The Impact of Formative Assessment on Sixth Grade Students' Conceptual Understanding of Mathematics

by

David J. Malone

Bachelor of Science in Education, Slippery Rock University, 1997

Master of Science in Education, Johns Hopkins University, 2002

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This dissertation was presented

by

David J. Malone

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and approved by

Dr. Keith Trahan, Visiting Scholar, Administrative and Policy Studies and Educational Foundations, Organizations, and Policy

Dr. Patrick Mannarino, Superintendent, North Hills School District

Dissertation Chair: Dr. Charlene Trovato, Clinical Associate Professor, Department of Teaching, Learning and Leading

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David J. Malone, EdD

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Conceptual understanding of mathematics is foundational for students as they develop higher-level mathematical skills. In the simplest of terms, conceptual understanding of mathematics means developing students' ability to understand and explain why, rather than how, a particular concept is applied. Traditionally, K-12 teachers of mathematics in the United States emphasize the procedural nature of mathematics rather than developing students' conceptual understanding of mathematics. Although there are many concepts embedded in the conceptual understanding of mathematics, Mathematical Reasoning (MR) appears most frequently in Common Core Standards of Mathematics. According to those standards, developing students' MR skills is an integral component to developing students' conceptual understanding of mathematics (Common Core State Standards Initiative, 2010). We know from international research that American students fall far behind their counterparts in other industrialized nations, both on standardized tests of mathematics achievement and on tests designed to measure students' abilities to apply their knowledge to solving novel and challenging problems (Richland, Stigler, & Holyoak, 2012). Research shows that the gap between U.S. students and those in other countries grows wider as students' progress from elementary school through high school (Boston & Smith, 2009). Raising the overall level of mathematical proficiency of students in the United States can be seen "as both a matter of national interest and moral imperative" (Loewnberg & Ball, 2003).

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Acknowledgements

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1.0 Introduction

The statement attributed to the Greek Philosopher Heraclitus of Ephesus that “change is the only constant in life” could appropriately be altered to say change is the only constant in education. Standards change, curricula change, assessments change, and students change at an almost constant rate in education. If teachers are not able to adapt and evolve with those changes, they are most certainly destined to struggle to be effective in the classroom.

Arguably, Common Core Standards can arguably be credited as the most significant change in education in recent time. Common Core Standards are an educational initiative from 2010 that detail what US students in grades K-12 should know in English language arts and mathematics at the conclusion of each academic grade level. The initiative was sponsored by the National Governors Association and the Council of Chief State School Officers. Common Core Standards of Mathematics emphasize in-depth conceptual understanding, fluency and application of skills and concepts (Common Core State Standards Initiative, 2010). These standards push students to understand math on a deeper level digging into the reasoning behind a problem rather than just the solution.

The standards don’t make provisions for the fact that many students and teachers experience difficulty with these skills (Schwols, Dempsey, & Kendall, 2013). Research shows one of the most challenging goals for mathematics educators is developing proficiency in students’ reasoning and proving skills (Stylianides, 2014). Providing professional development and instructional support for staff and students to be successful with these new standards has become a challenge and priority for school leaders.

1.1 Statement of the Problem

Students in grade six at Springdale Intermediate School in a rural area of North Eastern Ohio have performed below the 80 percent proficiency level in mathematics as measured on the American Institutes for Research (AIR) test in Ohio for three consecutive years. The three-year average score for sixth grade mathematics is 71.6 percent, which is well below the state standard of 80 percent.

Ohio State standardized tests changed with the implementation of the Common Core Standards in the 2014-2015 school year. Prior to the 2014-2015 school year, students took the Ohio Achievement Assessments (OAA) and consistently met the state standard. During the first year of Common Core implementation students took the Partnership for Assessment of Readiness for College and Careers (PARRC) test and again met the state standard. In the 2015-2016 school year when Ohio changed to the American Institute for Research (AIR) assessments, students in grade six no longer met the standards for sixth grade mathematics. Teachers stated frequently in staff meetings that they felt the AIR assessments were much more difficult and challenging for students. Student test results reinforce this notion.

AIR mathematical test items released by the State of Ohio revealed that students were missing questions that involve concepts with modeling and reasoning. Both Common Core Standards and AIR assessments place a much higher emphasis on modeling and reason than previous standards and assessments. Released test items provide a sampling of actual questions student saw on the test. The test in its entirety is not released. Of the 46 questions released from the 2018 spring administration of the AIR test, sixth grade students missed 12 questions on modeling and reasoning, with 76 percent of students earning zero points. Of the 50 questions were released from the 2019 Spring AIR test, students missed 11 questions involving modeling and reasoning, with 71 percent of the students earning zero points.

1.2 Purpose

Common Core Standards not only stress conceptual understanding of mathematics but also returning to organizing principles such as place value or the properties of operations to structure those ideas. When students do not develop conceptual understanding of these basic mathematical concepts they significantly limit their chances for success as they progress through middle and high school mathematics content. Clearly the concepts of modeling and reasoning are essential foundational skills for higher-level mathematics at all grade levels.

Common Core Standards define *modeling* as the ability of mathematically proficient students to apply the mathematics they know to solve problems arising in everyday life, society and the work place (Common Core State Standards Initiative, 2010). Proficiency in modeling requires students to analyze relationships mathematically and to draw conclusions. Additionally, modeling requires interpreting mathematical results in the context of the situation, reflecting on results, and improving the model if and when appropriate.

Reasoning is defined as mathematically proficient students' ability to notice if calculations are repeated, and to look both for general methods and short cuts. As students work to solve a problem, they maintain oversight of the process, while also attending to the details. They continually evaluate the reasonableness of their intermediate results.

Conceptual understanding of mathematics can be characterized as an attainment of expert-like fluency with the conceptual structure of a domain (Richland et al., 2012). Generally, such an understanding allows students to think generally with in the context of a specific mathematical area, enabling them to select appropriate procedures for each step when solving problems, make predictions about solutions, and construct new understandings of problem-solving strategies and how to apply them to future mathematical problems and concepts. Conceptual understanding is essential for students to develop and improve their skills with modeling and reasoning.

Classroom walk-throughs and teacher observation further reinforce the view that teachers do not emphasize opportunities to draw connections between mathematical procedures and concepts. Most instruction is linearly organized for factual recall and application of procedures without helping students to achieve a full understanding of how and why the procedures are used. Too much importance is placed on finding the correct answer rather than focusing on a deeper understanding of how and why the answer is correct. International consistently shows that U.S. students fall behind their counterparts in other industrialized nations on standardized tests and tests designed to measure students' ability to apply their knowledge to solving challenging problems (Richland et al., 2012). We also know this gap widens as students' progress through school, from elementary school through graduation from high school (Richland et al., 2012). Providing students with the type of instruction needed to address these concerns will require ongoing educator professional development. The professional development will need to provide opportunities that are transformative in nature, catalyzing changes in teachers' long-held beliefs about effective teaching and learning of mathematics (Boston & Smith, 2009). Raising the level of mathematical proficiency in the United States can be seen "as both a matter of national interest and moral imperative" (Loewenberg Ball, 2003).

Thus, the problem of practice is a lack of conceptual understanding of mathematics in grade six. This is evidenced by the significant number of questions students answer incorrectly in the modeling and reasoning category on state assessments. Students need to show growth on classroom administered performance assessments before they can be expected to show similar growth on state assessments. Lacking strong conceptual understanding of mathematical concepts not only limits students' opportunities for both academic and financial success later in life, but it also puts them at a significant disadvantage with their international counterparts in an increasingly global economy (Stigler & Hiebert, 1999). The problem is actionable in that teachers are given the opportunity to review desegregated testing data and afforded the time and opportunity to reflect on that data to allow for changes in instructional practices that emphasize conceptual teaching and learning. The problem is feasible in that there is a full school year to make changes for improvement. Using concepts of improvement science to address drivers related to this problem offer a strategic approach tied to the specific practices of assessment and pedagogy (Mintrop, 2016). Working toward solving this problem is forward thinking in that it will ultimately help students reach their full academic potential and become college and career ready, all of which are important to the mission of the Springdale Local Schools.

1.3 Inquiry Questions

Q1.To what degree will a formative assessment on high level mathematical tasks in sixth grade classrooms increase student scores on performance-based assessment items related to ratios and geometry?

Q2.What impact will teacher-directed formative assessment have on students' ability to model mathematical ideas and articulate their mathematical reasoning skills with questions related to ratios and geometry on statistically valid performance based-assessment items?

Q3.To what extent will students completing workspaces on a digital formative assessment tool at a 75 percent proficient or higher-level result in students answering related items on school administered diagnostic test items correctly?

2.0 Review of Literature

This literature review will focus on the following guiding questions: (1) What is Mathematical Reasoning, and how does it relate to conceptual understanding of mathematics? (2) Why should Mathematical Reasoning be a major point of emphasis with mathematics instruction beyond the fact that it is included in both NCTM standards and Common Core Math Standards? (3) How can mathematics teachers incorporate Mathematical Reasoning into their daily instruction to help strengthen students' conceptual understanding of mathematics?

2.1 Mathematical Reasoning and Conceptual Knowledge of Mathematics

Mathematical thinking is commonly divided into two categories: procedural knowledge and conceptual knowledge (Star, 2005). Most mathematics instruction and materials (text books and curriculums) focus on procedural knowledge, the steps used to solve a problem (Crooks & Alibali, 2014). Procedural knowledge is typically measured by how many correct answers a student obtains based on the procedures they use to arrive at those answers (Stigler & Hiebert, 1999). With this type of instruction, there is often little or no emphasis on the conceptual basis of the skill being learned (Stigler & Hiebert, 1999). Reform efforts reflected in the National Council of Teacher of Mathematics and in the Common Core Standards emphasize students ability to integrate conceptual and procedural knowledge (e.g., National Council of Teachers of Mathematics, 2000; National Governor's Association Center for Best Practices & Council of Chief State School Officers, 2010). Possessing conceptual knowledge has benefits above and beyond procedural skills (Crooks & Alibali, 2014). Conceptual knowledge has been shown to help students evaluate which procedure is appropriate when solving a problem (Smith, Bill, & Raith, 2018).

According to a framework proposed by Hatano and Inagaki (1986), “conceptual understanding of mathematics can be characterized as an attainment of an expert like fluency with the conceptual structure of a domain” (pg. 263). This level of understanding allows the learner to think generatively within the content area, enabling them to select the appropriate procedures for each step when solving new problems, make predictions about the structure of solutions and construct new understanding and problem-solving strategies (Richland et al., 2012).

