# DETECTION RESOURCE ALLOCATION IN ADVERSARIAL PROBLEMS

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## ABSTRACT

## DETECTION RESOURCE ALLOCATION IN ADVERSARIAL PROBLEMS

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We consider the problem of optimally allocating static and dynamic detection resources in order to detect or prevent evaders from reaching their destinations. The evaders may be terrorists or smugglers attempting to enter a facility or illegally cross a border. Examples of static detection resources include sensors that detect people and weapons, cameras and check points. In addition, examples of dynamic detection resources include guards at the borders and unmanned aerial vehicles. It is crucial to use these resources efficiently to increase the detection probabilities of evaders.

This study describes two different models built to allocate the available resources. In the first model, we seek an optimal allocation scheme in which only static detection resources are considered. Information asymmetry between the evader and the system designer is utilized and several risk criteria are analyzed. In the second model, both static and dynamic detection resources are considered. We determine an allocation scheme for the static detection resources and an inspection policy for the dynamic detection resources.

The models are built, solved and analyzed using integer programming, stochastic programming and game theory techniques. Structural properties of the models are explored and heuristic algorithms are developed to solve larger problem instances.



**Keywords:** Network interdiction, asymmetric information, perimeter inspection, inspection game, homeland security.

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# 1.0 INTRODUCTION

Terrorist attacks are a concern that must be addressed for both public and private institutions. In assessing terrorist attacks, the goal is to discover the terrorist before an attack occurs or install adequate security measures to deter an attack. One of the main approaches in detecting the attacks is to place sensors that are capable of detecting weapons. There are several types of sensors used to detect different weapons. Among these are sensors used to identify metallic weapons, ceramic weapons, plastic explosives, organic materials, chemical and biological agents. Some of these sensor types are discussed in [13, 15, 16, 32, 40, 43]. Sensors are also used widely in industry to secure buildings and plants, to detect people, vehicles, voice and/or motion. Some of the industrial applications of sensors are discussed in [23, 44]. Preventing smugglers' entry into the country is another security issue that must be addressed by public institutions. Similar sensors to the ones described above as well as other detection resources such as check points, guards and unmanned aerial vehicles are employed at the borders for this purpose.

The U.S. has allocated significant financial resources for the prevention and detection of terrorism and for border security. The budget of the Department of Homeland Security for fiscal year 2008 is $37.7 billion which includes approximately $10.2 billion for customs and border protection, $5 billion for immigration and customs enforcement, $6.4 billion for Transportation Security Administration, and $8.8 billion for the U.S. Coast Guard [1, 2]. The budget also proposes spending $178 million on improved radiological and screening equipment at the borders, and $865 million on new technology to strengthen explosives screening. Annual government expenses on sensors are estimated to be around $421 million [13]. The market size for devices to screen people is estimated to have grown from $590 million in 2001 to $800 million in 2003, and is expected to grow to $1.85 billion in 2010 [8].

To secure the borders, U.S. Customs and Border Protection plans to hire 3000 new border patrol agents bringing the total number to 17819 and 151 new pilots, air crew and specialists to control the border using air fleet; to invest $1 billion to build fences; and to invest $100 million to build new facilities such as checkpoints and remote forward operating bases [1].

Considering the quantity of financial resources invested in detection resources, it becomes imperative to use them effectively. One way to achieve this goal is to improve the design of allocation schemes for these resources. In designing an allocation scheme, the decision to be made is which resources to use and where to place them in order to minimize the risk of attack, or maximize the detection probability of the terrorist or the smuggler.

In order to analyze these allocation schemes we define two entities with conflicting objectives. The first entity is the evader (referred to as he), which may be an terrorist attempting to reach a destination or a smuggler attempting to smuggle materials through a border. The second entity is a protector (referred to as she) who attempts to detect the terrorist before they reach their destination or to prevent a smuggler from smuggling materials.

An evader must pass through regions to reach his destination. The path he follows may be probabilistic or deterministic. The protector allocates her available resources to the regions on the evader's path. Each detection device has a chance of detecting the evader if he passes through the corresponding region. The protector's decision is to determine the optimal allocation scheme based on her risk criteria, and the evader's decision is to select a path that minimize's his detection probability.

The allocation of detection resources to the regions have mainly been studied in two categories: network interdiction models and inspection game models. In the network interdiction models, first the protector allocates her available detection resources to the regions, then the evader selects his path considering these detection resources. Note that, the detection resources stay where they are located after the initial allocation, i.e., the detection resources in the network interdiction models are static. In the inspection game models, the protector devises an inspection policy which is comprised of deciding which regions and/or when to inspect, while the evader selects a policy deciding where and/or when and/or how much material to smuggle. Both parties make their decisions simultaneously without knowing the other's policy. Since the protector inspects different regions at different time periods, the detection resources are dynamic in the inspection game models. Namely, the network interdiction models deal with the allocation of static detection resources, while the inspection game models deal with the allocation of dynamic inspection resources.

We consider two different models to allocate the detection resources optimally. The first model is a shortest path network interdiction model that determines the optimal allocation of the static detection resources. In this model, we utilize information asymmetry between the evader and the protector in terms of the detection capabilities of the detection resources. In the model, protector's objective is to maximize the detection probability of the evader, while the evader's objective is to minimize his detection probability. We also consider an extension of this model in which the worst case detection probability should be greater than a threshold value. The second model is a combination of a network interdiction model and an inspection game model. In this model, first the protector allocates her available static detection resources to the regions, then the protector and the evader play an inspection game. In the inspection game part of the model, the protector determines which region to inspect at each period using his dynamic detection resources, and the evader determines how much material he should smuggle through each region. To objective of the protector is to minimize the amount of the materials smuggled using her static and dynamic detection resources, while the objective of the evader is to smuggle as much material as possible.

## 1.1 LITERATURE REVIEW

### 1.1.1 Network Interdiction Models

Network interdiction models consist of two entities with conflicting objectives. The evader operates in a network in order to optimize his objective function (maximize flow, minimize the length of the shortest path, etc.), and the protector attempts to negatively impact the evader's objective function by interdicting some of the arcs using limited interdiction resources. Two main categories of network interdiction models are shortest path network interdiction models and maximum flow network interdiction models.

Network interdiction has attracted some interest in literature. One of the earliest works in maximum flow network interdiction is by Wollmer [52] in which he studied the effects of removing arcs from a network. The objective of the problem is to find $k$ arcs which cause greatest decrease in the maximum flow from the source to the sink if they are removed from the network. The author proposed a finitely terminating algorithm that solves the problem optimally. However, the algorithm is only valid for planar networks. McMasters and Mustin [39] extended this study to allow partial decreases in the capacities of the arcs. The capacities of the arcs are decreased linearly with the amount of resource used, and there is a limited amount of resource to be used for interdicting the arcs. The network has to be planar in this case, too. They solved the problem using a finitely terminating algorithm based on the minimum cut problem. Wood [53] studied the maximum flow network interdiction problem for general networks. In this problem, interdiction of an arc removes the arc from the network. He formulated the problem as an integer program and solved using standard integer programming techniques. He showed that this problem is NP-complete. He also extended the problem to allow partial arc interdictions, multiple source and sink nodes, undirected arcs, multiple interdiction resources, and multiple commodities. In addition, he proposed valid inequalities that give stronger formulations for the problem.

Cormican et al. [21] studied stochastic extensions of maximum flow network interdiction problem. In the problem, the interdiction successes are binary, i.e., if an interdiction is successful, then the capacity of the related arc is decreased by a fixed amount. In this case, the problem is a two stage stochastic integer program. They solved the problem by using sequential approximation algorithms, and stochastic programming techniques. The authors made some extensions to the problem to allow binary arc capacities, uncertain arc capacities that can take on a finite number of nonnegative values, and multiple uncertain interdictions on an arc.

One of the earliest works about the shortest path network interdiction is by Fulkerson and Harding [27]. In the problem they studied, the lengths of the arcs are increased linearly with the amount of resource used. The objective is to maximize the shortest path between two nodes by interdicting the arcs using a limited amount of resources. The authors showed that this problem is equivalent to a minimum cost flow problem. Israeli and Wood [34] studied the same problem with binary interdiction effort. They formulated this problem as an integer program and solved using branch-and-bound techniques. They also employed Benders decomposition algorithm, developed super-valid inequalities which improve the efficiency of Benders decomposition, and presented a covering decomposition particularly useful for the problems in which interdictions destroy arcs. Bayrak and Bailey [12] extended this problem to investigate the case where there is information asymmetry between the protector and the evader in terms of the arc length perceptions. They formulated this problem as a non-linear mixed integer program, provided a mixed integer program reformulation, and solved using standard branch-and-bound techniques. They also provided an algorithm to accelerate the solution speed. Held et al. [31] considered the shortest path network interdiction problem with binary interdiction effort for the case where arc lengths are stochastic and with the objective of maximizing the probability of sufficient disruption. They solved the problem using a decomposition algorithm. This model is appropriate especially for the networks for which a failure may be catastrophic. Pan et al. [42] considered a network interdiction problem in which the evader attempts to avoid detection while the protector attempts to maximize the detection probability. This problem is equivalent to that of Israeli and Wood [34], but they extended it by allowing unknown origin/destination pairs.

Network interdiction problems also have been analyzed in the context of game theory. Washburn and Wood [50] studied a long term interdiction problem using game theory. A single evader attempts to traverse a path between two nodes, and a single protector attempts to detect the evader by setting up an inspection point on one of the arcs. There is a fixed probability of being detected on each arc known to both the evader and the protector. The evader's objective is to find a probabilistic path selection strategy that minimizes the probability of being detected, and protector's objective is to find a probabilistic arc inspection strategy that maximizes the detection probability. The authors showed that such strategies can be found in polynomial time by solving a min-cut problem. They also solved the problem with unknown origins and destinations, multiple protectors or evaders, undirected arcs, and node interdictions. Bailey et al. [7] presented a more general form of stochastic network interdiction in which the protector interdicts the state-action rewards in an adversary's stochastic dynamic programming network.

### 1.1.2 Inspection Game Models

Inspection games typically involve a violator (or a set of violators) attempting to gain benefit by violating laws (e.g., environmental disposal restrictions), and an inspector (or set of inspectors) attempting to prevent or minimize violations by conducting inspections. The inspector decides when and/or which regions to inspect, while the violator decides on the amount, type, location, and time of the violations. Thomas and Nisgav [47], Baston and Bostock [11], and Garnaev [29] investigated the problem of a patrol attempting to stop a smuggler who is attempting to ship a cargo of contraband across a border. Thomas and Nisgav [47] studied the case where the inspector has a speedboat with which he can patrol during $k$ of $n$ nights. The inspector determines which dates to patrol, and the smuggler decides which date to ship. Baston and Bostock [11] studied the case where the inspector has two boats which can patrol $k_1$ and $k_2$ of $n$ nights respectively. Garnaev [29] extended these studies to allow the inspector to have three boats, and determined optimal policies. Garnaev et al. [28] considered an inspection game in which an evader attempts to go from an origin node to a destination node on a graph of $n$ arcs within a time limit without being caught, whereas an inspector attempts to catch the evader making a restricted number of inspections. They described optimal strategies and provided the value of this discrete zero-sum inspection game. Ferguson and Melolidakis [24] considered an inspection game where the smuggler can act more than once. In their model, a smuggler attempts to smuggle $l$ units in $n$ days, whereas the inspector can inspect $k$ of these days. They found optimal policies explicitly for this problem using the results of previous studies.

Canty et al. [18] studied the inspection problem for a single location which stores large numbers of identical items. They modeled this problem as a two-person, sequential game. They parameterized the timely detection of illegal activity in terms of a critical time to detection, and derived equilibria, which provide inspection policies. They also discussed the necessary conditions for deterrence of illegal behavior. Filar [26] considered the problem of dynamic inspection of a number of facilities in different locations. The inspector travels from location to location and inspects the facilities he visits aiming to minimize the losses due to undetected violations and traveling cost. Filar formulated this problem as a noncooperative, single-controller, stochastic game. He showed that violators can be aggregated into a single violator under mild conditions, and Nash equilibrium exists for this problem. He also discussed the issue of the inspector's power to enforce such an equilibrium. Avenhaus and Kilgour [6] considered an inspection game with two inspectees and imperfect inspections in the sense that violations can be missed. They characterized the resource level adequate for deterrence. They showed that when the detection probabilities are increasing in inspection resources, it is possible to describe optimal allocation policies for both special and general cases. They also showed that when detection probabilities are convex inspection efforts should be concentrated on one inspectee, whereas when they are concave it should be spread deterministically over the inspectees. They proposed that a priori constraints on the distribution of inspection effort can result in significant inefficiencies.

Canty et al. [19] considered a critically time-dependent inspection problem which is modeled as two-person non-cooperative game. In the problem, over a reference time interval the inspector performs precisely $k$ inspections and the inspectee behaves illegally at most once. The inspections are assumed to incur both types of errors and first type error is known both by the inspector and the inspectee. They also considered the variants of the problem in which the inspectee may or may not be in a position to take advantage of information gained during the reference time interval. They investigated equilibria and determined conditions for the existence of deterring inspection strategies. Avenhaus and Canty [5] considered a sequential two-person inspection game in which inspections are carried out for timely detection of illegal activity on a finite, closed time interval and subject to first and second kind errors. In the model, the utilities of the inspector and inspectee are assumed to be linear in the detection time with time-independent false alarm costs. They obtained sets of Nash equilibria in which the inspectee behaves illegally or legally with probabilities one.

## 1.2 CONTRIBUTION

### 1.2.1 Network Interdiction with Asymmetric Information

In all of the previous network interdiction models, it was assumed that both the protector and the evader have the same knowledge about the network. Both decision makers know the arc lengths (or the probability distributions if the arc lengths are stochastic) before and after the interdiction. This amounts to an interdiction scheme based on worst-case analysis. In practice, the evader may not know the true arc lengths, and instead has estimates of them. He will select his optimal path based on these estimates which can lead to a path other than the true optimal path. We exploit this asymmetry in information to better utilize available resources. In the context of detection resource allocation problem, the arc lengths correspond to the detection probabilities. Therefore, information asymmetry in terms of arc lengths corresponds to information asymmetry in terms of detection probabilities. Namely, the protector knows the detection capabilities of the resources used since she is the one who installs them. However, the evader may not know the true detection capabilities of the resources, instead he will use his estimates to evaluate his objective function. By utilizing this incomplete information of the evader, the protector can increase the detection probability of the evader. Also, by making sensitivity analysis of the results, the protector can decide how much she should spend on intelligence about the estimates of the evader. Overall, this study allows us to analyze the risks and benefits of modeling information asymmetry between the protector and the evader.

This study is the first one to incorporate information asymmetry in the network interdiction models. Network interdiction problems are often complex and difficult to model since they include both the design and the execution phases. Including information asymmetry makes the problem even more complicated. We were able to model the problem as a nonlinear mixed integer program, and converted it to a mixed integer program using linearization techniques and exploiting some structural properties of the problem. We also developed an algorithm that decreases the computation time to solve the problem.

### 1.2.2 The Perimeter Inspection Game with Interdiction

Perimeter security problems have been analyzed using network interdiction models and inspection game models. Network interdiction models assist the protector in optimally allocating her static detection resources to the regions, whereas the inspection game models assist the protector in optimally allocating her dynamic detection resources to the regions and to develop inspection policies. For perimeter security problems, the protector will often have both static and dynamic detection resources. Not taking some of the detection resources into account while making allocation decisions would lead to suboptimal results. However, until now no study has taken both dynamic and static detection resources into account at the same time. This dissertation fills this gap between the network interdiction models and inspection game models, and provide a more complete model for perimeter security problems by taking both kinds of detection resources into account. This comprehensive model comprises the network design capability of the network interdiction models and dynamic nature of the inspection game models.

We modeled the perimeter security problem with the allocation of static and dynamic detection resources as non-linear mixed integer program by using the linear programming formulation of competitive Markov decision processes, and converted this complex formulation into a mixed integer program using linearization techniques. We investigated some structural properties of the problem to simplify the formulation even further. We also provided explicit solutions for some simple cases and developed a heuristic algorithm to solve the larger problem instances.

## 1.3 OVERVIEW OF THE DISSERTATION

The remainder of this dissertation is organized as follows:

Chapter 2 presents the shortest path network interdiction problem with information asymmetry. It analyzes the case where the evader does not know the true arc lengths, and uses his estimates of these arcs lengths, but the protector knows both the true arcs lengths and the evader's estimates of these arc lengths. The problem is defined, formulated and solution techniques are developed. Finally, the benefits and risks of this model is illustrated with computational examples.

Chapter 3 presents the perimeter inspection game with interdiction. This model is developed to help the protector to allocate her dynamic and static detection resources to minimize the amount of the materials smuggled by the evader. The problem is formulated, its structural properties are investigated and a heuristic algorithm is developed to solve the problem more efficiently, and finally computational examples are presented.

Chapter 4 extends the model developed in Chapter 2 by including a risk criterion in the model. The criterion is that the risk arising due to modeling asymmetric information should not be greater than a prespecified threshold value. This model is formulated and computational examples are given to analyze the risk and benefits of this model. Finally, a stochastic model is introduced in which there is a complete information asymmetry between the protector and the evader.

Chapter 5 analyzes the results obtained from the models in Chapter 2, Chapter 3 and Chapter 4, and derives conclusions based on these results. The chapter concludes with a brief explanation of future research directions that are suggested by this dissertation.

# 2.0 SHORTEST PATH NETWORK INTERDICTION WITH ASYMMETRIC INFORMATION

## 2.1 INTRODUCTION

In this chapter we will discuss the allocation of the static detection resources to the regions in order to maximize the detection probability of the evader. First, we will demonstrate the equivalence between this problem and shortest path network interdiction problems. Then, we will formulate the problem, develop solution methodologies and finally present computational examples to illustrate the effectiveness of the model developed.

In the problem we consider, there is an evader who is attempting to reach from an origin to a destination without being detected, and there is a protector who is trying to catch the evader before he reaches his destination. First, the protector allocates her available detection resources to the regions to increase the detection probability of the evader. Then, the evader takes the path on which his detection probability is minimum to reach the destination. The protector's decision is to determine to which regions to allocate the detection resources, and the evader's decision is to select the path that minimizes his detection probability. At each region the evader passes through, there is a chance that he will be caught. The probability that he will be caught at that region depends on whether there is a detection resource there or not. We represent the evader's detection probability at region $k$ with $p_k$ if there is no detection resource at that region, and with $\bar{p}_k$ if there is a detection resource at that region. Also, let $P$ represent the set of regions that the evader passes through.

Finally, let $x_k$ be the binary variable indicating whether or not there is a detection resource at region $k$. The probability that the evader will be detected can be calculated as follows,

$$\text{P(evader will be detected)} = 1 - \prod_{k \in P} [(1 - x_k)(1 - p_k) + x_k(1 - \bar{p}_k)]. \tag{2.1}$$

The protector wants to determine the values of $x_k$'s that maximize the probability in (2.1), while the evader wants to determine the set of regions $P$ that minimizes the same probability. The probability expression in (2.1) is highly non-linear since binary variables are multiplied. However, this expression can be converted to a linear expression by taking the logarithm of it. Since logarithm is a monotonic function, maximizing or minimizing an expression is equivalent to maximizing or minimizing the logarithm of the same expression. For this purpose we define $c_k$ and $d_k$ as follows,

$$\begin{aligned} c_k &= -\log(1 - p_k), \\ d_k &= \log(1 - p_k) - \log(1 - \bar{p}_k). \end{aligned}$$

Using these new definitions, we can express the logarithm of the detection probability as follows,

$$\log \text{P(evader will be detected)} = \sum_{k \in P} (c_k + x_k d_k). \tag{2.2}$$

The right-hand side of (2.2) is the length of the path the evader takes, with arc lengths $c_k + x_k d_k$. The length of the arc corresponding to region $k$ is $c_k$ if there is no detection resource at that region, and it is $c_k + d_k$ if there is a detection resource there. Using the equivalence between (2.1) and (2.2) we can restate the detection resource allocation problem as follows: An evader attempts to go from an origin to a destination by taking the shortest path, while a protector attempts to maximize the length of evader's shortest path by interdicting a subset of the arcs using his limited interdiction budget. This problem is known as the shortest path network interdiction problem (e.g., [12, 30, 34]).

Network interdiction problems consist of two entities with conflicting objectives. The evader operates in a network in order to optimize his objective function (maximize flow, minimize the length of the shortest path, etc.), and the protector attempts to worsen the evader's objective function by interdicting the arcs in order to increase the arc length, or decrease the arc capacity using limited interdiction resources. In shortest path network interdiction problems, an evader attempts to traverse the shortest path between the origin and the destination, while an protector attempts to maximize the length of this shortest path by interdicting network arcs using limited resources.

In this chapter, we consider an extension of the shortest path network interdiction problem, where there is information asymmetry between the protector and the evader about the network. We consider the case where the information asymmetry is in the arc lengths. Information asymmetry occurs frequently in network problems with more than one operator, since each operator has his/her own perception of the network and makes decisions accordingly. From the detection resource allocation problem perspective, the information asymmetry occurs in the detection capabilities of the resources. Typically, the protector would know the detection capabilities of the detection resources since she is one who installs them. However, the evader may not know the true detection probabilities of the detection resources. In order to decide which path to take from the origin to the destination, he will use his best available estimates of the detection probabilities. This will result in an information asymmetry between the protector and the evader. The information asymmetry may lead the evader to take a path that is suboptimal in terms of the overall detection probability. The protector may utilize the fact that the evader does not have the perfect information about the detection probabilities by changing his detection resource allocation scheme based on the estimates of the evader. We will provide a model that helps the protector to achieve this goal.

In previous interdiction models, it was assumed that both the protector and the evader have the same knowledge about the network. Both decision makers know the arc lengths (or the probability distributions if the arc lengths are stochastic) before and after the interdiction. This amounts to an interdiction scheme based on worst-case analysis. In practice, the evader may not know the true arc lengths, and instead has estimates of them. He will select his optimal path based on these estimates which can lead to a path other than the true optimal path. We exploit this asymmetry in information to better utilize available resources.

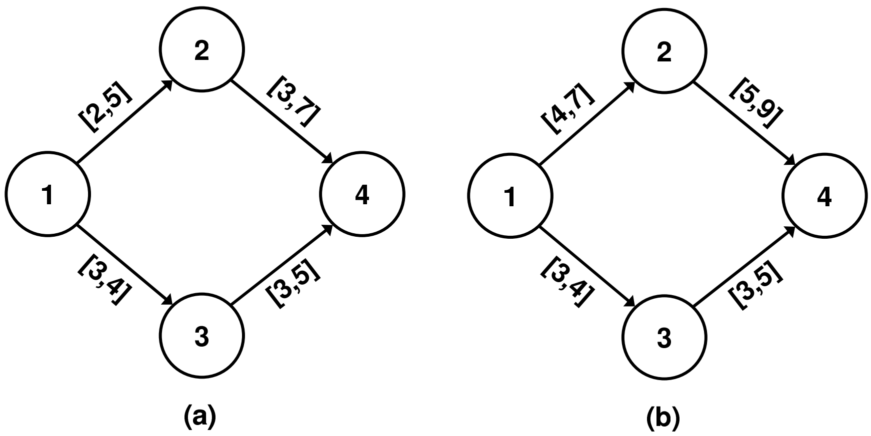


Figure 2.1: Asymmetric information networks

Assuming that the evader has the same information as the protector may lead to suboptimal solutions. We illustrate this with the following example. Consider the two networks in Figure 2.1, where network (a) is the true network and network (b) is the evader's perceived network. The numbers in brackets with each arc are the current length and the length if interdicted. In this example the evader overestimates the current and interdicted lengths of the arcs (1,2) and (2,4). Assume that as a result of budget constraints the protector can interdict only one arc. In standard network interdiction it is assumed that the evader has the same information as the protector. As a result, the optimal action is to interdict the arc (1,2) or (2,4) assuming that the evader will select the shortest path 1-3-4 with a length of six. After the interdiction, the evader will select this path based on his estimates of the arc lengths. However, if the protector utilized the estimates of the evader, then the optimal action would be to interdict the arc (3,4). After the interdiction of the arc (3,4), the evader will select the path 1-3-4. The length of the path will be eight in this case. As illustrated in the example above, knowing that the evader does not have full information about the network, the protector can increase the length of the path the evader traverses. As a result, including the estimates of the evader in the model may increase the optimal objective function value of the protector.

Asymmetric information is standard in game theory and bilevel programming. See [22, 36, 37, 38, 46] for some of the recent work. However, it has not received much attention in the network interdiction literature. The objective of this chapter is to introduce asymmetric information in the network interdiction literature and investigate its benefits and risks. In the problem we consider, we assume that the protector knows the evader's estimates of the arc lengths, and model the problem based on this assumption. However, if these estimates are incorrect, the protector incurs a risk by not interdicting certain arcs. This issue will be addressed in a later section. In our presentation, we follow a notation similar to Israeli and Wood [34]. We will present the remainder of the chapter using the terminology of the network interdiction problems rather than using the terminology of the detection resource allocation problem to avoid any confusion. The remainder of this chapter is organized as follows. In Section 2.2, we define and formulate the problem. In Section 2.3, we develop methodologies to solve the problem. In Section 2.4, we present a computational study, and provide concluding remarks in Section 2.5.

## 2.2 PROBLEM FORMULATION

### 2.2.1 Problem Definition

Let $G(N, A)$ be a directed graph composed of the node set $N = \{0, 1, \ldots, n\}$ and the arc set $A \subseteq \{(i, j) \mid i, j \in N\}$. We represent the nodes with indices $i$ or $j$. Nodes 0 and $n$ are the origin and destination nodes of the evader. We represent the arcs with arc number $k$, or the node pair $(i, j)$. The length of arc $k$ is $c_k$, and it is increased to $c_k + d_k$ when the arc is interdicted. Let $\bar{c}_k$ and $\bar{d}_k$ be the evader's estimates of $c_k$ and $d_k$, respectively. We also define $b_k$ as the amount of resource required to interdict arc $k$, and $B$ is the amount of available resource available for interdiction. We define $x_k$ as the variable representing whether or not arc $k$ is interdicted, and $y_k$ as the variable representing whether or not arc $k$ is on the path the evader traverses. As a result of the structure of the shortest path problem, $y_k$ is continuous rather than binary. Bold letters represent the vector forms of the corresponding parameters/variables. Finally, let $FS(i)$ be the forward star of node $i$ (set of arcs going out of node $i$), and let $RS(i)$ be the reverse star of node $i$ (set of arcs going into node $i$).

The problem is comprised of two phases. In the first phase, the protector interdicts a subset of the arcs using the available resources. In the second phase, the evader attempts to travel from node 0 to node $n$ in the interdicted network. The evader's objective is to minimize the length of the path he traverses based on his estimates of the arc lengths $\{\bar{\mathbf{c}}, \bar{\mathbf{d}}\}$. We assume he knows which arcs are interdicted. The protector's objective is to maximize the true length of the evader's path. Therefore, she uses $\mathbf{c}$ and $\mathbf{d}$ to evaluate the length of the evader's path, and determines an interdiction plan based on both $\{\mathbf{c}, \mathbf{d}\}$ and $\{\bar{\mathbf{c}}, \bar{\mathbf{d}}\}$.

### 2.2.2 Formulation

For a given interdiction plan $\hat{\mathbf{x}}$, the evader will determine his shortest path by solving the following problem:

$$[\text{SP}(\hat{\mathbf{x}})]: \quad \min_{\mathbf{y}} \sum_{k \in A} (\bar{c}_k + \bar{d}_k \hat{x}_k) y_k, \tag{2.3a}$$

$$\sum_{k \in FS(i)} y_k - \sum_{k \in RS(i)} y_k = \begin{cases} 1, & i = 0, \\ 0, & i = 1, 2, \dots, n-1, \\ -1, & i = n, \end{cases} \tag{2.3b}$$

$$y_k \geq 0, \quad \forall k \in A.$$

Note that in (2.3a) the perceived length of arc $k$ is $\bar{c}_k + \bar{d}_k \hat{x}_k$. Then the shortest path network interdiction problem with asymmetric information can be formulated as the following bilevel program:

$$
\begin{aligned}
\text{[SPNIA-BL]:} \quad \max_{\mathbf{x}} \sum_{k \in A} (c_k + d_k x_k) y_k^*, \qquad & & & \text{(2.4a)} \\
\sum_{k \in A} b_k x_k & \leq & B, & \text{(2.4b)} \\
x_k & \in & \{0,1\}, \quad \forall k \in A, &
\end{aligned}
$$

where,

$$
\mathbf{y}^* = \arg \left\{ \min_{\mathbf{y}} \sum_{k \in A} (\bar{c}_k + \bar{d}_k x_k) y_k \right\}, \tag{2.5a}
$$

$$
\begin{aligned}
\sum_{k \in FS(i)} y_k - \sum_{k \in RS(i)} y_k & = & \begin{cases} 1, & i = 0, \\ 0, & i = 1, 2, \ldots, n-1, \\ -1, & i = n, \end{cases} \qquad \text{(2.5b)} \\
y_k & \geq & 0, \quad \forall k \in A.
\end{aligned}
$$

The above bilevel problem consists of an outer optimization problem (2.4a) and (2.4b) associated with the protector, and an inner optimization problem (2.5a) and (2.5b) associated with the evader.

## 2.3 SOLUTION METHODOLOGY

Bilevel programming problems have been studied extensively in the literature. There are several exact and approximate solution algorithms to solve bilevel problems, such as extreme point algorithms [14, 17], branch and bound algorithms [9, 10, 41], descent methods [35, 49], penalty function methods [3, 51], and reformulation [4]. For a detailed review of the solution methods, see the papers by Vicente and Calamai [48] and Colson et al. [20]. We reformulate [SPNIA-BL] as a mixed integer nonlinear program, and provide a conversion to a mixed integer linear program. This allows us to solve the problem optimally, and handle problems of reasonable sizes.

### 2.3.1 MINLP Reformulation

Let $u_i$ be the dual variable corresponding to node $i$ in (2.3b). Then the dual problem of [SP($\hat{\mathbf{x}}$)] is:

$$[\text{SPD}(\hat{\mathbf{x}})]: \quad \max_{\mathbf{u}} u_0 - u_n, \tag{2.6a}$$

$$u_i - u_j \leq \bar{c}_k + \bar{d}_k \hat{x}_k, \quad \forall (i,j) = k \in A. \tag{2.6b}$$

In the above formulation, $u_0 - u_i$ represents the length of the shortest path from node 0 to node $i$ based on the arc lengths $\bar{c}_k + \bar{d}_k \hat{x}_k$. The protector evaluates the length of the path found in [SP($\hat{\mathbf{x}}$)] using the arc lengths $c_k + d_k \hat{x}_k$, resulting in the following problem:

$$[\text{SPNIA}(\hat{\mathbf{x}})]: \quad \max_{\mathbf{y},\mathbf{u}} \sum_{k \in A} (c_k + d_k \hat{x}_k) y_k, \tag{2.7a}$$

$$\sum_{k \in FS(i)} y_k - \sum_{k \in RS(i)} y_k = \begin{cases} 1, & i = 0, \\ 0, & i = 1, 2, \ldots, n-1, \\ -1, & i = n, \end{cases} \tag{2.7b}$$

$$u_i - u_j \leq \bar{c}_k + \bar{d}_k \hat{x}_k, \quad \forall (i,j) = k \in A, \tag{2.7c}$$

$$u_n - u_0 + \sum_{k \in A} (\bar{c}_k + \bar{d}_k \hat{x}_k) y_k = 0, \tag{2.7d}$$

$$y_k \geq 0, \quad \forall k \in A.$$

Note that (2.7b) is the same as (2.5b), and (2.7c) is the same as (2.6b). Also, (2.7d) states that the objective function values of [SP($\hat{\mathbf{x}}$)] and [SPD($\hat{\mathbf{x}}$)] should be equal. From duality theory we know that these two values will be equal only when $\mathbf{y}$ is an optimal solution for [SP($\hat{\mathbf{x}}$)], and $\mathbf{u}$ is an optimal solution for [SPD($\hat{\mathbf{x}}$)]. Therefore, any feasible solution for [SPNIA($\hat{\mathbf{x}}$)] will be an optimal solution for [SP($\hat{\mathbf{x}}$)] and [SPD($\hat{\mathbf{x}}$)]. Note that the objective function used in [SPNIA($\hat{\mathbf{x}}$)] does not affect this result. Since we want to evaluate the shortest path found in [SP($\hat{\mathbf{x}}$)] using the arc lengths $c_k + d_k \hat{x}_k$, we use the objective function (2.7a). As a result, for a given interdiction plan $\hat{\mathbf{x}}$, [SPNIA($\hat{\mathbf{x}}$)] will find the shortest path from node 0 to node $n$ using the arc lengths $\bar{c}_k + \bar{d}_k \hat{x}_k$, and evaluate it using the arc lengths $c_k + d_k \hat{x}_k$.

Finally, we release $\hat{\mathbf{x}}$ as a variable resulting in the formulation for our complete model of shortest path network interdiction with asymmetric information:

$$\text{[SPNIA]:} \quad \max_{\mathbf{x},\mathbf{y},\mathbf{u}} \sum_{k\in A}(c_k + d_k x_k) y_k, \tag{2.8a}$$

$$\sum_{k\in FS(i)} y_k - \sum_{k\in RS(i)} y_k = \begin{cases} 1, & i=0, \\ 0, & i=1,2,\ldots,n-1, \\ -1, & i=n, \end{cases} \tag{2.8b}$$

$$u_i - u_j - \bar{d}_k x_k \le \bar{c}_k, \quad \forall (i,j)=k\in A, \tag{2.8c}$$

$$u_n - u_0 + \sum_{k\in A}(\bar{c}_k + \bar{d}_k x_k) y_k = 0, \tag{2.8d}$$

$$y_k \ge 0, \quad \forall k\in A,$$

$$\mathbf{x} \in X,$$

where $X = \{\mathbf{x} \in \{0,1\}^{|A|} \mid \mathbf{b}^{\mathbf{T}}\mathbf{x} \le B\}$ is the set of feasible interdiction plans.

**Remark 1** For a given $\hat{\mathbf{x}}$, in the optimal solution of [SP($\hat{\mathbf{x}}$)], $\mathbf{y}$ will only take binary values even though it is not restricted as such. This fact may not always be valid for [SPNIA($\hat{\mathbf{x}}$)] since constraint (2.7d) links variables $\mathbf{y}$ and $\mathbf{u}$. The variables $\mathbf{y}$ can take non-integer values only when there are multiple optimal paths from node 0 to node $n$ that have the same lengths when evaluated both with the arc lengths $c_k + d_k x_k$ and $\bar{c}_k + \bar{d}_k x_k$. Even if this occurs, [SPNIA($\hat{\mathbf{x}}$)] will still give the correct optimal objective function value, and hence [SPNIA] will give the correct optimal $\mathbf{x}$ values. Subsequently, the corresponding integer $\mathbf{y}$ values can be computed easily by solving a shortest path problem.

**Remark 2** In the above formulation, if there are multiple shortest paths for the evader, then we assume that he will take the one which is longer when evaluated with the arc lengths $c_k + d_k x_k$. This can be interpreted as the cooperation of the evader with the protector (or the best case scenario) in case of multiple shortest paths. However, if he traverses another path, then the length of his path will be shorter than expected which results in an incorrect interdiction plan. We will also present another formulation for which this assumption is not necessary.

### 2.3.2 Linearization

[SPNIA] formulation is a MINLP which consists of quadratic terms in both the objective function and the constraints. One approach to solve this problem is to linearize the quadratic terms. For this purpose, we replace $y_k$ with two new variables $v_k$ and $w_k$: $v_k$ can take positive values if arc $k$ is not interdicted, and $w_k$ can take positive values if arc $k$ is interdicted. We can now reformulate the problem as follows:

$$\text{[SPNIA-L]:} \quad \max_{\mathbf{x},\mathbf{v},\mathbf{w},\mathbf{u}} \sum_{k\in A}(c_k v_k + (c_k + d_k)w_k), \tag{2.9a}$$

$$\sum_{k\in FS(i)} (v_k + w_k) - \sum_{k\in RS(i)} (v_k + w_k) = \begin{cases} 1, & i = 0, \\ 0, & i = 1, 2, \ldots, n-1, \\ -1, & i = n, \end{cases} \tag{2.9b}$$

$$u_i - u_j - \bar{d}_k x_k \leq \bar{c}_k, \quad \forall (i,j) = k \in A, \tag{2.9c}$$

$$u_n - u_0 + \sum_{k\in A}(\bar{c}_k v_k + (\bar{c}_k + \bar{d}_k)w_k) = 0, \tag{2.9d}$$

$$v_k + x_k \leq 1, \quad \forall k \in A, \tag{2.9e}$$

$$w_k - x_k \leq 0, \quad \forall k \in A, \tag{2.9f}$$

$$v_k, w_k \geq 0, \quad \forall k \in A,$$

$$\mathbf{x} \in X.$$

The validity of [SPNIA-L] is proven by the following theorem:

**Theorem 2.1** *Let* $(\mathbf{x}^*, \mathbf{v}^*, \mathbf{w}^*, \mathbf{u}^*)$ *be an optimal solution for [SPNIA-L], and let* $z^*$ *be the corresponding objective function value. Then* $(\mathbf{x}^*, \mathbf{y}^*, \mathbf{u}^*)$ *is also optimal in [SPNIA], and the corresponding objective function value is* $z^*$*, where* $y_k^* = \max\{v_k^*, w_k^*\}$ *for every arc.*

**Proof**. Constraints (2.9e) and (2.9f) guarantee that at most one of $v_k$ and $w_k$ will be positive for any $k$. When we set the other variable to 0, the positive one replaces $y_k$ in [SPNIA]. Therefore, for any given $\mathbf{x}$, (2.9a),(2.9b), (2.9c) and (2.9d) are equivalent to (2.8a), (2.8b), (2.8c) and (2.8d) respectively. Hence, [SPNIA] and [SPNIA-L] are equivalent, and the corresponding optimal solutions are also equivalent. Also, $y_k^* = \max\{v_k^*, w_k^*\}$, since $y_k = v_k$ when $w_k = 0$ $(x_k = 0)$, and $y_k = w_k$ when $v_k = 0$ $(x_k = 1)$. □

Now, [SPNIA-L] is a linear MIP, and can be solved using any branch-and-bound algorithm.

### 2.3.3 Alternative Formulation for Worst-case Scenario

As mentioned in Remark 2, in [SPNIA-L] if there are multiple shortest paths for the evader, we assume that he will take the one which is longer when evaluated with the actual arc lengths, which may be interpreted as the best case scenario. However, it is generally more reasonable to assume the worst case scenario. For this purpose, another formulation for the problem can be achieved by utilizing duality theory and penalty functions. We start with the [SPNI($\hat{\mathbf{x}}$)] formulation. Let $\pi$, $\alpha$ and $\gamma$ be the dual variables corresponding to (2.7b), (2.7c) and (2.7d), respectively. Then, the dual of [SPNI($\hat{\mathbf{x}}$)] is,

$$\text{[SPNI-D1}(\hat{\mathbf{x}})\text{]:} \quad \max_{\pi,\alpha,\gamma} \pi_0 - \pi_n + \sum_{k\in A} \alpha_k(\bar{c}_k + \hat{x}_k \bar{d}_k),$$

$$\begin{aligned}
\sum_{k\in FS(i)} \alpha_k - \sum_{k\in RS(i)} \alpha_k &= \begin{cases} \gamma \quad , & i = 0, \\ 0 \quad , & i = 1,\dots,n-1, \\ -\gamma \quad , & i = n, \end{cases} \\
\pi_i - \pi_j + (\bar{c}_k + \hat{x}_k \bar{d}_k)\gamma &\leq c_k + \hat{x}_k d_k, \quad \forall (i,j) = k \in A, \\
\alpha_k &\leq 0, \quad \forall k \in A.
\end{aligned}$$

If we define $\theta = -\gamma$ and $\beta_k = \frac{\alpha_k}{\gamma}$, we can restate [SPNI-D1($\hat{\mathbf{x}}$)] as follows,

$$\text{[SPNI-D2}(\hat{\mathbf{x}})\text{]:} \quad \max_{\pi,\beta,\theta} \pi_0 - \pi_n - \theta \sum_{k\in A} \beta_k(\bar{c}_k + \hat{x}_k \bar{d}_k),$$

$$\begin{aligned}
\sum_{k\in FS(i)} \beta_k - \sum_{k\in RS(i)} \beta_k &= \begin{cases} 1 \quad , & i = 0, \\ 0 \quad , & i = 1,\dots,n-1, \\ -1 \quad , & i = n, \end{cases} \\
\pi_i - \pi_j - (\bar{c}_k + \hat{x}_k \bar{d}_k)\theta &\leq c_k + \hat{x}_k d_k, \quad \forall (i,j) = k \in A, \\
\beta_k &\geq 0, \quad \forall k \in A.
\end{aligned}$$

Now, releasing $\hat{\mathbf{x}}$ as a variable, and fixing $\theta$ as a constant results in the following formulation,

$$
\text{[SPNI-D2}(\hat{\theta})\text{]:} \quad \max_{\pi,\beta,\mathbf{x}} \pi_0 - \pi_n - \hat{\theta} \sum_{k \in A} \beta_k(\bar{c}_k + x_k \bar{d}_k),
$$

$$
\begin{aligned}
\sum_{k \in FS(i)} \beta_k - \sum_{k \in RS(i)} \beta_k &= \begin{cases} 1 \quad , & i = 0, \\ 0 \quad , & i = 1, \ldots, n-1, \\ -1 \quad , & i = n, \end{cases} \\
\pi_i - \pi_j - (\hat{\theta}\bar{d}_k + d_k)x_k &\leq \hat{\theta}\bar{c}_k + c_k, \quad \forall (i,j) = k \in A, \\
\beta_k &\geq 0, \quad \forall k \in A, \\
\mathbf{x} &\in X.
\end{aligned}
$$

Finally, we replace $\beta_k$ with two new variables $v_k$ and $w_k$, and get the following formulation,

$$
\text{[SPNI-D}(\hat{\theta})\text{]:} \quad \max_{\mathbf{x},\mathbf{v},\mathbf{w},\pi} \pi_0 - \pi_n \quad - \quad \hat{\theta} \sum_{k \in A} (\bar{c}_k v_k + (\bar{c}_k + \bar{d}_k) w_k), \tag{2.10a}
$$

$$
\sum_{k \in FS(i)} (v_k + w_k) - \sum_{k \in RS(i)} (v_k + w_k) = \begin{cases} 1, & i = 0, \\ 0, & i = 1, 2, \ldots, n-1, \\ -1, & i = n, \end{cases} \tag{2.10b}
$$

$$
\pi_i - \pi_j - (\hat{\theta}\bar{d}_k + d_k)x_k \leq \hat{\theta}\bar{c}_k + c_k, \quad \forall (i,j) = k \in A, \tag{2.10c}
$$

$$
v_k + x_k \leq 1, \quad \forall k \in A, \tag{2.10d}
$$

$$
w_k - x_k \leq 0, \quad \forall k \in A, \tag{2.10e}
$$

$$
\begin{aligned}
v_k, w_k &\geq 0, \quad \forall k \in A, \\
\mathbf{x} &\in X.
\end{aligned}
$$

In the above formulation, constraint (2.10e) guarantees that $v_k$ replaces $\beta_k$ when $x_k = 0$, and similarly constraint (2.10d) guarantees that $w_k$ replaces $\beta_k$ when $x_k = 1$.

