APPLICATIONS OF EFFICIENT IMPORTANCE SAMPLING TO STOCHASTIC VOLATILITY MODELS

by

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First chapter of my dissertation uses an EGARCH method and a Stochastic Volatility (SV) method which relies upon Markov Chain Monte Carlo (MCMC) framework based on Efficient Importance Sampling (EIS) to model inflation volatility of Turkey. The strength of SV model lies in its success in explaining time varying and persistence volatility. This chapter uses the CPI index of Turkey as the inflation measure. The inflation series suffer from four exchange rate crisis in Turkey during this period. Therefore two different models are estimated for both EGARCH and SV models; with crisis dummies and without dummies. Comparison of different model results for EGARCH and SV models indicate the robustness problem for EGARCH and that SV model is far more robust than EGARCH.

Stochastic Volatility (SV) models typically exhibit short-term dynamics with high persistence. It follows that volatility is conceptually predictable. Since, however, it is not observable; the validation of SV forecasts raises non-trivial issues. In second chapter I propose a new test statistics to evaluate the validity of one-step-ahead forecasts of returns unconditionally on volatility. Specifically, I construct a Kolmogorov-Smirnov test statistic for the null hypothesis that the predicted cumulative distribution of return evaluated at observed values is uniform. Estimation of the SV model is based upon an Efficient Importance Sampling procedure. Applications of this test statistic to quarterly data for inflation in the U.S. and Turkey fully support the validity of one-step-ahead SV forecasts of inflation.

The basic SV model assumes that volatility is just explained by its first order lag. In the last chapter of my dissertation (coauthored with Jean-Francois Richard) we show that the difference between return and monthly moving average do granger-cause volatility. 35 S&P500 stock return applications from six different industries show that the difference parameter is both significant and addition of this variable to volatility equation affects both the persistence parameter and the standard deviation of volatility. Persistence increases with the inclusion of difference variable. Furthermore standard deviation of volatility decreases which is the indication of Granger-Causality. Likelihood-ratio (LR) test results also prove that the model improves when the difference variable is added.

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PREFACE

I am deeply grateful to my advisor Jean-Francois Richard for his guidance and encouragement throughout my graduate career. His assistance went beyond providing the advice usually associated with dissertation advisors. He provided all the support and guidance that enabled me to complete my dissertation. He taught me everything that I learned through my graduate carrer. He answered all my endless questions. He also suggested ideas for my dissertation which led to my graduation. He was always and still is more than an advisor. He still continues to support me. I am also thankful for all the advice and help I have received from Roman Liesenfeld. He also gave me the Gauss codes which he had written for his own research. Whenever I had problems with these codes he patiently answered my questions. Thanks to him I learned to program complex algorithms in Gauss. I am grateful for all his help and support.

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The least I can do, I want to dedicate this dissertation to everyone who has been a part of my life and provided me the support and love which have made it more joyful.

1.0 MODELLING INFLATION OF TURKEY: A COMPARISON OF EGARCH AND STOCHASTIC VOLATILITY MODELS

1.1 INTRODUCTION

Financial econometricians have shown increasing interest in the study of volatility models during the last two decades. Many papers compare the performance of different volatility models, and most concentrate on the estimation of phenomena such as stock returns, exchange rates, or interest rates. In this paper I compare the performance of EGARCH and SV models on the estimation of inflation volatility, using the case of Turkey.

Turkey provides a case study that is well suited to a comparison of the performance of EGARCH and SV models because the researcher can examine the Turkish economy's long horizon of high and variable inflation rates. Moreover, Turkey's four major exchange rate crises caused big jumps in the inflation rate. Within those events, a researcher can expect to find several outliers in the data set that will affect estimation results. The comparison of EGARCH and SV models on the inflation volatility of Turkey thus enables the researcher to examine the robustness of both models against outliers.

Policymakers generally agree that inflation is detrimental to economic growth. Friedman [17]states that inflation-uncertainty distorts relative prices and risks in nominal contracts. As inflation volatility becomes more unpredictable, investment and economic growth slow down. Because of such harmful effects, the estimating of inflation volatility is very important to the creation and implementation of government economic policies.

The original ARCH work by Nobel Laureate Robert Engel [14] concentrated on the estimation of inflation volatility in the United Kingdom. Researchers have also examined inflation volatility in order to understand the relationship between inflation and inflation uncertainty. Engel [15], Baillie et al. [3] and Berument and Dincer [5] all conducted notable studies of inflation uncertainty. Moreover, most research on inflation volatility explores a relationship between inflation and other economic phenomena such as labor market variables, output, or growth. For example, Rich and Tracy [34] examine the effect of inflation volatility on labor contracts. Nonetheless, even among the many studies focused on inflation uncertainty, research on the estimation of pure inflation volatility is limited. Thus, while examining the comparative strengths of leading methods of modeling inflation, this paper also offers a contribution to the literature on inflation volatility.

The key difference between the EGARCH and SV models is that the EGARCH model presents volatility as a deterministic process while SV models volatility as a random process. In the presence of outliers, EGARCH must adjust the coefficients to produce larger variances while the SV model needs only to increase the variance of errors in the volatility equation. Hence, it is easier for the SV model to deal with outliers. Even so, the estimation of the stochastic volatility model is not straightforward because volatility enters the inflation equation nonlinearly. It needs to be integrated from the likelihood function. This problem can easily be solved by using highly developed integrating techniques. In this paper, I use Efficient Importance Sampling, which was developed by Richard and Zhang [37]. I use two different model specifications for both EGARCH and SV models in order to examine the effects of outliers on estimation: a model with crisis dummies in the inflation equation as well as a model without crisis dummies.

Research that compares EGARCH and SV models shows that results from the two models in the absence of outliers are similar. In this paper, I investigate whether this similarity of results remains true when outliers occur in the data set. Comparison of results for each model under different specifications enables us to determine which model is more robust against outliers. Results from EGARCH model with Generalized Error Distribution (GED) of Nelson [33] indicates that there is a robustness problem for the EGARCH model when outliers occur. Based on these results, I also estimate EGARCH by using Student-t for error terms. Student-t distribution has fat tails, and fat tails provide greater flexibility in handling outliers. For these reasons, I compare SV to EGARCH with Student-t distribution when outliers are suspected. Although student-t distribution deals with outliers more successfully, the results still suggest that SV is more robust against outliers than the EGARCH model.

I organize this paper as follows. Section II presents the insights of the data. Section III, describes the EGARCH model. Section IV discusses estimation results for the EGARCH model. Section V introduces the SV model. Section VI presents estimation results for the SV model. Finally, Section VII concludes the discussion of the research for this paper.

1.2 INSIGHTS OF THE DATA

I use Turkey's monthly CPI index for the period from February 1982 to August 2005. The inflation series are obtained by using $ln(cpi_t/cpi_{t-1})$. Figure 1 in the Appendix B presents the inflation series. The graph indicates that the data set suffers from a trend problem. I also test for seasonality before eliminating the trend component. I do this by regressing the inflation series on its first order lag and 12 monthly dummies. Table 1 in Appendix B represents the estimation results for the seasonality test. Estimation results indicate that monthly dummies for January, May, June, July, September and October are significant at the 1% level. These results are reasonable and reflect the Turkish government's pattern of policy-making. The government launches its economic program in January. Announcements of agricultural sector prices are made in June and July. Finally, the government announces increases in spending for education in September and October.

In order to eliminate both the trend component and the seasonality factor, I use the following procedure. I let $x = t/T$ so that x lies in $(0, 1)$ interval. The trend polynomial $phi(l)$requires the properties of two extremums in $(0, 1)$ bound to capture an initial small positive trend followed by a small negative trend, then a positive trend, and finally a negative trend as well as a smooth landing for $x = 1$, which requires $phi(l) = phi(l)\prime = 0$. One such polynomial is the fifth degree detrending polynomial, $phi(x) = a{*}(x-1)^2+b{*}(x-1)^3+c{*}(x-1)^4+d{*}(x-1)^5$. Therefore, in order to eliminate both the trend component and seasonality, I regress the inflation series on twelve monthly dummies and $(x-1)^2$, $(x-1)^3$, $(x-1)^4$, $(x-1)^5$. The estimation results are given in Table 2 in Appendix B. Figure 2 also represents the final series after trend and seasonality are eliminated.

Four peak points remain in the data set: April 1984, December 1987, April 1994, and

March 2001. These peak points correspond to large increases in inflation caused by Turkey's four major exchange rate crises. In order to represent the effects of these peak points on EGARCH and SV model estimation, two models; one with crisis dummies in inflation equation and on without crisis dummies will be estimated. As we shall see, EGARCH model appears to be sensitive to these outliers when errors are assumed to be GED. On the other hand, SV model is more robust against outliers.

1.3 EGARCH

1.3.1 The Model

The EGARCH model, proposed by Nelson [33], allows for asymmetry in the responsiveness of inflation to inflation shocks and does not impose any non-negativity constraints.

The basic EGARCH model is formulated as follows:

$$\ln(h_t) = \omega + \sum_{i=1}^{q} \alpha_i g\left(z_{t-i}\right) + \sum_{j=1}^{p} \gamma_j \ln(h_{t-j}) \tag{1.1}$$

where

$$\begin{aligned} g(z_t) &= \theta z_t + \left[\left|z_t\right| - E\left|z_t\right|\right] \\ z_t &= \frac{\varepsilon_t}{\sqrt{h_t}} \end{aligned} \tag{1.2}$$

In this model h_t is the conditional variance and ε_t is the error term.

EGARCH models are commonly used in the literature to explain the volatility dynamics of interest rates, stock returns and exchange rates. Some well known papers are Brunner and Simon [9], Hu, Jiang and Tsoukalas [25] and Tse and Booth [42].

In this paper, in order to capture the effect outliers in the inflation series of Turkey, I use two different formulations for the inflation equation. In the first model inflation is explained by its first order lag.

$$\pi_t = \sum_{i=1}^{n} \alpha_i \pi_{t-i} + \varepsilon_t \tag{1.3}$$

where π_t is the inflation at time t and ε_t is the error term at time $t, t : 1 \rightarrow T$. First-order lag is chosen based on Akaike Information Criterion (AIC).

In the second model, inflation is explained by its first order lag and four crisis dummies which is given by

$$\begin{aligned}\pi_t &= \sum_{i=1}^{n} \alpha_i \pi_{t-i} + \lambda_1 DUMMY1 + \lambda_2 DUMMY2 + \lambda_3 DUMMY3 \\ &+ \lambda_4 DUMMY4 + \varepsilon_t\end{aligned} \tag{1.4}$$

where DUMMY1 represents the dummy variable for the crisis in April 1984, DUMMY2 is the dummy variable for the crisis in December 1987, DUMMY3 is the dummy variable for the crisis in April 1994, and DUMMY4 is the dummy variable for the crisis in March 2001.

I assume two different distributions for ε_t. Following Nelson [33], the first distribution is a general error distribution (GED) with mean zero and variance ${h_t}^2$. Because there are four outliers in the data set and fat tail distributions deal with the outliers more successfully, I also use a Student-t distribution with 3 degrees of freedom.

The specific conditional version of Equation (1) for both models is given by

$$\ln(h_t^2) = \beta_0 + \beta_1 \frac{|\varepsilon_{t-1}|}{h_{t-1}} + \beta_2 \frac{|\varepsilon_{t-2}|}{h_{t-2}} + \beta_3 \frac{\varepsilon_{t-1}}{h_{t-1}} + \beta_4 \ln(h_{t-1}^2) \tag{1.5}$$

In this specification β_4 represents the persistence parameter. Furthermore, β_3 is the leverage parameter. If it is significant, its sign characterizes the asymmetry of the conditional variance of inflation.

1.3.2 The Results

Two different sets of results are obtained for the EGARCH model. The first set represents the results under GED specification for the error term, ε_t. Table 3 in Appendix B presents the results for EGARCH(2,1) model without crisis dummies under GED specification. A second-order GARCH component and a first-order moving average ARCH term are chosen based on ARCH-LM statistics.

The results show that the persistence parameter, β_4, is significant at the 1% significance level and equal to 0.891. This indicates that volatility is highly persistent. Furthermore, the leverage parameter, β_3, is not significant, and this reflects the absence of asymmetry in the conditional variance of inflation. All other volatility equation parameters except β_1 are not significant.

Table 4 in Appendix B represents the results for the EGARCH model with crisis dummies under GED specification for ε_t. Results suggest that crisis dummies for April 1994, and March 2001 are significant at the 1% significance level. On the other hand, estimation results for volatility-equation parameters indicate a robustness problem for the EGARCH model against outliers. The persistence parameter of the EGARCH model with crisis dummies is negative and not significant. Furthermore, all other volatility equation parameters are insignificant when crisis dummies are added to the model. Comparison of log-likelihood values from both models (with and without crisis dummies) shows that adding crisis dummies improves the model.

The second sets of results for EGARCH(2,1) model is obtained by assuming a Student-t distribution with 3 degrees of freedom for the error term. Table 5 in Appendix B presents the results for the model without crisis dummies. Based on the results, the persistence parameter is equal to 0.888 and significant at the 1% significance level. Furthermore, the leverage parameter, β_3, is not significant. All other parameters except β_1 are insignificant. The log-likelihood value is larger than the log-likelihood value of EGARCH model without crisis dummies under GED assumption.

Table 6 in Appendix B represents the results for the model with crisis dummies. Estimation results indicate that all crisis dummies, except March 2001, are significant at the 5%-significance level. Moreover, the persistence parameter increases to 0.908 when crisis dummies are added to the model. However, β_1 becomes insignificant when inflation is also a function of crisis dummies. In terms of log-likelihood values, the model improves when crisis dummies are added to the inflation equation. Because this distribution has fat tails and deals with outliers more successfully, these results show that EGARCH is more robust against outliers when we assume Student-t distribution for error term.

1.4 STOCHASTIC VOLATILITY

1.4.1 The Model

The SV model was first introduced by Taylor [40], [41]. It arises from the mixture-of-distributions hypothesis in which it is assumed that the unobservable flow of price-relevant information drives volatility. Stochastic Volatility models account for time-varying and persistent volatility as well as for leptokurtosis in financial-return analysis. On the other hand, efficient estimation is less straightforward because of the nonlinearity of the latent-volatility process. The literature examines a variety of estimation procedures, including among others the Generalized Method of Moments (GMM) by Melino and Turnball [32], Quasi Maximum Likelihood (QML) by Harvey et al. [22], Markov Chain Monte Carlo (MCMC) by Jacquier et al. [26].

The basic SV model is given by

$$\begin{aligned} r_t &= \exp\left(\frac{\lambda_t}{2}\right)\varepsilon_t \\ \lambda_t &= \gamma + \delta\lambda_{t-1} + \nu\eta_t \end{aligned} \tag{1.6}$$

where r_t is return on day $t : 1 \rightarrow T$. The $\{\varepsilon_t\}$ and $\{\eta_t\}$ are mutually independent iid. Gaussian random variables with mean zero and unit variances. $\{\gamma, \delta, \nu\}$ are parameters to be estimated. δ is the persistence of the log volatility and if $|\delta| < 1$, we say that the returns are strictly stationary. The ν parameter is the standard deviation of the volatility shocks.

A second model for SV is also estimated by adding the crisis dummies into the inflation equation. The model is given by

$$\begin{aligned} r_t &= \alpha_1 DUMMY1 + \alpha_2 DUMMY2 + \alpha_3 DUMMY3 + \alpha_4 DUMMY4 \\ &\quad + \exp\left(\frac{\lambda_t}{2}\right)\varepsilon_t \\ \lambda_t &= \gamma + \delta\lambda_{t-1} + \nu\eta_t \end{aligned} \tag{1.7}$$

where $\{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$ are coefficients of dummy variables.

In order to deal with the nonlinearity of the model and its serial dependence, I used the Efficient Importance Sampling (hereafter EIS) procedure proposed by Richard and Zhang [37]. The EIS procedure is a Monte Carlo (MC) technique used for the evaluation of high-dimensional integrals. It relies upon a sequence of low-dimensional regressions to construct an auxiliary MC sampler, which produces highly accurate MC estimates of the likelihood.

I programmed the same procedure that Liesenfeld and Richard [31] used to estimate the SV model for daily data of IBM stock prices, S&P 500 price indexes, and the exchange rate for the US Dollar and the Deutsche Mark. The procedure is summarized below.

