# Pitch and Harmony in György Ligeti's Hamburg Concerto <br> and <br> Syzygy for String Quartet 


#### Abstract

by

\section*{Charles Corey} B.M. in Music Theory and Composition, Montclair State University, 2006 M.A. in music Theory and Composition, Montclair State University, 2007


> Submitted to the Graduate Faculty of the Department of Music in partial fulfillment of the requirements for the degree of Doctor of Philosophy

University of Pittsburgh
2011

## UNIVERSITY OF PITTSBURGH

 SCHOOL OF ARTS AND SCIENCESThis dissertation was presented
by

Charles Corey

It was defended on
March 21, 2011
and approved by
Roger Dannenberg, Ph.D., Associate Research Professor of Computer Science, Carnegie Mellon University

Eric Moe, Ph.D., Professor of Music, University of Pittsburgh
Amy Williams, Ph.D., Assistant Professor of Music, University of Pittsburgh
Dissertation Advisor: Mathew Rosenblum, Ph.D., Professor of Music, Chair, Department of Music, University of Pittsburgh

# Pitch and Harmony in György Ligeti’s Hamburg Concerto and <br> Syzygy for String Quartet 

Charles Corey, Ph.D.

University of Pittsburgh, 2011

Copyright © by Charles Corey
2011

# Pitch and Harmony in György Ligeti's Hamburg Concerto <br> and <br> Syzygy for String Quartet 

Charles Corey, Ph.D.
University of Pittsburgh, 2011

The analysis component of this dissertation focuses on intricate and complex pitch relationships in György Ligeti’s last work, the Hamburg Concerto. This piece uses two distinct tuning systems-twelve tone equal temperament and just intonation-throughout its seven movements. Often, these two systems are used simultaneously, creating complex harmonic relationships. This combination allows Ligeti to exploit the unique features of each system and explore their relationships to each other.

Ligeti's just intonation in the Hamburg Concerto comes mainly from the five French horns, who are instructed to keep their hands out of the bell to allow the instrument to sound its exact harmonics. The horns themselves, however, are tuned to varying different fundamentals, creating a constantly changing series of just-intoned pitches anchored above an equal-tempered bass. This method of generating justintoned intervals adds a second layer to the relationship between equal temperament and just intonation.

This paper focuses on creating ways to understand this relationship, and describing the ramifications of these tunings as they unfold throughout the piece. Ligeti very carefully crafts this work in a way that creates a balance between the systems.

Research done at the Paul Sacher Stiftung has uncovered a significant collection of errors in the published score. Clearing up these discrepancies allows for a much more accurate and more informed analysis. Throughout this dissertation, mistakes are corrected, and several aspects of the score are clarified. The tuning systems are described, and a likely tuning scheme for the horns is posited. (The analytical component of the dissertation delves into the many varying intervals which all fit into one interval class-a feature that is best explored when two distinct tuning systems are juxtaposed.) A language is created herein to better understand these pitch relationships that fit neither into equal temperament nor just intonation. The analysis clearly shows that very simple musical intervals turn out to be cornerstones of this piece, traceable throughout the entire Hamburg Concerto.

The composition, Syzygy for string quartet, is written in just intonation. Through four movements, the relationships evoked by the titles (always groups of homonyms) are examined and illuminated.

TABLE OF CONTENTS
PREFACE ..... X
1.0 INTRODUCTION ..... 1
1.1 GYÖRGY LIGETI'S HAMBURG CONCERTO ..... 1
1.2 MICROTONALITY IN THE WORKS OF GYÖRGY LIGETI ..... 2
1.2.1 Concert Românesc (1951) ..... 2
1.2.2 Requiem (1963-65) ..... 3
1.2.3 Cello Concerto (1966) ..... 4
1.2.4 Harmonies (1967) ..... 4
1.2.5 String Quartet No. 2 (1968) ..... 5
1.2.6 Ramifications (1968-69) ..... 5
1.2.7 Double Concerto (1972) ..... 6
1.2.8 Clocks and Clouds (1973) ..... 7
1.2.9 Passacaglia ungherese (1978) ..... 7
1.2.10 Trio (1984) ..... 8
1.2.11 Piano Concerto (1985-88) ..... 9
1.2.12 Violin Concerto (1990-92) ..... 9
1.2.13 Sonate (1991-94) ..... 10
1.2.14 Hamburg Concerto (1998-99, 2002) ..... 11
2.0 TUNING AND INTONATION IN GYÖRGY LIGETI'S HAMBURG CONCERTO ..... 12
2.1 TWELVE TONE EQUAL TEMPERAMENT ..... 12
2.2 JUST INTONATION ..... 13
2.3 TUNING SCHEMES ..... 17
3.0 ANALYSIS: PITCH AND HARMONY IN GYÖRGY LIGETI'S HAMBURG CONCERTO 21
3.1 INTRODUCTION ..... 21
3.2 I. "PRAELUDIUM" ..... 23
3.3 II. "SIGNALE, TANZ, CHORAL" ..... 39
3.4 III. "ARIA, AKSAK, HOKETUS" ..... 67
3.5 IV. "SOLO, INTERMEZZO, MIXTUR, KANON" ..... 88
3.6 V. "SPECTRA" ..... 102
3.7 VI. "CAPRICCIO". ..... 118
3.8 VII. "HYMNUS" ..... 135
3.9 CONCLUSION ..... 152
APPENDIX A. ARROWS AND ERRORS ..... 159
ARROWS ..... 160
ERRORS ..... 162
APPENDIX B. THE EIGHT HARMONIC SERIES USED IN THE HAMBURG CONCERTO ..... 166
APPENDIX C. INSTANCES OF UNISON INVOLVING THE HORNS ..... 168
ANNOTATED BIBLIOGRAPHY ..... 180
SYZYGY FOR STRING QUARTET ..... 183
PROGRAM NOTES ..... 184
NOTES FOR THE PERFORMERS ..... 186
I. CANON/CANNON ..... 191
II. POUR/PORE ..... 217
III. DESCENT/DISSENT ..... 236
IV. RAYS/RAZE/RAISE ..... 243

## LIST OF FIGURES

Figure 2.1—The first 16 pitches of the harmonic series of F .................................................................. 14
Figure 2.2—Ratios of intervals of a third in a just-intoned major scale ................................................... 15
Figure 2.3—Ratios of intervals of a fifth in a just-intoned major scale.................................................... 16
Figure 2.4—Valve combinations for the solo horn in Ligeti’s Hamburg Concerto .................................. 20
Figure 3.1—"Praeludium" measure 4 with cents values ......................................................................... 26
Figure 3.2-Major seconds and perfect fifths in the first four measures of "Praeludium" with cents values and, where appropriate, ratios or equal-tempered interval names ............................. 27

Figure 3.3-"Praeludium" measures 4-7 and 12-14: a comparison of the two ascents by the solo horn .. 35
Figure 3.4—"Praeludium" measures 17-20 with cents values................................................................. 37
Figure 3.5-"Signale, Tanz, Choral" measures 7 and 8 with cents values............................................... 41
Figure 3.6-"Signale, Tanz, Choral" measures 8-15 with cents values for motive one.......................44-45
Figure 3.7-Collapsing intervals in "Signale, Tanz, Choral" measures 9-10........................................... 47
Figure 3.8—Collapsing and expanding intervals in "Signale, Tanz, Choral" measures 11-13 .................. 47
Figure 3.9—Expanding and collapsing intervals in "Signale, Tanz, Choral" measures 13-15 .................. 48
Figure 3.10—"Signale, Tanz, Choral" measures 8-15 with cents values for motive two.....................50-51
Figure 3.11—"Signale, Tanz, Choral" measures 16-27 with partial numbers for the horns ..................... 56
Figure 3.12—Melodic equal-tempered intervals within phrase one of "Choral," measures 16-17 ............ 58
Figure 3.13—Melodic equal-tempered intervals within phrase two of "Choral," measures 18-20............ 59

Figure 3.14—Melodic equal-tempered intervals within phrases three and four of "Choral," measures
$\qquad$
Figure 3.15-Melodic equal-tempered intervals within phrase five of "Choral," measures 23-26.63
Figure 3.16-"Aria, Aksak, Hoketus" measures 1-11 with partial numbers for the solo horn ..... 68
Figure 3.17-"Aria, Aksak, Hoketus" measures 1-11 showing the solo horn and string relationship ..... 70
Figure 3.18-"Aria, Aksak, Hoketus" measures 12-26: horn hocket with partial numbers and
$\qquad$
Figure 3.19—"Aria, Aksak, Hoketus" measures 27-33: horn parts with partial numbers ..... 80
Figure 3.20-"Aria, Aksak, Hoketus" measures 27-31: horn hocket with partial numbers and instrument labels. ..... 83
Figure 3.21-"Aria, Aksak, Hoketus" last chord (measures 31-33) with cents values ..... 86
Figure 3.22—"Solo, Intermezzo, Mixtur, Kanon" measures 1-28 with cents values ..... 90
Figure 3.23-"Solo, Intermezzo, Mixtur, Kanon": bass line for measures 42-65 ..... 95
Figure 3.24-"Solo, Intermezzo, Mixtur, Kanon" measures 1-3 and 42-47: a comparison of "Solo" and "Mixtur" ..... 97
Figure 3.25-"Solo, Intermezzo, Mixtur, Kanon" measures 5-7 and 48-52: a comparison of "Solo" and "Mixtur" ..... 97
Figure 3.26-"Solo, Intermezzo, Mixtur, Kanon" measures 10-12 and 53-58: a comparison of "Solo" and "Mixtur" ..... 97
Figure 3.27-"Solo, Intermezzo, Mixtur, Kanon" measures 19-21 and 59-65: a comparison of "Solo"
and "Mixtur" ..... 98
Figure 3.28-"Solo, Intermezzo, Mixtur, Kanon": tone row used in "Kanon" ..... 100
Figure 3.29-"Spectra" measures 1-5 ..... 104
Figure 3.30-"Spectra" measure 4 with cents values. ..... 108
Figure 3.31-"Spectra" measure 15 with numbers of the simulated partials ..... 114
Figure 3.32-"Capriccio" measures 1-5 ..... 119
Figure 3.33-"Capriccio" measures 6-10. ..... 121
Figure 3.34-"Capriccio" measures 11-18 ..... 123
Figure 3.35-"Capriccio" measures 19-24 ..... 126
Figure 3.36-"Capriccio" measures 32-37: the lamento motif ..... 130
Figure 3.37-"Capriccio" measures 23-34: bass line foreshadowing the lamento motif. ..... 131
Figure 3.38-"Capriccio," measures 44-50 with cents values ..... 134
Figure 3.39—"Hymnus": horns (with partial numbers) and contrabass ..... 137
Figure 3.40—Melodic equal-tempered intervals within "Hymnus" ..... 143
Figure 3.41-"Signale, Tanz, Choral" measures 16-17 and "Hymnus" measures 1-4: a comparison ofthe first phrase of "Choral" to the corresponding measures in "Hymnus"...................... 145
Figure 3.42—"Signale, Tanz, Choral" measures 18-20 and "Hymnus" measures 4-7: a comparison of the second phrase of "Choral" to the corresponding measures in "Hymnus"147
Figure 3.43-"Signale, Tanz, Choral" measures 20-22 and "Hymnus" measures 8-11: a comparison of the third and fourth phrases of "Choral" to the corresponding measures in "Hymnus"... 149
Figure 3.44-"Signale, Tanz, Choral" measures 23-27 and "Hymnus" measures 12-15: a comparison of the fifth phrase of "Choral" to the corresponding measures in "Hymnus"151
Figure 3.45-The harmonic series of F from the fundamental through the nineteenth partial with all just-intoned perfect fifths and major seconds identified................................................. 152
Figure A.1—"Hymnus" Contrabass part with appropriate arrows 161
Figure B.1—The eight harmonic series used in the Hamburg Concerto with partial numbers and cents values.

## PREFACE

I am very grateful for the assistance given to me throughout the duration of this project. I would like to thank Amy Williams, Eric Moe, and Roger Dannenberg for their help in shaping this project into its current form. I would especially like to acknowledge Mathew Rosenblum for the work he has put into the entire process, from developing my initial ideas to helping me clarify and revise my prose. Thanks also to Roger Zahab and Trevor Björklund for their assistance with Syzygy.

Additional thanks to Johanna Blask, Dr. Heidy Zimmerman, and Evelyne Diendorf for their gracious assistance with my research at the Paul Sacher Stiftung in Basel, Switzerland. The time spent in those archives has proven invaluable to this project.

Finally, thank you to my family and my friends for years of support, and for the interest you have shown in my work.

### 1.0 INTRODUCTION

"It's hard to imagine what it could mean for young composers to immerse themselves in these works, to be versed in the problems posed by the Hamburg Concerto. What's important for me could mean life itself to them." ${ }^{1}$-György Kurtág

### 1.1 GYÖRGY LIGETI'S HAMBURG CONCERTO

György Ligeti's (1923-2006) Hamburg Concerto ${ }^{2}$ (1998-99, 2002) is the last piece he completed, although whether he considered it a finished work is unknown. ${ }^{3}$ The original version consisted of six movements: "Praeludium"; "Signale, Tanz, Choral"; "Aria, Aksak, Hoketus"; "Solo, Intermezzo, Mixtur, Kanon"; "Spectra"; and "Capriccio." In 2002, shortly after the premiere, Ligeti added a seventh movement, "Hymnus," but made no changes to the previous six. The concerto is for solo French horn and also features four obbligato natural horns. These five instruments are instructed to "always produce non-tempered natural harmonics; therefore the right hand must not correct the pitches." ${ }^{4}$ This system of just intonation is contrasted with twelve tone equal temperament played by the rest of the chamber orchestra, although at times the orchestra is also instructed to alter their pitches microtonally. The justintoned system of the horns and the equal-tempered system of the orchestra are heard against one another for the majority of the piece.

[^0]The Hamburg Concerto is one of Ligeti's most complicated uses of microtonality, as not only are the horns playing just-intoned pitches against the equal-tempered orchestra, but the horn players will change the key of their horns, which changes the fundamental and provides a new harmonic series for the horn player to utilize. A complete set of the eight harmonic series used in the Hamburg Concerto can be found in Appendix B.

### 1.2 MICROTONALITY IN THE WORKS OF GYÖRGY LIGETI

Ligeti's earliest usage of microtonality came in 1951 in his Concert Românesc. In this work the French horns are, at times, required to keep their right hand out of the bell while they play. This prevents the players from tempering the pitches with their hand, clearly showing Ligeti's intention to hear exact, justintoned pitches from the harmonic series. ${ }^{5}$ Interestingly, Ligeti's last microtonal work (and the subject of this paper), the Hamburg Concerto (1998-99, 2002), features the same microtonal system, expanded to include more notes from the harmonic series, and almost completely eliminating the use of valves. In the half-century between these two pieces, Ligeti wrote eleven other microtonal works and permitted microtones in one other piece after it was written. A brief discussion of each of these works follows.

### 1.2.1 Concert Românesc (1951)

Ligeti's Concert Românesc remains one of his most performed pieces, despite being a relatively early work. This piece is mainly equal-tempered, but at times the horns play non-tempered pitches which occur naturally in their harmonic series. The horn players do not temper the pitch with their right hand,

[^1]removing it from the bell entirely. Ligeti's intention here is to have the horns the pitches of the harmonic series. Similarly, the players do not change valves in these sections. This emulates the use of natural horns, and results in clear microtonal inflections against the twelve tone equal temperament of the rest of the piece.

In these natural horn sections, the horn players do not ascend far up the harmonic series, which means that the notes are relatively close to their equal-tempered counterparts. The result is that the microtonal pitches are clearly heard as being outside of equal temperament, but are also clearly heard as related to the equal-tempered notes. In part this is due to the position of these notes in the harmonic series, but it is also due to Ligeti's careful construction of these passages-the horn parts are often exposed and supported by long, soft sustained notes in the rest of the orchestra. This method of creating microtones through the use of harmonics (mainly in brass instruments, and especially in French horns) would become a favorite of Ligeti's.

### 1.2.2 Requiem (1963-65)

Ligeti's Requiem was not initially intended to be a microtonal piece, but after repeated rehearsals of a particularly difficult passage in the "Kyrie," Ligeti determined that imperfections in the choral parts, melodic or rhythmic, were acceptable and perhaps even enhanced the overall effect. He stated that the errors would result in "a kind of microtonality, dirty patches...these 'dirty patches’ are very important." ${ }^{6}$

While in actuality there is no microtonal system employed here (the microtones are created by the individual performers and would not be exactly the same between any two performances), when realized the piece does include pitches outside of twelve tone equal temperament in these "dirty patch" sections. Ligeti adds that "statistically speaking there will be hardly any difference between various performances; the smudginess both in intonation and in rhythm gives the same result, the same degree of 'dirtiness.' If

[^2]you have a sufficiently great number of parts then the various 'approximations' will cancel one another out." ${ }^{7}$

### 1.2.3 Cello Concerto (1966)

Following the inadvertent appearance of microtones in his Requiem, Ligeti returned to placing them intentionally. The Cello Concerto employs harmonics again to create microtones (this time using the higher string harmonics), and again these pitches are generally exposed. The cello soloist is the only performer to use microtones, sounding up to its fifteenth harmonic.

While the harmonics used in the horns in Concert Românesc were relatively low, the use of harmonics up to fifteen in the Cello Concerto results in pitches which deviate by nearly a quarter tone (the eleventh and thirteenth harmonics both result in approximately a quarter tone deviation from equal temperament). Because they are heard in the context of the harmonic series, these notes do not sound jarringly out of place, but as in Concert Românesc, they clearly add a microtonal element to the lines.

### 1.2.4 Harmonies (1967)

While the microtones in Harmonies were intentional from the inception of the piece, they have much in common with the microtones of the Requiem. Harmonies is written for mechanical organ, and the microtones are generated in two ways-first, the amount of air used in the organ is very low which prevents any stop from sounding at its intended pitch (each note will deviate by varying amounts), and second an assistant changes the organ stops gradually which, again, allows varying amounts of air into the organ but also creates microtonal friction as the stops engage and disengage slowly. To accentuate the

[^3]microtonal possibilities, at times a vacuum has been used to replace the bellows, and for the György Ligeti Edition recording, Ligeti himself manually controls the wind flow. ${ }^{8}$

### 1.2.5 String Quartet No. 2 (1968)

Ligeti's String Quartet No. 2 employs microtonal intervals of varying size in the second and third movements. His notation calls for approximate deviations which do not need to be any specific size, and typically are used as passing or neighbor tones which microtonally ornament his melodic lines. By the end of the third movement, these inflections have become articulated glissandi serving the same purpose.

In an interview in 1978, Ligeti stated, "it is perhaps my Second Quartet which reflects my ideas most clearly—where you would find all the different techniques I have used." ${ }^{9}$ While he is not specifically referring to microtonality here, it is certainly an important component of the piece, and clearly a major part of his style at the time-more than half of his microtonal works were written in the ten years between the Requiem and Clocks and Clouds (see below).

### 1.2.6 Ramifications (1968-69)

Ramifications, for either string orchestra or twelve solo strings (Ligeti's preference was the latter) involves two groups of string instruments tuned to different pitches (group II tunes to A 440, group I tunes to A 453 which is just over a quarter-tone higher). This results in an immediate microtonal climate which is maintained from beginning to end. The tuning of the two groups is constantly illuminated.

[^4]"Nowhere else does his music sound quite so like swarms of insects. Moreover, even simple chords sound rich. They 'smell high,' what Ligeti calls 'un goût faisandé,' as if putrefaction had set in." ${ }^{10}$

This simple tuning configuration provides access to a great range of pitch and timbral possibilities. Each group plays music that ranges from dense chromaticism to light, open material. This variety of material allows Ligeti to illustrate the results of this tuning in a variety of contexts and characters. The two separate groups of instruments always sound as though they are working together, inseparable parts of one musical machine which generates these microtonal intervals, rich chords, and swarms of insects with boldness and clarity.

### 1.2.7 Double Concerto (1972)

The Double Concerto for flute, oboe, and orchestra employs an interesting system. For every instrument that plays a microtonal passage (solo flute, solo oboe, flutes, oboes, trombone, strings), Ligeti provides detailed information about how these microtones should be reached on that instrument. Sometimes specific fingerings are given, sometimes there are indications to alter the embouchure, and sometimes there are indications that the player should find their own method (trombones and strings just adjust their position slightly to reach the affected pitches). He notes that the changes in intonation should be no more than a quarter tone, but gives no further information about the resulting pitch. Ligeti is very specific, however, about how these tones begin: any microtonal pitch should be attacked precisely, not arrived at via glissando.

While in some respects this method seems similar to the microtonal parameters of String Quartet No. 2, the goal in the Double Concerto is quite different. In the string quartet, the microtones were generally ornamentations of some kind, but in this work the microtones are actually a part of the melodic lines and the harmonic structures.
${ }^{10}$ Steinitz, 183.

### 1.2.8 Clocks and Clouds (1973)

In this piece Ligeti provides exact fingerings for the microtones in the flutes (like in the Double Concerto), and at times for clarinet 1 (with different fingerings for Boehm and German ${ }^{11}$ Systems). These fingerings usually approach a quarter tone, but always result in a noticeable change of pitch. Other instrumentalists and the singers are instructed at times to match the flutes or clarinet and at other times are given approximate microtonal deviations. Similar to the Double Concerto, these microtones are not functioning solely as ornamentations, but have important roles melodically and harmonically. The combination of extremely precise microtones and very loosely derived approximate microtones is unique to this piece. Currently this is the only of Ligeti's microtonal works that is not published.

### 1.2.9 Passacaglia ungherese (1978)

Passacaglia ungherese, a harpsichord solo, is the most problematic of Ligeti's microtonal pieces. While the instructions are very clear (it is to be played in a meantone temperament wherein all major thirds and minor sixths are just-intoned), they are impossible to execute. The system implied by these instructions is quarter-comma meantone, but because the passacaglia is very chromatic, every possible major third or minor sixth is played. In meantone temperament, not all intervals are necessarily playable. When the comma ${ }^{12}$ is distributed through the scale (in quarter-comma meantone, the fifths are all lowered by one quarter of the comma, or 5.3775 cents, allowing the thirds to sound just-intoned), a point is reached where the discrepancy needs to be rectified, and one of the fifths is tuned wildly larger than the rest. This interval is called the wolf fifth.

In Ligeti's intended tuning, there is no place for a tuner to hide the wolf fifth, and therefore it is impossible to have all of these thirds and sixths equally intoned. In equal temperament, this would not be

[^5]an issue, and this issue could be compensated for by many other instruments (most easily by non-fretted strings or electronics), but because this piece is for keyboard, the tuner must either tune eleven of the twelve thirds to precise just-intoned pitches or come up with a way to approximate Ligeti's tuning instructions (either through a different meantone or by using a system of well temperament). There are a number of viable solutions, any of which would provide nearly Ligeti's desired result, and each will create a different microtonal sound for the piece.

### 1.2.10 Trio (1984)

A clear predecessor to the Hamburg Concerto, and in the lineage of Concert Românesc, Trio for violin, horn, and piano uses non-tempered horn harmonics to generate nearly all of the microtonal material (the violin plays one microtonal pitch, simulating a seventh harmonic). This usage of horn harmonics is very similar to what happens in the Hamburg Concerto. The Trio often uses microtones as important melodic and harmonic content, but at times these harmonics are used more as a flourish or a collection of passing tones. Various valve combinations are used when the harmonics are played, allowing for a variety of microtonal inflections. Interestingly, while Ligeti specifies many places in the trio where the horn player must remove their hand from the bell to sound these natural harmonics, he also allows the horn player to utilize this technique in other sections of the piece as they wish. He indicates that "the natural horn technique can be used in other passages than those for which it is specified, for example throughout the whole first movement."13

The microtonal pitches are at times a part of the texture and at times a strongly contrasting feature, depending on how prominent the equal-tempered material is in the piano and violin and whether or not the horn is supporting the equal-tempered material with its line. Ideas from the Trio found their

[^6]way not only into the Hamburg Concerto, but also into the Études pour piano, most notably "Fanfares," which is strongly rooted in the second movement of the Trio.

### 1.2.11 Piano Concerto (1985-88)

Ligeti's Piano Concerto uses non-tempered harmonics in the horn as well as in the trumpet and the trombone in movements one, three, and five. Because of how strongly the rest of the piece is rooted in equal temperament, these pitches are a dramatic contrast to the other material, especially as the players ascend to higher harmonics which can be nearly a quarter tone away from the equal-tempered pitches. The dominance of the piano (and therefore of twelve tone equal temperament) could make it difficult to hear these microtonal lines clearly, but Ligeti presents them initially very carefully, choosing exposed places in the texture to introduce the microtonal lines so that they can be heard against the piano. As the piece continues on, these microtonal lines begin to be used as a clouding of the harmonic spectrum as well.

Separately, in the second movement an ocarina is played briefly and with some portamento, resulting in some indeterminate microtones. At the end of movement four the strings play quick passages using natural harmonics which frequently involve the seventh partial. This microtonal gesture passes quickly, but introduces the pacing and microtonality of the fifth movement, wherein the microtones in the brass are the densest and form the most powerful contrast to the equal-tempered pitch language.

### 1.2.12 Violin Concerto (1990-92)

In the Violin Concerto, Ligeti combines many of his previous microtonal systems in a very interesting manner, and adds new microtonal possibilities as well. Along with the Hamburg Concerto, this piece utilizes one of the most complex microtonal palettes in Ligeti's work. In the Violin Concerto, there are high string harmonics (as seen in the Cello Concerto), harmonics in the brass (as seen in Concert

Românesc, Trio, and the Piano Concerto), written indications for players to alter their pitches by a specific amount (as seen in Double Concerto, Clocks and Clouds, and Trio), and a scordatura (as seen in Ramifications) that puts two soloists from the orchestra into keys related to specific bass harmonics. These scordatura soloists also use high string harmonics, creating additional microtonal possibilities. Additionally, incidental microtones may arise from the percussionists when they play thematic material on slide whistles in various keys, or from some of the wind players when they play ocarinas and are instructed not to compensate for any intonation concerns which might arise (similar to the incidental microtones of the "Kyrie" from Ligeti's Requiem and the ocarina passage in the Piano Concerto).

The array of possibilities in this piece results in a constantly changing sound world wherein the microtonal colors always shift and evolve, and are frequently a part of the texture. Throughout the five movements, the microtones are used both harmonically and melodically, both in the foreground and in the background. These features of the Violin Concerto comprise some of the most interesting textural uses of microtonality that can be found in Ligeti's work.

### 1.2.13 Sonate (1991-94)

Sonate for solo viola uses microtones only in its first movement, "Hora lungă." In this movement Ligeti writes a combination of equal-tempered pitches and just-intoned pitches, some of which are a result of using high harmonics. The overall result is a just-intoned sound-the equal-tempered pitches are all close to just-intoned pitches, and the added microtonal inflections create passages based on the harmonic series. The movement ends with several measures using only natural harmonics, presenting (eventually) the harmonic series of C (the only string used in this movement) up to the sixteenth partial. Ligeti writes that "Hora lunğ," "evokes the spirit of Romanian folk music...however I do not write folklore or use folkloristic quotations, it is rather allusions which are made." ${ }^{14}$ While he does not specifically include the

[^7]use of microtones in his mention of such allusions, these inflections clearly strengthen the folk music sound in the context of this movement.

### 1.2.14 Hamburg Concerto (1998-99, 2002)

Ligeti's Hamburg Concerto, the last piece he completed, returns to a system of generating microtones via French horn harmonics. However, this piece is different in that these microtones are played by five horn players, at least four of whom are playing natural horns (the fifth, who is the soloist, sometimes plays a natural horn and sometimes plays a double horn, occasionally using valves). These natural horns are often in different keys, enabling different harmonic series to be heard against one another. Against this already complicated collection of pitch possibilities, the remainder of the chamber orchestra is playing in twelve tone equal temperament with occasional microtonal inflections. The various and complicated pitch relationships that result from these tuning combinations and their effects on the piece as a whole are the subject of this paper.

# 2.0 TUNING AND INTONATION IN GYÖRGY LIGETI’S HAMBURG CONCERTO 

"Ich habe kein fest geordnetes System geschaffen, sondern lasse die Klänge los, damitin Selbstorganisation-andere Arten von tonalen Zusammenhängen entstehn als die der Tradition. ${ }^{15}$-György Ligeti

One of the major issues with this piece comes as a direct result of the tuning systems used. Although Ligeti's directions for the horn players are very precise when it comes to how they should reach a particular note, the directions are lacking when it comes to the issue of how these horns are initially tuned-different fundamentals for the horns could drastically affect the sound of some of the passages in the Hamburg Concerto. These relationships will be examined after first looking more closely at the tuning systems involved in this piece.

### 2.1 TWELVE TONE EQUAL TEMPERAMENT

As the name implies, equal temperament is a division of the octave into exactly equal parts. The Western scale divides the octave into twelve parts, hence the term twelve tone equal temperament, each of these being further divided into 100 cents (which creates, essentially, a twelve hundred tone equal-tempered scale). The term temperament is used because the intervals are not "just intervals," or intervals that are

[^8]naturally occurring, they are instead tempered to fit into a specific tuning. For example, the just fifth is almost exactly 702 cents ( 701.955 cents), but the equal-tempered fifth is tempered to 700 cents. The tuner of a piano would have to tune the fifth by ear, and then detune (temper) it slightly, typically by listening to the beats created by being slightly out of tune.

There are several interesting results of an equal division of the octave that we tend to take for granted. For example, equal temperament is the only temperament that provides a tritone that exactly splits the octave in half and it is the only temperament that can provide a true whole-tone scale. By extension of these two features, it is also the only temperament with augmented triads and diminished tetrads which are exactly equal (and therefore do not contain any innate tonal implications-such implications change depending on context). There are several other equal temperaments as well-the quarter-tone scale, for example, is twenty-four tone equal temperament, and other equal temperaments such as thirteen tone, nineteen tone, thirty-six tone (the sixth tone scale), and fifty-three tone are also becoming more common.

Of course, equal temperament also comes with its own set of challenges, largely in the realm of intonation. A major third on an equal-tempered piano is, in fact, nearly 14 cents larger than that same major third would be in just intonation, and 14 cents is a very audible difference. The reason for this discrepancy will be explained in the following discussion of just intonation.

### 2.2 JUST INTONATION

Just intonation takes a different approach to tuning. Instead of tuning a particular interval and having to deal with the consequences for the rest of the pitches, just intonation bases its pitches from the harmonic series, where all the pitches are innately in tune. This provides a perfect fifth at exactly 701.955 cents, a major third at 386.314 cents, and a major seventh, the leading tone, at 1088.255 cents. These three examples alone show that cents will quickly become a complicated way to discuss just intonation.

However, because the pitches are exactly in tune with one another, the ratios of their frequencies are very useful. Many of the ratios for just intonation are called small-number ratios, as the less complicated the relationship is between the two pitches, the more readily they can be recognized as in tune. The frequency ratio of a perfect fifth is $3 / 2$-that is to say that the higher pitch vibrates three times for every two times the lower pitch vibrates. An easy way to make these calculations is to simply look at the harmonic series (see Figure 2.1).


Figure 2.1—The first $\mathbf{1 6}$ pitches of the harmonic series of $\mathbf{F}^{\mathbf{1 6}}$

In a chart of the harmonic series, the number of the pitch is also the ratio of its vibration to that of the fundamental. So instead of using the terms F, F, C, F, A, C, Eb, etc., just intonation ratios can be used: $1 / 1,1 / 1,3 / 2,1 / 1,5 / 4,3 / 2,7 / 4,1 / 1,9 / 8,5 / 4,11 / 8,3 / 2,13 / 8,7 / 4,15 / 8,1 / 1$. The intervals are always reduced to stay between one and two, or $1 / 1$ and $2 / 1$, to keep them within one octave ( $2 / 1$ is the ratio of an octave, as the higher note vibrates exactly twice as fast as the lower note). If the pitch cannot be reduced, then the ratio is either multiplied or divided by two, as this is an octave transposition. The pitches in this chart are useful to create several intervals and some chords, but not enough for something as simple as a major scale. Currently only $1 / 1,9 / 8,5 / 4,3 / 2$, and $15 / 8$ are present ( $F, G, A, C$, and $E$ ) and if the chart was limited to only the first three octaves, only $1 / 1,5 / 4$ and $3 / 2$ would be present. The rest of the notes come in from the harmonic series of two other pitches, the fourth and the fifth. The fifth is already present in the harmonic series of the fundamental; $3 / 2$ can then build its major triad and provide $3 / 2,15 / 8$ and $9 / 8$. These ratios are obtained by multiplying the new fundamental, $3 / 2$, by the intervals

[^9]present from the harmonic series of our tonic, in this case $1 / 1,5 / 4$ and $3 / 2$, and reducing the intervals to fit within the octave from $1 / 1$ to $2 / 1$. In order to find the pitch of the fourth, we move a fifth downward from the tonic, dividing $1 / 1$ by $3 / 2$ to arrive at $4 / 3$ (now instead of reducing, we have to double the interval to stay in the octave) and when $4 / 3$ is multiplied by $1 / 1,5 / 4$ and $3 / 2$, it yields $4 / 3,5 / 3$ and $1 / 1$ respectively.

Finally we have the just-intoned major scale: $1 / 1,9 / 8,5 / 4,4 / 3,3 / 2,5 / 3,15 / 8,1 / 1$.
To better understand what this scale contains, the individual intervals require closer examination. The intervals as compared to the tonic are all exactly just-intoned intervals. The interval from $1 / 1$ to $9 / 8$ is $9 / 8$, and $9 / 8$ is a just-intoned major second. The interval from $1 / 1$ to $5 / 4$ is $5 / 4$, and $5 / 4$ is a just-intoned major third. This pattern continues for the entire major scale. However, the interval from $9 / 8$ to $5 / 4$, a major second, is not $9 / 8$, but instead $10 / 9$ ( $5 / 4$ divided by $9 / 8$ ). This $10 / 9$ is slightly smaller than $9 / 8$, but is also a perfectly in tune major second. Some would differentiate the two by calling $9 / 8$ a major whole tone, and 10/9 a minor whole tone, but in just intonation the ratios give more information than names of this kind. Continuing through the scale, the interval from $5 / 4$ to $4 / 3$ is a $16 / 15$, an in tune minor second. The rest of the intervals of a second in the major scale are one of these three ( $16 / 15,10 / 9,9 / 8$ ). The interval from $4 / 3$ to $3 / 2$ is a $9 / 8$, from $3 / 2$ to $5 / 3$ is a $10 / 9$, from $5 / 3$ to $15 / 8$ is a $9 / 8$ and from $15 / 8$ to $1 / 1$ is a 16/15. Figures 2.2 and 2.3 examine the thirds and fifths in this same manner.


Figure 2.2—Ratios of intervals of a third in a just-intoned major scale


Figure 2.3—Ratios of intervals of a fifth in a just-intoned major scale

These two charts help show how the use of the two major seconds $10 / 9$ and $9 / 8$ manage to provide perfectly in tune major and minor thirds (5/4 and 6/5) and perfect fifths through much of the scale. The intervals that remain now are $32 / 27$ (a small minor third), 64/45 (one of many possible tritones, slightly larger than an equal-tempered tritone) and $40 / 27$ (a small fifth). The tritone and the diminished triad cannot be exact divisions of the octave into halves and fourths as in equal temperament, and they actually do carry a sense of a particular tonality, which an equal-tempered diminished triad cannot do without context from notes which come before it. The tritone created by just intonation is stronger for some scales than for others-common tritones include 11/8, 7/5, 45/32, 64/45, 10/7 and $16 / 11$. These intervals are all reciprocals of another of the intervals in this tritone collection; because the tritone does not exist in precisely the middle of the octave, each tritone has a complement to complete the octave.

The more complicated intervals in these figures are the $40 / 27$ (between $9 / 8$ and $5 / 3$ ) and the $32 / 27$ (between $9 / 8$ and $4 / 3$ ). Interestingly, if the $9 / 8$ is replaced with the other whole tone, $10 / 9$, now the intervals become $3 / 2$ and $6 / 5$. This difference will alter the tone by $81 / 80$ ( 21.5 cents, the syntonic comma mentioned in Chapter 1, which is nearly an eighth tone), but the change does not sound out of place or surprising. In fact, these types of small changes are the exact features that allow for common modulations in music, an excellent example of which can be found in Harry Partch's Genesis of a Music,
where he responds to a letter from Fox Strangways. ${ }^{17}$ On a larger and more complicated level, he has an entire chapter dedicated solely to the subject of resolution, ${ }^{18}$ and his book is an excellent place to find more in-depth information on just intonation.

### 2.3 TUNING SCHEMES

With this information, hypothetical tuning systems can be evaluated, and a probable tuning scheme can be determined. There are a number of different options for the initial tuning, each with their own set of ramifications for the piece as a whole. The standard option of tuning the A of each instrument might at first seem possible, but it will quickly prove problematic. The opening movement begins with the soloist playing a double horn, and horns 1 and 2 playing F horns. While it is not unusual for any of these instruments to tune to A , if these horns tune their non-tempered tenth partial A to the same A as the orchestra the F horns will be playing an F (their fundamental) 13.7 cents higher than any of the equaltempered orchestra members would play. The double horn would likely also tune the A on the F side of the horn, but this raises the question of what the relationship between the two sides of the horn should be, and later in the piece how the valves of the soloist's double horn should be tuned. These questions will be dealt with later.

Having the F horns play their fundamental 13.7 cents too high is bad enough, but the $\mathrm{E}^{b}$ horns would be in an even more awkward relationship. They would have to tune their eleventh partial A to the orchestra's A, resulting in an $E^{b} 48.7$ cents higher than an $E^{b}$ in the orchestra, which is nearly a quarter tone discrepancy. Clearly, tuning all the horns to A is not the best option.
${ }^{17}$ Harry Partch. Genesis of a Music, 2nd ed., (New York: Da Capo Press, 1974), 190-194.
${ }^{18}$ Ibid. 181-194.
${ }^{18}$ Ibid., 181-194.

