

Fluid Mechanics, Models, and Realism: Philosophy at the Boundaries of Fluid Systems

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Philosophy of science has long drawn conclusions about the relationships between laws, models, and theories from studies of physics. However, many canonical accounts of the epistemic roles of laws and the nature of theories derived their scientific content from either schematized or exotic physical theories. Neither Theory-T frameworks nor investigation on interpretations of quantum mechanics and relativity reflect a majority of physical theories in use. More recently, philosophers of physics have begun developing accounts based in versions of classical mechanics that are both homelier than the exotic physical theories and more mathematically rigorous than the Theory-T frameworks of the earlier canon. Some, including Morrison (1999, 2015), Rueger (2005), and Wilson (2017), have turned to the study of fluid flows as a way to unpack the complex relationships among laws, models, theories, and their implications for scientific realism.

One important result of this work is a resurgence of interest in the relationship between the differential equations that express mechanical laws and the boundary conditions that constrain the solutions to those equations. However, many of these accounts miss a crucial set of distinctions between the roles of mathematical boundary conditions modeling physical systems, and the roles of physical conditions at the boundary of the modeled system. In light of this systematic oversight, in this dissertation I show that there is a difference between boundary conditions and conditions at the boundary. I use that distinction to investigate the roles of boundary conditions in the models of fluid mechanics. I argue that boundary conditions are in some cases more lawlike than previously supposed, and that they can play unique roles in scientific explanations. Further, I show

that boundaries are inherently mesoscale features of physical systems, which provide explanations that cannot be inferred from microscale dynamics alone. Finally, I argue that an examination of the domain of application of boundary conditions supports a form of realism.

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Preface

This dissertation explores several aspects of the roles of boundary conditions in fluid mechanics. When this project was initially conceived, I had envisioned a single chapter on the topic of boundary conditions. As I began developing that chapter, I discovered that the topic was far richer than I had imagined, and soon boundary conditions became the guiding theme of the entire dissertation.

Chapter 1 explores the role of boundary conditions in fluid mechanics. I contrast the ways that philosophers and physicists have historically conceived of the role of boundary conditions in fluid modeling, and I argue that boundary conditions are in some cases more lawlike than previously supposed. In particular, they are capable of performing functions similar to that of laws in scientific explanations. Still, they cannot be restricted to any particular role in theories of explanation solely in virtue of the fact that they are boundary conditions. The way they function in explanations depends on their function in a scientific model.

Chapter 2 draws a distinction that has escaped attention in the philosophical literature: the distinction between boundary conditions and conditions at the boundary. This distinction was recognized by Brenner and Ganesan in the context of diffusive fluid systems. Though they draw the distinction as one between molecular models and fluid dynamics, I argue that the distinction is not tied to any particular theory or scale. Rather, the distinction depends only on whether or not the interactions between the fluid and the boundary are explicitly taken into account.

Chapter 3 shifts focus by looking at how the boundaries themselves figure into explanations. I draw a distinction between boundaries in the models of fluid dynamics and boundaries in the models of molecular dynamics. In molecular dynamics models, mesoscale

features of solid boundaries give rise to behaviors that cannot be explained only by looking at either the continuum level description of a fluid system or a molecular level description. I develop an extended example of the explanatory role of boundaries through a case study on the formation of nanobubbles on the interior of a pipe. The formation of nanobubbles, which determine slip properties of a flow, cannot be explained solely by the interactions described by molecular dynamics.

Chapter 4 investigates implications of the previous chapters' discussion for contemporary approaches to scientific realism. I introduce the concept of a model's domain of application in response to some antirealist critiques that have been directed at the no-slip boundary condition. Knowing how intervening in the conditions under which a model succeeds or fails gives us causal knowledge about the no-slip condition. I argue that this knowledge supports a local realism concerning the no-slip condition based on its domain of application. I contrast this local realism with scientific perspectivism.

While I think the arguments made in this dissertation are interesting in their own right, I believe further exploration of these ideas will prove even more rewarding. One of the major limitations of this dissertation is that I have chosen to focus not only just on the boundary conditions of fluid mechanics, but on a small subset of these. Further investigation into the nature of a wider array of boundary conditions from different areas of science might broaden the scope of the conclusions I have drawn here, or conversely, result in a different set of conclusions unique to the boundary conditions in those areas. Another avenue of further investigation is in the tradition of questioning standard accounts of lawhood. Although I have employed the language of laws and lawlikeness, I think this work is part of a school of thought that sees a strict delineation of laws and not-laws as ultimately fruitless. This work contributes to a general account of science that

dispenses with standard accounts of laws. Finally, while I have been as careful as possible in describing the science, someone with a keener mathematical insight than I have might be able to come to additional conclusions based on the unique mathematical features of individual boundary conditions.

This dissertation would not have been possible without support, criticism, suggestions, challenges, and encouragement from some very special people. Thanks first to my dissertation committee: Jim Woodward, Sandra Mitchell, Bob Batterman, and Porter Williams. They stuck with me throughout the long process of writing this. From initial conception to final revisions, their input has made a deep impact on my thinking and on the content of this dissertation. The ideas in here are also shaped by John Norton and Mark Wilson, whose guidance earlier in my graduate career at Pittsburgh helped shape my philosophical outlook. I am grateful to the members of the Philosophy faculty at the University of Washington who ignited my interest in philosophy of science: Cass Weller, Andrea Woody, Arthur Fine, Adam Moore, William Talbot, Gwynne Taraska, and Carole Lee. Without them, I do not know how I would have found this path. I am also grateful to my fellow graduate students Kathryn Tabb, Keith Bemer, Lei Jiang, Trey Boone, and Joe McCaffrey. We were all getting through it together, from late nights working at the cathedral to late nights playing pool. Thanks to my family, especially my parents Joseph and Eveline Sykora, whose unconditional love and support I can always count on, and my brother Bob Sykora, who has been an inspiration to me as I watch him forge his own path in life. Finally, thanks to Julia Sykora Bursten, who has always been exactly what I need. She was there to read drafts, to talk about ideas, and she was there to laugh with or cry with, when I needed that too. She is always there for me, absolutely unwavering, my safe home. And she walked with me on this journey, every single step.

To Oma and Opa

1.0 Boundary Conditions in Fluid Mechanical Models

Historically and presently, philosophers have tended to treat boundary conditions as relatively unimportant features of scientific modeling. Compared to scientific laws, which have been the focus of many philosophical accounts, boundary conditions are not usually even thought to be worth exploring on their own, receiving only passing mention. And with some notable exceptions, they have only played secondary roles in various philosophical endeavors like scientific explanation and accounts of causation. Usually, when boundary conditions are mentioned in such work, they are treated as merely contingent or even freely choosable parameters that are not even properly part of a theory. In virtue of their supposed contingency, they are treated as epistemically secondary as well. In contrast, knowledge about laws is thought to be justified based on not only empirical considerations, but also on knowledge of more fundamental physical facts. Moreover, laws of nature, and especially physical laws, are typically thought of as generating the epistemic and explanatory content in models of physical systems. In this chapter, I argue against this dominant view of the epistemic humility of boundary conditions. I show that, instead, there are characteristic parts of physics in which the boundary conditions play a variety of essential epistemic roles.

I will develop my account by taking a closer look at how boundary conditions function in the models of fluid dynamics, and at how they relate to the set of governing equations of fluid dynamics. In particular, I will look at the boundary condition that specifies the extent to which a fluid slips past a solid surface. In this context, boundary conditions often play a role just as important as the governing equations, and they have more in common with scientific laws than might be at first apparent. Because of these similarities, boundary conditions can play some

familiar epistemic roles that philosophers of science have typically ascribed to laws. However, boundary conditions are not laws. They can also play unique roles in explanations, different from the role of laws and different from the role of contingent facts. The upshot of all of this is that boundary conditions play an important and complex role in science that is often overlooked and obscured by their treatment in philosophical accounts. The case of fluid mechanics is instructive for boundary conditions generally, and similar lessons might be drawn for other areas of physics, as well as for other sciences. However, in light of their varied uses, it is also unlikely that a single account of boundary conditions will cover all cases.

1.1 Boundary Conditions in Philosophy of Science

References to boundary conditions have shown up in philosophical discussions of science for a long time, and in varied contexts. In discussions of explanation, causation, and confirmation, for example, boundary conditions have played a role. Often, they are found in conjunction with their more esteemed counterparts, scientific laws. But their precise role is less commonly examined, if at all. To make matters worse, philosophers have often used the term “boundary conditions” quite loosely, referring to a variety of things that are not in fact boundary conditions in the sense used by scientists. In some cases, philosophers have even used the phrase “boundary condition” metaphorically. In the web of belief metaphor that characterizes Quine’s epistemic outlook, for example, “total science is like a field of force whose boundary conditions are experience.” (Quine, 1953, p. 42) And although this metaphorical use is not particularly relevant to this discussion, it gives us an idea of how broadly the term “boundary condition” has been used.

Aside from such metaphorical usage, I can find two broad senses in which philosophers have employed the notion of a boundary condition. On the one hand, they appear in work on more general problems in philosophy of science. In these cases, we find claims about boundary conditions that are either wrong or so vague as to be unhelpful in really understanding their roles in science. On the other hand, they appear in more technical discussions by philosophers of physics. And while in these cases boundary conditions are used in technically correct ways, these discussions do not give insight into how boundary conditions should figure into larger philosophical discussion. I discuss each of these philosophical uses of boundary conditions in turn.

1.1.1 General Uses of Boundary Conditions in Philosophy of Science

Looking at the first sense, philosophers have used the concept of boundary conditions to refer vaguely to a variety of things. In some philosophical accounts of explanation, for instance, scientific laws do special work, and anything that is not a law — such as a boundary condition — often gets tossed into the same basket of epistemically unimportant support for the development of a law-centered explanation. Boundary conditions end up being used interchangeably with things like auxiliary hypotheses, background conditions, or the like. Look at, for example, Hempel's deductive-nomological model of explanation, where an explanation takes the form of a deductive argument in which the conclusion is the explanandum and the premises form the explanans. The explanans must include a law of nature, which is taken to be the major premise. But it must also include anything that is required to deduce the phenomenon to be explained. When talking about these additional premises, Hempel refers to “boundary conditions” and “outside influences” interchangeably. (Hempel, 1962, p. 107). He goes on to cite the example of such an outside influence: “a collision of Mars with and unexpected asteroid that affects its future orbit.” (Hempel,

1962, p. 116) . Elsewhere, he describes initial and boundary conditions (failing to distinguish between the two) as being cases of determining conditions, or “a set of statements asserting the occurrence of certain events C_1, \dots, C_n at certain times and places.” These statements form one part of an explanation, the other part being “a set of universal hypotheses,” which where we find the laws. (Hempel, 1965, p. 232) Thus boundary conditions are statements about particular events.

We can find discussions similar to Hempel’s scattered throughout the literature. Looking at another example, when Hilary Putnam is discussing the corroboration of theories (Putnam, 1974), he says that a theory is a set of laws. In contrast, statements like “all other bodies [in the solar system] exert forces small enough to be neglected” are not laws. Instead, that statement “is a statement about the ‘boundary conditions’ which obtain as a matter of fact in a particular system.” (p. 225) In this usage, “boundary conditions” function something like a *ceteris paribus* clause. They are contingent facts about a real system, but they interfere with the relations of interest. So by factoring them out, we can focus on the more important parts of the model. The relations of interest in this example are the laws that govern the motion of the planets in the solar system. In some broad sense, these “boundary conditions” do constrain the laws, but not in the more precise sense that I will explain in the next section.

Aside from the imprecision with which these philosophers have used the concept of boundary conditions, there are a few things to note about these kinds of usages. The thing that characterizes boundary conditions here is their role in explanations or corroboration of theories. That is, the boundary condition’s role is decided by the philosophical theory, not its role in the scientific model. But this way of identifying boundary conditions gets things reversed. In this type of philosophical approach to boundary conditions, boundary conditions are divorced from the scientific and mathematical domain in which they properly reside. In such cases, philosophers

make the mistake of attempting to conform scientific theory to philosophical theory, rather than developing a philosophical theory from careful study of the behavior of scientific theory.

Additionally, in this first mode of encountering boundary conditions philosophically, boundary conditions are always contrasted with laws (or lawlike statements), and the contrast favors laws over boundary conditions. Boundary conditions are demonstrated to be less important than laws along a variety of dimensions: they are smaller in scope, they are contingent rather than necessary, and they do not come with a background of theoretical support. Most importantly, they are highly variant under intervention, unlike stable, reliable laws.

Now it should be noted that the misuse of boundary conditions in an account of scientific explanation does not by itself refute that account. There are good reasons to make a distinction between lawlike statements and accidental statements, or a distinction between universal facts and particular facts. It is clear, for example, that Hempel has in mind distinct roles for both laws (and lawlike statements) and contingent facts about the world. But it is not accurate to use this distinction to characterize the explanatory or epistemic roles played by boundary conditions. It might be argued that as long as philosophers define what they mean and are consistent in their usage, using “boundary conditions” in this way is not harmful. After all, the focus of these accounts is not boundary conditions as such, but rather more general philosophical concerns. But “boundary condition” is a term of art used by mathematicians, physicists, engineers, etc. Misuse of “boundary conditions” in this sense not only minimizes their role in theories and takes for granted the rich theoretical progression that often underlies their use, but it also confuses what boundary conditions actually are, which thereby prevents philosophical discourse from making contact with scientific discourse. In the next section I will show how boundary condition lead a much richer life and do

much more work than has been assumed. When we see this, we will be able to better appreciate their role in both scientific and philosophical theories.

1.1.2 Boundary Conditions in Philosophy of Physics

The other way philosophers have used boundary conditions is in the context of more technical discussions found in philosophy of physics. In discussions of field theories, spacetimes, foundational issues in quantum mechanics, and other topics that rely on mathematical formalism, boundary conditions often come into play.¹ When setting up a field theory, for example, we must use the right boundary conditions in order to get things to behave nicely. For the most part, these uses are legitimate applications of the notion of a boundary condition. Boundary conditions are here treated as conditions that constrain the differential equations used. And more importantly, their role as boundary conditions is determined by their relation to other sets of equations.

However, what this treatment of boundary conditions gains in accuracy, it lacks in philosophical reflection. These accounts offer little insight into the nature of boundary conditions themselves, and are generally not connected to the issues surrounding more general concerns. This is not an oversight, but rather merely a change of focus: unlike the general accounts in the previous subsection, the accounts here are not intended to be accounts of the kinds of things boundary conditions are. So even though these authors use boundary conditions in technically better ways than Hempel, Putnam, and others, boundary conditions are not the focus of their discussion either. These usages of boundary conditions do not engage with the more general philosophical problems

¹ For plenty of these sorts of discussions, see Butterfield & Earman (2007)

like explanation or theory confirmation. And while these uses are not wrong, they usually do not emphasize boundary conditions' importance.

1.1.3 Other Treatments of Boundary Conditions

Philosophers are perhaps not entirely to blame for the minimization of the role of boundary conditions. Indeed, fluid mechanics textbooks often downplay the importance of boundary conditions. Entire chapters are dedicated to the derivation and even history of the governing equations, while boundary conditions are often introduced with perhaps a paragraph or two of explanation.² There are perhaps good reasons for this. The tools of fluid dynamics can be used (and taught) without going into details of why certain boundary conditions are used. And explanations for them require resources beyond fluid dynamics.

So neither of the common philosophical treatments of boundary conditions really gives us an analysis of the concept. There are some exceptions to the trend though. Mark Wilson (2017; 1990) explicitly addresses the relationship between differential equations and their boundary conditions in physics. George Ellis (2007) argues that there is difficulty in distinguishing between the laws of physics and boundary conditions in the cosmological context of the origin of the universe. And Mathias Frisch (2004) has made some similar points regarding initial conditions, namely that the distinction between purely contingent initial conditions and laws that are in some sense necessary is not as sharp as has been previously supposed. I see these as attempts to bring together the proper use of boundary conditions and general problems in philosophy of science. In the next section, I hope to add to these analyses by describing the role of boundary conditions in

² For example, see (Wendt, 2009).

certain scientific models and shedding some light on how they can be used in more general discussions as well.

1.2 Boundary Conditions at the Fluid-Surface Interface

Boundary conditions play an indispensable role in fluid mechanics, and so in order to develop a clearer philosophical picture of the roles that boundary conditions play in physical theory, I will use fluid mechanics as a foundational example. To better understand the epistemic and explanatory roles of boundary conditions in fluid dynamics, I will review some of the relevant physical theory, attending to the various governing equations and boundary conditions that enter the picture. Then, I will discuss just what roles boundary conditions actually play in that theory.

1.2.1 A Very Brief Overview of Fluid Dynamics

The governing equations of fluid dynamics are a system of differential equations that can be derived by applying some fundamental physical principles to fluid systems. The continuity equation can be derived from the principle of conservation of mass, the momentum equation can be derived from Newton's second law, and the energy equation can be derived from the principle of conservation of energy. (Anderson, 2009) These equations form a coupled system of non-linear partial differential equations, and each of these equations has different forms. The correct form to use in a given situation depends on the kind of system being modeled and the features of the system in which we are interested, as well as practical matters of computability. For example, each of the governing equations has a viscous form, which takes into account the phenomena of viscosity and

thermal conduction, and a non-viscous form, which does not take into account viscosity and thermal conduction. Additionally, each of the governing equations has a conservation and a non-conservation form. The conservation form looks at a finite control volume fixed in space through which the fluid moves, while the non-conservation form looks at a finite control volume that contains the same set of fluid particles and moves with the fluid. Historically, the momentum equations for viscous flow were called the Navier-Stokes equations, but in much of the modern literature, the use of this name has been expanded to include all of the flow equations. I will just refer to these as the governing equations of fluid mechanics.

These governing equations are used to describe fluid velocity, pressure density, and temperature with respect to space. But they are unable to describe a fluid system by themselves. They are only useful in conjunction with boundary conditions. And depending on the kind of system one is modeling, initial conditions might also be required, but I will not be considering initial conditions in this chapter. In the context of the governing equations of fluid mechanics, a boundary condition is a mathematical object that constrains solutions to those differential equations. Some typical boundary conditions types are Dirichlet boundary conditions which prescribe the value of unknowns on the boundary, Neumann boundary conditions which specify the normal gradient of the unknowns, and mixed boundary conditions which specify a combination of unknown quantities and their normal gradients. (Massoudi, 2007)

A set of differential equation and boundary conditions constitutes a boundary value problem, the solution to which must satisfy the boundary conditions. We must keep in mind this specific mathematical function of boundary conditions when we look at their role in models. This will become especially important in the next chapter, where I talk about the difference between boundary conditions and conditions at the boundary. While a boundary condition determines

solutions to equations at the boundary, its effects can be felt away from the boundary as well. The extent of this effect depends on details of the flow in question, but the choice of boundary conditions often affects the entire flow field.

1.2.2 Boundary Conditions in Fluid Dynamics: the Case of Slip

In fluid dynamics, boundary conditions are necessary in a wide variety of situations. For example, when modelling a fluid flow through a pipe, we might specify a constant flux of fluid through the beginning and end planes of the pipe. Any solution to the governing equations must satisfy these inlet and outlet boundary conditions, and a change in these conditions would change the solutions to the equations. Boundary conditions can also specify how the fluid behaves at the boundary between two fluids. For example, we could specify that at the boundary between the fluids, stress is continuous from one fluid to the other. But for the rest of this chapter, I will focus on boundary conditions that specify the behavior of a fluid at a solid boundary.

The boundary conditions that constrain the governing equations at the boundary between fluid and solid are some of the most common boundary conditions used in fluid-mechanical models. In most cases, the velocity component normal to the boundary is zero. Intuitively, this just assures that there is no transfer of mass across the boundary. But of much greater interest for us right now is the component of the fluid velocity tangential to the boundary. The tangential velocity of the fluid at the solid boundary is known as *slip*, since it measures the degree to which the fluid “slips” along the surface. In many situations, the boundary condition used is the *no-slip condition*, which requires that fluid elements directly on the surface of a solid boundary are stationary. That is, the fluid velocity goes to zero as it approaches the boundary.

The no-slip condition can be specified by setting the tangential velocity of the fluid to equal the velocity of the solid surface, which is zero if we are considering a stationary surface. But the no-slip condition can also be thought of as a special case of the Navier slip condition (Shu, Teo, & Chan, 2017):

$$u_s = \lambda \left. \frac{\partial u}{\partial x} \right|_s \quad \text{Equation 1 Navier slip condition}$$

The tangential velocity of the flow at the surface, u_s , is related to the shear rate, $\frac{\partial u}{\partial x}$, at the surface. Here, x is the normal from the surface pointing into the liquid, and λ denotes a parameter called slip length. The subscript s refers to the value at the surface. Slip length can be interpreted as a fictitious distance below the solid surface at which the no slip condition *would* be satisfied if the fluid did extend past the solid surface. If the slip length goes to zero, the no-slip condition is recovered. This is illustrated in figure 1, where the length of the arrows represents the velocity of the fluid at various distances from the boundary, and the shaded area on the bottom represents the solid boundary.

