Examining Gender-Based Associations of Parental Number Talk in the Home

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Parents' math engagement in the home is important for their children's math skill development prior to school entry (Daucourt et al., 2021). Both the frequency and type of parental number talk (discussions of number concepts) is related to children's math performance (Levine et al., 2010; Gunderson & Levine, 2011). However, previous research exploring numeracy engagement in the home focused primarily on mothers of young children, and particularly those with preschool- and school-aged children. This project aimed to investigate the joint and unique influences of mothers and fathers on their toddlers' number knowledge, by comparing the frequency and type of number talk mothers and fathers use with their toddlers during semi-structured interactions, and how these may predict children's number skills. In a sample of 133 children aged 2-3 years (M=30.8 months, SD=3.36 months, 52% female) and their mothers and fathers, we found that mothers and fathers of two- to three-year-olds used relatively similar proportions of number talk, $t(132) = 1.92$, $p = 0.057$, with mothers tending to use slightly more number talk. Additionally, both mothers and fathers most frequently used quantifiers, labeling, and counting when talking about numerical concepts, and unexpectedly, neither parents' frequency of number talk significantly predicted children's numeracy scores (assessed as performance in Point-to-X and Give-N tasks). Exploratory analyses suggested that fathers' use of labeling set sizes significantly predicted toddlers' number skills ($r = .22$, $p = .011$), and more work is needed to understand how this may contribute to children's number knowledge over time. Overall, this study presents novel findings in how mothers and fathers compare in their frequency and categorical use of number talk with their

toddlers.

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Preface

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1.0 Introduction

Distinct variability in children's early math performance has profound implications for a variety of long-term outcomes such as academic achievement, as well as income, likelihood of full-time employment, and financial and health status (Agarwal & Mazumder, 2013; Currie & Thomas, 1999; Jordan et al., 2006, 2009; Reyna & Brainerd, 2007; Trusty et al., 2000). During their toddler and preschool years, children develop a variety of critical skills, facilitating their ability to understand exact number symbols and their associated quantities (Libertus, Duong & Silver, 2020). Parents' roles within the home learning environment, and particularly their interactions surrounding math, play an important role in the development of math skills prior to children's school entry (Niklas et al., 2016; Napoli & Purpura, 2018). Relatedly, parents' gender and gender-related math associations are related to their children's beliefs and attitudes about math (Hildebrand et al., 2022). The current study aims to examine how parent-child interactions and the way parents speak with their children about math at an early age is related to children's math skill development and how these associations may differ based on parents' gender.

1.1 Early math skill development

The development of early math skills, prior to entering formal school, has been shown to predict children's later academic achievement (Duncan et al., 2007; Jordan et al., 2009). Distinct variability at the start of kindergarten in basic number skills and conventional arithmetic persists in growth rates of number sense through the year (Jordan et al., 2006). Moreover, individual differences in math skills emerge early in life. In work longitudinally examining this variability, infants' number sense at six months of age predicted their math performance three years later (Starr et al., 2013). Together, this leads to important questions about which factors may influence the development of math abilities prior to school entry.

1.2 Parental Math Support in the Home Environment

Previous work suggests that genetic, cognitive, and environmental influences all play a role in shaping children's math performance (De Smedt, 2020; Silver & Libertus, 2022). A particularly relevant environmental factor pertains to the home environment, in which parents can help to facilitate children's learning of math skills prior to school entry (Eason et al., 2022; Napoli & Purpura, 2018). Parent-child math engagement occurs in a variety of forms, including math activities, such as puzzles or number activity books, and participating in math-related discussions ("math talk") regardless of whether these occur in the context of math activities or other activities such as mealtimes. The frequency of parent-child math engagement is related to children's math performance, such that numeracy activities including number games, the practice of number skills, and use of number-related objects, as well as talk about numbers and numerical concepts ("number talk") positively predict children's math skills (LeFevre et al., 2009; Ramani et al., 2015; Daucourt et al., 2021; Levine et al., 2010; Silver et al., in press).

1.3 Parental Numeracy Engagement in the Home

Different types of parent-child math interactions and activities emphasize a variety of number skills to differing extents, which hold implications for children's math performance. Young children's number skills can relate to identifying number symbols, counting, cardinality and labeling set sizes, comparing set sizes, arithmetic, and recognizing numerical patterns. Moreover, formal teaching activities (i.e., direct teaching about numbers) were found to predict both preschool children's foundational concepts (i.e., counting and number identification) and their advanced numerical knowledge (i.e., cardinality, ordinal relations, and arithmetic), while informal activities (e.g., number games) only predicted children's foundational skills (Ramani et al., 2015). More specifically, the frequency of engagement in home math activities between parents and preschool children pertaining to the specific numerical subdomains of set size comparison, arithmetic, and patterning has been found to relate to children's math skills in the corresponding subdomains (Leyva, Libertus & McGregor, 2021).

Less is known about how different types of parent number talk (talk about different number concepts) may predict different number skills. Some work suggests parental number talk more robustly predicts children's understanding of advanced concepts (e.g., cardinality) compared to foundational concepts (e.g., number identification) (Ramani et al., 2015). Furthermore, another study found that parents' number talk that includes the discussion of larger quantities of objects is a better predictor of children's later cardinal number knowledge (Gunderson & Levine, 2011). Importantly, these early advanced counting and cardinality competencies have been shown to be more predictive of children's math achievement through elementary school compared to basic competencies (Nguyen et al., 2016).

1.4 Including Paternal Home Engagement

Critically, this past work focused primarily on the mothers of young children, and particularly those with preschool- and school-aged children. However, paternal influences are also important to consider in children's development, and this field of study is proliferating in general (Taraban & Shaw, 2018; Rolle et al., 2019). For example, fathers' support in the broader home learning environment and their warmth predicts higher math skills in children during toddlerhood and through kindergarten (Coley et al., 2011; Baker et al., 2017; 2018). When examining the effects of parental warmth and home learning stimulation more carefully, fathers' warmth and mothers' home learning stimulation mediated the relationship between poverty and children's math scores (Baker et al., 2018). Together, this suggests fathers' influence on the home learning environment should not be ignored.

However, only a few studies have begun to examine fathers' math-specific engagement, and studies have reported mixed results as to whether fathers differ from mothers in the frequency of their home math activities with their young children (Silver et al., 2023; del Rio et al., 2017, 2019). On one hand, some work suggests that mothers may be more involved than fathers in home math activities with their kindergarten children, while other work suggests no difference in math engagement frequency with their children (del Rio et al., 2019, 2017). Regardless of whether parents differ in their math activity frequency, in one study of preschool-aged children, fathers' involvement in math activities predicted children's math performance above mothers' involvement in math activities, particularly when mothers held less than a bachelor's degree (Foster et al., 2016). This work suggests that fathers' math engagement may have unique contributions for preschool-aged children's math learning. However, whether this is also the case for toddlers' math skills remains unclear.

