INVESTIGATIONS INTO CHARITABLE FUNDRAISING

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Charitable giving in the U.S. totaled more than $300 billion in 2009, amounting to about 2% of GDP. These organizations depend on fundraising activities to generate donations from individuals who provide three-quarters of the funding for charitable organizations.

Despite the size and scope of these operations, practical fundraising still relies heavily on rules-of-thumb and individual experience to design and run campaigns. These works aim to expand the understanding of fundraising through empirical and theoretical analysis.

Leadership giving is the first fundraising practice explored. Leadership gifts are funds collected privately by a charity prior to announcing the campaign and accepting donations from the public. "Seeds to Succeed" examines a theoretical model for leadership giving first put forth in Andreoni (1998). We implement his model in the laboratory and find that when fixed-costs are high leadership gifts can greatly increase the chances a project producing benefits for the public. Intriguingly, with low fixed-costs leadership giving can actually have a small negative effect on subsequent donations.

The second chapter, "Provision Point Mechanisms and the Over-provision of Public Goods", examines the use of contribution refunds by fundraisers. That rather simple tool of guarenteeing refunds theorectically provides fundraisiers the ability to extract a large amount of contributions. The result is that the expected outcome of the campaign is the collection of inefficiently large contributions. The predicted over-provision occurs in 82% of the time in our laboratory environment.

The final chapter, "Fundraising Goals", looks at the role of announcements at the start of campaigns. We theorize that announcements improve contributions by reducing donor's uncertainty about the project. Large improvements are possible with up to a 73% increase in contributions and a large increase in the donor base. Experimental data supports the prediction that announced goals increase contributions. Reducing uncertainty did not have the effect of further increasing contributions but led to greater coordination of contributions around the goal. The improved coordination significantly increases donor welfare under an uncertainty reduction.

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1.1 INTRODUCTION

A rule of thumb commonly followed by fundraisers is that past contributions are announced to future donors. This practice is perhaps most noteworthy in capital campaigns where the announcement of a substantial seed donation is used to launch the public phase of the campaign. The practice of sequential fundraising is intriguing in light of the analysis of voluntary provision of public goods provided by Varian (1994). Examining a model with continuous production of the public good, he compares the contributions that result when donations are made simultaneously versus sequentially. Recognizing that one donor's contribution is a perfect substitute for that of another, he demonstrates that sequential provision enables the initial donor to free ride off of subsequent donors, and as a result the overall provision in the sequential contribution game will be no greater than in the simultaneous one.[1]

This inconsistency between common fundraising practice and theoretical prediction has prompted researchers to identify conditions under which it may be optimal to raise funds

[1] Experimental investigations of the quasi-linear environment of Varian (1994) confirm the prediction of lower contributions in the sequential game (Andreoni et al., 2002; Gachter et al., 2010). As emphasized by Vesterlund (2003), this prediction relies on the somewhat unrealistic assumption that initial donors can commit to giving only once. Absent this assumption the contribution level is predicted to be the same with simultaneous and sequential moves. Thus the strict preference for sequential giving remains a puzzle in this case.

sequentially. Andreoni (1998), the first to propose an explanation, showed that a sequential fundraising strategy is preferable when there are fixed production costs. The reason is that, when no individual singlehandedly is willing to cover the fixed costs, simultaneous giving may result in both positive and zero provision equilibria. Thus fundraising campaigns that rely on simultaneous giving may get stuck in an equilibrium where donors fail to coordinate on a preferred positive provision outcome. Interestingly, a sequential fundraising strategy helps eliminate such inferior equilibria, as a large initial contribution secures that the fixed costs will be covered and that the good will be provided.

List and Lucking-Reiley (2002) use a field experiment to examine the prediction that followers respond positively to a large initial contribution. Raising funds for a number of $3000 computers, they sent out solicitations in which the initial contribution to the nonprofit institution varied between 10%, 33%, and 67% of the computer's cost. They find that the likelihood of contributing and the average amount contributed is greatest when 67% of the project funding had already been provided.[2] In fact a six-fold increase in contributions is seen when moving from the lowest to the highest seed amount. Qualitatively the results are very much in line with the prediction of Andreoni's model. However the results are also in line with the predictions made by a number of other models on sequential giving. For example, the increase in giving may also be explained by donors interpreting the initial contribution as a signal of the nonprofit's quality (Vesterlund, 2003).[3]

What distinguishes Andreoni's predictions from alternative models of sequential fundraising is the crucial role played by the presence of fixed production costs. Sequential giving is effective because it eliminates the inefficiencies that may arise as a result of fixed costs. Unfortunately in a field setting it is not straightforward to find a cause for which it is possible to vary the fixed costs while keeping all other characteristics constant. The objective of the present paper is to use laboratory experiments to examine the role of sequential giving in the presence and absence of fixed costs. By using the laboratory we can test a simplified environment and

[2] A series of field experiments also find that giving is influenced by the size of the initial contribution. For example, Frey and Meier (2004) show that contributions to charitable funds at the University of Zurich are affected by information on how many others donated in the past. In a campaign for a public radio station, Croson and Shang (2008) show that donations increase when a donor is informed that others have contributed more than he did in the past. Martin and Randal (2008) change the amount placed in an art gallery's donation box and show that average donations increase when it appears that others have given larger amounts.

[3] See also Potters et al. (2005, 2007); Andreoni (2006b); Komai et al. (2007).

determine if fixed costs play a critical role in the success of sequential giving. It is important to recognize that the question is not merely one of confirming previous evidence that sequential play improves efficiency in coordination games (see for example Weber et al. (2004)). With simultaneous play of the public good game there may be an incentive to contribute in excess of the preferred Nash equilibrium as it improves overall efficiency and alleviates strategic uncertainty, thus in the presence of fixed costs simultaneous play need not give rise to inefficient outcomes which sequential play can improve upon; furthermore absent fixed costs sequential play may in and of itself increase contributions thereby decreasing the likelihood that fixed cost play a critical role in explaining the frequent use of sequential fundraising.

Our study is designed to answer the following research questions. First, do sequential moves increase giving when there are no fixed costs? Second, do fixed costs give rise to inefficient outcomes under simultaneous provision? That is, do contributions decrease when we introduce fixed costs such that no individual has an incentive to single-handedly provide the good? Third, if such inefficient outcomes exist, does sequential play help eliminate these inefficiencies and increase the likelihood of providing the public good? Specifically do initial contributors respond to the coordinating role they hold in the sequential game, and do subsequent donors follow? Finally, to evaluate the extent to which the success of seed money depends on the presence of fixed costs, we ask whether the potential increase in contributions under sequential provision is sensitive to the size or even the presence of fixed costs.

Our results are supportive of the theory for high, but not for low fixed costs. Surprisingly, under simultaneous provision we find that the introduction of small fixed costs increases rather than decreases overall provision. Individuals seem uncertain of which equilibrium will be played and, at the risk of decreasing their payoff, they increase their contributions to ensure that the public good is provided. By facilitating coordination on the positive provision outcome, seed money effectively removes the strategic uncertainty and the risk of under-provision. Sequential contributions decrease to the predicted equilibrium level and fall below the greater-than-expected contributions in the simultaneous game. Consequently, our results show that sequential provision has no role when fixed costs are small. However, when fixed costs are high, contribution behavior is in line with the theoretical prediction: individuals often fail to provide the public good in the simultaneous game, and sequential provision successfully facilitates coordination and eliminates these undesirable outcomes. As a result, when fixed costs are high the likelihood of securing provision of the public good is much greater when contributions are made sequentially. The effect of sequential play on earnings is even greater than that seen absent fixed costs. Thus consistent with Andreoni's model we find that sequential moves play a unique coordinating role when there are (large) fixed costs.

The remainder of the paper is organized as follows. We first describe the theoretical insights in a simple example of Andreoni's model, and explain how the derived hypotheses helped shape our experimental design. The effect of sequential play under three different fixed cost treatments is presented in section 1.3. In section 1.4 we examine the interaction between sequential play and fixed costs of production. We conclude the paper in section 3.6.

1.2 EXPERIMENTAL DESIGN

Andreoni (1998) fully characterizes the equilibria of the contribution game with fixed costs. To demonstrate the insights of interest for this study, we start by presenting a simple two-person example of hismodel. This example has precisely the characteristics we want for our experiment and will serve as the basis for our design. We complete the section by describing the parameters and procedures used for the study.

1.2.1 Theory

Consider the following two-person voluntary contribution environment. A donor, $i = 1, 2$, has an endowment, w_i, which he must allocate between private consumption, x_i, and contributions to a public good, g_i. Let $c(g_i)$ denote i's cost of giving g_i and $r(G)$ i's benefit from a total contribution of $G = g_1 + g_2$.[4] Assuming that the price of the private good is 1, let i's

[4] The interest in examining public good provision is driven by the classic view that altruistically motivated giving can be viewed as voluntary contributions to a public good (see Becker (1974)). Rather than relying on altruistic preferences we instead follow Andreoni (1993) and induce preferences for a public good through the payoff function described earlier.

quasi-linear utility be given by

$$U_i(x_i, G) = w_i - c(g_i) + r(G).$$

Let the return from the public good equal m per unit contributed to the public good, provided that the total contribution exceeds a fixed cost of FC.

$$r(G) = \begin{cases} 0 & \text{if } G < FC \\ mG & \text{if } G \geq FC \end{cases}$$

Further assume that costs are convex and piecewise linear of the form

$$c(g_i) = \begin{cases} \alpha g_i & \text{if } g_i \in [0, l_{ne}] \\ \alpha l_{ne} + \beta(g_i - l_{ne}) & \text{if } g_i \in (l_{ne}, l_{pe}] \\ \alpha l_{ne} + \beta(g_i - l_{ne}) + \gamma(g_i - l_{pe}) & \text{if } g_i \in (l_{pe}, \bar{l}] \end{cases}$$

Thus the marginal cost of contributing is initially αa, then β, and finally γ. To secure an interior Nash equilibrium and an interior Pareto optimal outcome with $FC = 0$ assume that $0 < \alpha < m$, $m < \beta < 2$, $\gamma > 2m$, and that $0 < l_{ne} < l_{pe} < w_i$. In analyzing the game, let us start by characterizing the equilibria of the simultaneous game and describe how these change with the size of the fixed cost. For this purpose it will be beneficial to define the following two fixed cost levels: let FC_1 denote the fixed cost where the return to covering the fixed cost single-handedly equals the cost, i.e., $r(FC_1) = c(FC_1)$, and let FC_2 denote the fixed cost where the return from covering the fixed cost equals the cost of contributing an amount equal to half of the fixed cost, i.e., $r(FC_2) = c(FC_2/2)$. Absent fixed costs ($FC = 0$) the dominant strategy for each individual is to contribute l_{NE}, thus the equilibrium is $(g_1^\star, g_2^\star) = (l_{NE}, l_{NE})$. This remains the unique equilibrium outcome as long as individuals are willing to single-handedly cover the fixed cost, i.e., $FC < FC_1$. For higher fixed costs, i.e., $FC > FC_1$, a zero provision equilibrium arises. The reason is that when $FC > FC_1$ the best response to $g_i = 0$ is a contribution of $g_i = 0$; thus for a sufficiently high fixed cost, $(g_1^\star, g_2^\star) = (0, 0)$ is a Nash equilibrium of the simultaneous game. In fact zero provision is the unique equilibrium outcome when $FC > FC_2$. For intermediate value fixed costs, that is, when $FC_1 < FC < FC_2$, there are both zero and positive provision equilibria. Although all players would prefer positive provision, a failure to coordinate may trap contributors at zero provision. The role of seed money demonstrated by Andreoni (1998) arises when the fixed cost is in the intermediate range where the simultaneous game gives rise to multiple equilibria. He showed that while the simultaneous game may result in zero provision, such inefficiencies are eliminated with sequential play. The reason is that by providing a sufficiently large first donation the first mover can ensure that the second mover is willing to cover the remainder of the fixed cost. Thus for fixed costs in this intermediate range the fundraiser can secure positive provision by announcing the first donor's contribution.

1.2.2 Experimental parameters

We are interested in examining the effect of sequential giving for fixed costs in the intermediate range described above. In determining the interaction between fixed costs and sequential play we initially considered a simple 2×2 design, comparing simultaneous and sequential giving with and without fixed costs. In choosing the fixed costs we wanted to secure that the fixed cost, while high enough to give rise to positive and zero provision equilibria, was small enough that the positive provision equilibrium of the simultaneous game with fixed cost was the same as that absent fixed cost. However this resulted in a relatively low fixed cost and our investigation of this setting soon revealed that it also was of interest to examine the effect of sequential giving with a higher fixed cost. Thus we added two highcost treatments resulting in a 3×2 design: fixed cost $\in \{0, \text{low}, \text{high}\} \times$ play $\in \{\text{simultaneous}, \text{sequential}\}$.

Our design is based on the example presented above as it captures the critical features of Andreoni's model. Furthermore it is relatively simple and has characteristics that are desirable for our experimental design: an interior Nash equilibrium in dominant strategies and an interior Pareto optimal outcome.[5] Thus in contrast to the classic voluntary contribution mechanism (VCM) where the dominant strategy is to give nothing and the Pareto optimal outcome is to give everything, this design allows for participants not only to overcontribute but also to under-contribute. Furthermore, contributions are not limited to being inefficiently low but may also be inefficiently high. While previous studies have examined environments

[5] Menietti et al. (2009) examine a similar payoff structure.

	$FC = 0$	$FC = 6$	$FC = 8$
Simultaneous	(3,3)	(0,0) & (3,3)	(0,0),(3,5), (4,4), & (5,3)
Sequential	(3,3)	(1,5)	(2,6)

Table 1: Equilibrium predictions $(g_1^\star, g_2^\star)$.

in which both the Nash and Pareto optimal outcomes are interior, the attraction of our example is that we secure the Nash equilibrium in dominant strategies using piecewise linear payoffs, which are easily explained.[6]

The specific parameters chosen for the study were as follows. Participants interacted in a one-shot manner in groups of two. Provided that the fixed cost is covered, the marginal return per unit invested in the public account was 50 cents. The per unit cost of investing was 40 cents for units 1 to 3, 70 cents for units 4 through 7, and finally $1.10 for units 8 through 10. Thus the experimental parameters were: $m = 0.5$, $\alpha = 0.4$, $\beta = 0.7$, $\gamma = 1.1$, $l_{NE} = 3$, and $l_{PE} = 7$. Absent fixed costs it is a dominant strategy to contribute 3 units, and Pareto efficiency is achieved with each contributing 7 units.

As previously noted we selected the fixed cost to be so large that no individual had an incentive to cover the fixed cost single-handedly, yet small enough to secure both positive and zero provision equilibria of the simultaneous game. Furthermore in selecting the cost for the low-fixed-cost treatments we wanted to facilitate an easy comparison across treatments and selected a fixed cost for which the positive provision equilibrium was identical in the simultaneous games with and without fixed cost. A fixed cost of six satisfied these criteria. With $FC = 6$ it remains an equilibrium of the simultaneous game for each individual to contribute 3 units, yet if the other person contributes zero the best response is to contribute zero as well. This is because the cost of covering the fixed cost alone is $3.30 $(= 3 \times 0.4 + 3 \times 0.7)$ which

[6] See Laury and Holt (2008) for a review of the literature on VCMs with interior Nash equilibria. Our design also differs from the threshold models, be it for contributions or a minimum contributing set, where there is no return from exceeding the threshold, see, for example, Erev and Rapoport (1990); Dorsey (1992); Cooper and Stockman (2007); Coats et al. (2009); as well as the review by Croson and Marks (2000).

outweighs the benefit of \$3 $(= 6 \times 0.5)$. Thus with simultaneous play and $FC = 6$ there are two Nash equilibria-one that provides the public good and another that does not. Under sequential provision, however, the zero provision outcome is eliminated. The reason is that the first mover has an incentive to provide just enough to secure that the second mover will cover the remaining fixed costs.[7] Examining the second mover's incentives, we see that the second mover's best response is

$$g_2(g_1) = \begin{cases} 0 & \text{if } g_1 = 0 \\ 6 - g_1 & \text{if } g_1 \in \{1, 2\} \\ 3 & \text{if } g_1 \in [3, 10] \end{cases}$$

where g_1 denotes the first mover's contribution and g_2 the second mover's contribution. Thus, the first mover, by contributing 1 unit, can secure completion of the project and maximize her own payoff.

For the high fixed cost treatments we increased the cost to 8 units. Once again the fixed costs give rise to both zero and positive provision under simultaneous move. However in increasing the fixed cost beyond 6 units we also increase the number of positive provision equilibria of the simultaneous game. In particular there are now three Nash equilibria that secure provision: (3,5), (4,4) and (5,3). Introducing sequential play leads to a unique subgame perfect equilibrium of (2,6), thus sequential play not only eliminates the inefficient (0,0) equilibrium, it also eliminates the coordination problem associated with selecting one of the positive provision equilibria. The implications of the high-cost treatments are discussed in greater detail in subsection 1.3.3.

Our 3×2 design - $\{FC = 0, FC = 6, FC = 8\} \times \{\text{simultaneous play}, \text{sequential play}\}$ - gives rise to the predictions in table 1.

Of course, various forms of other-regarding preferences may give rise to deviations from the predicted equilibria.[8] Altruism may cause contributions to exceed the predicted contributions. The attraction of the fair and Pareto superior outcome may be so strong that we

[7] The characteristic of this subgame perfect equilibrium is similar to that of the ultimatum game where the proposer offers the smallest nonzero amount possible and the responder accepts. The equilibrium also resembles the quasi-linear settings by Andreoni et al. (2002) and Gachter et al. (2010), where there is a substantial first mover advantage.

[8] See Cooper and Kagel (Forthcoming) for a review of other-regarding preferences.

observe no inefficiencies in the simultaneous game with fixed costs. Reciprocity and inequality aversion may cause deviations in the sequential game where small initial contributions can be punished, while large contributions can be rewarded. In light of the many behavioral factors that may cause deviations from the equilibrium prediction, we refrain from assessing the model's predictive power by examining adherence to the predicted equilibria; instead we focus on the predicted comparative statics. Needless to say the role of sequential giving relies critically on inefficiencies arising with simultaneous giving, and when examining the comparative statics of sequential giving we therefore assume that there is a positive probability that simultaneous moves result in zero provision. In examining the comparative statics between and within the no fixed cost and fixed cost treatments we can answer the questions of interest. First, do sequential moves absent fixed cost increase contributions? Second, does the introduction of fixed cost gives rise to inefficiencies and decrease contributions. Third, comparing treatments (simultaneous versus sequential play) with fixed cost, do we find evidence that sequential play increases contributions and the likelihood of provision? Fourth, does sequential play have a unique coordinating role in the presence of fixed costs, and is such a role sensitive to the level of fixed costs?

1.2.3 Experimental procedures

The sessions were conducted at the Pittsburgh Experimental Economics Laboratory at the University of Pittsburgh. Three sessions were conducted for each of the initial four treatments ($FC = 0$ and $FC = 6$), and two sessions were conducted for each of the subsequent high fixed cost treatments ($FC = 8$). With 14 participants per session a total of 224 undergraduate students participated in the study. Each session proceeded as follows: First instructions and a payoff table were distributed.[9] Care was taken to make the payoff table as clear as possible. The payoffs to the participant and her group member are distinguished by color and location in each cell (see figure 1 for an example of the payoff table when $FC = 0$). The instructions were read out loud and a short quiz was given to gauge the participants' understanding. The quiz asked participants to use the payoff table to determine the payoffs earned by a partici-

[9] See section A.1 and section A.5, for the instructions and payoff tables.

Other Group Member

You		0	1	2	3	4	5	6	7	8	9	10
	0	4.0 4.0	4.5 4.1	5.0 4.2	5.5 4.3	6.0 4.1	6.5 3.9	7.0 3.7	7.5 3.5	8.0 2.9	8.5 2.3	9.0 1.7
	1	4.1 4.5	4.6 4.6	5.1 4.7	5.6 4.8	6.1 4.6	6.6 4.4	7.1 4.2	7.6 4.0	8.1 3.4	8.6 2.8	9.1 2.2
	2	4.2 5.0	4.7 5.1	5.2 5.2	5.7 5.3	6.2 5.1	6.7 4.9	7.2 4.7	7.7 4.5	8.2 3.9	8.7 3.3	9.2 2.7
	3	4.3 5.5	4.8 5.6	5.3 5.7	5.8 5.8	6.3 5.6	6.8 5.4	7.3 5.2	7.8 5.0	8.3 4.4	8.8 3.8	9.3 3.2
	4	4.1 6.0	4.6 6.1	5.1 6.2	5.6 6.3	6.1 6.1	6.6 5.9	7.1 5.7	7.6 5.5	8.1 4.9	8.6 4.3	9.1 3.7
You	5	3.9 6.5	4.4 6.6	4.9 6.7	5.4 6.8	5.9 6.6	6.4 6.4	6.9 6.2	7.4 6.0	7.9 5.4	8.4 4.8	8.9 4.2
	6	3.7 7.0	4.2 7.1	4.7 7.2	5.2 7.3	5.7 7.1	6.2 6.9	6.7 6.7	7.2 6.5	7.7 5.9	8.2 5.3	8.7 4.7
	7	3.5 7.5	4.0 7.6	4.5 7.7	5.0 7.8	5.5 7.6	6.0 7.4	6.5 7.2	7.0 7.0	7.5 6.4	8.0 5.8	8.5 5.2
	8	2.9 8.0	3.4 8.1	3.9 8.2	4.4 8.3	4.9 8.1	5.4 7.9	5.9 7.7	6.4 7.5	6.9 6.9	7.4 6.3	7.9 5.7
	9	2.3 8.5	2.8 8.6	3.3 8.7	3.8 8.8	4.3 8.6	4.8 8.4	5.3 8.2	5.8 8.0	6.3 7.4	6.8 6.8	7.3 6.2
	10	1.7 9.0	2.2 9.1	2.7 9.2	3.2 9.3	3.7 9.1	4.2 8.9	4.7 8.7	5.2 8.5	5.7 7.9	6.2 7.3	6.7 6.7

Figure 1: Payoff table with $FC = 0$.

pant and her group member for several combinations of contribution levels above and below the fixed cost level. To avoid priming the participants, the examples did not include focal outcomes, such as the Nash equilibrium and Pareto optimal outcome. The quiz questions were the same for all treatments, though the answers varied with the size of the fixed costs.

Once all participants had completed the quiz a solution key was distributed. The quiz answers were explained by an experimenter using a projection of the payoff table. Screen shots of the experimental software were shown and explained. The payoff table was displayed on all decision screens. Participants then began the portion of the experiment that counted for payment. They played 14 rounds of the public goods game. In each round each participant was randomly paired with another participant, was given a $4 endowment and the opportunity to invest any number of units between zero and ten in a public account.[10]

Contributions were either made "simultaneously" or "sequentially." Effectively decisions were made sequentially in both treatments with half the participants called "first movers" and the other half "second movers." However only in the sequential treatment was the second mover informed of the first mover's contribution before making her decision. The variation in information for the second mover was the only difference between the sequential and simultaneous treatments, which resulted in minimal variations in instructions and procedures between the two treatments.[11] The experimentwas programmed and conducted using the software z-Tree (Fischbacher, 2007). When the 14 rounds were completed, we randomly selected three rounds to count for payment.[12] Participants were then asked to complete a short questionnaire, following which they were paid in private and in cash. Sessions lasted approximately one hour and average earnings were $22, including a $5 show-up fee.

1.3 THE EFFECT OF SEQUENTIAL GIVING WITH AND WITHOUT FIXED COSTS

Our experiment is designed to examine the role of sequential fundraising in eliminating inefficient outcomes that may arise in the presence of fixed costs and simultaneous play. In reporting the results we first determine the effect sequential play may have absent fixed costs, we then see if the introduction of a fixed cost of six gives rise to inefficient outcomes when

[10] A consequence of our design is that a participant's cost can exceed their endowment; in effect they borrow against earnings from the group account. We made this clear in the instructions, and participants did not express any concerns about this aspect of the design. Throughout the experiment participants relied on the payoff table when making decisions. Thus their decisions appeared to be determined solely by final payoffs. Only one participant asked how purchases could exceed his endowment. The participant appeared satisfied with the explanation that the cost was taken out of his earnings from the group account. While we think it is unlikely, we cannot rule out that limited endowments rather than decreasing payoffs, restrained contributions.

[11] This procedure allows us to directly test the informational effect of sequential play and eliminates the possibility that sequencing alone can explain the results (see e.g., Cooper et al. (1993)). Potters et al. (2005, 2007) use a similar approach.

[12] This differs from the common approaches where either all or only one round count for payments. However we see no reason why this approach should be inferior and it allows us to keep the average payments in line with those commonly seen, and secures a transparent payoff table.

	All rounds 1-14	First seven 1-7	Last seven 8-14
Sequential	0.668	0.752	0.585
	(0.001)	(0.002)	(0.002)
Round	-0.030	-0.031	-0.060
	(0.001)	(0.238)	(0.017)
N	1176	588	588
Participants	84	84	84

Note: p-values are in parentheses.

Table 2: GLS random effects regression dependent variable: individual contribution, $FC = 0$.

contributions are made simultaneously, and whether sequential moves may help overcome such inefficiencies. We conclude the section by determining whether the answers to these questions are robust to an increase in the fixed cost.

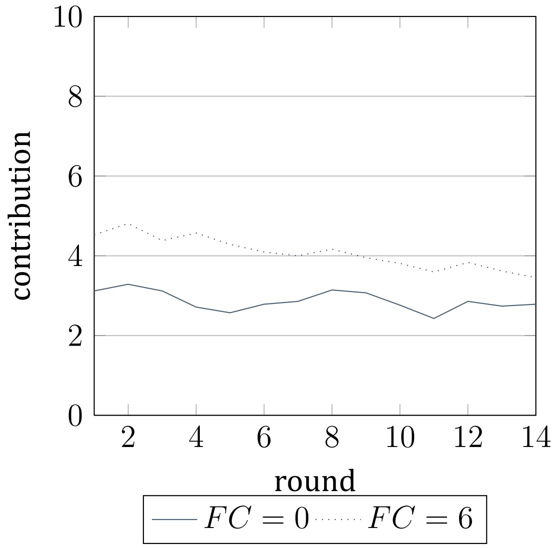
1.3.1 Contributions with zero fixed costs

Absent fixed costs the unique equilibrium prediction of both the sequential and simultaneous game is for each member of the two person group to contribute three units. Hence the first hypothesis is

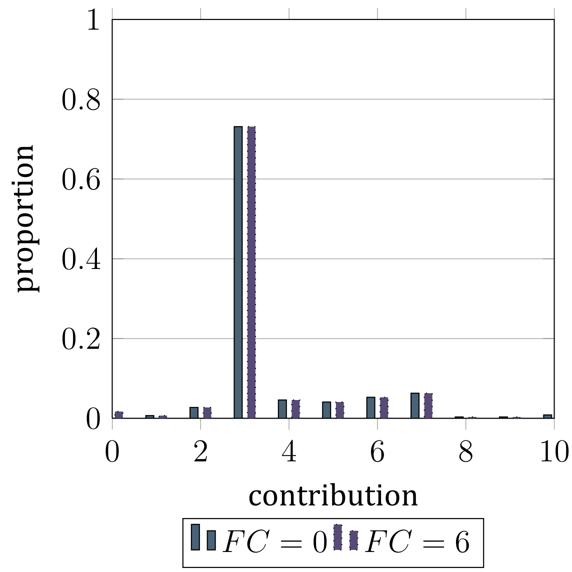
Hypothesis 1. *With zero fixed costs, sequential play has* no effect *on contributions.*

The average contributions for the simultaneous and sequential games with zero fixed costs are shown by round in Fig. 2. Focusing first on the simultaneous game, we note that average contributions are very close to the three-unit equilibrium prediction. With a mean contribution of 2.87 units we cannot reject that participants contribute the predicted amount (p=0.382).[13] Furthermore we do not see a substantial decrease in contributions over the

[13] To account for fact that each individual makes 14 decisions, the reported test statistics in our paper refer



(a) Mean Individual Contributions



(b) Probability mass plot of individual contribution

Figure 2: Simultaneous moves with fixed costs of zero and six.

course of the experiment – a sharp contrast to the behavior in the classic VCM game where contributions initially exceed the dominant strategy of zero giving and decrease over time.[14] Our data does however resemble that of previous VCMs in that the frequency of equilibrium play increases over the course of the experiment from 57% during the first half of the experiment to 66% during the second half of the experiment. The unusually high frequency of equilibrium play is most likely driven by the fact that we use a very simple piecewise linear cost function to secure an interior dominant strategy.[15]

While contributions in the simultaneous game are consistent with the equilibrium prediction, we see greater-than-predicted giving in the sequential game. In every round of the

the results from random effects regressions. Exceptions will be noted.

[14] A random effects regression of individual contributions on round shows that contributions decrease significantly over time, but the coefficient is small (-0.028, $p = 0.042$) in the simultaneous game and corresponds to no more than a one percent decrease in giving per round. See Ledyard (1995) for a review of commonly observed contribution patterns in the classic VCM.

[15] Previous examinations of interior equilibria in dominant strategies use the more complicated quadratic cost function and fail to see substantial equilibrium play (see Keser (1996); Sefton and Steinberg (1996); van Dijk et al. (2002) and Laury and Holt (2008) for a review). Menietti et al. (2009) use a linear payoff structure similar to that examined here and find substantial equilibrium play.

sequential game average contributions exceed the predicted contribution of 3. Indeed the mean contribution of 3.54 differs significantly from the prediction ($p = 0.00$). Note however that 73% of all decisions are at the predicted contribution of 3.

In describing the experimental design we hypothesized that reciprocity might cause behavior in the sequential game to deviate from the equilibrium prediction, and our data are consistent with this explanation. While our results do not show evidence of negative reciprocity there is some evidence of positive reciprocity.[16] When the first mover's contribution ranges between 0 and 3 units, second movers opt for the dominant strategy and contribute an average of 2.99 units. However the average secondmover contribution increases to 3.80 units when first movers give more than their dominant strategy. To assess the return from increasing first contributions by 1 unit, we use random effects to regress second mover contributions on that of the first mover. When first mover contributions range from 3 to 7 units we find that a one-unit increase in first mover contributions increases the second mover's contribution by 0.29 units. Although the positive coefficient is consistent with reciprocity the response is not large enough to make it payoff maximizing for first movers to deviate from their dominant strategy.[17] Nonetheless the incentive for first movers to give is greater with sequential play and average first mover contributions are found to be significantly higher in the sequential than simultaneous game (3.85 vs. 2.96, $p = 0.005$).

Comparing the sequential and simultaneous treatments with zero fixed costs we find a significant effect of sequential moves.[18] Using random effects Table 2 reports the results from regressing individual contributions on a "sequential" dummy that takes a value of 1 if the game is sequential and 0 otherwise, and a round number variable "round" which controls for changes in contributions over time, be it due to learning or changes in preferences.

[16] As noted by Charness and Rabin (2005) the experimental evidence of negative reciprocity is substantial whereas that on positive reciprocity is more limited. As demonstrated by Andreoni et al. (2003) the degree of both negative and positive reciprocity is however sensitive to the examined environment and the perceptions individuals have of a particular action. A contribution below the dominant strategy equilibrium is costly to the individual and may not be perceived as unkind.

[17] The net cost of contributing in the 4-7 unit range is 20 cents; thus it is payoff maximizing to increase first mover contributions by 1 unit if it generates an increase in second mover contributions of more than 0.4 units.

[18] This result is likely to be sensitive to the environment examined. In our study the equilibrium is symmetric and is predicted to be the same under sequential and simultaneous moves. In sharp contrast Andreoni et al. (2002) and Gachter et al. (2010) examine quasi-linear environments where contributions are predicted to decrease with sequential moves and where the asymmetric subgame perfect equilibrium predicts a substantial first-mover advantage. Both studies find evidence of lower sequential than simultaneous giving.

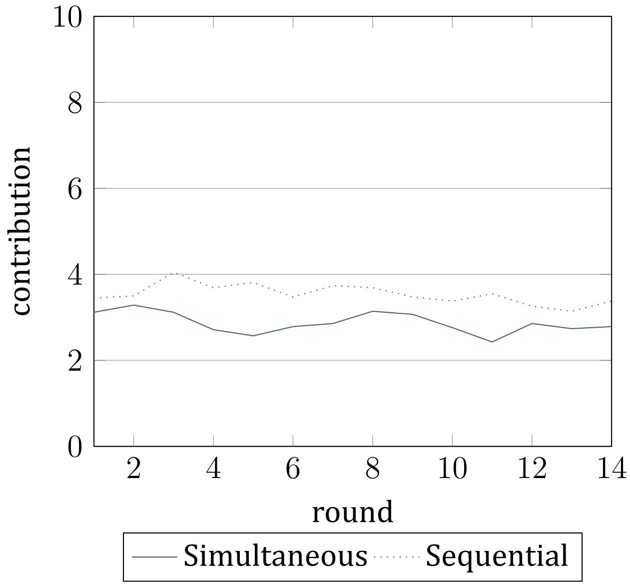


Figure 3: Fixed costs of zero, mean individual contributions.

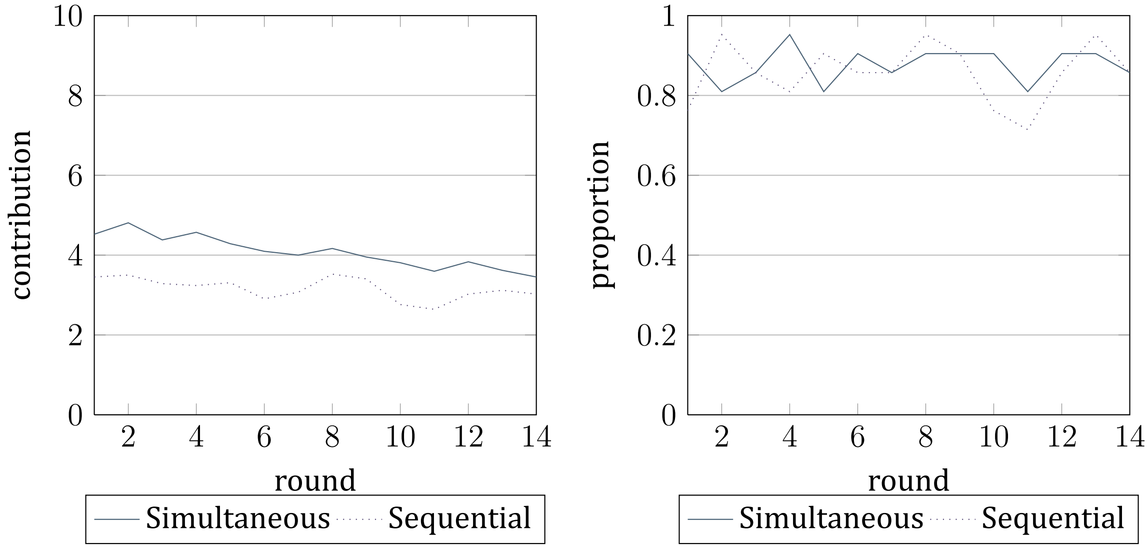
Table 2 shows that when pooling the sequential and simultaneous data we continue to see a slight decrease in contributions with round. While the decrease is significant overall and in the last seven rounds, it is not significantly different from zero during the first seven rounds. As expected from Fig. 2 sequential play is found to cause a significant and substantial 20% increase in contributions. This implies a 32 cent or 6% increase in earnings.[19] This positive effect is robust to breaking the data into the first-seven and last seven rounds. Hence we reject 1.[20] When fixed costs are zero, sequential play increases contributions.[21]

[19] For rounds 1-14 we get a constant of 5.67 ($p = 0.00$), coefficients of 0.32 (0.00) on sequential play, and -0.01 (0.10) on round. For rounds 1-7 the constant is 5.68 ($p = 0.00$), and the coefficients are 0.35 (0.00) on sequential play and -0.02 (0.30) on round. Finally, for rounds 8-14 the constant is 5.78 ($p = 0.00$) and the coefficients are 0.29 (0.00) on sequential play, and -0.017 (0.23) on round.

[20] Session level analysis generates the same result. Mean contributions in the three sequential sessions systematically exceed those of the three simultaneous sessions.

[21] Our results are robust to controlling for the correctness of the answers provided on the quiz. However the coefficient on the correctness of the quiz is never significant and including it has no qualitative (and most often no quantitative) effect on the estimated coefficients. An explanation for why a participant's initial ability to read the payoff table has no significant effect on behavior may be that the experimenter carefully reviewed and explained the quiz answers prior to the decision phase of the experiment.

