MECHANICS OF CRACK PROPAGATION IN CLAYS UNDER DYNAMIC LOADING

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Over-consolidated clays and shales forming part of the core section of earth dams, natural slopes, and clay deposits undergoing desiccation, can be subjected to dynamic loads from earthquakes, waves, traffic load, or machine vibrations. This research asks “what happens to fissured clays under dynamic loading?”.

Static and dynamic compression laboratory tests were done on fissured kaolinite clay specimens. The samples were subjected to uniaxial, biaxial and triaxial stress conditions. Crack propagation was analyzed using both Linear Elastic Fracture Mechanics and Discrete Element and Finite Difference methods. Crack propagation and failure in the samples was found to vary depending on the state of water distribution in the clay pores: the saturated-funicular state, the complete pendular state, or the partial pendular state. Finding that accepted theories of unsaturated soil mechanics don’t apply in latter two states, a new approach using the equivalent effective stress concept is presented and then be applied to all three states, showing that the strength of the clay is proportional to the equivalent effective stress in the soil.

Since tensile stresses at the tips of a crack produce additional propagation, a new apparatus was developed to study the strength of clays under tension, and it was found that the fissured clay failed specifically were shear and tensile stresses occurred within a region of high tension.
Also, subsequent dynamic compression tests revealed the crack stability thresholds (the compressive stress below which no crack propagation takes place in a fissured clay). The threshold appeared: almost constant in the complete and partial pendular states, but greatly diminished in the funicular-saturated state as water content increased. In the saturated-funicular state, the water flowed around the crack tip from zones receiving high compressive stress, to zones with highly-concentrated tensile stress. The higher the water content of the clay, the lower the tensile strength.
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1.0 STATEMENT OF THE PROBLEM

Over-consolidated clays and shales forming part of the core section of zoned earth dams and natural slopes have been found to exist in the fissured state (Bishop, 1967; Covarrubias, 1969; Duncan and Dunlop, 1969; Marsland, 1972; Morgenstern, 1977; Peterson et al. 1966; Rizkallah, 1977; Sherald, 1973; Skempton, 1964; Skempton and LaRochelle, 1965; Terzaghi, 1936; Vallejo, 1985; Williams and Jennings, 1977). According to Covarrubias (1969), fissures or cracks exist in the core section of earth dams as a result of one or more the following factors:

1. deformation of the materials in the dam or in the foundation due to their weight;
2. abrupt changes in the cross section of a valley;
3. large deformations caused by saturation of the materials in the dam;
4. excessively rapid filling of the reservoir that causes high rates of strain, especially of the materials undergoing substantial movement upon saturation;
5. large transient stresses caused by earthquakes;
6. large differences in stress-strain properties of materials in adjacent zones or layers.

In the case of stiff clays forming natural slopes, Williams and Jennings (1977) found that fissures develop as a result of a variety of processes, the most important of which are the following:

1. consolidation;
2. swelling of the clay as a result of a decrease in overburden pressure;
3. chemical reactions in the clay that induce volume distortions;
4. tectonic stresses;
5. desiccation of the clay;
6. weathering process inherited from bedrock; and
7. large lateral stresses.

The present research addresses the question of what happens to fissured soil deposits when they are subjected to dynamic loading such as that generated by earthquakes, wave action, traffic load, or machinery vibration.

1.1 HOW CRACKS AFFECT SOILS

Earlier research on fissured soil deposits and the effect that such fissures have on the behavior of these deposits have included the following:

- Skempton (1964) mentions that fissures in the over-consolidated London clay locally produce stress concentrations that can exceed the peak strength of the material, leading to the progressive failure of the slopes composed of those materials.

- The San Andreas strike-slip system of faults formed by several individual active faults affects much of the west coast of North America and has, over time, produced losses of many billions of dollars (Harbert, 2002). Seed et al. (1979) reports failure of earth dams after the 1906 San Francisco earthquake that was generated by one of those active faults. Figure 1 shows the location of several earth dams relative to the San Andreas Fault at the time of the earthquake.
Figure 1. Locations of several dams around the epicenter of the 1906 San Francisco earthquake started by the San Andreas Fault (Adapted from Seed, 1979).

- Sherard (1992) reports 15 cases of embankment dam cracking that have occurred around the world. He notes that 150 to 300 small dams (20 - 75 ft. high) are constructed in the United States each year and that most of the cracking in the dams in the U.S. results from embankment soils that are especially brittle and so susceptible to cracking. Figure 2 shows cracks developed in an earth dam after the Northridge earthquake in 1994.
• Clay deposits subjected to desiccation often develop intense cracking that extends deep below the surface. Arizona, Mexico City and Bogota are examples of areas particularly affected by such deep cracking. Vesga et al. (2003) found intensive deep cracking in the high plastic Bogota (Colombia) clay deposit that affects a flat area of 90000 ha located in a zone characterized by high seismic hazard: hundreds of kilometers of roadways and hundreds of small buildings in this area have been severely damaged or collapsed as a result of deep cracking caused by desiccation. Figure 3 shows some characteristics of such problems.

Figure 2. Cracks developed in an earth dam after the 1994 Northridge earthquake in Los Angeles (Picture from Los Angeles Department of Water and Power).
• The 1964 Alaska earthquake generated an enormous landslide (2593 m wide, 363 m long) in the Turnagain Heights of Anchorage where stiff clays with sand lenses formed the ground (Vallejo, 1988). He explains the failure process as one related to fracture mechanics and notes that in this case the lenses behaved like open cracks due to a drop in the vertical effective stress caused by sand liquefaction. Vallejo also states that the stress
concentrations around the lens tips produced tensile stresses that contributed to the catastrophic ground failure.

1.2 PREVIOUS RELATED RESEARCH ON CRACK PROPAGATION IN CLAYS

Vallejo (1985, 1986, 1987, 1988, 1989, 1991, 1993), Vallejo et al. (1993), and Vallejo and Shettima (1997) report several important findings related to the behavior of clays with pre-existing cracks. Vallejo and co-workers did several tests using rectangular kaolinite specimens with single or multiple cracks prepared in accordance with a special process that he developed; cracks of different orientations and specimens with different water contents were used. The specimens were subjected to monotonic uniaxial, biaxial, triaxial and shear stress fields. The researches used Linear Elastic Fracture Mechanics (LEFM) extensively to theoretically study the tension and compression stress concentrations around the cracks.

Four important conclusions can be derived from their findings as follows. (1) The critical pre-existing crack inclination, which corresponds to the condition of the lowest compression strength for crack propagation, varies between 45° and 60° with respect to the direction of the applied principal stress. (2) The maximum tangential stress criterion for a sharp crack of the type earlier proposed by Erdogan and Sih (1963) was used to predict the angle between the pre-existing crack plane and the crack propagation direction; this criterion was selected by Vallejo et al. (1995) as the closest to their findings between of all the criteria that were applied; (3) Fissures propagate as a result of constant compressive stresses (creep), which are much less than the crack-propagation compression strength of monotonically loaded clays (Vallejo and Shettima, 1997). (4) Multiple cracks will make the clay weaker, especially if superposition of tensile-stress concentration zones develop (Vallejo, 1993).
1.3 PREVIOUS RELATED RESEARCH ON STABILITY THRESHOLD

Lefebvre et al. (1988) studied the cyclic undrained resistance of non-fissured, intact saturated Hudson Bay clay and described the threshold as the stress level below which the soil suffers no failure regardless of the number of applied cycles. The researchers used the term cyclic stress ratio to describe this stress level which relates to both the applied triaxial cyclic stress and the triaxial compression strength of the intact clay. They found that for the saturated Hudson Bay clay, the stability threshold is defined by a cyclic stress ratio of between 0.60 and 0.65. In similar way, the stability threshold concept is applied in this research to the study of crack propagation in clays under dynamic loads.

1.4 CURRENT RESEARCH: STABILITY THRESHOLD APPLIED TO FISSURED CLAYS

The stability threshold research will now be extended to fissured clays. The laboratory tests for this research focused on specimens of fissured clays that were subjected to cyclic loads; the loads applied were just a fraction of the static loads that caused the failure of the clay. The research investigated the influence of the resulting fatigue on the propagation of cracks in a unsaturated kaolinite clay and on the threshold stress with respect to cyclic loads levels below which the cracks do not propagate. Samples of fissured clays were subjected to uniaxial, biaxial, and triaxial cyclic stress conditions. The cyclic stress ratio was defined as the ratio between the applied deviator dynamic vertical stress \( \sigma_d \) on a fissured specimen and the monotonic compression strength \( \sigma_u \) of a similar specimen having the same water content and crack geometry. The cyclic stress ratio is given as:

\[
r_d = \frac{\sigma_d}{\sigma_u}
\] 

(1)
The purpose of the dynamic-load testing was to find the threshold load (fraction of the static load), expressed as the cyclic stress ratio $r_{ds}$ at which, regardless of the number of cycles applied, there is no crack propagation in an unsaturated kaolinite clay subjected to dynamic loading conditions.
2.0 RESEARCH OBJECTIVES

2.1 GENERAL OBJECTIVE

The general objective of the proposed research was to study how laboratory-prepared unsaturated clay specimens with induced cracks respond to static and dynamic loads under uniaxial, biaxial, and triaxial stress conditions.

2.2 SPECIFIC OBJECTIVES

The specific objectives of the research program were as follows:

(1) To do a laboratory study of the mechanics of crack propagation in clays under static and dynamic loads in order to better understand the following three important factors:

(a) the effect of parameters such as inclination and number of cracks in the clay samples on crack propagation.

(b) the effect on crack propagation of the dynamic loading conditions (stress level, frequency, and number of cycles).

(c) the effect that soil parameters such as water content, degree of saturation, and suction pressure have on the crack propagation in the clay samples.

(2) To determine the cyclic axial load at which cracks in clay no longer propagate (i.e. the threshold load). This axial load was a cyclic load that was a fraction of the static
axial load at which cracks would propagate in the sample and was exerted on samples that were unconfined (uniaxial cyclic tests) or confined (biaxial or triaxial tests).

(3) To study the stress field around cracks that causes them to propagate under static or dynamic loads. This stress field was obtained by using LEFM theory along with two numerical methods: the Distinct Element Method (DEM) and the Finite Difference Method (FDM).
3.0 LABORATORY PROGRAM DEVELOPED

The laboratory testing program was completed utilizing specimens of clay consolidated from a slurry. The clay was a kaolinite similar to that used in previous research on crack propagation in soils completed earlier at the University of Pittsburgh (Vallejo, 1985, 1989, 1993), Vallejo et al (1993), Vallejo and Shettima (1997).

The tests were performed in three stages dealing with: characterization, monotonic loading, and dynamic loading. The monotonic and the dynamic loading included uniaxial, biaxial, and triaxial tests.

3.1 CLAY CHARACTERIZATION

The characterization tests included Atterbergh limits, specific gravity, one-dimensional consolidation, compaction, and the conventionally used water-retention curve. This last test evaluates the suction pressure and the water content of the clay. The hydraulic conductivity for the fully saturated soil was obtained from the consolidation test, using the one-dimensional consolidation theory (Terzaghi and Peck, 1948). The hydraulic conductivity of the water phase for the partially saturated soil was obtained using the methodology proposed by Fredlund and Rahardjo (1993), in which a relationship for deriving the unsaturated hydraulic conductivity from the saturated hydraulic conductivity and the water retention curve was proposed. The hydraulic conductivity of the water phase in an unsaturated soil decreases notably as the degree of saturation diminishes. As will be presented below, hydraulic conductivity must be taken into
account when explaining the behavior observed in the samples that were subjected to dynamic loading.

### 3.2 PREPARATION OF SPECIMENS

The specimens were prepared in accordance with the procedure summarized in Figure 4 immediately below.

The specific steps of the procedure were as follows:

1. Dry kaolinite was mixed with distilled water to yield a soft mass having a water content of about 40%.

2. The kaolinite slurry for each sample was consolidated over 24 hours in a mold under a vertical pressure of 30 KPa and in a controlled environment having a 75% relative humidity. Prismatic molds (7.5 cm x 7.5 cm x 2.5 cm) were used for the uniaxial, biaxial, and triaxial tests. Cylindrical molds (6.5 cm in diameter and 2 cm in thickness) were used for the indirect tensile tests. Bowtie-shape specimens (7.0 cm long, 3.5 cm wide and 2.0 cm at the neck) were used for the direct tensile tests. Vertical drainage was provided at the top and the bottom of the specimens. As a result of the consolidation stage, a water content reduction of about 2% was obtained.

3. The specimens were extracted from the molds and, immediately after, an artificial crack or group of cracks were made.
4. The specimens were subjected to a shrinkage stage by means of an air-drying process which continued until the estimated soil water content reached the desired level. During this step, the soil remained partially saturated and the water loss due to the evaporation process lowered the temperature of the specimens (a few degrees Celsius). The time needed to
complete the shrinkage test stage depended on the relative humidity in the laboratory; the higher the relative humidity, the greater the time required (generally around 24 to 48 hours).

5. Each specimen was stored in a plastic membrane for the minimum time required to reach a relative humidity equilibrium around it. When the relative humidity was stable, there was no vapor pressure transfer between the specimen and the air surrounding it in the plastic membrane. After suction pressure reached equilibrium the specimen’s temperature increased until it reached the temperature of the lab. The weight and volume of the specimen were measured (moist and dry unit weight, and the void ratio and degree of saturation were obtained). This step of the preparation lasted around 24 hours.

6. The specimen remained stored in the plastic membrane under equilibrium conditions and until it was tested under compression (the preparation of each specimen took approximately 3 to 4 days to complete).

3.3 CRACK GEOMETRIES

Cracks with inclination angles of 0°, 15°, 30°, 45°, 60° and 75° with respect to the horizontal were made in the prismatic specimens. Each crack had a length of 25 mm or 1.5 mm and a thickness of 1 mm. Figure 5 below shows the cracks geometries and the arrangements that were used in the case of multiple cracks (left step, right step, and combined).
3.4 MOISTURE CONTENT AND SUCTION PRESSURE

Moisture content levels of between 3% and 32% with intervals around them about 3% were used. As the actual moisture of each specimen could differ from the estimated level during the specimen-preparation stage, the actual moisture level was measured using an oven drying process once each test was done. Based on this, the suction pressure was then estimated from the water-retention curve of the soil at the beginning of the compression tests.

The clay specimens on which the tests were done were partially saturated. For this reason, some basic concepts related to unsaturated soil mechanics will now be reviewed in order to clarify the terms that will be used below to explain the behavior of the kaolinite clay. The most important of these parameters are suction pressure, water retention curve and hydraulic conductivity.
3.4.1 Pore suction pressure

The total suction pressure in the air-water pore phase of an unsaturated soil is defined by the psychrometric law (Alonso, 2000) as follows:

\[ \Psi = -\frac{RT\rho_w}{\omega_v}\ln(RH) \]  \hspace{1cm} (2)

where

\( \Psi \): Total suction

\( R \): Universal (molar) gas constant (8.31432 J/(mol \degree K))

\( T \): Absolute temperature; \( T=273.16+t^\circ \)

\( t^\circ \): temperature (\degree C)

\( \omega_v \): molecular mass of water vapor (18.016 kg/kmol)

\( \rho_w \): density of water

\( RH \): Relative humidity of the air in equilibrium with a liquid with suction \( s \).

Figure 6 which follows, shows the relationship derived from this equation. As can be seen, different temperatures have almost no effect on the total suction pressure.

The total suction (\( \Psi \)) is equal to the sum of the matric suction pressure (\( s \)) plus the osmotic suction pressure (\( \Psi_o \)) as follows (Alonso, 2000):

\[ \Psi = s + \Psi_o \]  \hspace{1cm} (3)

The matric suction is associated with the capillary phenomenon arising from the surface tension of water, and is defined (by Fredlund and Rahardjo, 1993) as follows:
where $s$ is the pore matric suction, $u_a$ is the pore air pressure, and $u_w$ is the pore water pressure.

The osmotic suction is related to the solutes dissolved in the water: the higher the concentration of solutes, the lower the equilibrium pressure. For pure water, the osmotic suction is zero.

Figure 6. Relative humidity versus total suction relationship (adapted from Fredlund and Rahardjo, 1993).

3.4.2 Measurement of pore suction pressure

Direct and indirect methods can be used for measuring the suction pressure in a soil. The total suction pressure can be obtained using the psychrometric law in cases when the relative humidity is known; the later can be obtained indirectly by the filter paper method or using a psychrometer (electronic sensor). Osmotic suction doest not seem to be sensitive to changes in the soil water content. As a result, a change in the total suction necessarily represents a change in the matric suction pressure (Fredlund and Rahardjo, 1993). On the other hand, since distilled water was
used for preparing the specimens for this research, the osmotic suction should be null; if this is the case, the total suction pressure ($\Psi$) equals the matric suction pressure ($s$). The psychrometer is especially useful for measuring high degrees of suction pressures. However, the psychrometer has one disadvantage in that its lag response time be too long to make it available for use in dynamic loading tests.

Matric suction pressure can be measured either directly by using tensiometers, which measure negative pressure, or by using the axis translation technique. Tensiometers are useable for low range suction pressures ($s<90$ kPa) due to the possibility of cavitation of the water in the tensiometer (Fredlund and Rahardjo, 1993). Use of this kind of sensor, however, was not appropriate in this research because is too big to be installed in the specimen.

