DESIGN AND PERFORMANCE ANALYSIS FOR LDPC CODED MODULATION IN MULTIUSER MIMO SYSTEMS

by

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The channel capacity can be greatly increased by using multiple transmit and receive antennas, which is usually called multi-input multi-output (MIMO) systems. Iterative processing has achieved near-capacity on a single-antenna Gaussian or Rayleigh fading channel. How to use the iterative technique to exploit the capacity potential in single-user and/or multiuser MIMO systems is of great interest. We propose a low-density parity-check (LDPC) coded modulation scheme in multiuser MIMO systems. The receiver can be regarded as a serially concatenated iterative detection and decoding scheme, where the LDPC decoder performs the role of outer decoder and the multiuser demapper does that of the inner decoder. For the proposed scheme, appropriate selection of a bit-to-symbol mapping is crucial to achieve a good performance, so we investigate and find the best mapping under various cases.

Analytical bound serves as a useful tool to assess system performance. The search for powerful codes has motivated the introduction of efficient bounding techniques tailored to some ensembles of codes. We then investigate combinatorial union bounding techniques for fast fading multiuser MIMO systems. The union upper bound on maximum likelihood (ML) decoding error probability provides a prediction for the system performance, with which the simulated system performance can be compared. Closed-form expression for the union bound is obtained, which can be evaluated efficiently by using a polynomial expansion. In addition, the constrained channel capacity and the threshold obtained from extrinsic information transfer (EXIT) chart can also serve as performance measures. Based on the analysis for fast fading case, we generalize the union upper bound to the block fading case.

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PREFACE

I would like to thank my advisor Professor Heung-No Lee, who worked with me closely in the area of wireless communications throughout the last four years. I express my sincere gratitude to him for his constant support, guidance and motivation.

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Finally, I would like to thank my parents for supporting me all the time.

1.0 INTRODUCTION

1.1 MULTI-INPUT MULTI-OUTPUT (MIMO) CHANNEL

Research on wireless communication systems attracts more and more interests. While previous wireless communication systems mainly provide voice service, current research focuses on providing service of voice, video and data access. The demanding requirement for the service in future wireless networks poses many challenges. Data rate should be high while the resource of the wireless system is limited. In particular, the spectrum is very scarce and expensive, so we need to maximize the data rate within a given bandwidth, that is, the spectrum efficiency.

One way of maximizing the spectrum efficiency is to increase the transmit power. However, unlike the conventional point-to-point communication channel, the overall throughput is interference limited in a wireless network. Improving the transmit power of one user cannot increase the spectrum efficiency of the whole wireless network, because the stronger transmit power, the stronger interference for other users. Without increasing the total transmit power, however, the spectrum efficiency can be greatly increased by using multiple transmit and receive antennas — instead of using a single antenna at both ends, which will be referred to as multi-input multi-output (MIMO) channels in this dissertation.

This is very promising for future wireless communication systems. In [1], Teletar investigates the capacity of MIMO channels, and shows that the capacity linearly increase with the number of antennas when the channels exhibit rich scattering. In [2], Foschini evaluates the capacity of MIMO channels with Monte Carlo simulation and also shows that large capacity can be obtained. This large spectral efficiency is based on the condition that a rich scattering environment provides independent transmission paths from each transmit antenna to each receive antenna. A transmission and reception strategy that exploits this structure achieves capacity on approximately $min(N_t, N_r)$ separate channels, where N_t is the number of transmit antennas and N_r is the number of receive antennas. Thus, capacity scales linearly with $min(N_t, N_r)$ relative to a system with just one transmit and one receive antenna.

To achieve any point close to the capacity curve, a symbol constellation with a Gaussian distribution is generally needed. However, to be practical, we restrict our attention to phase shift keying (PSK) or quadrature-amplitude modulation (QAM) constellations, in which case the capacity is referred to as "constrained capacity." Our calculation shows that constrained capacity still can be significantly increased by using multiple antennas. This means that the channel can support higher data rate by spreading the transmit power over all antennas. Consequently, MIMO systems can play an important role in the wireless systems.

1.2 TURBO PRINCIPLE

Most wireless systems employ error correcting coding to prevent the transmitted data from being corrupted by the channel. Coding introduces additional signal structures that can be utilized by decoding algorithms. Turbo-like codes are a broad class of powerful error correcting codes, which yields astonishing performance close to the Shannon information-theoretical capacity limits, yet enabling simple decoding algorithms. Followed by the discovery of the powerful turbo codes [3], iterative processing techniques have received considerable attention. Beyond its application to decoding, the iterative algorithm known as "turbo principle" [4] has been successfully applied to other parts of a digital receiver, like equalization [5], coded modulation [6], multiuser detection [7], and others.

Due to the invention of turbo code, interests of low-density parity-check (LDPC) codes have been rekindled [8], [9], which were proposed by Gallager in 1963 [10]. Gallager demonstrated desirable properties of these codes and proposed a practical iterative decoding algorithm for these codes. However, Gallager's poineering work was relatively ignored for about three decades until the recent advent of turbo codes in 1993. An LDPC code can be represented by its parity-check matrix, which alternatively can be represented by a bipartite graph [11]. When decoding is going on the graph, messages are passed from the bit nodes to the check nodes and then from the check nodes back to the bit nodes iteratively, which is often referred to as the message passing algorithm.

Iterative processing has achieved near-capacity on a single-antenna Gaussian or Rayleigh fading channel. How to use the iterative technique to exploit the capacity potential in single-user and/or multiuser MIMO systems is of great interest. In an attempt to approach the capacity limits of single-input single-output (SISO) channels, Narayanan and Stüber propose an iterative detection and decoding scheme with convolutional codes in [12]. Hochwald and Brink extend the iterative detection and decoding scheme to MIMO channel [13]. For multiuser systems, Wang and Poor propose iterative receiver of joint detection and decoding for coded code-division multiple-access (CDMA) systems in [14]. Based on the "turbo principle" and the characteristics of LDPC codes, we propose an LDPC coded modulation scheme for multiuser MIMO systems in [15]. As depicted in Fig.6, the receiver can be regarded as a serially concatenated iterative detection and decoding scheme, where the LDPC decoder performs the role of the outer decoder and the multiuser demapper does that of the inner decoder.

We identified that appropriate choice of bit-to-symbol map is crucial to achieve a good performance for this iterative multiuser demapping and decoding scheme. We investigate and find the optimal constellation mapping under various cases. One of our contributions in this area therefore is that we find the best mapping for LDPC coded modulation in single-user and multiuser MIMO systems. The best mapping for LDPC codes is Gray mapping while the best mapping for convolutional codes is anti-Gray mapping [16].

1.3 BOUNDING TECHNIQUE

The error performance analysis of coded modulation systems is very complex and difficult, especially to render an exact closed-form expression, while close-form measures are very useful to assess system performance. As specific codes are even harder to be evaluated, the performance of the ensembles of codes is considered in this dissertation. Fano [17] and Gallager [18] upper bounds were introduced as efficient tools to determine the error exponents of the ensembles of random codes. The advent of turbo codes and the rediscovery of LDPC codes have motivated the introduction of efficient bounding techniques tailored to some carefully chosen ensembles of codes. These bounds provide information up to the ultimate capacity limit, which means that they are suitable for long block codes.

As mentioned in Section 1.2, LDPC and turbo-like codes can approach the capacity limits when the block length of the codes grows into infinity. When these codes are applied to MIMO systems, the codes can exploit the diversity both in space and time domain, which means that the proposed concatenated LDPC coded modulation scheme is expected to perform at an operating point near the MIMO capacity if the length of the code is large. However, the length of the codes cannot be brought to infinity in practice. In current wireless local area network (WLAN) protocol *IEEE* 802.11n, for example, the length of the LDPC code is proposed to be about two thousand because of delay constraint. This means that the performance evaluation at a moderate length is of our interest. Hence, we use union bound as a useful means to evaluate MIMO system performance. The union bound is the summation of every pairwise error probability. Based on the distance distribution [19], the union bound can be calculated as the summation of distinct pairwise error probability, each of which is weighted by the multiplicity of codewords with the same distance.

Tight union bound techniques based on the Fano-Gallager's tilting measures have been investigated for SISO channels in [20], [21]. In this dissertation, we investigate the combinatorial union bounding techniques in single-user and multiuser MIMO systems [22]. The union upper bound on maximum likelihood (ML) decoding error probability for turbo-like or LDPC codes provides a performance prediction of the proposed transmission system although the ML decoding is usually prohibitively complex for long block codes. We derive union upper bounds on the ML detection using the distance distribution of the outer LDPC codes. Closed-form expressions are obtained which with specific SNRs and a constellation mapping rule can be evaluated efficiently by using a polynomial expansion.

1.4 OUTLINE OF DISSERTATION

The rest of the dissertation is divided into five parts.

In Chapter 2, to exploit the capacity potential of MIMO channel, we propose an LDPC coded modulation scheme with iterative demapping and decoding for multiuser MIMO systems. We study the mapping influence on the system performance for the proposed scheme. We tried various bit-to-symbol mappings and find that Gray mapping is the best one under different cases.

In Chapter 3, we first calculate the constrained capacity for single-user MIMO systems. Based on it, the constrained capacity region for multiuser MIMO systems is determined.

In Chapter 4, we derive the union upper bound of error probability for the proposed scheme in multiuser fast fading MIMO systems. Closed-form expression for the union bound is obtained, which can be evaluated efficiently by using a polynomial expansion. We compare the system performance with the union upper bound, the capacity limit, and the threshold obtained from EXIT chart analysis.

In Chapter 5, we generalize the union upper bound to multiuser block fading MIMO systems.

Chapter 6 contains our conclusions.

2.0 LDPC CODED MODULATION SCHEME FOR MULTIUSER MIMO SYSTEMS

2.1 MIMO SYSTEM

2.1.1 Channel model

We consider the wireless channel with N_t transmit and N_r receive antennas as shown in Figure 1. The fading coefficient h_{ij} is the complex path gain from the j-th transmit antenna to the i-th receive antenna. The coefficients are assumed to be independent identically distributed (i.i.d.) complex-valued Gaussian random variables. In Chapter 2, Chapter 3 and Chapter 4, we assume that each Gaussian variable is zero mean and unit variance with independent real and imaginary parts. This is intended to model Rayleigh fading channel. In Chapter 5, we assume that each Gaussian variable can be non-zero, which is intended to model Ricean channel. We can write the channel matrix as $\mathbf{h} = [h_{ij}] \in \mathbb{C}^{N_r \times N_t}$. The noise $\mathbf{n}$ is circularly symmetric complex Gaussian (CSCG) with zero mean and N_0 variance, and we write the noise as $\mathbf{n} = [n_i] \in \mathbb{C}^{N_r}$. The signal-to-noise ratio (SNR) is defined as the ratio of the received signal-energy to the energy of the noise whose one sided power spectral density is N_0. The received signal-energy is the symbol energy times the number of transmit antennas. Thus, the SNR is defined as $\text{SNR} = N_t E_s / N_0$.

Based on different fading environment, we first assume that the channel matrix $\mathbf{h}$ independently changes per channel-use, which is usually called the *fast fading* channel. In Chapter 2, Chapter 3 and Chapter 4, we will focus on the fast fading channel model. In chapter 5, we generalize the model to a *block fading* channel. For block fading channel, the channel coefficients matrix remains constant for T_D channel-uses. clearly, $1 \leqslant T_D \leqslant T$, where

T is the total number of channel-uses for transmission of one codeword. When $T_D = 1$, it becomes a fast fading channel; when $T_D = T$, it is a quasi-static fading channel. Throughout the dissertation, the channel state information (CSI) is assumed to be known at the receiver, but not at the transmitter.

In fast fading case, the received signal $\mathbf{y}_t$ at t-th channel-use can be written as

$$\mathbf{y}_t = \mathbf{h}^t \mathbf{x}_t + \mathbf{n}_t, \quad \text{for } t = 1, \cdots, T, \tag{2.1}$$

where we define the following vector variables:

$$\mathbf{y}_t := \begin{pmatrix} y_{1t} \\ \vdots \\ y_{N_r t} \end{pmatrix}, \mathbf{n}_t := \begin{pmatrix} n_{1t} \\ \vdots \\ n_{N_r t} \end{pmatrix}, \mathbf{x}_t := \begin{pmatrix} x_{1t} \\ \vdots \\ x_{N_t t} \end{pmatrix}, \bar{\mathbf{x}}_t := \frac{1}{\sqrt{E_s}} \mathbf{x}_t,$$

$$\mathbf{h}^t := \begin{pmatrix} h_{11}^t & \cdots & h_{1N_t}^t \\ \vdots & \ddots & \vdots \\ h_{N_r 1}^t & \cdots & h_{N_r N_t}^t \end{pmatrix},$$

where E_s is the symbol energy.

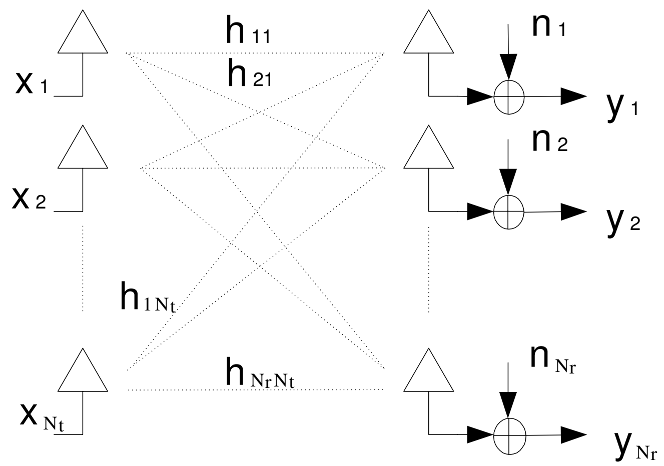

Figure 1: MIMO channel model

2.1.2 Channel capacity

Strictly speaking, the Shannon capacity is 0 for quasi-static fading channel, since for any given data rate, there is a strictly positive probability that the channel matrix $\mathbf{h}$ is too bad to support it, that is, a channel outage occurs. In this case, we can talk about a tradeoff between *outage probability* and supportable rate. Namely, given a transmission rate R, we can find $P_{out}(R)$ such that for any rate less than R and any δ there exists a code for which the error probability is less than δ for all but a set of $\mathbf{h}$ whose total probability is less than $P_{out}(R)$. The notion of *outage capacity* is studied in [1].

In this dissertation, we will mainly focus on the *ergodic capacity* of MIMO channels. In such a case, the channel is assumed to be fast fading. The ergodic capacity is the highest data rate that can be reliably transmitted by coding over infinite channel-uses. In the rest of this dissertation, we will refer to *capacity* as the *ergodic capacity.*

The capacity of MIMO channel is computed by Telatar in [1] and by Foschini in [2]. Assuming the fading coefficient matrix $\mathbf{h}$ is known to the receiver, the channel capacity of a MIMO system with N_t transmit antennas and N_r receive antennas is:

$$\begin{aligned}\mathbf{C} &= E\Big(\log_2 \det \left(I_{N_t} + \text{SNR } \mathbf{h}^*\mathbf{h}\right)\Big) \\ &= E\Big(\log_2 \det \left(I_{N_r} + \text{SNR } \mathbf{h}\mathbf{h}^*\right)\Big) \end{aligned} \tag{2.2}$$

At high SNR, the capacity is approximated as:

$$\mathbf{C} = min(N_t, N_r)\log(\text{SNR}) + o(\text{SNR}) \tag{2.3}$$

We observe that the MIMO channel capacity increases with SNR as $min(N_t, N_r)\log(SNR)$, in contrast to $\log(SNR)$ for single-antenna channels at high SNR. This result suggests that the multiple antenna channel can be viewed as $min(N_t, N_r)$ parallel spatial channels.

To achieve any point on the capacity curve, a symbol constellation that is the channel input with a Gaussian distribution is usually needed. However, to be practical, we restrict our attention to PSK or QAM constellations. We will refer to *capacity* as *constrained capacity* when the input is restricted to modulated constellation symbols.

2.2 LDPC CODE

LDPC codes have shown extreme success in communication system design due to their capacity approaching performance on SISO channels [23] and MIMO channels [13]. One of the most desirable characteristics of turbo-like and LDPC codes is that the complexity of the decoder grows linearly over the length of the code. This advantage can be used to attain a near capacity performance by using an extremely long block length code. When employed for MIMO channels, the soft-in soft-out message passing decoder can exploit the diversity benefits available in space, time and frequency domain. As the results, if the length of the outer code grows to infinity, the performance of the concatenated scheme can be brought close to the MIMO channel capacity. Our interest is to investigate the performance of LDPC codes in a multiuser MIMO system.

LDPC codes are codes specified by a parity-check matrix H that contains mostly 0's and relatively few 1's. In particular, an (N, J, K) LDPC code is a code of block length N with a parity-check matrix, where there are J 1's in a column and K 1's in a row. An (N, J, K) LDPC code can be represented by a bipartite graph as shown in Figure 2, whose code rate is $R_c = 1 - J/K$. Each edge in the graph is related to the non-zero entry of the parity-check matrix H.

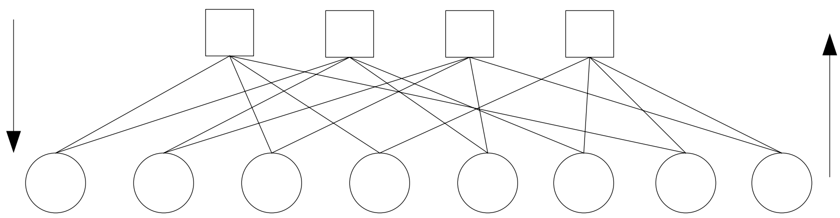

Figure 2: Bipartite graph of an (8, 2, 4) LDPC code.

The low-density matrix H of size $N(1 - R_c) \times N$ is randomly generated. By performing the Gaussian elimination, H matrix can be represented in a systematic way as $[I|P]$, where I is the identity matrix of size $N(1 - R_c) \times N(1 - R_c)$ and P is the parity check part of size $N(1 - R_c) \times NR_c$. The generator matrix is constructed as $[P'|I]$, where $'$ denotes transpose.

Codewords are generated by using the generator matrix, which is the same for the senders in our system.

Gallager proposed a probabilistic decoding algorithm for a LDPC code in [10]. Later Tanner constructed a bipartite graph and applied Gallager's decoding method on it. On the bipartite graph, there are two disjoint sets of nodes. One set of nodes, called bit nodes, corresponds to the bits in a codeword, and the other set of nodes, called check nodes, corresponds to the set of parity check equations. Connections between these two sets of nodes are determined by the parity check matrix. When decoding is going on, the message is passed from the bit nodes to the check nodes and then from the check nodes back to the bit nodes iteratively.

Based on the "turbo principle" and the characteristics of low density parity check (LDPC) codes, we propose a soft demapping method for modulation, which is combined with soft decoding for LDPC codes in an iterative manner in the multiuser MIMO system. The receiver can be regarded as a serially concatenated iterative decoding scheme, where the LDPC decoder performs the role of outer decoder and the demapping device does that of the inner decoder. Previous study [16] has shown that the appropriate choice of bit-to-symbol map is crucial to achieve a good performance in such a concatenated scheme for convolutional codes. In this chapter, we focus on studying the mapping influence and finding the best bit-to-symbol mapping for LDPC codes.

2.3 MULTIUSER MIMO SYSTEM MODEL

The concatenated transmission scheme with turbo-iterative demapping and decoding for single user is shown in Figure 3.

We further consider a multiuser MIMO system with two senders and one receiver. The simple multiuser system model is shown in Figure 4 and multiuser MIMO channel model is shown in Figure 5. The detailed system model is shown in Figure 6. Each sender is equipped with N_t transmit antennas, and the receiver with N_r receive antennas. When only one sender is active, the model is reduced to a single-user MIMO system. Furthermore, when $N_t = 1$

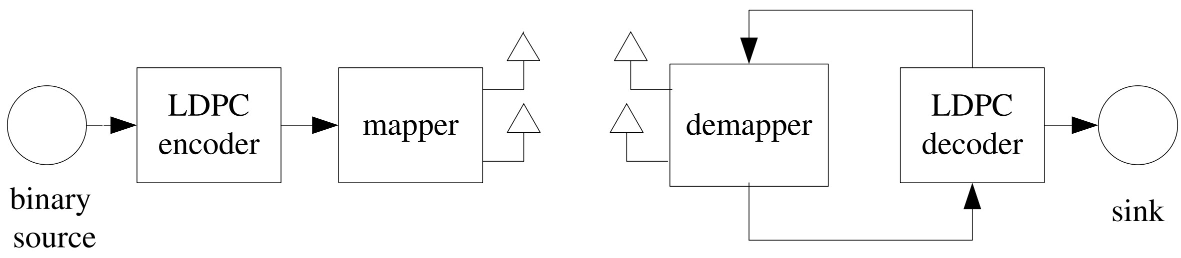

Figure 3: Single-user MIMO system model

and $N_r = 1$, it becomes a single-input single-output (SISO) Rayleigh fading channel. If we set all the fading coefficients equal to 1, it is just a additive white Gaussian noise (AWGN) channel.

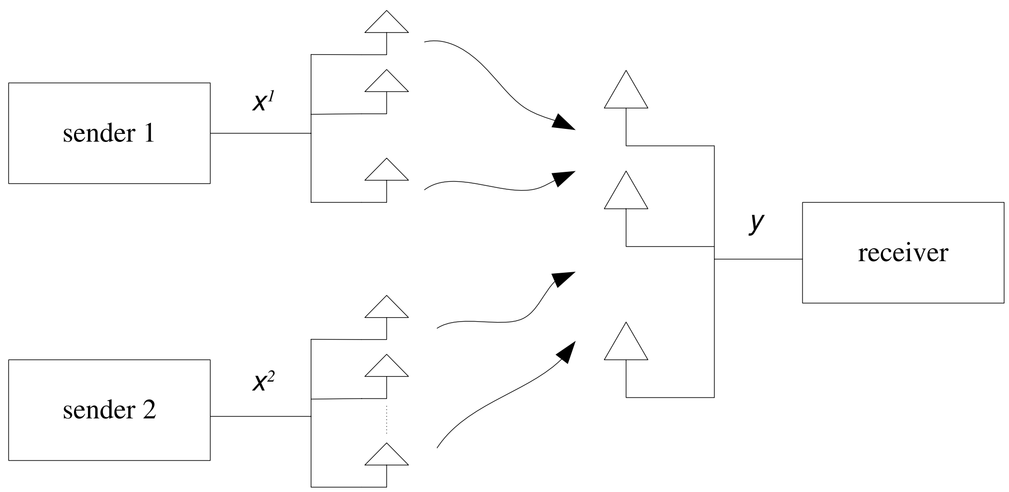

Figure 4: Simple multiuser MIMO system model.

At the transmitter, each LDPC encoder is combined with a space-time modulator. For each sender, the binary source is encoded as an LDPC code $\mathbf{c}^u = (c_1^u, \cdots, c_N^u)$, where $u = 1, 2$, is the user index and N is the block length of the code. The mapping device takes a group of $\log_2(M)$ coded bits and maps them into a constellation symbol, where M is the size of

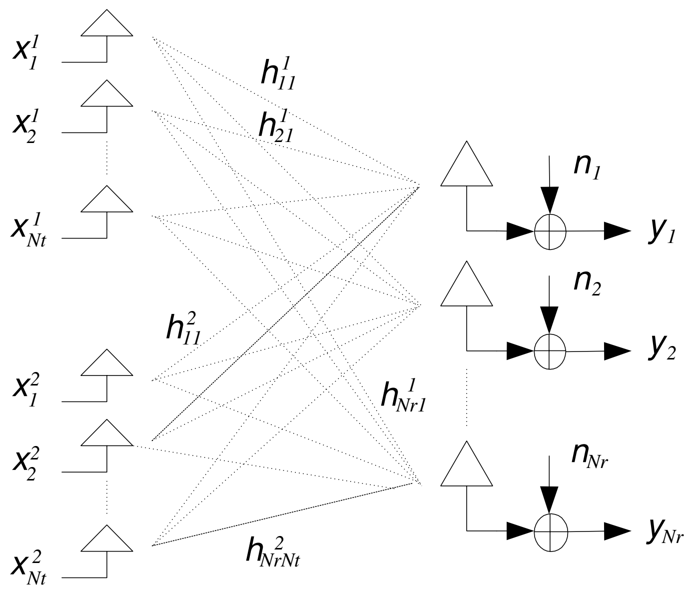

Figure 5: Multiuser MIMO channel model.

the constellation. Then, the mapping device transforms the symbol sequence into a space-time symbol matrix $\mathbf{x}^u$, i.e. the serial to parallel conversion of the symbol sequence without an explicit space-time coding. Two senders simultaneously transmit the space-time symbol vectors in each MIMO channel-use.