1.3.2 Contributions with low fixed costs



(a) Mean Individual Contributions (b) Likelihood of Provision

Figure 4: Fixed costs of six.

Having found that sequential play increases contributions in our zero-fixed-cost treatments, we continue our analysis to determine how behavior responds to the introduction of fixed costs. The primary question of interest is whether in the presence of fixed costs, sequential play causes an even greater increase in giving as it eliminates inefficient outcomes. Outcomes that may arise as a result of fixed costs in the simultaneous game. We begin by examining the response in our low-cost treatments where the fixed cost is six. To evaluate the potential role of sequential play we start by examining whether the introduction of low fixed costs causes coordination failure and zero provision outcomes in the simultaneous game. We compare contributions under simultaneous play when fixed costs are zero and six. As shown earlier, with fixed costs of six the simultaneous game admits two Nash equilibria: $(g_1^\star, g_2^\star) \in (0,0), (3,3)$. That is, an inefficient equilibrium with zero contribution emerges along with the previous equilibrium of three-unit contributions by each of the group members. Although the existence of an additional and inefficient equilibrium does not guarantee it will be played, this is an implicit assumption in Andreoni's argument for the role of sequen-

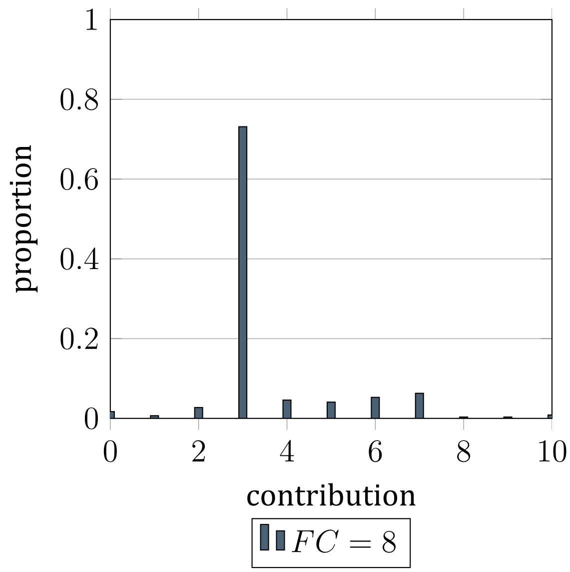


Figure 5: Probability density function of individual contributions simultaneous play and $FC = 8$.

	All rounds 1-14	First seven 1-7	Last seven 8-14
Sequential	-0.917	-1.129	-0.704
	(0.000)	(0.000)	(0.008)
Round	-0.064	-0.097	-0.082
	(0.000)	(0.004)	(0.002)
Constant	4.560	4.767	4.678
	(0.000)	(0.000)	(0.000)
N	1176	588	588
Participants	84	84	84

Note: p-values are in parentheses.

Table 3: GLS random effects regression dependent variable: individual contribution, $FC = 6$.

tial fundraising. If the inefficient equilibrium is played with some positive probability, average contributions are predicted to be lower with fixed costs of six. This comparative static prediction is summarized in the second hypothesis

Hypothesis 2. *Average contributions in the simultaneous game with fixed costs of six are smaller than with fixed costs of zero.*

Fig. 3 panel (a) demonstrates the mean contributions by round in the two simultaneous treatments ($FC = 0$ and $FC = 6$). With fixed cost the contribution pattern is in sharp contrast to the prediction. Rather than decreasing contributions, the introduction of low fixed costs is found to significantly increase contributions.[22] A random effects regression of individual contribution on round and a dummy variable ($FC = 6$) that takes a value of 1 for observations with fixed costs of six and 0 for observations with zero fixed cost reveals a positive and significant coefficient for the fixed cost dummy. All else equal, in the simultaneous game introducing a fixed cost of six increases individual contributions by 1.20 units.[23] Thus we reject hypothesis 2. To better understand the deviation from the predicted comparative static we examine the probability distribution of individual contributions. As seen in figure 2b the distribution with a fixed cost of six first-order stochastically dominates the distribution with a fixed cost of zero. Relative to the zero-fixed-cost treatment, we see a decrease in the number of contributions of less than 3 units and an increase in contributions between 4 and 7 units. Contributions in excess of the dominant strategy account for 26% of play when there are no fixed costs and increase to 55% when the fixed cost increases to six. Perhaps most importantly, and contrary to expectations, the presence of fixed costs is not found to increase the frequency of zero unit contributions. We conjecture that strategic uncertainty is the primary cause for the increase in contributions. Contributing all of the fixed costs happens to be a best response for a wide range of beliefs over the partner's contribution. Consider beliefs that only place weight on the partner selecting an action associated with the two Nash equilibria: contributing 0 or 3 units. If the subject is very certain to be matched with someone

[22] Session level data reveal the same contribution pattern: the simultaneous treatments with fixed costs of six systematically generate larger session averages than that observed with fixed costs of zero.

[23] A random effects regression of individual contributions for rounds 1-14 reveals coefficients of 1.20 on a $FC = 6$ dummy, -0.06 on round, and 3.32 as the constant. For rounds 1-7 the coefficient is 1.46 on $FC = 6$, -0.10 on round, and 3.31 as the constant. Finally, for rounds 8-14 the coefficient is 0.95 on $FC = 6$, -0.08 on round, and 3.70 as the constant. All p-values are smaller than 0.01.

contributing zero, the individual's best response is instead to contribute zero as well. Similarly, if she is very certain to be matched with someone contributing 3 units, the best response is to contribute three. However, if the likelihood of being matched with a zero contributor lies in the range of 40 to 80%, the best response is to contribute 6 units. Thus absent the ability to coordinate on one of the two Nash equilibria, individuals may benefit from single-handedly securing provision of the project. There are other examples in which a contribution of 6 units is a best response due to the tradeoff between risk of coordination failure and contribution costs. In fact there are a number of symmetric mixed strategy Nash equilibria that require that the individual contributes six with a positive probability.[24]

If the strategic uncertainty argument is correct, one would expect equilibrium play to increase as uncertainty about the strategies being employed diminishes. The data is consistent with an increase in equilibrium play. The effect of fixed costs is found to decrease from the first to the second half of the experiment, and over the course of the experiment the number of six-unit contributions decrease while the number of three-unit contributions increase. During the first seven rounds of the game, three- and six-unit contributions each account for 25% of all play. These numbers change for the latter half of the experiment, with 44% of all contributions at three and only 14% at six. Interestingly the frequency of zero contributions also decreases slightly over the course of the experiment. During the first and second half of the experiment a contribution of zero accounts for, respectively, 9 and 7% of overall contributions. We complete our analysis of the low-fixed-cost treatments by examining the effect of sequential play. With fixed costs of six the subgame perfect Nash equilibrium of the sequential game is $(g_1^\star, g_2^\star) = (1, 5)$: the first mover gives 1 unit while the second mover gives the remaining amount to cover the fixed cost, i.e., 5 units. From a theoretical viewpoint, the sequential game eliminates the inefficient Nash equilibrium outcome of zero provision, potentially increasing contributions (to an average of 3 units). This is summarized in the third hypothesis

[24] To see this formally, section A.3 presents the inequalities which determine whether or not contributing six is a best response. The set of inequalities contains 6 inequalities, which together define whether the strategy of contributing 6 units is a best response. The set of symmetric mixed strategy NE is {(0,3,6) with probability (0.4, 0.2, 0.4); (0,1,5,6) with probability (0.08, 0.36, 0.048, 0.512); (0,2,4,6) with probability (0.2, 0.15, 0.075, 0.575); (0,1,3,5,6) with probability (0.08, 0.24, 0.06, 0.036, 0.584); (0,2,3,4,6) with probability (0.2, 0.12, 0.024, 0.072, 0.584); (0,1,2,4,5,6) with probability (0.08, 0.096, 0.138, 0.0228, 0.02736, 0.63584); (0,1,2,3,4,,5,6) with probability (0.08, 0.096, 0.1152, 0.01824, 0.021888, 0.026266, 0.642406)}.

Hypothesis 3. *With fixed cost of six, sequential play increases contributions.*

Our results from the simultaneous game leave one skeptical that support for hypothesis 3 will be found in the low-fixed-cost environment. The limited evidence of inefficient outcomes in the simultaneous game with fixed costs leaves little room for sequential play to improve on the simultaneous outcomes. Furthermore, we argued that uncertainty with regard to the partner's play helped explain why fixed costs increased contributions in the simultaneous game. As this uncertainty is reduced in the sequential game, contributions may instead decrease to the equilibrium level. Figure 2a shows the mean individual contributions by round in the sequential and simultaneous game with low fixed costs. In contrast to the predicted comparative statics we see that mean contributions are lower with sequential play than with simultaneous play. Table 3 presents a random effects regression analysis of individual contributions for $FC = 6$. As before, the dependent variable is individual contribution and the explanatory variables are whether the game is sequential or simultaneous and the number of rounds. The effect of sequential play is found to be negative and significant. All else equal sequential play reduces individual contributions by almost 1 unit. Thus we reject hypothesis 3 with fixed costs of six, sequential play decreases the mean contribution.[25] The rejection of hypothesis 3 is not caused by behavior in the sequential game. In fact we cannot reject that the average contribution of 3.16 in the sequential game equals the predicted three-unit mean contribution ($p = 0.380$). Instead the deviation from the predicted comparative static (hypothesis 3) is driven by the higher-than-expected contributions in the simultaneous game. Before we draw any conclusions on the relative advantages of sequential versus simultaneous play, we should however examine the actual provision of the public good. After all, donors benefit from provision rather than contribution, thus contributions may be misleading when selecting between fundraising techniques. For example, it is possible that an individual gift of 5 units in the simultaneous game is matched with a contribution of zero causing the public good not to be provided. Figure 4b presents the fraction of cases in which the public good was provided, by round and by treatment (simultaneous versus sequential). The provision rate is high and in excess of 80% in both treatments. Despite the coordination problem asso-

[25] Session level data reveal the same contribution pattern, with the sequential treatments systematically generating lower session averages than those observed with simultaneous moves.

ciated with simultaneous giving, the 30% of contributions that are large enough to guarantee public good provision in the simultaneous game help secure similar provision rates in the two treatments. The high provision rate combined with the larger average contributions in the simultaneous treatment implies that individual earnings are slightly lower with sequential than simultaneous play. Using random effects to regress individual round earnings on a sequential treatment dummy and round number we find that sequential play reduces participant earnings by about 25 cents per round.[26] While this difference is significant it only corresponds to a 4% decrease in earnings. Contrary to the expectations we do not find evidence to suggest that participants on average get higher earnings in the sequential treatment when the fixed cost equals six.

1.3.3 Contributions with high fixed costs

Our analysis of contributions with a six-unit fixed cost did not show the expected increase in contributions from sequential play. This result was driven by the larger than expected contributions in the simultaneous game. We argued that strategic uncertainty and the associated risk of coordination failure could help explain this behavior. To illustrate this point we used an example of an individual who believes that her partner either contributes nothing or covers half of the fixed cost, and found that single-handedly covering the fixed cost is a best response for this individual as long as she believes the probability of the other group member contributing nothing is between 40 and 80%. The substantial probability range (40 to 80) makes the risk of coordination failure a real concern, and reduces the attractiveness of contributing less than six. For instance if the individual gives 3 units and the threshold is not met she will incur a loss of 1.2; however if she contributes 6 units herself, the worst that can happen is the partner contributing zero yielding a loss of 0.3.

Accounting for this type of strategic uncertainty renders contributions of six a best response. In this section we examine if behavior may be more in line with theory in a fixed-cost treatment where this type of strategic uncertainty does not make it a best response to single-

[26] For rounds 1-14 we get a constant of 5.97 ($p = 0.00$), coefficients of -0.25 (0.00) on sequential play, and -0.02 (0.03) on round. For rounds 1-7 the constant is 6.02 ($p = 0.00$), and the coefficients are -0.28 (0.01) on sequential play and -0.03 (0.25) on round. Finally, for rounds 8-14 the constant is 6.01 ($p = 0.00$) and the coefficients are -0.22 (0.00) on sequential play, and 0.02 (0.32) on round.

	All rounds 1-14	First seven 1-7	Last seven 8-14
$FC = 6$	-0.526	-0.526	-0.527
	(0.000)	-0.001	(0.000)
$FC = 8$	-1.091	-0.986	-1.091
	(0.000)	(0.000)	(0.000)
First mover	-0.453	-0.545	-0.362
	-0.001	-0.001	-0.011
$FC = 6 \times$ (First Mover)	0.385	0.389	0.381
	-0.039	-0.086	-0.049
$FC = 8 \times$ (First Mover)	1.322	1.272	1.372
	(0.000)	(0.000)	(0.000)
Round	-0.174	-0.029	-0.0136
	-0.006	-0.127	-0.432
Constant	6.148	6.229	6.067
	(0.000)	(0.000)	(0.000)
N	1568	784	784
Participants	112	112	112

Note: p-values are in parentheses.

Table 4: GLS random effects regression dependent variable: individual earnings.

handedly cover the cost. Specifically we examine an environment with an eight unit fixed cost where, given the belief that the other group member either contributes nothing or covers half the fixed cost, it is not a best response to contribute eight.[27] Recall that with a fixed cost of eight there is a unique subgame perfect equilibrium at (2,6), and four Nash equilibria of the simultaneous game: (3,5), (4,4), (5,3) and (0,0). As noted in Section 2, with fixed costs of eight there are two ways in which sequential play may increase contributions: first through the elimination of the zero contribution equilibrium, and second by alleviating the coordination problem associated with selecting one of the positive provision equilibria. If participants in the simultaneous game play the zero contribution equilibrium with some positive probability, then the comparative static of the low-fixed-cost treatment should still hold. Thus we test the following fourth hypothesis

Hypothesis 4. *With an eight-unit fixed cost, sequential play increases contributions.*

We first examine if there is coordination failure in the simultaneous game. The contribution distribution in the simultaneous game is shown in figure 5. As with fixed costs of six, a substantial fraction of contributions are found to cover half of the fixed cost (four), and a fair number of contributions are at the efficient level (seven). However, in sharp contrast to our earlier findings with fixed costs of six it is rare to see individual contributions that cover the fixed costs, and the modal choice now is to contribute nothing.[28] A third of all contributions are at zero units.[29] Thus behavior in the simultaneous game suggests that there is room for sequential play to improve outcomes. Figure 6b compares the mean individual contributions in the sequential and simultaneous game by round. Despite the high frequency of zero unit contributions in the simultaneous game, the mean contributions are found to be quite similar.

[27] If the agent assigns a probability p to the opponent playing 0 and the probability $1 - p$ to the opponent playing 4, then it is easily seen that there is no positive probability for which the agent would prefer giving 8 rather than either 0 or 4. Specifically if given only the choice of 0, 4 or 8, the agent will never give 8, and will give 0 when the probability that the opponent gives zero is more than 0.525, and will give four otherwise. Note that with a probability of 0.525 both 0 and 4 will give an expected payoff of 4, whereas the expected payoff from 8 is 3.85.

[28] A reviewer suggested that this need not result from differences in earnings, but may be a consequence of the low initial endowment. As noted in footnote 10 we are skeptical that participants used the costs rather than the final payoffs to make decisions. However if participants felt restricted by the costs then the comparative statics nonetheless shed light on the effect of sequential contributions in situations where budget constraints prevent donors from covering the fixed costs of a public project.

[29] The frequency of zero contributions increases from 31 to 35% between the first and second half of the experiment.

The similarity in mean contributions is further supported by a random effects regression of individual contributions on a sequential dummy and rounds. The coefficient on the sequential dummy is found to be small and insignificant whether it is examined overall, or during the first or second half of the experiment.[30] Thus contrary to hypothesis 4, sequential play does not significantly increase individual contributions. While sequential play is not found to increase mean contributions, the likelihood of providing the public good does increase substantially. As figure 6b illustrates, the difference in provision rates is large and persistent across the fourteen rounds of play. On average, sequential play almost doubles the likelihood of providing the public good from 40% when contributions are simultaneous, to 76% when contributions are sequential. Donors accrue substantial benefits from the increase in provision. Using random effects to regress round payoffs on a sequential dummy, we find that sequential play increases round earnings by approximately $1.20, a 27% increase.[31]

1.4 THE ROLE OF SEQUENTIAL MOVES WITH AND WITHOUT FIXED COST

Our results demonstrate that the response to sequential play is rather sensitive to the presence and level of fixed costs of production.

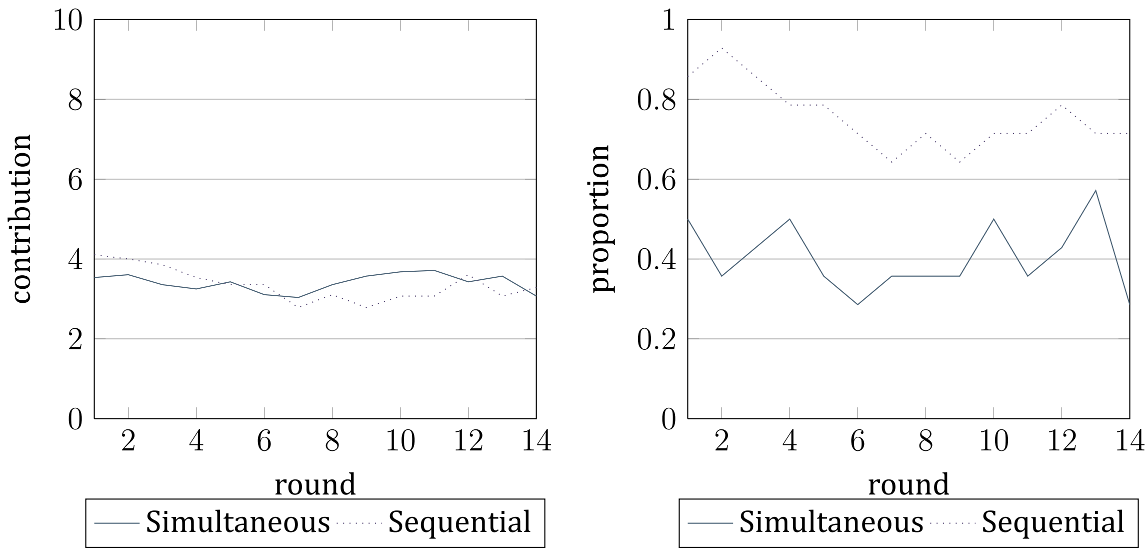
While sequential play decreased giving in the low-fixed-cost treatment, it increased giving in both the zero and high fixed cost treatments. With high fixed cost, sequential play increased earnings by 27%. By comparison, sequential play increased earnings by 6% when there were no fixed costs and decreased it by 4% when the fixed costs were low.[32] Thus our results confirm that sequential play is particularly effective under high fixed costs.

The differences in return from sequential play are largely driven by the larger than ex-

[30] The sequential coefficient equals -0.051 ($p = 0.91$) over all 14 rounds, 0.240 ($p = 0.61$) for the first seven rounds, and -0.342 ($p = 0.49$) for the last seven rounds.

[31] A random effects regression of individual earnings reveals for rounds 1-14 a constant of 4.41 ($p = 0.00$), coefficients of 1.18 (0.00) on sequential play, and -0.01 (0.39) on round. For rounds 1-7 the constant is 4.56 ($p = 0.00$), and the coefficients are 1.34 (0.00) on sequential play and -0.07 (0.10) on round. Finally, for rounds 8-14 the constant is 4.07 ($p = 0.00$) and the coefficients are 1.02 (0.00) on sequential play, and 0.03 (0.54) on round.

[32] A random effects regression reveals that the effect of sequential play is significant at the 0.02 level in each of the 3 fixed cost treatments; in addition the difference in response to sequential play is significant at the 0.00 level.



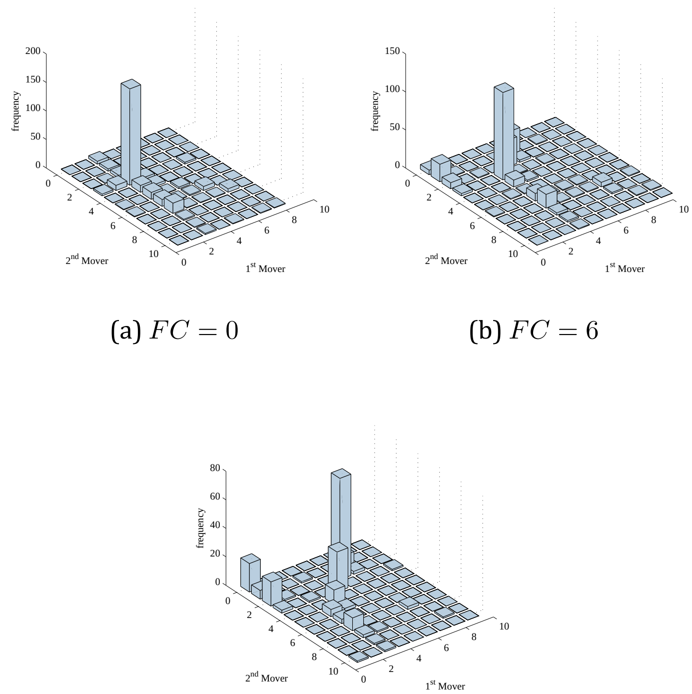
(a) Mean Individual Contributions

(b) Likelihood of Provision

Figure 6: Fixed costs of eight.

pected contributions in the simultaneous game with low fixed cost. In both the zero and high fixed cost treatment behavior of the simultaneous game was very much in line with equilibrium predictions. Absent fixed costs participants on average made the predicted 3 unit contribution, and under high fixed costs many individuals opted to not contribute which resulted in frequent failure to provide the public good. By contrast when the fixed cost was low individuals instead overcame strategic uncertainty by increasing contribution and single-handedly covering the fixed costs. Under low fixed costs simultaneous play only rarely resulted in zero provision outcomes.

While the emphasis has been on the surprising behavior under simultaneous play, it is important to note that adherence to equilibrium also is sensitive to the fixed cost when contributions are made sequentially. Figure 7 illustrates the behavior under the three sequential games reporting the observed combinations of first and second mover contributions. Figure 7a shows contributions in the no fixed cost treatment. Absent fixed cost there is substantial adherence to the highlighted subgame perfect equilibrium at (3, 3). The few deviations



(a) $FC = 0$

(b) $FC = 6$

(c) $FC = 8$

Figure 7: Contribution frequency.

from equilibrium suggest that contributions below the dominant strategy of 3 are not punished, whereas contributions in excess of 3 are sometimes rewarded. One reason why first mover contributions below three are not punished may be that it is costly for the first mover to give less than her dominant strategy, and in doing so she does not influence the second mover's payoff from giving. Introducing fixed costs gives rise to an asymmetric subgame perfect equilibrium with a substantial first mover advantage. This inequality may help explain why in panel figure 7b and figure 7c the fixed-cost-treatments result in greater deviations from the subgame perfect equilibrium. As predicted, contributions in the fixed-cost treatments generally cover the fixed cost. Provision rates are 86% with a fixed cost of six and 76% with a fixed cost of eight. However, there are large differences in the manner in which provision is secured. With a six-unit fixed cost, participants shy away from the highlighted subgame perfect equilibrium (1,5). Instead, the modal outcome is for the first and second mover to each contribute 3 units. In contrast, with fixed costs of eight the modal outcome is the highlighted subgame perfect equilibrium of (2,6). The difference in the frequency of equilibrium play is intriguing as in both cases the subgame perfect equilibrium involves the first player free riding off of the second player's desire to secure provision of the public good.[33]

Two factors may help explain the difference between the two sequential fixed-cost conditions ($FC = 6$, $FC = 8$): one is reciprocity and the other is trust. Reciprocity could be a factor since second movers may view contribution of one out of six as more unfair relatively to contribution of two out of eight; it may be easier for the second mover to accept the inequality associated with the subgame perfect equilibrium in the case of fixed costs of eight.[34] Indeed, with a fixed cost of six and an initial contribution of one, there is a 40% chance that the second mover selects a contribution which is insufficient to secure provision. By contrast, with a fixed cost of eight and an initial contribution of two there is only a 20% chance that the project fails to be provided. Despite the prediction that fixed cost gives rise to a first mover advantage, such an advantage only emerges in the high fixed cost treatments. Trust may also be a factor in explaining the difference in behavior across the fixed cost conditions: giving 1 unit risks 40 cents or 10% of the endowment, while contributing 2 units risk 80 cents or 20% of the endowment. Hence, it is possible that second movers interpret a two-unit contribution by the first mover as a stronger signal of trust compared with 1 unit contribution.

We investigate the difference in first mover advantage in Table 4. Pooling the earnings data from the three sequential treatments we note first that absent fixed cost there is a significant disadvantage to being a first mover, with first movers on average earning 53 cents less per round. Despite the prediction that fixed cost will introduce a first mover advantage, as seen by the first mover and the interaction term, we do not see such an effect under low

[33] Examining sequential public goods games, Andreoni et al. (2002); Cooper and Stockman (2007) and Gachter et al. (2010) also find that free riding by a first mover causes subsequent subjects to not give, even when it is a dominant strategy to do so. The asymmetric payoff outcome under high fixed cost may help explain why the likelihood of provision decreases in the high fixed cost treatment.

[34] Note that it is not only the perceived fairness of the equilibrium that may change when moving from a subgame perfect equilibrium of (1,5) to one of (2,6). The cost of punishing is also higher in the (2,6) equilibrium. Most games where distributional concerns may play a role have the characteristic that an improvement in fairness also increases the costs of punishment. Andreoni et al. (2003) is an exception as they keep the cost of punishment and rewards constant while allowing the distribution of payoffs to vary.

fixed cost. With fixed cost of six first movers earn 7 cents less than second movers, however this difference is not significant ($p = 0.60$). A significant and substantial first mover advantage is however seen when the fixed cost is eight, as the earnings of first movers on average exceed those of the second movers by 87 cents ($p = 0.00$). The advantage to the first mover is relatively robust over the first and second half of the experiment.

To summarize in the no and high fixed cost treatments behavior in both the sequential and simultaneous games is broadly consistent with the equilibrium predictions, however in the low-cost treatments we see substantial deviations in both the sequential and simultaneous games. On one hand strategic uncertainty appears to cause greater than predicted simultaneous giving when the fixed cost is low; on the other hand the tension associated with the substantial first mover advantage appears to move behavior away from the asymmetric subgame perfect equilibrium. Interestingly this asymmetry is more readily accepted when the fixed cost is high. The sensitivity to the size of the seed relative to the total fixed cost may suggest that fundraisers and initial donors should use caution when trying to exploit a potential first mover advantage.

1.5 CONCLUSIONS

Our study was designed to examine whether the frequent use of sequential fundraising and seed money contributions may be explained by the presence of fixed production costs. We find support for this claim for sufficiently high fixed costs, but not for low fixed costs. More specifically, the theoretical argument made by Andreoni (1998) is that in the presence of fixed costs, giving simultaneously to a public good may result in both positive and zero provision equilibria. Thus absent information on what others give, donors may get stuck in an inefficient equilibrium with zero provision of the public good. The attraction of sequential giving is that it eliminates such inefficient outcomes and guarantees provision of desirable public projects. Thus sequential fundraising is predicted to increase giving and individual payoffs.

For small fixed costs we do not find support for this claim; instead sequential play is shown to decrease both contributions and individual payoffs. The reason for this deviation from the predicted comparative statics is found in the simultaneous game where, surprisingly, the introduction of fixed costs increases rather than decreases contributions. The explanation for the larger than expected contributions is due to the coordination difficulties of the simultaneous game combined with the relatively low fixed costs. Interestingly, uncertainty over which equilibrium the partner is playing often makes it a best response to contribute an amount large enough to single-handedly cover the fixed cost. The sequential game, however, alleviates the coordination problem and participants can "safely" contribute less and still secure provision of the public good. Thus for low fixed costs we find that contributions in the simultaneous game exceeded those in the sequential game. While this result was not anticipated, it is not difficult to envision a case where the cost from contributing is so low and the benefit from provision is so high that individuals in a simultaneous move game will contribute an inefficiently large amount in order to secure the good.[35]

In the case of large fixed costs, behavior was found to be more in line with the theory. Although sequential play did not increase contributions, it did increase the likelihood of provision and individual earnings. As predicted, with simultaneous play many participants did not contribute to the public good, or failed to coordinate to meet the fixed costs level which is necessary to provide the good. With high fixed cost sequential play improved upon the simultaneous outcome through two channels: not only did it eliminate the zero contribution outcomes, it also eliminated the inefficiencies that result when participants fail to coordinate on one of the simultaneous game's multiple positive-contribution equilibria. Thus the success of sequential play with large fixed costs may partly be explained by the fact that the coordination problem is greater in this case.

While sequential play helps donors coordinate on positive provision outcomes, one needs to be wary of the risk associated with allowing for too low an initial contribution. The presence of fixed costs enables the first contributor to free ride off of the second contributor, and to fully extract the second mover's benefit from provision. Full exploitation of this advantage may cause second contributors to object to the unequal division of the burden and result in a failure to provide the public good. Examining the sequential game with both low and high

[35] Perhaps the excessive contributions seen in connections with the September 11 attacks in 2001 and the Asian tsunami in 2004 would have been smaller if donations had been made in a more sequential manner.

fixed costs, we found evidence to suggest that the success of the sequential play in our case was sensitive to the share of funds provided by the first contributor.

Research has proposed several explanations for why fundraisers rely on sequential solicitation strategies. Many of these explanations reduce the first contributor's inherent ability to free ride off of second contributors in a public good game.[36] By contrast, the introduction of fixed costs increases the first mover advantage inherent in the public good game, and a potential risk of sequential play is that provision may fail unless the fundraiser is successful in convincing initial contributors to donate a fair share. Perhaps this concern for equity helps explain why fundraisers have specific goals for how large seed money contributions need to be as a share of the overall fundraising goal.[37]

1.6 ACKNOWLEDGMENTS

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[36] For example, to signal that a charity is of high quality the first player will have to contribute an amount which is larger than what would have been needed had the charity been known to be of high quality.

[37] As noted in Andreoni (2006a); Lawson (2007) states "the lead gift should be at least 10% of the overall goal" (p. 756). Hartsook (1994) advises that "the leadership commitment ...should represent no less than 20% of the capital campaign goal" (p. 32).

2.0 PROVISION POINT MECHANISMS AND THE OVERPROVISION OF PUBLIC GOODS

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2.1 INTRODUCTION

Fundraising campaigns for non-profit causes often feature goals for total contributions. Sometimes these goals are binding in the sense that if the the campaign fails to raise total contributions that exceed the goal then pledges are not collected or contributions are refunded. For instance, Bagnoli and Mckee (1991) present a case from Manitoba, Canada, where the New Democratic Party in 1980 and 1985 sent letters to its larger contributors soliciting additional funds to mount an upcoming election campaign. The letters stipulated that a target had been set at $200,000 and that the New Democratic Party would refund all contributions if the target were not reached by a certain date. Both campaigns succeeded.[1] The ability of a fundraiser to collect pledges or offer refunds contingent on a goal for total contributions has been investigated both theoretically and experimentally, e.g., Admati and Perry (1991), Bagnoli and Lipman (1989), and Andreoni (1998).

Though collecting pledges contingent on a goal appears to be an innocuous power, it gives fundraisers the ability to artificially truncate the production function for the public good. If a fundraiser can commit to refunding contributions that fail to meet a threshold, then donors

[1] Other examples are presented in Bagnoli and Mckee (1991) as well as in Marks and Croson (1998) and Marks et al. (1999).

view production as having a step-wise structure. Contributions that fail to exceed the threshold do not increase production. As contributions cross the threshold, production of the public good increases discontinuously. Such an environment presents donors with incentives that are quite similar to those in Bagnoli and Lipman (1989). They show that a *discrete* public good can be efficiently provided by a fundraiser employing a refund rule. The fundraiser takes production of the public good as given and operates the mechanism to collect pledges and provide the public good. We aim to understand how a fundraiser with the ability to employ such thresholds chooses to use them and the resulting impact on provision of the public good.

The contribution game we study is similar to that commonly examined in the literature on private provision of public goods: A finite number of potential donors simultaneously allocate income between consumption of a private good and contributions to a public good, and individuals' utility depends only on the consumption levels of private and public goods.[2] With continuous production it is predicted that the result is under-provision of the public good. Added to this environment is a contribution maximizing fundraiser who can artificially *truncate* the production function. This truncation is secured by setting a threshold for total contributions, and by allowing donors to make contribution pledges contingent on the threshold being reached. Alternatively the fundraiser may collect contributions and refund them if they fall short of the goal.[3]

The primary result of the paper is that a fundraiser who can commit to such a strategy *always* chooses to set a threshold, and that the chosen threshold is "too high." At the equilibrium of this game more public good is produced than at any outcome in the core of the game; we consider this *over-provision*. This result does not strictly depend on the fundraiser committing to zero production of the public good if the threshold is not met or exceeded. We extend the model to allow the charity to run a second campaign if the threshold is not met. This campaign has no threshold, all contributions are accepted and used to produce the public good. At the equilbirium of this game more public good is produced than at *some* outcome in the core of the game; we consider this *not under-provison*.

[2] See for example the seminal work of Bergstrom et al. (1986)

[3] Marks et al. (1999) argue that these two mechanisms are isomorphic.

The equilibria that exhibit over-provision are not the unique Nash equilbria of the game. However, these over-provision equilibria are the only equilbria that are *subgame, undominated-perfect equilbria*. This refinement is adapted from the undominated-perfect equilibrium refinement from Bagnoli and Lipman (1989).

The behavioral assumptions of the refinement are not a theoretical curiousity. Experimental results strongly support the equilibrium refinements used to select the equilibrium outcome. When possible, inefficient over-provision occurred in 82 percent of cases overall. The cases in which provision did not occur appear to result from mis-coordination on how to reach the threshold, not a belief that over-provision would not occur..

The paper is organized as follows. In the next section, we present the model and describe the contribution game. Section 2.3 describes the main theoretical results. Then in section 2.4 we extend the model to allow for further contributions in the event the threshold is not reached and contributions are refunded. Section 3.4 discusses a laboratory experiment testing the models implications and presents the results. Finally, section 3.6 concludes.

2.2 THE MODEL

The model is one in which the underlying production technology of the public good is continuous, where a contribution maximizing fundraiser artificially truncates the production function by setting a minimum threshold for total donations. We call this game the *threshold game*. If the threshold is not reached, all contributions are returned to the respective donors. The outcomes that can be supported hinge on the outside option of the donors. In the simplest and most extreme case, the fundraiser commits to not producing the public good unless sufficient funds are raised, hence the outside option is no production and donors allocate all their endowment to the private good. At the other extreme, if the fundraiser cannot refuse any contributions then in the event that funds fall short of the threshold the outcomes coincide with those of a standard continuous voluntary contribution game. In this section we consider the simplest case where the fundraiser can commit to no production of the public good if the threshold is not met. In the next section we will consider a more general model that allows for another round contributions without a threshold in the event the threshold is not met.

2.2.1 Agents, Utility, and Production

There are $n+1$ agents in the model. A set of donors, I, who number $n \in \mathbf{N}$, and a fundraiser, c. The donors, indexed by $i \in I$, are endowed with $m_i \in \mathbf{R}_{++}$ of a private good. Each donor i values the private good and has altruistic preferences over the public good determined by her utility function, $u_i : [0, m_i] \times \mathbf{R}_+ \to \mathbf{R}$. In the expression $u_i(x, G)$, x denotes an amount of the private good and G an amount of the public good. We assume that u_i is strictly concave, and strictly monotone. Additionally, assume that both the public and private goods are normal goods for all donors.

Additionally, assume that production of the public good is a strictly increasing, continuous function of contributions. Hence without loss of generality, we treat the sum of contributions of the private good, $\sum_{i\in I} g_i$, as the public good itself.[4] The fundraiser seeks to maximize total contributions to the public good, $u_c : \prod_{i\in I}[0, m_i] \to \mathbf{R}$, $u_c(\mathbf{g}) = \iota'\mathbf{g}$.