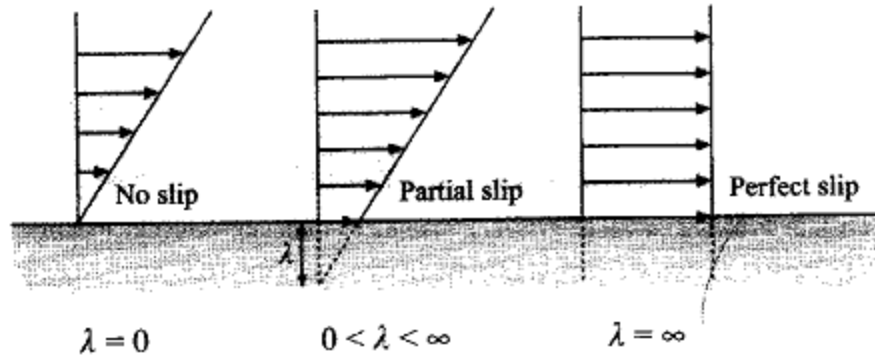


Figure 1 Interpretation of the slip length λ

Reprinted with permission from Lauga Brenner & Stone (2007)

Note that velocity changes continuously within the fluid, but in the partial and perfect slip cases ($0 < \lambda < \infty$ and $\lambda = \infty$, respectively), there is a discontinuous change in velocity in moving from the fluid to the solid boundary (which is assumed to be motionless).

Boundary conditions like slip are not just important at the boundaries. They help define the nature of a flow away from the boundary as well. The extent to which their effects are felt away from the boundary depends on the nature of the governing equations as well as physical parameters like viscosity. In general it is not possible to describe a flow without boundary conditions. Consider a fluid flowing through a channel. Figure 2 shows how the boundary conditions affect the rest of the flow.

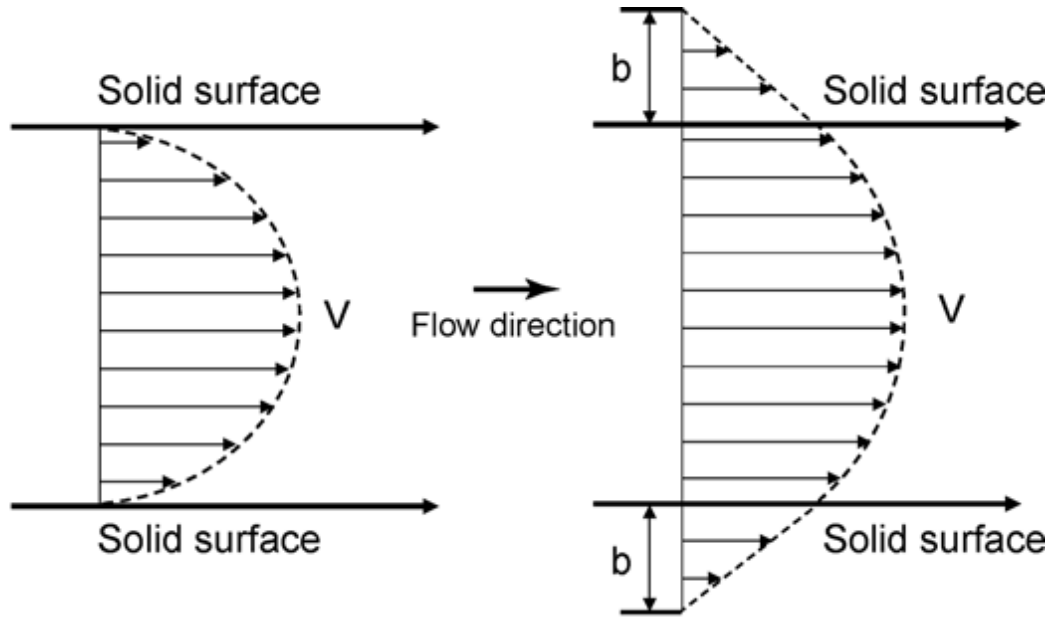


Figure 2 The effect of slip on fluid flow

Reprinted with permission from Jung & Bhushan (2010)

In figure 2, we see arrows representing velocity vectors, but other flow variables are affected as well. For example, a slip boundary condition, like the partial-slip boundary condition depicted in the right half of the figure, results in a greater flow rate than a no-slip condition (depicted in the left half of the figure). The flow rate, Q , is the volume of fluid that passes per unit time. This relationship can be quantified for a circular pipe by the equation:

$$\frac{Q(\lambda)}{Q_{NS}} = 1 + \frac{4\lambda}{a} \quad \text{Equation 2 Flow rate through circular pipe}$$

$Q(\lambda)$ is the flow rate with some slip length λ , Q_{NS} is the flow rate with the no-slip condition, and a is the radius of the pipe. The boundary condition has a dynamical impact throughout the system,

and so has more explanatory impact than supposed in the more generalized uses of boundary conditions

Often, the scale of the flow determines the extent to which the boundary condition affects the rest of the flow. For macroscopic flows, small amounts of slip can have a negligible effect on the rest of the flow. If there is an actual slip length of, say, 10^{-5} meters, a macroscopic flow can effectively be modeled using the no-slip condition. However, there exist flows that are small enough that the region of flow greatly affected by the boundary conditions represents a large percentage of the flow, but large enough to be accurately modeled as a continuous medium. These flows can still be described by the governing equations of fluid dynamics, but they are small enough that microscopic amounts slip make significant difference in flow parameters like velocity profile and flow rate. When working with these smaller systems on the micro- or nanoscale, even this small amount of slip can make a large difference in the flow rate.

Even for large scale flows, boundary conditions can play important roles. For example, when modeling the flow of air over an airfoil, the effects of viscosity of air are negligible for the regions of the flow away from the airfoil boundary. For these regions, the viscous terms can be dropped from the governing equations.

However, in order to capture the aerodynamics at the wing correctly, the velocity must tend to zero; the no-slip boundary condition must apply. This means that viscosity is important near the boundary. In order to accommodate the importance of viscosity at the boundary, aerodynamics models are separated into far-field, viscosity-free models and boundary-layer models with distinct governing equations intended to accommodate the need for a no-slip boundary condition. To model the air flow around an air foil accurately, and to predict the amount of lift it will experience accurately, the velocity gradient normal to the surface must be large in the boundary-layer region.

Such a gradient requires the fluid have viscosity. Without this boundary layer, the model would be unable to account for the amount of lift applied to the airfoil. The far-field regions are best modeled with the viscosity-free versions of the governing equations, but the boundary layer is modeled with equations for viscous fluids. (Grundmann, 2009). Without the boundary condition, we could not explain how an airfoil moves through the air and provides lift for aircraft.

1.3 Boundary Conditions and Laws

In the next three subsections, I will look at how boundary conditions of fluid dynamics compare to the laws of fluid dynamics. To do this, I will look at boundary conditions' degrees of invariance under intervention, the extent of their scope, and their levels of theoretical and empirical support. We will see that the degree to which boundary conditions are like laws varies from case to case.

1.3.1 Boundary Conditions as Invariant under Intervention

While there are differences between governing equations and boundary conditions, both are capable of playing similar roles in fluid mechanical models in terms of causation and explanation. In this section, I will argue that boundary conditions have some of the law-like features that the governing equations do, and so can play many of the roles of scientific laws in accounts of causation and explanation as well as give rise to unique issues of realism in models. My discussion is limited to laws in a particular domain of science, namely fluid dynamics. And although there are different conceptions of what it means to be a law, I do not have any particular

one in mind. But I will assume that if anything is a law of fluid dynamics, the governing equations are. And if the governing equations are the laws of fluid dynamics, then some boundary conditions should also be treated similarly to laws in many respects.

In establishing this similarity, the first thing to look at is the role boundary conditions play in models. Like the governing equations, boundary conditions are generalizations about a system. In the case of the slip boundary condition, it is a generalization about the system that relates the velocity of the fluid at the boundary to the shear rate at the boundary.

One of the key features in many accounts of law is that they are generalizations that are not merely accidental. One way of instantiating the non-accidental quality of lawlike generalizations is to frame them as capable of supporting counterfactuals, or alternatively to say that laws support confirmability by inductive inference. Like laws, boundary conditions represent stable relations between variables. This stability, encoded in the boundary conditions, allows generalizations of a non-accidental quality to be made.

There is of course much disagreement about what exactly makes a generalization law-like or non-accidental. But I have in mind an account of lawlikeness like that of Woodward (2003). On this account, laws (or lawlike things) are invariant under intervention (or possible intervention), over various ranges of background conditions. I think this has the advantage of avoiding unnecessary metaphysical discussion surrounding the nature of laws, and it captures the way laws are used by scientists in models, including those of fluid dynamics. On this view it does not matter if a generalization is genuinely a law of nature (whatever that means) as long as it is explanatory. The Navier slip condition is lawlike in this sense. And like laws, both the governing equations and boundary conditions are law-like in that they are not merely accidental generalizations. Further, if slip length does not depend on shear, this regularity is treated as a property of the fluid-solid pair,

(Lauga, Brenner, & Stone, 2007, p. 1232) akin to the possession of a material property like density or elasticity. The independence of slip length from shear for some fluid-solid pairs is a property of those systems, which depends essentially on the existence and qualities of the fluid boundary condition. Some influential views of laws, such as Dretske (1977), hold that laws are relations among properties, and in such a view the boundary condition would, in this instance, be a part of the law in virtue of being part of an important fluid property. Even Outside of such views, it is clear from this example that boundary conditions are not playing a role distinct from that of laws in the generation and constraining of fluid behavior.

One distinction that has been drawn between lawlike generalizations and other parts of explanation is that laws are stable or invariant, whereas other parts of explanation are highly variant and dependent on the particulars of a given situation. Historically, boundary conditions are treated as occupying the latter territory. However, while some boundary condition are variant under a wide range of interventions, this variance is sometimes overestimated in fluid dynamics, since it is often the boundaries themselves and not the boundary conditions that are variant. I discuss the distinction between defining boundaries and specifying boundary conditions at more length in later chapters, so for now suffice it to say that defining the shape of the boundary is not the same thing as defining the boundary conditions. An intervention that changes the shape of the channel through which a fluid flows changes the boundary, but not the boundary condition: the new channel shape will still be governed by the same slip conditions. This is an important distinction, and one which I believe most philosophical writing on boundary conditions has overlooked. In the case of slip in particular, it is highly invariant, and that is precisely why it is theoretically useful to the people who employ it in their models.

Not all boundary conditions are like the Navier slip condition. The slip boundary condition is very different from, say, an inlet condition that specifies the flow velocity at some region of a pipe. This region is the “beginning” of the flow, where the velocity of the flow must take a certain value. This boundary condition will influence the rest of the flow just as much as a wall boundary condition like slip. Mathematically, it might appear to have lawlike properties similar to the slip boundary condition. But in contrast to the wall boundary condition, this is something that can be intervened upon much more easily in practice. In a real system, we can intervene upon this boundary condition by, say, increasing the fluid pressure in the pipe, thereby increasing the velocity at the beginning of the flow. This is not to say that one cannot intervene upon the slip boundary condition. However, intervening on this sort of condition cannot be done directly. Rather one would have to change something else about the system, like the material that makes up the boundary or by changing the composition of the fluid. And changing the boundary conditions by changing the material of the boundary is like changing the parameters that go into the governing equations by changing the material the fluid is made of.

Using an interventionist account of lawlikeness, it is easy to see how boundary conditions can figure in explanations in more than merely circumstantial ways. Changes to boundary conditions are possible to see in the same way. However, the actual explanatory role of a boundary condition in a given explanation must be determined by paying attention to the details of the model. Not all boundary conditions have the same degree of invariance under intervention as the slip boundary conditions. For instance, consider a fluid flowing through a channel. The flow can be determined using the governing equations, boundary conditions, and the shapes of the boundaries themselves. In order to describe the flow field, we need boundary conditions besides the ones that determine what happens at the solid-fluid interface. For example, we need to know the velocity of

the fluid. This sort of boundary condition is often relatively easily intervened upon. We can adjust the flow of fluid through a pipe for example. Alternatively, consider the flow of air over a wing. We model the velocity of the wing by setting the boundary condition. In these instances, there is a lot of variation in the degree of variance under intervention, depending on what sort of boundary condition and what sort of explanatory setting is being considered.

1.3.2 The Scope of Boundary Conditions

The degree of variance under intervention leads directly to the issue of scope. It might be thought that the scope of boundary conditions might be a way to distinguish them from laws. Scientific laws tend to have relatively broad scope. It might be claimed that the scope of the governing equations, but not the scope of the various boundary conditions, is sufficiently large so as to do the explanatory work in causal explanations. Pincock, for instance, suggests that by contrast boundary conditions might not be thought of as part of a theory because a theory purports to have universal scope over its subject matter. (Pincock, 2011) It does seem to be true that the governing equations have a wider scope, since they are used in some form no matter what the fluid dynamical problem is. But kinds of limits to the scope of the governing equations are similar to the kinds of limits of scope on boundary conditions.

Further, there does not seem to be a principled way of deciding just when a regularity's scope is large enough to count as a law. Recall that the governing equations can take several forms. Whether one uses the viscous or non-viscous form depends on what kind of system is being modeled. Sometimes this choice involves using a simpler yet less realistic model. For example, there are fluids with very low viscosities that can be modeled as if they do not have any viscosity, because we can safely neglect the small amount of viscosity they do have. But this is not always

the case, since in some cases, there are actually no viscous forces active in the fluid. This is the case for superfluids, which have zero viscosity. It is also possible for viscous fluid flows to form “inviscid flow arrangements”, which are vortex-like fluid formations such that viscous forces vanish. (Runstedler, 2013) Taking another step back, the governing equations of fluid mechanics only hold when the continuum approximation holds. (Chen, Wang, & Xia, 2014, p. 114) The Knudsen number of a system is the ratio of the molecular mean free path to the system’s characteristic length. If a system’s Knudsen number is greater than some threshold (approximately 1), then the continuum governing equations cannot be used to characterize the system, and molecular dynamics must be used instead.

So the scope of the governing equations is not universal, and is limited by principled conditions. This makes the application of the governing equations structurally analogous to the application of slip boundary conditions: just as principled types of context determines which form of the governing equations to use, so do principled types of context determine which kind of slip boundary condition to use. While the scope of the governing equations is generally greater than that of boundary conditions, neither has universal scope over its subject matter. The scope of respective boundary conditions varies too. The scope of the no-slip condition is considerably smaller than the scope of the more general Navier slip condition. In any case, though, scope cannot be used to generate a categorical distinction between governing equations and boundary conditions.

1.3.3 The Epistemology of Boundary Conditions

Another apparent difference between the governing equations of fluid mechanics and boundary conditions is the theoretical support that the laws of fluid mechanics have. At first, it

seems that boundary conditions do not enjoy the same kind of theoretical support that the governing equations do. As noted above, these governing equations of fluid mechanics can be derived from some basic physical principles of conservation applied to fluid systems. In textbooks, there are often chapters devoted to deriving the governing equations from more fundamental physical principles, but only a few paragraphs or even a few sentences devoted to introducing boundary conditions like the no-slip condition. This might be because knowing which boundary conditions to use has historically had less to do with theory, and more to do with empirical methods. So while the governing equations had theoretical backing as well as empirical confirmation, the correct boundary conditions are phenomena for which there was not well established theoretical support. But while it is true that boundary conditions like the no-slip condition were developed in a more empirical way than the governing equations, this is a rather gross simplification. Evidence for boundary conditions has come via several different avenues, as other physical arguments have been made for their applicability.

The no-slip condition was the subject of controversy in the 18th and 19th centuries. Though it was unobservable at a macroscopic level, it played an essential role in predicting macroscopic flow quantities. The behavior of fluids directly on the boundary was, and still is, generally hard to observationally confirm. Some early evidence for the no-slip condition came from Bernoulli (1738) and Coulomb (1800). Bernoulli noticed large discrepancies between results measured for real fluids and results he calculated for ideal fluids. He recognized that real fluids could not slip freely over the surface of a solid body. Coulomb found that the resistance of a metal disk in a fluid was not appreciably altered when the disk was covered in grease (to lower resistance) or when the grease was covered with powdered sandstone (to increase resistance). In the 19th century, several hypotheses were put forward, without conclusive support for any of them. Gradually, as

experimental evidence accumulated, it became generally accepted that in most cases, there is no slip at the fluid-solid boundary. (Goldstein, p. 678)

In addition to experimental investigation, some theoretical arguments were made in support of the no-slip condition.³ For example, for geometrically similar systems, non-dimensional quantities like force coefficients depend only on Reynolds number, which is the ratio of inertial forces to viscous forces. If there were slip, then in addition to characteristic length d , another length, l , must be used to specify the thickness of the boundary layer of the fluid. So, when the dimensions of the system are manipulated, non-dimensional quantities would depend on l/d as well as Reynolds number. So unless l varies in proportion to d , which would be odd, the experimental evidence indicates that l is zero.

Additionally, it was argued that assuming slip would result in strange implications for differences in friction between a solid and a fluid, on the one hand, and the between two layers of fluid, on the other. Slip would imply that the friction between a fluid and a solid is infinitely less than the friction between two layers of fluid. Within a fluid, a shear stress between fluid elements produces a deformation, but velocity changes continuously. But slip between a fluid and a solid boundary would mean a discontinuity in velocity as you move from the fluid to the solid. (Goldstein, pp. 676-680)

Models of most ordinary fluids assume a no-slip boundary condition, as well as the condition that the component of the velocity normal to the wall is also equal to zero. The no-slip condition is usually assumed to be valid when the continuum assumption holds and the fluid is viscous. Otherwise slip might occur and another boundary condition must be used. For example, slip occurs in gas flows in systems with high Knudsen numbers, which have dimensions that are

³ See Goldstein (1938) for a summary of the history of the no-slip condition.

on the order of the mean free path of the gas molecules. In such systems, the continuum condition no longer holds, and statistical mechanics, rather than fluid dynamics, is the appropriate theory to use. Other examples where the no-slip condition should not be used are non-Newtonian fluid flows, which have a viscosity that is dependent on stress, and superfluids, which actually have zero viscosity. Finally, there are some conditions that allow for slip in fluids which would otherwise not display slip. For example, a liquid with a dissolved gas displays slip in some cases, though it depends on what the fluid is and what the gas is.

More recently, a wide variety of experiments are used to investigate slip phenomena. Although there is much to be learned about the microscopic conditions at the boundary, the no-slip condition is considered correct for ordinary viscous flows at the macroscopic scale. There are several methods used to detect slip. Indirect methods infer slip length λ by measuring some macroscopic quantity and using known equations of fluid mechanics to derive the result. (Cheng & Giordano, 2002) That is, any slip is estimated by way of the assumed effect of slip on some other macroscopic parameters. For example, recall the relationship between slip and flow rate mentioned earlier:

$$\frac{Q(\lambda)}{Q_{NS}} = 1 + \frac{4\lambda}{a} \quad \text{Equation 3 Flow rate through circular pipe}$$

A known pressure drop Δp is applied to a fluid in a small channel, and the resulting flow rate Q is measured. The degree to which the a slip boundary condition gives a flow rate $Q(\lambda)$ that is larger than the flow rate for a no-slip flow, Q_{NS} . Although such methods are still used, more recently, local methods are used to verify the existence of slip directly. For example, the method of particle image velocimetry uses small particles as tracers in a flow, and then uses optical methods to

measure their velocities and see whether the velocities extrapolate to zero at the boundary. (Pit, Hervert, & Leger, 1999)

There is still much to learn about the behavior of fluid near solid boundaries at smaller scales, and more recent investigations of slip have relied on computer simulations that model fluids at the molecular scale. And in addition to the historical arguments for the no-slip condition, there is more recent trend of justification for the no-slip (and slip) condition: molecular simulations. (Koplik & Banavar, 1995) These are computer simulations which model fluids as collections of discrete particles, and which use molecular models of the interactions between liquids and solids. Generally, for the fluid, these simulations use Newton's law of motion for single atoms in combination with some interaction potential to model the interaction between molecules. And for the boundary, a solid is modeled as a lattice, with some spring constant usually added to allow momentum transfer from the liquid. This method of molecular simulations sheds light on how we should think of the no-slip condition. The simulation itself is not a fluid dynamical model, so the results of such simulations must be interpreted in the continuum limit in order to be of use in fluid dynamical models. Here, we are using the (in some sense) more realistic model in order to derive information about the less realistic one. But while there are good empirically derived guidelines for when the condition applies and good molecular models that describe flows past a boundary at the micro scale, the exact mechanism that determines slip is still not totally understood. As the next chapter will show, we need to be careful in making inferences about boundary conditions by looking at molecular simulations.

