A NEURAL NETWORK APPROACH FOR MULTI-ATTRIBUTE PROCESS CONTROL WITH COMPARISON OF TWO CURRENT TECHNIQUES AND GUIDELEINES FOR PRACTICAL USE

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Both manufacturing and service industries deal with quality characteristics, which include not only variables but attributes as well. In the area of Quality Control there has been substantial research in the area of correlated variables (i.e. multivariate control charts); however, little work has been done in the area of correlated attributes. To control product or service quality of a multi-attribute process, several issues arise. A high number of false alarms (Type I error) occur and the probability of not detecting defects increases when the process is monitored by a set of uni-attribute control charts. Furthermore, plotting and monitoring several uni-attribute control charts makes additional work for quality personnel.

To date, a standard method for constructing a multi-attribute control chart has not been fully evaluated. In this research, three different techniques for simultaneously monitoring correlated process attributes have been compared: the normal approximation, the multivariate *np*-chart (MNP chart), and a new proposed Neural Network technique. The normal approximation is a technique of approximating multivariate binomial and

Poisson distributions as normal distributions. The multivariate *np* chart (MNP chart) is base on traditional Shewhart control charts designed for multiple attribute processes. Finally, a Backpropagation Neural Network technique has been developed for this research. Each technique should be capable of identifying an out-of-control process while considering all correlated attributes simultaneously.

To compare the three techniques an experiment was designed for two correlated attributes. The experiment consisted of three levels of proportion nonconforming *p*, three values of the correlation matrix, three sample sizes, and three magnitudes of shift of proportion nonconforming in either the positive or negative direction. Each technique was evaluated based on average run length and the number of replications of correctly identified given the direction of shifts (positive or negative). The resulting performances for all three techniques at their varied process conditions were presented and compared.

From this study, it has observed that no one technique outperforms the other two techniques for all process conditions. In order to select a suitable technique, a user must be knowledgeable about the nature of their process and understand the risks associated with committing Type I and II errors. Guidelines for how to best select and use multi-attribute process control techniques are provided.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

1.0 INTRODUCTION

Quality has been a concern in the manufacturing industry since 1700[1]. In present day, not only manufacturing but also service industries focus on product and service quality as main factors for customer satisfaction. To be competitive in the market, organizations must improve or at least maintain their product and service quality. Control charts, which are effective tools to monitor final product and process quality characteristics, were initially developed in 1924[2]. Since then, several kinds of control charts have been developed for different applications.

This research focuses on a relatively new area in quality control, namely that of the development and evaluation of multi-attribute control charts.

1.1 Quality Control Chart Applications

Quality control charts can be applied to almost any area within a company or organization, including manufacturing, process development, engineering design, finance and accounting, marketing and field service. Shewhart $\overline{X}$, S and R control charts are extensively used to monitor continuous process variables. For example, a steel sheet manufacturer considers sheet thickness as a major quality characteristic and monitors it via Shewhart control charts. However, there are many situations in which more than one variable is considered simultaneously. For instance, a bearing has inner and outer diameters to determine the part quality. As a second example, the operating temperature and pressure of a distillation column both affect the process yield. As an additional example, Jackson[3] presented the application of multivariate quality control in ballistic missile and photographic film examples.

In addition to the continuous type of quality characteristics mentioned, there is another data type of quality characteristic commonly referred to as discrete (or attribute data). For example, a rod diameter is measured as “go” or “no-go” (i.e. the diameter specification is given a “pass” or “do not pass”). Control charts constructed for discrete data are called attribute control charts. Examples of processes that apply attribute control charts include order taking (an example from the service sector) and integrated circuit board fabrication (an example from manufacturing). For the order taking example, the number of wrong orders taken is the attribute of interest, while in the integrated circuit board fabrication example the number of defects on a wafer is monitored. Similar to variable processes, attribute processes may involve more than one attribute. Many service industries work with multiple attributes to describe their quality characteristics. For instance, an airline company measures customer satisfaction as a function of the mannerisms of the flight attendants and the overall flight time. A healthcare provider may evaluate its performance by the number of service errors, and the number of negative comments received about doctors, nurses, and overall service.

1.2 Benefits of Multivariate/Multi-Attribute Process Control versus Univariate/Uni-Attribute Process Control

When a process involves more than one variable, two different types of control chart approaches can be selected, a single multivariate control chart or a set of univariate control charts. A multivariate control chart is more sensitive and economical than a set of univariate control charts. The number of false alarms (Type I error) decreases when a multivariate control chart replaces a set of univariate control charts. In addition, a multivariate chart shows out-of-control signals due to the joint effect of two or more correlated variables; however, a set of univariate control charts may not show any such signal because their individual effect may not be out-of-control. Lowry and Montgomery[4] have shown that, in general, a multivariate control chart has better sensitivity than a set of univariate control charts in monitoring multivariate quality processes. A multivariate control chart is easier to use than maintaining numerous univariate control charts since identifying an out-of-control sample in a multivariate control chart requires only one observation versus many in univariate control charts. Equivalently, monitoring simultaneous attributes via multi-attribute charts has similar benefits over monitoring several single attributes at one time.

1.3 Multi-Attribute Process Quality Control Approaches

When developing a control chart technique for multi-attributes, the following questions/statements should be considered. These questions/statements are adapted from multivariate quality control goals given by Jackson[5].

1. A single answer should be available to answer the question: "Is the process in control?"

2. An overall Type I error should be specified.

3. Techniques should take into account the relationships among the attributes.

4. Techniques should be available to answer the question: "If the process is out-of-control, what is the problem attribute?".

As service industries, which most often involve the use of attribute data, implement or improve upon quality programs, it has been found that control charts are the most common tools utilized[6]. As a result, the demand for effective techniques to monitor a process with multiple attributes simultaneously is increasing. However, little research has been done in this area. From the literature only three studies have been conducted; the first two studies focus on the statistical design of multi-attribute charts, and the third study focuses on the economic design. The two statistical design techniques are authored by Patel[7] and Lu *et al.*[8]. Patel suggests a multivariate normal approximation technique for multivariate binomial and Poisson distributed data. Lu *et al.* develop a multivariate *np*-chart (MNP chart) based on a Shewhart-type control chart to deal with multiple attribute processes. The economic design study is conducted by Jolayemi[9]. In his work, Jolayemi develops a *J* approximation technique to approximate the sum of independent binomial distributions, which have different proportion nonconforming.

The multi-attribute research mentioned specifies the probability of falsely identifying an in-control sample; however, neither author discusses how fast the control charts can detect an out-of-control sample. Lu *et al.* show that multivariate *np*-chart is more sensitive than a set of uni-attribute control charts but their conclusion is based on only a single numerical example. The literature lacks any discussion about how well the current multi-attribute process control techniques work on various values of proportion nonconforming, different magnitudes of mean(s) shift (shift of proportion nonconforming), and different values of the correlation matrix.

1.4 Research Objectives

The objective of this research is two-fold. The first objective is to develop a technique for monitoring a multi-attribute process. The proposed technique meets the objectives of stated in section 1.3, as well as requires smaller sample sizes than the current techniques described. This new technique is based on the use of backpropagation neural networks (BPNN), which has had many successes in the quality control arena. The second objective of this research is to conduct a comparison study among the two current statistical approaches (normal approximation and MNP chart) and the proposed neural network technique given different conditions of proportion nonconforming p, sample size n, correlation matrix, and direction and magnitude of mean(s) shift (shift of process's proportion nonconforming). Out-of-control average run length (ARL) and in-control ARL will be used as performance measures for the three comparisons. The number of replications of correctly identified directions of shifts (positive or negative) will be also considered in the performance comparison.

As a result of this research, guidelines have been developed for quality control engineers and administrators who intend to monitor their multiple attribute processes. Based on the guidelines, users can more easily select the most promising technique to satisfy their particular process conditions.

1.5 Research Contributions

From this research, a new technique using backpropagation neural network (BPNN) for monitoring multi-attribute processes has been developed and successfully evaluated. This technique is preferable for processes with small sample size. In addition, the new technique is able to identify the directions of shifts and this quality narrows down causes of the shifts. This research also presents how the current and proposed multi-attribute process control techniques perform in different process conditions. Finally, guidelines and benefits of using a particular technique versus the others are provided for particular users.

This document is organized in the following manner. Chapter 2 includes the literature review. Chapter 3 explains the various multi-attribute methodologies that were investigated. These include the above mentioned techniques as well as an investigation of other possible techniques. Chapter 4 provides an overview of the experimental design used to compare the three techniques and Chapter 5 discusses the performance measures used in the experiment. Chapter 6 describes how the code used to test the three techniques was verified and how the data generated for the experiment were validated. Chapter 7 provides the results of the experiment. Chapter 8 recommends how one might determine the best technique to employ given particular process conditions. Finally, Chapter 9 gives conclusions and contributions, and suggests directions for future research in the area of multi-attribute control charts.

2.0 LITERATURE REVIEW

Control charts have been used as tools for monitoring industrial and service related processes for decades. In general, control charts can be categorized into two groups by the type of quality characteristic. A quality characteristic, which is measured on a numerical scale, is called a variable. $\overline{X}$, *S* and *R* charts are broadly used to monitor the mean and variability of variables. However, not all quality characteristics can be measured numerically. This type of quality characteristic classifies an inspected item as either conforming or nonconforming to a particular specification. The latter quality characteristic is called an attribute. In the same manner, *p*, *np*, *c* and *u*-charts are extensively used to observe attribute means.

Depending on the nature of the process, either variable or attribute control charts may be used. Based on the needs of the customer, engineers select the type of control chart. Montgomery[10] suggests criteria for choosing the proper type of control chart. Advantages and disadvantages of attributes vs. variable control chart are also recommended[11]. Below are some advantages for using attributes control charts.

- Attribute control charts can provide joint quality characteristics, e.g. height, length and width, in one chart. Products are defined as nonconforming when any one characteristic fails to meet specifications. On the other hand, three separate variable control charts are needed if we consider the quality characteristics as variables.

- Attributes control charts consume less time and cost in inspection than variable control charts.

Disadvantages of attribute control charts include, but are not limited to, the following.

- Variable control charts can forewarn operators when the process is about to go out-of-control so that actions can be taken before any nonconforming products are actually produced. In contrast, attribute control charts will not indicate any such signal until the nonconforming products are produced.
- Attribute control charts require larger sample sizes than do variable controls charts to indicate a process shift.
- Attribute information does not provide potential causes; therefore remedial actions cannot be identified.

This literature review discusses various types of uni-attribute, multivariate control charts and current multi-attribute process control techniques. The last section of this Chapter presents neural network applications for control charts.

2.1 Uni-Attribute Control Charts

In addition to applications of attribute control charts in manufacturing processes, attribute control charts are very useful in service industries. One reason for the wide use of attribute control charts in service industries is that most of the quality characteristics are measured on a quality scale such as satisfied or unsatisfied. Palm *et al.* discussed control chart applications in relatively new areas such as the service industry[12]. Health care providers have applied attribute control charts to measure service quality and expense. Educational institutions are also one of latest areas in which attribute control charts are being implemented.[13] Several types of attribute control charts will be discussed next.

2.1.1 Control Chart for Proportion Nonconforming (*p*-chart)

The *p*-chart is used to monitor the proportion nonconforming. The proportion nonconforming is the ratio of number of nonconforming items in a population to the total number of items in the population. In service industries, the proportion nonconforming may be the ratio of number of unsatisfied customers to the total number of customers. The upper and lower control limits and centerline are calculated as follows.

$$ControlLimits = p \pm 3\sqrt{(p(1-p)/n)}\ , \qquad (2\text{-}1)$$

$$Centerline = p\ , \qquad (2\text{-}2)$$

where p and n are the proportion nonconforming and sample size respectively. The statistic $\overline{p}$ estimates p when p is unknown.

2.1.2 Control Chart for Number of Nonconforming Items (*np*-chart)

For the *p*-chart, operators convert the number of nonconforming items in the sample to proportion nonconforming. The conversion process can be discarded by switching to *np*-charts since numbers of nonconforming items from samples are plotted instead of the proportion nonconforming. There is one drawback in the *np*-chart. The control limits and centerline will change when the sample size varies. Control limits and centerline formulas are given below.

$$ControlLimits = np \pm 3\sqrt{np(1-p)}\ , \qquad (2\text{-}3)$$

$$Centerline = np, \qquad (2\text{-}4)$$

where p and n are the proportion nonconforming and sample size respectively.

2.1.3 Control Chart for the Number of Nonconformities (*c*-chart)

A *c*-chart is used when the number of defects or nonconformity in an item is of particular concern, such as the number of defective welds in 10 meters of oil pipeline, the number of defects in 100 m^2 of fabric, etc. In constructing a *c*-chart, the size of sample is called the area of opportunity. The area of opportunity may consist of a single unit or multiple units of an item. A constant size area of opportunity is required when *c*-chart is constructed. This control chart assumes that the underlying distribution of the occurrence of the nonconformities in a sample of constant size is Poisson. The centerline and control limits are given below.

$$ControlLimits = c \pm 3\sqrt{c}, \qquad (2\text{-}5)$$

$$Centerline = c, \qquad (2\text{-}6)$$

where c is the average number of nonconformity in an area of opportunity.

2.1.4 Control Chart for the Number of Nonconformities Per Unit (*u*-chart)

A *c*-chart is used when the sample size is constant. If the sample size changes from one sample to another, a *u*-chart is the proper tool. Even though the *u*-chart control limits change when sample size varies, the centerline remains constant. The centerline is the average number of nonconformities per unit.

$$Centerline = \bar{u} = \frac{\sum c_i}{\sum n_i}, \qquad (2\text{-}7)$$

where c_i and n_i are the number of nonconformities and sample size of the i^{th} sample. The control limits are given by the following equation.

$$ControlLimits = \bar{u} \pm 3\sqrt{(\bar{u} / n)} \tag{2-8}$$

2.1.5 Current Research Issues in Uni-Attribute Control Charts

A primary issue of discussion in attribute control charts is the appropriate sample size. The sample size should be selected to ensure that the normality assumption is not violated.[14] In the *p* and *np*-charts, when the proportion nonconforming is very small, sample size must be large. However, too large of a sample size causes a problem for a process with limited resources. Schwertman and Ryan suggest an alternative procedure called dual *np*-charts.[15] Dual *np*-charts are composed of two charts. One chart provides an early warning of quality deterioration and the other, a cumulative control chart, uses approximate normal theory properties. The first chart has a smaller sample size than the second chart. As a result, the control limits for the two charts are different.

Chen also discusses the use of large sample sizes, but adds information about the speed of detecting a shift in the mean.[16] In the *p*-chart, the lower control limit is always close to "0", which makes the probability of detecting decreases in *p* small. In order to have effective lower control limits, large samples sizes are required. Chen suggests two alternative charts, which are based on discrete probability integral transformations and arcsine transformations, respectively. He compared the alternative charts with the classical *p*-chart using three criteria: (1) the minimum sample size for effective lower control limits, (2) the closeness of the false alarm probabilities to the nominal values for both upper and lower control limits, and (3) the ability to detect a change in *p* right after the change occurs.

How fast a control chart detects the *p* shift, especially when the *p* shift is small, is another issue that has been discussed by several authors. The CUSUM chart is an alternative to the classical Shewhart *p*-chart. Reynolds and Stoumbos[17] developed two CUSUM charts. One chart is based on the binomial distribution in which variables are counted from the number of defective items in *n* sample size. The second chart is based on Bernoulli variables resulting from inspections of the individual items. Both CUSUM charts are faster in detecting small shifts in *p* than traditional Shewhart *p*-charts. Further, CUSUM charts are better than *p*-charts for detecting large shifts in *p*. In addition, Reynolds and Stoumbos provide the Sequential Probability Ratio Test (SPRT) chart, which provides faster detection of changes in *p* than CUSUM and classical *p*-charts; and the SPRT chart requires smaller sample sizes than CUSUM and classical *p*-charts in order to detect changes.[18] For processes where *p* is very small, such as parts-per-million, Nelson[19] introduces a new control chart as an alternative to the traditional *p* and *c*-charts in order to avoid a large sample size. The number of conforming items between two consecutive nonconforming items is counted, and is assumed to have an exponential distribution. A transformation is then applied to the exponential distribution to approximate a normal distribution.

For the *c*-chart, one of the interesting issues discussed in the literature is that the distribution is assumed Poisson. There are situations in which the occurrence of defects in an item of a process does not follow the Poisson distribution. For example, defects in integrated circuit board fabrication are often clustered such that they do not follow a

Poisson distribution. Therefore, using a *c*-chart results in more false alarms. Two methods, a Neyman-based control chart (a control chart on the Neyman-type A distribution) and fuzzy ART, are suggested by Su and Tong[20]. The Neyman-type A distribution is a member of the family of compound Poisson distributions. One drawback of the Neyman-based control chart is that it cannot be applied to large sample sizes.

