Voting and Information Aggregation in

Unicameral Parliamentary and

Semi-Presidential Democracies

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Abstract

This paper investigates legislation in parliamentary and semi-presidential democracies where the legislature and the president have formal role in legislation. A proposed law is first voted in the legislature and if it passes, comes to the consideration of the president. I study two prevalent legislative procedures: (i) Single-round legislation where the president’s action is final, (ii) Two-round legislation the president’s approval enacts the law but after his veto proposal returns to the legislature for rediscussion. In this setup I examine power balance and the efficiency of information aggregation. For this I build a model of strategic voting with incomplete information and analyze different ideological profiles of the president and the homogenous legislature. The president seems powerless in two-round legislation but in equilibrium there are instances he can change the legislation result. Power struggle arises only when the legislature is modernist and the president is conservative. If the legislature is conservative and the president is modernist, the president has no im-
pact on the outcome, but adversely affects informational efficiency. If they have the same ideological bias, the presidential institution is beneficial and the president’s existence provides full information aggregation with finite legislature size in single-round legislation. Above results can be generalized to heterogeneous legislature with two types, except full information aggregation is never achieved.

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1 Introduction

Modern representative democracies have different forms. Based on government formation and law-making, democratic systems can be categorized as presidential, semi-presidential and parliamentary democracies. Examples are United States, Brazil and S. Korea for presidential democracy and France, Portugal, Poland, Russia for semi-presidential democracy. Most European countries such as Germany, Spain, UK, Sweden and Turkey adopted parliamentary democracy. In addition representative democracies are also classified according to the composition of the legislature. Unicameral legislature is unitary and has only one house while bicameral legislature has two separate houses.

In a presidential democracy, the legislative or a committee initiates a law and then the legislative votes on the proposed bill to enact it. In a parliamentary democracy however the executive initiates a law and then the legislative i.e. the parliament votes on it. If the legislature does not approve the proposed law, then the process ends. If the legislature approves the proposed law then it comes to the consideration of the president. If the president also approves
then the law is enacted. However if the president vetoes, then depending on the constitution of the state either the law is abolished or the proposed law returns to the legislative for rediscussion. In the latter situation, if the legislative endorses the law again, then the law is enacted as the president does not have a second veto right. Legislation in semi-presidential democracies is similar to parliamentary democracies but the president in a semi-presidential system has more formal power and he can abolish the proposed law in the first place and even dismiss the executive.

Thus law making procedure differs in representative democracies. The process of voting in juries, committees and legislatures and information aggregation properties have been studied in the literature starting from the seminal contributions of Austen-Smith and Banks (1996). However the existing models of unicameral or bicameral legislatures implicitly assume that the legislature is itself decisive in enacting a law and thus do not take into account the role of the president. Then the starting point of this paper is to examine voting and legislation in parliamentary and semi-presidential democracies with considering the role of the president. To simplify analysis and get indicative results, I restrict attention to unicameral legislature. I study the two alternative legislative processes mentioned earlier: The veto of the president abolishes the proposed law or the veto of the president sends the proposal back to the legislature for voting second time.

In this setup one imminent question is how ideological profile of the legislature and the president affect voting or vetoing strategies. In particular do legislative members and the president act informatively (i.e. according to their own perception regarding the merits of the proposed law) or do they vote according to their ideological bias. Another aspect of this problem, as commonly
stressed in the literature, is the efficiency of the information aggregation. Does the existence of the president improve legislation and likelihood of correctly enacting beneficial laws or abolishing incompetent laws? And if so, how much. One can even question the necessity of the existence of the president. In light of Condercet Jury Theorem if ensembles aggregate information better, why do democracies need the institution of the president? It may further be the case that assigning one person to an influential role after the legislature harms the correct decision making. A possible defense is to have a power balance and to prevent the legislative from being dictatorial. This is satisfactory when the action of the president determines the fate of the proposal but the president seems to be powerless in two round process since the legislature can always approve the law in the second round and enact it. In this respect I investigate whether the president’s action make an impact or signal some information to the legislature in order to assess whether the president is really powerless in two round legislation.

To answer these questions I establish a model of strategic voting with the legislature and the president. Members of the legislature and the president receive a private and imperfect signal about the quality of the proposed bill. Each member of the legislature and the president have publicly known ideological orientation. In particular an actor can be either conservative who favors status quo, or modernist who favors new proposals.

How do the president and the legislature cast their votes? I call an actor votes informatively if he always follows his signal and cast the associated vote. I define full information aggregation equivalence when the probability of making correct decision in the actual game is be the same of the game played as if all information publicly known. Before solving the model, I first review the setting
in Austen-Smith and Banks (1996) where the unicameral homogenous legislature is deciding on its own without the president. I characterize the equilibria of single round and two round voting. Full information aggregation is achieved for a wider range of aggregation rule in two round voting compared to single round voting. Introducing a president that has the same ideology with the legislature provides full information aggregation in single round voting for a range of aggregation rules. Next I consider different ideological composition of the legislature and the president. When conservatives are more powerful in the legislature, i.e. they are a blocking coalition and the president is modernist, then the president does not have any role in the process. (effect on the outcome.) The legislative is basically deciding on its own. More interesting case is when modernists are dominant in the legislature and the president is conservative. In single round legislation there is a power struggle between the legislature and the president which decreases the efficiency of information aggregation. In two round legislation however the legislature becomes more powerful. Because power struggle is diminished significantly, the existence of the president does not have a negative impact on information aggregation and even his signal marginally improves (contributes towards) information efficiency. Here there are instances in which the president does not have an impact on the outcome but there are also instances in which he can change the outcome. Thus the president is not totally powerless in two round legislation.

Section 2 explains the relevant literature. In section 3, I describe the formal model and I perform a preliminary analysis in section 4. Section 5 characterizes equilibria of the model and provides main results of the article. Section 6 concludes.
2 Related Literature

This work contributes to two lines of research that develops after Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1997). There is a literature that studies two class voting which investigates decision making with two committees. Specifically, Maug and Yilmaz (2001) analyze the situation where two groups, seniors and juniors, inside the committee are voting simultaneously. More recently, Iaryzower (2008) studies voting between two sequential committees. He examines information transmission between committees and finds conditions for informative voting. Single round version of my legislation model (with unicameral legislature and the president) can be thought of a special case of sequential voting in which the receiving committee consists of a single person, the president but as I show, types of equilibria and the results are different. Moreover Iaryzower (2008) does not consider the case of two round legislation where the bill returns to the originating committee. Hence the setting here (in my research) is more suitable for examining parliamentary and semi-presidential democracies with (and there is) a focus on efficiency of information aggregation, power distribution and the role of the president. (between the legislature and the president.)

