ON THE VARIANCE OF ELECTRICITY PRICES IN DEREGULATED MARKETS

by

Claudio M. Ruibal

Civil Engineer, Universidad de la República, Uruguay, 1983

Master in Business Administration, Universidad de Montevideo, Uruguay, 1993

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SCHOOL OF ENGINEERING

This dissertation was presented

by

Claudio M. Ruibal

It was defended on

August 30, 2006

and approved by

Mainak Mazumdar, Ph. D., Professor, Department of Industrial Engineering

Jayant Rajgopal, Ph. D., Associate Professor, Department of Industrial Engineering

Kim LaScola Needy, Ph. D., Associate Professor, Department of Industrial Engineering

Uday Rajan, Ph. D., Associate Professor, Ross School of Business,

University of Michigan

Dissertation Director: Mainak Mazumdar, Ph. D., Professor, Department of Industrial Engineering

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Since 1990 many countries have started a deregulation process in the electricity wholesale market with a view to gaining in efficiency, lowering prices and encouraging investments. In most of the markets these objectives have been attained, but at the same time prices have shown high volatility. This is mainly due to certain unique characteristics of electricity: it cannot be easily stored; and the flow across lines is dependent on the laws of physics.

Electricity price variance has been studied very little. Variance is important for constructing prediction intervals for the price. And it is a key factor in pricing derivatives, which are used for energy risk management purposes.

A fundamental bid-based stochastic model is presented to predict electricity hourly prices and average price in a given period. The model captures both the economic and physical aspects of the pricing process, considering two sources of uncertainty: availability of the units and demand. This work is based on three oligopoly models —Bertrand, Cournot and Supply Function Equilibrium (SFE) due to Rudkevich, Duckworth, and Rosen— and obtains closed form expressions for expected value and variance of electricity hourly prices and average price.

Sensitivity analysis is performed on the number of firms, anticipated peak demand and price elasticity of demand. The results show that as the number of firms in the market

decreases, the expected values of prices increase by a significant amount. Variances for the Cournot model also increase. But the variances for the SFE model decrease, taking even smaller values than Bertrand's. Thus if the Rudkevich model is an accurate representation of the electricity market, the results show that an introduction of competition may decrease the expected value of prices but the variances may actually increase.

Price elasticity of demand severely affects expected values and variances in the Cournot model. So does the firms' anticipated peak demand in the SFE model. Market design and market rules should take these two parameters into account.

Finally, using a refinement of the model it has been demonstrated that an accurate temperature forecast can reduce significantly the prediction error of the electricity prices.

Keywords: Electricity Prices, Deregulated Electricity Markets, Electricity Price Variance, Cournot Model, Bertrand Model, Supply Function Equilibrium, Rudkevich and Duckworth and Rosen's Formula, Stochastic Load, Hourly Prices, Average Prices, Edgeworth Expansion, Method of Cumulants, Volatility, Energy Risk Management, Electricity Derivatives Prices, Value-at-Risk, Conditional Value-at-Risk.

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NOTATION AND ACRONYMS

Indices

i, j, k, l, m	Unit indices
r, s, t	Hour indices
u	Day index

Variables

Q_a	Actual traded quantity at settlement date
S_u	Spot price of energy at day u
t	Time (hours)

Parameters

c_i	Capacity of unit i
C_i	Sum of capacities of the first i units
d_i	Variable cost of generating unit i
f	Risk-free interest rate
H	Final hour for averaging price
I	Initial hour for averaging price
K	Strike or contractual price
M	Marginal unit at daily peak
n	Number of companies in the market
N	Number of generating units in the market
p_i	Proportion of time that generating unit i is up

q_i	Proportion of time that generating unit i is down
Q_c	Contractual quantity
T	Expiration or settlement date of a derivative (days)
w_t	Weight of the hourly load at time t for the weighted average
δ_i	defined as $\lambda_i + \mu_i$
λ_i^{-1}	Mean time to failure of generating unit i
μ_i^{-1}	Mean time to repair of generating unit i
μ_t	Mean of load at time t
ν	Return rate of an asset
ρ	Correlation coefficient
σ	Volatility
$\sigma_{r,t}$	Covariance between the loads at time r and t
σ_t^2	Variance of load at time t

Functions

$D(t, p)$	Total system demand at time t as a function of p
$D_p(t)$	Derivative of total system demand with respect to price p at time t
$d_{J(t)}$	Marginal cost at time t
$F_{u,T}$	Forward price of an asset at day u for the settlement date
$J(t)$	Marginal unit at time t
$L(t)$	Actual electricity load at time t
$\bar{L}_{J(t)}(t)$	Equivalent load at time t
$M_{u,T}^F$	Market value of a forward at day u
$p(t)$	Price of electricity at time t
$\bar{p}(I, H)$	Average price between hours I and H
$p_{ml}(r, t)$	Joint probability of $[J(r) > m, J(t) > l]$
$X_j(t)$	Excess of load not met by the available generated power up to generating unit j
$Y_i(t)$	Generating unit i state at hour t (=1 if working, =0 in case of outage)
$\rho_{ml}(r, t)$	Correlation coefficient between $X_m(r)$ and $X_l(t)$

Auxiliary variables and functions

z, z_1, z_2	Standard normal random variables
$\phi(z)$	Standard normal distribution probability density function
$\Phi(z)$	Standard normal cumulative distribution function
$\phi_2(z_1, z_2; \rho)$	Bivariate standard normal distribution probability density function

Acronyms

ARIMA	AutoRegressive Integrated Moving Average process
AR(1)	AutoRegressive process with 1-unit lag
CfD	Contract for difference
CVaR	Conditional Value at Risk
FOR	Forced Outage Rate
HHI	Herfindahl-Hirschmann Index
ISO	Independent System Operator
LDC	Load Duration Curve
LI	Lerner Index
LMP	Locational Marginal Price
LOLP	Loss of Load Probability
OTC	Over the counter
PCMI	Price-Cost Margin Index
PDFCR	Peak Demand to Full Capacity Ratio
PJM	Pennsylvania - New Jersey - Maryland Interconnection
PPP	Pool Purchase Price
PSP	Pool Selling Price
SFE	Supply Function Equilibrium
SMP	System Marginal Price
VaR	Value at Risk
VOLL	Value of Lost Load

PREFACE

To my beloved mother Marta Faral and in memory of my father Santiago Ruibal.

I must first confess that working on this dissertation over these years has brought me a great joy. It has given me the opportunity to meet many wonderful people and to get to know some of them very well. I have made a number of new friends in the course of these years. In fairness, all of them must be acknowledged because it is because of their friendship and support that I have been able to reach to this point.

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1.0 INTRODUCTION

This dissertation describes an engineering approach to a challenging economic problem related to electricity prices. Deregulating electricity markets is still in early stages. Consequently, markets are not yet mature enough and their behavior is not easily understood. The recently exhibited extreme volatility of prices provides an impetus for understanding the pricing process in order to be able to predict electricity prices more accurately. Obtaining estimates of the expected values of electricity prices is not sufficient for this purpose. At a minimum, estimates of the variances of prices are also necessary.

1.1 MOTIVATION

Many countries are in the process of restructuring and deregulating their electric power industry in order to introduce competition into the production markets. Electricity production was long considered a natural monopoly. However, decade-long experiments carried out by a number of countries as well as by some regions in the United States have shown that competition is quite feasible. According to economic theory, competition provides consumer benefits: low prices, reliable services, predictable bills and future value-added services. In short, competition offers efficiency which means that the right amount of electricity is produced by the cheapest generators and consumed by those customers who value it most. In deregulated markets electricity prices are set by the market itself for every hour. Since the deregulation process started, wholesale electricity prices have however shown a great amount of variability.

This variability is extreme compared with other markets. Some examples of volatilities[1] for daily prices follow (see Weron [88]):

- treasury bills and notes have a volatility of less than 0.5%
- stock indices have a moderate volatility of about 1-1.5%
- commodities like crude oil or natural gas have volatilities of 1.5-4%
- very volatile stocks have volatilities not exceeding 4%
- 2000 Nord Pool electricity price volatility was 11%
- 2000 California/Oregon Border(COB) electricity price volatility was 15%
- 2000 Cinergy electricity price volatility was 37%.

On one hand, large variability can be expected because of the special features of electricity: it cannot be stored, it has to be produced whenever it is needed, demand is stochastic, there are many physical limitations to production and transmission. On the other hand, the relative immaturity of the electricity markets also helps contribute to the variability. It is expected that the effect of the second factor may be reduced (and in fact it has been) as time passes by and people who influence markets learn from experience.

This variability brings uncertainty to the price. The exposure to uncertainty, that is when some expected result is affected by an unknown event, is called risk. As in other more mature markets, the presence of risk gives rise to a *derivatives* market emerging for the purpose of hedging risk. This is also the case in electricity markets. A sound derivative valuation is needed in order to make these markets work properly. The market value of a derivative is closely related to the volatility of spot prices.

A derivatives market is not self standing. It is closely linked to the spot market. They influence each other. Spot prices are affected by the existence of derivatives markets.

[1]Volatility is defined as the yearly normalized standard deviation of price returns. Price return is defined as the difference between prices over a period divided by the price at the beginning of the period.

The operating policies and strategic decisions of competing firms, under deregulation, are such that these tend to maximize profits. In order to do this, however, it is necessary to be able to predict prices. When predicting price one is interested in at least two important quantities: the expected value, that is the price one expects to see in the future, and the prediction error, that is how inaccurate the estimation may be.

Batlle [9] has classified the existing literature on electricity price models into three wide groups according to the methods they use: game[2] theory[3] models, time series models, and production cost models. To date it seems that no work has attempted to combine these models to get the best from each one in order to form a more comprehensive and complete model.

Game theory models have been used extensively. Their main advantage is capturing the bidding process but with no consideration of the complicated engineering production process.

Time series models are the most common because there are many well-known tools that can be used to analyze the historical data. But they ignore both the bidding and the engineering components of the pricing process. They are based on historical data from which a model is extracted, calibrated and validated. They lack the flexibility of adapting to structural changes like technology upgrades, increase in the number of competing firms and new rules. Any change requires a new model, or at least new parameters that need to be re-calibrated.

Production cost models are abundant in the literature about power markets because they were useful before the deregulation trend had started. In regulated markets, while the price is fixed, the firms want to predict cost. Cost is one of the ingredients of electricity price but not the only one. These models try to capture the engineering process but they do not portray the bidding behavior of competing firms.

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[2] A game is defined as a set of players who must independently choose among a set of strategies to optimize their individual payoff functions.

[3] Game theory is a combined branch of economics and mathematics that study the economic behavior.

Under the important assumption of symmetric markets (i.e. firms are identical) and without considering transmission constraints, this work attempts to develop a bottom-up stochastic model for electricity prices based both on game theory and production cost models. This model is used to get closed form expressions for the expected values and variances of hourly prices and average prices in the real-time (spot) market.

1.2 CONTRIBUTIONS OF THIS WORK

From a theoretical point of view, this work is perhaps a first attempt to compute the variance of electricity prices in deregulated markets using a bid-and-process-based stochastic model. The model itself is the main contribution of this work. It is flexible and adaptable to different supply systems, by calibrating the significant parameters. The conclusions are data depending.

It integrates the physical and engineering processes and the bidding strategies to define the price as a stochastic process. A supply model and a demand model capture the engineering process. Three different bidding behaviors are considered: Bertrand, Cournot and Supply Function Equilibrium (SFE). For each, the outcome is an expression of the hourly price as a function of stochastic variables, related to demand and supply, whose probability distributions can be ascertained. Thus the probability distribution of hourly prices can be obtained. The first two models (supply and demand) and the last three models constitute the warp and woof of the pricing process.

Under some assumptions, closed form expressions for the variance of both hourly real-time prices and daily average prices are found. The statistics of hourly real-time prices are useful from a very short-term perspective in order to ascertain the opportunity for offering energy from a specific unit, or deciding the supply bid function, or withdrawing a unit to do maintenance. The daily average price is useful for a longer term outlook, to make decisions about investments and to forecast profitability.

From a practical point of view, this work may help answer some of the following questions that market participants may have:

- How can a generation company forecast prices to make its production plan? Which electricity generating units will be called upon to produce on a given day? In order to carry out the maintenance plan or substitute some units it will be useful to know the variance of the merit-order index of the marginal unit.
- Will prices be attractive enough to promote investments so as to supply enough energy in the future? A competitive market must be capable of growing at the same pace as demand.
- What will the company's cash-flow be in a given period? Revenues depend on prices and on the amount of energy sold. In the computation of forecasted cash-flow the variance of prices has an important role.
- What is the financial risk level for a new firm entering the market? How can one hedge that risk? What will be the hedging cost? To cope with the volatility of prices financial markets have developed derivatives to hedge the risk. The design of the derivatives, and their costs, are mainly based on the variance of prices.
- How does the number of firms affect the mean and variance of price?
- What is the effect of price elasticity of demand[4] on price variability? Electricity demand elasticity is zero or very close to it because, in most of the markets where deregulation is in place, it is only the wholesale market that is subject to it, keeping a regulated price for end consumers. That means that the demand side is not sensitive to the variation in the wholesale price of electricity in the short run.

1.2.1 Approach

This is a bottom-up or process-based approach, which means that the models try to capture and integrate the dynamics of the generation process and the bidding process as well. This

[4]Price elasticity of demand is defined as the percentage change in quantity as a reaction to 1% change in price.

bottom-up approach should provide more accurate information than a statistical analysis of historical data because the composition of the participating generators in a given market will be ever-changing in number and technology. It also differs from a top-down perspective in which economic factors play the most important role.

The stochastic model proposed in this dissertation captures the uncertainty of both the load and the availability of the units. Due to the bidding rules prices depend heavily on the generation cost structure of the system and on the magnitude of hourly load. This aspect is crucial under deregulation, especially because the electricity prices have recently shown a large variation. Electricity companies need to make decisions under uncertainty. The more knowledge the participants have on the probability distribution of prices, the better off they will be to compete.

Few models combine a process-based fundamental as well as stochastic approach, and at the same time take into account the market equilibrium. None of the existing models has addressed directly the topic of variance of electricity prices. See subsection 4.1.4.

The model presented in this work improves other models in the following respects:

- it uses a more realistic modeling of supply curves,
- it includes the effect of ambient temperature on the load,
- it considers forced outages of generation units,
- it takes into account the market structure (number of firms, installed capacity and marginal cost),
- it is based on market equilibrium.

Game theory plays an important role in this work. The market equilibrium mentioned above proceeds from well known studies on oligopolistic games. Assumptions about the firms' behavior and knowledge of the market are made following standards of game theory.

This approach integrates the engineering aspects with the economic ones. There are many valuable papers in Economics studying the electricity markets under deregulation (see sections 4.2 and 4.3 below). It is a hot and current topic, perhaps because deregulation processes have recently started in many countries and they have shown signs of oligopolistic behavior. But they do not take into account the underlying engineering processes. In general they are theoretical studies on the market equilibrium. On the other hand, there are also some important research work on electricity production-cost based pricing models but they do not consider the bidding aspects of the process (see section 4.1.2 below).

1.2.2 Research objective

The objective here is to study the variance of the hourly real-time (spot) generation price, using a fundamental model, that includes physical (engineering) and economic aspects. It will study the propagation of uncertainty from demand and from the availability of generating units to the wholesale hourly electricity price, considering three economic models. It will also produce a closed form expression for the variance of the daily average real-time generation price. Finally a stochastic model will be used to look at the extent to which the error in predicted prices is reduced if an accurate temperature forecast is available.

1.2.3 Assumptions and limitations

The way that electricity prices are cleared in a deregulated market is extremely complex. Many factors intervene in the process:

- market rules (bidding patterns, schedule, cap price, derivatives),
- market structure (number and size of the firms, market power),
- demand elasticity,
- transmission infrastructure,
- reliability regulations (capacity reserve, ancillary services),
- fuel cost,
- unit commitment,

- forced outages,
- level of demand.

This work provides a simplified model in order to obtain several preliminary conclusions. The assumptions for this work are as follows.

In real power markets there are large generating companies which actually influence the wholesale electricity price, and there are many other small firms which are price-takers and constitute a fringe in the power market. These last ones usually do not own the marginal unit. The model only considers the large ones as competing firms. The model goes a step further and assumes that the competing firms are symmetric. This assumption is far removed from the real world but it is frequently made and needed for tractability purposes.

Transmission failures and transmission congestion are not included in the model because transmission is not a binding issue in the majority of situations (locations and hours). Fuel costs are random variables but the model assumes them to be deterministic because they do not change much in the short term.

1.3 ORGANIZATION OF THE WORK

The first three chapters introduce the electricity markets. Chapter 2 explains the characteristics of the power markets and the on-going deregulation trend. Chapter 3 covers fundamental aspects of risk management and how this study may provide a tool for formulating this important topic in the context of a young derivative market for electricity. Chapter 4 describes the basic economic theory for electricity prices and provides justification for the choice of the models used in this work.

Chapters 5 and 6 are the core of the analysis and develop the formulas for the expected value and variance of hourly price and the average daily price respectively. Chapter 7 gives numerical examples for the results obtained in chapters 5 and 6 using illustrative supply and demand models. A code written in Matlab is used to run the model.

Chapter 8 depicts a stochastic model of the load and considers the use of ambient temperature to forecast load more accurately. The objective is to show the extent of reduction in the error of the predicted prices. Finally, chapter 9 states the conclusions and recommendations for future work.

2.0 ELECTRICITY MARKETS

Electricity is a very peculiar commodity both from engineering and economic perspectives. There is no other commodity or product with similar characteristics. In order to understand the economic problem it is necessary to know how the markets operate and the underlying economic concepts. At the same time, it is important to get acquainted with the engineering aspects of electricity generation. Both topics are closely interlinked.

This chapter describes the characteristics of electricity markets that make them special and how they affect the pricing process. It also explains the deregulation trend in many countries and how this influences prices. Several different types of electricity markets are described. By the end of the chapter the object under study of this work is clearly defined.

2.1 WHY ARE ELECTRICITY MARKETS DIFFERENT?

2.1.1 The physics behind electricity

Electricity is not storable —at least it is not efficiently storable in great quantities. Its demand must always be met in real time. Many exogenous events can influence both demand and supply. Climatic events like high ambient temperature can change demand dramatically. A thunderstorm can damage transmission lines and consequently curtail supply. Unit outages and line congestion give rise to uncertainty of supply.

Another important aspect that needs to be brought into consideration is transmission congestion. Electricity follows physical transmission rules like:

- Electricity takes the path of least resistance.
- The transmission of power over the network is subject to a complex series of physical interactions (e.g., Kirchhoff's laws[1]).
- Electricity travels at the speed of light.

As a consequence, some paths may get congested putting at risk the security of infrastructure. Congestion may make it necessary to use more expensive units to supply energy to a specific place. The speed of transmission requires a permanent control over the system to avoid shortages or dangerous deviations (frequency, voltage). The time available to react with a corrective action is very short.

2.1.2 Strategic analysis of the electricity industry

Michael Porter's Five Forces model (Porter [58]) for industry analysis helps us to understand the electricity industry. The model assumes that a company is driven by five forces. In particular the price is affected by them. These forces are: level of rivalry, threats of substitutes, buyers' power, suppliers' power and barriers to entry (or, to the contrary, threats of entry).

Level of rivalry.

The experience on competition in commodity markets is vast and widespread. Under perfect competition[2] it is well known that price equals marginal cost[3]. But the electricity

[1] First Law: The current flow into any node in a circuit equals the current flow out.
Second Law: The voltage drops around any loop add up to zero.

[2] The market is under perfect competition when agents *act competitively*, have *well-behaved costs* and *good information*, and *free entry* brings the economic profit level to zero. To *act competitively* is to take the market price as given (agents are price takers). *Well-behaved costs* imply that short-run marginal cost increases with output and the average cost of production stops decreasing when a supplier's size reaches a moderate level. *Good information* means that market prices are publicly available. *Free entry* ensures that competitors are able to enter the market freely. Stoft [74], 1-5.2

[3] Marginal cost is defined as the cost of producing the last unit of output, or the cost of producing one

markets around the world which have already been deregulated are far from being a perfect market. In the transition from regulated monopolies towards deregulation, power markets show different levels of oligopoly[4] in which some evidence of market power[5] has been found. The special characteristics of the electricity market enumerated above make it relatively easy for generators to exercise market power as compared to producers of other goods. The market design (number and relative size of competing firms, auction type to clear the price) obviously influences the degree of rivalry too.

Threats of substitutes.

Electricity cannot be easily substituted, especially in the short term. Research is being conducted since long ago on substitutes for the main perishable sources of energy to produce electricity: coal, fuel and gas. But there is nothing competing with electricity.

Buyers' power.

Another characteristic of the electricity market is the very low price-elasticity of demand at least during certain hours. This means that a variation in price has almost no influence on the quantity to be sold. One explanation for this is that in most countries deregulation takes place in the wholesale market, whereas the retail market is still regulated. As a result the end user cannot react to the wholesale price and therefore cannot contribute to demand elasticity.

Suppliers' power.

Suppliers to the power generating companies are those that provide the "raw materials" which are also commodities in the majority of cases (coal, fuel, gas). The suppliers are scattered and have very little negotiating power. In other cases, nature is the supplier (hydro, wind).

more unit of output. Often, these two costs coincide.

[4] An oligopoly is a market dominated by a few sellers. Each of them can affect the market but does not control it. Each producer must consider the effect of a price change on the actions of the other producers.

[5] Market power is defined as the ability of a seller to reduce the output supplied to the market so as to raise the market price, and to do so profitably [Hunt [38], Glossary].

Barriers to entry.

Generating units are capital intensive which constitutes an entry barrier for new competitors to the market. And access to the grid is needed. The procedure to supply power to the grid also requires supervision on technical characteristics (voltage, frequency and synchronization).

As a result of all these facts, which are very difficult to change in the short term, electricity prices are very much dependent on the market design.

2.2 DEREGULATION TREND

Restructuring, competition and deregulation of power markets began less than two decades ago[6]. Power markets around the world are wending their way towards deregulation. The path is not easy. They are following the experience of the telecommunications and gas industries but they are finding different challenges along the way.

Most of the OECD[7] countries have reformed their power generation markets opening them up to free competition. This is already the case in Finland, Germany, New Zealand, Norway, Sweden, England and Wales in the UK and several states in the USA and Australia. In a few years they plan to open them up completely to include even the retail market. By the year 2006, more than 500 million people (and all large industrial users) in the OECD area will be able to choose their electricity supplier. This accounts for nearly 50% of the population of OECD countries.[8] Table 2.1 shows the global explosion of deregulation.

In the United States of America the situation is very different from state to state. There are three Independent System Operators (ISOs)[9] in the Northeast: PJM, New York and

[6] UK started the process in 1988; USA in 1992.
[7] Organisation for Economic Co-operation and Development
[8] cfr. International Energy Agency [40].
[9] System operator independent from control by any single market participant or group of participants.

Table 2.1: Deregulation by countries

Country	Year	Operator's Name
UK	1990	England & Wales Electricity Pool
Chile	1990	Centro de Despacho Económico de Carga
Argentina	1992	Mercado Eléctrico Mayorista (MEM)
Norway	1992	Nord Pool
Colombia	1995	Bolsa de Energía de Colombia
Sweden	1996	Nord Pool
New Zealand	1996	New Zealand Electricity Market (NZEM)
Australia	1998	National Electricity Market (NEM)
Spain	1998	Operadora del Mercado Español de Electricidad (OMEL)
Finland	1998	Nord Pool
US	1998	California Power Exchange (CalPX)
Netherlands	1999	Amsterdam Power Exchange (APX)
US	1999	New York ISO (NYISO)
Germany	2000	Leipzig Power Exchange (LPX)
Germany	2000	European Energy Exchange (EEX)
Denmark	2000	Nord Pool
Poland	2000	Towarowa Gielda Energii (Polish Power Exchange, PolPX)
US	2000	Pennsylvania-New Jersey-Maryland (PJM) Interconnection
UK	2001	UK Power Exchange (UKPX)
UK	2001	Automated Power Exchange (APX UK)
Slovenia	2001	Borzen
Poland	2002	Platforma Obrotu Energią Electryczną (POEE)
France	2002	Powernext
Austria	2002	Energy Exchange Austria (EXAA)
US	2003	ISO New England
Italy	2004	Italian Power Exchange (IPEX)
Czech Republic	2004	Operátor Trhu s Elektřinou (OTE)
US	2005	Midwest ISO (MISO)
Belgium	2006	Belgian Power Exchange (Belpex)

(Sources: Weron [88], and others)

New England which have full retail access[10] . California and Texas have their own ISOs. California has full retail access, but Texas does not —it is in process. Other states (Arizona, Ohio, Montana, Illinois, Michigan and Virginia) have begun reforms too but they do not have wholesale market institutions in place yet.[11]

At the inception of the deregulatory trend it was believed that an open market would behave as one in which perfect competition prevails. This has not been the case with the deregulated power markets. Rather their behavior has been closer to that of an oligopoly.

The goal of having a deregulated market is to improve efficiency on both the supply and the demand sides. Competition provides much stronger cost-minimizing incentives than regulated environments; it stimulates the creativity of suppliers to develop new energy-saving technologies and to make sounder investments. On the demand side it promotes energy conservation and ensures that electricity is consumed by the users who value it most.

The means to achieve efficiency through competition are *open access*, *restructuring* and *deregulation*. Every producer should have *open access* to transmission lines which means equal opportunity to sell the energy. *Restructuring* includes different actions to change existing companies: incorporation, privatizing, divesting. *Deregulation* means ceasing to regulate but not only by removing controls on prices and on the entry of competing suppliers but also creating the right environment for competition. Supportive market conditions must also be put into place to achieve efficiency.[12]

Deregulation also aims at stimulating investment. Electricity demand is growing at a rate of 2.7 % in the world, and 2.5 % in the USA (Figure 2.1)[13]. In order to be able to produce and deliver energy at the same growth rate, one needs to account for the fact that the installation of base load generation and transmission facilities requires a great deal of time.

Hunt [38], Glossary

[10]Retail access is the ability of different energy providers (retailers) to compete in the electricity market to sell residential, commercial or industrial customers power at unregulated rates. Hunt [38], Glossary

[11]cfr. Hunt [38], 273–277.

[12]cfr. Hunt [38], 5–8.

[13]Source: http://www.eia.doe.gov/emeu/iea/table62.html

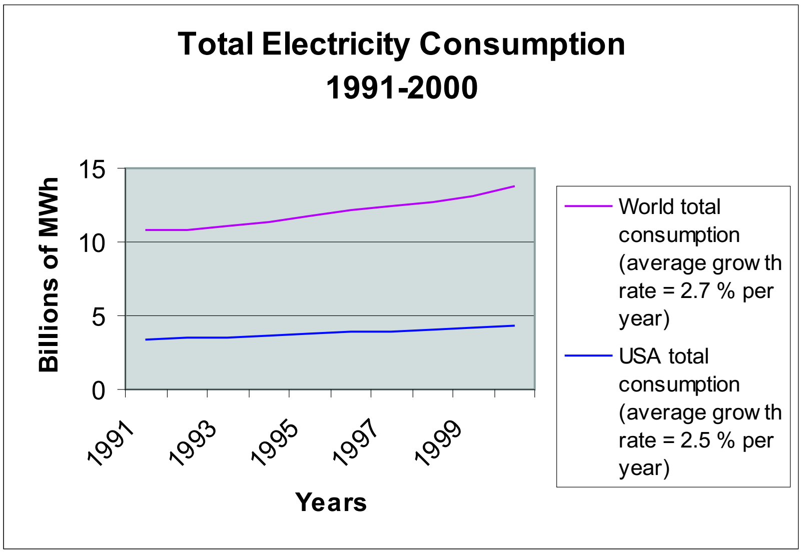


Figure 2.1: Total electricity consumption 1991 – 2000

2.3 THE ELECTRIC POWER SYSTEM

This section explains how the production, transmission and distribution of electrical energy occur. The knowledge of the engineering of electricity generation is necessary to understand the pricing process. As discussed earlier, electricity has special characteristics that deeply affect the market price.

Energy is produced by many different units that generate power from thermal, nuclear, hydro or wind energy. The specifications, performance, cost and capacity of these units are very different. The cheapest ones like nuclear, coal-fired and hydro units work continually satisfying the base load demand. For the load peaks, the more expensive generator units are called on to work in a merit order (based on respective price in order from cheapest to most expensive) as consumers demand more energy. Each unit has some technical aspects to deal with when they are called on to serve or to end serving. They require a period of time to start up and a period of time to shut down. This conditions the dispatching of units to serve. It may be more convenient to keep a specific unit running instead of shutting it down for a short time; or to start up a more expensive unit for a short time because a cheaper one requires more time to start up, for example.

The cost of producing energy may be broken down into two main types: fixed and variable costs. Fixed costs are those that do not depend on the amount of energy produced: start-up, shut-down, maintenance and depreciation costs. Variable costs , on the contrary, are proportional to the energy generated.

As demand needs to be met at every moment more units are called on to produce energy at the pace it is needed. All the properties mentioned above are relevant to decide which units will be called in and out, when, for how long, and the amount of energy to be produced by each. When electricity production was a monopoly or was regulated (or where it is still so), these decisions were taken in a centralized way, solving the so-called *unit commitment problem* together with the *economic dispatch problem*. There are many models and attempts to solve this very complex combinatorial optimization problem that have many variables and constraints and a non-linear objective function. The objective of the optimization problem is to minimize overall cost.

Under deregulation, the unit commitment problem still exists but its character has changed radically. First, there is no single omniscient decision- maker but many agents involved in the process with partial information. Second, the new objective function for each competitor is maximizing profits, not minimizing costs. So a new and very relevant variable comes into play: electricity price. Third, considering the characteristics of energy, especially the need to meet demand at every moment, another institution must take on an important role: the Independent System Operator (ISO). It is in charge of clearing the market spot price, scheduling the units and monitoring the system to meet demand continuously.