Let $r_t, t\colon 1 \rightarrow T$ is an n-dimensional vector of observable random variables and λ_t is a q-dimensional vector of latent variables. The ML procedure is based on the marginalized likelihood function

$$L(\theta; R) = \int f(R, \Lambda; \theta) d\Lambda \tag{1.8}$$

where $R = \{r_t\}_{t=1}^{T}$, $\Lambda = \{\lambda_t\}_{t=1}^{T}$ and θ is an unknown parameter vector. Equation (8) can be factorized as follows

$$L(\theta; R) = \int \prod_{t=1}^{T} f(r_t, \lambda_t \mid \Lambda_{t-1}, R_{t-1}, \theta) d\Lambda \tag{1.9}$$

where $R_t = \{r_\tau\}_{\tau=1}^{t}$ and $\Lambda_t = \{\lambda_\tau\}_{\tau=1}^{t}$ .The model implicitly assumes that r_t is independent of Λ_{t-1} conditional on (λ_t, R_{t-1}) with a density of $g(r_t \mid \lambda_t, R_{t-1}, \theta)$ and that λ_t has the conditional density of $p(\lambda_t \mid \Lambda_{t-1}, R_{t-1}, \theta)$. Whence, the likelihood can be written as

$$L(\theta; R) = \int \prod_{t=1}^{T} g(r_t \mid \lambda_t, R_{t-1}, \theta) p(\lambda_t \mid \Lambda_{t-1}, R_{t-1}, \theta) d\Lambda \tag{1.10}$$

The EIS procedure constructs a sequence of samplers that exploits the sample information on the $\lambda_t' s$ as conveyed by $r_t' s$. Let, $\{m(\lambda_t \mid \Lambda_{t-1}, a_t)\}$ denotes such a sequence of auxiliary samplers indexed by the auxiliary parameters $A = \{a_t\}_{t=1}^{T}$. Let $\{\overset{\sim(i)}{\lambda_t}(a_t)\}_{t=1}^{T}$ denotes a trajectory drawn from the sequence of auxiliary samplers. Let $a_{(t-1)} = \{a_s\}_{s=1}^{t-1}$. The corresponding MC estimate of the likelihood can be written as

$$\tilde{L}_N(\theta; R, A) = \frac{1}{N} \sum \left\{ \prod_{t=1}^{T} \frac{f(r_t, \overset{\sim(i)}{\lambda_t}(a_t) \mid \overset{\sim(i)}{\Lambda}_{t-1} a_{(t-1)}, R_{t-1}, \theta)}{m(\overset{\sim(i)}{\lambda_t}(a_t) \mid \overset{\sim(i)}{\Lambda}_{t-1} a_{(t-1)}, a_t)} \right\} \tag{1.11}$$

Obviously if $\Pi_t m(\lambda_t \mid \Lambda_{t-1}, a_t)$ were proportional to $\Pi_t f(r_t, \lambda_t \mid \Lambda_{t-1}, R_{t-1}, \theta)$ then the MC sampling variance would be equal to zero. More generally, EIS constructs density kernels $k(\Lambda_t; a_t)$ for $m(\lambda_t \mid \Lambda_{t-1}, a_t)$ which are global approximation for $f(r_t, \lambda_t \mid \Lambda_{t-1}, R_{t-1}, \theta).\chi(\Lambda_{t-1}, a_t)$.The relationship between m and k is given by

$$\begin{aligned} m(\lambda_t \quad | \quad \Lambda_{t-1}, a_t) = \frac{k(\lambda_t; a_t)}{\chi(\Lambda_{t-1}, a_t)} \\ \text{where } \chi(\Lambda_{t-1}, a_t) \quad = \quad \int k(\lambda_t; a_t) d\lambda_t \end{aligned} \tag{1.12}$$

Since $\chi(\Lambda_{t-1}; a_t)$ does not depend on λ_t , the EIS problem turns into that of solving a simple back-recursive sequence of low-dimensional least-square problems of the form

$$\begin{aligned} \hat{a}_t(\theta) \quad = \quad & \arg\min_{a_t} \sum_{i=1}^{N} \{ \ln[f(r_t, \tilde{\lambda}_t^{(i)}(\theta) \mid \tilde{\Lambda}_{t-1}^{(i)}(\theta), R_{t-1}, \theta) \\ & .\chi(\tilde{\Lambda}_{t-1}^{(i)}(\theta); \hat{a}_{t+1}(\theta))] - c_t - \ln k(\tilde{\Lambda}_t^{(i)}(\theta); a_t)\}^2 \end{aligned} \tag{1.13}$$

for $t : 1 \rightarrow T$, with $\chi(\Lambda_T; a_{t+1}) \equiv 1$. The $c_t' s$ are unknown log-proportionality constants to be estimated jointly with $a_t' s$. EIS likelihood estimates are then obtained by replacing $\{a_t\}_{t=1}^{T}$in equation 10 with $\{\hat{a}_t(\theta)\}_{t=1}^{T}$. A small number of EIS iterations are needed to obtain maximally efficient importance samplers only. Typically, Common Random Samplers(CRN) technique is used to provide the convergence to the fixed auxiliary parameter $\hat{a}_t$.

Finally, the estimates of θ are obtained by maximizing Equation (11) with respect to θ. The use of CRN technique also ensures the smoothness of the MC functional approximation in Equation (10).

Under our assumptions, the conditional density of r_t and λ_t are given by

$$\begin{aligned} g(r_t \quad | \quad \lambda_t, \theta) \propto \exp\left\{-\frac{1}{2}\left[r_t^2 \exp(-\lambda_t) + \lambda_t\right]\right\} \\ p(\lambda_t \quad | \quad \lambda_{t-1}, \theta) \propto \exp\left\{-\frac{1}{2\nu^2}\left(\lambda_t - \gamma - \delta\lambda_{t-1}\right)^2\right\} \end{aligned} \tag{1.14}$$

The next step is to parametrize the density kernel. Liesenfeld and Richard [31] suggests the following parametrization

$$k(\Lambda_t; a_t) = p(\lambda_t \mid \lambda_{t-1}, \theta)\zeta(\lambda_t, a_t) \tag{1.15}$$

where $\zeta(\lambda_t, a_t)$ is also a Gaussian density kernel. This specification offers the advantage that it eliminates p from the EIS auxiliary regressions. Since g only depends on λ_t , an appropriate choice for ζ is given by $\zeta(\lambda_t, a_t) = \exp(a_{1,t}\lambda_t + a_{2,t}\lambda_t^2)$.Whence, k is given by

$$\begin{aligned} k(\Lambda_t; a_t) \;\; &\propto \;\; \exp\{-\frac{1}{2}[\left(\frac{\gamma+\delta\lambda_{t-1}}{\nu}\right)^2 - 2\left(\frac{\gamma+\delta\lambda_{t-1}}{\nu^2} + a_{1,t}\right)\lambda_t \\ &+ \left(\frac{1}{\nu^2} - 2a_{2,t}\right)\lambda_t^2]\} \end{aligned} \tag{1.16}$$

The conditional mean and variance of λ_t on m are given by

$$\begin{aligned} \mu_t &= \sigma_t^2\left(\frac{\gamma+\delta\lambda_{t-1}}{\nu^2} + a_{1,t}\right) \\ \sigma_t^2 &= \frac{\nu^2}{1-2\nu^2 a_{2,t}} \end{aligned} \tag{1.17}$$

1.4.2 The Results

Table 7 in Appendix B presents the results for SV model without crisis dummies based on EIS.

Asymptotic errors are obtained from a numerical approximation to the Hessian and MC standard errors are computed from 10 ML-EIS estimations conducted under different sets of CRNs. These MC standard errors measure the numerical accuracy of the coefficient estimates, and the MC standard errors indicate that our results are numerically very accurate. The persistence parameter δ is highly significant and equal to 0.816.

For the second SV model with crisis dummies, Table 8 in Appendix B represents the estimation results.

The estimation results for volatility equation parameters are very similar to estimation results from the first model. The persistence parameter is a little higher than the persistence parameter of the first estimation and equal to 0.853. Moreover, the standard deviation of volatility decreases when dummy variables are added to the inflation equation. All crisis-dummy parameters are significant except the crisis dummies for March 2001. Finally, decreases in the log-likelihood value indicate that the model is improved when dummy variables are added to estimation. Moreover, the log-likelihood values are larger than log-likelihood values for all EGARCH models and the SV model without crisis dummies.

Filtering enables us to compute a sequence of standardized residuals . By checking the distributional properties of the standardized residuals, we can check whether our model is correctly specified. The standardized residuals are of the form:

$$z_t = [r_t - E(r_t \mid R_{t-1})]Var(r_t \mid R_{t-1})^{-\frac{1}{2}} \tag{1.18}$$

For our basic SV model, the mean and standard deviation of r_t conditional on R_{t-1} are zero and $E[exp(\lambda_t) \mid R_{t-1}]^{-1/2}$, respectively. The model is correctly specified if z_t has zero mean and unit variance and is uncorrelated in the first and second order moments.

Furthermore, to check for the distributional properties of r_t, I applied an approach used by Liesenfeld and Richard [31]. This approach requires computing $u_t = Pr(r_t \leq r_t^* \mid R_{t-1})$ in which r_t^* is the actual observed return. If the model is correctly specified, u_t is a serially independent random variable and follows a uniform distribution on $[0, 1]$. Thus, we can map u_t into a standard normal distribution by using the inverse of the standard normal distribution function. Therefore, we have

$$z_t^* = F_N^{-1}(u_t) \tag{1.19}$$

Correct specification requires z_t^* to be serially independent standardized normal random variables.

Table 9 in Appendix B represents the results for the diagnostic checks.

According to the Kolmogorov-Smirnov statistic, we cannot reject the null hypothesis of normality. Furthermore, because the kurtosis of the z_t^* is not considerably higher than 3, which is the benchmark for normality, normality cannot be rejected. From the Ljung-Box statistics for the squared residuals including 30 lags, we can conclude that the model successfully accounts for the autocorrelation in the inflation series. On the other hand, the Ljung-Box statistics for the residuals implies the need for including an autoregressive component in the return function of the SV model. All in all, these results suggest in general that the SV model accounts for the distributional properties of the inflation series.

A further analysis can be performed by comparing filtered volatility graphs of EGARCH and SV models under the two different settings. Filtered volatility is the mean of volatility computed by using information available on inflation up to time t-1. Figures 3, 4 and

5 represent the filtered volatility graphs of respectively the EGARCH model with GED, student-t distribution assumptions for error term, and the SV models when no crisis dummies exist in the inflation equation. In contrast, Figures 6, 7 and 8 show the volatility graphs for EGARCH and SV models respectively when dummy variables are added to the inflation equation. Filtered volatility graphs for both the EGARCH model under different error-term distributions and the SV model are similar for the model without dummies. Volatility during the April 1994 crisis has a stronger peak in the EGARCH model. For the model with crisis dummies, the filtered volatility graphs for the EGARCH model with Student-t distribution and the SV model are almost the same. On the other hand, the filtered volatility graph for the EGARCH model with the GED assumption represents the robustness problem against outliers.

1.5 CONCLUSION

This paper represents research results from a comparison of EGARCH and SV models for Turkey's inflation volatility. We use different error-term specifications for the EGARCH model of inflation volatility. Overall results suggest that the SV model is more robust than EGARCH models against outliers, which are the crisis dummies.

The main findings of the paper are as follows.

First, inflation data for Turkey suffers from trend and monthly-seasonality problems.

Second, after these problems are eliminated, the results of EGARCH estimation without exchange-market crisis dummies under error-term distribution assumptions are quite similar. In terms of SV model results, persistence is smaller than EGARCH models.

Third, when dummies are included in the model, EGARCH results under GED specification indicate a robustness problem against outliers. Furthermore, under Student-t distribution for error-terms, the robustness problem still remains because β_1 becomes insignificant when dummy variables are added.

Fourth, when we use the SV model with crisis dummies, persistence increases and standard deviation of volatility decreases. The volatility constant remains almost the same. Therefore, the SV model is more robust than both EGARCH specifications. Furthermore, log-likelihood values indicate that the SV model with crisis dummies is better than all other model specifications.

Finally, when the distributional properties of filtered values from the SV model are examined, the results show that the model successfully accounts for the serial correlation in the volatility of inflation and that inclusion of an autoregressive component in the return function might be needed.

The comparison of the estimation results from two models, EGARCH and SV, under two different settings clearly indicates that SV is more robust than EGARCH. With or without dummy variables, SV has a higher log-likelihood value than EGARCH. Furthermore, persistence parameter estimates are more plausible under SV model than EGARCH models.

2.0 FORECASTING INFLATION VOLATILITY: A STOCHASTIC VOLATILITY APPROACH

2.1 INTRODUCTION

As highlighted by the recent instability of world financial markets, volatility is a fundamental component of asset allocation. Specifically, investors need carefully to assess rates of return and volatility when making financial decisions. Much research in financial econometrics focuses on understanding the relationship between volatility or risk and return while often emphasizing volatility estimation. However, sound investment decisions require more than estimation. Investors also must analyze whether estimated relationships remain constant over time or instead change their dynamics. In spite of the growing need for such analysis, research into the forecasting of volatility, which is the primary focus of this paper, lags well behind many other topics that have a less direct bearing on investors' portfolios.

In general, most studies of volatility focus on first modeling and then forecasting its effects on specific economic phenomena, such as stock returns, exchange rates, etc. Researchers use a number of different approaches and methods in volatility modeling. The class of models known as Autoregressive Conditional Heteroscedastic (ARCH) models, invented by Nobel Laureate Robert Engel [15], remains the most widely used. The ARCH model characterizes the distribution of stochastic errors that are conditional on the realized values of a set of variables. Because this model can create problems in the higher order of the polynomials, researchers developed different extensions to the ARCH model. The Generalized ARCH (GARCH) model, which was developed by Bollerslev [8] and Taylor [40], defines volatility as a combination of polynomials in auto-correlated errors and polynomials in moving average term. This definition of the volatility structure resolves the shortcoming of the original

ARCH model in higher order polynomials. However, empirical analysis demonstrates that these models still possess shortcomings. The continued manifestation of such shortcomings motivated researchers to develop other extensions for the ARCH models. As one example, both ARCH and GARCH models assume that there is symmetry between the effects of positive and negative shocks to the return on volatility. However, in practice, this symmetry is violated because negative shocks have a greater effect than positive shocks. Noting this anomaly, several researchers tried to overcome it by allowing a leverage effect in the GARCH model, an effect which implies that volatility reacts asymmetrically to the negativity and positivity of the shocks. Among several extensions of GARCH models that allow asymmetry, the Exponential GARCH (EGARCH) model introduced by Nelson [33] is the most famous and widely used.

In addition to the ARCH family, an analyst can turn to several other tools for the modeling of volatility, including models of Implied Volatility, Historical Volatility and Stochastic Volatility. This paper examines the Stochastic Volatility Model, which Taylor [40], [41] first introduced. Researchers have given it attention in recent years because of its flexibility in modeling volatility.

The flexibility of the SV model finds most of its expression in the model's allowance for noise in the volatility function. The model does not force the innovations to have fat tails, in other words, to have more outliers, or require volatility persistence to be close to the value 1 in order to allow simultaneous occurrences of both high kurtosis and small autocorrelation. The existence of these additional error terms in the volatility equation permits the SV Model to be more flexible than ARCH family models.

Nonetheless, the estimation of the SV model is not a straightforward calculation. Because of the nonlinearity of latent or unobservable variables in the SV model, an estimation problem arises. In turn, that problem results in a likelihood function that depends upon high-dimensional integrals which I cannot evaluate with straightforward mathematical tools. Researchers use different methods to overcome this problem such as Generalized Method of Moments (GMM), the Quasi-Maximum Likelihood (QML) and the Markov Chain Monte Carlo (MCMC). My research employs the Maximum Likelihood (ML) based on Efficient Importance Sampling (EIS) by Richard and Zhang [37]. The EIS has numerous attractive features. One of its most important features is success in producing highly accurate Monte Carlo (MC) estimates. Furthermore, because it is used to evaluate the likelihood function itself, it can also be used for a full range of likelihood-based inference techniques, such as estimation, testing and Bayesian inference. Equally important, since its basic structure does not depend upon a specific model, changes in the model can be easily accommodated by minor changes in the algorithm. These characteristics of EIS make it attractive for the SV analysis.