Finally, boundary conditions like slip or no-slip are representational in the sense that they represent physical boundaries in the world. Boundary conditions like slip or no-slip along a solid surface or a continuous stress and velocity across two fluids with different viscosities both

represent physical boundaries in real fluid systems. On the other hand, some boundary conditions are not representative of physical boundaries in the world. Boundary conditions like inlet and outlet conditions do not necessarily represent physical boundaries in the world. Rather, they represent the fact that a model must start and end somewhere.

The upshot of this discussion, and especially of these last points regarding molecular simulations and the representational role of boundary conditions, is that boundary conditions can live lives just as rich as the governing equations. Not only are they necessary to make a fluid mechanical model work, but choosing the right one is not an arbitrary matter. Boundary conditions are not just the background conditions in which fluid dynamics is set. Rather, they are an integral part of the field. In the case of slip, there is empirical and theoretical support that seriously undermines the idea that this boundary condition is an entirely contingent matter of fact.

The three dimensions of lawlikeness that I have looked at above are not the only features of laws.⁴ They are not wholly independent. Indeed, the scope of a law or boundary condition might just be another way of talking about its degree of invariance under intervention. They are not wholly dependent on each other either. A given boundary condition might be more lawlike than a law according to one measure, and less lawlike according to another, when compared to a given law.⁵

⁴ See, for example, Mitchell (2000), which looks at the dimensions of stability, strength, and degree of abstraction of claims in biology.

⁵ Thanks to Porter Williams for pointing out this feature of the account.

1.4 Unique Roles for Boundary Conditions in Explanation

In the previous sections we saw that the difference between boundary conditions and laws is sometimes fuzzy. Boundary conditions that are more lawlike provide a much different purpose in explanation than boundary conditions that are less lawlike. This is relevant for theories of explanation that designate special roles for laws or lawlike statements. In these theories of explanation, a boundary condition cannot be prescribed a role simply in virtue of the fact that it is a boundary condition. Given the variety of boundary conditions, their particular explanatory roles depends on their role in scientific models. We can contrast the way a boundary condition like the Navier slip condition might take on a decidedly lawlike role in an explanation with the way a boundary condition like an inlet condition might take on the role of a determining condition or some particular fact.

For example, a deductive nomological model of explanation generally distinguishes between lawlike and non-lawlike premises. Hempel emphasizes the important role of laws in explanation: “the laws connect the explanandum event with the particular conditions cited in the explanans, and this is what confers upon the latter the status of explanatory (and, in some cases, causal) factors in regard to the phenomenon to be explained.” (Hempel, 1962) Boundary conditions like the Navier slip condition do indeed take on the role of laws in a deductive nomological explanation. That said, they can take on the role of particular conditions as well. An inlet boundary condition that specifies the fluid velocity at a particular region of the flow does this.

We can look at other theories of explanation, too. For example, take Woodward’s (2003) interventionist account. Here, the minimal condition for successful explanation is:

Suppose that M is an explanandum consisting in the statement that some variable Y takes the particular value y . Then an explanans E for M will consist of

(a) a generalization G relating changes in the value(s) of a variable X (where X may itself be a vector or n -tuple of variables X_i) and changes in Y , and (b) a statement (of initial or boundary conditions) that the variable X takes the particular value x . A necessary and sufficient condition for E to be (minimally) explanatory with respect to M is that (i) E and M be true or approximately so; (ii) according to G , Y takes the value y under an intervention in which X takes the value x ; (iii) there is some intervention that changes the value of X from x to x' where $x \neq x'$, with G correctly describing the value y' that Y would assume under this intervention, where $y \neq y'$. (Woodward, 2003, p. 203)

Now see how different boundary conditions play different roles under this condition. A boundary condition like the Navier slip condition relates shear rate to the amount of slip at the boundary. This sort of boundary condition functions like the generalization G . In contrast, a boundary condition like an inlet condition functions like a variable X that takes a particular value. It can be intervened upon, and via a generalization G (in this case the governing equations), another variable (some property of the flow like volume flux) would be changed.

At first blush it might seem that the difference between boundary conditions and laws is just a matter of degree. But while there are certain similarities, boundary conditions have their own unique properties as well. These properties allow them to do work that laws cannot. Of course, as a matter of mathematical fact, they are simply different sorts of things, and so have different functions in scientific models. They put a different kind of constraint on a system than differential equations. After all, there is a reason we make a distinction in the first place. They are just different sorts of mathematical objects. But their properties also allow for different conceptual roles in philosophical accounts. The degree to which a boundary condition is lawlike is not the most

important factor in the role of the boundary condition in an explanation. Their mathematical roles must be taken into account. And their particular places in fluid mechanical models make a difference too.

The big point I want to make is that the fact that a piece of a model is a boundary condition is by itself not a reason to assign it a particular role in an explanation. Instead, as I have shown throughout this chapter, boundary conditions play multiple important roles in scientific models and the explanations they generate. I believe, further, that boundary conditions also play roles in explanation that have not yet been identified in the literature and which cannot be understood merely by similarity to or difference from the roles played by laws.

One way that boundary conditions do unique work in explanation is in the way they constrain a system. Wilson (1990) characterizes the situation in terms of internal requirements and external requirements that are put on a system. For a given region with a boundary (the region is a piece of iron in Wilson's example), there is a differential equation that tells the system how to behave within the region. This equation would seem to place certain requirements on the system at the very edge of the region. But there are also external requirements put on the system by the boundary. And there is an apparent mismatch between the requirements of the differential equation and the requirements of these boundary conditions. Importantly, as Wilson also points out, physicists do not always give priority to the internal requirements. The most straightforward way of solving these sorts of conflicts is to let the inner requirement approach the outer requirement in the limit. When scientists model a material with a boundary, they often have prior knowledge about what sorts boundary conditions work for a material. And this knowledge is often independent of what we know about the internal requirements.

The scenario is similar to cases in fluid dynamics. Recall that while the governing equations had been derived from known conservation laws applied to fluid systems, our knowledge of the no slip boundary condition comes from a wide range of sources. Besides empirical confirmation, there was an argument from simplicity (since slip would seem require an extra parameter that depended on distance to the boundary) and molecular simulations. The way these two sorts of requirements interact is not a trivial matter. So the role of the boundary condition in explaining some feature of the system is not a matter of whether or not the boundary condition acts a covering law or play some other logical role. Instead, and following more closely with the treatment of boundary conditions in the philosophy of physics discussed above, the explanatory role played by boundary conditions in these cases is a product of the mathematical role they play in the systems of equations that constitute the fluid mechanical models. As I discuss in the remainder of this dissertation, and particularly in Chapters 2 and 3, the mathematical structure of boundary conditions is often irreducible to a set of philosophically-imposed structural constraints.

1.5 Conclusions

Boundary conditions are an often neglected feature in philosophical accounts of scientific explanation, scientific laws, causation, and reductionism. But they are often just as important as the equations they constrain. I have explored the role that boundary conditions play in fluid mechanical modeling. Understanding how boundary conditions function in actual scientific models should guide philosophical understanding of their role in explanation, etc. But since the role of the boundary conditions often depends on the details of the model, it is doubtful that a

single account will apply in all cases. But the hope is that a more technically correct understanding of boundary conditions can be used in discussions of more general issues in philosophy of science.

Philosophers have gotten their accounts of boundary conditions wrong, because they have been working backwards in some sense. They have been trying to fit the behavior of boundary conditions into a philosophical account first, before observing boundary conditions in their natural habitats in physics. But by looking at them in their natural habitat, we see that they do not always fit neatly into a particular philosophical theory. The difference between laws and boundary conditions depends on their role in the theory, often defined by their mathematical properties. Their role depends on their relationship to differential equations, not their role in explanations.

Boundary conditions are well defined mathematically, and their relation to differential equations is clear. But the fact that something is a boundary condition does not prescribe a particular role in a theory of explanation. Their philosophical role is less well defined and heavily context dependent. Some boundary conditions are highly variant under intervention, others are not. Some are only useful in very particular contexts, others have a scope almost as large as the governing equations. Some rely solely on empirical considerations for justification, others receive a lot of theoretical support. But this variety does not mean that we cannot assign philosophical roles to them; it just means we have to be careful when we do.

Boundary conditions have not only not gotten their due as explanantia, they have also been under appreciated as explananda. Attempts to explain boundary conditions have drawn on both top down and bottom up approaches. We already saw that historically explanations for boundary conditions have developed in rather piecemeal fashion, with evidence ranging from arguments about the length scales needed to characterize a fluid system, to inferences based on other

macroscopic fluid properties, to molecular simulations. The exact mechanism for slip and no slip conditions is still not well understood.

I began this chapter by outlining two ways in which philosophers of science have historically theorized about boundary conditions. My discussion of the slip boundary conditions in fluid dynamics aimed to show how neither of these modes of theorizing are adequate frameworks for capturing the varied explanatory roles played by boundary conditions in contemporary physical explanations. I have highlighted problems in philosophical understanding of the explanatory role of boundary conditions in order to motivate the account of boundary conditions that I develop in the chapters that follow. Unlike the philosophers I have argued against, I will begin from the boundary conditions first, aiming eventually to understand their role in physical and scientific explanation by inquiring after their place in other parts of the landscape of philosophy of science. In Chapter 2, I develop a contrast between boundary conditions and conditions at the boundary, in order to uncover a distinction that has gone as-yet unnoticed in the philosophy of science. In Chapter 3, I consider how the very scale at which boundaries exist in physical modeling impacts the epistemic and explanatory roles they can play. And in Chapter 4, I use the example of boundary conditions to make an argument for realism. Together, these pieces paint an alternate picture of the explanatory roles of boundary conditions.

2.0 The Difference between Boundary Conditions and Conditions at the Boundary

In the previous chapter, I discussed how boundary conditions work in fluid dynamics, explored some of their features, and noted how important they are in building scientific models. I argued that boundary conditions can play varied roles in scientific explanations. In this chapter I will look at a distinction that goes underappreciated or even unnoticed by both philosophers and scientists: the distinction between boundary conditions and conditions at the boundary. In contrast to boundary conditions, conditions at the boundary explicitly take into account the interactions between the fluid and the boundary. This distinction was first described by Howard Brenner and Venkat Ganesan (2000) in the context of models of fluid diffusion. They show how a molecular description of a fluid near a boundary does not necessarily translate to the correct fluid dynamics boundary condition. I will argue that conditions at the boundary are distinguished from boundary conditions in virtue of two features: 1) they explicitly describe interactions between the fluid and the solid boundary, and in virtue of this fact, 2) they cannot play the constraining roles of boundary conditions, as the latter were characterized in Chapter 1.

First I will explain a conceptual difference between boundary conditions and conditions at the boundary. Then I will present Brenner and Ganesan's case for making the distinction, where they show how the conditions at the boundary described by a molecular model do not correctly predict the boundary conditions of a corresponding continuum model. They make the distinction in terms of the difference between fluid dynamics and molecular dynamics, where the former accompany boundary conditions and the latter accompany conditions at the boundary. I will argue, contra Brenner and Ganesan, that this is not the important feature of the distinction. Rather, the important part of distinguishing boundary conditions from conditions at the boundary is the way

the two parts of scientific models take into account the interactions between fluid and boundary. As evidence, I will show that we can describe conditions at the boundary using continuum modeling, and we can describe boundary conditions using molecular modeling. There are constraints out on molecular dynamics systems that are boundary conditions, and there are conditions at the boundary in fluid dynamics. So the distinction between boundary conditions, on the one hands, and conditions at the boundary, on the other, is best made in terms of whether or not boundary interactions with fluid are explicitly taken into account.

The distinction has important consequences for philosophical accounts of scientific explanation and intertheory relations, as well as being important to the construction of scientific models. I will argue that boundary conditions and conditions at the boundary occupy different explanatory contexts. They play different roles in explanation because they have different explanatory targets. The explanatory shift is a result of the content of the condition at the boundary rather than its form. This is particularly relevant for issues surrounding intertheory relations. Given the difference between boundary conditions and conditions at the boundary, we must be careful when making inferences about the former based on the latter.

2.1 A Conceptual Difference

Before we look at the difference, we must clear up the way we are using the respective concepts. Right away, we can identify a conceptual difference between boundary conditions and conditions at the boundary. In a mathematical context, a boundary defines the area in which a differential equation operates. A set of differential equations and boundary conditions constitutes a boundary value problem, the solution to which must satisfy the boundary conditions. In this

context, a boundary condition is a mathematical entity with a specific mathematical function in a system of equations. For example, they can prescribe the value of unknowns on the boundary (Dirichlet boundary condition), specify the normal gradient of the unknowns (Neumann boundary conditions), or specify a combination of unknown quantities and their normal gradients (mixed boundary condition). (Massoudi, 2007) These systems of equations including boundary conditions are used in fluid dynamics to represent fluid flows. In fluid dynamics and physics generally, boundary conditions are indispensable parts of models. Recall some of the examples from the previous chapter, like liquid flowing through a channel or air flowing past an airfoil. The same set of governing equations are used to build models of very different looking flows. The difference in these flows depends differences in boundary conditions, alongside differences in the configuration of the boundaries themselves, and sometimes differences initial conditions. A set of boundary conditions in a flow model might represent how a fluid behaves at a physical boundary like a solid wall. The no-slip boundary condition, which requires that fluid at the boundary has zero velocity relative to the boundary, is an example of this. Boundary conditions can also define other parts of the flow as well, like an inlet condition of constant flow velocity into the system, where there is no corresponding physical boundary.

In contrast to a boundary condition, a condition at the boundary is a less well-defined concept. At first approximation, conditions at the boundary are what happens at or near the boundary of a physical system in the actual world. For our purposes, a boundary in this sense is the interface between a solid and a fluid. A condition at the boundary can be the temperature of the fluid near the boundary, or the movements of individual atoms in this region, or transfer of momentum between the solid and the fluid, or dielectric differences between the fluid and the

solid, or so on. Conditions at the boundary include the region near the boundary, but they may also include the boundary itself.

There is at first an obvious difference between boundary conditions and conditions at the boundary in virtue of the kinds of things they are. As I've described them above, a boundary condition is a mathematical object, while a condition at the boundary is a part of the actual world. That is, there is a difference in ontological kind, and it would be a category mistake to use them interchangeably. One concept applies to the world; the other applies to mathematically defined models. But this ontological difference is not the interesting distinction that I am trying to define. In order to see the distinction I am after, we have to be able to make the two concepts commensurable. To do that, we need to be able to choose one context, either the world or models, and talk about both concepts in that context. For this discussion, it will be easier to work in the context of models. Boundary conditions, being parts of a model, can be used to represent the world. In the same way that other equations which define a fluid model represent the world, so can boundary conditions. Likewise, features of the world can be represented in a model, and are often represented by pieces of mathematics. In this way, condition at the boundary, being features of the world, can also be parts of models. So conditions at the boundary can be represented in a model, and we can compare these concepts in meaningful ways. We can also compare the features of the world that these two concepts represent. Most of what follows has to do with scientific models. So unless specified otherwise, I will assume that both boundary conditions and conditions at the boundary are features of models. But we should keep in mind that all of these models represent the world.

Now that we can compare the two concepts in the context of a model, we can begin to see that they have different properties. Even though a condition at the boundary is now part of a model,

it is not the same thing as a boundary condition. For one thing, a condition at the boundary can be modeled using whatever theoretical apparatus is appropriate. For a fluid and a solid boundary, the theoretical apparatus is often a molecular dynamics model. Even if it were to be described in terms of fluid dynamics, we should not assume that it is a boundary condition, since a mere description is not a boundary condition unless it plays the specific mathematical role mentioned above. For example, we might give a description of the conditions at the boundary such that a fluid has zero velocity. But this is not the same thing as the no slip boundary condition. In a sense, they are both descriptions, but boundary conditions have additional mathematical infrastructure, and additional constraints, built into them. A boundary condition stands in a particular relationship to a system of equations, and so plays a particular role in a fluid model. In contrast, a condition at the boundary only needs to describe part of a flow; it does not need to constrain the rest of the flow the way a boundary condition does.

One way to see the difference is to look at the Navier slip condition itself:

$$u_s = \lambda \left. \frac{\partial u}{\partial x} \right|_s \quad \text{Equation 4 Navier slip condition}$$

We can think of the fluid velocity at the boundary, u_s , as a condition at the boundary. In the last chapter, I argued that boundary conditions have a more lawlike character than is often acknowledged. This boundary condition will hold in a wide variety of conditions. It becomes more specific when a slip length λ is specified. If other flow conditions are intervened upon, then the conditions at the boundary will change, even though the Navier slip condition still holds. The boundary condition is a generalization that holds, while the condition at the boundary is a result of

the application of that generalization. Since its function is to constrain solutions to differential equations, the way it represents the world is significantly different from conditions at the boundary.

So even if we make these concepts commensurable, a conceptual difference remains. But the difference between boundary conditions and conditions at the boundary goes even further this conceptual difference. As I will show in the next section, there is difference not just in the role, but in the content of the boundary conditions and conditions at the boundary.

2.2 A Difference in Content

When modeling fluids at the macroscopic scale, it is often sufficient to assume a no-slip condition. But if we are interested in modeling the smaller-scale behavior of the fluid, there is limited evidence from which to draw. For a long time, experiments relied only on inferences made from other macroscopic observables. Only relatively recently have direct detection techniques been used in slip experiments. There are difficulties in using either of these available experimental strategies to detect the degree of small scale slip. (Lauga, Brenner, & Stone, 2007)

Recently, computer simulations have become an increasingly powerful tool to examine fluids at the molecular scale. These molecular-scale simulations can give insight into unsolved questions in boundary phenomena in fluid dynamics. There are still open questions about the microscopic behavior of a fluid at a boundary and how this behavior translates to macroscopic behavior, but there are ways of making these two pictures, the continuous fluid and the discrete molecular, talk to each other. Roughly, in a molecular model, the mean molecular motion at a given distance y from the boundary can be averaged. If this value equals zero at $y=0$, then intuitively we can say that there is no slip at the boundary. We can define any amount of slip

greater than zero in an analogous manner. This ability to link a molecular model with a continuum model gives the impression that we can take the boundary condition of the continuum model as an accurate description of the fluid at the boundary. In some cases, it might be an accurate description. For example, the no slip condition typically holds for a viscous flow past a boundary, and a molecular model typically models the mean molecular motion as approaching zero at the wall. Using these sorts of connecting principles, there have been numerous attempts to predict and explain slip phenomena using molecular dynamics. (Thompson & Robbins, 1990; Barrat & Bocquet, 1999; Koplik & Banavar, 1998; Koplik & Banavar, 1995)

In response to these sorts of investigations, Brenner and Ganesan (2000) have presented an argument that it is not correct to use molecular dynamics to draw conclusions about the fluid dynamics that describe fluids at the continuum scale, using their distinction between boundary conditions and conditions at the boundary. The conceptual distinction between boundary conditions and conditions at the boundary becomes evident when the conditions at the boundary are described using tools of molecular dynamics simulations. When a matching procedure is used to try to turn this description into a boundary condition, it describes a lack of slip for the fluid at the boundary. However when one tries to apply Fick's law (discussed in more detail below), which is given in terms of fluid dynamics, to the same system, a slip condition is required. The result is that there is no clear translation procedure to derive a fluid dynamical boundary condition from a given set of conditions at the boundary. Moreover, the two likely candidates to do the translating offer conflicting suggestions about the character of the boundary condition that could result.

These models, the molecular dynamics model and the fluid dynamics model, both have a high degree of experimental and theoretical support. Yet when we treat the conditions at the boundary as boundary conditions, we run into an inconsistency. Specifically, the no slip boundary

condition should not be used in the continuum model, even though the molecular simulation model seems to call for it. The conclusion is that slip lengths should not be regarded as accurate representations of the conditions at the boundary, nor should the molecular conditions at the boundary be used to determine boundary conditions. Even if the no slip condition can be derived from a molecular simulation, it need not (and in some cases should not) apply to the continuum equations that govern the flow on the macroscopic scale.

