ESSAYS ON FISCAL POLICY

by

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I study selected fiscal policy issues both in the short-run and long-run using empirical methods. In Chapter 2, I investigate the effects of increases in U.S. government expenditures on the private sector in a real business cycle framework using maximum likelihood. I distinguish between the consumption and the investment expenditure components of government purchases. This makes the model sufficiently flexible to admit crowding-in and crowding-out effects of the two government expenditures separately. I show that a 1% increase in government consumption increases output by 1.5%, while a 1% increase in government investment increases output by 0.0085%. In Chapter 3, I analyze the international transmission of shocks to public expenditures in a two-country business cycle model of saving and investment under perfect international capital mobility. I test the model for its ability to account for some puzzles identified in the international real business cycle literature, specifically the low cross-country consumption correlations, and the positive correlation between national saving and investment over time. I show that a model with technology and government spending shocks quantitatively explains the low consumption correlations across countries and the high investment-saving correlation. In Chapter 4, we develop a new index which provides early warning signals of fiscal sustainability problems for advanced and emerging economies. We use the index to assess the build-up of fiscal stress over time since the mid-1990s in advanced and emerging economies. We show that fiscal stress has increased recently to record-high levels in advanced countries, reflecting raising solvency risks and financing needs. In emerging economies, we show that risks are lower than in mature economies owing to sounder fiscal fundamentals, but fiscal stress remains higher than before the crisis.
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PREFACE

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1.0 INTRODUCTION

In my research, I use empirical models and methodologies to address both short- and long-run fiscal policy issues. In the three chapters, I investigate how government spending affects the private sector over the business cycle, what types of government spending are more stimulating for private activity than others, and whether different types of government spending explain the synchronization of private activity across countries. Next, I turn to long-run considerations and investigate which fiscal indicators can signal fiscal vulnerability before a crisis occurs, and whether there exist differences in the vulnerability structure between emerging and advanced economies.

In Chapter 2, the effects of increases in U.S. government expenditures on the private sector are investigated in a real business cycle framework using maximum likelihood. Distinction is made between the consumption and the investment expenditure components of government purchases using CES specifications over composite consumption and capital. This makes the model sufficiently flexible to admit crowding-in and crowding-out effects of the two government expenditures separately. Parameter estimates show that both government consumption and capital complement private consumption and capital in the utility and production functions, respectively. This results in crowding-in of private activity: A 1% increase in government consumption increases private consumption by 0.55%, and output by 1.5%, while a 1% increase in government investment increases private investment by 0.04% and output by 0.0085%. Furthermore, I show that government expenditures help improve the model’s ability to account for labor market puzzles.

In Chapter 3, I analyze the international transmission of shocks to public expenditures in a two-country business cycle model of saving and investment under perfect international capital mobility. Distinction is made between the consumption and the investment expenditure components of government purchases using CES specifications over composite consumption and capital. The model is tested for its ability to account for some puzzles identified in the international real business
cycle literature, specifically the low cross-country consumption correlations, and the positive correlation between national saving and investment over time. A model with technology and government spending shocks quantitatively explains the low consumption correlations across countries and the high investment-saving correlation. Public expenditure shocks are shown to improve the model’s ability to match several moments in the data. Overall, I find in Chapters 2 and 3 that government demand shocks significantly improve real business cycle models’ performance both domestically and internationally.

The last Chapter has a similar fiscal policy flavor, but addresses different questions. In Chapter 4 (IMF working paper co-authored with Emanuele Baldacci, Iva Petrova, Nazim Belhocine, and Gabriela Dobrescu), we develop a new index which provides early warning signals of fiscal sustainability problems for advanced and emerging economies. Unlike previous studies, the index assesses the determinants of fiscal stress periods, covering public debt default as well as near-default events. The fiscal stress index depends on a parsimonious set of fiscal indicators, aggregated using the approach proposed by Kaminsky, Lizondo and Reinhart (1998). The index is used to assess the build up of fiscal stress over time since the mid-1990s in advanced and emerging economies. Fiscal stress has increased recently to record-high levels in advanced countries, reflecting raising solvency risks and financing needs. In emerging economies, risks are lower than in mature economies owing to sounder fiscal fundamentals, but fiscal stress remains higher than before the crisis.
2.0 PUBLIC EXPENDITURES IN AN RBC MODEL: A LIKELIHOOD EVALUATION OF CROWDING-IN AND CROWDING-OUT EFFECTS

2.1 INTRODUCTION

What are the effects of government expenditures on the private sector over the business cycle? Although the answer to this question is of significant policy relevance and is central to the macroeconomics literature, it remains open to debate both theoretically and empirically. Indeed, there is no agreement even on the qualitative effects concerning whether government spending crowds-in or crowds-out private consumption and investment and on the effects on real wages. Clear answers to these questions are crucial for understanding several policy issues, such as the effect of government spending on GDP, the effectiveness of stimulus packages in recessions, and distinguishing between competing predictions of alternative macroeconomic models.

Before reviewing the literature in detail, it is important to make an observation on the nature of government expenditures. Most of the work done on the effects of fiscal policy shocks either treats government spending as total government purchases (investment + consumption) or studies one of these two components separately. Since one contribution of this paper is to take this composition seriously by estimating the separate effects in a single model, this distinction is made clear in the next review section and throughout the chapter.

Starting with government consumption, dating at least to the rise of the Keynesian paradigm, there has been a vigorous debate on whether government consumption crowds-in or crowds-out private activity. A standard neoclassical view argues for crowding-out: Government consumption shocks induce negative wealth effects that crowd-out private consumption. A standard Keynesian view argues for crowding-in, via the multiplier effect. Recent empirical evidence gleaned from VARs suggests a tendency towards crowding-in; the purpose here is to investigate whether similar evidence
emerges from the empirical analysis of a structural (dynamic stochastic general equilibrium) model. As for government investment, the conventional view is that since government and private capital are likely to be complementary in production, crowding-in is likely to prevail in this dimension (e.g., see Aschauer, 1989). However, the empirical evidence on this view, based on estimates obtained using either production functions or VARs is mixed, and no clear consensus has emerged.

Here, I investigate the relationship between public and private consumption and investment expenditures in an RBC framework specified as sufficiently flexible to encompass the possibility of crowding-out or crowding-in along both dimensions. Along the consumption dimension, crowding-out can occur for two reasons: either via a standard wealth effect or via a channel wherein government consumption substitutes for private consumption in the representative household’s CES utility. But crowding-in is also possible due to two features of the model: first, through the same CES aggregator in the utility function, wherein government consumption can complement private consumption depending on the elasticity of substitution parameter; and second, through a non-separable functional utility form specified such that consumption and leisure are substitutes. For certain parameterizations of the model, each of these two features is capable of dominating the wealth effect, leading to crowding-in (e.g., see Bouakez and Rebei, 2007; and Linnemann, 2006).

Along the investment dimension, crowding-out can similarly occur either via a wealth effect, or if government capital substitutes for private capital in the CES production function. Crowding-in takes place if government capital complements private capital in the CES production function. Given the flexibility of the model, the nature of the relationship between government and private consumption and investment amounts to an empirical question, which I evaluate by estimating the model via maximum likelihood.

The maximum likelihood estimates I obtain indicate that both government consumption and investment crowd-in private consumption and investment. Specifically, a 1% increase in government consumption is found to increase private consumption by 0.55%, and output by 1.5%; while a 1% increase in government investment increases private investment by 0.04% and output by 0.0085%.

Pappa (2005) also distinguishes between government consumption and investment in an RBC model. This paper differs from his in many aspects. First, here the production function has a CES form, allowing government investment shocks to generate either a crowding-in or a crowding-out effect on private activity. In contrast, in Pappa’s model public capital enters the production function
directly, thereby inducing a strict crowding-in effect. Second, the goal here is to derive concluding results on crowding-in versus crowding-out effects on private consumption and investment via maximum likelihood. Pappa calibrates his model and compares it to a new-Keynesian version in order to shed light on the two models’ different predictions regarding the real wage.

The chapter proceeds as follows: Section 2.2 reviews the literature; Section 2.3 presents the model; and Section 2.4 discusses the data. Section 2.5 then presents calibration exercises designed to illustrate the model’s flexibility; there it is shown that alternative parameterizations can imply opposing results. Section 2.6 then presents ML estimates that provide a clear resolution of the alternate possibilities admitted under the model, yielding the crowding-in/crowding-out pattern described above. Section 2.7 concludes.

2.2 THE EVIDENCE

2.2.1 Government Consumption

The empirical literature on public spending effects has struggled to provide robust stylized facts. The main debate concerns (1) the specification of the reduced-form VAR model and (2) the approach used to identify fiscal policy shocks. Traditionally, two major approaches have been used to identify government shocks.

The first approach commonly referred to as narrative is based on the argument that increases in military spending are good indicators of unanticipated policy shifts, which motivates the use of dummy variables to identify unanticipated increases in military purchases in the U.S. economy. For example, Ramey and Shapiro (1998) use this narrative approach to identify three major episodes of military buildups in the US: the Korean War 1950:3, the Vietnam War 1965:1, and the Carter-Reagan defense buildup 1980:1. Studies following this approach find that real wages and private consumption tend to fall due to exogenous increases in defense purchases (e.g., see Ramey and Shapiro, 1998; Edelberg et al., 1999; Burnside et al., 2004; Fisher and Peters, 2009; and Barro and Redlick, 2010). Auerbach (2010) argues that this approach focuses on the effect of specific types of shocks and cannot be generalized to evaluate broader policy effects such as short-run recessionary periods.
The second strand employs an identification strategy under which government spending is assumed to be predetermined relative to innovations in other variables in the VAR within a quarter. Here, government expenditures consist of both consumption and investment components. These studies find that private consumption tends to rise after positive government spending shocks (e.g., see Fatas and Mihov, 2002; Blanchard and Perotti, 2002; Gali et al., 2007; and Mountford and Uhlig, 2009).

The sharp difference in results obtained using the narrative approach and standard VAR studies has led to a series of recent papers. For example, Kamps and Caldara (2008) explore the role of alternative identification approaches in generating differences in results. They find that private consumption and the real wage rise in all of their approaches, except when following the narrative approach. Ramey (2009) on the other hand argues that the difference in results arises due to a timing issue. Since it is implicitly assumed in VAR studies that government shocks are unanticipated by the private sector, the VAR might be missing the decline in consumption that occurred when the news was learned. She concludes that after adjusting for anticipations, private consumption falls. Tenhoffen and Wolff (2007) follow a similar approach, but allow future identified shocks to have an effect in standard VAR studies. They find that when anticipation is allowed, standard VARs indeed show a decrease in private consumption to defense spending shocks. However, they do not find that anticipation matters at all for other government spending components: private consumption still rises due to a government spending shock even when allowing for anticipation. Hence, there is still a clear debate concerning whether government spending shocks crowd-in or crowd-out private consumption.

Just as the empirical evidence has been mixed, so too have theoretical predictions. In standard RBC models, a representative infinitely-lived household is assumed to optimize under rational expectations. Government consumption shocks increase household incentives to work inducing negative wealth effects; this causes private consumption to decrease (e.g., see Christiano and Eichenbaum, 1992; Baxter and King, 1993). In the Keynesian tradition on the other hand, expansionary fiscal policies are thought to trigger increases in aggregate demand, triggering increases in production, employment and consumption via the multiplier effect (e.g., see Blanchard, 2001). Empirical evidence obtained using the narrative approach supports neoclassical economic theory, while the evidence from VARs seems to support predictions of the Keynesian tradition. Nonetheless, some
researchers have attempted to reconcile theory with evidence by introducing modifications both to neoclassical and New Keynesian models that admit the possibility that private consumption may increase following a government consumption shock. For example, Gali et al. (2007) construct a New Keynesian model wherein consumption rises in response to a public consumption shock. Their model features price stickiness, rule-of-thumb households and a monopolistic labor market. Linne-mann and Schabert (2004) augment a New Keynesian model with a utility function that depends on government consumption as well as private consumption. Linnemann (2006) and Bouakez and Rebei (2007) make the same attempt for the neoclassical model. The former introduces a non-separable utility specified wherein consumption and leisure are substitutes, and the latter assumes government consumption affects household utility through a CES aggregator. All these studies use DSGE models and show that private consumption can rise due to the modifications presented in the models.

Note that although the VAR literature uses total government expenditures to study the effect on private consumption, the literature using DSGE models summarized above focuses on government consumption shocks only and abstracts from the investment component. In the next subsection, I characterize a separate literature that focuses on the effects of public investment on the private sector.

2.2.2 Government Investment

This strand of literature stems from a series of studies by Aschauer, who in 1989 argued that decreases in public investment in the US can explain the post-1970 productivity slowdown. Following this work, alternate methodologies have been used to estimate the effects of government investment shocks on the private sector, all of them based on reduced-form specifications.

One strand focuses on aggregate production functions augmented to include the public capital stock as an input. For example, Aschauer (1989b) uses annual US data, Munnell (1990) uses pooled annual US state data in levels and Finn (1993) looks at the effects of different components of public capital. They all find positive effects of public capital on private investment and output. There are however noted problems following this approach, mainly since the direction of causality between government and private variables is ambiguous. In time series studies for example, government investment can be higher when growth is higher, depending on whether policies are procyclical or
counter cyclical. The same reasoning applies for state data in levels; richer states have more means to invest. Indeed, when the production function is estimated with US data in differences, or with state data with fixed effects, studies find zero or negative effects of public capital on private activity (e.g., see Tatom, 1991; Hulten and Schwab, 1991; Evans and Karras, 1994; and Hotz-Eakin, 1994).

The second strand of the literature employs VAR techniques. Kamps (2005) summarizes 20 VAR studies, nearly half of which consider the effects of government investment for the US, while others extend the analysis for other countries. The majority of these VAR studies consider a simple model with four or five variables including public capital or public investment, private capital or private investment, employment and output. The summary shows a clear divergence in results depending on the country in question and on whether the data is annual or quarterly. When annual data is used, these studies tend to find crowding-in effects (e.g., see Pereira, 2000 for the United States; and Afonso and Aubyn, 2009 for 14 EU countries). In VAR studies with quarterly data on the other hand, studies find insignificant or even negative effects of government investment on private investment and output (e.g., see Voss, 2002; and Perotti, 2004). Furthermore, the US and Canada seem to show insignificant or negative effects, while EU countries show positive effects on output (e.g., see Kamps, 2005). One reason results differ depending on the frequency of data used is that using annual data makes it difficult to analyze feedback effects from output to public investment. These studies show that the empirical evidence for government investment is mixed, and no clear consensus has emerged regarding its effects.

2.2.3 Discussion

The literature reviewed above has sought to identify the effects of (1) government total expenditures in VAR frameworks; (2) government consumption expenditures in DSGE frameworks; and (3) government investment expenditures in reduced-form frameworks. Here, I employ a DSGE model sufficiently rich and flexible to admit the possibility of both crowding-in or crowding-out effects for both government consumption and investment expenditures. I then use ML estimates to determine the pattern of effects that appear most plausible when the data are viewed through the lens of the model.

It is important to distinguish between the components of government expenditures in a structural model, because these shocks can generate very different macroeconomic effects. While both
shocks lead to the absorption of resources by the government, and thus generate negative wealth effects, other factors come into play that are distinct to each. In the public consumption case, depending on the size of the complementarity between leisure and consumption, and on whether public and private consumption are substitutes or complements, crowding-in may result in spite of the negative wealth effect.

As for government investment, it is distinct from government consumption in that it increases output directly in the production function. In this case, if the productivity of public capital is sufficiently large, this could in theory overcome the wealth effect inducing a rise in both private consumption and investment. In Pappa (2005), public capital crowds-in private capital automatically due to the specification of their model, while the model estimated here allows for both crowding-in and crowding-out effects. Further, here the model is estimated via maximum likelihood while Pappa’s results are sensitive to alternate parameterizations considered in a calibration exercise.

2.3 THE MODEL

2.3.1 Basic Features

The economy consists of a large number of identical households. The representative household’s objective is to maximize its expected discounted flow of utility given preferences defined over effective consumption and leisure during each period t=0,1,2,...:

$$\max_{c_t, l_t} U = E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, l_t)$$  \hspace{1cm} (2.1)

Here, $E_0$ is the expectations operator conditional on information available at time 0; $\beta \in (0, 1)$ is the discount factor; $u(.)$ is an instantaneous utility function; and $C_t$ and $l_t$ are levels of effective consumption and leisure. Preferences are given by:

$$u(C_t, l_t) = \frac{(C_t^\gamma l_t^{1-\gamma})^{1-\phi}}{1-\phi},$$  \hspace{1cm} (2.2)

where $\phi > 1$ is the coefficient of relative risk aversion, and $\gamma \in (0, 1)$ is consumption’s share relative to leisure of instantaneous utility. As Linneman (2005) shows, this utility specification implies
\( U_{cl} < 0 \) for \( \phi > 1 \), which is a necessary but not sufficient condition for government consumption to crowd-in private consumption.\(^1\) With this utility form, the negative wealth effect of a government consumption shock reduces leisure, but raises the marginal utility of private consumption. If this effect is sufficiently strong, private consumption will increase after the shock. The household maximizes utility derived from consumption and leisure, knowing that every period it is constrained with one unit of time to be allocated between two activities:

\[
1 = n_t + l_t. \tag{2.3}
\]

By investing \( i_{pt} \) units of output during period \( t \), the household increases the capital stock \( k_{pt+1} \) according to the law of motion

\[
k_{pt+1} = i_{pt} + (1 - \delta)k_{pt}, \tag{2.4}
\]

The aggregate public capital stock is likewise

\[
k_{gt+1} = i_{gt} + (1 - \delta)k_{gt}, \tag{2.5}
\]

where \( \delta \) is the depreciation rate for both capital stocks.

### 2.3.2 CES Aggregators

Effective consumption \( C_t \) is divided into private and public consumption according to the CES (constant elasticity of substitution) specification

\[
C_t = [\tau c_{pt}^{\psi} + (1 - \tau)c_{gt}^{\psi}]^{1/\psi}, \tag{2.6}
\]

where \( c_{pt} \) and \( c_{gt} \) denote private and public consumption, \( \frac{1}{\psi} > 0 \) is the elasticity of substitution, and \( \tau \) is a share parameter. The household’s production technology is the Cobb-Douglas specification

\[
y_t = z_t K_t^\alpha n_t^{1-\alpha}, \tag{2.7}
\]

where \( K_t \) and \( n_t \) denote the inputs of aggregate capital and labor needed for the production process, \( z_t \) is a productivity or technology shock, and \( \alpha \) is capital’s share of output. Aggregate capital \( K_t \) embeds private and public capital according to the CES specification

\[
K_t = [\omega k_{pt}^{\chi} + (1 - \omega)k_{gt}^{\chi}]^{1/\chi}, \tag{2.8}
\]

\(^1\)The utility form \( u(C_t, l_t) = \log(C_t) + \gamma \log(l_t) \), on the other hand, precludes such a relationship unless utility depends on government consumption.
where $\frac{1}{1-\chi} > 0$ is the elasticity of substitution and $\omega$ is a share parameter. Finally, the economy satisfies the income identity

$$y_t = c_{pt} + c_{gt} + i_{pt} + i_{gt}.$$  \hspace{1cm} (2.9)

Notice that when $\tau = 1$, government consumption reflects a pure resource drain on the economy, since it takes away resources but does not enter the utility function. For $\tau < 1$, and $\psi > \gamma(1 - \phi)$, it can be shown that a unit increase in $c_{gt}$ decreases the marginal utility of private consumption, so they act as substitutes; the opposite is true when $\psi < \gamma(1 - \phi)$, where they act as complements. Similar relationships hold for aggregate capital: When $\omega = 1$ public capital is no longer an input in production; when $\omega < 1$ and $\chi < \alpha$ public capital complements private capital; and if the productivity of public capital is sufficiently large, private consumption and investment will increase in response to an increase in public investment.

2.3.3 Shocks

To maintain proper alignment with the data, stationarity is induced in the model by eliminating trends. Specifically, $c_{pt}, i_{pt}, k_{pt}, c_{gt}, i_{gt},$ and $k_{gt}$ are normalized by the common growth rate given by $\frac{g}{1-\alpha}$, and all variables are expressed as deviations from steady states. Technology is specified according to the following specification:

$$\log(\tilde{z}_t) = (1 - \rho_z)\log(\bar{z}) + \rho_z \log(\tilde{z}_{t-1}) + \epsilon_{zt},$$ \hspace{1cm} (2.10)

where $\bar{z} > 0$, $\rho_z \in (-1, 1)$, and $\epsilon_{zt} \sim N(0, \sigma^2_z)$.