The shortest path network interdiction problem with asymmetric information can be solved optimally by solving [SPNI-D($\hat{\theta}$)] once the optimal value of $\hat{\theta}$ is known. We showed that any value of $\hat{\theta}$ that is greater than a threshold value is optimal for [SPNI-D($\hat{\theta}$)]. To prove this fact, we define $\sigma$ to be the length of the longest path from node 0 to node $n$ in $G(N, A)$ with the arc lengths $c_k + d_k$, and $\delta$ to be the least significant digit (e.g., 0.001) of the evader's estimates of arc costs, the $\bar{c}_k$ and $\bar{d}_k$. The validity of the [SPNI-D($\hat{\theta}$)] formulation is given in the following theorem:

**Theorem 2.2** *For any $\hat{\theta} > \frac{\sigma}{\delta}$, [SPNI-D($\hat{\theta}$)] solves the shortest path network interdiction problem with asymmetric information. If there are multiple shortest paths for the evader, the interdiction plan will be based on the evader's path with the shortest actual path length.*

**Proof.** For any given $\mathbf{x}$, [SPNI-D($\hat{\theta}$)] can be separated into two problems. Constraints (2.10b), (2.10d) and (2.10e) along with the corresponding part of the objective function form a shortest path problem from node 0 to node $n$ on $G(N, A)$ with the arc lengths $\hat{\theta}(\bar{c}_k + \bar{d}_k x_k)$, whereas constraint (2.10c) along with the corresponding part of the objective function form a shortest path problem from node 0 to node $n$ with the arc lengths $\hat{\theta}(\bar{c}_k + \bar{d}_k x_k) + c_k + d_k x_k$. In the latter problem, let $P_1$ and $P_2$ be two distinct paths from node 0 to node $n$. Assume that

$$\sum_{k \in P_1} [\hat{\theta}(\bar{c}_k + \bar{d}_k x_k) + c_k + d_k x_k] \leq \sum_{k \in P_2} [\hat{\theta}(\bar{c}_k + \bar{d}_k x_k) + c_k + d_k x_k].$$

Then,

$$\sum_{k \in P_1} \hat{\theta}(\bar{c}_k + \bar{d}_k x_k) - \sum_{k \in P_2} \hat{\theta}(\bar{c}_k + \bar{d}_k x_k) \leq \sum_{k \in P_2} (c_k + d_k x_k) - \sum_{k \in P_1} (c_k + d_k x_k) \leq \sigma,$$

and

$$\sum_{k \in P_1} (\bar{c}_k + \bar{d}_k x_k) - \sum_{k \in P_2} (\bar{c}_k + \bar{d}_k x_k) \leq \frac{\sigma}{\hat{\theta}} < \delta,$$

by assumption. Therefore,

$$\sum_{k \in P_1} (\bar{c}_k + \bar{d}_k x_k) - \sum_{k \in P_2} (\bar{c}_k + \bar{d}_k x_k) \leq 0,$$

since $\delta$ is the least significant digit of the $\bar{c}_k$ and $\bar{d}_k$. As a result, if $P_1$ is a shortest path on $G(N, A)$ with the arc lengths $\hat{\theta}(\bar{c}_k + \bar{d}_k x_k) + c_k + d_k x_k$, then it must also be a shortest path on $G(N, A)$ with the arc lengths $\hat{\theta}(\bar{c}_k + \bar{d}_k x_k)$. Therefore, the shortest path will be found based solely on the arc lengths $\hat{\theta}(\bar{c}_k + \bar{d}_k x_k)$, and if there is a tie, it will be broken in favor of shortest actual path length. Therefore, in the optimal solution of [SPNI-D($\hat{\theta}$)],

$$\pi_0 - \pi_n - \sum_{k \in A} \hat{\theta}(\bar{c}_k v_k + (\bar{c}_k + \bar{d}_k) w_k) = \sum_{k \in A} (c_k + d_k x_k).$$

Hence, [SPNI-D($\hat{\theta}$)] will find the shortest path based on the arc lengths $\bar{c}_k + \bar{d}_k x_k$, and evaluate its length using the arcs lengths $c_k + d_k x_k$. □

### 2.3.4 Algorithm to Solve [SPNI-D($\hat{\theta}$)]

We have presented two different formulations of the shortest path network interdiction problem, which handle the multiple shortest paths in different ways. [SPNIA-L] assumes evader cooperation when there is a tie, whereas [SPNI-D($\hat{\theta}$)] assumes the worst case scenario and breaks the tie in favor of the actual shortest path. In this sense, it is more reasonable to use [SPNI-D($\hat{\theta}$)]. However, it may be computationally cumbersome to use [SPNI-D($\hat{\theta}$)] if the least significant digit of the arc length estimates is very small, which leads to a large $\hat{\theta}$ value. Our computational experiments show that solving [SPNI-D($\hat{\theta}$)] requires significantly more time than solving [SPNIA-L]. For example, for $B = 5$, it takes 5 to 250 times more time (depending on the problem instance) to solve [SPNI-D($\hat{\theta}$)]. On the other hand, these two formulations almost always give the same solutions. Based on these observations, we present the following algorithm in an attempt to solve [SPNI-D($\hat{\theta}$)] more efficiently.

***Algorithm to solve [SPNI-D($\hat{\theta}$)]***

**Step 1** Solve [SPNIA-L]. Let $z^*$ be the optimal objective function value, and let $\mathbf{x}^*$ be the optimal interdiction plan.

**Step 2** Add

$$\pi_0 - \pi_n - \sum_{k \in A} \hat{\theta}(\bar{c}_k v_k + (\bar{c}_k + \bar{d}_k) w_k) \leq z^*$$

to [SPNI-D($\hat{\theta}$)], and solve this problem using $\mathbf{x}^*$ as the starting solution.

In the algorithm, we use the fact that the objective function value of [SPNI-D($\hat{\theta}$)] is always less than or equal to the objective function value of [SPNIA-L]. If the evader does not have multiple shortest paths, then these two values will be equal, and $\mathbf{x}^*$ will be optimal for [SPNI-D($\hat{\theta}$)], too. Therefore, the solver will terminate the solution at the first iteration. If the evader has multiple shortest paths, then $x^*$ may not be optimal. In this case, $z^*$ will be an upper bound for the objective function value of [SPNI-D($\hat{\theta}$)], and the solver will continue the solution procedure with a tight bound.

### 2.3.5 Benders Decomposition Algorithm

The network interdiction problem with asymmetric information is a two-stage problem. In the first stage the protector determines which arcs to interdict, and the second stage the evader determines which path to take. This two-stage nature of the problem offers that we can apply decomposition algorithms to solve the problem. The first decomposition algorithm that comes to mind is Benders decomposition algorithm. However, because of the non-linearities in the objective function, we cannot apply Benders decomposition algorithm to [SPNIA-BL] formulation of the problem. Therefore, we applied Benders decomposition algorithm to both [SPNIA-L] and [SPNI-D($\hat{\theta}$)] formulations, which are the linearized versions of [SPNIA-BL].

The algorithm is similar for both formulations, therefore we present it for only [SPNI-D($\hat{\theta}$)] formulation. To apply Benders decomposition algorithm we define the master problem and subproblem as follows, where $\epsilon$ is the acceptable optimality gap difference:

$$\begin{aligned}
\text{[RMP]:} \quad & \max_{x,\eta_0} \eta_0, \\
& \mathbf{x} \in X.
\end{aligned}$$

$$\begin{aligned}
\text{[SUB]:} \quad \eta_1 = \max_{\mathbf{v},\mathbf{w},\pi} \pi_0 - \pi_n \quad &- \quad \hat{\theta} \sum_{k \in A} (\bar{c}_k v_k + (\bar{c}_k + \bar{d}_k) w_k), && \text{(2.11a)} \\
\sum_{k \in FS(i)} (v_k + w_k) - \sum_{k \in RS(i)} (v_k + w_k) \quad &= \quad \begin{cases} 1, & i = 0, \\ 0, & i = 1, 2, \ldots, n-1, \\ -1, & i = n, \end{cases} && \text{(2.11b)} \\
\pi_i - \pi_j \quad &\leq \quad \hat{\theta}\bar{c}_k + c_k + (\hat{\theta}\bar{d}_k + d_k)\hat{x}_k, \quad \forall k \in A, && \text{(2.11c)} \\
v_k \quad &\leq \quad 1 - \hat{x}_k, \quad \forall k \in A, && \text{(2.11d)} \\
w_k \quad &\leq \quad \hat{x}_k, \quad \forall k \in A, && \text{(2.11e)} \\
v_k, w_k \quad &\geq \quad 0, \quad \forall k \in A.
\end{aligned}$$

Let $\mu^1$ , $\mu^2$ , $\mu^3$ and $\mu^4$ be the dual vectors corresponding to constraints (2.11b), (2.11c), (2.11d) and (2.11e) respectively. Then, we can apply Benders' decomposition algorithm as follows:

***Benders decomposition algorithm for [SPNI-D($\hat{\theta}$)]***

**Step 1** Set initial solution $\hat{x} = 0$, lower bound $LB \leftarrow 0$, and upper bound $UB \leftarrow \infty$.

**Step 2** Solve subproblem with current $\hat{x}$. Obtain new dual vectors $(\hat{\mu}^1, \hat{\mu}^2, \hat{\mu}^3, \hat{\mu}^4)$, and objective function value $\hat{\eta}_1$. Set $LB \leftarrow \max\{LB, \hat{\eta}_1\}$. If $UB - LB < \epsilon$, STOP.

**Step 3** Add

$$\eta_0 - \sum_{k \in A}(\hat{\theta}\bar{d}_k\hat{\mu}_k^2 + d_k\hat{\mu}_k^2 - \hat{\mu}_k^3 + \hat{\mu}_k^4)x_k \leq \sum_{k \in A}(\hat{\theta}\bar{c}_k\hat{\mu}_k^2 + c_k\hat{\mu}_k^2 + \hat{\mu}_k^3) + \hat{\mu}_0^1 - \hat{\mu}_n^1$$

to the RMP. Reoptimize the RMP,and obtain a new solution $\hat{x}$ and objective function value $\hat{\eta}_0$. Set $UB \leftarrow \hat{\eta}_0$. If $UB - LB < \epsilon$, STOP, otherwise return to **Step 2** with new $\hat{x}$.

Our computational studies showed that Benders decomposition algorithm does not perform well for neither [SPNIA-L] nor [SPNI-D($\hat{\theta}$)] in terms of running time. In most of the instances solving the mixed integer programming formulation directly takes less time than solving the problem using Benders decomposition algorithm. Bender decomposition algorithm does not perform well for [SPNI-D($\hat{\theta}$)] since there are terms including $\hat{\theta}$ both in the objective function and the constraints in the subproblem. This results in large gaps between the lower and upper bounds of the problem and the algorithm converges slowly. Even though there are not terms including $\hat{\theta}$ in the subproblem of the Benders decomposition algorithm for [SPNIA-L], the cuts added to the master problem are not very strong, and the algorithm converges slowly for this case, too. In order to improve the effectiveness of the Benders decomposition algorithm, strong valid inequalities should be added throughout the run, and we are working on this problem. However, these valid inequalities will not be discussed in this dissertation.

## 2.4 COMPUTATIONAL STUDY

In this section we study the computational effectiveness of the proposed algorithms. We also investigate the benefits and risks of modeling asymmetric information. We begin our analysis by detailing the test problem sets.

### 2.4.1 Test Problems and Environment

We created 7 different random directed networks. The networks are $n_1 \times n_2$ grid networks, where $n_1$ is the number of rows and $n_2$ is the number of columns. There exists an arc from each node at grid position $(r, c)$ to the nodes at grid positions $(r, c+1)$, $(r+1, c)$ and $(r+1, c+1)$ if there exists a node at the particular position. In these networks, the node $(1, 1)$ is the origin node and the node $(n_1, n_2)$ is the destination node. The number of rows, columns, nodes and arcs for the instances are given in Table 2.1.

Table 2.1: Network instances used

| Network | Rows | Columns | Nodes | Arcs |
|---|---|---|---|---|
| 1 | 6 | 8 | 48 | 113 |
| 2 | 6 | 10 | 60 | 141 |
| 3 | 6 | 12 | 72 | 169 |
| 4 | 8 | 8 | 64 | 161 |
| 5 | 8 | 10 | 80 | 201 |
| 6 | 8 | 12 | 96 | 241 |
| 7 | 10 | 10 | 100 | 261 |

For each network, we created 10 different instances by using different $\mathbf{c}, \mathbf{d}, \bar{\mathbf{c}}, \bar{\mathbf{d}}$ values. When presenting the results, we will give the average values for these 10 instances. The values of $c_k$ and $d_k$ are real numbers generated from Uniform(1,10). The values of $\bar{c}_k$ and $\bar{d}_k$ depend on the values of $c_k$, $d_k$ and $Q$, where $Q$ represents how precise the estimates of the evader are. The values of $\bar{c}_k$ are generated from Uniform$(c_k \cdot Q, c_k \cdot (2 - Q))$ and the values of $\bar{d}_k$ are generated similarly using $d_k$ instead of $c_k$. We tested three values of $Q$: 0.6, 0.7 and 0.8. We used a cardinality constraint as the budget constraint, i.e., $b_k = 1$ for every arc. We tested four different values of $B$: 5, 10, 15 and 20. We assumed that all arcs are interdictable. The parameters used in the test instances are summarized in Table 2.2.

We used CPLEX 9.0 with default settings to solve the test instances. The programs were coded in C, and run on an Intel Pentium IV computer with 3.05 GHz CPU and 512 MB of RAM.

Table 2.2: Parameters used in test instances

| Parameter | Value |
|---|---|
| $c_k$ | Uniform(1,10) |
| $d_k$ | Uniform(1,10) |
| $Q$ | 0.6, 0.7, 0.8 |
| $\bar{c}_k$ | Uniform$(c_k \cdot Q, c_k \cdot (2 - Q))$ |
| $\bar{d}_k$ | Uniform$(d_k \cdot Q, d_k \cdot (2 - Q))$ |
| $b_k$ | 1 |
| $B$ | 5,10,15,20 |

### 2.4.2 Computational Results

We first investigate the benefits of modeling asymmetric information. We compare our results with the case where there is no information asymmetry. We used the [MXSP-D] formulation used in Israeli and Wood [34] for comparison. Both formulations were solved using standard branch-and-bound algorithms of CPLEX 9.0. The results are presented in Appendix A, and summarized in Table 2.3 and Table 2.4. Table 2.3 summarizes the results for $B = 5$ and $B = 10$, whereas Table 2.4 summarizes the results for $B = 15$ and $B = 20$. In these tables, T represents the solution time in seconds for the [SPNI-D($\hat{\theta}$)] formulation. We used the algorithm explained in Section 3.2 to solve [SPNI-D($\hat{\theta}$)]. The runs are terminated after 3600 seconds, and numbers in parentheses represent the number of instances (out of 10) that are solved within 3600 seconds if not all instances are solved to optimality. Also, %I represents the percent increase in the objective function value when the information asymmetry is introduced, and is calculated as follows. Problem [MXSP-D] is solved using $\mathbf{c}$ and $\mathbf{d}$. This means that the protector plans the interdiction (calculates $\mathbf{x}^*$) assuming that the evader has the same information. Then the evader finds a shortest path using his estimates of the arc lengths $\bar{c}_k + \bar{d}_k x_k^*$. Finally, the true length of this path is calculated using the arc lengths $c_k + d_k x_k^*$. This value gives the length of the path the evader will take if the protector assumes that there is no information asymmetry. We define this value as $z_1^*$, and $z_2^*$ as the optimal objective function value of [SPNI-D($\hat{\theta}$)]. Then %I is calculated using the formula $\%\text{I} = \frac{z_2^* - z_1^*}{z_1^*} \cdot 100$. In problem [SPNI-D($\hat{\theta}$)], we assume that the protector knows the evader's estimates of the arc lengths. However, if this is not the case, then some important paths are left more vulnerable. As a result, if the protector assumes incorrect evader estimates, then a risk arises due to this lack of information. We represent this risk with %R, which is calculated as follows.

Problem [SPNI-D($\hat{\theta}$)] is solved, and an optimal interdiction plan $\mathbf{x}^*$ is determined. Then the true shortest path on this network is determined by using the arc lengths $c_k + x_k^* d_k$. Let the length of this path be $z_3^*$, and define $z_4^*$ as the optimal objective function value of [MXSP-D]. Then %R is calculated using the formula $\%\text{R} = \frac{z_4^* - z_3^*}{z_4^*} \cdot 100$. In Table 2.4, there are some blank entries for %I and %R, since not all instances are solved to optimality. A summary of Table 2.3 and Table 2.4 is presented in Table 2.5 by taking the averages of %I and %R over all network instances. Also, the summary of the results is illustrated in Figure 2.2.

Table 2.3: Comparison of the results for symmetric and asymmetric information for $B = 5$ and $B = 10$

| $Q$ | | 0.6 | | | 0.7 | | | 0.8 | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Network | $B$ | T | %I | %R | T | %I | %R | T | %I | %R |
| 1 | 5 | 0.4 | 4.8 | 5.6 | 0.4 | 3.4 | 5.6 | 0.3 | 2.1 | 1.9 |
| 2 | 5 | 0.5 | 9.0 | 4.9 | 0.9 | 3.9 | 3.9 | 6.7 | 2.2 | 3.1 |
| 3 | 5 | 0.9 | 9.7 | 5.7 | 1.4 | 4.8 | 5.6 | 1.3 | 2.5 | 4.1 |
| 4 | 5 | 1.0 | 8.1 | 5.6 | 0.9 | 6.0 | 5.5 | 1.2 | 2.1 | 1.7 |
| 5 | 5 | 2.3 | 8.1 | 6.9 | 1.4 | 6.2 | 4.4 | 7.5 | 1.8 | 3.6 |
| 6 | 5 | 2.2 | 7.4 | 4.2 | 2.1 | 5.8 | 3.5 | 1.6 | 1.8 | 3.0 |
| 7 | 5 | 3.1 | 8.5 | 7.0 | 2.8 | 3.7 | 4.0 | 2.1 | 1.5 | 1.5 |
| 1 | 10 | 2.1 | 7.1 | 5.4 | 1.5 | 4.8 | 4.7 | 1.0 | 2.3 | 2.3 |
| 2 | 10 | 4.3 | 12.2 | 6.1 | 8.1 | 8.9 | 3.9 | 18.4 | 5.3 | 2.9 |
| 3 | 10 | 10.6 | 12.2 | 9.1 | 68.2 | 6.9 | 5.6 | 36.5 | 4.8 | 4.3 |
| 4 | 10 | 8.5 | 8.3 | 7.2 | 9.3 | 5.5 | 7.8 | 9.3 | 3.5 | 3.8 |
| 5 | 10 | 33.9 | 10.8 | 8.3 | 24.4 | 6.7 | 5.4 | 27.1 | 3.2 | 5.0 |
| 6 | 10 | 35.4 | 9.3 | 6.4 | 64.5 | 6.7 | 3.9 | 71.4 | 2.7 | 3.7 |
| 7 | 10 | 170.7 | 11.4 | 6.8 | 99.7 | 6.7 | 4.2 | 42.7 | 3.2 | 3.2 |

Table 2.4: Comparison of the results for symmetric and asymmetric information for $B = 15$ and $B = 20$

| $Q$ | | 0.6 | | | 0.7 | | | 0.8 | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Network | $B$ | T | %I | %R | T | %I | %R | T | %I | %R |
| 1 | 15 | 3.5 | 7.3 | 6.4 | 4.8 | 4.3 | 5.2 | 4.1 | 2.6 | 2.0 |
| 2 | 15 | 36.0 | 12.5 | 10.4 | 50.1 | 8.5 | 5.9 | 110.8 | 4.4 | 4.3 |
| 3 | 15 | 151.0 | 12.6 | 11.0 | 164.3 | 9.3 | 7.6 | 123.2 | 5.1 | 4.5 |
| 4 | 15 | 87.3 | 13.2 | 9.9 | 149.7 | 10.6 | 6.6 | 94.2 | 5.1 | 3.8 |
| 5 | 15 | 263.4 | 11.7 | 9.5 | 207.3 | 7.2 | 6.7 | 301.4 | 4.3 | 4.1 |
| 6 | 15 | 444.2 | 13.7 | 10.2 | 752.6 | 8.2 | 5.7 | 543.3 | 3.6 | 5.0 |
| 7 | 15 | 770.4 | 13.1 | 10.1 | 927.5 | 8.4 | 5.1 | 508.5 | 4.4 | 2.5 |
| 1 | 20 | 11.5 | 7.6 | 6.0 | 8.2 | 4.5 | 4.3 | 5.0 | 3.3 | 3.3 |
| 2 | 20 | 140.0 | 12.5 | 9.5 | 169.0 | 9.3 | 7.1 | 459.8 | 5.8 | 4.9 |
| 3 | 20 | 320.2 | 13.6 | 11.4 | 767.7 | 9.5 | 6.9 | (7) | | |
| 4 | 20 | 312.4 | 11.4 | 8.8 | 426.1 | 7.9 | 8.7 | 682.7 | 3.9 | 5.0 |
| 5 | 20 | (7) | | | (6) | | | (6) | | |
| 6 | 20 | (6) | | | (5) | | | (6) | | |
| 7 | 20 | (3) | | | (4) | | | (2) | | |

Table 2.5: Summary of benefits and risks

| $Q$ | 0.6 | | 0.7 | | 0.8 | |
|---|---|---|---|---|---|---|
| $B$ | %I | %R | %I | %R | %I | %R |
| 5 | 7.9 | 5.7 | 4.8 | 4.6 | 2.0 | 2.7 |
| 10 | 10.2 | 7.0 | 6.6 | 5.1 | 3.6 | 3.6 |
| 15 | 12.0 | 9.6 | 8.0 | 6.1 | 4.2 | 3.8 |
| 20 | 11.3 | 8.9 | 7.8 | 6.7 | 4.3 | 4.4 |

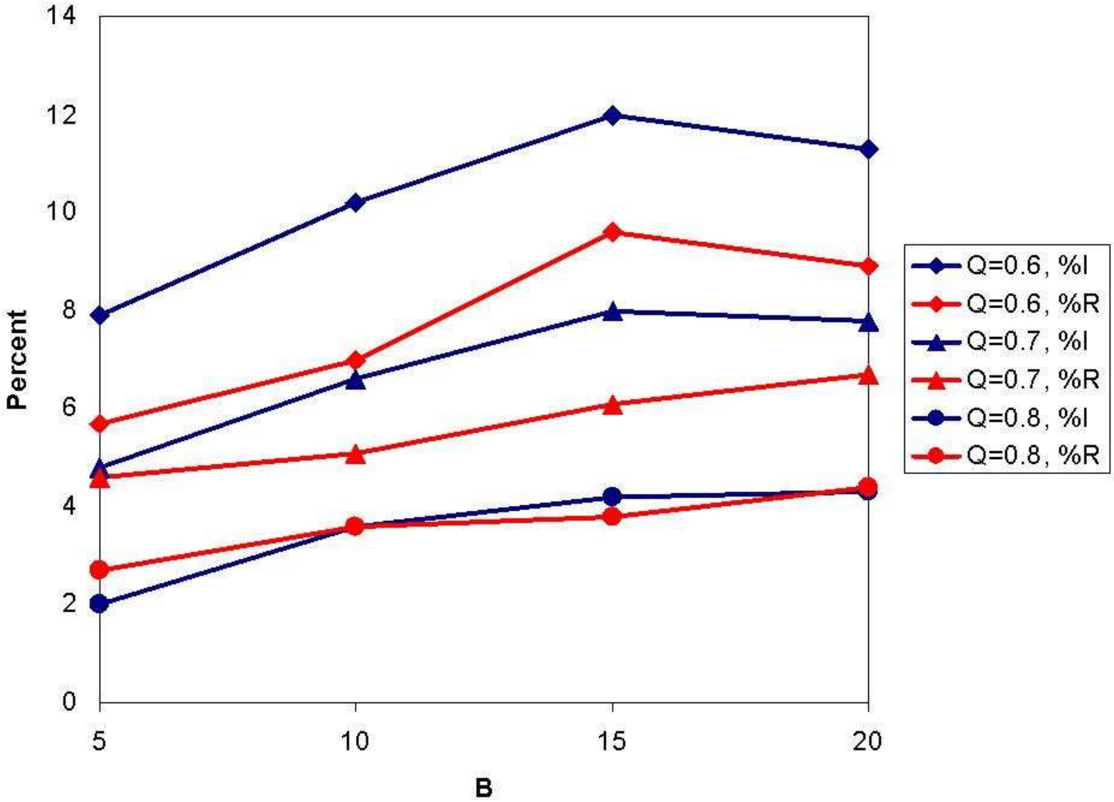


Figure 2.2: Summary of benefits and risks

On average, running times increase as the number of arcs increases and as the available interdiction resources increase. The benefits of modeling asymmetric information tend to increase as the available interdiction resources increase and as the accuracy of the evader's estimates decreases. As a result, if the protector knows that evader's estimates are not very accurate, it may be reasonable to allocate resources to gather better intelligence about the evader. On the other hand, there is a high correlation between the benefit and the risk taken. Therefore, if the protector does not have significant knowledge of the evader's estimates, a fixed estimate may not be preferable. If this is the case, the evader's estimates can be treated as random variables, and the problem can be modeled as a stochastic program. This approach will be explained in Chapter 4.

## 2.5 CONCLUSIONS

In this chapter we studied the detection resource allocation problem. We demonstrated the equivalence between this problem and the shortest path network interdiction problem. In the shortest path network interdiction problem, an evader attempts to go from an origin node to a destination node by taking the shortest path, and an protector attempts to maximize the length of the evader's shortest path by interdicting some of the arcs using limited resources. We extended this problem by assuming that there is an information asymmetry between the evader and the protector, i.e., the evader does not know the true lengths of the arcs. We formulated this problem as a mixed integer nonlinear bilevel programming problem and converted it to a linear MIP. We presented two different formulations that handle the cases where there are multiple shortest paths for the evader. One formulation assumes the worst case scenario, whereas the other assumes the best case scenario. We were able to solve the resulting MIP formulations by using standard branch-and-bound techniques. We also developed an algorithm to solve the problem more efficiently, and discussed the applicability of Benders decomposition algorithm for this problem. Finally, we presented the computational examples and discussed the benefits and risks associated with the modeling of symmetric information.

We demonstrated that the protector can gain benefit by including the information asymmetry between herself and the evader in the model. However, there are some risks in modeling the asymmetric information. The protector should be careful if the evader's estimates are close to the real values, or the protector does not know these estimates accurately.

## 3.0 THE PERIMETER INSPECTION GAME WITH INTERDICTION

### 3.1 INTRODUCTION

In this chapter, we present a model to assist in the allocation of static and dynamic resources for perimeter security. In the problem, the protector controls a perimeter by installing static detection resources and subsequently devising a dynamic inspection policy, whereas the evader attempts to smuggle contraband through the perimeter without being detected. An example of the perimeter is the border between two countries, in which case the static detection resources may be check points and cameras and the dynamic inspection resources may be guards and unmanned aerial vehicles. The problem of static resource allocation while accounting for the subsequent dynamic strategies can be viewed as the interdiction of a competitive Markov decision process (or a stochastic game) [45].

Interdiction models have classically been associated with the static interdiction of networks, while inspection game models have typically only dealt with the dynamic strategies of the protector and the smuggler. In this chapter, we fill the gap between static interdiction models and dynamic inspection game models to provide a more complete model for perimeter security problems.

In the network interdiction models, first the protector interdicts the network, and then the evader operates on the interdicted network. Whereas, in the inspection models, the protector and the evader play an inspection game, but the protector does not have the capability to alter the properties of the network. Namely, the interdiction models lack the dynamic nature of the inspection games, whereas the inspection games lack the network design capability of the protector in network interdiction models. In this chapter, we fill this gap by combining these two problems into a larger problem allowing the protector both to alter the network properties and operate on the interdicted network along with the evader. In our problem, the protector first installs static detection resources at some of the regions using an allocated budget. After the initial installation phase, the protector and the evader play an inspection game over an infinite time horizon.

The remainder of the chapter is organized as follows. In Section 3.2, we present the general framework for single player controlled long-run average reward stochastic games which are the basis for the perimeter inspection problem, and extend this framework to allow interdiction. We present the perimeter inspection problem with interdiction, explore some of the structural properties of this problem, and develop solution methodologies in Section 3.3. We finalize the chapter with computational illustrations in Section 3.4, and concluding remarks in Section 3.5.

## 3.2 STOCHASTIC GAMES WITH INTERDICTION

### 3.2.1 Single Player Controlled Long-run Average Reward Stochastic Games

A zero-sum, single player controlled stochastic game can be viewed as an extension of a standard Markov decision process where the reward is not only dependent on the current state and decision maker's action, but also on the simultaneous action selection of another person. Formally, following the notation of Filar and Vrieze [25], we assume a finite state-space $S$, where $|S| = n$, and a finite set of actions for player 1 in state $s \in S$, being $A^1(s)$, where $|A^1(s)| = m^1(s)$, and $A^2(s)$ for player 2, where $|A^2(s)| = m^2(s)$ . If actions $a^1 \in A^1(s)$ and $a^2 \in A^2(s)$ are selected by players 1 and 2 respectively, then player 1 receives a reward of $r(s, a^1, a^2)$ and player 2 incurs a cost of $-r(s, a^1, a^2)$. The system then transitions from state $s$ to state $s'$ according to probability $p(s'|s, a^1)$ based solely on player 1's action and the current state. This process is repeated over the infinite horizon of the game. Player 1's objective is to select a stationary policy $\mathbf{f}$ to maximize the long-run average reward from the game, where $\sum_{a^1 \in A^1(s)} f(s, a^1) = 1, \forall s \in S$. In other words, player 1 selects action $a^1 \in A^1(s)$ with probability $f(s, a^1)$ in state $s$. We can then define a transition probability under policy $\mathbf{f}$, $p(j|s,\mathbf{f}) = \sum_{a^1 \in A^1(s)} p(j|s,a^1) f(s,a^1)$. Similarly, player 2's objective is to select a policy $\mathbf{g}$ to minimize his average cost. The expected average payoff to player 1 for his policy $\mathbf{f}$ and player 2's policy $\mathbf{g}$ beginning in state $s \in S$ is given by

$$v(s,\mathbf{f},\mathbf{g}) = \lim_{T\to\infty} \left( \frac{1}{T+1} \sum_{t=0}^{T} E(R_t|\ s,\mathbf{f},\mathbf{g}), \right)$$

where $R_t$ is the reward received at period $t$ and $E(\cdot|\ s,\mathbf{f},\mathbf{g})$ is the expected value conditioned on the initial state $s$, and policies $\mathbf{f}$ and $\mathbf{g}$. We select the long-run average reward to capture the stationary behavior of the system. This criterion is appropriate for determining policies to minimize rates of non-critical smuggling, such as illegal narcotics or aliens. However, for critical single occurrence consequences, such as nuclear smuggling, other criteria are more appropriate, but will not be explored in this dissertation.

We say that stationary policies $\mathbf{f}^*$ and $\mathbf{g}^*$ are an optimal stationary policies for players 1 and 2 if

$$v(s,\mathbf{f},\mathbf{g}^*) \le v(s,\mathbf{f}^*,\mathbf{g}^*) \le v(s,\mathbf{f}^*,\mathbf{g})$$

for all $s \in S$ and stationary policies $\mathbf{f}$ and $\mathbf{g}$. We also define $\alpha \in \mathbb{R}^{|S|}$, where $\alpha(s) \ge 0$ and $\sum_{s\in S} \alpha(s) = 1$, as an initial probability distribution over the states. Under this framework, the zero-sum player 1 controlled long-run average reward game can be formulated as the following primal and dual linear programs [33].

**Primal:**

$$\begin{aligned}
&\min_{\mathbf{u},\mathbf{v},\mathbf{g}} \sum_{s\in S} \alpha(s) v(s), \\
u(s) + v(s) &\ge \sum_{a^2\in A^2} g(s,a^2) r(s,a^1,a^2) + \sum_{s'\in S} p(s'|s,a^1) u(s'),\ \forall s \in S, a^1 \in A^1, \\
v(s) &\ge \sum_{s'\in S} p(s'|s,a^1) v(s'),\ \forall s \in S, a^1 \in A^1, \\
\sum_{a^2\in A^2} g(s,a^2) &= 1,\ \forall s \in S, \\
\mathbf{g} &\ge 0.
\end{aligned}$$

The resulting optimal solution $(\mathbf{u}^*, \mathbf{v}^*, \mathbf{g}^*)$ provides the optimal long-run average reward vector for player 1, $\mathbf{v}^*$, and player 2's optimal policy, $\mathbf{g}^*$. Associating vectors $\mathbf{x}, \mathbf{y}$, and $\mathbf{z}$ with the above constraints yields the associated dual formulation.

**Dual:**

$$\begin{aligned} & \max_{\mathbf{x},\mathbf{y},\mathbf{z}} \sum_{s\in S} z(s), \\ \sum_{a^1\in A^1} x(s',a^1) - \sum_{s\in S}\sum_{a^1\in A^1} p(s'|s,a^1)x(s,a^1) &= 0, \ \forall s' \in S, \\ \sum_{a^1\in A^1} x(s',a^1) + \sum_{a^1\in A^1} y(s',a^1) - \sum_{s\in S}\sum_{a^1\in A^1} p(s'|s,a^1)y(s,a^1) &= \alpha(s'), \ \forall s' \in S, \\ \sum_{a^1\in A^1} r(s,a^1,a^2)x(s,a^1) &\geq z(s), \ \forall s \in S, a^2 \in A^2, \\ \mathbf{x}, \mathbf{y} &\geq 0. \end{aligned}$$

The resulting optimal solution $(\mathbf{x}^*, \mathbf{y}^*, \mathbf{z}^*)$ in the dual formulation provides player 1's optimal steady state and transient policies and the average long term reward for each state respectively. Player 1's policy can be determined by

$$f(s,a) = \frac{x(s,a)}{\sum_{a^1\in A^1} x(s,a^1)}, \ \forall s \in S, a \in A^1.$$

### 3.2.2 Single Player Controlled Stochastic Games with Interdiction

We now extend these formulations to permit player 1, prior to the beginning of the game, to allocate his constrained resources to impact the one-period rewards in the subsequent game. We call this action-reward interdiction and represent the interdiction plan with vector $\gamma$. We define $r(s,a^1,a^2,\gamma)$ to be the post-interdiction reward value for state-action set $(s,a^1,a^2)$ for interdiction vector $\gamma$. Since player 1 is interdicting the game, it will typically be true, although not necessary, that $r(s,a^1,a^2,\gamma) \geq r(s,a^1,a^2)$ for all $\gamma$. For example, binary interdiction results in

$$r(s,a^1,a^2,\gamma) = r(s,a^1,a^2) + t(s,a^1,a^2)\gamma(s,a^1,a^2),$$

where $t(s, a^1, a^2)$ is the increase in reward when state-action set $(s, a^1, a^2)$ is interdicted, and $\gamma(s, a^1, a^2)$ is the binary variable indicating whether or not state-action set $(s, a^1, a^2)$ is interdicted. This results in the following mixed-integer nonlinear formulation for the single player controlled game with initial interdiction.

$$
\begin{aligned}
&\max_{\gamma \in \Gamma} \max_{\mathbf{x},\mathbf{y},\mathbf{z}} \sum_{s \in S} z(s), \\
\sum_{a^1 \in A^1} x(s', a^1) - \sum_{s \in S} \sum_{a^1 \in A^1} p(s'|s, a^1) x(s, a^1) &= 0, \ \forall s' \in S, \\
\sum_{a^1 \in A^1} x(s', a^1) + \sum_{a^1 \in A^1} y(s', a^1) - \sum_{s \in S} \sum_{a^1 \in A^1} p(s'|s, a^1) y(s, a^1) &= \alpha(s'), \ \forall s' \in S, \\
\sum_{a^1 \in A^1} r(s, a^1, a^2, \gamma) x(s, a^1) &\geq z(s), \ \forall s \in S, a^2 \in A^2, \\
\mathbf{x}, \mathbf{y} &\geq 0,
\end{aligned}
$$

where $\Gamma$ represents the set of feasible interdiction plans.

## 3.3 PERIMETER INSPECTION PROBLEM WITH INTERDICTION

### 3.3.1 Problem Definition and Formulation

Using our presented framework in Section 3.2, we extend the work of Filar [26] on the traveling inspector problem to include the ability to initially install static detection resources at various locations, in addition to determining a dynamic policy for the protector. The problem we consider consists of two entities with conflicting objectives, a protector and an evader. The evader attempts to bring contraband into a country without being detected, while the protector attempts to minimize the amount of contraband smuggled by allocating her static and dynamic detection resources to the regions on the border.

The problem we model is composed of two stages. In the first stage, which is the design stage, the protector allocates his static detection resources to the regions. In the second stage, which is the execution stage, the protector and the evader play an inspection game over an infinite time horizon. The set of regions on the border are represented with

$N = \{1, 2, ..., n\}$. In the second stage, the protector selects the region to inspect for the next period, i.e., $A^1 = \{1, 2, ..., n\}$, and the evader decides how to allocate a total of $C$ discrete units of contraband to each region at each period, i.e., $A^2 = \{a^2 = (a_1^2, \ldots, a_n^2) | \; a_i^2 \in \{0, 1, \ldots, C\}, \forall i \in S$ and $\sum_{i=1}^n a_i^2 \leq C\}$. We assume that the protector inspects one and only one region at each period. The location of the protector at a given period can be considered as the state of game in the second stage, i.e., $S = \{1, 2, ..., n\}$. We assume that the protector and the evader make their decisions simultaneously, i.e., the protector decides which region to inspect in the next period and the evader decides how much contraband to attempt to smuggle through each region in the next period without knowing the other's decision. In addition, for clarity we assume that if the protector decides to inspect region $j$, then she will do so with certainty. This condition implies that if the protector is at region $s$ in the current period and she wants to inspect region $j$ in the next period, then $p(j|s, a^1, a^2) = 1$ if $j = a^1$ and $p(j|s, a^1, a^2) = 0$ if $j \neq a^1$ for every $a^2 \in A^2$. Hence, this is a game controlled, in a probabilistic sense, by the protector.

If any contraband is brought through region $s$ while under surveillance by the protector, then the protector has a probability $p_s$ of detecting the contraband and $q_s$ of not detecting, where $q_s = 1 - p_s$. Similarly, if a static detection resource is located in region $s$, it will detect the contraband with probability $\bar{p}_s$ and not detect with probability $\bar{q}_s$, where $\bar{q}_s = 1 - \bar{p}_s$. We assume that the detection probabilities of the static detection resources and the detection probabilities of the protector are independent, and they are also independent of the amount of contraband passing through the region.

The protector earns a reward of one unit for each unit of contraband confiscated and is charged with a penalty of $\lambda$ for each unit of contraband smuggled. In the remainder of the chapter we will use a value of one for $\lambda$, however, the results can be extended for any value of $\lambda$ with minimal effort. When the protector has inspected region $s$ in the current period, selects region $j$ to inspect for the next period, and the evader's contraband allocation to the regions is $a^2$; the protector's reward is the expected amount of contraband confiscated less the expected penalized amount of contraband successfully smuggled, less his travel cost.

Then for $\lambda = 1$, $a^1 = j$, and $a^2 = (a_1^2, a_2^2, ..., a_n^2)$,

$$\begin{aligned} r(s,j,a^2,\gamma) &= \gamma_j \left[ (1-\bar{q}_j q_j)\, a_j^2 - \bar{q}_j q_j a_j^2 \right] + (1-\gamma_j) \left[ p_j a_j^2 - q_j a_j^2 \right] + \\ & \sum_{\substack{i \in S \\ i \neq j}} [\gamma_i (\bar{p}_i a_i^2 - \bar{q}_i a_i^2) - (1-\gamma_i) a_i^2] - c(s,j), \end{aligned}$$

where $\gamma_i$ is a binary variable indicating whether a static detection resource has been placed at region $i$, and $c(s, j)$ is the cost of traveling from region $s$ to region $j$ for the protector. Note that the cost $c(s, j)$ should be defined in a way such that it is comparable to an amount of smuggled contraband. The appropriate definition of this value is highly problem dependent and is beyond the scope of this study. The placement of a static detection resource at a region is equivalent to interdicting the particular state and these two terms will be used interchangeably throughout the remainder the chapter. To represent $r(s, j, a^2, \gamma)$ more clearly, we define $k(j, a^2), l(j, a^2)$ and $m(s, j, a^2)$ as:

$$\begin{aligned} k(j,a^2) &= 2\bar{p}_j q_j a_j^2, \\ l(j,a^2) &= 2\bar{p}_j a_j^2, \\ m(s,j,a^2) &= 2p_j a_j^2 - \sum_{i=1}^{n} a_i^2 - c(s,j). \end{aligned}$$

Above, $k(j, a^2)$ represents the portion of $r(s, j, a^2, \gamma)$ related to region $j$ and dependent on the interdiction at region $j$, $l(j, a^2)$ represents the reward related to other regions and dependent on the interdiction at the corresponding region, and $m(s, j, a^2)$ represents the reward related to region $j$ but independent of the interdiction plan. This representation will make the reward function more tractable, and we rewrite $r(s, j, a^2, \gamma)$ as follows:

$$r(s,j,a^2,\gamma) = k(j,a^2)\gamma_j + \sum_{\substack{i \in S \\ i \neq j}} l(i,a^2)\gamma_i + m(s,j,a^2).$$

The problem described above fits in the framework of the single player controlled game described in Section 3.2. Also, note that the protector can reach from any region to all other regions, i.e., the underlying system is not a multichain Markov chain. Therefore, the variables corresponding to transient policies are unnecessary as is the probability distribution for the initial location of the protector. As a result, we can simplify the formulation of the perimeter inspection game with initial interdiction as follows:

$$\max_{\gamma\in\Gamma}\max_{\mathbf{x},\mathbf{z}}\sum_{s=1}^{n} z(s), \tag{3.1a}$$

$$\sum_{j=1}^{n} x(s,j) - \sum_{j=1}^{n} x(j,s) = 0,\ \forall s\in S, \tag{3.1b}$$

$$\sum_{s=1}^{n}\sum_{j=1}^{n} x(s,j) = 1, \tag{3.1c}$$

$$\sum_{j=1}^{n} r(s,j,a^2,\gamma)x(s,j) \geq z(s),\ \forall s\in S,\ a^2\in A^2, \tag{3.1d}$$

$$\mathbf{x} \geq 0,$$

where $\Gamma = \{\gamma | \sum_{i=1}^{n} b_i\gamma_i \leq B\}$ represents the set of feasible static detection resource installations, $b_i$ is the cost of installing a static detection resource at region $i$, and $B$ is the total budget available for static detection resource installation. Also, $x(s,j)$ represents the long term proportion of periods that the protector is at region $s$ and will visit region $j$. Therefore in the protector's policy, $f(s,j) = \frac{x(s,j)}{\sum_{i=1}^{n} x(s,i)}$ represents the probability that the protector will visit region $j$ in the next period if he is at region $s$ in the current period. Also, let $g(s,a^2)$ be the dual variable corresponding to region $s$ and action $a^2$ in constraint (3.1d). Then, $g(s,a^2)$ represents the probability that the evader should select action $a^2$ in the next period if the protector is at region $s$ in the current period.

If we state $r(s,j,a^2,\gamma)$ explicitly in the above formulation, we have the following formulation:

$$\max_{\gamma \in \Gamma} \max_{\mathbf{x},\mathbf{z}} \sum_{s=1}^{n} z(s), \tag{3.2a}$$

$$\sum_{j=1}^{n} x(s,j) - \sum_{j=1}^{n} x(j,s) = 0, \ \forall\, s \in S \tag{3.2b}$$

$$\sum_{i=1}^{n} \sum_{j=1}^{n} x(i,j) = 1, \tag{3.2c}$$

$$\sum_{j=1}^{n} [k(j,a^2)\gamma_j + \sum_{\substack{i \in S \\ i \neq j}} l(i,a^2)\gamma_i + m(s,j,a^2)] x(s,j) \geq z(s), \ \forall\, s \in S,\ a^2 \in A^2, \tag{3.2d}$$

$$\mathbf{x} \geq 0.$$

The above formulation is a mixed integer nonlinear program (MINLP) since there are nonlinear terms in constraint (3.2d). However, we linearize these terms by defining

$$\beta(i,s,j) = \gamma_i x(s,j)$$

for every pair of $\gamma_i$ and $x(s,j)$. Adding the following constraints will ensure that this equation holds:

$$\beta(i,s,j) \leq \gamma_i, \tag{3.3a}$$

$$\beta(i,s,j) \leq x(s,j), \tag{3.3b}$$

$$\gamma_i + x(s,j) \leq \beta(i,s,j) + 1. \tag{3.3c}$$

When $\gamma_i = 0$, (3.3a) guarantees that $\beta(i,s,j) = 0$, and when $\gamma_i = 1$, (3.3b) and (3.3c) guarantee that $\beta(i,s,j) = x(s,j)$. After making these changes, we have the following final linear mixed integer program (MIP), the "Linearized Perimeter Inspection Game with Interdiction":

[PI-L]:

$$\max_{\gamma\in\Gamma}\max_{\mathbf{x},\mathbf{z},\beta}\sum_{s=1}^{n} z(s),$$

$$\begin{aligned}
\sum_{j=1}^{n} x(s,j) \; - \; \sum_{i=1}^{n} x(i,s) &= 0, \ \forall\ s \in S, \\
\sum_{i=1}^{n}\sum_{j=1}^{n} x(i,j) &= 1, \\
\sum_{j=1}^{n}[k(j,a^2)\beta(j,s,j) + \sum_{\substack{i\in S\\ i\neq j}} l(i,a^2)\beta(i,s,j) + m(s,j,a^2)]x(s,j) &\geq z(s), \ \forall\ s \in S, \ a^2 \in A^2, \\
\beta(i,s,j) - \gamma_i &\leq 0, \ \forall i,s,j \in S, \\
\beta(i,s,j) - x(s,j) &\leq 0, \ \forall i,s,j \in S, \\
\gamma_i + x(s,j) - \beta(i,s,j) &\leq 1, \ \forall i,s,j \in S, \\
\mathbf{x} &\geq 0.
\end{aligned}$$

[PI-L] can be solved using any solver. However, as the amount of regions or the number of contraband increases, the number of possible actions for the evader becomes very large, which makes [PI-L] very difficult to solve in an acceptable time. Therefore, we investigate structural properties about the problem that can reduce the computational time. In addition, we develop a heuristic algorithm that provides near optimal solutions for large problem sizes which cannot be solved in a reasonable time, and provide an explicit solution for a simple case of the problem. We investigate the structural properties of the problem in Section 3.3.2, present a heuristic algorithm for large problem sizes in Section 3.3.3, and provide the explicit solution for a simple case in Section 3.3.4.

### 3.3.2 Structural Properties

When there are $C$ units of contraband to smuggle and $n$ regions that the evader can allocate these contraband, there are $\sum_{c=0}^{C}\binom{c+n-1}{c}$ possible allocations which means that the evader has $\sum_{c=0}^{C}\binom{c+n-1}{c}$ actions to choose from at each state. As $C$ becomes large, the number of actions for the evader becomes intractable making [PI-L] intractable. However, some of these actions may be dominated by others in the optimal policy of the evader. If we can determine a subset of the action space of the evader that covers the optimal policies, then we can reduce the solution time. For this purpose, we define $a^{2,0}$ as the action of the evader where he allocates zero contraband to all regions, i.e., $a_i^{2,0} = 0$, for $i = 1, 2, \ldots, n$. Additionally, we define $a^{2,j}$ as the action such that he allocates all $C$ units of contraband to region $j$, i.e., $a_j^{2,j} = C,\ a_i^{2,j} = 0,\ i \neq j$, for $j = 1, 2, \ldots, n$. Also, define $\bar{A}^2 = \{a^{2,k} : k = 0, \ldots, n\}$. Finally, define $\tilde{A}^2$ as the set of actions such that the sum of contraband sent through all regions are either $n$ or 0, i.e., $\tilde{A}^2 = \{\{a^{2,0}\} \bigcup \{a^2 : \sum_{i=1}^n a_i^2 = n\}\}$.