Notation. *We often need to refer to a vector, for example of contributions* $\mathbf{g} \in \prod_{i\in I}[0, m_i]$*, and the sum of its elements,* $\sum_{i\in I} g_i$*. We prefer the succinct notation,* $\iota'\mathbf{g} = \sum_{i\in I} g_i$*, where* ι *is an appropriately sized vector of ones.*

2.2.2 Agent's Decision Problems

The fundraiser chooses a threshold for contributions, $T \in \mathbf{R}_+$, such that any vector of contributions which totals less than T is refunded to donors. The provision level of the public good is then determined by

$$G\left(\iota'\mathbf{g}, T\right) = \begin{cases} \iota'\mathbf{g} & \text{if } \iota'\mathbf{g} \geq T \\ 0 & \text{otherwise} \end{cases}.$$

When the threshold is not met the contributions are refunded and no public good is provided.

[4] Any increasing production function can be viewed as a monotone transformation of the utilities and thus represents the same preferences.

Once a threshold has been selected by the fundraiser, the n donors simultaneously contribute to the public good, choosing $g_i \in [0, m_i]$. If the contributions, **g**, exceed the threshold T, $\iota'\mathbf{g} \geq T$, then the payoff to the fundraiser is $u_c(\mathbf{g}) = \iota'\mathbf{g}$ and each donor receives a payoff of $u_i(m_i - g_i, \iota'\mathbf{g})$. Otherwise no production of the public good occurs, the fundraiser receives a payoff of $u_c(\mathbf{g}) = 0$ and each donor receives a payoff of $u_i(m_i, 0)$.

2.3 EQUILIBRIUM OUTCOMES

2.3.1 Efficiency and Over-provision

Before developing the equilibrium results we need to be precise about what constitutes over-provision of the public good. Over-provision implies that the amount of public good produced is larger than the socially efficient amount of public good under the continuous production function; society could be made better off producing less public good. We assume that the socially efficient allocation is an element of the *core*, as defined in definition 1.[5]

Definition 1. *$s \subset I$ is said to be a* blocking coalition *of $\mathbf{g}' \in \prod_{i \in I}[0, m_i]$ if $\exists \mathbf{g} \in \prod_{i \in s}[0, m_i]$ such that*

1. *$u_j(x_j, \iota'\mathbf{g}) \geq u_j(m_j - g'_j, \iota'\mathbf{g}')$, for all $j \in s$,*
2. *$u_k(x_k, \iota'\mathbf{g}) > u_j(m_k - g'_k, \iota'\mathbf{g}')$, for at least one $k \in s$,*
3. *$\sum_{i \in s} x_i \leq \sum_{i \in s}(m_i - g_i)$.*[6]

The core *of threshold game X, $C(X) \subset \prod_{i \in I}[0, m_i]$, is the set of all contribution vectors not blocked by any coalition.*

We denote the least amount of public good provided at a core outcome by $C_{min} = \min_{\mathbf{g} \in C(X)} \iota'\mathbf{g}$. Similarly, we denote the maximum amount of public good provided at a core outcome by $C_{max} = \max_{\mathbf{g} \in C(X)} \iota'\mathbf{g}$. Provision, G, of the public good can be broken into three regions: *a*) $G \in [0, C_{min})$, *under-provides* the public good, *b*) $G \in [C_{min}, C_{max}]$, *does not under-provide* the public good, *c*) and $G \in (C_{max}, \infty)$, *over-provides* the public good.

[5] We adapt our definition of the core from Bagnoli and Lipman (1989).

[6] Note that redistribution of endowments is allowed; the bounds for consumption are based on the total endowment of the coalition.

2.3.2 Undominated-perfect, subgame equilibria

There are many Nash equilibria of the threshold games we examine. Our results employ an equilibrium refinement borrowed from Bagnoli and Lipman (1989), *undominated-perfect equilibrium*. We adapt their refinement to fit our sequential move environment.

The undominated-perfect equilibrium refinement is similar to trembling-hand perfection. However, trembles to strictly dominated strategies are not permissible. It is as though strictly dominated strategies are removed from the game, and then trembling-hand perfection is considered. Our subgame, undominated-perfect equilibrium strengthens subgame perfection by requiring the subgame equilibria to be undominated-perfect equilibria.

2.3.3 Equilibria

The main result, proposition 1, states that the unique outcome of a threshold game is one of over-provision of the public good. The addition of a fundraiser with the non-coercive ability to set thresholds and offer refunds not only corrects any inefficiency from too few contributions, but introduces a new inefficiency from extracting too many contributions. A formal discussion and proof of proposition 1 is placed in section A.4; here we give the intuition and illustrate a parametric example.

Proposition 1. *Let* $X = \left\{c, \{u_j, m_j\}_{j=1}^{n}\right\}$ *be a threshold game. Then there exists an equilibrium outcome,* $\{T, \mathbf{g}\} \in \left\{\mathbf{R}_+, \prod_{i\in I}[0, m_i]\right\}$*, in which total contributions,* $\iota'\mathbf{g}$*, are greater than in any other Nash equilibrium of any subgame.* $\{T, \mathbf{g}\}$ *is the unique subgame, undominated-perfect equilibrium outcome in any game in which a Nash equilibrium exists for every* $T' \in \mathbf{R}_+$*.*

Moreover, $\{T, \mathbf{g}\}$*, over-provides the public good.*

The existence of the equilibrium outcome, $\{T, \mathbf{g}\}$, is established by constructing indifference curves for the donors to find their maximum willingness-to-pay for a given level of the public good. With only two donors the equilibrium can be illustrated rather well. Figure 8 illustrates the indifference curves for a two player, symmetric threshold game. The x-axis marks the contributions of donor 1, and the y-axis marks the contributions of donor 2. In the diagram, a fixed level of total contributions would be a line of slope -1.

Each indifference curve marks the set of contributions that make the respective donor indifferent between contributing to the public good and consuming all of her endowment as private good. These are precisely the points where the donors' incentive compatibility constraint binds. A donor considering a contribution of g_i to meet the threshold, T, compares her utility if the threshold is met to her utility from no provision,

$$u_i(m_i - g_i, T) - u_i(m_i, 0) \geq 0. \tag{2.1}$$

When total contributions just meet the threshold, T, then contributing g_i is a best response as long as the left-hand side equation 2.1 is non-negative.

Finding the largest equilibrium threshold is then a matter of moving out from the origin until the indifference curves intersect; marked as $\{T, \mathbf{g}\}$ in figure 8. Our assumptions of continuity and monotonicity ensure that an intersection will occur. At such a point total contributions precisely equal the threshold and each donor is best responding to the threshold and the other contributions. If there are multiple such points (there is only one in figure 8) then the fundraiser selects the one farthest from the origin. They are sufficient, but not necessary conditions for the public good to be over-provided.

Over-provision at the equilibrium outcome, $\{T, \mathbf{g}\}$, results from the fact that donor surplus is fully extracted; each donor is indifferent between contributing and receiving the public good and giving nothing and consuming her endowment. By assumption, the zero provision outcome is not in the core of the game. It follows that each donor prefers any core contribution vector to the zero provision outcome. Hence there is always some positive donor surplus for contributions within the core. Hence T can be increased somewhat beyond any level of contributions in the core by extracting that surplus. Therefore, $T > C_{max}$ and the public good is over-provided in equilibrium.

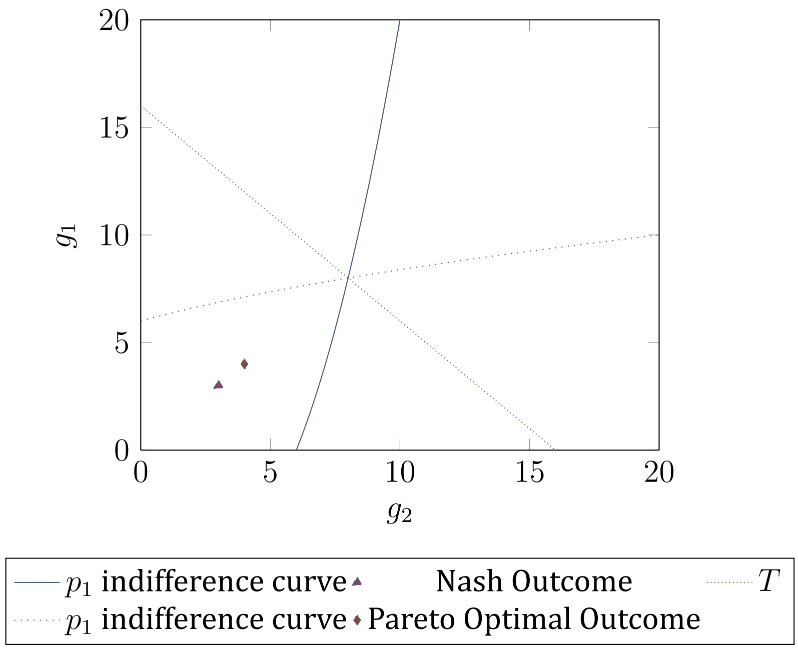
Proving that $\{T, \mathbf{g}\}$ is the unique subgame, undominated-perfect outcome is more subtle. The logic for two donors extends basically unchanged to any finite number of donors. Consider a threshold, $\tilde{T}$, that is just below the equilibrium T, so that the donors strictly prefer meeting the threshold to the no provision outcome.[7] This situation is illustrated in figure 9.

[7] The subgame for this threshold has the same structure as the economy in Bagnoli and Lipman (1989). The logic of the proof for establishing uniqueness in the subgame is the same and readers are encouraged to see their approach.

Any contribution vector that sums to the threshold is a trembling-hand perfect equilibrium outcome. At these points each donor's contribution is a strict best response and so each remains a best response for small enough trembles to other strategies. However, contributions of (0,0) are also trembling-hand perfect equilibrium outcomes. Since neither donor will meet the threshold on her own, contributing zero is a best response if the other donor also contributes zero. Contributing zero is also a strict best response if the other donor contributes more than the threshold. As in that case contributing any positive amount for more of the public good would be outweighed by the costs. Hence trembles that place weight on the other donor contributing in excess of the threshold can support the (0,0) outcome.

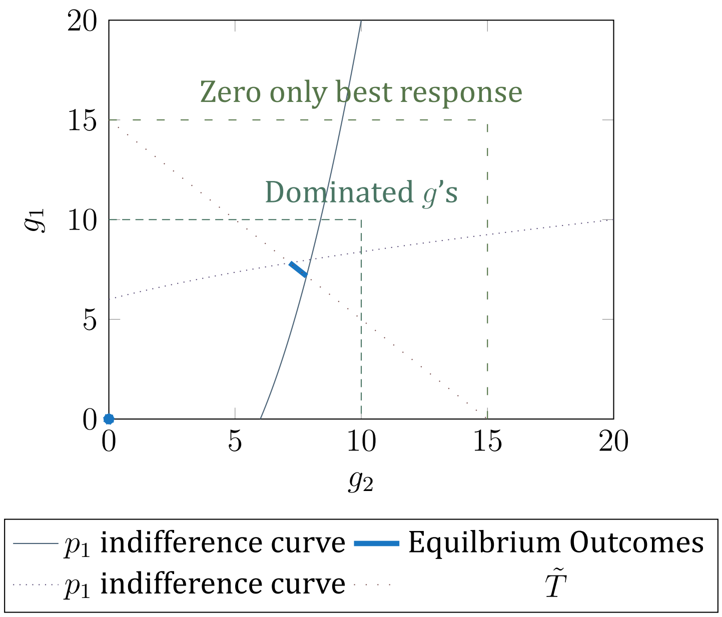
The trembles that support (0,0) as a trembling-hand perfect equilibrium outcome place weight on contributions that are strictly dominated; no donor is ever willing to contribute more than the threshold. If dominated contributions are removed from consideration, the possible outcomes are greatly reduced; as illustrated in figure 9b. On the set of undominated contributions, contributing zero is not a trembling-hand perfect equilibrium outcome. Contributing zero is never a strict best response to undominated contributions. The the donor does strictly better by contributing a positive amount and having a chance of getting more than the outside option $u(m, 0)$.

The above reasoning ensures that thresholds slightly below T will be met in any undominated-perfect equilibrium of the subgame. The uniqueness of the outcome $\{T, \mathbf{g}\}$ comes from the fundraiser maximizing collected contributions. Undominated-perfect equilibrium strategies in the subgame must meet thresholds that can be arbitrarily close to T. As no thresholds larger than T will be met, the only threshold that maximizes contributions is T. The uniqueness of the contribution vector, **g**, comes from extracting the whole surplus from each donor; there can be only one largest compatible contribution from each donor.

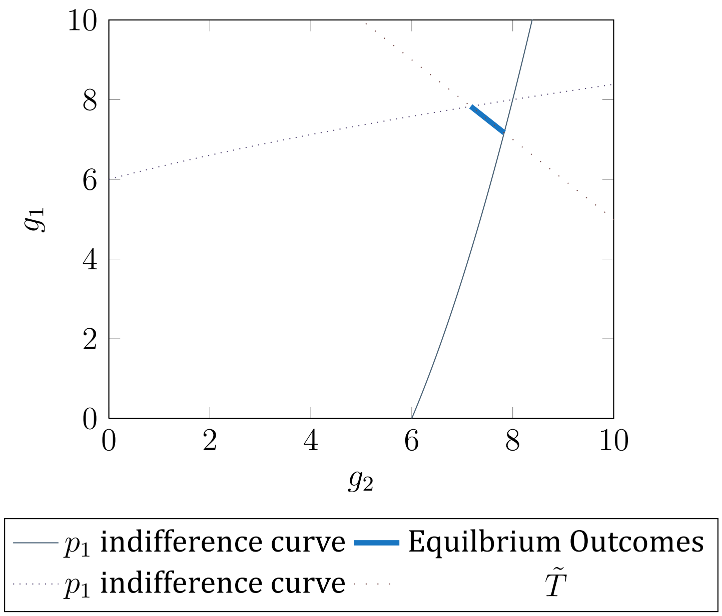


The origin is part of each donor's the indifference set, but is not connected to the indifference curves. The donors strictly prefer to consume some of their endowment as public good so it is only after contributing more than their preferred amount that utility falls back to the level at the origin. Hence, the indifference curves cut the axes away from the origin.

Figure 8: Two donor, symmetric threshold example.



(a) full outcome space



(b) outcome space with dominated strategies removed

Figure 9: Illustration of the removal of dominated contributions.

2.4 WEAKENING THE FUNDRAISER

The over-provision results thus far (proposition 1) hinge to some extent on the assumption of the zero-provision outcome being the alternative for donors. It may be unrealistic to assume that the fundraiser can commit to refusing any subsequent contributions; implying that if the fundraising campaign fails no public good is provided. Suppose instead that the "status quo" is to have the public good provided without active fundraising. The fundraiser sets no threshold, but merely collects contributions that are offered. Then the environment is that of a classic contribution game similar to that studied in Bergstrom et al. (1986).

The anticipated contribution for a simultaneous contribution game is the Nash equilibrium, $\mathbf{g}^\star$. The assumptions we have made thus far ensure that such a Nash equilibrium exists and is unique. Requiring subgame perfection of the outcome of the contribution game means we can fix its outcome to the Nash outcome. Then, given a threshold T, from the donors' perspective the provision of the public good follows,

$$G(\iota'\mathbf{g}, T) = \begin{cases} \iota'\mathbf{g} & \text{if } \iota'\mathbf{g} \geq T \\ \iota'\mathbf{g}^\star & \text{otherwise} \end{cases}.$$

Once a threshold has been selected by the fundraiser, the n donors simultaneously contribute to the public good, choosing $g_i \in [0, m_i]$. If the contributions, $\mathbf{g}$, exceed the threshold T, $\iota'\mathbf{g} \geq T$, then the payoff to the fundraiser is $u_c(\mathbf{g}) = \iota'\mathbf{g}$ and each donor receives a payoff of $u_i(m_i - g_i, \iota'\mathbf{g})$. Otherwise, the fundraiser receives a payoff of $u_c(\mathbf{g}^\star) = \iota'\mathbf{g}^\star$ and each donor receives a payoff of $u_i(m_i - g_i^\star, \iota'\mathbf{g}^\star)$. We call this game the *extended-threshold game*.

Proposition 2 states that the unique equilibrium outcome of an extended-threshold game does not under-provide the public good, i.e., the total collected, $\iota'\mathbf{g}$, is larger than C^{min}.[8] By improving the alternative available to donors, the limits on provision weaken. Over-provision is no longer guaranteed, only that the amount of public good will not be under-provided. Note that this is only a lower-bound on provision. For example, if there exists a Pareto optimal contribution vector then the core of the game reduces to a singleton, the Pareto optimal vector. Then over-provision is guaranteed.

[8] The result extends beyond the Nash alternative outcome, to any alternative that is not in the core of the corresponding threshold game.

Proposition 2. *Let* $X = \left\{c, \{u_j, m_j\}_{j=1}^{n}\right\}$ *be an extended-threshold game. Then there exists an equilibrium outcome,* $\{T, \mathbf{g}\} \in \left\{\mathbf{R}_+, \prod_{i\in I}[0, m_i]\right\}$*, in which total contributions,* $\iota'\mathbf{g}$*, are greater than in any other Nash equilibrium of any subgame.* $\{T, \mathbf{g}\}$ *is the unique subgame, undominated-perfect outcome in any game in which a Nash equilibrium exists for every* $T' \in \mathbf{R}_+$*.*

Moreover, $\{T, \mathbf{g}\}$*, does not under-provide, or over-provides the public good.*

The proof of proposition 2 is quite similar to the proof of proposition 1. The indifference conditions need to be adjusted to match the new alternative outcome,

$$u_i(m_i - g_i, T) - u_i(m_i - g_i^\star, \iota'\mathbf{g}^\star) \geq 0.$$

Then existence and uniqueness are established in the same manner. The result that $\{T, \mathbf{g}\}$, does not under-provide the public good follows from the fact that $\mathbf{g}^\star$ is always weakly inferior to a contribution vector in the core. There always exists a contribution vector in the core of the corresponding threshold game that is preferred to the Nash contribution by every donor. Then it is guaranteed that $\iota'\mathbf{g} > C^{min}$.

2.5 AN EXPERIMENT

While the existence of over-provision in these models relies on simple, robust mechanisms, the over-provision outcome coexists with zero provision outcomes that do not survive the undominated perfect equilibrium refinement. Refinements of this type are in some sense an assumption on behavior. We have assumed that agents prefer strategies that are robust to small errors in others' actions, and that such errors are confined to undominated strategies. It is perfectly rational for an agent who believes that the threshold will not be met to contribute zero, and if such beliefs are held in common they are self-reinforcing. If the refinement is not justified in practice, then the over-provision outcome may not occur or may occur so infrequently as to be safely ignored. The extent to which we can justify refining the set of equilibria and focusing on over-provision is an empirical matter we address with experiments.[9]

[9] The undominated perfect equilibrium refinement was experimentally examined in Bagnoli and Mckee (1991). The authors find strong support for the refinement in their environment; in which the public good

In order to examine whether and how likely over-provision occurs, we implemented an over-provision environment in the laboratory. We use a simple two treatment layout to test the implications of the theory. The comparison is between a control environment with no threshold and a treatment environment with an inefficiently large threshold. A between-subjects, factorial design was used; each subject only participated in one session and only one treatment was run each session.

2.5.1 Payoffs [10]

In order to examine the over-provision setting in the laboratory, it must be possible to clearly observe both Nash play and over-provision (contributions in excess of the social optimum). For this setting, the contributions that maximize total payout to the group of subjects are considered the socially efficient contributions. The payoff function we choose must then have interior Nash and socially optimal contribution levels. We achieve these solutions by using a continuous, piecewise-linear function. Importantly, we introduce concavity into the payoff function by manipulating the return to the private good, not the public good. With no threshold, this results in a dominant strategy to contribute a positive amount.

Subjects were matched into groups of two for each round of play. The individual's marginal cost to contributing to the public good is \$0.10 for the first three units, \$0.70 for units four to seven, and \$1.30 for units eight to twelve. Each unit contributed to the public good results in a \$0.50 return to each group member. The individual's return from contributing is greater than the cost from contributing for the first three units, but costs dominate thereafter. Hence the dominant strategy is to contribute three units. The return to the group for each unit contributed is $2 \times \$0.50 = \$1.00$. Hence the group return to contributions is positive up to the seventh unit. Then the payout maximizing contributions are for each group member to contribute seven units. Consuming all of the endowment as private good gives a payoff of \$4; the payout maximizing outcome is \$7.90.

We selected a sixteen unit threshold; two units above the socially efficient level of con-

comes in a single discrete amount. It has not been examined in our case of continuous provision with inefficiently large thresholds.

[10] Laury and Holt (2008) contains a detailed discussion of payoff functions with interior equilibria in public goods experiments.

tributions, fourteen units. If the threshold level is not achieved, contributions are refunded to the subjects. The refund rule creates a great deal of indifference in the payoffs, creating many equilibria in which the public good is not provided. The equilibria of interest lie along the threshold: (10, 6), (9, 7), (8, 8), (7, 9), and (6, 10). In these equilibria the public good is provided, in fact *over-provided* in the sense contributions exceed the payout maximizing contribution level. The full specification can be found in figure 10.

The payoffs are presented in a payoff table (see figure 24, in section A.5). For simplicity the payoff table only shows the subjects' own payoffs in each cell. [11] As the game is symmetric, the other group members' payoffs can be found by reversing the row and column indexes.

2.5.2 Experimental Procedure

The experiments were conducted at the Pittsburgh Experimental Economics Laboratory at the University of Pittsburgh. Three sessions with a threshold of sixteen and three sessions with no threshold were run with fourteen subjects per session for a total of eighty-four subjects. Fourteen rounds of our voluntary contribution mechanism were played in each session with subjects randomly rematched into groups between each round of play. Three of these rounds were randomly selected for payment; average earnings were twenty-six dollars including a five dollar show up fee.[12] Sessions lasted approximately one hour.

Each session had several steps. First the instructions and payoff table were distributed and read out loud. Then a short quiz on reading the payoff table was given to gauge the subjects' understanding. Once all subjects had completed the quiz, a solution key was distributed. The quiz answers were then read and demonstrated by the experimenters. Once the quiz solutions had been read, a practice version of the software was played. After two rounds of play in the practice software, the subjects began playing the fourteen rounds that counted for payment. Subjects were then paid in private and in cash.

[11] The high amount of equilibrium play seen in our experiment was surprising to us as experimenters. We suspected it may have been partly due to the payoff tables we used which only placed the subjects' own payoff in each cell. In order to check this conjecture we ran another two no threshold sessions with payoff tables that included the other group member's payoff as well. The distribution of contributions from this second wave of sessions appears similar to our earlier data in shape and magnitude (see figure 26). Kolmogorov-Smirov and rank sum tests for distributional differences were insignificant as well with p-values of 0.67 and 0.13 respectively.

[12] Three sessions were selected in order to raise subject earnings.

Let $a, b, \alpha, \beta, \delta, \gamma \in \mathbf{R}_{++}$ such that $\beta > \delta > \gamma$, and $K \in \mathbf{R}$.

Private good payoff

$$s(x_i) \equiv \begin{cases} \beta x_i & : x_i \in [0, a) \\ \delta x_i + (\beta - \delta)a & : x_i \in [a, b) \\ \gamma x_i + (\beta - \delta)a + (\delta - \gamma)b & : x_i \in [b, m_i] \end{cases}$$

Implementation used

$$= \begin{cases} 1.3x_i & : x_i \in [0, 5) \\ 0.7x_i + 2 & : x_i \in [5, 9) \\ 0.1x_i + 4.7 & : x_i \in [9, 12] \end{cases}$$

Full utility with threshold, T

$$u_i(g_i, g_{-i}; T) \equiv \begin{cases} s(w - g_i) + \alpha(g_i + g_{-i}) + K & : g_i + g_{-i} \geq T \\ s(w) + K & : g_i + g_{-i} < T \end{cases}$$

Implementation used

$$= \begin{cases} s(12 - g_i) + 0.5(g_i + g_{-i}) - 5.6 & : g_i + g_{-i} \geq 16 \\ s(12) - 5.6 & : g_i + g_{-i} < 16 \end{cases}$$

Figure 10: Piecewise Linear Utility with Diminishing Marginal Value of the Private Good

The quiz consists of reporting the payoffs earned by the subject and her other group member for several contribution levels above and below the threshold level.[13] The quiz questions were the same for both treatments, though the answers varied according to the underlying payoff function. Outcomes that we anticipated occurring in actual play (Nash outcomes and Pareto outcomes) were not included in the quiz to avoid priming the subjects. After all quizzes were completed a solution key was distributed. The solutions were then worked through on a payoff table, temporarily projected on a white-board for this purpose.

Once the solutions had been worked through, two practice rounds of the software were played. During the rounds subjects did not interact and the rounds did not count towards payment. The other player was fictional and was set to always contribute four units. The subjects were informed of this ahead of time and told that the purpose of the rounds was to familiarize them with the software interface.[14]

After the practice rounds the rounds that counted for payment were played. These rounds proceeded quickly; typically taking less than fifteen minutes. After the rounds had been completed a monitor was selected from among the subjects to select the rounds that counted for payment by drawing numbers from a "hat".

2.5.3 Results

2.5.3.1 No Threshold The Nash prediction for contributions in the no threshold condition is three units. Three is a strictly dominant strategy and is thus a best response to any beliefs about the other contribution in the one period game.[15] The Nash prediction for contributions is operationalized in hypothesis 5.

[13] One consequence of our design choices is that a subject's cost can exceed their endowment (the payoff received from contributing zero to the group account); in effect they borrow against earnings from the group account. While it is impossible to know how many subjects may have been troubled by this, quiz responses indicate that the majority of subjects understood how their contributions would translate into final earnings. Average scores on the quiz were 87% and 98% in each of the treatments with 55% and 88% of the subjects receiving perfect scores. We suspect that part of the discrepancy in scores is due to the fact that in the threshold treatment many of the payoffs are the same, \$4.00. Hence even carelessness in reading the payoff table often results in a correct answer.

[14] The experiment was programmed and conducted with z-Tree software (Fischbacher, 2007).

[15] Though subjects were randomly rematched to minimize any repeated game effects, if subjects believed future contributions could be affected by current contributions the best response may differ from three.

Hypothesis 5. *Mean individual contributions in no threshold treatment are three units, $\mu_{NT} = 3$.*

In order to test hypothesis 5 we form a 95% confidence interval of bootstrapped mean contributions. Bootstrapping the mean allows us to provide a nonparametric confidence interval. We use only contributions from round one in order to provide the most statistically conservative estimates. Round one has both the lowest amount of three unit contributions and only contains behavior that occurred prior to any interaction within the experiment. The bootstrapped mean contribution is 3.76 with a 95% percent confidence interval of $ci = [3.33, 4.29]$. As the interval does not contain 3 we can reject hypothesis 5, that mean individual contributions are three units.

Though hypothesis 5 is rejected, 95% of contributions in round 1 are between 3 and 7 units. These contributions are the Pareto improving contributions. This is consistent with subjects understanding and responding to the payoff structure.

The most striking aspect of the subjects' behavior is the large proportion of equilibrium play in all sessions. Figure 11 illustrates the distribution of actions pooled across all rounds and sessions in the no threshold treatment. Equilibrium contributions constitute 77 percent of all contributions in the no threshold sessions. Though play of the dominant strategy did increase over time to 86 percent, even in the first round 62 percent of actions in the control sessions were at the dominant strategy, a contribution of three units to the public good (see figure 12).

In addition to the high levels of equilibrium play, the typical pattern of contributions seen in linear VCM designs of over contribution in early rounds and significant decreases over time is only weakly matched in our data. A random effects regression of individual contributions in the no threshold treatment against a constant and round does indicate a significant decrease of -0.0214 units per round (see table 5). However, the relative magnitude of the change is small. The estimate amounts to less than one percent of the overall mean contribution per round.

Comparing our design with others in the literature suggests the source of the equilibrium behavior comes from the piecewise-linear payoffs.[16] Payoffs with interior dominant strategy

[16] Bracha et al. (2010) uses a similar payoff function and also shows significant equilibrium play.

equilibria were initially conjectured to induce greater equilibrium play and a number of studies have used quadratic payoffs with interior dominant strategy equilibria (see van Dijk et al. (2002); Falkinger et al. (2000); Gronberg et al. (2009); Keser (1996); Sefton and Steinberg (1996); Willinger and Ziegelmeyer (2001)). However none report equilibrium behavior in line with our findings. We conjecture that the simple presentation the piecewise linear form allows reduces subject confusion.

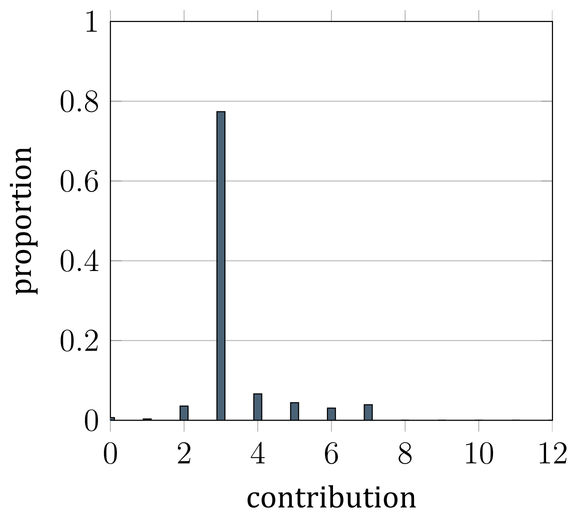


Figure 11: Distribution of individual contributions in no threshold, $T = 0$, treatments

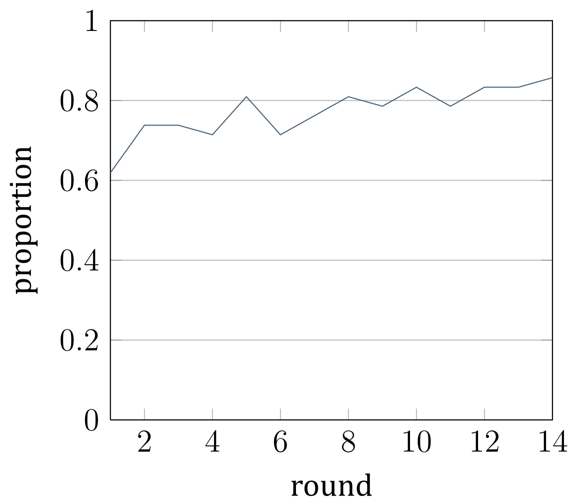


Figure 12: Equilibrium play, contributing 3 units, by round in no threshold treatment.

2.5.3.2 Threshold Nash analysis does not provide a sharp prediction for individual contributions in the presence of a threshold. In our threshold treatment, there are multiple threshold outcomes that theory predicts should survive the equilibrium refinement. However since all the outcomes lie along the threshold, total contributions are predicted to be 16 units. The prediction for total contributions is operationalized in hypothesis 6.

Hypothesis 6. *Mean total contributions in threshold treatment are sixteen units, $\mu_T = 16$.*

In order to test hypothesis 6 we again form a bootstrapped 95% confidence interval. As before we use only contributions from round one in order to provide the most statistically conservative estimates. Round one has both the lowest mean contributions and only contains behavior that occurred prior to any interaction within the experiment. Bootstrapped mean total contributions are 15.30 with a 95% percent confidence interval of $ci = [13.76, 16.48]$. As the interval contains 16 we cannot reject hypothesis 6, that mean total contributions are 16 units.

The distribution of contributions in the threshold treatment is quite concentrated (see figure 13). The most popular contribution, eight, constitutes 78 percent of all contributions. The proportion of 8 unit contributions rises from 48 percent in round one to 95 percent in round fourteen (see figure 14).

The time trend of mean contributions in the threshold treatment is flatter than in the no threshold case and shows no evidence of the decreases found in linear VCM data. A random effects regression of individual contributions in the threshold treatment shows a significant *increase* of 0.17 units per round (see table 5). As in the no threshold case this magnitude is small relative to the mean contribution, amounting to an increase of less than one percent of the overall mean contribution per round.

The overall pattern of outcomes strongly supports the equilibrium refinements used to predict the outcome. The undominated-perfect equilibrium refinement predicts that only outcomes that just meet the threshold should occur in equilibria. Outcomes that exceed the threshold are 71% of outcomes in round 1 and rise in proportion to 95% in round 14.

The stability of individual contributions in the threshold sessions belies the much more dynamic distribution of public good provision. The probability of the public good being provided in the threshold treatment shows an increasing trend across rounds from 62 percent in round one to 95 percent in round fourteen (see figure 15). The threshold itself appears to be quite focal for subjects. As figure 14 illustrates, 86 percent of contributions in round one could have been part of an equilibrium that lies on the threshold. By round three 95 percent of contributions could have been part of an equilibrium on the threshold and that proportion does not fall below 95 percent for the remainder of the rounds (hitting 100 percent in 7 out of 11 rounds). The strong draw of the threshold equilibria does not seem surprising given that it is where the highest payoffs are achieved and that the refund rule makes these relatively high contributions weakly dominant. Bracha et al. (2010) uses an experimental design with piecewise-linear payoffs and also finds a similar pattern of contributions with threshold public goods; a large concentration of contributions among threshold equilibria, and dynamic adjustment within the set. Subjects seem to identify the set of Pareto improving outcomes quite easily, but reaching an outcome within that set is more difficult.

Whatever the cause of the variance in responses in early rounds, the resulting “mismatching” of contributions causes a large proportion of outcomes to fail to meet the threshold and have zero provision of the public good. The probability that the public good is provided is 62 percent in round one, rising to 95 percent in round fourteen. The discontinuous nature of public good production causes mean provision of the public good to be much more varied within and across rounds when compared to the no threshold treatment (see figures 16 and 17).

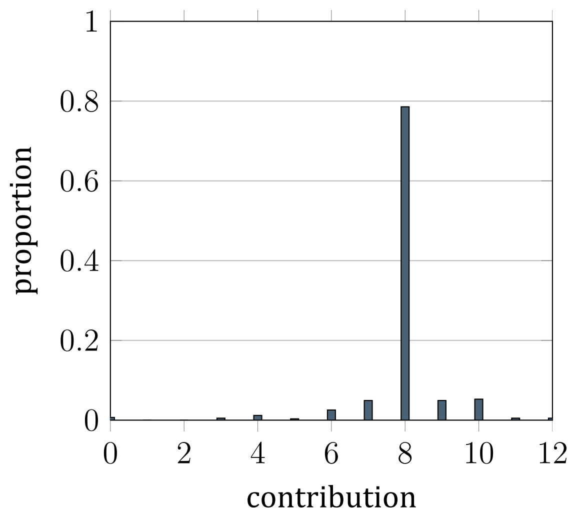


Figure 13: Distribution of individual contributions in threshold, $T = 16$, treatments

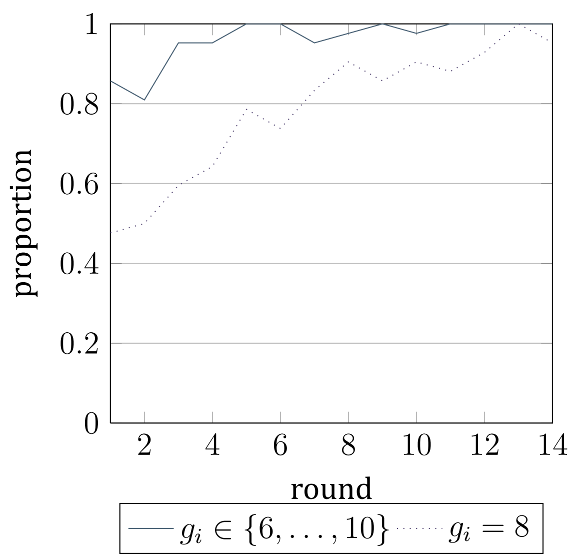


Figure 14: Equilibrium play by round in threshold treatment.

Regressor	No Threshold	Threshold
constant	3.50	6.31
	(0.00)	(0.00)
round	-0.02	0.12
	(0.04)	(0.00)

[a] p values in parenthesis.

Table 5: Random effects of individual contributions

2.5.4 Discussion

The experimental data strongly support the equilibrium refinements used to select the over-provision equilibrium. Total provision in the threshold sessions is statistically indistinguishable from the predicted 16 units. Even in the first round, almost all contributions could be part of a threshold equilibrium, with virtually no contributions at zero. This pattern is consistent with subjects' believing the other group member would contribute a similar moderate amount. A belief that appears to be justified based on the actual contributions.