The main goal of this paper is to forecast volatility, not to model it. As noted earlier, researchers use several different methods to estimate and forecast volatility, and numerous papers compare the performance in estimating and forecasting volatility among many types of models. When comparing forecasting accuracy, the main focus has been on ARCH and Implied Volatility models. Akgiray [1] states that forecasts based on the GARCH model are superior. Yet his conclusion appears to be outweighed by the greater number of research findings that favor the Implied Volatility model, as, for example, Day and Lewis [12] and Fleming [16]. I use the SV model based on EIS because of the flexibility and numerical accuracy of the method and the indication of its success in forecasting volatility as reported by several investigators. Bluhm and Yu [7] argue that SV should be used to forecast volatility of option prices. Furthermore, Hol and Koopman [24] state that the SV model outperforms the GARCH model when there is an absence of intraday volatility information. However, the amount of research supporting these assertions is limited. Although several commentators state that SV performs better than other volatility models, it remains difficult to conclude that the SV model provides the most accurate forecasts due to the limited amount of work on forecasting based on the SV model.

Application of the tool to real world data and the accuracy of the results are important parts of volatility research. The model gains its importance due to its success in application. Most research that came from the application of the SV and ARCH models focused on the forecasting of volatility of stock prices, currency exchange rates and other valuations of investments. Engel provided the original ARCH Model to equip analysts with a tool for measuring the dynamics of inflation. While preparing to investigate the performance of the SV model, I noted a dearth of work on inflation volatility. To address this issue, when seeking to test the model, I decided to direct my work to the forecasting of inflation volatility as a way of also widening the window of research on this important topic.

Most published research about forecasting inflation volatility investigates the relationship of the volatility to other economic phenomena, such as labor market variables and forecast outputs, as can be seen in the work of Giordani and Soderlind [21] and Rich and Tracy [34]. My work develops a discussion around the value of this analytical tool in forecasting the core phenomenon of inflation. Thus I focused research for this discussion on the value of the SV function as a tool that warrants attention for its success in forecasting the volatility of inflation. Although the primary emphasis of this paper rests upon the value of the SV Model based on EIS as a tool for forecasting inflation volatility, the search for a tool to validate the forecasting method drew my attention to the limited availability of such tools in the case of inflation.

In general, when assessing forecasting performances, researchers use Root Mean Squared Error (RMSE) or other similar measures, which work relatively well when applied to high-frequency data such as stock returns. The RMSE is defined as the distance of a data point from the fitted line, which, in this case, is the distance of realized volatility from the forecasted volatility point. The need for a realized volatility arises because volatility is not observable. Moreover, while unobservable, volatility can only be calculated for high-frequency data sets. Further complicating this analysis, points of measurement for inflation occur at a very low frequency – once a month as compared to several times a minute or thousands of times a month in the case of prices for transactions in financial markets. Therefore it is not possible to calculate a measure of realized volatility for inflation. This limitation represented a significant problem for assessing the validity of the forecasting method. Therefore, I proposed another method based on the empirical distribution of forecasted errors, extending earlier contributions by Liesenfeld and Richard [31], [30].

This paper is an empirical analysis of the SV model based on EIS when used for forecasting inflation volatility. The analysis uses a new tool for assessing the validity of the method for forecasting volatility and owes a special debt of gratitude to the work of Liesenfeld and Richard [31], [30]. The remainder of the paper is organized as follows. In Section II, I present the SV model and EIS method. In Section III, I explain the one-step ahead forecasting procedure. Then, I talk about U.S. data and results and Turkish data and results in Section IV and V, respectively. Finally, in Section VI, I offer a conclusion.

2.1.1 Stochastic Volatility and EIS

The basic SV model by Taylor [40], [41] is given by

$$\begin{aligned} r_t &= \exp(\lambda_t/2)\epsilon_t \\ \lambda_t &= \gamma + \delta\lambda_{t-1} + \nu\eta_t \end{aligned} \tag{2.1}$$

where r_t is the return on day $t : 1 \to T$. (γ, δ, ν) are the parameters to be estimated and the processes $\{\varepsilon_t\}$ and $\{\eta_t\}$ are mutually independent iid Gaussian random variables with zero means and unit variances. The unobserved log volatility λ_t follows an AR(1) process with the unobservable persistence parameter δ. If $|\delta| < 1$, the returns are strictly stationary. Finally, the standard deviation of volatility shocks is measured by $\nu > 0$.

In order to evaluate the likelihood associated with the returns, I need to integrate out the latent variable $\{\lambda_t\}$ from the joint density of the observed and latent variables. The λ_t latent variables are serially dependent and enter the model nonlinearly. Therefore, standard numerical integration techniques are not applicable to this high dimensional non-Gaussian integration problem. To overcome this problem, different methods are used in the literature, including, for example, the "Generalized Method of Moments" (GMM) by Melino and Turnbull [32], the "Quasi-Maximum Likelihood" (QML) by Harvey et al [22], "Markov Chain Monte Carlo" (MCMC) by Jacquier at al [26] and Kim et al [28].

In this paper, I use EIS to evaluate the likelihood function itself which is then used for inference and forecasting. The EIS procedure is a Monte Carlo (MC) technique which is used for efficient evaluation of high-dimensional integrals. See Richard and Zhang [37] for details. The procedure basically relies upon a sequence of low-dimensional regressions to construct an auxiliary MC sampler which produces highly accurate MC estimates of the likelihood. The procedure is summarized below.

Let r_t denote an n-dimensional vector of observable random variables and let λ_t denote a q-dimensional vector of latent variables, $t : 1 \to T$. The ML procedure is based on the marginalized likelihood function

$$L(\theta; R) = \int f(R, \Lambda; \theta) d\Lambda \tag{2.2}$$

where $R = \{r_t\}_{t=1}^{T}$, $\Lambda = \{\lambda_t\}_{t=1}^{T}$ and θ is an unknown parameter vector. Equation (21) can be factorized as follows

$$L(\theta; R) = \int \prod_{t=1}^{T} f(r_t, \lambda_t \mid \Lambda_{t-1}, R_{t-1}, \theta) d\Lambda \tag{2.3}$$

where $R_t = \{r_\tau\}_{\tau=1}^{t}$ and $\Lambda_t = \{\lambda_\tau\}_{\tau=1}^{t}$ .The joint density of $\lambda_t, r_t|\Lambda_{t-1}, R_{t-1}$ is then factorized into the product of the density of $r_t|\Lambda_t, R_{t-1}$ and that of $\lambda_t|\Lambda_{t-1}, R_{t-1}$. It is then assumed that r_t is independent of Λ_{t-1} given (λ_t, R_{t-1}). Whence, the likelihood can be written as

$$L(\theta; R) = \int \prod_{t=1}^{T} g(r_t \mid \lambda_t, R_{t-1}, \theta) p(\lambda_t \mid \Lambda_{t-1}, R_{t-1}, \theta) d\Lambda \tag{2.4}$$

The EIS procedure constructs a sequence of samplers that exploits the sample information on the $\lambda_t' s$ as conveyed by $r_t' s$. Let, $\{m(\lambda_t \mid \Lambda_{t-1}, a_t)\}$ denotes such a sequence of auxiliary samplers indexed by the auxiliary parameters $A = \{a_t\}_{t=1}^{T}$. Here, a_t is implicitly a function of (θ, R). While R is fixed, a new value of a_t will have to be computed for each θ . Let $\{\widetilde{\lambda}_t^{(i)}\}_{t=1}^{T}$ denote a trajectory drawn from a particular sequence of auxiliary samplers. The corresponding MC estimate of the likelihood can be written as

$$\widetilde{L}_N(\theta; R, A) = \frac{1}{N} \sum \left\{ \prod_{t=1}^{T} \frac{f(r_t, \widetilde{\lambda}_t^{(i)} \mid \widetilde{\Lambda}_{t-1}^{(i)}, R_{t-1}, \theta)}{m(\widetilde{\lambda}_t^{(i)} \mid \widetilde{\Lambda}_{t-1}^{(i)}, a_t)} \right\} \tag{2.5}$$

Obviously if $\Pi_t m(\lambda_t \mid \Lambda_{t-1}, a_t)$ were proportional to $\Pi_t f(r_t, \lambda_t \mid \Lambda_{t-1}, R_{t-1}, \theta)$ then the MC sampling variance of $\widetilde{L_N}$ would be equal to zero. More generally, $f(r_t, \lambda_t|\Lambda_{t-1}, R_{t-1}, \theta)$ is a function of Λ_t whose integral w.r.t. λ_t is unknown. Hence I can not expect it to be approximated by a density $m_t(\lambda_t|\Lambda_{t-1}, a_t)$.

EIS approximations are based upon density kernels rather than densities. Let $k_t(\lambda_t; a_t)$ denote such a kernel for $m_t(\lambda_t|\Lambda_{t-1}, a_t)$. The relationship between k_t and m_t is given by

$$
\begin{aligned}
m_t(\lambda_t|\Lambda_{t-1}, a_t) &= \frac{k_t(\lambda_t; a_t)}{\chi_t(\Lambda_{t-1}, a_t)} \\
\text{where } \chi_t(\Lambda_{t-1}, a_t) &= \int k_t(\lambda_t; a_t) d\lambda_t
\end{aligned} \tag{2.6}
$$

Kernels are to be selected in a such a way that χ_t has a known analytical expression. Note that $\chi_1(\Lambda_0, a_1) = \chi_1(a_1)$.Because $\chi(\Lambda_{t-1}; a_t)$ does not depend on λ_t it can be transferred back into the $t-1$ integral. Hence Equation (24) is rewritten as

$$
\widetilde{L}_N(\theta; R, A) = \chi_1(a_1).\frac{1}{N}\sum\left\{\prod_{t=1}^{T}\frac{f(r_t, \overset{\sim(i)}{\lambda_t} \mid \overset{\sim(i)}{\Lambda_{t-1}}, R_{t-1}, \theta).\chi_{t+1}(\widetilde{\Lambda}_t^{(i)}, a_{t+1})}{k_t(\widetilde{\lambda}_t^{(i)}; a_t)}\right\} \tag{2.7}
$$

The EIS problem becomes a matter of solving a simple back-recursive sequence of low-dimensional least-square problems of the form

$$
\begin{aligned}
\hat{a}_t &= \arg\min_{a_t}\sum_{i=1}^{N}\{ \ln[f(r_t, \overset{\sim(i)}{\lambda_t} \mid \overset{\sim(i)}{\Lambda_{t-1}}, R_{t-1}, \theta) \\
&\quad .\chi_{t+1}(\widetilde{\Lambda}_t^{(i)}; \widehat{a}_{t+1})] - c_t - \ln k(\overset{\sim(i)}{\Lambda_t}; a_t)\}^2
\end{aligned} \tag{2.8}
$$

for $t : 1 \rightarrow T$, with $\chi(\Lambda_T; a_{t+1}) \equiv 1$. The $c_t's$ are unknown log-proportionality constants to be estimated jointly with $a_t's$. EIS likelihood estimates are then obtained by replacing $\{a_t\}_{t=1}^{T}$in Equation (5) with $\{\widehat{a}_t(\theta)\}_{t=1}^{T}$. Since $\left\{\widetilde{\Lambda}^{(i)}\right\}_{i=1}^{N}$ themselves are draws from the EIS samplers, EIS fixed iterations are needed to obtain maximally efficient importance samplers. See Richard and Zhang [37] for details. Common Random Samplers(CRN) are used to smooth the convergence to the fixed auxiliary parameter $\widehat{a}_t$.

Finally, the estimates of θ are obtained by maximizing Equation (26) with respect to θ. The use of CRN technique also ensures the smoothness of the MC functional approximation in Equation (26). Next, I discuss the application of EIS to the SV model as defined in Equation (20).

Under our assumptions, the conditional density of r_t and λ_t are given by

$$\begin{aligned} g(r_t|\lambda_t,\theta) &\propto \exp\left\{-\frac{1}{2}\left[r_t^2\exp(-\lambda_t)+\lambda_t\right]\right\}, \\ p(\lambda_t|\lambda_{t-1},\theta) &\propto \exp\left\{-\frac{1}{2\nu^2}\left(\lambda_t-\gamma-\delta\lambda_{t-1}\right)^2\right\}, \end{aligned} \tag{2.9}$$

respectively. The next step is to parametrize the density kernel. Since the class of Gaussian densities is closed under multiplication, Liesenfeld and Richard [31] suggests the following parameterization

$$k(\Lambda_t;a_t) = p(\lambda_t \mid \lambda_{t-1},\theta)\zeta(\lambda_t,a_t) \tag{2.10}$$

where $\zeta(\lambda_t,a_t)$ is also a Gaussian density kernel. This specification offers the advantage that it eliminates p from the EIS auxiliary regressions. Since g only depends on λ_t , an appropriate choice for ζ is given by $\zeta(\lambda_t,a_t)=\exp(a_{1,t}\lambda_t-a_{2,t}\lambda_t^2)$.Hence,

$$\begin{aligned} k(\Lambda_t;a_t) \propto \ & \exp\{-\frac{1}{2}[\left(\frac{\gamma+\delta\lambda_{t-1}}{\nu}\right)^2-2\left(\frac{\gamma+\delta\lambda_{t-1}}{\nu^2}+a_{1,t}\right)\lambda_t \\ & +\left(\frac{1}{\nu^2}-2a_{2,t}\right)\lambda_t^2]\} \end{aligned} \tag{2.11}$$

The conditional mean and variance of λ_t on m are given by

$$\mu_t = \sigma_t^2\left(\frac{\gamma+\delta\lambda_{t-1}}{\nu^2}+a_{1,t}\right) \tag{2.12}$$

$$\sigma_t^2 = \frac{\nu^2}{1-2\nu^2 a_{2,t}} \tag{2.13}$$

and χ_t is given by

$$\chi(\lambda_{t-1},a_t) \propto \exp\left\{\frac{\mu_t^2}{2\sigma_t^2}-\frac{(\gamma+\delta\lambda_{t-1})^2}{2\nu^2}\right\} \tag{2.14}$$

2.1.2 One Step Ahead Forecasting Method

The forecasting procedure I use in this paper is based on the idea of filtering, whereby forecasts for period $t+1$ are based on observations from 1 to t. Hence I start estimation with a fixed number of observations and then increase the number of observations by one at a time. Because the EIS procedure runs backward, the model needs to be fully re-estimated each time new observations are added. Reruns of EIS under added dimensions are very fast because I use the previously computed EIS sampler as an initial sampler and augment it by initial samplers for the added dimensions. The forecasts which are calculated in the previous steps provide these initial samplers for the added dimensions.

The forecasting algorithm starts with the estimation of inflation by an AR(1) process given by

$$\pi_t = \alpha + \beta\pi_{t-1} + \varepsilon_t \tag{2.15}$$

where π_t is the inflation at time t. The corresponding OLS residuals $\hat{\varepsilon}_t s$,which are centered around their sample mean, are then standardized in order to produce stationary series (assuming $|\beta| < 1$)

$$r_t = \left(\frac{1}{\widehat{\sigma}}\right)\widehat{\varepsilon_t} \tag{2.16}$$

where $\widehat{\sigma}$ is the estimated standard deviation of $\widehat{\varepsilon_t}s$.

After the model is estimated, the forecasts for the volatility are given by

$$\widehat{\lambda_t} = \widehat{\gamma} + \widehat{\delta}\lambda_{t-1} + \widehat{\nu}\eta_t \tag{2.17}$$

Because volatility is an unobservable process, I use the latent variables produced by the EIS procedure as an approximation of the filtered distribution of volatility at time $t-1$. Using the same CRN as in the estimation procedure allows the analyst to compute a set of volatility forecasts, which provide a distribution for volatility at time t. Because the returns are products of the error terms by volatility, it is possible to construct a forecast of r_t as follows

$$\widehat{r_t} = \exp(\widehat{\lambda_t}/2)\epsilon_t, \tag{2.18}$$

using standardized Gaussian draws for ϵ_t.

Each time an observation is added the procedure starts over by reestimating the AR(1) process and the $\widehat{\varepsilon}_t s$.

Because the volatility cannot be observed, an evaluation of the forecasting performance requires special care. To address this limitation, I devised a new method for checking the validity of forecasts. This approach requires computing $\widehat{u_t} = F(\widehat{r_t|r_{t-1}}, \widehat{\theta}),\ t : 1 \rightarrow\rightarrow T$ where $\widehat{F()}$ denotes the forecasted distribution function and T_o the initial forecasting period. If the forecasts are correctly distributed, the $\widehat{u}_t' s$ follow a uniform distribution which implies a linear graph for their cumulative distribution.