At the receiver, the received signal at t-th channel-use is

$$\mathbf{y}_t = \mathbf{h}^t \mathbf{x}_t + \mathbf{n}_t, \quad \text{for } t = 1, \cdots, T, \tag{2.4}$$

where the vector variables are defined as following:

$$\mathbf{y}_t := \begin{pmatrix} y_{1t} \\ \vdots \\ y_{N_r t} \end{pmatrix}, \mathbf{n}_t := \begin{pmatrix} n_{1t} \\ \vdots \\ n_{N_r t} \end{pmatrix}, \mathbf{x}_t := \begin{pmatrix} \mathbf{x}_t^1 \\ \mathbf{x}_t^2 \end{pmatrix}, \mathbf{x}_t^u := \begin{pmatrix} x_{1t}^u \\ \vdots \\ x_{N_t t}^u \end{pmatrix}, \bar{\mathbf{x}}_t^u := \frac{1}{\sqrt{E_{s_u}}} \mathbf{x}_t^u,$$

$$\mathbf{h}^t := \begin{pmatrix} \mathbf{h}^{1t} & \mathbf{h}^{2t} \end{pmatrix} := \begin{pmatrix} h_{11}^{1t} & \cdots & h_{1N_t}^{1t} & h_{11}^{2t} & \cdots & h_{1N_t}^{2t} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ h_{N_r 1}^{1t} & \cdots & h_{N_r N_t}^{1t} & h_{N_r 1}^{2t} & \cdots & h_{N_r N_t}^{2t} \end{pmatrix},$$

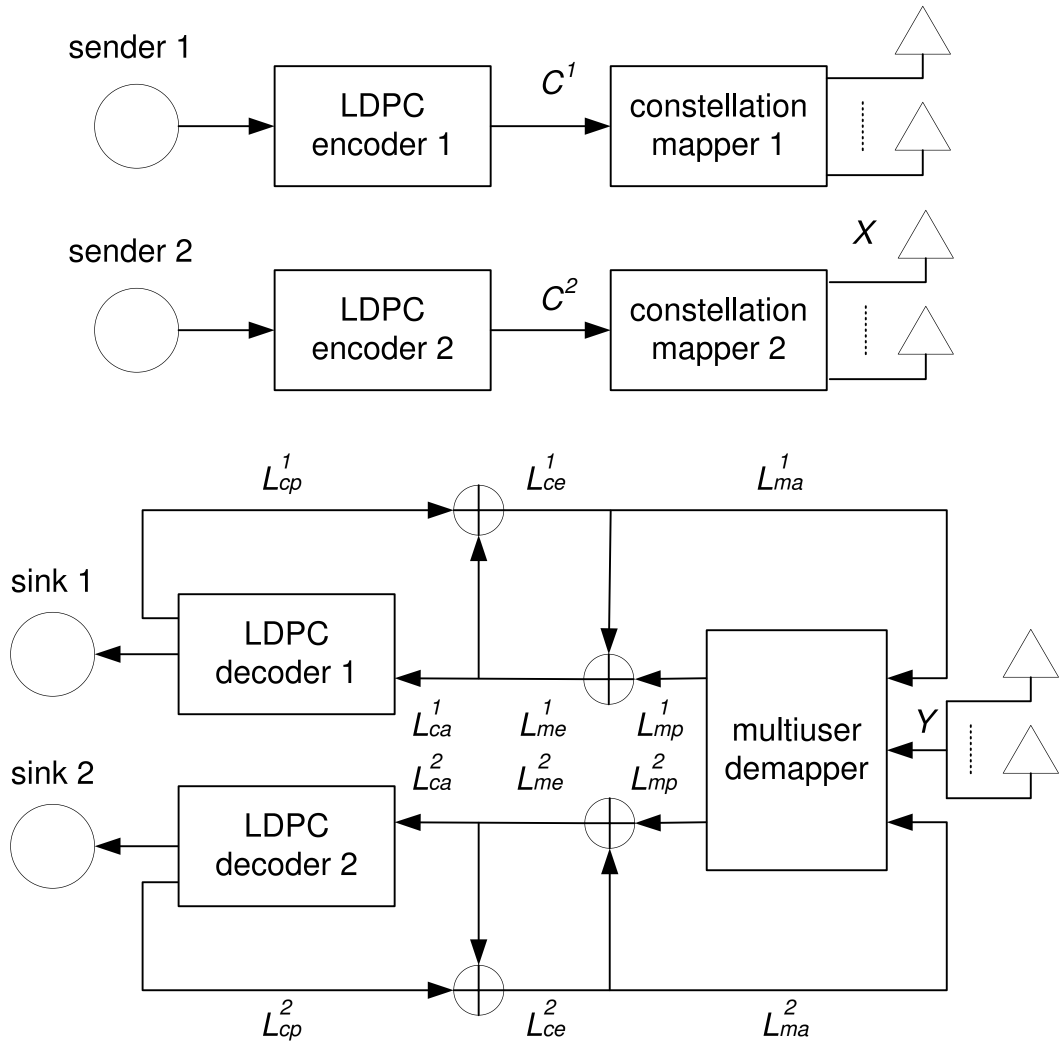

Figure 6: Detailed multiuser MIMO system model.

where $T = \frac{N}{N_t \log_2(M)}$ is the number of channel uses with an assumption that N is a multiple of $N_t \log_2(M)$ without loss of generality; E_{s_u} for $u = 1, 2$, are the symbol energies of user 1 and user 2, respectively. we assume that they are equal, i.e., $E_{s_u} = E_s$. Thus, this is a model for equal SNRs for the two senders. The matrix $\mathbf{h}^t$ is an $[N_r \times 2N_t]$ matrix with i.i.d. complex-valued Gaussian random elements. For each sender, the channel matrix $\mathbf{h}^{ut}$ for $u = 1, 2$ is the same as the described in the section 2.1.1. The SNR in multiuser MIMO systems is defined as the ratio of the received signal-energy per user to the energy of the noise whose one sided power spectral density is N_0. Thus, the SNR is defined as $\text{SNR} = N_t E_s / N_0$.

Referring to the receiver part of Fig.6, the received signal is iteratively demapped and decoded by mutually exchanging soft information between the inner multiuser demapper and the outer LDPC decoders. The demapper computes the posterior log-likelihood ratios (LLRs) L^u_{mp} for each coded bit by using the channel observation $\mathbf{y}$ and the prior information L^u_{ma}, which is zero initially. This posterior information L^u_{mp}, after subtracting the prior part L^u_{ma}, becomes the so-called "extrinsic information" L^u_{me}, which is passed to the corresponding LDPC decoder for the initialization of the LDPC message passing algorithm. The decoder then calculates the posterior LLRs L^u_{cp} as the output of the decoder. Subtracting the prior part L^u_{ca} forwarded from the demapper, we obtain the extrinsic information L^u_{ce} of the decoder, which is fed back to the multiuser demapper.

In this decoding scheme, there are two kinds of iterations: one is *super* iteration, a single iteration between demapping and decoding blocks; the other is *internal* iteration, an iteration in the LDPC decoder itself. It should be noted that in this system, the two independent codewords from the two independent senders are simultaneously demapped and decoded.

2.4 MULTIUSER ITERATIVE SOFT DEMAPPING AND DECODING

In this section, we will focus on the multiuser iterative soft demapping. For simplicity the script t is omitted in this section since the demapping algorithm is the same for any time t.

The multiuser demapper calculates the posterior probability on each unmapped bit in the received signal vector from both senders. We arrange all the bits in order from 0 to

$A = 2N_t \log_2(M) - 1$, where the first $N_t \log_2(M)$ bits are from the sender 1 and the second $N_t \log_2(M)$ bits from sender 2. We keep all calculations in log domain in this section.

Let $L_{mp}(c_k)$, $L_{ma}(c_k)$ and $L_{me}(c_k)$ denote the posterior probability, the prior probability and the extrinsic information on the k-th bit of the multiuser demapper, respectively; We name the collection of the corresponding probability of the first $N_t \log_2(M)$ bits as L^1_{mp}, L^1_{ma} and L^1_{me} and the second $N_t \log_2(M)$ bits as L^2_{mp}, L^2_{ma} and L^2_{me} that are shown in Fig.6; Let $\tau(\mathbf{c}_{0\cdots A,k})$ denote all the possible combinations of $c_0 \cdots c_A$ excluding c_k. By using the total probability theorem, the log ratios of the posterior probability on each bits of the demapper can be written as

$$L_{mp}(c_k) = \log \frac{\sum p\big(c_k = 1, \tau(\mathbf{c}_{0\cdots A,k})|\mathbf{y}\big)}{\sum p\big(c_k = 0, \tau(\mathbf{c}_{0\cdots A,k})|\mathbf{y}\big)}. \tag{2.5}$$

Because the parity check matrix H is randomly generated, it assures near independence until a convergence is reached. Thus, we can write the joint probabilities as the product of individual terms. Using Bayes' rule, (2.5) can be re-written as

$$\begin{aligned} L_{mp}(c_k) &= \log \frac{\sum p\big(\mathbf{y}|c_k = 1, \tau(\mathbf{c}_{0\cdots A,k})\big) p\big(c_k = 1, \tau(\mathbf{c}_{0\cdots A,k})\big)}{\sum p\big(\mathbf{y}|c_k = 0, \tau(\mathbf{c}_{0\cdots A,k})\big) p\big(c_k = 0, \tau(\mathbf{c}_{0\cdots A,k})\big)} \\ &= \log \frac{p(c_k = 1)}{p(c_k = 0)} + \log \frac{\sum p\big(\mathbf{y}|c_k = 1, \tau(\mathbf{c}_{0\cdots A,k})\big) p\big(\tau(\mathbf{c}_{0\cdots A,k})\big)}{\sum p\big(\mathbf{y}|c_k = 0, \tau(\mathbf{c}_{0\cdots A,k})\big) p\big(\tau(\mathbf{c}_{0\cdots A,k})\big)} \\ &= L_{ma}(c_k) + L_{me}(c_k), \end{aligned} \tag{2.6}$$

Let $\mathbf{c}_{bin(j,k)} = c_0 \cdots c_{k-1} c_{k+1} \cdots c_A$ be the binary decomposition of j such that $j = \sum_{i=0, i\neq k}^{A} c_i \, 2^{(i-u(i-k))}$, where $u(t) = 1, t \geqslant 0$ and $u(t) = 0, t < 0$; Let $B = 2^A - 1$. The extrinsic information on the k-th bit equals to

$$L_{me}(c_k) = \log \frac{\displaystyle\sum_{j=0}^{B} p\big(\mathbf{y}|c_k = 1, \mathbf{c}_{bin(j,k)}\big) \exp\Big(\sum_{\substack{i=0, i\neq k \\ c_i = 1}}^{A} L_{ma}\, c_i\Big)}{\displaystyle\sum_{j=0}^{B} p\big(\mathbf{y}|c_k = 0, \mathbf{c}_{bin(j,k)}\big) \exp\Big(\sum_{\substack{i=0, i\neq k \\ c_i = 1}}^{A} L_{ma}\, c_i\Big)}. \tag{2.7}$$

To calculate $L_{me}(c_k)$, we need the channel output $\mathbf{y}$. The transition probability can be expressed as

$$p(\mathbf{y}|\mathbf{x}, \mathbf{h}) = \frac{1}{(\pi N_o)^{n/2}} \exp(-\frac{1}{N_o} \|\mathbf{y} - \mathbf{hx}\|^2), \tag{2.8}$$

where $n = 1$ if the signal is real, otherwise, $n = 2$ if the signal is complex.

Let $\mathbf{x}_{k,1,j} = map(c_k = 1, \mathbf{c}_{bin(j,k)})$; $\mathbf{x}_{k,0,j} = map(c_k = 0, \mathbf{c}_{bin(j,k)})$. The function "*map*" transforms the included bits into a constellation symbol. Substituting (2.8) into (2.7), we obtain the extrinsic information as

$$L_{me}(c_k) = \log \frac{\sum_{j=0}^{B} \exp\Big(-\frac{1}{N_o}\|\mathbf{y} - \mathbf{h}\mathbf{x}_{k,1,j}\|^2 + \sum_{\substack{i=0, i\neq k \\ c_i=1}}^{A} L_{ma}\, c_i\Big)}{\sum_{j=0}^{B} \exp\Big(-\frac{1}{N_o}\|\mathbf{y} - \mathbf{h}\mathbf{x}_{k,0,j}\|^2 + \sum_{\substack{i=0, i\neq k \\ c_i=1}}^{A} L_{ma}\, c_i\Big)}. \tag{2.9}$$

This extrinsic information is used as the prior information for LDPC decoder. On the code-graph, we perform the message passing algorithm [11] to decode the LDPC code.

2.5 BEST MAPPING FOR LDPC CODED MODULATION SCHEME IN MULTIUSER MIMO SYSTEMS

In the proposed multiuser transmission-receive system, the factor to affect the system performance is the mapping device once the LDPC code is selected and the iterative processing is used. Appropriate selection of a bit-to-symbol mapping is crucial to achieve a good performance, so we try to find the best mapping under different cases. We use the rate $\frac{1}{2}$ $(1024, 3, 6)$ LDPC codes in simulations, and try out all the different mappings considered in [24], which include Gray, anti-Gray, natural, d21 and d23 maps. Figure 7 shows the five different mappings for 8-PSK signals.

The Hamming distance between neighboring constellation symbols is 1 for Gray mapping; the Hamming distance is largest between neighboring constellation symbols for anti-Gray mapping.

The mutual information between the transmitted constellation symbol and the received signal, referred to as symbol-wise mutual information, is independent of the applied mapping.

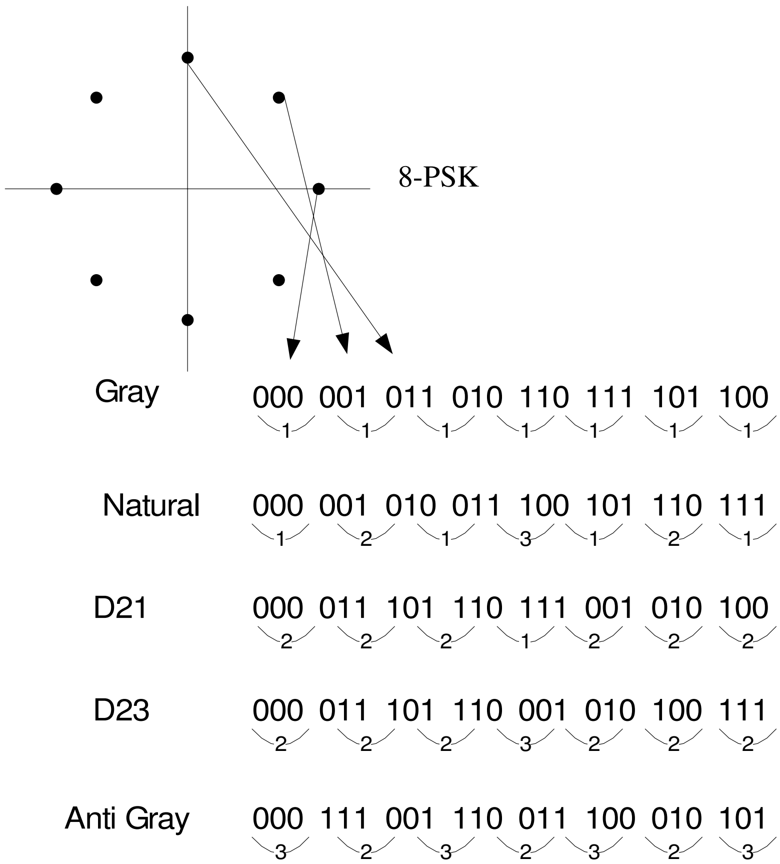

Figure 7: Five mappings for 8-PSK signal

The mapping influences the partition of the total amount of the symbol-wise mutual information. This partitioning of symbol-wise mutual information has an effect on the iterative demapping and decoding.

We simulate extensively on AWGN and Rayleigh fading channels, and also in single-user and multiuser MIMO systems. With simulation we try to determine the mapping which goes most well with the use of iterative demapping and decoding receiver. On AWGN and Rayleigh fading channels, we use 8-PSK modulation. To compare with the result for convolutional codes, two simulation schemes are performed:

- Simulation scheme I—*super* iteration with *internal* iteration
- Simulation scheme II—*super* iteration without *internal* iteration

We know that there is no *internal* iteration for convolutional codes and the anti-Gray mapping works best for convolutional codes on SISO channels [16]. We try to find the best mapping for simulation scheme I and II. The results shown in Figure 8 and Figure 9 indicate that the simulation scheme I works much better than the simulation scheme II.

From the figures, we can see that the Gray mapping is not as good as the anti-Gray mapping for simulation scheme II, which agrees to the conclusion for convolutional codes. This means that anti-Gray mapping works best when there is only *super* iteration. For simulation scheme I, the Gray mapping greatly improves the performance and outperforms the anti-Gray mapping. From the comparison we find that it is the best mapping among the five mappings we tested. Since simulation scheme I works better than scheme II, Gray mapping is the best one for the system.

The same results also hold in single-user and multiuser MIMO systems. In simulation, we still use $(1024, 3, 6)$ LDPC codes. They are mapped into 4-QAM signals and sequentially separated into N_t symbol streams for transmission. In multiuser MIMO systems, each sender simultaneously transmits N_t symbol streams to one receiver and transmits information at the same rate. Each sender is equipped with two transmit antennas and the receiver with two receive antennas. Since scheme I always works better than scheme II, we will compare the performance for simulation scheme I. Figure 10 and Figure 11 show the performance with Gray and anti-Gray mappings in single-user and multiuser MIMO systems, respectively.

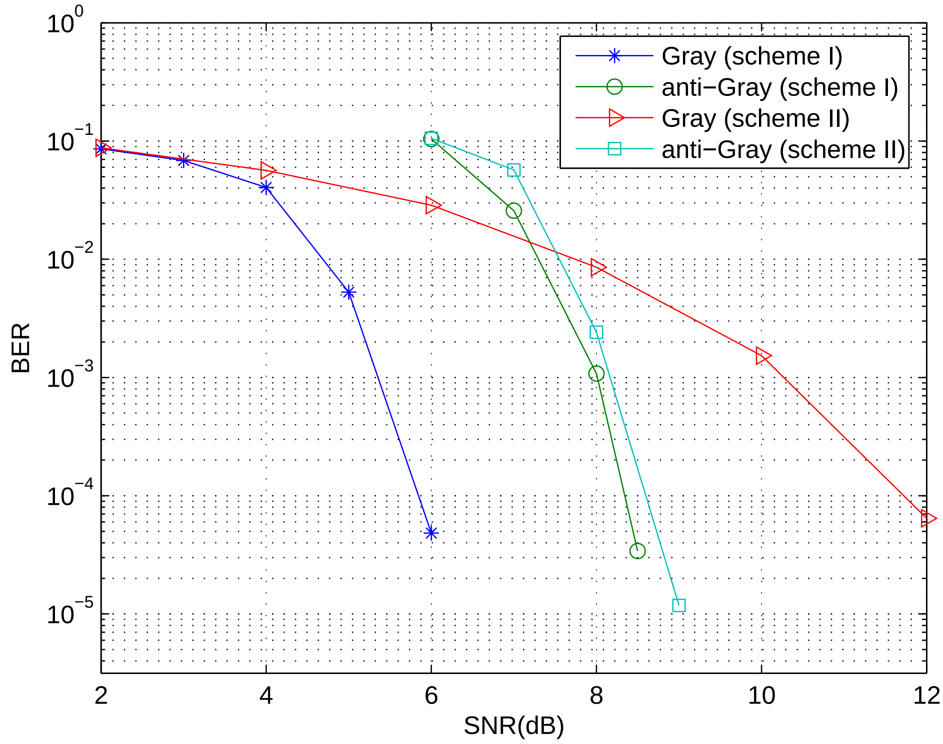

Figure 8: Performance comparison of different mappings on AWGN channel

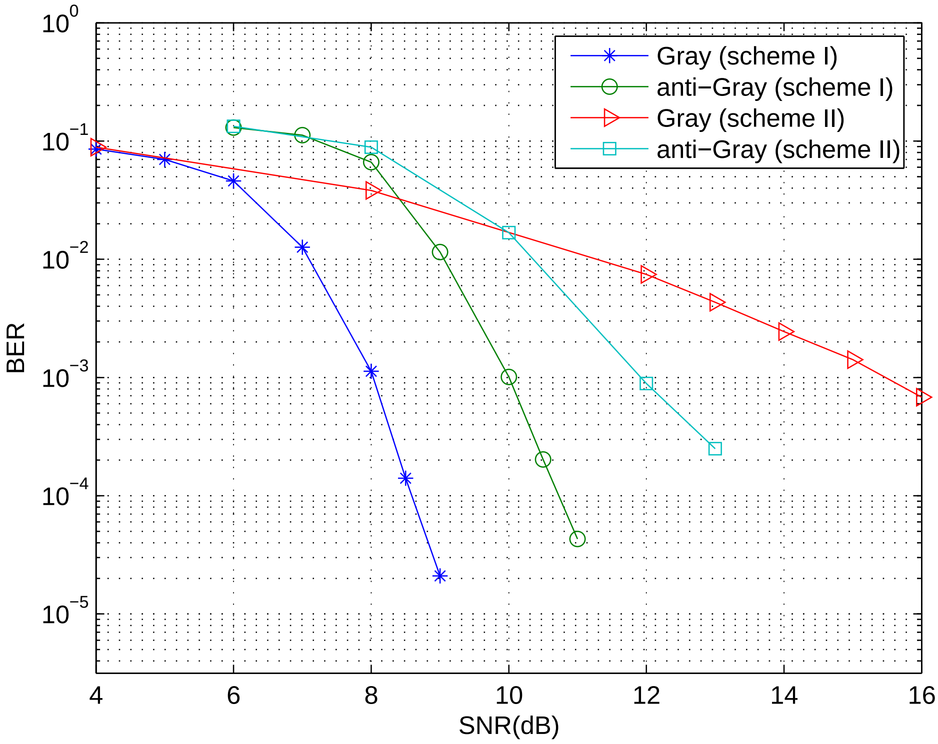

Figure 9: Performance comparison of different mappings on Rayleigh fading channel

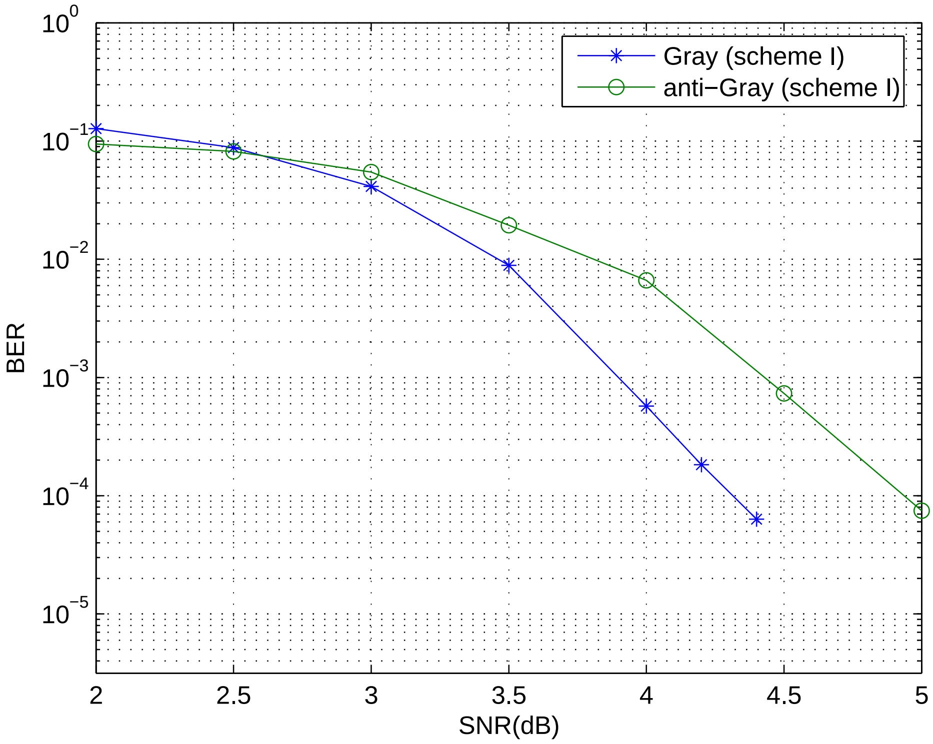

Figure 10: Performance comparison of different mappings on 2×2 MIMO channel

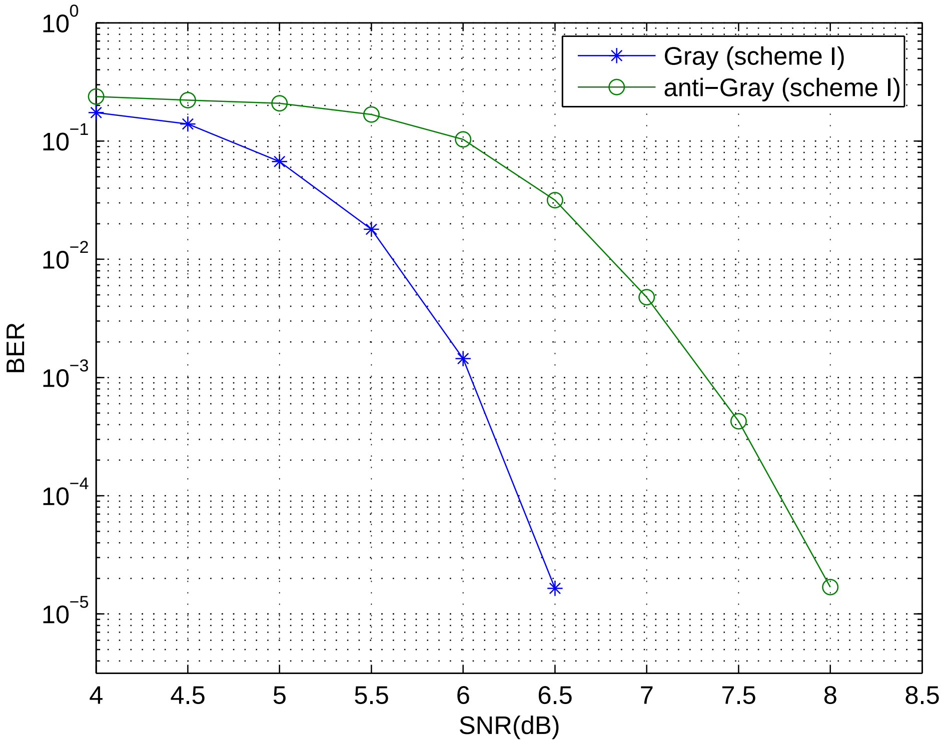

Figure 11: Performance comparison of different mappings in multiuser MIMO systems

From the comparison, we again find that the Gray mapping performs better than the anti-Gray mapping.

