Experimental Studies on Public Good Allocation with Agents

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Inefficiencies in private giving are a common occurrence in public good games. In this dissertation, we ask four questions: do subjects recognize the inefficiency due to the lack of coordination on group giving, is there a simple way to overcome this coordination problem, is there a mechanism for giving that improves group welfare that is preferred by subjects, and do subjects choose a predatory allocation strategy when given opportunities to discriminate. To investigate these questions, we design a public goods experiment where the contributions of each individual may be determined by a member of the group named the "agent." In the game with common wealth and preferences among the group, the dominant strategy for the agent is to choose the Pareto-efficient allocation. Thus, giving through an agent in this environment eliminates the group coordination problem seen in private giving. In the game with diversified wealth among the group, the dominant giving behavior for the agent becomes a predatory allocation strategy, where members of their own wealth group can free-ride off the contributions of the other wealth group.

Using public good games with both boundary and interior Nash equilibria, results from the agent treatment are contrasted with results from a no-agent treatment. In addition, efficiency and allocation decisions are compared between the equal-endowment experiment and the diversified wealth experiment. Subjects do recognize the inefficiency of individual giving to a

group, and therefore, higher contributions to the public good are observed under the agent treatment which improves social welfare.

In a third chapter is presented which models an industry with two differentiated firms producing a homogenous product priced by contracts. Each firm faces difficulty in pricing their product since they are competing in a market with two types of customers, "old" and "new." Two types of switching costs are considered, one explicit in the contract and another which is implicit. We examine how prices and fees in contracts are affected by the parameter of the model including the spread and expected value of this implicit switching cost.

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PREFACE

The majority of this dissertation is focused around two experiments I had the privilege to run under the supervision and encouragement of Professor John Duffy. Without his guidance throughout these projects, this dissertation would not have come to fruition. Thank you for your support.

In addition, I would like to thank Professor Lise Vesterlund for her advice that encompassed not just the intellectual work in this dissertation. I am so grateful for her support that aided me in finding answers to questions and motivation for completion.

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A big thank you goes out to my "moms of Pittsburgh," Annette DiDiano and Terry Waters, for providing a frequently-need listening ear and many laughs.

Finally, I would like to dedicate this dissertation to my family, who are the foundations I have stood upon in the last five years. To Momma, thank you for never letting me give up even in the midst of the hardest seasons of my life. To my husband Matthew, the word "thank you" cannot encompass the gratitude I feel for your persistent faith in me, your patience, and your unconditional love.

1.0 PRIVATE VS. PUBLIC PROVISION: AN EXPERIMENTAL INVESTIGATION

1.1 INTRODUCTION

1.1.1 Overview

How to most efficiently provide for society's welfare through the use of public goods is a question that has been on the minds of economists and politicians for decades. Of particular concern is whether society is better off when public goods are privately or publicly provided. Numerous arguments can be made for or against pubic provision. Although Pareto-efficient giving is virtually never observed in private giving games, coordination in giving has been consistently observed albeit at small amounts.

Previous experimental literature has looked into the mystery of persistent coordination in private giving with discoveries that subjects give above predicted outcomes due to kindness, confusion, or because they are simply cooperative types (Andreoni 1995a, Fischbacher and Gachter 2006). As subjects give less of their endowments to the public good with experience, the cooperation that was previously enjoyed within the group in early rounds becomes frustrated and ultimately fails. This cooperation failure, according to these authors, comes from "frustrated attempts at kindness," general confusion, and the interaction of conditional-cooperators with free-riders. Thus, the use of an agent making a group contribution decision eliminates frustration and solves the coordination failure seen in private giving treatments.

This paper is focused on a public good experiment which examines the possibility of efficient contribution through a group agent. Specifically, a giving structure is implemented utilizing agents who make a single allocation decision for all members of their group. By examining decisions made under private provision of a public good with those under agency, we are able to determine which is more efficient. Under a voting condition, we can also determine which is more popular. Subjects allocate tokens from their endowment into two accounts, a group account and a private account. The "twist" to the traditional public good game is that in the agent treatment a player's allocation decision to the group account is for the group as a whole.

Giving to public goods through an agent is theoretically efficient when every group member is endowed with the same wealth and unable to discriminate between individuals within the group. In the agent's allocation problem, the Pareto-optimal level of the public good is not only the optimal response for a subject's allocation decision, but it is also a dominant strategy. This Pareto-optimal outcome is achieved due to the agent's ability to make a single, binding contribution decision. This one decision eliminates the coordination problems often witnessed during private giving sessions. Thus, agency increases social welfare over private provision.

A natural question to ask is where do we see such agents? Agents make a single, binding allocation decision for their group; thus, in government an agent is a dictator. Agents are those who may make authoritative decisions that apply to a body of individuals under them or around them. Examples of these agents are a president of a company, a manager of a task force, or even a parent in their own household. In addition, there are certain environments where it is not efficient for every individual in a group to have a voice; these are cases where group size makes it difficult to achieve beneficial outcomes. We see these large group sizes in representative governments, corporations, and unions. Such situations call for individuals to delegate their decision power to an agent to speak or make decisions on their behalf. Specifically agency can be seen in the context of a class action lawsuit, where an individual plaintiff gives up their right to sue individually in order to allow a lawyer to pursue restitution for the group as a whole.

When looking at the benefits of agency in public good games, it is only natural to wonder why these public mechanisms are not used more frequently. Depending on the size of the group making a collective decision, having one centralized agent, such as the government, can bring about large inefficiencies in terms of administrative costs and special interest groups. One goal of this paper is to investigate what would happen if all of the costs of centralization were stripped away. Could efficiency in the public good be achieved if agents had a straightforward environment in which to make decisions?

This experiment will attempt to answer this question by comparing within-subject results of private giving and allocations with an agent to see if greater efficiency and higher social welfare are achieved with the use of an agent. Further, if subjects can vote for an allocation structure, which would be the majority outcome: agency or no-agency? In a voting treatment after subjects have experienced giving under both the no-agent and agent treatments, they are given an institutional choice, which is then imposed by majority rule. The allocation mechanism emerges endogenously in the voting treatment at the end each session.

The experimental design is comprised of two allocation structures which yield different solutions to the player's giving problem. In the previous public goods literature, most commonly the Nash equilibrium and Pareto-optimal outcome can be found on the boundary of a subject's decision space. In the experimental design, I provide results from a traditional boundary case as well as an interior case where the Nash equilibrium and Pareto-optimal outcome are interior to a subject's decision space. This interior design provides room for players to not only over-contribute but also under-contribute relative to both the Nash equilibrium and Pareto-efficient outcome. The addition of the interior design also helps address the question of whether subjects can achieve the Pareto-optimal outcome in an environment where the solution is not as obvious as in the boundary design. Agency, in theory, should solve all cooperative problems seen in private giving session, especially in the boundary solution. The interior design with its greater complexity provides a further test upon the cooperative power of the agency treatment. Will agents make less Pareto-optimal decisions in the interior solution than in the boundary due to its complexity?

Results from the experimental design are mostly as expected in both solution cases. Social welfare and efficiency increase under the agent treatment due to higher contributions than in no-agent treatments. In fact, by the end of the agent-treatment rounds, giving behavior approaches the respective Pareto-optimal outcomes, although more frequent Pareto-optimal outcomes are seen in the boundary solution. In the boundary case, giving behavior in the no-agent treatment deteriorates toward the Nash equilibrium of zero contribution. In the interior case, there is no general decline in giving during no-agent rounds, but instead giving converges to a level slightly above the Nash equilibrium. In both solution cases, voters always favor the agent contribution mechanism. This results in even more frequent Pareto-optimal contribution decisions.

1.1.2 Literature review

The literature on public good experiments is too large to summarize here. For an overall survey see Ledyard (1995).

The general trends in the literature about behavior in boundary VCM games are the following. Individuals and groups do not behave as predicted contributing positive amounts of their endowment to the group account when the Nash equilibrium prescribes zero contribution. On the other hand, they do not make Pareto-optimal contributions to the public good. Players in boundary VCM experiments usually invest amounts in between these two outcomes. It may be argued that subjects extract some gains from giving to the public good, but they cannot exploit them to the fullest extent because of poor coordination within the group and incentives to free ride.

Since economists have been interested in how to attain Pareto optimality in giving behavior in public good experiments, they have added various treatments to VCM games in an effort to facilitate efficient provision, eliminate free riding, and affect cooperation. These include methods of group pairing (Andreoni 1988; Croson, 1996; Andreoni and Croson, 1998), sequential moves and signaling (Gachter and Renner, 2004; Meidinger and Villeval, 2002; Moxnes and van der Heijden, 2003; Potters et al. working paper, Potters et al. 2005), varying group size (Issac and Walker, 1988b), pre-play communication (Issac and Walker, 1988a; Frey and Bohnet, 1996; Ostrom, Walker and Gardner, 1992; Bochet, Page and Putterman, in press), punishments and rewards (Andreoni et al., 2003; Fehr and Gachter, 2000; Masclet et al. 2003), asymmetry of information, status (Kumru and Vesterlund, working paper) and even dynamic sequences of contributing (Duffy et al., working paper). These mechanisms often make considerable improvements to efficient giving, but none of these has consistently brought about Pareto-optimal behavior in group decision making.

The effect of leaders (whom I call agents) and their assistance in aiding groups to make efficient choices has just begun to be investigated in public goods experimental literature. Potters, Sefton, and van der Heijden (2005) investigate the significance of leaders in a team-production experiment. They compare a revenue-sharing treatment to a treatment with a team leader in an experiment with team production. Under the revenue-sharing treatment, the total production is split evenly, whereas the leader has the power to implement their own allocation of the total production. They find that the "presence of a team leader results in a significant improvement in team performance." Their results show that leaders can improve efficiency in a group setting even when the leader has the discretionary reward power to take the entire team production for themselves or disperse it among the group. Their design is very similar to a traditional VCM design since it maintains incentives to free-ride. Having a leader does not remove this incentive unless they send appropriate signals through their allocations to help foster cooperation. The leaders in their experiment are different than the agents in my experiment. My agents' payoffs are tied up with their group decision, while in Potters et al., leaders can take the whole team production pie or encourage team production by their allocation. Their leaders are making multiple allocation decisions, as opposed to the agents in my experiment who make a single allocation decision for themselves as a part of the group. Their experiment has exciting results showing the benefits of a leader in a public good environment, but due to the differences in the payoff design and strategy for the leaders, their results have different implication than my experiment.

Duffy and Kim (2005) also exhibit the efficiency benefit of leaders (agents) in their experimental research in a predator-prey environment. Their results show that in treatments without a government agent, subjects coordinate on an inefficient equilibrium, but when they introduce a government agent to impose an irreversible group decision subjects can actually achieve a more efficient equilibrium. Thus, government agents who are given power to make a binding decision can help their group reach better outcomes.

In designing the mechanics of this experiment, I wanted to be able to compare my results to a design that had been already tested to replicate findings of similar studies. This would ensure that the behavioral results are a product of the design and not the laboratory surroundings. Thus, the technicalities of the interior-solution experimental design are derived from an interior public good experiment conducted by Laury, Walker, and Williams (1999).[1] In their design, each group consists of five members and both the Pareto optimal and Nash equilibrium solutions are interior to the decision space. These design features are important to my experiment not only for the benefit of comparison but also because they allow for testing of both under- and over-contribution to the public good. The group composition of five members also allows group voting to utilize majority rule to implement outcomes. In addition, the interior Nash and Pareto optimal solutions allow for an opportunity to watch contribution convergence. Will players converge on Nash equilibrium behavior from above, below or even at all? In their experiment, LWW see over-contribution above the Nash equilibrium to the group account in all treatments with slight convergence from above, although contribution levels were still below the Pareto optimum and never fully converge.

[1] References to the Laury, Walker, and Williams (1999) paper henceforth will be abbreviated LWW.

The agent design, though a simple change, can address some questions raised in the voluminous literature generated by the voluntary contribution mechanism. Is it possible for one small modification in mechanism design could overcome all the inefficiencies in group-giving behavior? Can group opinion affect decisions of representatives? Also, does the choice of whether the group chooses to be under the guidance of an agent bring consistently improved outcomes? If ever efficient outcomes are consistently to be seen in public good games, I would predict it would be in the environment of a single agent making a single, binding group contribution decision. If we do not see them here, then we may easily be lead to believe that there is a good amount of confusion associated with the setup or players understanding the nature of the game as shown by Andreoni (1995a).

1.2 EXPERIMENTAL DESIGN

1.2.1 Basic Design

The experiment follows a 2x3 design for a public good experiment. Each session consists of 60 decision rounds, 20 rounds of no-agent decisions, 20 rounds of agent decisions, and 20 rounds of voting decisions.[2] These three environments are examined in both interior and boundary solutions designs.

[2] The ordering between the first two treatments was reversed to check for ordering effects but the voting treatment remained as the last 20 rounds so subjects could decide which mechanism they would rather have after having experienced both the agent and no-agent treatments.

In all treatments, the group composition does not change across rounds, meaning the same group of $n = 5$ subjects play a finitely-repeated public good game over the course of the 60-round experimental session. Due to the slight complexity of the mathematics and search associated with the mechanics of the interior solution, the fixed pairings allow for increased cooperation and fewer distractions in strategy formation during the decision process.[3] The set up of each session as well as Nash equilibrium allocations are presented in Table 1. Both the interior and boundary designs consisted of 6 groups of 5 subjects for a total of 30 subjects per solution design.

Table 1: Session Treatment Designs and Contribution Equilibria

Session Name*	Rounds 1-20	Rounds 21-40	Rounds 41-60	NA Prediction	Agent Prediction
BoundaryNA	No Agent	Agent	Voting	0	125
BoundaryAN	Agent	No Agent	Voting		
InteriorNA	No Agent	Agent	Voting	20**	84
InteriorAN	Agent	No Agent	Voting		

*Each session consisted of 3 groups of 5 subjects and is abbreviated by its solution design and the first treatment round.

**Assumes each subject follows a symmetric giving strategy in order to achieve the Nash equilibrium of 100 tokens.

At the beginning of each round, each subject is gifted with 125 tokens to allocate between one of two accounts: the group public account and their own private account. Tokens cannot be carried between rounds. When the round starts, each individual is asked how many tokens they want to allocate to the group account. Tokens not contributed to the group account remain in a

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[3] Even though these fixed pairings allow for repeated game strategies to emerge among group members, behavior should not be affected by the matching strategy given the research done by Andreoni and Croson (2001). In addition, in looking at the results, it appears that the treatment variables and learning are driving the differences in giving behavior, but this remains an open question for further research.

subject's private account. The difference in the agent treatment is that each individual is asked to make an allocation decision for the group. In each round in the agent treatment, each group member makes a single decision about how many of every group member's tokens will be contributed to the group account. The agent's allocation decision is similar to a single, lump-sum tax applied to every group member, including themselves. After all subjects have made agent decisions, one group member is chosen at random to be the agent whose decision is then implemented for the group.

Thus, the total number of tokens in the group account in the agent treatment is five times the amount the agent allocated. For the no-agent treatment, the total number of tokens in the group account equals the sum of the tokens contributed individually by all group members.

In the last 20 rounds, subjects vote on whether they would like to have an agent make a group allocation decision or utilize the individual, no-agent mechanism. Any voting mechanism should lead to the same result, given that it is best response for every member of the group to choose agency. The voting method chosen in this experiment is majority rule in an effort to emulate a realistic environment which reflects how most group decisions are made. In addition, as shown by Bowen (1969) and Bergstrom (1979), majority voting brings about a Pareto-optimal provision of a single public good when all voters have equal wealth and marginal rates of substitution. Thus, the most frequent outcome of the voting treatment should be the agency mechanism with a Pareto-efficient allocation to the group account.

Through voting, the allocation mechanism becomes endogenous to the group. Group members can voice their opinion about what method they prefer. If three or more group members vote for one outcome, it is imposed. The numerical outcome of the vote is not revealed to participants in an effort to avoid influencing giving behavior. The outcome is only revealed by the allocation method which is implemented.

Participants were volunteers recruited from the University of Pittsburgh Experimental Economics Laboratory mailing list which include students and adults from the Pittsburgh area. All experimental sessions were conducted at the University of Pittsburgh Experimental Economics Laboratory.

1.2.2 Payoff Design

1.2.2.1 Boundary Design

In the no-agent treatment, the unique Nash equilibrium prediction is zero contribution for all marginal per capital return (MPCR) rates less than one. By contrast, in the agent treatment, the Nash equilibrium links to the Pareto optimum involving full contribution. At the beginning of each period, each of the *n* subjects is endowed with *e* tokens. In each period, the agent decides to allocate *t* of each individual's endowment to the public good. This allocation *t* is uniform across group members and therefore is not indexed by *i*. Once an agent makes a decision for the group, it is binding for all group members. Therefore, each player *i* acting as an agent must choose t to maximize his or her profits given by:

(1) $\pi_i = e - t + ant$ in each period, where a is the marginal per capita return to the public good, $0 < a < 1 < an$.

Maximizing (1) by t yields a boundary solution: $\frac{d\pi_i}{dt} = -1 + an > 0.$ Note that complete allocation of each group member's endowment ($t = e$) follows from $an > 1$. The Pareto optimum of complete giving to the experimental public good is the dominant strategy for the agent. Thus, the total number of tokens in the group account is *ne*. Note, by maximizing his own profit, the agent is maximizing each member of the group's profits since (1) does not depend on *i*. Due to the large increase in profits from the Pareto-optimal allocation strategy in the agent treatment, during voting rounds choosing agency is a dominant strategy for each group member.

Since the interior solution is designed around parameters set by LWW, the boundary solution design structural components should be similar for the sake of comparison. The most important structural design to match is the benefit of the Pareto optimal outcome over the Nash equilibrium outcome. Therefore, in the boundary design, the ratio of the token payoff in the Pareto optimum over the token-payoff in the Nash equilibrium is the same as it is in the interior design. This ratio of payoffs is the marginal per capita return (MPCR) of contributing to the group account since it will approximately measure the rate of return of the public good for each group member. To find this ratio of payoffs, I determine the payoffs each subject would receive in the interior design from playing a symmetric Nash equilibrium of 20 tokens each and a Pareto optimum of 84 tokens each. By taking the ratio of these payoffs, I find an MPCR of 0.318. Therefore, each token in the group account yields 2.2 points to every member of the group, and each token in a subject's private account yields them 0.7 points.

Looking more specifically into the payoff design in the boundary case, at the beginning of each period, each of the 5 subjects is endowed with 125 tokens. Thus, our specific agent's payoff function is:

$$\pi_i = 2.2(e - t) + 0.7(5t)$$

$$\pi_i = 2.2(125 - t) + 0.7(5t).$$

1.2.2.2 Interior Solution

Recall that the mechanics of the interior solution are derived from the Laury, Walker, and Williams (1999) paper utilizing the details of their Z125-DET treatment. The set up for the return to the group account for a subject is denoted by *F(.)*, the group size, the endowments for each member, as well as parts of the instructions and tables given to the subjects were all employed from their interior design.

This design adheres to the structure of a voluntary contribution mechanism where subjects decide how to allocate their endowment between their private and the group account. At the beginning of each period, each of the five subjects is endowed with 125 tokens. Each individual i decides how many of their tokens to allocate to the group account, g_i. Thus, the total number of tokens in the group account is $g_i + \sum_{j=1}^{4} g_j$, where $j \neq i$.

Table 2: Synopsis of Solution Design

	Boundary	Interior
F(X)*	1.59X	$6.25X-0.00625X^2$
F'(X)/N - MPCR	0.318	1.25-0.0025X
Group NE	0	100
Group PO	625	420
Indiv. NE	0	20**
Indiv. PO	125	84**

*X denotes the number of tokens in the group account and F(X) dictates a function which shows group earnings from the group account.

**The Nash and Pareto-optimal solutions in the interior design are in aggregate tokens. These individual token amounts hypothesize behavior if all players acted the same.

The return to the group account for a subject is characterized by *F(.)* which is a non-linear function with a declining benefit from the group account. Each subject receives $\frac{1}{n}$ or $\frac{1}{5}$ of total group account earnings. Therefore, each subject receives $\frac{F(g_i + \sum_j g_j)}{5}$ in points as their individual return from the group account. Each token remaining in their private account earns a subject one point.

Denote the number of tokens in the group account *X*. The total of group earnings from the group account is; $F(X) = 6.25X - 0.00625X^2$. Thus, the marginal social benefit from the group account is; $F'(X) = 6.25 - 0.0125X$ and the marginal per capita return (MPCR) is; $\frac{F'(X)}{N} = 1.25 - 0.0025X$.

Using the same variables from the boundary analysis, if an agent chooses an allocation of *t* for each group member, then the payoff function for each individual *i* is:

$$\pi_i = e - t + \frac{F(Nt)}{N} = 125 - t + \frac{F(5t)}{5}.$$

Figure 1 shows a graphic representation of the individual and group benefits and costs of placing a token in the group account. The marginal cost of allocating one token to the group account is the one point that could have been earned from placing the token in the private account. That same token can also be placed in the group account generates returns not only for the individual player in the form of their marginal private benefit but also to the group in the form of the marginal social benefit.

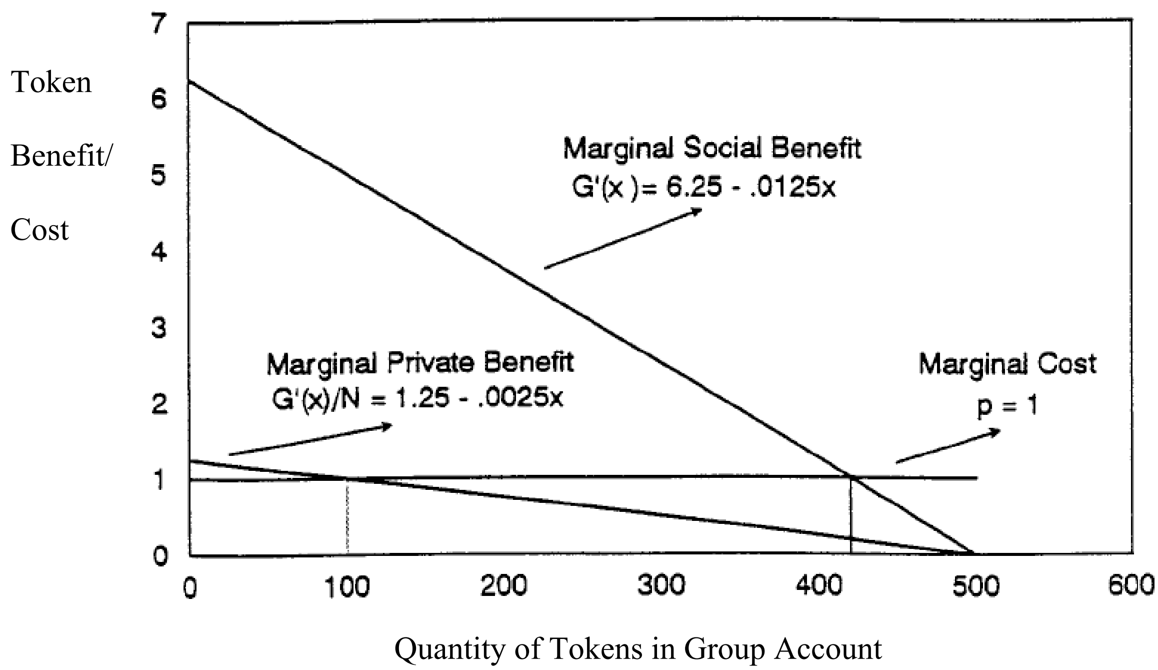

Figure 1: Interior Nash and Pareto Optimal Outcomes (from LWW)

Thus, setting the marginal cost equal to marginal private benefit yields a Nash equilibrium provision for the group of 100 tokens. Similarly, setting marginal cost equal to marginal social benefit generates a group Pareto optimum provision of 420 tokens. It is important to note that these solutions are in aggregate contributions. Therefore, there exists a continuum of individual best-response strategies to behavior on the part of their group members' decisions to reach these optimal contribution levels. All players have the same endowment of 125 tokens, and agents make a single allocation decision for the group. Therefore, all agents will seek to maximize social benefit. At the optimum, each agent should be making an allocation decision of 84 tokens for each member of his or her group. Higher allocations should be seen in the agent treatment than in the no-agent treatment, where if all subjects are behaving symmetrically, they would each give 20 tokens to the group account to form a Nash equilibrium.

The interior, Nash-equilibrium environment is more complex than the boundary solution. Subjects no longer face a Nash equilibrium that is unique on the individual level or involves a dominant strategy. Thus, there exists a coordination issue involved in reaching the unique aggregate Nash equilibrium since there are many individual Nash outcomes. Additionally, the complex MPCR in the non-linear case makes the calculation of the solutions more difficult than in the boundary condition.

Therefore, participants are given detailed information on the payoff structure of the experiment in the form of payoff tables in order to compare group and individual returns to specific contributions to the group account (see Appendix C). These are provided in addition to the written instructions and were given to each participant. The tables are reviewed by the experimenter along with the instructions before the beginning of the experiment. Included in these tables is information on the additional return to the group and the additional return to the individual for contribution to the group account as well as examples of earnings from the group account.

Subjects were given questionnaires in every treatment to test their understanding of the instructions and the environment. Participants demonstrated high levels of understanding of the interior and boundary environments. In addition, the questionnaire and answers were carefully reviewed with the subjects to further ensure understanding.

1.2.2.3 Payment

At the end of the experiment, participants were paid in private for their earnings in one randomly chosen period during their experimental session. Paying participants for one period rather than all of their decisions is chosen in order to avoid wealth effects throughout the experiment. Points are converted to dollars in the following fashion. In the interior-solution design 1 point = 0.05 dollars and in the boundary-solution design 1 point = 0.03 dollars. The difference in payment comes from the increased difficulty of the interior-solution case and consequently its longer sessions. Boundary-solution sessions often lasts between an hour to an hour and a half, and interior-solution sessions lasts approximately just under two hours. All participants also earn a five dollar participation bonus. Thus, average earnings are around \$13-15 in the boundary-solution sessions and \$19-21 in the interior-solution sessions.

1.3 PREDICTIONS

We predict that,

Hypothesis 1: *In treatments with an agent, the efficient allocation is more likely than in treatments without an agent.*

In no-agent rounds, the individual incentive to reach the efficient, Pareto-optimal group allocation does not exist, since subjects would prefer to free-ride off the contributions of their group members.[4] Because there is no incentive to give efficiently, Pareto optimal decisions should not be seen in no-agent rounds.

In contrast, each agent has both the individual and group incentive to allocate the efficient, Pareto-optimal amount of tokens to the group account, and thus, this efficient-giving behavior should be seen frequently throughout agent rounds. Although theory predicts that efficient allocation should be chosen in every round, the outcomes depend entirely on the rationality of the representative and their understanding of the payoffs of the game. Therefore, outcomes in the agent treatment may vary depending on subjects' understanding of the game. If

[4] Contributions are still necessary to reach the Nash equilibrium in the interior solution. Free-riding behavior still exists in this solution set up above Nash equilibrium contributions.

agents are not confident in making efficient allocations for the public good initially, then I would expect that as rounds continue, learning (either from experience or from other players) will take place, and players would gain confidence in making fully efficient choices. Especially in the boundary-solution case, I would expect almost every agent allocation decision to be 125 tokens for each member of the group by the end of the 20 rounds since it is more easily understood than the interior solution.

Corollary: Agent allocations should be higher than no-agent allocations.

Due to their increased incentives to make higher contributions, agents should make higher contributions to the group account. Additionally, in the agent treatment, there no longer exists the incentive to free ride, therefore higher contributions yield higher payoffs.

Hypothesis 2: The majority of votes will vote to have an agent make the group's allocation decision.

Due to predicted increased giving amounts and more frequent efficient allocations, individual and group profits in the agent treatment should be higher than profits in the no-agent treatment. If subjects expect payoffs similar to previous treatment rounds, they should follow the increase in profits with their vote to have an agent. Thus, we should see agency emerge as the giving mechanism of choice in every round during the voting treatment.

1.4 RESULTS

The interior no-agent decisions are similar to the experimental findings of LWW. Because the interior-solution design is based upon the previous work of LWW, it is important to compare the interior experimental findings with their results. The interior no-agent design mimics the Z125-

DET treatment from the LWW paper where they found an average contribution of 43.31 tokens with a median of 45 and a standard deviation of 33.07. With these sample estimates, a 95% confidence interval for the expectation of the population mean can be calculated yielding (39.57, 47.05). Their Z125-DET treatment should be compared to my interior no-agent decisions that occupy the first 20 rounds since subjects have not experienced or been influence by any other treatment during the course of the session. These no-agent first decisions have a slightly higher average contribution of 49.96 with a median of 44.5 and a standard deviation of 35.56. In constructing a 95% confidence interval of the population mean from my results, I find an interval of (45.96, 53.96) around the sample mean. Even though the interior no-agent sample mean does not lie in the confidence interval for LWW sample, the confidence intervals overlap, sharing contribution values in common. This implies that the results are similar within a 5% level of confidence, allowing us to draw the conclusion that behavior in the interior design is not driven by experimenter effects but by the experimental environment.

Giving in the boundary no-agent design closely mimicked results in previous public good games. The trends on giving during the no-agent boundary design follow the same patterns as seen in a host of public good experimental literature. Mainly, contributions start high and deteriorate toward the Nash equilibrium contribution of zero as rounds increase. In Figure 2, this behavioral trend can easily be identified.

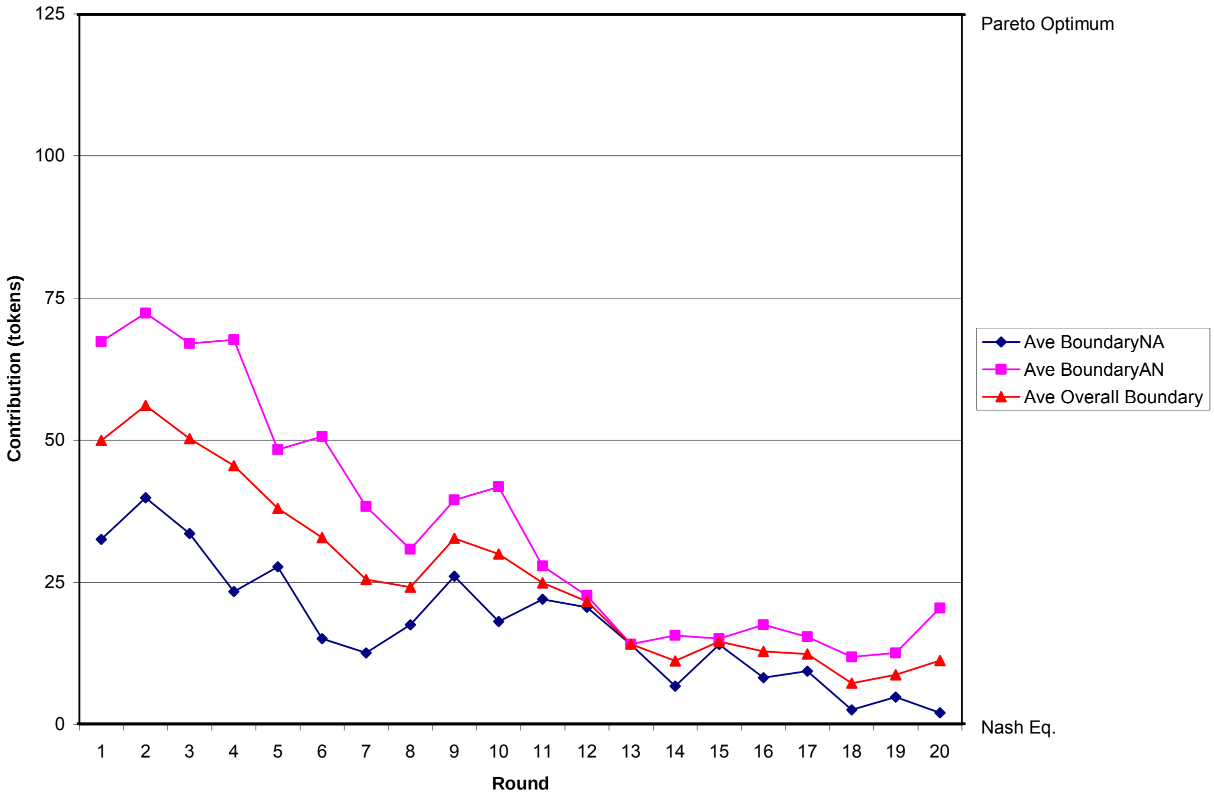

Figure 2: Boundary No-Agent Contributions by Session

The literature in linear public good games is quite large, thus I will compare my results to Issac, Walker, and Thomas (1984) which was the first public good experiment testing the effects of group size and different MPCR rates. Their experiment is a benchmark upon which a whole wealth of literature on public goods is founded. With a group size of 4 and an MPCR of 0.3, Issac, Walker, and Thomas showed average contribution rates among players of 19% over 10 rounds. Although group size in my experiment is 5 and the MPCR is 0.318, my results are comparable.[5]

The average contribution rate is 20% over 20 boundary no-agent rounds. At the bookends of that same session, IWT found a contribution rate of 43% in the first round and 17%

[5] Issac, Walker, and Thomas will henceforth be abbreviated as IWT.

in the last round. In the first no-agent round, the average contribution rate is 40%, and after 20 rounds the contribution rate drops to 9%. Although there are many design differences between my experiment and IWT, contribution rates for the standard linear contribution game are similar.