The axis translation technique is used only in the laboratory and involves a translation of the pore-air pressure within the sample. Pore-water pressure can be derived from a positive air pressure (Hilf, 1956) if the unsaturated soil specimen is placed in a closed chamber that has a high air entry disk, as depicted in Figure 7 below. The disk supports the specimen and acts as an interface between the air in the chamber and the water compartment. A positive air pressure level is applied to the chamber and the resulting water pressure is measured. The water pressure which would be positive can be measured through a common electronic pressure gauge. The difference between the positive air pressure ($u_a$) and the positive water pressure ($u_w$), which is defined as the matric suction ($s = u_a - u_w$), can then be obtained. If the soil is unsaturated, the air pressure applied to the chamber would be greater than the water pressure exerted by the clay. However, enough time must be allowed for the water pressure of the sample to stabilize after the air pressure has been applied.
Figure 7. Pressure plate apparatus for measuring negative pore-water pressures using the axis-translation technique (adapted from Fredlund and Rahardjo, 1993).

The high air entry disk is a porous ceramic material with very low permeability. When saturated, it will maintain this condition until a certain degree of suction has been applied; however if a higher level is applied, the disk will permit air to enter, and the technique cannot be used. Common air entry values disks are 200, 500, and 1500 kPa; thus for higher suction pressures the psychrometer should be used instead of the axis translation technique.

3.4.3 Water-retention curve

The water-retention curve (or soil-water characteristic curve) reflects the relationship between the amount of water in the soil (i.e., gravimetric or volumetric water content) and the degree of soil suction (Sillers et al, 2001). The water retention curve contains important information that has been used to derive unsaturated soil property functions for hydraulic conductivity, shear strength and volume change (Sillers et al, 2001).
Figure 8. Water-retention curve illustrating the regions of desaturation (adapted from Sillers et al, 2001).

Figure 8 shows three stages in the desaturation of a clay. The water content in the soil is reduced as the amount of suction is increased. In the capillary saturation zone, the soil remains essentially saturated due to the capillary forces; in this zone, the suction goes up to the air entry value (AEV), where air starts to enter the largest pores. In the desaturation zone, the liquid water within the pores is increasingly displaced by air; the zone ends at the residual water content ($\theta_r$) where the pore-water becomes essentially immobile within the soil matrix. Increases in soil suction do not result in significant changes in water content (Sillers et al, 2001).

3.4.4 Classical Theories of Shear Strength of Unsaturated Soils

The shear strength of unsaturated soils has traditionally been derived by one of two means. The first way is based on a relationship derived from the net pressure ($\sigma_f - u_a$) applied to the soil and
on the actual matric suction pressure \((u_a-u_w)\) to which the soil is subjected (Fredlund and Rahardjo, 1993); this equation is

\[
\tau_f = c' + (\sigma_f - u_a) \tan \phi' + (u_a - u_w) \tan \phi^b
\]  

(5)

where

\(c'\): Cohesion intercept
\(\sigma_f\): Normal stress at failure
\(u_a\): Air pressure
\(u_w\): Water pressure
\(\phi', \phi^b\): Friction angles

The other means commonly used to represent the shear strength of unsaturated soils is the following equation (Bishop and Blight, 1963):

\[
\tau_f = c' + [\sigma_f - u_a] + \chi(u_a - u_w)] \tan \phi'
\]  

(6)

where

\(\chi\): Parameter related to the degree of saturation of the soil

Although these means are commonly used, as will be seen later, the shear strength of the unsaturated kaolinite clay used in this research could not be expressed using either Equation 5 - or Equation 6. This is because it was found that over a certain level of suction pressure \((u_a-u_w)\), the strength of the material was not affected by the degree of suction or the strength reduced as
the suction was increased. As a result of these findings, the equivalent effective stress concept was used to explain the behavior of the unsaturated kaolinite clay and will be discussed in more detail below.

### 3.5 MONOTONIC LOADING TESTS

Uniaxial, biaxial and triaxial compression tests were carried out on the prismatic specimens described above. The suction pressure during the test was estimated based on measurements of the relative humidity around the specimen inside the plastic membrane. The amount of suction pressure was obtained indirectly by use of the psychrometric law, which relates the total suction to the relative humidity inside the soil.

![Figure 9. MTS hydraulic compression machine.](image)

The uniaxial, biaxial, and triaxial tests were done by using the MTS hydraulic compression machine at the University of Pittsburgh Geotechnical Laboratory. The equipment was updated by
this researcher with the data acquisition system DATAQ ID-194 (Dataq Instruments, 2003), which permits data acquisition at speeds up to 240 samples/second distributed in 4 channels; the system was connected to a computer through a serial interface RS-232. The data was stored and managed using the software program Windaq V2.23 (Dataq Instruments, 2003). Figure 9 shows the MTS machine.

3.5.1 Uniaxial compression tests
The uniaxial constant water content tests were done using both intact and pre-cracked specimens. Specimens with moisture contents of between 3% and 32% with intervals around 3% were used. The pre-cracks had inclination angles (α) of 0°, 15°, 30°, 45°, 60° and 75° with respect to the horizontal. The multiple crack arrangements that were applied --left step, right step and combined-- are shown in Figure 5.

3.5.2 Biaxial compression tests
A special chamber (see Figure 10 below) developed at the University of Pittsburgh (Dupin, 1986) was used for the biaxial constant water content tests. The specimens had a moisture content of approximately 15% and crack inclinations of 0°, 15°, 30°, 45°, 60°, and 75° with respect to the horizontal. Levels of intermediate normal stress of 0, 35 and 70 kPa were used; 70 kPa is the maximum stress level that can be applied in the chamber. Constant water contents type tests were carried out.
3.5.3 Triaxial compression tests

The triaxial constant water content tests were done using a conventional Geotest cell available in the Geotechnical Laboratory in the University of Pittsburgh. The specimens that were used had a moisture content of 15% and crack inclinations of 0°, 15°, 30°, 45°, 60°, and 75° with respect to the horizontal. Cell pressures of 210, 420, 630, 840, and 1050 kPa were used. Constant water content type test (Fredlund and Rahardjo, 1993) were implemented. Figure 11 shows the equipment configuration.

*Figure 10. Biaxial apparatus for monotonic and dynamic loading tests (Dupin, 1986).*
3.6 DYNAMIC LOADING TESTS

The MTS hydraulic compression machine described above was used for the dynamic loading tests. The purpose of these tests was to find the threshold cyclic stress ratio (fraction of the ultimate static strength) at which there is no crack propagation under dynamic loading conditions regardless of the number of applied cycles. The cyclic stress ratio $r_d$ was defined as the ratio between the applied dynamic vertical stress and the ultimate monotonic compression strength of a similar specimen having the same water content and the same crack inclination. Thus, the cyclic axial load was a fraction of the static load at which the specimen failed.
3.6.1 Uniaxial dynamic compression tests

Dynamic uniaxial compression tests were performed on fissured specimens having moisture contents of between 3% and 32% with intervals around 3%. The pre-cracks had inclination angles of 0°, 15°, 30°, 45°, 60°, and 75° with the horizontal. A sine wave loading type with a frequency of 1 Hz was applied to the specimens having different crack inclinations. In order to evaluate the influence of the loading frequency on the threshold, additional tests with loading frequencies of 0.5 Hz and 2 Hz were done on specimens with a crack inclined 30° with respect to the horizontal. The data acquisition system DATAQ ID-194 (Dataq Instruments, 2003) recorded the load and deformation of the samples occurred during the tests.

For all resulting combinations of water content and crack inclination angles, the level of the cyclic stress ratio $r_d$ was changed in order to determine the crack propagation threshold.

3.6.2 Biaxial dynamic compression tests

Compression tests were done using the biaxial chamber developed by Dupin (1986) and the MTS dynamic compression machine. The tests were performed on specimens having a crack inclined 45° with respect to the horizontal and a moisture content close to 15%. The tests were done using a lateral cell pressure of 70 kPa, the maximum lateral pressure allowed by the equipment.

3.6.3 Triaxial dynamic compression tests

The triaxial dynamic compression tests were done using a Geotest pressure cell and the MTS dynamic compression machine. The tests were performed on specimens having a crack inclined 45° with respect to the horizontal and a moisture content of close to 15%. The applied cell pressures used in the tests were 210, 420, and 630 kPa.
4.0 RESULTS OF LABORATORY PROGRAM

4.1 KAOLINITE CHARACTERIZATION

In order to characterize the kaolinite used to make the specimens for this research Atterbergh limits, specific gravity, compaction, one-dimensional consolidation and shrinkage tests were done. The results obtained are summarized in Table 1, below.

Table 1. Characterization Tests Results

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>RESULT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liquid Limit, $LL$</td>
<td>44%</td>
</tr>
<tr>
<td>Plastic Limit, $PL$</td>
<td>25%</td>
</tr>
<tr>
<td>Shrinkage Limit, $SL$</td>
<td>24%</td>
</tr>
<tr>
<td>Specific Gravity, $G_s$</td>
<td>2.5866</td>
</tr>
<tr>
<td>Optimum moisture content, $w_{opt}$</td>
<td>29%</td>
</tr>
<tr>
<td>Degree of saturation at optimum, $S_{r_{opt}}$</td>
<td>92.8%</td>
</tr>
<tr>
<td>Maximum dry unit weight, $\gamma_{d,max}$</td>
<td>14.3 kN/m$^3$</td>
</tr>
<tr>
<td>Saturated hydraulic conductivity, $k_w$</td>
<td>1.59941E-08 cm/s</td>
</tr>
</tbody>
</table>
The optimum moisture content and the maximum dry unit weight were obtained using the Harvard miniature compaction test; Figure 12 below contains the compaction graph and the saturation curve. It is important to note that the optimum moisture content shown in the figure corresponds to a degree of saturation of 92.8%. As will be seen later, when the kaolinite specimens that have a crack are subjected to dynamic loading, the worst behavior occur when they are close to saturation.

![Graph of compaction and saturation](image)

*Figure 12. Harvard miniature compaction test results.*

### 4.2 WATER-RETENTION CURVE

The water-retention curve is shown in Figure 13 below. This curve was established by use of the psychrometric law (i.e., measuring the relative humidity) and the axis-translation technique. The capillary saturation zone of the kaolinite was found to be 10 MPa, the desaturation zone was found to be between 10 and 100 MPa, and the residual saturation was found to occur for suction
pressures higher than 100 MPa. The degree of actual suction pressure acting on the specimens was derived from this graph after determining the moisture content (right axis) or degree of saturation (left axis) in the specimens.

![Water-retention curve for the kaolinite clay.](image)

4.3 HYDRAULIC CONDUCTIVITY

The hydraulic conductivity was obtained using the method developed by Fredlund and Rahardjo (1993), and is shown in Figure 14 below. The hydraulic conductivity in the clay directly relates the water content: the lower the water content (or degree of saturation), the lower the hydraulic conductivity and greater the difficulty for the water to permeate into the clay. This behavior is very important because the water remains nearly immobile when clay has a low water content, and, as a result, the water flow is very limited. Water flows with the highest velocity when the soil is saturated. As will be seen later, this behavior affects the crack propagation threshold when the clay is close to saturation. In such cases, the water flows from the high compressive stress...
zones through the high tensile stress zones close to the crack tip. This is the reason why the stability threshold is much lower for specimens with water content approaching saturation.

![Figure 14. Hydraulic conductivity of the unsaturated kaolinite clay.](image)

**Figure 14. Hydraulic conductivity of the unsaturated kaolinite clay.**

### 4.4 UNIAXIAL AND TRIAXIAL STRENGTH OF INTACT SPECIMENS

Variations in the uniaxial compression strength (ultimate strength) of the intact specimens as a function of the water content are shown in Figure 15 below. This figure indicates that three distinct zones can be identified from the laboratory results. Zone I represents the increase in soil strength as the water content is reduced from 32% to 26%. In Zone II, the strength reaches a maximum of 1200 kPa and remains almost constant for water contents of between 26% and 12%. Zone III shows a reduction in the soil’s strength as the water content is reduced.
Figure 15. Uniaxial compression tests on intact specimens subjected to a drying path. Zone I: the strength increases as water content is reduced. Zone II: constant strength. Zone III: the strength diminishes as the water content is reduced. (note: Zones are numbered from right to left here and elsewhere to reflect the drying path).

Triaxial tests were done using specimens with a moisture content of 15%. Chamber pressures of 210, 420, 630, 840 and 1050 kPa were applied. Figure 16 below shows the results obtained. The internal friction angle of the clay was found to be $\phi=25^\circ$ and the cohesion intercept was found to be 350 kPa.
Figure 16. Triaxial test results for unsaturated kaolinite clay specimens with moisture content of 15%.

4.5 UNIAXIAL MONOTONIC LOADING TESTS ON FISSURED SPECIMENS

Two stress levels were registered for each uniaxial compression loading test performed on fissured specimens. With the stress level of $\sigma_c$, crack propagation occurred in the front and rear faces of the specimen (see Figure 17 below). With the stress level of $\sigma_u$ – which is the maximum stress specimens’ were able to withstand – a shear failure plane appeared in the lateral faces of the specimen.
4.5.1 Stress level at crack propagation $\sigma_c$

The stress level at crack propagation $\sigma_c$ as a function of the water content that resulted from the uniaxial compression tests is shown in Figure 18. A total of 47 uniaxial tests were done. The three zones that were first observed with the uniaxial compression tests of the intact specimens are again present in fissured specimens. Zone I shows increments of the clay strength $\sigma_c$ as the water content is reduced from 32% to 26%. In Zone II, the strength $\sigma_c$ reaches a maximum value and remains almost constant for water contents between 26% and 12%. Zone III shows the reduction in clay strength $\sigma_c$ as the water content falls below 12%.

The stress $\sigma_c$ depends also on the inclination of the primary crack. Figure 19 shows $\sigma_c$ as a function of the primary crack inclination angle $\alpha$. The crack propagation stress $\sigma_c$ of specimens
having cracks inclined 45° or less was found to be remarkably lower than the propagation stress for specimens with inclination angles of 75° and 60° (see Figure 18 and Figure 19 below).

![Figure 18](image)

**Figure 18.** Uniaxial monotonic compression strength for pre-fissured specimens. (a) Crack propagation stress, $\sigma_c$. (b) Ultimate compression strength, $\sigma_u$. The crack inclination angle ($\alpha$) is measured with respect to the horizontal in both tests.

4.5.2 Uniaxial ultimate compressive strength $\sigma_u$

The ultimate uniaxial compressive strength $\sigma_u$ of the fissured specimens is presented as a function of the water content in Figure 18b. Zones I, II, and III are still applicable as described above. The ultimate compression strength for intact specimens and for specimens with a crack is very close (compare Figure 15 and Figure 18a and 18b). This indicates that the presence of the primary crack has no effect on the ultimate uniaxial compressive strength of the tested
specimens. As Figure 17 shows, a shear failure plane was observed in the lateral face of all of the specimens. This confirms that the two types of failure occur independently one of each other.

Figure 19. Relationship between the crack propagation stress $\sigma_c$ and the primary crack inclination angle, $\alpha$.

4.5.3 Stress-strain relationship under static loading of fissured specimens

Figure 20 shows several stress-strain relationships obtained using static loading tests of fissured specimens of kaolinite clay. Curves for specimens having different water contents and primary crack inclination angles are shown. The stress-strain relationships for all specimens having water contents below 24% are almost identical. However, the primary crack inclination angle appears to have no influence on the elastic moduli, at least for moisture contents below 24%. This observation is considered of particular interest and will be discussed in more detail later.
Figure 20. Stress-strain relationships from static loading tests of kaolinite specimens having different moisture contents and primary crack inclinations. Specimens having moisture contents below 24% have very similar elastic moduli.

4.5.4 Crack propagation angle $\delta$ and crack propagation path

Figure 21 shows the condition of two samples after crack propagation occurred and one sample which primary crack closed and didn’t propagate; one specimen had a water content of 4.5%, and the other two had water content of 30.6% and 32.4%. The specimen with the lower water content had a typical brittle behavior characterized by a primary crack that remains open after the secondary crack propagates. The specimen having moisture content of 30.6% had a typical ductile type I behavior: the primary crack closed after the propagation of the secondary crack.
took place. In this specimen, it was observed that the secondary crack finished its propagation after the primary crack closed.

The specimen having a water content of 32.4% demonstrated a ductile, Type II behavior—i.e., the crack closed and didn’t propagate. This is consistent with the more general finding that in some ductile specimens with primary crack inclination angles of 30º, 15º, or 0º with respect to the horizontal, no propagation occurred.

The crack in the brittle specimen propagated at an angle of 88º with respect to the plane of the primary crack while the crack, in the ductile type I specimen propagated at an angle of 92º.

![Figure 21.](image)

Figure 21. (a) Secondary tensile cracks in a brittle sample with moisture content of 4.5%. (b) Secondary tensile cracks in a ductile sample with moisture content of 30.6%; the primary crack closed after propagation occurred. (c) The primary crack closed and didn’t propagate in a ductile sample with moisture content of 32.4%. The failure that is shown occurred by shear.

