Netnomics (2008) 9:125–151
DOI 10.1007/s11066-009-9032-7

A study of secondary spectrum use using agent-based computational economics

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Accepted: 6 January 2009 / Published online: 23 January 2009

Abstract Wireless communication relies on access to the radiocommunications spectrum. A series of high-priced spectrum auctions has indicated that there is a scarcity or an inefficient use of spectrum resources. This paper presents an Agent-based Computational Economics (ACE) model designed to study the *secondary use* spectrum market, which improves spectrum usage by allowing incumbent spectrum users to lease unused portions of their assigned spectrum to third parties who could put them to a better use. In this paper, we are particularly interested in the conditions under which such a market is likely to emerge.

Keywords Spectrum management · Agent · Secondary market

1 Introduction

Innovative wireless communications services are among the most cutting edge technologies. During the past decade, the wireless communications industry has grown by orders of magnitude due to technological advances that allow widespread deployment and smaller, more reliable, and more affordable equipment. This explosive growth has stimulated the demand for access to spectrum and has consequently driven regulatory bodies and researchers to look into a more effective use of the radiocommunications spectrum.

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Measurement studies of the spectrum use in the FCC's Spectrum Policy Task Force (SPTF) report indicate that portions of spectrum are not in use in many geographical areas for significant periods of time [2]. In response to this, the FCC has taken steps to facilitate the development of markets for spectrum usage rights to permit spectrum to flow freely among users in response to economic demand [3]. These markets would allow incumbent spectrum users to lease unused portions of their assigned spectrum to third parties who could put them to better use. Leese et al. show that these kinds of trades enhance productive efficiency [8]. If a trade can be conducted with transparency and accountability, spectrum trading may impose a clear, market-based opportunity cost upon incumbents, thereby providing them with incentives to conserve spectrum. In the long run, the development of secondary markets includes a market for license trading and one for temporary access of existing licensed spectrum.

Spectrum trading as well as secondary use of spectrum raises numerous technical, institutional, economic, and strategic issues that merit investigation, many of which have been addressed in the literature [5, 6, 8, 10, 15]. These researchers have built technical, economic and techno-economic models to analyze questions related to spectrum commons and spectrum reuse using economic, game and auction theory. These papers have produced impressive theoretical results that provide significant advice to policymakers (for example, Rodriguez and Jondral show that Ultra-Wideband (UWB) Personal Area Networks (PANs) create no economic loss to incumbent carriers [15]).

This paper focuses on the secondary use of spectrum, defined as a temporal use of licensed spectrum owned by an incumbent who is distinct from the (temporary) spectrum user. We wish to extend previous work in this area to domains where agents have uncertainties about market demand, are only boundedly rational, and may act opportunistically. Thus, our approach has been to focus on the effects of the transaction costs that ensue under these conditions and the uncertainties due to potential interference [23–25]. In taking the transaction cost approach, we assert that the transaction costs associated with the market mechanism must be sufficiently low for secondary use to become practical. We expect that the adoption of secondary spectrum use will depend on technical factors (such as types of wireless services, coverage, and application requirements) and economic factors (such as number of market participants and price setting). In addition, different forms of secondary use (e.g., spot markets, long-term lease, Mobile Virtual Network Operator (MVNO), and others) are potentially suitable in some environments and for some wireless services but not in others.

The Transaction Cost Economics (TCE) framework has proven to be useful for studying the relationship between the characteristics of transactions and the suitable organizational forms for carrying them out. Instead of explicitly measuring transaction costs (which is quite difficult), TCE analyses assume that agents seek to minimize these costs by adopting various organizational forms. TCE identifies *markets* and *firms (hierarchy)* as two polar forms in which a transaction between buyer and supplier can take place [16]. Between these two extremes, several intermediate forms exist including different types of contractual arrangements, alliances, and joint ventures. These intermediate forms are categorized as *hybrid forms* and they produce the positive features of markets and hierarchy while reducing the negative aspects of both. The preferred choice of organizational form depends on a comparison of the transaction costs under each alternative. TCE provides a framework to assess the strengths and weaknesses of each organizational form under different scenarios through the analysis of transaction characteristics and the behaviors of economic agents. Empirical studies demonstrate that a number of economic activities across various industries are generally aligned with the TCE framework [1, 17, 19].

Like Grandblaise et al. [5], we seek to explicitly associate technical parameters and economic effects. Because we seek to incorporate only boundedly rational agents, we use Agent-based Computational Economics (ACE) as a tool to model the development of transactions in secondary spectrum use rather than analytical techniques. ACE is the computational study of economies modeled as dynamic systems of autonomous interacting agents [22]. The merits of ACE are often compared to the mainstream, neoclassical approach in economics. With neoclassical analysis, economic agents are typically assumed to be homogeneous and fully rational. The goal is largely to derive analytic closed-form solutions (i.e., equilibria) of an economic system. Accordingly, ACE researchers often argue that this conventional analysis is a top-down and deductive approach. For tractibility, neoclassical analyses tend to represent, simplified, stylized settings of the economic system. In contrast, ACE works from the bottom up by creating adaptive, heterogeneous, and autonomous agents who interact with one another in dynamic environments. The goal of ACE is not to derive closed-form solutions, but rather to observe and study the aggregate outcomes and the norms of behavior that are developed and sustained over time.

Applying these ideas to spectrum, we can see that, at present, there are two main methods to obtain spectrum access: (1) use the unlicensed spectrum band and (2) acquire an exclusive spectrum license. The focus of this paper is to examine what happens when the secondary use of spectrum is introduced as a new option for spectrum access. Secondary use can allow a portion of spectrum users who cannot afford an exclusive license to become secondary users. In particular, we are interested in how user and provider agents change their spectrum access and use options under a variety of scenarios.

The purpose of this paper is to present our ACE model for studying secondary use and to present the initial results from that model. In Section 2, we review the agent-based spectrum access model and discuss the model implementation with descriptions of spectrum access transactions, agents, environments, and the model's relationship to the TCE framework. Then, Section 3 describes several experiments and presents and discusses the results. Finally, in Section 4, we draw general conclusions and discuss future directions for this research.

2 An agent-based model for secondary use of radio spectrum

This section introduces a discrete-time agent-based economic model for spectrum access, which is described in more detail in [26]. We use the Recursive Porous Agent Simulation Toolkit (Repast) as an agent modeling toolkit [12]. The model consists of two types of economic agents: spectrum access consumers and spectrum access providers. A spectrum access consumer is a new spectrum user who is seeking to obtain spectrum access and so is a potential secondary user. A spectrum access provider is a spectrum license holder. The provider may become a primary user if s/he allows secondary use by leasing portions of spectrum to the consumer. Our goal is to explore the emerging behavior among agents when the secondary use of spectrum is introduced as an additional method to obtain spectrum access. More specifically, we focus on the questions of when and why consumers would choose secondary use of spectrum and in what form. This study involves identifying processes or scenarios that leads to outcomes through the repeated interactions of these autonomous agents.

2.1 Structure of spectrum access transactions

Transaction cost economics is used to identify the organizational forms in which spectrum access transactions can take place. TCE identifies two behaviors underlying the existence of transaction costs: bounded rationality and opportunism. These behaviors are used by TCE to explain the decisions made by trading partners about the choice of organizational forms for the transactions.

In order to gain insight into the effects of transaction costs on the structure of spectrum access transactions, our model incorporates both of these behaviors into the agents. In the case of bounded rationality, consumer and provider agents are subject to several kinds of limitations. Agents do not have complete information about the environment in which they operate. Each provider does not have access to consumer preferences and their potential opportunistic behaviors. Similarly, each consumer does not have information about the behaviors of providers or other consumers. An agent must observe the outcomes of the transactions, learn from the interactions, and adapt its action to the dynamic environment. In addition, agents have limited processing capacity, which is implemented by using a reinforcement learning algorithm (described in more detail in Section 2.3) which has minimal complexity.