Tuning all of the horns to a $\mathrm{B}^{b}$ is another option, one which would be far more successful. In this instance the double horn could tune the $\mathrm{B}^{b}$ side exactly to this $\mathrm{B}^{b}$ and its fundamental would be perfectly in tune. The F horns could tune their F to this $\mathrm{B}^{b}$, which would result in an F only 2 cents lower than an equal-tempered $F$. This discrepancy would be unnoticeable. Similarly the $E^{b}$ horns could tune their $B^{b}$ to the orchestra's $B^{b}$ and would have their fundamental $E^{b}$ only 2 cents higher than the orchestra's $E^{b}$. Since the four obbligato horn players are supposed to simply swap different crooks in and out to change keys, this tuning scheme would hold up well for the entire piece as long as all four obbligato horn players tuned their horns with their initial crooks fully onto the horn.

This tuning option is a reasonable system for the piece as a whole with only one major problemwhen all four horns are in the same key (or when horns 1 and 3 or horns 2 and 4 are in the same key) there will be a 4 cent discrepancy. While this is still a very small interval-nearly unnoticeable-the discrepancy would create a slight beating which would disrupt a lot of the perfect consonances Ligeti constructs at important points throughout the Hamburg Concerto.

The optimal tuning scheme, therefore, would be for the fundamentals of all horns to be tuned to equal-tempered pitches. This will result not only in horns of the same key always having the same pitches, but will also result in over one hundred instances of unison or other strong relationships between the horns and the orchestra, ${ }^{19}$ and will highlight the horn relationships in the second movement, "Signale, Tanz, Choral," wherein the horns are the only instruments playing until the last chord, and the obbligato horns are tuned in half steps.

As an extension of this tuning scheme, the solo horn will need to have both sides tuned an equaltempered perfect fourth apart and all valves tuned precisely to equal temperament so that the soloist is able to use any of eight fundamentals freely. Figure 2.4 shows clearly how such a tuning is possible. In this instance, valve three is used to replace the combination of valves one and two, allowing each

[^10]fundamental to have one independent tuning slide on the horn. Therefore, an open horn depressing valve two will lower the pitch exactly 100 cents, an open horn depressing valve one will lower the pitch exactly 200 cents and an open horn depressing valve three will lower the pitch exactly 300 cents. An open horn depressing the trigger will raise the pitch exactly 500 cents, and on the $B^{b}$ side of the horn, the valves can be tuned similarly. While this tuning process may be painstaking and not completely reliable, the following analysis, supported by the instances identified in Appendix C, will show that these relationships are what Ligeti had in mind.


Figure 2.4-Valve combinations for the solo horn in Ligeti's Hamburg Concerto ${ }^{20}$
${ }^{20}$ Note that this only shows the fundamental through the fourth partial for each harmonic series. For the rest of each harmonic series, see Appendix B.

# 3.0 ANALYSIS: PITCH AND HARMONY IN GYÖRGY LIGETI'S HAMBURG CONCERTO 

"This is the most decisive progression to a new harmonic world. Each horn plays its own tones. But their different tunings interfere with the harmony. In this way he transgresses the tempered tone system with the simplest means."21-György Kurtág

### 3.1 INTRODUCTION

György Ligeti's Hamburg Concerto is, in many ways, both a continuation of his microtonal excursions (following especially the Violin Concerto) as well as a summary of his compositional tendencies. Imitation and repetition-including one of Ligeti's most common musical devices, canon-are important frameworks of this piece, appearing both on the smallest and largest scales. These are just a small part of one of the primary features of the concerto-dualities. Some of the pairings in the Hamburg Concerto include: twelve tone equal temperament and just intonation; the solo horn and the horn section; the soloist and the orchestra; the five horns and the orchestra; repetition and imitation; repeated or used only once; major seconds and perfect fifths; major seconds and minor seconds; static and active; consistent or interrupted; slow and fast; and consonant and dissonant. These features all take part in shaping this work in some way, and although the elements of pitch and harmony are the primary focus of this paper, all of these dualities will be components of the following analysis.

This analysis is based specifically on Ligeti's score-in other words, the analysis is focusing on what Ligeti intended to hear and the mathematical relationships which would result from an exact
${ }^{21}$ Varga, 108.
realization of his score. ${ }^{22}$ The analysis is (generally) not considering issues such as intonational difficulties, challenges specific to certain instruments, or performance tendencies. For example, while string instruments will almost certainly tune closer to a just-intoned perfect fifth than an equal-tempered one (at less than 2 cents difference, this point is difficult to distinguish anyway), all stopped notes and open strings will be considered equal tempered. ${ }^{23}$ While the performers may have a tendency to tune parts of a chord (by ear) to something closer to just intonation, Ligeti's intention while conceiving of this piece was clearly for these notes to be heard in twelve tone equal temperament and the following analysis will assume such a performance.

Some of the standard musical terminology used in this paper will have a wider range of possible definitions. For example, the term consonant might be applied equally to the equal-tempered major third and the major third 13.7 cents lower which is the just-intoned major third, or $5 / 4$. Various other major thirds will also be considered consonant, although perhaps to differing degrees. The range of possible major thirds (or possible members of interval class 4) is expanded from only 400 cents to nearly anything between 350 cents and 450 cents. Depending on context, however, these boundaries might become even larger, or they may be more constricted. Context plays a significant role in the way these intervals sound, much like it would in a piece using just one tuning system. Unfamiliar intervals will become familiar over the course of the Hamburg Concerto, but even early in the piece, pitch relationships that fit within either tuning system are more stable, while those outside of either system may feel as though they need to resolve.

As mentioned above, this piece focuses on dualities such as the conflict between the tuning systems and the instrument groups, repetition and imitation, and various different interval classes. These elements can be seen clearly through all seven movements, and the gradual change that occurs within their functions is another of the interesting aspects of this work.

[^11]
### 3.2 I. "PRAELUDIUM"

Ligeti sculpts the opening section of the Hamburg Concerto brilliantly, creating a soundscape so full of tiny intervals and close voicings that the arrival of an open perfect fifth-from $B^{b}$ to $F$ in measure seven—sounds like a remarkable event. This arrival stands out despite the fact that the solo horn line is leading to the F for a measure and one-half (approximately ten seconds at tempo) if not longer. The way Ligeti sets up this arrival in the opening six bars is a clear example of how the microtonal possibilities he has created through his instrumentation help to shape the piece and add dramatic qualities to the music which otherwise might not be possible. In addition to demonstrating the interaction of the tuning systems, Ligeti also examines several other relationships in this piece, including imitation, interruption (which, interestingly, is a feature that will be imitated throughout the piece), and various pairings of instruments and interval classes. In this way, "Praeludium" makes an excellent introduction to this work—while it overtly demonstrates the interest within the tuning systems, it also introduces the listener to some of the other important features of the concerto in more subtle ways.

Through the entire first movement, and in fact through most of the Hamburg Concerto, there is a focus on two interval classes-2 and 5. That is, the major second and minor seventh, and the perfect fourth and perfect fifth. Of these, the major second and the perfect fifth are the most common in this movement. It is important to note that because of the unique combination of tuning systems in this piece, the interval of a major second contains a number of possibilities, which include the equal-tempered major second (200 cents), the just-intoned major second (203.9 cents, a ratio of 9/8), the just-intoned septimal whole tone ( 231.2 cents, a ratio of $8 / 7$ ) and many other values that arise from combining these tuning systems. In other words, these interval classes contain a significantly larger range of possibilities than they would in a strictly twelve tone equal temperament system. All of the intervals in interval class two, for example, will still technically be major seconds or minor sevenths, but the boundary of what constitutes a major second or minor seventh has been expanded.

From the very first bar of the piece, Ligeti challenges (and rewards) the listener's ear, with a justintoned major second ( $E^{b}$ to $F$, both on $E^{b}$ horns, which will sound a $9 / 8$ or 203.9 cents) starting out, and a slightly lower note added at the end of the bar—an $E^{b}$ played on an F horn. This third note is about 31.2 cents lower than the first $E^{b}$, and this interval creates an unstable, beating dissonance which seems to pull the $E^{b}$ simultaneously toward $F$ and $D^{b}$. In measure two, a second $F$ enters, played on an $F$ horn. This $F$ is about 4 cents lower than the first F-this relationship creates a different sensation. Most listeners cannot detect a difference of less than 6 cents, so these two variations of F will not sound out of tune, but they will create audible beats, which create a pulsing sensation. Of course, the other pitches sounding will mask these beats somewhat, but they still clearly add a new color to the harmony.

The $E^{b}$ horns (horns 3 and 4) fade out and reenter-by the end of measure three they are now sounding $D^{b}$ and $G$. The $D^{b}$ is nearly 31.2 cents lower than an equal-tempered $D^{b}$ would be, but that puts it exactly an equal-tempered major second below the $\mathrm{E}^{b}$ still sounding from horn 2 in F . This particular interval, like the interval between horns 1 and 4 in measure two, fits exactly into equal temperament, opening the way for the rest of the piece to explore many conflicts and compromises between the two systems at work.

The G, however, will sound 13.7 cents lower than an equal-tempered G, making the difference between it and the F that is still sounding 186.3 cents-noticeably flat of an equal-tempered major second. Through these two entrances, Ligeti manages to remove the beating created by the 4 cent difference between the F in horn 1 and the F in horn 3 , but replaces it with even more wavering dissonances. The intervals of the resulting whole tone tetrachord are: $D^{b}$ to $E^{b}-200$ cents (an equaltempered major second); $\mathrm{E}^{b}$ to $\mathrm{F}-231.2$ cents (a very large major second which is a just-intoned 8/7); and F to G—186.3 cents (a slightly small major second).

The beating that exists within this tetrachord is amplified dramatically in measure four when the solo horn, both flutes and both bassett horns enter. The most obvious reason for this increased beating is
that the flutes and bassett horns, all of whom are playing in twelve tone equal temperament, enter on $D^{b}$, $E^{b}, F$, and $G$ on top of the $D^{b}, E^{b}, F$, and $G$ being sustained by the natural horns. This introduces additional beating, providing a powerful (even at $\boldsymbol{p} \boldsymbol{p}$ ) change in timbre as well. With this entrance, there are now eight distinct major seconds sounding-the three described above, three equal-tempered major seconds between the woodwinds, and two major seconds created between the horns and the woodwinds.

At the same time that the woodwinds enter, the solo horn enters—playing the $B^{b}$ side of the double horn-on an $A^{b}$, the highest note so far, and this has a few interesting ramifications as well. First, there is a foreshadowing of the perfect fifth which is coming in measure seven-horn 4 in $E^{b}$ is playing a $D^{b}$ which is exactly an equal-tempered perfect fifth below this $A^{b}$ (and the second $F$ horn is playing an $E^{b}$ exactly an equal-tempered perfect fourth below). But even more interesting is the aggregate of these nine pitches. While flute 1 and horn 1 in F are both sounding the same F ( 500 cents ${ }^{24}$ ), the other seven players each have their own pitch. The structure of this sound mass contains a great deal of dissonance and some beating, bracketed by a perfect fifth in the horns that is unattainable by the equal-tempered instrumentstheir pitch center is nearly a sixth tone lower than equal temperament (see Figures 3.1 and 3.2). At the end of measure four, the four obbligato horns drop out and are replaced by the oboe playing an A in tune with the equal-tempered whole tone chord. This A adds another major second and causes the $\mathrm{A}^{b}$ of the solo horn to stick out; it is the only pitch sounding outside of equal temperament. Its location between the highest two notes not only illuminates the fact that this $\mathrm{A}^{b}$ is not in tune with the rest of the chord (both because $A^{b}$ does not fit into this whole tone configuration, and because it is 31.2 cents away from fitting into the tuning system), but also suggests that some sort of resolution is coming.

[^12]

Figure 3.1-"Praeludium" measure 4 with cents values


Figure 3.2—Major seconds and perfect fifths in the first four measures of "Praeludium" with cents values and, where appropriate, ratios or equal-tempered interval names ${ }^{25}$

Measure five continues to expand the registers, as the F horns reenter, with the first F horn now playing a B above the oboe and the second F horn playing a C below the second flute. The C is only 2 cents high, and therefore does not significantly clash with the equal-tempered whole tone chord in the strings. The B, however, is the eleventh harmonic of the F horn, and comes in nearly a quarter tone lower than an equal-tempered B, at 1051.3 cents. This adds a different sort of major second above the oboe's A, lying almost exactly between a major second and a minor second, but slightly closer to a major second. It is worth noting that this interval reinforces the importance of interval class 2-it sounds like a new type of dissonance that wants to resolve to a more stable major second. ${ }^{26}$ The same can be said for the seventh between this B and the low C in the second F horn that entered with it.
${ }^{25}$ Note that the 213.7 cent major second between horn 3 and the oboe is not presented harmonically-this interval may, however, be heard melodically.
${ }^{26}$ As mentioned in the introduction, these terms have broader definitions here. The interval being discussed is almost exactly a quarter tone away from a major second, but is also in tune in just intonation as a $12 / 11$. This interval, in this context, sounds dissonant, and wants to resolve to a larger type of major second which would be

This new bracket around the sound seems to pull in both directions, and shortly after it enters, the whole tone material leaves, replaced simply by horn 3 in $E^{b}$ playing the same $G$ from before ( 686.3 cents) and the bassoon, sounding an $F$ below the $C$ in the second $F$ horn. At the same time that these two instruments enter, the solo horn finally begins its ascent, moving to C. This moment continues to pull the pitch spectrum in both directions, finally expanding it beyond an octave. The G creates the most tension here—still noticeably flat, it does not quite allow for a perfect fifth or major second from the notes below, nor does it allow a consonant third or fourth with the B or C above (these are too small by 35 cents and too large by 17.6 cents, respectively). The two Cs are just more than an octave apart at 1201.9 cents, which creates a slight beating; the low F in the bassoon, which is dramatically the lowest note of the piece so far, sounds perfectly in tune (in just intonation) with the lower C, at an interval of 702 cents.

The obbligato horns and bassoon hold their note through the next measure while the soloist gradually ascends through the harmonics to D (slightly low at 186.3 cents-an equal-tempered perfect fifth above the $G$ in horn 3 ), $E$ (nearly a quarter tone low at 351.3 cents-an equal-tempered perfect fourth above the B in horn 1) and finally completes the ascent to F ( 502 cents, this is very difficult to distinguish from an equal-tempered F , like the one two octaves lower in the bassoon which is fading out).

Finally, in measure seven, the open fifth Ligeti has been preparing is reached. When the soloist reaches this high F , and while the bassoon is still holding on for a few more eighth notes, the strings, trombone, and flute enter playing open fifths between $B^{b}$ and F over three and one-half octaves. Violin 1 and the solo horn play the highest F together, and while there is a difference of two cents between the equal-tempered $F$ and the just-intoned $F$, this difference is too slight to make them sound out of tune (and the violinist may in fact come in on that higher F instinctively). While this does provide an exciting moment for the piece the dynamics have yet to be marked higher than $\boldsymbol{p p}$; in the very next measure most of these instruments drop out, leaving just a murmuring $B^{b}$ in the trombone and contrabass who are joined quarter tone (such as the 12/11) may be consonant intervals that function as a place of stability in the music.
halfway through the measure by the solo horn on that same $B^{b}$ (since the soloist is playing the $B^{b}$ side of the horn, all three pitches will be the same 1000 cents). This $\mathrm{B}^{b}$ is sustained alone until the middle of bar nine. This nearly clears out the entire register of the orchestra, allowing for a second buildup. This time the process and the result will be quite different.

While the discussion of cents will not always be so illuminating, these eight and one-half opening measures (about 1 minute and 15 seconds at tempo) are a great introduction to this type of discussion. The intervals created through the combination of these tuning systems are still fairly limited at this point in the piece, and yet the sonic result grabs the ear immediately. Interestingly, looking at the opening measures with a focus on texture and orchestration can show additional features about the basic shape of the piece that support this initial discussion.

From the beginning, the sound starts with just a single major second dyad between two horns and builds into a fairly dense cluster, loosely based around a whole tone configuration with various microtonal inflections. As these first four bars progress, each time the sound changes it is by the addition of a new instrument, not by a melodic move by an instrument that is already playing (when the $E^{b}$ horns drop out, they are out for either three or four eighth notes before they reenter-this cannot be heard as a melodic move, just a change in texture). The result of the opening three measures is a whole tone tetrachord with three major seconds of different sizes, which sounds similar to but noticeably different from an equaltempered whole tone tetrachord.

In measure four, there is an entrance of five instruments together (both flutes, both bassett horns and the solo horn) which is noteworthy for several reasons. Not only is this mass entrance a new feature, but it introduces new tone colors provided by the woodwinds. Additionally, the woodwinds come in playing the equal-tempered versions of the pitches that the horns are playing. This creates two different whole tone configurations, which gives a total of eight major seconds sounding within a total space of a slightly large tritone. Perhaps the most important aspect of this entrance, however, is how the solo horn
fits in-it plays the highest note yet in the piece, an $\mathrm{A}^{b}$, and this creates what will soon become an important interval-the perfect fifth, sounding between it and the $D^{b}$ being held over by horn 4 in $E^{b}$.

The fifth measure is the first time an instrument changes pitch without resting first. At the end of measure four, the obbligato horns drop out, leaving only the woodwinds and the solo horn, whose $A^{b}$ is a strong contrast in two ways-it does not fit into the whole tone cluster that the woodwinds are playing, and it is from a different tuning system, leaving it just over an eighth of a semitone lower than an equaltempered $A^{b}$ would be. As measure five begins, the solo horn begins its ascent from $A^{b}$ to $F$, which can also be seen as a move from a note which is well outside of the twelve tone equal temperament system to a note which lies within it. This ascent is illuminated by the thinning of the texture-of the seven other instruments which are playing here, five drop out and only two new instruments come in. Not only does this remove the thick whole tone material from the harmony, but this reduction also takes out the flutes, the bassett horns and the oboe. They are replaced with one natural horn and the bassoon, resulting in not only a thinner texture, but a more homogenous tone color. Once the horn begins its ascent with a move from $A^{b}$ to C, the harmony and timbre stay the same. This allows the soloist's ascent to D, E, and F to ring out above the rest of the ensemble, and when the F is finally reached it is met with a completely new color, the strings supplemented with flute 1 , and a brass instrument Ligeti has not used yet, the trombone. During this ascent, the horn begins by sustaining an interval of slightly more than an octave when it hits a C above the C being sustained by horn 2 in F . It then strikes a perfect fifth between itself on D and the G played by horn 3 in $E^{b}$, and a perfect fourth between its next note, $E$, and the $B$ played by horn 1 in $F$, but the most remarkable interval is the one that follows. When the solo horn reaches its F the orchestration changes-flute 1 returns and the listener is introduced to the trombone and strings. All of these instruments are playing $B^{b}$ or $F$, making a series of open perfect fifths which span three and one-half octaves. The musical material is then reduced down to the low $\mathrm{B}^{b}$ of the contrabass and trombone. This
pitch is joined by the horn halfway through measure nine, leaving a huge registral space available for the next buildup.

Whether the introduction is examined with cents in mind or not, the end result is similar. There is a focus on major seconds and perfect fifths of varying sizes and stabilities. Through the majority of this opening section, the major second is given priority, moving back and forth between several possible interval sizes, but at the end of these eight measures the importance of the perfect fifth becomes extremely clear. Once the perfect fifth has been established, the next sound Ligeti writes is a unison $\mathrm{B}^{\text {b }}$-the lowest note to this point of the movement by far-which serves to clear out all of the previous material before the next section begins, loosely imitating the opening gesture.

Once the $\mathrm{B}^{b}$ has settled in, the obbligato horns, the bassett horns, and the bassoon rejoin in the middle of measure nine. The horns bring back the earlier $\mathrm{D}^{b}, \mathrm{E}^{b}, \mathrm{~F}$, and G whole tone tetrachord, while the basset horns sound the $A$ and $B^{b}$ above, and the bassoon the $G^{b}$ below. The horns have the same collection of three different major seconds, but the woodwinds relate to that tetrachord in a different way-they stay completely outside of the horn harmony, creating their own dissonance, and again foreshadowing the upcoming expansion of the register. When the next group of instruments enters one measure later, Ligeti gives the listener a lot of new information.

The most apparent new change is a shift in dynamics. Until the middle of measure ten all instruments entered at $\boldsymbol{p p}$ and if the dynamic changed it was always by diminuendo. Here, however, most entrances are $\boldsymbol{f f} \boldsymbol{f p p}$ (or just $\boldsymbol{f f}$ in the case of the vibraphone), with the trumpet entering at $\boldsymbol{f p}$ and the solo horn at $\boldsymbol{p}$ with a crescendo to $\boldsymbol{m} \boldsymbol{f}$ over three eighth notes. As these instruments enter, the last sustaining low $\mathrm{B}^{b}$ (played by the trombone) descends to A and fades out. The second change is the new instrumentation-the trumpet and vibraphone enter for the first time, joined by the oboe and the solo horn, while the horns and the bassoon drop out and the trombone fades away. The collection of pitches is new as well, with the difficult to hear solo horn note (played $\boldsymbol{p}$ ) being the lowest, F below middle C . The
rest of the notes lie above middle C, sounding E, G\#, B, and C. This low F on the horn will sound to be in tune with the equal-tempered notes (it is only two cents high), so the collection provides a perfect fifth between the lowest and highest notes, a tritone reminiscent of the whole tone tetrachords, and an augmented triad-an expansion of the whole tone groupings.

But despite this jarring change in texture, harmony, and dynamics, the most important change comes two eighth notes later. Here the solo horn plays a melodic figure reminiscent of the initial ascent to F but much faster, much more disjunct, and with a descending leap of a major ninth added in. Instead of using only long values like the first melodic line, this melody begins with three short values which are followed by two long notes. The entirety of this melody is played over the same sustained $\mathrm{E}, \mathrm{G} \#, \mathrm{~B}, \mathrm{C}$ tetrachord.

This horn melody continues to feature interval classes 2 and 5, starting with a leap of a minor tenth to $A^{b}$ followed by a major third up to $C$, outlining a perfect twelfth (the $A^{b}$ in the middle is only one eighth note long, while the F and C are twice as long). From this C follows the descending major ninth, held for four eighths, and then an ascending perfect fifth which returns to another F , now one octave higher than where the horn started, held for six eighths.

As in measures five through seven, the horn melody triggers another event. The first time, a long sustained melody led to a sustained open fifth. Now the quick, leaping melody introduces more rapid melodic figures, starting with horn 1 in F in measure eleven. This passage goes up the harmonic series from partial five (A) to partial eleven (B) skipping only the eighth partial F on the way. The ascent from nine to ten and ten to eleven ( G to A and A to B ) end this part of the phrase with consecutive major seconds. This B is followed by an augmented eleventh leap down to F, and then a perfect fifth up to C. Partway through this line, the harmony changes with the E, F, G\#, and B being replaced by the strings playing $B, D \#, F \#, A, C \#$, and $E$ while the $C$ from the oboe is still being sustained. This new harmony is filled with major seconds and perfect fifths-ordered, it reads as $C, C^{\#}, D^{\#}, E, F^{\#}, A$, and $B$-totaling
four class 2 intervals and four class 5 intervals. The C from the oboe does not add to either set; when it fades out before the rest of the chord, this actually serves to clear up the harmony.

Halfway through this new chord, in the middle of measure twelve, there is a response to the melodic figure of horn 1 . The solo horn and horn 3 in $\mathrm{E}^{b}$ enter together exactly an equal-tempered perfect fifth apart-the solo horn is still playing the $B^{b}$ side of a double horn—and stay at that interval for several measures as they move through parallel melodic lines. These melodies open with a perfect fourth (the solo horn from $F$ to $B^{b}$, horn 3 from $B^{b}$ to $E^{b}$ ), and that perfect fourth is echoed an octave higher between notes five and six of the phrase. The first five notes are all eighth notes but the second perfect fourth triggers a slowing down of the melody. The horns move up through a series of major seconds of various sizes until they are a tritone higher on the eleventh partial ( $\mathrm{B}^{b}, \mathrm{C}, \mathrm{D}$, and E for the solo horn and $\mathrm{E}^{b}, \mathrm{~F}, \mathrm{G}$, and A for the horn 3). In the solo horn, this is exactly the same melodic line that led to the F and the open fifth in measure seven, but the note values here are no longer than half of the durations used the first time. Once this progression of major seconds begins, the harmony shifts. When the horns move to F and C, the woodwinds all enter together, on $\mathrm{A}, \mathrm{D}, \mathrm{G}, \mathrm{C}, \mathrm{F}$, and $\mathrm{B}^{\mathrm{b}}$ —an undeniably strong reinforcement of interval classes 5 and 2 (five and four instances, respectively). In this passage, Ligeti is clearly trying to persuade the listener to recall the open perfect fifths from measure seven. Two notes later when the horns hit A and E , the stacked fourths are replaced by C and G in the tubular bells, again reinforcing interval class 5 . By now the ear expects a familiar arrival point-the solo horn hitting F-to come next.

Although the solo horn does reach the high F in measure fourteen, the total result is not what the listener might expect; the open fifth of measure seven is nowhere to be seen. The $E^{b}$ horn, seemingly guaranteeing a similar result, has moved not to $\mathrm{B}^{b}$, but down to G . Instead of reaching a perfect fifth, Ligeti writes a minor seventh—yet another instance of interval class 2 . The notes of the tubular bells have been silenced and replaced by the strings, sounding $G, F, C, B^{b}$, and $E^{b}$. The open perfect fifth in measure seven has now turned into a stacking of perfect fifths which has four intervals of class 5 and
three more intervals of class 2. This surprising arrival point has the added element of two different types of G with a discrepancy of approximately 14.7 cents. See Figure 3.3 for a comparison of these two ascents.

This unexpected outcome is barely afforded a chance to settle down as the next bar, measure fifteen, starts with an explosive $\boldsymbol{f f f}$ (or $\boldsymbol{f f}$ for the timpanist) interruption on B and C\#. ${ }^{27}$ This major second/minor seventh interval stays between the two different voices for nearly three full measures, much like the perfect fifth between the horns leading up to the previous chord. Unlike the horn melody, however, this line speeds up drastically over the course of nearly three full measures, and constantly changes instrumentation. It begins with strings and bassoon (with the first note accented by timpani and trombone) and during its ascent the passage loses the bass, adds trumpet and oboe, adds flute, loses the cello, adds piccolo for the first time, and loses the viola. At the end, the melody is just violins 1 and 2 , flute 1 , and the piccolo-the oboe only lasts until the penultimate note.

This new melody, doubled at the major second, begins with an ascending minor seventh leap that is followed by three notes which outline another ascending minor seventh. A few notes later, there is a perfect fifth, and it is followed by a major second two notes later. As the passage continues to speed up, the intervals begin to condense, leading to a perfect fourth to start the second measure of this phrase, and shortly afterwards two pairs of major seconds followed by a surprising minor seventh. Leading into measure three-where the piccolo is introduced for the first time-is another ascending major second followed by a descending major second, and there are two last major seconds beginning two notes later before the melodic possibilities become so compressed that the remainder of the passage can only be chromatic. The line escalates and accelerates and, as suddenly as it began, this surprising new material is suddenly over, replaced in measure seventeen by a chord in the French horns and the bassett horns ( $\boldsymbol{p} \boldsymbol{p}$

[^13]

Figure 3.3-"Praeludium" measures 4-7 and 12-14: a comparison of the two ascents by the solo horn
and $\boldsymbol{p p p}$, respectively), accentuated by a $\boldsymbol{m} \boldsymbol{f}$ strike of the Rin (another term for singing bowl or Japanese temple bell-in this instance the Rin is not tuned to a specific pitch).

This new chord is reminiscent of the opening material-the $E^{b}$ horns sound $D^{b}$ and $E^{b}$, the $F$ horns sound F and C and the solo horn sounds an A ${ }^{\text {b }}$; the bassett horns add B and D. This new collection of pitches (from low to high: $D^{b}, E^{b}, F, A^{b}, B, C$, and $D$-see Figure 3.4) gives three major seconds (one slightly large), one slightly small perfect fourth, two perfect fifths and one slightly large minor seventh. In other words, there are four intervals from interval classes 2 and three from class 5, with a focus on the major second and perfect fifth. This chord is given more than a full measure to settle the dramatic energy of the previous material before the strings enter. Their entrance, just two measures before the end, adds one last new element-an instrument in the equal-tempered group altering its pitch to fit into the justintoned group.

The strings come in seemingly doubling the horns exactly, which means the instrument playing the $D^{b}$ must lower its pitch by 31.2 cents-in this instance, that instrument is the cello and it reaches this note via artificial harmonic. The bass is the other instrument that has an altered pitch, and at first glance it seems the bass's G\# will match the solo horn's $A^{b}$ one octave lower. ${ }^{28}$ However, Ligeti has written a different style of arrow for each note. ${ }^{29}$ The bass reaches its note using a natural harmonic to sound a $G \#$ on the E string-this method actually results in a pitch only 13.7 cents low, significantly different than the note sounded by the horn an octave above. Although the C played first by horn 1 and second by violin 1 actually varies by approximately 2 cents, just as in measure seven this difference is not noticeable to the listener, and will likely be compensated for by the violinist. Once the strings have entered, the horns begin to fade out-resolving the dissonance between the solo horn and the bass-and by the time the last
${ }^{28}$ In editing the score, George Benjamin felt this octave displacement was unintentional. In his letter, he wrote, "I am convinced the bass should perfectly match the horn solo here...the present pitch ruins the transition between 5 horns and strings." See Appendix A for more information.
${ }^{29}$ Ligeti's use of three different types of arrows here shows that the two pitches will be altered by different degrees. In my Figures, since cents values or partial numbers are given, only one type of arrow is used. See Appendix A for more information on the arrows in the score.


Figure 3.4-"Praeludium" measures 17-20 with cents values
measure begins only the bassett horns and the strings are sounding, still sustaining a similar version of the previous chord-now ordered $G^{\#}, D^{b}, E^{b}, F, B, D$, and C—and this too fades out over one measure to end the opening movement.

While "Praeludium" mainly serves to introduce the tuning system conflicts and demonstrate several of the interesting colors and textures that are possible in the ensemble, the underlying material that makes it work so well is the combination of major seconds and perfect fifths within and across tuning systems. In fact, this simple relationship is foreshadowed before the piece even begins by looking at the keys of the featured instruments of the piece as two horns are in $\mathrm{E}^{b}$, two horns are in F and one, the soloist, is in $\mathrm{B}^{b}$. Additionally, throughout the entire first movement, each time one of the horns enters, the entrance creates either a major second or a perfect fifth.

Of the two interval classes, interval class 2 definitely is the dominant interval class of "Praeludium": a major second begins the piece; the major second is featured in the whole tone material throughout the piece, a sudden move to a minor seventh tears apart the parallel perfect fifth melody of the horns in bar fourteen and follows it with a melody in parallel major seconds; in the ending chord the major seconds outnumber the perfect fifths three to two. Throughout the piece the listener is also treated to a variety of different types of major seconds and minor sevenths, not only by changes of register, but also by their size and how in tune they sound. Conversely, the role of interval class 5 (led by the perfect fifth) is to support the intervals of class 2 . The perfect fifth not only aids in creating intervals of interval class 2 , but also allows an escape from the early whole tone cycles, creates resting places in the music, helps to foreshadow new events, and when needed it serves as a middle ground to reconcile the two tunings. As for the other features of "Praeludium," the imitation of the opening measures in bars eight through fourteen is a very interesting moment for this movement, but also foreshadows similar events in many of the remaining movements, as well as large-scale relationships that exist within the Hamburg Concerto as a whole. The outburst that occurs in measure fifteen is the first example of interruptionsanother strong feature of this work, and one which will appear in several other movements.

### 3.3 II. "SIGNALE, TANZ, CHORAL"

Having introduced so many of the interesting aspects of the piece in the opening movement, Ligeti now examines just a small set of these possibilities. Where the first movement was a comparison between the orchestra and the horns with long-range imitation, this movement illustrates some of the possibilities that exist within the horns themselves, and does so with immediate imitation and canon. A wide range of intervals are explored, and the harmonic series is presented from partial two through sixteen, which is nearly the maximum range used in the entirety of the Hamburg Concerto (which uses from partial one to partial seventeen and simulates up to partial nineteen). This second movement is broken into three separate, clearly defined sections, and for the majority of "Signale, Tanz, Choral," the only instruments used are the horns.

Ligeti introduces a different tuning scheme for the horns in this movement. The soloist now is on a natural horn in F , as is obbligato horn 1 . Obbligato horns 2,3 , and 4 are tuned to $\mathrm{E}, \mathrm{E}^{b}$, and D respectively. This change allows Ligeti to employ aspects of the twelve tone equal-tempered system without having any one player use that system. In other words, this natural horn scheme and the homogenous sound created by the horns allows for the illusion of equal-tempered movement within a harmonic-based just intonation system.

Initially, this tuning scheme is used to provide interesting effects to the "Signal" of the solo horn. After the soloist plays the opening measure (starting on A , its fifth partial), it is echoed exactly by F horn 1 , starting a canon. Then the soloist extends the signal, and it is echoed again, now by horn 2 , playing in E. This (equal-tempered) half step lowering of the horn motive sets much of this movement in motion. In measures four through six, the soloist extends the signal a bit more, and horn 3 echoes the call again, but this time instead of using the same partials as the soloist, the $\mathrm{E}^{b}$ horn starts one pitch lower in the harmonic series. This has the overall effect of sounding the soloist's passage around a tritone lower (although depending on which harmonics are being used, this tritone approaches a perfect fourth). Before
this echo can finish, the soloist interrupts with a fourth version. Horn 4 in D wastes no time-it plays the signal at the same time as the soloist, abandoning the echo formula completely. Like horn 3, this horn also starts one partial lower, creating intervals ranging from a tritone to a perfect fifth. This interesting effect is complicated as the E horn enters just over a beat after this fourth version begins, adding a new shape to the melodic line, and additional intervals to this new harmonic language (see Figure 3.5). This version of the horn motive is cut short by another interruption-the obbligato horns play the same set of partials on their instruments, creating an equal-tempered chromatic cluster ascending the harmonic series through partials four, five, six, and nine. This bold gesture (starting at $\boldsymbol{f f f}$ and ending at $\boldsymbol{f f f f}$ ) ends the "Signale" portion of the movement.

Looking at the intervals created in measures seven and eight (before this point, instruments only played one at a time or at most overlapped with the sustained last note of the previous instrument's line) a wide range of intervals that are not readily available within twelve tone equal temperament or just intonation can be found. The first interval heard is 686.3 cents-a small perfect fifth. A listener can easily hear that this is too small to be a real equal-tempered perfect fifth but not quite small enough to sound like a tritone. The notes go by rather quickly here, so before the listener can reflect on that interval, the horns play intervals of 615.7 cents, 566.8 cents, 735.1 cents and a slightly longer-lasting interval of 647.4 cents. On this last interval, they are joined by the E horn, which sounds 1465 cents ( 265 cents plus an octave) below the solo horn, and 817.6 cents below the D horn. This note ends at the halfway point of measure seven.

Within this first half of measure seven, there are already two notes which, in rough terms, seem like perfect fifths of varying sizes, and three notes which seem like tritones of some sort. The E horn adds a very small minor tenth and a slightly large minor sixth. Not surprisingly, none of these intervals fit exactly with a just intonation ratio-the relationship of the equal-tempered tuning scheme to the harmonics is too complex. However, what does come out of this is the relationship of intervals outside of equal temperament. The intervals at 566.8 cents, 615.7 cents and 647.4 cents all still sound like tritones,


Figure 3.5-"Signale, Tanz, Choral" measures 7 and 8 with cents values
even though two of them are nearing a quarter tone discrepancy from their equal-tempered relative. The intervals closer to a perfect fifth- 686.3 cents and 731.5 cents-are not quite as easily accepted by the ear because of how pure the perfect fifth is, and how closely the equal-tempered scale approximates that fifth. These notes do, however, sound closer to a perfect fifth than a tritone or minor sixth.

The E horn complicates the issue drastically. When it enters in measure seven, its interval with the soloist is very close to a $7 / 6$, which would be a consonant minor third called the septimal third. However, this interval is slightly flat of the 7/6, and that potential dissonance is somewhat masked by the tritone between the F and D horns mentioned above, and the interval of a large minor sixth between this E horn and the D horn. The overall effect of this note is that of some strange diminished triad where the tritone is nearly too big and the thirds are uneven. Certainly it is a chord which cannot even be approximated in a system of equal-tempered pitches, although a just-intoned system could come reasonably close (a triad using $1 / 1,7 / 6$, and $16 / 11$ is very close to this chord-the third and fifth would be off by less than two cents each).

The most interesting combination of these tuning systems in this opening section might be the very last part-the interruption by the four obbligato horns in measure eight. As mentioned earlier, they all play the same partials, which results in a just-intoned passage being played with an equal-tempered harmony. All of the harmonic intervals are equal-tempered half steps of 100 cents, spanning a total of only 300 cents. The melodic intervals are all in the just-intoned system: first a $5 / 4$-at 386.3 cents it is a just-intoned major third; second a 6/5-at 315.6 cents this is a just-intoned minor third; and last is the 3/2—at 702 cents this is a just-intoned perfect fifth. ${ }^{30}$ This grouping of three just-intoned melodic

[^14]intervals played with an equal-tempered harmony creates a sound that lies outside the normal boundaries of either system-this is only possible through Ligeti's unique combination of the two.

This interruption leads into the "Tanz" (beginning on the third beat of measure eight), where another interesting aspect of the tuning is illuminated. Through this entire section, the horns always move stepwise (as much as possible in a system limited to harmonics), which shows some interesting relationships between the horns as an ensemble, and further enables Ligeti to feature the juxtaposition of the just-intoned and equal-tempered systems.

The dance starts halfway through measure eight with the solo horn and the D horn playing similar lines approximately a perfect fourth apart, but never exactly a perfect fourth apart—neither at 500 cents for equal temperament nor 498 cents for just intonation. The first four intervals are at 482.4 cents, 503.9 cents, 531.2 cents, and 503.9 cents again as the second notes are repeated (see Figure 3.6). ${ }^{31}$ This type of figure is the first of two components of the dance. The second, a reflection on (or imitation of) the "Signale" motive, ${ }^{32}$ enters early in the next bar (measure eight) in horn 2—a passage of quick ascending or descending notes (moving always by step) in a 4:3 rhythmic relationship to the first figure. While the first dance motive is nearly always played by two horns through this section, the second motive begins with just one horn but by the end of this section is played by three.