After the secondary crack started the propagation, the crack curved towards the vertical direction as Figure 21 shows. Similar behavior was observed for the other inclinations as well: the secondary crack propagation path curved to a direction parallel to the applied maximum compression stress.
The average crack propagation angle $\delta$ as a function of the primary crack inclination angle that was observed in the laboratory tests is presented in Figure 22. The MTS criterion proposed by Erdogan and Sih (1963) for the prediction of the crack propagation angle is also presented. As can be seen from this figure, the MTS criterion predicts very well the angle $\delta$ for primary crack inclination angles $\alpha$ of 15°, 30° and 45°, but not for the angles. A new criterion for the prediction of the crack propagation angle is proposed later in this research.

Figure 22. Secondary crack inclination angle $\delta$ derived from the MTS criterion and the laboratory results.
4.6 BIAXIAL MONOTONIC LOADING TESTS ON FISSURED SPECIMENS

The deviator stress at crack propagation \((\sigma_1 - \sigma_2)_c\) as a function of the primary crack inclination angle is shown in Figure 24 below. These results show that the weaker specimens had primary crack inclination angles of less than 45°. Figure 25 shows the deviator stress \((\sigma_1 - \sigma_2)_c\) as a function of the intermediate stress \(\sigma_2\) applied to the samples. It can be seen from this figure that no influence of \(\sigma_2\) can be noticed; \((\sigma_1 - \sigma_2)_c\) is almost constant. This is because the maximum lateral intermediate stress that can be applied using currently-available equipment is just 70 kPa – a stress too low to produce noticeable effects on the specimens of the type used in this study.
Figure 24. Biaxial compression tests results. The deviator stress at crack propagation \((\sigma_1 - \sigma_2)_c\) is shown as a function of the primary crack inclination angle, \(\alpha\).

Figure 25. Biaxial deviator stress at crack propagation as a function of the intermediate stress \(\sigma_2\).
4.7 TRIAXIAL MONOTONIC LOADING TESTS ON FISSURED SPECIMENS

Figure 26 below shows the results from the triaxial compression tests for specimens with a primary crack inclination angle $\alpha=45^\circ$. The difference between the deviator stress at crack propagation $(\sigma_1-\sigma_3)_c$ and the deviator stress for ultimate failure $(\sigma_1-\sigma_3)_u$ is shown in this figure. The deviator stresses became closer and closer as the chamber pressure $(\sigma_3)$ increased. For high values of $\sigma_3$, no crack propagation was observed.

![Triaxial tests results for a primary crack inclination angle $\alpha=45^\circ$.](image)

*Figure 26. Triaxial tests results for a primary crack inclination angle $\alpha=45^\circ$.***
4.8 UNIAXIAL DYNAMIC LOADING TESTS ON FISSURED SPECIMENS

A total of 101 dynamic uniaxial compression tests were performed on specimens with different primary crack inclination angles $\alpha$, moisture contents, and loading frequencies. The crack propagation threshold was established through these tests. In addition, the accumulated strains during the dynamic loading stages were also studied.

4.8.1 Crack propagation threshold derived from dynamic uniaxial compression tests

The cyclic stress ratio was earlier defined as the ratio between the applied dynamic vertical stress and the ultimate monotonic compression strength of a similar specimen (with the same water content and the same primary crack inclination). The purpose of the dynamic loading test was to find the threshold load (fraction of the static load) at which no crack propagation occurred under dynamic loading conditions regardless of the number of applied cycles.
The dynamic uniaxial loading tests were next developed for specimens with a single crack having inclination angles (\(\alpha\)) of 0°, 15°, 30°, 45°, 60°, and 75° and a loading frequency of 1 Hz. Additional tests were done using specimens with \(\alpha=30^\circ\) and frequencies of 0.5 and 2 Hz. Figure 28 to Figure 33 show the obtained results for a frequency of 1 Hz; in these figures, the cyclic stress ratio is expressed as a function of the water content in the specimens. The dark points in these figures represent the cyclic stress ratios \((r_d)\) for which crack propagation occurred; the clear points represent the cases for which no crack propagation was observed after 7200 cycles of load applications.

The general trend observed in these tests was that the crack stability threshold is almost constant regardless of the moisture content for water contents of less than 26%. For water content above 26%, the crack stability threshold diminishes noticeably.

Figure 34 compares the threshold value \((r_d)\) for all the primary crack inclination angles that were used. In this figure, \(r_d\) is shown as a function of the water content in the specimens. Depending on the value of \(\alpha\), the threshold value was found to be between 0.42 and 0.70 for moisture contents below 26%. The weakest crack resulted from one with \(\alpha=30^\circ\). The threshold diminishes dramatically for water contents higher than 28%; at a moisture content of 30%, the threshold was found to be as low as 0.15 for all inclinations of cracks.

The explanation and the analysis of these particular findings will be discussed later. It remains an interesting question why cracks inclined 30° or 15° with respect to the horizontal can be as weaker as or weaker than cracks inclined 45° (see Figure 35). On the other hand, the reduction in the crack stability threshold for high moisture contents is very significant. Complementary laboratory tests as well as theoretical and numerical analyses were done in
order to arrive at a satisfactory explanation of the dynamic behavior of fissured unsaturated clay specimens.

Figure 28. Crack stability threshold for specimens having a horizontal primary crack ($\alpha=0^\circ$).

Figure 29. Crack stability threshold for specimens having a crack with an inclination angle $\alpha=15^\circ$. 

Figure 29. Crack stability threshold for specimens having a crack with an inclination angle $\alpha=15^\circ$. 

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Figure 30. Crack stability threshold for specimens with a crack having an inclination angle $\alpha=30^\circ$.

Figure 31. Crack stability threshold for specimens with a crack having an inclination angle $\alpha=45^\circ$. 
Figure 32. Crack stability threshold for specimens with a crack having an inclination angle $\alpha = 60^\circ$.

Figure 33. Crack stability threshold for specimens having a crack with an inclination angle $\alpha = 75^\circ$.
Figure 34. Crack stability threshold ($r_d$) for uniaxial compression tests on specimens having different crack inclination angles $\alpha$.

Figure 35. Relationship between crack stability threshold and primary crack inclination angle ($\alpha$) for moisture content of 15%.
4.8.2 Fatigue of fissured specimens

The majority of specimens subjected to cyclic stress ratios above the threshold level developed a secondary crack caused by fatigue of the clay. Figure 36 shows the fatigue curve obtained from specimens having a primary crack at $\alpha=45^\circ$. The dynamic numerical analyses with DEM were calibrated using this curve.

![Figure 36. Relationship between cyclic stress ratio $r_d$ and number of cycles applied to produce crack propagation by fatigue of the clay for a primary crack inclination angle $\alpha=45^\circ$ and moisture contents of less than 26% (complete and incomplete pendular states).](image)

4.8.3 Dynamic strain during uniaxial compression tests of fissured specimens

Figure 37, below, shows a typical stress-strain curve obtained a dynamic load testing of a fissured specimen having a primary crack inclination angle $\alpha=30^\circ$ and a moisture content of 14%. The axial strains that occurred in the specimens as a response to the dynamic loading conditions were identified as elastic strains, plastic strains and total strains (Figure 38). Elastic strains in the specimen correspond to the recovered strain in each individual hysteresis loop.
Plastic strains, by contrast, are permanent and do not reverse when the cycles are applied. Total strains are the elastic strains plus the plastic strains for each applied cycle.

Figure 37 Stress-strain relationship for a specimen under dynamic uniaxial load. The moisture content of the sample is $\omega=14\%$, and the primary crack is inclined at $\alpha=30^\circ$. Only some initial and intermediate cycles are shown.
Figure 38. Elastic, plastic and total strains for a hysteresis loop $n$.

Figure 39 shows the accumulated strains during two typical dynamic uniaxial compression tests. The upper part of the figure shows the response of a specimen in which cracks did not propagate even after 15000 loading cycles were applied. In this case, the accumulated strain reached a maximum value, after which the sample hardened. It seems that if more cycles had been applied, the crack wouldn’t have propagated either since the elastic strain remained almost constant during the test.

The lower part of Figure 39 below, shows the strains in a typical specimen, in which the crack propagated after 3660 cycles. The accumulated total and plastic strains increased continuously with the application of cycles.
Figure 39. Accumulated axial strains during two different dynamic uniaxial compression tests. 
(a) The crack did not propagate, (b) the crack propagated. \( r_d \) is the cyclic stress ratio.

4.8.4 Threshold as a function of the loading frequency

The uniaxial dynamic testing for the tests mentioned above was done using a loading frequency of 1 Hz. However, additional tests were completed using loading frequencies of 0.5 and 2.0 Hz. Those specimens had a primary crack inclined at \( \alpha = 30^\circ \) and a water content of 15%. Figure 40
shows the results obtained. The crack propagation threshold was the same as in the case for the 1 Hz frequency. This means that the loading appears to have no influence on the crack propagation threshold.

Figure 40. Crack propagation threshold as a function of the water content and for different loading frequencies.

4.9 BIAXIAL DYNAMIC LOADING TESTS ON FISSURED SPECIMENS

The triaxial dynamic loading tests were done on specimens having a crack inclined at 45° with respect to the horizontal and a water content of ω=15%. However, for the biaxial tests, the threshold is defined as the ratio between the applied intermediate dynamic deviator stress (σ₁-σ₂)d and the ultimate deviator monotonic compression strength (σ₁-σ₂)u of a similar specimen (with the same water content and the same primary crack inclination). The purpose of the dynamic loading test was to find the threshold load (fraction of the static load) at which no crack propagation occurred under dynamic loading conditions regardless of the number of applied cycles.
The biaxial dynamic loading tests were performed using the maximum capacity of the available equipment to apply lateral loads. The maximum intermediate stress used for the tests was 70 kPa. Figure 41 below shows the cyclic stability threshold $r_d$ as a function of the intermediate chamber pressure $\sigma_2$. As can be noted from this figure, the intermediate stress have any apparent influence on the threshold. For the biaxial tests, the threshold is very similar to that of the uniaxial compression tests. The equipment used did not allow any significant difference to be found. It’s possible that the application of higher lateral loads would produce such a difference.

Figure 41. Crack stability threshold from biaxial tests for specimens with $\omega=15\%$ and primary crack inclined at $\alpha=45^\circ$.

4.10 TRIAXIAL DYNAMIC LOADING TESTS ON FISSURED SPECIMENS

The triaxial dynamic loading tests were done on specimens having a crack inclined at 45° with respect to the horizontal and a water content $\omega=15\%$. Figure 42 below represents the cyclic stability threshold $r_d$ as a function of the chamber pressure $\sigma_3$. For the triaxial tests the threshold...
was defined as the ratio between the applied dynamic deviator stress \((\sigma_1-\sigma_3)_d\) and the ultimate deviator monotonic compression strength \((\sigma_1-\sigma_3)_u\) of a similar specimen (with the same water content and the same primary crack inclination). As with the biaxial tests, the purpose of the dynamic loading test was to find the threshold load (fraction of the static load) at which no crack propagation occurred under dynamic loading conditions regardless of the number of applied cycles.

As can be seen from this figure, the threshold increased as the confining pressure \(\sigma_3\) increased. For \(\sigma_3>420\) kPa, no crack propagation occurred because the primary crack closed after some cyclic loading was applied.

![Figure 42. Crack stability threshold from triaxial tests for specimens with \(\omega=15\%\) and primary crack inclined at \(\alpha=45^\circ\).](image)

For \(\sigma_3 > 420\) kPa the crack closed and didn’t propagate.

Note: For all tests, the air pressure in the clay was, \(\sigma_a=0\).
5.0 LEFM THEORY AS APPLIED TO THE ANALYSIS OF CRACK PROPAGATION

According to the Linear Elastic Fracture Mechanics (LEFM) theory, a crack or fissure in clay can be stressed in three different modes illustrated in Figure 43 (Vallejo, 1994). With Mode I, the stress normal to the crack walls produces a different type of cracking in which the disarrangements of the crack surfaces are perpendicular to the plane of the crack. With Mode II, a type of cracking is produced by shear stresses along the crack plane causes the walls of the crack to slide with the crack plane. With Mode III, a tearing and cracking is caused by out of-plane shear stresses. Cracks can propagate in materials as a result of one or more of these modes (Vallejo, 1994).

Covarrubias (1969) used the three modes of fracture to investigate the propagation characteristics of cracks in cohesive materials used in earth barriers. The propagation of the cracks was found by Covarrubias to be result from the tensile stresses normal to the plane of the cracks (mode I type of loading). More recently, Lee et al. (1988), Fang et al. (1989), and Morris et al. (1992) have used LEFM theory to analyze the crack propagation in solids that result from tensile stresses (mode I type loading).
Vallejo (1985, 1986, 1987, 1988a, 1989) used the principle of LEFM theory to explain the failure mechanisms of rigid fissured kaolinite clay samples that had been subjected to compression and direct shear stress conditions. Failure resulted when stress concentrations induced by the tips of the fissures caused the propagation and interaction of the fissures in the samples. The fissures in the samples were subjected to a combination of Mode I and Mode II types of loading. Saada et al. (1985, 1994) applied LEFM theory to normally and over-consolidated clays with cracks tested under a combination of normal and shear loads (Modes I and II type of loading). Saada et al (1994) determined that the degree of consolidation as well as the degree of anisotropy of the clay samples effect the way the cracks propagated.

Figure 43. The three modes of cracking (Vallejo, 1994).
5.1 STRESSES AROUND A CRACK TIP

According to LEFM, the tangential stress ($\sigma_\theta$), the radial stress ($\sigma_r$) and the shear stress ($\tau_{r\theta}$) in the material located in the vicinity of a crack subjected to a mixed mode type of loading (Mode I plus Mode II) can be obtained from the following formulas (Vallejo, 1994)

\[
\sigma_\theta = \frac{1}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left[ K_I \left( \frac{\theta}{2} - \frac{3}{2} K_{II} \sin \theta \right) \right]
\]

\[
\sigma_r = \frac{1}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left[ K_I \left( 1 + \sin \frac{\theta}{2} \right) + \frac{3}{2} K_{II} \sin \theta - 2 K_{II} \tan \frac{\theta}{2} \right]
\]

\[
\tau_{r\theta} = \frac{1}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left[ K_I \sin \theta + K_{II} (3 \cos \theta - 1) \right]
\]

In the equations above, $r$ is the radius between the tip of the crack and a point in the clay surrounding the crack where the stresses are being measured, $\theta$ is the angle that the radius $r$ makes with the axis of the crack.

Figure 44, and $K_I$ and $K_{II}$ are the stress intensity factors for an open crack under Mode I or Mode II type of loading (Figure 43). The stress intensity factors (Vallejo, 1994) are represented as follows:

\[
K_I = 1.1215 \sigma_n \left( \frac{\pi c}{2} \right)^{1/2}
\]

(10)

and

\[
K_{II} = 1.1215 \tau \left( \frac{\pi c}{2} \right)^{1/2}
\]

(11)
Figure 44. a) Tangential, radial, and shear stresses around a crack tip. b) Crack propagation angle $\delta$.

where $\sigma_n$ is the normal stress that acts perpendicular to the plane of the open crack, $\tau_{r\theta}$ is the shear stress that acts parallel to the crack, and $c$ is the semi-length of the crack.

The above mentioned equations, formulated with LEFM were implemented through an electronic sheet using Mathcad (MathSoft Engineering & Education, Inc, 2001). Appendix A contains the detailed description of steps included in the electronic sheet. Figure 45 shows the isobars for the tangential stress $\sigma_\theta$ normalized with respect to an arbitrary stress. Positive isobars indicate compressive stresses and negative isobars indicate tensile stresses. Compressive stress concentrations can be seen on the right side of the crack tip and tensile stress concentrations on the left side.
Figure 45. Normalized tangential stress $\sigma_\theta$ around a crack tip; derived using LEFM. The stresses were normalized with respect to an arbitrary compression stress. To increase clarity of the image, the stresses inside the gray circle are not shown.

5.2 PRINCIPAL STRESSES

The principal stresses can be deducted from the tangential, radial, and shear stresses presented above. The maximum and minimum principal stresses are calculated as follows:

$$\sigma_1 = \frac{(\sigma_\theta + \sigma_r)}{2} + \sqrt{\left(\frac{\sigma_\theta - \sigma_r}{2}\right)^2 + (\tau_{r\theta})^2}$$  (12)

$$\sigma_3 = \frac{(\sigma_\theta + \sigma_r)}{2} - \sqrt{\left(\frac{\sigma_\theta - \sigma_r}{2}\right)^2 + (\tau_{r\theta})^2}$$  (13)
The center \( p \) and the radius \( q \) of the Mohr’s circle that represents the stresses at a point are calculated as

\[
p = \frac{(\sigma_1 + \sigma_3)}{2} \quad \text{(14)}
\]
\[
q = \frac{(\sigma_1 - \sigma_3)}{2} \quad \text{(15)}
\]

5.3 DIRECTION OF CRACK PROPAGATION

Erdogan and Sih (1963) have proposed the hypothesis that crack extension in fragile materials takes place in a direction in which \( \sigma_\theta \), given by Equation 7, reaches its maximum value – i.e., that crack will take place in a radial direction from the crack tip and that the direction of crack growth is normal to the direction of the maximum tangential stress \( \sigma_\theta \) (Vallejo, 1994). This theory is known as the maximum tangential stress criterion (MTS).

