Implementation of Metacognitive Practices in Secondary Mathematics to Increase Enrollment of Low SES Students in Advanced-Level Courses

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The purpose of this study was to determine effects of implementing metacognitive practices on the engagement of students in secondary mathematics courses. The study was guided by two primary inquiry questions that focused on the current implementation of metacognitive practices in the mathematics courses as well as the perceived change in student engagement while using metacognitive practices during problem-solving experiences.

This study analyzed qualitative data gather via: teacher interviews, observations of lessons, and focus group discussions. Teachers participated in workshops on metacognitive practices and then implemented them into three of their lessons. The data revealed that before the workshops, the teachers did not fully use metacognitive practices in their instruction. However, through the professional development, data suggest that the teachers were able to adjust their instruction to include metacognitive problem-solving frameworks. All three teachers in this study perceived an increase in their students' engagement while using metacognitive frameworks for problem solving, though all three teachers reported a significant time investment needed to properly implement the problem-solving framework. The study also proposes next steps on how to leverage the perceived engagement due implementing metacognitive practices for improving the system toward increasing the enrollment of low SES students in advanced-level mathematics courses.

Table of Contents

List of Tables

List of Figures

1.0 Introduction to the Problem of Practice

Enrollment in higher-level or Advanced Placement (AP) mathematics courses provide students with experiences similar to college-level courses as well as develop the necessary critical thinking skills needed for success in post-secondary education (Kettler & Hurst, 2017). Additionally, AP courses provide students with an opportunity to attain college credit and have a greater chance of gaining college acceptance with the highly regarded AP courses on their high school transcript (Warne, Larsen, Anderson, & Odasso, 2015). Students who take more rigorous coursework in high school have a greater chance of success in their post-secondary studies and this effect is magnified for economically disadvantaged students (Long, Conger, & Iatarola, 2012). Moreover, students who enroll in advanced level mathematics courses in high school have a greater chance of pursuing highly paid STEM careers (Sadler, Sonnert, & Hazari, 2014).

Studies have found that enrolling in advanced courses, like AP, lead to no difference in college enrollment rates when comparing low socioeconomic (SES) students and their higher SES peers, effectively eliminating the achievement gap in this aspect (Bailie & Wiseman, 2018). However, students coming from lower SES households face barriers that prevent academic achievement (Wyner, Bridgeland, & DiIulio, 2007). Though AP course enrollment presents many benefits, my district's mathematics department is experiencing a drop in enrollment in AP Calculus AB, AP Calculus BC, and AP Statistics.

1.1 Local Context

At my high school, located in Western Pennsylvania, our mathematics department has seen a 63% decrease of enrollment in our advanced math courses over the past four years with a negative effect on our economically disadvantaged student population. Our student body consists of 20% low SES(socioeconomic status) students (students who qualify for free or reduced lunch), but this portion of the population only represents 3% of students enrolled in advanced math courses. In the 2021-2022 school year, the student body contained 88% white, 4% Hispanic/Latino, 4% Multiracial, 3% Asian, 1 % Black/African American, and less than 1% Native American. All students take the Algebra 1 Keystone exam after completing their Algebra 1 course. In Pennsylvania, the Keystone exams are given after state-specified courses (Algebra 1, Biology, Literature) and are used for teacher and school accountability as well as graduation requirements for students looking to receive a high school diploma from a public school district.

Students who score “proficient” or “advanced” on this assessment are more prepared for higher-level mathematics courses. Over the past five years, 73% of our low SES students scored either proficient or advanced on their first attempt of the Algebra 1 Keystone exam, suggesting that they have the prerequisite mathematics ability to enroll in higher-level mathematics courses later in their high school career. This suggestion assumes that Keystone exam proficiency is an indicator of future success in advanced math courses. Though this assumption may be debatable across the state of Pennsylvania, local data suggests a correlation. Over the same time, 82% our non-disadvantaged population scored either proficient or advanced on their first attempt.

The mathematics department consists of six full-time math teachers (five in the fall of 2021 due to a maternity leave), who service 575 students in grades 9-12. These teachers consist of two females (7-12 years of experience each) and four males (25-32 years of experience each). Our students are required to take at least four math courses during their high school career as a graduation requirement.

These teachers specialize in specific areas of mathematics, but each teacher teaches a variety of students ranging from each of the four grades in the building. Our department offers ten different classes, with various leveled tracks including three AP courses. The leveled tracks consist of three levels (College Prep, Honors, and Advanced Placement). Students are placed in courses based on their previous performance in mathematics courses. Each course has its own prerequisites, but in general, students remain in their track, if they receive a grade between an A-C. They have the option to move up if they receive an A and are moved down if they receive a D.

During observations and walkthroughs, administrators observed a large portion of students disengaged in their math courses. This disengagement manifested in a) students not talking with their peers about the problems at hand; b) quickly giving up on issues when they face difficulty; c) not engaging in conversations about how the math concept relates to their lives; and d) a general lack of participation. Conversation with the mathematic teachers confirmed the perceived lack of engagement by the students. Teachers have recently expressed how they see their students as less engaged than they had been in the past. For the disengaged students, our math instruction is teacher-centered. The teachers do not conduct practical formative assessments of their students, nor do they use their assessments to guide their future teaching. These teachers also struggle to provide timely and meaningful feedback. Most assessment feedback consists of marking whether a problem is correct or not and does not provide students with any guidance for how to improve their learning.

Over the last few years, the mathematics department has made it a priority to tighten prerequisites for their courses. These prerequisites are consistent with other departments and are in place with hopes to ensure that students take courses where they will have the greatest chance of success. Though the prerequisites in the mathematics department are not different from other departments, they seem to have a different effect. These requirements contribute to an overall teacher attitude of making honors level and advanced mathematics courses exclusive and only available to a select portion of the student population.

Some mused that a decrease in enrollment in advanced courses could indicate a shift in the clientele of the student body, which is compounded by some teachers having a deficit view of students of low SES backgrounds. However, the consistent enrollment of advanced courses in other departments in our high school negates this musing. Though one may see some fluctuations in enrollment in the other departments' AP courses, none have the drastic misrepresentation of low SES students as seen in the mathematics department.

In many of the mathematics classrooms, most of the students who engage in their learning have historically performed well in mathematics. Students who come to the high school with an aversion to mathematics often do not change their opinion. Much of our math instruction depends on students' ability to complete necessary practice at home instead of in class. This can prove challenging for students who work after school to provide additional income for their family or students that need to provide childcare for their younger siblings so that their parent/guardian can work an additional shift. Forcing the bulk of the practice to happen at home in our math courses exposes the inequities our students face within their home environments. This leads to giving a portion of our student population, those with supportive home environments conducive to completing homework, a distinct advantage over students with additional responsibilities outside of school. This disadvantage marginalizes an already marginalized population in that students from low SES families do not have the same access to success as their peers in our mathematics curriculum because of the dependency placed on out-of-class practice.

1.1.1 COVID-19 and the Local Context

In March of 2020, education was forever changed with COVID-19 pandemic. The school district, like all other schools in the area and many across the country, closed its doors on March 13th, 2020 and moved students to a virtual learning format. Luckily, our district was already one-to-one with iPads and the district made a concerted effort to take mobile hotspots to homes to ensure that all students had access to internet and consequently access to their learning. Even with these measures, students and teachers experienced difficulties in navigating the new learning environment.

The school district remained in a remote setting for the remainder of the 2019-2020 and for the month of September to start the 2020-2021 school year. Starting in October, students attended school in a hybrid format where one half of the students were in school for two days a week and all students were virtual on Wednesday. This hybrid format continued until April of 2021. Since April of 2021, all students have had the opportunity to attend school in-person fulltime. This upheaval in the educational system has led to many challenges for both teachers and students, challenges that will not be discussed here. However, this is pertinent to the context in that the challenges of the COVID-19 pandemic have shown to affect students' engagement in their classes and their achievement in their courses in high school.

1.2 Problem of Practice

The mathematics department at my high school underserves a marginalized population by enrolling a disproportionately lower number of low SES students compared to their non-disadvantaged peers. This problem is complex and will take many small iterative changes to show significant improvement. The aim of this study and the drivers involved will be discussed in more detail in a later section, however it is important to understand a brief theory of improvement here. Fixing this problem will take time and consistent change in the direction of improvement. To move toward improving this problem, one cannot focus on the low enrollment itself, but the root causes to this problem. One of those root causes is a lack of student engagement.

Student engagement influences student achievement in mathematics classes. Researchers have found that students who exhibit either affective (interest in learning mathematics concepts), behavioral (what students do during the learning process, e.g. how they discuss concepts with peers), or cognitive (student perseverance during the problem-solving process) engagement achieved more in their secondary math courses than their disengaged counterparts (Fung, Tan, & Chen, 2018). Students can easily lose interest in mathematics and feel that it can be tedious (Burkett, 2002). Teachers find that students become frustrated when learning math and quickly give up on difficult tasks as a result of their disengagement (Skilling, Bobis, Martin, Anderson, & Way, 2016). This frustration can lead to a form of math anxiety that can continue to negatively affect student interest and performance which limits the number of students pursuing STEM-related fields in the United States. (Beilock & Maloney, 2015). To increase student enrollment in higher-level math courses, Dev (2016) found that students are more open to future learning opportunities when they have affective engagement (interested in learning) in mathematics. Student engagement in mathematics relates to their success in their problem-solving (Slavin, Lake,

& Groff, 2009) and having a process or method to follow during problem-solving experiences, like metacognitive strategies, can lead to student success (Yimer & Ellerton, 2010).

Metacognition is a process of evaluating one's understanding and thinking about how one thinks. Metacognition also refers to the awareness and regulation of learning and problem-solving actions (Özkubat, Karabulut, & Özmen, 2020). A student's ability to self-regulate through metacognitive practices leads to higher problem-solving achievement (Özcan, 2016) and leads to effective storage of problem-solving strategies into long-term memory (Selçuk, Sahin, & Açikgöz, 2010). However, many students experience difficulties solving advanced mathematical problems (Pol, Harskamp, Suhre, & Goedhart, 2008). The difficulties that students experience can either come from a lack of content knowledge or a lack of effective problem-solving skills, though Mathan & Koedinger (2018) pose that a lack of strategic reasoning more often leads to student difficulties than a lack of content knowledge. Students that exhibit greater problem-solving skills than their peers tend to possess self-regulating metacognitive practices (Pol, Harskamp, Suhre, & Goedhart, 2009).

Problem solving can involve a complex process of mapping conceptual and procedural knowledge to a variety of problem states, or checkpoints along the path to a solution (VanLehn, Siler, Murray, Yamauchi, & Baggett, 2003). Problem states allow students to verify their performance along their problem-solving journey and make corrections when needed. Content knowledge acquisition supports the problem-solving process, but students cannot effectively solve applied mathematics problems on content knowledge alone, they need an understanding and working ability of how to apply their knowledge (Mathan & Koedinger, 2018).

Many secondary mathematics classrooms present problem solving by demonstrating examples. Though students can learn through observing the demonstration of examples, learning increases when the demonstrations contain decreasing instruction that transfers the ownership to the student through using self-regulated metacognitive practices (Moreno, 2006). In contrast to observing proper problem-solving procedures, students acquire new knowledge when they arrive at an impasse and detect errors in their thinking (VanLehn et al., 2003). Impasses can rarely occur through observation but rather through student practice.

Secondary teachers do not often use metacognitive practices in their instruction (Dignath & Büttner, 2018). Initially, teachers exhibit reluctance toward introducing new instructional strategies into their classrooms, however, teachers that do implement new instructional strategies, such as metacognitive practices, find an increase in their motivation (Selçuk et al., 2010). The increase in teacher motivation may not be a primary consequence of the new strategy but a result of observing deeper student learning. Teachers become more motivated when they observe their students acquire new content knowledge and demonstrate success in their problem-solving experiences (Özcan, 2016).

Figure 1, below, attempts to simplify the theory behind the approach of this study. By training the mathematics teachers to implement metacognitive practices in their classrooms, teachers will teach their students how to properly develop, foster, and appropriately use metacognitive skills during their problem-solving. This practice will lead to an increase in metacognition in our mathematics students. Increased metacognition could better engage the students in their mathematical learning which will lead to greater student achievement. With great student achievement, more students will have the opportunity to enroll in advanced-level mathematics courses. If all students have a greater opportunity to enroll in advanced mathematics courses, then the students enrolled in those courses will become a better representation of the student population which will lead to more low SES students having the opportunity to enroll in these advanced classes and reap all the benefits of the classes discussed above.