The opening pattern of the "Tanz" section constantly plays with similar intervals that waver towards and away from being in tune in either of the tuning systems in use. The formula is simple enough: the horns begin on different partials in their harmonic series; during the passage, they always move stepwise through their harmonics, often in parallel motion, but not exclusively; when a new horn takes over for one which is currently playing, these relationships will change-the new horn may enter on any partial, but will still move by harmonic steps. Through this process, the listener is treated to a great

[^15]

Figure 3.6-"Signale, Tanz, Choral" measures 8-15 with cents values for motive one


Figure 3.6 continued
range of possible intervals from seconds through sevenths. These will range from an interval of slightly larger than three quarters of a whole tone (166.8 cents, beat two of measure eleven) to a very slightly large minor seventh (1002 cents, beat two of measure fourteen). There is one larger interval in this first motive on beat two of measure thirteen. This is the only time three horns are playing part of this motive together, and the distance between the outer two is a slightly large major tenth of 1602 cents. However, the D horn, which plays the highest note here, is entering this triad via a passage from the second motive of the dance section, and it is unlikely that a listener will realize that this arrival is the beginning of the D horn's motive one passage until well after this chord has sounded.

There is another interesting result of the structure of this motive aside from the wavering intervals discussed in the previous paragraph. This other aspect can be heard when the horns move in the same direction for several beats. The effect, first seen in measures nine and ten (between solo horn and horn 4 in D ), is the gradual collapsing or expanding of the interval between the horns as they continue through their passage. In this first instance, over the course of eight notes the interval decreases from 686.3 cents to 450.7 cents (see Figure 3.7). The next two instances are more dramatic: in measures eleven through thirteen, horns 1 in F and 2 in E move from an interval of 713.7 cents (the end of beat two in measure eleven) to an interval of 286.3 cents (beat two of measure twelve) and back past the starting point to an interval of 868.8 cents (beat two of measure thirteen, see Figure 3.8). The very next note begins a similar passage, again between the solo horn and horn 4 in D, which begins at 589.2 cents, expands to 1002 cents (beat two of measure fourteen) and contracts again beyond the starting point to 531.2 cents at the last chord of the dance (see Figure 3.9). A fourth example of this effect, much shorter in duration, can be found in measures ten and eleven between horn 1 in F and horn 2 in E, where over the course of just three notes, the interval expands from 717.5 cents to 913.7 cents. However, because it happens for only three notes, and the change is so large, it is hard to hear the process at work here.


Figure 3.7—Collapsing intervals in "Signale, Tanz, Choral" measures 9-10


Figure 3.8-Collapsing and expanding intervals in "Signale, Tanz, Choral" measures 11-13


Figure 3.9—Expanding and collapsing intervals in "Signale, Tanz, Choral" measures 13-15

Throughout this entire "Tanz" section, one of the F horns is always playing this motive, while the $E^{b}$ horn never plays any of it. Because the F horns are working together to provide the listener with a common ground for the other shifting intervals to be heard against, it is almost inevitable that the $E^{b}$ horn would be left out. It simply shares too many notes of its harmonic series ${ }^{33}$ with that of the F horn, and would not create as interesting of a sensation (either of wavering or of expanding and collapsing intervals) as the D or E horns can.

After the opening movement and the "Signale" section of movement two, the listener is fairly familiar with the sounds of the harmonic series. Ligeti continues to provide the listener with reminders of that pattern so that in instances like "Tanz" he is able to create different systems to exploit the harmonics in a way that the listener can hear. This time, the fact that the horns are merely ascending and descending

[^16]through the harmonic series is clear to the listener, and the shifting intervals that result can be heard clearly as part of the tuning system clash that Ligeti has designed.

The second element in the "Tanz," a series of quick ascents and descents, also plays with the listener's familiarity with the harmonic series (see Figure 3.10). These passages are played staccato, and the line frequently moves between horns. When the pattern of ascending or descending notes is brokeneither by changing direction or by a large leap-this break is caused by the entrance of a different horn, and therefore a change of the harmonic series (neither of the F horns interrupts the other in this manner). This passage, much like the opening material to which it is closely related, is set up as a canon. It is important to note that this passage is heard as a continuing rise and fall, not as a group of disparate ascents and descents. The way one horn connects to the end of the line of another horn is an interesting feature of this section.

Looking at Figure 3.10, it is reasonably easy to see how the process of changing horns is used throughout this section. The ascending figure begins as close to the beginning of measure nine as possible (this motive never begins on a downbeat) from the fourth partial of the E horn, and it continues up to the twelfth partial. Less than one beat later, the $E^{b}$ horn takes over the line, starting on its thirteenth partial. This is an interesting effect-the E horn's last note was a B that sounded 1102 cents and the thirteenth partial of $E^{b}$ is 1140.5 cents. ${ }^{34}$ This 38.5 cent change sticks out to the listener as an unusual interval for either tuning system, and here the $E^{b}$ horn descends from that thirteenth partial for nearly three full octaves to the second partial at 300 cents into the third beat of measure ten. This is picked up immediately by the E horn an octave and 100 cents above, who plays just partials four, five, six, and seven before joining the first motive. The D horn begins its ascent from the same partial as the E horn did as soon as the E horn switches.

[^17]

Figure 3.10-"Signale, Tanz, Choral" measures 8-15 with cents values for motive two


Figure 3.10 continued

This entrance of the D horn one beat later and one whole step lower than the E horn reinforces the canon, and extends the passage past where the E horn left off. The entrances of these two horns at 200 and 400 cents have surrounded the last note of the $E^{b}$ horn of 300 cents, which was played an octave lower. The D horn ascends from the fourth to the eleventh partial, and is interrupted there by the return of the $E^{b}$ horn. The $D$ horn's last note of 751.3 cents is followed by a note an octave and 949.3 cents lower-this is the second entrance which has a difference that does not fit in either tuning system—when the $E^{b}$ horn starts on its third partial and ascends to its fourteenth at 68.8 cents, just slightly larger than a perfect fourth above where the D horn left off.

The last three notes of this passage in the $E^{b}$ horn are at 1002 cents, 1140.5 cents, and 68.8 cents as mentioned above. This last note is the downbeat of measure twelve, and is immediately followed by the solo F horn playing a descending passage starting on its eleventh partial, 1051.3 cents. Again Ligeti writes in an interval which sounds out of place against the previous passage, but the descent quickly becomes familiar. Much like the $\mathrm{E}^{b}$ horn in the only other descending passage thus far, the soloist descends down to the second partial, sounding 500 cents. The D horn begins on its fifth partial (an octave and 86.3 cents higher ${ }^{35}$ ) and ascends much like it did following the $E^{b}$ horn earlier (measures ten to eleven) to a note seven partials higher. The twelfth partial, 902 cents, begins a segment of the first motive; the $E^{b}$ horn is left to take over and comes in almost exactly a quarter tone lower, on 851.3 cents. From here, its eleventh partial, it descends down to the second partial of 300 cents, exactly as the F horn did just over one measure earlier.

This entrance of the $\mathrm{E}^{b}$ horn is the last entrance that takes over from a previous horn player. When the next entrance is made, it is the E horn entering at 783.6 cents on its tenth partial. However, the $E^{b}$ horn is still finishing its descent when the E horn comes in-the canon begins to overlap other

[^18]iterations instead of presenting the ideas one at a time. From this entrance, in the last beat of measure thirteen, the second motive begins to mimic an element of the first: the gradual collapsing or expanding of the interval between the horns.

At first, the overlap of the two lines goes by so quickly it is hard to notice the similarity between the two components. Although the change is drastic, the overlap lasts for only two notes, and the shifting of the interval from 2184.3 cents to 2703.9 cents (from a minor seventh plus an octave to a minor third plus two octaves-which at a slow tempo could sound cadential) covers a very large distance; it also must contend with the first motive, whose change of interval is well established and more prominent. A listener might not even be aware that two horns are playing that line at this point.

In measure fourteen this is made much clearer. Two horns are playing together from the middle of beat one for a full beat where they are joined by a third horn for one additional beat. After three notes of just one horn, Ligeti writes a little more than a full beat of three horns, three notes of two horns, and three notes of three horns. The rapid thickening of this motive at the end of the dance section is accentuated by the crescendo from $\boldsymbol{m f}$ to $\boldsymbol{f f f}$.

During these last measures of the dance, the interval created when a new horn enters is much harder to detect because it is not playing the line alone. However, the rising and falling of this line is preserved and made slightly more obvious with the thickened texture. Starting halfway through beat one in measure fourteen, horns 1 and 2 play an ascending line together, with horn one starting on its third partial and horn two on its fifth, 784.3 cents higher. Over the next two beats, this interval decreases by more than a perfect fourth down to 215.7 cents. When horn 3 joins halfway through this ascent, it starts on its third partial, creating an interval of 1666.8 cents between itself and horn 1 and an interval of 2001.9 cents between itself and horn 2 (horns 1 and 2 are at an interval of 335.1 cents here). Over the next three notes, these intervals decrease to 1084.3 cents and 1300 cents (an equal-tempered minor ninth) at which point horns 1 and 2 drop out.

Horn 3 continues to ascend, and is rejoined in beat four by horns 1 and 2 approaching each other via contrary motion. They start 2500 cents apart and two notes later the interval is exactly 1300 cents. Slightly less than one beat later the interval is down to 131.2 cents. Two notes later, they have expanded to 1100 cents (an equal-tempered major seventh), and two notes after that, these horns are approaching the initial interval-they are at 2300 cents. Here horn 1 changes direction and ends moving in the same direction as horn 2 . When horns 1 and 2 enter, the ascent of horn 3 is continuing the diminishing interval size feature with the line of horn 2 . Over five notes, the interval changes from 1984.3 cents to 1100 cents. The contrary motion between horns 1 and 3 begins at an interval of 515.7 cents, moves to 200 cents (an equal-tempered major second) on the second note, and ends at 768.8 cents. Horn 3 ends the dance with a short descending passage from its twelfth partial down to its ninth, reinforcing the descending motion that horn 1 abandons. Horns 1, 2, and 3 all arrive on beat three—along with the solo horn and horn 4 playing motive one-on a loud chord lasting for one full beat, a comparatively long time based on the rest of this dance section.

This final chord of the "Tanz" section features two Fs, one at 500 cents and one at 503.9 cents (just like measure two of "Praeludium") a C and a D exactly 200 cents apart (at 1168.8 and 168.8 cents) and an A, the lowest note by a minor sixth, of 886.3 cents. The chord spans a total of 2013.7 cents, and the highest note, an F of 500 cents, is the highest pitch so far in this movement, and in fact will remain the highest pitch until the last chord, when higher notes are played by the flute, piccolo, and violin.

Through the dance, Ligeti directs the horns in such a way that several equal-tempered intervals are sounded as they continue up and down their respective harmonic series. The tuning of the five horns makes this possible, but also makes these relationships incredibly easy to avoid. The fact that Ligeti has worked out so many different places where an equal-tempered interval is sounded shows how important the integration of the two systems is to the language of this piece. As mentioned earlier, the end of the "Signale" section features the theme played by all four horns together, which creates an equal-tempered chromatic tone cluster. After this introduction to the dance, there are several measures without equaltempered harmonies, but in the last two bars there are seven instances of equal-tempered intervals,
ranging from 200 cents to 2500 cents. Just as "Signale" introduced the tuning of the horns one at a time and carefully illustrated some of the possibilities Ligeti had available, "Tanz" takes this same tuning and employs all five horns together. This allows for more adventurous melodic lines and more interesting harmonic shapes, which Ligeti further explores in the final section of movement two, "Choral."

This final section of "Signale, Tanz, Choral" is very different from the first two. It is a much slower, more intricate examination of the tunings in this movement. Because it is slow and sustained, the listener has more time to hear how Ligeti's system alters the possibilities of chorale-like music.

While the tuning affects the harmonies, the chords are still similar in sound to the equal-tempered versions. The chords in "Choral" create a modified ${ }^{36}$ chord progression in which the resulting chords do not have the same implied functions as equal-tempered chords may have. The first phrase works with a modified chord progression of: $D^{b^{M}} / C ; E^{b} m^{9} / G^{b}$ with no seventh ${ }^{37} ; C^{M_{7}} / G ; A m^{7}$ with a third that approximates a quarter tone ${ }^{38} ; \mathrm{B}^{\mathrm{M}_{7}} / \mathrm{A}^{\#}$; $\mathrm{E}^{b^{M 7}}$; and ends on an $\mathrm{F}^{\#^{0} 7} / \mathrm{A}$ chord which is twice the duration of the previous chords (see Figure 3.11). While the harmonic interest of this section is still guided by the tuning system, and the main melodic line is highlighted through dynamics (horn 2 in E is marked as the cantus firmus, and plays $\boldsymbol{m} \boldsymbol{f}$ while the other three horns are playing $\boldsymbol{p} \boldsymbol{p}$ ), the apparent equal-tempered movements of some voices is another important feature within this chorale. In fact, the two primary features of the chorale are the interesting intonation of the chords (many of which can only be created in this particular tuning scheme) and the use of equal-tempered intervals (melodically and harmonically) throughout the chorale. Combining these two features together is a complicated task, but one Ligeti has

[^19]carried out in a way that is very musical and also intensely mathematical. The description which follows should serve to illuminate both of these points.


Figure 3.11-"Signale, Tanz, Choral" measures 16-27 with partial numbers for the horns

Throughout the chorale, every chord change but one has at least one apparent equal-tempered move. This is created by different horns playing the same partial in consecutive chords (or partials that sound an octave apart). For example, in the first chord, horn 1 in F is playing an F on its eighth partial, horn 2 in $E$ is playing a $G^{\#}$ on its tenth partial, horn 3 in $E^{b}$ is playing a $D^{b}$ on its seventh partial, and horn 4 in D is playing a C on its seventh partial. In chord two, horn 1 in F stays on its eighth partial F , horn 2 in $E$ moves up to an $A^{\#}$ on its eleventh partial, horn 3 in $E^{b}$ moves up to its eighth partial $E^{b}$, and horn 4 in D leaps down to $\mathrm{F} \#$ on its fifth partial. There are a few interesting relationships here. First, there are two apparent equal-tempered movements. The F played by horn 1 in the first chord and the $E^{b}$ played by horn 3 in the second chord will be exactly an equal-tempered major second apart, and the G\# played by horn 2 in the first chord and the $\mathrm{F}^{\#}$ played by horn 4 in the second chord will be exactly an equaltempered major ninth apart. The second interesting relationship is that of equal-tempered intervals within the chords-in chord one, horns 3 and 4 are exactly an equal-tempered major second apart, and in chord two, horns 1 and 3 are exactly an equal-tempered major second apart. This helps to ground the progression through the inclusion of familiar intervals (these specific equal-tempered intervals have been important throughout the piece to this point, as they were also a major focus of the first movement) into these somewhat foreign harmonies. The fact that horn 1 remains on the same note could be seen as an equal-tempered relationship as well, but it is does not sound like there is a new type of relationship happening because neither the pitch nor the instrument has changed.

Several other equal-tempered relationships can be traced through the chorale section. Within the first seven chords mentioned earlier, there are ten melodic equal-tempered relationships (not counting unisons, see Figure 3.12), and seven harmonic equal-tempered relationships.


Figure 3.12—Melodic equal-tempered intervals within phrase one of "Choral," measures 16-17

The ten melodic equal-tempered intervals shown above illustrate a few important features. First, the octaves occurring in horn 3 in $E^{b}$ —while they might seem out of place in this diagram, these intervals are very important as the octave is the only interval that is exactly in tune (and, therefore, exactly the same) both in equal temperament and in just intonation. The unisons are not counted as mentioned above. The next interesting feature is surprisingly the lack of an equal-tempered interval between chords five and six. In the entire chorale section, this is the only point in which two chords are not connected by an equal-tempered movement. Finally, this collection of equal-tempered intervals actually creates an interesting equal-tempered melodic line in the middle of the chorale. From the $G \#$ in the first chord which is played by horn 2, there are several consecutive notes played on the fifth and tenth partials, creating a line of $G^{\#}, F^{\#}, G^{\natural}$, A, and $F^{\#}$ (or possibly $G^{\#}, F^{\#}, G^{\natural}, G^{\natural}$, and $F \#$, depending on whether the listener hears the A or the $G$ at the end of measure sixteen as taking over for the previous G ). The line is broken here-the following chord has no equal-tempered relationships at all with the chord on the downbeat of measure sixteen where this final $\mathrm{F}^{\sharp}$ is sounded. As mentioned above, this is the only place in the chorale where two consecutive chords do not have at least one equal-tempered interval.

Harmonically, there are seven equal-tempered intervals in this phrase, one in each chord. These seven intervals are: a minor second in chord one between horns 3 and 4; a major second in chord two between horns 1 and 3; a major seventh in chord three between horns 1 and 2; a major ninth in chord four between horns 1 and 3; a minor ninth in chord five between horns 2 and 3 ; a major seventh in chord six between horns 3 and 4; and a minor third in chord seven between horns 1 and 4 . One interesting aspect of this component of the progression is that at each new chord, one of the horns which had an equaltempered harmony in the previous chord will have an equal-tempered harmony in the new chord. This continues throughout the entire chorale between any two consecutive chords which contain equaltempered harmonies. However, as the chorale continues on, the equal-tempered harmonies become less common, making some of them stand out even more when they are used.

The second phrase, beginning in measure eighteen, uses a modified chord progression of: $E^{b^{M_{7}}} / D ; D^{b_{7}} / F ; C^{7^{b}} / G^{b}$ (French augmented sixth chord); $D^{M_{7}} ; \mathrm{Dm}^{7} / C$; $A^{9}$ with no seventh; and a $B^{7} / F^{\#}$ chord which is again twice as long as the other chords in the phrase. This phrase contains sixteen melodic equal-tempered intervals (see Figure 3.13), and is connected to the previous phrase with one as well (a descending minor ninth between horn 3 and horn 4). It also has significantly fewer harmonic equaltempered intervals-four to the previous phrase's seven.


Figure 3.13-Melodic equal-tempered intervals within phrase two of "Choral," measures 18-20

These melodic intervals form a few short equal-tempered melodies ( $G, G^{\#}, F^{\#}, A$, and $A$ on the fifth and tenth partials; $D, F, E$, and $D$ on the fourth and eighth partials; and either $D^{b}, C, D^{b}$ or $D^{b}, D^{\natural}, D^{b}$ on the seventh partial) weaving around and through the cantus firmus line and the various harmonies. This phrase connects to the previous phrase with a minor ninth between the $\mathrm{E}^{b}$ in horn 3 and the D in horn 4, although due to the rest between the phrases and the distance between these notes, it is difficult to hear this relationship. Ligeti avoids giving an equal-tempered relationship to the first note in the cantus firmus line in this phrase (horn 2 in E) to further distinguish it from the other three lines, and highlight its microtonal inflections.

As the phrase unfolds, the cantus firmus line becomes more integrated with the other lines, which is especially noteworthy after the first phrase where it had such a slight involvement with the other horns. Phrase one included only two equal-tempered intervals involving the cantus firmus (one at the very start and one right at the end) whereas phrase two includes nine.

Harmonically, there are just four instances of equal-tempered intervals, a minor second in chord two between horns 3 and 4, a major second in chord three between horns 1 and 3 , a major second in chord five between horns 2 and 4, and a minor third on the final chord between horns 1 and 4. As noted earlier, this is a shift from the first phrase where every chord had an equal-tempered interval within it somewhere. Also, the range of harmonic equal-tempered intervals has diminished-there are no compound equaltempered intervals in this phrase.

Phrases three and four (measures twenty through twenty-two) are each almost like half-phrases. Together they total seven chords-the same as each of the two previous phrases were on their own-but they each stand on their own in context. Phrase three uses the modified chord progression: $\mathrm{Fm}^{7} ; \mathrm{A}{ }^{\# 9} / \mathrm{C}^{\times}$ with no seventh; and finally $\mathrm{C}^{\mathrm{M}_{7}}$, held for a longer duration which the listener now recognizes as a sign that the phrase has ended. It is connected to the previous phrase via three equal-tempered intervals (a minor seventh between horns 3 and 1, a major ninth between horns 4 and 2 and a major seventh between horns 1 and 2 ) as well as one unison (in horn 3). Within this short phrase there are five equal-tempered
melodic intervals (see Figure 3.14) and two equal-tempered harmonic intervals (a minor seventh in chord one between horns 1 and 3 and a major seventh in chord three between horns 1 and 2). Due to the brevity of this phrase, no underlying equal-tempered melody can clearly be traced-no partial involved in the move from chord one to chord two is also involved in the move from chord two to chord three.


Figure 3.14—Melodic equal-tempered intervals within phrases three and four of "Choral," measures 20-22

The chords in phrase four are not as straightforward as in previous phrases-the language of the chorale is beginning to change. Phrase four has a modified chord progression of: Cm\#11/G with no seventh or ninth; $\mathrm{D}^{b} \mathrm{~m} / \mathrm{F}$; $\mathrm{Am}^{b} / \mathrm{C}$ with no seventh; and finally a comparatively stable $\mathrm{B}^{b 7} / \mathrm{F}$, again at twice the length of the previous chords. What phrase four does have in common with previous phrases is the distribution of equal-tempered intervals, which is similar to that of phrase three with nine equaltempered melodic intervals (see Figure 3.14) and two chords with equal-tempered harmonic intervals.

Phrase three connects to phrase four with an octave in horn 3 and a major second between horns 4 and 2. However, the more obvious interval between the two phrases is the just-intoned perfect fourth played by horn 2, not only because that horn is playing the loudest, but because it is the highest voice in both chords. It is difficult to trace an equal-tempered melodic line in this phrase (another similarity with
phrase three). This is partially due to its brevity, and partially due to the fact that the intervals between the only three notes that share a partial are a major seventh followed by a major fourteenth.

The harmonic intervals in this phrase are particularly interesting. The phrase starts with an equaltempered minor third between horns 1 and 4 in chord one, and no equal-tempered-intervals in chord two. Chord three has three equal-tempered intervals-a minor seventh between horns 1 and 3 , a major seventh between horns 3 and 4, and a major thirteenth between horns 1 and 4 . However, the note being played by horn 2 is less than 2 cents away from being in the same equal temperament as the other three horns, which would create three more equal-tempered intervals-a major tenth between horns 1 and 2, a tritone (as an augmented fourth) between horns 2 and 3, and a perfect fourth between horns 3 and 4 . Because this 2 cent difference is so miniscule, it does sound like this is an entirely equal-tempered chord. This is the first part of the chorale that features such a strong presence of equal temperament. It does not last long, however, as the final chord of this phrase has no equal-tempered harmonies within.

The final phrase, starting at measure twenty-three, is a bit longer than any of the others. The first five chords of its modified chord progression consist of the following: $\mathrm{F}^{7} / \mathrm{E}^{b}$ (here the fifth is between a C and a C $\#$ —the fifth will sound very large, but it will not result in a feeling of an augmented triad); $\mathrm{A}^{9}$ with no fifth; $C^{0}{ }^{M} ; D^{b} m^{M 7}$; and $C+{ }^{7} / B^{b}$. At this point in the phrase the bassoon enters, playing $\boldsymbol{p}$, allowing for five-note chords, which Ligeti uses often, and doublings, which he uses only once. The rest of the phrase from the bassoon entrance consists of the modified chords: $D^{b} 6^{b} 9 ; B m^{7^{b} 11} / F^{\#} ; G \#^{0} 9 / A ; B^{b^{b} 9}$ with no seventh-the root is played by both the bassoon and horn 3 ; and a chord twice as long as the previous chords which is an $A^{b 7^{b} 9}$ chord. This time, however, the increased duration does not signal the end of the phrase-there is another chord of that same doubled duration on the downbeat of measure twenty-six which is $\mathrm{Gm}^{11} \mathrm{~B}^{b}$ with no ninth. This chord is followed by a striking change in timbre, as the horns and bassoon drop out and are replaced by most of the orchestra, playing a chord built of eight
stacked perfect fourths (interval class 5) from G\# up to G, with a bass note of E. This chord is twice the duration of the previous chord, signaling that the movement has come to its end.

Within the first part of this final phrase (before the bassoon joins), there are only three harmonic equal-tempered intervals, and eight melodic equal-tempered intervals (see Figure 3.15). It connects to the previous phrase with three additional melodic equal-tempered intervals: a perfect fifteenth in horn 1 ; a minor second between horns 1 and 3; and a major seventh between horns 3 and 4. From where the bassoon enters until the horns and bassoon drop out, there are thirteen equal-tempered melodic intervals within the horns (see Figure 3.15), and five equal-tempered harmonic intervals within the horns. There are five additional equal-tempered harmonic intervals created between the horns and the bassoon.


Figure 3.15-Melodic equal-tempered intervals within phrase five of "Choral," measures 23-26

Between chords nine and ten of this phrase, the largest quantity of equal-tempered intervals can be found-there are six, including a major ninth between horn 1 and horn 3 , a minor second between horn 2 and horn 1, a major second between horn 2 and horn 4, a minor seventh between horn 3 and horn 1, a minor ninth between horn 3 and horn 4, and a major seventh between horn 4 and horn 3 . The last two
chords are on the other side of the scale, with only one equal-tempered interval connecting them: a major second from horn 3 to horn 1 .

The harmonic equal-tempered intervals within the horn section begin with a major sixteenth in the first chord between horn 1 and horn 3, and the following chord contains another major sixteenth a tritone higher between horn 2 and horn 4. Chord five (the last chord before the bassoon enters) contains an equal-tempered major second, and chord seven, the second chord with bassoon, contains a minor tenth between horns 1 and 4. Chord eight, like chord six, has no equal-tempered harmonic intervals, but chord nine has two: a minor third between horns 1 and 4, and a major seventh between horns 2 and 3 . As was the case in the previous phrase, this chord is interesting because the partials involved are eight, six, and twelve. Since these partials are separated by less than 2 cents, this entire chord will sound like it is equaltempered (and the bassoon, playing the lowest note—which doubles the highest note in this chord—will reinforce the perceived equal-tempered relationship). The last two chords each have one harmonic equaltempered relationship. Chord ten has another minor third between horns 1 and 4 , and chord eleven has an equal-tempered major second between horns 2 and 4 .

When the bassoon is considered along with the horns, there are an additional five equal-tempered intervals that can be found. Chord six, which does not have an equal-tempered relationship just within the horns, has a perfect eleventh between horn 4 and the bassoon. Chord seven has a major third between horn 3 and the bassoon, and chord eight which did not have an equal-tempered relationship within the horns, still does not have an equal-tempered relationship when the bassoon is considered. Chord nine has two, a major third between the bassoon and horn 4 and a perfect fifth between the bassoon and horn 1 . These three intervals form an equal-tempered major triad. The fourth chord has an equal-tempered perfect fifth between the bassoon and horn 3, and the last chord has no equal-tempered relationship between the horns and the bassoon.

Since the initial phrase, there has not been a strong equal-tempered melodic line amongst all of the chords. However, in the final phrase this feature returns. From chord two to chord four are two short equal-tempered melodies, one on the seventh partial between horns 3 and $4\left(\mathrm{D}^{b}, \mathrm{C}\right.$, and $\left.\mathrm{D}^{b}\right)$ and one on
the ninth partial between horns 1,2 , and $4(G, F \#$, and $E)$. These two lines are not in equal temperament with each other, though. The first sounds 31.2 cents lower than equal temperament, and the second sounds 3.9 cents higher. Near the end of the phrase are two more short equal-tempered melodic lines. From chord eight to chord ten on partials three and six, two possible melodies exist. One is between horns 2 and 4 (A, B, and A) which is made up of a major ninth and a major second, and the other is between horns 1,2 , and 4 (A, B, and C) which is made up of the same starting major ninth followed by a minor second. While these melodies also share an equal-tempered interval in the middle chord on partial twelve, that would require the first melodic interval to be over two octaves, and in this context that is extremely difficult to hear. The second line is between chords nine and eleven (the last chord before the rest of the orchestra takes over) and again it has two possibilities. First, between horns 1 and 3 ( $F, E^{b}$, and F), the melody consists of a major ninth and a major second. The second possible melody would include horns 1,3 , and $4\left(\mathrm{D}, \mathrm{E}^{\mathrm{b}}\right.$, and F ) and would consist of a major seventh and a major second.

Of course, the other equal-tempered melodic line that is easily traceable in this final phrase is the bassoon line, in which all of the notes are played in equal temperament. Just as the bassoon line reinforces the importance of these equal-tempered melodies or melodic intervals, the fact that a movement where only the horns play for the first twenty-three and one-half measures ends with a chord played by the orchestra in exact equal temperament (featuring stacks of intervals, which is easier to accomplish in equal temperament than in many other tuning systems) underlines the importance of the harmonic equal-tempered intervals in use throughout this chorale.

It becomes even clearer that these chords and harmonic relationships are important when the cantus firmus line is considered. This line is the foreground melody in the chorale; each of the first four phrases features either stepwise motion through the partials, or small leaps skipping only one partial. The fifth phrase breaks this pattern slightly, with a leap from the twelfth to the ninth partials near the start of that phrase and a leap from the sixth to the ninth partial near the end. These still are relatively small leaps in terms of the intervals they encompass (a perfect fourth and a perfect fifth, respectively) and have been
set up previously in the chorale with a jump from partial eight to partial six which is also a perfect fourth, and yet within the context of partials is a small leap. However, while the cantus firmus is carefully controlled (and all within the space of a major tenth), the other voices have large, dramatic leaps which they must downplay to fill their subservient roles at a $\boldsymbol{p} \boldsymbol{p}$ dynamic. Sixths, sevenths, and octaves are common intervals in these three parts, with even larger intervals such as ninths, tenths, twelfths and fourteenths involved. The largest leap occurs in horn four in measure twenty-four where there is a leap of two octaves and a tritone from a slightly flat $G \#$ down to $D$. The drastic features of these three lines help to underscore how important the harmony is, and how crucial it is that the E horn is allowed to play a smooth, uninterrupted line amidst all of this activity.

The "Choral" section ends the second movement with a strong example of not only how this hybrid tuning system can create sounds much like those of just intonation or twelve tone equal temperament but also how the combination created by the horns alone is an incredibly rich palette. The focus on incorporating equal-tempered intervals into this chorale is especially interesting, as it was in the preceding "Signale" and "Tanz," each of which show the same general features (exploring the tuning systems alone and in combination) in a different way. While "Praeludium" served mainly to display the pitch material and coloristic qualities of this orchestra as a whole, "Signale, Tanz, Choral" took that example farther and showed how the featured subset of this orchestra has a great capacity for creating musical interest without the support of the equal-tempered ensemble. This movement has also reinforced the importance of imitation and interruption, employing both of these features early in the movement. From here, Ligeti will continue to examine the potential relationships between the two tuning groups, but much of the piece will build on material from these opening two movements.

### 3.4 III. "ARIA, AKSAK, HOKETUS"

Movement three opens with a duet between the soloist-still playing a natural horn in F-and four bongos, with a string accompaniment in the background. The strings, playing stacks of fifths (again emphasizing interval class 5) ascend chromatically over two and one-half octaves, playing on every beat in a 9/8 time signature for eleven measures. This accompaniment occasionally aligns with the pitch being played by the horn soloist (see Figures 3.16 and 3.17), but is generally a separate gesture. The bongos are much more closely related to the horn solo, as this line fills in the sections where the horn is playing longer notes, creating a steady stream of eighth notes from measure two through measure eleven. The bongos imitate the rhythm and character of the faster sections of the horn part, forming a simple counterpoint between the two lines.

The opening, "Aria," introduces another new feature-the half stopped horn. Here, Ligeti finally allows the horn player to put their hand in the bell, but only for specific notes. The half stopped horn, also called three-quarters stopped, is typically used to lower the pitch by a half step and will somewhat darken the tone. ${ }^{39}$ Ligeti's use of arrows in this part is slightly confusing-all of the half stopped pitches have an arrow on them indicating they are approximately $30 \%$ lower than the note written which is the same notation he uses for the seventh and fourteenth harmonics, which sound 31.2 cents low. However, if the pitches are half stopped to exactly a semitone low, these arrows would not be entirely accurate. It seems in this passage the arrows are used to illustrate that this pitch will be lowered when the player puts their hand in the bell, but with the awareness that the precise value (in cents) by which the pitch will be lowered is going to vary. Regardless of this notational quandary, Ligeti is presenting the listener with another new method for subverting the tuning systems; this new method, like the previous ones, exists because of the way Ligeti has chosen to combine the systems to create a unique pitch world which-as he demonstrates again here-has potential for more possibilities than it initially may seem.

[^20]

Figure 3.16-"Aria, Aksak, Hoketus" measures 1-11 with partial numbers for the solo horn

Ligeti chooses to add three pitches to the harmonic series of F through the use of the half stopped horn: $\mathrm{G}^{b}$, $\mathrm{A}^{b}$, and $\mathrm{B}^{b}$. He also adds a new pitch within the harmonic series-the seventeenth partial. On the F horn, this is a $G^{b}$ at 605 cents. Throughout "Aria," major seconds play an important role as Ligeti often uses the space between partials seven and twelve.

The character of the horn and bongo duet makes it stick out much more than the chords of fifths in the strings do. However, the interaction between the horn and the strings provides some additional harmonic interest which again is partially due to the contrast of their pitch possibilities.

Figure 3.17 more clearly shows the relationship between the melody of the solo horn and the fifths played by the strings. The opening C of the horn melody is only 2 cents sharper than the $C$ an octave lower in the strings. On beat two of the first measure, the A in the horn ( 886.3 cents) is outside of the notes in the strings, and creates a strong dissonance with the $G^{\#}$ played by violin 1 and the $B$ played by the cello and the bass. The third chord has a similar issue, with the $A^{b}$ being sustained by the horn creating a dissonance against the $A^{\natural}$ in violin 1 , the $D$ in violin 2 and the $G$ in the viola.

As this opening "Aria" section continues, there are similar instances of consonance and dissonance throughout. Measure two begins with an F that fits into the stacks of fifths-although not in the proper octave-and is in perfect equal temperament with the pitches of the strings, and beat three of measure two features an exact just-intoned consonance-the $A$ in the horn is a $5 / 4$ (just-intoned major third) above the F in violin 2 , and only two cents away from being a just-intoned minor third below the C in violin 1 . This chord, despite the two cents, is likely to sound exactly like a just-intoned major triad. The $\mathrm{B}^{b}$ below it and the $\mathrm{E}^{b}$ in the bass creates something resembling a dominant seventh chord in third inversion with a just-intoned major triad and an equal-tempered minor seventh.


Figure 3.17—"Aria, Aksak, Hoketus" measures 1-11 showing the solo horn and string relationship

On the second beat of measure three, the horn takes the moving line back from the bongos and a consonance is heard here as the horn plays an F exactly one octave above the F of the cello, aligning it perfectly with the stack of fifths. Beat three of this measure has an equal-tempered version of the chord on beat three of the previous measure. The C in the horn, only two cents high, sounds like an equaltempered third of a major triad between the two violins and the horn, and the rest of the harmony lends it a dominant seventh sound.

While measure four begins with the horn dissonant against the strings, the C on beat two will fit into the stack of fifths (displaced by an octave). The A on beat three is 13.7 cents flatter than the A in the cello or bass, creating a tense dissonance between those instruments, and two beats later (beat two of measure five) the A is repeated an octave lower and is even more dissonant-the minor triad generated between the strings and the horn will contain a third lower than it would in either equal temperament or just intonation, and too high to be a septimal third (7/6) in just intonation. After a full measure of dissonance, beat three of measure five is in tune with the fifths, and creates a minor triad between the D and A of the violins (and as before, the bass could function as a minor seventh of the triad) because the F in the horn is in tune with the equal-tempered system.

Six beats of dissonant harmonization follow, occupying all of measures six and seven. This nearly resolves at the start of measure eight, but the tuning system interferes. The horn plays an A , as does violin 2, but they are separated by 13.7 cents which continues the dissonance, but to a lesser degree. The second beat is also dissonant, but the third beat is consonant once again, with the horn playing a C (at 2 cents) that creates a nearly equal-tempered triad between the A in the cello and the E in the viola. Above this triad, however, are some dissonances for the $\mathrm{C}-\mathrm{aB}$ in violin 2 and an $\mathrm{F} \#$ in violin 1.

Measure nine begins with another $C$ which is consonant with the upper strings, and this is followed by a chord that is the same as the second chord of measure five, but with all six instruments playing one octave higher; the A in the horn nearly fits into the upper voices as a minor third, but is too distant from equal temperament or any simple just-intoned relationship to sound consonant. This beat and the following three beats are dissonant, but at the end of measure ten the horn's ascent to the seventeenth partial reaches an eighth partial $F$ which fits into the $E^{b}, B^{b}, F$, and $C$ fifths of the strings.

The final measure of the aria begins with another dissonance, the second beat is somewhat less dissonant, and very consonant with the cello and bass (it is exactly a septimal whole tone-8/7, 231.2 cents-below the F in the cello, and a perfect 7/4-the harmonic seventh or septimal minor seventhabove the F in the bass). The final beat of the aria has the horn returning to equal temperament as it
arrives on $F$, clashing with the $\mathrm{F} \#$ in the cello and bass, but forming an equal-tempered major triad with the $C \#$ and $G \#$ in the viola and violin 2 and fitting in to a whole tone structure with the $D^{\#}$ of violin 1 . Finally, at the end of this third beat, the seventeenth partial $G^{b}$ is reached (only 5 cents higher than the $F^{\#}$ in the cello and the bass), and the next section begins immediately.

Through this "Aria," Ligeti continues to examine the relationship between his two tuning worlds, but here he is also adding slightly unpredictable pitches to add to the colors and melodic inflections available through the use of half stopped horn. This first section is also part of a progression from "Signale" (the first segment of movement two) to "Solo" (the first segment of movement four) wherein the solo horn plays completely alone. The use of half stopped horn is also part of a progression to fully stopped horn, again heard for the first time in the "Solo" of movement four.

This next section is actually a combination of the two ideas remaining from the title: "Aksak" and "Hoketus." While this section still uses nine eighth notes per bar, they are now divided into groups of four, three, and two. This persistent, uneven metric division is the aksak, or limping, rhythm, common in Ligeti's work. ${ }^{40}$ Against this rhythmic structure is a melody shared between the five horns, played in a hocketing line which gradually expands from one horn at a time to all five horns playing simultaneously. Occasionally this melody is joined or harmonized by other members of the ensemble.