2.2 Multivariate Control Charts

In most processes, more than one quality characteristic can affect the final product quality. In another words, multiple quality characteristics are monitored simultaneously. In such cases, engineers develop and monitor either several univariate control charts or a single multivariate control chart. The practice is similar for attribute control charts. One drawback of using several univariate control charts is that the probability of a Type I error (plotting the sample outside control limits when it is really in control) increases. An increased Type I error will result in a higher number of false alarms to occur. For example, consider a process that consists of two independent quality characteristics, x_1 and x_2, each plotted on separate control charts. Each individual chart has Type I error of 0.0027 given three sigma control limits. Assuming independence, the joint probability of plotting the sample in control limits when it is actually in control is (1-.0027)(1-.0027) = 0.9946. The overall Type I error of the two univariate control charts is 1- 0.9946 = 0.0054, which is two times larger than the 0.0027 Type I error of a single multivariate control chart. Therefore, if we have two independent quality characteristics and would like to maintain an overall Type I error of 0.0027, each individual chart Type I error needs to be adjusted to 0.001351. Consider the example where 10 variables are investigated instead of two. Type I error will increase to 0.026 (roughly a 10 times increase). As the number of quality characteristics increases the Type I error distortion becomes more severe.

If the quality characteristics are not independent, a more complex process must be employed to obtain the overall Type I error. Aparisi[21] provides control limits when the two variables, x_1 and x_2, follow a bivariate normal distribution. Using a multivariate control chart reduces the operating personnel's work by plotting only one chart instead of multiple charts. In addition, monitoring the process status in multivariate control charts is easier than univariate control charts. However, assignable causes of an out-of-control process are more easily defined by set of univariate control charts.

There are several standard statistical process control methods that can be used to monitor the processes with multiple variables, such as the Hotelling T^2 control chart, Principal Component Analysis (PCA), Partial Least Square (PLS), to name a few. The three techniques mentioned will be discussed.

2.2.1 Hotelling T^2 Control Chart

Hotelling[22] conducted the original work in multivariate control charts. The Hotelling T^2 control chart was developed to monitor process variables simultaneously and overcome the drawbacks associated with using several univariate control charts when variables are correlated. The underlying distribution of the quality characteristics for which the Hotelling T^2 is appropriate is multivariate normal; however, a small deviation from multivariate normal distribution will not affect the results severely. The procedure for constructing the control chart is similar in nature to other types of control charts. The procedure is composed of two phases. The objective of phase I is to obtain an in-control set of observations so that control limits can be established for phase II. Phase II uses the control chart derived in phase I to monitor whether the future process is in-control or not. Historical or new data (preliminary) collected from the process is used to generate a phase I control chart. A sample mean and variance are estimated. Samples that are shown to be out-of-control are investigated and deleted from the data set if assignable causes are found. A new control chart without these points is then developed. The estimated mean vector and covariance matrix are:

$$\bar{\bar{X}}_j = \frac{1}{m}\sum_{k=1}^{m}\bar{X}_{jk} \qquad j = 1, 2, \ldots, \upsilon \;\; \text{and} \;\; k = 1, 2, \ldots, m \tag{2-9}$$

$$\bar{X}_{jk} = \frac{1}{n}\sum_{i=1}^{n}X_{ijk} \qquad i = 1, 2, \ldots, n \tag{2-10}$$

$$\bar{S}_j^2 = \frac{1}{m}\sum_{k=1}^{m}S_{jk}^2 \qquad j = 1, 2, \ldots, \upsilon \;\; \text{and} \;\; k = 1, 2, \ldots, m \tag{2-11}$$

$$\bar{S}_{jh} = \frac{1}{m}\sum_{k=1}^{m}S_{jhk} \qquad j \neq h \tag{2-12}$$

where m is the number of samples collected for preliminary data, υ is the number of monitored variables and n is the sample size. The test statistic is given as

$$T^2 = n(\bar{x} - \bar{\bar{x}})' S^{-1}(\bar{x} - \bar{\bar{x}}) . \tag{2-13}$$

In phase I, control limits for T^2 control chart are

$$UpperControlLimit = \frac{\upsilon(m-1)(n-1)}{mn - m - \upsilon + 1}F_{\alpha,\upsilon,mn-m-\upsilon+1} , \tag{2-14}$$

$$LowerControlLimit = 0 . \tag{2-15}$$

where α is a specified significance level and F is F distribution. Once the chart is used for monitoring future observations (Phase II), the control limits are

$$UpperControlLimit = \frac{\upsilon(m+1)(n-1)}{mn-m-\upsilon+1} F_{\alpha,\upsilon,mn-m-\upsilon+1}, \tag{2-16}$$

$$LowerControlLimit = 0. \tag{2-17}$$

The Chi-square distribution with υ degrees of freedom, where υ is the number of variables, and a significance level of α can be used as the upper control limits for both phase I and II when the mean vector, variance and covariance matrix are estimated from a large number of preliminary samples.[23]

2.2.2 Principal Component Analysis (PCA)

Principal Component Analysis is a useful technique for multivariate statistical process control, particularly with large size data and correlated variables. The general concept of PCA is to reduce a data matrix's dimension from m to k ($k<m$). The reduced dimensional matrix accounts for the majority of variability in the original data. Principal Component Analysis calculates a vector, called the first principal component, which is a linear combination of the m measure variables. This line is the direction of maximum variance and is defined so as to minimize the orthogonal deviation from each data point. A unit vector, which defines the direction of a principal component, is called an eigenvector. The distance of each original data point, which is projected along i^{th} principal component, is called a z-score (z_i). The second component is obtained in the same way as the first principal component but it is fitted through the residual variation of the first component. Both the first and second components are orthogonal. This approach is continued until m principal components, which are orthogonal, are obtained. For large

data sets, it is often found that the first k components ($k << m$) explain the majority of the variation in the data matrix.

According to Jackson[24], Principal Component Analysis (PCA) can be useful in multivariate process quality control because it transforms a set of correlated variables to a new set of uncorrelated variables. Quality engineers can then plot individual control charts from the sets of uncorrelated variables. However, Type I error is increased when variables are monitored individually. Techniques, such as Hotelling T^2, can take care of this increased Type I error. From Jackson[25],

$$T^2 = y'y, \tag{2-18}$$

$$y = \begin{bmatrix} y_1 \\ y_i \\ : \\ y_k \end{bmatrix}, \tag{2-19}$$

$$y_i = z_i / \sqrt{l_i}, \tag{2-20}$$

where z_i is projected distance along i^{th} principal component of each original data point. The variance of z_i is eigenvalue l_i. A process is out of control if T^2 is larger than upper control limit where

$$UpperControlLimit = \frac{\upsilon(m-1)}{m-\upsilon} F_{\upsilon,m-\upsilon,\alpha}, \tag{2-21}$$

$$LowerControlLimit = 0, \tag{2-22}$$

and F is the F distribution.

If the process is out of control, y_i must be examined to provide the root causes of the out-of-control condition. One advantage of using PCA is that quality engineers only have to work with k instead of m variables ($k<m$).

2.2.3 Partial Least Squares (PLS)

Often one group of variables, *Y*, is of great importance, such as *product* quality, and should be included in the monitoring process. Unfortunately, these variables are measured much less frequently and accurately than the normal *process* variables, such as *X*. Therefore, a technique using process variables, *X*, to detect and predict the change of product quality variables, *Y*, is used. This technique, Partial Least Squares (PLS), is a regression method based on projecting a high dimensional space (*X,Y*) onto a lower dimensional space defined by two sets of latent variables from both *X* and *Y*. Wold[26] provides details on the use of PLS.

2.3 Multi-Attribute Control Charts

Multi-attribute control charts can be used to simultaneously monitor many attribute quality characteristics of a process. The objectives of multi-attribute control charts are the same as multivariate control charts. Examples of industries that can capitalize on the multi-attribute control charts are the airline, healthcare, and food service industries. An airline company may measure customer satisfaction as a function of the mannerisms of the stewards/stewardesses and the overall flight time. For a restaurant, the food quality and the waiter's behaviors are possible quality attributes.

Even though multi-attribute control charts can be as useful as multivariate control charts, little research has been published. Patel conducted one of the earliest multi-attribute control charts studies[27]. Patel developed quality control methods for multivariate binomial and multivariate Poisson distribution observations. The correlated attributes were monitored simultaneously. In addition, his work considered time-dependent samples. Two assumptions, normality and equal process variance, are drawbacks of Patel's method. Lu *et al.*[28] addressed the statistical design of multi-attribute control charts. This paper discussed a mechanism for developing a Shewhart-based control chart to deal with multiple attribute processes. The chart is called multivariate *np*-chart (MNP chart). The MNP chart is easy to implement and interpret. An X statistic, which is the weighted sum of the counts of nonconforming units for each quality characteristic in a sample, is introduced. Control limits are derived based on this X statistic and traditional Shewhart-based control charts. Naturally occurring correlations between attributes are also considered in the model. A comparison of MNP and individual *np*-charts in a numerical example (see Lu *et al.*) shows that MNP chart has less Type II errors than the individual *np*-charts since the correlation of attributes is taken into account by MNP chart. However, there is no discussion about the average run length of MNP charts in this work. In addition, MNP chart has not been compared to other multi-attribute process monitoring techniques.

Jolayemi[29] developed a model for an optimal design of multi-attribute control charts for processes with multiple assignable causes. The model addresses the economic design, which is a departure from the above two models. This model is based on a J approximation[30,31] and Gibra's model[32,33] for the design of *np*-charts. J approximation is applied to the model instead of the direct convolution method (sum of independent binomial variables with different values of proportion nonconforming) in order to reduce the model complexity. The model yields the optimal sample size, the sampling interval and the control limits of the control charts. By applying latent structure analysis, all attributes are considered locally independent within an assignable cause. All assignable causes are assumed to occur independently and non-overlapping. From *J* approximation, the distribution of the sum of *m* independent binomial distributions, $b(n, p_1)$, $b(n, p_2)$,…, $b(n, p_m)$, is well approximated by a single binomial distribution, $b(mn, \overline{p})$, where

$$\overline{p} = \frac{\sum_{i=1}^{m} p_i}{m} \quad . \tag{2-23}$$

Here p_i is the proportion of defects corresponding to the i^{th} binomial variable and *n* is the sample size. As a result, the distribution of the sum of the number of defective items is approximated by $b(mn, \overline{p})$ for a sample of size *n* from the process with respect to all *m* attributes. The upper and lower control limits are then calculated as shown below.

$$ControlLimits = nm\overline{p}_0 \pm k[nm\overline{p}_0(1-\overline{p}_0)]^{1/2} , \tag{2-24}$$

where $\overline{p}_0$ is the average in-control proportion nonconforming of all attributes and *k* is constant value (normally $k = 3$ is used).

No calculation of average run length is provided for this method since the author focused on the economic design of the multi-attribute control chart. Because some assignable causes may not result in locally independent attributes, a possible drawback of the above formula to monitor a multi-attribute process is the assumption of local independence within an assignable cause.

2.4 Neural Networks and Control Charts

Neural networks have been applied to statistical process control (SPC) since late 1980s. A principal reason for applying neural networks to SPC is to automate SPC chart interpretation. To date, the application of neural networks to SPC chart has focused primarily on univariate control charts.

2.4.1 Neural Networks for Univariate Control Charts

Zorriassatine and Tannock[34] categorized the literature into two problem classes: identification of structure change (change in process mean or variance) and pattern recognition. The problem of structure change has been researched by Pugh[35,36], Guo and Dooley[37], Smith[38], Stutzle[39], Cheng[40], Chang and Aw[41].

Pugh[42] developed a back propagation neural network with four layers to identify the structure change of SPC charts. The unnatural pattern of concern in this study is a sudden mean shift. The trained data is composed of non-shifted and shifted means either plus or minus k standard deviations. Results showed that the average run length for both the neural network and the $\overline{X}$ control chart with two standard deviation limits are roughly the same. Pugh[43] extended his work by including mean shifts from three different populations: a fixed shift, several uniform distributions, and a parabolic distribution. The network was improved by training with multiple shifts (known as contouring), which decreases the mean square error and training time. In addition, the model was trained with noise to increase the robustness of the neural network. The performance of the neural network was the same as and better than a $\overline{X}$ control chart with two standard deviation control limits in terms of Type I and Type II errors, respectively.

Cheng[44] studied performance comparisons between artificial neural networks and Shewhart-CUSUM schemes in detecting unnatural patterns of a process. Both sudden and gradual shifts in the process mean were considered. The average run length (ARL) was used as a performance measure. The results showed that the neural network approach provided better performance than the Shewhart-CUSUM chart in detecting abrupt shifts as well as trend patterns.

Chang and Aw[45] proposed a neural network with fuzzy logic called NF, to detect and classify mean shifts. The average run length and percent correct classification were used as the performance measures to compare NF with Shewhart $\overline{X}$ and CUSUM charts. Results indicated that the NF approach outperforms conventional $\overline{X}$ and CUSUM charts in terms of the ARL. The NF approach also has advantage over the $\overline{X}$ chart in identifying the magnitude of a shift.

Neural networks have also been used to study pattern recognition problems. For example, Hwarng and Hubele[46] developed back propagation networks to identify unnatural patterns on Shewhart $\overline{X}$ control charts. Analyses were performed to determine the best training patterns and network parameters (such as number of hidden layers). To do this, a 3^2 factorial experiment was conducted. Once the best network configuration was found, the capability of the back propagation network was determined. Instead of using a single neural network, Cheng[47] developed two different neural networks, a multilayer perceptron trained by back propagation and a modular neural network, to identify the unnatural patterns of control charts. The modular neural network consists of two to five local expert multilayer perceptron networks. The networks were presented with several unnatural patterns to include trend, cycle, systematic variation, mixture and sudden shift. A set of performance measures such as rate of target and an average run length index compared the two neural network approaches. The results showed that the modular neural network provides better recognition accuracy than back-propagation when there are strong interference effects.

The use of neural networks has been demonstrated as a successful tool for statistical process control pattern recognition. However, only one pattern at a time, such as mean shifts, cyclic and trend patterns, has been considered. Guh and Tannock[48] proposed a back propagation neural network model that investigates all patterns simultaneously. In addition to identifying an out of control pattern, a major function of SPC charts is to notify the parameters of the out-of-control patterns. Guh and Hsieh[49] conducted a study which concerned not only the recognition of abnormal patterns but also the parameters of the abnormal patterns, such as shift magnitude, trend slope and cycle length. Their proposed method is composed of two modules. The first module has a back propagation network for categorizing the patterns into normal, shift, trend and cycle. The second module includes three networks for estimating the parameters of the shift, trend and cycle. Chang and Ho[50] developed a combined neural networks control scheme for monitoring mean and variance shifts at the same time. This monitoring scheme is composed of two neural networks, one for detecting mean shift and the other for detecting variance shifts. A comparative study between the neural network approach and traditional SPC charts was conducted and performance measures used were average run length (ARL) and percent correct classification. The result of the study showed that the proposed neural network control scheme outperforms other SPC charts in the majority of situations for individual observations and subgroups with sample sizes of five.

There are several factors that affect the performance of a neural network model in detecting unnatural patterns. In the literature review of Zorriassatine and Tannock[51], they summarized the factors into two levels, neural network model construction and training. Significant factors in constructing a neural network model for SPC are:

- Neural network paradigm: Neural networks architectures such as multi-layer perceptron (MLP), radial basis function (RBF), learning vector quantization (LVQ), adaptive resonance theory (ART), auto-associative neural networks, and Kohonen self-organizing maps (SOM) have been implemented.
- Type of connection: Full or partial connection.
- Number of hidden layers: Guo and Dooley[52] concluded that there is no standard way to determine the number of hidden layers and recommended that either one or two hidden layers should be sufficient for almost any classification problem.
- Number of nodes: Input layer, hidden layer and output layer nodes.
    - Hidden layer nodes: Hwarng and Hubele[53] ran a 3^2 factorial experiment and concluded that the number of hidden layer nodes in a neural network statistical process control with back propagation architecture had no significant effect on either Type I or Type II errors of the network.
    - Output layer nodes: Normally, the number of output nodes is the same as the number of different classes that a neural network is trained to recognize. However, this is not always the case. For instance, it is possible to train a network with various magnitudes of shifts $\left(\pm 1\sigma, \pm 2\sigma and \pm 3\sigma\right)$, but the output pattern can be represented as a single node.
- Transfer or activation function: is a function that transforms the net input to a neuron into its activation. A transfer function can be linear or nonlinear.

Significant factors in training a neural network model for SPC pattern recognition are the following.

- Preprocessing data: The trained network should be able to recognize patterns with new process mean and standard deviation only if the training data are standardized. Subtracting the data by the mean and dividing the result with the standard deviation provides standardized data. Upon discovery and removal of an unnatural cause, the process must be reset and new mean and standard deviation calculated. Therefore, if standardization is not used new sets of training patterns need to be generated after every reset.
- Number of training examples: Cheng[54] recommended equal number of training examples for each unnatural pattern. Through experimentation, he showed that using small training and testing data sets produced undesirable results. On the contrary, too large a range can bias the network in detecting large process changes, thus making the network more complex. Large training data may be separated into several networks in order to reduce the size of training data per neural network. For instance, in the modular neural network (MNN) of Cheng[55] the training was organized into three-separated 'specialists' (known as local expert networks) each responsible for only a subset of the training cases.
- Presentation frequency of training examples to NN: In using neural networks, one should be aware of overtraining and undertraining if back propagation neural network is used. According to Hetch-Nelson[56] some networks such as self-organizing map (SOM) do not suffer from overtraining phenomena while others such as back propagation neural networks do.

