches suggested that they were on the same page as her in that respect. Additionally, the evidence from both Ms. Ellis's interviews and her students' focus group interviews showed that they all saw value in pursuing multiple solution paths, sharing those with one another and understanding and supporting one another.

It is important to note that Ms. Ellis demonstrated through her interview and artifact binder that she had a particular stance toward mathematical learning. She even noted her own shift from being hesitant to use the CMP2 curriculum. She noted struggling with it during the first year that she implemented it, partly because she came from "skill and drill." She went on, then, to value students' approaches, their ways of viewing problems and the ways in which they could support one another by sharing their own understandings. This view in itself points to her own epistemological shift that likely positioned her to support students to engage in sense making (Heyd-Metzuyanim et al., 2018; Watkins et al., 2017). She stated in interviews that she saw students' ideas as a central focus in her lessons. Further, she directed students when necessary to "Give [the student presenter] the attention she deserves" when they were not listening to her. She also asked questions of them just as she expected her students to. There was a lot socially that she did that consistently supported a sense making stance as opposed to one of "doing the lesson."

The fact that Ms. Ellis noted her own shift with the curriculum, and alluded to an epistemological shift within the mathematics more generally, suggested that she was uniquely positioned to support her students to engage in a similar shift. Had Ms. Ellis failed to see the value in students sharing their approaches or in talking to one another instead of asking her for help, the extent to which she could have framed the activity for sense making may have been more limited. If she had not viewed her students as capable of supporting and learning from one another, she likely would not have promoted it. Further, her students similarly pointed out the value they placed on seeing others' strategies and in working together to come to new understandings. That they seemed to align epistemologically with her suggested that she was successful in supporting such a shift.

5.1.3 Ms. Ellis's Framing Not for Sense Making

While Ms. Ellis framed for sense making in her classroom, she also framed in less productive ways. She did so particularly during times of confusion and during the 'Launch.' This aligns with the literature. In their co-planning session with a teacher, Heyd-Metzuyanim et al.

(2018) alluded to **doing** frames to begin the lesson. They made use of a beginning task for students to complete in order to better set them up for success during later portions of the lesson. Jackson et al. (2012) researched the 'Launch' phase of the lessons and found that when teachers did some work to establish the context of the problem (both mathematically and contextually), students were better positioned to be successful with the tasks on which they were working. Ms. Ellis's **doing** framing during the 'Launch' was oftentimes about soliciting students' prior knowledge or asking them what they knew about a given context. Therefore, her **doing** framing likely did not take away from student sense making during the 'Launch'.

However, the same cannot be said for the times during the 'Summarize' that Ms. Ellis framed the lesson as **doing**. In Excerpt 8 above with Keri and her approach to finding a fraction between one fifth and two fifths, Ms. Ellis seemed to invoke the **doing** frame out of her own confusion. It appeared to me that she was not sure why Keri had written 1.5=30% on the board. Her confusion was further evident in her private conversation with Keri after class, Excerpt 9. Instead of asking one of Keri's group members if they could support her explanation (as Ms. Ellis did in the following day's lesson), she invoked the **doing** frame. In so doing, Ms. Ellis took over Keri's thinking and also interrupted and ignored another student, Cole's, contribution. In this sense, she took away an opportunity for one of Keri's group members to help her, for Keri to make sense of Cole's question, and for the whole class to think about how Keri might have come to that equivalence. Ms. Ellis's **doing** framing turned the 'Summarize' phase of the lesson into a conversation between her and Keri.

In addition, at the end of that lesson, with Tiana and Lyla, Ms. Ellis again invoked the **doing** framing. As Tiana and Lyla attempted to find a fraction in between one tenth and one ninth, Ms. Ellis took over their thinking as well. She turned the interaction into a series of teacher questions that required one-word answers from the student presenters. In this instance, again, Ms. Ellis's framing limited the extent to which class members could make sense of the challenging problem. It is reasonable to suspect that if Ms. Ellis had been supporting the entire class throughout Lesson 5 to co-construct mathematical explanations that related to one another's strategies and to the lesson's goals, that the students would not have needed so much support when it came to the final problem of the day. That is, they may have developed a more complete understanding of the continuity of the number line, and strategies that did not involve converting fractions to percent equivalents by that point in the lesson.