First, we show that the evader either attempts to smuggle all $C$ units of contraband, or he does not attempt to smuggle any contraband at all.

**Theorem 3.1** *There is an optimal solution to [PI-L] such that* $\sum_{a^2 \in \tilde{A}^2} g(s, a^2) = 1$, $\forall s \in S$.

**Proof**. Let $\mathbf{f}^*$ be an optimal policy for the protector, and $\mathbf{g}^*$ be an optimal policy for the evader corresponding to $\mathbf{f}^*$. Let $\rho(s, j, \mathbf{g})$ represent the expected amount of contraband confiscated minus the expected amount of contraband successfully smuggled when the protector is at region $s$ and chooses to inspect region $j$ at the next period under the evader's policy $\mathbf{g}$. Then,

$$\rho(s, j, \mathbf{g}^*) = \sum_{a^2 \in A^2} \left[ 2\bar{p}_j p_j a_j^2 \gamma_j + \sum_{i=1}^n 2\bar{p}_i a_i^2 \gamma_i + 2\bar{p}_j a_j^2 - \sum_{i=1}^n a_i^2 \right] g^*(s, a^2).$$

Using the above equation, we can express the contribution of the state $s$ to the objective function in the optimal solution, $z^*(s)$ as follows:

$$
\begin{aligned}
z^*(s) &= \sum_{j=1}^{n} f^*(s,j)[\rho(s,j,\mathbf{g}^*) - c(s,j)] \\
&= \sum_{a^2\in A^2} \left\{ \sum_{j=1}^{n} f^*(s,j) 2\bar{p}_j p_j a_j^2 \gamma_j + \sum_{i=1}^{n} \left[ 2\bar{p}_i a_i^2 \gamma_i \sum_{j=1}^{n} f^*(s,j) \right] \right. \\
&\quad \left. + \sum_{j=1}^{n} f^*(s,j) 2\bar{p}_j a_j^2 - \sum_{i=1}^{n} \left[ a_i^2 \sum_{j=1}^{n} f^*(s,j) \right] \right\} g^*(s,a^2) - \sum_{j=1}^{n} f^*(s,j) c(s,j) \\
&= \sum_{a^2\in A^2} \left\{ \sum_{j=1}^{n} f^*(s,j) 2\bar{p}_j p_j a_j^2 \gamma_j + \sum_{i=1}^{n} 2\bar{p}_i a_i^2 \gamma_i + \sum_{j=1}^{n} f^*(s,j) 2\bar{p}_j a_j^2 - \sum_{i=1}^{n} a_i^2 \right\} g^*(s,a^2) \\
&\quad - \sum_{j=1}^{n} f^*(s,j) c(s,j) \\
&= \sum_{a^2\in A^2} \left\{ \sum_{j=1}^{n} a_j^2 [f^*(s,j) 2\bar{p}_j p_j \gamma_j + 2\bar{p}_j \gamma_j + f^*(s,j) 2\bar{p}_j - 1] \right\} g^*(s,a^2) - \sum_{j=1}^{n} f^*(s,j) c(s,j).
\end{aligned}
$$

Now, suppose that $g^*(s,a) > 0$ for some action $a$ such that $0 < \sum_{i=1}^{n} a_i = c < n$. Then,

$$
\sum_{j=1}^{n} a_j [f^*(s,j) 2\bar{p}_j p_j \gamma_j + 2\bar{p}_j \gamma_j + f^*(s,j) 2\bar{p}_j - 1] \leq 0.
$$

Otherwise, $z^*(s)$ can be decreased by increasing $g^*(s,a^{2,0})$ by $g^*(s,a)$ in the optimal solution and setting $g^*(s,a) = 0$. Then, there exist $j_1 \in S$ such that

$$
f^*(s,j_1) 2\bar{p}_{j_1} p_{j_1} \gamma_{j_1} + 2\bar{p}_{j_1} \gamma_{j_1} + f^*(s,j_1) 2\bar{p}_{j_1} - 1 \leq 0,
$$

since $a_j \geq 0, \ \forall j \in S$. Let's define $\bar{a}$ as the action such that

$$
\bar{a}_{j_1} = a_{j_1} + n - c; \ \bar{a}_j = a_j, \forall j \in S, j \neq j_1.
$$

Then,

$$
\sum_{j=1}^{n} \bar{a}_j [f^*(s,j) 2\bar{p}_j p_j \gamma_j + 2\bar{p}_j \gamma_j + f^*(s,j) 2\bar{p}_j - 1] \leq \sum_{j=1}^{n} a_j [f^*(s,j) 2\bar{p}_j p_j \gamma_j + 2\bar{p}_j \gamma_j + f^*(s,j) 2\bar{p}_j - 1].
$$

So, if we increase $g(s,\bar{a})$ by $g(s,a)$ and set $g(s,a)=0$, then $z^*(s)$ either decreases or remains the same. As a result, we can replace the actions whose total contraband allocation is less than $n$ with the actions whose total contraband allocation is equal to $n$ in the optimal solution. □

Using Theorem 3.1, the total number of actions for the evader can be reduced from $\sum_{c=0}^{C}\binom{c+n-1}{c}$ to $\binom{C+n-1}{C}$. Now, [PI-L] can be solved using actions in $\tilde{A}^2$ instead of using the actions in $A^2$. The resulting problem is much smaller, however it is still very large for large values of $C$. The problem can be further simplified with the following theorem which states that there is an optimal solution for evader in which he attempts to smuggle all of his contraband through one region or he does not attempt to smuggle any contraband at all.

**Theorem 3.2** *There is an optimal solution to [PI-L] such that* $\sum_{a^2\in\bar{A}^2} g(s,a^2)=1,\ \forall s\in S.$

**Proof**. Let $\mathbf{f}^*$ be an optimal policy for the protector, and $\mathbf{g}^*$ be an optimal policy for the evader corresponding to $\mathbf{f}^*$. Using the previous definition of $\rho(s,j,\mathbf{g})$,

$$\begin{aligned}\rho(s,j,\mathbf{g}^*) &= \sum_{a^2\in A^2}[2\bar{p}_j p_j a_j^2\gamma_j + \sum_{i=1}^{n} 2\bar{p}_i a_i^2\gamma_i + 2\bar{p}_j a_j^2 - \sum_{i=1}^{n} a_i^2] g^*(s,a^2)\\ &= \sum_{i=1}^{n}\sum_{a^2\in A^2} 2\bar{p}_i a_i^2\gamma_i g^*(s,a^2) + \sum_{a^2\in A^2}[2\bar{p}_j p_j a_j^2\gamma_j + 2\bar{p}_j a_j^2 - C] g^*(s,a^2)\end{aligned}$$

Let's define a policy $\bar{\mathbf{g}}$ for the evader which only consists of the actions in $\bar{A}^2$, and let the probabilities be as follows.

$$\bar{g}(s,a^{2,k}) = \frac{1}{C}\sum_{a^2\in A^2} a_k^2 g^*(s,a^2),\ \ k=1,2,\ldots,n.$$

Note that,

$$\sum_{k=1}^{n}\bar{g}(s,a^{2,k}) = \sum_{k=1}^{n}\frac{1}{C}\sum_{a^2\in A^2} a_k^2 g^*(s,a^2) = \frac{1}{C}\sum_{a^2\in A^2}\sum_{k=1}^{n} a_k^2 g^*(s,a^2) = \frac{1}{C}\sum_{a^2\in A^2} C g^*(s,a^2) = 1.$$

So, $\bar{\mathbf{g}}$ is a valid policy for the evader. Now, we can restate $\rho(s, j, \mathbf{g}^*)$ as follows.

$$\begin{aligned}
\rho(s, j, \mathbf{g}^*) &= \sum_{i=1}^{n} 2\bar{p}_i \gamma(i) C \bar{g}(s, a^{2,i}) + 2\bar{p}_j p_j \gamma(j) C + 2\bar{p}_j C - C \\
&= \sum_{i=1}^{n} 2\bar{p}_i \gamma(i) \sum_{k=1}^{n} a_i^{2,k} \bar{g}(s, a^{2,k}) + 2\bar{p}_j p_j \gamma(j) \sum_{k=1}^{n} a_j^{2,k} \bar{g}(s, a^{2,k}) \\
&\quad + 2\bar{p}_j \sum_{k=1}^{n} a_j^{2,k} \bar{g}(s, a^{2,k}) - \sum_{k=1}^{n} \bar{g}(s, a^{2,k}) \sum_{i=1}^{n} a_i^{2,k} \\
&= \sum_{k=1}^{n} \left[ \sum_{i=1}^{n} 2\bar{p}_i \gamma(i) a_i^{2,k} + 2\bar{p}_j p_j \gamma(j) a_j^{2,k} + 2\bar{p}_j a_j^{2,k} - \sum_{i=1}^{n} a_i^{2,k} \right] \bar{g}(s, a^{2,k}) \\
&= \rho(s, j, \bar{\mathbf{g}}).
\end{aligned}$$

Since $\rho(s, j, \mathbf{g}^*)$ is equivalent to $\rho(s, j, \bar{\mathbf{g}})$, and the traveling cost of the the protector is independent of the evader's policy, $\bar{\mathbf{g}}$ is also an optimal policy for the evader. □

Now, we can reformulate [PI-L] using actions in $\bar{A}^2$ only.

[PI-LR]:

$$\max_{\gamma \in \Gamma} \max_{\mathbf{x}, \mathbf{z}, \beta} \sum_{s=1}^{n} z(s),$$

$$\begin{aligned}
\sum_{j=1}^{n} x(s, j) - \sum_{i=1}^{n} x(i, s) &= 0, \ \forall\ s \in S, \\
\sum_{i=1}^{n} \sum_{j=1}^{n} x(i, j) &= 1, \\
\sum_{j=1}^{n} [k(j, a^2)\beta(j, s, j) + \sum_{\substack{i \in S \\ i \neq j}} l(i, a^2)\beta(i, s, j) + m(s, j, a^2)] x(s, j) &\geq z(s), \ \forall\ s \in S,\ a^2 \in \bar{A}^2, \\
\beta(i, s, j) - \gamma_i &\leq 0, \ \forall i, s, j \in S, \\
\beta(i, s, j) - x(s, j) &\leq 0, \ \forall i, s, j \in S, \\
\gamma_i + x(s, j) - \beta(i, s, j) &\leq 1, \ \forall i, s, j \in S, \\
\mathbf{x} &\geq 0.
\end{aligned}$$

The following theorem proves the equivalence between [PI-L] and [PI-LR] in terms of the optimal solutions.

**Theorem 3.3** *An optimal policy for the protector and the evader in [PI-LR] is also an optimal policy for the protector and the evader in [PI-L].*

**Proof**. Let let $\bar{z}$ be the optimal objective function value of [PI-LR]. Assume that there is an optimal solution for [PI-L] with an objective function value of $z^*$ such that $z^* < \bar{z}$. Let the optimal policies for the protector and the evader be $\bar{\mathbf{f}}$ and $\bar{\mathbf{g}}$ in [PI-LR]. Let the protector select policy $\bar{\mathbf{f}}$ in [PI-L], and let $\mathbf{g}$ be the optimal policy for the evader corresponding to $\bar{\mathbf{f}}$ and $z$ be the corresponding objective function value in [PI-L]. Obviously, $z \leq z^*$. By Theorem 3.2, $z = \bar{z}$. Then, $\bar{z} \leq z^*$, which is a contradiction with our assumption that $z^* < \bar{z}$. Therefore, $\bar{z} \leq z^*$. Also, [PI-LR] is a relaxation of [PI-L], therefore, $\bar{z} \geq z^*$. As a result, $\bar{z} = z^*$. Since any feasible solution in [PI-LR] is also feasible in [PI-L], any optimal policy for the protector and the evader in [PI-LR] is also an optimal policy for the protector and the evader in [PI-L]. □

Using Theorem 3.2 and Theorem 3.3, we converted our problem into a significantly smaller problem, resulting in much shorter solution times. Note that, even though [PI-LR] formulation limits the number of actions the evader can take, this does not mean that the evader has to take the action found using this formulation, there may be other optimal actions for the evader. However, no matter which optimal action the evader takes, the protector's optimal action found using [PI-LR] would still be optimal. Since the problem is formulated to maximize the protector's objective, solving [PI-LR] is equivalent to solving [PI-L].

### 3.3.3 Heuristic Algorithm

[PI-LR] formulation of the perimeter inspection game with interdiction can be solved within acceptable times for smaller problem instances (around 15 nodes) using standard solvers. However, solving larger problem instances optimally may require much longer times. Therefore, we provide a heuristic algorithm to solve the larger problem instances in shorter times.

The algorithm utilizes the fact that the static detection resource allocation scheme will primarily follow a greedy pattern. In the algorithm, we first start with a single static detection resource and determine the best region to allocate it by solving $n$ perimeter inspection games. We allocate the detection resource at the region that maximizes the objective function, and find the next best region to allocate a static detection resource by solving $n-1$ perimeter inspection games. The algorithm progresses until all static detection resources have been allocated to regions.

For a given interdiction plan $\gamma$, the perimeter inspection game can be formulated as:

$$
\begin{aligned}
\text{[PI-FI]:} \qquad & \max_{\mathbf{x},\mathbf{z}} \sum_{s=1}^{n} z(s), \\
\sum_{j=1}^{n} x(s,j) - \sum_{j=1}^{n} x(j,s) &= 0, \ \forall\, s \in S, \\
\sum_{i=1}^{n}\sum_{j=1}^{n} x(i,j) &= 1, \\
\sum_{j=1}^{n}[k(j,a^2)\gamma_j + \sum_{\substack{i \in S \\ i \neq j}} l(i,a^2)\gamma_i + m(s,j,a^2)]x(s,j) &\geq z(s), \ \forall\, s \in S, \ a^2 \in \bar{A}^2, \\
\mathbf{x} &\geq 0.
\end{aligned}
$$

We can formally state the algorithm as follows, where $AR$ represents the set of regions that are available to place a static detection resource:

### ***Heuristic algorithm to solve perimeter inspection game with interdiction***

1. *Initialization*
   - $AR = \{1, 2, \ldots, n\}$
   - $\gamma_i = 0$, for $i = 1, 2, \ldots, n$
2. *Iteration*
   - For $b = 1, \ldots, B$
     - For every $i \in AR$, temporarily set $\gamma_i = 1$ and solve [PI-FI]
     - Get $i$ with the best [PI-FI] optimal objective value and set $\gamma_i = 1$ , $AR = AR \backslash \{i\}$.

Note that the above algorithm is valid for the case where there is a limit on the number of available static detection resources. However, it can be generalized to allow other types of budget constraints with little effort.

### 3.3.4 Explicit Solution for a Simple Case

We provided [PI-LR] formulation of the problem which shortens the computational time of the problem considerably, and also provided a heuristic algorithm to solve the problem more efficiently with small errors. In this section we investigate solution methodologies that will reduce the solution time. With this in mind, we seek more insight about the optimal solution of the problem. We start with a simple case of the problem, state the optimal solution explicitly for this case, and propose future directions that would build on this solution by analyzing more general cases. First we consider the case where no static detection resource allocation is allowed and there is no travel cost between the regions.

Define $f(s)$ as the long run rate that the protector visits region $s$, i.e.,

$$f(s) = \sum_{j \in S} f(s,j).$$

Also, define

$$\rho_j = \frac{1}{p_j}, \ \forall j \in S,$$
$$\rho = \sum_{j \in S} \rho_j.$$

The following theorem states the optimal solution of [PI-LR] when there is no traveling cost and no static detection resource allocation allowed. For this case, we show that the optimal policy has the following form: the fraction of time the protector selects region $i$ versus region $j$ is inversely proportional to the ratio of the detection probabilities at these two regions.

**Theorem 3.4** *If $c(s,j) = 0, \forall s,j \in S$, and $B = 0$, then there is an optimal solution of [PI-LR] which satisfy the following equations:*

$$\begin{aligned}
\frac{f^*(s,i)}{f^*(s,j)} &= \frac{p_j}{p_i}, \ \forall s,i,j \in S, \\
\frac{g^*(s,i)}{g^*(s,j)} &= \frac{p_j}{p_i}, \ \forall s,i,j \in S, \\
\frac{f^*(i)}{f^*(j)} &= \frac{p_j}{p_i}, \ \forall i,j \in S, \\
\frac{z^*(i)}{z^*(j)} &= \frac{p_j}{p_i}, \ \forall i,j \in S.
\end{aligned}$$

**Proof**. The optimal values that satisfy the equations in the proposition are as follows:

$$\begin{aligned} f^*(s) &= \frac{\rho_s}{\rho}, \ \forall s \in S, \\ f^*(s,j) &= \frac{\rho_j}{\rho}, \ \forall s,j \in S, \\ g^*(s,j) &= \frac{\rho_j}{\rho}, \ \forall s,j \in S. \end{aligned}$$

$\mathbf{f}^*$ and $\mathbf{g}^*$ are optimal policies for the protector and the evader if

$$v(\mathbf{f}, \mathbf{g}^*) \le v(\mathbf{f}^*, \mathbf{g}^*) \le v(\mathbf{f}^*, \mathbf{g})$$

for any stationary policies $\mathbf{f}$ and $\mathbf{g}$. For policies $\mathbf{f}$ and $\mathbf{g}$, we can state the objective function value as

$$\begin{aligned} v(\mathbf{f}, \mathbf{g}) &= \sum_{s\in S} f(s) \left\{ \sum_{j\in S} Cg(s,j)[1 - f(s,j) + f(s,j)(1 - p_j - p_j)] \right\} \\ &= \sum_{s\in S} f(s) \left\{ \sum_{j\in S} Cg(s,j)[1 - 2f(s,j)p_j] \right\} \\ &= \sum_{s\in S} Cf(s) \left\{ \sum_{j\in S} g(s,j) - \sum_{j\in S} 2g(s,j)f(s,j)p_j \right\} \\ &= \sum_{s\in S} Cf(s) \left\{ 1 - \sum_{j\in S} 2g(s,j)f(s,j)p_j \right\} \\ &= C - 2C \sum_{s\in S} f(s) \left\{ \sum_{j\in S} g(s,j)f(s,j)p_j \right\} \end{aligned}$$

For the optimal policy $\mathbf{f}^*$ stated above and any policy $\mathbf{g}$,

$$\begin{aligned} v(\mathbf{f}^*, \mathbf{g}^*) - v(\mathbf{f}^*, \mathbf{g}) &= 2C \sum_{s\in S} f^*(s) \left\{ \sum_{j\in S} g(s,j)f^*(s,j)p_j \right\} - 2C \sum_{s\in S} f^*(s) \left\{ \sum_{j\in S} g^*(s,j)f^*(s,j)p_j \right\} \\ &= 2C \sum_{s\in S} f^*(s) \left\{ \sum_{j\in S} f^*(s,j)p_j[g(s,j) - g^*(s,j)] \right\} \\ &= 2C \sum_{s\in S} \frac{\rho_s}{\rho^2} \left\{ \sum_{j\in S} g(s,j) - \sum_{j\in S} g^*(s,j) \right\} \\ &= 0. \end{aligned}$$

Therefore,

$$v(\mathbf{f}^*, \mathbf{g}^*) \leq v(\mathbf{f}^*, \mathbf{g})$$

for any policy $\mathbf{g}$. Similarly, for the optimal policy $\mathbf{g}^*$ stated above, and any policy $\mathbf{f}$,

$$\begin{aligned}
v(\mathbf{f}^*, \mathbf{g}^*) - v(\mathbf{f}, \mathbf{g}^*) &= 2C \sum_{s \in S} f(s) \left\{ \sum_{j \in S} g^*(s,j) f(s,j) p_j \right\} - 2C \sum_{s \in S} f^*(s) \left\{ \sum_{j \in S} g^*(s,j) f^*(s,j) p_j \right\} \\
&= \frac{2C}{\rho} \left\{ \sum_{s \in S} f(s) \sum_{j \in S} f(s,j) - \sum_{s \in S} f^*(s) \sum_{j \in S} f^*(s,j) \right\} \\
&= \frac{2C}{\rho} \left\{ \sum_{s \in S} f(s) - \sum_{s \in S} f^*(s) \right\} \\
&= 0.
\end{aligned}$$

Therefore,

$$v(\mathbf{f}, \mathbf{g}^*) \leq v(\mathbf{f}^*, \mathbf{g}^*)$$

for any policy $\mathbf{f}$. As a result,

$$v(\mathbf{f}, \mathbf{g}^*) \leq v(\mathbf{f}^*, \mathbf{g}^*) \leq v(\mathbf{f}^*, \mathbf{g})$$

for any stationary policies $\mathbf{f}$ and $\mathbf{g}$. Hence, $\mathbf{f}^*$ and $\mathbf{g}^*$ are optimal stationary policies for the protector and the evader respectively. □

Using the findings in Theorem 3.4, algorithms can be developed to find the optimal solutions when interdiction is allowed, but there is no traveling cost between the regions. Finally, using these results a decomposition algorithm can be developed for the perimeter inspection problem with interdiction. These extensions are a direction for future research, and will not be discussed in this dissertation.

## 3.4 COMPUTATIONAL RESULTS

In this section we study the computational effectiveness of the proposed formulations. We begin our analysis by detailing the test problem sets.

### 3.4.1 Test Problems and Environment

Fully connected random networks were created in which nodes correspond to the regions and arc lengths correspond to the distances between the regions. Four values for the number of nodes in the network ($n$) were considered: 10, 15, 30 and 50. The distances between regions, $c(i, j)$, are real numbers generated randomly from Uniform(0.1, 0.5), and are the same in both directions. The detection probabilities by the protector at each region ($p_i$) are real numbers generated from Uniform(0.25, 0.75), and similarly the detection probabilities by the static detection resources at each region ($\bar{p}_i$) are real numbers generated from Uniform(0.1, 0.3). We assumed that the budget required to place a static detection resource at each region ($b_i$) was 1, and investigated two values for the total available budget ($B$): 3 and 5. Also, four values for the total number of contraband to smuggle ($C$) were considered: 3, 4, 5 and 6. Ten random instances were created for each $n$ using different random numbers to generate the distances between the regions and detection probabilities. When presenting the results, the average values of these ten instances are reported. The parameters used in the test instances are summarized in Table 3.1.

Table 3.1: Parameters used in test instances.

| Parameter | Value |
|---|---|
| $n$ | 10, 15, 30, 50 |
| $C$ | 3, 4, 5, 6 |
| $B$ | 3, 5 |
| $p_i$ | Uniform(0.25, 0.75) |
| $\bar{p}_i$ | Uniform(0.1, 0.3) |
| $c_(i, j)$ | Uniform(0.1, 0.5) |
| $b_i$ | 1 |

CPLEX 9.0 with default settings was used to solve the test instances. The programs were coded in C, and run on an Intel Pentium IV computer with 3.05 GHz CPU and 512 MB of RAM.

### 3.4.2 Computational Examples

We first investigate the computational effectiveness of the proposed formulations. We compare the running times of [PI-L] with running times of [PI-LR] for $n = 10$. We do not make the comparison for larger values of $n$ since instances could not be solved using [PI-L] in acceptable time periods for most of the instances (runs were terminated after 3 hours if an optimal solution could not be reached). Both formulations were solved using standard branch-and-bound algorithms of CPLEX 9.0. The results are listed in Appendix B, summarized in Table 3.2, and illustrated in Figure 3.4.2. In Table 3.2, %D represents the percent decrease in running time resulting from using [PI-LR] instead of [PI-L].

Table 3.2: Comparison of running times (in seconds) for [PI-L] and [PI-LR].

| $n$ | $C$ | $B$ | [PI-L] | [PI-LR] | %D |
|---|---|---|---|---|---|
| 10 | 3 | 3 | 134.5 | 23.5 | 82.6 |
| 10 | 3 | 5 | 117.7 | 21.5 | 81.7 |
| 10 | 4 | 3 | 444.4 | 23.7 | 94.7 |
| 10 | 4 | 5 | 414.7 | 20.3 | 95.1 |
| 10 | 5 | 3 | 2044.4 | 24.0 | 98.8 |
| 10 | 5 | 5 | 1766.2 | 21.6 | 98.8 |
| 10 | 6 | 3 | 8351.5 | 24.1 | 99.7 |
| 10 | 6 | 5 | 7293.2 | 21.6 | 99.7 |

From the table we observe that as $C$ increases, the running time for [PI-L] increases considerably, whereas the running time for [PI-LR] remains about the same. This is because the number of actions, hence number of constraints, increase considerably with $C$ for [PI-L], whereas they remain the same for any value of $C$ for [PI-LR]. Therefore, using [PI-LR]

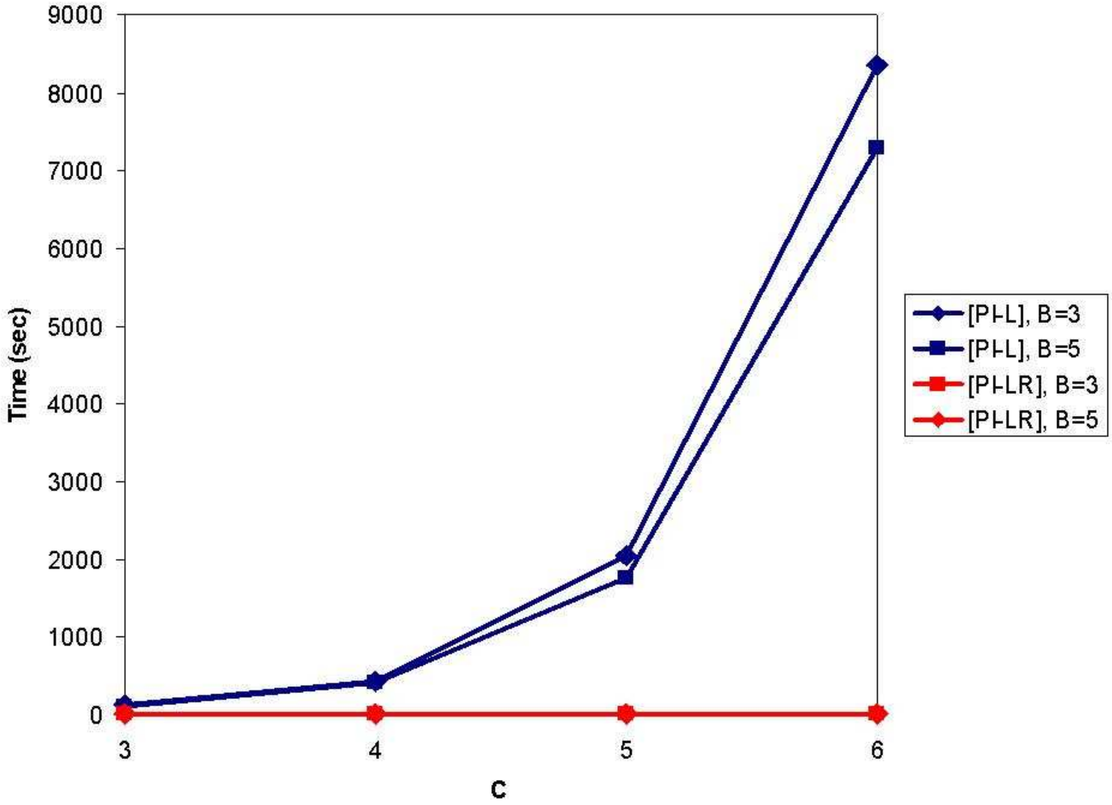


Figure 3.1: Comparison of running times for [PI-L] and [PI-LR]

formulation provides substantial savings in running times for large values of $C$.

Next, we investigate the effectiveness of the heuristic algorithm proposed in section 3.3.3 in terms of the running time and quality of the solutions. The results are listed in Appendix B, and summarized in Table 3.3. The results are reported only for $C = 3$ and 5 for simplicity since the results are similar for $C = 4$ and 6. In the table, %E represents the percent error (deviation from the optimal solution) for the heuristic algorithm. Columns corresponding to [PI-LR] and HA are the running times (in seconds) for the [PI-LR] formulation and the heuristic algorithm respectively. Cells corresponding to rows $n = 30$ and 50, and columns %E and [PI-LR] are left blank since [PI-LR] could not be solved to optimality within allocated time.

Table 3.3: Comparison of [PI-LR] and the heuristic algorithm

| $n$ | $C$ | $B$ | %E | [PI-LR] | HA |
|---|---|---|---|---|---|
| 10 | 3 | 3 | 0.02 | 23.5 | 0.10 |
| 10 | 3 | 5 | 0.01 | 21.5 | 0.14 |
| 10 | 5 | 3 | 0.02 | 24.0 | 0.09 |
| 10 | 5 | 5 | 0.02 | 21.6 | 0.13 |
| 15 | 3 | 3 | 0.18 | 945.9 | 0.47 |
| 15 | 3 | 5 | 2.52 | 3089.7 | 0.76 |
| 15 | 5 | 3 | 0.00 | 1150.3 | 0.46 |
| 15 | 5 | 5 | 0.00 | 3056.8 | 0.67 |
| 30 | 3 | 3 | | | 5.22 |
| 30 | 3 | 5 | | | 8.55 |
| 30 | 5 | 3 | | | 8.50 |
| 30 | 5 | 5 | | | 14.09 |
| 50 | 3 | 3 | | | 51.65 |
| 50 | 3 | 5 | | | 86.57 |
| 50 | 5 | 3 | | | 62.27 |
| 50 | 5 | 5 | | | 104.52 |

As can be seen from the table, the proposed heuristic algorithm provides significant time savings while providing solutions that are close to optimal solutions. Also, it makes it possible to solve the larger problem instances within acceptable times.

## 3.5 CONCLUSIONS

In this chapter, we studied the perimeter security problem with static and dynamic resource allocations. In the problem, an evader attempts to smuggle as much contraband as possible through a perimeter, and an protector attempts to minimize the number of contraband smuggled by allocating his static detection resources to some of the inspection points and later devising a dynamic inspection policy. The problem was formulated as a non-linear mixed integer program, and converted to a linear mixed integer program using linearization techniques. The size of the mixed integer program was decreased by exploiting some structural properties of the problem. Also, a heuristic algorithm was developed to solve the large problem instances in acceptable times with a small error, and the explicit solution for a simple case of the problem was provided. Finally, the effectiveness of the proposed formulations and the algorithm was investigated through computational examples.

The primary contribution of this chapter is that it combines two different approaches in perimeter security problems, static detection resource allocation problems and dynamic inspection games, in a single problem. The resulting problem takes into account both the design and execution phases of perimeter security, and provides a more comprehensive and heretofore unexplored formulation.

# 4.0 ASYMMETRIC NETWORK INTERDICTION UNDER RISK AND UNCERTAINTY

## 4.1 INTRODUCTION

In Chapter 2, we developed a model to assist the protector in allocating her static detection resources in order to increase the detection probability of the evader. In the model, it was assumed that there was information asymmetry between the protector and the evader in terms of the detection capabilities of the detection resources. The computational studies demonstrated that including this information asymmetry could increase the detection probability. However, there was also an additional risk associated with accounting for information asymmetry. This additional risk is the result of leaving more vulnerable regions with less detection resources due to the assumption that the evader would not pass through these regions. The benefits and risks associated with the modeling of information asymmetry was discussed through computational examples in Section 2.4, and the results were summarized in Table 2.3 and Table 2.4. As can be seen from these tables, the risk associated with modeling information asymmetry can be as high as the benefit associated with it. The model presented in Chapter 2 can be very useful if the protector has good intelligence regarding the evader's estimates. However, if this is not the case, she should be more conservative in her approach.

In this chapter we present two models that take the risk into account. In the first model, based on the model in Chapter 2, we include a constraint that bounds the risk arising from the modeling of the information asymmetry. In the second model, instead of assuming that the protector knows the evader's estimates, we let her to plan for a set of possible scenarios with various probabilities. In the latter model, there is a complete information asymmetry between the protector and the evader.

The remainder of this chapter is organized as follows. We formulate the asymmetric network interdiction model under risk, and present computational examples in Section 4.2. We define and formulate the asymmetric network interdiction model under uncertainty in Section 4.3, and make concluding remarks in Section 4.4.

## 4.2 ASYMMETRIC NETWORK INTERDICTION UNDER RISK

In the shortest path network interdiction model presented in Chapter 2, first the protector interdicts the arcs, then the evader finds the shortest path using his estimates of the arc lengths. The true length of this path is calculated to evaluate the performance of the interdiction plan. If the evader uses the true arc lengths as his estimates to find the shortest path, then he will find the true shortest path on the interdicted network. The length of this path may be shorter than the true shortest path if the protector interdicted the arcs based on the worst case scenario assuming that there was no information asymmetry between herself and the evader. This is the risk arising due to the assumption that the interdictor knows the evader's estimates of arc lengths. We want to limit this risk, i.e., we want the length of the true shortest path on the interdicted network to be greater than a threshold value. This threshold value can be a certain percentage (e.g. 95%) of the length of the shortest path found when the interdiction plan was calculated based on the worst case scenario.

### 4.2.1 Formulation

In this chapter, we use the definitions in Chapter 2. In addition to those, let $\bar{y}_k$ be the variable representing whether or not arc $k$ is on the true shortest path. As a result of the structure of the shortest path problem, $\bar{y}_k$ is continuous rather than binary. For a given interdiction plan $\hat{\mathbf{x}}$, the true shortest path in the network can be determined by solving the following problem:

$$\text{[TSP}(\hat{\mathbf{x}})]: \qquad \min_{\bar{\mathbf{y}}} \sum_{k \in A} (c_k + d_k \hat{x}_k)\bar{y}_k, \tag{4.1a}$$

$$\sum_{k \in FS(i)} \bar{y}_k - \sum_{k \in RS(i)} \bar{y}_k = \begin{cases} 1, & i = 0, \\ 0, & i = 1, 2, \ldots, n-1, \\ -1, & i = n, \end{cases} \tag{4.1b}$$

$$\bar{y}_k \geq 0, \quad \forall k \in A.$$

Let $\bar{u}_i$ be the dual variable corresponding to node $i$ in (4.1b). Then, the dual problem of [TSP($\hat{\mathbf{x}}$)] is:

$$\text{[TSPD}(\hat{\mathbf{x}})]: \qquad \max_{\bar{\mathbf{u}}} \bar{u}_0 - \bar{u}_n, \tag{4.2a}$$

$$\bar{u}_i - \bar{u}_j \leq c_k + d_k \hat{x}_k, \quad \forall (i,j) = k \in A. \tag{4.2b}$$

In the above formulation, $\bar{u}_0 - \bar{u}_i$ represents the length of the shortest path from node 0 to node $i$ based on the arc lengths $c_k + d_k\hat{x}_k$. The optimal objective function values of these two formulations are same and are equal to the length of the shortest path in $G(N, A)$ with arc lengths $c_k + d_k\hat{x}_k$. If we combine constraints (4.1b) and (4.2b) in a single formulation, and add a constraint stating that the objective function values (4.1a) and (4.2a) should be equal, then we convert the shortest path problem into a feasibility problem:

$$
\begin{aligned}
\text{[TSPF}(\hat{\mathbf{x}})]: \qquad \sum_{k \in FS(i)} \bar{y}_k - \sum_{k \in RS(i)} \bar{y}_k &= \begin{cases} 1, & i = 0, \\ 0, & i = 1, 2, \ldots, n-1, \\ -1, & i = n, \end{cases} \\
\bar{u}_i - \bar{u}_j &\leq c_k + d_k \hat{x}_k, \quad \forall (i,j) = k \in A, \\
\bar{u}_n - \bar{u}_0 + \sum_{k \in A} (c_k + d_k \hat{x}_k) \bar{y}_k &= 0, \\
\bar{y}_k &\geq 0, \quad \forall k \in A.
\end{aligned}
$$

Any feasible solution to [TSPF($\hat{\mathbf{x}}$)] gives a shortest path on $G(N, A)$ with arc lengths $c_k + d_k \hat{x}_k$. Note that, there is no objective function in the above formulation since it is a feasibility problem rather than an optimization problem. Any objective function value can be used with [TSPF($\hat{\mathbf{x}}$)]. Now, we can combine [TSPF($\hat{\mathbf{x}}$)] with [SPNI-L] formulation in Chapter 2, and add a constraint stating that the length of the true shortest path on the interdicted network should be greater than a threshold value, $\underline{z}$, and get the following formulation:

$$
\begin{aligned}
\text{[SPNIA-R]:} \quad & \max_{\mathbf{x},\mathbf{y},\bar{\mathbf{y}},\mathbf{u},\bar{\mathbf{u}}} \sum_{k\in A}(c_k + d_k x_k) y_k, && \text{(4.3a)}\\
\sum_{k\in FS(i)} y_k - \sum_{k\in RS(i)} y_k \;&=\; \begin{cases} 1, & i = 0,\\ 0, & i = 1,2,\ldots,n-1,\\ -1, & i = n, \end{cases} && \text{(4.3b)}\\
u_i - u_j - \bar{d}_k x_k \;&\le\; \bar{c}_k, \quad \forall (i,j) = k \in A, && \text{(4.3c)}\\
u_n - u_0 + \sum_{k\in A}(\bar{c}_k + \bar{d}_k x_k) y_k \;&=\; 0, && \text{(4.3d)}\\
\sum_{k\in FS(i)} \bar{y}_k - \sum_{k\in RS(i)} \bar{y}_k \;&=\; \begin{cases} 1, & i = 0,\\ 0, & i = 1,2,\ldots,n-1,\\ -1, & i = n, \end{cases} && \text{(4.3e)}\\
\bar{u}_i - \bar{u}_j - d_k x_k \;&\le\; c_k, \quad \forall (i,j) = k \in A, && \text{(4.3f)}\\
\bar{u}_n - \bar{u}_0 + \sum_{k\in A}(c_k + d_k x_k) \bar{y}_k \;&=\; 0, && \text{(4.3g)}\\
\bar{u}_0 - \bar{u}_n \;&\ge\; \underline{z}, && \text{(4.3h)}\\
y_k, \bar{y}_k \;&\ge\; 0, \quad \forall k \in A,\\
\mathbf{x} \;&\in\; X.
\end{aligned}
$$

[SPNIAR] is a non-linear mixed integer program since there are non-linear terms in the objective function in constraints (4.3d) and (4.3g). However, these terms can be linearized by defining new variables $v_k, w_k$ to replace $y_k$, and $\bar{v}_k, \bar{w}_k$ to replace $\bar{y}_k$ similar to the ones defined in Section 2.2. This results in the following linear mixed integer program:

$$
\text{[SPNIA-RL]:} \quad \max_{\mathbf{x},\mathbf{v},\mathbf{w},\bar{\mathbf{v}},\bar{\mathbf{w}},\mathbf{u},\bar{\mathbf{u}}} \sum_{k\in A}(c_k v_k + (c_k+d_k)w_k),
$$

$$
\begin{aligned}
\sum_{k\in FS(i)}(v_k+w_k) - \sum_{k\in RS(i)}(v_k+w_k) &= \begin{cases} 1, & i=0, \\ 0, & i=1,2,\ldots,n-1, \\ -1, & i=n, \end{cases} \\
u_i - u_j - \bar{d}_k x_k &\leq \bar{c}_k \quad \forall (i,j)=k\in A, \\
u_n - u_0 + \sum_{k\in A}(\bar{c}_k v_k + (\bar{c}_k+\bar{d}_k)w_k) &= 0, \\
v_k + x_k &\leq 1 \quad \forall k\in A, \\
w_k - x_k &\leq 0 \quad \forall k\in A, \\
\sum_{k\in FS(i)}(\bar{v}_k+\bar{w}_k) - \sum_{k\in RS(i)}(\bar{v}_k+\bar{w}_k) &= \begin{cases} 1, & i=0, \\ 0, & i=1,2,\ldots,n-1, \\ -1, & i=n, \end{cases} \\
\bar{u}_i - \bar{u}_j - d_k x_k &\leq c_k \quad \forall (i,j)=k\in A, \\
\bar{u}_n - \bar{u}_0 + \sum_{k\in A}(c_k\bar{v}_k + (c_k+d_k x_k)\bar{w}_k) &= 0, \\
\bar{u}_0 - \bar{u}_n &\geq \underline{z}, \\
\bar{v}_k + x_k &\leq 1 \quad \forall k\in A, \\
\bar{w}_k - x_k &\leq 0 \quad \forall k\in A, \\
v_k, w_k, \bar{v}_k, \bar{w}_k &\geq 0 \quad \forall k\in A, \\
\mathbf{x} &\in X.
\end{aligned}
$$

**Remark:** In the above formulation, if there are multiple shortest paths for the evader, then we assume that he will take the one which is longer when evaluated with the arc lengths $c_k + d_k x_k$. This can be interpreted as the cooperation of the evader with the protector (or the best case scenario) in case of multiple shortest paths. A similar formulation to [SPNI-D($\hat{\theta}$)] can be developed if the worst case scenario is assumed in the case of multiple shortest paths for the evader.

### 4.2.2 Computational Study

In this section we investigate the results of including the risk criterion presented in the model. We use the same data samples that were presented in Chapter 2, and compare the results of the formulations with and without the risk criterion.

We first compare the results of asymmetric network interdiction problem under risk with the results of network interdiction problem without information asymmetry. For this purpose, we compare the results of [SPNIA-L] formulation with the results of [MXSP-D] formulation used in Israeli and Wood [34]. Both formulations were solved using standard branch-and-bound algorithms of CPLEX 9.0. The results are presented in Appendix C, and summarized in Table 4.1. In the table, $Q$, T, %I and %R are as defined in Section 2.4, except that [SPNIA-L] formulation is used instead of [SPNI-D($\hat{\theta}$)]. In these examples, the threshold value for the true shortest path is calculated by taking the 95% of the length of the shortest path when no information asymmetry is assumed, i.e., $\underline{z} = 0.95z_4^*$.

A summary of Table 4.1 is given in Table 4.2, and is illustrated in Figure 4.1. As can be seen from the table and the figure, the benefit gained by modeling the information asymmetry is larger than the risk arising because of it most of the time. For example, for $Q = 0.6$ and $B = 5$, the benefit is 6.3%, whereas the risk is only 2.8%; and for $Q = 0.7$ and $B = 15$, the benefit is 7.1%, whereas the risk is only 3.7%. The difference between the benefit and the risk becomes more significant as the level of information the evader has ($Q$) decreases. If the protector knows that the evader's estimates are not very accurate, then she can reap a large reward with a reasonable amount of risk.

When we compare the results of this chapter with the results of Chapter 2, we see that the benefits of the model in this chapter is smaller than the benefits of the model in Chapter 2, but the difference is not that significant. However, the risk of the model in this chapter is much smaller than the risk of the model in Chapter 2. Therefore, if the protector knows the estimates of the evader, we suggest to use the models in Chapter 2; but if the level of confidence in these estimates is low, we suggest to use the model in Chapter 4.