- Presentation order of training patterns: There are two types of presentation order. The first is to randomly present all patterns and the second is present one pattern after another. Guo and Dooley[57] and Hwarng and Hubele[58] concluded that random selection of training data within each pattern classes produces faster convergence.

2.4.2 Neural Networks for Multivariate Statistical Process Control

Martin and Morris[59] proposed a fuzzy neural network as an alternative approach for identifying out-of-control causes in a multivariate process. Cause detection capability of a fuzzy neural network and principal component analysis were compared in a multivariate process of a Continuous Stirred Tank Reactor (CSTR). Eleven on-line process measurements and three controller outputs were monitored as input variables. Each variable was classified into three fuzzy sets: increased, steady and decreased. Output nodes included eleven fault types or causes of the process out-of-control.

Neural networks have also been applied to traditional multivariate statistical process control techniques. Wilson, Irwin and Lightbody[60] applied Radial Basis Function (RBF) networks to Partial Least Squares (PLS), an algorithm to monitor a multivariate process, in order to extend a linear to a nonlinear algorithm. Radial Basis Function (RBF) networks have also been used with Principal Component Analysis (PCA) for nonlinear correlated data.[61]

2.4.3 Neural Networks for Uni-Attribute Control Charts

In integrated circuit (IC) manufacturing processes, a *c*-chart is used to monitor the number of defects on each product item (wafer). A wafer's defects are assumed to occur independently and with equal chance in all locations if a *c*-chart is used. However, as a wafer size increases, defects on the wafer are no longer randomly distributed, but will tend to cluster. Monitoring clustered defects via a *c*-chart, which is based on Poisson distribution, results in a high number of false alarms. Su and Tong[62] propose a neural network-based procedure for monitoring clustered defects in IC fabrication. They apply fuzzy ART to find the number of clusters treating all defects in a particular cluster as one defect. As a result, the numbers of defects is reduced; and are distributed randomly. A *c*-chart is then constructed for monitoring the randomly distributed defects.

2.4.4 Neural Networks for Multi-Attribute Control Charts

As described above, there are numerous neural network papers in recognition of univariate control chart patterns and detection of multivariate process faults. There are also a few studies using neural networks for uni-attribute control charts. However, no research has been found to date that applies neural networks to the recognition of multi-attribute control chart mean shifts.

2.5 Interpretation of Out-of-Control Signals for Multivariate Control Charts

One of the major issues in multivariate control charts is the identification of assignable cause(s) of the out-of-control signals. Once a signal is generated, process variables, which contribute to the out-of-control process, need to be identified and adjusted to bring the process back to in-control status. Several techniques have been developed for interpreting out-of-control signals for multivariate processes. One of the simplest techniques is to view the corresponding univariate charts of a multivariate process to determine which variable is producing the assignable cause. However, some concerns arise in implementing this technique. First, when a process includes several variables, there are many univariate control charts to interpret. Second, the univariate control charts may not show any signal when the multivariate control chart detected a signal since the signal may be a function of several correlated variables. Third, the overall significance level of the simultaneous use of p univariate control charts is difficult to determine[63,64,65].

Principal component analysis (PCA) is an approach proposed by Jackson[66] to interpret out-of-control signals. Once the multivariate Shewhart chart (T^2-chart) identifies an out-of-control signal, T^2 statistic is decomposed into the sum of squares of independent principal components, linear combinations of the original variables. These components can be examined to understand why the process is out-of-control. However, it may be difficult to interpret these components meaningfully. Mason *et al.*[67] developed a series of orthogonal decompositions of the T^2 statistic. The orthogonal components can be easily interpreted. There are two types of components: unconditional and conditional. The unconditional component measures whether a variable is out-of-control. A signal from this component does not consider the relationship between the specified variable and the other variables. The conditional component explains that the out-of-control signal is a function of the relationship of various variables. For large amount of variables, the number of possible decompositions is large, but a suggested computing scheme can greatly reduce this computational effort.

Fuchs and Benjamini[68] proposed a control chart that presents univariate and multivariate statistics simultaneously. This chart is based on the T^2 control chart, but a sample plotted on the chart, which represents T^2 statistic, is replaced by a small bar chart. The bar chart contains the values of several univariate statistics.

Runger *et al.*[69] suggested decomposing the T^2 statistic into components that reflect the contribution of each individual variable. A contribution of the i^{th} variable is calculated by the deviation of the $T^2_{(i)}$ (the value of T^2 statistic for all process variables except the i^{th} variable) from the current T^2 (the value of T^2 statistic for all process variables). When an out-of-control process is indicated, the authors recommended focusing on the variables that have large deviations.

3.0 MULTI-ATTRIBUTE METHODOLOGIES

This Chapter presents, in detail, the various techniques for detecting out-of-control multi-attribute processes. The two current techniques the normal approximation of multivariate binomial distribution and the multivariate *np*-chart (MNP chart) are provided first. The proposed neural network approach is then discussed. Finally, several other possible techniques were investigated with regards to their feasibility and use in multi-attribute control charts. A critical review of these techniques is also provided.

3.1 Current Methods In Literature

3.1.1 Normal Approximation of Multivariate Binomial Distribution

Patel[70] proposed quality control techniques for multivariate binomial and Poisson distributions. His work included both time independent and time dependent (autocorrelation) samples. However, only the time independent technique was considered in this research since the generated data had weak autocorrelation (less than 0.20).

From Patel, when sample size n is large, the following statistic provides the basis for the control chart.

$$G = \left(X - \bar{x}\right)' S^{-1} \left(X - \bar{x}\right) \qquad (3\text{-}1)$$

G has an approximate Chi-square distribution with υ degrees of freedom where X is a random vector from a population of interest, υ is the number of attribute in the process, and S is an estimator of the population covariance matrix, Σ, which is assumed to remain unchanged from process to process.

The upper control limit of the control chart is the value of χ^2_α with υ degrees of freedom where α is a specified significance level. The lower control limit is “0”. One can use this χ^2 chart to monitor future observation vectors X, such that

$$X = \begin{bmatrix} X_1 \\ X_i \\ : \\ X_\upsilon \end{bmatrix}, \tag{3-2}$$

where υ is the number of attributes. X_i is the number of nonconforming units for the i^{th} attribute and has binomial distribution. If G has value greater than the upper control limit then the sample indicates an out-of-control process.

Patel stated that the estimated covariance matrix S may be singular or near singular (this might happen even when the number of samples exceeds the number of attributes). He also proposed a technique based on factor analysis to ensure that the estimated covariance matrix S is a non-singular matrix. However, he did not indicate whether the technique was only used for the singular estimated covariance matrix, or for all cases (both singular and non-singular S).

In this research, and in particular for this experimental design, we know that the estimated covariance matrices are non-singular (determinant of S are > 0). As a result, the technique of factor analysis was not considered.

3.1.2 Multivariate *np*-Chart (MNP chart)

Lu *et al.*[71] introduced an X statistic, which is the weighted sum of the counts of nonconforming units for each quality characteristic in a sample. Control limits are derived based on this X statistic and traditional Shewhart-based control charts. Naturally occurring correlations between attributes are also considered in the model. Control limits and centerline of the MNP chart are given by

$$ControLimits = n\sum_{j=1}^{m} d_j \sqrt{p_j} \pm 3\sqrt{n\{\sum_{j=1}^{m} d_j^2 (1-p_j) + 2\sum_{i<j}(d_i d_j \delta_{ij}\sqrt{(1-p_i)(1-p_j)}\}}$$

$$Centerline = n\sum_{j=1}^{m} d_j \sqrt{p_j}\ , \qquad (3\text{-}3)$$

where n is the sample size, d_j is the number of demerits that indicate the severity of the nonconformance in the quality characteristic *i*, p_j is the proportion nonconforming of the i^{th} quality characteristic and δ_{ij} is the correlation coefficient between the quality characteristics *i* and *j*.

The sample size n of the MNP chart should be selected based on the value of proportion nonconforming. If the proportion nonconforming is not small, the sample size of the MNP chart should be

$$n \geq \frac{3m}{\sum_i p_i}\,. \qquad (3\text{-}4)$$

If the proportion nonconforming is small, the sample size is

$$n \geq 9\frac{Trace(\sqrt{(I-P)(I-P)'}\,\Sigma')}{(\sqrt{P}I')^2}, \qquad (3\text{-}5)$$

where $I = [1,1,\ldots,1]_{mx1}$ is the unit vector, P is the fraction nonconforming vector and Σ is the correlation matrix of the attributes.

It should be noted that the degree of severity caused by each nonconformance is different from process to process. For instance, a nonconformance in one dimension may be more serious than in another dimension. A demerit system is therefore included in the model to remedy this potential problem. If all quality characteristics' nonconformance have the same level of severity, then $d_j = 1$. When unknown, the proportion nonconforming p_j and the correlation matrix must first be estimated from observed data. As with establishing traditional control charts, preliminary samples of roughly 25 with individual samples of size n are recommended for estimating the unknown parameters and constructing trial control limits.

The quality characteristic, which is the cause of the out-of-control signal, can be identified by a score statistic (Z), as shown

$$Z_{Di} = d_i[C_i - np_i]/\sqrt{p_i}\ , \tag{3-6}$$

where C_i is the count of nonconforming units with respect to quality characteristic i.

The quality characteristic with the largest positive Z_{Di} score is considered the major contributor to an upward shift in the process. Conversely, the smallest negative Z_{Di} score is considered the major contributor to a downward shift in the process.

3.2 Backpropagation Neural Networks

3.2.1 General Concept

Backpropagation networks appear often in pattern recognition and classification problems. The network is based on a gradient descent method that minimizes the total squared error of the output computed by the network. Given a set of historical (training) input data, the network produces output, which in turn is compared to the actual output.

The resulting error is then mapped back into the network through an adjustment of the network's weights. The objective is to get the best possible set of weights so that the outputs are close to the actual results for both the training and new data.

There are three stages in training a backpropagation network: (1) the forward feed of the input training pattern to the network, (2) the calculation of the associated error, and (3) the adjustments of the weights. After training, application of the net involves only the feed forward stage.

3.2.1.1 Architecture

A backpropagation neural network comprises of a multilayer neural network with one or two layers of hidden units. Figure 1 shows a sample architecture for a single hidden layer backpropagation neural network.

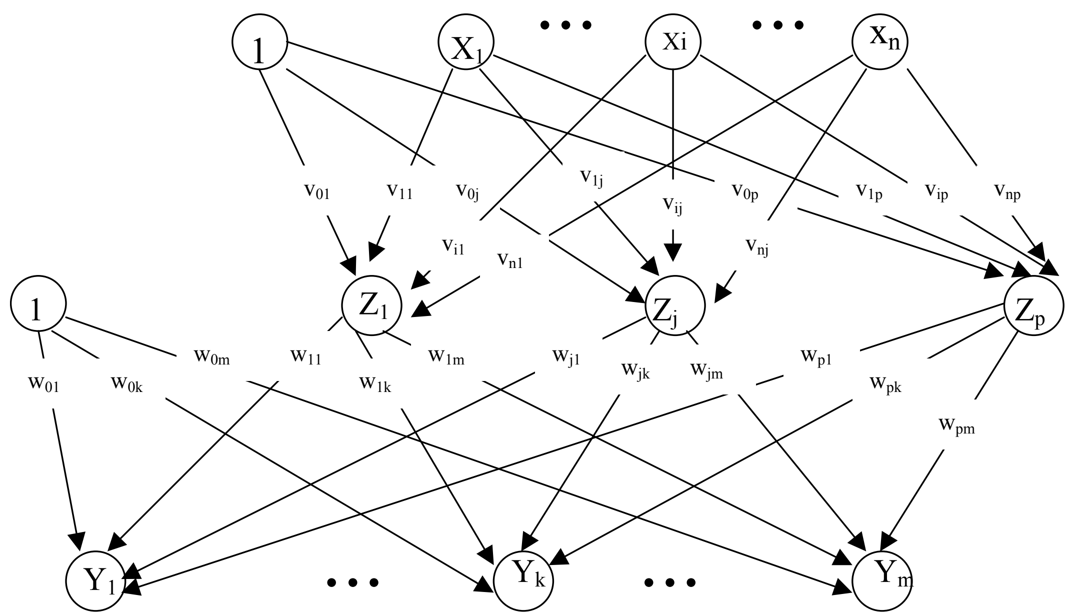

Figure 1 Backpropagation Neural Network with One Hidden Layer

3.2.1.2 Algorithm

As discussed, there are three stages involved in training a network by backpropagation. This section discusses the three-stage algorithm for a backpropagation neural network with one hidden layer.

During the feed forward stage, each input unit (X_i) receives an input signal that is broadcasted to each of the hidden units Z_1, Z_2, …, Z_p. Each hidden unit then computes its activation and sends its signal (z_j) to each output unit. Each output unit (Y_k) computes its own activation and sends its signal (y_k) to form the final response of the net for the particular input pattern.

At each hidden and output unit, net input is calculated from the summation of weights and input products of that unit. The output signal of a unit is computed from the net input to an activation function as follows. For hidden unit Z_j

$$z\_in_j = v_{0j} + \sum_i x_i v_{ij} \text{ and} \quad (3\text{-}7)$$

$$z_j = f(z\_in_j), \quad (3\text{-}8)$$

where z_in_j is the net input to hidden unit j, v_{0j} is the bias on hidden unit j, and z_j is the output signal of hidden unit j. For output unit Y_k

$$y\_in_k = w_{0k} + \sum_j z_j w_{jk} \text{ and} \quad (3\text{-}9)$$

$$y_k = f(y\_in_k), \quad (3\text{-}10)$$

where y_in_k is the net input to output unit k, w_{0k} is the bias on output unit k, and y_k is the output signal of output unit k.

Common activation functions for backpropagation network include but are not limited to binary sigmoid, bipolar sigmoid, etc. The type of data, especially the target values or output values, is a crucial factor when selecting the appropriate function.

During training, each output unit compares its activation y_k with its target value t_k to determine the associated error for that particular pattern. Based on this error, the factor δ_k (k = 1, …, m) is computed. δ_k is used to distribute the error at output unit Y_k to all units in the prior layer. It is also used to update the weights between the output and the hidden layer. In the same manner, the factor δ_j (j = 1, …, p) is computed for each hidden unit Z_j. It is not necessary to propagate the error back to the input layer, but δ_j is used to update the weights between the hidden layer and the input layer. The following equations show the calculation of an error and adjusted weight. For each output unit,

$$\delta_k = (t_k - y_k) f'(y\_in_k), \quad (3\text{-}11)$$

$$w_{jk}(new) = w_{jk}(old) + \Delta w_{jk}, \text{ and} \quad (3\text{-}12)$$

$$\Delta w_{jk} = \alpha \delta_k z_j, \quad (3\text{-}13)$$

where δ_k is the error factor of output unit *k*; t_k and y_k are the target and output patterns, respectively. The weight between the output unit *k* and the hidden unit *j* is w_{jk}. The learning rate is α. For each hidden unit,

$$\delta\_in_j = \sum_{k=1}^{m} \delta_k w_{jk} , \tag{3-14}$$

$$\delta_j = \delta\_in_j f'(z\_in_j) , \tag{3-15}$$

$$v_{ij}(new) = v_{ij}(old) + \Delta v_{ij} \text{ , and} \tag{3-16}$$

$$\Delta v_{ij} = \alpha \delta_j x_i , \tag{3-17}$$

where δ_j is error factor of hidden unit *j*. The weight between the hidden unit *j* and input unit *i* is v_{ij}.

3.2.2 Backpropagation Neural Network for Multi-Attribute Process Control

3.2.2.1 Architecture and Algorithm

A four-layer backpropagation network with two input nodes, two hidden layers each with ten nodes, and an output node was chosen for the architecture. The number of input nodes represents the number of attributes in a process; each node accepts a value from a particular attribute sample. Each input node is connected to the first layer of hidden nodes. Each node in the first hidden layer is connected to the second layer of hidden nodes, whereby each node is connected to the output node. The strength of each connection is stored as a weight. The network is trained by the *trainbgf* function using MATLAB version 6 release 12. This training function is an alternative to the gradient descent methods that produce fast optimization[72]. The hidden nodes transfer the sum of their input and weight products by using hyperbolic tangent functions. The output node uses the sum of its input and weight products as the result. Figure 2 shows the network architecture. In order to clearly see the connections, only one hidden node from the 1st layer shows its connection to all the nodes in the 2nd hidden layer.