Opportunistic behavior can take many forms. In this paper, we focus on the impact of interference (i.e., wireless signal interference) on both consumers and providers. To model this, we allow provider agents to overstate supply quantities (with a certain probability); i.e., they sometimes lease spectrum that could create interference with other consumers in order to generate more revenue. Likewise, consumers also have the potential to understate their demand quantities; i.e., specify less spectrum than what they actually need in order to reduce cost. It should be noted that overstating the supply might not

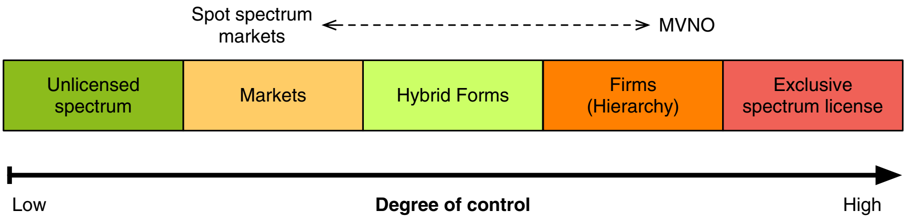

Fig. 1 Governance structures in secondary use of radio spectrum

always be the best strategy for providers. Depending on the price elasticity of consumers and the characteristics of the market, providers may instead understate their supply quantities in order to generate a higher market price. Understating the supply quantities, however, will not create interference for consumers. The implementation of opportunism in the agent-based model is further explained in Section 2.4.

Given the absence of widespread secondary use in practice, it is not clear how exactly the spectrum sharing via secondary use will take place. We expect that different forms of secondary use could affect the magnitude of the transaction costs and the level of uncertainty due to potential interference. These different forms of secondary use map to different choices of organizational form on the market-hierarchy continuum under the TCE framework. In the context of secondary use, an organizational form toward the market side implies a real-time spot spectrum market (or sharing formations of similar configuration), whereas the hierarchy side suggests formations similar to a MVNO.

We use *degree of control* as a common indicator to differentiate multiple forms of economic organization that can be used for organizing spectrum access transactions. Degree of control refers to the ability of the organizational form to contain opportunistic behaviors and to facilitate the compatibility of actions among transacting agents. As suggested by TCE [29], Fig. 1 illustrates that markets have a relatively low value for degree of control, hybrid forms possess intermediate values, and firms (hierarchy) have a relatively high value for degree of control.

Considering opportunistic behaviors and coordination capabilities in terms of their interference effects, using degree of control as an indicator allows us to incorporate traditional methods of spectrum access, namely unlicensed spectrum and exclusive license. Unlicensed spectrum does not provide interference protection, nor does it facilitate the coordination among unlicensed users. Harmful interference between unlicensed users or the tragedy of the commons that could render unlicensed spectrum unusable may occur.[1] Therefore, it has lower degree of control when compared to markets. On the other hand,

[1] We do not consider mesh networking scenarios as do Levine et al., so we see no capacity increases as the number of users increase [10].

a spectrum license gives a licensee exclusive access and a full interference protection. Thus, a license yields a higher degree of control than hierarchy.

2.2 Agent descriptions

Agents are autonomous entities capable of encapsulating their own data and behavioral methods. An agent's data may include its current inventory, the current market price, its utility function, etc., while the agent's behavioral methods include market protocols, pricing strategies, learning algorithms, etc. Some of the methods and data, such as protocols and price, can be publicly accessible, whereas others, such as inventory and strategies, should be contained within an agent. Agents also have communication capability and continuously exchange information with each other to achieve their own goals.

An agent in the spectrum access model is denoted as a spectrum user who utilizes the radio spectrum to provide a wireless service and/or application. Spectrum users are characterized by their spectrum needs, which in turn are influenced by their application requirements and network configurations. We adopt a utility-based approach to provide a unified framework for different spectrum-based applications and for spectrum users with different priority levels. Specifically, each spectrum user maintains its utility function as a perceived value (utility) of the received Quality of Service (QoS) of wireless transmission. We assume that the QoS depends on the received Signal to Interference-plus-Noise Ratio (SINR). We model the data rate as an increasing function of the received SINR. Different users may have different utilities from the same QoS, depending on their individual services and applications. Thus, various service requirements can be embedded in each user's utility function.

The communications system of each spectrum user is represented by a set of transmitters and receivers that belong to that user. Obviously, the direction of communications is from the transmitter to the receiver. Casting (e.g., unicast, multi-cast, and broadcast) is achieved by defining the pair or connectivity between transmitter(s) and receiver(s).

Spectrum users are further classified into two subclasses: consumer agents and provider agents. Consumer agents are spectrum users who are seeking spectrum access. Provider agents are spectrum users who hold exclusive spectrum licenses; hence they are the potential primary users. The remainder of this section describes the details of the consumer and provider agents, respectively.

Consumer agents are spectrum users who are actively seeking spectrum access but do not currently hold a spectrum license. Thus, they are the potential secondary spectrum users and are buyers in the market. As discussed in Section 2.1, we use *degree of control* (d) to identify the suitable organizational form(s) for a potential spectrum access transaction. Following Fig. 1, we assign values of degree of control so that consumers using the unlicensed spectrum will derive a degree of control of zero and those using the exclusive license will acquire a degree of control of one. It should be noted that these degree of control values should be interpreted as ordinal measures only as the objective here is to convey the relative ranking of different organizational forms. Consumer agents have three main methods to obtain spectrum access as follows:

- *Unlicensed spectrum:* When using unlicensed spectrum ($d_c = 0.0$),[2] a consumer agent randomly selects an operating frequency inside the unlicensed spectrum band at every time step of the model's execution. This implementation is similar to the frequency hopping technique currently used in the unlicensed band. It also implies that the consumer agent is free to select other spectrum access options as soon as the current time step ends. The price for using unlicensed spectrum is zero. The consumer, however, is taking the risk of being interfered with by other unlicensed spectrum users.
- *Secondary use:* If a consumer agent selects secondary use ($0.0 < d_c < 1.0$), the d_c value will represent the degree of control selected by the consumer. The provider must agree to the selected degree of control to be eligible for a transaction with this consumer. Therefore, consumer's d_c choice can control the outcome of the economic organization that will be used to organize the current spectrum access transaction. With the chosen value of d_c, the consumer agent will announce a spectrum access bid at period t (which is a time step in the model's execution) that includes the following information: the amount of spectrum (M_c^{total}), the degree of control ($d_{c,t}$), the access duration ($l_{c,t}$), and the bid price ($p_{c,t}^{bid}$). The buying bids of consumer agents and selling offers from provider agents will enter a competitive bidding process (see Section 2.5). We assume that the provider can arrange secondary access without interference over the entire simulation area. We also assume that the consumer pays an additional fixed cost that increases linearly with d_c for every secondary use transaction.
- *Exclusive license:* When selecting an exclusive license ($d_c = 1.0$), the consumer pays the license fee. The consumer agent is locked in and cannot select other spectrum access choices until the license expires.[3] We assume that when the consumer agent exercises the exclusive license option, s/he can expect an interference-free operation over the entire simulation area.

Given this array of choices for obtaining spectrum access, each consumer agent explores spectrum access options by repeatedly choosing among several combinations of {degree of control, access duration} or $\{d_c, l_c\}$ to satisfy his/her spectrum demand. For each spectrum transaction, the received utility (u) is

[2] d_c and d_p denote consumer's and provider's choices of *degree of control*, respectively.

[3] More generally, this is the cost of a technological alternative to secondary or unlicensed use, which could include wireline technology or the use of frequencies in a different band. In the more general case, license expiration can be taken to be the time in which the investment in the alternative technology is depreciated. The generalization of license cost should also include those costs associated with switching technologies.

the minimum received utility of all receivers of that consumer. Then, each consumer j calculates a surplus value ($s_{j,t}$) that s/he received as a function of the received utility ($u_{j,t}$) and the cost of spectrum access ($c_{j,t}$):

$$s_{j,t} = \alpha_{u,j} u_{j,t} - \alpha_{c,j} c_{j,t} \tag{1}$$

where $\alpha_{u,j}$ and $\alpha_{c,j}$ are the QoS sensitivity and the cost sensitivity of consumer j, respectively. The objective of the consumer is commonly assumed to maximize his/her surplus [27]. Accordingly, consumers use a learning algorithm to adapt their preferences on $d_{c_j,t}$ and $l_{c_j,t}$ based on the received surplus ($s_{j,t}$) from each transaction. The price update of the consumer's bid ($p^{bid}_{c_j,t}$) is discussed in Section 2.5.

