Detect, Practice, Repair Multiplication Fact Fluency Intervention for Middle School Students Identified with Learning Disabilities

by

Justin Lee Schwartz

Submitted to the Graduate Faculty of the

School of Education in partial fulfillment

of the requirements for the degree of

Doctor of Education

University of Pittsburgh

2023

UNIVERSITY OF PITTSBURGH

SCHOOL OF EDUCATION

This dissertation was presented

by

Justin Lee Schwartz

It was defended on

March 24, 2023

and approved by

Dr. Rachel E. Robertson, Associate Professor, Department of Teaching, Learning, and Leading

Dr. Mary-Kate Najarian, Director of User Success and Learning Technologies, Montgomery County Community College

Dissertation Director: Dr. Douglas E. Kostewicz, Associate Professor, Department of Teaching, Learning, and Leading

Detect, Practice, Repair Multiplication Fact Fluency Intervention for Middle School Students Identified with Learning Disabilities

Justin Lee Schwartz, EdD

University of Pittsburgh, 2023

This research study sought to identify a solution for building fact fluency for middle school aged students identified with LD. After analyzing a 1st – 12th grade independent school for students with LD's math curriculum, it was noted that basic fact fluency is not taught after grade five. This has shown to be problematic for students who enroll in the school in the later elementary school years or even in middle or high school who do not receive the needed fact fluency practice to become fully fluent in basic math facts. Therefore, it was decided to use a multiple probe design, consisting of baseline, intervention, and maintenance phases with staggered onset to examine the effects of Detect, Practice, Repair (DPR) on single-digit multiplication fact fluency for six identified sixth graders with LD. Fluency growth for each participant was tracked, as well as fluency maintenance once criterion was reached. The Detect stage utilized Xtra Math to create three distinct sets of multiplication facts (Set A, Set B, and Set C), ten problems each, made up of five unknown facts and five non-fluent known facts for each participant. Students worked through the Practice and Repair phases for one set at a time until criterion was reached, then moving onto the next set of facts. Results showed that five of the six participants successfully reached criterion for Set A, three of the six participants reached criterion for Set B, and one participant reached criterion for Set C, suggesting that DPR positively impacted fluency growth. However, fluency maintenance was not achieved for the majority of the participants. These findings help extend the research for fact fluency intervention for middle school aged students, as well as students identified with LD.

Table of Contents

List of Tables

List of Figures

Preface

Dedication

First and foremost, I want to express my love and gratitude to my amazing wife Christie, who has been my constant source of support and encouragement, which has sustained me through the long hours and endless revisions of this dissertation over the years.

To my amazing children Charlotte and Madelyn, who bring joy and meaning to my life every day. Your laughter, curiosity, and boundless energy remind me of the importance of completing my doctoral degree and inspire me to be a better person and role model for you both.

To my father Lee, who instilled in me the perseverance, hard work, and determination I needed to succeed. I am forever grateful for the countless hours you sacrificed throughout my educational career, supporting and motivating me along the way. You are the reason I decided to pursue what I love.

Finally, to my truly incredible mother Celeste, your strength and perseverance to succeed in your own educational pursuits have been a constant source of admiration, and I am forever grateful for all that you have done for me. You have always been and continue to be my role model and source of inspiration. Your unwavering strength, wisdom, and guidance have been invaluable in shaping my character and helping me overcome challenges. I can only hope that I may be a source of inspiration for my own children, just as you are for me. Without you, none of this would have been possible.

Thank you all for your love, support, and belief in me. This dissertation is dedicated to you.

Acknowledgement

I would like to express my deepest gratitude to the University of Pittsburgh for providing me with the opportunity to pursue my doctoral studies. The academic environment and resources provided by the university have been invaluable in shaping my academic and personal growth.

I would like to thank my dissertation committee, consisting of Professor Rachel Robertson and Mary-Kate Najarian, for their invaluable support, and feedback especially at the end of this journey. Their expertise, constructive criticism, and insightful suggestions have helped me to refine and improve my dissertation significantly.

I am particularly grateful to my advisor, Douglas Kostewicz, for his unwavering support, encouragement, and mentorship. His wealth of knowledge, experience, and expertise have been instrumental in shaping my research, and I am deeply grateful for his generous guidance and invaluable feedback throughout the entire process.

I am also indebted to the countless individuals who have generously given their time, expertise, and support, including my fellow colleagues, friends, and family. Their support and encouragement have been instrumental in making my academic journey a fulfilling and rewarding experience.

Finally, I would like to express my heartfelt appreciation to my family for their unwavering love, support, and encouragement throughout my academic journey. Their unwavering faith in me has been a constant source of inspiration and motivation, and I am deeply grateful.

1.0 Introduction

1.1 Problem of Practice

As a Special Education/ Upper School Mathematics teacher at a 1 – 12 school whom serves only students identified with learning differences, I am charged with the responsibility to develop and implement multisensory lessons using best practices and research-based instruction, which builds upon previous student learning, as well as filling gaps in students' mathematical knowledge.

The practice-related setting, a 1 – 12 independent school located in eastern Pennsylvania where I am employed, identifies itself as a research-to-practice-school, a leader in teaching techniques identified to foster learning for students with language-based learning differences, such as dyslexia, dysgraphia, and dyscalculia. The school's educational philosophy and ideologies embrace research-based intervention strategies and an art-based curriculum, especially in their lower and middle schools. Additionally, the school identifies itself as a rigorous college preparatory school, seeking to prepare all of its students with the knowledge, studentship skills, and self-advocacy skills needed to succeed in a university/ college atmosphere.

The school's 1 – 5 mathematical curriculum aligns with a constructivist approach to teaching. The spiraling mathematical curriculum exposes students to various mathematical strategies while cycling back to previously taught material at planned intervals. The curriculum has students practice multiple approaches to solving basic problems, aiming for students to be able to use their own strategies or their chosen strategy to solve future problems, believing it will promote a deeper level of conceptual understanding and foster greater mathematical reasoning skills. (Poncy et al., 2010).

My work in grades sixth through twelfth revealed a continuous problem area in our mathematics curriculum and teaching. In all grade levels students are taught numerous strategies to problem solving that are meant to enhance and improve students' mathematical reasoning and problem-solving skills (Poncy et al., 2010). In addition to problem solving, students are taught computational strategies in order to develop their basic math fact understanding and acquisition particularly in the lower grades. However, when analyzing the mathematical deficit in middle and upper school students identified as struggling in math, learning disability (LD) was a common theme across most students. Mentally computing basic math facts was one of the most common deficits held by these specific middle and upper school students. Most of these students were still using the counting strategies taught in the younger grades for basic fact acquisition in the areas of addition and subtraction, but when presented with basic multiplication fact problems they needed to use a calculator.

This problem led to questioning the mathematics curriculum and how math facts are taught throughout each grade. Across first through fifth grade, the math curriculum introduces multiple numeration strategies to develop students' number sense in order to foster mental calculation flexibility. Additionally, as students progress through grades, they learn conceptual strategies to calculate basic math facts, while completing weekly fact quizzes. The quizzes are timed and math fact fluency, accuracy plus speed, was stressed. However, after grade five, fact fluency was no longer a focus in middle and upper school math classes. This appears to be problematic for students who started attending the school in the later lower school grades, and did not receive enough fact fluency practice to become fully fluent in all basic fact calculations, as well as new students who start in sixth grade who come to the school without being fluent in their basic math facts.

Haring and Eaton (1978) identify the needed sequential steps for skill development, acquisition, fluency building, then generalization. Currently, math fact fluency building is only explicitly and consistently taught in the lower grades, 1 – 5. This could explain why middle and upper school students, in grades 6 - 12, continue to struggle with basic math facts and subsequently mathematics throughout their academic careers. Therefore, it is apparent that there is a need for a math fact fluency intervention for students identified as struggling to acquire facts and build fact fluency, particularly in the middle and upper school grades. Addressing this problem of practice will positively affect student skill generalization and overall academic success in the future.

The various stakeholders that are affected directly and indirectly consist of the students, parents and families of the students, teachers, and administrators at the school. Additionally, less directly affected stakeholders consist of other schools who serve students with learning differences and their populace, as well as constructivist mathematical curriculum developers focused on math fact development. Although, constructivist math curriculums are meant to instill greater mathematical understanding in students, and for some students this style of curriculum will do just that, for others, particularly students with learning differences or students struggling to acquire basic fact fluency, a constructivist teaching approach may need to be supplemented with an additional fact fluency intervention.

There is clearly a need to address this problem of practice. A multiplication fact intervention, which addresses accuracy and speed (fluency), needs to be identified. The identified intervention will have to be implemented class-wide, while addressing individual needs of all students. Additionally, the ideal intervention will not require too much daily implementation time, so as not to take away from content instructional time needed for the middle and upper school mathematics curriculums.

2.0 Review of Literature

Molding responsible and academically independent students is a fundamental goal of our educational system, specifically in the areas of reading and mathematics (Konard & Joseph, 2013). However, the results of The National Center for Education (2015) (NAEP) national mathematics assessment indicates that only 40% of fourth graders and 33% of eighth graders performed at or above proficient math level (NAEP, 2015). Acquiring proficient mathematical understanding and skills is important to students' future academic and occupational success. Thus, Ansari (2016) indicates that early numerical skills are a better predictor for academic achievement compared to early reading skills, and suggests that numerical skills are linked to future economic status and outcomes. Therefore, the possible reasons behind the national deficit in mathematics performance should be explored further.

There are two basic mathematical domains in which students can be identified as struggling in, computation/ calculation and problem solving/ reasoning (Fuchs et al., 2008). Mathematical computation/ calculation refers to the ability to mentally compute numbers using basic mathematical operations, such as addition, subtraction, multiplication, and division, but eventually continues onto higher levels of calculations (Gürbüz & Erdem, 2016). Challenges with mathematical computation may be linked to attention and processing deficits (Fuchs et al. 2008). Mathematical problem solving/ reasoning refers to the process used to come to a mathematical truth in order to solve a problem and is associated with language deficits (Fuchs et al. 2008; Gürbüz & Erdem, 2016). Therefore, this suggests computation and mathematical reasoning deficits are the two skill areas that should be addressed to improve the national mathematical performance.

Practitioners and researchers have used numerous methods, techniques, and interventions to address math difficulties in problem solving and computation. Examples of current curricular interventions to address math difficulties which focus on improving conceptual understanding and is used to improve both problem solving and computational skill development is hands-on experiential learning techniques, such as discovery learning, problem-based learning, and constructivist learning, instead of just focusing on basic story problems when beginning to teach problem solving skills (Poncy et al., 2010). The educators that prefer to use hands-on experiential learning to teach computational and problem solving skills generally believe in an unguided educational environment where students are able to construct their own meaning and use their own strategies to solve problems, which in turn is meant to promote a deeper level of understanding and further build students' independent mathematical computation and reasoning skills (Poncy et al., 2010).

Specifically, in mathematics, practitioners who adhere to using experiential learning to promote problem solving skills also allow students to practice multiple approaches to solving basic problems in addition to unguided discovery learning (Poncy et al. 2010). Furthermore, Poncy et al. (2010) argues that providing excessive guidance to students, especially during skill development, will impede their ability to use those learned skills independently.

However, Gürbüz and Erdem (2016) and Fuchs et al. (2008) suggest experiential learning techniques only promote mathematical reasoning and do not address the underlying reading difficulties. Therefore, to address students' underlying reading difficulties, which are closely associated with mathematical problem solving challenges, researchers focus on procedural interventions for problem solving. Procedural interventions are meant to instill a deeper understanding of mathematical language and provide a framework to solve specific problems that students experience, examples include Mercer, Mercer, and Pullen's (2011) RIDE intervention, as well as Owen's (2003) TINS (Hott et al., 2014; Poncy et al., 2010). These procedural interventions use acronyms to enable students, particularly struggling readers, to remember and follow specific steps when tackling and deciphering word problems (Hott et al., 2014).

Additionally, Gürbüz and Erdem (2016) also argue experiential learning techniques do not address mathematical computation difficulties, which are needed when problem solving in order to use mathematical truths to make logical conclusions. They suggest that mathematical computation/ mental computing play an important role in building mathematical reasoning skills. Moreover, Gersten, Jordan, and Flojo (2005) state that most students identified with a math disability exhibit challenges with basic math computation, such as simple addition facts, both in accuracy and retrieval speed. Therefore, it can be argued that improving mathematical computation skills will not only improve mental computation but also mathematical problem solving for most students identified with a learning disability (Fuchs et al., 2008; Gürbüz and Erdem, 2016).

Furthermore, it may be possible that addressing each skill individually, computation and reasoning, may not be enough for students identified with a learning difference. Students need to be fluent in their calculation and reasoning skills, which suggests fluency should be examined in relation to the national deficit in math performance. Fluency or behavioral fluency has been a term closely associated with skill mastery. Fluency has been defined as, "a fluid combination of accuracy plus speed to characterize competent performance" (Binder, 1996, p.164). Binder (1996) suggests that when a high rate of accurate performance is attained, a higher level of retention is achieved.

Poncy, McCallum, and Schmitt (2010) suggest one reason behind the national deficit in mathematics performance is students' inability to master basic fundamental mathematical skills, such as math fact fluency, which is identified as the ability to answer basic operational math fact problems with effortless speed and accuracy. In the area of mathematics, computational fact fluency is an essential skill, which affects a student's ability to compute and problem solve, and therefore deficits in fact fluency can inhibit their ability to meet national education standards as well as acquire advanced mathematical skills (Poncy, McCallum, and Schmitt, 2010). Therefore, it can be argued that obtaining fact fluency is an essential part of the learning process and is necessary to acquire skill proficiency and can positively impact computation and reasoning skills.

Mastery of computational fact fluency is particularly challenging for struggling students to attain, such as students identified with learning differences or at-risk for math failure/ achievement difficulties (Gürbüz &Erdem, 2016). According to the US Department of Education 38th Annual Report to Congress on the Implementation of the Individuals with Disabilities Education Act (IDEA, 2016), nearly 40% of all students diagnosed with a disability, ages 6 – 21, fall under the category of Specific Learning Disability (SLD). SLD accounts for approximately 8% of all students. The term learning disability refers to the difficulty a student has acquiring academic or functional skills, while possessing average to above average intelligence (Kauffman & Hallahan, 2011). SLD is a general classification of special education, comprised of disabilities in such areas as reading, writing, and mathematics.

The National Center for Education (2015) (NAEP) national mathematics assessments indicates a significant difference in performance between students identified with a disability compared to students without a disability. Sixteen percent of fourth graders and 8% of eighth graders identified as having a disability performed at or above proficient math level. This statistic is significantly worse than their nondisabled peers. Forty-four percent of fourth graders and 37% of eighth graders without disabilities performed at or above proficient math level on the national mathematics assessments (The National Center for Education, 2015).

Mathematical problem solving and computation (explained above) are two distinct areas in which students can have difficulty (Fuchs et al., 2008; Lyon, 1996). Therefore, students can be identified as having a specific learning disability in either problem solving or computation, or a combination of the two, which is supported by IDEA (2004). If left unaddressed, those difficulties may lead to a narrowing of future educational opportunities and potential occupational prospects (Poncy et al., 2010).

However, it's argued that achieving computational fact fluency can positively impact both computation and reasoning skills (Gersten et al., 2005; Poncy & Skinner, 2011). Therefore, effective interventions aimed at math fact computation are needed. Those may include procedural intervention, which only focus on acquiring math facts and possibly also focusing on increased responding rate, or conceptual interventions, which concentrate on improving operational understanding. An examination into current interventional practices and research focused on fundamental mathematical skills is needed, especially for students diagnosed with learning disabilities or identified as struggling math learners at risk for low math achievement.

2.0.1 Current Curricular Interventions to Address Computation Difficulties

Currently, standard math curricula only emphasize acquisition of basic math facts, using strategies such as finger counting, number lines, touchpoints, flash cards, fact families, and manipulatives (Poncy et al., 2010; Poncy & Skinner, 2011). These strategies are used to increase students' understanding of basic math facts, which is important, but these strategies can interfere with fluency, specifically for students with a learning difference, due to strategy dependence (Poncy & Skinner, 2011). Current standard curricula do not focus on increasing students' fact speed/ automaticity, which is paramount to attain computational fact fluency (Poncy et al., 2010). Computational fact fluency, which focuses on increasing fact accuracy and speed, is an important cornerstone of mathematical learning, affecting all areas of mathematics and is essential in everyday life (Koponen, Aro, Räsänen, & Ahonen, 2007; Poncy et al., 2010). It is an important skill that needs to be directly taught, particularly for students identified with learning disabilities (Gersten et al., 2005).

Poncy and Skinner (2011) suggest that students with disabilities have limited cognitive resources, which can affect their endurance and time needed with a challenging task. Therefore, when learning advanced mathematical concepts, students' focus should not be drawn away from advanced mathematical concepts to calculate basic facts. Poncy and Skinner (2011) also argue that if students solve basic facts slowly but accurately, they will have fewer opportunities to develop advanced math skills. Additionally, they are less likely to engage in math tasks, which can affect their academic achievement (Poncy & Skinner, 2011). Therefore, this suggests that fact acquisition is not sufficient for students identified with learning disabilities, and that fact fluency is crucial if not essential for mathematical growth and future academic achievement (Gersten et al., 2005; Poncy & Skinner, 2011).

2.0.2 Computational Fact Fluency Interventions

Haring and Eaton (1978) describe the skill development hierarchy, starting with the acquisition stage, which focuses on accuracy by using descriptions, demonstrations, and modeling. The following stage is the fluency building stage, which focuses on increasing the speed of accurate responding. The third stage is generalization, where the skills acquired are used to problem solve (Poncy & Skinner, 2011). In today's curricula, the second stage of fluency building is skipped; moving from acquisition to generalization, assuming fluency will be a byproduct of learning mathematical skills and problem solving (Poncy et al., 2010). This is not always the case, especially for those students identified with learning disabilities or students identified as at-risk for math failure (Gersten et al., 2005; Poncy et al., 2010).

Joseph et al. (2012) conducted a literature review on interventions used to improve the accuracy or fluent responding, specifically analyzing the intervention Cover, Copy, Compare (CCC) and variation studies for spelling and mathematics, as well as other content areas (i.e. geography and science) for students with and without disabilities. Within the review, twelve CCC/ CCC variation mathematical intervention studies were examined. Four dependent variables were identified, accuracy, fluency, maintenance, and social validity. Fluency was measured in nine of the twelve math studies, and were the only fluency studies out of all reviewed articles. It was concluded that overall CCC/ CCC variation interventions helped improve student academic performance, as well as improve fluent responding.

Burns et al. (2010), a meta-analysis, compared acquisition and fluency interventions for skill levels, frustration and instructional. Burns et al. (2010) findings suggest acquisition interventions are superior compared to fluency interventions for participants identified in the frustration level. Additionally, researchers also found that participants identified in the instructional level made little improvement when using acquisitions interventions. Therefore, researchers suggest additional research is needed in this area to better understand appropriate interventions for skill level. Moreover, Methe et al. (2012) conducted a meta-analysis on single-case math computational intervention studies to qualify the evidence for math computation interventions. Methe et al. (2012) concluded that there was a lack of experimental rigor in math computation intervention studies. Therefore, researchers suggest continued research is needed to bolster the evidence base.

The behavioral fluency paradigm indicates that increased speed combined with accuracy will promote mastery and improve skill maintenance (Binder, 1996). Therefore, interventions focusing to improve both speed and accuracy may be necessary to achieve fact mastery. In order to understand the efficacy of basic fact computation interventions for struggling math learners (i.e. students with learning disabilities and/ or students identified as being at-risk for math failure or math achievement difficulties), a systematic literature review was conducted, specifically targeting multiplication fact interventions. Multiplication fact intervention studies were chosen to review based on the specific need within the proposed case study setting.

The review of literature was undertaken to identify current interventional practices to improve multiplication fact computation and address the following questions:

1. What types of participants were included?
2. What types of interventions were used to affect multiplication fact computation (accuracy or fluency)?
3. What dependent variables were measured in the published intervention studies?
4. What study design types (single case or group) were used and what were the outcomes of the selected intervention studies by design type?

2.1 Methods

A comprehensive search was conducted to identify studies with students identified as learning disabled or having a learning disability (LD) or students identified as at-risk for math failure or math achievement difficulties, examining multiplication math fact fluency interventions, and their effects on student fact fluency (accuracy and/or rate).

The studies included in the current review were found using a computerized online search of two educational databases, PsycInfo and ERIC. The databases were systematically scanned using all combinations of the following key search terms to find relevant articles (math*, mathematic*, arithma*, computation*, computation* fluency, fact fluency, math fluency, instruct*, strate*, interven*, fluen*, speed, accura*, rate, learn* disabilit*, learn* disabl*, learn* deficit*, learn* impair*, math* disabilit*, at-risk, learn* difficult*, mild disabilit*, high incidence disabilit*, learn* disorder*), which yielded 1,433 articles. Additionally, Boolean operators (AND, OR) were used to combine a group of related terms together with other related terms (i.e. math or mathematic*or arithma* and computation*or computation* fluency or fact fluency or math fluency, etc.). The search was limited to articles published in peer reviewed journals, written in English, and all publication years were included.

