## List of Figures

Figure 1. Images of the semi-structured ball maze activity ........................................................ 19

Figure 2. Example stimuli for the non-symbolic number comparison task ................................. 32

Figure 3. Histogram of teachers' total math talk proportion scores ............................................ 44

Figure 4. ANCOVA results for teachers' math talk proportion scores ........................................ 49

### 2.3.1 Observations of Naturally Occurring Activities

During the observation, the observer classified the classroom activity minute-by-minute as one of the following categories: *Large-Group Instruction, Small Group Instruction, Free Play, Meal Time*, or *Transitions*. Definitions of these types of activities were adapted from the Teacher Observation in Preschools (TOP) Manual (Bilbrey, Vorhaus, Farran, & Shufelt, 2007). In general, the classroom activities were classified based on the actions of the children in the classroom. For example, if the children were seated at tables to eat a meal, and the participating teacher moved to another area of the classroom to prepare the next activity, *Meal Time* was still coded until the children finished eating their meal. Full descriptions of how observers classified each activity are described below.

**Large Group Instruction**. The observer classified the classroom activity as *Large Group Instruction* when the entire class of children was gathered for any type of learning activity that was directed by a teacher or specialist (e.g., music teacher). Learning activities were broadly defined to include activities involving math, reading, science, art, or music, as well as discussions about behavioral expectations or plans for the day (often called "morning meeting" or "circle time"). If some of the children were removed from the group for another purpose (e.g., for discipline; for speech therapy), *Large Group Instruction* was still coded if the majority of the children remained in the group. In classrooms with less than four children in total, *Small Group Instruction* was coded instead of *Large Group Instruction*.

**Small Group Instruction.** The observer classified the classroom activity as *Small Group Instruction* (sometimes called "center time") when at least one group of children was assigned to work on a specific task that was facilitated by the teacher participating in the study. Note that the remaining children in the classroom could either be engaging in free play or working in other small groups facilitated by other teachers. In Montessori classrooms, *Small Group Instruction* was coded during "Work Time" — the 1-2-hour period in which teachers gave tailored individual and small group lessons on a range of subjects.

**Free Play.** The observer classified the activity as *Free Play* (sometimes called "activity time" or "discovery time") if children were given choices about what to play with or how to play. *Free Play* occurred both indoors and outdoors. In one form of free play, children were assigned to a specific area but given freedom to choose how to play in that given area. In another form of free play, children were allowed to freely move throughout the classroom or outdoor space, even if teachers placed some restrictions on which toys or areas were available. *Free Play* was coded for the entire duration that children were playing, even if a teacher stayed in a certain area to facilitate activities with individual children or groups of children (e.g. reading a book; playing a board game).

**Meal Time.** The observer classified the activity as *Meal Time* when the children were gathered together to eat either breakfast, lunch, or a snack. *Meal Time* was coded as soon as the children started setting the tables, passing food, saying a prayer, or eating. *Meal Time* was coded for the entire duration that children were eating. If the teacher led the children in a cooking activity, the time the children spent cooking was coded as *Large* or *Small Group Instruction* (depending on the number of children), while the time the children spent eating what they cooked was coded as *Meal Time*. Activities to prepare for eating, such as washing hands or retrieving lunch boxes, were not coded as *Meal Time*, but rather as *Transition*s.

**Transitions.** The observer classified the classroom activity as *Transitions* when one activity had ended and the next activity had not yet begun. Examples of *Transitions* include time spent cleaning up activities or meals, calling children's names to move to the next activity, preparing materials for the next activity, performing personal care tasks (e.g., helping children wash hands before lunch or put on sunscreen before going outside), lining children up to leave the classroom, leaving the classroom to walk to another area of the school, welcoming children into the classroom at the start of the day, and preparing children to go home at the end of the day. If the children gradually transitioned from one activity to the next, *Transition* was coded until the majority of children started the next activity.

Not all teachers engaged in all activities on the day(s) of the observation: 6 teachers did not have *Large Group Instruction*, 19 did not have *Small Group Instruction*, and 13 did not have *Free Play*. Eighteen teachers engaged children in both *Small Group Instruction* and *Free Play*, 11 teachers had *Small Group Instruction* but not *Free Play*, 17 teachers had *Free Play* but not *Small Group Instruction*, and 2 teachers had neither *Small Group Instruction* nor *Free Play*. See Table 1 for the average amount of time teachers spent in each type of activity.

**Table 1. Descriptive statistics of observed classroom activities**

Number of teachers for whom each activity was observed and descriptive statistics on the duration, number of utterances, and number of utterances per minute for each type of classroom activity.

| | Type of Activity | | | | | |
|---|---|---|---|---|---|---|
| | Large Group Instruction | Small Group Instruction | Free Play | Meal Time | Transitions | Semi-Structured Ball Maze Activity |
| N | 42 | 29 | 35 | 48 | 48 | 48 |
| Duration in Minutes | | | | | | |
| *M* | 38 | 36 | 36 | 19 | 31 | 31 |
| *SD* | 20 | 19 | 19 | 11 | 15 | 14 |
| *Range* | 7–96 | 4–86 | 4–82 | 5–48 | 3–66 | 9–62 |
| Number of Utterances | | | | | | |
| *M* | 489.57 | 465.24 | 339.17 | 170.60 | 263.40 | 406.65 |
| *SD* | 318.00 | 257.77 | 194.55 | 100.47 | 141.04 | 195.79 |
| *Range* | 72–1774 | 21–1081 | 51–756 | 21–428 | 18–558 | 65–933 |
| Utterances per Minute | | | | | | |
| *M* | 12.70 | 13.11 | 9.78 | 9.00 | 8.88 | 13.37 |
| *SD* | 3.94 | 3.61 | 2.76 | 2.88 | 2.78 | 4.10 |
| *Range* | 3.6–21.79 | 5.25–21.86 | 3.03–14.91 | 2.63–15.18 | 3.89–15.71 | 2.21–22.21 |

### 2.3.2 Observation of the Semi-Structured Ball Maze Activity

We also audio recorded teachers' speech during the semi-structured ball maze activity using the same method described for the naturally occurring activities. For the semi-structured ball maze activity, we asked teachers to guide their preschoolers in playing with a novel toy brought by the researcher. The toy, a wooden ball run by Wonderworld (see Figure 1), included wooden track pieces, 85 interlocking orange blocks, 5 balls, and special trick pieces (e.g., ramps, a staircase, a funnel, flip pieces). The observer explained to the teacher that the track pieces and special trick pieces could be used to create a path for the balls, and that the orange interlocking blocks could be used to build the path at various heights. Teachers were given an example picture of a completed maze (see Figure 1) but were encouraged to guide their preschoolers in playing with the pieces in any way they thought best for their children. They were told to do the activity with at least one small group of children, but were encouraged to rotate groups of children if they had time. Some teachers were the only adult present in the classroom, and for feasibility reasons chose to do the semi-structured ball maze activity with the entire group of children at once. See Table 1 for descriptive statistics on the amount of time teachers spent doing the semi-structured ball maze activity in their classroom.

The observer classified the classroom activity as *Semi-Structured Ball Maze Activity* as soon as the teacher began to introduce the toy to their students and continued for the entire duration that one or more children were playing with the toy. Time spent cleaning up the toy was not coded as *Semi-Structured Ball Maze Activity*.

For the semi-structured ball maze activity, teachers were provided with wooden tracks pieces, interlocking orange blocks, 5 balls, and special trick pieces (left) and a laminated picture of an example structure (right).

**Figure 1. Images of the semi-structured ball maze activity**

### 2.3.3 Transcriptions and General Coding Procedure

The entire duration of each audio recording was transcribed by a trained research assistant and each transcription was verified by a second trained research assistant. The transcribers segmented teachers' speech into utterances. An utterance was defined as talk by one speaker that was bound by a transition in the speaker, a pause of more than two seconds, or grammatical closure (Pan, Rowe, Spier, & Tamis-Lemonda, 2004). Across the entire observation, teachers used 1797.29 utterances on average (*SD* = 521.04, *min* = 624, *max* = 3407) and 11.07 utterances per minute (*SD* = 2.58, *min* = 3.78, *max* = 17.29). See Table 1 for descriptive statistics on how many utterances teachers used during each of the different activities (*Large Group Instruction*, *Small Group Instruction, Free Play*, *Meal Time, Transitions,* or *Semi-Structured Ball Maze Activity*).

Utterances were then coded for math-relevant input ("math talk"). As a first pass, a trained research assistant read through 50% of the transcriptions and marked utterances that generally seemed to relate to math. From these marked utterances, we established a list of search terms — words indicating that an utterance potentially contained math talk (for example, words such as "three", "count", or "add"). See Table 2 for the full list of search terms.

**Table 2. Search terms for math talk coding**

A list of search terms for the first step of the math talk coding process

| Search Terms | | | | | | |
|---|---|---|---|---|---|---|
| Zero | Eleven | First | How much | Compare | Sum | Fraction |
| One | Twelve | Second | How many | Even | Difference | Half |
| Two | Thirteen | Third | Number | Order | Single | Quarter |
| Three | Fifteen | Fifth | Amount | Add | Double | Old |
| Four | Twenty | Ninth | More | Plus | Triple | Date |
| Five | Thirty | Twelfth | Most | Subtract | Ones | |
| Six | Forty | Twentieth | Less | Take away | Tens | |
| Seven | Fifty | Thirtieth | Least | Minus | Hundreds | |
| Eight | Hundred | Once | Equal | Multiply | Thousands | |
| Nine | Thousand | Twice | Different | Divide | Place value | |
| Ten | Million | Count | Same | Total | Place | |

All transcriptions were then coded for math talk following a two-step process. In the first step of the coding process, the researcher searched each transcript for the words listed in Table 2. Utterances that contained one or more of the search terms, used in a numeric sense, were selected for further coding in the second step of the coding process. The meaning of each selected word was determined from the context of the transcript. When selected words were not used in a numeric sense (e.g., using the word "order" in the phrase "order in the court", using the word "once" in the phrase "once upon a time", or using the word "one" in the context of "this one", "that one", "the one", or "which one"), they were eliminated from further math talk coding in the second step of the process.

In the second step of the coding process, the researcher examined each of the selected utterances to classify them into 13 different categories of math talk. These categories were compiled from a combination of those used in previous studies (Klibanoff et al., 2006; McCray & Chen, 2012; Rudd et al., 2008; Simpson & Linder, 2016). Definitions of each math talk category are described in section 2.3.4.

For 12 of the math talk categories, utterances were further classified as either elicitations or statements, resulting in 25 different types of math talk. Elicitations were characterized by any math-related attempts to encourage a child or group of children to respond either verbally or non-verbally. Elicitations could have included direct commands (e.g., "Count the blocks", "Give me three", "Point to the number seven"), indirect commands (e.g., "We should see who has the most", "Do you want to count?", "Let's add them together"), questions (e.g., "What number comes after nine?", "How many do you have all together?"), incomplete statements designed to elicit a response (e.g., "Four plus two is…"), or statements that were inferred as questions based on their tone (e.g., "That's twelve?"). Statements were defined as utterances that provided math-relevant information (e.g., “I see five cars”, “One, two, three”, “One plus one is two”, “Adding means we will put these together”) or commentary about how children used math-relevant information (e.g., “Counting was a good strategy”, “You just did addition!”).

In order to control for individual variability in teachers’ overall amount of speech, we examined all math talk as a proportion of math-related utterances to the total number of utterances. Note that single utterances could have been coded as more than one instance of math talk. For example, the phrase “We need seven blocks so let’s count them”, would be coded as one instance of a statement of cardinality and one instance of an elicitation of counting. If a teacher repeated the exact same phrasing over multiple utterances, each utterance received a code for math talk, but if a teacher repeated the exact same phrasing within a single utterance, it only received one code.

Ten out of the 48 transcripts (20%) were randomly selected to be independently double coded by a trained research assistant, and there was 86% agreement between the two coders. The codes from the first coder, the researcher, were used in all analyses.

### 2.3.4 Math Talk Categories

**Cardinality.** Utterances were coded as *Statements of Cardinality* if teachers used number words to state how many things were in a set. These utterances included labels for the number of items in a set (e.g., “I see *five* cars”), people in a group (e.g., “Sure, all *three* of you can help me”), actions in a sequence (e.g., “You jumped *two* times”, “You spun it *twice*”, “It looks like you only did it *once*”), or units of measurement (e.g., “There are *seven* days in a week”; “It is *fifty* degrees outside”; “You are *forty-one* inches tall”). Utterances that provided general commentary about cardinality (e.g., “Let me see *how many* we have”; “The spinner will tell you *how many* turns to take") were also coded as *Statements of Cardinality.* Some utterances received more than one *Statement of Cardinality* code. For example, the utterance "I see *five* dogs and *two* cats" received one *Statement of Cardinality* code for "*five* dogs" and one *Statement of Cardinality* code for "*two* cats". If a number word was used to state how many things were in a set after counting (e.g., "*One, two, three, four,* so there are *four* balls"), the utterance received one code for *Statement of Counting* (see below) and one code for *Statement of Cardinality*.