We found that the Gray mapping outperforms other mappings over a variety of channels for simulation scheme I, which is different from the results of convolutional codes.

3.0 CONSTRAINED CAPACITY CALCULATION FOR MULTIUSER MIMO SYSTEMS

In the Chapter 2, we propose an LDPC coded modulation scheme with iterative demapping and decoding, and find that Gray mapping performs best over AWGN and Rayleigh fading channels, and also in single-user and multiuser MIMO systems. To assess the system performance, we try to compare the system performance with the capacity limit and the upper bound. We will focus on calculating the constrained capacity in this chapter 3, and derive the upper bound in the next chapter 4.

The capacity of a channel is defined as the maximum mutual information between the channel input and output. Shannons capacity theorem shows that this maximum mutual information should be the maximum data rate that can be transmitted over the channel with arbitrarily small error probability. When the instantaneous channel state information (CSI) is known perfectly at the receiver, the capacity is the maximum mutual information averaged over all channel states.

In a multiuser system with U senders, capacity becomes a U-dimensional region, where the set of all rate vectors $(R_1, \cdots, R_U)$ are simultaneously achievable by all users.

To achieve any point on the capacity curve, a symbol constellation with a Gaussian distribution, is usually needed. In practice, we restrict our attention to PSK or QAM constellations. In the following sections, constrained capacity for single-user and multiuser MIMO systems will be calculated.

3.1 CONSTRAINED CAPACITY FOR SINGLE-USER MIMO SYSTEMS

In a single-user MIMO system, there is only one active sender in the system model. We denote the transmitted signal vector as $\mathbf{x}^u$,for $u = 1, 2$.

Proposition 1. *(Constrained capacity for MIMO channel with CSI)*

The constrained capacity for a single-user MIMO system with CSI is equal to

$$\mathbf{C_h} = \log M^{N_t} - \frac{1}{M^{N_t}} \sum_{k=1}^{M^{N_t}} E_{\mathbf{h}}\Big\{ E_{\mathbf{n}}\big\{ \log \sum_{i=1}^{M^{N_t}} \exp\big(-\frac{1}{2}(\mathbf{f}_{k,i}^T \Delta^{-1} \mathbf{f}_{k,i} - \mathbf{n}^T \Delta^{-1} \mathbf{n})\big)\big\}\Big\}, \tag{3.1}$$

where Δ is the covariance matrix of white Gaussian noise, $\mathbf{f}_{k,i} = \mathbf{h}\mathbf{x}_k^u + \mathbf{n} - \mathbf{h}\mathbf{x}_i^u$, and E denotes the expectation.

Proof : To calculate the capacity, we need to maximize the mutual information, averaged over all channel states

$$\begin{aligned} \mathbf{C_h} &= \max_{p(\mathbf{x}^u)} I(\mathbf{x}^u; \mathbf{y}) \\ &= \max_{p(\mathbf{x}^u)} E_{\mathbf{h}}\{I(\mathbf{x}^u; \mathbf{y}|\mathbf{h})\}, \quad u = 1, 2 \end{aligned} \tag{3.2}$$

We intend to use soft decoding in the receiver, so we study the channel with discrete-valued multilevel/phase input and continuous-valued output. Further we assume the equiprobability of input signals, and the maximization can be omitted. Then (3.2) can be written as

$$\begin{aligned} \mathbf{C_h} &= \max_{p(\mathbf{x}^u)} \iiint p(\mathbf{x}^u, \mathbf{y}|\mathbf{h}) p(\mathbf{h}) \log \frac{p(\mathbf{x}^u, \mathbf{y}|\mathbf{h})}{p(\mathbf{x}^u|\mathbf{h}) p(\mathbf{y}|\mathbf{h})} \, \mathrm{d}\mathbf{x}^u \, \mathrm{d}\mathbf{y} \, \mathrm{d}\mathbf{h} \\ &= \max_{p(\mathbf{x}^u)} \sum_{k=1}^{M^{N_t}} p(\mathbf{x}_k^u) \iint p(\mathbf{h}) p(\mathbf{y}|\mathbf{x}_k^u, \mathbf{h}) \log \frac{p(\mathbf{y}|\mathbf{x}_k^u, \mathbf{h})}{p(\mathbf{y}|\mathbf{h})} \, \mathrm{d}\mathbf{y} \, \mathrm{d}\mathbf{h} \\ &= \sum_{k=1}^{M^{N_t}} \frac{1}{M^{N_t}} \iint p(\mathbf{h}) p(\mathbf{y}|\mathbf{x}_k^u, \mathbf{h}) \log \frac{p(\mathbf{y}|\mathbf{x}_k^u, \mathbf{h})}{\sum_{i=1}^{M^{N_t}} p(\mathbf{y}|\mathbf{x}_i^u, \mathbf{h}) \frac{1}{M^{N_t}}} \, \mathrm{d}\mathbf{y} \, \mathrm{d}\mathbf{h} \\ &= \log M^{N_t} - \frac{1}{M^{N_t}} \sum_{k=1}^{M^{N_t}} E_{\mathbf{h}}\Big\{ E_{\mathbf{n}}\{\log \sum_{i=1}^{M^{N_t}} \frac{p(\mathbf{y}|\mathbf{x}_i^u, \mathbf{h})}{p(\mathbf{y}|\mathbf{x}_k^u, \mathbf{h})}\}\Big\}, \end{aligned}$$

where

$$\frac{p(\mathbf{y}|\mathbf{x}_i^u,\mathbf{h})}{p(\mathbf{y}|\mathbf{x}_k^u,\mathbf{h})} = \exp\big(-\frac{1}{2}(\mathbf{f}_{k,i}^T\Delta^{-1}\mathbf{f}_{k,i} - \mathbf{n}^T\Delta^{-1}\mathbf{n})\big). \qquad \square$$

In a certain mobile environment, the receiver cannot easily estimate the channel. For such a case, we derive the constrained capacity for unknown channel state both at the receiver and at the transmitter, which is given in the following corollary.

Corollary 1. *(Constrained capacity for MIMO channel without CSI)*

The constrained capacity for a single-user MIMO system without CSI is equal to

$$\mathbf{C}_{\bar{\mathbf{h}}} = \log M^{N_t} - \frac{1}{M^{N_t}}\sum_{k=1}^{M^{N_t}} E_{\mathbf{n}}\Big\{\log\sum_{i=1}^{M^{N_t}}\frac{E_{\mathbf{h}}\big\{\exp\big(-\frac{1}{2}\mathbf{f}_{k,i}^T\Delta^{-1}\mathbf{f}_{k,i}\big)\big\}}{E_{\mathbf{h}}\big\{\exp\big(-\frac{1}{2}\mathbf{n}^T\Delta^{-1}\mathbf{n}\big)\big\}}\Big\}. \tag{3.3}$$

3.2 CONSTRAINED CAPACITY REGION FOR MULTIUSER MIMO SYSTEMS

We now calculate the capacity region for multiuser MIMO systems. Let $\mathbf{S} \subseteq \{1, 2, \cdots, U\}$, where U is the number of senders. Let $\mathbf{S}^c$ denote the complement of $\mathbf{S}$. Let $R(\mathbf{S}) = \sum_{u\in\mathbf{S}} R_u$, and let $\mathbf{x}(\mathbf{S}) = \{\mathbf{x}^u : u \in \mathbf{S}\}$, where R_u is the achievable rate for user u. According to the basic information theory [25], we know that the capacity region for a multiuser system is the closure of the convex hull of the rate vectors satisfying the follows:

$$R(\mathbf{S}) \leqslant I\big(\mathbf{x}(\mathbf{S}); \mathbf{y}|\mathbf{x}(\mathbf{S}^c)\big),$$

for all $\mathbf{S} \subseteq \{1, 2, \cdots, U\}$ for some product distribution $p_1(\mathbf{x}^1)p_2(\mathbf{x}^2)\cdots p_U(\mathbf{x}^U)$ on $\mathbf{x}^1\mathbf{x}^2\cdots\mathbf{x}^U$. For example, the capacity region of a 2-user MIMO system is the closure of the convex hull of all (R_1, R_2) pairs satisfying

$$R_1 \leqslant I(\mathbf{x}^1; \mathbf{y}|\mathbf{x}^2),$$

$$R_2 \leqslant I(\mathbf{x}^2; \mathbf{y}|\mathbf{x}^1),$$

$$R_1 + R_2 \leqslant I(\mathbf{x}^1, \mathbf{x}^2; \mathbf{y}).$$

To determine the capacity region of a 2-user MIMO system, we need to calculate the capacities at corner points, that is, $I(\mathbf{x}^1; \mathbf{y}|\mathbf{x}^2)$, $I(\mathbf{x}^2; \mathbf{y}|\mathbf{x}^1)$ and $I(\mathbf{x}^1, \mathbf{x}^2; \mathbf{y})$. We calculate the sum mutual information $I(\mathbf{x}^1, \mathbf{x}^2; \mathbf{y})$ first.

Proposition 2. *(Constrained sum mutual information for 2-user MIMO systems)*

The constrained sum mutual information with CSI equals

$$I(\mathbf{x}^1, \mathbf{x}^2; \mathbf{y}) = 2\log M^{N_t} - \frac{1}{M^{2N_t}} \sum_{i,j=1}^{M^{N_t}} E_{\mathbf{h}}\Big\{ E_{\mathbf{n}}\big\{ \log \frac{\sum_{k,l=1}^{M^{N_t}} \exp(-\frac{1}{2}\mathbf{g}_{i,j,k,l}^T \Delta^{-1} \mathbf{g}_{i,j,k,l})}{\exp(-\frac{1}{2}\mathbf{n}^T \Delta^{-1} \mathbf{n})} \big\} \Big\}, \tag{3.4}$$

where $\mathbf{g}_{i,j,k,l} := \mathbf{h}\mathbf{x}_{i,j} + \mathbf{n} - \mathbf{h}\mathbf{x}_{k,l}$ *and* $\mathbf{x}_{i,j} := \begin{pmatrix} \mathbf{x}_i^1 \\ \mathbf{x}_j^2 \end{pmatrix}$.

Proof : The sum mutual information is the difference of entropies averaged over all channel states

$$\begin{aligned}
&I(\mathbf{x}^1, \mathbf{x}^2; \mathbf{y}) = E_{\mathbf{h}}\{I(\mathbf{x}^1, \mathbf{x}^2; \mathbf{y}|\mathbf{h})\} \\
&= E_{\mathbf{h}}\{\mathbf{H}(\mathbf{x}^1, \mathbf{x}^2|\mathbf{h}) - \mathbf{H}(\mathbf{x}^1, \mathbf{x}^2|\mathbf{y}, \mathbf{h})\} \\
&= E_{\mathbf{h}}\{\mathbf{H}(\mathbf{x}^1|\mathbf{h}) + \mathbf{H}(\mathbf{x}^2|\mathbf{x}^1, \mathbf{h})\} + \iiiint p(\mathbf{x}^1, \mathbf{x}^2, \mathbf{y}, \mathbf{h}) \log p(\mathbf{x}^1, \mathbf{x}^2|\mathbf{y}, \mathbf{h}) \mathrm{d}\mathbf{x}^1\, \mathrm{d}\mathbf{x}^2\, \mathrm{d}\mathbf{y}\, \mathrm{d}\mathbf{h} \\
&= \mathbf{H}(\mathbf{x}^1) + \mathbf{H}(\mathbf{x}^2) - \frac{1}{M^{2N_t}} \sum_{i,j=1}^{M^{N_t}} E_{\mathbf{h}}\Big\{ E_{\mathbf{y}}\big\{ \log \frac{\sum_{k,l=1}^{M^{N_t}} p(\mathbf{y}|\mathbf{x}_k^1, \mathbf{x}_l^2, \mathbf{h})}{p(\mathbf{y}|\mathbf{x}_i^1, \mathbf{x}_j^2, \mathbf{h})} \big\} \Big\} \\
&= 2\log M^{N_t} - \frac{1}{M^{2N_t}} \sum_{i,j=1}^{M^{N_t}} E_{\mathbf{h}}\Big\{ E_{\mathbf{n}}\big\{ \log \frac{\sum_{k,l=1}^{M^{N_t}} \exp(-\frac{1}{2}\mathbf{g}_{i,j,k,l}^T \Delta^{-1} \mathbf{g}_{i,j,k,l})}{\exp(-\frac{1}{2}\mathbf{n}^T \Delta^{-1} \mathbf{n})} \big\} \Big\}. \qquad \square
\end{aligned}$$

In the calculation, we assume that the input signals are equiprobable, and $\mathbf{x}^u$'s and $\mathbf{h}$ are mutually independent.

Since the conditional mutual information $I(\mathbf{x}^1; \mathbf{y}|\mathbf{x}^2)$ and $I(\mathbf{x}^2; \mathbf{y}|\mathbf{x}^1)$ are symmetric, we only need to calculate one of them, which is given in the following proposition.

Proposition 3. *(Constrained conditional mutual information for 2-user MIMO systems)*

The constrained conditional mutual information with CSI equals

$$I(\mathbf{x}^2;\mathbf{y}|\mathbf{x}^1) = \log M^{N_t} - \frac{1}{M^{2N_t}} \sum_{i,j=1}^{M^{N_t}} E_{\mathbf{h}}\Big\{ E_{\mathbf{n}}\{ \log \frac{\sum_{l=1}^{M^{N_t}} \exp(-\frac{1}{2}\mathbf{g}_{i,j,l}^T \Delta^{-1}\mathbf{g}_{i,j,l})}{\exp(-\frac{1}{2}\mathbf{n}^T\Delta^{-1}\mathbf{n})} \} \Big\}, \tag{3.5}$$

where $\mathbf{g}_{i,j,l} = \mathbf{hx}_{i,j} + \mathbf{n} - \mathbf{hx}_{i,l}$.

Proof : Proof is the same as Proposition 2.

3.3 CAPACITY CALCULATION RESULTS

The constrained capacities of the SISO channel for various constellations appear in Figure 12. Figure 13 and Figure 14 show the constrained capacities for various constellations in a single-user MIMO system with 2×2 antennas and 4×2 antennas, respectively. Here 2×2 antennas mean that the number of transmit antennas and receive antennas are both two. The constrained capacity calculation for single-user MIMO systems is also independently shown in [13].

From the comparison of Figure 12, Figure 13 and Figure 14, we can see that the capacity can be significantly increased by using multiple antennas. For example, for 4-QAM modulation, transmission rate of 2 bits/channel-use occurs at about SNR=20 dB for 1×1 antenna; For 2×2 antennas, the rate of 2 bits/channel-use happens about at SNR=2 dB, and for 4×2 antennas the SNR is about at 0 dB. To achieve the same rate, a MIMO system requires less power than a SISO sysetm. Also, we can compare the rates by fixing SNR. Fixing SNR=10 dB, the achievable rates are 1.75 bits/channel-use for 1×1 antenna, 3.75 bits/channel-use for 2×2 antennas, and 5.8 bits/channel-use for 4×2 antennas. This demonstrates that we can transmit signal with higher rate for MIMO systems.

Also, we compare the constrained capacity when CSI is known or unknown at the receiver. Figure 15 shows that the channel capacity is increased when the receiver can estimate the channel state information.

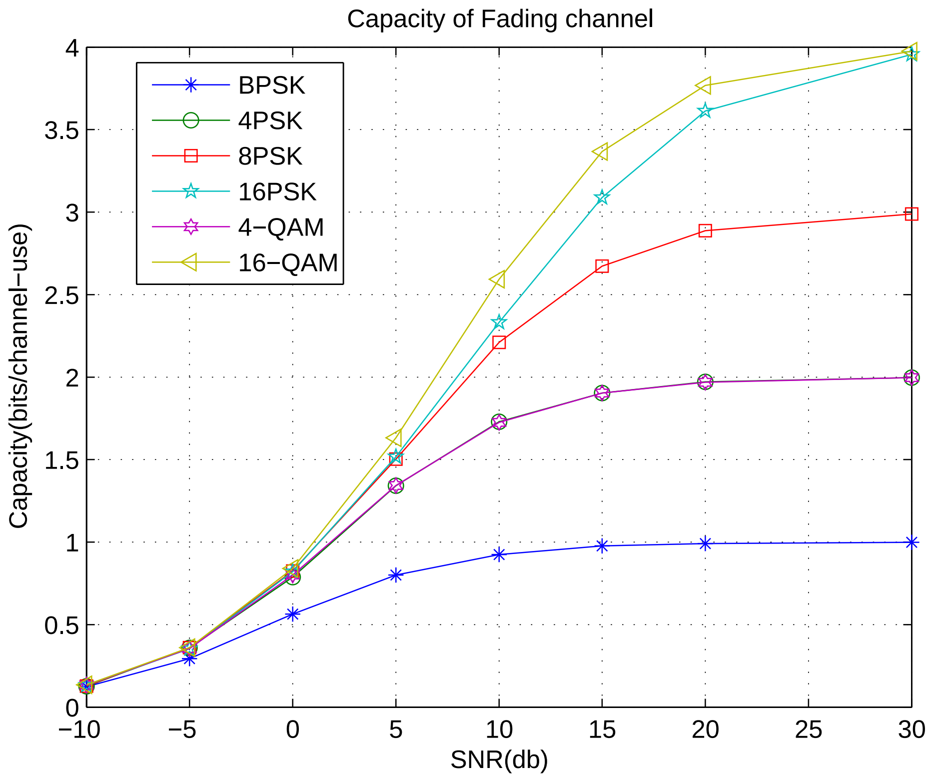

Figure 12: Channel capacity: 1×1 SISO channel

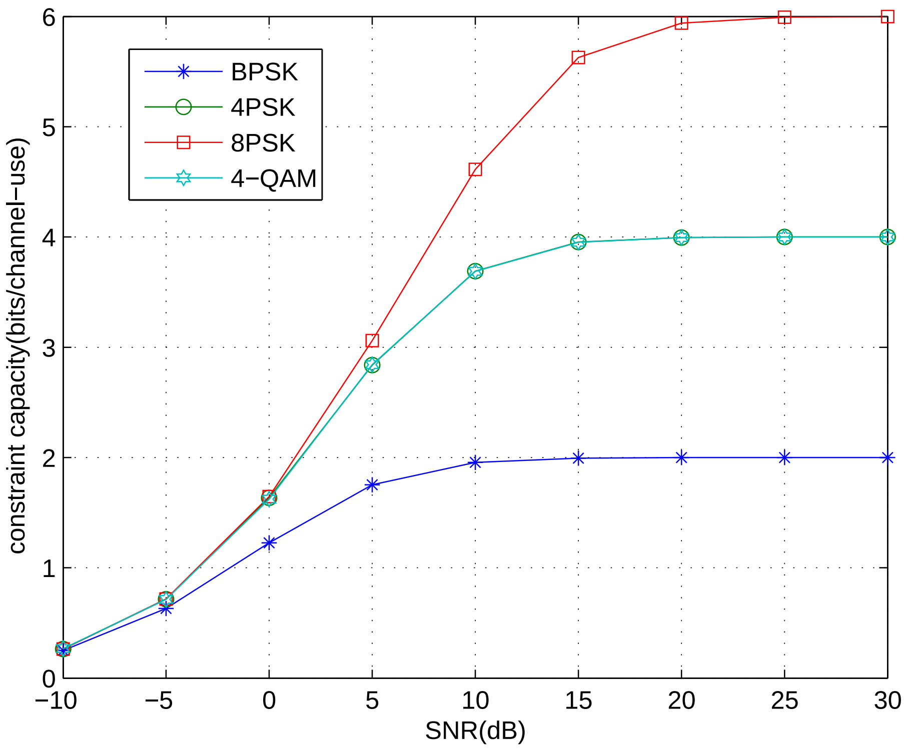

Figure 13: Channel capacity: 2×2 MIMO channel

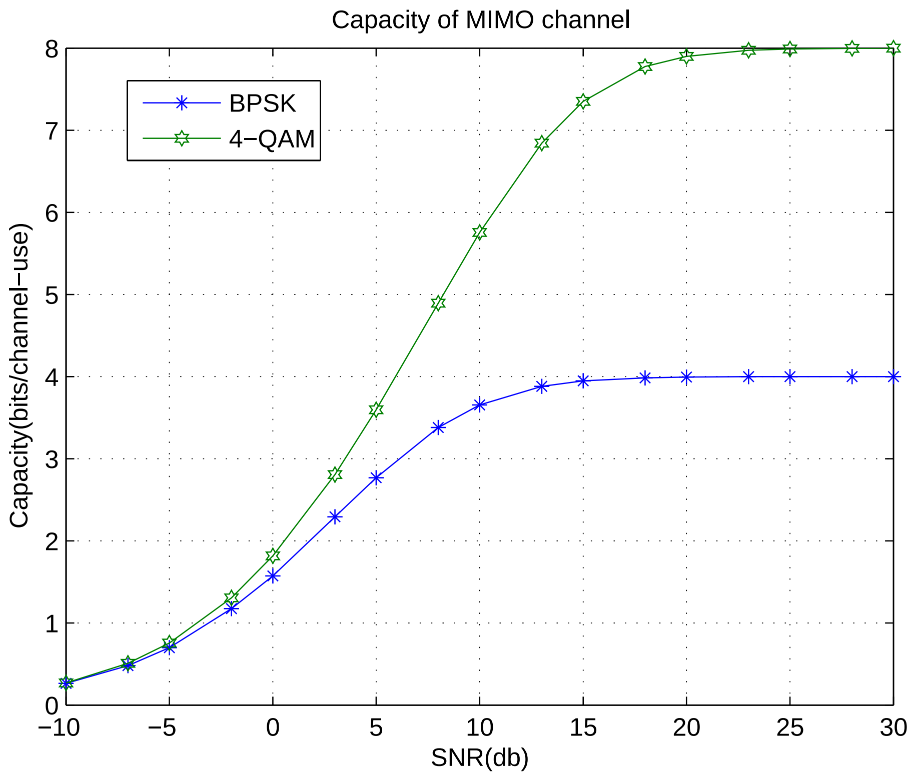

Figure 14: Channel capacity: 4×2 MIMO channel

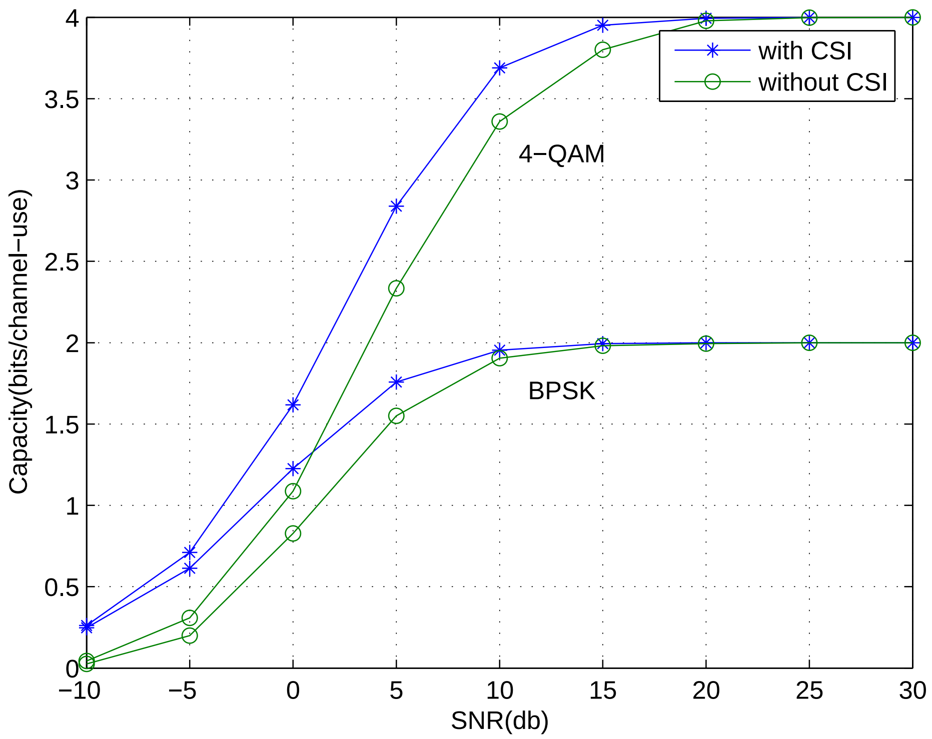

Figure 15: Capacity comparison: with CSI and without CSI for 2×2 MIMO channel

Figure 16 and Figure 17 demonstrate the constrained capacity regions for BPSK modulation and 4-QAM modulation, respectively. In this multiuser MIMO system, each sender is equipped with two transmit antennas and the receiver with two receive antennas.

