ESSAYS ON BEHAVIORAL PUBLIC ECONOMICS AND MICROECONOMIC THEORY

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ABSTRACT

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This dissertation consists of the three independent chapters in the areas of Public Economics and Microeconomic Theory. The first two chapters use experimental and computational techniques to address two important behavioral issues in Public Economics. In particular, the first chapter (with Lise Vesterlund) examines if concerns for status may help explain why fundraisers commonly announce past contributions to future donors. To answer this question we incorporate status concerns into the standard charitable giving model, and subsequently test the predicted comparative statics in the laboratory. Consistent with the economic prediction we find that low-status followers are likely to mimic contributions by high-status leaders and that this encourages high-status leaders to contribute. Contributions are therefore larger when individuals of high status contribute before rather than after those of low status. The second chapter (with Athanasios C. Thanopoulos) uses computational techniques to assess welfare implications of an unfunded social security system when individuals have self-control preferences. Our computation model demonstrates that the welfare costs of an unfunded social security system are substantially reduced when agents have self-control preferences. However, the positive effect of reducing self-control costs is not large enough to surpass its negative effect on capital accumulation. Finally, the third chapter (with Hadi Yektaş) of the dissertation examines an important and open mechanism design question. It characterizes the necessary conditions of optimal auction for multiple objects when agents are risk-averse. We show that the optimal auction is weakly efficient; in the sense that each

object is sold to a buyer who has high valuation for it, if such a buyer exists. The seller perfectly insures all buyers against the risk of losing the object(s) for which they have high valuation. While the buyers who have high valuation for both objects are compensated if they do not win either object; the buyers who have low valuation for both objects incur a positive payment in the same event. The objects are bundled to the same buyer if all buyers have low valuation for both objects, thus, independent auctions are not optimal.

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PREFACE

An endeavor of this magnitude is virtually impossible without the unflinching support of several individuals who assist in their own selfless way. I have been blessed to have had the support of several individuals who offered their unstinted support, especially in difficult times to ensure the successful culmination of this project.

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My mother shouldered tremendous responsibilities stoically for several years to enable me and all my siblings to realize their true potential in life. It was daunting to say the least, but it is a measure of her bigness and grace, that she regarded it as no more than her responsibility to do whatever was needed to ensure that we lacked nothing. The least I can do by way of acknowledgement is to dedicate this dissertation to her, something I do with the greatest pleasure.

1.0 THE EFFECT OF STATUS ON VOLUNTARY CONTRIBUTION (WITH LISE VESTERLUND)

1.1 INTRODUCTION

Many capital campaigns are launched by the announcement of a large initial contribution made by a well-known donor. These initial contributions often trigger contributions of others. For example, characteristic of Brook Astor's philanthropic endeavors is that others tend to copy her contribution after news about her donation. "When she gave one donation to the New York Library, for example, three other major gifts – from Bill Blass, Dorothy and Lewis B. Cullman, and Sandra and Fred Rose – all followed, with her generosity cited as the inspiration."[1]

As shown by Varian [81] the classical model of voluntary contributions cannot explain why fundraisers announce past contributions to future donors.[2] A possible explanation may be that initial contributions are used to signal the quality of the non-profit to subsequent contributors (Vesterlund [82]).[3] Interestingly such a model also gives rise to an optimal solicitation ordering, which appears consistent with the contribution ordering commonly observed. When the quality of the non-profit is unknown, high-quality organizations find it optimal to first solicit their wealthiest donor.[4] The reason is not simply that wealthy donors

[1] New York Times, March 30, 2002, p. A13.

[2] When contributions are made sequentially, the first mover can commit to a small initial contribution and effectively free ride of the contribution of the subsequent contributor. The contribution level in the sequential game is therefore no larger than that in the simultaneous game.

[3] Potters et al. [71] show experimentally that sequential moves result in larger overall contributions, and that signalling is a likely explanation for this increase in contributions. Potters et al. [72] show that a sequential contribution ordering will arise endogenously when there is uncertainty about the quality of the public good.

[4] To not reveal their type low-quality organizations may also first solicit wealthy donors.

are willing to pay more to investigate the charity, but also that they need to increase their contribution more to convince others that an organization is of high quality.

While imperfect information about the nonprofit's quality can help explain why some organizations first solicit the wealthy donors, it is important to note that this contribution ordering also is used by more established nonprofits, where there is less uncertainty about their quality. This suggests that signaling is unlikely to be the only reason for the observed solicitation ordering. An alternative explanation may be found by considering other characteristics of the initial donors. Lead contributors distinguish themselves not only by being wealthy, but also by being well-known and well-respected. In particular they are often seen as having a higher rank in the social hierarchy.[5] The objective of this paper is to examine if and how concerns for status may help explain why more well-known and respected individuals tend to contribute first.

Economists have come to recognize that status and concerns for status may affect both economic decisions and the allocation of resources. The literature is extensive, and the notion that individuals are concerned about their relative standing is not new.[6] Of particular interest have been the theoretical implications of such concerns. For example Frank [36] examines behavior when status is determined by one's ordinal rank in the distribution of consumption, income, or wealth. Fersthman and Weiss [32] study the role of social status in a general equilibrium framework. They show that changes in the demand for status may affect the wage structure, the level of aggregate output and economic welfare. Congleton [17] studies status-seeking games in which an individual's utility is not only determined by his absolute consumption, but also by his relative expenditure on status-seeking activities. He shows that some status-seeking activities may generate positive externalities, and that status acquisition need not be wasteful. More recently Hopkins and Kornienko [49] examine behavior when individuals care about their relative rank in the distribution of consumption of a "positional" good. They find that a result of such preferences is that too many resources are allocated to consumption of the positional good.[7]

[5] A person's status is a ranking in any hierarchy that is socially recognized.

[6] For early explorations of status see e.g., Smith [76], Veblen [83], and Becker [13].

[7] See Ball et al. [12] and Hopkins and Kornienko [49] for reviews of the status literature. The literature on social identity and prestige is also related. Harbaugh [44, 45] examines a model where an individual's contribution provides prestige, and he shows that a charity may increase total contributions by using a

While substantial theoretical work has been conducted to examine the potential effects of status, little work has been done to demonstrate its actual behavioral implications. One exception is Ball et al. [12]. Using experimental methods they induce status in the laboratory and examine prices in a competitive market where either the buyer or the seller side has low status, while the other side has high status. To secure that there is room for status to influence market outcomes, they use a box-design market, where a vertical overlap in supply and demand ensures that there are multiple equilibrium prices. They show that independent of which side of the market is held by the high-status agents, they always capture a greater share of the surplus.

Finding that concerns for status affect behavior in a competitive market lead us to expect that it also can influence many other behaviors, and we want to examine, theoretically and experimentally, if status concerns affect voluntary contributions to nonprofits. Following Ball et al. [12] we take the social ranking as given and assume that individuals prefer to associate with those of higher status, but not with those of lower status than themselves. We incorporate such status-motivated preferences into a voluntary-contribution model to examine if they give rise to an optimal-solicitation order, where high-status individuals give first. We then test the predicted comparative statics experimentally.

We consider a simple linear-contribution mechanism when members of a group differ in their status ranking. There are two members of each group, and they must each decide whether to contribute to a group activity. Contributing incurs a private cost, but generates a benefit for both group members. Consistent with the frequently observed contribution pattern, our model demonstrates that to maximize total contributions the optimal solicitation ordering is one where individuals of high-status contribute prior to, rather than after, those of low status. There are two reasons why this ordering is optimal. First, an initial contribution by those of high status subsequently encourages individuals of low status to contribute. Second, knowing that low-status contributors mimic the high-status contribution encourages the high-status person to give as well. The latter effect arises because the benefit of the sub-

category reporting plan. Andreoni and Petrie [5] alter the identification of the participants and information on their contribution in an experimental study. By doing so, they allow social effects such as pride, shame, social comparison and prestige to affect participants' decisions. They find that identity and information matter. Akerlof and Kranton [3] analyze the effects identity, i.e., a person's sense of self, has on economic outcomes.

sequent contribution may be sufficient to overcome the monetary and status cost associated with giving to the same organization as someone of low status.

We follow Ball et al.'s [12] status-inducement procedure to examine the extent to which the predicted comparative statics are good approximations for actual behavior. Having induced status in the laboratory we pair people in groups of two, where one participant has higher status than the other. Using a between-subject design we compare the effect on behavior of reversing the contribution order between the two participants. That is, in one treatment the leader has high status and the follower low status, and in the second treatment the contribution order is reversed.

Our experimental results are consistent with the predicted comparative statics. Low-status followers tend to mimic the high-status leaders' contributions, providing high-status leaders with a monetary incentive to give. In contrast, high-status followers are reluctant to mimic low-status leaders, and low-status leaders therefore do not have a monetary incentive to contribute. Many leaders appear to correctly anticipate these responses, and high-status leaders contribute substantially more than low-status leaders. The net effect is that total contributions increase by more than 80 percent when high-status participants contribute first.

The remainder of the chapter is organized as follows. In Section 1.2, we introduce a simple model for voluntary contributions and examine the interaction between contribution order and status. This model serves as the foundation for our experimental design, which is described in Section 1.3. The associated results are presented in Section 1.4, and Section 1.5 concludes.

1.2 A SIMPLE MODEL OF VOLUNTARY CONTRIBUTIONS WITH STATUS

There are a number of ways in which concerns for status can affect charitable giving. First, an individual's contribution may affect how she is ranked relative to other people, and hence a motivation for giving may be status acquisition. Second, an individual's status prior to giving may influence her contribution behavior. Although her contribution to the New York

Library may have enhanced her status, Brook Astor was already known to be the grand-dame of philanthropy prior to giving, and it is possible that this initial status influenced her contribution.

We focus on an environment where a status differential exists prior to the individual contributing. Taking the social hierarchy as given, we determine how such a differential may alter the predictions of the charitable-giving model. Of particular interest is whether, in the presence of status, fundraisers and donors prefer that contributions be made simultaneously or in sequence, and if so who they would prefer contribute first?

We will use a simple binary example to illustrate the effect concerns for status may have on voluntary contributions. Suppose that two potential contributors, Player A and Player B, each must allocate a unit endowment to either a private good ($g_i = 0$) or a public good ($g_i = 1$). If allocated to the private good the individual gets a return of one, while an allocation to the public good generates a return of m to both players. The individual's return from the interaction is given by

$$\pi_i = 1 - g_i + m(g_A + g_B), i \in \{A, B\}.$$ [8]

In addition to the return from the public and private good, we assume that individuals also are concerned about the status of the individuals they interact with. Following Ball et al. [12] we assume that individuals want to associate with people who have higher status than themselves and dislike associating with those of lower status.[9] Association can be prevented by not contributing to the same organization, and it can be secured by contributing to the same organization as someone else.[10] In addition to the monetary payoff, we assume that individual i's utility from contributing to the same organization as individual j, also is an

[8] The theoretical predictions extend to many alternative specifications. We focus on this simple binary case because it mirrors our experimental setting.

[9] Note that the predicted comparative statics do not depend on the latter part of this assumption. All that is required is that individuals prefer to associate with those of high status. Thus a model where the status component takes the form $e_i.max\{(s_j - s_i), 0\}.g_A.g_B$, results in the same predictions.

[10] Perhaps one explanation for why individuals like to associate with those of high status lie in the following statement made by Akerlof and Kranton [3]: "In a world of social difference, one of the most important economic decisions that an individual makes may be the type of person to be. Limits on this choice would also be critical determinants of economic behavior, opportunity, and well being." Thus individuals may not only make decisions on how much of the public good they want provided, but also about their identity. In particular, they decide whether to associate themselves with their opponents in the contribution game.

increasing function of the status term $S_{ij} = e_i.(s_j - s_i).g_i g_j$, where $s_i \in R_+$ denotes the individual's status score, and $e_i \in [0,1]$ the individual's status concern. Thus, the marginal return from giving is larger when contributing to charities with a high-status donor base, and the return is increasing in the amount given by these high-status donors. Combining the two elements we consider the simple quasi-linear utility function:

$$U_i = 1 - g_i + m(g_A + g_B) + e_i(s_j - s_i)g_A g_B, i \in \{A, B\}.$$[11]

The parameters of the model are assumed to be common knowledge. We focus on the case where ignoring the status term there is a social dilemma, i.e., $0.5 < m < 1$. Implying that when $e_i = 0, i \in \{A, B\}$, it is efficient for both to contribute, but independent of the contribution order neither individual will choose to do so.

Now let us examine behavior when participants are concerned about status, i.e., $e_i > 0$. Note first that independent of the solicitation order neither donor contributes when the other donor has the same status level as herself.[12] Consider instead the case where individual A has more status than individual B, i.e., $s_A > s_B$, and refer to individual A as the high-status agent and individual B as the low-status agent. We focus on the situation where individuals differ only in their individual status, that is, both players have the same individual-specific status concerns, normalizing $e_i = e = 1$. Thus low-status individuals are as eager to be with someone of high status as the high-status individuals are reluctant to be with someone of low status.

How should a contribution-maximizing fundraiser design his campaign in such an environment? Suppose he first solicits the low-status agent, and subsequently the high-status one. Since in this case the low-status contribution is taken as given, contributing is costly for the high-status follower and she allocates her endowment to the private good. Knowing that the high-status follower will not contribute, the low-status leader's return from giving is only m, and he too allocates his endowment to the private good. Thus no contributions

[11] Our intent is to illustrate the comparative statics that may result when individuals have status concerns. Such concerns may enter the utility function in a number of different ways. For our binary example we consider the particularly simple example U_i. In continuous contribution models interior solutions will often require that the status component is some $f(S_{ij})$, where $f' > 0$, and $f" < 0$.

[12] The zero contribution is due to the linear voluntary contribution model. Generally the contribution levels just reduce to those of the standard contribution model, thus if there is an interior equilibrium in the model without status, then there will also be one in the model where individuals do not differ in their status.

are made to the public good when the low-status agent is solicited first. The outcome is the same if the two agents simultaneously contribute to the public good, the reason is that once again the high-status agent takes the low-status contribution as given and therefore does not contribute. Therefore, when agents give simultaneously or the low-status agent goes first the equilibrium prediction is the same as in the classical linear contribution model (i.e., when $e_i = 0$).

Interestingly, this prediction does not hold when the high-status person is the first to give.[13] The reason is that a low-status follower mimics the high-status contribution when $s_A - s_B > 1 - m$, that is when the added benefit of being associated with someone of high status is sufficient to compensate for the cost of contributing. How does this influence the high-status leader? The follower's mimicking compensates the leader for the cost she experiences from contributing to the same charity as someone of low status. Specifically, conditional on a low-status follower mimicking her action, a high-status leader contributes if the follower's status is not too low and is compensated by the net return from the public good, specifically when $s_A - s_B < 2m - 1$.[14] When the difference in status between the two donors is neither too large nor too small, i.e., $2m - 1 > s_A - s_B > 1 - m$, we see that contributions can be secured by first soliciting the high-status person, then announcing the contribution, and asking the low-status person to give.

While participants of low status prefer to associate with those of higher status, their ability to do so is limited by the contribution ordering. In particular they are only able to do so in the sequential-move game where those of high-status give first.

Interestingly a status differential can give rise to an efficient outcome, where both individuals contribute to the public good. This suggests that in contrast to the common view that status acquisition is wasteful and decreases overall welfare, there may be cases where this need not be the case. In particular the resulting status differential may facilitate a contribution game, which generates welfare improvements that outweigh any status acquisition

[13] Romano and Yildirim [75] extend Varian's [81] analysis of the classical voluntary contribution model to more general preferences. Maintaining the assumption that leaders benefit from the follower's contribution, they show that contributions in the sequential game will be larger than in the simultaneous one as long as the best response function of the follower is increasing in the leader's contribution.

[14] This condition is not relevant when participants only aim to be with those of higher status, but do not care about being associated with those of lower status (e.g., $max\{(s_j - s_i), 0\}.g_A g_B$). In this case the leader's contribution is triggered as long as we consider a social dilemma where $2m - 1 > 0$.

costs. [15]

Our analysis of this simple example provides some important insights. Consistent with the frequently observed solicitation ordering, we show that status differentials may cause a contribution-maximizing fundraiser to have an optimal solicitation ordering, whereby he first solicits the high-status donor. Furthermore, such a contribution order can arise even in the absence of a fundraiser. The reason is that since the high-status individual is better off contributing and triggering the contributions of others, he will volunteer to go first.

1.3 EXPERIMENTAL DESIGN

To examine whether status differentials may be an alternative explanation for the frequently observed contribution ordering, we need to determine whether our predicted comparative statics are good approximations of actual behavior.

Using experimental methods we study behavior in a voluntary contribution game where some individuals have higher status than others.[16] A number of alternative approaches can be used to examine the effect. One possibility is to rely on individual characteristics that previously have been thought to be associated with high status, e.g., gender and height.[17] Another possibility is to induce status in the laboratory. There are several reasons why we choose the latter of these two approaches. First, in our study it is crucial that participants agree on who has high versus low status. However, individuals have several different status characteristics, and while we may be sorting them according to one characteristic, partici-

[15] The argument here differs from that of Congleton [17]. He demonstrates that acquisition of status need not be wasteful if status is acquired from investing in a good that has positive externalities. In contrast our results suggest that existing status differences may influence subsequent behavior and cause an improvement in welfare even when status acquisition is costly.

[16] The experiment was programmed and conducted using the software z-Tree (Fischbacher [35]).

[17] An example of a study that examines the effect of an individual's prior status is Bohnet and Hong [14]. Using individual characteristics to classify individual status (e.g., gender, race, religion), they find that while low- and high-status groups are equally unlikely to trust others, the motives for distrust differ. Individuals with high-status characteristics do not trust because they fear betrayal, and those with low-status characteristics do not trust for fear of inequality (but are indifferent to the risk of betrayal). Since individuals are not aware of the characteristics of their opponent in the study, they examine only the effect an individual's status has on own behavior. Glaeser et al. [38] measure an individuals status by characteristics such as whether you have a sexual partner, drink alcohol on weekends, and family education. They too find little effect of status on trusting behavior, but find that those of higher status elicit more trust worthiness.

pants may instead be focusing on a different characteristic. Second, even if individuals pay attention to the dimension by which we sort them, there may not be agreement on what constitutes high or low status. For example, individuals may disagree on how geeks or jocks rank in the social hierarchy. Third, and perhaps most importantly, commonly accepted status characteristics may not only be indicative of an individual having higher status, but also of them having different preferences. For example, while gender and height commonly are used to characterize an individual's status, others have argued that preferences also differ along these dimensions. For example, some studies have found males to be more risk seeking, less reciprocating, more trusting, and less altruistic than females.[18] If these characteristics are correct then a study using males (as high status) and females (as low status) may find results consistent with the predicted comparative statics simply because preferences differ systematically by gender. In particular, we may be mislead to interpret the results as suggesting that status influences behavior, when instead the results are caused by generous and reciprocating female followers being more likely to mimic the leader's action, and the risk seeking more trusting male leaders being more willing to make an initial contribution.[19]

To secure that participants in the laboratory jointly recognize the status differential, and to avoid that our results potentially are driven by differences in preferences, we follow the Ball et al. [12] procedure to induce a status differential among our participants.[20]

Mirroring our simple model we pair participants in groups of two, where one person has higher status than the other one. We study voluntary contributions in two different treatments that only differ in the participants' contribution order. In one treatment participants with high status contribute before those of low status, in the other, the contribution order

[18] E.g., Croson and Buchan [19] and Chaudhuri and Gangadharan [16] find that men exhibit greater trust and women show higher levels of reciprocity. Eckel and Grossman [26] find that women are more risk averse and Eckel et al. [27] find that females are more altruistic. Andreoni and Vesterlund [4] describe a more complex picture of gender differences in altruism. See Eckel and Grossmann [28, 29] as well as Croson and Gneezy [20] for reviews of experimental research on gender differences.

[19] Taller people have also been found to be less altruistic than shorter people (Harbaugh et al. [46]). Persico et al. [69] find that while taller men earn more than short ones, this correlation can be explained using height at age 16. Height as an adult does not add any additional explanatory power. They attribute this to the fact that taller adolescents report have larger social networks, which they hypothesize lead to the development of skills that are valuable in the labor market. Thus observed differences between short and tall people need not be due to a status differential.

[20] Inducing status differences gives participants a common experience thereby diminishing the possibility that there is disagreement on who has higher status.

is reversed. Thus members of the higher-status group are assigned to one of the two roles (leaders or followers) facing members of the other group in the other role.

We ran four sessions of each treatment with 12 participants in each session. A total of 96 participants were recruited from the Pittsburgh area and were randomly assigned to a treatment.[21] No participant was allowed to participate in more than one session of the experiment.

An identical protocol was used in each of the two treatments.[22] The protocol consisted of two parts. The first was a status-inducement exercise and the second a sequential voluntary-contribution game.

Upon arrival, participants were seated throughout the laboratory. They were given the first part of the status-inducement exercise, as they were asked to answer a trivia quiz which contained ten general knowledge questions with numerical answers.[23] Participants were told they would receive $5 for completing the quiz, and that their answers to the quiz would be used to determine their role in the experiment. Once everyone had completed the quiz and it was collected, an experimenter proceeded to hand out the instructions for the sequential voluntary-contribution game.

While one experimenter read the instructions for the voluntary-contribution part of the experiment, a second experimenter (who was seated towards the front of the room) reviewed the trivia quiz answers and determined which participants would be assigned to either a star-group (high-status) or a no-star-group (low-status). To secure that participants randomly were assigned to the star- versus no-star-group we used the size of the answer to the last question on the trivia quiz to determine their assignment. In half the sessions participants in the star-group consisted of the six participants who provided the largest numerical answers to the last question, and in the remaining sessions members of the star-group were those who provided the smallest numerical answer.[24] Participants were not told what the assignment

[21] The majority of the participants were undergraduate students at the University of Pittsburgh.

[22] A copy of the trivia quiz and the instructions for the experiment can be found in the Appendix A.

[23] It is generally argued that status may arise in any dimension individuals care about. One such dimension is knowledge, e.g., Nie et al. [68] argue that education is a good proxy for relative status.

[24] Our procedure differs slightly from that of Ball et al. [12] where the quiz consisted of 5 obscure economic questions with numerical answers, and assignment to the star-group was based on the sum of the five numerical answers. Surprisingly they find that status has less of an effect in an "awarded" than in an obviously random-status treatment. They argue that this most likely is because the test was considered

procedure was.

Once the instructions for the voluntary contribution game were reviewed participants were asked to calculate payoffs for the possible decisions that may occur in the game. The answers to these questions were then presented on the blackboard. Participants were allowed to ask questions by raising their hand and speaking to the experimenter in private. No communication among participants was allowed.

Having completed the instructions to the decision-making part of the experiment, we then held an award ceremony to assign participants to the two groups. We first called out the ID numbers for those who were assigned to the star-group. One by one they were invited to come to the front of the room where they were given a shiny black folder with a gold star as well as a congratulatory ribbon which they were asked to wear for the remainder of the experiment. A public applause was given once all six members of the star-group were standing at the front of the room. Members of the star-group where then seated in the two front rows of the laboratory. The walls of this section were marked by three large gold stars, and the individual computers had a gold-star sticker attached to the board. While seating members of the star-group, members of the no-star-group were asked to come and receive a yellow manila folder, and were then seated in the back two rows of the laboratory.

Once everyone was seated we reviewed the content of the folders. The content of the two types of folders were the same, both included a brief summary of instructions and a record sheet. We then read the summary of instructions and began the voluntary-contribution game. The game consisted of 12 contribution rounds. In each round, a star participant was anonymously and randomly paired with a no-star participant, under the stipulation that no participant was paired with another participant twice in a row, and that no two participants could be paired more than twice during a session.

In each round participants were given the choice between two actions A and B.[25] Choosing A gave the participant a \$1 payoff, while choosing B provided both players with a payoff of 75 cents. Choosing A corresponds to not contributing ($g_i = 0$) and choosing B corresponds to contributing ($g_i = 1$). After leaders made their decisions, followers observed the

unfair. We therefore choose to modify the questions such that they do not relate to economics, and may be considered more fair.

[25] The voluntary contribution game mirrors that of Potters et al. [71] when $m = 0.75$.

leaders' contributions and made their decisions. In one treatment members of the star-group were asked to contribute first, we refer to this as the Star-First treatment, in the other treatment members of the no-star-group were asked to contribute first, we refer to this as the Star-Second treatment. At the end of each round the participants were informed about the choices and the payoffs in their game, and they recorded this information on their record sheets.

At the end of the 12 rounds participants were asked to come to a back room where they were paid in private for their participation in the two parts of the experiment. Each session of the experiment lasted a little less than an hour and average earnings were $18.93 (with a minimum of $15.5 and a maximum of $23).

1.4 RESULTS

Our analyses of the data focus on testing the predicted comparative statics of our model. In particular we examine whether total contributions to the public good are larger when individuals of high status contribute first. We then examine the underlying dynamics of such a finding, in particular we see if low-status followers are more likely to mimic the leader's contribution, and whether in anticipation of such a response high-status leaders contribute more frequently than low-status leaders. While confirmatory answers to these questions will be seen as supportive of our simple model, one must keep in mind that the results are very sensitive to the status differential we manage to induce in the laboratory. The induced status differential depends crucially on the experimenter's ability to convince participants that there is a difference, and the resulting differential is likely to be minimal relative to those that arise in the real world.

Observing the participants' behaviors in the laboratory suggest that they did care about the status-inducement part of the experiment. They were anxious to get the results of the quiz, and those assigned to the star-group seemed very pleased with themselves, while those in the no-star-group did not. There is less evidence that they consciously thought about their assignment when making decisions in the voluntary-contribution part of the experiment. In our open-ended exit survey regarding their voluntary-contribution decisions, only one of our participants made reference to the star- vs. no-star assignment. The participants' behavior upon leaving the experiment suggests, however, that they still cared about their assigned role. While all participants were asked to leave their folders and other material by their computer, members of the star-group frequently brought their shiny folders and ribbons with them to receive payment, such behaviors were never observed among the no-star members.

Despite inducing a potentially small status differential, our results from the Star-First and Star-Second treatments strongly support the predicted comparative statics. Figure 1 shows the average group contribution per round. With two people in each group the maximum contribution is 2. We see that on average group contributions are 80% larger when high-status participants contribute first. As usual in public-good games, the frequency of contributions is larger in the first half than in the second half of the experiment, and this decrease in contributions is observed in both treatments. Note however that the difference between treatments does not decrease. Whether we look at the first or second half of the experiment, aggregate contributions remain larger in the Star-First treatments. Using each session as the unit of observation the conservative Mann-Whitney U-test reveals that these differences in aggregate contributions are statistically significant, whether we look at the entire experiment, or only the first half or second half of the experiment.[26]

Thus letting the high-status leader contribute first rather than last has a substantial and significant effect on aggregate contributions. Next we examine whether behaviors by followers and leaders are consistent with those predicted.

We start by examining the frequency by which followers mimic the leader's contribution. While not contributing remains the payoff-dominant strategy for the followers, Figure 2 shows that a number of followers nonetheless contribute and that such deviations are more common in the Star-First treatment. While low-status followers on average mimic high-status leaders 45% of the time, this only happens 30% of the time with high-status followers and low-status leaders. This difference is statistically significant, i.e., we reject the null hypothesis that the follower is less likely to mimic a leader contribution when the high-status participant gives first (one-sided p-value is .0786). In contrast, when the leader does not contribute, only

[26] The three one-sided p-values are no larger than 0.0571. A summary of the reported statistical tests can be found in Appendix B.

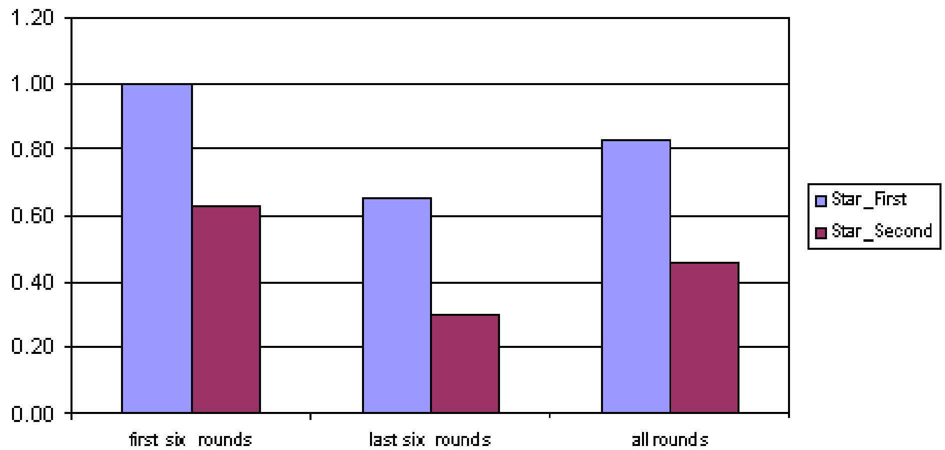

Figure 1: Average Group Contributions per Round

6% and 5% of the followers choose to contribute in Star-First and Star-Second treatments, respectively. Hence, by contributing a leader can increase the probability that a follower contributes by 39% in the Star-First treatment and by 25% in the Star-Second treatment. While this difference may appear small it implies that a payoff-maximizing leader prefers to contribute when she is of high status, but not when she is of low status. Since the cost of contributing is 25 cents, and each contribution by the follower generates a leader payoff of 75 cents, the leader is better off contributing as long as her contribution increases the probability that the follower contributes by 33%.

