ESSAYS ON STRATEGIC INFORMATION TRANSMISSION

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This dissertation consists of three chapters, each analyzes a model of strategic information transmission - or cheap talk - between an expert and a decision maker. The first chapter, "Expert Advice for Amateurs," analyzes a model in which the expert is perfectly informed and the decision maker partially informed. The decision maker can tell, privately, whether the state of the world is "high" or "low." The expert responds to this second layer of asymmetric information by providing less informative advice. For some types of decision maker, this negative, strategic effect outweighs the benefit of information - being informed makes them worse off. Information is always beneficial to the decision maker only when welfare is evaluated before the realization of his type.

The second chapter, "Challenged Authority," analyzes a model in which the expert is perfectly informed and the decision maker is of one of two types: uninformed or partially informed. The decision maker can reveal his private type to the expert before the expert communicates with him. The expert is susceptible to emotion: she becomes annoyed if she believes that her authority is challenged - when her access to information is not exclusive - and reacts to it by being less helpful to the decision maker. The expert's emotion affects communication. It can deter an informed decision maker from revealing himself, who otherwise would have done so to an emotion-free expert.

The third chapter, "Uncertain Expertise," analyzes a model in which the expert is imperfectly informed and the decision maker, uninformed, is also uncertain about how informed the expert is. The model, in which the expert's private type summarizes two aspects of her information status - her expertise and her information - can be transformed into a standard cheap-talk model with finite types. The equilibria of the former can be analyzed via those of the latter; the second-order imperfect information does not change the way in which strategic information transmission with imperfectly informed expert is analyzed. In a specialized information structure, it is found that an increase in the level of uncertainty over the expert's expertise makes communication more difficult.

TABLE OF CONTENTS

LIST OF FIGURES

PREFACE

The decision to pursue a Ph.D. was a life-changing one, and completing it was quite an accomplishment. There are many people I would like to express my gratitude to for making this happen. Let me start with the very beginning.

An academic career has long been a dream since I was seventeen in matriculation. But after leaving school for several years, it had become a remote one. Had it not been for that Chinese new year gathering in 2002, when Francis Lui - my life-long mentor since undergraduate years - encouraged me to give it a try, I would not have taken such a bold move. Francis seeded the "necessary condition" of my move, but it was my family who provided the "sufficient condition" for the whole event. I owe my wife, who gave up her promising career and what we have established in Hong Kong, to follow me to start a new life in a then unknown place. I owe my parents, who, in their senior years, encouraged their only child to leave them behind and pursue his dream.

I am indebted to my advisor, Andreas Blume, for his guidance and support since my first year in the program. I learned game theory from him, not by going through various definitions but by seeing him solving a bunch of apparent simple games on the board. At the end, I grasped all the important concepts. I met Lise Vesterlund in my second year and served as her research assistant in my third year and teaching assistant in my fifth year. During these years, I learned a lot from her about experimental economics and what are the essential characteristics of a good teacher. Her encouragement and support over the years were also indispensable for, if any, my success in the program. It was my fortune to have taken a course in accounting and information economics in my second year at Carnegie Mellon and met Pierre Liang. Pierre's generous support to students is beyond the boundary of schools. With his expertise in accounting information, he guided me to look at the problems I work on from a different angle. I audited Oliver Board's course in game theory in my third year. I am very grateful for his guidance and support on my dissertation research. The speed at which he figures out the intuition behind some complicated theorem is also something that I wish I could acquire someday.

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1.0 INTRODUCTION

Different states of information typically call for different decisions to be made, and a fundamental problem in organizing economic activities is the dispersed distribution of information among economic agents. As Hayek [23] observed some sixty years ago, "[the economic problem of society] is a problem of the utilization of knowledge not given to anyone in its totality." Since then, generations of economists have devoted their effort into studying alternative information settings that deviate from the neoclassical paradigm in which information is assumed to be perfect. Many important insights were obtained by analyzing various formal models where the problem of information manifests as *information asymmetries*: one party is assumed to have perfect access to information while no information is available to other parties.[1]

This simple dichotomy of information asymmetry, though facilitating a clean analysis, may not encompass all the information problems we face today. In the lemons model of Akerlof [2], for example, used-car owners know the exact quality of their cars whereas buyers are assumed to know only the average in the market. The consequence of this information asymmetry is that mutually beneficial trades may not be realized. Thanks to the advance of information technology, we now have, for example, the CARFAX Vehicle History Report which provides access to information on *particular* vehicles; the information inequality between sellers and buyers is reduced, and the result of market collapse may no longer be prevalent in used-cars markets today.

On the other hand, acquiring information is often costly. The distribution of information among economic agents is dictated by their relative cost of acquiring information. Information asymmetry is simply a short-cut for studying the whole problems (Maskin and Tirole [32], p.107). When we acknowledge the presence of costly information, we may have to take into consideration that even the informed parties may not have perfect access to information - acquiring perfect information will be an extremely costly enterprise.

[1] For the development of information economics, see, for example, the Nobel lecture by Stiglitz [47].

The papers in this dissertation are all motivated by the considerations above. It consists of three chapters, each of them takes one of the simplest possible forms of information problem as the starting point: there are two parties, one informed and one uninformed and whose interests are misaligned; the informed party transmits information to the uninformed, who then takes an action that affects the welfare of both. I extend on this strategic information transmission model of Crawford and Sobel [12] by relaxing the traditional dichotomy of information asymmetry.[2]

In the first chapter, "Expert Advice for Amateurs," I analyze strategic information transmission in which the original uninformed party has partial access to information. The doctor-patient relationship, in which the doctor is the *expert* with superior information or knowledge and the patient is the *decision maker* who decides on what action (e.g., treatment or life-style choices) to take, is used to illustrate the problem. The doctor is assumed to know perfectly the diagnosis of the patient. The patient does not observe his exact diagnosis but is able to form some idea about whether his condition is serious or not, which may be a consequence of being able to obtain information from other sources such as the Internet.

Formally, the above is modeled as the decision maker's ability to tell whether the state of the world is "high" or "low," whereas the expert observes its exact value which is a real number in the unit interval $[0, 1]$. The decision maker's definition of "high" and "low" is assumed to be unknown to the expert - a doctor may not be able to tell readily whether, given a diagnosis, the particular patient sitting in front of her considers his condition as serious or not. The relaxation of the traditional dichotomy creating another layer of information asymmetry - in this case the expert's being uninformed about the decision maker's definition of "high" and "low" - is indeed a recurring theme in this dissertation.

When the decision maker has information of his own, the influences of the information provided by the expert are attenuated: the decision maker can now compare and contrast what he knows with what the expert tells him. I find that in response the expert provides less informative advice to the decision maker. Given the misaligned interests between them, in order to be willing to provide information the expert has to be given certain room to exaggerate in her advice. When she is not as influential, she exaggerates more, and this leads to less information being transmitted.

For decision maker having access to certain information (e.g. patients with certain cutoff for "serious" and "not serious"), this negative, strategic effect outweighs the benefit of information -

[2] This model is also commonly referred to as model of *cheap talk* because the medium through which the informed party transmits her information - the messages - is costless that it does not affect payoffs. This is to be contrasted with the costly signaling model pioneered by Spence [45].

being informed makes them worse off. Information is always beneficial to the decision maker only when welfare is evaluated before the realization of his information type. The moral for patients is that, while the Internet as another channel of medical information is *on average* beneficial to them, some of them could indeed be worse off as a result.

The second chapter, "Challenged Authority," extends on the first chapter. The lesson from the first chapter is that the strategic response of the expert toward an informed decision maker is negative. In practice, there could be other factors that aggravate the situation. Another consequence of having an informed decision maker in place is that the expert no longer has exclusive access to information. In other words, she loses her authority as being the only one who is informed, and not all experts welcome such challenge. It is reported, for example, that some doctors are annoyed by their "Googler-patients" and become less helpful to them.[3] This chapter captures such behavioral aspect of communication with experts when informed decision makers are involved.

I consider a model in which the decision maker is of one of two types: uninformed or partially informed. The expert does not know whether the decision maker is informed, and here comes the additional layer of information asymmetry. There is, however, a channel of communication available for it: the decision maker can reveal his private type to the expert before the expert communicates with him, a consideration that is absent in the first chapter. On the other hand, the expert is susceptible to emotion, which is modeled via a larger extent of misaligned interests between the parties when the expert believes that she is facing an informed decision maker.

I find that despite the expert's different response to informed decision makers, there are scenarios under which the informed decision maker is willing to reveal himself to the expert. However, when the expert is emotional enough, the informed decision maker may as well pool with the uninformed decision maker, in effect shutting off the channel of communication from the decision maker to the expert.

The third chapter, "Uncertain Expertise," takes a somewhat different but related direction and is of a more methodological nature. We typically expect experts to be well informed, but, given that acquiring information is rarely costless, it is next to impossible for them to be perfectly informed. Doctors are certainly well-cultivated in medical knowledge, but it is an impractical task for them to keep abreast of every new finding in the medical arena. On the other hand, there are, in principle, an infinite number of ways in which one can be imperfectly informed. When an imperfectly informed expert is in place, it brings in an additional layer of information asymmetry

[3] See the various references in Chapter 1 and Chapter 2.

as to how, from the decision maker's perspective, informed the expert is. This chapter studies strategic information transmission when the decision maker is uncertain about the expert's level of expertise - when the two layers of information asymmetry reside on the decision maker's side.[4]

In the model, the expert's private type summarizes two aspects of her information status: her level of expertise and what she knows about the state of the world. Utilizing the quadratic payoffs commonly adopted in the literature, I show that the model can be transformed into a standard model with finite type space that is much more amenable to analysis with results readily available. Analyzing the equilibria of the standard model with one layer of information asymmetry sheds light on the equilibria of the original model with two layers. Putting this transformation into action, I find in a fairly general example that an increase in the level of uncertainty the decision maker faces over the expert's level of expertise makes communication more difficult.

[4]This is different from those in the first two chapters in which the information asymmetries are double-sided.

2.0 EXPERT ADVICE FOR AMATEURS

2.1 INTRODUCTION

With a great deal of information only a few clicks away, the boundary between experts and novices blurs. Once the privilege of experts, specialized knowledge is now widely available on encyclopedic websites and through search engines. In the medical arena, for example, websites such as *www.webmd.com* have rendered patients much more informed and sophisticated than their counterparts a decade ago. We can perhaps jump a step ahead by saying that novices no longer exist today, and *amateurs* - those who know but do not know enough to dispense with the help from experts - have emerged to fill the void.

Is evolving from novices into amateurs a welfare improvement for patients or other decision makers? Advocates for consumer education would respond affirmatively. The underlying proposition of consumer education is that more information leads to better decision.[1] Yet it is a well-known result in information economics that more information is not necessarily better. In the classic lemons model of Akerlof [2], for example, information, when asymmetrically distributed, can eliminate trades that are otherwise efficient. This chapter examines the effects of decision makers' information in strategic information transmission (Crawford and Sobel [12]), a setting that captures the relationship between experts and decision makers. It addresses the questions of whether in a strategic environment becoming an amateur improves welfare; how experts respond to informed clients; and in what sense consumer education is beneficial when strategic elements are present. It is documented that patient use of Internet information can worsen the physician-patient relationship (e.g., Ahmad et al. [1]), and they are mostly accounted for in psychological terms.[2] Another

[1] Ben Bernanke was quoted on the Fed Education website: "In today's complex financial markets, financial education is central to helping consumers make better decisions for themselves and their families." As one of its missions, the Bureau of Consumer Protection "empowers consumers with free information to help them exercise their rights and spot and avoid fraud and deception"; they believe "education is the first line of defense against fraud and deception; it can help you make well-informed decisions before you spend your money."

[2] One of the most common accounts for doctors' negative responses toward informed, sophisticated patients is that

contribution of this chapter is to provide a rational, strategic account that complements these behavioral explanations; this chapter shows that adverse effects of patients' information can also arise under doctors' rational behavior.[3]

I start with Crawford and Sobel's [12] model (the "CS model"). There is an expert (she), who, after privately observing the state of the world, sends a message (advice) to a decision maker (he). "Talk is cheap" - the advice itself does not directly affect payoffs. Based on the advice, the decision maker takes an action that affects both parties' payoffs. Interests between the parties are misaligned. Relative to the decision maker's, the expert's preferences are biased toward higher actions. The novelty of my model - which I call the *amateur model* - lies in the fact that the decision maker is an "amateur" who is partially informed. The decision maker does not directly observe the state but can tell whether a state is "high" or "low"; his definition of "high" and "low" is his private information and is unknown to the expert.

I utilize this setup to capture the way amateurs interpret information and advice, and this may be illustrated with the example of patient use of Internet information. Fox [18] of the Pew Internet & American Life Project reports that eight out of ten Internet users in the U.S., accounting for some 113 million adults, have searched for health information on the Internet; among them two-third searched about specific diseases. These users may have access to the same information available to the professionals (e.g., by using the *Google Scholar*). As amateurs, however, they typically lack the ability to interpret the information and sort out the relevant from the irrelevant. As one doctor puts it, "There's so much information (as well as misinformation) in medicine - and, yes, a lot of it can be Googled - that one major responsibility of an expert is to know what to ignore."[4] Without such an ability, a patient who resorts to online health information may only be able to self-diagnose roughly whether his condition could be serious ("high") or not ("low"). Moreover, even for the same set of information, two patients may interpret differently and arrive at different self-diagnoses.

Fox [18] reports that only one-third of the respondents mentioned their findings to the doctors; a doctor facing a "Googler-patient" is likely to find herself offering advice in the midst of some

they feel being challenged in their authority (e.g., Murray et al. [36]).

[3] Cheap-talk models are commonly used to study the interactions between legislators and lobbyists (e.g. Gilligan and Thomas [21] and Krishna and Morgan [29]) and between investors and investment advisors (e.g., Benabou and Laroque [7] and Morgan and Stocken [35]). The questions addressed here also apply to these arenas: legislators can now conduct their own research on the impacts of a proposed legislation without incurring a high cost; market data and analyst reports are now widely available online for retail investors.

[4] This controversial *Time Magazine* article [22], "When the Patient Is a Googler," was written by an orthopedist who reported his unpleasant experience with a "Googler-patient" whom he described as "brainsucker" and eventually decided not to treat. The article suggests that trust could be another behavioral factor behind the worsening physician-patient relationship.

information asymmetry as to what the patient knows. Suppose, driven by financial interests, the doctor reports a diagnosis that is biased toward inducing the patient to take more intense and expensive treatments than necessary.[5] A patient who believes that he is seriously ill may consider the biased diagnosis as a confirmation of his findings and proceed with an expensive treatment. Otherwise, he may request to take a less expensive treatment or even seek a second opinion.[6] For the same piece of advice, once decision makers have information of their own, it is inevitable that different responses will ensue.

To illustrate how different interpretations of information and advice arise in the model, suppose there are two types of patient, A and B. The diagnosis, observed only by the doctor, is represented by a point in $[0,1]$. The patients do not directly observe the exact diagnosis, but given his (limited) ability to interpret online health information, A will consider his condition as "not serious" when the underlying diagnosis lies in $[0,\frac{1}{3}]$ and "serious" when it lies in $[\frac{1}{3},1]$; B, on the other hand, has a different interpretation: he considers $[0,\frac{2}{3}]$ as "not serious" and $[\frac{2}{3},1]$ as "serious." Suppose the true, exact diagnosis is $\frac{1}{2}$ for both A and B. Then, A will consider himself "serious," in effect believing that his exact diagnosis can be anywhere in $[\frac{1}{3},1]$; B will consider himself "not serious," in effect believing that his exact diagnosis lies in $[0,\frac{2}{3}]$.

Suppose the doctor does not fully reveal the true diagnosis.[7] Instead, she provides a "vague" advice, suggesting that the true diagnosis lies in $[\frac{1}{4},\frac{7}{12}]$; she is not telling the exact truth but is not deceiving either because $[\frac{1}{4},\frac{7}{12}]$ contains $\frac{1}{2}$. In light of his knowledge, A will interpret the advice to mean that the exact diagnosis lies in $[\frac{1}{3},\frac{7}{12})$.[8] Since B's information is even vaguer than the advice, he will take the advice as it is. To A, the advice is a *complementary advice*; it adds on but does not replace his information. The same advice is, however, a *substituting advice* to B - it substitutes what B knows. Should the doctor provide a completely vague advice that the diagnosis can be anywhere in $[0,1]$, both A and B will ignore it as they both know better; the advice is a *redundant advice* to them.[9] Finally, suppose the doctor deceives with advice $[\frac{7}{8},1]$, which does not

[5]The supplier-induced demand hypothesis in health economics (Evans [15]) posits that doctors recommend more health care purchases than patients would buy if they had the same information. This coincides with the expert's bias in cheap-talk models. See the literature review (Section 2.7) for previous work that uses cheap-talk models to study the original neoclassical supplier-induced demand.

[6]Thirty percent of the respondents in Fox [18] indicated that online information led them to ask further questions to their doctors or seek a second opinion. In an article on *salon.com* [40], "Is There a Doctor in the Mouse?," the pediatrician-author mentions that some parents refused to vaccinate their children after being exposed to stories on autism websites about the dangers of vaccines to children.

[7]A biased expert in the CS model does not fully reveal her information in (any) equilibrium; such an equilibrium property carries over to the amateur model (see Section 2.4).

[8]The decision maker's "information," "knowledge" and "interpretations" are considered synonyms.

[9]This "babbling advice" will also be ignored by an uninformed decision maker. While I use it as an example of

contain the true $\frac{1}{2}$. While this advice will be considered by A as substituting, B can detect that it is a *false advice* because it contradicts with what he knows; this represents our model analog of "defense against fraud and deception."

Using the CS model, which features a novice, uninformed decision maker, as the benchmark, the first important result is that the expert in the amateur model provides less informative advice. A highly biased expert (bias $b \in [\frac{1}{6}, \frac{1}{4})$) provides no advice useful to an amateur, whereas the same expert will provide informative advice to a novice (an informative equilibrium exists in the CS model whenever $b < \frac{1}{4}$). A less biased expert ($b < \frac{1}{6}$), who does provide useful advice to an amateur, would have provided more informative advice had she lived in the CS model.[10] The information of some types of decision maker attenuates the expert's influence on their actions. In response, the expert adjusts her advice strategy. She induces the decision maker, whose type is unknown to her, to take actions that are more biased toward her preferred ones.[11] Given the misaligned interests, this adjustment in the expert's benefit harms the decision maker.

The result is analogous to that in Akerlof [2]. The information on what the decision maker knows - his definition of "high" and "low" - is asymmetrically distributed between the expert and the decision maker. The decision maker of course knows what he himself knows but the expert only knows the probability distribution, and this creates a second layer of information asymmetry. Consequently, "good" communication between the expert and the decision maker (in the CS model) is superseded by "bad" communication (in the amateur model). It is reported that some doctors become less helpful if the patient is a "Googler."[12] While there are certainly many factors behind the doctors' responses, this chapter provides a rational, strategic account for why doctors can become less helpful to informed patients under asymmetric information.

Without considering the expert's strategic response, as a decision problem under uncertainty more information always increase expected payoff. Comparing this direct benefit of information with the negative, strategic effect, information does improve the welfare of some decision makers, but there are decision makers who are worse off for being informed. *The evolution from novices*

redundant advice, there is non-babbling advice in the amateur model that can nevertheless be redundant.

[10] By "more informative," I mean precisely that a novice will benefit more from an advice that is customized for novice than under that that is customized for amateur.

[11] In the followings, I shall use the plural form "decision makers" to abbreviate "types of decision maker."

[12] In Fox and Fallows [19], for example, a patient responds to the survey: ".....they [doctors] became irritated with me for having knowledge of the condition beyond what they chose to share with me.....they became defensive and short with me when I would question.....what I had found out on my own on the Internet." Despite the incentives to protect their professional images, negative responses are also noted from the doctors' side (Ahmad et al. [1]): "So, if your patient [is] having a $15 visit, you're not going to sit for 15 minutes going through all this [patient's findings], you're going to get them out of the office."

into amateurs does not benefit every decision maker in a strategic environment. Nevertheless, when payoff is evaluated before the decision maker's type is realized, more information is always beneficial. An implication is that advocates would always consider patient or investor education as welfare improving; as an outside observer they by definition evaluate the benefit of information in an *ex-ante* sense. To certain individuals, however, more information can indeed backfire as the experience of the patient demonstrates.

Medical studies cited in this chapter report that patients in general consider online information to be helpful but doctors are either neutral or negative. The results in this chapter shed light on these apparent contrasting views: doctors are likely to evaluate the effects of online information in terms of how it affects patients' responses to their advice, whereas patients evaluate it in terms of its contributions to their overall health outcomes. By assessing the extents of welfare changes, the decision maker's information is found to make the most difference when the expert's bias is more extreme. The advice of a highly biased stock broker, who always keeps an eye on commission, may not be so useful anyway, and the investor should rely on his own for information. On the other hand, the negative response from a barely biased (altruistic) doctor will be limited, and the patient can benefit from more information without worrying too much about the negative, strategic effect.

On the technical side, the decision maker's information raises the issue of how specifications of off-equilibrium beliefs affect equilibria, a consideration absent in traditional cheap-talk models with uninformed decision maker. After describing the model in Section 2.2, I analyze off-equilibrium beliefs in Section 2.3. I then characterize equilibria in Section 2.4 (the general model) and Section 2.5 (the uniform-quadratic model). Section 2.6 analyzes the decision maker's welfare.

Decision makers with access to information have yet to be substantially incorporated into cheap-talk models. Two early exceptions were Seidmann [43] and Watson [48], followed by Olszewski [37]. With some alternations to otherwise standard cheap-talk games (e.g. different specifications of payoffs), they show that a decision maker's information can improve communication or even elicit full revelation of information from the expert. This chapter shows, quite differently, that, in the original cheap-talk model of Crawford and Sobel [12], a decision maker's information not only does not elicit full revelation but can indeed worsen communication. I postpone a more detailed discussion of the related literature to Section 2.7. Section 2.8 concludes with discussions of the results and possible extensions. All proofs in this chapter, as those in other chapters, are relegated to the Appendix.

2.2 THE MODEL

2.2.1 Players and payoffs

There are two players, an expert (e) and a decision maker (d). They are in a principal-agent relationship, with the expert being the agent and the decision maker the principal. The expert is perfectly informed about the state of the world, $\theta \in \Theta \equiv [0, 1]$; the state is distributed on Θ in accordance with a common-knowledge continuous distribution F with everywhere positive density f.[13] After privately observing θ, the expert sends a message (advice) $m \in M$ to the decision maker, who then takes an action $a \in \mathbb{R}$.[14]

The players' payoffs depend on states and actions: $U^e(a, \theta, b)$ and $U^d(a, \theta)$. The payoff functions are twice continuously differentiable and satisfy the following conditions: 1) $U^i_{11}(\cdot) < 0$, 2) $U^i_1(a, \theta, b) = 0$ for some $a \in \mathbb{R}$, and 3) $U^i_{12}(\cdot) > 0$, $i = e, d$. The first condition guarantees that the a that satisfies the second condition is, for a given pair (θ, b), a unique value that maximizes $U^i(\cdot)$. The third, a sorting condition similar to that in costly signalling models, further ensures that the maximizing value of a, called the *ideal actions* of the players, is strictly increasing in θ. The parameter $b > 0$ captures the bias of the expert's preferences relative to those of the decision maker. I impose two further assumptions that $U^d(a, \theta) \equiv U^e(a, \theta, 0)$ for all (a, θ) and $U^e_{13}(a, \theta, b) > 0$ everywhere; thus, the misaligned preferences between the players are represented as: for all θ, the ideal action of the expert, $a^e(\theta, b)$, is strictly higher than that of the decision maker, $a^d(\theta)$.

I shall analyze the model with the above general payoff and distribution function (the *general model*). For welfare analysis, however, I shall focus on the specialization where F is uniform and the payoff functions are in the following form:

$$U^e(a, \theta, b) \equiv -(a - (\theta + b))^2, \tag{2.1}$$

$$U^d(a, \theta) \equiv -(a - \theta)^2. \tag{2.2}$$

This is the *uniform-quadratic model* introduced by Crawford and Sobel [12], a specialization adopted in almost all subsequent work in cheap talk. Apart from its tractability, it offers a benchmark to compare the results in this chapter with those in the literature. Note that, under this specialization,

[13] Depending on the context, I shall interchangeably refer θ as the "state" or the "type of the expert."

[14] There is no restriction on M other than that it should be sufficiently large. For example, an infinite set will more than suffice.

the expert's ideal action $a^e(\theta, b) = \theta + b$ which is higher than the patient's $a(\theta) = \theta$ exactly by the amount of her bias b.

Remark for Physician-Patient Relationship. The state of the world, θ, can be interpreted as a diagnosis of a certain disease, with a more serious diagnosis represented by a higher θ. It is assumed that the doctor perfectly and privately observes the *true diagnosis*. She then delivers her *reported diagnosis*, m, to the patient. The reported diagnosis induces the patient to take certain action, a, which can be interpreted as a treatment or a medical procedure. Alternatively, it could also be interpreted as a lifestyle choice the patient adopts that has health consequences for him.

Taking the first interpretation of action as an example, a higher a corresponds to a more intense and expensive treatment. Consistent with the supplier-induced demand hypothesis in health economics (Evans [15]), the model says that the doctor always prefers more intense treatments than are ideal from the patient's perspective.[15] The doctor's preferences toward more intense treatment may reflect her cautiousness given that she does not bear the costs of the treatment. It can also be due to her financial interests in the treatment choices. For lifestyle choice, this can be interpreted as the doctor's preference over more disciplined lifestyle than is deemed optimal by the patient; doctors normally have a stricter standard than patients on what count as healthy lifestyles. In this case, b captures the doctor's "paternalistic bias."

That the reported diagnosis is a "cheap talk" and cannot be verified *ex post* could be due to the fact that the disease in question does not have an objective diagnosis (e.g., various emotional problems). Likewise, there is no objective criterion to evaluate a recommendation on lifestyle choice. Moreover, in reality, as is in the model, what the patient ultimately observes is his health outcome (payoff) and the remedies he received; he may never be able to observe exactly the true diagnosis. Thus, even if outcomes are bad, there is always a leeway favoring the doctors in which the patients face some identification issue on what really went wrong.[16]

2.2.2 Decision maker's information and types

The decision maker is partially informed: he does not directly observe θ but can tell whether θ belongs to the set of "high states" or "low states"; how he defines "high" and "low" remains his private information.[17] The decision maker first privately observes a parameter $t \in T \equiv [0, 1]$, called

[15]See Section 2.7 for a brief discussion of this major topic in health economics.

[16]In the model, a given pair $\bar{a}, \bar{U}^d$ can, for example for quadratic payoffs, be associated with two different θ.

[17]The choice of only "high" and "low" is guided by parsimony and tractability. The analysis will become exponentially complicated if only "medium" is added.

his *threshold*, that is commonly known to be uniformly distributed on T, independently of θ. A realized $t \in (0,1)$ divides $\Theta \equiv [0,1]$ into two non-degenerate intervals, one low $t_l \equiv [0,t)$ and one high $t_h \equiv [t,1]$. If, for a given t, the realized state $\theta < t$, the decision maker further receives a private signal $s = l \in \{l,h\}$; he then knows that $\theta \in t_l$. If, on the other hand, the realized $\theta \geq t$, the decision maker receives signal $s = h \in \{l,h\}$, and he will know that $\theta \in t_h$. The following summarizes the common-knowledge property of the decision maker's signal in terms of θ and t:

$$s(\theta|t) = \begin{cases} l, & \text{if } \theta < t, \\ h, & \text{if } \theta \geq t. \end{cases}$$

I call the decision maker an *amateur decision maker*; he knows less than an expert but almost always more than an uninformed, *novice decision maker*.[18] The set of thresholds T coupled with the set of signals $\{l,h\}$ defines the type space of the amateur decision maker, $T \times \{l,h\}$, with generic element t_s. I call t_h a *higher-interval* type with threshold t as t_h is an interval defining what the decision maker knows; similarly t_l is called the *low-interval* type.

Remark for Patients' Information. The patient searches on the Internet, gathers and interprets some information he finds. The information helps but he also faces the limitation of his amateur ability in interpreting it. He is unable to distinguish among all the potential diagnoses $\theta \in [0,t)$ and therefore lumps them together under the category of "not serious." Likewise, he lumps all the potential diagnoses $\theta \in [t,1]$ under the category of "serious." In this context, t has an intuitive, somewhat behavioral interpretation: a relatively high t means that the underlying diagnosis has to be very serious in order to be considered by the patient as serious - a patient with a high t is a "carefree" kind; on the other hand, a relatively low t means that the underlying diagnosis will be considered by the patient as serious even if it is not that serious - a patient with a low t is then a hypochondriac.

2.2.3 Timeline of the game

The game begins with nature drawing the threshold t and then proceeds with the realization of the state θ and the simultaneous generation of the signal s. The two players then update their beliefs about each other given the realized information. The decision maker updates his beliefs in terms

[18]The amateur knows nothing more than a novice in the measure-zero event that $t = 0$ or $t = 1$. Accordingly, a novice can be viewed as a special type of amateurs with $t = 0$ or 1; the CS model can thus be viewed as a special case of the amateur model.

of what interval type he is; the expert updates her beliefs in terms of what type of decision maker she could be facing.

The players then interact: the expert communicates with the decision maker by sending him a message m. The decision maker further updates his beliefs, taking into account his previously updated beliefs and any information contained in the message. The game ends with the decision maker's taking an action a and the distribution of payoffs to the players.

The followings dissect the players' beliefs before they interact. To the decision maker, the generation of s determines, given his realized t, whether he is a high-interval type t_h or a low-interval type t_l. They differ by their beliefs about θ:

$$\phi(\theta|t_h) = \begin{cases} 0, & \text{for } \theta \in [0,t) \\ \frac{f(\theta)}{1-F(t)}, & \text{for } \theta \in [t,1], \end{cases} \quad \text{and} \tag{2.3}$$

$$\phi(\theta|t_l) = \begin{cases} \frac{f(\theta)}{F(t)}, & \text{for } \theta \in [0,t), \\ 0, & \text{for } \theta \in [t,1]. \end{cases} \tag{2.4}$$

In a particular instance of the game, only one kind of interval types is realized as they are mutually exclusive. Nevertheless, the beliefs of both are relevant to the expert because she does not observe t and s and thus does not know whether the decision maker is a high-interval or low-interval type. In deciding what message to send, the expert takes into account the would-be responses of all possible interval types. Since the best responses of the decision makers in turn depend on their beliefs about θ and these beliefs differ among the decision maker's types, to the expert the decision maker's types are nothing more than a bunch of beliefs-types - beliefs on what the expert's own type is.