Government consumption and investment, $c_{gt}$ and $i_{gt}$ are assumed to follow similar laws of motion after detrending:

$$\log(\tilde{c}_{gt}) = (1 - \rho_{c_g})\log(\bar{c}_g) + \rho_{c_g} \log(\tilde{c}_{gt-1}) + \epsilon_{c gt},$$ \hspace{1cm} (2.11)

where $\rho_{c_g} \in (-1, 1)$ and $\epsilon_{c gt} \sim N(0, \sigma^2_{c_g})$;

$$\log(\tilde{i}_{gt}) = (1 - \rho_{i_g})\log(\bar{i}_g) + \rho_{i_g} \log(\tilde{i}_{gt-1}) + \epsilon_{i gt},$$ \hspace{1cm} (2.12)

where $\rho_{i_g} \in (-1, 1)$ and $\epsilon_{i gt} \sim N(0, \sigma^2_{i_g})$.

\hspace{1cm} \textsuperscript{2}Tildes represent detrended values, and bars denote steady states.
It is possible that government expenditures may respond endogenously to TFP innovations. This is not modeled directly here, but endogeneity will be allowed for indirectly by estimating potential non-zero correlations between the innovations: $\epsilon_z$ and $\epsilon_{cg}$, and $\epsilon_z$ and $\epsilon_{ig}$. Finally, government purchases are assumed to be entirely financed by lump-sum taxes:

$$c_{gt} + i_{gt} = T_t.$$  

(2.13)

### 2.3.4 Model Solution

The equilibrium allocations for the model are found by solving the planner’s problem of maximizing $U$ subject to the production function equation (2.7), the laws of motion for both forms of capital (2.4 and 2.5), the income identity (2.9), the resource constraint (2.3), the laws of motion for government expenditures (2.11 and 2.12), and the specification for technology (2.10).

To summarize, the vector denoting the collection of model variables is

$$x_t = \left[ \log \frac{y_t}{\bar{y}}, \frac{c_{pt}}{\bar{c}_p}, \frac{i_{pt}}{i_p}, \log \frac{n_t}{\bar{n}}, \log \frac{l_t}{\bar{l}}, \frac{k_{pt}}{k_p}, \frac{k_{gt}}{k_g}, \frac{z_t}{\bar{z}}, \log \frac{c_{gt}}{\bar{c}_g}, \log \frac{i_{gt}}{i_g} \right]',$$

and there are 18 structural parameters

$$\mu = [\alpha \beta \delta \phi \gamma \rho_z \sigma_z \rho_{cg} \sigma_{cg} \rho_{ig} \sigma_{ig} \tau \psi \omega \chi \text{ Corr}(\epsilon_z, \epsilon_{cg}) \text{ Corr}(\epsilon_z, \epsilon_{ig})].$$

The model does not have a closed-form solution. The system of equations is mapped into a linearized system by taking a log-linear approximation around the steady states. The approximate solution takes the form $x_{t+1} = F(\mu)x_t + G(\mu)\nu_{t+1}$, where $\nu_{t+1}$ denotes the collection of structural shocks. I construct impulse response functions to trace the reaction of endogenous variables over time, around the steady states, in response to innovations to the exogenous shocks: TFP, government consumption, and government investment.\(^3\)

### 2.4 DATA

In the data, private consumption is measured as non-durable goods and services, and private investment is measured as real private fixed investment; both are measured in chained 2000 dollars.

\(^3\)The implementation algorithm and computer code was supported by DeJong and Dave (2007).
These come from the Federal Reserve Bank of St. Louis' FRED database. Hours worked are measured as those worked in the nonfarm business sector, and come from the Bureau of Labor Statistics' (BLS) Establishment Survey.

I construct the data on government components using National Income and Product Accounts (NIPA) tables produced by the Bureau of Economic Analysis. This is because they have a measure for military consumption and investment that I deduct from the government series. This is done to make sure the model is well-aligned with the data: it is not clear how government consumption providing agents with utility can be mapped with “military consumption”, nor how public capital entering the production function can be mapped with “military investment”. Furthermore, the VAR literature discussed in the review section typically considers total government expenditures minus defense components. Figures 2.1 and 2.2 show plots of the military and non-military components of real raw government consumption and investment expenditures. Military consumption constitutes about 34% of total government consumption and military investment about 24% of total government investment. After removing defense expenditures, the mean of government consumption to total output is roughly 13% and the mean of government investment to output is 3%, where output is measured as $Y_t = C_{pt} + C_{gt} + I_{pt} + I_{gt}$.

![Figure 2.1: A Plot of U.S. Raw Government Consumption Decomposed in Two Components](image)

It is worthwhile to note here that government consumption and investment both include expen-
Figure 2.2: A Plot of U.S. Raw Government Investment Decomposed in Two Components

ditures on items such as general public service, public order and safety, economic affairs, housing and community services, health, recreation and culture, education, and income and security. However, government consumption measures the value of these goods and services provided, while government investment measures the value of the equipment, software and structures that the government uses in producing these goods and services.\(^4\) Hence, the measurement is well-aligned with the model, wherein public capital is measured as accumulated government investment.

The sample period considered is 1955:I through 2008:I. All the series are quarterly, expressed in per-capita terms and normalized by the potential aggregate labor force, using the civilian non-institutional population aged 16 years and over. All series are seasonally adjusted except for population. Trends are removed from the data by using the HP filter, which is sufficient for eliminating trends including time-varying trends. Figure A.1 in the Appendix illustrates data; solid blue lines are logged data values, and dashed lines are HP trends. Finally, although the model variables are detrended, information regarding steady states and relative sample values will be restored. This will provide critical identification for the parameter $\gamma$, where the fraction of hours

\(^4\)For example, government consumption in the case of education includes wages and salaries paid to employees working in the education sector, while government investment includes payments on desks, the physical school building, computers in labs, etc.
worked will be a $\frac{1}{3}$ in the steady state. This will be discussed further in Section 2.6.

2.5 MODEL IMPLICATIONS

Before estimating the model, and to provide intuition behind the implications of the model’s dynamics, I illustrate specific cases of how different parameterizations in the CES consumption and production functions carry alternative implications for the relationship between public and private variables. The set of parameter values is summarized in Table 2.1. The capital share parameter $\alpha$, the discount rate $\beta$, the depreciation rate $\delta$, and the coefficient of relative risk aversion $\phi$ are set to standard values, and $\gamma$ is chosen so that hours worked in the steady state is $\frac{1}{3}$. In calibrating the autocorrelations and standard deviations of the government shocks, I run OLS regressions based on the government consumption and investment series (Equations 8 and 9). I find the persistence parameters $\rho_{c_g}$ and $\rho_{i_g}$ to be low (0.684 and 0.683). Hansen and Wright (1992) calibrate $\rho_{c_g}$ to 0.96, but their sample period is shorter. Others (e.g., see Linneman, 2006) simply set it to 0.9 without discussing the reasons. Thus when calibrating the model in this way, a better representation of the data comes with a cost of a lower quantitative effect of a government spending shock on private activity. This will change with the estimated parameters I discuss later.

The standard deviation and autocorrelation of the technology shock were chosen to match those of output in the data (Table 2.2 reports the targets matched). The growth rate $g$ is calibrated to satisfy: $g = (1 - \alpha) \times \hat{g}_y$, where $\hat{g}_y$ is the sample estimate of the growth rate of output (0.00472 for the period considered). Finally, $\bar{c}_g / \bar{y}$ and $\bar{i}_g / \bar{y}$ were restricted to match the means found in the data.

2.5.1 A Government Consumption Shock

In order to isolate the effects of a government consumption shock, I set $\omega$ and $\chi$ to 1 in what follows so that public capital has no effect on aggregate capital. Note that if $\tau$ and $\psi$ are 1
### Table 2.1: Parameter Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.33</td>
<td>Capital share parameter</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.99</td>
<td>Discount factor</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.025</td>
<td>Depreciation rate of private and public capital</td>
</tr>
<tr>
<td>$\phi$</td>
<td>1.5</td>
<td>Coefficient of relative risk aversion</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.29</td>
<td>Consumption’s share relative to leisure of utility</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>0.87</td>
<td>Technology shock persistence parameter</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>0.0037</td>
<td>Standard Deviation of the technology shock</td>
</tr>
<tr>
<td>$\rho_{cq}$</td>
<td>0.684</td>
<td>Public consumption shock persistence parameter</td>
</tr>
<tr>
<td>$\sigma_{cq}$</td>
<td>0.010</td>
<td>Standard deviation of the public consumption shock</td>
</tr>
<tr>
<td>$\rho_{iq}$</td>
<td>0.683</td>
<td>Public investment shock persistence parameter</td>
</tr>
<tr>
<td>$\sigma_{iq}$</td>
<td>0.026</td>
<td>Standard deviation of the public investment shock</td>
</tr>
<tr>
<td>$g$</td>
<td>$(0.00472) \cdot (1 - \alpha)$</td>
<td>Growth Rate</td>
</tr>
</tbody>
</table>

### Table 2.2: Calibration Statistics: Matching Moments

<table>
<thead>
<tr>
<th>Target</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Autocorrelation of $Y$</td>
<td>0.89</td>
<td>0.89</td>
</tr>
<tr>
<td>Standard Deviation of $Y$</td>
<td>0.0120</td>
<td>0.0118</td>
</tr>
<tr>
<td>Autocorrelation of $C_q$</td>
<td>0.684</td>
<td>0.69</td>
</tr>
<tr>
<td>Standard Deviation of $C_q$</td>
<td>0.014</td>
<td>0.015</td>
</tr>
<tr>
<td>Autocorrelation of $I_q$</td>
<td>0.683</td>
<td>0.684</td>
</tr>
<tr>
<td>Standard Deviation of $I_q$</td>
<td>0.036</td>
<td>0.036</td>
</tr>
</tbody>
</table>
as well, government consumption does not enter household utility. An increase in government consumption reduces resources available to the private sector: Agents will respond by lowering private consumption and working more so that consumption does not drop by much. This is a well-known negative wealth effect documented, e.g. by Christiano and Eichenbaum (1992) and Baxter and King (1993).

If the household derives utility from government consumption (represented here using $\tau = 0.83$), the effect on private consumption depends on the elasticity of substitution.\(^5\) Figure 2.3 illustrates the difference between a resource drain ($\tau$ and $\psi = 1$) and a case where government consumption perfectly substitutes private consumption in the utility function ($\tau = 0.83$ and $\psi = 1$). Clearly, private consumption decreases more in the second case.

\(^5\)Public consumption constitutes about 17% of total consumption in the data.
Figure 2.4: Impulse Response Functions to a 1% Increase in Government Consumption: Perfect Substitutes with Effect on Utility (Dashed), Less than Perfect Substitutes (Dotted), and Complements (Solid)
in government consumption on the marginal utility of private consumption, which is given by

\[
\frac{\partial \tilde{\lambda}_t}{\partial \tilde{c}_{gt}} = \left[ \gamma (1 - \phi) - \psi \right] \frac{\partial \tilde{C}_t}{\partial \tilde{c}_{gt}} + \left[ (1 - \gamma)(1 - \phi) \right] \frac{\partial \tilde{l}_t}{\partial \tilde{c}_{gt}},
\]

(2.14)

where \( \lambda_t \) is the Lagrange multiplier associated with the budget constraint at time \( t \). We know that \( \frac{\partial \tilde{C}_t}{\partial \tilde{c}_{gt}} \) is positive and equals \( (1 - \tau) \ast (\frac{\bar{c}_g}{\bar{C}^*})^\psi \), and \( \frac{\partial \tilde{l}_t}{\partial \tilde{c}_{gt}} \) has been shown to be negative for the utility form used here.\(^6\)

Note that if \( \tau = 1 \), there are two ways government consumption can have effects in the model: first through the resource constraint which induces a negative wealth effect; second through the second part of equation 2.14 which is positive. Linnemann (2006) shows that the non-separable utility form used here provides a necessary but not sufficient condition to generate a crowding-in effect on private consumption. This is so because an increase in employment makes the household want to consume more (consumption and leisure are substitutes). Since this effect has to be strong enough to overcome the negative wealth effect, the model here is augmented with the possibility that agents derive utility from public consumption. For example, if \( \tau < 1 \) and \( \psi < \gamma (1 - \phi) \), public and private consumption are complements.\(^7\) In figure 2.4, the solid lines show that the model can generate a very strong quantitative complementarity effect: For a 1% increase in government spending, private consumption can increase up to 0.25%. As for the effect on private investment, the impulse response functions show that it decreases after the shock, since the government shock is not sufficiently persistent, in which case private consumption doesn’t fall enough for investment to rise.

Finally, Figures 2.5 and 2.6 consider the case where government and private consumption are complements and tracks different impulse responses according to different parameterizations of \( \phi \) (Figure 2.5), and \( \gamma \) (Figure 2.6). As \( \phi \) increases, the intertemporal elasticity of substitution decreases, implying lower changes in consumption in response to government shocks. Interestingly, private investment increases for lower elasticities of substitution. For example, in the case where \( \phi = 5 \), the household increases hours worked but doesn’t increase consumption by much, which allows for an increase in investment. Higher values of \( \gamma \) on the other hand are associated with lower relative utility derived of leisure. This results in increasing hours worked, which in turn makes it

---

\(^6\)See Linnemann (2006) for a detailed proof.

\(^7\)The first part of the right hand side of equation 2.14 would be positive as a result.
more productive for the agent to invest, generating higher increases in output.

Figure 2.5: Impulse Response Functions to a 1% Increase in Government Consumption ($\psi = -1$): $\phi = 1.5$ (Dashed), $\phi = 3.5$ (Dotted), and $\phi = 5$ (Solid)

2.5.2 A Government Investment Shock

In order to understand how shocks to government investment expenditures are different from shocks to government consumption expenditures, I set $\tau$ and $\psi$ to 1 and examine the effects of different values of the public capital CES parameters on private variables. If both $\omega$ and $\chi$ are 1, government investment behaves the same way as government consumption that doesn’t affect household utility: It acts as a resource drain and does not affect marginal products. However, government investment may be used by the private sector for its required purposes in a way that it either encourages or competes with private activity. I set $\omega$ to 0.83 making public capital an input in the aggregate production function. The effect of an increase in public capital on the marginal product of private capital is given by:

$$\frac{\partial \tilde{\lambda}_t}{\partial k_{gt}} = (\alpha - \chi) \frac{\partial \tilde{K}_t}{\partial k_{gt}} + (1 - \alpha) \frac{\partial \tilde{n}_t}{\partial k_{gt}},$$

(2.15)

\footnote{Like government consumption, government investment is 17\% of total investment in the data.}
where \( \frac{\partial K_t}{\partial k_{gt}} \) is positive and equals \((1 - \omega) * (\frac{K_g}{K})^\chi\). Public capital increases the productivity of private capital when \( \chi < \alpha \). If this effect is strong enough, government investment shocks crowd-in private activity. Figure 2.7 illustrates different parameterizations of \( \omega \) and \( \chi \). Private investment decreases the most when public capital enters the production function, and perfectly substitutes private capital (\( \omega = 0.83 \) and \( \chi = 1 \) as indicated by the dotted lines). Finally, I set \( \chi \) to \(-0.5\) to illustrate the crowding-in effect shown with the solid lines. The contractionary effect dominates in the earlier periods due to government absorption of resources. However, as the capital stock builds up, both private consumption and investment eventually increase.

2.6 MAXIMUM LIKELIHOOD ESTIMATION

The model has 18 parameters contained in \( \mu \) describing preferences, technology, and the stochastic behavior of the exogenous shocks. Since the representation of the model is linear, and stochastic
innovations normally distributed, the likelihood function is evaluated via the Kalman Filter. The observable variables of the model are taken as private consumption and investment, hours worked, and government consumption and investment, and denoted as $X_t$. Using the solution of the system, and given a specification for the parameter space $\mu$, I form the parameters of the state-space representation. Since there are more observable variables than structural shocks, various combinations of observable variables are predicted to be deterministic (e.g., see Ingram, Kocherlakota and Savin, 1994). This stochastic singularity problem is avoided by augmenting the observation equations of the Kalman Filter with measurement errors. The observation equation is given by $X_t = H(\mu)'x_t + u_t$, where $x_t$ is the full vector of model variables.

A first attempt to estimate the model using a simplex method reveals, as in Ireland (2001), that the data do not contain sufficient information to estimate all of the model’s parameters. Specifically, the parameters $\alpha$ and $\delta$ prove difficult to identify. To circumvent this problem, I fix $\alpha$ and $\delta$ to 0.33 and 0.025, respectively, prior to estimation. Additionally, I impose restrictions on the parameters

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9 This method has been used in DSGE models by McGrattan (1994) and Ireland (2001) among others.
10 These values are widely accepted in the RBC literature. Fortunately, they do not bear directly on the issue of
in order to match the relative steady states of private consumption, private investment, government consumption and government investment to output, as well as the steady state of hours worked found in the data. Moreover, each parameter is restricted to lie in an economically plausible interval. Three measurement errors (corresponding to the private investment, government consumption and government investment series) hit their zero lower bound, so they were fixed at zero as a result.

Finally, a simulated annealing method has been used in optimization. This algorithm has proved more likely than other methods to find the global optimum and deal with difficult functions, and with ridges and plateaus.\footnote{Indeed, based on 75 different initial starting values, the simulated annealing method found a better global optimum than a gradient-based method.} The basic idea behind why simulated annealing is more powerful than other methods lies in that it explores the entire surface of the likelihood function, and optimizes the function by making both uphill and downhill moves.\footnote{Goffe et al. (1994) test simulated annealing on four statistical models and show that it outperforms other methods in finding the global optimum. I make use of their Gauss code implementation in this paper.}

This estimation method revealed another problem: the CES shares hit boundary solutions for some starting values. As a result, the shares $\tau$ and $\omega$ were both fixed at 0.83, since government consumption and investment constitute around 17% of total consumption and investment, respectively. This should not be a problem since the focus of this paper is to uncover the crowding-in and crowding-out effects of government variables on the private sector, which are indicated by the elasticity of substitution parameters, and not the share parameters. These restrictions, as well as estimation results, are reported in Table 2.3. Further, Figure A.2 in the Appendix depicts the likelihood function around some of the estimated parameters, by changing one parameter at a time, while Figure A.3 shows the likelihood surface and corresponding contours by changing two parameters at a time, with a focus on the two most important parameters $\psi$ and $\chi$.\footnote{The likelihood surfaces corresponding to the remaining parameters are similar to those reported here and are available upon request.} Overall, the parameters seem to be well identified, with clear peaks around their estimates.