A power transmission system is sometimes referred to as a power grid. The arcs are wires and the nodes are either energy suppliers or consumers. Redundant paths and lines are provided so that power can be routed from any power plant to any load center, along a

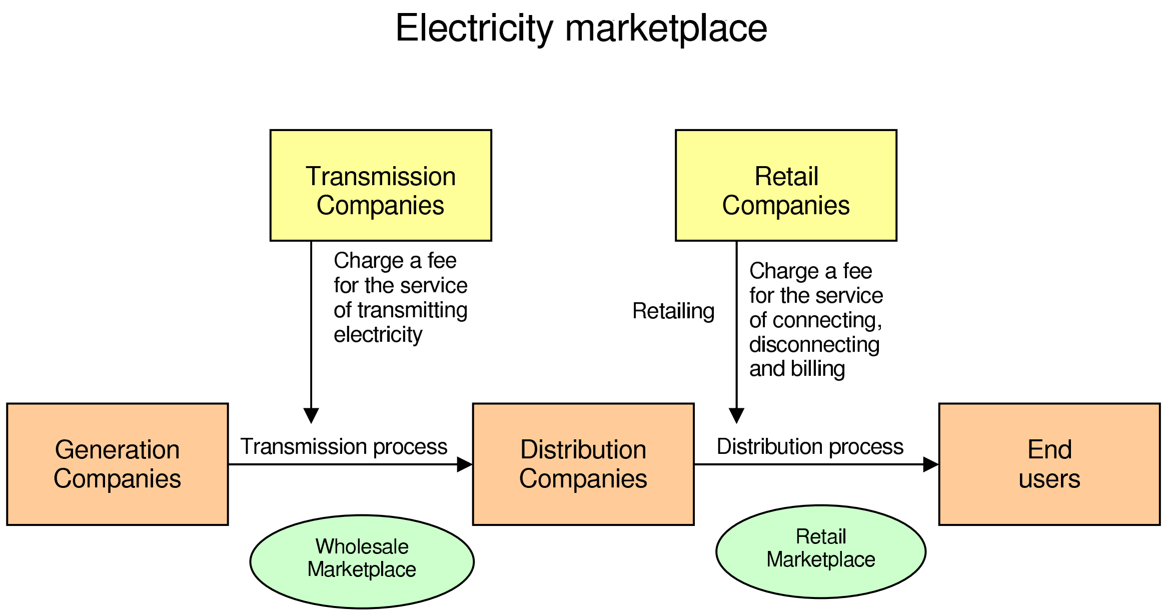


Figure 2.2: Electricity marketplace

variety of routes, based on the economics of the transmission path and the cost of power.

A first level of the grid is the *generation* level (25 kV). As soon as electricity is produced it is transformed into high voltage electricity (above 110 kV and up to 770 kV) to reduce energy losses in the *transmission* process over long distances through high-voltage lines. Transmission consists of delivering electricity from the power generation plants to large consumer points like cities or industrial parks. Once there, electricity voltage must be reduced for *distribution* purposes in the area to less than 50 kV. This voltage reduction is done in consecutive steps in stations, substations and small transformers at the city, neighborhood or block level, until the end consumer voltage (110 V or 220 V) is reached. The last step of the delivery of energy is *retailing* which consists of managing the connection, disconnection and billing of electricity consumers. See figure 2.2.

The market existing between generating companies and distribution companies, retailers or large consumers is called the *wholesale marketplace*. Delivery is made along transmission wires. Transmission companies usually neither buy nor sell energy, but charge a fee for transmitting electricity from one node to another. The marketplace existing between distribution companies/retailers and end users is called the *retail marketplace*. Electricity is delivered via distribution wires.

Electricity transmission and distribution are natural monopolies. Traditionally generation was also a monopoly everywhere. Although in many countries it is still so, in some others the generation industry has been opened up to different competing actors creating a wholesale market, a global trend that started in 1988 in the UK (see table 2.1). In some countries, retailing activity has also been opened up to a free market following the wholesale market trend, providing the opportunity for end consumers to select from different energy retail suppliers. This is known as "retail access" or "customer choice". This is the case of UK, New Zealand, Australia, Argentina, Norway, Sweden, Spain, Alberta and many states of the United States [14]. But in most other countries the retail price is still regulated. In all cases there are some intermediate actors that buy electricity at a spot price and sell it at fixed rates, who have to absorb the volatilities of the spot prices without being able to pass it on to their customers.

2.4 WHOLESALE POWER MARKETS

The challenge facing countries seeking deregulation, is how to design the new electricity markets (the number and size of participants and rules) such that the decisions made by the profit-maximizing independent companies contribute to an efficient market performance, reliable power supply and cheap prices. The first deregulated market was the England and Wales Electric Pool. Many others around the world followed. Each newcomer was looking at the experiences of the existing ones. Some of them subsequently needed to be reformed. This is the case with the England and Wales Electric Pool, which in 2001 adopted the New Electricity Trading Arrangements (NETA), substituting the old Pool Rules. The California

[14] See Hunt [38], chapter 3.

Power Exchange (CalPX) suspended trading on its markets in January 2001. In 2002, a new design started up, operated by the California Independent System Operator (CAISO). For details on market design Zhou, Grasso, and Niu [93] have given a very good review.

Deregulated electricity markets differ from one region or country to another in many ways: number and size of the competing firms, bidding rules and existence of different types of price. The following subsections present some market mechanisms.

2.4.1 Bilateral and mediated markets

There are two basic market types: *bilateral markets* and *mediated markets*. In *bilateral markets* buyers and sellers trade directly (or through brokers). In *mediated markets* there exists an intermediary who buys from the supplier and sells to the end-consumer. The most rudimentary type of mediated market is a dealer market. A dealer buys and sells at his own risk. There exist more organized forms of mediated markets — namely, exchanges and pools.

2.4.2 Exchange and pool

An *exchange* is a mediated, centralized market and provides security for traders. It is less flexible than a bilateral market because traders must follow specific rules. It utilizes auctions that give transparency to the market and constitutes a traditional method of competitive market implementation. It uses simple (one-part) bids: energy quantity and its price. Due to the lack of flexibility it can operate much cheaper, faster and closer to real-time than a bilateral market. But marginal cost is not the only cost that generating firms have. There are also start-up costs, no-load costs and ancillary services[15] among others. The drawback of an exchange is that a simple bid cannot capture this complexity.

[15] Those services are required to deliver electricity to end-users at stable frequencies and voltages; they include frequency regulation or control, spinning reserves, non-spinning reserves, and reactive supply/voltage control. (Hunt [38], Glossary)

A *pool*is also a mediated auction market characterized by the existence of side payments such as ancillary services and no-load costs. The side payments are useful to make up for costs other than marginal costs. Pools utilize complex bids (at least two-part bids): energy quantity and price, and other side items. Complex bids represent real costs better.

2.4.3 Pay-as-bid and marginal bid pricing

There are two main pricing methods: *pay-as-bid* and *marginal bid* pricing. In *pay-as-bid* pricing all the generators that bid less than the clearing price will operate and be paid as they bid. On the other hand, *marginal bid* pricing will pay the same clearing price to all the generators that run. The latter is the most widely-used methodology in deregulated power markets.

2.4.4 Day-ahead and real-time markets

Usually power markets are *two-settlement systems*. This means that the system operator runs two energy markets: a *day-ahead market* and a *real-time (spot) market*. The *day-ahead market* is essentially a forward contract market. It operates a day in advance of the *real-time market*. Transactions in the *day-ahead market* are cleared against *real-time spot prices* in the following way.

Suppose a day-ahead transaction for quantity Q_c is at strike price K. Later on the real-time transaction is for quantity Q_a and the spot price is S_T. In this case the supplier will be paid and the consumer will be charged the amount:

$$Q_c K + (Q_a - Q_c) S_T$$

2.4.5 PJM market

PJM is a marginal bid pricing pool that operates a day-ahead energy market and a real-time energy market, besides other markets like ancillary services and capacity. The day-ahead energy market is a forward market in which day-ahead locational marginal prices (LMPs) [16] are calculated for each hour of the next operating day based on generation offers, demand bids and bilateral transactions submitted to it. The real-time energy market is based on current day operations in which real-time locational marginal prices (LMPs) are calculated at five-minute intervals based on the actual system operating conditions. Technically PJM refers to demand bids as bids and to supply offers as offers. Figure 2.3[17] describes the PJM day-ahead market.

The day-ahead market matches supply with demand. The market is voluntary for the demand side. That is the buyers do not need to submit bids. Even if the load bid into the market is less than the PJM load forecast, PJM will commit units up to the forecast.

The re-bidding period, also called reliability run, is only for generation that was not selected in the day-ahead market when the results were posted at 4 pm. Generators that were not accepted in the day-ahead market are given an opportunity to re-bid if they like to. This changed bid is also the bid that will be carried into the real-time, should this unit be needed for energy. Loads cannot re-bid.

In the real-time market there is no call for new bids. Generators offered in the day-ahead market and in the re-bidding period carry through to the real-time market. Loads pay spot price if they need to purchase their energy from real-time markets. Real-time prices are calculated using the real-time flow of energy with the generation offers from the day-ahead market acting as a foundation.

[16] LMP are defined as the cost to serve the next MWh at a specific location. See page 26 for more details.

[17] Sources: PJM Training, PJM 101: The Basics: http://www.pjm.com/

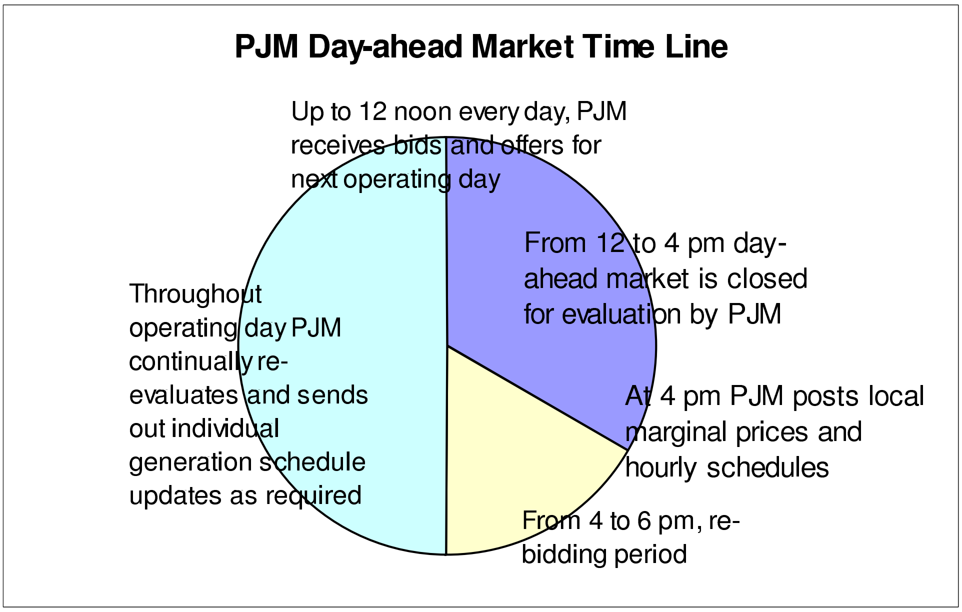


Figure 2.3: PJM Day-ahead Market Time Line

2.5 PRICING ENERGY

Over the past few years, deregulated electricity markets have gained a lot of experience regarding electricity prices and market configuration. They have succeeded in coordinating the daily system operations but the level and volatility of electricity prices have been far above expectations. There are two chapters on this topic in Ilic, Galiana, and Finck's *Power Systems Restructuring. Engineering and Economics* [39] from which I extract the main ideas of this section: Chapter 4 (by Green [31]) and Chapter 7 (by Graves, Read, Hanser, and Earle [29]).

Price spikes have been a problem since the beginning of regulation but, as time passes by, electricity markets have been able to reduce price spikes considerably. Figure 2.4[18] shows the average hourly locational marginal price during the period July-August 1999 in the PJM zone. There were many price spikes. But they have been dying down until 2004.

[18]Source: PJM, Energy Prices, 1999 LMP Duration Data & Graphs : http://www.pjm.com/

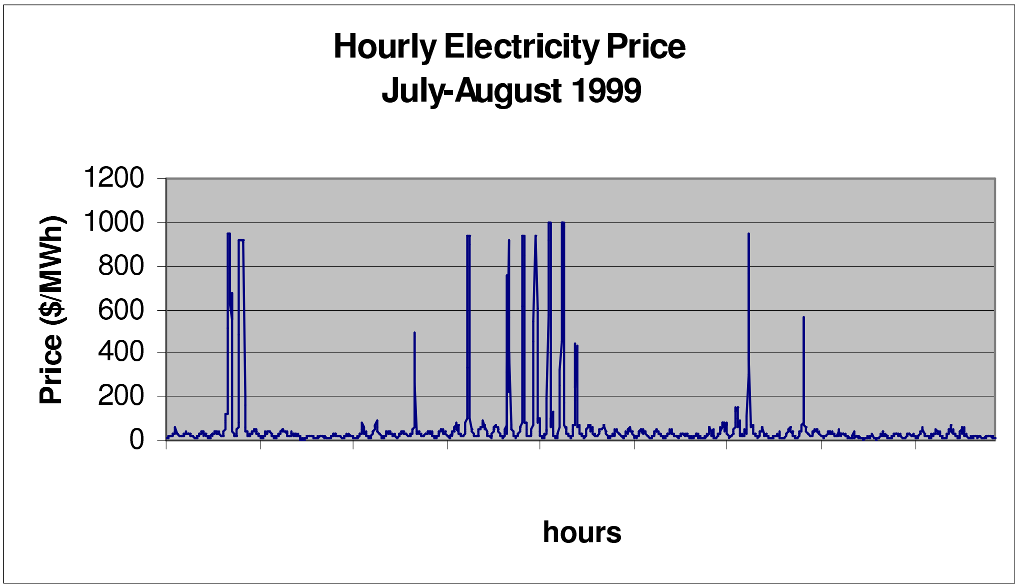


Figure 2.4: Electricity prices for July-August, 1999 (PJM market)

Nevertheless, in 2005 there were again many hours in which the PJM system price was above the $ 150 benchmark. Among other reasons this is due to significant increases in fuel cost for the marginal units and in load, geographic expansion of the PJM zone and a hotter summer. See table 2.2.

One tool used to cope with the volatility of prices is the existence of active and competitive forward markets such as the day-ahead market. It allows customers to insure themselves against price spikes. From the investor's point of view, forward contracting helps to mitigate the risks of building and maintaining new peaking capacity that may have only rare but significantly profitable use.[19]

As seen in subsection 2.4.2, there are two main pricing systems: a) one-part markets reflecting both marginal operating costs and capacity scarcity; and b) two-part markets having separate energy and capacity markets. Electricity Pool of England and Wales (UK) is an example of the first model and PJM Interconnection (USA) is an example of the second.

[19] See Graves, Read, Hanser, and Earle [29].

Table 2.2: Price spikes

	Hours with LMP above				
Year	$ 900	$ 700	$ 200	$ 150	Maximum price
1999	33	48	N/A	91	$ 999
2001	10	13	N/A	60	greater than $ 900
2002		1	N/A	20	less than $ 800
2003			1	11	$ 211
2004				5	$ 180
2005			35	234	N/A

(Source: PJM State of the Market 2002, 2003, 2004, and 2005 [56])

One-part markets tend to be more volatile than two-part markets. In the British power pool, since privatization (in 1990) and up to 1997, price has not risen significantly but volatility has risen dramatically.[20]

2.5.1 A one-part electricity market

Electricity Pool of England and Wales (the UK Pool) is a one-part market in which there is only one payment, without side payments. It defines two prices: the Pool Purchase Price (PPP) and the Pool Selling Price (PSP).

There are three components involved: the System Marginal Price (SMP), the Capacity Payment and a residual Uplift. PPP is the price (£/MWh) awarded by the Pool for electricity generated by generators and purchased at Grid Supply Points (GSPs). It is the sum of SMP plus the Capacity Payment. PSP is the price (£/MWh) which suppliers pay for their electricity, sold at Grid Supply Points (GSPs). It is the sum of PPP plus the residual Uplift. The Uplift is the difference between PSP and PPP covering reserve, constrained running,

[20] See Graves, Read, Hanser, and Earle [29].

forecasting errors, ancillary services and marginal plant adjustments. The Uplift is intended to recover costs not met by SMP.

The SMP is the basis for pricing. Between 1990/1 and 1995/6, SMP accounted for 85% of the demand-weighted Pool Selling Price (PSP). It is the marginal price of electricity, established day-ahead in the Unconstrained Schedule[21] through a matching of supply- and demand-side price and quantity bids in the wholesale market, settled every half-hour. SMP is equal to the *average* cost per MWh of the marginal generating unit. It includes incremental price (i.e., the actual marginal cost), start-up and no-load costs. No-load cost is the cost of running the unit unloaded.

$$\text{Marginal unit price} = \text{Incremental price } + \frac{\text{no-load cost} \times \text{duration} + \text{start-up cost}}{\text{total output}}$$

Capacity Payment is a component that encourages the reserve of capacity to prevent outages. Every MW of capacity which is declared available in a half-hour receives a capacity payment for that half-hour whether or not it is scheduled to be generated. It reflects the cost to society of an outage. This depends on the Loss of Load Probability (LOLP) [22] and the Value of Lost Load (VOLL)[23]. LOLP is calculated by the UK Pool to measure the risk of a power cut while the VOLL is set by the British government.[24]

$$\text{Capacity Payment} = LOLP \times (VOLL - SMP)$$

2.5.2 A two-part electricity market

PJM is a two-part market because it uses one price for electricity and one price for other services (ancillary services, capacity). It defines locational marginal price (LMP) as the cost to serve the next MWh at a specific location using the lowest price of all available generation

[21]The half hour by half hour schedule of generating units notionally required to meet forecast demand and reserve, which is produced the day ahead of trading, ignoring transmission constraints.

[22]Loss of Load Probability is the probability that the electricity system will have a service interruption due to a lack of generating capacity. (Hunt [38], Glossary)

[23]Value of Lost Load is the cost to end-use customers if power is cut off. (Hunt [38], Glossary)

[24]Sources: Green [31] and The Electricity Pool web site: www.elecpool.com

while observing all transmission limits. In other words, the marginal cost to provide energy at a specific location depends on marginal cost to operate generation, total load (demand) and cost of delivery on transmission system. Its components are:

$$
\begin{aligned}
LMP &= \text{Generation Marginal Cost} + \text{Transmission Congestion Costs} \\
&\quad + \text{Cost of Marginal Losses}
\end{aligned}
$$

LMPs are equal when the transmission system is unconstrained (ignoring loss component), and they vary by location when the transmission system is constrained. Generators get paid at generation bus LMP. Loads pay at load bus LMP. LMPs are settled every hour. (Source: PJM web site [56])

2.5.3 Congestion management

Electricity travels through a complex wired grid following the laws of physics. There is no way to control the path of electricity. If flow is larger than the line capacity the line will be overloaded. The only thing that can be done to prevent overloading a line is by asking some generators to produce less and others to produce more.

When the flow through a line (or many) reaches its transmission capacity a congestion event is said to have occurred. *Congestion management is the process of managing the use of the transmission system so* [that] *transmission capacity constraints are not violated* (Hunt [38], Glossary).

Congestion management is one of the toughest problems in electricity market design (Stoft [74], Section 1-2.2). There are many ways to handle this problem. The most efficient one is nodal pricing adopted by PJM, New York and New England ISOs. The following quote comes from 2004 PJM State of the Market [57]:

> *Congestion occurs when available, low-cost energy cannot be delivered to all loads as a result of limited transmission facilities. When the least cost available energy cannot be delivered*

to load in a transmission-constrained area, higher cost units in that area must be dispatched to meet the load.[25] *The result is that the price of energy in the constrained area is higher than elsewhere and congestion exists. Locational Marginal Prices (LMPs) reflect the cost of the lowest cost resources available to meet loads, taking into account the actual delivery constraints imposed by the transmission system. Thus LMP is an efficient way of pricing energy supply when transmission constraints exist. Congestion reflects this efficient pricing.* [26]

As the 2004 PJM State of the Market [57] shows in Section 6, Local Congestion (Figures 6-9 through 6-36), the congestion component of the LMP is very small: around 1% of the annual average LMP.

2.5.4 Market power and market concentration

In an oligopoly, price can be manipulated by a number of large companies exercising what is called market power. Market power can generally be defined as the ability of a particular seller, or group of sellers, to influence the prices of a product to their advantage over a sustained period of time.

There are two indices to measure the extent of market power. The Lerner Index (LI) and the Price-Cost Margin Index (PCMI) defined as follows. Both measure the degree to which the actual price of a product in a market deviates from the perfectly competitive price. The LI considers the deviation over the actual price, while the PCMI does so over the perfectly competitive price[27]. The definitions are:

$$LI = \frac{\text{Actual Price} - \text{Perfectly Competitive Price}}{\text{Actual Price}} \times 100\% \tag{2.1}$$

$$PCMI = \frac{\text{Actual Price} - \text{Perfectly Competitive Price}}{\text{Perfectly Competitive Price}} \times 100\% \tag{2.2}$$

[25] This is referred to as dispatching out of merit order. Merit order is the order of all generator offers from lowest to highest cost. Congestion occurs when loadings on transmission facilities mean that the next unit in merit order cannot be used and that a higher cost unit must be used in its place

[26] 2004 PJM State of the Market [57], Section 6

[27] The Perfectly Competitive Price is equal to the marginal cost of electricity generation.The US Department of Justice (DOJ) merger guidelines state that a market can be considered competitive if PCMI is below 5%.

The PCMI and the LI are connected in the following way:

$$LI = \frac{PCMI}{1 + PCMI} \tag{2.3}$$

Market concentration refers to what the market share distribution is like in a given market. It can be measured by the Herfindahl-Hirschmann Index (HHI), which is defined as:

$$HHI = \sum S_i^2 \tag{2.4}$$

where S_i is the share of each firm in the market expressed as a percentage. HHI ranges between a very small number for an extremely atomized market, and 10,000 for a monopoly. Note that 10,000 divided by HHI yields a number that can be interpreted as the equivalent number of identical-sized firms in the market.[28]

Market power and market concentration are obviously correlated. In general, the more unconcentrated a market, the less market power can be exercised.[29]

An instrument to mitigate market power is forward markets. They reduce the capability of dominant firms to manipulate prices in times of scarcity alleviating generation market concentration concerns. They also encourage greater competition.

2.6 SUMMARY

Electricity prices are very complex and may include different components in different markets according to the market design. In order to accomplish the objective of this work it is necessary to make certain assumptions.

[28]The Federal Energy Regulatory Commission (FERC) adopted the DOJ/Federal Trade Commission (FTC) guidelines which state that a market is "unconcentrated" if its HHI is less than 1,000, "moderately concentrated" if its HHI lies between 1,000 and 1,800, and "highly concentrated" if its HHI is greater than 1,800.

[29]See Rudkevich, Duckworth and Rosen [63].

This analysis considers real-time prices which are intrinsically the stochastic variables and present a high variability. The prices in this model are cleared in mediated markets with the marginal-bid pricing method. In a one-part electricity market like the UK Pool, the object of study is the SMP. In a two-part market like PJM Interconnection, it is the LMP when the system is unconstrained.

This work does not consider either transmission constraints, or transmission costs or costs from losses. Since transmission line congestion is not that common, the analysis given here provides a close approximation to reality. The analysis becomes much more complex if the transmission constraints and line losses are included in the model.

3.0 ENERGY RISK MANAGEMENT

Until deregulation started electricity prices were fixed by a regulator. There was no way nor need to predict prices: uncertainty did not exist. Under deregulation electricity prices became uncertain. Now prices need to be predicted as accurately as possible. When predicting prices one has to look for two measures: what she expects to see in the future; and how wrong she may be, that is how far the actual price in the future might differ from the expected value.

Being exposed to uncertainty implies that somebody may be adversely affected by a future unknown event. In that case it is said that he or she is facing risk. Risk is exposure to uncertainty. In recognition of the prevailing uncertainty energy markets have recently started to be transformed by derivatives and other instruments for risk management[1].

This chapter introduces some concepts on risk management and describes the use of derivatives in energy markets. It shows the strong dependence of the derivatives' prices in power markets on the expected value and variance of electricity prices, pointing out the importance of a deeper study of these quantities. In measuring risk variance of prices also plays an important role.

Its purpose is not to cover risk management extensively but to suggest several possible uses for this dissertation's derivations in this regard. Dragana Pilipović, in his *Energy Risk. Valuing and Managing Energy Derivatives* [55], deals deeply with risk management in energy markets.

[1]The process and tools used for evaluating, measuring and managing the various risks within a company's portfolio of financial, commodities and other assets

3.1 FINANCIAL MARKETS

Derivatives are financial tools to hedge risk. A *derivative* instrument is called derivative because its value *derives* from the value of some other variable (commodity, energy, stock or any other financial instrument). The latter is called the *underlier*. The underlier may be a cash instrument (stocks, commodities, interest rates) which has a value by itself or, in turn, may be another derivative.

Some derivatives or commodities are traded on established exchanges, like the New York Stock Exchange (NYSE) or the New York Mercantile Exchange (NYMEX), and are called *exchange traded*. The role of the exchange is to guarantee or facilitate the agreement settlement.

The derivatives themselves being holder's rights (and issuer's liabilities) have some market value. It is necessary to value or price a derivative then. This is especially important when an option[2] is issued, since the issuer will want to charge a reasonable price —what is called the premium— for the option. But after a derivative is issued, it has a market value which is not constant but depends on expected spot prices and its volatility.

3.1.1 Forward and futures contracts

The simplest hedging tools are *forward* and *futures* contracts. Both are derivatives in which two parties agree on a transaction that will take place some time in the future. It has four components: *i*) the underlier, *ii*) the notional (or contractual) amount, *iii*) the delivery (or strike) price and *iv*) the settlement (or expiration) date on which the transaction will take place. The difference is that a forward is done directly between two parties (it is referred to as *over-the-counter OTC*) while a futures is settled in an exchange.

[2] An option is a contract that gives one party the right, but not the obligation, to perform a specified transaction with another party.

In energy markets forward and futures are used to hedge the risk of price variation. They are called *Contracts for Difference* (CfD). Under a CfD, the buyer (long party) will pay and the seller (short party) will receive the amount

$$Q_c K + (Q_a - Q_c) S_T \tag{3.1}$$

or, equivalently,

$$Q_a S_T + Q_c (K - S_T) \tag{3.2}$$

where

Q_c and K are contract quantity and strike price respectively, and

Q_a and S_T are actual traded quantity and spot price at the settlement date T.

At settlement, the CfD has a market value for the buyer of

$$M^F_{T,T} = Q_c (S_T - K) \tag{3.3}$$

The quantities Q_c and K are fixed at the moment of signing the CfD but S_T is a random variable.

Prior to settlement the market value of the forward incorporates the concept of *forward price*. Forward and futures prices are directly tied to the spot price: they both are risk-adjusted and net cost-adjusted expectations of the spot prices at forward points in time. A graph of forward prices for different maturities is called a *forward curve*. When a forward contract is entered the strike price K is set equal to the forward price $F_{0,T}$ seen at time 0 for the settlement date T. That is $K = F_{0,T}$. Subsection 3.1.3 takes a closer look at forward prices.

At any time $u < T$, that is before settlement, the forward market value for the long party is given by

$$M^F_{u,T} = Q_c (F_{u,T} - K) e^{-f(T-u)} \tag{3.4}$$

where

f is the risk-free interest rate, and

$F_{u,T}$ is the forward price at time u for the settlement date T.

Note that at any time, the market value for the long party is positive if the forward price is greater than the strike price. Therefore, the forward price is needed for forward valuation.

3.1.2 Volatility

Volatility is a key factor in the behavior of the derivative price process. It depicts the magnitude of the randomness of the asset price and is used as an important input in the valuation and risk management of a portfolio[3]. In option pricing, for example, the width of the price distribution determines the probability that the option expires in-the-money[4] and how much intrinsic value it has. The wider the distribution, the more value the option has.

Price return is defined as the ratio between the difference between prices over a period and the price at the beginning of the period:

$$\frac{dS_u}{S_u} = \frac{S_{u+du}}{S_u} - 1 \tag{3.5}$$

Volatility σ is defined as the price returns' standard deviation normalized by time with time du expressed in years:

$$\sigma = \frac{StDev(dS_u/S_u)}{\sqrt{du}} \tag{3.6}$$

It is also the square root of the variance of price returns in a year. In other words, it is the square root of the annualized variance of price returns. The relation between volatility and variance of the spot price is given by

$$\sigma = \frac{\sqrt{Var_u[S_T]}}{S_u\sqrt{(T-u)}} \tag{3.7}$$

where

T is the settlement (or expiration) date

S_u is the option price at day u

$Var_u[S_T]$ is the variance of the option price at the expiration date, seen at day u.

[3] A collection of assets and financial positions based on such assets.

[4] An option expires in-the-money, when it has an intrinsic value at the settlement date.

There are three different volatilities to measure:

1. Historical volatilities are observed from historical data of spot prices. They provide us with information about the past.
2. Market-implied volatilities come from the expected price of options in the future. As one has the actual market option price, he can go backwards from the expected price in the future and calculate the volatility implied in that option price. This volatility depicts what the market considers the spot price will be at some point in the future. It gives information about the future.
3. Model-implied volatilities, as the term suggests, depend on the model in use and its parameters. Given the models for spot and forward prices, one can estimate the model-implied volatilities.

This work helps to asses the market-implied volatility, reversing the process. The beliefs of expected forward prices will not determine the volatility but the other way around: the volatilities will set the forward prices. This model, in finding the variance of prices, will provide a tool to compute the volatility through equation 3.7.

3.1.3 The forward price curve

Forward prices are key inputs to any derivatives pricing and risk management calculation. They are closely related to spot prices, but they differ. Pilipović [55] explains why forward prices are not —as a rule— equal to the expected spot prices.

Under some assumptions, at any given moment in time $u < T$, the forward price curve is

$$F_{u,T} = E_u[S_T]e^{-\lambda\sigma(T-u)} \tag{3.8}$$

where

$E_u[S_T]$ is the expected value of the spot price at expiration date T seen at day u,

λ is the market cost of risk defined by $\lambda = \dfrac{(\nu - f)}{\sigma}$(see Hull [37]),

ν is the expected return, continuously compound, and

σ is the implied volatility of the stock price.

The forward price is proportional to the expected stock price and is affected also by the volatility. This model by determining the expected value and variance may provide the tools to set the forward prices.

3.2 MEASURING RISK

3.2.1 Value-at-Risk and conditional Value-at-Risk

Volatility is the traditional measure of risk in financial markets. But it fails to point out the direction of the changes in price.

Value-at-Risk (VaR) is a measure of how much an investor can lose in a very bad scenario. Rather than look only at the expected return and the variance of return, it studies what is the most an investor can expect to lose —with 95% or 99% level of confidence— in a certain period (a day, a month, a year). It can be expressed in dollars (or in any other currency) or as a percentage.

VaR is becoming a very useful tool to measure risk, up to the point of being included into industry regulations, although technically it is not a risk measure because it does not fulfill the axiom of sub-additivity[5]. It looks at the 5% or 1% worst case scenarios of the profit-and-loss (P&L) probability distribution. For example, if 95% of the cases the profit is bigger than, say -4%, the VaR is -4% with 95% level of confidence.

Three methods are used to calculate VaR: from historical data, variance-covariance method and Monte Carlo simulation.

[5] A function is called a coherent risk measure if it is monotonous, sub-additive, positively homogeneous, and translation invariant. See Acerbi and Tasche [3] for an explanation of these terms

1. The historical data method assumes that the future will develop the same way as the past, from a risk perspective. VaR can be obtained from the worst case scenarios in the past.