From Figure 16 and Figure 17, we can see that the shape of capacity region changes from rectangle to pentagon, and from pentagon back to rectangle with SNR increasing. To simultaneously achieve high data rates for both senders, rectangle capacity region is desirable.

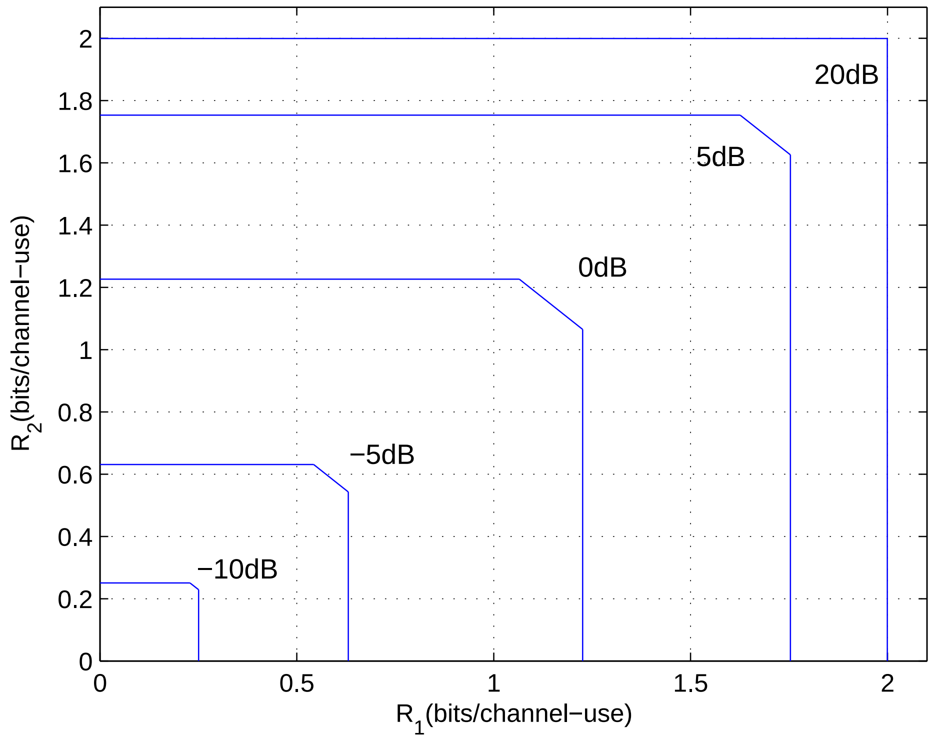

Figure 16: Capacity region of 2-user MIMO systems: BPSK modulation case

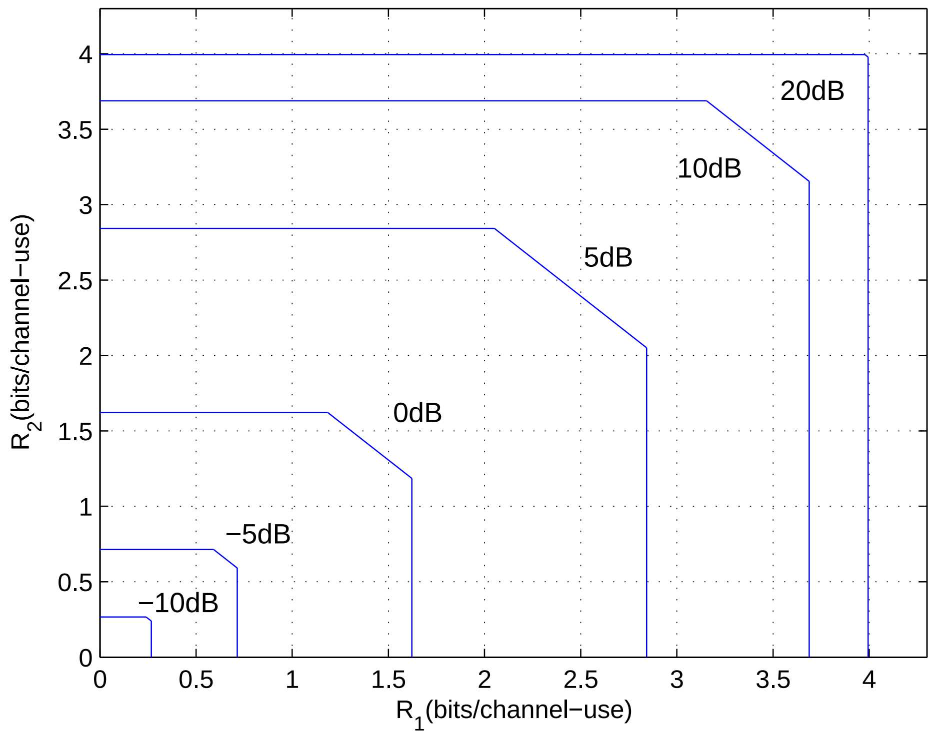

Figure 17: Capacity region of 2-user MIMO systems: 4-QAM modulation case

4.0 PERFORMANCE ANALYSIS FOR LDPC CODED MODULATION SCHEME IN FAST FADING MULTIUSER MIMO SYSTEMS

In this chapter, we derive a union upper bound for the LDPC coded modulation scheme in the multiuser MIMO system, which can be compared with the simulated system performance as a benchmark. We first discuss the properties of an ensemble of LDPC codes. Next the union upper bound is derived for multiuser MIMO systems. Finally, we investigate the union upper bound under a variety of scenarios.

4.1 DISCUSSION ON THE ENSEMBLE OF LDPC CODES

Error performance of coded modulation systems is very complex and difficult to obtain exact expressions. As specific codes are even harder to be evaluated, the performance of the ensembles of codes is considered in this research.

Both senders use the codes from the same ensemble; thus, they share the same properties of the ensemble of LDPC codes, which will be utilized in the following derivation of union upper bounds. The transformation of an LDPC code into a space-time code is shown in Figue 18.

Let $\mathcal{H}$ be the ensemble of the low-density parity-check matrices H, each of which defines an (N,J,K) LDPC code. Let $\mathcal{C}$ be the ensemble of (N,J,K) LDPC codes defined by $\mathcal{H}$. The ensemble $\mathcal{H}$ is closed under the column permutation. That is, a column permutation of a particular $H \in \mathcal{H}$ produces another low density parity check matrix belong to the same ensemble. Accordingly, a permutation of a certain codeword in a particular codebook is a codeword in another codebook in the ensemble $\mathcal{C}$. Let $\mathcal{C}_d$ be the set of all codewords with

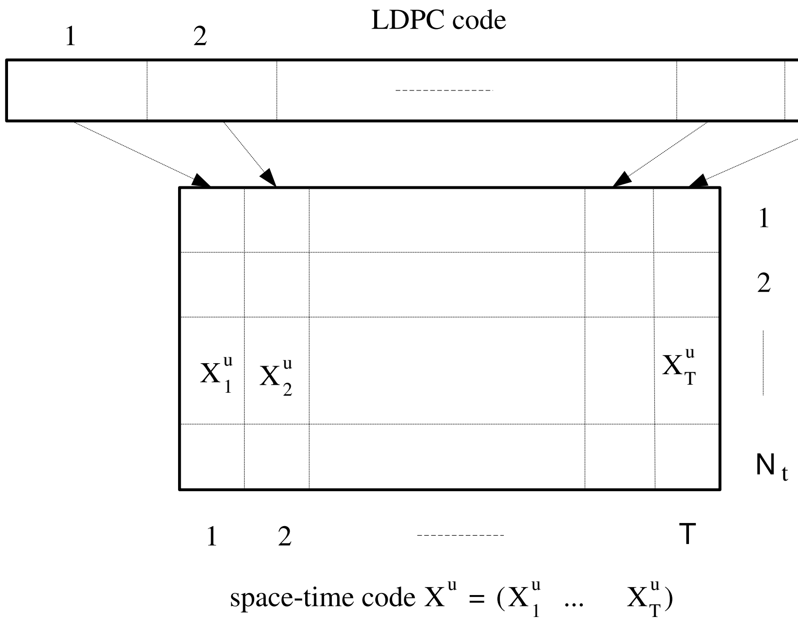

Figure 18: Transformation of an LDPC code into a space-time code.

Hamming distance d from any codebook in the ensemble $\mathcal{C}$, and denote the corresponding set of space-time symbol matrices as $\mathcal{X}_d$.

We assume that every code $C \in \mathcal{C}$ is equi-probably selectable; so does every codeword $\mathbf{c} \in C$. Thus, each bit within a randomly selected codeword in $\mathcal{C}_d$ can be modelled as a Bernouli distributed random variable with the parameter d/N. We will refer to this as the equiprobable property of the ensemble of LDPC codes.

Next we will introduce the notion of an ensemble-averaged distance distribution of LDPC codes. Litsyn and Shevelevin propose a number of ways to calculate the distance distributions in [19]. For a particular code $C \in \mathcal{C}$, the distance distribution is defined as:

$$\mathbf{S}(C) := \big(S_0(C) = 1, S_1(C), \cdots, S_N(C)\big),$$

where $S_d(C) = |\{\mathbf{c} \in C : \theta(\mathbf{c}) = d\}|$ for $d = 0, \cdots, N$, where $\theta(\cdot)$ denotes the Hamming weight and $|\cdot|$ denotes the cardinality of the set. Then, the ensemble-averaged distance distribution can be defined as:

$$\mathbf{S}(\mathcal{C}) := \big(S_0(\mathcal{C}), S_1(\mathcal{C}), \cdots, S_N(\mathcal{C})\big), \text{ where } S_d(\mathcal{C}) = \frac{1}{|\mathcal{C}|} \sum_{C \in \mathcal{C}} S_d(C) = \frac{|\mathcal{C}_d|}{|\mathcal{C}|}.$$

4.2 UNION UPPER BOUND FOR LDPC CODED MODULATION SCHEME

In this section, we derive a union upper bound for the LDPC coded modulation scheme in the multiuser MIMO system. We first calculate the pairwise error probability by averaging over the channel fading ensemble. Based on it, we further compute the pairwise error probability averaged over the column distance distribution, which will be discussed in detail in the following Section 4.2.2. The union upper bound is calculated by summing up the averaged pairwise error probabilities for the binary modulation scheme. Then, the derivation of union upper bound is extended to the case of an M-ary modulation scheme. Finally, we provide an example to illustrate the calculation of the union upper bound.

4.2.1 Pairwise error probability averaged over fast fading channel state

Let $\mathbf{x}_{(d_1,d_2)} := \begin{pmatrix} \mathbf{x}^1_{(d_1)} \\ \mathbf{x}^2_{(d_2)} \end{pmatrix}$ denote the space-time symbol matrix, where $\mathbf{x}^1_{(d_1)}$ is mapped from the codeword with Hamming weight d_1 for user 1 and $\mathbf{x}^2_{(d_2)}$ mapped from the codeword with Hamming weight d_2 for user 2. Assume the all-zero codewords are transmitted for both senders. We consider the probability of transmitting $\mathbf{x}_{(0,0)}$ in favor of deciding $\mathbf{x}_{(d_1,d_2)}$, conditioned on the channel state. This pairwise error probability based on ML detection can be upper bounded by using the Chernoff bound

$$p\big(\mathbf{x}_{(0,0)} \to \mathbf{x}_{(d_1,d_2)}|\mathbf{h}\big) \leqslant \exp\left(-\frac{d^2\big(\mathbf{x}_{(0,0)}\,,\mathbf{x}_{(d_1,d_2)}\big)}{4N_0}\right), \tag{4.1}$$

where

$$d^2\big(\mathbf{x}_{(0,0)}\,,\mathbf{x}_{(d_1,d_2)}\big) = \sum_{t=1}^{T} \|\mathbf{h}^t\mathbf{x}_{t,(0,0)} - \mathbf{h}^t\mathbf{x}_{t,(d_1,d_2)}\|_F^2;$$

$\mathbf{x}_{t,(0,0)}$ and $\mathbf{x}_{t,(d_1,d_2)}$ are the t-th column of $\mathbf{x}_{(0,0)}$ and $\mathbf{x}_{(d_1,d_2)}$ respectively; F denotes the Frobenius norm.

To calculate the upper bound on the pairwise error probability averaged over the channel state, we take the average of the R.H.S of (4.1) with respect to the channel fading matrix $\mathbf{h}$. For single-user MIMO systems, the pairwise error probability averaged on the channel state can refer to [26]. For multiuser MIMO systems, the detailed derivation of pairwise error probability averaged on channel state can refer to Appendix A. Then, the pairwise error probability can be bounded by

$$\begin{aligned} p\big(\mathbf{x}_{(0,0)} \to \mathbf{x}_{(d_1,d_2)}\big) &\leqslant \prod_{t=1}^{T} \left(1 + \frac{|\mathbf{x}_{t,(0,0)} - \mathbf{x}_{t,(d_1,d_2)}|^2}{4N_0}\right)^{-N_r} \\ &= \prod_{t=1}^{T} \left(1 + |\bar{\mathbf{x}}_{t,(0,0)} - \bar{\mathbf{x}}_{t,(d_1,d_2)}|^2 \rho\right)^{-N_r} \end{aligned} \tag{4.2}$$

where $\bar{\mathbf{x}}_{t,(0,0)} = \frac{1}{\sqrt{E_s}}\mathbf{x}_{t,(0,0)}$, $\bar{\mathbf{x}}_{t,(d_1,d_2)} = \frac{1}{\sqrt{E_s}}\mathbf{x}_{t,(d_1,d_2)}$ and $\rho = \frac{E_s}{4N_0}$.

To illustrate the reasoning clearly, we first consider the BPSK mapping case. Let $w_t^u := \frac{|\bar{\mathbf{x}}^u_{t,(0)} - \bar{\mathbf{x}}^u_{t,(d_u)}|^2}{4}$ denote the *weight* of the t-th column error of user u, for time epoch $t = 1, \cdots, T$. The weight w_t^u takes a value from the set of $\{0, \cdots, N_t\}$ and thus it has at most N_t+1 different values. Then, (4.2) can be re-written as

$$p\big(\mathbf{x}_{(0,0)} \to \mathbf{x}_{(d_1,d_2)}\big) \leqslant \prod_{t=1}^{T} \Big(1 + 4(w_t^1 + w_t^2)\rho\Big)^{-N_r}. \tag{4.3}$$

We notice that the pairwise error probability is determined by T *column weight pairs* (w_t^1, w_t^2).

4.2.2 Pairwise error probability averaged over column distance distribution

We observe that the columns with the identical column weight pair (w_t^1, w_t^2) result in the same term in the product of (4.3). Thus, we let $l_{i,j}$ denote the number of columns of the difference matrix $(\bar{\mathbf{x}}_{(0,0)} - \bar{\mathbf{x}}_{(d_1,d_2)})$, each of which has weight i from user 1 and weight j from user 2. That is,

$$l_{i,j} := \sum_{t=1}^{T} \mathbf{1}_t \tag{4.4}$$

where $\mathbf{1}_t$ is the indicator function of t, which is either 1 if $(w_t^1, w_t^2) = (i, j)$ or 0 otherwise. This operation is shown in Figure 19.

By grouping the identical terms, (4.3) can be further simplified as

$$p\big(\mathbf{x}_{(0,0)} \to \mathbf{x}_{(d_1,d_2)}\big) \leqslant \prod_{i=0}^{N_t} \prod_{j=0}^{N_t} \Big(1 + 4(i+j)\rho\Big)^{-N_r l_{i,j}}. \tag{4.5}$$

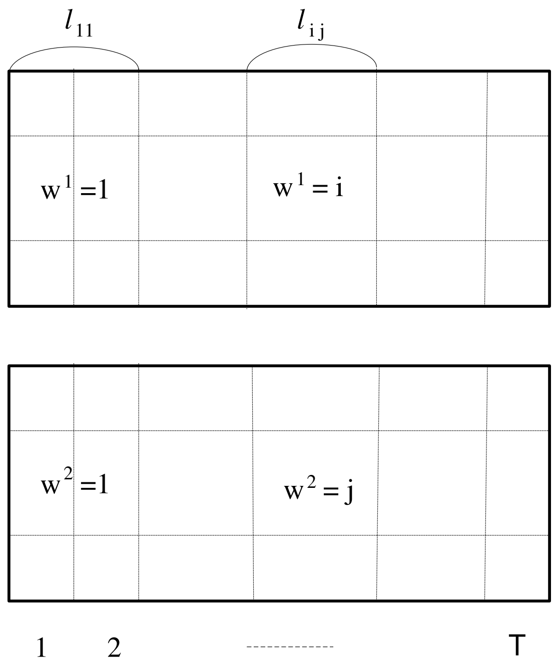

Figure 19: Grouping of the columns with the same weight pair.

Now we aim to compute the average pairwise error probability for any $\mathbf{x}_{(d_1,d_2)} \in \mathcal{X}_{d_1,d_2}$, where $\mathcal{X}_{d_1,d_2} := \begin{pmatrix} \mathcal{X}_{d_1} \\ \mathcal{X}_{d_2} \end{pmatrix}$. Collect $l_{i,j}$'s into an $(N_t+1) \times (N_t+1)$-matrix, which is denoted as $\mathbf{L}$. Let us name $\mathbf{L}$ as the *column distance distribution*(CD) matrix. Also, define $\mathcal{L}_{d_1,d_2}$ as the collection of all CD matrices, i.e.,

$$\mathcal{L}_{d_1,d_2} := \Big\{\mathbf{L} \,|\, l_{i,j} \in \{0,1,\cdots,T\}, \sum_{i=0}^{N_t}\sum_{j=0}^{N_t} l_{i,j} = T, \sum_{i=0}^{N_t} i l_{i,j} = d_1, \sum_{j=0}^{N_t} j l_{i,j} = d_2\Big\}.$$

That is, each CD matrix in the set $\mathcal{L}_{d_1,d_2}$ must satisfy the following constraints:

- $\sum_{i=0}^{N_t}\sum_{j=0}^{N_t} l_{i,j} = T$:
  the total number of columns should add up to T;
- $\sum_{i=0}^{N_t} i l_{i,j} = d_1$ and $\sum_{j=0}^{N_t} j l_{i,j} = d_2$:
  the weight of the first user's codeword and that of the second user's should add up to d_1 and d_2, respectively.

Applying the equiprobable property of the ensemble of the codes, the probability of a given $\mathbf{L}$ is the number of selections satisfying $\theta(\mathbf{c}^u) = d_u$ for a particular $\mathbf{L}$ divided by the total number of such selections for any $\mathbf{L}$. Using the usual combinatorial techniques, we obtain the probability distribution of $\mathbf{L}$ as:

$$p(\mathbf{L}) = \begin{cases} \binom{T}{\mathbf{L}} \prod_{i=0}^{N_t} \prod_{j=0}^{N_t} \binom{N_t}{i}^{l_{i,j}} \binom{N_t}{j}^{l_{i,j}} \prod_{u=1}^{2} \binom{N}{d_u}^{-1}, & \text{if } \mathbf{L} \in \mathcal{L}_{d_1,d_2} \\ 0, & \text{otherwise} \end{cases} \tag{4.6}$$

where $\binom{T}{\mathbf{L}}$ is the multinomial coefficient, i.e.,

$$\binom{T}{\mathbf{L}} = \frac{T\,!}{\prod_{i=0}^{N_t} \prod_{j=0}^{N_t} l_{i,j}\,!}.$$

The multinomial coefficient, $\binom{T}{\mathbf{L}}$, denotes the number of every possible way that the T distinct columns can be partitioned into $(N_t+1)^2$ unordered subsets. The (i,j)-th subset has $l_{i,j}$ columns, where (i,j) with $i = 0, \cdots, N_t; j = 0, \cdots, N_t$ is the index of the subset.

Thus, we obtain the upper bound of the pairwise error probability averaged over the *column distance distribution* as following:

$$\overline{p\big(\mathbf{x}_{(0,0)} \to \mathbf{x}_{(d_1,d_2)}\big)} = \sum_{\mathbf{L} \in \mathcal{L}_{d_1,d_2}} p\big(\mathbf{x}_{(0,0)} \to \mathbf{x}_{(d_1,d_2)}\big) p(\mathbf{L})$$

$$\leqslant \sum_{\mathbf{L} \in \mathcal{L}_{d_1,d_2}} \binom{T}{\mathbf{L}} \prod_{i=0}^{N_t} \prod_{j=0}^{N_t} \binom{N_t}{i}^{l_{i,j}} \binom{N_t}{j}^{l_{i,j}} \Big(1 + 4(i+j)\rho\Big)^{-N_r l_{i,j}} \prod_{u=1}^{2} \binom{N}{d_u}^{-1}. \tag{4.7}$$

4.2.3 Union upper bound for LDPC coded modulation in MIMO multiple access systems: BPSK case

The union bound on the word error probability for ML detection is to sum up the average pairwise error probabilities of (4.7), each of which is weighted by the ensemble-averaged distance distribution. In a MIMO multiple access system, a pairwise error happens if the decoder is in favor of $\mathbf{x}_{(d_1,d_2)}$, which is not equal to $\mathbf{x}_{(0,0)}$. Applying the ensemble-averaged distance distribution property, the average word error probability in a MIMO multiple access system can be upper bounded by

$$\overline{P_e} \leqslant \sum_{d_1=0}^{N}\sum_{d_2=0}^{N} S_{d_1}S_{d_2}\overline{p\big(\mathbf{x}_{(0,0)} \rightarrow \mathbf{x}_{(d_1,d_2)}\big)} - S_0^2\overline{p\big(\mathbf{x}_{(0,0)} \rightarrow \mathbf{x}_{(0,0)}\big)}, \tag{4.8}$$

where $S_{d_u} = S_{d_u}(\mathcal{C})$ for $u = 1, 2$.