The computation of standard errors was based on the Fisher information matrix.\footnote{Under some regularity conditions, the Fisher information matrix can be written as negative of the Hessian matrix.} Standard errors associated with parameter estimates were computed using 10,000 Monte Carlo simulations, wherein the variance covariance matrix was calculated using artificial data generated using the crowding-out versus crowding-in.
Table 2.3: Maximum Likelihood: Parameter Estimation Results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate*</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>0.9942</td>
<td>Discount factor</td>
</tr>
<tr>
<td>( \phi )</td>
<td>3.3863</td>
<td>Coefficient of relative risk aversion</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>0.9958</td>
<td>Consumption’s share relative to leisure of utility</td>
</tr>
<tr>
<td>( \rho_z )</td>
<td>0.9603</td>
<td>Technology shock persistence parameter</td>
</tr>
<tr>
<td>( \sigma_z )</td>
<td>0.0033</td>
<td>Standard Deviation of technology shock</td>
</tr>
<tr>
<td>( \rho_{eg} )</td>
<td>0.8739</td>
<td>Public consumption shock persistence parameter</td>
</tr>
<tr>
<td>( \sigma_{eg} )</td>
<td>0.0033</td>
<td>Standard deviation of public consumption shock</td>
</tr>
<tr>
<td>( \rho_{ig} )</td>
<td>0.7241</td>
<td>Public investment shock persistence parameter</td>
</tr>
<tr>
<td>( \sigma_{ig} )</td>
<td>0.0265</td>
<td>Standard deviation of public investment shock</td>
</tr>
<tr>
<td>( \psi )</td>
<td>-4.860</td>
<td>Substitution parameter - public and private consumption</td>
</tr>
<tr>
<td>( \chi )</td>
<td>-0.383</td>
<td>Substitution parameter - public and private capital</td>
</tr>
<tr>
<td>( \Sigma_{u_{cp}} )</td>
<td>0.0054</td>
<td>Private consumption - measurement error parameter</td>
</tr>
<tr>
<td>( \Sigma_{u_{in}} )</td>
<td>0.017</td>
<td>Hours worked - measurement error parameter</td>
</tr>
<tr>
<td>( \text{Corr}(\epsilon_z, \epsilon_{cg}) )</td>
<td>-0.297</td>
<td>Correlation between innovations to TFP and government consumption</td>
</tr>
<tr>
<td>( \text{Corr}(\epsilon_z, \epsilon_{ig}) )</td>
<td>0.250</td>
<td>Correlation between innovations to TFP and government investment</td>
</tr>
</tbody>
</table>

*: The restrictions imposed on the parameters are \( \beta \in (0, 1), \phi \in (0, 15), \gamma \in (0, 1), \rho_z, \rho_{eg}, \rho_{ig} \in (0, 1), \sigma_z, \sigma_{eg}, \sigma_{ig}, \Sigma_{u_{cp}}, \Sigma_{u_{jp}}, \Sigma_{u_{cq}}, \Sigma_{u_{ip}}, \Sigma_{u_{in}} \in (0, 0.05), \psi \text{ and } \chi \in (-5, 1), \text{ and } \text{Corr}(\epsilon_z, \epsilon_{cg}) \text{ and } \text{Corr}(\epsilon_z, \epsilon_{ig}) \in (-1, 1). \)
estimated parameter values. Standard errors were then simply computed as the square root of the diagonals of the variance covariance matrix. Finally, this leads to the classical inference procedure, wherein a 95% confidence interval based on the maximum likelihood estimator (MLE) is:

$$CI = [\mu_{MLE} - 2s.e., \mu_{MLE} + 2s.e.]$$

where s.e. corresponds to the standard errors of the residuals for government consumption and investment.

### 2.6.1 Implications for Crowding-in and Crowding-out Effects

#### 2.6.1.1 Government Consumption

Starting with standard parameters, the estimated value of $\beta$ of 0.995 ($s.e.s.1.06e - 007$) is precise and implies a quarterly discount rate of 0.6%. Regarding the estimate of the coefficient of relative risk aversion $\phi$, the estimated value here of 3.38 (4.44e−006) is higher than found in some other studies, but in line with papers such as DeJong and Ripoll (2006) who similarly estimate $\phi$ around 3.5. This estimated value indicates the agent’s desire to smooth consumption, and proves key in helping the model generate the increase in the real wage due to a government consumption shock, which is also observed in VAR studies. The high estimate of $\gamma$ of 0.995 (1.21e − 007) indicates that the agent places a high weight on consumption relative to leisure in instantaneous utility. This helps create the required volatility in hours worked to match the second moments related to labor market puzzles.

Turning to the parameters of the shocks, the technology and government consumption shocks are more persistent than their calibration counterparts, which implies amplified effects on the private sector. The persistence of government investment, on the other hand, is closer to its calibrated value. The innovations to TFP and government consumption and investment are found to be −0.29 (0.15) and 0.25 (0.20), respectively, indicating relatively small feedback effects from innovations to

---

15 This relies on the following theorem: For large iid samples of size n,

$$P(\mu_{MLE}|\mu_0) = N(\mu_0, [I_X(\mu_{MLE})]^{-1}) = N(\mu_0, [nI_{X_i}(\mu_{MLE})]^{-1}),$$

where $I$ represents the Fisher information matrix, $X$ is the data set used in estimation, and $X_i$ corresponds to the $i^{th}$ simulated data set.

16 This takes place after 8 quarters in the estimated impulse response functions presented in Figure 2.8.

17 Note that the steady state of hours worked is still 1/3.

18 This is discussed in detail in Subsection 2.6.2.
output to innovations to government spending.

Of special interest are the parameters related to substitutability/complementarity. These are $\psi$ for government consumption and $\chi$ for government investment. The approximate estimate of $\psi$ of $-4.86$ ($1.36e-005$) makes it much lower than $\gamma * (1 - \phi)$ implying a crowding-in effect, as shown in the impulse response functions in Figure 2.8. This estimate implies an elasticity of substitution of 0.17 between private and government consumption, meaning they are complements.

Earlier studies that looked at the substitutability between private and government consumption report mixed results depending on the functional utility form assumed. For example, Aschauer (1985) assumes a linear utility form and find the two types of spending to be substitutes, while Ni (1995) assumes a non-separable utility form and finds them to be complements. Here, a more general non-linear CES form is assumed, which encompasses both forms of utility mentioned. Bouakez and Rebei (2007) recently use a CES form and report an elasticity of 0.33. However, there is an additional amplification effect here through which government consumption can cause crowding-in. The assumption that utility is non-separable in consumption and leisure further creates interesting
dynamics in the labor market, as government consumption can affect private consumption through more than one channel. In addition to directly substituting or complementing private consumption, the effect on hours worked has further complementary effects on private consumption. Last, the parameters here are jointly estimated and depend on crucial parameters such as $\phi, \gamma,$ and $\chi$.

Overall, estimation results indicate a much stronger quantitative result than was shown in the calibration exercise. Specifically, one standard-deviation innovation to government consumption induces a 0.34% increase in government consumption. This estimated government shock causes private consumption to rise by about 0.2%. This amounts to a rough 0.55% increase in private consumption in response to a 1% increase in government consumption. Note that the maximum increase observed in the calibration exercise was around 0.25% for a 1% increase in government consumption.

There are two reasons behind this result. First, the substitutability between consumption and leisure is strong enough to overcome the absorption of resources; this is better illustrated by examining the response of hours worked, increasing to about 0.7%. Second, the complementarity between government consumption and private consumption in the CES function is also high, as indicated by $\psi$.

As for the effects on output, the impulse response functions reveal that output rises by 0.5%, implying an approximate 1.5% increase due to a 1% increase in government consumption. This result should not be surprising, as similar evidence arises from VAR estimates. For example, Gali et al. (2007) report high output multipliers, some of which are greater than 1, depending on the sample period, and how many variables are included in the VAR. Similarly, the New-Keynesian model they develop features higher than 1 output multipliers as well.

While the VAR literature concludes that private consumption is crowded-in by government consumption, it does now show clear results concerning the response of private investment to a government consumption shock. For example, Fatas and Mihov (2002) estimate a positive response while others such as Kamps and Caldara (2008), and Blanchard and Perotti (2002) estimate small or insignificant effects. The estimates here indicate that private investment persistently rises (by almost 2%), since the increase in hours worked makes investment affordable to the household. While this estimate strikes as being high, note that private investment in the data is typically seven times more volatile than private consumption. Second, had the estimate of $\psi$ been lower
(−2.86 instead of −4.86 for example), the increase in private investment would have been lower as well, but this would come at a cost of hindering the match of other second moments with their data counterparts.\textsuperscript{19}

2.6.1.2 Government Investment As for the government investment parameters, the approximate estimate of $\chi$ of $−0.38 \ (7.73e \ – \ 005)$ is lower than $\alpha$. This implies a complementarity effect in the CES production function, as indicated in Figure 2.9. Specifically, the estimated government shock of size 2.6\% causes private investment to rise by about 0.1\%. This amounts roughly to a 0.04\% increase in private investment in response to a 1\% increase in government investment. The effects on output are lower, where an increase of 0.0085\% is noted for a 1\% increase in government investment. The effects of government investment on the real wage are similar to those found for government consumption. The real wage increases in both cases, confirming the impulse responses found in VAR studies that use the sum of government consumption and investment as their measure for government spending.

![Figure 2.9: Impulse Response Functions to an Estimated Government Investment Shock with 95\% (Dashed) Confidence Intervals](image)

\textsuperscript{19}These are discussed in Subsection 2.6.2.
Earlier studies that looked at the effect of government capital on private activity have either used reduced-form regressions or VAR models, and find mixed results depending on the assumptions or identification method used. Aschauer (1989) and Munnell (1990) estimate very high values of the elasticity of output with respect to public capital (0.39 and 0.34, respectively). Lynde and Richmond (1992) find a smaller but still significant estimate of 0.2 using time series techniques. These studies however ignore state or time-effects. Once these are controlled for, papers find an estimate very close to zero (e.g., see Holtz-Eakin, 1994, Hulten and Schwab, 1991; and Tatom, 1991).

In this paper, I estimate the elasticity of substitution assuming a CES form, as well as use a structural model for that purpose. This takes into account possible non-linearities without imposing the assumption that public capital is a direct complementary input in production. The estimated elasticity parameter confirms the conventional wisdom on the noted crowding-in effects of public capital, via the channel where it encourages and complements private activity. However, the output elasticity with respect to public capital of 0.0085 is much smaller than found in studies such as Aschauer (1989) using the production function approach and Pereira (2001) using VAR. Contrary to the conventional wisdom, the findings presented here show that government consumption is more stimulating for output than government investment. One potential reason behind this result is that government capital takes a longer time to be built, consequently delaying the effects on output. The impulse response functions in Figure 2.9 confirm this intuition: Private capital does not start accumulating until the $12^{th}$ quarter.

Another potential reason is that the focus here is on aggregate spending. A large fraction of government consumption consists of spending on education (25%), which is found to be complementary to private activity in studies using disaggregated data (e.g., see Evans and Karras, 1994). Government investment’s largest component, on the other hand, is spending on economic affairs (35%), while education spending constitutes about 15% of total government investment.
2.6.2 Implications for Second Moments

Finally, it is of interest to look at statistics implied by the estimated model. Table 2.4 shows that they closely match those found in the data. The model accounts well for the volatility of private consumption, investment and even hours worked relative to output. It also replicates well the first-order autocorrelations of the series, and to a good extent the correlations across series. More importantly, the estimated model accounts well for a puzzle observed in RBC models, which is their inability to replicate some moments related to labor market behavior. Christiano and Eichenbaum (1992) and Hansen and Wright (1992) document that the prototypical RBC model cannot account for the low correlation between real wages and hours worked, or the high volatility of hours worked relative to real wages. Following Hansen and Wright (1992), average productivity is used as a proxy for real wages and denotes output divided by hours worked. This can be seen in the table, where the calibrated version of the model shows that the standard deviation of hours worked relative to the real wage is 0.51 and the correlation between these two series is high.

The reason behind why the estimated model fares so well is that an increase in government spending shifts the supply of labor, and hours worked increase while the real wage falls. These dynamics don’t arise when considering productivity shocks alone, since an increase for the demand of labor causes both hours worked and the real wage to increase. The labor supply channel is further magnified in the estimated model by the agent’s higher preference for consumption relative to leisure (γ = 0.995.)

---

20 Second moments were computed at the model's estimated parameters (e.g., see McGrattan, Rogerson, and Wright, 1997).

21 Hansen and Wright (1992) report that the empirical correlation between hours worked and average productivity lies between −0.35 and 0.10, depending on the series used for hours worked. The empirical correlation presented here is lower. First, they use GNP as their measure for output, while the sum of total consumption and investment is used here. Second, they use a shorter data sample. We perform a sensitivity analysis and show that the correlation between hours worked and productivity can differ depending on the definition of output used. When we use GNP as the output measure and the shorter data series of Hansen and Wright (1992) from 1955 through 1988, the correlation drops to -0.34. Their measure for the relative standard deviation between the two series on the other hand lies between 1.37 and 2.15, and is closer to that reported here.

22 In the calibrated model, I assume that government consumption and investment do not enter in the CES functions, and are simply resource drains.
Table 2.4: Actual, Calibrated, and Estimated Moments

<table>
<thead>
<tr>
<th>Measure</th>
<th>Data</th>
<th>Estimated Model</th>
<th>Calibrated Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{\ddot{y}}$</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>$\sigma_{\ddot{c}_p}$</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>$\sigma_{i_p}$</td>
<td>0.05</td>
<td>0.05</td>
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<td>$\sigma_{\ddot{c}_g}$</td>
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<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>$\sigma_{i_g}$</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
</tr>
</tbody>
</table>

| $\sigma_{\ddot{c}_g}/\sigma_{\ddot{y}}$ | 0.64   | 0.53            | 0.70             |
| $\sigma_{i_p}/\sigma_{\ddot{y}}$      | 4.06   | 4.12            | 2.92             |
| $\sigma_{\ddot{n}}/\sigma_{\ddot{y}}$ | 1.46   | 1.53            | 0.39             |
| $\sigma_{\ddot{c}_g}/\sigma_{\ddot{y}}$ | 1.26   | 0.59            | 1.14             |
| $\sigma_{i_g}/\sigma_{\ddot{y}}$      | 3.07   | 3.33            | 2.96             |
| $\sigma_{\ddot{n}}/\sigma_{\ddot{w}}$ | 1.70   | 0.92            | 0.51             |

| Corr($\ddot{y}, \ddot{y}^l$) | 0.89   | 0.88            | 0.89             |
| Corr($\ddot{c}_p, \ddot{c}_p^l$) | 0.85   | 0.95            | 0.98             |
| Corr($\ddot{i}_p, \ddot{i}_p^l$) | 0.90   | 0.85            | 0.84             |
| Corr($\ddot{n}, \ddot{n}^l$)      | 0.90   | 0.91            | 0.83             |
| Corr($\ddot{c}_g, \ddot{c}_g^l$) | 0.69   | 0.87            | 0.68             |
| Corr($\ddot{i}_g, \ddot{i}_g^l$) | 0.68   | 0.72            | 0.68             |

| Corr($\ddot{c}_p, \ddot{y}$)       | 0.90   | 0.92            | 0.83             |
| Corr($\ddot{i}_p, \ddot{y}$)      | 0.91   | 0.96            | 0.92             |
| Corr($\ddot{n}, \ddot{y}$)        | 0.82   | 0.18            | 0.73             |
| Corr($\ddot{c}_g, \ddot{y}$)      | 0.22   | 0.78            | 0.02             |
| Corr($\ddot{i}_g, \ddot{y}$)      | 0.32   | 0.04            | 0.01             |
| Corr($\ddot{n}, \ddot{w}$)        | -0.75  | -0.81           | 0.45             |

---

*a* $w$ refers to the real wage measured by output divided by hours worked following Hansen and Wright (1985). The superscript $l$ denotes lagged values.

*b* The statistics are based on logged and HP filtered quarterly data for the period 1955:1 - 2008:1.

*c* The parameters in Table 2.1 are used to calculate the moments based on the calibrated model. The CES parameters are all set to 1.
2.7 CONCLUSION

The empirical evidence on the effects of fiscal spending shocks can be explained through many mechanisms. The empirical evidence from VAR is mixed both for public consumption and public investment effects, and no clear resolution has emerged. In this paper, I presented a structural model flexible enough to generate either increases or decreases in private consumption and investment depending on the parameters used. When estimated, it shows crowding-in effects of public consumption and investment. Specifically, government consumption has higher multipliers than government investment: a 1% increase in government consumption increases private consumption by approximately 0.55%, and output by 1.5%, while a 1% increase in government investment increases private investment by 0.04% and output by 0.0085%. Additionally, the estimated model accounts for observed second moments in the data fairly well.

Finally, I have assumed in this paper that government expenditures were entirely financed by lump-sum taxation. Future work needs to relax this assumption, as distortionary taxation can have counterproductive effects on the private sector. Future work also needs to relax the Ricardian Equivalence assumption and estimate multipliers under different financing scenarios. Multipliers can also be estimated differently in expansions and in recessions under certain government feedback rules.
3.0 PUBLIC EXPENDITURES AND INTERNATIONAL BUSINESS CYCLE PUZZLES

3.1 INTRODUCTION

There are two well-known stylized facts observed in industrialized countries that an open real business cycle model fails to account for. First, current account balances are found to be small, that is national saving and investment move together (e.g., see Feldstein and Horioka henceforth FH; 1980). This finding has been documented as evidence against perfect capital mobility. We would expect that in a small open economy, additional savings should be invested around the world in countries where those savings can earn the highest expected rates of return, not necessarily affecting domestic investment. The second puzzle is that cross-country consumption correlations are found to be low relative to output correlations (e.g., see Backus, Kehoe and Kydland henceforth BKK; 1992 and Ambler, Cardia and Zimmerman; 2004). In theory, we would expect that even if output correlations are low across countries, consumption correlations should still be high because of international risk-sharing.

The international macroeconomics literature is not short of documentation and potential explanations to each of these puzzles. However, a few of these explanations allow for a government sector and none take the composition of government spending seriously, and the puzzles still remain. This paper explores the possibility that the public sector has a central role to understanding capital flows across countries. The question here is whether realistic government expenditure processes are quantitatively able to bring simulated private international consumption and investment-saving correlations in the range of what we observe in the data.

This paper shows that the observed international comovements can be rationalized by a standard dynamic stochastic general equilibrium (DSGE) model once two forms of government purchases are
allowed to generate aggregate fluctuations. In particular, government consumption and investment shocks are introduced in the model. Government consumption shocks were first introduced in real business cycle models by Christiano and Eichenbaum (1992) and Baxter and King (1993) because of their ability to help account for some aspects of the labor market. As for government investment shocks, Aschauer (1989a and 1989b) were the first to argue that public capital can be used as an input in aggregate production. Mazraani (2010) estimates in post-war US data that both government consumption and investment shocks have complementary effects on the private sector in the utility and production functions, respectively. These shocks are proved to help account for international business cycle puzzles in this paper.

The intuition behind the ability of government spending shocks to resolve these two puzzles is straightforward. Due to a government investment shock taking place in one country, private economic activity is boosted through the complementarity channel with private investment. Private investment and output and saving increase as a result: This helps explain the FH puzzle. Hence, saving and investment can be positively correlated due to an omitted factor other than technology shocks in a setting where capital is assumed to be mobile across the two countries. Government investment shocks are found to be as quantitatively able to explain the puzzle as technology shocks.

Government consumption purchases are used to explain the BKK puzzle. Wen (2007) argues that consumption demand shocks in a general equilibrium model can explain the low cross-country consumption correlations observed in the data. The argument is that preference shocks are country-specific causing consumption expenditures to be less synchronized across countries. However, in their setting shocks to consumption demand only come from preference changes which are unobservable in the data. This paper can be viewed as an attempt to give these preference shocks a name in order to quantitatively test the implied correlations. Here, government consumption shocks are used as a proxy for preference country-specific disturbances. Since these are observable in the data, consumption demand will be exogenously given as a result. These shocks are quantitatively able to explain that consumption correlations are low across countries.

The chapter proceeds as follows: Sections 3.2 and 3.3 review the empirical findings as well as potential explanations in the literature of the two puzzles summarized and clearly discusses the paper’s contribution; Section 3.4 presents the model; and Section 3.5 discusses the data. Section 3.6 presents the calibrated parameters, and Section 3.7 presents calibration exercises designed to
illustrate the model’s intuition. Section 3.8 then presents moments’ results and sensitivity checks. Section 3.9 concludes.

3.2 THE FH PUZZLE

Feldstein and Horioka estimated cross-country regressions of long period averages of investment on saving over the 1960-1974 period for 16 OECD countries and found the so-called retention coefficient to be close to unity. This sparked an immense literature testing this hypothesis for different countries and time spans, examining what the coefficient means, and providing various explanations for the mismatch with the theory. While authors such as Blanchard and Giavazzi (2002) claim that the relationship between saving and investment in the euro area has declined substantially over time, others including Feldstein (2005) argue that while the link has become weaker for the smaller OECD countries, it remains high for larger ones. Obstfeld and Rogoff (2000) and Kim (2001) similarly confirm the FH findings with more recent data.

Much of the literature has attempted to build models that can predict high investment-saving correlations as observed in the data, while maintaining the assumption that capital is perfectly mobile. Several studies have explained the puzzle with technology shocks (e.g., see Baxter and Crucini; 1993 and Mendoza; 1991). Other explanations have attributed it to high population growth (e.g., see Obstfeld; 1986), non-traded goods (e.g., see Tesar; 1993), asymmetric information (e.g., see Gordon and Bovenberg; 1996), and trade costs (e.g., see Obstfeld and Rogoff; 2000).