While movement two displayed many of the possibilities of the horn tunings, the horns themselves usually played for relatively long passages. In this section, the hocket is strongly featured, showing how the relationships between the horns can be exploited to allow even more intervals into Ligeti's pitch language. As a result, it is here where some of the most interesting and playful melodic lines of the Hamburg Concerto can be found. The hocketing lines pass through a multitude of tuning systems-by the second measure of this section, the tuning systems themselves are nearly completely obfuscated, and the ear picks up instead on the gradual evolution of brief melodic fragments into a full

[^21]canon. This change will be examined closely, as it is another way of employing the equal-tempered horn relationships that were examined in "Signale, Tanz, Choral."

The underlying metric structure of the aksak rhythm is played by a variety of instruments in small groups. The instruments change frequently, creating a cycle of different tone colors. Initially, the figure is played by two flutes, the bassoon, and the marimba, but within the first four measures, pizzicato strings and the bassett horns also play this line. The pitch aspect is kept fairly simple at the start of this section, alternating between open fifths of G to D and F to C . These are the only pitches used for the opening six measures of the accompaniment and they are always used in perfect fifth pairs.

Ligeti employs another new sound as the horns enter-they are using mutes (straight mutes, not hand muting). Tuned in half steps as in the previous movement, the horns use their different pitch possibilities to, as Ligeti writes in the score, "appear to play a continuous melodic line." ${ }^{41}$ To that end, he also instructs the horn players to be sure they "do not shorten the last note" ${ }^{42}$ of any of these short motives. The resultant melodic line, seen in Figure 3.18, combines equal temperament and just intonation in a more intricate exploration of the ideas used in the opening of movement two.

This melodic passage, the first two-thirds of the aksak section, spins long melodies out of tiny fragments which span both tuning systems. Figure 3.18 clearly shows how the melodic lines move between the instruments. The first four notes (in measure thirteen) have intervals of: 3/2 (a just-intoned perfect fifth) or 702 cents; $5 / 4$ plus 100 cents (a just-intoned major third and an equal-tempered half step) for a total of 486.3 cents; and 7/5 (one of several possible just-intoned tritones) at 582.5 cents. The next fragment consists of five notes, and is built mainly on just-intoned perfect fifths. However, when the D horn replaces the $E^{b}$ horn, an equal-tempered minor second is the resulting interval. These two notes involved in the equal-tempered half step, $B^{b}$ and $A$, are an important part of the next several groups of notes as the next four entrances begin on $B, B^{b}, A$, and $B^{\natural}$, all played on the sixth partial of one of the

[^22]obbligato horns. These last two entrances also overlap by one note, creating the first horn dyad of the hocketing sections-at the start of the sixteenth measure the $D$ horn is playing a $D$ when the $E$ horn enters on B 902 cents higher.


Figure 3.18-"Aria, Aksak, Hoketus" measures 12-26: horn hocket with partial numbers and instrument labels

These few opening measures actually encapsulate many of the important features of the hocket figure. For one, the horns play short melodic fragments (gradually increasing in length over the course of several measures) that occasionally overlap, creating dyads (which gradually become more frequent over the course of several measures). One other important aspect of the hocket is the relationship between the different horns as each participates in the hocketing line. Already in these few measures we have seen an equal-tempered relationship mid-phrase (measure fourteen), a relationship that is neither equal-tempered nor just-intoned (measure thirteen) and an interesting equal-tempered chromatic descent of the starting notes of each fragment. This $B^{\natural}$, $B^{b}$, and A descent, played on the sixth partial, is foreshadowed in measures thirteen and fourteen with the first note of the first fragment and the middle notes (where the horns change) of the second fragment, but is made overt in measures thirteen and fourteen.

The equal-tempered passage on the sixth partial continues through measure sixteen, reordered as $B^{\natural}, A, B^{b}$, and is somewhat hidden within the lines again. Similar equal-tempered punctuations can be found on the ninth partial in measures sixteen and seventeen, and on the seventh and tenth partials in measure seventeen. These equal-tempered relationships in measure seventeen are perhaps more interesting because they involve the exact same just-intoned interval (10/7) raised by an equal-tempered major second when it is played first by the D horn and followed by the E horn. The E horn concludes its line with a sixth partial B at the start of measure eighteen.

Up to this point the horns have had no notes longer than an eighth note. However, this B is held for the duration of a half note, which integrates it into the harmonic system played by the rest of the orchestra. That harmonic system changes at this point in the movement. The previous six measures used only the aforementioned perfect fifths G to D and F to C . Against this B , if the pattern continued, the orchestra would be playing $G$ and $D$, leaving a major triad to sound for the first four eighth notes of the measure. Instead, the orchestra changes its accompaniment and the resulting chord is an E minor triad. The final change that takes place at the start of this measure is that the trumpet and trombone, dormant
thus far in this movement, join in. They begin a slightly different aspect of the accompaniment-they have long sustained notes lasting more than one beat.

Halfway through this first (half note) beat, there is one more surprising event-the oboe joins the horn section's hocket. Previously this hocket was played only by the horns, and the only equal-tempered intervals that existed were the result of one horn entering on the same partial another horn had recently played. Here, the oboe enters on $\mathrm{B}^{b}$ and moves to $\mathrm{G}^{b}$, which leads into the $G$ played by the $\mathrm{E}^{b}$ horn on beat two. These two oboe notes (marked with ET in Figure 3.18, as there are no partial numbers involved and the notes will sound in equal temperament) are the only two notes in the hocket that are not played by one of the horns. This serves to further illuminate the changes taking place in the harmonic accompaniment at this point in the movement. When the $\mathrm{E}^{b}$ horn does enter, it is marked by the accompaniment switching mostly to F and C, but the trombone plays an A which it is instructed to play about $30 \%$ lower; this is also Ligeti's indication for the seventh harmonic. This is an interesting event as well, as this pitch does not create an in tune major third with the equal-tempered F (as shown in the "Aria" section). It results in an awkwardly out of tune major triad, and an A that may be in exact equal temperament with the last note of this beat—a $\mathrm{D}^{b}$ around 400 cents higher played by the $\mathrm{E}^{b}$ horn. The last beat of this measure, only a quarter note in length, contains another E minor triad in the accompanying voices, and the entrance of the F horn on a C .

Measure nineteen begins with a pair of dyads as the F horn and the $E^{b}$ horn overlap by two eighth notes, creating two major seconds- 186.3 cents and 200 cents. Beneath these dyads, the orchestra is playing an F major triad (without the altered A), and the bongos have entered for the first time in this section, playing a figure similar to the bongo lines from the "Aria." The second beat of measure nineteen contains an $\mathrm{F} \#$ minor triad in the accompaniment, and the third is a stack of fifths-C, G, and D. On the downbeat of beats two and three, no horns are playing, but the second eighth note of each beat has a horn playing on its twelfth partial, resulting in an ascending equal-tempered half step between the $E^{b}$ horn and the E horn.

In measure twenty (rehearsal letter M), the orchestra returns to playing open fifths that gradually descend while the bongos continue to play. The hocket figure is gradually increasing both in the length of the fragments and the number of horns involved in a particular fragment. The line that begins one eighth note before measure twenty, for example, lasts for nearly one full measure, and switches from the E horn to the F horn and back to the E horn-one dyad of 382.4 cents is created in the first switch (on the downbeat of measure twenty). Between measures twenty and twenty-one, the $F$ and $E^{b}$ horns are used in a shorter fragment, and this is followed by a melodic line which lasts again for nearly one full measure and involves all four obbligato horns, two of which (the E and D horns) play a pair of dyads to end the fragment at the start of measure twenty-two-first a tritone of 582.5 cents (interestingly, these two different fundamentals create nearly a just-intoned 7/5) and second a large minor sixth of 817.6 cents. Through these three measures, two possible equal-tempered lines can be traced-one on the seventh partial ( $C, D^{\natural}, D^{b}, E^{b}, D^{b}$, and $\left.D^{\natural}\right)$ found in the middle of the hocket register, and one on the fifth partial ( $G^{\natural}$, A, and $G^{\#}$ in measures twenty-one and twenty-two) which is easier to hear as these are the three lowest horn pitches in this passage.

The accompaniment figure gradually moves back towards the opening material (alternating between open fifths from G to D and F to C ). Of particular interest is the second beat of measure twentytwo. The horns and bongos stop playing just before the end of beat one, leaving the orchestra to play two perfect fifths alone-F to C and C to G. Interestingly, this is the first and only full beat (this time three eighth notes in length) since the horns entered in measure thirteen where no horns play. On beat three the solo F horn enters for the first time in this section, opening with a descending perfect fourth. This begins more than a measure of only the solo horn playing (with accompaniment) before the hocket resumes.

The accompaniment in measure twenty-three uses only fifths, and much like in the opening "Aria," the solo horn moves into and out of this harmony. On beat two the horn plays an A exactly a $5 / 4$ (just-intoned major third) above the F in the accompaniment, resulting in nearly a just-intoned F major triad (the fifth, being equal-tempered, is 2 cents too low to be a just-intoned fifth). On beat three, the horn
avoids the notes of the accompaniment, but does end on F, which adds another fifth to the orchestral harmony.

The accompaniment figure on beat one of measure twenty-four contains a stack of fifths including $B^{b}, F$, and $C$-on beat two, the accompaniment has returned to its initial pattern, but below this pattern the trombone and contrabass will add additional notes, often held for more than one beat-these additional low notes form lengthy descending lines containing few pitches.

In the horns, measure twenty-four begins with a dyad played between the solo horn and the D horn, as the obbligato horns return with their hocket figure. Although this dyad ( 633.2 cents) is not the first overlap in the horns, it is noteworthy because it involves the soloist, and this conflict between the soloist and the obbligato horns causes the hocket to falter as the horns begin to continuously overlap with one another. This, in fact, is the last actual dyad that will be heard for some time; the horns are now creating two distinct hocket lines. At the end of measure twenty-four, two hocket lines can be heard (see Figure 3.18). What seems initially like a series of overlapping fragments does not return to one voice for nearly a full measure, resulting in two lines moving contrapuntally and, more importantly, imitatively.

The last two horn notes of beat one, a seventh partial $D^{b}$ and a fifth partial G played by the $E^{b}$ horn, not only reflect on earlier equal-tempered lines that could be traced within the hocket (between partials ten and seven in measure seventeen and between partials seven and five in measures twenty and twenty-one), but also set up new equal-tempered lines on partials five and seven that will move between hocket lines in measures twenty-four through twenty-six. The line on the seventh partial contains $D^{b}, D^{\natural}$, $E^{b}, C, D^{b}, D^{\natural}, E^{b}, D^{b}$, and $E^{b}$, while the line on the fifth partial contains $G^{\natural}, G^{\#}, A, F^{\#}, G^{\#}, A, G^{\natural}$. Again, the seventh partial line takes place in the middle of the hocket register while the fifth partial line is nearer the bottom of that range, and perhaps easier to hear. One additional noteworthy fact about the fifth partial line is that two of these notes are presented consecutively (in measure twenty-five between the end of beat one in the D horn and beat two in the E horn). This creates an equal-tempered interval within the hocket line-something that has not been heard since the oboe joined the hocket in measure eighteen.

In measure twenty-six, the horn lines again move imitatively, with the D horn (which begins just before beat two) following the contour of the line of the F horn (which began just before beat one). In addition, the solo F horn enters in the middle of beat one with a series of sustained notes creating a third line which contrasts greatly to the faster hocket material. This line builds intensity in the hocket (especially when these sustained pitches are joined by sustained pitches in the obbligato F horn) which is released dramatically at the start of the following measure.

In measure twenty-seven, a new feature of this section is introduced-a series of canons (see Figure 3.19) that were foreshadowed by the imitation of the two hocket lines in the previous three bars. The accompaniment figure continues to alternate between the two perfect fifth dyads for two more measures, and at the start of measure twenty-seven the trombone and bass are sustaining their $\mathrm{A}^{b}$ for the first two beats and then moving to $G$ for the third beat and sustaining it for a comparatively long time (three more measures and a half note). The horns begin these canons by the middle of beat one; suddenly the interest in individual intervals is gone and the shape of the canonic passages catches the ear instead.

Initially there are two separate canons. First, the solo F horn sets up a canon with the obbligato F horn, which follows with exactly the same notes starting two eighth notes later. Second, the E horn starts a canon at the same time the obbligato F horn echoes the soloist, and the line played by the E horn is followed by both the $E^{b}$ horn and the D horn. The $\mathrm{E}^{b}$ horn and the D horn play the same partials, but due to the tuning, obviously will be a minor second and a major second lower, respectively. Like the first canon, the $E^{b}$ horn follows the $E$ horn after two eighth notes, and the $D$ horn follows an additional two eighth notes after the $\mathrm{E}^{b}$ horn. These two canons-one at a unison, one at a minor second-have similar contours in this first measure. In the second measure the canons change slightly; the measure begins with two horns-the first voice of each canon-entering at the same time, and the second voices of each canon are entering only an eighth note later. The third voice of the second canon still comes in four eighth notes after the first. Shortly after the canons begin, it is very apparent that there are some pitch differences as well.


Figure 3.19-"Aria, Aksak, Hoketus" measures 27-33: horn parts with partial numbers

The obbligato F horn follows the solo F horn for three notes and then changes its pitches. The obbligato horn continues to follow the contour of the soloist's line, but adds a few extra pitches, expanding the line by nearly fifty percent. The second canon is also somewhat altered-although the D horn follows the E horn's line exactly, the $E^{b}$ horn does not, changing the order of the first four notes, and truncating the phrase. By the end of its second measure, the canon has already been subverted, and it is only loosely followed for the rest of the movement.

At the end of measure of measure twenty-eight, the solo horn begins what might seem like another phrase that will be played in canon, but instead this is a new melodic line. The ascent in eighth notes from the tenth to the thirteenth partial leads into three longer pitches ascending to the sixteenth partial. The E horn, leading the second canon, plays a figure that spans one octave which is echoed by the $E^{b}$ horn playing one partial higher. This has the effect of the canon starting a large major second above where the $E$ horn began (the sixth partial $B^{b}$ in the $E^{b}$ horn is 215.7 cents higher than the fifth partial $G^{\#}$ in the E horn), and finishing a small minor second above where the E horn ended (the eleventh partial A in the $E^{b}$ horn is 65 cents higher than the tenth partial $G^{\#}$ in the $E$ horn) while not quite reaching the same one octave total. When the D horn follows in the next measure, it plays an abridgment of this line, starting on its fifth partial and ending on its eleventh, beginning an equal-tempered major second below the $E$ horn, ending an equal-tempered minor second below the $E^{b}$ horn, and surpassing the one octave total of the E horn by 165 cents, which is a just-intoned 11/10 (a small major second called Ptolemy's second).

At the start of measure twenty-nine, the accompaniment figure changes again. The bass continues to sustain the low G, but the rest of the accompaniment switches from open fifths to an E minor triad on beat one, an F major triad on beat two, and another E minor triad on beat three. The texture of the accompaniment is also thicker here. When the canon began the instrumentation gradually began to increase; at this point the figure is played by all of the woodwinds, the trumpet, the trombone, and all of the strings-only the percussionists are not playing. This is the largest simultaneous instrumentation Ligeti uses in the entirety of the Hamburg Concerto, although with most of the instruments playing $\boldsymbol{p} \boldsymbol{p}$, the overall effect is somewhat subdued. This measure includes the end of the soloist's ascent, and much of the canon described in the previous paragraph (see Figure 3.19). The obbligato F horn-now unable to follow the soloist's line-plays a sort of variation on the figure it played in the previous bar, keeping a similar contour, but shortening the phrase by one eighth note. On beat three of this bar, the F horn will play a third version of this figure, again shortened by one eighth note, and with a slightly different overall
shape. The accompaniment here is playing an E minor triad, above which is an E 11.7 cents lower being played on the fifteenth partial of the solo F horn.

During this measure (twenty-nine) the canon between the three lower horns completely breaks down. Instead of generally following what another horn is doing, each horn begins its own line. The horns play similar but varied versions of their lines until the second beat of measure thirty-one. During this time, the obbligato F horn joins the other obbligato horns in this role. At the start of measure thirty, the soloist is sustaining the same E from the previous measure while the accompaniment switches back to F major, still with a G in the bass. Halfway through this beat the soloist resolves to F, and on beat two, sustains that F while the accompaniment moves to $\mathrm{F}^{\#}$ minor with a G in the bass. On beat three, with the high F and low G still sustained, the accompaniment (now playing $\boldsymbol{m} \boldsymbol{f}$ or $\boldsymbol{f}$ ) moves to a $\mathrm{G}, \mathrm{C}$, and D -a group of fourths into which the F of the solo horn fits perfectly. This is sustained into measure thirty-one, and lasts for that entire first beat at which point all of these fourths drop out.

Although the hocket did seem to disappear in this section, long, cohesive phrases can be formed by combining the horn passages once the canons dissipate (see Figure 3.20). Up to four hocketing phrases can be found at one given time. Like the earlier hocket section, dyads occasionally appear within one line of the hocket. In this section, a total of three dyads are used over the course of five measures. This is similar to the frequency of dyads in the earlier hocket section wherein there were seven total dyads over fourteen measures.

These hocket lines follow the same general pattern as those earlier in the piece. The first hocket shown in Figure 3.20 (the last to enter) has two particularly interesting features. First, in measure twentynine, the hocket moves from the D horn to the E horn, via a small perfect fifth of 682.5 cents. The last three notes of the $D$ horn's passage use partials seven, nine, and eleven, while the first three notes of the $E$ horn's passage use partials seven, eight, and ten. The overall effect is that of a strange echo, where the second passage starts an equal-tempered major second higher, but ends a small minor second (65 cents) lower. The second interesting event in this first line is found in the second half. Over the last measure
and one beat, each horn plays a short, one beat figure which focuses on its eighth partial. This line creates a focus on this shifting eighth partial ( $\left.E^{b}, F, E^{\natural}, D, E^{b}\right)$ and has clear, strongly directed motion towards its return to $\mathrm{E}^{b}$.


Figure 3.20-"Aria, Aksak, Hoketus" measures 27-31: horn hocket with partial numbers and instrument labels

The second hocket displayed in Figure 3.20 has its own interesting features, the most obvious of which is the appearance of two dyads between the E and F horns in the middle of the line (found at the start of measure twenty-nine). These dyads form fairly large intervals (a major tenth of 1598 cents and a minor tenth of 1484.3 cents) in what is otherwise a relatively smooth hocket. The first note of each entrance into this line is the same partial as the penultimate note of the previous horn, except where these large leaps are, and in this case the first note after the dyads is the partial an octave below the last partial played by the previous horn. So while this results in a leap of an equal-tempered major seventh, it is closely related to the relationship of the other two instrument changes in this hocket.

The third line of Figure 3.20 actually contains two similar but separate hocket lines. The first of these is relatively short, involving only two horns. It has three distinct ascents, one from partial two to partial six on the F horn, one from partial five to partial nine on the F horn, and one from partial six a whole step lower on the $\mathrm{E}^{b}$ horn to partial eleven. After this first hocket, the second one has similar ascending features, but includes many leaps instead of simple, stepwise motion. The F horn begins again with an ascent from partial three to partial seven, but the partials are not played in order. This leaping is continued when the D horn enters on its sixth partial, which is just slightly higher than the F horn's fifth partial. The D horn, rather than leaping up and down, plays every other partial from six to twelve, and ends with the thirteenth partial, which is followed by the F horn now on its sixth partial, 1038.5 cents lower. This is followed by a leap to its ninth partial, and then the E horn finishes the phrase, entering on its seventh partial, leaping to its ninth (echoing both the F horn's six to nine motion and the D horn's ascent of every other partial) and resolving this phrase by step to its tenth partial, much like the D horn ended its part of the phrase.

The fourth line of Figure 3.20 contains the longest hocket, spanning just over four full measures. The opening of this hocket has the F horn transition to the E horn via a minor sixth of 802 cents. The E horn, nearly in canon with the F horn, plays the same partials reversing the order of the last two. After an eighth rest, the line continues in the solo F horn, with its ascent from partial nine to partial thirteen.

Although the F horn's arrival on the fourteenth partial an eighth note later is clearly where this ascent is heading, before that note is played the fourteenth partial seems to connect to the seventh partial of the E horn nearly an octave lower ( 1171.7 cents). The E horn plays a short ascent from partial seven to twelve. After this, the E horn plays partials eleven, ten, and twelve, and then repeats that figure one partial lower, playing ten, nine, and eleven. This last note creates the third dyad of this closing section, an 882.5 cent major sixth between the low $\mathrm{A}^{\#}$ in the E horn and the low $\mathrm{D}^{b}$ in the $\mathrm{E}^{b}$ horn. This is followed by the Eb horn playing a retrograde inversion of the E horn's figure on partials eight, ten, and nine. The E horn finishes its line with partials eleven and twelve, and this is followed by the F horn returning with partials seven and eight. This transition is an interval of a large perfect fifth (733.2 cents, nearly a just-intoned 84/55), and is followed by the F horn resolving this entire phrase back to $F$ where it began.

As mentioned above, the accompaniment figure exits at the end of the first beat of measure thirtyone. On beat two, a large chord-again built of fifths-is played by the bassoon, the obbligato horns, the crotales, and the strings (see Figure 3.21). The horns and bassoon drop out halfway through the chord, leaving a more stable stack of fifths, and the chord remaining in the strings and crotales ends the movement. The fifths initially are somewhat uneven due to the tuning of the horns. There are two different kinds of $\mathrm{F}^{\#}$ (the D horn playing its fifth partial at 586.3 cents and the crotales sounding 600 cents ${ }^{43}$ three octaves higher), two different kinds of A (the F horn playing its tenth partial at 886.3 cents and violin 1 playing a partial at 900 cents two octaves higher), two exactly equal versions of B (the bassoon playing 1100 cents and the crotales sounding 1100 cents four octaves higher), and a $\mathrm{D}^{b}$ in the $\mathrm{E}^{b}$ horn that does not quite form a perfect fifth with the $\mathrm{F}^{\#}$ below it, but comes close at 682.5 cents. The total effect is that this chord feels like an unresolved arrival point-another interruption, although more subdued-which is remedied by removing the horns and bassoon from the equation in the next measure.

[^23]This leaves a stack of seven fifths with a gap in the middle. An E is missing from the structure, which interestingly was one of the horn notes-the only one not creating a strange dissonance in the chord. This stack of fifths has a sense of stability, as they were used so frequently throughout this movement (and the previous two) that they take the place of a traditional resolution to a tonic.


Figure 3.21-"Aria, Aksak, Hoketus" last chord (measures 31-33) with cents values

As shown above, the melodic line in the "Aksak, Hoketus" section evolves over a short period of time from short two-note fragments into a series of canons and imitative lines shared between all five horns. This at first seems to contrast strongly with the opening "Aria" material, but in fact there are several similarities. Aside from a comparable character, they both feature the addition of pitches into the horn line. In the "Aria," the solo horn uses half stopping to accomplish this feat, but in the remainder of the movement, new notes for the melodic line are provided simply by having another horn sound the desired pitch. As the line begins to grow longer, this process becomes increasingly clear, as does the cohesive nature of this movement.

Throughout "Aria, Aksak, Hoketus," the problem of relating the horns to the equal-tempered orchestra is handled in various ways such as accompanying the opening horn and bongo duet with a stack of perfect fifths which ascends chromatically; accompanying the horn hocket with a generally static harmonic background, and changing that background into something more active and harmonically complex; incorporating the oboe briefly into the horn hocket; incorporating punctuated equal-tempered lines into the generally just-intoned hocket; and using imitative and canonic passages where the melodic line is just-intoned but the interval at which the imitation takes place is equal-tempered. With that said, Ligeti is always careful to create a solution that contains interesting melodic lines, generally light in character, which not only are directly a result of his tuning scheme, but are also a deliberate attempt to mask it.

### 3.5 IV. "SOLO, INTERMEZZO, MIXTUR, KANON"

The fourth movement, by far the longest of the seven, opens with a solo played on the double horn. This opening solo is the most clear and pure use of just intonation in the Hamburg Concerto, with the opening of the first movement close behind. The clarity of the melodic lines and the purity of the just-intoned intervals are colored by the shifting fundamental which adds a hint of equal temperament and a slight variation of timbre. As in the previous movement, "Aria, Aksak, Hoketus," this solo also shows Ligeti's intention of altering the tuning system. Here the tuning system is occasionally slighted simply by having the soloist switch fundamentals as mentioned above-the general characteristics of just intonation, however, are always present. These different fundamentals are one additional way Ligeti employs imitation-the soloist will play the same partials over different fundamentals to sound the same justintoned passage with different equal-tempered starting points, much like the opening of "Signale, Tanz, Choral."

In this opening solo, the soloist uses the valves on the horn to alter the fundamental-eight of the possible twelve are used: $D, E^{b}, E^{\natural}, F, G, A^{b}, A^{\natural}$, and $B^{b}$. As mentioned in Chapter 2, this allows each fundamental to be tuned exactly to equal temperament. ${ }^{44}$ There is a second new feature employed here as well, the use of stopped horn (mentioned briefly in the discussion of "Aria" from the third movement). The effect of stopped horn is quite different from that of the half stopped or three-quarters stopped horn which was used early in the previous movement. Instead of slightly lowering the pitch and having a slight effect on tone color, the end result of stopped horn (or fully stopped horn) is that the pitch is raised by a half step, and the tone is drastically different. ${ }^{45}$ Stopped horn is commonly used to create an echo

[^24]effect and Ligeti writes "(eco)" in his score above the stopped horn passages to make it clear this is the effect that he wants. ${ }^{46}$ This, too, references the imitation used on small and large scales throughout the Hamburg Concerto.

Figure 3.22 shows the entire solo section of the fourth movement. Played with some rubato, this section lasts over one minute, and is unaccompanied. As mentioned in the previous paragraph, the horn plays from eight different fundamentals, indicated in boxes above the staff. The relationships between pitches are typically kept very simple. The majority of intervals in this opening solo are just-intoned, and early on there are also several intervals that are either equal-tempered, or within just a few cents of being equal-tempered. As the solo continues on, some intervals that do not approximate either system are used as well, starting between measures six and seven, and becoming as common as the exact or nearly exact equal-tempered intervals by the end. The use of valves to create these intervals is a logical next step from "Hoketus," where a melodic line had to be passed between several horn players. Here, the soloist alone can access any of these pitches.

Measure one begins with the horn in $B^{b}$, playing just-intoned intervals of $7 / 5,9 / 7,11 / 9,11 / 10$, and $5 / 3$. While these may not be standard fare for an equal-tempered composition, by this point in the concerto, these intervals seem completely normal, and the intervals and contour of the line are simple and consonant. Measure two switches sides of the horn to the F fundamental, and they connect with a slightly large major second of 201.9 cents. This is followed by just-intoned intervals of $10 / 9,6 / 5$, and $12 / 11$. The next measure starts in F with an 11/10 interval connecting the two bars. The second note of measure three is back on the $B^{b}$ side of the horn 117.5 cents lower, after which is a $7 / 4$ and a $3 / 2$. In the following

Regardless, depending on the partial in question, which side of the horn is being used, and even which valves are used, the pitch will be affected by a different amount. Of course, the end result is also dependent on the dimensions of the particular horn and can be further altered somewhat by the player. Fortunately, the goal of this effect is simple, physics notwithstanding-the player is to play a stopped tone sounding the exact pitch notated. The player, generally, will transpose the note down a half step and stop the horn (creating the intended tone, and effectively raising the pitch by an equal-tempered minor second) to create this sound. Note that in Ligeti's score, and in my Figure 3.22, the transposition has been written in above the staff.
${ }^{46}$ Ligeti, Hamburgisches Konzert, 13.
measure, these last two pitches are echoed by a now stopped horn playing over an A fundamental. This results in a descent of $3 / 2$ and another ascent of the same.


Figure 3.22-"Solo, Intermezzo, Mixtur, Kanon" measures 1-28 with cents values
Much of the "Solo" is similar to these four opening measures. The intervals that involve one fundamental are, of course, just-intoned, and the intervals created between different fundamentals will be either equal-tempered or outside of both tuning systems. These stopped horn "echoes" are another important feature which is used throughout this opening. The next two measures (five and six) stay on an A fundamental, but the horn is open, which gives way to a different harmonic series than that of the stopped A fundamental. The move from a stopped sixth partial to an open seventh partial between measures four and five is approximately one and two-thirds semitones (166.8 cents). After this, all of measures five and six are just-intoned, and measure six connects to measure seven with an ascent of 384.3
cents (a small major third in equal temperament, but only two cents smaller than a just-intoned $5 / 4$ ) as the soloist changes fundamentals and moves up two partials.

From this E fundamental, the soloist plays the same three intervals contained in the very opening (7/5, 9/7, and 11/9) from a starting pitch an equal-tempered tritone lower. Measure eight switches back to the $B^{b}$ side of the horn with a downward leap of 765 cents. This is followed by an ascending $5 / 3$, and these two notes are then echoed on the stopped horn (with an A fundamental).

Measure ten moves to another new fundamental, G. Playing its twelfth partial, the horn plays a D 402 cents above the previous $B^{b}$, just slightly larger than an equal-tempered major third. The notes within this measure are all just-intoned. Measure eleven begins with the horn switching back to the A fundamental and the fifth partial, a $C \# 586.3$ cents higher than the $G$ at the end of measure ten. At the end of this measure, Ligeti changes the fundamental again-this time to $A^{b}$-and the soloist ends this measure on an $E^{b}$, the sixth partial, nearly an equal-tempered minor sixth below the previous note (801.9 cents). The $\mathrm{E}^{b}$ fundamental is used through the first note of measure thirteen, and for the second note of that bar the soloist returns to an A fundamental; the soloist's move from the twelfth partial of $\mathrm{A}^{b}$ to the eighth partial of A creates a tritone of 602 cents.

Measure fourteen remains on the $A$ fundamental, and measure fifteen switches back to an $E^{b}$ fundamental-the first note, a low G, is an exact equal-tempered tritone ( 600 cents) above the low $C \#$ at the end of measure fourteen. Measures sixteen and seventeen are stopped horn "echoes" of measures fourteen and fifteen-measure sixteen moves to an $A^{b}$ fundamental, and uses a fundamental of $D .^{47}$ Again the interval between these measures is an equal-tempered tritone.

[^25]Following a G.P. in measure eighteen, the soloist moves back to the $A^{b}$ fundamental. At the end of measure nineteen the fundamental moves to A—mirroring the first mid-measure fundamental change from measure eleven-resulting in a descent of 868.8 cents which is a very small major sixth. This A fundamental is used for the first half of measure twenty, at which point the soloist switches to $B^{b}$. The ninth partial of A moves to the eighth partial of $B^{b}$, resulting in a 103.9 cent minor second.

Measure twenty-one begins by moving the fundamental to $G$. The move from the fifth partial D at the end of measure twenty to the fifth partial B in twenty-one is exactly 300 cents, an equal-tempered minor third. The soloist's first few intervals are the same as those from measures one, seven, and twenty, and are followed by an equal-tempered major second from the eleventh harmonic of $G$ to the eleventh harmonic of A. The A fundamental is used for only this one note (nearly a quarter tone below $\mathrm{D}^{\#}$ ) before the fundamental moves to $B^{b}$ on the tenth partial, 65 cents lower. From this $D$, the soloist descends just over an octave to a $B^{b}$ at the end of the bar.

This $\mathrm{B}^{b}$ is followed by a C at the start of measure twenty-two which is 1386.3 cents higher. This C begins a section similar to the previous passages which included stopped horn. However, in this case the stopped horn is not used-the line continues to gradually slow down and generally descend in pitch to its final note, a second partial A. While the fundamental switches four times over these seven bars, no equal-tempered intervals are created.

This very clear use of just intonation shows Ligeti's interest in the natural sounds of the harmonic series, as well as his intention to increase the possibilities of this system, resulting in interesting colors and intervals. "Solo" is followed by a stark contrast: it is interrupted by a focus on exact equal temperament, which is used for the rest of the movement (over three minutes)—all five horns are tacet
process for the $D$ horn. Reaching the $E^{b}$, however, would require the soloist to raise the seventh partial $D$ by a much larger 131.2 cents. While this is certainly not out of the realm of possibilities, it is unnecessarily complicated when the use of a D fundamental would not only require the same adjustment for both pitches, but also would comply with conventional performance practices and would match exactly Ligeti's three prior uses of stopped horn in this solo passage as well as his use of stopped horn in movement six. See Appendix A for more information.
until movement five. While the soloist has provided a melodic shape which will reappear later in the movement, the actual pitch material of just intonation will be absent and replaced by a dense harmonization.

The next section of this movement, "Intermezzo," begins immediately after the soloist's A fades away-this A is interrupted by a $7 / 8$ figure played on the snare drum with the snares off. This $7 / 8$ figure, which introduces the short second section, continues as a hemiola over which the strings play a ferocious, homorhythmic passage (with the exception of the bass which does play all the quarter notes, but only plays the first of any group of eighth notes) which changes meter nearly every measure and-much like earlier passages in the piece-features a strong presence of stacks of fifths.

The strings enter at the second measure of the snare drum hemiola (measure thirty) playing measures of $9 / 8,7 / 8,9 / 8,9 / 8,7 / 8,3 / 4$ (functioning as an extension of the $7 / 8$ ), and $5 / 8$. All of these bars except the $3 / 4$ and $5 / 8$ bars begin with three eighth notes and conclude with quarter notes-the $3 / 4$ bar is only quarter notes and the $5 / 8$ bar is only eighth notes. The chords range from having three to six distinct pitches, and are mainly either tertian harmonies or stacks of fifths. Early in this section the stacked fifths are featured, and near the end the harmonies are mainly complex tertian chords, but in most measures both types of harmonies can be found. Often, a note in the middle of a stack of fifths or a tertian chord is omitted. ${ }^{48}$

Measure thirty-six, the $5 / 8$ bar, is not only the last bar of this homorhythm, but also the last bar of the snare drum hemiola. It is followed by five measures in $4 / 8$ time in which the bass (with some help from the viola), the bassoon, and both basset horns play a soft, low, rumbling passage leading into the third section, "Mixtur." No percussion is used within this brief passage, and harmonically the section is entirely built of tertian harmonies, most of which are seventh chords. The final chord (beat four of measure forty-one-D/A with the third and fifth doubled) is marked by the entrance of the trumpet and the trombone, both muted. The "Mixtur" section interrupts suddenly at the start of the following measure.

[^26]"Mixtur" immediately switches to a slower tempo and reintroduces the 7/8 hemiola-now split between the bass drum and the tambourine-on top of which the rest of the orchestra plays in a steady 4/8. Figure 3.23 shows the bass line for the "Mixtur" section. Above this bass Ligeti uses only a few chords which move in parallel motion for long passages of this section. This static harmonic field is fairly complex, and it serves to mask the melodic content. This is especially important, as the harmonic content here is strongly rooted in the possibilities of equal temperament-the just-intoned tuning system Ligeti is using would not be able to obscure this melodic line in the same way.

In measure forty-two, where "Mixtur" begins, the trombone is the lowest voice sounding. This first chord has six distinct pitches and one doubling. From low to high the chord reads: A in the trombone and as a sustained pitch in the contrabass; $\mathrm{F} \#$ in the trumpet; D in the bassoon; $\mathrm{A}^{b}$ in bassett horn 2; C in the oboe; E in bassett horn 1; and $\mathrm{F}^{\#}$ in both flutes. As in this chord, for the remainder of the opening six bars the flutes will always play in unison. The trumpet, however, immediately moves to its own pitch in the second chord.

The second and third chords are made up of seven pitches. Both of these chords, when ordered, are identified as $[0,2,4,5,6,8,9]$ with the bass note occupying the nine position. The major sixth above the bass is played by the flutes, and is always the highest note in this harmony. In chords four and five (the last chord of measure forty-two and the first chord of measure forty-three), both bassett horns play the same pitch, reducing the total chord to six notes, ordered as $[0,1,4,6,8,9]$ with the bass note still at nine. This second chord is a subset of the first, as was the chord on the downbeat of measure forty-two, [0,2,4,6,8,9] (again with the bass at nine). For the remainder of this first section (through the end of measure forty-seven) the chord will be consistently $[0,2,4,5,6,8,9]$ with the bass note at nine.

For the next five measures, forty-eight through fifty-two, Ligeti continues to use [0,2,4,5,6,8,9]. The bass note is still nine, and the highest pitch in the flute is still a major sixth above the bass. However, the instrumentation in this section is different. Instead of doubling the soprano line, Ligeti now doubles the bass-it is played by both the bassoon and the contrabass. Above the bass line, the other instruments
used are the cello, the viola, violin 2 , bassett horn 1 , violin 1 and flute 1 . This section, unlike the first, uses only the $[0,2,4,5,6,8,9]$ chord-it never switches to a subset.


Figure 3.23-"Solo, Intermezzo, Mixtur, Kanon": bass line for measures 42-65"

At rehearsal T (measure fifty-three) the instrumentation changes again-it is reduced to only a six-voice texture. The trombone resumes its bass role, above which the trumpet, bassett horn 2 , the oboe, flute 2 and flute 1 can be heard. Since there is one less voice, this section mandates a change to a new
${ }^{49}$ In Figure 3.23 whenever the Contrabass and Bassoon are playing in octaves instead of in unison, only the lower pitch (contrabass) is shown.
chord. From measure fifty-three through the end of measure fifty-eight, Ligeti uses [0,1,3,4,7,9] with the bass as four, and the soprano a minor third above (still the same interval class). This chord is also a subset of the [0,2,4,5,6,8,9] chord.

At the start of measure fifty-nine, the orchestration shifts back to the previous texture (except flute 2 is now playing and flute 1 is not) and resumes using the $[0,2,4,5,6,8,9]$ chord. This return also means the bass is back to 9 with the soprano a major sixth above. The last three measures of "Mixtur" move back to the six voice texture, but with a different chord-[ $0,1,4,6,8,9$ ], which was the same chord used for two consecutive (eighth note) beats at the start of "Mixtur." This time there is a change in the bass-soprano relationship, as the bass is on one with the soprano a perfect fifth above. At the end of measure sixty five there is an abrupt switch into "Kanon," a vastly different final section.

Throughout "Mixtur," the $[0,2,4,5,6,8,9]$ chord is used almost exclusively-it is only replaced with smaller subsets of the same chord. This harmony uses, among many other intervals, four intervals from interval class 2 and three from interval class 5 . Initially such a static harmony seems odd-since the opening measures of "Praeludium," Ligeti has been writing harmonies that shift and evolve-but in this instance it is used as a major feature of the equal-tempered pitch language. The transposition of the set is not possible in a just-intoned tuning system, and since the horns are absent from this part of the Hamburg Concerto, it is a good place to show one of the interesting features of equal temperament.