Table 4.1: Comparison of results for [MXSP-D] and [SPNIA-L]

| $Q$ | | 0.6 | | | 0.7 | | | 0.8 | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Network | $B$ | T | %I | %R | T | %I | %R | T | %I | %R |
| 1 | 5 | 0.6 | 4.3 | 2.2 | 0.7 | 2.5 | 2.3 | 0.8 | 1.9 | 1.3 |
| 2 | 5 | 0.9 | 6.8 | 2.7 | 1.2 | 5.1 | 2.6 | 1.2 | 3.5 | 2.7 |
| 3 | 5 | 1.8 | 7.1 | 3.4 | 3.1 | 3.6 | 3.3 | 2.7 | 2.1 | 2.3 |
| 4 | 5 | 2.1 | 7.1 | 3.2 | 2.3 | 4.5 | 2.9 | 2.1 | 2.0 | 1.8 |
| 5 | 5 | 3.4 | 5.2 | 3.2 | 2.6 | 5.7 | 3.9 | 2.5 | 1.3 | 2.6 |
| 6 | 5 | 4.2 | 6.7 | 2.2 | 3.6 | 5.6 | 2.2 | 4.2 | 2.6 | 1.6 |
| 7 | 5 | 5.7 | 7.0 | 2.6 | 4.6 | 3.7 | 2.4 | 4.1 | 1.9 | 2.2 |
| 1 | 10 | 3.1 | 6.1 | 4.1 | 2.7 | 3.9 | 2.4 | 1.9 | 2.6 | 1.8 |
| 2 | 10 | 12.6 | 11.0 | 4.3 | 12.5 | 8.9 | 3.8 | 39.2 | 5.0 | 2.7 |
| 3 | 10 | 33.7 | 10.1 | 3.9 | 125.6 | 5.9 | 3.0 | 86.2 | 4.6 | 3.4 |
| 4 | 10 | 17.9 | 8.1 | 3.1 | 22.3 | 4.6 | 2.8 | 17.7 | 4.3 | 2.5 |
| 5 | 10 | 46.6 | 8.6 | 4.0 | 29.2 | 5.9 | 3.7 | 58.9 | 3.4 | 3.7 |
| 6 | 10 | 104.0 | 8.7 | 3.4 | 158.4 | 4.9 | 3.2 | 183.4 | 2.6 | 2.6 |
| 7 | 10 | 246.6 | 9.8 | 3.9 | 113.9 | 7.0 | 3.0 | 90.3 | 3.3 | 2.8 |
| 1 | 15 | 5.3 | 6.6 | 3.1 | 8.1 | 3.6 | 3.0 | 8.3 | 2.1 | 2.0 |
| 2 | 15 | 77.1 | 9.8 | 4.2 | 130.2 | 7.7 | 3.9 | 184.8 | 4.5 | 3.5 |
| 3 | 15 | 615.7 | 9.6 | 4.0 | 703.0 | 7.5 | 4.1 | 769.2 | 4.0 | 3.8 |
| 4 | 15 | 140.5 | 11.5 | 4.4 | 173.5 | 8.6 | 4.0 | 134.3 | 5.4 | 2.3 |
| 5 | 15 | 466.5 | 10.7 | 4.3 | 367.0 | 7.4 | 3.5 | 433.7 | 4.5 | 3.4 |
| 6 | 15 | 1309.7 | 10.6 | 4.1 | 1488.8 | 7.2 | 3.4 | 1335.2 | 3.6 | 3.7 |
| 7 | 15 | 1508.8 | 11.1 | 4.3 | 998.7 | 7.7 | 4.1 | 1565.8 | 3.7 | 3.4 |

Table 4.2: Summary of benefits and risks.

| $Q$ | 0.6 | | 0.7 | | 0.8 | |
|---|---|---|---|---|---|---|
| $B$ | %I | %R | %I | %R | %I | %R |
| 5 | 6.3 | 2.8 | 4.4 | 2.8 | 2.0 | 2.1 |
| 10 | 8.9 | 3.8 | 5.9 | 3.1 | 3.6 | 2.8 |
| 15 | 10.0 | 4.1 | 7.1 | 3.7 | 4.0 | 3.2 |

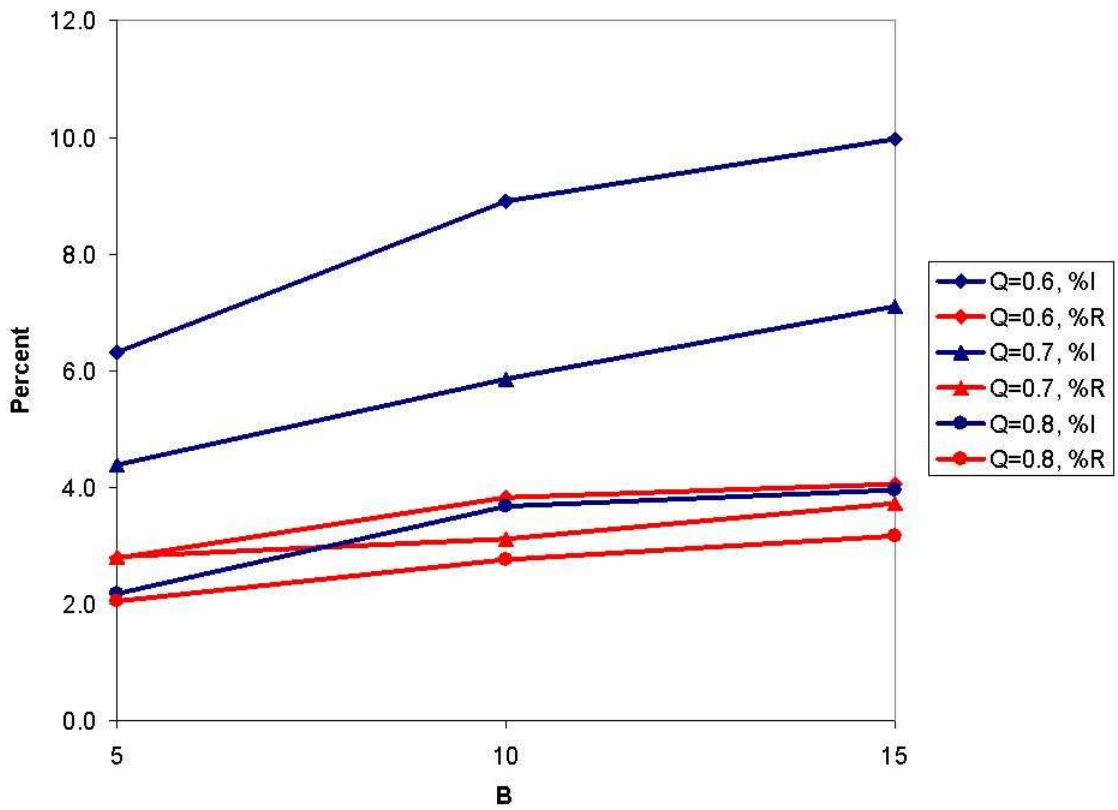


Figure 4.1: Summary of benefits and risks

## 4.3 ASYMMETRIC NETWORK INTERDICTION UNDER UNCERTAINTY

In the network interdiction models in Chapter 2 and in Section 4.2, we assumed that the evader does not know the true arc lengths, instead he uses estimates of these arc lengths to find the shortest path. On the other hand, it was assumed that the protector knows the values of these estimates. It was mentioned that a risk arises because of this assumption, and presented a model in Section 4.2 to limit this risk to beneath of given threshold. However, in certain cases, the protector may not know the exact values of the evader's estimates. For example, in the context of the resource allocation problem, there is an information asymmetry since the evader may not know the true detection probabilities. Also, there is an information asymmetry in the other direction, since the protector may not know the estimates of the evader. In this case, the protector may use the values that she thinks are most likely to represent the estimates of the evader, and analyze the problem using the models developed in Chapter 2 and Section 4.2; or instead of using point estimates, she can use a probability distribution to represent the estimates of the evader. In this case, the problem will become a two-stage stochastic programming problem.

Different approaches may be taken to model the problem as a stochastic program. The first approach may be risk based, i.e., the protector might want to minimize the probability that the length of the path that the evader will be less than a certain value. This approach is appropriate especially for the cases where a single occurrence of an event might be catastrophic. Another approach to model the problem as a stochastic program is to minimize the expected value of the shortest path that the evader will take. This approach is more appropriate for the cases where the event occurs many times, and the protector attempts to minimize the number of long-term occurrences. For illustration, we will present the stochastic programming formulation based on the latter approach. In this dissertation we will only provide the formulation, but will not formally analyze the model. Solutions methodologies for the provided formulation is left as future research.

The shortest path network interdiction problem with asymmetric information is composed of two stages. In the first stage, the protector interdicts the arcs. In the second stage, the evader realizes the arc lengths on the interdicted network, and finds the shortest path based on this realization. Let $\xi$ be a random event (scenario) that represents a realization of arc lengths by the evader, and let $\bar{c}_k(\xi)$ and $\bar{d}_k(\xi)$ be the estimates of $c_k$ and $d_k$ respectively by the evader for this scenario. Also, let $p(\xi)$ be the probability that scenario $\xi$ will occur. Finally, define variables $v_k(\xi), w_k(\xi)$ and $u_i(\xi)$ for every scenario $\xi$. In [SPNIA-L] formulation, if we replace the second stage variables and parameters with the corresponding random variables and parameters we will get the following stochastic programming formulation:

$$\text{[SPNIA-LS]:} \quad \max_{\mathbf{x},\mathbf{v},\mathbf{w},\mathbf{u}} \sum_{\xi} p(\xi)\{\sum_{k\in A}(c_k(\xi)v_k(\xi) + (c_k(\xi) + d_k(\xi))w_k(\xi))\},$$

$$\begin{aligned}
\sum_{k\in FS(i)} (v_k(\xi) + w_k(\xi)) - \sum_{k\in RS(i)} (v_k(\xi) + w_k(\xi)) &= \begin{cases} 1, & i = 0,\ \forall\xi, \\ 0, & i = 1, 2,\ \ldots, n-1, \forall\xi, \\ -1, & i = n,\ \forall\xi, \end{cases} \\
u_i(\xi) - u_j(\xi) - x_k\bar{d}_k(\xi) &\leq \bar{c}_k(\xi), \quad \forall (i,j) = k \in A,\ \forall\xi, \\
u_n(\xi) - u_0(\xi) + \sum_{k\in A}(\bar{c}_k(\xi)v_k(\xi) + (\bar{c}_k(\xi) + \bar{d}_k(\xi))w_k(\xi)) &= 0,\ \forall\xi, \\
v_k(\xi) + x_k &\leq 1, \quad \forall k \in A,\ \forall\xi, \\
w_k(\xi) - x_k &\leq 0, \quad \forall k \in A,\ \forall\xi, \\
v_k(\xi), w_k(\xi) &\geq 0, \quad \forall k \in A,\ \forall\xi, \\
\mathbf{x} &\in X.
\end{aligned}$$

A similar formulation can be presented for [SPNI-D($\hat{\theta}$)] by replacing the second stage variables with the corresponding random variables. [SPNIA-LS] formulation falls into the class of standard two-stage stochastic programs, and can be solved using standard stochastic programming techniques. However, the definition of $\xi$ and $p(\xi)$, and the solution methodology are left for future work.

## 4.4 CONCLUSIONS

In this chapter, we presented two approaches to handle the risk arising due to modeling information asymmetry in the network interdiction models. In the first approach, we modeled the problem in a way such that the risk is never greater than a certain value. The computational results showed that significant benefits could be gained while limiting the risk at a desired level. In the second approach, we modeled the problem as a two-stage stochastic program based on the expected value formulation.

The computational examples suggest that if the protector has a good intelligence about the evader and knows the evader's estimates, the models in Chapter 2 provide more benefit. However, if the protector does not know the evader's estimates, the model presented in this chapter is more appropriate and risk averse.

# 5.0 SUMMARY AND FUTURE RESEARCH

## 5.1 SUMMARY

This dissertation considered the problem of optimally allocating static and dynamic detection resources available in order to detect or prevent evaders from reaching their destinations. The evaders may be terrorists or smugglers attempting to enter a facility or illegally cross a border. Examples of static detection resources include sensors that detect people and weapons, cameras and check points, whereas examples of dynamic detection resources include guards at the borders and unmanned aerial vehicles. It is crucial to use these resources efficiently to increase the detection probabilities of evaders.

We analyzed the detection resource allocation problem in the context of network interdiction problems and inspection games. In the context of network interdiction problems, we considered the allocation of static detection resources and developed models to help the protector in allocating her detection resources under information asymmetry. The computational examples showed that there are benefits as well as risks associated with the modeling of information asymmetry. We proposed two models to deal with the risk. First model limits the risk to a threshold value, and the second model helps the protector to calculate the expected value of the risk under uncertainty. In the context of inspection games, we considered the allocation of both static and dynamic detection resources. We built a model that combines the inspection games with the network interdiction models. In this model, first the protector allocates her static dynamic resources to the regions, then the evader and the protector play an inspection game.

This dissertation is the first study to introduce the information asymmetry in the network interdiction models, and also the first one to combine the network interdiction models and inspection games to analyze the allocation of detection resources. With these contributions, the perimeter and facility security problems can be analyzed more thoroughly and comprehensively. Also, this dissertation proposes possibilities for a wide range of future research.

The models in this dissertation are built, solved and analyzed using integer programming, stochastic programming and game theory techniques. Structural properties of the models are explored and heuristic algorithms are developed to solve larger problem instances.

## 5.2 FUTURE RESEARCH

There are several possible extensions to this dissertation. Section 5.2.1 discusses several future research directions for the network interdiction problem with asymmetric information, and Section 5.2.2 discusses several future research direction for the perimeter security problem with interdiction.

### 5.2.1 Possible Extensions to the Network Interdiction Problem with Asymmetric Information

**5.2.1.1 Computational Considerations** In Chapter 2, we modeled the network interdiction problem with asymmetric information, and solved the problem using standard branch-and-bound algorithms. We also proposed how to employ Benders' decomposition algorithm for these models in Section 2.3.5. However, Benders' decomposition algorithm converged slowly. In order to improve the effectiveness of the Benders' decomposition algorithm, strong valid inequalities should be added throughout the run. These valid inequalities can be derived from the structural properties of the problem, and requires attention to solve larger problem instances effectively.

We provided a two-stage stochastic programming formulation for the network interdiction problem when there is a complete information asymmetry between the protector and the evader in Section 4.3. However, we did not develop solution methodologies to solve this formulation. L-shaped method based algorithms can be employed to solve this problem.

**5.2.1.2 Unknown Origin/Destination** In all of the problems discussed in this dissertation, we assumed that there is a single origin and a single destination, and they are known both by the evader and the protector. However, the protector may not know the origin and destination nodes of the evader, instead she may have a probability distribution for the origin/destination pairs (e.g. [42]). In this case, we have to incorporate the probabilities and corresponding variables arising from the stochasticity in the origin/destination pairs. This problem can be formulated as a two-stage stochastic program and solved using stochastic programming techniques.

**5.2.1.3 Unknown Sensor Locations** Another assumption made in all of the models discussed in this dissertation is that the evader knows the locations of the static detection resources allocated. This is typically a valid assumption , however detection resources can often be hidden from view. Hence, the evader will have less information about the network, and his capability to make better decisions will be reduced. The protector can utilize this source of information asymmetry and plan her detection resource allocation scheme accordingly.

### 5.2.2 Possible Extensions to the Perimeter Security Problem with Interdiction

**5.2.2.1 Multiple Protectors** In the perimeter security problem model presented in Chapter 3, there was a single protector that inspected $n$ regions. As $n$ becomes large, effectiveness of a single protector decreases, and the need for more than one protector arises. This generalization can be included in the model by expanding the state space of the problem and defining new traveling costs. However, this increase in model complexity will require further computational refinements.

**5.2.2.2 Non-zero Sum Games** In the execution phase of the perimeter security problem model presented in Chapter 3, we assumed that the game was zero-sum, i.e., we assumed that the cost to the protector was a reward for the evader. However, the evader does not benefit from the traveling cost of the protector. If we can model the problem to take this asymmetry into consideration, the game becomes a non-zero game. This results in a more realistic, but more complex model.

**5.2.2.3 Asymmetric Information** In the perimeter security problem model presented in Chapter 3, we assumed that the detection probabilities of the detection resources are known by both the protector and the evader. however, it is possible that the evader may not know the true detection probabilities. In this case, a similar approach to the ones in Chapter 2 and Chapter 4 can be employed to account for this information asymmetry. In this case, the problem will be more complicated and difficult to model. However, if successful, it can provide great benefits to the protector.

**5.2.2.4 Non-linear rewards** In the perimeter security problem model presented in Chapter 3, we assumed that the rewards were linearly dependent on the amount of the contraband crossing through a region. Also, we assumed that the detection probabilities did not depend on the amount of the contraband passing through the region. However, if these are not the case, the theorems developed in Chapter 3 would not be valid, and the problem would be more difficult to solve. Structural properties of the problem can be investigated for this more complicated model.

# APPENDIX A

# RESULTS OF THE RUNS FOR CHAPTER 2

Table A1: Results of the runs for $Q = 0.6$, $B = 5$ for networks 1,2,3,4

| Network | $Q$ | Instance | $B$ | $z_2^*$ | $z_3^*$ | $T$ | $z_1^*$ | $z_4^*$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.6 | 1 | 5 | 53.681 | 47.382 | 0.328 | 50.494 | 50.494 |
| 1 | 0.6 | 2 | 5 | 57.664 | 55.065 | 0.484 | 57.467 | 55.065 |
| 1 | 0.6 | 3 | 5 | 50.531 | 46.484 | 0.375 | 50.141 | 46.484 |
| 1 | 0.6 | 4 | 5 | 53.192 | 47.861 | 0.249 | 47.861 | 47.861 |
| 1 | 0.6 | 5 | 5 | 57.150 | 46.618 | 0.094 | 56.395 | 51.130 |
| 1 | 0.6 | 6 | 5 | 56.091 | 47.980 | 0.937 | 52.625 | 52.143 |
| 1 | 0.6 | 7 | 5 | 51.485 | 43.530 | 0.359 | 45.662 | 45.662 |
| 1 | 0.6 | 8 | 5 | 60.833 | 50.506 | 0.234 | 60.409 | 55.107 |
| 1 | 0.6 | 9 | 5 | 56.081 | 46.517 | 0.453 | 53.630 | 53.630 |
| 1 | 0.6 | 10 | 5 | 63.118 | 55.410 | 0.296 | 61.210 | 59.512 |
| 2 | 0.6 | 1 | 5 | 60.697 | 45.909 | 0.187 | 48.987 | 48.356 |
| 2 | 0.6 | 2 | 5 | 56.478 | 48.074 | 0.546 | 55.822 | 49.968 |
| 2 | 0.6 | 3 | 5 | 54.917 | 45.007 | 0.641 | 51.153 | 48.359 |
| 2 | 0.6 | 4 | 5 | 56.903 | 47.915 | 0.656 | 55.492 | 52.047 |
| 2 | 0.6 | 5 | 5 | 66.705 | 50.159 | 0.203 | 56.173 | 53.511 |
| 2 | 0.6 | 6 | 5 | 50.494 | 41.440 | 0.250 | 44.911 | 43.260 |
| 2 | 0.6 | 7 | 5 | 51.877 | 43.701 | 0.515 | 47.626 | 45.479 |
| 2 | 0.6 | 8 | 5 | 49.430 | 43.210 | 0.765 | 46.974 | 45.242 |
| 2 | 0.6 | 9 | 5 | 60.017 | 50.934 | 0.297 | 57.898 | 52.048 |
| 2 | 0.6 | 10 | 5 | 56.468 | 47.020 | 0.297 | 53.331 | 49.080 |
| 3 | 0.6 | 1 | 5 | 59.857 | 46.604 | 1.203 | 55.251 | 52.810 |
| 3 | 0.6 | 2 | 5 | 64.704 | 51.215 | 0.250 | 59.401 | 53.690 |
| 3 | 0.6 | 3 | 5 | 56.185 | 45.241 | 3.249 | 52.495 | 49.410 |
| 3 | 0.6 | 4 | 5 | 65.072 | 49.751 | 0.969 | 58.560 | 55.286 |
| 3 | 0.6 | 5 | 5 | 67.783 | 57.060 | 0.688 | 62.518 | 59.468 |
| 3 | 0.6 | 6 | 5 | 59.136 | 50.532 | 0.390 | 52.520 | 50.532 |
| 3 | 0.6 | 7 | 5 | 59.217 | 49.275 | 0.641 | 55.370 | 50.220 |
| 3 | 0.6 | 8 | 5 | 65.350 | 53.572 | 0.297 | 55.784 | 55.784 |
| 3 | 0.6 | 9 | 5 | 66.048 | 55.476 | 0.406 | 61.987 | 56.856 |
| 3 | 0.6 | 10 | 5 | 65.045 | 52.331 | 0.328 | 59.382 | 57.918 |
| 4 | 0.6 | 1 | 5 | 53.217 | 41.171 | 0.406 | 46.188 | 43.547 |
| 4 | 0.6 | 2 | 5 | 52.100 | 44.977 | 0.500 | 51.020 | 45.414 |
| 4 | 0.6 | 3 | 5 | 49.240 | 47.175 | 0.953 | 47.762 | 44.242 |
| 4 | 0.6 | 4 | 5 | 50.850 | 41.006 | 0.719 | 48.367 | 44.360 |
| 4 | 0.6 | 5 | 5 | 54.362 | 42.134 | 0.406 | 47.852 | 45.089 |
| 4 | 0.6 | 6 | 5 | 41.328 | 36.163 | 1.062 | 39.596 | 38.116 |
| 4 | 0.6 | 7 | 5 | 41.816 | 34.539 | 1.859 | 37.584 | 36.679 |
| 4 | 0.6 | 8 | 5 | 49.754 | 34.591 | 0.265 | 43.766 | 39.221 |
| 4 | 0.6 | 9 | 5 | 49.562 | 39.079 | 0.797 | 45.914 | 44.711 |
| 4 | 0.6 | 10 | 5 | 46.364 | 39.415 | 2.890 | 44.184 | 42.265 |

Table A2: Results of the runs for $Q = 0.6$, $B = 5$ for networks 5,6,7

| Network | $Q$ | Instance | $B$ | $z_2^*$ | $z_3^*$ | $T$ | $z_1^*$ | $z_4^*$ |
|---|---|---|---|---|---|---|---|---|
| 5 | 0.6 | 1 | 5 | 53.531 | 46.267 | 1.703 | 50.584 | 49.750 |
| 5 | 0.6 | 2 | 5 | 51.747 | 42.279 | 2.234 | 47.544 | 45.951 |
| 5 | 0.6 | 3 | 5 | 56.686 | 42.517 | 0.375 | 49.295 | 47.745 |
| 5 | 0.6 | 4 | 5 | 56.424 | 42.653 | 1.844 | 50.367 | 49.006 |
| 5 | 0.6 | 5 | 5 | 50.287 | 42.827 | 0.719 | 44.772 | 44.735 |
| 5 | 0.6 | 6 | 5 | 48.446 | 44.281 | 9.016 | 48.444 | 46.388 |
| 5 | 0.6 | 7 | 5 | 53.466 | 43.715 | 1.984 | 48.460 | 47.555 |
| 5 | 0.6 | 8 | 5 | 55.003 | 45.974 | 0.609 | 55.003 | 45.974 |
| 5 | 0.6 | 9 | 5 | 52.924 | 42.920 | 2.031 | 48.892 | 48.892 |
| 5 | 0.6 | 10 | 5 | 53.377 | 48.557 | 2.531 | 49.294 | 48.889 |
| 6 | 0.6 | 1 | 5 | 61.338 | 51.976 | 1.265 | 61.007 | 58.927 |
| 6 | 0.6 | 2 | 5 | 64.172 | 55.249 | 1.016 | 60.908 | 57.445 |
| 6 | 0.6 | 3 | 5 | 66.951 | 56.518 | 1.234 | 59.393 | 57.353 |
| 6 | 0.6 | 4 | 5 | 61.667 | 53.747 | 9.609 | 60.580 | 54.758 |
| 6 | 0.6 | 5 | 5 | 64.523 | 49.907 | 1.047 | 54.024 | 54.024 |
| 6 | 0.6 | 6 | 5 | 68.488 | 60.593 | 1.328 | 63.400 | 63.400 |
| 6 | 0.6 | 7 | 5 | 62.498 | 50.574 | 1.250 | 54.408 | 53.574 |
| 6 | 0.6 | 8 | 5 | 62.452 | 49.603 | 1.391 | 61.369 | 51.998 |
| 6 | 0.6 | 9 | 5 | 63.448 | 58.723 | 1.797 | 61.952 | 59.424 |
| 6 | 0.6 | 10 | 5 | 59.267 | 55.565 | 1.437 | 55.565 | 55.565 |
| 7 | 0.6 | 1 | 5 | 65.851 | 53.594 | 1.343 | 59.343 | 56.025 |
| 7 | 0.6 | 2 | 5 | 62.436 | 53.989 | 1.109 | 55.717 | 53.520 |
| 7 | 0.6 | 3 | 5 | 65.453 | 55.349 | 6.828 | 66.008 | 62.095 |
| 7 | 0.6 | 4 | 5 | 59.391 | 48.744 | 2.547 | 53.339 | 53.052 |
| 7 | 0.6 | 5 | 5 | 56.119 | 52.740 | 1.328 | 54.865 | 53.972 |
| 7 | 0.6 | 6 | 5 | 53.017 | 41.405 | 0.625 | 46.917 | 45.612 |
| 7 | 0.6 | 7 | 5 | 54.576 | 44.069 | 3.844 | 49.205 | 47.727 |
| 7 | 0.6 | 8 | 5 | 66.615 | 50.218 | 1.187 | 60.119 | 55.195 |
| 7 | 0.6 | 9 | 5 | 67.598 | 50.368 | 10.406 | 61.203 | 57.505 |
| 7 | 0.6 | 10 | 5 | 58.242 | 48.825 | 1.156 | 56.074 | 52.409 |

Table A3: Results of the runs for $Q = 0.7$, $B = 5$ for networks 1,2,3,4

| Network | $Q$ | Instance | $B$ | $z_2^*$ | $z_3^*$ | $T$ | $z_1^*$ | $z_4^*$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.7 | 1 | 5 | 51.487 | 44.160 | 0.281 | 50.494 | 50.494 |
| 1 | 0.7 | 2 | 5 | 57.664 | 55.065 | 0.265 | 57.467 | 55.065 |
| 1 | 0.7 | 3 | 5 | 48.745 | 45.924 | 0.563 | 46.484 | 46.484 |
| 1 | 0.7 | 4 | 5 | 50.101 | 42.223 | 0.313 | 47.861 | 47.861 |
| 1 | 0.7 | 5 | 5 | 51.749 | 49.011 | 0.171 | 51.130 | 51.130 |
| 1 | 0.7 | 6 | 5 | 53.927 | 49.995 | 0.984 | 52.625 | 52.143 |
| 1 | 0.7 | 7 | 5 | 48.621 | 44.885 | 0.453 | 45.662 | 45.662 |
| 1 | 0.7 | 8 | 5 | 60.409 | 50.506 | 0.124 | 56.737 | 55.107 |
| 1 | 0.7 | 9 | 5 | 56.081 | 46.517 | 0.234 | 53.630 | 53.630 |
| 1 | 0.7 | 10 | 5 | 61.640 | 60.237 | 0.250 | 61.210 | 59.512 |
| 2 | 0.7 | 1 | 5 | 57.306 | 48.156 | 0.234 | 48.987 | 48.356 |
| 2 | 0.7 | 2 | 5 | 53.928 | 48.074 | 0.610 | 52.340 | 49.968 |
| 2 | 0.7 | 3 | 5 | 51.397 | 48.059 | 0.531 | 51.153 | 48.359 |
| 2 | 0.7 | 4 | 5 | 55.065 | 44.410 | 0.453 | 55.492 | 52.047 |
| 2 | 0.7 | 5 | 5 | 58.738 | 49.250 | 2.438 | 56.173 | 53.511 |
| 2 | 0.7 | 6 | 5 | 45.150 | 43.499 | 2.906 | 44.911 | 43.260 |
| 2 | 0.7 | 7 | 5 | 50.062 | 43.701 | 0.313 | 47.626 | 45.479 |
| 2 | 0.7 | 8 | 5 | 47.698 | 43.210 | 0.656 | 45.242 | 45.242 |
| 2 | 0.7 | 9 | 5 | 57.898 | 52.048 | 0.344 | 57.898 | 52.048 |
| 2 | 0.7 | 10 | 5 | 53.196 | 47.407 | 0.484 | 51.084 | 49.080 |
| 3 | 0.7 | 1 | 5 | 59.296 | 47.436 | 0.343 | 55.251 | 52.810 |
| 3 | 0.7 | 2 | 5 | 59.383 | 49.979 | 1.984 | 55.919 | 53.690 |
| 3 | 0.7 | 3 | 5 | 53.968 | 44.174 | 1.312 | 52.495 | 49.410 |
| 3 | 0.7 | 4 | 5 | 62.059 | 52.179 | 1.156 | 58.560 | 55.286 |
| 3 | 0.7 | 5 | 5 | 64.938 | 57.060 | 1.281 | 62.518 | 59.468 |
| 3 | 0.7 | 6 | 5 | 54.102 | 50.450 | 1.859 | 52.520 | 50.532 |
| 3 | 0.7 | 7 | 5 | 54.120 | 46.462 | 1.875 | 50.367 | 50.220 |
| 3 | 0.7 | 8 | 5 | 57.202 | 53.572 | 2.906 | 55.784 | 55.784 |
| 3 | 0.7 | 9 | 5 | 63.929 | 55.476 | 0.391 | 61.987 | 56.856 |
| 3 | 0.7 | 10 | 5 | 62.948 | 55.095 | 0.484 | 59.382 | 57.918 |
| 4 | 0.7 | 1 | 5 | 49.424 | 42.326 | 0.656 | 46.188 | 43.547 |
| 4 | 0.7 | 2 | 5 | 49.147 | 44.977 | 0.672 | 46.356 | 45.414 |
| 4 | 0.7 | 3 | 5 | 46.211 | 41.129 | 1.062 | 44.581 | 44.242 |
| 4 | 0.7 | 4 | 5 | 49.566 | 40.756 | 0.437 | 45.246 | 44.360 |
| 4 | 0.7 | 5 | 5 | 50.946 | 41.727 | 0.375 | 47.852 | 45.089 |
| 4 | 0.7 | 6 | 5 | 39.657 | 33.006 | 2.266 | 39.596 | 38.116 |
| 4 | 0.7 | 7 | 5 | 41.217 | 36.948 | 1.453 | 37.584 | 36.679 |
| 4 | 0.7 | 8 | 5 | 47.515 | 37.713 | 0.250 | 43.766 | 39.221 |
| 4 | 0.7 | 9 | 5 | 47.207 | 42.199 | 1.062 | 45.914 | 44.711 |
| 4 | 0.7 | 10 | 5 | 45.861 | 39.591 | 0.859 | 43.379 | 42.265 |

Table A4: Results of the runs for $Q = 0.7$, $B = 5$ for networks 5,6,7

| Network | $Q$ | Instance | $B$ | $z_2^*$ | $z_3^*$ | $T$ | $z_1^*$ | $z_4^*$ |
|---|---|---|---|---|---|---|---|---|
| 5 | 0.7 | 1 | 5 | 51.003 | 47.917 | 1.312 | 49.874 | 49.750 |
| 5 | 0.7 | 2 | 5 | 50.197 | 42.310 | 2.187 | 46.066 | 45.951 |
| 5 | 0.7 | 3 | 5 | 52.839 | 46.503 | 1.265 | 49.295 | 47.745 |
| 5 | 0.7 | 4 | 5 | 53.420 | 46.751 | 2.406 | 50.367 | 49.006 |
| 5 | 0.7 | 5 | 5 | 50.287 | 42.827 | 0.156 | 44.772 | 44.735 |
| 5 | 0.7 | 6 | 5 | 48.444 | 46.388 | 1.031 | 48.444 | 46.388 |
| 5 | 0.7 | 7 | 5 | 50.087 | 43.715 | 2.375 | 48.460 | 47.555 |
| 5 | 0.7 | 8 | 5 | 52.187 | 43.887 | 0.984 | 47.963 | 45.974 |
| 5 | 0.7 | 9 | 5 | 51.041 | 45.220 | 1.438 | 48.892 | 48.892 |
| 5 | 0.7 | 10 | 5 | 53.377 | 48.274 | 0.578 | 49.294 | 48.889 |
| 6 | 0.7 | 1 | 5 | 59.648 | 57.366 | 1.640 | 59.648 | 58.927 |
| 6 | 0.7 | 2 | 5 | 61.887 | 57.445 | 1.078 | 59.993 | 57.445 |
| 6 | 0.7 | 3 | 5 | 66.153 | 53.800 | 0.547 | 59.393 | 57.353 |
| 6 | 0.7 | 4 | 5 | 60.580 | 53.747 | 5.312 | 57.767 | 54.758 |
| 6 | 0.7 | 5 | 5 | 60.916 | 53.782 | 0.500 | 54.024 | 54.024 |
| 6 | 0.7 | 6 | 5 | 64.552 | 57.292 | 6.312 | 63.400 | 63.400 |
| 6 | 0.7 | 7 | 5 | 57.547 | 48.623 | 1.890 | 54.408 | 53.574 |
| 6 | 0.7 | 8 | 5 | 60.248 | 49.603 | 1.593 | 53.873 | 51.998 |
| 6 | 0.7 | 9 | 5 | 61.952 | 59.424 | 1.250 | 61.952 | 59.424 |
| 6 | 0.7 | 10 | 5 | 59.267 | 55.565 | 0.719 | 55.565 | 55.565 |
| 7 | 0.7 | 1 | 5 | 63.422 | 54.884 | 1.016 | 59.343 | 56.025 |
| 7 | 0.7 | 2 | 5 | 59.594 | 53.021 | 1.187 | 55.717 | 53.520 |
| 7 | 0.7 | 3 | 5 | 64.159 | 62.095 | 3.156 | 66.008 | 62.095 |
| 7 | 0.7 | 4 | 5 | 56.915 | 50.316 | 2.047 | 53.339 | 53.052 |
| 7 | 0.7 | 5 | 5 | 54.865 | 53.972 | 0.593 | 54.865 | 53.972 |
| 7 | 0.7 | 6 | 5 | 50.594 | 42.898 | 0.953 | 46.917 | 45.612 |
| 7 | 0.7 | 7 | 5 | 49.721 | 44.453 | 4.438 | 47.727 | 47.727 |
| 7 | 0.7 | 8 | 5 | 59.019 | 50.906 | 2.172 | 56.812 | 55.195 |
| 7 | 0.7 | 9 | 5 | 63.212 | 54.791 | 11.266 | 61.203 | 57.505 |
| 7 | 0.7 | 10 | 5 | 56.208 | 48.825 | 1.375 | 56.074 | 52.409 |

Table A5: Results of the runs for $Q = 0.8$, $B = 5$ for networks 1,2,3,4

| Network | $Q$ | Instance | $B$ | $z_2^*$ | $z_3^*$ | $T$ | $z_1^*$ | $z_4^*$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.8 | 1 | 5 | 51.460 | 48.952 | 0.171 | 50.494 | 50.494 |
| 1 | 0.8 | 2 | 5 | 55.999 | 55.065 | 0.219 | 55.802 | 55.065 |
| 1 | 0.8 | 3 | 5 | 48.425 | 45.924 | 0.406 | 46.484 | 46.484 |
| 1 | 0.8 | 4 | 5 | 49.736 | 47.778 | 0.218 | 47.861 | 47.861 |
| 1 | 0.8 | 5 | 5 | 51.749 | 49.011 | 0.094 | 51.130 | 51.130 |
| 1 | 0.8 | 6 | 5 | 53.443 | 52.143 | 0.422 | 52.625 | 52.143 |
| 1 | 0.8 | 7 | 5 | 47.736 | 45.654 | 0.328 | 45.662 | 45.662 |
| 1 | 0.8 | 8 | 5 | 57.958 | 50.506 | 0.375 | 56.737 | 55.107 |
| 1 | 0.8 | 9 | 5 | 54.072 | 51.895 | 0.250 | 53.630 | 53.630 |
| 1 | 0.8 | 10 | 5 | 61.640 | 60.237 | 0.109 | 61.210 | 59.512 |
| 2 | 0.8 | 1 | 5 | 51.601 | 48.156 | 0.531 | 48.987 | 48.356 |
| 2 | 0.8 | 2 | 5 | 51.276 | 48.538 | 0.781 | 49.968 | 49.968 |
| 2 | 0.8 | 3 | 5 | 51.153 | 48.059 | 0.250 | 49.407 | 48.359 |
| 2 | 0.8 | 4 | 5 | 52.745 | 49.112 | 0.391 | 53.809 | 52.047 |
| 2 | 0.8 | 5 | 5 | 56.173 | 51.231 | 0.984 | 56.173 | 53.511 |
| 2 | 0.8 | 6 | 5 | 44.027 | 41.440 | 1.047 | 43.260 | 43.260 |
| 2 | 0.8 | 7 | 5 | 48.004 | 43.421 | 0.360 | 47.626 | 45.479 |
| 2 | 0.8 | 8 | 5 | 45.242 | 44.263 | 0.703 | 45.242 | 45.242 |
| 2 | 0.8 | 9 | 5 | 55.956 | 51.053 | 0.313 | 54.687 | 52.048 |
| 2 | 0.8 | 10 | 5 | 53.006 | 47.020 | 0.188 | 49.080 | 49.080 |
| 3 | 0.8 | 1 | 5 | 55.428 | 50.723 | 0.547 | 55.251 | 52.810 |
| 3 | 0.8 | 2 | 5 | 57.365 | 51.654 | 1.078 | 55.919 | 53.690 |
| 3 | 0.8 | 3 | 5 | 51.099 | 48.401 | 2.562 | 49.779 | 49.410 |
| 3 | 0.8 | 4 | 5 | 57.489 | 52.303 | 2.391 | 55.716 | 55.286 |
| 3 | 0.8 | 5 | 5 | 62.518 | 56.208 | 0.656 | 60.523 | 59.468 |
| 3 | 0.8 | 6 | 5 | 51.376 | 50.532 | 2.375 | 51.376 | 50.532 |
| 3 | 0.8 | 7 | 5 | 51.969 | 47.191 | 1.000 | 50.367 | 50.220 |
| 3 | 0.8 | 8 | 5 | 56.545 | 50.630 | 0.906 | 55.784 | 55.784 |
| 3 | 0.8 | 9 | 5 | 60.012 | 56.562 | 0.781 | 58.776 | 56.856 |
| 3 | 0.8 | 10 | 5 | 62.948 | 55.095 | 0.344 | 59.382 | 57.918 |
| 4 | 0.8 | 1 | 5 | 46.717 | 41.171 | 0.688 | 45.060 | 43.547 |
| 4 | 0.8 | 2 | 5 | 46.356 | 45.414 | 0.578 | 46.356 | 45.414 |
| 4 | 0.8 | 3 | 5 | 44.924 | 47.175 | 0.782 | 44.581 | 44.242 |
| 4 | 0.8 | 4 | 5 | 46.738 | 44.359 | 0.531 | 45.246 | 44.360 |
| 4 | 0.8 | 5 | 5 | 47.387 | 42.134 | 0.625 | 45.624 | 45.089 |
| 4 | 0.8 | 6 | 5 | 37.958 | 36.163 | 3.453 | 38.116 | 38.116 |
| 4 | 0.8 | 7 | 5 | 38.829 | 36.336 | 1.047 | 36.993 | 36.679 |
| 4 | 0.8 | 8 | 5 | 40.916 | 38.122 | 1.047 | 39.221 | 39.221 |
| 4 | 0.8 | 9 | 5 | 45.914 | 44.891 | 0.515 | 45.914 | 44.711 |
| 4 | 0.8 | 10 | 5 | 43.731 | 40.868 | 2.281 | 43.379 | 42.265 |

Table A6: Results of the runs for $Q = 0.8$, $B = 5$ for networks 5,6,7

| Network | $Q$ | Instance | $B$ | $z_2^*$ | $z_3^*$ | $T$ | $z_1^*$ | $z_4^*$ |
|---|---|---|---|---|---|---|---|---|
| 5 | 0.8 | 1 | 5 | 49.874 | 48.947 | 0.984 | 49.874 | 49.750 |
| 5 | 0.8 | 2 | 5 | 47.042 | 43.712 | 2.859 | 46.066 | 45.951 |
| 5 | 0.8 | 3 | 5 | 50.148 | 46.517 | 1.281 | 49.295 | 47.745 |
| 5 | 0.8 | 4 | 5 | 50.592 | 46.930 | 2.656 | 50.367 | 49.006 |
| 5 | 0.8 | 5 | 5 | 46.425 | 41.876 | 0.359 | 44.772 | 44.735 |
| 5 | 0.8 | 6 | 5 | 48.444 | 45.501 | 0.359 | 48.444 | 46.388 |
| 5 | 0.8 | 7 | 5 | 49.248 | 46.279 | 0.906 | 47.869 | 47.555 |
| 5 | 0.8 | 8 | 5 | 47.787 | 43.688 | 1.469 | 46.272 | 45.974 |
| 5 | 0.8 | 9 | 5 | 50.185 | 46.325 | 0.734 | 48.892 | 48.892 |
| 5 | 0.8 | 10 | 5 | 49.779 | 48.274 | 0.953 | 49.294 | 48.889 |
| 6 | 0.8 | 1 | 5 | 59.430 | 58.927 | 0.718 | 59.430 | 58.927 |
| 6 | 0.8 | 2 | 5 | 60.908 | 57.445 | 0.578 | 59.993 | 57.445 |
| 6 | 0.8 | 3 | 5 | 59.045 | 52.282 | 2.062 | 59.393 | 57.353 |
| 6 | 0.8 | 4 | 5 | 56.855 | 53.418 | 5.688 | 55.088 | 54.758 |
| 6 | 0.8 | 5 | 5 | 55.133 | 51.414 | 1.187 | 54.024 | 54.024 |
| 6 | 0.8 | 6 | 5 | 63.572 | 60.593 | 1.265 | 63.400 | 63.400 |
| 6 | 0.8 | 7 | 5 | 54.191 | 48.623 | 2.594 | 53.817 | 53.574 |
| 6 | 0.8 | 8 | 5 | 56.945 | 52.011 | 0.750 | 53.873 | 51.998 |
| 6 | 0.8 | 9 | 5 | 61.123 | 58.723 | 0.703 | 59.767 | 59.424 |
| 6 | 0.8 | 10 | 5 | 57.289 | 55.886 | 0.828 | 55.565 | 55.565 |
| 7 | 0.8 | 1 | 5 | 57.212 | 53.906 | 3.515 | 56.152 | 56.025 |
| 7 | 0.8 | 2 | 5 | 55.951 | 53.021 | 1.297 | 55.717 | 53.520 |
| 7 | 0.8 | 3 | 5 | 62.794 | 62.095 | 1.468 | 64.643 | 62.095 |
| 7 | 0.8 | 4 | 5 | 56.852 | 53.948 | 1.500 | 53.339 | 53.052 |
| 7 | 0.8 | 5 | 5 | 54.865 | 53.972 | 0.359 | 54.865 | 53.972 |
| 7 | 0.8 | 6 | 5 | 46.917 | 45.119 | 1.890 | 46.917 | 45.612 |
| 7 | 0.8 | 7 | 5 | 49.721 | 45.587 | 1.281 | 47.727 | 47.727 |
| 7 | 0.8 | 8 | 5 | 56.812 | 55.195 | 0.640 | 56.812 | 55.195 |
| 7 | 0.8 | 9 | 5 | 59.687 | 56.083 | 6.203 | 57.505 | 57.505 |
| 7 | 0.8 | 10 | 5 | 53.191 | 50.428 | 2.422 | 52.622 | 52.409 |

Table A7: Results of the runs for $Q = 0.6$, $B = 10$ for networks 1,2,3,4

| Network | $Q$ | Instance | $B$ | $z_2^*$ | $z_3^*$ | $T$ | $z_1^*$ | $z_4^*$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.6 | 1 | 10 | 65.732 | 55.549 | 0.484 | 62.390 | 59.549 |
| 1 | 0.6 | 2 | 10 | 65.471 | 59.755 | 8.093 | 65.224 | 62.822 |
| 1 | 0.6 | 3 | 10 | 62.278 | 54.834 | 4.249 | 60.383 | 57.565 |
| 1 | 0.6 | 4 | 10 | 64.162 | 56.679 | 1.749 | 58.529 | 58.529 |
| 1 | 0.6 | 5 | 10 | 69.127 | 55.475 | 0.234 | 58.697 | 58.697 |
| 1 | 0.6 | 6 | 10 | 65.668 | 55.442 | 1.031 | 61.026 | 59.822 |
| 1 | 0.6 | 7 | 10 | 63.243 | 55.079 | 0.499 | 57.853 | 57.582 |
| 1 | 0.6 | 8 | 10 | 72.778 | 57.724 | 0.656 | 68.513 | 63.774 |
| 1 | 0.6 | 9 | 10 | 67.153 | 59.619 | 2.890 | 62.522 | 62.522 |
| 1 | 0.6 | 10 | 10 | 72.908 | 67.103 | 0.968 | 70.005 | 69.032 |
| 2 | 0.6 | 1 | 10 | 69.453 | 51.851 | 1.703 | 58.083 | 56.643 |
| 2 | 0.6 | 2 | 10 | 65.080 | 51.347 | 1.625 | 60.066 | 55.541 |
| 2 | 0.6 | 3 | 10 | 67.768 | 55.271 | 0.687 | 63.743 | 58.682 |
| 2 | 0.6 | 4 | 10 | 68.602 | 53.803 | 2.765 | 63.109 | 58.794 |
| 2 | 0.6 | 5 | 10 | 72.662 | 54.565 | 5.468 | 66.705 | 60.036 |
| 2 | 0.6 | 6 | 10 | 57.253 | 46.879 | 5.421 | 54.777 | 48.126 |
| 2 | 0.6 | 7 | 10 | 63.090 | 50.176 | 4.343 | 53.376 | 52.312 |
| 2 | 0.6 | 8 | 10 | 61.738 | 50.850 | 6.984 | 52.457 | 52.457 |
| 2 | 0.6 | 9 | 10 | 65.615 | 55.484 | 13.578 | 59.193 | 58.002 |
| 2 | 0.6 | 10 | 10 | 66.500 | 51.594 | 0.781 | 56.126 | 55.701 |
| 3 | 0.6 | 1 | 10 | 74.404 | 55.929 | 4.546 | 62.373 | 60.868 |
| 3 | 0.6 | 2 | 10 | 73.320 | 53.556 | 3.078 | 67.202 | 61.142 |
| 3 | 0.6 | 3 | 10 | 70.583 | 56.815 | 18.843 | 60.925 | 60.556 |
| 3 | 0.6 | 4 | 10 | 75.432 | 58.067 | 9.875 | 66.995 | 63.877 |
| 3 | 0.6 | 5 | 10 | 77.360 | 60.511 | 8.203 | 67.597 | 67.597 |
| 3 | 0.6 | 6 | 10 | 65.268 | 52.543 | 40.750 | 60.770 | 55.770 |
| 3 | 0.6 | 7 | 10 | 70.675 | 49.275 | 1.203 | 63.466 | 56.390 |
| 3 | 0.6 | 8 | 10 | 75.206 | 56.915 | 10.078 | 65.620 | 63.638 |
| 3 | 0.6 | 9 | 10 | 73.373 | 59.477 | 6.999 | 71.254 | 64.580 |
| 3 | 0.6 | 10 | 10 | 76.223 | 59.910 | 1.453 | 66.458 | 65.327 |
| 4 | 0.6 | 1 | 10 | 59.532 | 46.180 | 22.921 | 58.306 | 50.817 |
| 4 | 0.6 | 2 | 10 | 62.283 | 47.660 | 4.531 | 57.664 | 52.439 |
| 4 | 0.6 | 3 | 10 | 62.033 | 53.757 | 2.843 | 62.033 | 53.892 |
| 4 | 0.6 | 4 | 10 | 59.775 | 50.584 | 18.281 | 57.228 | 52.852 |
| 4 | 0.6 | 5 | 10 | 62.693 | 42.917 | 3.843 | 56.418 | 52.734 |
| 4 | 0.6 | 6 | 10 | 48.309 | 40.347 | 17.718 | 45.790 | 42.876 |
| 4 | 0.6 | 7 | 10 | 53.909 | 42.582 | 2.015 | 46.461 | 45.556 |
| 4 | 0.6 | 8 | 10 | 60.472 | 45.810 | 1.171 | 54.172 | 47.832 |
| 4 | 0.6 | 9 | 10 | 58.323 | 47.299 | 7.718 | 53.354 | 53.040 |
| 4 | 0.6 | 10 | 10 | 55.503 | 45.488 | 4.187 | 48.351 | 46.913 |