Utterances were coded as *Elicitations of Cardinality* if teachers prompted the children to demonstrate their understanding of cardinality either verbally (e.g., "*How many* cars do you have?"; "*How much* money is that?"; "Do you have *three* of them?", "Did you get *one*?"; "Do you wear *one* of those?", "Name *two* colors") or through actions (e.g., "Can you give me *five* of those?", "We need *seven* blocks", "Put in *four* more", "Take *three* more bites and you can be done"; "Go find *seven* of them"; "Do it *twice*", "Go *one more* time", "Only pick *one* sticker").

**Equivalence.** *Statements of Equivalence* were coded when teachers described a numerical match between two or more discrete quantities. This category included utterances in which teachers directly stated that two or more amounts were equal (e.g., "You both have *three*"; "Look, you have the *same number*"; "This and this are the *same amount* of money"), mentioned one-to-one mappings (e.g., "Each child gets *one* cracker"), mentioned one-to-many mappings (e.g., "Each group has *four* children"), or gave a comment about equivalence (e.g., "We share by *dividing equally*"). Utterances that were coded as *Statements of Equivalence* could have included *Statement(s) of Cardinality*, if the utterance still indicated *Equivalence* after removing the *Statement(s) of Cardinality*. For example, the utterance "You have *four* and she has *four,* so you both have the same number" received two *Statement of Cardinality* codes and one *Statement of Equivalence* code. Statements that only indirectly described a numerical match, such as, "You have *two* and I have *two*", were coded as two *Statements of Cardinality* rather than a *Statement of Equivalence. Equivalence* was not coded when teachers described matches in continuous amounts (e.g., "You have the same *amount* of milk"), size (e.g., "They are the *same* size"), age (e.g., "You are both *four* years old"), or time (e.g., "You both get five *more* minutes"). The phrase "*One* at a time" was always coded as *Cardinality* rather than *Equivalence*.

*Elicitations of Equivalence* were coded when teachers asked children if there was a quantitative match between two or more discrete quantities (e.g., "Do we have the *same number* of crackers?"), to find a quantitative match (e.g., "We need another tower that has an *equal number* of blocks", "Which pile has the *same number*?"), or to perform a one-to-one or one-to-many mapping (e.g., "Give each friend *one* cookie"). Utterances that were coded as *Elicitations of Equivalence* could have included *Statement(s) of Cardinality*, if the utterance still indicated *Equivalence* after removing the *Statement(s) of Cardinality*. For example, the utterance "I have *two* and you have *two*, so do we have the same number?" received two *Statement of Cardinality* codes and one *Elicitation of Equivalence* code.

**Nonequivalence.** *Statements of Nonequivalence* were coded when teachers stated that two or more discrete quantities were unequal. For example, this category included utterances such as "No, you have *different numbers* of blocks", "You have *more*", "I have the *most*", "Oh no, you have *more* than *four* crackers", and "It looks like you have *double* that *amount*". Utterances that were coded as *Statements of Nonequivalence* could have included *Statement(s) of Cardinality*, if the utterance still indicated *Nonequivalence* after removing the *Statement(s) of Cardinality*. For example, the utterance "If you get *four* and she gets *three*, then you will have more" received two *Statement of Cardinality* codes and one *Statement of Nonequivalence* code. Statements that only indirectly described a numerical mismatch, such as, "You have *two* and he has *three*", were coded as two *Statements of Cardinality* rather than a *Statement of Nonequivalence. Nonequivalence* was not coded for utterances describing differences in continuous amounts (e.g., "Who has *more* milk?"), size (e.g., "Those are *different* heights"), age (e.g., "*Four* is older than *three*"), or time (e.g., "I gave you more than *three* minutes"; "I said it more than *two* times now").

*Elicitations of Nonequivalence* were coded when teachers asked children about two or more unequal discrete quantities (e.g., "Who has *more*?", "Who has the *most*?", "Is *nine* more than *seven*?"). Utterances that were coded as *Elicitations of Nonequivalence* could have included *Statement(s) of Cardinality*, if the utterance still indicated *Nonequivalence* after removing the *Statement(s) of Cardinality*. For example, the utterance "*Six* kids said yes and *two* kids said no, so which vote has *more*?" received two *Statement of Cardinality* codes and one *Elicitation of Nonequivalence* code.

**Counting.** Utterances were coded as *Statements of Counting* when the teacher recited count words in the proper order, either forwards (e.g., "*One, two, three, four*"; "*Seventeen, eighteen, nineteen, twenty"*; "*One, two, three,* eyes on me") or backwards (e.g., "*Three, two, one*"), counted how many in a set (e.g., "*One, two, three, four* chairs") or provided a comment about counting (e.g., "I *counted* all of the children that are here today"; "I am going to *count* to *three*"; "*Counting* was a great strategy"; "You've never *counted* to *four hundred* before"; "Every time I say a *number*, I point to *one* object to *count*"). Utterances that included counting were coded as one *Statement of Counting* regardless of the number of count words used. Number words used in the context of a count sequence did not receive additional codes for *Cardinality.* If teachers counted over a series of multiple utterances without using other words in-between (e.g., "*One*" — 2 second pause — "*Two*"), the counting sequence received only one code of *Statement of Counting*, which was assigned to the last utterance with a count word.

*Elicitations of Counting* were coded when teachers directly asked children to count (e.g., "*Count* them"; "*Count six more*"), indirectly suggested they count (e.g., "We should *count* them"; "We could *count* the blocks"; "We are going to *count*"; "Now, we will *count*"; "I need your help *counting*", "Are you ready to *count*?", "It might help if you *count* them"), began counting and paused in order to encourage children to continue (e.g., "*One, two*, ….."), or asked questions about counting (e.g., "You *counted* to *nine*?"; "What types of things can we *count*?").

**Ordering.** *Statements of Ordering* were coded when teachers referenced the order of numbers (e.g., "*Four* comes after *three*") or used ordinal words to describe sets of visible objects or people (e.g., "That is our *third* small group", "Turn to the *second* page", "Only the *first* letter in the word is capitalized"). Utterances with ordinal words used in a temporal sense (e.g., "*First*, we are going to paint", "That is the *second* time I told you to stop") were not coded. Because it was rare that a teacher would use more than one ordinal word in an utterance, each ordinal word received one *Statement of Ordering* code. Thus, in the rare instance that teachers said "*First, second, third*" to describe a set of visible objects or people, three *Statement of Ordering* codes were assigned. Reciting a list of number words in order was not coded as *Ordering* but rather as *Counting*. Using the terms "*seconds"* or "*thirds"* to describe meal servings were not coded as *Ordering*. *Statements of Ordering* that included number words received no additional codes for *Cardinality*.

*Elicitations of Ordering* were coded when teachers prompted children to complete number sequences (e.g., "What comes after *nine*?"; "Put these *numbers* in *order*") or demonstrate their understanding of ordinal words either verbally or through actions (e.g., "Take a

total of an equation after removing the *Statement(s) of Cardinality*. For example, the utterance "We had *two* apples and you just brought *three* more, so how many do we have now?" received two *Statement of Cardinality* codes and one *Elicitation of Calculation* code.

**Place Value.** *Statements of Place Value* were coded when the teacher referred to place value or the decomposition of at least a two-digit number (e.g., "There are *two tens* in *twenty*"). *Elicitations of Place Value* were coded when the teacher prompted the children to provide information about a place value or the decomposition of at least a two-digit number (e.g., "How many *hundreds* in that *number*?"). Number words used in the context of describing place value did not receive additional codes for *Cardinality*.

**Fractions.** *Statements of Fractions* were coded when the teacher used words to indicate parts of a whole (e.g., "The recipe calls for *one third* cup"), and *Elicitations of Fractions* were coded when the teachers prompted the children to provide information about parts of a whole (e.g., "Is that *half* or a *quarter* of the sandwich?"). Fraction words (i.e., "half", "third", "quarter") did not receive additional codes for *Cardinality*.

**Number Symbols.** *Statements of Number Symbols* were coded when a teacher labeled a written Arabic numeral (e.g., "Look, this is the number *five*"), described how to spell a number word (e.g., "*Ten* is spelled T- E- N"), or provided general commentary about number symbols ("We are doing a great job tracing our *numbers*"; "You just wrote the *number ten*"). A phrase such as "A *two* and a *three* together mean the number *twenty-three*" would receive three codes for Statements of Number Symbols (one code for *two*, one code for *three*, and one code for *twenty-three*). *Statements of Number Symbols* included talk about telling time if the children were involved in looking at or identifying number symbols on a clock, and included talk about the calendar if children were involved in looking at or identifying number symbols on the calendar. In cases in which it was unclear if teachers were using number words to label written Arabic numbers, *Cardinality* was coded instead of *Number Symbols*.

*Elicitations of Number Symbols* were coded when teachers prompted children to identify number symbols (e.g., “What *number* is this?”; “What *number* is a *one* and a *two* next to each other?”; “Look at the clock and tell me what is next to the number *two*”), write number symbols (e.g., “Can you write a *two* and a *zero* for February *twentieth*?”; “Trace the *three*”), or find number symbols (e.g., “Find the blue *nine*”; “Look at the *number twelve*”; “Where is the *seven*?”; “If you have a *number four*, put a bingo chip on it”; “Where did the *twenty-three* go?”).

**Age.** *Statements of Age* were coded when teachers used numbers to label ages (e.g., “You are *four* years old”; “Yes, I think your mom is *thirty*”; “Those children are in *fifth* grade”) and *Elicitations of Age* were coded when teachers asked children about ages (e.g., “How *old* are you?”; “Are you turning *three* or are you turning *four*?”). Number words used to indicate an age or grade received no additional codes for *Cardinality*.

**Dates.** *Statements of Dates* were coded when teachers used numbers to label the date (e.g., “Today is February *ninth*”) and *Elicitations of Dates* were coded when teachers asked children to identify the date (e.g., “What is today’s *date*?”). Number words used to indicate the date received no additional codes for *Cardinality*.

**Time.** *Statements of Time* were coded when teachers used numbers to state the time (e.g., “It is *ten thirty*”) or an amount of time (e.g., “We will go outside in *one* hour”, “*Three* more minutes”; “I think it will take longer than *fifteen* minutes”; “*One* second please”). *Elicitations of Time* were coded when teachers prompted children to give amounts of time (e.g., “Should we set the timer for *three* minutes or *four* minutes?”). Number words used to indicate time received no additional codes for *Cardinality*.

**Miscellaneous Names.** *Miscellaneous Names* were coded when teachers used numbers as names for actions (e.g., "Give me high *five*"), things (e.g., "The *counting* book"; "The *number* game"; "Find your *number* rod"; "Where is the *hundred* board?"), people (e.g., "You are the *counter* today"), groups (e.g., "Class *four*"), phone numbers or addresses. Number words used as names or labels received no additional codes for *Cardinality*.

## 2.4 Procedure for Part 2: Follow-up Session

After the observation, participating teachers completed a follow-up session either on the same day as the observation (n=20) or at a later date (n=28) either in their own classroom, a quiet room in their school, or public setting that was arranged at a convenient time (i.e., before school, during breaks, or after school). During the follow-up session, a researcher administered a battery of math, reading and language, and spatial tasks (see sections 2.4.1, 2.4.2, and 2.4.3 for details about each task). Sessions lasted approximately 1 hour. When possible, tasks were administered in the following order: Word Attack, Block Design, Non-symbolic Comparison, Reading Fluency, Calculation, Math Fluency, Reading Vocabulary. All but two of the tasks were drawn from the nationally normed Woodcock Johnson III Tests of Achievement (Woodcock, McGrew, & Mather, 2001) and were administered according to the directions in the manual. For each subtest from the Woodcock Johnson III Tests of Achievement, raw scores were converted into age-normed standardized scores.

### 2.4.1 Math Measures

Math skills were assessed using three different tasks. A non-symbolic number comparison task measured basic numerical estimation skills while two math subtests (Calculation; Math Fluency) from the Woodcock Johnson III Tests of Achievement measured more advanced math skills. Together, these three tasks took approximately twenty minutes to complete. Calculation and Math Fluency were combined into a composite measure for all analyses.

**Non-symbolic Number Comparison Task.** Teachers were administered a standard version of a non-symbolic number comparison task, similar to that used by Halberda et al. (2008), that was designed to measure approximate number estimation skills. All stimuli were created from the Psychological Assessment of Numerical Ability (Panamath; www.panamath.org) and presented using a custom-made Matlab script. In this task, teachers briefly viewed images of simultaneously presented arrays of blue and yellow dots on either side of a computer screen (i.e., yellow dots on the left half of the screen and blue dots on the right half of the screen; see example stimuli in Figure 3). They were instructed to select the color of the more numerous array by pressing either the key marked with a yellow sticker or the key marked with a blue sticker. After four practice trials, teachers were administered 150 test trials. On every trial, the image was preceded by a fixation cross for 500 ms. Each image appeared on the screen for 1,500 ms followed by a blank screen. Note that this presentation duration was too short for teachers to count the numbers of dots exactly. Teachers were able to select their response either during the image or during the blank screen. The next trial began immediately following their response. Overall accuracy (percentage of correct trials) was used to measure performance.