Looking more closely at Brenner and Ganesan's argument, we consider a fluid next to a solid boundary. There are two main length scales. On the one hand there is the molecular length scale l that characterizes both the scale of the fluid-fluid (f-f) interactions between the fluid molecules and the solid-fluid (s-f) interactions between the fluid molecules and the molecules of the solid boundary. These f-f lengths and s-f lengths are usually similar in magnitude, and so usually the same length scale l applies to both. On the other hand, there is the macroscopic length scale, L . This length scale is determined by the size of the apparatus being modeled. Typically, this length scale is defined by the distance between two planar walls that bound the fluid, or the radius of a channel through which the fluid flows. A molecular model must take into account the f-f and s-f length scales and the apparatus length scale L . However, due to computational limitations, current molecular models are limited to those for which L is on the same order of magnitude as l . So if modelers want to model a system more than a few intermolecular length scales in size, they must rely on continuum models.

Continuum models, by contrast, are built using governing equations that take into account the f-f forces, but not the s-f forces. The closer we get to a solid boundary, the more important the s-f forces become. Eventually, they cannot be ignored, and this is where boundary conditions like the no-slip condition are employed. As Brenner and Ganesan put it, "no single L -scale boundary

condition can ever fully compensate for the loss of detailed l -scale physical molecular information about the s-f (or possible even the s-s) interactions implicit in such a coarse-scale description.” (Brenner & Ganesan, 2000, p. 6881) In other words, the fluid dynamics that operate at the continuum scale simply do not have a way of accounting for the s-f forces described by the molecular model.

Brenner and Ganesan go on to describe a two dimensional fluid system that consists of a solute which diffuses down a concentration gradient in a solvent. The fluid is bounded by solid surfaces on the top and bottom of the diffusion cell. These are the surfaces where either a slip or no-slip boundary condition will hold. Meanwhile, the two side boundaries are solute reservoirs that supply their own boundary condition, namely that uniform solute concentrations are maintained at those boundaries. This maintains a constant concentration gradient which drives the diffusion, as molecules of solute move from high concentration to low concentration. The lengths of the boundaries determine the L length scale. We can then model the diffusive transport of the solute in two different ways. One is a continuum model that uses the assumptions of fluid mechanics. The other is a molecular model that posits Brownian particles that represent the solute molecules. Due to limits in computational power, the molecular model is only used to describe the region very close to the solid boundary.

The continuum description of the flow yields Fick’s law of diffusion, which describes the diffusive flux (the amount of fluid that flows through a unit area in a unit time) in relation to concentration. Fick’s law states that:

$$J = -D \frac{d\phi}{dx} \qquad \text{Equation 5 Fick’s law}$$

where J is the diffusion flux, which is the amount of substance that flows through a unit area in a unit time, D is the diffusion coefficient, ϕ is concentration, and x is position. It is important to note that the diffusion coefficient D is position independent. In order for the equations to correctly model the diffusion, they need to be constrained by certain boundary conditions that maintain a concentration gradient. Crucially, in order for Fick's law to correctly describe diffusive flux, the no slip condition does *not* hold. If it did hold, the result would be a conflict with the other boundary conditions that maintained the gradient.

The molecular model considers the transport of Brownian particles near a smooth solid boundary. The molecular motion is generated using a Langevin equation, which describes the stochastic motion of Brownian particles. The model produces a probability field which then corresponds to a density concentration field. This simulation using the molecular model yields a "no flux" condition at the boundary. That is, there is no fluid movement at the boundary. But note that this is *not* a boundary condition; it is a consequence of the molecular wall effects that fall out of the physics of the molecular simulation. The flow field in this lower level model has the property that both the normal and tangential (i.e. slip) components of the flow go to zero at the wall. This is the key result of the molecular model.

The crux of the discussion is this: while the continuum model requires a slip boundary condition, the molecular model tells us that the fluid at the wall does not slip. So if you were to try to derive boundary conditions from conditions at the boundary of the molecular simulations, you would not get the right boundary conditions needed to make the continuum equations work. Using this result, Brenner and Ganesan conclude that the boundary conditions imposed on the macroscale model cannot be derived by taking the conditions at the boundary that emerged from of the lower level model and interpreting them as boundary conditions. Lauga, Brenner, and Stone summarize,

“slip lengths should not be measured literally at the molecular scale but arise as the extrapolation, at the boundaries, of the far field hydrodynamic results.” (Lauga, Brenner, & Stone, 2007, p. 1228)

Brenner and Ganesan go on to use a singular perturbation analysis to arrive at a solution that is a composite of the microscale and macroscale solutions, composed of the microscale, or inner, solution and the macroscale, or outer, solution. The solution describes a singular layer near the wall, where Fick’s law is not valid. The boundary condition used on the macroscale equations was derived by matching the inner and outer solutions. So rather than imposing the condition derived from the molecular model, the outer limits of the inner solution are the boundary conditions that we must use on the Fick’s law equation. As Brenner and Ganesan put it, “it is precisely because this boundary condition is an asymptotic *matching* condition, rather than a *literal* condition prevailing at the actual physical solid-fluid boundary, that current molecular dynamics simulations furnish conditions at a boundary that are generally inconsistent with conventional continuum-mechanical boundary conditions.” (2000, p. 6881)

The sort of issue that Brenner and Ganesan highlight in this diffusion case is not unique to that particular case, or to diffusion more generally. Flow near contact lines is another place where there is apparent conflict between the boundary condition and the conditions at the boundary.⁶ Consider two immiscible fluids that are in contact with each other and with a solid surface. The line that forms the boundary between the two fluids is called the contact line, and it forms a contact angle with the solid. The two fluids can both be liquids or they could be a liquid and a gas. Think of a drop of water sitting on a solid surface. We observe the contact line move if more liquid is added to the drop, causing it to spread out, or if it is subjected to a force, as it would if it were on

⁶ This type of situation is of increasing practical interest, as it describes the conditions needed for vapor-liquid-solid growth methods of growing metal nanorods. Thanks to Julia Bursten for pointing this out.

an inclined surface and gravity pulled it down across the surface. The continuum description of moving contact lines must employ a slip boundary condition in order to match observations and avoid singularities that cannot be integrated over. Molecular simulations of moving contact lines has been unable to explain the boundary condition necessary to avoid this.

Since the Brenner and Ganesan article, there has been continued discussion regarding the use of molecular models to determine boundary conditions. For example, Denniston and Robbins (2001) have been able to use molecular models of convective-diffusive flows, in which diffusive flow is either dominant or on the same scale as convective flow, to produce flows consistent with Fick's law within one molecular diameter from the boundary. Despite this, Brenner and Ganesan's main philosophical point remains, namely that molecular dynamics explicitly takes into account fluid-solid interactions in a way that the boundary conditions of fluid dynamics cannot by themselves. This does not mean that we will never be able to use conditions at the boundary to explain boundary conditions. But it does mean that we must always take care when using conditions at the boundary to make inferences about boundary conditions.

The difference between boundary conditions and conditions at the boundary has concrete consequences for scientists who model fluid systems. Practically, in the course of modelling fluids, context will determine the best model to use. Whether modelers are interested in the fluid-solid interface for its own sake, or in the overall shape of the flow, often determines whether they use a molecular model or a continuum model. But there are contexts in which they need to pay attention to both. The smaller the system, the more the molecular conditions at the boundary matter. For instance, if modelers want to model a flow through a capillary with a radius on the order of tens of nanometers, then the conditions at the boundary will take up a larger proportion of the flow. So

it is sometimes important to not only get the boundary conditions that will describe the bulk flow correctly, but also to get the right conditions at the boundary.

When talking about experiments, scientists often seem to conflate boundary conditions and conditions at the boundary. Experiments used to investigate fluid flows near the boundary are often described in term of boundary conditions only. But the distinction is tacitly acknowledged when discussing results. For example, when a slip boundary condition is found to be appropriate, experimenters try to distinguish between microscopic slip that only happens at the scale of individual molecules, macroscale slip that is detectable at the continuum scale, and apparent slip. (Lauga, Brenner, & Stone, 2007)

2.2.1 Apparent Slip

Before I go any further, I need to comment on apparent slip. Apparent slip is when a slip boundary condition is the correct boundary condition to use in a model of a flow, even though the fluid at the surface satisfies the no slip condition. (Lauga, Brenner, & Stone, *Microfluidics: The No-Slip Boundary Condition*, p. 1229) This is usually because the region in which the no-slip condition holds is so small that it can be neglected when modeling macroscopic systems. In such cases, there is usually a layer of some other fluid, often a gas, between a fluid and a solid boundary. There are other known conditions that might cause apparent slip, such as heating that changes the viscosity of the fluid near the boundary or electrically charged fluids. But typically, an upper layer of fluid is moving over another fluid layer of lower viscosity, instead of the upper fluid itself slipping past the boundary. Stress must be continuous at the boundary between a liquid and a gas. And a difference in viscosities produces a difference of strain rates. But since the layer of gas is small, its shear rate is not used to estimate slip. Instead the shear rate of the upper layer fluid is

extrapolated all the way to the boundary surface. In modelling such flows, unless modelers are interested in the microscopic conditions near the boundary, a slip boundary condition is the often the best condition to use, since it will better represent the overall flow. If they were to try to model the thin layer of fluid, then a no slip boundary condition would have to be used for this boundary, and they would have to treat the boundary between the two fluids separately. This would require its own boundary condition. But in cases where they are not concerned with the thin layer, they can safely model the flow as if the upper fluid is just slipping past the boundary.

At first, it might seem like apparent slip is an example of the distinction I am trying to define. That is, it might seem like I am calling the no-slip region the “condition at the boundary”, while I am calling the apparent slip region the “boundary condition.” If this were the case, then it does seem like the boundary condition is not the same as the conditions at the boundary. It seems like we are using a slip boundary condition while the condition at the boundary is actually a zero velocity. But this is not the distinction I want to make. In cases of apparent slip, there is another boundary condition that is being suppressed for the sake of simplicity. This is not really the difference between a boundary condition and a condition at the boundary, but rather the difference between two boundary conditions. Instead of defining two boundary conditions (one at the boundary between the two fluids and one at the boundary between the bottom layer of fluid and the solid boundary), we have just defined one. Even if we know that the bottom layer of fluid should be constrained by the no slip condition, this is still a boundary condition and not a condition at the boundary.

However, the treatment of apparent slip does indicate that a boundary condition’s primary role is constraining differential equations, not describing the boundary. Unlike cases of apparent slip, Brenner and Ganesan’s example does not depend on layer of another fluid or other sources of

apparent slip, like heating that changes the viscosity of the fluid near the boundary. It is all a model of a single homogenous fluid. Their example comes only from comparing the molecular simulation and the continuum model of the same fluid.

2.3 Separating Differences in Theory and Scale from the Difference between Boundary

Conditions and Conditions at the Boundary

Brenner and Ganesan have presented the distinction between boundary conditions and conditions at the boundary in terms of fluid dynamics and molecular dynamics. They attribute the distinction to the macroscopic length scale present in the fluid dynamics but not the molecular dynamics. While they recognize that conditions at the boundary are characterized by the way they take into account interactions between the fluid and the solid, they do not distinguish this from the molecular model itself, or from the microscopic length scale. In this section, I will show how the distinction they make is not tied essentially to a particular model or a particular length scale. Not only do boundary conditions show up at the molecular scale, but conditions at the boundary show up at the continuum scale.

First, I will look at molecular models of boundary conditions that do not explicitly take into account the interactions between the fluid and the boundary. It is common for molecular dynamics simulations to build the solid boundary in the same way they build the fluid. They model both the fluid and the solid as discrete atoms or molecules that interact according to some interaction potential. While the fluid molecules are free to move around, the molecules that make up the solid are fixed to a lattice. This is not the only way to make a molecular dynamics model of a fluid near a solid boundary. Some molecular models do not build the boundary out of molecules.

Rather, they impose boundary conditions, which specify the motion of the molecules as they approach the wall. This is typically done using an accommodation coefficient, which is a way to quantify the behavior of fluid particles when they collide with a boundary. (Karniadakis, Beskok, & Aluru, 2005, p. 62) The tangential momentum accommodation coefficient σ_v is defined as:

$$\sigma_v = \frac{\tau_i - \tau_r}{\tau_i} \quad \text{Equation 6 Tangential momentum accommodation coefficient}$$

where τ_i is the tangential momentum of incoming molecules and τ_r is the tangential momentum of the reflected molecules. When $\sigma_v = 0$, there is completely specular reflection; the tangential velocity of the molecules stays the same, but the normal velocity is reversed. When $\sigma_v = 1$, there is completely diffuse reflection; the average tangential velocity of the molecules is zero. Other boundary conditions are a bounce-back reflection condition, in which both the tangential and the normal velocity components are reversed, and a Maxwellian reflection, in which reflected molecules follow a Maxwell-Boltzmann distribution that centers on the boundary velocity.

The important thing to see here is that the accommodation coefficient acts as a boundary condition. In contrast to the models that model the interactions via some interaction potential between the fluid molecules and the solid molecules, the information about the interactions are encoded in the accommodation coefficient. So we have boundary condition behavior in a molecular dynamics system.

In the other direction, there are also conditions at the boundary that take place at the scale of fluid dynamics. We can quantify wetting behavior of liquids on solid surfaces. Wetting is the ability of a liquid to stay in contact with a solid surface, resulting from the physicochemical properties of both the liquid and the solid. Wetting properties tell us about the two materials

interact. Here the interactions between the fluid and the solid are in terms of the respective materials' surface energies. For a liquid on a flat solid surface, the spreading coefficient S is:

$$S = \gamma_S - \gamma - \gamma_{LS} \quad \text{Equation 7 Spreading coefficient}$$

where γ_S is the solid surface energy, γ is the liquid surface energy, and γ_{LS} is the liquid solid interfacial energy. If $S > 0$, then the solid is said to be completely wetted as the liquid spreads spontaneously, and if $S < 0$, the surface is only partially wetted. In the case of partial wetting, a droplet of the liquid forms. Its shape is then characterized by a contact angle, which is the angle the liquid drop creates with the solid. The contact angle θ_c is given by Young's law:

$$\gamma \cos \theta_c = \gamma_S - \gamma_{LS} \quad \text{Equation 8 Young's law}$$

Again, γ_S is the solid surface energy, γ is the liquid surface energy, and γ_{LS} is the liquid solid interfacial energy. Contact angles tell us about the interaction between fluid and solid in a way that is related to slip. While there is no contact angle in a fluid flow, since contact angles are measured when a fluid drop is stationary, it represents a property of the fluid-solid pair. This is an example of conditions at the boundary present at the continuum scale fluid dynamics. It has been found that slip depends, to some degree and in some cases, on the wetting properties of a fluid on a solid.

In these cases, we are still in the domain of fluid mechanics, and thinking of fluids in terms of continuum mechanics. Yet according to the distinction we have made, spreading coefficients and contact are conditions at the boundary, not boundary conditions, because they are explicitly taking into account interactions between the boundary and the fluid. Of course they cannot take

into account the molecular interactions explicitly; molecules are just not part of the model, and wetting is simply not a molecular phenomenon. The wetting properties of a fluid and solid pair are not themselves boundary conditions, but they do correlate with boundary conditions. And like molecular conditions at the boundary we must be careful when making inferences to the correct boundary conditions. The interactions between these surface energies are related to boundary conditions, but are not themselves boundary conditions.

The upshot of this section is that the distinction that Brenner and Ganesan made does not depend on the theory or the scale being used to model the boundary. It is just that they explicitly take into account interactions between the fluid and the solid. This changes the explanatory context, which then allows them to play some explanatory roles, but prevents them from playing others.

2.4 The Explanatory Roles of Conditions at the Boundary

Conditions at the boundary take into account information about the boundary that boundary conditions do not. This allows them to play certain explanatory roles that boundary conditions cannot. It also means that conditions at the boundary cannot play certain roles that are played by boundary conditions, some of which I discussed in Chapter 1. These two components of models of systems with boundaries play different roles in explanation. This is not because either component occupies a particular role in an explanatory schema. Instead, it is because of the different roles they play in models and the relations they stand in with the rest of the parts of the model.

On the one hand, conditions at the boundary can provide explanations or rationales for boundary conditions. In the discussion above, I have highlighted cases when conditions at the boundary do not do this, in order to drive home the distinction. Nevertheless, studying the behavior of conditions at the boundary might help modelers make sense of the use of particular boundary conditions. Indeed, the investigations to which Brenner and Ganesan are responding is designed to do just that.

On the other hand, though, conditions at the boundary are not made to stand in the sorts of relations with the governing equations of fluid dynamics that boundary conditions do. Conditions at the boundary are the result of the interaction of fluid and boundary behavior. I argued in Chapter 1 that boundary conditions can stand in lawlike relations or be used as lawlike generalizations to constrain the solutions to the equations that describe a fluid system. Conditions at the boundary simply are not the kinds of things that can stand in such relations. Rather, as I have developed them here in the case of scientific modeling, conditions at the boundary arise as the consequences of applying lawlike generalizations to a molecular system. Under intervention on a fluid system, the conditions at the boundary are highly variant. They are a description of a system that results from applying lawlike generalizations, like Lennard-Jones interactions and steric wall interactions, to a molecular system. This is similar to a description of a fluid flow resulting from the application of the governing equations plus boundary conditions to a fluid system. Conditions at the boundary provide knowledge about what happens at the boundary, but they do not have the lawlike character of boundary conditions.

As highly contingent and context-sensitive features of a system, conditions at the boundary have more in common with accidental generalizations than lawlike generalizations. Boundary conditions are lawlike (at least in some ways) parts of solutions to boundary value problems. In

the diffusion example from Brenner and Ganesan, the boundary condition has some lawlike features. It displays these in conjunction with the laws governing a fluid system, in this case, Fick's law of diffusion. Conversely, the conditions at the boundary are the result of applying lawlike principles that describe the interactions between pairs of molecules. They are explained in terms of these principles along with the particular facts derived in the model.

A final point on how conditions at the boundary compare with boundary conditions in terms of evidential bases will close out this section. As a particular type of mathematical object, boundary conditions stand in a necessarily distinct kind of relationship to evidence than conditions at the boundary. Physical behavior of a particular sort might be evidence *for* the application of one boundary condition over another. For instance, a high viscosity flow will be evidence for the appropriateness of using the no-slip boundary condition. However, physical behavior will always be evidence *of* a particular set of conditions at the boundary.⁷ Nonetheless, there is not a clear line that distinguishes evidence for the application of a particular boundary condition from evidence of conditions at the boundary. The same observed physical behavior may offer evidence, or provide a rationale, for one or both type of condition.

Despite their differences, there is considerable overlap in the kinds of evidence that are gathered for each. There are, however, some tendencies in the ways each are confirmed. Experiments that infer the existence of slip from other macroscopic parameters speak to boundary conditions, but not to conditions at the boundary. As we have seen from our example, molecular simulations tell us about conditions at the boundary, but not necessarily boundary conditions. These models are then compared to other experiments that measure (instead of infer) fluid velocity as close to the boundary as possible. Experiments that use tracer particles, the closest way of

⁷ Thanks to Julia Bursten for pointing out this distinction.

measuring slip directly, potentially gives us information regarding both boundary conditions and conditions at the boundary. This serves to reinforce the initial discussion point in this section, which is that conditions at the boundary, in some cases anyway, might explain why a particular boundary condition is correct.

2.5 Reduction and Interttheory Relations

In addition to the general philosophical considerations I mentioned above, the distinction between boundary conditions and conditions at the boundary is relevant to intertheory relations generally, and to the question of reduction in particular. While I have just shown how they are distinct concepts, they are obviously related. And since our example included a molecular model and continuum model, old questions about theory reduction are raised. Basically, we have a macro level phenomenon that cannot yet be explained in terms of a micro level phenomenon. Even if we agree that in some sense fluid dynamics depends on the underlying molecular nature of fluids, it is not clear how those molecular facts produce the boundary conditions that we know should be used in fluid dynamics. The situation looks more complex than either a straightforward reduction or antireductionist account, especially given the ongoing research into slip boundary conditions. But the conceptual difference between the two concepts seems to remain, regardless of what molecular models will end up explaining.

There is an obvious antireductionist undertone in Brenner and Ganesan's example. We cannot derive the "upper level" fluid dynamics boundary condition from the "lower level" molecular model. In terms of explanation, molecular dynamics does not explain the fluid dynamics. However, a reductionist response might appeal to the incomplete nature of the theories

involved. The mechanism of boundary conditions is still not well understood, but we do know that in some sense, fluid mechanics relies on the underlying molecular nature of fluids. Surely, the reductionist says, advances in our understanding will allow future models to capture the fluid dynamics entirely. Brenner and Ganesan themselves start to say things in this direction. They point out that there is good evidence that the molecular model is a fairly accurate representation of what is happening at the boundary. Echoing certain sentiments found in the reductionism literature, they say “were such simulations ever to reach the stage where they were computationally capable of dealing with the $l/L \ll 1$ case, rather than being limited to the $l/L = O(1)$ case, the potential inconsistency would presumably disappear. In such circumstances, in order to capture the underlying physics (albeit “far” from boundaries in terms of the length scale l), one would no longer require the conventional f-f macroscale boundary conditions.” In other words, we could in principle dispense with a fluid dynamics model, and instead describe the entire system using the more accurate molecular model.