By applying(4.7) and defining $\alpha_{i,j}$ and Φ_{d_1,d_2}, (4.8) can be written as

$$\begin{aligned}\overline{P_e} &\leqslant \sum_{d_1=0}^{N}\sum_{d_2=0}^{N}\binom{N}{d_1}^{-1}\binom{N}{d_2}^{-1} S_{d_1}S_{d_2}\sum_{\mathbf{L}\in\mathcal{L}_{d_1,d_2}}\binom{T}{\mathbf{L}}\prod_{i=0}^{N_t}\prod_{j=0}^{N_t}(\alpha_{i,j})^{l_{i,j}} - S_0^2 \\ &= \sum_{d_1=0}^{N}\sum_{d_2=0}^{N}\binom{N}{d_1}^{-1}\binom{N}{d_2}^{-1} S_{d_1}S_{d_2}\Phi_{d_1,d_2} - S_0^2,\end{aligned} \tag{4.9}$$

where

$$\alpha_{i,j} := \binom{N_t}{i}\binom{N_t}{j}\Big(1 + 4(i+j)\rho\Big)^{-N_r}, \tag{4.10}$$

and

$$\Phi_{d_1,d_2} := \sum_{\mathbf{L}\in\mathcal{L}_{d_1,d_2}}\binom{T}{\mathbf{L}}\prod_{i=0}^{N_t}\prod_{j=0}^{N_t}(\alpha_{i,j})^{l_{i,j}}. \tag{4.11}$$

To evaluate the R.H.S of (4.9) efficiently, we resort to the method of a polynomial expansion. Let $\mathbf{L}$ denote a square matrix with $(N_t+1)\times(N_t+1)$ elements, the following equation holds:

$$\Big(\sum_{i=0}^{N_t}\sum_{j=0}^{N_t} x_{i,j}\Big)^T = \sum_{\mathbf{L}\in\mathcal{L}}\binom{T}{\mathbf{L}}\prod_{i=0}^{N_t}\prod_{j=0}^{N_t}(x_{i,j})^{l_{i,j}}, \tag{4.12}$$

where

$$\mathcal{L} := \Big\{\mathbf{L}\,|\, l_{i,j} \in \{0, \cdots, T\}, \sum_{i=0}^{N_t}\sum_{j=0}^{N_t} l_{i,j} = T\Big\}.$$

By applying (4.12), we get

$$\begin{aligned}\Big(\sum_{i=0}^{N_t}\sum_{j=0}^{N_t}\alpha_{i,j}\, y^i z^j\Big)^T &= \sum_{\mathbf{L}\in\mathcal{L}}\binom{T}{\mathbf{L}}\prod_{i=0}^{N_t}\prod_{j=0}^{N_t}\left(\alpha_{i,j}\, y^i z^j\right)^{l_{i,j}}\\ &= \sum_{\mathbf{L}\in\mathcal{L}}\binom{T}{\mathbf{L}} y^{\sum_{i=0}^{N_t} i\, l_{i,j}} z^{\sum_{j=0}^{N_t} j\, l_{i,j}}\prod_{i=0}^{N_t}\prod_{j=0}^{N_t}(\alpha_{i,j})^{l_{i,j}}\\ &= \sum_{d_1=0}^{N}\sum_{d_2=0}^{N}\sum_{\mathbf{L}\in\mathcal{L}_{d_1,d_2}}\binom{T}{\mathbf{L}}\prod_{i=0}^{N_t}\prod_{j=0}^{N_t}(\alpha_{i,j})^{l_{i,j}} y^{d_1} z^{d_2}\\ &= \sum_{d_1=0}^{N}\sum_{d_2=0}^{N}\Phi_{d_1,d_2} y^{d_1} z^{d_2}, \end{aligned} \tag{4.13}$$

where $N = N_t T$.

Then applying (4.13) to (4.9), Φ_{d_1,d_2} can be calculated by collecting the coefficients $\alpha_{i,j}$'s in $\left(\sum_{i=0}^{N_t}\sum_{j=0}^{N_t}\alpha_{i,j}\, y^i z^j\right)^T$.

Using the equiprobable property of the ensemble of the codes again, we can show that the union upper bound for bit error probability as the following:

$$\overline{P_b} \leqslant \sum_{d_1=0}^{N}\sum_{d_2=0}^{N}\frac{d_1}{N}\frac{d_2}{N}\binom{N}{d_1}^{-1}\binom{N}{d_2}^{-1} S_{d_1} S_{d_2}\Phi_{d_1,d_2}. \tag{4.14}$$

4.2.4 Union upper bound for LDPC coded modulation in MIMO multiple access systems: M-ary case

The previous analysis is based on LDPC space-time code with the BPSK modulation. For an M-ary modulation, we need to re-calculate the average pairwise error probability by re-considering the distribution of $\mathbf{L}$. We take the following approach.

Each entry of $\bar{\mathbf{x}}_{(d_1,d_2)}$ is selected from the M-ary symbols $\{s_i\}_{i=0}^{M-1}$. We have s_0 mapped from the string of all-zero bits of length $\log_2(M)$. Let δ_i denote the Hamming weight of the bits mapped to s_i, and r_i^u the number of s_i's contained in a particular column of $\bar{\mathbf{x}}^u_{(d_u)}$ for $u = 1, 2$. Collect δ_i's and r_i^u's into M-tuples, i.e., $\delta = (\delta_0, \cdots, \delta_{M-1})$ and $\mathbf{r}^u = (r_0^u, \cdots, r_{M-1}^u)$. Then, the column weight can be obtained by the inner product of the two, i.e., $w_t^u = \mathbf{r}^u \cdot \delta$, for $t = 1, \cdots, T$. Each column weight w_t^u takes a value from the set $\{0, \cdots, N_t \log_2(M)\}$.

Note the cardinality of this set is $N_t \log_2(M) + 1$. We also note that the column weight w_t^u is completely determined by $\mathbf{r}^u$ for a fixed constellation mapping rule (i.e. for a fixed δ).

We note that any two columns with an identical column weight pair (w_t^1, w_t^2) produce the same term in the product of the pairwise error probability. Thus, we denote $l_{\mathbf{r}^1,\mathbf{r}^2}$ as the number of the columns in the difference matrix $(\bar{\mathbf{x}}_{(0,0)} - \bar{\mathbf{x}}_{(d_1,d_2)})$, each of which has a weight tuple $\mathbf{r}^1$ from user 1 and a weight tuple $\mathbf{r}^2$ from user 2, i.e., it can be written as

$$l_{\mathbf{r}^1,\mathbf{r}^2} := \sum_{t=1}^{T} \mathbf{1}_t, \tag{4.15}$$

where $\mathbf{1}_t = 1$ if $(w_t^1, w_t^2) = (\mathbf{r}^1 \cdot \delta, \mathbf{r}^2 \cdot \delta)$ and 0 otherwise. By grouping the identical terms together, the expression of the upper bound on the pairwise error probability can be written as

$$p\big(\mathbf{x}_{(0,0)} \to \mathbf{x}_{(d_1,d_2)}\big) \leqslant \prod_{\mathbf{r}^1\in\mathcal{R}_1} \prod_{\mathbf{r}^2\in\mathcal{R}_2} \Big(1 + \sum_{u=1}^{2} \sum_{i=0}^{M-1} r_i^u |s_i - s_0|^2 \rho\Big)^{-N_r l_{\mathbf{r}^1,\mathbf{r}^2}} \tag{4.16}$$

We aim to compute the average pairwise error probability for any $\mathbf{x}_{(d_1,d_2)} \in \mathcal{X}_{d_1,d_2}$. Let us collect $l_{\mathbf{r}^1,\mathbf{r}^2}$'s into an $(N_t \log_2(M) + 1) \times (N_t \log_2(M) + 1)$-matrix $\mathbf{L}$. Define $\mathcal{L}_{d_1,d_2}$ as the collection of all such matrices that satisfy a set of constraints, i.e.,

$$\mathcal{L}_{d_1,d_2} := \Big\{\mathbf{L} \,|\, l_{\mathbf{r}^1,\mathbf{r}^2} \in \{0, 1, \cdots, T\}, \sum_{\mathbf{r}^1\in\mathcal{R}_1} \sum_{\mathbf{r}^2\in\mathcal{R}_2} l_{\mathbf{r}^1,\mathbf{r}^2} = T, \\ \sum_{\mathbf{r}^1\in\mathcal{R}_1} (\mathbf{r}^1 \cdot \delta)\, l_{\mathbf{r}^1,\mathbf{r}^2} = d_1, \sum_{\mathbf{r}^2\in\mathcal{R}_2} (\mathbf{r}^2 \cdot \delta)\, l_{\mathbf{r}^1,\mathbf{r}^2} = d_2\Big\},$$

where $\mathcal{R}_u = \big\{\mathbf{r}^u \,|\, r_i^u \in \{0, 1, \cdots, N_t\}, \sum_{i=0}^{M-1} r_i^u = N_t\big\}$ for $u = 1, 2$.

Then, the distribution of $\mathbf{L}$ is as following:

$$p\,(\mathbf{L}) = \begin{cases} \binom{T}{\mathbf{L}} \prod_{u=1}^{2} \binom{N}{d_u}^{-1} \prod_{\mathbf{r}^u\in\mathcal{R}_u} \binom{N_t}{\mathbf{r}^u}^{l_{\mathbf{r}^1,\mathbf{r}^2}}, & \text{if}\, \mathbf{L} \in \mathcal{L}_{d_1,d_2} \\ 0, & \text{otherwise} \end{cases} \tag{4.17}$$

where

$$\binom{T}{\mathbf{L}} = \frac{T\,!}{\prod_{\mathbf{r}^1\in\mathcal{R}_1} \prod_{\mathbf{r}^2\in\mathcal{R}_2} l_{\mathbf{r}^1,\mathbf{r}^2}\,!},$$

and

$$\binom{N_t}{\mathbf{r}^u} = \frac{N_t\,!}{\prod_{i=0}^{M-1} r_i^u\,!}.$$

Thus, the upper bound of the average pairwise error probability can be obtained as

$$\begin{aligned}
\overline{p\big(\mathbf{x}_{(0,0)} \to \mathbf{x}_{(d_1,d_2)}\big)} &= \sum_{\mathbf{L}\in\mathcal{L}_{d_1,d_2}} p\big(\mathbf{x}_{0,0} \to \mathbf{x}_{d_1,d_2}\big) p\left(\mathbf{L}\right) \\
&\leqslant \sum_{\mathbf{L}\in\mathcal{L}_{d_1,d_2}} \binom{T}{\mathbf{L}} \prod_{\mathbf{r}^1\in\mathcal{R}_1} \prod_{\mathbf{r}^2\in\mathcal{R}_2} \left(\binom{N_t}{\mathbf{r}^1}\binom{N_t}{\mathbf{r}^2}\right)^{l_{\mathbf{r}^1,\mathbf{r}^2}} \\
&\quad \times \Big(1 + \sum_{u=1}^{2}\sum_{i=0}^{M-1} r_i^u |s_i - s_0|^2 \rho\Big)^{-N_r l_{\mathbf{r}^1,\mathbf{r}^2}} \prod_{u=1}^{2} \binom{N}{d_u}^{-1}
\end{aligned} \tag{4.18}$$

By summing up the average pairwise error probabilities, we get the upper bound of the word error probability for multiuser space-time coded M-ary modulation:

$$\begin{aligned}
\overline{P_e} &\leqslant \sum_{d_1=0}^{N}\sum_{d_2=0}^{N} S_{d_1}S_{d_2}\overline{p\big(\mathbf{x}_{(0,0)} \to \mathbf{x}_{(d_1,d_2)}\big)} - S_0^2\overline{p\big(\mathbf{x}_{(0,0)} \to \mathbf{x}_{(0,0)}\big)} \\
&\leqslant \sum_{d_1=0}^{N}\sum_{d_2=0}^{N} \binom{N}{d_1}^{-1}\binom{N}{d_2}^{-1} S_{d_1}S_{d_2} \sum_{\mathbf{L}\in\mathcal{L}_{d_1,d_2}} \binom{T}{\mathbf{L}} \prod_{\mathbf{r}^1\in\mathcal{R}_1} \prod_{\mathbf{r}^2\in\mathcal{R}_2} \left(\beta_{\mathbf{r}^1,\mathbf{r}^2}\right)^{l_{\mathbf{r}^1,\mathbf{r}^2}} - S_0^2 \\
&= \sum_{d_1=0}^{N}\sum_{d_2=0}^{N} \binom{N}{d_1}^{-1}\binom{N}{d_2}^{-1} S_{d_1}S_{d_2}\Psi_{d_1,d_2} - S_0^2,
\end{aligned} \tag{4.19}$$

where

$$\beta_{\mathbf{r}^1,\mathbf{r}^2} := \binom{N_t}{\mathbf{r}^1}\binom{N_t}{\mathbf{r}^2}\Big(1 + \sum_{u=1}^{2}\sum_{i=0}^{M-1} r_i^u |s_i - s_0|^2\rho\Big)^{-N_r}, \tag{4.20}$$

and

$$\Psi_{d_1,d_2} := \sum_{\mathbf{L}\in\mathcal{L}_{d_1,d_2}} \binom{T}{\mathbf{L}} \prod_{\mathbf{r}^1\in\mathcal{R}_1} \prod_{\mathbf{r}^2\in\mathcal{R}_2} \left(\beta_{\mathbf{r}^1,\mathbf{r}^2}\right)^{l_{\mathbf{r}^1,\mathbf{r}^2}}. \tag{4.21}$$

By applying (4.12), we get:

$$\begin{aligned}
&\Big(\sum_{\mathbf{r}^1\in\mathcal{R}_1}\sum_{\mathbf{r}^2\in\mathcal{R}_2}\beta_{\mathbf{r}^1,\mathbf{r}^2}\,y^{(\mathbf{r}^1\cdot\delta)}z^{(\mathbf{r}^2\cdot\delta)}\Big)^T\\
&=\sum_{\mathbf{L}\in\mathcal{L}}\binom{T}{\mathbf{L}}\prod_{\mathbf{r}^1\in\mathcal{R}_1}\prod_{\mathbf{r}^2\in\mathcal{R}_2}\Big(\beta_{\mathbf{r}^1,\mathbf{r}^2}\,y^{(\mathbf{r}^1\cdot\delta)}z^{(\mathbf{r}^2\cdot\delta)}\Big)^{l_{\mathbf{r}^1,\mathbf{r}^2}}\\
&=\sum_{\mathbf{L}\in\mathcal{L}}\binom{T}{\mathbf{L}}\Big(y^{\sum_{\mathbf{r}^1\in\mathcal{R}_1}(\mathbf{r}^1\cdot\delta)\,l_{\mathbf{r}^1,\mathbf{r}^2}}\Big)\Big(z^{\sum_{\mathbf{r}^2\in\mathcal{R}_2}(\mathbf{r}^2\cdot\delta)\,l_{\mathbf{r}^1,\mathbf{r}^2}}\Big)\prod_{\mathbf{r}^1\in\mathcal{R}_1}\prod_{\mathbf{r}^2\in\mathcal{R}_2}\Big(\beta_{\mathbf{r}^1,\mathbf{r}^2}\Big)^{l_{\mathbf{r}^1,\mathbf{r}^2}}\\
&=\sum_{d_1=0}^{N}\sum_{d_2=0}^{N}\sum_{\mathbf{L}\in\mathcal{L}_{d_1,d_2}}\binom{T}{\mathbf{L}}\prod_{\mathbf{r}^1\in\mathcal{R}_1}\prod_{\mathbf{r}^2\in\mathcal{R}_2}\Big(\beta_{\mathbf{r}^1,\mathbf{r}^2}\Big)^{l_{\mathbf{r}^1,\mathbf{r}^2}}y^{d_1}z^{d_2}\\
&=\sum_{d_1=0}^{N}\sum_{d_2=0}^{N}\Psi_{d_1,d_2}y^{d_1}z^{d_2},
\end{aligned}\tag{4.22}$$

where $N = N_t T\log_2 M$ and

$$\mathcal{L} := \Big\{\mathbf{L}\,|\,l_{\mathbf{r}^1,\mathbf{r}^2}\in\{0,1,\cdots,T\},\ \sum_{\mathbf{r}^1\in\mathcal{R}_1}\sum_{\mathbf{r}^2\in\mathcal{R}_2}l_{\mathbf{r}^1,\mathbf{r}^2}=T\Big\}.$$

Applying (4.22) to (4.19), Ψ_{d_1,d_2} can be evaluated by collecting the coefficients $\beta_{\mathbf{r}^1,\mathbf{r}^2}$'s in $\Big(\sum_{\mathbf{r}^1\in\mathcal{R}_1}\sum_{\mathbf{r}^2\in\mathcal{R}_2}\beta_{\mathbf{r}^1,\mathbf{r}^2}\,y^{(\mathbf{r}^1\cdot\delta)}z^{(\mathbf{r}^2\cdot\delta)}\Big)^T$.

Then, the union upper bound for bit error probability for space-time coded M-ary modulation is:

$$\overline{P_b}\leqslant\sum_{d_1=0}^{N}\sum_{d_2=0}^{N}\frac{d_1}{N}\frac{d_2}{N}\binom{N}{d_1}^{-1}\binom{N}{d_2}^{-1}S_{d_1}S_{d_2}\Psi_{d_1,d_2}.\tag{4.23}$$

4.2.5 An illustrative example

We provide an example to illustrate how to calculate the union upper bound. Consider a MIMO multiple access system with two senders under BPSK modulation, each of which is equipped with two transmit antennas, and the receiver with two receive antennas. By applying (5.18), we first form the $[3\times 3]$ matrix with elements $\alpha_{i,j}$, which are obtained as

$$\alpha := \begin{pmatrix}\alpha_{0,0} & \alpha_{0,1} & \alpha_{0,2}\\ \alpha_{1,0} & \alpha_{1,1} & \alpha_{1,2}\\ \alpha_{2,0} & \alpha_{2,1} & \alpha_{2,2}\end{pmatrix} = \begin{pmatrix}1 & 2(1+4\rho)^{-2} & (1+8\rho)^{-2}\\ 2(1+4\rho)^{-2} & 4(1+8\rho)^{-2} & 2(1+12\rho)^{-2}\\ (1+8\rho)^{-2} & 2(1+12\rho)^{-2} & (1+16\rho)^{-2}\end{pmatrix}$$

We take $\log_2(T)$-fold two-dimensional convolution to obtain the $[(N+1)\times(N+1)]$ coefficient matrix $[\Phi_{d_1,d_2}]$. For an illustration, suppose $N = 4$, and thus $T = 2$.

$$\begin{aligned}
&\begin{pmatrix} \alpha_{0,0} & \alpha_{0,1}z & \alpha_{0,2}z^2 \\ \alpha_{1,0}y & \alpha_{1,1}yz & \alpha_{1,2}yz^2 \\ \alpha_{2,0}y^2 & \alpha_{2,1}y^2z & \alpha_{2,2}y^2z^2 \end{pmatrix} * \begin{pmatrix} \alpha_{0,0} & \alpha_{0,1}z & \alpha_{0,2}z^2 \\ \alpha_{1,0}y & \alpha_{1,1}yz & \alpha_{1,2}yz^2 \\ \alpha_{2,0}y^2 & \alpha_{2,1}y^2z & \alpha_{2,2}y^2z^2 \end{pmatrix} \\
&= \begin{pmatrix} \gamma_{0,0} & \gamma_{0,1}z & \gamma_{0,2}z^2 & \gamma_{0,3}z^3 & \gamma_{0,4}z^4 \\ \gamma_{1,0}y & \gamma_{1,1}yz & \gamma_{1,2}yz^2 & \gamma_{1,3}yz^3 & \gamma_{1,4}yz^4 \\ \gamma_{2,0}y^2 & \gamma_{2,1}y^2z & \gamma_{2,2}y^2z^2 & \gamma_{2,3}y^2z^3 & \gamma_{2,4}y^2z^4 \\ \gamma_{3,0}y^3 & \gamma_{3,1}y^3z & \gamma_{3,2}y^3z^2 & \gamma_{3,3}y^3z^3 & \gamma_{3,4}y^3z^4 \\ \gamma_{4,0}y^4 & \gamma_{4,1}y^4z & \gamma_{4,2}y^4z^2 & \gamma_{4,3}y^4z^3 & \gamma_{4,4}y^4z^4 \end{pmatrix} \\
&= \gamma \cdot V,
\end{aligned} \tag{4.24}$$

where $*$ and $\cdot$ denote two-dimensional convolution and dot product for matrices, respectively;
$$\gamma = \begin{pmatrix} \gamma_{0,0} & \gamma_{0,1} & \gamma_{0,2} & \gamma_{0,3} & \gamma_{0,4} \\ \gamma_{1,0} & \gamma_{1,1} & \gamma_{1,2} & \gamma_{1,3} & \gamma_{1,4} \\ \gamma_{2,0} & \gamma_{2,1} & \gamma_{2,2} & \gamma_{2,3} & \gamma_{2,4} \\ \gamma_{3,0} & \gamma_{3,1} & \gamma_{3,2} & \gamma_{3,3} & \gamma_{3,4} \\ \gamma_{4,0} & \gamma_{4,1} & \gamma_{4,2} & \gamma_{4,3} & \gamma_{4,4} \end{pmatrix}, \text{ and } V = \begin{pmatrix} 1 & z & z^2 & z^3 & z^4 \\ y & yz & yz^2 & yz^3 & yz^4 \\ y^2 & y^2z & y^2z^2 & y^2z^3 & y^2z^4 \\ y^3 & y^3z & y^3z^2 & y^3z^3 & y^3z^4 \\ y^4 & y^4z & y^4z^2 & y^4z^3 & y^4z^4 \end{pmatrix}.$$

We note that $\gamma = \alpha * \alpha$, which are exactly the coefficients of expanded $\left(\sum_{i=0}^{2}\sum_{j=0}^{2}\alpha_{i,j}\,y^i z^j\right)^2$. For $T > 2$, we repeat the convolution operation for $\log_2(T)$ times.

4.3 BOUND CALCULATION RESULTS AND COMPARISONS

In this section, we provide the simulation results of the LDPC coded modulation scheme for the multiuser MIMO system depicted in Fig.6 and illustrate the performance of the turbo-iterative multiuser detection and decoding processing receiver. We first study the effect of *super* iteration and *internal* iteration. Next we introduce the thresholds obtained by extrinsic information transfer (EXIT) chart analysis [27]-[29], which can be used as a performance limit. Then we compare the simulated system performance with the union upper bound, the constrained capacity and the threshold. Finally, we investigate the union upper bound and system performance for an LDPC coded modulation scheme with explicit space-time coding.

4.3.1 Study on the effect of super iteration and internal iteration

As mentioned in Section 2.3, there are two kinds of iterations for the receiver. We investigate the optimal ratio of the number of *super* iterations (NSI) to the number of *internal* iterations (NII), given the total number of iterations (TNI). In simulations, each sender is equipped with two transmit antennas and the receiver with two receive antennas. The regular LDPC (1024,3,6) codes are used.

For compact description of the simulation results, Table I tabulates the required SNRs to achieve bit-error-rate (BER) of 10^{-4} at each option for the (1024,3,6) LDPC code with BPSK modulation in a multiuser MIMO system. The TNIs considered are 30, 60 and 120. For each TNI, we vary the ratio between the NSI and the NII and make a notice on the best option. We note that the performance benefit is about 0.3dB when the TNI varies from 30 to 60; the benefit becomes about 0.1dB when the TNI varies from 60 to 120. At the TNI of 60, the best combination is found to be 6 *super* iterations and 10 *internal* iterations.

Table 1: Comparison of SNRs to achieve BER of 10^{-4}

TNI	30	30	60	60	60	120
NSI	3	6	3	6	12	6
NII	10	5	20	10	5	20
SNR (dB)	1.5	2.1	1.4	1.2	2.1	1.1

In addition, we find from extensive simulations that increasing the NSI is beneficial when the number of bits per channel use is increased. For example, using 4-QAM modulation and fixing the TNI at 60, it is better off to have the NSI increased to twelve while the NII decreased to five. We try three cases: NSI and NII are (12,5), (6,10) and (3,20). Figure

20, Figure 21 and Figure 22 are the system performances for above three cases for 4-QAM modulation in a multiuser MIMO system. In what follows, we fix the TNI at 60 and use the best ratio obtained for each constellation option.

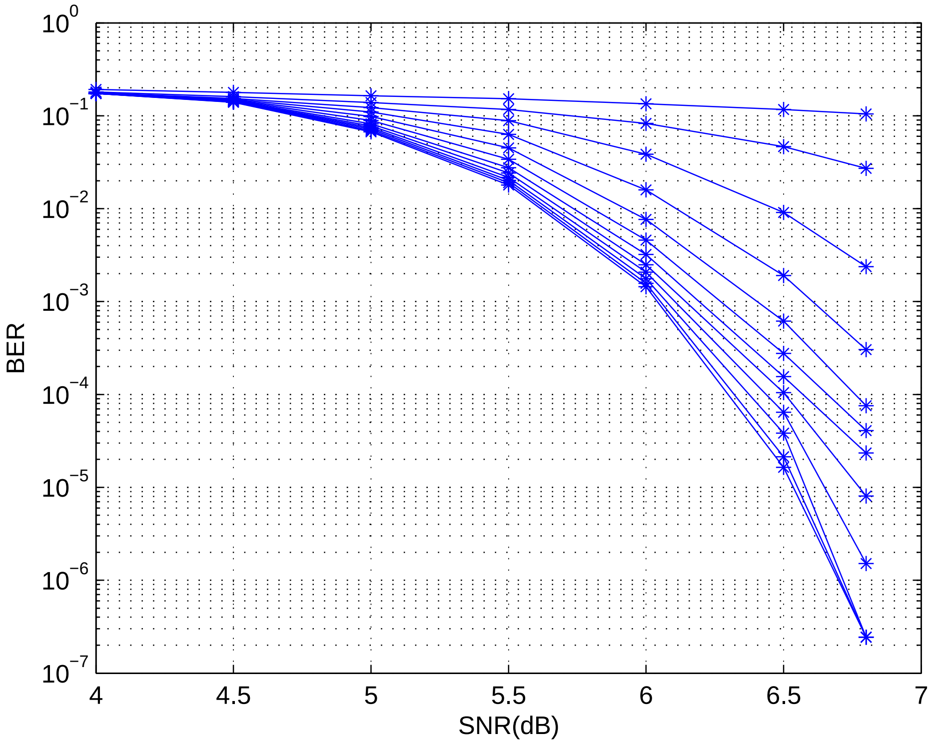

Figure 20: System performance for (NSI, NII)=(12, 5)

4.3.2 EXIT chart analysis

Besides bounding techniques, EXIT chart analysis is also an interesting tool to assess the system.