If we adopt this perspective, when the expert updates her beliefs about the decision maker after observing θ, she is in effect updating *her beliefs about what the decision maker's beliefs about her is*. In this sense, the amateur model is a model with *higher-order beliefs* in disguise. Classifying into whether the decision maker she is facing is a high-interval or low-interval type (i.e., whether the decision maker has received h or l), these updated beliefs of the expert are

$$\gamma(t_h|\theta) = \begin{cases} \frac{1}{\theta}, & \text{for } t \in [0,\theta], \\ 0, & \text{for } t \in (\theta,1], \end{cases} \quad \text{and} \tag{2.5}$$

$$\gamma(t_l|\theta) = \begin{cases} 0, & \text{for } t \in [0,\theta] \\ \frac{1}{1-\theta}, & \text{for } t \in (\theta,1]. \end{cases} \tag{2.6}$$

From the perspective of a type-θ expert, the probability that the decision maker has received h or l is, respectively, θ and $1-\theta$. Moreover, since a decision maker who has received h must have threshold $t \in [0,\theta]$, the expert attaches positive probability to facing a high-interval type t_h only for $t \in [0,\theta]$. Similar reasoning applies for her beliefs about the low-interval types t_l. Thus, even though t and θ are independent, the signal s, correlated with both of them, serves as an anchor for the expert to update his beliefs in terms of t.

2.2.4 Strategies and expert's expected payoff

A behavior strategy of the expert, $\sigma : \Theta \to \Delta M$, specifies the distribution of message she sends for each $\theta \in \Theta$. The decision maker's pure strategy, $\rho : M \times T \times \{l,h\} \to \mathbb{R}$, specifies for each combination of received message and interval type an action he chooses to take.[19] I denote $\Theta_\sigma(m)$ to be the set of θ for which the expert sends message m with positive probability under σ, i.e., $\Theta_\sigma(m) = \{\theta \in \Theta : \sigma(m|\theta) > 0\}$. When there are some messages in M that are not used under σ, I adopt the convention that $\Theta_\sigma(\cdot)$ is an empty set for those messages.

Since the expert does not observe t, the derivation of her expected payoff before a message is sent calls for consideration of all its possible values in $[0,1]$ and the corresponding interval types. Suppose an expert of type θ sends message m'. Using the interval types as the unit of analysis, her conditional expected payoff from sending m' if the decision maker is a high-interval type is $\int_0^1 U^e(\rho(m',t_h),\theta,b)\gamma(t_h|\theta)dt$. Similarly, her conditional expected payoff if the decision maker is a low-interval type is $\int_0^1 U^e(\rho(m',t_l),\theta,b)\gamma(t_l|\theta)dt$.

To arrive at her unconditional expected payoff, we take an average of the conditional ones weighted by their corresponding probabilities of occurrence, which is θ for high-interval type and $1-\theta$ for low-interval type. Using (2.5) and (2.6) for $\gamma(t_h|\theta)$ and $\gamma(t_l|\theta)$, the unconditional expected

[19] The condition $U^d_{11}(\cdot) < 0$ guarantees that only pure strategy will be adopted by the decision maker.

payoff is then

$$V^e(m',\theta,b) \equiv \int_0^\theta U^e(\rho(m',t_h),\theta,b)dt + \int_\theta^1 U^e(\rho(m',t_l),\theta,b)dt. \tag{2.7}$$

The expert's type divides $[0,1]$ into two intervals, and only one kind of interval types in each of $[0,\theta]$ and $(\theta,1]$ contributes to the expert's expected payoff.

2.3 BELIEFS, ADVICE INTERPRETATIONS, AND EQUILIBRIUM DEFINITION

The presence of an informed decision maker leads to an important difference between unused and out-of-equilibrium messages in the amateur model. This section analyzes this issue and lays down the equilibrium definition - perfect Bayesian equilibrium.

2.3.1 Decision maker's beliefs after communication

The decision maker's (post-communication) beliefs function, $\mu : M \times T \times \{l,h\} \to \Delta\Theta$, specifies for each combination of received message and interval type a density over Θ; upon receiving an advice, the decision maker interprets it in light of his knowledge to infer the posterior distribution of θ.

The fact that the decision maker updates his beliefs using two sources of information, one strategic and one not, creates an important difference between unused and out-of-equilibrium messages. In the CS model (as in signalling games in general), unused messages are synonyms for messages sent off the equilibrium paths, and vice versa.[20] Since in cheap-talk games messages do not directly affect payoffs, for every equilibrium (in which there may be unused messages) there exists another with the same equilibrium outcome in which all messages are used.[21] Insofar as outcomes are concerned, one can therefore restrict attention to equilibria with no unused message. Furthermore, since unused messages are the same as out-of-equilibrium messages, any specification of off-equilibrium beliefs will have no bite in ruling out certain equilibrium outcomes.[22]

A common approach to focus on equilibria with no unused message, as is originally used by Crawford and Sobel [12], is to assume that the expert adopts behavior strategy and randomizes

[20] In Crawford and Sobel's [12] own words, unused messages are "signals that are not in the support of the signaling rule used by some S[ender]-type in equilibrium."

[21] Equilibrium outcomes are defined as probability distributions from states to actions.

[22] See, for example, the discussion in Farrell [16].

uniformly over supports of distinct intervals that exhaust the message space. Under this strategy, Bayes's rule always applies for updating the decision maker's beliefs. *In the amateur model, however, even if the expert adopts such a behavior strategy, there can still be out-of-equilibrium messages*; despite the fact that every message is used by some expert-type in equilibrium, there exist cases where Bayes's rule does not apply.

Suppose there is an *informative equilibrium* with no unused message such that M is partitioned under the expert's strategy σ into at least two distinct and exhaustive sets. Given that every message is used and the equilibrium is informative, for every $m \in M$, $\Theta_\sigma(m)$ is a non-empty strict subset of Θ. Since the decision maker correctly anticipates σ in equilibrium, he takes every $m \in M$ to mean that $\theta \in \Theta_\sigma(m)$ for some $\theta \in \Theta$ - all messages he can potentially receive means something to him. Suppose the decision make is of type t'_s so that his beliefs before communications are $\phi(\theta|t'_s)$. When he receives message m', he updates his beliefs using $\phi(\theta|t'_s)$ for the Bayes's rule: $[\sigma(m'|\theta)\phi(\theta|t'_s)]/\int_0^1 \sigma(m'|\theta')\phi(\theta'|t'_s)d\theta'$. Since there is no unused message, for all $m \in M$, $\sigma(m|\theta) > 0$ for some $\theta \in [0,1]$; the usual scenario for Bayes's rule to be undefined (when $\sigma(m|\theta) = 0$, and thus $\sigma(m|\theta)\phi(\theta|t_s) = 0$ for all $\theta \in [0,1]$) will never appear here given all messages are used.

However, suppose in this equilibrium there is a type-θ' expert with $\theta' \in t'_s$. She deviates from σ by sending a message which, notwithstanding meaning something to the decision maker, contradicts with what he knows. More precisely, suppose θ' sends one of the *used* messages m'' such that $\Theta_\sigma(m'') \cap t'_s = \emptyset$.[23] In an attempt to apply the Bayes's rule, even though $\phi(\theta|t'_s)$ and $\sigma(m''|\theta)$ are positive for some $\theta \in [0,1]$, the decision maker will still find it not defined. This is because $\phi(\theta|t'_s) > 0$ only for $\theta \in t'_s$ and $\sigma(m''|\theta) > 0$ only for $\theta \in \Theta_\sigma(m'')$; by our choice of m'' the two sets contain no comment element, and thus the term $\sigma(m''|\theta)\phi(\theta|t'_s) = 0$ for all $\theta \in [0,1]$. This will never arise in the CS model with no unused message because, using the interval-type terminology, there is only one single type for novice which is Θ; for any used message m, $\Theta \cap \Theta_\sigma(m)$ can never be empty.

Let me summarize the discussion with the formal notion of information set. I start with the following lemma, which is true almost by definition:

Lemma 2.1. *A decision maker of type t_s is in an information set unreached in equilibrium if and only if he receives message $m \in M$ such that $\Theta_\sigma(m) \cap t_s = \emptyset$, where σ is an equilibrium strategy.*

[23] Let me further elaborate how this matches the plain language: type θ' *deviates by choosing this* m'' ($\theta' \notin \Theta_\sigma(m'')$) such that $\Theta_\sigma(m'') \cap t'_s = \emptyset$. The intersection being empty is not due to an empty $\Theta_\sigma(m'')$ (m'' *is a used message, it means something to the decision maker*) but because the two sets contain no common element (m'' *means something that contradicts with what the decision maker knows*).

Lemma 2.1 provides us with a link between the notion of information set and the notations used. Indeed, $\Theta_\sigma(m) \cap t_s$ is an information set of the decision maker. The next lemma summaries the discussion above:

Lemma 2.2. *Suppose there is an informative equilibrium in the amateur model with no unused message. There exists deviation of the expert from this equilibrium that leaves some types of decision maker in information sets unreached in equilibrium.*

Lemma 2.1 implies that we have to consider off-equilibrium beliefs whenever there are cases where, for some m and some t_s, $\Theta_\sigma(m) \cap t_s = \emptyset$. Since the equilibrium concept of perfect Bayesian equilibrium places no restriction on beliefs off the equilibrium paths, we are therefore free to specify any beliefs in these cases. While the cases of empty $\Theta_\sigma(m) \cap t_s$ arise for both unused and deviated messages, the amateur model shares a property of the CS model that allows us to focus on the latter for off-equilibrium beliefs:

Lemma 2.3. *In the amateur model, for every equilibrium (σ, ρ, μ) there exists another equilibrium (σ', ρ', μ') with no unused message of which the equilibrium outcome is equivalent to that of (σ, ρ, μ).*

Guided by Lemmas 2.1-2.3, the decision maker's post-communication beliefs can be specified as follows. Upon receiving m', a type-t_s decision maker updates his beliefs according to

$$\hat{\mu}(\theta|m', t_s) = \begin{cases} \frac{\sigma(m'|\theta)\phi(\theta|t_s)}{\int_0^1 \sigma(m'|\theta')\phi(\theta'|t_s)d\theta'}, & \text{if } \Theta_\sigma(m') \cap t_s \neq \emptyset, \\ \psi(\theta), & \text{if } \Theta_\sigma(m') \cap t_s = \emptyset,\ \Theta_\sigma(m') \neq \emptyset, \end{cases} \tag{2.8}$$

where $\psi(\theta)$ is any arbitrary density supported on $[0, 1]$.

2.3.2 Interpretations of advice

Switching language from "messages" to "advice," there are four types of advice that *a type of decision maker can receive*, namely, *substituting advice*, *complementary advice*, *redundant advice* and *false advice*. Similar to messages having no intrinsic meaning in cheap-talk models, an advice has no intrinsic type in the amateur model; the type of an advice is determined only with respect to how it is interpreted, i.e., who is receiving it. To a decision maker, the first two types of advice are useful whereas the latter two are not.[24] Except for false advice, all three types of advice can arise in equilibrium, and they are embodied in the case of non-empty $\Theta_\sigma(m') \cap t_s$ in (2.8).

[24]In this discussion, I use "a decision maker" to abbreviate "a type of decision maker."

A *redundant advice* is received when the expert provides advice to a decision maker who knows better than that; his updated beliefs in (2.8) reduce to his pre-communication beliefs $\phi(\theta|t_s)$. In other words, the decision maker adheres to his own information in deciding what action to take when a redundant advice is received.[25] *Substituting advice* is received when the advice provides more precise information that what a decision maker knows - it therefore substitutes the decision maker's knowledge. This manifests in (2.8) as his beliefs putting zero density on all but the support $\Theta_\sigma(m')$. When a *complementary advice* is received, neither the information provided by the expert nor a decision maker's information is more precise. Suppose a division manager (the expert) reports to her CEO (the decision maker) that an investment project can be either "medium" or "high" in terms of profitability. The CEO, however, figures out on his own that it can only be "low" or "medium." Neither information, {medium, high} nor {low, medium}, dominates the other, but the inclusions of "medium" in both make them complementary. In terms of beliefs, in the case of complementary advice, (2.8) will put positive density only on the intersection $\Theta_\sigma(m') \cap t_s$ which is a strict subset of either $\Theta_\sigma(m')$ or t_s.

It is well known that if the expert could commit before the state realizes to reveal all his information, both parties would be better off. Viewed in this light, *it is the absence of commitment opportunities that prevents the biased expert from fully revealing the truth.* If the manager indeed observes "medium" and includes "high" only for her own benefit, the CEO's own information facilitates a more effective communication because his interpretation of the manager's {medium, high} in light of his own {low, medium} allows him to pin down the true profitability of the project.

Finally, *false advice* is the same as the deviated message in Lemma 2.2. Representing the model counterpart of guard against fraud and deception, it happens when the expert provides (false) information that contradicts with what a decision knows. In general, we can use as in (2.8) any arbitrary density for the decision maker's beliefs when false advice is received, so long as it is defined on the same support as the prior of θ, i.e., Θ. There could, however, be more to consider.

[25]Strictly speaking, in order to have $\sigma(m'|\theta)\phi(\theta|t_s)/\int_0^1 \sigma(m'|\theta')\phi(\theta'|t_s)d\theta' = \phi(\theta|t_s)$ in the case of redundant advice, we have to assume in addition that $\sigma(m'|\theta)$ is uniform. But regardless, the idea that the decision maker will adhere to his own information applies.

2.3.3 Equilibrium definition

While F is the prior distribution of the state in the model, one can argue that for a decision maker of type t_s, his prior is the same as his pre-communication beliefs $\phi(\theta|t_s)$.[26] If one adopts this perspective, any off-equilibrium beliefs should be supported on t_s rather than on Θ. Indeed, it is intuitively unappealing that a decision maker will be contradicting himself by putting positive density on $\Theta \setminus t_s$ after being deceived. While the concept of perfect Bayesian equilibrium remains silent on what we should choose here, I let intuitiveness prevail and perform a minor refinement by replacing $\psi(\theta)$ with $\psi(\theta|t_s)$, an arbitrary density over t_s, for the equilibrium definition:[27]

Definition 2.1 (**Perfect Bayesian Equilibrium**). *A perfect Bayesian equilibrium of the amateur model is a pair of strategies (σ, ρ) and a set of beliefs μ such that*

1. *The expert maximizes her expected payoff given the decision maker's strategy: for all $\theta \in \Theta$, if $m \in \text{supp}[\sigma(\cdot|\theta)]$, then*

$$m \in \underset{m' \in M}{\text{argmax}} \left[\int_0^\theta U^e(\rho(m', t_h), \theta, b) dt + \int_\theta^1 U^e(\rho(m', t_l), \theta, b) dt \right],$$

2. *The decision maker maximizes his expected payoff given his beliefs and his interval type: for all $m \in M$ and all t_s, $t_s \in T \times \{l, h\}$,*

$$\rho(m, t_s) = \underset{a'}{\text{argmax}} \int_0^1 U^d(a', \theta) \mu(\theta|m, t_s) d\theta, \quad \textit{and}$$

3. *The decision maker updates his beliefs using Bayes's rule whenever possible, taking into account the expert's strategy and his interval type: for all t_s, $t_s \in T \times \{l, h\}$,*

$$\mu(\theta|m, t_s) = \begin{cases} \frac{\sigma(m|\theta)\phi(\theta|t_s)}{\int_0^1 \sigma(m|\theta')\phi(\theta'|t_s)d\theta'}, & \textit{if } \Theta_\sigma(m) \cap t_s \neq \emptyset, \\ \psi(\theta|t_s), & \textit{if } \Theta_\sigma(m) \cap t_s = \emptyset,\ \Theta_\sigma(m) \neq \emptyset, \end{cases}$$

where $\psi(\theta|t_s)$ is any density supported on t_s.

[26] Thus, this can be considered as different decision makers having different prior beliefs about θ in the amateur model. For information transmission models with heterogeneous priors on the senders' side, see, for example, Spector [44], Banerjee and Somanathan [5] and Che and Kartik [10].

[27] This shares the same spirit of equilibria with "support restrictions" which requires that the support of beliefs at an information set be a subset of that at the preceding information sets. See, for example, Madrigal et al. [31] for a discussion and Rubinstein [42] for an application.

2.4 EQUILIBRIUM CHARACTERIZATIONS I: GENERAL MODEL

Crawford and Sobel [12] show that there is no fully separating equilibrium when the decision maker is a novice - all equilibria are partition-equilibria in which the expert endogenously partitions $[0, 1]$ into a finite number of intervals and reveals limited information to the decision maker as to which interval contains the realized state. The first characterization is that such property of equilibria is preserved in the amateur model:

Proposition 2.1. *There exists no fully separating equilibrium in the amateur model.*

In a fully separating equilibrium, if there is any, the amateur's own information becomes useless because every equilibrium advice that he receives will substitute what he knows. The amateur will act in the exact same way as a novice in response to a fully separating strategy. Since we do not have fully separating equilibrium for novice, it follows that the amateur has to face the same fate.

The significance of Proposition 2.1 is two-fold. First, it demonstrates that, in contrast to some previous work on informed decision maker in information transmission game (Seidmann [43], Watson [48] and Olszewski [37]), the presence of an informed decision maker in the CS model does not facilitate full revelation of information. Second, it shows that, for the characterization of the decision maker's best responses, it is without loss of generality to consider strategies of the expert that partition the state space into intervals.

2.4.1 Induced actions: the decision maker's best responses

This subsection characterizes the decision maker's best responses when the expert adopts an "N-step strategy." It should be noted that we are yet to consider the equilibrium, so these strategies are not necessarily equilibrium ones.

Suppose there is a partition of $[0, 1]$, $\{\theta_i\}_{i=0}^{N}$, where $0 = \theta_0 < \theta_1 < \cdots < \theta_{N-1} < \theta_N = 1$. The partition divides $[0, 1]$ into N intervals, $\{I_i\}_{i=1}^{N}$, where $I_i = [\theta_{i-1}, \theta_i]$, $i = 1, \ldots, N$. The subset of the partitioning elements, $\{\theta_i\}_{i=1}^{N-1}$, are commonly called the *boundary types*, and I call the types in the interior of I_i the *interior types*. Suppose the expert adopts the following strategy:

Definition 2.2 (**N-Step Strategy**). *The expert partitions the message space M into N distinct and exhaustive sets, M_i, $i = 1, \ldots, N$. Upon observing the realization of the state, if $\theta \in I_i$, the expert randomizes uniformly over messages in M_i.*

Under this strategy, exactly N actions will be induced on a novice, one for each M_i. In the amateur model, however, the strategy will induce an *uncountable* number of actions. Some decision makers - those who interprets the expert's advice differently in light of their information - take different actions even when they receive the same message, and there are a continuum of them. Accordingly, the different actions they take also form a continuum.

Some of these actions are taken under the influence of the expert (when the advice is substituting or complementary), and some of them are taken based solely on the decision maker's own information (when the advice is redundant or false). I call the former set of actions *effectively induced actions*, the latter *ineffectively induced actions* and their union induced actions. Here comes the operational definition of effectively induced actions:

Definition 2.3 (**Effectively Induced Actions**). *An action a is said to be effectively induced by message m if the decision maker updates his beliefs $\mu(\theta|m, t_s)$ using Bayes's rule and there exists $\theta \in [0, 1]$ such that, in the decision maker's maximization problem of which a is the solution, $\mu(\theta|m, t_s) \neq \phi(\theta, t_s)$.*

In order for an action to qualify as effectively induced, it has to be taken after Bayes's rule is used, which distinguishes it from actions taken after an out-of-equilibrium false advice. Furthermore, the updating must generate beliefs that are different from the pre-communication ones, further distinguishing it from actions taken in response to a redundant advice.[28] The actions that are taken without satisfying Definition 2.3 are ineffectively induced actions.

The following proposition characterizes the situations in which a t_s will receive a substituting or complementary advice under an N-step strategy, which lead to effectively induced actions:

Proposition 2.2 (**Effectively Induced Actions**). *Under an N-step strategy of the expert, an effectively induced action is induced on a high-interval type t_h if and only if he receives*

1. *a complementary advice: his threshold $t \in I_i$, $i = 1, \ldots, N-1$ and he receives $m \in M_i$; or*
2. *a substituting advice: his threshold $t \in I_i$, $i = 1, \ldots, N-1$ and he receives $m \in M_j$, $i < j \leq N$.*

An effectively induced action is induced on a low-interval type t_l if and only if he receives

1. *a complementary advice: his threshold $t \in I_i$, $i = 2, \ldots, N$ and he receives $m \in M_i$; or*
2. *a substituting advice: his threshold $t \in I_i$, $i = 2, \ldots, N$ and he receives $m \in M_k$, $1 \leq k < i$.*

[28] The notion of effectively induced actions also allows us to define informative equilibrium alternatively as any equilibrium where effectively induced actions are taken by some types of decision maker.

Define

$$a(r,s) = \begin{cases} \underset{a'}{\operatorname{argmax}} \int_r^s U^d(a',\theta) f(\theta) d\theta, & \text{if } r < s, \\ a^d(\theta), & \text{if } r = s, \end{cases}$$

it is straightforward to verify under this characterization that the profile of actions effectively induced on t_h, $t \in I_i$, $i = 1, \ldots, N-1$, is

$$\rho(m, t_h) = \begin{cases} a(t, \theta_i), & \text{for } m \in M_i \text{ (complementary advice)}, \\ a(\theta_{j-1}, \theta_j), & \text{for } m \in M_j,\ i < j \leq N \text{ (substituting advice)}; \end{cases} \tag{2.9}$$

and that induced on t_l, $t \in I_i$, $i = 2, \ldots, N$ is

$$\rho(m, t_l) = \begin{cases} a(\theta_{i-1}, t), & \text{for } m \in M_i \text{ (complementary advice)}, \\ a(\theta_{k-1}, \theta_k), & \text{for } m \in M_k,\ 1 \leq k < i \text{ (substituting advice)}. \end{cases} \tag{2.10}$$

The actions effectively induced by substituting advice are taken when the decision maker interprets the advice as it is. Had he been a novice, he would have done the same. His information therefore does not change his response to the advice. On the other hand, the actions effectively induced by complementary advice are taken when the decision maker effectively combines his own information with the advice he receives. Without any information in his hands, a novice would have responded differently.

Ineffectively induced actions are taken when useless advice is received and ignored, either in the process of applying the Bayes's rule or when it is an out-of-equilibrium event. The situations that give rise to them are the "mirror image" of those of effectively induced actions:

Proposition 2.3 (**Ineffectively Induced Actions**). *Under an N-step strategy of the expert, an ineffectively induced action is induced on a high-interval type t_h if and only if he receives*

1. *a redundant advice: his threshold $t \in I_N$ and he receives $m \in M_N$;*
2. *a false advice: his threshold $t \in I_i$, $i = 2, \ldots, N$, and he receives $m \in M_k$, $1 \leq k < i$.*

An ineffectively induced action is induced on a low-interval type t_l if and only if he receives

1. *a redundant advice: his threshold $t \in I_1$ and he receives $m \in M_1$;*
2. *a false advice: his threshold $t \in I_i$, $i = 1, \ldots, N-1$, and he receives $m \in M_j$, $i < j \leq N$.*

The profile of actions ineffectively induced by redundant advice is

$$\rho(m, t_h) = a(t, 1), \quad \text{and} \tag{2.11}$$

$$\rho(m, t_l) = a(0, t). \tag{2.12}$$

The profile of actions induced by false advice will depend on the specifications of off-equilibrium beliefs. Although the actions are ineffectively induced, they are so exactly because the decision maker's information is put into use. A novice would have followed in these cases advice that is either deceptive or too vague. The amateur therefore benefits from his information when he takes ineffectively induced actions.

The following property of actions induced by a particular type of expert will be useful for considering the expert's equilibrium strategy.

Corollary 2.1. *Under an N-step strategy, every $\theta \in I_i$, $i = 1, \ldots, N$, induces a different set of actions. Furthermore, $\theta \in I_i$ induce the set of actions $a(\theta_{i-1}, t)$, $t \in I_i$, if and only if she is an interior type.*

2.4.2 Existence of equilibria

In the CS model, the condition that the boundary types are indifferent between sending messages in two adjacent message sets (e.g., θ_i is indifferent between M_i and M_{i+1}) is *always* sufficient (and necessary) for the existence of informative equilibrium. Moreover, the specifications of off-equilibrium beliefs have no impact on equilibrium whatsoever. It turns out that this is no longer the case in the amateur model.

When the *indifference condition* above is satisfied, it follows immediately in the CS model that 1) every boundary type θ_i will (weakly) prefer sending messages in M_i over any other messages; and 2) θ in the interior of every I_i will (strictly) prefer sending messages in M_i over any other messages. I call the first and the second conditions the incentive-compatibility condition for the boundary types and the interior types. When they are satisfied, no single type of the expert has incentives to deviate from the N-step strategy - it constitutes an equilibrium.

In the amateur model, however, even if the following set of equations that characterize the indifference condition holds:

$$V^e(M_i, \theta_i, b) = V^e(M_{i+1}, \theta_i, b), \quad i = 1, \ldots, N-1, \, \theta_0 = 0, \, \theta_N = 1, \tag{2.13}$$

depending on the specification of off-equilibrium beliefs, the incentive-compatibility conditions *may* not be satisfied; there could be a benefit of "lying" by providing false advice in the amateur model.

Suppose the expert's strategy is of three steps, i.e., $N = 3$, and it satisfies the indifference condition. Consider boundary type θ_1 in this proposed equilibrium (Figure 2.1).[29] If the realized θ equals θ_1, then all high-interval types of decision maker will have $t \leq \theta_1$ and all low-interval types will have $t > \theta_1$. If θ_1 sends $m \in M_1$ as is prescribed by the strategy, indicating to the decision maker that $\theta \in [0, \theta_1]$, then all high-interval types will take action $a(t, \theta_1)$, $t \in [0, \theta_1]$, and all low-interval types will take action $a(0, \theta_1)$ (the second line in Figure 2.1). These actions are all effectively induced. Given that θ_1 satisfies the indifference condition, she will be indifferent between inducing these actions and those induced by $m \in M_2$, which are $a(\theta_1, \theta_2)$ and $a(\theta_1, t)$, $t \in (\theta_1, \theta_2]$ (not shown in the figure).

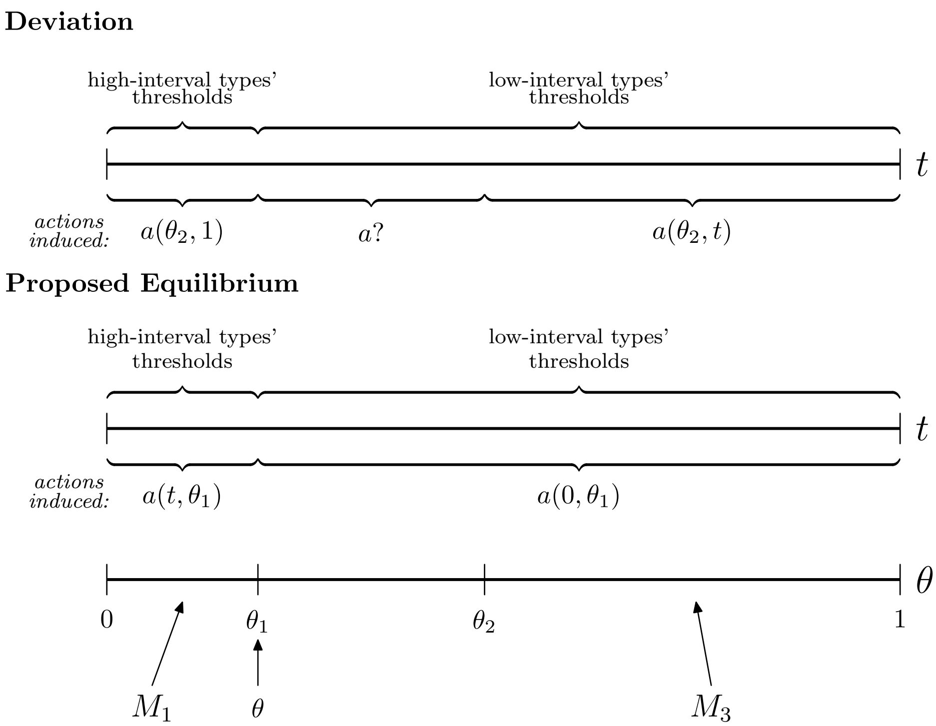


Figure 2.1: Incentives for deviations

Now, suppose the expert deviates from the strategy: she provides a false advice by sending $m \in M_3$, in effect telling the decision maker that $\theta \in [\theta_2, 1]$. All high-interval types will take

[29]There is benefit of lying for boundary types whenever $N \geq 3$. The benefit of lying also arises for interior types, which makes the indifference condition not always sufficient even for two-step equilibria. Indeed, in the CS model, incentive compatibility for the interior types is a consequence of that for the boundary types, and, as will be discussed below, this is also not true in the amateur model.

$a(\theta_2, 1)$ and all low-interval types with $t \in [\theta_2, 1]$ will take $a(\theta_2, t)$. These decision makers are not able to detect that the advice is false and are effectively induced to take actions. To the expert, these actions induced by the false advice are less favorable than those induced by M_1 (or M_2) because they are positioned farther away from her ideal action.

However, to the low-interval types with $t \in [\theta_1, \theta_2)$, the false advice can be detected because it contradicts with their information. To decide what action to take, ineffectively induced in this case, the off-equilibrium beliefs $\psi(\theta|t_l)$ come into play. Without any restriction, one can come up with beliefs so that these decision makers will take an action that is closer to (or even the same as) the expert's ideal action than $a(0, \theta_1)$ effectively induced in the proposed equilibrium. This creates a benefit of lying that is absent in the CS model. In equilibrium with more steps, it is conceivable that such benefit of lying can outweigh the cost of inducing unfavorable actions on some types of decision makers. Some types of expert may therefore have incentive to deviate from the strategy that satisfies the indifference condition. In effect, the expert lies on purpose so that some types of decision maker will take actions on their own, and the actions are favorable to her, yielding her a higher expected payoff.

