Two Conditions under which WDM Networks are Rearrangeably Nonblocking without Wavelength Interchangers

Richard A. Thompson
Telecommunications Program, University of Pittsburgh

Abstract – Any network's blocking characteristic depends on its path diversity. WDM networks have path diversity in the space division if the network's topology is rich in alternate routes *and* *in the wavelength division if the network's links have many wavelengths or can interchange wavelengths at some nodes. This paper shows that this costly wavelength interchange is not needed if the network's spatial topology is sufficiently rich, and shows that this spatial richness is defined by the classic Clos inequalities.*

1. Introduction

Do WDM networks need wavelength interchangers? Network designers must know because, however they're implemented, WLIs are expensive, expected to remain expensive, and don't scale over increasing numbers of wavelengths.

Some papers [1, e.g.] show how to minimize the number of WLIs in a WDM network, while others [2, e.g.] claim WLI is unnecessary (the minimum number of required WLIs is zero). This second case is seen if a WDM network has six billion wavelengths and each human is assigned a personal lifelong wavelength. Then, A hears B by tuning his receiver to B's wavelength. In such a network, WLI is unnecessary (even counterproductive). But, it's impractical; so, we remain in a quandary. Complicating matters, different papers make different assumptions including that some papers assign each wavelength as a two-way channel, while others, like this one, deal with unidirectional communications.

The principle issue is *path blocking,* the probability that a requested connection may be blocked by other connections in the network. Since blocking depends on a network's path diversity and WDM networks define paths in the space and wavelength divisions, a WDM network's *net path diversity* has two dimensions.

1. *Spatial diversity* requires the network topology be rich in alternate routes.
2. *Wavelength diversity* requires multiple wavelengths, which may require WLIs in some nodes.

These two diversity-types trade off. If a network's topology is *lean* (spatial paths have few alternate routes), then low blocking probability requires many wavelengths, usually with WLIs. If a network's topology is *rich* (many alternate spatial routes), then low blocking probability requires fewer wavelengths, and possibly no WLIs. This paper shows that a WDM network's spatial diversity can be rich enough so that the network will be rearrangeably nonblocking (RNB) using a minimum number of wavelengths and no WLIs; and it shows the conditions for determining this necessary spatial diversity.

Section 2 reviews the blocking hierarchy, the Clos meta-architecture, and the inequality that makes a Clos fabric rearrangeably nonblocking. Section 3 shows a logical transformation from a WDM network to the Clos meta-architecture. Section 4 presents and interprets the conditions for avoiding WLIs in WDM networks. Section 5 gives an example of a WDM network with $r = 5$ nodes. Section 6 discusses future work.

2. Review of Classic Switching Theory

Cost has always forced networks and switching fabrics to be so lean that connection requests occasionally block. But, new inexpensive dense channels may require nonblocking networks. Three similar issues must be distinguished.

1. *Called-party busy.* A requested connection may be denied because the destination port is busy receiving data from another transmitter over another connection. The new connection request is not considered to have been blocked.
2. *Access blocking* occurs if network ports are concentrated at exterior stages of switching. While calls may block at these concentrators, the network's interior may still be nonblocking for those calls that are not blocked at its edges.
3. *Path blocking.* In this kind of blocking, calls may be blocked by existing calls in any stage of a fabric, not just in the one-stage concentrators at its edges.

The classic blocking hierarchy [3] distinguishes four types of blocking.

1. *Strict-sense nonblocking (SSNB).* A fabric is nonblocking in the strict sense if there is a path between any two idle ports for any existing configuration of calls in the fabric, no matter how paths were selected for these existing calls.
2. *Wide-sense nonblocking (WSNB).* A fabric is nonblocking in the wide sense if path existence depends on using some given path-selection algorithm.
3. *Rearrangeably nonblocking (RNB).* A fabric is rearrangeably nonblocking if an idle port on the left can always connect to an arbitrary idle port on the right. However, it may be necessary to move existing calls to alternate paths. This paper focuses on RNB networks.
4. *Blocking.* A fabric is blocking if a path cannot be guaranteed.

Shown in Figure 1, the Clos meta-architecture [3] is a generalized modular three-stage switching fabric with perfect-shuffle wiring between its stages. The Clos meta-architecture has an equal number of input and output ports, with r switching modules in its left and right stages and m switching modules in its center stage. Each module in the left and right stages has n outer ports and m inner ports; each module in the center stage is *r-by-r.* There are $N = r \times n$ ports on each edge of the fabric and m paths between any input port and any output port (one path through each center-stage switch).