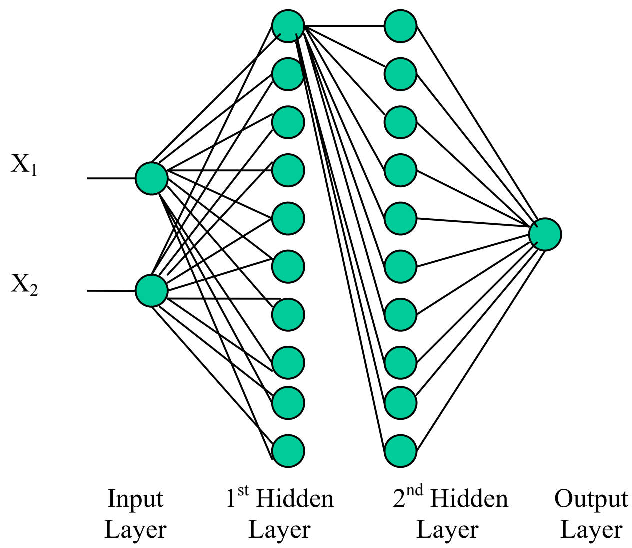

Figure 2 Back Propagation Neural Network Architecture

3.2.2.2 Preprocessing Data

Before training, inputs are scaled so that they always fall within a specified range. The function *premnmx* in MATLAB is used to scale the inputs to fall in a range [-1, 1]. The function stores minimum and maximum input values. Inputs are scaled by equation (3-18).

$$\text{Standardized Input} \quad = -1 + 2\left(\frac{Input - Min.Inputs}{Max.Inputs - Min.Inputs}\right) \qquad (3\text{-}18)$$

3.2.2.3 Training Data

Three populations were used to train and validate the network. The first population consisted of 100 samples of unshifted data. The other two populations consisted of 100 samples each with the both process's proportion nonconforming shifted to three standard deviations (one population in the positive direction and the other in the negative direction).

$$Standard Deviation = \sqrt{\frac{\overline{p}\left(1-\overline{p}\right)}{n}}, \qquad (3\text{-}19)$$

where $\overline{p}$ is the number of nonconforming items divided by sample size and n is the sample size.

The later two populations represented data from an out-of-control process. Three-fourths of the samples were used for training and one-fourth was used for validating the model. Each sample pattern consisted of an input vector and a corresponding output. The input vector comprised of two correlated binomial random variables drawn randomly from one of the three populations. The output was the process status. The output was set to one of three conditions. It was zero if the mean was in control, one (1) if the sample came from a positive out-of-control population, and negative one (-1) if the sample came from a negative out-of-control population. Input patterns were presented to the network in random order.

3.2.2.4 Cut-Value for In-Control and Out-of-Control Processes

The process status predicted from the network resulted in a continuous value that ranged from $-\infty$ to ∞. In order to compare this to the actual process status (that of "0" in control, “1” positive out-of-control, or “-1” negative out-of-control) cut-values were selected for the network outputs based on the specified in-control average run length (ARL). The in-control ARL for the backpropagation neural network (BPNN) technique was specified such that all the techniques (BPNN, MNP, and normal approximation) had similar in-control ARL. Two cut-values were defined. The first cut-value (CV1) was set to distinguish an out-of-control process with proportion nonconforming shifted in the positive direction, and the second cut-value (CV2) was set to differentiate an out-of-control process with proportion nonconforming shifted in the negative direction. CV1 was computed from the training data such that all three techniques result in an equivalent probability of indicating an out-of-control process in positive direction. By setting CV2 equal to negative CV1, all techniques have similar in-control ARL.

3.3 Other Techniques

One of the main objectives of the research was to find a technique(s) capable of detecting out-of-control and in-control processes. In other words, a technique with an ability to classify conforming and nonconforming data was sought. Since there are a large number of classification techniques in the literature, a preliminary study was conducted to determine if any were promising in the use of multi-attribute control charts. The study included discriminant analysis (for discrete and normally distributed variables), logistic regression (binary and multinomial), and neural networks (backpropagation and probabilistic neural network). This section discusses the preliminary results for each technique.

3.3.1 Discriminant Analysis

Discriminant analysis for discrete variables is used for classifying data that are multivariate dichotomous in nature. Solomon conducted a study entitled "Attitude Toward Science" on a sample of high school students, which comprised of 2 groups, high and low IQ scores[73]. His study included four dichotomous variables as follows:

X_1: The development of new ideas is the scientist's greatest source of satisfaction,

X_2: Scientists and engineering should be eliminated from the military,

X_3: The scientist will make his maximum contribution to society when he has freedom to work on problems that interest him, and

X_4: The monetary compensation of a Nobel prized winner in physics should be at least equal to that given popular entertainers.

Responses for each variable were either "1" (agree) or "0" (disagree). Individual student responses (independent variables) were presented in a binary form, for example 1011 indicates "agree" for X_1, X_3, and X_4, and "disagree" for X_2. Discriminant analysis for discrete variables was applied to classify students into high and lower IQ based on the four dichotomous variables.

The independent variables used in the discrete discriminant analysis are presented in the form that is different from the one generated for this research. The independent variables in the research are the number of nonconforming items from a sample size (not the individual sample). For instance, a hundred samples are drawn from a process with two attributes (A and B), independent variables are presented as [30, 25], which means out of a hundred samples there are 30 and 25 nonconforming items for attribute A and B respectively. Because of the varied form for presenting the independent variables, the discrete discriminant analysis technique was not a suitable technique for this research and thus not further investigated.

Discriminant analysis for normally distributed variables consists of two types of functions: linear and quadratic. Linear functions have the assumption of equal variance while quadratic functions do not. Both functions share the assumption that the variables are normally distributed.

One of the objectives presented in this research is to reduce the sample size. Consequently, the data generated for some the subsets (with small sample sizes) in the experiment does not follow a normal distribution. As a result, discriminant analysis techniques for normally distributed variables are not appropriate.

3.3.2 Logistic Regression

Logistic regression is one of the most extensively applied classification techniques to determine relationships among variables, specifically between a binary or polytomous response (or dependent variable) and one or more independent (or predictor) variables. The logistic function can be used with dichotomous independent variables or a combination of multivariate normal and dichotomous variables[74]. James *et al.*[75] conducted two studies of non-normal classification problems whereby logistic regression and linear discriminant analysis were compared, and found logistic regression using maximum likelihood outperformed linear discriminant analysis in both cases, but not by a large amount. Also, two studies by Halperin *et al.*[76] and Truett *et al.*[77] confirmed that logistic regression outperformed linear discriminant analysis for problems involving non-normal independent variables. Data (independent variables) generated in the research included either normal (in the case of large sample sizes) or binomial (in the case of small sample sizes) distributions. As a result, logistic regression technique was investigated further for the purposes of this research.

There are two major types of logistic regression: binary and multinomial (or polytomous). Binary logistic regression is appropriate for a dichotomous response while multinomial logistic regression is suitable for responses with more than two categories. The following equation describes the binary logistic function for responses “0” and “1”.

$$P(Y{=}0/\boldsymbol{X}) = e^u / (1 + e^u),\ P(Y{=}1/\boldsymbol{X}) = 1 / (1 + e^u), \tag{3-20}$$

where $P(Y{=}0/\mathbf{X})$ is the probability of predicting the response “0” given that $\boldsymbol{X}$ is the independent variable vector, and u is

$$u = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_p X_p. \tag{3-21}$$

For this regression X_i is the i^{th} independent variable, β_i is the i^{th} regression coefficient, β_0 is the intercept, and p is the number of independent variables.

Instead of using the least squares method, model parameters are estimated by the maximum likelihood method. The resulting estimated regression coefficients for the model can be interpreted in the same manner as ordinary least squares regression coefficients. However, difficulties arise when fitting a binary logistic regression if the conditional probability is not monotonic in the independent variables[78]. For example, if there is a high incidence for either low or high body weights, and there is a low incidence for intermediate body weights, the logistic function may not provide a good fit to the data. In this research, the probability of a process being out-of-control is high when the values of the independent variables (e.g. number of unsatisfied customers) are at either extremes (low or high); and the probability of a process being out-of-control is low (hence, it is in-control) when the values of the independent variables are not at either extremes. Multinomial logistic regression with three categories for the dependent variable overcomes the drawback when the conditional probability is not monotonic in the independent variables; therefore, it is an alternative technique to binary logistic regression.

Multinomial logistic regression is an extension of binary logistic regression for dependent variables with more than two categories. A nominal scaled dependent variable with three categories (0, 1, and 2) is discussed. In the three-category response model, there are two logit functions: one for $Y = 1$ versus $Y = 0$, and the other for $Y = 2$ versus $Y = 0$. In theory, any two pairwise logit comparisons of the responses can be used. The following equations depict the two logit functions.

$$g_1(x) = \ln\left[\frac{P(Y = 1/\mathbf{X})}{P(Y = 0/\mathbf{X})}\right]$$

$$= \beta_{10} + \beta_{11}X_1 + \beta_{12}X_2 + ... + \beta_{1p}X_p, \tag{3-22}$$

and

$$g_2(x) = \ln\left[\frac{P(Y = 2/\mathbf{X})}{P(Y = 0/\mathbf{X})}\right]$$

$$= \beta_{20} + \beta_{21}X_1 + \beta_{22}X_2 + ... + \beta_{2p}X_p. \tag{3-23}$$

For the equations presented, $\boldsymbol{X}$ is the vector that comprises the independent variables, X_i is the i^{th} independent variable (i = 1, 2,…, p), β_{ji} is the j^{th} logit function regression coefficient for the i^{th} independent variables (j = 1, 2), β_{j0} is the j^{th} logit function intercept, and p is the number of independent variables.

The conditional probabilities for the three responses are

$$P(Y=0/\mathbf{X})=\frac{1}{1+e^{g_1(x)}+e^{g_2(x)}},$$

$$P(Y=1/\mathbf{X})=\frac{e^{g_1(x)}}{1+e^{g_1(x)}+e^{g_2(x)}}, \text{ and}$$

$$P(Y=2/\mathbf{X})=\frac{e^{g_2(x)}}{1+e^{g_1(x)}+e^{g_2(x)}}. \qquad (3\text{-}24)$$

For the conditional probabilities, $P(Y=0/\mathbf{X})$, $P(Y=1/\mathbf{X})$, $P(Y=2/\mathbf{X})$ are the probability of predicting response “0”, “1”, and “2”, respectively, given that $\boldsymbol{X}$ is the independent variable vector.

To determine suitability in this research, a preliminary experiment was conducted comparing multinomial logistic regression, the MNP chart, and the normal approximation technique in monitoring multi-attribute processes. A process with proportion nonconforming 0.3 for each quality characteristic, correlation coefficient 0.8, and sample size 50 was used in the experiment.

The experiment included ten sets of data. Each data set comprised of training and test data. The dependent variable of the study was the process status, as measured by: (1) in-control, (2) out-of-control with process’s proportion nonconforming shifted in the positive direction, and (3) out-of-control with process’s proportion nonconforming shifted in the negative direction. The independent variables were multivariate binomial variables. The training data were used to calculate mean vector (of nonconforming items) and covariance matrix for the normal approximation technique, mean vector (of proportion nonconforming) and correlation matrix for the MNP chart, and the coefficients for the multinomial logistic regression technique. The in-control ARL of all techniques are calculated from the test data.

Table 1 shows in-control ARL for the three techniques where each training set consisted of 300 observations (i.e. 100 in-control, 100 out-of-control with process's proportion nonconforming shifted three standard deviations in the positive direction, and 100 out-of-control with process's proportion nonconforming shifted three standard deviations in the negative direction. The standard deviation was calculated from eq.3-19).

Table 1 In-control ARL of the MNP chart, the normal approximation, and the multinomial logistic regression techniques where the number of in-control and out-of-control observations in training set are equal

Techniques	Training/Test Set No.	In-control ARL
Normal Approximation	1	521
MNP chart		580
Multinomial Logistic		2
Normal Approximation	2	132
MNP chart		37
Multinomial Logistic		9
Normal Approximation	3	>1000
MNP chart		245
Multinomial Logistic		9
Normal Approximation	4	106
MNP chart		106
Multinomial Logistic		10
Normal Approximation	5	3
MNP chart		625
Multinomial Logistic		3
Normal Approximation	6	211
MNP chart		483
Multinomial Logistic		5
Normal Approximation	7	773
MNP chart		581
Multinomial Logistic		2
Normal Approximation	8	328
MNP chart		310
Multinomial Logistic		14
Normal Approximation	9	176
MNP chart		218
Multinomial Logistic		7
Normal Approximation	10	154
MNP chart		154
Multinomial Logistic		8

Table 1 shows that in-control average run length (ARL) for multinomial logistic regression is considerably less than the other techniques (statistically significant at *p-value* = 0.000). In another words, the multinomial logistic regression technique indicates a false alarm significantly more than the other two techniques.

Hosmer and Lemeshow indicated "classification is sensitive to the relative sizes of the two component groups and will always favor classification into the large group"[79]. That is, if there are, say considerably more 1's than 0's among the *Y* values, one would expect most of the $\hat{\pi}(x)$ (estimated conditional probability of *Y* given *X* independent variable vector) to be closer to 1 than to 0. To improve the in-control ARL of the multinomial logistic regression technique, additional in-control observations were added to the training data. A new set of training data was composed of 500 in-control observations and 200 out-of-control observations (100 each for positive and negative directional shifts). The comparison of in-control ARL for the three techniques is shown in Table 2. The table shows that the in-control ARL of the multinomial logistic regression technique improved after adding the additional in-control observations. However, they remained substantially less than the results from the other two techniques. This means the multinomial logistic regression technique has considerably higher false alarms than the other two techniques.

Additional in-control observations were continually added to the training data until the in-control ARL for multinomial logistic regression was close to the other techniques. Table 3 shows the in-control ARL of multinomial logistic regression, normal approximation, and the MNP chart when the training data consisted of 10,000 in-control observations and 200 out-of-control observations (100 each for positive and negative direction of shifts). In-control ARL for the three techniques are comparable (i.e. not significantly different at significance level of 0.05). This means we have 95% confidence that all techniques have the same probabilities of indicating a false alarm.

To ensure that the out-of-control ARL for the multinomial logistic regression does not increase when in-control ARL increases, a set of data with the proportion nonconforming of the process shifted three standard deviations in the positive direction were used as the test set. The results are shown in Table 4.

Table 2 In-control ARL of the MNP chart, the normal approximation, the multinomial logistic regression techniques when number of in-control and out-of-control observations in training set are unequal: 500 in-control observations and 200 out-of-control observations (100 each for positive and negative direction of shifts).

Techniques	Training/Test Set No.	In-control ARL
Normal Approximation	1	521
MNP chart		580
Multinomial Logistic		15
Normal Approximation	2	132
MNP chart		37
Multinomial Logistic		37
Normal Approximation	3	410
MNP chart		726
Multinomial Logistic		56
Normal Approximation	4	106
MNP chart		106
Multinomial Logistic		10
Normal Approximation	5	332
MNP chart		625
Multinomial Logistic		3
Normal Approximation	6	383
MNP chart		383
Multinomial Logistic		86
Normal Approximation	7	773
MNP chart		581
Multinomial Logistic		37
Normal Approximation	8	818
MNP chart		310
Multinomial Logistic		14
Normal Approximation	9	176
MNP chart		218
Multinomial Logistic		7
Normal Approximation	10	>1000
MNP chart		154
Multinomial Logistic		49

Table 3 In-control ARL of the MNP chart, the normal approximation, and the multinomial logistic regression techniques when number of in-control and out-of-control observations in training set are unequal: 10,000 in-control observations and 200 out-of-control observations (100 each for positive and negative direction of shifts).

Techniques	Training/Test Set No.	In-control ARL
Normal Approximation	1	924
MNP chart		271
Multinomial Logistic		271
Normal Approximation	2	1134
MNP chart		1256
Multinomial Logistic		1256
Normal Approximation	3	75
MNP chart		877
Multinomial Logistic		>2000
Normal Approximation	4	365
MNP chart		93
Multinomial Logistic		93
Normal Approximation	5	989
MNP chart		419
Multinomial Logistic		209
Normal Approximation	6	221
MNP chart		41
Multinomial Logistic		41
Normal Approximation	7	839
MNP chart		839
Multinomial Logistic		839
Normal Approximation	8	3
MNP chart		897
Multinomial Logistic		346
Normal Approximation	9	38
MNP chart		104
Multinomial Logistic		104
Normal Approximation	10	375
MNP chart		1407
Multinomial Logistic		451

Table 4 Out-of-control ARL of the MNP chart, the normal approximation, and the multinomial logistic regression techniques when number of in-control and out-of-control observations in training set are unequal: 10,000 in-control observations and 200 out-of-control observations (100 each for positive and negative direction of shifts).

