ESSAYS ON SOCIAL INSURANCE

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In the first essay, we analyze the welfare effects of an unfunded social security system. We do so using an overlapping generations economy wherein agents have self-control preferences, face mortality risk, individual income risk, and borrowing constraints. Given our specification of preferences, unfunded social security helps reduce the agents' temptation to consume in every period; consequently, the welfare costs it otherwise entails are substantially mitigated. While both social security and self-control when considered separately reduce welfare, their combination renders this effect considerably less severe. Moreover, if the cost of resisting temptation is very high, the introduction of social security might even improve welfare.

In the second essay I use a dynamic stochastic general equilibrium overlapping generations model to examine the relevance of unfunded social security in an environment where both CRRA and self-control agents co-exist. I identify conditions under which the existence of CRRA agents in the economy makes self-control agents better-off. I, therefore, conclude that temptation prevalence across individuals in the economy and temptation intensity within individuals can be considered to be substitutes in reducing the welfare cost associated with unfunded social security for self control agents.

In the third essay we analyze a fully funded social security system under the assumption that agents face temptation issues. Agents are required to save through individually managed Personal Security Accounts without, and with mandatory annuitization. When the analysis is restricted to CRRA preferences our results are congruent with the literature in indicating that the complete elimination of social security is the reform scenario that maximizes welfare. However, when self control preferences are introduced, and as the intensity

of self control becomes progressively more severe the "social security elimination" scenario loses ground very rapidly. In fact, in the case of very severe temptation the elimination of social security becomes the least desirable alternative. Under the light of the above findings, any reform proposal regarding the social security system should consider departures from standard preferences to preference specifications suitable for dealing with preference reversals.

TABLE OF CONTENTS

[1] We will henceforth refer to to the system presented in this section as *PSA*.
[2] We will henceforth refer to to the system presented in this section as *PSA+Annuity*.

LIST OF TABLES

LIST OF FIGURES

PREFACE

I am deeply grateful to my Academic Advisor David N. DeJong for his mentoring and support throughout the course of my doctoral studies. It wouldn't be an overstatement to say that virtually everything (those few things) I know about reading, refereeing and writing academic work is owed to him. Thanks to his ingenious guidance and unrivaled patience, I was fortunate enough to see a part of this dissertation published; his constant encouragement, motivation and insight were instrumental to this end, at a time I was lacking the necessary self-confidence. I would like to wholeheartedly thank him for trusting me, for being so meticulous and incisive yet fast and efficient, and for providing me with the right blend of flexibility and control that allowed me to successfully complete this work. I feel particularly proud and honored to be his student.

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thus, inadvertently, gifting me the most precious moments of my life. My son Constantinos deserves a special mention in that he instantly forced me to realize the very existence of the future, along with the most compelling rationale about the worthiness of lifelong planning, regardless of preferences.

Finally, having completed a dissertation on social insurance, I feel compelled to reveal my very own *sine qua non* full insurance throughout these years: my parents Constantinos D. Thanopoulos and Eleni Thanopoulou who have provided me with ample and unconditional support on all imaginable levels through the years, while daringly (but rightfully, I want to believe) ignoring all "moral hazard" considerations. I can only pledge them to do likewise for my own family.

Dedicating this dissertation to my parents is the absolute minimum I could do to express my appreciation and gratitude for their love, trust and support.

$A.K.\Theta.$

1.0 INTRODUCTION

This dissertation focuses on the study of various institutional arrangements pertaining to social insurance. The broad objective is the investigation of the welfare implications of different social security settings, pursued by examining competing assumptions about agents' preferences in both homogeneous and heterogeneous with respect to agents' preferences economies. The basic building block is a stochastic variant of an overlapping generations model (pioneered by Auerbach & Kotlikoff, 1987[3]) with long but finite-lived individuals who face random life-span and unemployment risk. More specifically, this model is modified along two principal dimensions:

(A) It is postulated that agents have self-control preferences, that is, at every period they run into the temptation to consume their entire wealth. As a consequence, agents featuring self-control preferences save at a lower rate than standard (CRRA) agents, in spite of being just as concerned with lifetime utility as their CRRA counterparts. The consequences of this behavioral assumption are thoroughly examined on both a PAYGO and a fully/partially funded social security system.

(B) In a model of unfunded social security, type heterogeneity is introduced by assuming that both self-control and CRRA agents co-exist in the economy.

The main results obtained are summarized in the following:

The central issue addressed in the first essay "*Social Security and Self-Control Preferences*" (co-authored with Cagri S. Kumru) is the welfare improving potential of unfunded social security under the assumption that agents have self-control preferences. Thus far, the relevant literature has echoed the concern that a PAYGO system can never be shown to be welfare improving, by means of any possible variation of our baseline model. For the sake of comparability with the literature, we use a variant of the same -industry standard- baseline model, but we assume that agents feature self-control preferences. We find that PAYGO social security helps reduce the agents' temptation to consume in every period. In stark contrast with the existing literature, the welfare losses are substantially mitigated. Remarkably, while both unfunded social security and self-control when considered separately reduce welfare, their combination renders their joint effect considerably less severe. Moreover, if the cost of resisting temptation is very high, the introduction of social security might even *improve* welfare.

The second essay "*Temptation Prevalence and Unfunded Social Security*" introduces heterogeneity in preferences in a dynamic stochastic general equilibrium overlapping generations model in order to examine the relevance of unfunded social security in an environment where both CRRA and self-control agents co-exist. Conditions (analogous to those in the first essay) under which the existence of CRRA agents in the economy makes self-control agents better-off are identified. Therefore, a *mixed* economy where there exist simultaneously agents with either CRRA or self-control preferences, allows to examine the extent to which "temptation prevalence" across individuals and temptation severity/intensity within individuals are substitutes in offsetting the adverse effects of unfunded social security. In that sense, this model nests the model in the first essay and provides a richer perspective over the mechanics of the interaction between unfunded social security and self-control preferences.

The main findings in the second essay indicate that, in an economy featuring both CRRA and self-control individuals, social security can be welfare improving for the latter, provided that the temptation those agents face is sufficiently severe. Moreover, the presence of CRRA agents lowers the documented in the literature threshold of self-control intensity that is required for social security to benefit self-control individuals in an "all-self-control" environment. This is due to the fact that the presence of CRRA agents slows down the capital decumulation process in the economy and mitigates the welfare cost that temptation elimination entails for their self-control counterparts.

It is worth noting that as these essays are calibrated to the US Economy, a notoriously dynamically efficient economy (Abel *et al,* 1989[1]) one should expect to see the above welfare improving results even more pronounced when calibrating to dynamically *in*efficient economies.

Having investigated the welfare improving potential of the aforemetioned assumptions on PAYGO social security, the third essay "*Social Security Reform and Temptation*" (co-authored with Cagri S. Kumru) emerges naturally as the concluding step of this dissertation. The impact of self-control preferences is investigated on a fully-funded social security setting in which agents save through Personal Security Accounts (PSAs), without, and with *mandatory* annuitization. Such a model allows to assess the welfare-enhancing potential of mandatory annuitization of accumulated PSA wealth at retirement. When the analysis is restricted to CRRA preferences the results in this essay are congruent with the literature in indicating that the complete elimination of social security is the reform scenario that maximizes welfare. However, when self-control preferences are introduced, and as the intensity of self-control becomes progressively more severe the "social security elimination" scenario loses ground very rapidly. In fact, in the case of very severe temptation the elimination of social security becomes the least desirable alternative. Under the light of the above findings, any reform proposal regarding the social security system should consider departures from standard preferences to preference specifications suitable for dealing with preference reversals.

There are some important caveats that the reader of this dissertation should be aware of:

- In this dissertation I abstract from considering the consequences of aggregate (or social) risk.
- In evaluating different policies I abstract from the extent to which they serve as a redistributive mechanism. In some parts of my work (mainly chapter 2) I do use a concave benefit function (which implies the aforementioned redistributive role) but in the majority of cases, I evaluate my policies using an unbounded benefits function. A political-economy model would be more suitable to deal with these effects properly.
- I abstract from examining any transitional dynamics. Instead I only focus on steady state analysis.

2.0 SOCIAL SECURITY AND SELF-CONTROL PREFERENCES

2.1 INTRODUCTION

[1]The economic benefits of an unfunded social security system are largely summarized in providing intra- and inter-generational risk sharing. Still, this is accomplished at the significant cost of encouraging early retirement, while it also entails very severe distortions in agents' labor supply and private savings decisions. The latter can be readily shown in an overlapping generations model where consumers inelastically supply labor (Diamond (1965)[7]): Since social security redistributes income from the young to the old generation by imposing a tax on current workers' income (payroll tax) -i.e. from a generation with low propensity to consume to a generation with a high propensity to consume- it lowers savings and consequently, the steady state capital stock. In addition, Auerbach & Kotlikoff (1987)[3], Imrohoroglu *et al.* (1995), and Hugget & Ventura (1999)[21], by using a large-scale overlapping generations model, show that an unfunded social security system's distortions in the amount of labor supply and capital accumulation exceed its benefits and hence its existence in an economy reduces overall welfare.

Interestingly, the redistribution mechanism of social security and its induced between-and-within generations allocation of risk is not the only factor that positively affects welfare: Potential idiosyncrasies in agents' preferences highlight yet another extremely important source of ambivalence with regard to the welfare implications of social security. Many studies, both theoretical and empirical have argued on the welfare gains that can be accrued thanks to social security when households lack the foresight to save adequately for their retirement. In particular, Imrohoroglu *et al.* (2003)[24] provide a concise review of the relevant

[1]This chapter is based on joint work with Cagri S. Kumru

literature, as well as an interesting discussion of the debate as to whether myopia is indeed empirically identified from e.g. unforeseen events and other factors that cause a sudden drop in consumption at retirement.

It is well documented in the experimental economics literature that subjects facing intertemporal choice problems often exhibit preference reversals, or that their preferences feature some kind of time inconsistency (Gul and Pesendorfer (2001[17], 2004a[18], 2004b)[19]). In a seminal paper Phelps & Pollak (1968)[31] introduce an intertemporal framework involving quasi-hyperbolic discounting (in lieu of exponential discounting) and utilize it in order to study intergenerational altruism (we shall henceforth refer to the preference structure developed by Phelps & Pollak (1968)[31] as "time-inconsistent preferences").[2]

In a recent study that enhances considerably the insights found in Feldstein (1985)[8], Imrohoroglu *et al.* (2003)[24] investigate the welfare effects of unfunded social security in an economy populated by agents with time-inconsistent preferences who suffer from inability to commit to future actions and hence, save inadequately. In Imrohoroglu *et al.*[24], there is a government that engages in savings on behalf of the quasi-hyperbolic discounters through the social security system. Their main findings are that: (1) quasi-hyperbolic discounters incur substantial welfare costs because of their time-inconsistent behavior, (2) to maintain old-age consumption, social security is not a good substitute for a perfect commitment technology, and (3) there is little room for social security in a world of quasi-hyperbolic discounters.

In spite of their theoretical appeal in providing an alternative that adequately explains observed patterns of behavior, quasi-hyperbolic discounting models entail a non-recursive structure that renders them computationally intractable. This is because quasi-hyperbolic discounting structure does not allow a desire for commitment to one's future actions.

Gul & Pesendorfer (2004a)[18] choose a different approach in their attempt to explain preference reversals. They develop self-control preferences that depend on what an agent actually consumes on one hand, and what would be the level of consumption that would explain the experimental phenomenon, on the other.[3] To this purpose, they introduce self-

[2] Laibson (1997)[30], Diamond and Koszegi (2003)[8], Krusell *et al.* (2002a)[26], and Krusell and Smith (2003)[28] analyze various macroeconomic models by using time-inconsistent preferences.

[3] We will henceforth use the terms "Gul and Pesendorfer preferences" and "Self-Control preferences" interchangeably. It is worth noting that "Gul and Pesendorfer" preferences is not the only available specification for self-control preferences in the literature.

control and temptation utilities, concepts that capture the trade off between the temptation to consume on the one hand, and the long-run self interest of the agent on the other. Under certain rationality assumptions, preferences over sets of actions are consistent with experimental evidence. In stark contrast to time-inconsistent preferences however, self-control preferences are time-consistent. In particular, it is assumed that the preferences governing behavior at time t differ from the preferences over continuation plans implied by the agent's first period preferences and choices prior to period t. In contrast, self-control preferences may already exhibit a desire for commitment.[4]

In this paper we explore the role of an unfunded social security system in a setting where agents have self-control preferences. To this purpose, we develop an overlapping generation model in which agents live up to the real age of 85. The economy consists of three sectors: agents, firms and a government. Agents have idiosyncratic income and face a mortality risk. They work up to the real age of 65 whenever they have an opportunity to work. When unemployed or retired, they are compensated by the government by unemployment insurance or retirement benefits respectively. In addition, they maintain positive asset holdings in order to insure against idiosyncratic income risks and low old-age consumption. Moreover, we assume that private credit markets (including annuities' markets) are closed. The government collects unemployment insurance and payroll taxes from workers to the purpose of financing its activities.

We compute the steady state equilibria under different social security replacement rates by calibrating our model economy to the U.S. economy. From previous studies we know that if an economy is populated by agents with constant relative risk aversion (CRRA) preferences i.e. neither facing a commitment nor a temptation problem, the introduction of an unfunded social security system reduces welfare (Imrohoroglu *et al.* (1995[22] and 2003[24])). The reason is that the insurance benefit of an unfunded social security system is dominated by its negative effect on agents' savings decisions. We also know that if an economy is populated by agents with time-inconsistent preferences, the introduction of social security still reduces welfare, although it provides an additional benefit as a commitment apparatus.

[4]See DeJong and Ripoll (2007)[4] and Krusell *et al.* (2002b)[27] for applications of self-control preferences in various macroeconomic models.

The reason is that the latter benefit along with the insurance benefit are dominated by social security's negative effect on agents' savings decisions (Imrohoroglu *et al.* (2003)[24]). Several interesting insights obtain in our setting: Social security indeed tends to reduce welfare. However, it is worth mentioning that social security is less detrimental to welfare under self-control preferences than it is under CRRA preferences. In addition, if the cost of resisting the temptation is very severe, the introduction of social security might even improve welfare. Controlling for all other factors we infer that this is due to our specification of preferences: Agents with self-control preferences face no commitment problem. Nonetheless, the cost of resisting the temptation associated with the exertion of self-control becomes very severe as wealth increases. In turn, this may impair overall savings in an economy. In our environment, an unfunded social security system has no role as a commitment apparatus but might play a role as a device to decrease available wealth when agents make their consumption-savings decisions.

We identify the underpinnings of our results with the impact social security has on agents' marginal propensity to consume. In the "traditional" setting where agents have CRRA preferences, the young have a low marginal propensity to consume while the old have a high marginal propensity to consume. This relation preserves a high rate of capital accumulation through higher savings during the young age. In contrast, in our environment the young face temptations that operate as impediments to their propensity to (privately) save. Alternatively, the agents' marginal propensity to consume is not as low as it is in the case of CRRA preferences. Accordingly, the cost of resisting temptation increases with the level of wealth. Inevitably, social security by being a mechanism that is bound to deprive agents from early consumption accomplishes at the same time to reduce the cost associated with the exertion of self-control and consequently to partially offset its adverse effect on welfare. Note that this effect is absent in environments where preferences do not allow agents the option to exert self-control as in Imrohoroglu *et al.* (2003)[24].[5]

[5] Diamond (2004)[3] and Diamond *et al.* (2005)[4] argue that the current unfunded social security system does not need radical reform and it is enough to put the system on stronger financial footing while improving the benefit structure at the same time. They state further that mandated savings make sense since many workers would not save enough for their old-age consumption. Our results are in line with those of Diamond in the following sense: When individuals are endowed with temptation, they substantially save less due to the burden of resisting the temptation. The current social security system helps agents to overcome the temptation problem and hence, the welfare cost of the system is not very large.

2.2 GUL & PESENDORFER SELF-CONTROL PREFERENCES

An alternative way of modelling self-control issues is a class of utility functions identified by Gul and Pesendorfer (2004a)[18]. They provide a time-consistent model that addresses the preference reversals that motivate the time inconsistency literature.

Consider a set B of consumption lotteries, and a two-period setting. Gul and Pesendorfer (2004a)[18] have shown that under a specific assumption on choice sets (set betweenness) combined with other standard axioms that yield the expected utility function $U(.)$ defined as

$$U(B) := \max_{p \in B} \int (u(c) + v(c))\, dp - \max_{p \in B} \int v(c) dp$$

represents the preference relation implied by the above axioms. The function $u(.)$ represents the agent's ranking over alternatives when he is committed to a single choice while when he is not committed to a single choice, his welfare is affected by the temptation utility represented by $v(.)$. Note that when B is a singleton, the terms involving $v(.)$ will vanish leaving only the $u(.)$ terms to represent preferences. However, if it is e.g. $B = \{c, c'\}$ with $u(c) > u(c')$ an agent will succumb to the temptation (that is, he will pick the commitment utility - reducing alternative, c') only if the latter provides a sufficiently high temptation utility $v(.)$ in the second period and offsets the fact that $u(c) > u(c')$, i.e. when

$$u(c') + v(c') > u(c) + v(c)$$

In this case the agent wishes he had only c as the available alternative, since under the presence of c', he cannot resist the temptation of choosing the latter.

When the above inequality is reversed, however, the agent will pick c in the second period, albeit at a cost of $v(c') - v(c)$.[6] We call the latter difference the "cost of self-control."

[6] To see that, note that for $B = \{c, c'\}$ and $u(c) > u(c')$ we would have that

$$U(\{c, c'\}) = \max_{\tilde{c} \in \{c,c'\}} (u(\tilde{c}) + v(\tilde{c})) - \max_{\tilde{c} \in \{c,c'\}} v(\tilde{c})$$

$$= u(c) + v(c) - v(c')$$

and since by assumption $v(c') > v(c)$ this means that

$$U(\{c, c'\}) = u(c) - [v(c') - v(c)]$$

In terms of the setting in the present paper, in every period a household faces a consumption - savings problem. Each period, our agents make a decision that yields a consumption for that period and wealth for the next. However, each period these agents face the temptation to consume all of their wealth, and hence, resisting to this temptation results in a self-control-related cost.

Under standard assumptions combined with the multi-period version of "set betweenness," we can represent self-control preferences in a recursive form for the purposes of our T period model which is presented in the next section.

The main difference between self-control preferences and time-inconsistent preferences is that the former do not imply dynamic inconsistency. Preferences are perfectly consistent. Agents can perfectly commit to future actions and do not regret their past actions. Moreover, self-control preferences allow agents to exercise self-control, an option not existing in time-inconsistent preferences. The difference in discounting is the source of preference reversals in the case of time-inconsistent preferences while it also explains why agents find immediate rewards tempting. Instead, Gul and Pesendorfer's self-control preferences assume that agents maximize a utility function that is a "compromise" between the standard utility (or "commitment" utility) and a "temptation" utility. Imrohoroglu *et al.* (2003)[24] considered a setting similar to ours and analyzed the consequences of time-inconsistent preferences while we follow the self-control paradigm in a similar finite-horizon setting. Gul and Pesendorfer (2004a)[18] showed that for finite decision problems a time-inconsistency model can be re-interpreted as a temptation model. In light of that we consider our work as an extension of Imrohoroglu *et al.* in that direction. The purpose of doing so is to check, *inter alia*, if our results encompass the ones of Imrohoroglu *et al.*[24] or if the fact that agents in our setting are capable of exercising self-control (an option not available in Imrohoroglu *et al.*[24]), alters their findings substantially.

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i.e. the utility of the choice c gets penalized by a positive number, the "cost of self-control." Note that in the case $v(c') < v(c)$ i.e. when there is congruence of the utility functions as to which alternative is the best, there is no temptation issue anymore; c is chosen at no penalty since the $v(.)$ terms in $U(\{c, c'\})$ cancel out.