Social welfare increases under the agent treatment compared to the no-agent treatment. The use of an agent to reduce the problems associated with free-riding can greatly increase overall social welfare. If the benefits to an agent are not appreciated or understood by the group, then the change in giving-mechanism will not be effective at benefiting the group.

Table 3: Comparison of Social Wealth Changes by Session (in average profits per player)

Session	No Agent	Agent	Voting
BoundaryNA	297.8	399.4	420.4
BoundaryAN	313.6	406.0	429.4
InteriorNA	304.8	325.7	328.3
InteriorAN	304.1	316.8	321.0

Changes in social welfare are measured in average profits per player as shown in Table 3. Overall, profits per player increase in the agent treatment over the no-agent treatment and rise even further from the agent treatment to the voting treatment. These increases in profit are especially statistically significant in the boundary design sessions using one-sided Wilcoxon signed rank tests comparing each player's average profits from the no-agent treatment to the agent treatment and also from the agent treatment to the voting treatment. We can reject the null hypothesis that average profits are the same in the no-agent and the agent treatment since the p-vale of this test is 0.000. The null hypothesis that average profits are the same in the boundary agent and voting rounds can also be rejected with a p-value of 0.000. These one-sided signed rank rests using data from the InteriorNA design session demonstrate results that reject the null hypothesis that average profits are the same in the no-agent treatment and the agent treatment with a p-value of 0.0176. In the InteriorAN sessions, the null hypothesis that no-agent and agent average profits are the same could not be rejected at the 10% level. In general, group members are earning higher profits in agent rounds increasing social welfare compared to no-agent rounds.

In the final rounds of the session, there is no "drop off" in interior design no-agent decisions while this drop-off in giving is present in the boundary no-agent decisions. As mentioned previously, in traditional private-giving games, contributions to the group account start off high and experience large declines toward the Nash equilibrium as rounds increase. Even though the Nash equilibrium of the interior solution calls for contributions to take place in all rounds, we would still expect the general allocation trend decline to hold as subjects learn from the experimental environment and attempt to coordinate contributions with other group members. Thus, we would expect to see higher than Nash giving in early rounds and a decline in giving toward the Nash equilibrium as the rounds elapse.

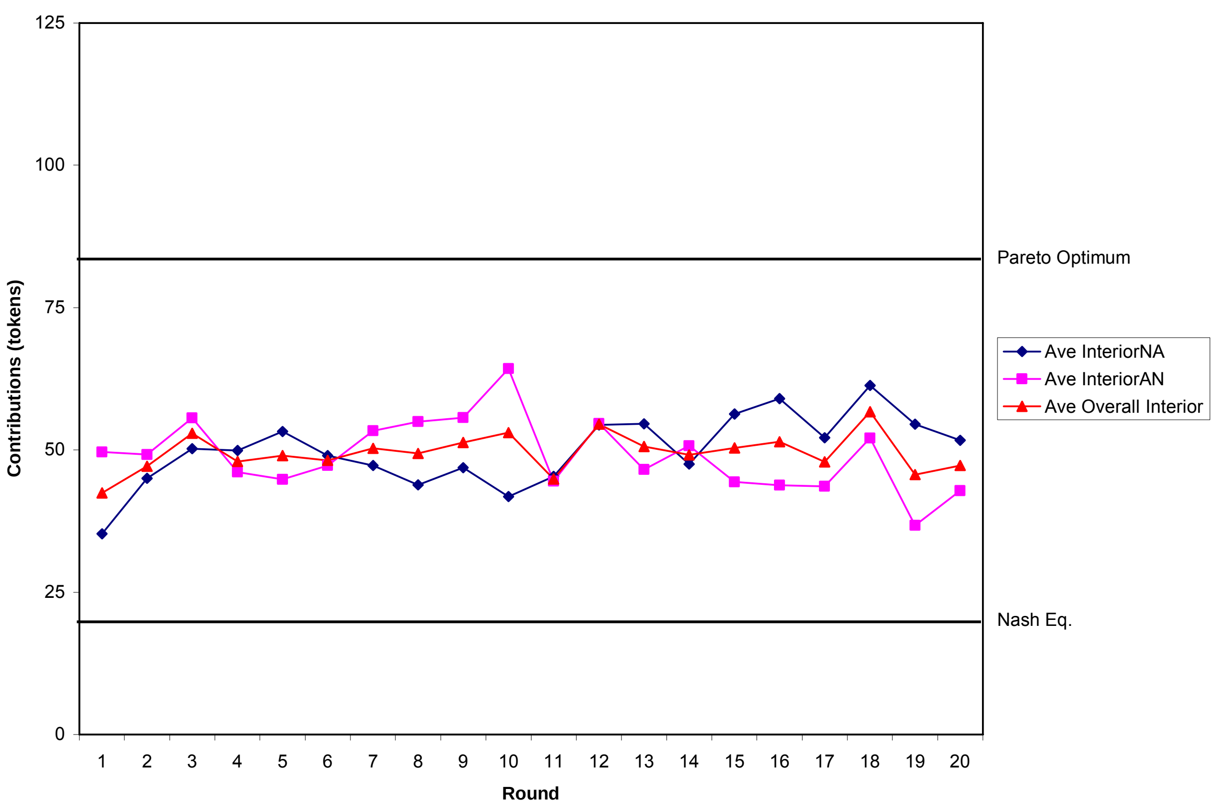

Figure 3: Interior No-Agent Contributions By Session

However in all interior-design treatments, there is no decline in contribution behavior in the final rounds of the no-agent treatment. No-agent giving behavior in the interior design sessions is shown in Figure 3. As the 20 rounds of no-agent decisions elapse in the interior design, individual contribution behavior does not significantly change. While in the boundary design, as the 20 rounds come to a close a large decline in contribution behavior is clearly visible. Some explanations for this deviation from typical contribution behavior in the interior-solution case are that group members might be continuing attempts to coordinate when the 20 rounds end, the overall lack of understanding of the payoff structure, or since the mathematic structure is more complicated than the boundary design, attempts at kindness may not have become frustrated yet. The environmental factors are simplified in the boundary design in which the standard drop-off toward Nash equilibrium behavior is observed.

Observation 1: Consistent with hypothesis 1, agent allocations are greater than no-agent allocations, specifically they are closer to the group efficient Pareto optimum.

Figure 4 shows the results of the pooled data between all agent and no-agent contribution decisions during the first and last five rounds of the first treatment in each session. The gap between the agent and no-agent allocations becomes even wider in the last 5 rounds of decisions than was already present in the first five rounds. This result is very dramatic in the boundary condition, as expected since the distance between the Pareto optimal and Nash equilibrium contribution level is greater in this design.

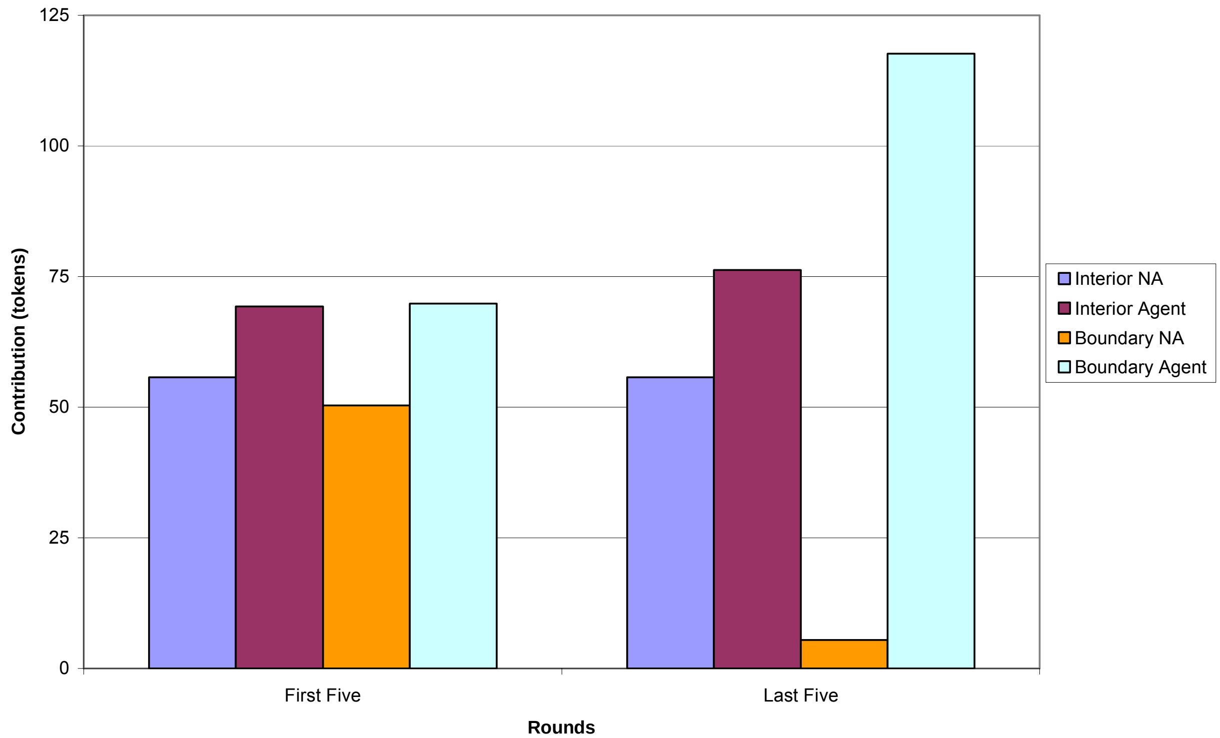

Figure 4: First Five vs. Last Five Rounds of First Treatment in all 4 session types

Although agent allocations are greater than no-agent allocations in all designs and rounds, they do not fully reach Pareto optimal levels as predicted would occur by the end of the treatment rounds. On average, agent allocations come quite close to group optimal levels in these results (within approximately 8 tokens in each solution design).

This gap between agent and no-agent giving decisions can also be seen in Table 4 which shows overall means, medians, and standard deviations in contribution behavior in both the interior and boundary cases. Agent allocation decisions approach Pareto optimality as seen especially when comparing results from the first 20 rounds of both agent and no-agent treatments. In both solution designs, a 95% confidence interval around the pooled means for agent decisions do not contain the solution's Pareto-optimal allocation. However, the medians for the interior pooled agent and boundary agent decisions exactly pinpoint the Pareto-efficient provision of the group account. These approximately Pareto-efficient allocations to the group account combined with greater social welfare shows that the agent treatment attains greater efficiency for the public good than the no-agent treatment.

Table 4: Data for Treatments

Treatment	# of Obs*	Mean	Median	Standard Dev.	Range
Interior					
No Agent (first)**	300	49.96	44.5	35.557	0 to 125
No Agent (pooled)	600	49.492	49.5	33.809	0 to 125
Agent (first)***	300	71.809	80	25.331	0 to 125
Agent (pooled)	600	74.177	84	23.243	0 to 125
Voting	600	73.037	84	24.006	0 to 110
Boundary					
No Agent (first)**	300	17.553	5	29.391	0 to 125
No Agent (pooled)	600	26.202	5	39.603	0 to 125
Agent (first)***	300	100.23	125	37.532	0 to 125
Agent (pooled)	600	99.187	125	39.488	0 to 125
Voting	600	114.99	125	28.89	0 to 125

*Number of observations = (# of rounds) * (# of replications) * (# of individuals in each group).
**Presents results from within no-agent treatment when no-agent decisions occupy first 20 rounds
***Presents results from within agent treatment when agent decisions occupy first 20 rounds

Further evidence of the difference between agent and no-agent allocation decisions can be confirmed by two-sided Wilcoxon Mann-Whitney (MW) tests, which reveal large differences between contribution mechanisms as seen in Table 5. Mann-Whitney rank sum tests form a null-hypothesis that two samples are drawn from the same population and therefore have equivalent probability distributions. I performed these MW tests to ensure that average individual contribution decisions from agent and no-agent rounds are from decidedly different samples and different populations. The p-values for the effect of agency on all interior and boundary pooled data are significant (0.000), meaning agent and no-agent decisions belong to decidedly different populations.[6]

Table 5: Two Sided Wilcoxon Mann-Whitney Test P-values

Boundary Contribution Decisions	**Agency**	**First**
Pooled Agent vs. NA	0.000	
Agent First vs. Second		0.518
No Agent First vs. Second		0.065
Interior Contribution Decisions	**Agency**	**First**
Pooled Agent vs. NA	0.000	
Agent First vs. Second		0.443
No Agent First vs. Second		0.917

In addition to these agent pooled MW test results, I find an order-effect in boundary no-agent decisions. The no-agent decisions that come first are significantly different from the no-agent decisions that come second. If the no-agent decisions occupy the second 20 rounds of the session, then the subjects have seen the value of increased giving to the public good under the agent treatment and therefore try to coordinate on higher values of giving throughout the 20 no-agent rounds. No other order effects are present.

Observation 2: Consistent with hypothesis 2, agency was the overwhelming majority outcome when choosing a mechanism of contribution to the public good.

Because of both the increased individual and social wealth in the agent treatments, I would expect that every player would prefer to have an agent make a contribution decision for the group instead of playing the no-agent treatment where group coordination problems are always prevalent. In the 600 opportunities for subjects to place a vote in the interior-design

[6] These p-values represent at what percentage of confidence one can reject the null hypothesis that the two samples are drawn from the same population.

treatments, agents are chosen 75.17% of the time. Agency is the majority voting outcome 96.67% of the time. "No-agent" is selected only by one group at the onset of the voting rounds for 4 rounds. Similarly, in the boundary-design treatments, there were also 600 opportunities for subjects to cast their vote, and agency is chosen 82.3% of the time. Agency is the majority voting outcome 99.2% of the time. Only once is "no-agent" selected in this design. Therefore, not only did subjects recognize the benefits of having a group agent, but they preferred to have an agent decide contribution behavior to the group account.

Agent decisions in voting rounds approach (and quite often attain) the Pareto optimal outcome. The contribution decisions made during voting rounds are even higher than previous agent contributions during the same experimental session. Reasons for the increased giving behavior are familiarity with the experimental design by the end of the experiment, learning from the previous 40 rounds, and a boost to agent confidence in decision making from the vote. This added certainty is accomplished through group approval. Thus, agents in these last 20 rounds, having the benefit of learning from the previous agent and no-agent treatments, are more likely to make group-beneficial decisions.

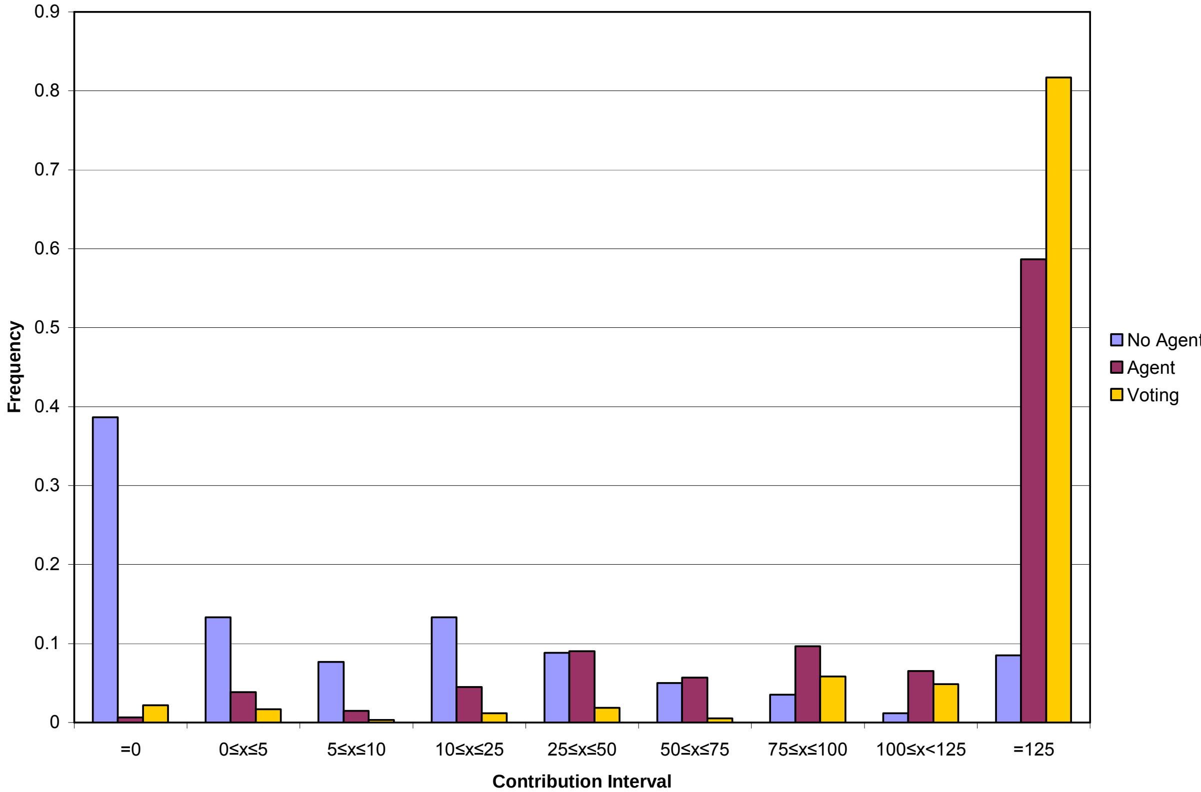

Figure 5: Boundary Contribution Choices by Frequency

For evidence of this increase in giving behavior in voting rounds, see Figures 5 and 6 which show the frequencies of contribution behavior by treatment in both the boundary and interior-solution designs. In both solution designs, Pareto-optimal allocations are more frequently chosen in the agent treatment than in the no-agent treatment (confirming hypothesis 1). In addition, the frequency of contributions in the voting treatment in the interval near the Pareto optimum of both designs is higher than in the agent condition. For further evidence of the increased optimal contribution behavior in the voting treatment, please see Figures 7 through 10 in Appendix D which show allocation averages by group and session in both the interior and boundary designs.

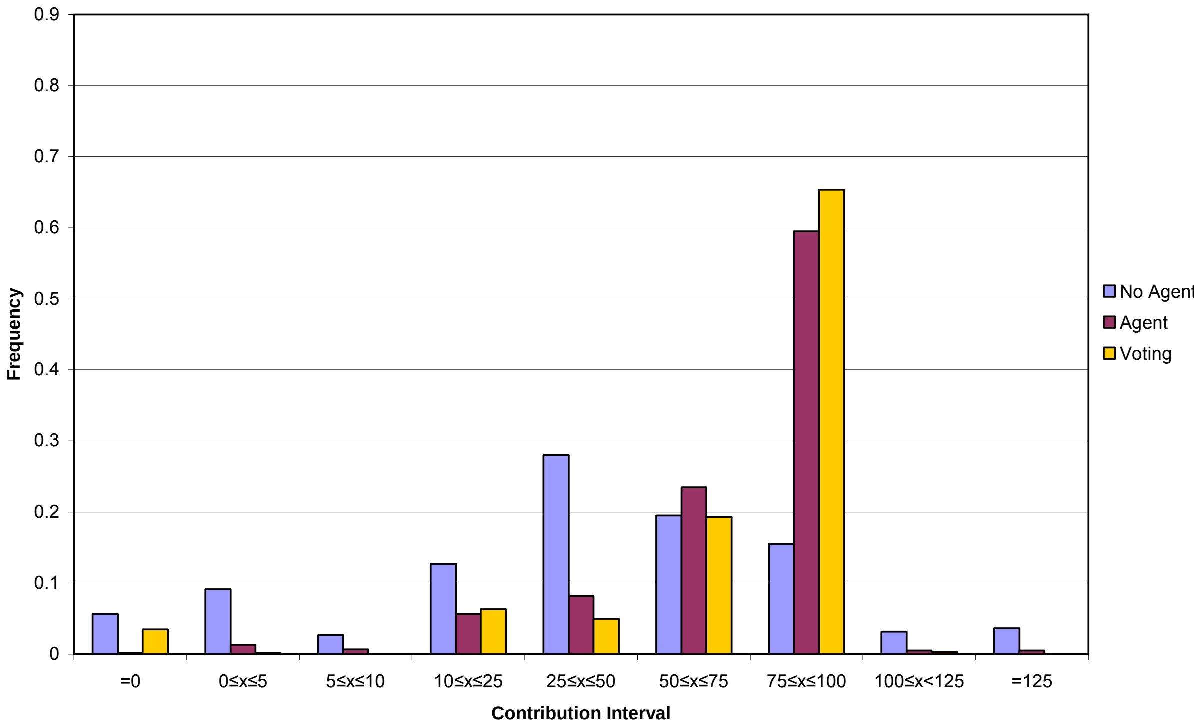

Figure 6: Interior Contribution Choices by Frequency

1.5 CONCLUSION

As seen here and in the earlier literature, when groups are privately giving to a public good, subjects do not achieve Pareto optimal outcomes either due to lack of understanding of the optimal choice or group coordination failure. In this experimental environment, using the agent mechanism, we remove the problem of free-riding, simplifying a subject's dominant strategy to an efficient allocation. Therefore, the agent treatment can easily test if subjects understand the benefit of Pareto-optimal giving or if there exists confusion in the experimental environment.

The optimal giving seen in agent rounds provides evidence against confusion being the culprit of efficient giving in private environments.

Agents in general made more optimal choices and improved group outcomes over individuals contributing to the pubic good. In both solutions, agents are more likely to make Pareto-optimal outcomes with higher frequency given the reinforcement of a group vote and the learning taking place throughout the experiment's 60 rounds.

As for the mechanism of contribution, overwhelmingly subjects back agency with their vote. Since the benefits of the public good can be best exploited under the agency mechanism, group members are willing to support it with their vote and pave the way for agents to make efficient group decisions.

Thus, opportunities to have a centralized agency make contribution decisions for the public good should increase efficiency in the allocation structure for the good. We could experience the added benefits of agency in tax structures by the government who decides how much each household must contribute for a given public program to succeed. The benefits of this agent system can also be found in other organizations that are able to make binding allocation decisions for a good that will be enjoyed by all its members. These organizations can be as small as a tour group deciding how much to tip their guide or a neighborhood deciding how much to contribute to form a neighborhood watch program.

This experiment is a first step in understanding the affect of agency (or dictatorship) on group opinion and decision-making. But the question remains, can the efficiency seen in agent rounds be carried over to an environment where there exists group heterogeneity? In order to address this question, we need to look further into who these agents are and what incentives they have to make their group decisions. Do agents in this experiment make efficient decisions because they cannot discriminate between group members? The experiment above is not equipped to answer these questions since all subjects are effectively the same, facing the same endowment and the same preferences, and the agent makes one allocation decision for all group members. Therefore, looking into the agent's problem and exploring some experimental extensions is a natural next move. Endowing the subjects with a distribution of wealth and allowing the agent to make an allocation decision for each wealth type should provide an opportunity for agents to discriminate among wealth types in the group while confronting issues of fairness. In creating disparity among group members in their wealth levels, it will be interesting to see if groups are able to attain Pareto-optimal outcomes and which mechanism for allocation, agent or no-agent, will be more popular among different wealth types.

These questions are interesting to answer because when we think about the frequency of our choices, there are a multitude of occasions when we do not make decisions for ourselves. In these circumstances, we can only hope that our agents are making the most optimal choices for the goods we collectively enjoy.

1.6 APPENDICES

1.6.1 Appendix A

The following is the instructions for the first twenty rounds of the interior no-agent treatment with the agent changes in brackets.

This is an experiment in decision making. The Department of Economics has provided funds for this research. During the course of the experiment, you will make a series of decisions.

You will be paid for participating, and the amount of money you earn depends on the decisions that you and the other participants make. At the end of today's session you will be paid privately and in cash for your decisions. Please do not talk to one another for the duration of the experiment.

The first phase of the experiment will consist of 20 rounds. When these rounds have elapsed, please wait for further instruction.

At the beginning of the experiment, everyone is randomly assigned to a group of 5 individuals in the first round. During the course of the experiment, your group composition does not change. You will be in a group with the same 4 other members for the experiment. All decisions you make in this experiment are anonymous; therefore, please do not reveal any of your decisions to any other participant.

At the beginning of each round, each of you will be gifted with 125 tokens. These tokens can be invested between two accounts: the group account and your own private account. When each round starts, you will be asked to make a decision [for the group] about how many tokens you would like to allocate to the group account. Tokens not contributed to the group account remain in your private account. The number of tokens in the group account equals the total of all the tokens contributed by all 5 members of your group at the end of the round.

[After each member of the group has made decisions about token allocations, then one member will be chosen at random to be the group agent and their decision about the group allocation will be implemented. Thus, the number of tokens in the group account equals the 5 times the amount the agent decides each person will contribute.]

An individual's earnings are determined from the number of tokens left in their private account and the total number of tokens in the group account. The total earnings from the group account are divided equally among all five individuals in the group.

For each token remaining in your private account at the end of the round, you earn 1 point. If you [the agent has] have decided to allocate *X* number of tokens to the group account, then you have *125-X* tokens remaining private account. These tokens earn you (*125-X*) points.

Earnings from the group account are decided according to an equation, which you do not need to know. Further explanation about earnings from the group account will follow.

Remember, the total number of tokens in the group account equals the total of all the tokens contributed by all members of your group. We'll denote the total number of group account tokens by *Y* [*5*X*].

Total (in points) from the group account are:

$$6.25Y - 0.0025Y^2$$

Your share in those earnings is one-fifth. Therefore, your individual earnings from the group accounts are (in points):

$$\frac{6.25Y - 0.0025Y^2}{5}$$

Attached are two tables to aid in understanding how this payoff function works with the 125 token gift given to each of you at the beginning of each period. The first shows group and individual earnings from various token allocations to the group account. The second shows examples of possible earnings from the group account.

Consider first the group's earnings from the group account. Table 1 *(which is table 8 in this text)* displays information on the group's per-token earnings from the group account. Beginning with the first token, each additional token allocated to the group account will increase the group's earnings from this account by a smaller amount than the token before it. For example, the 1st token allocated to the group account earns 6.238 points for the group as a whole. The 2nd token allocated to the group account earns an additional 6.225 points for the group as a whole. The 99th token allocated to the group account earns an additional 5.013 points for the group as a whole, and so on.

The 420th token allocated to the group account earns an additional 1.000 point for the group as a whole. Thus, the additional return to the group as a whole from the allocation of this token to the group account is the same as the return the individual who allocated this token would have received if it had been allocated to the private account. If more than 420 tokens are allocated to the group account, each additional token allocated to the group account increases earnings for the group as a whole by a smaller amount than the alternative option of allocating this token to the private account. The 500th token allocated to the group account earns a zero additional return to the group as a whole. Beyond 500 tokens, each additional token decreases earnings from the group account.

Now consider the individual's earnings from the group account. Table 1 *(which is table 9 in this text)* displays information on each individual's per-token earnings from the group account. Beginning with the first token, each additional token allocated to the group account will increase each individual's earnings from this account by a smaller amount than the token before it. For example, the 1st token allocated to the group account earns 1.248 points for each individual in the group. The 2nd token allocated to the group account earns an additional 1.245 points for each individual in the group. The 99th token allocated to the group account earns an additional 1.003 points for each individual in the group, and so on.

The 100th token allocated to the group account earns an additional 1.000 point for each individual in the group. Thus, the additional return to each individual from the allocation of this token to the group account is the same as the return the individual who allocated this token would have received if it had been allocated to the private account. If more than 100 tokens are allocated to the group account, each additional token allocated to the group account increases earnings for each individual in the group by a smaller amount than the alternative option of allocating this token to the private account. The 500th token allocated to the group account earns a zero additional return to each individual. Beyond 500 tokens, each additional token decreases earnings from the group account.

Consider how this information is related to the earnings information on your computer screen and on Table 2. The first 39 tokens allocated to the group account earn 234.2 points for the group as a whole. When an additional 39 tokens are allocated to the group account (for a total of 78 tokens), the group's earnings from this account increase 215.3 points. Thus the group's earnings increase by a smaller amount as more tokens are allocated to this account.

The first 39 tokens allocated to the group account earn 46.8 points for each individual in the group. When an additional 39 tokens are allocated to the group account (for a total of 78 tokens), each individual's earnings from the group account increases 43.1 points. Thus, each individual's earnings increase by a smaller amount as more tokens are allocated to this account.

An Example: In the first round, your contribution decision was 80 tokens. The other 4 members of your group made contribution decisions of 55, 104, 120, and 32. Therefore the group account has 80 + 55 + 104 + 120 + 32 = 391. Earnings from the group account are:

$$6.25(391) - 0.00625(391)^2 = 1488 \text{ points.}$$

As one of 5 group members, you will receive one-fifth of total group earnings. Therefore your share of total group earnings is 298 points.

This whole example is different in the agent treatment since only the agent's decision matters (see below).

[An Example: In the first round, your contribution decision as the agent of the group was 77 tokens. The other 4 members of your group made agent contribution decisions of 91, 104, 120, and 32. Only one agent will be randomly chosen from you group of 5, and after all decisions were made the member who chose 91 tokens was randomly picked to be the leader. Thus, everyone must contribute 91 tokens to the group account. Therefore the group account has 5*91 = 455 tokens. Total group earnings are:

$$6.25(455) - 0.00625(455)^2 = 1549.85 \text{ points.}$$

As one of 5 group members, you will receive one-fifth of total group earnings. Therefore your share of total group earnings is 309.97 points.]

Your total payoff for each round is the sum of your earnings from the private and the group account, and will indicated on your computer screen. Earnings from the group account depend only on the total number of tokens in that account.

Your earnings from the experiment will be drawn from the decisions from one round out chosen at random plus the $5 show up fee. The exchange rate for points to dollars is 1 point=3 cents. Thus, 300 points=$9. At the end of the experiment, you will be asked to come to the side room where you will be paid in private.