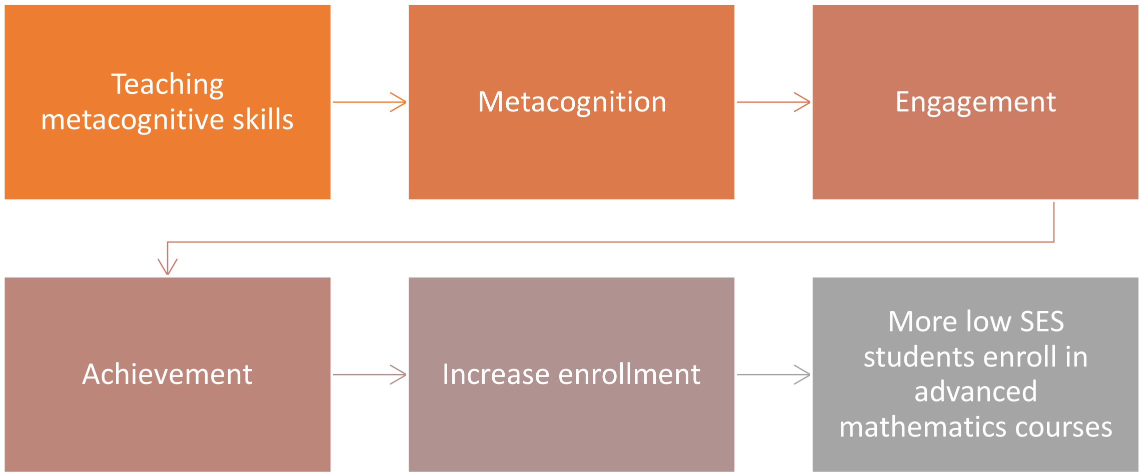

Figure 1 Theory of Improvement Graphic

1.3 Inquiry Questions

This study will be guided by the following questions:

1. To what extent are mathematics teachers at my high school currently using metacognitive practices in their gateway courses?
2. How do teachers implementing metacognitive practices in their classes perceive the effects of these changes on student engagement?

This theory, though simplified in a concise graphic, is anything but simple and the execution will take many changes in the direction of improvement.

1.4 Positionality Statement

This problem and context are especially important to me because of the investment I have in the community. First, I am a graduate of my high school, where I work, and received all my K-12 instruction from the school district. Because of the quality education I received from this district, it was always my professional goal to return to the school and be a part of providing quality education for the students of the community where I was raised. After teaching in a couple other districts, I was able to return to my current school where I taught mathematics and physics for six years. I transitioned out of the classroom as the assistant principal for four years and am currently in my first year as principal of my school. In addition to my vocation, I am also an active member of the community, and my own two children attend our district's elementary school. It is for these reasons that I am fully invested in the success of my community's high school, and I am willing to see the growth and change, started by this study, through to completion.

2.0 Review of Supporting Literature

2.1 Student Engagement

Many research studies have centered on student engagement and the factors that may lead to increased engagement. The literature supports the notion that student engagement leads to achievement. Though the need for student engagement in the classroom may be clear, engagement includes many contributing factors. The following will investigate these factors through the literature, specifically around secondary mathematics.

2.1.1 Affective, behavioral, and cognitive engagement

Fung et al. (2018) conducted research that focused on the relationship between student engagement and achievement in secondary mathematics. The study divided engagement into three domains: affective, behavioral, and cognitive engagement. The study showed that, generally, students with greater engagement experienced higher achievement. Additionally, students that showed higher engagement in two of the three domains showed even more significant performance than students with higher engagement in only one domain. The authors proposed that these findings support the need for a whole-school based approach to promote student achievement. Though the idea that student engagement leads to achievement may be elementary, solving the problem of engagement is complicated.

Openness in problem-solving and interest in mathematics learning contribute more to student achievement than general perceptions of schooling or perseverance in problem-solving

(Fung et al., 2018). Additionally, "…all three kinds of student engagement were positively related to their mathematics achievement" (p.826). However, students participating in activities in and out of school only made a small difference in student achievement. Cognitive engagement had the most significant effect on achievement. "[T]he results suggested that a whole-school approach that enhanced students' engagement in different aspects might be necessary to raise their mathematics achievement" (p. 827). The authors suggested that teachers, "need to intellectually challenge their students in class, extracurricular teachers need to encourage students to participate in exciting mathematics activities, and guidance teachers need to foster in students a sense of belonging to their schools" (p. 827). Teachers can engage students in their learning by promoting math-related clubs and activities available outside of the school day.

2.1.2 Student Adaptability

Students encounter new challenges each day, and those able to adapt to the new challenges can appropriately navigate the situation by accessing and utilizing their prior knowledge. Collie and Martin (2017) describe adaptability as "the capacity to adjust one's thoughts, behaviors, and emotions to manage changing, new, or uncertain demands" (p.355). Collie and Martin conducted a study that focused on the specific adaptability of students in their mathematics courses. Both students and teachers reported on observed adaptability throughout several challenging situations.

From their research, Collie and Martin (2017) showed that student-reported adaptability predicted student engagement as well as achievement in mathematics. The study found a positive correlation between student-reported adaptability and engagement as well as both student- and teacher-reported adaptability with achievement. This research demonstrates a need for students to develop adaptability so that they can handle a variety of situations that they may encounter in their future.

Collie and Martin's (2017) study calls practitioners to work toward improving student adaptability because of its close association with both student engagement and achievement in mathematics. A shift in the approach of teachers would require teaching students to recognize changing or new situations in a mathematics classroom. This ability to understand the change would allow students to shift their thoughts and behaviors to suit the situation. Students need to have the opportunity to practice adapting their problem-solving skills to challenging situations - opportunities that should arise in their mathematics classrooms.

Teachers play a significant role in a student’s engagement in learning. "What teachers think about engagement influences the teaching practices they adopt, their responses to students, and the efforts they make in the classroom" (Skilling et al., 2016, p. 545). Skilling et al. demonstrated that a teacher's self-efficacy, or the internal belief that one can accomplish a specific task, plays a role in their willingness to attempt new forms of intervention to engage their students. If a teacher does not think they will be successful with a new intervention, they are unlikely to try. Teachers that felt unable to engage their students were less likely to try new interventions in their classrooms.

2.2 Metacognition and Secondary Mathematics

Metacognition can has been defined in a variety of ways. Garofalo and Lester (1985) split the definition of metacognition into two parts: “(a) knowledge and beliefs about cognitive phenomena, and (b) the regulation and control of cognitive actions” (p. 163). Cognition refers to an understanding based on factual knowledge within a conceptual framework (Kahan, Cooper, &

Bethea, 2003), and metacognition is often referred to as the "think about thinking" (Flavell, 1979). Cognition and metacognition are tightly related in that "one way of viewing the relationship between them is that cognition is involved in doing, whereas metacognition is involved in choosing and planning what to do and monitoring what is being done" (Garofalo & Lester, 1985, p. 164). Metacognition also involves higher order thinking that enables understanding, analysis, and control of one's thinking while engaged in learning (Su, Ricci, & Mnatsakanian, 2016).

2.2.1 Knowledge of Cognition

The first aspect of metacognition that Garofalo and Lester (1985) illustrate is knowledge of cognition. The researchers split this aspect into three distinct parts: person, task, and strategy. Knowledge in the person category consists of what a problem-solver thinks about his-/herself and how he/she compares with peers. This category can also include the what the problem-solver believes about the concept they are learning, for instance, the preconceived notions a student has about mathematics.

Garofalo and Lester (1985) also describe how the knowledge in the task category describes what a problem-solver knows about a particular task. This includes any expectations they have based on past experiences as well as how difficult they perceive the problem to be. The task category leads to the third knowledge of cognition category of strategy. This metacognitive category consists of the problem-solvers knowledge of strategies, both general and specific, that can apply to a specific problem. The knowledge includes a problem-solver's understanding of how the strategies relate to the problem as well as its potential usefulness.

2.2.2 Regulation of Cognition

The second aspect of metacognition, regulation of cognition, describes how the problem-solver uses their prior knowledge to organize and monitor the strategies they choose to apply to the situation they wish to solve. Like the knowledge of cognition, the regulation of cognition consists of three categories (Garofalo & Lester, 1985). First, the planning category is where the problem-solver selects appropriate strategies and resources. This category can also include goal setting where the problem-solver articulates the primary goal they will set out to achieve (Dignath & Büttner, 2018). Through accessing prior knowledge, the problem-solver analyzes the problem and determines what additional resources are needed. Additionally, the problem-solver will make some intuitive estimation on the time it will take to complete the task at hand.

Monitoring is the second category of the regulation of cognition. In this phase, the problem-solver will begin to make conjectures, or predictions, about possible solutions. These conjectures will serve as guidance for the problem-solver (Garofalo & Lester, 1985). A key component of monitoring is pausing to re-evaluate and detect any errors that may occur. Through this re-evaluation, the problem-solver can determine if alternative strategies are needed to move forward successfully. One of the most important aspects of metacognition is when the problem-solver learns from their mistakes (Desoete, Roeyers, & De Clercq, 2003).

The third category of the regulation of cognition is evaluation. In evaluation, the problem-solver determines the reasonableness of the result they have and compare it to previously made predictions. Garofalo and Lester (1985) describe this process as the mechanism through which a learner will integrate their new knowledge into their prior knowledge and store the experience for use in a later problem-solving experience.

As illustrated in Figure 2, below, the three categories of the regulation of cognition should not be thought of as sequential. Instead, they are part of an iterative cycle where a problem-solver can move through the cycle multiple times during a problem-solving experience.

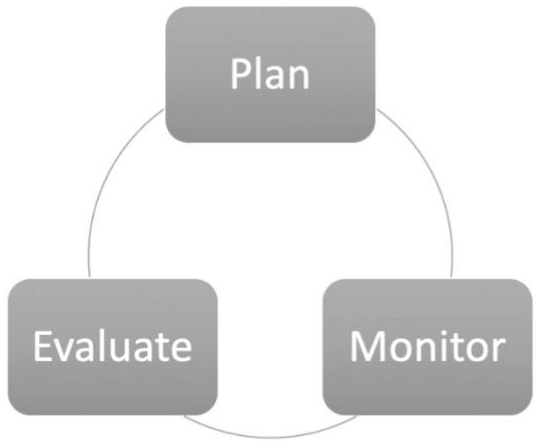

Figure 2 Depiction of the three stages of the regulation of cognition

2.2.3 Framework for Metacognitive Practices

Polya (1957) created a set of problem-solving techniques (summary in Appendix B). This exists as a framework for approaching any mathematical problem. This framework was the basis of work done by researchers such as Newman (1977) in studying the error analysis of students during their problem-solving process. In addition, Schoenfeld, (1985) expanded on this construct by studying how students articulate their problem-solving process and make sense of the decisions they make during their journey. Yimer and Ellerton (2010) built upon these researchers by developing a five-phase model for mathematical problem solving. The five phases *engagement, transformation-formulation, implementation, evaluation,* and *internalization* can be found with their descriptions in Appendix C. The model developed by Yimer and Ellerton (2010) provides a framework for problem-solvers to use while learning mathematics that applies the ideas and practice of metacognition.

2.2.4 Example of Metacognition at Work in Mathematics

To illustrate the aspects of metacognition discussed above, consider Figure 3. A student asked to complete task A would not require much planning or monitoring because the student would simply recognize the need to apply a known formula that allows the student to calculate the area of the triangle. Task B would require the student to access their prior knowledge and then devise a plan to use known strategies to solve the problem. However, this task would not require much regulation or monitoring since the plan devised would be rather straight forward and the combination of two known formulas. In contrast, task C would require a student to work through each of the phases of metacognition and possibly even repeat the cycle in Figure 1 multiple times until a reasonable result is reached. Task C does not require more metacognition simply because it is a more difficult question, in fact, the computation required is similar in all three tasks. The difference lies in the rigor and the required cognitive demand on the problem-solver. When problems are more cognitively demanding, students can increase their chances of success while implementing metacognitive practices (Desoete et al., 2003).

A. Find the area of a triangle that has a base of 7 cm and a height of 4 cm.
B. Alex wants to put a rectangular fence around his garden. He knows that one side of the rectangle needs to be 25 ft. If he wants to have an enclosed area of 500 ft^2, what length of fence will he need in total for the entire enclosure?
C. If you have a block of cheese (consider it to be a rectangular prism) and you slice off each corner, how many faces, edges, and vertices does the new figure have?