Next we examine whether leaders in the two treatments appear to anticipate the follower's response. Figure 3 illustrates the leader's contribution frequency in each treatment. As predicted high-status leaders are more likely to contribute than low-status leaders. While the contribution frequency is 55% among high-status leaders, it is only 33% among low-status leaders. Despite the decrease in contributions over the course of the game, the difference between the contribution frequencies remains substantial and in the $22-23$ percentage-point range. This difference is significant whether we examine the entire experiment, or only the first or second half of the experiment.[27] The larger contribution rate among high-status

[27] One-sided p-values for the three tests are no larger than 0.0429.

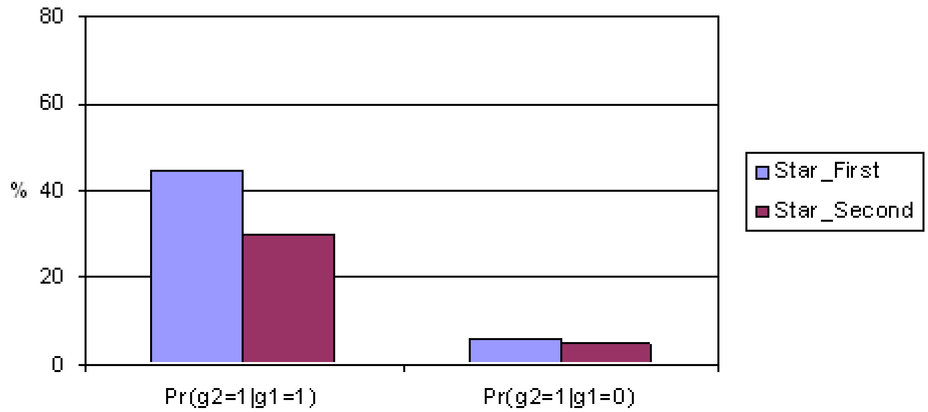

Figure 2: Conditional Contribution Probabilities

leaders suggests that they, from the very beginning of the game, have different expectations about the follower's response, and that this difference in expectations between the two treatments is maintained throughout the game.

While the larger leader contributions in the Star-First treatment are consistent with our predictions, they are not necessarily what we would have anticipated in light of past experimental results. For example, Hoffman and Spitzer [48] show that individuals who earn a role in a simple bargaining game feel entitled to that role and tend to make less generous offers. To the extent that our high-status leaders feel that they are entitled to their role of leaders, one may therefore have anticipated that they contribute less, rather than more, than low-status leaders.

Combined the higher leader-contribution rates and the more frequent mimicking in the Star-First treatment has substantial implications on the follower-contribution rates in the two treatments. These are illustrated in Figure 4. The follower-contribution frequency is much larger in the Star-First than Star-Second treatment. While low-status followers contribute 27% of the time, high-status followers only do so 13% of the time. This difference in contributions is significant.[28]

[28] One-sided p-values for all 12, the first six, or the last six rounds are no larger than 0.0571.

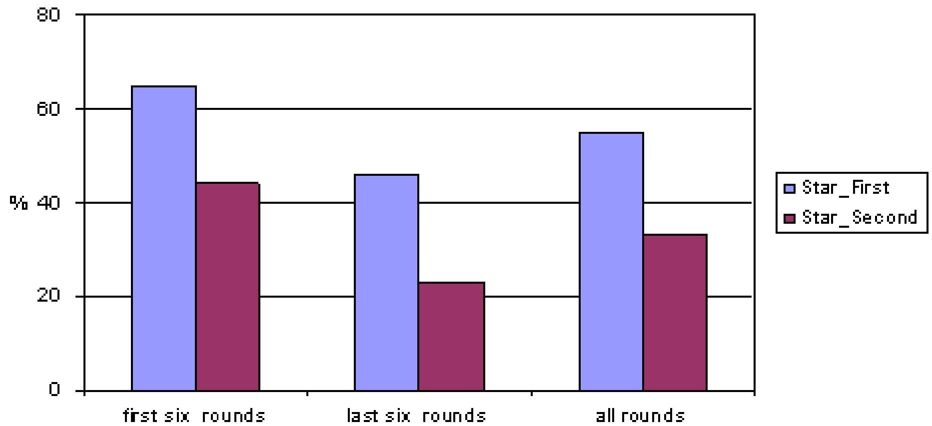

Figure 3: Frequency of Contributions by the Leader

Interestingly behavior in the Star-Second treatment is very similar to what has been seen in experiments absent status. Potters et al. [71] uses the same type of environment to analyze the effects of announcements on contributions when there is uncertainty about the quality of the public good. They observe that if there is no uncertainty and $m = 0.75$, the leader's contribution frequency is 27%, and conditional on the leader contributing the follower contributes 33.3% of the time. Their findings are very close to what we observe in the Star-Second treatment, where low-status leader's contribution frequency is 33%, and conditional on the leader contributing the high-status follower contributes 30% of the time. Thus while behavior in the Star-Second treatment differs from the economic prediction that there be no contributions, as predicted individual contributions are very similar to what is observed in an environment without status.

As an alternative explanation for our results it may be argued that perhaps the difference between treatments is caused by followers being likely to mimic behavior of those who are perceived to be more intelligent but not those who are less intelligent. In considering this explanation it is important to recognize that intelligence is considered an important status indicator. Thus if mimicking by low-status followers is driven by a desire to associate with those of greater intelligence then this interpretation is in line with our status model. This is

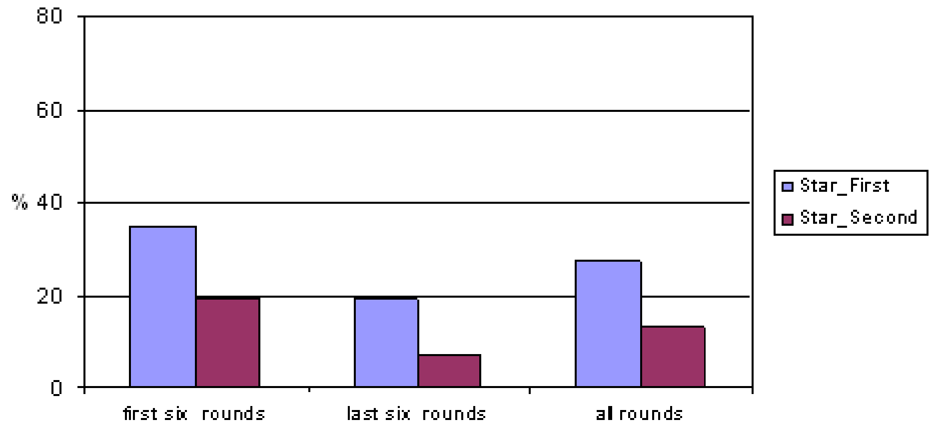

Figure 4: Frequency of Contributions by the Follower

however not the case if individuals mimic the high-status leaders because these are perceived to have a superior understanding of the game. There are a number of reasons why we do not think that the latter is a convincing explanation for our results. First, the examined game is an exceptionally simple one and it is hard to imagine that perceived superior performance on our unrelated trivia quiz would be seen as an indicator of greater game theoretic insights. Second, if our results are driven by participants mimicking behavior of those who are more intelligent, and not those who are less intelligent. Then we should observe treatment differences both when the leader does and does not contribute. However, when the leader fails to contribute, the contribution rate of the follower is $5 - 6\%$ independent of treatment.[29]

Another suggested explanation for our results is that perhaps high-status individuals simply feel more generous once they have been rewarded high status. Note however that while this explanation is consistent with the observation that high-status leaders give more, it is not consistent with the fact that high-status followers give less than low-status followers.

Next we examine the effect of status on individual earnings. A common view among economists has been that individuals acquire status not because they value status itself, but

[29]Similarly if participants perceived low-status leaders as being less intelligent then it is somewhat surprising that the behavior in our Star-Second treatment is so similar to a no-status treatment. Finally, if participants perceived members of the star group as being more intelligent, then we would have anticipated that such an effect would have been mentioned on the exit questionnaire.

rather that they seek it because high status allows higher incomes and better consumption opportunities (see e.g., Postlewaite [70]).[30] The prediction that high-status individuals earn more has been confirmed in a couple of studies. For example, Ball and Eckel [11] examine bargaining games and show larger high-status earnings. Ball et al. [12] show that this finding extends to competitive markets, where low-status individuals appear to bear the cost of associating themselves with the high-status individuals. Similarly Glaeser et al. [38] find evidence suggesting that individuals with high-status characteristics tend to extract larger rents from a voluntary non-market transaction, namely a trust game.

The prediction of our simple model, however, is not that high-status individuals have higher earnings than those of low status. While no earnings differential is predicted in the Star-Second treatment, differences may arise in the Star-First treatment. Specifically, when agents have heterogeneous status concerns it is entirely possible that earnings of the high-status leader be smaller than those of the low-status follower. The reason is that low-status followers only contribute when a high-status leader has already done so. Suppose that only some low-status followers are willing to mimic the contribution of a high-status leader, and that the proportion doing so is large enough to provide some leaders with an incentive to contribute. Such an environment will cause the average contributions of high-status leaders to be larger than those of low-status followers, and as a result individuals of high-status will on average earn less than those of low-status.

Table 1 reports average earnings from the two treatments. Focusing first on the Star-First treatment, we see high-status leaders earning less than their low-status followers.[31] Thus, contradictory to the common economic assumption, in the Star-First treatment high-status individuals earn less, rather than more, than those of low status. As pointed out above this finding is entirely in line with our simple model. In contrast the earnings differential found in the Star-Second treatment contradicts the prediction. Rather than finding no difference we see low-status leaders earning less than their high-status followers.[32] Independent of the treatment, both star- and no-star-participants earn more as followers.[33] This latter finding

[30] In contrast, biology and evolutionary psychology, proposes that people pursue status as an (emotional) goal in itself, independent of the other benefits status may engender.

[31] One-sided $p = 0.0143$ over the 12 rounds or the first six rounds, and $p = 0.0286$ for the last six rounds.

[32] One-sided $p = 0.0143$ over the 12 rounds or the first six rounds, and $p = 0.0429$ for the last six rounds.

[33] For star participants we can reject the hypothesis that leader earnings exceed those of followers over the

implies that in contrast to the theory high-status individuals will not necessarily volunteer to move first.[34]

Table 1: Average Earnings ($) per Participant per Session

	Leaders	Followers
Star-First	12.8	16.1
Star-Second	12.2	14.6

So given these results what contribution ordering should we anticipate?[35] To the extent that a contribution-maximizing fundraiser is in charge of a campaign it is clear that he first will solicit donors who have high social ranking. Interestingly this contribution ordering not only maximizes the contributions but also aggregate earnings. The sum of leader and follower earnings is significantly larger in the Star-First treatment.[36] This suggests that while an individual's status-seeking activities may be a wasteful use of productive resources, such activities may nonetheless be welfare improving. When individuals hold different ranks in a social hierarchy a properly selected solicitation ordering may give rise to contribution levels that are not achievable absent differential rankings.

12 rounds and the first six rounds (one-sided $p = 0.0286$ and $p = 0.0429$ respectively). We cannot reject this hypothesis during the last six rounds ($p = 0.1357$). For non-stars we reject the hypothesis that earnings are larger as a leader than follower overall and for both the first and second half of the experiment (one-sided $p = 0.0143$ in all three cases).

[34] Caution needs to be taken when comparing these two exogenously imposed contribution orders. As shown by Potters et al. [71] behavior may differ when the contribution order arises endogenously.

[35] In fact, fundraisers have one more alternative strategy which prescribes the simultaneous move of donors. However, Potters et al. [72] observe that when there is no uncertainty and $m = 0.75$, average contributions in the simultaneous-move and the sequential-move games are almost identical. As demonstrated earlier, the difference between contribution levels in the Star-Second treatment and Potters et al. sequential-move treatment is negligible. This may be seen as evidence that a fundraiser will not prefer simultaneous-move game in this environment.

[36] One-sided $p = 0.0143$ over the 12 rounds and the first six rounds, and $p = 0.0571$ for the last six rounds.

1.5 CONCLUSION

Fundraisers often start their campaigns by soliciting the wealthier, more recognized, and respected individuals in a community. We have examined whether concerns for status may help explain such a solicitation strategy. Assuming that individuals prefer to associate with those of higher status, we use a simple linear example to show that the observed solicitation strategy is likely to benefit all the involved parties. Asking individuals of high status to give before rather than after those of lower status will not only result in an increase in overall contributions, but also in an improvement in welfare. Thus the organization, the fundraiser, and the associated donors will benefit from there being a differential in social status. Interestingly this implies that costly status acquisition may be welfare enhancing.

Evidence from our laboratory experiment is consistent with the predicted dynamics of our model. When individuals with high status contribute first, low-status followers are likely to mimic the initial contribution, thereby providing the leader with a monetary incentive to give. In contrast if a low-status individual contributes first, the high-status follower is reluctant to mimic the initial contribution.

Despite inducing a small status differential in the laboratory, we nonetheless find that, provided the correct contribution ordering, such a differential can give rise to a welfare improving and large increase in total contributions. In light of the more substantial status differences that occur in the real world, fundraisers may therefore be well served to first solicit donors who have high status. Of course this is precisely what fundraisers tend to do. When asked why they start by soliciting the wealthier and more prominent individuals, their explanations tend to be that this strategy helps create enthusiasm around the campaign.[37] There is however no explanation provided for why this is the case. Our paper has shown both theoretically and experimentally that one such explanation may be that individuals like to associate with those who have higher status than themselves. When asked more specifically if the commonly used strategy may work because it enables subsequent donors to associate

[37] The chairman of the trustees of Johns Hopkins explains that the reason why the university asks donors for permission to announce their gifts is that "fundamentally we are all followers. If I can get somebody to be the leader, others will follow. I can leverage that gift many times over." The New York Times, February 2, 1997, p. 10. Our study demonstrates that this statement relies crucially on who is used as the leader.

with the initial donors, one fundraiser commented that indeed the strategy appears to work well when it enables new money to associate with old money.

2.0 SOCIAL SECURITY AND SELF-CONTROL PREFERENCES (WITH ATHANASIOS C. THANOPOULOS)

2.1 INTRODUCTION

Historically, the necessity of a social security system in the U.S. emerged as a consequence of the Great Depression for the purpose of inter-temporal distribution of the impact of economic crises. Since only certain generations experience adverse shocks, albeit not due exclusively to their own actions or choices, it has been considered as socially desirable (and coherence-promoting) to devise a way to share that risk among different generations.

Nowadays, a social security system is principally considered to be a mechanism that provides pension payments and associated benefits to retirees and their families. Naturally, the apparent merits it has for its beneficiaries are coupled with substantial costs for the economy: Thanks to the democratization of the aforementioned benefits but also due to adverse demographics, during the most recent decades social security spending has evolved into the largest item in the government budget.

Therefore unavoidably, social security has become the object of increasingly intense study by economists. There is an abundance of studies related to the importance of social security and its impact on welfare. The primary reason for this is its dramatically growing scale which triggers -but also never ceases to feed- an effervescent debate regarding the optimal allocation of the -anyway scarce- resources. That controversy stems from the huge monetary burden that the mere presence of a social security system entails for the society and the associated budget implications: In a world of limited resources and ever-rising budget deficits, the dilemma of directing resources either to social security or to alternative uses becomes more vital than ever.

During the second half of the twentieth century, several studies seem to emerge as direct or implicit offspring's of this debate. Most of these focus on the welfare implications of alternative social security systems in an economy. In the very core of this debate and deliberately touching critical social coherence issues, one can clearly identify the dilemma between an "unfunded" (Pay-As-You-Go) versus a "funded" social security system. In an unfunded system, resources are transferred statically from the working population to the concurrent retirees (inter-generational transfers). In contrast, a funded system prescribes a dynamic allocation of resources within the same generation (inter-temporal within the same generation transfers). While both systems rely on an external institution (e.g. government) in order to be implemented, their different logic and mechanics eventually induce entirely different allocations of risk. Therefore, their welfare implications may significantly differ because of this difference.

Departing from the general problematic of "unfunded versus funded" and henceforth confining our analysis to the former, the debate about the benefits and costs of social security also becomes more specific. While its economic benefits are largely summarized in providing intra- and inter-generational risk sharing, social security encourages early retirement and entails very severe distortions in agents' labor supply and private savings decisions.[1] The latter can be readily shown in an overlapping generations model where consumers inelastically supply labor (Diamond [22]): Since social security redistributes income from the young to the old generation by imposing a tax on current workers' income (payroll tax) -i.e. from a generation with low propensity to consume to a generation with a high propensity to consume- it lowers savings and consequently, the steady-state capital stock.[2] Auerbach and Kotlikoff [9], Imrohoroglu et al. [51], and Hugget and Ventura [50] comprehensively analyze the welfare implications of an unfunded social security system by using a large-scale overlapping generations model. All these studies show that an unfunded social security system's distortions in the amount of labor supply and capital accumulation exceeds it's

[1] The -inherent to the insurance market- adverse selection problem induces a very thin annuities market both in the U.S. and elsewhere. Hence, an unfunded social security system becomes an important tool for providing risk-sharing. See Imrohoroglu et al. [51] and Diamond et al. [25] for more discussion on the market for annuities.

[2] Reducing the steady state capital stock decreases welfare if it is below the "golden rule" level. However in the opposite case (when the economy is "dynamically inefficient") reducing capital improves welfare. Abel et al. [1] report that the US economy is dynamically efficient.

benefits and hence it's existence in an economy reduces overall welfare.

Interestingly, the redistribution mechanism of social security and it's induced between-and-within generations allocation of risk is not the only factor that positively affects welfare: Potential idiosyncrasies in agents' preferences highlight yet another extremely important source of ambivalence with regard to the welfare implications of social security. Many studies, both theoretical and empirical have argued on the welfare gains that can be accrued thanks to social security when households lack the foresight to save adequately for their retirement.[3]

Two different research approaches have provided theoretical machinery as well as empirical support that could serve in explaining the observed anomalies: It is well documented in the experimental economics literature that subjects facing inter-temporal choice problems often exhibit preference reversals, or that their preferences feature some kind of time inconsistency. In particular, when they are asked to choose between a smaller-earlier and a larger-later reward, they seem to prefer the earlier reward when it offered an immediate payoff, whereas the larger-later reward was preferred when both rewards were to be received with delay (Gul and Pesendorfer [41]). Furthermore, during the last decade, advances in the theory battlefield have elucidated the underlying factors that induce these anomalies. These being well-known issues in the psychology literature, it would only be a matter of time until they attracted economists' interest: Incorporating disproportionate discounting of the immediate future as an alternative to exponential discounting emerges through a series of studies. Laibson [58], in particular, uses a quasi-hyperbolic (or quasi-geometric) discounting structure to incorporate possible preference reversals into economic theory. A quasi-hyperbolic discounting structure provides the possibility to modify macroeconomic models which are otherwise inadequate to explain the observed facts in the data.[4] Models allowing quasi-hyperbolic discounting seem to better explain the empirical facts compared to models with the standard exponential discounting structure.

In a recent study that enhances considerably the insights found in Feldstein [31] by exploiting these new theoretical advances, Imrohoroglu et al. [53] investigate the welfare

[3] Imrohoroglu et al. [53] provide a concise review of the relevant literature, as well as an interesting discussion of the debate as to whether myopia is indeed empirically identified from e.g. unforeseen events and other factors that cause a sudden drop in consumption at retirement.

[4] For instance, Diamond and Koszegi [23], Krusell et al. [55], and Krusell and Smith [57] analyze various macroeconomic models by using Laibson-type preferences.

effects of unfunded social security in an economy populated by agents with Laibson-type preferences who suffer from inability to commit to future actions. Laibson [58] shows that an agent endowed with quasi-hyperbolic discounting saves less compared to an agent endowed with exponential discounting (e.g. an agent with CRRA preferences). The reason is that the introduction of quasi-hyperbolic discounting creates a conflict between different intertemporal-selves and the agent cannot commit to his own future actions. Although present-selves think that they save enough, future-selves regret the decisions made by the former-selves. In order to make the quasi-hyperbolic discounter to save as much as the exponential discounter, a commitment technology is required (note that the exponential discounters naturally have this technology). In Imrohoroglu et al., the government saves on behalf of the quasi-hyperbolic discounters through social security system. Their main findings are that: (1) quasi-hyperbolic discounters incur substantial welfare costs because of their time-inconsistent behavior, (2) to maintain old-age consumption, social security is not a good substitute for a perfect commitment technology, and (3) there is little room for social security in a world of quasi-hyperbolic discounters.

In spite of their theoretical appeal in providing an alternative that adequately explains observed patterns of behavior, quasi-hyperbolic discounting models entail a non-recursive structure that renders them computationally intractable. This is because quasi-hyperbolic discounting structure does not allow a desire for commitment to one's future actions.

Gul and Pesendorfer [41] choose a different approach in their attempt to explain preference reversals. They develop self-control preferences that depend on what an agent actually consumes on one hand, and what would be the level of consumption that would explain the experimental phenomenon, on the other.[5] To this purpose, they introduce self-control and temptation utilities, concepts that capture the trade-off between the temptation to consume on the one hand, and the long-run self interest of the agent on the other. Under certain rationality assumptions, preferences over sets of actions are consistent with experimental evidence as in Laibson [58]. In stark contrast to Laibson-type preferences however, self-control preferences are time-consistent. In particular (considering Laibson's framework for example),

[5] We will henceforth use the terms "Gul and Pesendorfer preferences" and "Self-Control preferences" interchangeably. It is worth noting that "Gul and Pesendorfer" preferences is not the only available specification for self-control preferences in the literature.

it is assumed that the preferences governing behavior at time t differ from the preferences over continuation plans implied by the agent's first period preferences and choices prior to period t. In contrast, self-control preferences may already exhibit a desire for commitment, which is not the case in Laibson.

Gul and Pesendorfer's self-control preferences have been utilized in different contexts in various macroeconomic models in an attempt to resolve well-known puzzles in the literature. DeJong and Ripoll [21] provide a concise asset pricing environment in order to investigate the potentiality of self-control preferences to account for the volatility of stock prices. They show that self-control preferences partially explain the level of price volatility that is observed in the data. Similarly, Krusell et al. [56] analyze a general equilibrium asset pricing model where a small subset of investors are tempted to save (not to consume). They contend that their model can help to better understand some aspects of wealth and asset pricing data.

In this paper we explore the role of an unfunded social security system in a setting where agents have self-control preferences. To this purpose, we develop an overlapping generation model in which agents live up to the real age of 85 (corresponds to the model age of 65). The economy consists of three sectors: agents, firms and a government. Agents have idiosyncratic income and face a mortality risk. They work up to the real age of 65 (corresponds to the model age of 45) whenever they have an opportunity to work. When unemployed or retired, they are compensated by the government by unemployment insurance or retirement benefits respectively. In addition, they maintain positive asset holdings in order to insure against idiosyncratic income risks and low old-age consumption. Moreover, we assume that private credit markets (including annuities' markets) are closed. The government collects unemployment insurance and payroll taxes from workers to the purpose of financing its activities.

We compute the steady-state equilibria under different social security replacement rates by calibrating our model economy to the U.S. economy. From previous studies we know that if an economy is populated by agents with constant relative risk aversion (CRRA) preferences i.e., neither facing a commitment nor a temptation problem, the introduction of an unfunded social security system reduces welfare. The reason is that the insurance benefit of an unfunded social security system is dominated by its negative effect on agents' savings decisions. Several interesting insights obtain in our setting: In congruence with Imrohoroglu et al. [53] social security indeed tends to reduce welfare. In particular, when we consider a utility function featuring a concave temptation component, the lifetime consumption path remains essentially invariant to departures from the CRRA case. Accordingly, social security remains equally detrimental to welfare under self control preferences as it is under CRRA preferences. However, in the case of a convex temptation function the degree to which this effect obtains is substantially lower. Controlling for all other factors we infer that this is due to our specification of preferences: Agents with self-control preferences face no commitment problem. Nonetheless, the cost of resisting the temptation associated with the exertion of self-control becomes very severe as wealth increases. In turn, this may impair overall savings in an economy. In our environment, an unfunded social security system has no role as a commitment apparatus but might play a role as a device to decrease available wealth when agents make their consumption-savings decisions.

We identify the underpinnings of our results with the impact social security has on agents' marginal propensity to consume. In the "traditional" setting where agents have CRRA preferences, the young have a low marginal propensity to consume while the old have a high marginal propensity to consume. This relation preserves a high rate of capital accumulation through higher savings during the young age. In contrast, in our environment the young face temptations that operate as impediments to their propensity to (privately) save. Alternatively, the agents' marginal propensity to consume is not as low as it is in the case of CRRA preferences. Accordingly, the cost of resisting temptation increases with the level of wealth. Inevitably, social security by being a mechanism that is bound to deprive agents from early consumption accomplishes at the same time to reduce the cost associated with the exertion of self-control and consequently to partially offset its adverse effect on welfare. Note that this effect is absent in environments where preferences do not allow agents the option to exert self-control as in Imrohoroglu et al. [53].[6]

[6] Diamond [24] argues that the current unfunded social security system does not need radical reform and it is enough to put the system on stronger financial footing while improving the benefit structure at the same time. He states further that mandated savings make sense since many workers would not save enough for their old-age consumption. Our results are in line with those of Diamond in the following sense: When agents are endowed with temptation, they substantially save less due to the burden of resisting the temptation. The current social security system helps agents to overcome the temptation problem and hence, the welfare cost of the system is not very large.

This chapter is organized as follows: The following section provides a concise introduction to self-control preferences and time inconsistency. We briefly present and compare the two theories and attempt to shed light on the different implications they have for the question at hand. Section 2.3 introduces the model economy. In Section 2.4, we define the parameter values of the model and explain the solution methodology. Section 2.5 presents the results and Section 2.6 concludes.

2.2 METHODS OF ACCOUNTING FOR SELF-CONTROL

In this section we briefly highlight the differences between the Gul-Pesendorfer type and the Laibson type of preferences as well as motivate the use of the former in this paper.

2.2.1 Laibson Type of Preferences

In our model preferences are defined over sequences of lifetime consumption, $\{c_1, ., c_k, ., c_T\}$. If agents in the economy have time consistent preferences, and a deterministic life-span equal to T, the utility ranking of the lifetime consumption sequences will not depend on their standpoint k : Ranking of these sequences will be invariant with respect to the time the ranking took place.

The essence of the Laibson type of preferences is that the aforementioned invariance result no longer holds: The discounting structure sets up a conflict between today's preferences and the preferences that will be held in the future, commonly labeled as a "preference reversal." For example, from today's perspective, the discount rate between two far-off periods, t and $t+1$, is the long-term low discount rate, while from the time t perspective, the discount rate between t and $t+1$ is the short-term high discount rate. This can be modeled by the following preference structure adapted to the purposes of our model from Laibson [58]:

$$U_t = E_t\left[u(c_t) + \beta \sum_{j=1}^{T-t} \delta^j u(c_{t+j})\right]$$

When $0 < \beta < 1$, the above discount structure can mimic the qualitative property of the generalized hyperbolic discount function (namely a function implying discount rates that decline as the discounted event moves further away in time, see e.g. Ainslie [2]), but at the same time maintain most of the analytical tractability of the exponential discount function. The preferences given in the above equation are dynamically inconsistent, in the sense that preferences at date t are inconsistent with preferences at date $t+1$.[7]

For an equivalent statement in terms of our setting, suppose that an agent of age k has preferences over lifetime consumption given by

$$U_k = \sum_{t=1}^{k-1} \delta_B^{t-k} u(c_t) + u(c_k) + \beta E_k \sum_{t=k+1}^{T} \delta_F^{t-k} u(c_t)$$

where δ_B is the backward looking discount factor and δ_F is the agent's forward looking discount factor. The above setting was considered in Imrohoroglu et al. [53] and in addition to the "Laibson effect" (which is achieved through the fact that $0 < \beta < 1$) assumes (through $\delta_B > \delta_F$) the "Feldstein effect" (Feldstein [31]) namely, that agents place less weight on the past than they would if $\delta_B = \delta_F$. Both of the effects will lead to regret in later periods.

Note that a major consequence of the Laibson effect is that the optimal policy functions derived at age k for ages $k' > k$ will no longer be optimal when the agent arrives at age k'; and in the absence of any commitment technology, the agent's future behavior will deviate from that prescribed by the earlier policy functions.

2.2.2 Gul & Pesendorfer Self-Control Preferences

An alternative way of modelling self-control issues is a class of utility functions identified by Gul and Pesendorfer [41]. They provide a time-consistent model that addresses the preference reversals that motivate the time inconsistency literature.

Consider a set B of consumption lotteries, and a two-period setting. Gul and Pesendorfer [41] have shown that under a specific assumption on choice sets (set betweenness) combined with other standard axioms that yield the expected utility function $U(.)$ defined as

$$U(B) := \max_{p \in B} \int (u(c) + v(c))\, dp - \max_{p \in B} \int v(c) dp$$

[7] To check this note that the MRS between periods $t+1$ and $t+2$ from the standpoint of the decision maker at time t is given by $\frac{u'(c_{t+1})}{\delta u'(c_{t+2})}$, which is not equal to the MRS between those same periods from the standpoint of the decision maker at $t+1$: $\frac{u'(c_{t+1})}{\beta\delta u'(c_{t+2})}$

represents the preference relation implied by the above axioms. The function $u(.)$ represents the agent's ranking over alternatives when he is committed to a single choice while when he is not committed to a single choice, his welfare is affected by the temptation utility represented by $v(.)$. Note that when B is a singleton, the terms involving $v(.)$ will vanish leaving only the $u(.)$ terms to represent preferences. However, if it is e.g., $B = \{c, c'\}$ with $u(c) > u(c')$ an agent will succumb to the temptation (that is, he will pick the commitment utility-reducing alternative, c') only if the latter provides a sufficiently high temptation utility $v(.)$ in the second period and offsets the fact that $u(c) > u(c')$, i.e., when

$$u(c') + v(c') > u(c) + v(c).$$

In this case the agent wishes he had only c as the available alternative, since under the presence of c', he cannot resist the temptation of choosing the latter.