Typically, the bit error rate (BER) chart of iterative decoding can be divided into three regions:

- the low SNR region with negligible iterative BER reduction. This means that iteration does not work in low SNR region.

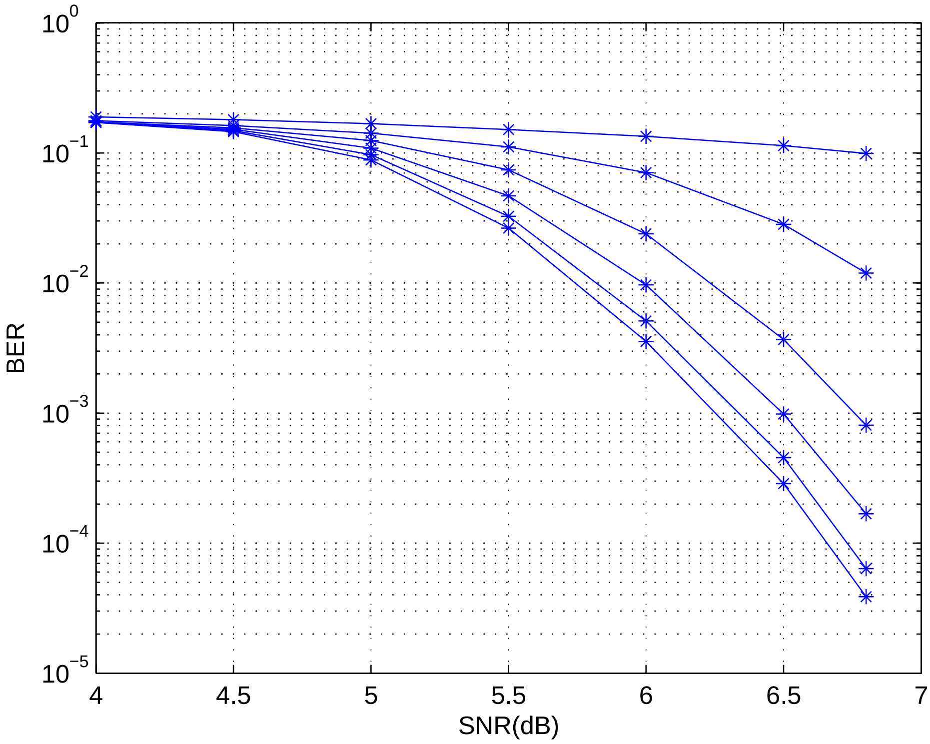

Figure 21: System performance for (NSI, NII)=(6, 10)

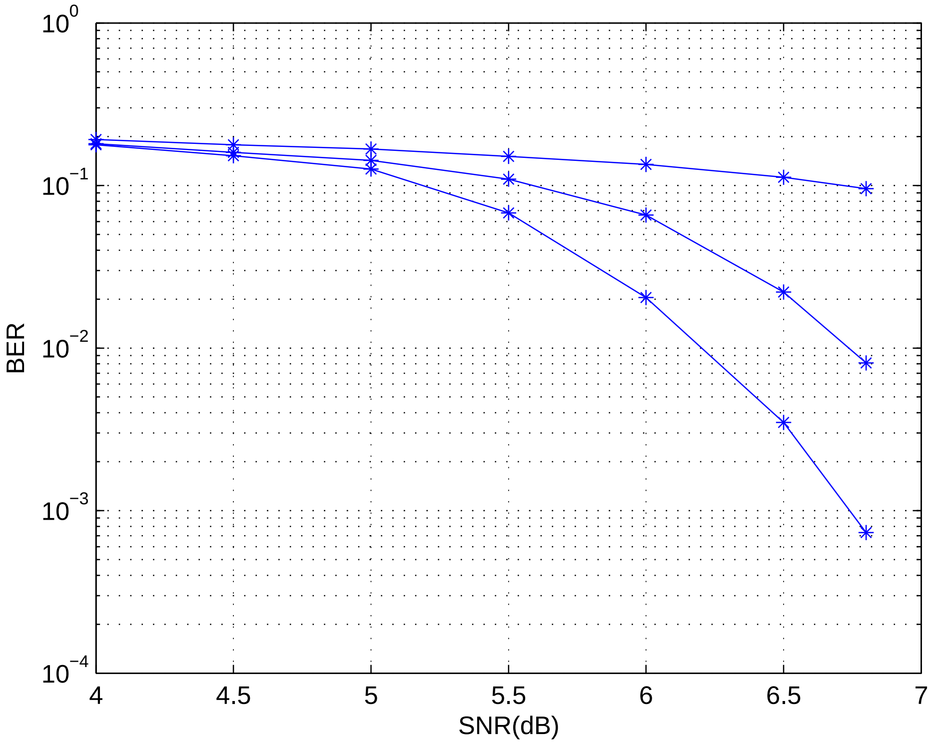

Figure 22: System performance for (NSI, NII)=(3, 20)

- the turbo cliff region with iterative BER reduction over iterations. This means that iteration start to work at this region.

- the BER floor region. BER can reach rather low after few iterations.

Using EXIT chart analysis, we try to find the SNR where turbo cliff happens, that is, the SNR at which the iteration start to work.

For EXIT chart analysis, we note that, there are at least two methods available in the literature [27]-[29]. In [28], the MIMO demapper is combined with the bit nodes of the LDPC decoder, which is treated as one entity for the EXIT chart analysis. The check node in the decoder is the other entity. In another approach, e.g. in [29], the MIMO demapper is treated as one entity for the EXIT chart analysis and the entire LDPC decoder as the other. We provide our EXIT results based on the latter method. That is, the multiuser-demapper is regarded as one entity and the LDPC decoder as the other. The latter method gives us a clear view of the separate effects of the demapper and the decoder. For the decoder transfer curve, a randomly selected (1024,3,6) LDPC code is used and the number of the LDPC internal iterations is fixed at ten. In particular, the mutual information at the output of the LDPC decoder is calculated from the extrinsic information obtained at the end of ten internal iterations; the mutual information at the input of the LDPC decoder is obtained from the prior information, which is as usual assumed to be Gaussian distributed. The relationship of mutual information between the input and the output of the decoder is used to generate the transfer characteristic curve for the LDPC decoder.

To investigate the nature of the iterative algorithm, both the multiuser dempper and LDPC decoder characteristics are plotted into a single chart. However, the axes are swapped for the transfer characteristics of the LDPC decoder. Figure 23 shows the EXIT chart of multiuser demapper and LDPC decoder with 4-QAM modulation.

The exchange of extrinsic information can be visualized as a decoding trajectory. For SNR=4 dB, the trajectory gets stuck after two iterations since the transfer curves intersect; For SNR=4.7 dB, the trajectory has just managed to sneak through the bottleneck. When SNR is greater than this threshold, the iterative algorithm starts to work.

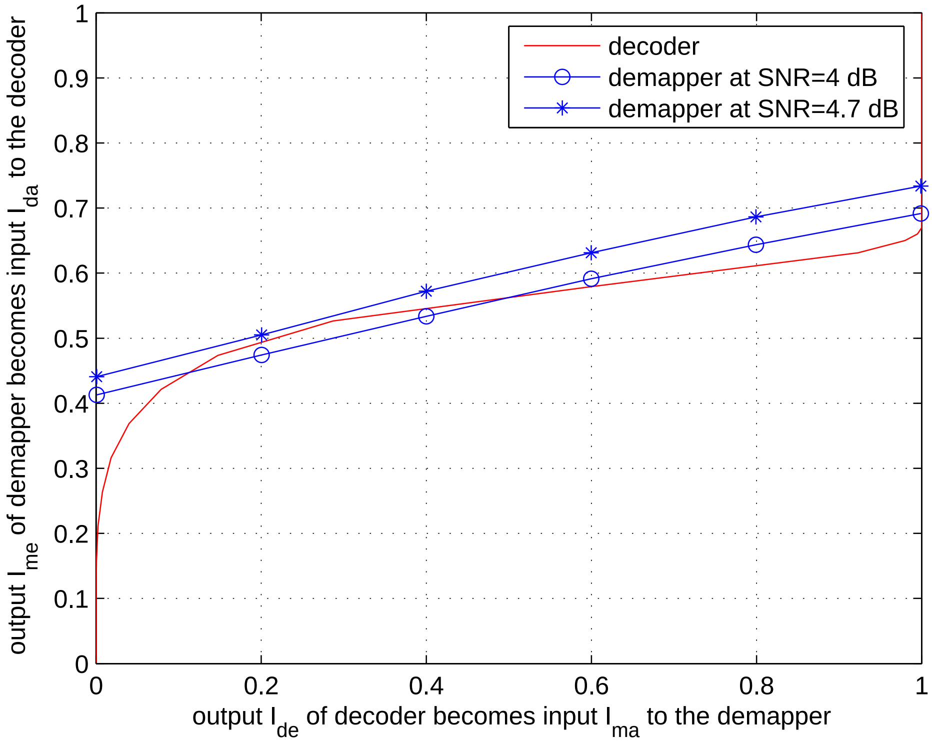

Figure 23: EXIT chart with transfer characteristics for a set of SNRs

4.3.3 Comparison of system performance with union upper bounds, constrained capacity and threshold from EXIT chart analysis

In this section, the simulated system performance results will be compared with the union upper bounds, as well as with well established performance prediction measures such as the constrained channel capacities and the threshold values obtained from EXIT chart analysis. For all simulations, the regular LDPC codes with the bit node degree 3 and the check node degree 6 are used, i.e., $J = 3, K = 6$. The rate R_c of this binary code is thus $1/2$. We simulate various block lengths, i.e., 256, 512 and 1024, to see how the bounds and the simulation results scale with increase in block length. In addition, each sender is equipped with two transmit antennas and the receiver with two receive antennas. Two different modulation cases, BPSK and 4-QAM with the Gray constellation mapping, are considered.

Fig.24 and Fig.25 show the comparison of performance curves with the upper bounds, the constrained capacities, and the thresholds from the EXIT chart analysis for single-user and multiuser MIMO systems, respectively. The constrained capacity is calculated from the Monte Carlo evaluation of the mutual information between the vector input and the vector output of the channel. Each entry of the vector input is assumed to be equally likely selected from a constellation such as BPSK and 4-QAM and the channel is assumed to be ergodic.

There are four performance curves in total in the two figures. The first two curves in Fig.24 are for the single-user MIMO system with BPSK and 4-QAM modulations. The next two curves in Fig.25 are for the multiuser MIMO system with BPSK and 4-QAM modulations. For convenience, we may refer to them as the first, the second, the third and the fourth scenarios respectively. Given the TNI at 60, the NSIs and NIIs are divided into four cases such as (3, 20), (6, 10), (6, 10), and (12, 5) for the four scenarios, respectively. Since both senders are equipped with two transmit antennas, the number of transmitted coded bits are 2, 4, 4 and 8 in one channel-use. The second and the third scenarios are similar in that they are both sending four coded bits per channel-use, and 4-QAM can be treated as two orthogonal BPSKs. Also note that in Fig.25, the x-axis is signal-to-noise ratio per user. If the two users are treated as one super-user, there will be a 3dB shift in SNR to the right. With this adjustment, we note that the second and the third scenarios indeed show a very

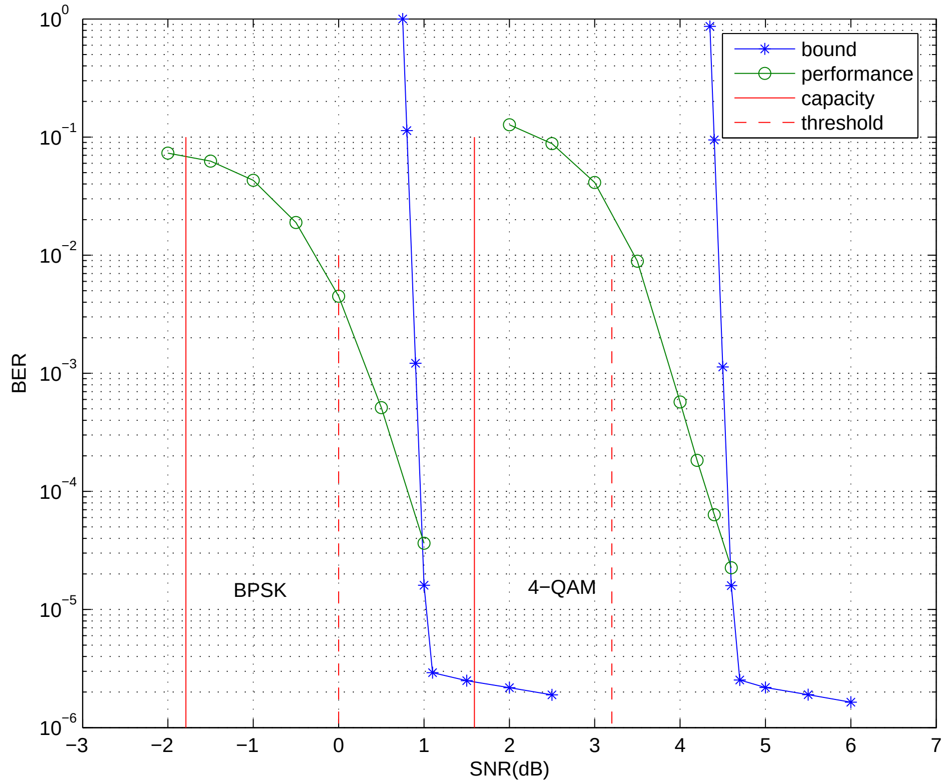

Figure 24: Comparison of performance, bounds, capacities and thresholds in single-user MIMO systems

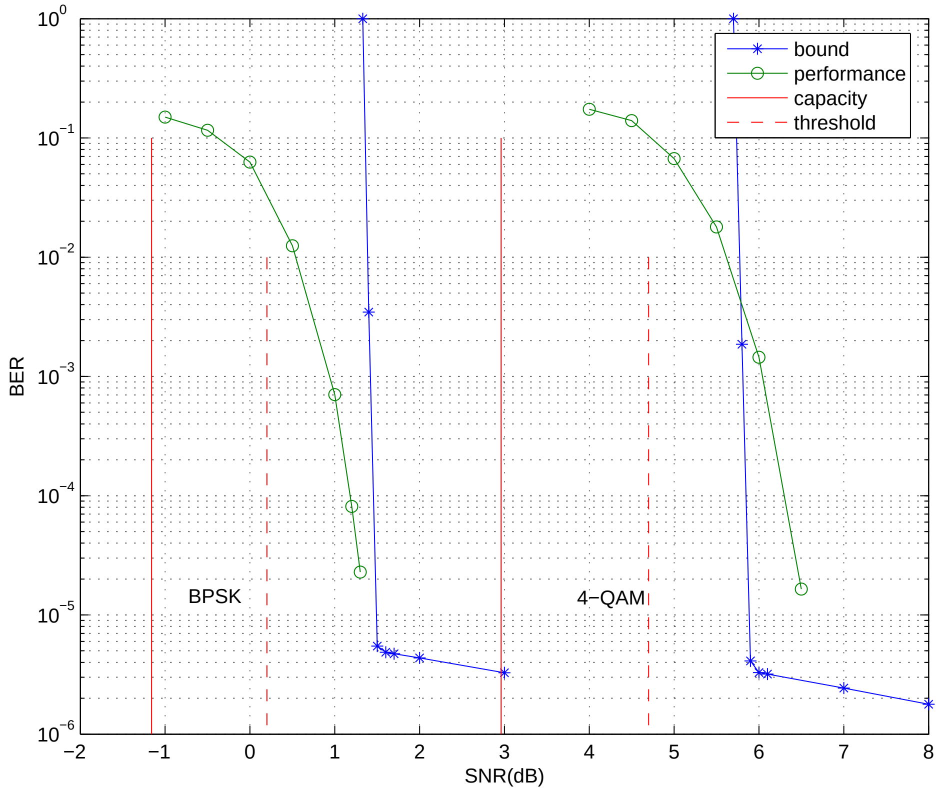

Figure 25: Comparison of performance, bounds, capacities and thresholds in multiuser MIMO systems

similar performance as expected. For the fourth scenario, we note that the simulation result goes above the union upper bound in high SNR region, which indicates that more iterations are needed, especially the internal iterations. We can see that the iterative processing in MIMO multiple access systems still has much potential to be improved.

From investigation of them, we note that each measure provides a different perspective. The union upper bound not only can predict the waterfall region for the iterative detection and decoding receiver, but also provide information on the error floor behavior of the ensemble of the codes. On the other hand, the threshold values from the EXIT chart analysis seem to indicate the starting point of the water-fall region.

Table II shows the results of upper bounds and simulated performances for different block lengths up to a thousand as well as for different number of antennas. From the table, we can see that both the bounds and the performance curves move toward the capacity limits as the block length increases. In fact, we note that the performance curves move toward the capacity limits faster than the union bounds do.

In addition, we investigate the number of multiplication required to evaluate the union upper bounds. The results for BPSK modulation for two users are tabulated in Table II. The required multiplications are for the convolution operation which is on the order of $O(N^{2U})$, where U is the number of users.

It is worthwhile to note that if the block length is further increased to a level beyond a thousand, both the threshold value from the EXIT chart and the waterfall SNR from system simulation would continue to converge to the capacity limit; while the union bound will converge only to a cut-off SNR. We believe that the cut-off SNR would be the SNR point at which the waterfall cliff occurs as the block length is increased. This opens up a future research direction for tight union bound techniques that continue to work beyond the cut-off SNR point. There are significant recent developments in this direction for single-input single-output channels [20], [21]. Finding tight union bounds for single-user and multiuser MIMO systems is an open research area.

Table 2: Comparison of bounds, performances and complexity of enumeration for bounds

Block length (bits)	No. of Tx and Rx antennas	Error floor of bound (log)	Bound gap from capacity (dB)	Performance gap from capacity (dB)	No. of mults for Φ_{d_1,d_2} $\left(O(N^{2U})\right)$
256	2×2	-4.45	2.8	3.6	2.8×10^8
512	2×2	-5.13	2.7	2.8	4.4×10^9
1024	2×2	-5.48	2.6	2.4	6.9×10^{10}
1024	4×4	-5.91	2.3	2.0	6.9×10^{10}

4.3.4 Performance and union upper bounds for LDPC coded modulation with Alamouti space-time transmission scheme

In previous LDPC coded modulation scheme, the mapping device transforms the symbol sequence into a space-time symbol matrix, i.e., the serial to parallel conversion of the symbol sequence without an explicit space-time coding, which is also called direct transmission scheme. We call this symbol matrix as space-time (ST)-I codeword.

Besides the above operation, scheme II further encodes the ST-I codeword with orthogonal space-time block code (OSTBC) encoder [30] as shown in Figure 26.

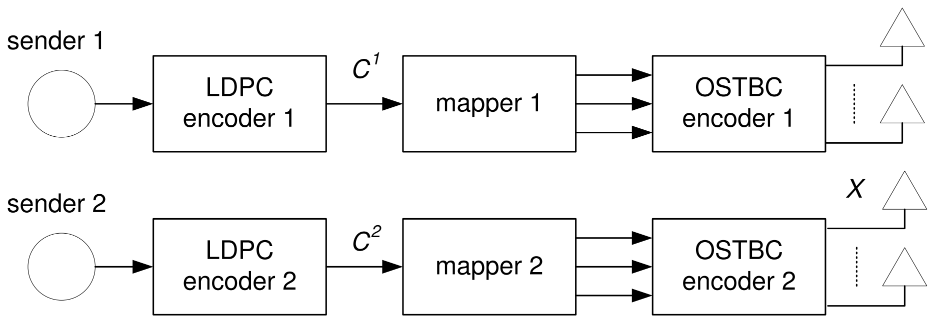

Figure 26: LDPC coded modulation with Alamouti space-time coding transmission scheme

For $N_t = 2$, the OSTBC coding scheme is actually the Alamouti scheme [31], that is,

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \Rightarrow \begin{pmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{pmatrix}$$

Then, we correspondingly call the encoded symbol matrix as ST-II codeword.

From the property of OSTBC, we notice that each leading column of sub-blocks of ST-II codeword is identical to the corresponding column of ST-I codeword. The rest $N_t - 1$ columns of each sub-block are the repetitions of leading column with operations of permutation, conjugation and negation. Since these operations do not affect column weights, they can be ignored for computing column distance distribution. Then, we can regard the ST-II codeword as N_t repetitions of ST-I codeword. So the pairwise error probability for scheme II becomes:

$$p\big(\mathbf{x}_{(0,0)} \to \mathbf{x}_{(d_1,d_2)}\big) \leqslant \prod_{\mathbf{r}^1 \in \mathcal{R}_1} \prod_{\mathbf{r}^2 \in \mathcal{R}_2} \Big(1 + \sum_{u=1}^{2} \sum_{i=0}^{M-1} r_i^u |s_i - s_0|^2 \rho\Big)^{-N_r N_t l_{\mathbf{r}^1,\mathbf{r}^2}}. \tag{4.25}$$

Using the same derivation as Scheme I, we can get the union upper bound on bit error probability for scheme II as following:

$$\overline{P_b} \leqslant \sum_{d_1=0}^{N} \sum_{d_2=0}^{N} \frac{d_1}{N} \frac{d_2}{N} \binom{N}{d_1}^{-1} \binom{N}{d_2}^{-1} S_{d_1} S_{d_2} \Psi_{d_1,d_2}, \tag{4.26}$$

where the definition of Ψ_{d_1,d_2} is same as scheme I, but here

$$\beta_{\mathbf{r}^1,\mathbf{r}^2} := \binom{N_t}{\mathbf{r}^1} \binom{N_t}{\mathbf{r}^2} \Big(1 + \sum_{u=1}^{2} \sum_{i=0}^{M-1} r_i^u |s_i - s_0|^2 \rho_u\Big)^{-N_r N_t}. \tag{4.27}$$

The comparison of the performance curves and bounds for LDPC coded modulation with Alamouti scheme is shown in Figure 27. In the simulation, a (512,3,6) LDPC code is used.

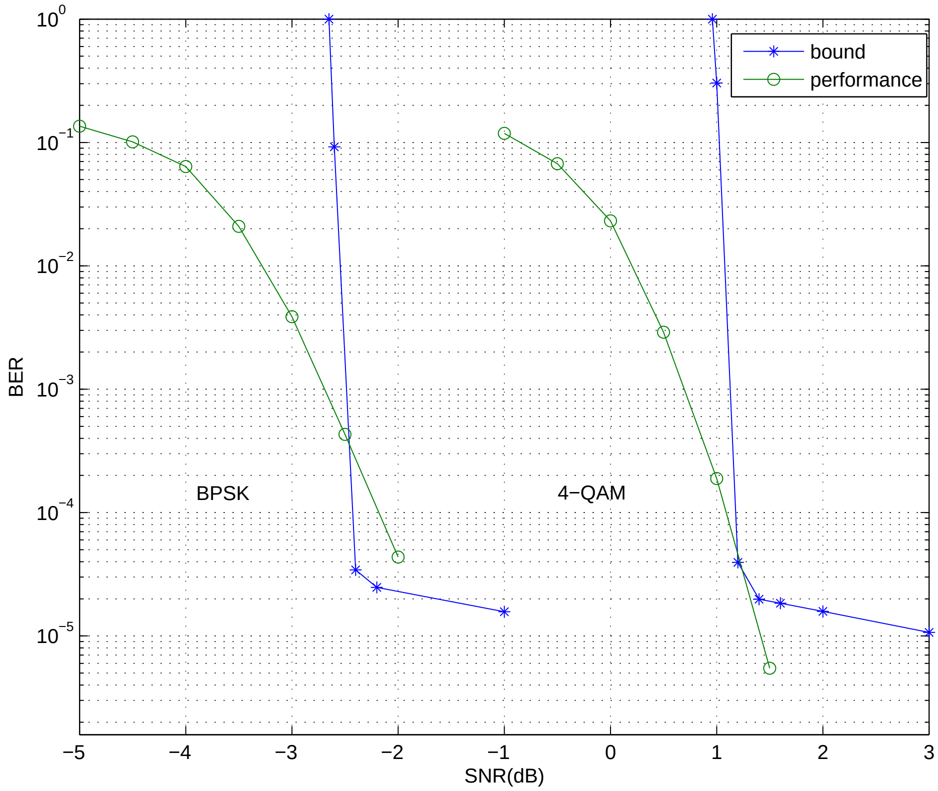

Figure 27: Comparison of performances and bounds for LDPC coded modulation with Alamouti scheme in multiuser MIMO systems

4.4 SUMMARY OF THE PERFORMANCE AND OTHER ANALYTICAL ASSESS MEASURES

The channel capacity can be significantly increased by using multiple transmit and multiple receive antennas. The LDPC coded modulation scheme with iterative demapping and decoding can be utilized to exploit the capacity potential available in the MIMO multiple access system. We proposed novel union upper bound techniques for the multiuser MIMO system. We provided some system simulation results which can be contrasted with the union upper bounds. It was shown that the union bounds can be used in combination with the EXIT chart analysis as a performance evaluation tool. It is worthy to note that while the EXIT chart analysis provides the threshold value which are only a single SNR point on the BER graph, the union bounds provide information on waterfall region as well as the error floor behavior.