The argument in this paper is that government investment shocks can have the same effect on private activity as technology shocks. Investment shocks that complement private investment increase productivity and output, causing the high saving-investment correlation. While Cardia (1991) explores the role of government expenditures in affecting the FH puzzle, their analysis abstracts away from the fact that government investment might have complementarity effects on the private sector. In their paper, government shocks are solely government consumption shocks that act as a resource drain on the economy. The distinction between government consumption and investment is important for the purposes of this paper since these two expenditures have very different effects on private activity. In particular, government investment shocks can be interpreted
3.3 THE BKK PUZZLE

In open economies, countries experience different shocks to their technologies. This might cause output fluctuations in different countries to be imperfectly correlated. With complete markets where agents can share risk internationally however, we expect to see a large correlation between consumption fluctuations across countries. Backus, Kehoe, and Kydland (1992) report that in the data, findings are reversed. They report that the consumption correlation between the US and Europe for example is 0.46, compared to an output correlation of 0.70 between the two countries. They attribute this finding to barriers of international trade in goods and assets.

Other explanations in the literature include fluctuations in non-traded goods (e.g., see Tesar; 1993), taste shocks (e.g., see Stockman and Tesar; 1995), trade costs (e.g., see Obstfeld and Rogoff; 2000), and demand shocks (e.g., see Wen; 2007). In this paper, a different approach is used wherein government consumption shocks providing agents with utility can similarly help explain the BKK puzzle. Since government consumption purchases cannot be relocated internationally, government consumption shocks create a wedge between domestic and foreign consumption. In that sense, it can be interpreted as the taste shock of Stockman and Tesar (1995), or the preference shock of Wen (2007).

The closest papers to the approach followed here are those of Marrinian (1999) and Roche (1996). In their setting, government consumption affects private utility in a linear fashion and different values of substitutability are explored under which the consumption correlations implied by the model can match those in the data. They find that government shocks can help explain the puzzle only if they are extremely persistent and if they are not strong substitutes with private consumption. Mazraani (2010) estimates however government consumption shocks to be strong complements with private consumption. In that paper, the complementarity effect is found to be highly amplified through the substitutability between leisure and private consumption. Since a high complementarity is needed to resolve the BKK puzzle, the format of that paper will be followed.
3.4 THE MODEL

The model employed extends that of Baxter and Crucini (1993) to include a government sector. There is a single consumption good produced in two countries. In addition, there are two forms of consumption and capital, both private and public.

3.4.1 Households

In the model, the economy in each country consists of a large number of identical households. The representative household’s objective in country \( i = h, f \) (home and foreign) is to maximize his expected discounted flow of utility given his preferences defined over effective consumption and leisure during each period \( t=0,1,2,... \), as described by the utility function:

\[
\max_{c_{ipt},l_{it}} U = E_0 \sum_{t=0}^{\infty} \beta^t u(C_{it}, l_{it}) \quad (3.1)
\]

Here, \( E_0 \) is the usual expectations operator conditional on the information available at time 0 and \( \beta \) is the discount factor, with \( \beta \in (0, 1) \). \( u(.) \) is an instantaneous utility function, and \( C_{it} \) and \( l_{it} \) are levels of effective consumption and leisure chosen in country \( i \) at time \( t \). Preferences are specified according to:

\[
u(C_{it}, l_{it}) = \left( \frac{C_{it}^{\gamma} l_{it}^{1-\gamma}}{1-\phi} \right)^{1-\phi}, \quad (3.2)
\]

where \( \phi > 1 \) is the coefficient of relative risk aversion, and \( \gamma \in (0, 1) \) is consumption’s share relative to leisure of instantaneous utility. Effective Consumption \( C_{it} \) is divided into private and public consumption according to a CES (constant elasticity of substitution) specification:

\[
C(c_{ipt}, c_{igt}) = [\tau c_{ipt}^{\psi} + (1-\tau)c_{igt}^{\psi}]^{1/\psi}, \quad (3.3)
\]

where \( \frac{1}{1-\psi} \) is the elasticity of substitution, \( \tau \), and \( 1-\tau \) are share parameters of private and public consumption, respectively.

The household in each country maximizes his utility derived from consumption and leisure, knowing that every period it is constrained with one unit of time to be allocated between two activities:

\[
1 = n_{it} + l_{it}. \quad (3.4)
\]
3.4.2 Firms and Government

The household’s production technology is a constant returns to scale Cobb-Douglas function, with a CES capital aggregator:

\[ y_{it} = z_{it} K_{it}^\alpha n_{it}^{1-\alpha}, \tag{3.5} \]

where \( K_{it} \) and \( n_{it} \) denote the inputs of aggregate capital and labor needed for the production process in country \( i \), \( z_{it} \) is a country-specific productivity disturbance, and \( \alpha \) is capital’s share of output. Aggregate capital \( K_{it} \) embeds private and public capital according to the CES specification

\[ K_{it} = [\omega k_{ipt}^\chi + (1 - \omega) k_{igt}^\chi]^{1/\chi}, \tag{3.6} \]

where \( \frac{1}{\chi} > 0 \) is the elasticity of substitution and \( \omega \) is a share parameter.

By investing \( i_{ipt} \) units of output during period \( t \), the household increases the capital stock \( k_{ipt+1} \) according to the law of motion:

\[ k_{ipt+1} = \eta\left(\frac{i_{ipt}}{k_{ipt}}\right) + (1 - \delta) k_{ipt}, \tag{3.7} \]

where \( \eta\left(\frac{i_{ipt}}{k_{ipt}}\right) \) determines the change in private capital stock (gross of depreciation) induced by investment spending. This adjustment cost is introduced to avoid large capital movements across borders following Baxter and Crucini (1993).\(^1\) We will assume that \( \eta' > 0 \), and \( \eta'' \leq 0 \), with \( \eta'\hat{\delta} = 1 \), and \( \eta(\hat{\delta}) = \hat{\delta} \).\(^2\) This will guarantee that there are no adjustment costs in the steady state. The aggregate public capital stock follows the law of motion:

\[ k_{igt+1} = i_{igt} + (1 - \delta) k_{igt}, \tag{3.8} \]

where \( \delta \) is the depreciation rate for both private and public capital stocks.

Since both countries produce the same good, the world resource constraint is given by:

\[ \pi y_{ht} + (1 - \pi) y_{ft} = \pi (c_{hpt} + c_{hgt} + i_{hpt} + i_{hgt}) \]

\[ + (1 - \pi) (c_{fpt} + c_{fgt} + i_{fpt} + i_{fgt}), \tag{3.9} \]

where we have assumed that \( TB_h + TB_f = 0 \), where \( TB_i \) is the trade balance in country \( i \), and where \( \pi \) is the fraction of agents living in the home country. Finally, I will also assume that government

---

\(^1\)It is shown in Baxter and Crucini (1993) that without these transaction costs, investment turns out to be too volatile compared to the data.

\(^2\)\( \hat{\delta} \) refers to \( \delta + g \), where \( g \) is the growth rate.
purchases are entirely financed by lump-sum taxes:

\[ c_{igt} + i_{igt} = T_{it}. \]  

(3.10)

### 3.4.3 Shocks

To maintain proper alignment with the data, stationarity is induced in the model by eliminating trends. Specifically, \( c_{ipt}, i_{ipt}, k_{ipt}, c_{igt}, i_{igt}, \) and \( k_{igt} \) are normalized by the common growth rate given by \( \frac{g}{1-\alpha} \), and all variables are expressed as deviations from steady states. The world economy is subject to a 6 x 1 vector of shocks: \( z_{ht}, z_{ft}, c_{hgt}, c_{fgt}, i_{hgt}, \) and \( i_{fgt} \). The logs of the shocks will be assumed to exhibit AR(1) fluctuations about a linear trend. Technology in the home country is specified according to:

\[ \log(\tilde{z}_{ht}) = (1 - \rho_{hz})\log(\bar{z}) + \rho_{hz}\log(\tilde{z}_{ht-1}) + \nu\log(\tilde{z}_{ft-1}) + \epsilon_{zht}, \]  

(3.11)

while technology in the foreign country is specified according to:

\[ \log(\tilde{z}_{ft}) = (1 - \rho_{fz})\log(\bar{z}) + \rho_{fz}\log(\tilde{z}_{ft-1}) + \nu\log(\tilde{z}_{ht-1}) + \epsilon_{zft}, \]  

(3.12)

where \( \bar{z} > 0, \rho_{iz} \in (-1,1), \) and \( \epsilon_{zit} \sim N(0,\sigma_{zit}^2). \)\(^3\) Under this specification, innovations to productivity in one country are permitted to spill to the other country, assuming the diffusion parameter \( \nu \) is non-zero. Additionally, technology shocks are allowed to be contemporaneously correlated across the two countries: \( \text{Corr}(\epsilon_{zht}, \epsilon_{zft}) = \epsilon. \)

Government consumption and investment, \( c_{igt} \) and \( i_{igt} \) are assumed to follow these laws of motion after detrending:

\[ \log(\tilde{c}_{igt}) = (1 - \rho_{cig})\log(\bar{c}_g) + \rho_{cig}\log(\tilde{c}_{igt-1}) + \epsilon_{cigt}, \]  

(3.13)

where \( \rho_{cig} \in (-1,1) \) and \( \epsilon_{cigt} \sim N(0,\sigma_{cig}^2); \)

\[ \log(\tilde{i}_{igt}) = (1 - \rho_{ig})\log(\bar{i}_g) + \rho_{ig}\log(\tilde{i}_{igt-1}) + \epsilon_{iigt}, \]  

(3.14)

where \( \rho_{ig} \in (-1,1) \) and \( \epsilon_{iigt} \sim N(0,\sigma_{ig}^2). \)

\(^3\)Tildes represent detrended values, and bars denote steady states.
3.4.4 Model Solution

In this model, financial markets are complete and financial capital is freely mobile across countries: agents in each country may buy contingent claims to insure against shocks. Since there is only one consumption good, the only motive for trade here is to smooth consumption profiles. The equilibrium allocations can be found by solving the world planner’s problem of maximizing $U$ subject to the constraints.

$$
\max E_t \left( \sum_{t=0}^{\infty} \pi \beta^t u(C_{ht}, l_{ht}) + (1 - \pi) \beta^t u(C_{ft}, l_{ft}) \right),
$$

where the social planner’s weights are set to be proportional to each country’s population size. In addition to the intertemporal and intratemporal trade-off decisions for each country, there will be an international risk-sharing condition wherein the social planner will allocate goods away from the low marginal utility to the high marginal utility country:

$$
\pi \left( \frac{C_{ht+1}}{C_{ht}} \right)^{\gamma(1-\phi)-\psi} \left( \frac{c_{ht+1}}{c_{ht}} \right)^{\psi-1} \left( \frac{l_{ht+1}}{l_{ht}} \right)^{(1-\gamma)(1-\psi)} =
\left( 1 - \pi \right) \left( \frac{C_{ft+1}}{C_{ft}} \right)^{\gamma(1-\phi)-\psi} \left( \frac{c_{ft+1}}{c_{ft}} \right)^{\psi-1} \left( \frac{l_{ft+1}}{l_{ft}} \right)^{(1-\gamma)(1-\psi)}
$$

The model does not have a closed-form solution. The system of equations is mapped into a linearized system by taking a log-linear approximation around the steady states. The approximate solution takes the form $x_{t+1} = F(\mu)x_t + G(\mu)\nu_{t+1}$, where $\nu_{t+1}$ denotes the collection of structural shocks. The model is then calibrated to compute international correlations to be compared with the data.

3.5 DATA AND EMPIRICAL REGULARITIES

The U.S. series are from the Federal Reserve Bank of St. Louis database (FRED). Hours worked for the U.S. comes from the Bureau of Labor Statistics’ (BLS) Establishment Survey. Data for the remaining countries on private consumption and investment, government consumption and investment, and hours worked are from the OECD database. Hours worked data was only available on an annual basis: I obtain quarterly data using interpolation. All series are quarterly, in chained volume estimates according to national reference year, seasonally adjusted, and converted to US
dollars using the PPP of the national reference year, and divided by working-age population. Population is available from the WDI database on an annual basis: I obtain quarterly data using interpolation. Saving is constructed as GDP minus private and public consumption. The data is HP-filtered and selected country statistics are reported for deviations from trend in Table 3.1. The countries considered are Australia, Canada, France, Japan, Korea, Norway, Switzerland, the UK, and the US. Country selection was based on availability of recent long time-series data. The time period considered for all the series is 1980:Q1 through 2008:Q1. The only exception for this time sample is the data on hours worked. The most comprehensive data within the sample were used. Hours worked data was available for all the countries in the sample except Japan.

Table 3.1: Standard Deviations Relative to Output (1980-2008)

<table>
<thead>
<tr>
<th>Country</th>
<th>$C_p$</th>
<th>$C_g$</th>
<th>$I$</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>0.67</td>
<td>1.11</td>
<td>3.67</td>
<td>0.56</td>
</tr>
<tr>
<td>Canada</td>
<td>0.80</td>
<td>0.64</td>
<td>2.64</td>
<td>0.18</td>
</tr>
<tr>
<td>France</td>
<td>0.92</td>
<td>0.69</td>
<td>3.05</td>
<td>0.72</td>
</tr>
<tr>
<td>Japan</td>
<td>0.73</td>
<td>0.80</td>
<td>2.40</td>
<td>-</td>
</tr>
<tr>
<td>Korea</td>
<td>1.43</td>
<td>0.60</td>
<td>2.68</td>
<td>0.20</td>
</tr>
<tr>
<td>Norway</td>
<td>1.20</td>
<td>1.02</td>
<td>4.28</td>
<td>0.60</td>
</tr>
<tr>
<td>Switzerland</td>
<td>0.59</td>
<td>1.06</td>
<td>2.67</td>
<td>0.30</td>
</tr>
<tr>
<td>UK</td>
<td>1.19</td>
<td>0.81</td>
<td>3.42</td>
<td>0.29</td>
</tr>
<tr>
<td>US</td>
<td>0.78</td>
<td>0.89</td>
<td>4.19</td>
<td>1.23</td>
</tr>
</tbody>
</table>

The only series for private consumption that is consistently available across countries on a quarterly basis is that of total (durables and non-durables) consumption. Hence the general empirical regularities reported below will be corresponding to total consumption including durables. This is typically the measure used in the literature.

Table 3.2 reports individual correlations for 9 OECD countries, Table 3.3 reports pairwise private consumption correlations, and Table 3.4 reports the same information for pairwise output
correlations. These tables show interesting regularities that confirm previous findings. These regularities are, by now, well documented in the literature but the statistics here confirm that these puzzles still exist even using recent data. Table 3.2 confirms that investment-saving correlations $Corr(S, I)$ are high. It also confirms the finding in Feldstein (2005): the correlation between saving and investment is lower for small countries than it is for larger ones (Norway and Korea in our sample). Tables 3.3 and 3.4 show that private consumption correlations across the countries in the sample are consistently lower than those of output.

Table 3.2: Domestic Correlations (1980-2008)

<table>
<thead>
<tr>
<th>Country</th>
<th>$(C_p, Y)$</th>
<th>$(C_g, Y)$</th>
<th>$(I, Y)$</th>
<th>$(n, Y)$</th>
<th>$(S, I)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>0.35</td>
<td>0.24</td>
<td>0.81</td>
<td>0.10</td>
<td>0.72</td>
</tr>
<tr>
<td>Canada</td>
<td>0.71</td>
<td>-0.28</td>
<td>0.99</td>
<td>0.54</td>
<td>0.62</td>
</tr>
<tr>
<td>France</td>
<td>0.76</td>
<td>-0.35</td>
<td>0.90</td>
<td>0.20</td>
<td>0.72</td>
</tr>
<tr>
<td>Japan</td>
<td>0.64</td>
<td>-0.06</td>
<td>0.82</td>
<td>-</td>
<td>0.73</td>
</tr>
<tr>
<td>Korea</td>
<td>0.83</td>
<td>0.08</td>
<td>0.82</td>
<td>0.69</td>
<td>0.21</td>
</tr>
<tr>
<td>Norway</td>
<td>0.53</td>
<td>-0.15</td>
<td>-0.03</td>
<td>0.23</td>
<td>-0.03</td>
</tr>
<tr>
<td>Switzerland</td>
<td>0.71</td>
<td>0.41</td>
<td>0.67</td>
<td>0.16</td>
<td>0.57</td>
</tr>
<tr>
<td>UK</td>
<td>0.85</td>
<td>-0.26</td>
<td>0.72</td>
<td>0.33</td>
<td>0.54</td>
</tr>
<tr>
<td>US</td>
<td>0.85</td>
<td>0.12</td>
<td>0.91</td>
<td>0.86</td>
<td>0.91</td>
</tr>
</tbody>
</table>

Finally, Table 3.5 compares the median found in our sample for relevant statistics to a data range found in the literature. The differences in these statistics arise because of using different countries or different time periods. Our cross-country correlations are all lower than those found in the literature. This is because we use the most recent data available. In the literature, lower correlations are typically found when using more recent samples. For example, the low estimates
### Table 3.3: Cross-Country Private Consumption Correlations (1980-2008)

<table>
<thead>
<tr>
<th>Country</th>
<th>Australia</th>
<th>Canada</th>
<th>France</th>
<th>Japan</th>
<th>Korea</th>
<th>Norway</th>
<th>Switzerland</th>
<th>UK</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>1.00</td>
<td>0.11</td>
<td>0.54</td>
<td>0.17</td>
<td>-0.12</td>
<td>-0.03</td>
<td>0.28</td>
<td>0.26</td>
<td>-0.001</td>
</tr>
<tr>
<td>Canada</td>
<td>1.00</td>
<td>0.01</td>
<td>-0.07</td>
<td>0.04</td>
<td>0.31</td>
<td>0.44</td>
<td>0.60</td>
<td>0.68</td>
<td></td>
</tr>
<tr>
<td>France</td>
<td>1.00</td>
<td>0.25</td>
<td>0.01</td>
<td>-0.13</td>
<td>0.53</td>
<td>0.37</td>
<td>0.02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Japan</td>
<td>1.00</td>
<td>0.40</td>
<td>-0.05</td>
<td>0.08</td>
<td>0.13</td>
<td></td>
<td>-0.08</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Korea</td>
<td>1.00</td>
<td>0.06</td>
<td>-0.08</td>
<td>0.02</td>
<td>-0.08</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Norway</td>
<td>1.00</td>
<td>0.11</td>
<td>-0.03</td>
<td>0.28</td>
<td></td>
<td></td>
<td>0.28</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Switzerland</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.30</td>
</tr>
<tr>
<td>UK</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.53</td>
<td></td>
</tr>
<tr>
<td>US</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.00</td>
</tr>
</tbody>
</table>

### Table 3.4: Cross-Country Output Correlations (1980-2008)

<table>
<thead>
<tr>
<th>Country</th>
<th>Australia</th>
<th>Canada</th>
<th>France</th>
<th>Japan</th>
<th>Korea</th>
<th>Norway</th>
<th>Switzerland</th>
<th>UK</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>1.00</td>
<td>0.73</td>
<td>0.07</td>
<td>0.07</td>
<td>-0.14</td>
<td>0.21</td>
<td>0.44</td>
<td>0.45</td>
<td>0.65</td>
</tr>
<tr>
<td>Canada</td>
<td>1.00</td>
<td>0.29</td>
<td>0.07</td>
<td>0.10</td>
<td>0.23</td>
<td>0.55</td>
<td>0.60</td>
<td>0.82</td>
<td></td>
</tr>
<tr>
<td>France</td>
<td>1.00</td>
<td>0.34</td>
<td>0.09</td>
<td>-0.13</td>
<td>0.61</td>
<td>0.43</td>
<td>0.19</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Japan</td>
<td>1.00</td>
<td>0.23</td>
<td>0.15</td>
<td>0.38</td>
<td>0.03</td>
<td>0.12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Korea</td>
<td>1.00</td>
<td>0.14</td>
<td>-0.14</td>
<td>0.27</td>
<td>-0.02</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Norway</td>
<td>1.00</td>
<td>0.23</td>
<td>0.10</td>
<td>0.40</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Switzerland</td>
<td>1.00</td>
<td>0.28</td>
<td>0.52</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>UK</td>
<td>1.00</td>
<td></td>
<td>0.49</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.00</td>
</tr>
</tbody>
</table>
correspond to the Kehoe and Perri (2002) sample covering data from 1970 to 1998, while the high estimates correspond to Baxter and Farr (2005) which uses data from 1961 to 1991. Therefore, it should not be surprising that our correlations are even lower than those of Kehoe and Perri (2002) since our sample covers the period from 1980 through 2008. The median we found will be used when comparing the model’s results to the data.