When the above inequality is reversed, however, the agent will pick c in the second period, albeit at a cost of $v(c') - v(c)$.[8] We call the latter difference the "*cost of self-control.*"

In terms of the setting in the present paper, in every period a household faces a consumption and savings problem. Each period, our agents make a decision that yields a consumption for that period and wealth for the next. However, each period these agents face the temptation to consume all of their wealth, and hence, resisting to this temptation results in a self-control-related cost.

Under standard assumptions combined with the multi-period version of "set betweenness," we can represent self-control preferences in a recursive form for the purposes of our T period model which is delegated to the next section.

[8] To see that, note that for $B = \{c, c'\}$ and $u(c) > u(c')$ we would have that

$$U(\{c, c'\}) = \max_{\tilde{c} \in \{c,c'\}} (u(\tilde{c}) + v(\tilde{c})) - \max_{\tilde{c} \in \{c,c'\}} v(\tilde{c}) =$$

$$= u(c) + v(c) - v(c')$$

and since by assumption $v(c') > v(c)$ this means that

$$U(\{c, c'\}) = u(c) - [v(c') - v(c)]$$

i.e. the utility of the choice c gets penalized by a positive number, the "cost of self-control". Note that in the case $v(c') < v(c)$ i.e. when there is congruance of the utility functions as to which alternative is the best, there is no temptation issue anymore; c is chosen at no penalty since the $v(.)$ terms in $U(\{c, c'\})$ cancel out.

2.2.3 Discussion

The main difference between the above models is that the model of Gul and Pesendorfer [41] does not imply dynamic inconsistency. Preferences are perfectly consistent. Moreover, it allows agents to exercise self-control, an option not existing in Laibson [58]. The difference in discounting was the source of preference reversals in Laibson and the explanation of why agents find immediate rewards tempting. Instead, Gul and Pesendorfer's explanation assumes that agents maximize a utility function that is a "compromise" between the standard utility (or "commitment" utility) and a "temptation" utility.[9]

Imrohoroglu et al. [53] considered a setting similar to ours and analyzed the consequences of time inconsistent preferences à la Laibson, while we follow the self-control paradigm in a similar finite-horizon setting. Gul and Pesendorfer [41] showed that for finite decision problems a time inconsistency model à la Laibson can be re-interpreted as a temptation model. In light of that we consider our work as an extension of Imrohoroglu et al. in that direction.

The purpose of doing so is to check, *inter alia,* if our results encompass the ones of Imrohoroglu et al. [53], or if the fact that agents in our setting are capable of exercising self-control (an option not available in Imrohoroglu et al.), alters their findings substantially.

2.3 A MODEL OF SOCIAL SECURITY

The model we consider in this section is quite standard in the social security literature. In particular, our model closely follows that of Imrohoroglu et al. [53].

2.3.1 The Environment

We consider a discrete time, stationary overlapping generations economy. Each period a new generation is born. Agents live a maximum of T periods. The population grows at a constant rate n. All agents face a probability (s_t) of surviving from age $t-1$ to t conditional

[9] Gul and Pesendorfer [40, 41, 42] thoroughly compare and contrast the differences and similarities between self-control and quasi-hyperbolic preferences.

on surviving up to age $t-1$. Since the economy is stationary, age t agents constitute a fraction μ_t of the population at any given date. The cohort shares ($\{\mu_t\}_{t=1}^{T}$) are given by

$$\mu_{t+1} = \frac{\mu_t s_{t+1}}{1+n}$$

where their sum is normalized to 1.

2.3.2 Preferences

Agents have self-control preferences. In every period they face the temptation to consume their entire wealth. Resisting temptation creates a self-control cost which is absent in the models with CRRA and quasi-hyperbolic preferences. We follow Gul and Pesendorfer [41] and DeJong and Ripoll [21] and model self-control preferences recursively. Let $W(x)$ denote the maximized value of the expected discounted objective function with state x. The utility function of an agent is as follows

$$W(x) = \max_{c}\{u(c) + v(c) + \beta EW(x')\} - \max_{\check{c}} v(\check{c}) \tag{2.1}$$

where E is the expectation operator; $u(.)$ and $v(.)$ are von Neumann-Morgenstern utility functions; $0 < \beta < 1$ is the discount factor; c is commitment consumption; $\check{c}$ is temptation consumption; and x' denotes next period state variable. As in the section above, $u(.)$ represents the momentary utility function and $v(.)$ represents temptation. In particular, $v(c) - \max_{\check{c}} v(\check{c})$ denotes the disutility of choosing consumption c instead of $\check{c}$. The concavity or convexity of $v(.)$ is quite important for our analysis.[10]

The momentary utility and convex temptation functions take the following forms:

$$u(c) = \frac{c^{1-\gamma} - 1}{1-\gamma} \tag{2.2}$$

$$v(c) = \lambda \frac{c^{\rho}}{\rho} \tag{2.3}$$

[10] Notice that if $v(.)$ is convex, we need to make sure that $v(.)+u(.)$ is stricly concave. In particular, $\gamma > 0$, $\rho > 1$ and $0 < \lambda < \gamma/(c^{\gamma+1}c^{\rho-2})$ guarantee that $u(.)$ is concave, $v(.)$ is convex and $u(.) + v(.)$ is strictly concave. When $v(.)$ is concave, one should show that $W(.)$ is strictly concave.

For the balanced growth rate considerations, the concave utility function is chosen as follows:

$$v(c) = \lambda u(c) \tag{2.4}$$

In the specification above, higher values of the scale parameter ($\lambda > 0$) imply an increase in the share of the temptation utility, i.e., a higher λ increases the importance of current consumption for an agent. The momentary utility function $u(.)$ features constant relative risk aversion (CRRA).

2.3.3 Budget Constraints

The exogenously given mandatory retirement age is t^*. Agents who are younger than age t^* face a stochastic employment opportunity. Agents that find a chance to work, inelastically supply one unit of labor.[11] We denote the employment state by $e \in \{0, 1\}$ where 0 and 1 denote unemployment and employment states respectively. The employment state follows a first order Markov process. Transition probabilities between current employment state e and next period employment state e' are denoted by the 2×2 matrix $\Pi(e', e) = [\pi_{k'k}]$ where $k', k = 0, 1$ and $\pi_{k'k} = \Pr\{e' = k' | e = k\}$.

An employed agent earns $w\epsilon_t$ where w denotes the wage rate in terms of the consumption good and ϵ_t denotes the efficiency index of an age t agent. If an agent is at the unemployment state, he receives unemployment insurance benefit equal to the fraction of employed wage ($\phi w \epsilon_t$) where ϕ is the unemployment insurance replacement ratio.

Agents retire at age t^* and receive a lump-sum social security benefit b. The social security benefit b is defined as a fraction θ of an average life time employed income, which is independent of an agent's employment history.

$$b = \begin{cases} 0 & \text{for } t = 1, 2, ..., t^* - 1; \\ \theta \frac{\sum_{t=1}^{t^*-1} w\epsilon_t}{t^*-1} & \text{for } t = t^*, t^* + 1, ..., T. \end{cases}$$

The disposable income of an agent at age t can be written as

[11] Adding labor-leisure choice into the model requires the modification of preferences in a way that agents are not only tempted by current consumption but also by current leisure.

$$q_t = \begin{cases} (1-\tau_s-\tau_u)w\epsilon_t & \text{for } t=1,2,...,t^*-1, \text{ if } e=1; \\ \phi w\epsilon_t & \text{for } t=1,2,...,t^*-1, \text{ if } e=0; \\ b & \text{for } t=t^*,t^*+1,...,T. \end{cases}$$

where τ_s and τ_u represent the social security tax rate and the unemployment insurance tax rate respectively.

We assume away private insurance market against the employment risk and private annuities market against the uncertain life span.[12] The only available device to smooth consumption across one's lifetime is the accumulation of assets in terms of physical capital. Agents cannot hold negative assets at any period.[13] Since death is certain at T and there is no bequest motive, the borrowing constraint can be stated as:[14]

$$\begin{cases} a_t \geq 0 & \text{for } t=1,...T-1 \\ a_t = 0 & \text{for } t=T \end{cases}$$

If agents in this economy die before age T, their remaining assets will be distributed to all of the survivors in a lump-sum fashion. Let η denote the equal amount of accidental bequests distributed to all remaining members of the society:

$$\eta = \sum_t \sum_a \sum_e \mu t \Lambda_t(a,e)(1-s_{t+1})a_t(a,e) \tag{2.5}$$

where $\Lambda(a,e)$ is the set of age dependent, time independent measure of agents.

[12] Although the annuities market exist in U.S., it is very thin (Imrohoroglu [51]). Hence, our assumption seems innocous. In our model, social security partially fullfills the role of the missing annuities market (it can be considered as mandotary annutization). Diamond et al. [25] analyze througly the relationship between annuities and individual welfare. He shows that full annuitization of wealth is optimal under certain conditions.

[13] In other words, an agent faces a borrowing (or liquidity) constraint. Given the size of private credit markets, this assumption may seem not so innocous. There are two main reasons behind this assumption: First, we would like to make careful comparision of our results with those of the existing social security literature and this assumption is the "industry standard." Second, when agents are not allowed to borrow against their future-income, this induces an additional boost in (private) savings for precautionary purposes, since agents may be/remain unemployed with a positive probability. It would be a fair question to explore the consequences of alleviating this constraint in our environment and allow borrowing against future income. In that case however, the ability to borrow would lower agents' marginal propensity to save (for precautionary reasons), thus implying that the effects of self-control and ability to borrow against future income are collinear, hence the effect of social security on savings due to self-control is non-identifiable. In a recent paper, Rojas and Urrita [74] show that adding endogenous borrowing constraint reduces the welfare cost of having a social security.

[14] Allowing bequest motive also changes welfare implications of social security system. Fuster et al. [37] make a welfare analysis of social security in a dynastic framework and show that steady state welfare increases with social security.

Hence, we can write the budget constraint of an agent as follows

$$a_t + c_t = (1+r)a_{t-1} + q_t + \eta, \tag{2.6}$$

$$a_t + \check{c}_t = (1+r)a_{t-1} + q_t + \eta \tag{2.7}$$

where r is the rate of return from the asset holdings.

2.3.4 Production Function

Firms have access to a constant returns-to-scale Cobb-Douglas technology that produces output (Y) by using labor input ($L = 0.94 \sum_{t=1}^{t^*-1} \mu_t \epsilon_t$) and capital input (K) which is rented from households:

$$Y = F(K, L) = AK^{\alpha} L^{(1-\alpha)} \tag{2.8}$$

where A represents the state of technology; $\alpha \in (0, 1)$ is the capital's share of output. Defining the capital-labor ratio as $\frac{K}{L}$, we can write the production function in the intensive form as follows:

$$y = f(k) = Ak^{\alpha}.$$

The technology parameter A grows at constant rate g and capital depreciates at a constant rate δ. Competitive firms in this economy maximize their profits by setting the real rate of return from asset holdings r and the real wage rate w according to the following:

$$r = A\alpha k^{\alpha-1} - \delta, \tag{2.9}$$

$$w = A(1-\alpha)k^{\alpha}. \tag{2.10}$$

2.3.5 Government

In our setting, the government's responsibility is limited to the task of administering the unemployment insurance and social security programs. The only constraint imposed on the government's behavior is to enforce self-financing of both the unemployment and social security programs. We restrict our attention to social security arrangements that are described by the pair $(\theta,\ \tau_s)$. The self-financing conditions are as follows:

$$\tau_s \sum_{t=1}^{t^*-1} \sum_a \mu_t \Lambda_t(a, e = 1) w \epsilon_t = \sum_{t=t^*}^{T} \sum_a \mu_t \Lambda_t(a, e) b \tag{2.11}$$

and

$$\tau_u \sum_{t=1}^{t^*-1} \sum_a \mu_t \Lambda_t(a, e = 1) w \epsilon_t = \sum_{t=1}^{t^*-1} \sum_a \mu_t \Lambda_t(a, e = 0) \phi w \epsilon_t \tag{2.12}$$

2.3.6 An Agent's Dynamic Program

We suppose that the temptation function $v(.)$ is strictly increasing, i.e., an agent is tempted to consume his entire wealth in each period. This implies that the agent maximizes the second part of equation (2.1) by holding zero asset for the next period, i.e., setting $a_t = 0$ in equation (2.7). In this economy, the agent's state vector x contains the current asset holdings and the employment state. Hence, we can write the agent's dynamic program for any arbitrary two period as follows

$$W(x) = \max_c \{u(c) + v(c) + \beta E_{s'} W(x')\} - v((1 + r)a + q + \eta) \tag{2.13}$$

subject to

$$a' + c = (1 + r)a + q + \eta,\ a' \geq 0,\ a_0 \text{ is given} \tag{2.14}$$

where $E_{s'}$ denotes the expectation over survival probabilities.

If the agent succumbs to a temptation and consumes his entire wealth, the term $v(c) - v((1 + r)a + q + \eta)$ in the equation above cancels out. When he resists to temptation and consumes less than his wealth, he faces a self-control cost at the amount of $v(c) - v((1 + r)a + q + \eta)$. The agent tries to balance his urge for current consumption $v(c)$ and long-term commitment utility $u(c) + \beta E_{s'} W(x')$.

2.3.7 Steady State Equilibrium

In our characterization of the steady state equilibrium, we follow Imrohoroglu et al. [53] and Huggett and Ventura [50].

Given a set of government policy $\{\theta, \phi, \tau_s, \tau_u\}$, a steady state recursive competitive equilibrium is a set of value functions $\{W_t(x)\}_{t=1}^T$, household's policy rules $\{a_t(x)\}_{t=1}^T$, time invariant measures of agents $\{\Lambda_t(x)\}_{t=1}^T$, wage and interest rate (w, r), and a lump sum distribution of accidental bequests η such that all of them satisfy the following:

- Factor prices (w, r) that are derived from the firm's first order conditions satisfy the equations (2.9) and (2.10).
- Given government policy set $\{\theta, \phi, \tau_s, \tau_u\}$, factor prices (w, r), and lump-sum transfer of accidental bequests η, an agent's policy rule $\{a_t(x)\}_{t=1}^T$ solves the agent's maximization problem (2.13) subject to the budget constraint (2.14).
- Aggregation holds,

$$K = \sum_t \sum_a \sum_e \mu_t \Lambda_t(x) a_{t-1}(x) \tag{2.15}$$

- The set of age-dependent, time-invariant measures of agents satisfies in every period t

$$\Lambda_t(x') = \sum_e \sum_{a:a'=a_t(x)} \Pi(e', e) \Lambda_{t-1}(x) \tag{2.16}$$

  where Λ_1 is given.
- The lump-sum distribution of accidental bequests η satisfies the equation (2.5).
- Both the social security system and the unemployment insurance benefit program are self-financing i.e., satisfy the equations (2.11) and (2.12) respectively.
- The market clears

$$\begin{aligned} & \sum_t \sum_a \sum_e \mu_t \Lambda_t(x)[a_t(x) + c_t(x)] \qquad (2.17) \\ = \; & Y + (1-\delta) \sum_t \sum_a \sum_e \mu_t \Lambda_t(x) a_{t-1}(x) \end{aligned}$$

2.4 CALIBRATION

In this section, we briefly define the parameter values of our model. Each period in our model corresponds to a year. We closely follow Imrohoroglu et al. [53] in order to be able to compare our results to those obtained there.

2.4.1 Demographic and Labor Market Parameters

Agents are born at a real life age of 21 (model age of 1) and they can live up to a maximum real life age of 85 (model age of 65). The population growth rate n is assumed to be equal to the average U.S. population growth rate between 1931-2003 which corresponds, on average, to 1.19% per year.[15] The sequence of conditional survival probabilities is the same as the Social Security Administration's sequence of survival probabilities for men in the year 2001. The mandatory retirement age is equal to 65 (model age 45). In order to set the efficiency index, we choose the average of Hansen's [43] estimation of median wage rates for males and females for each age group. We interpolate the data by using the Spline Method and normalize the interpolated data to average unity. The employment transition probabilities are chosen to be compatible with the average unemployment rate in the U.S. which is approximately equal to 0.06 between 1948 and 2003.[16] The implied employment transition matrix assumes the following form:

$$\Pi(e, e') = \begin{bmatrix} 0.94 & 0.06 \\ 0.94 & 0.06 \end{bmatrix}$$

2.4.2 Preference Parameters

We choose the values of preference parameters ρ, γ, λ and β in such a way that our model-economy's capital-output ratio matches that of the U.S. economy.

In the case where the temptation function $v(.)$ is convex, we choose to follow Imrohoroglu et al. [53] and DeJong and Ripoll [21], in letting γ be centered at 2 with a standard deviation 1, i.e., $\gamma = 2$ (1). In our benchmark calibration, we initially set $\gamma = 2$, and then check for

[15] The population data was obtained from the U. S. Census Bureau [80].
[16] The unemployment data are taken from the U. S. Department of Labor [79].

the robustness of our results by letting $\gamma = 3$. Holding γ constant, we choose different values of ρ *a priori,* and calculate the corresponding λ in such a way that $u(.) + v(.)$ stays a strictly concave function. For every triple ρ, γ, and λ, we search over the values of β that deliver the capital-output ratio which is compatible with its empirical counterpart. We assume that the social security replacement ratio is 40% and the unemployment replacement ratio is 25% during our search.

When the temptation function is concave, we follow DeJong and Ripoll [21] and set $\lambda = 0.0786(0.056)$

2.4.3 Production Parameters

The parameters describing the production-side of the economy are chosen to match the long-run features of the U.S. economy. Following Imrohoroglu [52, 53], we set the capital share of output α equal to 0.310 and the annual depreciation rate of physical capital equal to 0.069. The rate of technological progress g is assumed to be equal to 2.1%, which is the actual average growth rate of GDP per capita taken over the time interval from 1959 to 1994 (Hugget and Ventura [50]). The technology parameter A, can be chosen freely. In our calibration exercises, it is set equal to 1.01. All per capita quantities are assumed to grow at a balanced growth rate g.

2.4.4 Government

We set the unemployment insurance replacement ratio (ϕ) equal to 25% of the employed wage and allow the social security replacement ratio (θ) to vary between 0 and 1 in order to make welfare comparisons with different replacement ratios. Alternatively, we can choose the payroll tax rate (τ_s) and the unemployment insurance tax rate (τ_u) instead of the replacement ratios. Since the social security and the unemployment insurance benefits are self-financing, calibrating the replacement ratios will automatically pin-down the tax rates. This holds true because agents inelastically supply one unit of labor whenever they find an opportunity to work, and changes in tax rates do not affect their supply of labor.[17]

2.4.5 Solution Method

We use discrete-time, discrete-state optimization techniques to find a steady-state equilibrium of our hypothetical economy by using the aforementioned parameter values. Our solution method designedly resembles those of previous studies.[18] More precisely, we follow Imrohoroglu et al. [53] in order to be able to engage in a computational method-free evaluation/comparison of our results to theirs.

A discrete set of asset values (containing 4097 points) is created. The lower bound and upper bound of the set is chosen in such a way that the set never binds.[19] While the state space for working age agents comprises 4097×2 points, the state space for retired agents consists of only 4097×1 points. The discrete set of the control variable (consumption) contains 4097×1 points. We start with a guess about the aggregate capital stock and the level of accidental bequests and then solve agents' dynamic program by backward recursion. The time-invariant, age-dependent distribution of agents is obtained by forward recursion. After each loop, we calculate the new values for the accidental bequests and the capital stock. If the difference between the initial values and the new values exceed the tolerance value, we start a new loop with the new values. This procedure continues until we find values for the accidental bequests and the capital stock that are sufficiently close to their beginning-of-loop values.

2.5 RESULTS

There is a consensus in the literature about the adverse welfare implications of an unfunded social security system, which are mainly due to the distortions it impinges on capital accumulation and labor supply. In order to assess these welfare implications we use a compensating

[17] However, if we calibrate a model featuring labor-leisure choice, tax rates should be used instead of replacement rates.

[18] See Imrohoroglu et al. [51, 52, 53] and Hugget and Ventura [50].

[19] In particular, the lower bound is equal to 0 and the upper bound is equal to 60 times greater than the annual income of an employed agent.

variation measure, which is defined as the percentage by which consumption must be increased to compensate for the decrease in welfare generated by the presence of the social security.

In what follows, we present the results of our calibrations starting with a particular example where CRRA preferences (agents are immune from temptation) are used. Thereafter, we continue our analysis by allowing agents to have self-control preferences with a convex temptation function. Next, we proceed by conducting robustness tests and conclude this section by displaying our results when agents feature self-control preferences with a concave temptation function.[20]

2.5.1 CRRA Preferences

In our first calibration we use CRRA preferences and calibrate our economy so as to reach a capital-output ratio of approximately 2.5 under the assumption of a 40% social security replacement rate. The steady state features of this economy under alternative social security replacement rates are displayed in Table 9 ($\beta = 0.978, \gamma = 2, \lambda = 0$). Our findings in this case are congruent with those in Imrohoroglu et al. [52, 53]. Consumption, capital and output reach their highest levels when the social security replacement rate is zero.

The main intuition is that, despite the fact that social security provides insurance against life-time uncertainty (due to missing annuities market) and risk sharing among generations, its negative effect on capital accumulation makes it undesirable.[21] Table 9 provides evidence for that fact. It is worth noting that the level of consumption required to compensate the consumers (depicted in the last column of the table) increases in a disproportionately manner compared to a given increase in the social security replacement rate (θ).

2.5.2 Self-Control Preferences with a Convex Temptation Function

In this section we assume that agents feature self-control preferences with a convex temptation function. In order to demonstrate the quantitative significance of the temptation

[20] All tables of this section are delegated to the Appendix C.

[21] Since there is no labor-leisure decision in our model, social security system has an effect only on capital accumulation (saving).

parameter and its economic meaning, we calculate the quantity of steady state consumption which would be given up by an agent in order to escape from temptation. To this purpose, following DeJong and Ripoll [21], we obtain the value x such that

$$u(c^* - x) = u(c^*) + v(c^*) - v(\check{c})$$

where c^* is the steady-state value of the agent's actual consumption and $\check{c}$ is the steady-state value of temptation consumption. To isolate the effect of λ, the model is calibrated under zero social security replacement rate and all other parameters remain fixed at their CRRA case while ρ is chosen equal to 2. By increasing λ from 0 to 0.001, we observe that agents would be willing to forgo as much as 4.82% of their steady state consumption in order to eliminate temptation.[22]

This is an interesting result that highlights the forceful consequences of an arguably imperceptible departure from the CRRA preference specification. It underscores the welfare reducing role temptation (and the induced cost of self-control) plays in our model. Nonetheless, at the same time it validates our main intuition, namely, that social security may not be as detrimental to welfare as it has been generally argued in the literature.

Next our aim is to investigate whether there is any room for a social security system, when agents have self-control preferences. In our first calibration we use the same parameter values for β in order to measure the impact of temptation on savings under a 40% social security replacement rate. This example is a counter-factual in the sense that it does not yield capital output ratio around 2.5, but it serves as a device to better demonstrate the effect of self-control preferences on savings.

Tables 10, 11 and Figures 5, 6 show the steady state of an economy with self-control preferences under 40% replacement ratio. In particular, Table 10 is constructed holding all parameters of the utility function fixed in their CRRA values, except for λ, which is the parameter we vary. The value of parameter λ measures the strength of temptation towards current consumption. Higher values for this parameter corresponds to higher cost of exerting self-control. We notice that all variables but the interest rate decrease as λ increases (i.e. as

[22] DeJong and Ripoll (2006) report the analogous to their environment value as slightly above 5% of the steady state consumption when the scale of the temptation parameter is increased from 0 to 0.00286.

we depart from the CRRA case). In particular, the capital-output ratio decreases showing that the increase of λ triggers a process of dissavings. This process deprives the economy from future consumption capabilities. The latter point is congruent with what we observe in the consumption pattern as λ varies.

Figure 5 illustrates the aforementioned points. We plot lifetime consumption as a function of age. Even a casual glance suggests that an increase in the temptation intensity (λ) results in a abrupt departure from the consumption smoothing behavior of a CRRA agent. It is worth noting how dramatically the early high consumption pattern of a consumer with higher values of λ gets penalized in his retirement years compared to a CRRA consumer. As it could be expected, for a very low value of λ the observed pattern closely resembles that of CRRA.

Figure 6 provides additional support to our findings from the perspective of lifetime asset holdings. It is worth observing that the discrepancy in savings before retirement between different agents (in terms of λ) translates to the observed difference in consumption documented in Figure 5.

Table 11 is constructed holding β, and γ in their CRRA values and keeping λ fixed at 0.00009 under 40% social security replacement rate. Now, we only vary ρ which is a measure of the consumers' willingness to substitute current temptation consumption for future one. The higher ρ is the more the consumer prefers early to late temptation consumption which actually makes the self-control cost even more severe. This, in turn causes further dissavings and eventually lower steady state consumption for any value of λ.

Figure 7 illustrates our findings in terms of lifetime consumption. The clear difference in the observed consumption pattern manifests the impact of an increase in ρ.

Additional support is provided by Figure 8. Note that we observe that the impact of an increase in ρ on asset holdings is very similar to the impact of an increase in λ, which suggests that a given pattern of asset holdings is not uniquely identifiable by given (λ, ρ), but instead can be induced by different combinations of those two parameters.

Now that we are able to detect the effect of self-control preferences on savings, we can calibrate our benchmark economy to analyze the effect of a social security system on the entire economy.

Table 12 presents the features of various steady states of this economy. Our main point in this case is that an unfunded social security system serves an additional purpose to that of the provision of insurance against life-time uncertainty and intergenerational risk-sharing: It makes the cost of exerting self-control less burdensome by reducing the amount of wealth through taxing of the current income. One can speculate that if the unfunded social-security system's negative effect on savings is offset by its positive effect on the self-control cost, a certain level of social security replacement rate may generate larger benefits (through an increase of the level-of-capital channel) than the ones generated in the absence of social security. This additional benefit of the unfunded social security system is absent if an agent is not endowed with temptation.

The above is indeed a challenging remark. While both social security and self control, when considered separately, they have detrimental effects on welfare, their combination yields a noteworthy result: Welfare reduction is considerably less severe. The intuition behind this result lies in the following fact: Social security is a mechanism that deprives agents from early consumption. When agents face temptations, social security accomplishes at the same time to reduce the cost associated with the exertion of self-control and consequently to partially offset its adverse effect on welfare. Note that this effect is absent in environments where preferences do not allow agents the option to exert self-control. Not surprisingly, it is also absent in the case where the temptation component essentially does not modify the consumers' lifetime consumption paths.[23]

A careful comparison of Table 12 with Table 9 reveals that on the one hand social security decreases welfare both under the CRRA and the self-control preference specifications but on the other, the presence of self-control preferences seems to mitigate the welfare reducing effect of social security. This can be seen by directly comparing the compensation needed by a consumer facing temptation and the one needed by a CRRA consumer in order to offset the adverse welfare effects of social security. Although the scale of the temptation parameter (λ) is very small, the welfare cost of social security system is almost three times lower than that of the CRRA preference specification for a given social security replacement ratio (by comparing the last two columns of the two tables).

[23] See next section on "concave" temptation function.

Our findings parallel Imrohoroglu et al. [53] in that social security indeed entails welfare losses both under CRRA preferences and non-CRRA preferences and it is less severe under the latter. They used time-inconsistent (Laibson-type) preferences as their theoretical apparatus and concluded that only a negligible percentage of the whole population prefers a social security system. However, in their framework agents do not face a temptation problem (and consequently a cost of exerting self-control). Welfare issues stemming from their preference specification reduce to a commitment problem. Hence in their case, an unfunded social security system works only as a commitment device. Contrastingly, when consumers face temptation, social security is considerably less costly than in the case where consumers have CRRA preferences, precisely thanks to its additional benefit of reducing the temptation cost. This, in turn, mitigates the unfunded social security system's negative welfare effect.

A rather surprising result is displayed in Table 13 ($\beta = 1.0117, \gamma = 2, \lambda = 0.00065, \rho = 2$). The choice of a relatively large value λ, results in an increase in welfare as it can be seen in the last column. The meaning of negative values in the CV column is that there is a welfare cost associated with smaller values of social security replacement rate. That is, agents should be compensated for the absence of the social security system.

However, this rather controversial result is most probably due to the choice of a high β which is necessary in order for the targeted empirical capital-output ratio to be achieved. We by no means consider the above peculiar result as contrasting the literature. We rather believe that it further underscores the mitigating effect of the existence of a temptation component in the utility function as it is identified in our paper.

2.5.3 Robustness Check

The purpose of this section is to test the robustness of our results to different parameter settings. In the above calibration exercises, the risk aversion parameters γ and ρ are taken equal to 2. An increase in γ or ρ results in a smaller inter-temporal elasticity of substitution. The choice of γ and ρ naturally affects the choice of λ and in turn, the choice of all three parameters pin-down β.

Note, that what may seem at a first glance, a slight deviation from the parameter specification used in the main section ($\gamma = 2$ in Tables 12 and 13 compared to $\gamma = 3$ in Table 14, as well as $\rho = 2$ in Table 12 and 13 compared to $\rho = 3$ in Table 15), has a remarkable effect on the value of λ, compared to its value in the benchmark case (Table 12). As γ and ρ increase, λ essentially vanishes.