Techniques	Training/Test Set No.	Out-of-control ARL
Normal Approximation	1	4
MNP chart		1
Multinomial Logistic		1
Normal Approximation	2	3
MNP chart		2
Multinomial Logistic		2
Normal Approximation	3	2
MNP chart		1
Multinomial Logistic		1
Normal Approximation	4	1
MNP chart		1
Multinomial Logistic		1
Normal Approximation	5	1
MNP chart		1
Multinomial Logistic		1
Normal Approximation	6	8
MNP chart		1
Multinomial Logistic		8
Normal Approximation	7	6
MNP chart		1
Multinomial Logistic		1
Normal Approximation	8	2
MNP chart		2
Multinomial Logistic		2
Normal Approximation	9	1
MNP chart		1
Multinomial Logistic		1
Normal Approximation	10	5
MNP chart		3
Multinomial Logistic		3

The out-of-control ARL for the all techniques are not significantly different at significance level of 0.05.

From the preliminary results, multinomial logistic regression is a comparable technique to the MNP chart and the normal approximation technique when a large number of in-control observations are used for the coefficient estimation. However, the method of maximum likelihood used to estimate regression coefficients can produce poor results, or even fail to converge, for small data sets or data sets in which the average values of *Y* (nonconforming proportion) is close to zero or one. Maximum likelihood, which is most commonly used in logistic regression, performs well for large sample sizes[80].

In this research (and in practice) processes in which *Y* (nonconforming proportion) are close to zero is likely (e.g. 0.1 and 0.01). Therefore, the multinomial logistic regression method was not a suitable method for this research and thus excluded from further evaluation. However, further investigation using multinomial logistic regression for mutli-attribute processes in which the proportion nonconforming is not close to zero may be valuable.

3.3.3. Probabilistic Neural Network

Probabilistic neural network (PNN) is a feed forward neural network that uses statistical techniques as a foundation, that of Bayes decision strategy and nonparametric estimators of the data's probability density function. Figure 3 provides a paradigm of PNN with two categories.

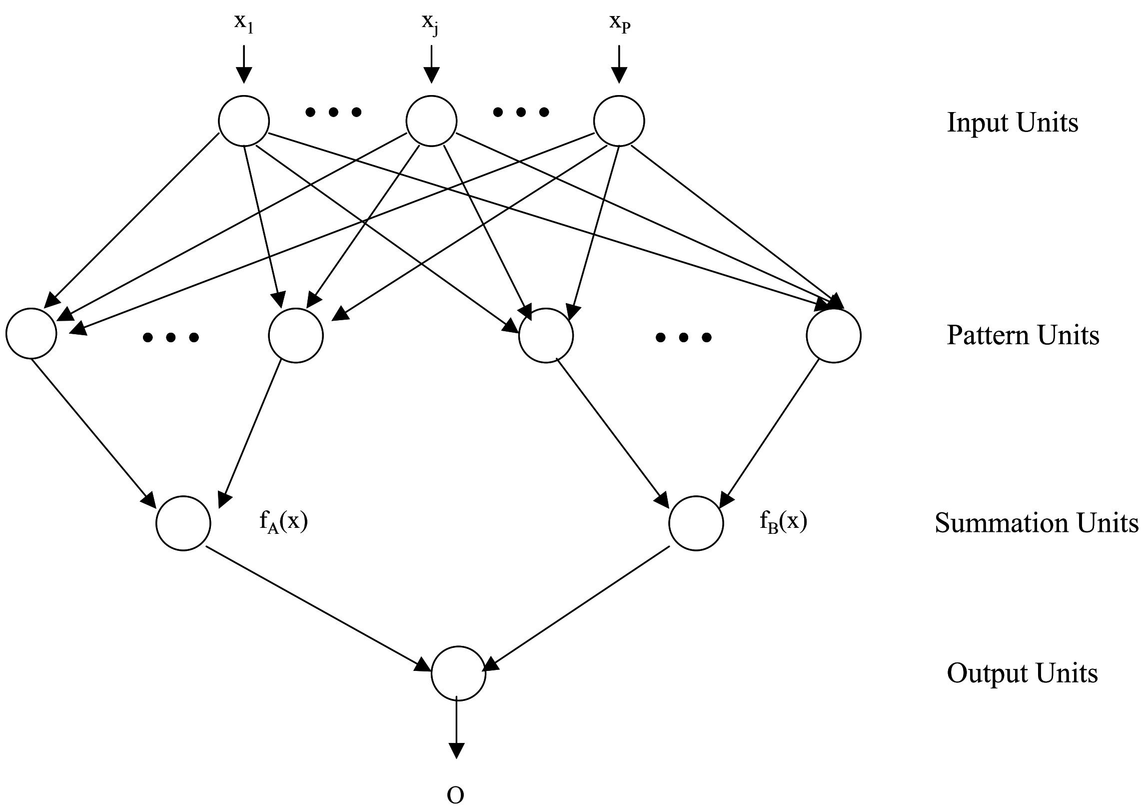

Figure 3 Probabilistic Neural Network Architecture For Two Categories

PNN comprises of four layers: input, pattern, summation, and output. The input layer includes *p* units (neurons) where *p* is the number of independent variables. The input units are merely distributions that supply the same input values to all of the pattern units. For each pattern unit a dot product is produced that consists of the input pattern vector ***X*** ($X = [X_1, X_2, \ldots, X_i, \ldots, X_p]$ where X_i is i^{th} independent variable) and a weight vector W_j (W_j is weight vector of the j^{th} pattern unit); and a nonlinear operation is then applied to the dot product. The resulting dot product from the pattern unit is $Z_j = X \cdot W_j$. The nonlinear operation used is

$$\exp[(Z_j - 1)/\sigma^2]. \tag{3-25}$$

The summation units sum the outputs of the pattern units that correspond to the category from which the training pattern was selected. The outputs from the summation units are the estimated probability density functions. The output units have two inputs. These units produce binary outputs. They have a single variable weight, *C*,

$$C = -\frac{h_B l_B}{h_A l_A} * \frac{n_A}{n_B}, \tag{3-25}$$

where n_A and n_B are the number of training patterns from categories A and B respectively, l_A and l_B are the loss functions of misclassification for categories A and B respectively, h_A and h_B are the priori probabilities for categories A and B respectively.

The ratio of losses is determined based on the severity or importance of the decision. If there is no reason to bias the decision, *C* may be simplify to –1[81].

To determine if PNN was a suitable method for multi-attribute control charts, a pilot study was conducted. This study included the dependent variable, process status, as measured by: (1) in-control, (2) out-of-control with both process's proportion nonconforming shifted in positive direction, and (3) out-of-control with both process's proportion nonconforming shifted in negative direction. The independent variables were the multivariate binomial variables (two correlated binomial distributed variables). To determine viability of the methodology on in-control and out-of-control ARLs, *C* was dissected into three ratios: number of training patterns (n_a/n_b), priori probabilities (h_a/h_b), and loss functions (l_a/l_b), for each category. When the three ratios equaled one, the probabilistic neural network resulted in a considerably (statistical significant at *p*-value 0.000) smaller in-control ARL than the normal approximation technique and the MNP chart. In-control ARL of PNN can be improved by adjusting the three ratios. However, caution should be exercised when increasing in-control ARL as it results in large out-of-control ARL. Also, the probability neural network uses the entire training set for each classification, which increases the time it takes to classify future observations. If the training set is large, probabilistic neural network classification time becomes much longer than feed-forward networks[82]. Therefore, this technique was not further investigated for this research due to its inefficiency.

3.3.4 Cumulative Sum Control Procedures

Cumulative sum control procedures (CUSUM procedures) are alternatives to univariate ($\overline{X}$ chart), uni-attribute (*p*-, *np*-, *c*-, and *u*-charts), and multivariate (i.e. Hotelling T^2) control charts when shifts in the process means are small. For univariate control charts, CUSUM procedures are far superior to the traditional $\overline{X}$ chart in detecting small process mean shifts (≤ one standard deviation), whereas they are quite competitive in detecting large process mean shifts[83]. Similarly, CUSUM procedures for binomial and nonconformance data outperform uni-attribute (*np*- and *c*-) charts in detecting small shifts while being competitive in detecting large shifts. Binomial and nonconformance data are required to be normally distributed (or approximated to be normally distributed) to employ CUSUM procedures. Both normal approximation and transformation by function can be used. CUSUM procedures can also be applied to multivariate processes, called multivariate CUSUM (MCUSUM) procedures.

Woodall and Ncube recommended using multiple univariate CUSUM charts for *p* variables[84]. It was shown that the CUSUM procedure is often preferred to Hotelling T^2 procedure for the case in which the quality characteristics are bivariate normal random variables. Healy suggested CUSUM procedures for detecting a shift in the mean vector and covariance matrix of the multivariate normal distribution[85]. However, his procedure is specific to the case that only mean shifts in a few known directions are to be expected. Crosier proposed two MCUSUM charts and compared them with the Hotelling T^2 chart[86]. Both MCUSUM charts gave faster detection of small shifts in the mean vector than Hotelling T^2 chart for multivariate normal distribution. Pignatiello and Runger proposed two more MCUSUM charts[87]. They compared the proposed CUSUM charts with Hotelling T^2 and multiple univariate CUSUM charts developed by Woodall and Ncube[88].

From the literature, CUSUM procedures have been shown to be effective in detecting small process mean shifts for variables, which are either normally distributed or transformed to be approximately normally distributed (e.g. such as binomial data in CUSUM procedure with *p* chart). However, no research has been found to date that applies CUSUM procedures to the control charts for multiple attributes. CUSUM procedures may be an alternative method in detecting mean shifts for multiple attributes processes. One possible disadvantage to using CUSUM procedures is that the calculation is too complicated. Because this research investigates shifts in the mean that are considered large (i.e. $\geq$ 1 standard deviation), MCUSUM charts for multi-attribute control charts were not investigated.

4.0 EVALUATION OF METHODOGIES: EXPERIMENTAL DESIGN

To compare the two current multi-attribute methods and the neural network approach through a common set of performance measures, an experimental design was created. This Chapter describes both the data used to compare the three methods and the experiment. Specifically, the experiment involves two correlated attributes, with varying proportion nonconforming, sample sizes and levels of correlation, as well as varying levels of shift in the attribute means (proportion nonconforming). In addition, an explanation about the number of replications needed and the assumptions made in the experiment are provided.

4.1 Data Generation

Data used in the overall experiment are generated based on algorithms suggested by Ong[89]. The algorithms generate bivariate binomial variables given marginal proportion nonconforming and correlation. The algorithms for generating positive and negative bivariate binomial variables are shown in Figure 4 and Figure 5, respectively.

Set $\phi = \sqrt{\delta_1 \delta_2 / (1-\delta_1)(1-\delta_2)}$, $\gamma = \dfrac{\rho_{xy}}{\rho_{xy} + \phi}$, $\alpha = \dfrac{\delta_1}{1-\gamma}$, and $\beta = \dfrac{\delta_2}{1-\gamma}$.

Generate $k \sim B(n, \gamma)$.

If $k = n$, return $x = 0$, $y = 0$.

Otherwise, generate $x \sim B(n-k, \alpha)$ and $y \sim B(n-k, \beta)$.

Figure 4 Ong's Algorithm for Positive Correlation (0 < ρ_{xy} ≤ 1)

Set $\phi' = \left(\dfrac{\delta_2}{1-\delta_2}\right)\sqrt{\delta_1 \delta_2 / (1-\delta_1)(1-\delta_2)}$, $\gamma = \dfrac{\rho_{xxy}}{\rho_{xy} - \phi'}$, $\tau = 1 - \dfrac{\delta_2}{1-\gamma}$,

$\theta = \dfrac{\delta_1}{1-\gamma}$, $\omega = \dfrac{\delta_1 \tau}{1-\delta_2}$, and $\delta = \dfrac{\delta_2}{1-\delta_2}$ where $0 < \omega < 1$ and $0 < \delta < 1$.

Generate $k \sim B(n, \delta_2)$.

If $k = n$, return $y = 0$. Generate $x \sim B(n, \theta)$.

If k = 0, generate $x \sim B(n, \omega)$ and $y = B(n, \delta)$.

Otherwise, generate $x = x_1 + x_2$, where $x_1 \sim B(k, \theta)$, $x_2 \sim B(n-k, \omega)$, and $y \sim B(n-k, \delta)$.

Figure 5 Ong's Algorithm for Negative Correlation ($-1 \leq \rho_{xy} < 0$)

For each algorithm, *x* and *y* are bivariate binomial variables. In addition, δ_1 and δ_2 are proportion nonconforming of *x* and *y* respectively, and ρ_{xy} is the correlation coefficient of *x* and *y*.

Both algorithms are used to generate both in-control and out-of-control data. To generate the in-control data for a particular set of parameters (sample size, level of correlation, etc.), the desired algorithm was applied to produce a set of data. To generate the out-of-control data for the same set of parameters, the same random number seed was applied, but a change in the proportion nonconforming. As a result, the out-of-control data is based on the in-control data, but has shifts in the attribute means (proportion nonconforming) reflected.

Each pair of bivariate binomial variables has a defining status (in-control or out-of-control). Status "0" is used for in-control sample while status "1" and "-1" are used for out-of-control samples with positive and negative shifts of attribute means, respectively.

4.2 The Experimental Design

Table 5 describes the experimental design of two positively correlated attributes. Three levels of proportion nonconforming (large, medium and small) are considered. Cut-off values among large, medium and small proportion nonconforming are not clearly defined in the quality control literature. As a result, small proportion nonconforming is chosen based on cumulative binomial probability tables. Weintraub[90] defines a small proportion nonconforming as values between 0.00001 and 0.01. A small p of 0.01 is selected for this experiment. For medium and large p, 0.1 and 0.3 were selected, respectively, as reasonable values to reflect proportion nonconforming in realistic processes.

If two attributes have proportion nonconforming values are considerably different (e.g. 0.3 and 0.01), the algorithm is limited in generating bivariate binomial variables, particularly when there is a strong level of correlation between the two variables. Consequently, some combinations of proportion nonconforming at some levels of correlation are not included in the experiment (see shaded areas in Table 5).

For each condition shown in Table 5, the experimental design is further expanded to include degrees of shift in proportion nonconforming, which are provided in Appendix A. There is a separate table for each condition represented in Table 5. For positively correlated attributes, the experimental conditions included simultaneous shifts in both of the proportion nonconforming in the same direction, either positive or negative. Two variables, which are highly correlated, are not expected to have shifts in the means in opposite directions[91]. The chance of a shift in an attribute mean (proportion nonconforming) while the other mean (proportion nonconforming) remains unchanged is rare because the two attributes are correlated at a very significant level[92]. Therefore, the experiment excludes these two circumstances.

Table 5 Experimental Design of Two Positively Correlated Attributes

Proportion Nonconforming		Test Data								
		Correlation Coef. = 0.8			Correlation Coef. = 0.5			Correlation Coef. = 0.2		
Attribute 1	Attribute 2	Normal Sample Size	MNP Chart Sample Size	Small Sample Size	Normal Sample Size	MNP Chart Sample Size	Small Sample Size	Normal Sample Size	MNP Chart Sample Size	Small Sample Size
large (0.3)	large (0.3)	50	10	10	50	10	10	50	10	10
large (0.3)	medium (0.1)							100	15	
large (0.3)	small (0.01)									
medium (0.1)	medium (0.1)	100	30	30	100	30	30	100	30	30
medium (0.1)	small (0.01)									
small (0.01)	small (0.01)	910	810	300	910	670	300	910	540	300

Two situations arose that further constrained the experimental design. First, in situations where data could not be generated by the algorithm, such particular experimental conditions could not be included in the experiment. In Appendix A, these situations are indicated on the tables with shaded regions. Second, a process mean shifted in a negative direction may result in a value less than “0” if one applies the normal approximation and uses a small sample size. As a result, this value is replaced with a new value, which is close to “0” so that methods can be compared with small sample sizes. In Appendix A, this situation is indicated on the tables with line-shaded regions.

For in-control proportion nonconforming of 0.3, values of 0.01 are used when shifts in the proportion nonconforming have negative values. Proportion nonconforming of 0.001 are used instead of the negative proportion nonconforming when a small sample size is applied to in-control proportion nonconforming of 0.1 and 0.01.

Values for the output are assigned as “0” if in-control, “1” for samples in which both attribute means have shifted in the positive direction, and “-1” for samples in which both attribute means have shifted in the negative direction.

When attributes are negatively correlated, shifts in the means move in different directions. For example, one attribute mean shifts in the positive direction and the other in the negative direction. As a result, a value for the sample output cannot be defined as a positive or negative shift (“1” or “-1”). Therefore, the experiment for two negatively correlated attributes was not conducted.

4.3 Sample Sizes

Three different sample sizes were used in this experiment. The first sample size used is based on estimating a normal distribution from a binomial distribution. The second sample size used is the recommended sample size for the MNP chart. The third sample size used tests the robustness of the three techniques under the condition that the probability of finding at least one nonconforming unit per sample is at least 0.95[93].

4.3.1 Sample Size #1 - Estimating Multivariate Normally Distributed Variables from a Multivariate Binomial Distribution

To approximate the normal distribution from the binomial distribution, Kenett and Zacks[94] recommend

$$n \geq \frac{9}{p(1-p)} . \qquad (4\text{-}1)$$

Given the varied values of proportion nonconforming *p,* different sample sizes resulted for the experiment.