During the experiment, you are not permitted to speak or communicate with the other participants. If you have a question while the experiment is going on, please raise your hand and one of us will come to your desk and answer it. At this time, do you have any questions about the instructions or procedures? If you have a question, please raise your hand and one of us will come to your seat and answer it.

1.6.2 Appendix B: Questionnaires

Questionnaire -- Part 1

1. (True or False) The group earns more when there are 78 tokens in the account than when there are 195 tokens in the account (hint: look at Table 2).[7]

2. (True or False) The group's additional earnings are higher with 140 tokens in the group account than with 380 tokens in the group account (hint: look at Table 1).[8]

3. What is your individual additional return for the 420th token placed in the group account?

4. (True or False) Players remain in the same group of 5 players in all rounds.

5. If the chosen agent of the group decides each individual contributes 81 tokens to the group account, how many tokens will be in the group account at the end of the period?

    a. How many tokens are in your private account?

6. (True or False) Each player makes an allocation decision for the group every round but only one allocation is chosen.

Questionnaire -- Part 2

1. If all of the other members of the group together allocate 185 tokens to the group account and you allocate 45, how many tokens will be in the group account at the end of the period?

    a. How many tokens are in your private account?

---

[7] which is table 8 in this text

[8] which is table 9 in this text

2. (True or False) Tokens in the group account are from the individual allocation decisions from each of the members in your group.

Questionnaire -- Part 3

1. (True or False) If 3 members of your group decide not to have an agent and 2 members decide to have an agent, then the group proceeds to make group allocation decisions and one is randomly selected to be implemented.

2. (True or False) If one outcome receives 3 or more votes, it is the majority outcome and will be implemented.

3. (True or False) No matter if the group decision outcome is for an agent, I make an allocation decision in every period.

1.6.3 Appendix C: Tables Given to Subjects

Tables 6 and 7 were given to subjects depending on whether it was given in a no-agent or agent environment. They were used to help aid subjects in understanding the interior solution environment. In addition, subjects were given table 8 to give examples of possible earnings from different allocations to the group account.

Table 6: Examples of possible earnings from the group account (in points)

Tokens in the group account	Additional tokens	Total group earnings	Group's additional earnings	Your 20% share of group earnings	Your additional earnings
0	-	0	-	0	-
39	39	234.2	234.2	46.84	46.84
78	39	449.5	215.3	89.9	43.06
117	39	645.7	196.2	129.14	39.24
156	39	822.9	177.2	164.58	35.44
195	39	981.1	158.2	196.22	31.64
234	39	1120.3	139.2	224.06	27.84
273	39	1240.4	120.1	248.08	24.02
313	39	1343.9	103.5	268.78	20.7
352	39	1425.6	81.7	285.12	16.34
391	39	1488.2	62.6	297.64	12.52
430	39	1531.9	43.7	306.38	8.74
469	39	1556.5	24.6	311.30	4.92
508	39	1562.1	5.6	312.42	1.12
547	39	1548.7	-13.4	309.74	-2.68
586	39	1516.3	-32.4	303.26	-6.48
625	39	1464.8	-51.5	292.96	-10.3

Table 7: No-Agent Per Token Earnings from the Group Account

Tokens allocated to the group account	Group's additional return	Individual's additional return (20% of group's return)	Tokens allocated to the group account	Group's additional return	Individual's additional return (20% of group's return)	Tokens allocated to the group account	Group's additional return	Individual's additional return (20% of group's return)
1	6.238	1.248	259	3.019	0.604	519	-0.231	-0.046
2	6.225	1.245	260	3.006	0.601	520	-0.244	-0.049
3	6.213	1.243	261	2.994	0.599	521	-0.256	-0.051
19	6.019	1.204	279	2.769	0.554	539	-0.481	-0.096
20	6.006	1.201	280	2.756	0.551	540	-0.494	-0.099
21	5.994	1.199	281	2.744	0.549	541	-0.506	-0.101
39	5.769	1.154	299	2.513	0.503	559	-0.731	-0.146
40	5.756	1.151	300	2.500	0.500	560	-0.744	-0.149
41	5.744	1.149	301	2.488	0.498	561	-0.756	-0.151
59	5.519	1.104	319	2.269	0.454	579	-0.981	-0.196
60	5.506	1.101	320	2.256	0.451	580	-0.994	-0.199
61	5.494	1.099	321	2.244	0.449	581	-1.006	-0.201
79	5.269	1.054	339	2.019	0.404	599	-1.231	-0.246
80	5.256	1.051	340	2.006	0.401	600	-1.244	-0.249
81	5.244	1.049	341	1.994	0.399	601	-1.256	-0.251
99	5.013	1.003	359	1.769	0.354	619	-1.481	-0.296
100	5.000	1.000	360	1.756	0.351	620	-1.494	-0.299
101	4.998	0.998	361	1.744	0.349	621	-1.506	-0.301
119	4.769	0.954	379	1.519	0.304	625	-1.556	-0.3112
120	4.756	0.951	380	1.506	0.301			
121	4.744	0.949	381	1.494	0.299			
139	4.519	0.904	399	1.263	0.253			
140	4.506	0.901	400	1.250	0.250			
141	4.494	0.899	401	1.238	0.248			
159	4.269	0.854	419	1.013	0.203			
160	4.256	0.851	420	1.000	0.200			
161	4.244	0.849	421	0.998	0.198			
179	4.019	0.804	439	0.769	0.154			
180	4.006	0.801	440	0.756	0.151			
181	3.994	0.799	441	0.744	0.149			
199	3.763	0.753	459	0.519	0.104			
200	3.750	0.750	460	0.506	0.101			
201	3.738	0.748	461	0.494	0.099			
219	3.519	0.7038	479	0.269	0.054			
220	3.506	0.7012	480	0.256	0.051			
221	3.494	0.6988	481	0.244	0.049			
239	3.269	0.6538	499	0.013	0.003			
240	3.256	0.6512	500	0.000	0.000			
241	3.244	0.6488	501	-0.013	-0.003			

Table 8: Agent Per Token Earnings from Group Account

Agent's allocation decision to group account	Tokens allocated to the group account	Group's additional return	Individual's additional return (20% of group's return)
-	1	6.238	1.248
1	5	6.194	1.239
2	10	6.131	1.226
4	20	6.006	1.201
8	40	5.756	1.151
12	60	5.506	1.101
16	80	5.256	1.051
20	100	5.000	1.000
24	120	4.756	0.951
28	140	4.506	0.901
32	160	4.256	0.851
36	180	4.006	0.801
40	200	3.750	0.750
44	220	3.506	0.701
48	240	3.256	0.651
52	260	3.006	0.601
56	280	2.756	0.551
60	300	2.500	0.500
64	320	2.256	0.451
68	340	2.006	0.401
72	360	1.756	0.351
76	380	1.506	0.301
80	400	1.250	0.250
84	420	1.000	0.200
88	440	0.756	0.151
92	460	0.506	0.101
96	480	0.256	0.051
100	500	0.000	0.000
104	520	-0.244	-0.049
108	540	-0.494	-0.099
112	560	-0.744	-0.149
116	580	-0.994	-0.199
120	600	-1.244	-0.249
124	620	-1.494	-0.299
125	625	-1.556	-0.311

1.6.4 Contribution Graphs by Group, Treatment, and Round

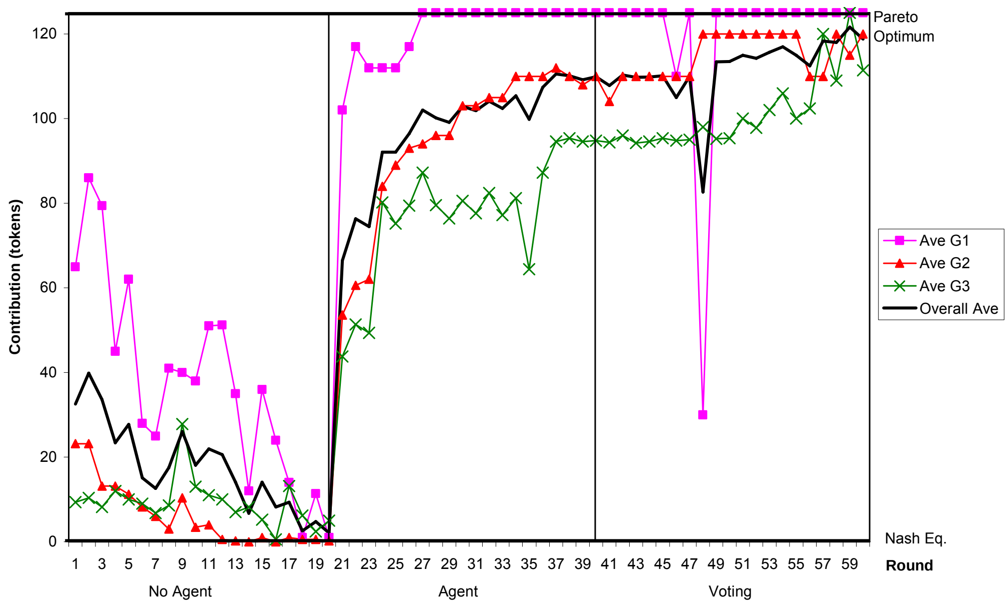

Figure 7: Boundary No-Agent First Contributions by Round

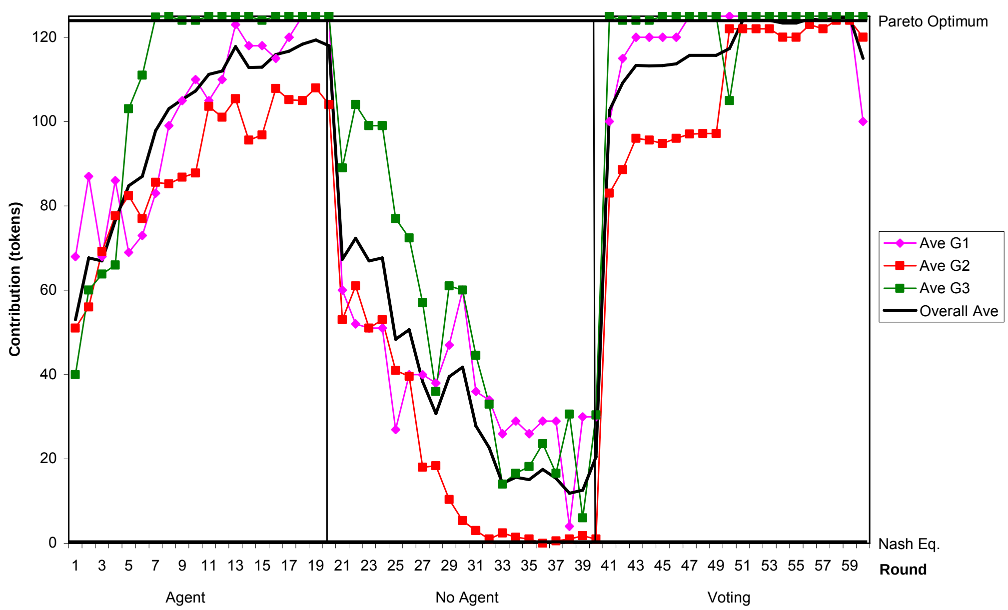

Figure 8: Boundary Agent First Contributions by Round

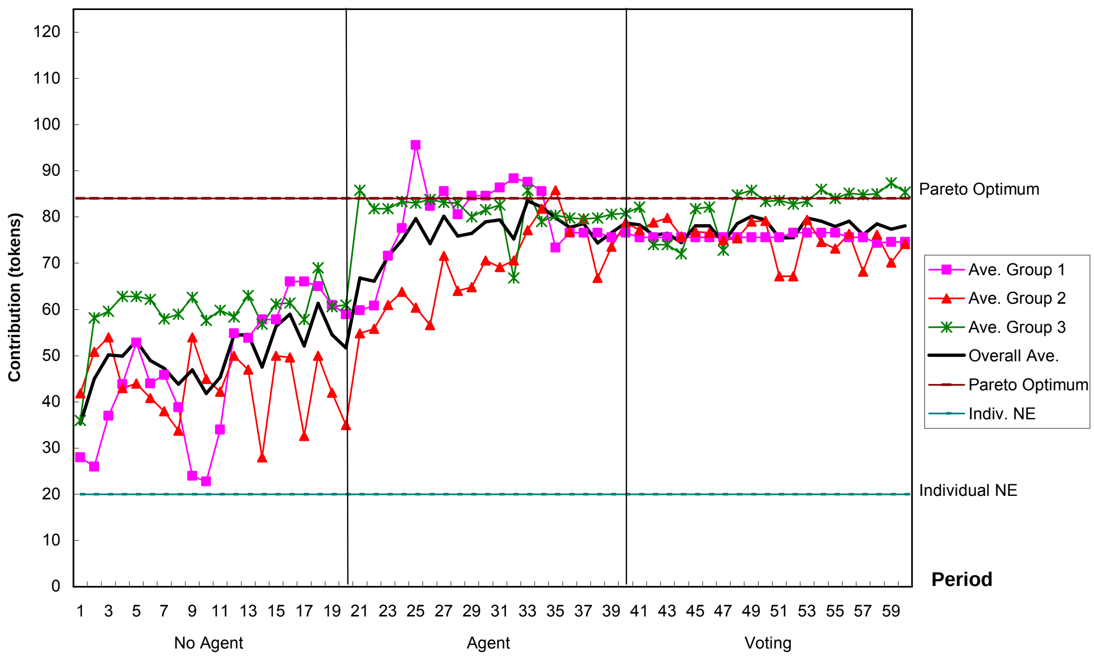

Figure 9: Interior No-Agent First Contributions by Round

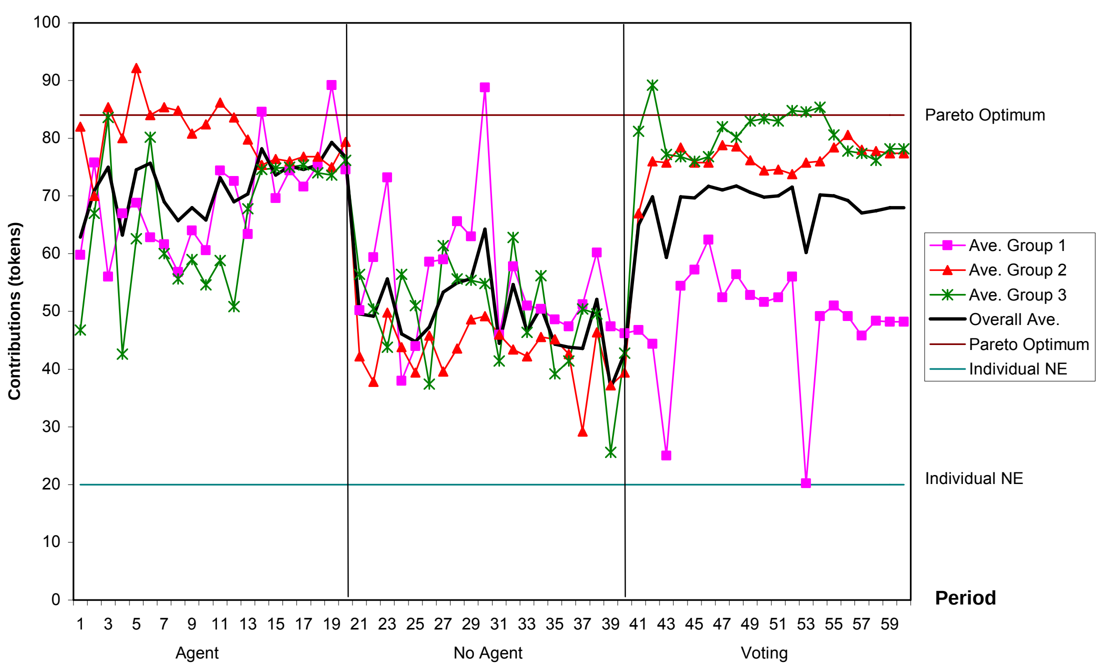

Figure 10: Interior Agent First Contributions by Round

2.0 WEALTH, HETEROGENEITY, EQUITY, AND PUBLIC GOOD PROVISION

2.1 INTRODUCTION

2.1.1 Overview

In the ongoing discussion over how to most efficiently provide for public goods, the question of fairness and who in society should bear the cost still is debated. When public goods are privately provided, no mechanism exists to ensure fairness of giving based on the level of wealth of the individual contributors. On the other hand, categorizing giving levels by wealth is possible in public provision of public goods. Thus, a query of equity through the public good develops. Is it possible to achieve equality in wealth through public provision of a public good when there exists group inequality in endowment levels and opportunities for discrimination? The following paper and experiment address this very question in an attempt to discover if equity is possible in public giving through group agents when predatory action is possible.

The issue of the efficiency of public provision versus private provision is addressed in a previous experiment on agent-giving to public goods (Wick 2008). In this paper, private giving is contrasted with giving under an agent treatment where agents make a single allocation decision for all members of their group. A voting treatment is also included to see which of these mechanisms, using a group agent or giving to the public good as individuals, is more popular. In this experiment all subjects had the same wealth; therefore, it is not clear how a player's wealth within the group may affect a subject's giving behavior both privately and as an agent. In addition, since agents are making a single contribution decision for all group members similar to a single lump-sum tax, they do not have the ability to discriminate giving levels among members of the group. Giving through these agents most frequently brings about Pareto-efficient provision of the public good. Therefore, the experiment in this paper provides a robustness check if the efficiency seen in agent rounds when wealth is equalized can be achieved with heterogeneity in the group.

The current experiment features a wealth distribution among the players of each group. This wealth inequality creates the basis on which the agent can choose different allocations for the separate wealth factions among their overall group. Therefore, the varied wealth levels will allow for the agent to differentiate contribution levels for each wealth group, permitting the agent to single themselves out of their contribution decisions. This bias in giving by the agent could be achieved by other means than creating wealth distinctions, but this design highlights questions of discrimination, predation, and equity. The similarity in all other design factors will allow the results to be compared across experiments.

With these changes in wealth and opportunities for discrimination among the group, new questions can be answered. They include: will agents be predatory in their allocation behavior by deciding their wealth group will not contribute to the public good? Or do agents consider issues of fairness and equity when making allocation decisions? Can efficiency be achieved in this environment with group heterogeneity? Also, due to their higher endowment, do the wealthy in the group privately provide more for the public good than the poor? In addition with the voting treatment, players will be able to voice their preference for the agent or no-agent treatment, and we can uncover if their vote is correlated with their wealth.

2.1.2 Literature Review

For an overall survey of public good experiments, see Ledyard (1995).

According to Samuelson (1954, 1955), using decentralized giving to a public good does not create an environment where an optimal social solution can be reached. In a private provision setting, "any one person can hope to snatch some selfish benefit in a way not possible under the self-policing competitive pricing of private goods" (Samuelson 1954). Thus, allowing for public provision through an agent can help to alleviate group coordination and cooperation problems seen in individual contribution experiments (Wick 2008). However, adding inequality among group members, such as varying levels of wealth, can create problems for cooperation. Heterogeneity among group members creates "psychological effects," reducing the tendency to cooperate and creating feelings of unfairness (Putnam 2000). Inequality weakens group cohesiveness, generating a decline in collective effectiveness and diminishing other regarding preferences (Kaplan et al. 1996, Wilkinson 1996, Knack and Keefer 1997, Kawachi et. al 1997, Putnum 2000).

For a review of social science literature on the costs of inequality, see Thorbecke and Charumilind (2002).

The dampening result of inequality on public good provision can be shown through empirical studies on factors influencing collective allocation. Alesina, Baqir, and Easterly (1999) find a significant association between degrees of heterogeneity (such as race or gender) and expenditures on public goods. Lindert (1996) and Moene and Wallerstein (2002) show that inequality across countries is correlated with lower public spending. In a study on contributions to local charities, Hochand and Rodgers (1973) show that giving is highly sensitive to the distribution of income in a community. Other studies that show group heterogeneity is negatively associated with the efficacy of collective action (Alesina and La Ferrara 2000, Cardenas 2003, Costa and Kahn 2003a, 2003b).

Looking through the literature on public good experiments where groups face some inequality among members, most often overall contributions to the group account are diminished in the presence of heterogeneity. Anderson, Mellor, and Milyo (2004) find that presence of inequality in subject-fixed payments reduces contributions regardless of relative position in the group. In situations where endowment inequality is present in the group, Buckley and Croson (2003) discover lower provision levels of the public good. In threshold public good games, both Bagnoli and McKee (1991) and Rapoport and Suleiman (1993) present results of reductions in contributions to the group account when endowments vary within the group.

As seen above, inequality or heterogeneity in group formation can often create problems for coordination in giving to a public good. But what are some causes for this difference in allocation behavior? In theory, the total supply of the public good is independent of the distribution of income or level of wealth inequality in a group (Warr, 1983; Bergstrom et al., 1986). In behavioral studies, Fehr and Schmidt (1999) discover that experimental subjects face asymmetric inequality aversion which may be the cause for lower contributions from the poor in private giving with varied wealth levels. Due to the decrease in giving from the poor, all else equal, the level of the public good is lower. Differences in wealth in a group can determine not only the extent of gains to be earned through the public good, but wealth can also have status value (Berger et al, 1972). Ball et al. (2001) finds that inequality in status affected the distribution of the group surplus with higher shares being received by those with higher status. In addition, Konrad (1994) finds that individuals have distortionary incentive to reduce their disposable income so as to shift the burden of provision to others. Thus, the wealthy in a group are more likely to spend their endowments on consummatory purchases in order have an excuse to not provide for the public good.

Subjects in environments where heterogeneity is present are shown to be concerned about issues of fairness. Research in psychology suggests that from childhood people can distinguish between outcomes of fairness and outcomes that serve their own interests (Messick and Sentis, 1983; Marwell and Ames , 1981; Wit, Wilke and Oppewal, 1992). Thus, in outcomes of fairness which support proportional contribution, the wealthy in a group are expected to contribute more in order to redistribute wealth through the public good (van Dijk and Wilke, 1994). In investigating choices over types of public good provision, Meltzer and Richard (1981 and 1983) show that, all else constant, an increase in inequality makes subjects more inclined to choose public expenditures with a redistributive effect. Using voting to choose allocation mechanisms, Clark (1998) investigates issues of fairness and proportionality and finds that, in over half the observations, subjects choose mechanisms that serve their own self-interest. Thus, the wealthy are more likely to choose allocation mechanisms that follow an equal giving rule to maximize their profits, and the poor are more likely to favor mechanisms focused on fairness such as proportional giving.

On one hand, the literature on group inequality in theory and in experiments suggests that heterogeneity in wealth among group members should generate questions of fairness and inequality aversion in private contributions to the group account. On the other hand, in investigating the theory of the agent's allocation dilemma, predatory allocation decisions based on wealth are observed frequently, diminishing the redistributive possibilities through the public good. Therefore, the question becomes: can experimental subjects overcome problems created by wealth heterogeneity in their group and exploit the benefit of an agent in providing for the public good or will predatory behavior frustrate the coordination role of agents.

2.2 EXPERIMENTAL DESIGN

2.2.1 Basic Design

The basic concept for this experiment is an extension of the previous work on agents in public good games allowing agents to make contribution decisions to a group account given that group members face a distribution of wealth. Thus, the design of the experiment is kept as close as possible to the original in order to compare results in contribution behavior, discrimination, and changes in wealth between experiments.

The experiment follows a 2x3x2 design for a public good experiment. Each design consists of 60 decision rounds, 20 rounds of no-agent decisions, 20 rounds of agent decisions, and 20 rounds of voting decisions.[9] These three environments are examined in both interior and boundary solutions designs. In addition, two wealth designs are implemented: one where the group had a majority of "wealthy" members and one where the group had a majority of "poor" members. In the design where the wealthy have the majority, the group has three high

[9] The ordering between the first two treatments was reversed to check for ordering effects but the voting treatment remained as the last 20 rounds so subjects could decide which mechanism they would rather have after having experienced both the agent and no-agent treatments.

endowment members and two low endowment members. It is referred to as the Rich design. In the design where the poor dominate, the group has two high endowment members and three low endowment members. It is referred to as the Poor design. Please see Table 9 for a layout of treatments during the experimental sessions.

In all treatments, the group composition does not change across rounds, meaning the same group of n = 5 subjects play a finitely-repeated public good game for the course of a 60-round experimental session. Due to the slight complexity of the mathematics and search associated with the mechanics of the interior solution, the fixed pairings allow for increased cooperation and fewer distractions in strategy formation during the decision process.[10]

The interior design consisted of 12 groups of 5 subjects for a total of 60 subjects, and the boundary design consisted of 8 groups of 5 subjects play their within-subject design for a total of 40 subjects. Each of the sessions listed in Table 9 was conducted once.

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[10] Even though these fixed pairings allow for repeated game strategies to emerge among group members, I do not believe behavior would be any different with random matching given the research done by Andreoni and Croson (2001). In addition, in looking at the results, it appears that the treatment variables and learning are driving the differences in giving behavior.

Table 9: Session Treatment Design

Session Name*	# Low Endow	# High Endow	Rounds 1-20	Rounds 21-40	Rounds 41-60
BoundaryRichNA	2	3	No Agent	Agent	Voting
BoundaryRichAN			Agent	No Agent	Voting
BoundaryPoorNA	3	2	No Agent	Agent	Voting
BoundaryPoorAN			Agent	No Agent	Voting
InteriorRichNA	2	3	No Agent	Agent	Voting
InteriorRichAN			Agent	No Agent	Voting
InteriorPoorNA	3	2	No Agent	Agent	Voting
InteriorPoorAN			Agent	No Agent	Voting

*Each interior session consisted of 3 groups of 5 subjects. Each boundary session consisted of 2 groups of 5 subjects.

At the beginning of each round, each group member is gifted with a number of tokens as his or her endowment. These tokens can be invested between two accounts: the group account and a subject's own private account. Each member of the group has a different amount of tokens gifted to them at the beginning of every period depending on their type, but overall group wealth is constant at 625 tokens. Low-endowed members in each group are given the title type 1, and high-endowed members are type 2. In the Rich wealth design, two type-1 group members have an endowment of 50 tokens, and three type-2 group members have an endowment of 175 tokens. In the Poor wealth design, three type-1 group members have an endowment of 75 tokens, and two type-2 group members have an endowment of 200 tokens. Both groups begin each round with wealth inequality given by a Gini coefficient of 0.24.[11] Tokens cannot be carried between rounds.

[11] A Gini coefficient measures inequality within a wealth distribution. It is a ratio between zero and one; the closer the coefficient is to zero the greater the wealth equity among the group.

When the round starts, each individual is asked how many tokens they would like to allocate to the group account. Tokens not contributed to the group account remain in a subject's private account. Thus, in the no-agent treatment, the group account consists of the sum of the tokens contributed by all five members. The difference in the agent treatment is each individual is asked to make an allocation decision for the group by deciding how many tokens both type-1 and type-2 members must contribute to the group account. The agent's allocation decision is mimics the tax bracket structure the US government imposes on wealth groups, where the specific amount of the tax depends on the individual's wealth level. Therefore in each round in the agent treatment, each group member makes decisions about how many of each type of group member's tokens will be contributed to the group account. Since the agent is included in one wealth group, their profit is affected by their allocation decision, which creates the differential-giving conundrum. In order to increase the agent's own profit, he or she should increase token allocations to the group account from the other wealth group and decrease allocations from their own wealth group.

After all subjects have made agent decisions, one group member is chosen at random to be the agent whose decision is then implemented for the group. The total number of tokens in the group account in the agent treatment is then decided from the random agent decision and the number of each type of group member in the treatment.

In the last 20 rounds, subjects vote on whether they would like to have an agent make a group allocation decision or utilize the individual, no-agent mechanism. The decision is imposed by majority rule. Therefore, the allocation mechanism becomes endogenous to the group. Even though agency should lead to more efficient outcomes increasing overall group wealth, agency would not be effective as an allocation method if it was not preferred above private giving.

Group members can voice their opinion about what method of giving they prefer. If three or more group members vote for one outcome, it is imposed. The numerical outcome of the vote is not revealed to participants so as not to influence giving behavior. The outcome is only revealed by allocation method which is implemented.

Participants were volunteers recruited from the University of Pittsburgh Experimental Economics Laboratory mailing list which include students and adults from the Pittsburgh area. All experimental sessions were conducted at the University of Pittsburgh Experimental Economics Laboratory.

2.2.2 Payoff Design

2.2.2.1 Boundary Solution

The no-agent treatment in the boundary condition leads to the typical, zero-contribution Nash equilibrium for both types of players, but theory suggests that behavior under agent decision making will be much different. In the previous experiment when each member of the group has equal wealth, the agent's single-allocation problem for the group leads to full contribution and Pareto optimality for all marginal per capita return rates (MPCR) less than 1 and greater than 0.2. The agent's problem changes with the addition of a wealth distribution and the ability to differentiate between two wealth groups. In this experiment, agents decide how much type-1 and type-2 players must allocate to the group account. To investigate this problem further, let's look at a type 1 agent's profit-maximization problem.

At the beginning of each period, each of the n_1 type-1 subjects is endowed with y_1 tokens and each of the n_2 type-2 subjects is endowed with y_2 tokens (where $n_1 + n_2 = 5$ and

$n_1 y_1 + n_2 y_2 = 625$). In each period, the representative decides to allocate t_1 of each type-1's endowment to the public good and t_2 of each type-2's endowment. These allocations, t_1 and t_2, are uniform across all subjects in their respective types; the agent's own endowment and profit are included in the decision he/she imposes for their own type. Therefore in each period, each type-1 subject acting as an agent must maximize his or her profits given by:

(1) $\pi_1 = y_1 - t_1 + a(n_1 t_1 + n_2 t_2)$, where a is the marginal per capita return from a contribution to the public good; $0 < a < n_1 a < 1 < a(n_1 + n_2)$.

In the previous experiment, since all group members were identical, the dominant strategy for any agent was to allocate all of each group member's tokens to the group account since they could not single themselves out of the group. By dividing the group into two wealth groups, each agent can now differentiate between their own wealth group and the other. Evidence of favoritism to their own wealth group can be seen in the solution to the maximization problem in (1). Even though the most-efficient, Pareto-optimal group outcome is for the agent to set $t_1 = y_1$ and $t_2 = y_2$, individual optimality in (1) allows for agents to isolate their own group and seek predatory action upon the opposite wealth group.

Because an agent can favor their own wealth group in their allocation designs, they will choose zero contribution for their group. Therefore, in order to maximize their profits from the group account, optimally the agent make allocations such that the opposite wealth group will bear the entire burden of the public good. A type-1 agent will set their own allocation t_1 for the public good such that $t_1 = 0$ since $\frac{d\pi_1}{dt_1} = -1 + an \; < 0$.[12] In addition, a type-1 agent will set t_2 equal to its maximum for the type-2 member of its group since $\frac{d\pi_1}{dt_2} = an_2 > 0$. A very similar profit maximization problem can be performed for type 2 agents to find that they would strategically choose $t_2 = 0$ and t_1 equal to its upper boundary. Thus, the total number of tokens in the group account is either $t_1 y_1$ or $t_2 y_2$ depending on which type of player is randomly chosen as the group agent. Either value, $t_1 y_1$ or $t_2 y_2$, are smaller than the total number of tokens allocated in the dominant strategy in the homogeneous wealth case.