Table 14 ($\beta = 0.999, \gamma = 3, \lambda = 0.00002, \rho = 2$) shows the features of the economy under new parameter values. It is evident that different parameter choices did not change the conclusion we reached above regarding the welfare reducing effect of social security. However, naturally, a higher degree of risk aversion leads to greater welfare loss for a given level of social security replacement rate.

In Table 15 ($\beta = 0.990, \gamma = 2, \lambda = 0.000001, \rho = 3$) we vary ρ while controlling for γ. Again, our results are qualitatively similar to our benchmark calibration.

2.5.4 Self-Control Preferences with a Concave Temptation Function

In this section we consider the case of a concave temptation function. DeJong and Ripoll [21] estimate the value of the parameter λ as 0.0786 with the standard deviation 0.056. In our calibration exercises, we set λ to its estimated mean (0.0786) and extreme values $(0.0226, 0.0786)$. One can readily notice that these parameter values are substantially higher than those of the convex temptation case.

Table 16 is constructed holding all parameters of the utility function fixed in their CRRA values, except for λ, which is the parameter we vary. Note that contrary to the convex temptation case, all relevant variables slightly deviate from their CRRA values. In addition, the optimal consumption pattern produced by the concave temptation function in Figure 9 is almost identical to that of the CRRA case.

In Figure 10, we observe that the concave temptation function produces an asset holdings pattern that sharply contrasts that observed in the convex temptation function case. The asset holdings pattern closely matches that of the CRRA case. These findings imply that the functional form for temptation utility dominates the magnitude of the scale parameter λ and as a consequence, concave temptation function produces very similar results to that of CRRA case.

Table 17 ($\beta = 0.984, \gamma = 2, \lambda = 0.0786$) presents the results of the benchmark calibration of this section. A casual comparison in the results obtained here with the ones obtained in the previous section (Tables 9, 12, and 13) suggests that agents facing a concave temptation function are still worse-off than in an economy without social security. A more careful look reveals the similarity between the CRRA and the concave temptation function cases in terms of the welfare losses associated with the presence of social security. Consequently, the welfare losses in the concave temptation function case are higher than those in the convex temptation function case and slightly lower than CRRA case.

This result seems intuitive if one recalls that when a utility function features a concave temptation component, the lifetime consumption and asset holding path remains essentially invariant to deviations from the CRRA case. Therefore, social security remains equally detrimental to welfare under self control preferences as it is under CRRA preferences.

In Table 18 ($\beta = 0.988, \gamma = 2, \lambda = 0.1346$), we use the highest acceptable value of λ. Even in this case, our results show that the cost of social security is slightly lower than that obtained in the CRRA case. Table 19 ($\beta = 0.999, \gamma = 3, \lambda = 0.0226$) is constructed for our robustness checking purposes. It demonstrates that varying the value of the parameter γ does not change the conclusion.

2.6 CONCLUSION

Social security related expenses is one of the largest expenditure items in the U.S. government's budget. As a result, there is an extensive literature regarding social security related issues. The costs and benefits of social security are well analyzed by many authors in the context of standard preferences: all of the studies with the exception of Imrohoroglu et al. [53] use CRRA preferences. Imrohoroglu et al. use quasi-hyperbolic preferences instead, and show that even in such a context where social security could be used as a commitment device, it turns out that social security does not improve welfare.

In the present paper, we assume that consumers have self-control preferences. In our environment, agents do not have a commitment problem but they instead face a temptation to consume all of their available wealth at each point in time.

Our methodology consists in implementing calibration techniques, similar to those used in the related literature, in order to simulate our economy and draw conclusions regarding the impact of social security on consumers' lifetime welfare. In doing so, we consider several variations of our specification of the temptation utility function (different degrees of convexity / concavity of the temptation function), and assess their influence separately, while at the same time compare it with the standard (CRRA) preferences case. Finally, we verify the numerical validity of our results by administering various robustness tests.

Our main findings can be summarized in the following: In a world where agents have self-control preferences social security generally decreases lifetime welfare. Interestingly however, we call attention to a challenging novelty which is due to our specification of self-control preferences: The presence of temptation considerably reduces the cost of social security. That is, indeed social security penalizes welfare but when the economy features agents with self-control preferences the above cost is substantially mitigated.

Moreover, in our calibrations we measure that the cost of temptation, namely, the amount of consumption that agents would be willing to relinquish in order to eliminate temptation is as high as 4.82% of their steady state consumption. Since this percentage corresponds to an insignificant deviation (increasing λ from 0 to 0.001) from the CRRA preference specification, it underscores the welfare reducing role temptation (and the induced cost of self-control) plays in our model. Nonetheless, at the same time it validates our main intuition, namely, that social security may not be as detrimental to welfare as it has been generally argued in the literature.

While both social security and self control, when considered separately, they have detrimental effects on welfare, their combination yields a noteworthy result: Welfare reduction is considerably less severe. The intuition behind this result lies in the following fact: Social security is a mechanism that deprives agents from early consumption. When agents face temptations, social security accomplishes at the same time to reduce the cost associated with the exertion of self-control and consequently to partially offset its adverse effect on welfare. It is worth noting that this effect is absent in environments where preferences do not allow agents the option to exert self-control or in contexts where the impact of temptation on lifetime consumption is moderate.

Table 2: Parameter Values of the Benchmark Calibration

Demographics	
Maximum possible life span T	65
Obligatory retirement age t^*	45
Growth rate of population n	1.19%
Conditional survival probabilities $\{s_t\}_{t=1}^{T}$	U.S. 2001
Labor efficiency profile $\{\epsilon_j\}_{t=1}^{t^*-1}$	Hansen [43]
Production	
Capital share of GDP α	0.310
Annual depreciation of capital stock δ	0.069
Annual per capita output growth rate g	2.1%
Markov Process for employment transition Π	$\begin{bmatrix} 0.94 & 0.06 \\ 0.94 & 0.06 \end{bmatrix}$
Preferences	
Annual discount factor of utility β	0.998
Scale factor of the temptation utility λ	0.000375
Risk aversion parameter γ	2.0
Risk loving parameter ρ	2.0
Government	
Unemployment insurance replacement ratio ϕ	0.25
Social security replacement ratio θ	$[0, 1]$

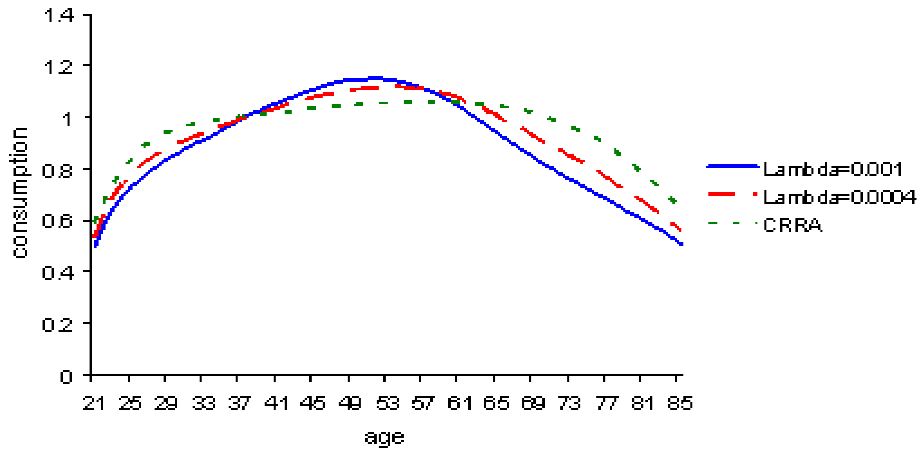

Figure 5: Optimal Consumption Choice When Temptation Function is Convex and Lambda Varies

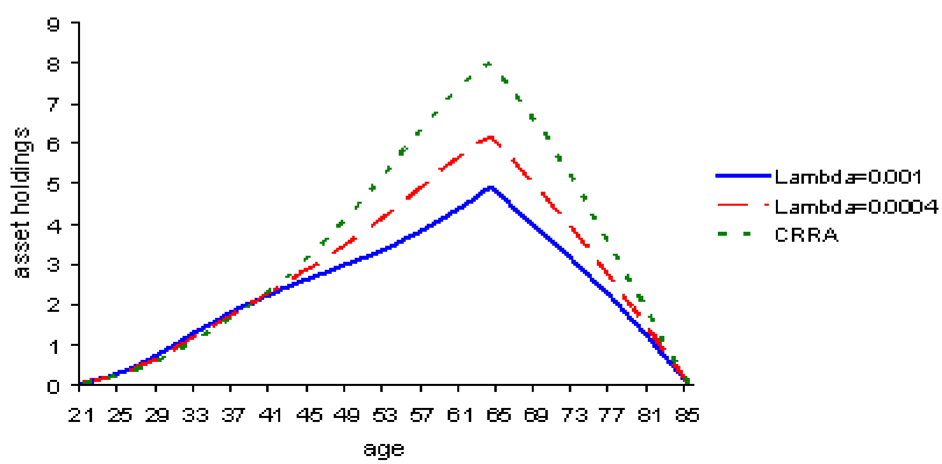

Figure 6: Optimal Asset Holding When Temptation Function is Convex and Lambda Varies

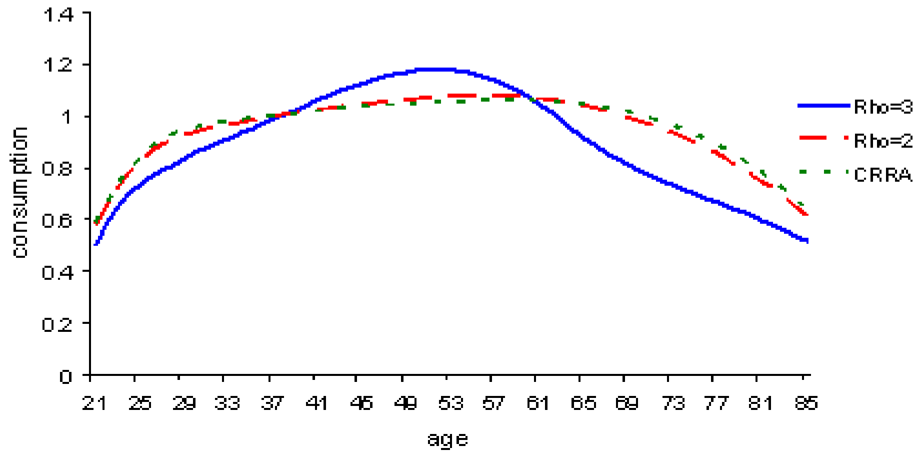

Figure 7: Optimal Consumption Choice When Temptation Function is Convex and Rho Varies

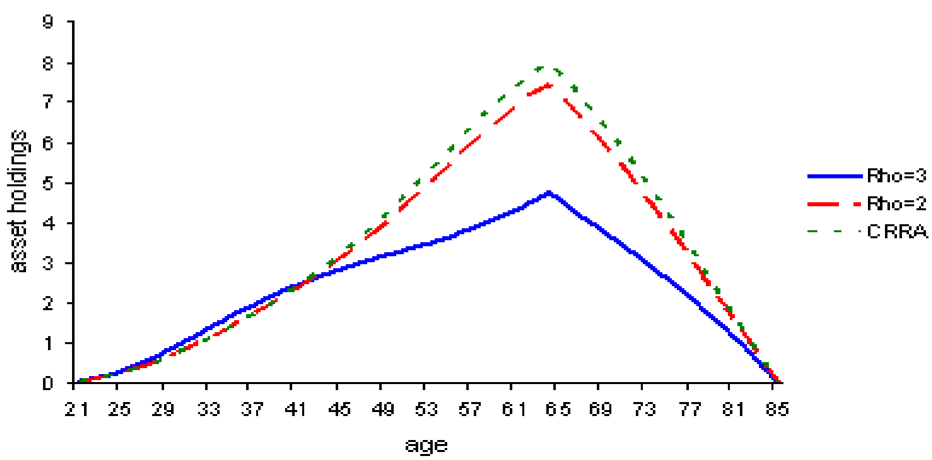

Figure 8: Optimal Asset Holding When Temptation Function is Convex and Rho Varies

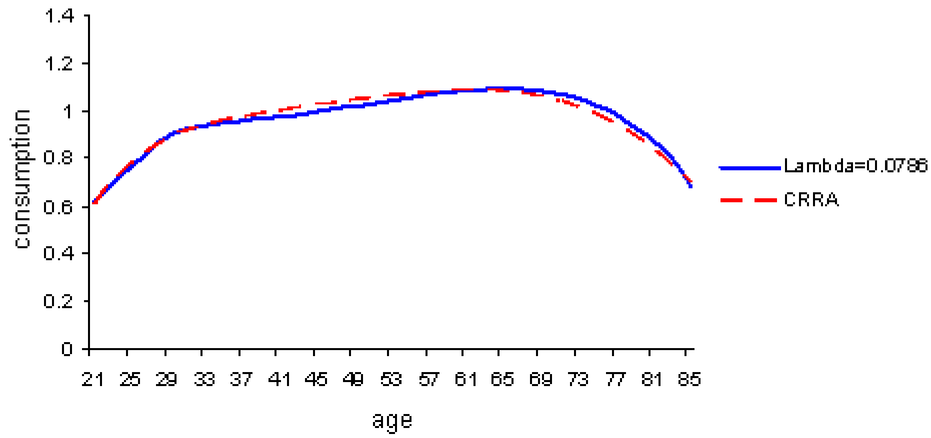

Figure 9: Optimal Consumption Choice When Temptation Function is Concave

Figure 10: Optimal Asset Holding When Temptation Function is Concave

3.0 OPTIMAL MULTI-OBJECT AUCTIONS WITH RISK-AVERSE BUYERS (WITH HADİ YEKTAŞ)

3.1 INTRODUCTION

Optimal selling mechanisms for multiple objects have been analyzed extensively due to their theoretical and practical (*e.g.*, the spectrum auctions, the used car auctions) importance.[1] One of the main assumptions in these studies is that buyers are risk-neutral. However, when there are many buyers and only a limited number of objects, buyers are faced with the risk of not getting the object(s). Such a risk is costly if buyers are risk-averse.[2] This is because while risk-neutral buyers' marginal utility of income does not change, risk-averse buyers' marginal utility of income may differ in the events of winning and losing. Yet, the effect of this kind of risk on the design of revenue maximizing auctions for multiple objects has not been studied, and this is the aim of the current paper.

We know from the single-object optimal auction literature that the environment with risk-averse buyers may deliver quite different results. Hence, it is natural to expect that the risk aversion assumption in the design of optimal multi-object auctions may provide new insights.

In his seminal work, Myerson [67] provides the framework for designing the revenue-maximizing auction. He shows that if a seller wishes to sell one indivisible object to one of n

[1] See for example, Harris and Raviv [47], Maskin and Riley [63], Levin [59], and Figueroa and Skreta [34].

[2] If the buyers are firms then it is more natural to assume that they are risk-averse, if the owners are non-diversified as it is the case in most family owned companies. Moreover, if the firms have liquidity constraints or are in a financial distress or if the tax system is non-linear, then the firms behave as if they are risk-averse as pointed out by Asplund [8]. He also states that the firms owned by risk-neutral shareholders may also behave in a risk-averse manner if the control of the firm is delegated to a risk-averse manager and his payment is linked to the firm's performance.

potential risk-neutral buyers with independently distributed private values, then it is optimal to give the object to the buyer who has the highest *virtual valuation* (not the actual valuation) that exceeds the seller's outside option.[3] Thus, the standard auctions, including the "high bid" and "English" auctions, with appropriately chosen reserve price are all optimal. He further shows that two auctions with the same allocation rule are revenue equivalent if the expected utility of each buyer in some benchmark case is the same, the celebrated *revenue equivalence theorem*. On the other hand, once the risk-neutrality assumption is relaxed, the standard auctions with appropriate reserve price neither generate the same expected revenue for the seller nor are they optimal. This result arises because of the conflict between the seller's desire to insure buyers against the risk and to exploit buyers' risk-bearing to screen them (Maskin and Riley [62]).

Moreover, if there is correlation between buyers' valuations then the seller can fully extract the informational rents using an efficient auction if buyers are risk-neutral (Crémer and McLean [18]). Yet, with risk-averse buyers the aforementioned result holds only if the correlation is sufficiently strong (Eső [30]).

With these comparisons at hand, in this paper, we aim to provide insights to answer the following two questions:

1. How can the optimal multi-object auction with risk-averse buyers be compared to that with risk-neutral buyers?
2. Which features of the optimal single-object auction carry over to the optimal multi-object auction when buyers are risk-averse?

In order to answer the first question, we build our model upon that of Armstrong [6] who characterizes the optimal auction for multiple objects for the case of risk-neutral buyers.[4] Making the analysis tractable with the assumption of binary distribution of types, he shows

[3] Virtual valuations are the adjusted valuations that take into account buyers' informational rents and, more precisely, are defined as $\psi_i(v_i) = v_i - [1 - F_i(v_i)]/f_i(v_i)$, if buyer i's valuation v_i is distributed according to cumulative distribution function $F_i(.)$ with associated density function $f_i(.)$.

[4] Armstrong [6] inherited his setting from Armstrong and Rochet [7], who study a principal-agent problem. Both of these papers and the current paper assume that buyers/agents have multi-dimensional private information and, in this regard, differ from the references mentioned in Footnote 1.

Manelli and Vincent [60, 61] also analyze an optimal selling scheme assuming multi-dimensional private information but different from the current paper they assume a single buyer.

that the optimal auction assigns each object to a buyer who has high valuation for it whenever there is such a buyer. Namely, the optimal auction is weakly efficient.[5] We show that this result is robust to relaxing the assumption of risk-neutral buyers.

When the buyers are risk-neutral, depending on the correlation between their valuations for the two objects, the optimal auction can take the form of two independent auctions, a bundling auction, or a mixed auction.[6,7] In contrast, if the buyers are risk-averse, we show that, the use of two independent auctions is not optimal because the objects must be sold to the same buyer if all buyers have low valuations for both objects.

To answer the second question, we first need to establish that our results, which we obtain in a binary model, are comparable with those of the current literature, which assumes continuous distribution of types. Therefore, in Section 3.2, we characterize the optimal single-object auction in the binary framework, imitating the work of Maskin and Riley [62].[8] We then show that the outcomes assigned to each possible valuation in the binary model resemble those assigned to the extreme values of the type space in the continuous model of Maskin and Riley [62]. Although we do not find new results in Section 3.2, this analogy helps us interpret the results we obtain in Section 3.3, which describes the properties of the revenue maximizing auctions for multiple goods. The seller perfectly insures buyers against the risk of losing the object(s) for which they have the high(est) valuation. The buyers who have high valuations for both objects are compensated if they can not obtain either

[5] Weak efficiency requires that each object is sold to the buyer with the highest valuation whenever it is sold. Some of the objects can be kept by the seller although there is a buyer who has valuation that exceeds that of the seller. For strong efficiency, on the other hand, the objects valued more highly by a buyer than the seller must always be sold. In this sense, the optimal auctions in Myerson [67] are weakly efficient.

[6] In all three forms, buyers have the same *expected* probability of winning the object(s) they have high valuation for. These forms differ only in the *expected* probability of winning the objects for which buyers have low valuation. In a mixed auction, a buyer who has low valuation, say, for object A but high valuation for object B, is assigned object A more often than a buyer who has low valuation for both objects. While independent auctions do not distinguish between these two types for object A, a bundling auction perfectly discriminates against the type that has low valuations for both objects.

[7] While Armstrong [6] assumes that all buyers have demand for both objects, in Avery and Hendershott [10], only one buyer wants both objects and the remaining buyers demand only single objects. Not surprisingly, the optimal auction in the latter may not be weakly efficient due to the good deal of asymmetry among buyers. Yet, even in that case, the optimal auction bundles the objects *probabilistically* for the multi-demand buyer.

[8] Matthews [64] studies the same problem as Maskin and Riley [62]. While the former assumes a particular form of utility function, namely CARA, and obtains necessary and sufficient conditions for an auction to be optimal, the latter consider different forms of risk aversion and characterize the properties of the optimal auction for all of these forms.

object. On the other hand, those who have low valuations for both objects must incur a positive payment if they lose both objects. The intuition as follows: The seller can confront risk-averse buyers with risk in order to screen them. There are two types of risk a buyer may face in the optimal auction. First, his payment contingent on winning and losing may be a random variable. Second, his marginal utility of income may differ when he wins and when he loses. The first kind of risk is absent for all type of buyers due to the CARA preference specification.[9] However, the second kind of risk is absent only for buyers who have high valuations for the object(s). The reason is that the seller wants to prevent high-type buyers from imitating low-type buyers. For this purpose, she confronts low-type buyers with risk by not offering insurance. This, in turn, implies that high-type buyers face with greater risk if they behave as if they are low-type. As a result, the seller prevents high-type buyers from not revealing their true valuations at the cost of extracting less revenue from low-type buyers. Since the benefit of preventing high-type buyers from not revealing their true valuations is larger than the cost of not offering complete insurance to low-type buyers, we observe that at the optimum the seller offers insurance to only high-type buyers. Moreover, at the optimum, the seller rewards the highest type buyers (i.e., buyers who have high valuations for both objects) by providing compensation but punishes the lowest type buyers (i.e., buyers who have low valuations for both objects) by making them pay penalty when they lose the both objects.

Finally, we comment on the solution methods used in this paper: In Section 3.2, we describe the optimal auction in *reduced form,* meaning we only construct an individual's *expected* probability of obtaining the object, rather than his *actual* probability as a function of all buyers' values. This technique is introduced by Matthews [64] and Maskin and Riley [62] in order to avoid the computational complexity that risk aversion involves. Yet, additional constraints must be imposed on the objective function in order to guarantee the existence of the *actual* probabilities, that is, in order to be able to *implement* the reduced form probabilities. Armstrong [6] is able to use the same method because, when buyers are risk-neutral, only the *marginal* probabilities of winning the objects matter for the buyers, as well

[9] As long as the structure of a utility function does not allow absolute risk aversion to increase too fast with income, the first kind of risk is irrelevant (Maskin and Riley [62]). Since the CARA preference specification is more tractable among others we choose this form as Matthews [64] and Eső [30].

as, for the seller.[10]

Yet, when buyers are risk-averse, the correlation between the events of winning object A and object B matters for the buyers and consequently for the seller. In this case, the conditions that one needs to impose in order to implement the reduced form probabilities cannot be easily determined. Therefore, in Section 3.3, we describe the optimal auction in the *non-reduced* form. That is, the actual probabilities of the events that a buyer can possibly face are constructed as functions of the entire type profile (as reported by all participating buyers). Since for the buyers (as well as for the seller) only expected probabilities matter, we also make use of the reduced form probabilities throughout our analysis.[11]

3.2 OPTIMAL SINGLE-OBJECT AUCTIONS

3.2.1 Description of the Problem

A single indivisible object is to be sold to one of $n \geq 2$ potential buyers, whose private valuations are distributed according to a discrete random variable v_i, which takes values v_H with probability $\alpha_H > 0$ and v_L with probability $\alpha_L > 0$ such that $\alpha_H + \alpha_L = 1$. Without loss of generality, we assume $v_H > v_L > 0$, so that v_H and v_L denote valuations of high-type and low-type buyers, respectively. Buyer valuations are distributed independently and identically. Buyers are risk-averse and have a constant measure of risk aversion (CARA). In particular, their preferences are represented by a utility function $u(\omega) = -\frac{e^{-r\omega}}{r}$, where $r(>0)$ measures the rate of risk aversion. Note that $u'(.) > 0$ and $u''(.) < 0$. Specifically, if a buyer with valuation v wins the object and incurs a *net* payment of τ then his utility is $u(v-\tau) = -\frac{e^{-r(v-\tau)}}{r}$. The seller is risk-neutral and her valuation for the object is zero. Both the seller and the buyers are expected utility maximizers.

The seller's problem is to design a selling scheme that maximizes her expected revenue.[12]

[10] The number of constraints that one needs to impose to implement the reduced form probabilities increases exponentially with the number of objects.

[11] In regard to the solution method, this paper is also related to Menicucci [65] which extends Armstrong [6] by allowing for a synergy if the two goods end up in the hands of the same buyer. He shows that in this case the optimal auction is likely to allocate the goods inefficiently.

[12] Milgrom [66] defines an auction to be a mechanism (scheme) to allocate resources among a group of bidders. Therefore, we will use these three terms interchangeably.

Such a scheme most generally consists of a message set, $M = M_1 \times \cdots \times M_n$, and an outcome function, $\psi : M \to \tilde{A}$, that maps the list of messages, $m \in M$, into a possibly random allocation $\tilde{a} \in \tilde{A} = \tilde{A}_1 \times \cdots \times \tilde{A}_n$.[13] Buyers' behavior is described by a Bayesian Nash equilibrium, $s = (s_1, ..., s_n)$, where $s_b : \Theta_b \to M_b$ is the equilibrium strategy of buyer b and $s_b(\theta_b)$ represents the message that maximizes the expected utility of buyer b with type θ_b assuming that all buyers other than him follow the equilibrium strategy.[14] Therefore, any selling scheme in a given equilibrium results in an outcome represented by $\psi(s_1(\theta_1), ..., s_n(\theta_n))$ when the buyers' type profile is $(\theta_1, ..., \theta_n)$.

Alternatively, when looking for the optimal selling scheme, we can restrict attention to the *revelation schemes* in which the message space is the type space, Θ. This is because any allocation, $\psi(s_1(\theta_1), ..., s_n(\theta_n))$, resulting from an equilibrium of an arbitrary selling scheme can also be obtained in a revelation scheme in which the outcome is determined via the composite function $\psi \circ s : \Theta \to \tilde{A}$ and truth-telling is an equilibrium (revelation principle).[15] Thus, the seller's problem can be reduced to finding the optimal revelation scheme in which the buyers are willing to participate (*individual rationality)* and have incentive to truthfully report their type (*incentive compatibility)*.

Given a profile of reports, a selling scheme must, most generally, assign each buyer a probability of winning, a payment in case he wins and another payment in case he loses. That is, the outcome is determined by functions of the form $\psi_b(m) = (p_b(m), \tilde{t}_b^w(m), \tilde{t}_b^l(m))$ for $b = 1, ..., n$, where tildes represent the possibility that the payment functions are random. Since there is only one object for sale, a feasible scheme must satisfy $\sum_{b=1}^n p_b(m_1, ..., m_n) \leq 1$ for all $(m_1, ..., m_n)$.

Given an equilibrium, we can calculate buyer b's expected probability of winning and his

[13] An allocation consists of a *decision* about who is going to get which object(s) and possibly negative monetary *transfers* from buyers to the seller.

[14] In this section, each type of a buyer corresponds to a possible valuation , namely $\Theta_j = \{v_H, v_L\}$ for all $j = 1, ..., n$, whereas, in the next section, there are four different types of buyers. That is, $\Theta_j = \{HH, HL, LH, LL\}$ for all $j = 1, ..., n$, where the first (second) letter in each type represents buyer j's value for object A (B).

[15] See Myerson [67].

expected random payments in case of winning and losing, respectively, as:

$$\rho_b(m_b) = E[p_b(m) \mid m_b] \tag{3.1}$$

$$\tilde{\tau}_b^w(m_b) = E[\tilde{t}_b^w(m) \mid m_b] \tag{3.2}$$

$$\tilde{\tau}_b^l(m_b) = E[\tilde{t}_b^l(m) \mid m_b]. \tag{3.3}$$

Since buyers are *ex ante* identical, only the schemes that treat them symmetrically need to be considered. This is because, for any asymmetric scheme, we can construct a symmetric scheme that generates the same revenue as the proposed asymmetric scheme. Symmetric schemes satisfy the following condition:

$$\text{For any } b, b' \in \{1, ..., n\} \text{ and any } m, m' \in M,$$
$$\psi_b(m) = \psi_{b'}(m')$$
$$\text{if } m_b = m'_{b'}, m_{b'} = m'_b, \text{ and for all } b'' \neq b, b' \; m_{b''} = m'_{b''}.$$

Therefore, in a symmetric scheme, the expected probability and the expected payments of two different buyers submitting the same message are equal. Hence, we can drop the subscript on each of the functions in (3.1) - (3.3). Describing a selling scheme from the perspective of an arbitrary buyer, using $\rho(.), \tilde{\tau}^w(.)$, $\tilde{\tau}^l(.)$, is called *reduced form representation.*

Three points need to be emphasized about our approach to solving the seller's problem. First, while using the Revelation Principle, we consider only the revelation schemes that satisfy two sets of conditions: individually rationality and incentive compatibility. Second, we construct the optimal auction in reduced form. We justify this by imposing another set of conditions called *implementability conditions.*[16] These conditions make sure that the

[16] Border [15] states the necessary and sufficient conditions, for the reduced form probabilities to be implementable. We include the proposition for easy reference:

Let (S, Υ) be a measurable space of possible types of bidders and $\lambda(.)$ be a probability measure on S. Define an auction to be a measurable function $p : S^n \to [0,1]^n$ satisfying $\sum_{i=1}^n p^i(s) \leq 1$ for all $s \in S^n$. Define an auction to be symmetric if $p^i(s)$ is independent of i. Given an auction, define

$$\rho^i(s_i) = \int_{S^{n-1}} p(s_1, ..., s_n) d\lambda(s_1, ., s_{i-1}, s_{i+1}, .., s_n)$$

to be the probability that a buyer i wins when he reports his type as s_i.