Hence the direction of crack propagation taking place when \( \theta \) reaches a value equal to \( \delta \) is given by

\[
\frac{d\sigma_\theta}{d\theta} = 0 \quad \text{(16)}
\]

and

\[
at \quad \theta = \delta \quad \frac{d^2 \sigma_\theta}{d\theta^2} < 0 \quad \text{(17)}
\]

where \( \delta \) is the value reached by \( \theta \) when crack propagation takes place (Figure 44).

After manipulating the above equations, the direction of crack propagation \( \theta \) can be obtained from the following equation
After replacing the values of $K_I$ and $K_{II}$ in the equation above, the relationship between $\delta$ and $\alpha$ is obtained

$$\cot \alpha \times \sin \delta + (3 \cos \delta - 1) = 0$$  \hspace{1cm} (19)

The direction of crack propagation follows the tensile concentration stresses shown in Figure 45 below. The relationship represented by the above equation is shown in Figure 46 below. Equation 18 applies to an open crack for which the stress intensity factors $K_I$ and $K_{II}$ are involved. If the crack closes, the stress intensity factor, $K_I$ becomes equal to zero (Broek, 1984), and Equation 19 can be written as

$$3 \cos \delta - 1 = 0$$  \hspace{1cm} (20)

*Figure 46. Secondary crack inclination angle $\delta$ predicted by the MTS criterion.*
6.0 TOOLS USED FOR NUMERICAL ANALYSIS OF CRACK PROPAGATION

In addition to the theories available for the analysis of crack propagation in clays, two geotechnical numerical analysis tools were applied to analyze specific aspects of the specimens having cracks: the Discrete Element Method (DEM) implemented using the computer program PFC$^{2D}$, and the Finite Difference Method (FDM) implemented using the computer program FLAC. Both software packages were supplied by the Itasca Consulting Group (2002, 2003).

6.1 DISCRETE-ELEMENT METHOD

The Discrete Element Method (DEM), developed by Cundall and Strack (1979), has been used extensively to study the behavior of materials such as granular soils and rocks when they are subjected to static or dynamic loads. The DEM has especially been used to study the response to loading of unbonded granular materials (Lobo-Guerrero and Vallejo, 2005). This method has also been used to simulate continuum materials such as rocks and clays if contact bonds between the DEM particles are implemented (Itasca Consulting Group, 2002).

Respected of the history, and recognizing that DEM has, it seems, likely never been used for this purpose, it was however used in this research in the study of crack propagation in stiff fissured clays. For each bond, PFC$^{2D}$ uses normal tensile and shear maximum loads as well as normal and shear stiffness, as it is shown in Figure 47 below (Itasca, 2002).
In general when a contact bond breaks, a separation between particles develops. This breakage takes place when the induced tensile force exceeds the tensile force that the bond can tolerate and a crack forms. If this bond was near the original crack in a simulated crack specimen, the original crack propagates in the location of the broken bond.

PFC\textsuperscript{2D} comes with a library with several functions that are very useful not only for solving specific problems but also for deriving new functions using the computer language FISH. In this study several subroutines were developed using FISH for the generation of the specimens with cracks and to simulate the dynamic loading of the specimens. These subroutines are included in the Appendix B.
6.2 FINITE-DIFFERENCE METHOD

In the field of geotechnical engineering differential equations have been used widely by researchers to find the theoretical solutions to many problems related to physics such as those dealing with the distribution of stresses, hydraulic flow, heat transfer, wave propagation, etc. Differential equations can be solved relatively easily using common mathematics when they are applied to one-dimensional problems (e.g., water flow in a permeameter). However, when they are applied to solve bi-dimensional problems that have different boundary conditions, the solutions to differential equations become very difficult. The finite difference method serves as the basis of numerical analysis tools which are used to solve numerically differential equations.

In this research, the computer program FLAC (Itasca Consulting Group, 2003) was used to study the fissured specimens. The program applies the FDM to solve many problems in engineering and allows the analysis of coupled problems such as the stress distribution and the flow of water in soil masses.
7.0 ANALYSIS OF THE BEHAVIOR OF FISSURED SPECIMENS SUBJECTED TO STATIC AND DYNAMIC LOADING

From the findings of static and dynamic loading tests of fissured unsaturated kaolinite clay specimens used in this research the following findings are of particular interest.

a) In the graph that shows the relationship between the crack propagation strength of the specimens $\sigma_c$ and their moisture content, three distinct zones were identified (see Figure 18). These zones are also visible in the ultimate strength vs. moisture content graphs (see Figure 15).

b) The most widely accepted theory for predicting the secondary crack propagation angle in fissured clays is the Maximum Tangential Stress criterion (MTS), proposed by Erdogan and Sih (1963). However, for primary crack inclination angles ($\alpha$) higher than $45^\circ$, this theory predicts propagation angles that are higher than laboratory test results (see Figure 46) (Vallejo et al., 1995).

c) As earlier noted the secondary cracks that propagates from the tips of the pre-existing primary crack was found to follow a direction that is parallel to the applied maximum compression stress (see Figure 21). This behavior has been observed before in other studies, but up to now, there has been no theory to explain the crack propagation path found in the specimens.

d) The cyclic crack stability threshold ($r_d$) revealed that the weaker specimens are those having a primary crack inclination angle $\alpha=30^\circ$ (see Figure 34).
e) The cyclic crack stability threshold was found to be closely related to the moisture content of the specimens. For specimens having water contents below 26%, the threshold was found to be almost constant, but for those over 26%, the threshold diminished in proportion to the moisture content – or – diminished as the moisture content increased.

To explain the above findings, it was necessary to develop an additional group of laboratory tests and new theoretical and numerical analyses. It was hoped that such new tests would support the findings a-e noted above. The laboratory tests included direct tensile tests as well as series of mixed direct tensile and shear tests (DTS) for which a new apparatus was developed. The laboratory tests were complemented by three-dimensional (3D) consolidation tests done using a second piece of equipment that was also specially developed for this research.

Combining the LEFM-derived theoretical analyses of stress concentrations around crack tips, numerical analyses with PFC\textsuperscript{2D} and FLAC, and numerous laboratory test results, a new criterion for predicting the crack propagation angle was developed. This will be discussed in more detail later.

**7.1 STUDY OF RELATIONSHIP BETWEEN $\sigma_c$ AND WATER CONTENT**

To analyze the three zones that were identified in the graph relating crack propagation strength $\sigma_c$ to moisture content (see Figure 18), new concepts regarding the mechanics of unsaturated soil mechanics are introduced and some results of the additional laboratory tests program developed for this research are presented.
7.1.1 Air and water distribution in pores of unsaturated clays

The air and water distribution in the pores of unsaturated clays can be used to explain the behavior of those clays when subjected to changes in stresses. German (1989) identified three forms of liquid distribution in the pores between packed particles: saturated-funicular and pendular states as shown in sections a-c of the Figure 48, respectively. It can be seen that all the voids are filled with liquid in the saturated state (Figure 48a). Air bubbles are present in the voids in the funicular state, and the liquid phase is continuous (Figure 48b). There is no continuity in the water phase in the pendular state (Figure 48c). A ring of liquid (a capillary neck) exists at the contact points between neighboring particles, but the rings do not touch one another (German, 1989).

![Figure 48. Stages of water distribution into a group of particles. a) Saturated state. b) Funicular state: air bubbles penetrate into the soil. c) Complete Pendular state: all contacts with meniscus. d) Partial Pendular state: some liquid bridges break.](image)
Theoretical analyzes from Pierrat and Caram (1997) and Hotta et al (1974) show that the capillary force between two particles that are not in physical contact can drop to zero if the water volume in the capillary neck is reduced but the distance between particles is greater than zero. On the other hand, using the scanning force microscopy (SFM), He et al (2001) found that the capillary neck between particles in physical contact can disappear altogether when the relative humidity (RH) of the vapor phase around the capillary neck is lowered sufficiently for below a critical value. As a result of these conditions under which capillary forces between particles can be lost, an additional stage was added to Figure 48 in order to show a partial pendular state for which there are disconnected liquid contacts within the group of particles.

As is mentioned later, each the above-mentioned states of water distribution inside the clay pores affects the overall strength of the clay. The clay behaves in different ways depending on distribution of air and water. For example, Vallejo (1989) mentions that when using uniaxial compression tests on brittle kaolinite clay specimens prepared in the laboratory, a reduction in the strength was found when the water content in the samples was reduced below $\omega=10\%$. Vallejo (1989) explained this behavior as a consequence of there have been more and more contacts between particles in the soil that lost their liquid contacts as the water content was reduced. His findings closely match the theoretical and experimental results mentioned above.

7.1.2 Inter-particle forces

7.1.2.1 Saturated-funicular state (accepted theory). This section will offer an overview of accepted theory regarding inter-particle forces caused by capillarity and then present a new concept regarding the behavior of specimens in partial pendular state when subjected to static and dynamic loading. As can be seen in Figure 48a, in a saturated state, the inter-particle attractive forces are due to the negative pore water pressure ($uw$) in the pores, and the Terzaghi’s
effective stress principle applies (Terzaghi and Peck, 1948); correspondingly, the inter-particle attractive force increases as the negative pore water pressure increases. The effective stress is expressed as

$$\sigma' = \sigma - u_w$$  \hspace{1cm} (21)

If the level of matric suction \( s = u_a - u_w \) directly above the air entry value (AEV) is increased, air bubbles penetrate the soil (Fredlund and Rahardjo, 1993) and a funicular state (Figure 48b) is produced. In such cases, there is continuity in the water phase, and there remains an effective stress in the soil, caused by the combined effects of the total stress and the confinement of the matric suction; the equation for this relationship is given by

$$\sigma' = \sigma + s$$  \hspace{1cm} (22)

When the air pore pressure \( u_a = 0 \) then Equation 22 is equal to Equation 21. As was seen before, when the soil is dried, the actual suction in the pores increases as a consequence of the water content having been reduced (see the water retention curve in Figure 13). In the saturated-funicular state, when the water content is reduced, both the effective stress increase and the shear strength increase as well.

7.1.2.2 Complete Pendular State (accepted theory). In a complete pendular state (Figure 48c) the water phase in the soil is not continuous and the liquid bridges don’t touch each other. As a result the effective stress cannot simply be attributed to the suction pressure in the meniscuses of the soil.

Several researches have analyzed theoretically the capillary force developed between two particles in the complete pendular state. Figure 49 offers an overview of studies of contact.
<table>
<thead>
<tr>
<th>CASE DESCRIPTION</th>
<th>AUTHORS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two equal-sized spheres in contact</td>
<td>Cho and Santamarina (2001)</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Two equal-sized, separated spheres</td>
<td>Pierrat and Caram (1995)</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Two different-sized spheres in contact</td>
<td>Mehrotra and Sastry (1979)</td>
</tr>
<tr>
<td></td>
<td></td>
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<tr>
<td>Two unequal-sized separated spheres</td>
<td>Hotta, Takeda and Inoya (1974)</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Sphere and a cone in contact</td>
<td>Ho et al (2000)</td>
</tr>
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<td></td>
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</tbody>
</table>

Figure 49. Theoretical cases for capillary contact force between two particles.
forces as they apply to spherical, planar, and conic-shaped particles. The various authors have used different mathematical approaches to reach their respective solutions, but the shared of their analyses is that the capillary force between two particles in a true-contact state tends to remain constant when the suction pressure in the meniscus is increased (identical results are obtained when the capillary neck volume is reduced and/or if the soil is dried).

The simplest case of capillary force is the force two equal-sized spheres in contact (Figure 50), in which the capillary force \( F \) that the meniscus in a pendular state produces on the particles results from two forces. One part of the attractive force is that produced by the unbalanced pressure (i.e., matric suction, \( s=\bar{u}_{a}-\bar{u}_{w} \)) acting on the cross sectional area of the meniscus. The second part is that produced by the liquid surface tension acting along the perimeter \( 2\pi r_2 \) of the capillary neck (Cho and Santamarina, 2001); \( F \) is given by

\[
F = s(\pi r_2^2) + T_s(2\pi r_2) \tag{23}
\]
The combined effect of the matric suction \( s \) and the surface tension \( Ts \) of the water (0.0727 N/m at 20ºC) is represented in Figure 51 below. The suction \( s \) and the surface tension \( Ts \) are related by the Laplace equation as follows

\[ s = Ts \left( \frac{1}{r_1} + \frac{1}{r_2} \right) \]

(24)

Figure 51. Liquid bridge between two equal-sized particles: a) Complete meniscus. b) Cross section showing the surface tension force acting on the perimeter. c) Suction pressure acting on the cross sectional area of the meniscus.

The Equation 23 shows a force \( F \) that remains constant as the actual suction pressure in the capillary neck is increased (as a result of the water content being reduced). If the contact forces inside the soil are constant when the actual suction pressure is changed, then the strength of the material should also be constant. As will be mentioned later, a constant uniaxial compression strength was observed in the kaolinite clay for an interval of water contents in the soil (different
water contents in the soil correspond to different matric suctions). This expected behavior coincides with earlier theoretical analyses of the complete pendular state.

### 7.1.2.3 Partial Pendular State (new concept)

Figure 52a below show two particles separated a distance d. Pierrat and Caram (1997) derived a mathematical expression to represent the capillary force for this case, which leads to the relationship shown in Figure 52b. If the two particles are in contact, the distance d is zero and the force F reaches the maximum value possible. If, however, particles are not in contact then the contact force can be zero for low values of the contact angle \( \theta_c \) (due to a low volume of the capillary liquid). As a result, the liquid bridge between particles can disappear and correspond to that partial pendular state. In addition, when capillary contact forces between particles are lost, the strength of the material of which the particles are part would diminish. Laboratory results shown below confirm that if the water content in the clay is reduced below a certain value, a resulting reduction in the overall strength can be expected. As was mentioned before, similar results were obtained by Vallejo (1989).

Another configuration that produces notable results is the one of a sphere and a plate (Figure 53), where once again, the capillary force can be deduced as a function of the suction pressure and the geometry of the liquid bridge. This would be the case of two plates having corner-to-face contact or edge-to-face contact. In cases like these, the capillary contact force can be zero even if the particles are separated from each other.

Contrary to the conclusion drawn from earlier research on this topic, it now appears that it is possible for the capillary neck between two particles in contact to disappear under two conditions: if there is a distance between the particles (as was explained above) or if the relative humidity \( (RH) \) in the vapor phase is low enough to produce this unexpected effect.
Figure 52. a) Two particles separated a distance d. b) Effect of the separation d on the capillary force (adapted from Pierrat and Caram (1995)).

Figure 54, below, shows the relationship found by He et al (2001) of the pull-off force needed to separate two particles on the RH of the air surrounding the capillary neck; four zones can be identified from the relationship. In Zone 1, the force between particles increases as the relative humidity decreases. The force reaches a maximum and remains almost constant in Zone 2 and reduces progressively in the Zone 3 until the capillary neck disappears (in Zone 4). For the final residual condition, the inter-particle forces are dominated by the van der Waals interactions, and there is not capillary action after the capillary neck breaks.
The above-mentioned conditions, which produce zero capillary force between particles will lead to a reduction in the equivalent effective stress in the surrounding soil that would, in turn, translate into a reduction in the strength and an increase in the volume of the soil.

As an illustration of the above, a simple test was done using the triaxial chamber (see Figure 55, below). A drop of water was inserted into the tip of the loading piston, and a piece of foam was suspended and maintained in contact with the water; then, the chamber was hermetically sealed, and a constant RH maintained (producing constant suction pressure in the foam). These conditions were maintained for 48 hr and then dry air was allowed to flow into the camber to reduce the RH. As a result, the capillary contact force was reduced, and the foam fell.
Figure 54. Relationship between the pull-off force (PF) between two nanoparticles having a capillary neck and the relative humidity of the vapor phase (RH) (Adapted from He et al, 2000).

Figure 55. Simple test using a triaxial chamber to demonstrate that inter-particle capillary force lessens or disappears altogether when RH is reduced.

(a) A drop of water in the piston tip that will provide the capillary forces.

(b) Foam suspended during 48 hr with suction pressure remaining constant in the triaxial chamber.

(c) The foam fell due to contact breakage after water evaporated (the suction was increased).
7.1.2.4 **Plate stacks.** Montmorillonite, kaolinite, and illite are clay minerals composed of plate-shaped particles (Lambe and Whitman, 1969). For parallel face-to-face plates, Cho and Santamarina (2001) developed a theoretical expression for the capillary force that depends on the matric suction pressure and the specific surface of the plates. According to them, the capillary force increases progressively as the suction pressure increases. This would occur with, e.g., a disperse structure in a clayey soil and could explain the high strength of high plasticity fine clays having low water content.

![Figure 56. Electron microscope picture of a kaolinite clay (adapted from Keller, 1985).](image)

The contact force between two plates arranged face to face can be very high, and depends mainly on the suction pressure in the capillary liquid and on the specific surface of the plates (Cho and Santamarina, 2001). The higher these two parameters, the greater the capillary force. However, clayey soils are composed of plates that can be arranged face to face and...
comprise/form stacks, as Figure 56 shows. The contact between two different plate stacks can be simulated by sphere-to-sphere or sphere-to-plate contacts, and lead to similar relationships, as *Figure 52* and *Figure 53* show. This suggests that the capillary force between two stacks of plates would not be as strong as between two individual plates resting face-to-face; in addition, as explained above, the capillary force in clays can be very low or even zero if the amount of water in the capillary neck between stacks is reduced.