5.0 PERFORMANCE ANALYSIS FOR LDPC CODED MODULATION SCHEME IN BLOCK FADING MULTIUSER MIMO SYSTEMS

Based on the analysis in Chapter 4, we generalize the derivation of the union upper bound to block fading case.

5.1 BLOCK FADING MULTIUSER MIMO SYSTEM MODEL

Consider a multiuser MIMO system as illustrated in Figure 6. A codeword of length N is mapped into a $[N_t \times T]$ space-time transmission matrix $\mathbf{x}^u$ for $u = 1, 2$ in a one-to-one relationship. T is the total number of channel-uses for a codeword to be transmitted. For the block fading channel, the channel coefficients matrix remains constant for T_D channel-uses as shown in Figure 28. We call T_D the *coherence time* of the channel. Then the number of blocks is $N_b = T/T_D$, which is assumed to be an integer without loss of generality.

The whole space-time codeword is composed from block space-time (BST) codewords. We regard the BST codeword as a super-symbol, which contains $N_t T_D \log(M)$ bits. Thus, we can consider an BST matrix constellation of size $Q = 2^{N_t T_D \log(M)}$. $\{S_p\}_{p=0}^{Q-1}$ are the super-symbols of the Q-ary BST constellation, where S_0 mapped from the string of all-zero bits of length $N_t T_D \log(M)$. There are a total number of N_b BST codewords within a space-time code, i.e., $\mathbf{x} = [\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_{N_b}]$, and $\mathbf{x}_j$ for $j = 1, \cdots, N_b$, takes a value S_p from the BST constellation. We denote $\bar{S}_p := \frac{1}{\sqrt{E_s}} S_p$ as the normalized BST super symbol.

In the block fading case, the received signal $\mathbf{y}_t$ at t-th BST block transmission can be written as:

$$\mathbf{y}_t = \mathbf{h}^t \mathbf{x}_t + \mathbf{n}_t, \quad \text{for } t = 1, \cdots, N_b, \tag{5.1}$$

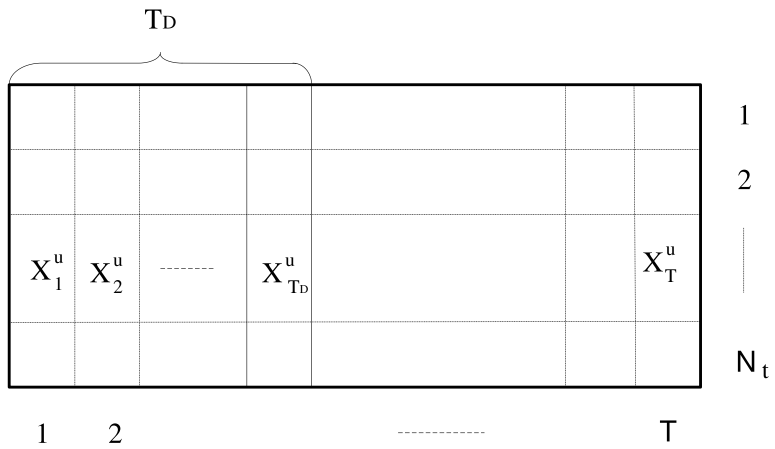

Figure 28: Block fading MIMO channel.

where we define the following vector variables:

$$\mathbf{y}_t := \begin{pmatrix} y_{11} & \cdots & y_{1T_D} \\ \vdots & \ddots & \vdots \\ y_{N_r1} & \cdots & y_{N_rT_D} \end{pmatrix}, \mathbf{n}_t := \begin{pmatrix} n_{11} & \cdots & n_{1T_D} \\ \vdots & \ddots & \vdots \\ n_{N_r1} & \cdots & n_{N_rT_D} \end{pmatrix},$$

$$\mathbf{x}_t := \begin{pmatrix} \mathbf{x}_t^1 \\ \mathbf{x}_t^2 \end{pmatrix}, \mathbf{x}_t^u := \begin{pmatrix} x_{11}^u & \cdots & x_{1T_D}^u \\ \vdots & \ddots & \vdots \\ x_{N_r1}^u & \cdots & x_{N_rT_D}^u \end{pmatrix}, \bar{\mathbf{x}}_t^u := \frac{1}{\sqrt{E_{s_u}}}\mathbf{x}_t^u,$$

$$\mathbf{h}^t := \begin{pmatrix} \mathbf{h}^{1t} & \mathbf{h}^{2t} \end{pmatrix} := \begin{pmatrix} h_{11}^{1t} & \cdots & h_{1N_t}^{1t} & h_{11}^{2t} & \cdots & h_{1N_t}^{2t} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ h_{N_r1}^{1t} & \cdots & h_{N_rN_t}^{1t} & h_{N_r1}^{2t} & \cdots & h_{N_rN_t}^{2t} \end{pmatrix}.$$

In this chapter, we also generalize the distribution of channel fading coefficients to Ricean distribution, not restricted to Rayleigh distribution. The channel fading matrix is held during each block of T_D channel-uses, and changes independently in another block.

5.2 PAIRWISE ERROR PROBABILITY AVERAGED OVER BLOCK CHANNEL FADING STATE AND BLOCK DISTANCE DISTRIBUTION

Inspired by the derivation of the union bound for M-ary LDPC coded modulation scheme in the fast fading case, we further derive the union bound in block fading case.

Let δ_p denote the Hamming weight of the bits mapped to super-symbol S_p for $p = 0, \cdots, Q-1$, and let $l_{p,q}$ denote the number of simultaneous appearance of S_p for user 1 and S_q for user 2 in the whole space-time code. From the definition, we note that following conditions are satisfied:

- $\sum_{p=0}^{Q-1} \sum_{q=0}^{Q-1} l_{p,q} = N_b$:
  the total number of blocks should add up to N_b;
- $\sum_{p=0}^{Q-1} \delta_p \, l_{p,q} = d_1$ and $\sum_{q=0}^{Q-1} \delta_q \, l_{p,q} = d_2$:
  the weight of the first user's codeword and that of the second user's should add up to d_1 and d_2, respectively.

Let $\mathbf{x}_{(d_1,d_2)} := \begin{pmatrix} \mathbf{x}^1_{(d_1)} \\ \mathbf{x}^2_{(d_2)} \end{pmatrix}$ denote the space-time symbol matrix, where $\mathbf{x}^1_{(d_1)}$ is mapped from the codeword with Hamming weight d_1 for user 1 and $\mathbf{x}^2_{(d_2)}$ mapped from the codeword with Hamming weight d_2 for user 2. The pairwise error probability based on ML detection can be upper bounded by using the Chernoff bound

$$p\big(\mathbf{x}_{(0,0)} \to \mathbf{x}_{(d_1,d_2)} | \mathbf{h}\big) \leqslant \exp\left(- \frac{d^2\big(\mathbf{x}_{(0,0)}\,, \mathbf{x}_{(d_1,d_2)}\big)}{4N_0} \right), \tag{5.2}$$

where

$$d^2\big(\mathbf{x}_{(0,0)}\,, \mathbf{x}_{(d_1,d_2)}\big) = \sum_{t=1}^{N_b} \|\mathbf{h}^t \mathbf{x}_{t,(0,0)} - \mathbf{h}^t \mathbf{x}_{t,(d_1,d_2)}\|_F^2;$$

$\mathbf{x}_{t,(0,0)}$ and $\mathbf{x}_{t,(d_1,d_2)}$ are the t-th block of $\mathbf{x}_{(0,0)}$ and $\mathbf{x}_{(d_1,d_2)}$ respectively; F denotes the Frobenius norm.

Therefore we can write the pairwise error probability conditioned on channel state as:

$$
\begin{aligned}
p\left(\mathbf{x}_{(0,0)} \rightarrow \mathbf{x}_{(d_1,d_2)}|\mathbf{h}\right) &\leqslant \prod_{t=1}^{N_b} p\left(\mathbf{x}_{t,(0,0)} \rightarrow \mathbf{x}_{t,(d_1,d_2)}|\mathbf{h}\right) \\
&= \prod_{p=0}^{Q-1}\prod_{q=0}^{Q-1} p\left(S_{(0,0)} \rightarrow S_{(p,q)}|\mathbf{h}\right)^{l_{p,q}}, \qquad (5.3)
\end{aligned}
$$

where $S_{(0,0)} = \begin{pmatrix} S_0 \\ S_0 \end{pmatrix}$ and $S_{(p,q)} = \begin{pmatrix} S_p \\ S_q \end{pmatrix}$.

Now our task is to calculate the block pairwise error probability $p\left(S_{(0,0)} \rightarrow S_{(p,q)}|\mathbf{h}\right)$.

$$
\begin{aligned}
p\left(S_{(0,0)} \rightarrow S_{(p,q)}|\mathbf{h}\right) &\leqslant \exp\left(-\frac{d^2\left(S_{(0,0)}\,, S_{(p,q)}\right)}{4N_0}\right) \\
&= \exp\left(-\frac{\|\mathbf{h}S_{(0,0)}\, - \mathbf{h}S_{(p,q)}\|_F^2}{4N_0}\right) \\
&= \exp\left(-\|\mathbf{h}\bar{S}_{(0,0)}\, - \mathbf{h}\bar{S}_{(p,q)}\|_F^2\,\rho\right) \qquad (5.4)
\end{aligned}
$$

where $\rho = \frac{E_s}{4N_0}$ and $\mathbf{h}$ is the fading coefficients matrix for some block.

Proposition 4. *(Block pairwise error probability)*

For the normalized transmitted BST super symbol matrix $\mathbf{c}$*, the block pairwise error probability of decoding in favor of* $\mathbf{e}$ *is bounded by*

$$
p\big(\mathbf{c} \rightarrow \mathbf{e}|\mathbf{h}\big) \leqslant \prod_{i=1}^{N_r}\prod_{j=1}^{2N_t} \frac{1}{1+\lambda_j\rho}\exp\Big(-\frac{K_{i,j}\lambda_j\rho}{1+\lambda_j\rho}\Big)
$$

over Ricean fading channel; and bounded by

$$
p\big(\mathbf{c} \rightarrow \mathbf{e}|\mathbf{h}\big) \leqslant \prod_{j=1}^{2N_t}\Big(\frac{1}{1+\lambda_j\rho}\Big)^{N_r}
$$

over Rayleigh fading channel, where $K_{i,j}$ is the Ricean factor and $\mathbf{c}$ and $\mathbf{e}$ are defined as following:

$$\mathbf{c} := \begin{pmatrix} \mathbf{c}^1 \\ \mathbf{c}^2 \end{pmatrix} := \begin{pmatrix} c^1_{11} & \cdots & c^1_{1T_D} \\ \vdots & \ddots & \vdots \\ c^1_{N_t1} & \cdots & c^1_{N_tT_D} \\ c^2_{11} & \cdots & c^2_{1T_D} \\ \vdots & \ddots & \vdots \\ c^2_{N_t1} & \cdots & c^2_{N_tT_D} \end{pmatrix},$$

where $\mathbf{c}^u := \begin{pmatrix} c^u_{11} & \cdots & c^u_{1T_D} \\ \vdots & \ddots & \vdots \\ c^u_{N_t1} & \cdots & c^u_{N_tT_D} \end{pmatrix}$*, for* $u = 1, 2$*;*

$$\mathbf{e} := \begin{pmatrix} \mathbf{e}^1 \\ \mathbf{e}^2 \end{pmatrix} := \begin{pmatrix} e^1_{11} & \cdots & e^1_{1T_D} \\ \vdots & \ddots & \vdots \\ e^1_{N_t1} & \cdots & e^1_{N_tT_D} \\ e^2_{11} & \cdots & e^2_{1T_D} \\ \vdots & \ddots & \vdots \\ e^2_{N_t1} & \cdots & e^2_{N_tT_D} \end{pmatrix},$$

where $\mathbf{e}^u := \begin{pmatrix} e^u_{11} & \cdots & e^u_{1T_D} \\ \vdots & \ddots & \vdots \\ e^u_{N_t1} & \cdots & e^u_{N_tT_D} \end{pmatrix}$*, for* $u = 1, 2$*.*

Proof : Over block fading channel, the received signal during some block can be expressed as

$$\mathbf{y} = \mathbf{hc} + \mathbf{n},$$

where

$$\mathbf{y} := \begin{pmatrix} y_{11} & \cdots & y_{1T_D} \\ \vdots & \ddots & \vdots \\ y_{N_r1} & \cdots & y_{N_rT_D} \end{pmatrix}, \mathbf{n} := \begin{pmatrix} n_{11} & \cdots & n_{1T_D} \\ \vdots & \ddots & \vdots \\ n_{N_r1} & \cdots & n_{N_rT_D} \end{pmatrix},$$

$$\mathbf{h} := \begin{pmatrix}\mathbf{h}^1 & \mathbf{h}^2\end{pmatrix} := \begin{pmatrix} h_{11}^1 & \cdots & h_{1N_t}^1 & h_{11}^2 & \cdots & h_{1N_t}^2 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ h_{N_r1}^1 & \cdots & h_{N_rN_t}^1 & h_{N_r1}^2 & \cdots & h_{N_rN_t}^2 \end{pmatrix}.$$

The probability of transmitting $\mathbf{c}$ and deciding in favor of $\mathbf{e}$ at the decoder is bounded by

$$\begin{aligned} p\big(\mathbf{c} \to \mathbf{e}|\mathbf{h}\big) &\leqslant \exp\bigg(-\frac{d^2\big(\mathbf{c}\,,\mathbf{e}\big)E_s}{4N_0}\bigg) \\ &= \exp\Big(-d^2\big(\mathbf{c}\,,\mathbf{e}\big)\rho\Big), \end{aligned} \tag{5.5}$$

where $d^2\big(\mathbf{c}\,,\mathbf{e}\big) = \|\mathbf{hc} - \mathbf{he}\|_F^2$; F denotes the Frobenius norm.

Now our task is to calculate $d^2\big(\mathbf{c}\,,\mathbf{e}\big)$. To calculate it, we first introduce the following Lemma.

Lemma 1. *Let* $\mathbf{A} = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} b_{11} & \cdots & b_{1k} \\ \vdots & \ddots & \vdots \\ b_{n1} & \cdots & b_{nk} \end{pmatrix}$, *then* $\|\mathbf{AB}\|_F^2 = \sum_{i=1}^m \mathbf{A}_i\mathbf{B}\mathbf{B}^*\mathbf{A}_i^*$, *where* $\mathbf{A}_i = (a_{i1} \cdots a_{in})$.

By applying Lemma 1, we get

$$\begin{aligned} d^2\big(\mathbf{c}\,,\mathbf{e}\big) &= \|\mathbf{h}(\mathbf{c} - \mathbf{e})\|_F^2 \\ &= \sum_{i=1}^{N_r} \mathbf{h}_i(\mathbf{c} - \mathbf{e})(\mathbf{c} - \mathbf{e})^*\mathbf{h}_i^*, \end{aligned}$$

where $\mathbf{h}_i = (h_{i1}^1, \cdots, h_{iN_t}^1 h_{i1}^2, \cdots, h_{iN_t}^2)$.

Let $\mathbf{B} := (\mathbf{c} - \mathbf{e})(\mathbf{c} - \mathbf{e})^*$. Clearly, $\mathbf{B}$ is Hermitian and non-negative definite. Thus, there exists a $2N_t \times 2N_t$ unitary matrix $\mathbf{U}$ such that $\mathbf{B} = \mathbf{UDU}^*$, where $\mathbf{D} = \begin{pmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_{2N_t} \end{pmatrix}$ is a diagonal matrix and its entries are made of eigenvalues of $\mathbf{B}$. Then,

$$\begin{aligned} d^2\big(\mathbf{c}\,,\mathbf{e}\big) &= \sum_{i=1}^{N_r} \mathbf{h}_i \mathbf{B} \mathbf{h}_i^* \\ &= \sum_{i=1}^{N_r} \mathbf{h}_i \mathbf{U} \mathbf{D} \mathbf{U}^* \mathbf{h}_i^* \\ &= \sum_{i=1}^{N_r} (\mathbf{h}_i \mathbf{U}) \mathbf{D} (\mathbf{h}_i \mathbf{U})^* \\ &= \sum_{i=1}^{N_r} \sum_{j=1}^{2N_t} \lambda_j |\mathbf{h}_i \mathbf{U}_j|^2 \\ &= \sum_{i=1}^{N_r} \sum_{j=1}^{2N_t} \lambda_j |\alpha_{i,j}|^2, \end{aligned} \tag{5.6}$$

where $\mathbf{U}_j = \begin{pmatrix} u_{1j} \\ \vdots \\ u_{2N_t j} \end{pmatrix}$ and $\alpha_{i,j} := \mathbf{h}_i \mathbf{U}_j$.

Since $\mathbf{B}$ is Hermitian and non-negative definite, λ's are real and non-negative, i.e., $\lambda_j \geqslant 0$, for $j = 1 \cdots 2N_t$. Since $\mathbf{U}$ is unitary, $\{\mathbf{U}_1, \cdots, \mathbf{U}_{2N_t}\}$ is an orthonormal basis of $\mathbb{C}^{2N_t}$ and $\alpha_{i,j}$ are independent complex Gaussian random variables with mean $E(\mathbf{h}_i \mathbf{U}_j)$ and variance 0.5 per dimension.

Let $K_{i,j} := |E(\alpha_{i,j})|^2 = |E(\mathbf{h}_i \mathbf{U}_j)|^2 = |E(\mathbf{h}_i)\mathbf{U}_j|^2$. Thus, $|\alpha_{i,j}|$ are independent Rician distributions with pdf

$$p(|\alpha_{i,j}|) = 2|\alpha_{i,j}| \exp\big(- |\alpha_{i,j}|^2 - K_{i,j}\big) I_0\Big(2|\alpha_{i,j}|\sqrt{K_{i,j}}\Big), \tag{5.7}$$

where $I_0(\cdot)$ is the zero-order modified Bessel function of the first kind.

Substituting (5.6) to (5.5), the block pairwise error probability is bounded by

$$\begin{aligned} p\big(\mathbf{c} \to \mathbf{e} | \mathbf{h}\big) &\leqslant \exp\Big(\sum_{i=1}^{N_r} \sum_{j=1}^{2N_t} \lambda_j |\alpha_{i,j}|^2 \rho \Big) \\ &= \prod_{i=1}^{N_r} \prod_{j=1}^{2N_t} \exp\big(- \lambda_j |\alpha_{i,j}|^2 \rho \big). \end{aligned} \tag{5.8}$$

Now we need to average over the channel state. Since the distribution of the channel state is known, we just do expectation with respect to the channel state using the following lemma.

Lemma 2. *For* $f(|x|) = \exp(-a|x|^2), a \geqslant 0, p(|x|) = 2|x|\exp\big(-|x|^2 - K\big)I_0\big(2|x|\sqrt{K}\big)$, $K = |E(x)|^2$, *the expectation of* $f(|x|)$, *i.e.,* $E\big(f(|x|)\big) = \frac{1}{1+a}\exp\Big(-\frac{Ka}{1+a}\Big)$.

By using Lemma 2, the average block pairwise error probability for Ricean fading channel can be written as

$$p\big(\mathbf{c} \to \mathbf{e}\big) \leqslant \prod_{i=1}^{N_r}\prod_{j=1}^{2N_t} \frac{1}{1+\lambda_j\rho}\exp\Big(-\frac{K_{i,j}\lambda_j\rho}{1+\lambda_j\rho}\Big). \tag{5.9}$$

For Rayleigh fading channel, $K_{i,j} = 0$. Then,

$$\begin{aligned} p\big(\mathbf{c} \to \mathbf{e}\big) &\leqslant \prod_{i=1}^{N_r}\prod_{j=1}^{2N_t} \frac{1}{1+\lambda_j\rho} \\ &= \prod_{j=1}^{2N_t}\Big(\frac{1}{1+\lambda_j\rho}\Big)^{N_r}. \qquad \square \end{aligned} \tag{5.10}$$

By using the Proposition, we can get the block pairwise error probability over Ricean fading channel as

$$p\left(S_{(0,0)} \to S_{(p,q)}\right) \leqslant \prod_{i=1}^{N_r}\prod_{j=1}^{2N_t} \frac{1}{1+\lambda_j^{(p,q)}\rho}\exp\Big(-\frac{K_{i,j}\lambda_j^{(p,q)}\rho}{1+\lambda_j^{(p,q)}\rho}\Big) \tag{5.11}$$

and over Rayleigh fading channel as

$$p\left(S_{(0,0)} \to S_{(p,q)}\right) \leqslant \prod_{j=1}^{2N_t}\Big(\frac{1}{1+\lambda_j^{(p,q)}\rho}\Big)^{N_r}, \tag{5.12}$$

where $\lambda_j^{(p,q)}$, for $j = 1, \cdots, 2N_t$, are the eigenvalues of
$(\bar{S}_{(0,0)} - \bar{S}_{(p,q)})(\bar{S}_{(0,0)} - \bar{S}_{(p,q)})^*$.

Then, the pairwise error probability over Ricean fading channel can be bounded by

$$\begin{aligned} p\left(\mathbf{x}_{(0,0)} \to \mathbf{x}_{(d_1,d_2)}\right) &\leqslant \prod_{p=0}^{Q-1}\prod_{q=0}^{Q-1} p\left(S_{(0,0)} \to S_{(p,q)}\right)^{l_{p,q}} \\ &= \prod_{p=0}^{Q-1}\prod_{q=0}^{Q-1}\Bigg(\prod_{i=1}^{N_r}\prod_{j=1}^{2N_t} \frac{1}{1+\lambda_j^{(p,q)}\rho}\exp\Big(-\frac{K_{i,j}\lambda_j^{(p,q)}\rho}{1+\lambda_j^{(p,q)}\rho}\Big)\Bigg)^{l_{p,q}}. \end{aligned} \tag{5.13}$$

Now we aim to compute the average pairwise error probability for any $\mathbf{x}_{(d_1,d_2)} \in \mathcal{X}_{d_1,d_2}$. Similar to the analysis for the fast fading case, collect $l_{p,q}$'s into a $Q \times Q$ matrix, which is denoted as $\mathbf{L}$. Name $\mathbf{L}$ as the *block distance distribution* (BD) matrix and define $\mathcal{L}_{d_1,d_2}$ as the collection of all BD matrices, i.e.,

$$\mathcal{L}_{d_1,d_2} := \Big\{\mathbf{L}\,|\,l_{p,q} \in \{0,1,\cdots,N_b\}, \sum_{p=0}^{Q-1}\sum_{q=0}^{Q-1} l_{p,q} = T, \sum_{p=0}^{Q-1} p\, l_{p,q} = d_1, \sum_{q=0}^{Q-1} q\, l_{p,q} = d_2\Big\}.$$

Using the usual combinatorial techniques, we obtain the probability distribution of $\mathbf{L}$ as:

$$p\left(\mathbf{L}\right) = \begin{cases} \binom{T}{\mathbf{L}} \prod_{u=1}^{2} \binom{N}{d_u}^{-1}, & \text{if}\, \mathbf{L} \in \mathcal{L}_{d_1,d_2} \\ 0, & \text{otherwise} \end{cases} \tag{5.14}$$

where $\binom{T}{\mathbf{L}}$ is the multinomial coefficient, i.e.,

$$\binom{T}{\mathbf{L}} = \frac{T\,!}{\prod_{i=0}^{N_t}\prod_{j=0}^{N_t} l_{i,j}\,!}.$$

Thus, we obtain the upper bound of the pairwise error probability averaged over the *block distance distribution* as following:

$$\begin{aligned} \overline{p\big(\mathbf{x}_{(0,0)} \to \mathbf{x}_{(d_1,d_2)}\big)} &= \sum_{\mathbf{L}\in\mathcal{L}_{d_1,d_2}} p\big(\mathbf{x}_{(0,0)} \to \mathbf{x}_{(d_1,d_2)}\big) p\left(\mathbf{L}\right) \\ &\leqslant \sum_{\mathbf{L}\in\mathcal{L}_{d_1,d_2}} \binom{T}{\mathbf{L}} \prod_{p=0}^{Q-1}\prod_{q=0}^{Q-1} \Big(\prod_{i=1}^{N_r}\prod_{j=1}^{2N_t} \frac{1}{1+\lambda_j^{(p,q)}\rho} \exp\Big(-\frac{K_{i,j}\lambda_j^{(p,q)}\rho}{1+\lambda_j^{(p,q)}\rho}\Big)\Big)^{l_{p,q}} \prod_{u=1}^{2} \binom{N}{d_u}^{-1}. \end{aligned} \tag{5.15}$$

5.3 UNION UPPER BOUND FOR LDPC CODED MODULATION SCHEME IN BLOCK FADING MULTIUSER MIMO SYSTEMS

We sum up the average pairwise error probabilities, each of which is weighted by the ensemble-averaged distance distribution. Applying the ensemble-averaged distance distribution property, the average word error probability in a MIMO multiple access system can be upper bounded by

$$\overline{P_e} \leqslant \sum_{d_1=0}^{N} \sum_{d_2=0}^{N} S_{d_1} S_{d_2} \overline{p\big(\mathbf{x}_{(0,0)} \to \mathbf{x}_{(d_1,d_2)}\big)} - S_0^2 \overline{p\big(\mathbf{x}_{(0,0)} \to \mathbf{x}_{(0,0)}\big)}, \tag{5.16}$$

where $S_{d_u} = S_{d_u}(\mathcal{C})$ for $u = 1, 2$.