In a classic inequality [3], Clos fabrics are RNB if $m \geq n$. So, each switch in the left and right stages must be *n-by-n*, at least. This inequality is a necessary condition due to *access blocking* in the outer stages. If there are fewer than n Internal ports on any outer switch, some port on the switch's outer edge might be blocked from reaching the heart of the fabric. Ignoring this access blocking, the inequality might be unnecessary to avoid path blocking in the network. Extremely important to this paper, a *second condition,* which is implied but rarely stated, is that an RNB Clos fabric also requires that all its component switching modules are at least RNB.

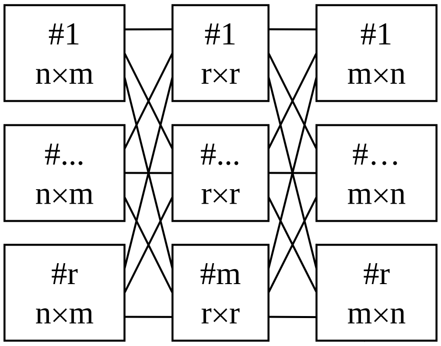

Figure 1. Clos Meta-Architecture.

In another classic inequality [3], Clos fabrics are SSNB if $m \geq 2n-1$. This inequality applies strictly to *path blocking,* so the SSNB case is different enough from the RNB case that the conditions by which WDM networks can be strict-sense nonblocking without WLI is postponed to later paper.

3. Transforming a WDM Network into the Clos Meta-Architecture

Consider an r-node WDM network whose inter-node fiber links are wavelength multiplexed. Figure 2 shows a *ring topology* with $r = 3$ nodes, each serving $n = 2$ clients (or LAN gateways) in its vicinity. In Figure 3 below, each of the $n = 2$ clients has a transceiver that is wired by a 2-way link to a mated transceiver in the serving node, where client connections are muxed/demuxed to/from wavelengths on inter-node links. Figure 3 shows that each node's internal switching fabric drops and adds these local wavelengths and performs tandem switching/routing in space and wavelength (possibly including WLI) for connections over inter-node WDM channels.

Similar to the Marcus transformation [3, 4], which transforms a Clos meta-architecture into the time-space-time fabric used in most digital telephone offices, the following seven steps transform Figure 2's multi-node WDM network into a logical Clos meta-architecture.

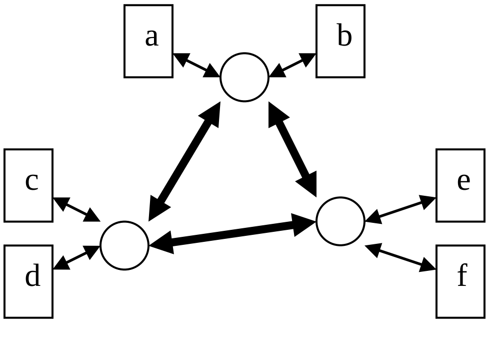

Figure 2. WDM Network.

1. At each of the *r* nodes, split each of its *n* clients; separating each client's transmitter and receiver. Move all *n*×*r* transmitters to the logical left edge of the figure and all *n*×*r* client receivers to the logical right edge.
2. Extract each node's physical wavelength mux/demux. Move the client-mux from inside each of the *r* nodes to the figure's logical mid-left, between the column of client transmitters and the logical network. Move the client-dmx from inside each of the *r* nodes to the figure's logical mid-right, between the column of client receivers and the logical network.
3. Split each of the *n*×*r* two-way client-to-node links into two one-way logical links. Wire each of the *n*×*r* client transmitters to its respective server-node mux and each of the *n*×*r* client receivers to its respective server-node dmx. Figure 4 shows how Client a, and its node, have been dissected so far.
4. Encapsulate the network topology in a logical box and make *m clones* (*m* is the maximum number of wavelengths on an inter-node link and each clone will represent the original network at its own wavelength). Place the column of clones in the center.
5. Replace each mux-to-fabric multi-WL physical link, which had been inside its respective node, by *m* logical single-wavelength links, each to its corresponding logical switch in each clone. Replace each dmx-to-fabric multi-wave-length physical link, which had been inside its respective node, by *m* logical single-wavelength links, each to its corresponding switch in each clone.
6. In Figure 5 below right, when the space-wavelength switches in the original nodes transform into one-clone-per-wavelength, any interchanged wavelengths in the original must cross between their respective clones. So, any WLIs in the original must become part of logical links connecting corresponding points in the respective clones, as shown by bold dotted lines in Figure 6 below left.
7. Steps 1-6 can be applied to any WDM network, but this seventh step departs from generality. To avoid interchanging wavelengths, we transform the network of Step 6 to the special case, with no WLIs, by eliminating all WLIs inside each of the *m* clones and eliminating any inter-clone links installed in Step 6.