The difficulty of each trial depended on the ratio between the number of yellow and blue dots. There were 30 trials for each of five ratio categories: 3:4 (e.g., 15 yellow dots and 20 blue dots), 4:5, 5:6, 7:8, 9:10. The side of the larger quantity (i.e., correct response) was counterbalanced across trials. To prevent participants from using perceptual cues other than number to determine the correct response, we included three trial types: Congruent (i.e., the array with the larger number had the larger cumulative area), Incongruent (i.e., the array with the smaller number had the larger cumulative area but both arrays had equal cumulative perimeter), and Neutral (i.e., the arrays had equal cumulative area). Each dot array contained between 12 and 36 dots, and dot size varied within single arrays (average dot diameter = 36 pixels; allowed variation = 20%). Two teachers had missing data on this task due to computer issues.

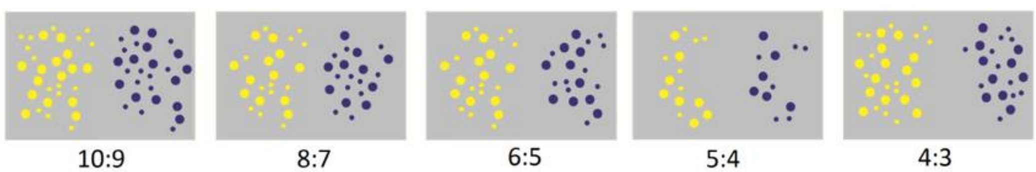



**Figure 2. Example stimuli for the non-symbolic number comparison task**

**Calculation.** On the Calculation subtest, teachers were asked to perform mathematical computations with no time restrictions. The problems involved arithmetic, geometric, and trigonometric operations and included negative numbers, percentages, decimals, and fractions. The problems increased in difficulty and administration stopped when the ceiling was established (6 consecutive incorrect items).

**Math Fluency.** The Math Fluency subtest measured the ability to accurately and rapidly solve simple addition, subtraction and multiplication problems within a time limit of three minutes. There were 160 problems that included numbers ranging from 1-10 (e.g., 2+9=__ ;

8x3=__). All teachers began with the first item and were told to work as quickly as possible without making mistakes. One teacher did not complete the math fluency subtest.

### 2.4.2 Reading and Language Measures

Reading and language skills were assessed using three different subtests (Word Attack, Reading Fluency, and Reading Vocabulary) from the Woodcock Johnson III Tests of Achievement (Woodcock et al., 2001). The Word Attack test measured basic phonological processing skills, while the Reading Fluency and Reading Vocabulary test measured more advanced aspects of reading and language skills. Together, these three tasks took approximately 15 minutes to administer. The Reading Fluency and Reading Vocabulary subtests were combined into a composite measure for all analyses.

**Phonological Processing.** The Word Attack subtest was used to measure teachers' phonological processing skills. Teachers were presented with a series of words that were unfamiliar (non-words or low-frequency words) but phonemically consistent with patterns in English orthography (e.g., "yosh", "quantric"). They were told to read each word silently to themselves and then to pronounce it smoothly. Trials were administered until the ceiling was reached (6 consecutive incorrect items).

**Reading Fluency.** On the Reading Fluency subtest, teachers were presented with 98 simple sentences (e.g., "Ice is hot."; "A man has two legs.") and were instructed to determine if the statement was true or false. All teachers began with the first sentence and were given three minutes to circle "true" or "false" for as many sentences as possible.

**Reading Vocabulary.** The Reading Vocabulary subtest measured the ability to supply appropriate meanings to written words. This subtest contained three sections: Synonyms (read a word and provide a synonym), Antonyms (read a word and provide an antonym), and Analogies (read three words of an analogy and provide a fourth word to complete the analogy; e.g., “generous is to stingy as verbose it to ___”). In each section, teachers were tested until the ceiling was established (4 consecutive incorrect items). Two teachers did not want to complete the Analogies portion of the assessment.

### 2.4.3 Spatial Measure

Teachers were administered the Block Design subtest from the second edition of the Wechsler Abbreviated Scale of Intelligence (WASI-II; Wechsler & Hsiao-pin, 2011). On each trial, teachers were given a set of identical red and white cubes and were asked to use the cubes to re-create the design presented on a two-dimensional card. Teachers had to complete each trial within a specified amount of time and were scored for the speed and accuracy of their response. Trials were administered until teachers had two consecutive scores of zero or completed all of the trials. Raw scores were converted into age-normed standardized scores. This task took 5-10 minutes to administer.

## 2.5 Procedure for Part 3: Questionnaire

After the follow-up session, teachers were given questionnaires that contained items about their typical classroom activities, curricular focus, beliefs and emotions about teaching and various academic subjects, demographic information and characteristics of their classroom (see sections 2.5.1 to 2.5.9 for further details and Appendix A for the full questionnaire). Teachers were given an option to complete the questionnaire items on a paper packet or online via Qualtrics. Some teachers chose to complete the questionnaires immediately after the follow-up session with the researcher present, while others chose to complete it on their own at a later time. Two teachers did not complete this questionnaire.

### 2.5.1 Reported Overall Frequency of Math and Reading Lessons

First, teachers reported how often the typical child in their class usually worked on lessons in math or reading and language arts on a 7-point scale (1 = *never,* 2 = *less than one time per week*, 3 = *1 day per week*, 4 = *2 days per week*, 5 = *3 days per week*, 6 = *4 days per week*, 7 = *5 days per week*). Second, teachers were asked to indicate how much time the typical child in their classroom would spend working on lessons or projects in math or reading and language arts on days that they work in those areas. They rated these questions on an 8-point scale (1 = *never*, 2 = *less than 30 minutes per day*, 3 = *30-60 minutes per day*, 4 = *1-1.5 hours per day,* 5 = *1.5-2 hours per day*, 6 = *2-2.5 hours per day*, 7 = *2.5-3 hours per day,* 8 = *3 or more hours per day*).

### 2.5.2 Reported Frequency of Specific Math and Reading Activities

On the questionnaire, teachers were asked to answer questions about how often they engaged their students in specific academic learning activities. The questionnaire was adapted from the Spring 2011 Kindergarten Teacher Questionnaire from the Early Childhood Longitudinal Study: Kindergarten Class of 2010-2011 (ECLS-K:2011: Tourangeau et al., 2015) by selecting only the items that were relevant for preschool classrooms. Teachers rated the frequency of how often they did certain academic activities with their students on a 6-point scale

devote to teaching, how much they like teaching, how much they value teaching, and how important teaching is to them on a 5-point scale (1 = *strongly disagree*, 5 = *strongly agree*).

### 2.5.5 Emotions about Teaching

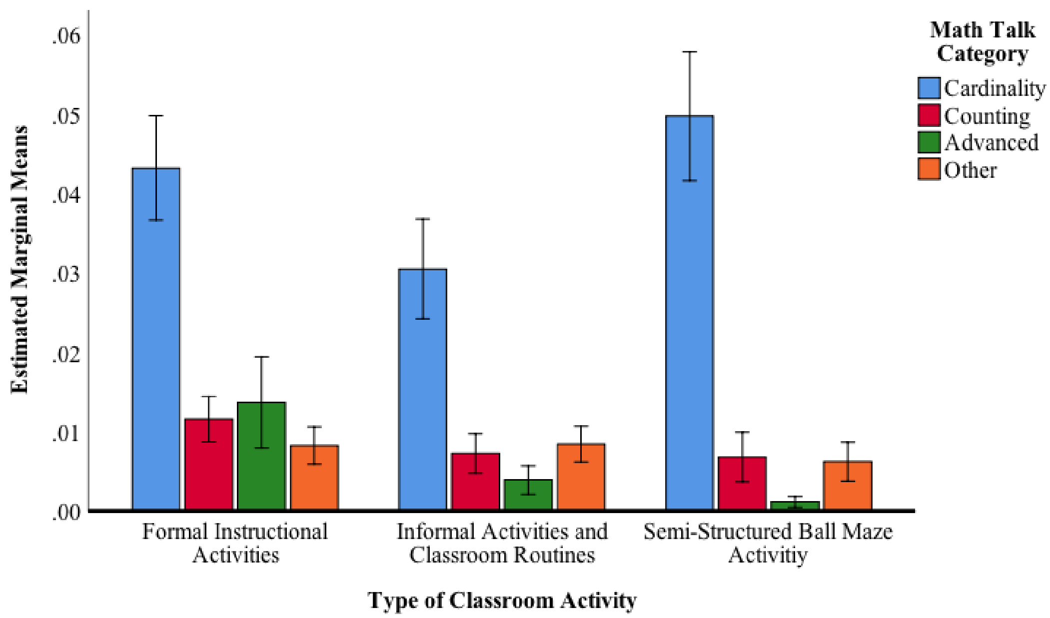
The Achievement Emotions Questionnaire for Teachers (AEQ-teacher; Frenzel, Pekrun, & Goetz, 2010) was used to evaluate teachers' most commonly experienced emotions related to teaching. Teachers used a 4-point scale (1 = *strongly disagree*, 4 = *strongly agree*) to rate five items (Cronbach's α = .91) related to enjoyment of teaching (e.g., "I gladly prepare and teach my lessons in this class"), four items (Cronbach's α = .85) related to anxiety during teaching (e.g., "I feel uneasy when I think about teaching"), and four items (Cronbach's α = .76) related to anger during teaching (e.g., "Teaching generally frustrates me").

### 2.5.6 Beliefs about Preschoolers' Math Development

Teachers completed the Mathematical Development Beliefs Survey (MDBS; Platas, 2015). The MDBS contained 40 items and was designed to measure teachers' beliefs concerning the (a) locus of generation of math knowledge (teacher- vs child-centered; 11 items; Cronbach's α = .86; e.g., "The teacher should play a central role in mathematical instruction"), (b) age-appropriateness of mathematics instruction (11 items; Cronbach's α = .90; e.g., "Very few preschoolers are ready for math"), (c) mathematical development as a primary goal of preschool education (8 items; Cronbach's α = .83; e.g., "Math activities are a very important part of the preschool experience"), and (d) level of confidence in providing mathematics instruction (10 items; Cronbach's α = .88; e.g., "I am unsure how to support math development for young

activity, $F(2, 66)=3.47$, $p=.037$, $\eta_{p2}=.1$, a significant main effect of math talk category, $F(1.86, 61.51)=4.69$, $p=.014$, $\eta_{p2}=.12$, and no significant interaction, $F(3.20, 105.49)=2.54$, $p=.060$, $\eta_{p2}=.07$ (see Figure 3).

The main effect of classroom activity revealed that teachers used the highest proportion of math talk during *Formal Instructional Activities* (*M*=.019, *SE*=.001), followed by the *Semi-Structured Ball Maze Activity* (*M*=.016, *SE*=.001), and then during *Informal Activities and Classroom Routines* (*M*=.013 *SE*=.001). Pairwise comparisons with Bonferroni corrections showed that teachers' used a significantly higher proportion of math talk during *Formal Instructional Activities* compared to *Informal Activities and Classroom Routines* (*M Difference*=.007, *SE*=.002, *p*=.002), but the difference between the *Semi-Structured Ball Maze Activity* and the other two types of naturally occurring classroom activities were not significant. The main effect of math talk category showed that *Cardinality* (*M*=.041, *SE*=.002) was by far the most frequently used category of math talk, followed by *Counting* (M=.009, SE=.001), *Other Math Talk* (M=.008, SE=.001) and then *Advanced Math Talk* (M=.006, SE=.001). Pairwise comparisons with Bonferroni corrections revealed that teachers' use of *Cardinality* was significantly higher than their use of *Counting* (M Difference=.033, SE=.002, p<.001), *Advanced Math Talk* (M Difference=.035, SE=.002, p<.001), and *Other Math Talk* (M Difference=.034, SE=.002, p<.001), but there were no significant differences among the other categories.




Differences in teachers' math talk proportions by type of classroom activity and type of math talk. Classroom size was evaluated at 13.28 and average age of the children was evaluated at 4 years. Error bars represent 95% confidence intervals.

**Figure 4. ANCOVA results for teachers' math talk proportion scores**

We also confirmed the main effect of classroom activity and type of math talk using non-parametric tests, since not all of the twelve math talk categories were normally distributed. A nonparametric Friedman test revealed a significant difference among the distributions of the three different types of classroom activities, $\chi^2(2)=6.72, p=.038$. Pairwise Friedman's tests revealed the same pattern of differences as the ANCOVA: math talk during *Formal Instructional Activities* (mean rank = 2.27) was significantly different than math talk during Informal Activities and Classroom Routines (mean rank = 1.73), but math talk during the *Semi-structured Ball Maze Activity* (mean rank = 2.00) was not significantly different than the other two categories. A nonparametric Friedman test also revealed a significant difference among the distributions of the four different types of math talk, $\chi^2(3)=85.44, p<.001$. Pairwise Friedman's tests showed the same pattern as the ANCOVA. *Cardinality* (mean rank = 3.97) was significantly different from the other three categories (*Counting* mean rank = 2.10; *Other Math Talk* mean rank = 2.03; *Advanced Math Talk* mean rank = 1.90), but there were no significant differences among the latter three categories.