The relevant literature was then narrowed using the inclusion criteria. The inclusion criteria were as follows: (a) a quantitative study that examined a computation fact intervention/s that focused on multiplication math fact responses (accuracy and/ or rate), (b) include school-aged students (ages 6 – 18), (c) include students identified as at-risk for math failure or math achievement difficulties, students identified with a LD or mathematics disability (MD), or students with ADHD (Attention Deficit Hyperactivity Disorder). Students identified with a LD, MD, or at-risk for math failure/ math achievement difficulties were included to identify what is currently being done to address deficits in fundamental mathematics skills, specifically math fact computation, for students who struggle to achieve fundamental math skill mastery (Fuchs et al., 2008; Gersten et al., 2005; Gürbüz and Erdem, 2016; Poncy & Skinner, 2011). Students identified as having ADHD were included because computation deficits are linked to attention and processing deficits, which is argued to influence computational fluency (Fuchs et al., 2008). In addition, studies had to employ an experimental group or single case design (such as multiple baseline or multiple probe design). Therefore, if some participants were focused on multiplication fact fluency, while others in the same study focused on fluency of another operation, like division, the study was included in the relevant literature.

Articles were excluded if published in non-peer reviewed journals, were written in a language other than English, or if classified as a qualitative study or descriptive concept paper. The relevant literature was limited using the exclusion criteria. The exclusion criteria were as follows: (a) a study that examined a computation fact intervention/s that only focused on addition, subtraction or division math fact responses (accuracy and/ or rate), (b) included participants that fell outside of school-age (6 - 18 years old), (c) included participants not identified as at-risk for math failure or math difficulties or with a disability other than a LD (e.g. physical disability, mental disorder, or autism spectrum disorder).

In order to determine if studies met inclusion and exclusion criteria for the literature review, all titles and abstracts were read through in their entirety. If an article was determined to be included by their title and abstract, then the entire article was read through to further determine if the article met all inclusion and exclusion criteria.

After inclusion and exclusion criteria were placed on identified studies, the search yielded fourteen relevant articles. Second, an ancestral search was conducted by reviewing all cited sources from the relevant articles in an attempt to identify any additional relevant studies that met inclusion criteria not previously found; one article was identified. Lastly, a hand search was conducted from the *Journal of School Psychology*, which produced one additional article (Nelson, Burns, Kanive, & Ysseldyke, 2013). The *Journal of School Psychology* was selected to conduct a hand search because the literature review found, described above, was published in the journal.

Overall, the literature search produced seventeen studies (Becker, McLaughlin, Weber, & Gower, 2009; Brady & Kubin, 2010; Burns, 2005; Burns, Kanive, & Degrande, 2010; Burns et al., 2015; Flores, Houchins, & Shippen, 2006; Glover, McLaughlin, Derby, & Gower, 2010; Hulac, Dejong, & Benson, 2012; Kanive, Nelson, Burns, & Ysseldyke, 2013; Leach, 2016; Mcintyre, Test, Cooke, & Beattie, 1991; Nelson, Burns, Kanive, & Ysseldyke, 2013; Ok & Bryant, 2015; Poncy, Skinner, & Axtell, 2010; Rhymer, Dittmer, Skinner, & Jackson, 2000; Skarr et al., 2014; Wood, Frank, & Wacker, 1998). Once the relevant literature was identified common themes were recognized and were used to code.

Characteristics coded in the relevant literature were, first, participant characteristics such as grade level or age, and disability (i.e., LD, ADHD, or at-risk for math achievement difficulties). Second, additional characteristics coded were dependent variables (single digit multiplication accuracy/ fluency, generalization, maintenance, and overall math achievement), intervention type (procedural or conceptual), and outcomes by designs of studies (single case vs. group). All studies reviewed contained at least one procedural fact computation treatment, however, some studies compared a fact fluency intervention to a conceptual intervention, which focused on improving multiplication understanding and knowledge.

2.2 Results

2.2.1 Participant Characteristics

Across the seventeen studies analyzed, 446 participants were included (see Table 1). Upon review, student grade levels were notably identified over student age.

2.2.1.1 Grade Level

The grade levels ranged from grade 1 (e.g., Burns et al., 2015) through grade 6 (e.g., Flores, Houchins, & Shippen, 2006) across all studies. However, the majority of students included in treatment groups fell in grades 3 – 5 (Becker, McLaughlin, Weber, & Gower, 2009; Brady & Kubin, 2010; Burns, 2005; Burns, Kanive, & Degrande, 2010; Burns et al., 2015; Hulac, Dejong, & Benson, 2012; Kanive, Nelson, Burns, & Ysseldyke, 2013; Leach, 2016; Mcintyre, Test, Cooke, & Beattie, 1991; Nelson, Burns, Kanive, & Ysseldyke, 2013; Ok & Bryant, 2015; Poncy, Skinner, & Axtell, 2010; Rhymer, Dittmer, Skinner, & Jackson, 2000; Skarr et al., 2014; Wood, Frank, & Wacker, 1998) and were included in 15 of the 17 studies (88%).

Students below third grade were only identified in 1 of the 17 studies, 6% (Burns et al., 2015). Burns et al. (2015) intervention study focuses on three participants, one in third grade working to master multiplication facts and two in second grade working to build addition fluency. The study was included within the review because the fluency intervention for the third grader met inclusion criteria. Students in grade 6 were included in 1 of the 17 studies, 6% (Flores et al., 2006). Additionally, 1 of the 17 studies, 6%, (Glover et al., 2010) did not report students' grade levels, but did report the students' ages, 11 and 12.

2.2.1.2 Disability Type

Students identified with LD or ADHD were included in nine out of the 17 studies, 53% (Becker et al., 2009; Brady & Kubin, 2010; Burns, 2005; Flores et al., 2006; Glover et al., 2010; Mcintyre, Test, Cooke, & Beattie, 1991; Ok & Bryant, 2015; Skarr et al., 2014; Wood et al., 1998). The remaining studies (*n*=8, 47%), contained students determined to be "at-risk" for math achievement difficulties or math failure, identified as performing below the 25th percentile on math achievement tests (Burns, Kanive, & Degrande, 2010; Burns et al., 2015; Hulac, Dejong, & Benson, 2012; Kanive et al., 2013; Leach, 2016; Nelson et al., 2013; Poncy, Skinner, & Axtell, 2010; Rhymer, Dittmer, Skinner, & Jackson, 2000).

2.2.2 Intervention Type

Table 1 reports data related to intervention type. Two intervention types were identified within the relevant studies reviewed, procedural and conceptual interventions. Procedural interventions focus on improving response accuracy or rate of single digit multiplication math facts, as well as improving generalization of skills. Conceptual interventions focus on the conceptual understanding of multiplication in order for students to better perform on single digit multiplication math facts or generalization of skills (Rittle-Johnson, Siegler, & Alibali, 2001).

As indicated within Table 1, 14 out of the 17 studies (82%) (Becker et al., 2009; Brady & Kubin, 2010; Burns, 2005; Burns, Kanive, & Degrande, 2010; Flores et al., 2006; Glover et al., 2010; Hulac, Dejong, & Benson, 2012; Leach, 2016; Mcintyre, Test, Cooke, & Beattie, 1991; Ok & Bryant, 2015; Poncy, Skinner, & Axtell, 2010; Rhymer, Dittmer, Skinner, & Jackson, 2000; Skarr et al., 2014; Wood et al., 1998) only analyzed a procedural intervention, focusing on building acquisition/ fluency, with some also focusing on maintenance of skills. For example, Becker et al.

(2009) conducted a single case study on one fourth grade student identified with a LD using the procedural intervention Cover, Copy, Compare (CCC). Becker et al. (2009) compared the CCC to CCC with error drill to identify if one of the interventions improved the growth rate of fact fluency for specific multiplication facts as well as decreased errors. For the CCC, the student was given ten multiplication fact problems on a CCC worksheet. The worksheet had four columns, one containing the problem and solution, the second for the student to copy the problem and solution into, the third for the student to copy the problem and the solution into by memory, and the fourth for the student to copy the problem and the solution into by memory three more times. The students were asked to copy a math fact while looking at the given math fact, cover up the math fact and write the math fact from memory, compare their written math fact to the given one, and finally write the math fact three more times. After the student completed the CCC worksheet they were given a timed one-minute probe with ninety problems.

The CCC with error drill intervention added an additional error drill after the one-minute probe was complete. The researcher identified errors on the probe and would correct them for the student. The student would correctly say a math fact aloud several times and then write it on a separate piece of paper. Once the student did this for all errors, they would say aloud the corrected errors again.

Similarly, Poncy, Skinner, and Axtell (2010) conducted a multiple-probe across tasks design single case study with seven third grade participants. The participants were identified as at-risk for math achievement difficulties. The study used a procedural intervention called Detect, Practice, Repair (DPR) in order to identify its effects on the students' multiplication fact fluency. DPR is made up of three phases, Detect, Practice (Cover, Copy, Compare: CCC), and Repair (one-minute sprint and self-graphing). In the Detect phase of the intervention, researchers first identified non-fluent multiplication facts for each student. After non-fluent facts were identified students moved on to the practice phase, where a CCC intervention was used with five identified facts. Finally, once the CCC intervention was complete the Repair phase began. Students completed a sprint, where they answered as many problems as possible in one minute on a sprint worksheet. The sprint worksheet contained ninety randomized single digit multiplication facts. Once the sprint was completed students graphed the number of problems they completed. Later, researchers identified digits per minute (dpm) correct for each of the sprints. Poncy, Skinner, and Axtell (2010) found that all seven participants dpm correct in the post assessment improved from the baseline for all three problem sets.

Three out of the 17 studies (18%) (Burns et al., 2015; Kanive et al., 2013; Nelson et al. 2013) compared procedural to conceptual interventions, focusing on multiplication fact fluency and/ or maintenance, as well as generalization skills. For example, Kanive et al. (2013) conducted a group study, which compared a computer-based intervention, Math Facts in a Flash (MFF), a computer-based program, which is meant to improve fact fluency developed by Van de Walle and Lovin (2006), was used in the study along with Fill the Chutes, Build It In Parts, and Broken Calculator. In MFF, students take a pretest to identify non-mastered facts, then the program has the students practice a specific set of problems until the student achieves mastery by reaching their goal on a forty problem timed test. Once a set of facts is mastered, the program moves the student onto the next set of non-mastered facts. The conceptual interventions, Fill the Chutes, Build the Parts, and Broken Calculator, used a model-led test explicit instruction approach to teaching multiplication facts. Fill the Chutes is an activity where students are given a multiplication fact problem and are asked to represent the problem by placing counters on rectangular "chutes" depicted on a piece of paper. Build It In Parts is an activity where students are instructed to use colored counters to represent a multiplication fact problem in as many ways as they can. Broken Calculator is an activity where students use a calculator to find the products of multiplication problems without using the multiplication key.

2.2.3 Dependent Variables and Measurement

Table 1 also shows data related to each study's dependent variable. Four areas of focus were identified: single digit multiplication accuracy/ fluency, generalization of skills, and maintenance of skills.

2.2.3.1 Single Digit Multiplication Accuracy/ Fluency

Four of the 17 studies (24%), measured single digit multiplication accuracy. The four studies measured accuracy using correct and/or incorrect verbal or written responses as their dependent variables (Glover et al., 2010; Nelson et al., 2013; Skarr et al., 2014; Wood et al., 1998). The studies only indicate correct responses given, not indicating time or speed of responses. Therefore, quick responding or improved speed could not be assumed and dependent variables were categorized as single digit multiplication accuracy, not fluency.

Eleven studies (65%) focused on fluency (accuracy and rate) using digit per minute (dpm) correct (Brady & Kubin, 2010; Burns, 2005; Burns et al., 2015; Hulac, Dejong, & Benson, 2012; Kanive et al., 2013; Mcintyre et al., 1991; Ok & Bryant, 2015; Poncy, Skinner, & Axtell, 2010; Rhymer, Dittmer, Skinner, & Jackson, 2000), correct responses per minute (Becker et al., 2009), or correct individual responses within three seconds as their dependent variable (Leach, 2016).

2.2.3.2 Generalization of Skills

Four out of the 17 studies (24%) (Burns, Kanive, and Degrande, 2010; Flores et al., 2006; Kanive et al., 2013; Nelson et al., 2013) assessed interventional effects of generalization probes (word problems probes, general math achievement test, or unknown math fact probes) as a secondary variable. The results varied between all four studies to improve generalization scores.

Both Kanive et al. (2013) and Nelson et al. (2013) found generalization (word problem probe scores) was not significantly affected by the multiplication fact interventions applied. Burns, Kanive, and Degrande (2010) found the fluency intervention, MFF, positively affected their treatment group, increasing the rate of improvement on generalization scores on a Star Math assessment, a general mathematics achievement test, when compared to the control group. However, generalization scores were not significantly different. Additionally, Flores et al. (2006) used timed unknown multiplication fact probes to measure generalization, and found that half of the participants performed better when using the conceptual intervention compared to using the procedural intervention. However, when given similar untimed generalization probes all participants performed better and scores were similar between both groups.

2.2.3.3 Maintenance of Skills

Four out of the 17 studies, 24%, (Flores et al., 2006; Hulac, Dejong, & Benson, 2012; Mcintyre et al., 1991; Poncy, Skinner, & Axtell, 2010) focused on maintenance of skills across time. Flores et al. (2006) used timed and untimed probes to assess maintenance of skills one and five weeks after the intervention phase, which furthered previous maintenance research that assesses maintenance after three weeks. Maintenance probes were made up of previously mastered math facts through the intervention. Hulac, Dejong, and Benson (2012) measured the effect of a self-administered fluency intervention (CCC) on math fact fluency growth and also assessed maintenance of skills two to three weeks after the intervention phase finished. Mcintyre, et al. (1991) used a multiple-probe-across-tasks design using Count-Bys, a skip counting intervention, for one fourth-grade student diagnosed with a LD. Researchers measured fact fluency growth for three multiplication fact families, eight, four, and seven. Additionally, maintenance data was taken for eights and fours. Poncy, Skinner, and Axtell (2010) also used a multiple-probe-across-tasks design, but used Detect, Practice, Repair (DPR), and adapted CCC intervention with added sprinting. The intervention used three different multiplication problem sets. Once a participant mastered the first problem set they moved onto the second, and then the third. While working on mastering the next problem set, maintenance data was collected for the previously mastered problem sets.

2.2.4 Outcomes by Design of the Studies

Table 1 reports the design type as well as outcomes for each study. Design types were divided between single case design (*n*=14, 82%) of the studies (Becker et al., 2009; Brady & Kubin, 2010; Burns, 2005; Burns et al. 2015; Flores et al., 2006, Glover et al., 2010; Hulac, Dejong, & Benson, 2012; Leach, 2016; Mcintyre et al., 1991; Ok & Bryant, 2015; Poncy, Skinner, & Axtell, 2010; Rhymer, Dittmer, Skinner, & Jackson, 2000; Skarr et al., 2014; Wood, Frank, & Wacker, 1998), and group design (n=3, 18%) of the studies (Burns, Kanive, & Degrande, 2010; Kanive et al., 2013; Nelson et al., 2013).

2.2.4.1 Single Case Studies

Fourteen single case design studies were identified within the literature (Becker et al., 2009; Brady & Kubin, 2010; Burns, 2005; Burns et al. 2015; Flores et al., 2006, Glover et al., 2010; Hulac, Dejong, & Benson, 2012; Leach, 2016; Mcintyre et al., 1991; Ok & Bryant, 2015; Poncy, Skinner, & Axtell, 2010; Rhymer, Dittmer, Skinner, & Jackson, 2000; Skarr et al., 2014; Wood, Frank, & Wacker, 1998). The majority of single case studies used multiple baseline (*n*=5) (Burns, 2005; Burns et al., 2015; Glover et al., 2010; Hulac, Dejong, & Benson, 2012; Skarr et al., 2014) or multiple probe designs (*n*=4) (Mcintyre et al., 1991; Ok & Bryant, 2015; Poncy, Skinner, & Axtell, 2010; Wood, Frank, & Wacker, 1998). The remaining single case designs used alternate treatment design (*n*=2) (Brady & Kubin, 2010; Rhymer, Dittmer, Skinner, & Jackson, 2000), AB design (*n*=1) (Leach, 2016), and ABCA design (*n*=2) (Becker et al., 2009; Flores et al., 2006) where researchers compared two interventions on pre and post assessments.

Three of the single case design studies (21%) reported substantial improvement or a significant increase in acquisition of multiplication math facts, using baseline to treatment data (Glover et al., 2010; Skarr et al., 2014; Wood, Frank, & Wacker, 1998). Seven studies (50%) reported improvement in multiplication fact fluency for the majority if not all participants, using baseline to treatment data (Becker et al., 2009; Burns, 2005; Leach, 2016; Mcintyre, Test, Cooke, & Beattie, 1991; Ok & Bryant, 2015; Poncy, Skinner, & Axtell, 2010; Rhymer, Dittmer, Skinner, & Jackson, 2000).

The remaining four single case design studies (29%) (Flores et al., 2006; Hulac, Dejong, & Benson, 2012; Mcintyre et al., 1991; Poncy, Skinner, & Axtell, 2010) focused on math fact fluency growth and interventional effects on maintenance of skills. Three of the four studies (Flores et al., 2006; Mcintyre et al., 1991; Poncy, Skinner, & Axtell, 2010) reported all participants demonstrated improved rate of fluency during intervention phases when compared to baseline phases.

Flores et al. (2006) showed that a combined intervention, Strategic Instructional Model (SIM), using conceptual understanding and procedural fluency creates a greater effect on maintenance of skills one week after treatment, with an increase between 16 and 24 dpm correct as well as untimed generalization probes with an increase between 39 and 43 dpm correct. Hulac, Dejong, and Benson (2012) showed that a fact fluency intervention, CCC, can solely positively impact maintenance of skills two to three weeks after the intervention finished. The researchers assessed maintenance for five of their eleven participants. Four of those five participants' fluency scores on maintenance probes showed that fact fluency was maintained throughout the maintenance phase.

Mcintyre et al. (1991) demonstrated that by using Count Bys, a skip counting multiplication fact intervention, could improve the fluency scores for the single participant. The intervention focused on three fact families, which were mastered by the participant by the end of the school year. Additionally, maintenance data was taken for two of the three fact families, eights and fours. The student demonstrated a mean score of 77 dpm correct during the maintenance phase for eights compared to a baseline mean score of 7.6 dpm correct. Also, the student demonstrated a mean score of 65 dpm correct during the maintenance phase for the fours compared to a baseline mean score of 17.6 dpm correct.

Poncy, Skinner, and Axtell (2010), conducted a sixteen day intervention, DPR, to measure fluency growth for three problem sets, as well as collecting maintenance data for each. However, the maintenance phases for the three problem sets differed due to the length of the intervention. Problem Set A's maintenance phase was conducted for six days, Problem Set B's maintenance phase was conducted for two days, and Problem Set C's maintenance phase was conducted for only one day. The researchers found that their participants' multiplication fact mean maintenance score 32.9 dpm correct improved from their mean baseline score of 20.9 dpm correct.

However, Hulac, Dejong, and Benson (2012), reported that seven of the 11 participants demonstrated an improved fluency rate during the intervention phase when compared to the baseline phase. However, two participants demonstrated greater fluency growth rate during the baseline phase compared to the intervention phase, but the intervention phase still showed a positive fluency rate. One participant showed very little growth during all phases of the intervention.

2.2.4.2 Group Design

Three randomized pretest–posttest control group design studies were identified within the literature (Burns, Kanive, & Degrande, 2010; Kanive et al., 2013; Nelson et al., 2013). All participants in the group designs were identified as at-risk for math achievement difficulties. Burns, Kanive, and Degrande (2010) measured the effects of a multiplication fact fluency intervention, MFF, on a Star Math achievement assessment. Outcomes show the treatment group achieved a larger increase in assessment scores compared to the control group. Researchers converted the overall mathematics scores to a normal curve equivalent (NCE), finding effective small to moderate effects for third graders (d = .34) and fourth graders (d = .44). The results indicate that improved fact fluency can affect overall math achievement.

Nelson et al. (2013), compared two interventions, Math Facts in a Flash (MFF), a computer-based math fact fluency intervention, and Times Tables the Fun Way (TTFW), a mnemonic strategic intervention using stories and pictures to help students remember math facts, on retention of multiplication math facts and achievement on a generalization (word problem) assessment. Kanive et al. (2013), also compared the effects of MFF to another intervention, a group of conceptual interventions developed by Van de Walle and Lovin (2006), Fill the Chutes, Build It In Parts, and Broken Calculator (described above), on multiplication fact fluency and achievement on a generalization (word problem) assessment. The participants in the two treatment groups were compared to a control group on a multiplication fact fluency and word-problem generalization assessments. Researchers found that MFF and Conceptual treatment groups outperformed the control group on generalization and retention fluency measures. The retention fluency mean score for the MFF group was found to be statistically significant when compared to the control group, but not when compared to the Conceptual group. Additionally, when comparing the Conceptual to the Control group, no statistical significance was found when comparing mean scores for retention fluency. Lastly, no statistical significance was found when comparing word-problem generalization mean scores between all groups.