Table 3.5: Summary of Data Moments

|---------|------------|------------------------|------------------------|

**Domestic**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{\hat{c}/}\sigma_{\hat{y}}$</td>
<td>0.78</td>
<td>0.76</td>
<td>0.79</td>
</tr>
<tr>
<td>$\sigma_{\hat{i}/}\sigma_{\hat{y}}$</td>
<td>4.73</td>
<td>2.98</td>
<td>3.27</td>
</tr>
<tr>
<td>$\sigma_{\hat{n}/}\sigma_{\hat{y}}$</td>
<td>1.23</td>
<td>0.63</td>
<td>0.85</td>
</tr>
</tbody>
</table>

**Corr($\hat{c}/,\hat{y}$) | 0.85 | 0.81 | 0.87 |
| **Corr($\hat{i}/,\hat{y}$) | 0.89 | 0.81 | 0.93 |
| **Corr($\hat{n}/,\hat{y}$) | 0.86 | 0.78 | 0.86 |
| **Corr($\hat{i}/,\hat{s}$) | 0.91 | 0.68 | 0.94 |

**US - Foreign Correlations**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>**Corr($\hat{y}/,\hat{y}$)</td>
<td>0.44</td>
<td>0.51</td>
<td>0.81</td>
</tr>
<tr>
<td>**Corr($\hat{c}/,\hat{c}$)</td>
<td>0.15</td>
<td>0.32</td>
<td>0.67</td>
</tr>
<tr>
<td>**Corr($\hat{n}/,\hat{n}$)</td>
<td>0.37</td>
<td>0.43</td>
<td>0.75</td>
</tr>
</tbody>
</table>

*This refers to US statistics in our sample for US Domestic, and to the median for cross-country correlations with the US for international statistics.*
3.6 CALIBRATION

Before providing quantitative results regarding the international puzzles, and to provide intuition behind the implications of the model’s dynamics, I first illustrate how different shocks carry alternative implications for variables’ movements. While Baxter and Crucini (1993) consider only cases where a shock takes place only in the home country, shocks to both countries are considered here. The model is calibrated to a situation where two countries have the same stochastic driving processes as the US and a small country. Since Canada is of size corresponding to the median size in our sample, it will be used as the foreign country. Since US output in the data is about ten times higher on average than a median country’s output in our sample, \( \pi \) is set to 0.91 such that the home country refers to the US.

Following the literature, the deep parameters of the model will be assumed identical in both countries, while heterogeneity is allowed in the exogenous forces’ specifications. These parameters are summarized in Table 3.6. The growth rate \( g \), the private capital share parameter \( \alpha \), the discount rate \( \beta \), the depreciation rate \( \delta \), and the coefficient of relative risk aversion \( \phi \) are set to values comparable with studies in the literature (e.g. see Baxter and Crucini, 1993; BKK, 1992; and Marrinan, 1999.) \( \gamma \) is chosen so that hours worked in the steady state is 1/3. As for the elasticity of the investment-capital ratio to Tobin’s Q, \( \zeta \), I calibrate it as the same value used in the benchmark specification of Baxter and Crucini (1993) and Baxter and Farr (2005).

In calibrating the autocorrelations and standard deviations of the government shocks, I run OLS regressions based on the government consumption and investment series of each country separately. As for the parameters of the productivity process, I follow the identification approach used by Baxter and Crucini (1993) and Wen (2007). Specifically, these were chosen to generate the persistence and autocorrelation of output observed in the data. Once government shocks are added, these parameters are recalibrated so that the residuals can explain the remaining persistence in output. Finally, \( c_g/\bar{y} \) and \( i_g/\bar{y} \) were restricted to match the means found in the data. These parameters are summarized in Table 3.7.
Table 3.6: Calibration: Deep Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.33</td>
<td>Private capital share parameter</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.99</td>
<td>Discount factor</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.025</td>
<td>Depreciation rate of private and public capital</td>
</tr>
<tr>
<td>$\phi$</td>
<td>2</td>
<td>Coefficient of relative risk aversion</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.28</td>
<td>Consumption’s share relative to leisure of utility</td>
</tr>
<tr>
<td>$g$</td>
<td>0.0052</td>
<td>Growth rate</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>15</td>
<td>Elasticity of I/K to Tobin’s Q</td>
</tr>
<tr>
<td>$\pi$</td>
<td>0.91</td>
<td>Home country share</td>
</tr>
</tbody>
</table>

Table 3.7: Calibration: Exogenous Shocks

<table>
<thead>
<tr>
<th>Parameter</th>
<th>US Value</th>
<th>Canada Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_{z}$</td>
<td>0.84</td>
<td>0.85</td>
<td>Technology shock persistence parameter</td>
</tr>
<tr>
<td>$\sigma_{e_{z}}$</td>
<td>0.0055</td>
<td>0.0035</td>
<td>Standard Deviation of the technology shock</td>
</tr>
<tr>
<td>$\rho_{c_{g}}$</td>
<td>0.77</td>
<td>0.69</td>
<td>Public consumption shock persistence parameter</td>
</tr>
<tr>
<td>$\sigma_{e_{c_{g}}}$</td>
<td>0.0069</td>
<td>0.0072</td>
<td>Standard deviation of the public consumption shock</td>
</tr>
<tr>
<td>$\rho_{i_{g}}$</td>
<td>0.69</td>
<td>0.82</td>
<td>Public investment shock persistence parameter</td>
</tr>
<tr>
<td>$\sigma_{e_{i_{g}}}$</td>
<td>0.021</td>
<td>0.019</td>
<td>Standard deviation of the public investment shock</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0</td>
<td>0</td>
<td>Spillover of technology shocks</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>0.258</td>
<td>0.258</td>
<td>Correlation of innovations to technology shocks</td>
</tr>
</tbody>
</table>
The literature has not reached a consensus on whether there is transmission of shocks from one country to another over time. While BKK (1992) estimate using Solow residuals that there is evidence of cross-country spillovers of shocks to technology, and that innovations to productivity are positively correlated across countries, Baxter and Crucini (1995) find that the transmission parameters are not statistically significant. In our benchmark economy, $\nu$ is set to zero, and I perform sensitivity analysis over this parameter later on. $\epsilon$ is set to 0.258 which is a value agreed upon in empirical studies.

In the benchmark specification of the model, I will assume that all the CES parameters are equal to one. As I change the CES parameters from one, the parameter $\gamma$ will be changed to ensure that hours worked in the steady state remain one third.

### 3.7 MODEL MECHANISMS

#### 3.7.1 A Technology Shock

Figure 3.1 shows impulse response functions to a 1% increase in technology in the home country under the benchmark specification. The technology shock raises domestic output, consumption, and investment at home due to a higher marginal product of private capital. This helps account for the high investment-saving correlation found in the data (e.g. see Baxter and Crucini, 1993). Investment is usually found to be very volatile in open economy models since it is easy to shift resources from one country to another. This is circumvented in the model used here by adding transaction costs to avoid large flows across borders. For example when $\zeta = 1000$, an extreme response of investment to international rate of return differentials is observed.\footnote{An infinite $\zeta$ is equivalent to no transaction costs in the model.} Foreign investment is then more than 5 times as volatile as output. If on the other hand, $\zeta = 1$, the volatility of foreign investment is less than one relative to output.

As for foreign variables, foreign output decreases since resources are switched to the more productive location (the home country). In the literature, foreign investment typically decreases. However, as shown in Baxter and Crucini (1993), as the persistence of the technology shock decreases, the incentive to move goods in response to the shock also decreases. Since the persistence
parameter for the US is found to be low here (0.84), foreign investment eventually increases. Hours worked increase at home and decrease abroad. This happens because of a substitution effect of leisure supported by risk-sharing and capital mobility. Even though foreign output decreases, foreign consumption rises. This is one aspect of how the benchmark economy differs from postwar international data, referred to as the BKK puzzle. In the data presented here, most consumption correlations across countries are less than 0.5, and a few are even negative. The model with technology shocks alone predicts an almost perfect correlation between the two countries’ consumption profiles.

3.7.2 A Government Investment Shock

In this paper I argue that, similarly to a productivity shock, an increase in home government investment causes a high correlation between investment and saving. But in order for this to take effect, government investment has to be complementary with private investment. The CES parameters are adopted from Mazraani (2010): $\omega$ is calibrated to 0.83 since the share of public investment to gross investment is 17% in US data, and $\chi$ is estimated to be −0.38 meaning public
and private capital are complements in the production function. Figure 3.2 shows impulse response functions to a 1% increase in home government investment in such a case. The shock at home raises output, and as the public capital stock builds up, it starts boosting private investment at home. Hours, saving, and consumption also increase. The effect on the foreign country is similar to the productivity shock’s effects: private investment moves to the more productive location, and agents insure themselves which allows them to increase private consumption even though foreign output decreases.

Figure 3.2: Impulse Response Functions to a 1% Increase in Home Government Investment: Complements

To see why government investment shocks need to have complementary effects in the model in order to help resolve the puzzle, an alternative version where government investment shocks substitute private investment is depicted in Figure 3.3 ($\chi = 1$). Under this scenario, not only does the shock cause an income effect through the resource constraint, but the drop in private investment at home is further amplified because of crowding-out. Agents in the two countries synchronize to work more, and consume and invest less. In this case, investment and saving don’t move in the same direction. This confirms the intuition behind using complementary government investment shocks qualitatively. The next section provides quantitative moments’ results, as well as sensitivity results to different values of $\chi$. 

49
3.7.3 A Government Consumption Shock

As with the technology shock, the resolution of the FH puzzle comes at the cost of high consumption correlations which contradicts the data. Government consumption shocks are added to the model as a result. The CES parameters are calibrated as follows: $\tau$ is calibrated to 0.83 and $\psi$ is estimated to be $-4.86$ implying that government consumption is a strong complement to private consumption in the utility function. Figure 3.4 shows impulse response functions to a 1% increase in government consumption using these assumptions.

Before explaining the dynamics of figure 3.4, it is worthwhile to examine the case when government consumption either doesn’t enter the utility function (Figure 3.5), or enters the utility function and perfectly substitutes private consumption at home (Figure 3.6). When government consumption does not affect private agents’ utility, it only has a wealth effect in the model. This increases hours, output and makes private consumption fall at home. This effect is shared equally by the foreign country. This can be better understood by examining the world resource constraint (equation 3.9). From the point of view of the social planner, a government consumption shock at
Figure 3.4: Impulse Response Functions to a 1% Increase in Home Government Consumption: Complements
home causes an income effect in the world economy. This is why consumption drops equally in the foreign country.

![Figure 3.5: Impulse Response Functions to a 1% Increase in Home Government Consumption: No Effect on Utility](image)

However, when government consumption perfectly substitutes private consumption (Figure 3.6), the reduction in private consumption at home is compensated for by the increase in government consumption at home, and home private consumption decreases more than before. The risk-sharing condition imposed by the social planner implies that resources are shifted from the low marginal utility country to the high marginal utility country. Since a government consumption shock that acts as a substitute to private consumption decreases the marginal utility of consumption at home, the foreign country does not share this effect equally with the home country. This helps break some of the correlation in consumption across countries. In Marrinan (1999), these are the only assumptions used for government consumption. With these assumptions, Marrinan does not get far in explaining the low consumption correlations observed in the data. In this paper, I assume that government consumption strongly complements private consumption such that it resembles the preference shock of Wen (2007). This will imply even lower correlations of consumption across countries.

In Figure 3.4, the government consumption shock at home increases labor supply at home.
due to the negative wealth effect. Owing to the high complementarity between government and private consumption however, government consumption shocks raise the marginal utility of private consumption at home. The social planner will then provide more resources to the home country until marginal utilities are equated again. This helps account for the BKK puzzle since preference shocks create a wedge between the two countries’ consumption profiles.

### 3.8 QUANTITATIVE RESULTS

Having illustrated the qualitative effects of how different shocks in the model help contribute to the puzzles’ resolutions, I now turn to present the quantitative moments implied by different specifications of the model. I consider five cases: a baseline situation with technology shocks alone and all the CES parameters turned to one, a case where only government investment shocks are present and the CES parameters of the government consumption shocks are one, a case where only...
government consumption shocks are present and the CES parameters of the government investment shocks are one, a case where only government consumption and investment shocks are present, and a case where all shocks take place simultaneously. The baseline situation is presented since our model encompasses the case studied in the literature and to facilitate comparison.

### 3.8.1 The Baseline Situation

Table 3.8 compares data statistics to the model’s predictions. The second column of the table shows results for the baseline case. The literature has well documented the behavior of several variables using international data. Consumption is typically found to be less volatile than output, while investment is about 3 to 4 times more volatile than output. Consumption, investment, and hours worked are procyclical. Investment and saving are highly correlated. Output, investment, consumption, and hours worked are all positively correlated across countries.

The baseline situation assumes only technology shocks are present. It is well known that a neoclassical two-country model with technology shocks generates negative cross-country correlations of output, investment, and hours worked, which is at odds with the data (e.g. see BKK, 1992.) It also generates an almost perfect correlation of consumption across countries. However, it correctly predicts that investment and saving are positively correlated domestically. While the correlation between investment and saving is 0.99 accounting for the FH puzzle, the cross-country consumption correlation is also 0.96 hurting the model’s ability to account for the BKK puzzle. Another shortcoming of technology shocks in two-country models is that they hurt the model’s ability to account for other moments of the data. For example, it incorrectly predicts that output and hours worked are negatively correlated.

So far, we have assumed that \( \nu = 0 \), meaning technology doesn’t spill across countries. We perform a sensitivity analysis over the value of \( \nu \). This is illustrated in Table 3.9. We get similar results to Baxter and Crucini (1993): Changing \( \nu \) has minor effects. The most notable change is that the output correlations across countries become more reasonable, and that the relative volatility of consumption to output increases, while that of investment decreases.
Table 3.8: Model Results

<table>
<thead>
<tr>
<th>Measure</th>
<th>Data</th>
<th>$z$ shocks</th>
<th>$i_g$ shocks</th>
<th>$c_g$ shocks</th>
<th>$c_g$ &amp; $i_g$ shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Domestic</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_{\tilde{c}<em>p}/\sigma</em>{\tilde{y}}$</td>
<td>0.78</td>
<td>0.70</td>
<td>0.77</td>
<td>0.99</td>
<td>0.91</td>
</tr>
<tr>
<td>$\sigma_{\tilde{i}}/\sigma_{\tilde{y}}$</td>
<td>4.73</td>
<td>2.58</td>
<td>3.01</td>
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<td>1.15</td>
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<tr>
<td>$\sigma_{\tilde{n}}/\sigma_{\tilde{y}}$</td>
<td>1.23</td>
<td>0.30</td>
<td>0.41</td>
<td>1.33</td>
<td>1.46</td>
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<tr>
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<td>0.94</td>
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<td><strong>US - Foreign Correlations</strong></td>
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<tr>
<td>$\text{Corr}(\tilde{y}_h,\tilde{y}_f)$</td>
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<td>$\text{Corr}(\tilde{n}_h,\tilde{n}_f)$</td>
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<td>-0.45</td>
<td>-0.35</td>
<td>-0.05</td>
<td>-0.07</td>
</tr>
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</table>

The predicted statistics are based on 10,000 simulations with sample length 100.
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<th>Measure</th>
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<th>$(\nu = 0.04)$</th>
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<td>$z$ shocks</td>
<td>$z$ shocks</td>
</tr>
<tr>
<td><strong>Domestic</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_{c_p}/\sigma_{\bar{y}}$</td>
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<td>0.70</td>
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<tr>
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</tr>
<tr>
<td>$\sigma_{n}/\sigma_{\bar{y}}$</td>
<td>1.23</td>
<td>0.30</td>
</tr>
<tr>
<td>Corr($\hat{c}_p, \bar{y}$)</td>
<td>0.85</td>
<td>0.92</td>
</tr>
<tr>
<td>Corr($\hat{i}_p, \bar{y}$)</td>
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<td>0.96</td>
</tr>
<tr>
<td>Corr($\bar{n}, \bar{y}$)</td>
<td>0.86</td>
<td>0.79</td>
</tr>
<tr>
<td>Corr($\hat{i}_h, \bar{s}_h$)</td>
<td>0.91</td>
<td>0.99</td>
</tr>
<tr>
<td><strong>US - Foreign Correlations</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corr($\bar{y}_h, \bar{y}_f$)</td>
<td>0.44</td>
<td>-0.34</td>
</tr>
<tr>
<td>Corr($\hat{c}_hp, \hat{c}_fp$)</td>
<td>0.15</td>
<td>0.96</td>
</tr>
<tr>
<td>Corr($\hat{i}_h, \hat{i}_f$)</td>
<td>0.25</td>
<td>0.36</td>
</tr>
<tr>
<td>Corr($\bar{n}_h, \bar{n}_f$)</td>
<td>0.37</td>
<td>-0.45</td>
</tr>
</tbody>
</table>
3.8.2 Government Investment Shocks

The hypothesis in this paper is that government investment shocks can perform as well as technology shocks in a two-country neoclassical model. Table 3.8 (third column) reports the moments for $\chi = -0.38$: Quantitative results confirm our initial intuition.

A model driven by government investment shocks alone predicts very similar moments to the model with technology shocks alone. It replicates the standard deviations of consumption and investment, and their correlations with output fairly well. It can also help account for the FH puzzle: $\text{Corr}(\tilde{i}_h, \tilde{s}_h)$ is found to be 0.94, which is similar to the moment predicted by technology shocks (0.99). The success of these shocks in matching cross-country correlations, however, is mixed. While the model correctly predicts that investment correlations are positive across countries, it predicts that output and hours worked are negatively correlated, while consumption is highly correlated. The intuition behind this result is similar to the technology shock’s case: due to a shock in the home country, agents in the foreign country will find it more productive to channel their resources towards the home country. Investment, saving, and hours worked go up at home, while they decrease in the foreign country. This explains the high saving-investment correlation. Since investment shocks do not enter agents’ utility, the risk-sharing condition implies that the planner will equate consumptions of the two countries. I show in the next subsection that government consumption shocks help circumvent this problem since they enter the utility function.

Government investment shocks also perform poorly with respect to the standard deviation of hours worked. Technology shocks predict the moment to be 0.30, while investment shocks predict it to be 0.41. This is far off from the moment found in the data of 1.23. It is well-known that a prototypical RBC model cannot account for labor market behavior (e.g., see Christiano and Eichenbaum, 1992; and Hansen and Wright, 1992). Since both government investment and technology shocks work through the supply dimension, they cannot help account for the behavior of hours worked in the data. I show in the next subsection that government consumption shocks working through the demand dimension can help with hours worked, as well as with cross-country consumption correlations.
3.8.3 Government Consumption Shocks

While Wen (2007) and Stockman and Tesar (1995) use preference or taste shocks in the utility function to explain the BKK puzzle, a limitation of their approach is that the shocks are unobservable. In their setting, it is assumed that these shocks follow random walk processes and their standard deviation is calibrated such that it matches that of output. It is also shown that the shocks have to be extremely persistent ($\sigma = 0.025$) in order to help account for the puzzle. In this paper, the attempt is to see whether government consumption shocks acting as “keynesian preference” shocks can be used as a proxy for the unobservable shocks and quantitatively resolve the puzzle.

In table 3.8 (column 4), it is shown that the model with government consumption shocks alone matches the volatility of hours worked better than the model with technology shocks. This is because the main channel through which government consumption shocks transmit is through hours worked. Due to these shocks that raise the marginal utility of private consumption at home, agents need to work harder in order to achieve the desired increase in consumption. For higher values of the coefficient of relative risk aversion, agents desire a more smooth consumption profile, and private investment can also increase. These effects cause home consumption, investment, and hours to be procyclical with output, which is what we find in the data. It is interesting to note that the volatility of private investment relative to output is predicted to be lower than found in the data. This is because we use transaction costs in the model which decrease the volatility of investment.