Finally, we used both non-parametric Spearman and Pearson partial correlations to examine the relations among the types of math talk within and across the different types of classroom activities, while controlling for classroom size and the average age of the children (see Table 5). Partial Spearman and Pearson correlations showed a similar pattern of results. Only the Spearman correlations are interpreted below.

First, we examined correlations within each type of classroom activity. Within *Formal Instructional Activities*, teachers' use of *Cardinality* was positively corelated with their use of *Counting*, and their use of *Counting* was positively corelated with their use of *Advanced Math Talk*. Within *Informal Activities and Classroom Routines*, teachers' math talk about *Cardinality* was positively correlated with their math talk about *Counting,* and *Advanced Math Talk* was correlated with *Other Math Talk.* Within the *Semi-Structured Ball Maze Activity*, teachers' math talk about *Cardinality* was positively correlated with their use of *Advanced Math Talk,* and their use of *Counting* was positively correlated with their *Advanced Math Talk*.

Second, we examined correlations across different types of classroom activities. There was a significant positive correlation between teachers' use of *Counting* during *Formal Instructional Activities* and their use of *Counting* during *Informal Activities and Classroom Routines.* There were no other significant correlations between math talk categories across the different classroom activities.

**Table 5. Correations among math talk proportion scores**

Correlations among math talk proportion scores in each math talk category during each type of classroom activity, controlling for classroom size and average age of the child. Partial Pearson correlations are reported above the diagonal and partial non-parametric Spearman correlations are reported below the diagonal.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1. Cardinality during Formal Instructional Activities | — | .34* | .22 | .28† | -.02 | .08 | -.07 | -.19 | .06 | .25 | -.1 | -.08 |
| 2. Counting during Formal Instructional Activities | .34* | — | .54*** | .25 | -.08 | .56*** | .04 | -.11 | -.09 | .05 | -.05 | -11 |
| 3. Advanced during Formal Instructional Activities | .23 | .41** | — | .1 | -.11 | .46** | .25 | -.07 | -.07 | -.07 | -.13 | -.1 |
| 4. Other during Formal Instructional Activities | .18 | .28† | .20 | — | .04 | .06 | -.07 | .08 | -.12 | .2 | .13 | .1 |
| 5. Cardinality during Informal Activities and Routines | -.03 | 0 | -.14 | .11 | — | .32* | .12 | .04 | .06 | .04 | .08 | .16 |
| 6. Counting during Informal Activities and Routines | .09 | .47** | .28† | .07 | .36* | — | .21 | -.06 | -.15 | -.09 | -.16 | -.02 |
| 7. Advanced during Informal Activities and Routines | -.09 | .05 | .26 | .01 | .23 | .21 | — | .24 | .04 | -.07 | -.02 | -.12 |
| 8. Other during Informal Activities and Routines | -.22 | -.11 | .13 | .02 | .09 | -.08 | .43** | — | .19 | .05 | .31 | .16 |
| 9. Cardinality during Semi-Structured Ball Maze Activity | .15 | -.1 | -.02 | -.22 | .11 | -.1 | .13 | .18 | — | .24 | .47** | .02 |
| 10. Counting during Semi-Structured Ball Maze Activity | .05 | .04 | 0 | .09 | .24 | -.01 | -.03 | .24 | .21 | — | .28† | .05 |
| 11. Advanced during Semi-Structured Ball Maze Activity | -.07 | .01 | -.16 | .19 | .16 | -.16 | -.02 | .27† | .42** | .47*** | — | .09 |
| 12. Other during Semi-Structured Ball Maze Activity | -.12 | -.01 | -.13 | .15 | .23 | -.03 | -.08 | .29† | .10 | .26† | .26† | — |

***p<.001; **p<.01; *p<.05; †p<.10

For the remainder of the analyses, we collapsed across the raw proportion scores for each type of math talk to examine how characteristics of the teachers related to their overall observed math talk over entire observation. Because there were so few math talk correlations across the different types of activities, we also performed analyses at the level of the classroom activity to examine how teacher factors separately related to their math talk during *Formal Instructional Activities* (*n*=45, *M*=.08, *SD*=.05; skewness=.69, *SE*=.35), *Informal Activities and Classroom Routines Activities* (*n*=47, *M*=.06, *SD*=.03; skewness=1.45, *SE*=.35), and the *Semi-Structured Ball Maze Activity* (*n*=48, *M*=.07, *SD*=.03; skewness=.13, *SE*=.34). Note that one score was removed from Informal Activities and Classroom Routines because it was over 3 standard deviations above the mean.

### 3.4 Teachers' Reported Classroom Activities and Math Talk

Does the math talk that we observed over the course of the observation relate to how much teachers report doing math in the classroom? On the questionnaire, teachers answered one question about how often the typical child in their classroom usually worked on lessons in math and one question about how often the typical child in their classroom usually worked on lessons in reading or language arts per week (see Table 6 for descriptive statistics). Their responses were binned into three categories (less than once per week, 1-3 days per week, or 4-5 days per week). We examined if there were differences in how much math talk teachers used during the different types of classroom activities according to how they responded to each item, controlling for classroom size and average age of the children. In the math domain, there was no significant main effect of teachers' response on their overall observed math talk, $F(2, 45)=2.84$, $p=.070$, nor

**Table 6. Descriptive statistics of teachers' reported classroom activities and lessons**

Descriptive statistics of teachers' survey responses about how much time children in their classroom spend on various activities and lessons.

| Construct | Subscales | Items | Scale | M | SD | Min | Max |
|---|---|---|---|---|---|---|---|
| Overall frequency of lessons per week | Math Lessons | 1 | 1 = *never*, 7 = *five days per week* | 5.67 | 1.77 | 1 | 7 |
| | Reading and Language Lessons | 1 | 1 = *never*, 7 = *five days per week* | 6 | 1.62 | 1 | 7 |
| Time spent on lessons per day | Math Lessons | 1 | 1 = *never*, 8 = *three or more hours per week* | 3.2 | 1.7 | 1 | 8 |
| | Reading and Language Lessons | 1 | 1 = *never*, 8 = *three or more hours per week* | 3.44 | 1.8 | 1 | 8 |
| Frequency of specific academic activities per week | Math Activities | 9 | 1 = *never*, 6 = *daily* | 4.16 | .93 | 1.44 | 5.67 |
| | Reading and Language Activities | 9 | 1 = *never*, 6 = *daily* | 4.08 | .94 | 1 | 6 |
| Frequency of specific academic skills taught per week | Focus on Math | 15 | 1 = *not taught*, 6 = *taught daily* | 3.29 | 1.09 | 1.33 | 5.47 |
| | Focus on Reading and Language | 12 | 1 = *not taught*, 6 = *taught daily* | 4.30 | .96 | 1.42 | 6 |

We then examined how teachers' use of math talk during the observation related to teachers' reports of how often they engaged students in specific math activities (e.g., calendar-related activities, using a number line), taught specific math skills (e.g., correspondence between number and quantity, writing numbers 1-10), engaged students in reading and language activities (e.g., practicing writing the letters of the alphabet, listening to the teacher read stories), or taught specific reading and language arts skills (e.g., alphabet and letter recognition, matching letters to sounds) per week (see Table 6 for descriptive statistics). Any scores that were more than three standard deviations from the mean were excluded from analyses. Partial correlations, controlling for child age and classroom size, are reported in Table 7. All four of the scales were highly correlated with one another.

Teachers' math talk during over the course of the entire observation was positively correlated with their reported frequency of math activities per week, teaching of math skills per week and their teaching of reading skills per week. These significant relations seemed to be driven by teachers' use of math talk during *Informal Activities and Routines*. Teachers who report doing more math activities and teaching more math skills per week, also used more math talk during *Informal Activities and Routines* during the observation. Teachers' math talk during *Informal Activities and Routines* was also positively correlated with their reported teaching of reading skills per week.

**Table 7. Correlations between teachers' reported actvities and observed math talk**

Partial correlations between teachers' reported classroom activities and their math talk proportion scores for the entire observation and each type of classroom activity separately, controlling for classroom size and average age of the children.

| | 2 | 3+ | 4 | 5+ | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| 1. Total Math Talk | .67*** | .52*** | .30* | .19 | .36* | .39** | .35* |
| 2. Math Talk during Formal Instruction | — | .11 | .03 | -.04 | .10 | .25 | .28† |
| 3. Math Talk during Informal Activities/Routines+ | | — | .09 | .24 | .34* | .33* | .35* |
| 4. Math Talk during Semi-Structured Ball Maze | | | — | .13 | .19 | -.04 | .08 |
| 5. Reported Frequency of Specific Reading Activities+ | | | | — | .75*** | .56*** | .71*** |
| 6. Reported Frequency of Teaching Reading Skills | | | | | — | .68* | .69*** |
| 7. Reported Frequency of Specific Math Activities | | | | | | — | .61*** |
| 8. Reported Frequency of Teaching Math Skills | | | | | | | — |

***$p$<.001; **$p$<.01; *$p$<.05; †$p$<.10; + Nonparametric Spearman correlations are reported for these variables because their distributions were highly skewed.

## 3.5 Teachers' Characteristics and their Math Talk

### 3.5.1 General Characteristics

We examined if teachers' years of experience, their level of education, and their general feelings towards teaching related to their math talk over the entire observation and during the three different types of classroom activities. Partial correlations, that controlled for classroom size and average age of the children, did not show significant correlations between teachers' years of experience and their total math talk, $r(43)=.11$, $p=.455$, math talk during *Formal*

*Instruction*, *r*(40)=.14, *p*=.372, *Informal Activities and Routines*, $r_s$(42)=.10, *p*=.513, or the *Semi-Structured Ball Maze Activity*, *r*(43)=.24, *p*=.144.

Next, we examined if teachers' level of education (less than Bachelor's degree, Bachelor's degree, or at least one year towards Master's degree) was related to their math talk. One-way ANOVAs, with classroom size and average age of the children as covariates, did not show a significant main effect of education level on math talk over the entire observation (*F*(2, 46)=1.35, *p*=.271, $\eta_{p2}$=.06), during *Formal Instruction* (*F*(2, 43)=.44, *p*=.645, $\eta_{p2}$=.02), or during the *Semi-Structured Ball Maze Activity* (*F*(2, 46)=1.30, *p*=.282, $\eta_{p2}$=.06). There was a significant main effect of education level on teachers' math talk during *Informal Activities and Classroom Routines* (*F*(2, 45)=4.56, *p*=.017, $\eta_{p2}$=.19). Pairwise comparisons with Bonferroni corrections showed that teachers who had a Bachelor's degree used a significantly higher proportion of math talk during *Informal Activities and Routines* (*M*=.07, *SE*=.01), compared to teachers with less than a Bachelor's degree (*M*=.03, *SE*=.01), controlling for classroom size and average age of the children, *M difference*=.04, *SE*=.01, *p*=.014. The comparisons to teachers with at least one year towards a Master's degree (*M*=.05, *SE*=.01) were not significant.

Finally, we examined if teachers' passion for teaching and their reported emotions while teaching (see Table 8 for descriptive statistics) were related to their observed math talk. A composite of teachers' emotions was calculated by averaging the items about enjoyment of teaching, anxiety during teaching (reversed scored), and anger during teaching (reversed scored) so that higher scores indicated more positive emotions (*M*=4.57, *SD*=.41, *range*=3.57-5). Controlling for classroom size and average age of the children, there were no significant partial Spearman correlations between teachers' passion for teaching and their math talk over the entire observation ($r_s$(42)=.17, *p*=.258), during *Formal Instruction* ($r_s$(39)=.10, *p*=.540), *Informal*

*Activities and Routines* ($r_s(41)=.11$, $p=.494$), or the *Semi-Structured Ball Maze Activity* ($r_s(42)=.19$, $p=.228$). Finally, there were no significant partial Spearman correlations between teachers' overall emotions towards teaching and their math talk over the entire observation, ($r_s(42)=.17$, $p=.272$), during *Formal Instruction*, $r_s(39)=.11$, $p=.502$, *Informal Activities and Routines*, $r_s(41)=.14$, $p=.383$, or the *Semi-Structured Ball Maze Activity*, $r_s(42)=.27$, $p=.079$.