In each study, the mean score for the MFF treatment group was significantly larger than control group. Kanive et al. (2013) reports the MFF mean score at 7.04 and standard deviation at 12.94 compared to the control with a mean score of 1.36 and a standard deviation of 5.27. Whereas Nelson et al. (2013) reports the MFF mean score at 7.10 with a standard deviation of 4.21 compared to the control's mean score of 5.13 with a standard deviation of 2.51. Additionally, both studies found no significant difference in word problem (application) mean scores between groups.

2.3 Discussion

The first objective of the literature review was to identify the types of students participating in the multiplication fact fluency intervention studies. The second was to identify the types of interventional studies being used to impact multiplication fact computation (accuracy and/ or fluency). The third objective was to identify what dependent variables were measured in the published intervention studies. Lastly, the fourth objective was to identify the outcomes of the selected intervention studies by design type.

2.3.1 Participants

The first research question addressed participant characteristics. Grade level and disability data was notable in the literature.

2.3.1.1 Grade Level

The majority of students included in treatment groups fell in grades 3 – 5 and were included in 15 of the 17 studies (88%), which corresponds to the assumed grade range associated with learning and mastering basic multiplication facts. The oldest participants were enrolled in the sixth grade. This is interesting to note because all school-aged students (6 – 18 years old) were a part of inclusion criteria. This points to an additional gap in the research for improving multiplication fact fluency for students in secondary education. The research indicates that multiplication fact fluency interventions can improve fact fluency, therefore, it could be argued that grade should not matter and success should still be achieved. However, this should be examined further to show this is the case.

2.3.1.2 Disability Type

Moreover, based on the research reviewed, the majority of studies (*n*=9, 53%) included participants identified with LD or ADHD, whereas the remaining studies (*n*=8, 47%) included students identified as being at-risk for math achievement difficulties. However, it is notable that students diagnosed with a mathematics disability (MD) were not specifically identified. It is possible that some of the students identified as having learning disabilities could have in fact had a learning disability in mathematics, but because it was not specifically stated it cannot be assumed. This points to a gap in the literature. The studies reviewed, generally show success with improving math fact responding (accuracy and/ or fluency), but would the same successes have occurred if the students included had a MD, arguably the most challenging type of student to reach?

2.3.2 Intervention Types

The second research question addressed intervention type. All 17 studies utilized a procedural multiplication fact intervention. Of the seventeen studies, ten (59%) used acquisition-based procedural interventions and seven (41%) used fluency-based procedural interventions. Therefore, the majority of intervention studies which investigated how to address computational multiplication deficits used acquisition-based interventions, which does not follow what is known about behavioral fluency and is contradictory to what is known about the hierarchy of achieving mastery. Therefore, this indicates that more research is needed to assess the success of multiplication fact fluency interventions on multiplication fact fluency growth. Additionally, three of the studies (18%) also compared a procedural intervention to a conceptual intervention. Researchers found that procedural interventions produced higher fluency growth rates when compared to conceptual interventions.

2.3.3 Dependent Variables and Measurement

The third research question addressed dependent variables. As stated above three dependent variable categories arose in the literature, single digit multiplication accuracy/ fluency, generalization, and maintenance.

2.3.3.1 Single Digit Multiplication Accuracy/ Fluency

Thirteen of the seventeen studies reviewed (*n*=13, 76%) defined fluency as a combination of accuracy and rate, measuring success by correct response in a certain amount of time. All of these studies looked to use interventions to improve dpm correct or correct responses in a specific amount of time. Four intervention studies (*n*=4, 24%) used interventions to improve acquisition (accurate responses) of facts and not fluency. Fifteen (*n*=15, 88%) intervention studies demonstrated success in positively impacting their dependent variables, accurate or fluent responses. It is important to note that the acquisition-based procedural interventions, which measured fact fluency growth, reported positive results in improving fluency. This is important because even though speed was not addressed in the acquisition-based interventions, rate improved. It could be argued that these acquisition-based interventions helped improve unknown fact acquisition/ accuracy, and therefore a participant's ability to answer more multiplication fact problems in the allotted time improved making it appear as rate improvement. The acquisition-based interventions did not address increasing speed and therefore increased speed of known facts cannot be assumed. An intervention that addresses both acquisition/ accuracy and speed, fluency, for unknown and known facts should be explored to see if improved fact fluency can be achieved.

Based on the research, computational fact fluency is an essential fundamental mathematical skill that needs to be taught (Gersten et al., 2005; Koponen, Aro, Räsänen, & Ahonen, 2007; Poncy et al., 2010). Only focusing on accuracy may only encourage strategy dependence (Poncy & Skinner, 2011). Therefore, only measuring math fact acquisition is not sufficient and will not ensure future math achievement. Both accuracy and speed are needed for future mathematical success.

2.3.3.2 Generalization

The research was inconsistent whether procedural or conceptual multiplication intervention positively impacted generalization scores, as well as the difference in improvement growth when comparing the two types of interventions. Therefore, it is not possible to state whether procedural or conceptual multiplication interventions will improve generalization scores or whether one is better than the other. This is interesting and further exploration is needed on how to actually improve generalization or word problems probe scores; possibly an intervention that focuses on multiplication as well as on reading skills.

2.3.3.3 Maintenance

Four studies (24%) measured maintenance of skills as a secondary dependent variable to single digit multiplication fact fluency. The four studies all reported that that multiplication fact fluency improved during intervention phases and was maintained during maintenance phases. It is important to point out that of the studies that measure maintenance of skills, the interventions used were only multiplication fact fluency interventions. Therefore, an argument can be made that an intervention that focuses on improving fluency can also positively impact retention/ maintenance of fluency for multiplication facts.

2.3.4 Study Design Types Outcomes

The fourth research questioned addressed study design type and outcomes. Based on the results, two study design categories were identified, single case and group design studies. The reviewed single case and group design studies found that multiplication computational interventions can positively affect accuracy or fluency of multiplication facts for students diagnosed with a LD or students at-risk for math failure or math achievement difficulties. Additionally, the studies that measured maintenance of skills (n=4) demonstrated that fluency growth can be maintained after fluency interventions are administered. Therefore, this indicates that math fact fluency interventions are useful tools to increase computational fluency scores, as well as help students maintain fluency growth after interventions.

However, the research reports the success of multiplication computational interventions on generalization assessments varies. Generalization was mostly assessed using word problem probes, and the results imply that there may be other causes for the unreliable success of the computational interventions, such as reading challenges (Fuchs et al., 2008; Gürbüz and Erdem, 2016). Additionally, accommodations such as a reader were not indicated in the literature and therefore cannot be assumed. In order to better assess generalization or mathematical reasoning, different types of assessments should be considered, or appropriate accommodations for a participant's learning profile should be applied.

2.3.5 Future Research

Based on the literature of multiplication fact interventions for students with disabilities and students at-risk for mathematics difficulties or math failure, future research directions are indicated. Although the majority of studies implied success based on their intervention outcomes, intervention types varied (accuracy and fluency) when assessing multiplication fact fluency. Therefore, future research should focus on the possible differences in improved fluency based on accuracy interventions and fluency interventions. Due to the variability of outcomes when assessing generalization, future research should focus on how best to improve generalization of skills; whether focusing on computational fluency versus mathematical concepts/ understanding better improves generalization, if a combination of both types of interventions is best, or if the outcome depends on a specific participant's learning profile.

Moreover, based on the literature it is recommended that future research on single-digit multiplication fact fluency for middle and high school students should be conducted. Only one study reviewed (Flores, Houchins, & Shippen, 2006) had middle school-aged participants (sixth grade). Most likely this was because basic multiplication fact acquisition and fluency is a skill taught and assessed at the elementary level as indicated by all of the reviewed studies. Additionally, it is recommended to use a procedural fact fluency intervention to improve and maintain fact fluency growth. The majority of studies were single case multiple baseline (*n*-=5) or multiple probe (*n*=4) designs, as well as the most common dependent variable measured was dpm correct.

2.3.6 Limitations of Literature Review

Several limitations exist within the literature review. Since the electronic search, ancestral search, and hand search are limiting in and of themselves because they were found using the specific chosen key terms, not all relevant articles measuring multiplication fact responses (accuracy and/ or fluency) could have been found. If specific intervention names were searched for it is possible that the search could have yielded more results, which could have provided further insight in addressing research questions. Moreover, additional articles could have been published since this literature review was conducted and could further the research for multiplication fact fluency. Additionally, an interobserver agreement (IOA) on inclusion or coding of articles was not conducted, and therefore articles could have been excluded based on researcher error.

2.3.7 Rationale for Study

Based on what is known about improving behavioral fluency, math fact interventions should use the behavioral fluency paradigm to increase both speed and accuracy, especially for students with disabilities or at-risk for math failure. Fluency will help bridge the gap between acquisition and generalization. As indicated by Gürbüz and Erdem (2016), mathematical computation/ mental computing significantly influences mathematical reasoning skills. In addition, Gersten, Jordan, and Flojo (2005) found that most students identified with MD have a mathematical calculation deficit, specifically with accuracy and retrieval speed. Therefore, based on the articles that met criterion within this review, there are indications that mathematical fluency interventions could help to mend the national achievement issue for students with disabilities, focusing on the majority of the equation, improving mathematical computation/ calculation skills.

In order to improve and maintain fact fluency, research points to using a fact fluency intervention which addresses both speed and accuracy. Additionally, based on the proposed setting, an independent 1 – 12 school serving only students identified with learning differences in eastern Pennsylvania, an intervention that can be individualized per student, plus be completed independently by students would be ideal. If students are able to complete the intervention independently, the intervention could be implemented class-wide as well as address individual student needs. Therefore, based on the research and the needs of the proposed setting, a study utilizing the fact fluency intervention, DPR, on single-digit multiplication fact fluency growth for middle school students diagnosed with LD could be valuable for future fact fluency instruction. Additionally, a secondary goal will be to track maintenance of fluency for mastered math facts to evaluate the lasting effects of the intervention. DPR was used in two of the studies in the reviewed literature (Hulac, Dejong, & Benson, 2012; Poncy, Skinner, & Axtell, 2010). Both studies reported that DPR positively impacted participants (14 of 18 participants) fact fluency growth, as well as their ability to maintain fluency for mastered multiplication facts.

3.0 Methods

3.1 Participants and Setting

The participants included within the study are six sixth grade middle school students identified with a learning disability (LD). The setting for the case study is a coeducational independent 1 – 12 school located in eastern Pennsylvania, who serves only students identified with language-based learning disabilities. The school identifies itself as a research-to-practice college preparatory school, using multisensory experiential teaching methodology, and takes a constructivist approach to teaching and learning, particularly in the lower elementary grades. Mathematics instruction combines guided hands-on experiential learning techniques in order to build meaningful independent strategies to promote deeper levels of understanding (Poncy et al., 2010), as well as, direct instruction for teaching basic mathematical facts and problem solving skills. The school's educational philosophy centers around student-centered education, which continually influences curriculum and teaching practices. Units and lessons usually are tailored toward current student needs and teachers use individualized teaching practices in an attempt to meet all students where they are.

The school has approximately 336 students from grades 1 - 12, with 94 students in their lower school (grades 1 – 5), 114 students in their middle school (grades 6 – 8), and 129 students in their upper school (grades 9 – 12). Additionally, 23% of the student population are students of color. Moreover, the school holds a low student to teacher ratio, 6:1 in the lower school and 8:1 in the middle and high school, which does not include teaching interns, who hold a teaching certificate and are working towards a Master's degree in education. Among the faculty, the average number of teaching years is eleven, with 78% of the faculty members holding advanced degrees, Master's and/ or Doctoral, 86% if faculty members currently seeking an advanced degree are included.

All of the participants that will be included in the study must be identified by their mathematics teacher as struggling with basic single-digit multiplication fact fluency. Additionally, all participants included must be diagnosed with a learning disability, which qualifies them for an Individualized Educational Plan (IEP)at the school. IEP's outline specific learning, academic, and behavioral goals, classroom accommodations, and/ or curriculum modifications for the identified student, usually diagnosed with a learning disorder.

The study will be conducted in the students' middle school math classroom during their scheduled math period. The number of weekly sessions will be dependent on participant availability based on the school's calendar, with the aim of four or five sessions per week. The study will use a sample of convenience. The students recruited to participate in the study attend the school where the researcher currently works. The researcher is one of the school's upper school mathematics teachers, teaching in grades 9 - 12. Due to the researcher's role within the school, he is aware of the students' learning profiles and their academic strengths and deficits. The students will qualify for the study if they currently are in six, seventh, or eight grade, are physically able to participate in the study (i.e. students do not have health, hearing, vision, or mobility issues hindering their performance in the study) and have a diagnosed learning disability. Additionally, a minimum of fifteen multiplication facts must be identified as unknown on a student's pre-assessment during the *Detect* phase of the intervention (*Detect* phase is described below). Lastly, only participants whose parents consent to their participation will be eligible to participate in the study.

3.2 Materials

Materials used to conduct the intervention will consist of students' assessment and intervention folders. Assessment folders will contain assessment worksheets with each student's identified thirty-six math facts, which is further explained below. Intervention folders will contain Cover, Copy, Compare (CCC) intervention packets and sprint worksheets, which are also further explained below. Additionally, on the outside of the intervention folder there will be a graph for students to mark the number of problems completed during Repair phases. Additional materials will include a countdown timer, pencils, and a computer. The computer will be used during the *Detect* phase (explained below). Students will complete a multiplication placement phase on the Xtra Math app, using the website xtramath.org (XtraMath, 2009).

The Xtra Math multiplication placement test will identify known non-fluent facts and unknown facts for each participant, and will be used in the intervention. All participants will be given a student login for the app and will complete the multiplication placement test. The placement test is made up of three placement quizzes, which vary in number of problems presented and is based on student performance. The Xtra Math app considers facts to be fluent if a student answers problems correctly within three seconds. Fluent facts are identified with a smiley face in the results section of the student report and will be green on the results multiplication chart. An example appears in Appendix B: Figure 12. Facts are considered non-fluent known facts if a student answers the problem correctly, but took between four and ten seconds to answer. Non-fluent known facts will be identified with a checkmark in the results section of the student report and are yellow on the results multiplication chart (Appendix B: Figure 13). Facts are considered unknown if a student answers the problem incorrectly or does not answer within the ten second window. Unknown facts will be identified with an X or an hourglass in the results section of the student report and are gray on the results multiplication chart (Appendix B: Figure 13). Additionally, Appendix B: Figure 11 Mastery Key, provides further explanation of how facts are identified as fluent, non-fluent known, and unknown within the Xtra Math app.

3.2.1 Assessment Packets

Baseline and maintenance data will be collected for each of the three problem sets using assessment probes, which are the same as the sprint probes administered during the *Repair* phase of DPR, each containing ten student specific multiplication facts (Appendix A: Figure 7). The baseline assessment probes will be administered prior to the start of the intervention until visual stability is achieved (at least three sessions). Additionally, assessment/ sprint probes will be administered regularly, during the intervention phase, for the current problem set being administered and randomly for the other two problem sets, in order to collect either baseline or maintenance data. For example, if a participant is currently working on problem set B and has already reached criterion for problem set A, but has yet to reach problem set C, sprint probes for set B will be collected daily during the *Repair* phase of the intervention, while maintenance data for set A and baseline data for set C will be collected randomly using assessment probes. The assessment/ sprint probe data will be collected and graphed to show treatment effects for each problem set.

3.2.2 Intervention Packets

The intervention packets will consist of CCC worksheets (Appendix A: Figure 8), sprint worksheets (Appendix A: Figure 9), and graphing worksheets (Appendix A: Figure 10). The CCC worksheets will have ten student-specific problems based on the results from the *Detect* phase, which will correlate to the sprint worksheets. Additionally, a self-graphing worksheet will be stapled to the front of the intervention folder for students to keep track of their progress.

3.3 Experimental Design and Dependent Variable

A multiple probe design (MP) (Horner & Baer, 1978), consisting of baseline, intervention, and maintenance phases with staggered onset will be used to examine the effects of Detect, Practice, Repair (DPR) on single-digit multiplication fact fluency for middle school students identified with a LD. There are numerous advantages of using a MP design to evaluate the effectiveness of an intervention. First, it will increase internal validity by demonstrating an intra-subject direct replication. Also, since returning to baseline is not necessary to examine control, an MP design lends itself perfectly to nonreversible interventions models, which is exactly the type of intervention DPR is. Lastly, the MP design creates a paradigm to progress monitor current goals as well as monitor maintenance of previously mastered skills (Ledford & Gast, 2018). Ideally, MP will show treatment effects once the intervention phase is implemented for each probe, which would establish the treatment effects are due to the intervention. The advantages of a MP design are comparable to a multiple baseline design, where treatment is staggered for each participant in order to demonstrate treatment effects due to the intervention being implemented (Ledford & Gast, 2018). However, an MP design was chosen because of time constraints to conduct the intervention, as well as the numerous examples in previous research utilizing a MP design to evaluate DPR (e.g., Axtell et al., 2009; Parkhurst et al., 2010; Poncy, Fontenelle, & Skinner, 2013; Poncy & Skinner, 2011; Poncy, Skinner, & Axtell, 2010).

The main dependent variable in this study will be multiplication fluency, measured as digit per minute (dpm) correct and errors per minute during sprints and assessment probes. There will be three tasks measured throughout the intervention, three different problem sets (Set A, Set B, Set C) each containing ten student specific single-digit multiplication facts. Dpm correct data will be collected for assessment probes (baseline and maintenance data) to monitor progress throughout the intervention for each problem set, as well as for sprint probes (intervention data) to monitor fluency growth until fluency criterion is reached. Deno and Mirkin (1977) identify performance levels for fact fluency as mastery, instructional, and frustration. Based on their work, ideally a middle school student would reach 40 dpm correct in order to be considered fluent; which corresponds to the previously identified fluency condition for the Xtra Math app. In order to be fluent a student must correctly answer a problem within three seconds. Therefore, if a participant reaches criterion of 50 dpm correct on a sprint probe during the *Repair* phase with 95% accuracy, then they will have reached criterion and will move onto the next problem set the following session. Students will continue to take assessment probes (baseline or maintenance) for all three problem sets during the entire study to track maintenance of skills. The assessment probes will be given randomly, and less frequently than sprint probes, in order to reduce participant frustration by being continually assessed on non-targeted facts, as well as to minimize testing effects. Additionally, baseline data for non-practiced problem sets will show if outside influences are affecting fluency growth. Once the study is complete, all data will be collected and analyzed in order to determine possible outcomes or conclusions.

This study will use the Deno and Mirkin's (1977) method to identify dpm correct. Dpm correct are defined by digits written in the correct place/ column. Additionally, digit reversals (i.e. the number three written backwards) will be counted as correct (Parkhurst et al., 2010; Poncy, Fontenelle, & Skinner, 2013; Poncy & Skinner, 2011; Poncy, Skinner, & Axtell, 2010). Digits that are incorrect or written in the wrong place/ column will be identified as errors.

3.4 Independent Variable

The independent variable used in the study will be Detect, Practice, Repair (DPR; Poncy & Skinner, 2006). DPR is an intervention approach that utilizes three phases in order to affect math fact fluency growth (Axtell et al., 2009; Parkhurst et al., 2010; Poncy, Fontenelle, & Skinner, 2013; Poncy & Skinner, 2011; Poncy, Skinner, & Axtell, 2010). The *Detect* phase identifies non-fluent math facts for participants. The *Practice* phase utilizes the intervention Cover, Copy, Compare (CCC) to repeatedly practice non-fluent facts in order for participants to acquire the facts. Poncy and Skinner (2006), the designers of DPR, decided CCC would be used for the *Practice* phase of the intervention. The *Repair* phase consists of a one-minute timed probe, presenting the facts from the *Practice* phase, for participants to work on increasing their responding speed and ultimately their fact fluency.

3.4.1 Detect Phase

The *Detect* phase identifies math facts that are considered fluent, non-fluent known facts, and unknown facts for each individual participant. In order to find this data, a math app, Xtra Math, will be used.

The Xtra Math app was chosen based on the work of Parkhurst et al. (2010). Parkhurst et al. (2010) furthered the research on DPR, specifically for the *Detect* phase of the intervention. Parkhurst et al. (2010) used researcher-made PowerPoint slides to identify the target multiplication facts for each participant. The slides presented individual multiplication fact problems for three seconds each. Participants were expected to write the answers for the multiplication fact problems on an answer sheet. Based on this research, XtraMath was chosen for the *Detect* phase for the study because similar to Parkhurst et al. (2010), Xtra Math also expects students to answer math facts within three seconds to demonstrate fluency. However, unlike Parkhurst et al. (2010), XtraMath allows students a total of ten seconds to answer a problem. Therefore, if a student knows a math fact but is not fluent, they are still able to demonstrate their knowledge, and Xtra Math makes special note that the fact is known but not fluent. Additionally, XtraMath is a free math fact fluency application program that is readily available for use by teachers. It is also a recommended supplemental application by the math curriculum used at the school, and therefore fits into their prescribed curriculum programming.

The researcher will use the data collected during the *Detect* phase to create three individualized problem sets for each participant. For each of the problem sets, assessment and intervention probes will be created by the researcher. On a single probe, each problem will be repeated four times, equaling 40 total problems. Multiplication problems will be randomly placed throughout the probes in eight rows of five. The same problem will not appear directly adjacent to itself or come sequentially after itself from one end of a column to the beginning of the next (Appendix A: Figure 9).