The power of Ms. Ellis's framing exhibited in her doing frame incident in Lesson 5 supported the above claim. Ms. Ellis was able to convince Keri that her strategy was not quite correct and that her thinking was incorrect. In fact, Keri was not incorrect, she was just having some trouble communicating her strategy. In failing to listen to Keri, even in her private conversation with her, Ms. Ellis was able to convince Keri that her thinking about the decimal was wrong. (See Excerpt 9.) She also took away Keri's chance to productively struggle through the explanation.

It is important to note that coding alone did not reveal these anomalies in the Lesson 5 'Summarize' phase. During this student, Keri's, presentation, Ms. Ellis's framing was coded as doing only four times. It was the context and substance of this interaction that made it stand out as an unusual instance. Because I did not consider Ms. Ellis's alignment to her students' framings and I did not systematically look for Ms. Ellis's misalignment to her own framings, I relied on the context to make this determination. This is a limitation of my data analysis process, but at the same time, it suggests a line of future inquiry. Berland & Hammer (2012a) noted at least one point in their analysis during which the teacher realigned with students' more productive framings to allow more arguing to take place with respect to the topic of study.

During these three interactions (Keri's Tiana's and Lyla's), there was evidence of some confusion and the **doing** frame was the teacher's response to that confusion – taking over the students' thinking and supporting their explanations by asking several one-word-answer questions. Because moments of confusion are one time during which students have the opportunity to engage in productive struggle, Ms. Ellis inhibited that aspect of sense making. However, in her perspective they did serve a purpose. Being that the lesson had run quite long by that point (well into the second day), Ms. Ellis possibly felt pressure to wrap up the lesson and move on. She also could have felt that her students needed support to make it through those explanations dealing with fractions for which percent-equivalents were less helpful. While not productive in terms of sense making, the **doing** frame did do something for moving the lesson toward its endpoint. That is to say, Ms. Ellis's actions were reasonable in a practical sense within the constraints of the class.

The implications of these less productive **doing** frames can be linked back to the literature as well. Ms. Ellis, a trained elementary school teacher, could have limited content knowledge in mathematics, even though she had taught sixth grade math for a few years at the time of data collection (Ball, Thames & Phelps, 2008). For example, maybe Ms. Ellis did not interpret the lesson's goal of using smaller partitions to find fractions in between fractions as being linked to the continuity of the number line. She could have just seen it as a more challenging problem because finding fractional equivalents with denominators that were the least common denominator did not result in *two* obvious fractions. If she did have that math-specific knowledge, maybe she would have recognized Keri's decimal-fraction hybrid more readily. The kinds of mathematical knowledge that teachers need to have to teach their students is different from pure knowledge of mathematics. Ball et al. (2008) pointed out that "teachers need to know mathematics in ways useful for, among other things, making mathematical sense of student work and choosing powerful ways of representing the subject so that it is understandable to students" (p. 404). Ms. Ellis exhibited difficulty in making sense of Keri's work. This was partly because Keri did not represent it entirely correctly, but she was not showing evidence of thinking about her work in terms of a common denominator of twenty. Ms. Ellis was trying to push her in that direction, though, possibly because that was a strategy that she anticipated students using.