**Table 8. Descriptive statistics of teachers' survey responses**

Descriptive statistics of teachers' survey responses about their passion for teaching, emotions during teaching, beliefs about preschoolers' math development, and their own feelings towards various academic subjects.

| Construct | Subscale | Items | Scale | M | SD | Min | Max |
|---|---|---|---|---|---|---|---|
| Passion about Teaching | | 4 | 1 = *strongly disagree*, 5 = *strongly agree* *(higher = more passion)* | 4.77 | .38 | 3.5 | 5 |
| Emotions about Teaching | Enjoyment of Teaching | 5 | 1 = *strongly disagree*, 5 = *strongly agree* *(higher = more enjoyment)* | 4.69 | .47 | 3.2 | 5 |
| | Anxiety during Teaching | 4 | 1 = *strongly disagree*, 5 = *strongly agree* *(higher = more anxiety)* | 1.47 | .48 | 1 | 2.25 |
| | Anger during Teaching | 4 | 1 = *strongly disagree*, 5 = *strongly agree* *(higher = more anger)* | 1.51 | .52 | 1 | 3 |
| Beliefs about the Locus of Generation of Math Knowledge | | 11 | 1 = *strongly disagree*, 6 = *strongly agree* *(higher = stronger belief that teachers are responsible to help children learn math)* | 3.12 | .95 | 1.27 | 5.45 |
| Beliefs about the Importance of Math in Preschool | Age-Appropriateness of Math Instruction | 11 | 1 = *strongly disagree*, 6 = *strongly agree* *(higher = stronger belief that math is age appropriate for preschool)* | 5.14 | .79 | 2.27 | 6 |
| | Math as the Primary Goal of Preschool | 8 | 1 = *strongly disagree*, 6 = *strongly agree* *(higher = stronger belief that math is a priority)* | 4.56 | .78 | 1.88 | 5.5 |
| Confidence in Ability to Teach Math | | 10 | 1 = *strongly disagree*, 6 = *strongly agree* *(higher = more confidence in ability to teach preschool math)* | 5.32 | .68 | 3.1 | 6 |
| Personal Value of Math | | 6 | 1 = *strongly disagree*, 4 = *strongly agree* *(higher = stronger personal value)* | 2.46 | .68 | 1 | 4 |
| Personal Value of English | | 6 | 1 = *strongly disagree*, 4 = *strongly agree;* *(higher = stronger personal value)* | 2.94 | .72 | 1 | 4 |
| Math Anxiety | | 30 | 1 = *not at all*, 5 = *very much* *(higher = more anxiety)* | 2.27 | .89 | 1 | 4.73 |

### 3.5.2 Beliefs about Preschool Math Instruction

On the questionnaire, teachers completed measures of their beliefs about math instruction in preschool (see Table 8 for descriptive statistics). Teachers' beliefs about the age-appropriateness of math and their beliefs about math as a primary goal of preschool tapped into very similar constructs and were combined into one measure of beliefs about the importance of preschool math in further analyses ($M$=4.85, $SD$=.75, range=2.08 to 5.70). Any scores that were more than three standard deviations from the mean were eliminated; one score was removed from the measure of teachers' confidence in their ability to teach math and one score was removed from teachers' beliefs about the importance of math.

We used partial correlations, controlling for classroom size, average age of the children, and teachers' level of education, to examine how teachers' beliefs about preschool math instruction related to their observed math talk. Teachers' beliefs about preschool math instruction did not significantly correlate with their observed math talk during the entire observation, or with their math talk during any of the three different types of classroom activities (see Table 9).

**Table 9. Correlations between teachers' characteristics and math talk**

Partial correlations between teachers' observed math talk and their beliefs about preschool math instruction, feelings towards math and English, and performance on the tasks. All models controlled for classroom size and average age of the children.

| | Total Math Talk | Math Talk during Formal Instruction | Math Talk during Informal Activities/Routines+ | Math Talk during Semi-Structured Ball Maze |
|---|---|---|---|---|
| Beliefs about the Locus of Math Instruction | -.01 | -.01 | -.11 | .25 |
| Beliefs about the Importance of Math in Preschool+ | .15 | -.09 | .28† | .07 |
| Confidence in ability to teach math+ | .01 | .14 | .01 | .03 |
| Personal Value of Math | .19 | .16 | .44** | -.25 |
| Personal Value of English | -.18 | .09 | .16 | -.13 |
| Anxiety about Math+ | -.23 | -.10 | -.10 | .18 |
| Woodcock Johnson Math Tasks | -.01 | -.01 | .12 | -.27† |
| Approximate Number Estimation Task | .20 | .09 | .05 | .01 |
| Woodcock Johnson Reading Tasks | -.07 | -.16 | .03 | .02 |
| Phonological Processing Task | .13 | .08 | -.08 | -.10 |
| Spatial Task | .21 | .05 | .37* | -.11 |

***$p$<.001; **$p$<.01; *$p$<.05; †$p$<.10; + Nonparametric Spearman correlations are reported for these variables because their distributions were skewed.

### 3.5.3 Personal Feelings towards and Comfort Level with Math

On the questionnaire, teachers completed measures of their own anxiety about math, personal value of math, and personal value of English (descriptive statistics are reported in Table 8). They also completed a number of math, reading and spatial tasks during the follow-up session (descriptive statistics are reported in Table 10). Any scores that were more than three standard deviations from the mean were eliminated in further analyses. One score was eliminated from the Woodcock Johnson reading measures, one from the phonological processing task, and one from the spatial task.

**Table 10. Descriptive statistics on teachers' performance on the tasks**

Descriptive statistics on teachers' performance on the math, reading and language, and spatial tasks.

| Domain | Task | Measure | M | SD | Min | Max |
|---|---|---|---|---|---|---|
| Woodcock Johnson Math | Calculation Subtest | Standardized Score | 98.56 | 12.02 | 73 | 123 |
| | Math Fluency Subtest | Standardized Score | 104.19 | 14.80 | 71 | 137 |
| Approximate Number Estimation | Non-symbolic Number Comparison | Proportion Correct | .79 | .06 | .65 | .89 |
| Woodcock Johnson Reading | Reading Fluency Subtest | Standardized Score | 108.38 | 17.11 | 84 | 182 |
| | Reading Vocabulary Subtest | Standardized Score | 96.67 | 16.08 | 71 | 186 |
| Phonological Processing | Word Attack Subtest | Standardized Score | 98.31 | 10.04 | 57 | 118 |
| Spatial | Block Design | Standardized Score | 48.13 | 10.06 | 17 | 69 |

First, we examined correlations among teachers' math-related factors, including their personal value of math, reported anxiety about math, and scores on the two math measures, the Woodcock Johnson combined math tasks and the measure of approximate numerical estimation (see full correlation table in Supplemental Table 5). The only significant correlation was between teachers' math anxiety and their personal value of math. Teachers who reported higher amounts of anxiety when doing math or thinking about math, also reported less personal value of math ($r_s(46)=-.57$, $p<.001$).

Second, we examined how teachers' math-related characteristics related to their math talk overall and during each type of classroom activity, controlling for classroom size and average age of the children (see Table 9). The only math-related characteristic that had a significant correlation with math talk was teachers' own personal value of math. Teachers who reported having a stronger value of math, also had a higher proportion of math talk during *Informal Activities and Routines*. Further, this relation seemed to be specific to teachers' value of math as the positive correlation remained even after controlling for teachers' reported value of English ($r(40)=.46$, $p=.002$).

### 3.5.4 Personal Feelings towards and Comfort Level with Other Subjects

Finally, we examined if any non-math factors, including teachers' own personal value of English and the reading and spatial measures, related to teachers' math talk over the entire observation and during each type of classroom activity (see Table 9). Controlling for classroom size and average age of the children, there were no significant correlations between teachers' value of English, phonological processing, or performance on Woodcock Johnson reading subtests, with any of the math talk variables. There was a significant positive correlation between teachers' performance on the spatial task and their math talk during *Informal Activities and Routines*.

# 4.0 Discussion

There were three main goals of this study: 1) to capture the frequencies and types of math talk preschool teachers use as they engage with their students during a range of classroom activities, 2) to test the relation between observed math talk and teacher reports of math activities in the classroom, and 3) to examine how characteristics of teachers relate to their use of math talk in the classroom. We discuss each finding in detail below.

## 4.1 Differences in Types of Math Talk

Across the entire duration of the classroom observation, we found that teachers used more talk about cardinality than any other type of math talk. This finding is consistent with prior studies showing utterances about cardinality to be by far the most common type of math talk in preschool classrooms (Klibanoff et al., 2006; Rudd et al., 2008; Simpson & Linder, 2016). Klibanoff and colleagues (2006), for example, found cardinality to make up 48% of all math talk and was used by all preschool teachers. We similarly found that the cardinality category made up 58% of all math talk instances and all teachers in our sample used the category at least once. Our teachers used significantly more math talk about cardinality compared to any of the other types of math talk, even after accounting for their overall amount of talk and controlling for classroom size and average age of the children.

While cardinality was by far the most frequent type of math talk, there was still a substantial amount of the other three types of math talk, although there were no significant differences among them. Counting made up 15% of all math talk instances and was used by all but one teacher in our sample, advanced math talk made up 15% of all math talk instances and was used by all but four teachers in our sample, and other math talk made up 12% of all math talk instances and was used by all but one teacher in our sample. Further, within each category there was variability across teachers. While teachers used on average 75.17 (*SD*=36.16) instances of math talk about cardinality, the range was from 19 to 164. Math talk about counting (*M*=19.98, *SD*=17.62, range=0-88) and advanced math talk (*M*=19.27, *SD*=24.86, range=0-99) also showed substantial variability.

The considerable amount of variability in math talk across a number of different categories raises questions about what types of math talk are most beneficial for preschoolers. In a sample of Kindergarten teachers, Boonen and colleagues (2011) examined the unique relations between teachers' types of math talk and various aspects of children's math skills. Teachers' speech was recorded during 30 minutes of math-related circle time activities and 30 minutes of non-math circle time activities. While there was an overall positive association between the teachers' total amount of math talk and growth in kindergarteners' performance on the math assessments, different types of math input benefitted different types of math skills. Teachers' talk about cardinality, for example, benefited kindergarteners' growth in number sense and in their ability to compare quantities. In contrast, teachers' talk about calculations negatively impacted children's number sense across time. The authors hypothesized that talk about cardinal values was developmentally appropriate, while talk about calculations may have been too advanced for children at the age of five. This study highlights the need for future research to carefully unpack the content of preschool teachers' math talk in order to examine the match between different types of math talk and what is developmentally appropriate for preschoolers at different ages and the specific group of students that teachers engage with at each moment in time.

### 4.2 Differences in Math Talk across different Classroom Activities

After controlling for classroom size and the average age of the children, we found that teachers used a significantly higher proportion of overall math talk during formal instruction than during informal activities/routines. In prior studies, researchers have suggested that large-group instruction, “circle time” in particular, includes a number of routines, activities, and songs that naturally lend themselves to math talk (e.g., counting the number of children in attendance, discussion of the calendar), which may be why we found higher proportions of math talk during formal instructional activities. On the one hand, it is possible that teachers choose to focus on math lessons during circle time or small groups because they want to make sure all children have access to the information. Simpson and Linder (2016) have shown that a majority of teachers’ math-related utterances during circle time are directed towards all children in the room rather than towards individual children at a time. On the other hand, informal activities/routines may provide teachers with more opportunities to tailor their math talk to specific children in the classroom rather than catering to a group of children who have varying levels of math knowledge.

Although the highest proportion of math talk occurred during formal instruction, we still extended the findings of previous studies by capturing incidental math talk that occurred outside of planned instructional activities. We showed that there are vast individual differences in teachers’ math talk during free play, meals, and transitions between activities. Capturing the math talk that occurred during informal activities/routines is important because teachers on average spent more time during free play, meals, and transitions (85.46 minutes) than in large-group and small-group instruction (73.95 minutes). Further, at least on the day(s) of the observation, not all of the teachers in our sample had a time of the day dedicated to formal instructional activities.

When we examined correlations between types of math talk during formal instruction with types of math talk during informal activities, there was only one significant correlation: teachers who used more counting during formal instruction were also more likely to use counting during informal activities/routines. The general lack of significant correlations between math talk during formal instruction with math talk during informal activities/routines supports the findings of McCray and Chen (2012). They found more math talk during circle time than during non-circle time activities. Here we find similar results with a larger sample of teachers, a longer period of classroom observation, and a more specific distinction between our types of classroom activities (formal teacher-directed instruction vs. less formal activities and classroom routines). The previous studies on the relation between teachers' math talk and growth in children's math knowledge focused mainly on teachers' math talk during circle time (Boonen et al., 2011; Klibanoff et al., 2006; McCray & Chen, 2012; Simpson & Linder, 2016). Given that math talk during formal instruction does not correlate with variability in teachers' math talk during other activities, it is important to design studies that observe teachers' math talk during a wide range of classroom activities while also measuring growth in students' math talk over time. Our discussion in sections 4.3 and 4.4 may help shed light on potential reasons for the inconsistencies in math talk across formal and informal classroom activities.