2.3 A MODEL OF SOCIAL SECURITY

The model we consider in this section is quite standard in the social security literature. In particular, our model closely follows that of Imrohoroglu *et al.* (2003)[24].

2.3.1 The Environment

We consider a discrete time, stationary overlapping generations economy. Each period a new generation is born. Agents live a maximum of T periods. The population grows at a constant rate n. All agents face a probability (s_t) of surviving from age $t-1$ to t conditional on surviving up to age $t-1$. Since the economy is stationary, age t agents constitute a fraction μ_t of the population at any given date. The cohort shares $(\{\mu_t\}_{t=1}^{T})$ are given by

$$\mu_{t+1} = \frac{\mu_t s_{t+1}}{1+n},$$

where their sum is normalized to 1.

2.3.2 Preferences

Agents have self-control preferences. In every period they face the temptation to consume their entire wealth. Resisting temptation creates a self-control cost which is absent in the models with CRRA and time-inconsistent preferences. We follow Gul & Pesendorfer (2004a)[18] and DeJong & Ripoll (2007)[4] and model self-control preferences recursively. Let $W(x)$ denote the maximized value of the expected discounted objective function with state x. The utility function of an agent is as follows:

$$W(x) = \max_{c}\{u(c) + v(c) + \beta E W(x')\} - \max_{\check{c}} v(\check{c}), \tag{2.1}$$

where E is the expectation operator; $u(.)$ and $v(.)$ are von Neumann-Morgenstern utility functions; $0 < \beta < 1$ is the discount factor; c is commitment consumption; $\check{c}$ is temptation consumption; and x' denotes next period state variable. As in the section above, $u(.)$ represents the momentary utility function and $v(.)$ represents temptation. In particular,

$v(c) - \max_{\check{c}} v(\check{c})$ denotes the disutility of choosing consumption c instead of $\check{c}$. The concavity or convexity of $v(.)$ is quite important for our analysis[7].

The momentary utility and convex temptation functions take the following forms,

$$u(c) = \frac{c^{1-\gamma} - 1}{1-\gamma} \tag{2.2}$$

and

$$v(c) = \lambda \frac{c^{\rho}}{\rho}. \tag{2.3}$$

respectively.

For the balanced growth rate considerations, the concave utility function is chosen as follows:

$$v(c) = \lambda u(c). \tag{2.4}$$

In the specification above, higher values of the scale parameter ($\lambda > 0$) imply an increase in the share of the temptation utility, i.e. a higher λ increases the importance of current consumption for an agent. The momentary utility function $u(.)$ features constant relative risk aversion.

2.3.3 Budget Constraints

The exogenously given mandatory retirement age is t^*. Agents who are younger than age t^* face a stochastic employment opportunity. Agents that find a chance to work, inelastically supply one unit of labor.[8] We denote the employment state by $e \in \{0, 1\}$ where 0 and 1 denote unemployment and employment states respectively. The employment state follows a first order Markov process. Transition probabilities between current employment state e and next period employment state e' are denoted by the 2×2 matrix $\Pi(e', e) = [\pi_{k'k}]$ where $k', k = 0, 1$ and $\pi_{k'k} = \Pr\{e' = k' | e = k\}$.

An employed agent earns $w\epsilon_t$ where w denotes the wage rate in terms of the consumption good and ϵ_t denotes the efficiency index of an age t agent. If an agent is at the unemployment

[7] Notice that if $v(.)$ is convex, we need to make sure that $v(.) + u(.)$ is stricly concave. In particular, $\gamma > 0$, $\rho > 1$ and $0 < \lambda < \gamma/(c^{\gamma+1}c^{\rho-2})$ guarantee that $u(.)$ is concave, $v(.)$ is convex and $u(.) + v(.)$ is strictly concave. When $v(.)$ is concave, one should show that $W(.)$ is strictly concave.

[8] Adding labor-leisure choice into the model requires the modification of preferences in a way that agents are not only tempted by current consumption but also by current leisure.

state, he receives unemployment insurance benefit equal to the fraction of employed wage ($\phi w \epsilon_t$) where ϕ is the unemployment insurance replacement ratio.

Agents retire at age t^* and receive a lump-sum social security benefit b. The social security benefit b is defined as a fraction θ of an average life time employed income, which is independent of an agent's employment history:

$$b = \begin{cases} 0 & \text{for } t = 1, 2, ..., t^* - 1; \\ \theta \frac{\sum_{t=1}^{t^*-1} w\epsilon_t}{t^*-1} & \text{for } t = t^*, t^* + 1, ..., T. \end{cases}$$

The disposable income of an agent at age t can be written as:

$$q_t = \begin{cases} (1 - \tau_s - \tau_u) w\epsilon_t & \text{for } t = 1, 2, ..., t^* - 1, \text{ if } e = 1; \\ \phi w\epsilon_t & \text{for } t = 1, 2, ..., t^* - 1, \text{ if } e = 0; \\ b & \text{for } t = t^*, t^* + 1, ..., T. \end{cases}$$

In the specification above τ_s and τ_u represent the social security tax rate and the unemployment insurance tax rate respectively.

We assume away private insurance market against the employment risk and private annuities market against the uncertain life span.[9] The only available device to smooth consumption across one's lifetime is the accumulation of assets in terms of physical capital. Agents cannot hold negative assets at any period.[10] Since death is certain at T and there is no bequest motive, the borrowing constraint can be stated as:[11]

$$\begin{cases} a_t \geq 0 & \text{for } t = 1, ..., T - 1; \\ a_t = 0 & \text{for } t = T. \end{cases}$$

[9] Although an annuity market exists in the U.S., it is very thin (Imrohoroglu, 1995 [22]). Hence, our assumption seems innocuous. In our model, social security partially fulfills the role of missing annuities' market (it can be considered as mandatory annuitization). Diamond *et al.* (2005) [3] analyze thoroughly the relationship between annuities and individual welfare. They shows that full annuitization of wealth is optimal under certain conditions.

[10] In other words, an agent faces a borrowing (or liquidity) constraint. There are two main reasons justifying this assumption: First, our desire for a careful numerical comparison of our results with those in the existing literature in which this assumption is a standard one. Second, constraining agents from borrowing against their future income induces an additional boost in (private) savings for precautionary purposes, as long as agents may be/remain unemployed with a positive probability. If we relaxed this constraint, the ability to borrow would lower agents' marginal propensity to save (for precautionary reasons). This would, in turn, render the effects of self-control and ability to borrow against future income highly correlated and hard to tell apart. As a result, the effect of social security on savings due to self-control would not be identifiable

[11] Allowing a bequest motive changes the welfare implications of social security. Fuster *et al.* (2003) [15] perform a welfare analysis of social security in a dynastic framework and show that steady state welfare increases with social security.

If agents in this economy die before age T, their remaining assets will be distributed to all of the survivors in a lump-sum fashion. Let η denote the equal amount of accidental bequests distributed to all remaining members of the society:

$$\eta = \sum_t \sum_a \sum_e \mu_t \Lambda_t(a,e)(1 - s_{t+1}) a_t(a,e), \tag{2.5}$$

where $\Lambda(a,e)$ is the set of age dependent, time independent measure of agents.

Hence, we can write the budget constraint of an agent as follows:

$$a_t + c_t = (1+r)a_{t-1} + q_t + \eta \tag{2.6}$$

and

$$a_t + \check{c}_t = (1+r)a_{t-1} + q_t + \eta, \tag{2.7}$$

where r is the rate of return from the asset holdings.

2.3.4 Production Function

Firms have access to a constant returns-to-scale Cobb-Douglas technology that produces output (Y) by using labor input ($L = 0.94 \sum_{t=1}^{t^*-1} \mu_t \epsilon_t$) and capital input (K) which is rented from households:

$$Y = F(K, L) = AK^{\alpha} L^{(1-\alpha)}, \tag{2.8}$$

where A represents the state of technology; $\alpha \in (0, 1)$ is the capital's share of output. Defining the capital-labor ratio as $\frac{K}{L}$, we can write the production function in the intensive form as follows:

$$y = f(k) = Ak^{\alpha}.$$

The technology parameter A grows at constant rate g and capital depreciates at a constant rate δ. Competitive firms in this economy maximize their profits by setting the real rate of return from asset holdings r and the real wage rate w according to the following:

$$r = A\alpha k^{\alpha-1} - \delta \tag{2.9}$$

and

$$w = A(1-\alpha)k^{\alpha}. \tag{2.10}$$

2.3.5 Government

In our setting, the government's responsibility is limited to the task of administering the unemployment insurance and social security programs. The only constraint imposed on the government's behavior is to enforce self-financing of both the unemployment and social security programs. We restrict our attention to social security arrangements that are described by the pair (θ, τ_s). The self-financing conditions are as follows:

$$\tau_s \sum_{t=1}^{t^*-1} \sum_{a} \mu_t \Lambda_t(a, e=1) w \epsilon_t = \sum_{t=t^*}^{T} \sum_{a} \mu_t \Lambda_t(a, e) b \tag{2.11}$$

and

$$\tau_u \sum_{t=1}^{t^*-1} \sum_{a} \mu_t \Lambda_t(a, e=1) w \epsilon_t = \sum_{t=1}^{t^*-1} \sum_{a} \mu_t \Lambda_t(a, e=0) \phi w \epsilon_t. \tag{2.12}$$

2.3.6 An Agent's Dynamic Program

We suppose that the temptation function $v(.)$ is strictly increasing, i.e. an agent is tempted to consume his entire wealth in each period. This implies that the agent maximizes the second part of equation (2.1) by holding zero asset for the next period, i.e. setting $a_t = 0$ in equation (2.7). In this economy, the agent's state vector x contains the current asset holdings and the employment state. Hence, we can write the agent's dynamic program for any arbitrary two period as follows:

$$W(x) = \max_{c}\{u(c) + v(c) + \beta E_{s'} W(x')\} - v((1+r)a + q + \eta) \tag{2.13}$$

subject to

$$a' + c = (1+r)a + q + \eta, \; a' \geq 0, \; a_0 \text{ is given}, \tag{2.14}$$

where $E_{s'}$ denotes the expectation over survival probabilities.

If the agent succumbs to a temptation and consumes his entire wealth, the term $v(c) - v((1+r)a+q+\eta)$ in equation (2.13) cancels out. When he resists to temptation and consumes less than his wealth, he faces a self-control cost at the amount of $v(c) - v((1+r)a + q + \eta)$. The agent tries to balance his urge for current consumption $v(c)$ and long-term commitment utility $u(c) + \beta E_{s'} W(x')$.

2.3.7 Steady State Equilibrium

In our characterization of the steady state equilibrium, we follow Imrohoroglu *et al.* (2003) [24] and Huggett & Ventura (1999) [21].

Given a set prescribing government policy $\{\theta, \phi, \tau_s, \tau_u\}$, a steady state recursive competitive equilibrium is a set of value functions $\{W_t(x)\}_{t=1}^{T}$, household's policy rules $\{a_t(x)\}_{t=1}^{T}$, time invariant measures of agents $\{\Lambda_t(x)\}_{t=1}^{T}$, wage and interest rate (w, r), and a lump sum distribution of accidental bequests η such that all of them satisfy the following:

- Factor prices (w, r) that are derived from the firm's first order conditions satisfy the equations (2.9) and (2.10).
- Given government policy set $\{\theta, \phi, \tau_s, \tau_u\}$, factor prices (w, r), and lump-sum transfer of accidental bequests η, an agent's policy rule $\{a_t(x)\}_{t=1}^{T}$ solves the agent's maximization problem (2.13) subject to the budget constraint (2.14).
- Aggregation holds:
$$K = \sum_t \sum_a \sum_e \mu_t \Lambda_t(x) a_{t-1}(x). \tag{2.15}$$
- The set of age-dependent, time-invariant measures of agents satisfies in every period t:
$$\Lambda_t(x') = \sum_e \sum_{a:a'=a_t(x)} \Pi(e', e) \Lambda_{t-1}(x), \tag{2.16}$$
where Λ_1 is given.
- The lump-sum distribution of accidental bequests η satisfies the equation (2.5).
- Both the social security system and the unemployment insurance benefit program are self-financing i.e. satisfy the equations (2.11) and (2.12) respectively.

- The market clears:

$$\begin{aligned} & \sum_t \sum_a \sum_e \mu_t \Lambda_t(x)[a_t(x) + c_t(x)] \\ = \quad & Y + (1-\delta) \sum_t \sum_a \sum_e \mu_t \Lambda_t(x) a_{t-1}(x). \end{aligned} \tag{2.17}$$

2.4 CALIBRATION

In this section, we briefly define the parameter values of our model. Each period in our model corresponds to a year. We closely follow Imrohoroglu *et al.* (2003)[24] in order to be able to compare our results to those obtained there.

2.4.1 Demographic and Labor Market Parameters

Agents are born at a real life age of 21 (model age of 1) and they can live up to a maximum real life age of 85 (model age of 65). The population growth rate n is assumed to be equal to the average U.S. population growth rate between 1931-2003 which corresponds, on average, to 1.19% per year.[12] The sequence of conditional survival probabilities is the same as the Social Security Administration's[34] sequence of survival probabilities for men in the year 2001. The mandatory retirement age is equal to 65 (model age 45). In order to set the efficiency index, we choose the average of Hansen's (1993) [20] estimation of median wage rates for males and females for each age group. We interpolate the data by using the Spline Method and normalize the interpolated data to average unity. The employment transition probabilities are chosen to be compatible with the average unemployment rate in the U.S. which is approximately equal to 0.06 between 1948 and 2003.[13] The implied employment transition matrix assumes the following form:

$$\Pi(e, e') = \begin{bmatrix} 0.94 & 0.06 \\ 0.94 & 0.06 \end{bmatrix}$$

[12] The population data was obtained from the U. S. Census Bureau.[34]

[13] The unemployment data are taken from the U. S. Department of Labor.[33]

2.4.2 Preference Parameters

We choose the values of preference parameters ρ, γ, λ and β in such a way that our model-economy's capital-output ratio matches that of the U.S. economy.

In the case where the temptation function $v(.)$ is convex, we choose to follow Imrohoroglu *et al.* (2003)[24] and DeJong & Ripoll (2007)[4], in letting γ be centered at 2 with a standard deviation 1, i.e. $\gamma = 2$ (1). In our benchmark calibration, we initially set $\gamma = 2$, and then check for the robustness of our results by letting $\gamma = 3$. Holding γ constant, we choose different values of ρ *a priori,* and calculate the corresponding λ in such a way that $u(.) + v(.)$ stays a strictly concave function. For every triple ρ, γ and λ, we search over the values of β that deliver the capital-output ratio which is compatible with its empirical counterpart. We assume that the social security replacement ratio is 40% and the unemployment replacement ratio is 25% during our search.

When the temptation function is concave, we follow DeJong & Ripoll (2007) [4] and set $\lambda = 0.0786(0.056)$.

2.4.3 Production Parameters

The parameters describing the production-side of the economy are chosen to match the long-run features of the U.S. economy. Following Imrohoroglu *et al.* (1998 [24], 2003 [24]), we set the capital share of output α equal to 0.310 and the annual depreciation rate of physical capital equal to 0.069. The rate of technological progress g is assumed to be equal to 2.1%, which is the actual average growth rate of GDP per capita taken over the time interval from 1959 to 1994 (Hugget & Ventura, 1999 [21]). The technology parameter A, can be chosen freely. In our calibration exercises, it is set equal to 1.01. All per capita quantities are assumed to grow at a balanced growth rate g.

2.4.4 Government

We set the unemployment insurance replacement ratio (ϕ) equal to 25% of the employed wage and allow the social security replacement ratio (θ) to vary between 0 and 1 in order to make welfare comparisons with different replacement ratios. Alternatively, we can choose the payroll tax rate (τ_s) and the unemployment insurance tax rate (τ_u) instead of the replacement ratios. Since the social security and the unemployment insurance benefits are self-financing, calibrating the replacement ratios will automatically pin-down the tax rates. This holds true because agents inelastically supply one unit of labor whenever they find an opportunity to work, and changes in tax rates do not affect their supply of labor.[14]

2.5 RESULTS

There is a consensus in the literature about the adverse welfare implications of an unfunded social security system, which are mainly due to the distortions it impinges on capital accumulation and labor supply. In order to assess these welfare implications we use a compensating variation measure, which is defined as the percentage by which consumption must be increased to compensate for the decrease in welfare generated by the presence of social security.

In what follows, we present the results of our calibrations starting with a particular example where CRRA preferences (agents are immune from temptation) are used. Thereafter, we continue our analysis by allowing agents to have self-control preferences.[15,16]

[14] However, if we calibrate a model featuring labor-leisure choice, tax rates should be used instead of replacement rates.

[15] We use discrete-time, discrete-state optimization techniques to find a steady-state equilibrium of our hypothetical economy by using the aforementioned parameter values. We follow Imrohoroglu *et al.* (2003) [24] in order to be able to engage in a computational method-free evaluation/comparison of our results to theirs. A discrete set of asset values (containing 4097 points) is created. The lower bound and upper bound of the set is chosen in a way that the set never binds. We start with a guess about the aggregate capital stock and the level of accidental bequests and then solve agents' dynamic program by backward recursion. The time-invariant, age-dependent distribution of agents is obtained by forward recursion. After each loop, we calculate the new values for the accidental bequests and the capital stock. If the difference between the initial values and the new values exceeds the tolerance value, we start a new loop with the new values. This procedure continues until we find values for the accidental bequests and the capital stock that are sufficiently close to their beginning-of-loop values.

[16] All tables of this section are presented in the Appendix.

2.5.1 CRRA Preferences

In our first calibration we use CRRA preferences and calibrate our economy so as to reach a capital-output ratio of approximately 2.5 under the assumption of a 40% social security replacement rate. The steady state features of this economy under alternative social security replacement rates are displayed in Table 2. Our findings in this case are congruent with those in Imrohoroglu *et al.* (1998, 2003)[24]. Consumption, capital and output reach their highest levels when the social security replacement rate is zero.

The main intuition is that, despite the fact that social security provides insurance against life-time uncertainty (due to missing annuities market) and risk sharing among generations, its negative effect on capital accumulation makes it undesirable.[17] Table 2 provides evidence for that fact. It is worth noting that the level of consumption required to compensate the consumers (depicted in the last column of the table) increases in a disproportionately manner compared to a given increase in the social security replacement rate (θ).

2.5.2 Self-Control Preferences

In this section we assume that agents feature self-control preferences with a convex temptation function. In order to demonstrate the quantitative significance of the temptation parameter and its economic meaning, we calculate the quantity of steady state consumption which would be given up by an agent in order to escape from temptation. To this purpose, following DeJong & Ripoll (2007)[4], we obtain the value x such that

$$u(c^* - x) = u(c^*) + v(c^*) - v(\check{c})$$

where c^* is the steady state value of the agent's actual consumption and $\check{c}$ is the steady state value of temptation consumption. To isolate the effect of λ, the model is calibrated under zero social security replacement rate and all other parameters remain fixed at their CRRA case while ρ is chosen equal to 2. By increasing λ from 0 to 0.001, we observe that agents would be willing to forgo as much as 4.82% of their steady state consumption in order to

[17] Since there is no labor-leisure decision in our model, social security system has an effect only on capital accumulation (saving).

eliminate temptation.[18]

This is an interesting result that highlights the forceful consequences of an arguably imperceptible departure from the CRRA preference specification. It underscores the welfare reducing role temptation (and the induced cost of self-control) plays in our model. Nonetheless, at the same time it validates our main intuition, namely, that social security may not be as detrimental to welfare as it has been generally argued in the literature.