The resulting logical network looks like Figure 6 without its WLI links. It has a column of *n*×*r* client transmitters on the extreme left and a column of *n*×*r* client receivers on the extreme right. Each client transmitter is wired to one of *r* server-node muxes, all stacked in the mid-left column. Each client receiver is wired to one of *r* server-node dmxes, all stacked in the mid-right column. Each of the *r* muxes is wired to the left edge of its corresponding switch in each of the *m* clones in the center (one clone per wavelength). Each of the *r* dmxes is wired to the right edge of its corresponding switch.

There is no logical difference between the logical network in Figure 6 and the original net-

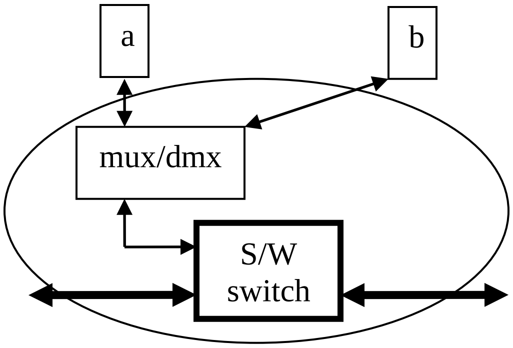

Figure 3. Inside a WDM Node.

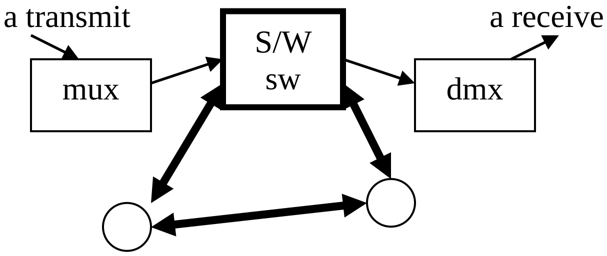

Figure 4. Dissecting 1 Client & 1 Node.

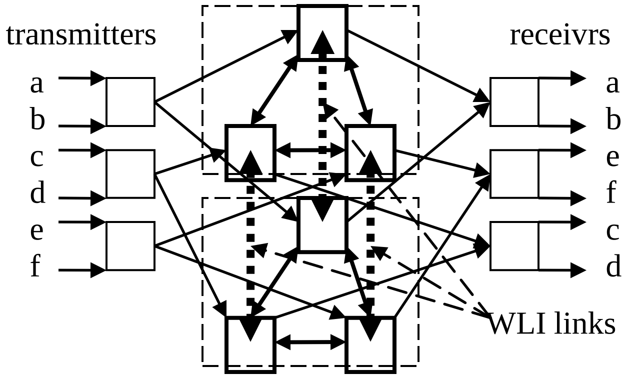

Figure 6. Transformed WDM Network.

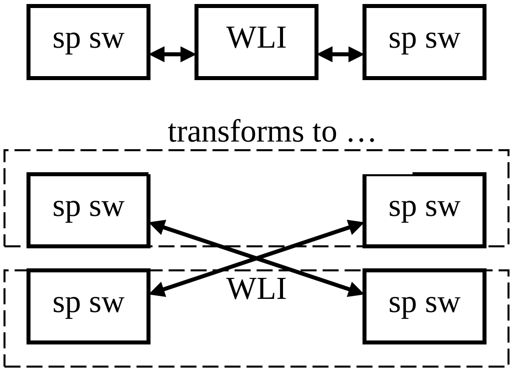

Figure 5. Logical WLI Links

work in Figure 2, except for WLIs. So, the network in Figure 6 without WLIs has the same blocking behavior as the network in Figure 2 if the latter had no WLIs. Now, compare Figure 6 to the Clos meta-architecture in Figure 1.

- With no WLIs, Figure 6's *m* logical single-WL clones correspond to the *m* switching modules in the center-stage of Figure 1's Clos meta-architecture.
- The *r* multiplexers in the column at the middle-left of Figure 6 correspond to the *r* switching modules in the left stage of the Clos meta-architecture in Figure 1. The *r* demultiplexers in the column at the middle right of Figure 6 correspond to the *r* switching modules in the right stage of the Clos meta-architecture in Figure 1.
- The $n \times r$ transmitters in the column at the extreme left of Figure 6 correspond to the $n \times r$ mouths on the left edge of the Clos meta-architecture in Figure 1. The $n \times r$ receivers in the column at the extreme right of Figure 6 correspond to the $n \times r$ ears on the Clos meta-architecture's right edge in Figure 1.

4. The Two Conditions

Summarizing the previous section, without WLI, the original multi-node network in Figure 2 is logically equivalent to its transformed logical network in Figure 6. And, without WLI, this logical network in Figure 6 is logically equivalent to the Clos meta-architecture in Figure 1. So, the Clos RNB conditions extend from Figure 1 to Figure 6 without WLI, and then to Figure 2 without WLI. Therefore:

.