3.4.2 Practice Phase

The *Practice* phase will use the Cover, Copy, Compare (CCC) intervention to improve student accuracy with the identified facts (Axtell et al., 2009; Parkhurst et al., 2010; Poncy, Fontenelle, & Skinner, 2013; Poncy & Skinner, 2011; Poncy, Skinner, & Axtell, 2010). The CCC intervention has shown to improve accurate and fluent responding for targeted math facts (e.g. Codding et al., 2006; Grafman & Cates, 2010; Lee & Tingstrom, 1994; Poncy, McCallum, & Schmitt, 2010; Poncy & Skinner, 2011; Poncy, Skinner, & Jaspers, 2006; Skinner et al., 1993). During CCC, there are generally five procedural steps students follow:

1. Look at a basic math fact problem with the answer (i.e., 3 x 5 = 15)
2. The student covers the problem by folding the paper over so that the problem is no longer visible.
3. The student writes the problem with the answer in the provided space next to the original problem.
4. The student unfolds the paper to uncover the original problem.
5. The student compares their written problem to the original problem given. If what they have written is correct, they move on to the next problem. If what they have written is incorrect, they cross out what they have written and repeat the steps for the problem again.

During this phase, ten facts will be identified for each student, five non-fluent known facts and five unknown facts. The CCC worksheets will consist of a 50:50 ratio of these non-fluent known to unknown facts (Shapiro, 1996). This is further explained below under the Procedures section. If all of a student's non-fluent-known facts are used in the CCC worksheets prior to the end of the intervention, fluent-known facts will be used in their place. Similarly, if a student's unknown facts are all used prior to the conclusion of the study, non-fluent-known facts will be used in their place. Individualized CCC worksheets will be created by the researcher (Set A, Set

B, Set C). Once the participants complete a CCC worksheet they will move onto the final phase, the *Repair* phase.

3.4.3 Repair Phase

During the *Repair* phase, participants will complete a one-minute sprint worksheet (Appendix A: Figure 9) of the problems used in the *Practice* phase. Sprint worksheets will correspond to the ten student specific problems on the CCC worksheets (Set A, Set B, Set C). Sprint worksheets will be similar to assessment probes (Appendix A: Figure 7), and will contain eight rows of five multiplication facts, totaling forty problems. Therefore, each of the ten facts, five non-fluent-known facts and five unknown facts, will be repeated four times each throughout the sprint worksheet. The multiplication fact problems will be randomly placed, using the same guidelines as in the Assessment Probes (i.e., the same problem will never follow itself nor appear above or below itself). Students will be given one minute to complete as many fact problems as possible. Dpm correct will be calculated by the researcher for every sprint worksheet completed by participants in order to track progress.

Lastly, graphing worksheets (Appendix A: Figure 10) will be included on the front of the intervention folder. Students will be expected to mark the number of problems completed during the one-minute sprint phase on the graph. The graph will visually represent students' progress during each task, and is meant to encourage fluency growth.

3.5 Procedures

3.5.1 IRB and Student Recruitment

The researcher will seek IRB approval to conduct the study, The Effects of Detect, Practice, Repair (DPR) Mathematics Fluency Intervention on Multiplication Fact Fluency for Middle School Students Diagnosed with Learning Disabilities. The researcher will follow all IRB procedures and comply with all practices and laws relating to the IRB. Additionally, a participant recruitment/ consent form will be sent to identified students' parents who qualify for the study (Appendix C). After all IRB requirements and all participant consent forms are filled out and signed by all parties, the study will commence.

3.5.2 Pre-Baseline Phase

During the pre-baseline phase of the intervention, the researcher will conduct the *Detect* phase using the Xtra Math app, with each participant. Once a participant completes the placement test, only facts identified as non-fluent known and unknown facts, yellow and gray fact problems, will be used in the intervention. Possible facts used in the intervention will be between two and nine. Facts containing a zero or a one will be excluded. The researcher will use the data to create individualized assessment and intervention packets for each participant to use in the *Practice* and *Repair* phases.

3.5.3 Baseline Phase

After the *Detect* phase, explained above, is complete and all multiplication facts are identified for each student, the baseline data will be collected. Students will complete assessment probes for all three problem sets (Set A, Set B, Set C) on a daily basis. Students will be given one minute to complete an assessment probe before moving onto the next. Probes will be completed in a random order for each student on all days given. Students will complete the assessment probes until visual stability of baseline data is recognized by the researcher for all problem sets. Once the visual stability is achieved the intervention phase will begin the following session.

3.5.4 Intervention Phase

3.5.4.1 Practice Phase

During the *Practice* phase, CCC will be implemented with the group of participants, by the researcher (Axtell et al., 2009; Parkhurst et al., 2010; Poncy, Fontenelle, & Skinner, 2013; Poncy & Skinner, 2011; Poncy, Skinner, & Axtell, 2010). Individualized CCC packets of worksheets will be provided to each participant, which will be in their personal intervention folders. On the CCC worksheets will be ten single digit multiplication problems (Appendix A: Figure 8). When the *Practice* phase (CCC) begins, the researcher will read a prepared script (Appendix D). When told to begin, participants will quietly read a problem out loud to themselves, then cover the problem by folding their paper over, writing the problem and answer in the provided space. After writing their response they will unfold their paper and compare their written response to the given problem. If a participant copies a problem correctly, they will move onto the next problem in the worksheet and repeat the procedure. If a participant copies a problem incorrectly, then they will cross out their written problem and repeat the CCC procedures. Participants will be given five minutes to complete as many of the problems as possible, which will be monitored by the researcher using a countdown timer (Poncy, Fontenelle, & Skinner, 2013; Poncy & Skinner, 2011). If a participant finishes a CCC worksheet containing all ten identified problems, they will work through an additional identical worksheet until the five minutes is up. Once the five-minute time concludes, students will be asked by the researcher to put down their pencils and put their CCC worksheets back in their intervention folders.

3.5.4.2 Repair Phase

Finally, students will be given their assessment folders by the researcher immediately following the *Practice* phase. The folders will contain the students individualized sprint worksheets. The researcher will pass out the assessment folders while collecting the intervention folders. Students will then be asked to pull out a sprint worksheet (Appendix A: Figure 9) from their intervention packet. Students will be given one minute to complete as many problems as possible. As described above, sprint worksheets will comprise of forty total problems, repeating each of the ten identified problems four times. Before the sprint begins, the researcher will read aloud a prepared script (Appendix D). The researcher will use a countdown timer to keep track of when the sprint is over. Once the sprint is finished, the researcher will read aloud the prepared script to the participants (Axtell et al., 2009; Poncy, Fontenelle, & Skinner, 2013; Poncy, Skinner, & Axtell, 2010). After participants finish graphing the number of completed problems and have placed their sprint worksheet back in their assessment folders, the researcher will come and collect the folders.

3.5.5 Maintenance Phase

As per the multiple probe design, maintenance data will be collected randomly throughout the intervention for problem sets that have been mastered by participants. Maintenance data will be collected from one-minute assessment probes (described in Materials section) to track maintained fluency. The purpose of collecting maintenance data is to better guarantee lasting effects of the intervention on math fact fluency.

3.5.6 Treatment/ Procedural Fidelity

In order to evaluate treatment fidelity, a script for the researcher to read aloud during intervention sessions was prepared (Appendix D). Additionally, the researcher plans to participate in all intervention sessions in order to assure the implementation integrity of the intervention, as well as follow exactly the prescribed procedures and script to ensure procedural fidelity during every intervention session.

3.5.7 Inter-Observer/ Inter-Scorer Agreement

To calculate inter-observer/ inter-scorer agreement, at least 25% of all assessment and sprint probes will be independently scored by a trained second party (middle school special education math teacher). Inter-observer/ inter-scorer agreement will be calculated by dividing the number of agreements on dpm correct for each probe scored by both researcher and teacher, by the total number of probes scored by both parties (agreements and disagreements), then multiplying the result by one hundred (Poncy, Fontenelle, & Skinner, 2013).

4.0 Findings

4.1 Introduction

The purpose of the study was to extend interventions primarily used in grades K-5 to improve math fluency to Middle School Age Students identified with Learning Disabled (LD) who have a current individualized educational program (IEP). The design interventions were extended to 6th grade middle school math students to better understand the impact of multiplication fact fluency interventions on multiplication fact fluency growth through procedural interventions. Specifically examining whether the intervention that addresses acquisition/accuracy and speed for both known and unknown facts improved multiplication fact fluency.

The main dependent variable in this study was multiplication fluency, measured as digit per minute (dpm) correct and errors per minute during sprints and assessment probes. The independent variable used in the study will be Detect, Practice, Repair (DPR). The Detect phase utilized Xtra Math app to identify level of multiplication fact fluency, the Practice phase utilized Copy, Cover, Compare (CCC) method to practice non-fluent single-digit multiplication facts, and the Repair phase was a one-minute timed probe, presenting the facts from the Practice phase which participants worked on to increase their multiplication facts response speed leading to their improvement in multiplication fact fluency.

4.2 Qualification of the Study Participants

Students were identified by their sixth-grade teacher to participate in the study. Participation was voluntary and participants could withdraw from the study at any time. Parents were sent information regarding the research study and participants whose parents signed the consent to participate were included in the study.

All students who participated in the study had a diagnosis of LD and had a current IEP. The first phase of the study, the Detect phase, used the Xtra Math App to determine single-digit multiplication fluency. Any student who demonstrated single-digit multiplication fluency is identified in the study as fluent in single-digit multiplication and is not included in the Practice and Repair phase of this study.

4.3 Quantitative Study

4.3.1 Detect, Practice, Repair

The study was conducted during a normal math classroom period and took approximately 7 minutes per class period. The researcher was in the classroom to distribute and collect the folders. The students worked independently and each phase, Detect, Practice, Repair, were timed by the researcher.

Each study participant who demonstrated single-digit multiplication deficiency during the Detect phase then participated in the Practice and Repair phases of the intervention. The Practice phase included three problem sets which students could move through at their own pace utilizing the Cover, Copy, Compare (CCC) intervention to repeatedly practice non-fluent facts to improve accuracy. During each meeting after the five-minute CCC, students participated in a one-minute timed probe, presenting the facts from the Practice phase. This allowed participants to work on increasing their responding speed and ultimately their fact fluency. Once a participant correctly answered 50 single digit multiplication problems within one minute at a 95% accuracy rate, they moved onto the next problem set at the next class meeting. Students also continued to take assessment probes (baseline or maintenance) for all three problem sets during the entire study to track maintenance of skills. The assessment probes were given less frequently than sprint probes.

4.3.2 Study Participant Results

During the Detect phase of the study two out of the eight participants demonstrated fluency for single digit multiplication facts and were deemed fluent. The remaining six participants participated in the remainder of the study. Below are the results of each study participant who participated in the Detect, Practice, Repair intervention of the research study. The three graphs per page for each student represent Set A, Set B, and Set C. Each graph's x-axis represents the day and the y-axis represents digits per minute (dpm). Each student participated in the study at the same time with the first four days of the study identifying the baseline resulting in visible stability. The dotted line on each student's graph separates baseline from intervention phase of the project. The point on the graph with no connecting lines represents the assessment probes at day 10, 19, and 31, representing maintenance before and after intervention. The study was conducted over 31 sessions. The sessions students were absent was noted.

4.3.2.1 Student 1

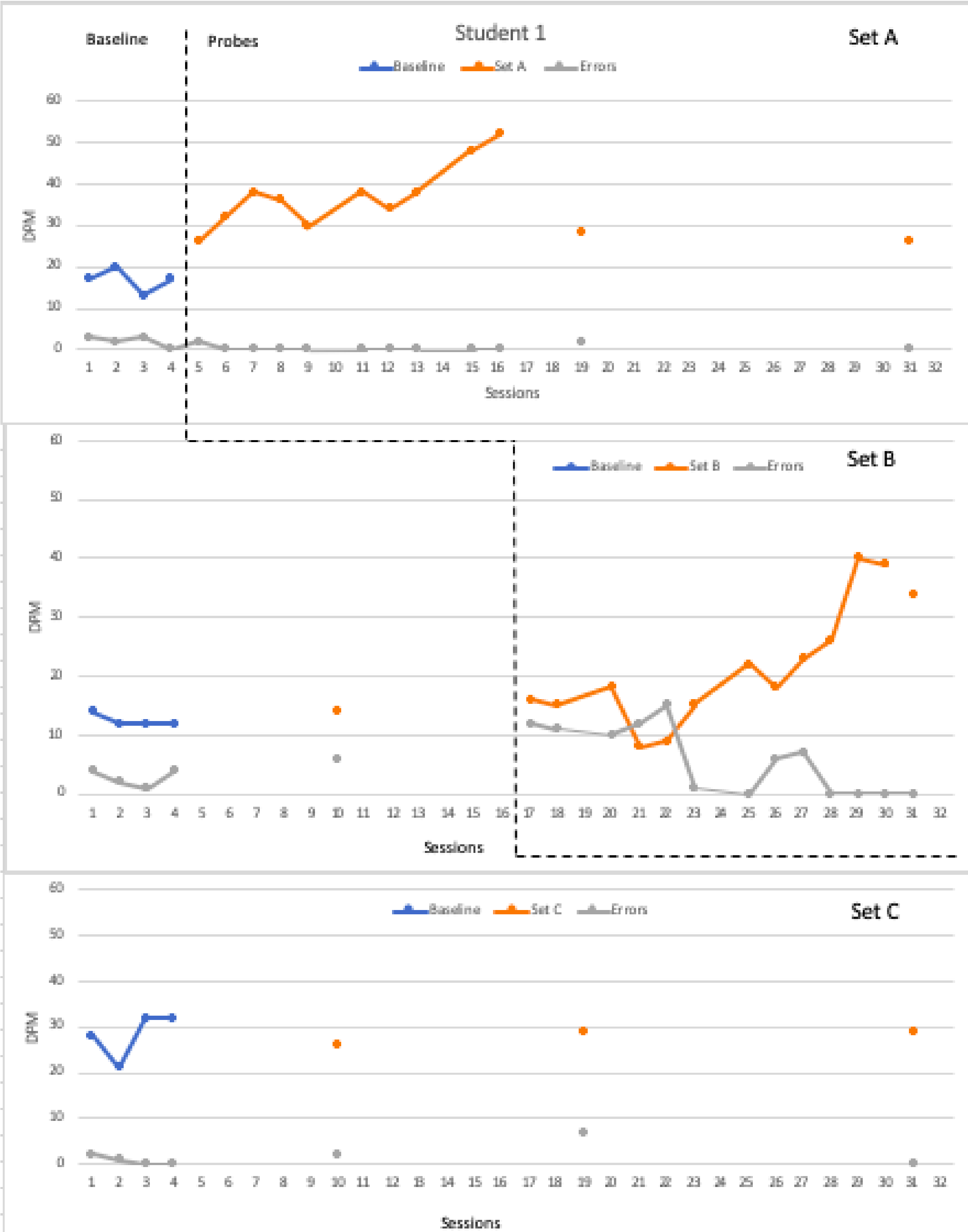


Figure 1: Student 1

Baseline for Problem Set A the student achieved dpm correct of 17, 20, 13, 17 and errors of 3, 2, 3, 0 with an average dpm correct of 16.75, with visible stability achieved. The intervention phase, days 5 through 15, the student participated 11 of the 12 days and achieved criterion of 52 dpm correct with 0 errors, however, one of the session days was used to collect baseline maintenance for Set B and Set C. The maintenance assessment during day 19 showed dpm correct of 29 with 2 errors and day 31 showed dpm correct of 26 with 0 errors.

Baseline for Problem Set B the student achieved dpm correct of 14, 12, 12, 12 and errors of 4, 2, 1, 4, with an average dpm correct of 12.50 with visible stability achieved. The maintenance assessment of the baseline during day 10 showed a dpm correct of 14 with 6 errors. The intervention phase, days 17 through 30, the student participated 13 of the 14 days and did not achieve criterion, however, one of the session days was used to collect maintenance for Set A and baseline maintenance data for Set C. During the first day of the intervention phase the student dpm correct was 16 with 12 errors and on day 30 the student's dpm correct was 39 with 0 errors.

Baseline for Problem Set C the student achieved dpm correct of 28, 21, 32, 32 and errors of 2, 1, 0, 0 with an average dpm correct of 28.25. The student did not reach the Practice and Repair phases for Problem Set C. The maintenance assessment of the baseline during day 10 showed dpm correct of 26 with 2 errors, day 19 showed dpm correct of 29 with 7 errors, and day 31 showed dpm correct of 29 with 0 errors.

4.3.2.2 Student 2

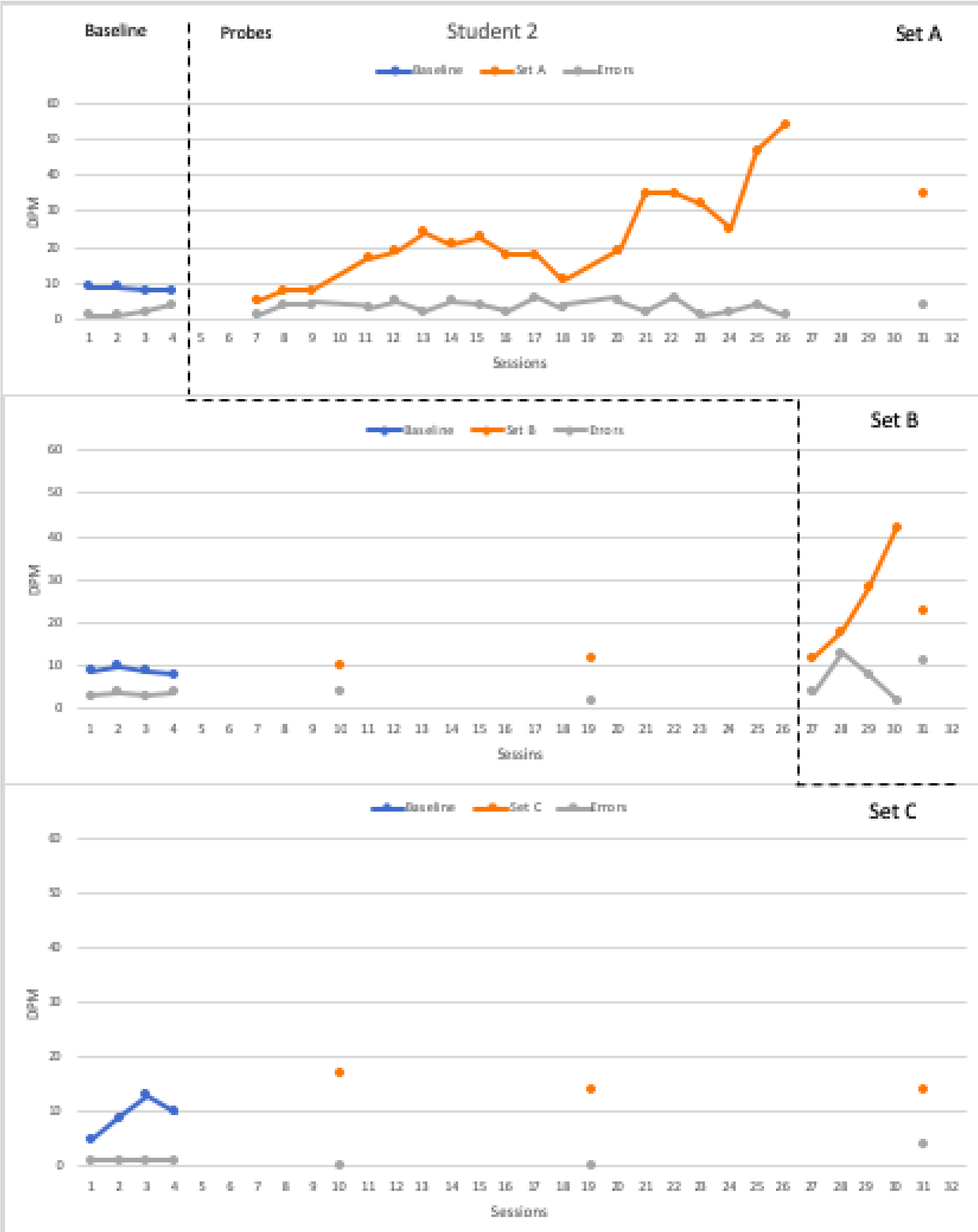


Figure 2: Student 2

Baseline for Problem Set A the student achieved dpm correct of 9, 9, 8, 8 and errors of 1, 1, 2, 4 with an average dpm correct of 8.50, with visible stability achieved. The intervention phase, days 5 through 26, the student participated 20 of the 22 days and achieved criterion of 54 dpm correct with 1 error, however, two of the session days were used to collect baseline maintenance for Set B and Set C. The maintenance assessment during day 31 showed dpm correct of 35 with 4 errors.

Baseline for Problem Set B the student achieved dpm correct of 9, 10, 9, 8 and errors of 3, 4, 3, 4, with an average dpm correct of 9 with visible stability achieved. The intervention phase, days 27 through 30, the student participated the 4 days and did not achieve criterion. During the first day of the intervention phase the student's dpm correct was 12 with 4 errors and on day 30 the student's dpm correct was 42 with 2 errors. The maintenance assessment of the baseline during day 10 showed dpm correct of 10 with 4 errors and day 19 showed dpm correct of 12 with 2 errors. The maintenance assessment during day 31 showed dpm correct of 23 with 11 errors.