Next our aim is to investigate whether there is any room for a social security system, when agents have self-control preferences. In our first calibration we use the same parameter values for β in order to measure the impact of temptation on savings, under a 40% social security replacement rate. This example is a counter-factual in the sense that it does not yield capital output ratio around 2.5, but it serves as a device to better demonstrate the effect of self-control preferences on savings.

Tables 3, 4 and Figures 1, 2 show the steady state of an economy with self-control preferences under 40% replacement ratio. In particular, Table 3 is constructed holding all parameters of the utility function fixed in their CRRA values, except for λ, which is the parameter we vary. The value of parameter λ measures the strength of temptation towards current consumption. Higher values for this parameter corresponds to higher cost of exerting self-control. We notice that all variables but the interest rate decrease as λ increases (i.e. as we depart from the CRRA case). In particular, the capital-output ratio decreases showing that the increase of λ triggers a process of dissavings. This process deprives the economy from future consumption capabilities. The latter point is congruent with what we observe in the consumption pattern as λ varies.

Figure 1 illustrates the aforementioned points. We plot lifetime consumption as a function of age. Even a casual glance suggests that an increase in the temptation intensity (λ) results in an abrupt departure from the consumption smoothing behavior of a CRRA agent. It is worth noting how dramatically the early high consumption pattern of a consumer with higher values of λ gets penalized in his retirement years compared to a CRRA consumer. As it could be expected, for a very low value of λ the observed pattern closely resembles that

[18] DeJong & Ripoll (2007) [4] report the analogous to their environment value as slightly above 5% of the steady state consumption when the scale of the temptation parameter is increased from 0 to 0.00286.

of CRRA.

Figure 2 provides additional support to our findings from the perspective of lifetime asset holdings. It is worth observing that the discrepancy in savings before retirement between different agents (in terms of λ) translates to the observed difference in consumption documented in Figure 1.

Table 4 is constructed holding β, and γ in their CRRA values and keeping λ fixed at 0.00009 under 40% social security replacement rate. Now, we only vary ρ which is a measure of the consumers' willingness to substitute current temptation consumption for future one. The higher ρ is the more the consumer prefers early to late temptation consumption which actually makes the self-control cost even more severe. This, in turn causes further dissavings and eventually lower steady state consumption for any value of λ.

Figure 3 illustrates our findings in terms of lifetime consumption. The clear difference in the observed consumption pattern manifests the impact of an increase in ρ.

Additional support is provided by Figure 44. Note that we observe that the impact of an increase in ρ on asset holdings is very similar to the impact of an increase in λ, which suggests that a given pattern of asset holdings is not uniquely identifiable by given (λ, ρ), but instead can be induced by different combinations of those two parameters.

Now that we are able to detect the effect of self-control preferences on savings, we can calibrate our benchmark economy to analyze the effect of a social security system on the entire economy.

Table 5 presents the features of various steady states of this economy. Our main point in this case is that an unfunded social security system serves an additional purpose to that of the provision of insurance against life-time uncertainty and intergenerational risk-sharing: It makes the cost of exerting self-control less burdensome by reducing the amount of wealth through taxing of the current income. One can speculate that if the unfunded social-security system's negative effect on savings is offset by its positive effect on the self-control cost, a certain level of social security replacement rate may generate larger benefits (through an increase of the level-of-capital channel) than the ones generated in the absence of social security. This additional benefit of the unfunded social security system is absent if an agent is not endowed with temptation.

While both social security and self-control, when considered separately, they have detrimental effects on welfare, their combination yields a noteworthy result: Welfare reduction is considerably less severe. The intuition behind this result lies in the following fact: Social security is a mechanism that deprives agents from early consumption. When agents face temptations, social security accomplishes at the same time to reduce the cost associated with the exertion of self-control and consequently to partially offset its adverse effect on welfare. Note that this effect is absent in environments where preferences do not allow agents the option to exert self-control. Not surprisingly, it is also absent in the case where the temptation component essentially does not modify the consumers' lifetime consumption paths. However, we shouldn't overlook the plausible scenario in which social security makes it more likely that agents exercise less self-control when they are young, given that retirement consumption is assured by the government.[19] It turns out that our quantitative exercises seem to indicate that the former intuition is more likely to prevail.

A careful comparison of Table 5 with Table 2 reveals that on the one hand social security decreases welfare both under the CRRA and the self-control preference specifications but on the other, the presence of self-control preferences seems to mitigate the welfare reducing effect of social security. This can be seen by directly comparing the compensation needed by a consumer facing temptation and the one needed by a CRRA consumer in order to offset the adverse welfare effects of social security. Although the scale of the temptation parameter (λ) is very small, the welfare cost of social security system is almost three times lower than that of the CRRA preference specification for a given social security replacement ratio (by comparing the last two columns of the two tables).

Our findings parallel Imrohoroglu *et al.* (2003) [24] in that social security indeed entails welfare losses both under CRRA preferences and non-CRRA preferences and it is less severe under the latter. They used time-inconsistent preferences as their theoretical apparatus and concluded that only a negligible percentage of the whole population prefers a social security system. However, in their framework agents do not face a temptation problem (and consequently a cost of exerting self-control). Welfare issues stemming from their preference

[19] We thank an anonymous referee for kindly drawing our attention to this scenario and enhancing our intuition.

specification reduce to a commitment problem. Hence in their case, an unfunded social security system works only as a commitment device. Contrastingly, when consumers face temptation, social security is considerably less costly than in the case where consumers have CRRA preferences, precisely thanks to its additional benefit of reducing the temptation cost. This, in turn, mitigates the unfunded social security system's negative welfare effect.

A rather surprising result is displayed in Table 6. The choice of a relatively large value λ, results in an increase in welfare as it can be seen in the last column. The meaning of negative values in the CV column is that there is a welfare cost associated with smaller values of social security replacement rate. That is, agents should be compensated for the absence of the social security system. Furthermore, a replacement rate of 60% maximizes welfare. This rather controversial result is most probably due to the choice of a high β $(= 1.011)$ which is necessary in order for the targeted empirical capital-output ratio to be achieved. We believe that this observation further underscores the mitigating effect of the existence of a temptation component in the utility function as it is identified in our paper. Although this result is most likely due to the choice of $\beta = 1.011$, it is an important as it highlights the potentially positive impact of social security when agents are highly tempted towards current consumption.[20,21,22]To our knowledge, this is the first paper that provides an environment and suitable conditions under which social security may *improve* welfare.

2.6 CONCLUSION

Expenses related to social security comprise one of the largest expenditure items in the U.S. government's budget. As a result, there is an extensive literature regarding social security related issues. The costs and benefits of social security are well analyzed by many authors in

[20]We thank an anonymous referee of the *Journal of Economic Dynamics and Control* for encouraging us to emphasize the importance of this result.

[21]Robustness tests of our results have been successfully performed and are available upon request.

[22]We have also calibrated our model economy to the U.S economy when agents feature a concave temptation function. In our calibration exercises, we use the value of the parameter λ equal to 0.0786 with the standard deviation equal to 0.056, as estimated by DeJong and Ripoll (2007) [4]. In this case, the life time consumption path remains essentially invariant to departures from the CRRA case. Accordingly, social security remains equally detrimental to welfare under self-control preferences as it is under CRRA preferences. Results are available upon request.

the context of standard preferences: all of the studies with the exception of Imrohoroglu *et al.* (2003) [24] use CRRA preferences. Imrohoroglu *et al.* use quasi-hyperbolic preferences instead, and show that even in such a context where social security could be used as a commitment device, it turns out that social security does not improve welfare.

In the present paper, we assume that consumers have self-control preferences. In our environment, agents do not have a commitment problem but they instead face a temptation to consume all of their available wealth at each point in time.

Our methodology consists in implementing calibration techniques, similar to those used in the related literature, in order to simulate our economy and draw conclusions regarding the impact of social security on consumers' lifetime welfare. In doing so, we consider several variations of our specification of the temptation utility function (different degrees of convexity / concavity of the temptation function), and assess their influence separately, while at the same time compare it with the standard (CRRA) preferences case. Finally, we verify the numerical validity of our results by administering various robustness tests.

Our main findings can be summarized in the following: In a world where agents have self-control preferences social security generally decreases lifetime welfare. Interestingly however, we call attention to a challenging novelty which is due to our specification of self-control preferences: The presence of temptation considerably reduces the cost of social security. That is, indeed social security penalizes welfare but when the economy features agents with self-control preferences the above cost is substantially mitigated. Moreover, should the cost of resisting temptation become very high, the introduction of social security may even improve welfare. To our knowledge, this is the first paper that provides an example of an environment under which social security may *improve* welfare.

Furthermore, in our calibrations we measure that the cost of temptation, namely, the amount of consumption that agents would be willing to relinquish in order to eliminate temptation is as high as 4.82% of their steady state consumption. Since this percentage corresponds to an insignificant deviation (increasing λ from 0 to 0.001) from the CRRA preference specification, it underscores the welfare reducing role temptation (and the induced cost of self-control) plays in our model. Nonetheless, at the same time it validates our main intuition, namely, that social security may not be as detrimental to welfare as it has been generally argued in the literature.

While both social security and self control, when considered separately, have detrimental effects on welfare, their combination yields a remarkable result: welfare reduction is considerably less severe. The intuition behind this result lies in the following fact: social security is a mechanism that deprives agents from early consumption. When agents face temptations, social security at the same time reduces the cost associated with the exertion of self-control and consequently partially offsets its adverse effect on welfare. It is worth noting that this effect is absent in environments wherein preferences do not allow agents the option to exert selfcontrol, or in contexts wherein the impact of temptation on lifetime consumption is moderate.

Table 1: Parameter Values of The Benchmark Calibration

Demographics	
Maximum possible life span T	65
Obligatory retirement age t^*	45
Growth rate of population n	1.19%
Conditional survival probabilities $\{s_t\}_{t=1}^{T}$	U.S. 2001
Labor efficiency profile $\{\epsilon_j\}_{t=1}^{t^*-1}$	Hansen (1993)
Production	
Capital share of GDP α	0.310
Annual depreciation of capital stock δ	0.069
Annual per capita output growth rate g	2.1%
Markov Process for employment transition Π	$\begin{bmatrix} 0.94 & 0.06 \\ 0.94 & 0.06 \end{bmatrix}$
Preferences	
Annual discount factor of utility β	0.998
Scale factor of the temptation utility λ	0.000375
Risk aversion parameter γ	2.0
Risk loving parameter ρ	2.0
Government	
Unemployment insurance replacement ratio ϕ	0.25
Social security replacement ratio θ	$[0, 1]$

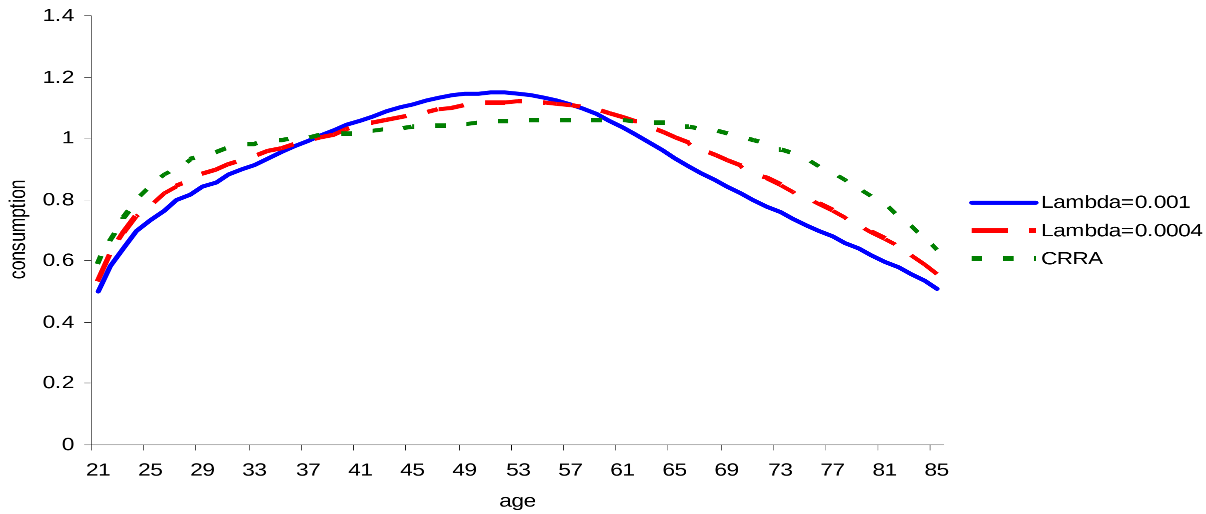

Figure 1: Optimal Consumption Choice-I

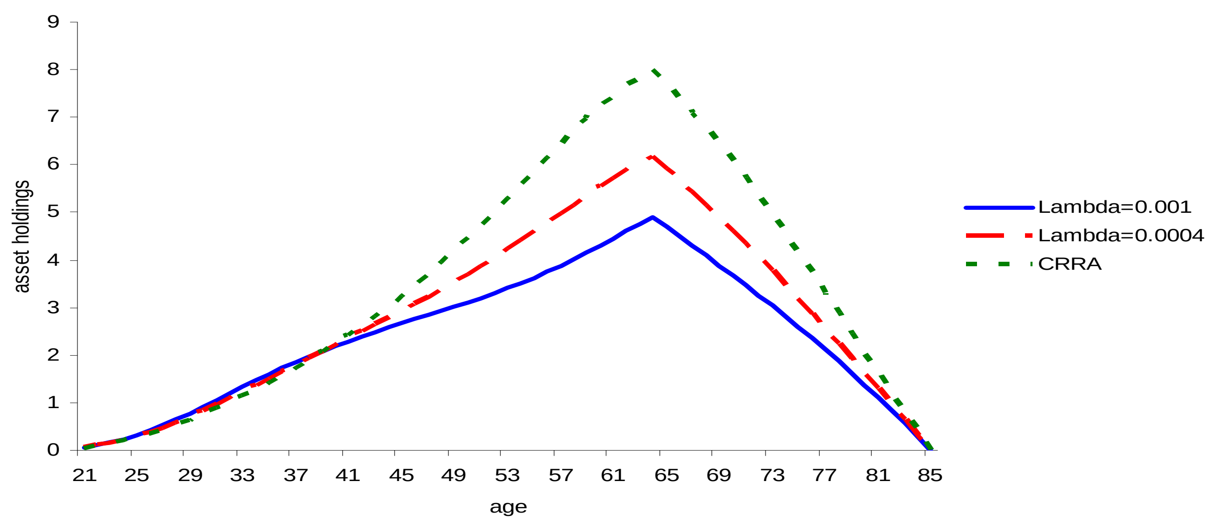

Figure 2: Optimal Asset Holding-I

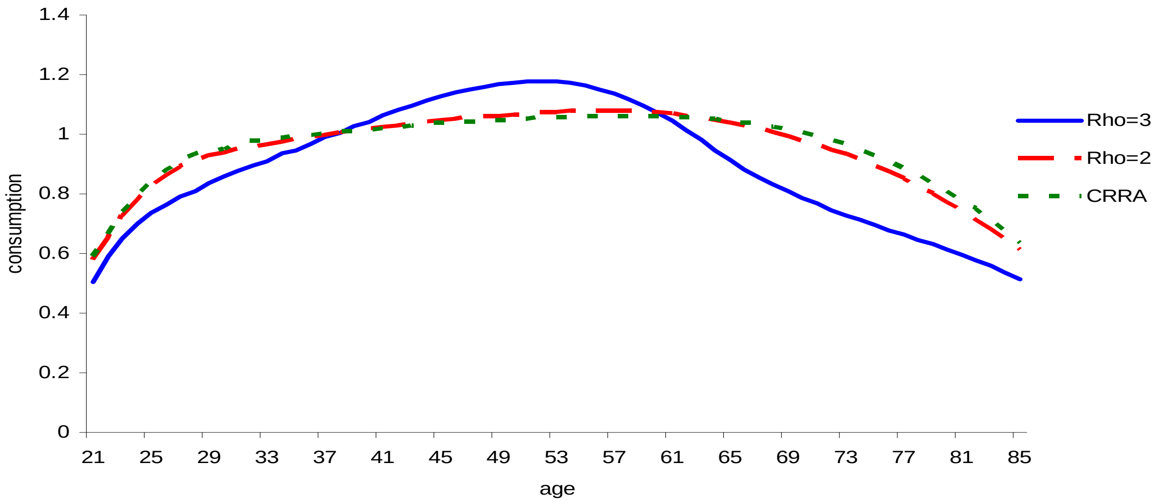

Figure 3: Optimal Consumption Choice-II

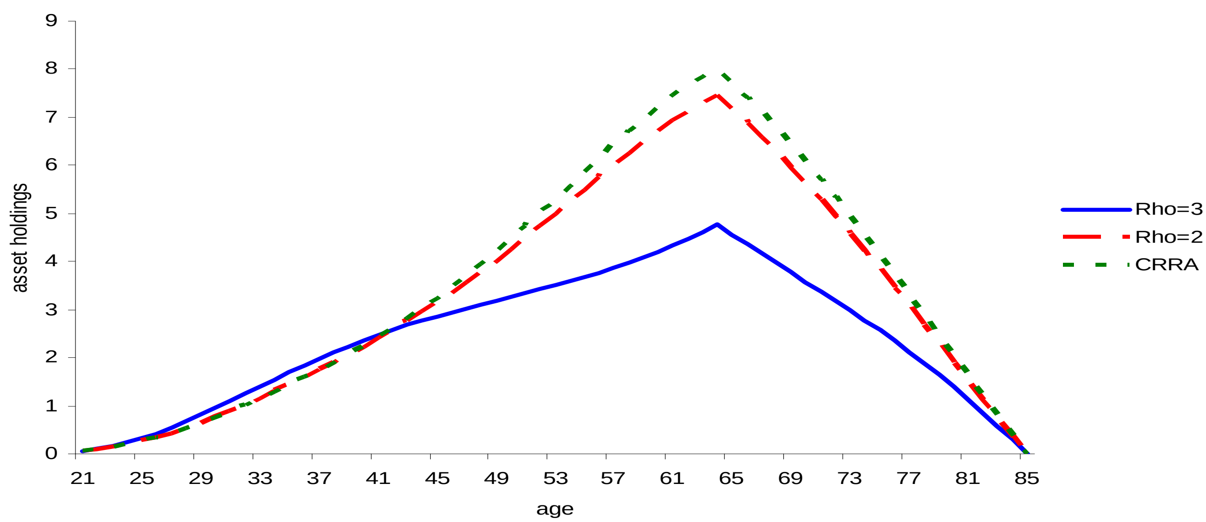

Figure 4: Optimal Asset Holding-II

Table 2: (beta=0.978, gamma=2, lambda=0)

θ	Y	K	C	K/Y	r	w	CV (%)
0	1.239	3.308	0.971	2.671	0.068	1.088	0.000
0.10	1.225	3.187	0.968	2.602	0.072	1.076	1.300
0.20	1.212	3.081	0.965	2.542	0.075	1.065	2.637
0.30	1.199	2.978	0.962	2.483	0.078	1.054	4.043
0.40	1.188	2.892	0.960	2.434	0.081	1.044	5.460
0.50	1.178	2.814	0.957	2.388	0.084	1.035	6.913
0.60	1.168	2.738	0.954	2.344	0.087	1.026	8.416
0.70	1.159	2.668	0.952	2.301	0.090	1.018	9.951
0.80	1.150	2.605	0.949	2.264	0.093	1.011	11.520
0.90	1.143	2.548	0.947	2.230	0.095	1.004	13.112
1	1.135	2.493	0.944	2.197	0.098	0.997	14.752

Table 3: (beta=0.978, gamma=2, rho=2)

λ	Y	K	C	K/Y	r	w
$CRRA$	1.188	2.892	0.960	2.434	0.081	1.044
$\lambda = 0.0004$	1.120	2.386	0.939	2.132	0.103	0.984
$\lambda = 0.001$	1.067	2.046	0.918	1.917	0.122	0.938

