Evaluating the Effects of an Algebraic Frequency-Building Intervention for Students With Disabilities

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Evidence-based instructional strategies in mathematics provide critical support related to academic achievement for students with disabilities. Data from national achievement assessments suggest that students with disabilities' academic growth in mathematics stagnates at the secondary level. Given that the majority of previously researched mathematics interventions focus on foundational and elementary mathematics skills, research on evidence-based practice with higher-level skills, such as algebra, may promote growth. Furthermore, algebra proficiency relates to increased educational and career opportunities. Current research on a core skill within algebra—solving equations—focuses primarily on the initial acquisition of the skill. In the current experiment, I designed a multiple-baseline study involving three middle school participants with specific learning disabilities to evaluate the effect of a frequency-building intervention on their rates of solving one-step equations. Results demonstrate immediate and significant positive effects on students' correct notations and drastic decreases in students' incorrect notations. Students generalized knowledge to untaught problem presentations. In pre- and post-think-alouds, students moved from using trial-and-error methods to using systematic procedures to solve equations. I discuss how the characteristics of the intervention supported students' overall improvement and discuss potential implications for both practice and future research.

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1.0 Introduction

Algebraic proficiency increases educational opportunities for students (Gaertner et al., 2014; Kilpatrick & Iszák, 2008; Morgatto, 2008). Algebra I provides the basic tools for students to attain more complex quantitative skills in rigorous math classes, such as algebra II and calculus (Usiskin, 1995). Students enrolled in algebra and subsequent classes are increasingly more likely to graduate from high school, enter college, and have higher grade point averages in college (Gaertner et al., 2014; Kim et al., 2015). Additionally, as science, technology, engineering, and mathematics, or STEM, jobs in the United States continue to grow rapidly, obtaining mastery of complex math skills can provide opportunities to obtain higher-wage, readily available jobs (Fayer et al., 2017).

Policymakers and education leaders in the United States, via initiatives such as the Common Core State Standards, centralize algebraic readiness and proficiency in their progression of mathematical skills (National Governor's Association, 2010). National organizations, such as the National Mathematics Advisory Panel (NMAP, 2008), direct recommendations toward the end goal of algebraic proficiency. Algebraic proficiency encompasses a variety of discrete and complex skills. Topics in algebra span the concepts of variables, polynomials, and functions (Bednarz & Janvier, 1996). As defined by the Common Core State Standards, algebraic proficiency also includes mastering problems involving expressions (e.g., $6x + 10x + 3$), equations (e.g., $2x + 3 = 11$), and inequalities (e.g., $6x > -12$) (National Governor' Association, 2010). States may also necessitate successful completion of algebra as a graduation requirement, directly linking future educational and career opportunities to algebraic competency (Kilpatrick & Iszák, 2008;

Morgatto, 2008). Given algebra's association with high school graduation and improved outcomes, researchers and scholars have often referred to algebra as a "gatekeeper" class (Lee & Mao, 2020).

Algebra's position as a gatekeeper may have negative consequences for students. Secondary students with disabilities have historically scored lower than their nondisabled peers on standardized mathematics assessments (Cortiella, 2011). In 2017, only 9% of eighth graders with disabilities attained a level of proficiency on grade-level materials (U.S. Department of Education, 2017). Data from the 2015 Trends in International Math and Science Study showed that lower-performing eighth graders demonstrated no measurable improvements by the time they reached their senior year (Provasnik et al., 2016). Furthermore, various policy initiatives, such as the NMAP's 2008 recommendations, suggest algebra should be offered in eighth grade (Rickles, 2013). However, students with disabilities have reduced odds of entering algebra in eighth grade (Faulkner et al., 2013). Thus, high-quality algebraic instruction for students with disabilities remains critical to increasing educational and professional opportunities.

To address the disparate mathematics performance of students with disabilities, researchers have investigated why students encounter math difficulties. Research detailing the fluency profiles of high school students with math difficulties revealed that students demonstrated computational fluency only on second grade– and third grade–level computations (Calhoon et al., 2007). For students with math difficulties, errors are also frequently related to gaps in students' procedural knowledge (Calhoon et al., 2007; Cawley & Miller, 1989). Algebraic proficiency necessitates both computational and procedural knowledge fluency, necessitating practitioners assess and consider both types of knowledge when developing instructional programming for students with disabilities.

Researchers have developed a variety of algebraic interventions specifically for students with disabilities. A recent review reported promising algebraic instructional practices for students with specific learning disabilities (Lee & Mao, 2020). Effective practices included the use of multiple representations and a sequence/range of examples, explicit instruction, heuristics, and student verbalizations (Lee & Mao, 2020). Reviews evaluating instruction for students with moderate to severe developmental disabilities similarly detail systematic and explicit instruction as efficacious practices in mathematics (Spooner et al., 2019). Additional research-supported practices include technology-aided instruction, graphic organizers, and the use of manipulatives (Spooner et al., 2019). However, researchers included secondary students less often than elementary students, and instructional practices did not frequently focus on foundational algebraic skills, such as solving equations (Spooner et al., 2019). Additionally, one review found that no interventions had sufficient, high-quality research to meet the criteria of being evidence based (Bone et al., 2021). Practices or strategies potentially efficacious include use of the concrete-representational-abstract (CRA) instructional sequence, manipulatives, schema-based instruction, peer-assisted learning strategies, and enhanced anchor instruction (Bone et al., 2021).

I reviewed empirical research that specifically targeted solving equations to focus precisely on the different ways a single skill has been taught and developed. A narrow focus on solving equations facilitated a deeper focus on the breadth of and similarities between interventions focused on solving equations. The results identified 11 studies. All studies contained multiple intervention components and targeted the initial acquisition of equation-solving skills. Consistent with previous, broader reviews, researchers used manipulatives, visual aids, explicit instruction, and systematic prompting (Bone et al., 2021; Lee & Mao, 2020; Spooner et al., 2019). Interventions targeted the breadth of equation types, focusing on one-step, two-step, and multiple-step equations. Researchers relied on task analysis to inform and guide the intervention design. Notably, researchers also frequently employed the task analysis as a basis for measurement of intervention efficacy, using the number of steps correctly completed. Other common dependent measures included the percent of number of correct problems completed and measures related to independence.

Students demonstrated higher accuracy in solving equations as a result of the interventions reviewed. However, interventions had an exclusive focus on accuracy, either via percent of correct problems or total number of correct problems. Researchers did not evaluate the speed at which students solved equations, and only two studies provided the exact time students took to complete the dependent variable assessment. Additionally, social validity data revealed student and teacher concerns about the feasibility of manipulative use. Given the many steps in the task analyses related to the use of manipulatives (e.g., “Move a red marker”), removing manipulatives past the acquisition stage will reduce steps in the task analysis and subsequently may increase students’ speed of responses.

Common Core State Standards prioritize the development of fluency, or speed and accuracy, across a variety of mathematical skills, including solving equations. Though Common Core conceptions of fluency focus narrowly on mathematics, fluent performance remains critical across academic domains. Behavioral fluency broadly describes the combination of both speed and accuracy in performing a specific behavior or behaviors (Binder, 1996; Stocker et al., 2019). Ensuring students achieve fluent responding can prevent cumulative dysfluency, a phenomenon that occurs when students struggle with complex skills due to a lack of fluent responding on prerequisite element skills (Binder, 1996). Cumulative dysfluency may explain the aforementioned plateau effect occurring when students reach eighth grade, especially given that descriptive research details students with mathematical difficulties having deficits related to their computational fluency (Binder, 1996; Calhoon et al., 2007; Provasnik et al., 2016).

In addition to preventing difficulties with acquiring complex skills, fluent responding is associated with various critical learning outcomes (Binder, 1996; Kubina & Yurich, 2012; McTiernan et al., 2016; Stocker et al., 2019). Critical learning outcomes include retention, endurance, application to novel or more complex activities, maintenance over time, and stability. For example, after kindergarteners improved their behavioral fluency related to element skills in spelling (e.g., seeing a letter and saying the associated sound, segmenting), they demonstrated application to an untaught skill: spelling (Kostewicz et al., 2020).

Interventions targeting behavioral fluency often employ systematic practice, or frequency building, as a primary method of improving students' rate of responding (Binder, 1996; Gist & Bulla, 2020; Kubina & Yurich, 2012). Frequency building refers to timed practice sessions in which a student repeatedly practices a specific behavior, working toward a performance standard indicative of fluent responding (Gist & Bulla, 2020; Kubina & Yurich, 2012). To support accurate responding, an interventionist may deliver performance feedback (Gist & Bulla, 2020; Kubina & Yurich, 2012). A systematic review of frequency-building interventions identified only four studies in the domain of mathematics; all four studies focused on simple computations not exceeding multiplication (Gist & Bulla, 2020). An additional review focusing broadly on behavioral fluency and mathematics also found no studies targeting algebra (Stocker et al., 2019). More recently, one study examined the effects of a self-managed intervention on students' behavioral fluency related to pre-algebraic tasks (Stocker & Kubina, 2021). Both reviews, as well as the most recent study, found improved student performance as a result of interventions targeting behavioral fluency.

I synthesize prior research on equation-solving interventions and prior research on frequency-building interventions. Using both literature bases, I will develop and evaluate an intervention specifically targeting behavioral fluency as it relates to solving equations. The evaluation explicitly supports students with disabilities, as little research exists to support the development of behavioral fluency for this population. In evaluating the results of the study, I ask and answer the following research question: *What are the effects of a frequency-building intervention on students with disabilities' rate of correct and incorrect notations while solving one-step equations?*

2.0 Literature Review

Mathematics interventions may be most efficacious when targeting specific, discrete skills based on student strengths and needs (Burns et al., 2010). As such, focusing on how algebraic interventions target specific algebraic knowledge can facilitate greater coherence between student need and intervention focus. For example, if students lack basic knowledge of a mathematical concept, interventions may target acquisition and initial concept development. Students struggling with fluent responding within a certain procedure, especially if such a skill lays the foundation for more complex skills, may require interventions specifically targeting fluent performance related to computational or procedural knowledge (Burns et al., 2010).

The present review focuses narrowly on interventions targeting a single skill: solving equations. By limiting the scope of the review, researchers can better analyze how features of the intervention relate to the measurement and efficacy of the intervention within the context of solving equations. A focused analysis can subsequently inform more nuanced development of effective frequency-building interventions targeting a critical skill within the broader context of a "gatekeeper" course, algebra. Furthermore, focusing on algebraic skills targets an area of research necessary for further investigation, given the dearth of established evidence-based practices and the consistent gap in algebraic performance for students with disabilities (Bone et al., 2021; Provasnik et al., 2016). Specifically, the present review seeks to understand features and purposes of intervention research focused on students' ability to solve equations via interrogation of the following sub-questions:

1. With whom and where have researchers conducted mathematic interventions targeting solving equations?

2. What are the intervention components included in interventions targeting solving equations?
3. How do researchers measure students' abilities to solve equations in intervention studies?
4. What is the efficacy of interventions targeting students' ability to solve equations?

2.1 Method

A three-step approach established a body of research for review. First, I identified three relevant databases based on their relevancy to the subject matter: PsycINFO, PsycARTICLES, and ERIC. I conducted searches using a Boolean search string (math* OR algebra* OR equat* OR inequalit*) AND (interven* OR method* OR teach* OR instruct* OR strat*) AND (student*) AND (disabilit*). Searches in PsycINFO and PsycARTICLES constrained results to search terms being located anywhere except the full text using the *noft* prefix. Searches in PsycINFO, PsycARTICLES, and ERIC also constrained results to peer-reviewed journal articles in English that were published between January 2001 and January 2021; the final search date was March 1, 2021. I constricted the date range given prior recommendations in systematic reviews (King et al., 2020). Searches in PsycARTICLES and PsycINFO returned a combined 1,506 articles for review. The search in ERIC returned 960 articles. After a review of titles and abstracts, a total of 109 full-text articles were obtained and subsequently assessed for eligibility.

In addition to electronic databases, I obtained relevant articles from hand searches of the journals *Remedial and Special Education* and *Exceptional Children*. Hand searches returned an additional 7 potential articles for inclusion. Finally, ancestral searches of included articles and relevant literature reviews (Bouck et al., 2019; Impecoven-Lind & Foegen, 2010; Jitendra et al., 2017; Lee & Mao, 2020; Spooner et al., 2019) provided additional articles. This method led to the retrieval of 3 additional potential articles not previously obtained. In total, 199 full-text articles met initial relevance criteria to assess for eligibility. Due to limitations in database features, duplicates were unable to be removed, and thus duplicate abstracts were read and evaluated for inclusion individually.

2.1.1 Inclusion Criteria

Initial searches and cursory reviews of relevance led to a more comprehensive review of abstracts and full texts for the 109 preliminary articles. The following inclusion criteria guided whether articles would be included in the final review:

1. The article was published in English in a peer-reviewed journal prior to January 31, 2021. Dissertations did not meet criteria.
2. The participants must be in grades K–12 (aged 3–21 years) and must have a documented disability qualifying them for an individual education plan (IEP) per federal law. Articles including students with no mentioned disability or described as having a math difficulty did not meet study criteria (e.g., Cuenca-Carlino et al., 2016).
3. The article included an empirical study, including experimental, quasi-experimental, and single-case designs. Case studies were excluded (e.g., Hord et al., 2018), as were studies that did not disaggregate data for students with disabilities (e.g., Rittle-Johnson & Star, 2007).
4. The dependent variable needed to contain a numerical output directly related to finding the solutions of equations. Dependent variables could include the number of steps completed in a task analysis or a measure of the solution to the equation itself. Studies

that combined measurement of multiple types of problems without disaggregating data related to equations were excluded.