As for the puzzles, it is shown that the model with government consumption shocks performs better in matching other moments of the data and in helping account for the BKK and the FH puzzle simultaneously. The cross-country consumption correlation is found to be low (-0.01 compared to a median value of 0.15). The intuition behind this result is straightforward: As the marginal utility of consumption increases at home, the social planner shifts resources from the foreign country until marginal utilities are equated. This causes the consumption profiles to be less synchronized across countries (Figure 3.4). While technology shocks alone predict an extreme negative correlation between output and hours worked across countries, government consumption shocks get closer to these moments. Finally, government consumption shocks alone similarly help bring investment-saving correlations closer to the data. As explained earlier, investment is procyclical with output; both saving and investment increase as a result.
3.8.4 Government Consumption and Investment Shocks

Next, we ask whether a model with government consumption and investment shocks combined can help account for the data better than a model with technology shocks alone. The results for this exercise are presented in the last column of Table 3.8. It is shown that a model with government consumption and investment shocks alone can do better than one with technology shocks in helping account for the FH and BKK puzzle simultaneously. The investment-saving correlation is found to be 0.97 which is close to the US value of 0.91. Further, the cross-country consumption correlation is found to be 0.02 which is also close to our median of 0.15. While the model we propose does not correctly predict the sign of the cross country correlations in output, hours, and investment, it performs better than technology shocks in matching those moments. The model also underpredicts the volatility of investment to output: Since we have included transaction costs in the model, this makes it costly to adjust capital.

3.8.5 Sensitivity Analysis

We check whether different values of $\chi$ and $\psi$ have different implications for the puzzles. In Table 3.10, we compare the moments for different values of $\chi$ and $\psi$. The second column of the table represents the baseline situation; the third considers the complements case; and the last considers a substitutes case. As discussed earlier using impulse response functions, the substitutes’ case cannot help account for the two puzzles simultaneously. Quantitative results reported in the last column confirm this intuition. While the model correctly predicts a high correlation between investment and saving, note that consumption and investment are now countercyclical. This is because investment decreases in the substitutes case. While saving increases due to investment shocks (see Figure 3.3), it decreases due to a consumption shock in the substitutes case (see Figure 3.6). Investment and saving both go down causing the high correlation as a result. For a lower $\psi$ of 0.5, cross-country consumption correlations are very high (0.74), which is at odds with the data.
<table>
<thead>
<tr>
<th>Measure</th>
<th>Data</th>
<th>Complements</th>
<th>Substitutes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi$ $\chi$</td>
<td>$\psi = -4.86, \chi = -0.38$</td>
<td>$\psi = 0.5, \chi = 0.5$</td>
<td></td>
</tr>
<tr>
<td>$z$ shocks</td>
<td>$c_g$ &amp; $i_g$ shocks</td>
<td>$c_g$ &amp; $i_g$ shocks</td>
<td></td>
</tr>
<tr>
<td>Domestic</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_{\tilde{c}<em>p}/\sigma</em>{\tilde{y}}$</td>
<td>0.78</td>
<td>0.70</td>
<td>0.91</td>
</tr>
<tr>
<td>$\sigma_{\tilde{i}}/\sigma_{\tilde{y}}$</td>
<td>4.73</td>
<td>2.58</td>
<td>1.15</td>
</tr>
<tr>
<td>$\sigma_{\tilde{n}}/\sigma_{\tilde{y}}$</td>
<td>1.23</td>
<td>0.30</td>
<td>1.46</td>
</tr>
<tr>
<td>$Corr(\tilde{c}_p, \tilde{y})$</td>
<td>0.85</td>
<td>0.92</td>
<td>0.99</td>
</tr>
<tr>
<td>$Corr(\tilde{i}, \tilde{y})$</td>
<td>0.89</td>
<td>0.96</td>
<td>0.91</td>
</tr>
<tr>
<td>$Corr(\tilde{n}, \tilde{y})$</td>
<td>0.86</td>
<td>0.79</td>
<td>0.99</td>
</tr>
<tr>
<td>$Corr(\tilde{i}_h, \tilde{s}_h)$</td>
<td>0.91</td>
<td>0.99</td>
<td>0.97</td>
</tr>
<tr>
<td>US - Foreign Correlations</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Corr(\tilde{y}_h, \tilde{y}_f)$</td>
<td>0.44</td>
<td>-0.34</td>
<td>-0.11</td>
</tr>
<tr>
<td>$Corr(\tilde{c}<em>{hp}, \tilde{c}</em>{fp})$</td>
<td>0.15</td>
<td>0.96</td>
<td>0.02</td>
</tr>
<tr>
<td>$Corr(\tilde{i}_h, \tilde{i}_f)$</td>
<td>0.25</td>
<td>0.36</td>
<td>-0.21</td>
</tr>
<tr>
<td>$Corr(\tilde{n}_h, \tilde{n}_f)$</td>
<td>0.37</td>
<td>-0.45</td>
<td>-0.07</td>
</tr>
</tbody>
</table>
3.8.6 All Shocks

Since technology shocks and government spending shocks predict opposing results for some data moments, it is likely that the data lies somewhere in between their predictions: we now examine the implications of adding the three shocks simultaneously. We still expect the puzzles to be resolved in this case since private activity is unsynchronized across countries even if shocks take place in both of them. Table 3.11 reports these results for the complements case. When adding technology shocks with government expenditure shocks, the contribution of technology shocks to the volatility of output changes. Government expenditure shocks contribute 61% to the variability of output in the US (65% in Canada), and technology shocks explain the remaining variability (39%). Note that in the setting of Wen (2007), technology shocks are only allowed to contribute 1% to the total variability in output since the preference shocks are of much higher magnitude than in our setting. Since government expenditure shocks have an amplification effect in our model, they can still resolve the puzzles quantitatively even with lower magnitude.

It is shown that the model augmented with government spending shocks (column 4) improves the performance of the model with technology shocks alone (column 2) along several dimensions. First, it helps reduce the cross-country consumption correlation (from 0.96 to 0.13), which helps account for the BKK puzzle. Second, it helps account for the FH puzzle: the investment-saving correlation is 0.99 compared to a value of 0.91 found in the data. Third, the model’s prediction for the volatility of hours worked improves (from 0.29 to 1.07). While the model augmented with government shocks does not correctly predict the sign of output cross-country correlations, it correctly predicts the sign of the cross-country investment correlation, and improves the model’s ability to match the cross-country correlation in hours worked.

3.8.7 External Empirical Validity

In this chapter, government consumption shocks are used as a proxy for taste or demand shocks, and government investment shocks are a proxy for a share of the productivity disturbances. I perform an exercise using the Kalman Filter to get external empirical validity for the model by comparing the model’s inferred behavior to the actual government data series. Specifically, any
Table 3.11: Model Results: All Shocks

<table>
<thead>
<tr>
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<th>Data</th>
<th>Complements</th>
<th>Complements</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>z shocks</td>
<td>c_g &amp; i_g shocks</td>
</tr>
<tr>
<td>$\psi = -4.86$, $\chi = -0.38$</td>
<td>$\psi = -4.86$, $\chi = -0.38$</td>
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<td></td>
</tr>
</tbody>
</table>

**Domestic**

<table>
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<tr>
<th>Measure</th>
<th>Data</th>
<th>Complements</th>
<th>Complements</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{c_p}/\sigma_{\tilde{y}}$</td>
<td>0.78</td>
<td>0.70</td>
<td>0.91</td>
</tr>
<tr>
<td>$\sigma_{i_p}/\sigma_{\tilde{y}}$</td>
<td>4.73</td>
<td>2.58</td>
<td>1.15</td>
</tr>
<tr>
<td>$\sigma_{\tilde{n}}/\sigma_{\tilde{y}}$</td>
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<td>0.30</td>
<td>1.46</td>
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<tr>
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<td>0.92</td>
<td>0.99</td>
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<td>0.96</td>
<td>0.91</td>
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<tr>
<td>$\text{Corr}(\tilde{n}, \tilde{y})$</td>
<td>0.86</td>
<td>0.79</td>
<td>0.99</td>
</tr>
<tr>
<td>$\text{Corr}(\tilde{i}_h, \tilde{s}_h)$</td>
<td>0.91</td>
<td>0.99</td>
<td>0.97</td>
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</table>

**US - Foreign Correlations**

<table>
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<th>Measure</th>
<th>Data</th>
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<th>Complements</th>
</tr>
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<td>$\text{Corr}(\tilde{y}_h, \tilde{y}_f)$</td>
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<tr>
<td>$\text{Corr}(\tilde{i}_h, \tilde{i}_f)$</td>
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<td>0.37</td>
<td>-0.45</td>
<td>-0.07</td>
</tr>
</tbody>
</table>
observable variables can be recovered from the data using the smoothing algorithm built into the Kalman Filter. Given the observable data on private consumption, investment, and hours worked, the unobservables for the government consumption and investment series are recovered. Table 3.12 shows the time-series in differences for the two pairs. The correlations between the unobservables and the government series are found to be positive (0.22 and 0.17 for government consumption, and investment, respectively.) Overall, this indicates positive empirical evidence that these shocks represent a portion of these disturbances.

![Smoothed consumption shock](image1)

![Smoothed investment shock](image2)

Figure 3.7: Smoothed Unobservable Series (Solid) and Actual HP-Filtered Data (Dashed)

### 3.9 CONCLUSION

This paper presents evidence that the FH and BKK puzzles still exist using very recent data. We construct a two-country, one good version of the neoclassical model and study the effects of government shocks in that setting. We show that government investment shocks alone in the model can help account for the FH puzzle quantitatively. This is achieved despite the fact that capital is assumed mobile across countries. We also show that government consumption shocks interpreted
as preference shocks which complement private consumption in the utility function help resolve the BKK puzzle. A model with government consumption and investment shocks combined can simultaneously explain the FH and BKK puzzle, and improves the performance of the model with technology shocks alone along several dimensions.
4.0 ASSESSING FISCAL STRESS (CO-AUTHORED WITH EMANUELE BALDACCI, IVA PETROVA, NAZIM BELHOCINE, AND GABRIELA DOBRESCU)

4.1 INTRODUCTION

Recent fiscal difficulties around the world brought to the fore the importance of assessing fiscal sustainability risks both in advanced and emerging economies. Based on the conceptual framework presented in Cottarelli (2011), these risks can lead to a sovereign debt rollover crisis in the absence of fiscal adjustment. Various factors can impact these fiscal sustainability risks, including: (i) whether current and projected fiscal policies are consistent with solvency and liquidity requirements (Baldacci, McHugh and Petrova, 2011); (ii) whether uncertainty around this baseline-reflecting shocks to macroeconomic assumptions, fiscal policy, and contingent liabilities-has heightened; and (iii) whether non fiscal factors (such as current account imbalances) and global financial market risk appetite have increased the likelihood of a fiscal crisis (IMF, 2011).

In this paper, we build a new index of fiscal stress that provides early warning signals of fiscal sustainability problems for advanced and emerging economies. Unlike previous studies, the analysis is not confined to sovereign debt default or near-default events. Fiscal crisis periods are defined as episodes of outright fiscal distress-public debt default/restructuring, need to access large-scale official/IMF support, hyperinflation-as well as extreme financing problems-spikes in sovereign bond spreads. In these cases, fiscal solvency is endangered and the government is forced to alter its policies to regain fiscal sustainability.

Another innovation of this paper is that the fiscal stress index is based on a set of indicators that measure the risk of fiscal sustainability based on current fiscal variables and their baseline projections using a consistent conceptual framework (Baldacci, McHugh and Petrova, 2011). For
each indicator, thresholds are estimated on the basis of a univariate procedure that maximizes the likelihood of predicting a fiscal crisis. The fiscal stress index measures the number of indicators exceeding these thresholds, weighted by their relative signaling power. The index can be used to assess the degree of fiscal stress in advanced and emerging market economies over time. Results show that fiscal stress risks remain elevated in advanced economies and well above the pre-crisis years. This owes to high solvency risks related to fiscal fundamentals and aging-related long-term budget pressures as well as record-high budget financing needs. Fiscal stress is lower for emerging economies, due to the rebuilding of fiscal buffers and more positive growth prospects than in mature economies. However, risks remain higher than in pre-crisis years also for these economies and point to continued vulnerabilities to shocks.

The rest of the paper is organized as follows. The next section surveys the literature on early warning systems, focusing on studies of fiscal crises. Section 4.3 elaborates the early warning methodology applied to developing the fiscal stress index. Section 4.4 describes the data used and main results, and Section 4.5 concludes.

4.2 LITERATURE REVIEW

There is an abundant literature on Early Warning System (EWS) models, mostly focused on currency and banking crises. These empirical studies differ according to: (i) the definition of crisis events; (ii) the methodology adopted; and (iii) the set of indicators used. Also country coverage tends to be limited by data quality, with only a few studies focusing on both advanced and emerging economies (and even in these cases limiting the analysis to relatively small samples).

Previous studies typically focused on financial crises, with a few papers assessing the risk of public debt default. In the latter studies, the definition of crisis events typically covers only tail events: for example, Detragiache and Spilimbergo (2001) define public debt crises as events of outright default or rescheduling, while Manasse, Roubini and Schimmelpfennig (2003) further add the provision of a large-scale official financing support to the definition of fiscal crises. However, extreme rollover problems are more common than public debt default episodes across advanced and emerging economies in the last decades. A broader definition of fiscal crises could provide better
information about changes in underlying fiscal sustainability risks, even in the absence of outright debt default (or near-default events triggering financial support of the official sector). In this paper, we define fiscal stress events to capture crisis episodes that encompass public debt default and near-default events, as well as severe deteriorations in the fiscal solvency risk outlook leading to fiscal sustainability risks (Cottarelli, 2011; IMF, 2011).

The empirical literature also differs with respect to the methodology used in the studies. Two approaches are common: the univariate “signaling” approach and the multivariate regression analysis of the crisis determinants.\(^1\) The “signaling” approach was proposed in a seminal paper by Kaminsky, Lizondo and Reinhart (1998) on determinants of currency crises. It entails using each potential indicator of crisis events separately, identifying critical thresholds that signal such events with the lowest prediction error, and then averaging the number of indicators exceeding this threshold into a composite index. This is based on weights proportional to the signaling power of each indicator. The methodology has been used in subsequent empirical studies, including to assess fiscal vulnerability indicators that help predict financial crises in emerging economies (Hemming, Kell and Schimmelpfennig, 2003) and to assess the risk of sudden stops (IMF, 2007). The multivariate regression approach uses panel regressions (probit or logit) with a binary dependent variable equal to one if a crisis occurs and zero otherwise. The impact of a set of determinants on the crisis probability is then derived by estimating the model and testing the coefficients’ significance. Berg and Patillo (1999) use this approach to predict currency crises and find that the crisis probability increases with changes in the predictive indicators.

Various studies have attempted to compare the performance of these two methods based on their success in correctly predicting crises. Berg and Patillo (1999) and Berg, Borensztein and Patillo (2005) find that the multivariate probit model outperforms the “signaling” approach both in-sample and in cross-country predictions, while the “signaling” approach has a better out-of-sample performance. Overall, no approach emerges as the clear winner and results depend on the type of crisis risk assessed.

In this paper, the “signaling” approach is used.\(^2\) This framework is relatively simple and allows for a transparent mapping from a large set of fiscal indicators into a composite index of fiscal sustainability risks, even in the absence of outright debt default (or near-default events triggering financial support of the official sector). In this paper, we define fiscal stress events to capture crisis episodes that encompass public debt default and near-default events, as well as severe deteriorations in the fiscal solvency risk outlook leading to fiscal sustainability risks (Cottarelli, 2011; IMF, 2011).

\(^1\)See Abiad (2003) for a survey, including other methodologies. Manasse, Roubini and Schimmelpfennig (2003) also use a non-parametric method based on binary recursive tree analysis to assess nonlinear combinations of factors affecting the likelihood of debt crises.

\(^2\)This is consistent with the method adopted by the IMF in the Early Warning Exercise (IMF, 2010a).
stress. Another advantage of the methodology is that it easily accommodates differences in data availability across variables, while using panel multivariate regression models would limit the number of predictive variables owing to data gaps. One limitation of this approach is that individual predictive variables cannot be tested for their conditional statistical significance. However, each variable contributes to the fiscal stress index with a weight proportional to its power in predicting a fiscal stress event.

The literature suggests several indicators that can help predict which countries are most vulnerable to banking crises. Frankel and Saravelos (2010) point to the importance of the level of international reserves, the real exchange rate and the current account in predicting financial crises. Similarly, IMF (2007) finds that external sector variables are important, in particular reserve coverage, the current account and external debt relative to exports.

Only a few studies focus on fiscal variables determinant of fiscal crises. While fiscal data are not as widely available as monetary or financial data, fiscal variables are also found to be relatively less powerful in predicting crises. Hemming and Petrie (2002) discuss fiscal vulnerability and potential fiscal indicators that might increase fiscal risks and Hemming, Hell and Schimmelpfennig (2003) use a large set of fiscal variables for 29 emerging economies over the period 1970-2000 to assess risks of currency, debt and banking crises. They find that the best fiscal indicators are short-term public debt, foreign-currency debt as well as other deficit measures.

In this paper, we rely on a parsimonious set of fiscal indicators that have been identified by Baldacci, McHugh and Petrova (2011) to measure fiscal sustainability risks under the medium-term scenario of the World Economic Outlook baseline projections. These indicators measure solvency risks based on current deficit and debt levels, and projected growth-adjusted interest rate on public debt. Indicators of long-term budget pressure associated with demographic aging, such as projected change in health care and pension expenditures, are also included. In addition to the solvency risk outlook, the framework also covers risks to fiscal sustainability stemming from sovereign asset and liability composition and financing requirements.

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3Data limitations and low degrees of freedom may limit the use of the multivariate approach in particular when the number of variables predicting a crisis is large.

4For example, Manasse, Roubini and Schimmelpfennig (2003) find that no fiscal variables are significant determinants of debt crises using a panel logit model in a sample of advanced and emerging economies. They find that the ratio of public debt to revenue is a better determinant of default risks when using a non-parametric method. Nonetheless, high values of these indicators are associated with sovereign debt crises in their findings only when other macroeconomic fundamentals are also weak.
4.3 METHODOLOGY

4.3.1 Fiscal Crisis Episodes

A fiscal crisis episode is identified in this study as a period of extreme government funding difficulties (Cottarelli, 2011). Funding pressures could arise as a result of public debt build-up, contingent liabilities that become outright fiscal costs, negative revenue shocks, or unaddressed demographic-related spending pressures. Financing constraints may also tighten due to market perception that the composition of public debt impedes the repayment capacity of the government. The surveyed literature suggests four types of criteria to capture such events: (i) debt default or restructuring; (ii) implicit default; (iii) recourse to exceptional official financing; and (iv) a sharp deterioration in market access.

Previous studies used a combination of the first three criteria to identify fiscal crises: public debt default or restructuring, hyperinflation, and large-scale IMF-supported programs. A limitation of this approach is that it misses fiscal distress episodes that are severe enough to alter the attainment of macroeconomic stability and growth but do not result in defaults or near-defaults. Fiscal crises can manifest themselves differently since the mid-1990s, with the development of bond markets and a lower reliance of countries on bank loans (see Pescatori and Sy, 2007). Notably, some episodes of severe difficulties may not trigger a debt default or restructuring and would not be captured by the standard definition used in the literature.

This paper combines the criteria above with indicators of severe spikes in financing costs to obtain a more comprehensive set of fiscal crisis events. To identify periods of public debt default, debt restructuring, and high levels of IMF financing support, the same definition is used for advanced and emerging economies. The definition of default follows Standard and Poor’s, which classifies a sovereign in default if it is not current on its debt obligations (including exchange offers, debt equity swaps, and buybacks for cash). Restructuring and rescheduling are defined as any operation which alters the original terms of the debt-creditor contract. Public debt defaults include both commercial and official creditors. Large IMF-supported programs are those with access above 100 percent of quota.\footnote{Starting in 2009, many high-access programs have exceeded this threshold. Changing the threshold for high-access programs does not alter the number of fiscal distress events significantly. Excluding precautionary arrangements from the definition does not change the results significantly, either.}

5 These are typically non-concessional loans and are provided as part of an ad-
justment program. Exceptional financing covers situations where near-default was avoided through large-scale IMF-supported programs.\textsuperscript{6}

Implicit domestic public debt defaults are identified by criteria for high inflation, differentiated between advanced and emerging economies. High inflation episodes are those where the inflation rate was above 35 percent per year in the case of advanced economies, and 500 percent per year for emerging economies.\textsuperscript{7} The threshold for advanced economies was chosen on the basis of the average haircut on public debt in case of external debt restructuring. This follows Sturzenegger et al. (2006) and aims to capture implicit domestic defaults. The threshold for emerging economies is based on results by Reinhart and Rogoff (2010).\textsuperscript{8}

Severe government bond yield pressures are also considered. This captures situations in which the government faces significant short-term market financing constraints.\textsuperscript{9} Periods when yield spreads exceeded two standard deviations above the country-specific mean were used to capture market financing pressure events for both advanced and emerging economies. In addition, for emerging economies periods were also included when the bond yield spreads exceeded 1,000 basis points (even if this level did not exceed two standard deviations from the mean) to capture countries that have exceptionally high credit risk spreads for long periods, reflecting high political risks and the consequences of past debt defaults (Pescatori and Sy, 2007).\textsuperscript{10}

The resulting definition of fiscal distress events for advanced and emerging economies is presented in Table 4.1. Annual data for 29 advanced economies and 52 emerging economies covering 1970-2010 are used to identify fiscal stress events.\textsuperscript{11} Data on debt default and restructuring were obtained from Standard and Poor. Information about exceptional IMF-supported programs is

\footnotesize{\textsuperscript{6}While a large set of distress events enhances the statistical robustness of the analysis, it could also weaken the predictive capacity of the model. Results for the sensitivity analysis of fiscal distress events to changes in the definition used in the text confirm that lowering the thresholds for identifying crisis episodes worsens the predictive ability of the model, in particular for emerging economies.