Though the threshold provides an unambiguous increase in expected contributions, a fundraiser contemplating using one must weigh the greater uncertainty in provision the threshold created. The uncertainty in provision is not just an issue for charities, as individual payoffs for subjects show a significant increasing trend of \$0.12 per round in a random effects regression, see table 6. Relative to the mean round payoff, that amounts to an increase of ≈ 2 percent per round or ≈ 24 percent over all rounds.

It seems reasonable to conjecture that the effect of thresholds could be made more predictable by introducing mechanisms to help solve the coordination problem at the threshold. While such a mechanism may need to be complex in order to handle heterogeneity among donors, there are indications in the data that even simple mechanisms could lead to improvements. A substantial majority of subjects contributing in the Pareto improving set (89 percent in round one) contribute at least eight units. Hence most do not appear to be attempting to get more than 50% of the surplus from crossing the threshold, but find equitable or even somewhat inequitable divisions acceptable.

2.6 CONCLUSION

Within economics, the anticipated outcome of an environment where a public good is paid for through voluntary contributions is one of under-provision of the public good, i.e., larger amounts of the public good would be Pareto improving. However, we have shown that for a large class of voluntary contribution games we should actually anticipate over-provision of

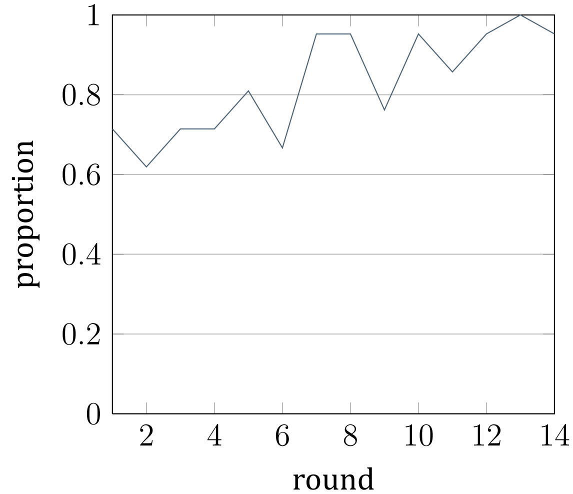


Figure 15: Provision probability by round

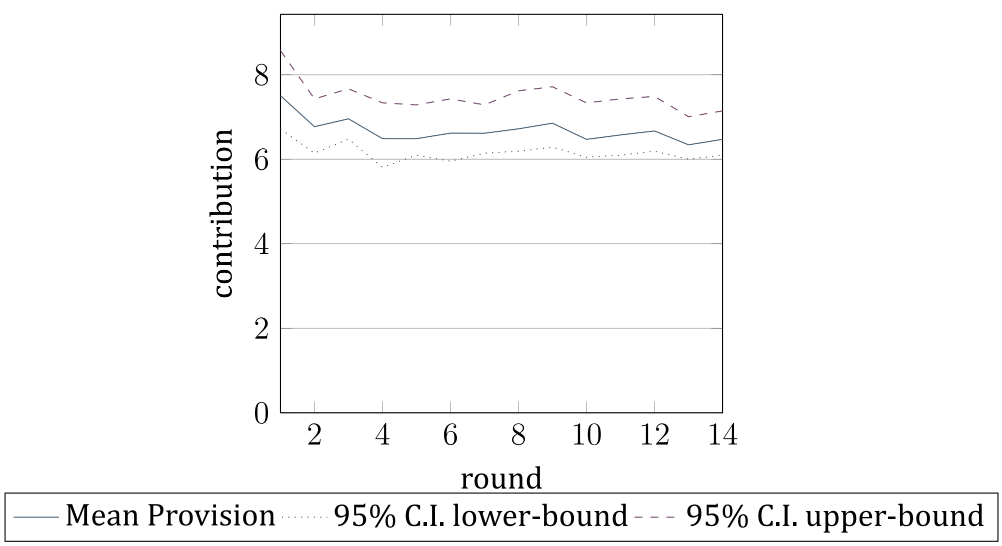


Figure 16: Mean public good provision by round

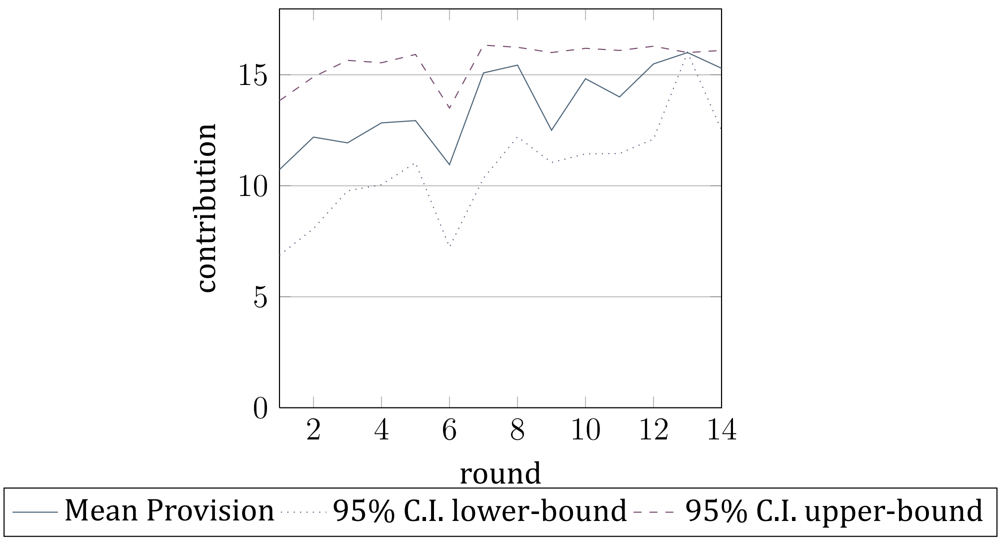


Figure 17: Mean public good provision by round

	No Threshold	Threshold
constant	3.50	7.83
	(0.00)	(0.00)
round	-0.02	0.17
	(0.00)	(0.00)

[a] p values in parenthesis.

Table 6: Random effects regression of individual payoff per round

the public good. As long as the fundraiser can set a threshold for contributions and commit to refunding contributions if the threshold is not met, then over-provision can occur.

Combining contribution refunds with a threshold for total contributions is a robust tool to achieve over-provision. In the laboratory, contributions hew to the theory extremely well, with over-provision of the public good occurring the vast majority of the time. Overall 96 percent of contributions were part of equilibria that secure provision above the Pareto efficient level.

Equilibrium predictions of behavior proved to be quite accurate in both treatment and control sessions of the experiment. The largest deviations from predicted outcomes occur in early rounds of play. The multiplicity of equilibria that secure provision seems to cause coordination problems for subjects, resulting in "mismatched" contributions and no provision of the public good. It seems likely that refined mechanisms that make coordination easier for subjects would greatly reduce these deviations.

The canonical voluntary contribution game and our extensions highlight the inefficiencies from free-riding behavior and the continuous nature of public good provision. However, as we have shown, these can be solved with a refund rule and threshold. Failure to provide the public good in our experiments appeared to be the result of coordination failure and not attempted free-riding. Even in the very simple environment of our experiments, miscoordination greatly reduced provision in early rounds of play. The most effective method of increasing provision of public goods may be to address informational and coordination problems that prevent donors from reaching a goal.

2.7 ACKNOWLEDGMENTS

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3.0 FUNDRAISING GOALS

3.1 INTRODUCTION

When charities and non-profits begin a fundraising campaign they often provide a goal for total donations and an overall objective, e.g., $200 million for a new hospital. These goals are a fairly ubiquitous part of fundraising, but they have not been studied in a game theoretic framework. We develop a model of fundraising in which a fundraiser sets a goal to increase contributions based on the risk attitudes of donors and production of the public good.

Goals play a particularly important role in capital campaigns (fundraising drives for particular capital expenses such as a building). Professional fundraisers suggest the first step in such campaigns should be to conduct a feasibility study to determine how much can be collected for a project (Krueger, 2010). This need for goals to be set with an understanding of possible donations is repeatedly mentioned by fundraising guides, with one fundraiser going so far as to say, "It is vitally important not to let 'the tail wag the dog' ". These statements suggest that goals are a strategic choice by charities. When selecting a goal for a campaign, charities are considering not only the physical production realities, but also the response from donors.

Current economic models of fundraising for discrete or discontinuous public goods allow goals to play a role in coordinating donations. Bagnoli and Lipman (1989) develop a model of discrete public goods in which incrementally increasing goals can achieve efficient provision of the public good. In more recent models, such as Andreoni (1998) or Marx and Matthews (2000), it is reasonable to interpret goals as identifying a point of discontinuity in production of the public good.[1] An announced goal could be used to coordinate donations in order to

[1] Note that these models do not specifically address announced goals. We believe this is the most reasonable

push total contributions beyond the discontinuity.

The role of goals is less clear in models of continuous public goods. Bergstrom et al. (1986) shows that a large class of public goods games have a unique equilibrium; eliminating any role for coordination. If goals for continuous public goods are to be studied as a fundraising tool, an extension to current public goods models is needed.

In order to expand the role of announced goals and the fundraisers who select them, goals need to influence donors' incentives. One possible capability suggested by observation of announcements for capital campaigns is information transmission. A new project announcement with a goal is typically accompanied by a fairly detailed description of the project. For example, the fundraising drive for a new hospital would not be complete without a location, scale model, and allocation of the space between offices, laboratories, and patient rooms.

Providing information about the public good with the goal may have an impact on contributions through an increase in the *tangibility* of the project. Cryder and Loewenstein (2011) provides an overview of how more tangible fundraising appeals generate greater contributions. Appeals that provide greater details on particular recipients of aid, or the impact of the particular contribution, consistently generate greater contributions. One example, provided in Cryder and Loewenstein (2011), is a campaign by Procter and Gamble to raise money toward eliminating newborn tetanus, a leading cause of newborn deaths in developing countries. The campaign was co-marketed with Proctor and Gamble's Pampers brand of disposable diapers. Part of the proceeds from each purchase of Pampers diapers went to providing vaccines to fight newborn tetanus. In South Africa, the campaign used the slogan "1 Pack = 1 Vaccine" and it was one of the most successful in the company's history, as measured by sales and consumer attitudes. A competing campaign launched in other countries with the slogan "1 Pack Will Help Eradicate Newborn Tetanus Globally" was much less successful.

A similar phenomenon is documented by studies examining how goals to motivate employees should be set by business managers. In many cases the impact of a goal on increasing productivity is related to the goal's specificity (Latham and Locke, 1991; Latham and Yukl, 1975). For example, a goal that specifies increasing production to 100 units a week is more effective than a goal of simply increasing production.

model counterpart to goals in these models.

The mechanism through which tangibility increases donations or effort is not entirely understood. In some cases providing information on particular recipients appears to increase demand for the public good by sparking an emotional reaction in the donor. The donor cares more about the recipient after being informed and so receives greater benefit from her donation. Another portion of the increase seems due to an increase in perceived *impact* of the donation. Both in the sense that the marginal productivity of a contribution may be higher, and that the giver's own donation is directly responsible for helping a recipient (Duncan, 2002). The donor does not necessarily care more about the recipient or cause, but she expects donations to be more effective and she benefits by knowing her own donation has helped.

Though the information in a goal announcement can influence giving in a number of ways, in our model the effect of the goal is limited to reducing uncertainty in production of the public good. The goal carries with it some assurance that a particular amount of the public good can be obtained for the goal level of contributions. As a consequence, the model only relies on risk-aversion among donors to motivate an increase in giving. Complementary impacts, like increasing the tangibility of the project, would make announcing goals even more effective as fundraising tools and would not change the model's implications. Risk-aversion can be seen as a proxy for other effects that can be heterogeneous among donors and increase a donor's expected utility if the goal is achieved.

Of course to reduce uncertainty at the goal requires that the public good itself is uncertain. Uncertainty in public goods may arise from the nature of the public good itself, or from the production process the charity employs. For example, consider the construction of a new hospital. Many design factors impact the final costs and benefits of the facility. The building may be located on costly urban land or at a cheaper satellite location. The design may be anything from a bog-standard commercial building to a unique, boldly architected design. From the perspective of donors, these possibilities create uncertainty about the provision of the public good to which they are being asked to contribute. At any given level of donations a donor does not know what trade-offs will be made to complete the building.

Uncertainty in production of the public good creates a unique role for announced goals. When the goal for the new building is announced with a site selected and a scale-model, the uncertainty for donors is reduced if the goal is reached. When contributions lie above or below the announced level, uncertainty will still be higher as donors and even the designers do not know what trade-offs will be made to bring costs into line. For risk-averse donors the reduced uncertainty at the goal brings additional benefits over other contribution levels. In theory, they should be willing to contribute somewhat more than they otherwise would in order to reach the announced goal. In turn, contribution maximizing fundraisers who foresee this advantage can adjust their plans accordingly and announce somewhat larger goals.

We model the goal setting process in a two-stage game in which the fundraiser moves before donors, setting a goal for donations. We dub the game, the *goal game*. With weak assumptions, theory guarantees that subgame-perfect equilibrium contributions increase when charities announce goals.

The greater contributions with an announced goal drive higher equilibrium payoffs for donors and the fundraiser. In order to investigate to what extent these comparative statics depend on the equilibrium assumptions, a laboratory implementation of the experiment is developed. Findings from the laboratory experiment indicate announced goals increase donations and the likelihood of donations reaching the goal level. Reducing the uncertainty at the goal level does not further increase these measures. However, donor behavior does change significantly as coordination on the symmetric goal outcome increases dramatically. Consequently, donors earn significantly more when playing the goal game.

Practical feasibility sharply limits the number of donors who can participate in a single game in the laboratory. We turn to computer simulations in order to better understand the comparative statics of the goal game with a large number of heterogeneously endowed donors. The simulations indicate the increases in total donations are substantial for charities announcing a goal; up to a 73% increase. More dramatically the set of donors increases from less than 1% of the population without a goal, to 100% when a goal is announced. These additional donors drive the increase in donations and provide more than 99% of the total donated when a goal is announced. The additional donations are drawn from all endowment levels, with up to 38% drawn from the two-thirds of donors with the smallest endowments.

In the following section our theoretical model of fundraising is developed. In section 3.3 theoretical results on the equilibrium outcomes are presented. Section 3.4 develops a laboratory implementation of the model and discusses the experimental findings. Numerical simulation results from goal-game implementations with large numbers of heterogeneous donors are presented in section 3.5. Finally, section 3.6 concludes.

3.2 MODEL

Our model can best be understood as a modification of the voluntary contribution game (VCG). There is a single charity which produces a public good through donations of a private good by a finite number of potential donors. The donors simultaneously choose an amount to donate to production of the public good from their endowment of the private good.

The important difference lies in the charity's action space. The charity possesses a technology to reduce the uncertainty of production at any positive level of donations, but *only* at one level.[2] For example, if the charity chooses to reduce the uncertainty at $1,000,000 then for total donations which lie in $[0, 1,000,000)$ or $(1,000,000, \infty)$ production does not benefit from reduced uncertainty. We term this point of reduced uncertainty a "goal". The charity moves before the donors, selecting the goal, which becomes common knowledge for the donors.

After the goal is selected, the donors allocate their endowments between a private and public good. The allocated donations are then used to produce the public good. If the goal has been met, the uncertainty in production of the public good is reduced. Otherwise the uncertainty in production of the public good is higher. The donors then consume their allocations of the private good and the realized provision of the public good.

We call the complete, two-stage game the *goal game*. Often we also need to make reference to the VCG that shares the common components of a goal game: utilities, donors, and production components. We refer to this related game as the *no goal game*.

Before proceeding we need to define some notation used in the remainder of the paper. We often need to sum the elements of vectors, usually donation vectors. We prefer the succinct

[2] Restricting the reduced uncertainty to a point is a technical restriction for locating equilibria. The restriction should not effect the set of outcomes in practical implementations. In equilibrium, the charity is extracting the greatest possible amount of donations from donors. The marginal rate of substitution between the public and private good is less than one for all donors under the non-goal distribution F. Suppose that uncertainty was reduced for all levels of donations greater than the announced goal. Only if the distribution F' is extreme enough to create increasing returns and a marginal rate of substitution greater than one will the equilibrium move beyond the goal. Otherwise, a goal at a single point can achieve the same amount of total donations.

notation, $\iota'\mathbf{d}$, where ι is a vector of ones and $\mathbf{d}$ a vector of donations. ι is not rigorously defined before use, but the context should make clear its length in particular cases.

3.2.1 Timing

The charity takes the first move in the game, selecting a non-negative real number to be the goal. Once the goal has been chosen it becomes public knowledge among the donors.

The donors have the next move. In the same manner as in the VCG, the donors simultaneously decide how to allocate their endowments between private and public consumption. After donations have been selected, the random input is realized (according to the reduced uncertainty distribution only if the goal is met), and production of the public good takes place. Note that each goal leads to a subgame of the goal game.

3.2.2 Agents, Utility, and Production

There are $n + 1$ agents in the model. A set of donors, I, who number $n \in \mathbf{N}$, and a charity, c. The donors, indexed by $i \in I$, are endowed with $m_i \in \mathbf{R}_{++}$ of a private good. Each donor i values the private good and has altruistic preferences over the public good determined by her utility function, $u_i : [0, m_i] \times \mathbf{R} \to \mathbf{R}$. In $u_i(x, G)$, x denotes an amount of the private good and G an amount of the public good. We assume that all donors have strictly concave utility functions as we are interested in an environment with risk averse agents.

The charity seeks to maximize total donations.[3] The public good is produced according to a production function with a random input, $G : \mathbf{R}_+ \times \Omega \to \mathbf{R}$, where $\Omega \subset \mathbf{R}$. In $G(D, \omega), D \in \mathbf{R}_+, \omega \in \Omega$, D denotes total donations of the private good and ω the realized value of the random input. We will assume that G is concave and strictly increasing in both arguments. The random input, $\omega \in \Omega$, has distribution function $F : \Omega \to [0, 1]$, when donations do not match the goal. When donations do match the goal, ω has distribution function $F' : \Omega \to [0, 1]$, which second-order stochastically dominates F.[4]

[3] Were the charity to maximize the public good none of the theoretical conclusions would change. However it should be noted that under concave production the charity would not necessarily be risk neutral.

[4] Importantly for the analysis that follows, we have implicitly assumed in the definitions that the distributions F, F' are independent of the level of production. This is to ensure the properties of the utility functions hold under expectation as well. Note that the implied distribution of the public good can vary with the level

G is a highly simplified stand-in for what are certainly more complicated production processes in the real world. The input ω is intended to fill the role of any uncertainty for the donors. The uncertainty may arise from some aspect of the production process the charity employs; for example, prices of materials, available quantities of materials, or weather. In some cases, ω may be thought of as capturing uncertainty in how concrete interventions translate into a public good that cannot be easily measured. For example, it is reasonable to think that donors care about the overall health of the population and that concern about particular interventions like hospital facilities is driven through this more general desire. [5]

3.2.3 Agent's Decision Problems

The charity must choose the level of private good donations which will face distribution F', $\tilde{D} \in \mathbf{R}_+$, referred to as *the goal*. For each goal, a donor must choose the amount of her private good endowment to donate to the charity for production of the public good.

3.2.3.1 Donors A donor's strategy maps every possible announcement to a donation, $s_i : \mathbf{R}_+ \to [0, m_i]$. Equilibrium conditions require that the strategies dictate best responses. The donor is choosing between donating the best response to others' donations and donating precisely the amount needed to match the goal. If the donation required to meet the goal is close enough to her best response donation, then the reduced uncertainty at the goal makes meeting the goal level the preferred choice.

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of production. For example, if ω models the uncertain price of an input available to a charity then F may be uniform from \$0.50 to \$1.50, $F \equiv U[0.5, 1.5]$. Then, assuming linear production, if total donations are \$10,000 then $G \sim [6666, 20000]$ whereas at total donations of \$100,000, $G \sim [66666, 200000]$. This example meets this assumption. However, if the distribution of prices depends on the amount purchased, i.e., $F(10000) = U[.5, 1.5]$ whereas $F(100000) = U[.75, 1.25]$, then this assumption is violated.

The assumption of independence is stronger than necessary for the results that follow. If the properties we assume of the utility functions hold under expectation with a dependent distribution the results still hold. For example some dependence on the level of production near zero contributions would be quite natural to model uncertainty associated with production. If the project cannot cause actual losses to the public good, some truncation of the random variable near the origin may be needed. As long as the truncated region lies below the outcome in the no goal game then the results will be unaffected.

[5] The goal of these production choices is to allow as broad a model of uncertain production as possible, while admitting the subtlest possible action by the charity. Hence second-order stochastic domination is the weakest requirement to ensure that risk averse donors prefer F' to F. The results that follow would still hold as long as donors strictly prefer F' to F. For example, if F' first-order stochastically dominates F, the results still hold.

Nothing is lost mathematically by "absorbing" the production and uncertainty into the utility function. In order to simplify notation, define

$$v_i(m_i - d_i, \tilde{D}; Z) \equiv \mathrm{E}_Z\, u_i(m_i - d_i, G(\tilde{D}, \epsilon)),$$

where E_Z denotes taking the expectation with respect to distribution $Z \in \{F, F'\}$.[6] Then given a goal of $\tilde{D} \in \mathbf{R}_+$ and donations from others, $\mathbf{d}_{-i}$, donor i's payoffs are

$$\pi(d_i, \tilde{D}, \mathbf{d}_{-i}) = \begin{cases} v_i(m_i - d_i, \tilde{D}; F') & \text{if } d_i = \tilde{D} - \left(\tilde{D} - \iota'\mathbf{d}_{-i}\right) \\ v_i(m_i - d_i, d_i + \iota'\mathbf{d}_{-i}; F) & \text{if } d_i \neq \tilde{D} - \left(\tilde{D} - \iota'\mathbf{d}_{-i}\right) \end{cases}.$$

Her best response solves

$$\max_{d_i \in [0, m_i]} \pi(d_i, \tilde{D}, \mathbf{d}_{-i}).$$

Let us take the opportunity to define a value function for the no-goal game,

$$w_i(\iota'\mathbf{d}_{-i}) \equiv \max_{d_i \in [0, m_i]} v_i(m_i - d_i, \iota'\mathbf{d}_{-i} + d; F).$$

3.2.3.2 The Charity The charity seeks to maximize total donations. Given strategies for each donor, s_i, the best response(s) of the charity solves

$$\max_{\tilde{D} \in \mathbf{R}_+} \sum_{i \in I} s_i(\tilde{D}).$$

The solutions we seek are subgame perfect equilibria. An equilibrium consists of a choice of goal by the charity and donor strategies, $\left(\tilde{D}, \mathbf{s}\right)$, such that every donor is best responding to other donations and the goal and that the charity is maximizing donations given the donor strategies. Subgame perfection further requires that $s_i(\tilde{D})$ solves the donor's problem for all $\tilde{D}$, even for goals not at the equilibrium.

[6] See subsection A.6.3 for a proof that v_i satisfies the same properties as u_i: is strictly increasing in both arguments, is strictly concave, and has a continuous maximum.

3.3 EQUILIBRIUM OUTCOMES

The focus of this section is on characterizing total donations in subgame perfect equilibria of the goal game; as opposed to equilibrium strategies. For simplicity we only examine pure-strategy, equilibrium outcomes. In other words, we only consider donor strategies where a goal is mapped to a single donation vector, as opposed to a random distribution over donation vectors.[7] Much of the analysis is fairly mathematical, the proofs and intermediate results have been placed in subsection A.6.2. In this section we state the main result as well as the intuition behind the proofs.

Proposition 3 is the main result of this section. It establishes the existence of subgame perfect total donations for the goal game that lie above the set of total donations collected in a Nash equilibrium of the no-goal game.

Proposition 3. *Let* $\left\{c, F, F', G, \{i_j, u_j, m_j\}_{j=1}^n\right\}$ *be a goal game. Then there exists at least one pure strategy, subgame perfect equilibrium,* $(\tilde{D}, \mathbf{d}) \in \mathbf{R}_+ \times \prod_{i\in I}[0, m_i]$*. Moreover, there exists an equilibrium of the no-goal game,* $\mathbf{d}^\star \in \prod_{i\in I}[0, m_i]$*, and* $\epsilon > 0$ *such that total contributions in every subgame perfect equilibrium are greater than* $\sum_{i=1}^n d_i^\star + \epsilon$*.*

As the goal game is a two stage game and we are imposing subgame perfection, our approach to proving proposition 3 uses backward induction. Given a goal, donors must reach a Nash equilibrium vector of donations in the subgame. These are the only donation vectors the charity considers when determining the goal to announce. Hence, the first steps in locating equilibria concern finding equilibrium total donations in the subgame for a given goal.

Identifying equilibrium total donations can be simplified by noting that incentives only differ from the no-goal game if total donations equal the announced goal. Hence, total donations in an equilibrium of the subgame either total to the announced goal or match an equilibrium of the no-goal game. Moreover, it is simple to check that any equilibrium of the no-goal game, $\mathbf{d}^\star$, is also equilibrium of the subgame when the announced goal coincides with the total donations, $\iota'\mathbf{d}^\star$. Therefore, the set of equilibrium total donations that match the announced

[7] Mixed-strategy equilibria must be discrete distributions over pure-strategy equilibrium vectors that meet the announced goal and a vector of best responses that do not match the goal. Realized donation vectors are bounded above by the largest elements of these vectors and below by the smallest elements of these donation vectors.

goal is the same as the unconstrained set of equilibrium total donations. We are free to impose the constraint that total donations equal the announced goal without effecting the results.

In the subgame, the choice of a donor when in an equilibrium is

Meet the goal	Or	Deviate to outside
$v_i(m_i - d_i, \tilde{D}; F')$		$w_i\left(\sum_{i=1}^{n} d_i\right)$

By imposing the constraint that total donations equal the announced goal, the choice of a donor in an equilibrium of the subgame can be rewritten.

Meet the goal	Or	Deviate to outside
$v_i(m_i - d_i, \tilde{D}; F')$		$w_i(\tilde{D} - d_i)$

Hence for any potential goal, the set of incentive compatible donations (that could form an equilibrium) can be found for each donor *independently*. If it happens that there exist incentive compatible donations that sum to the goal then those donations and goal are a potential equilibrium. This problem can be framed as a fixed-point problem and its solutions characterized. Note that many of these solutions require multiple donors to simultaneously increase donations. In these cases no single donor prefers reaching the goal on her own, and a Nash equilibrium in the no-goal game is still an equilibrium.

Subgame perfect strategies for the donors must dictate Nash equilibria in the subgame for every goal. Hence for every goal, $\tilde{D}$, donor's strategies will lead to contributions that meet the goal or a Nash equilibrium of the no-goal game, $\iota'\mathbf{s}(\tilde{D}) = \tilde{D}$ or $\mathbf{s}(\tilde{D}) = d^\star$, where $d^\star$ is a Nash equilibrium of the no-goal game.

Consider the case when the no-goal game has only one equilibria, $\mathbf{d}^\star \in \prod_{i\in I}[0, m_i]$. The charity will not announce any goals below $\iota'\mathbf{d}^\star$ as announcing an impossibly large goal will collect at least as much in donations. Additionally, there is some region around $\iota'\mathbf{d}^\star$ where all subgame perfect strategies *must* meet the goal. As donors are risk averse, goals close enough to $\iota'\mathbf{d}^\star$ are strictly preferred by at least one donor. Consequently, donors will always meet these goals rather than stay at $\iota'\mathbf{d}^\star$. No outcome strictly inside this region can be subgame perfect, as the charity could deviate by announcing a slightly larger goal that must be met by any subgame perfect strategies. Therefore total donations collected in any subgame perfect equilibrium are at least $\epsilon > 0$ more than $\iota'\mathbf{d}^\star$.

A short example using quadratic quasi-linear utility illustrates the process of locating an equilibrium outcome in a goal game. For simplicity, we will focus on interior equilibria in which every donor is contributing a positive amount. As mentioned, suppose n donors have quadratic quasi-linear utility with a bliss point at K units of the public good and a marginal return to private consumption of $\alpha > 0$, $u(m-d, G) = \alpha(m-d) - (K-G)^2$.[8] The public good is linear in donations and the uncertainty enters additively, $G(D, \omega) = D + \omega$. For the typical, non-goal, distribution of uncertainty, F, assume that the expected value is zero and that the second moment exists. The goal distribution of uncertainty, F', is actually just certainty, placing all the weight on the expected value, zero.

With these components specified, we can fold them together and write utility representations for a donor on and off a goal, $v(m-d_i, \tilde{D}; F') = \alpha(m-d_i) - (K-\tilde{D})^2$ and $v(m-d_i, \iota'\mathbf{d}_{-i}; F) = \alpha(m-d_i) - \mathrm{E}(K - \iota'\mathbf{d}_{-i} - d_i - \omega)^2$.

In order to solve the goal game, we first need to solve for the donor best response and value functions in the no-goal game. These can be found readily using standard techniques.

$$d_i^\star = K - \iota'\mathbf{d}_{-i} - \frac{\alpha}{2} \qquad\qquad w(\iota'\mathbf{d}_{-i}) = \alpha\left[m - K + \iota'\mathbf{d}_{-i}\right] + \frac{\alpha^2}{4} + \mathrm{E}\,\omega^2$$

The key equation to locate a goal game equilibrium is the incentive compatibility constraint for donors,

$$ICC(\tilde{D}, d_i, \iota'\mathbf{d}_{-i}) = v(m-d_i, \tilde{D}; F') - w(\iota'\mathbf{d}_{-i}) \geq 0.$$

The constraint compares the utility of meeting goal $\tilde{D}$ by contributing d_i, to contributing the best response to outside donations, $d_i^\star$ and not meeting the goal. Since it is safe to assume the announced goal is met in subgame perfect equilibria, $\tilde{D} = \iota'\mathbf{d}_{-i} + d_i$, we can rewrite the constraint as, $ICC(\tilde{D}, d) = v(m-d_i, \tilde{D}; F') - w(\tilde{D} - d_i)$. After algebraic simplification, this is equivalent to, $ICC(\tilde{D}, d) = -\tilde{D}^2 + (2K-\alpha)\tilde{D} - K^2 + \alpha K + \mathrm{E}\,\omega^2 - \frac{\alpha^2}{4}$.

[8] Note that the marginal utility of the public good approaches zero as $G \to K$. Since the marginal utility of the private good is greater than zero, K is beyond any Pareto efficient or Nash outcome of the no-goal game.

The incentive compatibility constraint can be solved for the maximum total donations that can be collected in a subgame perfect outcome, $D^{max} = K - \frac{\alpha}{2} + \sqrt{\mathrm{E}\,\omega^2}$. We can also solve the best response to find the outcome without a goal, $\iota'\mathbf{d} = K - \frac{\alpha}{2}$. Here the best possible gain to a charity using a goal, $\sqrt{\mathrm{E}\,\omega^2}$, is precisely the standard deviation of the uncertainty in production. For comparison, the Pareto optimal level of donations is $K - \frac{\alpha}{2n}$. Hence, if $\sqrt{\mathrm{E}\,\omega^2} \geq \frac{\alpha}{2}\left(\frac{n-1}{n}\right)$ the Pareto optimal allocation can be achieved.

As this example demonstrates, total donations increase when charities employ goals. The theory underlying this prediction relies on behavioral assumptions that may not be met in practice. In order investigate the behavioral aspects of the model, section 3.4 develops a laboratory implementation in order to determine if behavior is consistent with the predictions of the theory.

3.4 EXPERIMENT

Theory implies that more donations are collected in every subgame perfect equilibrium outcome of any goal game. These equilibrium outcomes rely heavily on the assumption of complete and common knowledge about the environment. Consider the situation a donor faces after an equilibrium goal has been announced. Reaching the goal will be Pareto improving, hence as long as she believes with a high enough probability that the goal will be reached, she will increase her donation to achieve it. However, if she doubts that the goal will be reached, she will not base her donation on achieving the goal and as a result will donate less.

The sensitivity of the outcome to beliefs suggests that equilibrium outcomes may require time to occur as beliefs about the strategic environment must converge first.[9] Instead of focusing directly on equilibirum outcomes, the focus of our analysis is on the comparative statics of the model. The model predicts that if a goal is announced that reduces the uncertainty of production, total donations increase, the probability of reaching the goal level increases, and

[9] Seely et al. (2005) reports on a modified voluntary contributions game with similar a equilibria structure. In their environment, one equilibrium entails high donations a common belief all donors are increasing donations, while another has small donations and the mutual belief other donors are also donating small amounts. The authors report that equilbrium outcomes occurred in a small minority of the cases and overall treatment effects were weak.

donor payoffs increase compared to the case without a goal.[10] These outcomes are of practical importance to fundraisers and donors regardless of whether behavior converges to an equilibrium outcome.

In order to test these implications, two laboratory treatments are needed. One treatment is a voluntary contribution game with an uncertain public good; the no-goal treatment. The second treatment implements an environment with a recommended level for total donations and a reduction in the uncertainty of production at the recommended level; the goal-with-reduced-uncertainty treatment. Comparing the total donations and probability of reaching the goal from these treatments can identify the presence and magnitude of the comparative statics.

The goal-with-reduced-uncertainty treatment changes two aspects of the environment from the no-goal treatment. A goal is announced and the uncertainty at the goal is reduced. While theory would predict no effect from a goal with no payoff consequences, experimenter demand or moral persuasion may generate an impact from the goal regardless of the reduction in uncertainty (Croson and Shang, 2008).[11] Distinguishing the effect of an announcement with no payoff consequences from the effect of reducing uncertainty requires a third treatment; the pure-goal treatment. The difference between the pure-goal treatment and the goal-with-reduced-uncertainty treatment can aid in isolating the impacts from reducing uncertainty and other mechanisms.

3.4.1 Design

In every treatment, subjects are organized into groups of two, and are randomly rematched each round of play for thirty rounds. Subjects begin each round endowed with \$7 which they may choose to "invest" in a group account at a cost of \$1.00 per unit. As required by theory, investment in the group account is uncertain. The uncertainty in investments is introduced as a binary lottery. When it will not result in negative amounts of the public good, a nega-

[10] Theoretically if a goal is announced, the goal is reached with certainty. If a no goal is announced, the goal is never met. Measuring the probability of reaching the goal allows for detecting smaller changes in likelihood.

[11] Our announced goals may be more properly characterized as "recommended play", though typically recommendations are directed at individuals not groups. The connection to this literature is discussed in with our findings in subsubsection 3.4.2.2.

tive realization of the lottery results in three units being removed from the group account. A positive realization results in three units being added to the group account. However if the amount in the group account is less than three units, a negative realization will result in negative amounts of the public good. In this region, the uncertainty is modified such that a negative realization results in the group account being reduced to zero. A positive realization doubles the amount in the group account. Thus, while it is costly to invest, at most the investment can be lost.

The payoffs in all treatments are a piecewise-linear function; calculated after the random adjustment is applied. For the first seven units in the group account every group member earns \$1.20 per unit. For units above seven, every group member earns \$0.40 per unit.

When taken together, the uncertainty and payoff functions interact such that in expectation, the payoffs in all treatments are piecewise-linear functions with interior Nash and Pareto optimal outcomes. In expectation, the first four units in the group account earn every group member \$1.20 per unit. For units between four and ten, each group member earns \$0.80 per unit. For any units above ten, each group member earns \$0.40 per unit.

In the treatment without a goal, the payoffs are as described above. Every subject is given a payoff table (*Earnings Table*) depicting the expected earnings from any investment combination and the difference between the expected earnings and the actual realization in any round. In the pure-goal treatment, the payoffs are identical, but the instructions include a recommendation to invest a total of eight units in the group account. Additionally, the diagonal of the payoff table where investments sum to eight units is shaded. The goal-with-reduced-uncertainty treatment adds to the pure-goal treatment by making the return at the recommended investment of eight units certain and modifying the uncertainty for investments beyond the goal. For investments from eight to ten units, the uncertainty is adjusted in a manner similar to that near zero investment. A negative realization of the lottery results in the amount in the group account being reduced to the recommended level of eight units. A positive realization of the lottery results in the "excess" amount, the units above the goal, being doubled and added to the group account. This structure ensures that any investment at least as large as the recommended level, ensures the group account will have at least the recommended level when payoffs are calculated. Table 7 summarizes the design choices for the experiment.

While the design does not have a unique Nash equilibrium, total donations are unique for risk averse subjects. In the treatments without a reduction in uncertainty at the announced goal, donations to the public good are predicted to total four units. In the treatment with a reduction in uncertainty, donations to the public good are predicted to total eight units, the recommended level. One caveat is that the Nash equilibrium predictions do not consider uncertainty over others' contributions, strategic uncertainty; just the uncertainty from the binary lottery we impose, environmental uncertainty. As uncertainty about others' actions almost certainly occurs in the laboratory, we focus on the comparative static implications of the Nash predictions.