What Ms. Ellis anticipated brings up another point from the literature. Ms. Ellis was using high cognitive demand tasks and such tasks are associated with "unique and unanticipated" solution strategies (Stein et al., 2008). Stein et al. (2008) noted this, but proposed an instructional strategy that would help teachers better support students' engagement with and making sense of such tasks, specifically during whole-class discussions. In fact, they proposed five related practices. *Anticipating* students' solution paths was the first of them, as well as thinking about supports to help students reach new understandings based on those solution paths. It seemed, in this interaction, that Ms. Ellis did not anticipate the solution of 1.5/5 as a fraction in between one fifth and two fifths. She actually commented to one of the original researchers (recorded in a content log) that the class she taught earlier in the day had used equivalent fractions instead of percentages to find their fractions in between fractions. Therefore, these students caught her off guard a bit. In the same note, she remarked at how much longer this class took to complete the task than her earlier class did. This additional contextual piece revealed that Ms. Ellis was: a) expecting the lesson to go faster and b) expecting students to use equivalent fractions instead of percentages. This evidence further suggested that Ms. Ellis did not anticipate Keri's strategy or solution. Of course teachers cannot be expected to anticipate any and all possible solution paths that their students might take in solving tasks. However, maybe Ms. Ellis could have been better prepared to deal with such unexpected strategies than to shift to the doing frame and take over student thinking.

To Ms. Ellis's credit, in her interview data she did allude to anticipating students' solution strategies. She stated, "Oh, the students are going to understand this and this is where they're gonna go with it. And then sometimes in come students who throw that idea out the window and you have to adapt quickly." She also mentioned the amount of planning and preparation that goes into teaching. Even though she had taught sixth grade math for a few years at that point, she still remarked about the amount of time she had to put into planning for her students to be successful. This evidence illustrated that Ms. Ellis was not a teacher who just needed to be told about Stein et al.'s (2008) *Five Practices*, though she showed that she could benefit from learning more about them, specifically *connecting*. She also did not need to merely be told not to funnel her students' thinking; she showed evidence of skill in that area quite often. What Ms. Ellis did need was to be better equipped with how to deal with the unexpected – something every teacher can expect to encounter. Part of the answer to that could rest in more content knowledge, or more content knowledge for teaching, but it might even extend beyond that (Ball et al., 2008).

While Ms. Ellis's framing in those instances infringed upon students' opportunities to make sense of the mathematics, she may have had other reasons for invoking the doing frame. The content logs revealed that Ms. Ellis's earlier class in the day had all used equivalent fractions throughout that task and made a lot more progress in the lesson. She also noted how long the recorded class was taking to get through the lesson. It is possible that Ms. Ellis felt pressed for time as a result of the lesson running so far into Lesson 6. Outside of these more practical possible explanations, Ms. Ellis may have simply been overwhelmed. Though she did talk about anticipating students' solution strategies (Stein et al., 2008), she did not seem to anticipate students finding the midpoint between two fractions and using a decimal to represent it (as they did with 1.5/5). The result of her not anticipating that solution and having been tutoring an individual student at the side board may have all come together as a cognitive overload (Choppin, McDuffie, Drake & Davis, 2018). In that moment, expecting her to productively frame the activity and resolve such confusion as she typically did (relying on classmates) might have been too much.

5.2 Framing versus Norms

Critics of framing who are familiar with work in mathematics education around norms and sociomathematical norms (Voigt, 1995; Yackel & Cobb, 1996) might see the work herein as a different way of talking about norms in the classroom. However, framing is quite different from norms. The differences are at least twofold: 1. Framing is a way in which the teacher can reinforce or cue a particular norm; and 2. It accounts for in-the-moment interactions – both the expected and the unexpected. That is, one might say that a norm in Ms. Ellis's class is that students go to the front of the room and present their strategies to the class. After that, it is normative for them to ask questions of one another and defend their approaches. However, there are particular actions, verbal and non-verbal, that cue that norm. If one prefers to call that aspect of classroom interaction a norm, I accounted for the verbal ways in which the teacher(s) cued those norms at particular times through their framing(s). An example of this occurred in Excerpt 4 when Ms. Ellis reinforced for Brad that he needed to be more articulate in explaining how he created ninths by folding a fraction strip. Ms. Ellis's framed the activity for Brad by saying, "You lost me after you created thirds. So, can you go back to the beginning? You created thirds and where did you go from there?" Her explicit verbal statement relative to what was going on in the interaction, supported Brad to be more articulate. This was not a norm that Brad simply adhered to on his own. Ms. Ellis's framing supported him to be more articulate and share his thinking.