Providers in the spectrum access model are spectrum users who currently own exclusive spectrum licenses. Therefore, they are the potential primary spectrum users and are sellers in the spectrum market. Similar to consumer agents, the provider agents' spectrum utilization is characterized by their application requirements and network configurations. We assume that the operating frequency of transmitters and receivers is within the bounds of the provider's spectrum license. The providers can sublease unused parts of their spectrum in terms of the number of BBUs and lease durations specified in consumer's spectrum access bids, subject to availability.[4]

Each provider agent selects a degree of control (d_p), which will represent the minimum threshold that is acceptable for the provider. The consumer must have an adequate degree of control ($d_c \geq d_p$) to be eligible for a transaction with this provider. Therefore, the choice of d_p controls the outcome of organization that will be used to manage this spectrum transaction. The range of d_p is between 0.0 and 1.0 as before, but the interpretation differs. For a transaction to take place, the minimum value of d_p must be equal to the minimum value of d_c. When a provider selects this minimum value, s/he indicates a willingness to conduct a transaction with any qualifying consumer agent. The maximum value of d_p is 1.0, which gives the provider an option not to participate in secondary use. This implementation is feasible because the maximum value of d_c for secondary use is always less than 1.0. With the selected value of d_p, the provider agent will announce a spectrum access offer at period t that includes the following information: the amount of shared spectrum ($M^{share}_{p,t}$), the minimum degree of control ($d_{p,t}$), the maximum lease duration ($l_{p,t}$), the frequency channel ($f_{p,t}$), and the ask price ($p^{ask}_{p,t}$).

The values of $M^{share}_{p,t}$, $l_{p,t}$, and $f_{p,t}$ may vary in each model time step depending on the characteristics of the provider's spectrum usage and the ongoing secondary use transactions. The offers from providers and the bids from consumers enter a competitive bidding process (see Section 2.5). The

[4]Thus, we implicitly assume that the provider uses some form of Frequency Division Multiple Access (FDMA) technology without loss of generality. We could imagine similar markets for time slots or noise levels in the case of Time- and Code- Division Multiple Access (i.e., TDMA and CDMA, respectively).

outcome will determine the transaction partner (consumer), the secondary use price (i.e., revenue from secondary use), and the organizational form (d) of the secondary use transaction. This information will be used by the provider to update the availability of spectrum in the subsequent offers.

With an array of degree of control choices, each provider agent repeatedly samples different d_p choices to find the best option for the current scenario. For each transaction, the provider calculates the received utility (u), which is the minimum received utility of all of the provider's receivers. Then, each provider k uses the received utility ($u_{k,t}$), the exclusive license fee ($C_{license}$), and the secondary use revenue to compute the surplus value ($s_{k,t}$):

$$s_{k,t} = \alpha_{u,k} u_{k,t} - \alpha_{c,k} C_{license} + revenue \tag{2}$$

where $\alpha_{u,k}$ and $\alpha_{c,k}$ are the QoS sensitivity and the cost sensitivity of provider k, respectively. Similar to consumer agents, the objective of providers is to maximize his/her surplus [27]. Accordingly, providers use a learning algorithm to adapt their preferences on $d_{p_k,t}$ based on the received surplus ($s_{k,t}$) from each transaction. The price update of provider's offer ($p^{ask}_{p_k,t}$) is discussed in Section 2.5.

2.3 Learning algorithms

Given the outcome of a transaction, both the consumer and provider agents adapt their future actions to maximize their own objectives (utilities). One of the popular learning algorithms used to capture the natural learning process of human and organization is *reinforcement learning*. The underlying concept of reinforcement learning is that the propensity to take any particular action should be increased or reinforced if it produces favorable outcomes and decreased if it leads to negative results [21]. The algorithm is decentralized and only requires information about an agent's past actions and the corresponding payoffs. The advantage of reinforcement learning is in its simplicity; it does not require high-level processing capabilities from the agent. Since reinforcement learning is relatively naive (in that it only uses local information), it is consistent with the notion of bounded rationality in TCE.

We implement a variation on this approach, called the *reinforcement comparison* method, which establishes a reference level of result (reference reward) and uses this reference value to evaluate the outcome of future transactions. The reinforcement comparison method maintains the propensity or preference for each action and uses the propensity values to calculate the probability that particular action will be chosen in the next transaction.

In the case of consumer agents, the action is the selection of $\{d_c \in \mathcal{D}_c, l_c \in \mathcal{L}_c\}$. The probability of consumer j selecting any d_c and l_c at period t_1 according to the softmax action selection rules is:

$$\rho_{c_j,t_1}(d_c, l_c) = \frac{\exp\left[q_{c_j,t_1}(d_c, l_c)\right]}{\sum\limits_{a \in \mathcal{D}_c, b \in \mathcal{L}_c} \exp\left[q_{c_j,t_1}(a, b)\right]} \tag{3}$$

The consumer obtains spectrum access based on the values of $\{d_{c_j,t_1}, l_{c_j,t_1}\}$ selected from this probability distribution. For example, if we assume that the transaction (i.e., unlicensed, secondary use, or exclusive license access) concludes at period t_2, the propensity of selecting d_{c_j,t_1} and l_{c_j,t_1} is updated by the difference between the received surplus (s_{j,t_2}) and the reference surplus (s^r_{j,t_1}):

$$q_{c_j,t_2}(d_{c_j,t_1}, l_{c_j,t_1}) = q_{c_j,t_1}(d_{c_j,t_1}, l_{c_j,t_1}) + \beta_{sc}(s_{j,t_2} - s^r_{j,t_1}) \tag{4}$$

Because of this update, the probability of selecting the same action on the next transaction will be increased if the received surplus is higher than the reference value, and decreased if the received surplus is lower. This adjustment is controlled by the step-size parameter β_{sc}.

After the propensity update in Eq. 4, the reference surplus is updated to include the received surplus (s_{j,t_2}). Instead of taking an average of all past surpluses as a new reference value, we weight the recent surpluses more heavily than the older ones. This exponential, recency-weighted average method is appropriate for a non-stationary environment [21]. The reference surplus update is as follows:

$$s^r_{j,t_2} = s^r_{j,t_1} + \alpha_{sc}(s_{j,t_2} - s^r_{j,t_1}) \tag{5}$$

Here, α_{sc} is the step-size parameter that controls the weight of the reference surplus update.

In addition to this learning process, we want to make sure that consumer agents explore the whole $\{d_c, l_c\}$ space and do not bind to certain solutions. We add a stochastic element into the model by assigning a small probability, δ_{c_j}, that consumer j will not follow the probability distribution $\rho_{c_j,t}(d_c, l_c)$ as suggested by the reinforcement learning in Eq. 3. Instead, consumer j will randomly select $\{d_c, l_c\}$ choices (i.e., using the uniform distribution):

$$\rho_{c_j,t}(d_c, l_c) = \begin{cases} \dfrac{\exp\left[q_{c_j,t}(d_c,l_c)\right]}{\sum_{a\in\mathcal{D}_c, b\in\mathcal{L}_c}\exp\left[q_{c_j,t}(a,b)\right]} & \text{with probability } 1-\delta_{c_j} \\ \dfrac{1}{\text{Total number of choices}} & \text{with probability } \delta_{c_j} \end{cases} \tag{6}$$

The learning process of provider agents is the selection of degree of control value ($d_p \in \mathcal{D}_p$) to control the organization of the spectrum access transaction. The process follows the reinforcement comparison method similar to the one we described for consumer agents.