These sorts of “in principle” appeals show up in the philosophical literature.⁸ And while the merits of these appeals can be debated, perhaps a more powerful argument can be found in current experimental practice, some of which does seem aimed at the explanation of boundary conditions in terms of molecular models. It is also the case that the governing equations of fluid mechanics implicitly only take into account the f-f forces. They do not take into account the s-f forces, either explicitly or implicitly. So by their very nature, the conditions at the boundary are external requirements that cannot be fully captured by the fluid dynamics. So taking a boundary condition expressed in terms of fluid dynamics to also be a literal description of the conditions at

⁸ See for example Norton (2012) and Butterfield (2011)

the boundary would be to take “too seriously”⁹ the continuum model like the one based on Fick’s laws.

But the fact is that today fluid dynamics models are required to explain the properties of fluids. For their part, despite their initial reductionist sentiments, Brenner and Ganesan admit: “However, that day appears to lie far in the future. And even then, it is difficult to imagine a scenario in which continuum mechanics would no longer prove useful for at least qualitatively interpreting the gross behavior of complex fluid-mechanical phenomena.” (p. 6881) But it is also possible that even if such a day comes, there will be explanations that depend essentially on the fluid dynamics models

To illustrate the challenge posed to reduction by the difference between boundary conditions and conditions at the boundary, I am going to return to a familiar theme: shear and slip. In a typical molecular dynamics model, the solid boundary is modeled as a set of fixed lattice points. The molecular model used to describe conditions at the boundary makes assumptions about the solid wall, necessary to capture s-f interactions. By contrast, in a boundary condition on a flow, all of the information about the boundary behavior is encoded in its effects on the fluid, which are represented using a single parameter, namely slip length. Slip behavior depends on a number of factors such as shear rate, wetting conditions, pressure, surface charge, surface roughness, and dissolved gas. (Lauga, Brenner, & Stone, p. 1234) If, on the other hand, we are engineering a pipe to produce a particular type of slip in the fluid it will channel, we might adjust the pipe’s material and surface conditions to manipulate wetting conditions, pressure, surface charge, or so on. Sometimes, the single-parameter boundary condition is all we need; other times, a more robust

⁹ In the sense of Callender (2001).

picture of the conditions at the boundary is necessary. One is simply not shorthand for the other. Instead, they occupy different but sometimes overlapping explanatory contexts.

To make the point in a more concrete context, recall that physicists distinguish between different kinds of slip. There is microscopic slip, which happens at the scale of individual molecules. There is actual continuum slip at a liquid-solid boundary. And there is apparent slip, due to motion of homogeneous boundaries. (Lauga, Brenner, & Stone, p. 1234)

Recall that the Navier slip condition is related to the shear rate at the boundary:

$$u_s = \lambda \left. \frac{\partial u}{\partial x} \right|_s \quad \text{Equation 9 Navier slip condition}$$

So even if we can identify the flow velocity with a molecular dynamics description, the actual velocity at the boundary depends on shear rate, which itself depends on the details of rest of the flow. To make matters more complicated, in some cases, it is not always just the fluid velocity (u_s) that depends on the shear rate. Experiment indicates that sometimes the slip length parameter (λ) itself depends on the shear rate, and the slip boundary condition becomes nonlinear.

The Navier slip condition is like the governing equations in this way. The equations themselves are quite general and need inputs in order to describe an actual system. Consider the momentum equation for incompressible fluid of constant velocity:

$$\rho \left[\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} * \nabla) \mathbf{V} \right] = -\nabla P + \mu \nabla^2 \mathbf{V} + \mathbf{B} \quad \text{Equation 10 Momentum equation}$$

Here, density (ρ), viscosity (μ), are parameters that depend on the nature of the fluid being described. The same law can describe very different flows depending on these parameters. And just like the laws, the same boundary conditions can describe different conditions at the boundary because of this dependence on the parameters of the flow. The conditions at the boundary depend on other features of the flow, like shear stress. Just as the governing equations need parameters to tell us anything about a flow, the boundary condition needs parameters to be specified as well. Just as the same governing equations produces different flows depending on their inputs. This is perhaps similar to the argument in Bishop (2008), since some conditions at the molecular level depend on large scale features of the flow.

The same boundary conditions can hold even though the underlying conditions at the boundary are different. This indicates the possibility of some kind of universal behavior. There are different ways to produce slip. Despite the underlying details not mattering, the boundary condition nevertheless can figure in an explanation of the fluid flow. There is reason to doubt that the underlying molecular picture can give us all of the explanations we want. There is a wide variety of phenomena that can give rise to slip. Whether these are all cases of apparent slip remains to be seen.

This discussion gives a more complex picture than a straightforward reductionist or antireductionist account. Instead, I hope to have shown that we are better off talking about intertheory relations by specifying what those relations are in particular cases. Of course, in some sense, the nature of the fluid depends on the nature of the molecules of which it is composed. But the large scale features of the flow determine variables, which in turn determine the conditions at the boundary, which can be described at a molecular level. If the slip condition depends on shear rate, then we have a complex system where the boundary condition influences the flow field, but

the flow field influenced the boundary condition. Since boundary conditions and conditions at the boundary are different sorts of things, it should not be surprising that there does not exist a straightforward reduction relation. Rather, it is an inappropriate extrapolation of a model beyond the domain for which it was constructed. If the reductionist is right that this takes fluid dynamics too seriously, then the antireductionist would also be right that this takes molecular dynamics too seriously. In each case, a concept is extended beyond its intended domain. It is not a matter of taking one view or the other “too seriously.” Obviously, any real fluid will be made up of molecules, and the continuum assumption breaks down at a certain scale. But at the same time, fluid dynamics is able to capture phenomena that molecular dynamics is unable to. If we are not interested in wall effects, but instead are interested in the fluid dynamics of a particular system, we would still turn to the continuum equations. They would still require boundary conditions to work. Boundary condition is a constraint on the flow, not a description. But there is a way in which increasing our understanding of the molecular picture helps us understand the fluid dynamic picture. They are complementary.

Even if future science explains the fluid picture in terms of the molecular picture, that will not change the central claim of this chapter. The practical difference between boundary conditions and conditions at the boundary is a result of their different roles in their respective theories, not the current state of the science. The conceptual difference means that conditions at the boundary and boundary conditions have different roles in scientific models. That is, even if we could derive boundary conditions from conditions at the boundary, these are two different kinds of things. As long as fluid dynamics exists as an independent field, so will this distinction. And it seems that fluid dynamics does provide explanations that molecular mechanics cannot. I argue that this distinction does not depend on the current state of the science. Because of the respective roles they

play in models, this remains a useful distinction. Learning the boundary conditions does not tell us what happens at the boundary, and knowing what happened at the boundary does not tell us what the correct boundary condition is. The boundary condition is not meant to be a description of the boundary. It is sometimes useful to use a matching procedure to connect the two models, but this is a way of relating the conditions at the boundary to the boundary conditions. This relation is not identity. But the direction research has taken is to attempt to connect the molecular model to the boundary conditions that are known to hold on the macroscopic scale.

Brenner and Ganesan note the difference between boundary conditions and conditions at the boundary is still important even when they seem to agree. In many cases, the boundary condition derived from the molecular conditions at the boundary matches the correct macroscale boundary condition. But given the difference between the two concepts, we cannot assume that the molecular model explains the continuum model. But in this case, the danger is drawing incorrect inferences in the molecular explanation of the boundary condition. Explanation of higher level phenomena in terms of lower level models is not straight forward, even if one can infer the boundary conditions from the conditions at the boundary.

2.6 Conclusions

Both conceptual analysis and practical modeling techniques of fluid mechanics support the distinction between boundary conditions and conditions at the boundary. This distinction has implications for philosophical debates surrounding explanation and intertheory relations. More generally, we should pay attention to the particular role that the parts of a model play. This chapter has depended on fluid dynamics to explain conditions at the boundary, and so the definition made

reference to the boundary between a fluid and a solid. But we can generalize this concept anywhere boundary conditions operate. Boundary conditions operate in a variety of contexts besides fluid dynamics. If we want to model a drum head (Wilson, 1990) or a violin (Bursten, 2019), the conditions at the boundary would not include a fluid.

In the next chapter, I will take a closer look at a particular aspect of conditions at the boundary: the boundary itself. We will see in closer detail just how conditions at the boundary arise, and how they depend on the form of the boundary. Despite the fact that the boundary is described at the molecular scale, the interactions that govern the molecules do not by themselves account for all of the behavior of the fluid.

3.0 Boundaries and Mesoscale Explanations

The previous chapters have focused on the roles of boundary conditions and conditions at the boundary. In those chapters, I focused on the relationship between boundary conditions and the governing equations of fluid dynamics, arguing that the latter are not able to tell us much unless they are accompanied by boundary conditions. I have not yet said much about the other important piece of the puzzle: the boundaries themselves. The nature of fluid-solid boundaries, and their role in scientific modeling and theorizing, will be the central concerns of this chapter. To address these concerns, I will develop two examples that illustrate applications of boundary modeling in fluid dynamics: aerodynamic modeling of projectile nosecones and the formation of nanobubbles in multiscale modeling of fluid-solid interfaces.

As I discussed in Chapters 1 and 2, fluid-solid boundaries are typically modeled with either fluid dynamics or molecular dynamics. In either case, modelers need to know something about the boundary. In the case of fluid dynamics, the shape of the surface that the fluid flows past will determine the shape and character of the flow. In the case of molecular dynamics, the description of the molecules of the solid boundary is just as important as the description of the fluid molecules in describing the conditions at the boundary. And while we must not conflate the boundary conditions and the conditions at the boundary, there is obviously some connection between the two. Molecular dynamics can capture boundary effects not found in the fluid dynamics, and these effects can explain why certain boundary conditions are appropriate at larger scales. But at the molecular scale too, the shape of the boundary can explain things about the boundary conditions at the fluid scale. That is, it is not just the interactions between individual molecules that explain

behavior at the fluid-dynamical scale; it is also the shape of the molecular boundary, as well other properties like the hydrophobicity of the molecules of the boundary.

In between the macroscale of fluid dynamics and the microscale of molecular dynamics simulations lies a mesoscale. Phenomena at this scale cannot be inferred from either the scale above it or below it. In this chapter, I will argue that some features of boundaries are mesoscale features that are necessary for explanations of larger-scale phenomena and which cannot be reduced to smaller-scale phenomena without losing their explanatory force. First, I will show how boundaries themselves figure into explanation of fluid flows at the scale of fluid dynamics. Then I will present a case of molecular dynamics simulations, in which modelers aim to model nanoscale bubble formation. This case illustrates the limits of molecular dynamics for modeling fluid-solid boundary phenomena. It turns out that the formation of nanobubbles cannot be explained without modeling certain mesoscale boundary features, not inferable directly from the molecular dynamics. This mesoscale feature prevents a completely reductive explanation, because such an explanation cannot be derived from the molecular dynamics alone. Rather, mesoscale features like these depend on a number of factors beyond the scope of the molecular dynamics simulation. From this result, I derive some general conclusions about the nature of fluid-solid boundaries as mesoscale phenomena.

3.1 How Boundaries Explain – Two Examples

As we saw in Chapter 1, the governing equations of fluid dynamics by themselves are not enough to describe a fluid system. Modelers also need to specify boundary conditions. These boundary conditions are essential in providing an explanation for properties of a fluid flow. I

showed that boundary conditions can play various roles in explanation, sometimes acting more like laws, sometimes acting more like contingent matters of fact, and often occupying some place between these two extremes. Regardless of the theory of explanation one favors, the role of boundary conditions in fluid models determines their role in explanations.

However, supplying the governing equations with boundary conditions is still not enough to describe a fluid system. We must also know *where* to apply these systems of equations. For this, we need to define the boundaries of the fluid system. As established in the previous chapter, boundaries are not the same thing as boundary conditions. A boundary condition constrains a differential equation by defining values of the solution at the boundaries, telling us what happens at the boundary. But the boundary itself defines the region governed by the differential equation and tells us where the boundary conditions obtain. Similarly, when we are dealing with conditions at the boundary, the boundary defines the region where the interactions between the fluid and the boundary take place. The difference is that when we are dealing with conditions at the boundary, the boundary must have additional properties that define its interactions with the fluid. Recall that it is these interactions that distinguish conditions at the boundary from boundary conditions.

In the following subsections, I look at two examples of how boundary conditions explain flow properties. In the first example, I show how boundaries themselves can explain properties of fluid flows in fluid mechanical models. These boundaries demarcate the regions in which the governing equations and the boundary conditions hold, respectively. Basically, the boundary describes the geometry of the system being modeled.

In the second example, I show how the boundary functions in a particular molecular model. In contrast to the first example, in this model, the boundary does more than describe the geometry of the system. It is a model that ascribes properties to individual molecules, both at the boundary

and in the fluid. These molecules interact, which results in the conditions at the boundary. Importantly, the properties of the molecules that make up the boundary are also necessary to explain the formation of nanobubbles.

As I illustrate below with the nanobubbles case, boundaries also generate characteristically mesoscale physical behaviors, which are not derivable either from the mere specification of boundary conditions or from the governing equations of fluid flow. Other philosophers of physics studying mesoscale physical behaviors have recently argued that sometimes lower-scale behaviors can and should be ignored or parameterized away in an instance of modeling. (Batterman, 2013; Wilson, 2017; Bursten, 2019) Through careful explications of physical modeling, they have shown how mesoscale modeling generates justification for ignoring some microscale physical behaviors. Most of this research acknowledges that boundaries play an important explanatory role in the models under consideration, but studying the exact explanatory role of boundaries in multiscale models is not the primary focus of those investigations. The nanobubbles case builds on this body research and expands it into a new domain to show that without an appreciation of the contribution of boundaries to fluid behavior it is impossible to understand certain features of fluid flow, such as slip. And while mesoscale features of physical systems are often important for many sorts of physical modeling, they are especially crucial in fluid mechanics, where a central goal of modeling is the design of systems to manipulate and control fluid flows. So in order to build a philosophical account of fluid dynamics, we need to understand the explanatory roles not only the governing equations and boundary conditions, but also the boundaries themselves. In models of fluid systems, boundary conditions are set in conjunction with specifying the geometry of a solid that the fluid flows past. By specifying even the mere spatial orientation of the boundaries, we can explain the shape of the flow, and by developing robust accounts of the mesoscale physics at the boundary,

certain observed behaviors of fluid systems and their mathematical models can be rationalized, explained, and, therefore, manipulated.

3.1.1 The Explanatory Role of Boundaries in Fluid Dynamics

Specifying the geometry of the boundary is necessary to describe a fluid flow in computational fluid dynamics. Suppose an engineer wants to design the nosecone for a vehicle that will travel through the atmosphere at hypersonic speeds, such as manned atmospheric reentry vehicle for lunar missions. The shape of the nosecone affects the properties and overall shape of the fluid flow. Nosecones can take a variety of shapes, including conic, elliptical, spherical, and parabolic. For example, an elliptical nosecone can be described like this:

$$y = R \sqrt{1 - \frac{x^2}{L^2}} \quad \text{Equation 11 Elliptical nosecone}$$

where L is the length of the nosecone and R is the radius of the base, x varies from the tip of the nosecone to L , and y is the radius at any point x . Nosecone design affects factors like atmospheric drag and aerodynamic heating, as well as the formation of a shock layer in front of the vehicle. In supersonic flows, a shockwave appears some distance in front of the nosecone. The location of the shockwave that precedes the nosecone is a direct result of the geometry of the nosecone, along with the governing equations and boundary conditions. (Anderson, Albacete, & Winkelmann, 1968) On the other hand, the vehicle's material surface and boundary-level interactions with the atmosphere provide information about how the vehicle will weather the heat generated by

shockwave. Both the geometry and the interfacial dynamics are important for the heating properties of the body. Importantly, though, the impact of the shockwave on the material properties of the solid body cannot be derived from *only* looking at the interfacial, molecular-level interactions. The geometry is an indispensable feature in the model. The shape of the nosecone of reentry vehicle or the shape of an airfoil are the sorts of features of bodies that are ripe for manipulation, and, therefore, which are often the subject of experimental investigation and simulations. As a result, they are also crucial explanantia in many explanations of the design of bodies intended for supersonic flight. When engineers design these things, they are limited in the kinds of things they can manipulate in their models. There are some things that physics of the situation decides for them: generally, they are stuck with the governing equations, which limits their ability to manipulate certain macroscopic details of the model. Similarly, the material of the object generally fixes the boundary condition and thus limits their ability to manipulate microscale surface-level interactions. But these engineers do have a lot of control over the mesoscale in their ability to manipulate the geometry of the nosecone or airfoil. And determining the effects of changing boundary geometry is often precisely the goal of these investigations. For example, it turns out, for bodies moving at hypersonic speeds through the atmosphere, that a blunt shape nosecone results in less aerodynamic heating than a thinner, sharp nosecone. This is due to the fluid flow characteristics as the fluid passes the nosecone. In effect, the blunt shape results in a cushion of air being pushed in front of the body, forcing the heated shock layer away from the body. So we can have two nose cone designs. The governing equations of fluid dynamics are the same in both cases. And in both cases, the same boundary condition is used. However, the difference is the geometry of the nosecone, which forms the boundary of the flow.

3.1.2 The Formation of Nanobubbles

Whereas the previous case study emphasized the role of geometry in the conditions at the boundary in computational fluid dynamics, here I look at how simulations are used to make inferences about fluid flows near solid boundaries. I use a case study on the formation of stable nanobubbles. In this case as well, the intertheory inferences that are made between the molecular dynamics (MD) and fluid models of nanobubbles rely on what I have identified as mesoscale features of the boundary.

Above, I identified the geometry of nosecones as a mesoscale feature in computational fluid dynamics, and I showed how certain modeling considerations made this feature one of special interest in designing bodies for supersonic flight. In the case of MD simulations below, I will show that surface features like roughness or layers of gas are mesoscale features. The philosophical point about explanation here is even stronger than in the last section. There I argued that the mesoscale conditions at the boundary are important because of their manipulability. Here I will show that the mesoscale features of the system, which are encoded by the conditions at the boundary, are explanatorily and predictively indispensable. That is, when trying to explain the macroscopic boundary conditions with the governing equations of molecular dynamics alone, the MD simulations often fail to predict what is observed at the macroscopic level.

Molecular simulations are used to predict and explain how nanobubbles form. In particular, they can provide an explanation for nanobubbles' formation and stability. While the result depends on the physics that governs the molecular interactions, it also depends on the configuration of the wall molecules. I will argue that this wall configuration represents the conditions at the boundary on this flow, that it is a mesoscale feature of the simulated fluid flow, and that it is indispensable to certain explanatory projects in fluid dynamics.

There are different ways to build an MD model. One way is to describe interactions between fluid molecules with a Lennard-Jones potential, which computes the energetic minimum between two interacting atoms or molecules; that is, it specifies a system's dynamics by finding the lowest-energy state of a system via summation over nearest-neighbor interactions. This model is moderately computationally intensive, in that each molecule requires its own state description. In Lennard-Jones models, as in many MD simulations, the interactions between the molecules of the flowing fluid and the molecules that make up the solid boundary are also governed by local, nearest-neighbor potentials. For these interactions, though, instead of freely moving around like the fluid molecules, though, the molecules of the solid boundary are fixed on a lattice, or coupled to a lattice with a spring constant. (Lauga & Stone, 2003)

In a model of this sort, the specification of the lattice spacing and average interatomic distance of molecules in the solid boundary constitute the boundary condition. In contrast, the dynamical interactions between the lattice-bound solid molecules and the freely flowing fluid molecules constitute the conditions at the boundary. The conditions at the boundary are dynamical parts of the computed flow, whereas the boundary conditions are not. Some of the features of the boundary that have an effect on these conditions at the boundary are (1) the geometry of the lattice, (2) any spring constant used to couple a boundary molecule to the lattice, (3) the defined Lennard-Jones potential for the interaction between the solid and fluid molecules, and (4) the depth of the solid boundary layer. These conditions are able to be represented in state descriptions of local Lennard-Jones interactions between fluid and solid molecules, but these features are necessarily derived from sources other than the Lennard-Jones potentials governing the fluid flow.