Many students in the United Sates lack conceptual understanding of mathematics. They struggle with comparative reasoning skills, often trying to apply previously memorized procedures (often incorrectly) and therefore never fully comprehending the content of the course in which they are enrolled (Stigler & Hiebert, 1999). This lack of knowledge often causes barriers that impede students’ progress towards higher-level degrees and careers that require the application of mathematical reasoning skills (Richland et al., 2012). Many schools are failing to teach students the conceptual basis for understanding mathematics that could support flexible transfer and generalization of skills. Many students are able to successfully graduate from K-12 schools without ever fully developing these skills but then are most often placed into developmental or remedial courses once they reach the university level. Often these courses create barriers for students’ success (Richland et al., 2012).

Typically, students are able to remember mathematical procedure well enough to pass tests in middle school and high school. After students stop taking mathematics courses their skills and reasoning ability degrade or were never fully acquired in the first place (Richland et al., 2012). This is mainly due to the procedural nature of instruction U.S. students are exposed to in K-12 schooling. Students must *struggle* with the conceptual structure of understanding mathematics, meaning they must expend effort to make sense of mathematics to figure something out that is not immediately obvious (Richland et al., 2012). Conceptual understanding increases students retention of difficult concepts and provides meaningful application of mathematics as compared to fact recall which is often quickly forgotten with the lack of meaningful understanding of concepts.

U.S. students often do not view mathematics as a system because their teachers do not stress opportunities to draw connections among mathematical procedures (Richland et al., 2012). Many students do not see mathematics as something they can reason their way through, so they often do not expand effort trying to connect procedures they are taught with fundamental concepts that help them understand mathematics as a meaningful system (Stigler & Hiebert, 1999). We know from international research that American students fall far behind their counterparts in other industrialized nations, both on standardized tests of mathematics achievement and on tests designed to measure students' abilities to apply their knowledge to solving novel and challenging problems (Richland et al., 2012). The gap between U.S. students and those in other countries grows wider as students progress elementary school through high school (Bailey, 2009).

A recent study (Lortie-Forgues & Siegler, 2017) illustrated the lack of conceptual understanding of mathematics in K-12 instruction by focusing on the conceptual knowledge of fraction arithmetic. Many school-aged students and their teachers lack conceptual knowledge of basic fraction arithmetic. Middle-school-aged students consistently possess weak understanding of multiplication and division of fractions below one even when they are able to correctly execute fraction arithmetic procedures, demonstrated by calculating correct answers and understanding the magnitudes of individual fractions. This situation is caused by instruction that lacks multiple embodiments of fractions. (Zhang, Clements, & Ellerton, 2015). Instructional approaches are generally traditional, emphasizing multiples and factors along with procedural understanding but without a variety of concrete models that emphasize conceptual understanding. When asked to estimate answers while multiplying fractions, students often apply the same concepts of estimation they use when multiplying whole numbers. They think that when multiplying two values, the answer will be larger than either of the two numbers being multiplied. Students often lack the basic understanding that there are an infinite number of fractional values between two whole numbers, and their value will actually get smaller when multiplied (Lortie-Forgues & Siegler, 2017).

Lortie-Forgues and Siegler (2017) found that middle school students and teachers demonstrated strong understanding of the magnitude of individual fractions between 0 and 1. Those students and teachers also demonstrated minimal understanding of the magnitudes produced by multiplication and division of fractions in the same range. They consistently predicted that multiplying two fractions below “1” would equal an answer greater than either factor, and dividing by a fraction less than “1” would equal an answer smaller than the fractions being divided. The results found with fractions closely parallel those found with decimals (Lortie-Forgues & Siegler, 2017). If students are to build a deeper understanding of fraction concepts, it is essential to let them become acquainted with a variety of models (Zhang et al., 2015). Math standards such as the Common Core Standards (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) explicitly state that teaching conceptual understanding in addition to procedures is a way to instill deeper and longer-lasting mathematical understanding. Thus, there is a widely held belief that conceptual knowledge plays an important role in mathematics learning (Crooks & Alibali, 2014).

Even though students can often correctly perform numerical calculations using all four operations of arithmetic they often lack the conceptual understanding of what they are actually doing (Byrnes & Waisk, 1991). This lack of understanding significantly limits students' estimation skills with very large and small numbers. Students often lack or are never taught conceptual understanding of mathematics because their teachers lack that understanding as well. Many teachers of middle school mathematics are K-8 certified and were not trained with nor do they possess strong conceptual understanding of mathematics (Lin et al., 2013). This lack of conceptual understanding by both students and their teachers can negatively affect their future success with not only mathematics but other academic content as well (Siegler et, al., 2012).

It is no secret that mathematical knowledge gained during school predicts academic, occupational, and financial success years later (Ritchie & Bates, 2013). Mathematics achievement at early ages can predict socioeconomic status (SES) later in life, even after statistically controlling for SES at birth, reading achievement, IQ, academic motivation, and years and level of education.

2.2 Common Core Standards of Mathematics

Common Core Standards were adopted by the Ohio Department of Education in 2010. The standards were a result of 48 states, two territories, and the District of Columbia's affiliation with the National Governors Association Center for Best Practices & Council of Chief State School Officers who shared the common goal that all students would graduate from high school prepared for college, career, and life regardless of where they live.

Research on mathematics education in high performing countries have consistently indicated that mathematics curriculum and instruction in the United States needs to be significantly more focused and coherent in order to improve mathematics achievement (Roth McDuffie et al., 2017). Common Core Standards are an attempt to address the problem that the U.S. math curriculum is “a mile wide and an inch deep” (Common Core State Standards Initiative, 2010, pg. 1). These standards are an attempt to bring clarity and specificity to what students are expected to learn in mathematics.

Mathematics concepts in the United States are most often taught form an emphasis on procedure and fact recall, with conceptual weakness in both areas (Lin et al., 2013). Common Core Math Standards stress conceptual understanding of key ideas by continually returning to organizing principles such as place value or properties of operations to structure those ideas (Common Core State Standards Initiative, 2010). The standards define what students are expected to *understand* as well as do in the study of mathematics. The standards require teachers to assess if the students have understood a concept. A hallmark of this understanding is asking students to explain or justify their answer and their thought process to arrive at their answer (Common Core State Standards Initiative, 2010).

The focus on preparing students for college and career readiness contained in the Common Core Math Standards requires students to demonstrate the ability to construct, interpret and use mathematical models. A mathematical model is a description of a system using mathematical concepts and language, to model conceptual understanding of mathematics, students need to be able to apply concepts they already know to solve problems that arise in everyday life. Students need to analyze those relationships mathematically in order to draw conclusions. Students also need to be able to reflect on those conclusions and interpret if the results make sense, possibly improving the model if it has not served its intended purpose (Common Core State Standards Initiative, 2010). Mathematical modeling is a way for students to demonstrate the immediate understanding of a concept as opposed to teaching a concept for use in the future.

Another important concept in Common Core Math Standards that goes along with and supports Mathematical Modeling is Mathematical Reasoning. Mathematical reasoning requires students to notice patterns and whether calculations are repeated. As students work to solve a problem, they must maintain oversight of the process while also focusing on the details (NCTM, 2009). They continually evaluate the reasonableness of their immediate results (Common Core State Standards Initiative, 2010). This ability along with modeling helps students to apply mathematics in everyday life.

Middle school teachers across the United States strongly agree that their current curriculum and instructional materials emphasize computational or procedural fluency, in contrast to an emphasis on conceptual understanding of mathematical concepts emphasized in Common Core Math Standards (Roth McDuffie et al., 2017). Many of the teachers in this study felt their materials are not aligned with Common Core Math Standards even when materials are labeled as aligned.

Common Core Math Standards have become high stakes because of the high-stake assessments that states have implemented to monitor student growth and achievement with the standards. Research has indicated that high-stakes assessments often lead to a narrowed and fragmented curriculum (Roth McDuffie et al., 2017). The results of these assessments is a stronger emphasis on test items and teacher-centered instruction rather than an emphasis on curricular concepts and conceptual understanding in many U.S. classrooms (Au, 2007).

Teachers' perceptions of Common Core Math Standards and supports provided to implement them influence their willingness to enact new forms of curriculum and instruction. According to findings reported in a study conducted by Roth McDuffie and colleagues (2017), teachers expressed a need for curriculum materials to be better aligned with Common Core Math Standards.

2.3 Mathematical Reasoning

Mathematical reasoning (MR) is listed as an important goal of instruction in both the Common Core Standards for Mathematics and the National Council of Teacher of Mathematics Standards (NCTM, 2009). Descriptions and definitions of MR within these standards, however, is very vague. Both sets of standards give little to no guidance on how to teach MR, as their purpose is to tell teachers *what* rather than *how* to teach. A great deal of disagreement on the definition of MR also exists within the mathematics education research community. The discourse on MR is not monolithic; it does not consist of a single voice (Jeannotte & Kieran, 2017). The term MR is often used by mathematicians and mathematics educators without any clarification or elaboration on its meaning (Jeannotte & Kieran, 2017).

Definitions vary from a classical approach of MR as the ability to apply deductive, inductive, and abductive reasoning to a more modern version with an emphasis on product versus process, with the focus on function of being able to explain the thought process of applying new knowledge (Jeannotte & Kieran, 2017). The lack of consistency in defining MR influences the inconsistencies with how educators attempt to teach MR within their individual classrooms.

From a sociocultural approach teachers need to possess the ability to not only be aware of the nature of the forms and processes of MR that they wish students to learn to participate in but also recognize when students are engaging in the desired aspects of reasoning (Jeannotte & Kieran, 2017). Sociocultural meaning how students are affected by circumstances surrounding them and how their behaviors are affected by other people, social institutions, and social forces such as socioeconomic status and gender bias. From a sociocultural view, MR can be seen as a discursive activity meaning MR might have different meaning for people in diverse situations or of diverse backgrounds. The Commognitive Framework by Sfard (2008, 2012) can help teachers to conceptualize MR as a discursive activity. Sfard (2008) defines *commognition* as "the term that encompasses thinking (individual cognition) and (interpersonal) communicating. As a combination of the words communication and cognition, it stresses the fact that these two processes are different (intrapersonal and interpersonal) manifestations of the same phenomenon" (p. 296). Commognition allows students to reflect on existing knowledge of constructed mathematical meaning and apply it to the construction of developing new meaning and mathematical understanding. This becomes sociocultural because students learn to develop their own meaning of MR rather than rely on preconceived notions influenced by external factors such as gender bias or student background.