We also included a semi-structured building activity in our study, in which we gave all teachers in the sample a novel toy for a small-group activity. After controlling for classroom size and average age of the children, the proportion of overall math talk during the semi-structured ball maze activity fell in between the average amount used during formal instruction and the amount used during informal activities/routines, though it was not significantly different from either of those two categories. The amount of math talk occurring during this activity is noteworthy given that math was not explicitly necessary to play with the toy, teachers were not given any math-related instructions, and they were unaware of the math-specific focus of our study. Most teachers incorporated cardinality, counting, and other math talk at least once, and about a third of the teachers even incorporated small amounts of advanced math talk. Because we only had audio-recordings of the activity and not videos, we were unable to assess details about how teachers and children completed the semi-structured activity that may be related to the amount of math talk teachers used during the activity. Details, such as whether or not they tried to make exactly the structure on the example image, the number of different structures they made, and if they were able to build a functional structure, should be assessed in any future studies using this task.

In the literature on parents' math talk with their children, semi-structured activities have been used to elicit certain types and amounts of math talk that rarely occur under naturalistic conditions (Casey et al., 2016; Ramani et al., 2015). As in the parent literature, we found high variability in teachers' use of math talk during the semi-structured ball maze activity and found that for some teachers, the ball maze activity elicited more math talk than the naturalistic classroom activities. Seventeen out of the 48 teachers exhibited the highest proportion of math talk during the semi-structured ball maze activity, while 19 teachers exhibited the highest proportion during formal instruction and 9 teachers exhibited the highest proportion during informal activities and routines. Further, there were no significant correlations between any of the types of math talk during the ball maze activity with any of the types of math talk during the naturalistic activities. These findings highlight the fact that some teachers who either rarely used math talk in the classroom, or who did not use much math talk on the particular day(s) of the observation, did use a high proportions of math input while completing the semi-structured ball maze activity.

There are some previous findings that suggest it may be important to examine teachers' math talk not just in terms of the types of classroom activities, but also in terms of the specific materials teachers are using. In a small sample of five Head Start teachers, Simpson and Linder (2016) found that preschool teachers' math talk was more likely to occur in some free play areas of the room (e.g., the math and science area) over others (e.g., the art area). Interestingly, the specific content of teachers' math talk also differed across contexts. When teachers talked to children in the pretend kitchen area their math talk was mostly limited to counting, but in the math and science area teachers used more complex types of math talk, such as talk about patterns and simple arithmetic. Replications with larger samples are needed to draw strong conclusions, but this preliminary study combined with our findings regarding the ball maze activity, suggests that teachers may provide different math-related learning opportunities depending on the materials that they have at their disposal during different activities. An interesting direction for future research would be to examine which teachers benefit the most from being given specific materials or semi-structured activities to prompt math-related talk.

### 4.3 Teachers' Observed Math Talk and their Reported Activities

We did not find any relation between teachers' observed math talk and their reports of how much time children in their classroom spend working on math lessons. However, we did find significant positive correlations between teachers' overall math talk with both their reported frequency of specific math activities and their reported frequency of teaching math skills. These findings are in line with a previous study that used the same questionnaire with a larger sample of Head Start teachers (Hindman, 2013). This study reported a small, but significant, correlation between teachers' reports of math activities and how much of their day involved either direct math instruction (e.g., talking about math content with the children) or indirect math instruction (e.g., setting up materials for children to learn about math through play). Our findings suggest that asking teachers to report how often they do specific math activities or teach certain math skills may be a better way to capture variability in their instruction rather than asking them to report the general amount of time children spend doing math lessons.

We also examined the associations separately for the three different types of classroom activities. Only teachers' math talk during informal activities/routines was significantly and positively correlated with their reported frequency of doing specific math activities or teaching math-related skills, though the correlations with formal instructional activities were marginally significant or trending in the positive direction.

There are at least two, not mutually exclusive reasons why teachers' reports of classroom activities could be more strongly related to their math talk during informal activities/routines compared to their math talk during formal instruction. First, it is possible that there is more day-to-day consistency across informal activities/routines than there is across formal instruction. During times of the day that are dedicated to formal instruction, including large-group and small-group, teachers often choose to teach a specific planned lesson (e.g., a science lesson on which objects float, the history of an upcoming holiday, fire safety, or words that start with the letter K). Because the topics of these lessons change daily, it is possible that our 93- to 226-minute observations over 1 or 2 days did not accurately capture the range of math talk that would have occurred during formal instructional activities over the course of multiple days. Indeed, for some items on the scale, most teachers reported doing the math activity, but few teachers reported doing it every single day. For example, while 38 teachers said they have used music to help children understand math concepts, only six teachers report doing it daily (4 teachers report doing it 3-4 times per week, 11 report doing it once or twice per week, and 17 teachers reported doing it do it 1-3 times per month). Perhaps some uses of math talk during informal activities/routines have more day-to-day consistency (e.g., always asking children to stand on numbered spots on the floor when lining up or specifying how many servings of snack to take each day). This potential theory could be tested using longitudinal studies of teachers interacting with their preschoolers across multiple days in order to examine the consistency of input children receive across different types of classroom activities.

Second, it is also possible that the questionnaire contained activities and skills that were more likely to be used or discussed during certain types of classroom activities. For example, teachers' reports of "working on calendar activities" may be more strongly related to their math talk during formal instruction, while their reports of "playing math-related games" may be more strongly related to their math talk during informal activities/routines, especially during free play. Follow-up studies could use item-level analyses to see if the questionnaires contained more items that were most likely to be used during informal activities/routines, rather than during formal instruction.

Item-level analyses could also help answer a number of more general concerns about the relation between teachers' reported activities and their observed math talk. First, there could be vast differences in the duration and quality of each activity that are not reflected in their responses on the questionnaire. For example, two teachers could have both reported working on calendar activities daily during circle time, but one could have done a daily calendar activity with one math-related utterance (e.g., "Today is March 4th") and the other with five math-related utterances (e.g. "Let's count the days on the calendar! One, two, three. Yesterday was March 3rd, so today must be what? Three plus one is four. Today must be March 4th"). Second, it is important to keep in mind that these questionnaires were originally developed for Kindergarten teachers. Although prior to data collection we eliminated some items that we thought were too advanced for preschoolers, there were still many items that the majority of teachers reported never doing, such as teaching children to "tell time" or "count by 2s, 5s, and 10s". Third, many of the items on the scales were related to aspects of math that were not captured by our math talk coding, i.e., "working with geometric manipulatives", "recognizing and naming geometric shapes", "organizing objects into subgroups according to a rule", "ordering objects by size", and "making, copying, or extending patterns". These concerns highlight the need for future directions to code a broader range of mathematically-relevant speech including talk about shapes and spatial relations, sequences, and patterns, and to unpack which individual items on the questionnaire explain more variability in teachers' math talk across settings.

### 4.4 Characteristics of Teachers and their Observed Math Talk

Prior work has found variability in math talk between preschool teachers holding the same educational credentials and even between teachers within the same schools or classrooms (Klibanoff et al., 2006; McCray & Chen, 2012; Rudd et al., 2008). In our study, we not only measured general characteristics of teachers' training (e.g., education level, years of experience), but we also included a number of specific math-related characteristics, including their beliefs about preschool math instruction and their own comfort with and feelings towards math.

In this sample, we found that certain characteristics of teachers relate to their use of math talk in the classroom, albeit with only their math talk during informal activities/routines. First, teachers with a Bachelor's degree, used more math talk during informal activities/routines than teachers without a Bachelor's degree, controlling for classroom size and average age of the children. Second, teachers who reported having a higher personal value of math also used more math talk during informal activities/routines, controlling for classroom size and average age of the children. Teachers with a high value of math endorsed statements such as "Mathematics is important to me personally", "I enjoy puzzling over mathematics", and "If I can learn something new in mathematics, I'm prepared to use my free time to do so". This scale had a good distribution of average scores, with 5 teachers strongly agreeing with the statements, 18 agreeing, 20 disagreeing, and 3 strongly disagreeing. Further, the relation between teachers' value of math and their observed math talk, remained after controlling for teachers' personal value of another academic, but non-math subject (i.e., English), suggesting that the association was specific to teachers' value of math. Finally, teachers' beliefs about the importance of math instruction in preschool classrooms followed a similar pattern of results, but the correlation between beliefs about importance and observed math talk during informal activities/routines was only marginally significant, controlling for classroom size and average age of the children.

Together, this pattern of results suggests a difference between how characteristics of teachers relate to their math talk during formal instruction and during informal activities/routines. It seems likely that teachers' use of math talk during formal instruction may be best explained by variables not measured in this study, such as which curriculum teachers use, how well they follow it, and expectations from the school. In line with previous studies, we did not find evidence that differences in teachers' math talk were largely driven by their belonging to certain schools. However, it is possible that school-level expectations or specific curriculum requirements differed between classrooms within the same school since very few schools in our study contained more than one classroom of children of the same age.

During informal activities/routines, teachers' math talk may be driven more so by their own backgrounds and personal preferences than the curriculum of the school. Teachers with Bachelor's degrees compared to teachers without Bachelor's degrees, may be better at capitalizing on everyday opportunities to help children learn math during free play, meals, or transitions between activities, as a result of more extensive professional development or training. Further, teachers who indicated a higher personal value of math, who reported enjoying thinking about math and doing math-related activities in their free time, may be more likely to knowingly or unknowingly initiate math-related conversations with students, incorporate math into everyday routines, or notice the math-related aspects of play during informal activities. This theory is supported by a recent finding showing that preschool teachers' reports of their own joy and interest in math predict their sensitivity to math content in hypothetical written scenarios of play-based situations in preschool classrooms (Anders & Rossbach, 2015). Our finding is also reflected in the parent literature. Because parents rarely engage their preschoolers in formal group instructional activities, their math-related interactions with their children are most aligned with our informal activities/routines category in preschool classrooms. In a study in which parents were asked why they preferred teaching their child language over math, 40% of parents mentioned that they themselves liked language more and felt more skilled at language than math (Cannon & Ginsburg, 2008). Thus, there is general support for the idea that teachers' math talk outside of formal, curriculum-driven activities, may be most influence by general aspects of their training as well as their own enjoyment of math.

We did not find any links between teachers' observed math talk and their beliefs about who is responsible for children's math learning (i.e., teachers' responsibility to set math goals vs. children construct their own math knowledge). In prior work, this measure was shown to be important for general teaching practices, but it had not yet been examined in relation to teachers' math talk (Platas, 2015). The lack of a statistical association between these two variables may be a result of heterogeneity in the responses on the questionnaire that depended on the specific items. For example, teachers who strongly endorsed statements such as "Teacher should play a central role in math instruction" and "Preschoolers learn math best through direct teaching", did not generally endorse statements such as "Math flashcards are appropriate" or "Math worksheets are appropriate for preschool", presumably because they believed those types of activities to be too advanced or formal for preschool. Prior work has shown that preschool teachers generally believe children learn math during everyday experiences (Chen, McCray, Adams, & Leow, 2013). Thus, future research is needed to examine if the relation between teachers' beliefs about the generation of math knowledge and their observed math talk, could be obscured by the questionnaire measuring two separable constructs.

We also did not find any relation between teachers' confidence in their ability to teach math to preschoolers nor their math anxiety with their observed math talk. As a group, teachers were very confident in their ability to teach math. Overall, across the items, 42 teachers "agreed" or "strongly agreed" with statements about having confidence in their ability to teach math, while only 1 teacher somewhat disagreed with the statements. Relatedly, the teachers in our sample did not report high levels of math anxiety. Only three teachers reported having "very much" or "much" math anxiety, with 12 teachers reporting a "fair amount" of math anxiety, 31 reporting "a little" math anxiety or "none at all". Because these were self-report measures, teachers could have felt pressure to indicate more confidence and more positive feelings towards math. However, their reports of math anxiety are aligned with their reported value of math and marginally correlated with their performance on the math tasks, making this explanation less likely. It is more likely that most teachers in our sample were confident in math but that these factors did not capture differences in how teachers used math in the classroom.

In this study, we found no evidence of an association between teachers' performance on any of the math tasks and their observed math talk in the classroom. Although research in the parent literature has found some evidence that parents with greater numerical estimation skills and greater self-reported math abilities tend to talk more about numbers with their kindergarteners when playing together (Elliott et al., 2017), it is possible that parents with greater math skills and more positive feelings towards math pass this disposition on to their children, who are then more likely to initiate math-related conversations during play. Further, the math skills measured in this study (i.e., the ability to solve advanced calculation problems and quickly recall arithmetic facts) are not likely to tap into the skills necessary to teach math in preschool.