Baseline for Problem Set C the student achieved dpm correct of 5, 9, 13, 10 and errors of 1, 1, 1, 1 with an average dpm correct of 9.25. This student did not reach the Practice and Repair phases for Problem Set C. The maintenance assessment of the baseline during day 10 showed dpm correct of 17 with 0 errors, day 19 showed dpm correct of 14 with 0 errors, and day 31 showed dpm correct of 14 with 4 errors.

4.3.2.3 Student 3

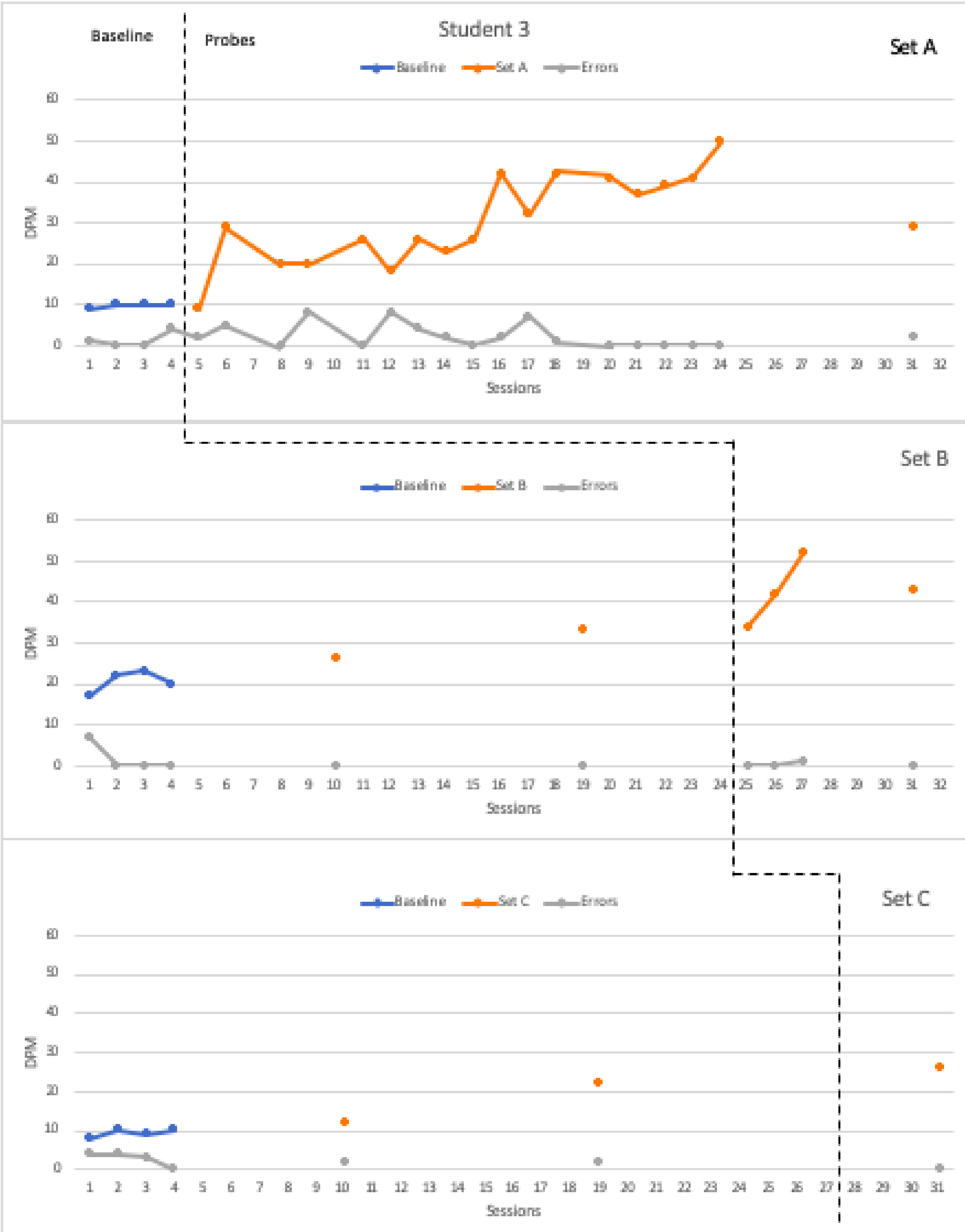


Figure 3: Student 3

Baseline for Problem Set A the student achieved dpm correct of 9, 10, 10, 10 and errors of 1, 0, 0, 4 with an average dpm correct of 9.75, with visible stability achieved. The intervention phase, days 5 through 24, the student participated 19 of the 20 days and achieved criterion dpm correct of 50 with 0 errors, however, two of the session days were used to collect baseline maintenance for Set B and Set C. The maintenance assessment during day 31 showed dpm correct of 29 with 2 errors.

Baseline for Problem Set B the student achieved dpm correct of 17, 22, 23, 20 and errors of 7, 0, 0, 0 with an average dpm of 20.5 with visible stability achieved. The intervention phase, days 25 through 27, the student participated the 3 days and achieved criterion dpm correct of 52 with 1 error. The baseline maintenance assessment during day 10 showed dpm correct of 26 with 0 errors and day 19 showed dpm correct of 33 with 0 errors. On day 31 Set B dpm correct was 43 with 0 errors.

Baseline for Problem Set C the student achieved dpm correct of 8, 10, 9, 8 and errors of 4, 4, 3, 0 with an average dpm correct of 8.75. This student did reach the Practice and Repair phases for Problem Set C, but was absent on day 28 through day 30 and was unable to begin the intervention phase for Problem Set C. The maintenance assessment of the baseline during day 10 showed dpm correct of 12 with 2 errors, day 19 showed dpm correct of 22 with 2 errors, and day 31 showed dpm correct of 26 with 0 errors.

4.3.2.4 Student 4

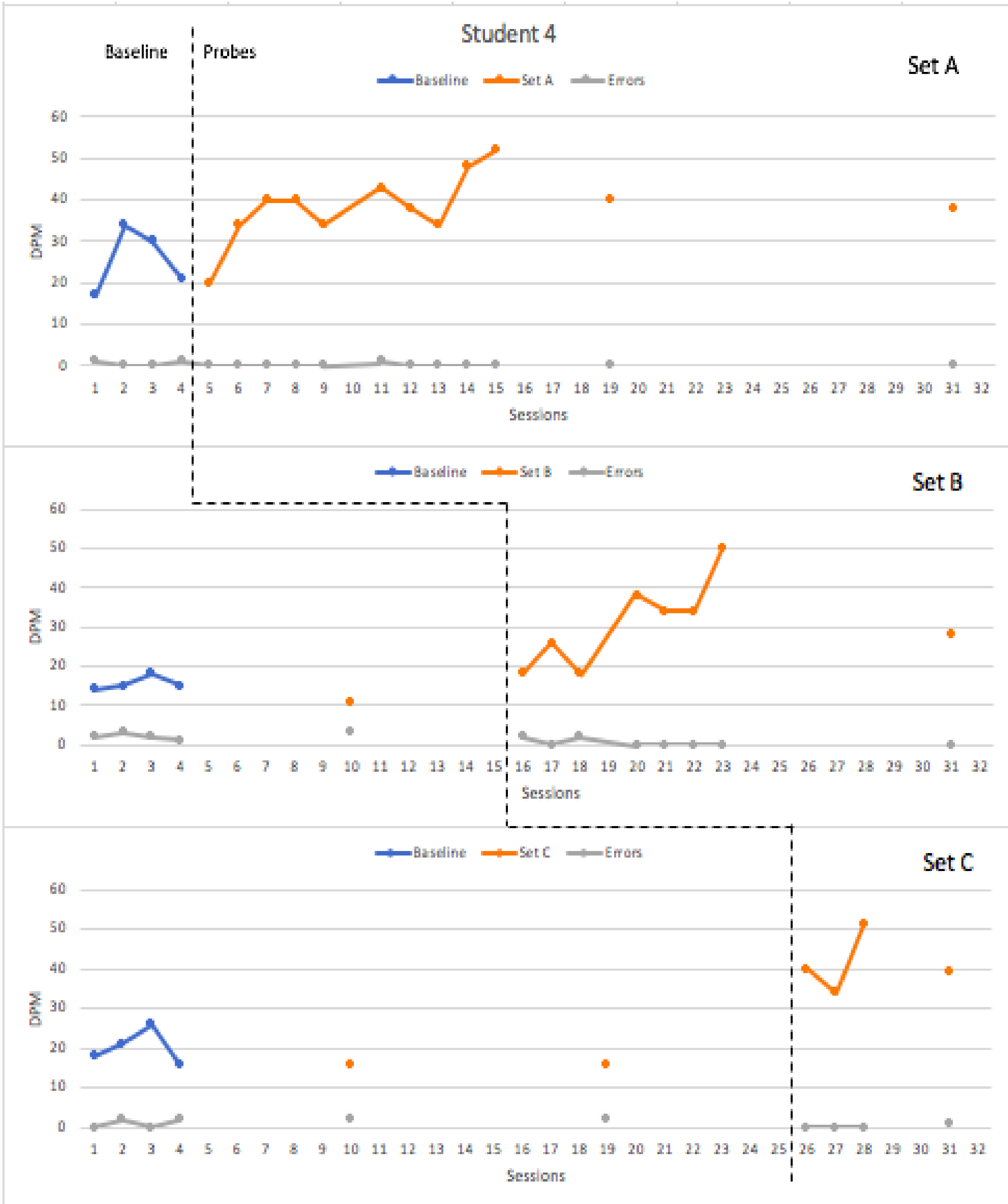


Figure 4: Student 4

Baseline for Problem Set A the student achieved dpm correct of 17, 34, 30, 21 and errors of 1, 0, 0, 1 with an average dpm correct of 25.5, with visible stability achieved. The intervention phase, days 5 through 15, the student participated 11 of the 11 days and achieved criterion of 52 with 0 errors, however, one of the session days was used to collect baseline maintenance for Set B and Set C. The maintenance assessment during day 19 showed dpm correct of 40 with 0 errors and week 31 showed dpm correct of 38 with 0 errors.

Baseline for Problem Set B the student achieved dpm correct of 14, 15, 18, 15 and errors of 2, 3, 2, 1 with an average dpm correct of 15.5 with visible stability achieved. The intervention phase, days 16 through 23, the student participated 8 of the 8 days and achieved criterion of 50 with 0 errors, however, one of the session days was used to collect maintenance for Set A and baseline maintenance data for Set C. The baseline maintenance assessment during day 10 showed a dpm correct of 11 with 3 errors. On day 31 Set B dpm correct was 28 with 0 errors.

Baseline for Problem Set C the student achieved dpm correct of 18, 21, 26, 16 and errors of 0, 2, 0, 2 with an average dpm correct of 20.25. The intervention phase, days 24 through 28, the student participated 3 of the 5 days and reached criterion of 51 dpm correct with 0 errors. The maintenance assessment of the baseline during day 10 showed dpm correct of 16 with 2 errors, and day 19 showed dpm correct of 16 with 2 errors. On day 31 Set C dpm correct was 39 with 1 error.

4.3.2.5 Student 5

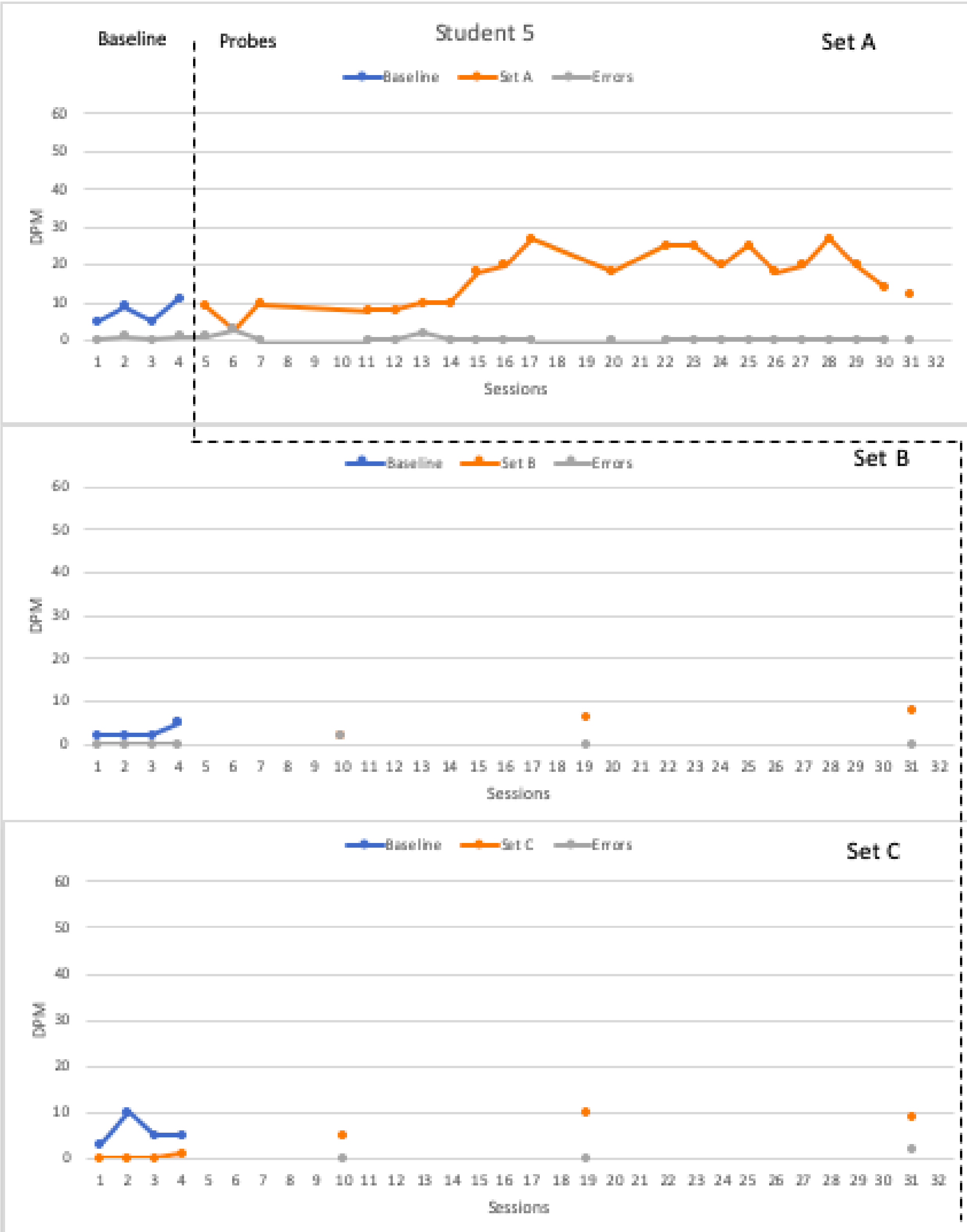


Figure 5: Student 5

Baseline for Problem Set A the student achieved dpm correct of 5, 9, 5, 11 and errors of 0, 1, 0, 1 with an average dpm correct of 7.5, with visible stability achieved. The intervention phase, days 5 through 30, the student participated 22 of the 26 days and did not achieved criterion, however, two of the session days were used to collect baseline maintenance for Set B and Set C. Day 1 the student achieved 9 dpm correct with 1 error and day 30 the student achieved 14 dpm correct with 0 errors. The student achieved a high of 27 dpm correct on days 17 and 28. On day 31 Set A dpm correct was 12 with 0 errors.

Baseline for Problem Set B the student achieved dpm correct of 2, 2, 2, 5 and errors of 0, 0, 0, 0 with an average dpm correct of 2.75 with visible stability achieved. This student did not reach the Practice and Repair phases for Problem Set B. The maintenance assessment of the baseline during day 10 showed dpm correct of 2 with 2 errors, day 19 showed dpm correct of 6 with 0 errors, and day 31 showed dpm correct of 8 with 0 errors.

Baseline for Problem Set C the student achieved dpm correct of 3, 10, 5, 5 and errors of 0, 0, 0, 1 with an average dpm correct of 5.75. The student did not reach the Practice and Repair phases for Problem Set C. The maintenance assessment of the baseline during day 10 showed dpm correct of 5 with 0 errors, day 19 showed dpm correct of 10 with 0 errors, and day 31 showed dpm correct of 9 with 2 errors.

4.3.2.6 Student 6

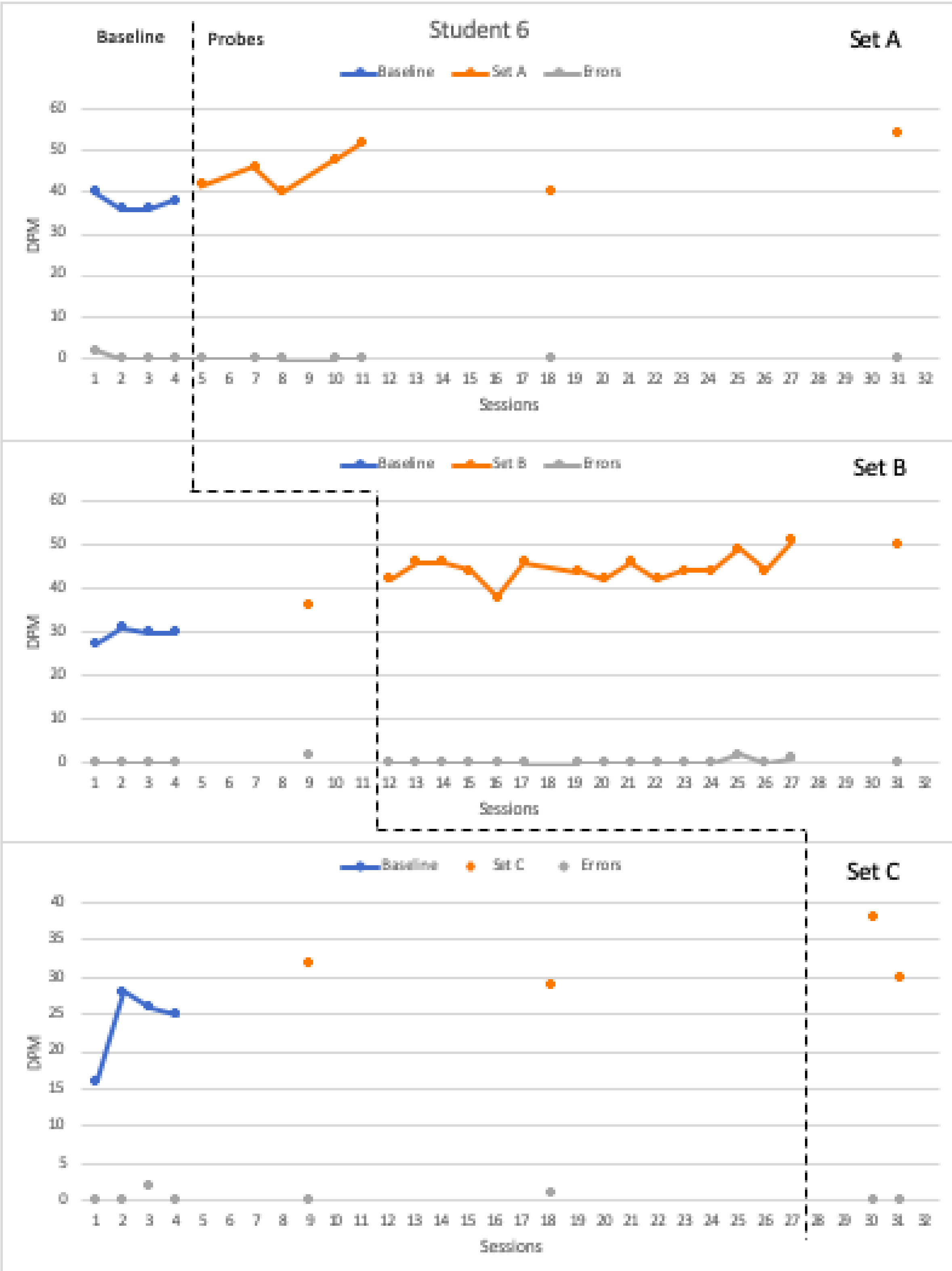


Figure 6: Student 6

Baseline for Problem Set A the student achieved dpm correct of 40, 36, 36, 38 and errors of 2, 0, 0, 0 with an average dpm correct of 37.5, with visible stability achieved. The intervention phase, days 5 through 11, the student participated 6 of the 7 days and achieved criterion of 52 dpm correct with 0 errors, however, one of the session days was used to collect baseline maintenance for Set B and Set C. The maintenance assessment during day 19 showed dpm correct of 40 with 0 errors and day 31 showed dpm correct of 54 with 0 errors.

Baseline for Problem Set B the student achieved dpm correct of 27, 31, 30, 30 and errors of 0, 0, 0, 0 with an average dpm correct of 29.5 with visible stability achieved. The intervention phase, days 12 through 27, the student participated 16 of the 16 days and achieved criterion of 51 dpm correct with 1 error, however, one of the session days was used to collect maintenance for Set B and baseline maintenance data for Set C. The baseline maintenance assessment during day 10 showed dpm correct of 36 with 2 errors. On day 31 Set B dpm correct was 50 with 0 errors.