To date, only two studies have compared the home math engagement of mothers and fathers of toddlers. Silver et al. (2023) specifically examined mothers' and fathers' self-reported home math activities with their toddlers and their relations to toddlers' number and spatial skills as well as their math language comprehension. They found that fathers' math engagement was predictive of both number and math language skills, while mothers' engagement was only related to toddlers' math language skills. However, this study measured home math engagement via parental report. Critically, males and females tend to differ in their biases on self-report measures. For example, prior work finds that women tend to underrate their team's academic performance, while men overrate their team's performance, even after receiving peer feedback (Scherpereel & Bowers, 2007). Other work finds that despite having similar math grades, high school girls reported lower math self-efficacy and were less likely to bridge the gap between subjective and objective ratings of test performance than their male peers (Zander et al., 2020).

One way to circumvent biases in self-reports would be to use direct observations. In the context of home math engagement, limited research exists that explicitly observes parents' conversations about math with their toddlers, and the way number-related discussions or patterns of speech may differ between mothers and fathers. Recently, Mix et al. (2024) were the first to longitudinally examine the relations between both mothers' and fathers' use of number talk in its frequency and complexity with their nine-month-old children with their children's number skills during preschool. Broadly, they found that mothers used more diverse number talk, and more abstract counting, equivalence/non-equivalence, and one-to-one correspondence, while only fathers' frequency of number talk was associated with their child's number skills. The current study aimed to examine whether similar unique contributions may arise concurrently in toddler-aged children using direct observation.

1.5 The Current Study

This project aimed to investigate the joint and unique relations of mothers and fathers on their toddlers' number knowledge, by comparing the frequency and type of number talk mothers and fathers use with their toddlers during semi-structured interactions, and how these may predict children's number skills. Specifically, the following project aims were be addressed:

1) In Aim 1, I examine whether mothers and fathers differ in their *overall frequency of number talk* to toddlers.
2) In Aim 2, I ask whether mothers and fathers differ in the *types of number talk* used with their toddlers.
3) In Aim 3, I ask whether there are unique relations between mothers' and fathers' overall frequency of number talk and their child's number skills.

2.0 Method

2.1 Participants

Participants for this study participated in a larger project investigating mothers' and fathers' influences on toddlers' math learning. Initially, to be eligible to participate in the larger project, participants had to speak only English and/or Spanish in the home and identify as either non-Hispanic/Latino and White, or Hispanic/Latino and White or another race. At later stages of recruitment, participants who spoke an additional language other than English or identified as another race were allowed to participate to increase the overall sample size of the study. Regardless, to participate, families were required to reside in the United States, and both parents had to be above 18 years of age and agree to participate in the study. Children were required to be between 24 and 36 months at the time of testing, born full term, and not have experienced any health or developmental problems since birth. In some cases, although families met the eligibility criteria, only one parent completed the study. Children who only had one participating parent were excluded from the current sample, such that each participating family for this study included the toddler and their biological mother and father. Thus, the final sample included 133 toddlers, between 24 and 36 months of age (M = 30.8 months, SD = 3.46 months) and their parents. Children's gender was reported by parents (53% female). Mothers reported their race and ethnicity as White, non-Hispanic/Latino (n = 112), Hispanic or Latino (n = 12), White and Hispanic/Latino (n = 5), Asian (n = 2), White and another race (n = 1), and another race, Hispanic/Latino (n = 1). Fathers reported their race and ethnicity as White, non-Hispanic/Latino (n = 113), Hispanic or Latino (n = 12), Asian (n = 2), Black or African American (n = 1), Native Hawaiian or other Pacific

Islander (n = 1), White and Asian (n = 1), White and Hispanic/Latino (n = 1), White and another race (n = 1), and multiple races and Hispanic/Latino (n = 1). During the completion of tasks, 89% of children used only English, 8% used both English and Spanish, and 4% only used Spanish. Parents' level of education ranged from having a high school degree to a graduate degree, and most parents were highly educated; 91% of mothers had at least a bachelor's degree and 83% of fathers had at least a bachelor's degree. An additional three families participated in the study but were excluded from analyses due to children being three months too young (n = 1) or parents' number talk being extreme outliers (+/- 3 SD from the mean; n = 2). All parents signed an informed consent form for participation, and each parent was compensated $50 for their time. Participants were not told that this study focused specifically on math, rather broadly related to parental support of children's early learning.

2.2 Procedure

Families were recruited primarily from three mid-Atlantic cities, and each child participated in two virtual study visits (one with each parent) where observations were collected of each parent-child dyad as well as measures of children's number knowledge. Parent-child interactions were observed via Zoom, as each parent-child dyad was provided with two images to discuss (one picture of an outdoor scene and one picture of an indoor scene) for as long as they desired. Children viewed each of the four images only once (Fig. 1), and each image and the order of their presentation was counterbalanced across dyads. They were given no prompt to induce math talk specifically and were only told to discuss the images as if they were pictures in a book for as long as they desired. To measure children's number skills, children also completed a Point-to-X and Give-N task after the parent-child interaction (see below for details). In addition to these measures of interest, parents also completed an assessment of their math skills and completed questionnaires reporting their home engagement with toddlers, and toddlers completed assessments of their spatial skills, but these data were not used in the current study.

2.3 Measures

2.3.1 Parental Number Talk

Parent-child interactions were recorded, and videos were transcribed verbatim at the utterance level by research assistants. An utterance was defined as a sequence of words separated via grammatical closure, intonation contour, or prolonged pausing, which can function as a natural completion of a thought or sentence (Rowe, 2012). After the first research assistant completed the transcriptions, a second research assistant checked all transcriptions for typos and any other errors. Transcripts were then searched for words that could constitute math language, and the type of math language was coded. This coding was then checked by a second research assistant. All words categorized as number talk were further categorized based on how the number talk was used in that utterance (Appendix A). Specifically, each utterance containing number talk was coded as: 1) using number symbols (when Arabic numerals or number words are used), 2) counting, 3) referencing or labeling set sizes, 4) ordinal relations (discussion of the order of numbers), 5) patterns, 6) comparing magnitudes of discrete quantities, 7) arithmetic operations, 8) quantifiers (referring to the approximate amount of discrete objects), and 9) other (instances of number talk that do not fall into one of the other categories). Utterances containing multiple types of number talk were coded for each category present in order to account for the full content and complexity of the utterance. For example, if a child replied to a counting prompt with “one two three butterflies,” this was coded as both counting and labeling a set size. Approximately twelve percent of the videos were independently coded by a second trained research assistant. Overall reliability was high, averaging agreement of 91.76% and $\kappa = .97$.