Furthermore, I can compute the $\widehat{u}_t'$ by using the probability that forecasted return is lower than observed return. Conditionally on λ_t, this probability is given by:

$$\begin{aligned} \widehat{u_t} &= \Pr(\widehat{r_t} < r_t|\lambda_t, R_{t-1}, \widehat{\theta}) = \Pr\left(\exp\left(\frac{\lambda_t}{2}\right)\epsilon_t < r_t|\lambda_t, \widehat{\theta}\right) \\ &= \Phi\left(r_t \exp\left(-\frac{\lambda_t}{2}\right)|\lambda_t, \widehat{\theta}\right) \end{aligned} \tag{2.19}$$

where $\widehat{r_t}$ denotes forecasted return for $t : T_o \rightarrow T$ and Φ is the cumulative distribution of the standardized normal. Then I have:

$$\begin{aligned} \widehat{u_t} &= \Pr(\widehat{r_t} < r_t|R_{t-1}, \widehat{\theta}) = \int \Phi\left(r_t \exp\left(-\frac{\lambda_t}{2}\right)|\lambda_t\right) f(\lambda_t|\lambda_{t-1}, \widehat{\theta}) \\ & \quad f(\lambda_{t-1}|Y_{t-1}, \widehat{\theta}) d\lambda_t d\lambda_{t-1} \\ &\simeq \frac{1}{N}\sum_{i=1}^{N}\Phi\left(r_t \exp\left(-\frac{\widetilde{\lambda}_t^{\,i}}{2}\right)\right) \end{aligned} \tag{2.20}$$

This probability gives us another representation for the u terms. Therefore, if the forecasts are valid the graph should be uniform again.

In order to provide a formal test of forecast validity, I compute the Kolmogorov-Smirnov statistic (KS) for the $\{\widehat{u_t}\}_{t=T_0}^{T}$ relative to the uniform distribution and rely upon Monte Carlo simulation to calibrate it. Specifically, I create 200 hundred fictitious data sets for each application based on the estimated coefficients for the whole sample. The fictitious data sets are simulated according to the Equation (20). The calibration process requires computing the Kolmogorov-Smirnov statistics of the fictitious data sets. The procedure follows as:

1) I rerun the estimation and forecasting procedure for each data set.

2) I compute $\{\widehat{u}_t\}_{t=T_0}^{T}$ for each estimation.

3) I calculate the Kolmogorov-Smirnov statistics for $\{\widehat{u}_t\}_{t=T_0}^{T}$ for each of the 200 fictitious data set and find the 5% critical value.

4) I compare the Kolmogorov-Smirnov statistic of my application and the critical value.

If my forecasts are valid, the Kolmogorov-Smirnov statistic of the application should be lower than the critical value (with probability 0.95).

2.2 U.S. INFLATION

2.2.1 Data

I use two different data sets to test the model. The first one is the U.S. inflation for the period January 1914-December 2006. Figure 8 in the Appendix C represents the graph of the U.S. inflation series. The data shows no apparent trend but the return for the current period is clearly highly correlated to the prior period's return. Autocorrelation is eliminated by the AR(1) estimation described above. Figure 9 in the Appendix C presents the graph of the corresponding $\widehat{\varepsilon}_t's$.

2.2.2 Results

In total, there are 1,115 data points for the U.S. inflation series. For the analysis, I have divided this data set into two segments. I use the first 799 points for the estimation of the volatility. Then I use the remaining 316 data points for filtered forecasts (T_0 = 800, T = 1115). Table 10 in the Appendix C presents the results for the first estimation. Asymptotic errors are obtained from a numerical approximation to the Hessian and MC standard errors are computed from 10 ML-EIS estimations conducted under different sets of CRNs. These MC standard errors measure the numerical accuracy of the coefficient estimates and indicates that our results are numerically very accurate. The results show that the persistence parameter δ is very close to 1, which implies that volatility is highly persistent. Table 11 in Appendix C represents the results with 1,114 data points. The result for the persistence δ is similar to the first persistence parameter. This second result, using 1,114 data points, also indicates high persistence. Note that, when the number of data points increases, the volatility intercept γ becomes smaller and the variance ν^2 becomes higher. The volatility in the graph is measured by the following formula;

$$E\left[e^{\lambda_t}\right] = e^{E(\lambda_t)+\frac{1}{2}\nu^2} \tag{2.21}$$

This function equals 16.48 for the first half of the data set. It equals 0.85 for the second half. These results are consistent with the shape of the inflation series in the graph.

As mentioned above, if the forecasting procedure is valid, one would expect the graph of $\widehat{F(r_t)}$ to be linear. Figure 10 in Appendix C represents the graph for $\{\widehat{u}_t\}_{t=T_0}^{T}$ which are computed according to Equation (39). This graph shows that SV model based on EIS is successful in out-of-sample forecasting of U.S. inflation volatility.

For the validity test based on the KS statistics, the critical value is equal to 0.113. The KS statistic for the application is 0.065 which is obviously smaller than the critical value.

2.3 TURKISH INFLATION

2.3.1 Data

I use the inflation series for Turkey between February 1982 and August 2005 for my second application. Figure 11 in Appendix C graphs the series. The shape of the graph indicates that the inflation series suffers from a trend problem. To eliminate the trend, I use the following approach. I assume a 4th order polynomial function for the trend component

$$\pi_t = \alpha + \beta x + \varphi x^2 + \delta x^3 + \epsilon x^4 + \varepsilon_t \tag{2.22}$$

where $x = \frac{t}{T}$. Because I do not want to extrapolate the trend beyond $t = T$, I impose the condition that the trend and its derivative are zero at $x = 1$. The corresponding restrictions are given by

$$\begin{aligned} \alpha + \beta + \varphi + \delta + \epsilon &= 0 \\ \beta + 2\varphi + 3\delta + 4\epsilon &= 0 \end{aligned} \tag{2.23}$$

I use these restrictions to eliminate α and β in Equation (42), which is then rewritten as

$$\pi_t = \varphi\left[(x^2 - 1) - 2(x - 1)\right] + \delta\left[(x^3 - 1) - 3(x - 1)\right] + \epsilon\left[(x^4 - 1) - 4(x - 1)\right] + \varepsilon_t \tag{2.24}$$

Next, I estimate this equation and subtract the estimated values for π_t from the actual values in order to obtain a detrended inflation series. Figure 12 in Appendix C shows the graph for the series after detrending.

The second step is to check for seasonality in the data set. I achieve this by regressing inflation on its first-order lag and 12 monthly dummies. Table 12 in Appendix C presents the results for this estimation. Monthly dummies for January, April, June, July, September and October are significant at the 5% significance level based on the p-values. These results are reasonable and reflect the pattern of government economic policymaking, which affects prices. The Turkish government launches its economic program in January, agricultural sector prices start to be announced during June and July and educational spending increases in September and October.

In order to eliminate seasonality, I regressed the series on these six monthly dummies and obtained the error terms from this regression. I checked whether this method succeeded in eliminating seasonality by regressing the seasonally adjusted inflation series on its first order lag and 12 monthly dummies. Table 13 in Appendix C presents the results, which indicate that seasonality is eliminated. The graph of the seasonally adjusted inflation series is presented in Figure 13 in Appendix C. There remain four peak points in this graph; April 1984, December 1987, April 1994 and March 2001. These spikes correspond to large increases in inflation caused by major exchange rate crises in Turkey. I used the following procedure to eliminate these peak points.

I introduce a dummy variable d_i for each peak points i ($i = 1, 2, 3, 4$) and construct the following auxiliary regression:

$$\begin{aligned} y_t^* &= y_t - d_i \\ y_t^* &= \alpha + \beta y_{t-1} + \varepsilon_t \\ y_{t+1} &= \alpha + \beta y_t^* + \varepsilon_{t+1} \end{aligned} \tag{2.25}$$

I rewrite Equation (44) as

$$\begin{aligned} (y_t - \alpha + \beta y_{t-1} + d_i)^2 &= \varepsilon_t^2 \\ (y_{t+1} - \alpha + \beta(y_t - d_i)) &= \varepsilon_{t+1}^2 \end{aligned} \tag{2.26}$$

Given (α, β), I minimize the sum of ε_t^2 and ε_{t+1}^2 with respect to d_i. The corresponding estimate of d_i is given by

$$\hat{d_i} = \frac{1}{1+\beta^2}\left[y_t - \alpha - \beta y_{t-1} - \beta y_{t+1} + \beta\alpha + \beta^2 y_t\right] \tag{2.27}$$

Since d_i depends on (α, β), I implement the following fixed point procedure: Given d_i, compute y_t^*; then regress y_t^* on y_{t-1} to obtain $(\widehat{\alpha}, \widehat{\beta})$ and compute a new value for d_i. I iterate this procedure until convergence. In the present case, two iterations suffice to achieve convergence.

Finally, the adjusted inflation series for Turkey suffers from the same autocorrelation problem as the U.S. inflation series. I eliminate the autocorrelation in the series by AR(1) estimation in the forecasting procedure. Figure 14 in Appendix C represents the graph of the final $\widehat{\varepsilon}_t$ for the Turkish data series.

2.3.2 Results

I first estimate the model by using the first 199 data points. Table 14 in Appendix C represents the results for this estimation. The estimation results for the Turkish data set are noticeably different from the results for the U.S. data set. The persistence parameter is lower, the volatility intercept γ is negative and the volatility variance ν^2 is higher. Table 15 in Appendix C presents the results with 281 data points. The persistence parameter δ is higher than the persistence parameter with 199 data points. Also, the constant parameter and the variance of the volatility are smaller. I can explain the estimation results by the increased stability of Turkish economy in recent years. The exchange rate crises from which Turkey suffered in the past caused Turkey's economy to be unstable, and this condition is evident in the inflation data. Since 2001 and the change in government, Turkey's economy became more stable with lower rates of unemployment and inflation. These changes in the Turkish economy are reflected in my results.

The estimation has two explicit results: when observations are added one at-the-time the persistence parameter δ increases while both the volatility intercept γ and the volatility variance ν^2 decreases. Forecasting results also reflect the varying stability of the Turkish economy. Figure 15 in Appendix C represents the graph for $\widehat{u}_t^{\prime} s$ which are computed according to Equation (39). It is less linear than that for the U.S. case. Nevertheless, the figure implies that out-of-sample forecasting by the SV model based on the EIS is successful.

The critical value of the KS statistic is equal to 0.234. The KS statistic for the application is 0.126 which is obviously much smaller than the critical value.

Furthermore, in order to investigate the power of KS-test I simulate return series with fat tails by using the filtered volatilities for Turkey. Before starting to simulate the data series I first of all recover the filtered volatilities of whole data set for Turkey. Then I simulate error terms of return equation by using Student-t distribution with 3,6 and 9 degrees of freedoms. Finally when we combine the filtered volatility series and error terms based on SV model formula, this provides new return series which has the same first and second moment with inflation series of Turkey but with fat tails. Next, I run the estimation and forecasting procedure for the simulated series and calculate the KS-statistics. If KS-test is a powerful tool for measuring the forecasting validity, we expect that the cumulative distribution graph for simulated series should mostly deviate from a straight line at both ends and also KS-statistics should be greater than KS-critical value. Furthermore, in order to calculate the probability of rejection, we create 200 different fat-tailed series for each degrees of freedom and apply this procedure for each one. Figure 16 in Appendix C represents the cumulative distribution for first fatter tailed series with 3 degrees of freedom. The graph deviates from a straight line at both ends as we expect. Moreover, the KS-statistics for the same series is equal to 0.256 which is greater than KS-critical value of 0.234. The probability of rejection for KS-test for 3, 6 and 9 degrees of freedom are 0.91, 0.80 and 0.67, respectively. Since fat-tails start to disappear and Student-t distribution converges to normal distribution when degrees of freedom increases, one shall expect the probability of ejection decrease when the degrees of freedom increases. Results reflect this expectation. Therefore, KS-test is a powerful tool for measuring the forecasting validity.

2.3.3 Conclusion

This paper presents research focused on forecasting inflation volatility by using the standard Stochastic Volatility model based on Efficient Importance Sampling. The main purpose of this paper is to evaluate the validity of the SV model for forecasting inflation volatility. Because inflation data is not a high frequency data set, it is not possible to calculate the realized volatility for inflation. Therefore I cannot compute a mean square error measure for inflation volatility. This paper represents an alternative procedure based on the forecasted error structure of the returns.

I used inflation data sets for the United States and Turkey to evaluate empirically the performance of the method. I summarize the contribution of this paper under three headings. First of all, this work validates the SV model as a tool for forecasting inflation volatility. Second, although inflation dynamics were the main concern of Engel's Nobel Prize-winning research, the actual use of volatility models to forecast inflation has captured very little attention among econometric researchers. This paper aims at filling this gap. Finally, I develop a validation procedure for volatility forecasting applied to low-frequency data sets, as an extension of work by Richard & Liesenfeld [31], [30].

The empirical results based upon the monthly inflation series of the U.S. and Turkey can be summarized as follows:

For the U.S. inflation series, the persistence parameter is very close to 1 both for 799 and 1,114 data points. Moreover, the volatility intercept decreases while variance of volatility increases as observations are added. In terms of forecasting results, my forecasting validity tests show that the SV model is successful in out-of-sample forecasting of inflation volatility of U.S.. For the Turkish data set, the persistence parameter is lower when the data set is smaller. Furthermore, the volatility intercept and volatility variance decrease when the data set grows larger. Although the forecasting results are not as strong as the results for U.S. inflation, the test results still support the validity of the SV model based on the EIS for forecasting inflation volatility.

3.0 DO RETURNS GRANGER-CAUSE VOLATILITY?

3.1 INTRODUCTION

Modeling volatility has been the focus of financial econometrics for the last two decades. The ARCH-family models, which were developed by Nobel Laureate Robert Engel [15], represent one well-known approach to volatility modeling. These models assume that conditional variance is a function of the squares of previous observations and past variations.

An important alternative to this framework, which is also the main focus of this paper, is the Stochastic Volatility (SV) model. This model was first introduced by Taylor[39], [41]. The SV model allows the conditional mean and the variance to be characterized by separate stochastic processes. The basic discrete SV model assumes that return is an exponential function of volatility and that volatility is an AR(1) process.

The SV model is more flexible than ARCH-family models because it allows for noise in the volatility function. As a result, the model does not force persistence to be close to 1 in order to allow simultaneous occurrences of small autocorrelation and high kurtosis. On the other hand, the basic SV model sometimes requires extensions or modifications in order to capture the properties of a return series better. For example, the conditional distribution of return does not need to be normal as assumed by the standard model. It also may be fat-tailed or skewed. Geweke [19] shows that SV performs poorly under a normality assumption when there are large outliers. This problem can be solved by allowing conditional distribution to have fat-tails. Furthermore, the standard SV model also assumes that volatility is only a function of its past values. In this paper we show that past values of return also have an impact on values of volatility at time t. There are some examples of models in volatility literature which suggest that return should be a part of the volatility equation. For example, in the standard GARCH(1,1) model the volatility is formulated as:

$$\lambda_t = a + b * (r_{t-1} - \mu)^2 + c * \lambda_{t-1} \tag{3.1}$$

where λ_t denotes the variance of return at time t, r_{t-1} is return at time $t-1$ and μ is the mean of return. This standard GARCH(1,1) model has been proven quite useful in finance. The key differences between the SV model used for this paper and the standard GARCH(1,1) model is: First, we replace μ by a moving average of return allowing for adjustment over time. And, then, second, we do not square the difference. Nevertheless, the success of the standard GARCH(1,1) model provides a motivation to explore the causality of returns on volatility in a traditional SV formulation. Furthermore, another model by Danielsson [11] also examines the causality between return and volatility. In his paper, the volatility equation of the SV model is also a function of lagged values of logged asset prices and absolute values of asset prices. He shows that the parameters of an asset price in the volatility function are significant. And these two papers provide a motivation to examine the Granger Causality between returns and volatility. In order to investigate this causality, we create a new model under the SV setting by adding an extra difference variable to the volatility equation.