4.3.2 Sample Size #2 - Recommended Sample Size for the MNP Chart

The sample sizes for the MNP chart are calculated based on the values of the proportion nonconforming. If the proportion nonconforming is moderate to large, the sample size is

$$n \geq \frac{3m}{\sum_i p_i} . \qquad (4\text{-}2)$$

If the proportion nonconforming is small, the sample size is

$$n \geq 9\frac{Trace(\sqrt{(I-P)(I-P)'}\,\Sigma')}{(\sqrt{P}I')^2}, \tag{4-3}$$

where $I = [1,1,\ldots,1]_{m\mathrm{x}1}$ is the unit vector, P is the fraction nonconforming vector and Σ is the correlation matrix of attributes.

4.3.3 Sample Size #3 - Satisfying the Condition of Finding at Least One Non-Conforming Item in a Sample

To be cost effective in maintaining control charts, it is common to pick the sample size to be small enough such that there is a high probability of finding at least one nonconforming item in a sample. Otherwise, we might find that the control limits are such that the presence of only one nonconforming unit in the sample would indicate an out-of-control signal.

To determine the sample size, the following calculation is used.

$$P\{Defects \geq 1\} \geq .95 \text{ or} \tag{4-4}$$

$$P\{np \geq 1\} \geq .95\,. \tag{4-5}$$

Using the Poisson approximation to the binomial distribution, we find from the cumulative Poisson table that $\lambda = np$ must exceed 3; therefore, n should be greater than or equal to $3/p$. As a result, for the experiment the sample sizes are 10, 30 and 300 for proportion nonconforming of 0.3, 0.1 and 0.01, respectively.

In case that the two attributes have different proportion nonconforming, the sample size is based on the larger value calculated.

4.4 Level of correlation

The level of correlation between two attributes varies in manufacturing and industry processes. To date, no literature has been found that supports what should be considered the minimum level of correlation necessary to employ a multi-attribute control chart. Preliminary analysis on the three multi-attribute chart techniques was conducted to observe the effects of correlation with regards to the performance measures. The results showed that different levels of correlation resulted in different in-control and out-of-control average run length. For this research three different levels of correlation were employed: strong = 0.8, moderate = 0.5, and weak = 0.2[95].

4.5 Number of Replications

Several replications must be performed for each experimental condition in order to obtain the specified precision of the performance measures. A relative error γ of 0.1 is used as a specified precision for each performance measure at a confidence level of 95 percent. The number of replications is obtained from a sequential procedure[96] as described in Figure 6.

Step 0: Make k_0 replications of the experiment and set $k_0 = k$,

Step 1: Compute $\overline{X}_k$ and $\delta(k,\alpha)$ from $X_1, X_2, ..., X_k$ where X is a performance measure from each replication. $X_1, X_2, ..., X_k$ is a sequence of IID random variables that need not be a normal.

$$\delta(k,\alpha) = t_{k-1,1-\alpha/2}\sqrt{S^2(k)/k} \qquad (4\text{-}6)$$

Step 2: If $\delta(k,\alpha)/\left|\overline{X}_k\right| \le \gamma'$, use $\overline{X}_k$ as the point estimate for μ and stop.

$$\gamma' = \frac{\gamma}{1+\gamma} \qquad (4\text{-}7)$$

Equivalently,

$$I(\alpha,\gamma) = [\overline{X}_k - \delta(k,\alpha), \overline{X}_k + \delta(k,\alpha)] \qquad (4\text{-}8)$$

is an approximate 100(1-α) percent confidence interval for μ with the desired precision. Otherwise, replace k by k+1, make an additional replication of the experiment, and go to step 1.

Figure 6 Procedure Used to Determine Number of Replications

4.6 Assumptions

To perform this experiment, the following assumptions were made.

1) The process under consideration has only two attribute quality characteristics.
2) Only changes in the attribute means are considered for this study. Correlation coefficients of attributes are assumed to be constant throughout the processes.
3) The normal approximation method assumes process variance is constant when the attribute means have shifted.
4) Changes in the attribute mean will remain until corrective actions have been taken.

5.0 PERFORMANCE MEASURES

To compare the three multi-attribute control chart techniques, a set of performance measures, in particular average run length and percent of correct classification, is suggested.

5.1 Average Run Length (ARL)

Average run length (ARL) is the average number of samples that must be taken before a sample indicates an out-of-control condition. Montgomery states, "If the process observations are uncorrelated, then for any Shewhart control chart, the ARL can be calculated easily from

$$ARL = \frac{1}{p}, \tag{5-1}$$

where p is the probability that any point exceeds the control limits."[97] There are two types of average run lengths: in-control and out-of-control.

In this research, the ARL for each experimental condition is a function of the number of replications. For each replication, the simulation run terminates when an out-of-control sample is detected[98,99].

5.1.1 In-Control Average Run Length

An in-control average run length (ARL_0) is the average number of samples that must be taken before a sample indicates an out-of-control condition when, in fact, the process is in control. For a Shewhart control chart with three-sigma control limits, the Type I error probability that a sample falls outside the control limits when the process is in control is 0.0027. Therefore, the in-control average run length is

$$ARL_0 = \frac{1}{p} = \frac{1}{.0027} = 370 \qquad (5\text{-}2)$$

This means, on average, for every 370 samples an out-of-control signal will occur when the process is actually in-control.

5.1.2 Out-Of-Control Average Run Length

An out-of-control average run length (ARL_1) is the average number of samples taken to detect a shift in the mean for a particular process, as described in Equation 5-3.

$$ARL_1 = \frac{1}{1-\beta}, \qquad (5\text{-}3)$$

where β is the probability of not detecting a shift on the first sample following the shift.

5.2 Percentage of Correct Classification

A classification table is used in addition to the ARL measures when the test data have the proportion nonconforming shifted. After an out-of-control sample is detected, a check is made to determine if the technique properly identified the direction of the shift.

6.0 MODEL VERIFICATION AND VALIDATION

The simulation model for this research was developed using MATLAB. The program was divided into three sub-modules: (1) data generation, (2) calculation of outputs for each technique, and (3) calculation of ARL for each technique. All sub-modules were verified and validated as discussed in the following sections.

6.1 Model Verification

The program was debugged in each sub-module. In addition, the second and third sub-module results were checked with hand calculations. In the second sub-module, only the outputs for the normal approximation and the MNP chart (the *G*-statistics for the normal approximation technique and the *X*-statistics for the MNP chart) were verified with the hand calculations. The BPNN technique outputs were not verified with the hand calculations since the network was large (i.e. doing a paper-pencil neural network was not feasible). All sub-modules were also verified by inputting different values of parameters (e.g. proportion nonconforming, level of correlation, and sample sizes) and observing if the outputs were reasonable.

6.2 Model Validation

In order to ensure that the data generated (or the input data for all techniques) have the specified proportion nonconforming and level of correlation, the outputs from the first sub-module were tested.

Data were generated for five process conditions with varied proportion nonconforming, correlation coefficients, and sample sizes. Each process condition had two data sets: training and testing. Each training and testing sets comprised of samples from the three process states: (1) in-control (2) out-of-control with proportion nonconforming shifted three standard deviations in the positive direction, and (3) out-of-control with proportion nonconforming shifted three standard deviations in the negative direction. Ten replications each with different random number seeds were tested for each condition. For each condition, three null hypotheses were tested:

- $p_1 = p_{10}$,
- $p_2 = p_{20}$, and
- correlation coefficient of samples = specified correlation coefficient.

For the null hypotheses, p_1 and p_2 are the sample proportion nonconforming for attribute 1 and attribute 2 respectively; and p_{10} and p_{20} are the specified proportion nonconforming for attribute 1 and attribute 2. A significance level of 0.05 was adjusted by three since three hypotheses were tested. Table B.1 through Table B.5 in Appendix B show the number of null hypothesis accepted for each process condition. The results showed that the sample proportion nonconforming are not significantly different from the specified proportion nonconforming for all process conditions. The correlation coefficients for some of the replications are significantly different from the specified correlation coefficients. As shown in Table B.4, this process condition (proportion nonconforming of 0.3, correlation coefficient of 0.8, shifts of proportion nonconforming in the negative direction, and sample sizes of 10) indicates that only 4 and 7 of the ten replications (training and test sets, respectively) showed no significant difference to the specified correlation coefficients. The data generated for this process condition, in which the correlation coefficients does not statistically equal the specified values, may have a potential effect on the ARL results. Chapter 7 discusses this issue in more detail for processes with small sample sizes (i.e. sample sizes #2 and #3).

7.0 RESULTS AND ANALYSES

This chapter discusses results and analyses of the results. The results are presented in three primary sections based on the sample sizes used in the experiment. The first sample size (sample size #1) is based on estimating multivariate normally distributed variables from a multivariate binomial distribution. The second sample size (sample size #2) is based on the recommended sample size necessary for the MNP Chart. The third sample size (sample size #3) is based on satisfying the condition of finding at least one non-conforming item in a sample. Within each primary section, there are multiple sub-sections designated according to the proportion of nonconforming (i.e. p_1 = 0.3 and p_2 = 0.3, p_1 = 0.1 and p_2 = 0.1, etc.). Within each of these sub-sections comparison between the three techniques is presented.

7.1 Sample Size #1 - Estimating Multivariate Normally Distributed Variables from a Multivariate Binomial Distribution

The following four sub-sections present for varied proportion nonconforming the pair-wise comparisons of the three techniques for large sample size.

7.1.1 p_1 = 0.3, p_2 = 0.3, Sample Sizes = 50 (Levels of Correlation: 0.8, 0.5, and 0.2)

Table 6 to Table 8 display the performances for the normal approximation technique, the MNP chart, and the BPNN technique for three of the experimental subsets (proportion nonconforming p_1 = 0.3 and p_2 = 0.3; a sample size #1 = 50; and levels of correlation = 0.80, 0.50, and 0.20, respectively). The tables (as well as Tables 9-14) present the number of replication runs in the simulation, the number of replications in which a particular technique detected a shift, the ARL and corresponding variance for each technique, and the performances of both the MNP chart and the BPNN techniques in correctly classifying the direction of the shifts (Note: the normal approximation technique does not have the quality of classifying the direction of the shifts). The number in parentheses in the upper half of the table presents the percentage of shifts detected (number of replication that detected shifts / number of replication run which is 2000 replications in this process condition). The number in parentheses shown in the lower half of the table presents the percentage of correct classification of shift direction (number of replication that correctly identify shift direction / number of replication run which is 2000 replications in this process condition).

Table 6 Performance of the Three Techniques for Experimantal Subset: p_1=0.3, p_2=0.3, Sample Sizes=50, Correlation Coefficient=0.8

	Shift									
	$+3s_1, +3s_2$	$+3s_1, +2s_2$	$+2s_1, +2s_2$	$+2s_1, +1s_2$	$+1s_1, +1s_2$	NoShift_NoShift	$-3s_1, -3s_2$	$-2s_1, -2s_2$	$-2s_1, -1s_2$	$-1s_1, -1s_2$
samples/replication	500	500	500	1000	1000	10000	500	1000	1000	2000
Replication	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
Shifts Detected (Reps.)										
Normal Approx.	2000	2000	2000	2000	2000	2000	2000	2000	2000	1971
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(98.55%)
MNP chart	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)
BPNN	2000	2000	2000	2000	2000	1999	2000	2000	1999	2000
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(99.95%)	(100%)	(100%)	(99.95%)	(100%)
ARL										
Normal Approx.	2.29	2.56	7.57	6.37	49.44	369.74	3.20	26.88	13.60	228.71
MNP chart	1.79	2.78	5.01	11.14	31.14	410.07	1.62	7.26	22.61	81.66
BPNN	1.81	2.83	5.11	11.74	32.49	430.60	1.58	6.94	21.78	77.62
ARL variance										
Normal Approx.	3.20	4.82	70.80	40.71	3950.00	244910.00	12.17	2299.90	350.21	79619.00
MNP chart	1.49	5.30	21.11	120.23	1195.10	182290.00	1.00	46.80	565.03	7875.90
BPNN	1.59	6.28	22.98	162.72	1483.70	348700.00	1.07	64.05	768.04	12715.00
Correct Classification of Shift Direction (Reps.)										
MNP chart	2000	2000	2000	2000	2000	n/a	2000	2000	2000	1996
(%)	(100%)	(100%)	(100%)	(100%)	(100%)		(100%)	(100%)	(100%)	(99.80%)
BPNN	2000	2000	2000	2000	1999	n/a	2000	2000	1999	1996
(%)	(100%)	(100%)	(100%)	(100%)	(99.95%)		(100%)	(100%)	(99.95%)	(99.80%)

Table 7 Performance of the Three Techniques for Experimental Subset: p_1=0.3,p_2=0.3, Sample Sizes=50, Correlation Coefficient=0.5

	Shift												
	$+3s_1, +3s_2$	$+3s_1, +2s_2$	$+3s_1, +1s_2$	$+2s_1, +2s_2$	$+2s_1, +1s_2$	$+1s_1, +1s_2$	NoShift_NoShift	$-3s_1, -3s_2$	$-3s_1, -2s_2$	$-3s_1, -1s_2$	$-2s_1, -2s_2$	$-2s_1, -1s_2$	$-1s_1, -1s_2$
samples/replication	500	500	1000	500	1000	1000	10000	500	500	1000	500	1000	2000
Replication	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
Shifts Detected (Reps.)													
Normal Approx.	2000	2000	2000	2000	2000	2000	1999	2000	2000	2000	2000	2000	1982
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(99.95%)	(100%)	(100%)	(100%)	(100%)	(100%)	(99.10%)
MNP chart	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)
BPNN	2000	2000	2000	2000	2000	2000	1998	2000	2000	2000	2000	2000	1997
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(99.90%)	(100%)	(100%)	(100%)	(100%)	(100%)	(99.85%)
ARL													
Normal Approx.	1.83	2.57	2.57	5.93	9.27	40.41	395.03	2.03	3.54	3.48	15.01	28.24	198.53
MNP chart	1.49	2.23	3.98	3.88	8.71	25.52	379.91	1.30	2.26	5.30	5.11	15.76	57.88
BPNN	1.51	2.23	4.07	3.83	8.76	26.91	435.68	1.32	2.26	5.70	5.09	16.37	61.96
ARL variance													
Normal Approx.	1.73	4.37	4.32	38.37	122.34	2180.20	253630.00	2.99	13.29	16.33	532.95	1811.70	79401.00
MNP chart	0.78	2.86	12.68	11.83	74.84	693.88	187200.00	0.41	3.09	26.76	24.86	298.16	4232.50
BPNN	0.84	3.11	15.70	13.50	89.34	1254.70	560590.00	0.51	3.36	47.99	28.03	550.94	9318.20
Correct Classification of Shift Direction (Reps.)													
MNP chart	2000	2000	2000	2000	2000	2000	n/a	2000	2000	2000	2000	2000	1996
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)		(100%)	(100%)	(100%)	(100%)	(100%)	(99.80%)
BPNN	2000	2000	2000	2000	2000	2000	n/a	2000	2000	2000	2000	1999	1997
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)		(100%)	(100%)	(100%)	(100%)	(99.95%)	(99.85%)

Table 8 Performance of the Three Techniques for Experimental Subset: p_1=0.3, p_2=0.3, Sample Sizes=50, Correlation Coefficient=0.2

	Shift												
	$+3s_1, +3s_2$	$+3s_1, +2s_2$	$+3s_1, +1s_2$	$+2s_1, +2s_2$	$+2s_1, +1s_2$	$+1s_1, +1s_2$	NoShift_NoShift	$-3s_1, -3s_2$	$-3s_1, -2s_2$	$-3s_1, -1s_2$	$-2s_1, -2s_2$	$-2s_1, -1s_2$	$-1s_1, -1s_2$
samples/replication	500	500	1000	500	1000	1000	10000	500	500	1000	500	1000	2000
Replication	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
Shifts Detected (Reps.)													
Normal Approx.	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	1995
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(99.75%)
MNP chart	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)
BPNN	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)
ARL													
Normal Approx.	1.51	2.10	2.78	4.15	8.05	31.49	415.22	1.38	2.40	3.63	7.70	22.41	137.02
MNP chart	1.32	1.76	2.92	2.88	6.39	20.32	385.28	1.12	1.62	3.38	3.37	10.32	41.30
BPNN	1.32	1.77	3.10	2.97	6.79	21.51	450.88	1.16	1.74	3.85	3.81	12.36	52.03
ARL variance													
Normal Approx.	0.81	2.70	5.84	17.07	77.70	1473.60	290200.00	0.67	6.62	18.04	92.77	1268.50	39285.19
MNP chart	0.43	1.46	6.25	5.81	42.33	485.92	204730.00	0.14	1.12	9.44	9.16	135.71	2298.46
BPNN	0.43	1.53	7.53	6.50	55.59	507.30	290840.00	0.20	1.43	14.41	12.59	231.57	4021.02
Correct Classification of Shift Direction (Reps.)													
MNP chart	2000	2000	2000	2000	1999	1998	n/a	2000	2000	2000	2000	2000	1999
(%)	(100%)	(100%)	(100%)	(100%)	(99.95%)	(99.90%)		(100%)	(100%)	(100%)	(100%)	(100%)	(99.95%)
BPNN	2000	2000	2000	2000	1999	1999	n/a	2000	2000	2000	2000	2000	2000
(%)	(100%)	(100%)	(100%)	(100%)	(99.95%)	(99.95%)		(100%)	(100%)	(100%)	(100%)	(100%)	(100%)

7.1.1.1 Comparing the BPNN to the Normal Approximation Techniques

Table 6 to Table 8 show for all the three correlation coefficients (0.8, 0.5, and 0.2) that the BPNN technique outperformed the normal approximation technique when the process is in-control. In general, the BPNN technique will indicate a false alarm much later than the normal approximation technique (e.g. 35 to 61 samples later).