2. The variance-covariance method assumes that the returns are normally distributed, with mean μ_R and standard deviation σ_R. This opens the path to use the standard deviation to assess the 1% or 5% worst case scenarios, using the well known formulas:
$\mu_R - 2.33\sigma_R$ for the VaR at 99% level of confidence and
$\mu_R - 1.65\sigma_R$ for the VaR at the 95% level of confidence.

3. Monte Carlo simulation involves developing a stochastic model for price and running it several times. This method allows the analyst to modify parameters that can be expected to change in the future. After running many trials, once again, the 1% or 5% worst case scenarios provided the VaR at respective levels of confidence.

Despite its popularity, one of the shortcomings of VaR is that it indicates the minimum loss attainable in the worst 1% or 5% of the scenarios. But it does not indicate how big the losses may be. For this reason, Rockafellar and Uryasev [61] [62] presented the conditional Value-at-Risk (CVaR). This measures the losses that may be in the tail of the probability distribution. It is defined, at a given confidence level, as the expected loss given that the loss is greater than or equal to the VaR. Once more, the probability distribution needs to be known or simulated. Otherwise, it is necessary to make assumptions of normality under certain conditions. CVaR also improves on VaR because it is a coherent risk measure.

With the assumption of normality the use of the expected value and variance of prices for determining VaR and CVaR for electricity markets can be illustrated as follows. A generating firm is considering entering in a forward contract to sell 10000 MWh at a strike price of \$30/MWh one month from now. By using the model presented in this work, the expected value of the daily average price for the termination date is found to be \$32/MWh and the standard deviation \$5/MWh. The VaR per MWh at 95% level of confidence is then $30 - (32 + 1.65 * 5) = \$ - 10.25$/MWh. As the notional amount is 10000 MWh, the total VaR at 95% level of confidence results $\$ - 102,500$.

The CVaR, being the mean of the loss given that the loss is greater than the VaR, can be computed by integration between $-\infty$ and VaR. Assuming normal distribution as before, it can be shown that CVaR for 95% level of confidence is $\mu_R - 2.05\sigma_R$ and for 99% level of confidence is $\mu_R - 2.64\sigma_R$. Similar tables to table 3.1 can also be constructed if the distribution is lognormal instead. In the example, the total CVaR at 95% level of confidence is $[30 - (32 + 2.05 * 5)] * 10000 = \$ - 122,500$.

Table 3.1: Values for VaR and CVaR for normal distribution

Confidence level	VaR	CVaR
90%	$\mu_R - 1.28\sigma_R$	$\mu_R - 1.76\sigma_R$
95%	$\mu_R - 1.65\sigma_R$	$\mu_R - 2.05\sigma_R$
99%	$\mu_R - 2.33\sigma_R$	$\mu_R - 2.64\sigma_R$

3.2.2 Expected returns – variance of return objective function

Markowitz [45] pointed out that the rule that investors maximize the expected value of discounted returns when choosing portfolios must be rejected. Between two alternatives with the same expected discounted return a rational agent will choose the one that has a smaller variance. And depending on the level of risk aversion, the agent may choose an alternative with a lower expected return but a smaller variance. The conclusion is that the investor will choose what the author calls the efficient expected value-variance combination.

As the cited paper shows the portfolio selection process has two stages. The first one starts with observations and experiences and produces beliefs about the future performance of securities. The second stage consists of the portfolio selection based on the beliefs.

To apply the Markowitz' expected value-variance objective function it is necessary to form the beliefs about the future performance, that is to know the expected value and variance. This work will help to get reasonable values for the expected values and variances.

3.3 ELECTRICITY MARKETS

Energy markets are very different from money markets. Among other causes, energy has many and complex fundamental price drivers that makes it hard to model them. Seasonality, storage and delivery issues, exogenous events (weather related, wars, etc.), regulation and centralization also affect the price. For more details see Pilipovic [55].

To give a snapshot on the development of derivatives in electricity markets, for example, forty-eight different derivatives on electricity are currently traded at the New York Mercantile Exchange (NYMEX): forty-seven different cash-settled futures [6] and one option.

3.3.1 The use of derivatives in electricity markets

A transmission company can conveniently hedge the energy cost by purchasing power several months ahead. The hedge eliminates price exposure. It is particulary suited to electricity markets because this purchase does not imply delivery in advance nor storage nor an initial outlay of funds to pay. In the case of a cash-settlement forward the hedge does not need to be put on with the ultimate supplier of energy - it can be done with any other counterpart. This also adapts itself well to electricity markets in which the energy is not delivered directly from the producer to the buyer but rather is done through the grid.

For example, a transmission company might enter into an OTC cash-settled option contract to hedge the wholesale electricity price. It buys energy at the pool as usual. Should it exercise the option the counterpart will not deliver electricity in exchange for payment. It will instead pay the transmission company the option's intrinsic value. In this manner, the transmission company is protected against rising electricity prices without changing the buying process at all. The day-ahead market is a forward market.

[6] In a *cash* or *financial settlement*, the underlier is not physically delivered. Instead, the derivative settles for an amount of money equal to what the derivative's market value would be at maturity/expiration if it were a physically settled derivative. By the contrary, a derivative instrument is *physically settled* if the underlier is to be physically delivered in exchange for a specified payment.

3.3.2 The need of the variance of electricity prices in risk management

The literature on energy risk management emphasizes the need to estimate, calibrate or obtain volatility of prices using procedures different from the approach given here. Skantze and Ilic [71] model the forward price as a function of the expected value and variance of spot prices. Denton, Palmer, Masiello, and Skantze [25] take volatility as a parameter of the model they use, that must be calibrated from historical data. Burger, Klar, Müller, and Schindlmayr [17] state that the volatilities can be calibrated with futures prices as historical or implied volatilities. Roark, Skantze, and Masiello [60] use Monte Carlo simulation to sample contracts for reserve from assumed distributions on prices.

Prices of derivatives are strongly related to volatility. There has been no work done yet on getting the volatility of electricity prices from a stochastic fundamental model. This work attempts to reverse the process that generates the market-implied volatilities. That is, estimate the volatilities first and obtain the future prices.

The extreme youth of energies markets, in comparison to money markets, makes the building of models more difficult. The lack of historical data complicates the process of valuing derivatives. The market is still very "illiquid" that means that the present-day market activity is quite small. There is not enough information on spot and forward prices to understand the price drivers and to test models.

3.4 SUMMARY

As energy options markets develop (especially electricity markets), this study provides insights into spot prices variance estimation, and hence volatility that can be used for derivatives' valuation and risk measure. Specifically, this model helps to asses the volatilities of prices, reverting the process of calculating the market-implied volatilities. Instead of starting from the future prices and deducing from them the market-implied volatilities, this study computes the volatilities and helps to determine the future prices consequently.

As described before, the forward prices are influenced by the expected value and the volatility of electricity spot prices. Concepts as VaR and CVaR are based on the probability distributions of electricity prices. In this matter, the model here presented is a useful tool to estimate these risk measures. Moreover, the estimation of expectation and variance of electricity prices are the first step for an investor to select investments following the expected value-variance rule.

4.0 MODELING ELECTRICITY PRICES IN COMPETITIVE MARKETS

Since deregulation started, researchers have made a great effort to study the behavior of electricity prices in the new open markets. The number of related papers and books that has been published in the past thirteen years is remarkable.

The objective of this chapter is to review the literature in the light of the work of this dissertation, and to select and adapt the models for this purpose. The analysis is focused in three significant models for oligopolies. By the end of the chapter the complete model proposed here is well defined.

4.1 REVIEW OF ELECTRICITY MARKET MODELS

4.1.1 Game theory, production cost and time series models

Following Batlle [9], electricity price models can be classified into three categories: game theory models, production-cost models and time series models.

Game theory models are concerned with the strategic behavior of the agents and its influence on price. Market equilibrium strategies are such that, given the strategies of the other players, any single firm is better off maintaining its strategy. This is known as the Nash equilibrium, defined by Fudenberg and Tirole [28], 1.2 as:

Nash equilibrium is a starting point of most applications of game theory. It is defined as a profile of strategies such that each player's strategy is an optimal response to the other players' strategies. (...)
Equilibrium is determined by the condition that all firms choose the action that is a best response to the anticipated play of their opponents. (...)
Nash equilibria are "consistent" predictions of how the game will be played, in the sense that if all players predict that a particular Nash equilibrium will occur then no player has an incentive to play differently. A Nash equilibrium, and only a Nash equilibrium, can have the property that the players can predict it, predict that their opponents predict it, and so on.

On the other hand, production-cost (or fundamental) models simulate the energy production and market operation mechanisms. They were developed for centralized power markets and have been extended to the reformed free markets.

Time series models perform statistical analysis on the price data as time series without capturing either the engineering or the economic aspects involved. Although this category is the weakest one, because it does not account for the richness and peculiarity of electricity markets, it uses well-developed statistical tools for analyzing the data. Mateo González, Muñoz San Roque, and García-González [47] provide a wide taxonomy of these models.

The special features of electricity markets mentioned above (see section 2.1) should be taken into consideration in the model when one is interested in explaining and measuring the variability of prices. Financial or economic aspects alone do not explain the price evolution thoroughly.

4.1.2 Production cost models

Methods for computing the expected production costs of a power generating system are well developed and documented: see Caramanis, Stremel, Fleck, and Daniel [19], Mazumdar [48], Mazumdar and Kapoor [49], Stremel, Jenkins, Babb, and Bayless [75]. There are two basic approaches to this computation. The first formulation is due to Baleriaux, Jamoulle, and de Guertechin [7] in which the time sequence in the chronological variation of the load is ignored and the computations are performed based on the load duration curve[1]. The second is the use of the chronological simulation models that explicitly trace the evolution of the system's states over time using the Monte Carlo method (Breipohl, Lee and Chiang [15]).

Ryan and Mazumdar [65] and [66] pointed out that even under a load duration curve (LDC) framework, the Baleriaux model is not capable of calculating the variance and higher order moments of production costs over a given time interval. In order to calculate such higher order statistics of the production costs, the Baleriaux model needs to be suitably enhanced so that the statistical dependence between the amounts of energy produced by different units for every hour within the study interval can be accounted for. These correlations can be evaluated only when the stochastic processes underlying the generator outages as well as the chronological load sequence are considered.

Using this model, several authors (Huang and Hobbs [36]; Kapoor and Mazumdar [42]; Lee, Lin, and Breipohl [44]; Ryan [64]; Shih and Mazumdar [69]; Shih, Mazumdar, and Bloom [68]) have provided analytical expressions for the mean and variance of the production costs as well as for the hourly average marginal costs over a given interval. This model has also been used in the Monte Carlo chronological simulation of production costs. In general, chronological models have great flexibility in modeling operating policies and constraints. However, they can also require substantial computational effort especially because the Monte Carlo method needs repeated runs of the random scenarios to obtain statistically significant estimates. Variance reduction methods to reduce the number of required Monte Carlo runs for the production simulation with explicit accounting of the chronological constraints were proposed by Mazumdar and Kapoor [50] and Valenzuela and Mazumdar [79]. Similar methods have also been proposed by Breipohl, Lee, Huang, and Feng [16] and by Marnay and Strauss [46].

[1] A load duration curve is the demand of all hours of the year, sorted from highest to lowest. In a load duration curve graph it is possible to read how many hours in a period of time (say, a year) the load is above or equal to a given amount.

4.1.3 Electricity market modeling trends

There is a comprehensive and original review on the modeling trends for the electricity market in the publication by Ventosa, Baíllo, Ramos, and Rivier [84]. It classifies the numerous papers under different criteria and describes the strengths, weaknesses and main uses of each group.

The three main stated trends are: optimization models, equilibrium models and simulation models. The first of these three focus on the profit maximization problem for one firm while the other two sets of models represent the overall market behavior of all the competing firms. Equilibrium models can handle simplified markets models. Simulation models, on the other hand, are more suitable for dealing with more complex problems.

The attractiveness of optimization models is that very well-known robust optimization algorithms exist to solve them. But the disadvantage lies in their not considering the reaction of competitors in the market to the firm's optimal strategy in the model. They are not suitable for medium- and long-term decisions.

Equilibrium models are the most numerous of the three. The many papers on these models are mainly based on two types of market equilibrium: Cournot equilibrium and Supply Function Equilibrium (SFE). In both cases the underlying concept is the Nash equilibrium mentioned earlier. Later on the two equilibria will be considered in greater details. In Cournot competition the players offer quantities while in SFE competition they offer supply curves (quantity-price). Cournot models are more tractable but the assumptions are less realistic. On the contrary, SFE models better capture the bid process but, in general, they give rise a system of differential equations which are much more difficult to solve. In a very few cases, however, is it possible to get a closed form solution.

These models have been used for many purposes that include market power analysis, market design, medium-term electricity pricing, economic planning, investment planning and congestion management.

Simulation models can handle complex equilibrium models which otherwise would need cumbersome mathematical calculation and computing time. They can also capture the iterative characteristic of electricity markets which provides players with the opportunity to learn from previous interactions and thus adjust their strategies.

4.1.4 Fundamental stochastic models

As it was mentioned in chapter 1, few models have a fundamental and stochastic approach at the same time.

One of these models is proposed by Skantze, Gubina, and Ilic [70]. Their model is founded on the assumption of inelastic demand and on an exponential supply function. The spot price at time t is given by

$$P_t = e^{aL_t + b_t} \tag{4.1}$$

where
a is a fixed parameter characterizing the bid curve slope (the same for every t),
L_t is the market clearing quantity in hour t and
b_t denotes the position or shift of the curve.

The stochasticity of load is modeled in L_t while that of supply is done in b_t. The factors included on the load side are seasonality, uncertainty, mean reversion and stochastic growth. The factors considered on the supply side are the stochastic availability of generation, uncertain fuel costs, unit commitment and import/export from and to other markets. They consider the expected value of the price but not the variance.

A second fundamental stochastic model is due to Vehviläinen and Pyykkönen [83]. This model considers the case of the Nordic market in which more than half of the production is hydro-electric power based and approximately one fourth is nuclear power, both of which have zero or very low variable costs. They model some fundamental factors separately and then combine them into a market equilibrium model. The factors under consideration are:

climate data (temperature and precipitation), hydro-balance (temperature below zero, snow-pack level, snow melting, hydro-inflow, hydro reservoir level, hydro spill), demand and base load supply.

For the market equilibrium they assume that demand is not elastic, so the supply price function gives the spot price at the level of inelastic demand. They use Monte Carlo simulation to obtain the distribution of the spot price.

4.1.5 The selection of a framework model

The research objective is to study the variance of electricity prices. Necessarily an appropriate price model must be selected. The following conclusions help to do it.

First, time series models were discarded because they fall short of capturing the internal characteristics of the production process and the market mechanisms.

Second, an integrated simplified game theory / production cost model is preferred to combine equilibrium aspects with the market and generation processes.

Third, the model to be used must necessarily be a probabilistic one. The model should recognize the different sources of uncertainty and propagate them on to the output price.

Fourth, in all the countries where deregulation is in place power markets are oligopolies.

Therefore, an imperfect-market equilibrium model matched with a stochastic production-cost model has been developed. The uncertainty sources chosen are demand and units' availability. This combined model considers the influence of economic issues in electricity pricing such as market power, capacity withholding, bidding strategies and market concentration.

This work uses three paradigmatic equilibrium models for imperfect-market (Bertrand, Cournot, and a specific SFE), which are described in section 4.2. A comparison of the three parallel approaches may provide useful information to market designers.

The specific SFE model selected is Rudkevich, Duckworth, and Rosen's formulation (see subsection 4.3.2) because of the following reasons:

- It represents a Nash equilibrium, which means that it is stable and predictable.
- It uses realistic stepwise supply functions.
- It considers a number of competing firms.
- It explains markups over marginal costs depending on demand level, daily peak demand, cost of supramarginal units which is intuitive or, at least, coherent.
- It gives a closed form expression for the electricity price, enabling derivation of the expected value and variance.
- It is possible to carry out sensitivity analysis on it.

One drawback of the formula is that it only applies to a symmetrical market consisting of identical firms. This assumption is not very realistic.

4.2 BASIC MODELS ON ELECTRICITY PRICING

In the current literature three major models are in use for (imperfect) electricity markets: the Bertrand model, Cournot model and Supply Function Equilibrium (SFE) model. Cournot and Bertrand models constitute the two often used paradigms of imperfect competition.

4.2.1 Bertrand model

In the Bertrand (1883) model firms compete in price. They simultaneously choose prices and then must produce enough output to meet demand after the price choices become known. In the assumption that each firm has enough capacity to meet demand, the Nash equilibrium price in this model is the marginal cost which is the same as the case of perfect competition.

One of the reasons to introduce competition into power markets is to reduce the price of electricity. It was thought that under competition the prices would drop to the marginal cost level. It is generally admitted that the design of the British Pool was based on the assumption that Bertrand competition would prevail. However, this is not what happened.

A first example of the use of Bertrand competition in electricity was proposed by Hobbs [34] for studying the restructuring of the industry in the US. The rationale for retaining this paradigm is as follows. Electricity cannot be stored. If a generator has extra capacity it will be interested in selling electricity if and only if the price is above the cost of production. It will thus be subject to short-term price competition, hence leading to a Bertrand assumption. The latter is equivalent to perfect competition. It supposes marginal cost pricing when supply and demand curves meet in a single location and all producers have the same marginal costs. However, empirical studies (Wolfram [92]) have shown that prices in some imperfect markets are sustained well above marginal costs.

4.2.2 Cournot model

The other basic non-cooperative equilibrium is the Cournot (1838) model. In this model competition is in quantities. Firms simultaneously choose the quantities they will produce, which they then sell at the market-clearing price (the price for which demand is met by supply). An auctioneer will clear the market equating demand and production.

The point made by the proponents of this model (Borenstein and Bushnell [13], Batstone [10], Wen and David [87]) is that a large proportion of energy transactions are done by long-term contracts for which the price is fixed. Taking away the amount of electricity contracted, the remaining demand for electricity is much more elastic than that of the whole market. Small variations in price will produce large changes in demand. So firms will choose the quantities that optimize their profit. Under these situations the Cournot model is a more accurate representation of the market. Since generation capacities present significant constraints in electricity markets, the assumption underlying the Bertrand model that competition is over prices and the firms have enough capacity to meet demand is not sustainable. Cournot models prevail over Bertrand models in the current literature on electricity markets.

4.2.3 Supply Function Equilibrium (SFE) models

A new model has been used in recent papers (Green and Newbery [32], Bolle [12], Newbery [52] [53], Rudkevich, Duckworth and Rosen [63], Visudhipan and Ilic [85] [86], Baldick, Grant and Kahn [5], Guan, Ho, and Pepyne [33], Baldick and Hogan [6], Baldick [4]). This approach is based upon the work of Klemperer and Meyer [43] and was applied to a pool model by Green and Newbery [32]. A supply function relates quantity to price. It shows the prices at which a firm is willing to sell different quantities of output. The SFE model applies very well to the market structure of many restructured electricity markets, such as New Zealand, Australia, Pennsylvania-New Jersey-Maryland Interconnection (PJM) and California Power Exchange. In these markets the bid format is precisely a supply function.

In this model competition is neither over price (as in Bertrand models) nor quantity (as in Cournot models) but in supply functions. Bertrand and Cournot models are limits of SFE models. The Bertrand model is the limiting case in which the supply function is constant in price for any quantity, which means that the producer is bidding a price at which it is willing to sell any quantity. On the other hand, the Cournot model is the limiting case in which the supply function is constant in quantity for any price, meaning that the producer is bidding quantity that will be sold at the market-clearing price.

The problem with the use of SFE models is that in general there is not a unique equilibrium. There are often an infinite number of solutions lying between the Cournot and Bernard equilibria, which represent their upper and lower limits in price respectively. The existence of many equilibria makes it difficult to predict the likely outcome of strategic interaction between players. There are some factors that reduce the range of feasible equilibria: uncertainty of demand and capacity constraints.

SFE models better explain the markups of electricity prices which empirical studies have shown to be above the Bertrand equilibrium but below the Cournot model. It is close to the Cournot equilibrium at peak time when capacities are almost saturated and close to the Bertrand equilibrium when there is a significant capacity excess.

4.3 MORE ON SUPPLY FUNCTION EQUILIBRIUM MODELS

4.3.1 Basic papers on Supply Function Equilibrium

The basic paper for Supply Function Equilibrium models is by Klemperer and Meyer [43]. They model an oligopoly facing uncertain demand in which each firm chooses as its strategy a supply function relating quantity to price. In the absence of uncertainty, there exists an enormous multiplicity of equilibria in supply functions, but uncertainty dramatically reduces the set of equilibria. Under uncertainty and considering a linear demand function and a linear marginal cost, they prove the existence of a unique Nash equilibrium in supply functions for a symmetric oligopoly[2] producing a homogenous good if the random exogenous shock has full support. The exogenous shock ξ is the random variable(s), not under our control (weather, failures) which is(are) the source of uncertainty. Having full support means that it can take any value with the restriction that demand $D(p, \xi) > 0$ and price $p > 0$.

Green and Newbery [32] apply the work of Klemperer and Meyer [43] to study the British electricity spot market at the time of the structural changes in 1992, and show that two dominant generating firms following Nash equilibrium strategies in supply schedules will price electricity with high markups over marginal cost. In their model they consider first a symmetric duopoly. In this case the symmetric solution is

$$\frac{dq}{dp} = \frac{q}{p - C'(q)} + D_p \tag{4.2}$$

[2] An oligopoly is symmetric if all the players have identical costs, capacity and knowledge.

in which

p is the spot price

q is the supply quantity

$C(q)$ is the cost of producing the quantity q

$C'(q)$ is its marginal cost

$D(p, \xi)$ is the demand curve given ξ

ξ is a random exogenous shock

D_p is the partial derivative of D with respect to p.

The meaning of the exogenous shock ξ is that the demand D as a function of price p can be modified by external causes like ambient temperature, humidity, lack or excess of any other energy source that are not under control. The demand curve can be shifted up or down because of these or other reasons that happen randomly. Klemperer and Meyer [43] assume that the second order partial derivative $D_{p\xi} = 0$ for all (p, ξ).

Equation (4.2) is central to the Supply Function Equilibrium theory on which this work is based. For this reason, the following paragraphs and figure 4.1 are taken from Green and Newbery [32] to justify equation (4.2).

Consider points (q, p) such that

$$C'(q) < p < C'(q) - \frac{q}{D_p} \tag{4.3}$$

Then at such points $0 < dq/dp < \infty$, and the trajectory of the differential equation through this point has a well-defined positive directional slope. It can be shown that all such trajectories pass through the origin, where they have the same slope. The next step is to consider the stationaries whose equations define the lower and upper limits in equation (4.3). Consider the first equation $p = C'(q)$. This is the supply schedule of a perfectly competitive firm, and along this curve (shown as the lower dotted line in figure 4.1), $dq/dp = \infty$, so $dp/dq = 0$. Any trajectory that intersects the lower stationary reaches it with horizontal slope at a point such as B in figure 4.1, and once it has crossed the stationary it will have a negative slope.
If the trajectory reaches the upper stationary (the dashed line in figure 4.1) at a point such as C, its slope there will be $dq/dp = 0$, so $dp/dq = \infty$. It will cross the stationary vertically and then bend back. The upper stationary has a simple interpretation as the Cournot supply

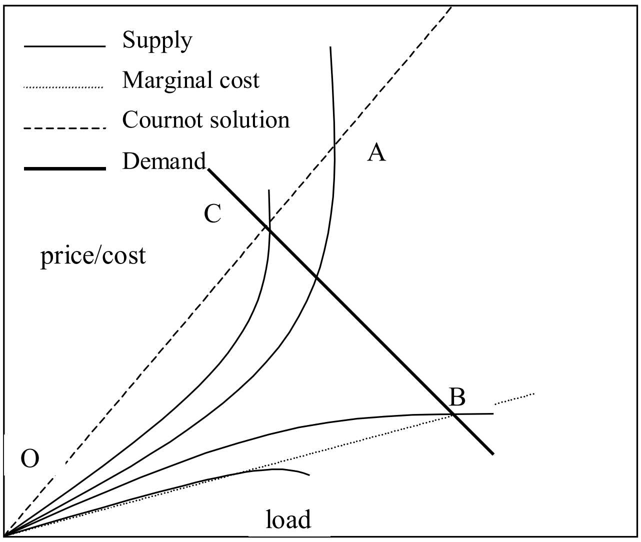


Figure 4.1: Supply function equilibrium solutions of Green and Newbery equation

schedule, for if firm j has unresponsive output k_j, then firm i is an effective monopolist with $q_i = D(p, \xi) - k_j$. The profit-maximizing choice of p satisfies

$$q_i + [p - C'(q_i)]D_p = 0 \tag{4.4}$$

or

$$p = C'(q) - \frac{q}{D_p} \tag{4.5}$$

In general, therefore, the duopoly supply schedule lies between the competitive and Cournot schedules along a trajectory such as OA in figure 4.1. Candidates for equilibrium supply schedules must not intersect either stationary over the range of possible price-output pairs.

The last sentence of the quote means that a point such as A (where the trajectory cross either (upper or lower) stationary must be outside the segments OC or OB respectively. In figure 4.1, if BC is the maximum demand, then all feasible solutions to equation (4.2) lie between the curves OC (maximum supply function) and OB (minimum supply function). Therefore, there are infinitely many solutions. As it was mentioned above, Klemperer and Meyer [43] proved uniqueness in the specific case of linear demand, linear marginal cost and full support for the exogenous shock. In figure 4.1, this means that the demand CB can be in any place. Note that if demand BC moves to the right (or up, which is the same), the set of of feasible solutions is reduced. In the limit to infinite there is only one solution.

The effect of supply constraints is to narrow the range of feasible equilibria. In the asymmetric case less output would be sold at a higher price.

To justify equation (4.5) note that the profit of firm i can be expressed as

$$\pi_i(p) = p[D(p,\xi) - k_j] - C(D(p,\xi) - k_j) \tag{4.6}$$

The first order condition is

$$\frac{d\pi_i(p)}{dp} = [D(p,\xi) - k_j] + pD_p - C'(D(p,\xi) - k_j)D_p = 0 \tag{4.7}$$

and considering that the residual demand for firm i is $q_i = D(p,\xi) - k_j$, last equation becomes

$$\frac{d\pi_i(p)}{dp} = q_i + pD_p - C'(q_i)D_p = 0 \tag{4.8}$$

which is the same equation (4.4)

4.3.2 A specific case of SFE: Rudkevich, Duckworth, and Rosen

The two preceding papers, Klemperer and Meyer [43] and Green and Newbery [32], were the basis for further research on application of SFE to electricity markets. Since their publications many authors have applied SFE to study different market scenarios.

Rudkevich, Duckworth and Rosen [63] calculated the electricity prices that would result from a pure pool market with identical profit-maximizing generating firms, bidding stepwise supply functions. They extended the theoretical concepts developed by Klemperer and Meyer [43] and Green and Newbery [32], and proposed a new formula for the instantaneous market-clearing price when generating firms adopt bidding strategies given by the Nash equilibrium using several assumptions:

- the generating firms are identical in size and have identical supply curves;
- the supply curves are stepwise;
- there is zero price elasticity of demand;
- generating firms have perfect information about one another's production cost curves;
- generating firms have equal accuracy in predicting demand.

The Nash equilibrium market-clearing price of electricity in a pool is a function of:

- the particular electric system's production cost curve (i.e., the size of the steps of capacity, and the increases in variable cost between these steps);
- the instantaneous demand for electricity;
- the maximum anticipated demand in the overall period for which bids are submitted;
- the number of identical generating firms bidding in the pool;

and is given by:

$$P(Q) = d_k + \sum_{j=k}^{m-1} (d_{j+1} - d_j) \left(\frac{Q}{C_j} \right)^{n-1} \tag{4.9}$$

where

P is the instantaneous market-clearing price of electricity in a given time interval.

Q is the instantaneous demand in a given time interval.

k is the dispatch order number of the generating unit that is on the margin in that time interval.

n is the number of identical firms.

d_k is the variable cost of the marginal unit.

j, d_j are respectively the dispatch order number and the variable cost of the generating units that are above the margin in that time interval and are expected to be on or below the margin in some other time interval during the 24-hour period.

m is the dispatch order number of the most expensive unit expected to run during the 24-hour period.

C_j is the total capacity of all generating units with dispatch order not exceeding j.

It is important to note that in the above formula Q is always less than C_j, which means that the effect on price of generating units beyond the marginal one decreases with the increase of the dispatch order number.

The formula (4.9), being a solution of a differential equation, needed a boundary condition. Rudkevich, Duckworth, and Rosen made the assumption that the price at peak demand is the marginal cost of the peak marginal unit, taking the lowest SFE possible.

Considering the nature of electricity markets, Rudkevich, Duckworth, and Rosen's formula is selected as a third model of market equilibrium for the following reasons:

- it assumes stepwise supply functions, as is the case in most actual markets,
- the structure of the market (e.g., number of firms) is reflected in the formula,
- it builds the price adding to the marginal cost other terms that depend on the generating units profile, on the daily peak demand and on the relation between the latter and the installed capacity.

4.4 THE COMPLETE MODEL

The objective of this work is to get an expression for the price variance considering uncertainty of supply and demand and market equilibrium. In deregulated markets prices result above marginal costs, contrary to what was originally thought. The reason is that power markets are not perfect: only a few players actually influence the price.

After reviewing how the electricity markets work (chapter 2), the need for tools for energy risk management (chapter 3) and the literature on electricity pricing models (the present chapter), this section defines the model with its attributes and limitations.

4.4.1 Supply model and the grid

It is assumed that power generation system consists of n symmetric firms which own a total of N generating units. Each one owns an identical set of N/n units. Each unit i in the set is assumed to have the following technical characteristics:

- variable cost d_i [\$/MWh]
- capacity c_i [MW]
- mean time to failure λ_i^{-1} [hour]
- mean time to repair μ_i^{-1} [hour]

Mean time to failure and mean time to repair are considered to be exponentially distributed. The steady state of the units is assumed.

This work admits the possibility of importing energy from an external market at a certain price. This source is modeled as an expensive dummy generating unit of unlimited capacity. This assumption assures that load is met at every moment.

Transmission constraints are not considered in this study. It can be seen as a one-node model in which there is only one price.

4.4.2 Market model

It is a pay-as-bid market. There exists a day-ahead market and a spot market. The firms simultaneously submit supply curve offers in the day-ahead market. A merit-order sorting based on the offers is used to dispatch the units to meet demand. There exists an Independent System Operator (ISO) that clears the day-ahead market by considering the offers and the expected demand. The offers stay the same for the spot market and are used to meet the actual demand the following day, setting the spot price.

4.4.3 Bidding strategies

Three bidding models are studied:

- the Bertrand model in which firms offer their marginal costs
- the Cournot model in which firms offer quantities that optimize the expected profit
- the SFE model in which firms offer a supply curve (quantity-price combination) based on the Rudkevich, Duckworth, and Rosen's equilibrium formula.