Relatedly, if it is normative for students to be asked to share and defend their approaches in Ms. Ellis's class, then what are the ways in which deviations from such a norm are discussed? Through my analysis of the teacher's and students' framings, I have found the moment-to-moment interactions that resulted in deviations from the typical interactional patterns. Ms. Pine revoking Keri's social authority in Excerpt 6 is one example, as well as Ms. Ellis taking away Keri's epistemic authority in Excerpt 8. Norms do not account for such subtleties in interactions nor for how to avoid such unproductive interactions in the future. My analysis of framing revealed the subtleties in explicit verbal statements that resulted in those less productive interactions.

5.3 Framing in Conjunction with Mathematics Instructional Practices

Ms. Ellis was engaged in mathematics instructional practices that are considered productive in the literature. She was using Accountable Talk in ways that supported students to talk about their own and others' ideas (Michaels, O'Connor & Resnick, 2008). She was also asking questions that probed students' thinking (Boaler & Brodie, 2004). Ms. Ellis showed evidence, too, of engaging in at least some of Stein et al.'s (2008) *Five Practices:* anticipating; monitoring; selecting; sequencing; and connecting. Ms. Ellis, through her interview data, said that she was *anticipating* students' solution strategies and difficulties with the day's task. She also *monitored* students' work during the 'Explore' phase of the lessons by walking around observing students' work and asking them questions. Though not evidenced in every lesson, Ms. Ellis at times also *selected* certain students to present their strategies to the class. She did not show evidence of *sequencing* students' presentations/sharing their strategies. Ms. Ellis also did not show much evidence of *connecting* students' responses to one another's or to the mathematics they were studying.

While those practices that were less evident in Ms. Ellis's work of teaching might be obvious to experienced researchers, it is reasonable to expect that others might not notice her lack of *connecting* as readily. For example, coaches or principals might see students talking to one another about the mathematics and think that Ms. Ellis is hitting the mark on all of the mathematics instructional practices. For example, in Excerpts 6 and 8, the students were talking to one another about each other's strategies. This was also evidenced each time that I coded **understanding others' strategies** and **explaining own strategy** framings. Without digging deeply into what mathematically the students were making sense of, seeing the lack of *connecting* could be difficult.

Not only did framing help reveal that Ms. Ellis was not connecting students' strategies to one another's or to the mathematics, it also revealed that her lack of framing for connecting inhibited students' opportunities to do so. When Ms. Ellis framed her students as the authority and framed the mathematical activity as co-constructing mathematical explanations, her students aligned. They took on the social and epistemic authority and they engaged in the mathematics by working together and supporting one another to reach more complete mathematical explanations. However, she did not frame the mathematics as co-constructing across multiple strategies or as connecting to underlying mathematical ideas or concepts. In turn, her students did not have the chance to align with that framing. That is to say, her framing mattered for her students' opportunities to think about the mathematics in particular ways.

My analysis of Ms. Ellis's framing revealed a kind of pattern in her work of framing. She would frame her students as the epistemic and social authority. She would then frame for co-constructing mathematical explanations by inviting students to ask questions of the student presenter and expecting that the student presenter answer those questions. Then what she did not do was frame each strategy in relation to the mathematics itself or to what other mathematics had been discussed up to that point in the lesson. This kind of pattern in her framing that was incomplete could be helpful for understanding how teachers might better engage in *connecting* students' responses.