Table A8: Results of the runs for $Q = 0.6$, $B = 10$ for networks 5,6,7

| Network | $Q$ | Instance | $B$ | $z_2^*$ | $z_3^*$ | $T$ | $z_1^*$ | $z_4^*$ |
|---|---|---|---|---|---|---|---|---|
| 5 | 0.6 | 1 | 10 | 64.612 | 53.094 | 12.656 | 57.562 | 56.734 |
| 5 | 0.6 | 2 | 10 | 65.661 | 50.204 | 7.984 | 55.321 | 53.423 |
| 5 | 0.6 | 3 | 10 | 63.898 | 49.729 | 38.859 | 56.117 | 55.298 |
| 5 | 0.6 | 4 | 10 | 64.017 | 46.273 | 63.358 | 62.424 | 56.976 |
| 5 | 0.6 | 5 | 10 | 62.083 | 44.735 | 2.828 | 53.044 | 52.586 |
| 5 | 0.6 | 6 | 10 | 59.309 | 47.690 | 2.343 | 54.075 | 51.964 |
| 5 | 0.6 | 7 | 10 | 60.445 | 51.916 | 184.858 | 57.627 | 55.564 |
| 5 | 0.6 | 8 | 10 | 63.446 | 49.906 | 7.171 | 58.638 | 51.329 |
| 5 | 0.6 | 9 | 10 | 64.142 | 53.104 | 7.499 | 56.262 | 56.089 |
| 5 | 0.6 | 10 | 10 | 64.034 | 53.975 | 14.531 | 59.714 | 56.260 |
| 6 | 0.6 | 1 | 10 | 71.853 | 62.107 | 30.124 | 68.475 | 65.749 |
| 6 | 0.6 | 2 | 10 | 74.179 | 62.650 | 39.765 | 70.694 | 65.231 |
| 6 | 0.6 | 3 | 10 | 80.277 | 60.222 | 10.453 | 74.337 | 67.434 |
| 6 | 0.6 | 4 | 10 | 70.943 | 57.918 | 52.421 | 64.733 | 62.054 |
| 6 | 0.6 | 5 | 10 | 75.799 | 60.072 | 12.656 | 66.422 | 64.621 |
| 6 | 0.6 | 6 | 10 | 76.149 | 67.694 | 22.515 | 71.505 | 68.630 |
| 6 | 0.6 | 7 | 10 | 73.141 | 54.832 | 49.202 | 61.791 | 61.200 |
| 6 | 0.6 | 8 | 10 | 71.841 | 53.490 | 94.311 | 69.694 | 58.940 |
| 6 | 0.6 | 9 | 10 | 75.492 | 61.905 | 21.890 | 68.002 | 66.065 |
| 6 | 0.6 | 10 | 10 | 71.364 | 59.737 | 21.577 | 63.254 | 61.365 |
| 7 | 0.6 | 1 | 10 | 76.697 | 64.455 | 2.891 | 73.890 | 66.508 |
| 7 | 0.6 | 2 | 10 | 73.047 | 58.507 | 29.468 | 64.113 | 61.398 |
| 7 | 0.6 | 3 | 10 | 75.859 | 64.718 | 176.545 | 69.360 | 68.045 |
| 7 | 0.6 | 4 | 10 | 71.218 | 57.667 | 2.484 | 63.741 | 61.092 |
| 7 | 0.6 | 5 | 10 | 66.510 | 59.644 | 70.655 | 61.572 | 61.572 |
| 7 | 0.6 | 6 | 10 | 61.101 | 46.950 | 9.062 | 53.126 | 52.165 |
| 7 | 0.6 | 7 | 10 | 64.826 | 53.701 | 83.390 | 58.697 | 53.830 |
| 7 | 0.6 | 8 | 10 | 75.098 | 54.525 | 221.139 | 63.701 | 62.150 |
| 7 | 0.6 | 9 | 10 | 76.516 | 54.342 | 1039.258 | 69.139 | 65.064 |
| 7 | 0.6 | 10 | 10 | 66.424 | 52.604 | 69.452 | 58.585 | 56.922 |

Table A9: Results of the runs for $Q = 0.7$, $B = 10$ for networks 1,2,3,4

| Network | $Q$ | Instance | $B$ | $z_2^*$ | $z_3^*$ | $T$ | $z_1^*$ | $z_4^*$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.7 | 1 | 10 | 64.963 | 55.779 | 0.296 | 59.549 | 59.549 |
| 1 | 0.7 | 2 | 10 | 65.224 | 62.822 | 1.390 | 65.224 | 62.822 |
| 1 | 0.7 | 3 | 10 | 59.984 | 56.977 | 4.874 | 58.897 | 57.565 |
| 1 | 0.7 | 4 | 10 | 63.186 | 55.950 | 0.734 | 58.529 | 58.529 |
| 1 | 0.7 | 5 | 10 | 65.716 | 55.536 | 0.312 | 58.697 | 58.697 |
| 1 | 0.7 | 6 | 10 | 63.020 | 59.349 | 1.656 | 61.026 | 59.822 |
| 1 | 0.7 | 7 | 10 | 59.383 | 54.329 | 1.374 | 57.853 | 57.582 |
| 1 | 0.7 | 8 | 10 | 70.896 | 56.556 | 0.562 | 68.513 | 63.774 |
| 1 | 0.7 | 9 | 10 | 67.153 | 59.619 | 0.468 | 62.522 | 62.522 |
| 1 | 0.7 | 10 | 10 | 70.005 | 63.605 | 3.093 | 70.005 | 69.032 |
| 2 | 0.7 | 1 | 10 | 64.565 | 54.897 | 2.640 | 58.083 | 56.643 |
| 2 | 0.7 | 2 | 10 | 62.230 | 53.897 | 7.203 | 57.471 | 55.541 |
| 2 | 0.7 | 3 | 10 | 63.743 | 55.271 | 2.343 | 59.214 | 58.682 |
| 2 | 0.7 | 4 | 10 | 64.790 | 56.431 | 10.843 | 63.109 | 58.794 |
| 2 | 0.7 | 5 | 10 | 69.451 | 56.962 | 4.937 | 60.512 | 60.036 |
| 2 | 0.7 | 6 | 10 | 54.777 | 46.324 | 11.984 | 51.737 | 48.126 |
| 2 | 0.7 | 7 | 10 | 58.520 | 50.484 | 12.484 | 53.376 | 52.312 |
| 2 | 0.7 | 8 | 10 | 56.040 | 50.014 | 23.265 | 52.457 | 52.457 |
| 2 | 0.7 | 9 | 10 | 64.322 | 56.984 | 3.281 | 58.220 | 58.002 |
| 2 | 0.7 | 10 | 10 | 62.522 | 53.222 | 1.890 | 56.126 | 55.701 |
| 3 | 0.7 | 1 | 10 | 67.922 | 57.906 | 14.171 | 62.373 | 60.868 |
| 3 | 0.7 | 2 | 10 | 68.948 | 55.026 | 7.312 | 63.720 | 61.142 |
| 3 | 0.7 | 3 | 10 | 66.773 | 52.153 | 28.531 | 60.925 | 60.556 |
| 3 | 0.7 | 4 | 10 | 70.601 | 57.989 | 22.296 | 66.995 | 63.877 |
| 3 | 0.7 | 5 | 10 | 74.361 | 64.836 | 8.499 | 67.597 | 67.597 |
| 3 | 0.7 | 6 | 10 | 60.588 | 51.943 | 583.918 | 57.730 | 55.770 |
| 3 | 0.7 | 7 | 10 | 65.524 | 58.448 | 4.124 | 63.466 | 56.390 |
| 3 | 0.7 | 8 | 10 | 71.773 | 60.222 | 7.562 | 65.620 | 63.638 |
| 3 | 0.7 | 9 | 10 | 71.254 | 62.875 | 5.046 | 71.254 | 64.580 |
| 3 | 0.7 | 10 | 10 | 72.790 | 63.490 | 3.406 | 66.458 | 65.327 |
| 4 | 0.7 | 1 | 10 | 57.034 | 47.224 | 4.406 | 55.483 | 50.817 |
| 4 | 0.7 | 2 | 10 | 57.664 | 49.580 | 11.218 | 56.186 | 52.439 |
| 4 | 0.7 | 3 | 10 | 57.309 | 50.222 | 5.499 | 54.396 | 53.892 |
| 4 | 0.7 | 4 | 10 | 58.172 | 47.639 | 2.109 | 54.308 | 52.852 |
| 4 | 0.7 | 5 | 10 | 61.340 | 45.089 | 2.140 | 56.418 | 52.734 |
| 4 | 0.7 | 6 | 10 | 46.745 | 40.347 | 10.531 | 43.754 | 42.876 |
| 4 | 0.7 | 7 | 10 | 49.136 | 43.148 | 13.656 | 46.461 | 45.556 |
| 4 | 0.7 | 8 | 10 | 55.799 | 45.179 | 2.656 | 54.172 | 47.832 |
| 4 | 0.7 | 9 | 10 | 55.066 | 47.466 | 16.703 | 53.354 | 53.040 |
| 4 | 0.7 | 10 | 10 | 52.123 | 43.787 | 23.609 | 47.652 | 46.913 |

Table A10: Results of the runs for $Q = 0.7$, $B = 10$ for networks 5,6,7

| Network | $Q$ | Instance | $B$ | $z_2^*$ | $z_3^*$ | $T$ | $z_1^*$ | $z_4^*$ |
|---|---|---|---|---|---|---|---|---|
| 5 | 0.7 | 1 | 10 | 62.114 | 54.245 | 5.328 | 57.562 | 56.734 |
| 5 | 0.7 | 2 | 10 | 59.031 | 50.204 | 67.437 | 53.843 | 53.423 |
| 5 | 0.7 | 3 | 10 | 60.282 | 49.729 | 25.671 | 56.117 | 55.298 |
| 5 | 0.7 | 4 | 10 | 62.424 | 54.704 | 13.281 | 57.991 | 56.976 |
| 5 | 0.7 | 5 | 10 | 59.432 | 48.455 | 1.687 | 53.044 | 52.586 |
| 5 | 0.7 | 6 | 10 | 56.741 | 49.623 | 9.218 | 52.501 | 51.964 |
| 5 | 0.7 | 7 | 10 | 58.158 | 51.571 | 126.592 | 57.627 | 55.564 |
| 5 | 0.7 | 8 | 10 | 59.811 | 49.605 | 8.109 | 58.638 | 51.329 |
| 5 | 0.7 | 9 | 10 | 60.530 | 54.515 | 23.046 | 56.262 | 56.089 |
| 5 | 0.7 | 10 | 10 | 61.904 | 53.975 | 6.390 | 59.714 | 56.260 |
| 6 | 0.7 | 1 | 10 | 68.475 | 62.686 | 51.249 | 68.475 | 65.749 |
| 6 | 0.7 | 2 | 10 | 70.694 | 64.301 | 50.484 | 65.772 | 65.231 |
| 6 | 0.7 | 3 | 10 | 78.859 | 63.830 | 3.999 | 72.727 | 67.434 |
| 6 | 0.7 | 4 | 10 | 66.493 | 57.918 | 193.296 | 64.733 | 62.054 |
| 6 | 0.7 | 5 | 10 | 73.446 | 60.072 | 3.671 | 66.422 | 64.621 |
| 6 | 0.7 | 6 | 10 | 73.384 | 67.575 | 66.718 | 71.505 | 68.630 |
| 6 | 0.7 | 7 | 10 | 68.417 | 58.170 | 80.890 | 61.791 | 61.200 |
| 6 | 0.7 | 8 | 10 | 69.123 | 55.300 | 45.452 | 59.925 | 58.940 |
| 6 | 0.7 | 9 | 10 | 70.243 | 65.020 | 57.609 | 68.002 | 66.065 |
| 6 | 0.7 | 10 | 10 | 66.639 | 61.365 | 81.046 | 63.254 | 61.365 |
| 7 | 0.7 | 1 | 10 | 73.890 | 64.077 | 6.296 | 70.816 | 66.508 |
| 7 | 0.7 | 2 | 10 | 67.824 | 61.398 | 38.453 | 64.113 | 61.398 |
| 7 | 0.7 | 3 | 10 | 73.196 | 64.843 | 85.634 | 69.360 | 68.045 |
| 7 | 0.7 | 4 | 10 | 66.630 | 57.667 | 23.486 | 61.092 | 61.092 |
| 7 | 0.7 | 5 | 10 | 63.936 | 59.531 | 22.533 | 61.572 | 61.572 |
| 7 | 0.7 | 6 | 10 | 56.803 | 45.825 | 32.894 | 53.126 | 52.165 |
| 7 | 0.7 | 7 | 10 | 59.706 | 51.256 | 241.503 | 54.522 | 53.830 |
| 7 | 0.7 | 8 | 10 | 70.695 | 62.343 | 25.876 | 63.701 | 62.150 |
| 7 | 0.7 | 9 | 10 | 71.301 | 62.785 | 515.666 | 69.139 | 65.064 |
| 7 | 0.7 | 10 | 10 | 63.185 | 54.106 | 48.265 | 58.585 | 56.922 |

Table A11: Results of the runs for $Q = 0.8$, $B = 10$ for networks 1,2,3,4

| Network | $Q$ | Instance | $B$ | $z_2^*$ | $z_3^*$ | $T$ | $z_1^*$ | $z_4^*$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.8 | 1 | 10 | 62.760 | 57.673 | 0.265 | 59.549 | 59.549 |
| 1 | 0.8 | 2 | 10 | 64.362 | 62.520 | 0.687 | 63.559 | 62.822 |
| 1 | 0.8 | 3 | 10 | 59.722 | 56.977 | 1.406 | 58.897 | 57.565 |
| 1 | 0.8 | 4 | 10 | 61.026 | 56.688 | 1.109 | 58.529 | 58.529 |
| 1 | 0.8 | 5 | 10 | 61.435 | 57.468 | 0.281 | 58.697 | 58.697 |
| 1 | 0.8 | 6 | 10 | 61.608 | 59.696 | 1.562 | 61.026 | 59.822 |
| 1 | 0.8 | 7 | 10 | 58.550 | 54.329 | 0.812 | 57.853 | 57.582 |
| 1 | 0.8 | 8 | 10 | 66.506 | 63.774 | 2.672 | 66.303 | 63.774 |
| 1 | 0.8 | 9 | 10 | 64.856 | 59.619 | 1.015 | 62.522 | 62.522 |
| 1 | 0.8 | 10 | 10 | 70.005 | 67.111 | 0.500 | 70.005 | 69.032 |
| 2 | 0.8 | 1 | 10 | 61.730 | 55.308 | 2.375 | 58.083 | 56.643 |
| 2 | 0.8 | 2 | 10 | 60.390 | 53.213 | 3.421 | 56.584 | 55.541 |
| 2 | 0.8 | 3 | 10 | 62.037 | 56.984 | 1.031 | 59.214 | 58.682 |
| 2 | 0.8 | 4 | 10 | 62.045 | 56.431 | 5.328 | 59.538 | 58.794 |
| 2 | 0.8 | 5 | 10 | 66.705 | 58.016 | 0.484 | 60.512 | 60.036 |
| 2 | 0.8 | 6 | 10 | 50.086 | 47.581 | 153.563 | 50.086 | 48.126 |
| 2 | 0.8 | 7 | 10 | 56.369 | 50.415 | 3.265 | 53.376 | 52.312 |
| 2 | 0.8 | 8 | 10 | 55.768 | 52.091 | 3.187 | 52.457 | 52.457 |
| 2 | 0.8 | 9 | 10 | 60.293 | 55.806 | 10.500 | 58.220 | 58.002 |
| 2 | 0.8 | 10 | 10 | 59.068 | 54.081 | 1.531 | 56.126 | 55.701 |
| 3 | 0.8 | 1 | 10 | 63.929 | 57.203 | 54.312 | 60.909 | 60.868 |
| 3 | 0.8 | 2 | 10 | 65.878 | 56.893 | 9.031 | 61.142 | 61.142 |
| 3 | 0.8 | 3 | 10 | 64.503 | 58.387 | 3.656 | 60.925 | 60.556 |
| 3 | 0.8 | 4 | 10 | 65.918 | 58.067 | 86.172 | 64.151 | 63.877 |
| 3 | 0.8 | 5 | 10 | 71.101 | 63.252 | 5.218 | 67.597 | 67.597 |
| 3 | 0.8 | 6 | 10 | 58.205 | 54.908 | 149.171 | 57.730 | 55.770 |
| 3 | 0.8 | 7 | 10 | 60.521 | 55.219 | 18.593 | 56.945 | 56.390 |
| 3 | 0.8 | 8 | 10 | 67.635 | 61.525 | 20.140 | 63.638 | 63.638 |
| 3 | 0.8 | 9 | 10 | 68.130 | 62.875 | 12.609 | 65.514 | 64.580 |
| 3 | 0.8 | 10 | 10 | 69.224 | 64.730 | 5.703 | 66.458 | 65.327 |
| 4 | 0.8 | 1 | 10 | 53.199 | 47.049 | 14.046 | 50.817 | 50.817 |
| 4 | 0.8 | 2 | 10 | 54.660 | 49.839 | 11.875 | 52.942 | 52.439 |
| 4 | 0.8 | 3 | 10 | 56.343 | 53.757 | 2.344 | 53.901 | 53.892 |
| 4 | 0.8 | 4 | 10 | 54.308 | 52.852 | 5.375 | 54.308 | 52.852 |
| 4 | 0.8 | 5 | 10 | 57.536 | 48.572 | 1.453 | 54.190 | 52.734 |
| 4 | 0.8 | 6 | 10 | 45.415 | 42.806 | 9.765 | 43.754 | 42.876 |
| 4 | 0.8 | 7 | 10 | 47.115 | 43.667 | 9.109 | 45.870 | 45.556 |
| 4 | 0.8 | 8 | 10 | 52.323 | 45.810 | 2.265 | 49.627 | 47.832 |
| 4 | 0.8 | 9 | 10 | 53.264 | 48.828 | 14.671 | 53.354 | 53.040 |
| 4 | 0.8 | 10 | 10 | 49.917 | 46.445 | 22.016 | 47.652 | 46.913 |

Table A12: Results of the runs for $Q = 0.8$, $B = 10$ for networks 5,6,7

| Network | $Q$ | Instance | $B$ | $z_2^*$ | $z_3^*$ | $T$ | $z_1^*$ | $z_4^*$ |
|---|---|---|---|---|---|---|---|---|
| 5 | 0.8 | 1 | 10 | 58.953 | 53.094 | 7.031 | 57.562 | 56.734 |
| 5 | 0.8 | 2 | 10 | 54.781 | 51.567 | 84.953 | 53.843 | 53.423 |
| 5 | 0.8 | 3 | 10 | 58.320 | 51.427 | 10.500 | 56.117 | 55.298 |
| 5 | 0.8 | 4 | 10 | 58.799 | 54.704 | 36.531 | 56.976 | 56.976 |
| 5 | 0.8 | 5 | 10 | 56.675 | 48.455 | 2.171 | 53.044 | 52.586 |
| 5 | 0.8 | 6 | 10 | 54.535 | 49.333 | 5.718 | 52.501 | 51.964 |
| 5 | 0.8 | 7 | 10 | 56.100 | 50.835 | 75.562 | 56.178 | 55.564 |
| 5 | 0.8 | 8 | 10 | 55.195 | 49.906 | 30.406 | 54.093 | 51.329 |
| 5 | 0.8 | 9 | 10 | 59.237 | 54.440 | 7.015 | 56.262 | 56.089 |
| 5 | 0.8 | 10 | 10 | 59.421 | 54.881 | 9.281 | 57.533 | 56.260 |
| 6 | 0.8 | 1 | 10 | 67.826 | 64.117 | 15.328 | 67.062 | 65.749 |
| 6 | 0.8 | 2 | 10 | 67.832 | 63.037 | 38.797 | 65.772 | 65.231 |
| 6 | 0.8 | 3 | 10 | 70.998 | 63.830 | 34.343 | 72.727 | 67.434 |
| 6 | 0.8 | 4 | 10 | 63.610 | 58.164 | 340.203 | 62.054 | 62.054 |
| 6 | 0.8 | 5 | 10 | 69.907 | 60.072 | 3.171 | 66.422 | 64.621 |
| 6 | 0.8 | 6 | 10 | 71.869 | 67.963 | 14.656 | 71.505 | 68.630 |
| 6 | 0.8 | 7 | 10 | 64.170 | 58.845 | 182.141 | 61.200 | 61.200 |
| 6 | 0.8 | 8 | 10 | 64.571 | 56.654 | 30.078 | 59.925 | 58.940 |
| 6 | 0.8 | 9 | 10 | 67.997 | 65.020 | 12.828 | 67.195 | 66.065 |
| 6 | 0.8 | 10 | 10 | 64.954 | 59.737 | 43.875 | 63.254 | 61.365 |
| 7 | 0.8 | 1 | 10 | 67.803 | 62.801 | 25.843 | 66.508 | 66.508 |
| 7 | 0.8 | 2 | 10 | 64.889 | 59.380 | 22.656 | 64.113 | 61.398 |
| 7 | 0.8 | 3 | 10 | 71.350 | 66.333 | 29.906 | 69.360 | 68.045 |
| 7 | 0.8 | 4 | 10 | 62.527 | 58.000 | 135.125 | 61.092 | 61.092 |
| 7 | 0.8 | 5 | 10 | 63.431 | 60.393 | 11.828 | 61.572 | 61.572 |
| 7 | 0.8 | 6 | 10 | 53.789 | 49.965 | 50.984 | 53.126 | 52.165 |
| 7 | 0.8 | 7 | 10 | 57.118 | 51.885 | 39.343 | 54.522 | 53.830 |
| 7 | 0.8 | 8 | 10 | 66.598 | 61.470 | 30.985 | 63.701 | 62.150 |
| 7 | 0.8 | 9 | 10 | 68.415 | 62.785 | 54.109 | 65.441 | 65.064 |
| 7 | 0.8 | 10 | 10 | 60.933 | 56.082 | 22.281 | 57.556 | 56.922 |

Table A13: Results of the runs for $Q = 0.6$, $B = 15$ for networks 1,2,3,4

| Network | $Q$ | Instance | $B$ | $z_2^*$ | $z_3^*$ | $T$ | $z_1^*$ | $z_4^*$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.6 | 1 | 15 | 71.649 | 62.376 | 3.000 | 68.503 | 66.376 |
| 1 | 0.6 | 2 | 15 | 75.133 | 67.400 | 2.375 | 68.981 | 68.675 |
| 1 | 0.6 | 3 | 15 | 72.097 | 61.713 | 7.593 | 67.605 | 66.521 |
| 1 | 0.6 | 4 | 15 | 73.423 | 60.069 | 0.469 | 65.043 | 65.043 |
| 1 | 0.6 | 5 | 15 | 74.565 | 61.544 | 0.718 | 71.154 | 64.585 |
| 1 | 0.6 | 6 | 15 | 73.718 | 62.801 | 0.375 | 65.095 | 64.048 |
| 1 | 0.6 | 7 | 15 | 69.563 | 63.057 | 0.828 | 66.770 | 64.048 |
| 1 | 0.6 | 8 | 15 | 82.153 | 63.774 | 1.343 | 76.907 | 70.034 |
| 1 | 0.6 | 9 | 15 | 75.719 | 58.087 | 2.609 | 70.157 | 69.297 |
| 1 | 0.6 | 10 | 15 | 78.609 | 69.030 | 15.829 | 76.447 | 74.733 |
| 2 | 0.6 | 1 | 15 | 73.950 | 57.061 | 63.002 | 65.313 | 64.019 |
| 2 | 0.6 | 2 | 15 | 70.153 | 53.832 | 68.345 | 66.474 | 61.183 |
| 2 | 0.6 | 3 | 15 | 77.055 | 58.682 | 0.390 | 67.768 | 65.697 |
| 2 | 0.6 | 4 | 15 | 75.240 | 58.187 | 18.297 | 67.096 | 64.898 |
| 2 | 0.6 | 5 | 15 | 79.313 | 55.276 | 28.922 | 67.304 | 66.130 |
| 2 | 0.6 | 6 | 15 | 64.288 | 50.421 | 0.890 | 57.501 | 53.637 |
| 2 | 0.6 | 7 | 15 | 70.578 | 50.720 | 4.718 | 60.585 | 57.894 |
| 2 | 0.6 | 8 | 15 | 67.456 | 52.091 | 158.050 | 59.608 | 57.876 |
| 2 | 0.6 | 9 | 15 | 71.810 | 58.506 | 15.734 | 65.906 | 61.900 |
| 2 | 0.6 | 10 | 15 | 70.896 | 54.081 | 3.188 | 63.078 | 60.184 |
| 3 | 0.6 | 1 | 15 | 78.476 | 61.194 | 667.614 | 69.489 | 67.045 |
| 3 | 0.6 | 2 | 15 | 77.564 | 58.216 | 154.734 | 72.028 | 67.370 |
| 3 | 0.6 | 3 | 15 | 82.601 | 59.827 | 7.828 | 69.817 | 67.910 |
| 3 | 0.6 | 4 | 15 | 83.987 | 58.067 | 60.250 | 74.612 | 70.689 |
| 3 | 0.6 | 5 | 15 | 84.278 | 66.550 | 91.968 | 75.635 | 72.883 |
| 3 | 0.6 | 6 | 15 | 71.092 | 56.214 | 113.437 | 61.656 | 60.367 |
| 3 | 0.6 | 7 | 15 | 74.975 | 53.931 | 127.515 | 67.015 | 62.376 |
| 3 | 0.6 | 8 | 15 | 82.197 | 63.218 | 151.937 | 71.846 | 71.846 |
| 3 | 0.6 | 9 | 15 | 79.461 | 65.194 | 130.656 | 75.506 | 70.113 |
| 3 | 0.6 | 10 | 15 | 82.905 | 63.644 | 7.953 | 71.408 | 70.721 |
| 4 | 0.6 | 1 | 15 | 65.898 | 49.282 | 169.890 | 58.306 | 55.532 |
| 4 | 0.6 | 2 | 15 | 70.100 | 54.175 | 16.140 | 58.756 | 57.695 |
| 4 | 0.6 | 3 | 15 | 69.667 | 52.334 | 65.406 | 69.122 | 60.841 |
| 4 | 0.6 | 4 | 15 | 66.564 | 53.647 | 48.312 | 59.590 | 58.072 |
| 4 | 0.6 | 5 | 15 | 67.089 | 49.287 | 29.765 | 59.076 | 56.454 |
| 4 | 0.6 | 6 | 15 | 56.247 | 44.786 | 43.421 | 48.031 | 47.329 |
| 4 | 0.6 | 7 | 15 | 61.694 | 47.839 | 2.703 | 55.162 | 51.721 |
| 4 | 0.6 | 8 | 15 | 67.383 | 49.097 | 18.578 | 53.416 | 53.161 |
| 4 | 0.6 | 9 | 15 | 64.836 | 48.680 | 47.203 | 57.350 | 56.396 |
| 4 | 0.6 | 10 | 15 | 58.847 | 44.946 | 449.937 | 55.598 | 51.618 |

Table A14: Results of the runs for $Q = 0.6$, $B = 15$ for networks 5,6,7

| Network | $Q$ | Instance | $B$ | $z_2^*$ | $z_3^*$ | $T$ | $z_1^*$ | $z_4^*$ |
|---|---|---|---|---|---|---|---|---|
| 5 | 0.6 | 1 | 15 | 71.013 | 58.809 | 314.796 | 65.743 | 60.815 |
| 5 | 0.6 | 2 | 15 | 71.429 | 53.993 | 178.406 | 59.817 | 58.854 |
| 5 | 0.6 | 3 | 15 | 72.645 | 52.411 | 267.671 | 61.241 | 60.422 |
| 5 | 0.6 | 4 | 15 | 71.469 | 54.704 | 340.796 | 63.424 | 62.236 |
| 5 | 0.6 | 5 | 15 | 68.597 | 48.692 | 13.281 | 60.452 | 56.690 |
| 5 | 0.6 | 6 | 15 | 65.869 | 51.207 | 1.984 | 56.903 | 55.032 |
| 5 | 0.6 | 7 | 15 | 67.941 | 55.126 | 967.718 | 66.342 | 61.662 |
| 5 | 0.6 | 8 | 15 | 68.415 | 50.441 | 335.750 | 63.446 | 55.947 |
| 5 | 0.6 | 9 | 15 | 71.956 | 58.933 | 26.468 | 63.897 | 61.198 |
| 5 | 0.6 | 10 | 15 | 70.218 | 53.560 | 159.875 | 66.126 | 61.378 |
| 6 | 0.6 | 1 | 15 | 81.846 | 65.534 | 78.812 | 74.020 | 71.294 |
| 6 | 0.6 | 2 | 15 | 81.127 | 64.758 | 686.734 | 72.605 | 70.882 |
| 6 | 0.6 | 3 | 15 | 90.095 | 62.045 | 111.437 | 77.805 | 74.749 |
| 6 | 0.6 | 4 | 15 | 76.485 | 63.316 | 1010.328 | 69.440 | 68.279 |
| 6 | 0.6 | 5 | 15 | 84.228 | 60.072 | 79.140 | 73.933 | 70.213 |
| 6 | 0.6 | 6 | 15 | 82.402 | 69.764 | 40.687 | 74.898 | 74.232 |
| 6 | 0.6 | 7 | 15 | 80.506 | 58.588 | 404.218 | 69.884 | 67.659 |
| 6 | 0.6 | 8 | 15 | 79.661 | 56.021 | 1862.671 | 65.244 | 64.033 |
| 6 | 0.6 | 9 | 15 | 83.186 | 66.905 | 63.860 | 71.728 | 70.248 |
| 6 | 0.6 | 10 | 15 | 78.402 | 60.971 | 288.000 | 70.292 | 67.341 |
| 7 | 0.6 | 1 | 15 | 83.187 | 67.844 | 67.109 | 77.697 | 70.726 |
| 7 | 0.6 | 2 | 15 | 78.810 | 60.439 | 2503.637 | 68.093 | 66.768 |
| 7 | 0.6 | 3 | 15 | 85.981 | 68.045 | 356.290 | 74.219 | 74.123 |
| 7 | 0.6 | 4 | 15 | 75.232 | 58.018 | 762.098 | 70.608 | 67.500 |
| 7 | 0.6 | 5 | 15 | 75.513 | 63.135 | 574.090 | 68.292 | 67.468 |
| 7 | 0.6 | 6 | 15 | 68.058 | 49.192 | 57.484 | 60.132 | 58.294 |
| 7 | 0.6 | 7 | 15 | 71.166 | 55.759 | 1132.021 | 61.242 | 57.853 |
| 7 | 0.6 | 8 | 15 | 82.467 | 60.045 | 368.096 | 70.695 | 68.508 |
| 7 | 0.6 | 9 | 15 | 85.521 | 60.015 | 1338.358 | 73.839 | 72.426 |
| 7 | 0.6 | 10 | 15 | 63.204 | 60.134 | 465.123 | 61.517 | 61.517 |

Table A15: Results of the runs for $Q = 0.7$, $B = 15$ for networks 1,2,3,4

| Network | $Q$ | Instance | $B$ | $z_2^*$ | $z_3^*$ | $T$ | $z_1^*$ | $z_4^*$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.7 | 1 | 15 | 70.225 | 63.013 | 1.188 | 66.454 | 66.376 |
| 1 | 0.7 | 2 | 15 | 71.890 | 67.766 | 8.562 | 68.981 | 68.675 |
| 1 | 0.7 | 3 | 15 | 69.409 | 63.830 | 9.516 | 67.605 | 66.521 |
| 1 | 0.7 | 4 | 15 | 70.686 | 59.101 | 2.390 | 65.043 | 65.043 |
| 1 | 0.7 | 5 | 15 | 71.544 | 60.657 | 0.203 | 71.154 | 64.585 |
| 1 | 0.7 | 6 | 15 | 71.150 | 62.655 | 0.594 | 64.048 | 64.048 |
| 1 | 0.7 | 7 | 15 | 67.649 | 58.942 | 1.125 | 66.770 | 64.048 |
| 1 | 0.7 | 8 | 15 | 80.152 | 66.600 | 0.578 | 76.907 | 70.034 |
| 1 | 0.7 | 9 | 15 | 72.644 | 63.817 | 19.828 | 70.157 | 69.297 |
| 1 | 0.7 | 10 | 15 | 77.280 | 72.519 | 3.688 | 76.447 | 74.733 |
| 2 | 0.7 | 1 | 15 | 71.096 | 60.249 | 13.578 | 65.313 | 64.019 |
| 2 | 0.7 | 2 | 15 | 67.373 | 58.862 | 65.797 | 62.992 | 61.183 |
| 2 | 0.7 | 3 | 15 | 73.752 | 60.213 | 0.828 | 67.768 | 65.697 |
| 2 | 0.7 | 4 | 15 | 72.494 | 60.471 | 23.906 | 67.096 | 64.898 |
| 2 | 0.7 | 5 | 15 | 75.042 | 61.855 | 11.031 | 67.304 | 66.130 |
| 2 | 0.7 | 6 | 15 | 60.615 | 50.002 | 83.422 | 57.501 | 53.637 |
| 2 | 0.7 | 7 | 15 | 66.932 | 54.674 | 8.781 | 60.585 | 57.894 |
| 2 | 0.7 | 8 | 15 | 62.992 | 56.375 | 291.250 | 57.876 | 57.876 |
| 2 | 0.7 | 9 | 15 | 69.298 | 58.242 | 18.391 | 65.906 | 61.900 |
| 2 | 0.7 | 10 | 15 | 67.322 | 56.270 | 1.844 | 60.926 | 60.184 |
| 3 | 0.7 | 1 | 15 | 74.404 | 59.455 | 637.969 | 69.489 | 67.045 |
| 3 | 0.7 | 2 | 15 | 74.494 | 63.198 | 199.328 | 68.546 | 67.370 |
| 3 | 0.7 | 3 | 15 | 77.029 | 62.270 | 65.391 | 69.817 | 67.910 |
| 3 | 0.7 | 4 | 15 | 78.231 | 63.877 | 259.906 | 74.612 | 70.689 |
| 3 | 0.7 | 5 | 15 | 80.778 | 70.100 | 57.672 | 75.635 | 72.883 |
| 3 | 0.7 | 6 | 15 | 79.134 | 71.235 | 86.331 | 77.123 | 73.152 |
| 3 | 0.7 | 7 | 15 | 72.245 | 59.337 | 158.145 | 64.025 | 62.376 |
| 3 | 0.7 | 8 | 15 | 78.873 | 63.218 | 97.658 | 71.846 | 71.846 |
| 3 | 0.7 | 9 | 15 | 77.678 | 69.516 | 18.625 | 70.113 | 70.113 |
| 3 | 0.7 | 10 | 15 | 80.249 | 62.397 | 10.719 | 71.408 | 70.721 |
| 4 | 0.7 | 1 | 15 | 62.129 | 49.724 | 250.788 | 55.532 | 55.532 |
| 4 | 0.7 | 2 | 15 | 64.802 | 55.794 | 80.471 | 58.756 | 57.695 |
| 4 | 0.7 | 3 | 15 | 63.636 | 57.433 | 412.792 | 61.607 | 60.841 |
| 4 | 0.7 | 4 | 15 | 65.477 | 53.653 | 51.424 | 58.072 | 58.072 |
| 4 | 0.7 | 5 | 15 | 65.831 | 49.762 | 8.797 | 59.076 | 56.454 |
| 4 | 0.7 | 6 | 15 | 53.679 | 44.786 | 11.344 | 48.031 | 47.329 |
| 4 | 0.7 | 7 | 15 | 58.565 | 51.474 | 13.032 | 52.161 | 51.721 |
| 4 | 0.7 | 8 | 15 | 62.449 | 48.932 | 25.547 | 53.416 | 53.161 |
| 4 | 0.7 | 9 | 15 | 60.415 | 55.303 | 569.499 | 56.575 | 56.396 |
| 4 | 0.7 | 10 | 15 | 56.963 | 45.767 | 85.299 | 52.538 | 51.618 |

Table A16: Results of the runs for $Q = 0.7$, $B = 15$ for networks 5,6,7

| Network | $Q$ | Instance | $B$ | $z_2^*$ | $z_3^*$ | $T$ | $z_1^*$ | $z_4^*$ |
|---|---|---|---|---|---|---|---|---|
| 5 | 0.7 | 1 | 15 | 67.220 | 59.409 | 151.926 | 65.743 | 60.815 |
| 5 | 0.7 | 2 | 15 | 65.992 | 54.762 | 1010.870 | 58.902 | 58.854 |
| 5 | 0.7 | 3 | 15 | 67.366 | 54.666 | 556.670 | 61.241 | 60.422 |
| 5 | 0.7 | 4 | 15 | 69.405 | 57.132 | 119.238 | 63.424 | 62.236 |
| 5 | 0.7 | 5 | 15 | 64.095 | 51.551 | 103.612 | 57.318 | 56.690 |
| 5 | 0.7 | 6 | 15 | 61.895 | 52.576 | 25.781 | 55.569 | 55.032 |
| 5 | 0.7 | 7 | 15 | 65.398 | 58.420 | 145.441 | 66.342 | 61.662 |
| 5 | 0.7 | 8 | 15 | 65.454 | 51.459 | 86.455 | 63.446 | 55.947 |
| 5 | 0.7 | 9 | 15 | 66.412 | 59.012 | 711.706 | 63.897 | 61.198 |
| 5 | 0.7 | 10 | 15 | 68.221 | 55.362 | 39.626 | 62.348 | 61.378 |
| 6 | 0.7 | 1 | 15 | 78.053 | 66.833 | 117.503 | 74.020 | 71.294 |
| 6 | 0.7 | 2 | 15 | 77.076 | 66.787 | 1751.014 | 71.002 | 70.882 |
| 6 | 0.7 | 3 | 15 | 86.761 | 69.132 | 72.674 | 77.805 | 74.749 |
| 6 | 0.7 | 4 | 15 | 73.348 | 67.152 | 725.940 | 69.440 | 68.279 |
| 6 | 0.7 | 5 | 15 | 78.943 | 63.792 | 194.302 | 73.933 | 70.213 |
| 6 | 0.7 | 6 | 15 | 80.166 | 69.764 | 63.424 | 74.898 | 74.232 |
| 6 | 0.7 | 7 | 15 | 75.459 | 62.889 | 1564.509 | 69.884 | 67.659 |
| 6 | 0.7 | 8 | 15 | 75.460 | 63.218 | 1100.090 | 65.244 | 64.033 |
| 6 | 0.7 | 9 | 15 | 76.584 | 67.533 | 1160.170 | 71.728 | 70.248 |
| 6 | 0.7 | 10 | 15 | 74.616 | 61.365 | 719.440 | 70.292 | 67.341 |
| 7 | 0.7 | 1 | 15 | 78.208 | 70.275 | 1510.602 | 74.623 | 70.726 |
| 7 | 0.7 | 2 | 15 | 74.395 | 62.392 | 880.475 | 68.093 | 66.768 |
| 7 | 0.7 | 3 | 15 | 80.486 | 69.386 | 1484.710 | 74.219 | 74.123 |
| 7 | 0.7 | 4 | 15 | 74.726 | 63.456 | 50.610 | 68.029 | 67.500 |
| 7 | 0.7 | 5 | 15 | 70.403 | 64.529 | 1400.129 | 68.292 | 67.468 |
| 7 | 0.7 | 6 | 15 | 63.051 | 50.803 | 194.614 | 60.132 | 58.294 |
| 7 | 0.7 | 7 | 15 | 66.805 | 56.885 | 1694.528 | 58.545 | 57.853 |
| 7 | 0.7 | 8 | 15 | 76.379 | 66.817 | 714.424 | 70.695 | 68.508 |
| 7 | 0.7 | 9 | 15 | 75.125 | 65.123 | 754.101 | 71.985 | 68.235 |
| 7 | 0.7 | 10 | 15 | 69.371 | 58.477 | 559.639 | 61.517 | 61.517 |

Table A17: Results of the runs for $Q = 0.8$, $B = 15$ for networks 1,2,3,4

| Network | $Q$ | Instance | $B$ | $z_2^*$ | $z_3^*$ | $T$ | $z_1^*$ | $z_4^*$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.8 | 1 | 15 | 68.022 | 63.826 | 1.093 | 66.454 | 66.376 |
| 1 | 0.8 | 2 | 15 | 69.708 | 68.532 | 15.907 | 68.981 | 68.675 |
| 1 | 0.8 | 3 | 15 | 67.605 | 66.283 | 6.485 | 67.605 | 66.521 |
| 1 | 0.8 | 4 | 15 | 66.918 | 62.191 | 7.094 | 65.043 | 65.043 |
| 1 | 0.8 | 5 | 15 | 67.067 | 64.585 | 0.515 | 67.067 | 64.585 |
| 1 | 0.8 | 6 | 15 | 68.818 | 62.821 | 1.046 | 64.048 | 64.048 |
| 1 | 0.8 | 7 | 15 | 66.543 | 63.905 | 0.578 | 65.520 | 64.048 |
| 1 | 0.8 | 8 | 15 | 77.110 | 67.832 | 0.672 | 71.251 | 70.034 |
| 1 | 0.8 | 9 | 15 | 71.126 | 66.915 | 6.828 | 70.157 | 69.297 |
| 1 | 0.8 | 10 | 15 | 77.175 | 72.792 | 0.578 | 76.447 | 74.733 |
| 2 | 0.8 | 1 | 15 | 67.652 | 61.308 | 52.689 | 65.313 | 64.019 |
| 2 | 0.8 | 2 | 15 | 66.474 | 59.402 | 6.469 | 62.992 | 61.183 |
| 2 | 0.8 | 3 | 15 | 71.071 | 62.624 | 2.390 | 67.768 | 65.697 |
| 2 | 0.8 | 4 | 15 | 68.798 | 63.729 | 185.223 | 67.096 | 64.898 |
| 2 | 0.8 | 5 | 15 | 72.125 | 62.277 | 5.656 | 66.146 | 66.130 |
| 2 | 0.8 | 6 | 15 | 56.227 | 51.702 | 761.270 | 55.850 | 53.637 |
| 2 | 0.8 | 7 | 15 | 63.090 | 55.255 | 8.891 | 60.585 | 57.894 |
| 2 | 0.8 | 8 | 15 | 61.738 | 53.532 | 32.329 | 57.876 | 57.876 |
| 2 | 0.8 | 9 | 15 | 66.152 | 59.682 | 48.220 | 65.906 | 61.900 |
| 2 | 0.8 | 10 | 15 | 64.666 | 57.457 | 7.078 | 60.926 | 60.184 |
| 3 | 0.8 | 1 | 15 | 71.217 | 63.597 | 249.381 | 68.025 | 67.045 |
| 3 | 0.8 | 2 | 15 | 72.560 | 63.198 | 23.375 | 68.546 | 67.370 |
| 3 | 0.8 | 3 | 15 | 72.345 | 63.668 | 188.630 | 69.817 | 67.910 |
| 3 | 0.8 | 4 | 15 | 75.218 | 66.270 | 94.393 | 72.119 | 70.689 |
| 3 | 0.8 | 5 | 15 | 77.782 | 70.100 | 32.579 | 73.946 | 72.883 |
| 3 | 0.8 | 6 | 15 | 67.094 | 60.287 | 320.009 | 64.025 | 62.376 |
| 3 | 0.8 | 7 | 15 | 70.231 | 64.123 | 185.145 | 68.237 | 65.859 |
| 3 | 0.8 | 8 | 15 | 75.479 | 69.845 | 87.049 | 71.846 | 71.846 |
| 3 | 0.8 | 9 | 15 | 73.676 | 69.605 | 85.236 | 70.113 | 70.113 |
| 3 | 0.8 | 10 | 15 | 76.541 | 66.446 | 29.797 | 71.408 | 70.721 |
| 4 | 0.8 | 1 | 15 | 58.665 | 51.350 | 245.319 | 55.532 | 55.532 |
| 4 | 0.8 | 2 | 15 | 62.357 | 55.935 | 9.141 | 58.756 | 57.695 |
| 4 | 0.8 | 3 | 15 | 62.823 | 57.831 | 15.203 | 61.112 | 60.841 |
| 4 | 0.8 | 4 | 15 | 61.070 | 56.832 | 26.860 | 58.072 | 58.072 |
| 4 | 0.8 | 5 | 15 | 61.740 | 52.725 | 49.767 | 59.076 | 56.454 |
| 4 | 0.8 | 6 | 15 | 50.060 | 46.866 | 89.518 | 48.031 | 47.329 |
| 4 | 0.8 | 7 | 15 | 54.518 | 50.880 | 46.970 | 52.161 | 51.721 |
| 4 | 0.8 | 8 | 15 | 58.038 | 48.932 | 28.126 | 53.416 | 53.161 |
| 4 | 0.8 | 9 | 15 | 58.415 | 54.949 | 386.666 | 56.575 | 56.396 |
| 4 | 0.8 | 10 | 15 | 55.598 | 51.187 | 22.563 | 52.538 | 51.618 |