4.4.4 Demand model

In the first approach (chapters 5 and 6), the hourly demand is considered a normal random variable with mean μ_t, standard deviation σ_t and covariances $\sigma_{r,t}$. σ_t is considered to be small enough with respect to μ_t to negate the possibility of a negative load. This approach is called load model 1.

In the second approach (chapter 8) hourly demand is modeled as a regression equation with temperature as an independent deterministic variable plus a remaining stochastic term for each hour. This last component is studied in two ways. The first one is considering each hourly remaining term as normally distributed but not independent. This is called load model 2. The second way is considering the remaining terms as a time series. This is called load model 3. It is assumed that temperature can be forecasted accurately for the next 24 hours. The time series component reflects the correlation between loads at consecutive hours and also between the same hours on the same weekdays of consecutive weeks.

For the numerical examples, this work considers demand and prices only for weekdays given that weekends and holidays have a different daily pattern. For the Cournot model, demand is assumed to be linear with respect to price and to have certain price elasticity. A range of elasticity values is considered. Instead of using the elasticity defined as a ratio that measures the change in quantity respect to change in price, the first derivative D_p of demand with respect to price, which also measures that relationship, is used. For the SFE model, the assumption is zero elasticity of demand.

4.4.5 Time line

At the moment of doing the calculations (hours, days, weeks or months in advance), the demand estimate, the temperature forecast and the steady state of the generating units are known. To make decisions on the offer curve on the day-ahead, a more accurate temperature and load forecast will be at hand for the following 24 hours. The model does not take into account the actual working status of the units, but the steady state of them.

The objective is to predict prices to make decisions. In the short- and medium-term (weeks, months) the estimates may be useful to schedule units' maintenance, to do cash-flow projections, and to make decisions on Contracts-for-Differences (CfD). In the very-short-term (tomorrow, days), pricing and unit commitment decisions will be based on this knowledge.

4.4.6 Price under study

This work models the spot (real time) price of energy. It may be the System Marginal Price (SMP) of a one-part market or the Locational Marginal Price (LMP) of a two-part market. Side payments or other components of the price are not the object of this study. Formulas for the expected value and variance of hourly prices (chapter 5) and of the average of hourly prices (chapter 6) are derived. The price average can be done in a 24-hour period, or for peak hours or for off-peak hours.

4.4.7 Limitations of the model

The most significant limitation is the assumption of symmetry that means that all the competing firms are supposed to be identical in number and characteristics of units, capacity, and knowledge of the market. This assumption is far removed from reality. It is necessary, though, in order to retain the closed form expressions. It can be partially justified by noting that only a few firms really influence price: those that usually owns the marginal unit. It is assumed that those are not the small firms, which are price-takers, but the large ones. So there is a first filter for small companies which provide energy for base load. Only the big firms are considered identical. Even if the firms were actually identical, the randomness of the availability of units would break the theoretical and assumed symmetry.

Another important limitation is that the model does not consider transmission constraints. Line congestion forces some units to stop generating because one or more lines are saturated. That means that the merit order is broken. Other more expensive units must be called on to supply energy to those nodes where the congested lines can not transmit electricity. This is the reason for having different prices in different locations: the Locational Marginal Prices (LMP). This limitation may be acceptable considering that this is the case only in a few locations during few hours. In PJM 2005 State of the Market [57], table 7.6 shows that in 2004 the congestion-event hours [3] were on the average 2.2% of the total annual hours, while in 2005 the average went up to 5.1%. In terms of prices the congestion component of the LMP is very small as it was in 2004: around 1%.[4]

A third limitation is that the unit commitment problem is neglected. In real markets, minimum up and down times have significant repercussion on the units that are called on to serve energy. Sometimes it is more economic to keep a more expensive unit running instead of turning it off, closing-down a cheaper one for a period because this has smaller start-up and shut-down costs or less technical constraints. The unit commitment problem is a challenging problem in itself. Attempting to consider it here would be quite difficult.

[3] The convention is that if congestion occurs for 20 minutes or more in an hour, the hour is congested.
[4] See figures 7.9 to 7.40 in PJM 2005 State of the Market [57].

A fourth one is that fuel costs are assumed to be deterministic in this model which in real word are also random variables. A plausible defense is that fuel prices do not have large variances, especially in the short-term.

4.5 SUMMARY

After reviewing the literature on electricity pricing and the different models that have been used, this chapter describes the model profile considered in this work highlighting its assumptions and limitations.

This model is unique; and contributes to the state of the art in the following respect:

- it fills in the gap in the literature combining a fundamental approach (both in the generating process and in the pricing process as well) with a stochastic outlook;
- the stochasticity includes uncertainty from the load and from the availability of the units;
- it considers Nash equilibrium solutions, which, according to game theory, are considered the prevailing outcomes for the prices;
- no other work has been done up to date, using an analytical model to compute the variance of electricity prices.

This work incorporates stochastic behavior to classic and modern theory on oligopolies done for deterministic scenarios. Thus, it enhances the formulas presented in this chapter to cover more realistic situations. The consideration of three bidding behaviors enriches the understanding of the model giving lower and upper bounds of prices and their variances. Furthermore, the use of closed form expressions enables to adapt the model to different markets and to change the system configuration in the same markets. The cost that is paid to get these insights are some assumptions that are removed from real markets, and constitute limitations to the model, especially those related to symmetry of firms.

5.0 MEAN AND VARIANCE OF THE HOURLY PRICE

In a deregulated market, price is a random variable resulting from various sources of uncertainty: demand, fuel costs, reliability of the generating units, bidding behavior, transmission congestion.

Utility managers classify the load (demand) in three main segments: base load, intermediate load and peak load. Base load is power that is used continuously and it is the cheapest: large coal-fired and nuclear stations usually supply it. Intermediate load is electricity needed for several hours a day, or even the whole day, but not every day. Utilities use more expensive units to provide intermediate load: some hydro plants and Combined Cycle Turbines. Peak load is electricity used to meet extreme demand. It is not needed very often, just a few hours a day or a week. The machines used to meet peak load are much more expensive: e.g., combustion turbines.

To estimate the profitability (measured as revenues minus costs) of base load units utility managers can rely on monthly or even annual average prices. Those units are expected to run without interruption. To calculate intermediate load units' profitability, a monthly average is not good enough. As these units run for several hours a day, but not every day, the generating companies may need to estimate the electricity price for the day(s) the units will be in use. Given that the volatility of electricity prices is high, the expected value alone will not be sufficient for purposes of prediction. At a minimum, the variance of the daily average will be also needed. Finally, for the profitability analysis of peak load units, the expected value and the variance of the hourly price must be at hand.

The study of the hourly prices are useful for scheduling maintenance of the individual units, and to decide on what kind of units are more needed: base, intermediate or peak load units.

This chapter focuses on the expected value and variance of hourly prices, to predict the price of electricity for a given hour. The following chapter studies the expected value and variance of the daily average price. The same techniques can be used for obtaining estimates for weekly or monthly average prices.

5.1 CONDITIONAL EXPRESSIONS FOR THE MEAN AND VARIANCE OF THE HOURLY PRICE

Ignoring unit commitment constraints, it is assumed that the system consists of $N+1$ generating units, which are dispatched in an ascending merit order , based on the production cost of each one. Utilities will offer energy (quantity and price), unit by unit, to the Independent System Operator (ISO). The latter will order the units by offered price, and dispatch the units from the cheapest to the more expensive ones, until the demand is met. This is the case with PJM and many other electricity markets.

Conditioning on the marginal unit[1] $J(t)$, the expected value of the price can be written as follows

$$E[p(t)] = \sum_{j=1}^{N+1} E[p(t)|J(t) = j]Pr[J(t) = j] \tag{5.1}$$

where j is the merit order index. $j = 1, 2, 3, ..., N + 1$.

The variance can be calculated as

$$Var[p(t)] = E[p(t)^2] - E^2[p(t)] \tag{5.2}$$

[1]Marginal unit is the last unit called on to produce electricity to meet demand.

where $E[p(t)^2]$ can be computed conditioning on $J(t)$, as was done before

$$E[p(t)^2] = \sum_{j=1}^{N+1} E[p(t)^2|J(t) = j]Pr[J(t) = j] \tag{5.3}$$

In order to get the expected value and the variance of the price at the time t the probability mass function of $J(t)$ is needed. The following section summarizes Valenzuela's work [76] on this topic.

5.2 PROBABILITY DISTRIBUTION OF THE MARGINAL UNIT

First note that

$$Pr[J(t) = j] = Pr[J(t) > j - 1] - Pr[J(t) > j] \tag{5.4}$$

Among many other sources of uncertainty which influence the randomness of $J(t)$, two are being considered: the load at time t, $L(t)$, and the availability of the generating units. The following random variables capture the availability of generators

$$Y_i(t) = \begin{cases} 1 & \text{if unit } i \text{ is up at time } t \\ 0 & \text{if unit } i \text{ is down at time } t \end{cases}$$

It is assumed that $Y_i(t)$ and $Y_j(t)$ are independent for $i \neq j$.

c_i is defined to be the capacity of the unit i. It follows that $\sum_{i=1}^{j} c_i Y_i(t)$ is the available capacity of the first j units at time t.

The model assumes a failure rate of unit i being λ_i and the repair rate being μ_i. The mean time to fail is consequently the inverse $\frac{1}{\lambda_i}$; and the mean time to repair is $\frac{1}{\mu_i}$.

Thus the steady-state proportion of time that the generating unit i is up is $p_i = \dfrac{\mu_i}{\lambda_i + \mu_i}$ and the complement proportion of time that the generating unit is down is $q_i = 1 - p_i = \dfrac{\lambda_i}{\lambda_i + \mu_i}$, also known as Forced Outage Rate (FOR) .

Defining $L(t)$ to be the load at time t, note that the events $[J(t) > j]$ and $[L(t) - \sum_{i=1}^{j} c_i Y_i(t) > 0]$ are equivalent. $J(t) > j$ means that the marginal unit is beyond unit j, which implies that the load $L(t)$ is larger than the available capacity up to unit j, $L(t) > \sum_{i=1}^{j} c_i Y_i(t)$. So

$$Pr[J(t) > j] = Pr[L(t) - \sum_{i=1}^{j} c_i Y_i(t) > 0] \tag{5.5}$$

An auxiliary variable is defined

$$X_j(t) = L(t) - \sum_{i=1}^{j} c_i Y_i(t) \tag{5.6}$$

with a cumulative distribution function $G_j(x; t) = Pr[X_j(t) \leq x]$.

Thus, $X_j(t)$ is the excess of load that is not being met by the available generated power up to generating unit j. It is assumed that $L(t)$ and $Y_i(t)$ are independent for all i. Equation (5.5) can be written as

$$\begin{aligned} Pr[J(t) > j] &= Pr[X_j(t) > 0] \\ &= 1 - G_j(0; t) \end{aligned} \tag{5.7}$$

And thus equation (5.4) reduces to

$$Pr\left[J\left(t\right) = j\right] = G_j\left(0; t\right) - G_{j-1}\left(0; t\right) \tag{5.8}$$

Assuming the random variable components of $X_j(t)$ to be independent, for a relatively large j, the distribution of $X_j(t)$ can be modeled as normal, by the Central Limit Theorem. But the normal approximation is not valid for small values of j, and may not be very accurate when computing the tail probabilities for any j. Valenzuela [76] shows that the Edgeworth expansion is a better approximation in these cases.

The Edgeworth expansion of the distribution function of $X_j(t)$ is given in Cramer [20]

$$\begin{aligned} G_j(x;t) &\cong \Phi(z) + \left[\frac{1}{6}\frac{K3_j}{K2_j(t)^{3/2}}(1-z^2) + \frac{1}{24}\frac{K4_j}{K2_j(t)^2}(3z-z^3) \right. \\ &\left. + \frac{1}{72}\frac{K3_j^2}{K2_j(t)^3}(-15z+10z^3-z^5)\right]\phi(z) \end{aligned} \quad (5.9)$$

where $z = z_j(x;t) = \dfrac{x - K1_j(t)}{\sqrt{K2_j(t)}}$, $\Phi(z)$ is the standard normal cumulative probability distribution function, $\phi(z)$ is the standard normal probability density function with mean zero and unit variance; and

$$K1_j(t) = E\left[X_j\left(t\right)\right] = \mu_t - \sum_{i=1}^{j} c_i p_i \quad (5.10)$$

$$K2_j(t) = Var\left[X_j\left(t\right)\right] = \sigma_t^2 + \sum_{i=1}^{j} c_i^2 p_i q_i \quad (5.11)$$

$$K3_j = \sum_{i=1}^{j} c_i^3 p_i q_i (p_i - q_i) \quad (5.12)$$

$$K4_j = \sum_{i=1}^{j} c_i^4 p_i q_i (p_i^2 - 4p_i q_i + q_i^2) \quad (5.13)$$

where μ_t and σ_t^2 are the mean and variance of $L(t)$ respectively; c_i is the nominal capacity of unit i; p_i is the proportion of time that unit i is up; and q_i is the proportion of time that unit i is down; $p_i + q_i = 1$.

p_i can be computed through $p_i = \dfrac{\mu_i}{\lambda_i + \mu_i}$ where λ_i^{-1} is the mean time to failure and μ_i^{-1} is the mean time to repair for unit i.

5.3 EXPECTED VALUE AND VARIANCE OF MARGINAL COST

For results to be later derived it is useful to compute the expected value and the variance of $d_{J(t)}$. They are calculated as follows conditioning on $J(t)$ once again

$$E[d_{J(t)}] = \sum_{j=1}^{N+1} E[d_{J(t)}|J(t) = j]Pr[J(t) = j] = \sum_{j=1}^{N+1} E[d_j]Pr[J(t) = j] \tag{5.14}$$

Similarly

$$Var[d_{J(t)}] = E[d^2_{J(t)}] - E[d_{J(t)}]^2 = \sum_{j=1}^{N+1} E[d^2_j]Pr[J(t) = j] - E^2[d_{J(t)}] \tag{5.15}$$

If d_j has a distribution, with mean $E[d_j]$ and variance $Var[d_j]$ the formulas above hold and equation (5.15) can be written as

$$Var[d_{J(t)}] = \sum_{j=1}^{N+1} \left\{Var[d_j] + E^2[d_j]\right\} Pr[J(t) = j] - E^2[d_{J(t)}] \tag{5.16}$$

In the rest of this work the d_j's will be considered as known and deterministic constants. So, equations (5.14) and (5.15) can be written as

$$E[d_{J(t)}] = \sum_{j=1}^{N+1} d_j Pr[J(t) = j] \tag{5.17}$$

$$Var[d_{J(t)}] = \sum_{j=1}^{N+1} d^2_j Pr[J(t) = j] - E^2[d_{J(t)}] \tag{5.18}$$

5.4 EXPECTED VALUE OF THE EQUIVALENT LOAD

In order to consider the uncertainty of the load and the reliability of the units as well, the random variable called equivalent load is defined as

$$\bar{L}_{J(t)}(t) = L(t) + \sum_{i=1}^{J(t)}[1 - Y_i(t)]c_i \tag{5.19}$$

The equivalent load is the load that could have been delivered if all the units up to the marginal were working. The expected value of the equivalent load is

$$E[\bar{L}_{J(t)}(t)] = \sum_{j=1}^{N+1} E[\bar{L}_{J(t)}(t)|J(t) = j]Pr[J(t) = j] \tag{5.20}$$

The cumulative capacity up to generating unit j is defined to be $C_j = \sum_{i=1}^{j} c_i$

Note that the events $[J(t) = j]$ and $[C_{j-1} < \bar{L}_{J(t)}(t) \leq C_j]$ are equivalent.

So,

$$\begin{aligned} E\left[\bar{L}_{J(t)}\left(t\right)|J\left(t\right) = j\right] &= E\left[\bar{L}_{J(t)}\left(t\right)|C_{j-1} < \bar{L}_{J(t)}\left(t\right) \leq C_j\right] \\ &= E\left[\bar{L}_j\left(t\right)|C_{j-1} < \bar{L}_j\left(t\right) \leq C_j\right] \end{aligned} \tag{5.21}$$

$\bar{L}_j\left(t\right)$ is defined as the equivalent load at time t assuming that $J\left(t\right) = j$.

An approximation of the expected value and variance of the equivalent load is described in appendix A.1, using Edgeworth formula. For the purpose of this study, a simpler approximation of $E[\bar{L}_{J(t)}(t)|J(t) = j]$ will be used. The reason for doing this is that none of them is an exact derivation; and the latter, being close enough, is much easier to compute.

There are many reasonable approximations to the equivalent load: C_{j-1}, $\dfrac{C_{j-1} + C_j}{2}$, C_j. The selected one in this work is

$$E[\bar{L}_{J(t)}(t)|J(t) = j] \cong C_j \tag{5.22}$$

This is justified by considering that, if $C_{j-1} < \bar{L}_{J(t)}(t) \leq C_j$, the difference $C_j - \bar{L}_{J(t)}(t)$ is smaller than c_j. In practice, a power market has many units, such that an error smaller than the capacity of a single unit is negligible, from a practical point of view. In appendix A.3, a table shows the difference between outputs of the model in chapters 5 and 6 in the two extreme cases: approximating $\bar{L}_{J(t)}(t)$ with C_j and with C_{j-1}. The differences turn to be very small, justifying the selected approximation.

Substituting equation (5.22) in equation (5.20), the following approximation holds

$$E[\bar{L}_{J(t)}(t)] \cong \sum_{j=1}^{N+1} C_j Pr[J(t) = j] \tag{5.23}$$

5.5 BERTRAND MODEL

Under the Bertrand model, the market-clearing price is the marginal cost. That is

$$p(t) = d_{J(t)} \tag{5.24}$$

So, using equation (5.17) the expected price at hour t is:

$$E[p(t)]_B = E[d_{J(t)}] = \sum_{j=1}^{N+1} d_j Pr[J(t) = j] \tag{5.25}$$

Similarly, the variance can be calculated using equation (5.18):

$$Var[p(t)]_B = Var[d_{J(t)}] = \sum_{j=1}^{N+1} d_j^2 Pr[J(t) = j] - E^2[p(t)]_B \tag{5.26}$$

5.6 COURNOT MODEL

Green and Newbery [32] derive the Cournot model formula for the basic case of a symmetric duopoly:

$$p(t)_{Cournot} = d_{J(t)} - \frac{q(t)}{D_p} \tag{5.27}$$

where $q(t) = \frac{L(t)}{2}$ and $D_p < 0$ is the derivative of the total system demand $D(t,p)$ with respect to price. Following Green [30], it is assumed in this work that the total demand $D(t,p)$ is a linear function of price. $L(t)$ is the demand realization at time t, which has to be met.

The expression for $q(t)$ is obtained based on the fact that firm i faces the residual demand $q_i(t) = L(t) - q_{-i}(t)$, where $q_{-i}(t)$ is the quantity provided by the rest of the competing firms. In equilibrium and under a symmetric duopoly, $q_i(t) = q_{-i}(t) = q(t)$, then $q(t) = \frac{L(t)}{2}$.

The formula (5.27) can be generalized for the case of n symmetric firms. In this case, the residual demand for firm i is also $q_i(t) = L(t) - q_{-i}(t)$. In equilibrium and under symmetry, $q(t) = L(t) - (n-1)q(t)$. Then, $q(t) = \frac{L(t)}{n}$.

Two more considerations must be made. The first is that when the reliability of the generating units is taken into consideration the symmetry is lost. The second is the need to distinguish between the quantity offered and the actual quantity produced. Denoting $q(t)$ as the quantity offered by each firm, actual supply is

$$nq(t) - \sum_{i=1}^{J(t)} [1 - Y_i(t)]c_i$$

At any time t, actual supply must equal actual demand. Therefore,

$$L(t) = nq(t) - \sum_{i=1}^{J(t)} [1 - Y_i(t)]c_i \tag{5.28}$$

Consequently, to capture the uncertainty of the state of the generating units in equation (5.27) the following expression is used

$$q(t) = \frac{L(t) + \sum_{i=1}^{J(t)} [1 - Y_i(t)] c_i}{n} = \frac{\bar{L}_{J(t)}(t)}{n} \tag{5.29}$$

as an approximation, because the symmetry has been lost.

Another way to visualize the last equation is by considering that companies must "supply" energy to account for the actual load plus the energy "lost" through outages, which is referred to the equivalent load. This heuristic formula is not rigorous, but it should be applicable for the purpose of determining the price under the merit order procedure used in the power markets. If a unit is out on forced outage, the ISO calls on the following one in the merit order. The effect on the price is exactly the same as a demand increase in the same amount as that of the capacity of the failed units.

When $D(p,t)$ is linear with respect to p, equation(5.27) can be generalized by

$$p(t)_C = d_{J(t)} - \frac{\bar{L}_{J(t)}(t)}{nD_p} \tag{5.30}$$

The expected value of $p(t)$ following the Cournot model is

$$E[p(t)]_C = E[d_{J(t)}] - E\left[\frac{\bar{L}_{J(t)}(t)}{nD_p}\right] = E[d_{J(t)}] - \frac{1}{nD_p} E[\bar{L}_{J(t)}(t)] \tag{5.31}$$

Substituting equation (5.17) and considering the approximation given by (5.23) in equation (5.31):

$$\begin{aligned} E[p(t)]_C &\cong \sum_{j=1}^{N+1} d_j Pr[J(t) = j] - \frac{1}{nD_p} \sum_{j=1}^{N+1} C_j Pr[J(t) = j] \\ &= \sum_{j=1}^{N+1} \left(d_j - \frac{C_j}{nD_p} \right) Pr[J(t) = j] \end{aligned} \tag{5.32}$$

The variance can be calculated using equation (5.2):

$$
\begin{aligned}
Var[p(t)]_C &= Var\left[d_{J(t)} - \frac{1}{nD_p}\bar{L}_{J(t)}(t)\right] \\
&= E\left[\left(d_{J(t)} - \frac{1}{nD_p}\bar{L}_{J(t)}(t)\right)^2\right] - E^2\left[d_{J(t)} - \frac{1}{nD_p}\bar{L}_{J(t)}(t)\right] \\
&= \sum_{j=1}^{N+1} E\left[\left(d_{J(t)} - \frac{1}{nD_p}\bar{L}_{J(t)}(t)\right)^2 |J(t) = j\right] Pr[J(t) = j] - E^2[p(t)]_C \\
&\cong \sum_{j=1}^{N+1}\left(d_j - \frac{C_j}{nD_p}\right)^2 Pr[J(t) = j] - E^2[p(t)]_C \qquad (5.33)
\end{aligned}
$$

5.7 SUPPLY FUNCTION EQUILIBRIUM MODEL

Rudkevich, Duckworth, and Rosen's [63] formula for deterministic load is as follows

$$p(t) = d_{J(t)} + \sum_{i=J(t)}^{M-1}(d_{i+1} - d_i)\left(\frac{L(t)}{C_i}\right)^{n-1} \qquad (5.34)$$

where M is the dispatch order number of the most expensive unit expected to run during the 24-hour period.

As discussed earlier, the outage of units has the same effect on price as a shift upwards of the load, in the same amount of the power that cannot be delivered. In order to consider the uncertainty of the load and the availability of the units simultaneously, the same procedure that has been used for the Cournot model is applied here: using the equivalent load $\bar{L}_{J(t)}(t)$ defined in equation (5.19) instead of the actual load $L_{J(t)}$.

The following heuristic formula comes from modifying equation (5.34)

$$p(t) = d_{J(t)} + \sum_{i=J(t)}^{M-1}(d_{i+1} - d_i)\left(\frac{\bar{L}_{J(t)}(t)}{C_i}\right)^{n-1} \qquad (5.35)$$

Substituting equation (5.35) into equation (5.1) and using the approximation given in equation (5.22) the expected value can be expressed by

$$
\begin{aligned}
E[p(t)]_R &= \sum_{j=1}^{N+1} E\left[d_{J(t)} + \sum_{i=J(t)}^{M-1} (d_{i+1} - d_i)\left(\frac{\bar{L}_{J(t)}(t)}{C_i}\right)^{n-1} |J(t) = j\right] Pr[J(t) = j] \\
&\cong \sum_{j=1}^{N+1} \left[d_j + \sum_{i=j}^{M-1} (d_{i+1} - d_i)\left(\frac{C_j}{C_i}\right)^{n-1}\right] Pr[J(t) = j] \qquad (5.36)
\end{aligned}
$$

Similarly, the variance can be calculated in (5.2) as

$$
\begin{aligned}
Var[p(t)]_R &= \sum_{j=1}^{N+1} E\left[\left[d_{J(t)} + \sum_{i=J(t)}^{M-1} (d_{i+1} - d_i)\left(\frac{\bar{L}_{J(t)}(t)}{C_i}\right)^{n-1}\right]^2 |J(t) = j\right] Pr[J(t) = j] \\
&\quad - E^2[p(t)]_R \qquad (5.37)
\end{aligned}
$$

Using the approximation given in (5.22) the variance results

$$
Var[p(t)]_R \cong \sum_{j=1}^{N+1} \left[d_j + \sum_{i=j}^{M-1} (d_{i+1} - d_i)\left(\frac{C_j}{C_i}\right)^{n-1}\right]^2 Pr[J(t) = j] - E^2[p(t)]_R \qquad (5.38)
$$

5.8 SOME COMMENTS ON RUDKEVICH, DUCKWORTH, AND ROSEN'S FORMULA

As Klemperer and Meyer [43] have shown, a supply function satisfying the definition of a Nash Equilibrium is generally not unique.

Rudkevich, Duckworth, and Rosen's formula is based on the assumption that the price at the peak will equal the marginal cost of the last unit used. In equation (5.35) M denotes the marginal unit at daily peak. M is then a random variable, with two sources of uncertainty: the peak load, and the state of the units Y_i at the peak hour.

The peak load is the maximum of $L(t)$, $t = 1, 2, 3, ..., 24$. Under the assumption that $L(t)$ follows a Gauss-Markov process, the distribution of the peak load has a quite complicated analytical form. Furthermore, the distribution of M is also quite difficult. As the choice of the SFE is arbitrary, the perception of what M is going to be is more useful than the actual distribution of M.

The second term of the right side of equation (5.35) varies from day to day. This means that each day the firms bid different supply functions, according to the forecasted peak unit. In practice, this is the case. The PJM website shows historical bid data, unit by unit. The supply function for each unit is not the same every day.

It is assumed that the firms can forecast M with great precision. Therefore, it will be considered to be a deterministic variable, based on historical data. According to the perception of M, the supply function bid will be greater or smaller. It is supposed that all the firms have the same ability to predict M. Rudkevich, Duckworth, and Rosen [63] take the lowest SFE, which intersects the marginal cost curve at the point of maximum anticipated demand for that day.

Looking at figure 5.1, if the anticipated maximum demand for the day is the line BC, the set of feasible solutions, as it was shown, is composed of curves passing through the origin O and crossing the demand between the points B and C like the curves OC' and OB'. The lowest SFE is then the curve OB that crosses the demand curve at point B with slope zero. B is the anticipated realization of the maximum demand for that day met by supply. Recall that the straight line OB is the Bertrand solution which is the marginal cost. In other words, the curve OB is the solution selected by Rudkevich, Duckworth, and Rosen.

On the other hand, the highest SFE is the supply schedule based on the assumption that each firm behaves as a monopolist in the particular hour of anticipated peak demand, capped by the maximum allowed price. This is point C in figure 5.1. The highest SFE is the curve OC, that crosses the demand curve at point C with infinite slope. Recall that the straight line OC is the Cournot solution.

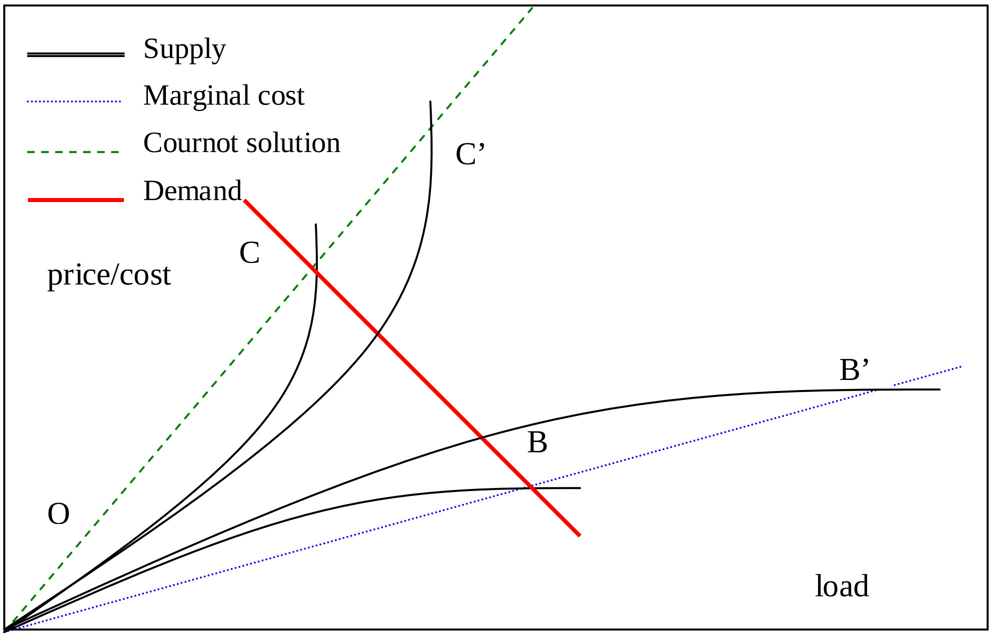


Figure 5.1: Rudkevich, Duckworth, and Rosen’s supply function equilibrium solution

Over time, in a repeated game, firms might employ a “tit-for-tat” pricing strategy, by gradually raising their bid prices. Raising prices can be seen as taking higher values of M, which can go up to N, the total number of generators in the market. In figure 5.1, the curve OB’ is a Rudkevich, Duckworth, and Rosen’s solution taking a higher M. In this way, Rudkevich, Duckworth, and Rosen’s formula has been generalized to cases other than the lowest SFE. The study includes sensitivity analysis on M to assess the repercussion on the expected value and on the variance of prices.

5.9 SUMMARY

This chapter finds expressions for the expected value and variance of hourly electricity prices, conditioning on the probability distribution of the marginal unit. The concept of equivalent load is used to capture the uncertainties of both the load and the reliability of units. It uses the fact that an outage of a unit is equivalent (for pricing purposes) to an increase of load in the same amount of energy as the capacity of the failed unit. The formulas explicitly use system parameters such as number of firms in the market, number of generating units, elasticity of demand, installed capacity, which allows us to perform sensitivity analysis and to adapt the model to a change in the system parameters.

6.0 AVERAGE ELECTRICITY PRICE

Average electricity prices are useful for medium-term decisions, in which the information on hourly prices is not that useful. The extreme volatility of electricity prices makes it difficult for the companies to survive in such an unpredictable environment. The firms rely on a reasonable forecast of the prices. For derivatives pricing, and to hedge the risk against significant changes in the market price of electricity, the average price for peak and off-peak hours are of interest. And so they are for making decisions about investments on capacity.