Production	$G(D,\omega) = \begin{cases} \text{min}\,\{\omega D, D+3\} & : \omega > 0 \\ \text{max}\,\{0, D-3\} & : \omega <= 0 \end{cases}$
Uncertainty	$\omega = \begin{cases} 0 & \text{with probability } \frac{1}{2} \\ 2 & \text{with probability } \frac{1}{2} \end{cases}$
Endowments	$m = \$7$
Group Size	$N = 2$
Payoffs	$\pi(x, G) \equiv s(G) + x$, $s(G) \equiv \begin{cases} 1.2G & : G \in [0,7] \\ 0.4G + 5.6 & : G \in (7,14] \end{cases}$

Table 7: Experimental Design

In order to collect information on the risk preferences of the subjects in our sessions, a risk assessment based on that of Holt and Laury (2002) was administered in all sessions. The risk assessment was given after the contribution game and subjects had no advance knowledge it would take place. Subjects were asked to choose between binary lotteries of \$2.00 or \$1.60 and \$3.85 and \$0.10.[12] The probability of the first amount occurring varied from zero to one

[12] These are the same amounts as the low payoff treatment in Holt and Laury (2002).

hundred percent. Eleven such choices were presented at once and subjects were asked to choose a probability to switch from the \$2.00/\$1.60 lottery to the \$3.85/\$0.10 lottery. The interface allowed them to change their decision as many times as they desired.

A total of nine sessions were conducted at the Pittsburgh Experimental Economics Laboratory. In total, 124 subjects were recruited from the subject pool at the Pittsburgh Experimental Economics Laboratory. It is a between-subjects design, each subject participated in one treatment. Subjects earned an average of \$18 with a guaranteed \$6 show-up fee. The experiment was conducted through software written in z-Tree (Fischbacher, 2007).

Each session lasted about one hour. At the start of each session, instructions were read aloud and questions were solicited from the subjects. Thirty rounds of the appropriate treatment were then played in the experimental software. Once the thirty rounds had been completed, a second set of instructions were handed out for a version of the risk assessment mechanism in Holt and Laury (2002). After the risk assessment was completed, subjects were paid in private and in cash, their earnings from the initial treatment, the risk assessment, and the show up fee.

3.4.2 Findings

In examining the comparative statics of the goal game we measure mean donations, the likelihood of reaching the goal level of eights units in donations, and mean donor earnings. These measures show a significant trend across rounds.[13] Most of the time trend is due to changes in the no-goal treatment. In the no-goal treatment mean donations fall about one unit over the thirty rounds. The results presented here use both the complete dataset as well as breaking out the last ten of the thirty rounds of data for analysis, presuming these rounds better match the behavior of experienced agents.

Random-effects regressions are used to establish the results in order to take into account the repeated actions of individuals. Probit regressions with random-effects errors are used for the results on likelihood. Our results are also not qualitatively changed by controlling for risk preferences using the collected measure. For simplicity, this control is left out of the

[13] See table 12 for regression results concerning the trend across time.

presented regressions.

In the next subsection (subsubsection 3.4.2.1), the results from reducing uncertainty of the public good at the goal level are compared to the results in the treatment without an announced goal. In subsubsection 3.4.2.2, the results from announcing a goal for total donations and reducing uncertainty of the public good at the goal level are compared to the results from announcing a goal with no payoff implications.

3.4.2.1 Introducing a Goal and Reduction in Uncertainty Introducing a goal with a reduction in uncertainty is predicted to increase all of the measures under examination. These predictions are operationalized in hypotheses 7, 8, and 9.

Hypothesis 7. *The mean donation to the public good in the no-goal treatment, μ_{ng}, is less than the mean donation in the goal-with-reduced-uncertainty treatment, μ_g.*

Hypothesis 8. *The likelihood of donations reaching the goal level of eight units ($d_1 + d_2 \geq 8$) in the no-goal treatment, p_{ng}, is less than the likelihood of reaching the goal level in the goal-with-reduced-uncertainty treatment, p_g.*

Hypothesis 9. *Mean donor earnings in the no-goal treatment, π_{ng}, are less than mean donor earnings in the goal-with-reduced-uncertainty treatment, π_g.*

The data do support hypotheses 7, 8, and 9.[14] All measures are significantly greater in the goal-with-uncertainty-reduction treatment than in the no-goal treatment. The gaps expand in the latter half of the experiment due to a stronger decrease in the no-goal treatment. Table 8 reports the results of random-effects regressions on the measures. Mean donations increase from 2.371 in the no-goal treatment to 2.950 in the goal-with-reduced-uncertainty treatment. Coefficient tests indicate these means are significantly different from the Nash predictions of two and four. However, donations in the no-goal treatment are only marginally different from the Nash prediction with a p-value of 0.076. Mean donations are 0.579 units higher in the goal-with-reduced-uncertainty treatment than the no-goal treatment. In relative terms, that is a 24 percent increase in mean donations over the treatment without a goal. As the coefficient is significantly different from zero with a p-value of 0.037, we can accept hypothesis 7.

[14] Session level, Wilcoxon-ranksum tests also support hypothesis 7. Mean donations are higher in the goal-with-reduced-uncertainty treatment with a p-value of 0.05.

	All Rounds	Last 10
Goal w/ Reduction	0.369	0.579
	(0.104)	(0.037)
Constant (No Goal)	2.744	2.371
	(0.000)	(0.000)
Total No. of Obs.	2460	820
Total No. of Agents	82	82

(a) Mean donations.

	All Rounds	Last 10
Goal w/ Reduction	0.267	0.497
	(0.044)	(0.003)
Constant (No Goal)	-0.755	-1.163
	(0.000)	(0.000)
Total No. of Obs.	2460	820
Total No. of Agents	82	82

(b) Likelihood of reaching the goal $(d_1 + d_2 \geq 8)$.

	All Rounds	Last 10
Goal w/ Reduction	0.540	1.680
	(0.000)	(0.000)
Constant (No Goal)	10.225	9.347
	(0.000)	(0.000)
Total No. of Obs.	2460	820
Total No. of Agents	82	82

(c) Donor earnings.

Table 8: Regression results comparing the no-goal and goal-with-reduced-uncertainty treatments (p-values reported in parentheses).

The likelihood of reaching the goal is far from the predicted levels of 0 and 1. The likelihood is 0.238 without an announced goal and 0.339 with a goal and reduction in uncertainty. Though the absolute change in magnitude is about 10%, in relative terms, that is a 43 percent increase in likelihood in the goal-with-reduced-uncertainty treatment. As the coefficient on the dummy variable for the goal-with-reduced-uncertainty treatment is significantly different from zero with a p-value of 0.003, we can accept hypothesis 8.

Donor earnings increase significantly in the goal-with-reduced-uncertainty treatment from \$9.35 to \$11.03 in the last ten rounds. The coefficient of the dummy variable for the goal-with-reduced-uncertainty treatment is significantly different from zero with a p-value of 0.000, thus we can accept hypothesis 9.

The data support the prediction that the introduction of a goal with reduced uncertainty increases donations and the likelihood of donations reaching the level of the announced goal. However, two things changed between these treatments: a goal was announced and the uncertainty in production was reduced at the goal level. In the next section the goal-with-reduced-uncertainty treatment is compared to the pure-goal treatment.

3.4.2.2 Introducing a Reduction in Uncertainty Though a goal with an associated reduction in uncertainty of production generates significant changes in donation behavior, it is not clear what changes are due to the reduction in uncertainty and what changes are due to the announcement of a goal. In order to distinguish these, our design incorporates a treatment in which a goal for total donations is announced that has no payoff consequences. Introducing a goal with a reduction in uncertainty is predicted to increase donations and increase the probability of reaching the goal level of donations. These predictions are operationalized in hypotheses 10, 11, and 12.

Hypothesis 10. *The mean donation to the public good in the pure-goal treatment, μ_{nr}, is less than the mean donation in the goal-with-reduced-uncertainty treatment, μ_g.*

Hypothesis 11. *The likelihood of reaching the goal level of eight units ($d_1 + d_2 \geq 8$) in the pure-goal treatment, p_{nr}, is less than likelihood of reaching the goal level in the goal-with-reduced-uncertainty treatment, p_g.*

Hypothesis 12. *Mean donor earnings in the no-goal treatment, π_{nr}, are less than mean donor earnings in the goal-with-reduced-uncertainty treatment, π_g.*

Somewhat surprisingly, the data do not support hypotheses 10 and 11. There is little separation of the measures over the thirty rounds, with the lines crossing multiple times. Table 9 reports results of the random-effects regressions. The random-effects estimate of the difference in mean donations between the treatments is a statistically insignificant -0.029 units; from 2.979 in the pure-goal treatment to 2.950 in the goal-with-reduced-uncertainty treatment. The coefficient is not significantly different from zero with a p-value of 0.929 and we cannot reject that $\mu_{nr} = \mu_g$. Similarly, the random-effects, probit estimated likelihood of reaching the goal level is 0.361 with an announced goal and no reduction in uncertainty, and 0.326 with a goal and reduction in uncertainty. The associated probit coefficient is not significantly different from zero with a p-value of 0.472. Hence we cannot reject that $p_{nr} = p_g$.

The data do support hypothesis 12. Earnings increase significantly with uncertainty reduced at the goal level from \$9.53 to \$11.03. Over all rounds, the increase with a reduction in uncertainty is estimated to be even larger than the increase over the no-goal treatment. As the coefficient on the dummy variable for the goal-with-reduced-uncertainty treatment is significantly different from zero with a p-value of 0.000, we can accept hypothesis 12.

The three outcome measures produce a puzzling picture of the treatment effects. Mean donations and the likelihood of provision are not significantly different under a reduction in uncertainty, suggesting that the announcement is driving the effect. Whereas, donor earnings are only significantly different under a reduction in uncertainty, suggesting that the uncertainty reduction is driving the effect. A more detailed examination of the underlying distribution of donations reveals the cause of these differing outcomes.

Figure 18a shows the distribution of donations in the treatments with and without a reduction in production uncertainty at the goal level. Reducing the uncertainty in production appears to significantly increase the proportion of donations at four units, half the announced goal. As shown in Figure 18b, the difference in four unit donations is maintained for almost all of the thirty rounds of play. The plot of the proportion of four unit contributions in the goal-with-reduced-uncertainty treatment remains above the plot of the proportion of four unit contributions in the pure-goal treatment and they never cross. Table 10 reports the results of

	All Rounds	Last 10
Goal w/ Reduction	0.030	-0.286
	(0.919)	(0.929)
Constant (Goal w/o Reduction)	3.083	2.979
	(0.000)	(0.000)
Total No. of Obs.	2460	820
Total No. of Agents	82	82

(a) Mean donations.

	All Rounds	Last 10
Goal w/ Reduction	0.001	-0.155
	(0.995)	(0.472)
Constant (Goal w/o Reduction)	-0.517	-0.570
	(0.000)	(0.000)
Total No. of Obs.	2460	820
Total No. of Agents	82	82

(b) Likelihood of reaching the goal $(d_1 + d_2 \geq 8)$.

	All Rounds	Last 10
Goal w/ Reduction	0.907	1.498
	(0.000)	(0.000)
Constant (Goal w/o Reduction)	9.858	9.526
	(0.000)	(0.000)
Total No. of Obs.	2460	820
Total No. of Agents	82	82

(c) Donor earnings.

Table 9: Regression results comparing the pure-goal and goal-with-reduced-uncertainty treatments (p-values reported in parentheses).

a random-effects, probit regression on the probability of donating four units. The regression estimate of the difference in likelihood of donations of four units is 0.303 from 0.183 in the treatment without a reduction in uncertainty to 0.485 in the treatment with a reduction in uncertainty. As the p-value on the coefficient of I_g is 0.007, we can accept that the difference is statistically significant.

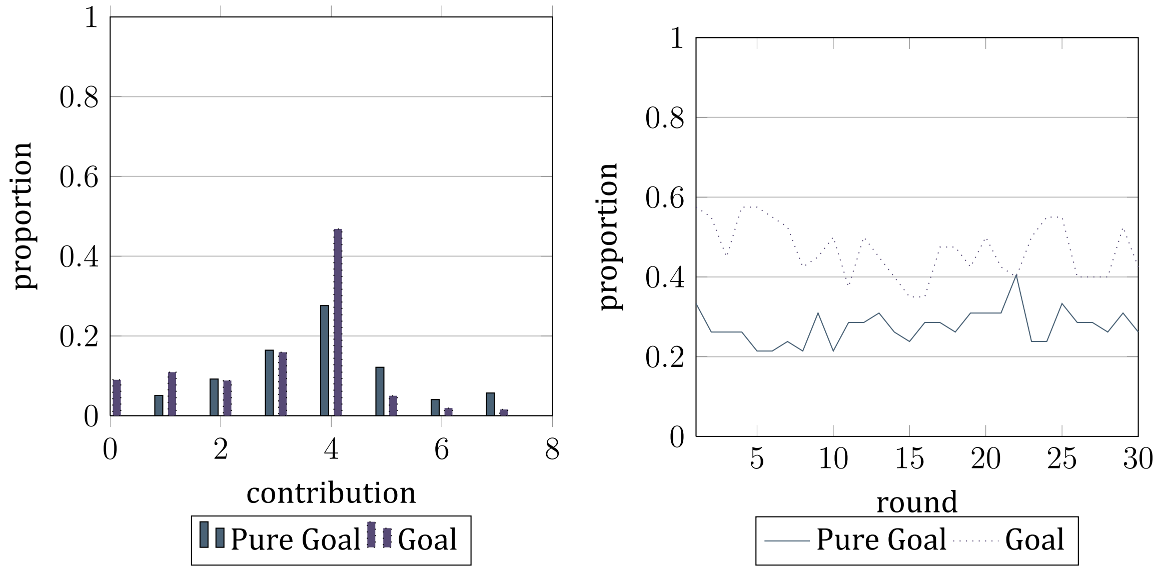
These results suggest that reducing the uncertainty at the goal is improving coordination on the symmetric goal outcome (4,4). The increased amount of symmetric play in the goal-with-reduced-uncertainty treatment drives the significantly improved earnings over the pure-goal treatment. Since payoffs are concave, asymmetric outcomes generate less total payout than symmetric outcomes with the same level of donations. Hence, even though average donations were not significantly different, the donations were being converted to dollars at a better rate.

In environments where goals are more costly, both in terms of earnings and the number of agents needed to coordinate, goals with reduced uncertainty may become increasing different from pure goals. In an environment with a larger number of agents and higher goal, the ability of a single agent to influence reaching the goal is diminished. Consequently, the improved coordination when uncertainty is reduced may allow donors to reach goals that are unreachable with only an announcement.

Our goal treatments (the pure-goal and goal-with-reduced-uncertainty treatments) have a number of similarities to experiments on selecting equilibria through recommending actions to subjects. The main difference is that in the literature on recommended play the recommendations are often for particular actions by individual subjects. Here the recommendation is only for total contributions and is announced to all subjects in a session. Since we used homogeneous groups of two subjects, the symmetric contribution profile to reach the recommended total contribution level is trivial to calculate. We conjecture that the impact of individual recommendations would be quite similar to recommending total contributions and hence it is worthwhile to compare our results with those from this literature.[15]

In contrast to earlier laboratory experiments, the recommendations induce significant in-

[15] The high incidence of four unit contributions in the goal-with-reduced-uncertainty treatment is consistent with the symmetric equilibrium being quite salient. This is consistent with the recommendation for total contributions being commonly understood by subjects as recommending the symmetric equilibrium.



(a) PMF of donations in the treatments with and without a reduction in uncertainty.

(b) Proportion of four unit donations across rounds in the treatments with and without a reduction in production uncertainty at the goal.

Figure 18: Donations in the pure-goal and goal-with-reduced-uncertainty treatments.

	All Rounds	Last 10
Goal w/ Reduction	0.870	0.844
	(0.007)	(0.047)
Constant (Goal w/o Reduction)	-0.905	-0.992
	(0.000)	(0.001)
Total No. of Obs.	2460	820
Total No. of Agents	82	82

Table 10: Random-effects, probit regression of the likelihood of donating four units from the last ten rounds of the treatments with and without an uncertainty reduction (p-values reported in parentheses).

creases in average contributions. Laboratory studies examining suggested contributions in public goods environments have found a limited impact on average contributions (Croson and Marks, 2001; Marks et al., 1999; Dale and Morgan, 2010; Seely et al., 2005). We conjecture the low levels of dominated contributions in the no-goal treatment (< 18% of contributions overall) may have made detecting an increase in average contributions easier. If the impact of recommendations falls mostly on those giving non-dominated, individually-motivated contributions then the impact on average contributions would be larger in our environment.

Though the difference in average contributions stands out, the improved coordination in the goal-with-reduced-uncertainty treatment is in line with other work in the literature on recommended play. Croson and Marks (2001); Marks et al. (1999); Seely et al. (2005) report significant increases in the frequency of the recommended contribution profile. Seely et al. (2005) find the largest effect when the recommended strategy is part of an equilibrium of the game, as is the case in the goal-with-reduced-uncertainty treatment.

Laboratory experiments do not scale well to models with a large number of donors. The physical limitations of a laboratory environment limit group size to a few dozen at most; far fewer in practice. In order to investigate the comparative statics of the goal game with a large number of donors we turn to numerical simulations. Section 3.5 details the implementation used and the results from numerical simulations with a large number of donors.

3.5 SIMULATIONS

Section 3.3 established that under weak assumptions charities increase total donations by announcing goals. Section 3.4 investigated the behavioral implications of the announced goals in small groups with homogeneous donors. However, large capital campaigns may have many donors with different resources. The comparative statics of goal game equilibria with large numbers of heterogeneous donors are not clear. In order to investigate such an environment we develop a parametric model and use simulations to get numerical estimates of equilibria in the goal game. These simulations are intended to supplement the theory, providing a more concrete context in which to understand it. Of particular interest are the potential benefits to fundraisers from employing a goal.

The two dimensions examined are

1. the increase in total donations generated and
2. the size of the set of contributors, ($d_i > 0$).

Generating more donations is certainly a desirable feature for fundraisers. As noted in Landry et al. (2006), the practical fundraising literature also places a great deal of emphasis on warm donor lists, i.e., lists of donors who have previously contributed. Apparently the emphasis is not misplaced. In List and Lucking-Reiley (2002), the authors explain that the professional fundraisers they contacted indicated campaigns directed at "cold" donors typically lose money. They are conducted in order to build lists of future donors. Hence as long as contributors provide some form of contact information with their donations (a name perhaps), a larger number of contributors is desirable.

Table 11 summarizes the choices made for the simulations. The production aspects of the simulation were selected for simplicity. The production of the public good is simply linear in donations with a random productivity parameter drawn on a uniform distribution that enters multiplicatively, i.e., ωD. It varies over an interval from 0.95 to 1.05, plus and minus 5 percent of the expected value, one. The distribution at the goal level is degenerate; the productivity parameter is simply the expected value, one.

The utility representation examined is a generalized constant relative risk-aversion model,

$$u(m_i - d_i, \omega D) = \frac{((m_i - d_i)^r (\omega D)^{1-r})^{1-\gamma}}{1-\gamma}.$$

It was chosen for a variety of reasons: it is well known, it is convenient to work with computationally, the degree of risk-aversion is simple to adjust, and there exist experimental estimates of γ (Holt and Laury, 2002). The familiar parameter $\gamma \in (0,1) \cup (1,\infty)$ determines the relative-risk aversion; higher values lead to greater risk aversion. The $r \in (0,1)$ parameter determines the relative utility of the private and public goods; higher values lead to greater utility from the private good. The best interpretation may be that r represents the optimal share of the private good for a donor in an individual consumption problem. Every donor has identical preferences when simulating a particular parameter combination. Six values of

r and six values of γ were used in the simulations for a total of thirty-six parameter combinations run. The values of r are spread roughly evenly across the parameter space of $(0, 1)$. The values of γ are spread roughly evenly across $(0, 1.1]$. The majority of values lie in $(0, 1)$ as experimental estimates of γ typically lie in that range (Holt and Laury, 2002; Harrison et al., 2007). Additionally, the no-goal game satisfies the conditions needed to have a unique equilibrium.[16] Hence, by locating the equilibrium without a goal and the goal game equilibrium with the most total donations, we can establish the interval of total donations within which all goal game equilibria lie.[17]

The simulations were run with nine endowment profiles drawn from a shifted χ_4^2 distribution. The endowments were drawn, then one was added to each endowment to ensure all endowments were large enough that $0 << m_i$, in order to avoid dividing by small values. The χ_4^2 distribution was used since it is well-studied, strictly positive, and roughly mimics typical income distributions. Goal and no goal (abbreviated as gg and ng) equilibria were then calculated for each endowment profile and parameter combination. Thus nine samples of each equilibria were available for every parameter combination.

The simulations do not scale well computationally. This limitation led us to use $N \in \{1000, 2000, 8000\}$. The simulations were run with differing numbers of agents purely to gauge how the size of the population affects the statistics of interest over these small populations. Implications of the limited population size are discussed with other robustness testing in subsection A.7.4. However, all tests indicate that the results either do not change with population or strengthen with larger populations. All results presented are drawn from the largest population, $N = 8000$.

In order to gauge the aspects of interest (the total donations collected and the size of the set of contributors) four measures were calculated. The improvement in total donations from

[16] This result comes from Bergstrom et al. (1986), see subsection A.7.5 for a discussion.

[17] In addition to comparing the goal equilibrium to the no goal equilibrium, it may be of interest to compare to the equilibrium of the no goal game where the uncertain input is always drawn from F' instead of F, i.e., the VCG described by the collection $\left\{C, F', G, \{i_j, u_j, m_j\}_{j=1}^{n}\right\}$. Under the simulation choices, the equilibrium donations of the no goal equilibrium are the same as those of the VCG $\left\{c, F', G, \{i_j, u_j, m_j\}_{j=1}^{n}\right\}$. In brief, the objective function is the certain CRRA utility function multiplied by a constant determined by the distribution of the random input. Hence the best response function is independent of the distribution and thus the total donations and distribution of donations are independent as well. As a result, the results we report for the no goal game are also results for the no goal game, $\left\{c, F', G, \{i_j, u_j, m_j\}_{j=1}^{n}\right\}$.

Production	ωD, linear in total donations
Uncertainty	$\omega \sim U[0.95, 1.05]$, uniform around 1
Endowments	$m_i' \sim \chi_4^2$, $m_i = m_i' + 1$
Number of Donors	$N \in \{1000, 2000, 8000\}$
Utility	$u(x,y) = \frac{(x^r y^{1-r})^{1-\gamma}}{1-\gamma}$
	$r \in \{0.1, 0.3, 0.5, 0.7, 0.9, 0.99\}$
	$\gamma \in \{0.1, 0.3, 0.64, 0.7, 0.9, 1.1\}$

Table 11: Simulation choices

announcing a goal is measured by the mean relative increase in total donations, $I_a = D_a^{gg}/D_a^{ng}$, for parameter combination a. Gauging the size of the set of contributors is more complicated. Directly using the number of agents with positive donations might be misleading. Due to strict risk aversion, every agent in the goal game model is induced to contribute in equilibrium, but the donations can get vanishingly small. Using the definition that a contributor donates a positive amount, the set of contributors in every goal game equilibrium will be the entire population. Hence, directly comparing the number of agents with positive donations does not convey much information. Instead we attempt to measure the distribution of donations within the population. Our reported measures are the differences between the proportion of donations coming from the poorest third, middle third, and wealthiest third of donors. Figure 19, depicts a typical plot of the distribution of donations.

The results suggest substantial benefits for charities announcing a goal. Under parameters least advantageous to an improvement ($r = 0.99, \gamma = 0.10$) donations increased 6 percent. Under parameters most advantageous to an improvement ($r = 0.10, \gamma = 1.10$) donations increased 73 percent.

The change in the distribution of donations is perhaps the most interesting. Without a

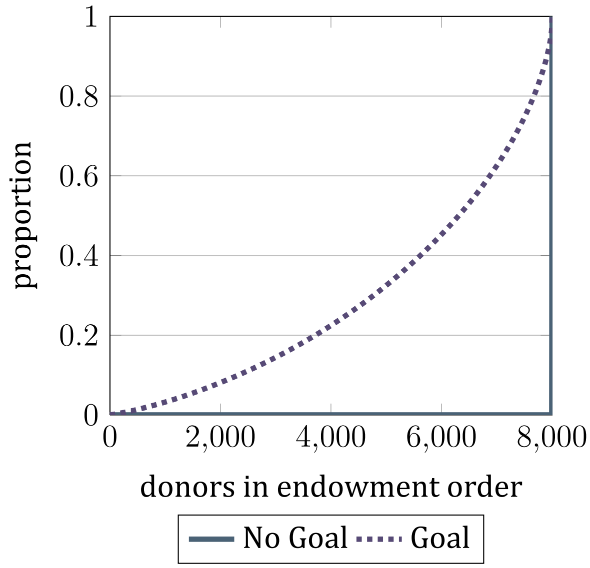
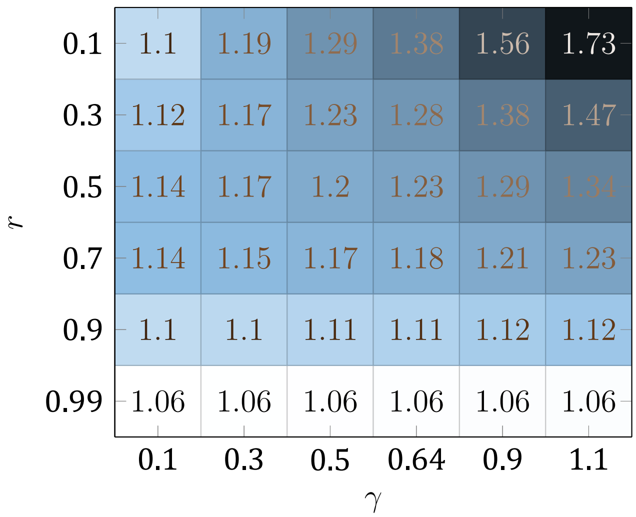


Figure 19: Plot of the mean distribution of donations at $\gamma = .64$, $r = .9$.

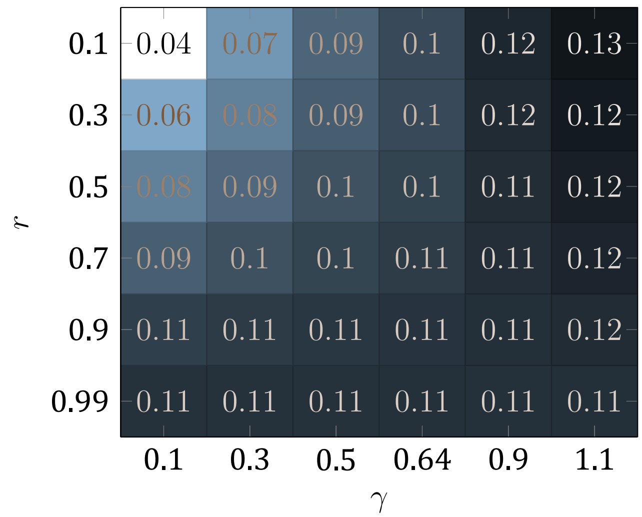
goal, only the wealthiest donors contribute; figure 19 is not atypical.[18] Hence the values in figures 20b and 20c are effectively the proportion of total donations in the goal game equilibrium. At almost all parameter combinations, the goal draws substantial donations from the poorer two-thirds of donors, $\approx 12 - 38$ percent. Interestingly this proportion is increasing in both γ and r. This is somewhat counter-intuitive as a higher r indicates less interest in the public good. It appears that a higher r drives down donations from the wealthiest donors relatively faster than the donations from moderate income donors.

These improvements are complementary, so it is not clear if the improvement in total donations is mostly driven by the increase in contributors or increases from those who contribute in the no goal game. In order to determine the source of changes, we examined the absolute differences in donations between the two equilibria. For all parameter combinations, those donors who contributed without a goal always give less when a goal is announced. Hence the improvement in total donations is universally driven by the increase in the set of contributors. The charity is able to increase contributions by limiting the amounts from

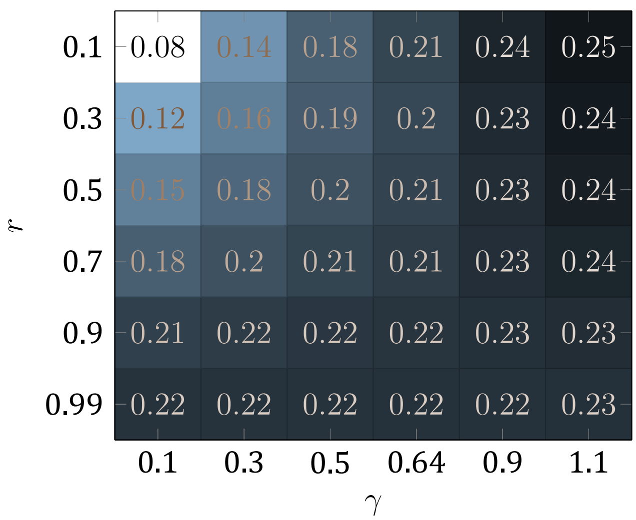
[18] Giving being restricted to the wealthiest individuals without a goal is true of public goods games more generally, see Andreoni (1988).



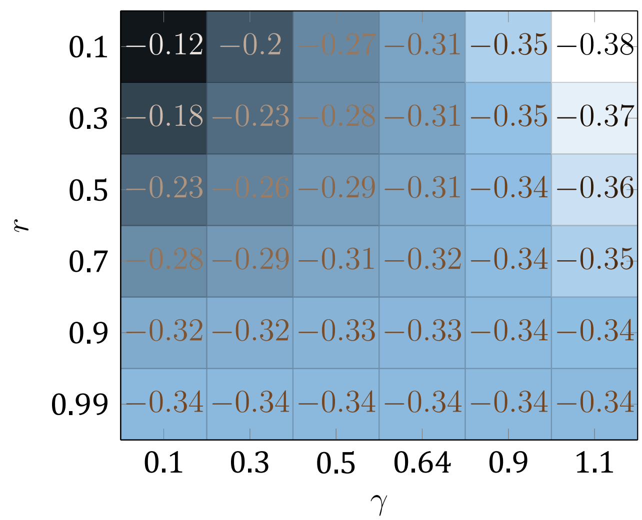
(a) Increase in donations from using a goal [a]



(b) $\mathcal{W}_a^{low,gg} - \mathcal{W}_a^{low,ng}$, difference in proportion of donations from the poorest third [b]



(c) $\mathcal{W}_a^{mid,gg} - \mathcal{W}_a^{mid,ng}$, difference in proportion of donations from the middle third[c]



(d) $\mathcal{W}_a^{high,gg} - \mathcal{W}_a^{high,ng}$, difference in proportion of donations from the wealthiest third [d]

[a]The mean relative increase in total donations from the no goal outcome to the goal game outcome.

[b]$\mathcal{W}_a^{low}$ is the mean proportion of total donations collected from the third of donors with the smallest endowments.

[c]$\mathcal{W}_a^{high}$ is the mean proportion of total donations collected from the third of donors with the largest endowments.

[d]$\mathcal{W}_a^{mid}$ is the mean proportion of total donations collected from the third of donors in neither of the above sets.

Figure 20: Simulation Results

wealthy contributors and gathering more from moderate income contributors.

3.6 CONCLUSION

We developed a model of charitable giving that explains the practice of charitable fundraisers setting goals, the goal game. The announced fundraising goals indicate a level of production of the public good that has reduced uncertainty. Under weak assumptions, subgame perfect equilibria always generate greater donations than the equivalent game without a goal. Results from a laboratory implementation of the goal game produced mixed results. Announcing a goal generates significantly more in donations and increases the likelihood of donations reaching the goal level. However, additionally reducing uncertainty at the goal does not further increase the benefits for fundraisers. Donor behavior does change significantly, concentrating donations at half the goal level and increasing donor earnings. We conjecture that these changes indicate a reduction in uncertainty may enable goals to maintain these benefits even in environments where reaching the goal is more difficult for an individual donor. Whereas the benefits of goals that are merely announced will fall. Numerical simulations were conducted in order to better understand the comparative statics of the goal game in an environment with large numbers of heterogeneous donors. The simulation data provides evidence of the effectiveness of exploiting risk aversion among donors; the benefits are substantial and robust under CRRA utility.

With this evidence as support, we conjecture that significant benefits to charities exist in more realistic settings. If better fundraising improves a charity's survival then the availability of significant benefits to charities that can exploit risk-aversion suggests that current fundraising techniques do in fact exploit it. No charity would leave these benefits on the table. The various tools used by fundraisers may exist in order to capture some of these benefits.

Extensions/modifications to the goal game should be straight-forward and may provide new theoretical justification for fundraising practices. To be clear, this discussion departs from firmly grounded theory into educated speculation. Possible avenues for extension include modifying the charity's ability to set goals, and creating more dynamic versions of the model. For example, in the current model the charity is fully aware of the donors' willingness to pay and can perfectly exploit it. It may be of interest to consider limiting the charity to setting goals it can achieve with a finite menu of possible donations. The menu could be interpreted as an analog to the donor "clubs" charities often have, i.e., silver members, gold members, platinum members, etc.

Many charities appear to announce a series of fundraising goals during a campaign; smaller intermediate goals as well as a target for total contributions. A sequential version of the goal game may be of interest in order to investigate the purpose of these repeated goals. We foresee no technical hurdles to extending the model in a manner similar to that of Marx and Matthews (2000), were donations can occur over several periods. Sequential giving may shrink the set of possible outcomes as initial donors will be able to select equilibria that they prefer. For example, suppose there are many possible donation profiles that sum to the charity's announced goal, but the first donor's contribution is unique to each profile. Then the first donor can in essence choose her preferred profile with her corresponding donation. Based on the simulation data, there may be fundraising incentives to have a particular order to donations in some contexts. In the simulations, the wealthiest donors gave less in the goal game equilibrium than in the no goal equilibrium, allowing the charity to extract more from the moderate income donors. Hence there may be incentives for fundraisers to pay particular attention to the wealthiest donors in order to target their donations.

A model with a dynamic charity may also be of interest. Consider the example of an National Public Radio listener being asked for contributions "to keep bringing you quality programs". She is familiar with current programming levels (at current funding levels), but may be uncertain about how the station would change if funding is reduced. Her reduced uncertainty at the current level of funding creates a discontinuity in her utility which gives her strong disincentives to have funding move away from that point. Setting current programing levels seems fundamentally different from the goal announcement used by the charity in the goal game. However, a model of charities including lump sum investment with the possibility of acquiring debt, and periodic funding drives may predict something similar. Suppose a charity wants to maximize an infinite stream of public good production and donors value an infinite stream of utilities earned in each period. The founders have an incentive to increase production early on, knowing that donors will want to keep that initial production level in each period. This mechanism may cause the initial outlays for the charity to far exceed what might be expected from one period donation levels.

To the extent that these extensions provide theoretical models of fundraising practices, the goal game is a useful foundation for further fundraising research.

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A.1 EXPERIMENTAL INSTRUCTIONS

A.1.1 Introduction

This is an experiment about decision making. There are fourteen people in this room participating in the experiment. You must not talk to the other participants or communicate with them in any way. If you have a question raise your hand and a monitor will come to where you are sitting to answer it.

The experiment consists of fourteen rounds. In each round you are randomly paired with one other participant. Your round earnings depend on the decisions made by you and by your group member for that round. Your decisions are anonymous; no one will be able to determine which decisions were made by you. At the end of the fourteen decision rounds we will randomly select three rounds for payment. You will be paid, in private and in cash, the sum of your earnings from the three selected rounds plus $5 for showing up to the experiment.

A.1.2 Investments

In each round you will be given $4. You can keep the $4 or you can invest in the group account. The cost of investing in the group account depends on the number of units you invest. The payoff from the group account depends on the sum invested by you and by your group member. Your earnings in each round will equal your initial $4 plus the payoff from the group account minus the cost of your individual investment.