Then ρ is implementable by a symmetric auction if and only if for each measurable set of types $A \in \Upsilon$, the following inequality is satisfied:

reduced form probability, $\rho(.)$, is *implementable*, that is, they make sure that there exists a symmetric auction with actual allocation probabilities, $p(.)$, which satisfies

$$\rho(m_b) = E[p(m) \mid m_b]. \tag{3.4}$$

Finally, we initially consider only the schemes in which the expected payments contingent on winning and losing are non-random. In other words, we first construct the optimal scheme within the class of schemes for which $\tilde{\tau}^w(.)$ and $\tilde{\tau}^l(.)$ are deterministic (hence, we drop the tildes over τ). We later establish that this scheme is also optimal among all selling schemes.

To summarize, the seller's problem is to construct the optimal revelation scheme, the reduced form of which can be represented by six variables, $\{\rho_i, \tau_i^w, \tau_i^l\}_{i=H,L}$, where $\rho_i \in [0,1]$ denotes the probability that a buyer wins the object when he reports a valuation of v_i, and $\tau_i^w, \tau_i^l \in \mathbb{R}$ denote the *net deterministic* payments that the same type of buyer incurs when he wins and loses the object, respectively.[17] As mentioned above three sets of conditions are imposed:

If a buyer with valuation v_i reports v_j then his utility is equal to $\rho_j u(v_i - \tau_j^w) + (1 - \rho_j)u(-\tau_j^l)$. Thus, buyers truthfully reveal their valuations if the auction satisfies the following two *incentive compatibility conditions*:

$$\begin{aligned} \rho_H u(v_H - \tau_H^w) + (1-\rho_H)u(-\tau_H^l) &\geq \rho_L u(v_H - \tau_L^w) + (1-\rho_L)u(-\tau_L^l) \\ \rho_L u(v_L - \tau_L^w) + (1-\rho_L)u(-\tau_L^l) &\geq \rho_H u(v_L - \tau_H^w) + (1-\rho_H)u(-\tau_H^l). \end{aligned}$$

Buyers are free to participate in the auction. Thus, participating buyers satisfy the *individual rationality conditions* of the form:

$$\begin{aligned} \rho_H u(v_H - \tau_H^w) + (1-\rho_H)u(-\tau_H^l) &\geq u(0) \\ \rho_L u(v_L - \tau_L^w) + (1-\rho_L)u(-\tau_L^l) &\geq u(0). \end{aligned}$$

$$\int_A \rho(s)d\lambda(s) \leq \frac{1-\lambda(A^c)^n}{n}$$

Furthermore, if S is a topological space and λ is a regular Borel probability on S, then Υ may be replaced by either the open subsets or the closed subsets of S.

[17] Alternatively, as in Matthews [64], we can use the bid function ($b(.) = \tau^w(.) - \tau^l(.)$) and bid submission or entry-fee function ($\tau(.) \equiv \tau^l(.)$) and search for the equivalent scheme $\{\rho_i, b_i, \tau_i\}_{i=H,L}$. The interpretation of this scheme as follows: A buyer must pay entry-fee $\tau(.)$ in order to submit bid $b(.)$. Submitting bid $b(.)$ gives the buyer a probability $\rho(.)$ of acquiring the object. In the event that the buyer acquires the object, he makes additional payment at the amount of his bid $b(.)$.

Finally, the *implementability conditions,* stated first in Border [15], take the following form in our binary model:

$$\begin{aligned} n(\alpha_L\rho_L + \alpha_H\rho_H) &\leq 1 && (IM_{\{H,L\}}) \\ n\alpha_H\rho_H &\leq 1-\alpha_L^n && (IM_{\{H\}}) \\ n\alpha_L\rho_L &\leq 1-\alpha_H^n. && (IM_{\{L\}}) \end{aligned}$$

One can interpret these conditions as follows: the probability with which the object is won by a buyer who belongs to a particular subset of the type space should be no greater than the probability that there *is* a buyer who belongs to that subset. Thus, these conditions are also called *resource constraints*.

The seller's revenue is the sum of the expected payments made by each buyer. Since buyers are *ex ante* identical the seller's revenue can be written in terms of the expected payments made by an arbitrary buyer (namely, the term in the bracket):

$$\pi = n[\alpha_H(\rho_H\tau_H^w + (1-\rho_H)\tau_H^l) + \alpha_L(\rho_L\tau_L^w + (1-\rho_L)\tau_L^l)].$$

To sum up, the seller's problem is to choose a reduced form scheme, $\{\rho_i, \tau_i^w, \tau_i^l\}_{i=H,L}$, that maximizes π subject to the two *incentive compatibility conditions*, the two *individual rationality conditions*, and the three *implementability conditions.*

For convenience, we define $c_i = e^{-rv_i}$ and $y_i^k = e^{r\tau_i^k}$. Note that, $0 < c_H < c_L < 1$ and $y_i^k > 0$ for all i and k. So, we can rewrite the seller's problem as

$$\max_{\{\rho_i, y_i^w, y_i^l\}_{i=H,L}} \pi = \frac{n}{r}[\alpha_H(\rho_H \ln y_H^w + (1-\rho_H)\ln y_H^l) + \alpha_L(\rho_L \ln y_L^w + (1-\rho_L)\ln y_L^l)] \quad (3.5)$$

subject to

$$\begin{aligned} \rho_H c_H y_H^w + (1-\rho_H) y_H^l &\leq \rho_L c_H y_L^w + (1-\rho_L) y_L^l && (IC_H) \\ \rho_L c_L y_L^w + (1-\rho_L) y_L^l &\leq \rho_H c_L y_H^w + (1-\rho_H) y_H^l && (IC_L) \\ \rho_H c_H y_H^w + (1-\rho_H) y_H^l &\leq 1 && (IR_H) \\ \rho_L c_L y_L^w + (1-\rho_L) y_L^l &\leq 1 && (IR_L) \\ n(\alpha_L\rho_L + \alpha_H\rho_H) &\leq 1 && (IM_{\{H,L\}}) \\ n\alpha_H\rho_H &\leq 1-\alpha_L^n && (IM_{\{H\}}) \\ n\alpha_L\rho_L &\leq 1-\alpha_H^n && (IM_{\{L\}}) \end{aligned}$$

and the non-negativity conditions $\rho_H, \rho_L \geq 0$.

For convenience, we refer to the left-hand side of the inequality in IR_H (IR_L) as D_H (D_L). Similarly, the right hand side of IC_H (IC_L) is referred to as D_H^L (D_L^H). The subscripts denote a buyer's actual type, whereas the superscripts denote the type he is imitating.

All proofs of the following section are delegated to the Appendix D.

3.2.2 Solution to the Problem

Since $c_L > c_H$, IC_H and IR_L together imply IR_H.[18] Hence, this condition is redundant. For now, we also ignore IC_L when we solve the seller's problem. That is, we first suppose that low-type buyers do not have the incentive to misrepresent their types. Later, we show that this is indeed the case.[19] The following lemma shows that it is optimal to make high-type buyers' incentive compatibility and low-type buyers' individual rationality constraints bind.

Lemma 1. *In the relaxed problem, where IC_L is ignored, the constraints IC_H and IR_L must be binding.*

The seller may want to increase her revenue by excluding low-type buyers from the auction if, for a given distribution of types, their valuation is small enough compared to that of high-type buyers.[20] This results in an inefficiency, because with positive probability the seller keeps the object, although all buyers value the object more highly than her. Inefficiency may also be due to a misallocation by the mechanism. To be consistent with Armstrong [6], we focus only on the latter kind inefficiency, by assuming that the goods are always sold, i.e. $\rho_L > 0$.[21] In other words, we search for weakly efficient auctions.

Lemma 2. *At the optimum, if the low-type buyers are not excluded from the auction, then IR_H must be slack.*

[18] $D_H \leq D_H^L \leq D_L \leq 1$, where the second inequality is due to $c_L > c_H$.

[19] The problem that ignores the downward incentive constraints is called relaxed problem. A solution to the relaxed problem is the solution for the full problem if it satisfies the ignored incentive constraints.

[20] Note that a monopolist solves the same problem as an auctioneer without having capacity (resource) constraints. Hence, as in the optimal auction design, when a monopolist implements second-degree price discrimination, it may be optimal in some circumstances not to sell the object(s) to low type-buyers.

[21] Clearly, high-type buyers should not be excluded from participating in the auction if revenue is maximized. That is, ρ_H must be strictly positive. If not, then the incentive conditions would imply $\rho_L c_L \leq \rho_L c_H$, and since $c_L > c_H$ this in turn would imply $\rho_L = 0$, meaning the good is not sold, at all. Yet, the seller can always guarantee a positive profit by posting a fixed price of $v_L > 0$.

Now, we can write the Lagrangian to the relaxed problem as:

$$\begin{aligned} \mathcal{L} &= \pi - \lambda_L(D_L - 1) - \mu_H(D_H - D_H^L) \\ &\quad -\phi_{\{H,L\}}(n\alpha_L\rho_L + n\alpha_H\rho_H - 1) - \phi_{\{H\}}(n\alpha_H\rho_H - 1 + \alpha_L^n) \\ &\quad -\phi_{\{L\}}(n\alpha_L\rho_L - 1 + \alpha_H^n) \end{aligned} \tag{3.6}$$

where λ_L and μ_H are the Lagrange multipliers on IR_L and IC_H, respectively, and $\phi_{\{H,L\}}$, $\phi_{\{H\}}$, and $\phi_{\{L\}}$ are the multipliers on the implementability conditions.

Since there is a single object, a high-type buyer faces the risk of losing the object to another high-type buyer. This creates a possibility that the marginal utility of income differs in the events of losing and winning. Hence, the seller can increase her profit and reward the high-type buyer for revealing his true valuation by removing the risk. This implies that the high-type buyer either does not make payment or is compensated when he loses. The following proposition shows that, at the optimum, the seller offers insurance to high-type buyers and makes their marginal utility of income same when they lose and when they win.

Proposition 3. *High-type buyers are fully insured against the risk.*

If the seller does not pay information rent to a high-type buyer ($\tau_H^w = v_H$), the perfect insurance requires that the seller sets the high-type buyer's payment contingent on losing equal to zero ($\tau_H^l = 0$) in order to keep him at the same level of marginal utility. However, when there is information gap between the seller and buyers, high-type buyers should receive information rents to be active. In this case (i.e.,$\tau_H^w < v_H$), perfect insurance requires that the seller compensates the high type buyer ($\tau_H^l > 0$).

Proposition 4. *High-type buyers are compensated if they lose the object.*

Using Proposition 3, we can write the seller's profit as

$$\pi = \frac{n}{r}[\alpha_H(\rho_H \ln \frac{1}{c_H} + \ln y_H^l) + \alpha_L(\rho_L \ln \frac{y_L^w}{y_L^l} + \ln y_L^l)] \tag{3.7}$$

Note that, since $0 < c_H < 1$, the seller's profit is strictly increasing with respect to ρ_H. Thus, given the values of other variables, ρ_H must be set as high as possible at the optimum. This implies that either $IM_{\{H\}}$ or $IM_{\{H,L\}}$, or both are binding.

The Kuhn-Tucker conditions with respect to y_L^w and y_L^l can be written as

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial y_L^w} &= \alpha_L \rho_L \frac{1}{y_L^w} - \lambda_L \rho_L c_L + \mu_H \rho_L c_H = 0 \\
\frac{\partial \mathcal{L}}{\partial y_L^l} &= \alpha_L (1-\rho_L) \frac{1}{y_L^l} - \lambda_L (1-\rho_L) + \mu_H (1-\rho_L) = 0.
\end{aligned}$$

Since $\alpha_L \rho_L \frac{1}{y_L^w} > 0$, these two equations together yield

$$\frac{y_L^w}{y_L^l} = \frac{\lambda_L - \mu_H}{\lambda_L c_L - \mu_H c_H}. \tag{3.8}$$

Note that the right-hand side of the equation (3.8) is smaller than $\frac{1}{c_H}$. So, we have

$$\frac{y_L^w}{y_L^l} < \frac{1}{c_H}. \tag{3.9}$$

This condition has the following implication: At the optimum, the iso-revenue curve must be flatter than the line corresponding to the implementability condition $IM_{\{H,L\}}$.[22]

Thus, $IM_{\{H\}}$ and $IM_{\{H,L\}}$ are both binding and the optimal allocation probabilities can be calculated as

$$\rho_H = \tfrac{1-\alpha_L^n}{n\alpha_H}; \quad \rho_L = \tfrac{\alpha_L^{n-1}}{n}$$

which are the points where the iso-revenue curve is tangent to the resource constraint set (Figure 11). Note that, $n\alpha_L \rho_L = \alpha_L^n$ means that the probability that the object is won by a low-type buyer is equal to the probability that all buyers are low-type. In other words, the object is won by a high-type buyer whenever there is one. Hence, the proposition follows.

Proposition 5. *The optimal auction is weakly efficient.*

We show that the seller insures high-type buyers. In fact, the seller can use the same strategy and extract more revenue from low-type buyers. However, the seller prefers to confront a low-type buyer with risk by making his marginal utility of income differs in the events of winning and losing in order to screen buyers. Hence, a high-type buyer who imitates the low-type buyer faces greater risk and prefers to reveal his own true valuation. At the optimum, the seller's gain from relaxing the high-type buyer's incentive constraint exceeds

[22]This condition is equivalent to $\frac{\alpha_L \ln(y_L^w/y_L^l)}{\alpha_H \ln(1/c_H)} < \frac{\alpha_L}{\alpha_H}$, where the left hand side of the inequality is slope of the iso-revenue curve and the right hand side is the slope of the line corresponding to the implementability condition $IM_{\{H,L\}}$.

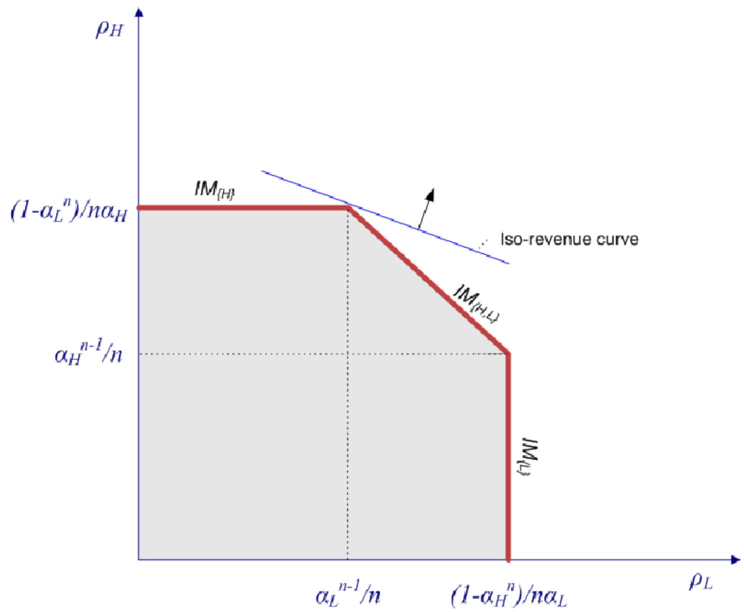

Figure 11: Optimal Single-Object Auction

the lost due to not offering insurance to the low-type buyer. As the proposition below shows that low-type buyers are better off if they obtain the object, on the contrary, high-type buyers have the same marginal utility of income when they win and lose.

Proposition 6. *Low-type buyers are better off winning than losing:* $c_L y_L^w < y_L^l$. *Moreover, in case of losing the object, low-type buyers incur a payment that is less than what they would pay if they win:* $1 < y_L^l < y_L^w$.

Next, we show that the solution to the relaxed problem also solves the full problem where IC_L is not ignored.

Proposition 7. *Low-type buyers do not have the incentive to misrepresent their type. That is,* IC_L *is slack.*

The reduced form of the revelation scheme that we've constructed above is optimal within the class of schemes in which the expected payments contingent on winning and losing are deterministic. Finally, we establish that making τ_i^w and τ_i^l random has a negative effect on seller's revenue.

Proposition 8. *If buyer preferences are represented by CARA, then, in an optimal auction, the payments, τ_i^w and τ_i^l, must be deterministic.*

The above proposition also implies that it is not profitable for the seller to condition the payments made by a buyer on the realizations of his opponents' types.

3.3 OPTIMAL MULTI-OBJECT AUCTIONS

3.3.1 Description of the Problem

Now, there are two non-identical objects, denoted by A and B, to be sold to $n \geq 2$ buyers. The seller's valuation for both objects is zero, whereas the buyers' valuations are random and described by a pair (v^A, v^B), where v^o denotes a buyer's valuation for object o. Suppose that $v^o \in \{v_H^o, v_L^o\}$, where the subscripts denote whether the buyer has high valuation (H) or low valuation (L) for object o. Thus, we assume $v_H^o - v_L^o > 0$. There are four types of buyer corresponding to the four possibilities $(v_H^A, v_H^B), (v_H^A, v_L^B), (v_L^A, v_H^B)$ and (v_L^A, v_L^B). Using a slightly shorter notation, we define the set of possible types as $\Theta = \{HH, HL, LH, LL\}$. A typical element of this set is denoted with ij, where i represents a buyer's valuation for object A and j represents his valuation for object B. Types are independently and identically distributed across buyers according to a probability measure α over Θ, so that the probability that a buyer is of type ij is represented by α_{ij}. The marginal probability that a buyer who has high valuation for object A is denoted with $\alpha_H^A = \alpha_{HH} + \alpha_{HL}$. Similarly, $\alpha_L^A = \alpha_{LH} + \alpha_{LL}$ denotes the marginal probability that the buyer who has low valuation for object A. In the same fashion, we define $\alpha_H^B = \alpha_{HH} + \alpha_{LH}$ and $\alpha_L^B = \alpha_{HL} + \alpha_{LL}$ to be the marginal probabilities that the buyer has high and low valuations for object B, respectively.

Each buyer is risk-averse and has preferences represented by the common CARA utility function of the form $u(\omega) = -\frac{e^{-r\omega}}{r}$, where $r > 0$. In the event that a buyer wins object(s) of a (total) value v and incurs a net payment τ, his utility is equal to $u(v - \tau)$. For example, if a buyer wins only object A when his valuation for that object is v_L^A and incurs a net payment τ^A then his utility is equal to $u(v_L^A - \tau^A)$. Similarly, if a buyer of type HL wins both objects and incurs a net payment τ^{AB} then his utility is $u(v_H^A + v_L^B - \tau^{AB})$. Both the seller and the buyers are expected utility maximizers.

The seller's problem is to design a selling scheme that maximizes her revenue. In view of the Revelation Principle, we solve this problem within the class of revelation schemes which satisfy incentive compatibility and individual rationality constraints.[23] Furthermore, among the revelation schemes, we focus only on the symmetric ones in which the buyers of the same type are treated the same.

Let n_{ij} be the number of buyers of type ij and $\eta = (n_{HH}, n_{HL}, n_{LH}, n_{LL})$ be the vector representing the profile of reports where $\sum_{ij\in\Theta} n_{ij} = n$. Then, a symmetric revelation scheme can most generally be described with two sets of rules:

- a *decision rule*, $p_{ij}^{k}(\eta)$, that assigns each type $ij \in \Theta$ probabilities of realizing possible events $k = A, B, AB, O$, for each profile of reports η. Given η, the decision rule must satisfy

$$\sum_{ij\in\Theta} n_{ij}[p_{ij}^{A}(\eta) + p_{ij}^{AB}(\eta)] \leq 1 \tag{3.10}$$

$$\sum_{ij\in\Theta} n_{ij}[p_{ij}^{B}(\eta) + p_{ij}^{AB}(\eta)] \leq 1 \tag{3.11}$$

$$n_{ij}[p_{ij}^{A}(\eta) + p_{ij}^{B}(\eta) + p_{ij}^{AB}(\eta) + p_{ij}^{O}(\eta)] = 1 \quad \forall ij \in \Theta \tag{3.12}$$

- a *payment rule*, $\tilde{t}_{ij}^{k}(\eta)$, that assigns each type $ij \in \Theta$ possibly random payments to be made to the seller at each possible event $k = A, B, AB, O$, for each profile of reports η.

The decision rule specifies with what probability a buyer b of type ij should realize v_i^A, v_j^B, $v_i^A + v_j^B$ or 0. We abuse the notation and list these four events respectively as:

A	- winning *only* object A
B	- winning *only* object B
AB	- winning *both* object A and object B
O	- winning neither object.

[23] Remember that in a revelation scheme, buyers will be asked to report their types.

As shown in Armstrong [6], the risk-neutral buyers are only interested in the marginal probabilities of winning the objects. For risk-averse buyers, on the other hand, the correlation between the events of winning object A and object B matters. The decision rule in the above specification takes this into consideration.

Note that $p_{ij}^{A}(\eta)+p_{ij}^{AB}(\eta)$ in (3.10) represents the marginal probability of winning object A which we shortly denote with $\hat{p}_{ij}^{A}(\eta)$. Similarly, $p_{ij}^{B}(\eta)+p_{ij}^{AB}(\eta)$, in (3.11), represents the marginal probability of obtaining object B which is denoted with $\hat{p}_{ij}^{B}(\eta)$. Thus, conditions (3.10) and (3.11) are the resource constraints representing the fact that there is only one unit of each object. Condition (3.12) states that the events A, B, AB and O are all inclusive.

Although the payment rule allows the seller to impose random payments, when we solve the seller's problem, we use $\tilde{t}_{ij}^{k}(\eta)=\tau_{ij}^{k}$ where $\tau_{ij}^{k}\in\mathbb{R}$ for all $ij\in\Theta$ and $k=A,B,AB,O$, and characterize the optimal scheme within the class of schemes which assigns deterministic payments. We show that imposing random payments to each type ij under each event k cannot improve the seller's revenue.

Define the ij-type buyer's expected probability of realizing the event $k=A,B,AB,O$ as

$$\rho_{ij}^{k}=\sum_{n_{HH}=0}^{n}\sum_{n_{HL}=0}^{n-n_{HH}}\sum_{n_{LH}=0}^{n-n_{HH}-n_{HL}}p_{ij}^{k}(n_{HH},n_{HL},n_{LH},n_{LL})\Psi\frac{n_{ij}}{\alpha_{ij}} \tag{3.13}$$

where $\Psi=\frac{(n-1)!\alpha_{HH}^{n_{HH}}\alpha_{HL}^{n_{HL}}\alpha_{LH}^{n_{LH}}\alpha_{LL}^{n_{LL}}}{n_{HH}!n_{HL}!n_{LH}!n_{LL}!}$. For any $n_{ij}>0$, the variable $\Psi\frac{n_{ij}}{\alpha_{ij}}$ denotes the probability that the buyer profile is $\eta=(n_{HH},n_{HL},n_{LH},n_{LL})$ given that there is one ij in that profile (of course, conditional on incentive constraints hold).[24]

The reduced form of a symmetric revelation scheme, then, can be represented by

$$\{\rho_{ij}^{A},\rho_{ij}^{B},\rho_{ij}^{AB},\rho_{ij}^{O},\tau_{ij}^{A},\tau_{ij}^{B},\tau_{ij}^{AB},\tau_{ij}^{O}\}_{ij\in\Theta}.$$

ρ_{ij}^{A} and ρ_{ij}^{B} are type ij's expected probability of winning object A or B, *alone*; whereas ρ_{ij}^{AB} is his probability of winning *both* objects. Apparently, $\rho_{ij}^{O}=1-\rho_{ij}^{A}-\rho_{ij}^{B}-\rho_{ij}^{AB}$ represents the probability of winning *neither* object. τ_{ij}^{k} is the net deterministic payment that type ij must incur if event k occurs.

[24] The multinomial distribution is used.

Then, the utility of a buyer of type ij who misrepresents his type as $i'j'$ is

$$\rho^A_{i'j'}u(v^A_i - \tau^A_{i'j'}) + \rho^B_{i'j'}u(v^B_j - \tau^B_{i'j'}) + \rho^{AB}_{i'j'}u(v^A_i + v^B_j - \tau^{AB}_{i'j'}) + \rho^O_{i'j'}u(-\tau^O_{i'j'}).$$

Let $c^o_i = e^{-rv^o_i}$ for $o = A, B$ and $i = H, L$ and $y^k_{ij} = e^{r\tau^k_{ij}}$ for $k \in K = \{A, B, AB, O\}$ and $ij \in \Theta$. Then a scheme is *individually rational* if, for each type $ij \in \Theta$,

$$D_{ij} \equiv \rho^A_{ij}c^A_i y^A_{ij} + \rho^B_{ij}c^B_j y^B_{ij} + \rho^{AB}_{ij}c^A_i c^B_j y^{AB}_{ij} + \rho^O_{ij}y^O_{ij} \leq 1.$$

An auction is *incentive compatible* if, for any $ij \in \Theta$ and $i'j' \in \Theta \setminus \{\, ij\}$,

$$D_{ij} \leq \rho^A_{i'j'}c^A_i y^A_{i'j'} + \rho^B_{i'j'}c^B_j y^B_{i'j'} + \rho^{AB}_{i'j'}c^A_i c^B_j y^{AB}_{i'j'} + \rho^O_{i'j'}y^O_{i'j'} \equiv D^{i'j'}_{ij}.$$

The seller's revenue can, then, be written in terms of the expected payment of an arbitrary buyer, namely the term in brackets:

$$\pi = n[\sum_{ij\in\Theta}\{\alpha_{ij}\sum_{k\in K}\rho^k_{ij}\tau^k_{ij}\}]. \tag{3.14}$$

Note that, $\tau^k_{ij} = \frac{1}{r}\ln y^k_{ij}$. Then, if the reduced form probabilities are *implementable* we can write the seller's problem in reduced form as:

$$\max_{\{\rho^k_{ij}, y^k_{ij}\}_{ij\in\Theta, k\in K}} \frac{n}{r}\sum_{ij\in\Theta}\{\alpha_{ij}\sum_{k\in K}\rho^k_{ij}\ln y^k_{ij}\} \tag{SP}$$

subject to

$$D_{ij} \leq 1 \qquad ij \in \Theta \tag{3.15}$$

$$D_{ij} \leq D^{i'j'}_{ij} \quad ij \in \Theta,\ i'j' \in \Theta \setminus \{ij\}. \tag{3.16}$$

Since the buyers are risk-averse, the correlation between the events of winning object A (namely, event $A \cup AB$) and object B (namely, event $B \cup AB$) matters for the buyers and also for the seller through (3.14), (3.15), and (3.16). Thus, Border's [15] theorem does not apply to this problem.[25] As it is mentioned in Armstrong [6], the conditions that we need to impose to ensure that the reduced form probabilities are implementable are not clear. For this reason, different from the previous section, we aim to construct the actual probabilities,

[25] See Footnote 16.

$p_{ij}^{k}(\eta)$, $\forall ij \in \Theta$, $k = A, B, AB$ and $\forall \eta$.[26] Given a payment rule, the optimality of a decision rule is analyzed as follows: For any modification of $p_{ij}^{k}(\eta)$, we first describe how expected probabilities ρ_{ij}^{k} is affected. Then, we figure out whether the incentive constraints in (3.16) and individual rationality constraints in (3.15) hold and whether the objective function (SP) increases after the modification. To demonstrate how this works, we borrow the following example from Menicucci [65]:

Suppose for a given profile of reports with $n_{HH} \geq 1$ and $n_{LH} \geq 1$ each type wins object A with probability $\frac{1}{n_{HH}}$ and each type LH wins object B with probability $\frac{\beta}{n_{LH}}(0 < \beta \leq 1)$. Note that from (3.13), this generates a contribution to ρ_{LH}^{B} equal to

$$\frac{\beta}{n_{LH}} \Psi \frac{n_{LH}}{\alpha_{LH}}.$$

Consider the possibility of reducing β by $\Delta\beta > 0$ while increasing the probability that the same buyer of type HH who wins the object A also wins the object B by $\Delta\beta$. Then,

$$\Delta\rho_{LH}^{B} = -\frac{\Delta\beta}{n_{LH}} \Psi \frac{n_{LH}}{\alpha_{LH}},$$

$$\Delta\rho_{HH}^{A} = -\frac{\Delta\beta}{n_{HH}} \Psi \frac{n_{HH}}{\alpha_{HH}} = -\Delta\rho_{HH}^{AB}.$$

So, $\Delta\rho_{HH}^{AB} = -\Delta\rho_{HH}^{A} = -\frac{\alpha_{LH}}{\alpha_{HH}} \Delta\rho_{LH}^{B}$. We can then evaluate the profitability of reducing β since the seller's profit function and the constraints are linear with respect to the expected probabilities.

In the next section, we solve the seller's problem in order to find the optimal auction. All proofs are presented in the Appendix E.

[26] Given ij and η, $p_{ij}^{O}(\eta)$ can be calculated using (3.12) once the values of $p_{ij}^{A}(\eta)$, $p_{ij}^{B}(\eta)$, and $p_{ij}^{AB}(\eta)$ are found.

3.3.2 Solution to the Problem

Before we attempt to solve the problem (SP), note that, since $0 < c_H < c_L$, incentive compatibility conditions imply that among the individual rationality conditions only the one corresponding to type LL matters.

Using the same approach as in Armstrong [6], we first solve the seller's problem considering only the five downward incentive constraints, which ensure that a buyer does not underreport his valuation for an object. To establish that a solution to the relaxed problem solves the full problem, it is required to show *ex post* that the remaining constraints are satisfied.