The above suggests that the existence of informative equilibria not only depends on whether the bias of the expert allows the indifference condition to hold, but also hinges on the specifications of off-equilibrium beliefs. I denote ψ to be the set of off-equilibrium beliefs of all interval types of decision maker: $\psi \equiv \bigcup_{\{t,s\} \in T \times \{l,h\}} \psi(\theta|t_s)$. The following proposition states that there is a set of off-equilibrium beliefs that, coupled with a mild assumption on the expert's payoff function, guarantees the sufficiency of the indifference condition:

Proposition 2.4. *There exists a set of off-equilibrium beliefs ψ^* such that, provided $U^e_{12}(\cdot)$ is sufficiently large, the boundary types $\{\theta_i\}_{i=1}^{N-1}$ that satisfy (2.13) always constitute an equilibrium in the general model.*

Proposition 2.4 in effect provides two additional sufficient conditions for the existence of equilibria; they are far from being necessary. The off-equilibrium beliefs ψ^* adopted in the proposition is roughly as follows. The decision maker divides $\Theta \equiv [0, 1]$ into different sets that coincide with the partition under the expert's N-step strategy. If his threshold $t \in (\theta_i, \theta_{i+1})$, upon receiving a false advice a high-interval incarnation of him will believe that θ is distributed on $[t, \theta_{i+1})$ and a low-interval incarnation will believe that θ is distributed on (θ_i, t). In other words, the decision maker's beliefs are contingent on the expert's strategy. The (correct in equilibrium) anticipation of σ is thus not only used by the decision maker to form his beliefs on the equilibrium paths using Bayes's rule but also to form his off-equilibrium beliefs.

As mentioned above, there are two incentive compatibility conditions behind the existence of equilibrium. The off-equilibrium beliefs specified in the proposition are sufficient for the condition to hold for the boundary types. However, since the interior types induce a set of actions not induced by the boundary types (Corollary 2.1), a sufficiently large $U_{12}^{e}(\cdot)$ comes into the picture to guarantee that incentive compatibility also holds for them. Figure 2.2 illustrates the rationale behind with an example of two-step equilibrium.

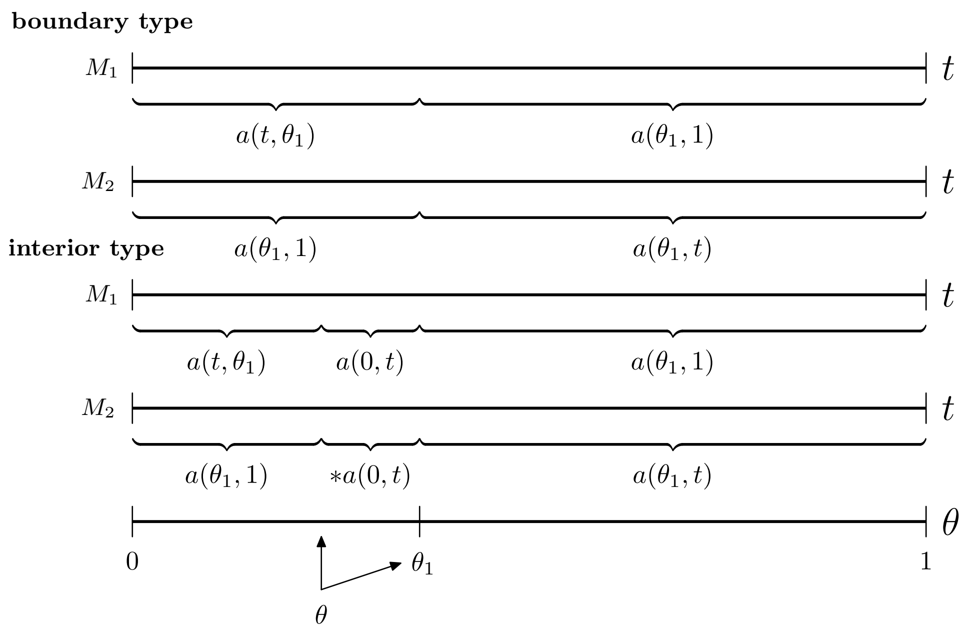


Figure 2.2: Actions induced by boundary and interior types

Given that the indifference condition holds, the boundary type θ_1's expected payoff from the profile of actions $a(0, \theta_1)$ and $a(t, \theta_1)$, $t \in [0, \theta_1]$, is the same from that from $a(\theta_1, 1)$ and $a(\theta_1, t)$, $t \in (\theta_1, 1]$ (the two upper lines). Consider the actions induced when the interior type θ sends messages in M_1 and M_2. If we compare the profile of actions in the lower pair of lines with those in the upper pair, we can see that they are the same except for $t \in (\theta, \theta_1)$. While θ_1 gives no false advice when she sends messages in either M_1 and M_2, there is one when θ sends $m \in M_2$. The specification of ψ^*, which allows incentive compatibility to hold for θ_1, (ineffectively) induce the action $a(0, t)$ (with asterisk) for the interior type if she sends messages in M_2, which is the same as the action effectively induced by $m \in M_1$.

If we could fix the profile of actions for θ_1 and decrease θ_1 to θ, the sorting condition would have guaranteed that θ strictly prefers to send messages in M_1 over M_2. However, when the expert's type changes, the profile of actions also changes, and, for $t \in (\theta, \theta_1)$, θ is indeed indifferent between M_1 and M_2. Thus, for incentive compatibility to hold, we have to ensure that θ prefers M_1 enough for $t \notin (\theta, \theta_1)$ so as to balance against her indifference for $t \in (\theta, \theta_1)$, and for this a sufficiently large $U^e_{12}(\cdot)$ is required.

A large $U^e_{12}(\cdot)$ means that the ideal action of a higher θ is sufficiently higher than that of a lower θ. This additional restriction is nothing but a strengthening of the already existing sorting condition. It is satisfied by the quadratic payoffs (2.1) and (2.2) in the uniform-quadratic model (in which $U^e_{12}(\cdot) = 2$). Thus, as the practice in most literature goes, the restriction does not impose too much of a limitation. Let me turn to the uniform-quadratic model now.

2.5 EQUILIBRIUM CHARACTERIZATIONS II: UNIFORM-QUADRATIC MODEL

In the uniform-quadratic model of Crawford and Sobel [12], the indifference condition reduces to a second-order linear difference equation. The availability of explicit solutions for linear difference equations allows a complete characterization of the relationship between the extent of information transmitted by the expert (N) and her level of bias (b). In the amateur model, however, the different specification of the expert's expected payoff turns the indifference condition into a non-linear second-order difference equation, which in general has no explicit solution.[30] For explicit results on how b relates to the properties of equilibria, I shall therefore provide a complete characterization only for two-step equilibria, in which the non-linear difference equation reduces to a solvable single equation.[31]

The off-equilibrium beliefs ψ^* in Proposition 2.4 have a special property in a two-step equilibrium. Under the prescription of ψ^*, when a false advice is received the decision maker's off-equilibrium beliefs are the same as his pre-communication beliefs. Thus, there is a consistency in how the decision maker behaves when he ignores the advice of the expert: regardless of whether the advice received is redundant or false, the decision maker relies on his own knowledge to take

[30] For conciseness, in this section I shall refer the uniform-quadratic specialization of the amateur model as simply the amateur model. The same applies for the uniform-quadratic specialization of the CS model.

[31] For the same reason of intractability, the welfare analysis in the next section will also focus on two-step equilibria.

the best decision he can.

The following characterizes the relationship between the existence of two-step (and thus informative) equilibria and the expert's bias:

Proposition 2.5. *In the amateur model, an informative equilibrium exists if and only if* $b < \frac{1}{6}$.

When $b \geq \frac{1}{6}$, what we are left with in the amateur model are the so-called "babbling equilibria" in which any advice from the expert is redundant. It is interesting to note that in the CS model, b has to be smaller than $\frac{1}{4}$ in order for an informative equilibrium to exist, and for $b \in (\frac{1}{12}, \frac{1}{4})$ the only informative equilibria are two-step (for the results that follow, a superscript "*cs*" will be added for the corresponding notations in CS model).[32] Incorporating this into Proposition 2.5 gives the following observations:

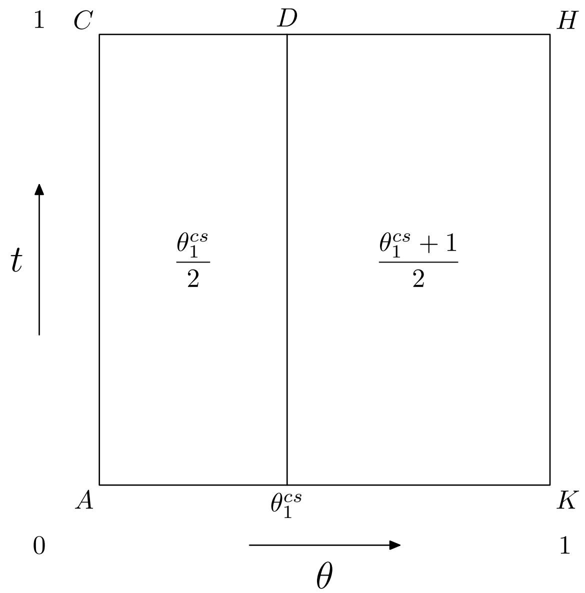
Corollary 2.2. *For* $b \in [\frac{1}{6}, \frac{1}{4})$, *an expert provides informative advice to a novice decision maker but babbles if the decision make is an amateur. For* $b \in (\frac{1}{12}, \frac{1}{6})$, *in which the most informative equilibria in both models have two steps, the boundary type in the amateur model* $\theta_1(b) < \theta_1^{cs}(b)$. *Furthermore, for* $b \in (\frac{1}{12}, \frac{1}{6})$, *the difference* $\theta_1^{cs}(b) - \theta_1(b) = b$.

Corollary 2.2 substantiates the claim that the expert responds strategically to her client's knowledge. More importantly, she responds in a negative way. When her bias is relatively high but not high enough for her to babble to a novice, she nevertheless babbles to an amateur. Even for lower level of bias so that she does provide informative advice to the amateur, her advice for the amateur will be less informative than that for a novice, in the sense that novices will prefer to make decision under the advice customized for them rather than that for the amateurs. Disentangling the contribution of the advice, the expert contributes less to an amateur's payoff than to a novice's, and such a negative differential aggravates as the expert becomes more biased.

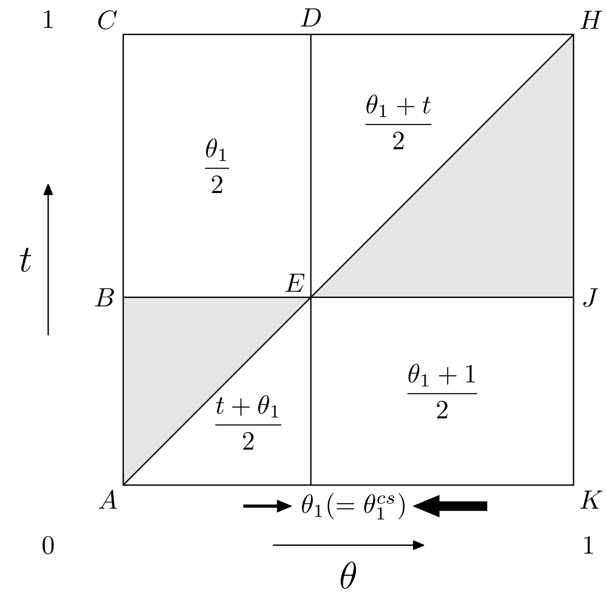
The information of some types of decision maker attenuates the influence of the expert on their actions. This change is, on average, unfavorable to the expert. To redeem the loss within the confine of her attenuated influence, she adjusts her strategy by providing less informative advice. Given the conflict of interests between the expert and the decision maker, such an adjustment in the expert's benefit harms the decision maker.

A more detailed explanation on why the expert adjusts her strategy in this way can be seen with the aid of Figure 2.3. The decision maker's threshold is measured vertically and the state horizontally. Consider first the CS model (Figure 2.3a). While the novice decision maker in

[32]There are also only two-step equilibria in the amateur model for b in this range.



(a) The CS Model



(b) The Amateur Model

Figure 2.3: Profile of induced actions

the CS model has only one single type, we can consider him as also having a threshold t but without an access to the signal s. In other words, we can imagine there is also a continuum of (single-dimensional) types for the novice decision maker, but since t by itself does not provide any information, all types of novice behave the same. Suppose the *equilibrium* boundary type in the CS model is θ_1^{cs}. The expert sends m_1 for $\theta \in [0, \theta_1^{cs}]$ and m_2 for $\theta \in [\theta_1^{cs}, 1]$.[33] Without any information on the decision maker's side, m_1 then induces a single action $\frac{\theta_1^{cs}}{2}$ and m_2 induces $\frac{\theta_1^{cs}+1}{2}$. Since θ_1^{cs} is an equilibrium boundary type, the expert will be indifferent between inducing these two actions.

Consider next the amateur model (Figure 2.3b). Suppose the boundary type in the amateur model, θ_1, sets out to coincide with θ_1^{cs}; she also sends m_1 for $\theta \in [0, \theta_1(= \theta_1^{cs})]$ and m_2 for $\theta \in [\theta_1(= \theta_1^{cs}), 1]$. Note first that there are two sets of amateur decision makers that θ_1 will never face, namely, t_l with $t < \theta_1$ and t_h with $t \geq \theta_1$ (the two gray triangles ABE and EHJ). Among the remaining types (represented by the area in white), some of them the expert has full influence (the two white rectangles $BCDE$ and $\theta_1 EJH$) and some of them on which her influence is attenuated (the two white triangles $AE\theta_1$ and EDH). Comparing the resulting induced actions shown in Figure 2.3b with those in Figure 2.3a, we can see that under m_1 some decision makers

[33]For illustrative purpose, I assume that the expert does not randomize and uses pure strategy.

take $\frac{\theta_1}{2} = \frac{\theta_1^{cs}}{2}$ and some take $\frac{t+\theta_1}{2} \geq \frac{\theta_1^{cs}}{2}$, $t \in [0, \theta_1]$. Accordingly, insofar as payoff-relevant actions are concerned, m_1 induce on average action higher than $\frac{\theta_1^{cs}}{2}$ in the amateur model. To maintain the indifference, a force is created pushing θ_1 above θ_1^{cs}. Applying a similar exercise to the rectangle $\theta_1 DHK$ on the right, one can see that there is another force pulling θ_1 below θ_1^{cs} since some decision makers will take $\frac{\theta_1+t}{2} \leq \frac{\theta_1+1}{2}$, $t \in (\theta_1, 1]$. The final position of θ_1 (i.e. the equilibrium θ_1) depends on the relative strength of these two forces. Should b approaches zero so that θ_1^{cs} approaches $\frac{1}{2}$, the two sides will tend to be symmetric. The two opposing forces will then balance out at $\theta_1 = \theta_1^{cs}$; θ^{cs} will indeed be the equilibrium boundary type in the amateur model.[34] However, for $b > 0$ so that $\theta_1^{cs} < \frac{1}{2}$ as in the figures, the pulling force will be stronger than the pushing force. Thus, θ_1^{cs} cannot be the equilibrium boundary type in the amateur model, and the net pulling force renders θ_1 lower than θ_1^{cs}. This also demonstrates why the distance between θ_1^{cs} and θ_1 increases with b.

An implication from this discussion is that the decision maker's information *per se* does not detriment his relationship with the expert - it only detriments his relationship with a *biased* expert; when interests are perfectly assigned, the expert will behave the same regardless of whether she faces a novice or an amateur.

Example 2.1: Three-Step Equilibria

Suppose $b = \frac{1}{15}$. The boundary types in the three-step equilibrium of the amateur model are $\{\bar{\theta}_1, \bar{\theta}_2\} = \{0.011, 0.307\}$.[35] The boundary types in the CS model are $\{\bar{\theta}_1^{cs}, \bar{\theta}_2^{cs}\} = \{0.067, 0.4\}$. Thus, for a given b and $N = N^{cs} = 3$, we have an order on the corresponding boundary types for amateur and novice: $\bar{\theta}_1 < \bar{\theta}_1^{cs}$ and $\bar{\theta}_2 < \bar{\theta}_2^{cs}$. The distances between them are $\bar{\theta}_1^{cs} - \bar{\theta}_1 = 0.056$ and $\bar{\theta}_2^{cs} - \bar{\theta}_2 = 0.093$.

Next, consider a lower level of bias $b = \frac{1}{20}$. The boundary types in the amateur model are $\{\hat{\theta}_1, \hat{\theta}_2\} = \{0.09, 0.404\}$ and those in the CS model are $\{\hat{\theta}_1^{cs}, \hat{\theta}_2^{cs}\} = \{0.133, 0.467\}$. The distances between them are: $\hat{\theta}_1^{cs} - \bar{\theta}_1 = 0.043$ and $\hat{\theta}_2^{cs} - \bar{\theta}_2 = 0.063$. Comparing with the first example, the change in the distances between the boundary types is consistent with Corollary 2.2 where they are strictly increasing in b.

[34] When b approaches zero, there are other equilibria in the CS model that have more steps, and the two-step equilibria is considered "implausible" because they are Pareto-dominated. I use them here just for an illustration.

[35] The numbers are approximations. All calculations in the examples can be found in the Appendix.

2.6 DECISION MAKER'S WELFARE

Several results so far provide fragmented pictures on how the decision maker's information relates to his welfare. Results in previous section suggest that, for a given piece of advice, different types of decision maker are impacted differently. Furthermore, the expert responds to the decision maker's knowledge in a way unfavorable to him. These are all due to the change in the strategic interaction between the parties. There is another piece of fragment, non-strategic in nature and opposing the above effect, that has not been considered: the welfare improvement due directly to the fact that the decision maker knows better as in the case when he makes decisions alone without help from the expert.

In this section, I weave all these fragments together and attempt to answer the main question of whether the evolution from novice to amateur is a welfare improving change for the decision maker. I shall restrict attention to two-step (and babbling) equilibria in the uniform-quadratic model. The decision maker's *ex-ante* expected payoff in two-step equilibria in the uniform-quadratic case of the CS model will be used as the yardstick of welfare comparisons. As such, the following analysis will be based on the cases where $b \in (\frac{1}{12}, \frac{1}{4})$ so that $N \leq 2$ in both models. As in the last section, an example pertaining to three-step equilibrium will also be given.

2.6.1 Stages of evaluating payoffs

The presence of two-sided private information entails an additional consideration regarding the stages of evaluating welfare. According to Holmström and Myerson [24], an *ex-ante* stage is a time interval in a model where no agent has received any private information, whereas in an *interim* stage *every* agent has received his private information but does not know that of the others. Applying this to the amateur model, it is obvious that the game is in an *ex-ante* stage before the realizations of t and θ, whereas after the expert knows her type and the decision maker knows his interval type, the game enters into an *interim* stage. If we are using the perspective that the interval type is a private signal of the decision maker that are determined simultaneously by t and θ, this will be the end of the story.

It will also be of interest to the welfare analysis, however, to consider a lapse between the realizations of t and θ, in particular when t is realized prior to θ. This is so because this represents an interesting situation where the decision maker has acquired a capacity to interpret information *but* *not yet make an interpretation.* A health-conscious person, who otherwise is completely healthy at the moment, may devote time and effort into studying the characteristics of several serious diseases just to save it for a rainy day. Similarly, an old-school CEO may decide to study an M.B.A. and develop general skills in evaluating investment projects so that in the future he will not be misled anymore by his staff graduated from top business schools. The welfare of them after they have acquired such abilities is to be evaluated in a time interval after t but before θ is realized.

However, according to Holmström and Myerson's [24] definition, when only t is realized, the game is still in an *ex-ante* stage because the other agent in the model - the expert - has yet to receive her private information, and thus not everyone in the model is privately informed. To distinguish this from the *ex-ante* stage before the realization of t, we need to invent some terminology: I shall call the time interval between the realizations of t and θ the post-t *ex-ante* stage.[36] Naturally, before t (and θ) is realized, it will called the pre-t *ex-ante* stage. Figure 2.4 visualizes the timeline under this new terminology.

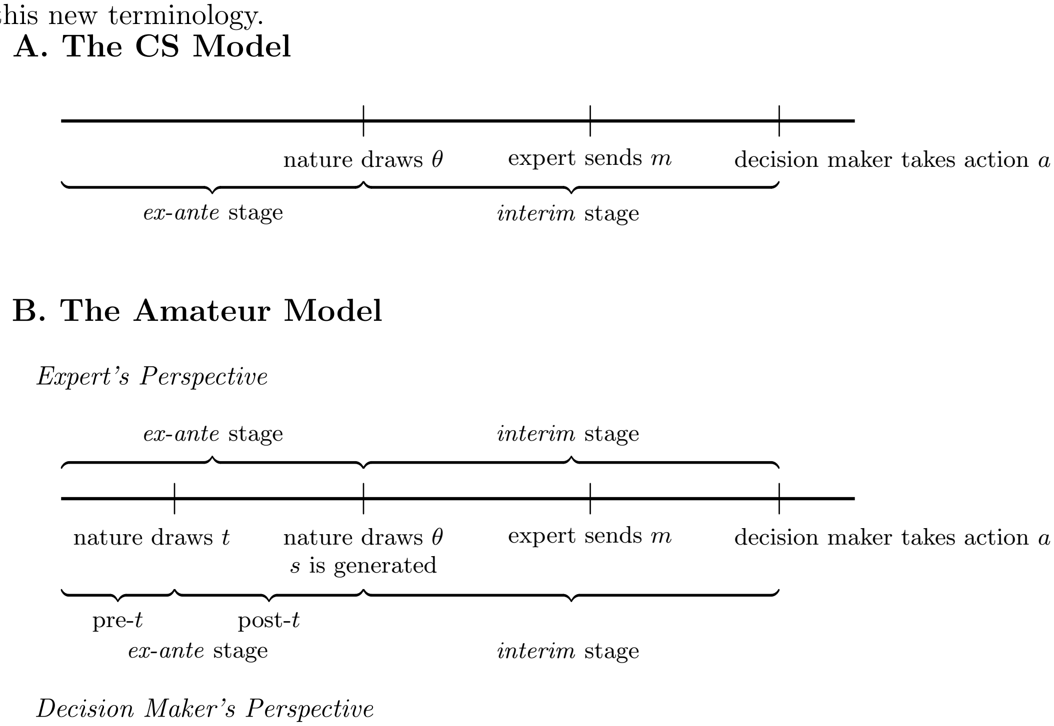


Figure 2.4: Stages of evaluating welfare

Finally, a few notations are in order before we delve into the analysis. I shall use $W^{cs}(b)$ to denote the expected payoff of the decision maker in the CS model when he interacts with an

[36]Since I always use *ex-ante* expected payoffs, pre-t or post-t, as the criterion to evaluate welfare, I omit "*ex-ante*" from this point on when I refer to the expected payoffs.

expert with bias b. Similar notations apply to the amateur model except for the use of t in the subscript indicating whether the expected payoffs are post-t or not. In particular, $W(b)$ is the pre-t expected payoff of an amateur decision maker who interacts with an expert with bias b, and $W_t(b)$ is the post-t counterpart of it. Note that $W^{cs}(b)$ will be *the* benchmark for evaluating whether an amateur decision maker is better off, regardless of whether the subject of comparison is pre-t or post-t (compare A. and B. in Figure 2.4).

2.6.2 Payoff comparisons

There are two opposing forces that contribute to the changes in the expected payoff of an amateur relative to a novice. Since decision makers with different thresholds interpret differently when information is realized or received, decision makers with different t should experience different levels of welfare changes compared to $W^{cs}(b)$, and it is conceivable that one of the forces may dominate the other for different cases. The following proposition indicates that this is indeed the case:

Proposition 2.6. *Consider two separate cases in the post-t ex-ante stage:*

1. *Suppose $b \in [\frac{1}{6}, \frac{1}{4})$ so that $\theta_1 = 0$. There exist $0 < \underline{t} < \overline{t} < 1$ such that, compared to his novice counterpart, an amateur is better off if and only if his threshold $t \in [\underline{t}, \overline{t}]$.*
2. *Suppose $b \in (\frac{1}{12}, \frac{1}{6})$ so that $\theta_1 > 0$,*
   i. *for $t \in (\theta_1, 1]$, there exist $\theta_1 < \underline{t}' < \overline{t}' < 1$ such that an amateur is better off if and only if his threshold $t \in [\underline{t}', \overline{t}']$, and*
   ii. *for $t \in [0, \theta_1]$, the amateur is always worse off.*

When $b \in [\frac{1}{6}, \frac{1}{4})$, the only equilibrium in the amateur model is babbling, while there are still two-step equilibria in the CS model. The welfare comparison in this case thus boils down to comparing the partition provided endogenously by the expert in the CS model and the partition generated exogenously by the decision maker's thresholds in the amateur model. In the uniform-quadratic setup, if an arbitrary interval in $[0, 1]$ is to be divided into two sub-intervals, the expected payoff (conditioned on θ lying within the interval) is a concave function of the dividing point, with the maximum achieved at the mid-point of the interval, which, in this case, is $\frac{1}{2}$. Since the endogenous $\theta_1^{cs} < \frac{1}{2}$ in the CS model, the decision maker in the amateur model is better off if and only if his t is closer to $\frac{1}{2}$ than θ_1^{cs} does, and this defines the interval $[\underline{t}, \overline{t}] = [\frac{1}{2} - 2b, \frac{1}{2} + 2b]$. Note also that this implies as the expert becomes more biased, this interval will be enlarged so that "more" decision makers will be better off.

The intuition behind case (2) is essentially the same, but now we are comparing partition with one element in the CS model (θ_1^{cs}) and partition with two elements in the amateur model, the exogenous t and the endogenous θ_1. By just comparing θ_1^{cs} and θ_1, the amateur is always worse off. But with the addition of t, cases vary. For $t \in (\theta_1, 1]$ in (2i), to compensate for the loss in payoff due to a lower θ_1, t must lie in a close-enough neighborhood of the mid-point, $\frac{\theta_1+1}{2}$, before an amateur can be better off. For t outside this range, however, the loss outweighs the gain, rendering these unlucky decision makers worse off even though they have a partition with one more element under their belts.[37] For $t \in [0, \theta_1]$ in (2ii), since 0 and θ_1 are close to each other, even when the decision maker's own information is most useful - when t lies squarely on the mid-point $\frac{\theta_1}{2}$ - it is not useful enough to counter the unfavorable response of the expert. As a result, all amateur decision makers with t in this range are worse off.

Despite the above, the following result offers a positive outlook when we "take an average" across these ts:

Proposition 2.7. *In the pre-t ex-ante stage, an amateur decision maker is always better off. Furthermore, the welfare improvement of an amateur relative to a novice, $W(b) - W^{cs}(b)$, first decreases and then increases as the expert becomes more biased.*

Thus, while some decision makers are better off and some are worse off, the cases of payoff improvement dominate the cases of reduction so that on average the decision maker's information pays off. As to why the welfare improvement is non-monotone in b, first note that a higher level of bias drives both θ_1 and θ_1^{cs} down so that the expected payoffs of both amateur and novice decrease. Initially, the negative impact on the amateur is relatively larger because, as b increases, $\theta_1^{cs}(b) - \theta_1(b)$ increases as well (Corollary 2.2), and thus the dominance of the amateur's payoff over that of the novice diminishes, explaining the decreasing $W(b) - W^{cs}(b)$. However, this only applies for $b < \frac{1}{6}$; when b eventually reaches $\frac{1}{6}$ so that θ_1 reaches 0, any further increases in b has no marginal impact on the amateur, and his payoff will be independent of b from that point on. For the novice, however, because $\theta_1^{cs}(\frac{1}{6}) > \theta_1(\frac{1}{6}) = 0$, when b reaches and goes beyond $\frac{1}{6}$, it still keeps driving down θ_1^{cs}, and thus the dominance of the amateur's payoff increases back as $\theta_1^{cs}(b) - \theta_1(b)$ decreases for $b \in (\frac{1}{6}, \frac{1}{4})$.[38]

[37] The exact values of the ts that define the intervals can be found in the Appendix.

[38] While our focus is for $b \in (\frac{1}{12}, \frac{1}{4})$, it is obvious from the above that when $b \geq \frac{1}{4}$, the dominance of the amateur's

Example 2.2: Payoff Comparisons in Three-Step Equilibrium

I shall demonstrate in this example the case where $b = \frac{1}{15}$. Recall from Example 2.1 that the boundary types in the three-step equilibrium in the amateur model are $\{\bar{\theta}_1, \bar{\theta}_2\} = \{0.011, 0.307\}$. Given that $b = \frac{1}{15}$, the decision maker's expected payoff in the CS model is $W^{cs}(\frac{1}{15}) = -0.021111$. Thus, an amateur decision maker is better off if and only if his expected payoff exceeds or equal to -0.021111.

It turns out, in the post-t *ex-ante* stage, all amateur decision makers with $t \in [0, \bar{\theta}_2]$ are worse off compared to their novice counterparts. For $t \in [\bar{\theta}_2, 1]$, some of them are better off and some of them worse off. In particular, only those decision makers with $t \in [0.390136, 0.916973]$ are better off. On the other hand, the decision maker is always better off in the pre-t *ex-ante* stage; the amateur's pre-t expected payoff is $-0.019959 > -0.021111$. The example demonstrates that the results for two-step equilibria carry over to three-step equilibrium.

2.7 RELATED LITERATURE

Among the papers which show that the decision maker's information can contribute to better communication, Seidmann [43] considers a model in which all expert's types share a common preference ordering over the decision maker's actions.[39] He shows with examples that, when the decision maker has private information, communications can be informative even though all expert's types prefer a particular action. Given the decision maker's information, messages can induce distributions of actions not ordered by stochastic dominance. This upsets the expert types' common preference ordering over individual actions and renders sorting possible in equilibrium.

Watson [48] considers a model where the expert only cares about the decision maker's actions. He shows that the correlation between the expert's types and the decision maker's types creates an incentive for full revelation of information - after observing her own type, the expert figures out from the correlation that the decision maker's information, when combined with a true report from her, will induce him to take an action that is favorable to her.

Olszewski [37] considers a model in which the expert wants to be perceived as honest in front

payoff will be independent of b because then we are simply comparing a novice and an amateur both making decision on their own.

[39] In the literature, the expert is commonly referred as the "sender" and the decision maker the "receiver." I shall adhere to the terminology of "expert" and "decision maker" in this literature review.

of an informed decision maker. Olszewski provides conditions on the information structure so that full revelation is the unique equilibrium, provided the expert's concern to be perceived as honest is strong enough. The decision maker's information therefore serves as a solution for multiplicity of equilibria commonly encountered in cheap-talk models.

A message in my model also induces a distribution of actions across the decision maker's types. But I show that in the original cheap-talk model of Crawford and Sobel [12], in which different types of expert exhibit different preferences over the decision maker's actions (i.e., the sorting condition is in place), the decision maker's information worsen communications, let alone achieving full revelation.

The paper that is closest to this chapter is Chen [11]. We both consider an informed decision maker in an otherwise standard CS model. Chen focuses on the possibility for the decision maker to communicate with the expert in the first round about what he knows. Chen shows that the decision maker cannot credibly communicate his information to the expert. She also shows that when, as in my model, the decision maker's information remains private, non-monotonic equilibria can arise. The welfare of an informed decision maker is, however, not explicitly considered in her paper.