When the process is out of control and when both proportion nonconforming have shifted in the same direction with the same magnitude, performance of the two techniques depended largely on the extent and direction of the shifts. Specifically, for large shifts (i.e. three standard deviations), the BPNN and normal approximation techniques performed equally for shifts in the positive direction. However, when both proportion nonconforming shifted in the negative direction, the performance of the BPNN technique compared to the normal approximation technique depended on the level of correlation. The stronger the level of correlation, the better the BPNN performed in comparison to the normal approximation technique. The BPNN technique outperformed the normal approximation for strongly correlated processes while the two techniques performed equally for moderately and weakly correlated processes. For medium (two standard deviations) and small (one standard deviation) shifts, the BPNN technique outperformed the normal approximation technique in both positive and negative shift directions. Also, the higher the level of correlation, the better the BPNN technique performed than the normal approximation technique.

When the magnitudes of the shifts were different and both proportion nonconforming shifted in the same direction (i.e. the proportion nonconforming of the first attribute shifted two standard deviations and the proportion nonconforming of the second attribute shifted one standard deviation) the normal approximation technique performed either better than or equally to BPNN for both positive and negative directions of shifts for strongly correlated processes. Figure 7 explains how the normal approximation technique detects the shifts. The shaded ellipse is the 99.7% confidence region of the two binomial distributed variables that are approximated by normal distributed variables for strongly correlated process while the non-shaded ellipse is the 99.7% confidence region for a weakly correlated process. Point A is the location where the process's proportion nonconforming shift with the same magnitude (in this case both proportion nonconforming shifted two standard deviations). Point B is the coordinate where the process's proportion nonconforming shift with different magnitudes (the proportion nonconforming of the first attribute shifted two standard deviations and the proportion nonconforming of the second attribute shifted one standard deviation). For the situation where the correlation between the two attributes is strong, the confidence region is narrow so the normal approximation technique detects Point B faster than the BPNN technique. In contrast, the BPNN technique detects point A earlier than the normal approximation technique. Processes in which the correlation between attributes is moderate (~0.50) and proportion nonconforming shifted in the positive direction similar results are produced but are not as obvious as the strongly correlated processes. The differences between the magnitudes of shifts for moderately correlated processes need to be higher than the ones for the strongly correlated processes in order to have this consequence (i.e. the proportion nonconforming of the first attribute shifted three standard deviations and the proportion nonconforming of the second attribute shifted one standard deviation). For moderately correlated processes with proportion nonconforming shifted in the negative direction, the normal approximation technique outperformed the BPNN technique when the proportion nonconforming of the first attribute shifted with three standard deviations and the proportion nonconforming of the second attribute shifted with one standard deviation. The BPNN technique outperformed the normal approximation technique for the other magnitudes of shifts. Weakly correlated processes, however, did not have the same results. BPNN performed the same or better than the normal approximation technique even though the magnitudes of shifts were different.

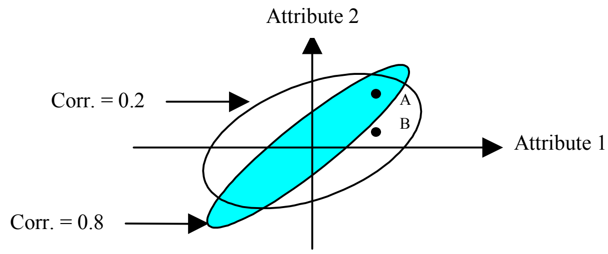

Figure 7 Confidence Regions of Normal Approximated Variables

7.1.1.2 Comparing the BPNN Technique to the MNP Chart

Table 6 through Table 8 show for the three correlation coefficients that BPNN technique outperformed the MNP chart when the process is in-control. In general the BPNN technique indicates a false alarm later than the MNP chart (i.e. 20 to 65 samples).

For all levels of correlation, the MNP chart and the BPNN technique performed similarly in detecting shifts when the process's proportion nonconforming shifted two to three standard deviations in either the positive or negative directions (i.e. both proportion nonconforming shifted in the same direction). The MNP chart was able to detect a shift only one sample faster than the BPNN technique when both proportion nonconforming shifted in the positive direction and the magnitudes were small (one standard deviation). When the shifts were small and in the negative direction, the level of correlation appeared to affect the results. The BPNN technique outperforms the MNP chart for strongly correlated processes. The MNP chart detected shifts faster than the BPNN technique given weak and moderate correlation; however, the BPNN technique identified the directions of shifts more correctly than the MNP chart one replication. These differences were considered negligible.

When the magnitudes of the shifts were different and both proportion nonconforming shifted in the positive direction, the BPNN technique and the MNP chart performed equally for all levels of correlation coefficient. When both proportion nonconforming shifted in the negative direction, both techniques performed equally for strongly and moderately correlated processes. For weakly correlated processes, the MNP chart outperformed the BPNN technique when the proportion nonconforming of the first attribute shifted with two standard deviations and the proportion nonconforming of the second attribute shifted with one standard deviation. Both techniques performed equally for the other magnitudes of shifts.

7.1.1.3 Comparing the MNP Chart to the Normal Approximation Technique

The MNP chart indicated a false alarm signal 41 samples later than the normal approximation technique when processes were strongly correlated. However, the normal approximation technique specified false alarms 16 and 30 samples later than the MNP chart when the processes were moderately and weakly correlated, respectively.

When processes were out-of-control, the performance of normal approximation technique compared to the MNP chart was similar to that of the comparison between the normal approximation technique and the BPNN.

7.1.2 $p_1 = 0.1$, $p_2 = 0.1$, Sample Sizes = 100 (Levels of Correlation: 0.8, 0.5, and 0.2)

Table 9 through Table 11 present the ARLs for the normal approximation technique, the MNP chart, and the BPNN technique for three of the experimental subsets (proportion nonconforming $p_1 = 0.1$ and $p_2 = 0.1$; a sample size #1 = 100; and levels of correlation = 0.80, 0.50, and 0.20, respectively).

Table 9 Performance of the Three Techniques for Experimental Subset: p_1=0.1, p_2=0.1, Sample Sizes=100, Correlation Coefficient=0.8

	Shift								
	$+3s_1, +3s_2$	$+3s_1, +2s_2$	$+2s_1, +2s_2$	$+2s_1, +1s_2$	$+1s_1, +1s_2$	NoShift_NoShift	$-3s_1, -3s_2$	$-2s_1, -2s_2$	$-1s_1, -1s_2$
samples/replication	500	500	500	1000	1000	10000	500	500	2000
Replication	2000	2000	2000	2000	2000	2000	2000	2000	2000
Shifts Detected (Reps.)									
Normal Approx.	2000	2000	2000	2000	2000	2000	1111	1200	1544
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(55.55%)	(60%)	(77.20%)
MNP chart	2000	2000	2000	2000	2000	2000	2000	2000	1991
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(99.55%)
BPNN	2000	2000	2000	2000	2000	2000	1998	1998	1974
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(99.90%)	(99.90%)	(98.70%)
ARL									
Normal Approx.	2.19	2.38	5.60	5.28	26.84	272.30	9.63	74.93	571.97
MNP chart	1.87	2.62	4.52	8.81	21.11	332.53	1.38	14.46	286.49
BPNN	1.89	2.77	4.62	9.55	22.66	362.26	1.28	11.86	185.72
ARL variance									
Normal Approx.	2.86	3.86	30.92	24.46	954.95	129350.00	2143.00	9745.00	260800.00
MNP chart	1.56	4.26	16.81	71.24	534.34	129900.00	0.56	254.24	101910.00
BPNN	1.65	5.60	20.70	115.19	701.16	260670.00	0.41	347.25	63950.00
Correct Classification of Shift Direction (Reps.)									
MNP chart	2000	2000	2000	2000	2000	n/a	2000	2000	1966
(%)	(100%)	(100%)	(100%)	(100%)	(100%)		(100%)	(100%)	(98.30%)
BPNN	2000	2000	2000	2000	2000	n/a	1998	1998	1964
(%)	(100%)	(100%)	(100%)	(100%)	(100%)		(99.90%)	(99.90%)	(98.20%)

Table 10 Performance of the Three Techniques for Experimental Subset: p_1=0.1, p_2=0.1, Sample Sizes=100, Correlation Coefficient=0.5

	Shift										
	$+3s_1, +3s_2$	$+3s_1, +2s_2$	$+3s_1, +1s_2$	$+2s_1, +2s_2$	$+2s_1, +1s_2$	$+1s_1, +1s_2$	NoShift_NoShift	$-3s_1, -3s_2$	$-2s_1, -2s_2$	$-2s_1, -1s_2$	$-1s_1, -1s_2$
samples/replication	500	500	1000	500	1000	1000	10000	500	500	1000	2000
Replication	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
Shifts Detected (Reps.)											
Normal Approx.	2000	2000	2000	2000	2000	2000	2000	1828	1761	1932	1490
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(91.40%)	(88.05%)	(96.60%)	(74.50%)
MNP chart	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	1999
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(99.95%)
BPNN	2000	2000	2000	2000	2000	2000	1997	1999	1999	1996	1979
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(99.85%)	(99.95%)	(99.95%)	(99.80%)	(98.95%)
ARL											
Normal Approx.	1.82	2.41	2.59	4.57	6.61	23.86	314.66	2.66	56.54	132.56	537.16
MNP chart	1.58	2.18	3.76	3.59	7.20	18.23	336.69	1.12	7.44	36.04	180.43
BPNN	1.63	2.29	3.93	3.84	7.88	19.99	400.77	1.15	8.89	48.25	201.13
ARL variance											
Normal Approx.	1.60	4.00	4.59	20.37	46.64	767.38	158640.00	179.33	7637.40	30648.00	258030.00
MNP chart	0.88	2.61	10.98	10.26	51.48	348.70	150240.00	0.13	55.77	1866.30	48734.00
BPNN	0.99	3.37	12.66	13.31	76.82	522.37	281070.00	0.23	152.41	5147.50	74155.00
Correct Classification of Shift Direction (Reps.)											
MNP chart	2000	2000	2000	2000	2000	2000	n/a	2000	2000	2000	1994
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)		(100%)	(100%)	(100%)	(99.70%)
BPNN	2000	2000	2000	2000	2000	2000	n/a	1999	1999	1996	1973
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)		(99.95%)	(99.95%)	(99.80%)	(98.65%)

Table 11 Performance of the Three Techniques for Experimental Subset: p_1=0.1, p_2=0.1, Sample Sizes=100, Correlation Coefficient=0.2

	Shift												
	$+3s_1, +3s_2$	$+3s_1, +2s_2$	$+3s_1, +1s_2$	$+2s_1, +2s_2$	$+2s_1, +1s_2$	$+1s_1, +1s_2$	NoShift_NoShift	$-3s_1, -3s_2$	$-3s_1, -2s_2$	$-3s_1, -1s_2$	$-2s_1, -2s_2$	$-2s_1, -1s_2$	$-1s_1, -1s_2$
samples/replication	500	500	1000	500	1000	1000	10000	500	500	1000	500	1000	2000
Replication	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
Shifts Detected (Reps.)													
Normal Approx.	2000	2000	2000	2000	2000	2000	2000	1999	1999	2000	1983	1942	1710
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(99.95%)	(99.95%)	(100%)	(99.15%)	(97.10%)	(85.50%)
MNP chart	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)
BPNN	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	1998	1994
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(99.90%)	(99.70%)
ARL													
Normal Approx.	1.55	2.02	2.57	3.52	6.06	19.98	266.76	1.28	4.95	15.70	24.58	106.70	420.59
MNP chart	1.37	1.81	2.91	2.92	5.54	15.05	306.32	1.02	1.49	4.15	3.97	16.54	103.80
BPNN	1.36	1.81	2.95	2.84	5.50	14.73	320.78	1.03	1.68	5.37	4.84	23.43	132.36
ARL variance													
Normal Approx.	0.84	2.08	5.30	9.74	35.38	541.08	117960.00	0.68	248.45	1384.10	2430.30	26331.00	212660.00
MNP chart	0.50	1.49	5.92	6.20	28.05	257.81	120570.00	0.02	0.82	17.09	15.49	382.31	22944.00
BPNN	0.48	1.57	6.47	5.70	29.53	251.30	232180.00	0.03	2.06	127.97	71.90	1498.30	32468.00
Correct Classification of Shift Direction (Reps.)													
MNP chart	2000	2000	2000	2000	2000	2000	n/a	2000	2000	2000	2000	2000	1998
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)		(100%)	(100%)	(100%)	(100%)	(100%)	(99.90%)
BPNN	2000	2000	2000	2000	2000	2000	n/a	2000	2000	2000	2000	1998	1990
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)		(100%)	(100%)	(100%)	(100%)	(99.90%)	(99.50%)

7.1.2.1 Comparing the BPNN to the Normal Approximation Techniques

Table 9 through Table 11 show for all the three correlation coefficients (0.8, 0.5, and 0.2) that the BPNN technique outperformed normal approximation when the process is in-control. In general, the BPNN technique will indicate a false alarm much later than the normal approximation technique (e.g. 54 to 90 samples later).

When the process is out of control and when both proportion nonconforming have shifted in the same direction with the same magnitude, performance of the two techniques depended largely on the extent and direction of the shifts. Specifically, for large (i.e. three standard deviations) and medium (i.e. two standard deviations) shifts, the BPNN and normal approximation techniques performed equally for shifts in the positive direction. However, when both proportion nonconforming shifted in the negative direction and with large magnitude, the BPNN technique outperformed the normal approximation technique for strongly and moderately correlated processes. Both techniques performed equally for weakly correlated processes. The BPNN technique outperformed the normal approximation technique for all levels of correlation coefficient when the proportion nonconforming shifted with medium magnitude and in the negative direction. For small (one standard deviation) shifts, the BPNN technique outperformed the normal approximation technique in both the positive and negative shift directions. When both proportion nonconforming shifted in the negative direction and with any magnitude, the performance of the BPNN technique compared to the normal approximation technique depended on the level of correlation. The higher the level of correlation, the better the BPNN technique performed in comparison to the normal approximation technique.

When the magnitudes of the shifts were different, the results were similar to the processes with large proportion nonconforming (p_1 = 0.3 and p_2 = 0.3); that is the normal approximation technique performed either better than or equally to the BPNN technique for positive direction of shifts for strongly and moderately correlated processes. Further, the BPNN technique performed the same as the normal approximation technique for weakly correlated processes. However, for shifts in the negative direction, the results were different from those of large proportion nonconforming. The BPNN technique outperformed the normal approximation technique for all magnitudes of shift.

Kramer and Jensen[100,101] and Jackson[102,103] discussed that χ^2 chart is directionally invariant. That is, the ARL performance of the technique is determined solely by the distance of the off-target mean from the on-target mean and not by the particular direction (or, location) of the mean. The directional invariant property is shown below.

$$\lambda^2(\mu) = (\mu - \mu_0)\Sigma^{-1}(\mu - \mu_0) \qquad (7\text{-}1)$$

where $\lambda(\mu)$ is the square root of the non-centrality parameter, μ is process mean vector at any time t, and μ_0 is in-control process mean vector. The normal approximation technique, which has G-square statistic approximated to χ^2 statistic, therefore, has the directional invariant property.

There is an assumption of equal covariance matrix in the invariant directional property. However, the assumption does not hold for the normal approximation technique in this research. The covariance matrix of processes with means (proportion nonconforming) shifted in negative direction was smaller than the covariance matrix of in-control processes. (The proportion nonconforming were close to zero when they were shifted in negative direction; therefore, the data could not be generated below zero values. As a result, the data generated had a small covariance matrix.) Consequently, the inverse matrix of the covariance of negatively shifted processes was larger than the inverse matrix of the covariance of in-control processes. The G-square statistic for the normal approximation technique was calculated based on the inverse covariance matrix of the in-control processes (due to the equal covariance matrix assumption). Therefore, the *G*-square statistics for processes with the negative direction of shift were smaller than what they should be. As a result, the processes with negatively shifted proportion nonconforming had larger ARLs than the processes with positively shifted proportion nonconforming.