This chapter derives formulas for the expected value and variance of average prices. The objective is to predict accurately the average price for a given day; or the average for some specific hours.

6.1 DAILY LOAD PROFILE

A load-weighted average is considered, as a more general approach. The simple average is a special case.

The weights of the hourly loads are given by

$$w_t = \frac{L(t)}{\sum_{t=I}^{H} L(t)}$$

where I is the initial hour and H is the final hour of the period under consideration.

The load-weighted average price for period I to H is

$$\bar{p}(I,H) = \sum_{t=I}^{H} w_t p(t) \tag{6.1}$$

If $I = 1$ and $H = 24$, $\bar{p}(I,H)$ is the load-weighted average daily price.

Note that w_t and $p(t)$ are correlated, w_t being also a random variable. Nevertheless, this work anticipates that, in general, w_t will have much less variability compared to p_t.

6.2 EXPECTED VALUE AND VARIANCE OF AVERAGE PRICE

The expected value is straightforward:

$$E[\bar{p}(I,H)] = \sum_{t=I}^{H} w_t E[p(t)] \tag{6.2}$$

using $E[p(t)]$ as derived in the preceding chapter for each bidding model.

The variance requires a more thorough analysis, because $p(r)$ and $p(t)$ are correlated for any pair r, t. Its formula is

$$Var[\bar{p}(I,H)] = \sum_{t=I}^{H} w_t^2 Var[p(t)] + 2 \sum_{t=I}^{H} \sum_{r=t+1}^{H} w_r w_t cov[p(r), p(t)] \tag{6.3}$$

where the expression for $Var[p(t)]$ has been derived in the preceding chapter for each bidding model. But still an expression for $cov[p(r), p(t)]$ is needed.

Using that

$$cov[p(r), p(t)] = E[p(r)p(t)] - E[p(r)]E[p(t)] \tag{6.4}$$

only a formula for $E[p(r)p(t)]$ needs to be found, given that $E[p(r)]$ and $E[p(t)]$ have been derived in the previous chapter. Conditioning on $J(r)$ and $J(t)$, the following expression holds

$$E[p(r)p(t)] = \sum_{m=1}^{N+1} \sum_{l=1}^{N+1} E[p(r)p(t)|J(r) = m, J(t) = l] Pr[J(r) = m, J(t) = l] \tag{6.5}$$

Sections 6.4, 6.5 and 6.6 give computations of $E[p(r)p(t)|J(r) = m, J(t) = l]$ for the three models under consideration.

Valenzuela [76] has obtained an approximate expression for $Pr[J(r) = m, J(t) = l]$ using the Edgeworth expansion formula. The following section summarizes his work.

6.3 MARGINAL UNIT'S BIVARIATE PROBABILITY DISTRIBUTION

Note that

$$\begin{aligned} Pr[J(r) = m, J(t) = l] \;\; = \;\; & Pr[J(r) > m-1, J(t) > l-1] \\ & - Pr[J(r) > m, J(t) > l-1] \\ & - Pr[J(r) > m-1, J(t) > l] \\ & + Pr[J(r) > m, J(t) > l] \end{aligned} \tag{6.6}$$

The events $[L(r) - \sum_{i=1}^{m} c_i Y_i(r) > 0, L(t) - \sum_{i=1}^{l} c_i Y_i(t) > 0]$ and $[J(r) > m, J(t) > l]$ are equivalent. Denoting by $p_{ml}(r,t)$ the joint probability of the two events, and using the variables $X_m(r)$ and $X_l(t)$ defined in equation (5.6), the following equalities hold

$$p_{ml}(r,t) = Pr[J(r) > m, J(t) > l] = Pr[X_m(r) > 0, X_l(t) > 0] \tag{6.7}$$

Using this notation, equation (6.6) becomes

$$Pr[J(r) = m, J(t) = l] = p_{m-1,l-1}(r,t) - p_{m,l-1}(r,t) - p_{m-1,l}(r,t) + p_{m,l}(r,t) \tag{6.8}$$

The joint probability distribution of $[X_m(r), X_l(t)]$ can be approximated by a bivariate normal distribution for relatively high values of m and n, because $L(r)$ and $Y_i(r)$ are assumed to be independent, for any given r. This normal approximation does not provide accurate answers when computing tail probabilities and for small values of m or l. A better approximation is obtained using the Edgeworth expansion.

Iyengar and Mazumdar [41] give the Edgeworth approximate expansion of the joint probability distribution of $[X_m(r), X_l(t)]$:

$$\begin{aligned} p_{ml}(r,t) \quad \cong \quad & \int_{a_m(r)}^{\infty}\int_{a_l(t)}^{\infty} \phi_2[z_1,z_2;\rho_{ml}(r,t)] \left\{ 1 + \frac{1}{6}\frac{K_{30}}{K_{20}^{\frac{3}{2}}} H_{30}[z_1,z_2;\rho_{ml}(r,t)] \right. \\ & + \frac{1}{2}\frac{K_{21}}{K_{20}K_{02}^{\frac{1}{2}}} H_{21}[z_1,z_2;\rho_{ml}(r,t)] + \frac{1}{2}\frac{K_{12}}{K_{20}^{\frac{1}{2}}K_{02}} H_{12}[z_1,z_2;\rho_{ml}(r,t)] \\ & \left. + \frac{1}{6}\frac{K_{03}}{K_{02}^{\frac{3}{2}}} H_{03}[z_1,z_2;\rho_{ml}(r,t)] \right\} dz_1 dz_2 \end{aligned} \tag{6.9}$$

where

$$a_m(r) = -\frac{K_{10}}{\sqrt{K_{20}}} \tag{6.10}$$

$$a_l(t) = -\frac{K_{01}}{\sqrt{K_{02}}} \tag{6.11}$$

$\rho_{ml}(r,t)$ is the correlation coefficient between $X_m(r)$ and $X_l(t)$ given by

$$\rho_{ml}(r,t) = \frac{K_{11}}{\sqrt{K_{20}K_{02}}} \tag{6.12}$$

$\phi_2[z_1, z_2; \rho]$ is the probability density function of the bivariate standard normal distribution with correlation coefficient ρ

$$\phi_2[z_1,z_2;\rho] = \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left[\frac{-1}{2(1-\rho^2)}\left(z_1^2 - 2\rho z_1 z_2 + z_2^2\right)\right] \tag{6.13}$$

K_{ij} is the bivariate cumulant of order (i, j) of $[X_m(r), X_l(t)]$ given by

$$K_{10} = E[X_m(r)] = \mu_r - \sum_{i=1}^{m} c_i p_i \tag{6.14}$$

$$K_{01} = E[X_l(t)] = \mu_t - \sum_{i=1}^{l} c_i p_i \tag{6.15}$$

$$K_{20} = Var[X_m(r)] = \sigma_r^2 + \sum_{i=1}^{m} c_i^2 p_i q_i \tag{6.16}$$

$$K_{02} = Var[X_l(t)] = \sigma_t^2 + \sum_{i=1}^{l} c_i^2 p_i q_i \tag{6.17}$$

$$K_{11} = Covar[X_m(r), X_l(t)] = \sigma_{r,t} + \sum_{i=1}^{\min(m,l)} c_i^2 p_i q_i e^{-\delta_i |t-r|} \tag{6.18}$$

$$K_{12} = K_{21} = \sum_{i=1}^{\min(m,l)} c_i^3 p_i q_i (p_i - q_i) e^{-\delta_i |t-r|} \tag{6.19}$$

$$K_{30} = \sum_{i=1}^{m} c_i^3 p_i q_i (p_i - q_i) \tag{6.20}$$

$$K_{03} = \sum_{i=1}^{l} c_i^3 p_i q_i (p_i - q_i) \tag{6.21}$$

where μ_r, μ_t, σ_r^2, σ_t^2 and $\sigma_{r,t}$ are the means, variances and covariances of the hourly loads.

The expressions for K_{11} and K_{12} hold only if the up and down times are independently exponentially distributed, as it is assumed in this work. In them, $\delta_i = \lambda_i + \mu_i$

Note that using the notation defined in equations (5.10) to (5.12) the following equalities hold:

$$\begin{aligned} K_{10} &= K1_m(r) \\ K_{01} &= K1_l(t) \\ K_{20} &= K2_m(r) \\ K_{02} &= K2_l(t) \\ K_{30} &= K3_m \\ K_{03} &= K3_l \end{aligned}$$

$H_{ij}[z_1, z_2; \rho]$ are the bivariate Hermite polynomials :

$$H_{30}[z_1, z_2; \rho] = \frac{(z_1 - \rho z_2)^3 - 3(z_1 - \rho z_2)(1 - \rho^2)}{(1 - \rho^2)^3} \tag{6.22}$$

$$H_{21}[z_1, z_2; \rho] = \frac{(z_1 - \rho z_2)^2(z_2 - \rho z_1) + 2\rho(1 - \rho^2)(z_1 - \rho z_2) - (1 - \rho^2)(z_2 - \rho z_1)}{(1 - \rho^2)^3} \tag{6.23}$$

$$H_{12}[z_1, z_2; \rho] = \frac{(z_2 - \rho z_1)^2(z_1 - \rho z_2) + 2\rho(1 - \rho^2)(z_2 - \rho z_1) - (1 - \rho^2)(z_1 - \rho z_2)}{(1 - \rho^2)^3} \tag{6.24}$$

$$H_{03}[z_1, z_2; \rho] = \frac{(z_2 - \rho z_1)^3 - 3(z_2 - \rho z_1)(1 - \rho^2)}{(1 - \rho^2)^3} \tag{6.25}$$

6.4 BERTRAND MODEL

The previous section provides an approximation of the bivariate probability distribution function for the marginal unit at two different hours $Pr[J(r) = m, J(t) = l]$. This section computes the variance of the average price for the Bertrand model.

Knowing that $p(t) = d_{J(t)}$ for Bertrand model from equation (5.24) the following equality holds

$$E[p(r)p(t)|J(r) = m, J(t) = l]_B = d_m d_l \tag{6.26}$$

Using this in equation (6.5) and the resulting expression in equation (6.4), the covariance of prices at two different hours is expressed by

$$cov[p(r), p(t)]_B = \sum_{m=1}^{N+1} \sum_{l=1}^{N+1} d_m d_l Pr[J(r) = m, J(t) = l] - E[p(r)]_B E[p(t)]_B \tag{6.27}$$

where $E[p(r)]_B$ and $E[p(t)]_B$ are derived as in equation (5.25).

Finally, the variance of the average price for the Bertrand model is obtained from equation (6.3) using

$$Var[\bar{p}(I,H)]_B = \sum_{t=I}^{H} w_t^2 Var[p(t)]_B + 2\sum_{t=I}^{H}\sum_{r=t+1}^{H} w_r w_t cov[p(r),p(t)]_B \tag{6.28}$$

where $Var[p(t)]_B$ is calculated using equation (5.26).

6.5 COURNOT MODEL

This section derives the variance of the average price for Cournot model, assuming that the demand $D(p,t)$ is linear with respect to p.

Using equation (5.30) and the approximation of equation (5.22), the following expressions hold

$$\begin{aligned} E[p(r)p(t)|J(r)=m,J(t)=l]_C &= E\left[d_{J(r)}d_{J(t)} - d_{J(r)}\frac{\bar{L}_{J(t)}(t)}{nD_p} - d_{J(t)}\frac{\bar{L}_{J(r)}(r)}{nD_p}\right. \\ &\quad \left. + \frac{\bar{L}_{J(r)}(r)\bar{L}_{J(t)}(t)}{n^2D_p^2}|J(r)=m,J(t)=l\right] \\ &= d_m d_l - \frac{d_m}{nD_p}E[\bar{L}_{J(t)}(t)|J(t)=l] \\ &\quad -\frac{d_l}{nD_p}E[\bar{L}_{J(r)}(r)|J(r)=m] \\ &\quad +\frac{1}{n^2D_p^2}E[\bar{L}_{J(r)}(r)\bar{L}_{J(t)}(t)|J(r)=m,J(t)=l] \\ &\cong d_m d_l - \frac{d_m C_l}{nD_p} - \frac{d_l C_m}{nD_p} + \frac{C_m C_l}{n^2D_p^2} \\ &= \left(d_m - \frac{C_m}{nD_p}\right)\left(d_l - \frac{C_l}{nD_p}\right) \end{aligned} \tag{6.29}$$

Using this in equation (6.5) and the resulting expression in equation (6.4) the covariance of prices at two different hours is expressed by

$$cov[p(r), p(t)]_C \cong \sum_{m=1}^{N+1} \sum_{l=1}^{N+1} \left(d_m - \frac{C_m}{nD_p}\right) \left(d_l - \frac{C_l}{nD_p}\right) Pr[J(r) = m, J(t) = l] - E[p(r)]_C E[p(t)]_C \tag{6.30}$$

where $E[p(r)]_C$ and $E[p(t)]_C$ are computed as in equation (5.31).

Finally, the variance of the average price for the Cournot model is obtained from equation (6.3) using

$$Var[\bar{p}(I, H)]_C = \sum_{t=I}^{H} w_t^2 Var[p(t)]_C + 2 \sum_{t=I}^{H} \sum_{r=t+1}^{H} w_r w_t cov[p(r), p(t)]_C \tag{6.31}$$

where $Var[p(t)]_C$ is calculated using equation (5.33).

6.6 SUPPLY FUNCTION EQUILIBRIUM MODEL

This section will compute the variance of the average price for the Rudkevich, Duckworth, and Rosen's model.

Using Rudkevich, Duckworth, and Rosen's formula (equation (5.35)) and the approximation of equation (5.22) as it has been done for the Cournot model, the following expression holds

$$E[p(r)p(t)|J(r) = m, J(t) = l]_R \cong \left[d_m + \sum_{i=m}^{M-1} (d_{i+1} - d_i) \left(\frac{C_m}{C_i}\right)^{n-1}\right] \times \left[d_l + \sum_{i=l}^{M-1} (d_{i+1} - d_i) \left(\frac{C_l}{C_i}\right)^{n-1}\right] \tag{6.32}$$

Using this in equation (6.5) and the resulting expression in equation (6.4) the covariance of prices at two different hours is expressed by

$$\begin{aligned} cov[p(r),p(t)]_R &= \sum_{m=1}^{N+1}\sum_{l=1}^{N+1}\left[d_m + \sum_{i=m}^{M-1}(d_{i+1}-d_i)\left(\frac{C_m}{C_i}\right)^{n-1}\right] \\ &\quad \times \left[d_l + \sum_{i=l}^{M-1}(d_{i+1}-d_i)\left(\frac{C_l}{C_i}\right)^{n-1}\right] Pr[J(r)=m, J(t)=l] \\ &\quad -E[p(r)]_R E[p(t)]_R \end{aligned} \tag{6.33}$$

where $E[p(r)]_R$ and $E[p(t)]_R$ are computed as in equation (5.36).

Finally, the variance of the average price for the Rudkevich, Duckworth, and Rosen's model is obtained from equation (6.3) using

$$Var[\bar{p}(I,H)]_R = \sum_{t=I}^{H} w_t^2 Var[p(t)]_R + 2\sum_{t=I}^{H}\sum_{r=t+1}^{H} w_r w_t cov[p(r),p(t)]_R \tag{6.34}$$

where $Var[p(t)]_R$ is calculated using equation (5.38).

6.7 SUMMARY

The expected value of the average price is straightforward and can be easily computed from section 6.2, using the expected values of hourly prices derived in the preceding chapter for each bidding model.

Sections 6.4, 6.5 and 6.6 present derivations for the variance of the average price for the three bidding models respectively. The derivations were done using the Edgeworth approximate expansion of the joint probability distribution $Pr[J(r) = m, J(t) = l]$, shown in section 6.3.

7.0 NUMERICAL RESULTS

This chapter provides numerical examples, using the formulas obtained in previous chapters. A supply model, an aggregate load model and the three bidding models are used. The objective is to compare results across the three bidding models. Sensitivity analysis is conducted on the number of firms in the market, on the price elasticity of demand and on the anticipated peak load, in order to obtain the conclusions.

7.1 SUPPLY MODEL

The system will comprise twelve identical sets of eight generators each. The total number of units in the system will be ninety-six. Table 7.1 shows the characteristics of the generating units: capacity, production cost, mean time to failure, mean time to repair, and the steady state proportion of time that it is able to generate power. The total nominal capacity is 18000 MW.

The model considers:

- that infinite amount of energy can be bought outside the system at $75/MWh.
- 4 ownership scenarios of the system: 3, 4, 6, and 12 identical firms, with 4, 3, 2, and 1 8-unit groups each respectively.
- that all the firms forecast the load with the same accuracy.

It is assumed that the generators are dispatched in a pre-arranged merit order, based on the offered prices. There exists a positive correlation between bids and production costs.

Table 7.1: Supply model

unit i	capacity c_i(MW)	mean time to fail λ_i^{-1}(hour)	mean time to repair μ_i^{-1}(hour)	energy cost d_i(\$/MWh)	1 - FOR p_i
1	400	1100	150	6.00	0,88
2	350	1150	100	11.40	0,92
3	150	960	40	11.40	0,96
4	150	1960	40	14.40	0,98
5	200	950	50	22.08	0,95
6	100	1200	50	23.00	0,96
7	50	2940	60	27.60	0,98
8	100	450	50	43.50	0,90

7.2 BIDDING MODELS

Three bidding strategies are considered: the Bertrand model, Cournot model and a specific case of Supply Function Equilibrium (SFE): Rudkevich, Duckworth, and Rosen's model. These strategies are the three primary equilibrium models of imperfect competition. They have in common the assumption that each competing firm seeks to maximize its profit by taking into account the market conditions, its own cost structure, and estimation of the behavior of the rivals. The key difference between the models is the strategic competing variable: price, quantity, or supply function, respectively. The choice of the strategy has an impact on the level of competition among the firms and the outcome of the equilibrium price.

7.3 AGGREGATE LOAD MODEL

Load data from PJM for weekdays of Spring 2002 (March 21 to June 20, 2002) is used in these illustrations. Table 7.2 shows the mean and standard deviation of the hourly load. It is assumed that hourly loads follow a normal probability distribution. Standard deviations are small enough with respect to the mean so that the probability of negative loads can be neglected. The data points used in the model were scaled by a factor of 0.75 to fit into the supply model.

Table 7.2: Actual aggregate load model

hour t	mean μ_t(MWh)	standard deviation σ_t(MWh)
1	24392	2323
2	23256	2088
3	22686	1917
4	22449	1815
5	22854	1769
6	24570	1812
7	27676	1981
8	30283	2072
9	31579	2256
10	32403	2613
11	33135	3091
12	33505	3598
13	33644	4061
14	33889	4519
15	33844	4895
16	33767	5140
17	33717	5166
18	33498	4942
19	33147	4476
20	32913	3973
21	33296	3541
22	32347	3432
23	29663	3079
24	26800	2651

For the Cournot model, a non-zero price elasticity of the demand is proposed. A linear demand function is used having the form $D(t,p) = a(t)+pD_p$ with $D_p < 0$ being deterministic and constant across all hours t, and $a(t)$ being a random variable different for every hour. For the Rudkwevich model, zero price elasticity of demand is required.

7.4 SENSITIVITY ANALYSIS

Keeping always the same 96-unit system, four scenarios of ownership were investigated:

- 12 identical firms with 8 units each,
- 6 identical firms with 16 units each,
- 4 identical firms with 24 units each, and
- 3 identical firms with 32 units each.

Sensitivity analysis was then performed on the parameters, in the range and with the reference values given in table 7.3. For the Cournot model, five cases of first derivative D_p of demand with respect to price were considered for sensitivity analysis purposes: $D_p = -300, -250, -200, -150, -100$ $(\text{MWh})^2/\$$, reflecting a descending order of price elasticity of demand . For the Rudkevich model, five beliefs about the anticipated peak demand were taken into account to measure sensitivity, expressed as a ratio (the peak-demand-to-full-capacity ratio, PDFCR) : 0.6, 0.7, 0.8, 0.9 and 1. Bertrand model results play the role of lower bound in all the cases, except for the variances where the Rudkevich model gives the lowest values. This issue will be commented upon later.

Table 7.3: Parameters considered for sensitivity analysis

Parameter	Range	Reference value
- number of firms	3 – 12	6
- demand slope $(\text{MWh})^2/\$$ (Cournot model)	-300 – -100	-200
- peak-demand-to-full-capacity ratio (PDFCR) (Rudkevich model)	0.6 – 1.0	0.8

7.5 RESULTS

The number of firms in the market affects the expected value of the price and the variance. In all the cases, when the number of firms increases, the results tend to the Bertrand solution. Especially, the cost and capacity of each unit influence the expected value and variance of price to an important extent.

7.5.1 Hourly prices

Figure 7.1 shows that the expected values and variances of hourly prices for the Cournot model follow a similar profile to the Bertrand model, but always stay above it. Only two cases of demand elasticity are shown (the highest and the lowest), to keep the graphics clear. The other three cases fall between them.

Both expected values and variances can reach high values when the elasticity of demand is low. For different values of demand elasticity, expectations and variances with twelve firms remain around one half those when there are only three firms in the market.

For the Rudkevich model, for low anticipated peak demand, market concentration does not affect that much the results in both expectations and variances. The expected values and variances of hourly prices for the Rudkevich model are shown in figure 7.2. Only two cases of anticipated peak load are given for the sake of clarity. The results for PDFCR=0.6 were equal or very close to the Bertrand solution; therefore they are not included.

In figure 7.2.a, for PDFCR=0.8 the curve of expected values is flatter than in the other cases, and between hour 9 and hour 22, the differences between ownership scenarios are very small. As was expected, all the prices are above Bertrand hourly prices. When the anticipated peak demand is high (close to 1), then the differences are striking. Rudkevich expected prices for low demand hours are more affected by the fluctuation in demand than the expected prices for peak hours. This produces the effect of leveling of prices.

Rudkevich model variances of hourly prices (figure 7.2.b) are less disparate for the different ownership scenarios for PDFCR=0.8. The lower the anticipated peak demand is, the closer the solutions are to Bertrand's curve.

Note that, for peak hours, except in the case of 12 firms, Rudkevich's variances are smaller than Bertrand's. The reason for this is that Rudkevich's supply functions have smaller slopes than the marginal cost at peak hours. This can be better illustrated by a diagram.

Figure 7.3.b shows examples[1] of Rudkevich supply functions, together with the marginal cost function (Bertrand solution). It is necessary to note that Rudkevich offer curves, for anticipated peak load ratio less than one, continue beyond the anticipated peak load coinciding with the marginal cost curve.

In the on-peak load range 19500-20500 MWh (labeled B) the slopes of the Rudkevich supply functions are lower than the slope of the marginal cost. Thus a fluctuation in the load at that range produces smaller variations in the prices for the Rudkevich model —for any value of the anticipated peak load— than for Bertrand's. On the contrary, in the off-peak load range 13500-14500 MWh (labeled A) some of the slopes of the Rudkevich supply functions are lower, while some are higher than the slope of the marginal cost. In this case, the variances under the Rudkevich model may be smaller or larger than those for Bertrand's.

In figure 7.3.a, a Cournot solution (for a specific value of D_p whose value is not important for this purpose) is shown. Note that the slope of the Cournot solution curve always has a higher slope than the marginal cost curve. Then the variances of hourly prices for the Cournot model remain always above those for the Bertrand model. Two vertical lines help to compare slopes for the same load level.

Furthermore, the fewer the number of firms in Rudkevich's model, the higher are the expected values, but the smaller are the variances, something which is not intuitive. This

[1]These examples do not come from the supply model that is used in this work. They come from other hypothetical supply model, but they are useful for illustrating the point of interest here.

can be illustrated by figure 7.4, in which Rudkevich supply functions can be compared for three firms and twelve firms. Especially for a large load, the curves for three firms are flatter than those for twelve. This explains that the variances for three firms are smaller than those for twelve.In conclusion, the Rudkevich model has smaller variances than any other model.

7.5.2 Average prices

Two 4-hour averages are studied: at on-peak hours and at off-peak hours. All the cases for Cournot and Rudkevich are shown in the graphics.

Average for hours 13 to 16: on-peak hours

Figure 7.5 shows the expected values and variances of average price between hours 13 and 16 for the three bidding models, with sensitivity analysis. Results for the Bertrand model are insensitive to all the parameters, and are taken as references.

As could be intuited, in all the cases the expected values increase when the number of firms decreases. When the number of firms is large, there is no oligopoly any more, and the behavior tends to the perfect competition case.

As also could be expected, Cournot average prices (figure 7.5.b) increase when demand is more inelastic (i.e., D_p decreases in absolute value). The increase may be very high with respect to Bertrand prices.

Rudkevich expected values (figure 7.5.a) increase with the daily peak load to full capacity ratio, implying that a bigger peak load for a given day will drag up all the hourly prices of that day. But markups are not that large for anticipated peak loads less than or equal to 90% of total capacity.

For the Cournot model, variances of average prices (figure 7.5.d) are always above the Bertrand model case. They also increase when the number of firms decreases, and when demand is more inelastic.

On the contrary, Rudkevich model variances of average prices (figure 7.5.c) have quite a different behavior. The first thing to point out is that the variances are below that for the Bertrand model for most of the chosen values of peak-demand-to-full-capacity ratio. The reason for this was explained in the preceding subsection. See figure 7.3.b.

Second, the variances increase with the number of firms. The explanation for this is again that when the number of firms increase, the market tends to the perfect competition scenario, so the variances get close to that of the Bertrand model. In addition, with fewer companies in the market, Rudkevich prices go up and flatten more quickly and, therefore, the slopes of the supply curves are smaller for relatively higher values of the peak-demand-to-full-capacity ratio. See figure 7.4.

Third, for values of peak-demand-to-full-capacity ratio PDFCR increasing from 0.6 to 0.9, the variances of average prices decrease in this range of on-peak load. The reason can be explained through figure 7.3.b. In the range 20500-22000 MWh (labeled C), the slope of the curve for anticipated peak load = 20000 MWh is the same as marginal cost curve's (Bertrands's)[2]. For anticipated peak load = 21000 MWh the slope has a very flat part between 20000 and 20500 MWh, and beyond this range, it follows the marginal cost curve. So, the variances are smaller than those for anticipated peak load = 20000 MWh. For anticipated peak load = 22000 MWh, the variances are even lower. But for anticipated peak load = 23000 MWh, variances may increase.

Average for hours 3 to 6: off-peak hours

Figure 7.6 shows the expected values and variances of average prices between hours 3 and 6 for the three models, including sensitivity analysis. The behavior of the expected values, for both models —Cournot and Rudkevich— (figures 7.6.a and .b) are similar to the average for hours 13 to 16, discussed above. Variances of average prices for the Cournot model (figure 7.6.d) show the same patterns as those of the average for hours 13 to 16, but they are much smaller.

[2]Rudkevich offer curves, for anticipated peak load ratio less than one, continue beyond the anticipated peak load coinciding with the marginal cost curve.

Once more, the variances of average prices for Rudkevich (figure 7.6.c) have a different behavior. This time, they are above the Bertrand model for most of the cases. This may be explained by the fact that Rudkevich offers for low levels of demand have steeper slopes than Bertrand. This can be seen in figure 7.3.b. At very low load level, say 6000-8000 MWh of the supply model used in the figure [3], the slopes of Rudkevich supply functions are higher than those of the marginal cost curve.

In figure 7.6.c variances of average prices are not monotonic with respect to the number of firms (the lines cross each other). The reason for this may be that the approximation methods that were used are not very accurate when the variances are small.

7.6 SUMMARY

The number of firms and the cost-capacity structure of the supply model play an important role when the expected values and variances are compared for the different models. The latter is reflected in the Bertrand solution operating as a benchmark in most of the cases.

Cournot solutions are very sensitive to the demand elasticity. The more elastic the market, the cheaper the prices and the smaller the variances. Electricity markets are rather inelastic; and it is not an easy task to change this. In part this is so, because the end consumer does not have the opportunity to choose, or is not affected by the changes of prices in the wholesale market. This way the consumers can not react to changes in prices that would provide elasticity to the wholesale market. As times passes by, more markets are offering open access to the end consumers making the market more elastic.

Rudkevich results have less pronounced fluctuations than Cournot outputs. The sensitivity around the anticipated peak load is more important for off-peak hours. The Rudkevich

[3] Be aware of that the supply model used in the figure is not the one used in this work, but the example is useful anyway.

model helps to level the prices across hours. The variances of hourly prices are usually smaller for the Rudkevich model than for the Bertrand model. On the contrary, variances for the Cournot model are always above Bertrand's. But the variances of average prices in the Rudkevich model follow different profiles. In peak hours, variances of average prices are lower than the Bertrand model. In off-peak hours variances are higher.

It seems that designing a wholesale power market to encourage Rudekvich model behavior, with a suitable number of competing firms, would result in a better performance of the market: lower prices and smaller variances. If the Rudkevich model is an accurate representation of the electricity market, the results show that an introduction of competition may decrease the expected value of prices but the variances may actually increase with the increase of the number of competing firms in the market.