Thus, the seller solves:

$$\begin{aligned} \max \quad & \alpha_{HH}\{\rho^A_{HH}\ln y^A_{HH} + \rho^B_{HH}\ln y^B_{HH} + \rho^{AB}_{HH}\ln y^{AB}_{HH} + \rho^O_{HH}\ln y^O_{HH}\} \\ & +\alpha_{HL}\{\rho^A_{HL}\ln y^A_{HL} + \rho^B_{HL}\ln y^B_{HL} + \rho^{AB}_{HL}\ln y^{AB}_{HL} + \rho^O_{HL}\ln y^O_{HL}\} \\ & +\alpha_{LH}\{\rho^A_{LH}\ln y^A_{LH} + \rho^B_{LH}\ln y^B_{LH} + \rho^{AB}_{LH}\ln y^{AB}_{LH} + \rho^O_{LH}\ln y^O_{LH}\} \\ & +\alpha_{LL}\{\rho^A_{LL}\ln y^A_{LL} + \rho^B_{LL}\ln y^B_{LL} + \rho^{AB}_{LL}\ln y^{AB}_{LL} + \rho^O_{LL}\ln y^O_{LL}\} \end{aligned} \tag{SP'}$$

subject to

$$\rho^A_{LL}c^A_L y^A_{LL} + \rho^B_{LL}c^B_L y^B_{LL} + \rho^{AB}_{LL}c^A_L c^B_L y^{AB}_{LL} + \rho^O_{LL}y^O_{LL} \le 1 \tag{IR_{LL}}$$

$$\begin{aligned} & \rho^A_{LH}c^A_L y^A_{LH} + \rho^B_{LH}c^B_H y^B_{LH} + \rho^{AB}_{LH}c^A_L c^B_H y^{AB}_{LH} + \rho^O_{LH}y^O_{LH} \\ \le \; & \rho^A_{LL}c^A_L y^A_{LL} + \rho^B_{LL}c^B_H y^B_{LL} + \rho^{AB}_{LL}c^A_L c^B_H y^{AB}_{LL} + \rho^O_{LL}y^O_{LL} \end{aligned} \tag{IC^{LL}_{LH}}$$

$$\begin{aligned} & \rho^A_{HL}c^A_H y^A_{HL} + \rho^B_{HL}c^B_L y^B_{HL} + \rho^{AB}_{HL}c^A_H c^B_L y^{AB}_{HL} + \rho^O_{HL}y^O_{HL} \\ \le \; & \rho^A_{LL}c^A_H y^A_{LL} + \rho^B_{LL}c^B_L y^B_{LL} + \rho^{AB}_{LL}c^A_H c^B_L y^{AB}_{LL} + \rho^O_{LL}y^O_{LL} \end{aligned} \tag{IC^{LL}_{HL}}$$

$$\begin{aligned} & \rho^A_{HH}c^A_H y^A_{HH} + \rho^B_{HH}c^B_H y^B_{HH} + \rho^{AB}_{HH}c^A_H c^B_H y^{AB}_{HH} + \rho^O_{HH}y^O_{HH} \\ \le \; & \rho^A_{LL}c^A_H y^A_{LL} + \rho^B_{LL}c^B_H y^B_{LL} + \rho^{AB}_{LL}c^A_H c^B_H y^{AB}_{LL} + \rho^O_{LL}y^O_{LL} \end{aligned} \tag{IC^{LL}_{HH}}$$

$$\begin{aligned} & \rho^A_{HH}c^A_H y^A_{HH} + \rho^B_{HH}c^B_H y^B_{HH} + \rho^{AB}_{HH}c^A_H c^B_H y^{AB}_{HH} + \rho^O_{HH}y^O_{HH} \\ \le \; & \rho^A_{LH}c^A_H y^A_{LH} + \rho^B_{LH}c^B_H y^B_{LH} + \rho^{AB}_{LH}c^A_H c^B_H y^{AB}_{LH} + \rho^O_{LH}y^O_{LH} \end{aligned} \tag{IC^{LH}_{HH}}$$

$$
\begin{aligned}
& \rho^A_{HH} c^A_H y^A_{HH} + \rho^B_{HH} c^B_H y^B_{HH} + \rho^{AB}_{HH} c^A_H c^B_H y^{AB}_{HH} + \rho^O_{HH} y^O_{HH} \\
\leq \; & \rho^A_{HL} c^A_H y^A_{HL} + \rho^B_{HL} c^B_H y^B_{HL} + \rho^{AB}_{HL} c^A_H c^B_H y^{AB}_{HL} + \rho^O_{HL} y^O_{HL}.
\end{aligned} \tag{IC^{HL}_{HH}}
$$

We first show that it is not optimal for the seller to assign random payments and determine which of the six constraints in the relaxed problem are binding at the optimum.

Proposition 9. *If the buyers preferences are represented by CARA then, in an optimal auction, the payments must be deterministic.*

Lemma 10. *At the optimum of the relaxed problem,* IR_{LL} *must be binding.*

Lemma 11. *At the optimum of the relaxed problem,* IC^{LL}_{LH} *and* IC^{LL}_{HL} *must be binding.*

Lemma 12. *At the optimum of the relaxed problem, at least one of* $IC^{LL}_{HH}, IC^{LH}_{HH}$ *and* IC^{HL}_{HH} *must be binding.*

By using the above lemmata, we first write the Lagrangian of the problem and derive its Kuhn-Tucker conditions with respect to payments. Then, the relation among payments is found by the help of the Kuhn-Tucker conditions.[27] Similarly to the single-object case, we show that each buyer who has high valuation for the object(s) is perfectly insured against not obtaining it (them). The intuition is similar to one explained in Section 3.2:[28]

It is clear that the seller can obtain the largest payments from buyers who have high valuation for the object(s). If there is not a resource (capacity) constraint, the seller can make a high-type buyer's probability of obtaining the object(s) equal to one in order to reward him for revealing his true valuation(s). However, when there is a resource constraint, the same rewarding strategy does not work because each high-type buyer may face the risk of losing the object(s) to another high-type buyer and hence, the marginal utility of income may differ in the events of losing and winning. The resource constrained seller, however, can reward a high-type buyer by offering perfect insurance and increase her revenue. Note that if buyers are risk-neutral, there is no insurance issue. In other words, when buyers are

[27] The details are presented in the Appendix E.

[28] In Section 3.2, we study the same problem as Matthews [64] and Maskin and Riley [62] by using the discrete distribution of buyer valuations and hence, our results and the intuition are similar to those of Matthews and Maskin and Riley. However, in this section we deviate from them not only due to the discrete distribution of valuations but also due to the number of objects.

risk-averse the seller has an additional tool to extract more revenue from buyers compared to risk-neutral environment.

Proposition 13. *Each buyer is perfectly insured against the risk of losing the object(s) for which he has high valuation.*

The proposition above states that the seller keeps a buyer's marginal utility of income at the same level for the object(s) for which he has high valuation in the events of winning and losing. This proposition also implies that since payments made in the event of winning cannot exceed a buyer's valuation for that object, a buyer either does not make any payment or is compensated when he loses the object(s) for which he has high valuation.

When it comes to LL-type buyers, the seller faces the following dilemma: to extract more revenue from the LL-type buyer by offering insurance and to exploit the high-type buyers' risk-bearing to screen them. At the optimum, the marginal benefit of exploiting high-type buyers' risk-bearing exceeds the marginal cost of not offering insurance to LL-type buyers. Moreover a LL-type buyer pays penalty when he loses both objects which further detects high-type buyers from behaving as if they are LL-type.

Proposition 14. *Suppose that at the optimum of the relaxed problem type LL is not excluded from the auction. Then, he incurs a positive payment if he loses both objects.*

With the help of the above propositions and lemmata, the seller's problem becomes the following:

$$\begin{aligned}
&\max[\alpha_{HH}\hat{\rho}^A_{HH} + \alpha_{HL}\hat{\rho}^A_{HL}]\ln\frac{1}{c^A_H} + [\alpha_{HH}\hat{\rho}^B_{HH} + \alpha_{LH}\hat{\rho}^B_{LH}]\ln\frac{1}{c^B_H} + \alpha_{HH}\ln y^O_{HH} \quad (SP^*)\\
&+\alpha_{HL}[\hat{\rho}^B_{HL}\ln y^B_{HL} + (1-\hat{\rho}^B_{HL})\ln y^O_{HL}] + \alpha_{LH}[\hat{\rho}^A_{LH}\ln y^A_{LH} + (1-\hat{\rho}^A_{LH})\ln y^O_{LH}]\\
&+\alpha_{LL}[\rho^A_{LL}\ln y^A_{LL} + \rho^B_{LL}\ln y^B_{LL} + \rho^{AB}_{LL}\ln y^{AB}_{LL} + \rho^O_{LL}\ln y^O_{LL}]
\end{aligned}$$

subject to

$$
\begin{aligned}
D_{LL} &= 1 \\
D_{LH}^{LL} &= \hat{\rho}_{LH}^{A} c_{L}^{A} y_{LH}^{A} + (1 - \hat{\rho}_{LH}^{A}) y_{LH}^{O} \\
D_{HL}^{LL} &= \hat{\rho}_{HL}^{B} c_{L}^{B} y_{HL}^{B} + (1 - \hat{\rho}_{HL}^{B}) y_{HL}^{O} \\
y_{HH}^{O} &= \min \left\{ \begin{array}{c} D_{HH}^{LL}, \\ \hat{\rho}_{LH}^{A} c_{H}^{A} y_{LH}^{A} + (1 - \hat{\rho}_{LH}^{A}) y_{LH}^{O}, \\ \hat{\rho}_{HL}^{B} c_{H}^{B} y_{HL}^{B} + (1 - \hat{\rho}_{HL}^{B}) y_{HL}^{O}. \end{array} \right\}
\end{aligned}
$$

where $\hat{\rho}_{ij}^{A} = \rho_{ij}^{A} + \rho_{ij}^{AB}$ and $\hat{\rho}_{ij}^{B} = \rho_{ij}^{B} + \rho_{ij}^{AB}$.

Thus, for the optimality of an auction only the following reduced form probabilities matter:

$$\{\hat{\rho}_{ij}^{A}, \hat{\rho}_{ij}^{B}\}_{ij=HH,HL,LH}, \{\rho_{LL}^{k}\}_{k=A,B,AB}.$$

Consider a mechanism where, for a given profile η, both objects are sold with probability one. Then, if the seller modifies the mechanism by increasing $p_{ij}^{k}(\eta)$ by $\frac{1}{n_{ij}} \varepsilon_{ij}^{k}$, the following condition must hold:

$$\sum_{ij \in \Theta} (\varepsilon_{ij}^{k} + \varepsilon_{ij}^{AB}) \leq 0 \text{ for } k = A, B.$$

After this modification, ρ_{ij}^{k} increases by $\frac{1}{\alpha_{ij}} \varepsilon_{ij}^{k} \Psi$.

We now establish that the solution to the relaxed problem is weakly efficient. That is, if there is a buyer with high valuation for an object then that object is never sold to a buyer who has low valuation for that object.

Proposition 15. *Let $\eta = (n_{HH}, n_{LH}, n_{HL}, n_{LL})$ be the profile of the participating buyers. Then, the solution to the relaxed problem satisfies the following two rules:*

i) For any η with $n_{HH} + n_{HL} > 0$, $n_{HH} \hat{p}_{HH}^{A}(\eta) + n_{HL} \hat{p}_{HL}^{A}(\eta) = 1$

ii) For any η with $n_{HH} + n_{LH} > 0$, $n_{HH} \hat{p}_{HH}^{B}(\eta) + n_{HL} \hat{p}_{LH}^{B}(\eta) = 1$.

Corollary 16. *At the optimum of the relaxed problem, reduced form probabilities satisfy*

i) $\alpha_{HH} \hat{\rho}_{HH}^{A} + \alpha_{HL} \hat{\rho}_{HL}^{A} = \frac{1}{n}(1 - (\alpha_{L}^{B})^{n})$ and

ii) $\alpha_{HH} \hat{\rho}_{HH}^{B} + \alpha_{LH} \hat{\rho}_{LH}^{B} = \frac{1}{n}(1 - (\alpha_{L}^{A})^{n})$.

The next lemma establishes that both objects are sold with probability one if a buyer's payment contingent on winning an object for which he has low valuation is larger than his payment contingent on losing the both objects.

Lemma 17. *If $y^A_{LH} > y^O_{LH}, y^B_{HL} > y^O_{HL}$, and $y^A_{LL}, y^B_{LL} > y^O_{LL}$ then the solution to the relaxed problem satisfies the following two rules:*

i) For any η with $n_{HH} + n_{HL} = 0$, $n_{LH}\hat{p}^A_{LH}(\eta) + n_{LL}\hat{p}^A_{LL}(\eta) = 1$

ii) For any η with $n_{HH} + n_{LH} = 0$, $n_{HL}\hat{p}^B_{HL}(\eta) + n_{LL}\hat{p}^B_{LL}(\eta) = 1$.

Corollary 18. *At the optimum of the relaxed problem, reduced form probabilities satisfy*

i) $\alpha_{LL}\hat{\rho}^A_{LL} + \alpha_{LH}\hat{\rho}^A_{LH} = \frac{1}{n}(\alpha^A_L)^n$ and

ii) $\alpha_{LL}\hat{\rho}^B_{LL} + \alpha_{HL}\hat{\rho}^B_{HL} = \frac{1}{n}(\alpha^B_L)^n$.

Since $D_{HH} = y^O_{HH} \leq 1$, when HH loses both objects he either does not pay anything (i.e. $y^O_{HH} = 1$) or he is compensated (i.e. $y^O_{HH} < 1$).

Proposition 19. *In any mechanism that solves the relaxed problem, if an HH type buyer loses both objects then he is compensated.*

The intuition of the above proposition is similar to that of Proposition 4. Since, at the optimum, the seller gives up some information rent to HH-type buyers, the perfect insurance requires the compensation for HH-type buyers when they lose both objects. This proposition is also interesting in the following sense. When there is a single-object and the buyer valuations are continuously distributed, the highest type is not only perfectly insured but also compensated when he loses the object (Maskin and Riley [62]). Although our model differs both in terms of the distribution of buyer valuations and the number of objects, we derive a similar result i.e., a buyer who has high valuation for both objects (the highest type) is compensated when he loses both objects.

Proposition 20. *In any mechanism that solves the relaxed problem, if all the buyers are of type LL (i.e., $n_{LL} = n$) then objects are bundled and each buyer wins the bundle with equal probability (i.e., $p^{AB}_{LL}(\eta) = \frac{1}{n}$).*

By the above proposition, the objects A and B must be bundled to the same LL-type buyer. In other words, it is not optimal to sell the goods separately, as in that case, with positive probability, the objects might end up in the hands of different LL-type buyers.

Since the seller probabilistically assesses the buyer valuations (i.e., only ex-ante probabilities of the type distributions matter) and never keeps the objects by assumption, there always exists a positive probability that LL-type buyers can obtain both objects. Therefore, the result of the above proposition can be generalized for any type profile η.

Proposition 21. *Independent auctions are not optimal.*

In Proposition 14, we show that the seller makes LL-type buyers pay the penalty when they lose the both objects in order to exploit high-type buyers' risk-bearings. Whenever there exists a positive probability that LL-type buyers can receive the objects, the seller can achieve the following two objectives by bundling the objects to one of the LL-type buyers. First, bundling removes the possibility that one LL-type buyer receives the object A and another LL-type buyer receives the object B. This, in turn, increases the probability that more LL-type buyers lose both objects and hence, that the seller increases her revenue by collecting more penalty fees (in the events of winning since the seller does not give up any information rent to a LL-type buyer, bundling and selling the objects separately generate the same revenue). Second, since bundling increases LL-type buyers' probability of losing both objects, high-type buyers who imitate LL-type buyers face with even greater risk than before. In other words, bundling enforces further the seller's objective for posing the penalty fee.

Note that when buyers are risk-neutral, the probability of obtaining both objects is simply the sum of the probabilities of obtaining the objects separately. Moreover, in this environment, the seller does not assign different payments conditional on winning and losing. Therefore, there is no issue of bundling the objects among the same type-buyers.

Lemma 22. *In any mechanism that solves the relaxed problem,*

i) if η is such that $n_{LH}, n_{LL} > 0$ and $n_{LH} + n_{LL} = n$, then object A is sold to an LH type buyer (i.e. $n_{LH}\hat{p}^A_{LH}(\eta) = 1$) if

$$\mu_{LH} < \left(\frac{\alpha_{HL}}{y^O_{HL}}\frac{y^O_{HH}}{\alpha_{HH}} + 1\right)\left(\frac{\alpha_{LL}}{y^O_{LL}}\frac{y^O_{LH}}{\alpha_{LH}} + 1\right)^{-1} \equiv \gamma_{LH}. \tag{†}$$

Otherwise, an LL type buyer gets object A (i.e. $n_{LL}\hat{p}^A_{LL}(\eta) = 1$).

ii) if η is such that $n_{HL}, n_{LL} > 0$ and $n_{HL} + n_{LL} = n$, then object B is sold to an HL type buyer (i.e. $n_{HL}\hat{p}^B_{HL}(\eta) = 1$) if

$$\mu_{HL} < (\frac{\alpha_{LH}}{y^O_{LH}}\frac{y^O_{HH}}{\alpha_{HH}} + 1)(\frac{\alpha_{LL}}{y^O_{LL}}\frac{y^O_{HL}}{\alpha_{HL}} + 1)^{-1} \equiv \gamma_{HL}. \tag{‡}$$

Otherwise, an LL type buyer gets object B (i.e. $n_{LL}\hat{p}^B_{LL}(\eta) = 1$).

Note that, $\gamma_{LH} \geq 1$ if and only if $\gamma_{HL} \geq 1$.

According to the previous lemma, in the optimal auction, if the excess payment that LH makes for the object A is larger than that of LL (namely, $\tau^A_{LH} - \tau^O_{LH} > \tau^A_{LL} - \tau^O_{LL}$), then LH wins the object A. Similarly, if the excess excess payment that HL makes makes for the object B is larger than that of LL ($\tau^B_{HL} - \tau^O_{HL} > \tau^B_{LL} - \tau^O_{LL}$), HL wins the object B.

We already establish that the seller allocates an object to a buyer who has high-valuation for that object whenever there exists such a buyer. If there is not a high-type buyer for a given object, the seller allocates the object $A(B)$ to $LH(HL)$ and LL-type buyers according to the following three cases (see Figure 12):

- $\gamma_{LH} + \gamma_{HL} \leq 1$ (Region A_1),
- $1 \leq \gamma_{LH} + \gamma_{HL} \leq 2$ (Region A_2),
- $2 \leq \gamma_{LH} + \gamma_{HL}$ (Region A_3).

The three cases listed above are analogous to those mentioned in Lemma 2 of Armstrong [6]: strong positive correlation, weak positive correlation, and negative correlation respectively. Armstrong shows that the seller allocates the object A (B) to LH (HL) and LL-type buyers as follows: if there is a strong correlation then the seller randomly allocates the object A (B) to LH (HL) and LL-type buyers (independent auction), if there is a weak correlation then the seller allocates the object A (B) to LH (HL)-type buyers (bundling auction), if the correlation is negative then the seller uses the mixed of bundling and independent auctions (mixed auction). Note that when buyers are risk-neutral, the seller allocates the objects according to the strength of correlation between values for two objects. However, in our environment, the relevant optimal payments ($y^O_{HH}, y^O_{HL}, y^O_{LH}, y^O_{LL}$) also affect the decision of the allocation probabilities. Given a strength of correlation, the relation between payments make easy (or difficult) to satisfy the above cases. For example, if $y^O_{HH}y^O_{LL} > y^O_{HL}y^O_{LH}$ then

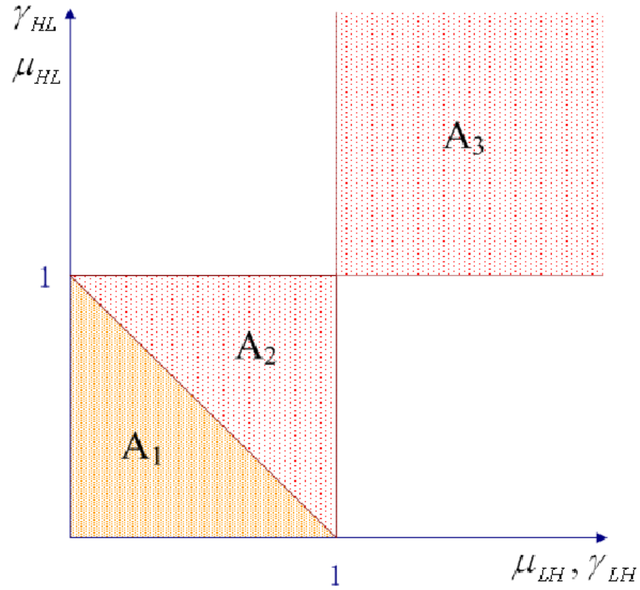

Figure 12: Possible Cases

it becomes difficult to satisfy the first condition but is easy to satisfy the third condition. Therefore, in order to find the allocation probabilities explicitly as in the risk-neutral case, one first needs to find the optimal payments. After that, by following the same methodology as in Armstrong [6], the allocation probabilities can be explicitly characterized.

3.3.3 Discussion

When buyers are risk-averse, two major difficulties arise in the solution of the problem: First, the buyers would not only be concerned about the marginal probabilities of obtaining the objects but also about the correlation between the events of obtaining the objects A and B together. Hence, the resource constraints of the risk-neutral case is not useful any more. We overcome this difficulty by using non-reduced form probabilities and finding their corresponding reduced-forms in our solution. Second, the number of variables to compute is considerably higher than that of the risk-neutral case. For each possible event and each possible type we are required to compute four allocation probabilities and their corresponding payments which makes a total of 32 variables, compared to the 8 variables of the risk-neutral case. However, we were able to characterize the fundamental features of the optimal auction under the relaxed problem. In other words, we established the necessary conditions of the optimal multi-object auctions: In terms of the payments, we show that the features of the single-object optimal auction is carried over to the multi-object optimal auctions i.e., the lowest type pays penalty, the high types are perfectly insured, and the highest type is compensated. We also establish the similarities and the differences between the characteristics of the optimal multi-object auctions with and without risk-averse buyers. Although no bundling issue occurs among the same types in risk-neutral environment, the seller bundles the objects to one of the LL-type buyers when the buyers are risk-averse (when there exists a positive probability that LL-type buyers receive the objects). It is also shown that the seller always gives the objects to a buyer who has high-valuation whenever such a buyer exists, as in the risk-neutral case. Moreover, in the absence of a high-type buyer for a given object, the seller allocates the object among the remaining types under similar conditions to those of the risk-neutral environment.

3.4 CONCLUSION

In a binary model, we show that when the buyers are risk-averse, the optimal auction is weakly efficient. That is, with probability one each object is sold to a buyer who has high valuation for it, if such a buyer exists. Each buyer is perfectly insured against the risk of losing the object(s) for which he has high valuation. Buyers who have high valuation for both objects are compensated if they can not win either object whereas, buyers who have low valuation for both objects incur a positive payment if they lose both objects. The objects are bundled when all buyers have low valuation for both objects, thus, independent auctions are not optimal.

In a more general framework, it has been shown that among all mechanisms for allocating multiple objects that are efficient, incentive compatible, and individually rational, the Vickrey-Clarke-Groves mechanism maximizes the expected revenue.[29] Thus, an optimal mechanism may not necessarily be efficient. The inefficiency results either because some

[29] For a clear and concise discussion of VCG mechanisms see Krishna [54].

types are *ex ante* excluded from participating the auction, or because of a misallocation. In this paper, we confined ourselves from the first kind of inefficiency, and showed that the latter kind of inefficiency does not occur in an optimal auction. Yet, this result is very sensitive to the assumption of binary distribution of types. Armstrong [6] shows that weak efficiency does not survive once the type space is made continuous.

The seller can exploit the risk bearing of the buyers, either by making their payments different or assigning random payments in the events of winning and losing. While the former improves the revenue because of the risk-aversion assumption the latter does not due to the CARA specification.

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APPENDIX A

EXPERIMENTAL INSTRUCTIONS

Preparation:

- Gold stars attached to the wall towards the front of the lab.
- Prepare folders with summary of instructions and record sheet (six shiny black folders with stars on the front and six yellow manila folders).

Arrival:

- Participants are seated in the lab.

Welcome and Consent:

- Thank you for coming.
- Before we begin we will hand out a consent form.
- Please read the consent form carefully, put your initials at the bottom of the first page and sign it at the bottom of the second page. Your signature will indicate your willingness to participate in the experiment.
- After you have signed the consent form we will come around to collect it.
- *Collect forms.*

Quiz:

- As indicated in the consent form there are two parts of this experiment.

- First, you will be asked to answer a quiz, and then you will participate in a decision-making experiment.
- We will start with the quiz [*hand out the quiz*].
- The answers you give on this quiz will determine to which one of two groups you will be assigned for the decision making part of the study. At the end of the experiment you will receive$5 for having completed the quiz.
- At the top of the quiz there is a yellow post-it note with an ID number on it. This is the number we will use to identify you in the experiment. Please remove your ID number and put it in a safe place.
- Please go ahead and answer the quiz.

* * * * * * * * * * * * * * * * * * * *

ID Number_________

Quiz

Please take a few moments to answer the quiz. If you do not know an answer please give your best guess. When you have completed the quiz, please turn it over and we will come around to collect it. Your score on the quiz determines to which one of two groups you will be assigned for the decision making part of the study. At the end of the experiment you will receive $5 for completing the quiz.

1. How many days are there in a non-leap year?
2. How many degrees Fahrenheit correspond to 0 degrees Celsius?
3. How many red stripes are there in the American flag?
4. How many days are there in the month of February during a leap year?
5. How many members are there of the U.S. Senate?
6. How many floors are there in the Cathedral of Learning?
7. What year was the University of Pittsburgh founded?
8. What are the costs of sending a one-ounce first-class letter within the United States?
9. How many acres is the main campus of the University of Pittsburgh?
10. How many millions of dollars did the University of Pittsburgh receive in research money from the National Institute of Health between 1995 and 2002?

- *Collect trivia quiz and hand out instructions.*
- *Score quiz in the back of the room-visible to the participants. The 50% who provide the largest numerical answers to question 10 are assigned to the star-group in half of sessions and assigned to the no-star group in the other half.*

Decision-Making Experiment:

- While we score your quiz we will go over the instructions for the decision making experiment. Please follow along as I read the instructions out loud.

* *

Instructions

This is an experiment about decision making. There are twelve people in this room participating in the experiment. Six participants will be given the role of 'first-mover,' the other six will be given the role of 'second-mover.' Your score on the quiz determines whether you are a first mover or a second mover. Your role will be the same throughout the experiment.

The experiment will consist of twelve rounds. In each round, each first-mover will be anonymously and randomly paired with a second-mover. This will be done in such a way that you will not be paired with the same person two rounds in a row. Nor will you be paired with the same person more than two times. You will never know the identity of the other person in your pair, nor will that person know your identity.

Choices and earnings

In each round you have to choose between two options: A or B. The other person in your pair also has to choose between options A and B. Your earnings in each round will depend on the decisions made by you and the person you are paired with for that round.

If you choose A, 100 cents are added to your earnings and 0 cents are added to the earnings of the person with whom you are paired. Likewise, if the person you are paired with chooses A, 100 cents are added to his or her earnings and 0 cents are added to your earnings.

If you choose B, 75 cents are added both to your earnings and to the earnings of the other person in your pair (irrespective of whether that person chooses A or B). Likewise, if the other person in your pair chooses B, 75 cents are added both to his or her earnings and to your earnings (irrespective of whether you choose A or B).

Procedure and information

In the first stage of a round the first-mover will enter a choice (A or B). Then, in the second stage, the second-mover will enter a choice (A or B). Before making his or her choice the second-mover will be informed of the first-mover's choice.

When all the second-movers have made their choices, the result of the round will be shown on your screen. The screen will list the choices made by you and the other person in your pair, and the amounts earned by you and the other person in your pair. You should then record this information on your Record Sheet.

You must not talk to the other participants or communicate with them in any way during the experiment. If, at any stage, you have any questions raise your hand and the experimenter will come to where you are sitting to answer them.

Quiz

To make sure everyone understands how earnings are calculated, we are going to ask you to complete a short quiz. Once everyone has completed the quiz we will go over the answers. If you finish the quiz early, please be patient. For each question you have to calculate earnings in a round for you and the other person in your pair. Please raise your hand if you have any questions.

	your earnings	other's earnings
1. You choose A and the person you are paired with chooses A?	– – ––	– – ––
2. You choose A and the person you are paired with chooses B?	– – ––	– – ––
3. You choose B and the person you are paired with chooses A?	– – ––	– – ––
4. You choose B and the person you are paired with chooses B?	– – ––	– – ––

* * * * * * * * * * * * * * * * * * * *

- *When all participants have completed the quiz go over the answers using the black board.*

Assignment to groups:

- We have completed the instructions for the decision-making part of the experiment, and based on the results of your quiz we will now assign you to be either a first or second mover in the experiment.

- Those who received the high score on the quiz are assigned to what we will refer to as the star-group. The members of the star-group will be first movers in the experiment. Individuals who received a low score will be assigned to the no-star-group and will be second movers in the experiment.
- We will first call out the ID numbers for those who received a high enough score to be part of the star-group.
- Once you hear your ID number called please come to the front of the class.
- Once you get up here Cagri will give you a folder with a summary of the instructions and your record sheet as well as a ribbon to congratulate you. Please wear this ribbon for the rest of the experiment. [hand out ribbons and shinny black folders]
- Please remain standing at the front of the room until all members of the star-group have been found.
- Call out ID numbers
- Let's give the Star-group a round of applause.
- Members of the star-group will be seated in the two front rows of the lab. If the no-star people could please come up and get your folders with a summary and record sheet (Cagri hand out yellow manila folders, Lise seat star-group towards front)
- If the members of the no-star-group can take a seat the last two rows. (Cagri direct them)
- Before we begin let us summarize the rules of the experiment by reading through the summary in your folder.