These properties of the boundary will influence the molecular interactions in the rest of the model, as well, because fluid molecules that enter the boundary region where they interact with

these conditions will depart that region having been impacted by the dynamical constraints in (1)–(4). Further, as might be expected, the MD model can explain elements of the fluid dynamics model. But the explanation is not fully reductionist, since it is not just the microscale physics of the free fluid molecules that determines the molecular flow. Additionally, these features of the boundary occupy a scale between the molecular scale model and the continuum scale model.

In order to see the unique role that these features play in explanation, we first need to look more closely at how MD simulations are constructed. Generally, MD simulations integrate numerically Newton’s law of motion for individual molecules:

$$m_i \frac{d^2 \mathbf{r}_i}{dt^2} = \sum_j \mathbf{F}_{ij} \quad \text{Equation 12 Newton's law of motion for single atoms}$$

Here, m_i is the atomic mass, r_i is the position of atom i , and F_{ij} is the intermolecular force between atoms i and j . The intermolecular force F_{ij} is given by:

$$\mathbf{F}_{ij} = -\nabla_i V_{ij} \quad \text{Equation 13 Intermolecular force}$$

where V_{ij} is the interaction potential. A commonly used interaction potential, which I described conceptually above, is the Lennard-Jones two-body potential:

$$V_{ij} = 4\varepsilon_{ij} \left[\frac{\sigma^{12}}{r_{ij}^{12}} - \frac{\sigma^6}{r_{ij}^6} \right] \quad \text{Equation 14 Lennard-Jones two-body potential}$$

Here ε_{ij} is the interaction strength, σ is the atomic size, r_{ij} is the distance between atoms i and j . This potential is useful for a wide variety of systems, and it can be modified to take into account different kinds of molecules. More complex potentials can be introduced, which might take into account things like many-body interactions or orientation-dependent interactions, but these are computationally more costly.

With these preliminaries established, I want to look more closely now at the case study on the formation of nanobubbles. Surface nanobubbles are less than 1 micrometer in height as measured from the solid surface with which they are in contact. In flows of liquids over solid boundaries, nanobubbles can form and affect the boundary conditions at the continuum scale. (Maali & Bhushan, 2013) The following summarizes a simulation of nanobubble formation run by Maheshwari et al. (2016). The simulation used a Lennard-Jones potential to define the interactions between the molecules. There were four types of molecules: two types of solid molecules (S and S_p), liquid molecules (L), and gas molecules (G). The L and G molecules could move freely, while the two solid molecules were fixed in a face-centered cubic (fcc) lattice, which represented the boundary. Importantly, spacing and orientation of this lattice were obtained from empirical considerations, not from information about the liquid or gas molecules. The values of the Lennard-Jones parameters are summarized in Table 1. Note especially the difference between interactions strengths ε for the hydrophobic solid (S) and the hydrophilic solid (S_p).

Table 1 Lennard-Jones parameters

i-j	σ_{ij} , nm	ϵ_{ij} , kJ/mol
S-L	0.34	1.8
S _p -L	0.34	1.5
S-G	0.40	2.0
S _p -G	0.40	5.0
L-G	0.40	1.55
G-G	0.46	0.8
L-L	0.34	3.0

In addition to the Lennard-Jones parameters, the saturation level of the gas in the liquid was specified. The saturation level ζ of the gas in the liquid is given by:

$$\zeta = \frac{C_{\infty}}{C_S} - 1 \quad \text{Equation 15 Saturation level}$$

where C_{∞} is the gas concentration and C_S is the gas solubility. If $\zeta > 0$ indicates gas oversaturation, while $\zeta < 0$ indicates gas undersaturation.¹⁰

¹⁰ Unlike some properties that can be inferred from lower scale phenomena, the higher scale properties of the boundary cannot be explained in virtue of being the aggregation of pairwise interactions described by the molecular scale physics. The saturation level of the gas in the liquid can be thought of as such an aggregation, since it depends only on the concentration and the solubility. The over- or undersaturation level of the gas does not depend on particular arrangements of the gas molecules. In that sense, saturation levels support reductive explanations.

Simulations were run with four different boundary configurations and saturation conditions. The simulation conditions were 1) homogenous solid surface and gas oversaturated liquid 2) hydrophobic solid surface heterogeneities and gas undersaturated liquid 3) hydrophilic solid surface heterogeneities and gas oversaturated liquid 4) hydrophobic surface heterogeneities and gas oversaturated liquid. The homogeneous boundary was made entirely of S molecules. The boundary with hydrophobic surface heterogeneities was made of mostly S molecules interspersed with regions of S_p molecules. The S_p molecules formed “pinning sites,” or areas of higher hydrophobicity than the S molecules. This means that the solid’s interaction strength with gas molecules was much higher than with liquid molecules. For the boundary with hydrophilic heterogeneities, the same boundary configuration was used, but the values of ϵ (SP-G) and ϵ (SP-L) were interchanged. The pinning sites should be interpreted as different kinds of molecules on a flat surface.

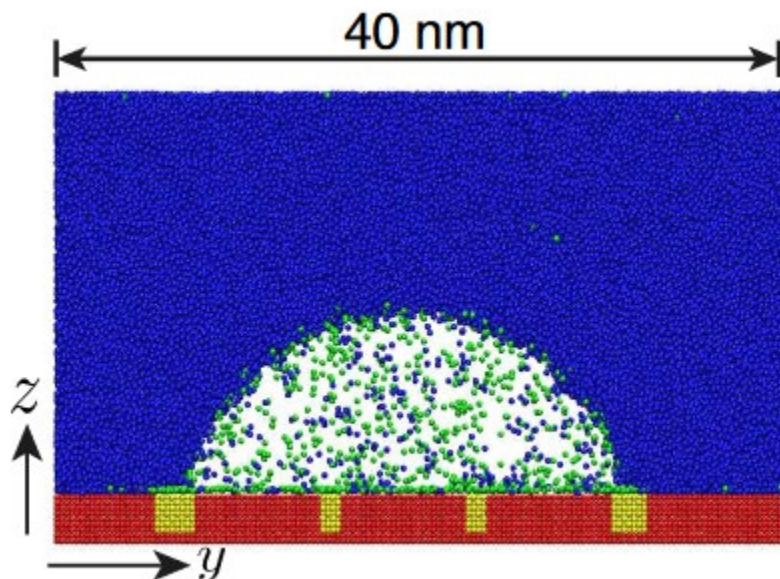


Figure 3 A typical simulation box with four kinds of particles

Reprinted with permission from Maheshwari, van der Hoef, Zhang, & Lohse (2016)

The central result is that the simulation produced stable nanobubbles only in the case of gas oversaturated liquid and hydrophobic heterogeneities. This result matches predictions made previously (Lohse & Zhang, 2015). Simulation with either undersaturation, a homogenous surface, or hydrophilic heterogeneities resulted in nanobubbles that were unstable. Saturation level alone are not enough to explain the persistence of nanobubbles. In similar conditions, but without the pinning sites of the solid surface, bulk nanobubbles dissolve in milliseconds. With pinning sites, they can last for days (Epstein & Plesset, 1950)

The results of this simulation demonstrate two of the causal factors that explain nanobubbles. One is the extent to which the gas has saturated the liquid. The other is the heterogeneities of the solid boundary. Both oversaturation and hydrophobic heterogeneities are required for the formation of stable nanobubbles. In contrast to the saturation of gas molecules, which I do take to be reductive, the heterogeneities are mesoscale features that do not yield a reductive explanation. The gas saturation can be explained in terms of the microscale physics, while the configuration of the boundary cannot.

The crucial thing to see here is that the formation of stable nanobubbles depends on features of the boundary that are composed of many molecules. Even though these features are built out of the molecules, and are subject to the molecular scale interactions, they are larger features. The pinning effect is not limited to chemical heterogeneities either. Pinning sites can be the result of geometrical heterogeneities as well. (Liu & Zhang, 2017) Such pinning sites represent surface roughness, and can also produce stable nanobubbles.

The solid boundaries of the MD simulation do more than just define the geometry of the region governed by the interaction potentials. They can have more features than the boundaries used in the computational fluid dynamics model discussed in the previous section. They are made

of molecules, like the molecules of the fluid, but with different sizes, masses, and interaction strengths. The strength of that interaction will depend on empirically-derived details about the molecules involved. But unlike the fluid molecules, the molecules of the boundary are not able to move around freely. They can be fixed in a lattice or attached to a lattice by a spring constant and allowed to oscillate. The nature of boundary also depends on how many layers of solid molecules there are in the boundary.

3.2 Two Senses of “Boundary”

Chapter 2 emphasized the distinction between boundary conditions and conditions at the boundary. The case studies above show that something else falls out of the distinction between boundary conditions and conditions at the boundary as well: each of these two concepts employs its own concept of boundary. Boundary conditions serve a specific mathematical purpose in fluid dynamics models. They constrain the solutions to differential equations by specifying the values that the solution must take at the boundary. In this way, they are an instance of what Wilson (2017) has called “physics avoidance”: they encode information about the interactions between the fluid and the boundary, but they do not explicitly describe the interactions. Conditions at the boundary, on the other hand, explicitly take into account interactions between the fluid and the boundary. This means that additional information about, and more detailed models of, boundary behaviors are required to determine how the fluid and boundary interact.

The boundary delineates the region where the governing equations operate and the region where the boundary condition obtains. As opposed to an instance of physics avoidance, this is closer to the sort of system-defining specification of a domain that Cartwright (1999) describes in

the construction of nomological machines and which Mitchell (2014; 2012; 2009) investigates in her studies on emergence in dynamic, self-organizing systems. As I will show below, for many cases of fluid modeling, defining the boundary requires more than specifying its geometry. The boundary interacts with the fluid, and those interactions need to be specified. In the following discussion of this chapter's case studies, my goals are to illustrate the limits of the physics-avoidance account in explaining the role of conditions at the boundary in fluid modeling, and to supplement accounts like Cartwright's and Mitchell's, which emphasize the importance of specifying domains of application in building physical explanations.

3.3 The Explanatory Role of Molecular Boundaries

The nanobubbles described above are important for explaining the boundary conditions at the macroscopic scale. As I argued in the last chapter, boundary conditions cannot always be inferred in a straightforward way from the conditions at the boundary. Molecular simulations often systematically underpredict, by an order of magnitude, the amount of slip inferred from macroscopic experiments. (Karniadakis, Beskok, & Aluru, p. 396) So it would seem that the simulations are failing to capture some feature of the physical system, and it is likely that mesoscale features of the boundary are such features. This illustrates the point I made above, that there are limits to the applicability of physics-avoidance strategies in fluid-dynamical modeling.

There are other explanations that rely on not just the interactions between the fluid and the boundary, but also on the boundary itself. For instance, de Gennes (2002) suggests that, for liquid flow, the boundary is not just the interface between solid and liquid. Instead, he shows how a thin film of gas between the solid and the liquid can produce the slip seen at the continuum level. This

suggests, following Cartwright and Mitchell, that an important part of building explanations from conditions at the boundary is clearly specifying the domain of application for those conditions. I discuss the importance of domains of application more in the next chapter.

Before I get to that discussion, though, I want to emphasize a how boundaries of the sort modeled in this case study play an essential role in explanations of macroscopic fluid dynamical behavior. In particular, conditions at the boundary can, and boundary conditions cannot, explain a particular unexpected phenomenon that interests computational fluid dynamics modelers: increased slip on rough surfaces. It is perhaps counterintuitive to think that a rough surface would provide more slip than a smooth one, yet that is what some models predict. For example, Richardson (1973) and later Jansons (1988) use only fluid dynamics to show how a macroscopic no-slip condition might emerge from a smaller scale slip condition. They model a fluid flowing past a boundary with small defects. Defects as small as 10^{-9} m can produce a slip length of only 10^{-5} m, which at larger scales is indistinguishable from a no-slip condition. But these studies are limited, since they do not explicitly take into account the interactions with the boundary. When we do take the microstructure of the boundary into account, a different picture emerges. If there are pockets of gas in tiny cracks on a solid surface, then they act as local stress-free boundaries, and a fluid would be flowing over alternating regions of slip and no-slip. This results in an effective partial slip condition at higher scales.

We saw that the formation and maintenance of these pockets of gas are explained using MD simulations of boundaries with varying degrees of surface heterogeneity. (Liu & Zhang, 2017; Maheshwari, van der Hoef, Zhang, & Lohse, 2016). On surfaces without heterogeneities, nanobubbles can form but are not stable. In contrast, on surfaces with heterogeneities, surface nanobubbles are stable. When a fluid can wet only the peaks of the rough surface, gas is trapped

in the grooves. This is known as a Cassie state. (Full surface wetting is known as a Wenzel state.) When a nanobubble nucleates in a groove of a rough surface, a local Cassie state is formed, that is, gas is trapped in the groove. The gas in the cavity can coalesce and the nanobubble can grow and join with other nanobubbles from adjacent cavities. These then become stable surface nanobubbles, which extend above the level of the peaks of the rough surface, ultimately forming a Wenzel state.

In this explanation of the causal role of gas pockets in the formation of nanobubbles, the key thing to see is that the boundary itself is playing an essential role in explaining the nanobubbles. Since the MD simulation explicitly takes into account the interactions between the fluid and the boundary, it describes the conditions at the boundary. The fact that the boundary has pockets of air explains the slip conditions observed at the continuum scale. Not only is the explanatory role of the boundary essential to these explanations; it is not reducible to either molecular dynamics or fluid mechanics alone. It lies between the two, imparting an essential set of mesoscale features on which both theories can draw.

The centrality of boundaries in explanations of slip via nanobubble formation builds on the earlier discussion of the explanatory role of nosecone geometry in supersonic rocket design. The earlier case showed that in fluid dynamics, conditions at the boundary are mesoscale and are important for achieving certain goals (i.e. making rockets that don't explode) consistent with explanatory projects. Here I have shown that conditions at the boundary, and boundaries themselves, can play explanatory roles that are not just important but indispensable given the modeling tools currently used. The failure of reduction in explaining nanobubble formation establishes this point.

3.3.1 Boundaries as Mesoscale

I will close this chapter with a final point on how these observations fit into recent discussion of multiscale modeling in philosophy of science. In particular, I believe this chapter's emphasis on the mesoscale nature of boundaries will contribute to a growing literature on mesoscale modeling and explanation. Mesoscale explanations have become a topic of some recent interest in the philosophy of physics. (Batterman, 2013; Wilson, 2017; Haueis, 2018) These middle-out approaches are not reductive. But they are not merely the result of universal behavior either. There are systematic reasons for their regularities, which distinguishes mesoscale explanations from universal or emergent ones. In fluid mechanics, mesoscale explanations frequently require another layer of explanation that has to do with the production of materials, whether by natural or human-controlled processes. She argues that tuning is a separate use of models, beyond explanation or prediction. However, tuning can also be explained, and in such explanations of why a pipe should be coated in such a way, or machined one way instead of another, the boundary plays a unique mesoscale explanatory role.

These mesoscale features cannot be explained by the physics that governs the molecular interactions. There is nothing special about the particular arrangement of the boundary molecules. The more important question is about why we should expect to see that kind of boundary. To borrow a familiar distinction, it is a type (ii) question, rather than a type (i). (Batterman, 2002, p. 23). What makes them effective tools in mesoscale explanations is not just that they happen to obtain, but they can be expected to obtain as well. A type (i) question would ask why a particular piece of glass has a particular surface roughness. However, no two pieces of glass are exactly the same at the molecular scale. Nevertheless, we can expect them to exhibit consistent behavior because glasses typically have consistent conditions at their boundaries. We can, for example,

expect a certain degree of roughness on pieces of glass on a certain type. An answer to this question would answer the type (ii) question. The roughness itself is not explained by the molecular constitution of the glass, even though it can be described by the molecular dynamics. There are mesoscale features that cannot be predicted by the molecular simulations, but that also cannot be inferred from fluid dynamics.

The prominence of mesoscale features makes straightforward reductive accounts of explanation difficult. The explanations for the mesoscale features do not come from the physics of the microscale model. The boundary is described in terms of the molecules and their interactions with fluid molecules. Further, boundaries fall into a larger class of mesoscale phenomena. The boundary heterogeneities do work similar to the mesoscale features of steel beams. The properties of steel at the macroscopic scale cannot be explained by the symmetric crystalline lattice structures. In between the microscopic scale and the macroscale, things like point defects, line defects, slip dislocations, and other properties appear that explain the properties of steel at the macroscale. (Batterman, 2013)

The mesoscale features of the boundary, and of conditions at the boundary, resist reductive explanations of macroscale fluid behavior. That is, the behavior of a fluid at a boundary at the macroscale cannot be explained entirely in terms of the lower scale physics. The presence of surface nanobubbles results in apparent slip. While not considered true slip, since the no-slip condition applies at the locations where the fluid is in contact with the solid boundary, the nanobubbles are too small to be characterized with standard continuum models. So the apparent slip is described as if it were actual partial slip. The nanobubbles responsible for this partial slip depend on the existence of the surface heterogeneities.

If the correct macroscale boundary condition depends on not only the microscale physics (such as the Lennard-Jones dynamics discussed above), but on the mesoscale features of the boundary, then a completely reductive explanation is out of reach. This appears highly likely from the previous sections' case study. In some sense, a molecular dynamics simulation of the conditions at the boundary can explain the boundary conditions, but only if the molecular dynamic simulation specifies mesoscale boundary features. The microscale model explains the macroscale, but only mediated by the mesoscale objects, the nanobubbles, and ultimately the boundary features that result in the nanobubbles. Complementarily, it is worth noting that the boundary in the fluid dynamics model is also not derived from the physics that govern the model. The governing equations plus the boundary conditions do not say anything about the shape of the boundary.

However, to close this point, it is worth emphasizing an important contrast, which I have so far only hinted at, between the specification of nosecone geometry in the first case study and the investigation on nanobubble formation in the second study. In each case, the modelers had relative freedom to choose the geometry of the boundary. The rocket modelers are able to choose a geometry that will optimize the flow in the manner useful to atmospheric reentry. The nanobubble modelers have freedom too, but they have different reasons for their choice. The purpose of their model is not to engineer, but to explain fluid flow. The heterogeneity built into their model is there, not because it optimizes some feature of the flow for manipulability, but because we can expect to find conditions like that in actual channels. This helps to make clear, too, why such a model ended up being particularly apt for use in developing the further mesoscale explanation of apparent slip via surface roughness.

One could ask whether the kinds of explanations furnished by the geometries and other features of the boundaries are genuinely *meso*-scale in nature.¹¹ To call something mesoscale implies that it happens as a scale *between* at least two other scales. The description of the pinning sites in the model is still a molecular scale description, after all.

Ultimately, the force of the explanation comes not from the scale of the features, but from the way they are formed. In the case of the shape of the nosecone, there is no type (ii) explanation, at least not in the same way there is for the molecular simulation. An explanation for the shape of the nosecone presumably stems from the choices of engineers who designed the nosecone. It is a notable feature of these systems, though, that the tuning or design feature being manipulated lies at a mesoscopic scale. As philosophical interest in mesoscale modeling continues to grow, I suspect that the connection between tunability and mesoscale modeling will be a rich area for further exploration.

3.4 Conclusions

Chapters 1 and 2 have painted a picture of boundary phenomena that is much more complex than has been assumed by philosophers. This chapter's discussion of boundaries as inherently mesoscale is meant to add to that complexity. Boundaries play different explanatory roles depending on the model in which they operate, but in both of the cases developed here, these roles slot boundaries in as mesoscale explanantia and illustrate how their roles in explanation are importantly distinct from the roles of governing laws and boundary conditions. The features of

¹¹ Thanks to Bob Batterman for raising this point.

fluid-solid boundaries can be described using the molecular level theory, but which the molecular theory by itself does not explain. Ultimately, there is a reason we systematically find certain types of mesoscopic features. And perhaps, the reductionist might claim, the reason is a lower level phenomenon. I share some sympathy with this view, maybe more than is fashionable, but I believe a better way to analyze these regularities is with an eye toward the implications for realism, rather than reduction, which will be the subject of the next chapter. And I must admit, while these boundaries are built out of the constituents of the microscale simulation, the form they take cannot be predicted from the microscale physics alone. However, these mesoscale boundaries are not mere contingent matters of fact either.