2.4 Opportunistic behavior modeling

Transaction costs emerge because transacting partners have the potential to behave opportunistically in order to gain an advantage in a trade. Here, we focus on the impact on interference of opportunistic transactions. An opportunistic consumer will understate the original demand quantity in order to reduce secondary use cost. When announcing a spectrum access bid, the original quantity M_c^{total} will be replaced with the new quantity, M_c^{total*}. The opportunistic consumer will operate in the provider's spectrum in excess of the quantity stated in the bid. Hence, the effect of a consumer's opportunism is potential interference to the provider's receivers. To simplify the model, we use $M_c^{total^*} = M_c^{total}/2$ for all opportunistic consumer agents.[5]

Opportunistic consumers can also learn from their opportunistic behaviors in the current transaction and decide whether they should act opportunistically in future transactions. We also use the reinforcement comparison method for this learning process. Let the opportunistic choice of consumer j, $o_{c_j,t}$, be 0 when consumer j decides not to become an opportunist and 1 otherwise. The probability that consumer j will act opportunistically at period t_1 is:

$$\rho_{c_j,t_1}(1) = \frac{\exp\left[q_{c_j,t_1}(1)\right]}{\exp\left[q_{c_j,t_1}(0)\right] + \exp\left[q_{c_j,t_1}(1)\right]} \tag{7}$$

For example, if we assume that, when the transaction concludes at period t_2, the propensity of making an opportunistic choice o_{c_j,t_1} in period t_1 is updated by the difference between the received profit (π_{j,t_2}) and the reference profit (π^r_{j,t_1}):

$$q_{c_j,t_2}(o_{c_j,t_1}) = q_{c_j,t_1}(o_{c_j,t_1}) + \beta_{oc}(\pi_{j,t_2} - \pi^r_{j,t_1}) \tag{8}$$

where β_{oc} is the step-size parameter that controls the rate of propensity update. The exponential, recency-weighted average method then is applied to the reference profit update:

$$\pi^r_{j,t_2} = \pi^r_{j,t_1} + \alpha_{oc}(\pi_{j,t_2} - \pi^r_{j,t_1}) \tag{9}$$

where α_{oc} controls the update weight. The profit from opportunistic behavior can be specified as follows:

- If consumer j does not act opportunistically (i.e., $o_{c_j,t_1} = 0$), there will be no gain from this action. Thus, the received opportunistic profit upon the ending of transaction is 0.0 ($\pi_{j,t_2} = 0.0$).
- If consumer j acts opportunistically (i.e., $o_{c_j,t_1} = 1$), the profit from this action will be in the form of a reduced cost of the spectrum access transaction. Thus, the received profit upon the ending of transaction, π_{j,t_2}, is set to the difference between the spectrum access cost with quantity M_c^{total} and the spectrum access cost with quantity $M_c^{total^*}$.
- If consumer j receives an interference complaint from his/her transacting provider, it implies that the transacting provider is suffering interference from one or more secondary use transactions. This situation will discourage the provider from participating in a secondary use transaction in the future. As a consequence, consumer j may have to rely on an exclusive license for spectrum access due to the limited availability of spectrum for

[5] Although we expect that opportunistic spectrum users would abuse the transaction with a small probability (e.g., 0.10), we select an aggressive margin to intensify the effects of opportunistic behavior in order to speed up the convergence of the agent-based model.

secondary use. Thus, upon receiving the interference complaint, consumer j will update the choice of acting opportunistically (i.e., $o_{c_j,t_1} = 1$) with the received profit, π_{j,t_2}, of negative exclusive license cost ($-C_{license}$).

Consumers also react to the opportunistic behavior of provider agents. We assume that a consumer agent can distinguish his/her transacting partners and develop as well as maintain basic trust information on each of his/her partners. Following Klos's agent-based modeling of trust [7], trust is defined as the ability to act according to expectation. In our case, consumer agents expect secondary use of spectrum without interference. We further assume that the consumer's trust in a particular provider increases with the number of consecutive transactions without interference when the consumer transacts with that provider. The development of consumer's trust is specified by the following equation [7]:

$$trust = T_{base} + (1 - T_{base})\left(1 - \frac{1}{xT_F + 1 - T_F}\right) \tag{10}$$

where T_{base} is the base-level of trust; x is the number of consecutive transactions without interference; and T_F is a parameter that controls trust development rate. From Eq. 10, the range of *trust* value is [0, 1]. We apply the following rules to trust modeling: when consumer j experiences interference while using the spectrum of provider k, consumer j's trust in provider k is reduced by half. If trust drops below T_{thres}, consumer j will transact with provider k with a probability T_{prob}; otherwise, consumer j will always transact with provider k. If consumer j does not experience interference while transacting with provider k, his/her trust in provider k starts to increase at the rate specified by Eq. 10.

An opportunistic provider will overstate the original supply quantity in order to generate more revenue. In doing this, the opportunistic provider will continue to announce a spectrum access offer even after all spectrum available for secondary use has been leased to other consumers. The consumer who transacts with this provider will operate in the frequencies that are being used by other consumer agents. Therefore, the effect of the provider's opportunism is potential interference to the consumer's receivers.

Opportunistic providers use reinforcement comparison to learn and maximize the profit from opportunistic behavior choices in a manner similar to the consumer agents. The profit from a provider's opportunistic behavior is described as follows:

- If provider k does not act opportunistically (i.e., $o_{p_k,t_1} = 0$), there will be no gain. Thus, the received opportunistic profit upon the ending of transaction is 0.0 ($\pi_{k,t_2} = 0.0$).
- If provider k acts opportunistically (i.e., $o_{p_k,t_1} = 1$), the profit from this action will be additional revenue from spectrum access offers in excess of the provider's truthful capacity. Thus, the received profit upon the ending of transaction, π_{k,t_2}, is the revenue of that opportunistic transaction.
- Provider k's spectrum offers may be rejected by a particular consumer, which implies that the consumer's trust in provider k is below T_{thres} because

the consumer has encountered interference in a previous transaction with provider k. With a low trust value, the consumer will avoid making secondary use transactions with provider k. As a result, provider k will receive fewer transactions and, hence, less secondary use revenue. Therefore, upon receiving a consumer's rejection of the spectrum offer, provider k will update the outcome of opportunistic behavior (i.e., o_{p_k,t_1}) with the received profit, π_{k,t_2}, equivalent to the negative revenue of the failed offer (i.e., the opportunity cost).

Providers also react to the opportunistic behavior of consumer agents. Here, providers expect an interference-free operation when they are sharing spectrum for secondary use. In contrast to the consumer's opportunistic model (where consumers maintain trust information on his/her partners), providers do not implement the trust model because it is difficult and costly for providers to identify the sources of interference (i.e., opportunistic consumers). This is especially true when each provider can serve multiple spectrum access consumers at the same time. Instead, provider k will issue an interference complaint to all consumers who are operating in provider k's spectrum as soon as s/he experiences interference while allowing secondary use to these consumers. In addition to this, provider k will choose the minimum degree of control, d_p, to limit the consumer's opportunistic behavior. Effectively, this removes spectrum from the secondary use market, which should have the effect of increasing prices to consumers, *ceteris paribus*, and decreasing their profits.

2.5 Spectrum leasing and pricing

We use an *auction* to determine the price of spectrum access transactions between consumer and provider agents. The *continuous double auction* (CDA) has been widely studied and is the dominant approach used in real-world equities and derivatives trading. In CDA, bids and asks are publicly announced and traded at any time during the trading period without relying on a central auctioneer [4]. Experiments indicate that the CDA mechanism can produce reliable price convergence close to theoretical equilibria [20]. In our agent-based model, we follow a form of CDA, the persistent shout double auction algorithm by Preist and Van Tol [13, 14]. In this auction, an agent may make an offer to buy or an offer to sell at any time. The offer is, however, persistent until the owner revises or removes it in response to other trading activities. Once bids and asks are met, they are removed from the transaction system and a trade takes place. Contracts between user and provider agents (as would be used in today's MVNO arrangements) are not within the range of choices offered to agents in this model.

Our objective in using CDA is to determine the equilibrium price of secondary use transactions given the current supply and demand for spectrum access. Although the negotiation in CDA may result in trades taking place at different prices and away from the equilibrium, several repetitions of the auction with the same bids and asks converge to the equilibrium price [14, 20]. Therefore, consumer and provider agents in the model will engage in a series of mock auctions before the real auction takes place in the final round. These mock auctions will allow agents to respond to market conditions and other trading activities before holding the final auction. Hence, the final auction will result in trades taking place closer to the equilibrium than a single auction [14].