* *

Summary

The rules of the experiment are as follows:

1. If you received a star for your performance on the quiz you are a first mover, if you did not receive a star you are a second mover.
2. You will be making decisions over 12 rounds. The sequence of each round is as follows:

a. Each first-mover is randomly paired with a second-mover.

b. The first-mover chooses between A and B.

c. The second mover is informed of the first mover's choice, and chooses between A and B.

d. Both the first-mover and the second-mover are informed of the results of the round and record them on their Record Sheet.

3. After round 12 the experiment ends and each participant is paid his or her accumulated earnings from the twelve rounds, plus $5 for completing the quiz. Payments are done in private and cash.

* *

- *Point to the first two rows* "you will be the first movers" and *point to the last two rows* "you will be the second movers."
- We are ready to begin the decision-making part of the experiment. At various times you will have to wait for others to make their decisions. When that happens please be patient. If you have a question at any time, just raise your hand. Be sure to click OK when you have finished reading the content on the screen.

APPENDIX B

EXPERIMENTAL RESULTS

Table 3: Average Contribution per Round

		g_1	g_2	G
All rounds	Star-First	.552	.274	.826
	Star-Second	.333	.128	.461
First 6 rounds	Star-First	.638	.354	.992
	Star-Second	.437	.187	.624
Last 6 rounds	Star-First	.458	.194	.652
	Star-Second	.231	.069	.300

Table 4: Average Earnings ($) per Round

		earnings for first mover	earnings for second mover	Total earnings
All rounds	Star-First	1.067	1.345	2.412
	Star-Second	1.013	1.217	2.230
First 6 rounds	Star-First	1.104	1.395	2.499
	Star-Second	1.031	1.281	2.312
Last 6 rounds	Star-First	1.031	1.295	2.326
	Star-Second	.994	1.154	2.148

Table 5: Conditional Probabilities (%) per Round

		$\Pr(g_2 = 1 \mid g_1 = 1)$	$\Pr(g_2 = 1 \mid g_1 = 0)$
All rounds	Star-First	45	6
	Star-Second	30	5
First 6 rounds	Star-First	52	6
	Star-Second	34	9
Last 6 rounds	Star-First	37	6
	Star-Second	undetermined	2

Table 6: Treatment Effects on Contributions: One-sided p-values for test that Star-First≤Star-Second (Mann-Whitney U-test)

A: First Contribution:	
All rounds	0.0143
First 6 rounds	0.0286
Last 6 rounds	0.0429
B: Second Contribution:	
All rounds	0.0143
First 6 rounds	0.0143
Last 6 rounds	0.0571
C: Total Contribution:	
All rounds	0.0143
First 6 rounds	0.0143
Last 6 rounds	0.0571

Table 7: Treatment Effects on Mimicking Behavior: One-sided p-values for test that Star-First≤Star-Second (Mann-Whitney U-test)

$\Pr(g_2 = 1 \mid g_1 = 1)$	
All rounds	0.0786
First 6 rounds	0.0571
Last 6 rounds	0.1357

Table 8: Treatment Effects on Average Earnings per Round: One-sided p-values for test that Star-First≤Star-Second (Mann-Whitney U-test)

A: Donor 1's Earnings	
All rounds	0.0571
First 6 rounds	0.0571
Last 6 rounds	0.1357
B: Second Contribution:	
All rounds	0.0143
First 6 rounds	0.0143
Last 6 rounds	0.0429
C: Total Contribution:	
All rounds	0.0143
First 6 rounds	0.0143
Last 6 rounds	0.0571

APPENDIX C

TABLES OF CHAPTER 2

Table 9: CRRA

θ	Y	K	C	K/Y	r	w	CV (%)
0	1.239	3.308	0.971	2.671	0.068	1.088	0.000
0.10	1.225	3.187	0.968	2.602	0.072	1.076	1.300
0.20	1.212	3.081	0.965	2.542	0.075	1.065	2.637
0.30	1.199	2.978	0.962	2.483	0.078	1.054	4.043
0.40	1.188	2.892	0.960	2.434	0.081	1.044	5.460
0.50	1.178	2.814	0.957	2.388	0.084	1.035	6.913
0.60	1.168	2.738	0.954	2.344	0.087	1.026	8.416
0.70	1.159	2.668	0.952	2.301	0.090	1.018	9.951
0.80	1.150	2.605	0.949	2.264	0.093	1.011	11.520
0.90	1.143	2.548	0.947	2.230	0.095	1.004	13.112
1	1.135	2.493	0.944	2.197	0.098	0.997	14.752

Table 10: Counter-factual Calibration I

λ	Y	K	C	K/Y	r	w
$CRRA$	1.188	2.892	0.960	2.434	0.081	1.044
$\lambda = 0.0004$	1.120	2.386	0.939	2.132	0.103	0.984
$\lambda = 0.001$	1.067	2.046	0.918	1.917	0.122	0.938

Table 11: Counter-factual Calibration II

λ	Y	K	C	K/Y	r	w
$CRRA$	1.188	2.892	0.960	2.434	0.081	1.044
$\rho = 2$	1.168	2.735	0.954	2.342	0.087	1.026
$\rho = 3$	1.075	2.091	0.922	1.946	0.119	0.944

Table 12: Convex Temptation Function I - Benchmark Calibration

θ	Y	K	C	K/Y	r	w	$CV(\%)$
0	1.229	3.224	0.969	2.623	0.071	1.080	0.000
0.10	1.216	3.116	0.966	2.562	0.074	1.068	0.294
0.20	1.204	3.015	0.963	2.504	0.077	1.058	0.705
0.30	1.193	2.932	0.961	2.457	0.080	1.048	1.208
0.40	1.184	2.856	0.958	2.412	0.083	1.040	1.795
0.50	1.174	2.783	0.956	2.370	0.086	1.031	2.462
0.60	1.165	2.714	0.953	2.329	0.088	1.023	3.205
0.70	1.157	2.652	0.951	2.292	0.091	1.016	4.008
0.80	1.150	2.600	0.949	2.262	0.093	1.010	4.856
0.90	1.143	2.548	0.947	2.230	0.095	1.004	5.770
1	1.136	2.499	0.944	2.200	0.097	0.998	6.743

Table 13: Convex Temptation Function II

θ	Y	K	C	K/Y	r	w	CV (%)
0	1.255	3.423	0.972	2.727	0.064	1.090	0
0.10	1.242	3.311	0.970	2.665	0.067	1.080	−0.867
0.20	1.230	3.210	0.968	2.608	0.070	1.069	−1.542
0.30	1.219	3.120	0.966	2.558	0.073	1.060	−2.304
0.40	1.209	3.033	0.963	2.508	0.076	1.050	−2.385
0.50	1.199	2.959	0.961	2.466	0.078	1.043	−2.599
0.60	1.190	2.888	0.959	2.425	0.082	1.034	−2.698
0.70	1.182	2.823	0.957	2.387	0.083	1.027	−2.696
0.80	1.174	2.764	0.955	2.352	0.085	1.021	−2.602
0.90	1.167	2.708	0.953	2.319	0.087	1.014	−2.421
1	1.161	2.660	0.951	2.291	0.089	1.008	−2.167

Table 14: Robustness Check I

θ	Y	K	C	K/Y	r	w	CV (%)
0	1.285	3.696	0.976	2.875	0.057	1.116	0
0.10	1.261	3.475	0.973	2.755	0.062	1.096	2.905
0.20	1.239	3.289	0.970	2.652	0.067	1.077	6.084
0.30	1.220	3.125	0.966	2.560	0.072	1.060	9.500
0.40	1.202	2.981	0.962	2.478	0.077	1.045	13.12
0.50	1.186	2.854	0.958	2.405	0.082	1.031	16.89
0.60	1.172	2.744	0.954	2.341	0.086	1.018	20.81
0.70	1.158	2.640	0.950	2.280	0.090	1.006	24.93
0.80	1.145	2.550	0.946	2.225	0.094	0.995	29.15
0.90	1.133	2.464	0.942	2.173	0.098	0.985	33.59
1	1.122	2.387	0.938	2.126	0.101	0.975	38.15

Table 15: Robustness Check II

θ	Y	K	C	K/Y	r	w	CV (%)
0	1.264	3.508	0.974	2.774	0.062	1.099	0
0.10	1.252	3.394	0.972	2.710	0.065	1.088	0.541
0.20	1.239	3.284	0.969	2.649	0.068	1.077	1.197
0.30	1.227	3.187	0.967	2.595	0.071	1.067	1.940
0.40	1.217	3.104	0.965	2.549	0.073	1.058	2.750
0.50	1.208	3.025	0.963	2.504	0.076	1.049	3.632
0.60	1.199	2.953	0.961	2.463	0.078	1.042	4.577
0.70	1.189	2.879	0.959	2.420	0.081	1.034	5.595
0.80	1.181	2.816	0.957	2.383	0.083	1.026	6.652
0.90	1.174	2.761	0.955	2.351	0.085	1.020	7.757
1	1.167	2.707	0.953	2.319	0.087	1.014	8.916

Table 16: Counter-factual Calibration III

λ	Y	K	C	K/Y	r	w
$CRRA$	1.188	2.892	0.960	2.434	0.081	1.044
$\lambda = 0.0786$	1.163	2.705	0.953	2.324	0.086	1.022
$\lambda = 0.1346$	1.145	2.567	0.947	2.241	0.094	1.006

Table 17: Concave Temptation Function I

θ	Y	K	C	K/Y	r	w	CV (%)
0	1.242	3.330	0.971	2.682	0.067	1.091	0.000
0.10	1.227	3.204	0.968	2.612	0.071	1.078	1.120
0.20	1.213	3.089	0.965	2.547	0.075	1.066	2.313
0.30	1.200	2.987	0.963	2.488	0.078	1.054	3.557
0.40	1.189	2.897	0.960	2.437	0.081	1.044	4.836
0.50	1.178	2.811	0.957	2.386	0.084	1.035	6.181
0.60	1.168	2.735	0.954	2.341	0.087	1.026	7.556
0.70	1.159	2.665	0.951	2.300	0.090	1.018	8.971
0.80	1.149	2.595	0.949	2.258	0.093	1.009	10.451
0.90	1.141	2.540	0.946	2.225	0.096	1.003	11.929
1	1.133	2.481	0.943	2.189	0.098	0.995	13.483

Table 18: Concave Temptation Function II

θ	Y	K	C	K/Y	r	w	CV (%)
0	1.244	3.352	0.971	2.694	0.067	1.093	0.000
0.10	1.228	3.219	0.969	2.620	0.071	1.079	0.983
0.20	1.214	3.098	0.966	2.551	0.074	1.066	2.046
0.30	1.201	2.995	0.963	2.493	0.078	1.055	3.157
0.40	1.189	2.897	0.960	2.437	0.081	1.044	4.353
0.50	1.178	2.810	0.957	2.385	0.084	1.035	5.586
0.60	1.168	2.732	0.954	2.340	0.087	1.026	6.864
0.70	1.158	2.657	0.951	2.295	0.091	1.017	8.210
0.80	1.149	2.592	0.948	2.257	0.093	1.009	9.583
0.90	1.141	2.533	0.946	2.221	0.096	1.002	10.994
1	1.132	2.475	0.943	2.185	0.099	0.995	12.461

Table 19: Robustness Check III

θ	Y	K	C	K/Y	r	w	$CV(\%)$
0	1.267	3.560	0.975	2.809	0.061	1.113	0.000
0.10	1.244	3.348	0.971	2.692	0.067	1.092	3.371
0.20	1.223	3.171	0.968	2.593	0.072	1.074	6.937
0.30	1.204	3.017	0.963	2.506	0.077	1.058	10.700
0.40	1.187	2.877	0.959	2.425	0.082	1.042	14.702
0.50	1.171	2.758	0.955	2.355	0.086	1.029	18.792
0.60	1.157	2.649	0.951	2.290	0.091	1.016	23.078
0.70	1.144	2.555	0.947	2.235	0.095	1.005	27.455
0.80	1.131	2.462	0.943	2.178	0.099	0.993	32.107
0.90	1.119	2.384	0.939	2.130	0.103	0.983	36.804
1	1.108	2.309	0.935	2.083	0.107	0.973	41.721

APPENDIX D

PROOFS OF SINGLE-OBJECT CASE

Proof of Lemma 1. Suppose first that IR_L is slack. Then, the seller can improve her revenue by increasing y_L^l by $\varepsilon = \frac{1-D_L}{2} > 0$. This would not violate any of the constraints of the relaxed problem. So, IR_L must be binding.

Suppose, next, that IC_H is slack. Then, again, the mechanism can be improved profitably, without violating any of the conditions considered in the relaxed problem. Namely, increasing y_H^l by $\varepsilon = \frac{D_H^L - D_H}{2} > 0$ improves the revenue. Hence, IC_H is also binding. □

Proof of Lemma 2. Suppose, by contradiction, that IR_H is binding. Then, we have $1 = D_H = D_H^L = D_L$, where the equalities are due to IR_H, IC_H, and IR_L, respectively. Yet, since low-type buyers are not excluded, this would contradict with $D_L - D_H^L = \rho_L(c_L - c_H)y_L^w > 0$. Hence, IR_H is slack. □

Proof of Proposition 3. Kuhn-Tucker conditions of (3.6) with respect to y_H^w and y_H^l yield

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial y_H^w} &= \alpha_H \rho_H \frac{1}{y_H^w} - \mu_H \rho_H c_H = 0 \\
\frac{\partial \mathcal{L}}{\partial y_H^l} &= \alpha_H (1-\rho_H) \frac{1}{y_H^l} - \mu_H (1-\rho_H) = 0
\end{aligned}$$

These two equations imply that $y_H^l = c_H y_H^w$ which is equal to the following equation: $\tau_H^l + \tau_H^w = v_H$. □

Proof of Proposition 4. Remember that IR_H is slack by Lemma 2. Using Proposition 3, we can rewrite this condition as

$$D_H = y_H^l < 1.$$

This is equivalent to $\tau_H^l < 0$, implying that, at the optimum, a high-type buyer is compensated when he loses the object. □

Proof of Proposition 6. Armed with the optimal values of ρ_H, and ρ_L, we now calculate the payments made by each type of buyer. Using IC_H, IR_L, and Proposition 3, we write the payments, y_L^w, y_L^l, and y_L^w, as

$$y_L^w = \tfrac{1-y_H^l}{\rho_L(c_L-c_H)}, \quad y_L^l = \tfrac{c_L y_H^l - c_H}{(1-\rho_L)(c_L-c_H)}, \quad y_H^w = \tfrac{y_H^l}{c_H}$$

where y_H^l is in

$$\arg\max_{y_H^l}\{\frac{n}{r}[\alpha_H(\rho_H \ln\frac{1}{c_H} + \ln y_H^l) + \alpha_L(\rho_L \ln(1-y_H^l) + (1-\rho_L)\ln(c_L y_H^l - c_H))]\}.$$

Equivalently, y_H^l solves the first-order condition of the form

$$\frac{\alpha_H}{y_H^l} + \frac{\alpha_H(1-\rho_L)c_L}{c_L y_H^l - c_H} - \frac{\alpha_L \rho_L}{1-y_H^l} = 0.$$

This equation can be rewritten as

$$c_L(y_H^l)^2 - \xi y_H^l + \alpha_H c_H = 0 \tag{D.1}$$

where $\xi = (1-\rho_L)(c_L + \alpha_H c_H) + \rho_L(c_H + \alpha_H c_L)$.

Since $0 < \rho_L < 1$ and $c_H < c_L$, $\xi > (c_H + \alpha_H c_L)$ must be true. Then, $\xi^2 - 4\alpha_H c_L c_H > (c_H + \alpha_H c_L)^2 - 4\alpha_H c_L c_H = (c_H - \alpha_H c_L)^2 \geq 0$. Thus, a solution to $(D.1)$ exists. Furthermore, if a buyer of type H loses the object he pays

$$y_H^l = \frac{\xi + \sqrt{\xi^2 - 4\alpha_H c_L c_H}}{2c_L}.$$

□

Proof of Proposition 7. We have already established above that IR_L and IC_H are binding and IR_H is slack. We only need to show that IC_L is slack. Equivalently, we need to show that $\rho_L y_L^w < \rho_H y_H^w$.[1] Plugging in the values of y_L^w and y_H^w gives

$$\frac{1 - y_H^l}{(c_L - c_H)} < \frac{\rho_H y_H^l}{c_H} \iff \frac{c_H}{\rho_H c_L + (1 - \rho_H)c_H} < y_H^l.$$

We substitute in the value of y_H^l to get

$$c_L c_H + \alpha_H[\rho_H c_L + (1 - \rho_H)c_H]^2 < \xi[\rho_H c_L + (1 - \rho_H)c_H].$$

Substituting in the value of ξ and using $IM_{\{H,L\}}$ yields

$$0 < c_L^2 \rho_H(n - 1) + c_H^2(1 - \rho_H) + c_L c_H[(2 - n)\rho_H - 1].$$

Now, we plug in the value of ρ_H and rewrite this condition as

$$0 < (1 - \alpha_L^n)[c_L^2(n - 1) - c_H^2 + c_L c_H(2 - n)] + (1 - \alpha_L)[c_H^2 n - c_L c_H n].$$

Since $c_H^2 n - c_L c_H n < 0$, we can replace $(1 - \alpha_L)$ with $(1 - \alpha_L^n)$ and get the following more restrictive condition

$$0 < (1 - \alpha_L^n)(n - 1)(c_L - c_H)^2,$$

which holds for any parameter values. Hence, IC_L must be slack. □

Proof of Proposition 8. Suppose that τ_i^w and τ_i^l [hence y_i^w and y_i^l] are stochastic. Replacing y_i^w and y_i^l with their expected values would not affect any of the incentive compatibility and individual rationality conditions (because buyers' utilities are linear with respect to these variables), but would strictly improve the seller's revenue (as revenue is concave with respect to y_i^w and y_i^l), which is a contradiction. □

[1] We add up IC_H (binding) and IC_L (slack).

APPENDIX E

PROOFS OF MULTI-OBJECT CASE

Proof of Proposition 9. The same argument applies as in the proof of Proposition 8. □

Proof of Lemma 10. Suppose that IR_{LL} is slack. Then, we have

$$D_{LL} \equiv \rho_{LL}^{A} c_{L}^{A} y_{LL}^{A} + \rho_{LL}^{B} c_{L}^{B} y_{LL}^{B} + \rho_{LL}^{AB} c_{L}^{A} c_{L}^{B} y_{LL}^{AB} + \rho_{LL}^{O} y_{LL}^{O} < 1.$$

Since number of buyers are larger than three and since buyers are treated symmetrically, each type's probability of losing both objects is positive i.e., $\rho_{LL}^{O} > 0$. Thus, an increase in y_{LL}^{O} by $\varepsilon/\rho_{LL}^{O}$ for $\varepsilon = (1 - D_{LL})/2 > 0$ strictly improves the seller's payoff. Note that, this modification on y_{LL}^{O} does not violate any of the constraints, yielding a contradiction.

Hence, IR_{LL} must be binding. □

Proof of Lemma 11. Suppose first that IC_{LH}^{LL} is slack. Then, we have

$$\begin{aligned} D_{LH} &\equiv \rho_{LH}^{A} c_{L}^{A} y_{LH}^{A} + \rho_{LH}^{B} c_{H}^{B} y_{LH}^{B} + \rho_{LH}^{AB} c_{L}^{A} c_{H}^{B} y_{LH}^{AB} + \rho_{LH}^{O} y_{LH}^{O} \\ &< \rho_{LL}^{A} c_{L}^{A} y_{LL}^{A} + \rho_{LL}^{B} c_{H}^{B} y_{LL}^{B} + \rho_{LL}^{AB} c_{L}^{A} c_{H}^{B} y_{LL}^{AB} + \rho_{LL}^{O} y_{LL}^{O} \equiv D_{LH}^{LL} \end{aligned}$$

Let $\varepsilon = (D_{LH}^{LL} - D_{LH})/2$. Since $\rho_{LH}^{O} > 0$, if we increase y_{LH}^{O} by $\varepsilon/\rho_{LH}^{O}$, the seller's payoff improves and none of the constraints are violated. This is a contradiction. So, IC_{LH}^{LL} must be binding.

Along the same lines, we can easily show that IC_{HL}^{LL} is binding, too. □

Proof of Lemma 12. Suppose that all three conditions are slack. Then, we have $D_{HH} < \min\{D_{HH}^{LL}, D_{HH}^{LH}, D_{HH}^{HL}\}$. Define $\varepsilon = (\min\{D_{HH}^{LL}, D_{HH}^{LH}, D_{HH}^{HL}\} - D_{HH})/2$. An increase in y_{HH}^{O} in the amount of $\varepsilon/\rho_{HH}^{O}$ improves the seller's payoff and does not violate any of the conditions. This is a contradiction. Therefore, at least one of these three conditions must be binding. $\square$

In order to provide the proofs of the remaining propositions and lemmata, we first need to write the Lagrangian of the relaxed problem (max SP' subject to IR_{LL}, IC_{LH}^{LL}, IC_{HL}^{LL}, IC_{HH}^{LL}, IC_{HH}^{LH}, and IC_{HH}^{HL}) and find its Kuhn-Tucker conditions. Since $D_{HH} = \min\{D_{HH}^{LL}, D_{HH}^{LH}, D_{HH}^{HL}\}$, we can replace HH type's incentive compatibility conditions with $D_{HH} = \mu_{LL} D_{HH}^{LL} + \mu_{LH} D_{HH}^{LH} + \mu_{HL} D_{HH}^{HL}$ where $\mu_{LL}, \mu_{LH}, \mu_{HL} \geq 0$ and $\mu_{LL} + \mu_{LH} + \mu_{HL} = 1$ provided that $\mu_{ij} = 0$ if and only if $D_{HH} < D_{HH}^{ij}$ (or equivalently, $\mu_{ij} > 0$ if and only if $D_{HH} = D_{HH}^{ij}$). Hence, the Lagrangian can be written as the following:

$\mathcal{L} = \alpha_{HH}(\rho_{HH}^{A} \ln y_{HH}^{A} + \rho_{HH}^{B} \ln y_{HH}^{B} + \rho_{HH}^{AB} \ln y_{HH}^{AB} + \rho_{HH}^{O} \ln y_{HH}^{O}) + \alpha_{HL}(\rho_{HL}^{A} \ln y_{HL}^{A} + \rho_{HL}^{B} \ln y_{HL}^{B} + \rho_{HL}^{AB} \ln y_{HL}^{AB} + \rho_{HL}^{O} \ln y_{HL}^{O})$

$+\alpha_{LH}(\rho_{LH}^{A} \ln y_{LH}^{A} + \rho_{LH}^{B} \ln y_{LH}^{B} + \rho_{LH}^{AB} \ln y_{LH}^{AB} + \rho_{LH}^{O} \ln y_{LH}^{O}) + \alpha_{LL}(\rho_{LL}^{A} \ln y_{LL}^{A} + \rho_{LL}^{B} \ln y_{LL}^{B} + \rho_{LL}^{AB} \ln y_{LL}^{AB} + \rho_{LL}^{O} \ln y_{LL}^{O})$

$+\lambda_{LL}(1 - \rho_{LL}^{A} c_{L}^{A} y_{LL}^{A} - \rho_{LL}^{B} c_{L}^{B} y_{LL}^{B} - \rho_{LL}^{AB} c_{L}^{A} c_{L}^{B} y_{LL}^{AB} - \rho_{LL}^{O} y_{LL}^{O})$

$+\lambda_{LH}[c_{L}^{A}(\rho_{LL}^{A} y_{LL}^{A} - \rho_{LH}^{A} y_{LH}^{A}) + c_{H}^{B}([\rho_{LL}^{B} y_{LL}^{B} - \rho_{LH}^{B} y_{LH}^{B}) + c_{L}^{A} c_{H}^{B}(\rho_{LL}^{AB} y_{LL}^{AB} - \rho_{LH}^{AB} y_{LH}^{AB}) + (\rho_{LL}^{O} y_{LL}^{O} - \rho_{LH}^{O} y_{LH}^{O})]$

$+\lambda_{HL}[c_{H}^{A}(\rho_{LL}^{A} y_{LL}^{A} - \rho_{HL}^{A} y_{HL}^{A}) + c_{L}^{B}(\rho_{LL}^{B} y_{LL}^{B} - \rho_{HL}^{B} y_{HL}^{B}) + c_{H}^{A} c_{L}^{B}(\rho_{LL}^{AB} y_{LL}^{AB} - \rho_{HL}^{AB} y_{HL}^{AB}) + (\rho_{LL}^{O} y_{LL}^{O} - \rho_{HL}^{O} y_{HL}^{O})]$

$+\lambda_{HH}\{\mu_{LL}[c_{H}^{A}(\rho_{LL}^{A} y_{LL}^{A} - \rho_{HH}^{A} y_{HH}^{A}) + c_{H}^{B}(\rho_{LL}^{B} y_{LL}^{B} - \rho_{HH}^{B} y_{HH}^{B}) + c_{H}^{A} c_{H}^{B}([\rho_{LL}^{AB} y_{LL}^{AB} - \rho_{HH}^{AB} y_{HH}^{AB}) + (\rho_{LL}^{O} y_{LL}^{O} - \rho_{HH}^{O} y_{HH}^{O})]$

$+\mu_{LH}[c_{H}^{A}(\rho_{LH}^{A} y_{LH}^{A} - \rho_{HH}^{A} y_{HH}^{A})] + c_{H}^{B}(\rho_{LH}^{B} y_{LH}^{B} - \rho_{HH}^{B} y_{HH}^{B}) + c_{H}^{A} c_{H}^{B}(\rho_{LH}^{AB} y_{LH}^{AB} - \rho_{HH}^{AB} y_{HH}^{AB}) + (\rho_{LH}^{O} y_{LH}^{O} - \rho_{HH}^{O} y_{HH}^{O})]$

$+\mu_{HL}[c_{H}^{A}(\rho_{HL}^{A} y_{HL}^{A} - \rho_{HH}^{A} y_{HH}^{A}) + c_{H}^{B}(\rho_{HL}^{B} y_{HL}^{B} - \rho_{HH}^{B} y_{HH}^{B}) + c_{H}^{A} c_{H}^{B}(\rho_{HL}^{AB} y_{HL}^{AB} - \rho_{HH}^{AB} y_{HH}^{AB}) + (\rho_{HL}^{O} y_{HL}^{O} - \rho_{HH}^{O} y_{HH}^{O})]\}$

Since the number of buyers participating in the auction is assumed to be larger than three and buyers of each type are treated the same in a symmetric auction, each type's probability of losing both objects is positive. That is $\rho_{ij}^{O} > 0$ for all $ij \in S$. Thus, using the four Kuhn-

Tucker conditions ($\frac{\partial \mathcal{L}}{\partial y^O_{HH}} = \rho^O_{HH}(\frac{\alpha_{HH}}{y^O_{HH}} - \lambda_{HH}) = 0; \frac{\partial \mathcal{L}}{\partial y^O_{LH}} = \rho^O_{LH}(\frac{\alpha_{LH}}{y^O_{LH}} + \lambda_{HH}\mu_{LH} - \lambda_{LH}) = 0; \frac{\partial \mathcal{L}}{\partial y^O_{HL}} = \rho^O_{HL}(\frac{\alpha_{HL}}{y^O_{HL}} + \lambda_{HH}\mu_{HL} - \lambda_{HL}) = 0; \frac{\partial \mathcal{L}}{\partial y^O_{LL}} = \rho^O_{LL}(\frac{\alpha_{LL}}{y^O_{LL}} - \lambda_{LL} + \lambda_{LH} + \lambda_{HL} + \lambda_{HH}\mu_{LL}) = 0$) we can solve for λ_{ij}s:

$$\begin{aligned}
\lambda_{HH} &= \frac{\alpha_{HH}}{y^O_{HH}} \\
\lambda_{HL} &= \frac{\alpha_{HL}}{y^O_{HL}} + \frac{\alpha_{HH}}{y^O_{HH}}\mu_{HL} \\
\lambda_{LH} &= \frac{\alpha_{LH}}{y^O_{LH}} + \frac{\alpha_{HH}}{y^O_{HH}}\mu_{LH} \\
\lambda_{LL} &= \frac{\alpha_{LL}}{y^O_{LL}} + \frac{\alpha_{LH}}{y^O_{LH}} + \frac{\alpha_{HL}}{y^O_{HL}} + \frac{\alpha_{HH}}{y^O_{HH}}.
\end{aligned}$$

The remaining Kuhn-Tucker conditions are of the following form:

$\frac{\partial \mathcal{L}}{\partial y^A_{HH}} = \rho^A_{HH}[\frac{\alpha_{HH}}{y^A_{HH}} - \lambda_{HH}c^A_H] = 0;$

$\frac{\partial \mathcal{L}}{\partial y^B_{HH}} = \rho^B_{HH}[\frac{\alpha_{HH}}{y^B_{HH}} - \lambda_{HH}c^B_H] = 0;$

$\frac{\partial \mathcal{L}}{\partial y^{AB}_{HH}} = \rho^{AB}_{HH}[\frac{\alpha_{HH}}{y^{AB}_{HH}} - \lambda_{HH}c^A_H c^B_H] = 0;$

$\frac{\partial \mathcal{L}}{\partial y^A_{HL}} = \rho^A_{HL}[\frac{\alpha_{HL}}{y^A_{HL}} - (\lambda_{HL} - \lambda_{HH}\mu_{HL})c^A_H] = 0;$