The structures of the signals represent another difference. In Chen's model, the decision maker's information pertains to a binary signal about the state of the world that satisfies the monotone likelihood ratio. On the other hand, the decision maker in my model can receive a continuum of signals. Most importantly, the decision maker's information in my model can confer on him an ability to detect whether a message is a deception or not. Such an ability to verify messages calls for the consideration of out-of-equilibrium (but used) messages which, to the best of my knowledge, do not appear in other cheap-talk models.

For that reason, this chapter is also related to the literature on verifiable messages or persuasion game (e.g., Milgrom and Roberts [34]). In this literature, messages being verifiable is an exogenous assumption imposed, whereas in my model verification of messages arise endogenously although it does not in equilibrium. In a sense, the amateur model can be viewed as a middle ground between these models and the pure cheap-talk models.

This chapter is also related to cheap-talk models with two senders. The receiver in these models has two sources of information, both of them strategic in nature, and it is found that the addition of one more sender can sometimes improve communications. For example, Krishna and Morgan [28] shows that when the interests of the two senders oppose, it will be beneficial to have both of them in place. There are also two sources of information in the amateur model, but one of them is replaced with non-strategic information access by the receiver himself. The results here therefore indicate that an additional source of information is not always useful. When it is not obtained from strategic sources, it can indeed worsen rather than improve communications.

In terms of applications in the doctor-patient relationship, the use of cheap-talk models centers around an important topic in health economics arisen under "physician agency" - the supplier-induced demand in which "the physician influences a patient's demand for care against the physician's interpretation of the best interest of the patient" (McGuire [33]). Dranove [14] was among the first to characterize the original neoclassical view of supplier-induced demand in terms of strategic interaction between doctors and patients under information asymmetry. Calcott [9] and De Jaegher and Jegers [13] explicitly use cheap-talk models to characterize the supplier-induced demand and study its properties.

Kőszegi [26][27] studies another aspect of doctor-patient relationship - the "emotional agency." In his cheap-talk model (he also considers another setting in which the "talk" can be verified), the doctor's reported diagnosis not only affects the patient's decision-making but also has emotional implications on him. To isolate the emotional concern from conflict of interests, Kőszegi shows that when a completely altruistic doctor takes into account the emotional effect of her reported diagnosis, the doctor tends to provide advice that is skewed toward "good news." My interpretations of the model in terms of doctor-patient relationship is largely the same as those in his papers.

2.8 CONCLUDING REMARKS

This chapter answers the question of how the information of a decision maker, who interacts with a biased expert, affects the decision outcomes. The question is asked in an apt time: recent developments in the Internet (e.g. information sharing as part of the Web 2.0) have created a group of amateurs who have access to specialized knowledge once available only to experts. Simply put, the analysis indicates that this development does not benefit every decision maker involved. When an amateur makes decision not just on his own but by consulting an expert, being informed could backfire: the expert responds strategically to the information asymmetry as to what her client knows and, as a result, provides less informative advice.

For some decision makers whose information is not that useful, this negative, strategic effect could outweigh the benefit of being informed, rendering them worse off by becoming an amateur. And the opposite side is that, in the case where an amateur enjoys a welfare improvement, it is the direct benefit of information that is playing the role. However, even taking into consideration the strategic behavior of experts, as *providing an channel for patients or investors to obtain information*, the Internet always contributes to their welfare: it is shown in this chapter that an amateur decision maker is *always* better off *only* in an *ex-ante* sense before his information is realized. The subtlety is that *after* a decision maker utilizes the "channel for information" and obtains certain knowledge from it, the fact that he has consulted the Internet before meeting with an expert does not necessarily guarantee that his final decision outcome will be improved.

Despite this qualification on the effects of decision makers' information, there is, however, no paradox in why the advocates for consumer education always argue that education improves welfare: as outside observers, they by definition can only evaluate its effects *ex-ante*. Amateurs' non-monotonic welfare improvement with respect to the expert's bias further suggests that educating the decision makers is more beneficial when the expert's bias is more extreme. This result is indeed quite intuitive: a more biased investment advisor may not be so helpful anyway, so the investor benefits by relying on his own for information; an altruistic doctor may not respond too negatively to an informed patient, and the patient can benefit from his own use of information without worrying too much about the negative, strategic effect.

The additional layer of information asymmetry worsens the communication between the expert and the decision maker, and a "redistribution effect" on welfare is created in which some decision makers are worse off due to the presence of others. As is the case for other instances of information asymmetry, some agents may have incentives to distinguish themselves from others. In the present context, some decision makers may even choose to be uninformed if they are given a choice and can credibly signal to the expert about it. A natural extension will be to provide an opportunity for the decision maker to talk to the expert before the expert offers advice, allowing the decision maker to reveal how much or whether he knows.[40]

Stigler was perhaps the first to formally consider information in economic models. In his classic paper on the economics of information (Stigler [46]), information (search) costs are explicitly considered. Later on until recently, costs of information were largely ignored in the profession.[41]

[40] Chen [11] considers this possibility. She shows that under the signal structure in her model the decision maker cannot credibly reveal his information to the expert.

[41] For recent work on costly information acquisitions, see, for example, Bergemann and Välimäki [8] for efficient mechanism design and Persico [41] for committee design.

To study the economic consequences of diverse distributions of information among different parties, a "shortcut" was made (Maskin and Tirole [32], p.107): one party is often assumed to have some information that is not available to the others - the asymmetry of information is exogenously imposed. When costs of information are present, as is the case in reality, such a dichotomy may no longer be appropriate; the information possed by different individuals should then be modeled in relative terms in reference to their different costs of acquiring information. While this paper does not explicitly consider information costs, it examines an environment in which information is distributed among the parties not in an all-or-nothing manner. I believe that the framework studied here will be useful for further studying the endogenization of information status of the players in models of strategic information transmission.

3.0 CHALLENGED AUTHORITY

3.1 INTRODUCTION

Knowledge and authority often go hand in hand. When a person is said to be an authority in certain subject, we take it to mean that he or she has a superior knowledge in the area - and vice versa. Not only is access to specialized knowledge a privilege of experts, but the professionals also carry the authority that comes with it. With the wide availability of information nowadays, however, experts' access to specialized knowledge is no longer exclusive. Consequently, their authority may be shaken or challenged. Not every authority welcomes challenges. In the doctor-patient relationship, for instance, it is documented that some doctors are annoyed by the "Googler-patients" (e.g., Ahmad et al. [1] and Murray et al. [36]).[1] Some of them may even become less helpful to their savvy clients.[2]

How an expert's aversion to dealing with informed clients affect the communication between the parties? This chapter attempts to analyze formally this aspect of communication with experts, extending on the strategic information transmission model of Crawford and Sobel's [12] (the "CS model").

"Knowledge" in the model refers to knowledge about the *state of the world*, represented by a random variable in the unit interval $[0,1]$. There are two players: an expert (she) and a decision maker (he). The expert is the authority: she has perfect knowledge about the state of the world in that she observes perfectly the realized random variable. The decision maker, who is to take an action that affects the payoffs of both parties, is of one of two types: uninformed or informed. The uninformed type knows nothing about the state except that it is uniformly distributed on $[0,1]$. The unit interval $[0,1]$ is divided for the informed type into two half intervals, $[0,\frac{1}{2})$ and $[\frac{1}{2},1]$; the

[1] See also Haig [22] for a first-hand report from a doctor who is annoyed by his Googler-patient.

[2] For example, the following response is noted from a doctor (Ahmad et al. [1]): "They're coming back [with Internet information]. It requires a little looking into. If you are tired, of course, you'll probably just fire them...if they're really belligerent." See also fn. 12 of Chapter 2.

informed decision maker has partial knowledge about the state in that he is able to tell whether it lies in $[0, \frac{1}{2})$ or $[\frac{1}{2}, 1]$. The decision maker's type is unknown to the expert, and her prior beliefs are that the two types are equally likely.

The expert and the decision maker have to coordinate on what action to take in each possible state of the world. The coordination is achieved through two rounds of communication. In the first round, the decision maker reports to the expert about his information type, i.e., whether he is informed or not. The decision maker's report is costless: the report itself does not affect his payoff nor that of the expert. It is typically not an easy task for an ignorant person to pretend, in front of experts, to be sophisticated. To capture this, the decision maker's report is assumed to be *partially verifiable* in which only the informed decision maker can report that he is informed. In the second round of communication, the expert advices the decision maker about the state of the world. She sends a costless, cheap-talk message to the decision maker, after which the decision maker takes an action.

The extent of coordination is limited by the misaligned interests between the parties. Relative to the decision maker's, the expert's preferences are biased toward higher actions. Crawford and Sobel [12] show that the larger is this bias, the "less helpful" the expert is to the decision maker in the sense that the expert provides less informative advice. I leverage on this interpretation to model how the expert becomes annoyed when she believes that her authority is challenged - when she is not the only one who has knowledge about the state - and responds to it by becoming less helpful. The expert's bias consists of two components: her *normal bias* and her *emotional bias*. The former can be considered as the same as that in the CS model. The latter is a new component in this model, and its magnitude depends on the expert's beliefs about whether the decision maker is informed: the more likely that she believes the decision maker is informed, the larger is her emotional bias. Since then the expert's payoffs depend on beliefs, the present model - which I call the *authority model* - is an instance of psychological games (e.g., Geanakoplos et al. [20] and Battigalli and Dufwenberg [6]).

I first analyze the continuation games after the expert receives the decision maker's report. The continuation games begin with the expert's posterior beliefs about whether the decision maker is informed, and there are three cases that are of particular interests: when the expert's beliefs about the presence of informed decision maker are 0, $\frac{1}{2}$ and 1. When, in the first case, the expert believes that the decision maker is uninformed, the continuation game reduces to the CS model. Suppressing, for comparison purposes, the expert's emotion so that her bias only consists of the normal bias, I find, using the first case of beliefs or the CS model as the benchmark, that more closely aligned interests between the parties are required for effective communication when the expert believes that the probability of facing an informed decision maker is $\frac{1}{2}$ or 1. Furthermore, among these two cases, a lower level of bias is required for the existence of informative equilibria when the expert is certain that she faces an informed decision maker than when her beliefs coincide with the prior of $\frac{1}{2}$.

The expert, with preferences biased toward higher actions, has to be allowed in equilibrium to exaggerate in her advice in order to be willing to provide information. When the expert believes that she is facing an informed decision maker, she knows that there is less room for her to exaggerate that will not be ignored, because the decision maker will compare and contrast what he knows with what he is told. Accordingly, for a range of bias in which the expert provides information to an uninformed decision maker, she will not provide any information when she is certain that the decision maker is informed. When the expert believes that an informed and an uninformed decision makers are equally likely, compared to facing entirely an informed decision maker she has, in expectation, more room for exaggeration - thus the less stringent requirement on the bias in this case.

Restricting, for simplicity, attention to the levels of (normal) bias so that the most informative equilibria in the first case of continuation games (the CS model) are of two-step, I find that there exists a range of normal bias in which the informed decision maker reveals himself to an emotion-free expert. However, such range is a strict subset of the range in which an emotion-free expert provides useful information to him. Thus, even though an emotion-free expert is willing to provide informative advice, the informed decision maker may not want to reveal himself. For normal bias in this range, the information provided by the emotion-free expert to the informed decision maker is in the form of partitioning *each* of $[0, \frac{1}{2})$ and $[\frac{1}{2}, 1]$ into two steps. If, on the other hand, the expert believes she faces an uninformed decision maker, she partitions the whole $[0, 1]$ into two steps. Despite the four steps in total in the former, they are so uneven that an informed decision maker prefers the latter.

The emotion of an expert can deter the informed decision maker from revealing himself who otherwise would have done so to an emotion-free expert. This happens when the expert's emotional bias is sensitive enough to her beliefs that she faces an informed decision maker. Naturally, in order for the informed decision maker to be willing to reveal himself, a higher normal bias should be accompanied by a less sensitive emotion.

The exposition of the rest of the chapter is as follows. Section 3.2 lays down the authority model. Section 3.3 provides the equilibrium analysis. Section 3.4 contains the literature review. Section 3.5 concludes.

3.2 THE MODEL

3.2.1 Players and information

There are two players: an expert (e) and a decision maker (d). The expert privately observes the state of the world, $\theta \in \Theta \equiv [0, 1]$, commonly known to be distributed uniformly on the support.[3] The decision maker does not observe θ, but, depending on his *information type*, k, he may have some knowledge about it. The decision maker's information type is a binary variable: k takes on value of 0 if the decision maker is uninformed and value of 1 if he is informed. The expert's prior beliefs, which are common knowledge, are that an informed type and an uninformed type are equally likely.

The decision maker's information type stipulates what he could possibly know about θ. In the event that he is uninformed, the decision maker knows nothing but that θ is uniformly distributed on $[0, 1]$. If, on the other hand, he is informed, the decision maker will be able to tell whether the realized θ lies in (or, more precisely, is distributed uniformly on) $[0, \frac{1}{2})$ or $[\frac{1}{2}, 1]$. This possible knowledge of the decision maker is represented by his *interval types*, $s \in \mathcal{T} \equiv \{o, l, h\}$, where $o = [0, 1]$, $l = [0, \frac{1}{2})$ and $h = [\frac{1}{2}, 1]$. It will be convenient to define a mapping between information types and interval types, $g : \{0, 1\} \rightarrow \mathcal{T}$, as follows:

$$g(k) = \begin{cases} \{o\}, & \text{if } k = 0, \\ \{l, h\}, & \text{if } k = 1. \end{cases}$$

I assume that the realization of k precedes that of θ, upon which, for $k = 1$, s is determined by the position of θ in relation to $\frac{1}{2}$.[4]

[3] As in Chapter 2, I shall interchangeably refer θ as the "state" or the "type of the expert."

[4] The uninformed type ($k = 0$) uniquely determines the interval type, o, without regard to the realization of θ.

3.2.2 Communication and beliefs

The players interact through communication with costless messages. The communication begins with the decision maker's reporting his information type, which is followed by the realization of the state. Since the realization of θ further determines the decision maker's interval type, the above timing implies that the decision maker reports before he observes his interval type. After receiving the decision maker's report and observing θ, the expert gives an advice to the decision maker by sending a message.

Since the decision maker's reports and the expert's messages are costless, the only effect of them is to influence the opponent's beliefs. The rest of the session analyzes the beliefs that they generate for each player. I begin with the decision maker's reports, which, while being costless, are not cheap-talk messages.

3.2.2.1 The decision maker's reports The decision maker sends a report $r \in \{r_0, r_1\}$ with literal meaning about his information type, where r_0 stands for "uninformed" and r_1 stands for "informed." I impose a partial verifiability constraint on the decision maker's reports: the set of report(s) available to information type k, $r(k)$, is

$$r(k) = \begin{cases} \{r_0\}, & \text{if } k = 0, \\ \{r_0, r_1\}, & \text{if } k = 1. \end{cases} \tag{3.1}$$

The partial verifiability constraint stipulates that only an informed decision maker can report that he is informed. And he has the option not to reveal it. This captures that it is typically not an easy task for an uninformed individual to claim, in front of experts, that he is informed.[5] On the other hand, the right to remain silence makes it relatively easy for an informed individual to make the claim that he is ignorant. Of course, such a claim lacks credibility, but that is an equilibrium issue.[6]

A reporting rule of the decision maker, $\alpha : \{0, 1\} \to \Delta\{r_0, r_1\}$ such that $\text{supp}(\alpha(\cdot|k)) \in r(k)$, $k = 0, 1$, specifies a probability of each report that he sends for each of his information type, subject to the partial verifiability constraint (3.1). The decision maker's report may influence the expert's beliefs about his information type. But since the expert observes θ - and thus whether it is above

[5]Even though the informed decision maker in the model reports before observing his interval type, we can imagine that he can credibly certify his access to information.

[6]This partial verifiability constraint is the same as that adopted for the sender (expert) in Austen-Smith [4]. See the literature review in Section 3.4.

or below $\frac{1}{2}$ - she will be able to form her beliefs directly on the interval types of the decision maker. Before communicating with the decision maker, the expert's beliefs about the interval types can be summarized as follows:

$$\gamma(o|\theta) = \frac{1}{2}, \tag{3.2}$$

$$\gamma(l|\theta) = \begin{cases} 0, & \text{if } \theta \geq \frac{1}{2} \\ \frac{1}{2}, & \text{if } \theta < \frac{1}{2}, \end{cases} \quad \text{and} \tag{3.3}$$

$$\gamma(h|\theta) = \begin{cases} \frac{1}{2}, & \text{if } \theta \geq \frac{1}{2}, \\ 0, & \text{if } \theta < \frac{1}{2}. \end{cases} \tag{3.4}$$

Upon receiving r, the expert updates on these beliefs. Her beliefs function, $\tau : \Theta \times \{r_0, r_1\} \to \Delta\mathcal{T}$, specifies for each combination of the state and the received report a probability over the interval types $\{o, l, h\}$. When the expert receives a report that is sent by some k, her beliefs function is defined as follows:

$$\tau(s|\theta, r) = \frac{\gamma(s|\theta)\alpha(r|g^{-1}(s))}{\sum_{s' \in \mathcal{T}} \gamma(s'|\theta)\alpha(r|g^{-1}(s'))}. \tag{3.5}$$

Suppose the informed type sends r_1 with probability one. Then, $\alpha(r_1|g^{-1}(h)) = \alpha(r_1|g^{-1}(l)) = \alpha(r_1|1) = 1$, and $\alpha(r_1|g^{-1}(o)) = \alpha(r_1|0) = 0$. Depending on whether θ is above or below $\frac{1}{2}$, the expert then assigns probability one to either l or h, and this is accounted for by $\gamma(s|\theta)$ in (3.3) and (3.4).[7]

3.2.2.2 The expert's advice After receiving the decision maker's report, the expert provides an advice to the decision maker by sending a cheap-talk message $m \in M$. Her behavior strategy, $\sigma : \Theta \times \{r_0, r_1\} \to \Delta M$, specifies the distribution of message she sends for each combination of the state and the received report. I denote $\Theta_\sigma(m)$ to be the set of θ for which the expert sends message m with positive probability under σ, i.e., $\Theta_\sigma(m) = \{\theta \in \Theta : \sigma(m|\theta, r) > 0\}$. When there are some messages in M that are not used under σ, I adopt the convention that $\Theta_\sigma(\cdot)$ is an empty set for those messages.

[7]Note that this conclusion is drawn by applying Bayes's rule for r_1 that is sent in an equilibrium. However, given the partial verifiability constraint, when r_1 is received as an out-of-equilibrium report, any reasonable off-equilibrium beliefs require the same conclusion. I shall adopt such off-equilibrium beliefs for r_1 for the expert.

Upon receiving a message from the expert, the decision maker updates his beliefs about θ. Contingent on his interval type, he uses one of the following beliefs (conditional densities) as the input for his update:

$$\phi(\theta|o) = 1, \quad \text{for } \theta \in [0,1], \tag{3.6}$$

$$\phi(\theta|l) = \begin{cases} 2, & \text{for } \theta \in [0, \frac{1}{2}), \\ 0, & \text{for } \theta \in [\frac{1}{2}, 1], \end{cases} \quad \text{and} \tag{3.7}$$

$$\phi(\theta|h) = \begin{cases} 0, & \text{for } \theta \in [0, \frac{1}{2}) \\ 2, & \text{for } \theta \in [\frac{1}{2}, 1]. \end{cases} \tag{3.8}$$

The decision maker's beliefs function, $\mu : M \times \mathcal{T} \times \{r_0, r_1\} \to \Delta\Theta$, specifies for each combination of received message, his interval type and his report sent in the first round of communication a density over Θ, and it is defined as follows:

$$\mu(\theta|m, s, r) = \begin{cases} \frac{\sigma(m|\theta, r)\phi(\theta|s)}{\int_0^1 \sigma(m|\theta', r)\phi(\theta'|s)d\theta'}, & \text{if } \Theta_\sigma(m) \cap s \neq \emptyset, \\ \psi(\theta|s), & \text{if } \Theta_\sigma(m) \cap s = \emptyset,\ \Theta_\sigma(m) \neq \emptyset, \end{cases} \tag{3.9}$$

where $\psi(\theta|s)$ is any arbitrary density supported on s. The beliefs function embodies the beliefs that an interval type can hold after receiving four types of advice, namely, *substituting advice*, *complementary advice*, *redundant advice* and *false advice*.[8] When the decision maker receives message that does not contradict with his knowledge about θ, i.e., the message is not a false advice (or $\Theta_\sigma(m) \cap s \neq \emptyset$), he updates his beliefs using Bayes's rule. Otherwise when a false advice is received, there is no restriction on what the decision maker's beliefs should be, except that it has to be supported only on s.

For notational clarity, I use $\psi(s)$ to denote the off-equilibrium beliefs, arising only under false advice, of interval type s. And I denote the off-equilibrium beliefs of both interval types by $\psi \equiv \psi(l) \cup \psi(h)$.

3.2.3 Actions and payoffs

After receiving the expert's advice, the decision maker ends the game by taking an action $a \in \mathbb{R}$. His action rule, $\rho : M \times \mathcal{T} \times \{r_0, r_1\} \to \mathbb{R}$, specifies for each combination of received message, his

[8] See Chapter 2 for a detailed discussion about these advice types.

interval type and his sent report an action he takes.[9] And the decision maker's reporting rule and action rule constitute his strategy.

The action (and the state) directly affects the players' payoffs, which are in the form of quadratic loss function:

$$U^e(a, \theta, b(\hat{\tau})) \equiv -(a - (\theta + b(\hat{\tau}))^2, \tag{3.10}$$

$$U^d(a, \theta) \equiv -(a - \theta)^2. \tag{3.11}$$

These payoff functions, commonly adopted in the literature of cheap talk, capture the misaligned interests between the parties: if the decision maker could observe θ, his ideal action, which gives him the highest payoff of zero, would be $a^d(\theta) = \theta$; for the same θ, however, the expert's ideal action will be $a^e(\theta, b(\hat{\tau})) = \theta + b(\hat{\tau})$.

The term $b(\hat{\tau})$ is referred to as the bias of the expert. Crawford and Sobel [12] show that the larger is the expert's bias, the "less helpful" the expert is to the decision maker in the sense that the expert provides less informative advice. I leverage on this interpretation to model how an expert becomes less helpful to an informed decision maker. The expert's bias $b(\hat{\tau})$ has two components and is assumed to take the following linear form: $b(\hat{\tau}) = b_0 + b_1\hat{\tau}$. The first component, $b_0 > 0$, is referred to as the expert's *normal bias*; it can be considered as the same as those in standard models.

The second component, $b_1\hat{\tau}$, which is beliefs-dependent, represents the expert's *emotional bias*, where $\hat{\tau}$ is her beliefs that the decision maker is informed.[10] The coefficient $0 \leq b_1 \leq 1$ - the expert's *emotional index* - quantifies the expert's susceptibility to being annoyed by informed decision makers and becoming less helpful to them. Thus, the more likely that the expert believes the decision maker is informed, the larger is her emotional bias. When $b_1 = 0$, the expert in question is *emotion-free*.

3.2.4 Equilibrium definition

The informed type ($k = 1$) can always send r_0 that he is uninformed, and he is willing to reveal himself truthfully only if it brings him a higher expected payoff. No such consideration is required for the uninformed type ($k = 0$) because, under the partial verifiability constraint, the uninformed

[9] That $U^d_{11}(\cdot) < 0$ guarantees that the decision maker will not randomize in his action.

[10] Thus, before any communication, $\hat{\tau} = \frac{1}{2}$, and after receiving the decision maker's report $\hat{\tau} = \tau(l|\theta, r) + \tau(h|\theta, r)$.

type is restrained from misreporting his type. Accordingly, the equilibrium definition needs only to cater for the reporting rule of the informed type. The expected payoff of an informed type who reports r is

$$\begin{aligned} V^d(r,1|\sigma) \equiv \int_0^t \left(\int_M U^d(\rho(m,l,r),\theta)\sigma(m|\theta,r)dm \right) d\theta \\ + \int_t^1 \left(\int_M U^d(\rho(m,h,r),\theta)\sigma(m|\theta,r)dm \right) d\theta. \end{aligned} \tag{3.12}$$

I adopt perfect Bayesian equilibrium as the equilibrium definition:

Definition 3.1 (Perfect Bayesian Equilibrium). *A perfect Bayesian equilibrium is a pair of strategies $(\sigma,(\alpha,\rho))$ and two sets of beliefs (τ,μ) such that*

1. *the informed decision maker chooses his reporting rule to maximize $V^d(r,1|\sigma)$ given the expert's strategy: if $r \in \operatorname{supp}(\alpha(\cdot|1))$, then*

$$r \in \operatorname*{argmax}_{r' \in \{r_0,r_1\}} V^d(r,1|\sigma),$$

2. *the expert maximizes her expected payoff given her beliefs and the decision maker's action rule: for all $\theta \in \Theta$ and all $r \in \{r_0,r_1\}$, if $m \in \operatorname{supp}(\sigma(\cdot|\theta,r))$, then*

$$m \in \operatorname*{argmax}_{m' \in M} \sum_{s \in \mathcal{T}} \tau(s|\theta,r) U^e(\rho(m',s,r),\theta,b(\hat{\tau})),$$

3. *the expert updates her beliefs using Bayes's rule whenever possible, taking into account the decision maker's reporting rule and her type: for all $r \in \{r_0,r_1\}$ and all $\theta \in \Theta$, $\tau(s|\theta,r)$ is defined as in (3.5) whenever it is so defined,*

4. *the decision maker maximizes his expected payoff given his beliefs: for all $m \in M$, all $s \in \mathcal{T}$ and each $r \in \{r_0,r_1\}$ sent,*

$$\rho(m,s,r) = \operatorname*{argmax}_{a'} \int_0^1 U^d(a',\theta)\mu(\theta|m,s,r)d\theta, \quad \textit{and}$$

5. *the decision maker updates his beliefs using Bayes's rule whenever possible, taking into account the expert's strategy and his interval type: for all $m \in M$, all $s \in \mathcal{T}$ and each $r \in \{r_0,r_1\}$ sent, $\mu(\theta|m,s,r)$ is defined as in (3.9).*

3.3 EQUILIBRIUM ANALYSIS

3.3.1 The continuation games

In the spirit of backward induction, I first analyze the continuation games after the expert receives the decision maker's report. The continuation games start with the expert's posterior beliefs that the decision maker is informed with probability $\hat{\tau}$. I provide the restrictions on the expert's bias for the existence of informative equilibria in the continuation games. For expositional purpose, the continuation games are categorized into two classes, when $\hat{\tau} < 1$ and when $\hat{\tau} = 1$. The former will be relevant for analyzing the entire game when the expert receives r_0, whereas the latter will be relevant when the expert receives r_1. In the followings, without further qualifications, *that an equilibrium exists means that there exist some (but not all) off-equilibrium beliefs that support the equilibrium.*

3.3.1.1 Expert's posterior beliefs: $\hat{\tau} < 1$ The following proposition stipulates the restrictions on the expert's bias so that the least informative equilibrium - two-step equilibrium in which $[0, 1]$ is endogenously partitioned into two intervals - exists.

Proposition 3.1. *Suppose $\hat{\tau} < 1$. An informative equilibrium exists in the continuation game if and only if $b(\hat{\tau}) < \frac{4-3\hat{\tau}}{16-8\hat{\tau}}$.*

When $\hat{\tau} = 0$, the continuation game reduces to the CS model. The proposition then reproduces the result that there is informative equilibrium if and only if the (normal) bias is strictly less than $\frac{1}{4}$. The continuation games in which $\hat{\tau} > 0$ differ fundamentally from the CS model in that off-equilibrium beliefs (of the decision maker) can have an impact on equilibria. Even when all available messages are used in equilibrium, which is assumed in this chapter and implicit in Definition 3.1, there still exist cases where off-equilibrium beliefs arise, and whether a particular strategy profile constitutes an equilibrium could depend on these beliefs.[11]

Consider a benchmark among the cases where $\hat{\tau} > 0$. Suppose that the opportunity for the decision maker to reveal his type is absent. Then, the expert's beliefs coincide with the prior that the informed and uninformed types are equally likely. With the right specification of off-equilibrium beliefs, there is an informative equilibrium in the continuation game if and only if $b(\frac{1}{2}) < \frac{5}{24}$. Proposition 3.1 can be applied to $\hat{\tau} = 1$ to find the corresponding restriction on bias.

[11] See Chapter 2 for the details of a related analysis.

However, there are more to consider when the expert is certain to face an informed decision maker, and a separate treatment is called for.

3.3.1.2 Expert's posterior beliefs: $\hat{\tau} = 1$ When the expert believes that she faces an informed decision maker with probability one, she knows that the influence of her advice can never cross the threshold $\frac{1}{2}$. If, for example, the expert sends a message revealing that $\theta \in [\frac{1}{3}, \frac{2}{3}]$, the beliefs that this message can generate on the informed decision maker will either be that $\theta \in [\frac{1}{3}, \frac{1}{2})$ (when the decision maker is of interval-type l) or $\theta \in [\frac{1}{2}, \frac{2}{3})$ (when the decision maker is of interval-type h). Accordingly, when $\hat{\tau} = 1$, the continuation game can be considered as a CS model with two truncated state spaces, $[0, \frac{1}{2})$ and $[\frac{1}{2}, 1]$, and the expert's strategy consists of partitioning each of the half intervals.

Since this is different from the traditional way in which the expert partitions the state space, a detailed treatment is in order. Suppose $[0, \frac{1}{2}]$ is partitioned by $\{\bar{\theta}_i\}_{i=0}^{N_0}$, where $0 = \bar{\theta}_0 < \bar{\theta}_1 \cdots < \bar{\theta}_{N_0-1} < \bar{\theta}_{N_0} = \frac{1}{2}$, and $[\frac{1}{2}, 1]$ is partitioned by $\{\hat{\theta}_i\}_{i=0}^{N_1}$, where $\frac{1}{2} = \hat{\theta}_0 < \hat{\theta}_1 \cdots < \hat{\theta}_{N_1-1} < \hat{\theta}_{N_1} = 1$. The partition $\{\bar{\theta}_i\}_{i=0}^{N_0}$ divides $[0, \frac{1}{2}]$ into N_0 intervals, $\{\bar{I}_i\}_{i=1}^{N_0}$, where $\bar{I}_i = [\bar{\theta}_{i-1}, \bar{\theta}_i]$, and $\{\hat{\theta}_i\}_{i=0}^{N_1}$ divides $[\frac{1}{2}, 1]$ into N_1 intervals, $\{\hat{I}_i\}_{i=1}^{N_1}$, where $\hat{I}_i = [\hat{\theta}_{i-1}, \hat{\theta}_i]$. Suppose the expert partitions the message space M into two distinct and exhaustive sets, $\bar{M}$ and $\hat{M}$, which are further partitioned, respectively, into N_0 distinct and exhaustive sets, $\bar{M}_i$, $i = 1, \ldots, N_0$, and into N_1 distinct and exhaustive sets, $\hat{M}_i$, $i = 1, \ldots, N_1$. Upon observing the realization of the state, if $\theta \in \bar{I}_i$ ($\theta \in \hat{I}_i$), the expert randomizes uniformly over messages in $\bar{M}_i$ ($\hat{M}_i$). Thus, under this strategy, the expert has a separate contingent plan for each of the half intervals. I call such strategy the *standard strategy* of the expert.