\textsuperscript{7}Sensitivity analysis was performed for alternative inflation rates. An inflation rate which exceeds 100 percent in the case of emerging economies does not significantly affect the results.

\textsuperscript{8}These authors prefer this benchmark to Cagan’s traditional definition of 50 percent inflation rate per month, because it allows for the use of annual data which are more widely available.

\textsuperscript{9}We separate periods of fiscal pressure into distinct events by assuming that there should be at least two years of no fiscal distress between separate events. In addition, only the start year of the event is considered as the actual fiscal distress year.

\textsuperscript{10}This threshold is not binding in the case of advanced economies, due to their traditionally lower sovereign bond spreads.

\textsuperscript{11}The advanced economies in the sample are those covered by the IMF’s Fiscal Monitor. The emerging economies are those covered in the Vulnerability Exercise for Emerging Market Economies (VEE) conducted by the IMF (2010). It should be noted that some countries have moved over time from emerging economy to advanced country status. In the paper, we classify these countries on the basis of the group they belong to in 2010.}
based on the IMF’s Finance Department database. Long-term domestic bond spreads and, where available, 5-year credit default swap (CDS) spreads are used to capture sovereign yield spikes in advanced economies. Data on spreads of long-term domestic bond spreads relative to comparable U.S. bonds are used for emerging economies. Sourced of data on sovereign bond yields at annual and monthly frequencies include the IMF’s International Financial Statistics (IFS), Bloomberg, and Datastream.

Table 4.1: Definition of Fiscal Crisis across Advanced and Emerging Market Economies

<table>
<thead>
<tr>
<th>Event</th>
<th>Criteria</th>
<th>Advanced Economies</th>
<th>Emerging Economies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Public debt default</td>
<td>failure to service debt as payments come due, as well as distressed debt exchanges</td>
<td>S&amp;P definition</td>
<td>S&amp;P definition</td>
</tr>
<tr>
<td>or restructuring</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Large financing</td>
<td>large IMF-supported program</td>
<td>access to 100 percent quota or more</td>
<td>access to 100 percent quota or more</td>
</tr>
<tr>
<td>Implicit/Internal Public debt default</td>
<td>high inflation rate</td>
<td>inflation greater than 35 percent per annum</td>
<td>inflation greater than 500 percent per annum</td>
</tr>
<tr>
<td>Extreme financing constraint of the sovereign</td>
<td>sovereign yield pressure</td>
<td>sovereign spreads greater than 1,000 basis points or 2 s.d. from country average</td>
<td>sovereign spreads greater than 1,000 basis points or 2 s.d. from country average</td>
</tr>
</tbody>
</table>

On the basis of the definition used in the paper, there were 41 fiscal distress events in advanced economies and 135 events in emerging economies (Table 4.2). Advanced countries’ events were identified mainly by government bond yield spikes, with only a few countries experiencing episodes of access to exceptional financing. Five countries experienced high-inflation events in the period; only 7 out of 29 countries had no crises. In contrast, fiscal stress events for emerging economies frequently involved multiple types of crises. About 60 percent of the cases relate to IMF-supported programs (79 events) and a third to outright defaults and restructuring (each 52 events). However, in the last decade fiscal stress events were increasingly identified through severe bond yield spikes in these economies.

The incidence of new fiscal stress events is clustered around specific periods (Figure 4.1). Prior to the recent financial crisis, several advanced economies experienced fiscal stress as a result of the oil boom of 1973 and the recession of the early 1990s. Many countries entered into fiscal distress
<table>
<thead>
<tr>
<th>Start of Crisis</th>
<th>Fiscal Stress Events</th>
<th>Default or Restructuring</th>
<th>IMF-Supported Program</th>
<th>High Inflation Pressure</th>
<th>Bond Yield</th>
<th>Duration of Fiscal Stress (in years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Advanced Economies</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1970-79</td>
<td>14</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>7</td>
<td>2.6</td>
</tr>
<tr>
<td>1980-89</td>
<td>8</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>5</td>
<td>2.5</td>
</tr>
<tr>
<td>1990-99</td>
<td>8</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>6</td>
<td>2.3</td>
</tr>
<tr>
<td>2000-10</td>
<td>11</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>11</td>
<td>2.6</td>
</tr>
<tr>
<td>Emerging Economies</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1970-79</td>
<td>15</td>
<td>8</td>
<td>9</td>
<td>1</td>
<td>0</td>
<td>3.1</td>
</tr>
<tr>
<td>1980-89</td>
<td>41</td>
<td>26</td>
<td>22</td>
<td>0</td>
<td>0</td>
<td>6.6</td>
</tr>
<tr>
<td>1990-99</td>
<td>37</td>
<td>12</td>
<td>20</td>
<td>5</td>
<td>5</td>
<td>2.6</td>
</tr>
<tr>
<td>2000-10</td>
<td>42</td>
<td>6</td>
<td>28</td>
<td>0</td>
<td>10</td>
<td>1.6</td>
</tr>
</tbody>
</table>
after the onset of the recent crisis in 2008, with a few more new crises occurring in 2009-10. Among emerging economies, fiscal stress events were clustered around the public debt crises in the early 1980s, the Latin American and Asian crises of the 1990s, and the recent global financial crisis.

Figure 4.1: Incidence of Fiscal Crises

Source: IMF International Financial Statistics; Bloomberg; Standard and Poor’s; and Authors’ calculations.

The length of fiscal stress is on average 2 and a half years in advanced economies, and 3 and a half years in emerging economies. As a result the incidence of fiscal crises may not correspond to the number of countries which experience fiscal stress in any given year (Figure 4.2). Therefore, in discussing the results we present the number of countries in fiscal stress in parallel with the incidence of fiscal crisis events.

As expected, our approach identifies more crisis events than other studies (Table 4.3). This stems from a more comprehensive definition of crisis events and from the larger sample used. The differences in the events identified in the paper and those identified by Reinhart and Rogoff (2010) arise mainly from the use of access to large IMF-supported programs and of government yield spikes. Lastly, the timing of crises also differs occasionally from other datasets, either because of the differences in definitions or because of the window required between two separate events.
Figure 4.2: Countries in Fiscal Stress

Source: IMF International Financial Statistics; Bloomberg; Standard and Poor’s; and Authors’ calculations.

Table 4.3: Summary Comparison of Events across Studies

<table>
<thead>
<tr>
<th></th>
<th>Fiscal Stress Index</th>
<th>IMF EWS</th>
<th>RR</th>
<th>HKS</th>
<th>MRS</th>
<th>LV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Events</td>
<td>176</td>
<td>28</td>
<td>48</td>
<td>16</td>
<td>32</td>
<td>22</td>
</tr>
<tr>
<td>Number of Common Events</td>
<td>20</td>
<td>30</td>
<td>13</td>
<td>33</td>
<td>33</td>
<td></td>
</tr>
<tr>
<td>Number of Countries</td>
<td>81</td>
<td>48</td>
<td>66</td>
<td>29</td>
<td>47</td>
<td>102</td>
</tr>
</tbody>
</table>

Events dated differently by only one year are considered common events. Events within our identified crisis durations are also considered common. IMF EWS refers to IMF (2007), RR to Reinhart and Rogoff (2010), HKS to Hemming, Kell and Schimmelpfenning (2003), MRS to Manasse, Roubini and Schimmelpfenning (2003), LV to Laeven and Valencia (2008). The number of total events and missed events in other studies do not add to 164 due to differences in the sample of countries and dates covered.
4.3.2 Fiscal Stress Thresholds

Indicator selection is based on the conceptual framework presented in Cottarelli (2011) and Baldacci, McHgh and Petrova (2011). The indicators are divided into three groups that can indicate risk over the short run, medium run, and long run. These are the basic fiscal variables, long-term fiscal trends, and asset and liability management. Basic fiscal variables indicate risks of government solvency: If the net present value of the future stream of primary fiscal balances is larger than the initial stock of public debt, the intertemporal budget constraint will be violated. The second dimension of risk refers to the medium run. If financing requirements are large, the risk that fiscal solvency concerns will lead to a rollover crisis is high. The composition of the government’s assets and liabilities may then amplify financing risks. Finally, fiscal solvency also depends on the extent to which long-term demographic and economic trends will put pressure on the budget. This is captured under the “long-term fiscal trends” group.

The estimation of fiscal stress thresholds for each indicator is based on the “signaling” approach (IMF, 2007; IMF, 2010). This consists of defining cut-off values for each fiscal indicator that discriminates between predicted crisis and non-crisis periods. To illustrate these methodologies, the true versus predicted occurrence of crises are reported in Table 4.4.

<table>
<thead>
<tr>
<th>State of the World</th>
<th>Crisis</th>
<th>No Crisis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal (crisis)</td>
<td>True Positive ($TP$)</td>
<td>False Positive ($FP$)</td>
</tr>
<tr>
<td>Predicted result</td>
<td>No signal (no crisis)</td>
<td>False Negative ($FN$)</td>
</tr>
<tr>
<td>Total</td>
<td>Total crises obs. $N_C$</td>
<td>Total non-crises obs. $N_{NC}$</td>
</tr>
</tbody>
</table>

The table shows the occurrence of type II errors ($FN(C)$) and type I errors ($FP(C)$). The objective is to find the optimal cut-off point $C^*$ for each indicator such that the occurrence of type I and type II errors is minimized. Formally one can define an indicator variable at time $t$, for each indicator $i$, $d_{it}$, for the following $j$ time periods as following:

$$d_{it} = \begin{cases} 
1 \forall j, & \text{if } i_{t-1} > C_i \\
0, & \text{otherwise}
\end{cases} \quad (4.1)$$
where \( i_t \) refers to a fiscal indicator and is a monotonically increasing function of crises probabilities and \( C_i \) represents a fixed cut-off for \( i_t \). As mentioned, the signaling window \( j \) is set to one year in the analysis.

Two methods are commonly used in the literature to determine the optimal value of \( C \): the minimization of the total misclassified errors and the maximization of the signal-to-noise ratio. Under the total misclassified errors (\( TME \)) method, for each cut-off point \( C_i \), the \( TME \) value for each indicator can be expressed as the sum of type I and type II errors,

\[
TME(C_i) = \frac{FN(C_i)}{N_{C_i}} + \frac{FP(C_i)}{N_{NC_i}}.
\] (4.2)

The optimal threshold \( C^*_i \) for each indicator is the value that minimizes \( TME(C_i) \). Due to the small number of fiscal crisis events relative to non-crisis periods, the \( TME \) methodology places greater weight on misclassifying fiscal crisis events, thereby yielding relatively conservative thresholds compared to other methods. The signal-to-noise (\( SNR \)) ratio can be defined as the ratio of the percentage of correctly classified crises observations (1-type II errors) to the percentage of incorrectly classified non-crises observations (type I errors). For each cut-off point \( C_i \), the \( SNR \) can be expressed as:

\[
SNR(C_i) = \frac{TP(C_i)/N_{C_i}}{FP(C_i)/N_{NC_i}}.
\] (4.3)

The optimal threshold \( C^*_i \) under this approach is the value that maximizes \( SNR(C_i) \).

If an indicator exceeds the cut-off level, the model issues a signal of an upcoming fiscal distress episode. The optimal cut-off point should balance the two types of statistical errors. The lower the threshold, the more signals the model will send (i.e., type II errors will decrease), but at the same time, the number of wrong signals rises (i.e., type I errors will increase). Using a higher threshold reduces the number of wrong signals, but at the expense of increasing the number of missed distress episodes.

\[12\] A fiscal indicator that crosses the optimal threshold in period \( t-1 \) signals a fiscal crisis in period \( t \), thus implying that the level of fiscal stress in the current period is determined by the values of the fiscal indicators in the previous period.
4.3.3 Fiscal Stress Index

A fiscal stress index is calculated based on the signaling power of each fiscal indicator. This entails two steps. In the first step, if an indicator crosses its calculated optimal cut-off, it is assigned a value of 1 and it is weighed proportionately to its predictive power. Each indicator’s weight in the group $w_{i,g}$ can be expressed as:

$$w_{i,g} = \frac{1 - TME(C^*_i)}{\sum_{\forall i \in g} (1 - TME(C^*_i))} \quad (4.4)$$

In the second step, the predictive power of the cluster indices is evaluated and the indicators are aggregated in the fiscal stress index based on their own predictive power and the predictive power of the cluster indices:

$$\text{Overall Index} = \sum_g \left( w_g \sum_{\forall i \in g} w_{i,g} d_i \right) \quad (4.5)$$

where $w_g$ is the weight of the group, and $d_i$ is a dummy that takes the value of 1 if the indicator is above (below) the threshold, and zero otherwise.

4.4 RESULTS

4.4.1 Data

The analysis uses 12 fiscal indicators (Baldacci, McHugh and Petrova, 2011),\(^{13}\) classified into three clusters: basic fiscal variables, long-term fiscal trends, and asset and liability management. The data were obtained from the IMF’s Fiscal Monitor, the IMF’s World Economic Outlook (WEO), the Bank of International Settlements (BIS), and United Nations databases. While some data are available for the period 1970-2010, most series start in the 1980s and are available for all countries only for the mid-1990s. Therefore, while the complete dataset is used to estimate the thresholds, the analysis focuses on the period after 1995.

\(^{13}\)The following fiscal indicators are included: the difference between GDP growth and the imputed interest rate on government debt ($r-g$); cyclically adjusted primary balance; general government gross debt to GDP; gross financing needs; short-term debt (on a remaining maturity basis) to total debt; debt denominated in foreign currencies to total debt for emerging economies/debt held by nonresidents to total debt for advanced economies; weighted average maturity of government debt; short-term external debt to international reserves for emerging economies; deviation of fertility rate from 2.1; old age dependency ratio; and long-term projections of the change in public pension and health expenditure. See Baldacci, McHugh and Petrova (2011) and IMF (2011) for a detailed definition of these indicators.
The analysis of the fiscal indicators reveals that the global financial crisis started in 2008 has triggered a pronounced deterioration in the basic fiscal variables (e.g., public debt to GDP ratio and the cyclically adjusted primary balance as a ratio of potential GDP) in advanced countries, leading also to a sharp upturn in gross financing needs (Figure 4.3). With long-term pension and health expenditure costs on an upward trend, risks of fiscal stress are expected to have increased in recent years.

In emerging economies, the basic fiscal indicators show that the deterioration in the cyclically adjusted primary balance had started before the outset of the crisis. Nonetheless, public debt to GDP has remained lower than historical levels. Asset and liability management variables have deteriorated since 2008, mostly on account of large deficit financing needs. However, financing conditions have also worsened, with short-term debt reaching levels seen during the Latin American and the Asian crises of the mid 1990s. Variables measuring long-term fiscal challenges are also trending up in emerging economies, but to a lesser extent than in advanced economies.

4.4.2 Indicator Thresholds and Weights

The estimation of the indicator thresholds is based on the performance of the TME and SNR approaches. The TME method performs better, in line with previous results in the literature (IMF, 2007). Nonetheless, adjustments to the TME methodology are necessary for several reasons. First, occasionally the TME solution is located close to the median of the distribution and in some case on the tail of the distribution where values of the indicators indicate low risk of fiscal distress. Second, trends and structural breaks in the data are likely over long time periods. Finally, data are reliably available only since the mid-1990s.

To maximize the predictive power of the indicator, the thresholds are estimated separately for advanced and emerging economies under the constraints that they are located on the risk-prone side of each indicator’s distribution relative to the 1995-2010 median (Figures 4.4 and 4.5). This is

As expected, the SNR approach leads to higher total errors; specifically, this method can force the solution to very high type II errors, yielding thresholds with very high percentages of missed crises and misclassified non-crises. In general, the indicators may not have an easily identified region that comes out of well-behaved CDFs, leading to corner solutions. Such cases were encountered when using a larger set of fiscal variables, including social spending and the slope of the yield curve. Solutions to such cases vary from using bootstrapping to assigning zero weights to the variables in the index. A preferred approach, however, was to use a parsimonious set of indicators, exhibiting well-behaved data properties.
Figure 4.3: Trends in Selected Fiscal Indicators

Sources: World Economic Outlook, Bank of International Settlements, Dealogic; and authors' calculations. See Appendix C.1 for the definition of fiscal indicators.
obtained by removing a few outliers\textsuperscript{16}, which allows more robust threshold estimation.\textsuperscript{17}

The estimated thresholds and the implied signaling power of the indicators determine the relative weight that a variable has in the fiscal stress index. Signaling power is defined as one minus the total error and it is a measure of the statistical power of the variable. As discussed in Section 4.2, predictive errors produced by EWS methodologies are typically non-negligible. The focus of the exercise, however, is on the relative performance of the fiscal variables and their role in detecting fiscal vulnerability. This is shown by the relative signal intensity for each variable (signaling power).

Information on the performance of individual indicators is presented in Tables 4.5 (advanced economies) and 4.6 (emerging economies). The first column shows the number of crises for which data on the indicator are available. The third column shows a measure of the tendency of indicators to issue good signals. This is defined as the number of good signals issued by the indicator as a percentage of total crises observations \((TP(C_i^+)/NC_i)\). The fourth column measures the performance of indicators regarding sending bad signals as a percentage of total non-crises observations \((FP(C_i^-)/N_{NC_i})\). The last column shows the value of the loss function minimized, which is the sum of type I and type II errors.

To put the model’s performance in perspective, Table 4.7 shows the overall performance of two leading methodologies in the literature used for currency crises. While the nature of vulnerabilities is different in our study, we show that our model’s performance is comparable to that typically found in the literature with respect to measures such as the percentage of pre-crisis observations correctly called, and the percentage of false alarms to total non-crises observations.

\textsuperscript{16}These are defined as observations with high absolute levels of the standardized score, based on subtracting the mean of the fiscal indicator distribution and dividing by its standard deviation.

\textsuperscript{17}Basic fiscal variables are subject to such adjustment, as well as gross financing needs and the fertility rate indicator. In some cases (weighted average maturity and debt held by nonresidents to total public debt for advanced economies) the threshold is located in the crisis-prone side of the distribution and it is selected without further adjustment of the data. These adjustments make some of the thresholds more plausible, while increasing the precision of the estimates. Including outliers in the estimation of the threshold would reduce the explanatory power of some variables. Excluding the indicators for which outliers were removed would, however, have a larger impact on the results.
Figure 4.4: Advanced Economies: Fiscal Indicator Medians and Thresholds

Note: Median for the period 1995-2010. See Appendix C.1 for the definition of fiscal indicators.

Source: Authors’ calculations.
Figure 4.5: Emerging Market Economies: Fiscal Indicator Medians and Thresholds

Note: Median for the period 1995-2010. See Appendix C.1 for the definition of fiscal indicators.