Thus, the payoff functions from an agent's decision for each type-1 player and each type-2 player are:

$$\pi_1 = 2.2(e_1 - t_1) + 0.7(t_1 n_1 + t_2 n_2),$$

$$\pi_2 = 2.2(e_2 - t_2) + 0.7(t_1 n_1 + t_2 n_2).$$[13]

This set of optimal strategies does not make for wealth-equalizing behavior on the part of rich members of a group. By acting in this predatory manner, type-2 agents, "the wealthy," will be taxing all of the poor's endowment to the public good and keeping all of their own endowment in their private accounts. These allocations will serve to widen further the gap between the rich and the poor. Therefore, when this favoritism is possible, it is especially

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[12] Recall $n_1 = 2,3$ depending on the wealth treatment and *a*, the MPCR, is 0.318. This MPCR is derived from a ratio of payoffs at the Nash equilibrium and Pareto optimal outcomes in the LWW paper and was used in the previous experimental paper. Therefore, keeping $a = 0.318$ will allow for comparisons between giving in this experiment and the previous one.

[13] Recall that $(e_1, e_2) = (50,175),(75,200)$ in the Rich and Poor wealth designs respectively.

interesting to investigate the question of equity in wealth while making agent decisions. If agents are giving according to the group Pareto optimum, each subject's profits would be equal, and there will longer exist a disparity between the wealthy and the poor. So this design begs the question: do all agents, poor and rich alike, act selfishly, keeping their own tokens and taxing their neighbors? Or do some look out for the well-being of the group as a whole?

In the last 20 rounds of each session, voting on a preferred mechanism of giving is conducted to see which is more favorable to the majority of group members. Private giving in no-agent rounds may frustrate players who would like to coordinate on higher group giving, but the advantages of agency may be limited if subjects follow predatory allocation strategies. Different wealth groups might prefer different mechanisms of giving, and thus these varied giving structures should arise from the majority wealth group, either the wealth or the poor, in the two wealth designs.

To investigate if subjects will opt for varied allocation mechanisms in voting rounds, we can examine their payoffs by type to find their preference. If we assume that in no-agent rounds, subjects follow the Nash-equilibrium strategy of zero contribution, and in agent rounds, subjects follow a predatory allocation strategy, then we can investigate extensive-form game trees of payoffs from different voting decisions as seen in Figure 11. At the end of each node are type-1 and type-2 players' payoffs in expectation from following this set of strategies.

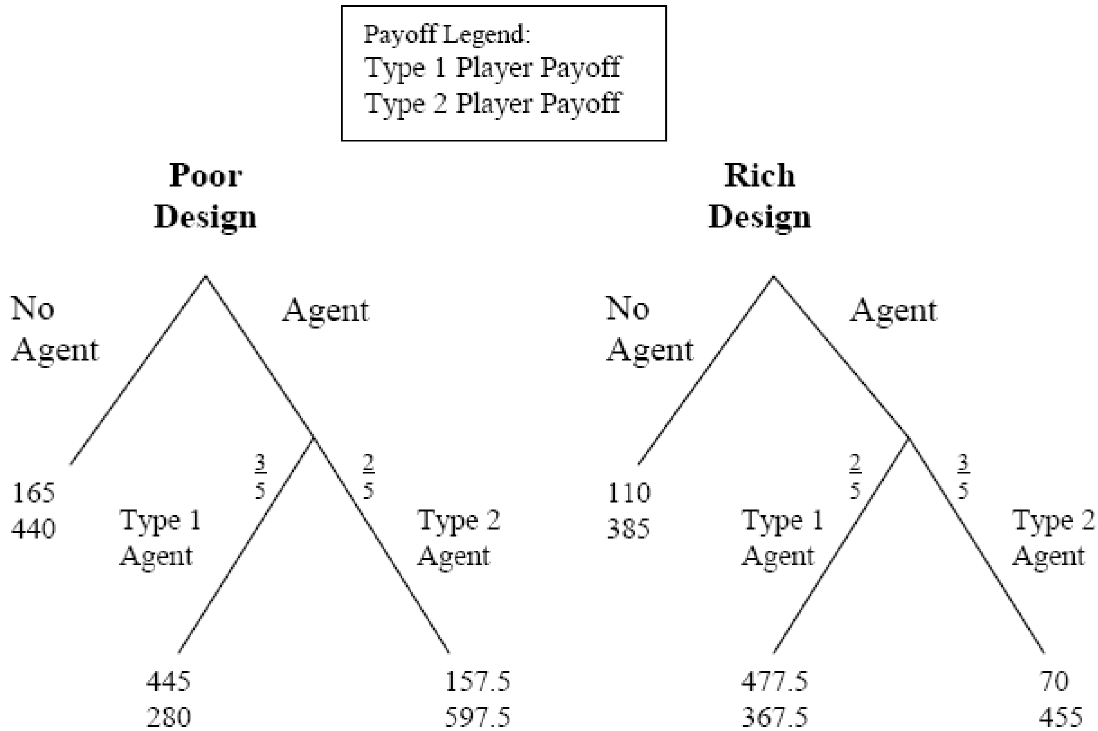

Figure 11: Extensive Form Game Trees of Type 1 and Type 2 Players' Voting Decisions in Boundary Wealth Designs

Agents can help to increase giving amounts to a public good, but in the case involving the ability to discriminate by wealth, agency does not always increase payoffs for each type of player. Given the probabilities that types will randomly be chosen as an agent, type-2 players in the Poor design strictly prefer the no-agent treatment to the agent treatment. All other types in the boundary wealth designs strictly prefer the agent treatment. Given that these wealthy players do not have the majority in the Poor design, agency should still arise as the giving mechanism of choice in this wealth design as well as the Rich design.

2.2.2.2 Interior Solution

Recall that the mechanics of the interior solution are derived from the Laury, Walker, and Williams (1999) paper utilizing the details of their Z125-DET treatment. The set up for the return to the group account for a subject denoted by the function *F(.)*, the group size, the endowments for each member, as well as parts of the instructions and tables given to the subjects were all employed from their interior design.

This design adheres to the structure of voluntary contribution mechanism where subjects decide how to allocate their endowment between their private and the group account. At the beginning of each period, each of 5 subjects in a group is endowed with different private amounts. Each individual i decides how many of their tokens to allocate to the group account, g_1. Thus, the total number of tokens in the group account is $g_i + \sum_{j=1}^{4} g_j$, where $j \neq i$.

The return the group account for a subject is given by *F(.)* which is a non-linear function with a declining benefit from the group account. Each subject receives $\frac{1}{n}$ or $\frac{1}{5}$ of total group account earnings. Therefore, each subjects receives $\frac{F(g_i + \sum_j g_j)}{5}$ in points as their individual return from the group account. Each token remaining in their private account earns a subject one point.

If we name the number of tokens in the group account X, then total group earnings from the group account is; ; $F(X) = 6.25X - 0.00625X^2$. Thus, the marginal social benefit from the group account is; $F'(X) = 6.25 - 0.0125X$ and the marginal per capita return (MPCR) is; $\frac{F'(X)}{N} = 1.25 - 0.0025X$.

Using the same variables as before in the boundary analysis, if an agent chooses an allocation of t_1 for type 1 group members and t_2 for type 2 group members, then the payoff functions for each type 1 player *i* and each type 2 player *j* are;

$$\pi_i = e_1 - t_1 + \frac{F(n_1 t_1 + n_2 t_2)}{N} = e_1 - t_1 + \frac{F(n_1 t_1 + n_2 t_2)}{5},$$ [14]

$$\pi_j = e_2 - t_2 + \frac{F(n_1 t_1 + n_2 t_2)}{N} = e_2 - t_2 + \frac{F(n_1 t_1 + n_2 t_2)}{5}.$$

Figure 1 shows a graphic representation of separate individual and group benefits and costs of placing a token in the group account. The marginal cost of allocating one token to the group account is the one point that could have been earned from placing the token in the private account. That token could also be placed in the group account which would yields returns not only for the individual player in the form of their marginal private benefit but also to the group in the form of the marginal social benefit.

Thus, setting the marginal cost equal to marginal private benefit yields a Nash equilibrium provision for the group of 100 tokens. Similarly, setting marginal cost equal to marginal social benefit generates a group Pareto optimum provision of 420 tokens. It is important to note that these solutions are in aggregate contributions. Therefore, there exist a continuum of individual best responses to behavior on the part of their group members' decisions to reach these contributions.

Even though the distribution of wealth has changed dramatically since the last experiment, the overall group wealth is the same and thus the aggregate Nash equilibrium is 100 tokens and the Pareto optimum is 420 tokens. Any combination of token allocations that achieves 100 tokens is dubbed a Nash equilibrium in this environment. However, any outcome

[14] Recall that $(e_1, e_2) = (50,175),(75,200)$ in the Rich and Poor wealth designs respectively.

without an agent will almost certainly favor the wealthy. Those with higher endowments in no-agent treatments are likely to hold onto the majority of their endowment in their private account and yield the same benefit from the group account as those with lower endowments. Similarly, if agents are making Pareto-optimal decisions, the group account has 420 tokens at the end of the round. It does not matter from whose account these tokens come. Thus, just as in the boundary solution, there is an incentive for agents to favor their wealth group and "tax" the wealth of the other toward the public good. Consequently, they will make the other wealth contingent contribute all or almost all of their tokens to the group account and keep all of their own to increase their earnings.

The interior, Nash-equilibrium environment is mathematically more complex than the boundary solution. Subjects no longer face a Nash equilibrium that is unique on the individual level or involves a dominant strategy. Thus, there exists a coordination issue involved in reaching the unique aggregate Nash equilibrium since there are many individual Nash outcomes. Additionally, the complex MPCR in the non-linear case makes the calculation of the solutions more difficult than in the boundary condition. Therefore, participants are given detailed information on the payoff structure of the experiment in the form of payoff tables in order to compare group and individual returns to specific contributions to the group account (see 1.6.3 Appendix C in previous chapter). These are provided in addition to the written instructions and were given to each participant. The tables are reviewed by the experimenter along with the instructions before the beginning of the experiment. Included in these tables is information on the additional return to the group and the additional return to the individual for contribution to the group account as well as examples of earnings from the group account.

Subjects are given questionnaires at the beginning every treatment to test their understanding of the instructions and the environment. Participants demonstrate high levels of understanding of the interior Nash environment. In addition, the questions and their answers are carefully reviewed with the subjects to further ensure understanding.

2.2.2.3 Payment

At the end of the experiment, participants are paid in private for their earnings in one randomly chosen period. Paying participants for one period rather than all of their decisions is chosen in order to avoid wealth effects throughout the experiment. Points are converted to dollars in the following fashion. In the interior-solution design, 1 point = 0.05 dollars and in the boundary-solution design 1 point = 0.03 dollars. The difference in payment comes from the increased difficulty of the interior-solution case and consequently its longer sessions. Boundary-solution sessions often lasts between an hour to an hour and a half, and interior-solution sessions lasts approximately just under two hours. One interior-solution session had to be cut short because it exceeded the two hours for which participants were recruited. All participants also earn a five dollar participation bonus. Thus, average earnings are around $13-15 in the boundary-solution sessions and $19-21 in the interior-solution sessions.

2.3 PREDICTIONS

Group members acting as agents are likely to place the entire burden of the public good on the wealth group in which they do not belong and "free-ride" off their contributions in both solution designs. The ability to distinguish between wealth types allows this predatory behavior to take place. One of the reasons for designing the experiment such that agents have ability to discriminate based on wealth levels is to discover which type of agent will approach taxing each wealth group in the most equitable way.

The ability to discriminate among wealth types will affect outcomes in the agent treatment, but we predict that the net effects of agency will be positive.

Hypothesis 1: Social welfare will increase with the use of an agent.

In Wick (2008), social welfare increased with the use of an agent due to the agent's ability to coordinate group giving to take advantage of the increasing gains from the group account. Although in this experiment agents will be making two allocation decisions with the ability to single themselves out through their wealth group, their group allocation decisions are likely to be higher than the sum of individual allocations in the no-agent treatment. This increase in allocations in the agent treatment is a result of not having to coordinate on giving with other group members as in the no-agent treatment. Even if agents follow a predatory allocation strategy, the balance in the group account will almost always be higher in the agent treatment.

Hypothesis 2: Because of the disparity of wealth among the group, the level of provision for the public good in the no-agent treatment will be lower than in the previous experiment with equalized endowments.

Although group members most likely will give positive token amounts to the public good in the no-agent treatment (especially in the interior solution case), the varying levels of wealth will cause group members to be less generous in their token allocations. The poor will look to the wealthy to contribute more to the group account because of their high endowments, and the wealthy will likely be tight-fisted due to a reluctance to let go of their wealth.

Hypothesis 3: In the no-agent treatment, the poor are as likely to provide for the public good as the wealthy.

Rapoport (1988) studied public good provision and the effect of asymmetry in endowments in an experimental setting. He found no effect of endowments on choice behavior in private contributions to the group account. He claims the reason this result was that low-endowed subjects are more inclined to contribute to public good because they have the most to gain from its provision. The rich, who have high endowments, can make high profits by keeping all their tokens in their private account, and thus when free-riding is present, these well-endowed subjects do not have as much to gain in profits from the group account.

Even though in equilibrium giving should only be taking place for the no-agent treatment in the interior solution condition, positive contributions will be made in the boundary conditions. Players may seek to extract some of the gains from group giving to the public good, and therefore they contribute positive amounts above the Nash equilibrium solution hoping other group members will play likewise. Thus, this positive contribution behavior is likely to continue in the boundary case without dependence on wealth.

In addition, given the design of the interior solution, if all subjects gave 20 tokens to the group account, the Nash equilibrium would be achieved. Since this 20 token allocation is possible for type 1s (the poor) to give in both wealth designs, both types of subjects are likely to contribute equally to the public good in the interior solution case. In addition, given the likely entitled attitude of the wealthy players from their high endowments, they may want to maintain a firm hold on their endowment from the beginning of each round. Thus, they might contribute only what is necessary to the public good to achieve a Nash equilibrium, not keeping in mind that they bear the majority of the wealth amongst the group.

Hypothesis 4: The Pareto-optimal outcome in the boundary condition of full contribution to the public good will not be attained due to agent predatory behavior.

Full contribution by every group member, no matter their type, is the Pareto-optimal outcome (PO) in the boundary solution setup. Given the solution to the agent's problem as seen above, attaining that PO will be very difficult since its choice by the agent is dominated by free-riding off the opposite wealth group's tokens, increasing the agent's profit. Thus, since the PO is no longer the dominant choice for the agent, tokens in the group account will fall below the PO in boundary agent rounds.

Hypothesis 5: By the end of agent decision rounds, both types of players will display predatory behavior, meaning the wealthy will place all of the poor group's endowment in the group account and none of their own, and the poor will do the same to the wealthy.

As we saw before, an agent's dominant strategy is to keep all their own to tokens in their own private account in the boundary condition. In addition, an agent following a dominant strategy would place all of the opposite wealth group's tokens into the group account. Therefore, agents in the interior design should act in a similar manner, placing all of the opposite wealth group's tokens into the group account up to the Pareto-optimal amount.

Hypothesis 6: Poor subjects will make more wealth-equalizing decisions as agents of the group.

Given their lower endowment at the start of each round, I predict that poorer group members will be more likely to make agent decisions that will equalize group wealth at the end of each round. As agents, the poor can only serve to better their situation over private giving where they are likely to collect small profits. However, this conclusion can be a little misleading since if the poor and the wealthy both follow a predatory allocation strategy, the allocations made by the poor will automatically appear more equitable. When the wealthy take predatory actions as agents upon the poor, the outcome only makes the existing wealth gap between the rich and the poor even wider.

Because of their low endowment, poor players can benefit greatly from agent rounds since the majority of their profits in any treatment are likely to come through the group account. Due to repeated game effects, low-endowed players should show less frequent predatory allocation action than their wealthy counterparts in an effort to prevent estranging themselves from the wealthy. Groups are matched for all 60 rounds of a session, and therefore players must think about how their allocation behavior might affect another group member's allocation behavior in future rounds.

2.4 RESULTS

Observation 1: In both the interior- and boundary- solution conditions, token allocations are lower in the no-agent treatments than when wealth was equalized among the group, confirming hypothesis 2.

This observation can be verified by comparing the contribution behavior across the experiments. This data is presented in Table 10. This table shows that every group account average under "SameW" is higher than under either of the two wealth designs in both the interior and boundary set up.[15] The only group allocation that is close to its same wealth levels is the

[15] This is an abbreviation "SameW" applies to results from my previous experiment (Wick 2008) where individuals faced a uniform wealth distribution in their group.

Boundary Poor wealth case where the difference in group account tokens with the SameW no-agent rounds is less than 25. Using paired, Wilcoxon sign-rank tests of average no-agent contributions by round, in all designs no-agent contributions with a wealth distribution do not come from the same distribution as those with equal wealth at the 2% level.[16] In addition, in both interior and boundary conditions, rich and poor wealth design no-agent contribution follow different distributions even at the 1% level.

Table 10: Comparing Provision of the Public Good in No-Agent Treatment under Wealth and Solution Conditions

	# of Obs	Ave NA Cont	Ave Group Tokens
INTERIOR			
SameW	600	49.5	247.5
Rich Design	600	37.8	188.8
Poor Design	600	29.4	147.1
BOUNDARY			
SameW	600	26.2	131
Rich Design	400	6.9	34.4
Poor Design	400	21.6	107.8

Adding a wealth distribution to the original experiment changed private giving behavior, specifically subjects are less inclined to give to the public good. The varying wealth levels created an inherent difference between the group players increased free-riding behavior in the group.

This observation diverges from theories developed by Bergstrom, Blume, and Varian (1986), who make predictions involving the level of the public good under private giving with a

[16] This non-parametric test checks the equality of matched pairs of observations using the Wilcoxon matched-pairs signed-ranks test. The null hypothesis is that both distributions are the same.

redistribution of wealth. In theorem 4 of their paper, they state that "In a Nash equilibrium, any change in the wealth distribution that leaves unchanged the aggregate wealth of current contributors will either increase or leave unchanged the equilibrium supply of the public good." The level of aggregate wealth (625 group tokens) is constant throughout both experiments. The group of non-contributors in all wealth and solution designs is quite small; most players contribute positive amounts to the public good. With the wealth redistribution in these experiments, the supply of the public good did not increase or stay the same, but rather it decreased.

Observation 2: Overall group surplus increases in the agent treatment over the no-agent treatment, thus confirming hypothesis 1.

Table 11: Average Group Profits by Treatment

Treatment	No Agent	Agent	Voting
BoundaryRich	1419.7	1693.3	1685.7
BoundaryPoor	1515.1	1875.5	1731.7
InteriorRich	1352.2	1515.2	1552.3
InteriorPoor	1234.8	1572.0	1661.0

Table 11 shows average group profits by treatment in all solution and wealth designs. In every instance, agent group profits are higher than no-agent group profits indicating greater overall surplus in the agent treatment. Using paired, one-sided, Wilcoxon sign-rank tests of average subject profits by treatment in each design, agent profits are higher than no-agent profits in every design at the 1% level. Average group profits are higher in the interior voting rounds but not in the boundary voting rounds. Using the same paired, sign-rank of average subject profits by treatment in each design, voting profits are only statistically higher than agent profits in the interior-poor design at the 5% level.

The addition of the wealth distribution does not affect the overall increased social benefit of the agent treatment over the no-agent treatment, but the wealth disparities among the group decrease social welfare in both solution conditions and in all treatments, as predicted. Average group profits are higher when wealth is equalized in the previous experiment.

Observation 3: In private provision, in most cases the poor provide close to 50% of the group public good, confirming hypothesis 3.

Table 12 helps to illustrate this observation by displaying subjects' giving statistics by wealth group in percentages of their endowment and of the wealth group's percentage of providing for the tokens in the group account. In any "No Agent" row, one can find what percentage of their endowment both low- and high-endowed subjects give to the group account, and how much their wealth groups' tokens account for the overall total of the group account's tokens.[17] In a Low Agent row, one can observe on average how type-1 players form their allocation structures as agents. This is accomplished by looking across the row at what percentage of their own type's endowment they allocate to the group account (Low%Endow) and what percentage of the opposite type's endowment do they allocate to the group account (High%Endow). Then in the last two columns, the percentages of provision by wealth group from the low agent decisions are displayed. The same analysis of a high agent behavior can be made in a High Agent row.

[17] Therefore, one can see by what percentage a low- or high-endowed wealth group provided for the public good.

Table 12: Giving Statistics as Percentage of Wealth

	Low%Endow	High%Endow	LowGroup% of GA***	HighGroup% of GA
Interior Poor Design:*				
NA Stats:	0.368	0.161	0.596	0.404
Low Agents:	0.311	0.762	0.330	0.810
High Agents:	0.753	0.287	0.620	0.380
Interior Rich Design:**				
	Low%Endow	High%Endow	LowGroup% of GA	HighGroup% of GA
NA Stats:	0.391	0.286	0.232	0.768
Low Agents:	0.147	0.807	0.040	0.960
High Agents:	0.622	0.432	0.435	0.688
Boundary Poor Design:*				
	Low%Endow	High%Endow	LowGroup% of GA	HighGroup% of GA
NA Stats:	0.253	0.127	0.429	0.571
Low Agents:	0.357	0.897	0.155	0.845
High Agents:	0.722	0.162	0.603	0.363
Boundary Rich Design:**				
	Low%Endow	High%Endow	LowGroup% of GA	HighGroup% of GA
NA Stats:	0.157	0.036	0.419	0.456
Low Agents:	0.210	0.769	0.071	0.917
High Agents:	0.680	0.318	0.603	0.397

* In the Poor wealth design, the wealthy have 1.78 times more wealth as the poor as a group in the TH treatment and each individual has 2.67 times more wealth than each poor individual. The poor have 36% of group wealth and the wealthy have 63%.

**In the Rich wealth design, wealthy have 5.25 times more wealth as the poor as a group in the TL treatment and each individual has 3.5 times more wealth than each poor individual. The poor have 16% of group wealth and the wealthy have 84%.

***These columns indicate what percentage of the group account's tokens came from the low or high endowment group.

As can be seen in third column Table 12, in all cases, the poor are providing large percentages of the tokens in the group account during no-agent treatments. Over all wealth and solution designs, the percentages of the tokens in the group account provided by the poor rival those of the wealthy. The poor in the Poor wealth design have 36% of the group endowment, and they providing for almost half of the public good and are also giving a higher percentage of their endowment. In the Rich wealth design, each one of the two poor individuals has $\frac{2}{7}$ths the endowment of one of three of their wealthy group counterparts, and the poor together only have 16% of the overall group wealth. Still even in the Rich interior case where the percentage of giving by the poor to the group account is the smallest, they are still giving close to one-fourth of the public good and a greater percentage of their endowment than the wealthy. Thus, the poor are making greater sacrifices from their small endowments to give privately to the public good than the wealthy because they stand to gain the most from a large collective group account.

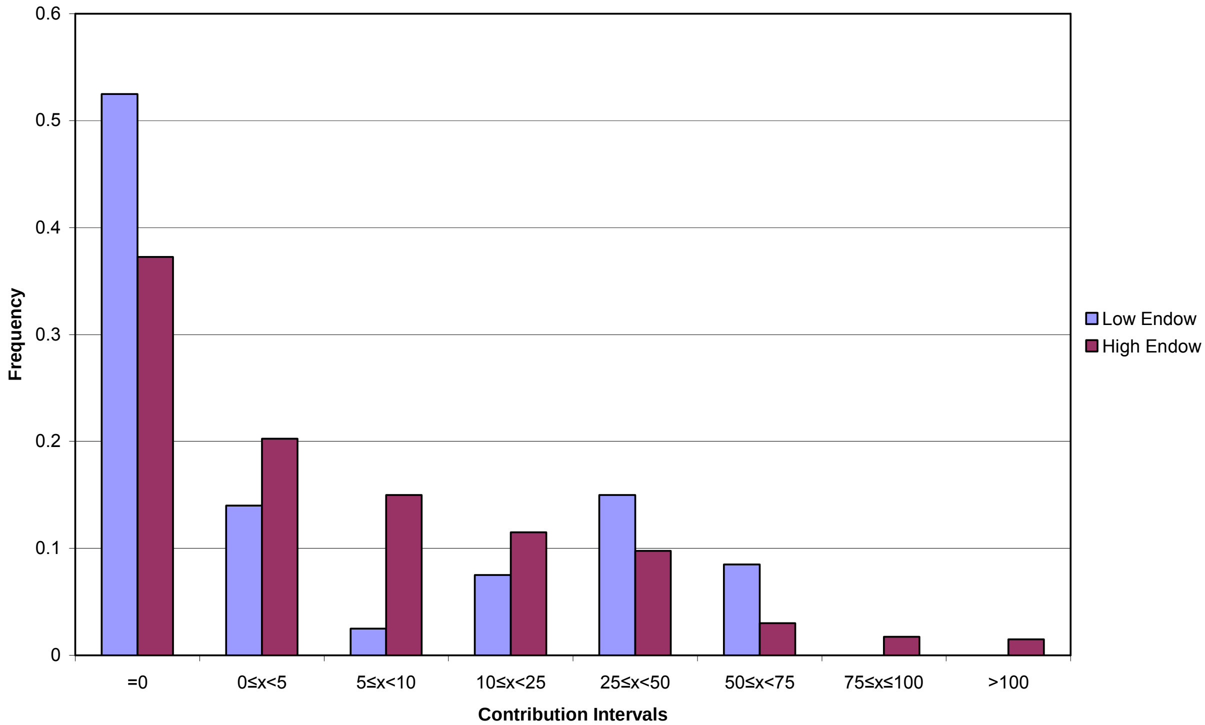

Figure 12: Pooled Boundary No Agent Contributions by Frequency

From this experiment, we cannot conclude that all contributors have greater wealth than non-contributors in private giving. Bergstrom, Blume, and Varian (1986), in theorem 5, state that with private provision of the public good, contributors have greater wealth than non-contributors in a Nash equilibrium. This experiment is not a true test of this theory, but it is interesting to note how behavior differs from predictions. As shown in Figures 12 and 13, non-contributors more frequently tend to be those with a low endowment, but the contribution decisions by both the wealthy and the poor does not seem to follow any clear distribution.

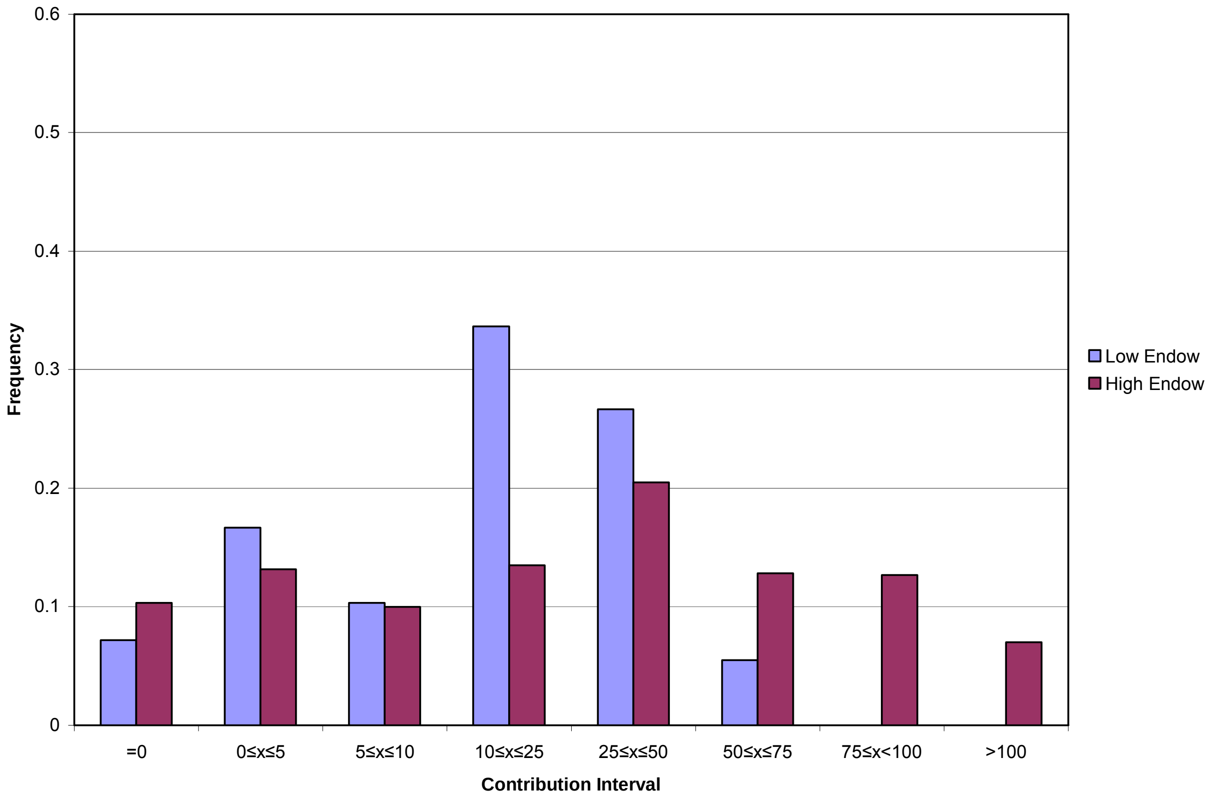

Figure 13: Pooled Interior No Agent Contributions by Frequency

In private giving, on average groups gave over the Nash equilibrium amount prescribed by each solution in both the interior and boundary conditions. Nash equilibrium giving in the interior solution is 100 tokens for the group and in the boundary solution is zero tokens. Giving in the no-agent rounds is above Nash levels but deteriorates as the rounds continue to approach the NE. The no-agent giving behavior is exhibited in Figure 14. This decline in giving is seen in the previous experiment only in the boundary no-agent rounds. As the interior solution no-agent rounds progressed in the previous experiment, there was little to no change in allocation behavior. Therefore this addition of wealth has some changing effect on giving behavior in this more complex environment.