Table 4: (beta=0.978, lambda=0.00009)

λ	Y	K	C	K/Y	r	w
$CRRA$	1.188	2.892	0.960	2.434	0.081	1.044
$\rho = 2$	1.168	2.735	0.954	2.342	0.087	1.026
$\rho = 3$	1.075	2.091	0.922	1.946	0.119	0.944

Table 5: (beta=0.998, gamma=2, lambda=0.000375, rho=2)

θ	Y	K	C	K/Y	r	w	$CV(\%)$
0	1.229	3.224	0.969	2.623	0.071	1.080	0.000
0.10	1.216	3.116	0.966	2.562	0.074	1.068	0.294
0.20	1.204	3.015	0.963	2.504	0.077	1.058	0.705
0.30	1.193	2.932	0.961	2.457	0.080	1.048	1.208
0.40	1.184	2.856	0.958	2.412	0.083	1.040	1.795
0.50	1.174	2.783	0.956	2.370	0.086	1.031	2.462
0.60	1.165	2.714	0.953	2.329	0.088	1.023	3.205
0.70	1.157	2.652	0.951	2.292	0.091	1.016	4.008
0.80	1.150	2.600	0.949	2.262	0.093	1.010	4.856
0.90	1.143	2.548	0.947	2.230	0.095	1.004	5.770
1	1.136	2.499	0.944	2.200	0.097	0.998	6.743

Table 6: (beta=1.0117, gamma=2, lambda=0.00065, rho=2)

θ	Y	K	C	K/Y	r	w	CV (%)
0	1.255	3.423	0.972	2.727	0.064	1.090	0
0.10	1.242	3.311	0.970	2.665	0.067	1.080	−0.867
0.20	1.230	3.210	0.968	2.608	0.070	1.069	−1.542
0.30	1.219	3.120	0.966	2.558	0.073	1.060	−2.304
0.40	1.209	3.033	0.963	2.508	0.076	1.050	−2.385
0.50	1.199	2.959	0.961	2.466	0.078	1.043	−2.599
0.60	1.190	2.888	0.959	2.425	0.082	1.034	−2.698
0.70	1.182	2.823	0.957	2.387	0.083	1.027	−2.696
0.80	1.174	2.764	0.955	2.352	0.085	1.021	−2.602
0.90	1.167	2.708	0.953	2.319	0.087	1.014	−2.421
1	1.161	2.660	0.951	2.291	0.089	1.008	−2.167

3.0 TEMPTATION PREVALENCE AND UNFUNDED SOCIAL SECURITY

3.1 INTRODUCTION

Unfunded social security has long been a topic of controversy, despite its beneficial role in providing intra- and intergenerational risk sharing. This is mainly due to the budget implications that its administration has, but also due to the distortions in labor supply and savings decisions. In a seminal paper, Diamond (1965)[7] questions the very core of existence of unfunded social security: in the context of an overlapping generations model where consumers inelastically supply labor, he shows that the redistribution of income among generations performed by imposing a social security payroll tax lowers savings and consequently, the steady state capital stock. Hence, it ends up lowering welfare. Likewise, Auerbach & Kotlikoff (1987) [3], (and later Hugget & Ventura (1999) [21]), by using a large-scale overlapping generations model, show that the costs associated with the distortions unfunded social security entails for the amount of labor supply and capital accumulation exceed its benefits and hence its existence in an economy reduces overall welfare.

Many subsequent studies (e.g. Storesletten *et al.* (1999) [32]) study different social security systems and typically compare welfare across alternative steady states, each corresponding to a stationary equilibrium matching a different social security system. Imrohoroglu *et al.*(1995) [24] emphasize the detrimental effects that unfunded social security has to the overall welfare in an economy. Interestingly, the possibility that alternative preference specifications, notably those dealing with preference reversals, are likely to bite in some cases was examined considerably later by Imrohoroglu *et al.*(2003) [24], in spite of the existing evidence from the experimental economics literature that preferences show some degree of time inconsistency and agents suffer from temptation.[1]

Imrohoroglu *et al.*(2003) [24] use time-inconsistent preferences in lieu of CRRA preferences, alas without identifying any significant impact of the preference structure on welfare. However later, Kumru & Thanopoulos (2008) [29] use self-control preferences to highlight that in a context of unfunded social security welfare may be critically affected by the preference specification, to the extent that in some cases unfunded social security may even improve welfare.

A follow-up research question pertains to the actual prevalence of this phenomenon in the general population. While Imrohoroglu *et al.*(2003)[24] and Kumru & Thanopoulos (2008)[29] analyze the welfare implications of preference specifications that give rise to preference reversals, they both do so assuming that the *entire* population features those non-standard preferences and compare it against a benchmark case where again *all* agents feature CRRA preferences.

In this paper I attempt to perform a fully-blown welfare analysis within the context of the family of models outlined above. I do so by enhancing the parameter space of my model by a parameter called "temptation prevalence". As it is the case in my main reference papers, in order to capture the agents' temptation towards current consumption, my model economies make use of the preference structure pioneered by Phelps & Pollak (1968)[31] and further elaborated by Gul & Pesendorfer (2004a) [17] to model self-control issues[2]. Naturally, my model specification closely follows that of Kumru & Thanopoulos (2008) [29] and Imrohoroglu *et al.*(2003) [24]. Likewise, my economic environment features uninsurable individual income shocks, borrowing constraints and missing annuity markets.

I introduce a "temptation prevalence" parameter by assuming that at the beginning of their lifetime, each individual's type is predetermined as either CRRA or self-control, meaning they feature either CRRA preferences or self-control preferences respectively throughout their lifespan. This bears the consequence that at each point in time a generation is born there will be a measure X of the new population featuring self-control preferences while the

[1] For a recent overview of studies that provide evidence that individuals indeed exhibit bias toward immediate gratification see Frederick *et al* (2002) [14].

[2] In a recent paper Dekel, Lipman, & Rustichini (2008)[6] propose a class of self-control preferences that nests Gul & Pesendorfer (2001, 2004a, 2004b)[16] preferences but contrary to the latter, takes account of the multidimensional nature of temptation and uncertainty about temptation.

remaining $(1 - X)$ will necessarily feature CRRA preferences.

My purpose is to investigate the consequences that a continuously varying proportion of self-control agents entails with regard to the welfare analysis performed in Kumru & Thanopoulos (2008) [29] and Imrohoroglu *et al.*(2003) [24]. In particular, considering the level of lifetime consumption that obtains in the model with CRRA preferences as my benchmark, I examine the extent to which this is penalized as the proportion of self-control agents in the population progressively increases, for a given intensity of self-control and a given social security replacement rate. A natural follow-up question on the findings in Kumru & Thanopoulos (2008) [29] would be the existence of a sufficiently severe temptation problem that would turn unfunded social security in an economy featuring some non-zero measure of CRRA agents welfare improving.

I aim to contribute to the debate on unfunded social security and further read into the ability of alternative preference specifications to make headway in resolving the current controversy about the suitability and scope of unfunded social security. Inquiring about the preference structure is very important for theoretical purposes because it enhances our understanding of the mechanics of similar models in the literature by providing an additional channel through which capital accumulation is distorted. As shown in Kumru & Thanopoulos (2008) [29] and Imrohoroglu *et al.*(2003) [24] the presence of slightly far-sighted or current consumption favored agents changes substantially the welfare implications of the system. In addition, investigating the impact of the more realistic scenario of mixed population (as opposed to a population consisting of "only CRRA" or "only self-control" agents) provides further insights with regard to the existence of settings where unfunded social security improves welfare even under the presence of some CRRA agents.

Therefore, my *mixed* economy where there exist simultaneously agents with either CRRA or self-control preferences, allows me to examine the extent to which "temptation prevalence" *across* individuals and temptation severity/intensity *within* individuals are substitutes in offsetting the adverse effects of unfunded social security. In that sense, our model nests previous relevant models (Kumru & Thanopoulos (2008) [29]) and provides a richer perspective over the mechanics of the interaction between unfunded social security and self-control preferences.

My findings indicate that indeed, in an economy featuring both CRRA and self-control individuals, social security can be welfare improving for the latter, provided that the temptation those agents face is sufficiently severe. Moreover, the presence of CRRA agents lowers the threshold documented in the literature (Kumru & Thanopoulos (2008) [29]) of self-control intensity required for social security to benefit self-control individuals in an "all-self-control" environment. This is due to the fact that the presence of CRRA agents slows down the capital decumulation process in the economy and mitigates the welfare cost that temptation elimination entails for their self-control counterparts. This entails an additional welfare burden to CRRA agents who become worse off not only due to the existence of a forced savings mechanism such as social security, but also because of the presence of their self-control counterparts in the economy. Note that our results are not sensitive to the unfunded character of social security; they are essentially driven by the underlying forced savings that such a mechanism entails.

It is worth noting that one of the most important aspects of this paper consists in the fact that a more general setting featuring type heterogeneity opens much richer interpretations pertaining to political economy and hence may naturally accomodate new modeling features[3]. An example includes a government featuring an objective function with preferences over the welfare of different types of agents; in turn, these preferences could be an endogenous function of the voting preferences of the agents' types. The trade offs arising from such a setting could provide an additional explanation as to why unfunded social security is so prevalent throughout the world. Note that such an explanation is not possible in an environment that abstracts from type heterogeneity, such as that analyzed by Kumru & Thanopoulos (2008) [29] .

3.2 A MODEL OF SOCIAL SECURITY

The model I consider in this section is a standard one in the social security literature. In particular, my model follows that of Kumru & Thanopoulos (2008) [29] and Imrohoroglu *et al.*(2003) [24].

[3] I thank John Duffy for pointing out this intuition to me.

3.2.1 Demographics

I consider a stationary overlapping generations economy in discrete time. Each period a new generation is born which is modeled to be n percent larger than the previous generation. Agents face lives of uncertain duration and some live through the maximum possible life span, denoted by J. At any given time t within their life-span, all agents have a (time-invariant) conditional probability $s_j \in (0,1)$ of surviving from age $j-1$ to j, conditional on having survived up to age $j-1$. Our stationary population assumption implies that age j agents constitute a fraction μ_j of the population at any given date. The cohort shares $\{\mu_j\}_{j=1}^{J}$ are given by

$$\mu_{j+1} = \frac{\mu_j s_{j+1}}{1+n}$$

while their sum is normalized to 1.

At birth, individuals make a draw from a distribution of types. An individual's "type" can be either $CRRA$, meaning they have constant relative risk aversion preferences, or SC meaning they have self-control preferences. At each point in time a generation is born there will be a time-invariant measure X of the new population featuring self-control preferences while obviously the remaining $(1-X)$ will feature CRRA preferences. Note that this implies that we treat X as an exogenously predetermined parameter in our model. I will henceforth call X the "temptation prevalence" in the economy.

3.2.2 Preferences

As noted in the previous paragraph, a proportion X of the agents in our economy feature self-control preferences and its complement feature CRRA preferences. self-control preferences prescribe that in every period the agent faces a temptation to consume their entire wealth. Resisting temptation gives rise to a self-control cost; note that the latter feature is absent in models with CRRA and quasi-hyperbolic preferences. I proceed to model self-control preferences recursively.

Let $W(x)$ denote the maximized value of the expected discounted objective function with state x. The utility function of an agent i is as follows:

$$W_i(x) = \max_c \{u(c) + \mathbf{1}_{\{i \in S_X\}} v(c) + \beta E W(x')\} - \mathbf{1}_{\{i \in S_X\}} \max_{\check{c}} v(\check{c}) \tag{3.1}$$

where E is the expectation operator; $u(.)$ and $v(.)$ are von Neumann-Morgenstern utility functions, $\mathbf{1}_{\{\}}$ is the indicator function while S_X denotes the set of self-control agents in the economy; $0 < \beta < 1$ is a discount factor; c represents the "commitment" consumption; $\check{c}$ is the "temptation" consumption; and x' denotes next period state variable. As in the section above, $u(.)$ represents the momentary utility function and $v(.)$ represents temptation. In particular, $v(c) - \max_{\check{c}} v(\check{c})$ denotes the disutility from choosing consumption c instead of $\check{c}$. The concavity or convexity of $v(.)$ turns out to be very important for our analysis.[4]

The momentary utility, convex temptation and concave temptation functions are assumed to take the following forms respectively:

$$u(c) = \frac{c^{1-\gamma} - 1}{1 - \gamma} \tag{3.2}$$

$$v(c) = \lambda \frac{c^{\rho}}{\rho} \tag{3.3}$$

$$v(c) = \lambda u(c) \tag{3.4}$$

In the specification above, higher values of the scale parameter $\lambda > 0$, imply an increase in the share of "temptation" utility, i.e. a higher λ increases the importance of current consumption for an agent. The momentary utility function $u(.)$ is a standard Constant Relative Risk Aversion (CRRA) form, $\gamma > 0$ measures the degree of relative risk aversion (and $1/\gamma$ the inter-temporal elasticity of substitution).

[4]Notice that if $v(.)$ is convex, we need to make sure that $v(.)+u(.)$ is stricly concave. In particular, $\gamma > 0$, $\rho > 1$ and $0 < \lambda < \gamma/(c^{\gamma+1}c^{\rho-2})$ guarantee that $u(.)$ is concave, $v(.)$ is convex and $u(.) + v(.)$ is strictly concave. When $v(.)$ is concave, one should show that $W(.)$ is strictly concave.

3.2.3 Production Function

Firms have access to a constant returns-to-scale Cobb-Douglas technology that produces output (Y_t) by using labor input ($L_t = 0.94 \sum_{j=1}^{j^*-1} \mu_j \epsilon_j$) and capital input (K_t) which is rented from households:

$$Y_t = F(K_t, L_t) = A_t K_t^{\alpha} L_t^{(1-\alpha)} \tag{3.5}$$

where A_t represents the state of technology; $\alpha \in (0, 1)$ is the capital's share of output, j^* denotes the compulsory retirement age and ϵ_j denotes the efficiency index of an age j agent. Defining the capital-labor ratio as $\frac{K_t}{L_t}$, we can write the production function in intensive form as follows:

$$y_t = f(k_t) = A_t k_t^{\alpha}$$

The technology parameter A_t grows at a constant rate g and capital depreciates at a constant rate δ.

Competitive firms in this economy maximize their profits by setting the real rate of return from asset holdings r and the real wage rate w according to the following:

$$r_t = A_t \alpha k_t^{\alpha-1} - \delta \tag{3.6}$$

$$w_t = A_t(1 - \alpha)k_t^{\alpha} \tag{3.7}$$

Since I am concerned only with steady state equlibrium , I henceforth drop the time subscript for the rest of the analysis.

3.2.4 Budget Constraints

The exogenously given mandatory retirement age is t^*. Agents who are younger than age t^* face a stochastic employment opportunity. Agents that find a chance to work, inelastically supply one unit of labor.[5] I denote the employment state by $e \in \{0, 1\}$ where 0 and 1

[5] Adding labor-leisure choice into the model requires the modification of preferences in a way that agents are not only tempted by current consumption but also by current leisure.

denote unemployment and employment states respectively. The employment state follows a first order Markov process. Transition probabilities between current employment state e and next period employment state e' are denoted by the 2×2 matrix $\Pi(e', e) = [\pi_{k'k}]$ where $k', k = 0, 1$ and $\pi_{k'k} = \Pr\{e' = k' | e = k\}$.

An employed agent earns $w\epsilon_t$ where w denotes the wage rate in terms of the consumption good and ϵ_t denotes the efficiency index of an age t agent. If an agent is at the unemployment state, he receives unemployment insurance benefit equal to the fraction of employed wage ($\phi w \epsilon_t$) where ϕ is the unemployment insurance replacement ratio.

Agents retire at age t^* and receive a lump-sum social security benefit b. The social security benefit b is defined as a fraction θ of an average life time employed income, which is independent of an agent's employment history[6]:

$$b = \begin{cases} 0 & \text{for } t = 1, 2, ..., t^* - 1; \\ \theta \frac{\sum_{t=1}^{t^*-1} w\epsilon_t}{t^*-1} & \text{for } t = t^*, t^* + 1, ..., T. \end{cases}$$

The above benefit formula is predominantly used in this paper since it is a standard on all other papers with which we compare our work. For the sake of completeness, I also use the following more realistic approximation of the actual social security benefits formula and compare our results with the case of linear benefits:

$$b = \begin{cases} 0 & \text{for } t = 1, 2, ..., t^* - 1; \\ \ln\left(\frac{\sum_{t=1}^{t^*-1} w\epsilon_t}{t^*-1}\right) & \text{for } t = t^*, t^* + 1, ..., T. \end{cases}$$

It is worth noting that the actual monthly social security benefit provided by the Social Security Administration is calculated according to a (continuous) piecewise linear concave function of the average indexed monthly earnings of an individual. The vector of slopes associated with this is $[0.9, 0.32, 0.15]$. My logarithmic specification is an excellent approximation of the above concave piecewise linear function, as it captures the essence of unfunded social security, while at the same time provides significant savings in terms of computational burden by capturing most of the information contained in those three parameters.

The disposable income of an agent at age t can be written as:

[6] Note that this formula entails unbounded benefits.

$$q_t = \begin{cases} (1-\tau_s-\tau_u)w\epsilon_t & \text{for } t=1,2,...,t^*-1, \text{ if } e=1; \\ \phi w\epsilon_t & \text{for } t=1,2,...,t^*-1, \text{ if } e=0; \\ b & \text{for } t=t^*,t^*+1,...,T. \end{cases}$$

In the specification above τ_s and τ_u represent the social security tax rate and the unemployment insurance tax rate respectively.

I assume away private insurance markets against the unemployment risk and private annuities markets against uncertain life-span. Note that, although an annuities market exists in the U.S., it is very thin (Imrohoroglu, 1995 [22]). Hence, my assumption seems innocuous. In my model, social security partially fulfills the role of the missing annuities market (it can be considered as mandatory annuitization) [7].

Consequently, in our model the only available device to smooth consumption across one's lifetime is the accumulation of assets in terms of physical capital. Agents cannot hold negative assets at any period. In other words, an agent faces a borrowing (or liquidity) constraint. There are two main reasons justifying this assumption: First, our desire for a careful numerical comparison of our results with those in the existing literature in which this assumption is a standard one. Second, constraining agents from borrowing against their future income induces an additional boost in (private) savings for precautionary purposes, as long as agents may be/remain unemployed with a positive probability. If we relaxed this constraint, the ability to borrow would lower agents' marginal propensity to save (for precautionary reasons). This would, in turn, render the effects of self-control and ability to borrow against future income highly correlated and hard to tell apart. As a result, the effect of social security on savings due to self-control would not be identifiable[8].

Since death is certain at T and there is no bequest motive, the borrowing constraint can be stated as:

$$\begin{cases} a_t \geq 0 & \text{for } t=1,...T-1; \\ a_t = 0 & \text{for } t=T. \end{cases}$$

Note the absence of a bequest motive in our economy. Allowing a bequest motive also

[7] Diamond *et al.* (2005)[11] analyze thoroughly the relationship between annuities and individual welfare. They show that full annuitization of wealth is optimal under certain conditions.

[8] In a recent paper, Rojas & Urrutia (2007) [32] show that adding an endogenous borrowing constraint reduces the welfare cost of social security.

changes the welfare implications of social security.[9]

If agents in this economy die before age T, their remaining assets will be distributed to all of the survivors in a lump-sum fashion. Let η denote the equal amount of accidental bequests distributed to all remaining members of the society:

$$\eta = \sum_t \sum_a \sum_e \mu_t \Lambda_t(a,e)(1 - s_{t+1}) a_t(a,e), \tag{3.8}$$

where $\Lambda(a,e)$ is the set of age dependent, time independent measures of agents.