2.4 Teaching Mathematical Reasoning

Once teachers develop and understanding of what MR, is they can begin to focus on how to teach it in their classrooms. According to the National Council of Teachers of Mathematics, reasoning and sense making should occur in classrooms on a daily basis (NCTM, 2009). Reasoning can take on many forms in the context of a classroom, including but not limited to explorations and conjectures, explanations and justifications of student thinking, and proofs and proving (NCTM, 2009). Sense making can be defined as developing an understanding of a situation, context, or concept by connecting it with existing knowledge, mirroring the first part of the Commognitive Framework (NCTM, 2009).

"In mathematics, the generation and validation of new knowledge frequently involves altering between two major activities: making generalizations, and developing argument" (Stylianides, 2010, pg. 39). In the early stages of this process students should look to arrange significant facts into meaningful patterns and use the pattern to make a conjecture. Mathematicians typically try to make sense of a conjecture by developing arguments for, or against, those conjectures. Some of these arguments may meet the standard for what is called a proof. A proof is the final product of the that process and is supported by other activities such as identifying patterns, developing conjectures and making arguments against the proof (Harel & Sowder, 1998).

Just as MR evokes different meanings among mathematicians and mathematics educators, so do terms like pattern, conjecture and proof when it comes to how teachers promote reasoning and problem solving skills (Stylianides, 2010). An analytic framework of reasoning-and-problem solving developed by Gabriel Stylianides (2010) can help develop clarity for classroom teacher.

Stylianides discussed the need to identify a pattern as part of the process of making a mathematical generalization. A pattern is a general mathematical relation that fits a given set of data. It is important to note that there will be an infinite number of patterns that will fit a given data set. Since there will be multiple patterns that will be mathematically legitimate students should consider *plausible* patterns. In identifying plausible patterns, it is impossible to provide conclusive mathematical evidence for the selection of one pattern over another. When a student can give mathematical evidence to support the selection of one pattern over that of another it is called a definite pattern (Stylianides, 2010).

When students make a conjecture, they make a reasonable hypothesis about a general mathematical relationship based on incomplete evidence (Reid, 2009). A hypothesis made by student must have reasoning and though behind it so it so can be distinguished from a guess. The process of making a conjecture and identifying a pattern is part of a more general activity Stylianides refers to as “making generalizations.” Teachers need to understand the important distinction between the two: when making a conjecture, the student formulates a hypothesis about a generalization whose domain extends beyond the domain of the problem the student has checked; pattern identification involves only the examined problem. Conjecturing is applying reasonability that a hypothesis will work in a different problem with the same concept (Reid, 2009).

The next component of Stylianides’s framework is developing arguments. The *Oxford American Dictionary* defines proof as “a demonstration of the truth of something” (1980, p. 535). In mathematics, a proof is a valid argument based on accepted truths for or against a mathematical claim (Knuth, 2002). Stylianides distinguished between two types of proof: “generic arguments”, and “demonstrations”. A generic argument is a proof that that is based on a particular case that is treated as a representative of the general case. Generic arguments provide students with a method to explain their thinking when they lack the mathematical vocabulary to do so. A demonstration is a proof that does not rely on the representation of a particular case. Proofs by counter example or contradiction are examples of a demonstration. Pictures can also be used in a demonstration proof.

When the criteria for a proof cannot be met, a student can develop a non-proof argument. There are two types of non-proof arguments, empirical arguments and rationales. An empirical argument is an argument that attempts to show the truth of a claim on the basis of the confirming evidence offered by the examination of a proper subset of all possible cases. Rationales are arguments for or against a mathematical claim that don’t meet the standard of proof. An argument is a rationale if it does not sufficiently justify one or more statements used in the arguments (Stylianides, 2010).

Proof is considered to be the central discipline of mathematics and the practice of mathematicians (Knuth, 2002). Traditionally, the concept of proof has been limited to instruction involving secondary geometry. Because of this limited exposure on using and applying proofs many students find the concept difficult (Chazan, 1993). Traditionally middle school math teachers throughout the United States who are not trained or licensed as secondary math teachers lack fundamental understanding of the concepts related to teaching poofs. According to the *Principles and Standards for School Mathematics* (NCTM 2000, pg. 56), instructional programs from prekindergarten through grade 12 should enable students to do the following:

- Recognize reasoning and proof as fundamental aspects of mathematics;
- Make and investigate mathematical conjectures;
- Develop and evaluate mathematic arguments and proofs;
- Select and use various types of reasoning and methods of proof (p. 56)

Because of these recommendations, proofs are expected to play a more significant role in mathematics instruction and materials designed for instruction.

Mathematicians are less interested in whether a conjecture is correct as they are in why it is correct. The primary role of proof in mathematics is to establish the truth of a result, and more importantly from an educational perspective, is the recognition of its role in developing an understanding of the underlying mathematics (Knuth, 2002). The concept of "why" something is correct is also deeply imbedded in the Common Core Standards of Mathematics which emphasize conceptual understanding of mathematics and were designed based on the Principles and Standards for School Mathematics developed by the National Council of Teachers of Mathematics (2000).

Developing MR skills will help students develop and understanding of how concepts are interrelated in the study of mathematics. Learning a concept involves not just knowing the meaning of the concept but also the multiple relationships that connect the concept to other ideas. Developing teaching strategies that present students with multiple representations of a problem will help students examine concept through a variety of lenses, with each lens helping to develop a clearer and deeper understanding of the concept (Tripathi, 2008).

Visual forms of representation have been found to play an important role in developing and nurturing problem-solving ability. Arcavi (2003) states, “Visualization is being recognized as a key component of reasoning, problem solving and even proving” (p. 441). Studies indicate that they ability to develop and use visual representational forms is valuable enough that it should become an integral part of mathematical learning (Tripathi, 2008).

Middle school is an appropriate time to develop students’ mathematical understanding with visual forms representing mathematical concepts. These types of representations can help students to bridge concepts that are more abstract in nature to a clearer, more concise concrete representation of the concept. Visualization strategies that play a role in developing mathematical thinking and reasoning skills include the following: 1. They can support and illustrate results that are more symbolic in nature; 2. They offer a possible way of resolving a conflict between correct solutions and incorrect intuitions; 3. They help students engage and recover conceptual underpinnings that may be bypassed by more formal and procedural solutions (Tripathi, 2008). Teachers who present problems that have strong visual content through a problem-solving activity will be able to improve their students’ MR skills and abilities over time (Arcavi, 2003).

2.5 The Need for Feedback

Formative assessment is a classroom practice that when carried out effectively can improve student learning (Black & William, 1998). Feedback can be motivating for students and help them to take ownership of their learning (Ebby & Petit, 2017). It also provides teachers with the opportunity to reflect on their teaching practices and student progress. It is important that feedback is not always in the form of a numerical grade illustrating right and wrong or it can become counterproductive (Ebby & Petit, 2017). According to Dylan William (2011), formative assessment lies in the sequence of two actions. The first is the learner recognizing that there is a gap between their present state of understanding a concept and a desired learning goal. The second is the action taken by the learner to close that gap. With feedback, there are complex links among the way in which the message is received, the way in which that perception motivates a selection among different courses of action, and the learning activity which may or may not follow (William, 2011).

For teachers of mathematics, the key to this process is making sense of and understanding student thinking in relationship to specific content (Ebby & Petit, 2017). This is where research on student learning of mathematics can support the use of formative assessment. Mathematics educators have made tremendous progress over the last decades with respect to conducting and synthesizing this research in the form of learning trajectories (Ebby & Petit, 2017).

A learning trajectory describes the progression of student thinking and use of strategies over time in terms of both conceptual understanding and procedural fluency of mathematical concepts (Ebby & Petit, 2017). Understanding this developmental process in terms of both conceptual understanding and efficiency allows teachers to assess where a student's understanding is on the trajectory and what the next instructional step should be to move the student's understanding and thinking forward (Ebby & Petit, 2017). Learning trajectories including counting, addition and subtraction, multiplicative thinking, measurement, geometry, proportional reasoning, and algebraic thinking are deeply imbedded in the Common Core Math Standards in the companion progressions documents, which explain the progression of topics across a number of grade levels (Common Core State Standards Initiative, 2010).

2.6 The Need for Professional Development

In mathematics education truths are established via proofs. It is this feature that sets mathematics apart from other content studied in K-12 schools. The proof concept makes the subject of mathematics cohere (Liu & Manouchehri, 2013). Both the Principles and Standards for School Mathematics (NCTM, 2000) and the Common Core Standards place tremendous emphasis on the need to assist students in developing their proving skills (Common Core State Standards Initiative, 2010). There is evidence that in many mathematics classrooms proofs and the proving process are taught procedurally instead of as a conceptual tool to develop mathematical reasoning skills (Liu & Manouchehri, 2013). As a consequence, many students tend to view proofs as a method for producing written work instead of a method for production of reliable explanations, or even means for understanding. There is evidence that understanding the role of mathematical proofs in establishing validity of arguments remains under developed at all grade levels further emphasizing the need for significant training and professional development (Liu & Manouchehri, 2013).

Studies indicate that students in the United States need to improve their ability to use cognitively demanding skills and apply facts and procedures in order to achieve at the level of higher performing countries (Boston & Smith, 2009). U.S. teachers often struggle to maintain task rigor and incorporate challenging and demanding tasks, that require students to reason during instruction because their training has emphasized fact recall and application of algorithms for sequential learning rather than conceptual understanding of mathematics (Weiss & Pasley, 2004). Students often spend most of their time in mathematics classrooms practicing procedures, regardless of the nature of the tasks they were given to complete.

According to Boston and Smith (2009) teachers who participated in an ongoing embedded professional initiative focused on selecting and using high-level cognitive tasks supporting conceptual understanding and mathematical reasoning significantly increased the "(a) the level of cognitive demand of their main instructional task and (b) their ability to maintain high-level cognitive demands of instructional tasks" (p. 125). Teachers were also more willing to use materials from standards-based curricula after participating in the professional development rather than conventional or traditional curricula (Llyod, 1999). The research presented in this study illustrates the importance of providing not only well planned but well implemented professional development to bring about a desired change in instructional practices. It is important to conduct research on the intended instructional task before planning and implementing professional development to ensure it is scientifically based and worth investing both institutional and financial resources. It is important to note that most changes no matter how necessary and worthwhile fail in the implementation stage (Boston & Smith, 2009). This reinforces the research conducted in this study on the importance of good professional development.