Figures 14 and 15: Average Sum Tokens in the Group Account in various designs across the first and last five rounds of both the no-agent and agent treatments

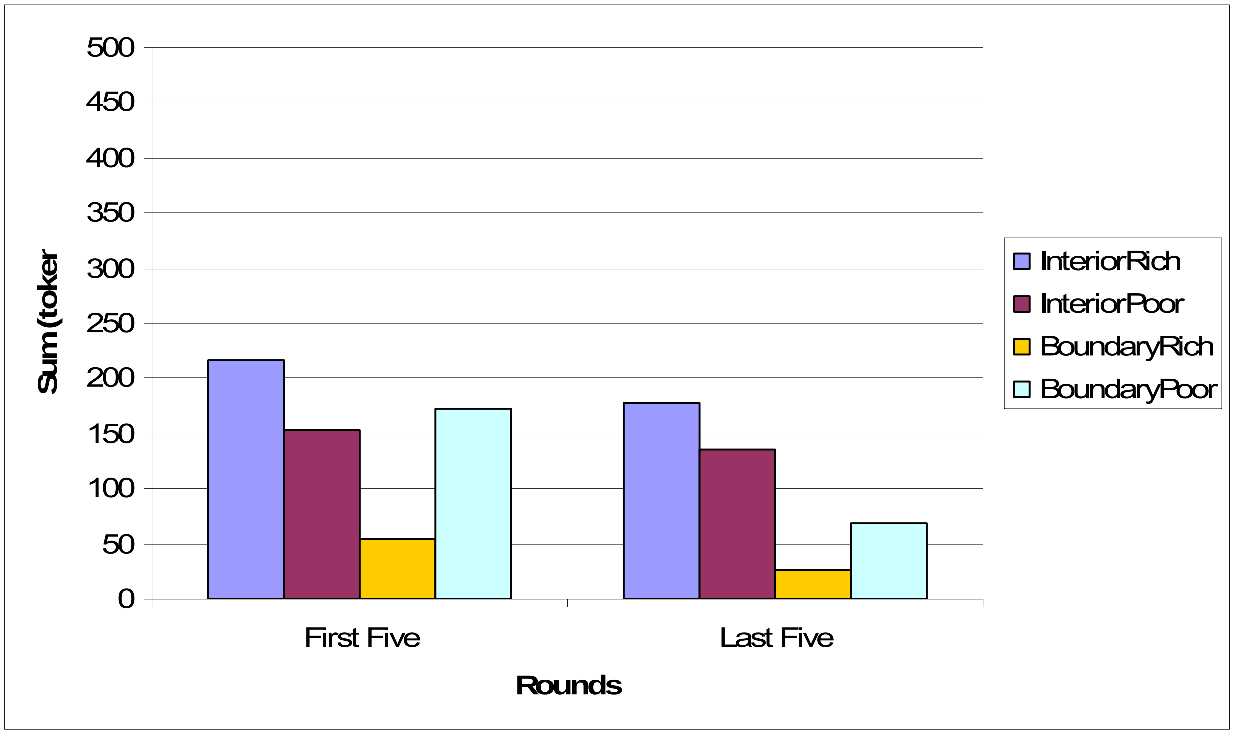

Figure 14: Average Token Sum in Group Account in No Agent Treatment

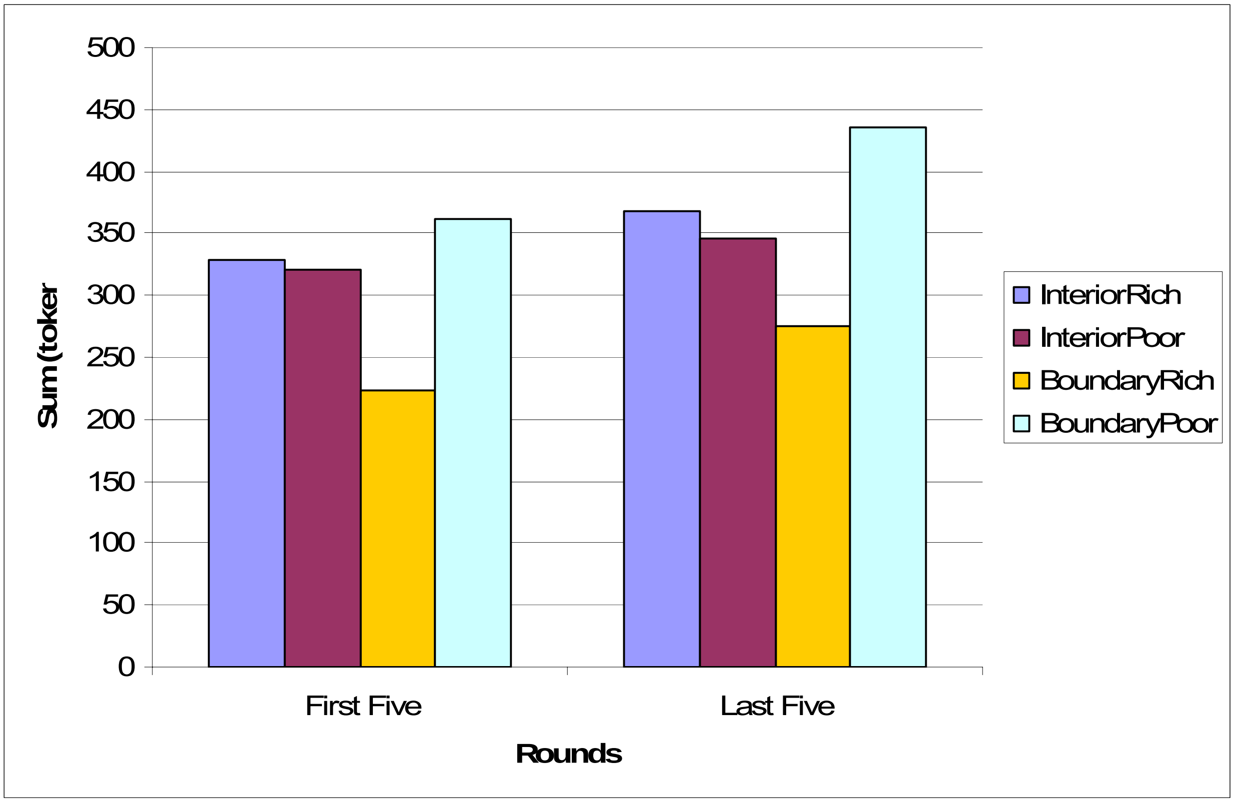

Figure 15: Average Token Sum in Group Account in Agent Treatment

Observation 4: In agent giving rounds, tokens in the group account did not reach Pareto optimum levels in either solution condition, confirming hypothesis 4.

In the interior solution, Pareto-optimal behavior for the group would be to have 420 tokens in the group account and in the boundary solution, optimality would be to have all of the group's 625 tokens in the group account. When wealth is equalized within the group, agents' allocations attain near Pareto-optimal levels in both the boundary and interior designs (Wick 2008). Giving in the boundary-agent rounds does not mimic what is seen in the previous experiment while interior-agent giving is quite similar. Allocations in the boundary-agent rounds are considerably lower than when wealth is equalized.

Allocation behavior in the first and last five agent rounds can be seen in Figure 15. The change in agent allocations from the first five rounds to the last five is very small in the interior design, implying that agents are making consistent allocation choices as rounds continue or are still searching for optimality. Even though these agent group allocation amounts are strongly similar to the equalized wealth experiment, they still do not attain Pareto-efficient levels.

In the boundary design, agent token allocations come quite short of group optimality. Agents are giving far less than the Pareto-optimal level of complete giving even as rounds continue due to the agent's incentive to use a predatory allocation structure. Even in the BoundaryPoor design where agent allocations are the highest, tokens in the group account are under 450 tokens, well below optimality of 625 tokens. Therefore, agents do not make Pareto-optimal allocations in either solution design.

Observation 5: In agency treatment rounds, most agents follow a simple predatory allocation structure, confirming hypothesis 5.

An agent would be following a simple predatory allocation structure if they allocated all of the tokens from the each member of the opposite wealth group into the group account and did not allocate any tokens for their own wealth group. Their own personal wealth group would keep all of their tokens in their own private account. An example of this predatory behavior would be if a low-endowment agent in the Rich wealth design chooses an allocation structure such that all type-1 group members (low endowment) would give zero to the group account and type-2 group members would give all 175 of their tokens to the group account.

By looking at the frequencies of agent contribution behavior seen in Figures 17 through 24 (see 2.6.2 Appendix B), we can examine how both poor and wealthy agents make allocation decisions for both their own and the opposite wealth group in each wealth and solution condition. We see that if the figure is showing decisions for low-endowment subjects then there is a concentration of frequencies of decision by the low agents at the low end of the allocation spectrum and a concentration of frequencies of decisions by high agents at the high end of the allocation spectrum. This type of predatory behavior can be observed on every graph, but it is observed more easily on Figures 23 and 24 which show the agent contributions from the Boundary Rich design.

One general consistency among the graphs is that the minority group, such as the high endowment subjects in the Poor wealth design, use this predatory allocation structure more frequently than the majority group. By comparing allocation structures across figures 17 and 18, we see that high agents in the Poor design choose very high allocations for the poor and low allocations for themselves by heavier frequencies than their wealth counterparts. This same comparison can be made in figures 21 and 22 to find that low agents seem to be more predatory than the high agents in the Rich wealth condition. This comparison can be taken to the boundary graphs as well.

Observation 6: As agents, the poor make more wealth-equalizing decisions than the wealthy due to the design of the experiment, but the poor are actually more predatory against the wealthy than vice versa, which does not confirm hypothesis 6.

Even though the wealthy have multiples of the poor's endowment both as a group and as individuals, when they are agents, they often follow the predatory strategy, increasing their profits by free-riding off the contributions of the poor. Agent allocation behavior in giving percentages of each wealth groups' endowment to the group account can be seen in Table 12 above. With the exception of the Interior Rich design, wealthy agents make decisions for the poor to provide 60% or greater of the public good when they have as little as 16-36% of group wealth.

The aggregate percentages of provision of the public good under low agents presented in table 12 show a more in-depth picture of allocations made by types of agents toward different wealth groups. Low agents are more inclined to place higher percentages of their opposing wealth groups' endowment into the group account than high agents. Low agents place a very high percentage (above 70%) of the wealthy's endowment toward the public good and, in almost every session, allocate higher percentages of the opposite wealth group's endowment to the public good than high agents. Low agents may be seeking a redistributive effect in wealth from their agent allocations to the group account. In a threshold public good setting, van Dijk and Grodzka (1992) find that low-endowed group members considered it fair for high-endowed members to contribute more to the group account.

Due to the design of the experiment, low-endowed agents seemingly make more equitable decisions in public good provision than those made by the wealthy. When the poor place the entirety of the wealthy's endowment into the public good, there is a considerable amount of the public profit to be split among the members of the group due to the large amount of tokens in the group account. Therefore, their predatory-agent actions lower group income inequality. On the other hand, predatory-agent decisions from the wealthy create a greater income gap than at the onset of the experiment.

As a result of the high-endowed agent decisions, as a group the wealthy become richer due to their predatory allocation behavior against the poor. As shown in Table 13, the profit gap between the wealthy and the poor can become quite wide with a high-endowed agent especially in the boundary solution case. The difference in profits between type-1 and type-2 players is much smaller with a low agent in charge, even though profits do not reach Pareto optimal levels.

Table 13: Average Profits for Types under Different Agents

Treatment	Low Agent Type 1*	Low Agent Type 2**	High Agent Type 1	High Agent Type 2
InteriorPoor	320.8	318.8	261.4	379.6
InteriorRich	340.6	327.2	238.4	318.8
BoundaryPoor	414.0	354.0	255.3	496.5
BoundaryRich	373.1	382.9	144.3	432.0

*Type 1 players are the low endowed group members.

**Type 2 players are the high endowed group members.

With Gini coefficient analysis, inequalities of wealth and income distributions from each type of agent decisions can be compared, and we can see further evidence that the poor seem to be "more fair" agents. Table 14 shows Gini coefficient analysis for each wealth and solution design for high agents, low agents and the no-agent treatment.[18] By comparing high and low agents in each wealth and solution design, low agents make more wealth equalizing decisions in every instance.[19] These results are significant at lower than the 1% level.[20] The Gini coefficient results from the no-agent rounds are varied. No-agent decisions are quite close to the starting Gini coefficient of 0.24 in the boundary solution design. In the interior design, though, the Gini coefficient is much lower but is still higher than for the agent treatments. The Gini coefficients for associated with agent-predatory action are presented in table 15. Because of the experiment design, a low agent's predatory strategy is more equalizing as shown by the lower Gini coefficient.

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[18] Recall at the beginning of each session, the wealth distribution amongst the players of the group is 0.24.

[19] Note: Perfect wealth and income equality are achieved as a Gini coefficient approaches zero.

[20] using one-tailed, equal variance t-tests

Table 14: Gini Coefficient Analysis

	High Agents		Low Agents		No Agent	
	# of Obs	Ave Gini	# of Obs	Ave Gini	# of Obs	Ave Gini
Boundary Rich	240	0.229	160	0.067	160	0.249
Boundary Poor	160	0.179	240	0.063		
Interior Rich	360	0.083	240	0.073	240	0.127
Interior Poor	240	0.099	360	0.052		

Table 15: Gini Coefficients for Agent Predatory Action

	High Agent Predatory Gini	**Low Agent** Predatory Gini
Boundary Rich	0.307	0.064
Boundary Poor	0.317	0.105
Interior Rich	0.193	0.036
Interior Poor	0.161	0.109

On the other hand, even though low-endowed agents are making more wealth-equalizing agency decisions, they are making more predatory decisions toward high endowed members of their group. In examining the agent decisions by percentages of endowment allocated to the group account by low agents versus high agents in all wealth treatments in table 12, low agents give a higher percentage of the high endowed members wealth to the group account than high agents give of low endowed members in every design. Therefore, as agents, low-endowed members are taking larger portions of the wealthy's endowment than vice versa. Thus, poor agents are more predatory toward the wealthy than the wealthy are to the poor.

The majority of decisions in the voting rounds were to elect an agent, although in most treatments the wealthy vote less frequently for an agent than the poor. Even though predatory behavior is observed in agent rounds, the majority of votes are still cast to use agency as a method of giving to the public good. Given that the incidence of an electing to use agency is 70% at the low end with the BoundaryPoor treatment and 97% at the high end with the

InteriorPoor treatment, players in general must believe that the benefits of using an agent outweigh the costs. As evidenced by their votes, the gains from using an agent to make a collective decision offset the risk of having an agent from an opposite wealth group use a predatory allocation scheme against your wealth group. The increase in profits from the use of an agent is a simple reason why most players would vote to use this method of giving to the public good.

The comparison of profits among treatments for both type 1 and type 2 players across treatments can be seen in Table 16. Only in the BoundaryPoor case do the wealthy profit considerably well in the no-agent treatment versus the agent treatment. In that same design, therefore, type 2 players do not vote for an agent with high percentages (only 24% as seen below in Table 16). Otherwise, players either do just as well or better in profits in the agent treatment versus the no-agent treatment, and the potential gains from the agent treatment are high therefore it is more likely to be voted for especially if you are a player in the majority group. What is also important to note from this chart is that wealth is redistributed among the group using any type of agent. The Gini coefficients in agent rounds are reduced from the starting coefficient of 0.24, pointing to an overall improvement in equality from the use of an agent no matter their wealth type.

Table 16: Comparison of Profits under Treatments by Type of Player

	Type 1		Type 2	
	No Agent	Agent	No Agent	Agent
InteriorPoor	199.1	295.1	319.7	345.1
InteriorRich	212.8	276.7	307.8	322.0
BoundaryPoor	197.0	352.6	459.5	408.9
BoundaryRich	195.4	195.6	397.9	398.0

As seen in Table 17 and Figure 16, type-1 players with a low endowment are generally more likely to vote for an agent than type-2 players who benefit from the security of their high endowment. Due to free-riding in private giving, the poor will only be profitable in the case of an agent who looks out for the interests of the entire group, therefore as a group they have very high incidences of voting for agency. Because the wealthy have the benefit of knowing that in private giving they can always keep their high endowment and still be somewhat profitable, they are less likely to vote for agency. The only exception is the Boundary Rich case where high endowed players voted for agency more than low endowed players by a slight margin. In addition, type-2 players (those with high endowments) in the Boundary Poor design vote to have an agent with far less frequency than any other treatment group, confirming that, as a majority, these players are following their dominant strategy according to Figure 1.

Table 17: Percentage of Votes of Each Type of Player Cast to Elect an Agent

Treatment	Overall %	Low Endow	High Endow	Incidence of Agent
InteriorPoor*	0.71	0.81	0.57	0.97
InteriorRich	0.61	0.70	0.54	0.71
BoundaryPoor	0.58	0.80	0.24	0.70
BoundaryRich	0.68	0.67	0.69	0.90

*Results from InteriorPoor with Agent rounds first had to be omitted since only 4 rounds of voting were completed.

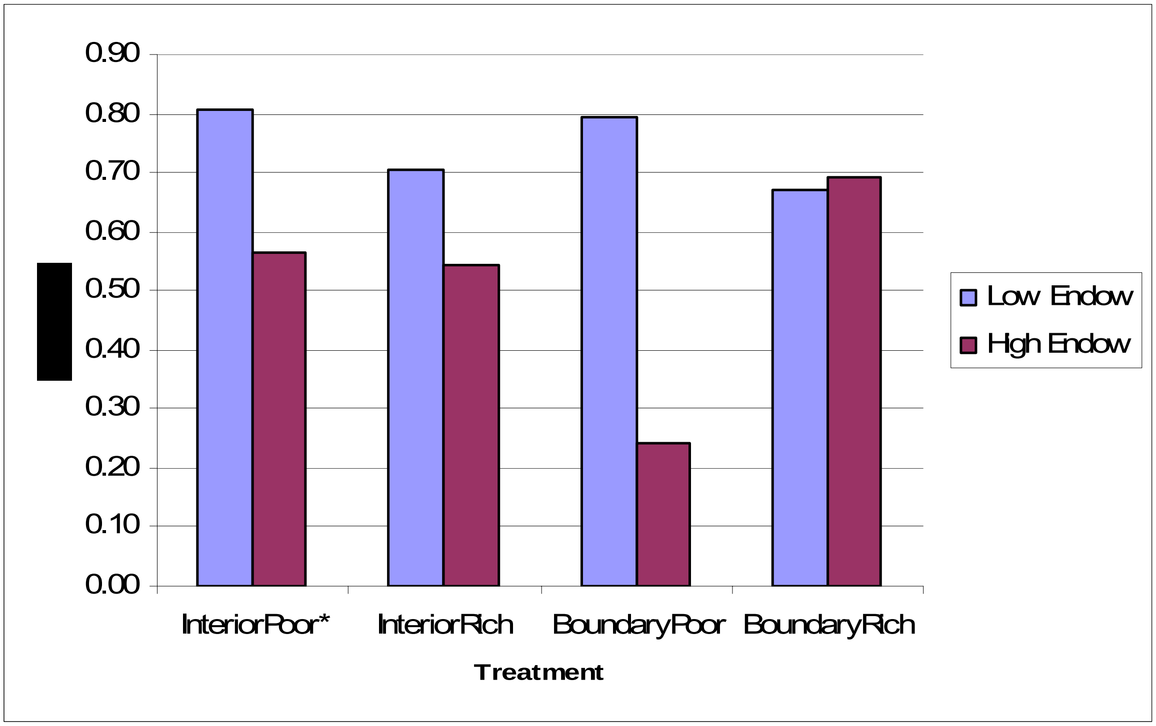

Figure 16: Percentage of Votes Cast to Elect an Agent by Type of Player

Table 18: Two-Sided Wilcoxon Mann-Whitney Test P-values

Boundary Design	**NA Decisions***	**Low Agent Decisions****	**High Agent Decisions**
Endowment (Pooled)	0.424	0.002	0.000
Endowment within first 20 rounds	0.545	0.000	0.000
Endowment Rich Design	0.175	0.006	0.001
Endowment Poor Design	0.396	0.031	0.000
Wealth Design	0.005	0.218	0.004
Order Effect of Treatment (Pooled)	0.040	0.081	0.839
Order Effect within Rich Design	0.544	0.472	0.384
Order Effect within Poor Design	0.013	0.070	0.184

Interior Design	**NA Decisions**	**Low Agent Decisions**	**High Agent Decisions**
Endowment (Pooled)	0.055	0.000	0.000
Endowment within first 20 rounds	0.663	0.004	0.001
Endowment Rich Design	0.031	0.000	0.000
Endowment Poor Design	0.767	0.000	0.000
Wealth Design	0.615	0.013	0.308
Order Effect of Treatment (Pooled)	0.008	0.767	0.690
Order Effect within Rich Design	0.152	0.165	0.443
Order Effect within Poor Design	0.036	0.340	0.141

*Compares no-agent decisions made by endowment type, across wealth designs, or by treatment sequence
**Compares agent decisions made for type 1 players by their endowment, by wealth design, or by treatment sequence.

Decisions made by type-1 players and type-2 players are, on the whole, different in not only agent treatments but also no-agent treatments as well as across wealth designs. Through Wilcoxon Mann-Whitney tests, average contribution decisions made by low-endowed players are compared to those made by high-endowed players to see if they belong to the same population with equivalent probability distributions. In addition, tests are run to see if there is an effect of placing the no-agent treatment first or second in the series of treatments in the within-subject design as well as if there was a difference in the decisions making between the wealth designs. The results of the two-sided Wilcoxon Mann-Whitney tests are shown in Table 18. The Wilcoxon Mann-Whitney rank sum tests show how the decisions in the three right hand columns are affected by the regressor in the first column of the table.

Table 18 shows decisions in no-agent treatment rounds are affected by variables causing differences in the distributions of private allocations to the group account. For instance, in the interior no-agent treatments, the pooled "endowment" variable is significant at the 6% level, meaning that type-1 private contributions are different from type-2 contributions. The pooled endowment variable is not significant in the boundary no-agent decisions, meaning that private contributions are not different by those with different wealth. When the endowment variable in no-agent rounds is broken down into different wealth designs, then it is not significant in the BoundaryRich, BoundaryPoor, and InteriorPoor setups. Looking further into no-agent decisions, endowment does not play a significant role in private allocations when no-agent decisions occupy the first twenty rounds of a session. Wealth design is only significant in the boundary design, meaning that BoundaryRich no-agent decisions are different than BoundaryPoor decisions. This wealth design effect is not present in the interior design. There does seem to be a significant pooled order effect in no-agent decisions at the 5% level, meaning that no-agent decisions in the first 20 rounds are different than those in the second 20 rounds after an agent treatment has been played. This order effect is also broken down into wealth designs to show if there is an order effect in no-agent decisions in each solution and wealth design.

The analysis of Table 18 can also be applied to agent rounds, specifically low-endowed agent decisions and high-endowed agent decisions. The size of a subject's endowment is significant in all agent decisions in every design and in every session. For both poor and wealthy agents, their allocation decisions are affected by their own wealth. The wealth design is significant only for boundary high-endowed agents and interior low-endowed agents. For these two groups, their decisions were affected by the overall wealth composition of their group. The pooled order effect is not significant for most agent decisions except low agent decisions in the boundary design. Only for these agents did their decisions vary by whether their agent decisions occupied the first twenty rounds of the experiment.

2.5 CONCLUSION

Agents in public good games have the opportunity to improve individual and social welfare through their coordinative allocations. Free-riding in private giving often creates inefficiency and poor social outcomes, and thus, agents present a natural alternative to individual giving environments. Social welfare does increase in this experiment when an agent is introduced, and thus, it is not surprising that agency is implemented in 70-90% of voting rounds.

Adding a wealth distribution among group members to test the robustness of agent efficiency over private giving creates a circumstance where choosing optimality as the agent is not optimal for the group. In general, agents faced with wealth heterogeneity in their group do not make Pareto-efficient allocation decisions. By identifying themselves by their wealth group, agents seek to increase their profits through the use of a predatory allocation strategy which places the majority, if not all, of the burden of the public good on the other wealth group. This predatory strategy on the part of a wealthy agent dramatically subtracts from the potential profits of those who started off already worse off, skewing group wealth even further. Even though a wealthy agent's predatory action may appear more unfair, poor agents actually take more predatory action than the wealthy by allocating greater percentages of their endowments to the group account.

Overall private contributions to the public good are smaller with a wealth distribution than when wealth is equalized, presenting a similar result to previous literature. However, the burden of provision for the public good in no-agent rounds is not affected by the wealth distribution. The poor in each group provide around half of the public good in most rounds, splitting the burden of the public good with the wealthy.

Introducing diversification in agent decisions through inequality in wealth among group members provides realistic extension to research in public-good decision making. The benefits of agency become entangled with issues of equity, wealth, and overall provision of the public good. The ability to show favoritism to the agent's own group is established with the addition of two allocation decisions. Combined with the wealth distribution addition, it is difficult to determine which addition might be causing some of these results.

A natural, next step to answering this question of predatory action, equity, and the benefit of an agent is to investigate agent giving when subjects are classified into groups without wealth context. Such an experiment would be very similar to this one except instead of wealth groups, subjects would be distinguished by some arbitrary name such as type A and type B. Then, agents would be able to single themselves out in order to show favoritism to their own group but would not be confronted with issues of wealth and equity. After this investigation, we will be able to discern which results are the effect of the ability to take predatory action or which are the result of wealth heterogeneity.

2.6 APPENDICES

2.6.1 Appendix A: Experiment Instructions, Questionnaires, and Tables

2.6.1.1 Experiment Instructions

The following instructions come from the first 20 rounds of the Boundary Poor wealth design where agent decisions occupy these beginning rounds. For the Rich wealth design and no-agent treatment design changes, only small changes to the instructions would occur. For an example of the interior instructions, please see Wick (2008).

This is an experiment in decision making. The Department of Economics has provided funds for this research. During the course of the experiment, you will make a series of decisions. You will be paid for participating, and the amount of money you earn depends on the decisions that you and the other participants make. At the end of today's session you will be paid privately and in cash for your decisions. Please do not talk to one another for the duration of the experiment.

The first phase of the experiment will consist of 20 rounds. When these rounds have elapsed, please wait for further instruction.

At the beginning of the experiment, everyone is randomly assigned to a group of 5 individuals in the first round. During the course of the experiment, your group composition does not change. You will be in a group with the same 4 other members for the experiment. All decisions you make in this experiment are anonymous; therefore, please do not reveal any of your decisions to any other participant.

At the beginning of each round, each of you will be gifted with a number of tokens in his or her private account. These tokens can be invested between two accounts: the group account and your own private account. Each member of the group will have a different amount of tokens gifted to them at the beginning of every period depending on their type. Your token gift at the beginning of each period is called your endowment. Three type-1 group members have an endowment of 75 tokens, and two type-2 group members have an endowment of 200 tokens.

When each round starts, you will be asked to make a decision as the agent of the group about how many tokens each member of your group must allocate to the group account. Thus, each of you will decide how many tokens both type-1 and type-2 group members must contribute to the group account. Tokens not contributed to the group account remain in your private account. The number of tokens in the group account equals the total amount of what the agent decides each person will contribute.

After each member of the group has made decisions about token allocations, then one member will be chosen at random to be the group agent and their decision about the group allocation will be implemented.

Your payment depends on the number of tokens remaining in your private account, and the total number of tokens contributed to the group account at the end of each round. For each token remaining in your private account at the end of the round, you earn 2.2 points. For each token in the group account, you and each member of your group will receive 0.7 points. Your total payoff for each round is the sum of your earnings from the private and the group account, and will indicated on your computer screen. Earnings from the group account depend only on the total number of tokens in that account.

Your earnings from the experiment will be drawn from the decisions from one round out chosen at random plus the $5 show up fee. The exchange rate for points to dollars is 1 point=2 cents. Thus, 300 points=$6. At the end of the experiment, you will be asked to come to the side room where you will be paid in private.

If you have a question while the experiment is going on, please raise your hand and one of us will come to your desk and answer it. At this time, do you have any questions about the instructions or procedures?

Questionnaire: We will now allow time to answer a questionnaire to make sure that all participants understand other important features of the instructions. Please fill it out now. Do not put your name on the questionnaire. We will then go over the relevant part of the instructions.

2.6.1.2 Questionnaires

The following is the questionnaire that would accompany the above instructions. Each set of instructions was followed by a similar set of questions based on the agent, no-agent, or voting treatment for those rounds as well as background conditions of the experiment.[21]

1. (True or False) Players remain in the same group of 5 players in all rounds of experiment.

[21] There was a set of instructions every 20 rounds as treatments changed.

2. If the chosen agent of the group decides each type 1 individual must contribute 30 tokens and each type 2 individual must contribute 125 tokens to the group account, how many tokens will be in the group account at the end of the period?

a. If you are a type 1 player, how many tokens remain in your private account?

b. If you are a type 2 player, how many tokens remain in your private account?

3. (True or False) Each player makes an allocation decision for the group every round but only one allocation is chosen.

2.6.1.3 Tables Given to Subjects

The tables given to subjects in this experiment are the same as Tables 6, 7, and 8 from the previous experiment.