5. Interventions in the study exclusively targeted how to solve algebraic equations. Interventions could target word problems with equations embedded or equations in isolation. Research was excluded if it focused exclusively on prerequisite skills, such as adding integers (Bouck et al., 2019).

2.1.2 Search Results

After reviewing each abstract, 11 studies met inclusion criteria. The majority (n = 10) came from electronic searches (Baker et al., 2015; Bouck et al., 2019; Chapman et al., 2019; Creech-Galloway et. al., 2013; Jimenez et al., 2008; Long et al., 2020; Satsangi et al., 2016; Satsangi, Hammer, & Evmenova, 2018; Satsangi, Hammer, & Hogan, 2018; Scheuermann et al., 2009), and 1 came from hand searches of relevant journals (Root & Browder, 2019).

2.1.3 Coding Procedure

To answer the first research question, I coded relevant demographic data of participants, such as total number included, age/grade, gender, and disability. Coding also recorded the setting of the intervention, including both school characteristics and the location and setting of the specific intervention. Codes relevant to the second research question included noting specific features of each intervention, including who provided the intervention, the dosage of the intervention, and the instructional components of each intervention. As inclusion criteria restricted content to focusing on finding solutions to equations, coding also noted the types of equations (e.g., one-step equations, two-step equations, multiple-step equations, equations within word problems, equations within geometric theorems). For example, problems that focused on students' solving the Pythagorean equation were noted.

Coding related to the third research question, measurement, focused on the dimensions of the primary dependent variable. Dimensions included the specific behavior measured, the measurement type (e.g., frequency, rate, percent accuracy), and any secondary or tertiary dependent variables included. Additionally, I recorded whether or not articles included measurement of social validity, fidelity of implementation, interobserver agreement, generalization, and/or maintenance. Finally, to measure effectiveness, I coded author description of results. For single-case design articles, I also conducted independent visual inspection to determine level, trend, and variability of performance.

2.2 Results

Tables 1–3 display descriptive characteristics of the 11 reviewed studies. Table 1 provides participant and school characteristics. Table 2 displays elements of the independent variable, including intervention components, setting of the intervention, delivery method, length per intervention session, and number of sessions. Table 3 details measurement and design characteristics, including the primary and additional dependent variables, the research design, and whether or not the design included measurement of maintenance, generalization, and social validity.

2.2.1 Research Question One: Participants and School Settings

In total, 46 students participated in the reviewed studies. Studies had an explicit focus on working with students with either learning disabilities (n = 23) or intellectual or developmental disabilities (n = 23). Participants also had co-occurring disabilities of autism spectrum disorder (n = 2), epilepsy (n = 1), attention-deficient/hyperactivity disorder (n = 2), and obsessive-compulsive disorder (n = 1). Researchers included more male (n = 32) than female (n = 14) students. Students ranged in age from 11 to 19 years old, and school grades of participants ranged from sixth grade to twelfth grade. Eight studies (72.7%) provided information about the race and ethnicity of students (Baker et al., 2015; Bouck et al., 2019; Long et al., 2020; Root & Browder, 2019; Satsangi et al., 2016; Satsangi, Hammer, & Evmenova, 2018; Satsangi, Hammer, & Hogan, 2018; Scheuermann et al., 2009). Within these studies, the majority of students were white (69.4%), with fewer Black (16.7%) and Hispanic (13.9%) students.

Researchers conducted interventions in both middle school (n = 5, 45.4%) and high school (n = 6, 54.5%) settings. Seven studies (63.6%) included additional geographic descriptors; researchers characterized schools as rural (n = 3), suburban (n = 2), and urban (n = 2). Researchers most commonly conducted interventions in traditional public schools (81.8%), with other interventions conducted in public charter schools (9.1%) or unspecified types of schools (9.1%).

2.2.2 Research Question Two: Independent Variable Characteristics

All interventions represented in the sample contained more than one discrete component. Researchers most frequently incorporated manipulatives into their multicomponent interventions (72.7%). Other common components included a system of systematic prompting (54.5%), task-analytic instruction (54.5%), and explicit instruction (54.5%). Other interventions incorporated visual aids (27.2%), real-life scenarios (18.2%), errorless learning procedures (9.1%), graphic organizers (9.1%), and technology (9.1%). Table 2 presents components discretely and uses labels as they are described by researchers within the study. However, some labels contain overlap within features. For example, visual aids and graphic organizers may have been presented via technology. Table 2 also provides the setting and format of the of the intervention, who delivered the intervention, and the dosage.

2.2.2.1 Manipulatives

Manipulatives included both concrete, physical objects being moved by students (Chapman et al., 2019; Jimenez et al., 2008; Long et al., 2020; Satsangi et al., 2016; Scheuermann et al., 2009) and virtual manipulatives manipulated via technology, such as iPads (Bouck et al., 2019; Long et al., 2020; Satsangi et al., 2016; Satsangi, Hammer, & Evmenova, 2018; Satsangi, Hammer, & Hogan, 2018). Two studies using concrete manipulatives used physical representations as part of an explicit sequence of instruction intending to move from concrete to representational to abstract forms of mathematical thinking (Jimenez et al., 2008; Scheuermann et al., 2009). Concrete manipulatives represented the first concrete step of instruction, which students completed before representing equations solely via abstract representation or numbers. Three studies explicitly studied the use of virtual manipulatives as a substitute for concrete manipulatives within the CRA sequence of instruction (Bouck et al., 2019; Satsangi, Hammer, & Evmenova, 2018; Satsangi, Hammer, & Hogan, 2018). Another study used virtual manipulatives as part of a comprehensive intervention package, which also included visual aids, task-analytic instruction, real-life scenarios, and systematic prompting (Chapman et al., 2019). In two studies, researchers explicitly compared the effectiveness of concrete and virtual manipulatives (Long et al., 2020; Satsangi et al., 2016).

2.2.2.2 Systematic Prompting

Interventions often used systematic prompting as a key component of the interventions. Systematic prompting involves consistent and planned interventionist responses to student responses during the intervention. Of the six studies that used prompting, researchers most frequently (n = 4, 66.6%) used a system of least-to-most prompts (SLP) during the intervention phase (Baker et al., 2015; Chapman et al., 2019; Long et al., 2020; Root & Browder, 2019). In SLP procedures, researchers created a hierarchy of least intrusive (e.g., gestural or indirect verbal) to more intrusive (e.g., physical modeling) prompts. In addition to SLP procedures during the intervention phase, some researchers (n = 2, 33.3%) used immediate, simultaneous prompts during the training phase (Baker et al., 2015; Root & Browder, 2019). Creech-Galloway et al. (2013) included a simultaneous prompting procedure, in which a controlling prompt was delivered immediately during each step in the task analytic.

Table 1 Participant and School Characteristics Across Reviewed Studies

Study	Number of Participants	Disabilities Included	Age	Gender	Race/Ethnicity	School Setting
Baker et al., 2015	3	ID	15, 15, 12	2 M, 1 F	3 W	MS, R, TP
Bouck et al., 2019	4	ID, ADHD, LD, OCD	14, 13, 15, 13	2 M, 2 F	4 W	MS, TP
Chapman et al., 2019	3	ID, ASD, E	14–15	3 M		HS, R, TP
Creech-Galloway et al., 2013	4	ID, SL	15–17	3 M, 1 F		HS
Jimenez et al., 2008	3	DD	15–17	2 M, 1 F		HS, U, TP
Long et al., 2020	3	ID, DD	15, 14, 14	2 M, 1 F	3 W	MS, R, TP
Root & Browder, 2019	3	ASD, ID	14, 12, 14	3 F	3 W	MS, U, TP
Satsangi et al., 2016	3	LD	17, 18, 19	3 M	3 B	HS, PC
Satsangi, Hammer, & Evmenova, 2018	3	LD	14, 16, 14	2 M, 1 F	2 H/L, 1 B	HS, S, TP
Satsangi, Hammer, & Hogan, 2018	3	LD	15, 15, 15	2 M, 1 F	3 H/L	HS, S, TP
Scheuermann et al., 2009	14	LD	11–14	10 M, 4 F	12 W, 2 B	MS, PC
REPORTED	*11/11 100%*	*11/11 100%*	*11/11 100%*	*11/11 100%*	*8/11 73%*	*11/11 100%*

Note. Information missing from manuscripts is shown with a blank cell. Participant demographic information for multiple students is provided in the order presented within manuscripts if provided for individual students. Ranges are provided if authors did not provide individual participant characteristics.

ADHD = attention-deficit/hyperactivity disorder; ASD = autism spectrum disorder; B = Black; DD = developmental disability; E = epilepsy; F = female; H/L = Hispanic/Latinx; HS = high school; ID = intellectual disability; LD = learning disability; M = male; MS = middle school; OCD = obsessive compulsive disorder; PC = public charter; R = rural; S = suburban; SL = speech/language disorder; TP = traditional public; U = urban; W = white

Table 2 Features of Independent Variables in Included Studies

Study	Intervention Components	Setting	Delivery	Length Per Session	Number of Sessions
Baker et al., 2015	GO, SP, TA	SC, I	TL	15 minutes	18–21
Bouck et al., 2019	EI, VM	OC, I	RL	61 minutes*	15–21
Chapman et al., 2019	CM, RLS, TA, VA, SP	SC, I	RL		39
Creech-Galloway et al., 2013	SP, TECH, TA	SC, I	TL	30 minutes	22–23
Jimenez et al., 2008	CM, TA, SP	SC, I	TL		35–48
Long et al., 2020	CM, SP, TA, VM	OC, I	RL	10–15 minutes	20–21
Root & Browder, 2019**	EI, SP, TA, VA, RLS	OC, I	RL	10–15 minutes	14–16
Satsangi et al., 2016	EI, CM, VM	OC, I	RL	15–20 minutes	30
Satsangi, Hammer, & Evmenova, 2018	EI, VM	OC, I	RL	45–60 minutes	16–18
Satsangi, Hammer, & Hogan, 2018	EI, VM	OC, I	RL	20–30 minutes	16–17
Scheuermann et al., 2009	EIR, CM, VA	SC, WG	TL	55 minutes	10
REPORTED	*11/11 100%*	*11/11 100%*	*11/11 100%*	*9/11 82%*	*11/11 100%*

Note. Information missing from manuscripts is shown with a blank cell.
CM = concrete manipulatives; EI = explicit instruction; EIR = explicit inquiry routine; GO = graphic organizer; I = individual (1:1); OC = outside classroom; RL = researcher-led; RLS = real-life scenarios; SC = special education classroom; SP = systematic prompting; TA = task-analytic sequence of instruction; TECH = technology support (iPad, computer, etc.); TL = teacher-led; VA = visual aids; VM = virtual manipulatives; WG = whole group (larger than five)
* Denotes length of instructional period; session length was not specified.
** Study described combination of components as "modified schema-based instruction," or MSBI.

Table 3 Measurement and Design Characteristics of Included Studies

Study	Dependent Variables	Generalization	Maintenance	Social Validity	Research Design
Baker et al., 2015	NSC: 1-step equations	●			MBL-P
Bouck et al., 2019	NPC: 1- and 2- step equations		●	●	MBL-B
Chapman et al., 2019	NSC: 1-step equations	●	●	●	MBL-P
Creech-Galloway et al., 2013	NSC: Pythagorean formula	●	●	●	MBL-P
Jimenez et al., 2008	NSC: 1-step equations		●		MBL-P
Long et al., 2020	PCP: 2-step equations, IND, T	●		●	AAT
Root & Browder, 2019	NSC: 1-step equations, NPC	●		●	MBL-P
Satsangi et al., 2016	PCP: multistep equations*, P, T			●	AT
Satsangi, Hammer, & Evmenova, 2018	PCP: multistep equations, IND, T	●	●	●	MBL-P
Satsangi, Hammer, & Hogan, 2018	PCP: multistep equations, IND, T		●	●	MBL-P
Scheuermann et al., 2009	WPT: multistep equations, CMT, SA	●	●		MBL-P

Note. Information missing or not provided from manuscripts is shown with a blank cell. Primary dependent variable is listed first; any secondary dependent variables are listed after.

AAT = adapted alternating treatments; AT = alternating treatments; CMT = concrete-manipulation test (demonstration of problem-solving with concrete manipulatives with oral provision of correct answer); IND = independence; MBL-B = multiple-baseline across behaviors; MBL-P = multiple-baseline across participants; NPC = number of problems completed (correctly); NSC = number of steps completed in task analysis (correctly, independently); P = number of prompts; PCP = percent of correct problems solved; SA = standardized assessment; T = task completion time; WPT = word problem test (measured graphical representation, mathematical representation, graphical processing, mathematical processing, correct solution)

* Multistep equations include one-step, two-step, and three-step or greater equations.