By applying(5.15) and defining $\alpha_{p,q}$ and Φ_{d_1,d_2}, (5.16) can be written as

$$\begin{aligned} \overline{P_e} &\leqslant \sum_{d_1=0}^{N} \sum_{d_2=0}^{N} \binom{N}{d_1}^{-1} \binom{N}{d_2}^{-1} S_{d_1} S_{d_2} \sum_{\mathbf{L} \in \mathcal{L}_{d_1,d_2}} \binom{T}{\mathbf{L}} \prod_{p=0}^{Q-1} \prod_{q=0}^{Q-1} (\alpha_{p,q})^{l_{p,q}} - S_0^2 \\ &= \sum_{d_1=0}^{N} \sum_{d_2=0}^{N} \binom{N}{d_1}^{-1} \binom{N}{d_2}^{-1} S_{d_1} S_{d_2} \Phi_{d_1,d_2} - S_0^2, \end{aligned} \tag{5.17}$$

where

$$\alpha_{p,q} := \prod_{i=1}^{N_r} \prod_{j=1}^{2N_t} \frac{1}{1+\lambda_j^{(p,q)}\rho} \exp\Big(- \frac{K_{i,j}\lambda_j^{(p,q)}\rho}{1+\lambda_j^{(p,q)}\rho} \Big), \tag{5.18}$$

and

$$\Phi_{d_1,d_2} := \sum_{\mathbf{L} \in \mathcal{L}_{d_1,d_2}} \binom{T}{\mathbf{L}} \prod_{p=0}^{Q-1} \prod_{q=0}^{Q-1} (\alpha_{p,q})^{l_{p,q}}. \tag{5.19}$$

Using the same method of polynomial expansion as in Chapter 4, (5.17) can be efficiently evaluated.

Then, the union upper bound for bit error probability on block fading channel is:

$$\overline{P_b} \leqslant \sum_{d_1=0}^{N} \sum_{d_2=0}^{N} \frac{d_1}{N} \frac{d_2}{N} \binom{N}{d_1}^{-1} \binom{N}{d_2}^{-1} S_{d_1} S_{d_2} \Psi_{d_1,d_2} \tag{5.20}$$

In this section, the simulated system performance results will be compared with the union upper bounds. For all simulations, the regular (1024, 3, 6) LDPC codes are used. The rate R_c of this binary code is thus 1/2. Each sender is equipped with two transmit antennas and the receiver with two receive antennas. Two different modulation cases, BPSK and 4-QAM with the Gray constellation mapping, are considered.

Fig.29 and Fig.30 show the comparison of performance curves with the upper bounds when $T_D = 2$ for single-user and multiuser MIMO systems, respectively.

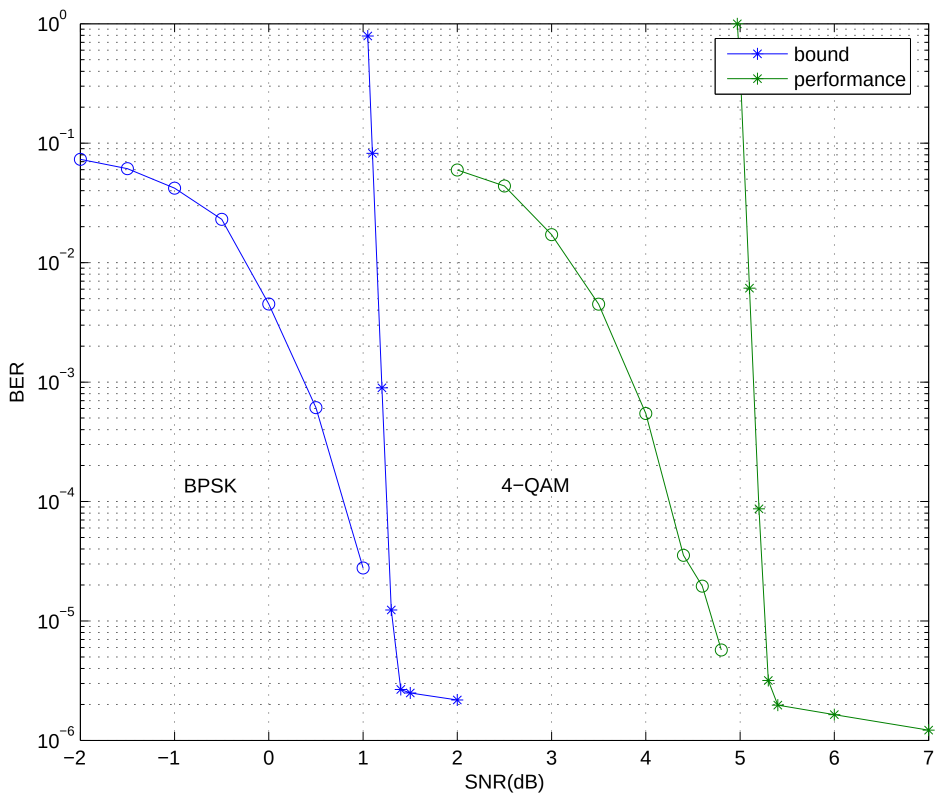

Figure 29: The comparison of performance and bounds in single-user MIMO systems

From the figures, we note that the union upper bounds on ML detection for block fading channel can accurately predict the waterfall region of the iterative detection and decoding

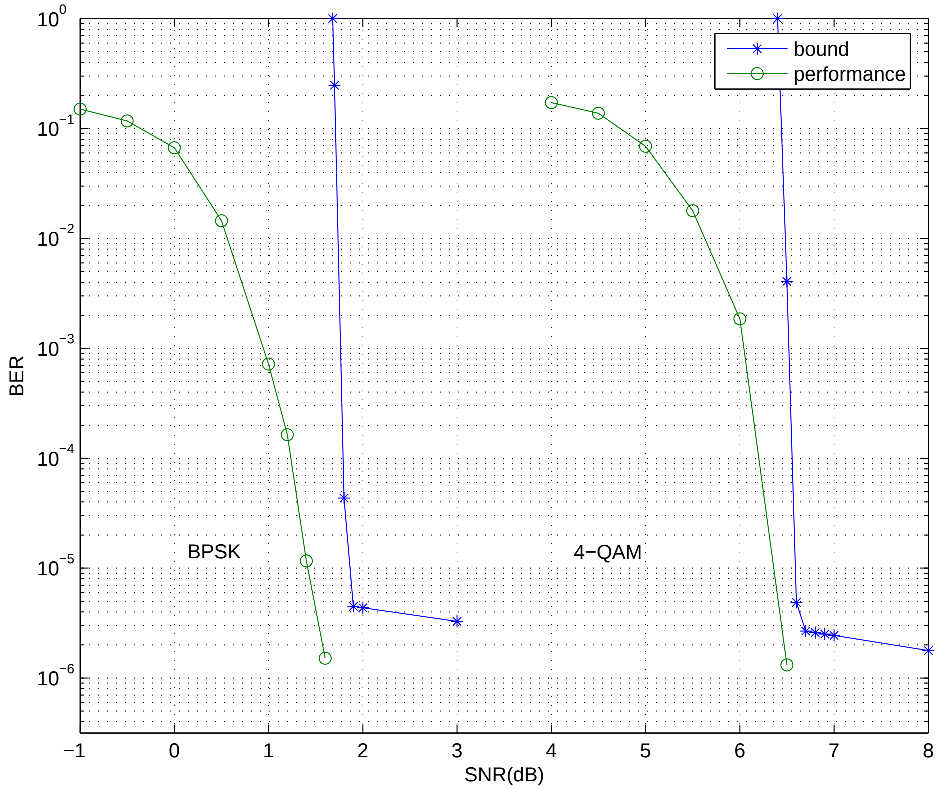

Figure 30: The comparison of performance and bounds in multiuser MIMO systems

receiver.

In addition, we study the effect when T_D increases. Fig.31 and Fig.32 show the change of curves with $T_D = 1, 2$ and 4 for single-user and multiuser MIMO systems, respectively.

From the figures, we note that the bounds move right much quicker than the performance curves when T_D increases. At $T_D = 4$, the union upper bound is loose for the performance curve. When $T_D = T$, the union upper bound is too loose and almost useless for the prediction of the performance. So our future work is to find a tight upper bound for quasi-static fading channel, i.e., when $T_D = T$.

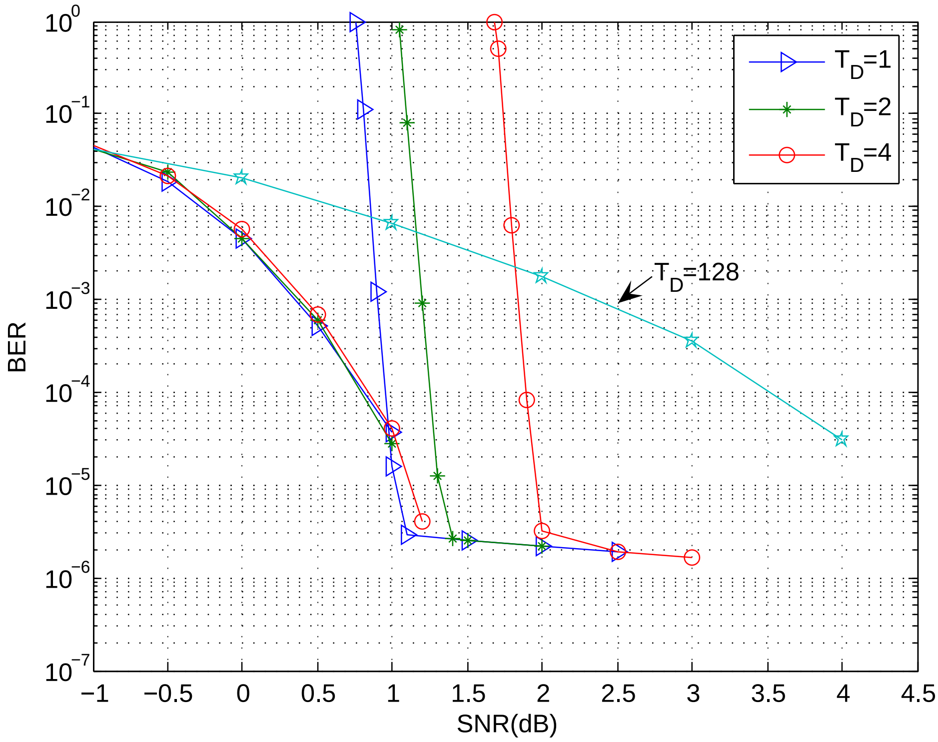

Figure 31: The comparison of performance and bounds for $T_D = 1, 2$ and 4 in single-user MIMO systems with BPSK modulation

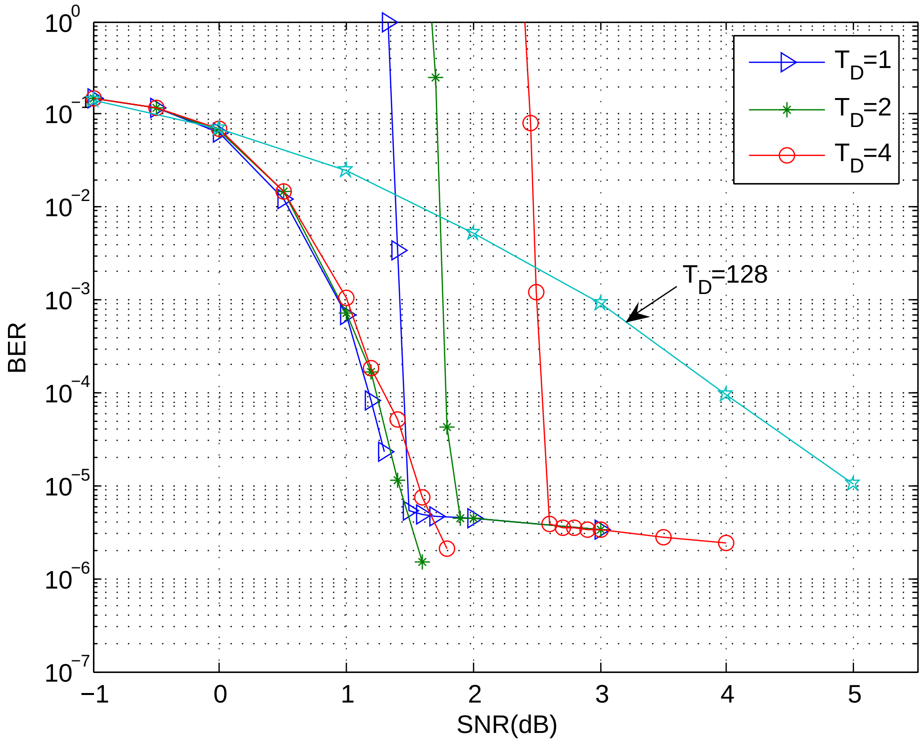

Figure 32: The comparison of performance and bounds for $T_D = 1, 2$ and 4 in multiuser MIMO systems with BPSK modulation

6.0 CONCLUSION

The channel capacity can be significantly increased by using multiple transmit and receive antennas. How to use the iterative technique to exploit the capacity potential in multiuser MIMO systems is of great interest. The purpose of this dissertation is to propose a transmission scheme to exploit the capacity potential in MIMO systems and provide upper bounds, capacity limits and thresholds obtained from EXIT chart analysis as the benchmark for the proposed scheme. The main contributions of this dissertation are summarized as follows:

- We propose an LDPC coded modulation scheme with iterative demapping and decoding for multiuser MIMO systems. We tried various bit-to-symbol mappings and find that Gray mapping is the best one for the proposed scheme in both SISO and MIMO systems, and also in both single-user and multiuser systems.
- We calculate the constrained capacity for single-user MIMO systems and compare the constrained capacity with CSI or without CSI. Based on the capacity calculation for single-user case, the constrained capacity region for multiuser MIMO systems is determined.
- We analyze the union upper bound of error probability for the proposed scheme in multiuser fast fading MIMO systems. Closed-form expression for the union bound is obtained, which can be evaluated efficiently by using a polynomial expansion. We compare the system performance with the union upper bound, the capacity limit and the threshold obtained from EXIT chart analysis. The result shows they are well matched.
- We generalize the union upper bound to multiuser block fading MIMO systems. The performance curve is well matched with the upper bound when the coherence time is not very large.

From the simulation results for the block fading case, we note that the union upper bound is loose when the coherence time is large. To find tight bound for the transmission scheme when the coherence time is large will be an interesting and challenging task in the future.

APPENDIX A

PROOF OF LEMMA 1

Lemma 1 *Let* $\mathbf{A} = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}, \mathbf{B} = \begin{pmatrix} b_{11} & \cdots & b_{1k} \\ \vdots & \ddots & \vdots \\ b_{n1} & \cdots & b_{nk} \end{pmatrix}$, *then* $\|\mathbf{AB}\|_F^2 = \sum_{i=1}^m \mathbf{A}_i \mathbf{B}\mathbf{B}^* \mathbf{A}_i^*$, *where* $\mathbf{A}_i = (a_{i1} \cdots a_{in})$.

Proof :

$$\begin{aligned} \|\mathbf{AB}\|_F^2 &= Tr\Big(\mathbf{AB}(\mathbf{AB})^*\Big) \\ &= Tr\Big(\mathbf{ABB}^*\mathbf{A}^*\Big) \\ &= \sum_{i=1}^m \mathbf{A}_i\mathbf{BB}^*\mathbf{A}_i^*. \qquad \square \end{aligned}$$

APPENDIX B

PROOF OF LEMMA 2

Lemma 2 *For* $f(|x|) = \exp(-a|x|^2), a \geqslant 0, p(|x|) = 2|x|\exp\big(-|x|^2 - K\big)I_0\big(2|x|\sqrt{K}\big)$, $K = |E(x)|^2$, *the expectation of* $f(|x|)$, *i.e.,* $E\big(f(|x|)\big) = \frac{1}{1+a}\exp\Big(-\frac{Ka}{1+a}\Big)$.

Proof: Let $y = |x|$. We utilize the following identity:

$$\int_0^\infty 2y\exp(-y^2 - K)I_0(2y\sqrt{K})\,\mathrm{d}y = 1. \tag{B.1}$$

Let $2y\sqrt{K} = z$. Then, $y = \frac{z}{2\sqrt{K}}$. (B.1) is transformed as

$$\int_0^\infty z\exp(-\frac{z^2}{4K})I_0(z)\,\mathrm{d}z = 2K\exp(K). \tag{B.2}$$

For the expectation of $f(|x|)$,

$$\begin{aligned}
E\big(f(|x|)\big) &= E\big(f(y)\big) \\
&= \int_0^\infty \exp(-ay^2)2y\exp(-y^2 - K)I_0(2y\sqrt{K})\,\mathrm{d}y \\
&= \frac{\exp(-K)}{2K}\int_0^\infty z\exp\Big(-\frac{(1+a)z^2}{4K}\Big)I_0(z)\,\mathrm{d}z \\
&= \frac{\exp(-K)}{2K}\int_0^\infty z\exp\Big(-\frac{z^2}{4K/(1+a)}\Big)I_0(z)\,\mathrm{d}z \\
&= \frac{\exp(-K)}{2K}\times 2\times\frac{K}{1+a}\exp\Big(\frac{K}{1+a}\Big) \\
&= \frac{1}{1+a}\exp\Big(-\frac{Ka}{1+a}\Big). \qquad \square
\end{aligned}$$

APPENDIX C

PROOF OF PAIRWISE ERROR PROBABILITY AVERAGED OVER FAST FADING CHANNEL STATE

The pairwise error probability based on ML detection can be upper bounded by using the Chernoff bound

$$p\big(\mathbf{x}_{(0,0)} \to \mathbf{x}_{(d_1,d_2)}|\mathbf{h}\big) \leqslant \exp\left(-\frac{d^2\big(\mathbf{x}_{(0,0)}\,,\mathbf{x}_{(d_1,d_2)}\big)}{4N_0}\right), \tag{C.1}$$

where

$$d^2\big(\mathbf{x}_{(0,0)}\,,\mathbf{x}_{(d_1,d_2)}\big) = \sum_{t=1}^{T} \|\mathbf{h}^t\mathbf{x}_{t,(0,0)} - \mathbf{h}^t\mathbf{x}_{t,(d_1,d_2)}\|_F^2;$$

$\mathbf{x}_{t,(0,0)}$ and $\mathbf{x}_{t,(d_1,d_2)}$ are the t-th column of $\mathbf{x}_{(0,0)}$ and $\mathbf{x}_{(d_1,d_2)}$ respectively; F denotes the Frobenius norm.

By applying Lemma 1, we get

$$d^2\big(\mathbf{x}_{(0,0)}\,,\mathbf{x}_{(d_1,d_2)}\big) = \sum_{t=1}^{T}\sum_{i=1}^{N_r} \mathbf{h}_i^t\big(\mathbf{x}_{t,(0,0)} - \mathbf{x}_{t,(d_1,d_2)}\big)\big(\mathbf{x}_{t,(0,0)} - \mathbf{x}_{t,(d_1,d_2)}\big)^*\big(\mathbf{h}_i^t\big)^*,$$

where $\mathbf{h}_i^t = (h_{i1}^{1t}, \cdots, h_{iN_t}^{1t}, h_{i1}^{2t}, \cdots, h_{iN_t}^{2t})$.

Let $\mathbf{B}_t := \big(\mathbf{x}_{t,(0,0)} - \mathbf{x}_{t,(d_1,d_2)}\big)\big(\mathbf{x}_{t,(0,0)} - \mathbf{x}_{t,(d_1,d_2)}\big)^*$. Clearly, $\mathbf{B}_t$ is Hermitian and non-negative definite. Thus, there exists a $2N_t \times 2N_t$ unitary matrix $\mathbf{U}_t$ such that $\mathbf{B}_t = \mathbf{U}_t\mathbf{D}_t\mathbf{U}_t^*$, where $\mathbf{D}_t$ is a diagonal matrix and its entries are made of the eigenvalues of $\mathbf{B}_t$. Then,

$$\begin{aligned} d^2\big(\mathbf{c}\,,\mathbf{e}\big) &= \sum_{t=1}^{T}\sum_{i=1}^{N_r}\mathbf{h}_i^t\mathbf{B}_t\big(\mathbf{h}_i^t\big)^* \\ &= \sum_{t=1}^{T}\sum_{i=1}^{N_r}\mathbf{h}_i^t\mathbf{U}_t\mathbf{D}_t\mathbf{U}_t^*\big(\mathbf{h}_i^t\big)^* \\ &= \sum_{t=1}^{T}\sum_{i=1}^{N_r}(\mathbf{h}_i^t\mathbf{U}_t)\mathbf{D}_t\big(\mathbf{h}_i^t\mathbf{U}_t\big)^* \\ &= \sum_{t=1}^{T}\sum_{i=1}^{N_r}\alpha_i^t\mathbf{D}_t\big(\alpha_i^t\big)^* \end{aligned}$$

where $\alpha_i^t = \mathbf{h}_i^t\mathbf{U}_t = (\alpha_{i1}^{1t}, \cdots, \alpha_{iN_t}^{1t}, \alpha_{i1}^{2t} \cdots, \alpha_{iN_t}^{2t})$.

Now we calculate the eigenvalues of $\mathbf{B}_t$ by using the following lemma.

Lemma 3. *For $B = b \times b^*$, where $b = \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix}$, there exists exact one eigenvalue that is $\lambda = |b|^2$.*

Applying the lemma, the diagonal matrix can be written as

$$\mathbf{D}_t = \begin{pmatrix} |\mathbf{x}_{t,(0,0)} - \mathbf{x}_{t,(d_1,d_2)}|^2 & & & \\ & 0 & & \\ & & \ddots & \\ & & & 0 \end{pmatrix},$$

and

$$\begin{aligned} d^2\big(\mathbf{c}\,,\mathbf{e}\big) &= \sum_{t=1}^{T}\sum_{i=1}^{N_r}\alpha_{i1}^{1t}|\mathbf{x}_{t,(0,0)} - \mathbf{x}_{t,(d_1,d_2)}|^2\big(\alpha_{i1}^{1t}\big)^* \\ &= \sum_{t=1}^{T}\sum_{i=1}^{N_r}|\mathbf{x}_{t,(0,0)} - \mathbf{x}_{t,(d_1,d_2)}|^2|\alpha_{i1}^{1t}|^2. \end{aligned}$$

Then, (C.1) can be written as

$$
\begin{aligned}
p\big(\mathbf{x}_{(0,0)} \to \mathbf{x}_{(d_1,d_2)}|\mathbf{h}\big) &\leqslant \exp\left(- \frac{\sum_{t=1}^{T}\sum_{i=1}^{N_r} |\mathbf{x}_{t,(0,0)} - \mathbf{x}_{t,(d_1,d_2)}|^2 |\alpha_{i1}^{1t}|^2}{4N_0} \right) \\
&= \prod_{t=1}^{T}\prod_{i=1}^{N_r} \exp\left(- \frac{|\mathbf{x}_{t,(0,0)} - \mathbf{x}_{t,(d_1,d_2)}|^2 |\alpha_{i1}^{1t}|^2}{4N_0} \right).
\end{aligned}
$$

By applying Lemma 2 for Rayleigh fading case, i.e. $K = 0$ case, we can get

$$
\begin{aligned}
p\big(\mathbf{x}_{(0,0)} \to \mathbf{x}_{(d_1,d_2)}|\mathbf{h}\big) &\leqslant \prod_{t=1}^{T}\prod_{i=1}^{N_r} \frac{1}{1 + \frac{|\mathbf{x}_{t,(0,0)} - \mathbf{x}_{t,(d_1,d_2)}|^2}{4N_0}} \\
&= \prod_{t=1}^{T} \left(1 + \frac{|\mathbf{x}_{t,(0,0)} - \mathbf{x}_{t,(d_1,d_2)}|^2}{4N_0} \right)^{-N_r}.
\end{aligned}
$$

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