While the harmonies of "Mixtur" provide one interesting aspect, the melodic content is perhaps even more intriguing, and it is here that the title becomes clear. The melodic passages, as shown in the bass line, are Ligeti's equal-tempered imitation of the opening section, "Solo." As mentioned above, this complex but relatively static harmonic content serves to somewhat obscure the relationship as a way to compensate for the discrepancies between the two tuning systems. Figures $3.24-3.27$ show four distinct lines from the opening "Solo" along with the matching line from "Mixtur." The measures of the solo that do not include eighth notes (the echoes and other measures with just a shorter note followed by a longer note, including the last two measures of the "Solo") are not replicated in "Mixtur."


Figure 3.24-"Solo, Intermezzo, Mixtur, Kanon" measures 1-3 and 42-47: a comparison of "Solo" and "Mixtur"


Figure 3.25-"Solo, Intermezzo, Mixtur, Kanon" measures 5-7 and 48-52: a comparison of "Solo" and "Mixtur"


Figure 3.26-"Solo, Intermezzo, Mixtur, Kanon" measures 10-12 and 53-58: a comparison of "Solo" and "Mixtur"


Figure 3.27-"Solo, Intermezzo, Mixtur, Kanon" measures 19-21 and 59-65: a comparison of "Solo" and "Mixtur"

The lines used in "Solo" and "Mixtur" are very similar, despite the differences of the tuning systems. The passages covered in Figure 3.24 use exactly the same interval classes, and their lines have identical contours. In Figure 3.25, the passages are similar but there are differences in the second fragment-the "Solo" used a descent of a $5 / 3$ (major sixth, interval class 3 ) at the end of measure six followed by an ascent of 384.3 (a small major third, interval class 4) to create an overall descent of 500 cents (an equal-tempered perfect fourth, interval class 5). In the corresponding section of "Mixtur," the bass line ascends a minor sixth (interval class 4) at the end of measure fifty and at the start of the next measure descends a minor second (interval class 1 ) to create a total ascent of a perfect fifth (interval class 5). Although the middle note of these two passages is different, the end result is simply an octave displacement.

The passage shown in Figure 3.26, like the passage in Figure 3.24, shows another identical match (in terms of interval classes and contour) between the two sections. The last passage, as seen in Figure 3.27, has a few discrepancies, starting immediately with the first pitch. The C in the solo horn in measure nineteen is an 11/10 ( 165 cents, interval class 2 ) below the last note of measure twelve, whereas the $G^{b}$ in the "Mixtur" bass line in measure fifty-nine is a minor third (interval class 3) below the last note of measure fifty-eight. From the first to the second note there is another difference: the line in "Solo" moves down a 10/9 (182.4 cents, interval class 2 ) while the line in "Mixtur" descends only a half step (interval class 1). Much like the difference that occurred in Figure 3.25, the overall result of these two
segments is the same-both lines descend by a major third. The "Solo" line descends by 11/9, or 347.4 cents—note that while this is technically closer to a minor third it will be perceived as a major third because it is approached by descents of two major seconds-while the "Mixtur" line descends by the equal-tempered major third of 400 cents. The third and final interval class difference between these two sections can be found a few notes later. Starting from the second note of measure sixty-one, the "Mixtur" bass line ascends by interval class 5 and follows this by a descent of interval class 2 , which results in an overall ascent of a minor third (interval class 3). In the corresponding section of "Solo," starting on the second note of measure twenty, the horn ascends by a 9/7 (435.1 cents, interval class 4 ) followed by a descent of 103.9 cents (interval class 1 ), which also results in an overall ascent of interval class 3 (331.2 cents). The remainder of the passage follows the same interval classes and contour (excepting octave displacements).

The melodic content of these two sections features mostly interval classes 2 and 4, which continues to reinforce the major second. Through this section of "Mixtur," there are nineteen instances of interval class 2, and the corresponding measures of "Solo" (not considering the sections of "Solo" that were not reproduced in "Mixtur") features twenty instances of interval class 2 . These sections each have a total of sixty-three intervals, which means just slightly less than one-third of the melodic intervals are from interval class 2. The notes of interval class 4 are also prominent-both "Solo" and "Mixtur" have seventeen instances of interval class 4 , which further illuminates the importance of the major second since interval class 4 is just a set of major seconds added together. These two considered together make up over half of the intervals in either section. This gives the passages a quality reminiscent of a whole tone scale, which is an especially interesting feature of the just-intoned system Ligeti has employed. From partial seven through partial eleven, the horn sounds four distinct major seconds- $8 / 7$ or 231.2 cents, $9 / 8$ or 203.9 cents, $10 / 9$ or 182.4 cents, and $11 / 10$ or 165 cents-and from partial eleven to partial twelve is a $12 / 11$ which, at 150.6 cents, has the interesting effect of not only nearly resembling another major second but also creating a sense that the eleventh partial note is resolving, creating an end point for the whole tone ascent.
"Kanon" enters quietly and at a furious pace, interrupting the comparatively long lines of "Mixtur." It consists of straight sixteenth notes, initially $\boldsymbol{p p}$ in the highest register of violins 1 and 2. This cascading passage is built on repeated fragments of a tone row ${ }^{50}$ (see Figure 3.28) which is repeated a total of three times over the course of approximately 60 seconds before the row begins to dissolve.


Figure 3.28-"Solo, Intermezzo, Mixtur, Kanon": tone row used in "Kanon"

This tone row is played in fragments of between five and twelve tones, and played in canon throughout. This makes the twelve tone aspect of "Kanon" very difficult to perceive—although the notes themselves pass by very rapidly, Ligeti exercises great patience in moving through the tone row. "Kanon" begins in measure sixty-six with the first five notes of the row repeated several times; the sixth note of the row is not introduced until measure eighty-one. Once this sixth note enters, the first note is removed from the passage. This pattern continues throughout the first two iterations of the tone row (measures sixty-six through 112). When the third iteration begins, it starts the same as the first two, with each instrument adding the $D^{b}$ at the end of the row and eliminating the $E$ at the beginning. Once the starting pitch has changed to C , however, the instruments continue to resume the row from C regardless of how many notes are added. The end result is that the repeated fragments gradually become longer until each one of twelve instruments is playing through the tone row in its entirety from C to E .

As the texture thickens the collection of harmonic intervals becomes less and less distinguishable so that by the time Ligeti arrives on a fully chromatic twelve note chord (which happens only twice, first as the last note of measure 121 and second as the first note of measure 123-these chords are voiced identically) it does not stick out from the rest of the chords. Once this fully chromatic chord has been

[^27]reached and the instruments are all playing the tone row in its entirety, Ligeti begins to dissolve the row, taking individual pitches back out and slowing the tempo. The first pitch Ligeti removes is $\mathrm{F}^{\#}$, followed by $B^{b}, D, G^{\#}, E, G$ and $B$. This leaves the instruments playing short passages of $C, F, D^{b}, E^{b}$, and $A$ from which Ligeti removes the $\mathrm{E}^{b}$ and replaces the E at the end. This final fragment-C, $\mathrm{F}, \mathrm{C} \#, \mathrm{~A}$, and E -is still played in canon by the strings for a total of four measures (from the fifth sixteenth note of measure 128 through the fourth sixteenth note of measure 132).

Starting over this remnant of the initial canon, the winds begin a new canon. The oboe is the first to play this line, starting on the downbeat of measure 130. Initially this line just reorders the pitches being played by the strings, but as soon as the strings stop playing their figure, this new line continues to descend. The line consists of two repeating elements: a downward leap of a fifth (typically perfect, but augmented and diminished are used) followed by an upward leap of a third (major or minor). After these thirds, typically, another descending fifth follows and the pattern continues. This canon lasts less than five measures before bottoming out in the bassett horns, bassoon and trombone, at which point the moving line is over.

When the strings finish with their figure the contrabass, cello and viola play $\boldsymbol{p p}$ harmonics sounding F (above middle C ), C and E , joining a suspended cymbal roll which has been playing a slow crescendo from $\boldsymbol{P P P P P P}$ for two measures, and is not even at $\boldsymbol{p p p p}$ when the strings begin these harmonics. When the moving line discussed above comes to an end, the cymbal and these strings are left to fade out to end the movement.

As the fourth movement was largely an equal-tempered movement, a densely chromatic canon seems a fitting way to conclude. This, certainly, is one of the numerous features of equal temperament that a just-intoned system cannot approach. The "Mixtur" section, combining equal-tempered pitches with the contour of the opening just-intoned "Solo" led into this "Kanon" nicely, bridging the gap between the tuning worlds and the harmonic worlds. "Intermezzo," built largely on the important interval class 5 also used "Mixtur" as a transition into "Kanon" via its more chromatic harmonic content. Within
the context of the fourth movement, "Kanon" functions as a strong conclusion to the progression away from just intonation. In fact, this is the farthest Ligeti gets from just intonation in the entire Hamburg Concerto, which is appropriate as this is the central movement of the piece and the longest of the seven. The following movement, "Spectra," is also very chromatic, but its chromaticism comes once again from a combination of tuning systems, and is heard though longer sustains and highlighted harmonic intervals.

### 3.6 V. "SPECTRA"

"Spectra" begins immediately after movement four, and quickly reintroduces the just-intoned sound world of the horns. While the previous movement strongly emphasized interval class 2 , in "Spectra" interval class 5 will be at the forefront, and is continually emphasized (this is shown in dramatic fashion by the final chord). Ligeti features two trichords in this movement, $[0,1,6]$ and $\left[0,5,6{ }^{51}\right.$, which serve to keep the perfect fourth and perfect fifth prominent. This trichord also shifts the attention from interval class 2 to interval class 1, as do the several chromatic passages and harmonies throughout this movement. This helps Ligeti direct attention to the complicated harmonic content of "Spectra," which he does for the entirety of the movement.

While emphasizing the very stable, simple relationship of interval class 5, this movement also explores the strange relationship between the tuning systems, "pursuing further than any other its 'spectral' progression of microharmonies, with deflected and tempered pitches in new and unsetting combinations." ${ }^{52}$ Throughout this exploration, a striking majority of the chords have one of two features,

[^28]either they are built mostly with stacks of thirds, or they feature a $[0,1,6]$ or $[0,5,6]$ relationship somewhere within the chord. Oftentimes, the harmonies utilize both of these features in the same chord.

At the start of this movement (attacca from movement four) all four obbligato players are playing E horns while the soloist is still using a double horn; for the opening two measures, the soloist is playing above a $B^{b}$ fundamental, but as in the opening section of movement four, the soloist will employ various valve combinations to alter the fundamental. The first three measures use only the horns, and in addition to the two harmonic features listed above, they also illuminate the equal-tempered relationship between the soloist and the obbligato horns. The meter stays the same throughout the entire movement, but is very irregular ( $16 / 8$ divided into $3 / 8,3 / 4,3 / 8$, and $2 / 4$ ) and there is nearly no sense of an underlying pulse. This further illuminates the importance of the harmonies employed in "Spectra."

The first chord (see Figure 3.29) can be seen as a ninth chord; it contains $G^{\#}, B, D, F$, and $A^{\#}$ (written as $\mathrm{B}^{b}$ ), and the four lowest notes are all in just-intoned relationships as they are all played by E horns. The solo horn is playing its $B^{b}$ at 1000 cents, 48.7 cents above where the $A^{\#}$ over an $E$ fundamental (the eleventh harmonic) would sound. This adds a complex element at the top of the chord, turning what would otherwise be a very simple, just-intoned chord into a combination of just intonation and equal temperament. This effect, initially found in the first chord of the movement, foreshadows much of the harmonic content of "Spectra."

The second chord is also built of thirds, now having both F and F\# at the top. The solo horn's move from the eighth to the sixth partial results in an equal-tempered tritone between the soloist and horn 4, also on its sixth partial-incorporating equal-tempered intervals into the horn lines is an important part of "Spectra," much like it was in "Signale, Tanz, Choral." The next chord is also build with thirds and also includes the first instance of an $[0,5,6]$ harmonic component between horn 4, horn 3 , and the solo


Figure 3.29—"Spectra" measures 1-5
horn as they play B, E, and F. ${ }^{53}$ These notes are played as either sixth or eighth partials, making the first instance of this component as close to equal temperament as is possible with this horn tuning. The following chord, starting on the third beat of the measure, eliminates the [0,5,6] element by moving the soloist from partial six to partial nine, but this move does maintain much of the tertian harmony-the resulting chord is B, D, E, G\# and C, which consists entirely of thirds (or sixths), and is followed by a chord of $\mathrm{E}, \mathrm{B}, \mathrm{D}, \mathrm{F}^{\#}$, and C on the last beat. While either an A or a G would create a full set of thirds, the missing third is strongly implied (in the form of $G^{\#}$ ) by the stability of the E horns. The final chord of this bar has a $[0,5,6]$ relationship between horn 2 , horn 1 , and the solo horn. This harmony is played with partials six and nine which are in the same simple relationship to each other as partials six and eight were two chords prior, and the tritone relationship is again exactly 600 cents between horn 1 and the solo horn, both playing partial nine.

Measure two begins with a chord again built of thirds, and moves to a chord that is built of thirds with one omitted ( $F^{\#}, A^{\#}, E, A^{b}, B$ ) and contains a $[0,1,6]$ between $E, A \#$, and $B$. This $[0,1,6]$ is sounded entirely by E horns on partials eight, eleven, and twelve. While it is now a clearly just-intoned relationship instead of a nearly equal-tempered one, the sound is still very similar; this is because these three partials form very simple relationships: $4 / 3$ from B to E, $11 / 8$ from E to $A \#$, and 11/6 from B to $A^{\#}$. The third chord, found at the end of beat two, involves the solo horn moving from a seventh partial $\mathrm{A}^{\mathrm{b}}$ to a tenth partial D which results in another tertian harmony omitting a third and has the same $[0,1,6]$ element as the previous chord. The final chord in this bar again uses the same three notes (played by different horns) for a $[0,1,6]$ chord, and is still built mainly with thirds.

The first chord of measure three again contains a mainly tertian harmony, but this time has both a $[0,1,6]$ chord and a $[0,5,6]$ chord. The $[0,1,6]$ chord is sustained from the previous measure, and the

[^29]$[0,5,6]$ chord is introduced when the solo horn changes its partial and fundamental, resulting in an $E^{b}$ as the twelfth partial of $A^{b}$ which creates a $[0,5,6]$ with the sustained eighth partial E and eleventh partial A\#. This instance of $[0,5,6]$ might be slightly less clear due to the combination of tuning systems involved, but it is still somewhat prominent because it shares two pitches with the [0,1,6] chord, and the major seventh between the $E$ and the $E^{b}$ is 1102 cents. The simple $11 / 8$ relationship between the $E$ and the $A \#$, and the nearly equal-tempered relationship between the $E$ and the $E^{b}$ compensate for the strange perfect fourth between the $E^{b}$ and the $A^{\#}$ which, at 550.7 cents, is off by nearly a quarter tone.

On beat two, the four E horns move to new notes, resulting in a completely tertian ninth chord, and eliminating the $[0,1,6]$ and $[0,5,6]$ harmonies. When the soloist moves to another new fundamental, F, partway through beat two, the resulting chord has fewer thirds-the $F \#, A, A \#, B, C \#$ harmony is voiced as closely as possible, and is built on some high partials (nine through thirteen) which further masks the tertian element. The chord beginning on beat four is not strongly tertian, but contains both $[0,1,6]$ and $[0,5,6]$ harmonies-the former between $G \#, A$, and $D$ featuring an equal-tempered minor second, a just-intoned tritone of $7 / 5$ ( 582.5 cents), and a 682.5 cent perfect fifth, and the latter between G\#, C\#, and D featuring three just-intoned intervals-the same 7/5, a small perfect fourth of 13/10 (454.2 cents—nearly a quarter tone low), and a large minor second of $14 / 13$ (128.3 cents). At the end of the measure, the soloist moves from $A$ to $B$, eliminating the $[0,1,6]$ chord, but slightly strengthening the tertian element. This chord does not last long, however, as all four of the E horns drop out on the downbeat of measure four, leaving only the soloist to sustain the eleventh partial B for the first beat.

Beat two of this fourth measure has several complex elements involved. For one, as the horn section re-enters, the winds and the rest of the brass enter as well. This chord involves three of the wind players (both bassett horns and flute 2) altering their pitches with microtonal inflections. The bassett horns enter on pitches matching exactly the notes of horns 3 and 1 from the end of measure three. The flute 2 pitch is also in a simple relationship-it enters a perfect fifth above the last note played by horn 4.

These three pitches along with three of the horn pitches—B on the solo horn, $A \#$ on horn $1, G \#$ on horn 3-lie outside of equal temperament, while the remaining six pitches (flute 1 , the oboe, the bassoon, horn 2 , horn 4, and the trombone) lie either exactly within equal temperament or, in the case of horn 2 , off by less than four cents. This creates a chord containing twelve total pitches, two of which are E and two of which are $\mathrm{F} \#$ (one is off by 3.9 cents); the remaining eight pitches are unique. This chord strongly demonstrates the conflict between the two tuning systems (see Figure 3.30).

Focusing first on the equal-tempered and nearly equal-tempered pitches, Ligeti has written $\mathrm{B}^{b}, \mathrm{E}$, $F$, and $F \#$, which contains one instance of a $[0,1,6]$ trichord. This chord is presented over a very large registral space, with the $\mathrm{B}^{b}$ being two and one-half octaves below the next closest pitch, the equaltempered E. The just-intoned pitches contain a tenth partial G\#, eleventh partial A\# and eleventh partial B in the horns, with the bassett horns simulating a thirteenth partial $D^{b}$ and a fourteenth partial $D^{\mathfrak{G}}$ and flute 2 simulating a tenth partial $D \#$. These are all presented within the range of a perfect fifth, and the total contains both a $[0,1,6]$ and a $[0,5,6]$-the former involving $D, D \#$, and $G \#$ (an equal-tempered perfect fifth, a just-intoned $7 / 5$ tritone, and a minor second of 117.5 cents), and the latter $G^{\#}, D^{b}$ and $D^{\natural}$ (a justintoned 7/5 tritone, a just-intoned small perfect fourth of $13 / 10$, and a just-intoned large minor second of 14/13 - the same chord was found at the end of measure three).

While neither of these groups individually can be thought of as a tertian harmony, when put together the chord could be rewritten as $B^{b}, D, F, A^{b}\left(G^{\#}\right), B^{\natural}, D^{\#}, F^{\#}, A^{\#}, C^{\#}\left(D^{b}\right)$, and $E$. Not only does this chord contain ten different pitches, it also uses nine (or eleven if you consider each $\mathrm{F} \#$ separately) different sized thirds: 368.8 cents (nearly a just-intoned $99 / 80$ ) between the $B^{b}$ in the trombone and the $D$ in bassett horn 1; 331.2 cents between the D in bassett horn 1 and the F in the oboe; 286.3 cents between the $F$ in the oboe and the $G \#$ in horn $3 ; 265$ cents between the $G \#$ in horn 3 and the $B$ in the solo horn; 435 cents (nearly a just-intoned 9/7) between the B in the solo horn and the $D \#$ in flute $2 ; 313.7$ cents between


Figure 3.30-"Spectra" measure 4 with cents values
the $D^{\#}$ in flute 2 and the $F \#$ in flute 1 or 317.6 (a just-intoned 19683/16384-the Pythagorean augmented second) cents between the $D \#$ in flute 2 and the $F \#$ in horn 2; 351.3 cents (nearly a just-intoned 49/40) between the $F^{\#}$ in flute 1 and the $A \#$ in horn 1 or 347.4 cents (a just-intoned 11/9) between the $F \#$ in horn 2 and the $A \#$ in horn $1 ; 289.2$ cents (a just-intoned 13/11) between the $A \#$ in horn 1 and the $D^{b}$ in bassett horn 2; and 459.5 cents (closer to a perfect fourth, but will still sound like a large major third) between the $D^{b}$ in bassett horn 2 and the F in the oboe.

This same chord also contains eight instances of $[0,1,6]$ or $[0,5,6]$ : the two mentioned above-a $[0,5,6]$ between $G \#, D^{b}$ and $D^{k}$ (a just-intoned 7/5 tritone, a just-intoned small perfect fourth of $13 / 10$, and a just-intoned large minor second of $14 / 13$ ) and a $[0,1,6]$ between $D, D^{\#}$, and $G \#$ (an equal-tempered perfect fifth, a just-intoned 7/5 tritone, and a minor second of 117.5 cents); a [0,5,6] between $A \#, D \#$ and E (the just-intoned tritone of $11 / 8$ or 551.3 cents and the 535 cent large perfect fourth leaves a tiny minor second of 13.7 cents, only slightly larger than an eighth tone); a [0,1,6] between $F, F \#$, and $B$ (with two slightly different options for $\mathrm{F}^{\#}$, the chord has an interesting quality-the tritone in either instance is $11 / 8$ with the other two intervals being either 451.3 cents and 100 cents with the $\mathrm{F}^{\#}$ in flute 1 or 447.4 cents and 103.9 cents with the $F^{\#}$ in horn 2 ); and finally two $[0,1,6]$ trichords and two $[0,5,6]$ trichords between E, F, A\# and B (both tritones here are just-intoned and both minor seconds are equal-tempered which leaves the perfect fourths off by about a quarter tone).

This extremely complex chord is followed by a dramatic shift to a simple, equal-tempered F minor triad, which is followed by an equal-tempered [0,1,6]. Ligeti's use of these two chords following the most complex harmony he has used in the Hamburg Concerto to this point overtly clarifies the importance of the triadic harmonies and the $[0,1,6]$ trichords in "Spectra." In the fifth measure the content is again relatively simple, with a mainly tertian harmony containing two $[0,1,6]$ trichords and one
$[0,5,6]$ trichord on beat two, followed by an equal-tempered minor second and an equal-tempered major seventh played between the soloist and horn 2 at the end of the measure.

As these opening five measures have shown, the use (and frequent juxtaposition) of these trichords and tertian harmonies are a major feature of the pitch content of "Spectra." These features will continue to be prevalent through the movement. The sixth measure begins with the horns playing a combination of a just-intoned major triad and a $[0,1,6]$ chord (the tritone is equal-tempered, the fourth is just-intoned, and the minor second is 102 cents). As more instruments enter throughout the measure, these two features are maintained, and more tertian chords and $[0,1,6$,$] and [0,5,6]$ trichords are added. ${ }^{54}$ In addition, partway into beat two, as horn 3 and the solo horn move to new pitches, ${ }^{55}$ a bowed cymbal enters, complicating the harmonic spectrum with an indeterminate pitch. On beat four, where both flutes enter, the chord is again a tertian construction that contains several $[0,1,6]$ and $[0,5,6]$ trichords: $a[0,1,6]$ trichord between the ninth partial C of the solo horn, the thirteenth partial C\# of horn 1, and the F\# in flute 2 (a minor second of only 36.6 cents, a large perfect fourth of 559.5 cents, and a nearly equal-tempered tritone of 596.1 cents); two $[0,1,6]$ trichords and two $[0,5,6]$ trichords between the same C and F \# along with the $B$ in horn 2 and the $F^{\natural}$ in bassett horn 2-these intervals are all either equal-tempered or within 4 cents of equal temperament; and two $[0,1,6]$ trichords and two $[0,5,6]$ trichords between the $E$ and $F$ in the bassett horns (the E is also in horn 4) and the eleventh partial $A \#$ in horn 3 and the $B$ in horn 2, which involves intervals which are all just-intoned or within 4 cents of being just-intoned.

[^30]The first beat of measure seven involves most of the notes from measure six being sustained, while flute 1 and horn 3 exchange pitches ${ }^{56}$ (altering the tuning slightly) and horn 1 moves to a D which reinforces the tertian harmony, as it is now the just-intoned minor seventh of the just-intoned dominant seventh chord in the horns. This move also eliminates one of the trichords (the [0,1,6] which involved horn 1) and makes the four trichords which involved the $A \#$ in horn 3 a little less stable, as they now have an equal-tempered $A \#$ from the flute, which is nearly a quarter tone higher. Starting on the second beat of this measure, the texture begins to thin out, and by the end of the bar just the bassett horns remain, playing a minor second.

In measure eight the cello and bass enter creating a $[0,1,6]$ trichord and a $[0,5,6]$ trichord (both equal-tempered) with the bassett horns. This chord changes gradually over the course of measure eight, and by the end of the bar, the solo horn, horn 1, bassett horn 1, the cello and the bass are playing a chord of $A, E, A^{b}, F$, and $F \#$ a chord that hints at the tertian harmonies which will appear in the following measure. The chord is sustained through the first beat of measure nine, but is replaced with a major seventh dyad on the second beat played by just the two horns. This moment is noteworthy as they are moving from a minor second on F and F\# to a major seventh on C and B-this echoes the end of measure five where two horns moved in a similar manner. From this second beat of measure nine until the end of measure eleven, the timbres of the brass instruments are highlighted, joined only by the contrabass.

On beat three of measure nine, the rest of the horns enter, resulting in a tertian chord spread out over two and one-half octaves-the harmonies in the rest of this measure are all tertian. At the start of measure ten this tertian harmony is maintained, but a $[0,1,6]$ trichord is also added with a just-intoned tritone and an equal-tempered minor second which results in a very small perfect fourth of 451.3 cents. On the second beat only three horns are playing, sounding an exposed $F, A \#$, and $B[0,5,6]$ trichord with a 12/11 minor second and nearly equal-tempered tritone (602 cents). The perfect fourth is the same 451.3

[^31]cents from the previous chord. From the end of the second beat of measure ten through all of measure eleven, the harmonies are tertian, and the trumpet, trombone and contrabass (doubling the trombone) join the texture over the course of these two measures. Initially the chord on beat two of measure eleven is somewhat complex, containing a nearly equal-tempered $[0,1,6]$ trichord, but on beat three this becomes a much simpler harmony resembling a $\mathrm{D} \#$ major triad with an equal-tempered perfect fifth in the horns and a third (played by the trumpet, the trombone, and the bass) that is very large, at 448.7 cents. This chord lasts for the rest of the measure.

Measure twelve begins entirely equal-tempered, with a simple C major triad sustained alone into beat two and joined by the horns, the crotales, and the flutes. Still loosely resembling a tertian harmony (or two separate ninth chords superimposed) this chord contains several $[0,1,6]$ and $[0,5,6]$ trichords-two nearly equal-tempered $[0,1,6]$ trichords, one $[0,1,6]$ trichord wherein all three intervals are small, one nearly equal-tempered $[0,5,6]$ trichord, and one $[0,5,6]$ trichord with a large fourth and tritone, but a slightly small minor second. At the end of this bar only the horns are playing-the harmony here is generally tertian, but very unstable as the pitches all lie very high in the harmonic series.

The thirteenth bar of "Spectra" begins with the horns and the oboe. The first chord is nearly tertian, and contains one $[0,1,6]$ harmony between the oboe's $A$, the low $A \#$ of horn 2 and the fifteenth partial $\mathrm{D}^{\#}$ of horn 1 (a minor second of 51.3 cents, a perfect fourth of $15 / 11$ which is 537 cents and a tritone of 588.3 cents). The second chord maintains the previous [0,1,6] trichord and adds a [0,5,6] (with a tritone of 602 cents, a minor second of a 12/11, and a perfect fourth of 451.3 cents).

On beat three of measure thirteen there is a drastic change in register, harmony, and color-a bass drum roll (starting $\boldsymbol{P p p p}$ with a gradual crescendo) is heard along with a chromatic chord of $A^{b}, A^{b}, B^{b}$, $B^{\natural}$, and C, played by the bass, trombone, bassoon, and cello, with the highest pitch being the cello's open fourth string C. This lasts until beat three of the following measure, at which point the chord descends further to F in the bass, G in the trombone, and A which is played by the solo horn as the first partial of the A fundamental (which is the fundamental itself or the pedal tone). This fundamental change is not
marked in the score, but is the only way to sound this pitch. ${ }^{57}$ This chord is sustained through the first beat of measure fifteen, where it changes to a major second dyad of $G^{b}$ and $A^{b}$ (the $A^{b}$ is played by the solo horn—again there is no fundamental change indicated, but it must move to $\mathrm{A}^{b^{58}}$ ) which is sustained through beats two and three. During this sustain the bass drum roll crescendos to $\mathbf{f f f f}$.

The following chord, starting on beat four of measure fifteen is extremely interesting. At a pianissimo dynamic, using no horns, Ligeti simulates a just-intoned chord from the fourth through the nineteenth partial without repeating any pitches (see Figure 3.31). The fundamental of this harmonic series would be the D in the middle of the bass clef, and the fourth harmonic D above this fundamental is played by muted trumpet. The third of the chord, partial five, is played by the oboe, and the fifth and seventh are played by $\mathrm{B}^{b}$ and $\mathrm{E}^{b}$ clarinets. This is the first time in the Hamburg Concerto where the clarinets (played by the bassett horn players) are used, and they are only used until the end of "Spectra." The ninth partial is played by the flute, the eleventh partial by the piccolo, ${ }^{59}$ and the thirteenth and fifteenth partials as harmonics on the cello and the viola. The seventeenth partial, heard only once before (in the second movement), is played by violin 2-this partial, 17/16, is 105 cents, very close to the equaltempered interval. The highest note here, the F played by violin 1, simulates the nineteenth partial19/16 is 297.5 cents, also extremely close to an equal-tempered interval (even more so than the 9/8). Within this ten-note, completely tertian chord, there are twelve instances of either $[0,1,6]$ or $[0,5,6]$, all just-intoned. This, of course, is just an incidental feature in this chord; the main interest here is the use of the just-intoned harmony in the winds and strings.

[^32]

Figure 3.31-"Spectra" measure 15 with numbers of the simulated partials

This stable, consonant harmony is interrupted by a harsh, fortissimo return to the conflict between just intonation and equal temperament. Within this next chord is a just-intoned $\mathrm{A}^{11}$ chord between both violins, the cello, and the viola. Violin 2 is sounding the root and the fifth (A and E), the cello is playing the third (a C\# slightly lowered), violin 1 is playing the seventh (a G lowered by approximately a sixth tone ${ }^{60}$ ), and the viola is playing the eleventh (a $\mathrm{D}^{\#}$ a quarter tone low)-the ninth is omitted. There are also equal-tempered pitches sounding-aside from the A in violin 2 , there is also an A in the tubular bells, a B in the tubular bells, ${ }^{61}$ an $\mathrm{F}^{\#}$ in flute 1 , and a G in the contrabass, all in equal temperament. This chord brashly reflects on the earlier content of "Spectra," containing a nearly just- intoned [0,5,6] trichord between $A$, the low $D^{\#}$, and $E$, and two $[0,5,6]$ trichords involving $C \#$ and $F \#$-one with the equaltempered $G$ in the bass creating a large perfect fourth, equal-tempered minor second, and large tritone, and one with the low G in violin 1 creating a large perfect fourth, small minor second, and small tritone.

From beat three of this bar until the end of the first beat of measure fourteen, the horns interrupt with lines ascending up the harmonic series, imitating the equal-tempered interruption of the orchestra in measure fifteen of "Praeludium." Within the four E horns, this results in a series of four-note chords which begins as a dominant seventh chord and ends as a collection of minor seconds-14/13, 15/14, and $16 / 15$. The solo horn, back to a $B^{b}$ fundamental, plays a similar line. The soloist starts lower than any of the four obbligato horns, yet ends higher than them all due to its fundamental being a tritone higher. The shifting of the relationships between these lines is another reference to movement two, where the "Tanz" section featured these collapsing intervals between the horns. These lines are surprising, as the horns move through this passage in eighth notes (much shorter than the majority of the durations in this

[^33]movement) at a faster tempo than the rest of the movement uses. They enter $\boldsymbol{f}$ and crescendo to $\boldsymbol{f f f}$ by the time they arrive at the second beat of measure seventeen.

This second beat of measure seventeen contains a $\boldsymbol{f f f}$ chord in the brass and clarinets. From low to high, this chord contains $B^{b}, D, E, F, G^{b}, G^{\natural}$, and A-a mostly tertian harmony with a completely equal-tempered $[0,1,6]$ trichord played between the $E^{b}$ clarinet's $A$, the trombone's $B^{b}$ and the $E$ of horns 1 and 3. This chord, with all instruments playing $\boldsymbol{f f f}$ lasts for all of beats two and three. The last chord of this movement begins on beat four of measure seventeen. Entering $\boldsymbol{\int} \iint f$ with no diminuendo, the woodwinds, strings, and trumpet play an open fifth from $D^{b}$ to $A^{b}$ that is very reminiscent of the first open fifth of the piece in the seventh measure of "Praeludium." This open fifth lasts for four full beats and ends "Spectra."

This final dyad is a strong reminder of interval class 5 , which has been nearly omnipresent in this movement. Aside from the complicated harmonic language, built frequently either with thirds or with intervals of interval class 1 (or smaller), the perfect fourth and perfect fifth have been the dominant feature of "Spectra." In the $[0,1,6]$ and $[0,5,6]$ trichords as well as the tertian harmonies, these intervals are prominent, showing up as crucial fourths in the trichords, and as fifths in the tertian harmonies. Aside from places where only one instrument is playing and chords which are just dyads of interval class 1 (or a collection of several pitches separated by half steps), only seven chords in the entire movement do not utilize interval class 5.

The movement is outlined by a few outstanding features. The beginning features three measures with the horns alone, which are then sharply offset by a return to solely equal-tempered pitches. When these systems combine, the spectrum is clouded further with the bowed cymbal. This is followed by a longer equal-tempered passage and a return to horns alone, which are eventually joined by the other brass instruments (and bass). The rest of the orchestra again joins in, and then disappears as we hear only the rumbling of extremely low pitches and the bass drum. This is followed by the simulated just-intoned
chord, then a clash of the tuning systems while the horns play the fastest material of this movement, and eventually the orchestra comes to rest on an open fifth. These highlights are sections which illuminate the tuning systems and their conflicts as well as where possibilities of one tuning system are imitated by the other, including not only the open fifths at the end as well as the orchestra's simulated just-intoned chord, but also a smaller component of "Spectra," the $[0,1,6]$ and $[0,5,6]$ trichords.

The $[0,1,6]$ and $[0,5,6]$ trichords have a few interesting features themselves. In addition to featuring interval class 5 , they have also clearly shifted the focus from interval class 2 to interval class 1 . While in earlier movements, the major second was an important feature of many chords and melodic lines, here interval class 2 is primarily a byproduct of tertian harmonies and stacks of perfect fifths-it is presented in isolation only twice, in consecutive chords, from the middle of measure fourteen through the third beat of measure fifteen. Interval class one, on the other hand, is presented in isolation twice as often (twice in measure five between the horns, on beat two of measure nine, and from the last beat of measure thirteen through the second beat of measure fourteen), and is a crucial component of one of the dominant harmonic features of "Spectra." Of course, interval class 1 is also created incidentally through some of the tertian harmonies.

The other interesting feature of these trichords is the different tunings through which they are presented. Sometimes these chords are presented entirely just-intoned. These chords have some interesting possibilities, as there are a few different ways the chords could be created. The entirely equaltempered trichords are not uncommon in this movement, but also are not the most interesting feature-the trichords which are a combination of equal temperament and just intonation, however, are very common and have a great range of outcomes. Within "Spectra," these trichords have been used in a way that makes intervals of less than a quarter tone or larger than 150 cents sound like a minor second, and have displayed perfect fourths that were more than 50 cents away from either an equal-tempered or justintoned perfect fourth. This works because of the tritone, which turns out to have a wide range of possibilities. In instances with a very large tritone, for example, the perfect fourth could be exactly a justintoned $4 / 3$, and the minor second nearly a quarter tone large at $12 / 11$, as in beat two of measure six. If
the tritone is nearly exactly equal-tempered, a minor second of only 36.6 cents and a large perfect fourth of 559.5 will also sound reasonable, as in the fourth beat of measure six. It is important to reiterate that these $[0,1,6]$ and $[0,5,6]$ trichords are not the same as equal-tempered trichords, and do not offer the same freedom of transposition. They do, however, accurately describe the harmonic content of much of "Spectra," and in so doing, they show that in "Spectra," Ligeti has expanded the borders of these interval classes to the widest points he will use in the Hamburg Concerto.

While a migration towards interval class 1 had been suggested through earlier movements, it has now clearly become the dominant interval class; within movement five, this proves to be a very useful replacement which generates a vast quantity of intensity and color in the pitch language. Through the two movements that follow, however, Ligeti is faced with the challenge of finding a way to create a clear balance between these interval classes and restore the familiarity of interval class 2.

### 3.7 VI. "CAPRICCIO"

"Capriccio," which was originally the final movement of the Hamburg Concerto (before Ligeti's revision), opens with a return to lighter, energetic material focusing mainly on the horns. It begins with the soloist playing a figure loosely resembling a horn call, recalling the opening of the second movement, "Signale, Tanz, Choral." This opening figure becomes the primary motivic material for much of the sixth movement (see Figure 3.32), as it is imitated constantly through the first half of this movement. Throughout "Capriccio," Ligeti uses material from earlier movements in different ways, highlighting some important aspects of the tunings in the process, and giving a very strong presence to the horn soloist.


Figure 3.32—"Capriccio" measures 1-5

One interesting aspect of the two opening measures is that the solo horn initially plays its C and F on the $B^{b}$ side of the horn, sounding 3.9 cents and 502 cents. In measure two, however, the soloist plays the $C$ and $F$ on the $F$ side of the horn, resulting in 2 cents and 500 cents. While in one respect this could be considered an attempt to make the horn sound more in tune with the bassett horns which have these same pitches two octaves lower, this difference, especially with the registral displacement, would be nearly unnoticeable to the listener. A more likely reason for the change would be that it adds a slightly different color, making this second instance somewhat darker than the first. Either way, these opening measures immediately return to placing an emphasis on the perfect fifth-not only is this interval a major component of the horn solo in the first two measures, but the first sustained interval (in measure two) is also a perfect fifth, and the $G^{M_{7}}$ chord which begins measure two in the strings is also built of two perfect fifths. This chord, reminiscent of the tertian harmonies from "Spectra" and the chords in "Choral" and
"Intermezzo," sets up much of the harmonic content of this movement-it features many tertian harmonies, often as seventh chords, and many perfect fifths. In the fourth measure, the strings again play a tertian chord featuring perfect fifths-this time there are four such intervals (C to G, G to D, E to B and B to $F^{\#}$ ) within a $C^{M^{\#} 11}$ chord. The minor second is also given emphasis early-it is the interval of the first horn dyad which occurs in measure three (the second is a perfect fifth), and in measure four there is a consistent equal-tempered minor second between the two horn lines.