If $N_0 \geq 2$, the equilibrium is said to be informative for l. Similarly, if $N_1 \geq 2$, it is informative for h. The following proposition stipulates the restriction on the expert's bias for which we have an informative equilibrium when $\hat{\tau} = 1$.

Proposition 3.2. *Suppose $\hat{\tau} = 1$. There exist in the continuation game equilibria informative for both or either one of l and h if and only if $b(1) < \frac{1}{8}$.*

To gain some perspective on the different restrictions on bias in the continuation games, consider an emotion-free expert. When the expert is devoid of emotion, her bias only has one component: the normal bias b_0. Comparing then the restrictions on bias - bias that is independent of the expert's beliefs - for informative equilibria when $\hat{\tau} = 0$, $\hat{\tau} = \frac{1}{2}$ and $\hat{\tau} = 1$, we arrive at the following observations:

1. when $b_0 \in [\frac{1}{8}, \frac{5}{24})$, an expert who is certain that she faces an informed decision maker babbles, whereas an expert believing that she faces an informed decision maker with probability $\frac{1}{2}$ or 0 provides informative advice; and
2. when $b_0 \in [\frac{5}{24}, \frac{1}{4})$, even the expert who believes that she faces an informed decision maker with probability $\frac{1}{2}$ babbles.

The threshold in the CS model remains that, when $b_0 \geq \frac{1}{4}$, no informative advice is provided for any beliefs of the expert. The expert, with preferences biased toward higher actions, has to be allowed in equilibrium to exaggerate in order to be willing to provide information. When the expert believes that she is facing an informed decision maker, she understands, as mentioned above, that her influence does not cross $\frac{1}{2}$. This means that there is less room for her to exaggerate that will not be ignored by the decision maker. Accordingly, for a range of bias ($b_0 \in [\frac{1}{8}, \frac{1}{4})$) that the expert is willing to provide information to an uninformed decision maker, not as much room for exaggeration when the decision maker is informed, she babbles. When the expert believes that an informed and an uninformed decision makers are equally likely, compared to facing entirely an informed decision maker she has, in expectation, more room for exaggeration; thus, for $b_0 \geq \frac{1}{8}$, so long as it is less than $\frac{5}{24}$, an informative equilibrium can be sustained.

Under the standard strategy, the expert has her contingent plan separately for each of the truncated state spaces. The two truncated state spaces, however, are not completely independent of each other: the possibility of sending false advice - when the information provided by the expert contradicts with what the decision maker knows - and the off-equilibrium beliefs that it generates link them together. Suppose, for some realized $\theta \in [0, \frac{1}{2})$, the expert deviates from a standard strategy by sending a message that is reserved for $\theta \in [\frac{1}{2}, 1]$, i.e., some $m \in \hat{M}$. Since the decision maker knows that $\theta \in [0, \frac{1}{2})$, the message $m \in \hat{M}$ contradicts his knowledge - it is a false advice to him. Whether a particular standard strategy constitutes an equilibrium depends in part on how the decision maker responds to false advice of this sort.

It turns out that, if the expert shares messages for the two truncated state space, there could be equilibrium that is free of any consideration of off-equilibrium beliefs. The sharing of messages can eliminate the possibility that the decision maker will receive false advice. And message sharing is defined as follows:

Definition 3.2 (Message Sharing). *A strategy of the expert is message sharing if there exists an*

$\bar{I}_i$ *(or* $\hat{I}_i$*) such that, upon observing* $\theta \in \bar{I}_i$ *(*$\theta \in \hat{I}_i$*), the expert randomizes uniformly over* $\bar{M}_i \cup \hat{M}_j$ *(*$\hat{M}_i \cup \bar{M}_j$*) for some* $\hat{M}_j \subset \hat{M}$ *(*$\bar{M}_j \subset \bar{M}$*).*

Definition 3.3 (**Strict Message Sharing**)**.** *A strategy of the expert is strict message sharing if* $N_0 = N_1$ *and, upon observing* $\theta \in \bar{I}_i \cup \hat{I}_i$*, the expert randomizes uniformly over* $\bar{M}_i \cup \hat{M}_i$ *for all* $i = 1, \ldots, N_0$*.*

If an equilibrium is constituted by a (strict) message-sharing strategy, the equilibrium is referred as (strict) message sharing. On the other hand, an equilibrium is a *ψ-robust equilibrium* if it does not depend on the off-equilibrium beliefs of both interval types. If it does not depend on the off-equilibrium beliefs of interval-type s, it is called a *$\psi(s)$-robust equilibrium.* The following proposition states the beliefs-free properties of message-sharing equilibrium:

Proposition 3.3. *If an equilibrium informative for both* l *and* h *is strict message sharing, then it is* ψ*-robust. If, for* $s = l, h$*, an equilibrium informative only for* s *is message sharing, then it is* $\psi(s)$*-robust.*

I illustrate (strict) message-sharing with two examples. In both examples, I assume that $b(1) = \frac{1}{10}$ so that the requirement for informative equilibria is satisfied. I begin with an equilibrium that is informative for both l and h:

Example 3.1

Suppose the expert partitions M into $\bar{M}$ and $\hat{M}$. The former is further partitioned into $\bar{M}_1$ and $\bar{M}_2$ and the latter into $\hat{M}_1$ and $\hat{M}_2$. Using the indifference condition to solve for the boundary types, who are indifferent between sending messages in two adjacent message sets, we have, for $b(1) = \frac{1}{10}$, the boundary type in $l = [0, \frac{1}{2})$ equal to $\frac{1}{20}$ and the boundary type in $h = [\frac{1}{2}, 1]$ equal to $\frac{11}{20}$. Accordingly, we have $\bar{I}_1 = [0, \frac{1}{20})$, $\bar{I}_2 = [\frac{1}{20}, \frac{1}{2})$, $\hat{I}_1 = [\frac{1}{2}, \frac{11}{20})$ and $\hat{I}_2 = [\frac{11}{20}, 1]$. Suppose, for $\theta \in \bar{I}_1 \cup \hat{I}_1$, the expert randomizes uniformly over $\bar{M}_1 \cup \hat{M}_1$, and, for $\theta \in \bar{I}_2 \cup \hat{I}_2$, the expert randomizes uniformly over $\bar{M}_2 \cup \hat{M}_2$.

When interval-type l receives $m \in \bar{M}_1 \cup \hat{M}_1$, given his own knowledge he will only cater for the possibility that $\theta \in \bar{I}_1$ and take action $a = \frac{1}{40}$. Similarly, he will take action $a = \frac{11}{40}$ for $m \in \bar{M}_2 \cup \hat{M}_2$. By the same token, interval-type h will take action $a = \frac{21}{40}$ for $m \in \bar{M}_1 \cup \hat{M}_1$ and take action $a = \frac{31}{40}$ for $m \in \bar{M}_2 \cup \hat{M}_2$.

The message space M is *effectively* partitioned into two distinct and exhaustive sets, and the expert is able to induce four different actions with them because a message can be interpreted differently by l and h. More importantly, the sets of messages used by $\theta \in l$ are exactly the same as those used by $\theta \in h$. When, for example, a $\theta \in l$ deviates to send a message used by another $\theta \in h$, under strict message-sharing that message is used by another $\theta \in l$. Thus, there is no message left that can generate off-equilibrium beliefs. By the indifference condition for the boundary type in l, the θ under consideration has then no incentive to deviate to send any message that is not prescribed to him.[12]

Example 3.2

Suppose the expert provides informative advice for l but babbles for h. The partition of l remains the same: $\bar{I}_1 = [0, \frac{1}{20})$ and $\bar{I}_2 = [\frac{1}{20}, \frac{1}{2})$, and the expert partitions $\bar{M}$ into $\bar{M}_1$ and $\bar{M}_2$ but keeps $\hat{M}$ intact. Suppose, for $\theta \in \bar{I}_1 \cup h$, the expert randomizes uniformly over $\bar{M}_1 \cup \hat{M}$. For $\theta \in \bar{I}_2$, the expert randomizes uniformly over the remaining set of messages: $\bar{M}_2$.

When interval-type l receives $m \in \bar{M}_1 \cup \hat{M}$, he will, as in Example 3.1, take action $a = \frac{1}{40}$. And upon receiving $m \in \bar{M}_2$, he will take action $a = \frac{11}{40}$. On the other hand, interval-type h will take action $a = \frac{3}{4}$ for $m \in \bar{M}_1 \cup \hat{M}$.

Similar to that in Example 3.1, there is "no room" for any $\theta \in l$ to deviate because every message used by $\theta \in l$ is also used by $\theta \in h$. However, $\bar{M}_2$ is not used by any $\theta \in h$; when a $\theta \in h$ sends $m \in \bar{M}_2$, h's action will depend on what his off-equilibrium beliefs, $\psi(h)$, are. If, for example, h adheres to his pre-communication beliefs, the above constitutes an equilibrium. Otherwise, there may exist a $\theta \in h$ who, by sending $m \in \bar{M}_2$, receives a higher payoff than that in the proposed equilibrium. Thus, under message-sharing that is not strict, the equilibrium is robust to the off-equilibrium beliefs of only one interval type.

Note that message-sharing equilibrium is to be distinguished from non-monotone equilibrium in, for example, Krishna and Morgan [30]. Equilibrium is referred as non-monotone when higher expert's types induce lower actions. Such a property on *outcome* does not exist in message-sharing equilibrium. The monotone outcome maintains in the authority model in that higher types induce higher actions. The expert includes in the messages some information that she knows will be ignored; it has no effect on the outcome and is included only for the sake of beliefs-free equilibrium.

The property of strict message-sharing may act as a selection criterion. There are three informative equilibria in Proposition 3.3: one informative for both l and h and one each for l and h. And there is always an equilibrium in which the expert babbles for both l and h. While there is

[12] According to Definition 3.3 the sharing of messages is symmetric across the half intervals: $\hat{I}_i$ shares messages in $\hat{M}_i \cup \bar{M}_i$ with $\bar{I}_i$. It should be clear from this example that this is not necessary for Proposition 3.3. The proposition is stated in this manner only for expositional convenience. So long as one interval in l shares messages with a unique interval in h and vice versa, the result holds.

no criterion to favor the most informative equilibrium over the babbling equilibrium, among the three informative equilibria, only the most informative is robust to any off-equilibrium beliefs of both interval types.

3.3.2 Decision maker's report strategy

This subsection analyzes the report strategy of the decision maker. In particular, I examine under what circumstances an informed decision maker will, in equilibrium, report his information type truthfully. The focus of the analysis is not to provide a complete characterization but to illustrate the underlying force at work.

This caveat generates several restrictions on the following analysis. First of all, to keep the analysis as simple as possible, I focus on the case where the normal bias $b_0 \in (\frac{1}{12}, \frac{1}{4})$. For a normal bias in this range, the most informative equilibria in the continuation game with $\hat{\tau} = 0$ is of two-step. In the continuation game with $\hat{\tau} = 1$, when $b_0 \in (\frac{1}{12}, \frac{1}{8})$ the most informative equilibria is of two-step for each of the half intervals. I also restrict attention, for a given bias level, to the most informative equilibria in the respective continuation games so that, as points of reference for the decision maker, he "gets the most out of his reporting strategy."

An informed decision maker is willing to distinguish himself only if doing so brings him a higher expected payoff. And the source of a higher expected payoff comes from that the expert may provide more information to an informed decision maker. According to Proposition 3.2, for $b_0 \in [\frac{1}{8}, \frac{1}{4})$, even an emotion-free expert provides no useful information to an informed decision maker; being identified as informed is completely counter-productive for eliciting useful advice from the expert, and the informed decision maker will not reveal himself. Even when $b_0 < \frac{1}{8}$ so that an emotion-free expert does provide informative advice to an informed decision maker in a "four-step" partition of $[0, 1]$ (two for each of the half intervals), the decision maker does not necessarily prefer it over a two-step partition reserved for his uninformed incarnation. And the consideration of the expert's emotion could further discourage the informed decision maker from distinguishing himself:

Proposition 3.4. *For $b_0 \in (\frac{1}{12}, \frac{1}{8}(\sqrt{3} - 1)]$, an informed decision maker reveals his information type truthfully to an emotion-free expert but not to an expert whose emotional index $b_1 > \beta(b_0)$, where $\beta(b_0)$ is strictly decreasing in b_0 with $\beta(\frac{1}{8}(\sqrt{3} - 1)) = 0$.*

Despite that an informed decision maker can elicit a "four-step" partition from the expert, the partition is so uneven that, in terms of expected payoff, it could be dominated by the two-step one.

For the former to dominate in the case of an emotion-free expert, the normal bias has to be down to $\frac{1}{8}(\sqrt{3}-1)$. And this does not guarantee that the informed decision maker will report truthfully if the expert is emotional enough. Naturally, a higher normal bias has to be accompanied by a lower emotional index in order for the informed decision maker to identify himself.

When the normal bias is slightly lower than $\frac{1}{12}$, the most informative equilibria in the continuation game with $\hat{\tau}=0$ will have three steps, whereas that in the continuation game with $\hat{\tau}=1$ will maintain its four steps in total even for an emotion-free expert. It is conceivable that, for normal bias lying below the range considered above, the informed decision maker may prefer not to identify himself. While I do not provide an analysis here, this suggests that truthful reporting by the informed decision maker may be non-monotone in the expert's bias; lower bias does not necessarily implies that the informed decision maker is more willing to distinguish himself. The exact value of the bias is of more importance.

Admittedly, the results above depend on the specification of the model; had, for example, the decision maker been informed differently, the results might well be different. The objective of this exercise is, however, to demonstrate with a formal model that an expert's aversion to dealing with an informed decision maker can detriment the communication between the parties. It not only discourages the decision maker from communicating with the expert, but it also can deter the expert from offering useful advice; even when the informed decision maker pools with the uninformed in their reporting (when, for example, $b_0 > \frac{1}{8}(\sqrt{3}-1)$), a corollary of Proposition 3.1 is that an expert averse enough to dealing with informed decision maker will simply refuse to help the decision maker.

3.4 RELATED LITERATURE

This chapter extends Crawford and Sobel [12] on two fronts. First, the decision maker has his own private type and is given an opportunity to communicate that with the expert. Second, the interaction between the parties take into consideration the behavioral element that the expert may be annoyed by an informed decision maker. Some of the previous literature in line with these extensions, in particular the first, has been reviewed in Chapter 2. Here, I focus on other work that has not been cited.

Papers that consider multi-round communication include but not limited to Aumann and Hart

[3] and Krishna and Morgan [30]. In Aummann and Hart, the players can talk as long as they want. They show that such a long conversation can improve communication outcomes. In Krishna and Morgan, the decision maker has no private information, and the purpose of the back-and-forth communication is more a coordination device for improving the communication from the expert to the decision maker.

The communication protocol of the decision maker in this chapter - the partial verification constraint - is the same as that in Austen-Smith [4]. In his paper, however, the constraint is imposed on the expert who may be uninformed and whose decision to become informed is endogenous. He shows that the possibility that the expert is uninformed can improve communication outcomes.

The payoff of the expert in the authority model depends on beliefs. Ottaviani and Sørensen [38] [39] study a cheap-talk model in which the expert also has beliefs-dependent payoff. The expert's payoff in their model depends on the decision maker's beliefs about the expert's ability - motivated as her reputation. They show that such a reputational concern doest not lead to full information revelation by the expert.

3.5 CONCLUDING REMARKS

In this chapter, I provide a simple framework to study an alternative aspect of the relationship between experts and decision makers. The expert in my model feels annoyed if her authority is challenged - when she is not the only one who is informed - and reacts to an informed decision maker by becoming less helpful. I model this via the bias of the expert: her bias consists of two components, her normal bias and her emotional bias, and the latter is increasing in her beliefs about whether the decision maker is informed. I find that the emotion of an expert can deter an informed decision maker from revealing himself who otherwise would have done so to an emotion-free expert.

This could have policy implications on the training of professionals such as doctors. The prevalence of amateurs, whose presence may challenge the authority of experts, is an irreversible trend. Experts may have to undergo a new set of lessons on how to deal with their informed clients, both professionally and emotionally. While there may be some intrinsic conflict of interests between the parties - the normal bias - that cannot be easily altered, resources directed to alleviate the conflicts that have emotional roots - the emotional index - may prove to be rewarding.

The results in this chapter are derived under a specific kind of information of the decision maker:

an even partition of the state space. A more general framework, in which the threshold can be different values in the unit interval, allows us to address how decision maker interprets information. A higher threshold, for example, can represent a decision maker who is more "conservative"; news have to be very good in order to be considered as "good news." On the other hand, a more "aggressive" decision maker may have a lower threshold. An interesting extension will be to study how a decision maker's behavioral altitude toward information interacts with the emotional aspect of the expert in affecting communication, in particular the decision maker's report strategy.

4.0 UNCERTAIN EXPERTISE

4.1 INTRODUCTION

Decision makers, who may lack the relevant information for deciding what action to take, turn to experts for advice. Patients consult doctors; investors seek advice from investment advisors; and legislators hold hearings with expert committees. These experts, while having the expertise that defines themselves, are by no means omniscient - compared to the decision makers, experts have better knowledge, but they are typically not perfectly informed. A medical practitioner, for example, is certainly well cultivated in her professional knowledge, but it is impractical for her to keep abreast of every single frontier findings that may be relevant to her role as a medical advice provider.

Unlike the case of perfect information in which there is only one kind of it, there could, in principle, be an infinite number of ways in which one can be imperfectly informed. Any useful information falling short of perfect information, be it more or less, all falls under the rubrics of imperfect information. Given the intangible nature of knowledge or information that precise certifications are usually not available, a natural consequence of experts being imperfectly informed is that decision makers may face some uncertainty over the experts' levels of expertise. Out of the many different kinds of imperfect information, a decision maker, more often than not, is imperfectly informed about which particular one the expert is equipped with. Such a *second-order imperfect information* is, I believe, a prevalent aspect of communication between experts and decision makers in real life.

This chapter proposes a tractable framework that can be used to analyze strategic information transmission (Crawford and Sobel [12]) with second-order imperfect information. Its objective is not to provide a complete analysis but to demonstrate that the same method of analysis in the literature of optimal information control (Ivanov [25] and Fischer and Stocken [17]) - when the decision maker knows or has control over the expert's imperfect information - can be employed to analyze the situations when the decision maker does not have such information.

There are two players: an expert (she) and a decision maker (he). The biased expert sends a cheap-talk message to the decision maker after observing, imperfectly, the state of the world, represented by a random variable drawn from the unit interval $[0, 1]$. Upon receiving the message, the decision maker, who does not observe the state, takes an action that affects the payoffs of both. The expert is imperfectly informed in that, from her perspective, the unit interval is partitioned into a finite set of non-degenerate intervals. She never observes the state itself but knows which interval contains the realized state. This interval becomes her *interval type*.

From the vantage point of the decision maker, the expert's partition is stochastic: the decision maker holds beliefs about but does not know exactly what the expert's partition is. The decision maker's uncertainty over the expert's expertise is thus captured by the expert's *expertise type*: the expert's partition becomes her private type where a finer partition corresponds to a higher level of expertise. I call this model the *uncertain expertise model*. The introduction of a second type for the expert - which captures the additional layer of information asymmetry the decision maker faces - is where the uncertain expertise model departs from those in the optimal information control literature.

Ivanov [25] shows explicitly that, when payoffs are quadratic and the expert knows that the state lies within an interval, only the conditional mean of the state is relevant for the expert's behavior. Leveraging on this observation, I transform the uncertain expertise model, in which the expert has two types, into one that is more amenable to analysis. In fact, with finitely many potential levels of expertise, the transformed model is the same as the Crawford and Sobel's [12] model (the "CS model") except that the transformed model has a finite type space. The equilibrium analysis of the transformed model, with results readily available, sheds light on the equilibrium properties of the uncertain expertise model.

To illustrate the essence of the transformation, suppose the state is uniformly distributed on $[0, 1]$, and the expert has two equally likely levels of expertise: one in which she is uninformed - the unit interval remains intact - and one in which $[0, 1]$ is divided into $[0, \frac{1}{2}]$ and $[\frac{1}{2}, 1]$.[1] An expert who knows, for example, that the state lies within $[0, \frac{1}{2}]$ behaves, under quadratic payoffs, as if she knew the state is $\frac{1}{4}$. Applying this observation to the other two intervals, the above information

[1]Throughout the chapter, I shall consider all intervals partitioned as closed intervals even though, for example, we can have here $[0, \frac{1}{2})$ and $[\frac{1}{2}, 1]$.

structure can then be transformed into one in which the expert's type space is $\{\frac{1}{4}, \frac{1}{2}, \frac{3}{4}\}$ with probability $\{\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\}$. Suppose the decision maker is to take an action that matches her expectation about the state. When, for example, the decision maker receives a message sent exclusively by the transformed type $\frac{1}{4}$, he takes action $\frac{1}{4}$. On the other hand, when he receives a message sent exclusively by interval type $[0, \frac{1}{2}]$, he takes an action believing that the state is uniformly distributed on $[0, \frac{1}{2}]$. The resulting action is again $\frac{1}{4}$, the expected value of the state given his beliefs. If an expert in the transformed model who knows that the state is $\frac{1}{4}$ is willing to send such a message, the same behavior will be expected from an expert in the uncertain expertise model who believes that the state lies within $[0, \frac{1}{2}]$.

Several properties of the equilibria of the uncertain expertise model are obtained from those of the transformed model. For example, Ivanov [25] and Fischer and Stocken [17] show that in a model in which the decision maker knows how imperfectly informed the expert is - a model that also yields a CS model with finite types - full revelation of information is possible. Such property carries over to the uncertain expertise model when the expert has finitely many potential levels of expertise. The introduction of uncertainty over the expert's level of expertise, which supposedly widens the information gap between the expert and the decision maker, does not by itself eliminate the possibility of full communication.

Yet, a larger set of potential levels of expertise does restrict such possibility. In a fairly general example of information structure, I show that the larger the set of potential levels of expertise, the smaller the expert's bias has to be in order to have full revelation of information to take place. A larger set of potential levels of expertise implies that there is a large number of types in the transformed model. Accordingly, more restrictions are in place for the absence of profitable deviations from a full-revelation equilibrium. If we relate the number of potential levels of expertise to the uncertainty faced by the decision maker, with a larger number associated with higher level of uncertainty, this suggests that, despite the possibility of full revelation, more uncertainty does make full communication more difficult. Finally, without any claim on generality, I demonstrate with a simple example that the communication-improvement result in the optimal information control literature may not be robust to a slight introduction of uncertainty over the expert's level of expertise.

The exposition of the rest of the chapter is as follows. Section 4.2 lays down the uncertain expertise model. Section 4.3 describes how the model can be transformed into a CS model with finite type space. Section 4.4 illustrates how the transformation allows us to study the equilibrium properties of the uncertain expertise model. Section 4.5 contains a brief literature review. Section 4.6 concludes.

4.2 THE MODEL

4.2.1 Players

There are two players: an expert (e) and a decision maker (d). They are in a principal-agent relationship: the decision maker relies on the expert, who has access to information, for advice on what action, $a \in \mathbb{R}$, to take.

There is a state of the world, $\theta \in \Theta \equiv [0, 1]$, commonly known to be uniformly distributed. The maximization of payoffs calls for different actions to be taken under different states of the world. The payoffs of the players are in the form of quadratic loss function:

$$U^e(a, \theta, b) \equiv -(a - (\theta + b))^2, \tag{4.1}$$

$$U^d(a, \theta) \equiv -(a - \theta)^2. \tag{4.2}$$

Had both players observed θ perfectly, the expert would have preferred the decision maker to take action $a^e(\theta, b) = \theta + b$, whereas the decision maker would have preferred $a^d(\theta) = \theta$. These *ideal actions* maximize the respective payoffs of the players, which are zero for the quadratic loss functions. For each θ, the ideal action of the expert exceeds that of the decision maker by $b > 0$; this captures the misaligned interests between the parties, and b is referred as the *bias* of the expert. Note also that the ideal actions are strictly increasing in θ, a consequence of that $U^i_{12}(\cdot) > 0$, $i = e, d$, which is a sorting condition similar to that in costly signalling models.

4.2.2 Information structure and expected payoffs

The expert sends a message $m \in M$ to the decision maker after obtaining her information about θ. The expert does not observe θ directly. Her information status is summarized in her *expertise type* and her *interval type*. The former refers to a partition of the state space $[0, 1]$. That the expert's partition is her private type captures the decision maker's uncertainty over the expert's level of expertise. The expert's interval type refers to an interval from her partition that contains the realized θ, and the expert's information about θ is limited to knowing that it is uniformly distributed on that interval.

Formally, a partition of $[0,1]$ is a finite set of points $\mathcal{P} \equiv \{t_i\}_{i=0}^n$, where $0 = t_0 < t_1 < \cdots < t_n = 1$. A partition $\mathcal{P}$ divides $[0,1]$ into a finite set of disjoint and exhaustive, non-degenerate intervals $\Omega_{\mathcal{P}} \equiv \{\omega_i\}_{i=1}^n$, where $\omega_i = [t_{i-1}, t_i]$. From the perspective of the decision maker, the expert's partition $\mathcal{P}$ - her expertise type - is a draw from a total of K partitions, $\{\mathcal{P}_k\}_{k=1}^K$. And the decision maker's beliefs are represented by a commonly known probability on $\{\mathcal{P}_k\}_{k=1}^K$, p. The set of K partitions has the property that $\mathcal{P}_{k+1}$ is strictly finer than $\mathcal{P}_k$, and a partition $\mathcal{P}$ is said to be strictly finer than $\mathcal{P}'$ if the latter is a strict subset of the former. A finer partition allows the expert to distinguish among a larger set of different realizations of θ, taken to represent that the expert in question has a higher level of expertise. And the above taken together captures the decision maker's uncertainty over what level of expertise the expert is equipped with. After nature sets off the game by drawing the expertise type, $\mathcal{P}$, it moves again by drawing $\theta \in [0,1]$ from the uniform distribution. To the expert, this second move of nature is equivalent to drawing a $\omega \in \Omega_{\mathcal{P}}$ - the ω that contains the realized θ. This ω becomes the expert's interval type.

In the above formulation, nature moves twice. Alternatively, and as a precursor of the transformation of the model, we can consider nature as moving just once, drawing for the expert a unique interval ω_i^k from all intervals in all K partitions, $\Omega \equiv \{\Omega_{\mathcal{P}_k}\}_{k=1}^K$, with probability $q_i^k \equiv \Pr(\omega_i^k) = p_k(t_i^k - t_{i-1}^k)$, where $p_k \equiv \Pr(\mathcal{P}_k)$.[2] Viewed in this light, the expertise type and the interval type can be subsumed into a single expertise-interval type. The type space of the expert is then Ω, with common prior probability q. In the analysis that follows, I shall adopt this perspective, and the pair (Ω, q) is called an *information structure* of the uncertain expertise model.

Since the expert does not observe θ, unlike that in the CS model the expert's payoff in (4.1) is not directly relevant to her behavior. I call the payoffs in (4.1) and (4.2) the players' *state-payoff* functions. The behavior of the expert is instead governed by the maximization of her *type-payoff* function, measuring her expected payoff conditioned on her imperfect information about θ:

$$U^e(a, b|\omega) \equiv E[U^e(a, \theta, b)|\theta \in \omega]. \tag{4.3}$$

[2] Indexing an interval by a pair (k, i) (i.e., ω_i^k is interval i from partition k) allows us to distinguish two intervals from different partitions even when they have the same end points.

It will be convenient to also define type-payoff function for the decision maker:

$$U^d(a|\omega) = E[U^d(a,\theta)|\theta \in \omega]. \tag{4.4}$$

4.2.3 Strategies and equilibrium definition

In the uncertain expertise model, a behavior strategy of the expert, $\alpha : \Omega \to \Delta M$, specifies the distribution of message she sends for each $\omega \in \Omega$. The decision maker's pure strategy, $\beta : M \to \mathbb{R}$, specifies for each message he receives an action he chooses to take.[3] His beliefs function, $\nu : M \to \Delta\Omega$, specifies for each message he receives a probability over Ω. The equilibrium definition is the perfect Bayesian equilibrium:

Definition 4.1 (**Perfect Bayesian Equilibrium: The Uncertain Expertise Model**). *A perfect Bayesian equilibrium of the uncertain expertise model is a pair of strategies (α, β) and a set of beliefs ν such that*

1. *the expert maximizes her payoff given the decision maker's strategy: for all $\omega \in \Omega$, if $m \in$ supp$[\alpha(\cdot|\omega)]$, then*

$$m \in \operatorname*{argmax}_{m' \in M} U^e(\beta(m), b|\omega),$$

2. *the decision maker maximizes his payoff given his beliefs: for all $m \in M$,*

$$\beta(m) = \operatorname*{argmax}_{a' \in \mathbb{R}} \sum_{\omega \in \Omega} U^d(a|\omega)\nu(\omega|m), \quad \textit{and}$$

3. *the decision maker updates his beliefs using Bayes's rule whenever possible, taking into account the expert's strategy:*

$$\nu(\omega|m) = \frac{q(\omega)\alpha(m|\omega)}{\sum\limits_{\omega' \in \Omega} q(\omega')\alpha(m|\omega')}.$$

The expert induces the decision maker to take certain actions with her messages. An action a is said to be induced by ω if $\int_{M_\alpha(a)} \alpha(m|\omega)dm > 0$, where $M_\alpha(a) \equiv \{m \in M : \beta(m) = a\}$, the set of messages such that the decision maker best responds by taking action a upon receiving any of it. If $\int_{M_\alpha(a)} \alpha(m|\omega)dm = 1$, ω is said to purely induce a.

[3]That $U^d_{11}(\cdot) < 0$ guarantees that only pure strategy will be used by the decision maker.