This research is also related to the literature on standard pivotal and signal pivotal voting motivations. Razin (2003) investigates signal pivotal voting in one round election and the winner implements his policy after the election. In Piketty (2000) voting takes place in two stages and the same electorate chooses between the previous policy and the alternative. Then Meirowitz and Shhotts (2009) consider pivotal and signaling motives in two period elections but the analysis is the information transmitted by voters and its effect on candidates’
positions in the second election. winning policy competes with another in the next round.

3 The Model

A proposed law is going to be accepted or rejected. If it is accepted (A) then it replaces the status quo otherwise the status quo (Q) remains. There is a unicameral legislature consisting of N members and the president. First the proposed law is voted in the legislature without abstention. The proposal passes the legislature if it receives R number of votes or more. R is an integer and identifies the aggregation rule. If the legislature does not pass the proposed law, the law is abolished and the process ends without any further action of the president. If the legislature passes the proposed law, then legislation continues with the president. The approval of the president will enact the law and end the legislation. In the case of the president’s veto, as indicated in the constitution of the state, either the proposed law is rejected or it returns to the legislature for voting. I call former procedure 1-round voting and the latter 2-round voting. In 2-round voting, if the legislature does not pass the law in the second round, then the law is rejected and the status quo remains. However if the proposed law passes the legislature again by receiving at least R votes, then the president has to pass the law and the law will be enacted.

There are two possible states of the world: The proposed law is beneficial (A) or not (Q). The true state of nature w is unknown. Members of the legislature and the president share a common prior belief that the true state is Q with probability π. In addition, each player receives an imperfect, private signal regarding the merits of the proposed law. The accuracy of the signal is q >
1/2 for all players and is independent of the state of the world. Because the environment is stochastic, given an information set $I$, an individual considers utility loss $u_{A,Q} < 0$ from type 1 error (passing poor law) and utility loss $u_{Q,A} < 0$ from type 2 error (blocking a beneficial law). Normalizing utility loss from making correct decision to zero, a player prefers outcome $Q$ if $\Pr\{w = A|I\} \cdot u_{Q,A} > \Pr\{w = Q|I\} \cdot u_{A,Q}$. Equivalently the player prefers status quo if $\Pr\{w = Q|I\} > \theta$ and prefers proposal to be implemented if $\Pr\{w = Q|I\} < \theta$, where $\theta = u_{Q,A}/(u_{Q,A} + u_{A,Q})$. $\theta \in (0, 1)$ is called threshold of reasonable doubt.

Both the members of the legislature and the president share a common objective in that they want to approve the law if it is beneficial and reject if it is poor. However they differ with respect to their threshold of reasonable doubt, $\theta$. In particular an individual can be of two type: Conservative who prefers status quo unless convinced enough for the merits of the proposed law, or a modernist who favors reforms unless really convinced that the status quo should be kept. Thus threshold of reasonable doubt of a conservative ($\theta_c$) is less than threshold of reasonable doubt of a modernist ($\theta_m$). The ideological biases of all players are common knowledge.

I assume $\Pr\{w = Q|\sigma_i = A\} > \theta_c$ and $\Pr\{w = Q|\sigma_i = Q\} < \theta_m$. These conditions ensure that an individual’s private signal, by itself, cannot overturn his preference against his bias and the individual seeks further evidence in addition to his own information to decide against his bias.

The solution concept is Perfect Bayesian equilibrium in symmetric, weakly undominated strategies. Although McLennan (2001) illustrates asymmetric mixed strategy equilibrium that yields superior informational efficiency prop-
erties, this class of equilibrium requires coordination among players. Therefore I commit to symmetric mixed strategy equilibria.

The criterion for efficiency is full information aggregation, that is an equilibrium is efficient if probability of correct decision is the same as if all signals were common knowledge.

The above game admits multiple equilibria that vary in the degree of players’ informative action. To analyze efficiency and information aggregation, I restrict attention to the most informative equilibria: The equilibria that yield both ex-ante greater probability of correct decision (matching true state) and lower probability of false decision in the Pareto sense inside all equilibria.

3.1 Standard and Signal Pivotal Motivations

In this setup a legislature member is guided by both standard-pivotal and signal-pivotal motivations while casting his vote. In single-round voting the member considers in the first step whether the proposed law will pass the legislature or not. So he should condition on the case he is just pivotal on the proposal being approved or abolished in the legislature. This is the standard-pivotal motive. At the same time he would also consider the effect of his vote on

1 (2009) shows that asymmetric mixed strategies can be implemented in an infinitely repeated game without coordination. However the game in my model proceed at most two round and voters condition on first round outcome. Thus coordination is still required to realize asymmetric equilibrium.

2 For instance if the legislature is homogenous there always exists an equilibrium where legislature members all unconditionally vote A or all unconditionally vote Q. However I eliminate such equilibria and other less informative equilibria.
the president’s approve or veto action (through the information transmitted by
the tally). This is the signal-pivotal motive and I call (designate) a legislature
member signal-pivotal when his vote just changes the action of the president.
In two-round legislation, similarly a legislature member has both standard-
pivotal and signal-pivotal motivates as in single-round voting. But now the
signaling phenomenon (motive) is more sophisticated. The member considers
being signal-pivotal on the final result through sending information to the
president and/or to other legislature members for second round. During second
round voting, members think of only being standard pivotal.