Figure 3 Mathematical Tasks

2.2.5 Benefits of Metacognition in Mathematics Classrooms

Using metacognitive strategies can provide benefits to students learning mathematics (Lan & Huy, 2021). Students able to use metacognition become more independent learners by developing an inner teacher that helps to guide them through their problem-solving process (Dignath & Büttner, 2018), which can lead to improved academic performance (Mapp & Kuttner, 2013). Using metacognition in mathematics classrooms can also engage disengaged learners (Hattie, 2009) and can even compensate for learning limitations experienced by students (Özcan, 2016). The success of using metacognitive strategies in mathematics classrooms is not limited to younger learners, but is also found to be successful in secondary mathematics (Desoete et al., 2003).

Artzt and Armour-Thomas (1998) studied teachers implementing metacognitive practices into their mathematics courses and found that the positive effects of metacognition were increased using cooperative learning. Additionally, Su et al., (2016) found that students refined their metacognitive skills by participating in solving problems that required higher-order thinking. These studies illustrate the role that a teacher need to take in fostering the metacognitive skills of their students.

2.2.6 Teachers and Metacognition

In the observation of both experienced and novice teachers, Artzt and Armour-Thomas (1998) found that students should be active participants in their problem-solving to reap the benefits of metacognitive practices. In these observations, the researchers found that students had the most success when the teacher facilitated the problem-solving experience instead of directing it. Mathematics teachers tend to value the idea of cognitive components of problem-solving, but often lack the knowledge of metacognition and are reluctant to promote it (Dignath & Büttner, 2018). Math teachers tend to teach cognitive strategies, like rote memorization, successful procedures, formulas, etc., but not metacognitive strategies (Ewijk & Büttner, 2013). Teachers need to "shift their vision of teaching from a solo act to interactional events that include students, the teacher, and the instructional task" (Tekkumru Kisa & Stein, 2015, p. 128) to help their students make sense of their learning and complete more demanding cognitive tasks.

Classrooms with high cognitive demand have common factors such as teachers attending to how students make sense of their mathematical tasks and how they make meaning of their experiences (Stein, Grover, & Henningsen, 1996). Additionally, teachers that are responsive to students' learning provide activities that allow students to demonstrate their thinking so that the teacher can help them learn to regulate their problem-solving processes (Levin, Hammer, Elby, & Coffey, 2013). However, using high-level tasks in the classroom does not guarantee complex cognitive reasoning by the students and they can experience difficulty in complex tasks (Stein et al., 1996). Stein et al. (1996) found that "the kinds of tasks that scholars and reformers have suggested as most essential for building students' capacities to think and reason mathematically are the very tasks that students had the most difficulty carrying out in a consistent manner" (p. 483).

During instruction, teachers can promote metacognition in their students in a variety of ways. Metacognition is not an innate skill, however, it is something that students can learn and refine through practice (Garofalo & Lester, 1985) First, teachers can indirectly promote self-regulated learning through encouraging cooperative learning and intervening when necessary

(Ewijk & Büttner, 2013). Cooperative learning has a significant impact on student learning (Slavin et al., 2009). During cooperative learning, teachers can facilitate the learning by listening and observing students share their thought processes with each other and providing guidance where needed.

In a case study of Railside High School, Boaler & Staples (2008) found that a multidimensional approach to mathematics had a positive effect on student learning. This approach called students to collaborate and promoted equity throughout the school. Students were more sensitive to their own and others' needs in their classroom, and teachers worked collaboratively with each other to continue promoting equity in their classrooms. In reflecting on the study, the researchers wrote, "The mathematical success shared by many students at Railside gave them access to mathematical careers, higher-level jobs, and more secure financial futures" (p.640).

Second, teachers can promote metacognitive skills in their students by having them activate prior knowledge (Artzt & Armour-Thomas, 1998). A teacher can prompt students to activate their prior knowledge by creating a cognitive conflict in a meaningful context. When a student is presented with a problem that provides them with a cognitive conflict that allows for multiple solutions, students will employ their metacognitive strategies to regulate their problem solving which will refine their skills.

Third, teachers can promote metacognition through problem-based learning (Pol et al., 2008). In this practice, teachers allow their students to take responsibility for structuring their learning by giving them freedom to make decisions. This is found to be most effective when a teacher finds the right balance between teacher-directed and self-directed learning (Pol et al., 2009).

Finally, mathematics teachers can promote metacognitive practices in their classrooms through using real-life contexts (Ewijk & Büttner, 2013). Students are more engaged in their learning when they feel the problems they are solving relate to their everyday lives (Skilling et al., 2016). By presenting problems in a real-life context, teachers are able to present the learning in diverse settings as well as provide a variety of ways to look at a problem (Ewijk & Büttner, 2013). As stated above, students need to have an active role in the problem-solving experience when employing metacognitive strategies, but this does not discredit the important role the teacher plays in the development of those metacognitive strategies.

2.3 Low-SES Students

2.3.1 Performance

There is no shortage of research on how a student's socioeconomic status affects their achievement in school. A direct relationship exists between a student's socioeconomic status and their academic achievement (Sirin, 2005). Though many factors affect academic achievement, socioeconomic status may be most effective in predicting a student's ability to succeed academically (Stewart, 2007). Even though low-SES can predict lower achievement when compared to non-disadvantaged peers, when given the right circumstances, low-SES students have the same opportunity for success as their peers (Sirin, 2005). Research finds that students from lower SES backgrounds are capable of achieving at the highest levels when the right supports are in place (Staats, 2016). Schools can provide these effective supports by first understanding and then attending to students' individual academic needs and goals (Hattie, 2009). When students have knowledge about the benefits of academic programs, they are more likely to receive support that allows them to make informed decisions about their academic future (Mapp & Kuttner, 2013). Schools need to provide support that attends to students need and enhances their strengths. Schools possess great power in influencing the success a student experiences. The experiences a student has in school can have lasting impressions on the student and their achievements in their academic future.

In a study of rural high school mathematics achievement gaps, Reeves (2012) found that "rural high school students in the United States (and elsewhere) have lower academic achievement than their nonrural counterparts" (p. 887). Reeves narrows his study to focus on the achievement gap present in mathematics, specifically in the final two years of high school. This study determined that the gap does not come from a variety of factors (availability of advanced math courses, evidence for track assignment by family background, and the quality of instruction), but from one: socioeconomic status. The study discovered that the gap did not appear in the 10^{th}-grade year; instead, it developed in the final two years of high school. The author proposes that this late development results from a lack of learning opportunities to learn advanced mathematics as opposed to deficiencies that the students accumulated in previous years. Reeves calls for family and close peers to aid in the motivation for students to take higher-level math courses and overcome the gap that exists between lower socioeconomic students and their peers.

2.3.2 Educator Bias

Biases are a natural part of being human and are not harmless on their own, however, the application of a bias, especially in an educational setting, can be harmful (Banaji & Greenwood, 2013). Teacher assumptions and biases can impede student growth and academic progress (Staats, 2016). Students look to their teachers for more than just the content they deliver. Students experience influence on their self-esteem and resilience from interactions they have with their teachers (Akin & Radford, 2018). Perceptions teachers have about their students can affect these interactions which can lead to an effect on their students' achievement. Students perform better when their teacher embraces their diversity and teaches to their strengths instead of trying a one-size-fits-all approach (Johnson & Willis, 2013).

2.4 Conclusion

Student success in secondary mathematics comes with many challenges and is, quite frankly, difficult. As the literature illustrates, a variety of factors play a role in student engagement and achievement in the secondary classroom as well as how the teacher approaches instruction. The problem of practice presented by this local context is a complex and intricate problem that possesses many contributing factors as well as opportunities for change. It is only through the research of the literature and the determination of the contributing factors of the problem that a practitioner can begin to develop a theory of improvement. The literature clearly demonstrates the need for the problem of practice to improve. Enrollment in advanced-level courses is believed to be paramount to future student success. This change needs to begin with the system and how we prepare our teachers to instruct in our secondary mathematics courses before students could enroll in advanced-level courses. Implementing a small change in how teachers use metacognitive practices in their mathematics classes can move the system in the needed direction to begin to fix the problem of practice.

3.0 Methods (Applied Inquiry Plan)

3.1 Definition of terms

Gateway Courses – these are courses that serve as the gateway or prerequisite for higher-level courses. In this context, this term will be used to describe courses that serve as a gateway to higher-level mathematics courses in the high school mathematics curriculum, specifically, Algebra 1, Algebra 2, and Geometry.

Metacognitive practices – these are practices used by students to think about their thinking in their learning. For this paper, metacognitive practices can be thought as, but are not limited to, students articulating their problem-solving ability, identifying where they went wrong in a problem-solving experience, and developing a plan to fix their thinking to avoid similar errors in the future.

Metacognitive tools – instructional tools that teachers can use to implement metacognitive practices in their lessons. Samples of these tools are in Appendices B-F.

3.2 Theory of Improvement & Driver Diagram

The previous review of literature demonstrated the importance of students enrolling in advanced-level mathematics courses during their high school experience. These data of my local context led me to my problem of practice (PoP): The mathematics department at in my high school underserves a marginalized population by enrolling a disproportionately lower number of low SES students compared to their non-disadvantaged peers.

My aim is to attempt to solve this problem by increasing the enrollment of the low SES student population in the advanced-level mathematics courses. This improvement will happen through targeted changes focused on the primary drivers of change that I will discuss below. From my system measures, I will determine not only if my change ideas promote positive change in these drivers, but also that I do not inadvertently cause negative change in other areas of my system as a result. Though seeing an increase in enrollment will take time, even years, there will be leading measures to help me determine if my change ideas are working. All the measures will help to determine if my iterative changes move my system one step closer to my aim.

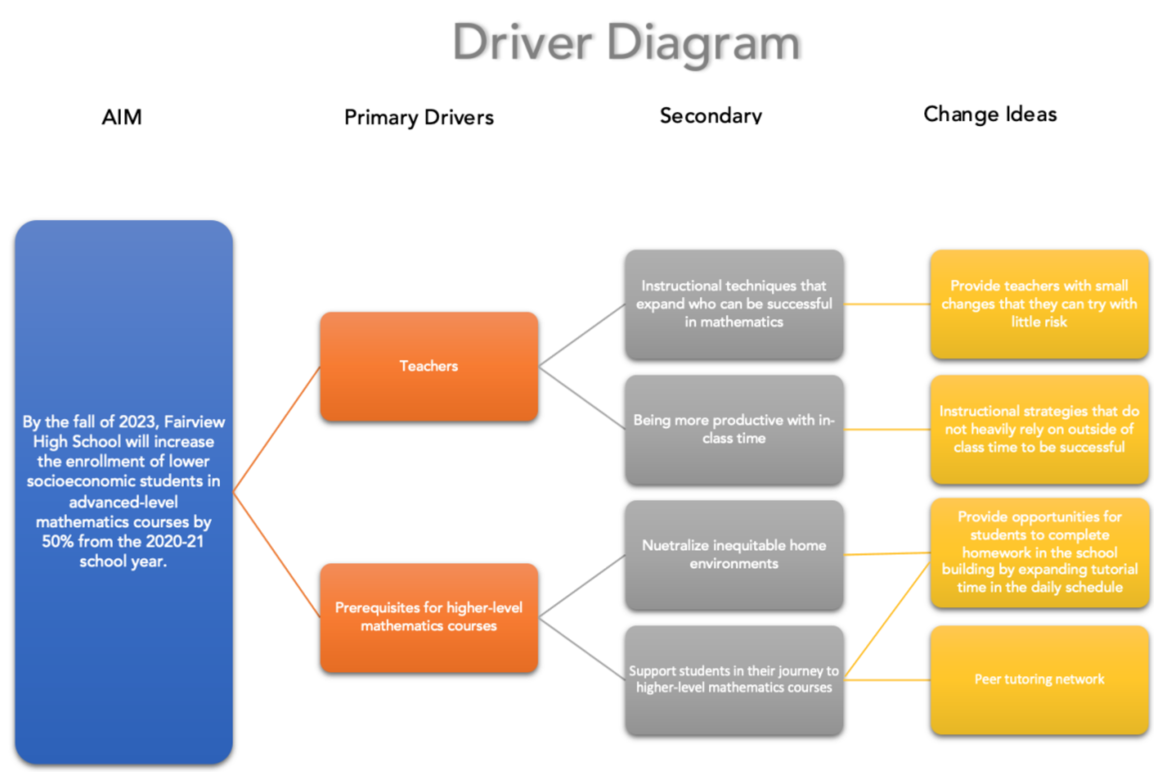

Figure 4 Driver Diagram

This PoP is a complex problem that needs multiple drivers to enact change toward accomplishing my aim. The two primary drivers entail two very important stakeholders in my PoP: teachers and students. Both teachers' perceptions of students' ability to have success in our mathematics curriculum as well as the students missing opportunities to enroll in higher-level mathematics courses contain areas in need of change.