4.3 THE TRANSFORMED MODEL

Implicit in considering Ω, the set of all intervals in all partitions, as the expert's type space is that her partition is not directly relevant for her strategies. Since it is the intervals, not the partitions, that are the expert's information sets, this is indeed consistent with the familiar notion that, in games with incomplete information, a strategy is a mapping from information sets to action space (message space for the expert). The analysis of the uncertain expertise model cannot, however, be performed as it is on Ω. The elements in Ω, being a set of intervals from different partitions, have no order relation. The properties of the state-payoff functions (4.1) and (4.2), including the indispensable sorting condition, are not preserved by the type-payoff functions (4.3) and (4.4) defined on Ω. To work around this, I leverage on a property of the quadratic payoffs to transform the model into one that is amenable to analysis.

4.3.1 Model transformation

Consider interval i from partition k, ω_i^k. Given that the state-payoff functions are quadratic, the expected payoff conditioned on $\theta \in \omega_i^k$ - i.e., the type-payoff functions (4.3) and (4.4) - can be decomposed into two components:

$$U^e(a, b|\omega_i^k) \equiv U^e(a, \bar{t}_i^k, b) - L(\omega_i^k), \tag{4.5}$$

$$U^d(a|\omega_i^k) \equiv U^d(a, \bar{t}_i^k) - L(\omega_i^k), \tag{4.6}$$

where $\bar{t}_i^k \equiv E(\theta|\theta \in \omega_i^k) = \frac{t_{i-1}^k + t_i^k}{2}$ is the conditional mean of θ, and $L(\omega_i^k) \equiv Var(\theta|\theta \in \omega_i^k) = \frac{(t_i^k - t_{i-1}^k)^2}{12}$ is the conditional variance.[4] Under quadratic payoffs, an expert who believes that θ is distributed uniformly on ω_i^k has an expected payoff that is equal to that of an expert who believes that $\theta = \bar{t}_i^k$, minus a positive term that is independent of a. It follows that an a that maximizes $U^e(a, b|\omega_i^k)$ will also maximize $U^e(a, \bar{t}_i^k, b)$; an expert of type ω_i^k will behave as if she knew that the state is $\bar{t}_i^k$.

A similar argument for the decision maker reveals that, for understanding equilibrium behavior in the uncertain expertise model, we can reuse the state-payoff functions $U^e(a, \cdot, b)$ and $U^d(a, \cdot)$,

[4]Ivanov [25] utilizes this decomposition to analyze the case when the decision maker has control over - and thus knows - how imperfectly informed the expert is. Also, note that the decomposition only requires that the state-payoffs are quadratic; even though the above expressions for conditional mean and conditional variance are derived based on uniformly distributed θ, it is not necessary for the decomposition.

with the types being conditional means derived from the expertise-interval types. Thus, define a function, $\tau : \Omega \to [0,1]$, such that, for all $\omega \in \Omega$, $\tau(\omega) = E(\theta|\theta \in \omega)$. For a given Ω, $T \equiv \tau(\Omega)$ is the derived type space, and the probability on T is $g \equiv q \circ \tau^{-1}$, where τ^{-1} is the inverse imagine of τ.[5] With a slight abuse of notations, I recycle to use t to denote a generic element of T. The main message of this chapter is that the *transformed model* with payoffs $U^e(a, \cdot, b)$ and $U^d(a, \cdot)$ and information structure (T, g) provides a convenient venue to study the equilibrium properties of the uncertain expertise model. The transformed model, taken on its own, is nothing but a CS model with finite type space. The next subsection establishes formally the relationship between the two models.

4.3.2 Equivalence

I begin by defining the strategies and equilibrium of the transformed model. A behavior strategy of the expert in the transformed model, $\sigma : T \to \Delta M$, specifies the distribution of message she sends for each $t \in T$. The decision maker's pure strategy, $\rho : M \to \mathbb{R}$, specifies for each message he receives an action he chooses to take. His beliefs function, $\mu : M \to \Delta T$, specifies for each message he receives a probability over T. An equilibrium of the transformed model is defined as:

Definition 4.2 (**Perfect Bayesian Equilibrium: The Transformed Model**). *A perfect Bayesian equilibrium of the transformed model is a pair of strategies (σ, ρ) and a set of beliefs μ such that*

1. *the expert maximizes her payoff given the decision maker's strategy: for all $t \in T$, if $m \in$ supp$[\sigma(\cdot|t)]$, then*
$$m \in \operatorname*{argmax}_{m' \in M} U^e(\rho(m), t, b),$$
2. *the decision maker maximizes his payoff given his beliefs:*
$$\rho(m) = \operatorname*{argmax}_{a' \in \mathbb{R}} \sum_{t \in T} U^d(a, t)\mu(t|m), \quad \textit{and}$$
3. *the decision maker updates his beliefs using Bayes's rule whenever possible, taking into account the expert's strategy:*
$$\mu(t|m) = \frac{g(t)\sigma(m|t)}{\sum\limits_{t' \in T} g(t')\sigma(m|t')}.$$

[5] In general, τ^{-1} is not an inverse function because there may exist two intervals, each from different partitions, that have the same conditional mean.

Induced actions and purely induced actions are defined similarly as in the uncertain expertise model. For an illustration of how the two models relate, consider first the following example, a more elaborate version of that in the Introduction:

Example 4.1

Suppose there are two equally likely partitions: one in which $[0,1]$ is intact, and one in which $[0,1]$ is divided into $[0,\frac{1}{2}]$ and $[\frac{1}{2},1]$. The transformation of this information structure is depicted in Figure 4.1. Suppose, in an equilibrium of the transformed model, $t_1 = \frac{1}{4}$ sends m_1, $t_2 = \frac{1}{2}$ sends m_2 and $t_3 = \frac{3}{4}$ sends m_3. The actions induced are then $a_1 = \frac{1}{4}$ for m_1, $a_2 = \frac{1}{2}$ for m_2 and $a_3 = \frac{3}{4}$ for m_3.

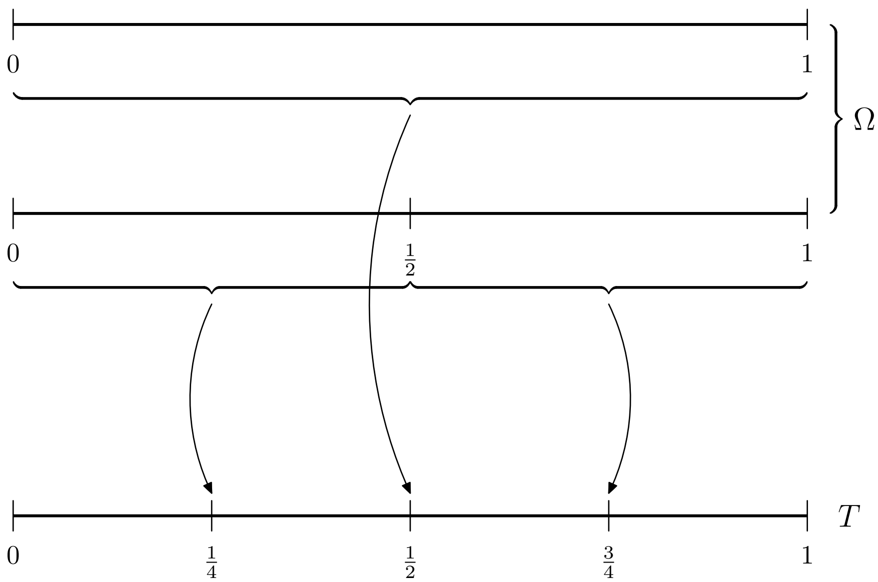


Figure 4.1: Transformation

Suppose, in the uncertain expertise model, the expert adopts the following strategy. Type $[0,\frac{1}{2}]$ sends m'_1, type $[0,1]$ sends m'_2 and type $[\frac{1}{2},1]$ sends m'_3. Upon receiving m'_1, since the message is sent exclusively by an expert who has information that θ is uniformly distributed on $[0,\frac{1}{2}]$, the decision maker updates his beliefs accordingly and takes action $a = \frac{1}{4}$. Similarly, he takes $a = \frac{1}{2}$ for m'_2 and $a = \frac{3}{4}$ for m'_3. Now, since the preference of type $[0,\frac{1}{2}]$ is the same as the preference of type $t_1 = \frac{1}{4}$ in the transformed model, that it is consistent with equilibrium for $t_1 = \frac{1}{4}$ to send m_1 and induce $\frac{1}{4}$ implies that type $[0,\frac{1}{2}]$ is willing to send m'_1 and induce the same action. The same is true for types $[0,1]$ and $[\frac{1}{2},1]$. And the strategy proposed for the expert in the uncertain expertise model indeed constitutes an equilibrium, in which the set of induced actions is the same as that in the transformed model.

The following proposition summarizes the essence of the above example:

Proposition 4.1. *Suppose, in the transformed model,* $(T, g) = (\tau(\Omega), q \circ \tau^{-1})$*. For any equilibrium in the transformed model in which the set of actions* A *is induced, there exists an equilibrium in the uncertain expertise model with information structure* (Ω, q) *in which the same set of actions* A *is induced. Furthermore,* $\omega \in \Omega$ *induces* a' *if and only if* $\tau(\omega) = t$*, where* t *is an element of the set that induces* a' *in the transformed model.*

In Ivanov [25], the analysis of strategic information transmission in which the decision maker has control over - and thus is informed of - the expert's information structure is conducted via a CS model with finite type space, which the transformed model is one of its kind. The presence of second-order imperfect information, when the decision maker faces an additional layer of information asymmetry on how informed the expert is, therefore does not impose a fundamental change in analyzing the game.

4.4 EQUILIBRIUM ANALYSIS: AN ILLUSTRATION OF THE TRANSFORMATION

The equilibrium analysis in this section consists of two parts. The objective is not to provide a complete analysis of any information structure in the uncertain expertise model but to illustrate how analyzing the transformed model sheds light on the equilibrium properties of the uncertain expertise model. In the first part, I adopt Ivanov's [25] characterization of induced actions and discuss the meaning for the uncertain expertise model. In the second part, I characterize the relationship between the expert's bias and the existence of informative equilibria in the transformed model. It provides a routine to solve for the requirement on the expert's bias for informative equilibria. It will then be applied to analyze a special case of information structure in the uncertain expertise model. A simple example illustrating how the communication-improvement result in Ivanov [25] may be sensitive to the introduction of uncertainty over the expert's expertise is also in order.

4.4.1 Induced actions

The transformed model is not a standalone object but derived from the uncertain expertise model with an underlying information structure. In the followings, I shall, however, assume otherwise. The analysis will proceed with an arbitrary information structure (T, g) for the transformed model, with the only restriction that T is finite; if certain properties hold for arbitrary information structures, it will also hold for the subset that is derived from the uncertain expertise model.

Induced actions in the transformed satisfy the following properties:

Proposition 4.2 (Ivanov [25]). *In any equilibrium of the transformed model, the set of induced actions, A, is finite. Furthermore, it satisfies the following properties:*

1. *If type t induces two actions a_i and a_j, then either $i = j - 1$ or $i = j + 1$;*
2. *The highest type,* $\max\{T\}$, *purely induces the highest action,* $\max\{A\}$*;*
3. *If type t induces action a, then any type $t' < t$ does not induce any action $a' > a$ and any type $t'' > t$ does not induce any action $a'' < a$;*
4. *If types t' and t'', $t' < t''$, induce action a, then any t such that $t' < t < t''$ purely induce a; and*
5. *If type t_i induces actions a_i and a_{i+1}, then t_{i+1} induces a_{i+1}.*

Any partition in the uncertain expertise model divides $[0, 1]$ into a finite number of intervals. When there are finitely many potential levels of expertise or, equivalently, the number of partitions is finite, the finiteness of the set of induced actions implies that full revelation of information can be achieved for the right level of expert's bias. The decision maker's uncertainty over the expert's level of expertise - an additional layer of information asymmetry imposed on the decision maker - does not by itself eliminate the possibility of full revelation. Note also that in the uncertain expertise model, full revelation of information is not necessarily the same as full separation; there may exist two types whose images under τ are the same. Full revelation only requires that each $\omega \in \Omega$ and its "equivalents," i.e., $\omega' \in \Omega$ such that $\tau(\omega') = \tau(\omega)$, send a distinct message.[6] Nevertheless, it is a full revelation of information in the sense that the expert reveals all information she has about θ.

The properties of the induced actions in the transformed model generate some intuitive observations for the uncertain expertise model. Property 2, for example, says that the highest action must be recommended by the expert with the highest level of expertise. And any expert with a lower level of expertise who ever recommends the highest action must share some common information

[6] For convenience, I shall assume in the followings that the expert does not mix over messages.

with the "most knowledgable" expert. Given that the expert with the highest level of expertise has the finest partition, the highest type in the transformed model - the type who induces the highest action - must be derived from the highest interval in this partition. If this highest interval in the finest partition has some equivalents in other less fine partitions, these equivalents will also induce the highest action.

Despite the monotonicity in Property 3, an expert with, for example, a higher level of expertise can recommend actions that are lower than the lowest action recommended by an expert with a lower level of expertise. When under no circumstance will an expert with certain level of expertise be "decisive" over recommending the highest action, Property 5 suggests that an expert with a higher level of expertise will be able to "make up her mind." If t_i, indifferent between inducing the highest and the next highest actions, is derived from the highest interval of a partition, then t_{i+1} must come from a finer partition.

4.4.2 Informative equilibria

Suppose there are N elements in the type space of the transformed model: $T = \{t_1, \ldots, t_N\}$. In equilibrium, whether these types pool with some others or each sends a distinct message depends on the relative magnitudes the expert's bias and how far the types lie apart. This subsection examines this relationship.

Property 4 of Proposition 4.2 suggests that any types that pool with each other in equilibrium must be in consecutive orders. Denote the set of consecutive types who send message m by $T(m)$. When H distinct messages are to be sent, $H \leq N$, there will be H sets of such types: $\mathcal{T} \equiv \{T(m_1), T(m_2), \ldots, T(m_H)\}$. Note that when $H = N$ in equilibrium, each of these sets will be a singleton, and there is a full-revelation equilibrium.

Fix a strategy for the expert in the transformed model such that we have $\mathcal{T} \equiv \{T(m_1), T(m_2), \ldots, T(m_H)\}$. When the decision maker receives message m_i, his best response is to take action

$$a(T(m_i)) = \sum_{t \in T(m_i)} \left[\frac{g(t)}{\sum_{t' \in T(m_i)} g(t')} \right] t.$$

For the strategy to constitute an equilibrium, no type in $T(m_i)$ should have incentive to send m_j, $j \neq i$. In other words, no type in $T(m_i)$ should strictly prefer $a(T(m_j))$ over $a(T(m_i))$. Property 3 of Proposition 4.2 suggests, however, that there is no need to check for profitable deviations for every single $t \in T(m_i)$: when the highest and the lowest types in $T(m_i)$ prefers to send m_i, the rest in the set will exhibit the same preference. Denote $\bar{t}_i = \max\{T(m_i)\}$ and $\underline{t}_i = \min\{T(m_i)\}$. In order for $\bar{t}_i$ to prefer sending m_i over m_{i+1}, we need, under quadratic payoffs,

$$\bar{t}_i + b \leq \frac{a(T(m_i)) + a(T(m_{i+1}))}{2}. \tag{4.7}$$

Similarly, in order for $\underline{t}_{i+1}$ to prefer sending m_{i+1} over m_i, we need

$$\underline{t}_{i+1} + b \geq \frac{a(T(m_i)) + a(T(m_{i+1}))}{2}. \tag{4.8}$$

Combining (4.7) and (4.8) - and note that they have to hold for all $i = 1, 2, \ldots, H-1$ - gives the following restriction on b for the existence of (informative) equilibria in the transformed model:

Proposition 4.3. *For any finite T in the transformed model, a collection of sets of types, $\mathcal{T} = \{T(m_1), T(m_2), ..., T(m_H)\}$, where all $t \in T(m_i)$ send message m_i, $i = 1, 2, ..., H$, constitutes an equilibrium if and only if*

$$\max_{i=1,\ldots,H-1}\left(\frac{a(T(m_i)) + a(T(m_{i+1}))}{2} - \underline{t}_{i+1}\right) \leq b \leq \min_{i=1,\ldots,H-1}\left(\frac{a(T(m_i)) + a(T(m_{i+1}))}{2} - \bar{t}_i\right). \tag{4.9}$$

Proposition 4.3 provides a routine to solve for the restriction on b for informative equilibria. As an illustration, suppose the uncertain expertise model has the following information structure, which I call the *uniform uncertain expertise*: there are K partitions in which the first partition keeps the unit interval intact, the second divides $[0, 1]$ into two equal intervals, the third into four equal intervals and so forth so that the Kth partition divides $[0, 1]$ into 2^{K-1} intervals. The realization of each of these partitions is equally likely. Under this set of partitions, Ω consists of a set of intervals each of which has no equivalent, and the type space of the transformed model has $2^K - 1$ elements. Note that the information structure underlying Example 4.1 is a special case of uniform uncertain expertise when $K = 2$.

I focus on the existence of a full-revelation equilibrium. In such an equilibrium, $H = |T| = 2^K - 1$. Since every $T(m_i)$ is a singleton, $\bar{t}_i = \underline{t}_i = t_i$ for all $i = 1, 2, \ldots, H$. Upon receiving message m_i, the decision maker best responds by taking action $a = t_i$. Given that $\frac{t_i + t_{i+1}}{2} < t_{i+1}$ and that $b > 0$, only the inequalities on the right in (4.9) need to be observed. Moreover, under the uniform uncertain expertise, the distances between two consecutive transformed types are uniform; if the inequality holds for i, it will also hold for $j \neq i$, and we can dispense with the min operator. This gives:

Corollary 4.1. *There exists a full-revelation equilibrium under uniform uncertain expertise if and only if* $b \leq \frac{1}{2^{K+1}}$.

The full-revelation equilibrium in Example 4.1 thus requires that $b \leq \frac{1}{8}$. If we vary K as a comparative statics, the above suggests that as the expert has a larger set of potential levels of expertise, a smaller bias is required for full-revelation equilibrium to exist. A larger set of potential levels of expertise implies that there is a larger number of types in the transformed model. Given that the distances between the consecutive types are uniform, they shrink when there are more types. Accordingly, a smaller b is required for the absence of profitable deviation. If we relate the decision maker's uncertainty over the expert's level of expertise to K so that when K increases there is a higher level of uncertainty, Corollary 4.1 suggests that an increase in the level of uncertainty makes full communication more difficult. When in the limit K approaches infinity, the expert will begin withholding information.

It may be constructive to compare the scenario when the decision maker knows, as in the optimal information control literature, how informed the expert is with that when uncertainty is present. Ivanov [25] shows that, when $b = \frac{1}{4}$ so that only babbling equilibria exist in the CS model, there exists informative equilibrium if the expert is imperfectly informed. In particular, informative communication can achieved when the expert's partition divides $[0, 1]$ into two uniform intervals, and this is controlled by and thus known to the decision maker. This can also be seen from applying Proposition 3. When types $[0, \frac{1}{2}]$ and $[\frac{1}{2}, 1]$ each sends a distinct message, the induced actions are $\frac{1}{4}$ and $\frac{3}{4}$. In order for the lower transformed type $t = \frac{1}{4}$ to prefer action $\frac{1}{4}$ over action $\frac{3}{4}$, we need $b \leq \frac{1}{2}(\frac{1}{4} + \frac{3}{4}) - \frac{1}{4} = \frac{1}{4}$. Consider, however, the following example in which some uncertainty over the expert's expertise is introduced:

Example 4.2

Suppose $b = \frac{1}{4}$, and the decision maker can largely control the expert's information structure as above, but with a small probability ϵ the expert is uninformed. The transformed types are the same as those in Example 1: $t_1 = \frac{1}{4}$, $t_2 = \frac{1}{2}$ and $t_3 = \frac{3}{4}$. But their probabilities are different: $g(t_1) = g(t_3) = \frac{1}{2}(1-\epsilon)$ and $g(t_2) = \epsilon$. It is obvious that there cannot be full-revelation equilibrium. It should also be immediately clear that there cannot be an equilibrium in which t_1 pools with t_2, and t_3 sends a distinct message; the inclusion of t_2 only slightly increases the lower induced actions, but its presence imposes a more stringent restriction for the absence of profitable deviations. Since pooling types must be in consecutive order, the only remaining candidate for informative equilibrium is that t_1 sends a distinct message and t_2 pools with t_3. The corresponding induced actions will then be $\frac{1}{4}$ and $\frac{3+\epsilon}{4+4\epsilon}$. For this to constitute an equilibrium, we require $b \leq \frac{1}{2}(\frac{1}{4} + \frac{3+\epsilon}{4+4\epsilon}) - \frac{1}{4} = \frac{1}{4+4\epsilon}$. Thus, no matter how small ϵ is, so long as it is positive we are left with babbling equilibrium for $b = \frac{1}{4}$.

While the example is by no means general, it demonstrates that the optimal information structure of the expert might be sensitive to the introduction of uncertainty over the expert's expertise or when the decision maker's control over the expert's information structure is not perfect.

4.5 RELATED LITERATURE

The optimal information control literature, repeatedly referred to in this chapter, focuses on how varying the quality of the expert's information may improve communication. This chapter, motivated by studying an aspect of communication with experts in real life, does not fall under this line of enquiry. Nevertheless, it is closely related to this literature because our information structure - partitional information - is the same as that used by the authors to model the quality of information. Fischer and Stocken [17] show that a less than perfect quality of expert's information, represented by a partition of the state space, can improve communication. Ivanov [25] extends their results in a more general setting and poses the question as a problem of organizational design, comparing information control with delegation.

As for previous work that addresses uncertainty over how informed the expert is, Austen-Smith [4] studies a cheap-talk game in which the sender (the expert) makes a private decision, before communicating with the receiver (the decision maker), on whether to acquire (perfect) information or not. He shows that the receiver's uncertainty over the binary information status of the sender can improve communication. Apart from his different information structure, he also allows the sender to send a separate message on her information status, which is not considered in this chapter.

In a different line of enquiry, Ottaviani and Sørensen [38] [39] consider strategic information transmission in which the expertise of the expert depends on her private ability type, which implies that the decision maker in their models is also uninformed about the expert's level of expertise. They focus on how the expert's concern for reputation, as being conceived as having a high ability, affects communication.

4.6 CONCLUDING REMARKS

In this chapter, I demonstrate that the same techniques for analyzing communication with expert who is known to be imperfectly informed can be applied to analyze the situations when the decision maker is uncertain about how informed the expert is. Leveraging on the quadratic payoffs commonly adopted in the cheap-talk literature, I show that the presence of second-order imperfect information does not change the fundamental way in which the game is analyzed.

Utilizing this finding, I show in a fairly general example that an increase in the level of uncertainty over the expert's level of expertise makes communication more difficult between the parties, even though full revelation of information is a possibility. I provide an example demonstrating that the optimal information structure in the literature of optimal information control may be sensitive to the introduction of a slight uncertainty over the expert's information structure. To investigate the extent of generality of this observation is, I believe, an interesting direction to extend this work.

APPENDIX

PROOFS

Proof of Lemma 2.1. For all m sent on the equilibrium paths, by definition $\Theta_\sigma(m)$ contains the realized θ. Furthermore, if the decision maker is of type t_s, the realized $\theta \in t_s$. Thus, for any message m that t_s receives in information sets reached in equilibrium, $\Theta_\sigma(m)$ and t_s contain at least one common element: the realized θ. Conversely, if $\Theta_\sigma(m)$ and t_s contain some common elements, then in equilibrium t_s receives m with positive probability. The lemma is phrased in the negative of the above.

□

Proof of Lemma 2.2. Suppose there is an informative equilibrium with no unused message under equilibrium strategy σ so that $\Theta_\sigma(m)$ is a strict non-empty subset of Θ for all $m \in M$. Consider an arbitrary $m \in M$. Suppose the realized state $\theta' \notin \Theta_\sigma(m)$ but the expert sends m for θ'; $\Theta_\sigma(m)$ being a strict subset of Θ ensures we can find such θ'. There exists t_s such that $\theta' \in t_s$ and $t_s \in \Theta \setminus \Theta_\sigma(m)$: if $\theta' > \sup \Theta_\sigma(m)$, choose a t_h with $t \in (\sup \Theta_\sigma(m), \theta')$, and if $\theta' < \inf \Theta_\sigma(m)$ choose a t_l with $t \in (\theta', \inf \Theta_\sigma(m))$; $\Theta_\sigma(m)$ being non-empty ensures that $\sup \Theta_\sigma(m)$ and $\inf \Theta_\sigma(m)$ are well-defined. When the chosen t_s receives m, because $\Theta_\sigma(m) \cap t_s = \emptyset$, he will, by Lemma 2.1, be in an information set unreached in equilibrium even though m is a used message.

□

Proof of Lemma 2.3. Consider an equilibrium (σ, ρ, μ). There may exist unused messages in this equilibrium; denote the set of unused messages by M_0. Now, construct another equilibrium (σ', ρ', μ') that has the same equilibrium outcome as (σ, ρ, μ). Consider an arbitrary message $m' \in M \setminus M_0$ that is sent under σ, and denote the resulting set of induced actions by $A(m')$. Note that since different types of decision maker may take different actions upon receiving the same message, $A(m')$ can be a non-singleton set and it also contains actions taken in pursuance of out-of-equilibrium messages. Suppose the expert is prescribed to randomizes under σ' over the support $M_0 \cup \{m'\}$ such that every $m \in M_0 \cup \{m'\}$ is used with positive probability. Upon receiving any message $m'' \in M_0 \cup \{m'\}$, every type of decision maker takes action that is the same as the response to m'; thus $A(m'') \equiv A(m')$. Given this, it is a best response for the expert to adopt the strategy σ', and (σ', ρ', μ') is an equilibrium that has the same outcome as (σ, ρ, μ) but every message in M is used with positive probability.

□

Proof of Proposition 2.1. Suppose, in the amateur model, σ is a fully separating strategy. Upon receiving $m \in M$, regardless of his interval type the decision maker's updated beliefs are always 1 at the θ for which m is sent. Thus, all types of decision maker will take the same action upon receiving m, and the expert's expected payoff in (2.7) becomes $V^e(m, \theta, b) = \int_0^1 U^e(\rho(m), \theta, b)dt = U^e(\rho(m), \theta, b)$ where $\rho(m) = \rho(m, t_h) = \rho(m, t_l)$ for all $t \in [0, 1]$. Hence, an amateur model with fully separating strategy reduces to a CS model with fully separating strategy. The fact that a fully separating strategy cannot constitute an equilibrium in the CS model rules out such possibility for the amateur model. Note that for this proof there is no need to consider off-equilibrium beliefs because there are incentives for deviations even along the paths occurring under σ in the proposed fully separating equilibrium.

□

Proof of Proposition 2.2. I begin by substantiating conditions 1 and 2 and appeal to Definition 2.3 for whether an action is effectively induced or not. Consider first the high-interval types. Suppose $t \in I_i$, $i = 1, \ldots, N-1$, and t_h receives $m \in M_j$, $j \leq N$, indicating that $\theta \in [\theta_{j-1}, \theta_j]$. For $j = i$, t_h's updated beliefs under Bayes's rule are $\mu(\theta|m, t_h) = f(\theta)/[\int_t^{\theta_i} f(\theta')d\theta']$ for $\theta \in [t, \theta_i]$ and zero elsewhere. For $j > i$, his updated beliefs are $\mu(\theta|m, t_h) = f(\theta)/[\int_{\theta_{j-1}}^{\theta_j} f(\theta')d\theta']$ for $\theta \in [\theta_{j-1}, \theta_j]$ and zero elsewhere. In both cases, it is obvious that there exists θ such that $\mu(\theta|m, t_h) \neq \phi(\theta|t_h)$. Thus, if a t_h with $t \in I_i$, $i = 1, \ldots, N-1$, receives $m \in M_j$, $i \leq j \leq N$, he will take an action that is effectively induced.

For the converse, I shall prove with contradiction. First of all, suppose $t \in I_N$, and t_h receives $m \in M_N$. His updated belief $\mu(\theta|m, t_h) = f(\theta)/[\int_t^1 f(\theta')d\theta']$ for $[t, 1]$ and zero elsewhere, and this is equivalent to $\phi(\theta|t_h)$ for all $\theta \in [0, 1]$. Thus, the action taken by t_h with $t \in I_N$ cannot be effectively induced. Next, suppose $t \in I_i$, $i = 1, \ldots, N-1$, and t_h receives $m \in M_j$, $j < i$. It is obvious that for $j < i$, $\Theta_\sigma(m) \cap t_h = \emptyset$, and thus by Lemma 2.1 Bayes's rule cannot be applied in updating his beliefs. Thus, for all messages $m \in M_j$, $j < i$, an effectively induced action cannot be induced on a t_h with $t \in I_i$.

Consider next the low-interval types. Suppose $t \in I_i$, $i = 2, \ldots, N$, and t_l receives $m \in M_k$, $k \geq 1$, indicating that $\theta \in [\theta_{k-1}, \theta_k]$. For $k = i$, t_l's updated beliefs under Bayes's rule are $\mu(\theta|m, t_l) = f(\theta)/[\int_{\theta_{i-1}}^{t} f(\theta')d\theta']$ for $\theta \in [\theta_{i-1}, t)$ and zero elsewhere. For $k < i$, his updated beliefs are $\mu(\theta|m, t_l) = f(\theta)/(\int_{\theta_{k-1}}^{\theta_k} f(\theta')d\theta']$ for $\theta \in [\theta_{k-1}, \theta_k)$ and zero elsewhere. There exists θ such that $\mu(\theta|m, t_l) \neq \phi(\theta|t_l)$ in both cases. Thus, if a t_l with $t \in I_i$, $i = 2, \ldots, N$, receives message $m \in M_k$, $1 \leq k \leq i$, he will take an action that is effectively induced.

For the converse, suppose first that $t \in I_1$, and t_l receives $m \in M_1$. His updated beliefs are $\mu(\theta|m, t_l) = f(\theta)/(\int_0^t f(\theta')d\theta')$ for $\theta \in [0, t)$ and zero elsewhere, and this is equivalent to $\phi(\theta|t_l)$ for all $\theta \in [0, 1]$. Thus, the action taken by t_l with $t \in I_1$ cannot be effectively induced. Next, suppose $t \in I_i$, $i = 2, \ldots, N$, and t_l receives message $m \in M_k$, $k > i$. Since for $k > i$, $\Theta_\sigma(m) \cap t_l = \emptyset$, Bayes's cannot be used. Thus, for all messages $m \in M_k$, $k > i$, an effectively induced action cannot be induced on a t_l with $t \in I_i$.