Hence, we can write the budget constraint of an agent as follows:

$$a_t + c_t = (1+r)a_{t-1} + q_t + \eta \tag{3.9}$$

and

$$a_t + \check{c}_t = (1+r)a_{t-1} + q_t + \eta, \tag{3.10}$$

where r is the rate of return from the asset holdings.

3.2.5 Government

In our setting, the government's responsibility is limited to the task of administering the unemployment insurance and social security programs. The only constraint imposed on the government's behavior is to enforce self-financing of both the unemployment and social security programs. We restrict our attention to social security arrangements that are described by the pair (θ, τ_s). The self-financing conditions are as follows:

$$\tau_s \sum_{t=1}^{t^*-1} \sum_a \mu_t \Lambda_t(a, e=1) w \epsilon_t = \sum_{t=t^*}^{T} \sum_a \mu_t \Lambda_t(a,e) b \tag{3.11}$$

and

$$\tau_u \sum_{t=1}^{t^*-1} \sum_a \mu_t \Lambda_t(a, e=1) w \epsilon_t = \sum_{t=1}^{t^*-1} \sum_a \mu_t \Lambda_t(a, e=0) \phi w \epsilon_t. \tag{3.12}$$

[9] Fuster *et al.* (2002)[15] perform a welfare analysis of social security in a dynastic framework and show that steady-state welfare increases with social security.

3.2.6 An Agent's Dynamic Program

We suppose that the temptation function $v(.)$ is strictly increasing, i.e. a self-control agent is tempted to consume his entire wealth in each period. This implies that a self-control agent maximizes the second part of equation (3.1) by holding zero asset for the next period, i.e. setting $a_t = 0$ in equation (3.10). In this economy, the agent's state vector x contains the current asset holdings and the employment state. Hence, agent i's Bellman's equation for any arbitrary two-periods span can be written as follows:

$$W_i(x) = \max_c \{u(c) + \mathbf{1}_{\{i \in S_X\}} v(c) + \beta E_{s'} W(x')\} - \mathbf{1}_{\{i \in S_X\}} v((1+r)a + q + \eta) \tag{3.13}$$

subject to

$$a' + c = (1+r)a + q + \eta,\ a' \geq 0,\ a_0 \text{ is given,} \tag{3.14}$$

where $E_{s'}$ denotes the expectation over survival probabilities.

In the case of a self-contol agent who succumbs to temptation and consumes his entire wealth, the term

$$\mathbf{1}_{\{i \in S_X\}} v(c) - \mathbf{1}_{\{i \in S_X\}} v((1+r)a + q + \eta)$$

in equation (3.13) cancels out. When, however, he resists to temptation and consumes less than his wealth, he incurs a self-control cost at the amount of $v(c) - v((1+r)a + q + \eta)$. A self-control agent tries to balance his urge for current consumption $v(c)$ and long-term commitment utility $u(c) + \beta E_{s'} W(x')$.

3.2.7 Steady State Equilibrium

Given a set prescribing government policy $\{\theta, \phi, \tau_s, \tau_u\}$, a steady state recursive competitive equilibrium is a set of value functions $\{W_t(x)\}_{t=1}^{T}$, household's policy rules $\{a_t(x)\}_{t=1}^{T}$, time invariant measures of agents $\{\Lambda_t(x)\}_{t=1}^{T}$, wage and interest rate (w, r), and a lump sum distribution of accidental bequests η such that all of them satisfy the following:

- Factor prices (w, r) that are derived from the firm's first order conditions satisfy the equations (3.6) and (3.7).
- Given government policy set $\{\theta, \phi, \tau_s, \tau_u\}$, factor prices (w, r), and lump-sum transfer of accidental bequests η, an agent's policy rule $\{a_t(x)\}_{t=1}^T$ solves the agent's maximization problem (3.13) subject to the budget constraint (3.14).
- Aggregation holds:

$$K = \sum_t \sum_a \sum_e \mu_t \Lambda_t(x) a_{t-1}(x). \tag{3.15}$$

- The set of age-dependent, time-invariant measures of agents satisfies in every period t:

$$\Lambda_t(x') = \sum_e \sum_{a:a'=a_t(x)} \Pi(e', e) \Lambda_{t-1}(x), \tag{3.16}$$

  where Λ_1 is given.
- The lump-sum distribution of accidental bequests η satisfies the equation (3.8).
- Both the social security system and the unemployment insurance benefit program are self-financing i.e. satisfy the equations (3.11) and (3.12) respectively.
- The market clears:

$$\begin{aligned} & \sum_t \sum_a \sum_e \mu_t \Lambda_t(x)[a_t(x) + c_t(x)] \\ = \quad & Y + (1 - \delta) \sum_t \sum_a \sum_e \mu_t \Lambda_t(x) a_{t-1}(x). \end{aligned} \tag{3.17}$$

3.3 CALIBRATION

In this section, we briefly define the parameter values of our model. Each period in our model corresponds to a calendar year.

3.3.1 Demographic and Labor Market Parameters

Agents are born at a real life age of 21 (model age of 1) and they can live up to a maximum real life age of 85 (model age of 65). The population growth rate n is assumed to be equal to the average U.S. population growth rate between 1931-2003 which corresponds, on average, to 1.19% per year.[10] The sequence of conditional survival probabilities is the same as the Social Security Administration's sequence of survival probabilities for men in the year 2001. The mandatory retirement age is equal to 65 (model age 45).

In order to set the efficiency index, we choose the average of Hansen's (1993)[20] estimation of median wage rates for males and females for each age group. We interpolate the data by using the Spline Method and normalize the interpolated data to average unity. The employment transition probabilities are chosen to be compatible with the average unemployment rate in the U.S. which is approximately equal to 0.06 between 1948 and 2003.[11] The implied employment transition matrix assumes the following form:

$$\Pi(e, e') = \begin{bmatrix} 0.94 & 0.06 \\ 0.94 & 0.06 \end{bmatrix}.$$

3.3.2 Preference Parameters

We choose the values of preference parameters ρ, γ, λ and β in such a way that our model economy's capital-output ratio matches that of the U.S. economy.

In the case where the temptation function $v(.)$ is convex, we choose to follow Imrohoroglu *et al.*(2003)[24] and DeJong & Ripoll (2007)[4], in letting γ be centered at 2 with a standard deviation 1, i.e., $\gamma = 2_{(1)}$. In our benchmark calibration, we initially set $\gamma = 2$, and then check the robustness of our results by letting $\gamma = 3$. Holding γ constant, we choose different values of ρ *a priori,* and calculate the corresponding λ in such a way that $u(.) + v(.)$ stays a strictly concave function. For every triple γ, ρ and λ, we search over the values of β that deliver the capital-output ratio which is compatible with its empirical counterpart. We

[10] The population data was obtained from the U. S. Census Bureau .[37]

[11] The unemployment data are taken from the U. S. Department of Labor.[36]

assume that the social security replacement ratio is 40% and the unemployment replacement ratio is 25% during our search.

When the temptation function is concave, we follow DeJong & Ripoll (2007)[4] and set $\lambda = 0.0786_{(0.056)}$

3.3.3 Production Parameters

The parameters describing the production side of the economy are chosen to match the long-run features of the U.S. economy. Following Imrohoroglu *et al.*(2003)[24], we set the capital share of output α equal to 0.310 and the annual depreciation rate of physical capital equal to 0.069. The rate of technological progress g is assumed to be equal to 2.1%, which is the actual average growth rate of GDP per capita taken over the time interval from 1959 to 1994 (Hugget & Ventura (1999)[21]). The technology parameter A, can be chosen freely. In our calibration exercises, it is set equal to 1.01. All per capita quantities are assumed to grow at a constant rate g.

3.3.4 Government

We set the unemployment insurance replacement ratio (ϕ) equal to 25% of the employed wage. In the benchmark case, we set the social security replacement ratio (θ) equal to 40%. Alternatively, we can choose the payroll tax rate (τ_s) and the unemployment insurance tax rate (τ_u) instead of the replacement ratios. Since the social security and the unemployment insurance benefits are self-financing, calibrating the replacement ratios will automatically pin-down the tax rates. This holds true because agents inelastically supply one unit of labor whenever they find an opportunity to work, and changes in tax rates do not affect their supply of labor.[12]

[12] However, if I calibrate a model featuring labor-leisure choice, tax rates should be used instead of replacement rates.

3.3.5 Solution Method

We use discrete-time, discrete-state optimization techniques to find a steady state equilibrium of our model economy by using the aforementioned parameter values. Our solution method designedly resembles those of previous studies.[13]

To this purpose we create a discrete set of asset values that contains 4097 points. The lower bound and upper bound of the set is chosen in such a way that the set boundaries never bind[14]. While the state space for working age agents comprises 4097×2 points, the state space for retired agents consists of only 4097×1 points (since there is no state transition after j^*).The discrete set of the control variable (consumption) contains 4097×1 points.

Our solution algorithm prescribes that we start with a guess about the aggregate capital stock and the level of accidental bequests and then proceed to solve agents' dynamic program by backward recursion. The time-invariant, age-dependent distribution of agents is obtained by forward recursion. After each loop, we calculate the new values for the accidental bequests and the capital stock. If the difference between the initial values and the new values exceeds the tolerance value, we start a new loop with the new values.

This procedure continues until we find values for the accidental bequests and the capital stock that are sufficiently close to their beginning-of-loop values.

3.4 RESULTS

In order to assess these welfare implications of unfunded social security we use a "compensating variation" measure, which is defined as the percentage by which consumption must be increased to compensate for the decrease in welfare generated by the presence of social security. Although this welfare decrease may be due to the distortions that unfunded social security inflicts on capital accumulation and labor supply, in what follows we will outline the conditions under, as well as the extent to, which the presence of self-control preferences

[13] See Auerbach & Kotlikoff (1987)[3] Imrohoroglu *et al.* (1995[22] and 2003[24]) and Hugget & Ventura (1999)[21].

[14] In particular, the lower bound is equal to 0 and the upper bound is equal to 60 times greater than the annual income of an employed agent.

offsets the above effects.

We consider our work as complementary to Kumru & Thanopoulos (2008)[29] who, however, examine exclusively the consequences of the *severity* of the self-control problem in an economy populated exclusively by self-control agents.

In contrast, we allow for *mixed* economies where agents can have either CRRA or self-control preferences, and this allows us to examine the extent to which "temptation prevalence" *across* individuals and temptation severity/intensity *within* individuals are substitutes in offsetting the adverse effects of unfunded social security.

Therefore, in that sense our model nests Kumru & Thanopoulos (2008)[29] as a special case corresponding to the case of "All-Self-Control" preferences presented below. In this section we report the results obtained by using the linear benefit function[15] instead of the logarithmic one. This is due to our wish to perform exact numerical comparison with our main reference paper, but also due to the fact that the results obtained by considering a concave benefit function are not qualitatively different, while the discrepancy in their respective numerical values is negligible. We provide an intuitive explanation for this observation here[16]:

In our model an individual works 94% of their lifetime. The working period is 45 years. Hence, an individual works on average 42 years. Since individual wage earnings do not change significantly year to year, averaging over the 35 highest years (as the Social Security Administration mandates with regards to the calculation of the average indexed monthly earnings) almost amounts to averaging over 45 years. However, if the duration of unemployment were sufficiently long, e.g., if an individual remained unemployed for 15 years and worked for 30 years, then averages in this case would matter. We don't allow for similarly long spans of unemployment in our model. In fact, in the absence of earning shocks in our model (which would give rise to much higher earnings heterogeneity), the degree of concavity of the benefits function becomes immaterial, even locally.

In the following subsections, we first examine the two extreme cases in our model, namely, the case where all agents feature CRRA preferences (of a given parametrization), followed

[15] As in the section outlining the budget constraints of this model.

[16] All numerical values from the use of the logarithmic benefit function are available upon request.

by the case where all agents have self-control preferences of a given parametrization featuring modest intensity of self-control. Naturally, we examine mixed economies in the third subsection.

3.4.1 The case of all-CRRA Preferences ($X = 0$)

Our first calibration makes use of CRRA preferences and our economy is calibrated in such a way as to reach a capital-output ratio of approximately 2.5 under the assumption of a 40% social security replacement rate. The steady state features of this economy under alternative social security replacement rates are displayed in Table 8. Our findings in this case are congruent with those in Imrohoroglu *et al.* (1998, 2003) and in Kumru & Thanopoulos (2008) [29] . Consumption, capital and output reach their highest levels when the social security replacement rate is zero, that is, under complete absence of unfunded social security.

The main intuition is that, despite the fact that social security provides insurance against life-time uncertainty (due to missing annuities market) and risk sharing among generations, its negative effect on capital accumulation prevails over these benefits and hence, makes it undesirable.[17] Table 8 provides evidence for that fact.

It is worth noting that the level of consumption required to compensate the consumers (depicted in the last column of the table) increases in a disproportionate manner compared to a given increase in the social security replacement rate (θ).

3.4.2 The case of all-self-control Preferences ($X = 1$)

This section draws heavily from Kumru & Thanopoulos (2008) [29] . We assume that agents feature self-control preferences with a convex temptation function. In order to demonstrate the quantitative significance of the temptation parameter and its economic meaning, we calculate the quantity of steady state consumption which would be given up by an agent in order to escape from temptation. As in DeJong & Ripoll (2007) [4], we obtain the value x such that

[17] Since there is no labor-leisure decision in our model, social security has an effect only on capital accumulation (saving).

$$u(c^* - x) = u(c^*) + v(c^*) - v(\check{c})$$

where c^* is the steady state value of the agent's actual consumption and $\check{c}$ is the steady state value of temptation consumption. To isolate the effect of λ, the model is calibrated under zero social security replacement rate and all other parameters remain fixed at their CRRA case while ρ is chosen equal to 2. By increasing λ from 0 to 0.001, we observe that agents would be willing to forgo as much as 4.82% of their steady state consumption in order to eliminate temptation[18].

Note the significant impact that a slight departure from the CRRA preference specification entails for welfare. It underscores the welfare reducing role temptation (and the induced cost of self-control) plays in my model. Nonetheless, at the same time it validates the intuition also developed in Kumru & Thanopoulos (2008) [29] , namely, that in the presence of self-control problems, unfunded social security may not be as detrimental to welfare as it had generally been argued in the literature. In order to see this, in my first calibration I use the same parameter values for β in order to measure the impact of temptation on savings, under a 40% social security replacement rate. This example is a counter-factual in the sense that it does not yield capital output ratio around 2.5, but it serves as a device to better demonstrate the effect of self-control preferences on savings.

Tables 9, 10[19] show the steady state of an economy with self-control preferences under 40% replacement ratio. In particular, Table 9 is constructed holding all parameters of the utility function fixed in their CRRA values, except for λ, which is the parameter I vary. The value of parameter λ measures the strength of temptation towards current consumption. Higher values for this parameter correspond to higher cost of exerting self-control. We notice that all variables but the interest rate decrease as λ increases (i.e. as we depart from the CRRA case). In particular, the capital-output ratio decreases showing that the increase of λ

[18] DeJong and Ripoll (2007) [4] report the analogous to their environment value as slightly above 5% of the steady state consumption when the scale of the temptation parameter is increased from 0 to 0.00286.

[19] Table 10 is constructed holding β, and γ in their CRRA values and keeping λ fixed at 0.00009 under 40% social security replacement rate. Now, we only vary ρ which is a measure of the consumers' willingness to substitute current temptation consumption for future one. The higher ρ is the more the consumer prefers early to late temptation consumption which actually makes the self-control cost even more severe. This, in turn causes further dissavings and eventually lower steady state consumption for any value of λ.

triggers a process of dissavings. This process deprives the economy from future consumption capabilities. The latter point is congruent with what we observe in the consumption pattern as λ varies.

3.4.3 The case of Mixed Preferences $(0 < X < 1)$

For the sake of comparison with our previous findings I will follow Kumru & Thanopoulos (2008) [29] and consider only self-control preferences with a convex temptation function. In terms of the model, this is summarized by setting the value of the parameter ρ equal to 2. With regards to the gradual introduction of self-control agents in the economy, my protocol consists in setting the initial value of X equal to 0.95 and progressively decreasing it (i.e. increasing the measure of CRRA agents in the economy) by increments of 0.05.

In what follows in this subsection, consumption refers to the steady state consumption of self-control agents in the economy unless explicitly stated otherwise.

Initially, for each intermediate value of X, I calculate the quantity of steady state consumption that a self-control agent would be willing to give up in order to escape from temptation[20]. As it was the case in the "all-self-control" economy of the preceding section, I isolate the effect of λ by calibrating the model under zero social security replacement rate ($\theta = 0$), $\rho = 2$, while keeping all other parameter values fixed at their CRRA case values. The results are shown in Table 11[21].

The last row of Table 11 displays the fraction of steady state consumption that a self-control agent is willing to relinquish in order to eliminate temptation. So, while in the case where $X = 1$ and λ increases from 0 to 0.001 the percentage of steady state consumption that self-control agents would be willing to give up is equal to 4.82% , the analogous percentage of a self-control agent in an economy populated only at a 50% ($X = 0.5$) by self-control agents is only equal to 2.31% and, not surprisingly, when the two thirds of the population have self-control preferences, we can see that the cost of temptation is as high as 4.17% of their steady state consumption.

[20] This is a similar to the case of $X = 1$ exercise with the difference that we now consider several intermediate values of $X \in [0, 1]$.

[21] In Table 11, all consumption numbers refer to self control agents with $\lambda = 0.001$.

This is due to the presence of CRRA agents in the economy who considerably halt the rate of capital decumulation in the economy and hence render the cost of temptation elimination more affordable to their self-control counterparts. An increase in the percentage of self-control individuals in the economy triggers a process of dissavings, in the same manner as, for a given X, an increase in λ does. This process deprives the economy from future consumption opportunities, something that becomes apparent in the consumption pattern depicted in all of the folowing Tables .

Next, we focus on the effects of a positive λ on consumption and welfare. For each intermediate value of X we repeat the analysis we did in the previous section regarding the case where $X = 1$. In particular, we initially set $\lambda = 0.00009$, $\rho = 2$ under a 40% social security replacement rate and hold β, and γ in their CRRA values. We start decreasing X by steps of 0.05 and report a summary of our results in Table 12.

We observe that the steady state consumption of self-control agents in the economy drops as the measure of self-control agents increases. However, *for a given* λ, this happens to a lesser degree, compared to the case where the economy is populated exclusively by self-control individuals.

Our next task is to perform the same experiment, for a higher level of λ, equal to 0.0004 (Table 13), and $\lambda = 0.001$ (Table 14).

Again, similar to Table 12 results obtain here as well: *for a given* λ, the drop in consumption is not as abrupt as it is the case in an "all-self-control" economy. Another angle to see this, is the amount of steady state capital in the economy[22], which turns out to be higher for a mixed economy, eventually allowing for more production/consumption opportunities.

Hence, for a given intensity of self-control, the introduction of self-control agents in the economy has both costs and benefits. The costs are associated with the acceleration of the process of dissavings in the economy while the benefits stem from the fact that an increasing number of agents are not adversely affected by unfunded social security to the extent that CRRA agents are.

A more careful look at the above tables allows us to obtain an estimate about the combined effects of λ and X. In particular, I can now consider X *as given* and see what

[22] Shown in the second row of Tables 12-13-14.

is the effect of an increase in the value of λ from 0.0009 (Table 12, entry $(1,3)$) to 0.001 (Table 14, entry $(1,3)$) on steady state consumption when e.g. $X = 0.5$, and compare it with the variation in steady state consumption due to the same numerical increase in λ shown in Kumru & Thanopoulos (2008) [29] (Tables 12 and 14, entries $(1,5)$). In Kumru & Thanopoulos (2008) [29] , consumption drops by 3.69%, while in our $X = 0.5$ scenario, it only drops by 2.82%[23].