Table A18: Results of the runs for $Q = 0.8$, $B = 15$ for networks 5,6,7

| Network | $Q$ | Instance | $B$ | $z_2^*$ | $z_3^*$ | $T$ | $z_1^*$ | $z_4^*$ |
|---|---|---|---|---|---|---|---|---|
| 5 | 0.8 | 1 | 15 | 65.343 | 59.409 | 41.126 | 62.328 | 60.815 |
| 5 | 0.8 | 2 | 15 | 61.722 | 54.371 | 614.500 | 58.902 | 58.854 |
| 5 | 0.8 | 3 | 15 | 64.079 | 57.265 | 202.692 | 61.241 | 60.422 |
| 5 | 0.8 | 4 | 15 | 65.449 | 58.157 | 180.520 | 62.315 | 62.236 |
| 5 | 0.8 | 5 | 15 | 61.583 | 55.758 | 74.549 | 57.318 | 56.690 |
| 5 | 0.8 | 6 | 15 | 59.327 | 52.576 | 13.859 | 55.569 | 55.032 |
| 5 | 0.8 | 7 | 15 | 62.349 | 58.667 | 961.134 | 62.330 | 61.662 |
| 5 | 0.8 | 8 | 15 | 61.995 | 54.660 | 35.751 | 58.901 | 55.947 |
| 5 | 0.8 | 9 | 15 | 63.897 | 59.836 | 292.679 | 62.751 | 61.198 |
| 5 | 0.8 | 10 | 15 | 64.013 | 59.213 | 567.546 | 62.348 | 61.378 |
| 6 | 0.8 | 1 | 15 | 74.295 | 69.293 | 460.074 | 72.607 | 71.294 |
| 6 | 0.8 | 2 | 15 | 73.989 | 67.858 | 264.553 | 71.002 | 70.882 |
| 6 | 0.8 | 3 | 15 | 79.827 | 69.132 | 270.741 | 77.805 | 74.749 |
| 6 | 0.8 | 4 | 15 | 70.138 | 65.295 | 1507.273 | 69.440 | 68.279 |
| 6 | 0.8 | 5 | 15 | 76.004 | 63.792 | 93.112 | 72.584 | 70.213 |
| 6 | 0.8 | 6 | 15 | 77.838 | 69.764 | 96.252 | 75.426 | 74.232 |
| 6 | 0.8 | 7 | 15 | 71.912 | 63.103 | 1325.643 | 69.884 | 67.659 |
| 6 | 0.8 | 8 | 15 | 69.886 | 61.722 | 1182.343 | 65.244 | 64.033 |
| 6 | 0.8 | 9 | 15 | 74.237 | 68.600 | 156.301 | 70.921 | 70.248 |
| 6 | 0.8 | 10 | 15 | 72.313 | 65.111 | 93.846 | 70.292 | 67.341 |
| 7 | 0.8 | 1 | 15 | 74.240 | 69.263 | 957.618 | 71.432 | 70.726 |
| 7 | 0.8 | 2 | 15 | 70.784 | 63.294 | 671.970 | 68.093 | 66.768 |
| 7 | 0.8 | 3 | 15 | 78.083 | 70.218 | 444.699 | 74.219 | 74.123 |
| 7 | 0.8 | 4 | 15 | 70.499 | 63.428 | 214.818 | 68.029 | 67.500 |
| 7 | 0.8 | 5 | 15 | 69.090 | 66.945 | 220.990 | 67.631 | 67.468 |
| 7 | 0.8 | 6 | 15 | 59.888 | 58.144 | 162.566 | 58.309 | 58.294 |
| 7 | 0.8 | 7 | 15 | 63.115 | 57.786 | 277.819 | 58.545 | 57.853 |
| 7 | 0.8 | 8 | 15 | 73.592 | 65.910 | 39.610 | 70.695 | 68.508 |
| 7 | 0.8 | 9 | 15 | 70.887 | 63.586 | 336.547 | 65.985 | 62.588 |
| 7 | 0.8 | 10 | 15 | 65.548 | 61.900 | 1638.651 | 61.517 | 61.517 |

Table A19: Results of the runs for $Q = 0.6$, $B = 20$ for networks 1,2,3,4

| Network | $Q$ | Instance | $B$ | $z_2^*$ | $z_3^*$ | $T$ | $z_1^*$ | $z_4^*$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.6 | 1 | 20 | 77.849 | 65.767 | 2.172 | 72.138 | 72.138 |
| 1 | 0.6 | 2 | 20 | 82.057 | 73.549 | 0.594 | 76.726 | 73.987 |
| 1 | 0.6 | 3 | 20 | 78.546 | 66.689 | 21.438 | 72.876 | 72.361 |
| 1 | 0.6 | 4 | 20 | 78.130 | 62.662 | 1.203 | 68.396 | 68.347 |
| 1 | 0.6 | 5 | 20 | 76.528 | 63.192 | 2.390 | 71.089 | 64.585 |
| 1 | 0.6 | 6 | 20 | 76.853 | 64.194 | 6.188 | 68.825 | 68.175 |
| 1 | 0.6 | 7 | 20 | 73.386 | 66.954 | 1.593 | 70.939 | 69.316 |
| 1 | 0.6 | 8 | 20 | 86.502 | 67.832 | 4.922 | 79.489 | 75.620 |
| 1 | 0.6 | 9 | 20 | 80.396 | 67.933 | 22.016 | 75.994 | 73.985 |
| 1 | 0.6 | 10 | 20 | 83.562 | 76.284 | 52.314 | 81.896 | 80.229 |
| 2 | 0.6 | 1 | 20 | 77.932 | 62.756 | 382.666 | 70.295 | 67.534 |
| 2 | 0.6 | 2 | 20 | 76.202 | 58.460 | 176.770 | 69.152 | 65.298 |
| 2 | 0.6 | 3 | 20 | 81.295 | 61.911 | 6.937 | 73.798 | 69.756 |
| 2 | 0.6 | 4 | 20 | 82.148 | 66.425 | 77.283 | 71.700 | 71.316 |
| 2 | 0.6 | 5 | 20 | 84.959 | 61.609 | 67.752 | 74.631 | 68.842 |
| 2 | 0.6 | 6 | 20 | 67.080 | 51.198 | 80.971 | 60.417 | 56.633 |
| 2 | 0.6 | 7 | 20 | 74.239 | 54.005 | 198.677 | 65.447 | 61.719 |
| 2 | 0.6 | 8 | 20 | 73.264 | 55.875 | 299.789 | 62.644 | 62.203 |
| 2 | 0.6 | 9 | 20 | 76.786 | 60.976 | 56.564 | 69.021 | 65.257 |
| 2 | 0.6 | 10 | 20 | 73.250 | 57.571 | 37.345 | 65.001 | 63.832 |
| 3 | 0.6 | 1 | 20 | 85.627 | 64.131 | 287.085 | 74.730 | 71.989 |
| 3 | 0.6 | 2 | 20 | 84.531 | 60.531 | 46.235 | 76.644 | 72.045 |
| 3 | 0.6 | 3 | 20 | 88.176 | 65.697 | 87.596 | 78.441 | 74.179 |
| 3 | 0.6 | 4 | 20 | 91.604 | 68.104 | 178.145 | 77.625 | 77.232 |
| 3 | 0.6 | 5 | 20 | 91.686 | 66.701 | 163.020 | 78.732 | 78.349 |
| 3 | 0.6 | 6 | 20 | 75.381 | 61.181 | 678.001 | 68.594 | 64.406 |
| 3 | 0.6 | 7 | 20 | 80.681 | 56.165 | 990.900 | 70.263 | 67.831 |
| 3 | 0.6 | 8 | 20 | 87.704 | 67.998 | 465.559 | 75.902 | 75.794 |
| 3 | 0.6 | 9 | 20 | 85.166 | 69.437 | 291.273 | 75.883 | 74.111 |
| 3 | 0.6 | 10 | 20 | 87.261 | 67.276 | 23.610 | 78.418 | 74.537 |
| 4 | 0.6 | 1 | 20 | 73.421 | 54.104 | 206.943 | 65.621 | 60.102 |
| 4 | 0.6 | 2 | 20 | 73.916 | 56.389 | 1721.278 | 66.173 | 61.602 |
| 4 | 0.6 | 3 | 20 | 75.966 | 56.594 | 223.693 | 69.935 | 63.186 |
| 4 | 0.6 | 4 | 20 | 70.472 | 58.114 | 1644.933 | 66.973 | 63.560 |
| 4 | 0.6 | 5 | 20 | 71.311 | 52.725 | 476.012 | 61.619 | 59.873 |
| 4 | 0.6 | 6 | 20 | 62.207 | 43.841 | 16.282 | 57.455 | 50.759 |
| 4 | 0.6 | 7 | 20 | 67.308 | 53.942 | 2.656 | 59.588 | 57.427 |
| 4 | 0.6 | 8 | 20 | 72.861 | 50.298 | 48.892 | 59.696 | 56.083 |
| 4 | 0.6 | 9 | 20 | 71.741 | 60.510 | 53.939 | 64.054 | 61.084 |
| 4 | 0.6 | 10 | 20 | 64.417 | 51.396 | 118.753 | 60.909 | 55.413 |

Table A20: Results of the runs for $Q = 0.6$, $B = 20$ for networks 5,6,7

| Network | $Q$ | Instance | $B$ | $z_2^*$ | $z_3^*$ | $T$ | $z_1^*$ | $z_4^*$ |
|---|---|---|---|---|---|---|---|---|
| 5 | 0.6 | 1 | 20 | 76.573 | 62.333 | 3599.691 | 70.681 | 66.194 |
| 5 | 0.6 | 2 | 20 | 76.978 | 57.579 | 1126.508 | 67.640 | 62.443 |
| 5 | 0.6 | 3 | 20 | 77.433 | 56.522 | 3598.322 | 67.000 | 65.740 |
| 5 | 0.6 | 4 | 20 | 75.540 | 57.984 | 3598.381 | 68.148 | 66.036 |
| 5 | 0.6 | 5 | 20 | 72.454 | 52.270 | 123.301 | 66.840 | 60.501 |
| 5 | 0.6 | 6 | 20 | 69.421 | 53.529 | 40.168 | 60.678 | 59.014 |
| 5 | 0.6 | 7 | 20 | 73.958 | 58.376 | 2685.351 | 67.418 | 66.177 |
| 5 | 0.6 | 8 | 20 | 74.537 | 56.344 | 460.576 | 64.170 | 60.181 |
| 5 | 0.6 | 9 | 20 | 79.186 | 58.394 | 13.368 | 68.820 | 65.187 |
| 5 | 0.6 | 10 | 20 | 75.136 | 57.947 | 922.073 | 66.360 | 65.688 |
| 6 | 0.6 | 1 | 20 | 90.157 | 72.956 | 101.357 | 81.811 | 76.401 |
| 6 | 0.6 | 2 | 20 | 86.035 | 66.758 | 3598.303 | 78.191 | 75.157 |
| 6 | 0.6 | 3 | 20 | 97.639 | 69.606 | 943.594 | 84.422 | 79.618 |
| 6 | 0.6 | 4 | 20 | 81.324 | 63.062 | 3598.304 | 73.265 | 72.881 |
| 6 | 0.6 | 5 | 20 | 89.081 | 63.792 | 1527.594 | 76.991 | 74.272 |
| 6 | 0.6 | 6 | 20 | 87.958 | 71.151 | 49.742 | 80.375 | 77.828 |
| 6 | 0.6 | 7 | 20 | 85.835 | 64.181 | 3598.303 | 76.386 | 72.586 |
| 6 | 0.6 | 8 | 20 | 86.026 | 66.973 | 3598.304 | 76.001 | 69.454 |
| 6 | 0.6 | 9 | 20 | 88.247 | 67.533 | 1789.032 | 76.453 | 73.164 |
| 6 | 0.6 | 10 | 20 | 84.899 | 66.658 | 3322.186 | 77.213 | 72.459 |
| 7 | 0.6 | 1 | 20 | 87.728 | 69.037 | 3440.707 | 80.460 | 74.068 |
| 7 | 0.6 | 2 | 20 | 82.787 | 62.897 | 3598.319 | 72.214 | 71.323 |
| 7 | 0.6 | 3 | 20 | 91.226 | 69.833 | 3598.304 | 80.096 | 77.400 |
| 7 | 0.6 | 4 | 20 | 80.805 | 62.310 | 3606.173 | 73.477 | 71.635 |
| 7 | 0.6 | 5 | 20 | 82.893 | 67.715 | 506.934 | 71.167 | 71.167 |
| 7 | 0.6 | 6 | 20 | 71.861 | 58.397 | 493.952 | 64.833 | 62.380 |
| 7 | 0.6 | 7 | 20 | 74.081 | 55.254 | 3603.588 | 65.423 | 61.192 |
| 7 | 0.6 | 8 | 20 | 88.151 | 65.343 | 3603.588 | 74.185 | 72.726 |
| 7 | 0.6 | 9 | 20 | 92.685 | 62.219 | 2496.364 | 78.487 | 76.647 |
| 7 | 0.6 | 10 | 20 | 77.734 | 61.713 | 3603.557 | 70.774 | 67.658 |

Table A21: Results of the runs for $Q = 0.7$, $B = 20$ for networks 1,2,3,4

| Network | $Q$ | Instance | $B$ | $z_2^*$ | $z_3^*$ | $T$ | $z_1^*$ | $z_4^*$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.7 | 1 | 20 | 74.790 | 65.381 | 9.181 | 72.138 | 72.138 |
| 1 | 0.7 | 2 | 20 | 77.882 | 72.868 | 7.194 | 76.726 | 73.987 |
| 1 | 0.7 | 3 | 20 | 74.956 | 68.605 | 41.963 | 72.876 | 72.361 |
| 1 | 0.7 | 4 | 20 | 75.241 | 62.293 | 4.192 | 68.396 | 68.347 |
| 1 | 0.7 | 5 | 20 | 73.333 | 63.537 | 0.376 | 71.089 | 64.585 |
| 1 | 0.7 | 6 | 20 | 75.076 | 65.526 | 1.236 | 68.825 | 68.175 |
| 1 | 0.7 | 7 | 20 | 73.386 | 69.316 | 1.189 | 70.939 | 69.316 |
| 1 | 0.7 | 8 | 20 | 84.020 | 71.140 | 2.330 | 79.489 | 75.620 |
| 1 | 0.7 | 9 | 20 | 79.138 | 71.808 | 10.276 | 75.994 | 73.985 |
| 1 | 0.7 | 10 | 20 | 82.981 | 77.547 | 3.941 | 81.896 | 80.229 |
| 2 | 0.7 | 1 | 20 | 75.351 | 62.756 | 83.488 | 70.295 | 67.534 |
| 2 | 0.7 | 2 | 20 | 72.398 | 60.907 | 118.929 | 65.670 | 65.298 |
| 2 | 0.7 | 3 | 20 | 77.537 | 64.032 | 91.965 | 71.062 | 69.756 |
| 2 | 0.7 | 4 | 20 | 79.962 | 65.927 | 5.271 | 71.700 | 71.316 |
| 2 | 0.7 | 5 | 20 | 80.119 | 61.609 | 82.080 | 74.631 | 68.842 |
| 2 | 0.7 | 6 | 20 | 64.288 | 52.445 | 459.324 | 57.516 | 56.633 |
| 2 | 0.7 | 7 | 20 | 70.397 | 57.080 | 149.756 | 64.216 | 61.719 |
| 2 | 0.7 | 8 | 20 | 70.258 | 60.236 | 450.147 | 66.338 | 63.719 |
| 2 | 0.7 | 9 | 20 | 71.594 | 63.901 | 420.271 | 65.325 | 65.257 |
| 2 | 0.7 | 10 | 20 | 69.788 | 59.670 | 112.266 | 65.001 | 63.832 |
| 3 | 0.7 | 1 | 20 | 80.470 | 68.940 | 1707.123 | 74.730 | 71.989 |
| 3 | 0.7 | 2 | 20 | 79.643 | 68.703 | 1431.619 | 73.162 | 72.045 |
| 3 | 0.7 | 3 | 20 | 84.363 | 69.260 | 210.534 | 78.441 | 74.179 |
| 3 | 0.7 | 4 | 20 | 88.591 | 70.689 | 22.882 | 77.625 | 77.232 |
| 3 | 0.7 | 5 | 20 | 88.186 | 72.883 | 60.418 | 78.732 | 78.349 |
| 3 | 0.7 | 6 | 20 | 84.891 | 67.289 | 125.968 | 77.224 | 73.187 |
| 3 | 0.7 | 7 | 20 | 77.189 | 64.159 | 129.518 | 70.263 | 67.831 |
| 3 | 0.7 | 8 | 20 | 84.056 | 66.405 | 432.438 | 75.902 | 75.794 |
| 3 | 0.7 | 9 | 20 | 80.801 | 69.215 | 2348.970 | 75.883 | 74.111 |
| 3 | 0.7 | 10 | 20 | 85.049 | 69.599 | 8.524 | 78.418 | 74.537 |
| 4 | 0.7 | 1 | 20 | 68.267 | 53.594 | 73.806 | 60.591 | 60.102 |
| 4 | 0.7 | 2 | 20 | 69.145 | 57.695 | 1363.271 | 64.009 | 61.602 |
| 4 | 0.7 | 3 | 20 | 70.660 | 58.722 | 479.469 | 65.676 | 63.186 |
| 4 | 0.7 | 4 | 20 | 68.596 | 57.181 | 608.377 | 63.964 | 63.560 |
| 4 | 0.7 | 5 | 20 | 69.320 | 51.801 | 97.017 | 61.619 | 59.873 |
| 4 | 0.7 | 6 | 20 | 57.958 | 48.045 | 53.020 | 57.455 | 50.759 |
| 4 | 0.7 | 7 | 20 | 63.161 | 52.912 | 137.635 | 59.588 | 57.427 |
| 4 | 0.7 | 8 | 20 | 67.383 | 49.383 | 47.343 | 59.696 | 56.083 |
| 4 | 0.7 | 9 | 20 | 66.582 | 57.493 | 813.186 | 64.054 | 61.084 |
| 4 | 0.7 | 10 | 20 | 61.915 | 50.853 | 475.277 | 57.459 | 55.413 |

Table A22: Results of the runs for $Q = 0.7$, $B = 20$ for networks 5,6,7

| Network | $Q$ | Instance | $B$ | $z_2^*$ | $z_3^*$ | $T$ | $z_1^*$ | $z_4^*$ |
|---|---|---|---|---|---|---|---|---|
| 5 | 0.7 | 1 | 20 | 73.543 | 64.338 | 679.602 | 69.184 | 66.194 |
| 5 | 0.7 | 2 | 20 | 71.163 | 58.816 | 3564.851 | 62.631 | 62.443 |
| 5 | 0.7 | 3 | 20 | 73.127 | 59.951 | 3594.777 | 67.000 | 65.740 |
| 5 | 0.7 | 4 | 20 | 73.125 | 60.585 | 3572.798 | 66.036 | 66.036 |
| 5 | 0.7 | 5 | 20 | 68.726 | 55.271 | 633.061 | 61.522 | 60.501 |
| 5 | 0.7 | 6 | 20 | 65.887 | 54.161 | 221.320 | 60.678 | 59.014 |
| 5 | 0.7 | 7 | 20 | 70.660 | 64.189 | 3597.359 | 67.418 | 66.177 |
| 5 | 0.7 | 8 | 20 | 70.459 | 55.681 | 662.508 | 64.170 | 60.181 |
| 5 | 0.7 | 9 | 20 | 72.521 | 61.050 | 1307.122 | 65.761 | 65.187 |
| 5 | 0.7 | 10 | 20 | 72.092 | 58.062 | 419.313 | 66.360 | 65.688 |
| 6 | 0.7 | 1 | 20 | 86.364 | 74.055 | 27.432 | 78.428 | 76.401 |
| 6 | 0.7 | 2 | 20 | 81.458 | 67.858 | 3597.358 | 76.713 | 75.157 |
| 6 | 0.7 | 3 | 20 | 93.378 | 73.593 | 357.891 | 84.422 | 79.618 |
| 6 | 0.7 | 4 | 20 | 79.077 | 68.163 | 3597.358 | 73.265 | 72.881 |
| 6 | 0.7 | 5 | 20 | 84.468 | 67.962 | 999.960 | 74.347 | 74.272 |
| 6 | 0.7 | 6 | 20 | 85.116 | 71.151 | 579.055 | 78.751 | 77.828 |
| 6 | 0.7 | 7 | 20 | 81.236 | 68.992 | 3597.358 | 76.386 | 72.586 |
| 6 | 0.7 | 8 | 20 | 81.751 | 67.179 | 1339.926 | 76.001 | 69.454 |
| 6 | 0.7 | 9 | 20 | 79.975 | 67.533 | 3597.405 | 74.547 | 73.164 |
| 6 | 0.7 | 10 | 20 | 79.653 | 66.658 | 3597.374 | 74.695 | 72.459 |
| 7 | 0.7 | 1 | 20 | 84.924 | 72.841 | 2540.284 | 77.386 | 74.068 |
| 7 | 0.7 | 2 | 20 | 78.810 | 66.758 | 3601.214 | 72.214 | 71.323 |
| 7 | 0.7 | 3 | 20 | 85.981 | 71.295 | 3601.183 | 77.400 | 77.400 |
| 7 | 0.7 | 4 | 20 | 78.214 | 66.137 | 3601.167 | 73.477 | 71.635 |
| 7 | 0.7 | 5 | 20 | 77.705 | 67.757 | 1380.457 | 71.167 | 71.167 |
| 7 | 0.7 | 6 | 20 | 68.460 | 58.823 | 582.984 | 64.833 | 62.380 |
| 7 | 0.7 | 7 | 20 | 70.509 | 58.753 | 3601.183 | 65.423 | 61.192 |
| 7 | 0.7 | 8 | 20 | 82.467 | 70.061 | 643.628 | 74.185 | 72.726 |
| 7 | 0.7 | 9 | 20 | 84.533 | 72.860 | 3601.293 | 78.487 | 76.647 |
| 7 | 0.7 | 10 | 20 | 73.601 | 62.584 | 3601.183 | 70.774 | 67.658 |

Table A23: Results of the runs for $Q = 0.8$, $B = 20$ for networks 1,2,3,4

| Network | $Q$ | Instance | $B$ | $z_2^*$ | $z_3^*$ | $T$ | $z_1^*$ | $z_4^*$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.8 | 1 | 20 | 73.425 | 70.329 | 1.641 | 72.138 | 72.138 |
| 1 | 0.8 | 2 | 20 | 75.864 | 71.944 | 9.737 | 74.324 | 73.987 |
| 1 | 0.8 | 3 | 20 | 74.053 | 71.772 | 7.487 | 72.876 | 72.361 |
| 1 | 0.8 | 4 | 20 | 71.321 | 64.767 | 13.911 | 68.396 | 68.347 |
| 1 | 0.8 | 5 | 20 | 69.798 | 64.585 | 0.188 | 67.002 | 64.585 |
| 1 | 0.8 | 6 | 20 | 71.926 | 65.526 | 4.627 | 68.825 | 68.175 |
| 1 | 0.8 | 7 | 20 | 70.939 | 66.771 | 0.954 | 69.896 | 69.316 |
| 1 | 0.8 | 8 | 20 | 81.459 | 71.140 | 7.190 | 76.210 | 75.620 |
| 1 | 0.8 | 9 | 20 | 77.869 | 68.628 | 3.454 | 73.985 | 73.985 |
| 1 | 0.8 | 10 | 20 | 82.661 | 79.221 | 0.485 | 81.896 | 80.229 |
| 2 | 0.8 | 1 | 20 | 71.976 | 64.084 | 524.480 | 68.831 | 67.534 |
| 2 | 0.8 | 2 | 20 | 70.015 | 62.875 | 48.828 | 65.670 | 65.298 |
| 2 | 0.8 | 3 | 20 | 74.540 | 67.277 | 66.647 | 71.062 | 69.756 |
| 2 | 0.8 | 4 | 20 | 75.132 | 68.788 | 482.342 | 71.700 | 71.316 |
| 2 | 0.8 | 5 | 20 | 77.189 | 66.430 | 3.149 | 70.684 | 68.842 |
| 2 | 0.8 | 6 | 20 | 60.915 | 52.854 | 2337.454 | 57.516 | 56.633 |
| 2 | 0.8 | 7 | 20 | 66.932 | 56.652 | 629.061 | 64.216 | 61.719 |
| 2 | 0.8 | 8 | 20 | 66.398 | 57.303 | 319.236 | 62.644 | 62.203 |
| 2 | 0.8 | 9 | 20 | 69.267 | 63.048 | 157.547 | 65.325 | 65.257 |
| 2 | 0.8 | 10 | 20 | 68.760 | 61.882 | 4.813 | 65.001 | 63.832 |
| 3 | 0.8 | 1 | 20 | 76.871 | 69.755 | 3600.400 | 73.266 | 71.989 |
| 3 | 0.8 | 2 | 20 | 77.648 | 68.703 | 116.887 | 73.162 | 72.045 |
| 3 | 0.8 | 3 | 20 | 80.578 | 71.158 | 29.800 | 78.441 | 74.179 |
| 3 | 0.8 | 4 | 20 | 81.971 | 73.052 | 449.781 | 77.625 | 77.232 |
| 3 | 0.8 | 5 | 20 | 84.278 | 73.702 | 23.378 | 78.732 | 78.349 |
| 3 | 0.8 | 6 | 20 | 67.514 | 60.188 | 3600.681 | 66.943 | 64.406 |
| 3 | 0.8 | 7 | 20 | 72.422 | 65.558 | 1748.523 | 70.263 | 67.831 |
| 3 | 0.8 | 8 | 20 | 79.896 | 70.991 | 3599.492 | 75.902 | 75.794 |
| 3 | 0.8 | 9 | 20 | 78.055 | 73.396 | 1396.362 | 75.883 | 74.111 |
| 3 | 0.8 | 10 | 20 | 81.341 | 71.793 | 70.941 | 76.760 | 74.537 |
| 4 | 0.8 | 1 | 20 | 62.718 | 55.677 | 1321.864 | 60.591 | 60.102 |
| 4 | 0.8 | 2 | 20 | 65.893 | 59.568 | 180.389 | 64.009 | 61.602 |
| 4 | 0.8 | 3 | 20 | 66.458 | 58.866 | 530.229 | 65.181 | 63.186 |
| 4 | 0.8 | 4 | 20 | 67.026 | 63.135 | 34.206 | 63.964 | 63.560 |
| 4 | 0.8 | 5 | 20 | 64.154 | 54.799 | 2063.143 | 61.619 | 59.873 |
| 4 | 0.8 | 6 | 20 | 54.283 | 47.058 | 296.132 | 51.537 | 50.759 |
| 4 | 0.8 | 7 | 20 | 60.032 | 53.942 | 137.370 | 59.588 | 57.427 |
| 4 | 0.8 | 8 | 20 | 62.587 | 56.083 | 37.940 | 56.338 | 56.083 |
| 4 | 0.8 | 9 | 20 | 63.637 | 57.991 | 1198.764 | 61.988 | 61.084 |
| 4 | 0.8 | 10 | 20 | 58.632 | 52.843 | 807.219 | 57.459 | 55.413 |

Table A24: Results of the runs for $Q = 0.8$, $B = 20$ for networks 5,6,7

| Network | $Q$ | Instance | $B$ | $z_2^*$ | $z_3^*$ | $T$ | $z_1^*$ | $z_4^*$ |
|---|---|---|---|---|---|---|---|---|
| 5 | 0.8 | 1 | 20 | 70.451 | 62.800 | 120.666 | 69.184 | 66.194 |
| 5 | 0.8 | 2 | 20 | 66.586 | 59.422 | 3600.307 | 62.631 | 62.443 |
| 5 | 0.8 | 3 | 20 | 68.885 | 61.330 | 3600.308 | 67.000 | 65.740 |
| 5 | 0.8 | 4 | 20 | 69.534 | 61.482 | 3600.308 | 66.036 | 66.036 |
| 5 | 0.8 | 5 | 20 | 66.840 | 57.796 | 58.770 | 61.522 | 60.501 |
| 5 | 0.8 | 6 | 20 | 63.873 | 56.776 | 39.159 | 60.678 | 59.014 |
| 5 | 0.8 | 7 | 20 | 67.683 | 64.189 | 3600.308 | 67.418 | 66.177 |
| 5 | 0.8 | 8 | 20 | 67.000 | 57.631 | 160.450 | 64.170 | 60.181 |
| 5 | 0.8 | 9 | 20 | 68.870 | 64.224 | 2638.468 | 65.761 | 65.187 |
| 5 | 0.8 | 10 | 20 | 69.212 | 63.355 | 1131.853 | 66.360 | 65.688 |
| 6 | 0.8 | 1 | 20 | 80.332 | 74.035 | 2221.702 | 78.210 | 76.401 |
| 6 | 0.8 | 2 | 20 | 79.838 | 72.722 | 498.465 | 76.713 | 75.157 |
| 6 | 0.8 | 3 | 20 | 84.553 | 72.531 | 3600.124 | 81.871 | 79.618 |
| 6 | 0.8 | 4 | 20 | 75.099 | 70.114 | 3600.124 | 73.265 | 72.881 |
| 6 | 0.8 | 5 | 20 | 80.853 | 72.298 | 646.063 | 74.347 | 74.272 |
| 6 | 0.8 | 6 | 20 | 83.022 | 75.534 | 162.957 | 79.279 | 77.828 |
| 6 | 0.8 | 7 | 20 | 77.621 | 68.992 | 2527.159 | 76.386 | 72.586 |
| 6 | 0.8 | 8 | 20 | 75.304 | 67.920 | 3600.123 | 70.133 | 69.454 |
| 6 | 0.8 | 9 | 20 | 77.183 | 70.407 | 3600.124 | 74.547 | 73.164 |
| 6 | 0.8 | 10 | 20 | 77.350 | 69.949 | 2707.054 | 74.695 | 72.459 |
| 7 | 0.8 | 1 | 20 | 79.189 | 73.124 | 3600.123 | 74.195 | 74.068 |
| 7 | 0.8 | 2 | 20 | 75.038 | 68.166 | 3600.123 | 72.214 | 71.323 |
| 7 | 0.8 | 3 | 20 | 82.772 | 74.238 | 3600.124 | 77.400 | 77.400 |
| 7 | 0.8 | 4 | 20 | 75.459 | 68.854 | 3600.124 | 73.477 | 71.635 |
| 7 | 0.8 | 5 | 20 | 73.739 | 68.452 | 3600.123 | 71.167 | 71.167 |
| 7 | 0.8 | 6 | 20 | 64.589 | 62.380 | 1976.207 | 63.010 | 62.380 |
| 7 | 0.8 | 7 | 20 | 67.211 | 61.823 | 3599.229 | 63.244 | 61.192 |
| 7 | 0.8 | 8 | 20 | 78.562 | 70.176 | 138.624 | 74.185 | 72.726 |
| 7 | 0.8 | 9 | 20 | 80.860 | 71.170 | 3599.733 | 78.487 | 76.647 |
| 7 | 0.8 | 10 | 20 | 69.792 | 65.801 | 3600.032 | 67.848 | 67.658 |

# APPENDIX B

# RESULTS OF THE RUNS FOR CHAPTER 3

Table B1: Results of the runs for $n = 10$, $C = 3, 4$

| $n$ | $C$ | $B$ | Instance | $z^*$ | [PI-L] | [PI-LR] |
|---|---|---|---|---|---|---|
| 10 | 3 | 3 | 1 | -2.780 | 122.1 | 19.7 |
| 10 | 3 | 3 | 2 | -2.778 | 134.0 | 22.2 |
| 10 | 3 | 3 | 3 | -2.871 | 146.8 | 24.6 |
| 10 | 3 | 3 | 4 | -2.741 | 145.4 | 20.2 |
| 10 | 3 | 3 | 5 | -2.684 | 112.4 | 27.5 |
| 10 | 3 | 3 | 6 | -2.742 | 142.8 | 23.3 |
| 10 | 3 | 3 | 7 | -2.724 | 128.1 | 24.5 |
| 10 | 3 | 3 | 8 | -2.856 | 118.5 | 25.7 |
| 10 | 3 | 3 | 9 | -2.729 | 158.2 | 24.5 |
| 10 | 3 | 3 | 10 | -2.810 | 136.7 | 22.4 |
| 10 | 3 | 5 | 1 | -2.483 | 81.6 | 18.0 |
| 10 | 3 | 5 | 2 | -2.480 | 167.4 | 18.9 |
| 10 | 3 | 5 | 3 | -2.636 | 124.4 | 24.3 |
| 10 | 3 | 5 | 4 | -2.392 | 95.6 | 16.0 |
| 10 | 3 | 5 | 5 | -2.430 | 96.5 | 20.1 |
| 10 | 3 | 5 | 6 | -2.531 | 119.0 | 26.2 |
| 10 | 3 | 5 | 7 | -2.475 | 91.6 | 22.9 |
| 10 | 3 | 5 | 8 | -2.628 | 144.2 | 23.9 |
| 10 | 3 | 5 | 9 | -2.475 | 150.8 | 24.4 |
| 10 | 3 | 5 | 10 | -2.548 | 106.2 | 20.3 |
| 10 | 4 | 3 | 1 | -3.627 | 335.8 | 19.1 |
| 10 | 4 | 3 | 2 | -3.620 | 450.7 | 22.1 |
| 10 | 4 | 3 | 3 | -3.745 | 386.2 | 20.3 |
| 10 | 4 | 3 | 4 | -3.571 | 490.0 | 20.7 |
| 10 | 4 | 3 | 5 | -3.495 | 448.9 | 33.1 |
| 10 | 4 | 3 | 6 | -3.568 | 377.0 | 24.1 |
| 10 | 4 | 3 | 7 | -3.554 | 379.2 | 28.9 |
| 10 | 4 | 3 | 8 | -3.732 | 517.6 | 23.5 |
| 10 | 4 | 3 | 9 | -3.563 | 594.7 | 23.6 |
| 10 | 4 | 3 | 10 | -3.666 | 463.7 | 21.6 |
| 10 | 4 | 5 | 1 | -3.235 | 210.5 | 16.6 |
| 10 | 4 | 5 | 2 | -3.229 | 521.2 | 20.7 |
| 10 | 4 | 5 | 3 | -3.431 | 431.6 | 22.2 |
| 10 | 4 | 5 | 4 | -3.116 | 353.8 | 14.5 |
| 10 | 4 | 5 | 5 | -3.166 | 453.6 | 24.7 |
| 10 | 4 | 5 | 6 | -3.284 | 431.8 | 22.5 |
| 10 | 4 | 5 | 7 | -3.234 | 290.0 | 18.9 |
| 10 | 4 | 5 | 8 | -3.428 | 421.9 | 19.6 |
| 10 | 4 | 5 | 9 | -3.223 | 595.3 | 24.2 |
| 10 | 4 | 5 | 10 | -3.324 | 437.4 | 19.7 |

Columns corresponding to [PI-L] and [PI-LR] represent the running times (in seconds) of the corresponding formulations.

Table B2: Results of the runs for $n = 10$, $C = 5, 6$

| $n$ | $C$ | $B$ | Instance | $z^*$ | [PI-L] | [PI-LR] |
|---|---|---|---|---|---|---|
| 10 | 5 | 3 | 1 | -4.473 | 1976.2 | 19.7 |
| 10 | 5 | 3 | 2 | -4.461 | 2001.4 | 20.8 |
| 10 | 5 | 3 | 3 | -4.618 | 2300.5 | 22.0 |
| 10 | 5 | 3 | 4 | -4.399 | 2055.6 | 25.8 |
| 10 | 5 | 3 | 5 | -4.306 | 1703.7 | 26.1 |
| 10 | 5 | 3 | 6 | -4.393 | 1709.0 | 24.2 |
| 10 | 5 | 3 | 7 | -4.383 | 1551.3 | 25.6 |
| 10 | 5 | 3 | 8 | -4.608 | 2364.5 | 26.3 |
| 10 | 5 | 3 | 9 | -4.398 | 2542.2 | 26.4 |
| 10 | 5 | 3 | 10 | -4.523 | 2239.4 | 23.5 |
| 10 | 5 | 5 | 1 | -3.988 | 1146.1 | 14.6 |
| 10 | 5 | 5 | 2 | -3.979 | 1569.5 | 22.1 |
| 10 | 5 | 5 | 3 | -4.225 | 1749.4 | 24.2 |
| 10 | 5 | 5 | 4 | -3.839 | 1844.7 | 17.8 |
| 10 | 5 | 5 | 5 | -3.901 | 1516.1 | 22.9 |
| 10 | 5 | 5 | 6 | -4.037 | 1318.8 | 24.7 |
| 10 | 5 | 5 | 7 | -3.994 | 1471.6 | 19.4 |
| 10 | 5 | 5 | 8 | -4.229 | 1898.1 | 22.7 |
| 10 | 5 | 5 | 9 | -3.969 | 2804.1 | 28.7 |
| 10 | 5 | 5 | 10 | -4.100 | 2343.8 | 19.1 |
| 10 | 6 | 3 | 1 | -5.320 | 6905.0 | 20.4 |
| 10 | 6 | 3 | 2 | -5.302 | 7323.2 | 24.3 |
| 10 | 6 | 3 | 3 | -5.492 | 8696.1 | 22.0 |
| 10 | 6 | 3 | 4 | -5.224 | 9040.4 | 24.3 |
| 10 | 6 | 3 | 5 | -5.117 | 7020.9 | 23.3 |
| 10 | 6 | 3 | 6 | -5.219 | 7038.3 | 25.4 |
| 10 | 6 | 3 | 7 | -5.213 | 8547.7 | 27.5 |
| 10 | 6 | 3 | 8 | -5.484 | 8803.8 | 25.0 |
| 10 | 6 | 3 | 9 | -5.232 | 10203.9 | 26.1 |
| 10 | 6 | 3 | 10 | -5.379 | 9335.6 | 22.5 |
| 10 | 6 | 5 | 1 | -4.740 | 3916.4 | 17.3 |
| 10 | 6 | 5 | 2 | -4.728 | 9773.2 | 18.7 |
| 10 | 6 | 5 | 3 | -5.020 | 7501.4 | 23.8 |
| 10 | 6 | 5 | 4 | -4.562 | 5709.3 | 20.8 |
| 10 | 6 | 5 | 5 | -4.637 | 7462.5 | 20.7 |
| 10 | 6 | 5 | 6 | -4.790 | 5831.6 | 25.0 |
| 10 | 6 | 5 | 7 | -4.753 | 8809.4 | 20.1 |
| 10 | 6 | 5 | 8 | -5.029 | 5606.3 | 23.0 |
| 10 | 6 | 5 | 9 | -4.714 | 10303.3 | 25.2 |
| 10 | 6 | 5 | 10 | -4.876 | 7518.2 | 21.0 |

Columns corresponding to [PI-L] and [PI-LR] represent the running times (in seconds) of the corresponding formulations.

Table B3: Results of the runs for $n = 10$ for [PI-LR] and HA

| $n$ | $C$ | $B$ | Instance | $z^*$ | [PI-LR] | $z^{HA}$ | HA |
|---|---|---|---|---|---|---|---|
| 10 | 3 | 3 | 1 | -2.780 | 19.73 | -2.780 | 0.09 |
| 10 | 3 | 3 | 2 | -2.778 | 22.23 | -2.778 | 0.11 |
| 10 | 3 | 3 | 3 | -2.871 | 24.59 | -2.878 | 0.13 |
| 10 | 3 | 3 | 4 | -2.741 | 20.23 | -2.741 | 0.11 |
| 10 | 3 | 3 | 5 | -2.684 | 27.50 | -2.684 | 0.09 |
| 10 | 3 | 3 | 6 | -2.742 | 23.28 | -2.742 | 0.09 |
| 10 | 3 | 3 | 7 | -2.724 | 24.52 | -2.724 | 0.08 |
| 10 | 3 | 3 | 8 | -2.856 | 25.66 | -2.856 | 0.09 |
| 10 | 3 | 3 | 9 | -2.729 | 24.48 | -2.729 | 0.09 |
| 10 | 3 | 3 | 10 | -2.810 | 22.38 | -2.810 | 0.11 |
| 10 | 3 | 5 | 1 | -2.483 | 18.05 | -2.483 | 0.14 |
| 10 | 3 | 5 | 2 | -2.480 | 18.94 | -2.480 | 0.14 |
| 10 | 3 | 5 | 3 | -2.636 | 24.34 | -2.636 | 0.16 |
| 10 | 3 | 5 | 4 | -2.392 | 16.05 | -2.392 | 0.14 |
| 10 | 3 | 5 | 5 | -2.430 | 20.09 | -2.432 | 0.13 |
| 10 | 3 | 5 | 6 | -2.531 | 26.16 | -2.531 | 0.14 |
| 10 | 3 | 5 | 7 | -2.475 | 22.92 | -2.475 | 0.13 |
| 10 | 3 | 5 | 8 | -2.628 | 23.89 | -2.628 | 0.16 |
| 10 | 3 | 5 | 9 | -2.475 | 24.36 | -2.475 | 0.14 |
| 10 | 3 | 5 | 10 | -2.548 | 20.34 | -2.548 | 0.16 |
| 10 | 5 | 3 | 1 | -4.473 | 19.70 | -4.473 | 0.09 |
| 10 | 5 | 3 | 2 | -4.461 | 20.77 | -4.461 | 0.09 |
| 10 | 5 | 3 | 3 | -4.618 | 22.00 | -4.624 | 0.09 |
| 10 | 5 | 3 | 4 | -4.399 | 25.83 | -4.400 | 0.09 |
| 10 | 5 | 3 | 5 | -4.306 | 26.09 | -4.306 | 0.08 |
| 10 | 5 | 3 | 6 | -4.393 | 24.19 | -4.393 | 0.09 |
| 10 | 5 | 3 | 7 | -4.383 | 25.63 | -4.383 | 0.08 |
| 10 | 5 | 3 | 8 | -4.608 | 26.30 | -4.608 | 0.09 |
| 10 | 5 | 3 | 9 | -4.398 | 26.39 | -4.398 | 0.09 |
| 10 | 5 | 3 | 10 | -4.523 | 23.47 | -4.523 | 0.09 |
| 10 | 5 | 5 | 1 | -3.988 | 14.61 | -3.988 | 0.14 |
| 10 | 5 | 5 | 2 | -3.979 | 22.11 | -3.979 | 0.14 |
| 10 | 5 | 5 | 3 | -4.225 | 24.20 | -4.225 | 0.14 |
| 10 | 5 | 5 | 4 | -3.839 | 17.80 | -3.839 | 0.13 |
| 10 | 5 | 5 | 5 | -3.901 | 22.89 | -3.908 | 0.13 |
| 10 | 5 | 5 | 6 | -4.037 | 24.66 | -4.037 | 0.13 |
| 10 | 5 | 5 | 7 | -3.994 | 19.41 | -3.994 | 0.13 |
| 10 | 5 | 5 | 8 | -4.229 | 22.72 | -4.229 | 0.14 |
| 10 | 5 | 5 | 9 | -3.969 | 28.70 | -3.969 | 0.13 |
| 10 | 5 | 5 | 10 | -4.100 | 19.09 | -4.100 | 0.13 |

Columns corresponding to [PI-LR] and HA represent the running times (in seconds) of [PI-LR] and the heuristic algorithm, $z^*$ is the optimal objective function value, and $z^{HA}$ is the objective function value found by the heuristic algorithm.