□

Proof of Proposition 2.3. Since there are only two types of induced actions, effectively induced and ineffectively induced, and they are mutually exclusive, conditions 1 and 2 follow directly from the proof of conditions 1 and 2 in Proposition 2.2 where necessities in Proposition 2.2 become sufficiencies in this proposition, and vice versa.

□

Proof of Corollary 2.1. Consider two arbitrary types $\theta' < \theta''$ in I_i, $i = 1, \ldots, N$, who send $m \in M_i$ under the prescribed N-step strategy. First note that, for decision makers with $t \in (\theta', \theta'')$, they are t_l if the realized expert's type is θ' and they are t_h if the realized expert's type is θ''. It then follows directly from Propositions 2.2 and 2.3 that the actions induced on these decision makers by θ' are different from those induced by θ''. Suppose that $\theta'' = \theta_i$. Note that, for decision makers with $t \in I_i$, they are all t_h if θ_i is the realized expert's type. Thus, the actions induced on t_l for $t \in I_i$, $a(\theta_{i-1}, t)$, will not be induced by θ_i, whereas they are induced by θ' on t_l for $t \in (\theta', \theta_i) \subset I_i$ with a complementary advice.

□

Proof of Proposition 2.4. The outline of the proof is as follows. First of all, I shall construct

ψ^* and state the cases of the expert's payoff $V^e(m,\theta,b)$ under ψ^*. I shall then show that, if $\psi=\psi^*$ and $U^e_{12}(\cdot)$ is sufficiently large, then (2.13) is sufficient for the following to always hold in the general model: for all $\theta\in[\theta_{i-1},\theta_i]$,

$$V^e(M_i,\theta,b)=\max_j V^e(M_j,\theta,b),\ i,j=1,\ldots,N. \tag{A.1}$$

The set of off-equilibrium beliefs ψ^* is constructed as follows. Suppose there exists a monotone solution, $\{\theta_1,\ldots,\theta_{N-1}\}\subset(\theta_0,\theta_N)\equiv(0,1)$, to (2.13) (which will be shown below), if a high-interval type t_h with $t\in(\theta_i,\theta_{i+1})$, $i=0,\ldots,N-1$, detects a false advice, his beliefs are that θ is distributed on $[t,\theta_{i+1})$ with density $\frac{f(\theta)}{F(t+\theta_{i+1})-F(t)}$ and zero elsewhere; if a low-interval type t_l with $t\in(\theta_i,\theta_{i+1})$ detects a false advice, his beliefs are that θ is distributed on (θ_i,t) with density $\frac{f(\theta)}{F(t)-F(t-\theta_i)}$ and zero elsewhere. Using Propositions 2.2 and 2.3 and ψ^*, the profile of expected payoffs of $\theta\in[\theta_{i-1},\theta_i]$ is

$$V^e(m,\theta,b)=\begin{cases}
\displaystyle\int_0^{\theta_{k-1}}U^e(a(\theta_{k-1},\theta_k),\theta,b)dt & \\
\displaystyle\qquad+\sum_{r=k-1}^{i-1}\int_{\theta_r}^{\theta_{r+1}}U^e(a(t,\theta_{r+1}),\theta,b)dt & \\
\displaystyle\qquad+\int_{\theta_i}^{\theta}U^e(a(t,\theta_{i+1}),\theta,b)dt & \text{if } m\in M_k,\ 1\le k<i,\\
\displaystyle\qquad+\int_{\theta}^{1}U^e(a(\theta_{k-1},\theta_k),\theta,b)dt, & \\
\displaystyle\int_0^{\theta_{i-1}}U^e(a(\theta_{i-1},\theta_i),\theta,b)dt & \\
\displaystyle\qquad+\int_{\theta_{i-1}}^{\theta}U^e(a(t,\theta_i),\theta,b)dt & \\
\displaystyle\qquad+\int_{\theta}^{\theta_i}U^e(a(\theta_{i-1},t),\theta,b)dt & \text{if } m\in M_i,\\
\displaystyle\qquad+\int_{\theta_i}^{1}U^e(a(\theta_{i-1},\theta_i),\theta,b)dt, & \\
\displaystyle\int_0^{\theta}U^e(a(\theta_{j-1},\theta_j),\theta,b)dt & \\
\displaystyle\qquad+\int_{\theta}^{\theta_i}U^e(a(\theta_{i-1},t),\theta,b)dt & \\
\displaystyle\qquad+\sum_{r=i}^{j-1}\int_{\theta_r}^{\theta_{r+1}}U^e(a(\theta_r,t),\theta,b)dt & \text{if } m\in M_j,\ i<j\le N.\\
\displaystyle\qquad+\int_{\theta_j}^{1}U^e(a(\theta_{j-1},\theta_j),\theta,b)dt, &
\end{cases} \tag{A.2}$$

Using the second and the third cases in (A.2), the expected payoff for θ_i to send $m\in M_i$ and

$m \in M_{i+1}$ are, respectively,

$$\int_0^{\theta_{i-1}} U^e(a(\theta_{i-1},\theta_i),\theta_i,b)dt + \int_{\theta_{i-1}}^{\theta_i} U^e(a(t,\theta_i),\theta_i,b)dt + \int_{\theta_i}^1 U^e(a(\theta_{i-1},\theta_i),\theta_i,b)dt \tag{A.3}$$

$$\int_0^{\theta_i} U^e(a(\theta_i,\theta_{i+1}),\theta_i,b)dt + \int_{\theta_i}^{\theta_{i+1}} U^e(a(\theta_i,t),\theta_i,b)dt + \int_{\theta_{i+1}}^1 U^e(a(\theta_i,\theta_{i+1}),\theta_i,b)dt. \tag{A.4}$$

Thus, the indifference condition (2.13) becomes the following second-order difference equation:

$$\begin{aligned} V(\theta_{i-1},\theta_i,\theta_{i+1},b) \equiv \int_0^{\theta_{i-1}} &[U^e(a(\theta_i,\theta_{i+1}),\theta_i,b) - U^e(a(\theta_{i-1},\theta_i),\theta_i,b)]dt \\ &+ \int_{\theta_{i-1}}^{\theta_i} [U^e(a(\theta_i,\theta_{i+1}),\theta_i,b) - U^e(a(t,\theta_i),\theta_i,b)]dt \\ &+ \int_{\theta_i}^{\theta_{i+1}} [U^e(a(\theta_i,t),\theta_i,b) - U^e(a(\theta_{i-1},\theta_i),\theta_i,b)]dt \\ &+ \int_{\theta_{i+1}}^1 [U^e(a(\theta_i,\theta_{i+1}),\theta_i,b) - U^e(a(\theta_{i-1},\theta_i),\theta_i,b)]dt = 0, \end{aligned} \tag{A.5}$$

$i = 1,\ldots,N-1$, $\theta_0 = 0$, $\theta_N = 1$. Suppose there is a strictly increasing partition, $\theta_0,\ldots,\theta_i$, that satisfies (A.5). Note that for θ_{i-1} and θ_i from the partition,

$$\begin{aligned} \frac{\partial V^e(\theta_{i-1},\theta_i,\theta',b)}{\partial \theta'} = \int_0^{\theta_i} & U_1^e(a(\theta_i,\theta'),\theta_i,b)\left[\frac{\partial a(\theta_i,\theta')}{\partial \theta'}\right]dt \\ &+ \int_{\theta'}^1 U_1^e(a(\theta_i,\theta'),\theta_i,b)\left[\frac{\partial a(\theta_i,\theta')}{\partial \theta'}\right]dt. \end{aligned} \tag{A.6}$$

Since $U_{11}^e(\cdot) < 0$ and $a(\cdot,\cdot)$ is strictly increasing in the second argument, it is clear from (A.6) that $V(\theta_{i-1},\theta_i,\theta_{i+1},b)$ strictly decreases eventually for $\theta' \geq \theta_i$. Given that $V(\theta_{i-1},\theta_i,\theta_i,b) > 0$, the continuity of $V(\theta_{i-1},\theta_i,\theta',b)$ in θ' (followed from the continuities of $U^e(\cdot)$ and $a(\cdot)$) implies that there exists a unique $\theta_{i+1} > \theta_i$ that satisfies (A.5).

Turning to incentive compatibility, I begin by showing that (A.1) holds for θ_i, $i = 1,\ldots,N-1$, that satisfy (A.5). If $N = 2$, there exists no other set of messages that θ_i can send so that (A.1) is satisfied vacuously. So, consider $N \geq 3$. Suppose θ_i sends message $m \in M_{i+n}$, $2 \leq n \leq N-i$. Then, from the third case in (A.2) her expected payoff is

$$\begin{aligned} \int_0^{\theta_i} U^e(a(\theta_{i+n-1},\theta_{i+n}),\theta_i,b)dt + \sum_{r=i}^{i+n-2} \int_{\theta_r}^{\theta_{r+1}} & U^e(a(\theta_r,t),\theta_i,b)dt \\ + \int_{\theta_{i+n-1}}^{\theta_{i+n}} U^e(a(\theta_{i+n-1},t),\theta_i,b)dt + \int_{\theta_{i+n}}^1 & U^e(a(\theta_{i+n-1},\theta_{i+n}),\theta_i,b)dt. \end{aligned} \tag{A.7}$$

Subtracting (A.7) from (A.4), we have

$$\begin{aligned} D_1 = &\int_0^{\theta_i} [U^e(a(\theta_i,\theta_{i+1}),\theta_i,b) - U^e(a(\theta_{i+n-1},\theta_{i+n}),\theta_i,b)]dt \\ &+ \int_{\theta_i}^{\theta_{i+1}} [U^e(a(\theta_i,t),\theta_i,b) - U^e(\theta_i,t),\theta_i,b)]dt \\ &+ \sum_{r=i+1}^{i+n-2} \int_{\theta_r}^{\theta_{r+1}} [U^e(a(\theta_i,\theta_{i+1}),\theta_i,b) - U^e(a(\theta_r,t),\theta_i,b)]dt \\ &+ \int_{\theta_{i+n-1}}^{\theta_{i+n}} [U^e(a(\theta_i,\theta_{i+1}),\theta_i,b) - U^e(a(\theta_{i+n-1},t),\theta_i,b)]dt \\ &+ \int_{\theta_{i+n}}^{1} [U^e(a(\theta_i,\theta_{i+1}),\theta_i,b) - U^e(a(\theta_{i+n-1},\theta_{i+n}),\theta_i,b)]dt. \end{aligned}$$

Note that (A.5) implies that the expert's ideal action $a^e(\theta_i,b) \in (a(\theta_{i-1},\theta_i), a(\theta_i,\theta_{i+1}))$. Since $a(\theta_{i+n-1},\theta_{i+n}) > a(\theta_i,\theta_{i+1})$, we have $a(\theta_{i+n-1},t) > a(\theta_i,\theta_{i+1})$ for $t \in (\theta_{i+n-1},\theta_{i+n})$, and $a(\theta_j,t) > a(\theta_i,\theta_{i+1})$ for $t \in (\theta_j,\theta_{j+1})$, $j = i+1,\ldots,i+n-2$, given $U^e_{11}(\cdot) < 0$ and the maximum of $U^e(a,\theta_i,b)$ is achieved for $a \in (a(\theta_{i-1},\theta_i), a(\theta_i,\theta_{i+1}))$, the first, third, fourth and fifth terms are positive. Also, the second term vanishes. Thus, $D_1 > 0$.

Next, suppose θ_i sends $m \in M_{i-\eta}$, $1 \le \eta \le i-1$. From the first case in (A.2), her expected payoff is

$$\begin{aligned} &\int_0^{\theta_{i-\eta-1}} U^e(a(\theta_{i-\eta-1},\theta_{i-\eta}),\theta_i,b)dt + \int_{\theta_{i-\eta-1}}^{\theta_{i-\eta}} U^e(a(t,\theta_{i-\eta}),\theta_i,b)dt \\ &\quad + \sum_{r=i-\eta}^{i-1} \int_{\theta_r}^{\theta_{r+1}} U^e(a(t,\theta_{r+1}),\theta_i,b)dt + \int_{\theta_i}^{1} U^e(a(\theta_{i-\eta-1},\theta_{i-\eta}),\theta_i,b)dt. \end{aligned} \tag{A.8}$$

Subtracting (A.8) from (A.4), we have

$$\begin{aligned} D_2 = &\int_0^{\theta_{i-\eta-1}} [U^e(a(\theta_{i-1},\theta_i),\theta_i,b) - U^e(a(\theta_{i-\eta-1},\theta_{i-\eta}),\theta_i,b)]dt \\ &+ \int_{\theta_{i-\eta-1}}^{\theta_{i-\eta}} [U^e(a(\theta_{i-1},\theta_i),\theta_i,b) - U^e(a(t,\theta_{i-\eta}),\theta_i,b)]dt \\ &+ \sum_{r=i-\eta}^{i-2} \int_{\theta_r}^{\theta_{r+1}} [U^e(a(\theta_{i-1},\theta_i),\theta_i,b) - U^e(a(t,\theta_{r+1}),\theta_i,b)]dt \\ &+ \int_{\theta_{i-1}}^{\theta_i} [U^e(a(t,\theta_i),\theta_i,b) - U^e(a(t,\theta_i),\theta_i,b)]dt \\ &+ \int_{\theta_i}^{1} [U^e(a(\theta_{i-1},\theta_i),\theta_i,b) - U^e(a(\theta_{i-\eta-1},\theta_{i-\eta}),\theta_i,b)]dt. \end{aligned}$$

Similar to the above, since $a(\theta_{i-1},\theta_i) > a(\theta_{i-\eta-1},\theta_{i-\eta})$, $a(\theta_{i-1},\theta_i) > a(t,\theta_{i-\eta})$ for $t \in (\theta_{i-\eta-1},\theta_{i-\eta})$, and $a(\theta_{i-1},\theta_i) > a(t,\theta_{j+1})$, for $t \in (\theta_j,\theta_{j+1})$, $j = i-\eta,\dots,i-2$, the first, second, third and fifth terms are positive, while the fourth term vanishes. Thus, $D_2 > 0$. That $D_1 > 0$ and $D_2 > 0$ imply that (A.1) holds for θ_i, $i = 1,\dots,N-1$.

I show next that given (A.5) and for sufficiently large $U^e_{12}(\cdot)$, all $\theta \in (\theta_{i-1},\theta_i)$ prefer sending $m \in M_i$ over $m \in M_{i+1}$, and all $\theta \in (\theta_i,\theta_{i+1})$ prefer sending $m \in M_{i+1}$ over $m \in M_i$, $i = 1,\dots,N-1$, so that (A.1) holds for all interior θ. Consider an arbitrary $\theta \in (\theta_{i-1},\theta_i)$. From the third case in (A.2), her expected payoff from sending $m \in M_{i+1}$ is

$$\begin{aligned}\int_0^{\theta} U^e(a(\theta_i,\theta_{i+1}),\theta_i,b)dt + \int_{\theta}^{\theta_i} U^e(a(\theta_{i-1},t),\theta_i,b)dt \\ + \int_{\theta_i}^{\theta_{i+1}} U^e(a(\theta_i,t),\theta_i,b)dt + \int_{\theta_{i+1}}^{1} U^e(a(\theta_i,\theta_{i+1}),\theta_i,b)dt.\end{aligned} \tag{A.9}$$

Subtracting his expected payoff from sending $m \in M_i$ in (A.2) from (A.9), we have

$$\begin{aligned}D_3 = &\int_0^{\theta_{i-1}} [U^e(a(\theta_i,\theta_{i+1}),\theta,b) - U^e(a(\theta_{i-1},\theta_i),\theta,b)]dt \\ &+ \int_{\theta_{i-1}}^{\theta} [U^e(a(\theta_i,\theta_{i+1}),\theta,b) - U^e(a(t,\theta_i),\theta,b)]dt \\ &+ \int_{\theta}^{\theta_i} [U^e(a(\theta_{i-1},t),\theta,b) - U^e(a(\theta_{i-1},t),\theta,b)]dt \\ &+ \int_{\theta_i}^{\theta_{i+1}} [U^e(a(\theta_i,t),\theta,b) - U^e(a(\theta_{i-1},\theta_i),\theta,b)]dt \\ &+ \int_{\theta_{i+1}}^{1} [U^e(a(\theta_i,\theta_{i+1}),\theta,b) - U^e(a(\theta_{i-1},\theta_i),\theta,b)]dt.\end{aligned} \tag{A.10}$$

Differentiating D_3 with respect to θ gives

$$\begin{aligned}\frac{\partial D_3}{\partial \theta} = &\int_0^{\theta_{i-1}} \frac{\partial[U^e(a(\theta_i,\theta_{i+1}),\theta,b) - U^e(a(\theta_{i-1},\theta_i),\theta,b)]}{\partial \theta}dt \\ &+ \int_{\theta_{i-1}}^{\theta} \frac{\partial[U^e(a(\theta_i,\theta_{i+1}),\theta,b) - U^e(a(t,\theta_i),\theta,b)]}{\partial \theta}dt \\ &+ \int_{\theta_i}^{\theta_{i+1}} \frac{\partial[U^e(a(\theta_i,t),\theta,b) - U^e(a(\theta_{i-1},\theta_i),\theta,b)]}{\partial \theta}dt \\ &+ \int_{\theta_{i+1}}^{1} \frac{\partial[U^e(a(\theta_i,\theta_{i+1}),\theta,b) - U^e(a(\theta_{i-1},\theta_i),\theta,b)]}{\partial \theta}dt \\ &+ [U^e(a(\theta_i,\theta_{i+1}),\theta,b) - U^e(a(\theta,\theta_i),\theta,b)]\theta.\end{aligned}$$

Since $a(\theta_i, \theta_{i+1}) > a(\theta_{i-1}, \theta_i)$, $a(\theta_i, \theta_{i+1}) > a(t, \theta_i)$ for $t \in (\theta_{i-1}, \theta)$, and $a(\theta_i, t) > a(\theta_{i-1}, \theta_i)$ for $t \in (\theta_i, \theta_{i+1})$, $U^e_{12}(\cdot) > 0$ implies that the first four terms are positive. The last term, derived under Leibniz rule that θ is in the range of integration of the second term in (A.10), is, however, negative. Thus, when θ decreases from θ_i, there are negative effects on D_3 from the first four term and a positive effect from the last term. However, if $U^e_{12}(\cdot)$ is sufficiently large at θ, the negative effects outweighs the positive effect. Thus, a sufficiently large $U^e_{12}(\cdot)$ guarantees, given (A.5), $D_3 \leq 0$ for θ. Take the minimum value of $U^e_{12}(\cdot)$ that satisfies this to be $c^{\uparrow}_{\theta}$. Note that $c^{\uparrow}_{\theta}$ is finite.

Consider next an arbitrary $\theta \in (\theta_i, \theta_{i+1})$. From the first case in (A.2), her expected payoff from sending $m \in M_i$ is

$$\begin{aligned}\int_0^{\theta_{i-1}} U^e(a(\theta_{i-1}, \theta_i), \theta, b)dt + \int_{\theta_{i-1}}^{\theta_i} U^e(a(t, \theta_i), \theta, b)dt \\ + \int_{\theta_i}^{\theta} U^e(a(t, \theta_{i+1}), \theta, b)dt + \int_{\theta}^{1} U^e(a(\theta_{i-1}, \theta_i), \theta, b)dt.\end{aligned} \tag{A.11}$$

Subtracting (A.11) from the expected payoff from sending $m \in M_{i+1}$ in (A.2), we have

$$\begin{aligned}D_4 = &\int_0^{\theta_{i-1}} [U^e(a(\theta_i, \theta_{i+1}), \theta, b) - U^e(a(\theta_{i-1}, \theta_i), \theta, b)]dt \\ &+ \int_{\theta_{i-1}}^{\theta_i} [U^e(a(\theta_i, \theta_{i+1}), \theta, b) - U^e(a(t, \theta_i), \theta, b)]dt \\ &+ \int_{\theta_i}^{\theta} [U^e(a(t, \theta_{i+1}), \theta, b) - U^e(a(t, \theta_{i+1}), \theta, b)]dt \\ &+ \int_{\theta}^{\theta_{i+1}} [U^e(a(\theta_i, t), \theta, b) - U^e(a(\theta_{i-1}, \theta_i), \theta, b)]dt \\ &+ \int_{\theta_{i+1}}^{1} [U^e(a(\theta_i, \theta_{i+1}), \theta, b) - U^e(a(\theta_{i-1}, \theta_i), \theta, b)]dt.\end{aligned} \tag{A.12}$$

Differentiating D_4 with respect to θ gives

$$\begin{aligned}\frac{\partial D_4}{\partial \theta} = &\int_0^{\theta_{i-1}} \frac{\partial [U^e(a(\theta_i, \theta_{i+1}), \theta, b) - U^e(a(\theta_{i-1}, \theta_i), \theta, b)]}{\partial \theta} dt \\ &+ \int_{\theta_{i-1}}^{\theta_i} \frac{\partial [U^e(a(\theta_i, \theta_{i+1}), \theta, b) - U^e(a(t, \theta_i), \theta, b)]}{\partial \theta} dt \\ &+ \int_{\theta}^{\theta_{i+1}} \frac{\partial [U^e(a(\theta_i, t), \theta, b) - U^e(a(\theta_{i-1}, \theta_i), \theta, b)]}{\partial \theta} dt \\ &+ \int_{\theta_{i+1}}^{1} \frac{\partial [U^e(a(\theta_i, \theta_{i+1}), \theta, b) - U^e(a(\theta_{i-1}, \theta_i), \theta, b)]}{\partial \theta} dt \\ &- [U^e(a(\theta_i, \theta), \theta, b) - U^e(a(\theta_{i-1}, \theta_i), \theta, b)]\theta.\end{aligned}$$

Since $a(\theta_i, \theta_{i+1}) > a(\theta_{i-1}, \theta_i)$, $a(\theta_i, \theta_{i+1}) > a(t, \theta_i)$ for $t \in (\theta_{i-1}, \theta)$, and $a(\theta_i, t) > a(\theta_{i-1}, \theta_i)$ for $t \in (\theta_i, \theta_{i+1})$, $U_{12}^e(\cdot) > 0$ implies that the first four terms are positive. While the last term is negative, similar to the above, if $U_{12}^e(\cdot)$ is sufficiently large at θ, the positive effects on D_4 from the first four terms outweigh the negative effect from the last term; thus, for sufficiently large $U_{12}^e(\cdot)$, (A.5) implies $D_4 \geq 0$. Take the minimum value of $U_{12}^e(\cdot)$ that satisfies this to be $c_\theta^\downarrow$. Similarly, $c_\theta^\downarrow$ is finite. Let $c^{*\downarrow} = \max_{1\leq i\leq N-1}(\sup_{\theta\in[0,1]} c_\theta^\downarrow)$. Further let $c^* = \max\{c^{*\uparrow}, c^{*\downarrow}\}$. For sufficiently large $U_{12}^e(\cdot)$ such that $U_{12}^e(\cdot) \geq c^*$, (A.1) holds for all interior θ.

□

Proof of Proposition 2.5. First, note that in a two-step equilibrium of the uniform-quadratic model, the profile of effectively induced actions is

$$\rho(m, t_h) = \begin{cases} \frac{t+\theta_1}{2}, & \text{for } t \in I_1 \text{ and } m \in M_1, \\ \frac{\theta_1+1}{2}, & \text{for } t \in I_1 \text{ and } m \in M_2, \end{cases} \qquad \text{and}$$

$$\rho(m, t_l) = \begin{cases} \frac{\theta_1}{2}, & \text{for } t \in I_2 \text{ and } m \in M_1, \\ \frac{\theta_1+t}{2}, & \text{for } t \in I_2 \text{ and } m \in M_2, \end{cases}$$

and that of ineffectively induced action is

$$\begin{aligned} \rho(m, t_h) &= \frac{t+1}{2}, \quad \text{for } t \in I_2 \text{ and } m \in M_1 \text{ or } m \in M_2, \text{ and} \\ \rho(m, t_l) &= \frac{t}{2}, \quad \text{for } t \in I_1 \text{ and } m \in M \text{ or } m \in M_2. \end{aligned}$$

Using these induced actions, the only equation in the indifference condition becomes

$$\begin{aligned} &\int_0^{\theta_1} \left(\left[\frac{t+\theta_1}{2} - (\theta_1 + b) \right]^2 - \left[\frac{\theta_1 + 1}{2} - (\theta_1 + b) \right]^2 \right) dt \\ &\quad + \int_{\theta_1}^1 \left(\left[\frac{\theta_1}{2} - (\theta_1 + b) \right]^2 - \left[\frac{\theta_1 + t}{2} - (\theta_1 + b) \right]^2 \right) dt = 0. \end{aligned} \tag{A.13}$$

Solving (A.13) for θ_1 gives $\theta_1 = \frac{1}{2}(1 - 6b)$. An informative equilibrium requires $\theta_1 > 0$, and this is satisfied if and only if $b < \frac{1}{6}$.

I verify in the followings that so long as (A.13) is satisfied, it is a best response for all interior θ to follow the strategy. Subtracting, for $\theta \in [0, \theta_1)$, the expected payoff from sending $m \in M_1$ from that from sending $m \in M_2$, we have, after imposing the equilibrium condition $\theta_1 = \frac{1}{2}(1 - 6b)$,

$$D_5 = \frac{\theta(27\theta - 10\theta^2 - 3) + b(9 + 96\theta - 30\theta^2) + 36b^2(4 - 3\theta) - 324b^3 - 4}{24}.$$

I show that $D_5 \leq 0$ for all $(\theta, b) \in [0, \frac{1}{2}(1-6b)] \times (0, \frac{1}{6})$. Differentiating D_5 with respect to θ twice, we have $\frac{\partial^2 D_5}{\partial \theta^2} = \frac{9-10(b+\theta)}{4} > 0$ for $(\theta, b) \in [0, \frac{1}{2}(1-6b)] \times (0, \frac{1}{6})$. Thus, D_5 is convex in θ for all $b \in (0, \frac{1}{6})$. If the value of D_5 at the boundaries of $[0, \frac{1}{2}(1-6b)]$ is non-positive for all $b \in (0, \frac{1}{6})$, then $D_5 \leq 0$ for all $(\theta, b) \in [0, \frac{1}{2}(1-6b)] \times (0, \frac{1}{6})$. By the indifference condition, $D_5 = 0$ when $\theta = \frac{1}{2}(1-6b)$. When $\theta = 0$, $D_5 = \frac{(1+6b)(33b-54b^2-4)}{24}$, and this can easily be verified to be negative for all $b \in (0, \frac{1}{6})$.

Next, subtracting, for $\theta \in [\theta_1, 1]$, the expected payoff from sending $m \in M_1$ from that from sending $m \in M_2$, we have

$$D_6 = \frac{(1-\theta)[10\theta^2 + 7\theta + 30b(1+\theta) - 5] + (1-6b)[9\theta + 3b(5+6\theta) + 54b^2 - 5]}{24}.$$

I show that $D_6 \geq 0$ for all $(\theta, b) \in [\frac{1}{2}(1-6b), 1] \times (0, \frac{1}{6})$. Differentiating D_6 with respect to θ twice, we have $\frac{\partial^2 D_6}{\partial \theta^2} = \frac{1-10(b+\theta)}{4} < 0$ for $(\theta, b) \in [\frac{1}{2}(1-6b), 1] \times (0, \frac{1}{6})$. Thus, D_6 is concave in θ for all $b \in (0, \frac{1}{6})$. If the value of D_6 at the boundaries of $[\frac{1}{2}(1-6b), 1]$ is non-negative for all $b \in (0, \frac{1}{6})$, then $D_6 \geq 0$ for all $(\theta, b) \in [\frac{1}{2}(1-6b), 1] \times (0, \frac{1}{6})$. By the indifference condition, $D_6 = 0$ when $\theta = \frac{1}{2}(1-6b)$. When $\theta = 1$, $D_6 = \frac{(1-6b)(33b+54b^2+4)}{24}$, and this can easily be verified to be positive for all $b \in (0, \frac{1}{6})$.

$\square$

Proof of Corollary 2.2. Note that in two-step equilibria of CS model's uniform-quadratic case, $\theta_1^{cs} = \frac{1}{2}(1-4b)$. The results follow directly by comparing this with $\theta_1 = \frac{1}{2}(1-6b)$.

$\square$

Calculation for Example 2.1. I shall demonstrate for the case of $b = \frac{1}{15}$; the case of $b = \frac{1}{20}$, which is almost the same, will be skipped. The objective of this calculation is to solve the following simultaneous equations for $0 < \theta_1 < \theta_2 < 1$:

$$
\begin{aligned}
&\int_0^{\theta_1}\left(\left[\frac{t+\theta_1}{2}-\left(\theta_1+\frac{1}{15}\right)\right]^2-\left[\frac{\theta_1+\theta_2}{2}-\left(\theta_1+\frac{1}{15}\right)\right]^2\right)dt\\
&\quad+\int_{\theta_1}^{\theta_2}\left(\left[\frac{\theta_1}{2}-\left(\theta_1+\frac{1}{15}\right)\right]^2-\left[\frac{\theta_1+t}{2}-\left(\theta_1+\frac{1}{15}\right)\right]^2\right)dt\\
&\quad+\int_{\theta_2}^{1}\left(\left[\frac{\theta_1}{2}-\left(\theta_1+\frac{1}{15}\right)\right]^2-\left[\frac{\theta_1+\theta_2}{2}-\left(\theta_1+\frac{1}{15}\right)\right]^2\right)dt=0;
\end{aligned}
\tag{A.14}
$$

$$
\begin{aligned}
&\int_0^{\theta_1}\left(\left[\frac{\theta_1+\theta_2}{2}-\left(\theta_2+\frac{1}{15}\right)\right]^2-\left[\frac{\theta_2+1}{2}-\left(\theta_2+\frac{1}{15}\right)\right]^2\right)dt\\
&\quad+\int_{\theta_1}^{\theta_2}\left(\left[\frac{t+\theta_2}{2}-\left(\theta_2+\frac{1}{15}\right)\right]^2-\left[\frac{\theta_2+1}{2}-\left(\theta_2+\frac{1}{15}\right)\right]^2\right)dt\\
&\quad+\int_{\theta_2}^{1}\left(\left[\frac{\theta_1+\theta_2}{2}-\left(\theta_2+\frac{1}{15}\right)\right]^2-\left[\frac{\theta_2+t}{2}-\left(\theta_2+\frac{1}{15}\right)\right]^2\right)dt=0,
\end{aligned}
\tag{A.15}
$$

and, given the solution, verify that incentive compatibility is satisfied for all interior types.