Baseline for Problem Set C the student achieved dpm correct of 16, 28, 26, 25 and errors of 0, 0, 2, 0 with an average dpm correct of 23.75. The intervention phase, days 28 through 30, the student participated 1 of the 3 days and did not achieve criterion. The maintenance assessment of the baseline during day 10 showed dpm correct of 32 with 0 errors, and day 19 showed dpm correct of 29 with 1 error. On day 31 Set C dpm correct was 30 with 0 errors.

4.4 Study Findings

4.4.1 Summary

Below a summary table is presented of the overall study. The table depicts, by Problem Set and student, the baseline average, if criterion of 50 dpm correct was reached, day criterion was reached, the dpm correct and errors achieved when reaching criterion, and the maintenance data on the final day, Day 31, of the study (dpm correct and errors).

Table 1: Intervention Summary Table

Set A					
Student	Baseline Average	Criterion Reached	Day	Dpm Correct, Errors	Day 31: Dpm Correct, Errors
1	16.75	Yes	15	52, 0	26, 0
2	8.5	Yes	26	54, 1	35, 4
3	9.75	Yes	24	50, 0	29, 2
4	25.25	Yes	15	52, 0	38, 0
5	7.5	No			12, 0
6	37.5	Yes	11	52, 0	54, 0
Set B					
Student	Baseline Average	Criterion Reached	Day	Dpm Correct, Errors	Day 31: Dpm Correct, Errors
1	12.5	No			34, 0
2	9	No			12, 2
3	20.5	Yes	27	52, 1	43, 0
4	15.5	Yes	23	50, 0	28, 0
5	2.75	No			8, 0
6	29.5	Yes	27	51, 1	50, 0
Set C					
Student	Baseline Average	Criterion Reached	Day	Dpm Correct, Errors	Day 31: Dpm Correct, Errors
1	28.25	No			29, 0
2	9.25	No			14, 4
3	8.75	No			26, 0
4	20.25	Yes	28	51, 0	39, 1
5	5.75	No			9, 2
6	28.25	No			30, 0

4.4.1.1 Set A

Five out of six students reached criterion of 50 dpm correct. Student 6 achieved a baseline average above 37 dpm correct and reached criterion on day 11 of the intervention, which was the fastest out of all participants. Additionally, Student 6 maintenance data for Set A revealed that they maintained fact fluency, completing 54 dpm correct with zero errors on day 31. Student 1 and Student 4 both achieved a baseline average above 16 dpm correct and reached criterion on day 15 of the intervention. Both Student 1 and Student 4's dpm correct for Set A decreased under the desired fluency rate of 40 dpm correct by the final day of the study, day 31. Student 2 and Student 3 achieved a baseline average below 10 dpm correct and reached criterion on days 26 and 24 of the intervention. Based on their lower dpm correct baseline it took both students approximately 10 more days to reach criterion than Student 1 and Student 4. Additionally, Student 2 and Student 3 also did not maintain the desired fluency rate of 40 dpm correct. Student 5 achieved a baseline average below 8 dpm correct and ultimately did not reach criterion of 50 dpm correct for Set A. Additionally, Student 5's final dpm correct score was 12 with zero errors, only 4.5 dpm correct over their baseline average.

4.4.1.2 Set B

Three out of six students reached criterion of 50 dpm correct. Student 6 achieved a baseline average of 29.5 dpm correct, the highest of all participants for Set B. Student 6 reached criterion on day 27, taking 15 intervention sessions since reaching criterion for Set A. Additionally, during the intervention phase for Set B, Student 6 completed between 38 and 49 dpm correct until reaching criterion on day 27. Student 3 achieved a baseline average of 20.5 dpm correct and reached criterion on day 27, only taking three intervention sessions to reach criterion. Based on Student 3's maintenance data, they maintained fact fluency completing 43 dpm correct with zero errors on day 31. Student 4 achieved a baseline average of 15.5 dpm correct and reached criterion on day 23 of the intervention, which was seven intervention sessions after reaching criterion on Set A. Additionally, Student 4 maintenance data for Set B revealed that they did not maintain fact fluency of 40 dpm correct, completing 28 dpm correct with zero errors on day 31.

Student 1, Student 2, and Student 5 did not reach criterion for Set B. Student 1 completed as high as 40 dpm correct with 0 errors during the intervention phase for Set B, but scored 34 dpm correct with zero errors on maintenance assessment on day 31. Student 2 completed as high as 42 dpm correct with 2 errors during the intervention phase for Set B, but scored 23 dpm correct with 11 errors on maintenance assessment on day 31. Student 5 never began the intervention phase for Set B since they did not reach criterion for Set A. On the final maintenance assessment on day 31, Student 5 completed 8 dpm correct with zero errors.

4.4.1.3 Set C

One out of six students reached criterion of 50 dpm correct. Student 4 achieved a baseline average of 20.25 dpm correct, and reached criterion on day 28, taking 3 intervention sessions since reaching criterion for Set B. Additionally, during the intervention phase for Set C, Student 4 completed 40 and 34 dpm correct on the two previous intervention sessions until completing 51 dpm correct with zero errors on day 28. Based on Student 4's maintenance data, they almost maintained fact fluency completing 39 dpm correct with one error on day 31.

Student 1, Student 2, Student 3, Student 5, and Student 6 did not reach criterion for Set C. Student 1 never began the intervention phase for Set C since they never reached criterion for Set B. On the final maintenance assessment on day 31, Student 1 completed 29 dpm correct with zero errors. Student 2 never began the intervention phase for Set C since they never reached criterion for Set B. On the final maintenance assessment on day 31, Student 2 completed 14 dpm correct with four errors. Student 3, even though reaching criterion for Set B, never began the intervention phase for Set C due to absence. On the final maintenance assessment on day 31, Student 3 completed 26 dpm correct with zero errors. Student 5 never began the intervention phase for Set C since they did not reach criterion for Set A. On the final maintenance assessment on day

31, Student 5 completed 9 dpm correct with two errors. Student 6 participated in one intervention session for Set C, completing 38 dpm correct with 0 errors. On the final maintenance assessment on day 31, Student 6 completed 30 dpm correct with zero errors.

The following chapter will include reflections on the intervention effectiveness and maintenance, the study limitations, and recommendations for future research.

5.0 Discussion

Research suggests that students are struggling to achieve proficient math level (NAEP, 2015). One identified reason for this is students' insufficient computational fact fluency (Poncy, McCallum, and Schmitt, 2010). Therefore, it is important to develop research-based computational fact fluency interventions, particularly for those students who are identified as struggling to attain computational fact fluency, such as students identified with learning differences (Gersten et al., 2005; Gürbüz and Erdem, 2016). Attaining computational fact fluency can positively impact both computation and reasoning skills for students identified with learning differences (Fuchs et al., 2008; Gersten et al., 2005; Poncy & Skinner, 2011). Therefore, it is important to continue to study computational fact fluency interventions to identify interventions which positively increase fluency rate.

The current study sought to determine what effect Detect, Practice, Repair (DPR) would have on computational fact fluency growth for middle school students identified with learning differences. Additionally, the current study tracked fluency maintenance for facts that were mastered through the DPR intervention.

5.1 Intervention Effectiveness

Results indicate that DPR intervention was effective in increasing participants digits per minute (dpm) correct during the intervention phases compared to baseline data for five of the six participants, which suggests that DPR may be an effective intervention to improve computational fluency for LD middle school students. Over the course of the current study, five of the six participants reached fluency criterion for Set A, three reached criterion for Set B, and one participant reached criterion for Set C. Only a single participant, Student 5, did not reach criterion for Set A. However, during the intervention phases for Set A, Student 5 did experience increased dpm correct when compared to their baseline data. The results, increasing dpm correct, of the current study align with the two DPR studies found through the literature review (Hulac, Dejong, and Benson, 2012; Poncy, Skinner, and Axtell, 2010).

Moreover, the results of the study validate the findings of previous research, suggesting the DPR procedure was effective in improving math fact fluency. It was suggested the detect phase of DPR was what made it unique when compared to other fact fluency interventions (Axtell et al., 2009; Hulac, Dejong, and Benson, 2012; Poncy, Skinner, and Axtell, 2010). In the current study, just as in previous studies, DPR provided students with an individualized intervention through the Detect phase. However, it should be noted that unlike previous studies, which used a timed group assessment to detect unknown math facts for each student, the current study used a different tool, Xtra Math. Xtra Math allowed the researcher to identify unknown and nonfluent known facts to be used in the Repair phase of the intervention. This differs from past research using the DPR intervention, where only unknown math facts were identified.

Results of the current study indicated that DPR was an overall successful intervention for improving multiplication fact fluency. The findings extend the research for multiplication fact fluency interventions, suggesting that DPR can increase dpm correct for middle school aged students identified with LD. However, progress for attaining multiplication fact fluency per Set appeared to be slow compared to other found DPR studies (Hulac, Dejong, and Benson, 2012; Poncy, Skinner, and Axtell, 2010). Additionally, one participant in the study, Student 5, although made fluency growth throughout the study never reached fluency criterion for Set A, which was intriguing.

When examining the participant data, it is notable that the participants with lower average baseline data took more sessions to reach criterion. Therefore, of the five participants who reached criterion for Set A, two of which, Student 1 and Student 2, did not reach criterion for Set B. However, both participants showed increased dpm correct during the intervention phase for Set B and were trending towards criterion. This suggests that if the study continued and ran longer it could be assumed that these participants would have reached criterion for Set B, and therefore also suggests that if the study continued more participants may have reached criterion for Set B and Set C.

However, when analyzing the participants' psychoeducational testing it is noteworthy that all students' working memory and processing speed fell at or below the 50th percentile, but all fell within the average range except for one participant. Research indicates that deficits in working memory as well as processing speed may directly correlate to basic math skills, which include fact fluency (DeStefano and LeFevre, 2004; Formoso et al., 2018; Kaufmann, 2002). Four of the six participants' processing speed fell between the 27th and 37th percentile, which is on the lower end of the average range. Processing speed could suggest why participants appear to take longer to reach fluency criterion than previous studies indicated, and why the majority of participants did not reach fluency criterion for all three math fact Sets (DeStefano and LeFevre, 2004; Kaufmann, 2002). Student 4, however, did reach fluency criterion of all three math facts Sets. Their psychoeducational testing indicated that their processing speed fell at the 50th percentile, which may indicate why they were able to reach criterion for all three math fact Sets in the intervention. Additionally, Student 5, who did not reach criterion for any math fact Set, had a processing speed at the 1st percentile, as indicated on the psychoeducational testing. Their processing speed falls far below the average range and may have been a factor in their inability to reach fluency criterion.

5.2 Intervention Maintenance

Computational fact fluency maintenance data was collected throughout the DPR intervention on sessions ten, nineteen, and thirty-one. The collected computational fact fluency maintenance data for mastered facts did not indicate that fluency, forty dpm correct, was maintained by the majority of the participants. Only one participant, Student 6, maintained fluency for Set A and two participants maintained fluency for Set B, Student 3 and Student 6. However, it should be noted that both Student 3 and Student 6 reached criterion for Set B four sessions prior to the final maintenance data collection. The computational fact fluency maintenance data indicated that DPR intervention was unsuccessful for maintaining fact fluency for four other participants, Student 1, Student 2, Student 3, and Student 4, which reached criterion for Set A. Additionally, Student 4 did not maintain fact fluency for Set B and Set C as well.

These findings conflict with the maintenance data found by Poncy et al. (2010) and Hulac et al. (2012). Poncy et al. (2010), studied DPR and its effects on multiplication fact fluency for non-special education third graders. They found that all participants reached criterion and maintained fact fluency growth throughout the study, which is inconsistent with Hulac et al. (2012) where maintenance data varied between participants. Hulac et al. (2012) found that two of the five participants (non-special education fourth graders) where maintenance data was collected maintained fluency, while one participant’s maintenance data fell lower than their baseline data, and two other participants' maintenance data was higher than the data collected during the intervention phase.

The current study's maintenance data contradicts Poncy et al. (2010) findings, while consistently indicating a lack of maintenance for four of the six participants, which differs from the varied maintenance data from Hulac et al. (2012). It should be noted that in the current study the four participants who exhibited a lack of computational fact fluency maintenance showed substantial improvement when compared to their baseline data, even though their maintenance scores were under the fluency criteria of forty dpm correct. However, caution should be used when interpreting the maintenance data of the current study considering the limited number of data points.

Additionally, as noted above, when evaluating participant psychoeducational testing the working memory scores for the majority of participants fell on the low end of the average range. Formoso et al. (2018) suggests that deficits in working memories as well as processing speed may negatively affect retrieval and storage of basic math facts into long term memory, which may relate to fact fluency maintenance data in the current study.

5.3 Limitation

This study furthered the research for computational fluency intervention particularly their effect on fluency growth for students identified with a learning difference. However, some limitations should be considered when analyzing the data and results. A limitation of the current study could be the procedural integrity of the intervention. DPR is an intervention which relies on students independently following the procedural directions. Even though the researcher surveyed all participants while the DPR intervention took place, correcting procedural mistakes, procedural integrity could not be always guaranteed for all participants.

Additionally, even though computational fluency improved throughout the DPR intervention for all participants, two of the participants appeared to make faster progress on sprint probes after completing Set A. When comparing Student 2 and Student 3, visually the two participants have similar data. Student 2 achieved fluency criterion on the twenty-sixth day of the study for Set A. However, for Set B they made significant fluency growth in just four sessions, starting from twelve dpm correct and achieving forty-two dpm correct on the fourth session. Student 3 made similar progress as Student 2, reaching criterion for Set A on day twenty-four of the study. Then, Student 3 only needed three sessions to reach criterion for Set B. Additionally, Student 3's baseline maintenance data for Set B and Set C trends up, showing slight increases in dpm correct without utilizing the intervention phase of the DPR intervention. The data for these two participants may point to a secondary limitation of the study. The data suggests that there may have been other factors aiding participants in achieving increased fluency scores. Possible factors could be students' willingness to apply testing strategies, such as skipping unknown math fact problems instead of thinking about the possible answer or writing one known digit for a two-digit answer.

5.4 Future Research

When considering the current study, there are a few directions future research could take to continue the research on DPR for students with learning differences to increase computational fact fluency. Students' feelings about the DPR intervention should be monitored throughout the intervention. It was observed by the researcher that early on participants expressed a dislike for the intervention and their lack of progress, until they felt they were making fluency growth and felt successful. Participants' feelings towards the intervention could have negatively impacted their motivation and fluency growth in the beginning of the intervention, which in turn could have affected the results. Therefore, future research could focus on how the number of problems presented in each Set affects students' motivation and overall fluency growth. Would less than ten problems yield better results? If there were less problems for participants to focus on at a time, would they have achieved criterion faster? And therefore, felt successful, which could have improved their overall feelings toward the DPR intervention as well as their overall success with reaching criterion on each set? Also, if more than ten problems were used for each Set would DPR have yielded different results than what was found?

Additionally, future research should address the validity of the study and its effects on the acquisition of computational fluency and maintenance for middle school aged LD students, as well as across all grade levels including LD students in elementary and high school. Moreover, based on the current study, future studies should consider focusing on the length of time it takes LD students to achieve computational fluency and maintenance. Evaluating the current study's results suggest that if the study continued to run more participants would have possibly reached criterion for all three data sets. Also, based on the participants in the study, future research may want to focus on how processing speed and working memory affect the acquisition of computational fluency as well as maintaining fact fluency over time for students with and without LD. Longitudinal studies using DPR should evaluate the effects of computational fluency acquisition, specifically for LD students, on students' mathematical confidence and their feelings towards math, as well as their future mathematical achievement.

5.5 Conclusion

Regardless of the limitations, the results suggest that the Detect, Practice, Repair (DPR) intervention improves computational fact fluency. This study adds to the current research on computational fact fluency interventions by studying the effects of DPR on multiplication fact fluency growth for middle school aged LD students in a classroom with small student to teacher ratio. Results showed five of the six participants reached criterion for at least one of the data sets. However, the maintenance data collected only showed one of the six participants maintained fluency for one of the three Sets, while another of the participants maintained fluency for two of the three Sets. Previous research indicated that fluency was maintained throughout DPR (Hulac, Dejong, and Benson, 2012; Poncy, Skinner, and Axtell, 2010). Therefore, future studies should focus on strategies not only to improve computational fact fluency, but also to improve fluency maintenance for LD students. More studies focusing on improving learning for challenged learners, such as students with LD, are needed to continue to better the teaching practice and interventions for the most needy students.

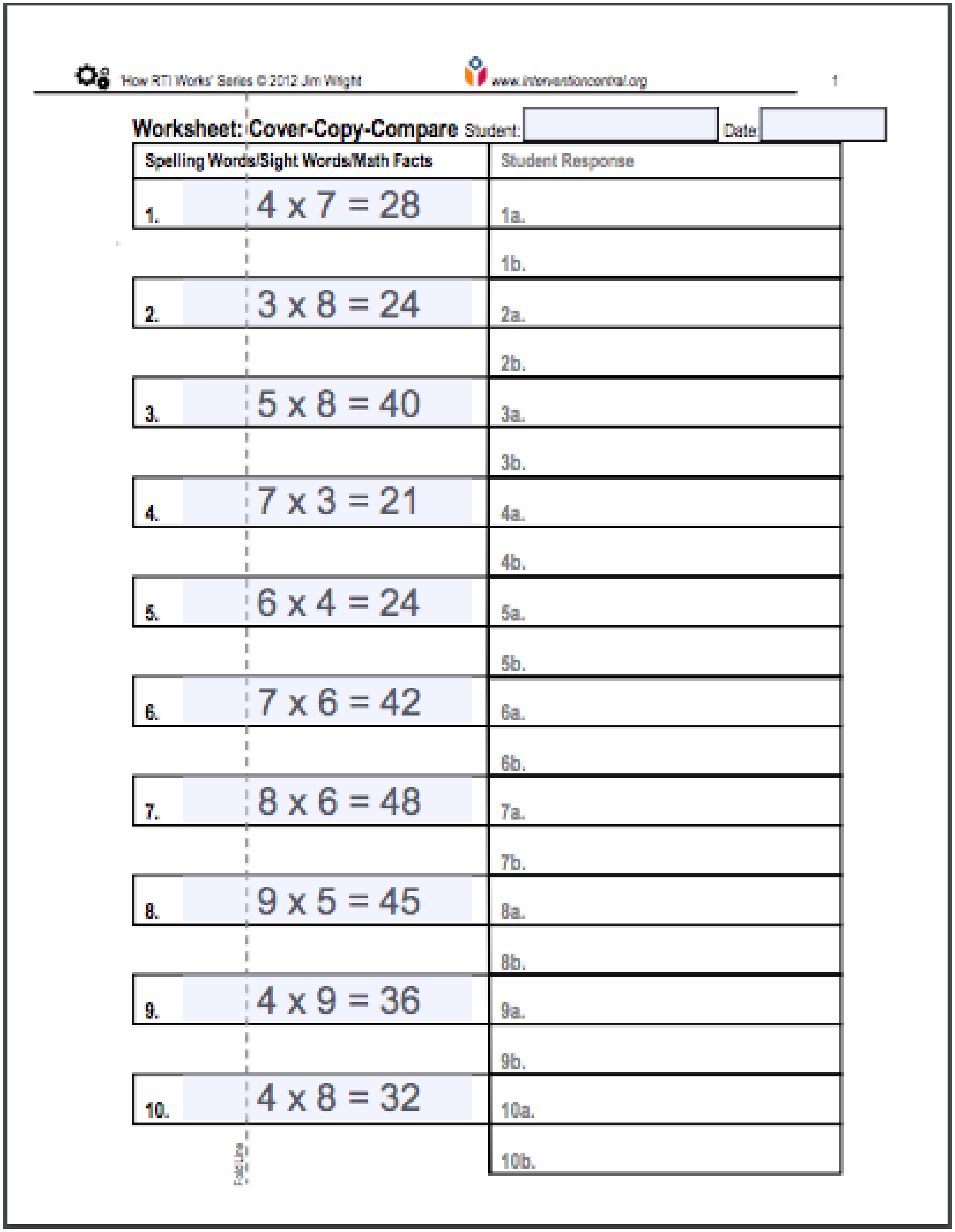
Appendix A Students Worksheets

Name:____ Date:_______________

Assessment Probe: Set A

8 x 6 =	6 x 4 =	3 x 8 =	4 x 9 =	4 x 8 =
4 x 9 =	4 x 8 =	9 x 5 =	4 x 8 =	3 x 8 =
4 x 7 =	7 x 6 =	6 x 4 =	8 x 6 =	5 x 8 =
6 x 4 =	7 x 3 =	7 x 6 =	3 x 8 =	9 x 5 =
7 x 6 =	8 x 6 =	4 x 7 =	7 x 6 =	5 x 8 =
7 x 3 =	6 x 4 =	5 x 8 =	9 x 5 =	4 x 7 =
8 x 6 =	7 x 3 =	4 x 7 =	4 x 9 =	5 x 8 =
4 x 8 =	5 x 8 =	3 x 8 =	7 x 3 =	9 x 5 =

Figure 7: Assessment Probe



'How RTI Works' Series © 2012 Jim Wright www.interventioncentral.org 1

Worksheet: Cover-Copy-Compare Student: Date:

Spelling Words/Sight Words/Math Facts	Student Response
1. 4 x 7 = 28	1a.
	1b.
2. 3 x 8 = 24	2a.
	2b.
3. 5 x 8 = 40	3a.
	3b.
4. 7 x 3 = 21	4a.
	4b.
5. 6 x 4 = 24	5a.
	5b.
6. 7 x 6 = 42	6a.
	6b.
7. 8 x 6 = 48	7a.
	7b.
8. 9 x 5 = 45	8a.
	8b.
9. 4 x 9 = 36	9a.
	9b.
10. 4 x 8 = 32	10a.
	10b.