2.7 Conclusion

"One of the most fundamental goals of mathematical education is to provide rational answers in response to why questions, in other words to develop reasoning skills" (Bozkus & Ayvaz, 2018, p. 17). Mathematics entails discovering patterns, reasoning, making predictions, motivated thinking, and reaching conclusions. A student cannot begin to solve a problem unless they understand what is being asked of them. Mathematical reasoning is essential to formulate justifications, develop mathematical arguments, and choose and utilize a variety of representations (NCTM, 200). Mathematical reasoning is similarly fundamental in solving a problem and questioning the validity of an argument (NCTM, 2000). Therefore, mathematical reasoning is considered foundational to doing mathematics (NCTM, 2000).

3.0 Applied Inquiry Plan

3.1 Inquiry Setting

The Springdale Local School System, is one of over 600 cities, local and exempted village school districts in the state and 19 in the county, provides education to 1,033 K-12 students. It is located in the southeast corner of Mahoning County in northeastern Ohio approximately 10 miles southeast of the City of Youngstown and 75 miles southeast of the City of Cleveland. The School District and County border the State of Pennsylvania.

The School District's 2020 population was 6,447 (according to U.S. Census data). Its area is approximately 36 square miles. Land use, as measured by the assessed value of real property, is presented in the following table.

Table 1. District Real Property

Residential	72.9%
Commercial/Industrial	13.5%
Public Utility	3.6%
Agricultural	10%

The next table illustrates wealth indicators compared to Mahoning County and the State of Ohio.

Table 2. Wealth Indicators

2020 US Census Bureau Estimates	Springdale LSD	Mahoning County	Ohio
Median Value of Owner-Occupied Homes Estimates	$161,300	$105,400	$145,700
Median Household Incomes	$60,323	$46,042	$56,602
Per Capita Income Estimates	$33,210	$28,378	$31,552
Poverty Level (% of all people)	11.80%	17.50%	14.00%

The Springdale Local School System is best described as a rural school district. As of the September 2019 headcount enrollment, 1,033 students were enrolled in the District's three schools (one elementary school, one intermediate school and one high school). An additional 24 district students attend classes at the Mahoning County Career and Technical Center. For the last academic year, the average class size was 19.7 students, and the average pupil/teacher ratio was 16:1. The District employs (full- and part-time) 78 professional staff members and 34 non-teaching and support staff employees. The School District currently has 12 buses, 10 picking up students daily for school and two spare buses.

The following table illustrates district student demographic data for the 2019-2020 school year.

Table 3. Student Demographics

White, Non-Hispanic	94%
Multiracial	2%
Hispanic	3.4%
Students with Disabilities	10.5%
Economically Disadvantaged	42%

The percentage of students in the Springdale Local Schools who qualify for a free or reduced lunch is approximately 42 percent. Springdale Local Schools generate a failure report quarterly for each building. If a student receives a grade of a "D" or "F," they will appear on that report. Springdale Local Intermediate School had 27 students appear on that report for the second quarter grade reporting cycle. Of the 27 students on the report, 20 of them or 74 percent, qualified for a free or reduced lunch. This clearly illustrates that not all students are reaching their maximum potential in academic, social, emotional, and physical development as described by the district mission statement. It also clearly shows that there is a strong separation of income class as related to academic achievement.

Twice a year, the Springdale Local Schools hold parent teacher conferences. During the February conferences at the Intermediate School 55 parents attended a conference with at least one teacher. Of those 55 parents, only 12 of them, or 22 percent were from families who qualify for a free or reduced lunch. The majority of the students who are struggling academically live in poverty (as defined by the federal government) and for a variety of reasons are additionally challenged by having parents who are not involved with the school. Of the 12 parents who are considered low income, only three were invited or encouraged by teachers to attend a conference because their son or daughter appeared on the failure report.

There is a direct correlation between attendance and academic achievement (Roby, 2004). The state of Ohio considers a student to be habitually absent if they miss 65 or more hours in a school year with or without a legitimate excuse. Springdale Local schools sends out an attendance warning letter at 30 hours. Thus far, 51 letters have been sent to 36 students (or 70 percent) have been for students who are considered income poor.

3.2 Inquiry Stakeholders

3.2.1 Board of Education

The Springdale Local Board of Education is made up of five elected members who each serve a four-year term. The election cycle for board members occurs every two years. In odd years, two members are up for re-election; in even years, three members are up for re-election. Currently the board is comprised of three women and two men. One board member has served three terms while the other four are all serving their first term. Four of the board members have students currently enrolled in the district. Three have students in the Intermediate School (one in sixth grade); the other board member's child is currently in the High School. This board member also has two children who recently graduated from the district. All four board members' children either currently have or have in the past been enrolled in Mrs. Kohler's sixth grade Math class. So, they all have a relationship and an experience with the sixth-grade math teacher, curriculum, and state standardized test.

3.2.2 Tom Cazvac, District Superintendent

Mr. Cazvac has been with the Springfield Local School District for 23 years. For nineteen of those years, he served as the district's elementary principal and for the past four as superintendent. Before his administration experience, he was a middle school choir teacher with a different district. He is the instructional leader for the district, supervising its three building principals and various other department heads. He reports directly to the Springdale Local Board of Education.

3.2.3 David Malone, Intermediate School Principal:

Mr. Malone has been with the Springdale Local School District for 15 years. For 13 of those years he has been as the intermediate school principal; for two years prior was the high school assistant principal. Before his administrative experience, Mr. Malone was a seventh-grade math teacher in a different district. He has twenty-three total years' experience as an educator. As building principal, Mr. Malone is the instructional leader for grades five through eight. He supervises and evaluates all teachers in his building. His experience as both a principal and math teacher have given him detailed insight into mathematical reasoning.

3.2.4 Kelly Kohler, 6^{th} Grade Math Teacher:

Mrs. Kohler has been teaching for 23 years. The past 17 years have been as a sixth-grade math teacher. Mrs. Kohler incorporates hands on student-centered instruction on a daily basis in her classroom. Before the state adoption of the Common Core Math Standards and the Ohio AIR test, Mrs. Kohler's students consistently performed well on the Ohio Achievement Math Assessment. Since the adoption of the new standards and test her students have not met the state standard of 80 percent passing rate. Mrs. Kohler is the only sixth grade Math Teacher at Springfield Local Intermediate School.

3.2.5 Angela Smith, 5^{th} & 6^{th} Grade Special Education Intervention Specialist:

Mrs. Smith has 24 of teaching experience. She has spent the past four years as a Special Education Intervention Support Teacher serving students in grades five and six. Mrs. Smith provides learning support and academic accommodations to students in and outside of the sixth-grade math classroom.

3.2.6 Laura McDonald, Director of Special Services:

Mrs. McDonald has been the Director of Special Services for the past four years. She has 17 years of experience and served as an elementary special education cognitively delayed teacher with the district for 13 years. There are seven special education students currently mainstreamed in sixth grade math. None of them passed the fifth-grade AIR Test. The previous year, there were six Special Education mainstreamed students in sixth grade math and only one of them passed the sixth-grade Math AIR Test.

3.2.7 6th Grade Parents

Parents play a vital role in the success of their children by providing support and nurturing outside of school. Currently, parents have a lot of mixed feelings about the Common Core Standards and Standardized Testing. Parents are often resistant to Common Core Standards because they are unfamiliar with them and the concepts they emphasize. As a whole the Springdale Local Schools have done a poor job educating parents about the Common Core Math Standards.

3.2.8 6th Grade Students:

Ultimately the students are the most important stakeholders in this Problem of Practice. Their learning and success is the responsibility of the all of the above-mentioned stakeholders.

3.2.9 Stakeholder Roles

The power structure of the Springdale Local Schools is that of a typical school system. The Superintendent is the instructional leader of the district and ultimately holds the most power from an administrative standpoint. That power is demonstrated through the evaluation process of the building principals and support for their academic and instructional initiatives. The superintendent has to answer to the board of education as far as the performance of the building principal, teachers, and most importantly, the students. The board of education holds the most power but fortunately has operated in a very supportive manner empowering the district leadership to address this problem of practice. The board will ultimately have to answer to the parents and court of public opinion on the success or lack thereof of the school district

The sixth-grade math teacher and building principal play the largest role in identifying the problem and working through developing possible solutions. All stakeholders are interwoven together at different levels of the problem, but the building instructional team of the math department and building principal play the largest role in developing this problem of practice. It will take the collaborative efforts of all stakeholders to address this problem of practice.

3.3 Inquiry Approach

The purpose of this inquiry focuses on the effect formative assessment has on sixth grade students' conceptual understanding of mathematics, specifically in the areas of mathematical reasoning and modeling.

Lack of useable data on individual student performance and progress at the classroom level can be a significant roadblock to improving instruction and learning. Appropriate instructional decisions and allocation of school resources often hinge on the degree to which relevant data on student performance are available (Nelson, Van Norman, & Lackner, 2016). Having such data on a daily basis allows teachers to make changes to instruction for students experiencing difficulty and to provide meaningful feedback for improvement (Ysseldyke & Bolt, 2007). One goal of this inquiry was to help instructional leaders develop a better understanding of how to support students' learning with conceptual understanding of mathematics and how to make important instructional decisions to impact student achievement.

3.4 Instrumentation

During this inquiry, four performance based-assessment items on ratios and two on geometry were administered to sixth grade students. Students also took a grade-level mathematics diagnostic test at both the beginning and end of the inquiry. These diagnostic tests are given quarterly during the school year but two of them took place during the inquiry. Scores on these diagnostic tests help educators predict which students are likely to meet proficiency criteria on state assessment. Data were also analyzed from two different types of formative assessments. Both student responses and teacher feedback from quick writes and a digital formative assessment tool were collected and analyzed to show growth over time.