7.1.2.5 **SEM photographs of the kaolinite clay used in the research.** Scanning Electron Microscope (SEM) photographs were taken of samples of a number of kaolinate clay specimens used in this research. *Figure 57*, below, shows plate stacks of different sizes, which conform the clay. The differences in plate size, like the stack-to-stack contact mentioned earlier, could greatly influence the amount of capillary force between plates. This difference in plate size is illustrated in *Figure 6*, below, which shows large plates surrounding small plates. If, in fact, this apparent configuration is accurate, the distance between the large plates would be constant, and a pendular state could also exist simultaneously. In such a state, if the soil were dried, the resulting capillary neck could produce a capillary force of zero between particles even though the large-plate-to-small-plate contact would still have a functioning capillary force.
Figure 57. SEM pictures of the kaolinite clay used in the research. Plate-shaped particles of different sizes can be observed (Courtesy of Dept. of Materials Science and Engineering, University of Pittsburgh).
7.1.3 Equivalent effective stress concept

The effective stress concept is useful for deriving the shear strength of saturated soils but is not longer applicable when the soil is in pendular states. The equivalent effective stress is now used instead to measure the effect of water distribution patterns in clay and, as such, relates to the effect of suction within the clay.

For two particles with a capillary force acting between them, the equivalent effective stress $\sigma'_{eq}$ due to capillarity can be defined as the capillary force between the particles ($F$) divided by the effective afferent area that corresponds to that contact ($A_{eff}$). The equivalent effective stress becomes

$$\sigma'_{eq} = \frac{F}{A_{eff}}$$  \hspace{1cm} (25)

One of the problems associated with the above is that the force $F$ depends not only on the surface water tension ($T_s$) but also on geometric conditions from each water bridge and from each particle in the soil; however, this information is not easy to obtain for a group of particles. Knowing the actual suction pressure in the meniscuses of a soil is not enough to establish the
shear strength of the entire group of particles; rather, the geometry of each meniscus in the soil, i.e., shape, arrangement, and distance of particles, is also needed.

Instead of using the actual matric suction pressure acting on the meniscuses in a soil to study the behavior of unsaturated soil, Atkinson and Nocilla (2003) proposed the concept of equivalent suction pressure ($s_{eq}$) or equivalent pore pressure ($-u_{eq}$), which represents the mean stress caused by the capillary forces between individual particles.

Indirect Brazilian tension tests and direct tensile tests were also used to measure the equivalent suction pressure in the clay. As there is no confinement in these tests, the total stress was zero and the equivalent suction pressure ($s_{eq}$) was equal to the equivalent effective stress in the soil $\sigma_{eq}$.

The equivalent effective stress in a cross section of a soil is defined as

$$\sigma_{eq} = \frac{\sum F_c}{A}$$

(26)

where $F_c$ is the component of a capillary inter-particle force that is parallel to the considered direction of $\sigma_{eq}$ and $A$ is the total area of the section of the soil. When the equivalent effective stress is known, the shear strength can also be obtained. The shear strength, in such a case would be the equivalent effective stress multiplied by the coefficient of friction between the particles.

As the equivalent effective stress $\sigma_{eq}$ depends on the attractive force between particles, the three states of water distribution in the pores would react as follows:

i. for soil in the saturated-funicular state, the strength of a clay would increase as the water content is reduced.
ii. for soil in the complete pendular state, the strength of a clay would remain constant regardless of whether if the water content is changed or not, and

iii. for soil in the partial pendular state, the strength of a clay would diminish as the water content is reduced.

7.1.4 Direct and indirect tensile tests to understand how cracks propagate in clays

To determine the shear strength of unsaturated soils some authors have used the concept of equivalent suction pressure in soils determined through the indirect Brazilian tension tests (Atkinson and Nocilla, 2003; Vesga and Vallejo, 2005). This concept is very helpful in understanding the shear strength of soils subjected to suction. The theories and concepts of traditional unsaturated soil mechanics have been criticized due to their complexity (having too long equations in which too many variables are involved). However, by introducing the equivalent effective stress concept a new, simpler way to understand the behavior of unsaturated soils becomes possible. Using this concept, it is now possible to extend and apply the traditional effective stress concept proposed by Terzaghi (Terzaghi and Peck, 1948) more easily and to facilitate interpretations.

Even so, since cracks in clays propagate as a result of tensile stresses (Vallejo, 1987), it is also important to know the tensile strength ($\sigma_t$) of the clay having different moisture contents. Direct and indirect tensile tests were done to measure this.

The direct tensile tests were done on bowtie-shaped specimens. Figure 59, below, shows the dimensions of the specimens and describes the equipment that was created for this research. As can be seen in the figure the specimen is subjected to a tensile force through two Plexiglass clamps that are pulled apart by a tensile force actuator. Digital data regarding measurements of
Figure 59. Direct tensile test. (a) Dimensions of sample. (b) Direct tensile test frame.
force and deformation were obtained using the data acquisition system DATAQ ID-194 (Dataq Instruments, 2003) and stored in a computer. The measurements of tensile strength ($\sigma_t$) from the direct tensile tests were based on the ratio between the force applied to produce the failure of the specimen under tension and the cross-sectional area of the specimen neck.

The Brazilian test is an indirect type test that is used to determine the tensile strength of rocks (ASTM D3967-95a). This test uses a cylindrical sample of diameter $D$ and length $L$ that is diametrically compressed by two point loads ($P$). The tensile strength $\sigma_t$ of the cylindrical sample is obtained from $\sigma_t = 2P/\pi LD$.

Figure 60. Indirect Brazilian tension test.

An unconfined compression test machine was used to apply the compressive force; digital data of axial load and deformation were recorded using the acquisition digital data unit (ADU) at University of Pittsburgh.
The specimens for the direct and indirect tensile tests had moisture contents between 3% and 32%, with intervals of around 3%. The bowtie-shaped or cylindrical specimens were prepared following the same procedure for mixing and consolidation that had earlier been used for the prismatic specimens.

Figure 61, below, shows the results of the direct and indirect tensile tests, which measure the equivalent effective stress in clay having different water contents. In this figure, three clear zones can be identified as was the case in the test of uniaxial compression strength seen before. Zone I corresponds to the saturated-funicular state for moisture contents higher that 26%, Zone II corresponds to the complete pendular state for moisture contents between 12% and 26%, and the Zone III to the partial pendular state for moisture contents below 12%. The results show clearly that the equivalent effective stress concept adequately explains the strength of a clay.

Figure 61. Tensile strength of unsaturated kaolinite clay from indirect Brazilian tension tests and direct tensile tests.
The tensile strength obtained with the Brazilian tests was found to be lower than that obtained with direct tensile tests. Compressed zones develop on the top and the bottom of the sample, close to the contact areas with the bearing plates. As a result, both the height and diameter of the sample are reduced once the sample is subjected to tension. Figure 62, above, shows a ductile sample that failed to develop a tensile crack anywhere in its entire face.

It can be seen that with a corrected, reduced diameter, the resulting tensile strength increases proportionally. Figure 62 also compares the results of the direct (a) and indirect (b) tensile tests; the results of the direct tensile tests and the corrected Brazilian tests are close.

In order to study the effect of the loading rate, stress-controlled direct-tensile tests were performed using small load increments; each load level was maintained until no more deformation in the sample occurred. The tensile strength found was very similar to that obtained using high loading rates. These results indicate that it is possible to obtain the same equivalent effective stress faster and less expensive by doing direct tensile tests on kaolinite clay at high strain rates.

In this research, the equivalent effective stress is the equivalent confinement applied to samples during the uniaxial compression tests and would be equal to the minor principal equivalent effective stress applied \((\sigma_{3eq}=\sigma_i)\). The values of uniaxial strength presented in Figure 64, below, become the deviator stresses \((\sigma_u=\sigma_{1eq} - \sigma_{3eq})\), and the equivalent major principal stress can then be deduced as a function of the tensile strength \((\sigma_{1eq}=\sigma_u + \sigma_i)\); this interpretation makes it possible to obtain the \(q-p\) plot at failure (Figure 64) by using the following equations.
\[
p = \frac{\sigma'_1 + \sigma'_3}{2} = \frac{\sigma_u + 2\sigma_t}{2}
\]  
(27)

\[
q = \frac{\sigma'_1 - \sigma'_3}{2} = \frac{\sigma_u}{2}
\]  
(28)

The data points shown in Figure 65 were derived from the values of the uniaxial compression strength \((\sigma_u)\) and the correspondent tensile strength \((\sigma_t)\) at the same water content obtained from Figure 64. The equivalent angle of internal friction \((\phi_{eq})\) was established from the value of \(\alpha\) using the relationship \(\tan(\alpha) = \sin(\phi); \phi_{eq} = 27^\circ\). As Figure 65 indicates, this angle is very similar to the effective friction angle of the kaolinite clay used in the research \((\phi' = 25^\circ)\).

Figure 62. a) Result of a direct tensile test. b) Result of a Kaolinite clay specimen after an indirect Brazilian tension test.
Figure 63. Failed ductile specimen from an indirect Brazilian tension test.

Figure 64. Uniaxial strength ($\sigma_u$), equivalent effective stress or tensile strength ($\sigma_t$), and derived shear strength ($\tau_f$) of kaolinite samples.

d_{corr} = 0.6 \ d_0

d_{corr}: Corrected diameter

d_0: Sample of initial diameter
3.1.5 Three-dimensional consolidation test due to suction stresses

The compressibility of unsaturated soils is not a topic directly related with the subject of this research. However, three-dimensional (3D) consolidation tests were done on unsaturated specimens subjected to different levels of suction – since through these tests, it was possible to validate the equivalent effective stress concept described above. This concept is applicable to determine not only the shear strength but also the compressibility and consolidation of unsaturated clays. If the changes in strength of an unsaturated clay are due to changes in the equivalent effective stress (which measure the effect of water-distribution patterns in clay), those changes of $\sigma'_{eq}$ should also be reflected in void ratio changes. This hypothesis, which was earlier stated, was confirmed through the 3D consolidation tests.

The consolidation tests using prismatic kaolinite clay specimens were performed applying different suction levels inside the soil through a controlled drying process. Since the actual suction pressure in the clay acts isotropically, the 3D deformation of the specimens had to
be measured. A special but simple apparatus was made by this researcher in order to measure the changes of the dimensions of the sample during the test, through which the changes in the void ratio were calculated. The samples were prepared using the same procedure for the prismatic samples as was earlier used with the fissured specimens, but in this case, no significant reduction in the initial sample’s moisture content was allowed.

The specimen was placed in a 3D consolidation chamber that was developed by the researcher. Shown in Figure 66, below, the chamber. It consists on a hermetically closed plastic container 50 cm x 50 cm x 70 cm with two lateral sealable windows. In order to produce the desired reductions in the water content of the specimen, dry air was blown through the windows with a fan. The windows remained closed until the specimens reached equilibrium at each water content level that was induced.

![3D consolidation chamber](image)

Figure 66. a) 3D consolidation chamber used for the tests. b) Interior of the chamber and measurement devices.
The weight of the specimens was controlled with a 0.1 grams precision electronic balance. The deformation of the samples was measured using five LVDT strain gages (arranged next to each other in normal directions), which were connected to an acquisition data unit (ADU) for automatic readings. Each moisture level induced was maintained (with chamber windows closed) over enough time (24 to 36 hours) to produce constant dimensions throughout the sample.

7.1.5.1 Suction compressibility curve. To study the compressibility of the unsaturated kaolinite clay, the void ratio \(e\) was calculated using the phase relationships between unit weight \(\gamma\), moisture content \(\omega\), and specific gravity (Gs) (Terzaghi and Peck, 1948). The parameters \(\gamma\) and \(e\) were obtained from the specimen actual dimensions and weight, which were recorded as the specimen dried inside the chamber.

Figure 67 shows the compressibility graph for the kaolinite clay subjected to controlled drying process, beginning with an initial water content \(\omega=33\%\) and ending with a final water content \(\omega=3\%\). Reductions of the water content were allowed by decrements of between 1% and 2%. The figure was matched to the equivalent effective stress of the kaolinite as a function of its water content that was presented above. The equivalent effective stress was determined from the direct tensile tests.
Three zones can be identified in Figure 66. The void ratio (e) diminishes quickly in the saturated-funicular state (Zone I) due to the suction generated by the drying process; in this zone the equivalent effective stress increases as the water content is reduced.

In the complete pendular state (Zone II), the void ratio doesn’t change noticeably but reaches a minimum value ($e_{\text{min}}=0.78$) of a water content of approximately $\omega=25\%$. By definition, the shrinkage limit is the moisture content below which further loss of water by evaporation does not reduce its volume (Terzaghi and Peck, 1948); the results from this test support this statement completely. The constant void ratio in the clay coincides with an almost constant equivalent effective stress in Zone II.

In the partial pendular state characterizing Zone III (see Figure 67) the volume of the sample increased as the sample was dried. In other words –and contrary to the findings of most
studies on this topic, it was found that the clay expanded as a result of drying. Generally, the expansion of clays is associated with an increase in the water content but, as was found in this study, an expansion through a drying process can occur instead.

The above two relationships ($e$ vs $\omega$ and $\sigma'_{eq}$ vs $\omega$) which are shown in Figure 67 lead to the relationship between the void ratio and the equivalent effective stress shown in Figure 68, below. The void ratio diminishes as the clay is dried and then reaches a minimum value as the equivalent effective stress is increased to a maximum value. After the maximum equivalent effective stress is reached ($\sigma'_{max}$) and the drying process is continued, the $\sigma'_{eq}$ diminishes and the soil becomes over-consolidated by desiccation; thus the over-consolidation ratio becomes $OCR=\sigma'_{eq}/\sigma'_{max}$.

![Figure 68. Three-dimensional compressibility curve of unsaturated kaolinite clay subjected to a drying process.](image-url)
These findings validate very well the concept of the equivalent effective stress as it applies to unsaturated soils. The shear strength as well as the compressibility of the kaolinite can be estimated using this concept that can be applied in a similar way as Terzaghi’s effective stress principle.

7.1.5.2 Consolidation curves. Three typical consolidation curves obtained from the 3D consolidation test are presented in Figure 69, below; the top curve corresponds to the saturated-funicular state, the middle one to the complete pendular state, and the bottom one to the partial pendular state. In the saturated-funicular state, the specimen’s volume decreased continuously until equilibrium was reached at the end of consolidation; the highest reduction rate was obtained while drying was permitted (Figure 69a). In the complete pendular state, the sample’s volume increased while drying was permitted; after this time, the volume decreased again until equilibrium was reached; the net volume change was negligible (see Figure 69b, below).

When the drying process was started, the outside edges of the specimen dried first, with the interior drying more slowly as water drained to the exterior and then evaporated from the clay. At this point, it’s possible for a sample to simultaneously be in both a partial pendular state (close to its edges/contours) and a complete pendular state (in the interior). Surprisingly, it was noted that the soil in the partial pendular areas first expanded as it lost the liquid contacts (the equivalent effective stresses decreased) but then contracted again when the water moving from the center of the specimen reestablished some contacts that had previously been lost.

The volume increased continuously when the specimen was in the partial pendular state (Figure 69c). In this state, the water coming from the center of the specimen was not enough to reestablish capillary contacts and the equivalent effective stress decreased commensurate with corresponding increase in volume.
Figure 69. Consolidation curves of unsaturated kaolinite clay subjected to a drying process. Arrows indicate when drying stopped – and the point after which the sample's weight no longer changed.

This unexpected behavior is especially important for analyzing geotechnical projects involving shallow foundations as well as pavements on clayey soils. In these cases, atmospheric changes caused by precipitation, wind or vegetation – such as changes in relative humidity
changes due to evapo-transpiration – can produce important volume changes because of variations in equivalent effective stress within the soil. Simply stated, the clay could shrink or expand if the water content were reduced.

7.1.6 The three zones of capillarity

The three Zones I, II and III that were identified in the graph relating the crack propagation, stress $\sigma_c$, and moisture content of the fissured specimens correspond to a high degree to zones for the equivalent effective stress that is developed in the clay as a product of the saturated-funicular state, complete pendular state, and partial pendular state, respectively.

![Graph](image)

**Figure 70.** Uniaxial monotonic compression strength for pre-fissured specimens. (a) Crack propagation stress, $\sigma_c$. (b) Ultimate compression strength, $\sigma_u$. The crack inclination angle ($\alpha$) is measured with respect to the horizontal.
Figure 70 shows these graphs but also takes into account the states identified above. The results observed in the fissured specimens of kaolinite clay can be explained satisfactorily by using the equivalent effective stress concept.

The moisture content at the limit between the saturated-funicular state and complete pendular state is 26%. Since the 3D consolidation results that are presented in Figure 67 show that the void ratio in the clay is almost constant in the complete pendular state, this limit should match the shrinkage limit as it is defined by Terzaghi and Peck (1948). Table 1 shows that the shrinkage limit of the clay as determined in accordance with the ASTM standard is 24%. These results agree reasonably well.

The other water content limit that should be considered is the pendular limit, which is defined as the water content below which the clay lose capillary contacts between particles as the clay is dried. For the particular case of the kaolinite clay used in this research, the pendular limit is 12%.