Fold Line

Figure 8: CCC Worksheet

Name:______________________________ Date:____________

Sprint: Set A

7 x 3 =	5 x 8 =	9 x 5 =	6 x 4 =	4 x 7 =
8 x 6 =	4 x 7 =	4 x 9 =	7 x 3 =	5 x 8 =
4 x 9 =	9 x 5 =	4 x 8 =	8 x 6 =	3 x 8 =
4 x 7 =	6 x 4 =	8 x 6 =	7 x 6 =	5 x 8 =
8 x 4 =	3 x 8 =	4 x 9 =	6 x 4 =	4 x 8 =
7 x 6 =	4 x 7 =	7 x 6 =	8 x 6 =	5 x 8 =
4 x 8 =	3 x 8 =	7 x 3 =	5 x 8 =	9 x 5 =
6 x 4 =	7 x 6 =	3 x 8 =	7 x 3 =	9 x 5 =

Figure 9: Sprint Probe

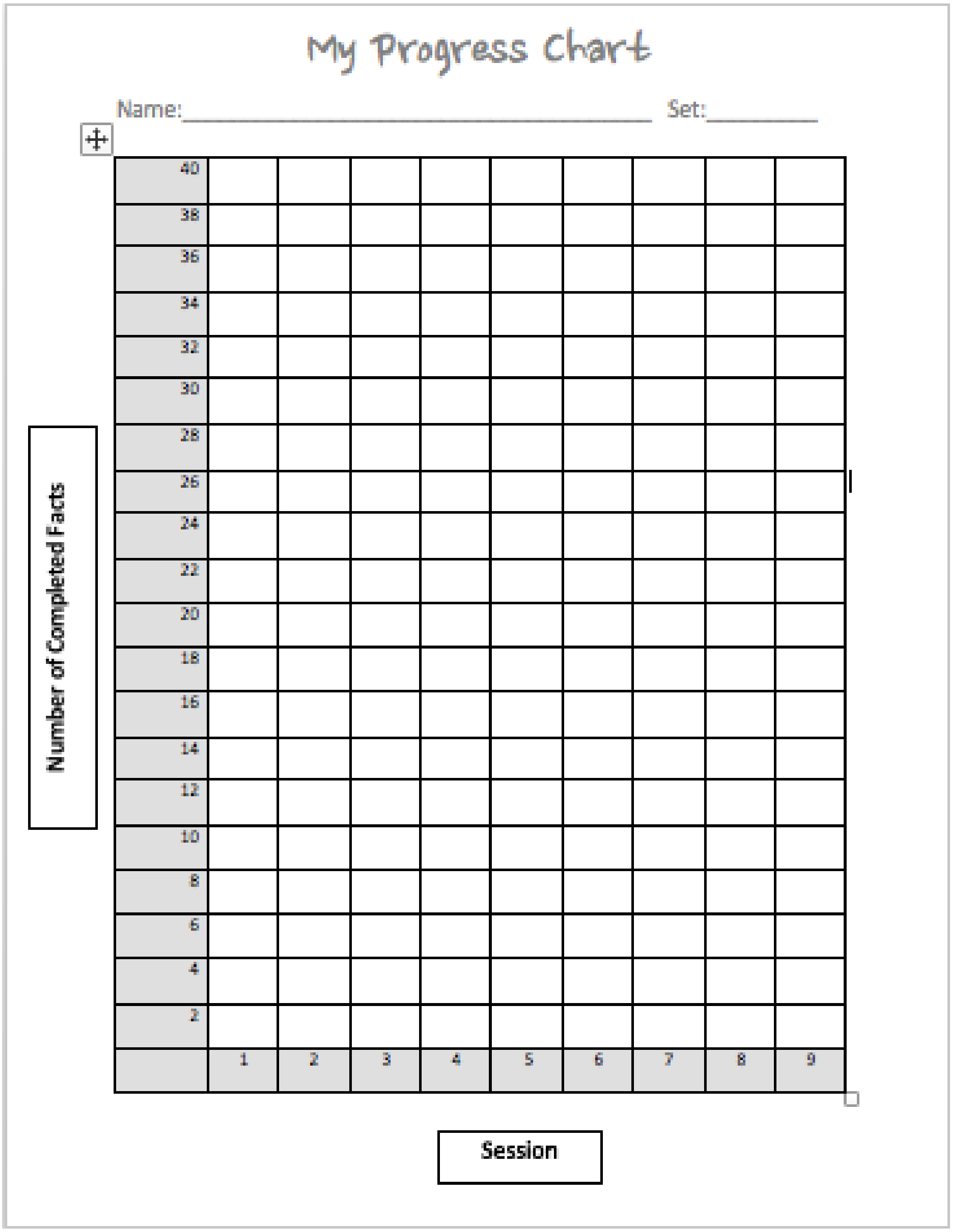


Figure 10: Graphing Worksheet

Appendix B Xtra Math App

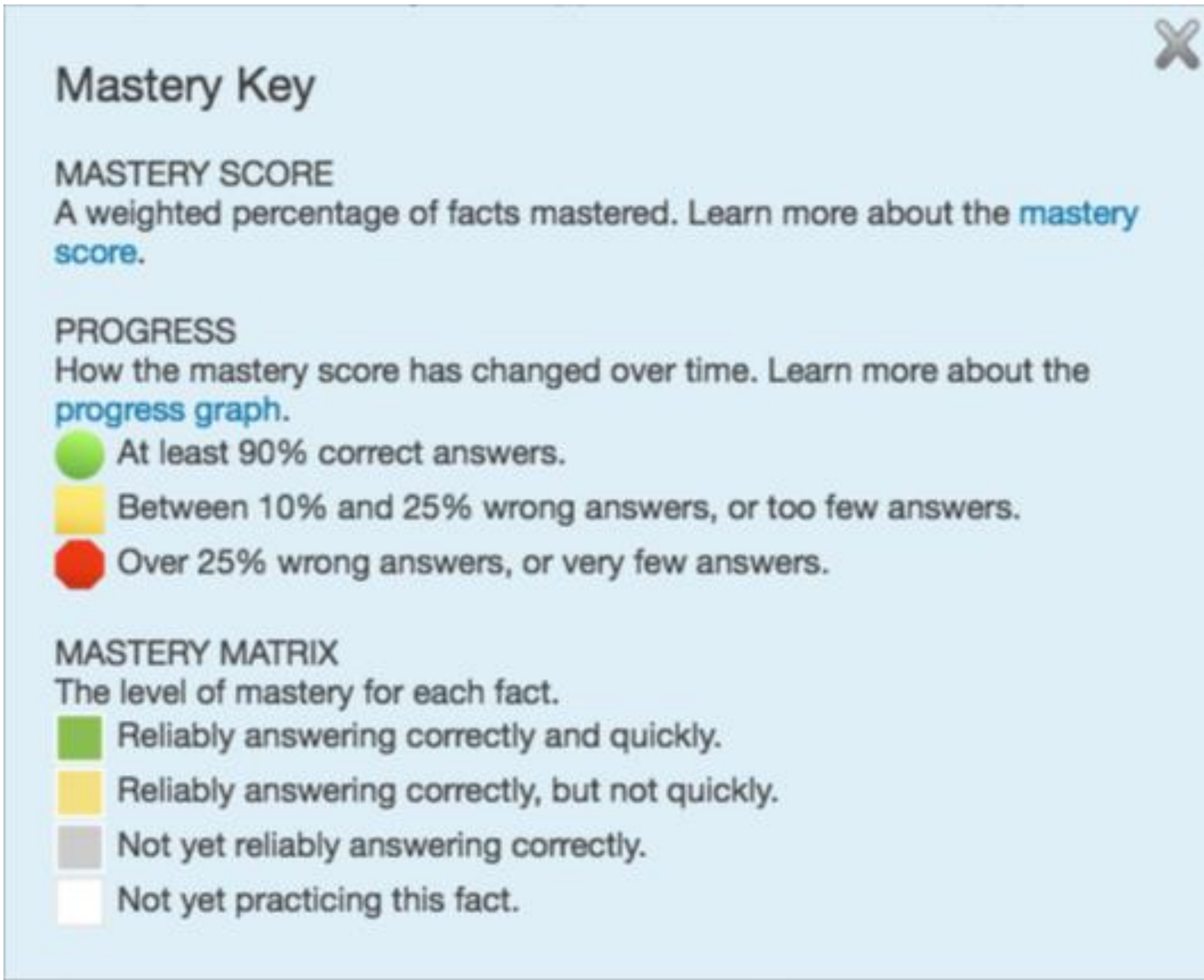


Figure 11 Mastery Key

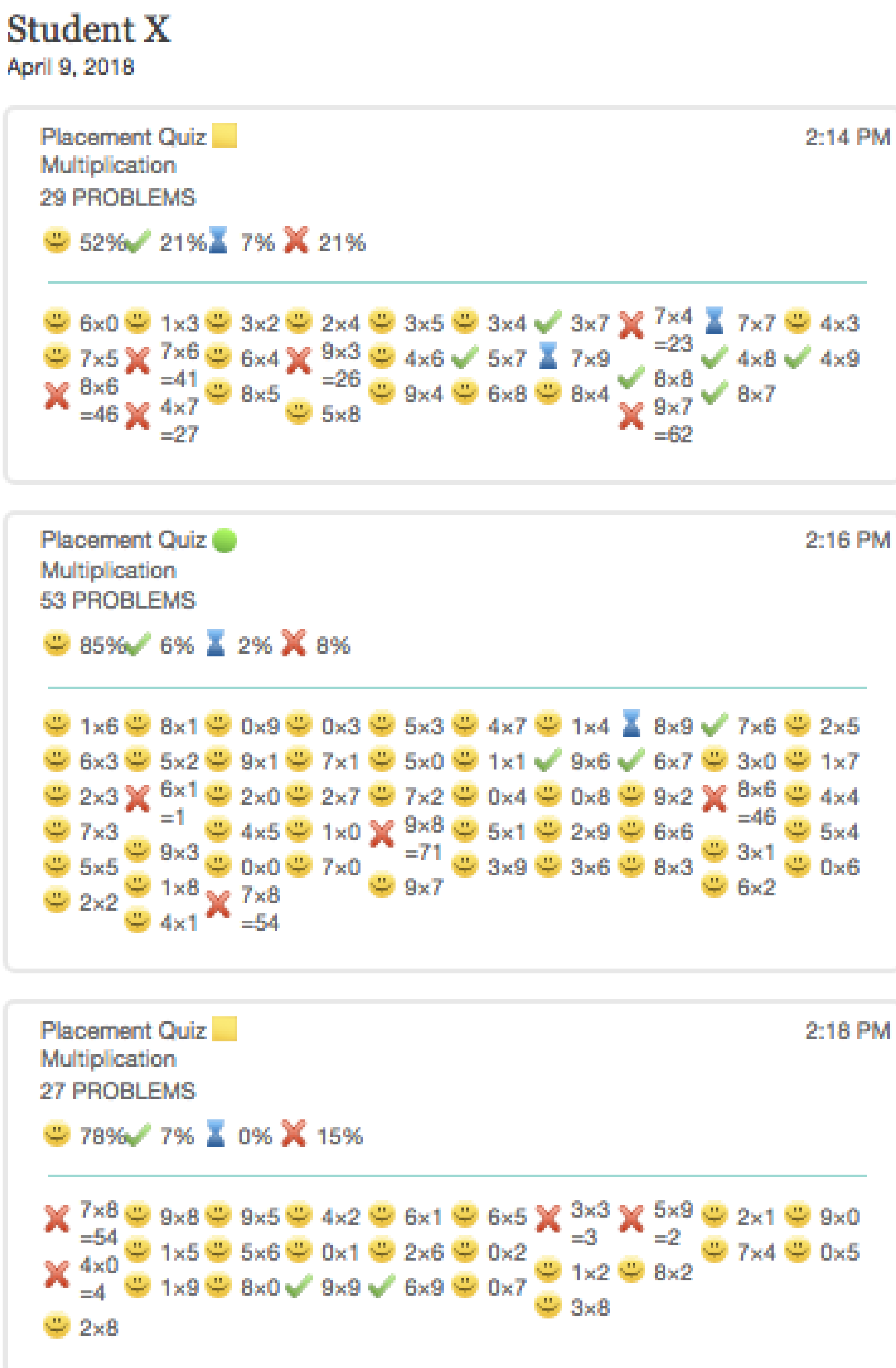


Figure 12 Results

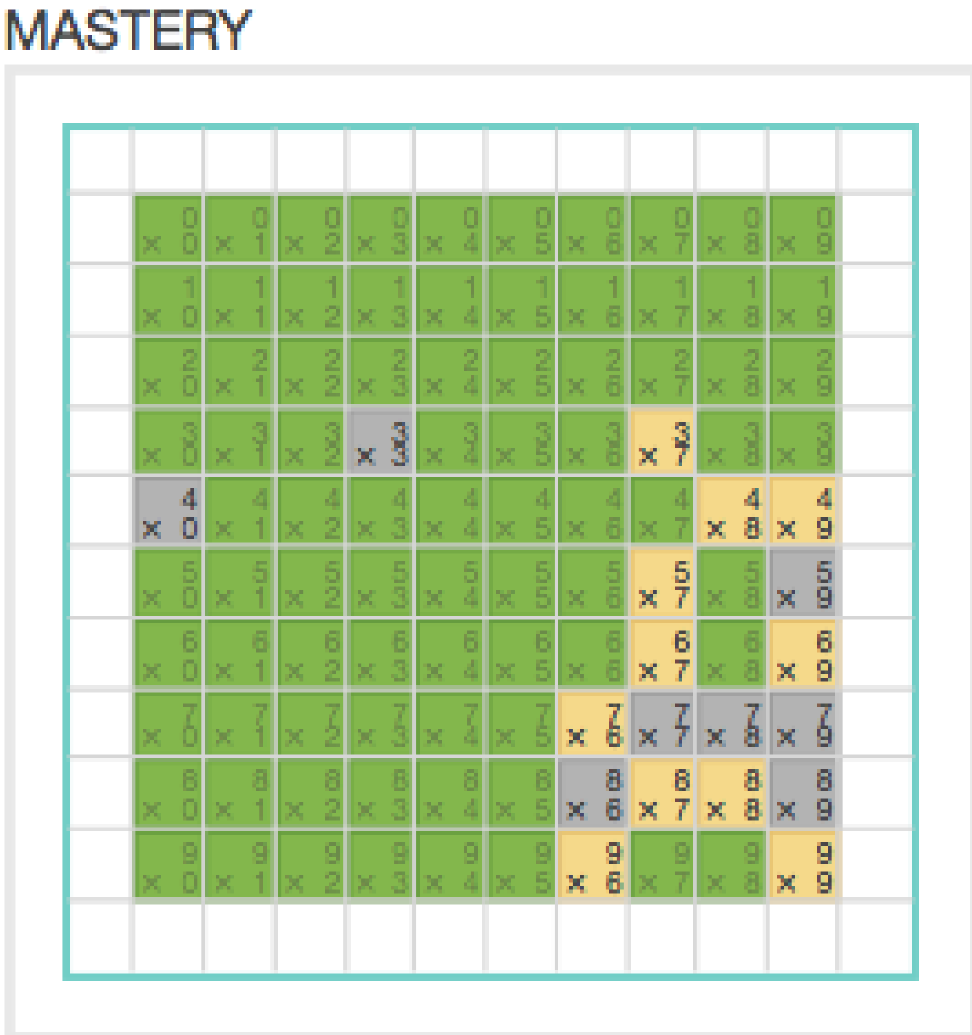


Figure 13 Results Multuplication Chart

Appendix C

STUDY TITLE:
The Effects of Detect, Practice, Repair (DPR) Mathematics Fluency Intervention on Multiplication Fact Fluency for Middle School Students Diagnosed with Learning Disabilities

PRINCIPAL INVESTIGATOR CONTACT INFORMATION:
Justin Schwartz
309 Sugar Bush Ct.
Mullica Hill, NJ 08062
JLS375@pitt.edu

Anastasia Kokina
5145 Wesley W. Posvar Hall
University of Pittsburgh
Pittsburgh, PA 15260
kokina@pitt.edu

Contact either Justin Schwartz or Anastasia Kokina with any questions.

PURPOSE:
This study is being conducted to satisfy graduation requirements for Justin Schwartz to complete his Doctorate in Education: Special Education at the University of Pittsburgh. The intervention will evaluate the success of a multiplication math fact fluency (accuracy and speed) intervention, Detect, Practice, Repair (DPR), for middle school students diagnosed with a learning disability. The potential participant is being asked to participate in the study because of their documented diagnosis of a learning disability, as well as being identified as needing an increase in their multiplication math fact fluency. There are five to eight subjects participating in the study and the study will be conducted over two to three months.

RESEARCH ACTIVITIES:
Participants will first complete a multiplication placement test on an app called XtraMath. The placement will identify the specific math facts the participant needs to improve. After the specific facts are identified, thirty facts will be chosen for each participant and be separated into groups of ten. Once a participant masters a set of ten facts they will move onto the next set of ten facts. Ideally, each participant will master all thirty multiplication math facts. Accuracy and speed growth, as well as a participants' ability to remember previously learned facts will be measured throughout the intervention.

STUDY BENEFITS:
Potentially, this study may improve a participant's ability to accurately and quickly recall specific multiplication facts.

PRIVACY and CONFIDENTIALITY:
Students will not be identified by name or other identifiable information in any publication or presentation at a scientific meeting unless parents sign a separate form giving permission.

Also, it is important to note that per University of Pittsburgh policy all research records must be maintained for at least 7 years following final reporting or publication of a project. For projects involving children, records must be maintained for 5 years past age of majority (age 23 per PA State law) after study participation ends.

WITHDRAWAL FROM STUDY PARTICIPATION:
Participants can, at any time withdraw from this research study. This means that he/ she will also be withdrawn from further participation in this research study. Any identifiable research obtained as part of this study prior to the date that you withdrew your consent will continue to be used and disclosed by the investigator.

To formally withdraw from this research study, participants should provide a written and dated notice of this decision to the principal investigator of this research study at the email address listed on the first page of this form.

It is possible that the participant may be removed from the research study by the researcher if, for example, consistent attendance at daily study sessions is not achieved.

VOLUNTARY PARTICIPATION:
Your child's participation in this research study is entirely voluntary. You may want to discuss this study with them before agreeing to participate. If there are any words you do not understand, feel free to ask. The investigator will be available to answer your current and future questions.

Whether or not you provide your consent for participation in this research study will have no effect on your current or future relationship with the AIM Academy.

CONSENT TO PARTICIPATE:

The above information has been explained to me and all of my current questions have been answered. I understand that I am encouraged to ask questions, voice concerns or complaints about any aspect of this research study during the course of this study, and that such future questions, concerns or complaints will be answered by a qualified individual or by the investigators listed on the first page of this consent document at the email address given. I understand that I may always request that my questions, concerns or complaints be addressed by a listed investigator. I understand that I may contact the Human Subjects Protection Advocate of the IRB Office, University of Pittsburgh (1-866-212-2668) to discuss problems, concerns, and questions; obtain information; offer input; or discuss situations that occurred during my participation. By signing this form I agree to participate/ my child participate in this research study. A copy of this consent form will be given to me.

______________________________ __________________

Participant's Signature Date

______________________________ __________________
Parent/Guardian's Signature Date

VERIFICATION OF EXPLANATION:

I certify that I have carefully explained the purpose and nature of this research study to the above-named participant in appropriate language. He/she has had an opportunity to discuss it with me in detail. I have answered all his/her questions and he/she has provided affirmative agreement (i.e., assent) to participate in this study.

______________________________ __________________
Investigator's Signature Date

INVESTIGATOR CERTIFICATION:

I certify that I have explained the nature and purpose of this research study to the above-named individual(s), and I have discussed the potential benefits and possible risks of study participation. Any questions the individual(s) have about this study have been answered, and we will always be available to address future questions, concerns or complaints as they arise. I further certify that no research component of this protocol was begun until after this consent form was signed.

___________________________________ _______________________
Printed Name of Person Obtaining Consent Role in Research Study

_________________________________ ______________________
Signature of Person Obtaining Consent Date

Appendix D

CCC Procedure and Script

Researcher:

1. (Have a countdown timer)
2. Pass out the student specific intervention folders containing CCC worksheets.
3. Ask the students to take out a CCC worksheet and place it face down on their desk.
4. Make sure all students have a writing utensil (pencil).
5. Read the following script:
    1. At the beginning of the intervention:
        1. "When I say begin, please turn over your worksheets and use the Cover, Copy, Compare procedures you have learned to complete the worksheet. If you finish your worksheet before the time is up, please take out another worksheet from your folder and continue working until the time is up. Ready, begin."
    2. At the end of the intervention:
        1. "The time is up. Please place the worksheets back in your folders. I will come around to collect them while I pass out your assessment folders."