As a result of the nonlinearity of latent variables the estimation of the SV model is not straightforward. However, several different methods overcome this problem. The methods are the Generalized Method of Moments (GMM) by Melino and Turnbull [32], the Quasi-Maximum Likelihood (QML) by Harvey et al. [22], the Efficient Method of Moments (EMM) applied by Gallant et al. [18] and the Markov Chain Monte Carlo (MCMC) procedure by Jacquier et al. [26] and Kim et al. [28]. A detailed survey and comparison of these methods can be found in Ghysels et al. [20] and Anderson et al. [2].

In this paper, we use a Maximum Likelihood (ML) approach based upon the Efficient Importance Sampling (EIS) procedure by Richard and Zhang [37] to estimate the SV model with a modification in volatility equation. EIS is a Monte Carlo (MC) technique that is mainly used for efficient evaluation of high dimensional integrals. It is ideally suited for the computation of likelihood in the SV model. This technique depends upon a sequence of simple low-dimensional regression, which, in turn, provides a global approximation of the integrand. Finally, the MC sampler provided by this approximation produces highly accurate

MC likelihood estimates. Furthermore, because the EIS procedure is generic, it is easy to adapt it to modifications in the SV model. Thus, we adapt it to the modification of the SV model in this paper.

The rest of the paper is organized as follows. In Section 2, we briefly review the basic version of the SV model, introduce the SV model with the return variable, and also explain the EIS procedure. In Section 3, we explain how the return variable is chosen and how it is formulated. Section 4 shows the application results on 35 different S&P 500 stock returns. Finally, in Section 5, we summarize our results and conclusions.

3.2 STOCHASTIC VOLATILITY MODEL AND METHODOLOGY

3.2.1 The Model

The standard SV model by Taylor is formulated as

$$\begin{aligned} r_{it} &= \exp(\lambda_{it}/2)\varepsilon_{it} \\ \lambda_{it} &= \gamma_i + \delta_i \lambda_{it-1} + \nu_i \eta_{it} \end{aligned} \tag{3.2}$$

where r_t represents the return on day $t : 1 \rightarrow T$. $\{\varepsilon_{it}\}$ and $\{\eta_{it}\}$ are mutually independent iid

Gaussian random variables with mean zero and unit variances. $\{\gamma_i, \delta_i, \nu_i\}$ are the parameters to be estimated.

The unobserved log-volatility λ_{it} follows an AR(1) process with persistence parameter δ_i. The returns are strictly stationary if $|\delta_i| < 1$.Finally, ν_i represents the standard deviation of volatility shocks and $\nu_i > 0$.

The model assumes that volatility is a latent or unobservable process. In other words, unobservable events on the same day explain volatility.

In this paper, we are looking for Granger-type causality where addition of an extra variable, which is the return variable, provides a reduction in the standard deviation of volatility. Because volatility is a latent process in a SV model (and standard Granger Causality tests require both variables to be observed), testing the causality between return and volatility is not possible with a standard Granger Causality test. For this reason, we use a different approach to test the causality between return and volatility. In this approach we estimate the standard SV model first. Then we introduce the return variable to the volatility equation in the SV model and estimate it. If the coefficient of the return variable is significant, the addition of the return variable reduces the standard deviation of volatility. And, if the Likelihood-ratio test results are significant, then we conclude that return "Granger-causes" volatility.

The methodology of this paper requires adding a new return variable to the volatility equation to see whether return does Granger-cause volatility. We modify the basic SV model by adding first lag of the return variable to the volatility equation. Then our model is given by

$$\begin{aligned} r_{it} &= \exp(\lambda_{it}/2)\varepsilon_{it} \\ \lambda_{it} &= \gamma_i + \delta_i \lambda_{it-1} + \beta_i x_{it-1} + \nu_i \eta_{it} \end{aligned} \tag{3.3}$$

where the parameters to be estimated are $(\gamma_i, \delta_i, \beta_i, \nu_i)$. If this new variable, the return variable, does Granger-cause volatility, then coefficient β should be significant and the standard deviation of volatility ν should decrease.

3.2.2 Efficient Importance Sampling

The evaluation of likelihood of the observed return r_{it}'s require us to integrate out latent or unobservable variable λ_t's. However the integration problem is not straightforward. It cannot be solved by standard integration techniques because λ_t is serially dependent. It enters into the model nonlinearly. As noted in the introduction, we use the EIS technique to overcome this problem

Let $f(R_i, \Lambda_i; \theta_i)$ represent the joint density of $R_i = \{r_{it}\}_{t=1}^{T}$ and $\Lambda_i = \{\lambda_{it}\}_{t=1}^{T}$, indexed by unknown parameter vector θ_i. Then the likelihood function associated with this joint density is given by

$$L(\theta_i, R_i) = \int f(R_i, \Lambda_i; \theta_i) \, d\Lambda_i \tag{3.4}$$

where L is Txq dimensional integral.

This integral can be factorized into sequence of conditional density functions $f(\cdot)$ for (r_{it}, λ_{it}) given $(R_{it-1}, \Lambda_{it-1})$. We can rewrite the likelihood function as

$$L(\theta_i, R_i) = \int \prod_{t=1}^{T} f(r_{it}, \lambda_{it} | R_{it-1}, \Lambda_{it-1}; \theta_i) \, d\Lambda_i \tag{3.5}$$

based upon the factorization.

Furthermore, we can rewrite the joint density as a function of conditional density $g(\cdot)$ of r_{it} and conditional density $p(\cdot)$ of λ_{it} given $(\Lambda_{it-1}, R_{it-1})$ as

$$f(r_{it}, \lambda_{it} | R_{it-1}, \Lambda_{it-1}; \theta_i) = g(r_{it} | \lambda_{it}, R_{it-1}, \theta_i) \, p(\lambda_{it} | \Lambda_{it-1}, R_{it-1}, \theta_i) \tag{3.6}$$

Under the standard SV model $g(\cdot)$ is a conditional Gaussian density and $p(\cdot)$ is the density for the Gaussian AR process of volatility.

A natural MC technique ignores that the observation of R_i conveys critical information about underlying latent process Λ_i since trajectories are just drawn from process $p(\cdot)$. This causes high inefficiency of MC estimator. To resolve this problem, EIS searches for samplers that exploits the sample information λ_{it}'s as conveyed by r_{it}'s. Let, $\{m(\lambda_{it} | \Lambda_{it-1}, a_{it})\}_{t=1}^{T}$ denote a sequence of auxiliary samplers which is indexed by auxiliary parameters $A_i = \{a_{it}\}_{t=1}^{T}$.

Then the likelihood function can be written as

$$L(\theta_i; R_i) = \int \prod_{t=1}^{T} \frac{f(r_{it}, \lambda_{it} | R_{it-1}, \Lambda_{it-1}; \theta_i)}{m(\lambda_{it} | \Lambda_{it-1}, a_{it})} \prod_{t=1}^{T} m(\lambda_{it} | \Lambda_{it-1}, a_{it}) \, d\Lambda_i \tag{3.7}$$

which produces the corresponding importance sampling estimate of likelihood as

$$\widetilde{L_N}\left(\theta_i; R_i, A_i\right) = \frac{1}{N}\sum_{j=1}^{N}\left\{\prod_{t=1}^{T}\frac{f\left(r_{it}, \widetilde{\lambda_{it}}^{(j)}(a_{it})|R_{it-1}, \widetilde{\Lambda_{it-1}}^{(j)}(a_{it-1}); \theta_i\right)}{m\left(\widetilde{\lambda_{it}}^{(j)}(a_{it})|\widetilde{\Lambda_{it-1}}^{(j)}(a_{it-1}), a_{it}\right)}\right\} \tag{3.8}$$

where $\left\{\widetilde{\lambda_{it}}^{(j)}(a_{it})\right\}_{t=1}^{T}$ denotes a trajectory drawn from the auxiliary samplers $m(\cdot)$.

EIS aims at selecting values of $\{a_{it}\}_{t=1}^{T}$ which provides a good match between the denominator and nominator in Equation 7 which will minimize the MC sampling variance of $\widetilde{L_N}$. To achieve the minimization, EIS constructs a functional approximation $k(\Lambda_{it}; a_{it})$ for the conditional joint density which is analytically integrable with respect to λ_{it}. Then $m(\lambda_{it}|\Lambda_{it-1}, a_{it})$ is given by

$$m(\lambda_{it}|\Lambda_{it-1}, a_{it}) = \frac{k(\Lambda_{it}; a_{it})}{\chi(\Lambda_{it-1}; a_{it})} \tag{3.9}$$

where $\chi(\Lambda_{it-1}; a_{it}) = \int k(\Lambda_{it}; a_{it})\, d\lambda_{it}$. Since $\chi(\Lambda_{it-1}; a_{it})$ does not depend on λ_{it} it can be transferred back into the period t-1 minimization subproblem. Therefore, the problem turns back into solving a simple back-recursive sequence of low-dimensional least squares problem of the form

$$\begin{aligned}\widehat{a_t}(\theta) \quad = \quad & \arg\min_{a_t}\sum_{j=1}^{N}[(\ln\ f\left(r_{it}, \widetilde{\lambda_{it}}^{(j)}(\theta)|R_{it-1}, \widetilde{\Lambda_{it-1}}^{(j)}(\theta); \theta_i\right) \\ & .\chi\left(\widetilde{\Lambda_{it}}^{(j)}; \widehat{a}_{it+1}\right)] - c_{it} - \ln k\left(\widetilde{\Lambda_{it}}^{(j)}; \widehat{a}_{it}\right))^2\end{aligned} \tag{3.10}$$

for $t: T \to 1$, with $\chi(\Lambda_{iT}; a_{iT+1}) \equiv 1$ and c_{it}'s are unknown constants to be estimated jointly with the a_{it}'s.

Nevertheless, in order to produce maximally efficient importance samplers just a small number of EIS iterations is required. To provide the convergence of auxiliary parameters $\widehat{a}_{it}$, we apply Common Random Numbers (CRNs) technique.

Finally, the ML-EIS estimates of θ are obtained by maximizing Equation 7 with respect to θ.

A detailed implementation of EIS for the SV model in Section 4 is given in Appendix.

3.2.3 The Return Variable

As mentioned earlier, in this paper we use 35 different S&P 500 stock returns from six different sectors. We investigated the effect of lagged values of different return variables on volatility. For example, we tried the first lag of the deviation of return from its mean to the volatility equation. Moreover, we tried using the deviation of return from its monthly moving average as well as its absolute value. To compare the effect of these variables on the model, we utilized the following procedures for each different extra variable candidate

1) We regress the filtered volatilities of an individual stock return on its first-order lag and calculated the residuals for this estimation.

2) We regress the first lag-of-return variable on the first-order lag of the filtered volatilities and calculated the residuals from this regression.

3) We regress the residuals from the first estimation on the residuals from the second regression.

These estimation results could provide the coefficients of the difference parameter. However, they would be based on a mis-specified model because the filtered volatilities are obtained by using the standard SV model.

The comparison of these estimation results for different return-variable candidates suggests that the deviation of return from its monthly moving average, which we call the difference variable, has the highest effect on volatility. Tables 16, 17 and 18 show the estimation results of the final regression of residuals for Coca-Cola, American Express, and Bristol-Myers Squibb. Regression results represent that estimated coefficients of final regression are significant. This indicates a relationship between the difference variable and the volatility. Furthermore, if we compare these initial results with the results of ML estimation, we see that the results are close to each other in the standard deviations as well as the point estimates. Therefore, these initial estimation results were useful to the investigation before I ran the full EIS-ML. Furthermore, this similarity between initial estimation and final EIS-ML results is true for all 35 stocks.

Next, we formulate the return variable before adding it to the volatility equation. This return variable is formulated as

$$x_{it} = r_{it} - \overline{r_{it}} \tag{3.11}$$

where r_{it} is stock return $i : 1 \rightarrow 35$ and $t : 1 \rightarrow T$. $\overline{r_{it}}$ represents the monthly moving average of return i at time t.We do not calculate the moving averages by using the standard moving average calculation which uses observations from $t-11$ to $t+11$. Because our moving average should depend on past values, we use observations from t-22 up to t. Furthermore, by using the moving instead of the mean average (which is used in the standard GARCH(1,1) model), we allow the mean to vary over time. Because we also tested the deviation of return from its mean when choosing the return variable, the comparison of the deviation of return from both its mean and its monthly moving average, as additional variables to the volatility equation, suggests that deviation from the monthly moving average has a stronger impact on volatility.

3.3 APPLICATIONS

For the application of the model, we use 35 different daily S&P 500 stock prices form six different sectors between January 2nd 1990 and October 31st 2008. The model is estimated for Coca-Cola, Hershey, Proctor & Gamble and Walmart from the consumer staples sector; Chevron, Sunoco, ConocoPhillips and Exxon from the energy sector; American Express, Bank of America, CitiBank, JP Morgan and Wells Fargo from the finance sector; Abbott, Amgen, Bristol-Myers Squibb, Johnson & Johnson, Merck, Pfizer, Schering & Plough and Wyeth from the health sector; 3M, Boeing, Caterpillar, GE, Masco and Southwest Airlines from the industrials sector; and Apple, Hewlett Packard, Intel, IBM, Micron, Motorola, Oracle and Java from the information technologies sector.

Stock returns are calculated by using formula

$$r_{it} = 100. \ln(s_{it}/s_{it-1}) \tag{3.12}$$

where s_{it} is daily stock return for return $i : 1 \rightarrow 35$ and$t : 1 \rightarrow 4750$.

Table 19 through 24 in Appendix C presents the estimation results under the standard SV model and SV model with the difference variable for each industry. Numbers in parentheses represent the asymptotic standard deviations. Mean and standard deviation are the parameters' means and standard deviations respectively.

For the consumer staples sector, the β parameter changes between -0.02 and -0.073 and is significant for all stock returns except Hershey. Furthermore, persistence parameter δ increases and the standard deviation of volatility ν decreases when the difference variable is added and significant.

For the energy sector, the difference parameter changes between -0.043 and -0.102. It is significant for all returns. Persistence parameter β increases and standard deviation of volatility ν decreases under the proposed model.

For the finance sector, the difference parameter β has the range of (-0.048,-0.068) and is significant for all returns. In terms of the persistence parameter and the standard deviation of volatility respectively, results again indicate increase and decrease.

For the health sector, the range of difference parameter is similar to the energy sector, which is between -0.027 and -0.110. The difference parameter is significant for all stocks. The persistence parameter increases when the difference variable is added. In terms of the standard deviation of volatility, there is a decrease, except in the case of Merck.

For the industrials sector, the difference parameter range is again similar to the energy and health sectors. It changes between -0.019 and -0.110. This sector has two stocks with insignificant difference parameters, 3M and Masco. The persistence parameter δ increases and the standard deviation of volatility ν decreases when the difference variable is added for all stocks except Masco.

Finally, among all sectors, the information technology sector has the widest difference in parameter range. The β parameter changes between -0.004 and -0.135 and it is not significant for Apple. Furthermore, adding the difference variable into the volatility equation causes an increase and decrease in the persistence parameter and standard deviation of volatility, respectively.

Almost all individual estimation results indicate that the difference parameter is significant, which shows that return does "Granger-cause" volatility. Moreover, when we add the difference variable into the volatility equation, persistence increases and the standard deviation of volatility decreases.

To represent the effect of new variables on persistence, we draw the filtered volatility graphs of Coca-Cola and Bristol-Myers Squibb. They are obtained by standard SV model estimation and the SV model with the difference variable estimation (for a small period after we observe a large x_{t-1}, which is the difference variable). Filtered volatility is the mean of volatility at time t computed by using information available on the returns up to time t-1. For Coca-Cola we observe that the 145th observation is large enough to examine the difference between two filtered volatility series from the two models. Figure 17 in Appendix D shows the filtered volatility series of 14 points after the large difference variable is observed at 145th point for Coca-Cola. For Bristol-Squibb-Myers we observe the large x_{t-1} at the 1892nd observation. Figure 18 in Appendix D also represents the filtered volatility series of 13 points after the 1892nd point by using respectively the standard SV model and the SV model with the difference variable for Bristol- Myers Squibb..

Since the coefficient of difference variable is negative when there is a large positive x_{t-1} we should expect that filtered volatilities from the SV model with the difference variable should be lower than filtered volatilities from the standard SV model. As noted for Coca-Cola, we observe a large positive x_{t-1} at the 145th and the 1892nd observations for Bristol-Squibb-Myers. Starting one point ahead of these observation points, the filtered volatilities graphs clearly represent that filtered volatilities are lower when the difference variable is added to the model.