### 7.2 STUDY OF THE CRACK-PROPAGATION

Theoretical and numerical analyses are presented below for the study of the crack propagation angle $\delta$ formed between a primary crack and a secondary propagated crack. The theoretical analyses were done using the Linear Elastic Fracture Mechanics (LEFM) and a new laboratory test called the Mixed Direct Tensile and Shear test (DTS). The numerical analyses were done using the Discrete Element Method (DEM).
7.2.1 Proximity-to-failure concept

Bourne and Willemse (2001) analyzed the brittle failure around faults using the proximity to failure concept. Figure 71, below, was prepared to illustrate this concept. Figure 71 shows a straight line failure envelope by shear (Mohr-Coulomb) and a straight line failure envelope by tension. In the same figure, an arbitrary Mohr’s circle that represents a state of stress at a point located close to the crack tip is shown. The state of stresses in the figure corresponds to a moment at which no failure has yet occurred and so the material is still continuous (i.e., no crack developed).

\[
\begin{align*}
v &= tp \sin(\phi) - q \\
w &= \sigma_t - \sigma_3 \end{align*}
\]

*Figure 71. Proximity-to-failure concept. v is the proximity by shear, and w is the proximity by tension.*

The proximity-to-failure concept proposed by Bourne and Willemse (2001) basically states that the failure of the material would occur by the failure mode having its failure envelope closest to the Mohr’s circle of stresses.
In Figure 71, the proximity to failure by shear is represented by the magnitude of the variable \( v \), and the proximity to tensile failure is represented by \( w \). The expressions for \( v \) and \( w \) are

\[
v = tp \times \sin(\phi) - q \tag{29}
\]

\[
w = \sigma_t - \sigma_3 \tag{30}
\]

The proximity-by-tension concept is very similar to the maximum tangential stress criterion (MTS) proposed by Erdogan and Sih (1963). In this concept, \( v \) can also be presented as a function of the cohesion intercept of the material. In this research, the proximity-to-failure concept was applied in order to predict the crack propagation angle formed between a primary crack and its secondary crack. The concept was also used to identify the effect of the inclination angle of the primary crack on the crack propagation stress \( \sigma_c \).

In this study, the proximity-to-failure concept was extended to the Mixed Tensile and Shear Stress (MTSS) criterion which enables better understanding of the behavior of fissured clays under compression. In order to apply this criterion, it was first necessary to determine the failure envelope of the clay when subjected to tension. This was done through the DTS tests described below.

### 7.2.2 Mixed direct-tensile and direct-shear (DTS) test

A series of DTS tests were carried out to determine the failure envelope of the kaolinite clay so that the proximity to failure concept could (then) be applied. Kezdi and Horvath (1973) presented some failure envelopes for soils subjected to low confining pressures (see Figure 72 below); in order to determine the envelope for the unsaturated kaolinite clay, the triaxial test results shown above (Figure 16) were complemented with the DTS test results below.
The Direct Tensile and Direct Shear (DTS) test apparatus was developed for the researcher at the University of Pittsburgh. It is a simple device which permits the mixed tensile and shear strength of clays to be measured. The DTS apparatus is composed of two clamps through which a transverse force is applied to a pre-tensioned, bowtie-shaped specimen. The transverse force applies a shear stress to the neck of the specimen, and the force is increased until failure occurs. Figure 73 and Figure 74 show the dimensions of the sample and the DTS equipment (see Appendix C also).

Figure 72. Failure envelopes used in soil mechanics (adapted from Kezdi and Horvath, 1973).
Figure 73. (a) Sample dimensions. (b) Loading clamps. (c) Direct tensile frame for DTS device.
Figure 74. Direct tensile and shear (DTS) apparatus (patent pending) developed in the University of Pittsburgh for this research. The Gauges are connected to a data acquisition system that send the digital information to the computer.

The pull-off force is applied by a tensile force hydraulic actuator as shown in Figure 73c, above. Figure 75, below, shows the loading stages of the DTS test. First (step a), the specimen is assembled and clamps are applied. Next (step b) a selected tensile force is applied to the specimen and maintained at a constant level. Finally (step c), a shear force is applied and increased until sample fails.
Figure 75. Loading stages in the DTS test: a) Specimen and clamps are assembled. b) Tensile forcing stage \(T\). c) Application of the shear force \(S_f\) to failure.

\[
\begin{align*}
\sigma_y &= \frac{T}{A_s}, \quad \sigma_x = 0 \\
\tau_{yx} &= \frac{S_f}{A_s} \\
A_s &= \text{Neck’s cross sectional area}
\end{align*}
\]

Figure 76. Stresses in the sample neck in the DTS test.
The dimensions of the failed specimen were measured after each test in order to obtain the cross sectional area and assess the applied normal and shear stresses. The tensile stress in the horizontal plane of the neck is given by \( \sigma_y = \frac{T}{A_s} \) and the shear stress is given by \( \tau_{yx} = \frac{S_f}{A_s} \), where \( A_s \) is the cross sectional area of the neck (see Figure 76). The normal horizontal is stress \( \sigma_x = 0 \) since there is not confinement pressure. Figure 77, below, shows the applied stresses represented in a Mohr’s diagram. The pole is located along the \( \tau \) vertical axis. This point will be very important in obtaining the theoretical failure plane inclination angle.

![Mohr's diagram representation of the applied stresses in the specimen's neck.](image)

**Figure 77.** Mohr’s diagram representation of the applied stresses in the specimen’s neck.

Figure 78, below, shows the relationship between the applied vertical stress and the shear stress at failure. A good linear relationship can be observed and also that as higher the tensile stress applied the lower the shear stress to produce failure.
Figure 78. Relationship between vertical normal stress ($\sigma_y$) and horizontal shear stress ($\tau_{yx}$).

The Mohr’s circles of stresses that are presented in Figure 79, below, show the four circles at failure close to tangency at a common point that equals the tensile strength of the material. This indicates that the shear stress applied at failure doesn’t, in fact, produce the failure by shear but rather the failure by tension. This is a very important finding since could be applied not only to the study of crack propagation in clays but to the study of other geotechnical problems, e.g., stability of slopes, foundations in previously excavated areas. More research can be developed using the DTS device to study the shear strength of soils under tension, a topic no considered before in the practice.
Five constant water content triaxial tests (in which air pressure inside the soil was equal to the atmospheric pressure) were done on prismatic kaolinite samples prepared with the same procedure described above for the bow-tie shaped specimens described above. The results from these tests and from the DTS tests are presented in Figure 80, below. In total there are results from nine tests that closely match results for a failure envelope that is a combination of a straight (frictional) portion with an inclination angle equal to the internal friction angle of the kaolinite and a circular sector in the tension zone.

Figure 79. Mohr’s diagram representation of the stresses at failure in the neck of the specimen.
Figure 80. Mohr's diagram of four DTS tests and five constant water content triaxial tests done using an unsaturated kaolinite clay with water content \( w = 15\% \). The Mohr’s circles for DTS are those in which \( \sigma_3 \) is negative.

Vesga and Vallejo (2005) have shown that this kaolinite clay at a \( \omega = 15\% \) has a water distribution characteristic of complete pendular state, in which the water inside the soil is practically unmovable when stress increments are applied, and in which, therefore, no water pressure changes are expected during the tests. This similarity is the reason why the straight portion of the failure envelope (the frictional component) fits very well with the internal friction angle of the kaolinite.

Figure 81, below, shows the detail of this Mohr-Coulomb-Tensile failure envelope near the origin of the stresses. The Mohr circle that represents the state of failure by shear and by tension at the same time has a center at \( p_t \) and a radius \( q_t \) that can be derived from the geometry of this arrangement as follows

\[
p_t = \sigma_t - q_t
\]  

(31)
Replacing Equation 27 in Equation 26

\[ p_t = \frac{\sigma_t - t \sin(\phi)}{1 - \sin(\phi)} \] (33)

Replacing Equation 28 in Equation 27

\[ q_t = \frac{(t - \sigma_t) \sin(\phi)}{1 - \sin(\phi)} \] (34)

In soil mechanics, it is very common to use the cohesion intercept \( c \) (in this particular case, \( c \) is due to the suction inside the clay) thus, Equations 33 and 34 can be expressed as

\[ p_t = \frac{\sigma_t - c \cos(\phi)}{1 - \sin(\phi)} \] (35)

\[ q_t = \frac{c \cos(\phi) - \sigma_t \sin(\phi)}{1 - \sin(\phi)} \] (36)

By using the above expressions, it is now possible to define the Mohr-Coulomb-Tensile failure envelope of a soil when the angle of internal friction, the cohesion intercept, and the tensile strength are known.
7.2.3 New MTSS criterion for crack-propagation angle $\delta$

The Mixed Tensile and Shear Stress criterion is a new criterion proposed to predict the crack-propagation angle $\delta$, which forms between a pre-existing primary crack and an associated secondary crack. This criterion is based on the proximity to failure by tension and shear.

Using the Equations 35 and 36 it is possible to define the Mohr-Coulomb-Tensile failure envelope of a soil having an angle of internal friction, a cohesion intercept, and a tensile strength. These parameters can be obtained from triaxial tests, direct tensile tests and DTS tests.

After the failure envelope is determined, the proximity to failure concept is next applied. The material is analyzed before it fails. Figure 82 shows the Mohr-Coulomb-Tensile curved envelope and three Mohr’s circles that represent different states of stresses around and close to a

Figure 81. Detail of the Mohr-Coulomb-Tensile failure envelope that is a combined straight line and a sector of a circle. The circle represents a state of stress at failure that occurs simultaneously by shear and by tension.
crack tip. The proximity to failure ($w$) of the circle 1 is by tension. The proximity to failure ($v$) of circle 2 is by shear.

Figure 82. Proximity to the curved Mohr-Coulomb-Tensile envelope.

In addition to the two conditions above, it is suggested that circle 3 represents another state of stresses very critical because it involves both proximity by tension and by shear at the same time. This new hypothesis which is based on the above concept, is proposed for determining the direction of crack propagation for the primary pre-existing crack. According with this hypothesis, crack propagation will begin at the point closest to the crack tip, where propagation of the primary crack began, and which, therefore, also has a unique combination of stresses that will cause the material to fail by tension and by shear at the same time. In other
words, this point must have a state of stresses with combined proximities such that \( w = v \). The hypothesis is called Mixed Tension and Shear Stress (MTSS) criterion.

Figure 83, below, shows the angle of crack propagation \( \delta \) as a function of the primary crack inclination angle \( \alpha \) for specimens that are subjected to uniaxial compression stress. The results of both laboratory tests and theoretical analyses are shown. As can be seen in this figure, the MTSS criteria correspond very well to the laboratory results obtained, especially for the intermediate and high-inclination angles of the primary crack (\( \alpha > 30^\circ \)).

![Figure 83. Crack propagation angle \( \delta \) obtained from laboratory tests results and from proximity theories.](image)

7.2.4 Theoretical incremental analysis of the secondary crack

The propagation path followed by the secondary crack was analyzed using the theoretical Mixed Tension and Shear Strength criterion (MTSS) presented immediately above and the incremental path analysis proposed by Sih and de Oliveira (1984). The authors proposed a methodology for
analyzing the secondary crack propagation path of pre-cracked specimens of titanium when subjected to tension.

**Figure 84. Incremental crack growth initiated from an inclined primary crack.**

At the moment at which the crack propagation begins, new crack tips are developed (see Figure 84 above). The new tips are separated a given distance from the former crack tips and a new, equivalent, straight crack is formed approximates the actual shape.

With incremental analysis, a new crack with a new length and orientation that depends on the length and orientation of the previous one immediately before is presumed. The process is repeated again and again with successive small length increments of the secondary propagated crack.

The theoretical propagation angle was obtained from the MTSS criterion by applying the incremental analysis method. Figure 85 shows the results obtained both in laboratory tests and by using the MTSS criterion. As the figure shows, the MTSS criterion predicts the propagation path of the secondary crack very well.
7.2.5 DEM analysis of crack propagation

DEM numerical analysis technique was used to simulate propagation of cracks in addition to the theoretical analysis presented in the last section. Cases of single primary crack at inclinations of 0°, 15°, 30°, 45°, 60° and 75° with respect to the horizontal as well as many primary cracks (left step, right step, and multiple cracks) were implemented. The results of both, the numerical and theoretical analysis, correspond very well with the laboratory tests results. The DEM parameters used for the computer simulations are shown in Table 1. Ten thousand disks were used to simulate the samples as Figure 86 shows.

7.2.5.1 Primary single crack. For primary crack inclinations of α=30° to 75°, the DEM simulation indicated that cracks propagated in the form of secondary cracks that developed at the tips of the primary crack (Figure 87 to Figure 90). In most cases, the new cracks developed in a

Figure 85. Experimental and theoretical secondary crack propagation path in specimens of kaolinite clay with a pre-existing crack that were subjected to uniaxial compression.
direction that could be predicted using the MTSS criterion. The secondary cracks propagated in a direction that was parallel to the direction of the uniaxial compressive stress, \( \sigma_c \). Laboratory tests performed as part of this study yielded similar results.

Table 2. Parameters used in DEM's simulations

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</tr>
<tr>
<td>Velocity of the bottom horizontal wall</td>
<td>0</td>
</tr>
</tbody>
</table>
Figure 86. Initial configuration of DEM’s specimen for $\alpha=30^\circ$. 

$\sigma_c$: Compressive Stress
Figure 87. Laboratory test results and simulations with DEM for $\alpha=75^\circ$.

Figure 88. Laboratory test results and simulations with DEM for $\alpha=60^\circ$. 

a) $\alpha=75^\circ$, Laboratory.  b) $\alpha=75^\circ$, DEM simulation.

Primary Crack  Secondary Crack

a) $\alpha=60^\circ$, Laboratory.  b) $\alpha=60^\circ$, DEM simulation.

Figure 88. Laboratory test results and simulations with DEM for $\alpha=60^\circ$. 

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Figure 89. Laboratory test results and simulations with DEM for $\alpha=45^\circ$.

Figure 90. Laboratory test results and simulations with DEM for $\alpha=30^\circ$. 

Figure 91. Laboratory test results and simulations with DEM for $\alpha=15^\circ$.

a) $\alpha=15^\circ$, Laboratory.  
b) $\alpha=15^\circ$, DEM simulation.

Figure 91. Laboratory test results and simulations with DEM for $\alpha=0^\circ$.

a) $\alpha=0^\circ$, Laboratory.  
b) $\alpha=0^\circ$, DEM simulation.
For the samples with cracks inclined at $\alpha=0^\circ$ and $15^\circ$, the DEM simulation indicated that secondary cracks form on the edges of the original cracks. The points from which the secondary cracks developed were those undergoing the largest tensile forces. These cracks also grew in a direction parallel to the $\sigma_c$ direction. The laboratory tests also produced results similar to those obtained with the DEM simulations (Figure 91 and Figure 91 above).

**7.2.5.2 Uniaxial stress-strain relationships and uniaxial stress level for crack propagation.** Figure 92, below, shows the uniaxial stress-strain curve measured in a specimen with a crack inclined at $\alpha=30^\circ$ as well as the level of secondary crack formation developed in the as the uniaxial compressive stress, $\sigma_c$, was increased. The uniaxial stress level $\sigma_c$ at the moment when the secondary crack formed in the sample is also indicated.
Figure 92. Stress – strain curve and secondary crack stages for a primary crack having $\alpha=30^\circ$. 
Figure 93. Stress-strain relationships and crack propagation stress level.

a) Stress – strain relationships and crack propagation stresses.

b) Normalized vertical stress level at crack propagation.

Figure 93. Stress-strain relationships and crack propagation stress level.
Figure 93, above, compares the stress-strain relationship curves as well as relationships between the normalized $\sigma_c$ values needed for secondary crack formation and the inclination of the original crack, $\alpha$, in the samples for both the DEM simulations and the laboratory tests. The $\sigma_c$ values were normalized with respect to the value of $\sigma_c$ obtained for crack propagation in the sample with a crack inclined at 45 degrees. Figure 93b shows that the DEM simulations predicted very well the value of $\sigma_c$ at which the laboratory samples developed secondary cracks. The DEM simulations also accurately predicted that the samples with an original crack inclined at 45 degrees would be the weakest of all the samples.

7.2.5.3 Forces in the specimens before crack propagation occurs. This research revealed that the maximum force concentrations for crack inclination angles $\alpha=30^\circ$ to $75^\circ$ occur around the crack tips. Figure 94, below, shows the forces in the crack tip for $\alpha=30^\circ$ just before crack propagation. The black lines represent the compression forces in the material and the red ones the tensile forces; the coarser the line the stronger the force that is acting at that point. The maximum compressive force on the crack tips was equal to 1098 N. The maximum tensile force that developed near the tip of the sample was equal to 237 N. Based on this finding, it appears that the point where crack propagation starts is the one at which maximum tensile force occurs (Figure 87 to Figure 90).

For the samples with cracks with inclination angles of $\alpha=15^\circ$ and $0^\circ$, high compressive and tensile forces were found to occur near the tips of the cracks; however, the highest tensile force producing secondary crack formation in these samples took place not on the tips but on the edges of the cracks (Figure 91 and Figure 91).
Figure 94. Stress concentrations around crack tip before crack propagation ($\alpha = 30^\circ$); the applied vertical compression stress is $\sigma_c = 562$ kPa.