3.5 Research Methods and Design

Table 4. Inquiry Questions and Methodology

Question	Data Collection	Data Analysis
To what degree will formative assessment on high level mathematical tasks in 6^{th} grade classrooms increase student scores on performance-based assessment items related ratios and geometry.	Four performance based assessment items from the Institute for Learning on ratios were administered and scored (2) during the instructional unit and (2) after completion. (2) Geometry performance assessment items were analyzed as well.	These items were used because they have been scored and validated to be statistically significant and undergone reliability checks by measurement.com. The assessment items results were entered into an Excel spreadsheet. Graphs were created to show growth over time. The geometry items were scored to see if

Table 4 continued

		students could reason and model with the concepts of ratio in a different context. The results were also entered into an Excel spreadsheet. Graphs were created to show growth over time. This observational data was analyzed using descriptive statistics.
What impact will teacher directed formative assessment have on students' ability to model mathematical ideas and articulate their mathematical reasoning skills with questions related to ratios and geometry on statically valid performance-based assessment items?	Teacher written responses (feedback) to both quick writes and assessing and advancing questions.	Responses were coded using a predetermined codebook. Results were analyzed to see if feedback and guidance were impacting assessment. Teacher feedback was analyzed for feedback on solutions with additional guidance or solutions only. Students' diagnostic scores for those who received solution with guidance feedback were compared with diagnostic scores for students who received solution only feedback. Data was entered into Excel and graphed to see if there was a difference in test scores compared to the type of feedback the student received.
Will students completing workspaces on a digital formative assessment tool at a proficient or higher-level result in students answering related items on school administered diagnostic test items correctly?	Student completed workspaces in a digital formative assessment tool in the areas of ratios and geometry and corresponding items on school administered diagnostic tests.	Student scores on the digital formative assessment tool were compared to scores in the same two areas on a statistically valid school administered performance based diagnostic test to see if there was a relationship. The results were also entered into Excel and graphed to show growth over time. This observational data was analyzed using descriptive statistics.

Diagnostic assessments were administered quarterly during the school year. These diagnostics were administered in a digital format and could be completed by students either in school or at home. The first and last diagnostic assessments were used as a pre-and-post assessment for this inquiry cycle. The diagnostic assessment is an adaptive digital tool that provides teachers with an actionable insight into the needs of students. Diagnostic results offer a complete picture of student performance and growth related to standards, specifically mathematical modeling and reasoning. Diagnostic assessment data helps inform teachers if students are making progress related to grade level learning standards and were measured on state AIR assessments administered in May.

Four performance-based assessment items on ratios from the Institute for Learning of the University of Pittsburgh were administered and scored. Instructional problems involving ratios provide students the opportunity to apply mathematical reasoning skills. Students were given the opportunity to represent, analyze, and look for patterns and relationships; make sense of concepts; and develop, support, and defend claims and argument, all elements of mathematical reasoning. Two of the items were administered and scored during the instructional unit on ratios and two after completion of the unit. Two geometry assessment items were also analyzed as well.

The performance-based assessment items were used because they have been scored and validated to be statistically significant and have undergone reliability checks by measurement.com. The assessment items results were entered into Excel and graphed to show growth over time. The geometry items were scored to see if students could reason and model with the concepts of ratio in a different context. The results were also entered into Excel to be graphed to show growth over time. The observational data was analyzed using descriptive statistics.

Teacher-directed formative assessment items in the form of quick writes were administered during the units on ratio and geometry. Quick writes are focused math prompts that direct students to write about a specific mathematical idea or relationship associated with a specific lesson. Students can complete them independently in class following a lesson, much like an exit slip. The purpose is to provide students the opportunity to engage in critical thinking about their own understanding of important mathematical ideas or relationship. This activity also gives teachers a widow into student thinking and understanding of concepts and the opportunity to provide feedback and guidance.

Results were analyzed to see if feedback and guidance impacted assessment. Teacher feedback was analyzed for feedback on solutions with additional guidance or solutions only. If the feedback contained additional guidance, it received a code of (1). If the feedback was only on the solution being correct or incorrect, it received a code of (2). Students' diagnostic scores with solution with guidance feedback (2) was compared with students' diagnostic scores with solution only feedback (1). Data was entered into Excel and graphed to see if there was a difference in test scores compared to the type of feedback the student received.

During the 2020-2021 school year, a formative digital assessment tool was used in grade 6 mathematics classes with a goal of three to five days a week from September to May during the 2020-2021 school year. The formative assessment tool provided personalized and constructive guidance. The tool gave step-by-step examples that guided students through sample problems, describing each step, rephrasing or redirecting questions, and homing in on parts of the problem proving to be difficult for the student. By using the formative assessment tool, teachers analyzed every aspect of the students' work to better understand where they were and what they were thinking in order to better inform instructional delivery. The fact that the tool was in a digital format gave teachers real-time data to help effectively guide student learning.

The digital assessment tool that was used is called MATHia by Carnegie Learning. MATHia is Carnegie Learning's online software program and deploys artificial intelligence to actually teach math. By providing targeted coaching and adapting to student thinking, MATHia mirrors a human tutor with more complexity and precision than any other math software. MATHia gives the user real time data to effectively guide student learning.

It has the ability, through a variety of different reporting options, to provide a day-to-day view of a student's progress, and also has the ability to predict where the student will end up at the end of the school year.

Student scores on the digital formative assessment tool were compared to scores in the same two areas on a statistically valid school administered performance based diagnostic test to see if there was a relationship. The results were also entered into Excel to be graphed to show growth over time. This observational data was analyzed using descriptive statistics.

4.0 Results

This section presents and describes the finding of the inquiry, including performance-based assessment (PBA) items, quick writes, school administered diagnostic tests, and scores from a digital formative assessment program. Specifically, the data answers the following questions that guided the inquiry:

1. To what degree will formative assessment on high level mathematical tasks in sixth grade classrooms increase student scores on performance-based assessment items related ratios and geometry?
2. What impact will teacher-directed formative assessment have on students' ability to model mathematical ideas and articulate their mathematical reasoning skills with questions related to ratios and geometry on statically valid performance-based assessment items and on school administered diagnostic test items in the area of sixth grade mathematics?
3. Will students completing workspaces on a digital formative assessment tool at a proficient or higher-level result in students answering related items on school administered diagnostic test items correctly?

4.1 Summary of PBA Performance Over Time

Scores from 48 students across six performance-based items were analyzed for growth over time. The six items were chronologically administered, with the first four items being on ratios and the final two items being on geometry. Growth is considered in three ways:

- Growth from PBA 1 to PBA 6
- Correlation Between PBA Score Change and Relative Frequency of Additional Feedback
- Correlation Between APLSE and Diagnostic Measures

Summary statistics and plots are provided. The Friedman test was used for the first two growth types to test for statistically significant changes over time in PBA score, as well as to determine which time points show significant differences. This test is used in place of a one-way repeated measures ANOVA due to severe violations of ANOVA assumptions.

Each PBA has a maximum score of seven points. The distribution of scores on each of the six PBAs is provided below. Frequencies are provided with corresponding percentages in parentheses.

Table 5. Distribution of PBAs

Score	PBA 1	PBA 2	PBA 3	PBA 4	PBA 5	PBA 6
0	0 (0%)	2 (4%)	0 (0%)	0 (0%)	0 (0%)	0 (0%)
1	2 (4%)	0 (0%)	0 (0%)	0 (0%)	0 (0%)	0 (0%)
2	5 (10%)	1 (2%)	0 (0%)	0 (0%)	0 (0%)	0 (0%)
3	6 (13%)	1 (2%)	0 (0%)	0 (0%)	0 (0%)	0 (0%)
4	8 (17%)	1 (2%)	0 (0%)	0 (0%)	0 (0%)	0 (0%)
5	4 (8%)	8 (17%)	6 (13%)	6 (13%)	5 (10%)	0 (0%)
6	11 (23%)	8 (17%)	9 (19%)	6 (13%)	22 (46%)	24 (50%)
7	12 (25%)	27 (56%)	33 (69%)	36 (75%)	21 (44%)	24 (50%)
Median (IQR)	5 (3-6.75)	7 (5-7)	7 (6-7)	7 (6.25-7)	6 (6-7)	6.5 (6-7)

Median/IQR is provided since the score distributions are nearly all heavily negatively skewed. Two visualizations are provided to view the distributions of scores across the different PBAs. The first (Figure 1) is a boxplot, and the second (Figure 2) is a heatmap.