In addition to the horn call quality of the melodic line, these opening measures show many other references to "Signale": the solo horn line being echoed by one or more of the obbligato horns; fragments of the line being played by other horns in other keys; and fragments of the line being played in minor seconds. Unlike "Signale," however, the rest of the orchestra is also involved, which leads to equaltempered interjections and juxtapositions against this motive; the first instance of this comes in measure six, where a short interruption (see Figure 3.33) featuring three perfect fifths and one major second both melodically and harmonically is played by the xylophone, bassett horn 1 (joined by bassett horn 2 for the last note), the oboe, flute 1 , and the piccolo. This leads directly into a longer line in measures seven and eight played by the solo horn which ascends from the first through the eleventh partial over the $\mathrm{B}^{b}$ fundamental. This ascent is marked with a few leaps, but has a strong upwards motion and its arrival on a low E is marked by a collection of just-intoned and equal-tempered perfect fifths (the E in the solo horn is a quarter tone off from creating an interval of interval class 5 with the trombone B).

This ascent is echoed by the solo horn playing another two measure ascent (in measures nine and ten) from the second through the thirteenth partial of F-during this ascent it is joined by all four of the obbligato horns (horns 1 and 3 are in F, horns 2 and 4 are in E). This time, the soloist plays only ascending notes, as do horns 1,3 , and 4 -horn 2 has one descent from partial six to partial five, but otherwise is also ascending. This creates another set of intervals which continue to get smaller, much like the end of "Spectra" and the second section of movement two, "Tanz." At the end of this ascent the horns arrive at a chord containing an E in horn 4 ( 400 cents, joined by the bass and bassoon also at 400 cents),


Figure 3.33-"Capriccio" measures 6-10
an F in horn 3 ( 500 cents), a G ${ }^{\#}$ in horn 2 ( 786.3 cents), a B in horn 1 ( 1051.1 cents), and a D in the solo horn ( 140.5 cents). This chord has an equal-tempered minor second at the bottom, a just-intoned major third between horns 2 and 4, and a just-intoned tritone between horns 1 and 3. The soloist's D creates a just-intoned 13/8 (a minor sixth which is more than a quarter tone low) with horn 3 and a 13/11 (small minor third) with horn 1.

After this second ascent, the solo horn and strings continue imitating the opening material with a variation which begins in measure eleven (see Figure 3.34). The soloist's line—which at times is joined by the marimba-is embellished with grace notes and additional leaps; under this line the strings play more frequently than they did at the start of the movement, adding extra punctuation to the horn line. The light character and whimsical use of grace notes is a clear reflection of the title of this movement.

When the strummed pizzicato resumes in the strings on the downbeat of measure eleven, it emphasizes the return to this material. The chord heard on the downbeat contains four perfect fifths, and is followed by the soloist outlining more instances of interval class $5-\mathrm{F}$ to C and F to $\mathrm{B}^{\mathrm{b}}$. Measure twelve begins with another group of fifths in the strings, and through the rest of that measure the strings continue to play chords containing perfect fifths (the first and last chords also contain major seconds). When the strings finish this line in measure twelve, it is on an open fifth between $E^{b}$ and $B^{b}$. Above these chords, the soloist continues to embellish a relatively simple line with grace notes and leaps, and is joined for most of measure eleven by the marimba playing the equal-tempered equivalents of the horn's pitches-the differences are 2 cents, 31.2 cents, 13.7 cents, and 3.9 cents respectively.

In the thirteenth measure the soloist plays alone. It starts with a rapid ascent and this is followed by a generally descending passage which is continued in the next bar and joined again by the marimba, playing dyads where the soloist has grace notes. The pitch differences here are 13.7 cents (on the grace note), 3.9 cents, 31.2 cents, 0 cents (on the grace note-they sound a unison), 13.7 cents, and 2 cents. The strings play once in this measure, off the beat-with the notes still sounding in the marimba and the solo horn they create a collection of major seconds on $B^{b}, C$, and $D$.



Figure 3.34-"Capriccio" measures 11-18

Measure fifteen is where the soloist's line ends, but the figure is immediately picked up by the obbligato horns. While the texture thickens here, and changes drastically at the end of the measure, the imitation of the motivic material is still heard clearly. This measure goes by very quickly; the change from one line to three lines at the start and the change from horns to woodwinds in the middle are the aspects that are most readily heard-the harmonic content of this measure is difficult to perceive, as is the fact that horns 3 and 4 are working in hocket, recalling the third movement. Harmonically there are eleven instances of interval class 5 and two of interval class 2 in this measure. These fifths are heard in equal temperament, just intonation, and as intervals outside of both tuning systems; the seconds are also heard in equal temperament and as intervals outside of these systems, but are not played just-intoned in this bar.

To this point in "Capriccio," the two tuning systems are being merged together at an increasing rate. At the start of this movement, the horns carry the majority of the melodic and rhythmic interest and these lines are punctuated by the strings and bassett horns. Once the orchestra joins the moving line in measure six, Ligeti begins integrating the equal-tempered instruments into these melodic figures. The marimba doubling the soloist's line in equal temperament (and therefore creating the various tiny intervals mentioned above) is the first component of this integration, but by measure fifteen, many members of the orchestra pick up the line as the horns trailed off. In the next few measures, these two tuning systems will become even more integrated as the strings-which were strongly outside of the horn lines at the start of the movement-will also join the horn melody.

The sixteenth measure of "Capriccio" opens with two consecutive chords in the strings. The first contains only one perfect fifth and no major seconds, but the second has three perfect fifths and one major second. This chord is followed by the solo horn beginning another leap-based descent while a simplified version of the line, consisting mainly of major seconds, is played by the strings and the vibraphone. The last note of the string and vibraphone descent is joined by the trumpet at a unison and the trombone a perfect fifth below. The intervals created between this line and the horn line are: 2 cents and 496.1 cents against the F; 113.7 cents (on the grace note), 48.7 cents, and 600 cents against the E; 13.7 cents and
631.2 cents against the D; 0 cents (on the grace note), 203.9 cents, and 498 cents against the $\mathrm{B}^{b}$; on that same dyad 200 cents (on the grace note), 3.9 cents and 698 cents against the C; and 31.2 cents and 200 cents against the $A^{b}$. These last two intervals compared to the trombone's $C \#$ yield 668.8 cents and 900 cents. The horn plays its last two pitches and their grace notes alone.

Measure eighteen contains an augmentation of the last four notes of the solo horn melody from the second half of the previous bar (with their grace notes), played by horns 1 and 2 . Horn 1 plays the same partials for most of the bar, sounding a perfect fourth lower, while horn 2 plays the same contour as horn 1 using slightly lower partials, creating a changing relationship between the two lines (this is similar to the expanding and collapsing intervals of "Signale, Tanz, Choral," but on a much faster and more drastic scale). Beneath these lines, the orchestra is harmonizing this passage with perfect fifths. These fifths are presented in equal temperament on the second and third eighth notes of the measure, and in just intonation over the course of the full measure, with horn 3 playing a C on the downbeat and an F at the end of the bar. The rhythmic augmentation used in this measure begins a transition to the second section of movement six which lasts for seven measures.

The solo horn plays over an A fundamental in measure nineteen (see Figure 3.35)—this is the only time in "Capriccio" that the soloist uses any valves, and the last time it happens in the Hamburg Concerto. This creates a perfect fourth relationship between the soloist and horn 2, who play the same line for most of the measure-this is an imitation of a section of the opening movement, "Praeludium," where a similar ascending line is played at the interval of a perfect fifth (measures twelve through fourteen, see Figure 3.3). At the end of this measure, the soloist moves to the thirteenth partial $F \#$, while horn 2 plays a twelfth partial B, again similar to the same section of "Praeludium," where the horns moved to different partials at the end of their ascent. They are joined by the crotales, flutes, horns 1 and 3 , and violin 1 , which results in one true perfect fifth ( $C \#$ to $G \#$, the other three intervals resembling


Figure 3.35-"Capriccio" measures 19-24
perfect fifths will actually sound like minor sixths or tritones) ${ }^{62}$, and three distinct major seconds of varying sizes—from A to B is 233.7 cents, from B to C\# is 198 cents, and from C\# to $D^{\#}$ is 186.3 cents (the major second from $F \#$ to $G \#$ is 259.5 cents which is off by more than a quarter tone, and closer to a minor third). Much of this oddly dissonant chord resolves in measure twenty, resulting in a juxtaposition of an $\mathrm{F}^{7} / \mathrm{A}$ chord and an $\mathrm{F}^{\#}$ minor triad with both fifths being equal-tempered, both chords sharing the same A of 886.3 cents (resulting in a nearly just-intoned major third and a small minor third) and a justintoned minor seventh.

The second half of measure twenty has the low A replaced by an equal-tempered A in the bass and the bassoon, creating an equal-tempered perfect fifth with the E above it played by horn 4, which in turn sounds a just-intoned perfect fifth with the B played by horn 2; this B creates a 202 cent major second with the $A$. The soloist, still in $B^{b}$, plays a seventh partial $A^{b}$ which disrupts the otherwise stable harmony. At the start of the following measure, the soloist makes a rapid, stepwise ascent from this seventh partial to the thirteenth. This thirteenth partial G ( 640.5 cents) is met with a seventh partial $\mathrm{E}^{b}$ in horn 3 ( 268.8 cents), a tenth partial A in horn 1 ( 886.3 cents), and an eleventh partial $A \#$ in horn 2 (951.3 cents). These three obbligato horn notes are doubled by the strings, with the cello matching horn 3, the viola matching horn 1 and violin 2 matching horn 2 . This chord, somewhat reminiscent of the harmonies from "Spectra," has a small perfect fifth of 682.5 cents between horns 2 and 3 and a large major second of 245.8 cents between horn 1 and the solo horn.

In measure twenty-two, this chord moves briefly to a slightly more stable harmony before horns 2 and 3 play ascents similar to the soloist's ascent at the start of the previous bar, leading into a very dense harmony which interrupts these ascending lines in measures twenty-three and twenty-four-a $\boldsymbol{f f f}$ chord (see Figure 3.35), that consists of thirteen distinct pitches (the C in horn 1 is only 2 cents higher than the

[^34]equal-tempered C in the cello and trumpet, and in this dense harmony this difference will not be heard). This chord is spread throughout nearly the entire orchestra-only the bassett horns and one percussionist are not playing here-and is more than fully chromatic, containing ten pitches in or near equal temperament (omitting $C \#$ and $A$ ) with three pitches outside of that tuning system-an $A \#$ nearly a quarter tone low, an $A^{b}$ nearly a sixth tone low, and an $A^{b}$ nearly a sixth tone low.

The transition to the second section of "Capriccio" is complete at this point, having changed the character and harmonic content of the movement completely. Following this intense, two measure chord, the bassett horns enter with a $\boldsymbol{m p}$ open perfect fifth, and all other instruments except the bass drop out. The bass sustains its $B^{b} \boldsymbol{p p}$ creating a $G^{b}$ major triad, and in measure twenty-six this chord is joined by a $\boldsymbol{p} \boldsymbol{p}$ C major triad in the cello, the viola and violin 2. The juxtaposition of these two triads sets up the slower pacing of this part of the movement, as well as the new harmonic content. While the harmonies here still feature perfect fifths and major seconds, they are now sustained for longer periods of time and presented alone instead of as background to the more active lines.

From measure twenty-seven through measure twenty-nine, there is a thickening texture that mostly involves the brass. These three chords begin as a combination of equal temperament and just intonation, but by the end of these three measures the horns have moved to playing pitches that are in or near equal temperament. Through this section, each chord has no less than two perfect fifths and three major seconds (of varying sizes) and the horns and flutes play some of their pitches frullato, which will be a major component of the last movement.

Following the nearly equal-tempered chord in measure twenty-nine is a fully equal-tempered chord in measure thirty, played by the woodwinds and the strings (the trombone is also playing, but is doubling the bass and dying away in this bar). Like the previous chord, it contains four perfect fifths and two major seconds. It also involves the cello and violin 2 playing rapid arpeggiations of their pitch material, adding a murmuring effect in the background. In measure thirty-one, two distinct harmonies are use-the cello and violin 2 change their pitches first, continuing their murmuring through this measure;
once that chord has been sounded, the bass and woodwinds change their notes. Both of these harmonies contain three perfect fifths and two major seconds.

The following measure contains the beginning of the only use of Ligeti's lamento motif in the Hamburg Concerto. The lamento motif (a term Ligeti himself used) is found in many of Ligeti's works, and can be identified by "three or four of the following attributes...it is a three-phrase melody, the third phrase of longer duration...each phrase descends stepwise in semitones and whole tones, interspersed with upward leaps...notes of greater expressive significance (e.g. immediately after the upward leaps) are intensified harmonically...[and] different versions of the formula similarly adopt strict rhythmic taleae." ${ }^{63}$ This particular instance of the lamento motif is particularly unique, as it is the only place in Ligeti's oeuvre this figure is played outside of twelve tone equal temperament. ${ }^{64}$

Beginning in measure thirty-two, the lamento motif is heard in the solo horn, the trumpet, the trombone, and the glockenspiel (see Figure 3.36). The trumpet and the trombone begin playing a perfect fourth apart, with the solo horn a major second above the trombone, and a perfect fifth above the trumpet. At the end of the first phrase of the lamento motif, the solo horn and the trombone are in a unison a perfect fourth above the trumpet. The glockenspiel initially is doubling the solo horn and the trombone, but as the horn descends through its harmonic series, the glockenspiel cannot relate to its pitches, and ends up approximating them in the second bar, and playing a whole step lower in the third bar when the solo horn and the trombone are playing a unison. This results in four chords containing: a perfect fourth, a perfect fifth, and a major second; a perfect fourth, a perfect fifth, a major second, a small perfect fifth ( 651.3 cents) a small major second ( 151.3 cents), and a small minor second ( 48.7 cents); a perfect fourth, a perfect fifth, a minor second a small tritone ( 586.3 cents), a small minor second ( 86.3 cents), and a tiny interval of 13.7 cents; and finally a perfect fourth, a major third, and a minor second. In the second phrase of the lamento motif, these same four chords are repeated, and it ends with a B in the trombone, a

[^35]$B^{b}$ (equal-tempered) in the horn, an F in the trumpet, and the glockenspiel doubling the $B$ and $B^{b}$. This results in a $[0,5,6]$ trichord, reflecting again on the previous movement, "Spectra."


Figure 3.36-"Capriccio" measures 32-37: the lamento motif

Based on the criteria for an instance of the lamento motif, this instance is missing its third phrase, and nothing in the following several measures involves a half step or whole step descent (the next is seven measures later). However, this is still a clear usage of this figure, especially based on its placement in the movement-it begins shortly after the end of the first section, and is one of the last strong gestures in the piece. This movement, initially, was the last movement of the Hamburg Concerto, which means this motif is in an especially prominent place. There is one further aspect to strengthen Ligeti's use of the lamento motif here-it is foreshadowed by a long chromatic descent by the contrabass and, at times, the trombone beginning in measure twenty-four with the last chord of the first section of "Capriccio" (see Figure 3.37). This descent ends at the end of the first phrase of the lamento motif, and during those twelve measures it encompasses a tritone (from $B^{b}$ down to $E$ ).


Figure 3.37-"Capriccio" measures 23-34: bass line foreshadowing the lamento motif

The first phrase of the lamento motif begins in measure thirty-two while the viola and violin 1 joining the murmuring arpeggiations of the cello and violin 2. The bass, flute 2, and flute 1 are playing F, $A^{b}$, and $C$ respectively, and these notes are sustained through measure thirty-three, but the arpeggiating strings change chords each time the melodic line moves. ${ }^{65}$ These arpeggiations begin with all four strings sharing the same pitches ( $C, E^{b}$, and $A^{b}$ ), but at times these lines have contrasting pitches-at the end of the second measure, for example the cello and viola are arpeggiating a $B^{b}$ minor triad and the violins are arpeggiating a $B^{b}$ major triad. The third measure of the lamento motif, measure thirty-four, involves the bass and flutes moving to an A minor triad, while the strings arpeggiate a complex $\mathrm{Am}^{\mathrm{b}} \mathrm{7}^{\mathrm{b}} 911 / \mathrm{E}$ chord that resolves to a more stable and diatonic $\mathrm{Am}^{11} / \mathrm{E}$. Interestingly, as this first phrase of the lamento motif ends by descending half step (or small whole step in the horn), the resolution in the strings is by ascending half step.

[^36]For the second phrase of the lamento motif, the sustained notes are gone and the only sound aside from the motif itself is the arpeggiation in the strings. This arpeggiation now changes chords more frequently: measure thirty-five begins with a $\mathrm{Cm}^{7}$ chord and moves to a C major triad; measure thirty-six begins with a complicated harmony resembling a thirteenth chord which gradually turns into a minor triad; and measure thirty seven, the end of the lamento motif, involves four comparatively simple chords as the pitch activity of the murmuring strings becomes simplified. The final chord, on the last eighth note of the bar, takes place after the instruments playing the lamento motif (the solo horn, the trumpet, the trombone, and the glockenspiel) have stopped playing, and here the strings are joined by the bass drum and obbligato horns. The horns are playing two different minor thirds: a 7/6 between the $C$ and low $E^{b}$ in the $F$ horns and a $6 / 5$ between the low $G^{\#}$ and $B$ in the $E$ horns. Together, this chord resembles a $G^{\#}$ triad with both a major and a minor third, and a small perfect fifth of 682.5 cents. The horns sustain this chord into measure thirty-eight, and are joined by a slightly low D played by the solo horn an equal-tempered tritone above the $G \#$ in horn 4 . The strings cease their arpeggiations here as well and end on $\mathrm{C}, \mathrm{G}, \mathrm{B}$, and D, creating two perfect fifths and one major second.

Halfway through measure thirty-eight, the solo horn moves up to an F (an equal-tempered tritone above horn 2), and at the end of the bar the strings drop out and all four obbligato players stop their horns ${ }^{66}$-this is the only place in the Hamburg Concerto where the obbligato players employ stopped horn. The soloist continues to ascend, reaching a fourteenth partial $A^{b}$ in measure forty where the only other sound is the bass drum. This low $\mathrm{A}^{b}$ lasts into measure forty-one, where it is joined by both flutes and three strings, creating a harmony with three perfect fifths, a small fifth (from $C \#$ to $A^{b}$, 668.8 cents) and two major seconds. Shortly after these instruments enter the horn moves to an eleventh partial E, complicating the chord further.

[^37]Measure forty-three contains simply a perfect fifth (reflecting again on previous important moments such as measure seven of "Praeludium" or the end of "Spectra") between the solo horn's F and the $\mathrm{B}^{b}$ below it played by the oboe, while the bass drum continues its $\boldsymbol{p p}$ roll. Measures forty-four through forty-six contain the final chord of "Capriccio" (see Figure 3.38), which includes four justintoned pitches-an eleventh partial B played by the soloist over the F fundamental, an F \# harmonic played on the D string by the contrabass, a D harmonic played on the G string in the cello, and an $\mathrm{F} \#$ harmonic played on the G string in the viola-and one equal-tempered pitch, the A in the oboe.

This last chord contains two perfect fifths-a slightly small perfect fifth between the oboe and the cello ( 698 cents), a large perfect fifth between the $B$ in the solo horn and the $F \#$ in the viola ${ }^{67}$ and bass (these intervals are 737 cents and 735 cents, respectively, but as the viola and bass pitches are only 2 cents apart they will sound like they are the same pitch, and this will sound like one interval). There is also a very small major second ( 151.3 cents) between the oboe and the horn. When this chord dies out, the bass drum is left to do the same over the next four measures.

Throughout much of "Capriccio," Ligeti focuses on combining material which is light in character (like that of movement two) with simple harmonic lines that fade in and out of the background. Towards the end, the lamento motif changes the mood of the movement, reflecting more on "Spectra" or even "Praeludium," an interesting combination since "Capriccio" also serves to resolve the tension between interval classes 1 and 2—a gradual shift from interval class 2 to interval class 1 was a long-term change that took place over the course of the opening five movements. Within "Capriccio," these two characters (the lighter opening material and the lamento motif) are presented as a dramatic contrast, much like the material in movements two, three, and four where the titles even suggest there will be strong

[^38]formal divisions. In "Capriccio," this is not hinted at in the title, but the appearance of the lamento motif is a strong signal that a new section of the piece is beginning, and indeed that proves to be the case.


Figure 3.38-"Capriccio," measures 44-50 with cents values

Initially this was the last movement of the Hamburg Concerto-"Hymnus" was not written until four years later. Therefore, it makes sense to take a brief look back at this point to see how "Capriccio" would function as an ending, and as it turns out, this ending would make a lot of sense. "Spectra," and to a lesser extent "Solo, Intermezzo, Mixtur, Kanon" before it, broke down the emphasis on interval class 2, and shifted the focus to interval class 1, concluding a process which began early in the "Signale, Tanz, Choral." The ending section is similar to some of the content in "Spectra" and "Praeludium," while the opening is strongly related to "Signale, Tanz, Choral." Both formally (through the use of previous
material, both overtly and subtly, and the resolution of that material) and harmonically (as the progression from interval classes 2 and 5 to interval classes 1 and 5 is reversed and resolved) Ligeti constructed a movement which ties in well with previous movements and returns, eventually, to the pitch language which began the Hamburg Concerto.

### 3.8 VII. "HYMNUS"

"Hymnus," written in 2002, was added to the Hamburg Concerto after the initial six movements had been premiered and published. Only a small subset of the orchestra is used in this movement-the four obbligato horns (horns 1 and 3 in F, horns 2 and 3 in E), one percussionist playing a bowed cymbal, and all five strings-and all of these instruments play from beginning to end.

There are two basic types of material in this movement-one is static and unchanging, and the other is a slow moving line. The static material is played by the percussionist, violins 1 and 2 , the viola, and the cello. This line, unchanging for the entire fifteen measures, is similar to some of the material of "Spectra," due to the interesting harmonic content. Unlike much of "Spectra," however, this line has a significant amount of indeterminacy. While the viola and the cello each sustain one pitch the entire time (a major second-the viola plays a C , the cello plays the D above) both violins are instructed to bow, tremolo, between the bridge and the tailpiece on any string. This results in high tones which have unpredictable pitches, and also may cause the open strings on the other side of the bridge to vibrate quietly, adding soft tones of G, D, A, or E. Lastly, the bowed cymbal (also found, briefly, in measures six and seven of "Spectra") adds another indeterminate element to this passage, as the pitch of the cymbal is also unknown; the resulting pitch not only varies based on the size and weight of the cymbal, but also the bow pressure and angle. Typically a bowed cymbal will create a very dense spectrum of overtones as well. Because the cymbal is to be played $\boldsymbol{p} \boldsymbol{p}$ throughout this movement the density of the overtones will
be limited somewhat and the resulting pitch may be clearer. The result is still the same, however, one or more indeterminate pitches will sound.

In contrast to these variable elements and the sustained major second, the obbligato horns and contrabass play a slow moving line. This line features all four obbligato horns playing muted (again, using a straight mute, not hand muting) and frullato throughout, while the bass, playing pizzicato, doubles either horn 3 or horn 4 for the entire movement. ${ }^{68}$ This use of frullato was foreshadowed somewhat by its presence in "Capriccio."

In addition to its relationships with movements five and six, "Hymnus" shares many features with movement two. The most obvious relationship is that "Hymnus," in its entirely, is an imitation of the entire "Choral" section of movement two-both of these are primarily focused on the obbligato horns playing a series of chords. These chords, much like the chords in "Choral," contain a series of equaltempered melodies and harmonies interwoven into the material from the harmonic series. In addition, elements of "Tanz" can be found as well-many times the horns move in a parallel motion, creating expanding or collapsing intervals between their lines. At times all four horns move in parallel motion, while at other times only subsets of the horn section have this relationship.

Much like in "Choral," "Hymnus" can be broken into a series of harmonies which loosely resemble equal-tempered chords. Unlike "Choral," however, the phrasing is not made clear in this movement. Ligeti ensures this both by omitting rests and by writing a series of slurs which overlap the slurs of other horn lines. This creates a sense of constant motion towards the end of the movement (see Figure 3.39). ${ }^{69}$

[^39]

Figure 3.39-"Hymnus": horns (with partial numbers) and contrabass

The metric pattern of "Hymnus" is simple-one measure of two plus three followed by one measure of three plus two-and repeats from the first measure until the end without changing. This movement begins with the bass exactly doubling horn 4 one octave lower, and this relationship continues for the first seven and one-half measures. Through the first seven measures of "Hymnus," the horn lines have varying contours, but unlike "Choral," the lines are generally smooth-the largest leap used is only 782.5 cents (a small augmented fifth in horn 1 between measures four and five). The harmonies in the section, again like "Choral" (and to a lesser extent "Spectra"), are generally tertian harmonies shaded by the tuning systems. The difference the tuning makes is illuminated more here than it was in "Choral" because the harmonies more frequently involve higher partials. For example, measure one begins with a $\mathrm{D}^{\sigma_{7}} / \mathrm{A}^{b}$ chord which features a just-intoned tritone and perfect fifth, but the second chord in measure one is an $\mathrm{Em}^{7}$ chord with a low minor seventh. While the three pitches that form the triad are relatively close to equal temperament (the fifth again is just-intoned), the seventh is a thirteenth partial D which sounds 140.5 cents. ${ }^{70}$

Measure two also begins with a more stable harmony than it ends with-the opening $\mathrm{B}^{7}$ chord has again a just-intoned perfect fifth and tritone, but the $\mathrm{E}^{7} / \mathrm{D}$ chord at the end of the measure has a fifth of $22 / 15$, which is more than a third tone low (its tritone, however, is just-intoned). Measure three uses the same two chords as measure one with different voicings. This relationship-where the second chord of the measure is less stable than the first-will continue through the end of measure seven.

In measure four, the horns begin with a somewhat unstable $\mathrm{D}^{7}$ chord (the tuning results in a large third and a large fifth, but the tritone between the third and seventh is nearly equal-tempered) that is

[^40]followed by a less stable $E^{\mathrm{M}_{7}}$ chord with a fifth that is nearly a quarter tone low. ${ }^{71}$ Measure five also begins with a somewhat unstable chord—a $C^{\varnothing 7}$ chord which uses minor thirds of 7/6 and 11/9 (nearly a quarter tone between the major and minor thirds at 347.4 cents), but like the previous bar this is still more stable than the following harmony-a triad which sounds neither major nor minor (the perfect fifth is slightly large at 713.7 cents, the third of the chord is 354.2 cents above the root and 359.5 cents below the fifth), with a seventh that does not help clarify the harmonic content (it lies between an equal-tempered and a just-intoned minor seventh at 986.3 cents, and creates a large, nearly just-intoned tritone of 645.8 cents with the third of the chord, but avoids a dominant sound.

Measure six begins with another return to the first chord $\left(D^{\phi_{7}} / A^{b}\right)$, again in a different voicing, but this time it moves to a different chord which involves two just-intoned major thirds of $5 / 4$ separated by a nearly equal-tempered tritone. ${ }^{72}$ Measure seven begins with a $C^{7} / B^{b}$ chord which has a generally equal-tempered triad and a seventh that is lower than the just-intoned minor seventh by approximately a sixth tone. This is followed by a strange and unstable chord which actually lies between a diminished seventh and a half-diminished seventh, as the seventh of this chord is a $C \#$ played as a thirteenth partial in horn 2. This thirteenth partial seventh creates the following intervals: a minor seventh of 971.7 cents (nearly a just-intoned minor seventh of 7/4); a tritone of 636.6 cents (a just-intoned 13/9); and a third of 354.2 cents, approximating a quarter tone. In other words, this $C \#$ creates the seventh required for a halfdiminished chord, the diminished fifth required for a fully diminished chord, and a third which, at nearly a quarter tone between a major third and a minor third, does not provide any clarification about which type of harmony is being used here.

Measure eight begins with a $\mathrm{D}^{\alpha_{7}}$ chord (another revoicing of the chord which began measures one, three, and six). On the fourth beat of measure eight, the bass begins to double horn 3 instead of horn

[^41]4, and does so at the unison instead of the octave. This beat also marks the approximate middle of this movement, and is the point at which the four horns begin their final ascent to their sixteenth partials. Entering this fourth beat, the horns play a $\mathrm{D}^{7} / \mathrm{C}$ chord which is about as stable as the previous chord-this is a change from the opening seven bars where the second chord was always noticeably less stable. Between the two chords of measure eight, horns 1,2 , and 3 descend in parallel by one partial expanding the intervals between them. Horn 4, however, sustains its pitch; this is the first time in "Hymnus" a horn pitch has been sustained between two chords.

Measure nine begins the process of collapsing this $\mathrm{D}^{7} / \mathrm{C}$ chord into just an equal-tempered minor second. From this chord through the first chord of measure eleven, the chords are all relatively stable, especially in comparison to the first half of the movement. The first chord in measure nine also contains the first exact equal-tempered harmonic interval-the major seventh between horns 3 and 4. Measure ten actually begins with a suspension in horn 2-its thirteenth partial C\# drastically complicates the first chord of the measure, but fits well into the second chord where it would function as the seventh (only slightly larger than a just-intoned 7/4).

Beginning in measure eleven, these harmonic relationships become increasingly complex as the horns approach the minor second at the end of "Hymnus"; by the first chord of measure thirteen, all four horns are playing within the space of a small major third. At the end of measure eleven, the horns and the violins (still bowing tremolo behind the bridge) begin to crescendo. (The bass, the bowed cymbal, and the string harmonics played by the viola and the cello are instructed to maintain their dynamic level.)

The second chord of measure twelve involves the second equal-tempered interval of the movement-a minor second between horn 1 and horn 4, both playing their thirteenth partials. From here until the end of the movement, there will be at least one equal-tempered interval between the horns at all times-a significant change from the first two-thirds of the piece in which there was a total of just one equal-tempered interval. The chord on this beat contains a small major second of 189.2 cents, an equaltempered minor second, and a small minor third of 259.5 cents.

Measure thirteen begins with the bass doubling horn 4 at the unison, and (as mentioned above) all four horns playing within the space of a small major third. This chord contains two equal-tempered minor seconds, the lowest notes of these are separated by a just-intoned 7/6, as are the highest notes of these seconds-the interval between these pairs of seconds is 166.9 cents. All four horns ascend one partial into the two following chords. This maintains the equal-tempered minor seconds between horns 1 and 2 and horns 3 and 4, and reduces the interval between these pairs to 147.7 cents at the end of measure thirteen (here the horns of the same key are a just-intoned 15/13 apart at 247.7 cents), and 131.2 cents to start measure fourteen (here the horns of the same key are a just-intoned $8 / 7$ apart). At this point all horns are playing $\boldsymbol{f f}$ and are continuing their crescendos, and the violins are now marked crescendo molto (possibile). The interval between these equal-tempered minor seconds further decreases at the end of measure fourteen; horns 1 and 2 sustain their sixteenth partial F and E , while horns 3 and 4 ascend to their fifteenth partial, narrowing the interval between horn 2 and 3 to just 11.7 cents as the F horns and E horns are a just-intoned 16/15 apart.

In measure fifteen horn 4 joins horn 2 on the sixteenth partial but horn 3 descends to its fourteenth partial, now creating an $8 / 7$ with horn 1 and an interval of 131.2 cents with both E horns. The bass is still doubling horn 4 in this measure, making this measure the only point in "Hymnus" where the bass is not playing the lowest pitch (the bass, like horns 2 and 4, is 131.2 cents above horn 3 ). This is necessary, however, to allow the bass to continue its ascent; the last time the contrabass played a descending interval was in measure eight-the same point at which the horns began their ascent. The bass does switch on the final chord to doubling horn 3 at the unison, allowing it to play one final ascending interval as both the bass and horn 3 move to the F being sustained by horn 1 . This again has the bass playing a pitch which is not the lowest, it is 100 cents above horns 2 and 4. All four horns are now sounding their sixteenth partials, and have moved their just-intoned material into equal temperament.

Harmonically, "Hymnus" begins by alternating stable and unstable harmonies, and this carries on for the first half of the movement. From measure eight until the end, however, there is a gradual shift
from very stable harmonies to extremely unstable harmonies built around minor seconds. Aside from the harmony, however, there is a second aspect of the chords of "Hymnus" which merits discussion-the equal-tempered melodic lines that weave their way through the harmonic progression (see Figure 3.40). Unlike "Choral," all four horns are playing at the same dynamic in this movement, which makes these internal melodies somewhat easier to hear. To counter that, Ligeti has written even shorter equaltempered lines-only two of them last longer than three chords. Additionally, due to the tuning scheme, the melodic lines consist solely of minor seconds, major sevenths, perfect octaves, and minor ninths. When the melodies are based on these larger intervals the lines become even more difficult to hear. Early in "Hymnus" these lines overlap one another constantly, but they are later presented only one at a time.

Figure 3.40 shows the equal-tempered melodic lines (there are thirty-two equal-tempered melodic intervals, including octaves) as well as instances of unison between two horns (there are four). This second feature, possible only because pairs of horns share the same fundamental, is shown via lines of a lighter shade, and is included in this diagram because it serves to elongate some of the melodic fragments, and in one instance is used in the middle of what might be considered an equal-tempered melodic line. These melodic lines, as mentioned above, are shorter and often built around large leaps-for example, the first melodic line which begins in measure two sounds A, G\#, and A, but this last interval is a descending major seventh. The next line is at the start of measure three, and uses two major ninths to create the line C, B, and C. Other short melodic lines include: A, G\#, and A (measures five and six); E, F, F, and E with the F passed between horn 1 and horn 3 (measures five through seven); and C, B, and C (measures six and seven). The two longest lines happen concurrently: starting in measure seven is $A, G^{\#}, A, G \#, A$ and $G^{\#}$ using all major sevenths; and starting one beat later in measure eight is $\mathrm{C}, \mathrm{C}, \mathrm{C}, \mathrm{B}$, and C , using octaves and major sevenths.


Figure 3.40—Melodic equal-tempered intervals within "Hymnus"

The first chord of measure nine is where the first harmonic equal-tempered interval is used in the horn chorale (a major seventh), which triggers the disintegration of the melodic equal-tempered intervals. After these two comparatively long lines, there is only one more equal-tempered melodic interval. Two chords later equal-tempered harmonic intervals return, and are used in every chord for the remainder of the piece.

While "Hymnus" does resemble "Choral" in terms of its construction, there are also a number of similarities between these two segments in the melodic content (see Figures 3.41-3.44), which gradually dissolve over the course of "Hymnus." The tuning of horns 3 and 4 differs between the two movements, which allows Ligeti to reharmonize the earlier melody. Because of how exposed the melodic material was in "Choral," this technique works well-the new version of that line is still recognizable despite the changes of harmony and tone color.

Within the first phrase (see Figure 3.41), the melodic line of the cantus firmus in "Choral" has a nearly identical contour to the horn 1 line (the highest pitched line at this point) in "Hymnus" over the same number of chords. The melodic line in "Choral" moves by intervals of an ascending 11/10 (165 cents), an ascending 12/11 ( 150.6 cents), an ascending 13/12 (138.6 cents), a descending 13/12 (138.6 cents), a descending $12 / 11$ ( 150.6 cents), and a descending $11 / 9$ ( 347.4 cents). In "Hymnus," the comparable melodic line moves by intervals of an ascending 13/12 (138.6 cents), an ascending 14/13 ( 128.3 cents), an ascending $15 / 14$ ( 119.4 cents), an ascending 16/15 (111.7 cents), a descending 16/13 ( 359.5 cents) and a descending 13/12 ( 138.6 cents). These lines are similar in that they both begin with intervals approximating the quarter tone between a major second and a minor second, and they both decrease these intervals by moving stepwise towards the apex of the phrase.

The first major difference is found between the fourth and fifth pitches where the "Choral" melody descends and the "Hymnus" melody ascends. This can be understood as the fourth pitch resolving to the fifth. Even without a harmonic context, the fifteenth partial in the harmonic series has a tendency to pull upwards towards the sixteenth, and the thirteenth partial has a tendency to descend to the twelfth—both of these tendencies are followed in this section of the melodic line. The other major


Figure 3.41-"Signale, Tanz, Choral" measures 16-17 and "Hymnus" measures 1-4: a comparison of the first phrase of "Choral" to the corresponding measures in "Hymnus"
difference is found within the last two intervals. In "Choral" the last three notes of the first melodic line involve first an interval which is nearly a quarter tone between a major second and a minor second and second an interval which is nearly a quarter tone between a major third and a minor third. These same two intervals are used in the same location of "Hymnus," but their order is reversed. Again this is to create a resolution-in "Choral" the line descends to an eleventh partial and leaps down to a ninth which
is more stable. In "Hymnus" the leap happens first, and arrives on the thirteenth partial which is followed by a descent to the more stable twelfth partial.

Harmonically there are few similarities, but two of these are found at the very start of "Hymnus." The first two chords in "Choral" and "Hymnus" both involve the root ascending by the exact same major second, an $8 / 7$ from a seventh partial up to an eighth partial. The second similarity is within the second chord itself-both sections of the piece use a chord with a generally minor sonority in which the melodic note is not a perfect fit.

The second phrases of "Choral" and "Hymnus" (see Figure 3.42) are also fairly similar, though slightly less so. The melodic line in "Choral" (generally the horn 2 line, marked cantus firmus, but the horn 1 and horn 3 lines stick out in this phrase when they are playing higher notes) contains a descending 11/10 ( 165 cents), a descending $5 / 4$ ( 386.3 cents), an ascending $9 / 8$ ( 203.9 cents), an ascending small minor third from horn 2 to horn 1 ( 282.4 cents), an ascending 11/10 (165 cents), and a descending small minor sixth from horn 1 to horn 3 ( 751.3 cents). In "Hymnus," the melodic line is also taken from the highest pitched horn; because they are all playing the same dynamics and frullato, they become nearly indistinguishable from one another. This line contains a descending small major third from horn 1 to horn 2 ( 389.2 cents), an ascending 13/11 (289.2 cents), an ascending 14/13 (128.3 cents), an ascending 15/14 (119.4 cents), an ascending 16/15 (111.7 cents), and a descending 16/13 (359.5 cents)—a similar melody to the first phrase of "Hymnus."