2.6.2 Appendix B: Figures

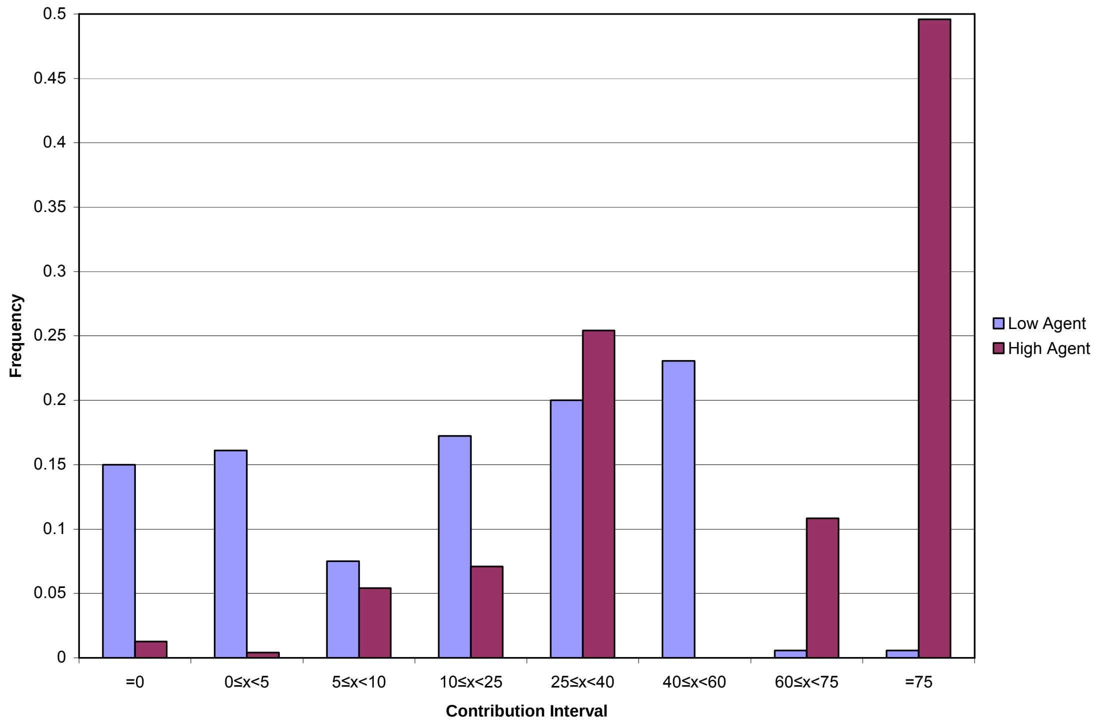

Figure 17: InteriorPoor Agent Contributions for Low Endowment Subjects by Frequency

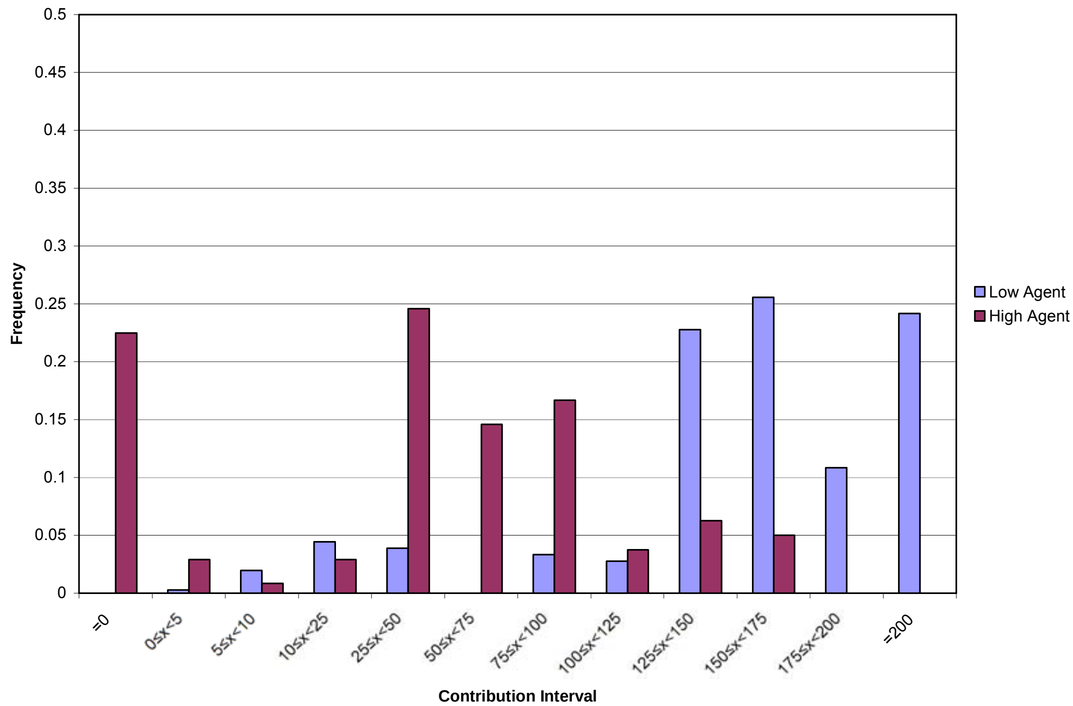

Figure 18: InteriorPoor Agent Contributions for High Endowment Subjects by Frequency

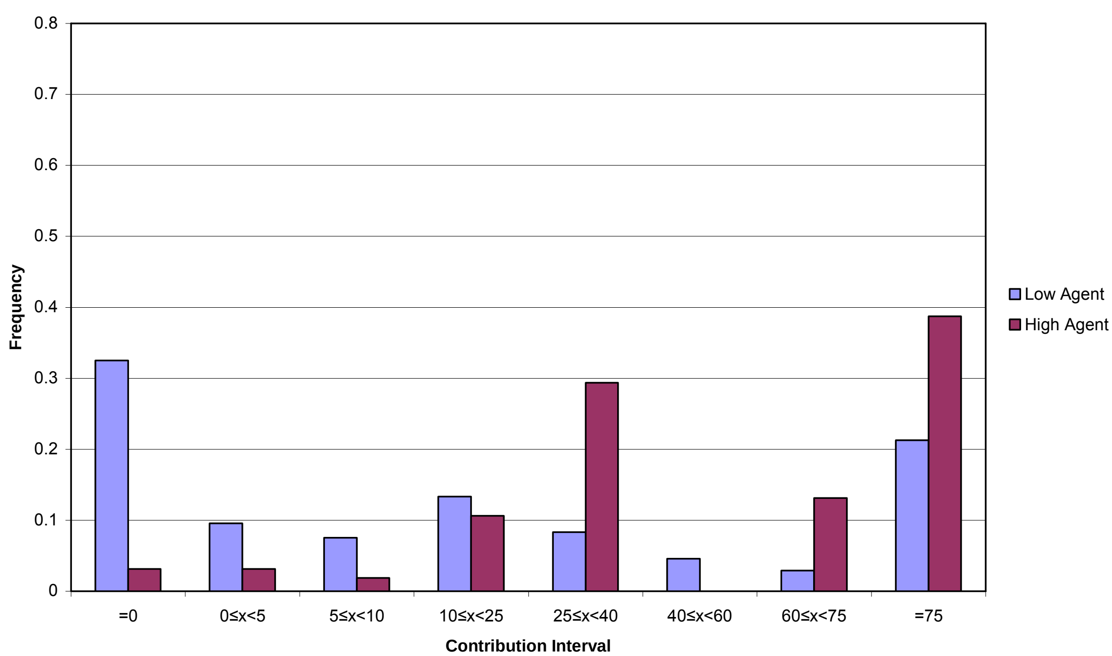

Figure 19: BoundaryPoor Agent Contributions for Low Endowment Subjects by Frequency

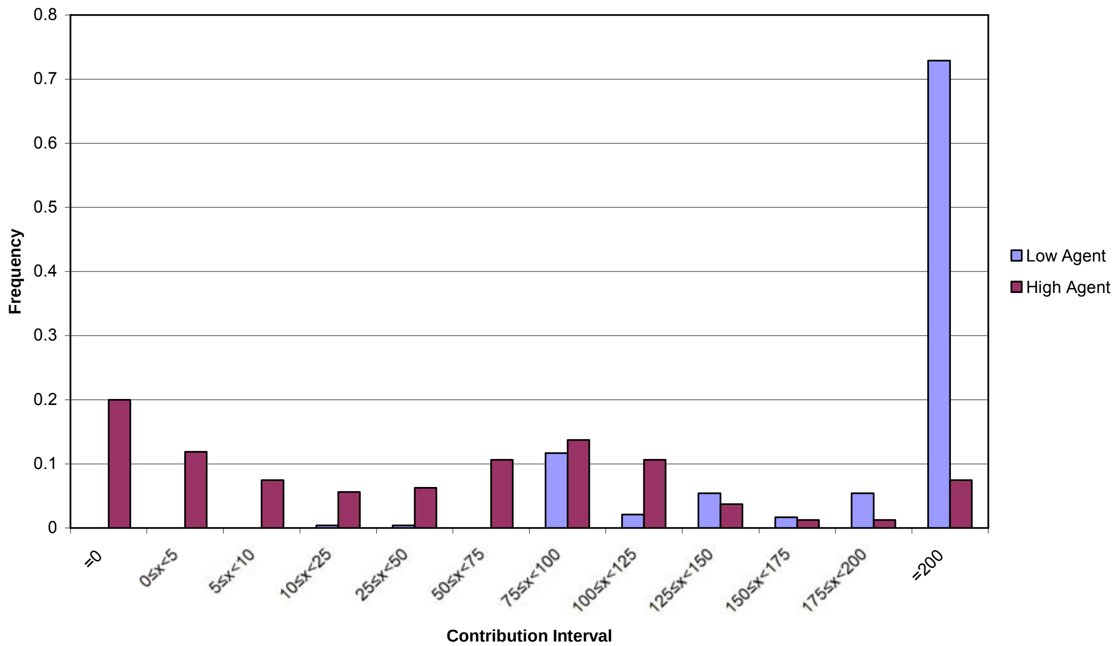

Figure 20: BoundaryPoor Agent Contributions for High Endowment Subjects by Frequency

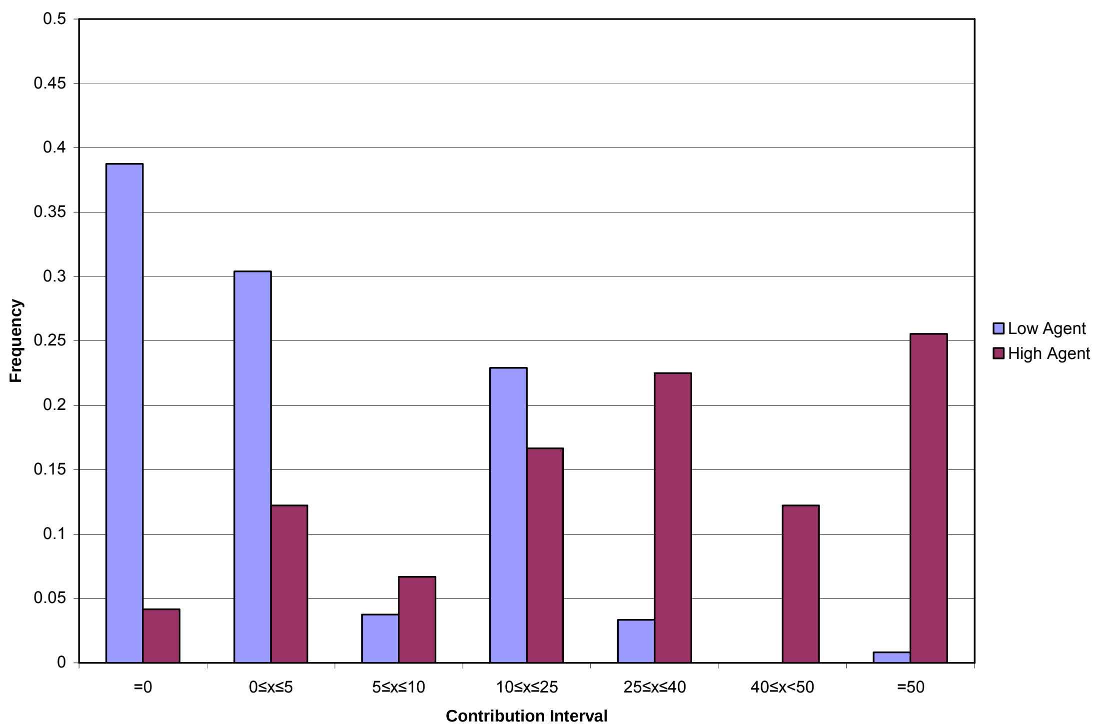

Figure 21: InteriorRich Agent Contributions for Low Endowment Subjects by Frequency

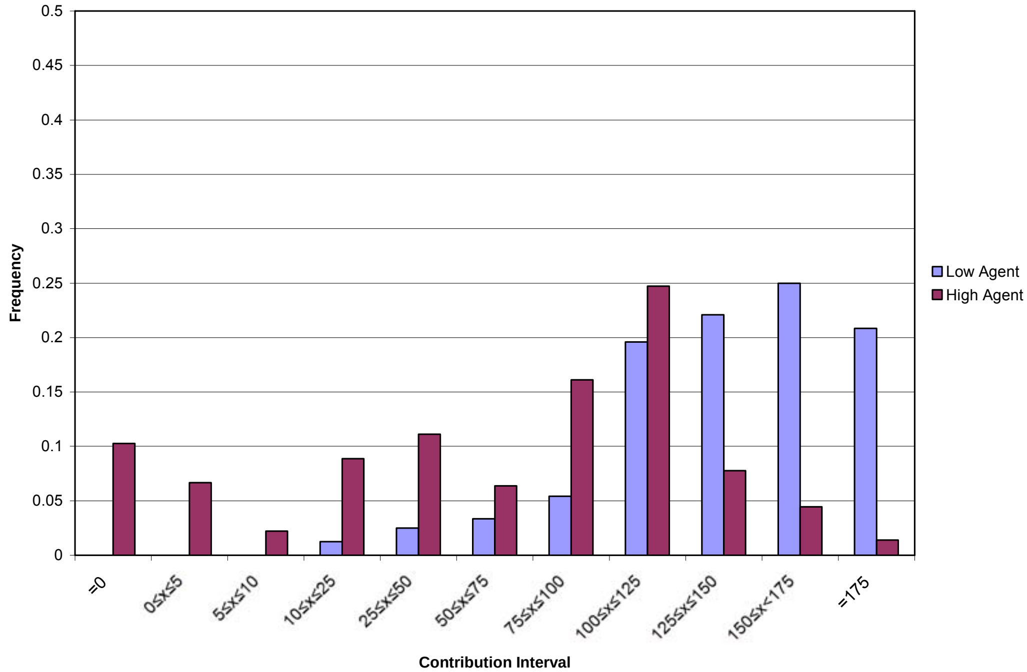

Figure 22: InteriorRich Agent Contributions for High Endowment Subjects by Frequency

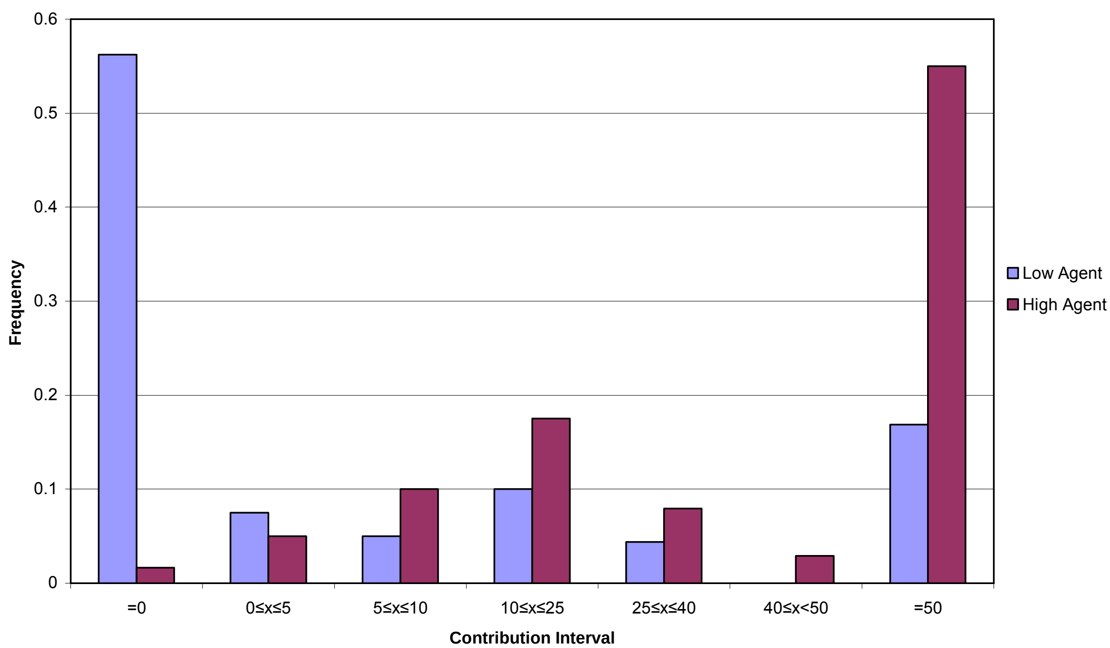

Figure 23: BoundaryRich Agent Contributions for Low Endowment Subjects by Frequency

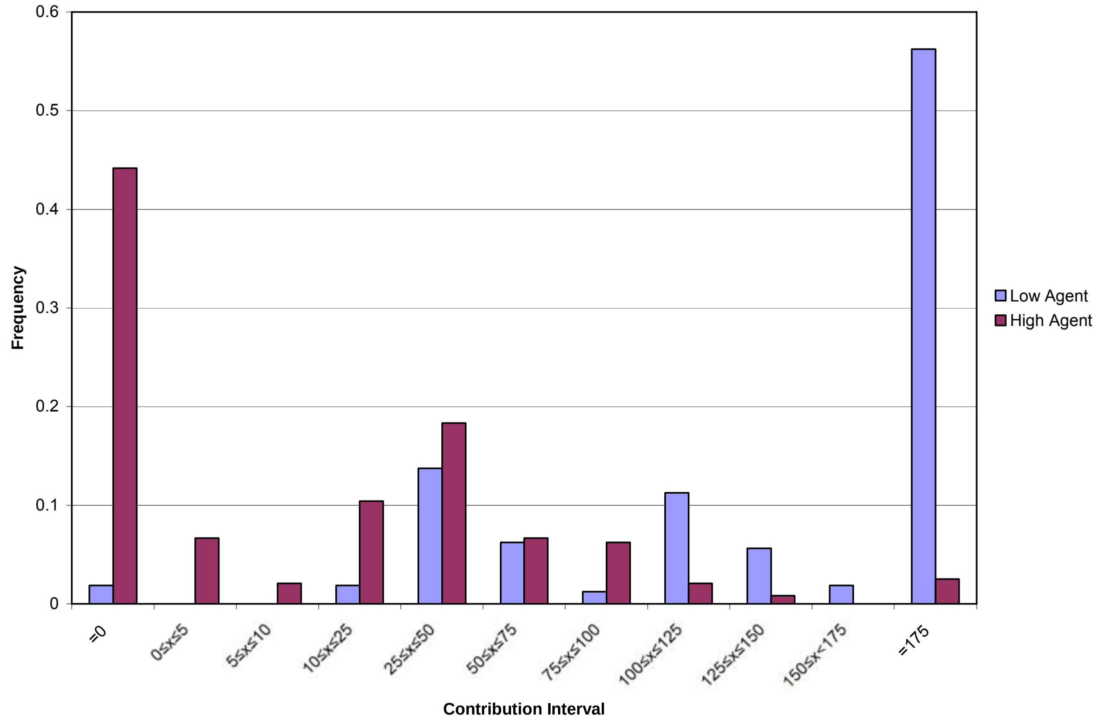

Figure 24: BoundaryRich Agent Contributions for High Endowment Subjects by Frequency

3.0 SWITCHING COSTS IN CONTRACT INDUSTRIES WITH TWO TYPES OF CUSTOMERS

3.1 INTRODUCTION

A switching cost is an expense incurred by an individual for changing from whom they purchase a given product. Switching costs can have serious implications for markets. They are often artificially created by companies to create loyalty, yet they can have significant backlashes in efficiency such as preventing entry of new firms and keeping prices high above the competitive price. In the cellular telephone industry before November 2003, when a customer decided to switch their cellular provider, they incurred a large implicit switching cost of losing your current telephone number on top of set up costs with the new company. In addition, any switching customer might face an explicit contract-break fee and implicit costs of learning a new network. Examples of switching costs are prevalent in most industries because firms try to hold onto their customers' business to increase their profits. Paul Klemperer has a long list of reasons for how these switching costs induce brand loyalty in his 1995 article.

For companies, the existence of switching costs on the part of their customers has very desirable properties. They keep customers locked in to purchasing their products, allowing them to charge a higher price and create more profits. In theory, this occurs because switching costs reduce consumers' price elasticity of demand. Paul Klemperer has an extensive literature on the theory of switching costs and their attractive qualities to companies. Klemperer not only comments on the seemingly positive aspects of switching costs but also on their draw backs especially with respect to the decrease in consumers' surplus and social welfare (1987b). Using a symmetric equilibrium argument, Klemperer's general model calls attention to non-cooperative behavior which in the presence of this expense to switch leads to outcomes that appear collusive. This is a result that is observed in this model as well.

The main problem of creating switching costs to establish loyalty and increase profits is that firms are extracting away from consumer surplus and creating social dead weight losses by not allowing for efficient outcomes since switching costs deter entry of new firms who price competitively and possibly drive the market price to competitive equilibrium (Klemperer 1987a, 1988).

In this theoretical paper, I model an industry that is faced with contracts as a source of pricing strategy. This paper investigates a duopoly faced with designing contracts to maximize their profits based on the behavior of their competitor. The contract itself presents its own form of switching cost because breaking the agreement early causes the consumer to have to pay the provider a termination fee, an explicit cost to switch. When consumers are locked into a price for the duration of a contract, then a company can practice price discrimination, charging them a higher price (as specified in the contract) than new consumers who are offered a lower price to capture their business.

Klemperer proves in a multi-period model that once customers are locked into a price, they face higher prices in early periods than new customers in subsequent periods (1995). In an industry where a firm cannot use discriminatory practices in any period, companies face a trade-off, whether to continue charging a high price to locked-in consumers or to charge a lower price in order to capture new consumers as well as maintaining the old ones (Klemperer 1995). Furthermore, Klemperer finds that prices to both new and old customers are higher than they are in a market without switching costs. In the present paper, I analyze similar issues in an environment where firms can sign contracts with customers, who vary in the extent of the switching costs they experience.

The contract framework provides another source of trouble for markets with switching costs in terms of efficiency. Specifically, they create the problem preventing competition among firms in a market to drive competition to the fullest extent. Aghion and Bolton see these contracts as creating negative externalities that allow the incumbent to extract some of the entrant's surplus and therefore create a barrier to entry (1987). These barriers from switching costs are not discussed in this paper.

Moreover, companies try to prey on their competitors' customers by proposing luring attractive introductory offers to lock them into contract at a high price (Doyle 1986). We see examples of this in the cell phone industry where a customer must pay a high price in a contract for a certain number of minutes but might get "pulled" in by an offer of a free cell phone. Doyle is arguing that companies often pay a customer's setup costs, such as the cost of a new phone, in order to attract them to sign a contract. Chen calls this "paying customers to switch" (1997). A company frequently gives customers a fixed amount of money or an introductory offer in order to entice them to switch. These luring payments can play a significant part in explaining pricing, but I do not specifically study them in this paper since they add one more pricing affect to an already complex model.

Chen's results in his 1997 paper are closely related to what is studied in this paper, with a few exceptions. In Chen’s paper, he considers a two-period model where in the first period new customers can be “paid” to purchase from a company who offers a luring, low price. In the second period, prices increase, and profits for the firm suffer due to offering these introductory prices. There are no contracts with associated fees or customers locked-in to purchasing from a particular producer; he considers the same pool of customers during two periods.

In contract markets, there exist groups that are locked into previous prices and therefore have associated switching costs from changing providers. In addition, there are new customers who have not signed contracts and therefore face no such switching costs. At any given period of time, firms cannot tell if a customer approaching them for service is a buyer who has previously consumed the product from another provider or if he is a new buyer who has never purchased from the producer. Therefore, firms can charge one set of prices to a group of old customers who are locked into their contracts and another set of prices to their incoming customers who are shopping around for a deal.

This fight over new customers and those whose contracts have expired creates a great deal of competition in markets with switching costs. It is in these cases that the competition can become quite intense. Firms can gain profits by continuing to charge high prices even when facing a group of new customers. Therefore, old customers never find it appealing to switch to another option. In addition, firms can gain profits by cutting price to drastically in a new period that they gain all new customers and old customers would switch from their current provider, lured by the low price.

When a company is making the decision based on this trade-off, it is faced with maximizing its total expected profits by trying to keep as many of their old customers as possible. They want to make sure they keep their customers marginally happy at some fixed utility value. This approach has been taken by Farrell and Shapiro (1989) in trying to discover what is the optimal contract that a firm can offer given that it has to try to keep customers marginally happy with their product. What they neglected to study in their paper is how new customers' entry and different types of switching costs affect those optimal contracts to be offered in every period.

My goal in this paper is to model an environment with both new and old customers where buyers face implicit switching costs, a private signal, and explicit costs specified in a customer's contract. Sellers maximize expected profits by choosing the contract terms with both new and old customers. The firms compete in an infinitely repeated game. I characterize the properties of the steady-state of this dynamic environment derived from a Hotelling model of competition between two differentiated firms.

The results obtained in this paper reinforce many of those obtained in the literature. There exist collusive outcomes where the firms can take advantage of their contacted old customers and extract their entire surplus just as shown in Klemperer's 1987 paper. I also find a higher price charged to old customers than to new customers, just like Klemperer found in his 1995 paper. I also characterize another equilibrium which is Pareto-inferior from the perspective of the firms. In this equilibrium, as the expected value of customers' implicit switching costs decreases, the prices and fees charged to old customers decrease. This result implies that when customers find it easier to switch among firms, price competition intensifies.

The equilibrium prices and fees charged to both old and new consumers are independent of the number of them in the market. Their number only influences the profit of the firm. This is to mirror many industries which have very small (approaching zero) costs for each additional customer they have on their service. This result is possibly implied by the demand structure assumed in the model, where customers demand the good with perfect inelasticity.

In my model, there exists a dead weight loss to society due to these switching costs and the actual switching taking place in the inferior equilibrium. On the other hand, the collusive outcomes, which allow the firm to exact the entirety of the representative consumer surplus in a period, are Pareto optimal because they maximize social welfare. Since demand is assumed to be perfectly inelastic in the model, the results may change with a different demand structure.

3.2 THEORETICAL MODEL

3.2.1 Companies' Pricing Profiles

p price to old customers locked into contracts

q fee to break contract if old customer leaves

m price to new customers which may include explicit setup costs

3.2.2 Representative Company *i* pricing strategy

n locked in customers

φ_i probability of customer switching (This can also be thought of as the proportion of old customers who switch from using i's product at the current contract prices p_i and q_i.) Therefore, $(1-\varphi_i)$ is firm i's market share of old customers.

r pool of new customers

λ_i proportion of new customers that i captures

y location of the firm on a line reflecting the distribution of preferences of customers

x customer's implicit switching cost, unknown to the company. The distribution of the implicit switching cost is common knowledge.

The probability that a customer switches is a function of his implicit switching cost and the prices and explicit costs set by each firm. Therefore, when a seller chooses a contract strategy, it affects their own profits and indirectly the probability that a given customer switches to a different provider. This is an important aspect of the setup because the choices made in the contract strategy of a seller have both direct effects (profits) and indirect effects (loyalty and customer switching). The properties of the new customers who choose each firm is endogenously determined in the model by the prices the firms select.

3.2.2.1 For New Customers

λ_i, the proportion of new customers captured with by firm *i*, is determined by a simple, one-dimensional Hotelling model, with *i* located at one of the extremes, as follows:

If $m_i + \alpha q_i + ty \leq m_j + \alpha q_j + t(1-y)$, a customer prefers *i* over *j*. Consumers in a Hotelling model have preferences for both the utility obtained from a particular company's characteristics (such as prices and fees) as well as its geographic location in relation to their preferences. t is a transportation cost associated with a customer's preferences for geographic location and is incurred they sign a contract with a particular company located a specified distance from their preferences.

Therefore, $\lambda_i = \frac{1}{2} + \frac{(m_j + \alpha q_j) - (m_i + \alpha q_1)}{2t}$, by solving for the indifferent new customer.

α measures an uncertainty factor, for consumers are not certain a particular company will line up with their preferences in the future. Since new customers are choosing in one period a company with whom to sign a contract for more than one period, they face uncertainty about how the company's product will align with their preferences. They might decide they do not like the product at some point in their contract, and therefore be forced to incur a break fee to later switch companies.

3.2.2.2 For Old Customers

When considering the choices of old customers, the analysis become a little more complicated since the probability that a firm keeps an old customer depends on the prices they are charged in their contract, how their current company aligns with their preferences, how much the other firm is charging to new customers, how well the other firm aligns with their preferences both now and in the future, and how much it costs them to switch. For old customers to switch, they incur their contract break fee plus their implicit switching cost x.

The implicit switching cost x is distributed uniform from $[0,\theta]$. This is a model of two symmetric firms competing for both old customers, who have signed contracts in the past, and new customers, who are completely new to the market. Also, these two firms are differentiated providers of a good in the market. Therefore, this model is centered a two-dimensional Hotelling model with the implicit switching costs on the x-axis and the placement of the firms along the y-axis. For simplicity, assume that firm i is located at 0 and firm j is located at 1. See Figure 25 for graphic representation. One of the questions I attempt to answer in this paper is how changes in the distribution of implicit switching cost affect the equilibrium prices.

The probability that firm i keeps an old customer ($1-\varphi_i$) is affected by both x, the customer's implicit switching costs, and y, his location, as follows:

If $p_i + ty \le m_j + \alpha q_j + q_i + (1-y)t + x$, then an old customer prefers i over j.

If this inequality is satisfied, then an old customer is not willing to switch from their current provider firm *i* to firm *j*. The price in their contract plus the total transportation cost of where the firm in located has to be less than the price that *j* is offering new customers plus the transportation cost to choose firm *j*, the contract break fee from *i*, the total aversion of signing a new contract for *j*, and that customer's individual private switching cost signal. Solving for *y* yields the set of customers who stay with *i* as follows:

$$y \leq \frac{1}{2} + \frac{x}{2t} + \frac{\mathrm{m_j} + \alpha \mathrm{q_j} + \mathrm{q_i} - \mathrm{p_i}}{2t}$$

Thus, the model becomes a Hotelling model in two dimensions. Old customers make their staying or switching decisions based on two factors outside the control of the firm: the location of the firm in their preferences, *y*, and their implicit switching cost, *x*.

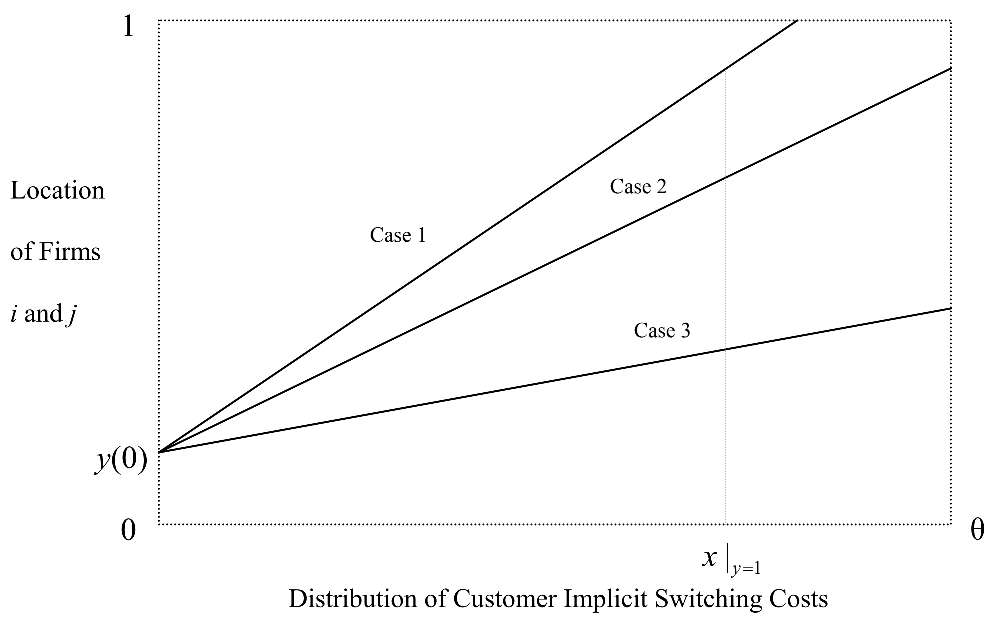

Figure 25: Two Dimensional Hotelling Model for Old Customers

An upward-sloping line in Figure 25 represents those customers who are indifferent between switching or staying with *i*. It shows where the group of old customers are indifferent between choosing firm *i* or firm *j* based on their implicit switching cost, which is a private signal to the customer, and the location of each company.

The proportion of old customers a firm *i* keeps, ($1-\varphi_i$), is the area under the line of indifference in each case. The proportion of old customers that switch from firm *i*, φ_i, is the area above the line of indifference in each case.

The y-intercept, *y(0),* is chosen to be positive for the purposes of this graph. The intercept could take on any value in [-1,1]. It is necessary for $1 \geq y(0) \geq -1$ since this is a Hotelling model of a duopoly where it is exogenously given that the distance between the two firms is 1. The intercept in this model cannot exceed the distance set between the duopolists. The positive intercept means that there is a proportion of customers having the lowest switching costs who continue choose firm *i* because of its location in relation to their preferences. A negative intercept means that a given proportion of customers with the lowest switching costs choose to switch from provider *i* because of its location in relation to their preferences.

Using the above inequality for *y* from the old customer's purchasing decision, I can solve for the intercepts of this two-dimensional Hotelling model. In the above inequality, *y* is a function of *x* so I continue to use that notation in order to mathematically solve for the values of the intercepts of the Hotelling model.

$$y(0) = \frac{1}{2} + \frac{\mathrm{m_j} + \alpha \mathrm{q_j} + \mathrm{q_i} - \mathrm{p_i}}{2t}$$

$$y(\theta) = \frac{1}{2} + \frac{\theta}{2t} + \frac{\mathrm{m_j} + \alpha \mathrm{q_j} + \mathrm{q_i} - \mathrm{p_i}}{2t}$$

Assume $0 < y(0) < 1$ (as depicted in Figure 25 above)

$$-t < \mathrm{m_j} + \alpha \mathrm{q_j} + \mathrm{q_i} - \mathrm{p_i} < t$$

Call $\mathrm{m_j} + \alpha \mathrm{q_j} + \mathrm{q_i} - \mathrm{p_i} \equiv R_{ij}$

$-t < R_{ij} < t$

The values of R_{ij} are important in differentiating among the different cases for each equilibrium.

Figure 25 depicts the three different cases that might arise in describing the indifference line of old consumers. Case 1 is such that those who are old customers of firm *i* with the highest implicit switching costs are always deterred from switching. Case 2 allows those old customers with the highest values of switching costs in the distribution to be exactly indifferent between switching. Case 3 allows a proportion of old customers locked into firm *i*'s contracts with the highest costs to switch to firm *j*. This case is a bit counter-intuitive and works out mathematically to take the same shape as case 1. Therefore, we restrict our attention to the case where customers at the top of the distribution of switching costs always remain loyal to their contracts and their current provider, case 1.