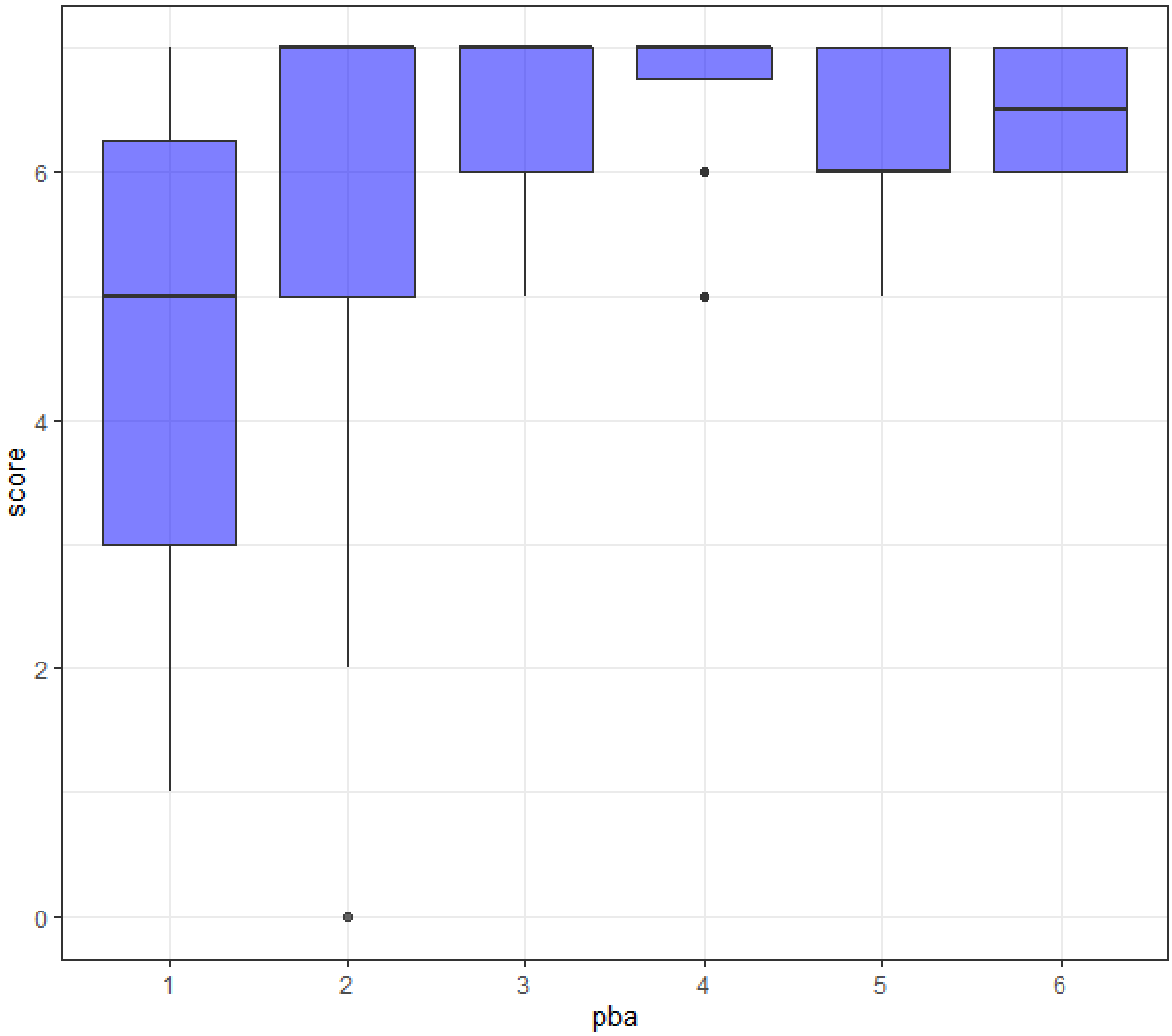

Figure 1. Box Plot of PBA Distribution

The boxplot displays the middle 50 percent of the distribution in the rectangle, with extended lines coming out from them to cover values going all the way to the minimum and maximum values (with the exception of outliers, which are represented as single points). An increasing trend in scores is apparent when looking at the first few PBAs. The score distribution appears to stabilize from PBA 3 to PBA 6. The heatmap below provides more granular information regarding the distributions of scores across PBAs.

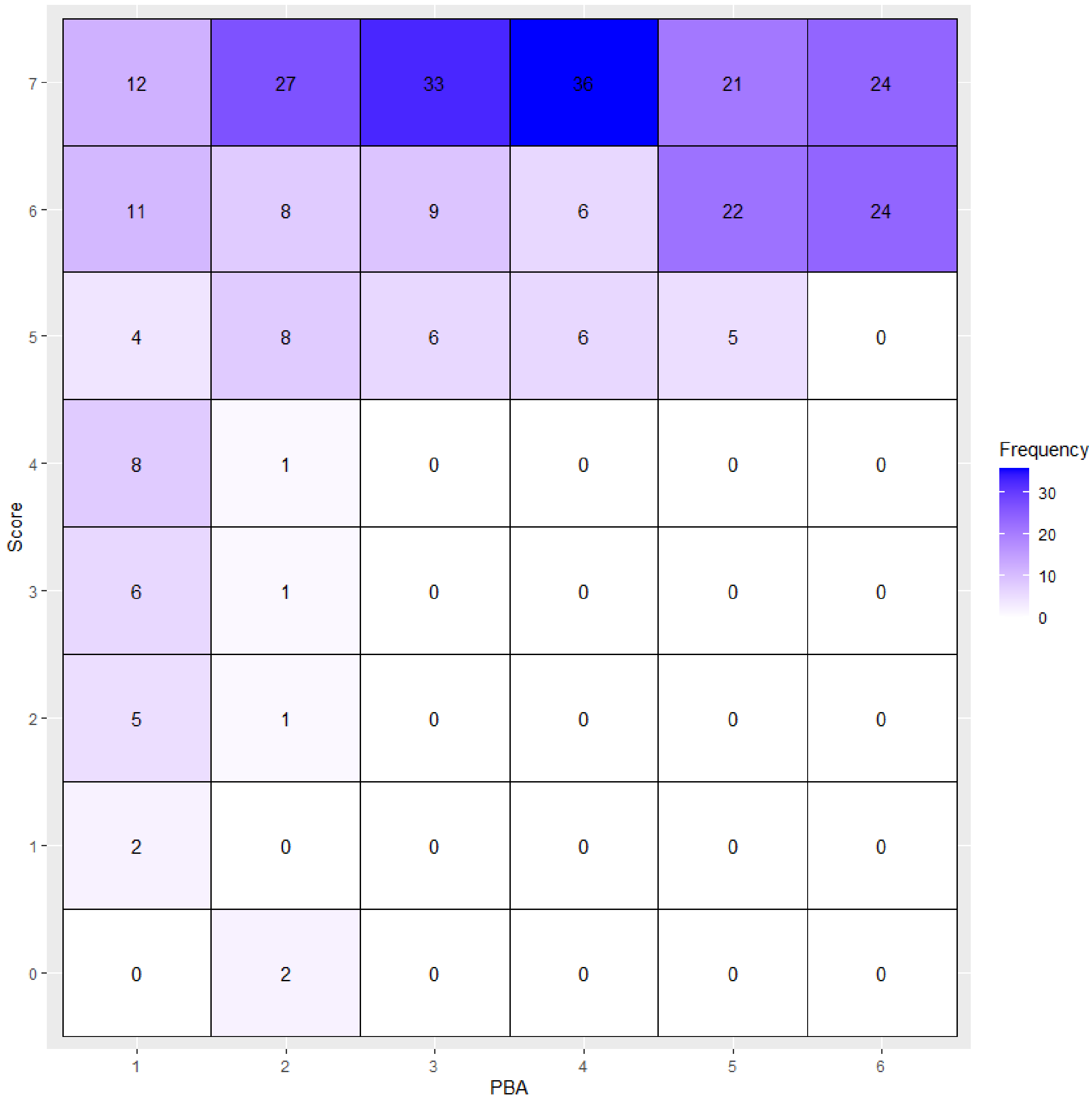

Figure 2. Heatmap of PBA Distribution

On this plot, the frequencies are listed at each possible combination of score (y-axis) and PBA (x-axis). The larger the frequency, the darker the shade of blue for that particular PBA/Score combination. The lower the frequency, the closer to white the shaded cell will be. From this plot, it is again easy to see the quick increase in scores. However, the subtle differences among PBAs 3-7 are easier to see. Specifically, the reader can see from PBA 2 to PBA 4 a slightly increasing trend towards more perfect scores. However, for PBA 5 and PBA 6, the scores become roughly split between 6 and 7. All of the students scored a five or higher on both PBA 4 and PBA 5. The difference is most noticeable in the number of students scoring a six or seven on PBA 4 compared to PBA 5 (see Table 6), resulting in a change of $p < .001$ to $p = .011$ with a result being $p < .05$ showing the only other significantly different change. This sudden shift could be due to switching from ratio to geometry items. Answering questions related to geometry requires students not only to apply mathematical reasoning skills but also to model their reasoning. This may have impacted the change in scores from PBA 4 to PBA 5.

Next, growth for these items was tested for statistical significance. Friedman's test was used to test for an overall change in scores over the entire span of PBAs, with follow-up Wilcoxon Signed Rank pairwise comparisons to test for differences between pairs of PBAs. The pairwise comparisons are adjusted using the Bonferroni correction for multiple comparisons.

Result:

- PBA scores were found to differ significantly across time, $\chi^2_F(5) = 50.332$, $p < .001$

Follow-up pairwise comparisons are summarized in Table 6 below:

Table 6. Pairwise Comparisons

PBA Pair	1	2	3	4	5	6
1	-					
2	.037*	-				
3	< .001***	1.000	-			
4	< .001***	0.743	1.000	-		
5	.011*	1.000	1.000	1.000	-	
6	< .001***	1.000	1.000	1.000	1.000	-

* $p < .05$
** $p < .01$
*** $p < .001$

PBA 1 had a significantly lower score than each of the other PBAs. PBAs 2, 3, 4, 5, and 6 were not significantly different from one another. This suggests a significant increase in scores from PBA 1 to PBA 2, and then any change in distribution of scores following was not statistically significant.

4.2 Summary of Teacher Feedback on Quick Writes Across Time

For each student, the type of feedback that was given at each quick write (QW) was one of the following: solution only or solution with additional guidance. It is interesting to note that the students who had the correct solution did not receive feedback with additional guidance. The frequency and percentage of feedback that provided additional guidance is summarized for each QW below:

Table 7. Frequency of Feedback Type

Quick Write	Frequency (%)
1	9 (19%)
2	31 (65%)
3	19 (40%)
4	9 (19%)
5	9 (19%)
6	15 (31%)
7	21 (44%)
8	15 (31%)
9	15 (31%)
10	14(29%)

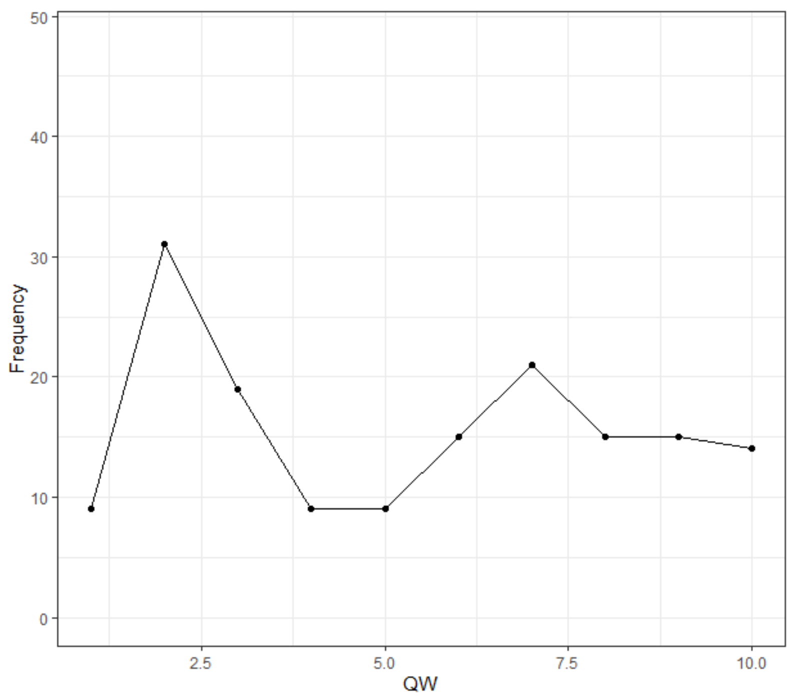

Figure 3. Line Graph of Feedback Frequency

The frequency of students receiving additional feedback ranged from 9 to 31 across the various QWs (19%-65%).

4.3 Correlation Between PBA Score Change and Relative Frequency of Additional Feedback on Quick Writes

After completing each of the six PBAs, students received teacher feedback on each individual PBA based on a scoring rubric. Students completed quick writes, which were based on classroom instruction that took place between each of the six PBAs. Classroom instruction was related to the mathematical content contained in both PBAs and quick writes. Quick writes are a type of formative assessment providing information that allows the teacher not only to provide student feedback but also to reflect on their instruction.