2.2.2.3 Task-Analytic Instruction

Six studies explicitly included a task analysis to guide the components of the intervention (Baker et al., 2015; Chapman et al., 2019; Creech-Galloway et al., 2013, Jimenez et al., 2008; Long et al., 2020; Root & Browder, 2019). Two task analyses included explicitly identical steps (Chapman et al., 2019; Jimenez et al., 2008). Otherwise, task analyses varied in content and length. Variance often related to intervention components. When working with manipulatives, task analyses included the movement of markers (Chapman et al., 2019; Jimenez et al., 2008). However, another study using manipulatives and task-analytic instruction did not incorporate the movement of manipulatives as part of the task analysis (Long et al., 2020). Other task analyses included steps potentially unnecessary for all students, such as reading the equation aloud (Baker et al., 2015). While the majority of task analyses (five of six) contained 10 or fewer steps, one study (Creech-Galloway et al., 2013) contained 32 steps. The additional number of steps related to the use of a calculator; researchers labeled each button pressed on the calculator as a discrete step in the task analysis.

2.2.2.4 Explicit Instruction

Researchers described using elements of explicit instruction in six studies (Bouck et al., 2019; Root & Browder, 2019; Satsangi et al., 2016; Satsangi, Hammer, & Evmenova, 2018; Satsangi, Hammer, & Hogan, 2018; Scheuermann et al., 2009). Explicit instruction occurred either during a training lesson prior to intervention (Root & Browder, 2019; Satsangi et al., 2016; Satsangi, Hammer, & Evmenova, 2018; Satsangi, Hammer, & Hogan, 2018) or throughout the duration of the intervention (Bouck et al., 2019; Scheuermann et al., 2009). Descriptions of explicit instruction varied. Researchers described explicit instruction as using the model-lead-test approach (Root & Browder, 2019), a guided process using handouts, whiteboards, and virtual or concrete manipulatives (Satsangi et al., 2016; Satsangi, Hammer, & Evmenova, 2018; Satsangi, Hammer, & Hogan, 2018), or simply engaging in modeling and guided practice with feedback (Bouck et al., 2019). Scheuermann et al. (2009) named the form of explicit instruction used the "explicit inquiry routine," which incorporated a series of questions beginning with "Tell me how," progressing to "Tell your neighbor how," and ending with "Tell yourself how." Root and Browder (2019) labeled their combination of explicit instruction and virtual manipulatives "modified schema-based instruction."

2.2.2.5 Setting and Delivery

Researchers most frequently conducted and studied interventions delivered outside of the classroom (n = 6). Locations outside of the classroom included office spaces, testing rooms, and tables in the hallway. The remainder of studies (n = 5) investigated an intervention delivered within the classroom setting. Only one study (Scheuermann et al., 2009) detailed an intervention delivered in a whole-group format; the remainder of studies (n = 10) detailed interventions delivered in a 1:1 format. Researchers (n = 7) delivered the intervention more frequently than classroom teachers (n = 4).

2.2.2.6 Dosage

Nine studies reported the length of individual intervention sessions (Baker et al., 2015; Bouck et al., 2019; Creech-Galloway et al., 2013; Long et al., 2020; Root & Browder, 2019; Satsangi et al., 2016; Satsangi, Hammer, & Evmenova, 2018; Satsangi, Hammer, & Hogan, 2018; Scheuermann et al., 2019). Sessions ranged from 10 minutes to 61 minutes in length, with the most frequent lengths including 15 minutes (n = 4) and over 30 minutes (Bouck et al., 2019; Satsangi et al., 2018a; Scheuermann et al., 2009). The total number of sessions, as reported on graphs within the studies, varied greatly. Researchers reported interventions spanning a minimum of 10 sessions and a maximum of 48 sessions. Given data were derived from graphs; ungraphed sessions, such as training sessions, may be excluded from the range.

2.2.3 Research Question 3: Dependent Variable Characteristics

To be included in the present review, dependent variables had to directly relate to a numerical output related to solving equations. Equations ranged from one-step equations to multistep equations. Students solved problems either in isolation (n = 7) or within word problems or real-life scenarios (n = 4). Numerical output related to the primary dependent variable differed across studies. Most often, researchers measured effectiveness by measuring the number of steps completed independently and correctly in a task analysis related to solving equations (Baker et al., 2015; Chapman et al., 2019; Creech-Galloway et al., 2013; Jimenez et al., 2008; Root & Browder, 2019). As mentioned previously, task analyses varied in both content and length, and thus had ceilings ranging anywhere from 9 to 32 steps. Task analyses with fewer steps often solely focused on the written steps to solve equations, while longer task analyses often incorporated steps for using physical or virtual manipulatives. Task analyses generally did contain observable, measurable behaviors, although some verbs, such as the word "represent," could be vague out of the context of the procedures. For a complete list of the task analyses included in research, see Appendix A.

Researchers also frequently used the percent of correct problems solved (Long et al., 2020; Satsangi et al., 2016; Satsangi, Hammer, & Evmenova, 2018; Satsangi, Hammer, & Hogan, 2018). Researchers provided a fixed number of problems during the assessment portion, ranging from 5 to 10 problems per probe. Other primary dependent variables included the raw number of problems completed correctly (as opposed to percent of problems completed) (Bouck et al., 2019) and a researcher-created word problem test measuring students' ability to accurately represent, process, and solve problems both graphically and mathematically (Scheuermann et al., 2009).

Studies also included secondary dependent variables. Secondary variables often focused on ensuring students could independently solve problems. Such variables included the number of prompts provided (Satsangi et al., 2016) and the number of steps or problems completed independently (Long et al., 2020; Satsangi, Hammer, & Evmenova, 2018; Satsangi, Hammer, & Hogan, 2018). Additionally, four studies measured time by collecting the duration of the assessment (Long et al., 2020; Satsangi et al., 2016; Satsangi, Hammer, & Evmenova, 2018; Satsangi, Hammer, & Hogan, 2018). One study used both a standardized assessment and a researcher-created concrete-manipulation test, an assessment of whether the student could orally describe and demonstrate manipulation (Scheuermann et al., 2009).

Table 3 displays the studies that collected generalization, maintenance, and social validity data. To qualify as having collected maintenance data, studies must have explicitly referenced assessing student performance after the end of the intervention. Researchers typically conceptualized generalization in two differing categories. Some researchers measured generalization via student performance with removed scaffolds (Baker et al., 2015; Long et al., 2020; Root & Browder, 2019; Satsangi et al., 2018a); other researchers conceptualized generalization as student performance on untaught or more difficult problems (Chapman et al., 2019; Creech-Galloway et al., 2013; Scheuermann et al., 2009). Social validity data incorporated a mix of Likert scales, open-ended questions and interviews, and closed-ended questions and interviews.

All the included studies used single-case methodology to assess change in the dependent variable over time. The majority (n = 8, 72.7%) assessed changes via a multiple-baseline across participants (MBL-P) design (Baker et al., 2015; Chapman et al., 2019; Creech-Galloway et al., 2013; Jimenez et al., 2008; Root & Browder, 2019; Satsangi, Hammer, & Evmenova, 2018; Satsangi, Hammer, & Hogan, 2018; Scheuermann et al., 2009). Bouck et al. (2019) used a multiple-baseline across behaviors (MBL-B) design, differentiating between one-step division, two-step addition, and two-step subtraction problems as separate and discrete behaviors. The two studies comparing virtual to concrete manipulatives (Long et al., 2020; Satsangi et al., 2016) employed an alternating treatments design to assess the differential effects of each manipulative.

2.2.4 Research Question 4: Efficacy of Interventions

Given that the included studies used a single-case design to demonstrate effects of the independent variable, I used visual analysis of graphical displays of data to assess within- and between-phase changes in the dependent variable (Kratochwill et al., 2013). In addition, the researcher assessed the immediacy of the effect of the intervention on student behavior, the amount of overlap between data points, and consistency of the patterns of student responding (Kratchowill et al., 2013). All studies included in the current review broadly demonstrated interventions that led to students improving their ability to accurately solve equations, as evidenced by increased levels of accurate student responding from baseline to intervention.

2.2.4.1 Baseline

All studies incorporated a baseline phase in their graphs. Students demonstrated low levels of accurate responding during baseline. Many students (n = 19, 41.3%) did not solve one equation correctly, independently, or accurately. Additionally, approximately one-third of students solved less than 20% of problems accurately. Baseline performance generally remained stable across the entirety of the phase.

2.2.4.2 Training

One study (Long et al., 2020) displayed the training phase within their data display. During training, all students demonstrated immediate increases from lower levels of accuracy to 100% accuracy on the first data point, which represented the use of virtual manipulatives. Student performance related to virtual manipulatives remained stable at 100% accuracy. Data points representing student performance during training using concrete manipulatives were relatively lower and more variable.

2.2.4.3 Intervention

In four studies investigating manipulatives (Bouck et al., 2019; Long et al., 2020; Satsangi et al., 2016; Satsangi et al., 2018a), data displays of the intervention phase indicate that student performance hit the ceiling of performance immediately or within one session. Students also mostly maintained stable, responding at the ceiling within the four studies. Little to no overlap occurred between the baseline and intervention phases. Two of these studies compared performance between concrete and virtual manipulatives using alternating treatment designs (Long et al., 2020; Satsangi et al., 2016). In these studies, students did demonstrate some variable performance based on whether they used virtual or concrete manipulatives. Data from students in Satsangi et al.'s (2016) study displayed slightly more variable data for virtual manipulatives across students, while Long et al.'s (2020) data demonstrated some variability for both concrete and virtual manipulatives.

Student performance changed more gradually across a different four studies (Chapman et al., 2019; Jimenez et al., 2008; Root & Browder, 2019; Scheuermann et al., 2009). Instead of an immediate change in performance, student performance started at lower levels and demonstrated an upward trend across the intervention phase. Little to no overlap between baseline and intervention existed. By the end of the intervention, students within the four studies all demonstrated levels of responding at the ceiling for performance. For some students, the trend line had little variability (Root & Browder, 2019; Scheuermann et al., 2009). Other students' trends demonstrated greater initial variability, which sometimes persisted throughout the intervention phase (Chapman et al., 2019; Jimenez et al., 2008). The interventions used across the four studies varied. Chapman et al. (2019) explicitly replicated much of the Jimenez et al. (2008) intervention, which used task analysis paired with concrete manipulatives and systematic prompting. Chapman et al. (2019) added real-life scenarios and visual aids. Root and Browder (2019) investigated an intervention with five different components: explicit instruction, systematic prompting, task-analytic instruction, visual aids, and real-life scenarios. Scheuermann et al. (2009) used an explicit inquiry routine with concrete manipulatives and visual aids.

In the remaining three studies, performance across students within each study was more variable. Students in Baker et al.'s (2015) study, which used graphic organizers, task analysis, and systematic prompting, largely demonstrated steady, gradual performance. However, despite no overlapping data, two students never consistently attained the ceiling of performance, and one student demonstrated variable performance at lower levels, comparatively. Two students in Satsangi et al.'s (2018b) intervention using virtual manipulatives demonstrated immediate increases in level but a more gradual and variable performance than another student, who immediately and consistently performed at the ceiling for responding. In Creech-Galloway et al.'s

(2013) study, which used iPad technology paired with task-analytic instruction and systematic prompting, three of four students demonstrated gradual and less variable increases in responding. One student, despite increasing their level from baseline, remained highly variable around a level of 50% accuracy.

Although not graphed, studies that also measured time as a dependent variable show consistent trends. Students ranged from completing 1.1 problems per minute to completing 2.5 problems per minute. In studies that compared virtual and concrete manipulatives, students spent more time using concrete manipulatives than virtual manipulatives (Long et al., 2020; Satsangi et al., 2016). Both studies reporting duration for baseline and intervention reported longer durations during intervention (Satsangi, Hammer, & Evmenova, 2018; Satsangi, Hammer, & Hogan, 2018). Researchers aggregated duration across phases; as such, trends across duration could not be ascertained.

2.2.4.4 Maintenance

Over half (n = 7, 63.6%) of the reviewed studies collected maintenance data (Bouck et al., 2019; Chapman et al., 2019; Creech-Galloway et al., 2019; Jimenez et al., 2008; Satsangi, Hammer, & Evmenova, 2018; Satsangi, Hammer, & Hogan, 2018; Scheuermann et al., 2009). Students generally maintained levels similar to intervention. In three studies, no decrease in level occurred (Creech-Galloway et al., 2013; Jimenez et al., 2008; Satsangi et al., 2018a). In another three studies (Bouck et al., 2019; Chapman et al., 2019; Satsangi et al., 2018b), student performance initially dropped, then returned to comparable levels to intervention after one session. Initial drops ranged from decreases of 40% to decreases of 60%. Students in Scheuermann et al.'s (2009) study also performed at lower levels in maintenance; however, they did not recover to their intervention performance levels.

2.2.4.5 Generalization

Researchers collected data on generalization in a majority of studies (n = 7, 63.6%). Generalization measures included either removal of scaffolds/fading elements in the intervention (Baker et al., 2015; Long et al., 2020; Root & Browder, 2019; Satsangi et al., 2018a) or presentation of novel problem types (Chapman et al., 2019; Creech-Galloway et al., 2013; Scheuermann et al., 2009). Two studies faded use of manipulatives (Long et al., 2020; Satsangi et al., 2018), while other studies removed specific visual aids (Baker et al., 2015; Root & Browder, 2019). Students struggled to generalize when scaffolds were removed, with student performance falling to lower levels of accuracy or steps completed than during the intervention. Student performance on generalization to new problem types remained at comparable levels to the intervention (Chapman et al., 2019; Creech-Galloway et al., 2013; Scheuermann et al., 2009).