Examining framing, beyond just instructional practices, showed, in-the-moment, what happened when *anticipating* students' solutions was not enough. Instead of being open to hearing Keri's strategy and the reasoning behind it, Ms. Ellis simply assumed she was wrong. She went on to try to correct her thinking. The way that Ms. Ellis dealt with Keri's unforeseen answer of 1.5/5 revealed an area of teacher practice worthy of further study. Addressing what happens when teachers themselves are confused or encounter student thinking that is unexpected could help better prepare teachers for such instances.

So, the literature on mathematics instructional practices was obviously significant to this analysis, but it was not helpful in explaining everything that was going on in the class. The practices are a broader stroke to looking at teaching practices, but framing revealed the more subtle details within those practices, how they functioned within this classroom, and what some of their limitations might be.

5.4 Limitations

As with any study, this one had its limitations. First of all, it was a secondary analysis of data that was previously collected without a mathematics instructional focus. The lessons were recorded on non-sequential days and the interview questions were not related to the mathematics specifically. Relatedly, I had limited understanding of Ms. Ellis's instructional goals. I only had access to the curriculum-stated goals. As a result of the secondary analysis as well, I did not have any data on students' learning. What did they learn in this classroom? What did they know when they came in to Ms. Ellis's class – both socially and mathematically? I also do not know Ms. Ellis's professional development history. I know she participated in Accountable Talk® professional development, but the extent to which she was supported outside of that, or even the number of sessions she attended was not known. Ms. Ellis pointed to her own shift with respect to the CMP2 curriculum. Knowing a bit more about how that shift was supported, who she attributed it to, and what specifically she might say was her stance on mathematics learning previously would all be helpful pieces of information for better understanding this classroom.

Outside of those background pieces of information, this framing analysis was done independently. That is, other studies that examined framing in the literature were done with multi-person teams who could talk about how they were interpreting what it was that was going on in interactions to reach consensus. I did not have that in this study due to limitations in funding and time. I also did not have the opportunity to compare Ms. Ellis's framing to another teacher's framing. That is, having a comparative case study would have been more informative than this single explanatory case study. In addition for making claims about framing, it would have been helpful to know exactly what Ms. Ellis did at the beginning of the year. How she framed the mathematical activity from the first day of school could have shed more light on how her students came to expansively frame their social activity, how they came to engage in group work so readily, and how they were supported to see asking for help as a productive practice without a stigma attached. Another limitation is my data reduction strategy. I privileged lessons in which the entire 'Launch' was recorded. Had I dropped that stipulation in reducing my sample, I could have ended up with a different sub-sample for my microanalysis. Ms. Pine's role in this class brought some limitations. Because Ms. Pine did some of the work of framing and even led some of the 'Summarize' phase, the extent to which I can make claims about Ms. Ellis's role in the framing is limited. However, I still accounted for Ms. Pine's framings, so I still coded all teacher framing within the subsample.

5.5 Implications

5.5.1 Implications for Future Research

Analyzing teachers' framing as a support for sense making activity in comparison to others is one line of future inquiry. Using framing to analyze teachers' framing for sense making, and mathematics more generally, could support this field of knowledge. In addition, considering teachers' pedagogical histories would help further explain how one teacher might be better positioned to frame for sense making than another. Framing research in mathematics education has been linked to equity and providing more equitable opportunities for students in schools and classrooms (Hand & Gresalfi, 2015; Hand et al., 2013; Selling, 2016). Further studies that examine the links between teachers' framing more generally in classrooms and opportunities for students to engage in meaningful mathematics would add to that knowledge base, as well as better inform teacher practice.

Within the same vein of studying framing within teachers' classrooms, specifically making use of these and other findings related to teachers' expansive framing is a future direction to take (Engle, 2006; Engle et al., 2012; Selling, 2016). Ms. Ellis's students' focus group interviews revealed that her expansive framing of their social skills in class had an impact on the ways in which they saw their participation within it. They saw value in what they were engaged in together beyond "doing the lesson." If Ms. Ellis had expansively framed the mathematics of study more often in this dataset, it is reasonable to wonder how that might have impacted them.