In order to assess whether a relationship exists between PBA score change (from first to final PBA) and feedback received, two variables were created:

1. A score change variable, which is recorded as the difference between PBA 6 and PBA 1.
2. A relative frequency variable, which is recorded as the percentage of times a student received additional guidance feedback (recorded for each student).

The association between these two variables was assessed via a scatterplot and corresponding correlation measure.

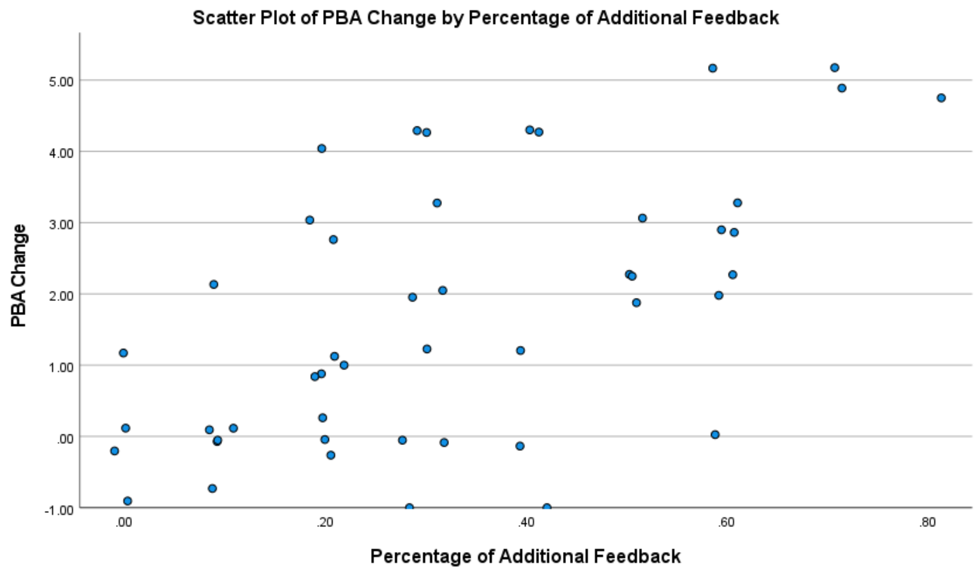

Figure 4. Scatter Plot of PBA Change of Additional Feedback

A slight jitter was used when plotting the points in this scatterplot since PBA change only has a limited number of discrete outcomes. Also, due to the ordinal nature of the change variable, a Spearman correlation was used to quantify magnitude and direction of the potential linear relationship between the two variables. This was found to be $r = .573$, $p < .001$, inferring a significant as well as positive and strong relationship between additional guidance received in feedback and PBA change. In summary, more additional feedback tends to correspond with larger increases in PBA scores from PBA 1 to PBA 6.

4.4 Correlation Between APLSE and Diagnostic Measures

The Adaptive Personalized Learning Score (APLSE) is a predictive report that displays student progress over time. The APLSE score comes from a digital formative assessment tool called MATHia, created by Carnegie Learning. The score is based on the amount of work completed, the time taken to complete it, and student performance (including content mastery, hint requests, and errors). A student who scores between 495 and 564 on the statistically valid school administered diagnostic test is performing at grade level. A student APLSE score of 75 percent or higher means that they are performing at grade level and predicted to score at a proficient level on the end-of-year state mandated AIR assessment.

In order to assess the relationship between APLSE (formative assessment) and diagnostic measures (summative assessment), scatterplots and corresponding correlation measures were created for the following:

- APLSE and Diagnostic 1
- APLSE and Diagnostic 2
- APLSE and change in Diagnostic Score

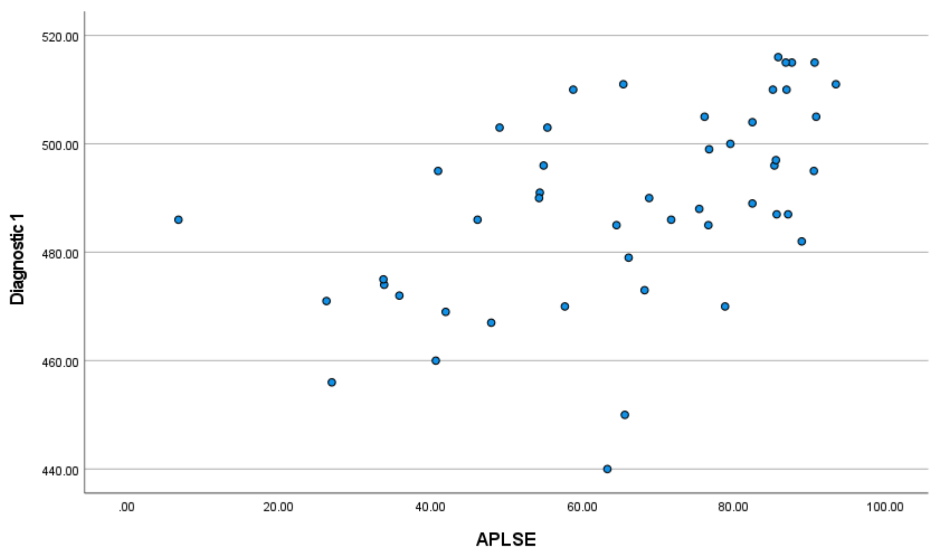

Figure 5. APLSE Score Compared to Diagnostic 1

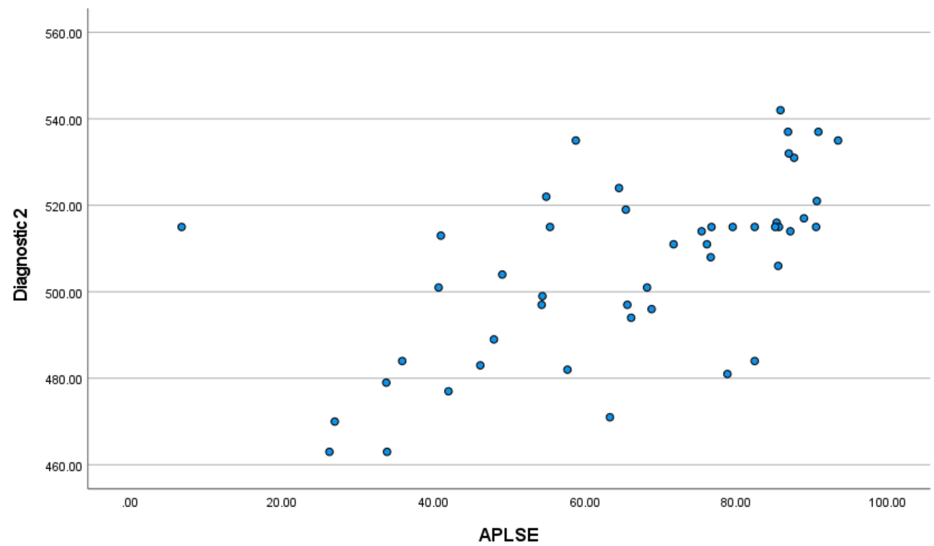

Figure 6. APLSE Compared to Diagnostic 2

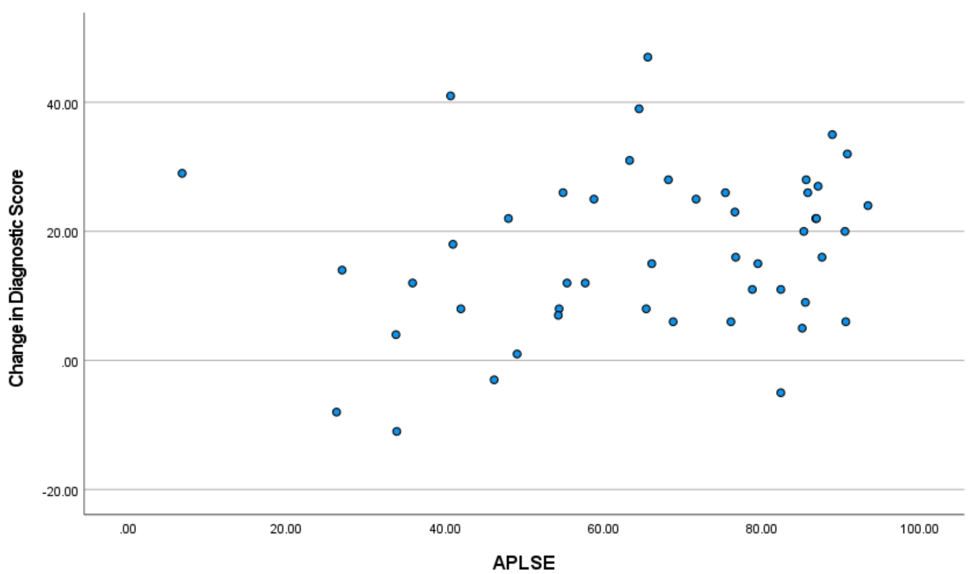

Figure 7. APLSE Compared to Change in Diagnostic Score

Pearson Correlation:

- APLSE and Diagnostic 1:
  - $r = .518$, $p < .001$

- APLSE and Diagnostic 2:
    - $r = .603$, $p < .001$
- APLSE and Change in Diagnostic Score:
    - $r = .226$, $p = .123$

In summary, APLSE is significantly correlated with both Diagnostic 1 and Diagnostic 2 scores. In each case, that relationship is positive and implies larger APLSE scores correspond to higher diagnostic scores in general. However, APLSE was found not to be significantly correlated with the change in score between Diagnostic 1 and Diagnostic 2.

5.0 Discussion

This inquiry attempted to measure the effectiveness of formative assessment on sixth grade students' conceptual understanding of mathematics, specifically in the areas of mathematical reasoning and modeling. AIR mathematical test items released by the State of Ohio revealed that sixth grade students at Springdale Intermediate School were missing questions that involve concepts with modeling and reasoning. Both Common Core Standards and AIR assessments place a much higher emphasis on modeling and reasoning than previous standards and assessments.

During the inquiry, students took a total of six performance-based assessment (PBA) items, four on ratios and two with geometry. Ratios and geometry were chosen because they would give students the opportunity to use both reasoning and modeling skills. Students received teacher feedback on each PBA based on a rubric that incorporated seven points from both sixth-grade mathematical content standards and mathematical practice standards. Research shows one of the most challenging goals for mathematics educators is developing proficiency in students' reasoning and proving skills (Stylianides, 2014).