So, the presence of CRRA agents reduces the cost in terms of lost steady state consumption for their self-control counterparts. Moreover, the aforementioned results confirm our intuition that the smaller the number of self-control agents, the more they benefit from CRRA agents' consumption smoothing and capital accumulation behavior.

To see this, in the Table 15 that follows I report consumption values for CRRA agents using parameters corresponding to those used in Table 14. Recall that in order to isolate the effect of injecting self-control agents of a *given* self-control intensity ($\lambda = 0.001$) in the economy, the model is calibrated under the assumption that $\theta = 0.4$, $\rho = 2$, and all other parameter values remain fixed at their CRRA values.

Table 15 shows that the consumption levels of CRRA agents are penalized even more than the corresponding levels of self-control agents shown in Table 14. This is not a surprising result: we already know (by Kumru & Thanopoulos (2008) [29] and Imrohoroglu *et al.*(2003) [24]) that CRRA agents were expected to be worse off under any scenario involving unfunded social security relative to a scenario featuring the absence of social security. What is new here is that CRRA agents seem to "subsidize" self-control agents' welfare. To see that, first note that more capital in the economy gives rise to higher wages for all agents. However, this also implies a lower return on capital, so the net effect is ambiguous. But recall also that we have imposed an accidental bequest condition which means that at any point in time, a CRRA agent who dies will leave a higher "accidental bequest" than a self-control agent. Hence the total amount of capital that returns to the economy through this channel comes predominantly from CRRA agents, but is redistributed equally to all surviving agents regardless of type. Thus, the ambiguity is eliminated.

In summary, my results indicate that the process of "injecting" self-control agents in the

[23] For $X = 0.1$ the drop is equal to 0.96% , and for $X = 0.95$ the drop is equal to 3.28%

economy has, to some extent, commensurable consequences as the process of increasing the severity of temptation in an economy populated only by self-control agents, as in Kumru & Thanopoulos (2008) [29] . When agents face temptations, social security accomplishes at the same time to reduce the cost associated with the exertion of self-control and consequently to partially offset its adverse effect on welfare. On the other hand, the simultaneous existence of a CRRA subset in the population slows down the proccess of capital decumulation and mitigates welfare reduction for self-control agents.

3.5 CONCLUSIONS

There is an open question in the public finance literature with regards to the conditions under which unfunded social security can be beneficial for some individuals in the economy, if at all. Early research, both theoretical and empirical, mostly with agents featuring CRRA preferences, argued that PAYGO social security can only be detrimental for welfare, a result that comes in stark contrast with the actual prevalence of unfunded social security systems across the world.

Recent attempts to reconcile empirical observation with economic theory, showed that alternative preference specifications may substantially mitigate the welfare reducing potential of unfunded social security even when it comes through the exact same channels identified in the literature that advocates the elimination of social security. Notably, when individuals feature self-control preferences it was shown that PAYGO social security may even be welfare improving under certain conditions.

In this paper I built on the theory advocating the conditional usefulness of unfunded social security in an environment where all, or some individuals in my model economy face the temptation to consume their entire wealth.

My contribution to the literature consists in addressing and quantitatively assessing the prevalence of self-control preferences across the population in the economy and not only in examining the impact of the severity of temptation within individuals (intensity of self-control). I allow therefore, for *mixed* economies where agents can have either CRRA or self-control preferences, and this allows me to examine the extent to which "temptation prevalence" *across* individuals and temptation severity/intensity *within* individuals are substitutes in offsetting the adverse effects of unfunded social security. In that sense, my model nests previous relevant models (Kumru & Thanopoulos (2008) [29]) and provides a richer perspective over the mechanics of the interaction between unfunded social security and self-control preferences.

I quantify the cost of temptation in an environment where both CRRA and self-control agents co-exist when for several values of the temptation prevalence parameter, X. Not surprisingly, this cost of temptation, namely, the amount of consumption that self-control agents would be willing to relinquish in order to eliminate temptation ranges from 0.46% of their steady state consumption when the CRRA agents constitute 90% of the population, to 4.17% when they are only at 5%. Naturally, those percentages are lower than the corresponding level under the assumption of an economy populated exclusively by self-control agents ($X = 1$) which is 4.82%.

I find that indeed, in an economy featuring both CRRA and self-control individuals, social security can be welfare improving for the latter, provided that those agents face sufficiently severe temptation. Moreover, the presence of CRRA agents lowers the threshold of self-control intensity required for social security to benefit self-control individuals in an "all-self-control" environment. This is due to the fact that the presence of CRRA agents slows down the capital decumulation process in the economy and mitigates the welfare cost that temptation elimination entails for their self-control counterparts. Finally, I also find that this entails an additional welfare burden to CRRA agents who become worse off not only due to the existence of a forced savings mechanism such as social security but also because of the presence of their self-control counterparts in the economy. Note that my results are not sensitive to the unfunded character of social security; they are essentially driven by the underlying forced savings such a mechanism entails.

As noted, one of the most important aspects of my generalization consists in the fact that a setting featuring type heterogeneity is open to more general interpretations pertaining to political economy and hence may naturally accomodate new modeling features. An example includes a government featuring an objective function with preferences over the welfare of different types of agents; in turn, these preferences could be an endogenous function of the voting preferences of the agents' types. The trade offs arising from such a setting could provide an additional explanation as to why unfunded social security is so prevalent throughout the world. Note that such an explanation is not possible in an environment that abstracts from type heterogeneity, such as that analyzed by Kumru & Thanopoulos (2008) [29] .

Table 7: Parameter Values of The Benchmark Calibration

Demographics	
Maximum possible life span T	65
Obligatory retirement age t^*	45
Growth rate of population n	1.19%
Conditional survival probabilities $\{s_t\}_{t=1}^{T}$	U.S. 2001
Labor efficiency profile $\{\epsilon_j\}_{t=1}^{t^*-1}$	Hansen (1993)
Production	
Capital share of GDP α	0.310
Annual depreciation of capital stock δ	0.069
Annual per capita output growth rate g	2.1%
Markov Process for employment transition Π	$\begin{bmatrix} 0.94 & 0.06 \\ 0.94 & 0.06 \end{bmatrix}$
Preferences	
Annual discount factor of utility β	0.998
Scale factor of the temptation utility λ	0.000375
Risk aversion parameter γ	2.0
Risk loving parameter ρ	2.0
Government	
Unemployment insurance replacement ratio ϕ	0.25
Social security replacement ratio θ	$[0, 1]$
Compulsory deposit rate κ	0.027

Table 8: (beta=0.978, gamma=2, lambda=0)

θ	Y	K	C	K/Y	r	w	CV (%)
0	1.239	3.308	0.971	2.671	0.068	1.088	0.000
0.10	1.225	3.187	0.968	2.602	0.072	1.076	1.300
0.20	1.212	3.081	0.965	2.542	0.075	1.065	2.637
0.30	1.199	2.978	0.962	2.483	0.078	1.054	4.043
0.40	1.188	2.892	0.960	2.434	0.081	1.044	5.460
0.50	1.178	2.814	0.957	2.388	0.084	1.035	6.913
0.60	1.168	2.738	0.954	2.344	0.087	1.026	8.416
0.70	1.159	2.668	0.952	2.301	0.090	1.018	9.951
0.80	1.150	2.605	0.949	2.264	0.093	1.011	11.520
0.90	1.143	2.548	0.947	2.230	0.095	1.004	13.112
1	1.135	2.493	0.944	2.197	0.098	0.997	14.752

Table 9: (beta=0.978, gamma=2, rho=2)

λ	Y	K	C	K/Y	r	w
$CRRA$	1.188	2.892	0.960	2.434	0.081	1.044
$\lambda = 0.0004$	1.120	2.386	0.939	2.132	0.103	0.984
$\lambda = 0.001$	1.067	2.046	0.918	1.917	0.122	0.938

Table 10: (beta=0.978, lambda=0.00009)

λ	Y	K	C	K/Y	r	w
$CRRA$	1.188	2.892	0.960	2.434	0.081	1.044
$\rho = 2$	1.168	2.735	0.954	2.342	0.087	1.026
$\rho = 3$	1.075	2.091	0.922	1.946	0.119	0.944

Table 11: (beta=0.978, theta=0, rho=2, lambda: 0 to 0.001)

$X:$	$CRRA$	0.1	0.3	0.5	0.7	0.95	SC
C	N/A	0.9552	0.9506	0.9459	0.9332	0.9249	0.9201
%	0	0.461	1.175	2.309	4.173	4.744	4.821

Table 12: (beta=0.978, lambda=0.00009)

$X:$	$CRRA$	0.1	0.5	0.95	SC
C	N/A	0.9622	0.9588	0.9543	0.9541
K	2.892	2.878	2.813	2.748	2.735

Table 13: (beta=0.978, lambda=0.0004)

$X:$	$CRRA$	0.1	0.5	0.95	SC
C	N/A	0.9591	0.9437	0.9402	0.9391
K	2.892	2.805	2.609	2.487	2.391

Table 14: (beta=0.978, lambda=0.001)

$X:$	$CRRA$	0.1	0.5	0.95	SC
C	N/A	0.9529	0.9317	0.923	0.9189
K	2.892	2.701	2.423	2.246	2.046

Table 15: (beta=0.978, theta=0.4, rho=2, lambda: 0 to 0.001)

$X:$	$CRRA$	0.1	0.3	0.5	0.7	0.95	SC
C	0.9642	0.9432	0.9345	0.9198	0.9089	0.8817	N/A

4.0 SOCIAL SECURITY REFORM AND TEMPTATION

4.1 INTRODUCTION

[1]There is an abundance of studies related to the importance of social security and its impact on welfare. The primary reason for this is its dramatically growing scale which triggers renewed academic debate regarding the optimal allocation of the available resources. This controversy stems from the huge monetary burden that the mere presence and administration of a social security system entails for the society and the associated budget implications: Old age, disability, unemployment and health insurance policies have evolved into the most expensive items on government budgets.

Most of the studies that seem to emerge as a direct or implicit offspring of this debate focus on the welfare implications of alternative social security schemes in an economy. In the very core of this debate one can clearly identify the dilemma between an "unfunded" (Pay-As-You-Go) versus a "funded" social security system. In an unfunded system, resources are transferred statically from the working population to the concurrent retirees (inter-generational transfers). In contrast, a funded system prescribes a dynamic allocation of resources within the same generation (inter-temporal within the same generation transfers). While both systems rely on an external institution (e.g. government) in order to be implemented, their different logic and mechanics eventually induce entirely different risk sharing properties as well as savings incentives. Therefore, their welfare implications may significantly differ because of this difference.

Population ageing as a result of the declining population growth rate and decreasing birth rate have challenged enormously the sustainability of a PAYG system and called for a

[1]This chapter is based on joint work with Cagri S. Kumru

minimization of the fiscal burden through tax reforms and benefits restructuring. As a result, there are numerous studies suggesting alternative institutional arrangements that could be more robust to adverse demographic shocks. However, as much as converting an unfunded system to a fully (or partially) funded one may seem a plausible solution, in most cases the transition costs associated with such a reform make it prohibitively costly (De Nardi *et al.*(1995) [5]).

The welfare implications of social security are well identified in the relevant literature.[2] Several studies (e.g. Storesletten *et al.* (1999) [32]) comparing different social security systems typically compare welfare across alternative steady states, each corresponding to a stationary equilibrium with a different social security system. Focusing only on unfunded social security, Imrohoroglu *et al.*(1995)[24] emphasize the detrimental effects that such an arrangement has to the overall welfare in a country.

However, all the above studies ignore alternative preference specifications that may be binding in several cases: Imrohoroglu *et al.*(2003) [24] use time-inconsistent preferences and later Kumru & Thanopoulos (2008) [29] use self-control preferences to highlight that in a context of unfunded social security welfare may be critically affected by the preference specification. Furthermore, the experimental economics literature documents some evidence that preferences show some degree of time inconsistency and agents suffer from temptation, a fact that further supports the results in the aforementioned studies.[3]

In this study, we would like to quantitatively assess the welfare implications of the reform of the current *unfunded* social security system into a partially funded or fully privatized one, under the assumption that individuals face self-control problems. We proceed by assessing in terms of welfare a hybrid (partially funded) social security system under the alternative hypotheses of self-control or CRRA preferences. The apparatus by means of which we model departures from unfunded social security is a Personal Security Account (PSA). Within that class of "funded" models, we investigate two competing scenarios involving PSAs: one *without* annuitization and an alternative one that prescribes a mandatory annuitization of

[2]The interest in the welfare implications of a social security system has been sparked with the seminal work of Diamond (1965) [7]. The first quantitative model that assessed the welfare implications of the system was created by Auerbach & Kotlikoff (1987) [3].

[3]For a recent overview of studies that provide evidence that individuals indeed exhibit bias toward immediate gratification see Frederick et al. (2002) [14].

retirement benefits.

Moreover, in order to capture our agents' temptation towards current consumption, our model economies make use of the preference structure pioneered by Phelps & Pollak (1968) [31] and further elaborated by Gul & Pesendorfer (2004a) [17] to model self-control issues. Gul & Pesendorfer (2004a) [17] identified a particular class of utility functions that provide a time-*consistent* model suitable for addressing the preference reversals that motivated the time inconsistency literature. The key theme here is that self-control preferences assume that agents maximize a utility function that is a 'compromise' between the standard utility (or 'commitment' utility) and a 'temptation' utility. The conflicting ways by which agents derive utility in this setting, is the device through which the trade off between the temptation to consume on the one hand, and the long-run self interest of the agent on the other is captured. The main benefit is that self-control preferences remain perfectly time-consistent and, contrary to time-inconsistent preferences, allow agents in our model to commit.

With the exception of the aforementioned difference in the specification of preferences, our model specification follows that of Imrohoroglu *et al.*(2003) [24] and Fuster *et al.*(2005) [16]. Furthermore, our economic environment features uninsurable individual income shocks, borrowing constraints and missing annuity markets.

We aim to contribute to the debate on the reform of social security. Our augmented model allows us to look at the welfare gains or losses due to the reforms from a different angle. In particular, it allows us to assess the welfare-enhancing potential of mandatory savings versus mandatory annuitization of accumulated PSA wealth at retirement. For the sake of comparability of our results, the particular specification of those alternative policies is purposely chosen to match the proposals analyzed in the literature (Storesletten *et al.* (1999)[32] and Fuster *et al.*(2005)[16]), as well as those featuring in the reform recommendations made by the 1997 Advisory Council on Social Security.

Moreover, changing the preference structure is very important for theoretical purposes because it enhances our understanding of the mechanics of similar models in the literature by providing an additional channel through which capital accumulation is distorted. Equally importantly, an augmented preference structure is also essential for providing a comprehensive comparison framework for policy makers in their evaluation of various proposals. As shown in Imrohoroglu *et al.*(2003) [24] and Kumru & Thanopoulos (2008) [29] the presence of slightly far-sighted or current consumption favored agents changes substantially the welfare implications of the system.

Our results in this paper highlight the important role that self-control preferences play, especially with regards to the kind of reform that could generate the highest welfare. While if we restrict our analysis to CRRA preferences our findings initially confirm those in other studies (e.g., Fuster *et al.*(2005)[16]), i.e., we also find that the complete elimination of any social security is the policy that maximizes welfare, we come across a drastic alteration of our results when self-control preferences are introduced: the elimination of social security still remains, alas marginally, the most desirable policy from the point of view of welfare, it is nevertheless not *as* desirable with respect to our alternative reform scenarios as it is under CRRA preferences. That is, the presence of self-control preferences renders the elimination scenario less appealing compared to the examined alternatives.

A noteworthy result is that, unlike Fuster *et al.*(2005)[16], a personal security account with mandatory annuitization of retirement benefits performs worse than a reform scenario that allows a PSA without annuitization. This is due to the fact that under our parametrization (required to converge to the basic stylized facts of the US economy) the stream of benefits that mandatory annuitization entails exposes individuals with self-control preferences to a respective stream of repeated costs to resist temptation. In our setting, the shape of the temptation function is such that the total temptation cost from a stream of benefits exceeds the temptation cost from a lump-sum payout at the time of retirement.

Along the same lines, our research indicates that the clear-cut advantage of the elimination scenario so salient under CRRA preferences fades away as self-control becomes gradually more severe. Our robustness tests confirm this finding: in the case of severe temptation the CRRA pecking order is completely reversed, and the elimination of social security becomes the least desirable scenario.

Under the light of the above findings, any reform proposal regarding the social security system should consider departures from standard preferences to preference specifications suitable for dealing with preference reversals.

4.2 A MODEL OF SOCIAL SECURITY

The model we consider in this section is quite standard in the social security literature. In particular, our model follows that of Imrohoroglu *et al.*(2003) [24].

4.2.1 The Environment

We consider a stationary overlapping generations economy in discrete time. Each period a new generation is born which is modeled to be n percent larger than the previous generation. Agents face lives of uncertain duration and some live through the maximum possible life span, denoted by J. At any given time j within their life-span, all agents have a (time-invariant) conditional probability $s_j \in (0,1)$ of surviving from age $j-1$ to j, conditional on having survived up to age $j-1$. Our stationary population assumption implies that age j agents constitute a fraction μ_j of the population at any given date. The cohort shares $\{\mu_j\}_{j=1}^{J}$ are given by

$$\mu_{j+1} = \frac{\mu_j s_{j+1}}{1+n}$$

while their sum is normalized to 1.

4.2.2 Preferences

Agents in our economy feature self-control preferences. That is, their preferences are such that in every period they induce a temptation to consume their entire wealth. Resisting temptation gives rise to a self-control cost; note that the latter feature is absent in models with CRRA and quasi-hyperbolic preferences. We follow Gul & Pesendorfer (2004a) [17] and DeJong & Ripoll (2007) [4] and proceed to model self-control preferences recursively.

Let $W(x)$ denote the maximized value of the expected discounted objective function with state x. The utility function of an agent is as follows:

$$W(x) = \max_{c}\{u(c) + v(c) + \beta EW(x')\} - \max_{\check{c}} v(\check{c}) \tag{4.1}$$

where E is the expectation operator; $u(.)$ and $v(.)$ are von Neumann-Morgenstern utility functions; $0 < \beta < 1$ is a discount factor; c represents the "commitment" consumption; $\check{c}$ is the "temptation" consumption; and x' denotes next period state variable. As in the section above, $u(.)$ represents the momentary utility function and $v(.)$ represents temptation. In particular, $v(c) - \max_{\check{c}} v(\check{c})$ denotes the disutility from choosing consumption c instead of $\check{c}$. The concavity or convexity of $v(.)$ turns out to be very important for our analysis.[4]

The momentary utility, convex temptation and concave temptation functions are assumed to take the following forms respectively:

$$u(c) = \frac{c^{1-\gamma} - 1}{1-\gamma} \tag{4.2}$$

$$v(c) = \lambda \frac{c^{\rho}}{\rho} \tag{4.3}$$

$$v(c) = \lambda u(c) \tag{4.4}$$

In the specification above, higher values of the scale parameter $\lambda > 0$, imply an increase in the share of "temptation" utility, i.e. a higher λ increases the importance of current consumption for an agent. The momentary utility function $u(.)$ is a standard Constant Relative Risk Aversion (CRRA) form, $\gamma > 0$ measures the degree of relative risk aversion (and $1/\gamma$ the inter-temporal elasticity of substitution).

4.2.3 Production Function

Firms have access to a constant returns-to-scale Cobb-Douglas technology that produces output (Y_t) by using labor input $(L_t = 0.94 \sum_{j=1}^{j^*-1} \mu_j \epsilon_j\ )$,[5] and capital input (K_t) which is rented from households:

$$Y_t = F(K_t, L_t) = A_t K_t^{\alpha} L_t^{(1-\alpha)} \tag{4.5}$$

[4] Notice that if $v(.)$ is convex, we need to make sure that $v(.)+u(.)$ is stricly concave. In particular, $\gamma > 0$, $\rho > 1$ and $0 < \lambda < \gamma/(c^{\gamma+1}c^{\rho-2})$ guarantee that $u(.)$ is concave, $v(.)$ is convex and $u(.) + v(.)$ is strictly concave. When $v(.)$ is concave, one should show that $W(.)$ is strictly concave.