Recall that our agent-based model for secondary use operates on a set of conditions that includes degree of control, lease duration, and amount of spectrum. These factors place constraints on secondary use transactions in addition to the reservation price. Let R, D, L, and M denote reservation price, degree of control, lease duration, and amount of spectrum, respectively. Also, let b_i and a_j represent bid i and ask j, respectively. To determine the eligibility of a consumer's bids and a provider's asks, the following conditions are tested:

- *Reservation price*: $R_{b_i} \geq \min_j(R_{a_j})$ and $R_{a_j} \leq \max_i(R_{b_i})$
- *Degree of control*: $D_{b_i} \geq \min_j(D_{a_j})$ and $D_{a_j} \geq \min_i(D_{b_i})$
- *Lease duration*: $L_{b_i} \leq \max_j(L_{a_j})$ and $L_{a_j} \geq \min_i(L_{b_i})$
- *Amount of spectrum*: $M_{b_i} \leq \max_j(M_{a_j})$ and $M_{a_j} \geq \min_i(M_{b_i})$

Each bid must have a least one ask that satisfies all four constraints to be considered eligible (and *vice versa*). All ineligible bids and asks will be removed from the auction process.[6]

In addition to the four constraints, a consumer agent may decline to transact with a particular provider agent based on the their level of trust, as discussed earlier. In this case, the transaction between these two agents will occur with a probability T_{prob}, otherwise the provider's ask will be removed.

The agent's algorithm in the auction has minimal complexity. It consists of simple heuristics and uses simple learning rules. Each agent i maintains a profit margin, μ_i, which governs the buying/selling price relative to the agent's reservation price, R_i. The individual agent calculates the current price, p_i, as follows:

$$p_i = R_i(1 + \mu_i) \tag{11}$$

For a provider agent, the price p_i represents the minimum price at which the provider will make a trade. The profit margin for providers must lie in $[0, \infty)$. This implies that a provider's margin can be raised by increasing μ_i and lowered by decreasing μ_i. In the case of a consumer agent, the price p_i represents the maximum price at which the consumer is willing to pay. The profit margin for consumers is in the range $[-1, 0]$. Therefore, a consumer's profit margin can be increased by decreasing μ_i and reduced by increasing μ_i.

Each agent begins with a random profit margin. The profit margin μ_i is then adjusted dynamically in response to the actions of other agents and the trading activities. If the agent sets the profit margin too low, it will not make

[6] Because there are four constraints to be matched, we assume that sufficient market thickness exists. We will examine this assumption in future research.

as much profit as it could. On the other hand, if the profit margin is too high, it may be undercut by other agents and will not be able to secure a deal. The decision to increase or decrease the profit margin is based on the objective of maintaining a competitive bid/ask price compared to other agents. In order to do this, an agent maintains a target price, τ_i ($\tau_i = R_i(1 + \mu_i)$), and follows the simple heuristics: if trades are not taking place in the current trading period, an agent should set its target to the most competitive bid/ask. Thus, the target price should be slightly better than the agent's competition. If, however, trades are taking place, an agent should set its target to be slightly better (i.e., higher profit) than the best price that can result in a transaction. Here, the agent anticipates that it could have asked even a higher profit and still secured a deal.

In our model, τ_i denotes a target price, b_{max} represents the highest bid, a_{min} the lowest ask in the current trading round, and r is a small random variable. The following pseudocode summarizes the heuristic for consumer agents.

if $a_{min} > b_{max}$ **then**
  $\tau_i = (1 + r)b_{max}$
else
  $\tau_i = (1 - r)a_{min}$
end if

The heuristic for provider agents is as follows:

if $a_{min} > b_{max}$ **then**
  $\tau_i = (1 - r)a_{min}$
else
  $\tau_i = (1 + r)b_{max}$
end if

Following Preist and Van Tol [13, 14], we implement r as a random variable uniformly distributed over the range [0, 0.2].

Agents who are not participating in an auction (i.e., they are successfully engaged in a trade or they do not wish to participate at the current period) also continuously observe the activities of other agents. In this case, they have little or no incentive to lower their profit margins. However, if the trade activities suggest that they could benefit from raising the profit margin, they would do so. Thus, both active and inactive agents can increase their profit margins, but only active agents can reduce their margin in response to the current condition.

With the target price τ_i, an agent adjusts his/her current price p_i towards the target using the Widrow-Hoff with momentum learning rule, which is an adaptation algorithm used in back-propagation in neural networks [18]. The algorithm specifies two parameters: the learning rate coefficient β and the momentum γ. Given the price $p_i(t)$ at time t, the price at time $t + 1$ is given by:

$$p_i(t + 1) = \gamma p_i(t) + (1 - \gamma)\beta(\tau_i(t) - p_i(t)) \tag{12}$$

Here, the price will move towards the target with the speed determined by β. The momentum γ is used to reduce oscillation in the price adjustment. In our experiments, the values of β and γ are set to 0.3 and 0.05, respectively [13, 14].

As we mention earlier, consumer and provider agents will participate in a series of trading periods (i.e., mock auctions) and the real auction will take place in the final period, when the price has stabilized. We fix the number of mock auctions at 500 auctions.

3 Results and discussion

In all experiments, each spectrum user is represented by a wireless system that consists of a set of wireless transmitters and receivers. We assume that a consumer's wireless system is a two-way infrastructure network that consists of one base station and five wireless clients. In contrast, a provider's wireless system is modeled as a one-way infrastructure network, since we assume that primary users (i.e., provider agents) share the spectrum of their downlink channels. The default settings for the number of base stations (transmitters) and clients (receivers) of provider agents are 0.5625 transmitter per sq. km and ten receivers per transmitter, which represents a medium-level density of transmitters and receivers. In both the consumer and provider cases, base stations are randomly placed in the environment and wireless clients are randomly positioned inside the coverage area of their base station. An example configuration of a consumer's wireless system is shown in Fig. 2. The locations of the base stations and the wireless clients are also randomly changed in every run.

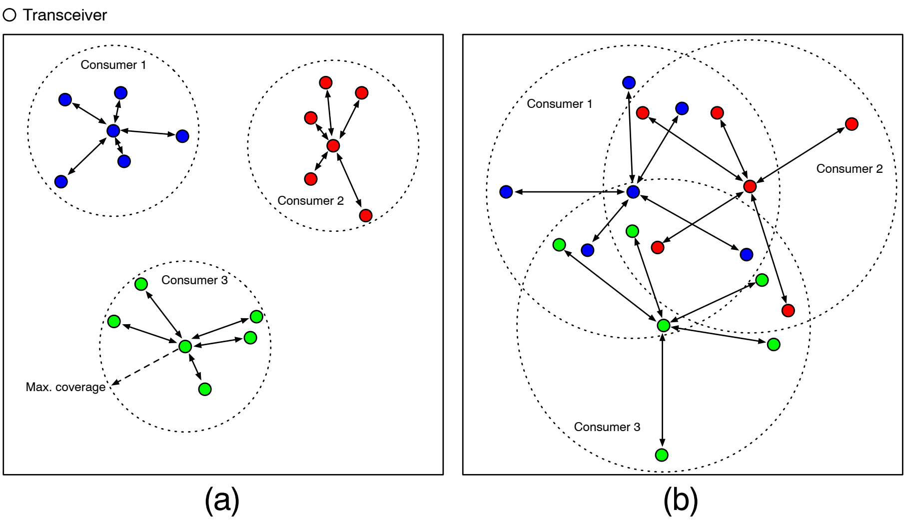

Fig. 2 Example of three consumer agents: **a** With small coverage area and **b** With large coverage area

Transmit power is calculated to provide reliable communications (i.e., satisfy receiver's SINR requirement) for any receiver inside the transmitter's coverage area. The utility function of both consumers and providers is defined as:

$$\mathcal{U}(\gamma) = \begin{cases} 1 - \exp\left(-(\gamma - \gamma^*)/\eta\right) & \text{if } \gamma \geq \gamma^* \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

where γ is the received SINR, γ^* is the SINR threshold value, and $\eta(> 0)$ is the parameter that can be varied to obtain different levels of concavity and correspondingly different levels of application requirements. To run the simulation, we had to set the model's parameters. For the kinds of questions we chose to address, the model area had to be of a size to accommodate multiple cells that could be made to overlap or not to overlap, based on the path loss coefficient and the transmitter power. We chose to model the 1900 MHz band, which is used for licensed 3G mobile services; this band is also close to the unlicensed 2400 MHz band, so a single path loss exponent (set to 3.5) can be used. An overall simulation area of 16 sq. km (i.e., a square 4 km on each side) was of the appropriate size to contain several cells, whose radius we vary in different experiments; Fig. 2 shows two different configurations of the simulated system. Additionally, for the results reported in this paper, we assume: that each spectrum user has a hard requirement on the received SINR with $\gamma^* = 15$ dB and $\eta = 0.001$, that both the consumer's and the provider's receivers have a sensitivity of 100 dBm, and that all spectrum users may behave opportunistically. The results are presented as average values over multiple runs.