Parents’ overall number talk frequency was tallied as the total number of utterances in which parents used number talk. We also tallied the frequency of parents’ use of each category of number talk (total number of utterances using number symbols, using counting, etc.). Finally, to account for parents’ overall amount of talking during interactions, we tallied their total number of utterances used.

2.3.2 Child Number Skills

Children completed two number-skill assessments, and scores from the tasks were averaged to create a composite score of children’s number skills.

2.3.2.1 Give-N

The Give-N task (Wynn 1990, 1992; Silver et al, 2023) assesses children’s exact cardinal number knowledge by asking children to produce a specific number of objects. Parents supplied children with a plate and a set of six spoons. Researchers drew the child’s attention to an animal puppet named Ellie that they were told was hungry for ice cream. Children were asked to give the researcher’s animal puppet different numbers of spoons (e.g., “Can you give Ellie three scoops of ice cream?”), across eight trials. Children were asked to create sets ranging from one to four, twice each, in a pseudo-randomized order. Once children were prompted, they placed their selected spoons onto the plate in front of them. Once children finished adding spoons to the plate, researchers asked “Is that [number]?” If children indicated “yes”, the researcher held the puppet to the webcam and said “Oh good! Yum yum yum!” If children indicated “no”, the researcher stated, “Well Ellie wanted [number] scoops. Can you give Ellie [number] spoons?” Children were allowed to update their response once, and upon doing so, the experimenter held the puppet to the webcam and said “Oh good! Yum yum yum!” Responses were coded for accuracy based on the number of spoons placed on the plate after children confirmed their response, and the percentage of correct responses was recorded. Videos of children’s participation were coded by trained researchers to identify the number of spoons children produced for each trial. A second researcher coded 20% of videos to ensure reliability; and inter-coder reliability was excellent. Agreement for each trial ranged from 94.87% to 100% (κ's from 0.72 to 1.00).

2.3.2.2 Point-to-X

The Point-to-X task (Silver et al. 2021, 2023) assesses children’s number word comprehension. During this task, children viewed two images on their screen and were asked to point to one. In two practice trials, children viewed common objects, and were prompted to point to one (e.g., “Which has a ball?”). In twelve number-word trials, children were then shown a series of images containing two sets of identical stimuli, differing only in number (e.g., five pears and three pears), and were asked to point to one of the images (e.g., “Which has five pears?”). Responses were coded based on accuracy, and the percentage of correct responses was recorded. Videos of children’s participation were coded by trained research assistants to identify which image children pointed to for each trial. A second researcher double-coded 20% of videos to assess inter-coder reliability. Inter-coder reliability was excellent; agreement for each trial ranged from 90.24% to 100% (κ's from 0.81 to 1.00).

2.3.3 Family Demographics

Family demographics were collected via a parent questionnaire including their race and ethnicity. Parents also reported their child's preferred language of task administration, which we dichotomized as English-only (1) or Spanish-only or a combination of English and Spanish (0). Parents also reported their highest level of education, which we dichotomized as having at least a college degree (1) or less than a college degree (0). Additionally, parents reported the gender of their child, which we dummy coded to reflect female (1) or male (0). Finally, parents reported their child's birthdate, which we used to calculate children's age in months at the date of the first testing session.

3.0 Data Analytic Plan

First, I conducted a t-test to examine whether mothers and fathers differ in their overall talk (amount of total utterances regardless of whether they pertain to number concepts or not) to decide whether it was necessary to consider proportions of parents' number talk out of overall talk, for each research question, or raw frequencies.

I then examined whether mothers and fathers differed in their overall number talk (Aim 1). If mothers and fathers significantly differed in their overall talk, I planned to complete a t-test, examining differences in mothers' and fathers' frequency of number talk to their toddlers as a proportion of their total talk. If they did not significantly differ in their overall talk, I planned to conduct this analysis examining the raw counts of mothers' and fathers' frequency of overall number talk. Due to mixed reports of mothers' and fathers' home math engagement with their children (Silver et al., 2023, del Rio et al., 2017, 2019), it is unclear whether mothers or fathers may use a higher frequency of number talk or whether mothers and fathers will not differ.

To examine whether mothers and fathers differ in the *types of number talk* used with their toddlers (Aim 2), I first examined patterns of most frequent number talk categories used by mothers and fathers, as either a proportion of each number category out of total talk (if mothers and fathers significantly differ in overall talk), or as a proportion of each number category out of total number talk (if mothers and fathers do not significantly differ in overall talk). Descriptively, I looked at which category/categories is/are used most frequently by mothers and fathers. Then, I compared the overall amount of each category's use between parents using a MANOVA, using parent gender as the independent variable and number talk categories as the dependent variables. Due to the novelty of examining specific number talk category usage between parents, it was unclear which types of number talk parents use most frequently with their toddlers and how mothers and fathers may differ in their use of different types of number talk.

Finally, to test whether there are unique relations between mothers' and fathers' overall frequency of number talk and their child's number skills (Aim 3), we ran a step-wise regression model to examine how the frequency of mothers' and fathers' overall number talk frequency relates to children's number skills above and beyond a number of covariates that frequently relate to children's math skills. Specifically, I examined whether mothers' and fathers' frequency of number talk predict additional variance in children's number knowledge composite score, above and beyond parents' overall talk (total number of utterances), and demographic variables, including child age, child gender, language of task interaction, and mothers' and fathers' level of education. In the first step of these analyses, I always added baseline covariates. In a first model, I added mothers' number talk in the second step to test whether mothers' number talk accounts for significantly more variance in their child's number skills above that of baseline covariates. In a second model, I again started with baseline covariates in the first step, and then added fathers' number talk in the second step. If there is a main effect of either parent's number talk on children's number skills above and beyond the covariates, I would then run an additional model, adding covariates to the first step, and then both parents' number talk simultaneously to the second. Doing this would allow me to examine whether the unique contributions of each parent hold when controlling for the effects of the other. I hypothesized that parents' use of number talk will positively predict their children's number skills above and beyond parents' overall talk and demographic covariates (Daucourt et al., 2021; Silver et al., in press), but I do not have specific hypotheses about whether mothers' and fathers' number talk will uniquely predict children's number skills.