Table B4: Results of the runs for $n = 15$ for [PI-LR] and HA

| $n$ | $C$ | $B$ | Instance | $z^*$ | [PI-LR] | $z^{HA}$ | HA |
|---|---|---|---|---|---|---|---|
| 15 | 3 | 3 | 1 | -3.000 | 789.79 | -3.000 | 0.45 |
| 15 | 3 | 3 | 2 | -3.000 | 1065.68 | -3.000 | 0.49 |
| 15 | 3 | 3 | 3 | -3.000 | 715.29 | -3.000 | 0.44 |
| 15 | 3 | 3 | 4 | -2.990 | 1082.32 | -3.000 | 0.47 |
| 15 | 3 | 3 | 5 | -2.974 | 971.73 | -3.000 | 0.47 |
| 15 | 3 | 3 | 6 | -2.985 | 1118.40 | -3.000 | 0.50 |
| 15 | 3 | 3 | 7 | -3.000 | 977.59 | -3.000 | 0.47 |
| 15 | 3 | 3 | 8 | -3.000 | 714.95 | -3.000 | 0.42 |
| 15 | 3 | 3 | 9 | -2.996 | 1249.46 | -3.000 | 0.50 |
| 15 | 3 | 3 | 10 | -3.000 | 773.96 | -3.000 | 0.45 |
| 15 | 3 | 5 | 1 | -2.943 | 2466.11 | -3.000 | 0.75 |
| 15 | 3 | 5 | 2 | -2.935 | 3552.89 | -3.000 | 0.78 |
| 15 | 3 | 5 | 3 | -2.948 | 2551.87 | -3.000 | 0.70 |
| 15 | 3 | 5 | 4 | -2.900 | 3250.08 | -3.000 | 0.77 |
| 15 | 3 | 5 | 5 | -2.880 | 2954.44 | -3.000 | 0.75 |
| 15 | 3 | 5 | 6 | -2.900 | 3553.77 | -3.000 | 0.83 |
| 15 | 3 | 5 | 7 | -2.927 | 3191.03 | -3.000 | 0.78 |
| 15 | 3 | 5 | 8 | -2.966 | 2472.81 | -3.000 | 0.67 |
| 15 | 3 | 5 | 9 | -2.921 | 4485.74 | -2.996 | 0.83 |
| 15 | 3 | 5 | 10 | -2.944 | 2418.70 | -3.000 | 0.72 |
| 15 | 5 | 3 | 1 | -4.877 | 1080.82 | -4.877 | 0.50 |
| 15 | 5 | 3 | 2 | -4.827 | 1140.56 | -4.827 | 0.44 |
| 15 | 5 | 3 | 3 | -4.881 | 1143.65 | -4.881 | 0.50 |
| 15 | 5 | 3 | 4 | -4.806 | 1171.84 | -4.806 | 0.45 |
| 15 | 5 | 3 | 5 | -4.761 | 1073.73 | -4.761 | 0.39 |
| 15 | 5 | 3 | 6 | -4.785 | 1146.35 | -4.785 | 0.42 |
| 15 | 5 | 3 | 7 | -4.826 | 1130.21 | -4.826 | 0.45 |
| 15 | 5 | 3 | 8 | -4.894 | 1135.37 | -4.894 | 0.52 |
| 15 | 5 | 3 | 9 | -4.817 | 1407.73 | -4.817 | 0.44 |
| 15 | 5 | 3 | 10 | -4.863 | 1072.94 | -4.863 | 0.49 |
| 15 | 5 | 5 | 1 | -4.739 | 2559.80 | -4.739 | 0.72 |
| 15 | 5 | 5 | 2 | -4.698 | 3802.29 | -4.698 | 0.66 |
| 15 | 5 | 5 | 3 | -4.740 | 2340.37 | -4.740 | 0.72 |
| 15 | 5 | 5 | 4 | -4.662 | 2869.68 | -4.662 | 0.66 |
| 15 | 5 | 5 | 5 | -4.613 | 2846.81 | -4.613 | 0.58 |
| 15 | 5 | 5 | 6 | -4.647 | 3356.30 | -4.647 | 0.61 |
| 15 | 5 | 5 | 7 | -4.697 | 3069.07 | -4.697 | 0.66 |
| 15 | 5 | 5 | 8 | -4.774 | 2836.70 | -4.774 | 0.75 |
| 15 | 5 | 5 | 9 | -4.698 | 4138.95 | -4.698 | 0.63 |
| 15 | 5 | 5 | 10 | -4.733 | 2748.29 | -4.733 | 0.69 |

Columns corresponding to [PI-LR] and HA represent the running times (in seconds) of [PI-LR] and the heuristic algorithm, $z^*$ is the optimal objective function value, and $z^{HA}$ is the objective function value found by the heuristic algorithm.

Table B5: Results of the runs for $n = 30$ for [PI-LR] and HA

| $n$ | $C$ | $B$ | Instance | $z^*$ | [PI-LR] | $z^{HA}$ | HA |
|---|---|---|---|---|---|---|---|
| 30 | 3 | 3 | 1 | | | -3.000 | 9.06 |
| 30 | 3 | 3 | 2 | | | -2.986 | 8.59 |
| 30 | 3 | 3 | 3 | | | -3.000 | 9.24 |
| 30 | 3 | 3 | 4 | | | -3.000 | 7.98 |
| 30 | 3 | 3 | 5 | | | -2.985 | 8.32 |
| 30 | 3 | 3 | 6 | | | -2.982 | 8.42 |
| 30 | 3 | 3 | 7 | | | -2.986 | 7.92 |
| 30 | 3 | 3 | 8 | | | -2.987 | 8.95 |
| 30 | 3 | 3 | 9 | | | -3.000 | 8.40 |
| 30 | 3 | 3 | 10 | | | -3.000 | 8.60 |
| 30 | 3 | 5 | 1 | | | -3.000 | 5.36 |
| 30 | 3 | 5 | 2 | | | -2.974 | 4.93 |
| 30 | 3 | 5 | 3 | | | -2.974 | 5.81 |
| 30 | 3 | 5 | 4 | | | -3.000 | 4.85 |
| 30 | 3 | 5 | 5 | | | -3.000 | 4.97 |
| 30 | 3 | 5 | 6 | | | -2.980 | 5.53 |
| 30 | 3 | 5 | 7 | | | -2.973 | 4.85 |
| 30 | 3 | 5 | 8 | | | -3.000 | 5.56 |
| 30 | 3 | 5 | 9 | | | -2.986 | 5.05 |
| 30 | 3 | 5 | 10 | | | -2.972 | 5.24 |
| 30 | 5 | 3 | 1 | | | -5.000 | 14.35 |
| 30 | 5 | 3 | 2 | | | -5.000 | 14.88 |
| 30 | 5 | 3 | 3 | | | -4.978 | 13.59 |
| 30 | 5 | 3 | 4 | | | -4.975 | 15.85 |
| 30 | 5 | 3 | 5 | | | -4.981 | 14.31 |
| 30 | 5 | 3 | 6 | | | -5.000 | 15.18 |
| 30 | 5 | 3 | 7 | | | -5.000 | 13.37 |
| 30 | 5 | 3 | 8 | | | -5.000 | 11.99 |
| 30 | 5 | 3 | 9 | | | -4.988 | 15.34 |
| 30 | 5 | 3 | 10 | | | -4.984 | 12.08 |
| 30 | 5 | 5 | 1 | | | -4.987 | 8.18 |
| 30 | 5 | 5 | 2 | | | -4.972 | 9.18 |
| 30 | 5 | 5 | 3 | | | -4.970 | 8.07 |
| 30 | 5 | 5 | 4 | | | -4.989 | 9.49 |
| 30 | 5 | 5 | 5 | | | -4.973 | 8.48 |
| 30 | 5 | 5 | 6 | | | -4.983 | 8.51 |
| 30 | 5 | 5 | 7 | | | -4.978 | 8.65 |
| 30 | 5 | 5 | 8 | | | -5.000 | 7.53 |
| 30 | 5 | 5 | 9 | | | -4.978 | 9.59 |
| 30 | 5 | 5 | 10 | | | -5.000 | 7.34 |

Columns corresponding to [PI-LR] and HA represent the running times (in seconds) of [PI-LR] and the heuristic algorithm, $z^*$ is the optimal objective function value, and $z^{HA}$ is the objective function value found by the heuristic algorithm.

Table B6: Results of the runs for $n = 50$ for [PI-LR] and HA

| $n$ | $C$ | $B$ | Instance | $z^*$ | [PI-LR] | $z^{HA}$ | HA |
|---|---|---|---|---|---|---|---|
| 50 | 3 | 3 | 1 | | | -3.000 | 89.14 |
| 50 | 3 | 3 | 2 | | | -3.000 | 93.75 |
| 50 | 3 | 3 | 3 | | | -2.982 | 85.55 |
| 50 | 3 | 3 | 4 | | | -3.000 | 82.70 |
| 50 | 3 | 3 | 5 | | | -2.989 | 91.29 |
| 50 | 3 | 3 | 6 | | | -2.986 | 88.15 |
| 50 | 3 | 3 | 7 | | | -2.986 | 83.67 |
| 50 | 3 | 3 | 8 | | | -2.982 | 84.73 |
| 50 | 3 | 3 | 9 | | | -3.000 | 86.92 |
| 50 | 3 | 3 | 10 | | | -3.000 | 79.85 |
| 50 | 3 | 5 | 1 | | | -3.000 | 54.18 |
| 50 | 3 | 5 | 2 | | | -2.987 | 56.02 |
| 50 | 3 | 5 | 3 | | | -3.000 | 51.09 |
| 50 | 3 | 5 | 4 | | | -3.000 | 49.61 |
| 50 | 3 | 5 | 5 | | | -2.989 | 52.99 |
| 50 | 3 | 5 | 6 | | | -2.984 | 51.68 |
| 50 | 3 | 5 | 7 | | | -3.000 | 48.80 |
| 50 | 3 | 5 | 8 | | | -3.000 | 51.32 |
| 50 | 3 | 5 | 9 | | | -2.990 | 52.51 |
| 50 | 3 | 5 | 10 | | | -2.985 | 48.33 |
| 50 | 5 | 3 | 1 | | | -4.983 | 108.76 |
| 50 | 5 | 3 | 2 | | | -5.000 | 103.58 |
| 50 | 5 | 3 | 3 | | | -5.000 | 108.71 |
| 50 | 5 | 3 | 4 | | | -4.984 | 96.49 |
| 50 | 5 | 3 | 5 | | | -4.985 | 99.95 |
| 50 | 5 | 3 | 6 | | | -4.982 | 100.19 |
| 50 | 5 | 3 | 7 | | | -4.990 | 107.98 |
| 50 | 5 | 3 | 8 | | | -4.980 | 108.12 |
| 50 | 5 | 3 | 9 | | | -5.000 | 104.63 |
| 50 | 5 | 3 | 10 | | | -4.982 | 106.75 |
| 50 | 5 | 5 | 1 | | | -5.000 | 63.06 |
| 50 | 5 | 5 | 2 | | | -5.000 | 61.77 |
| 50 | 5 | 5 | 3 | | | -4.983 | 63.90 |
| 50 | 5 | 5 | 4 | | | -5.000 | 57.92 |
| 50 | 5 | 5 | 5 | | | -4.987 | 60.66 |
| 50 | 5 | 5 | 6 | | | -5.000 | 60.38 |
| 50 | 5 | 5 | 7 | | | -5.000 | 63.72 |
| 50 | 5 | 5 | 8 | | | -5.000 | 63.44 |
| 50 | 5 | 5 | 9 | | | -4.988 | 64.51 |
| 50 | 5 | 5 | 10 | | | -4.987 | 63.37 |

Columns corresponding to [PI-LR] and HA represent the running times (in seconds) of [PI-LR] and the heuristic algorithm, $z^*$ is the optimal objective function value, and $z^{HA}$ is the objective function value found by the heuristic algorithm.

# APPENDIX C

# RESULTS OF THE RUNS FOR CHAPTER 4

Table C1: Results of the runs for $Q = 0.6$, $B = 5$ for networks 1,2,3,4

| Network | $Q$ | Instance | $B$ | $z_2^*$ | $z_3^*$ | $T$ | $z_1^*$ | $z_4^*$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.6 | 1 | 5 | 53.681 | 48.113 | 0.594 | 50.494 | 50.494 |
| 1 | 0.6 | 2 | 5 | 57.664 | 55.065 | 0.812 | 57.467 | 55.065 |
| 1 | 0.6 | 3 | 5 | 50.531 | 46.484 | 0.765 | 50.141 | 46.484 |
| 1 | 0.6 | 4 | 5 | 53.192 | 47.861 | 0.421 | 47.861 | 47.861 |
| 1 | 0.6 | 5 | 5 | 56.395 | 50.56 | 0.171 | 56.395 | 51.130 |
| 1 | 0.6 | 6 | 5 | 56.091 | 49.995 | 1.421 | 52.625 | 52.143 |
| 1 | 0.6 | 7 | 5 | 51.485 | 43.53 | 0.562 | 45.662 | 45.662 |
| 1 | 0.6 | 8 | 5 | 56.94 | 55.107 | 0.516 | 60.409 | 55.107 |
| 1 | 0.6 | 9 | 5 | 55.305 | 51.346 | 0.687 | 53.630 | 53.630 |
| 1 | 0.6 | 10 | 5 | 62.888 | 57.565 | 0.515 | 61.210 | 59.512 |
| 2 | 0.6 | 1 | 5 | 57.306 | 48.156 | 1.171 | 48.987 | 48.356 |
| 2 | 0.6 | 2 | 5 | 56.478 | 48.074 | 1.187 | 55.822 | 49.968 |
| 2 | 0.6 | 3 | 5 | 52.922 | 48.059 | 0.703 | 51.153 | 48.359 |
| 2 | 0.6 | 4 | 5 | 54.428 | 49.112 | 1.375 | 55.492 | 52.047 |
| 2 | 0.6 | 5 | 5 | 66.705 | 52.701 | 0.734 | 56.173 | 53.511 |
| 2 | 0.6 | 6 | 5 | 50.494 | 41.44 | 0.421 | 44.911 | 43.260 |
| 2 | 0.6 | 7 | 5 | 51.233 | 43.668 | 1.046 | 47.626 | 45.479 |
| 2 | 0.6 | 8 | 5 | 49.43 | 43.21 | 1.328 | 46.974 | 45.242 |
| 2 | 0.6 | 9 | 5 | 60.017 | 50.934 | 0.562 | 57.898 | 52.048 |
| 2 | 0.6 | 10 | 5 | 56.468 | 47.02 | 0.703 | 53.331 | 49.080 |
| 3 | 0.6 | 1 | 5 | 57.1 | 50.827 | 4.484 | 55.251 | 52.810 |
| 3 | 0.6 | 2 | 5 | 64.704 | 51.215 | 0.921 | 59.401 | 53.690 |
| 3 | 0.6 | 3 | 5 | 55.278 | 47.119 | 3.406 | 52.495 | 49.410 |
| 3 | 0.6 | 4 | 5 | 62.059 | 52.93 | 3.531 | 58.560 | 55.286 |
| 3 | 0.6 | 5 | 5 | 67.783 | 57.06 | 1.156 | 62.518 | 59.468 |
| 3 | 0.6 | 6 | 5 | 59.136 | 50.532 | 0.688 | 52.520 | 50.532 |
| 3 | 0.6 | 7 | 5 | 59.217 | 49.275 | 1.531 | 55.370 | 50.220 |
| 3 | 0.6 | 8 | 5 | 65.35 | 53.572 | 0.656 | 55.784 | 55.784 |
| 3 | 0.6 | 9 | 5 | 66.048 | 55.476 | 0.75 | 61.987 | 56.856 |
| 3 | 0.6 | 10 | 5 | 64.181 | 55.35 | 0.765 | 59.382 | 57.918 |
| 4 | 0.6 | 1 | 5 | 50.265 | 42.326 | 3.14 | 46.188 | 43.547 |
| 4 | 0.6 | 2 | 5 | 52.1 | 44.977 | 0.89 | 51.020 | 45.414 |
| 4 | 0.6 | 3 | 5 | 49.24 | 43.362 | 1.062 | 47.762 | 44.242 |
| 4 | 0.6 | 4 | 5 | 50.85 | 42.835 | 1.375 | 48.367 | 44.360 |
| 4 | 0.6 | 5 | 5 | 54.362 | 43.181 | 0.89 | 47.852 | 45.089 |
| 4 | 0.6 | 6 | 5 | 41.328 | 36.163 | 1.843 | 39.596 | 38.116 |
| 4 | 0.6 | 7 | 5 | 41.816 | 35.176 | 3.156 | 37.584 | 36.679 |
| 4 | 0.6 | 8 | 5 | 47.515 | 37.713 | 1.046 | 43.766 | 39.221 |
| 4 | 0.6 | 9 | 5 | 46.935 | 42.682 | 1.187 | 45.914 | 44.711 |
| 4 | 0.6 | 10 | 5 | 45.861 | 40.868 | 6.406 | 44.184 | 42.265 |

Table C2: Results of the runs for $Q = 0.6$, $B = 5$ for networks 5,6,7

| Network | $Q$ | Instance | $B$ | $z_2^*$ | $z_3^*$ | $T$ | $z_1^*$ | $z_4^*$ |
|---|---|---|---|---|---|---|---|---|
| 5 | 0.6 | 1 | 5 | 51.456 | 47.641 | 3.313 | 50.584 | 49.750 |
| 5 | 0.6 | 2 | 5 | 51.706 | 43.712 | 4.843 | 47.544 | 45.951 |
| 5 | 0.6 | 3 | 5 | 52.839 | 46.503 | 2.234 | 49.295 | 47.745 |
| 5 | 0.6 | 4 | 5 | 56.424 | 46.751 | 3.188 | 50.367 | 49.006 |
| 5 | 0.6 | 5 | 5 | 50.287 | 42.827 | 0.296 | 44.772 | 44.735 |
| 5 | 0.6 | 6 | 5 | 48.446 | 44.281 | 6.719 | 48.444 | 46.388 |
| 5 | 0.6 | 7 | 5 | 50.473 | 45.936 | 7.546 | 48.460 | 47.555 |
| 5 | 0.6 | 8 | 5 | 55.003 | 45.974 | 1.093 | 55.003 | 45.974 |
| 5 | 0.6 | 9 | 5 | 49.748 | 46.648 | 2.937 | 48.892 | 48.892 |
| 5 | 0.6 | 10 | 5 | 53.377 | 48.269 | 1.531 | 49.294 | 48.889 |
| 6 | 0.6 | 1 | 5 | 61.007 | 58.184 | 2.515 | 61.007 | 58.927 |
| 6 | 0.6 | 2 | 5 | 64.172 | 55.249 | 2.015 | 60.908 | 57.445 |
| 6 | 0.6 | 3 | 5 | 66.951 | 56.518 | 1.484 | 59.393 | 57.353 |
| 6 | 0.6 | 4 | 5 | 61.667 | 53.747 | 16.406 | 60.580 | 54.758 |
| 6 | 0.6 | 5 | 5 | 60.916 | 53.259 | 0.968 | 54.024 | 54.024 |
| 6 | 0.6 | 6 | 5 | 68.488 | 60.593 | 2.687 | 63.400 | 63.400 |
| 6 | 0.6 | 7 | 5 | 59.028 | 49.668 | 7.812 | 54.408 | 53.574 |
| 6 | 0.6 | 8 | 5 | 62.452 | 49.603 | 2.812 | 61.369 | 51.998 |
| 6 | 0.6 | 9 | 5 | 63.448 | 58.723 | 2.515 | 61.952 | 59.424 |
| 6 | 0.6 | 10 | 5 | 59.267 | 53.576 | 3.093 | 55.565 | 55.565 |
| 7 | 0.6 | 1 | 5 | 65.851 | 53.594 | 3.109 | 59.343 | 56.025 |
| 7 | 0.6 | 2 | 5 | 62.436 | 51.479 | 1.984 | 55.717 | 53.520 |
| 7 | 0.6 | 3 | 5 | 64.159 | 59.982 | 7.796 | 66.008 | 62.095 |
| 7 | 0.6 | 4 | 5 | 59.391 | 52.256 | 5.828 | 53.339 | 53.052 |
| 7 | 0.6 | 5 | 5 | 56.119 | 52.74 | 1.25 | 54.865 | 53.972 |
| 7 | 0.6 | 6 | 5 | 48.675 | 45.017 | 6.187 | 46.917 | 45.612 |
| 7 | 0.6 | 7 | 5 | 51.462 | 44.165 | 11.719 | 49.205 | 47.727 |
| 7 | 0.6 | 8 | 5 | 66.615 | 54.515 | 2.109 | 60.119 | 55.195 |
| 7 | 0.6 | 9 | 5 | 65.837 | 56.083 | 12.796 | 61.203 | 57.505 |
| 7 | 0.6 | 10 | 5 | 56.359 | 48.825 | 4.547 | 56.074 | 52.409 |

Table C3: Results of the runs for $Q = 0.7$, $B = 5$ for networks 1,2,3,4

| Network | $Q$ | Instance | $B$ | $z_2^*$ | $z_3^*$ | $T$ | $z_1^*$ | $z_4^*$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.7 | 1 | 5 | 51.46 | 48.952 | 0.5 | 50.494 | 50.494 |
| 1 | 0.7 | 2 | 5 | 57.664 | 55.065 | 0.718 | 57.467 | 55.065 |
| 1 | 0.7 | 3 | 5 | 48.745 | 45.924 | 1.046 | 46.484 | 46.484 |
| 1 | 0.7 | 4 | 5 | 49.736 | 47.134 | 0.64 | 47.861 | 47.861 |
| 1 | 0.7 | 5 | 5 | 51.749 | 49.011 | 0.203 | 51.130 | 51.130 |
| 1 | 0.7 | 6 | 5 | 53.927 | 49.995 | 1.468 | 52.625 | 52.143 |
| 1 | 0.7 | 7 | 5 | 48.621 | 44.885 | 0.875 | 45.662 | 45.662 |
| 1 | 0.7 | 8 | 5 | 56.94 | 55.107 | 0.468 | 56.737 | 55.107 |
| 1 | 0.7 | 9 | 5 | 55.305 | 51.346 | 0.625 | 53.630 | 53.630 |
| 1 | 0.7 | 10 | 5 | 61.64 | 57.611 | 0.515 | 61.210 | 59.512 |
| 2 | 0.7 | 1 | 5 | 57.306 | 48.156 | 0.438 | 48.987 | 48.356 |
| 2 | 0.7 | 2 | 5 | 53.928 | 48.074 | 0.984 | 52.340 | 49.968 |
| 2 | 0.7 | 3 | 5 | 51.397 | 48.059 | 0.843 | 51.153 | 48.359 |
| 2 | 0.7 | 4 | 5 | 54.428 | 49.112 | 0.562 | 55.492 | 52.047 |
| 2 | 0.7 | 5 | 5 | 58.738 | 50.935 | 2.625 | 56.173 | 53.511 |
| 2 | 0.7 | 6 | 5 | 45.15 | 42.147 | 3.656 | 44.911 | 43.260 |
| 2 | 0.7 | 7 | 5 | 50.062 | 43.701 | 0.703 | 47.626 | 45.479 |
| 2 | 0.7 | 8 | 5 | 47.698 | 43.21 | 0.859 | 45.242 | 45.242 |
| 2 | 0.7 | 9 | 5 | 57.898 | 52.048 | 0.391 | 57.898 | 52.048 |
| 2 | 0.7 | 10 | 5 | 53.196 | 47.407 | 1.14 | 51.084 | 49.080 |
| 3 | 0.7 | 1 | 5 | 55.428 | 50.723 | 3.421 | 55.251 | 52.810 |
| 3 | 0.7 | 2 | 5 | 58.823 | 51.215 | 6.718 | 55.919 | 53.690 |
| 3 | 0.7 | 3 | 5 | 53.664 | 47.15 | 3.109 | 52.495 | 49.410 |
| 3 | 0.7 | 4 | 5 | 61.017 | 52.605 | 2.812 | 58.560 | 55.286 |
| 3 | 0.7 | 5 | 5 | 64.938 | 57.06 | 2 | 62.518 | 59.468 |
| 3 | 0.7 | 6 | 5 | 54.102 | 50.45 | 4.015 | 52.520 | 50.532 |
| 3 | 0.7 | 7 | 5 | 53.312 | 48.162 | 4.015 | 50.367 | 50.220 |
| 3 | 0.7 | 8 | 5 | 57.202 | 53.572 | 3.546 | 55.784 | 55.784 |
| 3 | 0.7 | 9 | 5 | 63.929 | 55.476 | 0.765 | 61.987 | 56.856 |
| 3 | 0.7 | 10 | 5 | 62.948 | 57.349 | 0.671 | 59.382 | 57.918 |
| 4 | 0.7 | 1 | 5 | 49.424 | 42.326 | 1.109 | 46.188 | 43.547 |
| 4 | 0.7 | 2 | 5 | 49.147 | 44.977 | 1.171 | 46.356 | 45.414 |
| 4 | 0.7 | 3 | 5 | 44.924 | 43.362 | 1.875 | 44.581 | 44.242 |
| 4 | 0.7 | 4 | 5 | 49.566 | 42.968 | 1.687 | 45.246 | 44.360 |
| 4 | 0.7 | 5 | 5 | 50.946 | 43.961 | 0.921 | 47.852 | 45.089 |
| 4 | 0.7 | 6 | 5 | 39.596 | 36.565 | 3.171 | 39.596 | 38.116 |
| 4 | 0.7 | 7 | 5 | 41.217 | 35.176 | 2.421 | 37.584 | 36.679 |
| 4 | 0.7 | 8 | 5 | 47.515 | 37.713 | 0.546 | 43.766 | 39.221 |
| 4 | 0.7 | 9 | 5 | 46.935 | 42.682 | 1.094 | 45.914 | 44.711 |
| 4 | 0.7 | 10 | 5 | 44.05 | 40.825 | 9.39 | 43.379 | 42.265 |

Table C4: Results of the runs for $Q = 0.7$, $B = 5$ for networks 5,6,7

| Network | $Q$ | Instance | $B$ | $z_2^*$ | $z_3^*$ | $T$ | $z_1^*$ | $z_4^*$ |
|---|---|---|---|---|---|---|---|---|
| 5 | 0.7 | 1 | 5 | 51.003 | 47.917 | 2.218 | 49.874 | 49.750 |
| 5 | 0.7 | 2 | 5 | 49.327 | 42.665 | 6.531 | 46.066 | 45.951 |
| 5 | 0.7 | 3 | 5 | 52.839 | 46.125 | 2.109 | 49.295 | 47.745 |
| 5 | 0.7 | 4 | 5 | 53.42 | 46.751 | 3.796 | 50.367 | 49.006 |
| 5 | 0.7 | 5 | 5 | 50.287 | 42.827 | 0.25 | 44.772 | 44.735 |
| 5 | 0.7 | 6 | 5 | 48.444 | 44.671 | 1.921 | 48.444 | 46.388 |
| 5 | 0.7 | 7 | 5 | 49.839 | 45.936 | 4.671 | 48.460 | 47.555 |
| 5 | 0.7 | 8 | 5 | 52.187 | 43.887 | 1.422 | 47.963 | 45.974 |
| 5 | 0.7 | 9 | 5 | 49.748 | 46.648 | 2.281 | 48.892 | 48.892 |
| 5 | 0.7 | 10 | 5 | 53.377 | 47.642 | 0.859 | 49.294 | 48.889 |
| 6 | 0.7 | 1 | 5 | 59.648 | 58.184 | 2.328 | 59.648 | 58.927 |
| 6 | 0.7 | 2 | 5 | 61.887 | 56.119 | 2.171 | 59.993 | 57.445 |
| 6 | 0.7 | 3 | 5 | 65.527 | 54.713 | 1.125 | 59.393 | 57.353 |
| 6 | 0.7 | 4 | 5 | 60.58 | 53.747 | 6.656 | 57.767 | 54.758 |
| 6 | 0.7 | 5 | 5 | 60.916 | 53.259 | 0.812 | 54.024 | 54.024 |
| 6 | 0.7 | 6 | 5 | 64.552 | 58.621 | 9.062 | 63.400 | 63.400 |
| 6 | 0.7 | 7 | 5 | 56.028 | 50.574 | 5.609 | 54.408 | 53.574 |
| 6 | 0.7 | 8 | 5 | 60.248 | 50.654 | 3.812 | 53.873 | 51.998 |
| 6 | 0.7 | 9 | 5 | 61.952 | 59.424 | 2.266 | 61.952 | 59.424 |
| 6 | 0.7 | 10 | 5 | 59.267 | 53.576 | 1.968 | 55.565 | 55.565 |
| 7 | 0.7 | 1 | 5 | 63.422 | 54.884 | 2.312 | 59.343 | 56.025 |
| 7 | 0.7 | 2 | 5 | 58.128 | 50.859 | 3.031 | 55.717 | 53.520 |
| 7 | 0.7 | 3 | 5 | 64.159 | 59.982 | 6.125 | 66.008 | 62.095 |
| 7 | 0.7 | 4 | 5 | 56.852 | 52.256 | 5.515 | 53.339 | 53.052 |
| 7 | 0.7 | 5 | 5 | 54.865 | 53.972 | 1.234 | 54.865 | 53.972 |
| 7 | 0.7 | 6 | 5 | 48.675 | 45.017 | 3.968 | 46.917 | 45.612 |
| 7 | 0.7 | 7 | 5 | 49.721 | 44.165 | 6.078 | 47.727 | 47.727 |
| 7 | 0.7 | 8 | 5 | 56.812 | 55.114 | 2.593 | 56.812 | 55.195 |
| 7 | 0.7 | 9 | 5 | 63.212 | 54.791 | 12.718 | 61.203 | 57.505 |
| 7 | 0.7 | 10 | 5 | 56.208 | 48.825 | 2.828 | 56.074 | 52.409 |

Table C5: Results of the runs for $Q = 0.8$, $B = 5$ for networks 1,2,3,4

| Network | $Q$ | Instance | $B$ | $z_2^*$ | $z_3^*$ | $T$ | $z_1^*$ | $z_4^*$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.8 | 1 | 5 | 51.46 | 48.952 | 0.328 | 50.494 | 50.494 |
| 1 | 0.8 | 2 | 5 | 55.999 | 55.065 | 0.5 | 55.802 | 55.065 |
| 1 | 0.8 | 3 | 5 | 48.425 | 45.924 | 0.687 | 46.484 | 46.484 |
| 1 | 0.8 | 4 | 5 | 49.736 | 47.778 | 0.468 | 47.861 | 47.861 |
| 1 | 0.8 | 5 | 5 | 51.749 | 49.011 | 0.187 | 51.130 | 51.130 |
| 1 | 0.8 | 6 | 5 | 53.443 | 52.143 | 3.765 | 52.625 | 52.143 |
| 1 | 0.8 | 7 | 5 | 47.736 | 45.654 | 0.812 | 45.662 | 45.662 |
| 1 | 0.8 | 8 | 5 | 56.94 | 55.107 | 0.281 | 56.737 | 55.107 |
| 1 | 0.8 | 9 | 5 | 54.072 | 51.895 | 0.485 | 53.630 | 53.630 |
| 1 | 0.8 | 10 | 5 | 61.64 | 58.95 | 0.359 | 61.210 | 59.512 |
| 2 | 0.8 | 1 | 5 | 51.601 | 48.156 | 1.25 | 48.987 | 48.356 |
| 2 | 0.8 | 2 | 5 | 51.276 | 48.538 | 1.468 | 49.968 | 49.968 |
| 2 | 0.8 | 3 | 5 | 51.153 | 48.059 | 0.64 | 49.407 | 48.359 |
| 2 | 0.8 | 4 | 5 | 52.745 | 49.112 | 0.687 | 53.809 | 52.047 |
| 2 | 0.8 | 5 | 5 | 56.173 | 51.36 | 2.203 | 56.173 | 53.511 |
| 2 | 0.8 | 6 | 5 | 44.027 | 41.44 | 2.921 | 43.260 | 43.260 |
| 2 | 0.8 | 7 | 5 | 48.004 | 43.421 | 0.734 | 47.626 | 45.479 |
| 2 | 0.8 | 8 | 5 | 45.242 | 44.263 | 1.093 | 45.242 | 45.242 |
| 2 | 0.8 | 9 | 5 | 55.956 | 51.053 | 0.656 | 54.687 | 52.048 |
| 2 | 0.8 | 10 | 5 | 53.006 | 47.02 | 0.64 | 49.080 | 49.080 |
| 3 | 0.8 | 1 | 5 | 55.428 | 50.723 | 1.203 | 55.251 | 52.810 |
| 3 | 0.8 | 2 | 5 | 57.365 | 51.654 | 2.421 | 55.919 | 53.690 |
| 3 | 0.8 | 3 | 5 | 51.099 | 48.401 | 3.687 | 49.779 | 49.410 |
| 3 | 0.8 | 4 | 5 | 57.383 | 54.453 | 4.156 | 55.716 | 55.286 |
| 3 | 0.8 | 5 | 5 | 61.534 | 57.06 | 1.656 | 60.523 | 59.468 |
| 3 | 0.8 | 6 | 5 | 51.376 | 50.532 | 5.109 | 51.376 | 50.532 |
| 3 | 0.8 | 7 | 5 | 51.614 | 48.995 | 3.609 | 50.367 | 50.220 |
| 3 | 0.8 | 8 | 5 | 55.784 | 55.784 | 2.796 | 55.784 | 55.784 |
| 3 | 0.8 | 9 | 5 | 60.012 | 56.562 | 1.828 | 58.776 | 56.856 |
| 3 | 0.8 | 10 | 5 | 62.948 | 55.095 | 0.718 | 59.382 | 57.918 |
| 4 | 0.8 | 1 | 5 | 46.717 | 42.326 | 1.5 | 45.060 | 43.547 |
| 4 | 0.8 | 2 | 5 | 46.356 | 45.414 | 0.875 | 46.356 | 45.414 |
| 4 | 0.8 | 3 | 5 | 44.924 | 43.362 | 1 | 44.581 | 44.242 |
| 4 | 0.8 | 4 | 5 | 46.738 | 44.359 | 1.484 | 45.246 | 44.360 |
| 4 | 0.8 | 5 | 5 | 46.564 | 43.181 | 1.703 | 45.624 | 45.089 |
| 4 | 0.8 | 6 | 5 | 37.958 | 37.326 | 5.25 | 38.116 | 38.116 |
| 4 | 0.8 | 7 | 5 | 38.829 | 35.767 | 2.421 | 36.993 | 36.679 |
| 4 | 0.8 | 8 | 5 | 40.916 | 37.713 | 2.156 | 39.221 | 39.221 |
| 4 | 0.8 | 9 | 5 | 45.914 | 44.702 | 0.921 | 45.914 | 44.711 |
| 4 | 0.8 | 10 | 5 | 43.731 | 40.868 | 3.234 | 43.379 | 42.265 |

Table C6: Results of the runs for $Q = 0.8$, $B = 5$ for networks 5,6,7

| Network | $Q$ | Instance | $B$ | $z_2^*$ | $z_3^*$ | $T$ | $z_1^*$ | $z_4^*$ |
|---|---|---|---|---|---|---|---|---|
| 5 | 0.8 | 1 | 5 | 49.874 | 49.75 | 2.187 | 49.874 | 49.750 |
| 5 | 0.8 | 2 | 5 | 47.042 | 43.712 | 5.297 | 46.066 | 45.951 |
| 5 | 0.8 | 3 | 5 | 50.148 | 46.517 | 1.984 | 49.295 | 47.745 |
| 5 | 0.8 | 4 | 5 | 50.592 | 46.93 | 5.312 | 50.367 | 49.006 |
| 5 | 0.8 | 5 | 5 | 45.079 | 42.827 | 0.765 | 44.772 | 44.735 |
| 5 | 0.8 | 6 | 5 | 48.444 | 46.388 | 0.859 | 48.444 | 46.388 |
| 5 | 0.8 | 7 | 5 | 49.248 | 46.279 | 2.312 | 47.869 | 47.555 |
| 5 | 0.8 | 8 | 5 | 47.787 | 43.688 | 2.375 | 46.272 | 45.974 |
| 5 | 0.8 | 9 | 5 | 49.093 | 47.733 | 1.937 | 48.892 | 48.892 |
| 5 | 0.8 | 10 | 5 | 49.779 | 47.642 | 2.468 | 49.294 | 48.889 |
| 6 | 0.8 | 1 | 5 | 59.43 | 58.047 | 1.562 | 59.430 | 58.927 |
| 6 | 0.8 | 2 | 5 | 60.908 | 57.445 | 1.328 | 59.993 | 57.445 |
| 6 | 0.8 | 3 | 5 | 58.901 | 56.376 | 4 | 59.393 | 57.353 |
| 6 | 0.8 | 4 | 5 | 56.855 | 53.418 | 16.171 | 55.088 | 54.758 |
| 6 | 0.8 | 5 | 5 | 54.378 | 53.256 | 1.938 | 54.024 | 54.024 |
| 6 | 0.8 | 6 | 5 | 63.572 | 60.577 | 3.281 | 63.400 | 63.400 |
| 6 | 0.8 | 7 | 5 | 54.018 | 50.574 | 7.625 | 53.817 | 53.574 |
| 6 | 0.8 | 8 | 5 | 56.945 | 51.139 | 2.125 | 53.873 | 51.998 |
| 6 | 0.8 | 9 | 5 | 61.123 | 58.723 | 1.625 | 59.767 | 59.424 |
| 6 | 0.8 | 10 | 5 | 57.289 | 53.038 | 2.64 | 55.565 | 55.565 |
| 7 | 0.8 | 1 | 5 | 57.212 | 53.953 | 8.64 | 56.152 | 56.025 |
| 7 | 0.8 | 2 | 5 | 55.951 | 50.859 | 3.515 | 55.717 | 53.520 |
| 7 | 0.8 | 3 | 5 | 62.794 | 59.982 | 3.531 | 64.643 | 62.095 |
| 7 | 0.8 | 4 | 5 | 56.852 | 52.256 | 2.281 | 53.339 | 53.052 |
| 7 | 0.8 | 5 | 5 | 54.865 | 53.972 | 0.875 | 54.865 | 53.972 |
| 7 | 0.8 | 6 | 5 | 46.917 | 44.267 | 2.625 | 46.917 | 45.612 |
| 7 | 0.8 | 7 | 5 | 49.721 | 44.165 | 2.015 | 47.727 | 47.727 |
| 7 | 0.8 | 8 | 5 | 56.812 | 55.114 | 1.484 | 56.812 | 55.195 |
| 7 | 0.8 | 9 | 5 | 59.687 | 56.083 | 10.39 | 57.505 | 57.505 |
| 7 | 0.8 | 10 | 5 | 53.191 | 50.428 | 5.188 | 52.622 | 52.409 |

Table C7: Results of the runs for $Q = 0.6$, $B = 10$ for networks 1,2,3,4

| Network | $Q$ | Instance | $B$ | $z_2^*$ | $z_3^*$ | $T$ | $z_1^*$ | $z_4^*$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.6 | 1 | 10 | 64.881 | 57.25 | 2.609 | 62.390 | 59.549 |
| 1 | 0.6 | 2 | 10 | 65.471 | 59.755 | 8.875 | 65.224 | 62.822 |
| 1 | 0.6 | 3 | 10 | 62.278 | 54.834 | 6.328 | 60.383 | 57.565 |
| 1 | 0.6 | 4 | 10 | 64.162 | 56.679 | 2.171 | 58.529 | 58.529 |
| 1 | 0.6 | 5 | 10 | 67.289 | 55.828 | 0.89 | 58.697 | 58.697 |
| 1 | 0.6 | 6 | 10 | 65.184 | 57.506 | 3.5 | 61.026 | 59.822 |
| 1 | 0.6 | 7 | 10 | 63.243 | 55.079 | 1.343 | 57.853 | 57.582 |
| 1 | 0.6 | 8 | 10 | 69.954 | 61.782 | 1.015 | 68.513 | 63.774 |
| 1 | 0.6 | 9 | 10 | 67.153 | 59.619 | 1.687 | 62.522 | 62.522 |
| 1 | 0.6 | 10 | 10 | 72.908 | 65.398 | 2.25 | 70.005 | 69.032 |
| 2 | 0.6 | 1 | 10 | 67.956 | 54.983 | 9.171 | 58.083 | 56.643 |
| 2 | 0.6 | 2 | 10 | 64.456 | 53.215 | 8.75 | 60.066 | 55.541 |
| 2 | 0.6 | 3 | 10 | 66.614 | 55.891 | 2.203 | 63.743 | 58.682 |
| 2 | 0.6 | 4 | 10 | 65.474 | 56.355 | 38.969 | 63.109 | 58.794 |
| 2 | 0.6 | 5 | 10 | 70.101 | 57.222 | 19.422 | 66.705 | 60.036 |
| 2 | 0.6 | 6 | 10 | 57.253 | 45.827 | 19.125 | 54.777 | 48.126 |
| 2 | 0.6 | 7 | 10 | 63.09 | 49.917 | 4.14 | 53.376 | 52.312 |
| 2 | 0.6 | 8 | 10 | 61.738 | 50.138 | 10.265 | 52.457 | 52.457 |
| 2 | 0.6 | 9 | 10 | 65.615 | 55.484 | 12.39 | 59.193 | 58.002 |
| 2 | 0.6 | 10 | 10 | 65.984 | 53.214 | 1.687 | 56.126 | 55.701 |
| 3 | 0.6 | 1 | 10 | 70.744 | 58.262 | 75.828 | 62.373 | 60.868 |
| 3 | 0.6 | 2 | 10 | 69.932 | 58.441 | 35.781 | 67.202 | 61.142 |
| 3 | 0.6 | 3 | 10 | 70.583 | 58.387 | 6.671 | 60.925 | 60.556 |
| 3 | 0.6 | 4 | 10 | 71.737 | 60.769 | 59.89 | 66.995 | 63.877 |
| 3 | 0.6 | 5 | 10 | 76.54 | 64.836 | 8.421 | 67.597 | 67.597 |
| 3 | 0.6 | 6 | 10 | 64.713 | 54.278 | 78.89 | 60.770 | 55.770 |
| 3 | 0.6 | 7 | 10 | 70.675 | 53.834 | 1.89 | 63.466 | 56.390 |
| 3 | 0.6 | 8 | 10 | 73.353 | 61.525 | 48.5 | 65.620 | 63.638 |
| 3 | 0.6 | 9 | 10 | 73.373 | 62.536 | 6.531 | 71.254 | 64.580 |
| 3 | 0.6 | 10 | 10 | 73.135 | 62.543 | 14.64 | 66.458 | 65.327 |
| 4 | 0.6 | 1 | 10 | 58.368 | 49.226 | 37.89 | 58.306 | 50.817 |
| 4 | 0.6 | 2 | 10 | 62.283 | 50.009 | 12.328 | 57.664 | 52.439 |
| 4 | 0.6 | 3 | 10 | 62.033 | 53.451 | 6.781 | 62.033 | 53.892 |
| 4 | 0.6 | 4 | 10 | 59.775 | 50.584 | 7.828 | 57.228 | 52.852 |
| 4 | 0.6 | 5 | 10 | 60.75 | 51.619 | 5.422 | 56.418 | 52.734 |
| 4 | 0.6 | 6 | 10 | 47.779 | 41.714 | 38.296 | 45.790 | 42.876 |
| 4 | 0.6 | 7 | 10 | 53.909 | 44.796 | 4.453 | 46.461 | 45.556 |
| 4 | 0.6 | 8 | 10 | 60.472 | 46.279 | 2 | 54.172 | 47.832 |
| 4 | 0.6 | 9 | 10 | 55.468 | 49.089 | 51.265 | 53.354 | 53.040 |
| 4 | 0.6 | 10 | 10 | 55.503 | 44.572 | 12.547 | 48.351 | 46.913 |

Table C8: Results of the runs for $Q = 0.6$, $B = 10$ for networks 5,6,7

| Network | $Q$ | Instance | $B$ | $z_2^*$ | $z_3^*$ | $T$ | $z_1^*$ | $z_4^*$ |
|---|---|---|---|---|---|---|---|---|
| 5 | 0.6 | 1 | 10 | 64.612 | 54.367 | 14.515 | 57.562 | 56.734 |
| 5 | 0.6 | 2 | 10 | 62.176 | 51.884 | 24.938 | 55.321 | 53.423 |
| 5 | 0.6 | 3 | 10 | 63.898 | 52.842 | 11.516 | 56.117 | 55.298 |
| 5 | 0.6 | 4 | 10 | 62.424 | 54.128 | 62.484 | 62.424 | 56.976 |
| 5 | 0.6 | 5 | 10 | 59.21 | 50.157 | 6.89 | 53.044 | 52.586 |
| 5 | 0.6 | 6 | 10 | 59.309 | 49.67 | 3.5 | 54.075 | 51.964 |
| 5 | 0.6 | 7 | 10 | 58.978 | 52.289 | 282.296 | 57.627 | 55.564 |
| 5 | 0.6 | 8 | 10 | 63.446 | 49.906 | 11.093 | 58.638 | 51.329 |
| 5 | 0.6 | 9 | 10 | 62.212 | 54.44 | 23.89 | 56.262 | 56.089 |
| 5 | 0.6 | 10 | 10 | 64.034 | 53.975 | 24.656 | 59.714 | 56.260 |
| 6 | 0.6 | 1 | 10 | 71.853 | 64.172 | 55.437 | 68.475 | 65.749 |
| 6 | 0.6 | 2 | 10 | 74.179 | 62.355 | 30.984 | 70.694 | 65.231 |
| 6 | 0.6 | 3 | 10 | 76.749 | 66.336 | 26.5 | 74.337 | 67.434 |
| 6 | 0.6 | 4 | 10 | 67.939 | 59.024 | 416.875 | 64.733 | 62.054 |
| 6 | 0.6 | 5 | 10 | 75.055 | 61.718 | 10.671 | 66.422 | 64.621 |
| 6 | 0.6 | 6 | 10 | 76.149 | 67.963 | 45.64 | 71.505 | 68.630 |
| 6 | 0.6 | 7 | 10 | 71.674 | 56.976 | 266.781 | 61.791 | 61.200 |
| 6 | 0.6 | 8 | 10 | 71.633 | 56.116 | 121.375 | 69.694 | 58.940 |
| 6 | 0.6 | 9 | 10 | 73.785 | 63.309 | 27.468 | 68.002 | 66.065 |
| 6 | 0.6 | 10 | 10 | 71.364 | 59.737 | 38.015 | 63.254 | 61.365 |
| 7 | 0.6 | 1 | 10 | 76.697 | 64.455 | 5.578 | 73.890 | 66.508 |
| 7 | 0.6 | 2 | 10 | 71.394 | 58.47 | 49.203 | 64.113 | 61.398 |
| 7 | 0.6 | 3 | 10 | 75.737 | 64.962 | 69.078 | 69.360 | 68.045 |
| 7 | 0.6 | 4 | 10 | 71.218 | 57.667 | 10.343 | 63.741 | 61.092 |
| 7 | 0.6 | 5 | 10 | 66.51 | 59.569 | 90.609 | 61.572 | 61.572 |
| 7 | 0.6 | 6 | 10 | 58.746 | 49.714 | 71.468 | 53.126 | 52.165 |
| 7 | 0.6 | 7 | 10 | 62.869 | 50.554 | 217.563 | 58.697 | 53.830 |
| 7 | 0.6 | 8 | 10 | 73.592 | 59.96 | 247.891 | 63.701 | 62.150 |
| 7 | 0.6 | 9 | 10 | 73.751 | 61.619 | 1194.39 | 69.139 | 65.064 |
| 7 | 0.6 | 10 | 10 | 63.859 | 54.145 | 510.296 | 58.585 | 56.922 |