3.1 Emergence of secondary use

The objective of this set of experiments is to study the behavior of consumer agents in terms of their selections of degree of control (d_c) in response to the existence of secondary use, number of consumers in the environment (N_C), and spectrum access characteristics of consumer agents.

Thus, there are two scenarios in this set of experiments. The first scenario examines the case where secondary use of spectrum is not permitted (which is the current policy scenario). In this scenario, each consumer agent must either use unlicensed spectrum or acquire an exclusive license in order to obtain spectrum access. The second scenario presents the case where secondary use is introduced, and consumers are allowed to choose intermediate values for degree of control.

Figure 3 shows the percentage of spectrum access options for different maximum coverage (D_{max}) and QoS sensitivity (α_u) of five consumer spectrum users under the first scenario and the second scenario for three different cell sizes. Since the secondary use option is not allowed in the first scenario, spectrum access choices are limited to unlicensed spectrum and exclusive licenses and the degree of control value for each consumer is either 0.0 or 1.0.

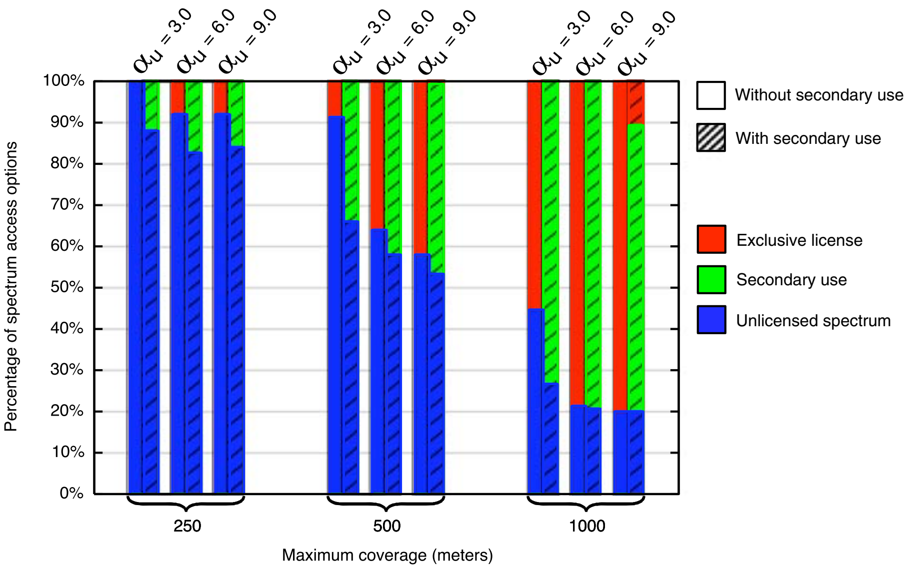

Fig. 3 Percentage of spectrum access options for 5 consumers

In Fig. 3, all consumers operate in the unlicensed spectrum when cell sizes are small ($D_{max} = 250m$) and α_u is 3.0. As the QoS sensitivity (α_u) increases, the negative effects of interference in the unlicensed band start to outweigh the cost of an exclusive license. In other words, those consumers with stringent QoS requirements have less tolerance for interference. As a result, some of the consumer agents switch to exclusive licenses. The percentage of exclusive license is, however, less than 10%.

This figure also shows an increasing use of exclusive licenses as the maximum coverage of each consumer cell increases. The larger coverage requires a transmitter to increase its transmit power, producing higher interference for other unlicensed spectrum users. At D_{max} of 500 m, there is evidence of exclusive license usage as early as α_u of 3.0. As we increase α_u, the effects of larger coverage result in more than 35% and 40% of exclusive license use for α_u values of 6.0 and 9.0, respectively. The results continue to follow this trend at D_{max} of 1000 m. It should be noted that although the unlicensed band can accommodate at least five consumers ($U/M_c^{total} = 50/10 = 5$), the results show that the exclusive license choice existed in most of the scenarios. Such an outcome is not unexpected, considering that the unlicensed spectrum does not facilitate any coordination among the unlicensed users. As a result, operating frequencies of unlicensed users may overlap and create interference. These interference effects can be so severe that some agents opt for exclusive licenses even when the unlicensed band is not yet fully congested.

Figures 4 and 5 present the results where N_C is 13 and 19, respectively. With an increase in N_C, the unlicensed spectrum gets congested more easily. The figures show an increase in the number of consumers operating in the same

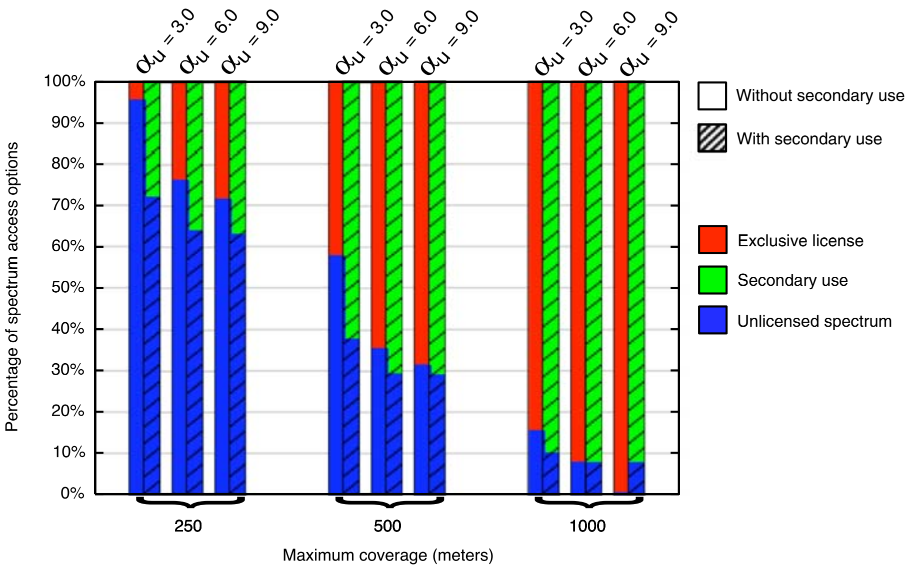

Fig. 4 Percentage of spectrum access options for 13 consumers

area, which yields a higher use of exclusive licenses (as expected). With 13 consumer agents in Fig. 4, the use of exclusive licenses appears at all parameter levels, starting from 4.62% ($D_{max} = 250$, $\alpha_u = 3.0$) to the highest value of 99.74% ($D_{max} = 1000$, $\alpha_u = 9.0$). These percentages are even higher when we

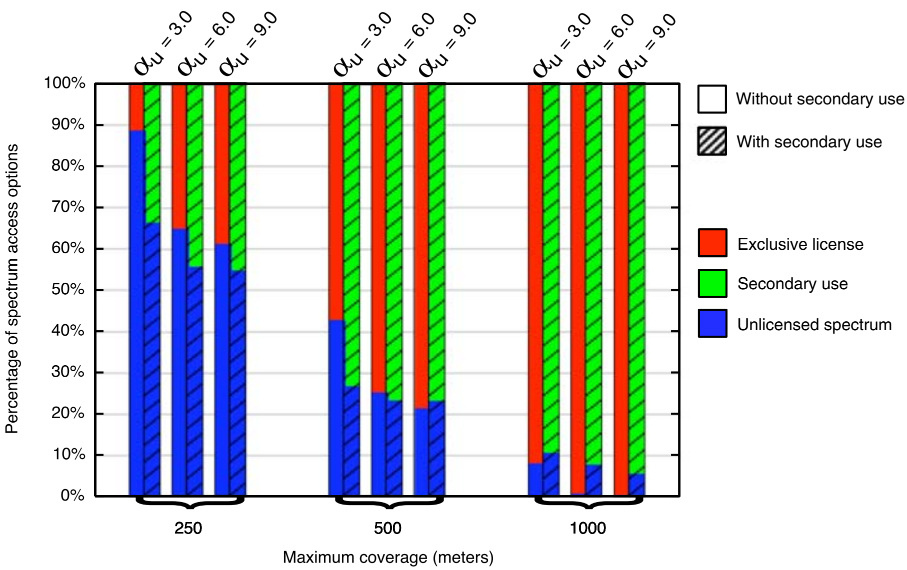

Fig. 5 Percentage of spectrum access options for 19 consumers

have 19 consumer agents (Fig. 5), ranging from 11.58% ($D_{max} = 250, \alpha_u = 3.0$) to 100% ($D_{max} = 1000, \alpha_u = 9.0$).