Source: Authors’ calculations.
Table 4.5: Advanced Economies: Thresholds and Relative Weights of Fiscal Indicators

<table>
<thead>
<tr>
<th>Indicator</th>
<th>Crisis obs.</th>
<th>Threshold</th>
<th>Good Signals</th>
<th>Bad Signals</th>
<th>Loss function</th>
<th>Index</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Basic Fiscal Variables</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>r-g (5 year average)</td>
<td>21</td>
<td>3.6</td>
<td>71</td>
<td>48</td>
<td>87</td>
<td>77</td>
<td>14.9</td>
</tr>
<tr>
<td>General government gross debt (% GDP)</td>
<td>15</td>
<td>72.2</td>
<td>33</td>
<td>22</td>
<td>89</td>
<td>7.3</td>
<td></td>
</tr>
<tr>
<td>Cyclically adjusted primary balance (% potential GDP)</td>
<td>5</td>
<td>-4.2</td>
<td>60</td>
<td>45</td>
<td>85</td>
<td>9.4</td>
<td></td>
</tr>
<tr>
<td><strong>Asset and Liability Management</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gross financing needs (% GDP)</td>
<td>6</td>
<td>17.2</td>
<td>83</td>
<td>45</td>
<td>62</td>
<td>48.2</td>
<td>24.6</td>
</tr>
<tr>
<td>Share of short term debt (ratio of total debt)</td>
<td>15</td>
<td>9.1</td>
<td>100</td>
<td>96</td>
<td>96</td>
<td>2.8</td>
<td></td>
</tr>
<tr>
<td>Debt held by nonresidents (% total debt)</td>
<td>7</td>
<td>83.6</td>
<td>21</td>
<td>6</td>
<td>85</td>
<td>10.1</td>
<td></td>
</tr>
<tr>
<td>Weighted average maturity of general government debt (years)</td>
<td>18</td>
<td>3.9</td>
<td>7</td>
<td>9</td>
<td>92</td>
<td>5.0</td>
<td></td>
</tr>
<tr>
<td><strong>Long-Term Fiscal Trend</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fertility rate (deviation from 2.1)</td>
<td>31</td>
<td>0.64</td>
<td>29</td>
<td>25</td>
<td>96</td>
<td>2.4</td>
<td></td>
</tr>
<tr>
<td>Long-Term projections of public health expenditure (% GDP)</td>
<td>28</td>
<td>4.5</td>
<td>68</td>
<td>53</td>
<td>85</td>
<td>9.4</td>
<td></td>
</tr>
<tr>
<td>Long-Term projections of public pension expenditure (% GDP)</td>
<td>21</td>
<td>6.2</td>
<td>24</td>
<td>9</td>
<td>85</td>
<td>9.6</td>
<td></td>
</tr>
<tr>
<td>Dependency ratio</td>
<td>37</td>
<td>36</td>
<td>14</td>
<td>6</td>
<td>92</td>
<td>4.5</td>
<td></td>
</tr>
<tr>
<td><strong>Overall Index</strong></td>
<td>39</td>
<td>0.15</td>
<td>49</td>
<td>26</td>
<td>77</td>
<td>83</td>
<td></td>
</tr>
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</table>

Table 4.6: Emerging Economies: Thresholds and Relative Weights of Fiscal Indicators

<table>
<thead>
<tr>
<th>Indicator</th>
<th>Crisis obs.</th>
<th>Threshold</th>
<th>Good Signals</th>
<th>Bad Signals</th>
<th>Loss function</th>
<th>Index</th>
<th>Weight</th>
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</thead>
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<tr>
<td><strong>Basic Fiscal Variables</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>r-g (5 year average)</td>
<td>52</td>
<td>1.1</td>
<td>75</td>
<td>58</td>
<td>83</td>
<td>96</td>
<td>11.3</td>
</tr>
<tr>
<td>General government gross debt (% GDP)</td>
<td>20</td>
<td>42.8</td>
<td>65</td>
<td>61</td>
<td>96</td>
<td>96</td>
<td>2.5</td>
</tr>
<tr>
<td>Cyclically adjusted primary balance (% potential GDP)</td>
<td>60</td>
<td>-0.5</td>
<td>48</td>
<td>33</td>
<td>85</td>
<td>85</td>
<td>9.9</td>
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<td><strong>Asset and Liability Management</strong></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gross financing needs (% GDP)</td>
<td>29</td>
<td>20.6</td>
<td>31</td>
<td>27</td>
<td>96</td>
<td>96</td>
<td>2.8</td>
</tr>
<tr>
<td>Share of short term debt (ratio of total debt)</td>
<td>45</td>
<td>44.0</td>
<td>29</td>
<td>15</td>
<td>86</td>
<td>86</td>
<td>9.2</td>
</tr>
<tr>
<td>Debt denominated in foreign currencies (% total debt)</td>
<td>52</td>
<td>40.3</td>
<td>85</td>
<td>70</td>
<td>85</td>
<td>85</td>
<td>9.9</td>
</tr>
<tr>
<td>Weighted average maturity of general government debt (years)</td>
<td>40</td>
<td>2.3</td>
<td>15</td>
<td>11</td>
<td>96</td>
<td>96</td>
<td>2.6</td>
</tr>
<tr>
<td>Short term external debt (% gross international reserves)</td>
<td>101</td>
<td>61.8</td>
<td>72</td>
<td>43</td>
<td>71</td>
<td>71</td>
<td>19.1</td>
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<tr>
<td><strong>Long-Term Fiscal Trend</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fertility rate (deviation from 2.1)</td>
<td>60</td>
<td>1.3</td>
<td>95</td>
<td>87</td>
<td>92</td>
<td>92</td>
<td>5.2</td>
</tr>
<tr>
<td>Long-Term projections of public health expenditure (% GDP)</td>
<td>28</td>
<td>2.7</td>
<td>50</td>
<td>38</td>
<td>88</td>
<td>88</td>
<td>8.2</td>
</tr>
<tr>
<td>Long-Term projections of public pension expenditure (% GDP)</td>
<td>5</td>
<td>4.0</td>
<td>80</td>
<td>60</td>
<td>80</td>
<td>80</td>
<td>13.4</td>
</tr>
<tr>
<td>Dependency ratio</td>
<td>81</td>
<td>16.1</td>
<td>49</td>
<td>40</td>
<td>91</td>
<td>91</td>
<td>5.9</td>
</tr>
<tr>
<td><strong>Overall Index</strong></td>
<td>99</td>
<td>0.47</td>
<td>64</td>
<td>33</td>
<td>69</td>
<td>69</td>
<td></td>
</tr>
</tbody>
</table>

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Table 4.7: Comparing Model Performance

<table>
<thead>
<tr>
<th>Study</th>
<th>Good</th>
<th>Bad</th>
<th>Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current Study - Advanced Economies</td>
<td>49</td>
<td>26</td>
<td>77</td>
</tr>
<tr>
<td>Current Study - Emerging Markets</td>
<td>64</td>
<td>33</td>
<td>69</td>
</tr>
<tr>
<td>KLR (1998) - Signaling Approach(^a)</td>
<td>41</td>
<td>15</td>
<td>74</td>
</tr>
<tr>
<td>Berg and Patillo (1999) - Probit Model</td>
<td>47</td>
<td>13</td>
<td>66</td>
</tr>
</tbody>
</table>

\(^a\)These numbers are taken from Berg and Patillo (1999).

The top predictors of fiscal stress are different for advanced and emerging economies.\(^{18}\) In the advanced economies, government rollover pressures are associated with the size of financing needs and fiscal solvency concerns, while for emerging economies liquidity constraints are the main signal of fiscal stress. This finding underlies the different economic structure and weaknesses that characterize these countries. When advanced economies are vulnerable to market financing shocks, this is generally in response to evidence of an unsustainable debt path. With about one third of the fiscal stress index determined by international liquidity and the currency composition of government debt, emerging economies are more exposed to “original sin” problems and spillovers from financial markets.\(^{19}\)

A logit regression is used to assess the ability of the fiscal stress index to provide early warning signals on fiscal sustainability risks. This is done by plotting the fiscal stress index and the probability of entering into fiscal stress (and remaining in stress after an episode has started). The fiscal stress index components are all significant determinants of fiscal stress episodes. The correlation is higher with basic fiscal variables, whereas the other components of the index have a lower

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\(^{18}\)In both advanced countries and emerging economies the nature of fiscal sustainability risks has changed over time as population aging has emerged as a key fiscal risk and economies have gained market access. This is reflected in the changes over time in the weights that the indicators have on the fiscal stress index when they are calculated for sub-periods of time.

\(^{19}\)”Original sin” is the inability of emerging market economies to finance externally in domestic currency (Eichengreen, B., Hausmann, R., Panizza, U., 2002). With a small domestic investor base, a government that resorts to heavy external borrowing is exposed to substantial foreign currency risk.
correlation—although their coefficients are highly significant (Figure 4.6).  

Figure 4.6: Advanced Economies: Probability of Fiscal Crisis at Different Levels of the Fiscal Stress Index

Note: Cumulative marginal effect of the fiscal stress index and its components with 95-percent confidence bands.

Source: Authors’ estimations.

In emerging economies, the relationship between fiscal crises and the fiscal stress index follows a similar pattern, with a narrower confidence interval than for advanced countries (Figure 4.7). The correlation between the fiscal stress index and probability of experiencing a fiscal crisis is driven primarily by the asset and liability management variables for these countries.

Notice that the dependent variable in this case includes the periods of fiscal stress after the first year in which a crisis occurs. This is different from the definition of fiscal stress episodes used for the construction of the index and can help assess how the index helps predict the level of risk once the event has occurred. This also explains why basic fiscal variables have a stronger weight in the regression results than in the fiscal stress index, as they are associated with more persistent stress spells.

Multivariate logit regressions confirm these results, with basic fiscal variables having the largest marginal effect in advanced economies, and the asset and liability management component having the largest marginal effect in emerging market economies.
Figure 4.7: Emerging Economies: Probability of Fiscal Crisis at Different Levels of the Fiscal Stress Index

Note: Cumulative marginal effect of the fiscal stress index and its components with 95-percent confidence bands.

Source: Authors’ estimations.
4.4.3 Fiscal Stress Index Trends

Fiscal stress has increased more rapidly in advanced than in emerging economies. In 2011, the fiscal stress index-weighted with countries’ PPP-GDP- is higher in advanced countries (Figures 4.8 and 4.9). Overall, in advanced economies the fiscal stress index has doubled since 2006 and is at record-high levels. In contrast, in emerging economies the fiscal stress index is elevated, but still slightly below the peak experienced during the financial crises of the late 1990s.

Figure 4.8: Fiscal Stress Index, 1995-2011

Note: PPP-GDP weights used to calculate the weighted average index.

Source: Authors’ calculations.

Decomposing the fiscal stress index for advanced economies reveals that its increase since the mid-2000s is a result of a sharp deterioration in the basic fiscal variables-mainly debt to GDP and the cyclically adjusted primary balance (Figure 4.10). The asset and liability management component has also peaked, contributing for about half of the increase in the index. Long-term fiscal indicators have also exerted continuous pressure on the fiscal stress index. In emerging economies, the main factors behind the increase in the fiscal stress index have been the basic fiscal variables, followed by the long-term fiscal trends. The asset and liability management component-mostly due to declining short-term debt to international reserves-has kept the index from increasing further.

Focusing on the regional differences (Figure 4.11), in advanced economies, the fiscal stress index is highest in North America, although the peak levels of the index are observed in peripheral euro

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22 The unweighted average index is useful in gauging how the index fares compared to the incidence of fiscal crisis and the number of countries in fiscal stress. The weighted average is useful in assessing the systemic importance of
Figure 4.9: Fiscal Stress Index, 1995-2011

Note: PPP-GDP weights used to calculate the weighted average index.

Source: Authors’ calculations.

Figure 4.10: Contribution of the Fiscal Stress Index Components, 1996-2011

Note: Unweighted fiscal stress index. It measures the change in the index compared to the base year in percent.

Source: Authors’ calculations.
countries. In emerging economies, the fiscal stress index is markedly higher in Emerging Europe, followed by countries in the Middle East and North Africa.

![Figure 4.11: Fiscal Stress Index Levels by Region, 2011](image)

Note: Unweighted fiscal stress index.
Source: Authors’ calculations.

In the last five years, the index has increased sharply in North America (Figure 4.12). This is mainly due to deterioration in the cyclically adjusted primary balance and a sharp increase in debt and gross financing needs. While in Asia and the Pacific the index has increased the least, it has been on an upward trend for the last 15 years. This is due to underlying demographic trends, putting pressure on the long-term fiscal component of the index, as well as rising debt and large gross financing needs.

In emerging economies, over the last five years the index has increased the most in Latin American countries, due to peaking cyclically adjusted primary deficits, in a few cases accompanied by declining debt maturity and international reserve coverage of short-term debt. In Emerging

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23 As the two indices are constructed independently and therefore also have different levels, using a common scaling year helps compare the behavior of fiscal stress across country groupings. The trends in the indices, when commonly scaled, are indicative of the developments in fiscal vulnerability among the two groups.
Europe, the index has remained elevated throughout 1996-2011. This is not only due to the solvency indicators, but also worsening asset and liability management risks—high ratio of foreign currency denominated debt and low reserve coverage of short-term debt—plus growing concerns about the long-term fiscal outlook.

4.5 CONCLUSIONS

The fiscal stress index presented in this paper provides a signaling tool to assess exposure to fiscal sustainability risks and helps identify the factors underlying changes in fiscal stress risks. However, like similar early warning tools, the stress index does not attempt to predict crises, which are typically triggered by a combination of economic, financial, or political shocks. While signaling tools like the fiscal stress index presented here are important to assess vulnerabilities, they should be complemented by judgment-based approaches.
This paper calculates thresholds that identify the likelihood of fiscal stress for a large set of fiscal variables. These thresholds are based on an EWS methodology and are used to construct a summary index of fiscal sustainability risks for advanced economies and emerging markets. In contrast with previous studies, the fiscal stress index relies on a broader definition of crisis episodes, consistent with the conceptual framework developed by Cottarelli (2011). In calculating the fiscal stress index, this paper uses a parsimonious set of fiscal indicators proposed by Baldacci, McHugh and Petrova (2011).

The fiscal stress index is calculated for a large sample of advanced and emerging economies during 1995-2011. Results show that in advanced countries the top predictors of fiscal stress are indicators of gross financing needs and fiscal solvency risks. In emerging economies, the best predictors of fiscal stress are risks associated with public debt structure and exposure to spillovers from financial markets. Fiscal stress risk has increased dramatically across the world as a consequence of the global financial crisis. Risks are higher in advanced economies than in emerging economies, but remain higher than before the crisis in the latter group. North America and Europe are the regions were fiscal stress risks are highest.

There is scope for further extensions based on the analysis presented in this paper. In particular, bootstrapping methods could be used to gauge the uncertainty surrounding the point estimates. Another avenue of further research is to conduct the analysis using thresholds based on country-specific distributions (as in Hemming et al., 2003) instead of using an overall threshold, in order to control for country-specific characteristics. Using time-specific effects could also prove useful in view of the common factors that affect many countries during periods of global contagion.
APPENDIX A

FIGURES

A.1 DATA FIGURES WITH THE HP-TREND

Figure A1: A Plot of U.S. Logged Private Consumption With The HP Trend

A.2 LIKELIHOOD CONTOUR PLOTS
Figure A2: A Plot of U.S. Logged Private Investment With The HP Trend

Figure A3: A Plot of U.S. Logged Hours Worked With The HP Trend
Figure A4: A Plot of U.S. Logged Government Components With The HP Trend
Figure A5: One-Dimensional Plots of the Likelihood Function around Estimated Parameters
Figure A6: Two-Dimensional Plots of the Likelihood Function around Estimated Parameters
APPENDIX B

MODEL SOLUTION

B.1 THE NON-LINEAR SYSTEM

\[
\left(\frac{1 - \gamma}{\gamma}\right) \frac{\tilde{C}_t}{\tau c_{pt-1}} l_t = (1 - \alpha) \tilde{z}_t \left(\frac{\tilde{K}_t}{n_t}\right) ^{\alpha}
\]

\[
c_{pt-1} \tilde{C}_t^{\gamma(1-\phi) - \psi(1-\gamma)(1-\phi)} = \beta E_t \left\{ \left(1 + \frac{g}{1 - \alpha}\right)^{\gamma(1-\phi) - 1} c_{pt+1}^{\gamma(1-\phi) - \psi(1-\gamma)(1-\phi)} \right\} \left\{ \alpha \omega \tilde{z}_{t+1} \left(\frac{n_{t+1}}{K_{t+1}}\right)^{1-\alpha} \left(\frac{K_{t+1}}{k_{pt+1}}\right)^{1-\chi} + (1 - \delta) \right\}
\]

\[
\tilde{y}_t = \tilde{z}_t \tilde{K}_t^{\alpha} n_t^{1-\alpha}
\]

\[
\tilde{y}_t = \tilde{c}_{pt} + \tilde{c}_{gt} + \tilde{i}_{pt} + \tilde{i}_{gt}
\]

\[
\left(1 + \frac{g}{1 - \alpha}\right) \tilde{k}_{pt+1} = \tilde{i}_{pt} + (1 - \delta) \tilde{k}_{pt}
\]

\[
\left(1 + \frac{g}{1 - \alpha}\right) \tilde{k}_{gt+1} = \tilde{i}_{gt} + (1 - \delta) \tilde{k}_{gt}
\]

\[1 = n_t + l_t\]

\[
\log(\tilde{z}_t) = (1 - \rho_z) \log(\tilde{z}) + \rho_z \log(\tilde{z}_{t-1}) + \epsilon_{zt}
\]

\[
\log(\tilde{c}_{gt}) = (1 - \rho_{cg}) \log(\tilde{c}_g) + \rho_{cg} \log(\tilde{c}_{gt-1}) + \epsilon_{cgt}
\]

\[
\log(\tilde{i}_{gt}) = (1 - \rho_{ig}) \log(\tilde{i}_g) + \rho_{ig} \log(\tilde{i}_{gt-1}) + \epsilon_{igt}
\]

Note that \(\tilde{C}_t\) denotes \(\left[\tau c_{pt} + (1 - \tau) c_{gt}\right]^{1/\psi}\), and \(\tilde{K}_t\) denotes \(\left[\omega \tilde{k}_{pt} + (1 - \omega) \tilde{k}_{gt}\right]^{1/\chi}\).
APPENDIX C

DATA SOURCES

C.1 FISCAL INDICATORS

Debt default: Period of domestic or foreign bonded and bank debt default (available from Standard and Poor’s 1970-2008).

IMF-supported programs: Period of IMF-supported program exceeding 100 percent of IMF member’s quota (available from IMF 1970-2010).

Inflation rate: In percent: period during which inflation exceeds 35 percent per annum for AE, or 500 percent per annum for EMs (available from IMF/IFS 1970-2010).

Bond yield pressure: Government bond spreads (relative to 10-year US Treasury bond) exceeding 2 standard deviations above the historical annual mean of the country, or 1000 basis points on annual basis; or at least 6 months in a year based on monthly data (available from IMF/IFS 1970-2010).

r-g (5-year average): Imputed interest rate on general government debt, deflated by the GDP deflator, minus real GDP growth rate. Five year forward moving average (available from WEO 1971-2010 for advanced economies, and 1985-2010 for emerging economies).


**Total fertility rate:** The average number of children per woman (available from UN 1970-2010).

**Old age dependency ratio:** 20 years ahead projections of the ratio of population over 65, divided by the number of adults (available from UN 1970-2010).

**Long-term projections of the change in public pension expenditure:** Expressed in percent of GDP, the change in projected expenditures 30 year ahead relative to the base year (available from IMF staff estimates 1980-2010).

**Long-term projections of the change in public health expenditure:** Expressed in percent of GDP, the change in projected expenditures 30 year ahead relative to the base year (available from IMF staff estimates 1979-2010 for advanced economies, and 1995-2010 for emerging economies).

**Current gross financing need:** Projected general government overall balance plus general government debt with a maturity of one year or less. Expressed in percent of GDP (available from WEO,VEE 1990-2010 for advanced economies, and 1993-2010 for emerging economies).

**Share of short-term debt as a ratio of total debt:** Short-term debt is defined as general government debt with remaining maturity of one year or less. Total debt is general government gross debt (available from BIS 1989-2010).

**Debt denominated in foreign currencies:** General government debt, expressed in terms of total debt (available from WEO,VEE 1990-2010 for emerging economies).

**Debt held by non-residents as a proportion of total debt:** Includes both domestic and foreign currency debt issued. Expressed as a proportion of total debt (available from JEDH,WEO 1980-2010 for advanced economies).

**Weighted average maturity of general government debt:** Historical data calculated by staff; current data available in Bloomberg (available from Bloomberg; Dealogic 1980-2010).

**Short-term external debt to international reserves:** Short-term debt is defined as general government debt with remaining maturity of one year or less (available from WEO,IFS 1990-2010 for advanced economies, and 1970-2010 for emerging economies).
BIBLIOGRAPHY