4.0 Boundary Conditions, Domains of Application, and an Argument for Realism

In the previous chapters, I showed that there is a conceptual difference between boundary conditions and conditions at the boundary, and that this difference produces models that seem to make incompatible claims about how fluids behave near a solid boundary. This kind of incompatibility brings to light a problem for scientific realists, since under most standard accounts of scientific realism, at most only one of the models can be an accurate description of the world. One of the responses to this problem is that these models are complementary, not contradictory. That is, they each represent part of the target system rather than the whole system. And while I think this is largely correct, merely showing that different models correctly model different parts of the world leaves out a significant part of the story. It is not just the part of the world being modeled that makes the difference, but also the conditions in which the model is employed. In this chapter, I take a closer look at the experimental investigation of boundary conditions at a fluid-solid boundary, namely investigation of the no-slip condition. This will be the basis for an argument for realism from examination of the conditions under which a model breaks down. So instead of incompatible models leading to antirealism, I will argue that it is precisely this incompatibility that is evidence for realism.

I begin with brief remarks on the interpretation of the no-slip condition under a canonical antirealism, the constructive empiricism of Bas van Fraassen. These remarks motivate some of my long-standing concerns about how the no-slip condition might slot into a realist or empiricist ontology. In order to resolve these concerns, I then turn to contemporary considerations on models and realism, since the no-slip condition is investigated through modeling more often than through direct experimental probing. I look at the apparent inconsistency between models that emerges

when we look at fluid flows from different perspectives and, following Margaret Morrison (2015), highlight complementarity of models, as a possible means of recovering realism. I will build on the idea of complementary models to look at how models, generally, have a domain of application. We are able to know not just where these domains begin and end, but also how they relate to the model in question in messy, technical detail that generates a warrant for belief in the reality of some of the model's parts. Specifically, we are able to learn about how the conditions that define a domain of application are causally linked to the success or failure of the model. I will argue that a realist stance toward the no-slip condition is supported by experiments that increase understanding of the conditions that define its domain of application. To do this, I will look at the experimental methods used to examine slip phenomena, and at the results of these methods. How parts of models relate to their respective domain of application gives us evidence for realism. The results of these methods are not merely verdicts as to whether or not the no slip condition was able to make a prediction successfully. Rather, they produce causal knowledge of the conditions under which the no slip condition holds. Although I will make this argument on the basis of fluid dynamics, I believe an analogous pattern of reasoning can be found in a variety of other contexts. Since our epistemic attitude depends in part on a model's domain of application, the kind of realism supported by this argument is a local realism.

4.1 Observability, Realism, and Slip

Fluid dynamics might seem like a strange setting for the scientific realism debate to take place. After all, this is the physics of the water going down the drain of your sink, not some exotic subatomic particles or miniature black holes created in the Large Hadron Collider. In fluid

dynamics, the issue of realism does not surround canonical unobservable entities like electrons. However, there are parts of fluid models that are subject to anti-realist arguments. I will review two avenues of antirealist argument that apply here.¹²

One standard challenge to realism depends on the distinction between observable and unobservable parts of theory. Traditionally, this distinction has depended on human sensory capacities. This challenge can be traced back to form of instrumentalism associated with the Vienna Circle (Carnap, Hempel, et al.). According to this kind of account, terms for unobservables by themselves are not meaningful. Rather they are instruments for predicting behavior of observables. While this particular line of antirealist thought has fallen out of fashion, more recently, constructive empiricism has continued this line of thinking. Owing in particular to van Fraassen (1980), constructive empiricism claims that the aim of science is empirical adequacy, not truth. On this view a theory or model is empirically adequate “exactly if what it says about the observable things and events in the world, is true.” (van Fraassen, 1980, p. 12)

There are features of fluid flow that are unobservable in van Fraassen’s sense. Slip phenomena generally, and the no-slip condition in particular, are hypotheses that have been useful in building fluid models for centuries. Yet they remain unobservable in the sense employed by constructive empiricism. By these standards, even so called “direct” experimental practices cannot be said to observe slip, and there is currently no way to directly observe what happens at the boundary of a fluid flow, in the region within a few micrometers of the boundary.

¹² I am limiting my discussion to the lines of antirealist argument that are relevant to my argument. So I will not be discussing arguments based on historical considerations (e.g. Kuhn, 1962) or social constructivism (e.g. Latour & Woolgar, 1986; Pickering, 1984). My argument simply does not address these positions.

While the argument to follow will not try to break down the distinction between observable and unobservable, the purported unobservability of the slip is still important. Even if there is no principled way to draw a line between the observable and unobservable, there does seem to be degrees of how directly a phenomenon can be detected. It is relevant to the discussion that the part of the model in question requires a significant degree of experimental and theoretical apparatus to detect.

There are three general ways of investigating slip phenomena. I bring this up because none of the three ways provides an observation, in the sense of the constructive empiricists, of fluid flows at the boundary. And since the no-slip condition cannot be observed, it is the sort of thing that realism debate targets. The first way to investigate slip is through indirect methods, which infer slip length by measuring some macroscopic quantity that stands in an already known relation to slip length. These are essentially the same sorts of experiments used to confirm slip on the macroscopic scale, but more recently, they can be carried out in nanochannels. (Karniadakis, Beskok, & Aluru, 2005)

The second way to investigate slip is through local methods, which attempt to measure slip more directly, via for example high-resolution microscopy and the use of tracer particles. And while these methods do not rely on inference from other macroscopic parameters, they are not actually considered direct. (Shu, Teo, & Chan, 2017, p. 15) As it was recently explained in a review of work in this area, “[n]o experimental technique is able to distinguish between the two pictures [i.e. true slip and apparent slip] and to directly measure the motion of molecular layers of liquids close to a surface.” (Neto, Evans, Bonaccorso, Butt, & Craig, 2005, p. 2885) While these methods are considered more direct, there are still complications that prevent us from actually observing flow variables right at the boundary. Tracer particles might also be subject to electrical effects that

cause them to either stick to the wall or be repelled from it. So there is still enough inference involved to prevent this from being an observation of the boundary conditions.

The third way slip is investigated is through molecular dynamics simulations, which simulate the motions of individual molecules at a liquid-solid boundary. These results can be interpreted in the continuum limit, and then applied as a boundary condition. As we saw in Chapter 2, care must be taken when inferring boundary conditions from these molecular dynamics simulations. While there is debate about the relationship between simulations and experiment, the results of these simulations are often treated as evidence in the same sense as experimental results.¹³ Neto et al. note that “the slip lengths estimated by simulations are normally much smaller than those measured in physical experiments.” (Neto, Evans, Bonaccorso, Butt, & Craig, 2005, p. 2879)

None of these three ways of investigating slip phenomena provides an actual observation of fluid flows at the boundary. The no-slip condition cannot be observed by any present means, and current research tends away from methods of investigating slip that are likely to result in a direct observation. So, claims about slip are at most empirically adequate, in the constructive-empiricist framework. However, slip is not an “exotic” phenomenon that must be cordoned off in the laboratory under pristine conditions—it happens in soda cans and bathtubs. It is exactly the sort of mundane phenomenon that we interact with daily, which scientific realists want to safeguard as part of the real world. Moreover, it is their reality that motivates researchers to investigate them in order to put them to use in technologies. So empirical adequacy as a resolution for slip is unsatisfying at best, and at worst it could run a risk of missing the rationale and goals of

¹³ For further discussion of the epistemic role of computer simulations relative to experiments in contemporary science, see Winsberg (2010).

research. In my argument below on the Domain of Application, I will show that some of the probes of slip phenomena in slip research in fact generate reasons to be realists about slip. That argument needs some groundwork about how philosophical accounts of scientific models factor in to contemporary views on scientific realism, so before I develop it, I want to lay out a challenge to antirealism specifically from studying modeling practices in fluid dynamics.

4.2 Realism and Models

While there is a challenge to the truth of the no-slip condition that stems from the ways we experimentally detect slip, there is a further challenge to realism that does not come from the distinction between observable and unobservable entities. Instead, this challenge comes from scientific modeling practices. (Cartwright, 1983; Godfrey-Smith, 2006; Wimsatt, 2007; Morgan & Morrison, 1999; Teller, 2001) Successful models often have non-representational features, which are chosen for pragmatic reasons, such as mathematical tractability. They often make free use of idealizations, and so these models represent systems that do not exist or could not possibly exist. Such a model-based approach to science is often thought to support some kind of scientific antirealism. And these sorts of non-realistic models might make for better explanations, severing the link between truth and explanation. With respect to scientific laws, for example, Cartwright (1983) argues for severing the connection between explanation and truth. She points to cases of highly idealized models that are very dissimilar from their target systems, and are explanatory precisely because of their idealized nature. Conversely, more realistic models fail to be explanatory.

The focus on modeling practice also shifts the focus away from entities. It is not unobservability, but rather approximations, idealizations, and fictions used in successful models that are reason to doubt their truth. Some have argued that such approximations, idealizations, and fictions can be eliminated by providing the models with more detail. (Laymon, 1985) On these accounts, the addition of these previously suppressed details results in more realistic models, as evidenced by their improved predictions. In response, it has been pointed out that the practice of de-idealizing models does not always improve the model (Cartwright, 1989), and such de-idealizations do not reflect actual scientific practice. (Hartmann, 1998)

The modeling practices of fluid dynamics are an ideal target for this challenge to realism. Rueger (2005) and Morrison (2015) note the variety of fluid mechanical models that serve different explanatory purposes. The use of models in fluid dynamics seems to support a model-based approach to science. When modeling fluid systems, scientists have a collection of modeling tools that they can employ when constructing models. Problems are typically solved by using the tools of the theory to build a model of a fluid system. As we saw in previous chapters, depending on the details of the target system, the modeler must choose an appropriate set of boundary conditions. But there are more decisions to make than just the boundary conditions. Recall that there are different versions of the governing equations as well: each one has a viscous and non-viscous form as well as a conservation and non-conservation form. In practice, none of these versions is regarded as the unequivocal best equation for describing fluid systems. Rather, the details of the target system determine which tools are the best for that particular system.

Given the distinction between boundary conditions and conditions at the boundary explored in the previous chapter, I want to focus on one argument in particular: the incompatible models argument. (Morrison, 2000) This argument is an extension of the more general antirealist

argument from modeling practices. Scientists often use models that represent incompatible claims about the same target system. Since they assign incompatible properties to the same target system, these models imply a contradiction. Therefore, at most one of the models can be an accurate representation of the target system.

We saw this kind of incompatibility when I explored the difference between boundary conditions and conditions at the boundary. When modeling the boundary condition of a diffusive system, a slip condition is correct, but when modeling the conditions at the boundary, the fluid does not slip relative to the boundary. The challenge to the realist, then, is what to make of this discrepancy.

4.2.1 Responses to the Modeling Challenge

Some like Giere (2006), Rueger (2005), van Fraassen (2008), and Callebaut (2012) respond to the problem of incompatible models with perspectivism. Since there is no “view from nowhere”, this line of thinking goes, the world can only be represented from some perspective or other, and any attempt to step outside of a perspective is fruitless. And so, the perspectivist argues, our knowledge of the world is limited to the way things seem from some perspective or other. On this view, truth is relativized to a perspective, and even though Giere thinks of perspectivism as a form of realism, it is hard to see how it qualifies as such. According to Giere’s perspectivism, the best we can do in making claims about ontology is justify claims such as, “According to this highly confirmed theory (or reliable instrument), the world seems to be roughly such and such.” In contrast to this, we cannot justify claims like “This theory (or instrument) provides us with a complete and literally correct picture of the world itself.” (Giere, 2006, p. 6)

I agree that a “complete and literally correct picture” is not attainable, but Giere seems to be missing some middle ground here. Realism is compatible with the claim that we can have knowledge about how the world is (not merely how it seems according to a given theory), even if this knowledge is incomplete and approximate in places. Further, the notion of a perspective relies heavily on analogy to different spatial perspectives on an object, and it is not clear how well this analogy tracks with scientific modeling practice. Van Fraassen makes heavy use of this visual perspective metaphor, as does Rueger. Giere relies heavily on the case of our visual perception of color. Morrison (2015) seems to agree that perspectivism is unduly epistemically modest. She grants that in some cases, multiple incompatible models are an indication that we are not justified in believing that the models truly represent. An example of this is the case of incompatible models of atomic nuclei. Some models behave classically, as in the liquid drop model, while other models behave quantum mechanically, as in the shell model. These differences in models of atomic nuclei are differences on a fundamental level. They represent structure of the nucleus as responsible for different kinds of behavior. Morrison cites this incompatibility as evidence against the representational veracity of either model. She notes that in the context of our current state of knowledge of the atomic nucleus, these models are largely phenomenological. Given the state of the evidence, there is not a single model of the nucleus about which we are warranted to be realists.

In contrast to models of atomic nuclei, Morrison cites various models of fluid mechanics as examples of models that seem incompatible, but which do not warrant antirealism, or even some kind of perspectivism. These models differ from those of the atomic nucleus, because these are complementary, rather than contradictory. That is, while none of the models gives an accurate representation of an entire fluid system, each one gives an accurate representation of some part of the fluid system. What makes this work in the case of fluid mechanics, but not models of the atomic

nuclei, is that the underlying assumptions about fluids are the same across all of the models. In the fluid mechanics case, the models represent different kinds of flows or different parts of the same flow. “The models can only ever give an approximate description and with a particular set of empirical constraints are valid only for certain flows or ranges of flows.” (Morrison, 2015, p. 173)

I am in general agreement with Morrison’s approach. Indeed, it seems such a rather straightforward approach that one wonders why such incompatible models would trouble a realist in the first place. I would emphasize that complementarity is not just about different parts of flows, but can also be related to scale. That is, one model is meant to capture some feature of the flow at a large scale, and another model is meant to capture some feature of the flow at a smaller scale. This sort of complementarity can be used to resolve the apparent incompatibility between modeling boundary conditions and conditions at the boundary. Different models get things right in under different conditions.

However, the fact that we can interpret the models of fluid mechanics as complementary does not tell us much more than perspectivism does if it does not also explain how different conditions give rise to different models, or how different models are linked. Perspectives are dependent upon an experimental setup. This is true of both complementary and contradictory models. In complementary models, there is an explanation for why the models disagree. Knowing that a model fails for a certain perspective is not enough. We need to know why it fails. Answering this question, I will contend, often generates more reasons to be a realist than antirealist about particular models in fluid mechanics.

Models are complementary not only in terms of the part of the world they represent. They are also complementary with respect to the other conditions that are present in a given system. Complementary models are complementary because of the domain of application. The reason they

get things right is not a matter of perspective, but rather of the experimental conditions under which they were confirmed. Understanding the conditions under which a model is valid helps explain why the models are complementary.

The next section considers the results of recent experimental investigations of the no slip boundary condition. These experiments do not only tell us the conditions under which the no slip condition is valid, but they also give us insight into why it fails under other conditions. I will argue that this insight helps justify our belief in the no slip condition when it is valid. The truth of the models is relative to conditions determined by experiments. It gives us knowledge of under what conditions a boundary condition applies, where it fails, and why it fails. So instead of painting an anti-realist picture, the variety of models instead give a more robust argument for realism.

This will explain how models are complementary, which goes toward how we can continue to be realists when different models say apparently incompatible things. In effect, a model is accurate relative to a particular part of a system, but it is also true relative to other conditions that are present in the target system. This depends not on perspectives, but on the material conditions. While this might not address all of the challenges to realism that stem from modeling practices, it provides an explanation for how models can be complementary.

4.3 The Domain of Application

Virtually every scientific model has a domain of application. Save perhaps models at the most fundamental level of physics (fields or superstrings or whatever the case may be), no model is valid without exception. Scientific models are useful under some conditions, and not others. The

conditions under which a model accurately describes a target system I call a domain of application. To see what I mean by domain of application, I will look at some examples.

Consider the progression from classical (Newtonian) mechanics to relativistic mechanics. Even though Cartwright (1983) has convincingly argued that classical mechanics is technically false except under very particular conditions which never actually obtain anyway, it seems strange to say that classical mechanics is simply false. We know that classical mechanics is a very good approximation (good enough for successful engineering) under certain conditions. While it provides a very good approximation in the limit of low velocities and low gravity, we find that as a given system departs from this domain, the approximation of classical mechanics gets worse and worse. Since we understand how classical mechanics arises in certain conditions, we should be more confident in its approximate truth in those conditions. By learning that relativistic mechanics reduces to classical mechanics, we are more justified in our belief in the approximate truth of classical mechanics under low velocity and low gravity conditions.

The governing equations of fluid dynamics have a domain of application. Generally, fluid systems can be categorized by Knudsen number (Kn). There are four flow regimes based on Knudsen number of the flow. (Shu, Teo, & Chan, 2017) The Knudsen number is the ratio of molecular mean free path length to a representative physical length scale. In the no-slip regime ($Kn < 0.001$), the governing equations of fluid dynamics along with the no-slip condition are valid. In the slip flow regime ($0.001 < Kn < 0.1$), the governing equations are valid but there is slip at the boundary. Intuitively, we can think of a fluid that is rarified enough that molecules bounce along the solid surface, and where interactions with other fluid molecules are less frequent. In the transition regime ($0.1 < Kn < 10$), the governing equations of fluid dynamics begin to become questionable as the continuum assumption is no longer able to capture all of the relevant behaviors

of the system. In the free-molecular flow regime ($Kn > 10$), the continuum assumption is no longer of any use, and molecular dynamics must be used to describe the system. So once we leave the no-slip regime, not only does the no-slip condition break down, but so do the basic assumptions of fluid dynamics, generally.

In fact, dimensionless numbers, like Knudsen numbers, which relate relative magnitudes of fluid properties are a good place to look for this kind of evidence for realism. For example, the Reynolds number (Re) of a fluid flow relates inertial forces to viscous forces. It can be used to predict when the transition from laminar to turbulent flow occurs. Mach number (Ma) relates flow velocity to the local speed of sound. A flow's Mach number determines the character to the governing equations that describe the flow. For Mach numbers less than 1, the governing equations are elliptic. If it is greater than 1, the governing equations are hyperbolic.

The important thing to note is not just that we can identify the domain of application. The important thing is that we can have *causal knowledge* of how manipulating certain conditions affects the model in question. These domains of application are essential in understanding how models complement each other. While the conclusions I have drawn in the other chapters have been limited to the boundary conditions of fluid dynamics, I think the pattern of reasoning here can be applied quite generally. Almost every theory has some domain of validity, which defines the domain of application of its associated models. So almost every model fails under some circumstances. These failures are often places to look for new physics. The fact that these domains exist, and the models that explain them, are themselves interesting empirical facts that had to be discovered.

Models characterized by classical or relativistic mechanics and models characterized by the governing equations of fluid dynamics are relatively large classes of models. In the next

section, I will show how the concept of a domain of application applies to more particular parts of models, resulting in a domain of application for a smaller class of models. In particular, I will look at the domain of application of models that include the no-slip boundary condition.

4.3.1 Domain of Application of the No-Slip Condition

The previous section looked at domains of application for relatively broad classes of models. But the concept of a domain of application extends to smaller parts of models as well. This results in the possibility of fine-grained cleaving of domains of application. The domain of application of the no-slip condition is limited to a subset of the domain of application of the broader theory in which it is contained. Within the domain of application of fluid dynamics models, generally, the domain of particular parts of fluid models is further limited. Like the velocity limits the domain of special relativity and Knudsen numbers limit the domain of fluid dynamics, other factors put limits on smaller parts of the models as well. The appropriate boundary conditions depend on the conditions of the fluid flow.

The no-slip condition is further limited to fluids that have non-zero viscosity. Almost all real fluids have some viscosity, even if fluids with very little viscosity can be modeled as if they have zero viscosity. Actual zero viscosity is only seen at extremely low temperatures in fluids known as superfluids. Whether or not we model a fluid as viscous determines which governing equations we use. Recall from Chapter 1 that the governing equations have both viscous and non-viscous forms. For a viscous flow in which the no-slip conditions holds, both the normal and tangential velocities go to zero at the boundary. For an inviscid flow, only the normal velocity goes to zero.

The no-slip condition is also limited to Newtonian fluids. Roughly, Newtonian fluids are fluids with a constant viscosity that is independent of stress.¹⁴ Non-Newtonian fluids not only display slip, but require changes to the governing equations for describing the entire flow. (Schowalter, 1988) For example, a shear-thickening liquid is a liquid whose viscosity increases with shear rate.¹⁵ To describe the flow of a shear-thickening liquid, the form of the governing equation must take into account that viscosity is dependent on shear rate, rather than constant.

The failure of the no-slip condition in high Knudsen, non-Newtonian, or non-viscous contexts corresponds to the failure of other theoretical assumptions as well. Outside of slip and no-slip Knudsen regimes, the continuum assumption of fluid dynamics is no longer accurate, nor are the governing equations. And modelling inviscid or non-Newtonian fluids also require alterations to the governing equations. So it should not be surprising that common boundary conditions also fail under these conditions. However, not all failures of the no-slip condition are accompanied by failures of other parts of fluid dynamics. There are other limitations to the domain of the no-slip condition's application, which are specific to the no-slip condition. Understanding these limitations will allow us to isolate the casual factors that explain slip phenomena.