4 Preliminary Analysis

Before solving the actual model, it is useful to look at an analogous problem
to Austen-Smith and Banks (1996). Consider a single round legislation in
which all the members of the legislature have the same ideological bias \( \theta \)
and there is no president. That is the legislature is homogenous and deciding
autonomously. To find the equilibrium, think about the voting decision of an
individual. The only situation in which the vote of a legislature member effects
the outcome is the tally of votes (number of approval votes) \( t_{-i} \) except him is
exactly \( R-1 \). Therefore while casting vote, a member conditions on the case he
is pivotal, i.e. the tally of votes except him is exactly R-1. I define informative
voting as the pure strategy that votes Q when having Q signal and votes A
when having A signal. The information set of a voter while casting a vote is
his private signal \( \sigma_i \) and the fact that he is pivotal. Let \( P_Q(R) \) and \( P_A(R) \)
denote the likelihood of true state being Q condition on the information set
with Q and A signals, respectively: \( P_Q(R) = \Pr \{ w = Q | t_{-i} = R - 1, \sigma_i = Q \} \)
and \( P_A(R) = \Pr\{w = Q|t_{-i} = R - 1, \sigma_i = A\} \). Then member \( i \) will vote informatively if \( P_A(R) < \theta < P_Q(R) \). Let \( K_\theta^N \) be the set of aggregation rules that satisfy this condition, \( K_\theta^N = \{R : P_A(R) < \theta < P_Q(R)\} \).

**Proposition 1 (Austen-Smith and Banks (1996))** In single round voting, legislature members vote informatively if and only if \( R \in K_\theta^N \). If \( R > \overline{K}_\theta \), then in equilibrium \( P_A(R) < \theta = P_Q(R) \). If \( R < \underline{K}_\theta \), then in equilibrium \( P_A(R) = \theta < P_Q(R) \).

Thus informative voting is equilibrium only for specific aggregation rule(s). By definition, full information aggregation occurs in informative voting equilibrium. Note that the set of critical values \( K_\theta^N \) is a function of the ideological bias and may contain one or more elements depending on size of the legislature. Let \( \underline{K}_\theta^N = \min\{K_\theta^N\} \) and \( \overline{K}_\theta^N = \max\{K_\theta^N\} \) denote smallest and largest rules that allow informative voting. If \( R > \overline{K}_\theta \), (in equilibrium,) voters with \( A \) signal will vote \( A \) with probability 1 but voters with \( Q \) signal will randomize and vote \( Q \) with probability \( \alpha \in (0, 1) \). Since the threshold is relatively high, voters tend to favor towards \( A \). Similarly if \( R < \underline{K}_\theta \), voters with \( Q \) signal will vote \( Q \) with probability 1 but voters with \( A \) signal will randomize and vote \( A \) with probability \( \beta \in (0, 1) \). Equilibrium beliefs are consistent with actions and randomizing types are indifferent: (Action probabilities \( \alpha \) and \( \beta \) are derived from equilibrium beliefs)

So when \( R \notin K_\theta^N \), voters neglect some of their private information and the resulting equilibrium is inefficient.
4.1 Two Round Voting in Legislature

Now suppose the homogenous legislature is voting in two rounds. If the proposed law doesn’t pass in the first round, it is abolished. If it passes, then the legislature votes again in the second round after observing the realized tally after the first round. Outcome in the second round determines the destiny of the proposal. If \( R \leq \bar{k}_N \), there exists an equilibrium where legislature members vote informatively in the first round, and based on the realized tally, cast their votes in the second round. Because all signals are revealed in the first round, full information aggregation is achieved.

**Proposition 2** In the two round voting model of homogenous legislature without a president, if \( R \leq \bar{k}_N \), there exists an equilibrium in which all members of the legislature vote informatively in the first round. If \( R > \bar{k}_N \), then there is no equilibrium in which the legislature votes informatively in either first or second round.

To get the intuition, when \( R \leq \bar{k}_N \), the threshold does not constitute a real hurdle against informative voting in the first round because decision will be finalized in the next round. Since there is no conflict of interest among members, everyone has an incentive to vote informatively in the first round to increase informational efficiency. Thus adding second round provides full information aggregation also for \( R < \bar{k}_N \). This mechanism does not work however when \( R > \bar{k}_N \). The threshold is relatively high and prevents members from voting informatively in the first round. Full information equivalence cannot be achieved in this case.
The next proposition describes the strategy of the legislature members in the second round. Notice that the second round is reached only if the tally $t$ in the first round is greater than or equal to $R$.

**Proposition 3** Suppose the legislature is voting in two rounds without the president and $R \leq \bar{k}_\theta^N$. If the realized tally in the first round is $t \geq \bar{k}_\theta^N$, then all legislature members will vote $A$ in the second round. If $R \leq t \leq \bar{k}_\theta^N - 1$ then all legislature members will vote $Q$ in the second round. If $t \in (\max\{R, \bar{k}_\theta^N\}, \bar{k}_\theta^N - 1)$ then legislature members will also vote informatively in the second round and the realized tally will not change. Whether $t \geq R$ or $t < R$ will determine the legislation.

**Proposition 4** Suppose the legislature is voting in two rounds without the president. If $R > \bar{k}_\theta^N$ and the proposed law passes the first round then all members will vote $A$ in the second round and the law will be enacted with unanimity.

5 Main Results

After the preliminary analysis with legislature on its own, we are ready to introduce the president. First, consider the case where the legislature is homogenous and the president has the same ideological bias with the legislature. That is there is no conflict and all players have the same threshold of reasonable doubt $\theta_l = \theta_p = \theta$. 

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5.1 Single round legislation

Let $k_0$ be as defined before for a homogenous autonomous legislature without a president. Because the interests of the president and the legislature are perfectly aligned, the actual threshold does not constitute a real barrier against informative voting in legislature as long as $R < k_0^N$. Legislature members already have an incentive to vote informatively when $R \in (k_0^N, \bar{k}_0^N)$. Then if $R \leq \bar{k}_0^N$, there exists an equilibrium in which legislature members vote informatively and the president decides on behalf of all players condition on the realized tally $t$ and his own signal $\sigma_p$. In particular, the president will approve if $\Pr\{\omega = Q|t, \sigma_p\} < \theta$.

**Proposition 5** If $R \leq \bar{k}_0^N$ and all players have the same ideological bias, then in single round legislation there exists an equilibrium in which all legislature members vote informatively. If the realized tally is $t$, then the president having Q signal will approve the proposal if $t \geq \bar{k}_0^{N+1}$ and veto otherwise. The president having A signal will approve if $t \geq l_0^{N+1} - 1$ and veto otherwise.

Because all players’ signals are taken into account in decision, the resulting equilibrium fully aggregates information. Recall that in single round legislation without president, informative voting is equilibrium only if $R \in K_0^N$ so thanks to the existence of the president, informative voting and efficiency are also achieved when $R < k_0^N$. However when $R > \bar{k}_0^N$ there is no equilibrium in which the legislature votes informatively. In the most informative equilibrium, strategy of a legislature member is the same as his strategy in single round legislation without president (as described by Remark 1). The president always approves the law (even when he has Q signal) if the proposal passes the
legislature.