2.2.4.6 Social Validity

Most studies (n = 8, 72.7%) collected social validity data (Bouck et al., 2019; Chapman et al., 2019; Creech-Galloway et al., 2013; Long et al., 2020; Root & Browder, 2019; Satsangi et al., 2016; Satsangi, Hammer, & Evmenova, 2018; Satsangi, Hammer, & Hogan, 2018). On closed-ended scales provided to students, students reported feeling that the intervention was easy, valuable, and enjoyable, and that it targeted valuable skills (Chapman et al., 2019; Root & Browder, 2019). Researchers also collected open-ended responses in four studies (Bouck et al., 2019; Long et al., 2020; Satsangi, Hammer, & Evmenova, 2018; Satsangi, Hammer, & Hogan, 2018). Across open-ended responses, students found the interventions enjoyable and helpful. Students found that manipulatives supported their learning (Long et al., 2020; Satsangi et al., 2018a), but also felt they did not need manipulatives postintervention (Satsangi et al., 2018b). Preferences on concrete and virtual manipulatives varied by individual, although students did raise concerns about the length of time necessary to use concrete manipulatives (Long et al., 2020; Satsangi et al., 2016).

Two studies also included teachers in social validity data collection. In terms of closed-ended rating scales, Chapman et al. (2019) found that teachers felt their intervention helped students with real-world skills, but they were undecided on whether they would use it. In Satsangi et al.'s (2018b) open-ended interviews, teachers expressed surprise at the results and liked the virtual manipulatives for 1:1 or small-group instruction, but also expressed that using virtual manipulatives for whole-group instruction may pose significant challenges.

2.3 Discussion

Studies included in the present review detail effective procedures for teaching equations to students with developmental disabilities and learning disabilities. Studies focused on initial acquisition of the skill, and they often incorporated multiple intervention components. The most common components embraced research-validated, high-quality instructional practices, such as visual or concrete manipulatives, explicit instruction, task analysis, and systematic prompting (Bone et al., 2021; Lee & Mao., 2020; Spooner et al., 2019). Features of reviewed studies highlight important areas for further consideration, including the emphasis on acquisition, features of dependent variable measurement, and variance across task analyses.

Within the reviewed literature, students frequently demonstrated low, stable responding at baseline, suggesting that students had little prior knowledge of or ability to complete the skills (Baker et al., 2015; Bouck et al., 2019; Creech-Galloway et al., 2013; Long et al., 2020; Root & Browder, 2019; Satsangi et al., 2016; Satsangi, Hammer, & Evmenova, 2018; Satsangi, Hammer,

& Hogan, 2018; Scheuermann et al., 2019). Researchers also explicitly grounded studies in acquisition via the CRA instructional sequence. Research indicates that researchers have most frequently used and studied the CRA sequence for students with learning disabilities in mathematics (Watt et al., 2016). Within the CRA sequence, students learn algebraic principles first with concrete representations (e.g., concrete or virtual manipulatives), then progress toward abstract application of concepts with number representations (Witzel, 2005). Modifications of the CRA sequence, such as the virtual-representational-abstract (VRA) and virtual-abstract (VA) instructional sequences, replace concrete, physical manipulatives with virtual, technology-based manipulatives. Ultimately, the CRA, VRA, and VA sequences place emphasis on developing a conceptual understanding of mathematical processes alongside procedural problem-solving (Witzel, 2005).

Numerous studies have found CRA to be effective for acquisition, positing that a dual focus on conceptual and procedural knowledge helps students form explicit links between the abstract and the concrete (Agrawal & Morin, 2016). Conceptual knowledge refers to the underlying principles, both abstract and general, that create a certain mathematical idea (Rittle-Johnson et al., 2001, 2015). Procedural knowledge relates to the understanding of steps necessary to complete a specific task (Rittle-Johnson et al., 2015). The use of manipulatives within CRA interventions emphasizes the conceptual logic behind more abstract processes used to solve equations. Task-analytic instruction, graphic organizers, and systematic prompting may more closely target procedural knowledge and the steps a student must take to solve equations.

Researchers highlight CRA as valuable, given that current theories of mathematical knowledge suggest a bidirectional nature between conceptual and procedural knowledge (Rittle-Johnson et al., 2015). However, the question of how conceptual and procedural knowledge should be taught (i.e., distinctly, or if sequentially, in what order) remains to be empirically evaluated (Rittle-Johnson et al., 2015). Although the general success of CRA in the reviewed studies may provide evidence for simultaneous teaching, in some studies, students struggled to accurately complete problems without visual scaffolds (Baker et al., 2015; Long et al., 2020; Root & Browder, 2019; Satsangi et al., 2018a). Furthermore, researchers did not assess whether students promoted deep concept knowledge or whether the manipulatives merely provided scaffolding to complete procedures.

Dependent variable measurement evaluated acquisition most frequently via the number or percent of steps correctly completed in the task analysis. Measurement also focused on the percent or number of problems correctly completed. Notably, only four of the reviewed studies included a measure of time as a secondary or tertiary dependent variable (Long et al., 2020; Satsangi et al., 2016; Satsangi, Hammer, & Evmenova, 2018; Satsangi, Hammer, & Hogan, 2018). Within these four studies, the rate of problem completion ranged from about 1.5 minutes to 2 minutes per problem.

Although acquisition of a skill represents the first step in learning, accuracy measures alone may not sufficiently represent mastery of a skill (Binder, 1996). Thus, by removing time as a component within dependent variable measurement, researchers may overstate student mastery or may not support a student's meaningful, socially valid acquisition of the skill. Standard aims for fluent equation-solving do not presently exist in the literature, although the times provided in research studies likely fall under the aims for fluent responding. Behavioral fluency, or the combination of performing a skill quickly and accurately, can reinforce conceptual knowledge and support attainment of critical learning outcomes (Binder, 1996; Stocker et al., 2019. Critical learning outcomes include improved retention, greater endurance, application to more complex problems, maintenance across time, and stability (McTiernan et al., 2016; Stocker et al., 2019 Strømgren et al., 2014). Fluent responding may also promote deep procedural knowledge, characterized by flexibility in the application of procedures to challenging or novel problems (Star, 2005). If students are expected to build on simple equation-solving skills to apply them to more complex mathematical problems, a lack of fluent responding may inhibit attainment of complex skills. Without complete dependent variable assessment, which includes time, critical information remains missing from the current literature base.

The task analysis within each intervention highlights the specific procedures for equation-solving taught within the reviewed studies. Additionally, researchers most frequently used completed steps in the task analysis as the primary dependent variable within studies. Appendix A Table 4 outlines the task analyses used in each study. Task analyses varied greatly in both quantity of steps and content. First, the quantity of steps ranged from 9 to 32 across studies. A smaller number of steps, such as in Chapman et al. (2019) and Long et al. (2020), may create an artificial ceiling for student responding, especially when students are expected to only complete a fixed number of problems. Longer task analyses, such as Creech-Galloway et al. (2013)'s task analysis containing 32 steps, may include arbitrary steps such as "clear calculator," which could unnecessarily inflate accurate student responding.

Furthermore, steps related to using manipulatives, such as "counting the number of items in a container" (Chapman et al., 2019), emphasize a specific focus on using manipulatives, not on extrapolating those skills for abstract use with notations. Thus, if the end goal is to promote abstract knowledge, task analyses grounded in manipulative use may prevent students from generalizing absent the presence of manipulatives. While accurately completed steps can measure how well a student is using manipulatives, training loosely using more abstract steps could be one avenue to move students’ past acquisition and toward fluent responding (Stokes & Baer, 1977). Manipulatives may slow student responding and prevent procedural knowledge from being attained. Given that participants also raised social validity concerns about the feasibility of using manipulatives, considering how task analyses can readily translate to the next step of instruction can support true mastery of equation-solving skills.

Given the positive results across studies, interventions included in the current review present promising strategies for teaching students with disabilities how to solve equations. Researchers should continue to expand the populations and settings in which researchers implement interventions. Researchers should also consider how to move students away from the acquisition stages of learning. Students voiced a decreased need to use manipulatives, and so interventions to build on accuracy and support the development of fluent responding without manipulatives could be a valuable next instructional step. Additionally, given that generalization data indicated students struggled to maintain accurate responding with the removal of scaffolds, training on task analyses may be more beneficial if task analyses are initially generalized and not specific to manipulative use.

2.3.1 Limitations

The present literature review facilitated a novel approach by looking at how researchers study teaching a critical foundational skill within algebra: solving equations. A narrow focus on equation-solving facilitates a more nuanced look at elements of the independent and dependent variables that can facilitate future research. However, the narrow inclusion criteria may have inadvertently missed articles that include teaching equation-solving to students with disabilities. Furthermore, making comparisons across disability categories, rather than focusing on a single disability category, may create complicated comparisons. Inclusion criteria did not specify quality indicators prior to inclusion, which could also lead to the inclusion of articles that do not actually evaluate experimental effects or functional relationships between the independent and dependent variables, potentially inflating the success of the reviewed interventions (Kennedy, 2005).

3.0 Method

Using the prior literature review, as well as research explicitly targeting behavioral fluency in mathematics, I implemented a MBL-P design. The design evaluated the effects of a practice intervention on correct and incorrect notations while solving one-step equations for students with disabilities. The study ran for 54 calendar days from early March 2022 to the end of April 2022. I analyzed the primary dependent variable via statistics derived from the use of the standard celeration chart (SCC).

3.1 Setting

I conducted the current research project in a public charter school specializing in serving students with disabilities in a midsize Midwestern city. The school includes grades two through eight and enrolls students based on a lottery system. In 2021, 310 students were enrolled with a 6:1 student-to-teacher ratio. The majority (64.84%) of students were White, with fewer Black (25.25%), multiracial (7.98%), Hispanic (2.33%), and Asian (< 1%) students. The school received Title I funding, and 100% of the students were eligible for free or reduced lunch. A unique characteristic of the school was a dedicated additional intervention period for intensive reading and math supports provided to all students. Students received either four days of reading intervention and two days of math intervention, or vice versa, as determined by student need.

3.2 Participants

A total of three students participated in the current study, which received institutional review board (IRB) approval under the exempt determination (See Appendix B). I identified participants through a two-step process. First, I provided general criteria to a school-based Board-Certified Behavior Analyst (BCBA) who served as the liaison for the project. Criteria indicated that students must have a diagnosed disability via an IEP that specified math as an area of need. Criteria also stipulated students must not engage in behaviors that would prevent them from participating in the study. The BCBA provided criteria to math intervention teachers in grades six, seven, and eight and the middle school mathematics instructional coach. After meeting initial criteria, I disseminated IRB-approved informational letters and consent forms (see Appendix C) to the parents or caregivers of the students. Upon attaining consent from the parents or caregivers, I met with students to describe the IRB-approved script (see Appendix D) and obtain assent from students. After obtaining consent and assent, I screened students to ensure the presence of prerequisite skills necessary for participation (see Procedures). Three students met study criteria.

Alyssa was a 12-year-old Black female in the sixth grade. Alyssa qualified for special education services via a specific learning disability in mathematics. Alyssa received four days of intensive mathematics intervention per six-day class rotation at the school. Jacob was a 12-year-old White male in the sixth grade. Jacob qualified for special education services via a diagnosis of specific learning disability (SLD) in reading and mathematics. Jacob received four days of intensive mathematics intervention per six-day class rotation at the school. Kyleigh was a 13-year-old White female in the seventh grade. Kyleigh qualified for special education services via a diagnosis of SLD in reading and mathematics. Kyleigh received two days of intensive mathematics intervention per six-day class rotation at the school.

3.3 Materials

I created materials for screening and frequency-building sessions based on the implementation guidelines in *The Precision Teaching Implementation Manual* (Kubina, 2019) and in conjunction with experts in precision teaching.

3.3.1 Screening Assessments Sheets

Screening assessments contained two different sections. The first section contained single-digit addition, subtraction, multiplication, and division problems (i.e., the basic 390 math facts). Each sheet contained approximately 80 problems and included only one type of operation per sheet (see Appendix E for an example of one addition sheet). I randomly distributed problems across each sheet.

The second section of screening assessments contained 12 one-step equations and was identical to the dependent variable assessment sheet and the sheets used during frequency building (see Appendix F for an example). I created sheets by compiling all possible one-step equations for each basic math fact. Then, I distributed math facts across four bins to ensure one bin did not contain a disproportionate number of equations with a specific number (e.g., problems requiring multiplication or division with the number 8 were distributed evenly across bins). I also evenly distributed problem difficulty across sheets (e.g., one sheet would not contain two problems where the student only had to add 1 to each side). Then, I created sheets using approximately the same number of one-step addition, subtraction, multiplication, and division problems on each page. I used each bin to create four different sheets, for a total of 16 unique sheets. I completed timed assessments on each sheet to ensure equivalence of difficulty.

3.3.2 Instructional and Think-Aloud Materials

Instructional and think-aloud materials consisted of two whiteboards, two markers, a voice recorder, a stopwatch, and practice sheets. The initial practice sheet included vocabulary definitions, examples, and non-examples of two terms: inverse operations and variables (see Appendix G). Underneath each definition, I included eight problems where students had to identify the vocabulary term in context (see Appendix H). Four guided practice sheets were used for instruction of the task analysis (see Independent Variable below), with one practice sheet per type of problem (e.g., one-step addition, one-step subtraction). The task analysis practice sheets contained one section for the instructor to model each problem type and have the student complete the identical steps on a whiteboard. The back of the sheet included eight independent practice problems. Appendix I displays both types of instruction sheets.