When secondary use is included as an option (in the second scenario), the role of provider agents (spectrum incumbents) comes into play. Each provider holds an exclusive license and shares an unused amount of spectrum with consumers for a period of time. We assume that the number of provider agents is 19. The results of this scenario are displayed in the cross-hatch pattern in Figs. 3–5. Consider Fig. 3: when D_{max} is 250m and α_u is 3.0, all consumer agents choose unlicensed spectrum when secondary use is not permitted. With secondary use, the results show that 12% of consumers choose secondary use. Here, the utility and cost of exercising the secondary use option outweighs the utility of using the unlicensed spectrum. This outcome implies that secondary use is a viable alternative for spectrum access when an exclusive license is considered to be too expensive. The percentage of secondary use is also higher when α_u is increased to 6.0, as expected. The difference between α_u of 6.0 and 9.0 is less noticeable because agents who are suffering interference in the unlicensed band have already chosen secondary use.

The amount of secondary use also increases with the maximum coverage. As the coverage increases, consumer agents are more likely to encounter interference from other agents using the unlicensed spectrum. Therefore, some of them switch to secondary use as an alternative to the crowded unlicensed band and the expensive exclusive license.

The results also show that when the secondary use option is introduced, the demand for exclusive licenses is completely eliminated except for one case. In this case ($N_C = 5$, $D_{max} = 1000$, and $\alpha_u = 9.0$), the percentages of exclusive license and secondary use are 10.67% and 69.33%, respectively. These results are the consequence of opportunism and small numbers bargaining [16]. Since the number of consumers is only five and each consumer has the potential to behave opportunistically, there is insufficient competition to drive secondary use transactions. The trace data confirms that exactly one consumer agent can occupy the unlicensed band with acceptable QoS. The other four only select secondary use in some runs or only use exclusive licenses in other runs. In the runs in which all four consumers chose exclusive licenses, the data reveals that providers suffer interference from the opportunistic consumers. As a result, they elect not to participate in secondary use by choosing a high value of degree of control (d_p). The average d_p in these runs is greater than 0.95. Hence, the four consumers have no choice but to use exclusive licenses. This situation does not occur when the number of consumers is higher, as in Figs. 4 and 5.

Another point of interest is the percentage of unlicensed spectrum use when secondary use is allowed. At D_{max} of 1000 m, Fig. 5 shows that the percentage of unlicensed spectrum is higher than without secondary use. In such scenarios, where a large number of consumers have strict QoS requirements and the coverage is also large, the unlicensed spectrum becomes over-saturated due to high contention for the shared spectrum resources. Without the secondary use option, this tragedy of the commons renders the unlicensed band unusable. As a result, unlicensed use drops to 0.53% at α_u of 6.0 and down to 0% at α_u of

9.0. On the other hand, when secondary use is introduced, some consumers opt for secondary use, making others experience less contention for the unlicensed spectrum. Thus, the unlicensed band becomes usable again. In this case, the occupant of the unlicensed spectrum is often the consumer agent who is the first mover into the unlicensed band (which raises interesting political economy questions that are outside the scope of this paper).

These experiments demonstrate that the agent-based model can produce outcomes corresponding to what might be expected in real-world spectrum use. Unlicensed spectrum can accommodate a limited number of spectrum users without causing excessive interference. In addition, spectrum users whose application requires a large geographical coverage or demand a high-level QoS tend to acquire exclusive licenses for spectrum access. Finally, the introduction of secondary use allows a portion of consumers who cannot afford exclusive licenses to become secondary users. Although we did not perform welfare analysis, this outcome is suggestive of an increase in overall welfare because both primary users (who now profit from otherwise unutilized spectrum) and secondary users (who now incur lower costs to provide the same service) are better off.

3.2 Feasible regions of secondary spectrum markets

In this set of experiments, we study the feasibility of the secondary spectrum market in terms of the number of participants. This can be taken as one measure of market thickness [11]. We perform experiments on the number of consumer (N_C) and provider (N_P) agents and measure the percentage of consumers selecting secondary use as the spectrum access option in each experiment.

Figure 6 presents the results of percentage of secondary use for different numbers of consumers and providers in the environment. The figure shows that secondary use increases as the number of consumers and providers

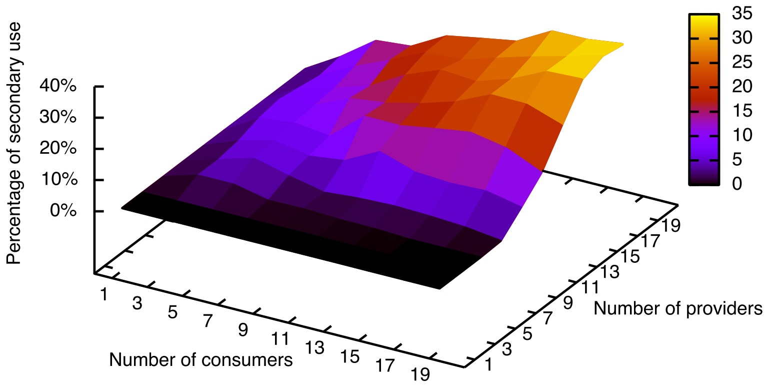

Fig. 6 Percentage of secondary use vs. number of consumer and provider agents

increases. With a small number of providers, there is a lack of viable competition. Thus, these providers enjoy monopoly advantages. They can charge the most profitable price that the consumers would accept (i.e., near or at consumer's reservation price). A small number of providers also limit the amount of spectrum available for secondary use. In addition to this, a small number of participants, whether they are consumers or providers, can engage in opportunistic behaviors and result in interference among agents without significant consequence. This is in contrast to the situation where there is a large number of participants. As Williamson [16] observed: "...rivalry among large numbers of bidders will render opportunistic inclinations ineffectual. Parties who attempt to secure gains by strategic posturing will find that such behavior is nonviable." As a result, consumers may find the secondary use option undesirable when the number of participants in secondary use is low.

Figure 7 is a contour plot of the percentage of secondary use in Fig. 6. The contour lines demonstrate that consumer agents are taking advantage of secondary use at a 5% level, even if there are a small number of consumers in the environment ($N_C = 3$). However, the 5% secondary use only emerged when there were at least seven provider agents in the environment. At $N_C = 9$, the secondary use reached a 20% level with N_P of at least 15. The percentage did not go beyond 20% because the unlicensed spectrum can accommodate the other 80% without harmful interference. As the number of consumers increases, the percentage of secondary use becomes higher because of the congestion in the unlicensed band. The higher percentage also requires a higher number of providers in the environment. For example, secondary use can reach a 30% level when there are at least 15 consumers and 15 providers.