The similarities here are definitely more subtle, but the contour, with one exception, is the same. That exception, between notes two and three, involves a descent in "Choral" and an ascent in "Hymnus," after which both lines ascend for three notes and descend to end the phrase. These ascending lines have a similar quality despite the different intervals, and the descending intervals which end the phrases are nearly complementary-both are approximately a quarter tone away from being part of interval class 4 . Harmonically, this second phrase of "Choral" begins with an $E^{b_{7}}$ chord, and at this point in "Hymnus" there is an $\mathrm{E}^{\mathrm{M}_{7}}$ chord. Also, the penultimate chord of this section is similar-in "Choral" a dominant
chord is used, while in "Hymnus" it is a major triad with a ninth. While they have a different sound, the chords both are grounded with a major triad.


Figure 3.42-"Signale, Tanz, Choral" measures 18-20 and "Hymnus" measures 4-7: a comparison of the second phrase of "Choral" to the corresponding measures in "Hymnus"

While the third phrase of "Choral" has only three chords, the corresponding segment of "Hymnus" has four (see Figure 3.43). This added chord serves an important function-it allows the longer note values of "Hymnus" to correspond to the long notes in "Choral" which were used to signify the end of the phrase. The intervals in common are three ascending pitches-in "Choral" the intervals, all in horn 2, are 11/10 (165 cents) and 12/11 (150.6 cents), in "Hymnus" (starting on the second chord of measure eight) the intervals are 10/9 (182.4 cents) and 6/5 (315.6 cents). Both of these lines resolve to a twelfth partial, and when the first chord of measure eight is included, the "Hymnus" melody and the "Choral" melody both begin their phrases on a partial ten (and the "Hymnus" line will begin with a 10/9 descent). This creates two very similar melodies, mainly (or entirely) ascending, and covering a total interval of a $6 / 5$. Due to the addition of one chord in "Hymnus," the harmonic relationships are absent in this phrase.

The fourth phrase is the last phrase that contains a strong relationship between "Hymnus" and "Choral." Tracing the melodic line in "Choral" shows that it begins in horn 2 for two pitches and the last two pitches are in horn 4. The intervals include an ascending 10/9 (182.4 cents), an ascending slightly large minor second (115.7 cents) and a descending 12/11 (150.6 cents). In "Hymnus," the highest moving pitches can all be found in horn 4 (horn 2 has higher pitches, but they are sustained), and the intervals here are an ascending 11/10 (165 cents), an ascending 12/11 (150.6 cents) and an ascending 13/12 (138.6 cents). These intervals are all similar in size, with a difference in contour at the end of the line. In fact, since all of the horns are playing stepwise ascents here, a similar line can also be traced through horn 1 which plays an ascending 9/8 (203.9 cents), an ascending 10/9 (182.4 cents) and an ascending 11/10 (165 cents).

Harmonically, there are two final similarities between the fourth phrase of "Choral" and the corresponding measures of "Hymnus," both involving the penultimate chord. The roots of the second chords of these sections of "Choral" and of "Hymnus" both descend by a major third (written as a diminished fourth) of exactly 366.8 cents and the resulting chord in both movements features a minor sonority (Am ${ }^{9}$ in "Choral," $\mathrm{Bm}^{7}$ in "Hymnus").


Figure 3.43-"Signale, Tanz, Choral" measures 20-22 and "Hymnus" measures 8-11: a comparison of the third and fourth phrases of "Choral" to the corresponding measures in "Hymnus"

As shown in Figure 3.44, there is little similarity between the final phrase of "Choral" and the ending of "Hymnus." This lack of a relationship is emphasized by Ligeti with the extreme change of dynamics at the end of "Hymnus" which contrasts strongly with the static dynamic shape in "Choral." After adhering relatively closely to the shapes and ideas of "Choral," a different ending shape was necessary to provide closure to the piece. The end of "Choral" introduces the bassoon and later many
other members of the orchestra as the registral space expands to one of the widest ranges used in the Hamburg Concerto, and this arrival chord is played very softly. There is also a strong downward trend in the bassoon line which is hinted at in the horns. In "Hymnus," the opposite effect occurs-all moving lines ascend to a very tiny range of possible destinations, and the piece ends with all of the determinate pitches within the space of a minor seventh playing extremely loudly.

By adding "Hymnus" to the Hamburg Concerto, Ligeti created a much stronger sense of closure to this work. "Capriccio," while having similarities to several previous movements, was very strongly related to "Signale" and "Tanz" from the second movement. To a lesser degree, there was a relationship between "Spectra" and "Praeludium." The original ending did provide a sense of closure, but due to its relationship with "Signale, Tanz, Choral," the relationship was incomplete. "Hymnus" serves to finish this aspect of the large-scale form of the Hamburg Concerto by reflecting on the final segment of movement two-it allows the imitation of "Signale, Tanz, Choral" (begun in movement six) to be concluded.

By the end of "Hymnus," Ligeti has also reflected on the opening of "Praeludium"-the initial 9/8 major second has been inverted into the final interval created by the determinate pitches-an equaltempered minor seventh. The major second itself is also present; it is played between the viola and the cello throughout "Hymnus." The other important interval class, interval class 5, is also present-it can be heard between the highest horn note, F, and the lowest string note, C. The minor second between the horns which is the most prominent interval of the final chord of the Hamburg Concerto is a reminder of the conflict that existed between interval class 2 and interval class 1 as a major feature of this work. This, too, provides a more stable resolution than that of "Capriccio," as the prominence of interval class 1 in "Spectra" seemed to indicate it would remain an important component of the pitch language.


Figure 3.44-"Signale, Tanz, Choral" measures 23-27 and "Hymnus" measures 12-15: a comparison of the fifth phrase of "Choral" to the corresponding measures in "Hymnus"

### 3.9 CONCLUSION

The pitch content of the Hamburg Concerto is built, as shown above, on interval classes 2 and 5, with interval class 1 creating a conflict with interval class 2 through much of the piece. The prominence of these intervals is partially a result of the system Ligeti used to generate microtones-these interval classes are found throughout the harmonic series, and are among the most recognizable intervals when the series is heard in order.


Figure 3.45-The harmonic series of $F$ from the fundamental through the nineteenth partial with all justintoned perfect fifths and major seconds identified

Figure 3.45 shows the harmonic series up to the highest partial Ligeti uses in the Hamburg Concerto (the nineteenth partial is actually only simulated in "Spectra") and identifies the six just-intoned perfect fifths and ten just-intoned major seconds which can be found within these first nineteen pitches. Of these intervals, three of the fifths are distinct (partials two to three, six to nine, and ten to fifteen) and the other three are repeats of one of these intervals, while eight of the seconds are distinct with only two repetitions. These fifths are all exactly just-intoned perfect fifths with the same $3 / 2$ ratio. All eight of the distinct major seconds, however, are a slightly different size. The largest, 15/13, is 247.7 cents while the smallest, 12/11, is 150.6 cents. The collection of major seconds from partials seven through twelve create something resembling a whole tone scale. From the interval between the twelfth and thirteenth partial up through the interval between the thirty-fourth and thirty-fifth partials are twenty-three different minor seconds ranging from 138.6 cents to 50.2 cents, but due to their higher location in the harmonic series they do not have the prominence that the major seconds and perfect fifths share. The fact that these
intervals are less prominent in the harmonic series might explain, to a small extent, why these intervals become important so much later in the Hamburg Concerto than the intervals of interval classes 2 and 5once the stable and common intervals of the harmonic series have been established, Ligeti then moves on to include the next category.

Aside from their occurrence within the harmonic series, Ligeti also creates these major second and perfect fifth relationships with the tuning schemes of the horns, and of course within the orchestra itself. As shown over the course of this chapter, interval class 5 is generally a constant and can be found frequently throughout every movement. Interval class 2 is used frequently early in the piece, and in some movements is in contest with interval class 1 . Interval class 1 is the more prominent interval in only one movement, "Spectra," but afterwards it becomes a more frequent component of the pitch language, even as interval class 2 returns to prominence. Ligeti creates a great variety of sounds while focusing on the interactions of these elements.

One of the most critical applications of the interval class conflicts in the Hamburg Concerto is the microtonal system itself. The combination of the harmonic-based just intonation system and twelve tone equal temperament allows Ligeti the option of several different sized perfect fourths and fifths and an endless range of seconds and sevenths. Throughout the piece-and especially when the conflict between interval classes 1 and 2 is illuminated-Ligeti finds ways to use major or minor seconds that approximate a quarter tone in a way that makes sense to the ear and fits within his harmonic and melodic structures. These intervals occur in small numbers within the harmonic series but are readily available when these harmonics are sounded against equal-tempered pitches or harmonics from different fundamentals.

The various sizes of minor seconds are combined with various perfect fourths for much of "Spectra" as components of the $[0,1,6]$ or $[0,5,6]$ trichords, creating a collection of tritones as a result. These tritones are another important interval throughout this work, but are especially illuminated in the fifth movement where they actually help to stabilize the minor seconds and perfect fourths. The interval of a tritone has a very wide range of possibilities because of how narrow the boundaries of the perfect fourth and perfect fifth are, and this allows tritones to function in many different senses throughout the

Hamburg Concerto. Outside of "Spectra," tritones are found in a number of other important locations including: the fourth measure of "Praeludium" when the horn whole tone tetrachord and the equaltempered whole tone tetrachord are juxtaposed-this is the first time the two tuning systems are heard together; the hocketing section of "Aria, Aksak, Hoketus" which includes tritones as common melodic intervals; the bass line foreshadowing the lamento motif in "Capriccio" which encompasses exactly an equal-tempered tritone; and the harmonic structure of "Hymnus" in which every chord in the first eight measures includes a tritone of some kind, as do more than half of the chords in the next four measures. The tritones frequently are used to illuminate other important intervals, and this is the case for all of the above instances. Sometimes these tritones help highlight perfect fifths, but more frequently tritones point to the minor and major seconds-for example, as they are removed from the harmonic language of "Hymnus," this points to the narrowing of the harmonic spectrum which results in the horns playing an equal-tempered minor second, and when the first tritone in the piece is introduced in "Praeludium," it arrives as the result of stacking major seconds.

The potential applications of the wide array of major seconds (several just-intoned major seconds of which Ligeti uses six, the equal-tempered major second, and major seconds that are created solely by the conflict between these systems) are the focus of the Hamburg Concerto literally from the first sound to the last, and the different gradations of these intervals provide new sounds and new colors for the entirety of this work. The perfect fifths have a more constant function; due to the similarity between the just-intoned and equal-tempered perfect fifths, they are used as a grounding sonority which runs throughout the piece functioning as a stable harmonic interval and often a point of rest or finality. Ligeti's usage of these important interval classes is definitely a major component of the pitch structure of the Hamburg Concerto, and one which is used to provide a sense of cohesion between the two tuning worlds.

These intervals are one of the many aspects mentioned in the introduction. Another which has featured prominently in this work is that of imitation. On a small scale, imitation has been seen in canons, but it has also been seen between larger phrases or sections of movements. Imitation has even
been used between movements, and in this way it is an important part of the overall structure of the concerto. For example, the entirety of "Hymnus" imitates "Choral" from movement two, and the interruption near the end of "Praeludium" is imitated near the end of "Spectra." These interruptions form another important feature of the Hamburg Concerto, as they appear in nearly every movement (only "Hymnus" is lacking a true interruption, but if it is considered a partner of "Capriccio" the beginning of "Hymnus" could function in a similar manner to previous interruptions). These interruptions often appear near the end of movements, or at places where one of the sections named in the title is switching to another, giving them an important formal role.

The balance between these tuning worlds is, of course, a major component of the work as well. The central movement of the piece, "Solo, Intermezzo, Mixtur, Kanon," is the most segregated-it begins with primarily just-intoned lines, and ends with a fully chromatic equal-tempered canon. The other six movements involve the juxtaposition of these tuning systems. In "Praeludium" the juxtaposition is fairly simple; both tuning systems are used together to achieve the same basic goals-the arrival of the open perfect fifths. The relationship is also reasonably simple in "Signale, Tanz, Choral," where the majority of the movement uses horns only. Here, the contrast between the tuning systems is highlighted in the form of the different fundamentals the horns are playing from. While each line must be just-intoned, the harmonic relationships can be equal-tempered, just-intoned, or completely separated from either. Of course, this was also the case in "Praeludium," but here, because the horns are exposed, these relationships are illuminated.

At the start of "Aria, Aksak, Hoketus," the relationship between the tuning systems is almost set aside. The strings are playing an accompaniment in equal temperament while the solo horn is playing a duet with the bongos. The tuning systems interact through this section, but the main interest of this opening is found within each system, not in the relationship between them. The rest of the movement also uses this type of relationship, but with slightly more interaction between the parts. Having had a fully integrated movement (with both tuning systems constantly working together), a partially integrated movement (in which the relationship between the tuning systems is present but the systems are not
presented against one another), and a slightly integrated movement (where both systems are present but essentially each is playing their own lines and figures), the complete separation of the tuning systems in the fourth movement is the only natural outcome of this process.
"Spectra" follows by completely reintegrating both tuning systems to the extent that each takes on some characteristics of the other (the equal-tempered instruments altering their pitches to simulate justintoned chords; the soloist changing its fundamentals through equal temperament; the horns creating trichords that are strongly associated with equal temperament; the equal-tempered instruments featuring tertian harmonies native to just intonation). "Capriccio" continues to have both systems integrated by having them essentially share the same material for much of the movement. The last movement, "Hymnus," keeps the tuning systems separated (in fact, equal temperament has all but disappeared) and clouds the pitch language with sounds of indeterminate pitch.

Compared to the process between the first and the fourth movement, these last three are much more static. After the complete separation featured in movement four, the tuning systems return to being fully integrated in "Spectra" and remain that way in "Capriccio." The change is that the relationship has diminished slightly (as the two tuning systems borrow less from each other in "Capriccio," although it does still happen), and it continues to diminish in "Hymnus" as the movement is almost entirely just-intoned-the equal-tempered minor second between the fundamentals of the horns is the only strong equal-tempered relationship left, and while initially this is difficult to perceive, by the end of the movement this interval is the most prominent sound.

Finally, there is the issue of the horns themselves. While the major interest of the piece takes place in the contrast between the tuning systems-either in terms of the juxtaposition of the two systems or the contrast between the harmonic series of differing (equal-tempered) fundamentals-the horns also shape much of the piece. Initially they are tuned in $F$ (horns 1 and 2 ), $E^{b}$ (horns 3 and 4) and $B^{b}$ (the solo horn), which hints at the major second and perfect fifth relationships that will become very important. In this opening movement, the horns remain open throughout. This allows Ligeti access to the just-intoned
pitches, but it also means he is saving the techniques of stopped and half stopped horn for later in the piece. The use of three fundamentals similarly means that Ligeti can wait until later in the piece to use more chromatic materials between the horns, which he does in the second movement when the horns are tuned to $\mathrm{F}, \mathrm{E}, \mathrm{E}^{\mathrm{b}}$, and D .

In the third movement, Ligeti further increases the pitch possibilities with the half stopped horn in the opening "Aria." Here, the soloist is playing in F, but adds three additional pitches by lowering the tone a half step. He adds additional pitches, and the sound of mutes, to the melodic line in the second section, "Aksak, Hoketus," as Ligeti creates lengthy hocketing lines which involve up to four different fundamentals, and therefore have a vast range of pitch possibilities. This is expanded further in the opening of movement four when the soloist begins employing the valves (as well as stopped horn).

For the remainder of the Hamburg Concerto, Ligeti does not add to the pitch possibilities of the horns, but he does change the way they interact in order to maximize the effects of this pitch language. In "Spectra," all four obbligato horns are playing E horns, but the soloist is constantly shifting the fundamental, resulting in several different relationships with these E horns, not to mention the rest of the orchestra. In "Capriccio," the horns play a sort of culmination of all of the previous horn ideas-there are hocketing passages, parallel passages, different fundamentals (and changing fundamentals by the soloist), stopped horn, and moments with the horns alone as well as moments where the horns interact with the orchestra. The horns also add the effect of frullato (which is not intended to alter the pitch, just the tone), and the soloist plays Ligeti's lamento motif in just intonation.

Ligeti does not include the horn soloist in "Hymnus" (just as "Solo, Intermezzo, Mixtur, Kanon" did not include the obbligato horns), but does make use of the frullato sound, and he requires all horns to play with mutes, as he did in the third movement. The final sound of the Hamburg Concerto is a loud, brassy, frullato minor second, high in the horn register, supported mainly by indeterminate pitches and soft harmonics. This contrasts very strongly with the opening which, using only horns, was a subtle exploration of different major seconds, and was suddenly joined by the orchestra playing only equal-
tempered major seconds. The combination of the two systems for the first time in this piece is a very striking moment, yet at the end of the piece the opposite happens as all four horns compress their intervals-be they just-intoned, equal-tempered, or outside of either system—into a forceful minor second.

The shape of the Hamburg Concerto as a whole is elaborated by the two devices mentioned earlier-imitation and interruption. Early in "Praeludium," the idea of imitation is present. The equaltempered instruments first enter imitating the just-intoned instruments, and the first arrival point (measure seven) is followed by an imitation of the opening measures. This imitation creates the expectation of a similar arrival, but the actual arrival point is much different, and is followed by a forceful interruption. These imitations and interruptions function as dramatic events throughout the work. Ligeti is constantly using imitation to set up expectations which may be met or subverted, often by an interruption. The interruptions themselves follow a similar course in the concerto, as some interruptions imitate those from earlier movements. Interestingly, this relationship is found in the tuning systems as well. Equal temperament and just intonation are used to imitate one another from the start of the piece, and often one system interrupts the other. This is true of some larger interruptions of the piece as well-in the first movement equal-temperament interrupts the horns, and in the fifth just-intonation interrupts the orchestra.

The intrigue of the tuning systems is grounded in the relationship between the horns and the orchestra-a relationship which is strengthened by the interplay of various components of interval classes 2 and 5. As the horns build their own language, increasing the possibilities of both pitch and color, the orchestra borrows an increasing amount (while always maintaining its primary identity as a twelve tone equal-tempered entity) of its pitch material from the horns. With the two tuning systems both at play through the majority of the piece, it is a fitting conclusion that the final movement includes a significant amount of indeterminate pitch material as well-this helps to completely obscure the discrepancies between the two systems. Throughout the Hamburg Concerto, the interactions of these tuning systems are actually guided by the pitch relationships they generate, and the material they borrow from one another.

## APPENDIX A

## ARROWS AND ERRORS

Throughout the published score of Ligeti's Hamburg Concerto there are several errors, most of which pertain to the arrows which are used to approximate the microtonal pitches played by the horns and other instruments. The first section of this appendix will show where arrows are missing or misused in instruments besides the French horns. The arrows in the French horn parts are frequently erroneously placed, but since it will not affect the resulting sound they will not be mentioned here. The second section of this appendix will address all additional errors in the score. These errors have been found by examining Ligeti's manuscripts, his correspondence with Schott Musik International, and George Benjamin's correspondence with Schott. Some of these errors have been mentioned in Chapter 3 when they were relevant to the analysis; those errors and all others will be addressed herein. Errors that were present in the first publication and have been fixed for this publication will not be discussed.


#### Abstract

ARROWS

Ligeti uses three distinct types of arrows in this piece: $\downarrow$ Indicates notes that are approximately " $15 \%$ lower."73 This includes the fifth, tenth, and fifteenth partials. † Indicates notes that are approximately " $30 \%$ lower." ${ }^{74}$ This includes the seventh and fourteenth partials. $\frac{1}{\forall}$ Indicates notes that are approximately " $50 \%$ lower." ${ }^{75}$ This includes the eleventh and thirteenth partials.

Additionally, several of the horn partials do not require arrows because they either lie within equal temperament or are less than 5 cents away from an equal-tempered pitch. This includes the first, second, third, fourth, sixth, eighth, ninth, twelfth, sixteenth, seventeenth, eighteenth, and nineteenth partials. Misuse of arrows in the horn parts is not a significant problem-while this might serve to confuse someone reading the score, the players themselves will approach their thirteenth partial, for example, the same way regardless of what type of arrow is present. When arrows are missing in a string instrument part where the note is reached via natural harmonic, this is similarly not an issue for the same reason. However, if arrows are missing or if the wrong type of arrow is used in any other instance where a microtonal inflection is needed, the resulting pitch will not correspond to what Ligeti intended.

The first major problem involving an arrow comes in measure sixteen of "Spectra." Violin 1 has the open-headed arrow in the score, suggesting the G should be played nearly a quarter tone lower. This arrow is incorrect-the arrow in Ligeti's manuscript is the closed-headed arrow, suggesting a G played approximately $30 \%$ lower, like a seventh partial. This arrow results in a G which fits into the harmony of

^[ ${ }^{73}$ Ligeti, Hamburgisches Konzert, 1. ${ }^{74}$ Ibid. ${ }^{75}$ Ibid. ]


the cello, the viola, and violin 2 as the seventh of the $\mathrm{A}^{11}$ chord. In context, this pitch makes more sense as the just-intoned and equal-tempered elements of the movement are clashing here, and this contrast is much stronger if the just-intoned elements have simple, consonant relationships which the simulation of a seventh partial would provide.

There are two issues involving arrows in "Capriccio." The first occurs in measure thirty-eight in horn 4. While most of the horn arrows will not be addressed, in this instance the arrow should be written on a stopped note. The A which begins at the end of this measure and lasts through measure thirty-nine should have an open down arrow, since it is a stopped fifth partial, and the resultant pitch is intended to be an A of 886.3 cents. In measure forty-four, the viola is playing an $\mathrm{F}^{\#}$ with no arrow. This F \# is reached via natural harmonic on the $G$ string, which will result in a pitch of 688.3 cents (a just-intoned $15 / 8$ above that G , the same as a horn sounding its fifteenth partial). In previous passages, Ligeti has used an open arrow when a horn sounded its fifteenth partial-the same should be done here.

Finally, there are several arrows missing in the contrabass part of "Hymnus." As mentioned in the analysis, the contrabass is always doubling either horn 3 or horn 4 at the octave or at the unison. Therefore, the bass part needs to have arrows on fourteen of its thirty pitches; arrows only appear on two pitches in the score. The corrected bass part can be seen in Figure A.1.


Figure A.1-"Hymnus" Contrabass part with appropriate arrows

## ERRORS

Aside from the many arrows which are incorrectly (and inconsistently) used throughout the score, there are several other errors that can be found. Those errors which directly impacted the analysis in Chapter 3, they were discussed as part of the analysis. A full list, including any previously mentioned errors, is provided here.

There are two errors in "Praeludium":

- Measure six, solo horn: The 10 written above the C on beat two should be written above the $D$ on beat three to show that the $D$ is the tenth partial of the $B^{b}$ horn.
- Measure fifteen (rehearsal C), timpani: On the downbeat, the dyad should only be a single note--C\#. This was written in the manuscript as a C\# with a marcato marking ( $*$ ) above it and a staccatissimo marking ( 4 ) below it. This staccatissimo marking was mistranscribed as a G because it crossed the bottom line of the staff. The timpani C\# should match exactly the trombone B in terms of articulations. There should be no G.

Additionally, George Benjamin added a note regarding the contrabass part in measures nineteen and twenty (the last chord). He felt the bass part should have been one octave higher so that its $\mathrm{G}^{\#}$ would be in the same octave as the solo horn's $A^{b}$. This would copy the relationship between the four higher strings and the four obbligato horns, and create a smoother transition. The harmonic the contrabass is playing would result in a $G^{\#}$ which is 17.5 cents lower than the solo horn's $A^{b}$. The higher $G^{\#}$ is theoretically attainable via natural harmonic, although quite difficult to produce. Replacing it with an artificial harmonic could result in the bass and the solo horn sounding exactly the same pitch.

There is one error in "Signale, Tanz, Choral":

- Measure eighteen, horn 1: The half note F should have a tenuto marking.

There is one error in "Aria, Aksak, Hoketus":

- Measure eleven, solo horn: There should be a 15 written above the low $E^{\natural}$ in the middle of beat two. While this does not actually change the resultant sound, Ligeti marked this change in his correspondence.

There are six errors in "Solo, Intermezzo, Mixtur, Kanon":

- Measure eight, solo horn: The symbol to play the horn open $\left({ }^{\circ}\right)$ is written on the downbeat of this measure, but is unnecessary.
- Measure ten, solo horn: The symbol to play the horn open $\left({ }^{\circ}\right)$ should be on the downbeat of this measure.
- Measure seventeen, solo horn: The valve indication should read "in Re" so that this measure results in sounding the exact pitches of measure fifteen with the horn stopped. As it is marked currently ("in Mi") approximations will be possible, but in order to get exactly the same notes (which happens every other time stopped horn is used in "Solo") the soloist should play over the D fundamental.
- Measure nineteen, solo horn: The symbol to play the horn open ( ${ }^{\circ}$ ) should be on the downbeat of this measure.
- Measure ninety-five, violin 2: The third note of this measure should be an E, not an F as written.
- Measure 132, flute 1: The first note after the sixteenth rest should have a tenuto marking. This was noted by George Benjamin.

There are three errors in "Spectra" in addition to the one arrow concern addressed above:

- Measure six, horn 3: An 11 should be written above the A\# on the third beat, not a 10 .
- Measure seven, horn 3: A 10 should be written above the G\# on the downbeat.
- Measures fifteen through nineteen (rehearsal EE, all of page twenty-eight): The flute 1 and piccolo staves are in the wrong order. Here the piccolo is written above flute 1 , but in all other instances flute 1 is written above the piccolo.

There are two errors in Capriccio in addition to the two arrow concerns addressed above:

- Measure fifteen (rehearsal HH), horn 1: The C that is the last sixteenth note of beat one should not be accented. This was noted by George Benjamin.
- Measure twenty-four, cello: There should be no ties into the following measure (in which the cello does not play).

There are nine errors in "Hymnus" in addition to the contrabass arrow concerns addressed above:

- Measure three, horn 3: There should be a breath mark at the end of the measure.
- Measure four, horn 1: The thirteenth partial $D$ that begins on beat four should actually be an eleventh partial B.
- Measure seven, horn 3: There should be a breath mark at the end of the measure.
- Measure eleven, horn 3: The breath mark that is at the beginning of measure twelve should actually be at the end of measure eleven.
- Measure twelve, horns 1 and 3: The breath mark that is at the beginning of measure thirteen should actually be at the end of measure twelve.
- Measures twelve through fourteen, horns 1, 2, and 3: The dynamics here should all match exactly the dynamics of horn 4 . The spacing was not preserved in these other three parts. The $\boldsymbol{m} \boldsymbol{p}$ should occur before the downbeat of measure thirteen and the $\boldsymbol{f}$ should occur before the downbeat of measure fourteen.
- Measure thirteen, horn 2: There should be a breath mark after beat two.
- Measure thirteen, horns 1, 2, and 3: The breath mark that is at the beginning of measure fourteen should actually be at the end of measure thirteen.
- Measure fourteen, horn 3: The breath mark that is at the beginning of measure fifteen should actually be at the end of measure fourteen.


## APPENDIX B

## THE EIGHT HARMONIC SERIES USED IN THE HAMBURG CONCERTO

There are eight distinct harmonic series used in the Hamburg Concerto. They are presented here (see Figure B.1) from the fundamental up to the sixteenth partial or higher when additional partials are used in this piece.


Figure B.1—The eight harmonic series used in the Hamburg Concerto with partial numbers and cents values

## APPENDIX C

## INSTANCES OF UNISON INVOLVING THE HORNS

As mentioned in Chapter 2, finding unisons between the horns and the orchestra was one of the important factors involved in determining the tuning scheme of the horns. The unisons created between the horns and the orchestra that are a result of equal-tempered tuning of the horn fundamentals are mentioned here. Near unisons (intervals of 5 cents or less), and other important effects of this tuning are also mentioned. There are a total of 129 unisons or other strong relationships in the Hamburg Concerto with the equaltempered fundamental tuning scheme. Unisons created when a member of the orchestra alters its pitch to match the horns will not be considered here-the player could, theoretically, match the horn pitch regardless of the tuning system used. Also, unisons between two horns of the same fundamental will not be considered-they would sound a unison regardless of the tuning system. Near unisons or other important relationships between the horns, however, will be mentioned.

There are thirteen instances of unison or a different strong relationship in "Praeludium," wherein horns 1 and 2 are in $F$, horns 3 and 4 are in $E^{b}$, and the solo horn uses only the $B^{b}$ side of the double horn:

- Measure four, beat two: The F in horn 1 is the same as the F in flute 1.
- Measure five, beat four: The C in horn 2 is 2 cents higher than the C in the solo horn.
- Measure seven, beat two: The F in the solo horn is 2 cents higher than the F in flute 1 , the bassoon, violin 1, and the cello.
- Measure eight, beat three: The $B^{b}$ in the solo horn is the same as the $B^{b}$ in the trombone and the contrabass.
- Measure ten, beat three: The C in the solo horn at the end of beat three is 2 cents higher than the C in the oboe and the vibraphone.
- Measures twelve through fourteen: The solo horn and horn 3 play an extended passage at exactly an equal-tempered perfect fifth.
- Measure thirteen, beat two: The F in horn 3 is 3.9 cents higher than the F in flute 2 .
- Measure thirteen, beat two: The C in the solo horn is 3.9 cents higher than the C in the oboe.
- Measure fourteen, beat two: The F in the solo horn is 2 cents higher than the F in the cello
- Measure fifteen, beat two: The F in the solo horn is 2 cents higher than the F in the bassoon and the contrabass.
- Measure nineteen: The C in horn 1 is 2 cents higher than the C in violin 1.
- Measure nineteen: The F in horn 2 is the same as the F in violin 2.
- Measure nineteen: The $E^{b}$ in horn 2 is the same as the $E^{b}$ in the viola.

There are no direct instances of unison or a different strong relationship between the horns and the orchestra in "Signale, Tanz, Choral." However, as shown in Chapter 3, the equal-tempered relationships between the horns themselves-horn 1 and the solo horn are in F , horn 2 is in E , horn 3 is in $E^{b}$, and horn 4 is in D-are the main feature of this movement (for example, the equal-tempered displacement of the echoes in "Signale," or the chromatic cluster in measure eight which ascends via a just-intoned arpeggiation). These relationships strengthen the case for this tuning system in a different way. The equal-tempered moments that exist in "Signale, Tanz, Chorale" would not be possible if any other tuning scheme were used-the result would instead be a collection of intervals of various sizes, which would be a weaker contrast to the just-intoned passages.

There are forty-eight instances of unison or a different strong relationship in "Aria, Aksak, Hoketus," in which the solo horn and horn 1 are in F , horn 2 is in E , horn 3 is in $\mathrm{E}^{\mathrm{b}}$, and horn 4 is in D :

- Measure one: The C in the solo horn is 2 cents higher than the C in violin 2.
- Measure one: The F in the solo horn at the end of beat one is the same as the F in the viola at the start of beat one.
- Measure two, beat two: The F in the solo horn at the end of beat two is the same as the F in violin 2 at the start of beat three.
- Measure three, beat two: The F in the solo horn is the same as the F in the cello and the contrabass.
- Measure three, beat two: The $G$ in the solo horn in the middle of beat two is 3.9 cents higher than the G in violin 2 at the start of beat two.
- Measure five: The F in the solo horn in the middle of beat one is the same as the F in the viola at the start of beat one.
- Measure five, beat three: The G in the solo horn in the middle of beat three is 3.9 cents higher than the G in the viola at the start of beat three.
- Measure eight, beat two: The F in the solo horn in the middle of beat two is the same as the F in violin 1 at the start of beat two.
- Measure nine: The C in the solo horn is 2 cents higher than the C in violin 2.
- Measure ten, beat three: The F in the solo horn is the same as the F in violin 2.
- Measure eleven, beat three: The $\mathrm{G}^{b}$ in the solo horn at the end of beat three is 5 cents higher than the $F \#$ in the cello and the contrabass at the start of beat three.
- Measure sixteen: The D in horn 4 is the same as the D in flute 1 , bassett horn 1 , and the marimba.
- Measure nineteen: The F in horn 1 is the same as the $F$ in bassett horn 2 , the viola, and the cello.
- Measure twenty-three: The C in the solo horn in the middle of beat one is 2 cents higher than the C in the trombone (and the C in the cello and the contrabass at the start of beat one).
- Measure twenty-three: The G in the solo horn at the end of beat one is 3.9 cents higher than the $G$ in the trumpet.
- Measure twenty-four, beat three: The F in the solo horn at the end of beat one is the same as the F in bassett horn 2 and the marimba.
- Measure twenty-five: The G in the solo horn in the middle of beat one is 3.9 cents higher than the G in bassett horn 2 and the marimba.
- Measure twenty-five, beat three: The F in horn 1 at the end of beat three is the same as the F at the start of beat 1 in the trombone.
- Measure twenty-six: The C in horn 1 at the end of beat one is 2 cents higher than the C in the flutes and the marimba.
- Measure twenty-six, beat two: The $D$ in horn 4 is the same as the $D$ in the flutes and the marimba.
- Measure twenty-six, beat three: The C in the solo horn is 3 cents higher than the C in the flutes and the marimba.
- Measure twenty-seven, beat two: The F in horn 1 is the same as the F in flute 2, bassett horn 2 , violin 2 , and the cello.
- Measure twenty-seven, beat two: The C in the solo horn at the end of beat two is 2 cents higher than the C in flute 1 and bassett horn 1 (and the C in violin 1 and the viola at the start of beat two).
- Measure twenty-seven, beat three: The G in horn 1 is 3.9 cents higher than the $G$ in flute 2 , bassett horn 2, the trombone, violin 2, the cello, and the contrabass.
- Measure twenty-seven, beat three: The D in horn 4 is the same as the D in flute 1 , bassett horn 1 , violin 1 , and the viola.
- Measure twenty-eight: The F in the solo horn is the same as the F in flute 2, the oboe, bassett horn 2 , violin 2 , and the cello.
- Measure twenty-eight: The C in the solo horn in the middle of beat one is 2 cents higher than the C in flute 1 , bassett horn 1 , violin 1 , and the viola.
- Measure twenty-eight: The F in horn 1 in the middle of beat one is the same as the F in flute 2 , the oboe, bassett horn 2 , violin 2 , and the cello.
- Measure twenty-eight: The C in horn 1 in the middle of beat one is 2 cents higher than the C in flute 1 , bassett horn 1 , violin 1 , and the viola.
- Measure twenty-eight: The F in the solo horn in the middle of beat one is the same as the F in flute 2, the oboe, bassett horn 2, violin 2, and the cello.
- Measure twenty-eight: The F in horn 1 at the end of beat one is the same as the F in flute 2 , the oboe, bassett horn 2, violin 2, and the cello.
- Measure twenty-eight, beat two: The D in horn 4 is the same as the D in the flutes, bassett horn 1 , violin 1 , and the viola.
- Measure twenty-nine: The E in horn 2 is the same as the E in the trombone.
- Measure twenty-nine: The E at in horn 4 at the end of beat one is 3.9 cents higher than the E in the trombone.
- Measure twenty-nine: The G in horn 1 at the end of beat one is 3.9 cents higher than the G in bassett horn 1, the trumpet, and the contrabass (and the G in violin 2 at the start of beat one).
- Measure twenty-nine, beat two: The C in horn 1 is 2 cents higher than the C in flute 1 , the oboe, the bassoon, and the viola.
- Measure twenty-nine, beat two: The A in horn 4 at the end of beat two is 2 cents higher than the A in the trumpet (and the A in the violins at the start of beat two).
- Measure thirty: The F in horn 1 is the same as the F in flute 1, bassett horn 1, the trombone, violin 2, the viola, and the cello.
- Measure thirty: The C in horn 1 in the middle of beat one is two cents higher than the C in the oboe, bassett horn 2, and the bassoon (and the C in the cello at the start of beat one).
- Measure thirty: The A in horn 4 in the middle of beat one is 2 cents higher than the A in flute 2 and the trumpet (and the A at the start of beat one and beat two in violin 1 and the viola).
- Measure thirty: The F in the solo horn in the middle of beat one is the same as the F in flute 1 , bassett horn 1 , the trombone, violin 2 , the viola, and the cello.
- Measure thirty: The A in horn 4 at the end of beat one is 2 cents higher than the A in flute 2 and the trumpet (and the A at the start of beat one and beat two in violin 1 and the viola).
- Measure thirty, beat two: The $F \#$ in horn 2 is 3.9 cents higher than the $F \#$ in flute 1 , bassett horn 1, the trombone, violin 2, the viola, and the cello.
- Measure thirty, beat two: The A in horn 4 at the end of beat two is 2 cents higher than the A in flute 2 and the trumpet (and the A at the start of beat two in violin 1 and the viola).
- Measure thirty, beat three: The C in horn 1 at the end of beat three is 2 cents higher than the C in flute 2, bassett horn 2, and the trumpet (and the C in the viola at the start of beat three).
- Measure thirty, beat three: The F in horn 3 at the end of beat three is 2 cents higher than the F in the solo horn.
- Measure thirty-one: The G in horn 1 is 3.9 cents higher than the G in flute 1 , bassett horn 1 , the trombone, and the contrabass.
- Measure thirty-one: The D in horn 4 is the same as the D in the oboe and the bassoon.

There is only one instance of unison in "Solo, Intermezzo, Mixtur, Kanon," since the majority of the movement does not involve the horns. The relationship is a bit tenuous and hard to hear, but the last note of the opening horn solo (measure twenty-eight) is an A which is the same as the A played by both violins and the cello in measure thirty (measure twenty-nine contains only snare drum). However, the equal-tempered relationships created by the solo horn player using valves on the double horn (see Figure 2.4 and Appendix B) are important to the opening section, "Solo."