4.0 Results

4.1 Mothers' and Fathers' Overall Number Talk Frequency (Aim 1)

First, we tested whether mothers and fathers differ in their overall talk, as a count of total utterances, in order to decide whether it was necessary to use either raw number talk frequencies or proportions of number talk for subsequent analyses. We found that mothers' total length of interaction in minutes (M = 7.25 min, SD = 3.88) was significantly longer than that of fathers' (M = 6.35 min, SD = 3.25), $t(132) = 2.54$, $p = .012$. In addition, mothers' overall number of utterances (M = 159.89, SD = 90.54) was significantly greater than that of fathers (M = 135.74, SD = 70.05), $t(132) = 3.02$, $p = 0.003$. Thus, the proportion of number talk out of overall talk for each parent was calculated for all further analyses.

We first examined mothers' and fathers' frequency of number talk as a proportion of their total talk (Table 1). Both mothers and fathers varied considerably in their number concepts used per utterance, with mothers using slightly more number talk compared to fathers, $t(132) = 1.92$, $p = 0.057$.

Table 1.

Parents' number talk frequencies

Variable	***M***	***SD***	**Min**	**Max**
Mothers' overall talk (raw frequency)	159.89	90.54	39	560
Fathers' overall talk (raw frequency)	135.74	70.05	29	390
Mothers' number talk (raw frequency)	22.93	17.32	0	85
Fathers' number talk (raw frequency)	17.76	14.72	0	69
Mothers' number talk (proportion of total utterances)	.14	.07	0	.37
Fathers' number talk (proportion of total utterances)	.12	.08	0	.33

4.2 Mothers' and Fathers' Type of Number Talk

We then examined parents' use of the different categories of number talk. Table 2 contains descriptive statistics for the most frequent number talk categories used by mothers and fathers. We found that both mothers and fathers most frequently used quantifiers, followed by labeling, and counting with their children. Next, we used a MANOVA to compare whether mothers and fathers differed significantly in their usage of the different number talk categories. Since use of arithmetic, patterns, and ordinal relations were very infrequent amongst parents, these frequencies were combined with the other category. The MANOVA was significant, F(6.0, 259.0) = 2.87, p = .010, suggesting that mothers and fathers differed in their use of number talk categories broadly. Thus, we performed a series of exploratory analyses to compare the frequencies of each number talk category between mothers and fathers. The categories of number talk were not normally distributed, so we employed Wilcoxon signed rank tests. Mothers counted significantly more often than fathers, z = 2.91, *p* = .004, used significantly more ordinal relations, z = 3.16, *p* = .002, more comparisons, z = 2.98, *p* = .003, and more other number talk, z = 1.97, *p* = .049. Parents did not significantly differ in any other category.

Table 2.
Proportions of number talk category use

	Mothers		Fathers		Difference between parents
Number talk category	***M***	***SD***	***M***	***SD***	***z***
Number symbol	.002	.006	.003	.01	1.18
Counting	.04	.04	.02	.04	2.91**
Labeling	.05	.03	.04	.04	0.83
Ordinal relations	.0006	.003	0.00	0.00	3.16**
Pattern	0.00	0.00	.0003	.004	-1.00
Comparing magnitudes	.001	.003	.0004	.002	2.98**
Arithmetic	.0001	.001	.0004	.002	-1.15
Quantifier	.05	.03	.05	.04	0.25
Other	.002	.006	.001	.004	1.97*
Other, arithmetic, patterns, ordinal relations	.003	.006	.002	.006	2.02*

* *p* < .05, ** *p* < .01

4.3 Associations Between Parents' Number Talk Frequency and Children's Number Skills (Aim 3)

Finally, we tested whether there were unique relations between mothers' and fathers' overall frequency of number talk and their child's number skills. Our sample had considerable variability in toddlers' number skills (M = .51, SD = .20), and we were curious whether parents' use of number talk would predict these individual differences. We first ran pairwise correlations and found that fathers' overall proportion of number talk was significantly correlated with children's number skills, $r = .17$, $p = 0.045$, but mothers' number talk was not, $r = .03$, $p = .737$. Next, we examined whether these relations would hold above and beyond several relevant covariates (Table 3).

In the first model, examining the relation between mothers' number talk and their child's number skills above that of baseline covariates, we found no significant main effect of mothers' proportion of number talk. We did, however, observe significant effects for language of task administration and child age, such that children administered tasks in English and children who were older tended to perform better in the number skills assessments.

We then turned to examining these relations for fathers. In the second model, fathers' proportion of number talk was only marginally significantly related to their child's number skills ($p = .100$), while we found significant main effects of language and child age (such that children administered tasks in English and children who were older tended to perform better). We also observed a marginally significant main effect of fathers' level of education, such that toddlers whose fathers had earned at least a Bachelor's degree tended to perform better in the number skills assessments than toddlers whose fathers did not complete a Bachelor's. Because there was no significant main effect of either parent's number talk on children's number skills above and beyond the covariates, the proposed additional model testing parents' unique contributions was not needed.

Table 3.

Regression analysis predicting children's number knowledge

Variable	Model 1 Baseline: Mothers	Model 1: Mothers	Model 2 Baseline: Fathers	Model 2: Fathers
	B (SE)	*B (SE)*	*B (SE)*	*B (SE)*
Proportion of number talk	-	0.12 (0.21)	-	0.33 (0.20) †
Child age	0.02 (0.00) ***	0.02 (0.00) ***	0.02 (0.00) ***	0.02 (0.00) ***
Child gender	0.02 (0.03)	0.01 (0.03)	0.01 (0.03)	0.01 (0.03)
Language of task administration	0.24 (0.05) ***	0.25 (0.05) ***	0.25 (0.05) ***	0.24 (0.05) ***
Parent level of education	0.02 (0.05)	0.02 (0.05)	0.07 (0.04) †	0.08 (0.04) †
Constant	-0.34 (0.15) *	-0.36 (0.16) *	-0.39 (0.15) **	-0.41 (0.15) **
Model R^2	.28	.28	.29	.31
Model F	$F(4, 128) = 12.25$ ***	$F(5, 127) = 9.82$ ***	$F(4, 128) = 13.36$ ***	$F(5, 127) = 11.38$ ***

† *p < .10, * p < .05, ** p < .01, *** p < .001*

4.4 Exploratory Analysis

As mothers and fathers tended to use similar categories of number talk most frequently, we were interested in examining whether use of these specific categories would pose unique contributions to children's number knowledge. Using pairwise correlations, we found that mothers' use of counting was significantly, but negatively, related to toddlers' number skills (r = -0.18, p = .037) and fathers' use of labeling set sizes was positively correlated with children's number skills (r = .22, p = .011). While the association between mothers' counting and toddlers' number skills did not hold in regression above controls for covariates, we found that fathers' use of labeling continued to be a significant predictor of toddlers' number skills even when controlling for children's age, gender, language of task administration, and fathers' education (p = .010) and adding fathers' labeling to the baseline model explained significantly more variance in toddlers' number skill, (change in R^2 = .04, F (1, 127) = 6.85, p = .010).