5.5.2 Implications for Teacher Practice

This study demonstrated that teachers' work of framing mattered for students' engagement in sense making activity. Framing, however, is a theoretical construct that might present challenges when working with teachers. Exploring whether or not addressing teachers' framings with them is appropriate is one future direction. If framing is not an adequate way to talk about teachers' practice with them, expectations or explicitness might be acceptable alternatives. Supporting teachers to continually make their expectations explicit for all of their students could support their students to engage in more sense making activity (Selling, 2016). Further, sticking to those expectations is even better. Just as Ms. Ellis reinforced the epistemological framing when students tried to slip into the doing framing helped students better understand what her expectations of them were. Related to making one's expectations known, teachers could benefit from thinking about what it means to be successful in their classrooms. Is it enough to ask questions? Is it enough to merely share a solution strategy (Ball, 2001)? Defining success in ways that support meaningful engagement with the mathematics practices and making progress toward the mathematical goals seems necessary.

Related to defining success in teachers' classrooms, the messaging of professional development is important. Supporting teachers to change their practice may need to focus more on teachers' own epistemological shifts than on specific teacher practices (Watkins et al., 2017). Ms. Ellis alluded to her own epistemological shift with respect to the curriculum she was using. She also noted initial resistance to it. Other mathematics education researchers have hinted at this same idea, though not necessarily in terms of epistemological shifts. Heyd-Metzuyanim et al. (2018) and Smith (2000) both referred to the very different roles that teachers and students must embody to engage in dialogic instruction in ways that address the underlying intentions of such approaches. Teacher professional development may need to emphasize such a shift for teachers through messaging or other supports so that they have an understanding of what the ultimate goal is of engaging in particular instructional practices. Ms. Ellis referenced "skill and drill." If that is what teachers know, how can they be supported to think about math differently – to epistemologically shift to viewing mathematics as a sense-making endeavor? Further, how can they be supported to foster such a shift in their own students?

Beyond that, supporting teachers to see more in their work than just instructional practices is important. Ms. Ellis *anticipated* students' solutions, but at times, that was not enough to fully prepare her for all of the ways that her students approached the tasks. Beyond expecting the unexpected at times, what is a teacher to do when it happens? How can they make the most out of those instances to keep students' thinking central and to support mathematical progress?

A final note is that I had difficulty at times discerning whether a student's or teacher's contribution was addressing social aspects of framing or epistemological ones. The literature shows that the interplay of the two is to be expected. Berland and Hammer (2012a) noted that students rely on both aspects of teachers' epistemic and social authority as they decide how to make sense of what is taking place in an interaction. Watkins, et al. (2017) noted the same in stating that they did not view epistemological framing as a separate idea that was independent of other framing components. These notes in the literature together with my findings, suggest that teacher professional development that attends to both social and epistemic aspects of teachers' practice in relation to more dialogic instructional approaches is likely necessary.

5.6 Conclusions

My analysis showed that framing mattered in Ms. Ellis's classroom. Ms. Ellis's view of mathematics as more than just answers to be gotten was not enough to support students in fully making sense of the mathematics in ways that linked to broader mathematical goals. Teacher awareness of, and work toward, the goal of a lesson is still important teacher work (Stein et al., 2008). Within the literature, lack of teacher progress toward the mathematical goals is nothing new. What my analysis showed, though, was that Ms. Ellis was engaged in high quality mathematics instructional practices. The IQA scores supported this claim, as did other aspects of Ms. Ellis's practice. That is, there were a lot of aspects of Ms. Ellis's practice that were good and checked boxes – she *anticipated* students' solutions, she *monitored* them during the 'Explore' phase, she gave them the authority, she did not evaluate their work (Stein et al., 2008). Still there was more that she could have done to better support students' sense making. Ms. Ellis showed that her students were engaging in more than merely "show and tell" during the 'Summarize' phase of her lessons (Ball, 2001; Wood & Turner-Vorbeck, 2001).