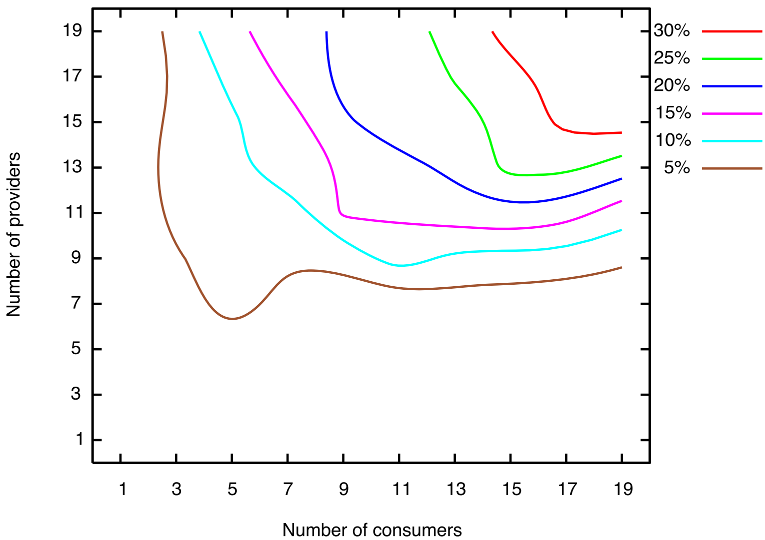

Fig. 7 Contour lines of the percentage of secondary use

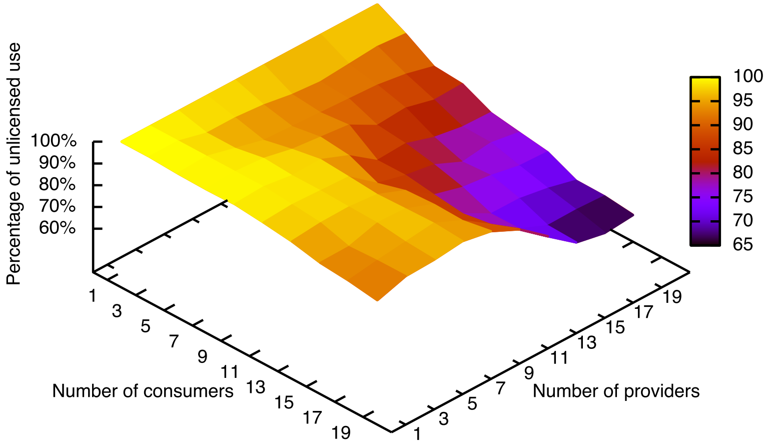

Fig. 8 Percentage of unlicensed use vs. number of consumer and provider agents

Figures 8 and 9 display the results of the other two spectrum access options, unlicensed spectrum and exclusive license, respectively. The results of unlicensed spectrum usage show that a small number of consumers (i.e., $N_C = 1$ and 3) select the unlicensed spectrum on average more than 90% of the time because they can coexist without interference. As N_C increases, the unlicensed spectrum gets crowded and the effects of interference starts to outweigh the zero price, so consumer agents seek other alternatives. From Fig. 8, the deviation from unlicensed spectrum (i.e., the percentage drops) occurs in two cases: (1) when the number of consumers and providers increases or; (2) when only the number of consumers increases.

The first case can be explained by comparing Fig. 8 to Fig. 6. The increases in N_C and N_P result in a higher percentage of secondary use while unlicensed spectrum usage is reduced. The drop in the percentage of unlicensed usage

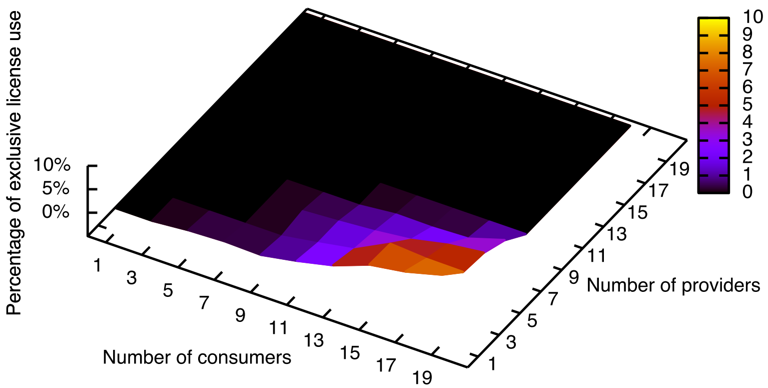

Fig. 9 Percentage of exclusive license use vs. number of consumer and provider agents

is particularly noticeable when $N_P \geq 9$. Similarly, a comparison of Fig. 8 and Fig. 9 describes the second case. The increase in N_C alone when the number of providers is low (i.e., less than nine in most cases) leads to an increasing use of exclusive licenses. This outcome derives from two factors: (1) the unlicensed spectrum is crowded resulting in interference among users and; (2) secondary use is undesirable due to a small number of providers. The data shows that consumers start using exclusive licenses from $N_C = 5$. The percentage increases noticeably when $N_C \geq 11$ and reaches as high as 9.82% at $N_C = 19$.

To evaluate the significance of these qualitative conclusions, we performed statistical testing using multiple regression analysis, with the percentage of secondary use as a dependent variable and the number of consumers (N_C) and the number of providers (N_P) as the independent variables. Table 1 presents two regression models. Model 1 uses N_C and N_P as independent variables without considering the interaction effect. The result shows that both N_C and N_P have a significant effect on the percentage of secondary use. The R^2 value for this model is .452. Once we incorporate the interaction term, $N_C \times N_P$, into the regression, Model 2 achieves a higher R^2 of .567. From Table 1, the regression equation for Model 2 is

$$\begin{aligned}\text{Percentage of secondary use} &= 2.235 - 0.602 N_C + 0.136 N_C N_P \\ &= 2.235 + (-0.602 + 0.136 N_P) N_C \qquad (14)\end{aligned}$$

Both N_C and $N_C \times N_P$ are significant in this model, while N_P is not significant. Since the dependent variable measures the choices of consumers, the change in N_P alone does not affect the percentage of secondary use. However, the increases in both N_C and N_P result in a higher percentage of secondary use, which confirms our analysis of Figs. 6–9. The number of consumers alone also significantly influences the secondary use because the capacity and interference effects in the unlicensed spectrum varies with the number of consumers. The

Table 1 Regression analysis of percentage of secondary use on number of consumers and providers

Variable	Model 1		Model 2	
	b	Beta	b	Beta
N_C	.754*** (.031)	.327	−.602*** (.056)	−.261
N_P	1.357*** (.031)	.588	.000 (.056)	.000
$N_C \times N_P$			.136*** (.005)	.898
Constant	−11.326		2.235	
Adjusted R^2	.452		.567	

$N = 3000$; b = unstandardized regression coefficient with standard error in parentheses; *Beta* = standardized regression coefficient
$^{*}p \leq .05$, $^{**}p \leq .01$, $^{***}p \leq .001$

regression in Eq. 14 shows that the increase in N_C will produce a higher percentage of secondary use (i.e., the coefficient of N_C will become positive) when N_P is at least five.

In summary, the statistical analysis confirms our previous conclusion that the secondary use of spectrum only emerges when there are a large number of participants.

4 Conclusion

Increased demand for spectrum has caused regulators to consider a variety of strategies to improve access to spectrum. This has included auctioning licenses for more spectrum, seeking approaches to spectrum trading and increasing unlicensed spectrum. Simultaneously, many countries are seeking ways of stimulating innovation in wireless devices. This has led to new technologies, such as UWB, and policies, such as "open access" (see, for instance, Wilkie [28]), which open markets. It is far from clear which approach (or portfolio of approaches) will meet social goals (see, for example, [8, 9]).

Beyond limited cases, such as MVNOs, policymakers have no concrete information about what the consequences of changes in spectrum policy might be, especially when new mechanisms, such as market-based secondary use, are introduced. Researchers have risen to the occasion to provide some advice (see, for instance Rodriguez [15]). We set out to build a platform that enables researchers to understand the effects of complex user behavior in markets (such as secondary use) that have not previously existed. The purpose of this paper is to describe this agent-based computational economics (ACE) model in conjunction with TCE which we optimized to study the secondary use of radio spectrum.

In this application of the ACE model, we study the effect of secondary use as a new spectrum access option. The results presented here show that the secondary use is a viable alternative for spectrum users who find the exclusive licenses too expensive or the unlicensed band too crowded. The fact that the main results of this paper are intuitively satisfying is a validation of the model that we developed.

It should be noted that auction-based secondary use only emerges when there is sufficient market thickness. We have not yet studied the lower bound of market thickness, or liquidity, which will be necessary for markets to function. This condition creates a competitive market force to drive secondary use transactions and suppresses opportunistic behaviors of the secondary use participants.

These results have some practical significance. We have seen liberalization in primary markets for spectrum (that is, initial assignment through auctions), but we have yet to see a viable secondary market. This secondary market is critical both for innovation and for encouraging license holders to continually evaluate the choice between investments spectrum and technology. This research provides a first step toward understanding what a viable spectrum market might look like, even if it is focused specifically on the question of secondary use. Clearly, the scope of the network (i.e., cell radius) is an important factor for considering which approach to take. Also critical is the market structure of spectrum holders.

Forthcoming papers will report on additional results from this model. Furthermore, we are in the process of extending this model to examine not just secondary use but also spectrum markets that could include permanent transfers of licenses between users. The questions of interest in this extended model include examining the impact of different market forms, emergence of brokers and band managers, minimum levels of liquidity for effective operation of spectrum markets and the emergence of derivatives.

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