In order to summarize the effects of the difference variable on volatility, we compare likelihood values under two different models for each return series as a final test. We use a LR-test to examine if there is an improvement in the model when we add the difference variable to the volatility equation. Tables 25 through 30 in Appendix D represent the likelihood values for each return series among sectors for the two models and the LR-test results. The results suggest that the model is improved and that there is causality between return and volatility except for those stocks with an insignificant difference parameter.

Table 31 in Appendix D shows the variance-covariance structure between parameters among the returns for all 35 stocks. This table represents the common feedback structure of SV model parameters among 35 stocks. Furthermore, the bivariate plot of δ and β, Figure 19 in Appendix D, also reflects that, when the difference parameter β is added, the persistence parameter δ increases.

The final step of this paper is a joint EIS-ML estimation. Here the parameters for each stock are assumed to be iid draws from a common four-dimensional distribution. We introduce a re-parameterization in order to avoid the problem of δ's being no larger than one, produce a more reasonable joint distribution, simplify the correlation structure, and produce neater bivariate graphs. This re-parametrization is given by

$$\begin{aligned} \alpha^* &= \frac{\alpha}{1-\delta} \\ \delta^* &= \ln(\frac{1-\delta}{\delta}) \\ \beta^* &= \beta \\ \nu^* &= \frac{\nu}{\sqrt{1-\delta^2}} \end{aligned} \tag{3.13}$$

As we noted above, this re-parameterization will simplify the correlation structure and simplify the common four-dimensional distribution

3.4 CONCLUSION

The standard SV model assumes that volatility is explained only by its first order lag. This paper presents research focused on examining the causality between return and volatility in the SV model. The causality is given by adding a return variable to the equation, which modifies the volatility equation in the standard SV model. The choice of this return variable is carried out by examining the partial correlation between the first-order lag of filtered volatilities and first-order lag of return variables. The examination of different return variables suggests that using the first-order lag of the difference between return and its past monthly moving average as the return variable provides the greatest improvement in the model.

After analyzing 35 different S&P 500 stock returns from six different sectors (consumer staples, finance, energy, health, industrials, and information technology), the empirical results obtained in this paper can be summarized as follows:

First, the estimation results indicate that for more than 30 stocks, the difference parameter is significant. Furthermore, when the difference variable is added to the volatility equation, the persistence parameter increases and, more importantly, the standard deviation of volatility decreases. The reduction in the standard deviation of volatility and the significant difference parameter together prove the existence of Granger-causality between return and volatility.

Second, the examination of filtered volatility graphs from the SV model with the difference variable also shows that filtered volatilities decrease after a high and positive observation for the difference variable.

Finally, the LR-test results represent that the model is improved for stocks except Hershey, 3M, Masco, Pfizer, and Apple. The final investigation will also be done by a joint EIS-ML estimation, where the parameters for each stock are assumed to be iid draws from a common four-dimensional distribution. In order to simplify the correlation structure between parameters, a re-parameterization will be introduced.

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APPENDIX A

IMPLEMENTATION OF EIS FOR STOCHASTIC VOLATILITY MODEL WITH DIFFERENCE PARAMETER

This appendix represents the functional forms of EIS implementation for the SV model with extra difference parameter in the volatility equation given by Equation .(4):

Let the integrating constant of $k_t\left(\lambda_{it}, \lambda_{it-1}; \widehat{a}_{it}\right)$ w.r.t. λ_{it} be formulated as

$$\chi_{t+1}\left(\lambda_{it}; \widehat{a}_{it+1}\right) = \exp -\frac{1}{2}(p_{it+1}\lambda_{it}^2 - 2q_{it+1}\lambda_{it} + r_{it+1}) \qquad \text{(A.1)}$$

where $(p_{t+1}, q_{t+1}, r_{t+1})$ are the appropriate functions of the EIS auxiliary parameter $\widehat{a}_{it+1}$ which will be obtained by backward recursions.$\chi_{T+1} = 1$, values of p_i, q_i and r_i at $T+1$ are equal to 0. Let the EIS approximation for the product of density function of return be denoted as:

$$k_t^1(\lambda_{it}, \widehat{a}_{it}) = \exp -\frac{1}{2}(\widehat{b}_{it}\lambda_{it}^2 - 2\widehat{c}_{it}\lambda_{it}) \qquad \text{(A.2)}$$

The EIS auxiliary parameter $\widehat{a}_{it}$ is describes as $\widehat{a}_{it} = \left(\widehat{b}_{it}, \widehat{c}_{it}\right)$. Then the EIS kernel can be represented as

$$k_t(\lambda_{it}, \lambda_{it-1}, a_{it}) = k_t^1(\lambda_{it}, \widehat{a}_{it})p\left(\lambda_{it}|\Lambda_{it-1}, R_{it-1}, \theta_i\right)\chi_{t+1}\left(\lambda_{it}; \widehat{a}_{it+1}\right) \qquad \text{(A.3)}$$

Furthermore, the conditional densities for return and volatility are defined as:

$$g\left(r_{it}|\lambda_{it}, R_{it-1}, \theta_i\right) \sim \exp -\frac{1}{2}\left(\lambda_{it} + r_{it}^2 \exp(\lambda_{it})\right) \qquad \text{(A.5)}$$

$$p\left(\lambda_{it}|\Lambda_{it-1},R_{it-1},\theta_i\right)=\frac{1}{\sqrt{2}v_i}\exp-\frac{1}{2}\left(\left(\lambda_{it}-\gamma_i-\delta_i\lambda_{it-1}-\beta_i x_{it-1}\right)^2/\nu_i^2\right) \tag{A.6}$$

If we combine together,A.1, A.2. and A.5. we have

$$\begin{aligned}-2\ln k_t^1(\lambda_{it},\lambda_{it-1},\widehat{a}_{it}) &= \lambda_{it}^2\left(\widehat{b}_{it}+1/\nu_i^2\right)-2\lambda_{it}\left(\widehat{c}_{it}+(\gamma_i+\delta_i\lambda_{it-1}+\beta_i x_{it-1})/\nu_i^2\right) \\ &+(\gamma_i^2+\delta_i^2\lambda_{it-1}^2+\beta_i^2x_{it-1}^2+2\lambda_{it-1}\left(\gamma_i\delta_i+\delta_i\beta_i x_{it-1}\right) \\ &+2\gamma_i\beta i x_{it-1})/\nu_i^2 \\ &+p_{it+1}\lambda_{it}^2-2q_{it+1}\lambda_{it}+r_{it+1}\end{aligned} \tag{A.7}$$

I we rewrite the equation as follows

$$\begin{aligned}-2\ln k_t^1(\lambda_{it},\lambda_{it-1},\widehat{a}_{it}) &= \lambda_{it}^2A_{it}-2\lambda_{it}B_{it}+(\gamma_i^2+\delta_i^2\lambda_{it-1}^2+\beta_i^2x_{it-1}^2) \\ &+2\lambda_{it-1}\left(\gamma_i\delta_i+\delta_i\beta_i x_{it-1}\right) \\ &+2\gamma_i\beta_i x_{it-1})/\nu_i^2 \\ &+p_{it+1}\lambda_{it}^2-2q_{it+1}\lambda_{it}+r_{it+1}\end{aligned} \tag{A.8}$$

$$A_{it}=\widehat{b}_{it}+1/\nu_i^2 \tag{A.9}$$

$$B_{it}=\widehat{c}_{it}+(\gamma_i+\delta_i\lambda_{it-1}+\beta_i x_{it-1})/\nu_i^2 \tag{A.10}$$

It immediately follows that the EIS sampler for $\lambda_{it}|\lambda_{it-1}$ is given by

$$m_t\left(\lambda_{it}|\lambda_{it-1};\widehat{a}_{it}\right)\sim N\left(A_{it}^{-1}B_{it},A_{it}^{-1}\right) \tag{A.11}$$

We can obtain the log-integrating constant by re-grouping all the remaining factors in A.5 and is therefore in form introduced in A.1 together with

$$p_{it}=\delta_i^2/\nu_i^2-A_{it}\delta^2 \tag{A.12}$$

$$q_{it} = (\gamma_i\delta_i + \delta_i\beta_i x_{it-1})/\nu_i^2 - A_{it}(\widehat{c}_{it} + \gamma_i + \beta_i x_{it-1}) \tag{A.13}$$

$$\begin{aligned} r_{it} &= (\gamma_i^2 + 2\gamma_i\beta_i x_{it-1} + \beta_i^2 x_{it-1}^2)/\nu_i^2 \\ &\quad -(\widehat{c}_{it} + \gamma_i + \beta_i x_{it-1})A_{it}(\widehat{c}_{it} + \gamma_i + \beta_i x_{it-1}) \end{aligned} \tag{A.14}$$

Hence equations (A.9)-(A.10) and (A.12)-(A.14) fully characterize the EIS recursion whereby the coefficients $(p_{t+1}, q_{t+1}, r_{t+1})$ are combined with the period t EIS coefficients $\left(\widehat{b}_{it}; \widehat{c}_{it}\right)$ in order to produce (back recursively) the coefficients $(A_{it}; B_{it})$ characterizing the EIS-sampling densities.

Based on these the EIS steps can be described as follows:

Step 1. Generate N independent trajectories from the initial sampler $m\left(\lambda_{it}|\lambda_{it-1}; a_t^{(0)}\right)$. Such a sequence can be found by e.g. using a Taylor Series Approximation (TSA) in λ_{it} for conditional density of return around its mean which is equal to zero. Replacing the resulting TSA values with $\left(\widehat{b}_{it}; \widehat{c}_{it}\right)$ in equations (A.9) and (A.10) provides the initial samplers together with the recursions described above.

Step 2. Now we can use these trajectories for solving the back recursive LS problem defined in Equation 12. This requires to run for each period t the following linear regression

$$\ln g\left(r_{it}|\widetilde{\lambda_{it}}^{(i)}\right) = \text{constant} - \frac{1}{2}b_{it}\widetilde{\lambda}_{it}^{(i)2} + c_{it}\widetilde{\lambda}_{it}^{(i)} + \zeta_{it}^{(i)} \tag{15}$$

where $\zeta_{it}^{(i)}$ represents the regression error term.

Step 3. Use the LS estimates $\widehat{b}_{it}$ and $\widehat{c}_{it}$ obtained in Step 2 to construct back-recursively the sequence of EIS-sampling densities as given by Equation (A.11) together with the recursions (A.9)-(A.10) and (A.12)-(A.14).

Step 4. Use N independent trajectories from auxiliary samplers constructed in Step 3 and then repeat Step 2 and 3 to compute EIS-MC estimate of the likelihood function.

APPENDIX B

FIGURES AND TABLES OF CHAPTER 1

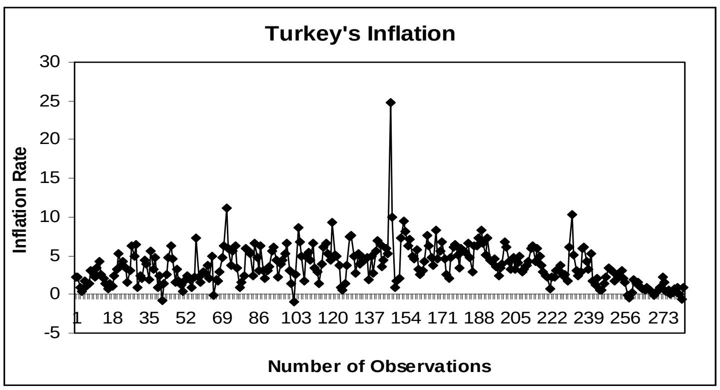

Figure 1: Inflation Series of Turkey

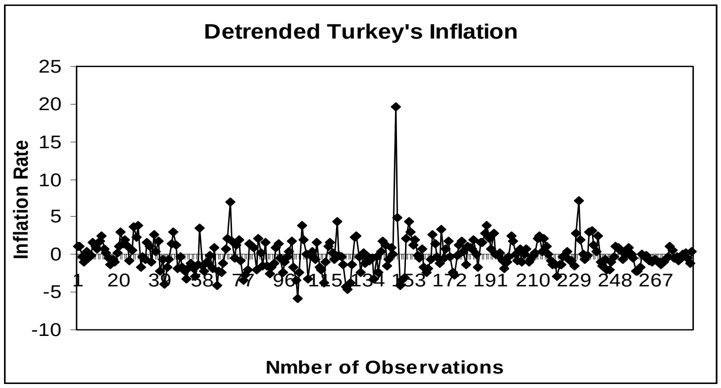

Figure 2: The Detrended and Deseasonalized Inflation Series of Turkey.

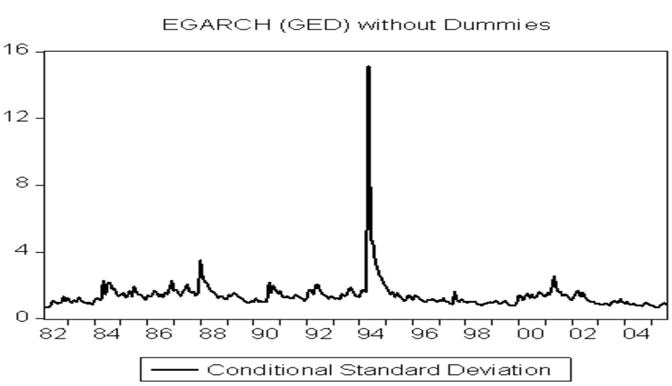

Figure 3: Filtered Volatilities from EGARCH Model (GED) without Dummies

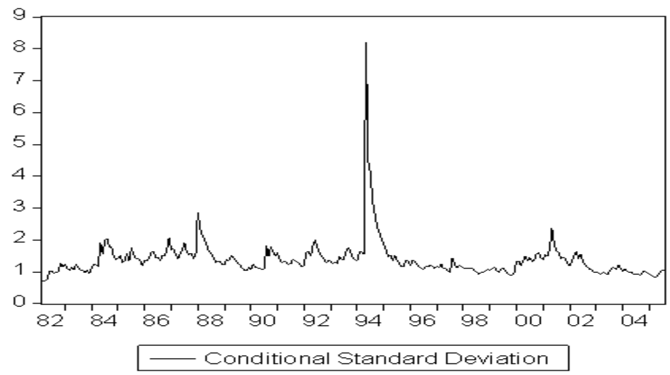

Figure 4: Filtered Volatilities tiwh EGARCH Model without dummies

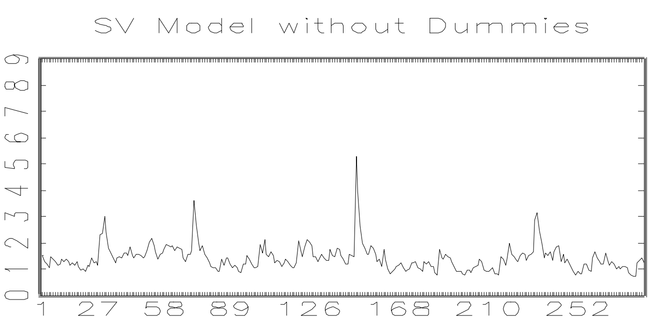

Figure 5: Filtered Volatilities with SV Model without Dummies

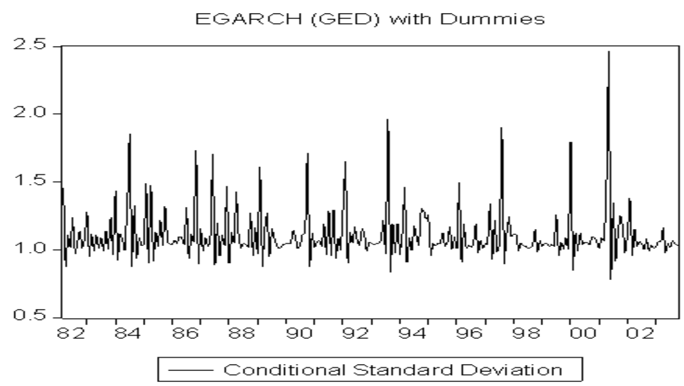

Figure 6: Filtered Volatilitilies from EGARCH (GED) Model with Dummies

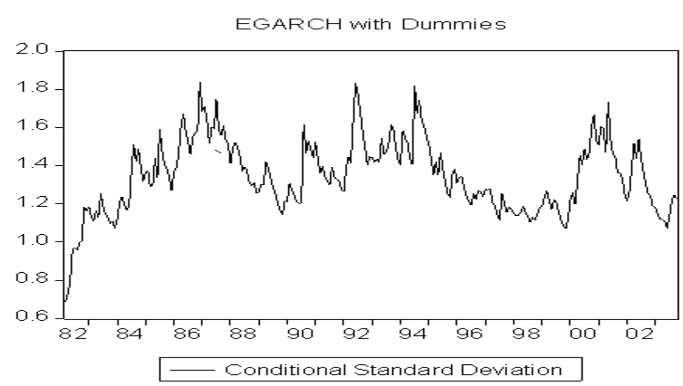

Figure 7: Filtered Volatilities from EGARCH Model with Dummies

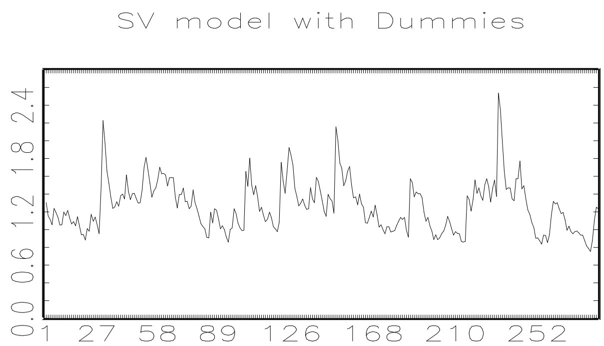

Figure 8: Filtered Volatilities from SV Model with Dummies

	Coefficients	**t-statistics**	**P-value**
Inflation(-1)	0.341	7.720	0.000
January	1.146	3.966	0.000
February	−0.694	−2.376	0.018
March	0.161	0.571	0.568
April	0.445	1.542	0.124
May	−0.994	−3.452	0.000
June	−1.775	−6.358	0.000
July	−1.061	−3.589	0.000
August	−0.324	−1.108	0.268
September	1.546	5.296	0.000
October	1.471	5.011	0.000
November	0.023	0.076	0.939
December	−0.584	−2.014	0.045
R-squared	0.623		