Table C9: Results of the runs for $Q = 0.7$, $B = 10$ for networks 1,2,3,4

| Network | $Q$ | Instance | $B$ | $z_2^*$ | $z_3^*$ | $T$ | $z_1^*$ | $z_4^*$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.7 | 1 | 10 | 62.76 | 57.673 | 2.171 | 59.549 | 59.549 |
| 1 | 0.7 | 2 | 10 | 65.224 | 62.822 | 2.703 | 65.224 | 62.822 |
| 1 | 0.7 | 3 | 10 | 59.984 | 56.977 | 7.64 | 58.897 | 57.565 |
| 1 | 0.7 | 4 | 10 | 63.186 | 55.95 | 1 | 58.529 | 58.529 |
| 1 | 0.7 | 5 | 10 | 62.492 | 56.497 | 1.671 | 58.697 | 58.697 |
| 1 | 0.7 | 6 | 10 | 63.02 | 59.696 | 3.046 | 61.026 | 59.822 |
| 1 | 0.7 | 7 | 10 | 59.331 | 56.379 | 2.406 | 57.853 | 57.582 |
| 1 | 0.7 | 8 | 10 | 69.33 | 61.782 | 1.078 | 68.513 | 63.774 |
| 1 | 0.7 | 9 | 10 | 67.153 | 59.619 | 0.671 | 62.522 | 62.522 |
| 1 | 0.7 | 10 | 10 | 70.005 | 67.111 | 5.015 | 70.005 | 69.032 |
| 2 | 0.7 | 1 | 10 | 64.565 | 54.602 | 8.015 | 58.083 | 56.643 |
| 2 | 0.7 | 2 | 10 | 62.23 | 53.215 | 7.359 | 57.471 | 55.541 |
| 2 | 0.7 | 3 | 10 | 62.037 | 56.984 | 8.296 | 59.214 | 58.682 |
| 2 | 0.7 | 4 | 10 | 64.79 | 56.355 | 23.703 | 63.109 | 58.794 |
| 2 | 0.7 | 5 | 10 | 69.428 | 57.316 | 10.562 | 60.512 | 60.036 |
| 2 | 0.7 | 6 | 10 | 54.777 | 46.324 | 23.813 | 51.737 | 48.126 |
| 2 | 0.7 | 7 | 10 | 58.52 | 49.917 | 12.312 | 53.376 | 52.312 |
| 2 | 0.7 | 8 | 10 | 56.04 | 50.014 | 24.843 | 52.457 | 52.457 |
| 2 | 0.7 | 9 | 10 | 64.322 | 56.984 | 4.687 | 58.220 | 58.002 |
| 2 | 0.7 | 10 | 10 | 62.522 | 53.222 | 1.781 | 56.126 | 55.701 |
| 3 | 0.7 | 1 | 10 | 67.922 | 57.906 | 24.671 | 62.373 | 60.868 |
| 3 | 0.7 | 2 | 10 | 66.74 | 58.737 | 104.359 | 63.720 | 61.142 |
| 3 | 0.7 | 3 | 10 | 65.011 | 58.387 | 33.875 | 60.925 | 60.556 |
| 3 | 0.7 | 4 | 10 | 69.676 | 61.083 | 53.046 | 66.995 | 63.877 |
| 3 | 0.7 | 5 | 10 | 74.361 | 64.836 | 13.796 | 67.597 | 67.597 |
| 3 | 0.7 | 6 | 10 | 60.355 | 54.523 | 962.921 | 57.730 | 55.770 |
| 3 | 0.7 | 7 | 10 | 65.524 | 55.462 | 17.437 | 63.466 | 56.390 |
| 3 | 0.7 | 8 | 10 | 71.068 | 62.875 | 28.062 | 65.620 | 63.638 |
| 3 | 0.7 | 9 | 10 | 71.254 | 63.891 | 9.687 | 71.254 | 64.580 |
| 3 | 0.7 | 10 | 10 | 72.79 | 63.49 | 8.14 | 66.458 | 65.327 |
| 4 | 0.7 | 1 | 10 | 55.152 | 50.817 | 25.265 | 55.483 | 50.817 |
| 4 | 0.7 | 2 | 10 | 57.664 | 51.416 | 16.796 | 56.186 | 52.439 |
| 4 | 0.7 | 3 | 10 | 56.595 | 53.451 | 9.14 | 54.396 | 53.892 |
| 4 | 0.7 | 4 | 10 | 56.871 | 50.53 | 9.5 | 54.308 | 52.852 |
| 4 | 0.7 | 5 | 10 | 57.259 | 51.836 | 10.218 | 56.418 | 52.734 |
| 4 | 0.7 | 6 | 10 | 46.061 | 41.244 | 40.921 | 43.754 | 42.876 |
| 4 | 0.7 | 7 | 10 | 48.958 | 43.625 | 16.265 | 46.461 | 45.556 |
| 4 | 0.7 | 8 | 10 | 55.799 | 45.81 | 4.656 | 54.172 | 47.832 |
| 4 | 0.7 | 9 | 10 | 53.634 | 49.369 | 42.078 | 53.354 | 53.040 |
| 4 | 0.7 | 10 | 10 | 52.123 | 45.197 | 48.516 | 47.652 | 46.913 |

Table C10: Results of the runs for $Q = 0.7$, $B = 10$ for networks 5,6,7

| Network | $Q$ | Instance | $B$ | $z_2^*$ | $z_3^*$ | $T$ | $z_1^*$ | $z_4^*$ |
|---|---|---|---|---|---|---|---|---|
| 5 | 0.7 | 1 | 10 | 62.114 | 54.245 | 9.203 | 57.562 | 56.734 |
| 5 | 0.7 | 2 | 10 | 58.683 | 51.133 | 54.375 | 53.843 | 53.423 |
| 5 | 0.7 | 3 | 10 | 60.259 | 52.817 | 30.813 | 56.117 | 55.298 |
| 5 | 0.7 | 4 | 10 | 62.424 | 55.985 | 16.328 | 57.991 | 56.976 |
| 5 | 0.7 | 5 | 10 | 56.675 | 50.168 | 6.625 | 53.044 | 52.586 |
| 5 | 0.7 | 6 | 10 | 56.741 | 49.628 | 10.406 | 52.501 | 51.964 |
| 5 | 0.7 | 7 | 10 | 58.122 | 53.252 | 114.89 | 57.627 | 55.564 |
| 5 | 0.7 | 8 | 10 | 59.811 | 49.605 | 18.437 | 58.638 | 51.329 |
| 5 | 0.7 | 9 | 10 | 60.53 | 54.44 | 21.515 | 56.262 | 56.089 |
| 5 | 0.7 | 10 | 10 | 61.904 | 53.975 | 9.546 | 59.714 | 56.260 |
| 6 | 0.7 | 1 | 10 | 68.475 | 64.117 | 76.593 | 68.475 | 65.749 |
| 6 | 0.7 | 2 | 10 | 69.602 | 63.037 | 139.297 | 65.772 | 65.231 |
| 6 | 0.7 | 3 | 10 | 74.921 | 65.575 | 26.031 | 72.727 | 67.434 |
| 6 | 0.7 | 4 | 10 | 65.447 | 59.318 | 915.515 | 64.733 | 62.054 |
| 6 | 0.7 | 5 | 10 | 70.911 | 62.327 | 14.656 | 66.422 | 64.621 |
| 6 | 0.7 | 6 | 10 | 73.384 | 67.963 | 54.75 | 71.505 | 68.630 |
| 6 | 0.7 | 7 | 10 | 68.417 | 57.652 | 104.406 | 61.791 | 61.200 |
| 6 | 0.7 | 8 | 10 | 69.123 | 56.251 | 66.203 | 59.925 | 58.940 |
| 6 | 0.7 | 9 | 10 | 70.243 | 63.309 | 34.343 | 68.002 | 66.065 |
| 6 | 0.7 | 10 | 10 | 66.626 | 59.4 | 152.234 | 63.254 | 61.365 |
| 7 | 0.7 | 1 | 10 | 73.89 | 62.229 | 6.328 | 70.816 | 66.508 |
| 7 | 0.7 | 2 | 10 | 67.824 | 59.882 | 43.984 | 64.113 | 61.398 |
| 7 | 0.7 | 3 | 10 | 73.196 | 64.843 | 104.734 | 69.360 | 68.045 |
| 7 | 0.7 | 4 | 10 | 66.63 | 57.667 | 33.921 | 61.092 | 61.092 |
| 7 | 0.7 | 5 | 10 | 63.936 | 59.927 | 59.328 | 61.572 | 61.572 |
| 7 | 0.7 | 6 | 10 | 55.415 | 50.094 | 123.468 | 53.126 | 52.165 |
| 7 | 0.7 | 7 | 10 | 58.657 | 51.369 | 293.921 | 54.522 | 53.830 |
| 7 | 0.7 | 8 | 10 | 70.695 | 61.754 | 39.562 | 63.701 | 62.150 |
| 7 | 0.7 | 9 | 10 | 71.301 | 62.785 | 284.015 | 69.139 | 65.064 |
| 7 | 0.7 | 10 | 10 | 63.051 | 55.567 | 149.297 | 58.585 | 56.922 |

Table C11: Results of the runs for $Q = 0.8$, $B = 10$ for networks 1,2,3,4

| Network | $Q$ | Instance | $B$ | $z_2^*$ | $z_3^*$ | $T$ | $z_1^*$ | $z_4^*$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.8 | 1 | 10 | 62.76 | 57.673 | 0.61 | 59.549 | 59.549 |
| 1 | 0.8 | 2 | 10 | 64.362 | 62.52 | 1.546 | 63.559 | 62.822 |
| 1 | 0.8 | 3 | 10 | 59.722 | 56.977 | 3.609 | 58.897 | 57.565 |
| 1 | 0.8 | 4 | 10 | 61.026 | 57.056 | 1.89 | 58.529 | 58.529 |
| 1 | 0.8 | 5 | 10 | 61.435 | 57.468 | 0.625 | 58.697 | 58.697 |
| 1 | 0.8 | 6 | 10 | 61.608 | 59.349 | 3.047 | 61.026 | 59.822 |
| 1 | 0.8 | 7 | 10 | 58.438 | 56.379 | 1.718 | 57.853 | 57.582 |
| 1 | 0.8 | 8 | 10 | 66.506 | 63.774 | 2.375 | 66.303 | 63.774 |
| 1 | 0.8 | 9 | 10 | 64.856 | 59.619 | 1.89 | 62.522 | 62.522 |
| 1 | 0.8 | 10 | 10 | 70.005 | 67.111 | 1.234 | 70.005 | 69.032 |
| 2 | 0.8 | 1 | 10 | 61.73 | 55.308 | 5.89 | 58.083 | 56.643 |
| 2 | 0.8 | 2 | 10 | 60.39 | 53.154 | 3.468 | 56.584 | 55.541 |
| 2 | 0.8 | 3 | 10 | 62.037 | 56.984 | 2.625 | 59.214 | 58.682 |
| 2 | 0.8 | 4 | 10 | 62.045 | 58.718 | 15.094 | 59.538 | 58.794 |
| 2 | 0.8 | 5 | 10 | 66.705 | 58.016 | 1.125 | 60.512 | 60.036 |
| 2 | 0.8 | 6 | 10 | 50.086 | 47.074 | 318.375 | 50.086 | 48.126 |
| 2 | 0.8 | 7 | 10 | 56.369 | 50.415 | 8.468 | 53.376 | 52.312 |
| 2 | 0.8 | 8 | 10 | 55.768 | 51.859 | 8.25 | 52.457 | 52.457 |
| 2 | 0.8 | 9 | 10 | 60.293 | 55.806 | 25.937 | 58.220 | 58.002 |
| 2 | 0.8 | 10 | 10 | 59.068 | 53.469 | 2.828 | 56.126 | 55.701 |
| 3 | 0.8 | 1 | 10 | 63.551 | 58.756 | 163.125 | 60.909 | 60.868 |
| 3 | 0.8 | 2 | 10 | 65.87 | 58.737 | 13.906 | 61.142 | 61.142 |
| 3 | 0.8 | 3 | 10 | 64.503 | 58.387 | 8 | 60.925 | 60.556 |
| 3 | 0.8 | 4 | 10 | 65.491 | 61.2 | 214.015 | 64.151 | 63.877 |
| 3 | 0.8 | 5 | 10 | 71.101 | 65.839 | 11.016 | 67.597 | 67.597 |
| 3 | 0.8 | 6 | 10 | 58.205 | 54.523 | 314.796 | 57.730 | 55.770 |
| 3 | 0.8 | 7 | 10 | 60.225 | 53.676 | 59.296 | 56.945 | 56.390 |
| 3 | 0.8 | 8 | 10 | 67.635 | 61.117 | 37.968 | 63.638 | 63.638 |
| 3 | 0.8 | 9 | 10 | 68.13 | 62.875 | 16.938 | 65.514 | 64.580 |
| 3 | 0.8 | 10 | 10 | 69.224 | 63.625 | 22.953 | 66.458 | 65.327 |
| 4 | 0.8 | 1 | 10 | 53.199 | 48.568 | 21.109 | 50.817 | 50.817 |
| 4 | 0.8 | 2 | 10 | 54.66 | 49.839 | 23.421 | 52.942 | 52.439 |
| 4 | 0.8 | 3 | 10 | 56.343 | 53.451 | 2.796 | 53.901 | 53.892 |
| 4 | 0.8 | 4 | 10 | 54.308 | 51.785 | 11.39 | 54.308 | 52.852 |
| 4 | 0.8 | 5 | 10 | 56.418 | 51.535 | 6.25 | 54.190 | 52.734 |
| 4 | 0.8 | 6 | 10 | 45.415 | 42.519 | 17.843 | 43.754 | 42.876 |
| 4 | 0.8 | 7 | 10 | 47.115 | 45.213 | 15.109 | 45.870 | 45.556 |
| 4 | 0.8 | 8 | 10 | 52.323 | 46.087 | 4.812 | 49.627 | 47.832 |
| 4 | 0.8 | 9 | 10 | 53.264 | 48.828 | 30.078 | 53.354 | 53.040 |
| 4 | 0.8 | 10 | 10 | 49.917 | 46.445 | 44.656 | 47.652 | 46.913 |

Table C12: Results of the runs for $Q = 0.8$, $B = 10$ for networks 5,6,7

| Network | $Q$ | Instance | $B$ | $z_2^*$ | $z_3^*$ | $T$ | $z_1^*$ | $z_4^*$ |
|---|---|---|---|---|---|---|---|---|
| 5 | 0.8 | 1 | 10 | 58.953 | 53.898 | 15.671 | 57.562 | 56.734 |
| 5 | 0.8 | 2 | 10 | 54.781 | 51.874 | 90.375 | 53.843 | 53.423 |
| 5 | 0.8 | 3 | 10 | 58.32 | 52.842 | 16.546 | 56.117 | 55.298 |
| 5 | 0.8 | 4 | 10 | 58.799 | 54.704 | 56.812 | 56.976 | 56.976 |
| 5 | 0.8 | 5 | 10 | 56.675 | 50.168 | 3.375 | 53.044 | 52.586 |
| 5 | 0.8 | 6 | 10 | 54.335 | 49.864 | 14.25 | 52.501 | 51.964 |
| 5 | 0.8 | 7 | 10 | 55.766 | 52.856 | 303.093 | 56.178 | 55.564 |
| 5 | 0.8 | 8 | 10 | 55.195 | 49.906 | 48.265 | 54.093 | 51.329 |
| 5 | 0.8 | 9 | 10 | 59.237 | 54.44 | 27.187 | 56.262 | 56.089 |
| 5 | 0.8 | 10 | 10 | 59.421 | 54.881 | 13.531 | 57.533 | 56.260 |
| 6 | 0.8 | 1 | 10 | 67.826 | 65.749 | 31.312 | 67.062 | 65.749 |
| 6 | 0.8 | 2 | 10 | 67.832 | 62.104 | 79.937 | 65.772 | 65.231 |
| 6 | 0.8 | 3 | 10 | 70.988 | 66.877 | 30.469 | 72.727 | 67.434 |
| 6 | 0.8 | 4 | 10 | 63.61 | 59.369 | 1183.125 | 62.054 | 62.054 |
| 6 | 0.8 | 5 | 10 | 67.996 | 63.468 | 13.203 | 66.422 | 64.621 |
| 6 | 0.8 | 6 | 10 | 71.869 | 67.575 | 30.765 | 71.505 | 68.630 |
| 6 | 0.8 | 7 | 10 | 64.17 | 58.845 | 251.406 | 61.200 | 61.200 |
| 6 | 0.8 | 8 | 10 | 64.571 | 56.654 | 70.484 | 59.925 | 58.940 |
| 6 | 0.8 | 9 | 10 | 67.997 | 62.852 | 20.921 | 67.195 | 66.065 |
| 6 | 0.8 | 10 | 10 | 64.954 | 59.737 | 121.985 | 63.254 | 61.365 |
| 7 | 0.8 | 1 | 10 | 67.803 | 62.801 | 58.75 | 66.508 | 66.508 |
| 7 | 0.8 | 2 | 10 | 64.889 | 58.663 | 44.188 | 64.113 | 61.398 |
| 7 | 0.8 | 3 | 10 | 71.35 | 66.333 | 55.531 | 69.360 | 68.045 |
| 7 | 0.8 | 4 | 10 | 62.527 | 57.638 | 193.531 | 61.092 | 61.092 |
| 7 | 0.8 | 5 | 10 | 63.431 | 60.393 | 27.5 | 61.572 | 61.572 |
| 7 | 0.8 | 6 | 10 | 53.789 | 49.965 | 87.156 | 53.126 | 52.165 |
| 7 | 0.8 | 7 | 10 | 57.118 | 51.705 | 83.234 | 54.522 | 53.830 |
| 7 | 0.8 | 8 | 10 | 66.598 | 61.724 | 53.015 | 63.701 | 62.150 |
| 7 | 0.8 | 9 | 10 | 68.415 | 62.785 | 147.171 | 65.441 | 65.064 |
| 7 | 0.8 | 10 | 10 | 60.084 | 55.645 | 152.968 | 57.556 | 56.922 |

Table C13: Results of the runs for $Q = 0.6$, $B = 15$ for networks 1,2,3,4

| Network | $Q$ | Instance | $B$ | $z_2^*$ | $z_3^*$ | $T$ | $z_1^*$ | $z_4^*$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.6 | 1 | 15 | 70.994 | 63.363 | 10.921 | 68.503 | 66.376 |
| 1 | 0.6 | 2 | 15 | 75.133 | 67.4 | 8.296 | 68.981 | 68.675 |
| 1 | 0.6 | 3 | 15 | 72.097 | 63.831 | 9.812 | 67.605 | 66.521 |
| 1 | 0.6 | 4 | 15 | 71.662 | 61.809 | 2.515 | 65.043 | 65.043 |
| 1 | 0.6 | 5 | 15 | 74.565 | 63.537 | 0.859 | 71.154 | 64.585 |
| 1 | 0.6 | 6 | 15 | 73.718 | 62.801 | 0.687 | 65.095 | 64.048 |
| 1 | 0.6 | 7 | 15 | 69.563 | 63.057 | 1.312 | 66.770 | 64.048 |
| 1 | 0.6 | 8 | 15 | 81.459 | 67.832 | 0.953 | 76.907 | 70.034 |
| 1 | 0.6 | 9 | 15 | 74.244 | 67.694 | 4.906 | 70.157 | 69.297 |
| 1 | 0.6 | 10 | 15 | 78.609 | 71.218 | 12.328 | 76.447 | 74.733 |
| 2 | 0.6 | 1 | 15 | 71.096 | 61.308 | 160.437 | 65.313 | 64.019 |
| 2 | 0.6 | 2 | 15 | 69.391 | 58.177 | 92.375 | 66.474 | 61.183 |
| 2 | 0.6 | 3 | 15 | 71.526 | 62.624 | 30 | 67.768 | 65.697 |
| 2 | 0.6 | 4 | 15 | 75.132 | 62.125 | 49.562 | 67.096 | 64.898 |
| 2 | 0.6 | 5 | 15 | 75.408 | 62.277 | 84.609 | 67.304 | 66.130 |
| 2 | 0.6 | 6 | 15 | 62.95 | 51.198 | 40.437 | 57.501 | 53.637 |
| 2 | 0.6 | 7 | 15 | 66.932 | 55.484 | 108.218 | 60.585 | 57.894 |
| 2 | 0.6 | 8 | 15 | 67.456 | 55.414 | 146.734 | 59.608 | 57.876 |
| 2 | 0.6 | 9 | 15 | 71.81 | 60.103 | 35.828 | 65.906 | 61.900 |
| 2 | 0.6 | 10 | 15 | 69.466 | 57.53 | 22.625 | 63.078 | 60.184 |
| 3 | 0.6 | 1 | 15 | 78.476 | 64.412 | 435.546 | 69.489 | 67.045 |
| 3 | 0.6 | 2 | 15 | 75.63 | 64.779 | 431.781 | 72.028 | 67.370 |
| 3 | 0.6 | 3 | 15 | 80.071 | 65.088 | 109.453 | 69.817 | 67.910 |
| 3 | 0.6 | 4 | 15 | 79.723 | 67.805 | 673.437 | 74.612 | 70.689 |
| 3 | 0.6 | 5 | 15 | 83.373 | 69.441 | 65.109 | 75.635 | 72.883 |
| 3 | 0.6 | 6 | 15 | 68.757 | 57.461 | 3600.031 | 61.656 | 60.367 |
| 3 | 0.6 | 7 | 15 | 73.374 | 60.406 | 251.062 | 67.015 | 62.376 |
| 3 | 0.6 | 8 | 15 | 80.689 | 66.932 | 322.985 | 71.846 | 71.846 |
| 3 | 0.6 | 9 | 15 | 79.461 | 67.725 | 100.985 | 75.506 | 70.113 |
| 3 | 0.6 | 10 | 15 | 80.289 | 68.438 | 166.781 | 71.408 | 70.721 |
| 4 | 0.6 | 1 | 15 | 65.621 | 54.228 | 165.719 | 58.306 | 55.532 |
| 4 | 0.6 | 2 | 15 | 70.1 | 54.897 | 73.719 | 58.756 | 57.695 |
| 4 | 0.6 | 3 | 15 | 68.199 | 56.315 | 49.391 | 69.122 | 60.841 |
| 4 | 0.6 | 4 | 15 | 64.121 | 55.807 | 292.235 | 59.590 | 58.072 |
| 4 | 0.6 | 5 | 15 | 65.831 | 53.275 | 50.141 | 59.076 | 56.454 |
| 4 | 0.6 | 6 | 15 | 56.247 | 44.786 | 29.813 | 48.031 | 47.329 |
| 4 | 0.6 | 7 | 15 | 60.032 | 49.749 | 31.422 | 55.162 | 51.721 |
| 4 | 0.6 | 8 | 15 | 67.122 | 50.533 | 14.125 | 53.416 | 53.161 |
| 4 | 0.6 | 9 | 15 | 63.008 | 53.732 | 252.969 | 57.350 | 56.396 |
| 4 | 0.6 | 10 | 15 | 57.728 | 49.147 | 445.766 | 55.598 | 51.618 |

Table C14: Results of the runs for $Q = 0.6$, $B = 15$ for networks 5,6,7

| Network | $Q$ | Instance | $B$ | $z_2^*$ | $z_3^*$ | $T$ | $z_1^*$ | $z_4^*$ |
|---|---|---|---|---|---|---|---|---|
| 5 | 0.6 | 1 | 15 | 71.013 | 59.246 | 353.625 | 65.743 | 60.815 |
| 5 | 0.6 | 2 | 15 | 70.832 | 56.3 | 145.61 | 59.817 | 58.854 |
| 5 | 0.6 | 3 | 15 | 71.133 | 57.433 | 504.031 | 61.241 | 60.422 |
| 5 | 0.6 | 4 | 15 | 69.405 | 58.933 | 1156.766 | 63.424 | 62.236 |
| 5 | 0.6 | 5 | 15 | 66.84 | 54.504 | 27.344 | 60.452 | 56.690 |
| 5 | 0.6 | 6 | 15 | 64.67 | 52.7 | 116.296 | 56.903 | 55.032 |
| 5 | 0.6 | 7 | 15 | 65.943 | 57.586 | 1349.235 | 66.342 | 61.662 |
| 5 | 0.6 | 8 | 15 | 67.721 | 53.222 | 607.328 | 63.446 | 55.947 |
| 5 | 0.6 | 9 | 15 | 71.956 | 58.933 | 35.61 | 63.897 | 61.198 |
| 5 | 0.6 | 10 | 15 | 68.819 | 58.463 | 368.828 | 66.126 | 61.378 |
| 6 | 0.6 | 1 | 15 | 80.66 | 68.424 | 563.672 | 74.020 | 71.294 |
| 6 | 0.6 | 2 | 15 | 80.561 | 67.872 | 439.406 | 72.605 | 70.882 |
| 6 | 0.6 | 3 | 15 | 85.476 | 71.778 | 347.016 | 77.805 | 74.749 |
| 6 | 0.6 | 4 | 15 | 73.862 | 64.792 | 3600.031 | 69.440 | 68.279 |
| 6 | 0.6 | 5 | 15 | 80.802 | 67.942 | 275.328 | 73.933 | 70.213 |
| 6 | 0.6 | 6 | 15 | 82.402 | 70.098 | 61.828 | 74.898 | 74.232 |
| 6 | 0.6 | 7 | 15 | 76.635 | 65.284 | 3600.031 | 69.884 | 67.659 |
| 6 | 0.6 | 8 | 15 | 78.206 | 60.948 | 3600.031 | 65.244 | 64.033 |
| 6 | 0.6 | 9 | 15 | 81.124 | 66.793 | 277.609 | 71.728 | 70.248 |
| 6 | 0.6 | 10 | 15 | 78.402 | 64.605 | 331.766 | 70.292 | 67.341 |
| 7 | 0.6 | 1 | 15 | 83.187 | 67.329 | 121.812 | 77.697 | 70.726 |
| 7 | 0.6 | 2 | 15 | 76.547 | 64.054 | 1127.782 | 68.093 | 66.768 |
| 7 | 0.6 | 3 | 15 | 83.071 | 70.52 | 1755.203 | 74.219 | 74.123 |
| 7 | 0.6 | 4 | 15 | 74.726 | 64.513 | 762.75 | 70.608 | 67.500 |
| 7 | 0.6 | 5 | 15 | 75.076 | 64.939 | 972.172 | 68.292 | 67.468 |
| 7 | 0.6 | 6 | 15 | 66.594 | 55.874 | 33.937 | 60.132 | 58.294 |
| 7 | 0.6 | 7 | 15 | 68.17 | 55.438 | 3600.031 | 61.242 | 57.853 |
| 7 | 0.6 | 8 | 15 | 79.57 | 65.777 | 639.141 | 70.695 | 68.508 |
| 7 | 0.6 | 9 | 15 | 82.669 | 69.084 | 2475.141 | 73.839 | 72.426 |
| 7 | 0.6 | 10 | 15 | 71.388 | 58.496 | 3600.031 | 61.517 | 61.517 |

Table C15: Results of the runs for $Q = 0.7$, $B = 15$ for networks 1,2,3,4

| Network | $Q$ | Instance | $B$ | $z_2^*$ | $z_3^*$ | $T$ | $z_1^*$ | $z_4^*$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.7 | 1 | 15 | 68.022 | 63.123 | 20.313 | 66.454 | 66.376 |
| 1 | 0.7 | 2 | 15 | 71.89 | 67.766 | 19.5 | 68.981 | 68.675 |
| 1 | 0.7 | 3 | 15 | 69.409 | 63.83 | 12.438 | 67.605 | 66.521 |
| 1 | 0.7 | 4 | 15 | 69.7 | 62.464 | 3.61 | 65.043 | 65.043 |
| 1 | 0.7 | 5 | 15 | 71.544 | 62.653 | 0.547 | 71.154 | 64.585 |
| 1 | 0.7 | 6 | 15 | 71.15 | 62.821 | 1.813 | 64.048 | 64.048 |
| 1 | 0.7 | 7 | 15 | 66.77 | 64.048 | 2.985 | 66.770 | 64.048 |
| 1 | 0.7 | 8 | 15 | 80.152 | 67.832 | 0.641 | 76.907 | 70.034 |
| 1 | 0.7 | 9 | 15 | 72.396 | 66.915 | 10.797 | 70.157 | 69.297 |
| 1 | 0.7 | 10 | 15 | 77.28 | 71.629 | 8.296 | 76.447 | 74.733 |
| 2 | 0.7 | 1 | 15 | 71.096 | 61.308 | 21.641 | 65.313 | 64.019 |
| 2 | 0.7 | 2 | 15 | 67.134 | 58.72 | 97.718 | 62.992 | 61.183 |
| 2 | 0.7 | 3 | 15 | 71.526 | 62.624 | 15.75 | 67.768 | 65.697 |
| 2 | 0.7 | 4 | 15 | 72.227 | 62.125 | 92.672 | 67.096 | 64.898 |
| 2 | 0.7 | 5 | 15 | 74.272 | 62.277 | 45.062 | 67.304 | 66.130 |
| 2 | 0.7 | 6 | 15 | 59.673 | 51.537 | 607.969 | 57.501 | 53.637 |
| 2 | 0.7 | 7 | 15 | 65.189 | 55.263 | 47.188 | 60.585 | 57.894 |
| 2 | 0.7 | 8 | 15 | 62.992 | 56.375 | 325.39 | 57.876 | 57.876 |
| 2 | 0.7 | 9 | 15 | 68.58 | 59.682 | 44.078 | 65.906 | 61.900 |
| 2 | 0.7 | 10 | 15 | 67.322 | 58.076 | 4.141 | 60.926 | 60.184 |
| 3 | 0.7 | 1 | 15 | 73.927 | 63.756 | 1032.469 | 69.489 | 67.045 |
| 3 | 0.7 | 2 | 15 | 73.32 | 64.47 | 290.719 | 68.546 | 67.370 |
| 3 | 0.7 | 3 | 15 | 73.534 | 64.598 | 673.531 | 69.817 | 67.910 |
| 3 | 0.7 | 4 | 15 | 76.923 | 67.758 | 729.516 | 74.612 | 70.689 |
| 3 | 0.7 | 5 | 15 | 80.778 | 70.1 | 75.063 | 75.635 | 72.883 |
| 3 | 0.7 | 6 | 15 | 66.722 | 57.461 | 3600.032 | 77.123 | 73.152 |
| 3 | 0.7 | 7 | 15 | 70.675 | 59.61 | 382.031 | 64.025 | 62.376 |
| 3 | 0.7 | 8 | 15 | 78.638 | 68.613 | 162.11 | 71.846 | 71.846 |
| 3 | 0.7 | 9 | 15 | 77.678 | 68.631 | 31.86 | 70.113 | 70.113 |
| 3 | 0.7 | 10 | 15 | 79.733 | 67.319 | 52.312 | 71.408 | 70.721 |
| 4 | 0.7 | 1 | 15 | 61.304 | 53.594 | 185.36 | 55.532 | 55.532 |
| 4 | 0.7 | 2 | 15 | 64.802 | 55.794 | 114.422 | 58.756 | 57.695 |
| 4 | 0.7 | 3 | 15 | 63.636 | 56.523 | 125.469 | 61.607 | 60.841 |
| 4 | 0.7 | 4 | 15 | 61.74 | 55.182 | 281.5 | 58.072 | 58.072 |
| 4 | 0.7 | 5 | 15 | 63.647 | 53.264 | 87.344 | 59.076 | 56.454 |
| 4 | 0.7 | 6 | 15 | 53.679 | 44.786 | 18.125 | 48.031 | 47.329 |
| 4 | 0.7 | 7 | 15 | 58.565 | 49.749 | 23.578 | 52.161 | 51.721 |
| 4 | 0.7 | 8 | 15 | 61.48 | 50.533 | 44.813 | 53.416 | 53.161 |
| 4 | 0.7 | 9 | 15 | 60.415 | 55.303 | 496.391 | 56.575 | 56.396 |
| 4 | 0.7 | 10 | 15 | 56.381 | 49.714 | 358.063 | 52.538 | 51.618 |

Table C16: Results of the runs for $Q = 0.7$, $B = 15$ for networks 5,6,7

| Network | $Q$ | Instance | $B$ | $z_2^*$ | $z_3^*$ | $T$ | $z_1^*$ | $z_4^*$ |
|---|---|---|---|---|---|---|---|---|
| 5 | 0.7 | 1 | 15 | 67.22 | 59.409 | 250.375 | 65.743 | 60.815 |
| 5 | 0.7 | 2 | 15 | 65.395 | 56.402 | 593.25 | 58.902 | 58.854 |
| 5 | 0.7 | 3 | 15 | 67.366 | 57.677 | 343.125 | 61.241 | 60.422 |
| 5 | 0.7 | 4 | 15 | 68.86 | 60.335 | 169.75 | 63.424 | 62.236 |
| 5 | 0.7 | 5 | 15 | 64.083 | 54.454 | 68.578 | 57.318 | 56.690 |
| 5 | 0.7 | 6 | 15 | 61.895 | 52.576 | 40.672 | 55.569 | 55.032 |
| 5 | 0.7 | 7 | 15 | 65.398 | 58.42 | 278.781 | 66.342 | 61.662 |
| 5 | 0.7 | 8 | 15 | 64.975 | 53.925 | 217.922 | 63.446 | 55.947 |
| 5 | 0.7 | 9 | 15 | 66.412 | 60.429 | 633.219 | 63.897 | 61.198 |
| 5 | 0.7 | 10 | 15 | 65.897 | 58.486 | 1073.969 | 62.348 | 61.378 |
| 6 | 0.7 | 1 | 15 | 78.053 | 68.424 | 186.265 | 74.020 | 71.294 |
| 6 | 0.7 | 2 | 15 | 76.033 | 67.858 | 2207.172 | 71.002 | 70.882 |
| 6 | 0.7 | 3 | 15 | 84.34 | 72.336 | 113.656 | 77.805 | 74.749 |
| 6 | 0.7 | 4 | 15 | 73.348 | 65.871 | 636.61 | 69.440 | 68.279 |
| 6 | 0.7 | 5 | 15 | 76.92 | 67.717 | 556.61 | 73.933 | 70.213 |
| 6 | 0.7 | 6 | 15 | 80.166 | 70.098 | 158.61 | 74.898 | 74.232 |
| 6 | 0.7 | 7 | 15 | 73.992 | 65.454 | 3600.062 | 69.884 | 67.659 |
| 6 | 0.7 | 8 | 15 | 74.853 | 62.306 | 2531.906 | 65.244 | 64.033 |
| 6 | 0.7 | 9 | 15 | 75.478 | 67.859 | 1296.687 | 71.728 | 70.248 |
| 6 | 0.7 | 10 | 15 | 73.361 | 65.707 | 3600.031 | 70.292 | 67.341 |
| 7 | 0.7 | 1 | 15 | 78.208 | 67.336 | 1707.656 | 74.623 | 70.726 |
| 7 | 0.7 | 2 | 15 | 73.047 | 63.468 | 1187.453 | 68.093 | 66.768 |
| 7 | 0.7 | 3 | 15 | 79.876 | 70.419 | 1630.547 | 74.219 | 74.123 |
| 7 | 0.7 | 4 | 15 | 74.726 | 65.411 | 105.235 | 68.029 | 67.500 |
| 7 | 0.7 | 5 | 15 | 70.403 | 64.529 | 1553.86 | 68.292 | 67.468 |
| 7 | 0.7 | 6 | 15 | 63.051 | 55.38 | 104.688 | 60.132 | 58.294 |
| 7 | 0.7 | 7 | 15 | 65.411 | 55.131 | 1455.444 | 58.545 | 57.853 |
| 7 | 0.7 | 8 | 15 | 76.379 | 66.817 | 298.582 | 70.695 | 68.508 |
| 7 | 0.7 | 9 | 15 | 78.465 | 69.313 | 1037.275 | 71.985 | 68.235 |
| 7 | 0.7 | 10 | 15 | 69.231 | 59.652 | 906.39 | 61.517 | 61.517 |

Table C17: Results of the runs for $Q = 0.8$, $B = 15$ for networks 1,2,3,4

| Network | $Q$ | Instance | $B$ | $z_2^*$ | $z_3^*$ | $T$ | $z_1^*$ | $z_4^*$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.8 | 1 | 15 | 68.022 | 63.826 | 4.75 | 66.454 | 66.376 |
| 1 | 0.8 | 2 | 15 | 69.708 | 68.532 | 39.735 | 68.981 | 68.675 |
| 1 | 0.8 | 3 | 15 | 67.605 | 66.283 | 12.188 | 67.605 | 66.521 |
| 1 | 0.8 | 4 | 15 | 66.918 | 62.191 | 9.219 | 65.043 | 65.043 |
| 1 | 0.8 | 5 | 15 | 67.067 | 64.585 | 0.75 | 67.067 | 64.585 |
| 1 | 0.8 | 6 | 15 | 68.818 | 62.821 | 2.281 | 64.048 | 64.048 |
| 1 | 0.8 | 7 | 15 | 66.543 | 63.905 | 1.531 | 65.520 | 64.048 |
| 1 | 0.8 | 8 | 15 | 77.11 | 67.832 | 1.672 | 71.251 | 70.034 |
| 1 | 0.8 | 9 | 15 | 71.126 | 66.915 | 8.25 | 70.157 | 69.297 |
| 1 | 0.8 | 10 | 15 | 77.175 | 72.792 | 2.61 | 76.447 | 74.733 |
| 2 | 0.8 | 1 | 15 | 67.652 | 61.308 | 122.235 | 65.313 | 64.019 |
| 2 | 0.8 | 2 | 15 | 66.474 | 58.72 | 8.375 | 62.992 | 61.183 |
| 2 | 0.8 | 3 | 15 | 71.071 | 62.624 | 7.406 | 67.768 | 65.697 |
| 2 | 0.8 | 4 | 15 | 68.798 | 63.729 | 441.781 | 67.096 | 64.898 |
| 2 | 0.8 | 5 | 15 | 72.125 | 62.277 | 9.578 | 66.146 | 66.130 |
| 2 | 0.8 | 6 | 15 | 56.227 | 51.537 | 1037.5 | 55.850 | 53.637 |
| 2 | 0.8 | 7 | 15 | 62.073 | 56.07 | 71.031 | 60.585 | 57.894 |
| 2 | 0.8 | 8 | 15 | 61.738 | 56.323 | 91.828 | 57.876 | 57.876 |
| 2 | 0.8 | 9 | 15 | 66.152 | 60.611 | 53.391 | 65.906 | 61.900 |
| 2 | 0.8 | 10 | 15 | 64.666 | 57.457 | 5.36 | 60.926 | 60.184 |
| 3 | 0.8 | 1 | 15 | 71.217 | 64.412 | 944.344 | 68.025 | 67.045 |
| 3 | 0.8 | 2 | 15 | 71.386 | 64.47 | 166.985 | 68.546 | 67.370 |
| 3 | 0.8 | 3 | 15 | 70.584 | 65.088 | 1158.125 | 69.817 | 67.910 |
| 3 | 0.8 | 4 | 15 | 75.218 | 67.164 | 267.485 | 72.119 | 70.689 |
| 3 | 0.8 | 5 | 15 | 77.782 | 70.1 | 65.031 | 73.946 | 72.883 |
| 3 | 0.8 | 6 | 15 | 63.051 | 57.417 | 3600.063 | 64.025 | 62.376 |
| 3 | 0.8 | 7 | 15 | 66.524 | 60.197 | 1189.406 | 68.237 | 65.859 |
| 3 | 0.8 | 8 | 15 | 75.479 | 68.394 | 124.078 | 71.846 | 71.846 |
| 3 | 0.8 | 9 | 15 | 73.676 | 68.976 | 123.703 | 70.113 | 70.113 |
| 3 | 0.8 | 10 | 15 | 76.279 | 67.667 | 52.906 | 71.408 | 70.721 |
| 4 | 0.8 | 1 | 15 | 58.658 | 53.661 | 246.266 | 55.532 | 55.532 |
| 4 | 0.8 | 2 | 15 | 62.357 | 55.935 | 24.641 | 58.756 | 57.695 |
| 4 | 0.8 | 3 | 15 | 62.823 | 57.735 | 19.61 | 61.112 | 60.841 |
| 4 | 0.8 | 4 | 15 | 61.07 | 56.832 | 65.485 | 58.072 | 58.072 |
| 4 | 0.8 | 5 | 15 | 61.34 | 54.445 | 173.922 | 59.076 | 56.454 |
| 4 | 0.8 | 6 | 15 | 50.06 | 46.866 | 196.656 | 48.031 | 47.329 |
| 4 | 0.8 | 7 | 15 | 54.518 | 51.721 | 85.485 | 52.161 | 51.721 |
| 4 | 0.8 | 8 | 15 | 58.038 | 51.083 | 44.406 | 53.416 | 53.161 |
| 4 | 0.8 | 9 | 15 | 58.415 | 54.134 | 420.375 | 56.575 | 56.396 |
| 4 | 0.8 | 10 | 15 | 55.598 | 51.187 | 65.86 | 52.538 | 51.618 |

Table C18: Results of the runs for $Q = 0.8$, $B = 15$ for networks 5,6,7

| Network | $Q$ | Instance | $B$ | $z_2^*$ | $z_3^*$ | $T$ | $z_1^*$ | $z_4^*$ |
|---|---|---|---|---|---|---|---|---|
| 5 | 0.8 | 1 | 15 | 65.343 | 59.83 | 147.594 | 62.328 | 60.815 |
| 5 | 0.8 | 2 | 15 | 61.722 | 56.714 | 423.156 | 58.902 | 58.854 |
| 5 | 0.8 | 3 | 15 | 64.079 | 58.081 | 372.625 | 61.241 | 60.422 |
| 5 | 0.8 | 4 | 15 | 65.449 | 59.643 | 440.735 | 62.315 | 62.236 |
| 5 | 0.8 | 5 | 15 | 61.147 | 54.454 | 140.594 | 57.318 | 56.690 |
| 5 | 0.8 | 6 | 15 | 59.327 | 52.576 | 96.594 | 55.569 | 55.032 |
| 5 | 0.8 | 7 | 15 | 62.349 | 57.586 | 1428.25 | 62.330 | 61.662 |
| 5 | 0.8 | 8 | 15 | 61.995 | 54.394 | 112.594 | 58.901 | 55.947 |
| 5 | 0.8 | 9 | 15 | 63.897 | 59.836 | 565.813 | 62.751 | 61.198 |
| 5 | 0.8 | 10 | 15 | 64.013 | 59.194 | 609.516 | 62.348 | 61.378 |
| 6 | 0.8 | 1 | 15 | 74.295 | 69.293 | 1291.328 | 72.607 | 71.294 |
| 6 | 0.8 | 2 | 15 | 73.989 | 67.858 | 380.765 | 71.002 | 70.882 |
| 6 | 0.8 | 3 | 15 | 77.08 | 71.116 | 846.593 | 77.805 | 74.749 |
| 6 | 0.8 | 4 | 15 | 70.138 | 65.871 | 3460.375 | 69.440 | 68.279 |
| 6 | 0.8 | 5 | 15 | 75.567 | 68.128 | 224.812 | 72.584 | 70.213 |
| 6 | 0.8 | 6 | 15 | 77.838 | 70.098 | 185.063 | 75.426 | 74.232 |
| 6 | 0.8 | 7 | 15 | 70.222 | 64.277 | 3600.031 | 69.884 | 67.659 |
| 6 | 0.8 | 8 | 15 | 69.886 | 61.394 | 2779.953 | 65.244 | 64.033 |
| 6 | 0.8 | 9 | 15 | 74.237 | 68.352 | 180.656 | 70.921 | 70.248 |
| 6 | 0.8 | 10 | 15 | 72.313 | 65.111 | 402.141 | 70.292 | 67.341 |
| 7 | 0.8 | 1 | 15 | 74.24 | 67.336 | 1822.393 | 71.432 | 70.726 |
| 7 | 0.8 | 2 | 15 | 70.784 | 63.257 | 648.738 | 68.093 | 66.768 |
| 7 | 0.8 | 3 | 15 | 78.083 | 70.426 | 676.329 | 74.219 | 74.123 |
| 7 | 0.8 | 4 | 15 | 69.032 | 64.795 | 2163.188 | 68.029 | 67.500 |
| 7 | 0.8 | 5 | 15 | 69.09 | 66.945 | 711.625 | 67.631 | 67.468 |
| 7 | 0.8 | 6 | 15 | 59.888 | 57.738 | 220.531 | 58.309 | 58.294 |
| 7 | 0.8 | 7 | 15 | 61.751 | 56.252 | 3600.031 | 58.545 | 57.853 |
| 7 | 0.8 | 8 | 15 | 73.592 | 65.321 | 68.109 | 70.695 | 68.508 |
| 7 | 0.8 | 9 | 15 | 73.973 | 69.38 | 2416.61 | 65.985 | 62.588 |
| 7 | 0.8 | 10 | 15 | 65.548 | 60.086 | 3330.485 | 61.517 | 61.517 |

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