5.0 Discussion

The current study aimed to investigate the various contributions of mothers and fathers for their toddlers' number knowledge, by comparing the frequency and type of number talk mothers and fathers use with their toddlers during observed interactions, and how these may predict children's number skills. Our results indicate that both mothers and fathers use a considerable amount of number talk, while mothers tend to use slightly more number talk compared to fathers. In addition, mothers and fathers frequently used similar categories of number talk, favoring use of quantifiers, labeling set sizes, and counting. We examined whether each parent's use of number talk relates to children's number skills, and unexpectedly found no main effects for either mothers or fathers above key baseline covariates. These findings add to our understanding of the home math environment and add important nuance to prior findings.

5.1 Considerable Variability in the Frequency of Number Talk Across Parents

Prior research examining parents' number talk frequency finds differences in parents' use of number talk, likely due to the observational contexts. During parent-child interactions designed specifically to elicit math talk, Ramani et al. (2015) and Mix et al. (2024) found number talk proportions ranging on average from 19 to 43%, depending on strictly numerical or non-number inclusion of quantitative codes. In our semi-structured parent-child interaction task, we find less frequent number talk (12 to 14% number concepts per utterance), but this is expected due to the lack of specific prompts to induce number talk. Entirely naturalistic interactions have yielded significantly less parent number talk overall, ranging on average from 0.5 to 1.3% of word tokens or instances of total utterances (Gunderson & Levine, 2011; Levine et al., 2010).

Here, we find that although mothers use more number talk overall, mothers and fathers of two- to three-year-olds used relatively similar proportions of number talk when accounting for the total amount of utterances they used during interactions with their toddlers. This aligns with prior findings from Mix et al. (2024), who were the first to observe fathers' number-related parent-child interactions with their infants. They observed similar frequency and complexity of number talk utterances between mothers and fathers, but more diverse number talk utterances across an entire play session in mothers compared to fathers. While we coded for every instance of number talk throughout each parent-child interaction, the study by Mix et al. (2024) instead coded interactions in 30-second intervals, using three different measures of number talk (i.e., frequency, complexity, and diversity). Our results align with their frequency measures, during which mothers and fathers used similar amounts and types of number talk. Future work may account for a measure of timing and examine how complexity and diversity of numerical categories vary across intervals within each interaction.

5.2 Number Talk Categories Across Mothers and Fathers

Prior work establishes some variability in parents' most frequently used types of number talk with young children. In one study with their nine-month-old children, parents most often used non-numerical quantitative terms, one-to-one correspondence, cardinality, and counting, while in a study of 14- to 30-month-olds, parents most frequently used cardinality, particularly of small sets, and counting (Mix et al., 2024; Gunderson & Levine, 2011; Levine et al., 2010). In another study with three- to five-year old children, parents tended to most frequently identify numerals and use cardinality (Ramani et al., 2015). The current study examined parents' number talk with two- to three-year-old toddlers, and our findings closely align with the previous studies. In our sample, parents used a range of each evaluated number talk category but continued to favor use of similar categories of number talk including quantifiers, labeling set sizes (cardinality), and counting.

Critically, this finding remains similar between mothers and fathers, suggesting that children receive relatively consistent numerical input at this age (at least in contexts similar to the shared picture discussion), as they begin to master important skills including the stable order principle and one-to-one correspondence. Moreover, when comparing categorical use between mothers and fathers, mothers tended to use more counting, ordinal relations, comparison of magnitudes, and other forms of number talk. Similarly, Mix et al. (2024) found that mothers used more abstract counting and equivalence/non-equivalence (akin to magnitude comparison), suggesting that mothers' number talk diversity was driven specifically by their use of counting and comparisons. Interestingly, in the current sample fathers did not provide significantly more input in any number talk category, implying some variability within parent-child interactions between parents, despite use of similar categories most frequently.

5.3 Associations of Parents' Number Talk and Toddlers' Number Skills

Despite considerable use of number talk overall, we found no significant main effects of parents' overall number talk on children's numeracy scores. This is unexpected given findings from prior work indicating that parents' number talk positively predicted children's number knowledge longitudinally (Gunderson & Levine, 2011; Levine et al., 2010; Mix et al., 2024).

Importantly, Mix et al. (2024) only observed this association amongst fathers' (but not mothers') number talk frequency and children's numeracy scores. This aligns with the observed marginal effect of fathers' number talk on children's numeracy skills, and particularly in their use of labeling. While this suggests that parents may uniquely contribute to their child's number knowledge, the latter finding should be interpreted with caution due to the extent of our analytic comparisons. Future work may test for differences in cardinality-related talk specifically in order to examine potential unique effects between mothers and fathers.

Moreover, one reason for our null findings may be the concurrent measurement of parents' number talk and children's numeracy skills. It is possible that variations in numeracy input over time may accrue and thus only yield associations with children's skills later in development. Ramani et al. (2015) also examined preschool-aged children's numeracy skills concurrently and found significant positive predictors specifically in only parents' advanced number talk with their child's advanced number skills (e.g., cardinal relations and ordinality). Since the children in our sample were younger, what constitutes advanced number concepts may not be the same as for preschoolers. Instead, it is possible that numbers above a certain threshold may be considered advanced number talk and this threshold may vary between children. Alternatively, as we did not include testing measures for other foundational numeracy concepts, it is possible that other number skills not measured here would more closely align with these types of discussions. Future work should account for this by including additional measures of children's foundational and advanced numeracy skills.

In addition, children's frequency and categorical use of number talk should also be considered. For example, in the aforementioned study, Ramani et al. (2015) found that both children's foundational and advanced number talk related to their respective number skills. This was also corroborated longitudinally by children's overall number talk and their later cardinal number skills (Levine et al., 2010). Thus, our results may also be implicated in children's responsiveness and/or their spontaneous attention to number concepts within presented stimuli.