Table 1: Regression of Turkey's Inflation on Monthly Dummies before Seasonal Adjustment

	Coefficients	**t-statistics**	**P-value**
Inflation(-1)	0.361	8.344	0.000
January	−0.093	−0.348	0.727
February	−0.325	−1.215	0.225
March	−0.141	−0.539	0.590
April	0.187	0.698	0.485
May	−0.293	−1.108	0.268
June	−0.134	−0.511	0.609
July	−0.111	−0.423	0.672
August	−0.052	−0.198	0.842
September	0.081	0.304	0.761
October	−0.011	−0.042	0.966
November	−0.112	−0.418	0.675
December	0.369	1.377	0.169
R-squared	0.510		

Table 2: Regression of Inflation Series of Turkey on Monthly Dummies after Seasonal Adjustment

	Coefficients	**P-value**
Inflation(-1)	0.4716	0.000
β_0	-0.1888	0.071
β_1	0.5083	0.018
β_2	-0.1452	0.042
β_3	-0.0902	0.043
β_4	0.8912	0.000
Log-likelihood	-441.27	

Table 3: EGARCH (GED) Results without Dummy Variables

	Coefficients	**P-value**
Inflation(-1)	0.319	0.000
DUMMY1	4.608	0.625
DUMMY2	8.009	0.033
DUMMY3	18.515	0.000
DUMMY4	1.514	0.010
β_0	0.032	0.917
$\beta 1$	0.257	0.182
β_2	0.082	0.708
β_3	0.191	0.112
β_4	-0.497	0.230
Log-likelihood	-413.03	

Table 4: EGARCH (GED)Results with Dummy Variables

	Coefficients	**P-value**
Inflation(-1)	0.4818	0.000
β_0	-0.1610	0.070
β_1	0.4530	0.012
β_2	-0.1094	0.511
β_3	-0.0753	0.396
β_4	0.8883	0.000
Log-likelihood	-436.56	

Table 5: EGARCH Results without Dummy Variables

	Coefficients	**P-value**
Inflation(-1)	0.392	0.000
DUMMY1	4.848	0.007
DUMMY2	7.820	0.041
DUMMY3	18.797	0.000
DUMMY4	3.387	0.069
β_0	-0.076	0.427
$\beta 1$	0.324	0.153
β_2	-0.110	0.640
β_3	-0.105	0.290
β_4	0.908	0.000
Log-likelihood	-396.02	

Table 6: EGARCH Results with Dummy Variables

	Coefficients	**Asympt. stand. err.**	**MC stand. err.**
γ	-0.0465	0.03	0.0005
δ	0.8160	0.05	0.0007
ν	0.4506	0.09	0.0023
Log-likelihood	-380.53		0.0640

Table 7: SV Model Results without Dummy Variables

	Coefficients	**Asympt stand. err.**	**MC stand. err.**
α_1	3.8465	1.05	0.0002
α_2	0.1554	0.82	0.0002
α_3	0.0859	1.26	0.0001
α_4	2.2106	0.87	0.0004
γ	-0.0569	-0.05	0.0005
δ	0.8531	0.85	0.0006
ν	0.4357	0.43	0.0021
Log-likelihood	-374.95		0.053

Table 8: SV Model Results with Dummy Variables

	Inflation
Skewness	0.3486
Kurtosis	3.1956
$KS(z^*)$	0.0505 (0.23)
$Q_{30}(z^*)$	183.56 (0.00)
$Q_{30}(z)$	160.13 (0.00)
$Q_{30}(z^{*2})$	3.15 (1.00)
$Q_{30}(z^2)$	26.226 (0.66)

Table 9: Results for Diagnostic Checks

APPENDIX C

FIGURES AND TABLES OF CHAPTER 2

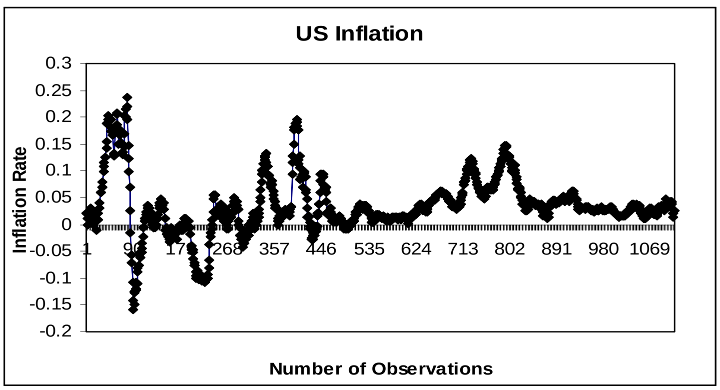

Figure 9: U.S. Inflation Series

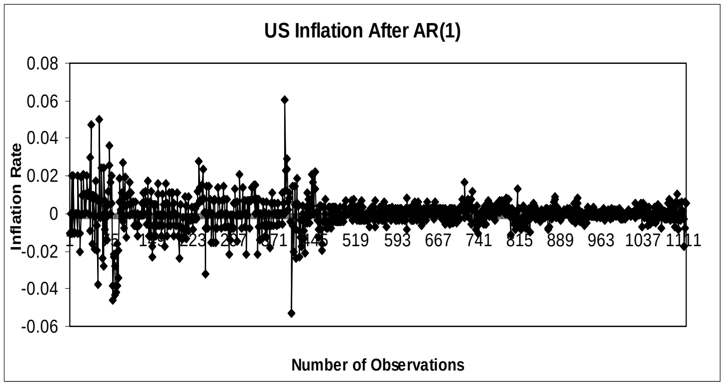

Figure 10: Graph of U.S Inflation Series after Autocorrelation is Eliminated

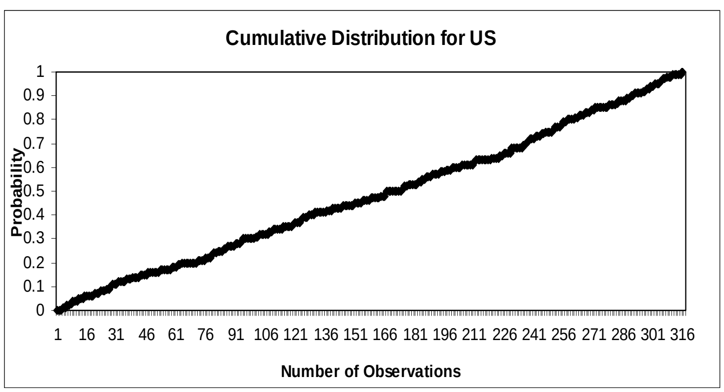

Figure 11: Cumulative Empricial Distribution Graph for U.S.

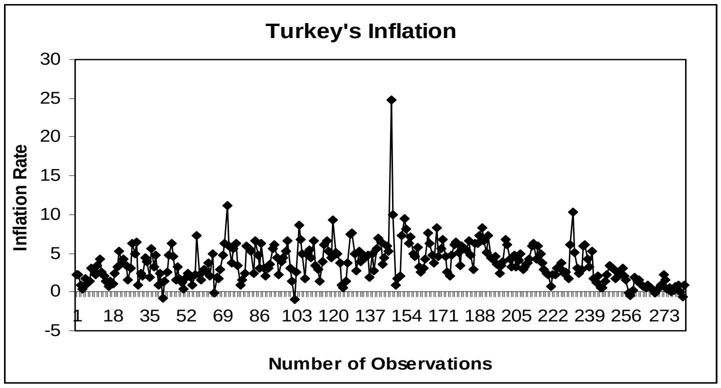

Figure 12: Inflation Series of Turkey

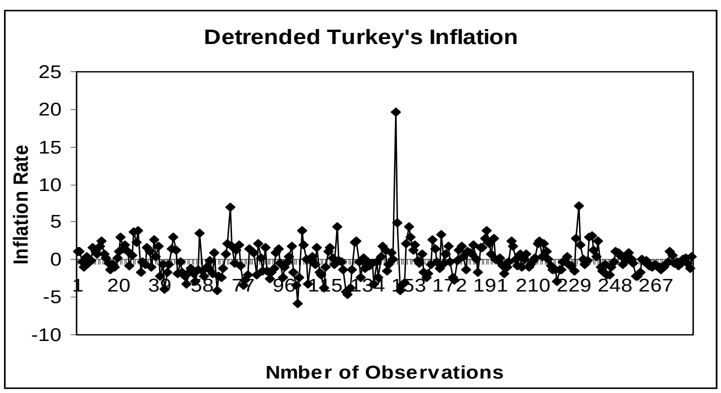

Figure 13: The Detrended Inflation Series of Turkey.

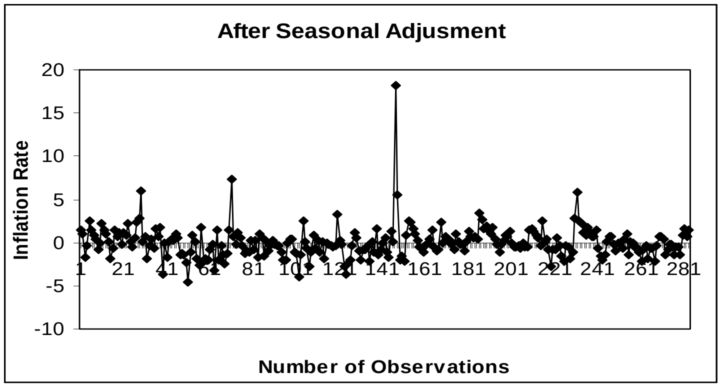

Figure 14: Inflation Series of Turkey after Seasonal Adjustment.

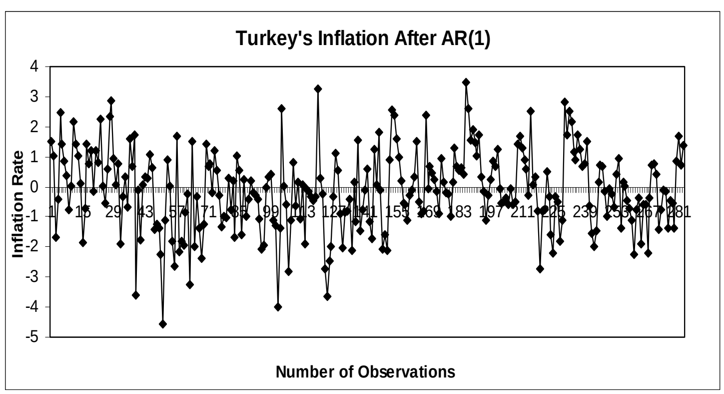

Figure 15: Inflation Series of Turkey after Autocorrelation is Eliminated.

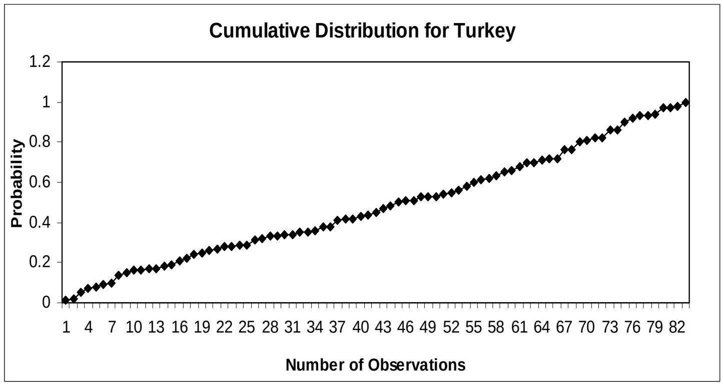

Figure 16: Cumulative Empricial Distribution Graph for Turkey.

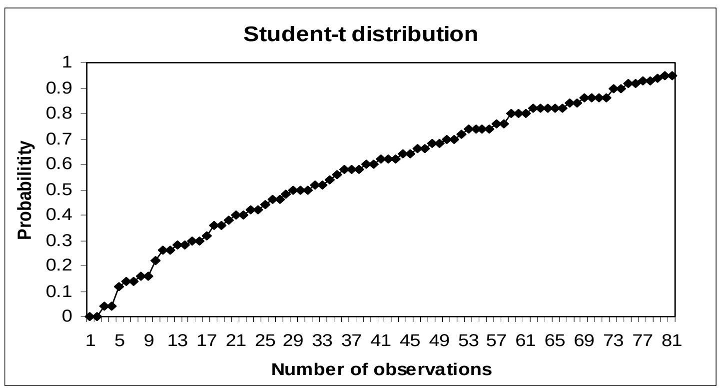

Figure 17: Cumulative Distribution Graph for the First Simulated Series by Using Student-t Distribution.