Sprint Procedure and Script

Researcher:

1. (Have a countdown timer)

2. Pass out the student specific assessment folders containing sprint worksheets.
3. Ask the students to take out a sprint worksheet and place it face down on their desk.
4. Make sure all students have a writing utensil (pencil).
5. Read the following script:
    1. At the beginning of the sprint:
        1. "When I say begin, please turn over your worksheets. You will be given one minute to answer as many problems as possible. Start at the top left corner and answer problems from left to right. If you are struggling to remember an answer, skipped the problem and come back to it later if possible. Ready, begin."
    2. At the end of the intervention:
        1. "The time is up. Please count the number of problems you've answered and mark that number on your graph on the outside of your assessment folder. After you have finished, please place the worksheet back in your folder. I will come around to collect folders when you finish."

Table 2: Multiplication Fact Fluency Intervention Studies for Students with LD or at Risk

Study	Students	Independent Variables	Dependent Variables	Design	Results
Becker, McLaughlin, Weber, & Gower, 2009	One female in fourth grade with LD	CCC and CCC with error drill. (Procedural intervention)	Correct responses and incorrect responses per minute. (single digit multiplication fluency)	ABCA single case study design	CCC produced an increase in corrects and a decrease in errors; CCC with error drill improved corrects and errors further declined
Brady & Kubin, 2010	Three males; One in fourth grade and two males in fifth grade; All with ADHD, two with LD	Endurance building compared to whole time practice trials (Procedural intervention)	Number of correct and incorrect dpm (written). (single digit multiplication fluency)	Single Case Alternating Treatment Design: endurance building practice (3x 20 sec., 10 sec feedback) and whole-time practice (1min, 30 sec feedback), then 1min math fact assessment	Endurance building practice trials produced higher level of learning compared to whole time practice trials
Burns, 2005	Three males age eight in third grade with LD	IR (Procedural intervention)	dpm correct on single digit multiplication facts CBM. (single digit multiplication fluency)	Single Case Multiple baseline design	All participants' scores immediately increased after treatment intervention was introduced after baseline; implies consistent and reliable treatment effects
Burns, Kanive, & Degrande, 2010	216 third and fourth grade students; at risk for math achievement difficulties (below 25th percentile)	MFF (Procedural intervention)	Star Math achievement test score covered to NCE (student at risk status: below 25th NCE) (generalization of skills)	Group Design: For MFF: Master a level move on to the next level until mastered Mastered: answering all 40 problems correctly in 2 min	Treatment group had larger increases in scores compared to control.
Burns et al., 2015	Three students; two first grade students and one third grade student; students identified as at risk for math achievement difficulties	Conceptual interventions (modeling, fill in chutes, build in parts, bowl of facts) and a Procedural Intervention (incremental rehearsal)	dpm correct. (single digit multiplication fluency)	Single Case Multiple-baseline and multiple interventions design with staggered onset.	Prescribed intervention produced more growth than contra-indicated intervention
Flores, Houchins, & Shippen, 2006	Four sixth grade students with LD	CTD (Procedural intervention) compared to SIM (Procedural intervention)	dpm correct and digits correct untimed generalization (unknown math facts) and maintenance (1 week & 5 week) probes.	Single Case ABCA design comparing fact fluency vs. conceptual interventions	SIM produced more dpm correct for two students on timed generalization and maintenance probes and more digits correct for all students on untimed generalization and maintenance probes.
Glover, McLaughlin, Derby, & Gower, 2010	Two males ages 11 and 12 (one with LD and one with LD and ADHD)	DI flashcard system with back three for errors (multiplication for one student, division for one student)	Correct responses. (single digit multiplication accuracy, generalization of skills, and maintenance of skills)	Single Case Multiple baseline design across 3 multiplication fact sheets	Both participants' scores immediately increased after treatment intervention for Set 1 and 2; generalization occurred in Set 3; implies treatment effects.
Hulac, Dejong, & Benson, 2012	11 fourth grade students: 3 male and 8 female; at risk for math achievement difficulties	SAFI (Procedural intervention)	Multiplication CBM: digits correct per 2 minutes with intervention and maintenance phases. (single digit multiplication fluency and maintenance of skills)	Single case multiple baseline design	7/11 of the participants demonstrated faster growth rate during intervention phase versus baseline phase; 2/11 participants showed greater growth rate during baseline than intervention phase, but intervention phase still showed positive growth; 1/11

					participants showed very little growth during any phase.
Kanive, Nelson, Burns, & Ysseldyke, 2013	90 fourth and fifth grade students; at risk for math achievement difficulties (below 25th percentile)	Computer-based intervention: MFF (Procedural intervention) compared to Conceptual interventions (activities developed by Van de Walle and Lovin (2006), fill the chutes, build the parts, and broken calculator)	dpm correct and word problem score. (single digit multiplication fluency and (generalization of skills)	Group Design: Pre and posttest for fluency (2 min): Easy CBM 18 word problems (20-30 min) 3 groups	MFF and Conceptual intervention treatment group out performed control group on generalization and retention fluency measures (dpm correct); MFF treatment group's mean score was significantly larger than control group; No significant difference between conceptual group and control group as well as MFF group and Conceptual Group; No significant difference between all groups on word problem mean scores
Leach, 2016	1 fourth grade student at-risk for math achievement difficulties and being assessed for special education; eligibility during study	1-on-1 High-probability instructional sequences and explicit, systematic, intensive instruction (Procedural intervention)	Correct verbal responses within three seconds. (single digit multiplication fluency)	Single case AB design: single subject, pre/ post assessment	At the end of a 4 week intervention period, the participant reached mastery, 80/80 single digit multiplication problems, from a baseline score of 56/80.
Mcintyre, Test, Cooke, & Beattie, 1991	1 fourth grader identified with a LD	Count-By intervention (skip counting): using a model-lead-test procedure (Procedural intervention)	dpm correct for daily fact family probes (i.e. 4s, 7s, 8s) and pre and post mixed problems probes. (single digit multiplication fluency and maintenance of skills)	Single case multiple probe design: single subject	The participant showed positive growth on each fact family from baseline to intervention phase and was maintained. Mixed probe baseline score: 6 dpm correct; posttest score: 40 dpm correct
Nelson, Burns, Kanive, & Ysseldyke, 2013	90 third and fourth grade students; at risk for math achievement difficulties (below 25th percentile)	Mnemonic strategic intervention: TTFW (Procedural intervention) compared to Computer-delivered intervention: MFF (Procedural intervention)	Correct responses on retention measure (20 single digit multiplication facts) and application measure (18 word problems) (single digit multiplication accuracy and (generalization of skills)	Group Design	MFF and TTFW treatment groups out performed control group on math fact fluency; MFF group's mean score was significantly larger than control group; No significant difference between TTFW group and control group, as well as MFF group and TTFW group; No significant difference between all groups on word problem mean scores
Ok & Bryant, 2015	4 fifth grade students, 2 males and 2 females, with LD	Explicit, strategic, intervention with iPad practice on multiplication fact fluency for factors of 4 and 8 and strategy use (doubling strategy) (Procedural intervention)	dpm correct and strategy use. (single digit multiplication fluency)	Single case multiple probe design	All 4 participants made positive growth in dpm correct and demonstrated 100% use of doubling strategy after intervention
Poncy, Skinner, & Axtell, 2010	7 third grade students:5 females, 2 males; at risk for math achievement difficulties	DPR (Procedural intervention)	dpm correct: baseline, intervention, and maintenance phases. (single digit multiplication fluency and maintenance of skills)	Single case multiple probe design	7/7 participants dpm correct increase for baseline to intervention phase and are maintained
Rhymer, Dittmer,	4 fourth grade students: 3	Multicomponent intervention: peer-	Problems correct per minute.	Single case alternate treatment design	3/4 of the participants showed slight increases in problems

Skinner, & Jackson, 2000	males and 1 female; at risk for math achievement difficulties (below 25th percentile)	tutoring, timing procedures, and positive-practice overcorrection (Procedural intervention)	(single digit multiplication fluency)		correct per minute after timing procedures, and positive-practice overcorrection intervention phase; 2/4 of the participants performed better when the tutee rather than the tutor
Skarr et al., 2014	Three students; One male age 8 (3rd grade), one female age 10 (5th grade), and one male age 11(5th grade) with LD	DI Flashcard and racetrack procedures intervention (Procedural intervention)	Correct verbal responses. (single digit multiplication accuracy)	Single Case: A single subject multiple baseline design across three sets of facts showed a clear functional relationship between the DI flashcard combined with the math racetrack procedures and increased mastery of multiplication facts.	All participants' scores immediately increased after treatment intervention was introduced after baseline; implies consistent and reliable treatment effects
Wood, Frank, & Wacker, 1998	Three males age 10 with LD; two 4th graders, one 5th grader	Instructional Package: sequence, associative learning, mnemonic procedures, strategic learning, self-instruction training (Procedural intervention)	Correct responses on multiplication facts test. This study design was selected to answer the following questions: (a) Did instruction in a particular strategy result in immediate improvement for only specific multiplication facts, and (b) was performance maintained on previously trained multiplication facts as successive strategies were introduced? (single digit multiplication accuracy)	Single Case multiple probe design: replications across students were used to analyze the effects of each instructional strategy on each specific category of multiplication facts.	All participants' scores immediately increased after each instructional strategy was introduced; implies consistent and reliable treatment effects

Note. CCC = Copy, Cover, Compare, MFF = Math Facts in a Flash, NCE = Normal curve equivalent, dpm = digits per minute, CTD = Constant Time Delay, SIM = Strategic Instructional Model, DI = direct instruction, TTFW = Times Tables the Fun Way, SAFI = Self-administered folding in technique, DPR = Detect, Practice, Repair, LD = Learning disability, CBM = Curriculum-based measure, IR = Incremental Rehearsal, ADHD = Attention deficit hyperactivity disorder

Bibliography

Ansari, D. (2016, March). No more math wars. *The Education Digest*, *81*(7), 4-9.

Axtell, P. K., McCallum, R. S., Bell, S. M., & Poncy, B. (2009). Developing math automaticity using a classwide fluency building procedure for middle school students: A preliminary study. *Psychology in the Schools, 46*, 526– 538.

Becker, A., McLaughlin, T., Weber, K. P., & Gower, J. (2009). The effects of cover, copy, and compare with and without additional error drill on multiplication fact fluency and accuracy. Electronic Journal of Research in Educational Psychology, 7(18), 747-760.

Binder, C. (1996). Behavioral fluency: Evolution of a new paradigm. The Behavior Analyst, 19(2), 163–197.

Brady, K. K., & Kubina, R. M. (2010). Endurance of multiplication fact fluency for students with attention deficit hyperactivity disorder. *Behavior Modification*, *34*(2), 79-93.

Burns, M. K. (2005, August). Using incremental rehearsal to increase fluency of single digit multiplication facts with children identified as learning disabled in mathematics computation. *Education and Treatment of Children*, *28*(3), 237-249.

Burns, M. K., Codding, R. S., Boice, C. H., & Lukito, G. (2010). Meta-analysis of acquisition and fluency math interventions with instructional and frustration level skills: Evidence for a skill-by-treatment interaction. *School Psychology Review, 39*(1), 69-83.

Burns, M. K., Kanive, R., & Degrande, M. (2010). Effect of a computer-delivered math fact intervention as a supplemental intervention for math in third and fourth grades. Remedial and Special Education, 33(3), 184-191.

Burns, M. K., Walick, C., Simonson, G. R., Dominguez, L., Harelstad, L., Kincaid, A., & Nelson, G. S. (2015). Using a conceptual understanding and procedural fluency heuristic to target math interventions with students in early elementary. *Learning Disabilities Research & Practice*, *30*(2), 52-60.

Codding, R. S., Chan-Iannetta, L., Palmer, M., & Lukito, G. (2009). Examining a classwide application of cover-copy-compare with and without goal setting to enhance mathematics fluency. *School Psychology Quarterly, 24*(3), 173-185.

Collier, M. (1922). Learning To Multiply Fractions. *School Science and Mathematics*, *22*(4), 324-329.

Deno, S. L., & Mirkin, P. (1977). Data-based program modification: A manual. Reston, VA: Council for Exceptional Children.

DeStefano, D., & LeFevre, J.-A. (2004). The role of working memory in mental arithmetic. European Journal of Cognitive Psychology, 16(3), 353–386.

Flores, M. M., Houchins, D. E., & Shippen, M. E. (2006). The effects of constant time delay and strategic instruction on students with learning disabilities' maintenances and generalization. *International Journal of Special Education*, *21*(3), 45-57.

Formoso, J., Injoque-Ricle, I., Barreyro, J.-P., Calero, A., Jacubovich, S., & Burín, D. I. (2018). Mathematical cognition, working memory, and processing speed in children. Cognition, Brain, Behavior, 22 (2018) 59-86.

Fuchs, L. S., Fuchs, D., Stuebing, K., Fletcher, J. M., Hamlett, C. L., & Lambert, W. (2008). Problem solving and computational skill: Are they shared or distinct aspects of mathematical cognition? Journal of Educational Psychology, 100(1), 30-47.

Gersten, R., Jordan, N. C., & Flojo, J. R. (2005). Early identification and interventions for students with mathematics difficulties. *Journal of Learning Disabilities*, *38*(4), 293-304.

Glover, P., McLaughlin, T., Derby, K., & Gower, J. (2010, September). Using a direct instruction flashcard system with two students with learning disabilities. *Electronic Journal of Research in Educational Psychology*, *8*(2), 457-472.

Grafman, J. M., & Cates, G. L. (2010). The differential effects of two self-managed math instruction procedures: Cover, Copy, and Compare versus Copy, Cover, and Compare. *Psychology In The Schools, 47*(2), 153-165.

Gürbüz, R., & Erdem, E. (2016). Relationship between mental computation and mathematical reasoning. Cogent Education, 3(1).

Horner, R. D., & Baer, D. M. (1978). Multiple-probe technique: a variation on the multiple baseline. *Journal of Applied Behavior Analysis*, *11*(1), 189–196.

Hott, B. L., Isbell, L., & Laura, L. O. (2014, December). Strategies and interventions to support students with mathematics disabilities. Retrieved January 29, 2018, from https://www.bing.com/cr?IG=7900F91F34894F50852D3143FEE0C0CC&CID=29B4A90C90AD607F22F8A28A91026138&rd=1&h=9iuhqZhw8MlbAt-w-1rnIVAZduGpml-gI4KjELMIH7E&v=1&r=https%3a%2f%2fwww.council-for-learning-disabilities.org%2fwp-content%2fuploads%2f2014%2f12%2fMath_Disabilities_Support.pdf&p=DevEx,5067.1

Hulac, D. M., Dejong, K., & Benson, N. (2012). Can students run their own interventions?: A self-administered math fluency intervention. *Psychology in the Schools, 49*(6), 526-538.

Johanning, D. I. (2013). Developing algorithms for adding and subtracting fractions. *Mathematics Teaching in the Middle School*, *18*(9), 527.

Joseph, L. M., Konrad, M., Cates, G., Vajcner, T., Eveleigh, E., & Fishley, K. M. (2012).A meta-analytic review of the cover-copy-compare and variations of this self-management procedure. *Psychology in the Schools*, *49*(2), 122-136.

Kanive, R., Nelson, P. M., Burns, M. K., & Ysseldyke, J. (2013). Comparison of the effects of computer-based practice and conceptual understanding interventions on mathematics fact retention and generalization. *The Journal of Educational Research*, *107*(2), 83-89.

Kauffman, J. M., & Hallahan, D. P. (2011). *Handbook of Special Education*. New York: Routledge.

Kaufmann, L. (2002). More evidence for the role of the central executive in retrieving arithmetic facts? A Case study of severe developmental dyscalculia. *Journal of Clinical and Experimental Neuropsychology*, 24(3), 302–310.

Koponen, T., Aro, T., Räsänen, P., & Ahonen, T. (2007). Language-based retrievaldifficulties in arithmetic: A single case intervention study comparing two children with SLI. *Educational and Child Psychology*, *24*(2), 98-107.

Lamberg, T., & Wiest, L. R. (2015). Dividing fractions using an area model: A look at in-service teachers' understanding. *Mathematics Teacher Education and Development*, *17*(1), 30-43.

Leach, D. (2016). Using high-probability instructional sequences and explicit instruction to teach multiplication facts. *Intervention in School and Clinic, 52*(2), 102-107.

Ledford, J. R., & Gast, D. L. (2018). Single case research methodology: Applications in special education and behavioral sciences. New York, NY: Routledge.

Lee, M. J., & Tingstrom, D. H. (1994). A group math intervention: The modification of cover, copy, and compare for group application. Psychology in the Schools, 31(2), 133-145.

Lyon, G. R. (1996). Learning Disabilities. The Future of Children, 6(1), 54-76.

Methe, S. A., Kilgus, S. P., Neiman, C., & Chris Riley-tillman, T. (2012). Meta-analysis of interventions for basic mathematics computation in single-case research. *Journal of Behavioral Education, 21*(3), 230-253.

Mcintyre, S. B., Test, D. W., Cooke, N. L., & Beattie, J. (1991). Using count-bys to increase multiplication facts fluency. *Learning Disability Quarterly, 14*(2), 82-88.

NAEP - 2015 Mathematics & Reading Assessments. (n.d.). Retrieved November 09, 2016, from http://www.nationsreportcard.gov/reading_math_2015/

Nelson, P. M., Burns, M. K., Kanive, R., & Ysseldyke, J. E. (2013). Comparison of a math fact rehearsal and a mnemonic strategy approach for improving math fact fluency. Journal of School Psychology, 51(6), 659-667.

Ok, M. W., & Bryant, D. P. (2015). Effects of a strategic intervention with ipad practice on the multiplication fact performance of fifth-grade students with learning disabilities. Learning Disability Quarterly, 39(3), 146-158.

Parkhurst, J., Skinner, C. H., Yaw, J., Poncy, B., Adcock, W., & Luna, E. (2010). Efficient class-wide remediation: Using technology to identify idiosyncratic math facts for additional automaticity drills. *International Journal of Behavioral Consultation and Therapy, 6*, 111–123.

Poncy, B. C., Fontelle, S. F., IV, & Skinner, C. H. (2013). Using detect, practice, and repair (DPR) to differentiate and individualize math fact instruction in a class-wide setting. *Journal of Behavioral Education*, *22*, 211– 228.

Poncy, B. C., Mccallum, E., & Schmitt, A. J. (2010). A comparison of behavioral and constructivist interventions for increasing math-fact fluency in a second-grade classroom. Psychology in the Schools, 47(9), 917-930.

Poncy, B. C., & Skinner, C. H. (2006). Implementation Guidelines: Detect, Practice, and Repair: The Effects of a Classwide Intervention on Elementary Students' Math-Fact Fluency. *Journal of Evidence-Based Practices for Schools*, *7*(1), 69-72

Poncy, B. C., & Skinner, C. H. (2011). Combining class-wide cover, copy, and compare (CCC) with an interdependent group contingency to enhance addition- fact fluency in a first-grade classroom. *Journal of Applied School Psychology*, *27*, 1– 20.

Poncy, B. C., & Skinner, C. H. (2011). Enhancing first-grade students' addition-fact fluency using classwide cover, copy, and compare, a sprint, and group rewards. *Journal of Applied School Psychology*, *27*(1), 1-20.

Poncy, B. C., Skinner, C. H., & Axtell, P. K. (2010). An investigation of detect, practice, and repair to remedy math-fact deficits in a group of third-grade students. *Psychology in the Schools, 47*(4), 342-353.

Poncy, B. C., Skinner, C. H., & Jaspers, K. E. (2007). Evaluating and comparing interventions designed to enhance math fact accuracy and fluency: Cover, copy, and compare versus taped problems. Journal of Behavioral Education, 16(1), 27-37.

Rhymer, K. N., Dittmer, K. I., Skinner, C. H., & Jackson, B. (2000). Effectiveness of a multi-component treatment for improving mathematics fluency. School Psychology Quarterly, 15(1), 40-51.

Rittle-Johnson, B., Siegler, R. S., & Alibali, M. W. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. *Journal of Educational Psychology*, *93*(2), 346-362.

Skarr, A., Zielinski, K., Ruwe, K., Sharp, H., Williams, R. L., & McLaughlin, T. F. (2014). The effects of direct instruction flashcard and math racetrack procedures on mastery of basic multiplication facts by three elementary school Students. *Education and Treatment of Children*, *37*(1), 77-93.

Skinner, C.H., Bamberg, H.W., Smith, E.S., Powell, S.S. (1993). Cognitive cover, copy, and compare: Subvocal responding to increase rates of accurate division responding. *Remedial and Special Education, 14*(1), 49-56.

US Department of Education 38th Annual Report to Congress on the Implementation of the Individuals with Disabilities Education Act (2016), 1-320.

Wood, D. K., Frank, A. R., & Wacker, D. P. (1998). Teaching multiplication facts to students with learning disabilities. *Journal of Applied Behavior Analysis*, *31*(3), 323-338.

XtraMath (2009). Retrieved from https://xtramath.org/