A WDM network is rearrangeably nonblocking without wavelength interchange if:

1. **each nodes' multiplexers satisfy the RNB Clos inequality *($m \geq n$)* and**
2. **the network's spatial topology is at least rearrangeably nonblocking.**

A WDM network will be RNB if it has minimal path diversity, as provided in the space <u>and/or</u> wavelength divisions. The two conditions in the box above specify (1) the mux in each node and (2) the network's spatial topology. For a WDM network to be RNB without WLI, the Clos inequality requires that the number of wavelengths in each node's mux and dmx must equal at least the number of clients. If this inequality isn't satisfied, a Clos meta-architecture has *access blocking* in its outer-stage concentrators. In a corresponding WDM network, transmitting and receiving clients cannot connect if they cannot find a wavelength in their serving node's mux/dmx.

This result is obvious to anyone familiar with the Clos inequalities; so it is <u>not</u> the most significant result. The second requirement in the box above is more relevant.

A Clos meta-architecture is RNB if each internal module is at least RNB. Each center module in Figure 1 must be RNB, so each clone (without WLI) in Figure 6 must be RNB. So, the basic spatial topology (ignoring WDM) of the network in Figure 2 must be at least RNB. The blocking characteristic of a WDM network's spatial topology is determined by a three-step process.

1. Assign a single representative client, with separated transmitter and receiver, to each of the *r* nodes.

2. Convert the network's two-way links into pair-wise one-way links.
3. Verify that the spatial topology is at least RNB for these *r* clients.

5. Example

Consider a five-node network in a ring topology; a pentagon [5]. Label each node's representative client as: A, B, C, D, and E, clockwise. With unidirectional connections, how many of the 5! = 120 permutations can be connected without blocking? Answer: 119. The only permutation that blocks is (ACEBD), in permutation-cycle notation. A straight-line sketch is a five-pointed star or *pentagram,* said to represent Satan's hoof-print [6]. So, while more than 99% of its permutations can be connected, the pentagon topology is *devilishly* not RNB.

Consider a WDM network in a pentagon topology, with *n* clients per node and *m=n* wavelengths per link (to prevent access blocking). If the *n* clients at A transmit to the *n* clients at C, and these *n* clients at C transmit to the *n* clients at E, and so forth in an *n-way* pentagram, the WDM network blocks, but only for this pentagram permutation.

The pentagon's topology can be made RNB, including the pentagram permutation, by adding links, which increases its spatial path diversity. The pentagon topology is unblocked by adding one more two-way link.

- If added *diagonally,* for example between E and B, the network is 100% RNB. But, since we wouldn't need the A-E link, we remove it and the network still has five two-way links; but in a different topology.
- If the extra link is added in parallel with an existing link, we double the capacity between any two adjacent nodes. Then, all permutations can be connected while retaining the pentagon topology.

A similar result is seen on a pentagonal WDM network. Adding a second link, diagonally or in parallel, increases the spatial diversity enough to allow RNB without WLI. But, WDM gives us another implementation; instead of adding a parallel edge link, we can double an edge link's capacity by giving it *2n* wavelengths. But, this approach requires WLIs in two of the nodes.

6. Future Work

The previous example shows that a pentagonal WDM network is RNB without WLI if network links have enough wavelengths and its spatial topology is RNB. But, this paper presents only sufficient conditions for WDM networks to be RNB without WLI; they may not be necessary conditions in general. Furthermore, SSNB must still be investigated. But, an even more glaring part of this work is still unfinished.

The second requirement in the box at the top of the previous page hasn't always been relevant because:

(1) in Clos' original work, the modules were assumed to be classic crossbar switches (which are SSNB), and
(2) in Benes' work, RNB components are nested recursively [3].

This requirement is important now, but we have never had a general procedure to determine whether a multi-node mesh network topology is non-blocking. This is a hard problem, which I am working on; but I urge others to help.

References

[1] X Qin & Y Yang, *"Nonblocking WDM Switching Networks with Full and Limited Wavelength Conversion,"* Conf. on Computer Communications and Networks, 2001.

[2] BJ Jaumard, C Meyer, & X Yu, *"When is Wavelength Conversion Contributing to Reducing the Blocking Rate?"* IEEE ComSoc GlobeCom, 2005.

[3] RA Thompson, *Telephone Switching Systems,* Artech House, 2000.

[4] MJ Marcus, *"Space-Time Equivalents in Connecting Networks,"* IEEE ComSoc Int. Conference on Communications, 1970.

[5] RA Thompson, *"Network Topologies that Illustrate Network Blocking and Wavelength Interchange,"* ITERA Conference, 2007.

[6] D Brown, *The Da Vinci Code,* Random House, 2003.