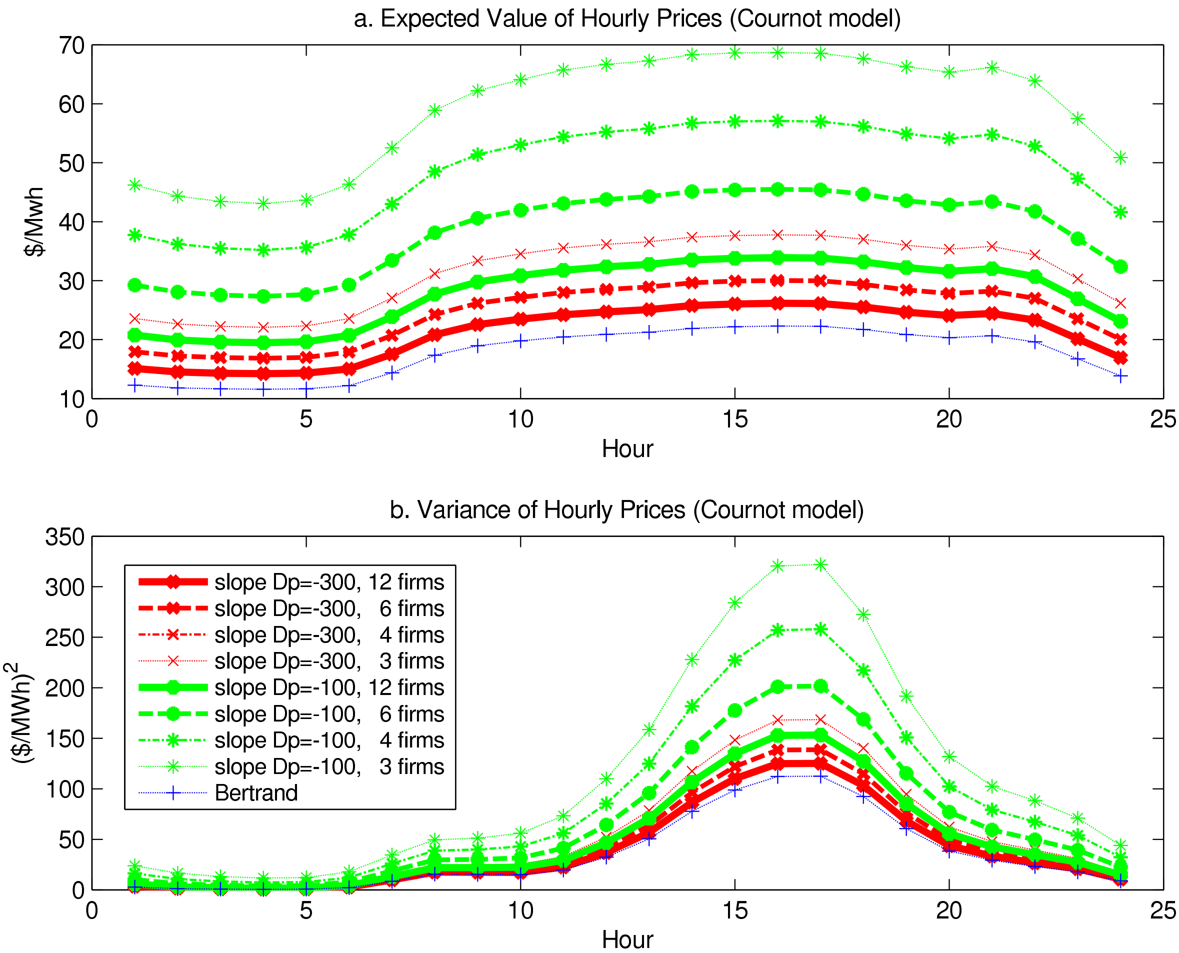


Figure 7.1: Expected values and variances of hourly prices (Cournot model)

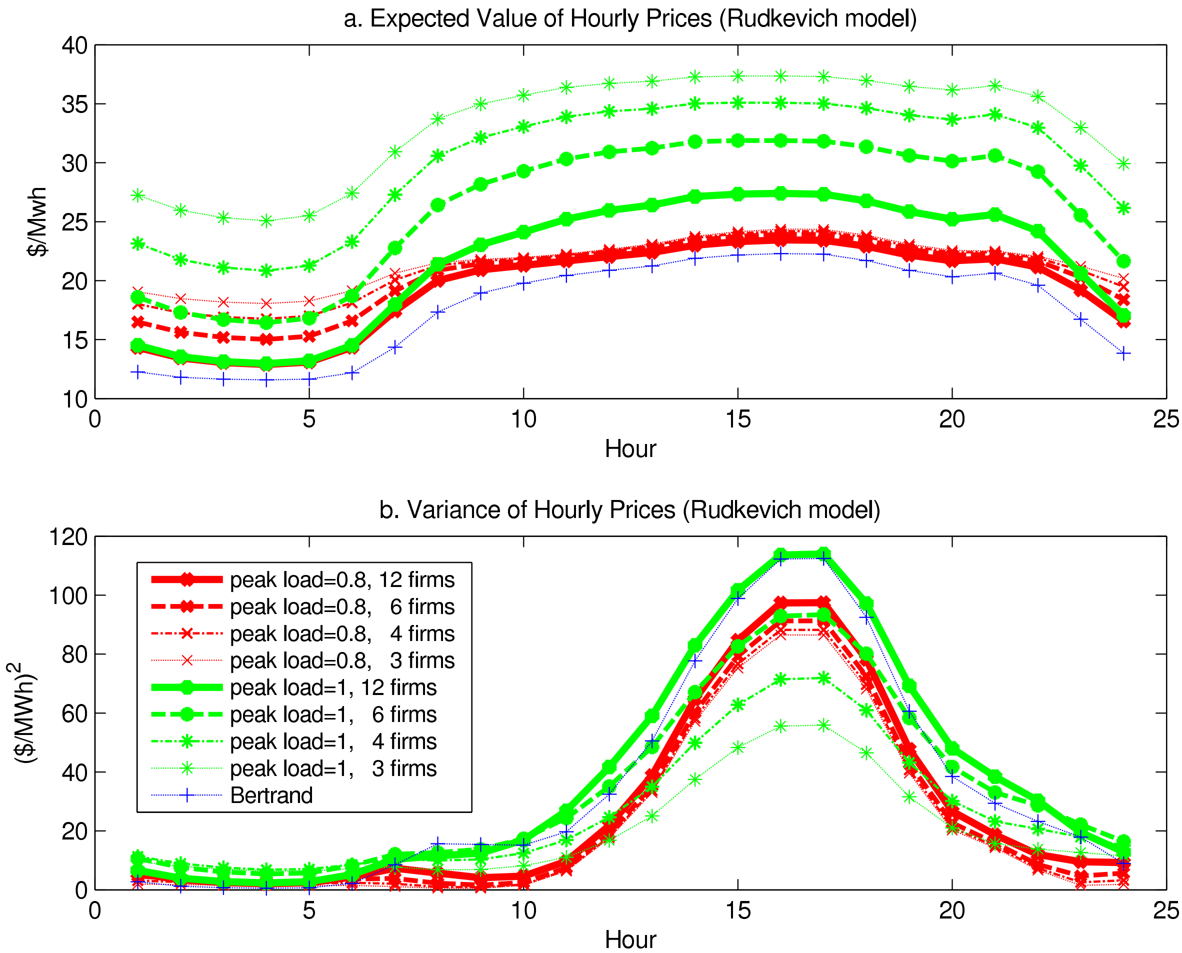


Figure 7.2: Expected values and variances of hourly prices (Rudkevich model)

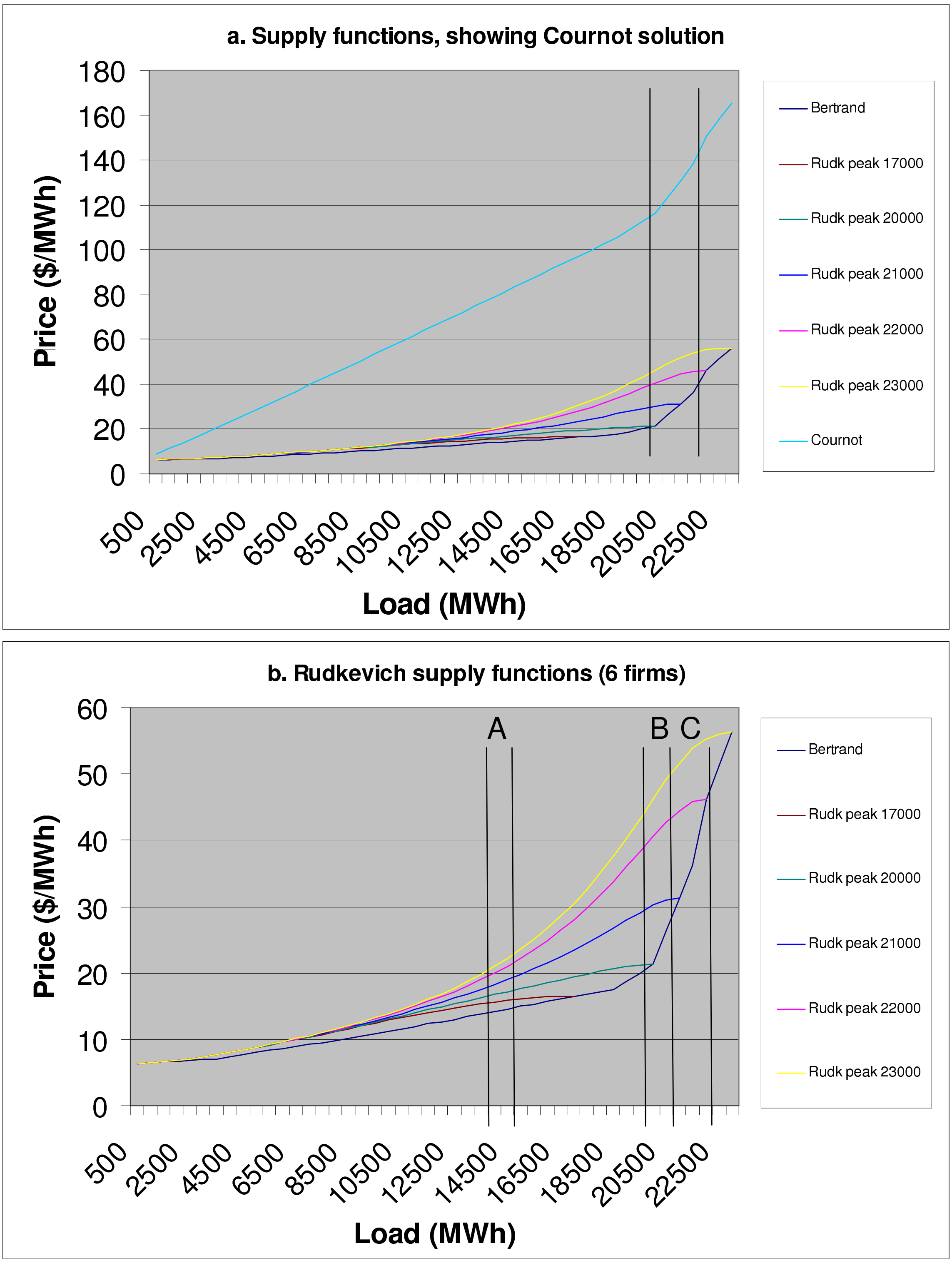
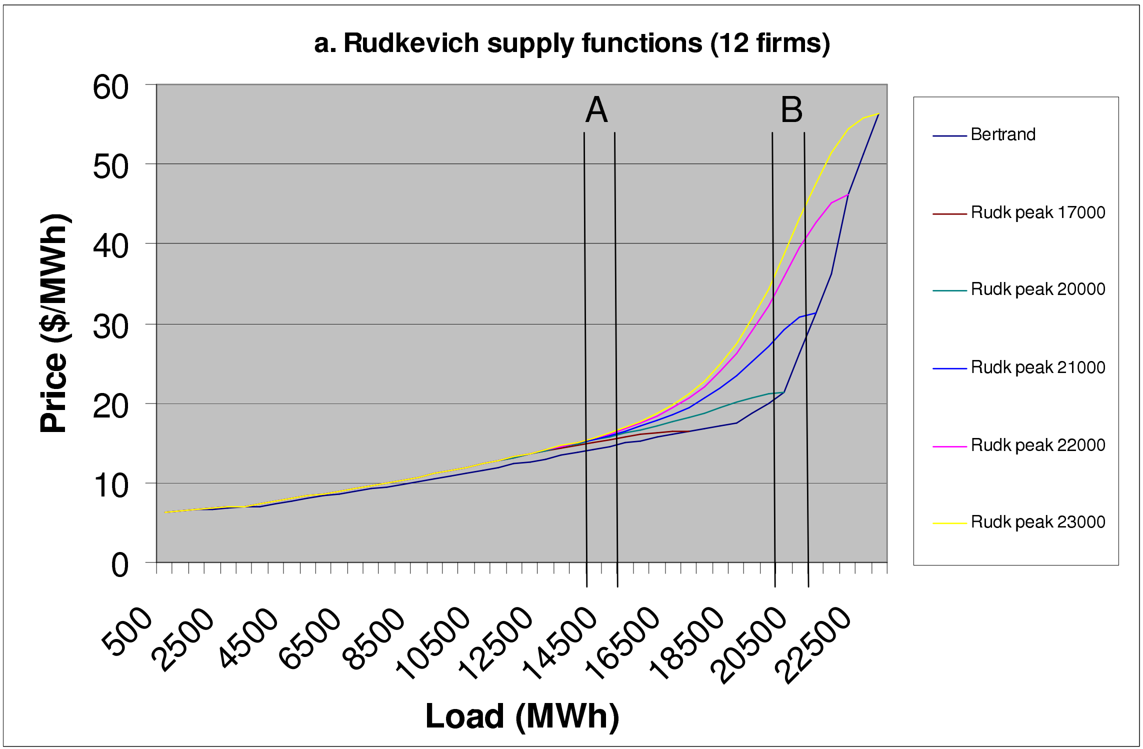


Figure 7.3: Supply functions for 6 firms



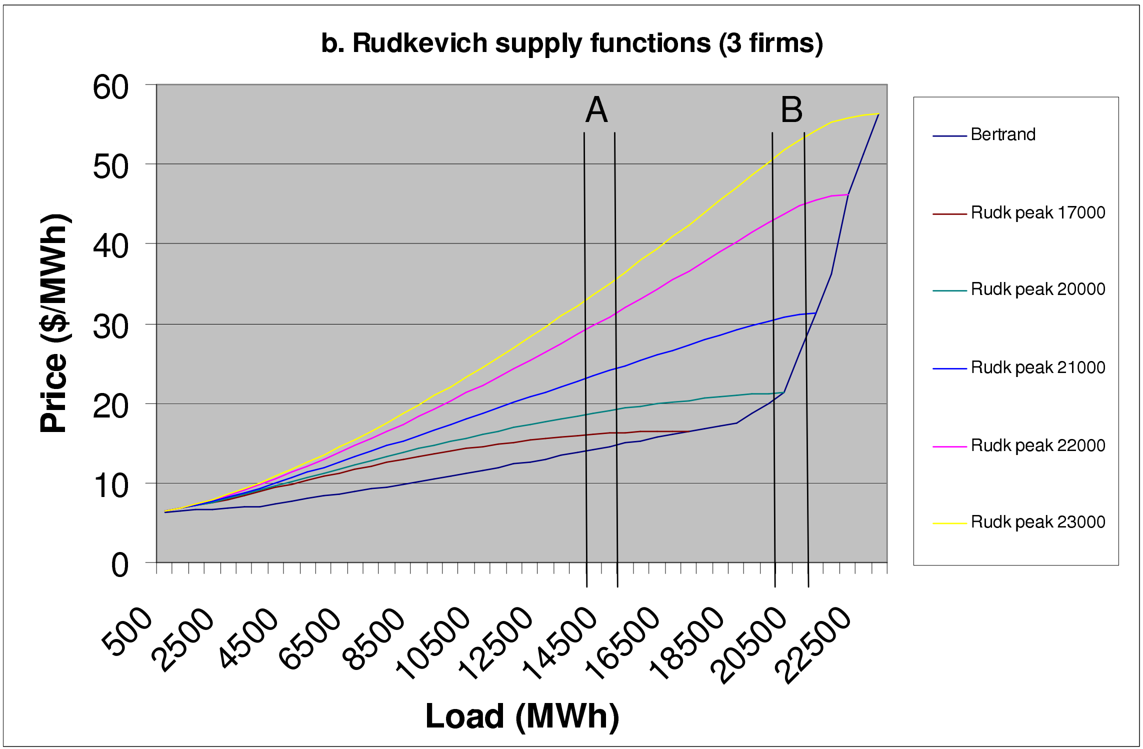


Figure 7.4: Rudkevich supply functions for 12 and 3 firms

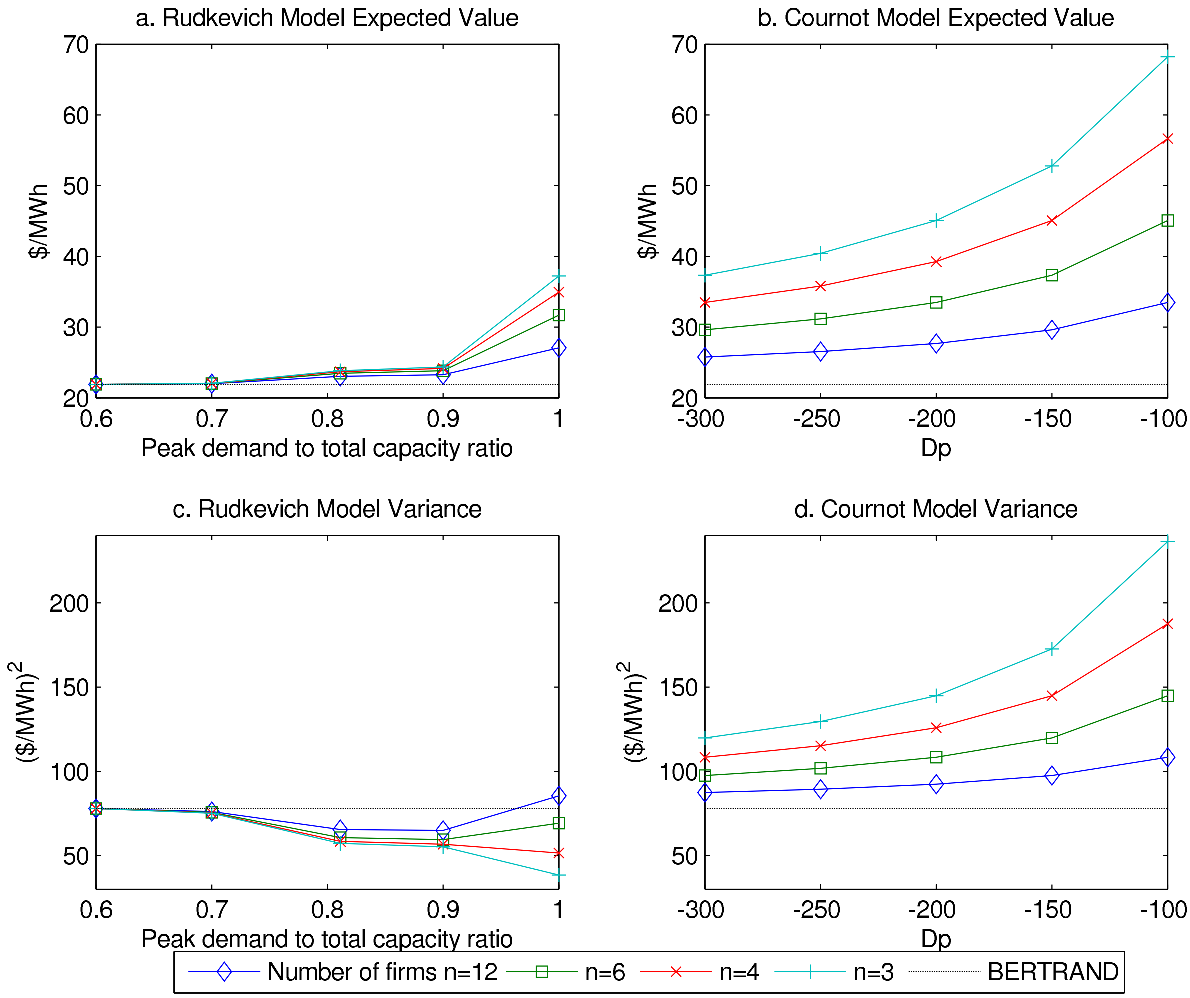


Figure 7.5: Expected values and variances of average prices between hours 13 and 16

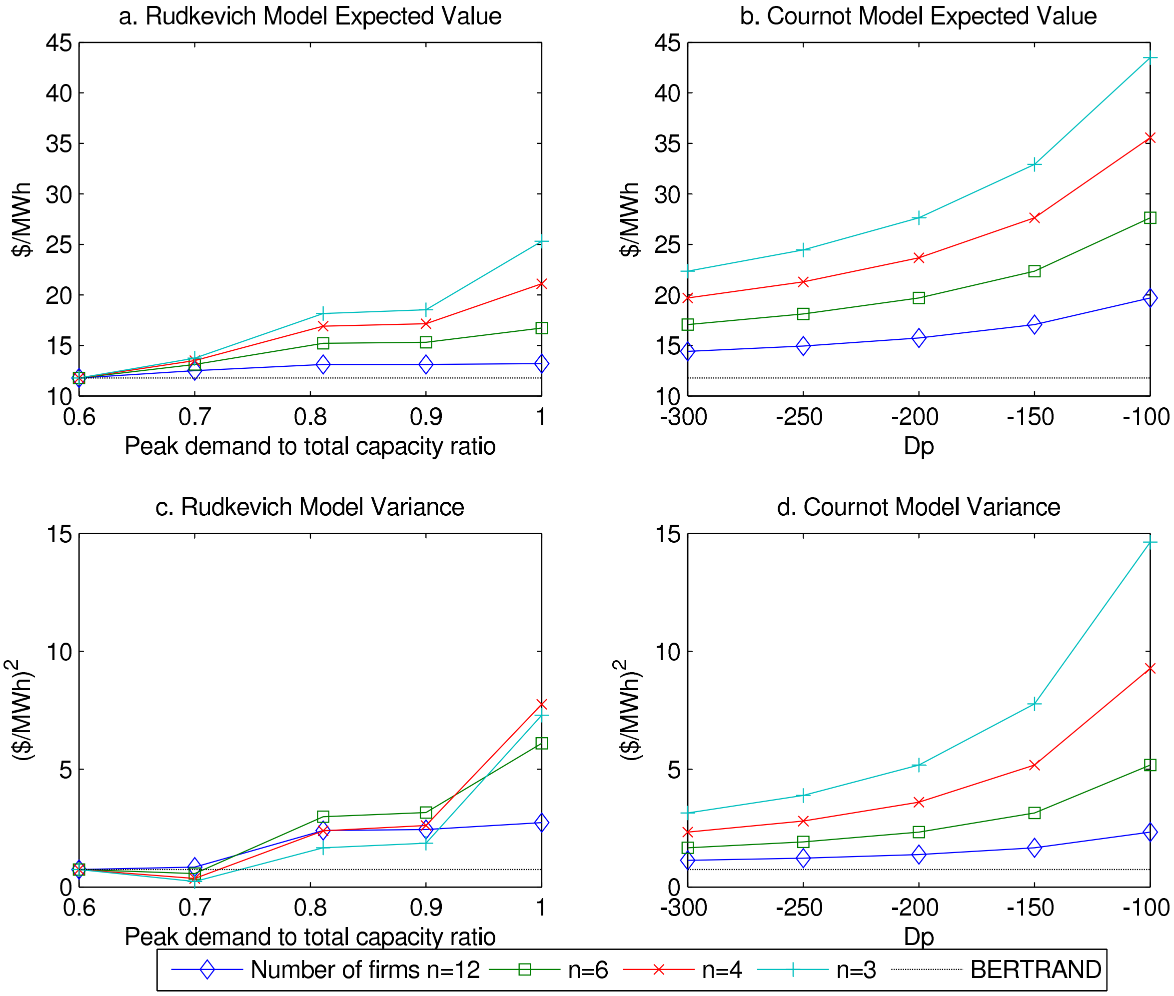


Figure 7.6: Expected values and variances of average prices between hours 3 and 6

8.0 STOCHASTIC MODEL OF THE LOAD

The sources of uncertainty that this work considers are the fluctuations of demand and the availability of the units. A question arises at this moment: Can we predict electricity prices more accurately if we can better explain the variability of demand?

Some previous work (see Valenzuela, Mazumdar and Kapoor [81]) showed that part of the load variance can be explained by the effect of temperature. This chapter studies how much the predicted quantities of the expected values and variances of hourly prices and average price change when one takes into account the information available on temperature. A stochastic model is used: the same as that in Valenzuela [76], who calibrated it based on a data set containing hourly load and temperature readings for weekdays during March to September 1996.

Three load models are considered here. The first one takes the hourly loads $L(t)$ as normally distributed and not independent, as in chapters 5 and 6. The second uses the information on temperature to explain the load and its variation. The third, in addition to using the previous information, considers explicitly the correlation of the load from one hour to the next using a time series approach.

8.1 REGRESSION EQUATIONS FOR HOURLY LOAD WITH TEMPERATURE AS INDEPENDENT VARIABLE

Ambient temperature is an important factor affecting the magnitude of the short-term variation in the load as Valenzuela and Mazumdar [77] and Valenzuela, Mazumdar, and Kapoor [81] show. This and the following sections investigate the change in the expected value and variance of electricity price when the effect of temperature is taken into account. The model is the same as in Valenzuela [76], and it is used in two steps.

This section covers the first step, in which the effect of temperature is removed from the load model; and the remaining terms $x(t)$ for each t are considered normally distributed. In the second step, it uses the full stochastic model by Valenzuela [76].

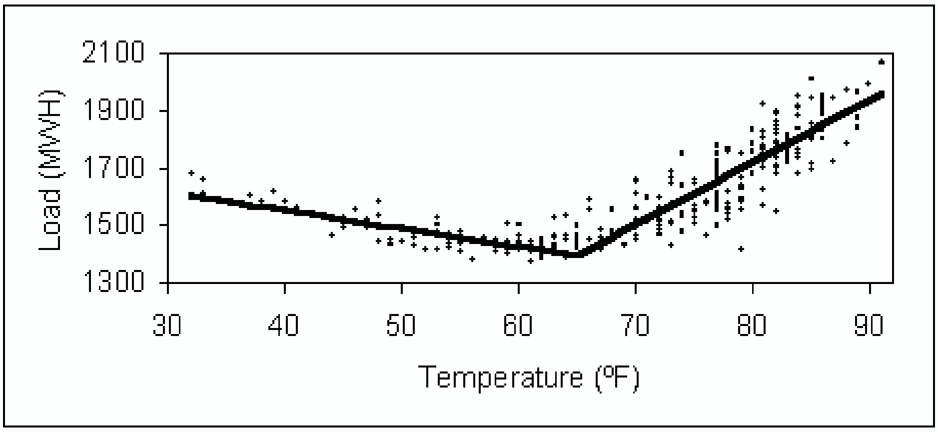


Figure 8.1: Demand versus temperature at noon in NE United States (1995-96)

Based on plots like the one shown in figure 8.1[1], for each hour t of a 24-hour period, Valenzuela found the following regression equations in which the hourly load $L(t)$ is the response and the hourly temperature τ_t(°F) is the independent variable.

$$L(t) = \beta_{0,t} + \beta_{1,t}\tau_t + \beta_{2,t}(\tau_t - 65)\delta(\tau_t) + x(t) \qquad t = 1, 2, ..., 24 \tag{8.1}$$

[1]Source: Valenzuela [76]

Where $\delta(\tau_t)$ is defined as

$$\delta(\tau_t) = \begin{cases} 0 \text{ if } \tau_t \leq 65 \\ 1 \text{ if } \tau_t > 65 \end{cases} \tag{8.2}$$

The data set was used to estimate the regression coefficients $\beta_{\cdot,t}$ shown in table 8.1.

Table 8.1: Least-square estimates of regression coefficients

hour t	$\beta_{0,t}$ (MWh)	$\beta_{1,t}$ (MWh/°F)	$\beta_{2,t}$ (MWh/°F)
1	1266.488	-3.043	24.923
2	1228.883	-3.098	23.214
3	1200.436	-2.784	21.825
4	1210.148	-3.110	23.743
5	1252.080	-3.378	25.518
6	1396.442	-4.181	26.683
7	1615.254	-5.040	24.353
8	1730.754	-4.980	25.635
9	1722.957	-4.150	25.587
10	1725.510	-4.286	25.880
11	1779.210	-5.160	26.604
12	1810.045	-5.940	27.871
13	1801.092	-6.194	27.849
14	1828.385	-6.812	28.760
15	1837.352	-7.484	29.619
16	1860.203	-8.327	30.908
17	1890.890	-8.823	30.870
18	1973.484	-10.534	33.116
19	2046.137	-11.728	33.575
20	2055.195	-11.765	33.010
21	2003.196	-10.099	32.082
22	1699.140	-4.486	28.454
23	1524.169	-3.820	28.715
24	1392.461	-3.821	28.062

First the effect of temperature is subtracted from the load, to compute the expected value and variance of the remaining $x(t)$ for each t. The new expected value and variance of hourly load are given by

$$E[L(t)] = \beta_{0,t} + \beta_{1,t}\tau_t + \beta_{2,t}(\tau_t - 65)\delta(\tau_t) + E[x(t)] \tag{8.3}$$

$$Var[L(t)] = Var[x(t)] \tag{8.4}$$

One more thing is needed to work with the new model: the computation of the covariance $\sigma_{r,t}$ between the load at time t and the load at time r, used in equation (6.18). It is assumed that the temperature can be forecasted accurately, so temperature is considered as a deterministic variable. Then, all the randomness comes from the term $x(t)$, and the covariance is then

$$Covar\left[L(t), L(r)\right] = Covar\left[x(t), x(r)\right] \tag{8.5}$$

The $Covar[x(t), x(r)]$ are computed from the historical data set, once the effect of temperature is removed.

Given a temperature forecast, the new expected values, variances and covariances of the load computed as above in this section, are used as μ_t, σ_t, and $\sigma_{r,t}$ in the model of chapters 5 and 6. Thus, different expected values and variances of prices are obtained.

8.2 TIME SERIES FOR HOURLY LOAD WITH TEMPERATURE AS INDEPENDENT VARIABLE

As a second step, the complete model by Valenzuela [76] is used. The hourly load $L(t)$ is described by equations (8.1) and (8.2) in which $x(t)$ is a time series —from now on denoted by the symbol x_t— following an ARIMA (1,120,0) process of the form

$$x_t = x_{t-120} + \rho(x_{t-1} - x_{t-121}) + z_t \tag{8.6}$$

where z_t is Gaussian white noise with mean zero and variance $\sigma_z^2 = 2032.55$ (MWh)^2, and the estimated ρ, the autocorrelation coefficient for a 1-hour lag, is .879.

The load can be predicted using the following procedure for $t = 1, 2, \ldots, 24$: 1. Based on forecasted temperature, compute its effect on hourly load. 2. Compute $\hat{x}_t$ as a time series, based on previous data. 3. Compute $\hat{L}(t)$ as the sum of steps 1 and 2.

8.3 MEAN AND VARIANCE OF THE HOURLY LOAD USING THE TIME SERIES REPRESENTATION

For each $t = 1, 2, ..., 24$, $\hat{L}(t)$, the expected value of $L(t)$ can be calculated given that the values of the time series are known up to time $t = 0$.

$$\hat{L}(t) = \beta_{0,t} + \beta_{1,t}\tau_t + \beta_{2,t}(\tau_t - 65)\delta(\tau_t) + \hat{x}_t \qquad t = 1, 2, ..., 24 \tag{8.7}$$

where $\hat{x}_t$ can be recursively computed by

$$\hat{x}_t = x_{t-120} + \rho[\hat{x}_{t-1} - x_{t-121}] \tag{8.8}$$

The recursion provides the following expression with $\hat{x}_0 = x_0$ and x_{t-120} being known terms

$$E[x_t] = \hat{x}_t = x_{t-120} + \rho^t(x_0 - x_{-120}) \tag{8.9}$$

$Var[x_t]$ can be recursively computed by

$$Var\,[x_t] = \rho^2 Var\,[x_{t-1}] + \sigma_z^2 \tag{8.10}$$

Following the recursive computation and knowing that $Var\,[x_0] = 0$ because x_0 is known, the variance of x_t can be expressed by

$$Var[x_t] = \sum_{i=0}^{t-1} \rho^{2i}\sigma_z^2 \tag{8.11}$$

Note that the limit for t going to infinity is

$$\lim_{t\to\infty} Var[x_t] = \frac{\sigma_z^2}{1-\rho^2} \tag{8.12}$$

as Box and Jenkins [14] show for an AR(1) process.