A more promising direction for future research may be to instead measure the specific knowledge teachers need to teach math at their particular instructional level, including knowledge of the math content, math-related teaching practices, and developmental processes for learning math (Ball, Hill, & Bass, 2005). McCray and Chen (2012) assessed preschool teachers' content knowledge by presenting them with classroom-based scenarios, such as a description of two children playing together in the block area. They then asked teachers a series of questions, such as "What kinds of math do you see in this play?" or "What might you say to help the children also see that math?". Teachers' scores from these interviews not only related to teachers' total amount of math language during an hour of recorded activities in the classroom, but also preschoolers' gains in math knowledge from the beginning to the end of the school year. When children begin to play in mathematically relevant ways (e.g., saying that there are not enough beds for the baby dolls), teachers' knowledge of key math principles such as one-to-one correspondence may prompt them to ask questions encouraging elaboration (e.g., "How do you know?") or questions to advance children's thinking to the next level (e.g., "Well, how many more beds do you need?"). These are the types of interactions that could explain differences in students' growth in mathematics over the school year. An important next step will be to expand this work using a number of aspects of the current study. First, researchers could examine the relations between preschool teachers' content knowledge and their use of math talk over multiple types of instructional and informal classroom activities. Second, researchers could examine the intercorrelations between teachers' content knowledge, education level, beliefs about math instruction, and their own personal value of math, as well as the relative strengths of those four variables for predicting teachers' observed math talk.

will be critical to conduct follow up studies that include teachers working in a wider range of preschool settings, including publicly funded programs. It should also be noted this study is correlational in nature and claims about causal relations between characteristics of teachers and their observed math talk in the classroom cannot be made from the evidence gathered thus far. Although we focused on potential explanations for variability in teachers' math talk, we also recognize the additional influences of variables not included in this study, including child-level factors (e.g., initial math ability at the start of the school year, attention, executive function), family-level characteristics (e.g., parental math talk, the home numeracy environment, socio-economic status) and school-level factors (e.g., type of center, curriculum, access to resources). Most importantly, as discussed above, children themselves play a role in initiating their teachers' math talk through their own talk and through their activity choices. A challenge for future research will be to simultaneously measure both teachers' and children's math talk in the classroom in order to examine how teachers tailor their math talk to the individual needs of their students and to examine the role that children play in eliciting different types of math talk from their teachers across different classroom activities.

A lack of early experiences in childhood that support mathematical development have long-term consequences for children's later math skills and educational attainment (Geary, 2000; Jordan et al., 2009). The National Association for the Education of Young Children and The National Council of Teachers of Mathematics have called for improvements in high-quality math education for preschoolers to ensure that all children enter elementary school with the skills necessary to succeed in math (NAEYC & NCTM Joint Position Statement, 2010). In this study, we add support to the general finding in the literature, that there is tremendous variability in the amount of math talk that occurs across preschool classrooms, that is not explained by teachers' overall amount of talk, classroom size, or the average age of the children. We also extended this work by 1) conducting a lengthier observation of preschool teachers interacting with their students across a range of naturally occurring and semi-structured classroom activities, 2) correlating observed math talk to teacher reports of the frequency of math activities in the classroom, and 3) examining how characteristics of teachers, beyond their level of education, explain individual variability in their use of math talk across different activities.

Findings from this study should serve to inform subsequent work that additionally measures children's math knowledge at the beginning and end of the school year, examines how math talk across different classroom settings affects children's math knowledge, and analyzes if teachers' math talk mediates the relation between math-specific characteristics of the teachers and the math skills of the children in their classroom. Intervention studies that use professional development training for in-service teachers are additionally needed to examine the malleability of teachers' math talk. There is some preliminary evidence that "naptime meetings", in which teachers received instruction on how to use more and varied types of math talk, increase their use of math talk in the classroom (Trawick-Smith, Oski, DePaolis, Krause, & Zebrowski, 2016). Perhaps, the most successful interventions will result from professional development about the potential carry-over effects of teachers' own feelings towards math and training programs to help teachers take advantage of "teachable moments" during everyday activities and routines in the classroom.

## Appendix A Questionnaire Items

**Math Instructional Activities**
Prompt: How often do children in your class do each of the following math activities?

1. Count out loud
2. Work with geometric manipulatives
3. Work with counting manipulatives to learn basic operations
4. Play math related games
5. Use music to understand math concepts
6. Use creative movement or creative drama to understand math concepts
7. Work with rulers, measuring cups, spoons, or other measuring instruments
8. Engage in calendar-related activities
9. Use a number line to understand number concepts

**Math Curricular Focus**
Prompt: For this school year as a whole, please indicate how often each of the following math skills are taught in your class or classes.

1. Correspondence between number and quantity
2. Writing all numbers between 1 and 10
3. Counting by 2s, 5s, and 10s
4. Recognizing and naming geometric shapes
5. Identifying relative quantity (e.g. equal, less, more, least, most)
6. Sorting objects in subgroups according to a rule
7. Ordering objects by size or other properties
8. Making, copying, or extending patterns
9. Adding single-digit numbers
10. Subtracting single-digit numbers
11. Reading two-digit numbers
12. Ordinal numbers (e.g. first, second, third)
13. Using measuring instruments accurately
14. Telling time
15. Estimating quantities

**Reading and Language Arts Activities**
Prompt: How often do children in your class do each of the following reading and language arts activities?

1. Practice writing the letters of the alphabet
2. Discuss new or difficult vocabulary
3. Work on phonics
4. Listen to you read stories where they see the print (e.g., Big books)
5. Listen to you read stories but they don’t see the print
6. Retell stories
7. Write with encouragement to use invented spellings, if needed

2. Mathematics is important to me personally.
3. It is important to me personally to be a good mathematician.
4. I enjoy puzzling over mathematics.
5. When I’m working on a mathematics problem, I sometimes don’t notice time passing.
6. If I can learn something new in mathematics, I'm prepared to use my free time to do so.

**English Expectancy-Value**

Prompt: Please rate how much you personally agree or disagree with the following statements:

1. I'm really keen to learn a lot in English.
2. English is important to me personally.
3. It is important to me personally to be good at English.
4. I enjoy puzzling over English problems.
5. When I’m working on a English problem, I sometimes don’t notice time passing.
6. If I can learn something new in English, I'm prepared to use my free time to do so.

**Beliefs about Math Instruction: Locus of Instruction**

Prompt: Please rate how much you personally agree or disagree with the following statements:

1. *Preschoolers learn mathematics without teachers
2. *In preschool, children construct mathematical knowledge
3. Math flashcards are appropriate
4. The teacher should play a central role in mathematical instruction
5. Preschoolers learn mathematics best through direct teaching
6. In preschool, children should learn specific procedures
7. Teachers should help children memorize number facts
8. Preschool teachers are responsible for right answer
9. Math worksheets are appropriate for preschoolers
10. Teachers should show preschoolers the correct way
11. Preschoolers memorize verbal counting

* indicates reverse scored item

**Age-Appropriateness**

Prompt: Please rate how much you personally agree or disagree with the following statements:

1. Teachers can help preschoolers learn mathematics
2. *It is better to wait until kindergarten for math instruction
3. *Mathematical activities are an inappropriate use of time
4. *Math is confusing to preschoolers
5. *Academic subjects such as mathematics are too advanced
6. *Very few preschoolers are ready for math
7. Preschoolers are capable of learning math
8. Most preschoolers are ready for participation in math
9. Children are ready for math activities in preschool
10. Mathematical activities are age appropriate for preschoolers
11. Math is a worthwhile and necessary subject for preschoolers

* indicates reverse scored item

**Math Development as Primary Goal**

Prompt: Please rate how much you personally agree or disagree with the following statements:

1. *Social and emotional development is a primary goal of preschool
2. *Preschool math will weaken preschoolers' self confidence
3. *Preschool children are not socially or emotionally ready for math
4. *Math activities mean socio-emotional development is neglected
5. Math is an important part of the preschool curriculum
6. Math activities are good opportunities to develop social skills
7. Math activities are a very important part of the preschool
8. Development in academics such as math is goal of preschool

* indicates reverse scored item

**Confidence in Instruction**

Prompt: Please rate how much you personally agree or disagree with the following statements:

1. *Teaching mathematics to preschoolers would be uncomfortable
2. *I am unsure how to support math development for young children
3. *Math would be a difficult subject for me to teach in preschool
4. *I do not know enough math to teach it in preschool
5. *I do not know how to teach math to preschoolers
6. I am knowledgeable enough to teach math in preschool
7. I can think of many math activities
8. Math would be easy for me to incorporate into preschool curricula
9. I can create effective math activities for preschoolers
10. I know how to support math learning in preschool

* indicates reverse scored item

**Math Anxiety**

Prompt: Please rate how much you personally agree or disagree with the following statements:

1. Taking an examination (final) in a mathematics course
2. Thinking about an upcoming mathematics test one week before
3. Thinking about an upcoming mathematics test one day before.
4. Thinking about an upcoming mathematics test one hour before.
5. Thinking about an upcoming mathematics test five minutes before.
6. Waiting to get a mathematics test returned in which you expected to do well.
7. Receiving your final mathematics grade in the mail.
8. Realizing that you have to take a number of mathematics classes to fulfill the requirements in your major.
9. Being given a "pop" quiz in a mathematics class.
10. Studying for a mathematics test.
11. Taking the mathematics section of a college entrance examination.
12. Taking an examination (quiz) in a mathematics course.
13. Picking up the mathematics textbook to begin working on a homework assignment.
14. Being given a homework assignment of many difficult problems, which is due the next class meeting.
15. Getting ready to study for a mathematics test.
16. Dividing a five-digit number by a two-digit number in private with pencil and paper.

**Supplemental Table 3. Raw instances of math talk**

Descriptive statistics of teachers' raw instances of math talk for each type of math talk during each type of classroom activity.

| | Classroom Activity Type | | |
|---|---|---|---|
| | Formal Instructional Activities *N = 45* | Informal Activities and Classroom Routines *N = 48* | Semi-Structured Ball Maze Activity *N = 48* |
| Cardinality and Equivalence Math Talk | | | |
| *M* | 33.56 | 23.00 | 20.71 |
| *SD* | 27.98 | 21.91 | 13.81 |
| *Range* | 1 – 199 | 4 – 132 | 0 – 52 |
| Counting and Ordering Math Talk | | | |
| *M* | 11.11 | 6.02 | 3.54 |
| *SD* | 13.79 | 7.48 | 4.89 |
| *Range* | 0 – 73 | 0 – 41 | 0 – 18 |
| Advanced Math Talk | | | |
| *M* | 15.44 | 4.15 | .65 |
| *SD* | 24.75 | 7.41 | 1.42 |
| *Range* | 0 – 99 | 0 – 35 | 0 – 8 |
| Other Math Talk | | | |
| *M* | 6.8 | 6.56 | 3.02 |
| *SD* | 762 | 7.39 | 3.19 |
| *Range* | 0 – 32 | 0 – 31 | 0 – 11 |

**Supplemental Table 4. Raw proportion scores of math talk**

Descriptive statistics of teachers' raw math talk proportion scores for each type of math talk during each type of classroom activity.

| | Classroom Activity Type | | |
|---|---|---|---|
| | Formal Instructional Activities *N = 45* | Informal Activities and Classroom Routines *N = 48* | Semi-Structured Ball Maze Activity *N = 48* |
| Cardinality and Equivalence Math Talk/Utterances | | | |
| *M* | .046 | .035 | .051 |
| *SD* | .029 | .025 | .024 |
| *Range* | .004 – .190 | .008 – .116 | 0 – .100 |
| Counting and Ordering Math Talk/Utterances | | | |
| *M* | .013 | .008 | .007 |
| *SD* | .011 | .008 | .010 |
| *Range* | 0 – .049 | 0 – .036 | 0 – .039 |
| Advanced Math Talk/Utterances | | | |
| *M* | .015 | .006 | .001 |
| *SD* | .023 | .01 | .003 |
| *Range* | 0 – .098 | 0 – .049 | 0 – .013 |
| Other Math Talk/Utterances | | | |
| *M* | .008 | .009 | .007 |
| *SD* | .007 | .008 | .007 |
| *Range* | 0 – .038 | 0 – .043 | 0 – .030 |

**Supplemental Table 5. Correlations between teacher measures**

Correlations between teachers' feelings towards math and English and performance on the tasks.

| | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| 1. Personal Value of Math | .62*** | -.59*** | .27† | .12 | -.06 | .15 | .32* |
| 2. Personal Value of English | — | -.24 | .21 | -.04 | .12 | .17 | .19 |
| 3. Anxiety about Math+ | | — | -.25† | -.25 | 0 | -.39* | -.29† |
| 4. Woodcock Johnson Math Tasks | | | — | -.08 | .36* | .50*** | .32* |
| 5. Approximate Number Estimation Task | | | | — | -.08 | .02 | .26† |
| 6. Woodcock Johnson Reading Tasks | | | | | — | .33* | .23 |
| 7. Phonological Processing Task | | | | | | — | .29† |
| 8. Spatial Task | | | | | | | — |

***$p$<.001; **$p$<.01; *$p$<.05; †$p$<.10 + Nonparametric Spearman correlations are reported for this variable because the distribution was skewed.