[5] j^* denotes the compulsory retirement age and ϵ_j denotes the efficiency index of an age j agent.

where A_t represents the state of technology; $\alpha \in (0, 1)$ is the capital's share of output. Defining the capital-labor ratio as $\frac{K_t}{L_t}$, we can write the production function in the intensive form as follows:

$$y_t = f(k_t) = A_t k_t^{\alpha}$$

The technology parameter A_t grows at a constant rate g and capital depreciates at a constant rate δ. Competitive firms in this economy maximize their profits by setting the real rate of return from asset holdings r and the real wage rate w according to the following:

$$r_t = A_t \alpha k_t^{\alpha-1} - \delta \tag{4.6}$$

$$w_t = A_t(1-\alpha)k_t^{\alpha} \tag{4.7}$$

Since we are concerned only with the behavior of steady state equlibria , we will henceforth drop the time subscript for the rest of the analysis.

4.2.4 Budget Constraints

The exogenously given mandatory retirement age is j^*. Agents who are younger than age j^* face a stochastic employment opportunity at each period $j < j^*$. Individuals who find an employment opportunity, supply inelastically one unit of labor.[6] We denote the employment state variable by $e \in \{0, 1\}$ where 0 and 1 denote unemployment and employment states respectively. Furthermore, we postulate that the employment state follows a first-order Markov process. The transition probability distribution between the current employment state e and next period's employment state e' is represented by the 2×2 matrix $\Pi(e', e) = [\pi_{k'k}]$ where $k', k = 0, 1$ and $\pi_{k'k} = \Pr\{e' = k' | e = k\}$.

An employed ($e = 1$) agent earns $w\epsilon_j$ where w denotes the wage rate per efficiency unit of labor in terms of the consumption good and ϵ_j denotes the efficiency index of an age j agent. If, on the other hand, an agent is unemployed ($e = 0$), he receives an unemployment

[6] Adding labor-leisure choice into the model requires the modification of preferences in a way that agents are not only tempted by current consumption but also by current leisure.

insurance benefit equal to a fraction ϕ of the wage of an employed agent, resulting in the amount $\phi w \epsilon_j$; ϕ is the unemployment insurance replacement ratio.

The disposable (after-tax) income of an agent at age j can be written as

$$q_t = \begin{cases} (1-\tau_s-\tau_u)w\epsilon_j & \text{for } j=1,2,...,j^*-1, \text{ if } e=1; \\ \phi w \epsilon_j & \text{for } j=1,2,...,j^*-1, \text{ if } e=0; \\ b_j & \text{for } j=j^*, j^*+1,...,J. \end{cases}$$

where τ_s and τ_u represent the social security payroll tax rate and the unemployment insurance tax rate respectively.[7]

We assume away any private insurance markets against unemployment risk and private annuities' markets against the risk of outliving one's own resources[8]. The only available device to smooth consumption across one's lifetime is the accumulation of assets in terms of physical capital. Agents cannot hold negative assets at any period.[9] Since death is certain at J and there is no bequest motive, the borrowing constraint can be stated as:[10]

$$\begin{cases} a_j \geq 0 & \text{for } j=1,...J-1 \\ a_j = 0 & \text{for } j=J \end{cases}$$

[7] Note that in the US social security system both retirement and unemployment benefits are taxed (Diamond & Orszag (2005) [11]). For the sake of less cumbersome notation we refrain from incorporating those taxes in our analysis.

[8] Although a market for private annuities exists in the U.S. it is nevertheless very thin (Feldstein & Liebman (2002) [13], Imrohoroglu *et al.* (1995) [22]). Hence, our assumption seems innocuous. In our model, social security partially fulfills the role of the missing annuities' market (it can be considered as mandatory annutization). Diamond *et al.* (2005) [11] analyze thoroughly the relationship between annuities and individual welfare. They show that full annuitization of wealth is optimal under certain conditions.

[9] In other words, an agent faces a borrowing (or liquidity) constraint. Given the size of private credit markets, this assumption may not seem so innocuous. There are two main reasons behind this assumption: First, we would like to engage in a careful comparison of our results with those of the existing social security literature where this assumption is a standard one. Second, the fact that agents are not allowed to borrow against their future income induces an additional boost in (private) savings for precautionary purposes, since they may become/remain unemployed with a positive probability. It would be a fair question to explore the consequences of alleviating this constraint in our environment and allow borrowing against future income. In that case however, the ability to borrow would lower agents' marginal propensity to save (for precautionary reasons), thus implying that the effects of self-control and ability to borrow against future income are highly correlated; consequently, the effect of social security on savings due to self-control will be non-identifiable. In a recent paper, Rojas & Urrutia (2007) [32] show that adding an endogenous borrowing constraint reduces the welfare cost of having social security.

[10] Allowing a bequest motive does change the welfare implications of a social security system. Fuster *et al.* (2003) [15] perform a welfare analysis of social security in a dynastic framework and show that steady state welfare increases with social security.

We can now proceed to write the growth-adjusted budget constraint of an agent as follows

$$(1+g)a_j + \kappa w \epsilon_j + c_t = (1+r)a_{j-1} + q_j + \eta \tag{4.8}$$

$$(1+g)a_j + \kappa w \epsilon_j + \check{c}_j = (1+r)a_{j-1} + q_j + \eta \tag{4.9}$$

where r is the rate of return from asset holdings and η denotes the amount of accidental bequests, to be distributed equally to all alive members of the society. The compulsory contribution/deposit rate for personal security accounts (PSA) κ is greater than zero for employed individuals if PSAs exist in the economy, otherwise, $\kappa = 0$. The details for the law of motion of the PSAs are given in the next section.

4.2.5 Social Security and Fiscal Policy

We consider three distinct social security arrangements: Pay-As-You-Go, Personal Security Accounts *without* annuitization, and Personal Security Accounts *with* mandatory annuitization. We follow Fuster *et al.*(2005) [16] as far as our modelling of Personal Security Accounts is concerned.

4.2.5.1 PAYG A PAYG system in our model economy resembles to that of the US Economy. Agents retire at age j^* and receive social security benefit b. The social security benefit b is defined as a fraction θ of an average life-time employed income, which is, notably, independent of a particular agent's employment history.

$$b = \theta \frac{\sum_{j=1}^{j^*-1} w\epsilon_j}{j^*-1} \quad \text{for } j = j^*, j^*+1, ..., J.$$

While technology grows at rate g, the pension payments remain constant during retirement. Therefore, the growth-adjusted pension payments are given as follows:

$$b_j = \frac{b}{(1+g)^{j-j^*}} \quad \text{for } j = j^*, j^*+1, ..., J.$$

The social security system is self-financing and the government administers the program. We restrict our attention to social security arrangements that are described by the pair (θ, τ_s). The self-financing conditions are as follows:

$$\tau_s \sum_{j=1}^{j^*-1} \sum_a \mu_j \Lambda_j(a, e = 1) w \epsilon_j = \sum_{j=j^*}^{J} \sum_a \mu_j \Lambda_j(a, e) b_j, \tag{4.10}$$

where $\Lambda_j(a, e)$ is the set of age dependent, time invariant measures of individuals.

If agents in this economy die before age J, their remaining assets will be distributed evenly to all of the survivors in a lump-sum fashion as follows:

$$\eta = \sum_j \sum_a \sum_e \mu_j \Lambda_j(a, e)(1 - s_{j+1}) a_j(a, e) \tag{4.11}$$

4.2.5.2 Personal Security Accounts without Annuitization[11] In this section, we consider a two-tier social security system. The first tier is a PAYG-financed universal flat pension benefit. The amount of benefit is set as 10% of the per capita gross domestic product (GDP). As in the above benchmark case, the first tier is self-financing and administered by the government.

$$\tau_s \sum_{j=1}^{j^*-1} \sum_a \mu_j \Lambda_j(a, e = 1) w \epsilon_j = \sum_{j=j^*}^{J} \sum_a \mu_j \Lambda_j(a, e) b^*$$

where $b^* = 0.10y$ and $y = w/(1 - \alpha)$.

Hence, the equilibrium tax rate can be found by using the equation below:

$$\tau_s = \frac{0.10}{1 - \alpha} \frac{\sum_{j=j^*}^{J} \sum_a \mu_j \Lambda_j(a, e)}{\sum_{j=1}^{j^*-1} \sum_a \mu_j \Lambda_j(a, e = 1) \epsilon_j}$$

The second tier of retirement benefits is financed through mandatory savings. In this setting, employed individuals are required to deposit 2.7% of their earnings into their privately managed personal savings account. The funds on PSA earn the tax-free rate of return on capital and they cannot be withdrawn until the mandatory retirement age. Individuals'

[11] We will henceforth refer to to the system presented in this section as *PSA*.

second-tier benefits depend on the funds accumulated in their PSAs. The law of motion for the PSA is the following:

$$\begin{aligned} (1+g)\varkappa_{j+1} &= (1+r)\varkappa_j + \kappa w \epsilon_j, \\ \varkappa_1 &= 0, \end{aligned} \tag{4.12}$$

where $\kappa = 0.027$ is the required deposit rate and $\varkappa_{j+1}$ denotes an age $j+1$ individual's accumulated PSA funds. Individuals receive a lump-sum transfer of funds accumulated in their PSAs when they are retired at age j^*. Hence, retirement benefits can be written as follows:

$$b_j = \begin{cases} b^* + (1+r)\varkappa_{j^*}, \text{ for } j = j^* \\ \frac{b^*}{(1+g)^{j-j^*}}, \text{ otherwise} \end{cases}$$

If an individual dies before one's retirement, the accumulated funds in his PSA account are distributed to the remaining members of the society in a lump-sum fashion along with their assets:

$$\eta = \sum_j \sum_a \sum_e \mu_j \Lambda_j(a,e)(1 - s_{j+1})[a_j(a,e) + \varkappa_j(e)] \tag{4.13}$$

4.2.5.3 Personal Security Accounts with Mandatory Annuitization[12] We consider a two-tier pension system here as well. While the first tier is exactly the same as that of the previous section, the second tier differs. The government annuitizes funds accumulated in individuals' PSAs and hence, individuals receive annuity payments equal to $b' = p(1+r)\varkappa_{j^*}$, where p denotes the proportion, i.e., the amount of an annuity payment received by an individual, which is obviously proportional to the accumulated funds in his PSA account, and is determined endogenously.

The government sets the proportion p in such a way that the expected present value of the aggregate annuity payments of the generation that retires now $(\bar{E})$ is equal to the funds held in PSAs, including the funds of individuals from the same generation who died

[12] We will henceforth refer to to the system presented in this section as *PSA+Annuity.*

before their retirement. The expected present value of the aggregate annuity payments of the generation that retires now is equal to the following:

$$\bar{E} = p(1+r)\varkappa_{j^*} \sum_{j=j^*}^{J} (1+r)^{j^*-j} \prod_{i=1}^{j-1} \mu_i.$$

The aggregate funds of the individuals from the same generation who survived to retirement and who died before their retirement are given as

$$(1+r)\varkappa_{j^*} \prod_{i=1}^{j^*-1} \mu_i$$

and

$$(1+r) \sum_{j=2}^{j^*} (\frac{1+r}{1+g})^{j^*-j} \varkappa_j (1-\mu_{j-1}) \prod_{i=1}^{j-2} \mu_i$$

respectively. Hence, the proportion p can be written as follows:

$$p = \frac{(1+r)\varkappa_{j^*} \prod_{i=1}^{j^*-1} \mu_i + (1+r) \sum_{j=2}^{j^*} (\frac{1+r}{1+g})^{j^*-j} \varkappa_j (1-\mu_{j-1}) \prod_{i=1}^{j-2} \mu_i}{(1+r)\varkappa_{j^*} \sum_{j=j^*}^{J} (1+r)^{j^*-j} \prod_{i=1}^{j-1} \mu_i} \tag{4.14}$$

Because individuals' survival rates do not depend on their types in our model economy, the above return is actuarially fair, it could be offered by private firms and there is no need for government intervention. Yet, in our model, the private annuity market is closed by assumption and hence the government has to administer it. If individuals' survival rates were type-dependent, the above equation would yield an actuarially not fair premium. This, in turn, would call for government intervention because adverse selection issues would cause a market breakdown.

The law of motion of the funds aggregated in PSA by the government is the following:

$$(1+n)(1+g)\digamma_{t+1} = (1+r)\digamma_t + \kappa w L - B',$$

where $\kappa = 0.027$ is the social security tax rate, $L = 0.94 \sum_{j=1}^{j^*-1} \mu_j \epsilon_j$ is the aggregate labor supply, and $B' = \sum_{j=j^*}^{J} b'(1+g)^{j^*-j} \mu_j$ is the aggregate annuity payment at time t.

Hence, the expression for retirement benefits featuring PSAs under a mandatory annuitization scheme can be written as follows:

$$b_j = \frac{b^* + b'}{(1+g)^{j-j^*}} \text{ for } j = j^*, ..., J.$$

In addition to a social security program, the government also runs a self-financing unemployment program. The self-financing condition for the unemployment insurance program is as follows:

$$\tau_u \sum_{j=1}^{j^*-1} \sum_a \mu_j \Lambda_j(a, e=1) w \epsilon_j = \sum_{j=1}^{j^*-1} \sum_a \mu_j \Lambda_j(a, e=0) \phi w \epsilon_j, \tag{4.15}$$

where ϕ is the unemployment insurance replacement rate.

4.2.6 An Agent's Dynamic Program

We assume that the temptation function $v(.)$ is strictly increasing, i.e., an agent is tempted to consume his entire wealth in each period. This implies that the agent maximizes the second part of equation (4.1) by holding zero assets for the next period, i.e., setting $a_j = 0$ in equation (4.9). In this economy, the agent's state vector x contains the current asset holdings and the employment state. Hence, we can write the agent's dynamic program for any arbitrary two periods as follows:

$$W(x) = \max_c \{u(c) + v(c) + \beta E_{s'} W(x')\} - v((1+r)a + q + \eta) \tag{4.16}$$

subject to

$$a' + c = (1+r)a + q + \eta,\ a' \geq 0,\ a_0 \text{ is given.} \tag{4.17}$$

where $E_{s'}$ denotes the expectation over survival probabilities.

If the agent succumbs to a temptation and consumes his entire wealth, the term $v(c) - v((1+r)a + q + \eta)$ in the equation above cancels out. When he resists to temptation and consumes less than his wealth, he faces a self-control cost at the amount of $v(c) - v((1 + r)a + q + \eta)$. The agent tries to balance his urge for utility from current consumption $v(c)$ and long-term commitment utility $u(c) + \beta E_{s'} W(x')$.

4.2.7 Steady State Equilibrium

In our characterization of the steady state equilibrium, we follow Imrohoroglu *et al.*(2003)[24] and Hugget & Ventura (1999)[21].

Given a set of government policy $\{\theta, \phi, \tau_s, \tau_u, \kappa\}$, a steady state recursive competitive equilibrium is a set of value functions $\{W_j(x)\}_{j=1}^{J}$, household policy rules $\{a_j(x)\}_{j=1}^{J}$, time-invariant measures of agents $\{\Lambda_j(x)\}_{j=1}^{J}$, wage and interest rate (w, r), and a lump-sum distribution of accidental bequests η such that all of them satisfy the following:

- Factor prices (w, r) that are derived from the firm's first-order conditions satisfy equations (4.6) and (4.7).
- Given the government policy set $\{\theta, \phi, \tau_s, \tau_u, \kappa\}$, the factor prices (w, r), and the lump-sum transfer of accidental bequests η, an agent's policy rule $\{a_j(x)\}_{j=1}^{J}$ solves the agent's maximization problem (4.16) subject to the budget constraint (4.17).
- Aggregation holds,
$$K = \sum_j \sum_a \sum_e \mu_j \Lambda_j(x)[a_{j-1}(x) + \varkappa_j]. \tag{4.18}$$
- The set of age-dependent, time-invariant measures of agents satisfies in every period t
$$\Lambda_j(x') = \sum_e \sum_a \Pi(e', e)\Lambda_{t-1}(x) \tag{4.19}$$
  where Λ_1 is given.
- The lump-sum distribution of accidental bequests η satisfies the equation (4.11) if the PAYG or *PSA+Annuity* programs are in use, or else it satisfies the equation (4.13) if the *PSA* program is in use.
- Both the social security system and the unemployment insurance benefit program are self-financing.
- The market clears
$$\begin{aligned} & \sum_j \sum_a \sum_e \mu_j \Lambda_j(x)[a_j(x) + \varkappa_j(x) + c_j(x)] \qquad (4.20) \\ = \quad & Y + (1-\delta)\sum_j \sum_a \sum_e \mu_j \Lambda_j(x)[a_{j-1}(x) + \varkappa_{j-1}(x)] \end{aligned}$$

4.3 CALIBRATION

In this section, we briefly define the parameter values of our model. Each period in our model corresponds to a calendar year.

4.3.1 Demographic and Labor Market Parameters

Agents are born at a real life age of 21(model age of 1) and they can live up to a maximum real life age of 85 (model age of 65). The population growth rate n is assumed to be equal to the average U.S. population growth rate between 1931-2003 which corresponds, on average, to 1.19% per year[13]. The sequence of conditional survival probabilities is the same as the Social Security Administration's sequence of survival probabilities for men in the year 2001. The mandatory retirement age is equal to 65 (model age 45). In order to set the efficiency index, we choose the average of Hansen's (1993)[20] estimation of median wage rates for males and females for each age group. We interpolate the data by using the Spline Method and normalize the interpolated data to average unity. The employment transition probabilities are chosen to be compatible with the average unemployment rate in the U.S. which is approximately equal to 0.06 between 1948 and 2003.[14] The implied employment transition matrix assumes the following form:

$$\Pi(e, e') = \begin{bmatrix} 0.94 & 0.06 \\ 0.94 & 0.06 \end{bmatrix}.$$

4.3.2 Preference Parameters

We choose the values of preference parameters ρ, γ, λ and β in such a way that our model economy's capital-output ratio matches that of the U.S. economy.

In the case where the temptation function $v(.)$ is convex, we choose to follow Imrohoroglu *et al.*(2003) [24] and DeJong & Ripoll (2007) [4], in letting γ be centered at 2 with a standard deviation 1, i.e., $\gamma = 2_{(1)}$. In our benchmark calibration, we initially set $\gamma = 2$, and then

[13] The population data were obtained from the U. S. Census Bureau (U.S. Census Bureau (2006)) [37] .

[14] The unemployment data are taken from the U. S. Department of Labor (U.S. Department of Labor (2006)). [36]

check the robustness of our results by letting $\gamma = 3$. Holding γ constant, we choose different values of ρ *a priori,* and calculate the corresponding λ in such a way that $u(.) + v(.)$ stays a strictly concave function. For every triple ρ, γ and λ, we search over the values of β that deliver the capital-output ratio which is compatible with its empirical counterpart. We assume that the social security replacement ratio is 50% and the unemployment replacement ratio is 25% during our search.

When the temptation function is concave, we follow DeJong & Ripoll (2007) [4] and set $\lambda = 0.0786_{(0.056)}$

4.3.3 Production Parameters

The parameters describing the production side of the economy are chosen to match the long-run features of the U.S. economy. Following Imrohoroglu *et al.*(2003)[24], we set the capital share of output α equal to 0.310 and the annual depreciation rate of physical capital equal to 0.069. The rate of technological progress g is assumed to be equal to 2.1%, which is the actual average growth rate of GDP per capita taken over the time interval from 1959 to 1994 (Hugget & Ventura (1999) [20]. The technology parameter A, can be chosen freely. In our calibration exercises, it is set equal to 1.01. All per capita quantities are assumed to grow at a constant rate g.