Currently in our mathematics department, many of the teachers have a preconceived model of what a student should look like to have success in our mathematics curriculum. These perceptions typically result from their years of experience and noting the common attributes that our successful mathematics student exhibit. Teachers then assume that future students need to possess these same qualities for them to have equal success. Attributes such as, having a proclivity for mathematical reasoning and having a support structure that allows students to complete most of the practice at home with the support of their parents. Teachers noticing successful attributes of students does not present a problem. However, the assumption that future students need to have these same qualities does. Though one could argue that these qualities are proven to be successful, this line of thinking unintentionally introduces bias against a portion of our population that is not privileged in the same way. A growing portion of our population does not have the same support structure at home to support our mathematics department's reliance on meaningful practice needing to happen at home with a built-in support structure from their family. Many of our low-SES students do not have the same opportunities for substantial practice at home because of a variety of reasons reaching from providing childcare to younger siblings to going to work after school to help support their families' finances. Whatever the reason, an already marginalized portion of our population continues to be marginalized when it comes to success in our mathematics classes. The mathematics teachers of my context often meet the fact that not all students have an opportunity to complete their much-needed practice at home with an attitude of indifference and a claim that if a student wants to be successful in math, they need to do their homework.

The limiting perception that a student needs to complete necessary practice at home for success in our mathematics curriculum presents a challenging problem. This idea of this practice is not flawed itself, just the assumption that this is the only option for success. Not all students have the same support structure at home and a school should provide equal opportunities for all students to succeed. This misguided perception and assumption by the mathematics teachers can afford to change – a difference that will lead to increased access to advanced-level mathematics for one of our marginalized populations. Teachers' perceptions can move toward change in two specific areas: the need for stepping out of their comfort zone instructionally while understanding the diverse circumstances that students find themselves in and how multiple pathways to success in mathematics can exist.

Our mathematics teachers hesitate to step out of their comfort zone because of a fear of fear of failure and additional pressure for their students to perform well on standardized tests (i.e., Keystone exams, SAT, AP Exams, etc.). The district does not intentionally place additional pressure on teachers over standardized test, however, to claim the pressure isn't real would be an oversight. Historically our students perform very well on standardized tests, so naturally, teachers feel pressure to maintain their elevated status. This pressure feeds a fear to not try new ideas. Research shows that teachers willing to adjust something as simple as how they present their objectives for a lesson can greatly affect their students' attitudes and perceptions of mathematics (Akinsola & Olowojaiye, 2008). Our math teachers are reluctant to attempt progressive instructional strategies and often meet the idea with a sentiment that they should teach their students math the same way they learned math, which is multiple decades ago for much of the department. Traditional instructional strategies in mathematics have proven successful, however, they do not always meet the increasing diversity of needs of our students.

To provide a change in this area, the teachers will need to take on small changes in their instructional strategies that come with little risk. Additionally, the administration will need to reinforce the idea that they allow teachers to try new things, and even fail, while supporting them along their journey. Much learning can occur for both the teacher and students in a failed attempt of implementing a new instructional strategy. These new instructional strategies should focus on ways to allow for students to master content during the class period and not rely so heavily on the need for students to complete practice outside of class. Not to say that homework should be banned altogether, but to have opportunities for students to practice, and receive feedback, all within the instructional time provided.

With any proposed change, the practitioner needs to know how the change will improve the system. The change ideas described above center around the primary driver for change of the mathematics teachers' perceptions that there only exists one specific model for a successful math student. By implementing a small, low-risk instructional strategy change, teachers can begin to feel more comfortable about trying new things and overcome their fear of failure. Also, with these new strategies focusing on allowing students to practice, receive feedback, and demonstrate their learning all within the provided class time, teachers can start to see how many different types of students can be successful in a mathematics curriculum. These changes in their instructional strategies can allow teachers to become more efficient with their time with students in class (secondary driver). This can then lead to improvement in the perceptions of teachers on what it takes for a student to be successful (primary driver). These improvements in the drivers can lead to more student success in our mathematics curriculum leading to increased enrollment in the advanced-level courses. An increase in enrollment from our general student population will lead to more low SES students enrolling in upper-level mathematics courses creating a more representative sampling of our student body.

In conjunction with the misguided perceptions of our mathematics teachers, students miss opportunities to enroll in higher-level mathematics courses because they do not meet the prerequisite course requirements. Each course in our mathematics curriculum has prerequisites that depend on performance in previous mathematics courses. The intent of prerequisites is to ensure that students will take courses in which they have the greatest chance for success. Prerequisites are necessary to ensure that students have mastered the required content before entering a course. Through several years of evaluations and comparison with other departments in the school and other mathematics departments from other districts, our school has determined that our current prerequisites lie within an appropriate level of expectations for our students. However, we do have students missing the mark to enroll in upper-level mathematics courses while still maintaining a desire to take those courses. Currently, our mathematics department does not provide alternative options for students that do not meet the requirements but want to still take a higher-level mathematics course. This barrier also affects our low SES population and prevents some students from enrolling in the higher-level mathematics courses – providing another area for change in our system. Though it would be easy to just eliminate the prerequisites altogether and have open enrollment, an approach like this would not address the fact that prior knowledge is needed to have success in higher-level mathematics courses. Alternative pathways need to be explored to provide students with appropriate assistance to meet the set requirements which gives them the ability to enroll as well as possess the necessary prior knowledge for success in the higher-level courses.

As mentioned above, our students do not have the same home environments. Some students have support structures in place that support further practice and learning at home, where others do not. The school can do little to change these inequitable home environments, however, it can provide support to overcome some of the inequities these differences present. For instance, the school can be more intentional with the schedule that students have during their time in the building. A school cannot change the environments that student go home to, but it can control the environment that exist within the walls of the school building. Providing additional time for students to complete independent work and meet with teachers for instructional help will provide a change that moves the school closer to my aim described above.

3.3 Intervention

To take steps toward my aim, I focused on the change idea of implementing a low-risk instructional strategy (Dignath & Büttner, 2018) by incorporating metacognitive practices in the gateway courses of the mathematics curriculum. I incorporated the new instructional practices by using a four-step workshop model outlined in Appendix A. Teachers of the gateway courses participated in the workshops. The professional development trained the teachers to implement metacognitive tools (Desoete, 2003) in their courses in a designated unit. The concept behind metacognitive tools may not be foreign to the mathematics teachers, however, the use of these tools in their lessons were new. Therefore, the workshops served as development to properly implement the metacognitive tools into their lessons (Dignath & Büttner, 2018).

In the workshops, the teachers discussed the benefits of metacognitive practices in secondary mathematics according to the literature. The study took a practice-oriented approach in that the goal of the participation was for the gateway teachers to determine if these benefits they discussed from literature could be realized in our context. The purpose of this approach was to increase buy-in from the teachers in trying something new (Selçuk et al., 2010). During the workshops, the teachers identified a unit where they wanted to implement metacognitive practices in their lessons. The teachers identified the units and developed their own versions of a metacognitive framework from the Metacognitive Problem-Solving Process (MPSP) found in Appendix C. Each teacher's version of the MPSP had all of the five components, but each teacher chose to organize it and present it in their own way without deviating from the intent or purpose of the framework.

Table 1 Timeline of Intervention

Time Frame	Activity
Winter of 2021-2022	• Introduce the Metacognition Workshops to mathematics teachers • Introduce the evaluation of the workshops to the teachers
Spring 2022	• Have teachers implement curated lessons from the workshops in their gateway courses • Collect data from teachers • Compile and analyze data

3.4 Participants

The change idea of incorporating metacognitive tools in gateway courses will involve three teachers in the mathematics department. Only these three teachers instruct the identified gateway courses during the spring semester of the 2021-2022 school year. One teacher is female (9 years' experience) and the other two are males (29- and 32-years' experience). Below, in Table 2, the three participants are identified with pseudonyms and some demographic information.

Table 2 Participants

Name	Gender	Ethnicity	Years of experience (Years in current position)	Gateway courses taught
Jordan	Female	White	9 (6)	Geometry, Algebra 1
Carl	Male	White	32 (24)	Algebra 1
Lenny	Male	White	29 (26)	Algebra 2

As part of the workshops outlined in Appendix A, the teachers selected a unit in which they implemented metacognitive tools into lessons they design in completion of the workshops.

3.5 Data Collection

The data collection for my study is an adaptation of Artzt and Armour-Thomas (1998). In their study, the researchers looked at how metacognition was incorporated in the problem-solving practices of seven teachers of varying experience levels. For my study, I applied their qualitative approach to gathering and analyzing data from my three teacher participants. In their study, Artzt and Armour-Thomas (1998) studied the metacognitive processes of the teachers in their planning and instructional practices. My study however, shifted the use of metacognitive practice to their instructional strategies. I studied how teachers implementing metacognitive practices in their instruction of problem-solving as well as providing a framework from which their students can use these metacognitive practices in their own problem-solving experiences.

For this study, teachers participated voluntarily. All three teachers of the gateway courses agreed to participate in the metacognitive process workshops. These teachers participated in a pre-implementation interview (Appendix I) (Kokka, personal conversation, November 12, 2021) in the first workshop. After teachers complete the third workshop, they implemented the metacognitive problem-solving process (Appendix C) in their purposefully designed lessons. During those lessons, they were observed (Ewijk & Büttner, 2013) and the observation form in Appendix J was completed for each lesson (Kokka, personal conversation, November 12, 2021). The researcher conducted a teacher-debriefing interview after each observed lesson and used post-observation teacher debriefing questions (Kokka, personal conversation, November 12, 2021) in Appendix M to facilitate discussions. During workshop four, the teachers evaluated their implementation through answering the focus group questions in Appendix M. All interviews, debriefing discussion, and focus group discussions were audio recorded with the permission of the participants.

3.6 Data Analysis

Table 3 Data Analysis Matrix

Inquiry Question	Data collected	Analysis of data
To what extent are mathematics teachers at my high school currently using metacognitive practices in their gateway courses	• Teacher interviews • Classroom observations	Identify common themes in the teacher interviews and classroom observations on the use of metacognitive practices in mathematics courses.
How do teachers implementing metacognitive practices in their classes perceive the effects of these changes on student engagement?	• Classroom observations • Teacher debriefing • Focus group questions	Identify common themes in the classroom observations, teacher debriefing, and focus group discussions on the effect of implementing metacognitive practices in the gateway mathematics courses.

I analyzed the data through the lens provided by Garofalo and Lester (1985) and their study of metacognition and cognitive monitoring. The data pertaining to how students are using the metacognitive practices were categorized by how they fit within two main areas developed by Garofalo and Lester (1985): knowledge of cognition and regulation of cognition. Other data were categorized in how the implementation of these metacognitive practices affect the perceived engagement of the students (Dignath & Büttner, 2018). Student engagement was analyzed within three distinct areas: affective, behavioral, and cognitive engagement (Fung et al., 2018). This analysis consisted of coding the qualitative data (Saldaña, 2021) collected through classroom observation, teacher interviews, and focus group discussions of implementing metacognitive practices. The data was labeled with categories that led to emerging themes that answered the inquiry questions described above.

3.7 Safeguards

The University of Pittsburgh Human Research Protection Board (IRB) reviewed this study and gave approval that it meets the regulatory requirements for exempt research under 45 CFR 46.104.

4.0 Results

4.1 Emerging patterns

Interviews with teachers were recorded, with their consent, using the Apple App, Voice Memo. These recordings were then transferred to a secure personal computer and the original files deleted off the mobile recording device. From the audio, the principal researcher transcribed the interviews verbatim. The transcriptions allowed for open coding (Saldaña, 2021). Additionally, the field notes taken during classroom observations were also coded using the same technique. Applying the practice of "concept coding" as described by Saldaña (2021) allowed the categorization of data into three distinct components. Below describes the three components, with their subcategories, and how they tie into the inquiry questions.

Table 4 Categories and Descriptions

Categories	Sub-categories	Description	Inquiry Question
Knowledge of cognition	Person	What the problem-solver believes about oneself, how he/she learns, beliefs about mathematics, and position as a learner compared to peers.	To what extent are mathematics teachers at my high school currently using metacognitive practices in their gateway courses
	Task	The knowledge of the problem-solver regarding the task as well as the expectations and perceived difficulty of the problem at hand.	
	Strategy	The knowledge of cognitive procedures that are useful in solving the given problem along with which procedure	

		would be most efficient to use.	
Regulation of cognition	Planning	Involves analyzing the problem for purpose of invoking memory, selecting strategies, and determining if additional resources are necessary. This includes goal setting, recall of relevant prior knowledge, and time management.	
	Monitoring	Involves making predictions, pausing to reevaluate, and de-bugging. Monitoring could result in choosing an alternative strategy.	
	Evaluation	Determining the reasonableness of the result, considering initial predictions, and integrating the newly formed knowledge into prior knowledge.	
Engagement	Affective	The interest a student shows in learning mathematics.	How do the teachers implementing metacognitive practices in their classes perceive the effect of these changes on student engagement?
	Behavioral	What a student does during the learning process and how the student discusses the topics with peers.	
	Cognitive	The student's perseverance through the problem-solving process	

From the coding of data into the categories and sub-categories described above, themes emerged. The emerging themes were organized into two distinct contexts: teachers and students. Below is an examination of the results that led to the emerging themes.