## References

Anders, Y., & Rossbach, H. G. (2015). Preschool teachers sensitivity to mathematics in childrens play: The influence of math-related school experiences, emotional attitudes, and pedagogical beliefs. *Journal of Research in Childhood Education*, *29*(3), 305–322. https://doi.org/10.1080/02568543.2015.1040564

Bachman, H. J., Degol, J. L., Elliott, L., Scharphorn, L., El Nokali, N. E., & Palmer, K. M. (2018). Preschool Math Exposure in Private Center-Based Care and Low-SES Children's Math Development. *Early Education and Development*, *29*(3), 417–434. https://doi.org/10.1080/10409289.2017.1406245

Ball, D. L., Hill, H. C., & Bass, H. (2005). Knowing Mathematics for Teaching. *American Educator*, (Fall), 14-22,43-46. https://doi.org/10.1016/j.cedpsych.2006.02.001

Beilock, S. L., Gunderson, E. A., Ramirez, G., & Levine, S. C. (2010). Female teachers' math anxiety affects girls' math achievement. *Proceedings of the National Academy of Sciences*, *107*(5), 1860–1863.

Berch, D. B. (2005). Making sense of number sense: implications for children with mathematical disabilities. *Journal of Learning Disabilities*, *38*(4), 333–339. https://doi.org/10.1177/00222194050380040901

Bilbrey, C., Vorhaus, B., Farran, D. C., & Shufelt, S. (2007). Teacher observation in prekindergarten classrooms. *Unpublished Manuscript. Nashville, TN: Vanderbilt University*.

Blevins-Knabe, B., Austin, A. B., Musun, L., Eddy, A., & Jones, R. M. (2000). Family Home Care Providers' and Parents' Beliefs and Practices Concerning Mathematics with Young Children. *Early Child Development and Care*, *165*(1), 41–58. https://doi.org/10.1080/0300443001650104

Boonen, A. J. H., Kolkman, M. E., & Kroesbergen, E. H. (2011). The relation between teachers' math talk and the acquisition of number sense within kindergarten classrooms. *Journal of School Psychology*, *49*(3), 281–299. https://doi.org/10.1016/j.jsp.2011.03.002

Cannon, J., & Ginsburg, H. P. (2008). "Doing the Math": Maternal Beliefs About Early Mathematics Versus Language Learning. *Early Education & Development*, *19*(2), 238–260. https://doi.org/10.1080/10409280801963913

Casey, B. M., Lombardi, C. M., Thomson, D., Nguyen, H. N., Paz, M., Theriault, C. A., & Dearing, E. (2016). Maternal Support of Children's Early Numerical Concept Learning Predicts Preschool and First-Grade Math Achievement. *Child Development*. https://doi.org/10.1111/cdev.12676

Chen, J. Q., McCray, J., Adams, M., & Leow, C. (2013). A Survey Study of Early Childhood Teachers' Beliefs and Confidence about Teaching Early Math. *Early Childhood Education Journal*, 1–11. https://doi.org/10.1007/s10643-013-0619-0

Claessens, A., Duncan, G., & Engel, M. (2009). Kindergarten skills and fifth-grade achievement: Evidence from the ECLS-K. *Economics of Education Review*, *28*(4), 415–427.

Cohen, D. K., & Ball, D. L. (1990). Relations between policy and practice: A commentary. *Educational Evaluation and Policy Analysis*, *12*(3), 331–338.

Duncan, G. J., Dowsett, C. J., Claessens, A., Magnuson, K., Huston, A. C., Klebanov, P., … Japel, C. (2007). School Readiness and Later Achievement. *Dev Psychol*, *43*(6), 1428–1446. https://doi.org/10.1037/0012-1649.43.6.1428

Durkin, K., Shire, B., Riem, R., Crowther, R. D., & Rutter, D. R. (1986). The social and linguistic context of early number word use. *British Journal of Developmental Psychology*, *4*(3), 269–288.

Elliott, L., Braham, E. J., & Libertus, M. E. (2017). Understanding sources of individual variability in parents' number talk with young children. *Journal of Experimental Child Psychology*, *159*, 1–15. https://doi.org/10.1016/j.jecp.2017.01.011

Frenzel, A. C., Pekrun, R., & Goetz, T. (2010). Achievement Emotions Questionnaire for teachers (AEQ-teacher)—User's manual. *University of Munich: Department of Psychology*.

Geary, D. C. (2000). From infancy to adulthood: The development of numerical abilities. *European Child and Adolescent Psychiatry*, *9*(SUPPL. 2), 11–17. https://doi.org/10.1007/s007870070004

Geary, D. C. (2015). Development and Measurement of Preschoolers' Quantitative Knowledge. *Mathematical Thinking and Learning*, *17*(2–3), 237–243. https://doi.org/10.1080/10986065.2015.1016823

Halberda, J., Mazzocco, M. M. M., & Feigenson, L. (2008). Individual differences in non-verbal number acuity correlate with maths achievement. *Nature*, *455*(October), 665–668. https://doi.org/10.1038/nature07246

Hindman, A. H. (2013). Mathematics instruction in Head Start: Nature, extent, and contributions to children's learning. *Journal of Applied Developmental Psychology*, *34*(5), 230–240. https://doi.org/10.1016/j.appdev.2013.04.003

Jordan, N. C., Kaplan, D., Locuniak, M. N., & Ramineni, C. (2007). Predicting First-Grade Math Achievement from Developmental Number Sense Trajectories. *Learning Disabilities Research & Practice*, *22*(1), 36–46. https://doi.org/10.1111/j.1540-5826.2007.00229.x

Jordan, N. C., Kaplan, D., Ramineni, C., & Locuniak, M. N. (2009). Early math matters: kindergarten number competence and later mathematics outcomes. *Developmental Psychology*, *45*(3), 850–867. https://doi.org/10.1037/a0014939

Klibanoff, R. S., Levine, S. C., Huttenlocher, J., Vasilyeva, M., & Hedges, L. V. (2006). Preschool children's mathematical knowledge: The effect of teacher "math talk.". *Developmental Psychology*, *42*(1), 59–69. https://doi.org/10.1037/0012-1649.42.1.59

Levine, S. C., Suriyakham, L. W., Rowe, M. L., Huttenlocher, J., & Gunderson, E. A. (2010). What counts in the development of young children's number knowledge? *Developmental Psychology*, *46*(5), 1309. https://doi.org/10.1037/a0019671

Locuniak, M. N., & Jordan, N. C. (2008). Using Kindergarten Number Sense to Predict Calculation Fluency in Second Grade. *Journal of Learning Disabilities*, *41*(5), 451–459. https://doi.org/10.1177/0022219408321126

Marsh, H. W., Trautwein, U., Lüdtke, O., Köller, O., & Baumert, J. (2005). Academic self-concept, interest, grades, and standardized test scores: Reciprocal effects models of causal ordering. *Child Development*, *76*(2), 397–416.

McCray, J. S., & Chen, J.-Q. (2012). Pedagogical content knowledge for preschool mathematics: Construct validity of a new teacher interview. *Journal of Research in Childhood Education*, *26*(3), 291–307. https://doi.org/10.1080/02568543.2012.685123

Missall, K., Hojnoski, R. L., Caskie, G. I. L., & Repasky, P. (2014). Home Numeracy Environments of Preschoolers: Examining Relations Among Mathematical Activities, Parent Mathematical Beliefs, and Early Mathematical Skills. *Early Education and Development*, *26*(December 2014), 356–376. https://doi.org/10.1080/10409289.2015.968243

Mix, K. S., Sandhofer, C. M., Moore, J. A., & Russell, C. (2012). Acquisition of the cardinal word principle: The role of input. *Early Childhood Research Quarterly*, *27*(2), 274–283. https://doi.org/10.1016/j.ecresq.2011.10.003

Musun-miller, L., & Blevins-Knabe, B. (1998). Adults' Beliefs about Children and Mathematics : How Important is it and How do Children Learn about it ? *Early Development and Parenting*, *7*(April 1997), 191–202. https://doi.org/10.1002/(SICI)1099-0917(199812)7:4<191::AID-EDP181>3.0.CO;2-I

National Association for the Education of Young Children/National Council of Teachers of Mathematics. (2010). Early childhood mathematics: Promoting good beginnings. *... the National Coun-Cil of Teachers of Mathematics ...*, 1–21. Retrieved from http://scholar.google.com/scholar?hl=en&btnG=Search&q=intitle:Early+Childhood+Mathematics+:+Promoting+Good+Beginnings#4

National Research Council. (2009). *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity* (C. T. Cross, T. A. Woods, & H. Schweingruber, eds.). https://doi.org/10.17226/12519

Pan, B. A., Rowe, M. L., Spier, E., & Tamis-Lemonda, C. (2004). Measuring productive vocabulary of toddlers in low-income families: Concurrent and predictive validity of three sources of data. *Journal of Child Language*, *31*(3), 587–608.

https://doi.org/10.1017/S0305000904006270

Parsons, S., & Bynner, J. (2005). Does Numeracy Matter More ? *National Research and Development Centre for Adult Literacy and Numeracy*, 1–37. https://doi.org/1905188090

Platas, L. M. (2015). The Mathematical Development Beliefs Survey: Validity and reliability of a measure of preschool teachers' beliefs about the learning and teaching of early mathematics. *Journal of Early Rhildhood Research*, *13*(3), 295–310. https://doi.org/10.1177/1476718X14523746

Pruden, S. M., Levine, S. C., & Huttenlocher, J. (2011). Children's spatial thinking: does talk about the spatial world matter? *Developmental Science*, *14*(6), 1417–1430. https://doi.org/10.1111/j.1467-7687.2011.01088.x

Ramani, G. B., Rowe, M. L., Eason, S. H., & Leech, K. A. (2015). Math talk during informal learning activities in Head Start families. *Cognitive Development*, *35*, 15–33. https://doi.org/10.1016/j.cogdev.2014.11.002

Rudd, L. C., Lambert, M. C., Satterwhite, M., & Zaier, A. (2008). Mathematical language in early childhood settings: What really counts? *Early Childhood Education Journal*, *36*, 75–80. https://doi.org/10.1007/s10643-008-0246-3

Simpson, A., & Linder, S. M. (2016). The indirect effect of children's gender on early childhood educators' mathematical talk. *Teaching and Teacher Education*, *54*, 44–53. https://doi.org/10.1016/j.tate.2015.11.011

Suinn, R. M., & Richardson, F. (1971). Anxiety management training: A nonspecific behavior therapy program for anxiety control. *Behavior Therapy*, *2*(4), 498–510.

Suinn, R. M., & Winston, E. H. (2003). The mathematics anxiety rating scale, a brief version: psychometric data. *Psychological Reports*, *92*(1), 167–173. https://doi.org/10.2466/pr0.2003.92.1.167

Susperreguy, M. I., & Davis-Kean, P. E. (2016). Maternal math talk in the home and math skills in preschool children. *Early Education and Development*, *9289*(April), 1–17. https://doi.org/10.1080/10409289.2016.1148480

Torbeyns, J., Gilmore, C., & Verschaffel, L. (2015). The Acquisition of Preschool Mathematical Abilities: Theoretical, Methodological and Educational Considerations. *Mathematical Thinking and Learning*, *17*(2–3), 99–115. https://doi.org/10.1080/10986065.2015.1016810

Tourangeau, K., Nord, C., Lê, T., Sorongon, A. G., Hagedorn, M. C., Daly, P., & Najarian, M. (2015). Early Childhood Longitudinal Study, Kindergarten Class of 2010-11 (ECLS-K: 2011). User's Manual for the ECLS-K: 2011 Kindergarten Data File and Electronic Codebook, Public Version. NCES 2015-074. *National Center for Education Statistics*.

Trautwein, U., Marsh, H. W., Nagengast, B., Lüdtke, O., Nagy, G., & Jonkmann, K. (2012). Probing for the multiplicative term in modern expectancy–value theory: A latent interaction modeling study. *Journal of Educational Psychology*, *104*(3), 763.

Trawick-Smith, J., Oski, H., DePaolis, K., Krause, K., & Zebrowski, A. (2016). Naptime data meetings to increase the math talk of early care and education providers. *Journal of Early Childhood Teacher Education*, *37*(2), 157–174.

Vallerand, R. J., & Houlfort, N. (2003). Passion at work. *Emerging Perspectives on Values in Organizations*, 175–204.

Vygotsky, L. (1978). Interaction between learning and development. *Readings on the Development of Children*, *23*(3), 34–41.

Watts, T. W., Duncan, G. J., Siegler, R. S., & Davis-Kean, P. E. (2015). What's Past is Prologue: Relations Between Early Mathematics Knowledge and High School Achievement. *HHS Public Acces*, *6*(2), 356–372. https://doi.org/10.1007/s12671-013-0269-8.Moving

Wechsler, D., & Hsiao-pin, C. (2011). NCS Pearson; San Antonio, TX: 2011. *WASI-II: Wechsler Abbreviated Scale of Intelligence*.

Woodcock, R. W., McGrew, K. S., & Mather, N. (2001). *Woodcock–Johnson III Tests of Achievement (3rd ed.)*. Itasca, IL: Riverside Publishing Company.