$\frac{\partial \mathcal{L}}{\partial y^B_{HL}} = \rho^B_{HL}[\frac{\alpha_{HL}}{y^B_{HL}} - (\lambda_{HL}c^B_L - \lambda_{HH}\mu_{HL}c^B_H)] = 0;$

$\frac{\partial \mathcal{L}}{\partial y^{AB}_{HL}} = \rho^{AB}_{HL}[\frac{\alpha_{HL}}{y^{AB}_{HL}} - c^A_H(\lambda_{HL}c^B_L - \lambda_{HH}\mu_{HL}c^B_H)] = 0;$

$\frac{\partial \mathcal{L}}{\partial y^A_{LH}} = \rho^A_{LH}[\frac{\alpha_{LH}}{y^A_{LH}} - (\lambda_{LH}c^A_L - \lambda_{HH}\mu_{LH}c^A_H)] = 0;$

$\frac{\partial \mathcal{L}}{\partial y^B_{LH}} = \rho^B_{LH}[\frac{\alpha_{LH}}{y^B_{LH}} - (\lambda_{LH} - \lambda_{HH}\mu_{LH})c^B_H] = 0;$

$\frac{\partial \mathcal{L}}{\partial y^{AB}_{LH}} = \rho^{AB}_{LH}[\frac{\alpha_{LH}}{y^{AB}_{LH}} - c^B_H(\lambda_{LH}c^A_L - \lambda_{HH}\mu_{LH}c^A_H)] = 0;$

$\frac{\partial \mathcal{L}}{\partial y^A_{LL}} = \rho^A_{LL}[\frac{\alpha_{LL}}{y^A_{LL}} - c^A_L(\lambda_{LL} - \lambda_{LH}) + c^A_H(\lambda_{HL} + \lambda_{HH}\mu_{LL})] = 0;$

$\frac{\partial \mathcal{L}}{\partial y^B_{LL}} = \rho^B_{LL}[\frac{\alpha_{LL}}{y^B_{LL}} - c^B_L(\lambda_{LL} - \lambda_{HL}) + c^B_H(\lambda_{LH} + \lambda_{HH}\mu_{LL})] = 0;$

$\frac{\partial \mathcal{L}}{\partial y^{AB}_{LL}} = \rho^{AB}_{LL}[\frac{\alpha_{LL}}{y^{AB}_{LL}} - c^A_L(\lambda_{LL}c^B_L - \lambda_{LH}c^B_H) + c^A_H(\lambda_{HL}c^B_L + \lambda_{HH}\mu_{LL}c^B_H)] = 0.$

By plugging the values of λ_{ij}s into the above equations, we can write Kuhn-Tucker conditions with respect to y^k_{ij} for $k = A, B, AB$ and $ij \in \Theta$ as:

$$
\begin{aligned}
\rho^A_{HH}\alpha_{HH}[y^O_{HH} - y^A_{HH}c^A_H] &= 0 && \text{(a)}\\
\rho^B_{HH}\alpha_{HH}[y^O_{HH} - y^B_{HH}c^B_H] &= 0 && \text{(b)}\\
\rho^{AB}_{HH}\alpha_{HH}[y^O_{HH} - y^{AB}_{HH}c^A_H c^B_H] &= 0 && \text{(c)}\\
\rho^A_{HL}\alpha_{HL}[y^O_{HL} - y^A_{HL}c^A_H] &= 0 && \text{(d)}\\
\rho^B_{HL}[\frac{\alpha_{HL}}{y^B_{HL}} - \frac{\alpha_{HH}}{y^O_{HH}}\mu_{HL}(c^B_L - c^B_H) - \frac{\alpha_{HL}}{y^O_{HL}}c^B_L] &= 0 && \text{(e)}\\
\rho^{AB}_{HL}[\frac{\alpha_{HL}}{y^{AB}_{HL}} - c^A_H(\frac{\alpha_{HH}}{y^O_{HH}}\mu_{HL}(c^B_L - c^B_H) + \frac{\alpha_{HL}}{y^O_{HL}}c^B_L)] &= 0 && \text{(f)}\\
\rho^A_{LH}[\frac{\alpha_{LH}}{y^A_{LH}} - \frac{\alpha_{HH}}{y^O_{HH}}\mu_{LH}(c^A_L - c^A_H) - \frac{\alpha_{LH}}{y^O_{LH}}c^A_L] &= 0 && \text{(g)}\\
\rho^B_{LH}\alpha_{LH}[y^O_{LH} - y^B_{LH}c^B_H] &= 0 && \text{(h)}\\
\rho^{AB}_{LH}[\frac{\alpha_{LH}}{y^{AB}_{LH}} - c^B_H(\frac{\alpha_{HH}}{y^O_{HH}}\mu_{LH}(c^A_L - c^A_H) + \frac{\alpha_{LH}}{y^O_{LH}}c^A_L)] &= 0 && \text{(i)}\\
\rho^A_{LL}[\frac{\alpha_{LL}}{y^A_{LL}} - \frac{\alpha_{LL}}{y^O_{LL}}c^A_L - \{\frac{\alpha_{HL}}{y^O_{HL}} + \frac{\alpha_{HH}}{y^O_{HH}}(\mu_{HL} + \mu_{LL})\}(c^A_L - c^A_H)] &= 0 && \text{(j)}\\
\rho^B_{LL}[\frac{\alpha_{LL}}{y^B_{LL}} - \frac{\alpha_{LL}}{y^O_{LL}}c^B_L - \{\frac{\alpha_{LH}}{y^O_{LH}} + \frac{\alpha_{HH}}{y^O_{HH}}(\mu_{LH} + \mu_{LL})\}(c^B_L - c^B_H)] &= 0 && \text{(k)}\\
\rho^{AB}_{LL}[\frac{\alpha_{LL}}{y^{AB}_{LL}} - \frac{\alpha_{LL}}{y^O_{LL}}c^A_L c^B_L - \frac{\alpha_{LH}}{y^O_{LH}}c^A_L(c^B_L - c^B_H) - \frac{\alpha_{HL}}{y^O_{HL}}c^B_L(c^A_L - c^A_H) & \\
-\frac{\alpha_{HH}}{y^O_{HH}}(c^A_L c^B_L - \mu_{LH}c^A_L c^B_H - \mu_{HL}c^A_H c^B_L - \mu_{LL}c^A_H c^B_H)] &= 0. && \text{(l)}
\end{aligned}
$$

Proof of Propositon 13. Note that, the equations (a)-(l) are of the form $\rho^k_{ij}\Omega = 0$. We can use them to solve for y^k_{ij} for $ij \in \Theta$ and $k = A, B, AB$ by implicitly assuming that $\rho^k_{ij} = 0$. This is without loss of generality, because each of these y^k_{ij}s appears with the corresponding ρ^k_{ij} everywhere in the problem. Thus, if $\rho^k_{ij} = 0$ for a type ij and for an event k then the value of y^k_{ij} does not matter in the solution. However, if $\rho^k_{ij} > 0$ then $\Omega = 0$ must be true. Thus, equations (a)-(d) and (h) respectively yield:

$$
\begin{aligned}
&y^A_{HH} = \tfrac{y^O_{HH}}{c^A_H}, \quad y^B_{HH} = \tfrac{y^O_{HH}}{c^B_H}, \quad y^{AB}_{HH} = \tfrac{y^O_{HH}}{c^A_H c^B_H},\\
&y^A_{HL} = \tfrac{y^O_{HL}}{c^A_H}, \quad y^B_{LH} = \tfrac{y^O_{LH}}{c^B_H}.
\end{aligned}
$$

Similarly, the pairs '(e), (f)' and '(g), (i)' respectively give:

$$
y^{AB}_{HL} = \tfrac{y^B_{HL}}{c^A_H}, \quad y^{AB}_{LH} = \tfrac{y^A_{LH}}{c^B_H}.
$$

These two sets of equations imply that the excess payment that a buyer makes for an object for which he has high valuation is equal to his valuation for that object. In other words, each buyer is perfectly insured against the risk of losing the object(s) for which he has high valuation. $\square$

Proof of Proposition 14. Similarly, equations (e),(g),(j),(k) and (l) can be used to solve for $y^B_{HL}, y^A_{LH}, y^A_{LL}, y^B_{LL}$, and y^{AB}_{LL} respectively.

$$
\begin{aligned}
\frac{\alpha_{LH}}{y^A_{LH}} &= \frac{\alpha_{LH}}{y^O_{LH}} c^A_L + \frac{\alpha_{HH}}{y^O_{HH}} \mu_{LH}(c^A_L - c^A_H) \\
\frac{\alpha_{HL}}{y^B_{HL}} &= \frac{\alpha_{HL}}{y^O_{HL}} c^B_L + \frac{\alpha_{HH}}{y^O_{HH}} \mu_{HL}(c^B_L - c^B_H) \\
\frac{\alpha_{LL}}{y^A_{LL}} &= \frac{\alpha_{LL}}{y^O_{LL}} c^A_L + \frac{\alpha_{HL}}{y^O_{HL}} (c^A_L - c^A_H) + \frac{\alpha_{HH}}{y^O_{HH}} (\mu_{HL} + \mu_{LL})(c^A_L - c^A_H) \\
\frac{\alpha_{LL}}{y^B_{LL}} &= \frac{\alpha_{LL}}{y^O_{LL}} c^B_L + \frac{\alpha_{LH}}{y^O_{LH}} (c^B_L - c^B_H) + \frac{\alpha_{HH}}{y^O_{HH}} (\mu_{LH} + \mu_{LL})(c^B_L - c^B_H) \\
\frac{\alpha_{LL}}{y^{AB}_{LL}} &= \frac{\alpha_{LL}}{y^O_{LL}} c^A_L c^B_L + \frac{\alpha_{LH}}{y^O_{LH}} c^A_L (c^B_L - c^B_H) + \frac{\alpha_{HL}}{y^O_{HL}} c^B_L (c^A_L - c^A_H) \\
&+ \frac{\alpha_{HH}}{y^O_{HH}} (c^A_L c^B_L - \mu_{LH} c^A_L c^B_H - \mu_{HL} c^A_H c^B_L - \mu_{LL} c^A_H c^B_H)
\end{aligned}
$$

Using the last three equations, one can write

$$
y^A_{LL} = \frac{y^O_{LL}}{c^A_L + \varepsilon_1}, \quad y^B_{LL} = \frac{y^O_{LL}}{c^B_L + \varepsilon_2}, \quad y^{AB}_{LL} = \frac{y^O_{LL}}{c^A_L c^B_L + \varepsilon_3}
$$

for some $\varepsilon_1, \varepsilon_2, \varepsilon_3 > 0$. We plug these values into LL's individual rationality constraint to get

$$
y^O_{LL}(1 - \rho^A_{LL} \frac{\varepsilon_1}{c^A_L + \varepsilon_1} - \rho^B_{LL} \frac{\varepsilon_2}{c^B_L + \varepsilon_2} - \rho^{AB}_{LL} \frac{\varepsilon_3}{c^A_L c^B_L + \varepsilon_3}) = 1.
$$

Note that, the term in the parenthesis is less than one if LL gets either or both objects. Thus, if $\rho^O_{LL} \neq 1$ then $y^O_{LL} > 1$ (hence, $\tau^O_{LL} > 0$) must be true. $\square$

In the following proofs we use the Lagrangian of the problem (SP*) and its corresponding constraints.

Proof of Proposition 15. i) Let η be such that $n_{HH}+n_{HL}>0$ and without loss of generality assume that $n_{HH}>0$. Suppose by contradiction that $n_{HH}\hat{p}^A_{HH}(\eta)+n_{HL}\hat{p}^A_{HL}(\eta)<1$. Let $\varepsilon\leq 1-n_{HH}\hat{p}^A_{HH}(\eta)-n_{HL}\hat{p}^A_{HL}(\eta)$.

There are three possibilities that we need to consider:

- $n_{LH}+n_{LL}=0$:

In this case, modify the mechanism by increasing $p^A_{HH}(\eta)$ by $\frac{\varepsilon}{n_{HH}}$. This would increase $\hat{\rho}^A_{HH}$ by $\Psi\frac{\varepsilon}{\alpha_{HH}}$. Change in the Lagrangian can be calculated as $\Psi\varepsilon\ln\frac{1}{c_H}>0$. This is a contradiction.

- $n_{LH}\hat{p}^A_{LH}(\eta)>0$:

Now, we show that decreasing $\hat{p}^A_{LH}(\eta)$ by $\frac{\varepsilon}{n_{LH}}$ and increasing $\hat{p}^A_{HH}(\eta)$ by $\frac{\varepsilon}{n_{HH}}$ is profitable for some $\varepsilon<n_{LH}\hat{p}^A_{LH}(\eta)$. After this modification, $\hat{\rho}^A_{LH}$ decreases by $\Psi\frac{\varepsilon}{\alpha_{LH}}$ and $\hat{\rho}^A_{HH}$ increases by $\Psi\frac{\varepsilon}{\alpha_{HH}}$.[1] We calculate the change in the Lagrangian as

$$\begin{aligned}\Delta\mathcal{L} &= \Psi\varepsilon\{\ln\frac{1}{c^A_H}-\ln\frac{y^A_{LH}}{y^O_{LH}}+\lambda_{LH}[c^A_L\frac{y^A_{LH}}{\alpha_{LH}}-\frac{y^O_{LH}}{\alpha_{LH}}]-\lambda_{HH}\mu_{LH}[c^A_H\frac{y^A_{LH}}{\alpha_{LH}}-\frac{y^O_{LH}}{\alpha_{LH}}]\}\\ &= \Psi\varepsilon\ln\frac{y^O_{LH}}{c^A_H y^A_{LH}}\end{aligned}$$

which is positive since $y^O_{LH}>c_H y^A_{LH}$.

- $n_{LL}\hat{p}^A_{LL}(\eta)>0$ and $n_{LH}\hat{p}^A_{LH}(\eta)=0$:

Suppose first that $n_{LL}p^A_{LL}(\eta)>0$. Then, decrease $p^A_{LL}(\eta)$ by $\frac{\varepsilon}{n_{LL}}$ and increase $p^A_{HH}(\eta)$ by $\frac{\varepsilon}{n_{HH}}$ for some $\varepsilon<n_{LL}p^A_{LL}(\eta)$. This would decrease ρ^A_{LL} by $\Psi\frac{\varepsilon}{\alpha_{LL}}$ and increase $\hat{\rho}^A_{HH}$ by $\Psi\frac{\varepsilon}{\alpha_{HH}}$. Then, the Lagrangian changes by

$$\begin{aligned}\Delta\mathcal{L} &= \Psi\varepsilon\{\ln\frac{1}{c^A_H}-\ln\frac{y^A_{LL}}{y^O_{LL}}+(\lambda_{LL}-\lambda_{LH})[\frac{c^A_L y^A_{LL}}{\alpha_{LL}}-\frac{y^O_{LL}}{\alpha_{LL}}]-(\lambda_{HL}+\lambda_{HH}\mu_{LL})[\frac{c^A_H y^A_{LL}}{\alpha_{LL}}-\frac{y^O_{LL}}{\alpha_{LL}}]\}\\ &= \Psi\varepsilon\ln\frac{y^O_{LL}}{c^A_H y^A_{LL}}>0.\end{aligned}$$

Suppose now that $n_{LL}p^A_{LL}(\eta)=0$. Then, $n_{LL}p^{AB}_{LL}(\eta)>0$ must be true. We show that the following modification is profitable: decrease $p^{AB}_{LL}(\eta)$ by $\frac{\varepsilon}{n_{LL}}$ and increase $p^{AB}_{HH}(\eta)$ by $\frac{\varepsilon}{n_{HH}}$

[1] $\hat{p}^A_{LH}(\eta)$ can be decreased either by decreasing $p^A_{LH}(\eta)$ or $p^{AB}_{LH}(\eta)$. If the former is positive then we decrease $p^A_{LH}(\eta)$ (and increase $p^A_{HH}(\eta)$). If the former is zero, however, $p^{AB}_{LH}(\eta)$ should be decreased (and in response $p^{AB}_{HH}(\eta)$ should be increased). In this case, marginal probabilities of winning A and B are affected for both types HH and LH. Yet, each modification has the same effect on the Lagrangian.

for some $\varepsilon < n_{LL}p_{LL}^{AB}(\eta)$. This would decrease ρ_{LL}^{AB} by $\Psi\frac{\varepsilon}{\alpha_{LL}}$ and increase $\hat{\rho}_{HH}^{A}$ and $\hat{\rho}_{HH}^{B}$ by $\Psi\frac{\varepsilon}{\alpha_{HH}}$. As a result, the Lagrangian increases by

$$\begin{aligned}
\Delta\mathcal{L} &= \Psi\varepsilon\{\ln\frac{1}{c_H^A} + \ln\frac{1}{c_H^B} - \ln\frac{y_{LL}^{AB}}{y_{LL}^{O}} + \lambda_{LL}[\frac{c_L^A c_L^B y_{LL}^{AB}}{\alpha_{LL}} - \frac{y_{LL}^{O}}{\alpha_{LL}}] \\
&\quad -\lambda_{LH}[\frac{c_L^A c_H^B y_{LL}^{AB}}{\alpha_{LL}} - \frac{y_{LL}^{O}}{\alpha_{LL}}] - \lambda_{LH}[\frac{c_H^A c_L^B y_{LL}^{AB}}{\alpha_{LL}} - \frac{y_{LL}^{O}}{\alpha_{LL}}] \\
&\quad -\lambda_{HH}\mu_{LL}[\frac{c_H^A c_H^B y_{LL}^{AB}}{\alpha_{LL}} - \frac{y_{LL}^{O}}{\alpha_{LL}}]\} \\
&= \Psi\varepsilon\ln\frac{y_{LL}^{O}}{c_H^A c_H^B y_{LL}^{AB}} > 0.
\end{aligned}$$

Thus, we conclude that if η is such that $n_{HH} + n_{HL} > 0$ then $n_{HH}\hat{p}_{HH}^{A}(\eta) + n_{HL}\hat{p}_{HL}^{A}(\eta) = 1$.

We can prove the part ii of the lemma along the same lines. $\square$

Proof of Corollary 16. We prove only the part i. Proof of the part ii is similar. (3.13) implies that

$$\begin{aligned}
\alpha_{HH}\hat{\rho}_{HH}^{A} &= \sum_{n_{HH}=0}^{n}\sum_{n_{HL}=0}^{n-n_{HH}}\sum_{n_{LH}=0}^{n-n_{HH}-n_{HL}} n_{HH}\hat{p}_{HH}^{A}(\eta)\Psi \\
\alpha_{HL}\hat{\rho}_{HL}^{A} &= \sum_{n_{HH}=0}^{n}\sum_{n_{HL}=0}^{n-n_{HH}}\sum_{n_{LH}=0}^{n-n_{HH}-n_{HL}} n_{HL}\hat{p}_{HL}^{A}(\eta)\Psi.
\end{aligned}$$

Adding these two equalities and multiplying both sides with n gives

$$\begin{aligned}
n[\alpha_{HH}\hat{\rho}_{HH}^{A} + \alpha_{HL}\hat{\rho}_{HL}^{A}] &= \sum_{n_{HH}=0}^{n}\sum_{n_{HL}=0}^{n-n_{HH}}\sum_{n_{LH}=0}^{n-n_{HH}-n_{HL}} [n_{HH}\hat{p}_{HH}^{A}(\eta) + n_{HL}\hat{p}_{HL}^{A}(\eta)]n\Psi \\
&= \sum_{n_{HH}=0}^{n}\sum_{n_{HL}=0}^{n-n_{HH}}\sum_{n_{LH}=0}^{n-n_{HH}-n_{HL}} n\Psi - \sum_{n_{LH}=0}^{n}\frac{n!\alpha_{LH}^{n_{LH}}\alpha_{LL}^{n-n_{LH}}}{n_{LH}!(n-n_{LH})!} \\
&= 1 - (\alpha_{LH} + \alpha_{LL})^{n}.
\end{aligned}$$

The second equality follows from the part i of Proposition 15. $\square$

Proof of Lemma 17. $i)$ Suppose the following profile: $n_{HH} + n_{HL} = 0$ and $n_{LH}\hat{p}^A_{LH}(\eta) + n_{LL}\hat{p}^A_{LL}(\eta) < 1$. Let $\varepsilon < 1 - n_{LH}\hat{p}^A_{LH}(\eta) - n_{LL}\hat{p}^A_{LL}(\eta)$. There are two cases that we need to consider:

- $n_{LH} > 0$: Increase $\hat{p}^A_{LH}(\eta)$ by $\frac{\varepsilon}{n_{LH}}$ which, in turn, increases $\hat{\rho}^A_{LH}$ by $\Psi\frac{\varepsilon}{\alpha_{LH}}$. Hence, the Lagrangian changes by

$$\begin{aligned}
\Delta\mathcal{L} &= \Psi\varepsilon\{\ln\frac{y^A_{LH}}{y^O_{LH}} + \lambda_{LH}[-c^A_L\frac{y^A_{LH}}{\alpha_{LH}} + \frac{y^O_{LH}}{\alpha_{LH}}] + \lambda_{HH}\mu_{LH}[c^A_H\frac{y^A_{LH}}{\alpha_{LH}} - \frac{y^O_{LH}}{\alpha_{LH}}]\} \\
&= \Psi\varepsilon\ln\frac{y^A_{LH}}{y^O_{LH}}
\end{aligned}$$

which is positive if $y^A_{LH} > y^O_{LH}$ or $\frac{c^A_L - c^A_H}{1-c^A_L} < \frac{\alpha_{LH}}{y^O_{LH}}\frac{y^O_{HH}}{\alpha_{HH}}\frac{1}{\mu_{LH}}$.

- $n_{LH} = 0$: A profitable modification would be to increase $p^A_{LL}(\eta)$ by $\frac{\varepsilon}{n_{LL}}$ which,in turn, increases ρ^A_{LL} by $\Psi\frac{\varepsilon}{\alpha_{LL}}$. Hence, the Lagrangian changes by

$$\begin{aligned}
\Delta\mathcal{L} &= \Psi\varepsilon\{\ln\frac{y^A_{LL}}{y^O_{LL}} - (\lambda_{LL} - \lambda_{LH})[\frac{c^A_L y^A_{LL}}{\alpha_{LL}} - \frac{y^O_{LL}}{\alpha_{LL}}] + (\lambda_{HL} + \lambda_{HH}\mu_{LL})[\frac{c^A_H y^A_{LL}}{\alpha_{LL}} - \frac{y^O_{LL}}{\alpha_{LL}}]\} \\
&= \Psi\varepsilon\ln\frac{y^A_{LL}}{y^O_{LL}}
\end{aligned}$$

which is positive if $y^A_{LL} > y^O_{LL}$ or $\frac{c^A_L - c^A_H}{1-c^A_L} < \frac{\alpha_{LL}}{y^O_{LL}}(\frac{\alpha_{HL}}{y^O_{HL}} + \frac{\alpha_{HH}}{y^O_{HH}}(1-\mu_{LH}))^{-1}$.

$ii)$ Along the same lines of the previous part, one can easily show that this part holds if $y^B_{HL} > y^O_{HL}$ and $y^B_{LL} > y^O_{LL}$ or if $\frac{c^B_L - c^B_H}{1-c^B_L} < \min\{\frac{\alpha_{HL}}{y^O_{HL}}\frac{y^O_{HH}}{\alpha_{HH}}\frac{1}{\mu_{HL}}, \frac{\alpha_{LL}}{y^O_{LL}}(\frac{\alpha_{LH}}{y^O_{LH}} + \frac{\alpha_{HH}}{y^O_{HH}}(1-\mu_{HL}))^{-1}\}$. □

Proof of Corollary 18. The proof follows directly from the previous lemma and the equation (3.13). □

Proof of Proposition 19. Suppose, for now, that HH is not compensated. Then $y^O_{HH} = 1$. Since $c^A_H < c^A_L$ and $c^B_H < c^B_L$, we have $1 = y^O_{HH} \leq D^{ij}_{HH} \leq D_{ij} \leq 1$ for $ij = LL, LH, HL$ where the first inequality is due to IC^{ij}_{HH} and the last inequality is the individual rationality constraint. Therefore, all individual rationality constraints are binding and $D_{ij} = D^{ij}_{HH} = 1$ for $ij = LL, LH, HL$. Moreover, since $D_{ij} - D^{ij}_{HH} = 0$, we have

$$\begin{aligned}
\rho^A_{LL}(c^A_L - c^A_H)y^A_{LL} + \rho^B_{LL}(c^B_L - c^B_H)y^B_{LL} + \rho^{AB}_{LL}(c^A_L c^B_L - c^A_H c^B_H)y^{AB}_{LL} &= 0, \\
\hat{\rho}^A_{LH}(c^A_L - c^A_H)y^A_{LH} &= 0, \\
\hat{\rho}^B_{HL}(c^B_L - c^B_H)y^B_{HL} &= 0.
\end{aligned}$$

Each term in these equations is nonnegative, therefore $\rho^A_{LL} = \rho^B_{LL} = \rho^{AB}_{LL} = \hat{\rho}^A_{LH} = \hat{\rho}^B_{HL} = 0$ must be true. This contradicts with the previous corollary because $\alpha_{LL}\hat{\rho}^A_{LL} + \alpha_{LH}\hat{\rho}^A_{LH} > 0$. $\square$

Proof of Proposition 20. Suppose, by contradiction, that for some profile η with $n_{LL} = n$ and $p^{AB}_{LL}(\eta) < \frac{1}{n}$. Since the both objects are sold with probability one, $p^A_{LL}(\eta) = p^B_{LL}(\eta) > 0$. Let $\varepsilon < 1 - np^{AB}_{LL}(\eta)$. Consider modifying the mechanism by decreasing both $p^A_{LL}(\eta)$ and $p^B_{LL}(\eta)$ by $\frac{\varepsilon}{n}$ and increasing $p^{AB}_{LL}(\eta)$ by $\frac{\varepsilon}{n}$. This would imply $\Delta\rho^{AB}_{LL} = -\Delta\rho^A_{LL} = -\Delta\rho^B_{LL} = \Psi\frac{\varepsilon}{\alpha_{LL}}$. Hence, the Lagrangian changes by

$$\Delta\mathcal{L} = \Psi\varepsilon \ln \frac{y^O_{LL} y^{AB}_{LL}}{y^A_{LL} y^B_{LL}}$$

which is positive if $y^O_{LL} y^{AB}_{LL} > y^A_{LL} y^B_{LL}$ or, equivalently, if

$$\frac{\alpha_{LL}}{y^A_{LL}}\frac{\alpha_{LL}}{y^B_{LL}} > \frac{\alpha_{LL}}{y^O_{LL}}\frac{\alpha_{LL}}{y^{AB}_{LL}} \iff (\lambda_{LH}\lambda_{HL} + \lambda_{LL}\lambda_{HH}\mu_{LL})(c^A_L - c^A_H)(c^B_L - c^B_H) > 0.$$

Since the last inequality holds for any parameter values, this modification is profitable. Thus, we conclude that if all the buyers are of type LL then the objects are bundled and each buyer gets the bundle with equal probability. $\square$

Proof of Lemma 22. i) Suppose the following profile for $\eta : n_{LH}, n_{LL} > 0$, $n_{LH} + n_{LL} = n$, and $n_{LH}\hat{p}^A_{LH}(\eta) < 1$. Since the object A is sold with probability one, $p^A_{LL}(\eta)$ must be positive. Let $\varepsilon < n_{LL} p^A_{LL}(\eta)$. Now, consider modifying the mechanism by decreasing $p^A_{LL}(\eta)$ by $\frac{\varepsilon}{n_{LL}}$ and increasing $\hat{p}^A_{LH}(\eta)$ by $\frac{\varepsilon}{n_{LH}}$. This, would decrease $\hat{\rho}^A_{LL}$ by $\frac{\Psi\varepsilon}{\alpha_{LL}}$ and increase $\hat{\rho}^A_{LH}$ by $\frac{\Psi\varepsilon}{\alpha_{LH}}$. As a result, the Lagrangian changes by

$$\Delta\mathcal{L} = \Psi\varepsilon \ln \frac{y^A_{LH} y^O_{LL}}{y^O_{LH} y^A_{LL}}.$$

This is positive if $y^A_{LH} y^O_{LL} > y^O_{LH} y^A_{LL}$ or, equivalently, if $\frac{\alpha_{LH}}{y^O_{LH}}\frac{\alpha_{LL}}{y^A_{LL}} > \frac{\alpha_{LH}}{y^A_{LH}}\frac{\alpha_{LL}}{y^O_{LL}}$. Using the Kuhn-Tucker conditions, we can rewrite this inequality as

$$\begin{aligned}(\lambda_{LH} - \lambda_{HH}\mu_{LH})[c^A_L(\lambda_{LL} - \lambda_{LH}) - c^A_H(\lambda_{HL} + \lambda_{HH}\mu_{LL})] &> \\ (c^A_L\lambda_{LH} - c^A_H\lambda_{HH}\mu_{LH})(\lambda_{LL} - \lambda_{LH} - \lambda_{HL} - \lambda_{HH}\mu_{LL}).&\end{aligned}$$

After some manipulation, we get

$$\begin{aligned} \lambda_{LH}(\lambda_{HL} + \lambda_{HH}\mu_{LL}) &> \lambda_{HH}\mu_{LH}(\lambda_{LL} - \lambda_{LH}) \\ (\frac{\alpha_{HL}}{y^O_{HL}}\frac{y^O_{HH}}{\alpha_{HH}} + 1)(\frac{\alpha_{LL}}{y^O_{LL}}\frac{y^O_{LH}}{\alpha_{LH}} + 1)^{-1} &> \mu_{LH}. \end{aligned}$$

Proof of the part ii is similar. □