3.3.3 Baseline and Intervention Materials

Baseline and intervention materials included a stopwatch, pencil, and intervention script, as well as the dependent variable assessment sheets and frequency-building sheets (described above). I sequenced and stored dependent variable assessment sheets and frequency-building sheets in individual binders. I used the daily data recording sheet to record students' correct and incorrect notations before transferring data to a Microsoft Excel spreadsheet for graphing purposes (see Appendix J).

3.3.4 Generalization Materials

I created the generalization probe with eight problems. The problems varied the presentation of one-step equations by changing the order of the variable (e.g., 8 = x + 2; 3 + x = 6). I also included different presentations of the operations (e.g., 6x instead of x * 6). Generalization probes did not require students to complete any additional steps, as modifications related only to the presentation of problems.

3.4 Dependent Variable

The primary dependent variable included correct and incorrect notations per minute. A correct response included any legible notation written in the correct location. I defined correct location as notations that followed steps in the task analysis for solving one-step equations. Operation signs (an x for multiplication, - for minus sign, ÷for division, + for addition sign) and numbers counted discretely; for example, writing –6 underneath each side of the equation x + 6 = 10 counted as four correct notations: two for each minus sign and two for each 6. In total, each problem had a minimum of seven opportunities for correct notations. Students could write numbers backwards but had to include the correct number—for example, after subtracting 10 – 6, students had to write the number 4 for the response to be marked correct; writing the number 5 was labeled as an incorrect response. Additionally, numbers or notations written in the incorrect location, such as on the wrong side of the equation, were marked as incorrect. Partial problems were counted, such as responses leading up to the correct answer, but not having the final correct answer, were counted as correct or incorrect. I did not count skipped problems or problems with no notations as incorrect or correct notations. I measured the dependent variable via the dependent variable assessment sheet.

Other studied variables included student verbalizations in the think-aloud activities. I recorded and transcribed student verbalizations and compared changes across pre– and post–think-aloud activities. I also evaluated students' ability to generalize to varied presentations of problems using the generalization sheets described above.

3.5 Independent Variable

The independent variable consists of a multicomponent frequency-building intervention package. Components, described below, include instruction and frequency-building sessions.

3.5.1 Instruction

Instruction consisted of explicit instruction, modeling, guided practice, independent practice, and feedback. Vocabulary instruction included the concepts of inverse operations and variables. Task analysis guided explicit instruction, modeling, and practice. Task analysis for one-step equations included the following seven steps:

1. Write notation for inverse operation below operation in the problem.
2. Write number being moved to opposite side of equation below the equation.
3. Write notation for inverse operation on opposite side of equation.
4. Write number being moved below opposite side of equation.
5. Write variable.

6. Write equal sign.
7. Write correct number opposite equal sign.

Instruction started with a stated objective: to solve one-step equations accurately by finding the value of the unknown, or the variable. I explained that the variable is the letter in the problem that represents the unknown, and that the goal of solving an equation is to isolate the variable. After explaining examples and non-examples, students had to identify examples of the variable in context in five consecutive trials with 100% accuracy. I then explained the concept of inverse operations by showing examples and non-examples. I checked for understanding by presenting the student with an operation and asking them to write the inverse operation on their board. I then presented the operation within the context of an expression and repeated the process with students. Students had to independently identify the correct inverse operation in the context of an equation in five consecutive trials with 100% accuracy. When the student answered incorrectly in either the variable or inverse operation vocabulary instruction, I provided the student with the correct answer verbally and in writing and asked that they repeat the correct answer both verbally and in written form.

Next, I reviewed the seven-step task analysis, which remains the same regardless of the operation. I read aloud the task analysis for each operation and modeled the step on the righthand side of the worksheet. After this modeling, the student completed the identical problem on a whiteboard. Then, I provided the student with a problem and guided them through the problem using a least-to-most system of prompting. If students made a computational error, I provided feedback by stating the entire problem and the correct answer, and had students repeat the problem and answer. If the student made an error related to the task analysis, I identified the error by connecting their error to the step of the task analysis, provided the correct answer, and had them repeat the correct answer. After completing the guided practice for a single operation, students completed eight independent practice problems. Students had to complete at least seven of eight problems correctly to exit the instruction phase for a given operation. The process was repeated across each operation.

3.5.2 Frequency-Building Sessions

I began each frequency-building session with a dependent variable assessment, followed by three one-minute sessions of practice and feedback. The frequency-building session also included goal-setting components. The dependent variable assessment was identical to the baseline assessment. I reminded each student to work as quickly as possible, and to work from left to right. After the student completed the dependent variable assessment, I provided positive statements about the student's effort in completing the assessment.

Next, students engaged in three one-minute frequency-building sessions, each followed immediately by the provision of feedback. At the beginning of each one-minute session, I stated the objective of the frequency-building sessions. I provided their prior correct number of notations in the dependent variable assessment and explained to the student that they would be working as fast as they could to advance past their prior correct notations. I started the timer. After each student had worked for one minute, I prompted them to stop working and provided feedback. Length of feedback varied based on the number of student errors and lasted no longer than one minute per one-minute session. I started each feedback session by providing behavior-specific praise on a correct problem. Then, I pointed to each incorrect problem. If the error was computational, I stated the problem and the correct answer, and had the student state the problem and correct answer. If the problem was related to a different step in the task analysis, I stated the step and the correct notation, and had the student repeat the step and the correct notation. The process repeated immediately after each session. If the student exceeded their prior performance during the frequency-building sessions, I provided specific praise for the number of notations. In its entirety, including assessment and frequency building, the intervention lasted an average of 8.25 minutes.

3.6 Research Design and Data Analysis

The study used a multiple-probe MBL-P design. I began baseline probes with all students at the start of the study. I collected baseline data on each student for five days. After each student's fifth day, I used metrics derived from the SCC to guide decision-making. Specifically, I used the improvement index, which describes the overall improvement or betterment of the dependent variable (Kubina, 2019). Improvement indices can quantify overall worsening or betterment of behavior (Kubina, 2019). The student with the lowest decelerating improvement index entered first, and subsequent entries into the intervention followed this rule. Calculation of the improvement index is described below. I collected data on students who entered the intervention for a minimum of five days before having the next student enter. Students not in instruction remained in baseline. I collected baseline probes for students every third school day; I did not continue baseline daily to prevent students from practicing incorrectly. I also did not collect baseline data twice in one week, and student absences and/or schedule changes in the school sometimes lengthened the time between baseline probes. Students needed to have an improvement index of x 1.25 (or a 25% increase) for the next student to enter into instruction and then intervention. If the student hit the performance standard and had a minimum of five data points, I exited students from the intervention phase. Otherwise, students remained in the study for at least

10 intervention days. I attempted to conduct intervention days consecutively for each student, although interruptions to the school calendar and/or student absences sometimes interfered.

I conducted data analysis via calculations derived from the SCC. The SCC uses standardized notations and displays to support objective quantification of data beyond the scope of visual analysis (Kubina, 2019). Analysis occurred both within and between conditions, so long as sufficient data points existed to adequately represent the pattern of behavior (Kubina, 2019). *Within conditions* refers to the analysis of data within either the baseline or intervention phase, while *between conditions* compares values derived from student performance in baseline and intervention.

Within conditions, I considered the level, celeration, and improvement index values. I calculated level by finding the geometric mean due to statistical advantages (Clark-Carter, 2005). Level calculations communicated the average level of responding in a phase. Celeration values represented the change in frequency of a behavior in time divided by time, or how fast the behavior changed (Johnston & Pennypacker, 2009; Kennedy, 2005; Kubina, 2019). Celeration encompasses both acceleration (increasing frequency, represented with an x symbol) and deceleration (decreasing frequency, represented with a ÷ symbol) of correct and incorrect notations. To calculate improvement index within a condition, celeration of incorrect and correct notations were compared. If both incorrect and correct notation celerations had the same signs, I divided the larger value by the smaller value; if the signs were different, I multiplied. The sign was assigned based on how the overall performance was changing.

Between conditions, I compared level, celeration, and improvement index. I compared level change between conditions by dividing the larger value by the smaller value and using the sign that represents the change (x for increase, ÷ for decrease). Level change captured differences in geometric means between conditions. A meaningful change was x2 or greater (Kubina, 2019). I calculated the celeration multiplier to compare celeration between conditions. The celeration multiplier represented the speed change between conditions; it provides critical information given the emphasis on behavioral fluency within the current study. I calculated the celeration multiplier to compare conditions using the same formula described for the improvement index. Similarly, I used the same formula to calculate changes between conditions within the improvement index. Comparison of the improvement index accounts for both correct and incorrect notations, and produced the improvement index change (Kubina, 2019).

3.7 Procedures

3.7.1 Screening

During screening, participants had 30 seconds to complete as many single-digit problems as possible on a sheet with a single operation. I repeated the process individually with students four times across each operation. I started by asking participants to work from left to right and to work as quickly as possible, then began the timer. After 30 seconds, the timer beeped, and I prompted students to put the pencil down. Alyssa completed 21 correct digits per minute on a 30-second screening probe with no incorrect notations. Alyssa completed fewer subtraction (15), multiplication (14), and division (7) problems per 30 seconds, but made no errors. Jacob completed 25 correct addition problems, with 3 incorrect notations, and 26 correct multiplication notations, with 0 incorrect notations, per 30 seconds. Jacob completed 16 correct digits and 1 incorrect digit per 30 seconds on subtraction problems, and 9 correct digits with 3 incorrect digits per 30 seconds on division problems. Kylie demonstrated relative strengths in addition fluency during screening for the study, scoring 25 correct digits per 30-second timing, with only 1 incorrect digit. Kylie scored fewer correct digits per 30 seconds on multiplication (13), subtraction (11), and division (10), and 19 incorrect notations, indicating she was at the instructional level with the task. On the assessment of single-digit addition and multiplication problems, obtaining 40–50 correct digits per 30 seconds indicates achievement of the performance standard (Kubina & Yurich, 2012). On the assessment of single-digit subtraction and division problems, obtaining 30–45 correct digits per minute indicates achievement of the performance standard (Kubina & Yurich, 2012). Although no student achieved the performance standard on the single-digit probes, I ensured students fell in the middle 50th percentile range, such that they were not at the frustration level of instruction.

After completing the 30-second assessment across each operation, I explained the second screening assessment to students. I told students that they would have 60 seconds to complete as many one-step equations as possible, and I reminded students to work in order from left to right. I also prompted students to show their work. I began the timer and asked students to put their pencil down after they had stopped. On the dependent variable assessment, Alyssa completed 10 correct notations and 1 incorrect notation. Jacob wrote 6 correct notations and 20 incorrect notations. Kyleigh wrote 13 correct notations and 9 incorrect notations. To determine the performance standard for the second assessment, I asked three different fluent performers to solve the screener. Based on their performance, I identified an initial performance standard of 60–80 correct notations per minute. I evaluated student performance to identify whether students were able to solve at least two problems correctly, even if they did not have the correct corresponding notations. If students demonstrated they could solve at least two problems correctly in 60 seconds, I admitted them into the study.

3.7.2 Baseline

During baseline, students were given one randomized problem sheet. I instructed students to work as fast as they could for one minute, from left to right, until they heard a beep from the timer. I also instructed students to show their work for each problem. I told students to start and started a timer. After 60 seconds, the timer beeped. I provided no feedback to students. I provided praise for student completion of the probe. After I collected five days of baseline data, I determined who entered the intervention, per the description above. Other students remained in baseline.

3.7.3 Instruction and Preintervention Think-Aloud

Instruction and the preintervention think-aloud lasted one day. Prior to instruction, I provided the final baseline probe. Next, I administered the preintervention think-aloud assessments. For the preintervention think-aloud, I used a whiteboard with a single-step equation. I prompted each student to read the problem aloud and solve the problem, describing each step and explaining why they were doing each step. To ensure students provided adequate verbalization, I prompted with either "What did you do?" if the student did not describe the step they had completed or "Why did you do that step?" if they did not provide rationale. No further prompts were given, and prompts were given only once. I audio recorded students while they completed the problem. I provided praise upon completion of the think-aloud. Afterward, I provided initial instruction. Each student required only one session of instruction, which lasted an average of 26 minutes and 20 seconds.

3.7.4 Frequency-Building Sessions

I conducted frequency-building sessions on the next day after students successfully completed instruction. Each day of intervention began with the student completing the dependent variable assessment. The dependent variable assessment was followed by the frequency-building practice, described above as the independent variable. At the end of each session, I provided specific praise if the student exceeded their prior performance on the dependent variable assessment. Intervention length ranged from 7 to 10 minutes.

3.7.5 Postintervention Think-Aloud and Generalization

Upon students' exiting out of the intervention, I asked them to complete the think-aloud on a whiteboard. The problem was identical to the problem provided in the preintervention think-aloud activity, as were the prompts provided. I recorded audio of student verbalizations. Next, I asked students to complete the generalization sheet. I did not time the generalization probe.

3.8 Dependent Variable Assessment (Accuracy)

The procedure provides a permanent product of correct notations per minute. Student notations, or observed values, were compared to true values to minimize measurement error (Johnston & Pennypacker, 2009). True values can be ascertained because problems have an objectively correct answer. I scored correct and incorrect values based on the description of the dependent variable.