There are twenty instances of unison or a different strong relationship in "Spectra," wherein the four obbligato horns are in E, and the solo horn is playing a double horn and frequently switching fundamentals. Most of these instances are found near the end of the movement:

- Measure four, beat two: The $\mathrm{F}^{\#}$ in horn 2 is 3.9 cents higher than the F \# in flute 1.
- Measure four, beat two: The E in horn 4 is the same as the E in the bassoon.
- Measure twelve, beat two: The F\# in horn 4 in the middle of beat two is 3.9 cents higher than the F \# in the crotales and the contrabass.
- Measure thirteen, beat three: The B in horn 2 is 2 cents higher than the B in the contrabass at the start of beat four.
- Measure sixteen, beat three: The E in horn 1 in the middle of beat three is the same as the E in violin 2.
- Measure sixteen, beat three: The B in horn 2 in the middle of beat three is 2 cents higher than the B in the tubular bells.
- Measure sixteen, beat three: The $F \#$ in horn 1 at the end of beat three is 3.9 cents higher than the F \# in flute 1 .
- Measure sixteen, beat three: The E in horn 3 at the end of beat three is the same as the E in violin 2.
- Measure sixteen, beat three: The B in horn 4 at the end of beat three is 2 cents higher than the $B$ in the tubular bells.
- Measure sixteen, beat four: The E in horn 2 is the same as the E in violin 2.
- Measure sixteen, beat four: The $F \#$ in horn 3 is 3.9 cents higher than the $F^{\#}$ in flute 1 .
- Measure sixteen, beat four: The $F \#$ in horn 2 in the middle of beat four is 3.9 cents higher than the F \# in flute 1.
- Measure sixteen, beat four: The E in horn 4 in the middle of beat four is the same as the E in violin 2.
- Measure sixteen, beat four: The B in horn 1 in the middle of beat four is 2 cents higher than the B in the tubular bells.
- Measure sixteen, beat four: The $F \#$ in horn 4 in the middle of beat four is 3.9 cents higher than the $\mathrm{F}^{\#}$ in flute 1.
- Measure sixteen, beat four: The B in horn 3 at the end of beat four is 2 cents higher than the $B$ in the tubular bells.
- Measure seventeen: The B in horn 2 is 2 cents higher than the B in the tubular bells.
- Measure seventeen: The B in horn 4 in the middle of beat one is 2 cents higher than the $B$ in the tubular bells.
- Measure seventeen: The E in horn 1 at the end of beat one is the same as the E in violin 2.
- Measure seventeen, beat two: The E in horn 3 is the same as the E in violin 2 at the end of beat one.

There are thirty-five instances of unison or a different strong relationship in "Capriccio," where horns 1 and 3 are in F, horns 2 and 4 are in E, and the solo horn is playing a double horn playing mostly in F or $\mathrm{B}^{b}$, but uses an A fundamental for one passage as well:

- Measure two: The C in the solo horn in the middle of beat one is 2 cents higher than the C in bassett horn 1 and the viola.
- Measure two: The F in the solo horn in the middle of beat one is the same as the F in bassett horn 2, the cello, and the contrabass.
- Measure three, beat four: The G in horn 1 at the end of beat four is 3.9 cents higher than the G in the violins, the viola, and the cello at the start of measure four.
- Measure four: The C in horn 1 in the middle of the measure is 2 cents higher than the C in the viola (and the C in the viola, the cello, and the bass at the start of the measure).
- Measure four: The B in horn 2 in the middle of the measure is 2 cents higher than the $B$ in the viola (and the B in the violins, the viola, and the cello at the start of the measure).
- Measure four: The F in horn 1 in the middle of the measure is the same as the F in the cello one note previous.
- Measure seven: The $B^{b}$ in the solo horn is the same as the $B^{b}$ in the oboe and the contrabass.
- Measure nine: The F in the solo horn is the same as the F in the cello and the contrabass.
- Measure ten: The E in horn 4 near the end of the measure is the same as the E in the bassoon and the contrabass.
- Measure eleven: The F in the solo horn at the end of beat one is 2 cents higher than the F in the violins, the viola, and the cello.
- Measure twelve: The C in the solo horn near the start of the measure is 3.9 cents higher than the C in the viola, the cello, and the contrabass.
- Measure twelve: The F in the solo horn in the middle of the measure is the 2 cents higher than the F in the marimba.
- Measure twelve: The C in the solo horn at the end of the measure is 3.9 cents higher than the C in the marimba (and the C in the viola and the cello one note previous).
- Measure thirteen: The $B^{b}$ in the solo horn is the same as the $B^{b}$ in the violins, the viola, and the cello.
- Measure fourteen: The C in the solo horn is 3.9 cents higher than the C in the marimba and violin 2.
- Measure fourteen, beat two: The F in the solo horn at the end of beat two is 2 cents higher than the F in the marimba.
- Measure sixteen, beat two: The F in the solo horn is 2 cents higher than the F in the marimba, the violins, the viola, and the cello.
- Measure seventeen: The $B^{b}$ grace note in the solo horn in the middle of beat one is the same as the $\mathrm{B}^{b}$ in the marimba, violin 2 , and the cello.
- Measure seventeen: The C in the solo horn is 3.9 cents higher than the C in the marimba, violin 1, and the viola.
- Measure nineteen: The solo horn and horn 2 play an extended passage at exactly an equaltempered perfect fourth.
- Measure twenty, beat two: The E in horn 4 is the same as the E in the contrabass.
- Measure twenty-two, beat two: The F in horn 3 near the end of beat two is the same as the F in the bassoon, the cello, and the contrabass.
- Measures twenty-three and twenty-four: The C in horn 1 is 2 cents higher than the C in the trumpet and the cello.
- Measure twenty-seven: The C in horn 1 is 2 cents higher than the C in bassett horn 1 and the C in flute two in the following measure.
- Measure twenty-seven: The F in horn 3 is the same as the F in bassett horn 2.
- Measure twenty-nine: The G in horn 1 is 3.9 cents higher than the G in the violins and the viola.
- Measure twenty-nine: The C in horn 3 is 2 cents higher than the C in the viola and the cello.
- Measures thirty-two through thirty-seven: The solo horn is moving towards an exact unison with the glockenspiel.
- Measure thirty-two: The F in the solo horn is 2 cents higher than the F in the glockenspiel and the contrabass.
- Measure thirty-four: The $C$ in the solo horn is 3.9 cents higher than the $C$ in flute 1 , the trombone, and the glockenspiel.
- Measure thirty-five: The F in the solo horn is 2 cents higher than the F in the glockenspiel.
- Measure thirty-six, beat two: The C in the solo horn is 3.9 cents higher than the C in the trombone and the glockenspiel.
- Measure thirty-seven: The $B^{b}$ in the solo horn is the same as the $A \#$ in the glockenspiel and the $B^{b}$ in the violins, the viola, and the cello.
- Measure thirty-eight: The B in horn 2 is 2 cents higher than the $B$ in the violins, the viola, and the contrabass.
- Measure thirty-eight: The C in horn 3 is 2 cents higher than the C in the viola.

There are twelve instances of unison or a different strong relationship in "Hymnus," where horns 1 and 3 are in $F$ and horns 2 and 4 are in $E$ :

- Measure one: The C in horn 1 is two cents higher than the C in the viola.
- Measure three: The C in horn 3 is two cents higher than the C in the viola.
- Measure four: The C in horn 1 is two cents higher than the C in the viola.
- Measure five: The C in horn 3 is two cents higher than the C in the viola.
- Measure six: The C in horn 3 is two cents higher than the C in the viola.
- Measure seven: The C in horn 3 is two cents higher than the C in the viola.
- Measure eight: The C in horn 1 is two cents higher than the C in the viola.
- Measure eight, beat two: The C in horn 3 is two cents higher than the C in the viola.
- Measure nine: The C in horn 1 is two cents higher than the C in the viola.
- Measure ten: The C in horn 3 is two cents higher than the C in the viola.
- Measure twelve: The C in horn 1 is two cents higher than the C in the viola.
- Measure thirteen: The C in horn 3 is two cents higher than the C in the viola.


## ANNOTATED BIBLIOGRAPHY

Benade, Arthur H. Fundamentals of Musical Acoustics. New York, New York: Dover, 1990.
Benade's book looks at many aspects of acoustics including the properties of brass instruments. His book also examines the acoustical properties of different tunings and temperaments including both twelve tone equal temperament and just intonation.

Hill, Douglas. Extended Techniques for the Horn: A Practical Handbook for Students, Performers and Composers. 2nd ed. Miami, Florida: Warner Brothers Publications, 1996.

Hill's book, as the title might suggest, discusses a series of extended techniques, and some fairly standard ones. His discussion of these techniques may at times supplement what Ligeti has included in his score. While the Hamburg Concerto does not rely heavily on extended techniques, it does employ techniques like stopped horn and flutter-tonguing, and these are discussed quite clearly in Hill’s book.

Ligeti, György. Gesammelte Schriften Band 2, ed. Monika Lichtenfeld. Publications from the Paul Sacher Foundation, Band 10. Mainz, Germany: Schott, 2007.

The second of the two-volume (German) Gesammelte Schriften, this is a collection of writings by Ligeti, the last section of which contains writings about his own music. Of these, two are about his Hamburgisches Konzert, one being his introductory notes for the premiere, and the other being the CD liner notes for the Teldec recording of the concerto on "The Ligeti Project Vol. 4."
. Hamburgisches Konzert. Mainz, Germany: Schott, 2004.
Ligeti's Hamburgisches Konzert or Hamburg Concerto, the focus of this paper, was written in 1998-1999, and revised in 2002, making it the last piece he worked on (as far as we know) before his death in 2006. The Hamburg Concerto consists of seven movements: "Praeludium"; "Signale, Tanz, Choral"; "Aria, Aksak, Hoketus"; "Solo, Intermezzo, Mixtur, Kanon"; "Spectra"; "Capriccio"; "Hymnus." The total duration is about 15". The Hamburg Concerto is for solo horn and an orchestra which four natural horns. These horns use the naturally occurring just-intoned pitches of their harmonic series while the rest of the orchestra (typically) plays in twelve tone equal temperament.
$\qquad$ . Sonate. Mainz, Germany: Schott, 2001.
Sonate is a six movement work for solo viola which uses just intonation in its opening movement and twelve tone equal temperament for the remaining five. Despite being a solo work, this piece encapsulates many of Ligeti's previous experiments and stylistic tendencies, and has strong connections to several works including the Hamburg Concerto.
$\qquad$ . Trio. Mainz, Germany: Schott, 1984.
Ligeti's Trio for Violin, Horn and Piano uses the same method of tuning systems as the Hamburg Concerto, relying on the horn for its just-intoned harmonics, while the piano plays in twelve tone equal temperament, and the violin, while typically also in twelve tone equal temperament, does briefly join the horn's pitch world.

Ligeti, György, Péter Várnai, Josef Häusler, and Claude Samuel. Ligeti in Conversation. London, England: Eulenburg Books, 1983.

This book, a collection of four separate interviews, contains information about many of Ligeti's works from before 1980. The interviews also focus on a number of other musical topics. For this paper, they are useful for providing insight into his early microtonal experiments.

Partch, Harry. Genesis of a Music, 2nd ed. New York: Da Capo Press, 1974.
Genesis of a Music is an excellent resource for information not only regarding Partch's life, instruments, and compositions, but also regarding historical and contemporary uses of just intonation and other tuning systems.

Schuller, Gunther. Horn Technique. 2nd ed. New York, New York: Oxford University Press, 1992.
Schuller's Horn Technique examines the opposite end of the spectrum from Hill's book, looking at standard practice techniques and the effort that the horn player must put forth to be successful. His chapter "Some notes for composers and conductors" is also useful for this paper, as it discusses issues such as the challenges horn players face when two or more players are playing high harmonics slightly out of tune with one another, which happens often (and intentionally) in Ligeti's concerto.

Shaffer, Kris. Overtones, Intervals, and Generative Transformations in György Ligeti’s Hamburg Concerto. Yale University, 2006.

Shaffer's paper discusses one of the processes involved in Ligeti's pitch selection for the horns in the Hamburg Concerto. He does a brief analysis of some sections of the piece to show how his analytical system works, but unfortunately oversimplifies the pitch content in the process. His examination of the horn language provides an interesting look at one of the many complex components of this piece, but ignores many other aspects of the tuning systems and notation in the process.

Steinitz, Richard. György Ligeti: Music of the Imagination. Boston, Massachusetts: Northeastern University Press, 2003.

Steinitz's book might be at this point the definitive resource on Ligeti's music as a whole, as well as a very accurate biography. Steinitz had the benefit of having much of his work checked and edited by Ligeti himself. In this book, Steinitz discusses the Hamburg Concerto, in its original six-movement form and in its revised seven-movement form, and mentions Ligeti's plans to include an eighth. He discusses some of the musical influences involved with the composition of this piece, including Ligeti's fondness for the horn.

Varga, Bálint András, ed. György Kurtág: Three Interviews and Ligeti Homages. Rochester, New York: University of Rochester Press, 2009.

As the title suggests, Varga's work is a compilation of three separate interviews with György Kurtág followed by a chapter titled "Mementos of a Friendship: György Kurtág on György Ligeti" in which Kurtág shares stories about Ligeti’s life and their friendship. Kurtág also discusses much of Ligeti's music, including the Hamburg Concerto. This is followed by a very brief biography of Kurtág.

# Syzygy <br> for string quartet 

## 1. Canon / Cannon <br> 3I. Four/fore <br> III. Descent/Dissent <br> IV. Pays/Rare/Rase

## Syzygy <br> Program Notes

Syzygy is an exploration, in four movements, of homonyms. Syzygy itself is a homonym, applicable to several fields including abstract algebra, zoology, medicine, astronomy, and poetry. These last two categories are where the term most closely fits with this composition. In poetry, a syzygy is similar to an elision, combining two metric feet into one. There are other types of syzygy in poetry as well, however, including something similar to alliteration, and one involving symmetrical patterns of verse form. In astronomy, syzygy refers to the alignment of multiple celestial bodies, such as what occurs during an eclipse. Throughout Syzygy, the homonyms in the title of each movement are displayed, and relationships between them are constructed. The different aspects of the title are juxtaposed and separated, resulting in movements which explore multiple ideas simultaneously, forming a cohesive whole.
"Canon/Cannon," the opening movement, is indeed a musical canon. From beginning to end, the piece is played in canon, although the type of canon and the number of voices will vary. The "Cannon" component is taken not from artillery, but from billiards, wherein cannon is an antiquated term for carom or ricochet. This ricochet is useful to illuminate the form of the canon-as the instruments head in one direction only to rebound quickly in the other, the processes of canon and cannon can be heard.
"Pour/Pore" involves an outpour or downpour of notes from beginning to end. Within this cascading pitch material are tiny spaces, or pores. Through these pores, other, slower material finds its way in, and through these same pores this slower material is forced out again.
"Descent/Dissent" involves something like a ground bass which gradually descends one octave. It is often presented in stacks of perfect fifths, all of which follow this same contour. The dissent comes from the voices not playing the descending line-these instruments pull against both the clear perfect
fifths with other harmonies, and the descending motion with ascending lines. Eventually, a compromise is reached.
"Rays/Raze/Raise" ends Syzygy. The rays are slow moving beams which pass through one another creating shimmering harmonies. This ethereal sound world is razed-torn down quickly and scraped away-and leaves behind musical fragments which are raised, or grown, into their own section of the movement. From here they are raised yet again, this time showing an ascent of pitch combined with the fragmented material.

Syzygy is written in expandable just intonation, a tuning system that allows the harmonic interest of the perfect fifths in "Descent/Dissent" or the shimmering harmonies of "Rays/Raze/Raise" to come through in a clear and more intricate manner than equal-temperament would allow. The system also permits very tiny intervals to take on important roles, as is the case in "Canon/Cannon" and "Pour/Pore." These intervals and harmonic relationships will be evident through much of the piece, as microtonal inflections of the musical lines are always present.

## Syzygy <br> Notes for the Performers

## Notation

Syzygy is written in expandable just intonation, employing a total of eighty distinct pitches per octave. The exact pitch of each note is indicated by the ratio below it. These numbers indicate the ratio of the frequency of that note to the $1 / 1$ below it, which in Syzygy is G ( 392 Hz ). These pitches are approximated on the staff-they are notated to the nearest eighth tone.

## Accidentals

The accidentals used apply for the entire measure in one octave only. The given ratios apply to all octaves and last until a new ratio is given.

In some instances, there are two accidentals that reflect the same approximate deviation-this is to simplify the notation as much as possible. Note that in some cases multiple pitches can be written with the same accidentals. In these cases, and all others, the ratio will provide further clarification. A list of all eighty pitches can be found at the end of these notes.

The accidentals used are:

| one semitone plus one eighth tone sharp (approximately +125 cents) |
| :---: |
| one semitone sharp (approximately +100 cents) |
| three eighth tones sharp (approximately +75 cents) |
| three eighth tones sharp (approximately +75 cents) |
| one quarter tone sharp (approximately +50 cents) |
| one eighth tone sharp (approximately +25 cents) |
| one eighth tone sharp (approximately +25 cents) |
| natural (approximately 0 cents of deviation) |
| one eighth tone flat (approximately - 25 cents) |
| one eighth tone flat (approximately - 25 cents) |
| one quarter tone flat (approximately -50 cents) |
| three eighth tones flat (approximately -75 cents) |
| three eighth tones flat (approximately -75 cents) |
| one semitone flat (approximately -100 cents) |
| one semitone plus one eighth tone flat (approximately -125 cents) |
| three quarter tones flat (approximately -150 cents) |

Note that the arrows are actual accidentals, not modifiers signaling the pitch should be raised or lowered. Where these accidentals follow a flat or sharp note, the arrow will always precede a natural sign, for additional clarity.

## Harmonics

The ratios given for any harmonic, natural or artificial, applies to the sounding pitch. All artificial harmonics in Syzygy sound two octaves above the stopped pitch, utilizing the node at the perfect fourth. Natural harmonics should be played on the string indicated to ensure the proper intonation.

## Glissandi

All glissandi should begin immediately on the beat on which they are notated and should take up the entire duration of that note. If this note is tied into, do not rearticulate the start of the glissando. Also do not rearticulate the arrival pitch. When possible, avoid changing bows during the glissando. In the case of pizzicato glissandi, it is possible that the string will no longer be sounding when the arrival pitch is reached. Even in this case, do not rearticulate the arrival pitch.

## Pizzicato

The $\mathbf{H}----\mid$ marking (only in the first movement) indicates the notes should be hammered onto, like a guitar. The notes under that slur will not be rearticulated, just struck with the fingers of the left hand. The resulting sound may be very quiet. In the fourth movement, a similar looking passage is actually to be played on adjacent strings. These strings are indicated where appropriate.

## Tuning

All instruments are tuned normally. This results in the following ratios for the open strings:

Cello:

| I | A | $9 / 8$ | 220.5 Hz | $($ A $=441)$ |
| :--- | :--- | :--- | :--- | :--- |
| II | D | $3 / 2$ | 147 Hz |  |
| III | G | $1 / 1$ | 98 Hz |  |
| IV | C | $4 / 3$ | 65.33 Hz |  |

Viola:

| I | A | $9 / 8$ | 441 Hz |
| :--- | :--- | :--- | :--- |
| II | D | $3 / 2$ | 294 Hz |
| III | G | $1 / 1$ | 196 Hz |
| IV | C | $4 / 3$ | 130.67 Hz |

Violins:

| I | E | $27 / 16$ | 661.5 Hz |
| :--- | :--- | :--- | :--- |
| II | A | $9 / 8$ | 441 Hz |
| III | D | $3 / 2$ | 294 Hz |
| IV | G | $1 / 1$ | 196 Hz |

A tuning CD is available which provides a 15 -second sample of each tone, starting with $1 / 1$, or G , 196 Hz . The following page provides the track listing, which is also the chromatic list of all pitches found in this piece. If using the tuning CD in practice, tuning to A 441 Hz (9/8, track 19) is recommended.

Throughout Syzygy, all ratios of 9/8 correspond to the tuning A or an octave displacement thereof. Similarly, all ratios of $3 / 2$ correspond to the D of the open string or an octave displacement, all ratios of $1 / 1$ correspond to the $G$ of the open string or an octave displacement, etc.

| Track 1 | 1/1 | 196.00 Hz | Track 41 | 10/7 | 280.00 Hz |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Track 2 | 81/80 | 198.45 Hz | Track 42 | 13/9 | 283.11 Hz |
| Track 3 | 64/63 | 199.11 Hz | Track 43 | 16/11 | 285.09 Hz |
| Track 4 | 33/32 | 202.13 Hz | Track 44 | 22/15 | 287.47 Hz |
| Track 5 | 28/27 | 203.26 Hz | Track 45 | 28/19 | 288.84 Hz |
| Track 6 | 25/24 | 204.17 Hz | Track 46 | 40/27 | 290.37 Hz |
| Track 7 | 21/20 | 205.80 Hz | Track 47 | 3/2 | 294.00 Hz |
| Track 8 | 19/18 | 206.89 Hz | Track 48 | 32/21 | 298.67 Hz |
| Track 9 | 18/17 | 207.53 Hz | Track 49 | 14/9 | 304.89 Hz |
| Track 10 | 17/16 | 208.25 Hz | Track 50 | 25/16 | 306.25 Hz |
| Track 11 | 16/15 | 209.07 Hz | Track 51 | 11/7 | 308.00 Hz |
| Track 12 | 15/14 | 210.00 Hz | Track 52 | 128/81 | 309.73 Hz |
| Track 13 | 14/13 | 211.08 Hz | Track 53 | 51/32 | 312.38 Hz |
| Track 14 | 13/12 | 212.33 Hz | Track 54 | 8/5 | 313.60 Hz |
| Track 15 | 12/11 | 213.82 Hz | Track 55 | 13/8 | 318.50 Hz |
| Track 16 | 11/10 | 215.60 Hz | Track 56 | 18/11 | 320.73 Hz |
| Track 17 | 10/9 | 217.78 Hz | Track 57 | 224/135 | 352.21 Hz |
| Track 18 | 28/25 | 219.52 Hz | Track 58 | 5/3 | 326.67 Hz |
| Track 19 | 9/8 | 220.50 Hz | Track 59 | 27/16 | 330.75 Hz |
| Track 20 | 8/7 | 224.00 Hz | Track 60 | 12/7 | 336.00 Hz |
| Track 21 | 7/6 | 228.67 Hz | Track 61 | 121/70 | 338.80 Hz |
| Track 22 | 13/11 | 231.64 Hz | Track 62 | 7/4 | 343.00 Hz |
| Track 23 | 32/27 | 232.30 Hz | Track 63 | 16/9 | 348.44 Hz |
| Track 24 | 6/5 | 235.20 Hz | Track 64 | 25/14 | 350.00 Hz |
| Track 25 | 11/9 | 239.56 Hz | Track 65 | 9/5 | 352.80 Hz |
| Track 26 | 16/13 | 241.23 Hz | Track 66 | 20/11 | 356.36 Hz |
| Track 27 | 56/45 | 243.91 Hz | Track 67 | 11/6 | 359.33 Hz |
| Track 28 | 5/4 | 245.00 Hz | Track 68 | 13/7 | 364.00 Hz |
| Track 29 | 81/64 | 248.06 Hz | Track 69 | 28/15 | 356.87 Hz |
| Track 30 | 14/11 | 249.45 Hz | Track 70 | 15/8 | 367.50 Hz |
| Track 31 | 9/7 | 252.00 Hz | Track 71 | 32/17 | 368.94 Hz |
| Track 32 | 21/16 | 257.25 Hz | Track 72 | 17/9 | 370.22 Hz |
| Track 33 | 4/3 | 261.33 Hz | Track 73 | 36/19 | 371.37 Hz |
| Track 34 | 27/20 | 264.60 Hz | Track 74 | 40/21 | 373.33 Hz |
| Track 35 | 19/14 | 266.00 Hz | Track 75 | 48/25 | 376.32 Hz |
| Track 36 | 15/11 | 267.27 Hz | Track 76 | 27/14 | 378.00 Hz |
| Track 37 | 11/8 | 269.50 Hz | Track 77 | 64/33 | 380.12 Hz |
| Track 38 | 112/81 | 271.01 Hz | Track 78 | 35/18 | 381.11 Hz |
| Track 39 | 18/13 | 271.38 Hz | Track 79 | 63/32 | 385.88 Hz |
| Track 40 | 7/5 | 274.40 Hz | Track 80 | 160/81 | 387.16 Hz |
















F





poco accel. . - $\quad-\quad-\quad-\quad . \quad . \quad . \quad$.































## C
























III. Descent/Dissent
$d=\mathbf{6 6}$ poco rubato, sempre poco espressivo













A
















L






















[^0]:    ${ }^{1}$ Bálint András Varga, ed. György Kurtág: Three Interviews and Ligeti Homages (Rochester, New York: University of Rochester Press, 2009), 114.
    ${ }^{2}$ The title Hamburg Concerto will be used throughout this paper. However, this piece is also correctly identified as Hamburgisches Konzert, Horn Concerto, Konzert für Horn solo und Kammerorchester, and Kürtverseny.
    ${ }^{3}$ There were plans and sketches for an eighth movement, but one was never written.
    ${ }^{4}$ György Ligeti. Hamburgisches Konzert (Mainz, Germany: Schott, 2004), 1.

[^1]:    ${ }^{5}$ Note that the terms overtone series and harmonic series express related, but different ideas. The harmonic series is the exact series of frequencies that would sound over a given fundamental with simple mathematical relationshipsthe harmonics are integer multiples of the fundamental. The overtone series is any set of frequencies that are produced along with the fundamental. Overtones are not necessarily integer multiples of the fundamental due to the physical properties of the instrument and the contributions of the performer.

[^2]:    ${ }^{6}$ György Ligeti, Péter Várnai, Josef Häusler, and Claude Samuel. Ligeti in Conversation (London, England: Eulenberg Books, 1983), 53.

[^3]:    ${ }^{7}$ Ibid., 55.

[^4]:    ${ }^{8}$ Richard Steinitz. György Ligeti: Music of the Imagination (Boston, Massachusetts: Northeastern University Press, 2003), 181-182.
    ${ }^{9}$ Ligeti, Ligeti in Conversation, 13.

[^5]:    ${ }^{11}$ Also called the Oehler system.
    ${ }^{12}$ In this instance, the syntonic comma, which has a ratio of $81 / 80$, and a cents value of 21.51.

[^6]:    ${ }^{13}$ György Ligeti. Trio (Mainz, Germany: Schott, 1984), 4.

[^7]:    ${ }^{14}$ György Ligeti. Sonate (Mainz, Germany: Schott, 2001), 5.

[^8]:    15 "I have not established a firm, regulated system, but rather I have let go of the sounds, so-through their own organization-tonal relationships outside of the tradition will arise." György Ligeti. Gesammelte Schriften Band 2 (Mainz, Germany: Schott, 2007), 312.

[^9]:    ${ }^{16}$ Note that the arrows (which are used here in a similar manner to Ligeti's usage in the Hamburg Concerto) do not indicate that the pitches are out of tune. They simply show their difference from the pitches of equal temperament.

[^10]:    ${ }^{19}$ See Appendix C for a complete list of these relationships.

[^11]:    ${ }^{22}$ Other interesting analytical perspectives could include an analysis of the pitches that will be produced regardless of Ligeti's intentions (and taking the unique acoustical characteristics of the instruments into account), or an analysis of what the listener can perceive of the interplay of the two tuning systems.
    ${ }^{23}$ String harmonics will be considered just-intoned intervals above the equal-tempered open string.

[^12]:    ${ }^{24}$ In this paper, all cents values for specific pitches are based on an equal-tempered C having the value of 0 (or 1200) cents.

[^13]:    ${ }^{27}$ Here is the first of several significant errors in the score. At rehearsal C (measure 15), the timpani line shows a C ${ }^{\#}$ and the G below it. This G does not belong. In his manuscript Ligeti wrote $\mathrm{C}^{\#}$ with a marcato marking ( A ) above and a staccatissimo marking ( $\boldsymbol{\Lambda}$ ) below, just like the markings on the B in the trombone. The staccatissimo marking crosses the bottom line of the staff in his manuscript, which led to it being mistranscribed as a G. For information on this and other errors in the score, please see Appendix A.

[^14]:    ${ }^{30}$ The cents values being used are rounded to the nearest tenth of a cent. For this reason, a just-intoned perfect fifth is usually called 702 cents in this paper. The real value, however, is 701.955 . When two of these $3 / 2$ intervals are stacked, the resulting 9/8 is 203.91 cents-rounded in this paper to 203.9. In Figure 3.5, the perfect fifth which ends the interruption is still exactly a $3 / 2$-the addition of the two intervals is what makes this example appear to be 0.1 cents smaller. The same is true for the $6 / 5$ interval. While it seems that the value should be 315.7 cents based on the cents values given, when the numbers are not rounded the interval is actually from a $5 / 4$ at 386.314 to a $3 / 2$ at 701.955 which results in the $6 / 5$ being 315.641 - just slightly too small to round up to 315.7 . These issues are simply a complication of rounding the cents, although they also provide a good reason for using ratios when discussing just intonation.

[^15]:    ${ }^{31}$ In Kris Shaffer’s paper, Overtones, Intervals, and Generative Transformations in György Ligeti’s Hamburg Concerto, he simplifies the intervals in this passage and ones similar to it. In his paper, his cents values are taken from approximations in Ligeti's score in which the cents values are rounded off to the nearest 5 cents. This leads to some faulty assumptions, as many of the intervals he assigns ratios to in his paper are not actually just-intoned intervals.
    ${ }^{32}$ For more information about the structure of this movement, see Steinitz, 357.

[^16]:    ${ }^{33}$ See Appendix B for charts of the showing the harmonic series of all fundamentals used in the Hamburg Concerto.

[^17]:    ${ }^{34}$ Ligeti notates the thirteenth harmonic a whole step above the twelfth with a down arrow which signifies it will sound about a quarter tone low. In this instance, the pitch at 1140.5 cents is written as C, even though it is in fact closer to B.

[^18]:    ${ }^{35}$ Although 86.3 cents is not a common just-intoned or equal-tempered interval, it is interesting to note that from 200 cents (the tonic of the D horn), the 500 cents which the D horn ends on is an equal-tempered minor third above, and the 586.3 cents on which the D horn begins its passage is a just-intoned major third above.

[^19]:    ${ }^{36}$ Since these are not equal-tempered chords, a slightly different sound results from this tuning. However, the overall progression is similar to what it would be in equal temperament.
    ${ }^{37}$ Enharmonic respellings are used frequently in these chord descriptions so that the root of the chord is always spelled the same in this paper as it is in Ligeti's score. The enharmonic spellings are necessary in the score to ensure that all pitches fall logically into the harmonic series of the horn.
    ${ }^{38}$ Although the third is written as a $C^{\#}$, it is approximately a quarter tone low because it is played as a thirteenth partial; the root is also low because it is played as a fifth partial. This results in a third of 354.5 cents, close to the quarter tone between a major and minor third. The third above this $C \#$ is also near a quarter tone, but more clearly a major third- 363.4 cents. The total fifth is slightly large at 717.6 cents and the minor seventh is an equal-tempered 1000 cents. The fifth and seventh in this chord will sound relatively in tune, which will result in the quarter tone third sounding like a large minor third and the 363.4 cent third sounding like a major third.

[^20]:    ${ }^{39}$ Gunther Schuller. Horn Technique. 2nd ed. (New York, New York: Oxford University Press, 1992), 57-65.

[^21]:    ${ }^{40}$ Steinitz, 258, 357.

[^22]:    ${ }^{41}$ Ligeti, Hamburgisches Konzert, 9.
    ${ }^{42}$ Ibid.

[^23]:    ${ }^{43}$ Although crotales tend to be tuned to $A=442 \mathrm{~Hz}$ instead of $\mathrm{A}=440 \mathrm{~Hz}$, this is not universally true, and it is similarly not a guarantee that the orchestra itself will tune to $A=440 \mathrm{~Hz}$. While an instrument tuned to $\mathrm{A}=442 \mathrm{~Hz}$ would be 7.8 cents higher than an instrument tuned to $\mathrm{A}=440 \mathrm{~Hz}$, such a discrepancy cannot be assumed in this case. Regardless, the difference in pitch-especially over so many octaves-would be very difficult to hear. So while this is one unknowable variable in the equation, after working through the problem the point is moot.

[^24]:    ${ }^{44}$ Figure 2.4 on page 20 shows how this collection of fundamentals allows each note to have its own tuning slide, which enables the player to get the fundamentals exactly in tune with equal temperament, preserving the simultaneity of the two tuning systems even in this passage where only one instrument is playing. These fundamentals use open, second valve, first valve, and third valve on both the F and $\mathrm{B}^{b}$ sides of the double horn.
    ${ }^{45}$ While the idea of stopped horn is to sound the written pitch with a stopped tone, the physics of the effect are often debated. Possible explanations could include that stopping the horn makes the horn behave as a shorter pipe closed on both ends, instead of a longer pipe closed on one end, or that the fundamental is lowered by a half step and the player (trying to stay on the written pitch) sounds the next higher partial, lowered by this same half step.

[^25]:    ${ }^{47}$ In the score, this measure is errantly marked "In Mi," suggesting an E fundamental. In the measure being echoed, the soloist plays the tenth and eighth partials on an $E^{b}$ fundamental, sounding $G$ and $E^{b}$. If a fundamental of $D$ is used for the echo—as shown above-the player will need to raise the notes $\mathrm{F} \#$ (tenth partial) and D (eighth partial) exactly 100 cents (an equal-tempered half step) to correctly echo the original. Were an E fundamental to be used, the player would need to raise a ninth harmonic $F^{\#}$ by 96.1 cents to match the $G$, which is comparably simple to the

[^26]:    ${ }^{48}$ The omission of one fifth emphasizes the major second relationship inherent in the stacking of perfect fifths.

[^27]:    ${ }^{50}$ There is another error in the score in this section. In measure ninety-five, the third note of the violin 2 part is written as an F, but clearly should be an E. See Appendix A for more information.

[^28]:    ${ }^{51}$ These trichords are at times equal-tempered, at times just-intoned, and frequently a combination of the two. Therefore, these sets do not have the same meaning as they do in equal-tempered music, especially when it comes to transposition. They are simply a convenient and established tool for identifying chords with the featured intervals of this movement.
    ${ }^{52}$ Steinitz, 357.

[^29]:    ${ }^{53}$ The obbligato horns move in parallel between their first and second pitches, and this type of motion is frequent throughout this movement. This is an example of the collapsing (or expanding) intervals created between the horns moving in the same direction, which was an important feature in "Tanz" from movement two.

[^30]:    ${ }^{54}$ The $[0,1,6]$ and $[0,5,6]$ trichords created at the end of beat two of measure six have a minor second which is a justintoned $12 / 11$ between horn 2 and horn 3 . While the $12 / 11$ is usually considered a major second, in this instance the other intervals of the trichords (the perfect fourth and the tritone) are stronger, creating the sense that this $12 / 11$ is actually a minor second. The $[0,1,6]$ trichord has a strong $4 / 3$ just-intoned perfect fourth and a reasonable tritone which leaves the ear to consider the space between as a minor second. The $[0,5,6]$ trichord has a very strong (nearly equal-tempered) tritone and a small perfect fourth which results in a more complicated chord. However, the 451.3 cent interval will still sound like a fourth, leaving the $12 / 11$ to again resemble a minor second. These relationships are common throughout "Spectra."
    ${ }^{55}$ The horn 3 part has this low A\# labeled as the tenth partial, but it should actually be the eleventh. See Appendix A for more information.

[^31]:    ${ }^{56}$ This note in horn 3 should be labeled as the tenth partial. See Appendix A for more information.

[^32]:    ${ }^{57}$ Presumably Ligeti has left the fundamental change notation out in these sections because the horn player will know there is only one way to sound this pitch. See Appendices A and B for more information.
    ${ }^{58}$ See Appendices A and B for more information.
    ${ }^{59}$ Through the rest of the piece, the piccolo (played by the flute 2 player) is written below the flute staff, but here the order is reversed. This error was addressed by Ligeti, but was not changed in the final publication. See Appendix A for more information.

[^33]:    ${ }^{60}$ In the score, violin 1 has an arrow indicating that this G should be played approximately a quarter tone low, but in the manuscript the arrow indicates it should be played approximately $30 \%$ lower-Ligeti's indication for the seventh partial. The simulation of the seventh partial makes much more sense in this context, as it creates a fully justintoned eleventh chord which clashes with several equal-tempered pitches.
    ${ }^{61}$ While this B on most instruments could be considered the ninth of an A ${ }^{11}$ chord, the pitch spectrum of the tubular bells is somewhat complex, and does not fit well into the harmonic series. It makes more sense to consider this, and the A of the tubular bells, as equal-tempered pitches separate from the just-intoned chord.

[^34]:    ${ }^{62}$ In this chord, the B to $F \#$ fifth is actually only 638.5 cents and sounds more like a tritone, the $F \#$ to $C \#$ fifth is 759.5 cents and sounds closer to a small minor sixth than a perfect fifth, and the $G \#$ to $D \#$ perfect fifth is 668.8 cents, which will sound much more like a large tritone than a perfect fifth.

[^35]:    ${ }^{63}$ Steinitz, 294.
    ${ }^{64}$ Ibid., 357.

[^36]:    ${ }^{65}$ Not all of the string instruments change to new chords at the same time. This creates a slight bit of overlap at the start of some chords and at the end of others. The chords to which I refer are the arrival points once all instruments have completed their chord change.

[^37]:    ${ }^{66}$ The A played by horn 4 here should have an arrow on it in the score, but does not. Since it is moving from a fifth partial to a stopped note exactly 100 cents higher, this note will still be 13.7 cents below the equal-tempered pitch. See Appendix A for more information.

[^38]:    ${ }^{67}$ This pitch does not have an arrow in the score, but it should. The $F \#$ harmonic on a $G$ string results in a $15 / 8$, which is 11.7 cents below the written pitch. Ligeti uses down arrows for all fifteenth partial notes in the horns, and the same convention should have been followed here. See Appendix A for more information.

[^39]:    ${ }^{68}$ Note that the bass part is missing several arrows in the score which would clarify this relationship. See Appendix A for more information.
    ${ }^{69}$ In Figure 3.39, breath marks and arrows missing from the score have been added. Additionally, the dynamics have been corrected (in the score, only horn 4 has the proper dynamic markings in measures 11-14). See Appendix A for more information.

[^40]:    ${ }^{70}$ Although pitches sounded on the thirteenth partial are more than a quarter tone low compared to equal temperament, when they are used to sound the a minor seventh of a chord they are less problematic. In this instance, the minor seventh of E should be 168.8 cents. This low D at 140.5 cents will more closely resemble a minor seventh than a major sixth (around 84.4 cents) due to the context.

[^41]:    ${ }^{71}$ There is another crucial error in the score here-the score indicates that horn 1 should play a thirteenth partial D at the end of measure four, but Ligeti wrote an eleventh partial B. See Appendix A for more information.
    ${ }^{72}$ The resulting sound here is similar to a strangely-tuned French augmented sixth chord.