The new values change the probability distribution function of the marginal unit of section 5.2. The coefficients $K1_j(t)$ in equation (5.10) and $K2_j(t)$ in equation (5.11) are modified because μ_t and σ_t changed respectively. $K3_j$ and $K4_j$ remain the same. Section 8.6 presents an example.

Therefore, new results for the expected values and variances of hourly prices are obtained. Section 8.8 discusses the numerical results.

8.4 COVARIANCE BETWEEN LOADS AT DIFFERENT HOURS USING THE TIME SERIES REPRESENTATION

Covariance of x_t and x_r is necessary to compute the variance of average prices, and it is given by

$$Covar\,[x_t, x_r] = E[x_t x_r] - E[x_t]E[x_r] \tag{8.13}$$

Without loss of generality, let r be such that $r = t + k$, with $k >= 0$. From equation (8.8) the following equalities hold

$$\begin{aligned} E[x_r] &= E[x_{t+k}] = x_{t+k-120} + \rho(E[x_{t+k-1}] - x_{t+k-121}) \\ &= x_{t+k-120} + \rho^2(E[x_{t+k-2}] - x_{t+k-122}) \\ &\dots \\ &= x_{t+k-120} + \rho^k(E[x_t] - x_{t-120}) \end{aligned} \tag{8.14}$$

with $x_{t+k-120}$ being a known term, because $t + k$ can be 24 at most, and it is assumed that the time series' terms x_i are known for $i \leq 0$

Extending (8.6) recursively for x_r, x_r can be written

$$x_r = x_{t+k-120} + \rho^k(x_t - x_{t-120}) + \rho^{k-1}z_{t+1} + \rho^{k-2}z_{t+2} + \dots + \rho z_{t+k-1} + z_{t+k} \tag{8.15}$$

Then, considering that $x_{t+k-120}$ and x_{t-120} are known terms,

$$\begin{aligned} E[x_t x_{t+k}] &= E[x_t[x_{t+k-120} + \rho^k(x_t - x_{t-120} + \sum_{i=0}^{k-1} \rho^i z_{t+k-i})]] \\ &= E[x_t]x_{t+k-120} + \rho^k E[x_t^2] - \rho^k E[x_t]x_{t-120} \end{aligned} \tag{8.16}$$

Therefore, the covariance is

$$\begin{aligned} Covar[x_t, x_r] &= E[x_t]x_{t+k-120} + \rho^k E[x_t^2] - \rho^k E[x_t]x_{t-120} \\ &\quad - E[x_t]x_{t+k-120} - \rho^k E[x_t]^2 + \rho^k E[x_t]x_{t-120} \\ &= \rho^k(E[x_t^2] - E[x_t]^2) \\ &= \rho^k Var[x_t] \\ &= \rho^k \sum_{i=0}^{t-1} \rho^{2i}\sigma_z^2 \qquad r = t+k, \quad k \geq 0 \end{aligned} \tag{8.17}$$

The limit when t goes to infinity is the well known result

$$\lim_{t\to\infty} Covar[x_t, x_{t+k}] = \rho^k \frac{\sigma_z^2}{1-\rho^2} \tag{8.18}$$

The new expected values, variances and covariances of hourly loads, found in section 8.3 and in this section, are used as in chapter 6, to compute the expected value and variance of the average price using this new stochastic model of load. From equations (6.14) to (6.18), using the new estimates, the new cumulants K_{10}, K_{01}, K_{20}, K_{02} and K_{11} are derived. On the contrary, K_{12}, K_{30} and K_{03} remain unchanged. Consequently, equations (6.10), (6.11) and (6.12) provide the new $a_m(r)$, $a_l(t)$ and $\rho_{ml}(r,t)$ respectively.

A new bivariate probability distribution function of the marginal unit at two different hours $Pr[J(r) = m, J(t) = l]$ is obtained through equation (6.8), with the new values of $p_{ml}(r,t)$ computed using equation (6.9). Section 8.7 presents an example. Therefore, this model provides different results for the expected value and variance of average price. Section 8.8 discusses the numerical results.

8.5 NUMERICAL COMPARISON OF EXPECTED VALUES AND VARIANCES OF HOURLY LOADS

In this and the following sections, the model of chapters 5 and 6 uses the load and temperature data sets from which Valenzuela obtained the regression coefficients $\beta_{.,t}$, the autocorrelation coefficient ρ of the time series and the variance σ_z^2 of the Gaussian white noise. In the present chapter, the supply model is composed of six sets of eight generating units with the characteristics of table 7.1, instead of twelve sets, as in the previous chapter. The are forty-eight units in the system, having a maximum capacity of 9000 MW. The Valenzuela model (mean and standard deviation of the load and of the remaining terms x_t, the regression coefficients $\beta_{.,t}$ and the standard deviation σ_z of the Gaussian noise) is scaled by a factor = 4, to fit into the supply system. As a numerical example, the expected value and variance of the hourly load for a specific day (09/20/96) are computed using these three models, and are compared to the actual hourly load of that day.

Model 1 considers that $L(t)$ comes from a normal distribution for each t, as in section 7.3. The $L(t)$ are not independent. The mean μ_t, the standard deviation σ_t and the covariances $\sigma_{r,t}$ were obtained from the historical data set of load $L(t)$.

Model 2 uses the known effect of temperature and the remaining term $x(t)$ has a normal distribution for each t, as in section 8.1. The $x(t)$ are not independent. The mean μ_t, the standard deviation σ_t and covariances $\sigma_{r,t}$ were obtained using equations (8.3)-(8.5) with historical data of temperature and load.

Model 3 also takes into account the temperature effect but considers x_t as an ARIMA process, as in sections 8.2, 8.3 and 8.4. The mean μ_t, the standard deviation σ_t and co-variances $\sigma_{r,t}$ were obtained using equations (8.9), (8.11), and (8.17) respectively with the historical data on temperature and load.

Tables 8.2 and 8.3 give the expected values and variances of hourly load using the three models, for a given forecasted temperature, and the actual load for 09/20/96. Figure 8.2

graphically show the same information. As expected, the variances for model 2 turn out to be smaller for every hour than those for model 1.

Model 1 show large variances, especially for on-peak hours. Model 2 and 3, on the contrary, have smaller variances, which proves that a great part of the variance of the hourly load is explained by the temperature effect. This is the main reason for considering models 2 and 3 as refinements of model 1.

Models 2 and 3 are close to each other for the expected values as is expected. But there are significant differences for the variances. Model 3 fails to capture the cyclic nature of the load. For some hours (e.g., 4 to 15) the results of method 3 are more accurate than the outputs of method 2. This is because the remaining term $x(t)$ of load, after removing the effect of temperature, show a smaller variance than the theoretical variance of the time series model. Furthermore, for some hours (e.g., 6 to 10) even the variance of the load is smaller than that of the time series model. See table 8.3.

Model 3 was calculated as if the load at hour 24 of the preceding day was known. This means that the model predicts the loads for the following 24 hours. This may not be the actual case, in practice. PJM market, for example, requires the firms to submit the bids at noon of the preceding day. That is, the firms have to predict the loads for the following 36 hours, although they will use only the last 24 hours to decide on the bids. Consequently, the expected values and variances of model 3 will change.

Model 3 does not present advantages over model 2 to study the load variance. The two parameters of the time series x_t (the autocorrelation coefficient ρ and the white noise variance σ_z^2) which define the variance in equation (8.11) contain less information than the data set on $x(t)$.

8.6 PROBABILITY DISTRIBUTION OF THE MARGINAL UNIT

With the data used in the preceding section, the probability distributions of the marginal unit under the three load models are computed here.

Figures 8.3 - 8.5 show three sets of probability density functions of the marginal unit for each hour t, corresponding to the different load models:

1. computed as in section 5.2 using the load data for every hour,
2. considering the temperature effect and computed as in section 8.1, and
3. computed as a time series, accounting for the effect of temperature, as in section 8.3.

The surface of model 1 appears to be lower and wider than the others, showing a larger variance.

Looking more deeply into it, figure 8.6 shows the probability density functions of the marginal unit at hours 7 and 17. Each graph shows three curves corresponding to the three load models mentioned above.

At hour 7, model 3 provides a higher expected value. It does not seem to make a big difference in the variances of the load. At hour 17, model 1 shows a larger variance.

8.7 MARGINAL UNIT'S BIVARIATE PROBABILITY DISTRIBUTION

There are $\dfrac{23 * 24}{2} = 276$ bivariate probability distribution functions; and it is not practical to show all of them. As an example, figures 8.7 - 8.9 depict the joint probability distribution function of the marginal units at hours 7 and 12. Model 1 depicts a lower and wider surface than models 2 and 3, which is consistent. The three surfaces are moved to the left, showing that the load is usually bigger at 12 than at 7.

8.8 RESULTS

After running the three load models, the outputs are compared to extract some conclusions. Figure 8.10 shows the expected values and variances of hourly prices under three load models and three bidding models. Rudkevich model was selected with a peak-load-to-full-capacity ratio of 80% and Cournot model was selected with a demand-to-price slope of $D_p = -200(\mathrm{MWh})^2/\$$. Rudkevich model's expected values are close to Cournot model's for low demand hours, and closer to Bertrand model's for peak hours. There do not appear to be great differences between load models. On the contrary, the variances of hourly prices show a huge difference between load models. Variances at peak hours are extreme for model 1, being twice to five times larger than in the other two models. Across bidding models, Rudkevich model's variances for hours 6 to 23 are half of Cournot model's.

Figure 8.11 depicts the expected values and variances of average prices between hour 13 and hour 18 for the three load models and the three bidding models. In this case, all Rudkevich's and Cournot's scenarios are shown. There are no big changes in expected values of average prices across load models. Rudkevich model's expected value of average price increases a lot for a forecasted peak demand close to full capacity. Also, in this case, variances of average price are much larger for model 1.

8.9 SUMMARY

As was anticipated, temperature plays an important role in the expected value and variance of hourly prices and average prices. Forecasting temperature accurately can reduce the variance of prices considerably.

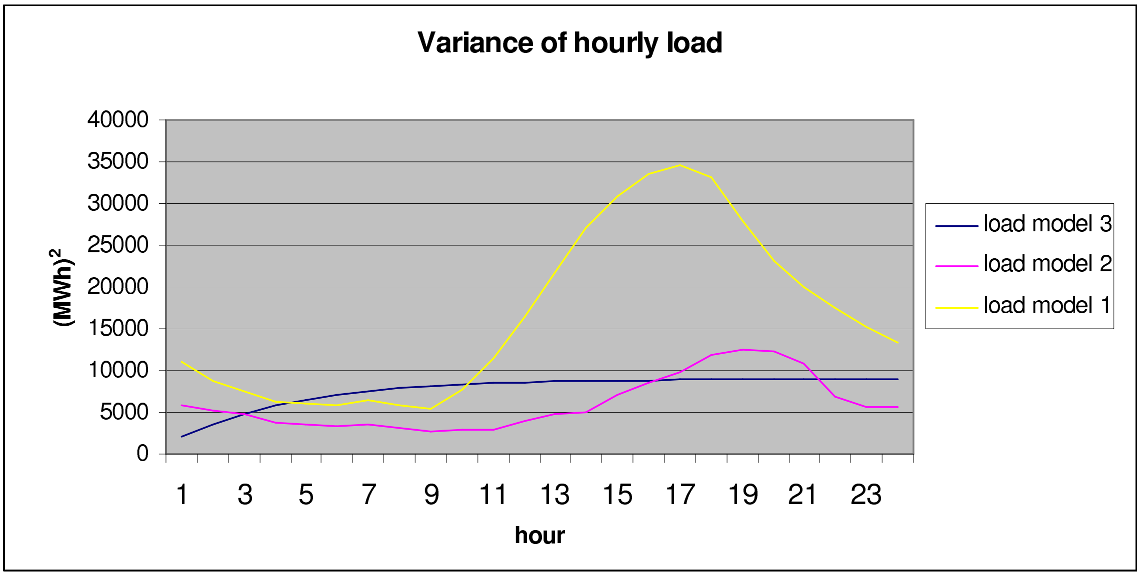
Time series analysis of the load, somehow underestimates its cyclic nature. For example, from historical data it appears that the variance of prices at early hours in the morning is very small. For these specific hours, the time series analysis does not provide more accurate estimates. The reason is that the data set contains more information hour by hour than can be captured by a single time series model.

Table 8.2: Expected value of hourly load for 09/20/96 (3 models)

Hour t	Temperature τ_t °F	Actual load MWh	Load model 1 $E[L(t)]$ MWh	Load model 2 $E[L(t)]$ MWh	Load model 3 $E[L(t)]$ MWh
1	59	1087	1185	1104	1110
2	57	1041	1129	1069	1081
3	56	1028	1103	1059	1067
4	54	996	1092	1058	1074
5	53	1035	1115	1090	1084
6	52	1139	1214	1198	1239
7	50	1325	1382	1385	1463
8	52	1440	1504	1493	1549
9	55	1456	1559	1514	1600
10	59	1469	1581	1490	1561
11	64	1501	1616	1467	1521
12	68	1504	1632	1508	1575
13	72	1497	1630	1568	1604
14	73	1505	1644	1582	1575
15	77	1476	1629	1637	1635
16	77	1470	1616	1611	1634
17	79	1436	1612	1645	1619
18	77	1415	1591	1579	1573
19	75	1374	1565	1518	1533
20	70	1399	1544	1414	1496
21	63	1408	1569	1384	1435
22	61	1379	1583	1443	1413
23	59	1242	1431	1315	1285
24	58	1116	1283	1183	1119

Table 8.3: Variance of hourly load for 09/20/96 (3 models)

Hour t	Load model 1 $Var[L(t)]$ (MWh)^2	Load model 2 $Var[x(t)]$ (MWh)^2	Load model 3 $Var[x_t]$ (MWh)^2
1	11050	5930	2027
2	8810	5132	3597
3	7546	4739	4814
4	6303	3805	5757
5	6065	3616	6488
6	5748	3376	7054
7	6433	3571	7493
8	5890	3049	7833
9	5337	2649	8096
10	7791	3017	8301
11	11382	2966	8459
12	16558	3865	8581
13	21638	4800	8676
14	27061	5084	8750
15	30767	6993	8807
16	33479	8626	8851
17	34680	9787	8885
18	33134	11899	8912
19	27981	12551	8933
20	23138	12303	8949
21	19910	10747	8961
22	17563	6914	8970
23	15263	5580	8978
24	13308	5665	8984



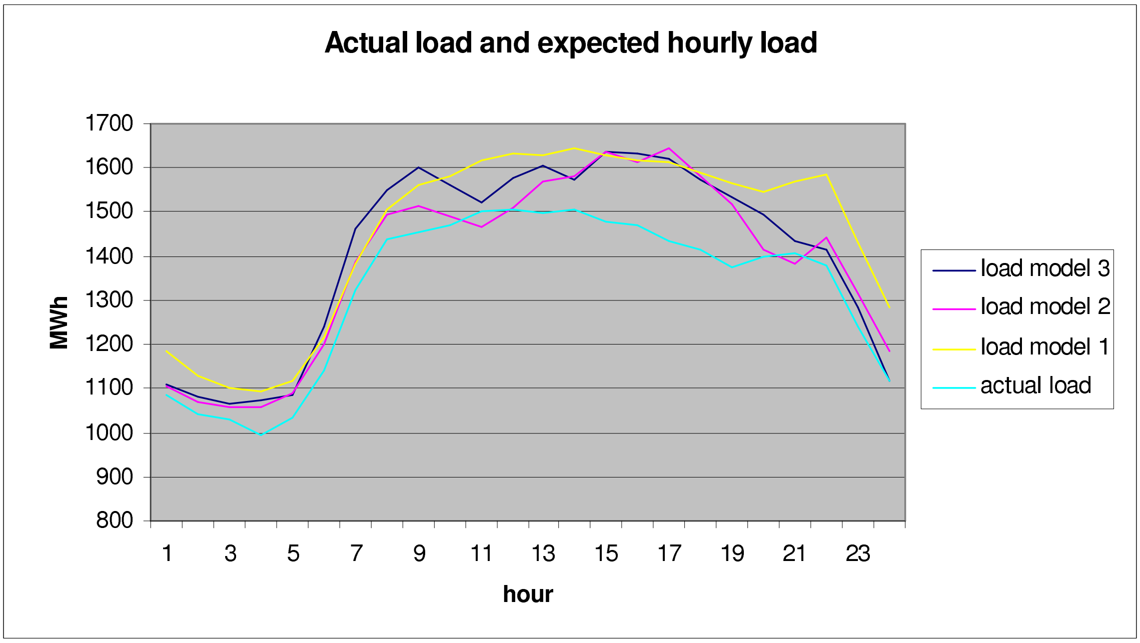


Figure 8.2: Expected values and variances of hourly load for 09/20/96 (3 models)

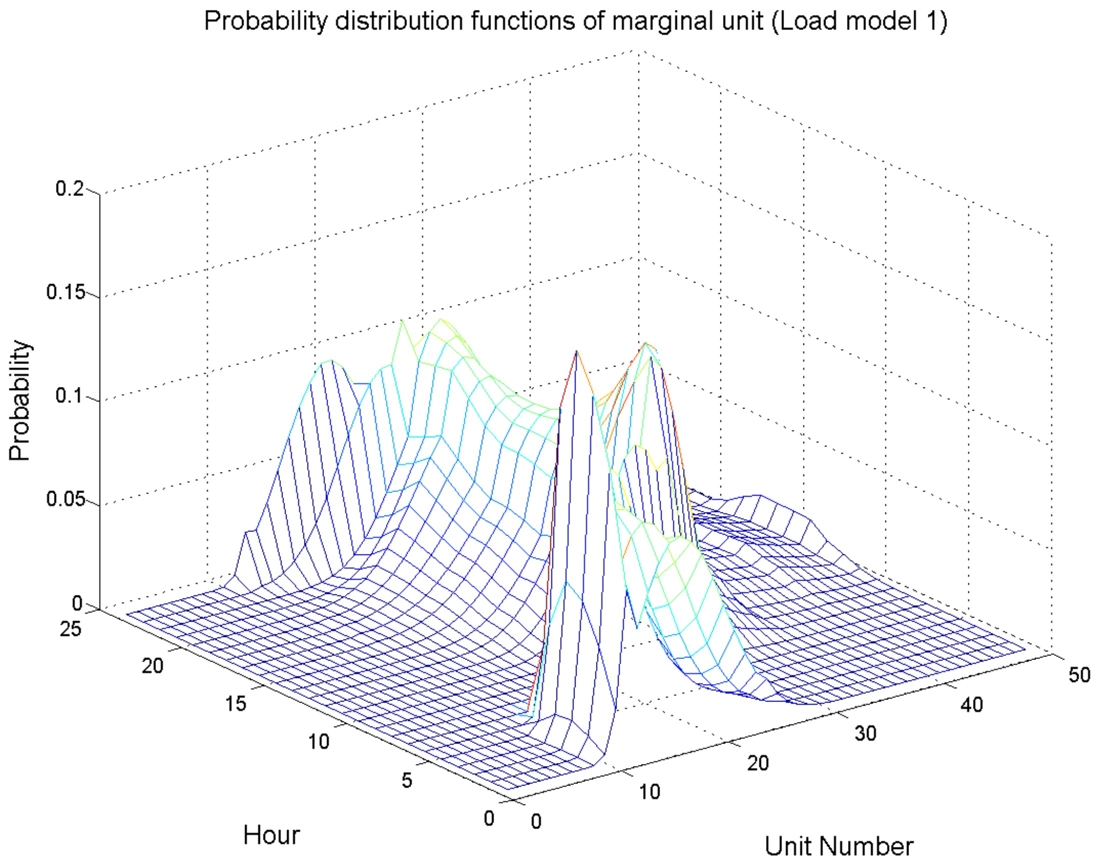


Figure 8.3: Probability distribution functions of marginal unit (Load model 1)

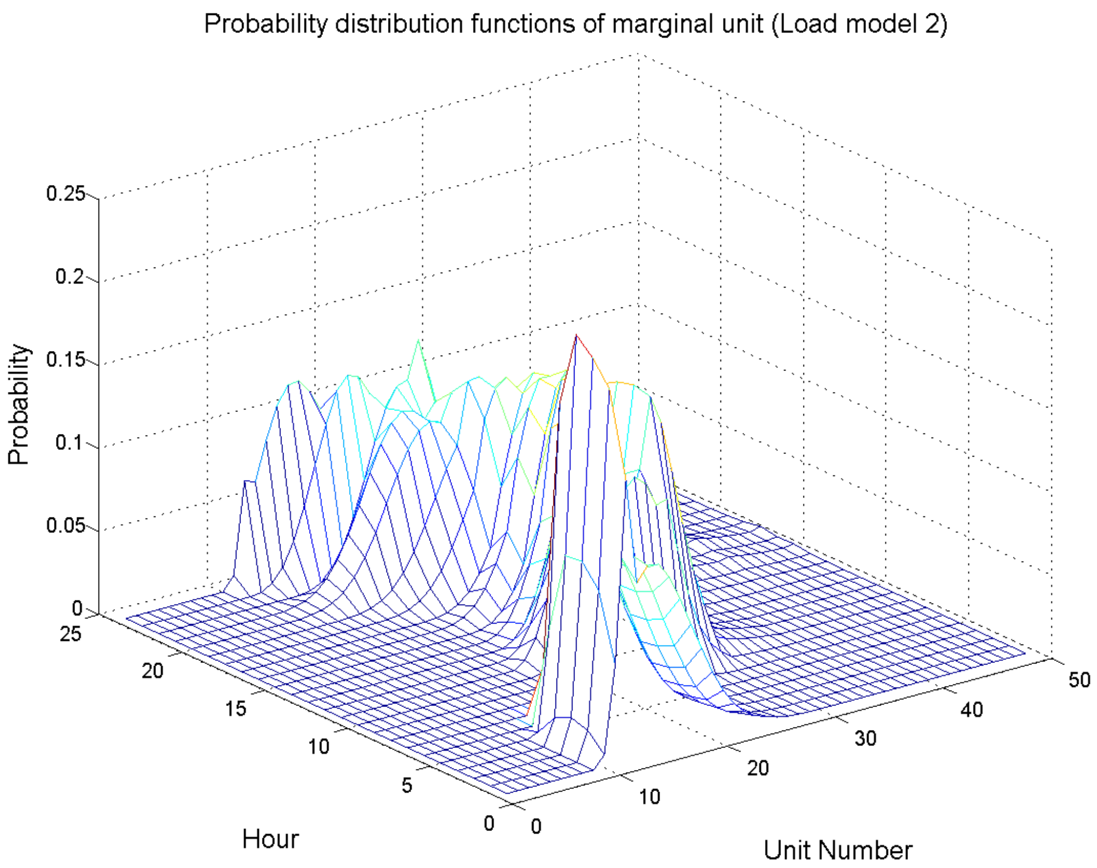


Figure 8.4: Probability distribution functions of marginal unit (Load model 2)

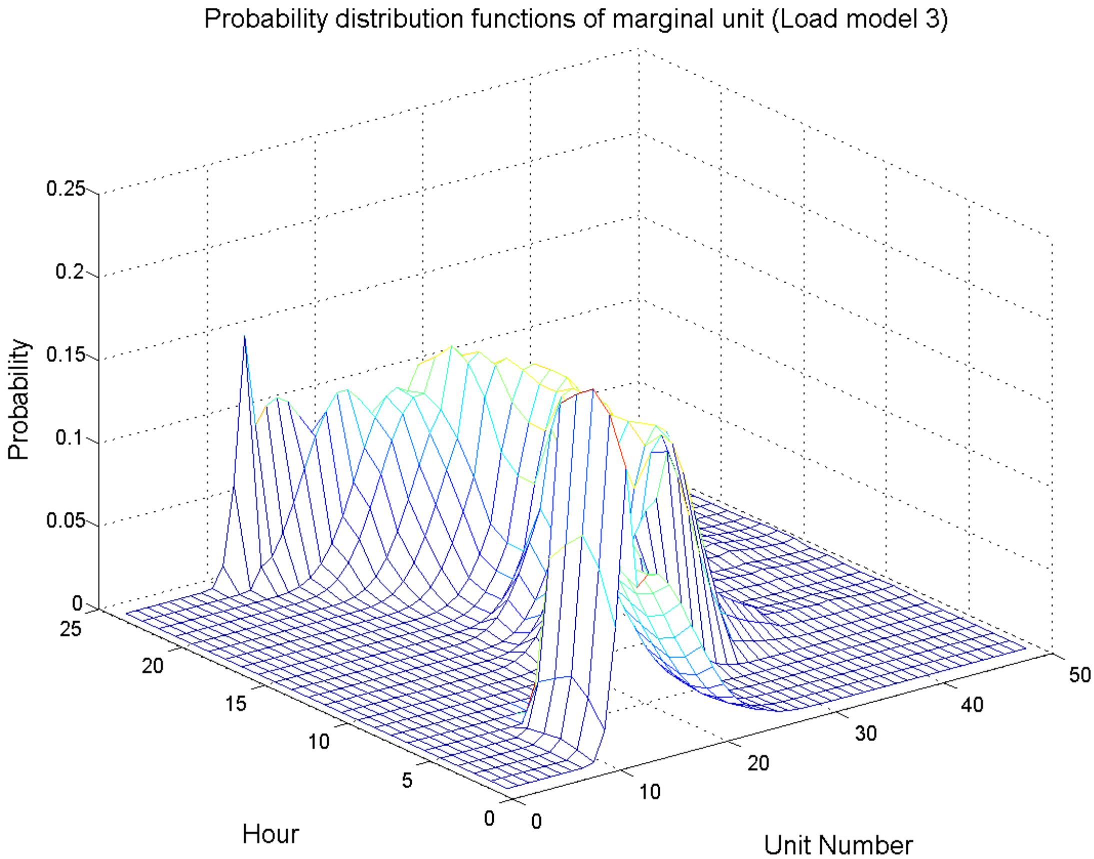
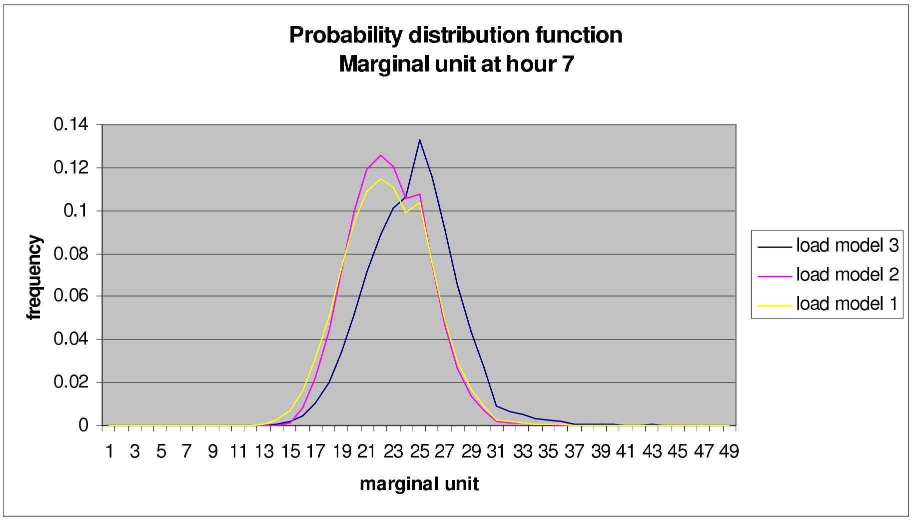


Figure 8.5: Probability distribution functions of marginal unit (Load model 3)



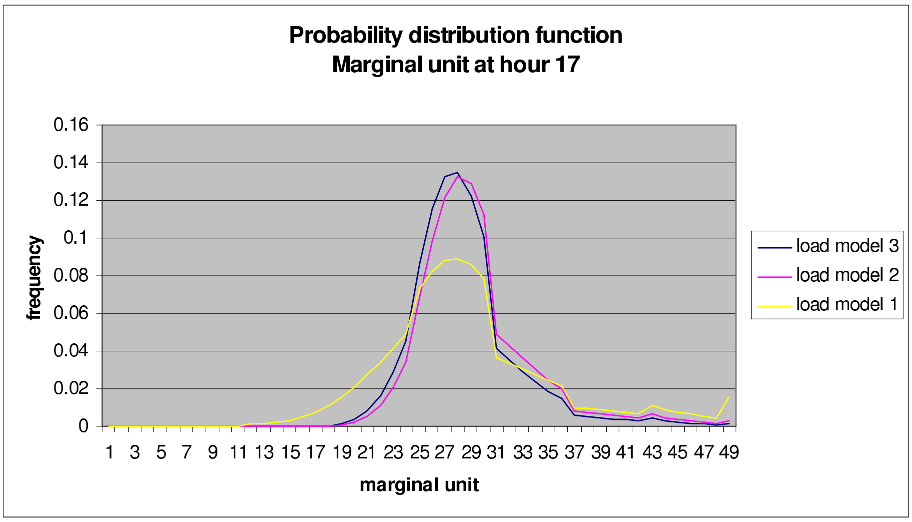


Figure 8.6: Probability distribution functions of marginal unit at two hours

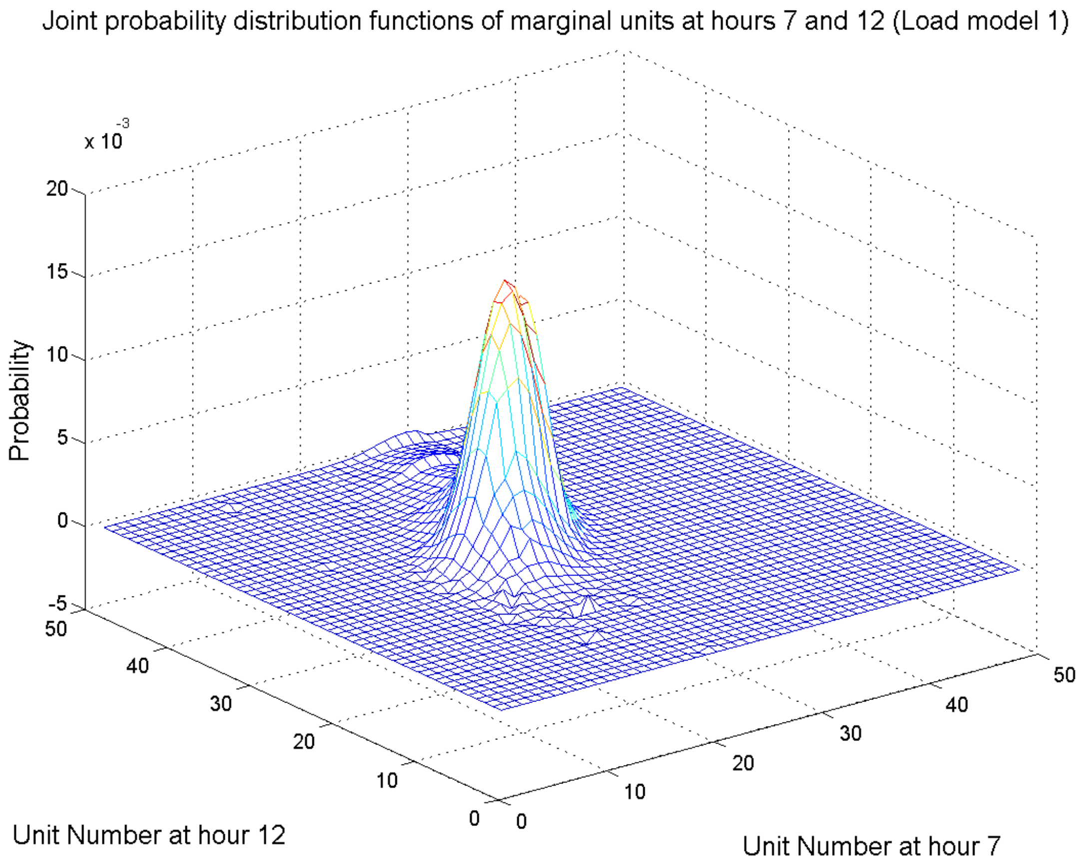


Figure 8.7: Joint probability distribution function of marginal units (Load model 1)