A.1.3 Payoff from the group account

[*Threshold*: Provided the total amount invested by you and by your group member equals or exceeds 6 units, you and your group member will each get a payoff of 50 cents per unit invested in the group account. Thus if a total of 4 units are invested in the group account, then neither you nor your group member will get a payoff from the group account.] [*No Threshold*: You and your group member will each get a payoff of 50 cents per unit invested in the group account. Thus if a total of 4 units are invested in the group account, then you and your group member will each get a payoff of 4x0.5=$2 from the group account.] If you and your group member invests a total of 20 units in the group account, then you and your group member will each get a payoff of 20x0.5=$10. Your payoff from the group account depends only on the total amount invested in the group account by you and your group member.

A.1.4 Cost of investing in the group account

The cost of investing in the group account depends on the number of units you invest. If you invest 3 units or less the cost per unit invested is 40 cents. Every unit you invest between 4 and 7 units will cost you 70 cents per unit. Finally, every unit you invest in excess of 7 will cost you $1.10 per unit. If you invest 9 units your costs are 40 cents per unit for each of the first three units (3x0.4=$1.2), 70 cents per unit for the fourth through seventh unit (4x0.7 = $2.8), and $1.1 per unit for the eighth through the ninth (2x1.1=$2.2). Thus the total cost of your nine unit investment is 1.2+2.8+2.2=$6.2. You can at most invest 10 units in the group account. With you and your group member each investing anywhere from 0 to 10 units, the total investment in the group account will range from 0 to 20 units.

A.1.5 Earnings

To determine the round earnings from the possible investments please take a look at the payoff table. Your earnings are reported in blue at the top left corner of each cell, the earnings to your group member are reported in orange at the lower right corner of each cell. Let us examine two examples to better understand the payoff table and how earnings are determined.

A.1.5.1 Example 1: [*Threshold*: Suppose you invest 2 units and your group member invests 0 units. With a total investment of 2 units, the investment in the group account is below the threshold and you and your group member will each get $0 from the group account. The per unit cost of your 2-unit investment is 40 cents for a total cost of 80 cents. Thus your earnings from this round equal $4 plus $0 from the group account minus your cost of $0.8 for a total of $3.2. Since your group member has zero investment cost he/she earns $4. These earnings are shown in the 0-column and 2-row cell, your earnings of $3.2 are listed in blue, and the $4 earnings to your group member is listed in orange. If the investment by your group member increases to 1 unit the payoff from the group account does not change because the total investment is still less than 6; while your earnings stay constant at $3.2 the earnings of your group member decreases by 40 cents from $4.0 to $3.6 to cover the 40 cent investment cost (see the 1-column and 2-row cell). If instead you increase your investment by 6 units, then the total investment of 8 is above the threshold and the payoff from the group account increases from $0 to $4 (0.5x8). However the cost of the additional investment is $4.3(=1x0.4+4x0.7+1x1.1). Therefore your earnings decrease by $0.3 dollars from $3.2 to $2.9, and the earnings to your group member increase by $4 from $4 to $8 (see the 0-column and 8-row cell).] [*No Threshold*: Suppose you invest 2 units and your group member invests 0 units. With a total investment of 2 units, you and your group member will each get 2x0.5=$1 from the group account. The per unit cost of your 2-unit investment is 40 cents for a total cost of 80 cents. Thus your earnings from this round equal $4 plus $1 from the group account minus your cost of $0.8 for a total of $4.2. Since your group member has zero investment cost he/she earns $5. These earnings are shown in the 0-column and 2-row cell, your earnings of $4.2 are listed in blue, and the $5 earnings to your group member is listed in orange. If the investment by your group member increases to 1 unit the payoff from the group account increases by $0.5; while your earnings increases by $0.5 to $4.7 the earnings of your group member increases by 10 cents from $5.0 to $5.1 to cover the 40 cent investment cost (see the 1-column and 2-row cell). If instead you increase your investment by 6 units, the payoff from the group account increases from $1 to $4 (0.5x8). However the cost of the additional investment is $4.3(=1x0.4+4x0.7+1x1.1). Therefore your earnings decrease by $1.3 dollars from $4.2 to $2.9, and the earnings to your group member increase by $3 from $5 to $8 (see the 0-column and 8-row cell).]

A.1.5.2 Example 2: Suppose you invest 8 and your group member invests 6. With a total investment of 14 units, you each earn $7 (14x0.5) from the group account. Your investment costs for the 8 units are: 40 cents per unit for each of the first three units (3x0.4=$1.2), 70 cents per unit for the fourth through seventh unit (4x0.7 = $2.8), and $1.1 for the eighth unit. Thus the total cost of your eight unit investment is 1.2+2.8+1.1=$5.1. As shown in the 6-

column and 8-row cell you earn 4+7-5.1=$5.9, and your group member earns $7.7. Increasing your investment by one unit increases the payoff from the group account by 50 cents and costs you $1.1. Thus as seen in 6-column and 9-row cell, your earnings decrease by 60 cents (0.5-1.1=-$0.6) to $5.3, your group member' earnings increase by 50 cents from $7.7 to $8.2.

A.1.6 Order of Investments

Seven participants will have the role of first mover, the other seven will have the role of second mover. The computer randomly assigns you to be either first or second mover. You are informed of your role at the beginning of the experiment, and you remain in this role throughout the experiment. Your role will appear at the top of your screen. It will either say "You are a FIRST mover" or "You are a SECOND mover", depending on your role. In each round, each first mover will be anonymously and randomly paired with a second mover. In subsequent rounds you are randomly paired with a new participant. In the first stage of a round the first mover decides how much to invest. Then, in the second stage, the second mover decides how much to invest. Before making his or her investment decision the second mover will not be informed of the first mover's investment decision.

A.1.7 Summary

In making your investment decisions, you will benefit from looking at the payoff table, or from recalling how earnings are determined.

1. In each round your earnings equal $4 plus your group-account payoff minus your investment costs.
2. First movers are randomly paired with second movers in each round. First movers make their investment decisions first, and second movers make their investment decisions second. The second mover will not be informed of the first mover's investment prior to making his or her decision.
3. [*Threshold*: Provided the total amount invested by you and by your group member equals or exceeds 6 units, the per unit payoff from the group account is 50 cents.][*No Threshold*:

the per unit payoff from the group account is 50 cents.] That is $0.5 x [the investment by you + the investment by your group member].

4. The cost per investment unit is:
   a. 40 cents per unit between 1-3
   b. 70 cents per unit between 4-7
   c. $1.1 per unit between 8-10

Before we begin the experiment we want to make sure that you know how to read the payoff table. We therefore ask you to take a little quiz to help you understand the payoffs. Once you have finished the quiz, we will go over the correct answers. Your answers to the quiz will not influence your earnings.

A.2 PAYOFF TABLES

		Other Group Member										
		0	1	2	3	4	5	6	7	8	9	10
	0	4.0 4.0	4.5 4.1	5.0 4.2	5.5 4.3	6.0 4.1	6.5 3.9	7.0 3.7	7.5 3.5	8.0 2.9	8.5 2.3	9.0 1.7
	1	4.1 4.5	4.6 4.6	5.1 4.7	5.6 4.8	6.1 4.6	6.6 4.4	7.1 4.2	7.6 4.0	8.1 3.4	8.6 2.8	9.1 2.2
	2	4.2 5.0	4.7 5.1	5.2 5.2	5.7 5.3	6.2 5.1	6.7 4.9	7.2 4.7	7.7 4.5	8.2 3.9	8.7 3.3	9.2 2.7
	3	4.3 5.5	4.8 5.6	5.3 5.7	5.8 5.8	6.3 5.6	6.8 5.4	7.3 5.2	7.8 5.0	8.3 4.4	8.8 3.8	9.3 3.2
	4	4.1 6.0	4.6 6.1	5.1 6.2	5.6 6.3	6.1 6.1	6.6 5.9	7.1 5.7	7.6 5.5	8.1 4.9	8.6 4.3	9.1 3.7
You	5	3.9 6.5	4.4 6.6	4.9 6.7	5.4 6.8	5.9 6.6	6.4 6.4	6.9 6.2	7.4 6.0	7.9 5.4	8.4 4.8	8.9 4.2
	6	3.7 7.0	4.2 7.1	4.7 7.2	5.2 7.3	5.7 7.1	6.2 6.9	6.7 6.7	7.2 6.5	7.7 5.9	8.2 5.3	8.7 4.7
	7	3.5 7.5	4.0 7.6	4.5 7.7	5.0 7.8	5.5 7.6	6.0 7.4	6.5 7.2	7.0 7.0	7.5 6.4	8.0 5.8	8.5 5.2
	8	2.9 8.0	3.4 8.1	3.9 8.2	4.4 8.3	4.9 8.1	5.4 7.9	5.9 7.7	6.4 7.5	6.9 6.9	7.4 6.3	7.9 5.7
	9	2.3 8.5	2.8 8.6	3.3 8.7	3.8 8.8	4.3 8.6	4.8 8.4	5.3 8.2	5.8 8.0	6.3 7.4	6.8 6.8	7.3 6.2
	10	1.7 9.0	2.2 9.1	2.7 9.2	3.2 9.3	3.7 9.1	4.2 8.9	4.7 8.7	5.2 8.5	5.7 7.9	6.2 7.3	6.7 6.7

Figure 21: Payoff table with $FC = 0$.

		Other Group Member										
		0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
	0	4.0 / *4.0*	4.0 / *3.6*	4.0 / *3.2*	4.0 / *2.8*	4.0 / *2.1*	4.0 / *1.4*	7.0 / *3.7*	7.5 / *3.5*	8.0 / *2.9*	8.5 / *2.3*	9.0 / *1.7*
	1	3.6 / *4.0*	3.6 / *3.6*	3.6 / *3.2*	3.6 / *2.8*	3.6 / *2.1*	6.6 / *4.4*	7.1 / *4.2*	7.6 / *4.0*	8.1 / *3.4*	8.6 / *2.8*	9.1 / *2.2*
	2	3.2 / *4.0*	3.2 / *3.6*	3.2 / *3.2*	3.2 / *2.8*	6.2 / *5.1*	6.7 / *4.9*	7.2 / *4.7*	7.7 / *4.5*	8.2 / *3.9*	8.7 / *3.3*	9.2 / *2.7*
	3	2.8 / *4.0*	2.8 / *3.6*	2.8 / *3.2*	5.8 / *5.8*	6.3 / *5.6*	6.8 / *5.4*	7.3 / *5.2*	7.8 / *5.0*	8.3 / *4.4*	8.8 / *3.8*	9.3 / *3.2*
	4	2.1 / *4.0*	2.1 / *3.6*	5.1 / *6.2*	5.6 / *6.3*	6.1 / *6.1*	6.6 / *5.9*	7.1 / *5.7*	7.6 / *5.5*	8.1 / *4.9*	8.6 / *4.3*	9.1 / *3.7*
You	**5**	1.4 / *4.0*	4.4 / *6.6*	4.9 / *6.7*	5.4 / *6.8*	5.9 / *6.6*	6.4 / *6.4*	6.9 / *6.2*	7.4 / *6.0*	7.9 / *5.4*	8.4 / *4.8*	8.9 / *4.2*
	6	3.7 / *7.0*	4.2 / *7.1*	4.7 / *7.2*	5.2 / *7.3*	5.7 / *7.1*	6.2 / *6.9*	6.7 / *6.7*	7.2 / *6.5*	7.7 / *5.9*	8.2 / *5.3*	8.7 / *4.7*
	7	3.5 / *7.5*	4.0 / *7.6*	4.5 / *7.7*	5.0 / *7.8*	5.5 / *7.6*	6.0 / *7.4*	6.5 / *7.2*	7.0 / *7.0*	7.5 / *6.4*	8.0 / *5.8*	8.5 / *5.2*
	8	2.9 / *8.0*	3.4 / *8.1*	3.9 / *8.2*	4.4 / *8.3*	4.9 / *8.1*	5.4 / *7.9*	5.9 / *7.7*	6.4 / *7.5*	6.9 / *6.9*	7.4 / *6.3*	7.9 / *5.7*
	9	2.3 / *8.5*	2.8 / *8.6*	3.3 / *8.7*	3.8 / *8.8*	4.3 / *8.6*	4.8 / *8.4*	5.3 / *8.2*	5.8 / *8.0*	6.3 / *7.4*	6.8 / *6.8*	7.3 / *6.2*
	10	1.7 / *9.0*	2.2 / *9.1*	2.7 / *9.2*	3.2 / *9.3*	3.7 / *9.1*	4.2 / *8.9*	4.7 / *8.7*	5.2 / *8.5*	5.7 / *7.9*	6.2 / *7.3*	6.7 / *6.7*

Figure 22: Payoff table with $FC = 6$.

Other Group Member

	0	1	2	3	4	5	6	7	8	9	10
0	4.0 / 4.0	4.0 / 3.6	4.0 / 3.2	4.0 / 2.8	4.0 / 2.1	4.0 / 1.4	7.0 / 3.7	7.5 / 3.5	8.0 / 2.9	8.5 / 2.3	9.0 / 1.7
1	3.6 / 4.0	3.6 / 3.6	3.6 / 3.2	3.6 / 2.8	3.6 / 2.1	6.6 / 4.4	7.1 / 4.2	7.6 / 4.0	8.1 / 3.4	8.6 / 2.8	9.1 / 2.2
2	3.2 / 4.0	3.2 / 3.6	3.2 / 3.2	3.2 / 2.8	6.2 / 5.1	6.7 / 4.9	7.2 / 4.7	7.7 / 4.5	8.2 / 3.9	8.7 / 3.3	9.2 / 2.7
3	2.8 / 4.0	2.8 / 3.6	2.8 / 3.2	5.8 / 5.8	6.3 / 5.6	6.8 / 5.4	7.3 / 5.2	7.8 / 5.0	8.3 / 4.4	8.8 / 3.8	9.3 / 3.2
4	2.1 / 4.0	2.1 / 3.6	5.1 / 6.2	5.6 / 6.3	6.1 / 6.1	6.6 / 5.9	7.1 / 5.7	7.6 / 5.5	8.1 / 4.9	8.6 / 4.3	9.1 / 3.7
You 5	1.4 / 4.0	4.4 / 6.6	4.9 / 6.7	5.4 / 6.8	5.9 / 6.6	6.4 / 6.4	6.9 / 6.2	7.4 / 6.0	7.9 / 5.4	8.4 / 4.8	8.9 / 4.2
6	3.7 / 7.0	4.2 / 7.1	4.7 / 7.2	5.2 / 7.3	5.7 / 7.1	6.2 / 6.9	6.7 / 6.7	7.2 / 6.5	7.7 / 5.9	8.2 / 5.3	8.7 / 4.7
7	3.5 / 7.5	4.0 / 7.6	4.5 / 7.7	5.0 / 7.8	5.5 / 7.6	6.0 / 7.4	6.5 / 7.2	7.0 / 7.0	7.5 / 6.4	8.0 / 5.8	8.5 / 5.2
8	2.9 / 8.0	3.4 / 8.1	3.9 / 8.2	4.4 / 8.3	4.9 / 8.1	5.4 / 7.9	5.9 / 7.7	6.4 / 7.5	6.9 / 6.9	7.4 / 6.3	7.9 / 5.7
9	2.3 / 8.5	2.8 / 8.6	3.3 / 8.7	3.8 / 8.8	4.3 / 8.6	4.8 / 8.4	5.3 / 8.2	5.8 / 8.0	6.3 / 7.4	6.8 / 6.8	7.3 / 6.2
10	1.7 / 9.0	2.2 / 9.1	2.7 / 9.2	3.2 / 9.3	3.7 / 9.1	4.2 / 8.9	4.7 / 8.7	5.2 / 8.5	5.7 / 7.9	6.2 / 7.3	6.7 / 6.7

Figure 23: Payoff table with $FC = 8$.

A.3 MIXED-STRATEGY NASH EQUILIBRIA

Below are the individual payoffs as a function of individual giving.

$$\pi_i(g_i = 6) = \sum_{g_{-i}=0}^{10} \Pr(g_{-i}) \cdot \frac{1}{2}(6 + g_{-i}) - 3.3$$
$$\pi_i(g_i = 5) = \sum_{g_{-i}=1}^{10} \Pr(g_{-i}) \cdot \frac{1}{2}(6 + g_{-i}) - 2.6$$
$$\pi_i(g_i = 4) = \sum_{g_{-i}=2}^{10} \Pr(g_{-i}) \cdot \frac{1}{2}(6 + g_{-i}) - 1.9$$
$$\pi_i(g_i = 3) = \sum_{g_{-i}=3}^{10} \Pr(g_{-i}) \cdot \frac{1}{2}(6 + g_{-i}) - 1.2$$
$$\pi_i(g_i = 2) = \sum_{g_{-i}=4}^{10} \Pr(g_{-i}) \cdot \frac{1}{2}(6 + g_{-i}) - 0.8$$
$$\pi_i(g_i = 1) = \sum_{g_{-i}=5}^{10} \Pr(g_{-i}) \cdot \frac{1}{2}(6 + g_{-i}) - 0.4$$
$$\pi_i(g_i = 0) = \sum_{g_{-i}=6}^{10} \Pr(g_{-i}) \cdot \frac{1}{2}(6 + g_{-i}) - 0$$

As is clear from these equations, the benefit in contributing 6 units is that whatever is the contribution of the opponent (g_{-i}), there will be a positive payoff from the public account. The disadvantage is, of course, the cost of 3.3 units. Contributing less than 6, say 5 units, reduce the benefit from public account payoff in case the opponent contributes zero units. The plus side is that contributing 5 units costs only 2.6. Comparing the payoff in both cases, we find the conditions when contributing 6 is a best response. That is, when is it the case that it yields greater payoff than the other contribution levels (we do not consider contributing more than 6, since these are clearly dominated strategies). In writing these inequalities below we use to be the probability the opponent contributing x units,i.e. Pr(g_{-i}).

$$
\begin{aligned}
2.6p_0 &> 0.2 && 6 > 5 \\
2p_0 + 2.5p_1 &> 0.4 && 6 > 4 \\
1.5p_0 + 2p_1 + 2.5p_2 &> 0.6 && 6 > 3 \\
p_0 + 1.5p_1 + 2p_2 + 2.5p_3 &> 0.5 && 6 > 2 \\
0.5p_0 + p_1 + 1.5p_2 + 2p_3 + 2.5p_4 &> 0.4 && 6 > 1 \\
0.5p_1 + p_2 + 1.5p_3 + 2p_4 + 2.5p_5 &> 0.3 && 6 > 0
\end{aligned}
$$

A.4 PROOFS OF PROPOSITIONS

A.4.1 Proposition 1

Definition 2. *A threshold game, $\left\{c, I, \{u_j, m_j\}_{j=1}^{n}\right\}$, consists of:*

1. *a charity, c, which monopolizes production of the public good,*
2. *a set of $n \in$ **N** donors, I,*
3. *a vector of endowments of a private good, $\{m_i\}_{i\in I}$, such that each donor $i \in I$ is endowed with some positive amount of a private good, $m_i \in$ **R**$_{++}$, $\sum_{i\in I} m_i = M$,*
4. *a set of continuous utility functions, $\{u_i\}_{i\in I}$, $u_i : [0, m_i] \times$ **R** $\rightarrow$ **R**, $u_i(x, G)$, which depend on consumption of a private good, x, and total contributions of the public good, G, such that each donor $i \in I$ is associated with u_i. u_i is increasing in both arguments.*

*The game has a continuum of information sets, but of only two types. At the first information set the charity, c, chooses a threshold for donations to the public good, $T \in$ **R**$_{+}$.*

Each possible goal determines the information set to which the game then moves. At the second information set, the donors, I, with full knowledge of T, simultaneously choose how to allocate their endowments between private consumption and donations to the charity for production of the public good, $g_i \in [0, m_i]$.

After donations are allocated, production of the public good takes place. The donors receive a payoff from their allocations of private good and the realized quantity of public good, i.e., $u_i\left(m_i - d_i, \sum_{i\in I} g_i\right)$*. The charity receives a payoff from the the realized quantity of public good,* $\sum_{i\in I} g_i$*.*

Definition 3. *Define three functions:*

- $\gamma_i : [0, \iota'\mathbf{m}] \to [0, m_i]$, $\gamma_i(T) \equiv \sup\{d \in [0, m_i] \,|u_i(m_i - d_i, T) - u_i(m_i, 0) \geq 0\}$.
- $\Gamma : [0, \iota'\mathbf{m}] \to \prod_{i\in I}[0, m_i]$, $\Gamma_i(T) \equiv \gamma_i(T)$.
- $\Lambda : [0, \iota'\mathbf{m}] \to [0, \iota'\mathbf{m}]$, $\Lambda(T) \equiv \iota'\Gamma(T)$.

Lemma 1. Γ *is a monotone function, i.e., if* $T < T'$ *then* $\Gamma(T) \leq \Gamma(T')$*.*

Proof. The proof follows by showing that γ_i is monotone which extends to Γ.

Let $T < T'$. Since u_i is increasing, $u_i(m_i - \gamma_i(T), T) < u_i(m_i - \gamma_i(T), T')$. Then $u_i(m_i - \gamma_i(T), G(T')) - u_i(m_i, 0) > 0$. By construction γ_i is the largest individual contribution, g for which $u_i(m_i - g, T') - u_i(m_i, 0) \geq 0$ holds. Hence, $\gamma_i(T') \geq \gamma_i(T)$.

Since i was chosen arbitrarily, this holds for all $i \in I$. Therefore, $\Gamma(T) \leq \Gamma(T')$ and the lemma holds. □

Lemma 2. *Let* $\mathbf{g}^\star \in \prod_{i\in I}[0, m_i]$ *be a Nash equilibrium vector of donations in the equivalent VCG.* Λ *has a greatest fixed-point in* $[\iota'\mathbf{g}^\star, \iota'\mathbf{m}]$*.*

Proof. The proof follows from Lemma 1 and Tarski's fixed-point theorem.

Since $\mathbf{g}^\star$ is at a Nash equilibrium it must be the case that $u_i(m_i - d_i, \iota'\mathbf{g}^\star) - u_i(m_i, 0) \geq 0$. Hence $\gamma_i(\iota'\mathbf{g}^\star) \geq g_i^\star$ for all $i \in I$. Then $\Gamma(\iota'\mathbf{g}^\star) \geq \mathbf{g}^\star$ and $\Lambda(\iota'\mathbf{g}^\star) \geq \iota'\mathbf{g}^\star$. It follows from Lemma 1 that Λ is monotone increasing. Thus, Λ maps $[\iota'\mathbf{g}^\star, \iota'\mathbf{m}]$ into itself. Then Tarski's fixed-point theorem implies that Λ has a greatest fixed-point in $[\iota'\mathbf{g}^\star, \iota'\mathbf{m}]$ and the lemma holds. □

Proposition 1. *Let* $X = \left\{c, G, I, \{u_j, m_j\}_{j=1}^n\right\}$ *be a threshold game. Then there exists a Nash equilibrium outcome,* $\left\{\tilde{T}, \tilde{\mathbf{g}}\right\}$*, where* $\tilde{\mathbf{g}}$ *is a Nash equilibrium of the subgame starting at* $\tilde{T}$*, and* $\iota'\tilde{\mathbf{g}} > \iota'\mathbf{g}$ *for any other Nash equilibrium,* $\mathbf{g}$*, of any subgame.*

Proof. Let $T \in \mathbf{R}_+$ be the greatest fixed-point of Λ in $[\iota'\mathbf{g}^\star, \iota'\mathbf{m}]$. Let $\mathbf{g} = \Gamma(T)$. First we show that g_i is a best response for the i^{th} donor in the subgame.

Any donation $g' < g_i$ will result in the threshold not being met and no production of the public good. The i^{th} donor would then receive $u_i(m_i, 0)$. By the construction of Γ, $u_i(m_i - g_i, T) - u_i(m_i, 0) \geq 0$. Hence g_i is at least as good as any $g' < d_i$.

Suppose there exists a $g' > g_i$ such that $u_i(m_i - g', T - g_i + g') - u_i(m_i - g_i, T) > 0$. Then the threshold $T - g_i + g' > \tilde{D}$ could be supported by the donations $\mathbf{g}'$ where $g'_j = g_j$ for all $j \neq i$ and $g_i = g'$. However $T - g_i + g'$ would then be a fixed-point of Λ which contradicts the fact that T is the greatest fixed-point. Hence our supposition is incorrect and there does not exist a $g' > g_i$ such that $u_i(m_i - g', T - g_i + g') - u_i(m_i - g_i, T) > 0$. Hence g_i is at least as good as any $g' > g_i$, and is a best response for donor i. As we chose i arbitrarily, $\mathbf{g} = \Gamma(T)$ is a best response for all donors.

Now consider a Nash equilibrium outcome, $\mathbf{g}'$, in another subgame beginning with threshold $T' \neq T$. If $\iota'\mathbf{g}' < T'$ then no public good is provided and the proposition holds with $\tilde{T} = T$ and $\mathbf{g} = \mathbf{g}$. If $\iota'\mathbf{g}' = T'$ then T' is a less than or equal to a fixed-point of Λ and is thus $T' \leq T$ and the proposition holds with $\tilde{T} = T$ and $\mathbf{g} = \mathbf{g}$. Finally, consider $\iota'\mathbf{g}' > T'$. Then it must be the case that $\mathbf{g}'$ is a Nash equilibrium vector of donations in the equivalent VCG. Thus by Lemma 2 $T' \leq T$ and the proposition holds with $\tilde{T} = T$ and $\mathbf{g} = \mathbf{g}$. $\square$

Proposition 2. *Let $X = \left\{c, I, \{u_j, m_j\}_{j=1}^{n}\right\}$ be a threshold game such that a Nash equilibrium exists for every threshold. Then $\tilde{T} \in \mathbf{R}_+$, is the unique threshold that is part of a subgame, undominated-perfect equilibrium and total equilibrium contributions equal $\tilde{T}$.*

Proof. Let $\{T, \mathbf{g}\} \in \left\{\mathbf{R}_+, \left(\prod_{i\in I}[0, m_i]\right)^{\mathbf{R}_+}\right\}$ be a subgame perfect equilibrium.

By construction $u_i(m_i - \tilde{g}_i, \tilde{T}) - u_i(m_i, 0) \geq 0$ for all $i \in I$. By assumption u_i is strictly increasing and continuous, hence there exists $\epsilon > 0$ and $\mathbf{g}' \in B(\tilde{\mathbf{g}}(\tilde{T}), \epsilon)$ such that $u_i(m_i - g'_i, \iota'\mathbf{g}') - u_i(m_i, 0) > 0$ for all $i \in I$. The subgame at $\iota'\mathbf{g}'$ is precisely the model of Bagnoli and Lipman (1989) for which they show that contributions which sum to $\iota'\mathbf{g}'$ are undominated-perfect equilibria. Then no total contributions below $\iota'\mathbf{g}'$ can be subgame perfect or the fundraiser would have a profitable deviation. Since $\mathbf{g}'$ can be arbitrarily close to $\tilde{T}$, $\mathbf{g}(T) \geq \tilde{T}$. By construction, there is no Nash equilibrium of the subgame, $\mathbf{g}^\star \in \prod_{i\in I}[0, m_i]$ such that $\iota'\mathbf{g}^\star > \tilde{T}$. Hence the donors' strategies must dictate $\mathbf{g}(T) \leq \tilde{T}$ for all $T \in \mathbf{R}$. Therefore $\mathbf{g}(T) = \tilde{T}$ and $T = \tilde{T}$. $\square$

Proposition 3. $\left\{\tilde{T}, \mathbf{g}\right\}$*, generates more public good than any vector of donations that Pareto dominates the zero provision outcome.*

Proof. Let $\mathbf{g}$ be a vector of donations that Pareto dominates the zero provision outcome. Then $\iota'\mathbf{g} \leq \Lambda(\iota'\mathbf{g})$ and there exists a fixed-point of Λ, T, such that $\iota'\mathbf{g} \leq T$. As $\tilde{T} \geq T$ the proposition holds. $\square$

Corollary 1. $\left\{\tilde{T}, \mathbf{T}\right\}$*, generates more public good than any socially optimal vector of donations that Pareto dominates the zero provision outcome.*

A.4.2 Proposition 2

A.5 PAYOFF TABLES

A.6 THEORY

A.6.1 Game Definitions

In this subsection are the formal, working definitions for the voluntary contribution game (VCG), goal game, and the no goal game. However, these definitions should closely match readers expectations and be in no way surprising.

Definition 4. *Let*

1. *a set of* $n \in \mathbf{N}$ *donors,* I*,*
2. *a set of endowments of a private good,* $\{m_i\}_{i \in I}$*, such that each donor* $i \in I$ *is endowed with some positive amount of a private good,* $m_i \in \mathbf{R}_{++}$*,* $\sum_{i \in I} m_i = M$*,*
3. *a set of utility functions,* $\{u_i\}_{i \in I}$*,* $u_i : [0, m_i] \times \mathbf{R} \to \mathbf{R}$*,* $u_i(x, G)$*, which depends on consumption of a private good,* x*, and total production of the public good,* G*, such that each donor* $i \in I$ *is associated with* u_i*.*
4. *and* $G : \mathbf{R}_+ \to \mathbf{R}$ *be the production function the public good.*

Your Investment	Investment by Other Group Member												
	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
0	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
1	4.4	4.9	5.4	5.9	6.4	6.9	7.4	7.9	8.4	8.9	9.4	9.9	10.4
2	4.8	5.3	5.8	6.3	6.8	7.3	7.8	8.3	8.8	9.3	9.8	10.3	10.8
3	5.2	5.7	6.2	6.7	7.2	7.7	8.2	8.7	9.2	9.7	10.2	10.7	11.2
4	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0	10.5	11.0
5	4.8	5.3	5.8	6.3	6.8	7.3	7.8	8.3	8.8	9.3	9.8	10.3	10.8
6	4.6	5.1	5.6	6.1	6.6	7.1	7.6	8.1	8.6	9.1	9.6	10.1	10.6
7	4.4	4.9	5.4	5.9	6.4	6.9	7.4	7.9	8.4	8.9	9.4	9.9	10.4
8	3.6	4.1	4.6	5.1	5.6	6.1	6.6	7.1	7.6	8.1	8.6	9.1	9.6
9	2.8	3.3	3.8	4.3	4.8	5.3	5.8	6.3	6.8	7.3	7.8	8.3	8.8
10	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0
11	1.2	1.7	2.2	2.7	3.2	3.7	4.2	4.7	5.2	5.7	6.2	6.7	7.2
12	0.4	0.9	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9	5.4	5.9	6.4

Figure 24: Payoff table given to subjects in the no threshold condition

Given $\left\{I, G, \{u_j, m_j\}_{j=1}^{n}\right\}$, *the* voluntary contribution game *(VCG) is the game where each agent* i, *chooses* $d_i \in [0, m_i]$ *to donate to the production of a public good. The payoff to agent* i *is then* $u_i(m_i - d_i, G(\sum_{i \in I} d_i))$.

Definition 5. *A goal game,* $\left\{c, F, F', G, I, \{u_j, m_j\}_{j=1}^{n}\right\}$, *consists of:*

1. *a charity,* c, *which monopolizes production of the public good,*
2. *a set of* $n \in \mathbf{N}$ *donors,* I,
3. *a set of endowments of a private good,* $\{m_i\}_{i \in I}$, *such that each donor* $i \in I$ *is endowed with some positive amount of a private good,* $m_i \in \mathbf{R}_{++}$, $\sum_{i \in I} m_i = M$,

Payoff Chart

Other Group Member

You	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
0	**4.0** *4.0*	**4.5** *4.4*	**5.0** *4.8*	**5.5** *5.2*	**6.0** *5.0*	**6.5** *4.8*	**7.0** *4.6*	**7.5** *4.4*	**8.0** *3.6*	**8.5** *2.8*	**9.0** *2.0*
1	**4.4** *4.5*	**4.9** *4.9*	**5.4** *5.3*	**5.9** *5.7*	**6.4** *5.5*	**6.9** *5.3*	**7.4** *5.1*	**7.9** *4.9*	**8.4** *4.1*	**8.9** *3.3*	**9.4** *2.5*
2	**4.8** *5.0*	**5.3** *5.4*	**5.8** *5.8*	**6.3** *6.2*	**6.8** *6.0*	**7.3** *5.8*	**7.8** *5.6*	**8.3** *5.4*	**8.8** *4.6*	**9.3** *3.8*	**9.8** *3.0*
3	**5.2** *5.5*	**5.7** *5.9*	**6.2** *6.3*	**6.7** *6.7*	**7.2** *6.5*	**7.7** *6.3*	**8.2** *6.1*	**8.7** *5.9*	**9.2** *5.1*	**9.7** *4.3*	**10.2** *3.5*
4	**5.0** *6.0*	**5.5** *6.4*	**6.0** *6.8*	**6.5** *7.2*	**7.0** *7.0*	**7.5** *6.8*	**8.0** *6.6*	**8.5** *6.4*	**9.0** *5.6*	**9.5** *4.8*	**10.0** *4.0*
5	**4.8** *6.5*	**5.3** *6.9*	**5.8** *7.3*	**6.3** *7.7*	**6.8** *7.5*	**7.3** *7.3*	**7.8** *7.1*	**8.3** *6.9*	**8.8** *6.1*	**9.3** *5.3*	**9.8** *4.5*
6	**4.6** *7.0*	**5.1** *7.4*	**5.6** *7.8*	**6.1** *8.2*	**6.6** *8.0*	**7.1** *7.8*	**7.6** *7.6*	**8.1** *7.4*	**8.6** *6.6*	**9.1** *5.8*	**9.6** *5.0*
7	**4.4** *7.5*	**4.9** *7.9*	**5.4** *8.3*	**5.9** *8.7*	**6.4** *8.5*	**6.9** *8.3*	**7.4** *8.1*	**7.9** *7.9*	**8.4** *7.1*	**8.9** *6.3*	**9.4** *5.5*
8	**3.6** *8.0*	**4.1** *8.4*	**4.6** *8.8*	**5.1** *9.2*	**5.6** *9.0*	**6.1** *8.8*	**6.6** *8.6*	**7.1** *8.4*	**7.6** *7.6*	**8.1** *6.8*	**8.6** *6.0*
9	**2.8** *8.5*	**3.3** *8.9*	**3.8** *9.3*	**4.3** *9.7*	**4.8** *9.5*	**5.3** *9.3*	**5.8** *9.1*	**6.3** *8.9*	**6.8** *8.1*	**7.3** *7.3*	**7.8** *6.5*
10	**2.0** *9.0*	**2.5** *9.4*	**3.0** *9.8*	**3.5** *10.2*	**4.0** *10.0*	**4.5** *9.8*	**5.0** *9.6*	**5.5** *9.4*	**6.0** *8.6*	**6.5** *7.8*	**7.0** *7.0*

Figure 25: Payoff table given to subjects when re-running the no threshold condition

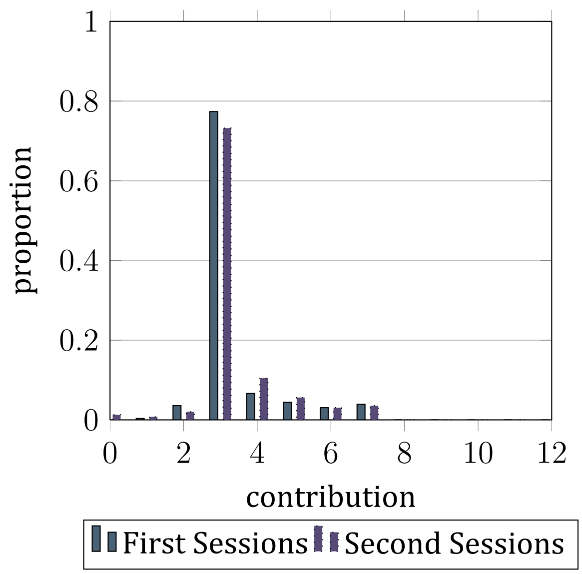

Figure 26: Distribution of individual contributions in no threshold treat, for first and second wave of sessions

4. *a production function for the public good,* $G : \mathbf{R}_+ \times \Omega \to \mathbf{R}$*,* $\Omega \subset \mathbf{R}$*,* $G(D, \omega)$*, which depends on total donations of the private good,* D*, and a random input,* ω*,*
5. *a distribution function* $F : \Omega \to [0, 1]$*, for the random production input* ω*,*
6. *a distribution function* $F' : \Omega \to [0, 1]$*, for the random production input* ω*, which second-order stochastically dominates* F*,*
7. *a set of utility functions,* $\{u_i\}_{i \in I}$*,* $u_i : [0, m_i] \times \mathbf{R} \to \mathbf{R}$*,* $u_i(x, G)$*, which depends on consumption of a private good,* x*, and total production of the public good,* G*, such that each donor* $i \in I$ *is associated with* u_i*.*

The game has a continuum of information sets, but of only two types. At the first information set the charity, c*, chooses a goal for donations to the public good,* $\tilde{D} \in \mathbf{R}_+$*.*

Each possible goal determines the information set to which the game then moves. At the second information set, the donors, I*, with full knowledge of* $\tilde{D}$*, simultaneously choose how to allocate their endowments between private consumption and donations to the charity for production of the public good,* $d_i \in [0, m_i]$*.*

After donations are allocated, production of the public good takes place. If total donations equal the goal, $\sum_{i \in I} d_i = \tilde{D}$, then the random input, ω, is drawn from distribution F'. Otherwise, the random input, ω, is drawn from distribution F. The donors receive a payoff from their allocations of private good and the realized quantity of public good, i.e., $u_i\left(m_i - d_i, G(\sum_{i \in I} d_i, \omega)\right)$. *The charity receives a payoff from the total collected donations,* $\sum_{i \in I} d_i$.