5.4 Limitations, Future Directions, and Conclusions

There are several limitations to consider regarding this study. The generalizability of the present findings remain limited as this study was conducted with a largely White, non-Hispanic/Latino, highly educated, and mostly English-speaking population. In addition, participants were only included in the study if both the child's mother and father completed all procedural elements. Future research should account for these participatory differences and more heavily consider factors such as ethnicity, parent education, and SES and their effects on findings. We particularly found significant language effects in relation to children's number knowledge, indicating a need to include more equivalent representation of English and Spanish speakers in order to make any meaningful comparisons related to linguistic differences in number talk quality (e.g., there were very few children who did not receive the tasks in English, making it difficult to draw conclusions about these significant differences).

Moreover, these findings do not relate to any causal conclusions, as correlational data was employed. Prior work finds that a picture book intervention increasing parental number talk, particularly related to small numbers, improved children's number knowledge (Gibson, Gunderson, & Levine, 2020). This suggests causal associations between the variables of interest in our study but is impossible to conclude from our data. Future work may expand on these types of interventions to target other forms of number talk amongst parents, particularly examining both concurrent relations and longitudinal effects in children.

Importantly, this study only included measures of children's cardinal number knowledge, formed as a composite between their Point-to-X and Give-N scores. Future work may include additional measures of children's number knowledge, particularly examining children's other foundational number concepts in order to attain a broader sense of the role parental number talk plays at these early ages.

Future work may also build on this study to examine parental and child variations in early number talk based on child gender. While we found no effects of child gender on their numeracy performance, as early as 5 to 7 years old, children begin to associate math more strongly with males as compared to females; this likely contributes to the higher rates of math anxiety and lower math concept reported by girls beginning in elementary school, despite holding comparable test scores (Cvencek et al., 2011; Maloney & Beilock, 2012; Hildebrand et. al., 2022). Previous work suggests that parents' math-gender associations seem to correlate with both their sons' and daughters' beliefs even prior to children entering school, and it has been found that by the end of kindergarten, boys seem to begin scoring higher in number skills compared to their female counterparts (Hildebrand et. al., 2022; Jordan et. al., 2006; LeFevre et. al., 2009). While the effects of early cognitive stimulation from mothers and fathers on children's math scores hold even when controlling for child gender (Cook et al., 2011), it is unclear how each parent-child interaction incorporates number talk (i.e., whether parents talk differently about number with their sons and daughters), and how children's gender may predict children's early number knowledge, which may disproportionately affect girls prior to entering formal school (Hildebrand et. al., 2022). Examining children's number talk broadly, and in relation to their (and parent) gender, is an important avenue of further research, as this may differentially contribute to the development of their number knowledge early on.

Overall, this study presents novel findings in how mothers and fathers compare in their frequency and categorical use of number talk with their toddlers. The results indicate that parents use fairly similar amounts and types of number talk with their young children, and more work is needed to understand how this may contribute to their child's number skills broadly.

6.0 Figures

6.1 Figure 1

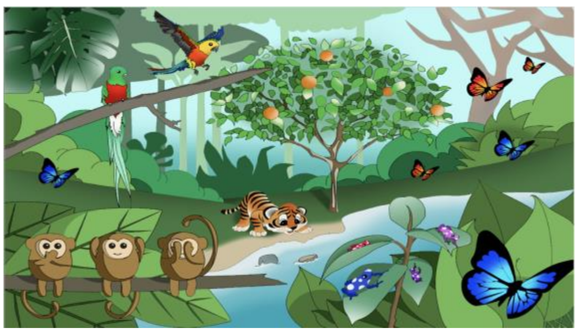
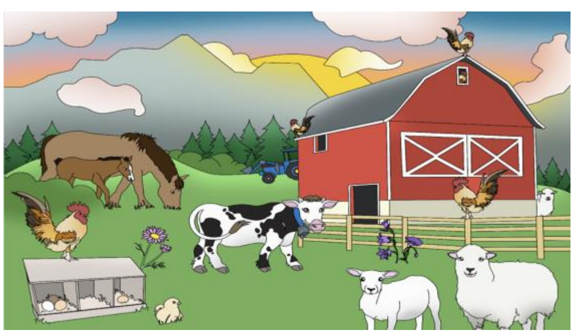
Outdoor	Indoor
Jungle Image 	*Kitchen Image* 
Barnyard Image 	*Living Room Image* 

Figure 1 Images discussed by parent-child dyads

Appendix A

Appendix A.1 Number Talk Pragmatics Coding Manual

Run the “EDL2_insert_numbertalk_categories.rb” script on the Checked Category Coding datavyu file. Now save the file as “ID_Task_Categories_Checked_Number”

To code number talk, you will answer a series of questions for each cell.

First:

Should this cell have even been pulled out by the automated script?

Here is the list of categories pulled by the batch script that may contain number talk:

Functions: count, add, subtract, multiply, divide, sum, take away, plus, minus, times, [how] many, [how] much, where

Ordinals: once, twice, first, second, third, single, double, next, last, before, after

Magnitude and Comparison: big, little, small, large, tiny, miniscule, enormous, huge, gigantic, giant, teeny, itsy-bitsy, itty-bitty, long, short, tall, wide, narrow, thick, thin, skinny, fat, deep, shallow, full, empty, filled up (adjective), whole, all, both, part, bit, more, less, most, least, a lot, a little, much, many, enough, some, each, every, half, third, quarter, fifth, none, inch, foot, mile, centimeter, equal, same, fragment, fraction, piece, section, segment, and all alternate forms of the these words (i.e. -er / -est)

Numbers: zero, one, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve, -teen, -twenty, -thirty, -forty, -fifty, -sixty, -seventy, -eighty, -ninety, -hundred, -thousand, -million, plural forms of numbers

Superordinates: size, length, height, width, depth, volume, capacity, area, measure, location, position, direction, distance, orientation, amount, room, space, pattern, design, sequence, order, number, shape, time

For all cells that may contain number talk, you will go through and categorize the type of number talk present in the "numbertalk_category" column.

Note: Some utterances may contain multiple words that could be number talk, and so may generate multiple cells for number talk coding. <u>Code all of the number talk categories for the utterance in the first generated cell and delete the redundant cells</u>. Make sure that you look out for instances where two people were talking at the same time before you delete redundancies.

Appendix A.1.1 How was the number talk in this utterance used?

Now we are going to move to categorizing at the utterance level.

Number talk can be used in multiple different ways and, critically, the same number word can be used in different ways depending on the rest of the utterance (e.g., "let's count to four," "I have four more than you," "what does the number four look like," and "how much is four plus three" all included the number word "four" but use it in different ways).