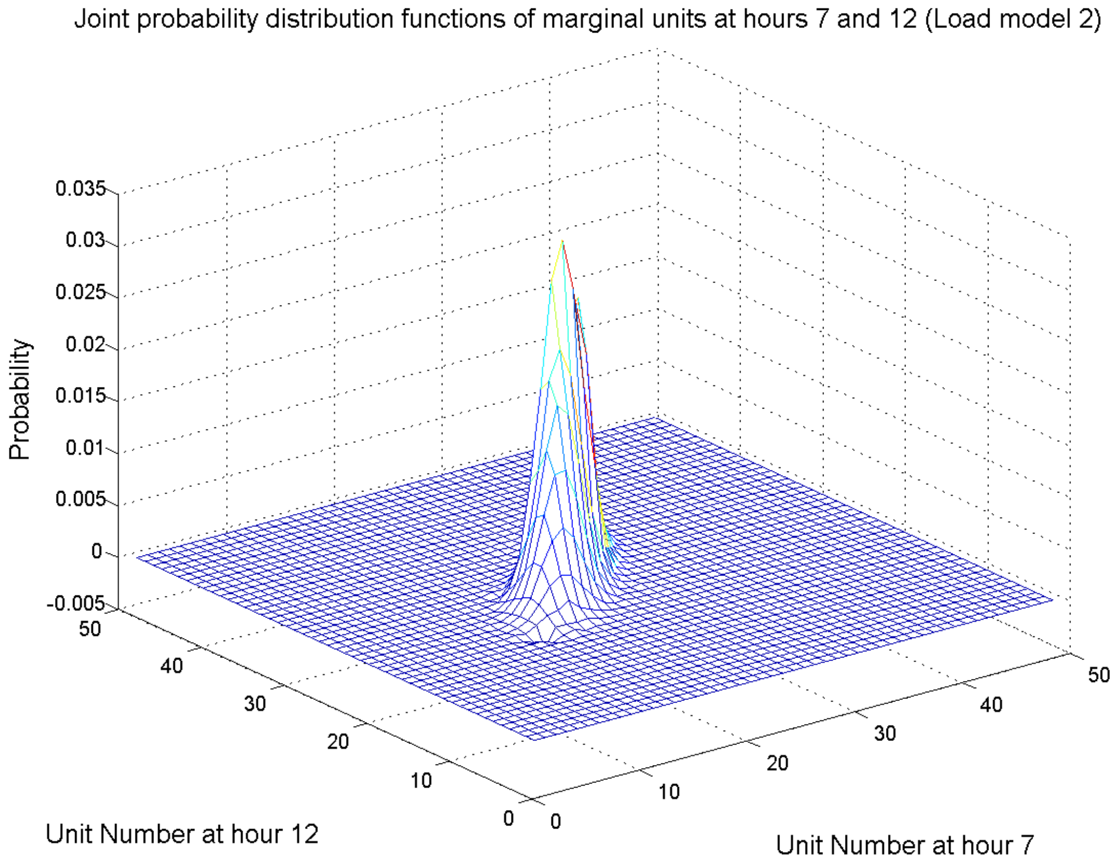


Figure 8.8: Joint probability distribution function of marginal units (Load model 2)

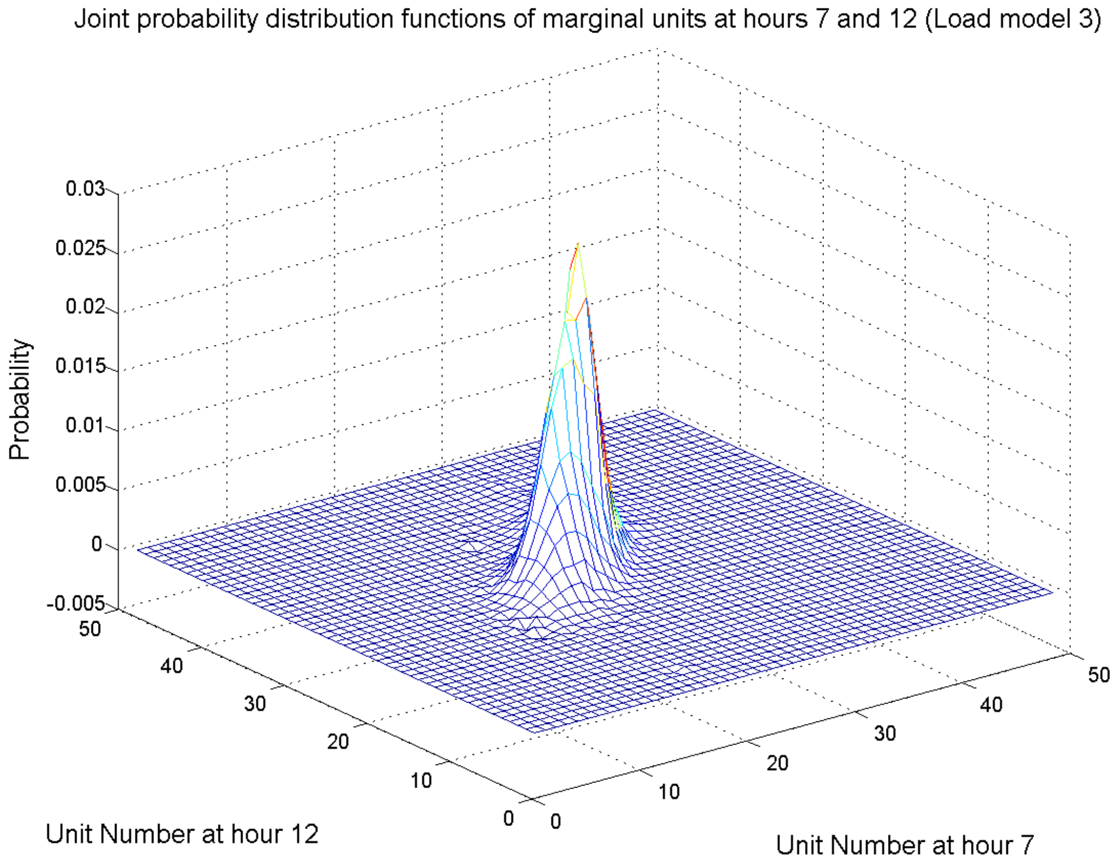


Figure 8.9: Joint probability distribution function of marginal units (Load model 3)

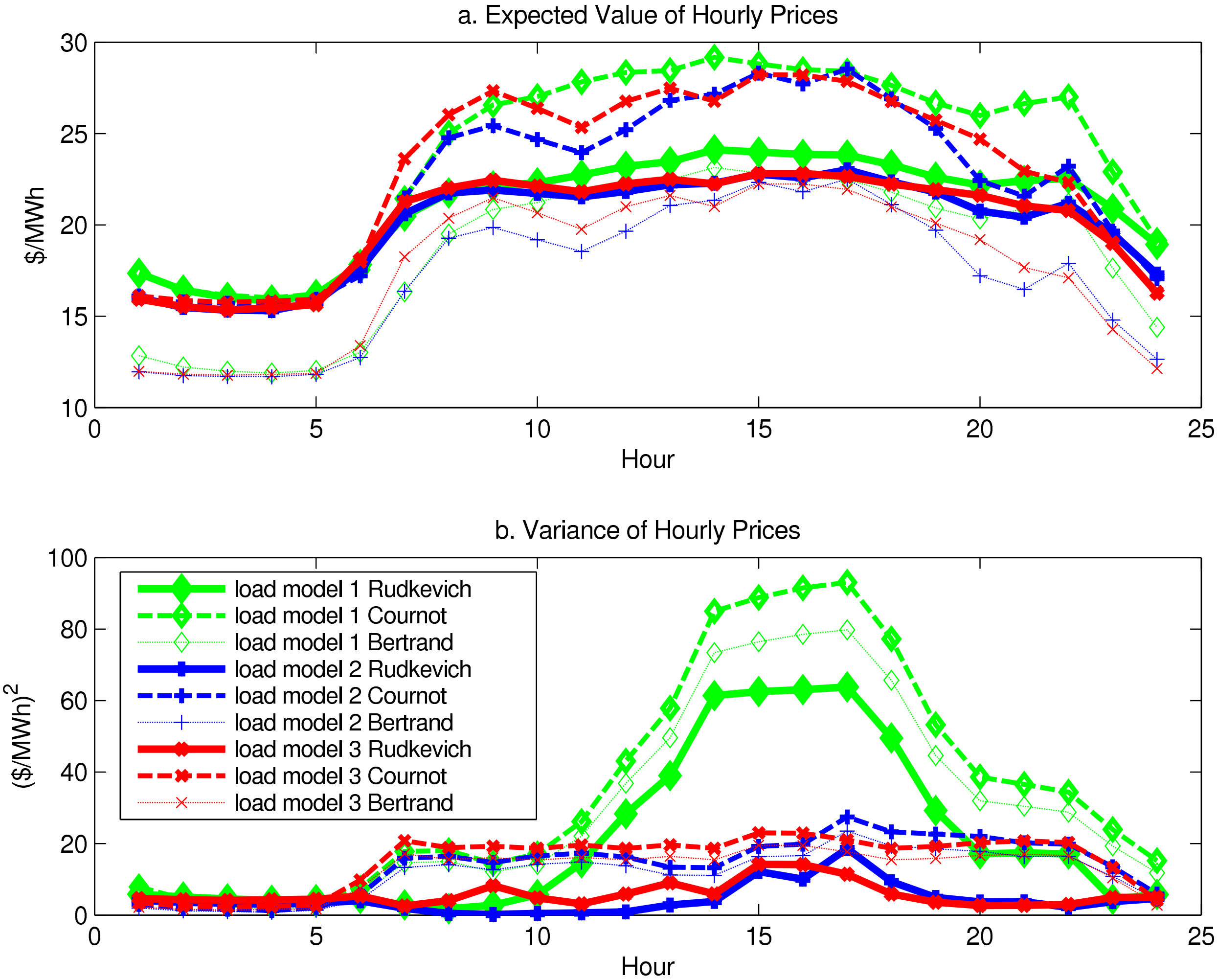


Figure 8.10: Expected values and variances of hourly prices

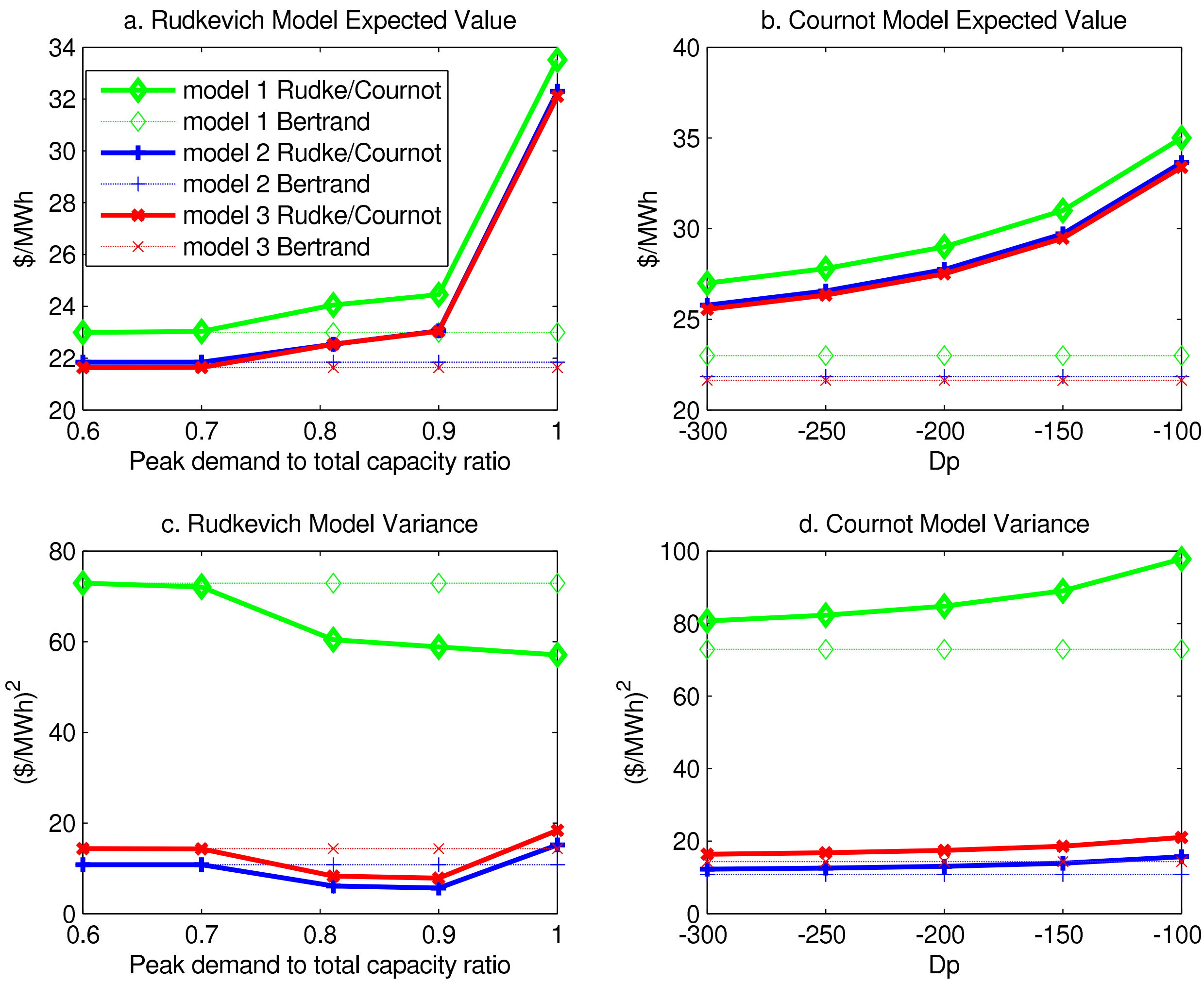


Figure 8.11: Expected values and variances of average prices

9.0 CONCLUSIONS AND DIRECTIONS FOR FUTURE RESEARCH

9.1 SUMMARY

The high volatility that wholesale electricity prices have shown in deregulated markets makes it necessary for electricity generating companies and electricity distribution firms to predict at least the expected value and variance of electricity prices, in order to make better decisions and to survive in such a challenging environment. A reasonable estimate of variance is needed as well to hedge the risk against significant changes in the market prices of electricity.

Deregulation, a global trend that started twenty-six years ago, was intended to provide efficiency to the market: lower prices, sound investments and better service. Contrary to expectations, electricity prices under deregulation have been above the marginal cost instead of being equal to it. The electricity markets which have been opened to competition have turned out to be oligopolies. This fact explains to some degree the high prices and their large variability.

Other issues that affect the price and its variance are the peculiarities of the power industry. Electricity can not be stored easily and in great quantities. Hence, it must be produced and delivered instantly. Transmission has specific physical laws. Electricity takes the path of less resistance and travels at the speed of light. Weather and other climatic events, wars, severe changes in the prices of oil and other factors that cannot be controlled, influence the power price.

The exposure of participants in the wholesale power market to price variability, makes it convenient to put in place an energy derivatives market, to hedge the risk associated with the uncertainty. This gives rise to the need for derivatives valuation and risk measurement. Consequently, it is also necessary to estimate beforehand the variance of prices.

For the reasons mentioned above, there have been many attempts to model electricity prices. Chapter 4 presents the modeling trends and a classification of the different models that exist in the literature. An important aspect to be considered is the behavior of the competing firms which react to the market rules. Three paradigmatic models for oligopolies are studied: the Bertrand model, the Cournot model, and the Supply Function Equilibrium (SFE) model. Rudkevich, Duckworth, and Rosen's solution was chosen as the a SFE because it is a realistic stepwise supply function and it has a nice closed form solution to the theoretical model's differential equation.

A fundamental stochastic model, combining the engineering process of production and the economic process of bidding and clearing price, is presented in this work to obtain the expected values and variances of hourly and average electricity prices. Some assumptions (which are also limitations to the work) are needed to get a useful model. The most important one is considering that the n firms competing in the market are identical.

The core of the dissertation lies in chapters 5 and 6 in which closed form expressions for the expected values and variances for hourly and average prices are presented respectively for the three bidding models in each case.

Chapter 5 relies mainly on the probability distribution function of the marginal unit. Based on this, the expected values and variances of marginal cost and of the hourly prices for the three bidding models are derived.

Chapter 6 presents the expressions for prices averaged over specified periods. Similar to the previous chapter, the basis of this chapter is the bivariate probability distribution function of the marginal unit at two different times.

A code written in Matlab is used to run the model. Chapter 7 depicts the numerical data for the inputs and for the outputs with the range of parameters to conduct sensitivity analysis. The conclusions are presented in the following section.

Finally, a refinement of the model is given in chapter 8. The effect of temperature on the load is studied, and it turns out that temperature can explain a great portion of the variance of prices. Two models are shown as alternatives to the original one of chapters 5 and 6. In the first of these two models, the temperature effect is removed from the load by regression equations and the remaining stochastic term is considered normally distributed but not independent. The second studies this term as a time series making explicit the autocorrelation of the data. Once again, the program written in Matlab is used to obtain numerical examples and to derive the conclusions presented below.

9.2 CONCLUSIONS

The numerical results of chapter 7 are rich enough to derive some conclusions in the following respects: price behavior with regard to market concentration, price reaction to demand elasticity and installed capacity.

Computations made using the models of chapter 8 allow us to arrive at some conclusions on the effect of temperature on expected prices and variances, and to assess the usefulness of a time series analysis of the load.

9.2.1 Market concentration

Market concentration is an important factor in the determination of the expected value and the variance of hourly and average prices, especially in the Cournot model for all values of demand elasticity. In the Rudkevich model, at on-peak hours and for a not very high anticipated peak demand, market concentration does not affect prices as much as it does at off-peak hours. With a small number of firms in the market prices tend to level off across hours. In the Rudkevich model, the greater the number of firms, the lower the prices are and the greater the variances.

What should be the most suitable and feasible number of firms in the market to assure efficiency? This question is important in the designing of the deregulated markets and its response depends on the cost-capacity structure and the market size. This work provides insights to answer this question.

9.2.2 Price reaction to demand elasticity

The Cournot model helps us to understand and to measure the effect of price elasticity of demand. As is to be expected, a large elasticity brings the prices down and the variances as well. A significant part of the demand is totally inelastic because it is needed not matter at what price. The remaining part of the demand shows more elasticity. A key factor is to design the market structure in such a way that it provides this elasticity. In order to do this, it is necessary to allow the end consumers to react to different prices in the wholesale market even though they buy energy in the retail market. This change should be carefully considered by the market designers as an important part of the deregulation process.

9.2.3 Installed capacity

The Rudkevich model has the advantage of showing the effect of the entire supply system on the prices. Prices are affected by the cost-capacity structure of the market, even by those units that are not running in a given hour. It is clear that if the market has much more capacity than needed it can assure a better service because it has a lot of energy reserve, and the buyers will appreciate that up to a certain point. Eventually, the firms will charge a bit more to compensate for the investment on the excess capacity. Even for a market which does not have a large excess capacity, the perception of the firms about the daily peak demand affects the price, under the Rudkevich model. The difference between an anticipated peak demand of 90% and of 100% turns out to be important. Then, the question to think about is how to influence the firms' beliefs.

9.2.4 Effect of temperature on the expected prices and variances

Temperature can explain in great part the variance of the load. In the example shown in chapter 8, for on-peak hours temperature explains up to 75% of the variance of the load and, what is more useful for this work, of the variance of the prices. In calibrating a good model to a real market the reduction of variance on prices can be important. This reduction in variance and therefore in the prediction error, may help companies to forecast prices more accurately and to make better decisions.

9.2.5 Time series analysis of the load

In addition to considering the effect of temperature a time series analysis was performed, to study the correlation between hourly loads. The resulting expected values and variances of the prices do no differ significantly from those obtained using just the temperature effect. Temperature plays a more important role in determining the hourly load than the load in the preceding hour. What is also true is that the hourly temperature are very correlated between them. Thus, there does not appear much reason to perform a rigorous time series analysis of the remaining term for this purpose.

9.3 DIRECTIONS FOR FUTURE RESEARCH

This work can be extended in a number of ways, towards more realistic situations by increasing its complexity. Some recommendations follow.

Calibrating the model for a real market. With real data from the cost-capacity structure, the actual offers, the actual hourly loads and the hourly prices, it will be possible to choose the most suitable model for that market.

Incorporating fuel cost as another source of uncertainty. In the model, the fuel costs were considered deterministic variables because their variability is not that great. But, in fact, they are random variables that add uncertainty to the process.

Extending the model for asymmetric firms. In this case there will not be closed formulas to work with but it will perhaps be possible to get results with optimization or simulation techniques.

Incorporating transmission constraints. In fact, the prices in a market are location dependent. Due to transmission constraints, prices in one node may differ from prices in another node. The merit order is then broken. Optimization techniques are appropriate for this problem.

Incorporating the unit commitment problem. The merit order is also broken several times because of technical aspects of the generating units such as start-up costs and start-up and shut-down times that make it more efficient to keep running more expensive units.

APPENDIX

THE EQUIVALENT LOAD

A.1 EXPECTED VALUE AND VARIANCE OF THE EQUIVALENT LOAD USING EDGEWORTH APPROXIMATION

The cumulative distribution function of the random variable $\bar{L}_j(t)$ is denoted by $F_j(x;t)$, and the probability density function of $\bar{L}_j(t)$ by $f_j(x;t)$.

Note that the cumulative distribution functions $F_j(x;t)$ and $G_j(0;t)$ (the latter defined in section 5.2 as the cumulative distribution function of $X_j(t)$) are linked between them because the correspondent random variables $\bar{L}_j(t)$ and $X_j(t)$ are obviously related. In the following appendix, the formula for $Pr[J(t) = j]$ is derived using the two cumulative distribution functions, showing the relationship between them.

Using that

$$E\left[\bar{L}_j(t)\,|C_{j-1} < \bar{L}_j(t) \le C_j\right] = \frac{\int\limits_{C_{j-1}}^{C_j} x f_j(x;t)dx}{Pr\left[C_{j-1} < \bar{L}_j(t) \le C_j\right]} \tag{.1}$$

equation (5.20) becomes

$$E\left[\bar{L}_{J(t)}(t)\right] = \sum_{j=1}^{N+1} \frac{\int\limits_{C_{j-1}}^{C_j} x f_j(x;t)\,dx}{Pr\left[C_{j-1} < \bar{L}_j(t) \leq C_j\right]} Pr\left[J(t) = j\right] \tag{.2}$$

Because $Pr\left[J(t) = j\right] = Pr\left[C_{j-1} < \bar{L}(j) \leq C_j\right]$ equation (.2) can be written as

$$E\left[\bar{L}_{J(t)}(t)\right] = \sum_{j=1}^{N+1} \int\limits_{C_{j-1}}^{C_j} x f_j(x;t)\,dx \tag{.3}$$

To compute $f_j(x;t)$, the Edgeworth approximation given in Cramer [20] is used

$$\begin{aligned} f_j(x;t) \;\cong\; & \frac{1}{K2_j(t)^{1/2}}\phi(z)\left[1 + \frac{1}{6}\frac{K3_j}{K2_j(t)^{3/2}}(z^3 - 3z) + \frac{1}{24}\frac{K4_j}{K2_j(t)^2}(z^4 - 6z^2 + 3)\right. \\ & \left. + \frac{1}{72}\frac{K3_j^2}{K2_j(t)^3}(z^6 - 15z^4 + 45z^2 - 15)\right] \end{aligned} \tag{.4}$$

where

$$z = z_j(x;t) = \frac{x - \bar{K}1_j(t)}{\sqrt{K2_j(t)}} \tag{.5}$$

$\bar{K}1_j(t) = \mu_t + \sum_{i=1}^{j} c_i q_i$, and $K2_j(t)$, $K3_j$ and $K4_j$ are the same as defined in equations (5.11) to (5.13).

A similar procedure is used to compute the variance

$$Var\left[\bar{L}_j(t)^2\right] = E\left[\bar{L}_{J(t)}(t)^2\right] - E^2\left[\bar{L}_{J(t)}(t)\right] \tag{.6}$$

where

$$E\left[\bar{L}_{J(t)}(t)^2\right] = \sum_{j=1}^{N+1} E\left[\bar{L}_{J(t)}(t)^2 | J(t) = j\right] Pr\left[J(t) = j\right] \tag{.7}$$

The conditional expectation of $\bar{L}_j(t)^2$ is

$$\begin{aligned} E\left[\bar{L}_{J(t)}(t)^2 | J(t) = j\right] &= E\left[\bar{L}_{J(t)}(t)^2 | C_{j-1} < \bar{L}_{J(t)}(t) \leq C_j\right] \\ &= \frac{\int\limits_{C_{j-1}}^{C_j} x^2 f_j(x;t)dx}{Pr\left[C_{j-1} < \bar{L}_j \leq C_j\right]} \end{aligned} \tag{.8}$$

Equation (.7) becomes

$$E\left[\bar{L}_{J(t)}(t)^2\right] = \sum_{j=1}^{N+1} \int\limits_{C_{j-1}}^{C_j} x^2 f_j(x;t)dx \tag{.9}$$

Using this expression in equation (.6), the variance is given by

$$Var\left[\bar{L}_{J(t)}(t)\right] = \sum_{j=1}^{N+1} \int\limits_{C_{j-1}}^{C_j} x^2 f_j(x;t)dx - \left[\sum_{j=1}^{N+1} \int\limits_{C_{j-1}}^{C_j} x f_j(x;t)dx\right]^2 \tag{.10}$$

A point to note is that with due patience equations (.3) and (.10) can be evaluated in a closed form.

A.2 RELATION BETWEEN EQUIVALENT LOAD AND EXCESS OF LOAD NOT MET

In the preceding appendix, it was mentioned that $F_j(x;t)$ and $G_j(0;t)$, cumulative probability distribution functions of $\bar{L}_j(t)$ and $X_j(t)$ respectively, are related. In this appendix, two formulas are derived for $Pr[J(t) = j]$ using both cumulative distribution functions, and showing the relationship between them.

It can be shown that $G_j(0;t) = F_j(C_j;t)$

Recall that

$$\begin{aligned} G_j(0;t) &= Pr\left[X_j(t) \leq 0\right] \\ &= Pr\left[L(t) - \sum_{i=1}^{j} c_i Y_i(t) \leq 0\right] \\ &= Pr\left[L(t) - \sum_{i=1}^{j} c_i Y_i(t) + \sum_{i=1}^{j} c_i \leq C_j\right] \\ &= Pr\left[\bar{L}_j(t) \leq C_j\right] \\ &= F_j(C_j;t) \end{aligned}$$

Similarly $G_{j-1}(0;t) = Pr\left[\bar{L}_{j-1}(t) \leq C_{j-1}\right]$.

Note that $J(t) = j$ implies $Y_j(t) = 1$. Therefore, given that $J(t) = j$, then

$$\bar{L}_j(t) = L(t) + \sum_{i=1}^{j}(1 - Y_i(t))c_i = L(t) + \sum_{i=1}^{j-1}(1 - Y_i(t))c_i = \bar{L}_{j-1}(t)$$

So

$$\begin{aligned} G_{j-1}(0;t) &= Pr\left[\bar{L}_{j-1}(t) \leq C_{j-1}\right] \\ &= Pr\left[\bar{L}_j(t) \leq C_{j-1}\right] \\ &= F_j(C_{j-1};t) \end{aligned}$$

And

$$\begin{aligned} Pr\left[J(t) = j\right] &= G_j(0;t) - G_{j-1}(0;t) \\ &= F_j(C_j;t) - F_j(C_{j-1};t) \\ &= Pr\left[C_{j-1} < \bar{L}_j(t) \leq C_j\right] \end{aligned}$$

A.3 EQUIVALENT LOAD APPROXIMATION

In section 5.4 an approximation of $\bar{L}_{J(t)}(t)$ was performed to make the computations easier. Table A1 shows the results of running the model in chapters 5 and 6 using the two possible limits of $\bar{L}_{J(t)}(t)$, namely C_j and C_{j-1}, for the 96-unit system. The parameter considered for the Cournot model is a demand slope of -200 (MWh)^2/\$. For the Rudkevich model, the capacity used at peak demand is assumed to be 81%. Expected values and variances of average price between hours 14 and 15, and of prices at hours 5 and 20 are shown for both models. The 96 units are supposed to be owned by 6 identical firms. Bertrand model's results are not affected by the approximation.

Differences are very small in absolute values, as it can be seen, and also in percentage. Thus the use of the approximation (namely, replacement of $\bar{L}_{J(t)}(t)$ by C_j) appears to be justified. The use of C_j gives always the upper bound to the expected value. It is not necessarily the case for the variance.

Table A1: Justification of the use of an approximate equivalent load

	using C_j	using C_{j-1}	difference	%
Expected Value of Average Price	(\$/MWh)			
Cournot	33.64	33.45	0.19	0.6%
Rudkevich	23.58	23.49	0.09	0.4%
Variance of Average Price	$(\$/\text{MWh})^2$			
Cournot	117.39	110.81	6.58	5.6%
Rudkevich	67.35	68.16	-0.81	-1.2%
Expected Value of Hourly Prices	(\$/MWh)			
hour 5				
Cournot	19.65	19.47	0.18	0.9%
Rudkevich	15.29	14.95	0.34	2.2%
hour 20				
Cournot	31.58	31.43	0.15	0.5%
Rudkevich	22.16	22.04	0.12	0.5%
Variance of Hourly Prices	$(\$/\text{MWh})^2$			
hour 5				
Cournot	1.98	2.09	-0.11	-5.6%
Rudkevich	2.82	2.83	-0.01	-0.4%
hour 20				
Cournot	55.67	54.08	1.59	2.9%
Rudkevich	22.76	23.33	-0.57	-2.5%

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