Definition 6. *Given a goal game, $\left\{C, F, F', G, I, \{u_j, m_j\}_{j=1}^{n}\right\}$, the* no goal game *is a VCG where the set of donors, endowments, and utilities are identical to their goal game counterparts. Additionally, the public good production function in the no goal game matches the goal game production function, with the random input always having distribution F. Hence a no goal game can be described by a collection $\left\{F, G, I, \{u_j, m_j\}_{j=1}^{n}\right\}$.*

A.6.2 Mathematical Results

This subsection presents the proofs and intermediate results required to prove Proposition 3. We begin by stating the model assumptions, equilibrium definitions, and defining some commonly used notations. Then proofs showing that the goal game is well-defined, and characterizing necessary conditions for equilibria follow in A.6.4. The results that directly lead to Proposition 3 are found in A.6.2.2 and A.6.2.3.

The model assumptions are gathered together in Assumption 1.

Assumption 1. *Let $\left\{c, F, F', G, I, \{u_j, m_j\}_{j=1}^{n}\right\}$ be a goal game. We assume the following:*

1. *u_i is strictly concave for all $i \in I$.*
2. *G is concave and strictly increasing in both arguments.*
3. *$\left\{F, F', G, \{u_i\}_{i \in I}\right\}$ are such that for all $i \in I$,*

$$\mathrm{E}_F\, u_i\left(m_i - d_i, G(D, \omega)\right) < \infty, \text{ and}$$
$$\mathrm{E}_{F'}\, u_i\left(m_i - d_i, G(D, \omega)\right) < \infty, \text{ for all } d_i \in [0, m_i], D \in [0, M], i \in I.$$

4. *$\left\{F, F', G, \{u_i\}_{i \in I}\right\}$ are such that for all $i \in I$, the donor objective functions have a* continuous maximum*.*

For the remainder of this section the results will reference a representative goal game which we define here; labelling some objects that reoccur frequently in the results that follow. Additional mathematical structures will need to be defined, but we place their definitions closer to the results that require them.

Definition 7. *Let* $X = \left\{c, F, F', G, \{i_j, u_j, m_j\}_{j=1}^{n}\right\}$ *be a goal game satisfying Assumption 1. Let*

$$\mathbf{br} : \mathbf{R}_+ \times \prod_{i \in I}[0, m_i] \to P\left(\prod_{i \in I}[0, m_i]\right), br_i(\tilde{D}, \mathbf{d}) \subset [0, m_i],$$

be the best response of the i^{th} *donor to announcement* $\tilde{D}$ *and donations* **d** *in the goal game, and*

$$\mathbf{br}^{\star} : \prod_{i \in I}[0, m_i] \to P\left(\prod_{i \in I}[0, m_i]\right), br_i(\mathbf{d}) \subset [0, m_i],$$

be the best response of the i^{th} *donor to donations* **d** *in the no goal game.*

Let $\mathcal{A}$ *be the set of Nash equilibria of the no goal game.*

Let $M = \sum_{i \in I} m_i$*, be the total amount of private good available.*

Before proceeding through the results it is worthwhile to be precise about the equilibria, and equilibrium outcomes we are seeking. The following definitions define the subgame perfect equilibrium, and subgame perfect outcomes in the context of the goal game.

Definition 8. *A subgame perfect equilibrium of the goal game* X *is a collection of the charities announced goal,* $\tilde{D} \in \mathbf{R}_+$*, and donor strategies* $\mathbf{s} \in \left(\prod_{i \in I}[0, m_i]\right)^{\mathbf{R}_+}$ *such that* $(\tilde{D}, \mathbf{s})$ *is a Nash equilibrium:*

1. *the donors are best responding to each other and the charity,* $\mathbf{s}(\tilde{D}) \in \mathbf{br}(\tilde{D}, \mathbf{d})$*,*
2. *the charity is best responding to the donor's strategies,* $\tilde{D} \in \underset{D' \in \mathbf{R}_+}{\text{argmax}}\ \iota' \mathbf{s}(D')$

and moreover the donors strategies dictate best responses even off the equilibrium path,

3 $\mathbf{s}(D') \in \mathbf{br}(D', \mathbf{s}(D')), \forall D' \in \mathbf{R}_+$.

Definition 9. A subgame perfect outcome *of the goal game* X *is a donation vector* $\mathbf{d} \in \prod_{i \in I}[0, m_i]$*, such that there exists a subgame perfect equilibrium* $(\tilde{D}, \mathbf{s}) \in \mathbf{R}_+ \times \left(\prod_{i \in I}[0, m_i]\right)^{\mathbf{R}_+}$ *in which* $\mathbf{s}(\tilde{D}) = \mathbf{d}$*. Moreover,* $D = \iota' \mathbf{d}$ *is* a level of total donations supported in a subgame perfect equilibrium.

A.6.2.1 Preliminaries First some minor results in order to show that our problem is well-defined. The existence of solutions for the donor's problem and equilibria for the subgame need to be established. The proofs are straight-forward and so are placed in A.6.4. The conclusions are summarized in Lemma 3.

Lemma 3. *1. a solution exists to the i^{th} donor's problem for all goals $\tilde{D} \in \mathbf{R}_+$ and other donations $\mathbf{d}_{-i} \in \prod_{j=1,j\neq i}^{n}[0, m_j]$,*

2. there exists at least one Nash equilibrium vector of donations, $\mathbf{d}^\star \in \prod_{i=1}^{n}[0, m_i]$, for the no goal game,

3. and for every announced goal, $\tilde{D} \in \mathbf{R}_+$, there exists a Nash equilibrium of the subgame.

The following two Lemmas (4 and 5) establish necessary conditions for subgame perfect equilibria of the goal game and that equilibria from the game without a goal are incentive compatible in the goal game. Whereas many of the other results are heavily mathematical, these have the most "economic" style proofs and are probably the most important.

Lemma 4 provides some structure to the set of subgame perfect equilibria. It implies that equilibria can be classified into two types: those where the total donations equal the announcement, and those where total donations equal the amount in a Nash outcome of the game without a goal. By Lemma 3, our assumptions imply that an equilibrium of the subgame exists for every announcement. Thus announcements either "succeed" with total contributions equaling the announcement, or "fail" with total contributions equaling a no goal outcome. Note that both outcomes may be equilibria of the subgame. If the announcement is significantly larger than a no goal outcome, then no one donor will find it optimal to supply the difference. Then both strategies that lead to an no goal equilibrium outcome and strategies that lead to meeting the announcement satisfy the best response constraints.

Lemma 5 has two important implications. The first is that announcing the total donations in an equilibrium of the no goal game can succeed in the goal game. This implies that a number of important sets are nonempty. The second is that an incentive compatible donation vector exists for any goal that equals a Nash outcome of the no goal game. Hence by considering all goals which can be supported by incentive compatible donations, we are also considering all equilibrium total donations.

Lemma 4. *Given a goal* $\tilde{D} \in \mathbf{R}_+$*, let* $\mathbf{d} \in \prod_{i \in I}[0, m_i]$*, be a subgame perfect, equilibrium outcome of donations for the subgame. Then if* $\mathbf{d} \notin \mathcal{A}$*,* $\iota'\mathbf{d} = \tilde{D}$*.*

Proof. Suppose the lemma does not hold. Then there exists an equilibrium outcome of the subgame, $\mathbf{d}' \in \prod_{i \in I}[0, m_i]$, where the amount collected does not equal the goal, $\iota'\mathbf{d}' \neq D$ and $\mathbf{d}' \notin \mathcal{A}$.

Since the goal is not matched and $\mathbf{d}'$ is an equilibrium outcome of the subgame, the donors must be best responding to the no goal problem as well,

$$\mathbf{d}' \in \mathbf{br}(D, \mathbf{d}') = \mathbf{br}^{\star}(\mathbf{d}').$$

Hence $\mathbf{d}'$ is a Nash equilibrium in the game without a goal. Thus, $\mathbf{d}' \in \mathcal{A}$ by definition. However, this contradicts the assumption that $\mathbf{d}' \notin \mathcal{A}$. Hence, it must be that $\mathbf{d}'$ does not exist. Therefore, the lemma holds. □

Lemma 5. *An equilibrium of the game without a goal,* $\mathbf{d}^{\star}$*, in combination with the goal which equals the total donations,* $\iota'\mathbf{d}^{\star}$*,* $(\iota'\mathbf{d}^{\star}, \mathbf{d}^{\star})$*, is a Nash equilibrium of the subgame.*

Proof. This follows from the observation that $d_i^{\star} = \iota'\mathbf{d}^{\star} - \iota'\mathbf{d}_{-i}, \forall i \in I$. Hence the no goal best-response coincides with the donation level needed to reach the goal. Since $\mathbf{d}^{\star}$ is a Nash equilibrium of the game without a goal,

$$v_i\left(m_i - d_i, d_i + \iota'\mathbf{d}_{-i}; F\right) \leq v_i\left(m_i - d_i^{\star}, d_i^{\star} + \iota'\mathbf{d}_{-i}; F\right), \forall d_i \in [0, m_i].$$

By the risk-aversion assumption,

$$v_i\left(m_i - d_i^{\star}, d_i^{\star} + \iota'\mathbf{d}_{-i}; F\right) < v_i\left(m_i - d_i^{\star}, d_i^{\star} + \iota'\mathbf{d}_{-i}; F'\right).$$

Thus,

$$v_i\left(m_i - d, d_i + \iota'\mathbf{d}_{-i}; F\right) < v_i\left(m_i - d_i^{\star}, d_i^{\star} + \iota'\mathbf{d}_{-i}; F'\right), \forall d_i \in [0, m_i].$$

Hence, $d_i^{\star}$ remains a best-response for all $i \in I$. Therefore, $(\iota'\mathbf{d}^{\star}, \mathbf{d}^{\star})$ is a Nash equilibrium of the subgame. □

Lemma 6 is needed to help locate the lower bound for equilibrium outcomes in the goal game. It establishes that announcing a goal close enough to a Nash equilibrium outcome will "disrupt" the equilibrium under any subgame perfect donor strategies. The result follows from the assumption of strictly risk averse donors. Hence, for each donor there is an open ball of donations around an equilibrium of the no goal game where the donor strictly prefers meeting the goal herself.

Lemma 6. *Let* $\mathbf{d}^\star \in \prod_{i\in I}[0, m_i]$ *be a Nash equilibrium of the game without a goal. Then there exists an interval,* $(\iota'\mathbf{d}^\star - \epsilon^L(\mathbf{d}^\star), \iota'\mathbf{d}^\star + \epsilon^U(\mathbf{d}^\star))$, $\epsilon^L, \epsilon^U > 0$, *such that for a goal* $\tilde{D} \in (\iota'\mathbf{d}^\star - \epsilon^L, \iota'\mathbf{d}^\star + \epsilon^U)\setminus\{\iota'\mathbf{d}^\star\}$, $\epsilon^L(\mathbf{d}^\star), \epsilon^U(\mathbf{d}^\star) > 0$, $\mathbf{d}^\star$ *is not a subgame perfect outcome of the goal game. Moreover,* $\mathbf{d}^\star$ *is a subgame perfect outcome of the goal game for goals outside that interval, i.e.,* $\tilde{D} \in \mathbf{R}_+ \setminus (\iota'\mathbf{d}^\star - \epsilon^L(\mathbf{d}^\star), \iota'\mathbf{d}^\star + \epsilon^U(\mathbf{d}^\star))$.

Proof. The proof follows from the strict concavity of the donor objective functions. Define $g_i : \mathbf{R}_+ \to \mathbf{R}$ be defined by

$$g_i(\epsilon) = v_i(m_i - d_i^\star - \epsilon, \iota'\mathbf{d}^\star + \epsilon; F') - v_i(m_i - d_i^\star, \iota'\mathbf{d}^\star; F).$$

Note that the second term of the definition is just a constant in this context. Hence g_i inherits strict concavity and continuity from v_i. Thus $\{\epsilon \in \mathbf{R}_+ \,|g_i(\epsilon) > 0\}$ is nonempty, convex, and open, i.e., an interval $(\epsilon_i^L, \epsilon_i^U)$. On its complement $\mathbf{R}_+ \setminus X_i$, $g_i \leq 0$.

Let $\epsilon^L = \max\left\{\epsilon_1^L, \epsilon_2^L, \ldots, \epsilon_n^L\right\}$ and $\epsilon^U = \max\left\{\epsilon_1^U, \epsilon_2^U, \ldots, \epsilon_n^U\right\}$. Consider a goal$D \in (\iota'\mathbf{d}^\star - \epsilon^L, \iota'\mathbf{d}^\star + \epsilon^U)\setminus\{\iota'\mathbf{d}^\star\}$. Let $\epsilon = D - \iota'\mathbf{d}^\star$. By construction $0 < |\epsilon| < \max\left\{\epsilon^L, \epsilon^U\right\}$. Hence there exists at least one $i \in I$ such that $v_i(m_i - d_i^\star - \epsilon, D; F') - v_i(m_i - d_i^\star, \iota'\mathbf{d}^\star; F) > 0$. Thus $d_i^\star$ is not a best response for i, and $\iota'\mathbf{d}^\star$ is not a subgame perfect outcome.

Similarly, consider a goal $D' \in \mathbf{R}_+ \setminus (\iota'\mathbf{d}^\star - \epsilon^L, \iota'\mathbf{d}^\star + \epsilon^U)$. Since $\mathbf{d}^\star$ is a Nash equilibrium of the no goal game it must be the case that for all $i \in I$, $v_i(m_i - d_i^\star, \iota'\mathbf{d}^\star; F) \geq v_i(m_i - d, \iota'\mathbf{d}_{-i}^\star; F), \forall d \in [0, m_i]$.

As v_i is strictly concave, $d_i^\star$ is the unique maximizer. Let $\epsilon' = D' - \iota'\mathbf{d}^\star$. By construction $|\epsilon'| \geq \max\left\{\epsilon^L, \epsilon^U\right\}$. Hence $g_i \leq 0$ for all $i \in I$. Thus for all $i \in I$, $v_i(m_i - d_i^\star - \epsilon, D; F') \leq v_i(m_i - d_i^\star, \iota'\mathbf{d}^\star; F)$. As such, $d_i^\star$ is a best response for all $i \in I$. Therefore $\mathbf{d}^\star \in \mathbf{br}(D', \mathbf{d}^\star)$ and the lemma holds. □

A.6.2.2 Donor Constraints Lemmas 4 and 5 provide the key tools to characterizing incentive compatible donations. Since equilibria only occur at the announcement level, the donor's problem, *in equilibrium*, can be greatly simplified. Consider the choice of a donor when in an equilibrium, $(\tilde{D}, \mathbf{d})$,

Meet the goal	Or	Deviate to outside
$v_i(m_i - d_i, \tilde{D}; F')$		$w_i(\iota' \mathbf{d}_{-i})$

The necessary condition that equilibrium donations must equal the goal means that the outside option of the donor can be valued in equilibrium without explicitly considering other donations.

Meet the goal	Or	Deviate to outside
$v_i(m_i - d_i, \tilde{D}; F')$		$w_i(\tilde{D} - d_i)$

Hence for any potential goal the set of incentive compatible donations (that could form an equilibrium) can be found for each donor *independently*. If it happens that there exist incentive compatible donations that sum to the goal then those donations and goal are a potential equilibrium. This problem can be framed as a fixed-point problem and its solutions characterized.

In order to further analyze those goals that can succeed it is helpful to define indicator functions for goals that are incentive compatible for donors to meet. Something akin to,

$$ICC_i(\tilde{D}, d) = v_i(m_i - d, \tilde{D}; F') - w_i(\tilde{D} - d).$$

Unfortunately, when $\tilde{D} < d$ or $d > m_i$ this is not defined. In these regions meeting the announcement is not feasible and hence not useful, so ICC_i needs to be extended with negative values. Setting $ICC_i = -1$ in the infeasible regions would suffice, but then it is no longer continuous. Since continuity simplifies the proofs that follow, ICC_i is extended with a continuous function that takes on negative values.[1]

[1] Tietze's extension theorem can be used to show that such an extension exists, see A.6.5 for a proof.

Definition 10. *Choose a donor $i \in I$. Construct,*

$$ICC_i : \mathbf{R}^2_+ \to \mathbf{R},$$

$$ICC_i(\tilde{D}, d_i) \equiv \begin{cases} v_i(m_i - d_i, \tilde{D}; F') - w_i(\tilde{D} - d_i) & : d \leq m_i \text{ and } \tilde{D} \geq d_i \\ g_i(\tilde{D}, d_i) & : \text{ otherwise} \end{cases}$$

where $g < 0$ and is such that ICC_i is continuous. If $ICC_i(\tilde{D}, d_i) > 0$ then contributing d_i to meet the goal is strictly preferred. If $ICC_i(\tilde{D}, d_i) < 0$ contributing $d_i^\star \in br_i^\star(\tilde{D} - d_i)$ is preferred. If $ICC_i(\tilde{D}, d_i) = 0$ then she is indifferent between contributing $d_i^\star$ and d_i.

The results in this subsection use properties of the incentive compatibility constraint to establish compactness on sets successively closer to the set of subgame perfect outcomes. Linking these sets are three correspondences which we define in Definition 11. First, the possible outcomes from announcing a particular goal are characterized. It follows quickly from the previous results that these outcomes form a compact set. Compactness is then extended to the set of outcomes from all relevant goals. The extension relies on establishing that the correspondence linking goals to outcomes is upper hemi-continuous. The end result of this process is a set $\mathcal{S}$ that is slightly larger than the set of subgame perfect outcomes.

Definition 11. *Define $\underline{\mathbf{d}}$ be a Nash equilibrium of the game without a goal which generates the least total donations, i.e., $\underline{\mathbf{d}} \in \underset{\mathbf{d} \in \mathcal{A}}{\text{argmin}}\, \iota' \mathbf{d}$.*

Define three correspondences that link the incentive compatibility constraints of individual donors to compatible levels of total donations.

- $\gamma_i : [\iota' \underline{\mathbf{d}}, M] \to \mathcal{P}([0, m_i])$, $\gamma_i(\tilde{D}) \equiv \left\{ d \in [0, m_i] \middle| ICC_i(\tilde{D}, d) \geq 0 \right\}$, *the set of compatible donations for donor i given goal $\tilde{D}$.*
- $\Gamma : [\iota' \underline{\mathbf{d}}, M] \to \mathcal{P}\left(\prod_{i \in I} [0, m_i]\right)$, $\Gamma(\tilde{D}) \equiv \prod_{i \in I} \gamma_i(\tilde{D})$, *the set of compatible donation vectors given goal $\tilde{D}$.*
- $\Lambda : [\iota' \underline{\mathbf{d}}, M] \to [0, M]$, $\Lambda(\tilde{D}) \equiv \left\{ D \in [0, M] \middle| \exists \mathbf{d} \in \Gamma(\tilde{D}), \text{ s.t. } \sum_{i \in I} d_i = D \right\}$, *the set of levels of total donations that are compatible given goal $\tilde{D}$.*

These enable us to define the set, $\mathcal{S} \equiv \{ D \in [\iota' \underline{\mathbf{d}}, M] \,|\, D \in \Lambda(D) \}$ the set of fixed-points of Λ.

Lemma 7. Γ *is upper hemi-continuous.*

Proof. The proof follows by showing that Γ is a compact correspondence and then establishing the conditions under which it is upper hemi-continuous.

Claim: Γ is a compact correspondence.

▪ *Proof of Claim:* Note that ICC_i is continuous. Then the pre-image $ICC_i^{-1}([0,\infty))$ is closed. As $ICC_i^{-1}([0,\infty)) \subset [0,m_i]$, it is bounded as well. Hence $\gamma_i = ICC_i^{-1}([0,\infty))$ is compact. Thus γ_i is a compact correspondence and Γ as the product of compact correspondences is a compact correspondence as well. ▪

Let $\{D_n\}_{n\in\mathbf{N}}, D_n \in [\iota'\underline{\mathbf{d}}, M], \quad \forall n \in \mathbf{N}, D_n \to D$ be a convergent sequence. Let $\{y_n\}_{n\in\mathbf{N}}, y_n \in \Gamma(D_n), \quad \forall n \in \mathbf{N}$, be a sequence.

In order to prove the lemma, it suffices to show that:

1. $\Gamma(D) \neq \emptyset$, for all $D \in [\iota'\underline{\mathbf{d}}, M]$.
2. $\{y_n\}$ has a convergent subsequence, $\{y_{n_k}\}$,
   such that $\lim y_{n_k} = y \in \Gamma(D)$.

Claim: $\Gamma(D) \neq \emptyset$, for all $D \in [\iota'\underline{\mathbf{d}}, M]$.

▪ *Proof of Claim:* Let $D \in [\iota'\underline{\mathbf{d}}, M]$.

Consider the incentive compatibility constraint for a donor with endowment $m_i \in \mathbf{R}_{++}$ and no goal best response function, $br_i^\star$, when evaluating a donation of $d \in [0,m_i]$. If the donation coincides with the best response when there is no goal, $br_i^\star(D-d) = d$, then the assumption of risk aversion ensures d is incentive compatible. Then $ICC_i(D,d) > 0$ and $\gamma_i(D) \neq \emptyset$. Hence it is sufficient to show that there exists $d \in [0,D]$ such that $br_i^\star(D-d) = d$ for the claim to hold.

Note that $br_i^\star$ is decreasing in outside donations. Additionally, note that $\iota'\underline{\mathbf{d}}_{-i} < D$. Hence $br_i^\star(D) \leq br_i^\star(\iota'\underline{\mathbf{d}}_{-i}) \leq D$. Let $d = D - \iota'\underline{\mathbf{d}}_{-i}$. Then $br_i^\star(D-d) = br_i^\star(\iota'\underline{\mathbf{d}}_{-i}) = \underline{d}_i \leq d$. As $br_i^\star$ is continuous, it must be the case that $br_i^\star(D-d) = d$ will be satisfied for at least one $d \in [0, D - \iota'\underline{\mathbf{d}}_{-i}]$. Hence $ICC_i(D, d_i^\star) > 0$, and $\gamma_i(D) \neq \emptyset$. Since i was chosen arbitrarily this holds for all $i \in I$. Therefore $\Gamma(D) \neq \emptyset$ for all $D \in [\iota'\underline{\mathbf{d}}, M]$, and the claim holds. ▪

Claim: There exists $\{y_{n_k}\}$, a subsequence of $\{y_n\}$, such that $\lim y_{n_k} = y \in \prod_{i\in I}[0,m_i]$.

▪ *Proof of Claim:* By the definition of $\{y_n\}_{n\in\mathbf{N}}$. $y_n \in \Gamma(D_n)$ for all $n \in \mathbf{N}$. Hence $y_n \in \prod_{i\in I}[0, m_i]$ for all $n \in \mathbf{N}$. As $\prod_{i\in I}[0, m_i]$ is compact there exists $\{y_{n_k}\}$, a subsequence of $\{y_n\}$, such that $\lim y_{n_k} = y \in \prod_{i\in I}[0, m_i]$. Therefore the claim holds. ▪

Claim: Let $\{z_n\}_{n\in\mathbf{N}}$, $z_n \in \Gamma(D_n)$ for all $n \in \mathbf{N}$, be a convergent sequence. Then $\lim z_n = z \in \Gamma(D)$.

▪ *Proof of Claim:* By the definition of Γ, it must be the case that $ICC_i(D_n, z_{ni}) \geq 0$ for all $n \in \mathbf{N}$. By the continuity of ICC_i, $\lim ICC_i(D_n, z_{ni}) = ICC_i(D, z_i) \geq 0$. Hence, $z_i \in \gamma_i(D)$ for all $i \in I$ and $z \in \Gamma(D)$. Therefore the claim holds. ▪

The first claim establishes condition 1 for upper hemi-continuity. By the second claim $\{y_n\}$ has a convergent subsequence, $\{y_{n_k}\}$. By the third claim that subsequence must converge to an element of $\Gamma(D)$, $\lim y_{n_k} = y \in \Gamma(D)$. Hence, together they show condition 2 for upper hemi-continuity. Therefore the lemma holds. □

Lemma 8. *Λ is upper hemi-continuous.*

Proof. The lemma follows from the upper hemi-continuity of Γ as $\Lambda(D)$ is a linear transformation of $\Gamma(D)$. □

Lemma 9. *$\mathcal{S}$ is nonempty and compact.*

Proof. Lemma 5 implies there exists at least one fixed-point of Λ, namely the total donations in a Nash equilibrium of the no goal game. Hence, $\mathcal{S} \neq \emptyset$.

Let $\mathcal{B} \equiv \{(D, D') \in [\iota'\underline{\mathbf{d}}, M]^2 \,|\, D' \in \Lambda(D)\}$, be the graph of incentive compatible total donations and $\mathcal{I} = \{(D, D') \in [0, M]^2 \,|\, D = D'\}$ be the graph of the $45°$ line. Observe that $D \in \mathcal{S} \Leftrightarrow (D, D) \in \mathcal{B} \cap \mathcal{I}$, the intersection of the graph of Λ and the $45°$ line. The upper hemi-continuity of Λ, established in Lemma 8, implies that $\mathcal{B}$ is closed. By construction $\mathcal{I}$ is closed. It also follows from their construction that both $\mathcal{B}$ and $\mathcal{I}$ are bounded. Thus, $\mathcal{B} \cap \mathcal{I}$ is the intersection of closed and bounded sets and is itself closed and bounded. Hence $\mathcal{B} \cap \mathcal{I}$ is a compact set. Then by virtue of their equivalence, $\mathcal{S}$ is also compact. Hence, $\mathcal{S}$ is nonempty and compact and the claim holds. □

A.6.2.3 Charity Constraints In this subsection the charity's maximization constraint is used to refine the set of outcomes developed from the donor's incentive compatibility constraints. The objects defined in Definition 12 act as outside options for the donors and the charity. $\iota'\mathbf{d}_0$ is the largest Nash equilibrium outcome of the no goal game that the charity can force the donors to reach. No subgame perfect outcome of the goal game can collect fewer donations, or the charity could deviate and receive $\iota'\mathbf{d}_0$ in donations. On the other hand, the donors "worst threat" is $\iota'\mathbf{d}_L$. Nothing below $\iota'\mathbf{d}_L + \epsilon^U(\mathbf{d}_L)$ can be a subgame perfect equilibrium as the subgame perfection constraint on strategies ensures there is some goal that generates greater donations for the charity. The set of subgame perfect outcomes, $\mathbb{S}$, is reached by adjusting the lower bound of the set of incentive compatible outcomes, $\mathcal{S}$, to account for these outside options.

The special case of a unique Nash equilibrium in the no goal game is simplest to describe. With a unique equilibrium, $\mathbf{d}_0 = \mathbf{d}_L$. The charity can always announce an impossible goal, e.g., $2M$, and receive $\iota'\mathbf{d}_0$, as it is the only equilibrium of the subgame. With a unique equilibrium the subgame perfection constraints on donors' strategies are particularly strong. All outcomes must be either the no goal Nash outcome, $\iota'\mathbf{d}_0$, or meet the announced goal. Hence, for announcements in the region defined in Lemma 6, $[\iota'\mathbf{d}_0, \iota'\mathbf{d}_0 + \epsilon^U(\mathbf{d}_0))$ where $\mathbf{d}_0$ is not subgame perfect, the only possibility is meeting the announced goal. Thus donors are compelled to meet a goal of $\iota'\mathbf{d}_0 + \epsilon^U(\mathbf{d}_0)$ with any subgame perfect strategies.

Definition 12. *Define $\mathbb{S} \subset \mathbf{R}_+$, to be the set of all total donations supported in a subgame perfect equilibrium of X.*

By Lemma 3 a Nash equilibrium of the subgame exists for every goal. The equilibrium either sums to the announced goal or is a Nash equilibrium of the no goal game. Define $\mathbf{h} : \mathbf{R}_+ \to \prod_{i\in I}[0, m_i]$ to be a function which maps each announced goal to an equilibrium vector such that $\mathbf{h}(D) \in \underset{\mathbf{d}\in\mathcal{A}}{\text{argmin}}\, \iota'\mathbf{d}$, or $\underline{\mathbf{d}}$ if such a vector does not exist. Then define:

$$D_0 \in \underset{D\in\mathbf{R}_+}{\text{argmax}}\, \iota'\mathbf{h}(D), \text{ and } \mathbf{d}_0 = \mathbf{h}(D_0),$$

$$\mathbf{d}_L \in \underset{\mathbf{d}\in\mathcal{A}}{\text{argmin}}\, \iota'\mathbf{d} + \epsilon^U(\mathbf{d}), \text{ where } \epsilon^U \text{ is as defined in Lemma 6,}$$

$$D^{min} = \max\left\{\iota'\mathbf{d}_L + \epsilon^U(\mathbf{d}_L), \iota'\mathbf{d}_0\right\}.$$

Lastly, define $\mathcal{S}' = \mathcal{S} \cap [D^{min}, \infty)$.

Lemma 10. $\mathbb{S} \subset \mathcal{S}'$.

Proof. Let $D \in \mathbb{S}$.

Lemmas 4 and 5 imply that any subgame perfect equilibrium donation vector can be supported by a goal that equals its sum, i.e., total donations equal the goal. Hence there exist subgame perfect strategies, $\mathbf{s} \in \left(\prod_{i\in I}[0, m_i]\right)^{\mathbf{R}_+}$, such that $\mathbf{s}(D) = \mathbf{d}$, $\iota'\mathbf{d} = D$.

Claim: $D \geq D^{min}$.

▪ *Proof of Claim:* From its definition, $\iota'\mathbf{d}_0$ is a lower bound on the level of donations the charity can extract from donors in a subgame perfect equilibrium. Otherwise, the charity could announce D_0 and be assured of collecting at least $\iota'\mathbf{d}_0$. Hence $D \geq \iota'\mathbf{d}_0$.

Now consider $\iota'\mathbf{d}_L + \epsilon^U(\mathbf{d}_L)$. Suppose that $D < \iota'\mathbf{d}_L + \epsilon^U(\mathbf{d}_L)$. Since D is subgame perfect, the donor strategies, $\mathbf{s}$, must map all goals greater than D to equilibria that generate less than D. Let one of these equilibria be $\mathbf{d}'$. By the definition of $\mathbf{d}_L$, it must be the case that $\iota'\mathbf{d}_L + \epsilon^U(\mathbf{d}_L) < \iota'\mathbf{d}' + \epsilon^U(\mathbf{d}')$. Hence the quantities are in the following order, $\iota'\mathbf{d}' \leq D < \iota'\mathbf{d}_L + \epsilon^U(\mathbf{d}_L) < \iota'\mathbf{d}' + \epsilon^U(\mathbf{d}')$. However, goals in the interval $[D, \iota'\mathbf{d}' + \epsilon^U(\mathbf{d}'))$ cannot be mapped to $\mathbf{d}'$. As this holds for all such equilibria, $\mathbf{d}'$ does not exist. Hence, the supposition is incorrect and $D \geq \iota'\mathbf{d}_L + \epsilon^U(\mathbf{d}_L)$.

As $D \geq \iota'\mathbf{d}_0$ and $D \geq \iota'\mathbf{d}_L + \epsilon^U(\mathbf{d}_L)$, the claim holds. ▪

As $\mathbf{s}$ is subgame perfect, $\mathbf{d} \in \mathbf{br}(D, \mathbf{d})$, which implies that $\mathbf{d}$ is incentive compatible for donors given goal D. By construction $\Lambda(D)$ is precisely the set of total donations which are incentive compatible with goal $D \in [\iota'\underline{\mathbf{d}}, M]$. Hence, $D = \iota'\mathbf{d} \in \Lambda(D)$. Thus, D is a fixed-point of Λ and is therefore an element of $\mathcal{S}'$. As D was chosen arbitrarily, $\mathbb{S} \subset \mathcal{S}'$. □

Lemma 11. $\mathcal{S}' \subset \mathbb{S}$.

Proof. The proof follows by constructing a parameterized class of subgame perfect strategies that can support any level of donations in $\mathcal{S}'$.

By the construction of $\mathcal{S}$, for a goal $D \in \mathcal{S}$, an incentive compatible donation vector, $\mathbf{d} \in \prod_{i\in I}[0, m_i]$, exists such that $\iota\mathbf{d} = D$. Let $\mathbf{g} : \mathcal{S} \to \prod_{i\in I}[0, m_i]$ be a function which maps each goal in $\mathcal{S}$ to such a vector of donations. Define the parameterized class of donor strategies,

$\mathbf{s}(D;\bar{D})$, $\bar{D} \in \mathcal{S}'$.

$$\mathbf{s}(D;\bar{D}) = \begin{cases} \mathbf{h}(D) & : D \in \mathbf{R}_+ \setminus \mathcal{S} \\ \mathbf{g}(D) & : D \in \mathcal{S} \text{ and } D \leq \bar{D} \\ \mathbf{d}_L & : D \in \mathcal{S} \text{ and } D > \bar{D} \end{cases} \cdot$$

By construction $\mathbf{s}$ is a subgame perfect strategy for the donors.

The lemma can be directly demonstrated with $\mathbf{s}$. Given the strategies, $\mathbf{s}(\cdot;\tilde{D})$, any goal D, in the interval $[0, \iota'\mathbf{d}_0)$ or $(\bar{D}, \infty)$ generates total donations strictly less than or equal to $\iota'\mathbf{d}_0$. Hence given the strategies, $\mathbf{s}(\cdot;\bar{D})$, the charity maximizes donations by announcing goal $\bar{D}$, $\iota'\mathbf{s}(\bar{D},\bar{D}) = \bar{D}$. Thus, $\left(\bar{D}, \mathbf{s}(\cdot;\bar{D})\right)$ is a subgame perfect equilibrium of the goal game X. Hence $\bar{D}$ is a level of total donations supported in a subgame perfect equilibrium; $\bar{D} \in \mathbb{S}$. As $\bar{D}$ was chosen arbitrarily from $\mathcal{S}'$, $\mathcal{S}' \subset \mathbb{S}$. □

Proposition 4. *The set of subgame perfect equilibrium outcomes, $\mathbb{S}$ is precisely $\mathcal{S}'$ and it is nonempty and compact.*

Proof. The proof follows from the previous lemmas. Lemma 10 and 11 imply that $\mathbb{S} = \mathcal{S}'$.

Compactness follows from the compactness of $\mathcal{S}$ as $\mathcal{S}' = \mathcal{S} \cap [D^{min}, \infty)$. By Lemma 5 $\mathcal{S}$ is nonempty, as D^{min} is an incentive compatible outcome and is thus in $\mathcal{S}$. Therefore the proposition holds. □

A.6.3 Equivalence of u_i and v_i

Rather than working with both utility functions, u_i, over the private and public goods, and production function G, over donations of the private good, it is often preferable to work with a single equivalent utility function v_i, over the private good and donations of the private good. The following proof establishes that such a function exists and has all the assumed properties of u_i.

Lemma 12. *Let $\left\{c, F, F', G, \{i_j, u_j, m_j\}_{j=1}^n\right\}$ be a goal game. Let the function v_i be defined by*

$$v_i(x, y; F) \equiv \mathrm{E}_F\, u_i(x, G(y, \omega)).$$

Then v_i:

1. *is strictly increasing in both arguments,*
2. *is a strictly concave function over the private good and donations of the private good,*
3. *has a continuous maximum.*

Proof. In order to establish that v_i is strictly increasing in the private good choose $x_1 < x_2 \in \mathbf{R}_+$. By the assumption that u_i is strictly increasing,

$$u_i(x_1, G(y, \omega)) < u_i(x_2, G(y, \omega)), \forall \omega \in \Omega.$$

As this holds for all ω,

$$\mathrm{E}_Z\, u_i(x_1, G(y, \omega)) < \mathrm{E}_Z\, u_i(x_2, G(y, \omega)).$$
$$v_i(x_1, y; Z) < v_i(x_2, y; Z).$$

Hence v_i is strictly increasing in the private good.