4.2 Results for Inquiry Question 1

Inquiry Question 1: To what extent are mathematics teachers at my high school currently using metacognitive practices in their gateway courses?

To answer this question, data were gathered from classroom observations before the professional development workshops and the implementation of metacognitive practices and during the implementation. These data were triangulated with teacher interviews and focus group discussions. Below, Table 5, displays a summary of the emerging themes that pertain to the first inquiry question.

Table 5 Emerging Themes of Metacognition

Metacognition	Component	Before implementation	After implementation
Knowledge of cognition	Person	Teachers modeled a positive attitude toward problem-solving	Teachers projected confidence in their problem-solving process
		Students were quick to give up on problem-solving when challenged	Students gained confidence and were less likely to give up on challenging problems
	Task	Teachers provided opportunities and prerequisite knowledge to students within their lessons	Teacher provided tasks that required a higher cognitive demand for students
		Students can access their prior knowledge in a limited capacity	Students were able to quickly identify the task at hand and what prior knowledge was helpful
	Strategy	Teachers demonstrated a variety of problem-solving procedures that students could use in their future experiences	Teachers helped students to develop a framework that students could apply to future problems

		Students accessed previous experiences to solve new problems	Students handled a wider variety of problems that reached beyond their previous experiences
Knowledge of Regulation	Planning	Teachers demonstrated examples of analyzing and solving problems	Teachers presented clear frameworks for students to apply to all future problem-solving experiences
		Students could solve problems like examples they were shown, but struggled to develop plans for unfamiliar situations	Students used learned frameworks that allowed them to better access their prior knowledge
	Monitoring	Teachers did not ask students, nor did they model making predictions in their problem-solving	Teachers gave students frameworks and tools to help them regulate and monitor their progress
		Students showed very little evidence of regulating their problem-solving	Students made predictions, reevaluated their strategies, and detected errors along the way
	Evaluation	Teachers rarely asked students to consider the reasonableness of their solutions, nor did the model reflection in their own practice	Teachers modeled reflection, discussed the reasonableness of results, and considered initial predictions
		Students showed very little evidence of evaluating their solutions or integrating their learning to new knowledge	Students evaluated and reflected on their solutions and integrated their experiences into new knowledge

4.2.1 Knowledge of cognition

4.2.1.1 Person

As explored above, Garofalo and Lester (1985) describe the personal aspect of knowledge of cognition as what the problem-solver believes about oneself, how he/she learns, beliefs about mathematics, and position as a learner compared to peers. Before teachers participated in the metacognition workshops, observations showed teachers modeling positive attitudes toward problem-solving by the way they demonstrated example problems and gave students insight into the procedure they would use to solve a problem. Teachers used phrases like, "I know I can solve this problem, because we have seen something similar to this in the past," and then relate the problem to previously shared experience with the students in class.

Students would emulate this attitude in their own problem solving, however, students had trouble when confronted with problems that were dissimilar to previously demonstrated examples. In these situations, students gave up quickly and stopped solving the problem. Students were observed to attempt a problem once, but if they did not recognize the problem as something they had seen in the past, they did not continue to a solution. One student was observed saying, "I have never seen anything like this…I don't know how to do this." This student's sentiment was shared by many of his peers.

In a collective discussion with the three teachers before implementing metacognitive practices, the primary researcher asked the teachers to describe one aspect of their classroom that they would change if they had the power. Jordan responded,

> I wish I could give my students more resilience. They so often give up on problems at the first sign of struggle. They just don't have the perseverance necessary to be successful and I think it comes from a lack of confidence.

After making this comment, the other two teachers agreed and stated that they share in this as a main issue in their classroom as well.

After the workshop, differences in teachers' attitudes were not observed, however, two of the three teachers did comment on a slight increase in their confidence in presenting problem-solving activities to their students while using a metacognitive framework. Carl stated that he felt more confident that he was reaching his students better because he was using the metacognitive framework. After this comment, Lenny agreed and stated that he felt similar. Jordan reported no change in her confidence in teaching problem-solving due to using the framework.

All three teachers reported that they noticed an increase in the confidence of their students when using a metacognitive framework. When students developed a plan, they were observed to get farther in their problem-solving experience than when they approached problems without a plan. Lenny reported that the day after he implemented the metacognitive framework in his lesson, all students completed their homework, and most of the class completed the practices problems correctly. The teacher reported this as a surprise since he expected students to have trouble with those problems and this class typically has several students that do not complete their homework due to giving up on problems they do not know how to start. The teacher stated,

> I was able to observe some of my students work on their homework together and I was surprised at how quickly they used the framework I showed them in class to set up their problems and they were able to complete the problems because they had a method for getting started.

The teacher also received feedback from his students stating that the framework made them feel like they could handle problems that were not familiar.

4.2.1.2 Task

The knowledge a problem-solver possesses regarding the task as well as the expectation of the perceived difficulty of the problem contributes to the problem-solvers ability to understand the problem at hand (Garofalo & Lester, 1985). Before implementation, teachers provided opportunities for students by modeling sample problems at the board and demonstrated how to access prior knowledge to solve the problem. When students were asked to do the same on their own, they had trouble when presented problems that did not look like the example demonstrated by the teacher. Carl stated, “If I give students a problem like my example, they can do it, but if I try to give them something different, some just won’t do it.” When asked to elaborate on why the students would not complete the unfamiliar problem, Carl reported that some of his students struggled to make the connections to their prior knowledge. He suggested that the struggle primarily stemmed from the student’s inability to properly understand the problem.

After the implementation, the teachers began to use sample problems that increased the cognitive demand of the students. Jordan mentioned, “I felt like the [metacognitive] framework forced me to up my game with the depth of knowledge I was requiring of my students.” The teacher explained that she felt like she needed to increase the cognitive demand of her problems to properly use the framework. She also noted that when she increased this cognitive demand, students did not struggle with the increased rigor as much as she would have expected. She postulated that the framework helped the students to recognize what the problem was asking and led to more student success. Students were observed to recognize what the problem was asking and apply their prior knowledge to their solution while using the metacognitive framework.

4.2.1.3 Strategy

Students need strategy to help them identify what cognitive procedures are useful in a problem-solving experience (Garofalo & Lester, 1985). Before implementing the MPSP within their lessons, the three mathematics teachers demonstrated a variety of problem-solving procedures that students could use in their future experiences. These experiences allowed students to build a knowledge base that they could then call on in their problem-solving experiences. As described above, students experienced trouble if they were asked to solve a problem that was not apparently like an example they had seen in the past. The three gateway teachers did try to give their students a variety of experiences, however, they could never provide a comprehensive repertoire.

After the implementation, teachers helped students develop a framework on how to approach future problems instead of relying on past practice. Teachers approached their examples as practice for using the framework instead of practice of an ensemble of procedures for students to refer to in the future. While using the framework, students were able to attempt a wider variety of problems. “One of the biggest take-aways from this lesson was that I felt like my students could attempt more problems than they could last unit,” the Carl said in a discussion about implementing the metacognitive framework in one of his lessons and comparing it to lessons from the past.

4.2.2 Knowledge of Regulation

4.2.2.1 Planning

After a student understands a problem, they begin planning their approach by analyzing the problem for the purposes of invoking memory of past experiences, select strategies, and determine if additional resources are necessary. Before the workshops on metacognitive practices, gateway teachers demonstrated examples of analyzing and solving problems and gave student strategies that they could use in future problem-solving experiences. However, the demonstrations did not include making predictions or goal setting as described in popular literature (Garofalo & Lester, 1985) (Yimer & Ellerton, 2010). Students in the gateway courses could develop plans for solving assigned problems, but these plans did not include predictions and students had trouble with problems dissimilar to demonstrated examples.

In using the MPSP, the teachers in the gateway courses presented a method for students to use for all future problems. Teachers presented the framework as a tool for students to use to help them develop a plan. Lenny used a model described in Appendix F that helped his students to organize the given information of the problem and develop a plan for solving. “This framework helped my students get started. Too often my students get stuck and don’t know how to start a problem, but having this model helped them to get moving,” said Lenny when describing the benefits of his students using the GUESS model (Appendix F) to set up their problem-solving plan. Additionally, Jordan recounted, “I feel like when my students have a well-defined plan, they can recall their prior knowledge. Having a defined framework to follow helped my students to access their prior knowledge.”

4.2.2.2 Monitoring

As a student carries out his/her plan, the student will make predictions, pause to reevaluate, and detect errors along the way that could lead the problem solver to alternative strategies (Garofalo & Lester, 1985). Before implementation, mathematics gateway teachers did not promote much monitoring within their problem-solving practices. When asked to describe monitoring in their teaching, one teacher replied, “I do not really talk to students about monitoring their thinking during their process…I do not make predictions when I am demonstrating, nor do I ask the student to make predictions."

Students in the gateway courses before implementing the metacognitive framework showed little evidence of regulating their problem-solving. "One thing my students really struggle with is finding where they go wrong in their problem solving," stated Carl. He continued by saying, "Often they don't even know they made a mistake because they do not stop and check."

After implementing the metacognitive framework, the teachers noticed students detecting their errors more frequently. "I noticed students talking to each other about their mistakes," recalled Lenny. Observations revealed that when students were using the MPSP, they would take time to pause and check for errors in their process along the way. If a student identified an error, he or she would look to an alternative path. However, there was an incident in which a Geometry student made a mistake in his problem-solving process that went undetected until the end when the teacher checked the student's work. In this incident, the teacher redirected the student to use the metacognitive problem-solving framework and reminded the student of the monitoring piece of the framework. This need for redirection to the implementation portion of the MPSP was evident in most of the observed classes. "The monitoring part is something that I have not focused on much in the past…my students needed direction on that," expressed Lenny when discussing how is students used the MPSP. Carl and Jordan agreed with this statement. With redirection students used the MPSP to make predictions, reevaluate their strategies, and detect errors during their problem-solving process.

4.2.2.3 Evaluation

Before the implementation of the MPSP, teachers rarely asked students to consider the reasonableness of their solution, nor did they model reflection. Carl stated, "I know that I probably should, but I do not talk about reflecting much in my class." Students also showed very little evidence of evaluating their solutions. Students showed motivation to get an answer and then move on to the next problem. There was also no evidence of students checking their predictions since students did not make predictions before the MPSP was implemented.

After implementation, teachers modeled reflection when demonstrating examples by saying comments like, "when we get this result, what does it mean?" or "what does this tell us about what we thought we were trying to find?" Teachers would refer to the initial conditions of the stem of the problem as well as the predictions made during the engagement phase of the MPSP. Students used the MPSP to evaluate and reflect on their solution and integrate their experiences into new knowledge.

While solving a practice problem with a partner in an Algebra 2 class, one student mentioned to the other, "why would we get a result like this, it is nowhere close to what we guessed at the beginning?" This question caused the two students to pause and reflect on their process. After a few moments, one of the students proposed that maybe they set up something wrong in their initial conditions. Through further deliberation, the students were able to agree upon an alternative strategy that eventually led them to a result that was more reasonable. Another group of students was working on a Geometry proof while using the MPSP. When the two finished the proof and reflected on their result, one student exclaimed how he finally made a connection with the postulates they were working with in that lesson and the concepts were making sense to him.

When the teachers were asked in the focus group about how their students reflected in their implementation of the MPSP, there was a consistent agreement with all three teachers seeing the benefit in having students reflect. One stated, "I feel like when the students pause to reflect on what they did, they learn more deeply. I could see evidence in this when they attempted future assessments – they showed they understood the content." The three teachers did report varying degree of difficulty with getting students to reflect. Carl, the Algebra 1 teacher, expressed the most frustration with getting students to comply with this part and Lenny, the Algebra 2 teacher, reported the least frustration. All three teachers agreed that students became more comfortable with reflecting on their results the more they practiced it and they predicted it would continue to get easier in the future if they all continued to require students to reflect as a part of their problem-solving process.

4.3 Results for Inquiry Question 2

Inquiry Question 2: How do teachers implementing metacognitive practices in their classes perceive the effects of these changes on student engagement?