Figure 95. Comparison of results between DEM and LAB for the crack propagation angle, $\alpha$. 

Primary crack inclination angle ($\alpha^\circ$)
Figure 95 shows the relationship between the angle \((\delta)\) formed between the plane of the original crack and the secondary crack. This figure shows that the DEM predicted very well the angle \(\delta\) that the secondary cracks later developed in the laboratory samples.

7.2.5.4 **Tensile and shear forces at the tip of propagated crack.** Figure 96, below, represents tensile and shear forces that are applied at contacts between particles located close to a crack tip. Green lines and black lines represent tensile and compressive forces respectively. As shown at the bottom of Figure 96, force lines having a different direction with respect to the axis that formed between the center of particles represent the presence of a shear component. The propagated crack is also shown. The contact bonds approaching breakage are subjected to both a normal tensile force and a shear force. This finding supports the MTSS theoretical criterion, which was based on the concept of proximity to failure by tension and shear described in Section 7.2.3 above.
Figure 96. Detail of crack propagation in a DEM simulation. Compressive, tensile, and shear forces between particles are shown. The shear and tensile forces act together to produce breakage of the next contact (i.e., light-gray contact forces in the contacts that are going to break are inclined).

7.2.5.5 Primary multiple-cracks. Simulations of specimens having many cracks were: left step, right step, and multiple cracks. For the left-step primary crack arrangement, the tension zones produced both the propagation of a primary crack toward a common point and a secondary linking crack as Figure 97, below, shows. For the right step, the primary cracks behaved independently, with no links between cracks (see Figure 98). In the section/sections containing
multiple cracks, the crack having the inclination $\alpha=30^\circ$ propagated the most. However, the linkage between upper and lower rows via cracks located in the center of the specimen didn’t develop as was earlier suggested by the laboratory test.

Figure 97. DEM simulation of left-step primary cracks.

Figure 98. DEM simulation of right-step primary cracks.
7.3 STUDY OF DYNAMIC BEHAVIOR OF FISSURED SPECIMENS

7.3.1 Water flow in fissured clay samples under dynamic loading

The relationship between the crack stability threshold \( r_d \) of the samples determined through the uniaxial dynamic tests was earlier presented, in Figure 28 to Figure 35. As indicated here, the crack stability threshold, was found to be fairly constant for samples with low and intermediate water contents but then dropped dramatically in samples of high water content. This result can be explained by referring to the flow of water in the samples during the testing program.

When a fissured clay is subjected to dynamic compression, and zones of tension and compression develop around a crack, it is expected that the water in the specimen will flow from the zones of compression to the zones of tension. Figure 100, below, shows the results of a FLAC simulation of pore water pressure generation and the resulting water flow around a crack tip. The zones subjected to negative pore pressure adsorb water expelled by the zones that were subjected to compression. In this research, it became apparent that these compression zones that
can serve as a source of water can be found not only in the zones around the crack tip but also in other zones as well (see Figure 100c, below).

As was seen before (see Figure 70) the tensile strength of clays in saturated-funicular state decreases as the moisture content increases. With the increase in water content in the zones subjected to tension as more and more cycles are applied to the samples, a lowering of the tensile strength in the clay occurs.

Figure 101, below, shows the effect of those water content changes in the clay when moisture content decreases (path 1 in Figure 101) the clay will have higher strength. When moisture content increases (path 2) the clay could fail and cause the primary crack to propagate.

The water content transfer within the samples – was found to be possible only when the clay is in a saturated-funicular state for which the water phase is continuous. However, for the complete and partial pendular states (Figure 48), the liquid meniscuses are disconnected from to each other and so water flow cannot occur. As a result of variable transfer of water content, the cyclic stress ratio $r_d$ of the crack propagation threshold was found to diminish as the water content in the clay increased (see Figure 102).
Figure 100. FLAC simulation of pore pressures and water flow in a fissured clay specimen when the clay is very close to saturation and is subjected to a vertical stress $\sigma_v$ that is lower than $\sigma_c$.

(a) Detailed view of the crack tip. Both equipotential lines and flow lines are shown.

(b) Equipotential lines only

(c) Flow lines only

Figure 100. FLAC simulation of pore pressures and water flow in a fissured clay specimen when the clay is very close to saturation and is subjected to a vertical stress $\sigma_v$ that is lower than $\sigma_c$. 
Figure 101. Effect of changes in the water content on the crack propagation due to water flow around the crack. Water content path 1 (decrease of $\omega$) would not cause crack propagation. Water content path 2 (increase of $\omega$) would produce crack propagation.

If water content decreases, the crack would not propagate even if the stress level remains constant.

If water content increases, the crack could propagate regardless of whether the stress level remains constant.

$\alpha=45^\circ$, Cyclic Stress Ratio: $r=\sigma d/\sigma u$

Figure 102. Zones of crack propagation under dynamic loading which are produced by fatigue in the clay or by mixed fatigue and water flow around the crack tip.
7.3.2 DEM analysis of dynamic loading of fissured specimens

Using DEM analysis, it was possible to simulate the dynamic loading of fissured specimens. Figure 103, below, shows the stress-strain graph obtained from a DEM simulation of one such specimen. In this case, the dynamic loading was applied by changing the velocity of the top wall of the specimen.

Figure 103. DEM-created stress-strain graph of simulated specimen using DEM and having a primary crack inclined at $\alpha=45^\circ$. 
The contact bond option included with the PFC$^{2D}$ computer program presumes that a material is elastic but it breaks when a certain degree of force greater than its contact bond is applied. Consequently, in this research, the bond resistance was reduced by applying an additional bonding-reduction factor ($f_d$) to the larger force level in the specimen for each loading cycle. Various expressions of the bonding reduction factor were then tested until the results from DEM closely matched the fatigue curve of the specimen having a primary crack inclination angle $\alpha=45^\circ$, which was shown earlier, in Figure 36. The best results were obtained using the bonding reduction factor shown in Figure 104, below. Here, it can been seen that the residual bonding $b_r$ is the minimum contact bond level in the material below which no additional reduction of strength occurs regardless of the number of cycles applied. The fatigue curve shown in Figure 105, below, that is derived from the DEM simulation, matches these results very well.

$$f_d = b_r + \left(1 - b_r\right) \frac{1}{\sqrt{N}}$$

*b_r*: Residual bonding  
*N*: Number of cycles applied

*Figure 104. Bonding reduction factor $f_d$ used in DEM fatigue simulations.*
7.3.3 Velocity of crack propagation

Figure 106 shows the crack growth velocity in three specimens subjected to dynamic testing in the laboratory and having a primary crack inclined at $\alpha=45^\circ$. As the cyclic stress ratio $r_d$ was increased, the velocity of crack propagation increased as well. Two cracks presented stable growth, and one presented unstable growth. Crack growth is considered stable when the crack stops growing after a certain number of load cycles are applied; crack growth is considered unstable when the crack quickly increases in length and propagates throughout all the faces of the specimen until it reaches a vertical direction parallel to the direction of the maximum uniaxial stress that was applied.

The stable and unstable growth can be explained using the stress intensity factors $K_I$ and $K_{II}$ included in Equations 10 and 11 (page 57). Figure 107, below, shows the relationship between the product $K_I \times K_{II}$ and the crack inclination angle $\alpha$. The top line was derived using the
stress level at crack propagation $\sigma_c$, obtained from the static loading tests of the fissured specimens in the laboratory ($K_{lc} \times K_{hc}$). The top line of the shadowed area in Figure 107 represents the product $K_I \times K_{II}$ obtained using the stress level of the crack stability threshold. It was found that below this level, the cracks don’t propagate regardless of the number of cycles applied.

Figure 106. Velocity of crack propagation in laboratory dynamic tests on specimens having a primary crack inclination angle $\alpha=45^\circ$. The cracks demonstrated stable crack growth, reached a maximum length, and then stopped before propagating through the specimen.

Figure 107 also shows two paths which were obtained theoretically, replacing the crack angle $\alpha_{eq}$ – which is variable as the crack propagates, (see Figure 108, below)- and two arbitrary stress levels from the stress intensity factors derived with Equations 10 and 11. Path 1 represents a case of unstable crack growth and corresponds to the highest stress level adopted. This path
doesn’t enter the stable crack region delimited by the crack stability threshold. Path 2, by contrast, represents a case of stable crack growth because $K_I \times K_{II}$ becomes low enough for the path to enter the stable crack region.

**Stress intensity factors paths:**

Path 1 - Unstable crack growth. This path won’t stop until it reaches a vertical direction.
Path 2 - Stable crack growth. Propagation will stop when the path reaches the stable crack region at the point circled.

*Figure 107. Product of stress intensity factors $K_I \times K_{II}$ vs. crack inclination angle.*
Figure 108. Variation in relative crack length and equivalent crack inclination angle as a crack grows. The primary crack inclination angle is $\alpha=45^\circ$. These results were obtained using the incremental analysis method.
8.0 FURTHER RELATED RESEARCH

Further research that could be derived from this thesis can be divided into two major areas of study: the behavior of unsaturated soils, and the behavior of cracks in clays.

Possible ideas for additional research related to the behavior of unsaturated soils could include the following.

1) The equivalent effective stress concept and the three-dimensional consolidation of unsaturated clays subjected to changes in moisture content could be studied using different soils, such as kaolinite and montmorillonite mixed in different proportions in the laboratory.

2) The DTS test proved to be a good way to establish the strength of soils under tension. Similar tests could be implemented for other soils, using the device developed for this research, which, combined with the results of the triaxial constant water content tests, would allow the tensile-Mohr-Coulomb failure envelope to be established.

3) The laboratory tests for this research were implemented using a drying path only. The validity of the water distribution states and the related equivalent effective stress should be confirmed for other water content paths as well.

4) Two of the major concerns within the area of traditional unsaturated soil mechanics relate to the expansibility and collapsibility of clays. This researcher believes that perhaps this expansibility occurs primarily in those unsaturated soils in which the water distribution is in a saturated-funicular state—in other words, that the expansion of the clay resulting
from increased water content reduces the equivalent effective stress. It's also possible that the collapsibility could be related to soils that are in a partial pendular state because the clay volume would diminish if water content were increased (i.e., the equivalent effective stress is increased).

Ideas for additional research related to the study of cracks in clays might include the following.

a) To date, fissured clays, particularly specimens with a primary-induced crack, have been used to study different aspects of crack propagation. New research could be developed to study the mechanisms that lead to the development of the crack itself and, especially, those related to the drying of clays.

b) As part of this study, dynamic uniaxial, biaxial, and triaxial compression tests were implemented in order to study the crack stability threshold in samples having primary cracks inclined at different angles. Since clay deposits subjected to desiccation develop vertical cracks, and since these deposits can also be subjected to earthquake waves composed mainly of shear horizontal waves, additional dynamic tests should be developed to study the often-serious effect of these waves.

c) DEM could be used to study not only the static behavior of soils with cracks but also the behavior of fissured clays subjected to dynamic loading. Additional research using this numerical tool could be developed and applied specifically to the study of fatigue.
9.0 IMPLICATIONS OF THE RESEARCH TO ENGINEERING THEORY AND PRACTICE

Many geotechnical problems are related to the behavior of unsaturated soils and more specifically to the formation and the presence of cracks in clays. Many regions that have those problems are located in areas subjected to intense seismic activity.

The State of California has a large number of active faults within its territory. Some of the active faults are located near earth dams, and when these faults become active many small tremors develop (300 tremors per day of the magnitude $M \leq 3$ on the Richter scale, Seed (1979)). These tremors exert stresses on the dams and if the clay core of any of these dams have cracks, the cracks could propagate, and interact, causing forming continuous channels through which water can move at high velocities (piping). This research has shown that under certain moisture conditions, cracks can propagate under even low levels of dynamic cyclic stresses. As shown in this research the degree of cyclic stress needed for crack propagation depends on the water content of the underlying clay: the higher the water content, the lower the cyclic stress causing crack propagation. Thus in addition to other possible areas of application this research will may eventually lead to improvements in the design of the core of earth dams that are known to develop cracks during their construction.

In the construction of dams, it is mandatory, in order to meet legal standards, that the degree of compaction of clay cores be nearly close to the maximum dry unit weight, and that the compaction water content be close to the optimum moisture content, as determined by the
reference compaction test in the laboratory. This research has shown that if the moisture content of the clay core is prepared at this level of water content or higher, cracks in the clay will propagate and detrimentally affect the strength of the clay mass that forms the core of the dams.
10.0 CONCLUSIONS

In this research, the static and dynamic behavior of laboratory-prepared unsaturated clay specimens with induced cracks was studied by subjecting them to uniaxial, biaxial, and triaxial stress conditions.

The major conclusions that can be reached from this research are the following:

1) The inclination of the primary induced crack was found to greatly influence the behavior under static and dynamic conditions of the fissured specimens. Specifically, it was found that cracks with inclination angles between 30° and 45° with respect to the horizontal are weaker than cracks with higher or lower angles.

2) The water content and degree of saturation in the clay specimens were found to play a very important role in the behavior of the clay. It was found that three states of water distribution in the pores such as saturated-funicular state, complete pendular state and partial pendular state conform a key to understand the behavior of fissured clays when subjected to static and dynamic loading. The most important findings regarding the strength and compressibility of clay for each state were as follows:
   a) in the saturated-funicular state, the capillary water phase in the clay is continuous and reducing the water content makes the clay stronger and the void ratio to reduce. In this state, the particular degree of suction in the soil influences in an important way the behavior of the clay.
b) in the complete pendular state, the liquid contacts between particles don’t touch each other, and the corresponding strength and compressibility of the clay are nearly constant. Changing the suction pressure inside the clay has no influence on its behavior.

c) in the partial pendular state, the clay is weaker when the water content is reduced because liquid contacts from the contacts between particles are lost. It was found that when fewer particles had contacts, the number of capillary forces acting on the clay will be lower as well. In this state, lower strength and larger volume is reached by the clay when the suction pressure in the soil is increased.

d) the water content at the limit between saturated-funicular state and the complete pendular state is the shrinkage limit of the clay. On the other hand, the limit between complete pendular state and partial pendular state was defined as the pendular limit of the clay.

3) The equivalent effective stress concept was applied to measure the effect of the water-distribution pattern in the pores of unsaturated clay. Direct tensile tests were used to measure the equivalent effective stress in the laboratory.

4) The concept of equivalent effective stress proved to be very useful in understanding how fissured clays behaved under dynamic and static loading. Contrary to longstanding theory in unsaturated soil mechanics, suction pressure was not found to increase—and, in fact, was found to have little influence on the strength of the clay for samples in the complete pendular and partial pendular states. For samples in the saturated-funicular state, however, suction pressure did more clearly influence the strength of the clay.
5) The Direct-Tensile and Direct-Shear (DTS) test apparatus was designed and implemented to ascertain the failure envelope of clays when they are subjected to tension. Based on the results of these tests the Tensile-Mohr-Coulomb failure envelope of unsaturated clay was established. This failure envelope was found to play an important role in the behavior of fissured clays subjected to compression.

6) The Linear Elastic Fracture Mechanics (LEFM) was implemented successfully to study and understand the stress concentrations produced around crack tips of clay samples. Using the stresses obtained from LEFM and the Mohr’s diagram of stresses, a new criterion to study the crack propagation in clays was proposed.

7) A new criterion for predicting the crack-propagation angle of fissured clays was presented. The “Mixed Tensile and Shear Stress” (MTSS) criterion, as it is known, allowed this researcher to predict that the crack will propagate in the direction through which the combined effect of tensile and shear stresses in the soil produce the state of stresses most close to the tensile-Mohr-Coulomb curved failure envelope. Using this criterion, it was shown that not only the tensile stresses but rather the combined effect of the tensile and shear stresses can determine the propagation of cracks in clays.

8) The Discrete Element Method (DEM) was found to be very useful for simulating crack propagation in clay, producing computer simulations that closely matched laboratory tests results.

9) A crack stability threshold in fissured clay specimens subjected to dynamic uniaxial, biaxial, and triaxial stress test conditions was established. This threshold was found to be lower for uniaxial and biaxial tests than for triaxial tests.
10) The crack-stability threshold was found to vary depending on the water content and the water distribution in the pores of the clay sample. For complete and partial pendular states, the threshold remained nearly/fairly constant even if the water content was changed. For the saturated-funicular state the threshold decreased considerably as higher water content in the clay was present. It was found that the stress concentrations around crack tips produced very high pore pressure changes in the soil. This may be related to the fact that the water flows toward the areas around the crack that are subjected to negative pore water pressures. The increase in water content increment produced a reduction in the tensile and shear strength of the clay, which permitted the easier crack propagation as the dynamic loading was applied.

11) The crack-stability threshold using triaxial tests was found to increase steadily and commensurable with the confining pressure. For a water content of 15% and cell pressures higher than 420 kPa, the crack didn’t propagate because it closed and the clay behaved as an intact material.

To verify the initial results reported here and investigate a number of geotechnical problems related to crack propagation in clays, further research topics related to the study of crack propagation in clays under static and dynamic loading have been proposed.
APPENDIX A

APLICATION OF LINEAR ELASTIC FRACTURE MECHANICS USING MATHCAD®

In this appendix sample of the application of the Linear Elastic Fracture Mechanics to study the stresses around crack tips is included. The electronic sheet is implemented as follows:

Mathcad sheet for calculation of stresses around a crack tip. 
Reference: Equations and diagrams were adapted from: 
Geotechnique 37, No. 1, 69-82. 