**Proposition 6** If $R > \bar{k}_\theta^N$ and all players have the same ideological bias, then in single round legislation the most informative equilibrium is legislature members vote $A$ if they have $A$ signal, and vote $Q$ with probability $\alpha_R \in (0, 1)$ if they have $Q$ signal. They randomize with $(\alpha_R, 1 - \alpha_R)$ so that $P_A(R) < \theta = P_Q(R)$. In equilibrium, the president, regardless of his signal, always approves and enacts the law once it passes the legislature.

This means the existence of the president is not sufficient for informative voting and the equilibrium is inefficient. Moreover the president’s signal has no value in terms of information aggregation because the president neglects his own signal.

5.2 Two round legislation

The analysis and the equilibria are somehow analogous to single round legislation with president. When $R \leq \bar{k}_\theta^N$, then there exists an equilibrium in which legislature members vote informatively in the first round. But for $R > \bar{k}_\theta^N$, informative voting of legislature is never realized in equilibrium. The president takes an action based on the realized tally $t$ and his own signal $\sigma_p$. For both ranges of $R$, his strategy is exactly the same as single round legislation with corresponding $R$, examined above. If $R > \bar{k}_\theta^N$, second round is never reached in equilibrium. When $R \leq \bar{k}_\theta^N$, a legislature member’s second round vote depends on the realized tally $t$ in the first round.

**Proposition 7** In two round legislation with president, if $R \leq \bar{k}_\theta^N$ and all players have the same ideological bias, then there exists an equilibrium in which
all legislature members vote informatively in the first round. If the realized tally is \( t \), then the president having \( Q \) signal will approve the proposal if
\[
t \geq k_{\theta}^{N+1}
\]
and veto otherwise. The president having \( A \) signal will approve if
\[
t \geq k_{\theta}^{N+1} - 1
\]
and veto otherwise. In the case of president’s veto:

If \( R < k_{\theta}^{N+1} \) and the realized tally \( t \) in the first round is \( t \in (R, k_{\theta}^{N+1} - 1) \) then all legislature members will vote \( Q \) in the second round and the law will be abolished.

For \( t \in (\max\{R, k_{\theta}^{N+1}\}, k_{\theta}^{N+1} - 1) \) a legislative member’s strategy in the second round is as specified by Proposition 1. The outcome will be determined by ex-post realization of votes and signals.

Thus when \( R \leq k_{\theta}^{N} \), the president’s action affects the result of the legislation. In equilibrium all legislature members’ and the president’s signals are fully utilized and full information aggregation is achieved in legislation. However recall that the legislature already attains full informational equivalence by two round voting without the president. Then in terms of efficiency, the existence of the president has benefit of only one more additional signal.

But as the next proposition states, for \( R > k_{\theta}^{N} \), informative voting of legislature is never realized in equilibrium. In the most informative equilibrium, strategy of a legislature member in the first round is the same of his strategy in single round legislation without president. The president always approves the law if the proposal passes the legislature. Therefore in two round legislation, when \( R > k_{\theta}^{N} \), the president has no role in decision making and his signal has no value for informational efficiency.

**Proposition 8** In two round legislation with president, if \( R > k_{\theta}^{N} \) and all
players have the same ideological bias, then there is no equilibrium in which all legislature members vote informatively in either round. Legislature member’s first round strategy is as specified in Proposition 1. If the proposal passes the legislature in the first round, the president will always approve and enact the law, regardless of his signal. Therefore in equilibrium second round is never reached.

6 Ideological Heterogeneity

What will happen if the legislature and the president have different ideological orientations? I first examine the situation where the legislature is conservative and the president is modernist. Next I examine modernist legislature with conservative president. To model ideological heterogeneity, I introduce two types: Conservative and modernist. \( \theta_c \) and \( \theta_m \) denote threshold of reasonable doubt of conservative and modernist players, respectively. I assume and are not very close to each other so that \( K_{\theta_c} \) and \( K_{\theta_m} \) are disjoint. Therefore condition on the same information set, if one type is voting informatively, the other type is voting uninformatively.

When I introduced the president in the last section, I initially assumed he has the same ideology with the legislature. Next I examine the situations where the ideological bias of the president is different from that of the legislature.

6.1 Conservative Legislature with a Modernist President

Consider the setting where all members of the legislature are conservative and the president is modernist. (Let denote threshold of reasonable doubt of
legislature member and the president, respectively.) Then \( \theta_L = \theta_c < \theta_P = \theta_m \).

Observe that in single-round legislation the conservative legislature has the first-mover advantage over the president, because in the case of a conflict i.e. the legislature doesn’t find the law beneficial enough while the president does, then the legislature can abolish the law and finish the process in the first place. This argument characterizes the power balance in single-round legislation: in the unique equilibrium, the legislature is fully decisive and legislature members are voting as if there is no president.

**Proposition 9**  *If the legislature members are conservative and the president is modernist, then there exists a unique equilibrium in single round legislation for all values of \( R \). While casting his vote, a legislature member thinks as if there is no president and conditions his vote on being pivotal on \((R-1)\). If the law passes the legislature, the president always approves it, regardless of his own signal.*

Intuitively if a conservative legislature passes a law, then the modernist president is already willing to accept it. So the president always approves the law even if he has Q signal. This suggests that the president has a no impact on the outcome. The president’s private signal does not have any contribution to information aggregation either since he completely ignores his own signal. Full information aggregation is achieved only when \( R \in K_{\theta_c}^N \). Compared to autonomous legislature voting single-round (Comparing single-round legislation with and without president) suggests (implies) that the existence of a relatively modernist president has no effect on output or informational efficiency when the legislature is conservative.
How do strategies and equilibria change in two-round legislation? As Proposition 11 states, the equilibrium in two-round legislation is unique and analogous to equilibrium in single-round legislation. Legislature members in the first round condition on being pivotal on \((R - 1)\) and the president always approves the law once it passes the legislature. The second round is never reached in equilibrium. As in single-round legislation, the legislature is again fully decisive on the outcome, but now the legislature members consider the existence of the president while forming their strategies. Recall that in two-round legislation without the president, a homogenous legislature votes informatively in the first round when \(R \leq k_{R,c}^N\). With the modernist president however, informative voting is no longer achieved when \(R < k_{R,c}^N\). To see why, suppose \(R\) and the legislature is voting informatively in the first round. It may be the case that the realized tally is greater than or equal to \(R\), but the tally is low that the legislature prefers proposal not implemented but high enough that the president prefers to be implemented. Since the proposed law passes the legislature, the president would approve and enact it. Thus a legislature member does not have an incentive to vote informatively when \(R\), meaning that the existence of the president negatively affects information aggregation. This is the drawback of power struggle originating from ideological divergence.