This is not a criticism of Ms. Ellis or of her teaching. In an urban setting with a diverse population, she supported her students to engage in mathematics as more than an answer-getting endeavor. That she was engaged in so many productive mathematics instructional practices made her a case worthy of deeper analyses. What that analysis revealed, was a more nuanced view of teaching practices. What framing did in this analysis was provide another layer to the ways in which teachers might support students' sense-making activity. Personally what this study helped me see was just how much there is to observe in classroom interactions. Having evaluated pre-service teachers previously, I am not sure that I would have been able to articulate the fact that her students were not making mathematical progress if I were witnessing it live, in real time. There were so many great things happening on Ms. Ellis's part and on the part of her students' that I think it is reasonable to expect that outsiders might be blinded by those high points in her practice.

Appendix A IQA Rubrics Applied to Lessons

List of Nine IQA Rubrics Applied to All Lessons (Boston & Wolf, 2006)

1) Potential of the Task
2) Implementation of the Task
3) Student Discussion Following the Task
4) Participation
5) Teacher Questioning
6) Teacher's Linking
7) Students' Linking
8) Asking (Teacher Press)
9) Providing (Student Responses)

Appendix B Mathematical Tasks and Goals from CMP2

B.1 Lesson 1 Task and Mathematical Goals

Task (Lappan et al., 2009, p. 40):

Problem 1.2 Folding Fraction Strips

A. 1. Use strips of paper that are $8\frac{1}{2}$ inches long. Fold the strips to show halves, thirds, fourths, fifths, sixths, eighths, ninths, tenths, and twelfths. Mark the folds so you can see them better.

2. What strategies did you use to fold your strips?

B. 1. How could you use the halves strip to fold eighths?

2. How could you use the halves strip to fold twelfths?

C. What fraction strips can you make if you start with a thirds strip?

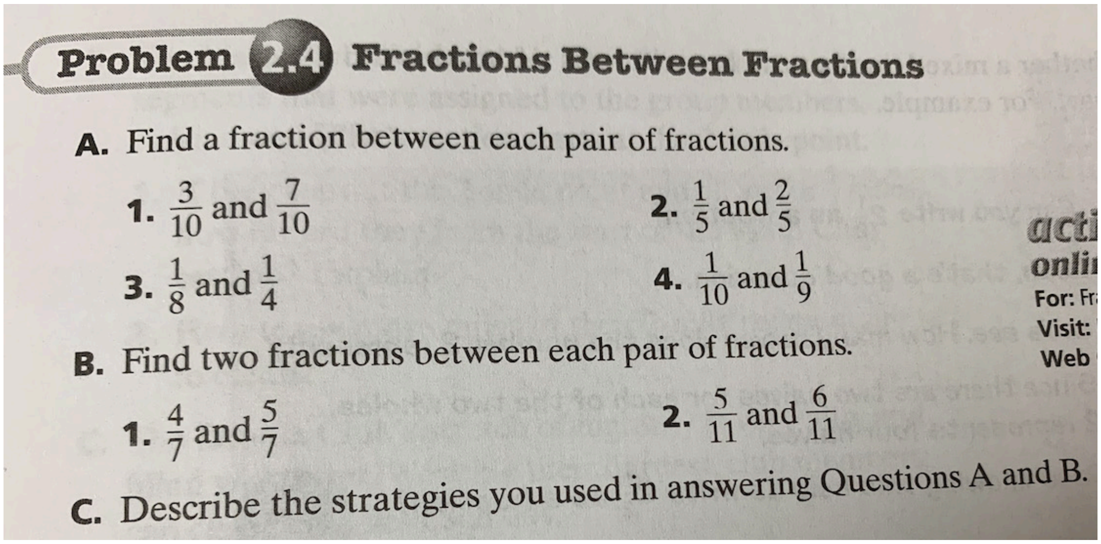
D. Which of the fraction strips you folded have at least one mark that lines up with the marks on the twelfths strip?