3.9 Procedural Fidelity

I created a checklist with explicit steps that corresponded to each session. The checklist was visible and present for all sessions. I also audio recorded 25% of the data collected in baseline and intervention, and scored procedural fidelity. I calculated procedural fidelity for a single session by dividing the number of steps observed by the total steps possible and multiplying by 100. In addition to identifying procedural fidelity, I took notes about any special or extenuating circumstances (e.g., student illness, student fatigue). All intervention sessions demonstrated 100% fidelity. One baseline session did not include the direction for students to work from left to right, for 80% fidelity, although the student performance did not change from previous baseline sessions (i.e., the student still worked from left to right). All other baseline sessions were completed with 100% procedural fidelity. I noted no personal extenuating circumstances, although the time of day of the intervention varied when intervention sessions occurred on days of statewide standardized testing sessions. Recorded intervention sessions lasted an average of 8.28 minutes, excluding time walking to and from the intervention space.

3.10 Social Validity

I measured social validity via a Likert scale survey, which I delivered to students participating in the intervention condition. Per Wolf (1978) and Horner et al. (2005), social validity questions asked, in audience-appropriate language, whether the outcomes were socially important, whether the magnitude of change was important, whether implementation was practical and time- and cost-effective, and whether the intervention was likely to continue after the research project was finished. Additional open-ended questions targeted what parts of the intervention were acceptable to students and teachers, and what parts were not. Using a Likert scale enabled a quantitative reporting of social validity that will better enable replications to compare social validity across time.

4.0 Results

Figure 1 is an SCC illustrating correct and incorrect notations per minute on dependent variable assessments across baseline and intervention for Alyssa, Jacob, and Kyleigh. Dots correspond to correct notations, and incorrect notations are noted with X's. The vertical axes refer to the number of notations per minute scaled logarithmically. The x axes represent calendar days scaled linearly. Dotted lines represent celeration lines derived from a linear regression formula.

4.1 Within-Condition Analysis

Table 4 depicts the level (geometric mean), celeration, bounce, and improvement index in baseline and intervention conditions for Alyssa, Jacob, and Kylie. In baseline, Alyssa and Kyleigh both had higher levels of correct notations than incorrect notations, with Jacob displaying the inverse. All three students had accelerating correct and incorrect notations per minute, except for Alyssa who had decelerating corrects (÷ 1.34). Bounce scores were similar for both correct and incorrect notations for Alyssa and Jacob. Kyleigh displayed greater bounce for incorrect notations (x 2.30) rather than correct notations.

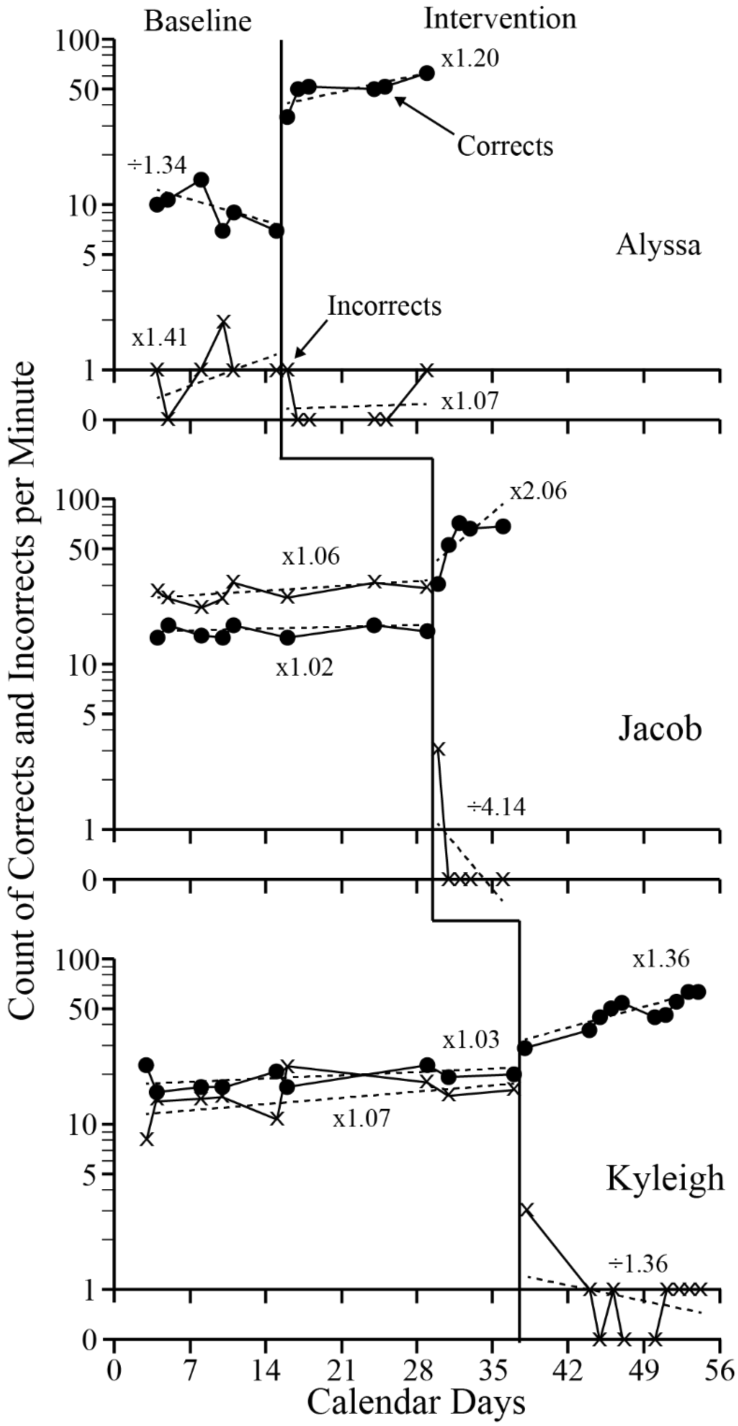

Figure 1 Standard Celeration Chart Displaying Correct and Incorrect Notations Across Participants

Table 4 Within-Condition Measures of Student Notations

		Baseline				Intervention			
Student	NT	Level	Cel.	Bounce	I.I	Level	Cel.	Bounce	I.I.
Alyssa	C	9.38	÷ 1.34	× 1.80	÷ 1.89	47.13	× 1.20	× 1.40	× 1.12
	I	0.00	× 1.41	× 2.00		0.00	× 1.07	× 2.00	
Jacob	C	15.78	× 1.02	× 1.20	÷ 1.04	54.37	× 2.06	× 1.60	× 8.53
	I	26.67	× 1.06	× 1.40		0.00	÷ 4.14	× 1.00	
Kyleigh	C	18.92	× 1.03	× 1.60	÷ 1.04	46.23	× 1.36	× 1.50	× 1.85
	I	14.43	× 1.07	× 2.30		0.00	÷ 1.36	× 2.70	

C = correct; Cel. = celeration; I = incorrect; I.I. = improvement index; NT = notation type

As an overall picture, all three students had declining improvement index scores in baseline, meaning that the overall quality of their performance was deteriorating across the condition.

During intervention, all three students showed consistency with performance. Alyssa, Jacob, and Kyleigh had correct notations-per-minute levels between 46 and 54, and 0 incorrect notations per minute. Jacob (x 2.06 and ÷ 4.14) and Kyleigh (x 1.36 and ÷ 1.36) had accelerating correct notations and decelerating incorrect notations. Alyssa also had accelerating correct notations (x 1.20) but slightly accelerating incorrect notations. Such a combination of scores resulted in improving improvement index measures for Alyssa (x 1.12), Jacob (x 8.53), and Kyleigh (x 1.85), or, in each case, the quality of performance improved during the intervention.

4.2 Between-Condition Analysis

Table 5 displays between-condition metrics for both correct and incorrect notations for each student. The frequency multiplier quantifies the immediacy of change, or jump, from baseline to intervention (Kubina, 2019). Students showed immediate and lasting intervention effects. Alyssa (x 4.86), Jacob (x 1.88), and Kyleigh (x 1.30) all had jumps up from baseline to intervention for correct notations. Other than Alyssa, who remained at 0 for incorrect notations, Jacob and Kyleigh had tremendous jumps down of ÷ 9.67 and ÷ 5.33 for incorrect notations. Comparing levels between conditions, all students had increasing level multipliers for correct notations ranging from x 2.44 (Kyleigh) to x 5.02 (Alyssa). Incorrect level multipliers could not be calculated, as levels dropped to 0.

Table 5 Between-Condition Measures of Student Notations

		Baseline to Intervention				
Student	NT	Frequency Multiplier	Level Multiplier	Bounce Change	Celeration Multiplier	Improvement Index Change
Alyssa	C	× 4.86	× 5.02	÷ 1.29	× 1.61	× 2.11
	I	× 1.00		1.00	÷ 1.32	
Jacob	C	× 1.88	× 3.45	× 1.33	× 2.02	× 8.87
	I	÷ 9.67		÷ 1.40	÷ 3.39	
Kyleigh	C	× 1.30	× 2.44	÷ 1.07	× 1.32	× 1.92
	I	÷ 5.33		× 1.17	÷ 1.46	

C = correct; Cel. = celeration; I = incorrect; NT = notation type

A few of the measured behaviors saw decreasing bounce change scores, suggesting a tightening of stimulus control due to the intervention. Bounce change measures for correct notations for Alyssa (÷ 1.29) and Kyleigh (÷ 1.07) and incorrect notations for Jacob (÷ 1.40) all showed decreases. Incorrect notations for Kyleigh (x 1.17) and correct notations for Jacob (x 1.33) did become more variable in the intervention. Unlike bounce change scores, all three students had consistent celeration multiplier scores for both measures and strong improvement index change scores.

Following instruction, Alyssa (x 1.61), Jacob (x 2.02), and Kyleigh (x 1.32) had accelerating correct notations per minute as compared to baseline. As correct notations accelerated, incorrect notations turned down or decelerated for each student as well, ranging from ÷ 1.32 (Alyssa) to ÷ 3.39 (Jacob). Looking at the improvement index change measures, which consider the change in quality from baseline to intervention, all three students showed dramatic improvement. Two of the three students (Alyssa and Kyleigh) showed an impressive approximately 100% improvement in quality. Rather than just impressive, Jacob's x 8.87 improvement index change measure, or an almost 900% improvement in performance quality, was massive.

4.3 Pre– and Post–Think-Aloud

In the pre–think-alouds, both Jacob and Alyssa described their method of solving the problem as "x + 8 = 15." Jacob described his process as asking himself what number added to 8 equaled 15. Alyssa's process involved similar trial and error. In her pre–think-aloud, she stated, "8 minus 8 plus 15 minus 15. Minus 2 is 2 is 13 and then 13 and then 8 will become a 10 minus 15, 3. Wait. No, no" before eventually arriving at the correct value of x. Kyleigh described without much detail, stating, "I minus the number," with no further explanation or detail.

After the intervention, all students increased the detail with which they described the process to the solution. Both Jacob and Alyssa described the steps in the task analysis. Jacob said, "I minus 7 because it is the inverse act," then went on to demonstrate doing so on both sides. Alyssa said, "Minus 8, minus 8, x equals 7," verbally noting each step. Kyleigh significantly increased her explanation, stating, "This becomes a minus" in reference to the addition sign, then adding, “You put 8 under. . . . Then you also do it for the answer side. . . . 15 minus 8 . . . which would be 7, and that’s the answer for x.”

4.4 Generalization

The generalization probe contained eight problems with various presentations of single-step equations. Alyssa completed seven of eight problems correctly, with her error relating to computation. Jacob also completed seven of eight problems correctly. His error was related to the presentation of x with a fraction bar to represent division, as opposed to the traditional division sign (÷) used in the dependent variable assessment. Kyleigh completed all generalization problems correctly.

4.5 Social Validity

All three participants responded to the social validity questionnaire, which included closed- and open-ended questions. Closed-ended questions used a five-point Likert scale. Average student responses indicated the intervention did make meaningful change in their behavior related to the skill: solving one-step equations (m = 4.0). Open-ended responses indicated students felt these skills would be important for “real-life skills.” On closed-ended questions, students reported that the intervention resulted in them feeling significantly more confident in solving one-step equations (m = 4.67). Students felt slightly less strongly about the intervention being practical and time efficient (m = 4.33). When asked whether they felt the intervention would be valuable in the context of their classroom, Jacob shared that only one of his teachers focused on speed, so he thought they might want to incorporate it. Both Alyssa and Kyleigh connected the intervention to problems they faced in their math class, with Alyssa explaining that "it helped with the thing we are working on in class." Generally, students responded that they would recommend the intervention to other students (m = 5.0).

5.0 Discussion

Promoting algebraic achievement supports for students facilitates access and success in advanced mathematics courses (Fayer et al., 2017; Gaertner et al., 2014; Kim et al., 2015). Standardized assessment data suggest students with disabilities struggle to make meaningful growth in mathematics beyond eighth grade, which coincides with more advanced mathematics (Cortiella, 2011; Provasnik et al., 2016). As with other subjects such as reading, strong knowledge of and fluency related to foundational skills supports achievement in higher levels of mathematics (Lin & Powell, 2022; Powell et al., 2019). Therefore, the current study evaluated an intervention focused on improving students' fluent responding on a key mathematic skill: the ability to solve one-step algebraic equations. My formal research question asked, "What are the effects of a frequency-building intervention on students with disabilities' rate of correct and incorrect notations while solving one-step equations?"