**Proposition 10** If the legislature members are conservative and the president is modernist, there exists a unique equilibrium in two-round legislation for all values of \(R\). The first round strategies of the legislature members and the president are the same as corresponding ones in single-round legislation, stated in Proposition 10. While casting his vote, a legislature member thinks as if there is no president and conditions his vote on being pivotal on \((R-1)\). If the law passes the legislature, the president always approves it, regardless of his
own signal.

Because the modernist president is ready to approve proposals that the legislature is not convinced enough, the legislature members mainly consider first round voting rather than second round. Anticipating that legislation will be finalized in the first round, legislature members condition on being pivotal on R-1 and they don’t relegate their decision to the second round. Note that as in the case of single-round legislation, the president does not have any real role in decision making and his signal does not have any informational value. Full information aggregation is achieved only when $R \in K^N_N$.

6.2 Modernist Legislature with a Conservative President

A more exciting situation is the legislature consists of modernist members and the president is conservative. That is There is a power struggle between the legislature and the president, at the same time information signaling. Power struggle arises in the case the modernist legislature members want the proposal to be implemented but the conservative president rejects as he is not convinced about the merits of the new law. Then in single-round legislation, the conservative president is powerful since his veto abolishes the law. In two-round legislation however, the legislature becomes more powerful since it can always endorse and enact the law in the second round. Formal results confirm this analysis.

In single-round legislation, the unique equilibrium is such that the legislature is voting as if the aggregation rule is unanimity. The actual rule therefore becomes irrelevant. If the proposal passes the legislature, the president ran-
domizes between approving and vetoing the law. To understand the equilibrium strategies, suppose the legislature members condition on effective rule $R'$ (may be different from actual rule $R$) and $R' < N$. If ex-post realization of tally is low i.e. $t = R'$, $R''$ then the president’s posterior belief will be $\dot{E}$. In this situation the president will veto but the legislature wants the law to be enacted. The fact that the realized tally is greater than or equal to $R'$, weakly impact influences legislature members more toward A or doesn’t change their idea at all. Thus a legislature member would deviate and exaggerate A by randomizing more toward A vote. This is equivalent to increasing the effective aggregation rule to. However the president will update his posterior and if realized tally is $\dot{E}$, conflict of interest will still happen. Then the legislature member will emphasize more and more toward A vote by increasing effective rule until it reaches upper limit N. Observe that the legislature member will not exaggerate further by increasing effective tally to $N=1$ or higher. This would be compensated by a decrease the president’s approval probability, yet increase the risk of approving poor laws. In single-round legislation legislature members take into account both standard and signal pivotal motivations.

**Proposition 11** In a setting where the legislature members are all modernist and the president is conservative, then single-round legislation exhibits unique equilibrium: A legislature member votes as if the legislature is autonomous and the effective aggregation rule is unanimity i.e. $R = N$. If the proposed law passes the legislature, the president with a Q signal approves the law with probability $\psi \in (0,1)$ and the president with an A signal approves the law with probability $\gamma \in (0,1)$ where $\gamma > \psi$.

As the effective tally becomes unanimity, legislature members tend to neglect their signal more and the efficiency of information aggregation decreases
compared to the case without president. Thus the existence of the president negatively affects information aggregation. If this game were played with all signals common knowledge, the president would collect all signals and approve only very beneficial proposals, consistent with his ideology. Likewise the legislature would eliminate really poor laws in the first place. Thus in the original game with private signals, the probability of enacting very poor laws and eliminating very beneficial laws is greater. The original legislation may yield different outcome compared to the one with public signals so by definition full information aggregation is never achieved.

6.2.1 Two-round legislation

In two-round legislation, the equilibrium depends on the range of R. In order to better express the equilibria, I define the trigger strategy for the president as follows: When the president has Q signal, he vetoes with probability 1 and when he has A signal, he approves with probability $\delta \in (0, 1)$. As $\delta$ becomes closer to 1, president’s action becomes more informative and in the case of veto, the legislature members attribute ex-post greater probability $\Pr\{\omega = Q|\text{veto}\}$ to president being Q type. So $\delta$ measures the accuracy of the signal that the president transmits and hence the degree of influence of his veto on the legislature in the second round. (Here $\delta$ measures the accuracy of the signal that the president transmits to the legislature in the second round.)

From the discussion in autonomous legislature voting two rounds, it is straightforward to see that when $R \leq \overline{k}^N_{qm}$, the aggregation rule is not a barrier to informative voting, thanks to the existence of second round. As Proposition 13 illustrates, there exists an equilibrium in which the legislature members
vote informatively in the first round when \( R \leq \overline{R}_{\theta_m} \). The president’s action may change the legislation outcome only when the realized tally in the first round is at the critical lower bound of informative voting \( t_1 = \overline{k}_{\theta_m}^N \) and the parameters are such that one additional voter alters the informative voting incentive at this lower bound. Then the president will transmit accurate enough Q signal to the legislature by utilizing an appropriate trigger strategy and law will be abolished if second round is reached. For all other situations i.e. \( t_1 = \overline{k}_{\theta_m}^N = \overline{k}_{\theta_m}^{N+1} \) or higher tally the president’s veto does not effect legislature members’ actions in the second round so the president cannot change the legislation outcome. In fact for high realizations of tally, depending on his type the president will be convinced and always approves the law.