Below, Table 6, displays the emerging themes on the preserved engagement of students during the implementation of the MPSP.

Table 6 Emerging Themes of Engagement

Type of engagement	Observation summary	Teacher report summary
Affective	Students demonstrated interest in learning mathematics	Students demonstrated more confidence in their problem-solving ability
Behavioral	Students engaged in meaningful discussions with their peers	Students engaged in more meaningful conversations than what was typical
Cognitive	Students demonstrated perseverance when they approached difficult problems	Students gave up less often than what was typical

4.4 Engagement

4.4.1 Affective Engagement

Interest in learning mathematics can present itself in multiple ways (Fung et al., 2018). During the implementation of the MPSP, all three teachers reported an increased interest in their students in the lesson. Carl noted that students who would normally stay quiet in his class were participating and asking insightful questions during the lessons with MPSP. Jordan stated, “My students were more receptive to the framework than I anticipated. It was almost like they were looking for something that would make their lives easier.” Lenny added, “My students showed more interest in completing their practice problems when they had developed a plan using the [MPSP].” All three teachers reported similarly in that they saw more students interested in learning while implementing the MPSP compared to previous lessons with those classes as well as similar lessons in their past teaching careers.

Observations of the lessons confirmed the teachers’ perception of student interest. However, not all students demonstrated their interest in learning during the lesson. No students were observed to demonstrate disinterest, however, there were students that seemed less engaged than their peers based on their lack of participation (talking with their peers about the concepts and answering teacher questions during the lesson). In the nine observed lessons, fewer than three students per class (average class size: 20) showed this lack of participation during the lesson. These students still completed their work; however, they were noticeably less engaged than their peers. When teachers were asked about these students during the debriefing interviews, the teachers responses varied. Jordan reported that for the student in question, she has struggled to fully engage the student in the lessons and what was observed was “a decent day for that student.” Carl replied with stating that there is always room for improvement, but overall, he felt that the students showed more interest than normal. Finally, Lenny acknowledged that he would have liked to have seen more interest out of a couple of his students, but he did notice an increase in interest in most of his class.

4.4.2 Behavioral Engagement

During observation, students engaged in conversation with their peers about the concepts of the lesson. While developing a plan to solve their problem, students engaged in conversations about their prior knowledge and what types of similar problems they had experienced in the past that could help them in this situation. Students also conversed with their peers about the reasonableness of their results and how they aligned to the predictions the made during the engagement portion of the MPSP. Students practiced articulating their thought process with each other and refined their own process through the discussions. Students also used their discussions with peers to identify errors in each other's problem-solving process. All students were engaged in a conversation with at least one of their peers at some point during each of the nine lessons. Like above, there were a few students that engage in conversation less than others, but all had the opportunity and participated at least once during the lesson.

Lenny reported, "I was happy to hear so much 'math talk' during the lesson." He stated that the conversations that he observed his students having were more meaningful than usual in that students were engaged in conversation about the problem at hand and talking about math concepts that pertained to solving the problem. Lenny added that he felt like he had a better understanding of what the students knew when he could listen in on their conversations with their peers. When reflecting on his lesson in Algebra 1, Carl said, "even if students were not talking about the right concepts for the problem, they were at least talking about math…I can work with that." The teacher continued to discuss how when students are talking about math, he can help to guide them in the right direction. He reported that using the MPSP allowed more opportunities for students to engage in conversation with their peers.

The geometry teacher reflected on the type of questioning she uses in her classroom and stated,

> When using the metacognitive framework and teaching my students to use it, I am forced to use increase questioning, which meant I was talking to more students during the lesson than normal. This meant that more students were engaged in the lesson, and I had a better idea of what my students knew through my formative assessments.

She continued to describe how she noticed that her students were engaged in conversation with their peers about the lesson's concepts and the MPSP. From the meaningful conversation, she noticed that students quickly identified errors and were able to correct them through the course of the conversation.

4.4.3 Cognitive Engagement

Before the implementation of the MPSP, all three teachers identified a need for students to develop more resilience in their problem-solving in that the teachers observe students giving up quickly in their problem-solving experiences when they face adversity. During the nine lessons, students had multiple opportunities for adversity. At each impasse, students could choose to give up or persevere to a solution. In each lesson, most students would work until a solution. Few students were observed to give up before completion of the problem at hand. Of the nine classes, students were observed to give up before a solution in five of them. Of those five classes, the greatest number of students that did not complete the problem during a lesson was in an Algebra 1 class where four students (18% of the class) quit solving the problem before arriving at a solution.

When the three teachers were asked to reflect on the cognitive engagement of their students during the implementation of the MPSP, all three reported that they noticed an increase in their students' engagement. Lenny stated,

> I was impressed that all students were able to get to a solution and no one gave up. This is rare in my class. There are usually a few that have trouble setting up the problem and give up before getting started.

The teacher continued to state, "even if their solution was wrong, they at least saw the problem through to completion, which is an improvement." Carl added, "I thought that my students showed more perseverance in their problem solving. Some still gave up, but they all seemed to get farther." The geometry teacher supported this with her reporting that her students showed more resilience in their problem solving than usual when they were using the MPSP framework.

4.4.4 Challenges

The observations of the three teachers implementing the MPSP showed they were confident in their ability to use the framework in their demonstrations as well as instruct students to use the framework in their own problem-solving. However, through individual teacher interviews and focus group discussions, the three teachers willingly described challenges they had with implementing the framework. All three teachers agreed that implementing the framework took them out of their comfort zone. "I am confident in how I teach problem-solving, but I was a nervous to introduce this framework because it was slightly different to what I am used to," Carl said while reflecting on his lesson.

Though all three teachers agreed that they were uncomfortable at first, they also expressed that they quickly got over this feeling as the lessons progressed. Jordan expressed,

> I felt a little uncomfortable at first because this was my first time approaching the lesson this way, but I became more comfortable as I noticed how it was helping the students to understand and be successful in their problem-solving.

The teacher continued to propose that she would gain comfort in using this framework as she continued to use it with her students and assumed it would become second nature to her presently.

In reflecting on their experiences, the teachers were asked to comment on what they saw as the biggest challenge with implementing the MPSP framework in their lessons. All three teachers quickly responded with the time that implementing this framework takes. "Since this is new to my students, it took time to teach them how to use it," mentioned Jordan. "I am always worried about spending extra time on new things because of the Keystone exams," expressed Carl. "My students took a while at first to get the hang of using [the MPSP framework]," added Lenny. Each teacher described how implementing the framework took more time than instructing a lesson of the same content without using the MPSP.

4.4.5 Metacognitive Practices in the Local Context

Secondary mathematics teachers do not often use metacognitive practices in their instruction (Dignath & Büttner, 2018) because of their reluctance to try new instructional strategies (Selçuk et al., 2010). However, teachers are motivated by their students experiencing success in their problem solving (Özcan, 2016). This research supports the finding within the context of the gateway course teachers.

Below, Figure 5, illustrated the two main categories of metacognitive problem-solving with the included sub-categories.

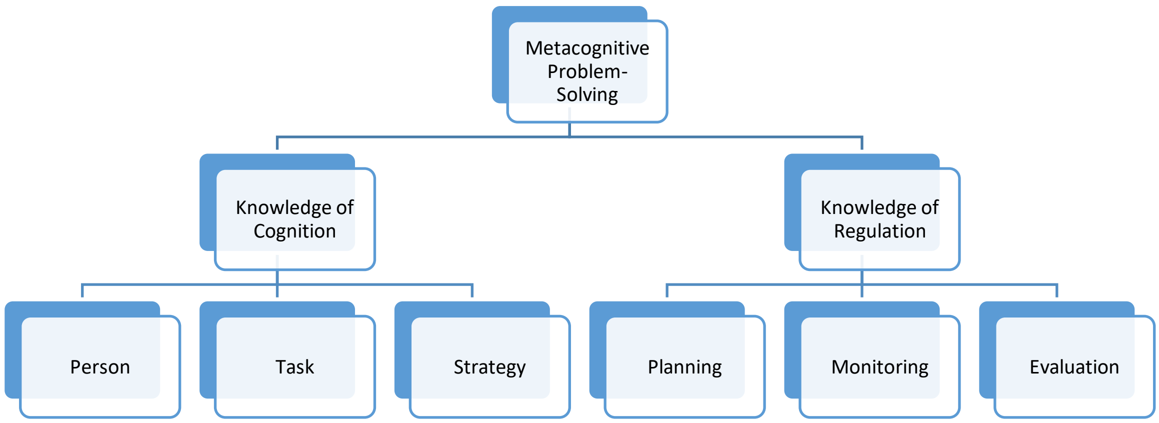

Figure 5 Metacognitive Problem-Solving

These components of metacognition are seen within the five phases of the MPSP. Figure 6 illustrates the phases of the MPSP.

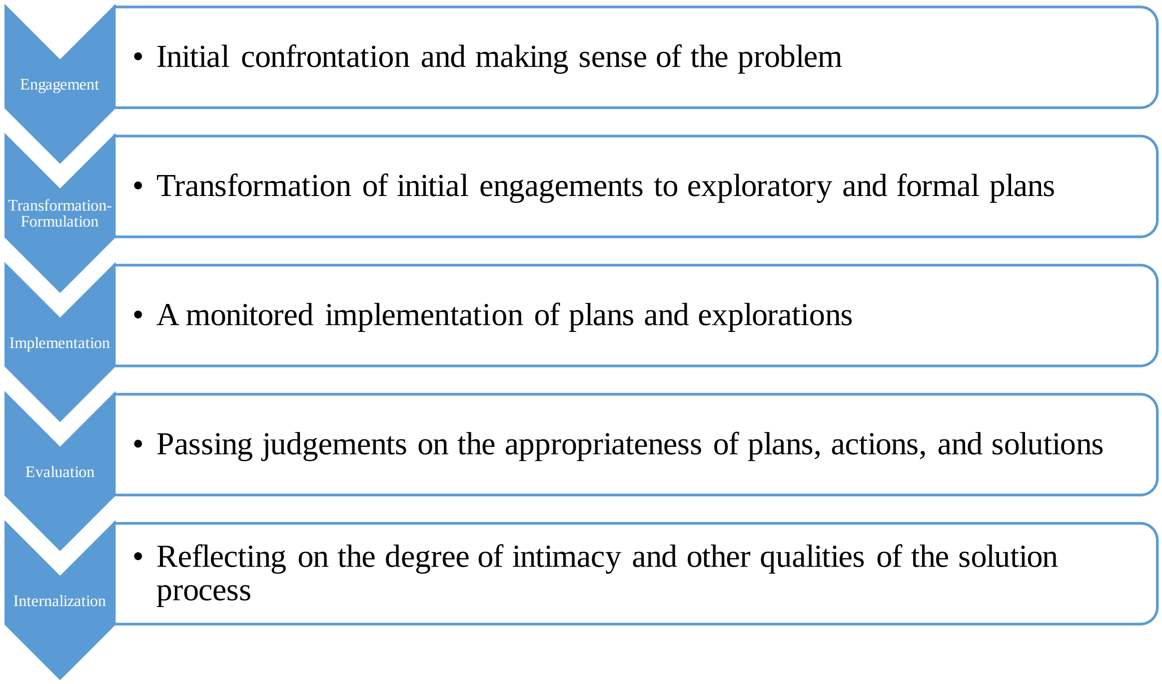

Figure 6 Phases of the MPSP

4.4.5.1 Knowledge of Cognition in the Gateway Course Instruction

Before implementation of the MPSP framework, the gateway teachers demonstrated knowledge of cognition in their instruction. Teachers modeled positive attitudes toward problem solving and this translated to their students. Teachers also provided opportunities and prerequisite knowledge so that students would be prepared for future problem-solving experiences. Finally, the gateway teachers demonstrated a wide variety of problem-solving experiences that provided opportunities for students to access their prior knowledge. However, teachers reported trouble with their students' motivation in their problem solving and proposed that this lack of motivation stemmed from a lack of confidence and resilience resulting in students giving up too quickly on challenging problem-solving experiences.

After discussing the phases of the MPSP in the pre-implementation interview, the three teachers reflected on their instruction and then engaged in a discussion about how they incorporate the idea of the metacognitive framework within their daily lessons. Jordan stated, “I feel like I do a good job of getting students through the first few phases, but I do not do much with monitoring and reflecting.” This statement is a fitting summary of the three teachers before the implementation of the MPSP in their lessons.

During the implementation of the MPSP, the teachers reported an increase in student motivation and problem-solving achievement (Özcan, 2016). This led to a gained confidence in the students which allowed them to persevere in their problem solving. Additionally, students were able to access their prior knowledge and develop a strategy to approach their problems. Teachers demonstrated the importance of knowledge of cognition in the lessons and students translated that into their problem-solving experiences.