Overall results suggest strong experimental effects between the intervention and students' correct and incorrect notations (Kazdin, 2011). Generalization probes suggest students used their knowledge to flexibly respond to varied presentations of single-step equations in which they were not explicitly trained in the intervention. Qualitative data from the pre– and post–think-alouds imply students changed their orientation to equations, moving from a trial-and-error approach to a more systematic use of algebraic procedures. Finally, social validity data indicated students would recommend the intervention to other students, citing its relevance to their classwork and increased confidence from consistent practice.

While the review of research indicates previous equation-solving interventions were generally effective, the current intervention demonstrates student improvement in not only accuracy but also in speed. Reviewed research did not include speed within the primary dependent variable, and researchers who did include time as a measured variable reported much slower rates of responding than in the present study. Current research on algebraic equation-solving interventions also frequently used manipulatives or concrete representations to support concept knowledge (Bouck et al., 2019; Chapman et al., 2019; Jimenez et al., 2008; Long et al., 2020; Satsangi et al., 2016; Satsangi, Hammer, & Evmenova, 2018; Satsangi, Hammer, & Hogan, 2018; Scheuermann et al., 2009). While the use of manipulatives is impactful for concept development, the long-term feasibility of using manipulatives may inhibit student performance as algebraic procedures become more complex. Specifically, manipulatives may slow down student responding and create prompt dependency on manipulatives, as potentially indicated by reported difficulties in generalization in the reviewed research. Additionally, the current intervention achieved dramatic improvements using a shorter intervention duration (less than 10 minutes) across only 5–10 sessions. Because of the emphasis on rate of responding, the current study was not focused on the same dependent variable. Subsequently, findings from the current study more closely align with another body of research: frequency building.

The current results mirror findings from other research on effective frequency building and instruction for skills such as words read (e.g., Lambe et al., 2015) and sentence writing (e.g., Datchuk et al., 2015). In addition to the direct benefit (i.e., more correct notations and fewer incorrect notations), all three students displayed positive side benefits, such as proper responding to a wide variety of single-step untainted equations. Again, students engaged in an effective, purposeful practice, and exhibited similar accurate responses to untrained situations (e.g., Kostewicz et al., 2020).

The significant positive results of the current study may be explained by the characteristics of the frequency-building intervention. The specific characteristics of practice are traditional features within precision teaching, which is a system of measurement designed to support educators in using science to make data-based decisions related to teaching and student learning (Evans et al., 2021; Lindsley, 1972, 1990). In precision teaching, four actions completed in a cyclical nature guide decision-making: *pinpointing* observable and measurable behavior, *recording* via daily measurement, *changing* behavior through the iterative process of analysis and decision-making, and *trying again* by continuing to strive for positive learning outcomes (Kubina & Yurich, 2012).

Unlike precision teaching in practice, which centralizes the learner's individual needs and skills. my pinpoints were predetermined based on specific criteria outlined in the research design. However, I used screening procedures that considered both prerequisite element skills, such as solving single-digit addition problems, and familiarity with the target of the intervention (i.e., single-step equations). Such procedures supported the identification of students at the instructional level, for whom frequency building would be most appropriate. Researchers suggest that a skill-by-treatment interaction may improve the effectiveness of interventions (Burns et al., 2010). Thus, the appropriateness of the intervention for the specific students, given their skill level, may have bolstered the effectiveness of the current intervention.

Within the design and delivery of the frequency-building intervention, I clearly defined, or pinpointed, each behavior using task analysis for the procedure of solving single-step equations. Each behavior had a consistent learning channel, or mode of stimulus and response (Kubina & Yurich, 2012). Specifically, students *saw* a written problem in a specific format and *wrote* in response to form a pinpoint of *see* notation/*write* notation. The current task analysis is different from prior literature, which often included steps that required attention to different modes of presentation. For example, prior interventions using manipulatives required students to both see the problem/write answers and see markers/touch markers (Jimenez et al., 2008). Because learning channels may be independent of each other, keeping response forms consistent across the task analysis, via clearly pinpointed behaviors, may have accelerated learning (Lindsley, 1996).

Prior to the intervention, as self-described in the pre–think-aloud, students prompted themselves via covert speech, showing little written work. Students also showed little written work on baseline probes, despite prompts to do so. In the intervention, because behavior was clearly defined, taught, and reinforced, students relied less on covert speech and guesswork as they did in the pre–think-aloud. Even in non-routine problems, strategies of "guess and check" have demonstrated little value in supporting students who struggle in mathematics (Arslan & Altun, 2007). Instead of engaging in guesswork, students wrote their own visual prompts to guide them through the systematic use of an algorithm, in which each written notation served as the prompt for the next written notation. Ultimately, the steps of the procedure became a complex behavior chain (Noell et al., 2011). I established stimulus control for the complex chain by modeling: pairing the steps of the task analysis, presented orally and textually, to the corresponding written behavior, in explicit instruction. After students demonstrated success with the task analysis, I faded the task analysis, and students then relied only on their own visual prompts.

Having each step in the task analysis available for visual inspection facilitated more precise corrective feedback. Consider the problem $x * 3 = 9$. If a student merely wrote $x = 6$, I would not know the precise nature of their error. The student could have correctly used division but erred in the computation; the student also could have erroneously used subtraction. However, when students write each step to their problem, the teacher no longer has to guess at the nature of the error. Instead, the teacher can directly address the precise student error, allowing for more improved formative assessment and more efficient behavior change (Beesley et al., 2018). As mentioned previously, feedback related either to computation errors or to the steps of the task analysis. In the post–think-aloud, students more readily used language mirroring the task analysis in their explanation. The language could have been bolstered by the precise feedback. I also delivered feedback on the task immediately upon completing each minute of timing, and thus the immediacy of feedback may also count for the significant drops in errors for two students in the study (Hattie & Timperley, 2007).

The frequency-building intervention also emphasized the speed of student responses, within both *recording* and *changing* the student behavior. Related to recording, the measurement of the dependent variable was not constrained by artificial ceilings, thus allowing for a more accurate understanding of the rate of change of the behavior. The inclusion of time as a feature of measurement also supports a view of mastery extending beyond mere accuracy (Binder, 1996). Including fluent performance as a component of mastery supports the attainment of critical learning outcomes related to fluency, such as endurance, generalization, and application (Binder, 1996; Stocker et al., 2019). Given the sequential nature of mathematics, developing fluent performance on foundational skills such as solving equations can more readily support students' using those skills flexibly as part of more complex problem-solving processes (Spooner et al., 2019; Stocker et al., 2019). For example, solving a one-step equation can be thought of as the last step in the chain of solving both two- and multistep equations, and can be easily generalized to solving inequalities.

I also centralized timing, or speed of student responses, in the regular purpose statement and goal-setting procedures. In notes regarding implementation, students responded positively and enthusiastically about trying to improve, sometimes asking questions immediately upon completion regarding whether or not they exceeded their past performance. Thus, the explicit emphasis on timing during regular and consistent practice led to high-quality practice focused on improving speed.

Pre– and post–think-aloud data may provide insight into the type of mathematics knowledge being used by students. On the surface, the emphasis on notations corresponding to the task analysis may appear to be related only to procedural knowledge. However, in student think-alouds postintervention, students shared concept knowledge related to the procedures (e.g., explaining the use of inverse operations) that they did not possess prior to the intervention. The potential deepening of conceptual knowledge emerging from the practice of procedures may support the iterative theory of procedural and conceptual knowledge development (Rittle-Johnson et al., 2015). Utilizing self-explanation, such as in the think-alouds, also aligns with related recommendations for supporting procedural transfer (Rittle-Johnson et al., 2015).

5.1 Limitations

Some limitations exist in the current study. First, due to external constraints on the length of the study, I did not collect maintenance data. Maintenance data may provide further insight into whether or not the specific performance standard of 60 correct notations per minute accurately captures fluent performance, or whether a higher performance standard may be warranted to result in maintenance of the skill. Second, the current study used a specific task analysis and counted incorrect notations as those that did not follow a specific procedural format. Other procedures for solving equations, with more or fewer notations, may also yield more positive results and necessitate different performance standards. Additionally, although procedural flexibility remains important to consider in problem-solving (Rittle-Johnson, 2017; Star & Newton, 2009), the current study's primary research question did not aim to tackle the development of procedural flexibility, but rather the speed and accuracy with which students were able to use a specific procedure. However, generalization data suggest students did demonstrate flexibility without being explicitly taught new procedures, but claims should be tempered given the current research design and primary focus. Finally, disruptions in the school schedule, including days where the school had standardized testing, resulted in the intervention sometimes being delivered at varied times of the day, resulting in potential confounding variables.

5.2 Implications

The current study has critical implications for algebraic instructional practices for students with learning disabilities. First, when teaching skills, educators should pinpoint and clearly define behaviors in the procedures for solving equations (Kubina & Yurich, 2012). Then, teachers should consider how students can demonstrate those behaviors in a consistent learning channel (e.g., see-write). Having students overtly perform behaviors in a way that can be measured via "showing their work" can enhance the quality of feedback by allowing teachers to provide precise corrections versus making assumptions about covert behaviors.

Second, educators should also consider how to incorporate timing into their interventions after the initial acquisition of a given skill. Students in the present study had some familiarity with equations. After even a brief exposure to a concept, the current study suggests that even a small dosage of 5–10 interventions across two to three weeks can support dramatic improvements in the speed and accuracy of students' solving equations. While the literature suggests that manipulatives and visual representations may support initial acquisition, students had not previously been exposed to the CRA sequence or manipulatives as they relate to solving equations. Thus, while no "best" order can be derived from the present study, the results indicate that introducing timing prior to interventions targeting conceptual knowledge still provides immense benefits for students.

The current intervention was delivered in a 1:1 format, which may be readily translated to existing tier 3 interventions in the context of a multitiered system of supports. However, incorporating timing, as well as automating or providing worked answers, also has a basis in the literature. For example, cover-copy-compare may be equally effective at promoting students' fluent performance (Codding et al., 2009). Thus, a lack of 1:1 availability of students should not preclude the focus on improving student speed.

Finally, given the broad success of frequency building across a variety of educational skills, mathematics educators may consider how to expand the practice beyond procedures for solving equations. Researchers have employed frequency building in pre-algebraic skills related to order of operations (Stocker & Kubina, 2021). By considering clearly defined behaviors within algebraic procedures, educators can incorporate frequency building across a variety of topics to support student achievement.

5.3 Future Directions

The current study extends the study of algebraic interventions for students with disabilities. Furthermore, I extend the literature base of precision teaching and, more specifically, frequency building to application beyond simple computations, such as addition, subtraction, and multiplication (Gist & Bulla, 2020). Given the sequential nature of procedural knowledge in mathematics, additional research should extend the current investigation to two-step and multistep equations. In addition to asking questions about the effects of frequency building on incorrect and correct notations, research may also be designed to answer questions about how fluent performance may relate to generalization and application to novel problem types. Such research could bolster not only the importance of behavior fluency, but also may bolster theories that posit the iterative theory of procedural and conceptual knowledge in mathematics (Rittle-Johnson et al., 2015). To better understand how procedural and conceptual knowledge is developed by frequency building, validated and previously researched measures of such knowledge may be incorporated into future iterations.

Additional research may investigate the dosage of the intervention. The current study used daily (when possible) practice, but future research may investigate whether similar results emerge from varying the length, time, and amount of practice. Similar investigations in reading fluency (e.g., Ross & Begeny, 2015) may provide relevant models. Scaling up frequency-building research to randomized controlled trial designs may also provide significant insight into the broad effectiveness across varied populations. Similar research has scaled up fluency-based methods grounded in precision teaching with significant success (Greene et al., 2018; Johnson & Street, 2004; Sawyer et al., 2021). Barriers to scaling up research may include the specific nature of 1:1 feedback delivered in the current intervention, as well as the availability of personnel to deliver feedback individually. Therefore, pursuing studies comparing the efficacy of 1:1 delivery to whole-group delivery using self-directed feedback methods remains a critical step in further isolating active ingredients and better understanding how to maximize positive results.

The graphical display of results in the current study provided unique advantages and should be used in future research. Instead of relying on traditional visual analysis, the current study used the SCC to guide both instructional decision-making and the evaluation of efficacy. Use of the SCC prevented the manipulation or inaccurate depiction of graphs. Graphs that do not adhere to graphical standards remain prevalent in special education single-case research and bias interpretation of the results (Kubina et al., 2021). Future use of the SCC in research could more readily allow for comparisons across studies, especially related to the development of students' fluent performance.

Finally, perceptions of timing as being related to math anxiety may need to be further explored, understood, and countered to promote widespread uptake of frequency-building procedures. Researchers posit that mathematics anxiety influences performance and development of higher-level math skills (Dowker et al., 2016). Some researchers associate mathematics anxiety with timed tests, arguing explicitly for the removal of timing in favor of methods that instead promote number sense (Boaler, 2015). Educators may thus be cautious of using timing, given concerns about the hypothesized relationship between timing and mathematics anxiety. However, mathematics anxiety may also result from a lack of practice or adequate skill development; thus, removing timed practice may exacerbate mathematics anxiety (Dowker et al., 2016). In sum, timed practice may help students develop skills that would lessen math anxiety by achieving fluent levels of performance (VanDerHeyden & Codding, 2020). Better understanding teacher conceptions of timing and the impacts of professional learning on misconceptions related to timing may improve social validity and narrow the research-to-practice gap.