¹⁴ More technically, in Newtonian fluids, the stress tensor, which consists of the normal components and the viscous stress tensor, is a linear function of the velocity gradient. (Karniadakis, Beskok, & Aluru, 2005, p. 52)

¹⁵ A common example of a shear-thickening liquid is “oobleck,” a mixture of cornstarch and water that has become a canonical kitchen science experiment for children.

4.3.2 Domains of Application and the Reality of Slip

The domain of application for the no-slip condition can be reduced to an even finer grained description. For most macroscopic applications, the no-slip condition works well in the domain of viscous Newtonian fluid flows. However, even in flows that satisfy these conditions, it is still an open question as to whether the velocity of a fluid flow literally goes to zero at the boundary. In previous chapters, I characterized the function of boundary conditions as constraining the behavior of the governing equations. However, they do also describe a part of the system. The no-slip condition constrains the solutions to a particular flow, and in doing so, it also describes the velocity of the fluid at the boundary as zero. We can ask the question of whether that description is literally true. At the macroscopic scale, fluid flows certainly behave as if it is true. But there are reasons to look beyond the macroscopic scale, and the question of slip takes on much more practical importance when looking at small systems. In flows through passages with diameters on the order of micrometers or nanometers, boundary effects become much more important, as the region near the boundary represents a significant proportion of the overall flow. Even a small amount of slip can have a large effect on the flow. There is evidence that fluid dynamics can describe flows through channels as small as 10 molecular diameters. Any smaller than that, though, and molecular dynamics must be used to describe the flow. (Karniadakis, Beskok, & Aluru, 2005) While the no-slip condition seems to get things right at the macroscopic level, it is still an open question as to whether it holds at smaller scales, even for Newtonian fluids.

Despite the no-slip condition being the default textbook boundary condition for Newtonian flows, the actual behavior of fluids near a solid boundary is still not entirely understood. While the no-slip condition is useful in predicting flow fields, the question of whether or not we should interpret it realistically still seems to be open. Lauga, Brenner and Stone (2007, p. 1220) write that

“a century of experimental results in liquids and theories derived assuming the no-slip boundary condition (i.e., $\lambda = 0$) had the consequence that today many textbooks of fluid dynamics fail to mention that the no-slip boundary condition remains an assumption.” Similarly, Shu, Teo, and Chan (2017, p. 2) point out that “the no-slip boundary condition originated as an assumption without any fundamental basis.” Further, Neto et al. (2005, p. 2859) note that the no-slip condition “has been applied successfully to model many macroscopic experiments, but has no microscopic justification.” They go on to say that by the “mid-20th century [...] it had been unanimously accepted that even if slip occurred, it would have been detected using only experimental techniques with resolution far beyond that available at the time.” (p. 2864) Despite our incomplete knowledge of slip conditions, the ongoing experimental investigation of slip has produced a great deal of information about the conditions under which slip occurs.

The no-slip condition depends on a number of factors that are important for implementing the correct fluid dynamics generally. But the domain of application of the no-slip condition is even further limited by other factors such as surface roughness, dissolved gas and bubbles, wetting properties, shear rate, and electrical properties. (Lauga, Brenner, & Stone, 2007). These factors do not affect fluid models as broadly as Knudsen numbers or viscosity do, so they are better suited to investigating the no-slip condition itself. Even in the no-slip regime, we can ask if no-slip condition ever literally obtains. Under some conditions, there appears to be slip in Newtonian fluids. Experimental investigations seek to not only quantify this slip but explain it as well. In most cases, it is concluded that the reason is apparent slip. Regardless of whether it is actual slip at the molecular scale or merely apparent slip, the results indicate that slip is dependent on a number of physical parameters. I will look at how some of these factors affect slip, in order to illustrate the

point that experiments and models investigating the limits of no-slip generate evidence in favor of a realist interpretation of the no-slip condition.

One factor that affects slip is the wetting properties of the fluid on the surface. The wetting behavior of a fluid can be characterized with a contact angle. The contact angle quantifies how hydrophobic or hydrophilic a surface is. There is a correlation between the contact angle and the degree of slip. This makes intuitive sense, as one could imagine fluid slipping more readily across a hydrophobic surface rather than a hydrophilic one. And indeed there does seem to be a connection between the wetting properties of a fluid-solid pair and whether or not slip behavior is observed. In the case of wetting behavior, models have been developed to predict slip based on contact angle. One of these models that has been proposed is the Tolstoi model (Blake, 1990):

$$\frac{\lambda}{\sigma} \sim \exp \left[\frac{\alpha \sigma^2 \gamma (1 - \cos \theta_c)}{k_B T} \right] - 1 \quad \text{Equation 16 Tolstoi model}$$

where λ is slip length, σ is a characteristic molecular length, α is a dimensionless geometrical parameter of order one, γ is the liquid surface tension, θ_c is the contact angle, k_B is Boltzmann's constant, and T is temperature. Models like this, which quantify the relationship between slip and contact angle, are robust evidence for a causal relationship of an interventionist sort.

Another factor that affects the amount of slip observed at a fluid-solid boundary is the surface roughness of the solid. The effect of surface roughness is more difficult to quantify than the effects of wetting properties. Experiments have shown that depending on the kind of fluid and the kind of surface, surface roughness can increase liquid friction. While the precise mechanism is not fully understood, it is thought that local irregularities in the flow cause the dissipation of mechanical energy. But even though the exact mechanism is not known, both physical experiments

and simulations have been able to quantify the degree of roughness necessary to produce the no-slip condition at a macroscopic level.

Zhu & Granick (2002) find that as surface roughness increases, so does agreement with the no slip condition. For a given surface roughness, slip can be induced when the shear rate of the fluid flow reaches some critical level. However, this critical shear rate diverges when surface roughness exceeds a root mean square (rms) roughness of approximately 6 nanometers, where rms roughness is the calculated root mean square of the surface's microscopic peaks and valleys.

Molecular dynamics models attempt to describe the degree of slip under various surface roughness conditions. For example, Koplik, Banavar, & Willemsen (1989) find that for certain kinds of flows, molecular roughness give rise to a no-slip boundary condition. However, Galea & Attard (2004) find that in other circumstances, rough surfaces lead to slip. This might be due to surface energies causing spontaneous dewetting of the solid surface, producing a hydrophobic state.

These experimental and simulation results work in concert with models derived from theoretical understanding. For example, Jansons (1988) calculated that, under certain roughness conditions, very small amounts of surface roughness produce a slip condition that will approximate the no-slip condition at the macroscopic level. In particular, one defect on the order of 10^{-9} m per 10^{-7} m² will produce a slip length of 10^{-5} m. Similarly, Casado-Diaz, Fernandez, & Simon (2003) calculate that for a characteristic length scale L and surface features of size a , as $a/L \rightarrow 0$, the velocity at the surface approaches 0. That is, the no-slip condition is recovered. These calculations result from the description of viscous dissipation. This is the transformation of kinetic energy to heat energy in turbulent flows.

The experiments regarding the effects of surface roughness on slip paint a complex picture. The factors that affects slip are not independent of each other either. Both surface roughness and wetting properties affect the formation of gas bubbles.

Surface roughness is connected with another condition that affects slip: the formation of gas between the surface and the fluid. Shu, Teo, & Chan (2017) find that slip might depend not just on whether the surface is rough, but also on *how* the fluid contacts a rough surface. On the one hand, the fluid can fill the crevices on a rough surface (Wenzel state). On the other hand, the fluid can sit above the crevices (Cassie state). Which of these two states the fluid has depends on the properties of the fluid involved, but generally a Wenzel state results in less slip. It is thought that any slip is lost via viscous dissipation. Conversely, a Cassie state results in significant slip. A Cassie state can result in pockets of gas that form between peaks of the microscopically rough surface. This way, the fluid flows over alternating regions of no-slip (solid) and slip (gas).

If instead of a collection of gas pockets or bubbles, there is a layer of gas between a liquid and a solid, the depth and viscosity of the gas layer can be used to predict apparent slip:

$$\lambda = h \left(\frac{\mu_1}{\mu_2} - 1 \right) \quad \text{Equation 17 Apparent slip length}$$

where λ is slip length, h is the height of the gas layer, and μ_1 and μ_2 are the viscosities of the liquid and the gas, respectively. Relations such as these give us deeper insight into the causal relationship between the no-slip condition and the condition under which it holds.

Rather than falsifying the no-slip condition, experiments that explore these relationships are taken as evidence of the truth of no-slip, *under certain conditions*. These experiments do more than either confirm or disconfirm the no-slip boundary condition. They help define the no-slip

condition's domain of application. In so doing, they incidentally offer reasons to hold a realist attitude toward no-slip. By framing in its edges more carefully, the resulting picture is more clearly defined.

Importantly, there is no critical experiment here to once and for all establish the truth of the no-slip condition. This is an example of a consensus emerging from contemporary experimental work. As more theoretical frameworks, more modeling techniques, more mathematical, simulation, and experimental methods pile on to the study of slip, they have uncovered a significant class of situations in the real world to which no-slip applies. Using Michelangelo's famous analogy, it is a sort of realism that is achieved by cutting out the excess marble and revealing the sculpture that was already there.

In this section, I have shown how experiments at the boundaries of the no-slip condition generate rationales for why no-slip should be trusted in cases where it is employed. Under a received view of the relation between models and theories, due largely to Cartwright, Suarez, and Shomar (1995) and Cartwright (1999), and supplemented by Woodward's (2003) interventionist account of causal explanation, this line of scientific investigation has revealed causal relationships between a variety of flow features and the no-slip boundary condition. Intervening on the conditions of the fluid system has a systematic effect on the effectiveness of the no-slip condition. Further, we have good theoretical reasons for why these relationships should hold.

4.4 Reasons to be Realists

Despite models giving conflicting descriptions of the world, I follow Morrison in understanding these models as complementary. In defining a model's domain of application, I take

a closer look at just how models can be complementary. Understanding the causal relationship between a model and a domain of application allows us to make stronger statement about the truth of the aspect of the model in question.

In contrast to most arguments for realism, which focus on a theory's success, this one relies on its failures as well. To summarize: the no-slip condition is a highly successful feature of fluid modeling practices. Despite this success, it is still unobservable, and so gives rise to doubts that it is literally true. There is evidence that it fails under some conditions, and experiments have provided insight into those conditions. Based on our best understanding of the results of these experiments, we are able to learn how intervening on those conditions affects the no-slip condition. Thus we gain causal knowledge about the failure of the no-slip condition. Boundary conditions display some degree of invariance under interventions, and there is a pattern of causal dependence that is exploited.

The kinds of relations that emerge from the experimental evidence give us a means of differentiating the strength of the evidence. Some evidence gives more robust relationships between the no-slip condition and the conditions under which it fails. When there is a quantifiable relationship between a variable and the degree of slip, as there is in the case of gas layers, we have more robust causal knowledge. In contrast, when the evidence does not yield a definite relationship, or the evidence is ambiguous, the inferred causal connection is not as strong. The effect of surface roughness, for example, is not easily quantifiable. At present, I think the evidence gives us reason to be realists about the no-slip condition. Further investigation might strengthen the causal relationship between certain flow conditions and the no-slip condition. If they do, then we have more evidence to be realists about the no-slip condition in its intended domain. If not, then we have evidence to be skeptical of the literal truth of the no-slip condition.

Some features of this pattern of reasoning should be familiar to realists. It is, in some ways, an extension of some other arguments for realism. This could be thought of as a form of inference to the best explanation. The reality of the no-slip condition under certain conditions is part of the explanation for the casual relationships we find. The thing that distinguishes this account is the emphasis on explanations of failure. The standard inference-to-the-best-explanation argument for realism focuses on the success of theories, so understanding the conditions under which a boundary condition is successful provides evidence for that boundary condition. But on my account, understanding the conditions under which it fails also provides evidence. If a model makes good predictions in some conditions, but not others, this alone is not enough to be a realist about it. But if we can understand the reason for its failure in those other conditions, then we have reason to be confident it its truth for the conditions in which it works well.

The focus on intervention should also bring to mind the slogan of the entity realism proposed by Ian Hacking as a response, in part, to his dissatisfaction with empirical adequacy: “If you can spray them, then they are real.” (Hacking, 1983, p. 23) If we can manipulate an entity, even one that is not directly observable, that manipulation counts as evidence that the entity is real. Unsurprisingly, I am sympathetic to this response. I see this discussion as an extension of Hacking’s view: in my argument, it is not successful manipulation of an *entity* that is doing the work, but rather manipulation of a *variable*, more generally conceived. The objects of manipulation are the variables like surface roughness, the presence of gas layers, and surface wettability. Their manipulation not only “saves” them, it also gives evidence that their causal nexus—that is, the no-slip condition—is real.

I want to make one more remark about my argument for the reality of the no-slip condition. In the realism literature, philosophers enjoy contrasting real things with “mere” phenomena. That

artificially limits the sorts of things that are candidates for reality to objects and entities. Part of what I am arguing here is that other stuff that populates models, the variables and parameters that don't neatly latch on to entities and objects, should also be candidates for reality. The causal powers of the no-slip condition, the manipulability they generate, the ways that manipulability is studied through direct experimentation and modeling, and especially the careful study of the domain of application of no-slip models: these are all evidence for a Hacking-style abduction to the reality of the no-slip condition itself, not merely the reality of the fluid or the pipe.

The idea that a model's domain of application assists in generating rationales for realism about certain phenomena encoded in the model is not just important for cases of apparently conflicting models. Recall that the antirealist argument from incompatible models grows out of more general concerns (e.g. Cartwright) about modeling practices. The fact that models are idealized or only obtain under very particular conditions does not warrant antirealism, as long as we understand how deviations from those conditions affect their approximation to the world by affecting their domains of application. It might be the case that the no-slip condition very rarely literally obtains. But it is a very good approximation under some conditions.

4.4.1 Local Realism

Finally, I will say a little bit about the kind of realism this argument supports. It does not support a global realism. That is, it does not show that we can make a generalized inference from our successful theories to their truth. Instead, it supports a more local realism, in the spirit of (Wimsatt, 2007). Not all models should be regarded as literally true. But this should not lead us to conclude that we cannot tell which models represent the world accurately and which models do not. This should allow us to identify which models should be regarded as literally (approximately)

true, rather than merely phenomenological. This argument depends on more than just the predictive or explanatory success of the model. Rather, the details of its success and failures inform our epistemic attitude. Almost every theory has a domain of application, outside of which it breaks down.

There are apparent similarities between what I am calling local realism and some versions of perspectivism. And it might be thought that the two views are in fact compatible. Giere (2006), for example, considers his position to be genuinely realist. And Callebaut (2012) even labels Wimsatt's local realism as a variety of perspectivism. For my own purposes, if there are versions of perspectivism that are compatible with local realism, then so much the better for both views. However, the versions of perspectivism put forward by at least Giere and van Fraassen do not claim the level of epistemic support for the unconditionalized truth of some models that local realism does.

The sort of local realism that I am arguing for is epistemically more ambitious than perspectivism. Recall that on Giere's characterization of perspectivism, the strongest claim we can make about what we know about the world is: "According to this highly confirmed theory (or reliable instrument), the world seems to be roughly such and such." (Giere, 2006, p. 6) In place of this, I posit that we can make the stronger claim: due to this highly confirmed theory, model, or reliable instrument, under certain specifiable physical conditions, this part of the world *is* roughly such and such. The scope of the truth being claimed in this view extends beyond perspectivism's mere seemings and beyond the model itself, although it is obtained through the model.

In Rueger's perspectivism (2005), properties of a system like viscosity are not "intrinsic." Instead, they are "relational." Similar to Giere's view, in Rueger's view, we have knowledge about what a system *looks* like rather than knowledge about what it *is* like. As Rueger puts it, "from this

perspective, the system looks as if it has intrinsic property x , and from that perspective, the system looks like it has property y .” (2005, p. 580) On my account the relation is not between a model and the target system, but between the phenomenon being modeled and the physical conditions under which it occurs or not.

As I suggested above, the mundane and human-scale nature of slip makes it the sort of phenomenon that it is easy to be a realist about. For a given system, there is slip, or there is not. If one model includes slip, and another model of the system does not, perspectivism would simply consider these models two different perspectives. But the representational successes and failures of each model can serve as further ways of comparing and evaluating the models, so that we can move beyond merely perspectival approaches. The correctness of a model is determined by facts about the system it is supposed to represent. And as the argument from the domain of application of the no-slip condition shows, these facts can apply to fine-grained parts of models. Looking at how the no-slip condition depends on the conditions of a flow shows how modelers can identify and explore particular elements of a model. While the model depends on a variety of assumptions and background knowledge, the experiments are (contra Quine) effective ways of questioning a particular part of a model. Experimenters are able to isolate the boundary condition. There is perhaps an in-principle argument to be made about the inseparability of the no-slip condition from the models that contains it: a boundary condition always works with other parts of a model, and the inability to directly observe the velocity at the boundary means that the model necessarily requires other assumptions to derive empirical consequences. But in practice, these are held fixed. The piecemeal fashion in which the no-slip condition is confirmed is evidence for a local realism. Rather than an argument about successful theories, generally, the debate over realism takes place at the level of individual models.

This pattern of reasoning should be used in conjunction with other evidence for realism with respect to particular theories or models. They are not only predictively successful, but they are able to be investigated via multiple means of detection. They are also subject to more direct means of detection, to the point of being called observable.

The upshot of the above considerations is that there has been a deep failure in much of the literature on scientific realism so far. The use of scientific models is a problem if the realist makes an argument based on a general scheme from explanatory or predictive success to truth. These sorts of virtues should be thought of as characteristic of true theories, and surely do carry epistemic weight; we should count these as evidence that some entity or property is real. But by themselves, these virtues are not enough evidence to distinguish models that give a realistic representation from those that are instrumentally useful but not realistic. Instead, the details of precisely how the entities and properties are being detected should inform the epistemic attitude we take towards them.

I think the above is an indication that the only defensible realism is a local, as opposed to global, realism. We are not warranted in making claims like “our best (most successful, explanatory, well-confirmed) scientific theories are true.” Since the details of confirmation and detection are varied, we can only proceed on a case by case basis. Then the argument for local realism with respect to a given entity or property depends, not on an abstract confirmation scheme (e.g. theory T predicts O, O obtains, therefore T receives some degree of confirmation), but on the details of experimental detection and the surrounding theoretical support. This echoes Giere’s rationale for perspectivism, but the domains of application are not perspectives. They are facts about the world.

Finally, the thing that a perspectivism like Giere's gets right is that it relativizes our epistemic attitude toward a model to certain conditions. Despite rejection of global realism, I think we can say something about the way we confirm our models with respect to the experimental settings in which they are confirmed. Ultimately our reason for believing the truth of any theory is the experimental evidence, and we run into trouble when we separate our theories from the conditions in which they are confirmed. We run into trouble when we make claims like "the world is roughly such and such" instead of "the world is roughly such and such, *when such and such conditions obtain.*" Compare this to Giere's formulation of perspectival realism, according to which, we should not treat our well confirmed scientific models as completely objective representations of our world. Instead of trying to justify claims like "This theory (or instrument) provides us with a complete and literally correct picture of the world itself," the best we can do is to justify claims like, "According to this highly confirmed theory (or reliable instrument), the world seems to be roughly such and such." (Giere, 2006, p. 6) But I am not talking about perspectives here; I am talking about differences in the world. For example, we often treat systems at different spatial scales. But a spatial scale is not a perspective. It is true that systems do *look* different at different scales, but the behavior we see when we look is the dominant behavior at that scale whether we are looking or not.

This is how we get the pluralism, and still retain local realism. Instead of saying the world is thus and so from some perspective, we can say that the world is thus and so, given certain conditions. These conditions are not a matter of perspective, but are objective features of the world.

4.5 Conclusions

In examining the experimental investigation of slip phenomena, I have argued that we find a pattern of reasoning that helps inform our epistemic attitude toward some unobservable parts of our models. Understanding the limits of our theories should increase our belief that they are true, more than the existence of those limits should decrease the belief that they are true. This paints a picture that is in line with both realist interpretations and actual modeling practices.

I have argued that the modeling practices surrounding the no-slip condition support the complementary nature of some models. The epistemic attitude we take towards a model depends on its domain of application. Once we take that into account, apparent incompatibilities are not a threat to realism, as long as we understand that realism does not commit us to the view that a model is absolutely true.

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