Conventions:
The global (far field) normal and shear stresses applied to the body in the same orientation of the crack plane are:

\[ \tau_n = \sigma_a \cdot \sin(2\cdot\alpha) + \tau_a \cdot \cos(2\cdot\alpha) \]

\[ \tau_n = 0.5 \]

\[ \sigma_n = \frac{(\sigma_a + \sigma_b)}{2} + \frac{(\sigma_a - \sigma_b)}{2} \cdot \cos(2\cdot\alpha) - \tau_a \cdot \sin(2\cdot\alpha) \]

\[ \sigma_n = 0.5 \]

\[ \sigma_p = \frac{(\sigma_a + \sigma_b)}{2} - \frac{(\sigma_a - \sigma_b)}{2} \cdot \cos(2\cdot\alpha) + \tau_a \cdot \sin(2\cdot\alpha) \]
Global stress field:
\( \sigma_p = 0.5 \)
\( k_1 := \sigma n \sqrt{c} \)
\( k_2 := \tau n \sqrt{c} \)
\( \frac{k_1}{k_2} = 1 \)
\( r := 0.00001 \)
The radius is constant
\( \theta_1 := -\pi \)
\( \theta_{i,1} := \theta_i + \frac{\pi}{36} \)

Variation of the angle \( \theta \)
\( i := 1..72 \)

Stresses from LEFM:
\[
\begin{align*}
\sigma_x &= \frac{k_1}{\sqrt{2r}} \cos \left( \frac{\theta_i}{2} \right) \left( 1 - \sin \left( \frac{\theta_i}{2} \right) \cdot \sin \left( \frac{3 \theta_i}{2} \right) \right) - \frac{k_2}{\sqrt{2r}} \sin \left( \frac{\theta_i}{2} \right) \left( 2 + \cos \left( \frac{\theta_i}{2} \right) \cdot \cos \left( \frac{3 \theta_i}{2} \right) \right) \\
\sigma_y &= \frac{k_1}{\sqrt{2r}} \cos \left( \frac{\theta_i}{2} \right) \left( 1 + \sin \left( \frac{\theta_i}{2} \right) \cdot \sin \left( \frac{3 \theta_i}{2} \right) \right) + \frac{k_2}{\sqrt{2r}} \sin \left( \frac{\theta_i}{2} \right) \cdot \cos \left( \frac{\theta_i}{2} \right) \cdot \cos \left( \frac{3 \theta_i}{2} \right) \\
\tau_{xy} &= \frac{k_1}{\sqrt{2r}} \cos \left( \frac{\theta_i}{2} \right) \cdot \sin \left( \frac{\theta_i}{2} \right) \cdot \cos \left( \frac{3 \theta_i}{2} \right) + \frac{k_2}{\sqrt{2r}} \cos \left( \frac{\theta_i}{2} \right) \cdot \left( 1 - \sin \left( \frac{\theta_i}{2} \right) \cdot \sin \left( \frac{3 \theta_i}{2} \right) \right)
\end{align*}
\]

Fracture mechanics stress equations don't include the global stress field applied. The following equations involve the combined applied stresses (due to crack tip and the global far field)
\[
\begin{align*}
\sigma_x &= \sigma_x + \sigma_p \\
\sigma_y &= \sigma_y + \sigma_n \\
\tau_{xy} &= \tau_{xy} - \tau_n
\end{align*}
\]
Principal stresses:

\[ \sigma_1 := \frac{(\sigma_x + \sigma_y)}{2} + \sqrt{\left(\frac{(\sigma_x - \sigma_y)}{2}\right)^2 + (\tau_{xy})^2} \]

\[ \sigma_3 := \frac{(\sigma_x + \sigma_y)}{2} - \sqrt{\left(\frac{(\sigma_x - \sigma_y)}{2}\right)^2 + (\tau_{xy})^2} \]

\[ p := \frac{(\sigma_1 + \sigma_3)}{2} \]

\[ q := \frac{(\sigma_1 - \sigma_3)}{2} \]
MOHR-COULOMB PLASTICITY ANALYSIS:
Select friction angle and tensile origin point $t$:

\[ \phi = 25 \text{ deg} \]
\[ t = -80 \text{c} \]

Select tensile strength:

\[ \sigma_t = -34 \text{c} \]

Explanation of variables:

Location of the shear-tensile circle envelope (see Section 7.2.2):

\[ pt := \frac{\sigma_t - t \cdot \sin(\phi)}{1 - \sin(\phi)} \]
\[ qt := \frac{(t - \sigma_t) \sin(\phi)}{1 - \sin(\phi)} \]

Auxiliary angles for failure envelope graph:

\[ \mu_1 := 29 \text{c} \]
\[ m := 1..14 \]
\[ \mu_{m+1} := \mu_m + 5 \]
\[ \mu := \mu \cdot \text{deg} \]

The Mohr-Coulomb failure envelope is:

\[ \tau_f(\sigma) := (\sigma - t) \tan(\phi) \]
\[ \sigma_f(\mu) := pt + qt \cdot \cos(\mu) \]
\[ \tau(\mu) := qt \cdot \sin(\mu) \]

Select some points for the analyses:

\[ f := 53 \]
\[ \begin{align*}
\theta_f &= 80 \quad \text{deg} \\
g &= 55 \\
h &= 57 \\
\theta_g &= 90 \quad \text{deg} \\
\theta_h &= 100 \quad \text{deg}
\end{align*} \]

Mohr equations of stresses for the above selected points:

\[ \begin{align*}
s_{\theta f}(\theta) &= p_f + q_f \cdot \cos(\theta) \\
t_{\theta f}(\theta) &= q_f \cdot \sin(\theta) \\
s_{\theta g}(\theta) &= p_g + q_g \cdot \cos(\theta) \\
t_{\theta g}(\theta) &= q_g \cdot \sin(\theta) \\
s_{\theta h}(\theta) &= p_h + q_h \cdot \cos(\theta) \\
t_{\theta h}(\theta) &= q_h \cdot \sin(\theta)
\end{align*} \]

\[ \alpha = 45 \quad \text{deg} \]

**MOHR CIRCLES:**

\[ \sigma_{\theta i} := \frac{k_1}{4\sqrt{2}r} \left( 5 \cdot \cos \left( \frac{\theta_i}{2} \right) - \cos \left( 3 \cdot \frac{\theta_i}{2} \right) \right) + \frac{k_2}{4\sqrt{2}r} \left( -5 \cdot \sin \left( \frac{\theta_i}{2} \right) + 3 \cdot \sin \left( 3 \cdot \frac{\theta_i}{2} \right) \right) \]

\[ \sigma_{\theta i} := \frac{k_1}{4\sqrt{2}r} \left( 3 \cdot \cos \left( \frac{\theta_i}{2} \right) + \cos \left( 3 \cdot \frac{\theta_i}{2} \right) \right) + \frac{k_2}{4\sqrt{2}r} \left( -3 \cdot \sin \left( \frac{\theta_i}{2} \right) - 3 \cdot \sin \left( 3 \cdot \frac{\theta_i}{2} \right) \right) \]

\[ \tau_{\theta i} := \frac{k_1}{4\sqrt{2}r} \left( \sin \left( \frac{\theta_i}{2} \right) + \sin \left( 3 \cdot \frac{\theta_i}{2} \right) \right) + \frac{k_2}{4\sqrt{2}r} \left( \cos \left( \frac{\theta_i}{2} \right) + 3 \cdot \cos \left( 3 \cdot \frac{\theta_i}{2} \right) \right) \]
\[
\alpha = 45 \text{ deg}
\]

For checking purposes

\[
\sigma_1 := \left(\sigma_{\theta\theta i} + \sigma_{rr i}\right) - \frac{\left(\sigma_{\theta\theta i} - \sigma_{rr i}\right)^2}{2} + \left(\sigma_{\theta i} \tau_i \right)^2
\]

\[
\sigma_3 := \left(\sigma_{\theta\theta i} + \sigma_{rr i}\right) + \frac{\left(\sigma_{\theta\theta i} - \sigma_{rr i}\right)^2}{2} + \left(\sigma_{\theta i} \tau_i \right)^2
\]
Explanation diagram:
Plasticity:

Shear failure criteria
The minimum $v$ value is the most critical

$v_i := (-t_i + p_i) \sin(\phi) - q_i$
$v := -v$
Proximity by tensile strength failure criteria:
\[ \sigma_{\text{ten}}(\theta) := \sigma_t \]

Proximity to failure by tension.
\[ w_i := \sigma_t - \sigma_2 \]
MTSS Criterion (see Section 7.2.2):
Hypothesis: the crack propagates by the point at which the proximities by tension and shear are the same. In other words, the tension-shear failure envelope and the Mohr circle that represents that critical state are concentric.

For this case \( \frac{\alpha}{\text{deg}} = 45 \) \( \theta_c := 90^\circ \)

For \( \alpha=45 \) deg both, the MTSS criterion and MTS criterion, the critical angle is \( \theta_c=90 \) deg. For other angles the results are different. See Figure 83.
APPENDIX B

EXAMPLES OF FISH® SUBRUTINES USED FOR SIMULATING FISSURED SPECIMENS OF CLAY SUBJECTED TO STATIC AND DYNAMIC LOADING

SUBRUTINE No. 1

;45 deg crack Brittle. STATIC LOADING.

def eraseballs ;Erasing balls forming the crack

i=1
ballini=4926
ballerasefor=ballini
ballerasebak=ballini
ballini2=ballini+1
ballini3=ballini-1

command
    del ball range id ballini2,ballini3
endcommand

loop while i<11
    n=100
    eraseballscrack
    eraseballscrack
    n=101
    eraseballscrack
endloop
end

def eraseballscrack
    ballerasefor=ballerasefor+n
    balleraseforaux1=ballerasefor+1
    balleraseforaux2=ballerasefor-1
    ;balleraseforaux3=ballerasefor-2
SUBRUTINE No. 2

; 45 deg crack Brittle. DYNAMIC LOADING

set dt dscale

def eraseballs ; Erasing balls in the crack

numcycle=0
risingflag=1
ncurrentbond=1000
normalbond=1000
cbreakflag=0
dynstress=3.95e5
i=1
ballini=4926
ballerasefor=ballini
ballerasebak=ballini
ballini2=ballini+1
ballini3=ballini-1

command
del ball range id ballini2,ballini3
endcommand

loop while i<11
n=100
eraseballscrack
eraseballscrack
n=101
eraseballscrack
endloop
end

def eraseballscrack

ballerasefor=ballerasefor+n
balleraseforaux1=ballerasefor+1
balleraseforaux2=ballerasefor-1
;balleraseforaux3=ballerasefor-2

ballerasebak=ballerasebak-n
ballerasebakaux1=ballerasebak+1
ballerasebakaux2=ballerasebak-1
;ballerasebakaux3=ballerasebak-2

i=i+1
command
del ball ballerasefor
del ball balleraseforaux1
del ball balleraseforaux2
; del ball balleraseforaux3

del ball ballerasebak
del ball ballerasebakaux1
end_if

end

Def VertStrain
  WallLength=0.174
  Yinitial=0.2
  VForce=w_yfob(find_wall(3))
  VertStrain=-w_y(find_wall(3))/Yinitial
  VertStress=VForce/WallLength
end

set fishcall FC_BOND_DEL cbreak ; call cbreak when cbond breaks
eraseballs

wall id=1 nodes (0,0) (0.174,0)
;wall id=2 nodes (0.174,0) (0.174,0.20)
wall id=3 nodes (0.174,0.20) (0,0.20)
;wall id=4 nodes (0,0.20) (0,0)

wall id 1 kn=1e8 ks=0
;wall id 2 kn=1e8 ks=0
wall id 3 kn=1e8 ks=0
;wall id 4 kn=1e8 ks=0

prop ks=1e8 kn=1e8
prop dens=1000 n_bond=1e3 s_bond=2e3

plot create specimen
plot add ball lblue
plot add wall black
;plot add cforce black
plot show

set dt dscale

history VertStress VertStrain
hist Normal Shear

plot create StresStrainGraph
plot add hist 1 vs 2
plot show

wall id 3 yvel=-1e-6
;wall id 2 yvel=-1e-8
SUBRUTINE No. 2

; 45 deg crack Brittle. DYNAMIC LOADING

set dt dscale
def eraseballs        ;Erasing balls in the crack

numcycle=0
risingflag=1
ncurrentbond=1000
normalbond=1000
cbreakflag=0
dynstress=3.95e5

i=1
ballini=4926
ballerasefor=ballini
ballerasebak=ballini

ballini2=ballini+1
ballini3=ballini-1

command
del ball range id ballini2,ballini3
endcommand

loop while i<11

n=100
eraseballscrack
eraseballscrack
n=101
eraseballscrack
endloop
end

def eraseballscrack

ballerasefor=ballerasefor+n
balleraseforaux1=ballerasefor+1
balleraseforaux2=ballerasefor-1
;balleraseforaux3=ballerasefor-2

ballerasebak=ballerasebak-n
ballerasebakaux1=ballerasebak+1
ballerasebakaux2=ballerasebak-1
;ballerasebakaux3=ballerasebak-2

i=i+1

command
del ball ballerasefor
del ball balleraseforaux1
del ball balleraseforaux2
;del ball balleraseforaux3
del ball ballerasebak
del ball ballerasebakaux1

del ball ballerasebakaux1
del ball ballerasebakaux2
;/del ball ballerasebakaux3

;del ball range id 4117,4119
;del ball range id 4018

endcommand

endponential

end ; end eraseballscrack


call C:\disco1\itascaplots\pfc2d\Ref\fishcall.FIS ;Load Macros
def cbreak
    cbreakflag=1
    cbond=fc_arg(0)  ; contact address
    fmode = fc_arg(1)  ; fracture mode
    pointercb1=c_ball1(cbond)
    pointercb2=c_ball2(cbond)
    if fmode=0 then
        ii=out ('break normal')
        ball1=b_id(pointercb1)
        ball2=b_id(pointercb2)
        Normal=c_nforce(cbond)
        Shear=c_sforce(cbond)
        command
        prop c_index=1 range id ball1
        prop c_index=1 range id ball2
        endcommand
    else
        ii=out('break shear')
        ball1=b_id(pointercb1)
        ball2=b_id(pointercb2)
        command
        prop c_index=2 range id ball1
        prop c_index=2 range id ball2
    ;pause
    endcommand
end_if
end
def assignvel

VForce=w_yfob(find_wall(3))
VertStress=VForce/0.174

if vertstress<=1 then
    command
    wall id 3 yvel=-1e-6
    endcommand
    risingflag=1
end_if
if VertStress>=dynstress then
    numcycle=numcycle+1
    risingflag=0
    maxtension=0
    cp=contact_head
    loop while cp # null
        if c_nforce(cp)<maxtension then
            maxtension=c_nforce(cp)
            maxcp=cp
        end_if
        cp=c_next(cp)
    end_loop
    c_nstrength(maxcp)=normalbond*(0.42+0.58/n^0.5)
    ;c_nstrength(maxcp)=normalbond*(1-0.2*(ln(numcycle))^0.5)
    ncurrentbond=c_nstrength(maxcp)
    c_sstrength(maxcp)=2*ncurrentbond
    wall3=find_wall(3)
    w_yvel(wall3)=1e-6
end_if

if vertstress<=dynstress then
    if risingflag=1 then
        command
        wall id 3 yvel=-1e-6
        endcommand
    else
        command
        wall id 3 yvel=1e-6
        endcommand
    end_if
end_if

end

Def VertStrain
    WallLength=0.174
    Yinitial=0.2
    VForce=w_yfob(find_wall(3))
    VertStrain=-w_y(find_wall(3))/Yinitial
    VertStress=VForce/WallLength
end

set fishcall FC_BOND_DEL cbreak    ;call cbreak when cbond breaks
set fishcall FC_CYC_TOP assignvel   ;call assignvel when new cycle is started
; the velocity is controlled
eraseballs

wall id=1 nodes (0,0) (0.174,0)
; wall id=2 nodes (0.174,0) (0.174,0.20)
wall id=3 nodes (0.174,0.20) (0,0.20)
; wall id=4 nodes (0,0.20) (0,0)

wall id 1 kn=1e8 ks=0
; wall id 2 kn=1e8 ks=0
wall id 3 kn=1e8 ks=0
; wall id 4 kn=1e8 ks=0

prop ks=1e8 kn=1e8
prop dens=1000 n_bond=1e3 s_bond=2e3

plot create specimen
plot add ball lblue
plot add wall black
; plot add cforce black
plot show

history VertStress VertStrain Vforce numcycle ball1
hist Normal Shear

plot create StresStrainGraph
plot add hist 1 vs 2
plot show

assignvel
APPENDIX C

THE DIRECT-TENSILE AND DIRECT-SHEAR TEST (DTS) APPARATUS

This appendix shows details of the Direct-Tensile and Direct-Shear Test (DTS) apparatus developed for this research (patent pending).
Figure C.1 - DTS Horizontal shear force actuator. (a) Partially assembled perspective; the secondary reaction plates were not drawn for simplicity. $S_f$ is the shear force to failure (b) Plan view.
Figure C.2 - Details of the DTS device.