5.4 Conclusion

The present study demonstrated powerful, positive effects of a frequency-building intervention on three participants' ability to quickly and accurately solve one-step equations. The efficient and dramatic improvements provide evidence that frequency building can be successfully embedded into higher-level math tasks for students with disabilities. Requiring students to write each step of the task analysis within the algebraic procedure created a sensitive measure of behavior and facilitated the delivery of precise feedback. Data indicated students generalized their skills to varied presentations of one-step equations. Future research may continue to consider how to understand the relationship between frequency building and generalization or application. Expanding the work to other higher-level algebraic processes would also target a much-needed area of research for students with disabilities in secondary mathematics.

Appendix A Description of Task Analyses

Author	Number of Steps	Task Analysis
Baker et al., 2015	10	1. Student will read the equation aloud. 2. Student will match the given variables and function with the corresponding colored box. 3. Student will move the function and flip it to the corresponding colored box. 4. Student will move the number variable to the opposite side of the equation. 5. Student will type in calculator the new equation. 6. Student will write the answer in the gray text box. 7. Student will check answer by placing the original equation in corresponding box. 8. Student will type in calculator equation with answer for Y. 9. Student will write in answer from calculator. 10. Student will confirm if answer in corresponding white text box matches original sum.
Chapman et al., 2019	9	1. Pointing to the sum on a visual aid of the equation when asked, "How many objects do you need?" 2. Moving a red marker to the sum on the equation 3. Counting the number of items in a container and finding this known number on the equation when asked, "How many objects do you already have?" 4. Moving a green marker to the known number on the number line 5. Counting to the sum with materials when asked, "How many more objects will you need to get?" 6. Selecting the number counted 7. Putting correct number for x on the equation 8. Puts correct number in container 9. Solving for x by writing the number
Creech-Galloway et al., 2013	32	1. Label side "a" on triangle. 2. Label side "b" on triangle. 3. Label side "c" on triangle. 4. Plug "a" into equation. 5. Put 2 (squared) by "a." 6. Plug "b" into equation. 7. Put 2 (squared) by "b." 8. Write "+." 9. Write "=."

		10. Write “c2.” 11. Put value “a” into calculator. 12. Square “a” using calculator. 13. Record answer. 14. Clear calculator. 15. Put value “b” into calculator. 16. Square “b” using calculator. 17. Record answer. 18. Clear calculator. 19. Write “+.” 20. Write “=.” 21. Write “c2.” 22. Put “a” squared in calculator. 23. Press “+.” 24. Put “b” squared in calculator. 25. Press “=.” 26. Write answer down. 27. Write “=.” 28. Write “c.” 29. Press square root sign on calculator. 30. Write down answer. 31. Write “=.” 32. Write “c.”
Jimenez et al., 2008	9	1. Pointing to the sum on a visual aid of the equation when asked, “How many objects do you need?” 2. Moving a red marker to the sum on the equation 3. Counting the number of items in a container and finding this known number on the equation when asked, “How many objects do you already have?” 4. Moving a green marker to the known number on the number line 5. Counting to the sum with materials when asked, “How many more objects will you need to get?” 6. Selecting the number counted 7. Putting correct number for x on the equation 8. Puts correct number in container 9. Solving for x by writing the number
Long et al., 2020	9	1. Represent x. 2. Represent the constant (if applicable). 3. Represent the sum/difference/product. 4. Bring out the inverse of the constant (get x by itself). 5. Apply inverse to the other side. 6. Simplify. 7. Evenly distribute. 8. Write down the answer. 9. Clear tiles.

Root & Browder, 2019	10	1. Read problem. 2. Circle groups. 3. Label equation. 4. Circle numbers. 5. Fill-in equation. 6. + or – 7. Use my rule. 8. Make sets. 9. Solve. 10. Write answer.

Appendix B Exempt Determination

University of Pittsburgh | Institutional Review Board
Office of Research Protections

EXEMPT DETERMINATION

Date:	November 23, 2021
IRB:	STUDY21110019
PI:	Olivia Enders
Title:	Evaluating the Effects of an Algebraic Frequency-Building Intervention for Students with Disabilities
Funding:	None

The Institutional Review Board reviewed and determined the above referenced study meets the regulatory requirements for exempt research under 45 CFR 46.104.

Determination Documentation

Determination Date:	11/23/2021
Exempt Category:	(1) Educational settings

Determinations:	No research activities may begin until a Modification is submitted to and approved by the Pitt IRB providing documentation of site permisson from an Administrator level employee of the educational site.
Approved Documents:	• STUDY21110019 Social Validity Interview Questions.docx, Category: Data Collection; • STUDY21110019 Student Info Sheet.docx, Category: Data Collection; • STUDY21110019 Daily Data Recording.docx, Category: Data Collection; • Enders HRP-720 - WORKSHEET - Exemption_Educational Strategies_Version_0.01.docx, Category: IRB Protocol; • Enders Introductory Script for Students.docx, Category: Recruitment Materials; • Enders Introductory Script for Teachers.docx, Category: Recruitment Materials; • STUDY21110019 - Parent Consent Letter_Version_0.01.docx, Category: Recruitment Materials;

If you have any questions, please contact the University of Pittsburgh IRB Coordinator, Amy Fuhrman.

Please take a moment to complete our Satisfaction Survey as we appreciate your feedback.

Appendix C Parent/Caregiver Informational Letter and Consent Form

University of Pittsburgh | School of Education

Dear Parent,

Hello! My name is Olivia Enders, and I am a doctoral candidate at the University of Pittsburgh School of Education. I am conducting a research study on interventions targeting algebraic equations. I am interested in studying how to help students solve equations problems faster, and more accurately. [Your student] has been recommended by [NAME] as being a student who can benefit from participation in this study. If you provide permission, [student] will work with me during their [intervention block and/or math intervention period] for 1-3 sessions of forty-minute interventions, and afterwards, approximately ten minutes per day, to practice their single-step equation solving skills. I will be evaluating how timed practice effects their problem-solving, their application to more difficult problems, and their thinking about mathematics equations. Practice will consist of working in timed trials, and being provided feedback on their work. I will also be taking two audio recordings of students describing how they solve problems, at both the beginning and the end of the study. There is no compensation provided for research. The research project is estimated to last for approximately 10 weeks. You can withdraw your permission to have your child from the study at any time.

Confidentiality and security are an utmost concern when I conduct research. It is important to me that you fully understand that your child's identity will be kept as confidential as possible. Upon admission into the study, your child's name will be removed and replaced with a unique identification number. Audio recordings will also be stored with this unique identification number. While there is always a risk of breach of confidentiality, all files and data collection documents will be kept in locked drawers or on locked folders in a password protected laptop. As a result of participation, your child may benefit by improving their ability to quickly and accurately solve math problems.

Your child's participation in this project is voluntary, and your child can withdraw from the study at any time. Participation will not affect his/her grade. If you have any questions or concerns about the study, please feel free to contact me, Olivia Enders, at (717)-574-8735 or at OGE2@pitt.edu. If you have questions about your rights as a research subject, please contact the Human Subjects Protection Advocate at the University of Pittsburgh IRB Office, 1-866-212-2668.

If you would like your child to participate, please sign the attached form. I look forward to the potential to work with [NAME].

Sincerely,

Olivia G. Enders, M.Ed, BCBA
Graduate Student Researcher
University of Pittsburgh School of Education

Parental Permission for Child to Participate in Research

You are being asked to provide permission for your child to participate in a research study.

Before you agree, the investigator must tell your child about (i) the purposes, procedures, and duration of the research; (ii) any procedures which are experimental; (iii) any reasonably foreseeable risks, discomforts, and benefits of the research; (iv) any potentially beneficial alternative procedures or treatments; and (v) how confidentiality will be maintained.

Where applicable, the investigator must also tell your child about (i) any available compensation or medical treatment if injury occurs; (ii) the possibility of unforeseeable risks; (iii) circumstances when the investigator may halt your child's participation; (iv) any added costs to your child; (v) what happens if your child decides to stop participating; (vi) when your child will be told about new findings which may affect your willingness to participate; and (vii) how many people will be in the study.

If your child agrees to participate, your child must be given a signed copy of this document and a written summary of the research.

You may contact Olivia Enders at OGE2@pitt.edu any time you have questions about the research.

You may contact the Human Subjects Protection Advocate at the University of Pittsburgh Institutional Review Board at 1-866-212-2668 if you have questions about your rights as a research subject.

Your child's participation in this research study is voluntary, and your child will not be penalized or lose benefits if you refuse to participate or decide to stop.

By signing this document it means that the research study has been described to your child orally, and that your child voluntarily agrees to participate.

________________________________ ____________

Signature of Parent/Guardian Date

Appendix D Research Project Script

Research Project: Evaluating the Effects of an Algebraic Frequency-Building Intervention for Students with Disabilities

Introductory Script

Hi, good (morning/afternoon)! The purpose of this research project is to figure out how to best help you in pre-algebra/algebra. I want to see how short periods of practice can help you solve equations faster and more accurately! You have been chosen by your (teacher/principal) to be considered to be a part of this study. Your parent's permission is required for participation in this study. The first few days of this project will take the entire intervention period, about 45 minutes. Afterwards, the project will last about 10 minutes each day, during which you will be practicing If you would like to continue to be in this project, you might improve your speed and accuracy in algebra. There is a risk that there will be a breach of confidentiality, however, we will take precautions to protect against this risk. We won't be sharing your name with anyone and your participation won't hurt your grade in class. We will maintain your confidentiality by assigning you a number and using that number on your work. Your initial referral information will be stored electronically in a password-protected file on a password-protected computer, and will be destroyed upon completion of the study. You will use your number on our daily work. You don't have to be in this study, and you can decide that you do not want to be in the study at any time. Your work may be used for future research, but it won't have any identifying information on that work. The study is being conducted by Ms. Olivia Enders, who can be reached at 7175748735, if you have any questions.

Appendix E Screening Assessment Addition Sheet Example

1-BF-A1 Slice 16.3

AIM: 80-100 digits correct with less than 2 digits incorrect/60 secs

5 + 5	8 + 5	1 + 3	4 + 1	7 + 9	8 + 1	9 + 1	2 + 8	2 + 6	2 + 1	(15)
7 + 5	4 + 9	6 + 8	3 + 6	2 + 2	6 + 5	5 + 9	3 + 7	7 + 0	9 + 0	(31)
5 + 8	8 + 4	3 + 2	0 + 4	6 + 3	3 + 3	5 + 1	6 + 7	4 + 8	7 + 8	(46)
7 + 7	5 + 0	6 + 1	4 + 4	0 + 8	9 + 7	8 + 2	4 + 7	5 + 7	5 + 3	(61)
6 + 6	3 + 8	3 + 4	2 + 3	6 + 2	5 + 4	4 + 3	3 + 9	8 + 3	6 + 9	(76)
8 + 7	4 + 6	0 + 6	0 + 2	2 + 7	3 + 5	7 + 4	2 + 5	4 + 5	7 + 2	(89)
8 + 8	5 + 6	6 + 4	3 + 0	3 + 1	7 + 3	5 + 2	9 + 3	9 + 6	1 + 2	(105)
1 + 4	7 + 6	7 + 1	2 + 9	9 + 2	1 + 0	9 + 4	8 + 6	9 + 5	4 + 2	(121)

Appendix F Screening Assessment Example

BxSx

$x \div 4 = 4$	$x + 3 = 8$	$x - 2 = 2$	$x \cdot 9 = 27$

(29)

$x \cdot 9 = 45$	$x + 6 = 7$	$x - 8 = 8$	$x \div 5 = 9$

(59)

$x - 3 = 5$	$x \cdot 4 = 32$	$x \div 5 = 1$	$x + 8 = 14$

(88)

Appendix G Initial Practice Sheet: Vocabulary Definitions

Vocabulary To Know

Word	**Definition**	**Example**	**Non-Example**
Variable	A symbol, usually a letter, that represents a number or quantity we do not know	$x + 8 = 7$ $8x$ $y - 3 = 8$	$8 + 2 = 1$ $8(5 + 4)$ ½
Inverse operation	An operation that reverses the effect of another operation	Addition is the inverse of subtraction. Subtraction is the inverse of addition. Multiplication is the inverse of division. Division is the inverse of multiplication.	Multiplication is NOT the inverse for addition. Division is NOT the inverse for subtraction.

Appendix H Initial Practice Sheet: Vocabulary in Context

<u>Vocabulary Check</u>

Circle the variables in the problems below. Some problems may not have variables.

5x 3 – 2 x + 6 6x - 3

4 * 5 x ÷ 8 3 – x ¼

Write the inverse operation for each expression below.

X – 5 _____ x * 3 ____ x ÷ 12 ____ x + 9 _____

x ÷ 2 _____ x + 10 ____ X – 1 ____ x * 10 ____

Appendix I Instruction Sheets

Steps for solving one-step addition equations:

1. Write notation for inverse operation (subtraction) below operation in the problem.	
2. Write number being moved to opposite side of equation below the equation.	
3. Write notation for inverse operation on opposite side of equation.	
4. Write number being moved below opposite side of equation.	
5. Write variable.	
6. Write equals sign.	
7. Write correct number (answer) to new.	

Practicing Independently (Addition)

$x + 5 = 7$

$x + 5 = 11$

$x + 3 = 8$

$x + 2 = 7$

$x + 4 = 13$

$x + 3 = 10$

$x + 8 = 16$

$x + 8 = 9$

Appendix J Daily Data Recording

STUDY21110019: Daily Data Recording

Student Unique Participant ID: ___________

Date	Phase	Correct Notations	Incorrect Notations

Phases Key
B= Baseline = Intervention

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