Mathematical Goals (as stated in Lappan et al., 2009, p. 27):

> "Develop strategies to partition fraction strips for halves, thirds, fourths, fifths, sixths, eighths, ninths, tenths and twelfths
>
> Explore the role of the numerator and denominator and the part-to-whole nature of fractions
>
> Investigate equivalent fractions that result from different partitioning strategies "

B.2 Lesson 5 Task and Mathematical Goals Task

Task: (Lappan et al., 2009, p. 40):



Problem 2.4 Fractions Between Fractions

A. Find a fraction between each pair of fractions.

1. $\frac{3}{10}$ and $\frac{7}{10}$ **2.** $\frac{1}{5}$ and $\frac{2}{5}$

3. $\frac{1}{8}$ and $\frac{1}{4}$ **4.** $\frac{1}{10}$ and $\frac{1}{9}$

B. Find two fractions between each pair of fractions.

1. $\frac{4}{7}$ and $\frac{5}{7}$ **2.** $\frac{5}{11}$ and $\frac{6}{11}$

C. Describe the strategies you used in answering Questions A and B.

Mathematical Goals (as stated in Lappan et al., 2009, p. 65):

"Develop a strategy for finding a fraction between any two given fractions

Begin to recognize that by using smaller partitions one can always find a fraction between two given fractions"

B.3 Lesson 6 Task and Mathematical Goals

Task (Lappan et al., 2009, p. 72-73):

Problem 2.5 Naming Fractions Greater Than 1

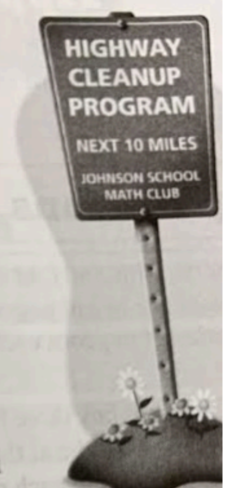
Each student activity group at Johnson School agreed to pick up litter along a 10-mile stretch of highway.

For each problem, use a number line to show what the problem describes and how you solved it. Show your answers as both a mixed number and an improper fraction.

A. Kate and Julianna are in the Marching Band. They work together to clean a section of highway that is $\frac{9}{4}$ miles long. Write this length as a mixed number.

B. The Math Club divided their 10-mile section into 2-mile segments that were assigned to the group members. Adrian and Ellie's section starts at the 2-mile point.

1. If they start at the 2-mile point and clean for $\frac{5}{3}$ miles, how far are they from the start of the Math Club section? Explain.
2. How many more miles of their 2-mile segment are left to clean?



C. The Drama Club's stretch of highway is very hilly and filled with litter. Working their hardest, club members can clean $1\frac{2}{3}$ miles each day.

1. How far will they be at the end of the second day?
2. At this rate, how many days will it take them to clean their 10-mile section?
3. Jacqueline says that in four days they can clean $\frac{19}{3}$ miles. Thomas says they can clean $6\frac{2}{3}$ miles in four days. Who is right? Why?

D. The 10 miles assigned to the Chess Club start at the 10-mile point and go to the 20-mile point. When the Chess Club members have cleaned $\frac{5}{8}$ of their 10-mile section, between which miles will they be?

E. The Gardening Club has a section of highway between the 20- and 30-mile points. The club members set their goal for the first day to reach the 24-mile point. What fraction of the Gardening Club's total distance do they plan to cover on the first day?

Mathematical Goals (as stated in Lappan et al., 2009, p. 71):

"Understand the underlying structure of fractions greater than one

Develop meaningful strategies for representing fraction amounts larger than one as both mixed numbers and improper fractions

Build understanding of fractions as numbers that measure lengths between whole numbers"

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