At the utterance level:

Go through each identified number utterance and fill in the presence or absence of each category of number talk. If the utterance contains an instance of that category, enter y in that code. If it does not contain an instance of that category, enter n.

Categories:

<symbol> (Number symbol)

<counting> (Counting)

<labeling> (Labeling set sizes)

<ord_rel> (Ordinal relations)

<pattern> (Patterns)

<comp_mag> (Comparing magnitudes)

<arithmetic> (Arithmetic)

<quantifier> (Non-Spatial Quantifiers)

<other> (Other)

<notnumber> (Non-numerical)

Appendix Table 1 Summary of Number Talk Pragmatics Codes

<table>
<tr><th></th><th></th><th>Variable</th><th>Code</th><th>Definition</th></tr>
<tr><td rowspan="10">How is the number talk used in this context?</td><td>1</td><td>Number Symbols</td><td><symbol></td><td>Number symbols occur any time the parent or child uses Arabic numerals or number words</td></tr>
<tr><td>2</td><td>Counting</td><td><counting></td><td>Counting occurs whenever an utterance involves reciting or referencing the count list</td></tr>
<tr><td>3</td><td>Labeling Set Sizes</td><td><labeling></td><td>Labeling Set Sizes occurs whenever the speaker is referencing cardinal values, including labeling values or asking about values</td></tr>
<tr><td>4</td><td>Ordinal Relations</td><td><ord_rel></td><td>Ordinal Relations include discussion of the order of numbers</td></tr>
<tr><td>5</td><td>Patterns</td><td><pattern></td><td>Patterns include utterances that identify common repeating elements of objects or people using numerical information</td></tr>
<tr><td>6</td><td>Comparing Magnitudes</td><td><comp_mag></td><td>Comparing Magnitudes occurs when the speaker is describing or identifying a numerical match or mismatch between two or more discrete quantities</td></tr>
<tr><td>7</td><td>Arithmetic</td><td><arithmetic></td><td>Arithmetic refers to any utterance referencing or requiring the use of operations, such as adding or subtracting</td></tr>
<tr><td>8</td><td>Quantifiers</td><td><quantifier></td><td>Quantifiers refer to any utterance where the approximate amount of discrete, countable objects is being discussed</td></tr>
<tr><td>9</td><td>Other</td><td><other></td><td>Other should be coded for any other use of number talk that is mathematical but does not fall into the above categories</td></tr>
<tr><td>10</td><td>Non-Numerical</td><td><notnumber></td><td>Utterances that that in one context might convey numerical concepts but in the current context do not convey numerical concepts</td></tr>
</table>

Bibliography

Agarwal, S., & Mazumder, B. (2013). Cognitive Abilities and Household Financial Decision Making. American Economic Journal: Applied Economics, 5(1), 193–207.

Baker, C.E. (2017). Father-Son Relationships in Ethnically Diverse Families: Links To Boys' Cognitive and Social Emotional Development in Preschool. *J Child Fam Stud*, 2335–2345. https://doi.org/10.1007/s10826-017-0743-3

Baker, C.E. (2018) When daddy comes to school: father–school involvement and children's academic and social–emotional skills, Early Child Development and Care, 188:2, 208-219, DOI: 10.1080/03004430.2016.1211118

Baker C. E., Kainz K. L., Reynolds E. R. (2018). Family poverty, family processes and children's preschool achievement: understanding the unique role of fathers. *J. Child Fam. Stud.* 27, 1242–1251. 10.1007/s10826-017-0947-6

Coley R. L., Lewin-Bizan S., Carrano J. (2011). Does early paternal parenting promote low-income children's long-term cognitive skills? *J. Fam. Issues* 32, 1522–1542. 10.1177/0192513X11402175

Cook, G.A., Roggman, L.A. & Boyce, L.K. (2011) Fathers' and mothers' cognitive stimulation in early play with toddlers: Predictors of 5th grade reading and math, Family Science, 2:2, 131-145, DOI: 10.1080/19424620.2011.640559

Currie, J., & Thomas, D. (1999). Early test scores, socioeconomic status and future outcomes (NBER Working Paper No. 6943). Cambridge, MA: National Bureau of Economic Research.

Cvencek, D., Meltzof, A. N., & Greenwald, A. G. Math–gender stereotypes in elementary school children. Child Development, 2011, 82(3): 766–779.

Daucourt, M. C., Napoli, A. R., Quinn, J. M., Wood, S. G., & Hart, S. A. (2021). The home math environment and math achievement: A meta-analysis. Psychological Bulletin, 147(6), 565.

del Río, M. F., Strasser, K., Cvencek, D., Susperreguy, M. I., and Meltzoff, A. N. (2019). Chilean kindergarten children's beliefs about mathematics: family matters. *Dev. Psychol.* 55, 687–702. doi: 10.1037/dev0000658

del Río, M. F., Susperreguy, M. I., Strasser, K., and Salinas, V. (2017). Distinct influences of mothers and fathers on kindergartners' numeracy performance: the role of math anxiety, home numeracy practices, and numeracy expectations. *Early Educ. Dev.* 28, 939–955. doi: 10.1080/10409289.2017.1331662

De Smedt, B. (2020). Sources of variability in mathematical development. *Educational neuroscience: Development across the life span* (pp. 169–194). https://doi.org/10.4324/9781003016830-10

Duncan, G. J., Dowsett, C. J., Claessens, A., Magnuson, K., Huston, A. C., Klebanov, P., et al. (2007). School readiness and later achievement. *Dev. Psychol.* 43, 1428–1446. doi: 10.1037/0012-1649.43.6.1428

Eason, S. H., Scalise, N. R., Berkowitz, T., Ramani, G. B., & Levine, S. C. (2022). Widening the lens of family math engagement: a conceptual framework and systematic review. Developmental Review, 66, 101046.

Foster, T.D., Froyen, L.C., Skibbe, L.E. *et al.* Fathers' and mothers' home learning environments and children's early academic outcomes. *Read Writ* 29, 1845–1863 (2016). https://doi.org/10.1007/s11145-016-9655-7

Gibson, D. J., Gunderson, E. A., & Levine, S. C. (2020). Causal effects of parent number talk on preschoolers' number knowledge. Child Development, 91(6), e1162–e1177. doi:10.1111/cdev.13423

Gunderson, E. A., and Levine, S. C. (2011). Some types of parent number talk count more than others: relations between parents' input and children's cardinal-number knowledge. *Dev. Sci.* 14, 1021–1032. doi: 10.1111/ j.1467-7687.2011.01050.x

Hildebrand L., Posid T., Moss-Racusin C.A., Hymes L., Cordes S. (2022). Does my daughter like math? Relations between parent and child math attitudes and beliefs. *Dev Sci*. 26(1):e13243. doi: 10.1111/desc.13243.