4.4.5.2 Knowledge of Regulation in the Gateway Courses

The three teachers identified that the regulation of cognition was an area where they were deficient in the pre-implementation interviews. Observations before the implementation of the metacognitive framework confirmed the teachers’ identification. Before the MPSP, students were not making predictions, they were not looking for errors within their reasoning, they were not reflecting on their process, and they were not evaluating the reasonableness of their results. Students mostly learned problem-solving procedures from their teachers’ demonstrations and performed those same procedures on their practice problems, leading to frustration by the students when they approached a problem dissimilar to the demonstrated examples. The frustration then led to students giving up and not completing the problem to a solution.

During the implementation of the MPSP, teachers demonstrated a framework that gave their students more than just procedures to learn, but a methodology that they could apply to their future problem-solving experiences. The teachers model regulation in their own examples and showed students how to reflect in their own work. From this instructional shift, students were able to apply this framework to a variety of problem-solving opportunities and use the framework to increase their rate of completion as well as their error detection.

4.4.5.3 Summary of Metacognitive Practices in the Local Context

Before the professional development, teachers were using portions of a metacognitive framework, but not its full potential. Like what is found in the literature (Ewijk & Büttner, 2013), the gateway teachers centered their instruction around cognition and not metacognition. This was due, in part, to a lack of awareness of the benefits that metacognition can bring to a secondary mathematics course (Dignath & Büttner, 2018). The gateway teachers had used cooperative learning in their lessons and knew of their benefits (Slavin et al., 2009), but did not leverage the benefit of cooperative learning with the use of metacognition (Ewijk & Büttner, 2013).

After the implementation of the metacognitive framework, the teachers used the framework within their instruction on problem-solving. In their instruction:

- they had students activate their prior knowledge (Artzt & Armour-Thomas, 1998)
- they gave students opportunities to demonstrate their thought process so that the teacher could help them regulate (Levin et al., 2013)
- continued to use cooperative learning (Ewijk & Büttner, 2013) (Slavin et al., 2009)
- allowed their lessons to be more student-centered (Pol et al., 2008)
- promoted reflection within their lessons to help student to internalize their learning (Yimer & Ellerton, 2010)

4.4.6 Perceived Engagement

The gateway teachers identified their students' engagement as their primary concern and the priority in what they would like to fix in their classrooms. They were eager to determine if implementing metacognitive practices would help them with this problem. Their eagerness, along with the low risk of the intervention helped to promote buy-in from the teachers. The motivation of the teachers likely contributed to the success of the implementation.

All three teachers reported increased engagement of their students in all three areas: affective, behavioral, and cognitive (Fung et al., 2018). The increase varied between teachers and between the lessons they taught, but they perceived an increase in the students' engagement. Teachers reported an increase in students' confidence in solving problems (Beilock & Maloney, 2015). They attributed this increased confidence to students understanding the problem better while using the framework (VanLehn et al., 2003). Carl reported, "My students showed that they understood the problem better because the framework helped them organize what they knew about the problem and get started." The increased confidence in the students led to more students completing their problems through to a solution. Even if the students did not arrive at the correct answer, by working further into the problem, the teachers felt they could better assist the students in their learning.

The way students talk to each other about their learning can shed insight on how and what students are learning (Su et al., 2016). This is especially true in secondary mathematics. Teachers gain this insight by observing how the students talk about the concepts with their peers. For students to have meaningful conversations, teachers need to provide opportunities for students to work together. This cooperative learning creates a more student-centered environment. In reflecting on how implementing the MPSP led him to use more cooperative learning in his class, Lenny said,

> At first, I felt like I was not doing my job because I was not talking as much as I normally do, but when I walked around and listened to what the students were saying to each other, I felt like I had a better understanding of what they knew.

Implementing the MPSP does not require cooperative learning, however, the framework does lend itself to using more cooperative learning. Students can realize additional benefits when metacognitive practices are combined with student-centered instruction (Artzt & Armour-Thomas, 1998) (Moreno, 2006).

The way a teacher questions influences the engagement of students (Desoete et al., 2003). Implementing the MPSP forced the teachers to be intentional with the questions they asked. Carl reported that he found himself needing to ask more leading questions with his students during the implementation which allowed for more students to participate in the lesson. Jordan described how she felt that her questioning was at a higher level of cognitive demand during her lessons. She noticed how this engaged the higher-achieving students more and brought some of her lower-achieving students to a new level of understanding. Though questioning practices were not the focus of this study, all three teachers reported their questioning improved in quality (higher cognitive level) and quantity while implementing the MPSP which led to more engaged students (Su et al., 2016).

The gateway teachers wanted their students to develop more resilience in their problem-solving. The teachers were discouraged with how quickly their students would give up on problem at the first sign of difficulty (Mathan & Koedinger, 2018). Though this implementation of a metacognitive framework did not create a classroom of resilient learners, it did have a positive effect on the perseverance that the students showed in their problem-solving experiences (Collie & Martin, 2017). All three teachers reported an increase in problem completion and noticed students giving up less. They attributed this to the increased confidence students showed when they had the framework to fall back on (Mapp & Kuttner, 2013). Jordan reported, “I feel like the biggest benefit was that my students had the framework to fall back on when they did not know what to do.” Lenny added, “The framework gave them questions they could ask themselves so that they could figure things out on their own.” Both accounts provide evidence that the MPSP helped the students to become more independent learners (Dignath & Büttner, 2018).

5.0 Learning and Actions

The following chapter will discuss the learning actions from this study. First, it will discuss the key finding relevant to the change sought on the problem of practice as well as explain the strengths and weaknesses of the change. Second, this chapter will look to the next steps and implications of this study in the place of practice and beyond.

5.1 Discussion

The analyses of the data collected are framed within the context of the two inquiry questions that guided the study:

1. To what extent are mathematics teachers at my high school currently using metacognitive practices in their gateway courses?
2. How do teachers implementing metacognitive practices in their classes perceive the effects of these changes on student engagement?

The purpose of this discussion is to help not only the teachers of the local context within this study, but also other educators that wish to implement metacognitive practices within their own mathematics courses to promote student engagement.

5.1.1 Limitations

The results of this study presented evidence for benefits of implementing metacognitive practices within the context of the mathematics gateway courses. However, this study did come with some limitations. Limitations do not negate the effects, rather they keep the effects within an appropriate perspective as to not over- or under-state any direct influence. This section will surely miss a limitation or two, however, it will attempt to highlight a couple of the most prominent limitations of this study.

First, this study focused on the perceived engagement of students by the teachers. Though teachers are the experts in their classroom, their perceptions could miss something that is happening under the surface. Having observations of the students conducted by someone other than they teacher helps to guard against this limitation, but still may not be comprehensive. To dive further into the effects of the MPSP on student engagement, one could bring in the student perspective to give further insight into how the metacognitive practices affect the students' engagement.

Second, implementing metacognitive practices takes a considerable amount of time for teachers to dedicate to their students to learning how to use the framework properly. All three teachers reported this as the only major disadvantage. They worry that the time they spend on teaching the students how to use the MPSP properly is time that could be spent on continuing their curriculum. However, all three teachers did report that they thought they could see the benefit from the time invested in their student engagement and learning, but that benefit did not replace the underlying worry of getting behind in their curriculum.

Third, one cannot ignore the effect that the COVID-19 pandemic had on this study. As briefly described above, the pandemic contributed to a context with disengaged students and teachers wanting to find something that could help. One could even describe this context as desperate. Both teachers and students were desperate to find a win after two years of unexpected losses. This desperation may have contributed to the teacher buy-in to use this new strategy in their classroom. They may have perceived a small increase in their students' engagement and because they wanted this to change for the better so badly, they may have overstated the effect. Also, students were looking for a win. Navigating continued advancement through their schooling without the same level of competence that peers before them received could contribute to a longing for something to help them in their learning. Receiving a framework that helped them navigate their unknown experiences could have been a welcomed addition to their repertoire. These conjectures are not meant to downplay the effects of metacognitive practices in secondary mathematics or to go against the literature of benefits, but to serve as a warning that the same study done in a different context may yield slightly differing results due to the contributing factors of the context during the time of this study.

5.2 Next Steps and Implications

Referring to the theory of improvement in Figure 1, teaching metacognitive skills leads to greater metacognition in students which leads to greater engagement. This study presented evidence that implementing the metacognitive practice in the gateway courses increased the engagement of the students. However, this is simply an intermediate step in the desired aim of increasing the enrollment of low-SES students in advanced-level mathematics courses - a small change in the direction of improvement. For this change to continue to move the system toward the improvement desired in the aim, a few considerations will need to be addressed.

First, the teachers reported the time it takes to teach students how to use metacognitive frameworks in their problem-solving. In discussion with the teacher, this effect seemed to relate to the age of the student. For instance, the younger, Algebra 1, students took the most time to learn the framework and the oldest, Algebra 2, students took the least amount of time. This evidence suggests that as the students mature, they can grasp the framework more quickly. The teachers postulated that if they continued to teach this framework, it would only get easier for the Geometry and Algebra 2 teachers because they would not have to introduce the framework to the students for the first time. To further help with this, the system would benefit from introducing some of these metacognitive practices before the students reach high school. This would allow the Algebra 1 teacher to also benefit from not having to teach the framework for the first time to his students.

Second, teachers will need to continue to be supported in their effort. As discussed above, the buy-in from the teachers helped to contribute to the success of implementing the MPSP. Without the buy-in from the teachers, additional challenges could have interfered with the positive change experienced in the classroom. Giving the teachers agency in how they implemented the framework contributed to their willingness to participate. Additionally, by taking the approach of determining if metacognitive practices are something that could work within the local context, teachers were afforded the opportunity to fail. They did not feel pressure to perform, and this allowed them to try something that may have made them nervous at the start. This idea of teacher buy-in may be the best hidden lesson of this study that could apply to future studies outside of the realm of metacognitive practices. When a school wants to see their teachers change something, giving them agency and the ability to fail safely can increase their buy-in to the change.

Finally, this study presents multiple paths for future study. As mentioned above, gaining the students' perspective could yield valuable insights that could aid a school in increasing the engagement of mathematics students. Additionally, one could study the effects of the MPSP on student achievement. As illustrated in the theory of improvement and the literature, engagement leads to achievement and a study of the direct effects could yield helpful results. Ultimately, the aim of addressing the problem of practice consists of increasing enrollment of low-SES students in advanced-level mathematics courses. To study this, one would need to look at the system longitudinally and study the effects of continued implementation of metacognitive practices has on the enrollment. This would take multiple years but is something that I will continue to pursue since I am invested in the improvement of the school district.

6.0 Reflections and Recommendations

This study clearly demonstrated a benefit of implementing metacognitive practices on student engagement. However, the study also illuminated perspectives on improvement, in general, as well as my role as a leader seeking to improve my system. I can take these lessons with me into the rest of my career as an agent for positive change.

As a leader, I can often identify a problem and even determine a solution on how to fix the problem. However, the solution involves others, and they may not share my vision. Therefore, it is my charge, as the leader, to cast the shared vision of improvement to my staff. The need for a shared vision was evident in this study. To move my three teachers toward improvement, I needed to take the time to share my vision with them and allow them to be part of the process.

Teacher agency is a vital part to change. Allowing the teachers to be a part of the process and not prescribing the change that I wanted them to carry out helped to promote buy-in and it reminded me that I do not have to have all the answers as the leader – I just need to provide the space for my staff to develop the answers, which leads me to collaboration. Too often, secondary teachers are on an island and are left to work independently of their peers. This study reminded me of the power of providing time for teachers to collaborate during their professional development. Like what we would like to see in our classrooms (a student-centered learning environment), allowing teachers to take an active and collaborative role in their development leads to greater professional growth.

Teachers also need to know that they are in a welcoming environment that affords failure. In this study, I noticed all three teachers were very concerned about covering the entirety of the curriculum and concerned that they did not have room for missteps. It was challenging to overcome this and reassure the teachers that they could try something new, and we would treat it as a learning experience whether it was successful or not.