**Proposition 12** Consider two round legislation where the legislative members are modernist and the president is conservative. When \( R \leq \overline{R}_{\theta_m}^N \), there exists an equilibrium in which all legislature members vote informatively in the first round when \( R \leq \overline{R}_{\theta_m}^N \). If the realized tally is \( t_1 = \overline{k}_{\theta_m}^N \) and \( \overline{k}_{\theta_m}^{N+1} = \overline{k}_{\theta_m}^N + 1 \) then the president adopts trigger strategy where \( \delta \) is such that \( \Pr\{\omega = Q|\sigma = A, t_1 = k_{\theta_m}^N, \text{veto}\} \) is just above \( \theta_m \). However if \( t_1 = \overline{k}_{\theta_m}^N \) and \( \overline{k}_{\theta_m}^{N+1} = \overline{k}_{\theta_m}^N \) the president will veto with probability 1. For all other tally values, the A type president will approve when \( t_1 \geq \overline{k}_{\theta_c}^{N+1} - 1 \) and veto otherwise; Q type president will approve when \( t_1 \geq \overline{F}_{\theta_c}^{N+1} \) and veto otherwise. The legislature member’s second round strategy is:
Comparing the equilibria in two-round legislation with and without president, I find that the existence of the president matters only when \( t_1 = k_{\theta_m}^N \) and \( k_{\theta_m}^{N+1} = k_{\theta_m}^N + 1 \). Because the legislature already achieves informative voting on its own in two-round legislation, the value of the president’s existence is the amount of information he transmits to the legislature provided his action affects the outcome. Thus the president contributes less than one signal to information aggregation. Note that as the next Remark states, full information aggregation is generally attained in this setting. (The two exceptions are when \( t_1 = k_{\theta_m}^N = k_{\theta_m}^{N+1} \). In such situations, the probability that the legislation concludes with A (or respectively Q) would be different if the game were played with all players’ signals common knowledge.)

**Remark 13** If the game were played with all players’ signals common knowledge, the probability that the legislation concludes with A (or respectively Q) would differ only for two possible cases: (i) the president has A signal, realized tally \( t_1 = k_{\theta_m}^N \) and \( K_{\theta_m}^N = K_{\theta_m}^{N+1} \) is a singleton (ii) the president has A signal, realized tally \( t_1 = k_{\theta_m}^N \) and \( k_{\theta_m}^{N+1} = k_{\theta_m}^N + 1 \).

When the aggregation rule is \( R > k_{\theta_m}^N \), the legislature member has to condition on \( R \) in the first round and thus informative voting is no longer incentive.
compatible. In the most informative equilibrium, the legislature member’s first round strategy is the same as his strategy in single-round legislation with autonomous legislature, as explained in Section . If the proposal passes the first round but the realized tally is low (in a low range), then the president will use an appropriate trigger strategy to send Q signal to the legislature. The president transmits just enough accuracy Q signal such that (transmitting) greater accuracy Q signal would to make himself worse off as it requires randomizing more towards approving, but the accuracy of the transmitted Q signal is high enough to ensure that ex-ante enough number of Q signal legislature members vote Q in the second round to abolish the proposal. (The A type president will randomize such that he transmits accurate enough Q signal to legislature to ensure that ex-ante enough number of Q signal legislature members vote Q in the second round to abolish the proposal. The president does not transmit so high accuracy Q signal since doing so requires randomizing more towards approving which makes him worse off, but not so high to make president worse off because of randomizing more towards approving). Thus the president may change the ex-post outcome in favor of himself if the second round is reached. However when the realized tally in the first round exceeds a critical level, the legislature members will not consider the president’s action and all vote A in the second round. So the president cannot change the legislation outcome and he always vetoes in this case. (the president cannot change the legislation outcome since legislature members will always vote A in the second round and don’t consider the president’s action. So the president always vetoes in this case.) As the realized tally further increases, the president actually become more and more convinced about the proposal and after some tally value he always approve the proposed law. But note that these equilibrium strategies heavily depend on parameters and the aggregation rule R. In particular if R
is high, members’ votes are not so informative so even at the unanimous tally (t=N), the president is not convinced and he does not always approve the law.

**Proposition 14** Consider two round legislation where the legislative members are modernist and the president is conservative. When $R > R_{\theta_{\text{dr}}}^N$, in the most informative equilibrium, the first round strategy of legislature members are the same of their strategy in single round legislation with autonomous legislature as illustrated in Proposition 1. If the realized tally is $t_1 \in [R, R_1]$ for some $R_1 \in [R, N]$ then the president adopts trigger strategy with $\delta(t_1)$. Depending on parameters, there may be a higher range of realized tally $t_1 \in [R_1 + 1, R_2]$ for which the president unconditionally vetoes. If so, there may exist, depending on parameters, a further higher range of tally $t_1 \in [R_2 + 1, R_3]$ where the president takes informative action and maybe another range of tally $t_1 \in [R_3 + 1, N]$ he always approves the law. If second round is reached and $t_1 > R_1$, all members will vote A in the second round. If $t_1 \in [R, R_1]$, ex-post realization of signals and first round votes will determine second round actions and the legislation result.

The president may affect the outcome (The president has a real role in decision making) only when the proposed law passes the legislature in the first round and the realized tally is low enough $t_1 \in [R, R_1]$. Whether the president actually changes the outcome depends on ex-post realization of legislature members’ signals and the tally in the first round. The president’s existence has a value of less than one signal because he sends imperfect Q signal to legislature when his action is influential. Full information aggregation is never achieved.
It would be natural to investigate more general cases where the legislature is ideologically heterogeneous and the president has different ideological biases. I consider two different ideological groups, conservatives and modernists in the legislature and a president with any ideological bias, not necessarily one of the types in the legislature.

Suppose there are \( N_c \) number of conservative members with \( \theta = \theta_c \) and \( N_m \) number of modernist members with \( \theta = \theta_m \) in the legislature. I assume the ideological bias of two types are not very close to each other that is . Modernists are more powerful in the legislature if they can always pass the proposed law in the legislature on their own, namely if they are winning coalition , on the contrary conservatives are more powerful if they can always block the law i.e. Observe that if modernists are not a winning coalition then conservatives are more powerful, hence one group is always more powerful in the legislature. As Proposition 15 states, this is crucial in characterizing equilibrium behavior.

In single round legislation, there is no equilibrium in which all members vote informatively. Independent of the aggregation rule and the president’s ideology, less powerful group in the legislature will always vote its bias and the powerful group will vote as if it is decisive on the legislature with effective aggregation rule. Full information aggregation is never achieved since members of one group do not reveal their private information.