Students also completed 10 quick writes during instruction on ratios and geometry. The students received teacher feedback the day after they were completed. Quick writes were analyzed and compared to PBAs based on feedback on solutions with additional guidance or solution only.

The final measure students were evaluated on was a digital formative assessment tool called MATHia by Carnegie Learning. Students' MATHia scores were compared to a school administered diagnostic test.

5.1 Reasoning and Modeling on PBAs

The first inquiry question was used to determine to what degree formative assessment on high-level mathematical tasks in sixth grade classrooms increase student scores on PBA items related to ratios and geometry. Students showed some improvement with their scores on the six PBAs.

Most notable was the increase in growth on PBA 1 compared to all other PBAs. The growth from PBA 1 compared to PBA 2 was statistically the most significant, with a p-value of .037. Anything below .05 is considered to be significantly significant, meaning the difference is sizable enough to be significant and not random. The difference gets even larger further out, as noted in Table 7; the p-values get so small they are close to zero. The score dipped a little with the first geometry PBA (#5), with a p-value of .011 compared to all others of .001. As discussed in Chapter Four, the score change may have been impacted by the transition to geometry requiring student to apply both mathematical reason and modeling skills for the first time. This indicates that PBA 1 compared to any other time point showed a significant difference in scores. The distribution of scores that follows was not statistically significant.

The early significant increase in score indicates that teacher feedback impacted student scores. Students continued to receive teacher feedback not only on the PBAs but also on quick writes, which were completed during instruction on ratios and geometry. Teacher feedback on PBAs, although statistically significant, was not as significant as it was on the quick writes. This illustrates that feedback given frequently rather than spread out over longer periods of time is most impactful. Feedback can be motivating for students and help them to take ownership of their learning (Ebby & Petit, 2017). It also provides teachers with the opportunity to reflect on their teaching practices and student progress. The fact that scores remained consistently high after PBA

1 indicate that not only did the students benefit from feedback, the teacher was able to reflect and adjust instruction.

5.2 The Impact of Teacher Directed Formative Assessment

The second inquiry question was used to determine what impact teacher-directed formative assessment would have on students' ability to model mathematical ideas and articulate their mathematical reasoning skills with questions related to ratios and geometry on statically valid performance-based assessment items. Students completed a total of 10 quick writes during instruction on ratios and geometry. The quick writes took place simultaneously with the PBAs and MATHia digital assessment tool.

The frequency of students receiving feedback on solutions with additional guidance ranged from 9 to 31 across the various quick writes, with an average of 19 percent to 65 percent as illustrated in Table 8. The association between PBAs and feedback on solutions with additional guidance was assessed via a scatterplot and corresponding correlation measure as illustrated in Figure 4. The relationship between the two variables was found to be $r=.573$, $p < .001$, inferring a significant as well as positive and strong relationship between feedback on solution with additional guidance and PBA change. In summary, more feedback tends to correspond with larger increases in PBA scores. This also indicates the strong impact teacher-directed feedback has on student achievement.

5.3 The Impact of Digital Formative Assessment

The final inquiry question was used to determine if students completing workspaces on a digital formative assessment tool at a proficient or higher level resulted in students correctly answering related items on school administered diagnostic test items. The students' results from a school-administered diagnostic test taken at both the beginning (September) of the school year and the end (May) was compared to students' APLSE scores on MATHia, which was used three to five days a week throughout the school year (September to May).

Student APLSE scores were significantly correlated with the results from both diagnostic one and two. In each case, that relationship was found to be positive and implies that larger APLSE scores correspond to higher diagnostic scores as illustrated in Figures 5 and 6. However, Figure 7 indicates APLSE scores were found not to be significantly correlated with the change in score between diagnostic one and two. Teachers use both diagnostic scores and APLSE scores to monitor student growth and to target student intervention. It was interesting to note a positive correlation between the two, meaning that teachers will have accurate information to help make important decisions concerning students.

5.4 Limitations of This Study

This study, like all studies, had its limitations. First, the study started in the fall of the 2020-2021 school year during the COVID 19 pandemic. Not only was the district faced with the challenge of opening school for in-person instruction, but teachers were also learning to develop digital synchronous instruction for students who chose to be at-home learners. The district was also in only the second year of implementing a new middle school mathematics curriculum rich in conceptual understanding of mathematics. The first year of implementation was impacted by students being sent home in March due to the pandemic and not returning to in person learning until the next school year.

Another limitation of the study was the unusually small class size of the sixth grade. There were only 63 students. Fifteen of those students chose to be at home digital leaners, leaving 48 students present for in-person instruction. Only in-person learners were included in the study, leaving an unusually small number of data points to analyze.

Finally, there is only one sixth grade math instructor at Springdale Intermediate School. This limited the study to the impact of one teacher's formative assessment response to students.

5.5 Implications for Future Research

Due to the limited size of the study, impact measured by the inquiry questions was relatively small in size. Conducting the same study with a larger sample size of students or additional grade levels may result in an even larger impact of formative assessment on mathematics instruction. It would be beneficial in the future to analyze and compare data from multiple grade levels and different instructors to see the overall impact on the entire building level mathematics program. Those results would be beneficial in making district wide data-based decisions on mathematics instruction.

5.6 Implications for Future Practice

The principal investigator studied a relatively small sample of students for a short period of time, with a limited number of mathematical concepts requiring students to apply reasoning skills. However, the study did find that formative assessment has an impact on conceptual understanding of mathematics. In the future, the investigator plans on repeating the inquiry with the new sixth grade students and expanding it to seventh grade, which will not only include students analyzed in this study but also the students who were not included because they were at-home leaners.

The same PBA assessment items and quick writes could be used with the sixth grades, and two years of data could be compared. The results could also be combined to create a larger sample size for analysis rather than compared for differences. Both data sets would provide interesting and valuable information to help inform instruction, which is one of the essential elements of formative assessment discussed in the review of literature.

Expanding the study to the seventh grade would give the investigator the opportunity to compare student results across two school years with more challenging math concepts. It would also beatifical to use PBAs and quick writes from different content strands other than ratios and geometry in order to investigate the impact of formative assessment across a broader mathematics content area.

In expanding the study further and moving forward to a new school year (one not significantly impacted by the COVID-19 pandemic), it will be essential to meet students where they are currently performing and not where we think they should be performing. Many students will enter the upcoming school year with significant learning loss due to disruptions caused by the pandemic. The use of formative assessment and digital instruction tools will allow teachers to use data and assessment to drive instruction.

Perhaps moving forward, intensive coaching and professional development on both conceptual understanding of mathematics and formative assessment will help bridge the learning loss created by the pandemic. Formative assessment is a classroom practice that, when carried out effectively, can improve student learning (Black & William, 1998). It also provides teachers with the opportunity to reflect on their teaching practices and student progress. According to Boston and Smith (2009), teachers who participated in an ongoing embedded professional initiative focused on selecting and using high-level cognitive tasks supporting conceptual understanding and mathematical reasoning significantly increased (p. 125). It is important to note that most changes, no matter how necessary and worthwhile, fail in the implementation stage (Boston & Smith, 2009). This reinforces the importance of intensive coaching and professional development to help bring about a desired outcome.

The feedback given to students in this study typically occurred when a student answered a question incorrectly or did not understand a particular mathematical concept. Perhaps coaching and professional develop on using formative assessment and feedback to provide positive reinforcement rather than just when a student answers incorrectly will yield more significant gains than illustrated by this study. The use of digital assessment tools such as the ones mentioned in this study should allow teachers to gain time to focus on feedback rather than just grading. The combination of both will provide valuable data to inform and drive instruction.

5.7 Note to the Reader

The entire study took place during the COVID-19 pandemic school year of 2020-2021. The students included in the study were in-person learners attending school five days a week with a normal schedule. However, it is important to note that the students missed an entire quarter of instruction the previous year. Instruction was delivered digitally to students to complete the 2019-2020 school year, but it would be naïve to think that it was as beneficial as being in school with a teacher. The primary investigator, staff, and students experienced a multitude of disruptions during the school year due to the COVID-19 pandemic. Fortunately, neither the primary investigator or the teacher was directly affected by the pandemic; however, some of the students were affected, causing them to miss school and to make up work included in this study upon return.

5.8 Conclusion

Students' demonstration of proficiency on state standardized tests required by the Every Student Succeeds Act and previously No Child Left Behind are not only a challenge for the Springdale Local School System but every district throughout the nation. As a result, school districts across the nation are faced with many challenging decisions concerning instructional practices to impact student learning. The results of this inquiry indicate that formative assessment can positively and significantly impact student achievement, specifically with conceptual understanding of sixth grade mathematics.

The inquiry results were more significant for teacher-created formative assessment than with a digital formative assessment tool, once again illustrating the fact we all know to be true: the impact a teacher has on student achievement. Raising the level of mathematical proficiency in the United States can be seen “as both a matter of national interest and moral imperative” (Loewenberg Ball, 2003). This study proves the simple notion that a teacher providing meaning feedback can help improve the level of mathematical proficiency in a specific grade level at an individual building one student at a time. Formative assessment is a classroom practice that, when carried out effectively, can improve student learning (Black & William, 1998).

Although the digital assessment tool did not have as significant of an impact, it still resulted in a positive correlation with student diagnostic scores. As schools such as Springdale are increasingly focused on improvement of the overall achievement of students, and there is a need to find methods to implement evidence-based instructional practices, useable data on individual students and progress at the classroom level is an asset to decision makers and school leaders. Diagnostic and digital formative assessment tools such as MATHia become even more valuable due to the instantaneous availability of useable data. Such systems enable professionals to engage in data-driven instructional decision making and are effective in addressing the diversity of academic skills evidenced by the students now attending U.S. schools (Ysseldyke & Bolt, 2007). School leaders will increasingly be accountable to use data to make decisions. Appropriate instructional decisions and the allocation of school resources hinge on the degree to which relevant data on student performance are available (Nelson et al., 2016).

In conclusion, every mathematics classroom could benefit from the use of formative assessment practices. This study showed that it is possible to identify an area of academic need through data analysis, such as a lack of conceptual understanding in mathematics, and impact student achievement through the use of formative assessment providing meaningful feedback to students. It also shows the importance of human interaction and the impact a teacher makes on student learning. This is an important take away as schools all over the nation emerge from the COVID-19 pandemic and continue to make difficult decisions with both in person and digital learning.

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