As I continue to lead in my context, I will take what I have learned from this study and translate it to my everyday practice. The fact is the work is not yet complete. My aim is to increase the enrollment of low-SES students in our advanced-level mathematics courses. This study was the first step in achieving my aim but there is still much work to be done. As I move forward toward my aim, I make the following recommendations for my place of practice as well as any other leaders in a similar context looking to make the same improvement:

1. Teachers need to have an active role in the improvement of the system. Real change cannot happen without the buy-in of those involved. Metacognitive practices can provide positive effects, but the teachers implementing them need to have a vested interest.
2. Engagement is just the start to fixing the problem of enrollment. It is necessary, but only the first step. The mathematics teachers need to continue to engage their students to increase their achievement. The school needs to support this effort by putting additional supports in place for students to succeed.
3. To maximize the effects of metacognitive practices in mathematics courses, the district would be wise to begin introducing metacognitive frameworks within the middle school setting. Currently the middle school is evaluating their mathematics curriculum and finding a way to incorporate the metacognitive frameworks would benefit our students and their future success.
4. Change takes time. We will not realize our aim overnight. It will take concerted effort to continue to take small steps by iterative changes that move the system

closer to the end goal. The high school needs to continue to hire the best teachers possible when positions become open as well as support the teachers through collaborative professional development.

Moving forward, I hope that each change undertook to move toward the aim is actionable and within the scope of influence of the researcher. My wish is that this study can provide insight to a foundation of improvement for the mathematics department of my school as well as any future researchers that hope to induce similar change in their place of practice.

Appendix A Workshop Outlines

- Workshop 1 – What is Metacognition?
  - Metacognition
    - Who, what, when?
    - Knowledge of cognition
    - Regulation of cognition
    - Benefits of using metacognition in mathematics courses
    - Effect on education and mathematics instructions
  - Metacognition and secondary mathematics
    - What do metacognitive practices look like in a high school math class?
    - How do teachers support students using metacognitive practices?
  - Where will we go from here?
    - Look ahead at specific tools
    - Implement in a designated unit
      - Duration of 3 weeks
      - Each teacher will be observed once per week
      - Debrief session after each observation
    - Evaluate the process
  - Reflect on current metacognitive practices of our students
    - Teacher discussion
    - Complete pre-implementation teacher interview (Appendix I)
- Workshop 2 – Specific Tools of Metacognitive Practices for Secondary Mathematics

    - Present several tools
        - Polya Problem-Solving Techniques (Appendix B)
        - Metacognitive Problem-Solving Process (Appendix C)
        - Assessment Correction Sheet (Appendix D)
        - Sample Documented Problem Solving (Appendix E)
        - Sample Problem Solving Using the GUESS Model (Appendix F)
        - Sample Partner Explanation Essay Activity (Appendix G)
        - Sample Concept Mapping (Appendix H)
        - Teacher-provided tool
    - Discuss initial idea of how teachers can use these tools
- Workshop 3 – Designing Lessons with Metacognitive Practices
    - Teachers choose a unit where they will implement metacognitive practices
    - Teachers choose a tool or tools to incorporate in a future lesson
    - Discuss planning with colleagues and provide feedback on lessons
    - Discuss how teachers will support students while implementing these practices
- Workshop 4 – Evaluation
    - Collect teacher data on feedback from students
    - Focus group questions (Appendix M)

Appendix B Polya's Problem Solving Techniques

Polya's Problem Solving Techniques

From the 1957 book *How to Solve It* by George Polya.

The Four Principles of Polya's problem solving techniques
1: UNDERSTAND the problem
2: Devise a Plan
3: Carry out the Plan
4: Look Back

Understand the problem

Teachers ask the students the following:

- Do you understand all the words used in the problem?
- What are you asked to find?
- What is unknown?
- What is the condition?
- Can you restate the problem in your own words?
- Can you think of a picture or diagram that might help you understand the problem?
- Is there enough information to enable you to find the solution?

Devise a plan

There are many reasonable ways to approach a problem. Through practice, a problem solver develops the ability to choose the appropriate method. Potential methods:

- Guess and check
- Look for a pattern
- Make a list
- Draw a picture
- Eliminate possibilities
- Create and solve a simpler problem
- Use a model
- Use symmetry
- Work backwards
- Use a formula
- Solve an equation
- Use direct or indirect reasoning
- Consider special cases
- Be creative

While developing a plan:

- Find the connection between the data and the unknown
- Have you seen a problem like this before? Can you use a previous problem as a guide?
- Do you know a theorem that is useful?
- Did you use all the data? Did you use the whole condition? Have you considered all the essential parts of the problem?

Carry out the Plan

Once the plan is devised, this step can be easy, given that the problem solver possesses the necessary skills to complete the plan. Needing to go back to the previous step and devising a new plan is possible and sometime necessary.

Look back

Take time to reflect on the experience. This can allow the problem solver to catch any errors as well as make connections to the learning. Reflection allows the learner to make connections with experience and prior knowledge so that the learner can access this experience in the future when necessary.

Appendix C The Metacognitive Problem-Solving Process (MPSP)

Metacognitive Problem-Solving Process

This process is an adaptation from the work of Yimer and Ellerton (2010) which was an extension on the work of Polya (1957).

Engagement

Initial confrontation and making sense of the problem

- Initial understanding of the problem
  - Write down main ideas
  - Draw a diagram
- Analysis of information
  - Make sense of the information
  - Identify key ideas
  - Determine the relevant information for solving the problem
- Reflecting on the problem
  - Assessing familiarity
  - Recall similar problems
  - Assessing the degree of difficulty
  - Recall the pertinent prior knowledge
  - Recall applicable mathematical concept, law, or theorem

Transformation-Formulation

Transformation of initial ENGAGEMENTS to exploratory and formal plans

- Exploring
  - Using specific cases or numbers to visualize the situation in the problem
- Conjecturing
  - Based on specific observations and previous experiences, make a prediction
- Reflection
  - Reflect on predictions and explorations to determine if they are feasible
- Formulate a plan
  - Devise a strategy to test predictions
- Feasibility
  - Reflect on the feasibility of your plan

Implementation

A monitored implementation of plans and explorations

- Exploring key feature of plan
  - Breaking down your plan into manageable parts
- Assessing the plan with the conditions and requirements set by the problem

- Performing the plan
    - Taking action by computing or analyzing
- Reflecting on the actions taken
    - Are they appropriate?
    - Are adjustments needed?

Evaluation

Passing judgements on the appropriateness of plans, actions, and solutions

- Rereading the problem to determine if the result answers the original question
- Assessing the plan for consistency with the key features of the problem as well as for possible errors in computation or analysis
- Assessing for reasonableness of the result
- Making a decision to accept or reject the solution

Internalization

Reflecting on the degree of intimacy and other qualities of the solution process

- Reflecting on the entire solution process
- Identifying critical features in the process
- Evaluating the solution process for adaptability in other situations, different ways of reaching the same solution
- Reflecting on the mathematical rigor involved, one's confidence in the process, and degree of satisfaction

Appendix D Assessment Correction Sheet

Name: ____ Date: ________________

Assessment Correction Sheet

Question What is the question that I missed?	
My Answer How did I arrive at my answer? What led me to my conclusions? What was my line of thinking?	
Incorrect Portion What portion of my reasoning was incorrect? Where did I go wrong in my problem-solving process?	
Correction What can I correct? Where do need to make changes in my conceptual understanding?	
Future What will I do in the future to prevent a similar mistake? What will I look for or avoid?	

Appendix E Sample Documented Problem Solving

For the following problem, in addition to finding a solution, write a short essay describing the thought process in which you used to arrive at your solution. Explain the mental processes you went through in solving the problem.

A baton twirler tosses a baton into the air. The baton leaves the twirler's hand 6 feet above the ground and has an initial vertical velocity of 45 feet per second. The twirler catches the baton when it falls back to a height of 5 feet. For how long is the baton in the air?

Appendix F Sample Problem Using the GUESS Model

For the following word problem, use the GUESS (Given, Unknown, Equation, Substitute, Simplify/Solve) model to organize the information and solve.

You have $10,000 to invest. You want to invest the money in a stock mutual fund, a bind mutual fund, and a money market fund. The expected annual returns for the funds are 10%, 7% and 5% respectively. You want your investment to obtain an overall annual return of 8%. A financial planner recommends that you invest the same amount in stocks as in bonds and the money market combined. How much should you invest in each fund?

G

U

E

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S

Once the problem is solved, write a sentence or two for each of the sections of the GUESS model describing your thought process and stating why you chose to write what you did.

Sample Partner Essay Explanation Activity

In this lesson students split into two groups, A and B. Each group is given a separate problem using quadratic functions in real life. Each student writes a plan of how to solve their problem but does not derive a solution. In this plan, each student puts the mental process they would go through, to solve the problem, on paper. The students are to use an adaptation of the GUESS model in essay form, without rewriting the original problem. Once this task is complete, the students find a partner from the opposite group and switch essays. The students then use their partner's essay to solve the problem without seeing the original word problem.

Group A (only shown to group A)

The equation $h = 0.019s^2$ gives the height h (in feet) of the largest ocean waves when the wind speed is s knots. How fast is the wind blowing if the largest waves are 15 feet high?

Group B (only shown to group B)

The aspect ratio of a TV screen is the ratio of the screen's width to its height. For most TVs, the aspect ratio is 4:3. The measurement of a TV is the length of its diagonal. What are the width and height of the screen for a 27 in TV?

Appendix G Sample Concept Mapping Activity

Concept Map

Create a concept map using the materials provided: poster paper, cards with vocabulary words and terms.

You will be given cards with the following words or terms: quadratic function; parabola; factoring; quadratic equation; zero of a function; square root; complex number; completing the square; quadratic formula; discriminant.

Arrange these cards on the poster paper in a logical order with the starting word: quadratic functions. If you would like to add any cards with your own words or phrases, feel free. Connect each of the cards with connections phrases.

You will present your concept map to me.

Concept question: Using your concept map, explain how quadratic functions are solved and what the solution of a quadratic function means.

Appendix H Pre-implementation Teacher Interview

Teachers will be interviewed before implementing metacognitive practices in their gateway courses. The following questions will guide the interviewer during the interview. However, the interviewer is not limited to these questions alone, instead the interviewer will use the following to begin discussion and add follow-up and extension questions as needed.

1. As we progress through this interview, I ask you to consider your gateway courses (Algebra 1, Algebra 2, and Geometry). Talk to me about your students problem-solving skills? Do you find them to be sufficient? Do you find them to be lacking in any way? Please explain.
2. Do you feel your students are equipped with the appropriate problem-solving skills to be successful in your class? Please explain.
3. How well can students articulate their problem-solving process why solving problems in your course?
4. How well can students identify where they went wrong during a problem-solving experience?
5. Do you see students making the same mistakes repeatedly? Do you see evidence of students learning from their mistakes?
6. How engaged are your students? Would you like this to change?
7. Research shows that metacognition plays an influential role in student engagement and students can increase their metacognition through incorporating metacognitive practices in their daily mathematics problem-solving. How do you feel about helping me determine if this can be an effective practice?

Appendix I Teacher Observation Guiding Questions

The researcher will observe each teacher three times during the intervention period. The following is a list of guiding questions to help record applicable observations.

Teacher: __________________ Course: ___________________Lesson: ______________________

1. How engaged are students in the lesson?

2. How is the teacher modeling the metacognitive practices?

3. How are students using the practice?

4. How well are students articulating their thought process?

5. How well are students detecting their errors in their process?

6. Are students making the same mistakes repeatedly?

7. What are other observations?

For the 2nd and 3rd observations:

8. What changes do you observe?

9. Have students shown growth in their implementation of metacognitive practices? Explain.

Appendix J Post-observation Teacher Debriefing

After each observation. The researcher will debrief with the teacher to gather data on how the teacher perceived the lesson. The following will guide this discussion with the teacher.

1. Please keep in mind that there are no right answers to the questions I am about to ask and I want you to help me evaluate if this technique is something that can be effective in our context. With that said, tell me how you thought the lesson went?
2. What do you think went well? What do you think did not?
3. How comfortable did you feel implementing metacognitive tools in class?
4. Did you notice an effect on your students' engagement?
5. What feedback have you received from students on these new instructional strategies?
6. What will you do differently next time?
7. How has this experience affected your instructional practices for the future?

Appendix K Teacher Focus Group Questions

These questions are used to facilitate discussion during the final workshop for teacher and as an evaluation of the implementation of the metacognitive tools in their gateway courses (Algebra 1, Algebra 2, or Geometry).

1. Thank you for being a part of these workshops and implementing metacognitive tools into your lessons. Talk to me about your experience.
2. What did you find most difficult? What do you find easy?
3. What are some benefits of using the metacognitive tools that you observed? What are some disadvantages?
4. How do you think this implementation affected your students' problem-solving skills?
5. Did you see any change in your students' engagement in their learning while implementing the tools? Please explain.
6. Has using these metacognitive practices changed your viewpoint on your mathematics course at all? Explain why or why not.
7. After reflecting on your experience, what would you do differently? What advice would you give to a teacher interested in trying a similar strategy?
8. What feedback have you gathered from your students regarding using metacognitive practices in your course?

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