3.2.3 Firm *i*'s Problem

$$MAX_{p_i,q_i,m_i} \quad \pi_i = \sum_{l=0}^{n} \binom{n}{l} (1-\phi_i)^l \phi_i^{n-l} \left[{}^{l}p_i l + q_i(n-l)\right] + \lambda_i m_i r$$

General maximization problem:

$$MAX_{p_i,q_i,m_i} \quad \pi_i = \frac{n}{\theta}(1-\phi_i)(p_i - q_i) + nq_i + \lambda_i m_i r$$

Since the old customers are already locked into contracts at p_i and q_i, they are making choices about whether to stay with their current provider. In firm *i*'s problem above, the company keeps an old customer, *l*, with probability $(1-\varphi_i)$ and therefore obtains the contract price net of marginal cost which is assumed to be zero. If an old customer decides to opt out of their contract, which occurs with probability φ_i, he has to pay firm *i* a break fee q_i. Firm *i* captures a proportion λ_i of new customers; thus, the profit from new customers is amount of customers attracted to firm *i* at their contracted price, m_i.

The firm's problem is set up as a profit maximization problem with both new and old customers aggregated into one period. Thus, the firm is trying to decide what to offer those who are approaching them for contracts today, who become old customers in the near future, at the same time as deciding contract prices to the next set of customers who will come to them after these "old" customers are locked into contracts. Each firm is forward looking in its contract pricing decisions, and therefore is simultaneously making decisions for contracts that are given at different points in time. Therefore at one stage of this infinitely-repeated game, firms are designing contracts for both types of customers who approach them sequentially.

The "old" customers were not always old, but in this model when the firm was deciding what prices to set for their group of consumers it was also deciding the pricing scheme for their next group of customers. In effect the firm is making its multi-period decisions simultaneously just as if it had a high discount rate (very close to one). In effect, these periods are very small since as soon as a customer signs a contract, he becomes an old customer due to the fact he is "locked-in" to consuming from his contracted firm. The price at which a customer signs a contract is the going price they face in all periods unless he decides either to resign with the same company when his term is up or to switch to another company and become its new customer. Firms are trying to find steady-state pricing behavior for this infinitely-repeated game in a market where they are facing two types of consumers who contract with them at different points in time.

3.3 SYMMETRIC EQUILIBRIUM ANALYSIS

In this section, I show that this model has three symmetric equilibria: two collusive equilibria, and one which is Pareto inferior with an associated dead weight loss.[22] First, we have to be able to find intersection points and boundaries on Figure 25, which determine our various cases in this paper, to further characterize the profit maximization problem of the firm.

3.3.1 Cases 1 and 2 - $y(\theta) \geq 1$

This condition $y(\theta) \geq 1$ implies that the line of indifference of the Hotelling model from Figure 25 either intersects the upper-right hand corner of the figure (where $x = \theta$, $y = 1$) or intersects the upper boundary of the figure.

$$\Rightarrow R_{ij} \geq t - \theta$$

This property is derived by looking at the equation above for $y(\theta)$.

If $2t \geq \theta$ or $(t\text{-}\theta \geq -t) \Rightarrow t\text{-}\theta \leq R_{ij} < t$.

If $2t < \theta$ or $(t\text{-}\theta < -t) \Rightarrow -t < R_{ij} < t$.

These properties are derived by looking at the conditions for the assumption that $0 < y(0) < 1$.

Thus, $x|_{y=1} = t - [m_j + \alpha q_j + q_i - p_i] = t - R_{ij}$.

[22] Asymmetric equilibria are not considered due to the complexity of the model.

3.3.2 Case 3 - y(θ) < 1

This condition, $y(\theta) < 1$, implies that the line of indifference of the Hotelling model from Figure 25 intersects the right boundary of the figure.

$$\Rightarrow -t < R_{ij} < t - \theta, \text{ possible only if } 2t > \theta$$

Also, we need to know the market share or proportion of old customers a firm keeps, $(1-\varphi_i)$, in the maximization problem. This proportion for firm i is calculated by looking at the area under the lines of indifference from Figure 25 since these are the customers who decide not to switch.

3.3.3 Market Share of Old Customers $(1-\phi_i)$ for Cases 1 and 2

The market share of old customers is defined as the area under the lines in Figure 25. They are those customers who do not switch from using our representative firm i's product.

$$\frac{[y(0)+1]}{2}x|_{y=1} + (\theta - x)|_{y=1}$$
$$\left(\frac{3}{2} + \frac{R_{ij}}{2t}\right)\left[\frac{t-R_{ij}}{2}\right] + (\theta - t + R_{ij})$$

3.3.4 Profit Maximization for Cases 1 and 2

$$E\Pi = \left[\left(\frac{3}{2} + \frac{R_{ij}}{2t}\right)\left[\frac{t-R_{ij}}{2}\right] + (\theta - t + R_{ij})\right]\frac{(p_i-q_i)n}{\theta} + nq_i + rm_i\left(\frac{1}{2} + \frac{(m_j+\alpha q_j)-(m_i+\alpha q_i)}{2t}\right)$$

3.3.5 General First-Order and Second-Order Conditions

$$\frac{\partial E\Pi}{\partial p_i} = \frac{n}{\theta}\left[\theta - \frac{(t-R_{ij})^2}{4t}\right] - \frac{n}{\theta}(p_i - q_i)\left[\frac{t-R_{ij}}{2t}\right]$$

$$\frac{\partial^2 E\Pi}{\partial p_i^2} = -\frac{2n}{\theta}\left[\frac{t-R_{ij}}{2t}\right] - \frac{n}{2t\theta}(p_i - q_i) < 0$$

since $n, \theta, t, (p_i - q_i), (t - R_{ij})$ are all assumed to be > 0.

Therefore, the second order condition is generally satisfied for cases 1 and 2. As we see in a later in section 3.8, under certain parameter restrictions, case 1 has a divergent second order condition, meaning its second derivative is positive at the equilibrium solution which is a violation of the second order condition. Therefore, case 1 does not survive as an inferior equilibrium of this model.

$$\frac{\partial E\Pi}{\partial q_i} = -\frac{\partial E\Pi}{\partial p_i} + n - \frac{\alpha r m_i}{2t} \text{ (Equation 1)}$$

3.3.5.1 Solving First- and Second-Order Conditions Explicitly

$$\frac{\partial E\Pi}{\partial p_i} = 0 \Rightarrow p_i - q_i = \frac{2t\theta}{t-R_{ij}} - \frac{t-R_{ij}}{2} \text{ (Equation 2)}$$

$$\text{If } \frac{\partial E\Pi}{\partial p_i} = 0, \text{ then } \frac{\partial E\Pi}{\partial q_i} \left\{ \begin{array}{l} > 0 \text{ if } n > \frac{\alpha r m_i}{2t} \\ < 0 \text{ if } n < \frac{\alpha r m_i}{2t} \end{array} \right\}$$

Thus, when the solution for p_i is interior to the firm's decision space, the first-order condition for the break fee is positive or negative depending on the size of the old customer population. If the set of old customers is quite large with respect to the pool of new customers, then the firm's profit increases as the break fee increases. The firm has more to gain from high break fees in an effort to retain their locked-in customers.

3.4 FIRST COLLUSIVE SYMMETRIC EQUILIBRIUM

If $n > \frac{\alpha r m_i}{2t}$, then the number of old customers outweighs the rate at which a firm can gain new customers in this symmetric model, which depends on how uncertain customers are about how firms align with their preferences in the future, the transportation cost, and the price that firm *i* charges its new comers. Since the number of old customers is relatively big in comparison to the rate of gaining new customers, each firm wants to choose the maximum q_i for their contracts to induce high consumer loyalty to their product. Meaning, old customers do not want to switch from their current provider to pay the break fee.

Therefore one of two above conditions from cases 1 and 2 apply:

$$\left\{ \begin{array}{c} t - \theta \leq R_{ij} < t \text{ if } 2t \geq \theta \\ -t < R_{ij} < t \text{ if } 2t < \theta \end{array} \right\}$$

Thus in both cases, a firm wants to choose maximum q_i such that R_{ij} reaches the limit of its maximum value ($R_{ij} = t$).

If $R_{ij} = t$, then $\frac{dE\prod}{dp_i} = n > 0$. Since there are always increasing returns to increasing the price, then the firm sets p_i at its greatest possible value.

If firms are going to exploit the highest price from their old customers, they set $p_i = v\text{-}t$ where *v* is the maximum reservation price of a representative customer. So the firm is charging a price such that all old customers still want to buy from them because the price is such that it subtracts the highest possible transportation cost. Therefore the farthest away customer from firm *i*, who would incur the entire transportation cost of *t*, still purchases from them since they are indifferent between their product and switching to *j*. The first order conditions become:

At $R_{ij} = t$,

$$\frac{dE\prod}{dp_i} = n$$
$$\frac{dE\prod}{dq_i} = -\frac{\alpha r m_i}{2t} < 0$$

Therefore, i wants to choose the smallest break fee, q_i, consistent with this new region.

Proposition 1: There exists a collusive equilibrium to this model where an old customer does not switch from their current provider at their given contract.

For the mathematic derivation of the equilibrium, please see 3.8.1 Appendix A.

Characteristics of this collusive symmetric equilibrium:

$$p_i = v - t$$
$$m_i = m_j = t$$
$$q = \frac{v-t}{1+\alpha}$$
$$E\prod = (v-t)n + \frac{rt}{2}$$
$$y(0) = 1$$

The graphic representation of this equilibrium in the two-dimensional Hotelling model is seen in Figure 26.

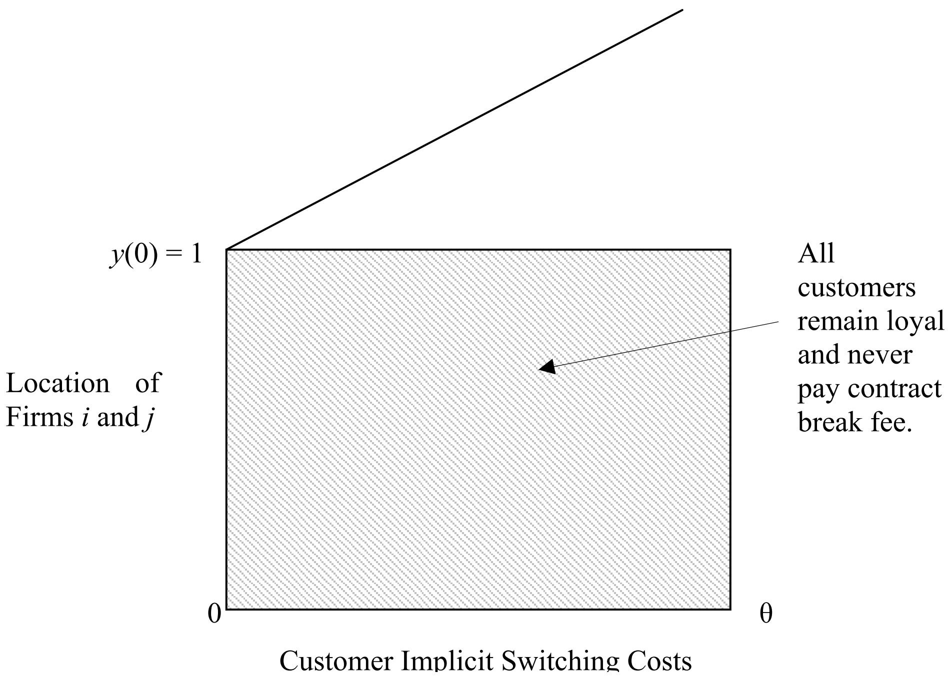

Figure 26: First Collusive Outcome

Since the area under the line of indifference is those who do not switch from their current provider (firm *i)*, in this equilibrium no one switches, and each firm gets $p = v - t$ from all old customers and $m_i = m_j = t$ from $\frac{1}{2}r$.

In this equilibrium, both firms are pursuing a collusive equilibrium. To their old customers, these duopolists are acting together as a monopoly in this Hotelling model and achieving the highest possible profit from the market. Therefore, this equilibrium has some attractive properties that most models with switching costs do not posses. There is no dead weight loss since all consumer welfare is transferred directly into the producer's profit. These duopolists are extracting all the possible surplus rents from their old customers. From an efficiency standpoint, this collusive outcome without switching is Pareto optimal. Also, due to the lack of switching taking place in equilibrium, the equilibrium is very stable.

3.5 SECOND COLLUSIVE SYMMETRIC OUTCOME

This equilibrium is also involves a collusive outcome between firms *i* and *j*, but this equilibrium brings about an exact opposite result from the first collusive outcome. To best illustrate this equilibrium, see the Hotelling model for this equilibrium Figure 27.

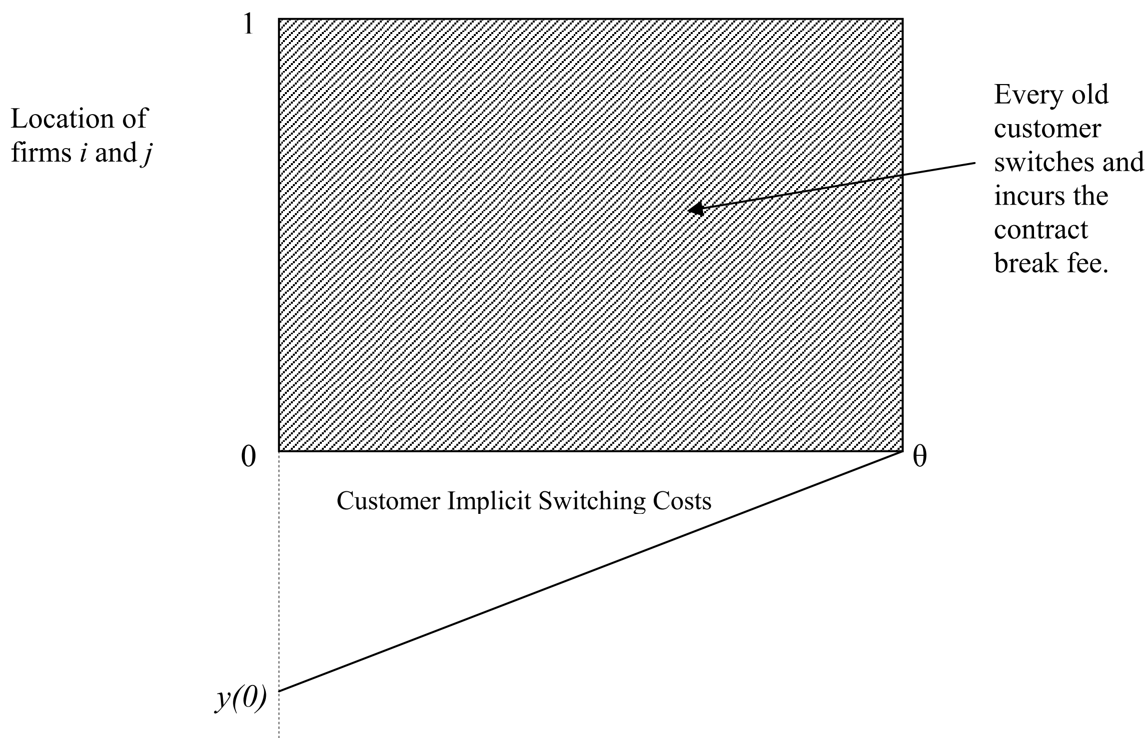

Figure 27: Second Collusive Outcome

As seen above in Figure 27, from the perspective of firm *i*, all of its old customers switch from using their product to provider *j* since the entirety of the area of the two-dimensional Hotelling model lies above the line of indifference.

Proposition 2: There exists a collusive equilibrium to this model where every old customer switches from their current provider at their given contract as seen in Figure 27.

For the mathematic derivation of the equilibrium, please see 3.8.2 Appendix B.

Thus, the characterization of this second collusive symmetric equilibrium is:

$$
\begin{aligned}
&p = (1+\alpha)(v-t)+4t \\
&m = t \\
&q = v-t \\
&E\prod = (v-t)n + \frac{rt}{2}
\end{aligned}
$$

Since every customer switches from their old providers and pays the break fee, each firm chooses $q = v - t$ to maximize their profit.

The difference from this equilibrium to the first is that it induces all old customers to switch companies in every period and therefore pay the break fee to their current provider. Compared to the results from this equilibrium, the first Pareto-optimal equilibrium set symmetric lower prices and lower break fees such that the old, loyal customers are not "lured" away by the pricing scheme to new customers from the competitor. When prices to old customers and explicit switching costs (break fees) are both higher, all customers are lured by the low prices ($m_j = m_i = t$) to new customers by the competitor, and therefore each old customer is willing to pay the break fee to switch companies in every period.

This result is not only different to the first in terms of the prices charged but also in terms of the large amounts of switching taking place. In the first collusive equilibrium, customers remain loyal to their contracts, and in this equilibrium, we see customers being completely disloyal to their contracts. Customers never pay their switching costs in the first equilibrium, but in this equilibrium, they always do. Since customers are paying their explicit and implicit switching costs, they have a smaller surplus which can be extracted by the firms. Therefore, this equilibrium is inferior to the first collusive outcome. Despite its inferiority, this equilibrium is still collusive since firms can symmetrically impose switching costs to increase profits and reach a collusive monopolistic outcome.

3.6 PARETO INFERIOR SYMMETRIC EQUILIBRIUM

I restrict attention to the region where $\theta > 3t$ so that only one case of the Pareto inferior symmetric equilibrium survives (case 2 from Figure 25). Cases 1 and 3 have divergent contract solutions under these conditions, meaning their second derivatives are positive at the equilibrium solution. Under these restrictions, I can characterize a Pareto-inferior equilibrium with switching where only some customers switch, creating a positive dead weight loss. This equilibrium is also inferior from the point of view of the firm because it brings lower profits and prices than the collusive equilibria. The details of the solution are as follows. For details on cases 1 and 3, see 3.8.3 and 3.8.5 Appendices C and E respectively.

3.6.1 If $2t < \theta \Rightarrow R_{ij} = -t$

This condition comes from the lower bound of R_{ij} when $y(\theta) \leq 1$ and $2t < \theta$. I have chosen the lower bound of R_{ij} since because when $y(\theta) \leq 1$, the firm keeps more old customers than it loses.[23] Under these conditions, I also assume $n > \frac{\alpha r m_i}{2t}$ just as in the first collusive equilibrium. Therefore, the number of old customers outweighs the rate at which a firm can gain new

[23] More customers stay with their product than who switch to the competitor. See Figure 28 and explanation below.

customers in this symmetric model. The rate at which a firm can gain new customers depends on how uncertain customers are about how firms will align with their preferences in the future, the transportation cost, and the price that firm *i* charges its new comers.

Proposition 3: There exists an equilibrium in this region which is inferior to the collusive equilibria from the point of view of the firm.

In 3.8.4 Appendix D, I derive the equilibrium and show its inferiority from the point of view of the firm.

The characterization of the inferior equilibrium in case 2 is:

$$p = \frac{\theta - 3t}{\alpha} + \theta - t$$
$$q = \frac{\theta - 3t}{\alpha}$$
$$m = t$$
$$E\Pi = \frac{n(\theta - t)^2}{\theta} + n\left(\frac{\theta - 3t}{\alpha}\right) + \frac{rt}{2}$$

Since $\theta > 3t$, then both $p,q > 0$.

The following comparative statics with respect to the break fee *q* and *p* hold:

$$\frac{\partial q,p}{\partial t} < 0,\ \frac{\partial q,p}{\partial \theta} > 0,\ \frac{\partial q}{\partial \alpha} < 0,\ \frac{\partial p}{\partial \alpha} < 0 \text{ since } \theta > 3t$$

Therefore, the explicit switching cost and price for each individual customer varies inversely with the transportation cost, *t*, and the uncertainty factor, α, but it varies directly with θ. This is a logical explanation of behavior in equilibrium. This form of the inferior equilibrium is plausible mostly because of the comparative statics with respect to θ. Firms do not know what kind of inconvenience individual customers face to switch companies to their product, but they do know the distribution of these implicit costs. As the distribution of these costs, θ, increases, companies can take advantage of the fact that it is a large hassle for their customers to switch from consuming their product and charge them higher prices to consume their product. But if an exogenous change occurs that causes θ to decrease, then companies have to compete more heavily to keep their customers and charge lower prices and contract fees.[24] Therefore making switching between companies less of a hassle creates more aggressive competition amongst firm in prices and fees for customers.

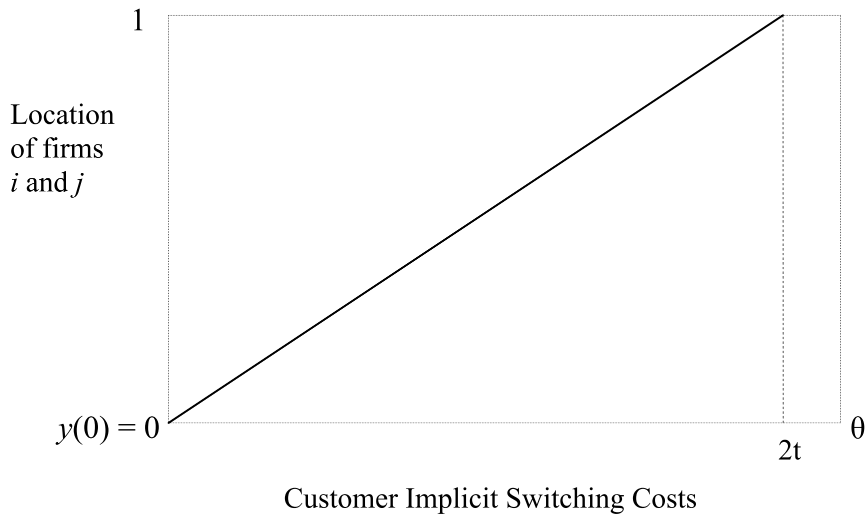

Figure 28: Inferior Equilibrium Case 2

What one may notice about this graphic representation of this Hotelling model versus the collusive outcomes seen earlier in this paper, is that there is some area above the line of indifference and the area below the line of indifference. This is highly important, for this represents the amount of loyalty and disloyalty in the market. The area above the indifference line shows the amount of switching in the equilibrium, and it is this switching which brings about the inferiority of this equilibrium because it creates dead weight losses to society.

[24] An example on an exogenous change on θ is the portability of cellular telephone numbers between carriers in 2003.

Collusion may not be sustainable if firms follow trigger strategies to punish their counterparts in this market. Therefore, the inferior outcome can survive as an equilibrium in spite of the fact that there is a much more desirable outcome under collusion in the market from the point of view of the firm. *i* and *j* can view this option as their contract pricing option if their competitor refuses to cooperate in the stage game. Though it is not as efficient or as profitable as the collusive equilibrium, this inferior outcome can provide a credible threat to cooperation if firms have a high enough discount rate over periods of the stage game. It is an unprofitable deviation if firms do not discount the future heavily.

3.6.2 Implications of Inferior Equilibrium

For the derivation of case 2 as the dominant equilibrium under the parameter restriction, please see 3.8.6 Appendix F.

3.6.2.1 Amount of Switching in Inferior Equilibrium

This section looks to the amount of switching that the inferior equilibrium.

The amount of switching, call it γ, in Case 2 is equal to φ_i or the area above the indifference line.

$$\gamma = \phi_{i|_{Case2}} = t$$

Since $t > 0$ is a assumed condition of this model, there is always a positive amount of switching in this inferior equilibrium. More switching occurs as the transportation cost increases or as customers choose companies that are not closest to their preferences.

3.6.2.2 Dead Weight Loss Associated with Switching

Each old customer has reservation value of v, new customers have a reservation value of β, and the maximum attainable expected profit for each firm is $E\prod = n(v-t)+\frac{rt}{2}$.[25] Therefore, we can calculate deadweight losses by investigating the total welfare in case 2 versus the Pareto-optimal equilibrium.

Proposition 4: There exists a positive dead weight loss to society due to switching in the inferior equilibrium.

For complete mathematic derivation, please see 3.8.7 Appendix G.

Total Possible Social Surplus Ω-

$$\Omega = r\beta + n\,(2v - t)$$

Total Surplus in Case 2 γ-

$$\gamma = n\,(v - \theta + t) + \frac{n(\theta-3t)}{\alpha} + r\beta + \frac{(\theta-t)^2 2n}{\theta}$$

Dead Weight Loss ξ -

$$\xi = n\left(\theta + v - 2t - \frac{(\theta-t)^2}{\theta} - \frac{(\theta-3t)}{\alpha}\right)$$

Positive Dead Weight Loss $\xi > 0$?

$$v > \frac{(\theta-t)^2}{\theta} + \frac{n(\theta-3t)}{\alpha} + 2t - \theta$$

If v satisfies $v > \frac{(\theta-t)^2}{\theta} + \left(\frac{\theta-3t}{\alpha}\right) + t$, then the collusive equilibrium brings more profit to the firm.[26] If we assume that colluding in a duopoly brings more profits to each firm, then this inequality is always satisfied.

Therefore, there is a positive dead weight loss in the inferior equilibrium. A cost to society exists with these contractual switching costs in place. This dead weight loss exists since

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[25] as seen in Pareto-optimal equilibria

[26] as shown in 3.8.7 Appendix D

if customers decide to switch and pay their switching costs, then firms cannot extract their surplus to the same extent. If customers pay their implicit switching cost, it becomes not only a source of loss to the customer but also to the firms since neither of them receive that cost as revenue. If collusion breaks down, then the industry finds itself at this inferior equilibrium which creates an overall loss from switching.

3.6.2.3 Trigger Strategy

For it to be profitable for firms to use this inferior equilibrium as a trigger strategy (credible threat), they have to have high discount rate for the future. If collusion succeeds, then firms enjoy $E\prod = n(v-t)+\frac{rt}{2}$, call it $\frac{\prod^m}{2}$, in every period. If a firm becomes greedy and decides to under cut their competitor's price to capture all the profits in the market Π^m, its competitor might price according to the inferior equilibrium to punish the deviator with corresponding profits $E\prod = \frac{n(\theta-t)^2}{\theta}+n\left(\frac{\theta-3t}{\alpha}\right)+\frac{rt}{2}$ which we call $\prod^{\inf}$. Therefore, for this trigger strategy to be profitable the discount rate, δ, has to be such that: $\delta > \frac{\prod^m}{2(\prod^m-\prod^{\inf})}$. For mathematic derivation, please see 3.8.8 Appendix H. Since I have assumed that firms have a high discount rate (close to one) in order to aggregate their pricing behavior decisions for new and old customers, this inequality should be satisfied.

3.6.2.4 Coordination Failure

The possibility does exist that the market may start out pricing according to the inferior equilibrium, even though it is more profitable for both firms to collude. In this situation, *i* and *j* may find themselves in coordination failure and would like to move to the collusive outcome to achieve higher profits. To do this, either *i* or *j* (for simplicity let us assume that *i* without loss of generality) has to take the lead and set prices and fees according to one of the collusive equilibria in order to signal to its counterpart that they are willing to price high if *j* is willing to price high. Firm *i*, the "leading firm," takes a big hit to its revenues because it suffers zero profits for the period it is trying to signal to *j*. Firm *i* only finds it beneficial to take this "hit" against their expected profits if its discount factor satisfies the following: $\delta > \frac{2\prod^{\inf}}{\prod^{m}}$.[27] It is always beneficial for firm *j* to follow firm *i*'s lead since it receives monopoly profits in the period that *i* is sending a collusive signal and collusive profits from that point on, which are always higher than the inferior equilibrium profits by assumption.

$1 > \delta > \frac{2\prod^{\inf}}{\prod^{m}}$, $2\prod^{\inf} < \prod^{m}$ may be a strong assumption to place in this model. It may not be possible for firms to reach collusion if they are "stuck" in inferior equilibrium pricing since a firm never has a high enough discount factor to take the lead toward the collusive outcome. If $2\prod^{\inf} < \prod^{m}$, then it is possible for firms to reach a collusive outcome after being in coordination failure if δ is high enough for one firm to take the lead and send the signal. Since both firms are symmetric, if δ is high enough to meet this condition requirement then both firms are willing to take the lead, and coordination failure never occurs.

[27] For mathematic derivation, please see 3.8.9 Appendix I.

3.7 RESULTS AND CONCLUSION

Switching costs, whether explicitly set in contracts or implicitly inherent in the industry, bring about market inefficiencies when a collusive outcome is not pursued by those within the market. Even though the Pareto-optimal collusive equilibrium is attractive to the firm and from overall social welfare analysis, it is also problematic since customers retain no surplus. Therefore, each equilibrium outcome has benefits and drawbacks.

In this model of expected profit aggregated to one period of the stage game, there exist three symmetric equilibria in a duopolists' market producing a homogeneous product. They can either collude in prices that keep their customers completely loyal to their product in every period, or they can collude in prices that induce entire customer disloyalty. If the discount rate is high enough, then it is profitable for firms to follow a trigger strategy in the multi-stage game to induce cooperation with their competitor. If firms choose pricing that allows some switching to take place, then there is a positive dead weight loss to society, and they do not achieve their highest possible profits. On the other hand, customers do not have their entire rent extracted in the inferior equilibrium, and therefore they enjoy some of the surplus unlike under the collusive outcomes. Therefore, customers always prefer the inferior equilibrium in this model.

An interesting extension to this model would change the demand structure for the contracted product. The market in this model has a fixed demand with perfect inelasticity. To extend the model, one should allow for a downward-sloping demand schedule, creating elasticity in choosing a firm contract.

Even though this inferior equilibrium does not seem appealing to the firm in terms of profit, it may turn out empirically that it is the pricing scheme that is most often chosen by leaders in industries. It is possible that we see switching behavior that is neither all or nothing on the part of consumers which would lead us to believe that firms are choosing pricing equilibria that might be inferior in these markets. In addition, there could be other factors that induce customers to switch providers of a good. Thus, there needs to be an empirical analysis to track cellular telephone market industry to answer these questions along with finding out what really happens to prices when the distribution of implicit switching costs decreases.

If a market makes it easier to switch between producers of a product, does it really make that industry more competitive? This model says yes under the conditions of the inferior equilibrium. An empirical analysis of pricing and customer behavior pre- and post- November 2003, when portability of cellular telephone numbers became effective, will hopefully be able to answer the question of whether these markets are actually exploiting their full power of collusion and if making markets more flexible will enhance competition, consumer surplus, and possible entry of new firms into the market.

3.8 APPENDICES

3.8.1 Appendix A:

Proposition 1: There exists a collusive equilibrium to this model where an old customer does not switch from their current provider at their given contract.

Mathematic Derivation and Proof:

If $n > \frac{\alpha r m_i}{2t}$ choose the maximum q_i

$$\left\{ \begin{array}{c} t - \theta < R_{ij} < t \text{ if } 2t > \theta \\ -t < R_{ij} < t \text{ if } 2t < \theta \end{array} \right\}$$

Choose maximum q_i such that $R_{ij} = t$

If $R_{ij} = t$, the profit maximization problem becomes:

$$E\Pi \quad p_i n + r m_i \left[\frac{1}{2} + \frac{(m_j + \alpha q_j) - (m_i + \alpha q_i)}{2t} \right]$$

Therefore $\frac{\partial E\Pi}{\partial p_i} = n > 0$

The firm would want to choose biggest p_i possible $\Rightarrow$

$p_i = v - t$ where v is the maximum reservation price of the indifferent customer

At $R_{ij} = t$

$$\frac{\partial E\Pi}{\partial p_i} = n$$

$$\frac{\partial E\Pi}{\partial q_i} = -\frac{\alpha r m_i}{2t} < 0$$

Choose the smallest q_i consistent with this new region

$p_i = v - t$, $m_i = m_j = t$

Since $R_{ij} = t \Rightarrow$

$$m_j + \alpha q_j + q_i - p_i = t$$

$$q(1 + \alpha) = p$$

$$q = \frac{v - t}{1 + \alpha}$$

$$E\Pi = (v - t)n + \frac{rt}{2}$$

$$y(0) = \frac{1}{2} + \frac{m_j + (1 + \alpha)q - p}{2t}$$

$$y(0) = \frac{1}{2} + \frac{t + (v - t) - (v - t)}{2t} = 1$$

$$y(\theta) = \frac{1}{2} + \frac{\theta}{2t} + \frac{m_j + (1 + \alpha)q - p}{2t}$$

$$y(\theta) = \frac{1}{2} + \frac{\theta}{2t} + \frac{t - (v - t) - (v - t)}{2t} = 1 + \frac{\theta}{2t} > 1$$

Therefore, Figure 26 follows from these derivations.