There is more than one solution to these equations, and the only one that satisfies $0<\theta_1<\theta_2<1$ is $\bar{\theta}_1=0.011122$ and $\bar{\theta}_2=0.307109$. Subtracting, for $\theta\in[0,\bar{\theta}_1)$, the expected payoff from sending $m\in M_1$ from that from sending $m\in M_2$ gives $D_7=-0.002871+0.2550076\theta+0.2765556\theta^2-0.4166667\theta^3$, and it is straightforward to verify that $\partial D_7/\partial\theta>0$ for $\theta\in[0,\bar{\theta}_1)$ so that, given, (A.14), $D_7\leq 0$ for θ in this range. Subtracting, for $\theta\in[\bar{\theta}_1,\bar{\theta}_2)$ the expected payoff from sending $m\in M_2$ from that from sending $m\in M_3$ gives $D_8=-0.119569+0.0996639\theta+1.228334\theta^2-1.41667\theta^3$, and it can be shown that $\partial D_8/\partial\theta>0$ for $\theta\in[\bar{\theta}_1,\bar{\theta}_2)$ so that, given, (A.15), $D_8\leq 0$ for θ in this range.

Next, subtracting, for $\theta\in[\bar{\theta}_2,1]$, the expected payoff from sending $m\in M_2$ from that from sending $m\in M_3$ gives $D_9=-0.222724+0.747777\theta+0.0545666\theta^2-0.41667\theta^3$. While the second derivative $\partial^2 D_9/\partial\theta^2<0$ for $\theta\in[\bar{\theta}_2,1]$, it can easily be verified that $D_9\geq 0$ for $\theta=\bar{\theta}_2$ and $\theta=1$. Thus, $D_9\geq 0$ for all $\theta\in[\bar{\theta}_2,1]$. Finally, subtracting, for $\theta\in[\bar{\theta}_1,\bar{\theta}_2)$, the expected payoff from sending $m\in M_1$ from that from sending $m\in M_2$ gives $D_{10}=-0.002924+0.2633056\theta-0.03055528\theta^2-0.4166667\theta^3$, and it can be shown that $\partial D_{10}/\partial\theta>0$ for $\theta\in[\bar{\theta}_1,\bar{\theta}_2)$ so that, given, (A.14), $D_{10}\geq 0$ for θ in this range. Hence, incentive compatibility is satisfied for all interior θ.

□

Proof of Proposition 2.7. First of all, the novice decision maker's expected payoff in the CS model is

$$
W^{cs}(b)=-\frac{1}{12I^{cs}(b)^2}-\frac{b^2(I^{cs}(b)^2-1)}{3},
\tag{A.16}
$$

where $I^{cs}(b)$ is, for a given b, the highest number of intervals partitioned by the expert. Since $I^{cs}(b) = 2$ for $b \in (\frac{1}{12}, \frac{1}{4})$, the expected payoff of the novice is $W^{cs}(b) = -\frac{1}{48} - b^2$.

In the amateur model, the decision maker's expected payoff if his $t \in (\theta_1, 1]$ is

$$W_t(b) = \int_0^{\theta_1} -\left(\frac{\theta_1}{2} - \theta\right)^2 d\theta + \int_{\theta_1}^{t} -\left(\frac{\theta_1 + t}{2} - \theta\right)^2 d\theta + \int_t^1 -\left(\frac{t+1}{2} - \theta\right)^2 d\theta. \tag{A.17}$$

First, consider case (1). When $\theta_1 = 0$, the first term (A.17) vanishes, and, after integration, the expected payoff becomes $W_t(b) = -\frac{1}{12}(3t^2 - 3t + 1)$. Thus, the amateur is better off if and only if $W_t(b) - W^{cs}(b) = -\frac{1}{12}(3t^2 - 3t + 1) - (-\frac{1}{48} - b^2) \geq 0$ or $t \in [\underline{t}, \overline{t}] = [\frac{1}{2} - 2b, \frac{1}{2} + 2b]$.

Consider next case (2i). When $\theta_1 > 0$ and if $t \in (\theta_1, 1]$, using the equilibrium condition that $\theta_1 = \frac{1}{2}(1 - 6b)$, the expected payoff in (A.17) becomes $W_t(b) = -\frac{1}{48} - b^2 + A(t, b)$, where

$$A(t, b) = \frac{t(3 - 2t) + 12bt(1 - t) + 4b^2(4 - 9t) - 1}{16}.$$

Thus, an amateur is better off if and only if $A(t, b) \geq 0$. Note that since for all $b \in (\frac{1}{12}, \frac{1}{6})$, $A(\frac{1}{2}(1 - 6b), b) = A(1, b) = -\frac{5b^2}{4} < 0$, $\frac{\partial^2 A(t,b)}{\partial t^2} < 0$, and $A_{\max}(b) = \max_{t' \in [\frac{1}{2}(1-6b), 1]} A(t', b) = \frac{1+18b-52b^2-216b^3}{128} > 0$, solving the equation $A(t, b) = 0$ shows that $A(t, b) \geq 0$ if and only if $t \in [\underline{t}', \overline{t}']$, where

$$\underline{t}' = \frac{3 + 12b - 36b^2 - \sqrt{1 + 24b + 56b^2 - 96b^3 + 1296b^4}}{4(1 + 6b)} > \frac{1}{2}(1 - 6b), \text{ and}$$
$$\overline{t}' = \frac{3 + 12b - 36b^2 + \sqrt{1 + 24b + 56b^2 - 96b^3 + 1296b^4}}{4(1 + 6b)} < 1,$$

Finally, consider case (2ii). If $t \in [0, \theta_1]$, $\theta_1 > 0$, the amateur's expected payoff is

$$\begin{aligned} W_t(b) &= \int_0^{t} -\left(\frac{t}{2} - \theta\right)^2 d\theta + \int_{t}^{\theta_1} -\left(\frac{\theta_1 + t}{2} - \theta\right)^2 d\theta + \int_{\theta_1}^1 -\left(\frac{\theta_1+1}{2} - \theta\right)^2 d\theta \\ &= -\frac{1}{48} - b^2 + B(t, b), \end{aligned} \tag{A.18}$$

where

$$B(t, b) = \frac{t(1 - 2t) - 12bt(1 - t) - 4b^2(5 - 9t)}{16}.$$

An amateur is therefore better off if and only if $B(t, b) \geq 0$. Note that similar to the above, $B(0, b) = B(\frac{1}{2}(1 - 6b), b) = -\frac{5b^2}{4} < 0$ and $\frac{\partial^2 B(t,b)}{\partial t^2} < 0$ for $b \in (\frac{1}{24}, \frac{1}{6})$. However, it can shown that, for $b \in (\frac{1}{12}, \frac{1}{6})$, $B_{\max}(b) = \max_{t' \in [0, \frac{1}{2}(1-6b)]} B(t', b) = \frac{1-18b-52b^2-216b^3}{128} < 0$. Thus, every threshold $t \in [0, \frac{1}{2}(1-6b)]$ is strictly worse off.

□

Proof of Proposition 2.8. Consider first the case where $b \in [\frac{1}{6}, \frac{1}{4})$. Integrating, with respect to t, $W_t(b) = -\frac{1}{12}(3t^2 - 3t + 1)$ gives the pre-t expected payoff $W(b) = -\frac{1}{24}$. Thus, an amateur is better off if and only if $W(b) \geq W^{cs}(b)$ or $b \geq \frac{1}{4\sqrt{3}}$, which is satisfied for $b \in [\frac{1}{6}, \frac{1}{4})$. Furthermore, it is straightforward to check that $\frac{\partial[W(b) - W^{cs}(b)]}{\partial b} > 0$ for b in this range.

For $b \in (\frac{1}{12}, \frac{1}{6})$, integrating (A.17) and (A.18) with respect to t in the respective ranges and summing them up gives $W(b) = -\frac{1+72b^2-432b^4}{64}$. It is straightforward to check that $W(b) \geq W^{cs}(b)$ for all $b \in (\frac{1}{12}, \frac{1}{6})$. Differentiating $[W(b) - W^{cs}(b)]$ with respect to b gives

$$\frac{\partial[W(b) - W^{cs}(b)]}{\partial b} = -\frac{1}{64}(144b - 1728b^3) + 2b.$$

Since the second partial derivative $\frac{\partial^2[W(b) - W^{cs}(b)]}{\partial b} = 162b > 0$ for $b > 0$, solving the first-order condition gives the minimizing value $b_{min} = \frac{1}{6\sqrt{3}} \in (\frac{1}{12}, \frac{1}{6})$. Thus, the welfare improvement decreases and then increases for b in this range. Note also that $-\frac{1+72b^2-432b^4}{64} = -\frac{1}{24}$ for $b = \frac{1}{6}$. Hence, $W(b) - W^{cs}(b)$ is continuous and decreases and then increases in b for $b \in (\frac{1}{12}, \frac{1}{4})$.

□

Calculation for Example 2.2. First, recall from Example 2.1 that, when $b = \frac{1}{15}$, the boundary types in the three-step equilibrium in the amateur model are $\{\bar{\theta}_1, \bar{\theta}_2\} = \{0.011122, 0.307109\}$. Also, when $b = \frac{1}{15}$, the equilibrium in the CS model with the highest number of steps is three-step equilibrium. Accordingly, by substituting $I^{cs} = 3$ and $b = \frac{1}{15}$ into (A.16), the novice's expected payoff is $W^{cs}(\frac{1}{15}) = -0.021111$.

Thus, for $t \in [0, \bar{\theta}_1]$, the amateur is better off if and only if

$$\begin{aligned} W_t\left(\frac{1}{15}\right) - W^{cs}\left(\frac{1}{15}\right) &= \int_0^t -\left(\frac{t}{2} - \theta\right)^2 d\theta + \int_t^{\bar{\theta}_1} -\left(\frac{t + \bar{\theta}_1}{2} - \theta\right)^2 d\theta \\ &\quad + \int_{\bar{\theta}_1}^{\bar{\theta}_2} -\left(\frac{\bar{\theta}_1 + \bar{\theta}_2}{2} - \theta\right)^2 d\theta + \int_{\bar{\theta}_2}^{1} -\left(\frac{\bar{\theta}_2 + 1}{2} - \theta\right)^2 d\theta \\ &\quad + 0.021111 \\ &= -\frac{t^3}{12} + 0.083333(t - 0.011122)(t^2 - 0.022244t + 0.000124) \\ &\quad - 0.008771 \geq 0. \end{aligned} \tag{A.19}$$

However, since $\frac{\partial^2[W_t(\frac{1}{15})-W^{cs}(\frac{1}{15})]}{\partial t^2} < 0$ for $t \in [0, \bar{\theta}_1]$ and the maximum of $W_t(\frac{1}{15}) - W^{cs}(\frac{1}{15})$ for $t \in [0, \bar{\theta}_1]$, $[W_t(\frac{1}{15}) - W^{cs}(\frac{1}{15})]_{|t=0.00561} = -0.008771 < 0$, (A.19) cannot be satisfied, and thus the amateur is worse off.

Consider next $t \in (\bar{\theta}_1, \bar{\theta}_2]$. The amateur is better off if and only if

$$\begin{aligned} W_t\left(\frac{1}{15}\right) - W^{cs}\left(\frac{1}{15}\right) = &\int_0^{\bar{\theta}_1} -\left(\frac{\bar{\theta}_1}{2} - \theta\right)^2 d\theta + \int_{\bar{\theta}_1}^{t} -\left(\frac{\bar{\theta}_1 + t}{2} - \theta\right)^2 d\theta \\ &+ \int_t^{\bar{\theta}_2} -\left(\frac{t + \bar{\theta}_2}{2} - \theta\right)^2 d\theta + \int_{\bar{\theta}_2}^{1} -\left(\frac{\bar{\theta}_2 + 1}{2} - \theta\right)^2 d\theta \\ &+ 0.021111 \\ = &\, t^3 - 0.073997t^2 + 0.023548t - 0.009024 \geq 0. \end{aligned} \tag{A.20}$$

Similar to the above, $\frac{\partial^2[W_t(\frac{1}{15})-W^{cs}(\frac{1}{15})]}{\partial t^2} < 0$ for $t \in (\bar{\theta}_1, \bar{\theta}_2]$, and the maximum of $W_t(\frac{1}{15}) - W^{cs}(\frac{1}{15})$ for $t \in (\bar{\theta}_1, \bar{\theta}_2]$, $[W_t(\frac{1}{15}) - W^{cs}(\frac{1}{15})]_{|t=0.159116} = -0.007151 < 0$. Thus, (A.20) cannot be satisfied, and the amateur is worse off.

For $t \in (\bar{\theta}_2, 1]$, the amateur is better off if and only if

$$\begin{aligned} W_t\left(\frac{1}{15}\right) - W^{cs}\left(\frac{1}{15}\right) = &\int_0^{\bar{\theta}_1} -\left(\frac{\bar{\theta}_1}{2} - \theta\right)^2 d\theta + \int_{\bar{\theta}_1}^{\bar{\theta}_2} -\left(\frac{\bar{\theta}_1 + \bar{\theta}_2}{2} - \theta\right)^2 d\theta \\ &+ \int_{\bar{\theta}_2}^{t} -\left(\frac{\bar{\theta}_2 + t}{2} - \theta\right)^2 d\theta + \int_t^{1} -\left(\frac{t + 1}{2} - \theta\right)^2 d\theta \\ &+ 0.021111 \\ = &-\frac{(1-t)^3}{12} - 0.083333(t - 0.307111) \\ &\times (t^2 - 0.614216t + 0.094315) + 0.018950 \geq 0. \end{aligned} \tag{A.21}$$

Note that $\frac{\partial^2[W_t(\frac{1}{15})-W^{cs}(\frac{1}{15})]}{\partial t^2} < 0$ for $t \in (\bar{\theta}_2, 1]$, but the maximum of $W_t(\frac{1}{15}) - W^{cs}(\frac{1}{15})$ for $t \in (\bar{\theta}_2, 1]$, $[W_t(\frac{1}{15}) - W^{cs}(\frac{1}{15})]_{|t=0.653555} = 0.012020 > 0$. Solving the equation $W_t(\frac{1}{15}) - W^{cs}(\frac{1}{15}) = 0$ in (A.21) gives that the amateur is better off if and only if his threshold $t \in [0.390136, 0.916973]$.

Finally, the pre-t expected payoff of the amateur, derived by integrating the expressions in (A.19)-(A.21) with respect to t in their respective ranges and then summing them up, is $W(\frac{1}{15}) = -0.019959 > -0.021111$. Thus, in the pre-$t$ *ex-ante* stage, the amateur decision maker is always better off.

□

Proof of Proposition 3.1. It suffices to focus on two-step equilibria. Suppose the boundary type $\theta' < \frac{1}{2}$. Then, the indifference condition is

$$-\left[\frac{\theta'}{2} - (\theta' + b(\hat{\tau}))\right]^2 = -\hat{\tau}\left[\frac{\theta' + \frac{1}{2}}{2} - (\theta' + b(\hat{\tau}))\right]^2 - (1-\hat{\tau})\left[\frac{\theta' + 1}{2} - (\theta' + b(\hat{\tau}))\right]^2. \tag{A.22}$$

Solving (A.22) gives $\theta' = \frac{3\hat{\tau} - 8b(\hat{\tau})(\hat{\tau}-2) - 4}{4(\hat{\tau}-2)}$. Thus, given $0 \leq \hat{\tau} \leq 1$, $\theta' > 0$ if and only if $b(\hat{\tau}) < \frac{4-3\hat{\tau}}{16-8\hat{\tau}}$. Suppose the boundary type $\theta' > \frac{1}{2}$. Then, the indifference condition is

$$-\hat{\tau}\left[\frac{\frac{1}{2} + \theta'}{2} - (\theta' + b(\hat{\tau}))\right]^2 - (1-\hat{\tau})\left[\frac{\theta'}{2} - (\theta' + b(\hat{\tau}))\right]^2 = -\left[\frac{\theta' + 1}{2} - (\theta' + b(\hat{\tau}))\right]^2. \tag{A.23}$$

Solving (A.23) gives $\theta' = \frac{\hat{\tau} - 8b(\hat{\tau})(\hat{\tau}-2) - 4}{4(\hat{\tau}-2)}$. Thus, given $0 \leq \hat{\tau} \leq 1$, $\theta' > \frac{1}{2}$ if and only if $b(\hat{\tau}) < \frac{\hat{\tau}}{16-8\hat{\tau}}$. Since $\frac{4-3\hat{\tau}}{16-8\hat{\tau}} \geq \frac{\hat{\tau}}{16-8\hat{\tau}}$ for $0 \leq \hat{\tau} \leq 1$, $\frac{4-3\hat{\tau}}{16-8\hat{\tau}}$ is the maximum threshold. This proves the necessity. For sufficiency, it suffices to focus on the case where $\theta' < \frac{1}{2}$. Suppose interval-type h holds off-equilibrium beliefs that coincide with his pre-communication beliefs. Then, it is obvious that h has no incentive to send a false advice. And note that interval-type l will not receive a false advice given $\theta' < \frac{1}{2}$.

□

Proof of Proposition 3.2. For equilibria informative for both l and h, it suffices to focus on equilibria in which there are two steps in each of $[0, \frac{1}{2})$ and $[\frac{1}{2}, 1]$. Consider an arbitrary real interval $[t_0, t_1]$. Partitioning $[t_0, t_1]$ by θ', the indifference condition is

$$-\left[\frac{t_0 + \theta'}{2} - (\theta' + b(1))\right]^2 = -\left[\frac{\theta' + t_1}{2} - (\theta' + b(1))\right]^2. \tag{A.24}$$

Solving (A.24) gives $\theta' = \frac{1}{2}(t_0 + t_1 - 4b(1))$. Thus, $\theta' > t_0$ if and only if $b(1) < \frac{1}{4}(t_1 - t_0)$. Since the length of the half-intervals is $\frac{1}{2}$, there are two steps in each of them, with the corresponding boundary types $\bar{\theta}_1 = \frac{1}{4} - 2b(1)$ and $\hat{\theta}_1 = \frac{3}{4} - 2b(1)$, if and only if $b(1) < \frac{1}{8}$. This proves the necessity. For sufficiency, suppose interval-type l holds off-equilibrium beliefs that θ is distributed uniformly over $[0, \frac{1}{4} - 2b(1)]$, and h holds off-equilibrium beliefs that θ is distributed uniformly over $[\frac{1}{2}, \frac{3}{4} - 2b(1)]$. Then, it is obvious that neither l nor h has a strict incentive to send a false advice because then the off-equilibrium actions coincide with the equilibrium ones. The proof for equilibria informative only for one of the interval types follows straightforwardly.

□

Proof of Proposition 3.3. For every strict message-sharing equilibrium to be ψ-robust, note that, if the expert's strategy in question is strict message sharing, then, for all $m \in M$ and all $s = l, h$, $\Theta_\sigma(m) \cap s \neq \emptyset$. The proof for equilibria informative for either one of l and h is similar.

$\square$

Proof of Proposition 3.4. The proof is by construction. Suppose $\alpha(1) = r_1$. Upon observing r_1, the expert starts the continuation game with $\hat{\tau} = 1$. From the proof of Proposition 3.2, the boundary types in the expert's equilibrium partition are $\bar{\theta}_1 = \frac{1}{4} - 2b(1)$ and $\hat{\theta}_1 = \frac{3}{4} - 2b(1)$, where $b(1) = b_0 + b_1$. An informed decision maker's expected payoff from this partition, which is equivalent to his expected payoff from sending $r = r_1$, is $V^d(r_1, 1|\sigma) = -\frac{1}{192} - (b_0 + b_1)^2$.

On the other hand, upon receiving r_0, the expert starts the continuation game with $\hat{\tau} = 0$, which is equivalent to the CS model. The boundary type in the CS model is $\theta_1 = \frac{1}{2}(1 - 4b(0))$, where $b(0) = b_0$. An informed decision maker's expected payoff from this partition, which is equivalent to his expected payoff from deviating to send $r = r_0$, is $V^d(r_0, 1|\sigma) = -\frac{1}{48}(1 - 6b_0(1 - 4b_0))$.

For the informed decision maker to have incentive to report his type, we need $V^d(r_1, 1|\sigma) \geq V^d(r_0, 1|\sigma)$, which holds for $b_0 \in (\frac{1}{12}, \frac{1}{8}(\sqrt{3} - 1)]$ and $b_1 \leq \beta(b_0) \equiv \frac{1}{8}\sqrt{1 - 8b_0(1 - 4b_0)} - b_0$.

$\square$

Proof of Proposition 4.1. Suppose there is an equilibrium of the transformed model with information structure $(T, g) = (\tau(\Omega), q \circ \tau^{-1})$ in which the set of actions A is induced. Denote the set of types that induces $a \in A$ by $T(a)$. Note that all $t \in T(a)$ sends message $m \in M_\sigma(a)$ in the equilibrium. Construct an equilibrium for the uncertain expertise model with information structure (Ω, q) as follows. Denote the set of $\omega \in \Omega$ such that $\tau(\omega) = t' \in T$ by $\Omega(t')$. Consider an arbitrary $a' \in A$. Suppose, in the uncertain expertise model, all $\omega \in \{\Omega(t) : t \in T(a')\}$ send message $m \in M_\sigma(a')$, which is available to them. Upon receiving any message $m' \in M_\sigma(a')$, the decision maker in the uncertain expertise model updates his beliefs in such a way that $\sum_{\omega \in \Omega(t)} \nu(\omega|m') \equiv \mu(t|m')$. Thus, $\sum_{\omega \in \Omega} U^d(a|\omega)\nu(\omega|m')$ is equivalent to $\sum_{t \in T} U^d(a, t)\mu(t|m')$ minus a positive term that is independent of a; that $a' = \text{argmax}_{a \in \mathbb{R}} \sum_{t \in T} U^d(a, t)\mu(t|m')$ implies $a' = \text{argmax}_{a \in \mathbb{R}} \sum_{\omega \in \Omega} U^d(a|\omega)\nu(\omega|m')$, and a' is induced by $\omega \in \{\Omega(t) : t \in T(a')\}$ in the uncertain expertise model. Suppose the above applies to all $a \in A$. Given that $U^e(a, b|\omega)$ is equivalent to $U^e(a, t, b)$ minus a positive term independent of a, that in the transformed model, for all $t \in T(a')$, $U^e(a', t, b) \geq U^e(a, t, b)$ for all $a \in A$, $a \neq a'$ implies that in the uncertain expertise model, for all $\omega \in \{\Omega(t) : t \in T(a')\}$, $U^e(a', b|\omega) \geq U^e(a, b|\omega)$ for all $a \in A$, $a \neq a'$; thus, it is a best response for the expert in the uncertain expertise model to do what is prescribed as above. Note that by definition there does not exist $\omega \in \Omega$ such that $\tau(\omega) \neq t$ for some $t \in T$; an equilibrium of the uncertain expertise model is thus constructed in which the exact same A is induced.

Necessity in the last statement of the proposition holds in the above construction. For sufficiency, suppose there exists an ω, $\tau(\omega) \notin T(a')$, that induces a' in the above constructed equilibrium. This implies that there exist a $t \notin T(a')$ that induces a' in the equilibrium of the transformed model, a contradiction.

□

Proof of Proposition 4.2. The proof is essentially the same as those of Lemma 1 and Lemma 8 of Ivanov [25]. For completeness, I reproduce the proofs here. I start with the finiteness of A. Suppose a and a', $a < a'$, are two induced actions in an equilibrium of the transformed model. This implies that there exist a $t \in T$ and a $t' \in T$ that induce, respectively, a and a'. To these two types, $U^e(a,t) \geq U^e(a',t)$ and $U^e(a',t') \geq U^e(a,t')$; since b is irrelevant for the arguments in this proof, for clarity I suppress it in the expert's payoff function. Given that $U^e(\cdot,\cdot)$ is continuous, this implies that there exists a $\bar{\theta} \in [0,1]$ such that $U^e(a,\bar{\theta}) = U^e(a',\bar{\theta})$. Define $a^i(\bar{\theta}) \equiv \operatorname{argmax}_{a' \in \mathbb{R}} U^i(a,\bar{\theta})$, $i = e,d$. That $U^e_{11} < 0$ implies that $a < a^e(\bar{\theta}) < a'$. I show that a is not induced by any $t'' < \bar{\theta}$. Note that, under the sorting condition, for $t'' > \bar{\theta}$, $U^e(a',t'') - U^e(a,t'') > U^e(a',\bar{\theta}) - U^e(a,\bar{\theta}) = 0$, implying that $U^e(a',t'') > U^e(a,t'')$. Thus, any $t'' > \bar{\theta}$ does not induce a. Similarly, it can be shown that a' is not induced by any $t'' < \bar{\theta}$. This implies that, for all $t > \bar{\theta}$, $\sigma(m|t) = 0$ a.e. on $M_\sigma(a)$, and, for all $t < \bar{\theta}$, $\sigma(m|t) = 0$ a.e. on $M_\sigma(a')$. Next, I show that $a \leq a^d(\bar{\theta})$. Suppose not. Then, given that $a^d(\cdot)$ is strictly increasing, for any $t'' < \bar{\theta}$, $a^d(t'') < a^d(\bar{\theta}) < a$. Since $U^d_{11} < 0$ and, by definition, $U^d(a^d(\bar{\theta}),\bar{\theta}) > U^d(a,\bar{\theta})$, $U^d(a^d(\bar{\theta}),t) > U^d(a,t)$ for all $t \leq \bar{\theta}$. Given that $\sigma(\cdot|t) = 0$ a.e. on $M_\sigma(a)$ for $t > \bar{\theta}$, for $m \in M_\sigma(a)$, it follows that

$$\sum_{t\in T}\left(\frac{g(t)\sigma(m|t)}{\sum_{t'\in T} g(t')\sigma(m|t')}\right)U^d(a^d(\bar{\theta}),t) > \sum_{t\in T}\left(\frac{(g(t)\sigma(m|t)}{\sum_{t'\in T}(g(t')\sigma(m|t')}\right)U^d(a,t),$$

and this contradicts that a is the decision maker's best response to m. Similarly, it can be shown that $a^d(\bar{\theta}) \leq a'$. Thus, we have $a < a^e(\bar{\theta}) < a'$ and $a \leq a^d(\bar{\theta}) \leq a'$. Since $a^e(\theta) > a^d(\theta)$ for all $\theta \in [0,1]$, the two inequalities imply that there exists an $\epsilon > 0$ such that $a' - a \geq \epsilon$. Since the set of actions induced in equilibrium is bounded by $a^d(0)$ and $a^d(1)$, it follows that the set of actions induced in equilibrium is finite.

I proceed to prove the five properties of the induced actions. For Property 1, suppose, on the contrary, that t induces a_i and a_j where $j \geq i+2$. This implies that $U^e(a_i,t) = U^e(a_j,t) \geq U^e(a_k,t)$ for all $a \in A$. Since $a_{i+1} = \lambda a_i + (1-\lambda)a_j$ for some $\lambda \in (0,1)$, that $U^e_{11} < 0$ yields a contradiction that $U^e(a_{i+1}) > \lambda U^e(a_i,t) + (1-\lambda)U^e(a_j,t) = U^e(a_i,t)$.

For Property 2, note that given $\rho(m) = \sum_{t \in T}\left(\frac{g(t)\sigma(m|t)}{\sum_{t' \in T} g(t')\sigma(m|t')}\right)t$, $t_1 \leq \rho(m) \leq \max\{T\}$ for all $m \in M$. In other words, $\max\{A\} \leq \max\{T\} < \max\{T\} + b$. Since $U^e(a, \max\{T\})$ is strictly increasing for $a \leq \max\{T\} + b$, $U^e(\max\{A\}, \max\{T\}) > U^e(a, \max\{T\})$ for all $a \in A$, and $\max\{T\}$ purely induces $\max\{A\}$.

For Property 3, suppose, on the contrary, that t induces a, and there exists $t' < t$ that induces $a' > a$. This implies that $U^e(a,t) \geq U^e(a',t)$ and $U^e(a',t') \geq U^e(a,t')$. The two inequalities further imply that $U^e(a',t') - U^e(a,t') \geq 0 \geq U^e(a',t) - U^e(a,t)$. This contradicts the sorting condition that $U^e_{12} > 0$.

For Property 4, note that, if t' and t'', $t' < t''$, induce a, then, by Property 3, any t such that $t' < t < t''$ does not induce $a' > a$ or $a'' < a$. This implies t purely induces a.

Finally, for Property 5, suppose, on the contrary that t_i induces a_i and a_{i+1}, but t_{i+1} does not induce a_{i+1}. By Property 3, any $t < t_i$ does not induce a_{i+1}, and t_{i+1} does not induce a_i. Thus, by assumption, t_{i+1} induces some $a > a_{i+1}$. Applying Property 3 again implies that any $t > t_{i+1}$ does not induce a_{i+1}. Invoking the assumption again implies that any $t > t_i$ does not induce a_{i+1}. Thus, any $t \neq t_i$ does not induce a_{i+1}, and this implies that $a_{i+1} = t_i$. Since t_i induces both a_i and a_{i+1} which implies that $U^e(a_i,t_i) = U^e(a_{i+1},t_i)$, $a_i < t_i + b < a_{i+1}$. Given $b > 0$, this yields a contradiction that $t_i > t_i + b$.

□

Proof of Proposition 4.3. In the main text.

□

Proof of Corollary 4.1. Under uniform uncertain expertise, the type space of the uncertain expertise model is $\Omega \equiv \{\{[\frac{n-1}{2^{i-1}}, \frac{n}{2^{i-1}}]\}_{n=1}^{2^{i-1}}\}_{i=1}^{K}$. Applying the function τ to Ω gives the type space of the transformed model:

$$\begin{aligned} T \equiv \tau(\Omega) &\equiv \left\{ \frac{2n-1}{2^i} \middle| n = 1, ..., 2^{i-1}; i = 1, ..., K \right\} \\ &\equiv \left\{ \frac{(2n-1)2^{K-i}}{2^K} \middle| n = 1, ..., 2^{i-1}; i = 1, ..., K \right\} \\ &\equiv \left\{ \frac{i}{2^K} \middle| i = 1, ..., 2^K - 1 \right\}. \end{aligned}$$

In a full-revelation equilibrium of the transformed model, $t_i = \underline{t}_i = \bar{t}_i = a(T(m_i)) = \frac{i}{2^K}$. Since $\frac{a(T(m_i))+a(T(m_{i+1}))}{2} - t_{i+1} = \frac{2i+1}{2^{K+1}} - \frac{i+1}{2^K} < 0$ and $\frac{a(T(m_i))+a(T(m_{i+1}))}{2} - t_i = \frac{1}{2^{K+1}}$ for all $i = 1, 2, \ldots, H-1$, that $b > 0$ implies that the inequality (4.9) reduces to $b \leq \frac{1}{2^{K+1}}$.

$\square$

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