4.3.4 Government

We set the unemployment insurance replacement ratio (ϕ) equal to 25% of the employed wage. In the benchmark case, we set the social security replacement ratio (θ) equal to 50%. Alternatively, we can choose the payroll tax rate (τ_s) and the unemployment insurance tax rate (τ_u) instead of the replacement ratios. Since the social security and the unemployment insurance benefits are self-financing, calibrating the replacement ratios will automatically pin-down the tax rates. This holds true because agents inelastically supply one unit of labor whenever they find an opportunity to work, and changes in tax rates do not affect their supply of labor.[15] In the PSA cases, the first tier universal benefits are set equal to 10% of per capita GDP and the social security tax rate for second tier benefits κ is set equal to 2.7%.

4.3.5 Solution Method

We use discrete-time, discrete-state optimization techniques to find a steady state equilibrium of our hypothetical economy by using the aforementioned parameter values. Our solution method designedly resembles those of previous studies.[16]

A discrete set of asset values (containing 4097 points) is created. The lower bound and upper bound of the set are chosen in such a way that the set never binds.[17] While the state space for working age agents comprises 4097×2 points, the state space for retired agents consists of only 4097×1 points (since there is no state transition after j^*).The discrete set of the control variable (consumption) contains 4097×1 points. We start with a guess about the aggregate capital stock and the level of accidental bequests and then solve agents' dynamic program by backward recursion. The time-invariant, age-dependent distribution of agents is obtained by forward recursion. After each loop, we calculate the new values for the accidental bequests and the capital stock. If the difference between the initial values and the new values exceed the tolerance value, we start a new loop with the new values. This procedure continues until we find values for the accidental bequests and the capital stock that are sufficiently close to their beginning-of-loop values.

4.4 RESULTS

In this section we first calibrate our model economy to the US data under the assumption that individuals have CRRA preferences and a PAYG system that is similar to that of the US is in use. We choose the social security replacement rate as 40% since it corresponds

[15] However, if we calibrate a model featuring labor-leisure choice, tax rates should be used instead of replacement rates.

[16] See Imrohoroglu et al. (1995 [22] and 2003 [24]) and Hugget & Ventura(1999)[21].

[17] In particular, the lower bound is equal to 0 and the upper bound is equal to 60 times greater than the annual income of an employed agent.

to the current average social security replacement rate in the US. Our aim is to gradually converge to the long term average of the US capital-output ratio of 2.5. Subsequently, we proceed to look at the implications of three reform proposals:

The first reform proposal postulates the substitution of the current social security system by a two-tier scheme: a universal PAYG-financed basic pension combined with a Personal Security Account that does not require annuitization of benefits at retirement.

The second reform proposal is similar to the first one except that it prescribes a mandatory annuitization of the funds accumulated in PSA accounts.

Finally, the third reform proposal suggests the complete removal of the social security system; in terms of our model, this amounts to postulating that the social security replacement ratio is equal to zero.

The effects that the three competing scenarios have on economic aggregates, as well as their welfare implications are given in Table 17 for the case of a CRRA population. The welfare implications of a given policy are measured in terms of the average expected utility (EU). The average expected utility of a policy is given in the last column of Table 17. Both our first reform proposal (featuring as "*PSA*" in Table 17) and our second one (featuring as "*PSA + Annuity*" in Table 17) prescribe a specific (given) rate of mandatory savings. We assume that the mandatory savings rate, denoted by κ, is equal to 2.7% of the gross individual labor income. As a consequence, individuals' payments to the social security system across *PAYG*, *PSA*, and *PSA + Annuity* policies are all equalized. It is a well established result in the literature that funded or partially funded social security systems induce an increase in capital accumulation as well as in other aggregate variables such as consumption and output. (e.g., Storesletten *et al.* (1999) [32]). However, the exact magnitudes of increase of those aggregate variables are sensitively dependent to the features of the model. As demonstrated in Fuster *et al.*(2005) [16] the increases in the absolute levels of aggregate variables are more moderate in an economy populated by altruistic individuals compared to an economy without altruistic individuals. Previous studies (Imrohoroglu *et al.*(1995) [24], Imrohoroglu *et al.*(2003) [24], and Hugget & Ventura (1999) [20]) also demonstrated that decreasing the social security replacement rate improves the social welfare whenever individuals have CRRA preferences. Their key finding was that the complete elimination of a PAYG system would deliver the highest social welfare.

Our results in Table 17 are consistent with those of previous studies mentioned above. Since both *PSA* and *PSA + Annuity* reform proposals entail less distortion on savings, something that comes as a consequence of a low level of payroll tax, the level of capital stock in both cases is higher than that of a *PAYG* system. Not surprisingly, the complete elimination of a PAYG system induces a higher capital stock compared to that of *PAYG*. Capital stock levels in *PSA* and *PSA+Annuity* reforms are higher than in the *elimination* scenario because individuals cannot borrow against the funds accumulated in their PSA accounts, even if it might be optimal to do so for consumption smoothing purposes. Interestingly, and unlike Fuster *et al.*(2005) [16], the *PSA* reform generates higher capital stock compared to the *PSA + Annuity* reform.

An immediate interpretation of this discrepancy lies in the fact that under the *PSA + Annuity* scenario analyzed in Fuster *et al.*(2005) [16] the funds of the deceased are invested in the capital market while under the *PSA* scenario they are transferred to the estates, resulting in a higher capital stock in the former scenario. In our model instead, annuity payments force individuals to reduce their post-retirement savings and hence, the gain in the level of capital stock from the increased investment in the capital market is offset by the reduced post-retirement savings. The complete *elimination* of social security scenario induces the highest welfare improvement followed by the *PSA* reform. This happens because the *elimination* reform allows individuals to engage in consumption smoothing more efficiently than the other proposed reforms.

The large and growing literature on time inconsistency and self-control issues suggests that preference specifications capturing individuals' self-control problems might provide richer insights to the observed behavior of individuals (for example see Frederick *et al.*(2002)). Needless to say, both time inconsistent preferences à la Phelps & Pollak (1968) [31] and self-control preferences à la Gul & Pesendorfer (2004a) [17] have been extensively used in an attempt to shed light to various macroeconomic problems, with issues pertaining to social security being among the most prominent ones.

More specifically, while Imrohoroglu *et al.*(2003) [24] analyze the welfare implications of the elimination of a PAYG-type social security system in an economy populated by individuals with time inconsistent preferences, Kumru & Thanopoulos (2008) [29] analyze the same policy proposal in an economy populated by individuals with self-control preferences. The latter study concludes that the welfare implications of the elimination of the PAYG system may vary drastically even if individuals' self-control problems differ only to a small extent.

Although the elimination of a PAYG social security system is one policy option, transforming the system into a partially or fully-funded one could very well be policy alternatives worth investigating. The types of *PSA* and *PSA+ Annuity* policies we analyze in this paper are similar to the ones analyzed Storesletten *et al.* (1999) [32] and Fuster *et al.*(2005) [16] and those proposed in the reform recommendations made by the 1997 Advisory Council on Social Security[18]. In what follows, we proceed to examine the implications of the above reform proposals under the assumption that individuals have self-control preferences. Before presenting our results for the benchmark case, we first explain the behavioral implications of the existence of temptation and then quantify the effects of temptation on economic aggregates.

Following DeJong & Ripoll (2007)[4], Kumru & Thanopoulos (2008) [29] calculated the quantity of steady state consumption that would have to be given up by a self-control individual in order to escape from temptation; Kumru & Thanopoulos (2008) [29] showed that individuals would be willing to forgo as much as 4.82% of their steady state consumption in order to eliminate temptation. This clearly demonstrates the welfare reducing effect that a self-control problem entails.

Next, we quantify steady state levels of capital accumulation and consumption under the assumption that individuals have self-control preferences. By keeping the annual discount factor at its CRRA level, we first fix ρ at 2 and vary λ then we fix λ at 0.00009 and vary ρ. Tables 18 and 19 display our results for both cases. The results are the same as those in Kumru & Thanopoulos (2008)[29] . Table 3 demonstrates that an increase in the intensity of temptation distorts capital accumulation severely and causes a reduction in steady state consumption. Similarly, Table 4 shows the effects of an increase in an individual's willingness

[18]Please see Fuster *et al* 2005 [16] for a detailed exposition of the reform proposals regarding to the US social security system.

to substitute current temptation consumption with future temptation consumption. Higher values of ρ mean that individuals prefer more current temptation consumption. Not surprisingly, higher values of ρ result in a reduction in the steady state level of capital, which in turn causes a reduction in the steady state level of consumption.

In this section we present our results for an economy in which agents have self-control preferences and we compare the effects of the aforementioned three reform proposals on both economic aggregates and social welfare. Our model provides the opportunity to compare the effects of two opposite factors on savings (capital accumulation). While the existence/availability of PSAs causes an increase in savings, individuals with self-control preferences save less in order to escape from the huge self-control cost associated with resisting to temptation once the accumulated assets become available. We present our results in Table 20. The effects of the three reform proposals on economic aggregates and social welfare are similar to those we documented in Table 17 for an economy populated by individuals featuring CRRA preferences. In particular, the highest level of capital stock is reached under the *PSA* reform scenario whereas the highest level of average expected utility is achieved under the complete *elimination* scenario. The mechanisms that deliver these results are the same as those we explained in the case of the CRRA case above. However, a careful look at Table 20 reveals that, unlike the CRRA case, the values of average expected utilities across our four scenarios are very close to each other.

For the sake of a more concise comparison, we created Table 21; this table presents the levels of capital stock and average expected utility under the three alternative reform proposals for both CRRA and self-control cases. In this table the values of the level of capital stock and the average expected utility induced by the CRRA model are normalized to 100 and -100 respectively. Table 21 reveals that only the *elimination* and *PSA* reform scenarios yield higher welfare in the case agents feature self-control preferences. However, the welfare benefits of these scenarios are substantially lower compared to the case where agents have CRRA preferences. The intuition is the following: Individuals with self-control preferences are tempted to consume their entire wealth each period. Payroll taxes help those individuals to reduce their cost of exerting self-control in order to resist temptation. This additional benefit of a PAYG system creates welfare improvement.

Still, this additional benefit of a PAYG system is not large enough to exceed the benefits associated with eliminating the entire social security system. The PSA reforms provide the same additional benefit to working age individuals as the PAYG system. Although under CRRA preferences the *elimination* scenario results in a welfare gain that is considerably higher than the PSA scenario, the welfare benefits for both the *elimination* and *PSA* scenarios are almost identical under self-control preferences. Table 21 demonstrates an intriguing result: *PSA + Annuity* reduces the overall welfare. This is a surprising result. Fuster *et al.*(2005)[16] showed that *PSA+Annuity* on average generated the highest welfare in an altruistic economy in which individuals have CRRA preferences. In our environment the *PSA+Annuity* reform does not work. The reason why this obtains is simple: after retirement the *PSA+Annuity* reform provides more consumption opportunities, and hence higher exposure to temptation than what an individual with a self-control problem prefers. This higher the retirement benefit is the more it exacerbates the retired individuals' self-control costs and the more likely it becomes for it to exceed the additional benefit provided to the working age agents. More specifically, the stream of benefits that mandatory annuitization entails exposes individuals with self-control preferences to a respective stream of repeated costs to resist temptation. In our setting, the shape of the temptation function is such that the total temptation cost from a stream of benefits exceeds the temptation cost from a lump-sum payoff at the time of retirement.

Finally we check the robustness of our results for the case where individuals face an extremely severe self-control problem. For this purpose we set the risk-aversion parameter of the momentary utility function, γ, equal to 2 and we set the temptation function parameters ρ and λ equal to 2 and 0.00065 respectively. We choose $\beta = 1.0117$ in order to converge to the long-term capital-output ratio of the US economy when a PAYG system is in use[19]. Table 22 below presents the results of our robustness test.

As it can be seen in Table 22, in contrast to our previous results, the current *PAYG* system dominates all other systems by generating the highest overall welfare, while the

[19] Further robustness checks were conducted by using various parameter choices. All these exercises confirm our results and they are available upon request. We also conduct exercises by using a concave tamptation function. As long as the self-control problem is not very severe, our results do not significantly deviate from the CRRA case. In order to keep the number of pages reasonable we don't present the results here. They are also available upon request.

elimination reform creates the lowest level of welfare. Since *PSA* and *PSA + Annuity* reforms provide the same additional benefit as *PAYG*, it is not surprising that in the case of severe temptation they end-up performing better than the *elimination* scenario.

Still, higher retirement benefits increase retirees' self-control cost and hence both *PSA* and *PSA + Annuity* reforms underperform with regards to welfare compared to PAYG in this environment. A higher value of the time preference parameter, β, might be effective in delivering the above results. Nevertheless, these results are important in that they blatantly manifest how sensitive are the welfare consequences of various reform proposals to little deviations from the CRRA case.

In summary, we show that if an economy is populated by individuals with self-control problems, the gain from the two reform proposals is not as large as what we observed in the CRRA case. More interestingly the *PSA + Annuity* reform proposal even decreases welfare in comparison to the PAYG system.

4.5 CONCLUSIONS

Population aging due to low birth and morbidity rates and the resulting expansion of social security benefits have prompted lively debates around the long-term viability of unfunded social security.[20] Several reform proposals are being discussed for the US Economy and other industrialized countries, most of which converge to a common resultant: social security must be reformed in the direction of a funded rather than an unfunded system.

In this paper we examine departures from a Pay-As-You-Go social security system towards a system consisting of two parts: A "defined benefits" component and a "defined contributions" one. This is implemented by means of annexing a (individually managed) Personal Security Account to a universal (PAYG-based) pension system.

We quantitatively assess the attractiveness (i.e., the welfare enhancing potential) of such a funded scheme in an economy populated by agents that face self-control issues. To this purpose, we use two different benchmarks: a economy featuring PAYG social security popu-

[20]See US Social Security Administration.2008. Fast facts and figures about social security. <http://www.ssa.gov> [35]

lated by self-control agents (Kumru & Thanopoulos (2008)[29]) and an economy featuring funded (as in this paper) social security but populated by CRRA agents. Furthermore, we refine our analysis by investigating the relative appeal of two distinct scenarios: PSAs without annuitization and PSAs with mandatory annuitization.

Our analysis suggests that the availability of PSAs increases savings "in general". This is principally true for CRRA agents. However, it is very ambiguous whether self-control agents will be better off with a genuine PAYG system, or with a funded system of the analyzed hybrid structure that prescribes mandatory annuitization, as it eventually depends on the intensity of the self-control problem.

Our results in this paper highlight the important role that self-control preferences play, especially with regards to the kind of reform that could generate the highest welfare. While when the analysis is restricted to CRRA preferences our findings initially confirm those in other studies (e.g., Fuster *et al.*(2005)[16]), i.e., we also find that the complete elimination of any social security is the policy that maximizes welfare, when self-control preferences are introduced instead, we come across a drastic alteration of our results: the elimination of social security still remains, alas very marginally, the most desirable policy from the point of view of welfare maximization, it is nevertheless not *as* desirable with respect to our alternative reform scenarios as it is under CRRA preferences.

A noteworthy result is that, unlike Fuster *et al.*(2005)[16], the *PSA + Annuity* scenario performs worse than the reform scenario that allows a PSA without mandatory annuitization. It is true that in the latter case, the prospect of a lump-sum PSA payout to self-control agents may entail an infinitely cumbersome cost of resisting temptation, but this eventully hinges on the curvature of the temptation function. Interestingly enough, even when PSA benefits are annuitized, this amounts to a stream of smaller in magnitude temptation problems in the post-retirement period. It turns out that in our setting, the shape of the temptation function is such that the total temptation cost from a stream of benefits exceeds the temptation cost from a lump-sum payout at the time of retirement. Hence, mandatory annuitization becomes less desirable to our self-control agents.

In conclusion, our research indicates that the incontestable advantage of the *elimination* scenario under CRRA preferences rapidly vanishes as self-control is introduced and becomes progressively more severe. Our robustness tests confirm this finding: in the case of severe temptation the CRRA pecking order is completely reversed, and the elimination of social security becomes the least desirable reform scenario.

Table 16: Parameter Values of The Benchmark Calibration

Demographics	
Maximum possible life span T	65
Obligatory retirement age t^*	45
Growth rate of population n	1.19%
Conditional survival probabilities $\{s_t\}_{t=1}^{T}$	U.S. 2001
Labor efficiency profile $\{\epsilon_j\}_{t=1}^{t^*-1}$	Hansen (1993)
Production	
Capital share of GDP α	0.310
Annual depreciation of capital stock δ	0.069
Annual per capita output growth rate g	2.1%
Markov Process for employment transition Π	$\begin{bmatrix} 0.94 & 0.06 \\ 0.94 & 0.06 \end{bmatrix}$
Preferences	
Annual discount factor of utility β	0.998
Scale factor of the temptation utility λ	0.000375
Risk aversion parameter γ	2.0
Risk loving parameter ρ	2.0
Government	
Unemployment insurance replacement ratio ϕ	0.25
Social security replacement ratio θ	$[0, 1]$
Compulsory deposit rate κ	0.027

Table 17: CRRA Preferences

	$\tau_s(\%)$	$\kappa(\%)$	K	Y	C	K/Y	$r(\%)$	EU
PAYG	4.58	0	3.082	1.228	0.974	2.511	7.95	-26.114
PSA	1.88	2.70	3.691	1.298	0.961	2.843	6.51	-25.608
PSA+Annuity	1.88	2.70	3.590	1.287	0.949	2.789	6.72	-25.936
Elimination	0	0	3.562	1.284	0.984	2.774	6.78	-24.932

Table 18: (beta=0.978, gamma=2, rho=2)

λ	Y	K	C	K/Y	r	w
$CRRA$	1.188	2.892	0.960	2.434	0.081	1.044
$\lambda = 0.0004$	1.120	2.386	0.939	2.132	0.103	0.984
$\lambda = 0.001$	1.067	2.046	0.918	1.917	0.122	0.938

Table 19: (beta=0.978, lambda=0.00009)

λ	Y	K	C	K/Y	r	w
$CRRA$	1.188	2.892	0.960	2.434	0.081	1.044
$\rho = 2$	1.168	2.735	0.954	2.342	0.087	1.026
$\rho = 3$	1.075	2.091	0.922	1.946	0.119	0.944

Table 20: Self-Control Preferences

	$\tau_s(\%)$	$\kappa(\%)$	K	Y	C	K/Y	$r(\%)$	EU
PAYG	4.58	0	2.765	1.187	0.960	2.330	8.91	-34.666
PSA	1.88	2.70	3.410	1.267	0.954	2.692	7.12	-34.288
PSA+Annuity	1.88	2.70	3.330	1.257	0.939	2.648	7.31	-34.748
Elimination	0	0	3.109	1.231	0.969	2.526	7.88	–34.244

Table 21: Comparison

	CRRA		Self-Control	
	K	EU	K	EU
PAYG	100	-100	100	-100
PSA	119.760	-98.062	123.327	-98.910
PSA+Annuity	116.483	-99.318	119.349	-100.237
Elimination	115.574	-95.500	112.441	-98.783

Table 22: Severe Temptation

	$\tau_s(\%)$	$\kappa(\%)$	K	Y	C	K/Y	$r(\%)$	EU
PAYG	4.58	0	2.783	1.189	0.961	2.340	8.86	-47.626
PSA	1.88	2.70	3.436	1.270	0.955	2.706	7.06	-48.202
PSA+Annuity	1.88	2.70	3.365	1.262	0.940	2.668	7.22	-48.812
Elimination	0	0	3.094	1.229	0.968	2.517	7.92	–48.979

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