	Coefficients	**Asymptotic Std. Err.**	**MC Std. Err.**
γ	0.0012	0.004	0.0001
δ	0.9911	0.005	0.0004
ν	0.1986	0.042	0.0021

Table 10: Initial SV Model Estimation Results for U.S

	Coefficients	**Asymptotic Std. Err.**	**MC Std. Err.**
γ	0.0007	0.003	0.0001
δ	0.9918	0.004	0.0003
ν	0.2108	0.035	0.0018

Table 11: Final SV Model Estimation Results for U.S

	Coefficients	**t-statistics**	**p-value**
Inflation(-1)	0.357	6.290	0.000
January	1.173	3.118	0.002
February	−0.691	−1.816	0.070
March	0.210	0.572	0.567
April	1.283	3.490	0.000
May	−0.972	−2.591	0.013
June	−1.726	−4.679	0.000
July	−0.990	−2.583	0.000
August	−0.258	−0.679	0.497
September	1.587	4.184	0.000
October	1.476	3.861	0.000
November	0.016	0.040	0.967
December	−0.573	−1.518	0.130
R-squared	0.369		

Table 12: Regression of Turkey's Inflation on Monthly Dummies before Seasonal Adjustment

	Coefficients	**t-statistics**	**p-value**
Inflation(-1)	0.101	1.682	0.093
January	-0.071	-0.178	0.858
February	-0.280	-0.699	0.485
March	0.050	-0.126	0.899
April	0.021	0.054	0.956
May	-0.510	-1.302	0.194
June	0.000	0.000	0.999
July	0.050	0.128	0.897
August	-0.855	-2.184	0.029
September	0.004	0.011	0.091
October	0.094	0.234	0.814
November	0.704	1.760	0.79
December	-0.321	-0.803	0.422
R-squared	0.045		

Table 13: Regression of Turkey's Inflation on Monthly Dummies after Seasonal Adjustment

	Coefficients	**Asymptotic Std. Err.**	**MC Std. Err.**
γ	-0.0402	0.031	0.0007
δ	0.7655	0.052	0.0009
ν	0.4939	0.091	0.0035

Table 14: Intial SV Model Estimation Results for Turkey

	Coefficients	**Asymptotic Std. Err.**	**MC Std. Err.**
γ	-0.0572	0.025	0.0006
δ	0.8792	0.036	0.0007
ν	0.4457	0.077	0.0031

Table 15: Final SV Model Estimation Results for Turkey

APPENDIX D

FIGURES AND TABLES OF CHAPTER 3

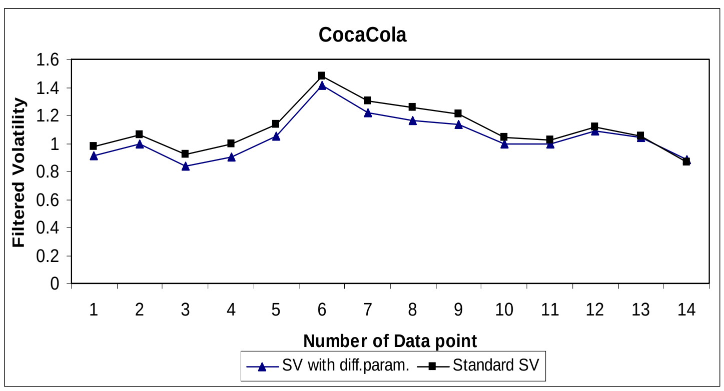

Figure 18: Filtered Volatility Series for CocaCola

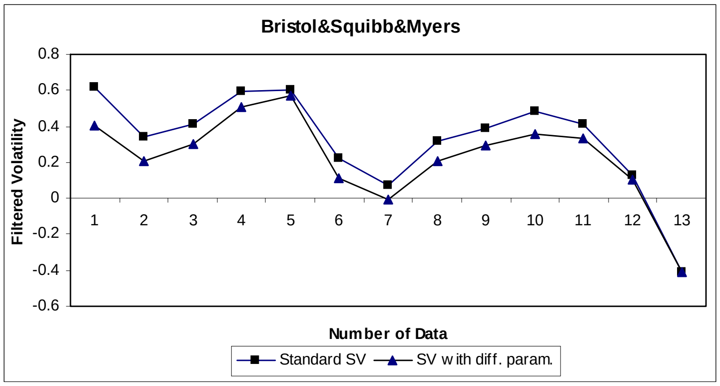

Figure 19: Filtered Volatility Series for Bristol-Squibb-Myers.

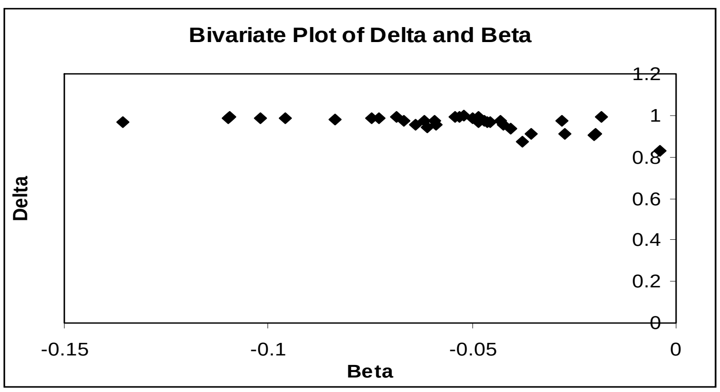

Figure 20: Bivariate Plot of Delta and Beta for 35 Stocks

Results for Final Regression		
	Coefficient	**Standard Dev.**
residuals-reg2	-0.021	0.003
R-squared	0.153	

Table 16: Estimation Results of Residual Regression for CocaCola

Results for Final Regression		
	Coefficient	**Standard Dev.**
residuals-reg2	−0.010	0.002
R-squared	0.181	

Table 17: Estimation Results of Residual Regression for American Express

Results for Final Regression		
	Coefficient	**Standard Dev.**
residuals-reg2	−0.019	0.004
R-squared	0.194	

Table 18: Estimation Results of Residual Regression for Bristol-Squibb-Myers

	Standard SV Model			SV Model with Difference Var.			
Stocks	γ	δ	ν	γ	δ	β	ν
CocaCola	−0.009 (0.003)	0.982 (0.0045)	0.166 (0.0176)	−0.007 (0.0030)	0.986 (0.0041)	−0.073 (0.0126)	0.146 (0.0175)
Hershey	−0.057 (0.011)	0.904 (0.0152)	0.381 (0.0325)	−0.058 (0.0108)	0.904 (0.0152)	−0.020 (0.0182)	0.382 (00319)
Proctor & Gamble	−0.015 (0.004)	0.973 (0.0064)	0.196 (0.0215)	−0.014 (0.0049)	0.976 (0.0066)	−0.059 (0.0149)	0.182 (0.0240)
Walmart	−0.001 (0.001)	0.990 (0.0031)	0.104 (0.0142)	0.000 (0.0013)	0.993 (0.0025)	−0.054 (0.0101)	0.089 (0.0123)
Mean	−0.020	0.962	0.212	−0.020	0.965	−0.062	0.199
Standard Deviation	0.025	0.039	0.119	0.026	0.039	0.020	0.119

Table 19: Estimation Results for Consumer Staples Sector

	Standard SV Model			SV Model with Difference Var.			
Stocks	γ	δ	ν	γ	δ	β	ν
Chevron	−0.015 (0.0050)	0.980 (0.0058)	0.126 (0.0159)	−0.012 (0.0041)	0.984 (0.0047)	−0.096 (0.0141)	0.107 (0.0137)
Sunoco	−0.009 (0.0039)	0.966 (0.0086)	0.192 (0.0252)	−0.007 (0.0034)	0.973 (0.0077)	−0.043 (0.0121)	0.170 (0.0250)
ConocoPhillips	−0.012 (0.0041)	0.973 (0.0086)	0.137 (0.0187)	−0.010 (0.0035)	0.976 (0.0063)	−0.067 (0.0126)	0.125 (0.0162)
Exxon	−0.014 (0.0049)	0.983 (0.0050)	0.129 (0.0161)	−0.011 (0.0041)	0.986 (0.0042)	−0.102 (0.0145)	0.113 (0.0142)
Mean	-0.020	0.962	0.211	−0.019	0.980	−0.077	0.129
Standard Deviation	0.025	0.039	0.119	0.026	0.006	0.027	0.028

Table 20: Estimation Results for Energy Sector

	Standard SV Model			SV Model with Difference Var.			
Stocks	γ	δ	ν	γ	δ	β	ν
American Express	−0.003 (0.0024)	0.986 (0.0034)	0.154 (0.0164)	−0.002 (0.0022)	0.989 (0.0031)	−0.053 (0.0115)	0.138 (0.0158)
Bank of America	−0.006 (0.0030)	0.986 (0.0035)	0.178 (0.0166)	−0.005 (0.0032)	0.989 (0.0036)	−0.050 (0.0104)	0.167 (0.0166)
CitiBank	−0.006 (0.0028)	0.989 (0.0031)	0.147 (0.0157)	−0.002 (0.0018)	0.995 (0.0010)	−0.069 (0.0127)	0.122 (0.0145)
JP Morgan	−0.003 (0.0022)	0.988 (0.0033)	0.155 (0.0167)	−0.001 (0.0022)	0.992 (0.0032)	−0.049 (0.0099)	0.139 (0.0159)
Wells Fargo	0.000 (0.0015)	0.993 (0.0026)	0.124 (0.0140)	0.000 (0.0012)	0.996 (0.0012)	−0.052 (0.0098)	0.110 (0.0119)
Mean	−0.004	0.988	0.152	−0.002	0.992	−0.054	0.135
Standard Deviation	0.003	0.003	0.019	0.002	0.003	0.008	0.021

Table 21: Estimation Results for Finance Sector

	Standard SV Model			SV Model with Difference Var.			
Stocks	γ	δ	ν	γ	δ	β	ν
Abbott	−0.019 (0.0054)	0.968 (0.0079)	0.182 (0.0222)	−0.017 (0.0054)	0.969 (0.0076)	−0.045 (0.0159)	0.179 (0.0220)
Amgen	0.004 (0.0034)	0.965 (0.0069)	0.227 (0.0210)	0.001 (0.0037)	0.966 (0.0066)	−0.048 (0.0117)	0.224 (0.0205)
Bristol&Squibb&Myers	−0.018 (0.0052)	0.970 (0.0062)	0.213 (0.0207)	−0.017 (0.0052)	0.972 (0.0063)	−0.062 (0.0159)	0.207 (0.0215)
Johnson&Johnson	−0.017 (0.0049)	0.980 (0.0045)	0.165 (0.0167)	−0.014 (0.0046)	0.985 (0.0040)	−0.110 (0.0158)	0.147 (0.0159)
Merck	−0.072 (0.0140)	0.874 (0.0201)	0.392 (0.0322)	−0.072 (0.0013)	0.875 (0.0178)	−0.038 (0.0173)	0.392 (0.0318)
Pfizer	−0.002 (0.0026)	0.970 (0.0063)	0.178 (0.0182)	−0.001 (0.0027)	0.972 (0.0064)	−0.028 (0.0119)	0.174 (0.0186)
Schering&Plough	−0.011 (0.0041)	0.963 (0.0074)	0.226 (0.0216)	−0.010 (0.0043)	0.966 (0.0074)	−0.046 (0.0131)	0.216 (0.0236)
Wyeth	−0.018 (0.0051)	0.968 (0.0069)	0.231 (0.0235)	−0.015 (0.0051)	0.971 (0.0067)	−0.047 (0.0151)	0.223 (0.0237)
Mean	−0.019	0.957	0.226	−0.018	0.960	−0.053	0.220
Standard Deviation	0.023	0.034	0.071	0.023	0.035	0.048	0.075

Table 22: Estimation Results for Health Sector

	Standard SV Model			SV Model with Difference Var..			
Stocks	γ	δ	ν	γ	δ	β	ν
3M	-0.092 (0.0173)	0.906 (0.0161)	0.348 (0.0330)	-0.089 (0.0164)	0.909 (0.0154)	-0.027 (0.0226)	0.343 (0.0312)
Boeing	-0.022 (0.0066)	0.946 (0.0109)	0.262 (0.0293)	-0.018 (0.0062)	0.956 (0.0108)	-0.059 (0.0142)	0.233 (0.0293)
Caterpillar	-0.028 (0.0073)	0.903 (0.0174)	0.313 (0.0320)	-0.026 (0.0068)	0.911 (0.0172)	-0.036 (0.0151)	0.300 (0.0303)
GE	-0.073 (0.0064)	0.991 (0.0071)	0.134 (0.0150)	-0.004 (0.0023)	0.994 (0.0023)	-0.110 (0.0128)	0.096 (0.0124)
Masco	-0.031 (0.0084)	0.913 (0.0154)	0.363 (0.0340)	-0.031 (0.0079)	0.913 (0.0137)	-0.020 (0.0158)	0.364 (0.0335)
Southwest Airlines	0.007 (0.0037)	0.933 (0.0119)	0.262 (0.0262)	0.006 (0.0040)	0.934 (0.0122)	-0.041 (0.0124)	0.260 (0.0260)
Mean	-0.040	0.932	0.280	-0.027	0.936	-0.049	0.266
Standard Deviation	0.036	0.033	0.083	0.033	0.034	0.033	0.097

Table 23: Estimation Results for Industrials Sector

	Standard SV Model			SV Model with Difference Var.			
Stocks	γ	δ	ν	γ	δ	β	ν
Apple	−0.083 (0.0158)	0.831 (0.0260)	0.460 (0.0411)	−0.083 (0.0154)	0.830 (0.0232)	−0.004 (0.0172)	0.461 (0.0339)
Hewlett Packard	−0.055 (0.0110)	0.941 (0.0102)	0.298 (0.0264)	−0.054 (0.0104)	0.943 (0.0093)	−0.061 (0.0206)	0.294 (0.0243)
Intel	−0.014 (0.0042)	0.982 (0.0050)	0.143 (0.0175)	−0.011 (0.0038)	0.984 (0.0044)	−0.075 (0.0144)	0.130 (0.0168)
IBM	−0.055 (0.0119)	0.964 (0.0072)	0.242 (0.0229)	−0.045 (0.0106)	0.970 (0.0063)	−0.136 (0.0236)	0.222 (0.0217)
Micron	−0.001 (0.0012)	0.992 (0.0011)	0.121 (0.0213)	0.000 (0.0012)	0.994 (0.0024)	−0.018 (0.0089)	0.081 (0.0118)
Motorola	−0.041 (0.0091)	0.950 (0.0094)	0.278 (0.0258)	−0.038 (0.0083)	0.953 (0.0086)	−0.064 (0.0181)	0.269 (0.0246)
Oracle	−0.009 (0.0041)	0.974 (0.0060)	0.222 (0.0227)	−0.007 (0.0036)	0.980 (0.0051)	−0.084 (0.0132)	0.193 (0.0217)
Java	−0.013 (0.0050)	0.948 (0.0104)	0.258 (0.0275)	−0.011 (0.0048)	0.953 (0.0099)	−0.042 (0.0141)	0.245 (0.0262)
Mean	−0.034	0.948	0.253	−0.031	0.951	−0.060	0.237
Standard Deviation	0.029	0.050	0.104	0.029	0.052	0.041	0.115

Table 24: Estimation Results for Information Technology Sector

Stocks	**LL-standard SV**	**LL-SV with diff. variable.**	**LR-test**
CocaCola	−5702.28	−5686.27	32.02
Hershey	−5707.99	−5707.82	0.34
Proctor & Gamble	−5569.58	−5561.93	15.30
Walmart	−6676.04	−6661.91	28.26

Table 25: Log-likelihood Values for Consumer Staples Sector

Stocks	LL-standard SV	LL-SV with diff. variable	LR-test
Chevron	−4995.70	−4976.32	38.76
Sunoco	−6309.99	−6304.99	10.00
ConocoPhillips	−5794.81	−5780.93	27.76
Exxon	−4800.30	−4783.99	32.62

Table 26: Log-likelihood Values for Energy Sector

Stocks	LL-standard SV	LL-SV with diff. variable	LR-test
American Express	−6315.69	−6305.30	20.78
Bank of America	−5741.38	−5731.06	20.64
CitiBank	−5380.22	−5365.51	29.42
JP Morgan	−6496.49	−6484.86	23.26
Wells Fargo	−6373.22	−6359.97	26.50

Table 27: Log-likelihood Values for Finance Sector

Stocks	LL-standard SV	LL-SV with diff. variable	LR-test
Abbott	−5589.03	−5576.97	24.12
Amgen	−7105.79	−7089.53	32.52
Bristol-Squibb-Myers	−5488.35	−5480.90	14.9
Johnson & Johnson	−4739.49	−4718.44	42.1
Merck	−5757.24	−5745.73	23.02
Pfizer	−6770.65	−6769.82	1.66
Schering & Plough	−6276.21	−6267.47	17.48
Wyeth	−5735.63	−5729.87	11.52

Table 28: Log-likelihood Values for Health Sector

Stocks	LL-standard SV	LL-SV with diff. variable	LR-test
3M	−4760.24	−4760.26	0.04
Boeing	−6020.08	−6011.91	16.34
Caterpillar	−6356.55	−6344.68	23.74
GE	−5083.66	−5050.75	65.82
Masco	−6287.67	−6287.64	0.06
Southwest Airlines	−7232.27	−7225.22	14.1

Table 29: Log-likelihood Values for Industrials Sector

Stocks	LL-standard SV	LL-SV with diff. variable	LR-test
Apple	−6018.50	−6018.76	0.52
Hewlett Packard	−4826.30	−4822.72	7.16
Intel	−5154.83	−5146.88	15.9
IBM	−3351.23	−3338.70	25.06
Micron	−7021.04	−7001.44	39.2
Motorola	−5126.90	−5120.51	12.78
Oracle	−6098.83	−6080.89	35.88
Java	−6437.10	−6424.80	24.6

Table 30: Log-likelihood Values for Information Technology Sector

	α	δ	β	ν
α	0.00065			
δ	0.00065	0.00140		
β	-0.00011	-0.00058	0.00079	
ν	-0.00158	-0.00335	0.00137	0.00889

Table 31: Variance-Covariance Matrix of Model Parameters