**Proposition 15** Consider a heterogeneous legislature with \( N_c \) conservative and \( N_m = N - N_c \) modernist members. In single round legislation, regardless of the president’s ideology and aggregation rule \( R \),
(i) If modernists are winning coalition \((N_m \geq R)\), then conservatives will always vote \(Q\) and modernists will act as if they are decisive in the legislature, as in Proposition 1.

(ii) If conservatives are a blocking coalition \((N_c > N - R)\), then modernists will always vote \(A\) and conservatives will act as if they are decisive in the legislature with effective aggregation rule \(\bar{R} = R - N_m\)

To see this result, there is no situation in which both types condition on being pivotal on the same aggregation rule \(R\) and both take some level of informative action; one type always votes its bias. Using similar logic, (I find that) in two round legislation, the less powerful group in the legislature always votes its bias in the first round. In the second round of legislation (when reached) if the powerful group votes its bias or against its bias as a pure strategy then it is decisive on the outcome. The only situation where the less powerful group may affect the result is when the dominant group is voting informatively or partially informatively. But as suggested by Proposition 16, the less powerful group is always voting its bias in this case. Then the powerful type adjusts (updates) the effective aggregation rule accordingly. Thus the powerful group is decisive on the legislature in both rounds. Again as in single round legislation, full information aggregation is never achieved.

**Proposition 16** Consider two round legislation with a president and a heterogeneous legislature with \(N_c\) conservative and \(N_m = N - N_c\) modernist members. In equilibrium, in the first round the less powerful group will always vote its bias and the more powerful group will act as if it determines the legislature outcome. single round legislation, regardless of the president's ideology and aggregation rule \(R\),
(i) If modernists are winning coalition \((N_m \geq R)\), then in the first round of the game all conservatives vote \(Q\) and modernists act as if the legislature is homogenous and consists of only themselves. In the second round (if reached) when modernists are voting informatively or voting \(Q\) then conservatives vote \(Q\) with probability 1. If modernists are voting \(A\) in the second round then depending on the information set, conservatives’ strategy can be always vote \(Q\) or always vote \(A\) or vote informatively or partially informatively. If in the second round conservatives are voting \(A\) or voting informatively or partially informatively then modernists must be voting \(A\) with probability 1.

(ii) If conservatives are a blocking coalition \((N_c > N - R)\), then in the first round of the game all modernists vote \(A\) and conservatives act as if the legislature consists of only themselves and the aggregation rule \((R - N_m)\). In the second round (if reached) when conservatives are voting informatively or voting \(A\) then modernists vote \(A\) with probability 1. If conservatives are voting \(Q\) in the second round then depending on the information set, modernists’ strategy can be always vote \(A\) or always vote \(Q\) or vote informatively or partially informatively. If in the second round modernists are voting \(Q\) or voting informatively or partially informatively then conservatives must be voting \(Q\) with probability 1. Conservatives take the effective aggregation rule \(\overline{R} = R - N_m\) in the second round too.

In light of Propositions 15 and 16, the less powerful group in the legislature does not have a real role in decision making. Thus without loss of generality we can solely consider the more powerful group in the legislature with effective aggregation rule. In other words, we can think of the legislature as homogenous. Then depending on whether the president is more conservative or more modernist or same ideology compared to the powerful group in the legislature,
the problem reduces to one of the cases studied before.

8 Asymmetric Equilibrium

Throughout the paper, I have performed the analysis using symmetric equilibrium. In this section I briefly investigate asymmetric equilibria of the single-round and two-round legislation game. I define asymmetric equilibrium as legislature members with the same ideology do not necessarily use the same strategy. Although other types of asymmetric equilibria exist, I concentrate (focus) on a particular type of asymmetric equilibrium in which among the same type, some players (one subgroup) vote informatively, only one player votes partially informatively and all others vote un informatively. I name (call) this polarized asymmetric equilibrium. As Proposition 17 shows, given the true state of nature polarized equilibrium yields greater likelihood of correct decision and lowest likelihood of wrong decision compared to all other asymmetric equilibria. Note that for the same committee and aggregation rule, there may be more than one polarized equilibrium. The most informationally efficient polarized equilibrium is the one in which greatest number of players voting informatively.

**Proposition 17** Consider single round legislation with an autonomous legislature (there is no president). The legislature consists of $N$ members all having the same threshold of reasonable doubt $\theta$ and the aggregation rule is $R$. If $R \in K^N_0$, there is a trivial polarized equilibrium in which all members vote informatively. If $\exists \tilde{N}$ such that $R \in K^\tilde{N}_0$ and $\tilde{N} < N$ then there exists an asymmetric equilibrium (polarized equilibrium) in which $\tilde{N}$ number of members (set $I$) vote informatively, $N - \tilde{N} - 1$ number of members uninform-
ditionally vote $Q$ and the remaining one member uses the following strategy: He votes $Q$ if he has $Q$ signal and randomizes if he has $A$ signal so that 

$$P_A(R) = \Pr\{w = Q|t_{-i} = R - 1, \sigma_i = A, i \in I\} \text{ is infinitesimally less than } \theta. \text{ If } \exists \tilde{N} \text{ such that } R \in K_{\tilde{\theta}}^{\tilde{N}} \text{ and } \tilde{N} > N \text{ and there is } a > 0 \text{ such that } (R - a) \in K_{\theta}^{N-a} \text{ then in the polarized equilibrium } (N - a) \text{ number of members (set } I) \text{ vote informatively, } (a - 1) \text{ number of members unconditionally vote } A \text{ and the remaining one member uses the following strategy: He votes } A \text{ if he has } A \text{ signal and randomizes if he has } Q \text{ signal so that }$$

$$P_Q(R) = \Pr\{w = Q|t_{-i} = R - 1, \sigma_i = Q, i \in I\} \text{ is infinitesimally larger than } \theta.$$

**Remark 18** For fixed $R$ and $N$, there is a polarized equilibrium corresponding to each committee of particular size $\tilde{N}$ for which $R$ is a critical rule for informative voting, namely $R \in K_{\tilde{\theta}}^{\tilde{N}}$. Among these the polarized equilibrium with greatest number of players voting informatively provides higher probability of correct decision and lower probability of wrong decision making compared to all other asymmetric equilibria.