In order to establish that v_i is strictly increasing in donations choose $y_1 < y_2 \in \mathbf{R}_+$. By the assumption that u_i and G are strictly increasing,

$$u_i(x_1, G(y_1, \omega)) < u_i(x_1, G(y_2, \omega)), \forall \omega \in \Omega.$$

As this holds for all ω,

$$\mathrm{E}_Z\, u_i(x_1, G(y_1, \omega)) < \mathrm{E}_Z\, u_i(x_1, G(y_2, \omega)).$$
$$v_i(x_1, y_1; Z) < v_i(x_1, y_2; Z).$$

Hence v_i is strictly increasing in in donations.

In order to establish that v_i is strictly concave, fix $x_1, x_2, y_1, y_2 \in \mathbf{R}_+$, $\omega \in \Omega$, $\alpha \in (0, 1)$. By the assumption of strict concavity of u_i,

$$\alpha u_i\left(x_1, G(y_1, \omega)\right) + (1-\alpha) u_i\left(x_2, G(y_2, \omega)\right)$$
$$< u_i\left(\alpha x_1 + (1-\alpha)x_2, \alpha G\left(y_1, \omega\right) + (1-\alpha) G\left(y_2, \omega\right)\right).$$

Then, by the assumption of concavity of G,

$$\alpha u_i\left(x_1, G(y_1,\omega)\right) + (1-\alpha)u_i\left(x_2, G(y_2,\omega)\right)$$
$$< u_i\left(\alpha x_1 + (1-\alpha)x_2, G\left(\alpha y_1 + (1-\alpha)y_2, \omega\right)\right).$$

Since this holds for all $\omega \in \Omega$,

$$\mathrm{E}_Z\left\{\alpha u_i\left(x_1, G(y_1,\omega)\right) + (1-\alpha)u_i\left(x_2, G(y_2,\omega)\right)\right\}$$
$$< \mathrm{E}_Z\, u_i\left(\alpha x_1 + (1-\alpha)x_2, G\left(\alpha y_1 + (1-\alpha)y_2, \omega\right)\right).$$
$$\Rightarrow \alpha\, \mathrm{E}_Z\, u_i\left(x_1, G(y_1,\omega)\right) + (1-\alpha)\, \mathrm{E}\, u_i\left(x_2, G(y_2,\omega)\right)$$
$$< \mathrm{E}_Z\, u_i\left(\alpha x_1 + (1-\alpha)x_2, G\left(\alpha y_1 + (1-\alpha)y_2, \omega\right)\right)$$
$$\iff \alpha v_i(x_1, y_1; F) + (1-\alpha)v_i(x_2, y_2; Z)$$
$$< v_i(\alpha x_1 + (1-\alpha)x_2, \alpha y_1 + (1-\alpha)y_2; Z).$$

Hence, v_i satisfies the strict concavity assumption.

Since v_i is merely a relabelling of the donor objective function, the assumption that F, F', G, and u_i are such that the donor objective function has a continuous maximum applies directly. □

A.6.4 Preliminaries

The proofs in this section establish the results found in Lemma 3.

Lemma 13. *Let* $\left\{C, F, F', G, \{i_j, u_j, m_j\}_{j=1}^{n}\right\}$ *be a goal game satisfying Assumption 1. Then a solution exists to the* i^{th} *donor's problem for all goals* $\tilde{D} \in \mathbf{R}_+$ *and other donations* $\mathbf{d}_{-i} \in \prod_{j=1, j\neq i}^{n}[0, M - m_j]$.

Proof. Consider $i \in I$. Since u_i is strictly concave over a convex set it is also continuous. If $m_i < \tilde{D} - D_{-i}$, the problem is to maximize a continuous function over a compact set. Hence a solution exists. So consider the situation where $m_i \geq \tilde{D} - D_{-i}$. A solution may only fail to exist if the maximizer of $\mathrm{E}_F\, u_i\left[m_i - d_i, G(d_i + D_{-i}, \omega)\right]$, $d_i^\star$, coincides with the amount needed to reach the announced level, $\tilde{D} - D_{-i}$. It is then possible that $\mathrm{E}_F\, u_i\left[m_i - d_i^\star, G(d_i^\star + D_{-i}, \omega)\right] > \mathrm{E}_{F'}\, u_i\left[m_i - d_i^\star, G(d_i^\star + D_{-i}, \omega)\right]$. However, by the risk aversion assumption on u_i this would be a contradiction. Therefore, a solution exists. As i was chosen arbitrarily the lemma holds. □

Lemma 14. *Let* $\{i_j, u_j, m_j\}_{j=1}^n$ *be a voluntary contribution game. If* u_i *is strictly concave for all* $i \in I$ *then there exists at least one Nash equilibrium in the game. Moreover, it follows that there exists at least one Nash equilibrium in every no goal game.*

Proof. The proof follows the approach used in Bergstrom et al. (1986). Existence follows from Brouwer's fixed point theorem.

Since each u_i is strictly concave over a convex set each is also continuous. As the strategy sets are compact and convex, it follows that the best response function, $\mathbf{br}^\star : \prod_{i\in I}[0, m_i] \to \prod_{i\in I}[0, m_i]$, is continuous and single valued. As $\prod_{i\in I}[0, m_i]$ is compact and convex, $\mathbf{br}^\star$ maps a compact, convex set into itself and thus has at least one fixed point. This fixed point is by definition a Nash equilibrium.

As shown in Lemma 12, in a goal game it is equivalent to consider a set of strictly concave functions $\{v_i\}_{i=1}^n$, instead of $\{u_i\}_{i=1}^n$ and G. Hence the above argument applies to all no goal games as well. Therefore, the lemma holds. $\square$

Lemma 15. *Let* $\left\{C, F, F', G, I, \{u_j, m_j\}_{j=1}^n\right\}$ *be a goal game satisfying Assumption 1. Then the payoff to donor* $i \in I$*, given by*

$$\pi_i^{gg}(d_i, \mathbf{d}_{-i}, \tilde{D}) = \begin{cases} v_i(m_i - d_i, \iota'\mathbf{d}_{-i} + d_i; F) & : d_i \neq \tilde{D} - \iota'\mathbf{d}_{-i} \\ v_i(m_i - d_i, \tilde{D}; F') & : d_i = \tilde{D} - \iota'\mathbf{d}_{-i} \end{cases}.$$

is upper semi-continuous in d_i.

Proof. Since v_i is continuous in d_i, $\pi_i^{gg}(d_i, \mathbf{d}_{-i}, \tilde{D})$ is upper semi-continuous for all $d_i \neq \tilde{D} - \iota'\mathbf{d}_{-i}$. It may only fail to be upper semi-continuous at $d' = \tilde{D} - \iota'\mathbf{d}_{-i}$. The assumption of risk aversion ensures this does not occur. Let $\{d_n\}_{n=\mathbf{N}} \subset [0, m_i]$ be a sequence which converges to $d' = \tilde{D} - \iota'\mathbf{d}_{-i}$, i.e., $\lim d_n = d'$. Strict risk aversion implies that $v_i(m_i - d_n, \tilde{D}; F) < v_i(m_i - d_n, \tilde{D}; F')$ for all $n \in \mathbf{N}$. By construction, $v_i(m_i - d_n, \tilde{D}; F) \leq \pi_i^{gg}(d_n, \mathbf{d}_{-i}, \tilde{D}) \leq v_i(m_i - d_n, \tilde{D}; F')$ for all $n \in \mathbf{N}$. Hence by the continuity of v_i, $\lim \pi_i^{gg}(d_n, \mathbf{d}_{-i}, \tilde{D}) \leq \lim v_i(m_i - d_n, \tilde{D}; F') = v_i(m_i - d', \tilde{D}; F') = \pi_i^{gg}(d', \mathbf{d}_{-i}, \tilde{D})$. Therefore $\pi_i^{gg}(d_i, \mathbf{d}_{-i}, \tilde{D})$ is upper semi-continuous in d_i. $\square$

Lemma 16. *Let* $\left\{C, F, F', G, I, \{u_j, m_j\}_{j=1}^n\right\}$ *be a goal game satisfying Assumption 1. For every announced goal,* $\tilde{D} \in \mathbf{R}_+$*, there exists a Nash equilibrium of the subgame.*

Proof. The proof is a direct application of Theorem 12.3 in found in Fudenberg and Tirole (1991). The action space for each $i \in I$, $[0, m_i]$, is a nonempty, convex, and compact subset of **R**. By assumption the payoffs have a continuous maximum as defined in Fudenberg and Tirole (1991). By Lemma 15 the payoffs are upper semi-continuous in the donor's own actions. Hence, the no goal game satisfies the prerequisites of Theorem 12.3. Therefore, there exists a pure strategy Nash equilibrium in the subgame. □

A.6.5 Extending the Incentive Compatibility Indicator

It is easier to work with the incentive compatibility constraints for donors in the goal game if the indicator function for compatibility is continuous. Continuity follows immediately from the continuity of the utility functions in the regions of the action space that are feasible. However, we also need to define the indicator function over infeasible regions. As these regions are infeasible, the actions they contain are incompatible and thus it is safe to set the indicator to a negative value Extending the function into these regions *while maintaining continuity* has to be handled with some care. Luckily showing that such an extension can be made is a straight-forward application of Tietze's extension theorem.

Lemma 17. *Let* $m_i > 0$*, and* $\tilde{D}, d \in \mathbf{R}_+$*. Define* $C_i = \left\{(\tilde{D}, d) \in \mathbf{R}_+^2 \,\middle|\, m_i \geq d \text{ and } \tilde{D} \geq d\right\}$*. Define* $ICC_i^0 : C_i \to \mathbf{R}$ *as*

$$ICC_i^0(\tilde{D}, d) = v^i(m_i - d, \tilde{D}; F') - w_i(\tilde{D} - d).$$

There exists a continuous extension of ICC_i^0*,* $ICC_i : \mathbf{R}_+^2$*, such that*

$$ICC_i(\tilde{D}, d) \equiv \begin{cases} ICC_i^0(\tilde{D}, d) & : (\tilde{D}, d) \in C_i \\ g_i(\tilde{D}, d) & : \text{ otherwise} \end{cases}$$

where $g_i < 0$ *and continuous.*

Proof. The proof follows by constructing a suitable g_i through applying Tietze's extension theorem.

Note that as $\mathbf{R}^2$ is a normal space and ICC_i^0 is continuous, Tietze's extension theorem implies that there exists a continuous extension to all of $\mathbf{R}^2$,

$$Q(\tilde{D}, d) \equiv \begin{cases} ICC_i^0(\tilde{D}, d) & : (\tilde{D}, d) \in C_i \\ \bar{g}_i(\tilde{D}, d) & : \text{ otherwise} \end{cases}$$

where $\bar{g}_i : \mathbf{R}^2 \to \mathbf{R}$ is continuous.

Construct $\tilde{g}_i : \mathbf{R}^2 \to (-\infty, 0)$ by

$$\tilde{g}_i(\tilde{D}, d) = -|\bar{g}_i(\tilde{D}, d)| + \min\left\{\tilde{D}, m_i\right\} - d. \tag{1}$$

Since taking the absolute value and adding the quantity $\min\left\{\tilde{D}, m_i\right\} - d$ are continuous transformations to the continuous function $\bar{g}_i$, $\tilde{g}_i$ is continuous. Note that $-|\bar{g}_i(\mathbf{x})| \leq 0$ and that on the domain $\mathbf{R}^2 \setminus C_i$, $\min\left\{\tilde{D}, m_i\right\} - d < 0$. Therefore

$$ICC_i(\tilde{D}, d) \equiv \begin{cases} ICC_i^0(\tilde{D}, d) & : (\tilde{D}, d) \in C_i \\ \tilde{g}_i(\tilde{D}, d) & : \text{ otherwise} \end{cases}$$

is an extension of ICC_i^0 satisfying the lemma. □

A.7 SIMULATION DETAILS

A.7.1 Theory

We begin the simulation details with a general proposition that provides sufficient conditions for the largest outcome of the goal game to occur where the incentive compatibility constraints bind for all donors. The practical value of this is that the largest outcome can be located by finding the fixed-point of the incentive compatibility constraints. Locating fixed-points is a well-studied problem in numerical analysis and so greatly simplifies locating equilibria of the goal game. After discussing the details of the simulations in Section 3.5 we show that they satisfy Proposition 5.

Proposition 5. *Let* $X = \left\{C, F, F', G, \{i_j, u_j, m_j\}_{j=1}^{n}\right\}$ *be a goal game. Define the functions,* $d_i^{max} : \mathbf{R}_+ \to [0, m_i]$*,* $D^{max} : \mathbf{R}_+ \to [0, M]$ *by*

$$d_i^{max}(D) = \max \gamma_i(D),$$

$$D^{max}(D) = \max \Gamma(D).$$

This is a well-defined function since γ_i *is shown to be a compact correspondence in Proposition 7. Assume* D^{max} *is lower semi-continuous on* $[\iota'\mathbf{d}^\star, M]$*. Then the largest total donations collected in subgame perfect equilibrium is* $\tilde{D} = D_i^{max}(\tilde{D})$*.*

Proof. The proof follows by contradiction. Suppose that D^{max} is not on the upper contour of Γ, i.e., $D^{max} \neq \sum_{i \in I} d_i^{max}(D^{max})$. By construction $D^{max}(\tilde{D}) \geq \tilde{D}$. By the supposition this inequality must in fact be strict, $D^{max}(\tilde{D}) > \tilde{D}$. Note that D^{max} is bounded above by M, hence at some point $D \in [\iota'\mathbf{d}^\star, M]$, $D^{max}(D) \leq D$. By assumption D^{max} is lower semi-continuous, hence it only decreases continuously. Thus there must exist some interval $[D_l, D_u] \subset (\iota'\mathbf{d}^\star, M]$ on which D^{max} is continuous, such that $D^{max}(D_l) > D_l$ and $D^{max}(D_u) \leq D_u$. Hence there exists a fixed-point of D^{max} on $[D_l, D_u]$, D_0. However we have reached a contradiction as $D_0 > \tilde{D}$ and is itself a subgame perfect outcome of the goal game X. Therefore, our supposition in incorrect and the proposition holds. $\square$

A.7.2 CRRA Simulation

This section provides the details of finding the functions needed to locate equilibria in simulations with a generalized constant relative risk aversion (CRRA) utility function. In addition to the standard parameter, $\gamma \in (0,1)$, that determines the relative risk aversion, there is $r \in (0,1)$ which determines the relative importance of the private and public goods.

$$u(x,y) = \frac{(x^r y^{1-r})^{1-\gamma}}{1-\gamma}$$

For the remainder of this section we will be considering a goal game, $\left\{C, F, F', G, \{i_j, u_j, m_j\}_{j=1}^{n}\right\}$, where $G(D,\omega) = \omega D$, and u_i has a generalized CRRA form for all $i \in I$. Note that under CRRA utility Inada conditions ensure that $x = 0$ is never an optimal choice.

First we can combine the production and utility functions to simplify the analysis. Below the linear production function, $y = \omega D$, is substituted into the utility and the results simplified.

$$\mathrm{E}\, u\left[x, \omega D\right] = \frac{1}{1-\gamma}\left[x^r D^{1-r}\right]^{1-\gamma} \mathrm{E}\, \omega^{(1-r)(1-\gamma)}$$

Let $K_F \equiv \mathrm{E}_F\, \omega^{(1-r)(1-\gamma)}$.

$$= K_F u_i(x,D)$$

Denote this function by

$$v(x,D;F) = \frac{K_F}{1-\gamma}\left[x^r D^{1-r}\right]^{1-\gamma}$$

The end result is just a constant, K_F, derived from the distribution of ω multiplied by the non-stochastic utility function. It actually plays no role in determining the donations in the no goal game; it only modifies the utility of donors at the solution.

A.7.2.1 No Goal Best Response

Standard constrained optimization yields the best response function.

$$d_i^\star(\mathbf{d}) = \max\{(1-r)m_i - rD_{-i}, 0\}$$

Note that the demand for donations when $D_{-i} = 0$ is $f_i(m_i) = (1-r)m$ and $f_i'(m_i) \in (0,1)$. This implies that the no goal game has a unique Nash equilibrium as shown in Bergstrom et al. (1986). See A.7.5 for a discussion and statement of the proof.

The best response can be solved for the equilibrium vector of donations.

$$\begin{aligned} d^\star &= (1-r)m_i - r\iota'\mathbf{d}^\star_{-i} \\ [(1-r)I + r\iota\iota']\,\mathbf{d}^\star &= (1-r)\mathbf{m} \end{aligned}$$

It can be shown that,

$$[(1-r)I + r\iota\iota']^{-1} = \frac{1}{((n-1)r+1)(1-r)}\left\{[(n-1)r+1]\,I - r\iota\iota'\right\}$$

Then

$$\begin{aligned} \mathbf{d}^\star &= \frac{1}{((n-1)r+1)}\left\{[(n-1)r+1]\,I - r\iota\iota'\right\}\mathbf{m} \\ d_i^\star &= m_i - \frac{r}{(n-1)r+1}\sum_{j\in I} m_j. \end{aligned} \tag{$\star\star$}$$

Note that this only holds for those donors giving strictly positive amounts. A donor gives a positive amount if and only if $m_i > \frac{r}{(n-1)r+1}\sum_{j\in I} m_j$. Hence the number of donors contributing is the fixed-point of

$$n = \sharp\left\{m_i > \frac{r}{(n-1)r+1}\sum_{j\in I_n} m_j\right\}.$$

Also note that if a donor with endowment m is contributing then any donors with endowments greater than m must also be contributing. The algorithm we use relies on these properties to locate the equilibrium. With the donors sorted in endowment order we repeatedly calculate $\sharp\left\{m_i > \frac{r}{(n-1)r+1}\sum_{j\in I_n} m_j\right\}$ until the fixed-point is located. Then ($\star\star$) is used to determine the equilibrium.

Substituting in the best response gives the value function for the i^{th} donor in the no goal game,

$$
\begin{aligned}
w_i(D_{-i}) &= v\left(m_i - d_i^\star(D_{-i}), d_i^\star(D_{-i}) + D_{-i}; F\right) \\
= &\begin{cases} \frac{K_F}{1-\gamma}\left(r^r(1-r)^{1-r}\right)^{1-\gamma}(m_i + D_{-i})^{1-\gamma} & : (1-r)m_i - rD_{-i} > 0 \\ \frac{K_F}{1-\gamma}\left[m_i^r D_{-i}^{1-r}\right]^{1-\gamma} & : \text{ otherwise} \end{cases}
\end{aligned}
$$

A.7.3 Goal Game Equilibria

The key equation needed for finding the goal game equilibrium is the individual's incentive compatibility constraint (ICC).

$$
\begin{aligned}
ICC_i(d, D) &= v(m_i - d, D; F') - w_i(D - d) \\
&= \begin{cases} f_1^i(d, D) & : (1-r)m_i - r(D-d) > 0 \\ f_2^i(d, D) & : \text{ otherwise} \end{cases}
\end{aligned}
$$

The pieces f_1^i, f_2^i are defined by,

$$
\begin{aligned}
f_1^i(d, D) &= \frac{1}{1-\gamma}\left\{\left[(m_i - d)^r D^{1-r}\right]^{1-\gamma} \ldots\right. \\
&\left. -K_F\left(r^r(1-r)^{1-r}\right)^{1-\gamma}(m_i + D - d)^{1-\gamma}\right\} \\
f_2^i(d, D) &= \frac{1}{1-\gamma}\left\{\left[(m_i - d)^r D^{1-r}\right]^{1-\gamma} \ldots\right. \\
&\left. -K_F\left[m_i^r (D-d)^{1-r}\right]^{1-\gamma}\right\}
\end{aligned}
$$

Note that ICC_i is only defined when $d \leq m_i$ and $d \leq D$. We will assume these hold for this section. They are feasibility constraints and assuming they hold does not cause problems. In practice ICC_i can be augmented to be some negative value outside the feasible region.

For the purposes of finding equilibria we are really only concerned with where $\{d \in [0, D] \cap [0, m_i] | ICC_i(d, D) \geq 0\}$. Simpler functions can be found that are equivalent to ICC_i in this role.

$$\begin{aligned} & f_1^i(d, D) \geq 0 \\ \Leftrightarrow \quad & (m - d_i)^r D^{1-r} - K_F^{1/(1-\gamma)} r^r (1-r)^{1-r} (m + D - d) \geq 0 \\ & f_2^i(d, D) \geq 0 \\ \Leftrightarrow \quad & (m - d)^r D^{1-r} - K_F^{1/(1-\gamma)} \cdot m^r (D - d)^{1-r} \geq 0 \end{aligned}$$

Allow us to reuse our notation, and denote these simpler functions as f_1^i, and f_2^i.

In determining the equilibrium of the goal game we need to locate $\max \left\{ d \in [0, D] \cap [0, m_i] | \tilde{F}_i(d, D) \geq 0 \right\}$, which will be at a $d \in [0, D] \cap [0, m_i]$ such that $\tilde{f}_1^i(d, D) = 0$ or $\tilde{f}_2^i(d, D) = 0$. Unfortunately, there are no closed-form solutions to the roots of $\tilde{f}_1^i$, and $\tilde{f}_2^i$. Hence, numerical root finding techniques are used in the simulations. It is shown in Lemma 19 that the region of incentive compatible regions defined by these functions is always convex. The algorithm we employ in our simulations uses that fact by first locating the maximum value of the incentive compatibility constraint to use as a lower bound for the root. This ensures only the largest root is found.

Lemma 18. *Let F be a uniform distribution around one, i.e., with $a > 0$*

$$F(x) = \begin{cases} 0 : x \in (-\infty, 1-a) \\ \frac{x+a-1}{2a} : x \in [1-a, 1+a] \\ 1 : x \in (1+a, \infty) \end{cases} .$$

Then $K_F = \mathrm{E}_F\, \omega^{(1-r)(1-\gamma)} < 1$ for all $\gamma, r \in (0, 1)$.

Proof. The proof follows by computation. First we locate the extrema of K_F.

$$\begin{aligned} K_F &= \mathrm{E}_F\, \omega^{(1-r)(1-\gamma)} \\ &= \frac{1}{2a} \int_{1-a}^{1+a} \omega^{(1-r)(1-\gamma)} d\omega \end{aligned}$$

Let $z = (1-r)(1-\gamma)$.

$$= \frac{1}{2a}\int_{1-a}^{1+a} \omega^z d\omega.$$

The second derivative of K_F with respect to z is

$$\frac{\partial^2 K_F}{\partial z^2} = \frac{1}{2a}\int_{1-a}^{1+a} \omega^z \left(\log \omega\right)^2 d\omega.$$

As $\frac{\partial^2 K_F}{\partial z^2}$ is strictly positive the extreme values of K_F occur at the extreme values of z. Since $z \in (0,1)$,

$$\frac{1}{2a}\int_{1-a}^{1+a} \omega d\omega = 1$$
$$\frac{1}{2a}\int_{1-a}^{1+a} d\omega = 1.$$

Therefore $K_F < 1$ and the lemma holds. □

Lemma 19. *Let* $X = \left\{C, F, F', G, \{i_j, u_j, m_j\}_{j=1}^n\right\}$ *be a goal game. Assume that,*

$$u_j(x,y) = \frac{1}{1-\gamma}\left(x^r y^{1-r}\right)^{1-\gamma},$$

$$F(x) = \begin{cases} 0 : x \in (-\infty, 1-a) \\ \frac{x+a-1}{2a} : x \in [1-a, 1+a] \\ 1 : x \in (1+a, \infty) \end{cases},$$

$$F'(x) = \begin{cases} 0 : x \in (-\infty, 1) \\ 1 : x \in [1, \infty) \end{cases},$$

where $\gamma, r, a \in (0,1)$. *Then for all* $D \in [\iota' d^\star, M]$, $A(D) = \{d \in [0, m_i] \,|ICC_i(d,D) \geq 0\}$ *is convex for all* $i \in I$. *Moreover,* $ICC_i(d,D) > 0$ *for all* $d \in A^\circ$.

Proof. Let $D \in [\iota' d^\star, M]$, and $d_1, d_2 \in \{d \in [0, m_i] \,|ICC_i(d,D) \geq 0\}$. Without loss of generality assume that $d_1 < d_2$. Due to the piecewise nature of ICC_i there are several cases to consider:

1. $ICC_i(d_1, D_0) = f_1(d_1, D_0)$ and $ICC_i(d_2, D_0) = f_1(d_2, D_0)$,

2. $ICC_i(d_1, D_0) = f_1(d_1, D_0)$ and $ICC_i(d_2, D_0) = f_2(d_2, D_0)$,
3. $ICC_i(d_1, D_0) = f_2(d_1, D_0)$ and $ICC_i(d_2, D_0) = f_2(d_2, D_0)$.
4. $ICC_i(d_1, D_0) = f_2(d_1, D_0)$ and $ICC_i(d_2, D_0) = f_1(d_2, D_0)$,

By construction if $ICC_i(d, D) = f_2(d, D)$ then $ICC_i(d', D) = f_2(d', D)$ for all $d' < d$. Likewise, if $ICC_i(d, D) = f_1(d, D)$ then $ICC_i(d', D) = f_1(d', D)$ for all $d' > d$. Hence we can reduce the cases to consider to:

1. $ICC_i(d_1, D_0) = f_1(d_1, D_0)$ and $ICC_i(d_2, D_0) = f_1(d_2, D_0)$,
2. $ICC_i(d_1, D_0) = f_2(d_1, D_0)$ and $ICC_i(d_2, D_0) = f_2(d_2, D_0)$.
3. $ICC_i(d_1, D_0) = f_2(d_1, D_0)$ and $ICC_i(d_2, D_0) = f_1(d_2, D_0)$,

We will consider each case in turn.

Suppose case 1 occurs, $ICC_i(d_1, D_0) = f_1(d_1, D_0)$ and $ICC_i(d_2, D_0) = f_1(d_2, D_0)$. The Hessian of f_1 is

$$H[f_1] = -r(1-r)\begin{pmatrix}(m-d)^{r-2}D^{1-r} & (m-d)^{r-1}D^{-r} \\ (m-d)^{r-1}D^{-r} & (m-d)^{r}D^{-r-1}\end{pmatrix}$$

The parameter assumptions ensure that $H[f_1]$ is negative definite. Hence f_1 is strictly concave and the lemma holds.

Suppose case 2 occurs, $ICC_i(d_1, D_0) = f_2(d_1, D_0)$ and $ICC_i(d_2, D_0) = f_2(d_2, D_0)$. Note that

$$\frac{\partial f_2}{\partial d} = \frac{-1}{(D-d)(m-d)}\left[r(D-d)\cdot(m-d)^r D^{1-r} - (1-r)(m-d)\cdot Cm^r(D-d)^{1-r}\right].$$

The domain constraint on f_2 is $(1-r)m < r(D-d)$. Hence $(1-r)(m-d) < r(D-d)$ as well. Then $f_2 > 0$ implies that $\frac{\partial f_2}{\partial d} < 0$. Thus, for all $d \in [0, d_2)$ $f_2(d, D) > 0$ and the lemma holds.

Suppose case 3 occurs, $ICC_i(d_1, D_0) = f_2(d_1, D_0)$ and $ICC_i(d_2, D_0) = f_1(d_2, D_0)$. In this case it must be that the transition point between the functions, $d_t = D - \frac{1-r}{r}m$, lies in $[d_1, d_2]$. If $f_1(d_t, D) = f_2(d_t, D) \geq 0$ then the previous cases ensure that the lemma holds. Thus consider $f_1(d_t, D) = f_2(d_t, D) < 0$.

As shown in case 1, f_1 is strictly concave in d. Combined with the fact that $f_1(d_2, D) > 0$ and $f_1(d_t, D) < 0$ it must be that $\frac{\partial f_1}{\partial d}(d_t) > 0$. Hence,

$$\begin{aligned}
\frac{\partial f_1}{\partial d_t} = r^r(1-r)^{1-r}K_F^{\frac{1}{1-\gamma}} - r\left(\frac{D}{m-d_t}\right)^{1-r} &> 0 \\
r^r(1-r)^{1-r}K_f^{\frac{1}{1-\gamma}} &> r\left(\frac{D}{m-d_t}\right)^{1-r} \\
\left(\frac{1-r}{r}\right)^{1-r}K_f^{\frac{1}{1-\gamma}} &> D^{1-r}(m-d_t)^{r-1}.
\end{aligned}$$

As $d_t \in [d_1, d_2]$, it must be that $d_t > 0$. Hence, $D > \frac{1-r}{r}m$. Substituting this into the condition derived from the derivative, ($\star$), implies

$$\begin{aligned}
\left(\frac{1-r}{r}\right)^{1-r}K_f^{\frac{1}{1-\gamma}} &> D^{1-r}(m-d_t)^{r-1} \\
&> \left(\frac{1-r}{r}\right)^{1-r}\left(\frac{m}{m-d_t}\right)^{1-r} \\
&\Rightarrow \\
K_f^{\frac{1}{1-\gamma}} &> \left(\frac{m}{m-d_t}\right)^{1-r} > 1.
\end{aligned}$$

By Lemma 18, $K_F < 1$ and we have reached a contradiction. Hence it must be that case that $f_1(d_t, D) = f_2(d_t, D) \geq 0$ and the lemma holds. □

Lemma 20. *Let $X = \left\{C, F, F', G, \{i_j, u_j, m_j\}_{j=1}^n\right\}$ be a goal game. Assume that,*

$$\begin{aligned}
u_j(x,y) &= \frac{1}{1-\gamma}\left(x^r y^{1-r}\right)^{1-\gamma}, \\
F(x) &= \begin{cases} 0 : x \in (-\infty, 1-a) \\ \frac{x+a-1}{2a} : x \in [1-a, 1+a] \\ 1 : x \in (1+a, \infty) \end{cases}, \\
F'(x) &= \begin{cases} 0 : x \in (-\infty, 1) \\ 1 : x \in [1, \infty) \end{cases},
\end{aligned}$$

where $\gamma, r, a \in (0,1)$. Then D^{max} is lower semi-continuous on $[\iota'\mathbf{d}^\star, M]$.

Proof. The proof follows by contradiction. Suppose that D^{max} is not lower-semi continuous on $[\iota'\mathbf{d}^\star, M]$. Then there exists a $D_0 \in [\iota'\mathbf{d}^\star, M]$ such that

$$\liminf_{D \to D_0} D^{max}(D) < D^{max}(D_0).$$

Then there exists some $i \in I$ such that

$$\liminf_{D \to D_0} d_i^{max}(D) < d_i^{max}(D_0).$$

Note that $d_i^{max}(D) \in \gamma_i(D)$ for all $D \in [\iota'\mathbf{d}^\star, M]$. Since γ_i is upper hemi-continuous by Lemma 7, $\liminf_{D \to D_0} d_i^{max}(D) \in \gamma_i(D_0)$. Hence $ICC_i(\liminf_{D \to D_0} d_i^{max}(D), D_0) \geq 0$.

Let $d_2 = d_i^{max}(D_0)$ and $d_1 = \liminf_{D \to D_0} d_i^{max}(D)$. By Lemma 19, $ICC_i(\bar{d}, D_0) > 0$ where $\bar{d} = \frac{d_1 + d_2}{2}$. Since ICC_i is continuous there exists an $\epsilon > 0$ such that $ICC_i(\bar{d}, D) > 0$ for all $D \in B(D_0, \epsilon)$. However, as $\bar{d} > d_1$ this contradicts the fact that $d_2 = \liminf_{D \to D_0} d_i^{max}(D)$. Hence the supposition is incorrect and the lemma holds. □

A.7.4 Simulation Robustness

In order to investigate goal game equilibria we calculated numerous subgame perfect equilibria under various parameters. Computational constraints make simulating large populations difficult and thus we used population sizes of $N \in \{1000, 2000, 8000\}$. If our statistical measures on the goal game equilibria vary wildly, or appear to become insignificant outside of our tested populations, it would diminish their impact. In order to check our measures we conducted a nonparametric test for trends of our statistical measures across population size. At each parameter combination a Jonckheere-Terpstra test was used to check for a positive trend. This test extends the Wilcoxon rank sum test to multiple treatments. The null hypothesis is that there is no treatment effect, medians are equal in each cell, $H_0 : \theta_{1000} = \theta_{2000} = \theta_{8000}$. The alternative hypothesis is that there is a positive trend, $H_1 : \theta_{1000} \leq \theta_{2000} \leq \theta_{8000}$, where at least one inequality is strict. With nine population draws at each parameter combination and population level, our sample size is nine for all tests.

Our measure of the increase in collected donations, I_a, showed strong evidence of a positive trend with populations. Across all parameter combinations the largest p-value is 2.87×10^{-7}. Hence it appears that our conclusions strengthen with larger populations.

Our measures of the donation distribution, $\mathcal{W}_a^{low}$, $\mathcal{W}_a^{mid}$, and $\mathcal{W}_a^{high}$, show no evidence of a trend with population. For all the measures, across all parameter combinations, the p-values lie in $[0.35, 0.66]$. The lack of a trend is consistent with the distributional measures being primarily functions of the population distribution. We conjecture that even the smallest population tested, $N = 1000$, is a good approximation of the χ_4^2 distribution used for endowments. Hence, there is little trend with population size at these sizes.

A.7.5 Unique Nash Equilibrium in VCG

This section provides conditions under which the Nash equilibrium of a VCG is unique. The proof of uniqueness is provided as theorem 3 in Bergstrom et al. (1986). We repeat the theorem here for convenience, but do not include the proof.

The needed assumptions require some context. Following the discussion in Bergstrom et al. (1986), consider a donor's problem in a VCG,

$$\max_{x_i, d_i} \mathrm{E}\, u_i\left(x_i, G(d_i + D_{-i}, \omega)\right)$$
$$\text{s.t. } x_i + d_i \leq m_i.$$

By adding D_{-i} to the budget constraint the problem can be turned into the equivalent problem,

$$\max_{x_i, D} \mathrm{E}\, u_i\left(x_i, G(D, \omega)\right)$$
$$\text{s.t. } x_i + D \leq m_i + D_{-i}$$
$$D \geq D_{-i}.$$

This problem is very similar to a standard consumer choice problem where wealth is $m_i + D_{-i}$,

$$\max_{x_i, D} \mathrm{E}\, u_i\left(x_i, G(D, \omega)\right)$$
$$\text{s.t. } x_i + D \leq m_i + D_{-i}.$$

Assume that there exists a demand function for donations in the consumer choice problem, $f_i : \mathbf{R}_+ \to \mathbf{R}_+$, $f_i(m_i + D_{-i})$. As discussed in Bergstrom et al. (1986), the demand for donations in the VCG is then $d_i^\star(m_i, D_{-i}) = \max\{f_i(m_i + D_{-i}) - D_{-i}, 0\}$. In order to ensure a unique VCG equilibrium we need to assume that f_i is differentiable and has $0 < f_i' < 1$.

Proposition 6. *Let $\{i_j, u_j, m_j\}_{j=1}^n$ be a voluntary contribution game. If there exists a differentiable demand function, f_i, such that $f_i' \in (0, 1)$, which solves the consumer choice problem,*

$$\max_{x_i,G} u_i\left(x_i, G\right)$$
$$\text{s.t. } x_i + G \leq m_i.$$

then there is a unique Nash equilibrium with a unique quantity of the public good and a unique set of consumers.

Proof. Shown in Bergstrom et al. (1986), page 34. □

A.8 EXPERIMENT

A.8.1 Time Trend

Individual Donations

	constant	round	round$\times I_{nr}$	round$\times I_g$
coefficient	3.272	-0.033	0.025	0.016
p-value	(0.000)	(0.000)	(0.019)	(0.110)

Probability of Reaching Goal ($d_1 + d_2 \geq 8$)

	constant	round	round$\times I_{nr}$	round$\times I_g$
coefficient	0.386	-0.008	0.008	0.004
p-value	(0.000)	(0.000)	(0.000)	(0.042)

Table 12: Random-effects regressions of mean donations and likelihood of reaching the goal level against round ($N = 124$, $t = 30$).