3.8.2 Appendix B

Proposition 2: There exists a collusive equilibrium to this model where every old customer switches from their current provider at their given contract as seen in Figure 27.

Mathematic Derivation:

To best describe this equilibrium, we need to analyze its properties.

$$y(0) \geq -1 \Rightarrow R_{ij} > -3t$$

It is necessary for $y(0) \geq -1$ since this is a Hotelling model of a duopoly where it is exogenously given that the distance between the two firms is 1. The intercept in this model cannot exceed the distance set between our duopolists.

$$y(\theta) \leq 0 \Rightarrow R_{ij} \leq -(t+\theta)$$

This is a necessary condition for our Hotelling model equilibrium to be of the form as seen in Figure 27.

The expected profit in this case becomes:

$$E\Pi \quad q_i n + r m_i \left[\frac{1}{2} + \frac{(m_j+\alpha q_j)-(m_i+\alpha q_i)}{2t}\right]$$

First order conditions with respect to q_i:

$$\frac{\partial E\Pi}{\partial q_i} = n - \frac{\alpha r m_i}{2t}$$

At optimum: $m_i = m_j = t$

Necessary condition for equilibrium of this maximization to exist -

$$\frac{\partial E\Pi}{\partial q_i} = n - \frac{\alpha r}{2} \geq 0$$

In this scenario, there are increasing returns to a higher q_i so the firm wants to set it at its highest possible value.

Therefore, $q_i = v - t$ where v is reservation value of customer. Thus, R_{ij} needs to be set at its smallest possible value (exactly opposite of the first collusive outcome when q_i is set at is smallest possible value).

$$R_{ij} = m_j + \alpha q_j + q_i - p_i = -3t$$
$$t + (1+\alpha)q - p = -3t$$
$$(1+\alpha)q - p = -4t$$
$$p = (1+\alpha)(v-t) + 4t$$
$$q = v - t$$
$$m = t$$
$$E\Pi = n(v-t) + \tfrac{rt}{2}$$

In terms of expected profit this equilibrium is the same as the first symmetric Pareto-optimal equilibrium.

$$y(0) = \tfrac{1}{2} + \tfrac{m_j + (1+\alpha)q - p}{2t}$$
$$y(0) = \tfrac{1}{2} + \tfrac{t + (1+\alpha)(v-t) - (1+\alpha)(v-t) - 4t}{2t} = -1$$
$$y(\theta) = \tfrac{1}{2} + \tfrac{\theta}{2t} + \tfrac{m_j + (1+\alpha)q - p}{2t}$$

$y(\theta) = \frac{1}{2} + \frac{\theta}{2t} - \frac{3t}{2t} = \frac{\theta}{2t} - 1 = \frac{\theta - 2t}{2t} < 0$ since $\theta \leq 2t$ is an assumption of this equilibrium following from $-3t \leq R_{ij} \leq -(t+\theta)$.

Therefore the set up of Figure 27 follows from this derivation. Everybody switches to competitor but pays the breakup fee (v - t).

Thus, the characterization of this second collusive equilibrium is:

$$p = (1+\alpha)(v-t) + 4t$$

$$q = v - t$$
$$m = t$$
$$E\Pi = n(v-t) + \tfrac{rt}{2}$$

Expected profit this equilibrium is the same as the first symmetric Pareto-optimal equilibrium.

3.8.3 Appendix C

3.8.3.1 If $2t > \theta \Rightarrow R_{ij} = t - \theta$

Proposition: There exists an equilibrium in this region which is inferior to the collusive equilibria from the point of view of the firm.

The characterization of the inferior equilibrium in case 1 is (for mathematic details see below):

$$p = \frac{2t}{\alpha}(1+\alpha) - \frac{\theta}{2\alpha}(\alpha+3)$$
$$q = \frac{4t-3\theta}{2\alpha}$$
$$m = t$$
$$E\Pi = \frac{n(4t-\theta)^2}{t} + \frac{n(4t-3\theta)}{2\alpha} + \frac{rt}{2}$$

Logically, a firm wants to set both positive prices and explicit switching costs in order to induce loyalty and have greater profits. Thus, we impose more constrictions on the parameters of the model.

for $q > 0$ then $t > \frac{3}{4}\theta$ necessarily

for $p > 0$ then $t > \frac{(\alpha+3)\theta}{4(1+\alpha)}$ necessarily (if $\alpha > 0$, then $t > \frac{3}{4}\theta$ is a more binding condition)

With these restrictions in place, the following comparative statics with respect to the break fee q and p hold:

$$\frac{\partial q,p}{\partial t} \geq 0, \ \frac{\partial q,p}{\partial \theta} < 0, \ \frac{\partial q,p}{\partial \alpha} > 0$$

Therefore, the explicit switching cost and price for each individual customer varies directly with the transportation cost, t, and the uncertainty factor, α, but it varies inversely with θ. These comparative static results are counter-intuitive. One expects a firm to compensate a customer who is facing more uncertainty (an increase in α) with lower prices and fees, not higher ones as implied by these results. Similarly, as transportation costs increase, one expects firms to compensate customers whose preferences are less fully aligned with the product they are selling to set lower prices. But the most counter-intuitive result is investigating the effects of θ. As the distribution of consumer implicit switching costs shrinks, then companies increase their contract break fees in order to keep their old customers from switching since it costs each of them less in inconvenience, opportunity, and learning to do so. In this equilibrium, as the distribution of customer implicit switching costs shrinks, then the market becomes less competitive. One expects prices to fall and switching between providers to increase since it is "less costly" for each customer to explore different firms for lower prices when they sign contracts.

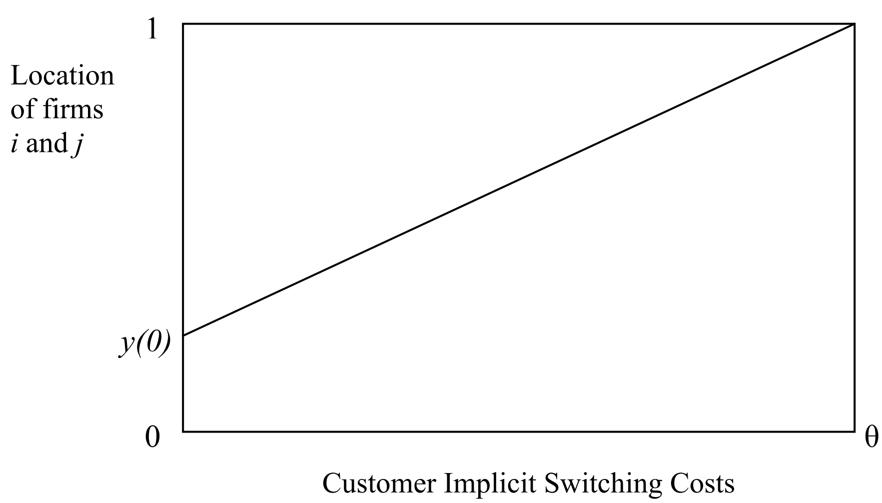

Figure 29: Inferior Symmetric Equilibrium Case 1

Mathematic Derivation:

Conditions - $2t > \theta \Rightarrow R_{ij} = t - \theta$

Returning to the general first order conditions seen Equation 2:

$$p_i - q_i = 2t - \frac{\theta}{2}$$
$$R_{ij} \equiv m_j + \alpha q_j + q_i - p_i = t - \theta$$
$$m_j + \alpha q_j - 2t + \frac{\theta}{2} = t - \theta$$
$$m_j + \alpha q_j = 3t - \frac{3}{2}\theta = \frac{3}{2}(2t - \theta)$$

$$MAX_{m_i} \quad rm_i \left(\frac{1}{2} + \frac{(m_j + \alpha q_j) - (m_i + \alpha q_i)}{2t} \right)$$
Solution - $m_i = m_j = t$
$$t + \alpha q_j = 3t - \frac{3}{2}\theta$$
$$\alpha q_j = 2t - \frac{3}{2}\theta$$
$$q_j = \frac{2t}{\alpha} - \frac{3}{2}\frac{\theta}{\alpha}$$
Add in symmetry $\Rightarrow q_i = q_j = \frac{2t}{\alpha} - \frac{3}{2}\frac{\theta}{\alpha}$

$$\left\{ \begin{array}{l} \text{if } t > \frac{3}{4}\theta, \ q_j > 0 \\ \text{if } t < \frac{3}{4}\theta, \ q_j < 0 \end{array} \right\}$$
$$p_i = 2t - \frac{\theta}{2} + \frac{2t}{\alpha} - \frac{3}{2}\frac{\theta}{\alpha}$$
$$p_i = \frac{2t}{\alpha}(1 + \alpha) - \frac{\theta}{2\alpha}(\alpha + 3)$$

$$p_i \left\{ \begin{array}{l} > 0 \text{ if } t > \frac{(\alpha+3)\theta}{4(1+\alpha)} \\ > 0 \text{ if } t < \frac{(\alpha+3)\theta}{4(1+\alpha)} \end{array} \right\}$$

$$\frac{\partial q, p}{\partial t} \geq 0, \ \frac{\partial q, p}{\partial \theta} < 0$$

$$\frac{\partial q, p}{\partial \alpha} \left\{ \begin{array}{l} < 0 \text{ when } q > 0 \\ > 0 \text{ when } q < 0 \end{array} \right\}$$

From the point of view of the firm, this equilibrium is inferior to the two collusive symmetric equilibria:

$$\begin{bmatrix} q & q = \frac{4t-3\theta}{2\alpha} < \frac{v-t}{1+\alpha} & q = \frac{4t-3\theta}{2\alpha} < v-t \\ p & p = \frac{2t}{\alpha}(1+\alpha) - \frac{\theta}{2\alpha}(\alpha+3) < v-t & p = \frac{2t}{\alpha}(1+\alpha) - \frac{\theta}{2\alpha}(\alpha+3) < (1+\alpha)(v-t)+4t \end{bmatrix}$$

These inequalities hold if:

$t(3\alpha+2) - \frac{\theta}{2}(\alpha+3) < v$

$E\Pi = \frac{\left(2t-\frac{\theta}{2}\right)n}{2}\left[\frac{3}{2} + \frac{t-\theta}{2t}\right] + n\left(\frac{2t}{\alpha} - \frac{3\theta}{2\alpha}\right) + \frac{rt}{2} < n(v-t) + \frac{rt}{2}$

$E\Pi = \frac{(4t-\theta)^2}{t} + \frac{(4t-3\theta)}{2\alpha} + t < v$

also need $t > \frac{3}{4}\theta$, $n < \frac{r\alpha}{2}$

To get an idea of what the graphic representation of this equilibrium as seen in Figure 29:

If $n < \frac{rm_i\alpha}{2t} = \frac{r\alpha}{2}$

$y(0) = \frac{1}{2} + \frac{m_j+(1+\alpha)q-p}{2t}$

$y(0) = \frac{1}{2} + \frac{t+(1+\alpha)q-2t-+\frac{\theta}{2}-q}{2t}$

$y(0) = \frac{1}{2} + \frac{t+\alpha\left(\frac{2t}{\alpha}-\frac{3}{2\alpha}\theta\right)-2t+\frac{\theta}{2}}{2t} = 1 - \frac{\theta}{2t} > 0$ since $2t > \theta$ is an assumption of this case

$y(\theta) = \frac{1}{2} + \frac{\theta}{2t} + \frac{m_j+(1+\alpha)q-p}{2t} = 1$

$x_{|_{y=1}} = t - R_{ij} = t - t + \theta = \theta$

3.8.4 Appendix D

Proposition 3: There exists an equilibrium in this region which is inferior to the collusive equilibria from the point of view of the firm.

If $2t < \theta \Rightarrow R_{ij} = -t$

Returning to the general solution in equation 2:

$$p_i - q_i = \theta - t$$
$$\frac{\partial E\Pi}{\partial q_i} = n - \frac{r\alpha t}{2t} = n - \frac{r\alpha}{2} < 0$$
$$R_{ij} \equiv m_j + \alpha q_j + q_i - p_i = -t$$
$$m_j + \alpha q_j - \theta + t = -t$$
$$m_j + \alpha q_j = \theta - 2t$$
$$\alpha q_j = \theta - 3t$$
$$q_j = q_i = \frac{\theta - 3t}{\alpha}$$

For $q > 0$, then $\theta > 3t$.

$$p_i = \frac{\theta - 3t}{\alpha} + \theta - t = \frac{\theta(1+\alpha) - (3+\alpha)t}{\alpha}$$
$$p_i \left\{ \begin{array}{l} > 0 \text{ if } \theta > \left(\frac{3+\alpha}{1+\alpha}\right) t \\ < 0 \text{ if } \theta < \left(\frac{3+\alpha}{1+\alpha}\right) t \end{array} \right\}$$
$$m_j = m_i = t$$

$$y(0) = \frac{1}{2} + \frac{m_j + (1+\alpha)q - p}{2t}$$
$$y(0) = \frac{1}{2} + \frac{t + (1+\alpha)\frac{\theta - 3t}{\alpha} - \frac{\theta - 3t}{\alpha} - \theta + t}{2t}$$
$$y(0) = 0$$
$$y(\theta) = \frac{1}{2} + \frac{\theta}{2t} + \frac{m_j + (1+\alpha)q - p}{2t}$$
$$y(\theta) = \frac{\theta}{2t} > 1$$
$$x_{|_{y=1}} = t - R_{ij} = 2t$$

$$E\Pi = \frac{(\theta - t)n}{\theta}\left[t + \theta - 2t\right] + n\left(\frac{\theta - 3t}{\alpha}\right) + \frac{rt}{2}$$
$$E\Pi = \frac{(\theta - t)^2 n}{\theta} + n\left(\frac{\theta - 3t}{\alpha}\right) + \frac{rt}{2}$$
$$n < \frac{\alpha r}{2},\ \theta > 3t$$

This equilibrium is inferior to the previous two because:

$$\begin{bmatrix} q & q = \frac{\theta - 3t}{\alpha} < \frac{v - t}{1+\alpha} & q = \frac{\theta - 3t}{\alpha} < v - t \\ p & p = \frac{\theta - 3t}{\alpha} + \theta - t < v - t & p = \frac{\theta - 3t}{\alpha} + \theta - t < (1+\alpha)(v - t) + 4t \end{bmatrix}$$

These inequalities hold if:

$$\frac{\theta - 3t}{\alpha} + \theta < v$$
$$E\Pi = \frac{(\theta - t)n}{\theta}\left[t + \theta - 2t\right] + n\left(\frac{\theta - 3t}{\alpha}\right) + \frac{rt}{2} < n(v - t) + \frac{rt}{2}$$
$$E\Pi = \frac{(\theta - t)^2}{\theta} + \left(\frac{\theta - 3t}{\alpha}\right) + t < v$$

If $n < \frac{\alpha r}{2}$, $\theta > 3t$, and v satisfies these restrictions, then this is an inferior equilibrium to the first two from the point of view of the firm.

3.8.5 Appendix E

3.8.5.1 $-t < R_{ij} < t - \theta$ and $2t > \theta$

Proposition: There exists an equilibrium in this region which is inferior to the collusive equilibria from the point of view of the firm.

For mathematic derivation of the equilibrium and to show its inferiority from the point of view of the firm, please see below.

The characterization of the inferior equilibrium in case 3 is:

$$p = \frac{2t}{\alpha}(1+\alpha) - \frac{\theta}{2\alpha}(\alpha+3)$$
$$q = \frac{2t}{\alpha} - \frac{3}{2\alpha}\theta$$
$$m = t$$
$$E\Pi = n\left(2t + \frac{\theta}{2} - \frac{\theta^2}{4t}\right) + n\left(\frac{2t}{\alpha} - \frac{3\theta}{2\alpha}\right) + \frac{rt}{2}$$

Just as in Case 1, for p and q to be positive the following conditions must be satisfied:

$$\text{for } q > 0 \text{ then } t > \tfrac{3}{4}\theta \text{ necessarily}$$
$$\text{for } p > 0 \text{ then } t > \tfrac{(\alpha+3)\theta}{4(1+\alpha)} \text{ necessarily}$$

Also, since the solutions for p and q are the same for Cases 1 and 3, the comparative statics hold with respect to θ, α, and t hold.

Conditions: $-t < R_{ij} < t - \theta$ and $2t > \theta$

3.8.5.2 Market Share of Old Customers ($1-\varphi_i$) $\rightarrow$ the area under the curve in Figure 30

$$\theta y(0) + \tfrac{1}{2}\theta\left[y(\theta) - y(0)\right]$$

$$\theta\left[\frac{y(\theta)+y(0)}{2}\right]$$

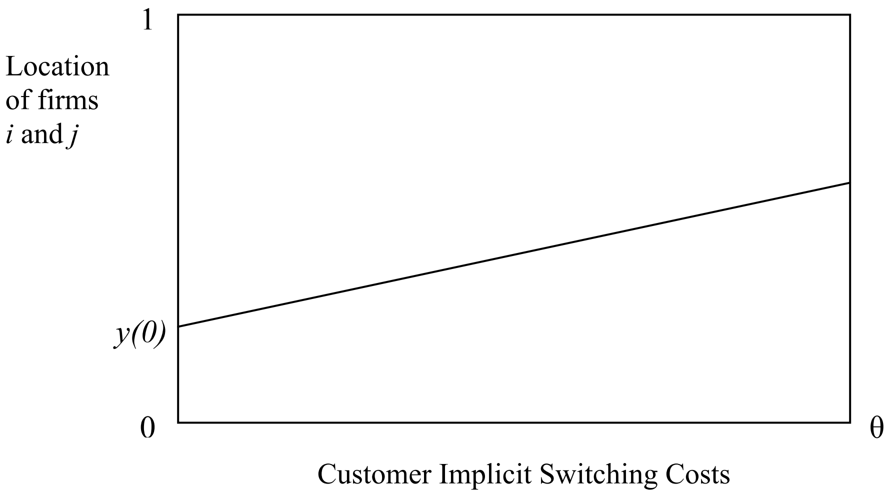

Figure 30: Inferior Symmetric Equilibrium Case 3

3.8.5.3 Profit Maximization for Case 3

$$E\Pi=\left[\theta\left(\frac{y(\theta)+y(0)}{2}\right)\right]\frac{(p_i-q_i)n}{\theta}+nq_i+rm_i\left(\frac{1}{2}+\frac{(m_j+\alpha q_j)-(m_i+\alpha q_i)}{2t}\right)$$

$$y(\theta)=\frac{1}{2}+\frac{\theta}{2t}+\frac{R_{ij}}{2t}$$
$$y(0)=\frac{1}{2}+\frac{R_{ij}}{2t}$$
$$\frac{y(\theta)+y(0)}{2}=\frac{1}{2}+\frac{\theta}{4t}+\frac{R_{ij}}{2t}$$
$$E\Pi=\left[\frac{1}{2}+\frac{R_{ij}}{2t}+\frac{\theta}{4t}\right](p_i-q_i)\,n+nq_i+rm_i\left(\frac{1}{2}+\frac{(m_j+\alpha q_j)-(m_i+\alpha q_i)}{2t}\right)$$

3.8.5.4 General First and Second Order Conditions

$$\frac{\partial E\Pi}{\partial p_i}=n\left[\frac{1}{2}+\frac{R_{ij}}{2t}+\frac{\theta}{4t}-\frac{(p_i-q_i)}{2t}\right]$$

$$\frac{\partial E\Pi}{\partial q_i} = -\frac{\partial E\Pi}{\partial p_i} + n - \frac{\alpha r m_i}{2t} \text{ (Equation 1)}$$

$$\frac{\partial E\Pi}{\partial p_i} = 0, \ \frac{\partial E\Pi}{\partial q_i} \left\{ \begin{array}{c} > 0 \text{ if } n > \frac{r m_i \alpha}{2t} \Rightarrow R_{ij} = t - \theta \\ < 0 \text{ if } n < \frac{r m_i \alpha}{2t} \Rightarrow R_{ij} = -t \end{array} \right\}$$

$$p_i - q_i = t + t - \theta + \frac{\theta}{2} = 2t - \frac{\theta}{2}$$
$$t - \theta = R_{ij} = m_j + \alpha q_j + q_i - p_i$$
$$t - \theta = t + \alpha q_j - 2t + \frac{\theta}{2}$$

Add in symmetry:

$$\alpha q = 2t - \frac{3}{2}\theta$$
$$q = \frac{2t}{\alpha} - \frac{3}{2\alpha}\theta$$

Therefore $p = 2t - \frac{\theta}{2} + q$

$$p = \frac{2t}{\alpha}(1+\alpha) - \frac{\theta}{2\alpha}(\alpha + 3)$$

$$E\Pi = n\left(2t + \frac{\theta}{2} - \frac{\theta^2}{4t}\right) + n\left(\frac{2t}{\alpha} - \frac{3\theta}{2\alpha}\right) + \frac{rt}{2}$$

Following similar analysis from the previous cases of the inferior equilibrium, this case is also not as desirable to the firm with respect to pricing, fees and profits.

This equilibrium is inferior to the previous two because:

$$\begin{bmatrix} q & q = \frac{4t-3\theta}{2\alpha} < \frac{v-t}{1+\alpha} & q = \frac{4t-3\theta}{2\alpha} < v - t \\ p & p = \frac{2t}{\alpha}(1+\alpha) - \frac{\theta}{2\alpha}(\alpha+3) < v - t & p = \frac{2t}{\alpha}(1+\alpha) - \frac{\theta}{2\alpha}(\alpha+3) < (1+\alpha)(v-t) + 4t \end{bmatrix}$$

These inequalities hold if:

$$t(3\alpha + 2) - \frac{\theta}{2}(\alpha + 3) < v$$

$$E\Pi = n\left(2t + \frac{\theta}{2} - \frac{\theta^2}{4t}\right) + n\left(\frac{2t}{\alpha} - \frac{3\theta}{2\alpha}\right) + \frac{rt}{2} < n(v-t) + \frac{rt}{2}$$
$$E\Pi = \frac{t}{\alpha}(3\alpha + 2) + \frac{\theta}{2\alpha}(\alpha + 3) - \frac{\theta^2}{4t} < v$$
$$t > \frac{3}{4}\theta \text{ and } n > \frac{r\alpha}{2}$$

$$y(0) = \frac{1}{2} + \frac{m_j + (1+\alpha)q - p}{2t}$$
$$y(0) = \frac{1}{2} + \frac{t + (1+\alpha)q - 2t - + \frac{\theta}{2} - q}{2t}$$
$$y(0) = \frac{1}{2} + \frac{t + \alpha\left(\frac{2t}{\alpha} - \frac{3}{2\alpha}\theta\right) - 2t + \frac{\theta}{2}}{2t} = 1 - \frac{\theta}{2t} < 0$$
$$y(\theta) = \frac{1}{2} + \frac{\theta}{2t} + \frac{m_j + (1+\alpha)q - p}{2t} = 1$$

$$x_{|_{y=1}} = t - R_{ij} = t - t + \theta = \theta$$

3.8.6 Appendix F

3.8.6.1 Case 2 Dominant Inferior Equilibrium

Proposition: The form of the inferior equilibrium in case 2 dominates the other cases of the inferior equilibria. The reasons for this are, from the point of view of the firm, case 2 brings more profit and therefore is the pricing scheme chosen more frequently in a one-shot game. In addition, the p and q associated with case 2 bring divergent solutions for cases 1 and 3.

Under one assumption, I show that case 1 and 3, which are mathematically equivalent, never occur in this inferior equilibrium. Firms always choose the contract set up in case 2 if it brings them more expected profit than either case 1 or 3. If $\theta > 3t$, which is a necessary condition for $p, q > 0$ in case 2, then case 2 dominates the inferior equilibrium.

Comparing profit of case 2 with case 1:

$$\frac{(\theta-t)^2}{\theta} + \left(\frac{\theta-3t}{\alpha}\right) > \frac{(4t-\theta)^2}{t} + \frac{(4t-3\theta)}{2\alpha}$$
$$\frac{(\theta-t)^2}{\theta} > \frac{(4t-\theta)^2}{t} + \frac{10t-5\theta}{2\alpha}$$

Comparing profit of case 2 with case 3:

$$\frac{(\theta-t)^2}{\theta} + \left(\frac{\theta-3t}{\alpha}\right) > 2t + \frac{\theta}{2} - \frac{\theta^2}{4t} + \frac{(4t-3\theta)}{2\alpha}$$
$$\frac{(\theta-t)^2}{\theta} > 2t + \frac{\theta}{2} - \frac{\theta^2}{4t} + \frac{10t-5\theta}{2\alpha}$$

Under this condition ($\theta > 3t$), $\frac{(\theta-t)^2}{\theta} > \frac{(4t-\theta)^2}{t}$, $\frac{(\theta-t)^2}{\theta} > 2t + \frac{\theta}{2} - \frac{\theta^2}{4t}$, and $\frac{10t-5\theta}{2\alpha} < 0$. Therefore the expected profit in case 2 would dominate; each firm would prefer to set contract pricing schemes under case 2 than the other cases.

Also, at the solution for *p, q*, and *m* for case 2, cases 1 and 3 have divergent solutions and therefore are never chosen in equilibrium.

Since Cases 1 and 3 are the same mathematically, I show that Case 1 has a divergent solution.

Check divergent solution:

Case 1 - $\frac{\partial^2 E\Pi}{\partial p_i^2} = -\frac{n}{\theta}\left[\frac{t-R_{ij}}{2t}\right] - \frac{n}{\theta}\left[\frac{t-R_{ij}}{2t}\right] - \frac{n}{2t\theta}(p_i - q_i)$

$R_{ij} = t - \theta, (p_i - q_i) = 2t - \frac{\theta}{2}$

$$\frac{\partial^2 E\Pi}{\partial p_i^2} = -\frac{2n}{\theta}\left[\frac{\theta}{2t}\right] - \frac{n}{2t\theta}\left(2t - \frac{\theta}{2}\right) = -\frac{n}{t} - \frac{n}{\theta} + \frac{n}{4t}$$

$\frac{\partial^2 E\Pi}{\partial p_i^2} < 0$? for second order condition to be satisfied

$$-\frac{n}{t} - \frac{n}{\theta} + \frac{n}{4t} < 0$$
$$n\left(\frac{1}{3t} - \frac{1}{\theta}\right) < 0$$
$$\frac{1}{3t} - \frac{1}{\theta} < 0 \Rightarrow \theta < 3t$$

Therefore if $\theta > 3t$ as in case 2, the solution for case 1 is divergent, and case 2 is optimal.

3.8.7 Appendix G

Proposition 7: There exists a positive dead weight loss to society due to switching in the inferior equilibrium.

Total Social Surplus Ω-

$$\Omega = n(v-(v-t)) + r(\beta - t) + 2n(v-t) + rt$$

$$\Omega = r\beta + n\,(2v-t)$$

Total Surplus in Case 2 δ-

$$\delta = n(v - (\tfrac{\theta-3t}{\alpha} + \theta - t)) + r(\beta - t) + \tfrac{(\theta-t)^2 2n}{\theta} + 2n\left(\tfrac{\theta-3t}{\alpha}\right) + rt$$

$$\delta = n\,(v-\theta+t) + \tfrac{n(\theta-3t)}{\alpha} + r\beta + \tfrac{(\theta-t)^2 2n}{\theta}$$

Dead Weight Loss ξ -

$$\xi = r\beta + n\,(2v-t) - \left[n\,(v-\theta+t) + \tfrac{n(\theta-3t)}{\alpha} + r\beta + \tfrac{(\theta-t)^2 2n}{\theta}\right]$$

$$\xi = n\theta + nv - 2nt - \tfrac{(\theta-t)^2 n}{\theta} - \tfrac{n(\theta-3t)}{\alpha}$$

$$\xi = n\left(\theta + v - 2t - \tfrac{(\theta-t)^2}{\theta} - \tfrac{(\theta-3t)}{\alpha}\right)$$

Positive Dead Weight Loss $\xi > 0$?

$$n\left(\theta + v - 2t - \tfrac{(\theta-t)^2}{\theta} - \tfrac{(\theta-3t)}{\alpha}\right) > 0$$

$$\theta + v - 2t - \tfrac{(\theta-t)^2}{\theta} - \tfrac{(\theta-3t)}{\alpha} > 0$$

$$v > \tfrac{(\theta-t)^2}{\theta} + \tfrac{n(\theta-3t)}{\alpha} + 2t - \theta$$

If v satisfies $v > \frac{(\theta-t)^2}{\theta} + \left(\frac{\theta-3t}{\alpha}\right) + t$, then the collusive equilibrium brings more profit to the firm. If we will assume that colluding in a duopoly will bring more profits to each firm, then this inequality will always be satisfied. Thus, there is a positive dead weight loss to society from this inferior equilibrium because:

$\frac{(\theta-t)^2}{\theta} + \frac{n(\theta-3t)}{\alpha} + 2t - \theta < \frac{(\theta-t)^2}{\theta} + \left(\frac{\theta-3t}{\alpha}\right) + t$ since it is assumed in case 2 that $\theta > 2t$.

3.8.8 Appendix H

The mathematic derivation for the discount rate necessary for the trigger strategy to be profitable:

$$\left[\frac{\Pi^m}{2}\right]\left(1+\delta+\delta^2+...\right) > \Pi^m + \Pi^{Inf}\left(\delta+\delta^2+...\right)$$
$$\left[\frac{\Pi^m}{2}\right]\left(\frac{1}{1-\delta}\right) > \Pi^m + \Pi^{Inf}\left(\frac{\delta}{1-\delta}\right)$$
$$\frac{\Pi^m}{2} > (1-\delta)\,\Pi^m + \delta\Pi^{Inf}$$
$$\frac{\Pi^m}{2} < \delta(\Pi^m - \Pi^{Inf})$$
$$\delta > \frac{\Pi^m}{2(\Pi^m - \Pi^{Inf})}$$

For $1 > \delta > \frac{\Pi^m}{2(\Pi^m - \Pi^{Inf})}$, $\Pi^m > \frac{\Pi^m}{2} + \Pi^{Inf}$ which should be easily satisfied since $\Pi^{Inf} < \frac{\Pi^m}{2}$ should always be the case otherwise there is no benefit to collusion just as seen in the section on dead weight loss.

3.8.9 Appendix I

The mathematic derivation for the discount rate necessary for one firm to take the lead to get out of coordination failure to the collusive outcome:

$$\Pi^{Inf}\left(1+\delta+\delta^2...\right) < \frac{\Pi^m}{2}\left(\delta+\delta^2...\right)$$
$$\Pi^{Inf}\left(\frac{1}{1-\delta}\right) < \frac{\Pi^m}{2}\left(\frac{\delta}{1-\delta}\right)$$
$$\Pi^{Inf} < \delta\frac{\Pi^m}{2}$$
$$\delta > \frac{2\Pi^{Inf}}{\Pi^m}$$

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