In polarized asymmetric equilibrium, intuitively some players vote unconditionally one alternative to adjust the effective aggregation rule so that other players have incentive to vote informatively. And one player uses partial informative strategy to increase information efficiency as much as possible without breaking others’ incentive. Now what asymmetric equilibria do single-round and two-round legislation exhibit? First note that the symmetric pure strategy equilibria are also a special type of asymmetric equilibria and polarized equilibria. So any symmetric equilibrium of the single-round or two-round legislation game in which legislature members are voting informatively and symmetric equilibria in which they all vote $Q$ or all vote $A$ are also asymmetric equilibria.
of the legislation game. Proposition 19 shows that, in addition, there exists an analogous polarized asymmetric equilibrium corresponding to each symmetric mixed strategy equilibrium of single-round or two-round game (legislation) in the sense that the strategy of the president is identical (remains unchanged) in both types of equilibria. Yet the ex-ante probability of making correct decision in polarized asymmetric equilibria is always greater than symmetric (mixed strategy) equilibria.

**Proposition 19** Suppose there is a symmetric equilibrium of single-round or two-round legislation where legislature members use mixed strategy in the first round (as stated in Proposition 1). Then for all ideological profile of players there exists a polarized equilibrium of the game with the president using the same strategy as in the symmetric equilibrium. However polarized equilibrium yields ex-ante higher probability of correct decision and lower probability of wrong decision. Moreover ex-post outcomes of the two equilibria may differ even with the same realization of signals.

Therefore all symmetric equilibria of single-round or two-round legislation are implemented (achievable) by asymmetric equilibria. There are however asymmetric equilibria not implemented (achievable) by symmetric equilibrium. The president’s strategy in such asymmetric equilibria are outside the scope of symmetric equilibria. But as far as the most efficient polarized equilibrium is concerned, the set of asymmetric equilibria coincides with the most efficient symmetric equilibria.

**Proposition 20** In single-round and two-round legislation, for all ideological profiles of the legislature and the president, the president’s strategy in the most efficient asymmetric equilibrium is the same as his strategy in the most effi-
cient symmetric equilibrium. But even with the same realization of signals the
decision of the legislature may differ under analogous symmetric and asym-
metric equilibria unless the legislature members utilize the same (pure) strategy
in two types of equilibrium. Ex-post outcome of the legislation is assured to be
the same only if all players’ strategies under two equilibrium are the same and
all players use pure strategies.

Thus the set of efficient asymmetric equilibria and efficient symmetric equi-
libria are analogous to each other. Nevertheless ex-ante probability of correct
decision making in analogous symmetric and asymmetric equilibrium is the
same only when all legislature members’ strategies are identical. This is pos-
sible if all legislature members use pure strategy in all rounds of the game i.e.
informative or uninformative. Ex-post outcome of the legislation also depends
on the president’s action.

9 Conclusion

In this paper I have studied a model of single and two round legislation in par-
liamentary and semi-presidential democracies. I have analyzed informational
efficiency and the role of the president in the legislation process. (Generally)
the president is more powerful in single-round voting than in two-round vot-
ing. The reverse is true (applies/happens) for the legislature. But even in
two-round voting the president is not totally powerless: There are instances
where the president can change the result of the legislation as in modernist
legislature with a conservative president and as in legislature and the president
same ideology. Power struggle occurs only when the legislature is modernist
and the president is conservative. In single-round legislation, the existence of
conservative president significantly decreases the likelihood of correct decision making. However if legislation is two round, the president does not have a negative effect on information aggregation efficiency. When the legislature is conservative and the president is modernist, the president has no impact on the outcome in single-round or two-round legislation. But in two-round legislation his existence hinders (disturbs) full information aggregation because legislature members would vote informatively for a range of R without the president. If the legislature and the president have the same ideology, there is no conflict of interest and the presidential institution is beneficial. Especially in single-round legislation, for a range of R rules, the existence of president provides full information aggregation for finite legislature size which the legislature cannot achieve by itself.

In two-round legislation, (note that) with the appropriate R rule, the legislature already attains full information aggregation without the president. For the same range of R rule, full informational efficiency (equivalence) is still attained in two-round legislation when the legislature and the president have the same ideology or when the legislature is modernist and the president is conservative. Thus in these settings the value of the president’s existence is the amount of information he contributes to information aggregation which is (in equilibrium) less than (at most) one signal.

The discussion above assumes a homogenous legislature but the results are generalizable to heterogeneous legislature as well. Section shows that in the case of heterogeneous legislature, the powerful group behaves as if the legislature is composed of only themselves with an adapted aggregation rule. So as far as equilibrium characteristics and the president are concerned, there is no loss of generality in assuming a homogenous legislature. However hetero-
geneous legislature never achieves full informational efficiency since votes of
the less powerful group members never affect the legislation process and their
private information (of less powerful group) are not utilized in the decision.

For all ideological profiles and aggregation rules R, the probability of the pres-
ident’s approval and the likelihood of the legislature passing the proposal in
the second round (if reached) are both weakly increasing in the tally of votes
(in favor of the proposal) in the first round. This is an evidence for the sig-
naling phenomenon. Realized tally in the legislature is a relevant information
for the president and the legislature members themselves in second round
voting (when applicable). Notice however that the legislature members must
also condition on the aggregation rule hence they also have standard-pivotal
motives. These points are parallel to Iaryczower (2008) that studies voting
and signaling in sequential committees. My single-round legislation model is
a special case of Iaryczower (2008) where the receiving committee consists of
single member. However in Iaryczower (2008) committees are heterogeneous
and not ideologically polarized. Thus he doesn’t investigate alternative ide-
ological profiles of two committees and strategic interaction among them (as in
this paper). Rather he classifies voting equilibria and gives conditions for in-
formative voting in both committees. Moreover he examines only single-round
voting. As I demonstrate equilibria of single-round legislation for various ide-
ological profiles of two sequential units (actors), I conjecture that the qualitatice
description of these equilibria will continue to hold for equivalent ideological
profiles of two sequential homogenous committees.

I hope my analysis is helpful to understand power balance between institu-
tions and information transmission in the real legislative processes of parlia-
mentary and semi-presidential democracies. The findings regarding the pres-
ident’s equilibrium action (veto or approval) and whether second round is reached in two-round legislation constitute testable hypothesis for empirical data on legislation as well as experimental economics. Another potential extension is optimal mechanism to reveal and utilize private information of all players to maximize likelihood of correct decision making. I leave these two promising topics for future research.

Acknowledgements

This is the Acknowledgements section. This section is placed at the end of the article, just before the references.

References