Jordan, N. C., Kaplan, D., Ramineni, C., & Locuniak, M. N. (2009). Early math matters: Kindergarten number competence and later mathematics outcomes. *Developmental Psychology*, *45*(3), 850–867. https://doi.org/10.1037/a0014939

LeFevre, J.-A., Skwarchuk, S.-L., Smith-Chant, B. L., Fast, L., Kamawar, D., & Bisanz, J. (2009). Home numeracy experiences and children's math performance in the early school years. *Canadian Journal of Behavioural Science/ Revue Canadienne Des Sciences Du Comportement*, *41*, 55–66. doi:10.1037/a0014532

Levine, S. C., Suriyakham, L. W., Rowe, M. L., Huttenlocher, J., & Gunderson, E. A. (2010). What counts in the development of young children's number knowledge?. *Developmental psychology*, *46*(5), 1309–1319. https://doi.org/10.1037/a0019671

Leyva D, Libertus ME, McGregor R. (2021). Relations between Subdomains of Home Math Activities and Corresponding Math Skills in 4-Year-Old Children. *Education Sciences*. 11(10):594. https://doi.org/10.3390/educsci11100594

Libertus, M. E., Duong, S., & Silver, A. M. (2020). Mathematical cognition. In J. B. Benson (Ed.), Encyclopedia of infant and early childhood development (Vol. 2, 2nd ed., pp. 311–318). Elsevier.

Maloney, E. A., & Beilock, S. L. (2012). Math anxiety: Who has it, why it develops, and how to guard against it. *Trends in Cognitive Sciences*, *16*(8), 404–406. https://doi.org/10.1016/j.tics.2012.06.008

Mix, K.S., et al. (2024). Number Input in Mothers and Fathers of 9-Month-Olds, Journal of Cognition and Development, DOI: 10.1080/15248372.2024.2303760

Napoli, A. R., & Purpura, D. J. (2018). The home literacy and numeracy environment in preschool: Cross-domain relations of parent–child practices and child outcomes. Journal of Experimental Child Psychology, 166, 581–603. https://doi.org/10.1016/ j.jecp.2017.10.002.

Nguyen, T., Watts, T. W., Duncan, G. J., Clements, D. H., Sarama, J. S., Wolfe, C., & Spitler, M. E. (2016). Which Preschool Mathematics Competencies Are Most Predictive of Fifth Grade Achievement?. *Early childhood research quarterly*, *36*, 550–560. https://doi.org/10.1016/j.ecresq.2016.02.003

Niklas, F., Cohrssen, C., & Tayler, C. (2016a). Improving preschoolers' numerical abilities by enhancing the home numeracy environment. Early Education and Development, 27(3), 372–383. https://doi.org/10.1080/10409289.2015.1076676.

Ramani, G. B., Rowe, M. L., Eason, S. H., & Leech, K. A. (2015). Math talk during informal learning activities in head start families. *Cognitive Development*, *35*, 15–33. https://doi.org/10.1016/j.cogdev.2014.11.002

Reyna, V. F., & Brainerd, C. J. (2007). The importance of mathematics in health and human judgment: Numeracy, risk communication, and medical decision making. Learning and Individual Differences, 17(2), 147–159.

Rollè, L., Gullotta, G., Trombetta, T., Curti, L., Gerino, E., Brustia, P., & Caldarera, A. M. (2019). Father Involvement and Cognitive Development in Early and Middle Childhood: A Systematic Review. *Frontiers in psychology*, *10*, 2405. https://doi.org/10.3389/fpsyg.2019.02405

Rowe, M. L. (2012). A longitudinal investigation of the role of quantity and quality of child-directed speech in vocabulary development: Child-directed speech and vocabulary. *Child Development*, *83*(5), 1762–1774. https://doi.org/10.1111/j.1467-8624.2012.01805.x

Scherpereel, Christopher M. and Bowers, Mary Y. (2008). Back to the future: Gender differences in self-ratings of team performance criteria? *Developments in Business Simulation and Experiential Learning, 35*, 170-179.

Silver, A. M., et al. (2021). Measuring emerging number knowledge in toddlers. *Front. Psychol.* 12:703598. doi: 10.3389/fpsyg.2021.703598

Silver, A.M., Libertus, M.E. Environmental influences on mathematics performance in early childhood. *Nat Rev Psychol* 1, 407–418 (2022). https://doi.org/10.1038/s44159-022-00061-z

Silver, A. M., et al. (2023). Investigating associations between parent engagement and toddlers' mathematics performance. *British Journal of Developmental Psychology*. 00:1-34. doi:10.1111/bjdp.12459

Silver, A. M., Chen, Y., Smith, D. K., Tamis-LeMonda, C. S., Cabrera, N., & Libertus, M. E. (2023). Mothers' and fathers' engagement in math activities with their toddler sons and daughters: The moderating role of parental math beliefs. *Frontiers in Psychology*, *14*. https://doi.org/10.3389/fpsyg.2023.1124056

Silver, A. M., Alvarez-Vargas, D., Bailey, D. H., & Libertus, M. E. (in press). Assessing the association between parents' math talk and children's math performance: A preregistered meta-analysis. *Journal of Experimental Child Psychology*.

Starr, A., Libertus, M. E., & Brannon, E. M. (2013). Number sense in infancy predicts mathematical abilities in childhood. Proceedings of the National Academy of Sciences, 110(45), 18116-18120.

Taraban, L., & Shaw, D. S. (2018). Parenting in context: Revisiting Belsky's classic process of parenting model in early childhood. Developmental Review, 48, 55-81.

Wynn, K. (1990). Children's understanding of counting. *Cognition* 36, 155–193. doi: 10.1016/0010-0277(90)90003-3

Wynn, K. (1992). Children's acquisition of the number words and the counting system. *Cogn. Psychol.* 24, 220–251. doi: 10.1016/0010-0285(92)90008-P

Zander, L., Höhne, E., Harms, S., Pfost, M., & Hornsey, M. J. (2020). When Grades Are High but Self-Efficacy Is Low: Unpacking the Confidence Gap Between Girls and Boys in Mathematics. *Frontiers in psychology*, *11*, 552355. https://doi.org/10.3389/fpsyg.2020.552355