When proportion nonconforming for in-control processes were smaller the covariance matrix of the processes with proportion nonconforming shifted in negative direction was also smaller. Therefore, for processes which have medium proportion nonconforming and shifts in negative direction, the ARLs resulted from the normal approximation technique were large compared to the BPNN technique.

7.1.2.2 Comparing the BPNN Technique to the MNP Chart

Table 9 through Table 11 show for all the three correlation coefficients (0.8, 0.5, and 0.2) that the BPNN technique outperformed the MNP chart when the process is in-control. In general, the BPNN technique will indicate a false alarm much later than the MNP chart (e.g. 14 to 64 samples later).

When process's proportion nonconforming shifted with two to three standard deviations in the positive direction, the BPNN technique and the MNP chart performed the same for all levels of correlation. For processes with small magnitude of shifts in positive direction, the MNP chart only detected one sample faster than the BPNN technique for strongly and moderately correlated processes. Their performances were the same for weakly correlated processes.

When both proportion nonconforming shifted with three standard deviations in the negative direction, both techniques performed equally. For processes with both proportion nonconforming shifted two standard deviations and in the negative direction (Table 9), the BPNN detected shifts 3 ARL samples faster than the MNP chart (15 and 12 for the MNP chart and the BPNN technique respectively). However, the MNP chart could indicate shifts 2 additional replications over the BPNN technique. If one assumed that the BPNN technique detected the shifts for those 2 replications, the ARL for each of those replications would be at least 500 (since there were 500 samples for each replication). This would result in a new ARL equal to 12.3 for the BPNN technique. Given this supposition, the BPNN technique would outperform the MNP chart as its ARL is smaller.

Furthermore, processes with both proportion nonconforming shifted one standard deviation and in the negative direction, the BPNN technique indicated shifts 101 ARL samples faster than the MNP chart for strongly correlated processes (287 and 186 for the MNP chart and the BPNN technique respectively), as shown in Table 9. However, the MNP chart could indicate shifts 17 additional replications over the BPNN technique. If one assumed that the BPNN technique detected the shifts for those 17 replications, the new ARL would be at least 201 for the BPNN technique. Therefore, the BPNN still outperformed the MNP chart.

The MNP chart outperformed the BPNN technique for moderately and weakly correlated processes for shifts of one and two standard deviations in the negative direction as shown in Table 10 and Table 11.

When the magnitudes of the shifts of proportion nonconforming were different and both proportion nonconforming shifted in the positive direction, the BPNN technique and the MNP chart performed equally for all levels of correlation coefficient. When the proportion nonconforming of the first attribute shifted three standard deviations and the proportion nonconforming of the second attribute shifted two standard deviation and both proportion nonconforming shifted in the negative direction, both techniques performed equally. For the other magnitudes of shifts in the negative direction, the MNP chart outperformed the BPNN technique.

7.1.2.3 Comparing the MNP Chart to the Normal Approximation Technique

Table 9 through Table 11 show for all the three correlation coefficients (0.8, 0.5, and 0.2) that the MNP chart outperformed the normal approximation technique when the process is in-control. In general, the MNP chart will indicate a false alarm much later than the normal approximation technique (e.g. 12 to 40 samples later).

When processes were out-of-control, the performance of normal approximation technique compared to the MNP chart was similar to that of the comparison between the normal approximation technique and the BPNN.

7.1.3 p_1 = 0.01, p_2 = 0.01, Sample Sizes = 910 (Levels of Correlation: 0.8, 0.5, and 0.2)

Table 12 through Table 14 present the ARLs for the normal approximation technique, the MNP chart, and the BPNN technique for three of the experimental subsets (proportion nonconforming p_1 = 0.01 and p_2 = 0.01; a sample size #1 = 910; and levels of correlation = 0.80, 0.50, and 0.20, respectively).

Table 12 Performance of the Three Techniques for Experimental Subset: p_1=0.01, p_2=0.01, Sample Sizes=910, Correlation Coefficient=0.8

	Shift									
	$+3s_1, +3s_2$	$+3s_1, +2s_2$	$+3s_1, +1s_2$	$+2s_1, +2s_2$	$+2s_1, +1s_2$	$+1s_1, +1s_2$	NoShift_NoShift	$-3s_1, -3s_2$	$-2s_1, -2s_2$	$-1s_1, -1s_2$
samples/replication	500	500	1000	500	1000	1000	10000	500	500	2000
Replication	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
Shifts Detected (Reps.)										
Normal Approx.	2000	2000	2000	2000	2000	2000	2000	225	339	1141
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(11.25%)	(16.95%)	(57.05%)
MNP chart	2000	2000	2000	2000	2000	2000	2000	2000	2000	1821
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(91.05%)
BPNN	2000	2000	2000	2000	2000	2000	1999	1980	1980	1936
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(99.95%)	(99%)	(99%)	(96.80%)
ARL										
Normal Approx.	2.17	2.40	1.71	5.24	4.88	24.26	252.02	1.27	97.13	725.14
MNP chart	1.92	2.69	4.20	4.33	8.17	19.93	295.92	1.09	25.36	501.88
BPNN	1.91	2.69	4.40	4.33	8.35	20.00	303.27	1.05	13.58	272.29
ARL variance										
Normal Approx.	2.90	4.01	1.34	24.82	22.16	778.44	105120.00	4.96	17069.00	302800.00
MNP chart	1.90	4.66	13.61	14.45	64.92	375.32	120720.00	0.10	901.96	207880.00
BPNN	1.94	4.91	20.94	16.27	72.83	449.16	178610.00	0.06	299.05	102760.00
Correct Classification of Shift Direction (Reps.)										
MNP chart	2000	2000	2000	2000	2000	2000	n/a	2000	2000	1776
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)		(100%)	(100%)	(88.80%)
BPNN	2000	2000	2000	2000	2000	1999	n/a	1980	1980	1910
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(99.95%)		(99%)	(99%)	(95.50%)

Table 13 Performance of the Three Techniques for Experimental Subset: p_1=0.01, p_2=0.01, Sample Sizes=910, Correlation Coefficient=0.5

	Shift										
	$+3s_1, +3s_2$	$+3s_1, +2s_2$	$+3s_1, +1s_2$	$+2s_1, +2s_2$	$+2s_1, +1s_2$	$+1s_1, +1s_2$	NoShift_NoShift	$-3s_1, -3s_2$	$-2s_1, -2s_2$	$-2s_1, -1s_2$	$-1s_1, -1s_2$
samples/replication	500	500	1000	500	1000	1000	10000	500	500	1000	2000
Replication	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
Shifts Detected (Reps.)											
Normal Approx.	2000	2000	2000	2000	2000	2000	2000	1009	1011	1768	1015
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(50.45%)	(50.55%)	(88.40%)	(50.75%)
MNP chart	2000	2000	2000	2000	2000	2000	2000	2000	2000	1999	1962
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(99.95%)	(98.10%)
BPNN	2000	2000	2000	2000	2000	2000	2000	1997	1997	1986	1936
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(99.85%)	(99.85%)	(99.30%)	(96.80%)
ARL											
Normal Approx.	1.86	2.34	2.37	4.24	6.14	19.83	271.08	1.09	58.28	222.76	681.77
MNP chart	1.61	2.20	3.51	3.49	6.76	16.83	279.19	1.01	10.50	62.22	307.29
BPNN	1.61	2.19	3.59	3.44	6.84	16.55	299.53	1.01	10.85	64.04	269.17
ARL variance											
Normal Approx.	1.74	3.70	3.93	15.94	35.43	471.73	114020.00	0.11	6107.50	54981.00	306610.00
MNP chart	1.03	2.99	10.00	9.85	42.64	305.45	103620.00	0.01	153.72	7701.50	112670.00
BPNN	1.11	3.13	11.98	10.21	47.28	371.11	215730.00	0.01	358.26	10216.00	106960.00
Correct Classification of Shift Direction (Reps.)											
MNP chart	2000	2000	2000	2000	2000	2000	n/a	2000	2000	1999	1945
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)		(100%)	(100%)	(99.95%)	(97.25%)
BPNN	2000	2000	2000	2000	2000	2000	n/a	1997	1997	1986	1927
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)		(99.85%)	(99.85%)	(99.30%)	(96.35%)

Table 14 Performance of the Three Techniques for Experimental Subset: p_1=0.01, p_2=0.01, Sample Sizes=910, Correlation Coefficient=0.2

	Shift										
	$+3s_1, +3s_2$	$+3s_1, +2s_2$	$+3s_1, +1s_2$	$+2s_1, +2s_2$	$+2s_1, +1s_2$	$+1s_1, +1s_2$	NoShift_NoShift	$-3s_1, -3s_2$	$-2s_1, -2s_2$	$-2s_1, -1s_2$	$-1s_1, -1s_2$
samples/replication	500	500	1000	500	1000	1000	10000	500	500	1000	2000
Replication	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
Shifts Detected (Reps.)											
Normal Approx.	2000	2000	2000	2000	2000	2000	2000	1894	1848	1755	1344
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(94.70%)	(92.40%)	(87.75%)	(67.20%)
MNP chart	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	1991
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(99.55%)
BPNN	2000	2000	2000	2000	2000	2000	2000	2000	2000	1998	1958
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(99.90%)	(97.90%)
ARL											
Normal Approx.	1.53	2.01	2.48	3.26	5.61	16.44	255.56	1.03	44.56	166.67	547.68
MNP chart	1.37	1.81	2.85	2.68	5.41	13.55	286.13	1.00	4.54	23.97	153.51
BPNN	1.39	1.85	2.91	2.77	5.69	14.00	308.73	1.00	6.90	38.33	251.05
ARL variance											
Normal Approx.	0.92	2.24	3.98	8.24	30.84	330.25	127780.00	0.04	6827.90	47338.00	279840.00
MNP chart	0.52	1.50	5.58	4.79	23.93	217.80	113840.00	0.00	20.35	1215.10	42363.00
BPNN	0.56	1.76	6.58	5.16	29.70	229.43	181170.00	0.00	238.05	3718.60	95495.00
Correct Classification of Shift Direction (Reps.)											
MNP chart	2000	2000	2000	2000	2000	2000	n/a	2000	2000	2000	1990
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)		(100%)	(100%)	(100%)	(99.50%)
BPNN	2000	2000	2000	2000	2000	2000	n/a	2000	2000	1998	1953
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)		(100%)	(100%)	(99.90%)	(97.65%)

7.1.3.1 Comparing the BPNN to the Normal Approximation Techniques

Table 12 through Table 14 show for all the three correlation coefficients (0.8, 0.5, and 0.2) that the BPNN technique outperformed the normal approximation technique when the process is in-control. In general, the BPNN technique will indicate a false alarm much later than the normal approximation technique (e.g. 28 to 53 samples later).

When the process is out of control and when both proportion nonconforming have shifted in the same direction with the same magnitude, performance of the two techniques depended largely on the extent and direction of the shifts. Specifically, for large shifts (i.e. three standard deviations) and medium shifts (i.e. two standard deviations), the BPNN and the normal approximation techniques performed equally for shifts in the positive direction. However, when both proportion nonconforming shifted in the negative direction, the BPNN technique outperformed the normal approximation technique. For small (one standard deviation) shifts, the BPNN technique outperformed the normal approximation technique in both the positive and negative shift directions. When both proportion nonconforming shifted in the negative direction and with any magnitude, the performance of the BPNN technique compared to the normal approximation technique depended on the level of correlation. The higher the level of correlation, the better the BPNN technique performed in comparison to the normal approximation technique.

When the magnitudes of the shifts were different, the results were similar to the processes with large proportion nonconforming ($p_1 = 0.3$ and $p_2 = 0.3$); that is the normal approximation technique performed either better than or equally to the BPNN for positive direction of shifts for strongly and moderately correlated processes. Further, BPNN performed the same as the normal approximation technique for weakly correlated processes. However, for shifts in the negative direction, the results were different from those of large proportion nonconforming. The BPNN technique outperformed the normal approximation technique for all magnitudes of shift.

7.1.3.2 Comparing the BPNN Technique to the MNP Chart

Table 12 through Table 14 show for all the three correlation coefficients (0.8, 0.5, and 0.2) that the BPNN technique outperformed the MNP chart when the process is in-control. In general, the BPNN technique will indicate a false alarm much later than the MNP chart (e.g. 8 to 22 samples later).

When both proportion nonconforming shifted in positive direction, the BPNN technique and the MNP chart performed the same for all levels of correlation coefficient.

When both proportion nonconforming shifted with three standard deviations in negative direction, the level of correlation appeared to affect the results. For strongly correlated processes (Table 12), both techniques had the same ARL (~1.0), but the MNP chart could correctly indicate shifts in the means 20 additional replications over the BPNN technique. If one assumed that the BPNN technique detected the shifts for those 20 replications, the ARL for each of those replications would be at least 500 (since there were 500 samples for each replication). This would result in a new ARL equal to 6 for the BPNN technique. Given this supposition, the MNP chart would outperform the BPNN technique as its ARL is smaller. For moderately correlated processes (Table 13), both techniques had the same ARL (~1.0). But, again, the BPNN technique could not indicate shifts for three of the 2000 replications while the MNP chart indicated shifts for all replications. If one assumed that the BPNN technique detected the shifts for those three replications, the new ARL would be at least 1.76 for the BPNN technique. Therefore, one might conclude that the MNP chart performed equal to or better than the BPNN technique. For weakly correlated processes, the two techniques performed equally.

When both proportion nonconforming shifted with two standard deviations in negative direction, the level of correlation appeared to affect the results. For strongly correlated processes (Table 12), the BPNN technique detected shifts 12 ARL samples faster than the MNP chart (26 and 14 for the MNP chart and the BPNN technique respectively). However, the MNP chart could correctly indicate shifts in the mean 20 additional replications over the BPNN technique. If one assumed that the BPNN technique detected the shifts for those 20 replications, the new ARL would be at least 18 for the BPNN technique. Therefore, the BPNN technique still outperformed the MNP chart. For moderately correlated processes (Table 13), both techniques had the same ARL (~11). But the BPNN technique could not indicate the shifts for three of the 2000 replications while the MNP chart indicated shifts for all replications. If one assumed that the BPNN technique detected the shifts for those three replications, the new ARL would be at least 11.5 for the BPNN technique. Therefore, one might conclude that the MNP chart performed equal to or better than the BPNN technique. For weakly correlated processes, the MNP chart outperformed the BPNN technique.

Furthermore, processes with both proportion nonconforming shifted one standard deviation and in the negative direction, the BPNN outperformed the MNP chart for strongly correlated processes. For moderately correlated process (Table 13), the BPNN technique detected shifts 38 ARL samples faster than the MNP chart (308 and 270 for the MNP chart and the BPNN technique respectively). However, the MNP chart could indicate shifts 26 additional replications over the BPNN technique. If one assumed that the BPNN technique detected the shifts for those 26 replications, the new ARL would be at least 293 for the BPNN technique. Therefore, the BPNN technique still outperformed the MNP chart. The MNP chart outperformed the BPNN technique for weakly correlated processes.

When the magnitudes of the shifts of proportion nonconforming were different and both proportion nonconforming shifted in the negative direction, the MNP chart outperformed the BPNN technique.

7.1.3.3 Comparing the MNP Chart to the Normal Approximation Technique

Table 12 through Table 14 show for all the three correlation coefficients (0.8, 0.5, and 0.2) that the MNP chart outperformed the normal approximation technique when the process is in-control. In general, the MNP chart will indicate a false alarm much later than the normal approximation technique (e.g. 8 to 43 samples later).

When processes were out-of-control, the performance of normal approximation technique compared to the MNP chart was similar to that of the comparison between the normal approximation technique and the BPNN, but the numbers of samples to ARL are different.

7.1.4 p_1 = 0.3, p_2 = 0.1, Sample Sizes = 100 (Levels of Correlation: 0.2)

Table 15 presents the ARLs for the normal approximation technique, the MNP chart, and the BPNN technique for an experimental subset (proportion nonconforming p_1 = 0.3 and p_2 = 0.1; a sample size #1 = 100; and levels of correlation = 0.20).

Table 15 Performance of the Three Techniques for Experimental Subset: p_1=0.3, p_2=0.1, Sample Sizes=100, Correlation Coefficient=0.2

	Shift																
	$+3s_1, +3s_2$	$+3s_1, +2s_2$	$+3s_1, +1s_2$	$+2s_1, +3s_2$	$+2s_1, +2s_2$	$+2s_1, +1s_2$	$+1s_1, +3s_2$	$+1s_1, +2s_2$	$+1s_1, +1s_2$	NoShift_NoShift	$-3s_1, -3s_2$	$-3s_1, -2s_2$	$-3s_1, -1s_2$	$-2s_1, -2s_2$	$-2s_1, -1s_2$	$-1s_1, -2s_2$	$-1s_1, -1s_2$
samples/replication	500	500	500	500	500	500	500	500	500	10000	500	500	1000	500	2000	2000	2000
Replication	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
Shifts Detected (Reps.)																	
Normal Approx.	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	1994
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(99.70%)
MNP chart	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)
BPNN	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	1998
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(99.90%)
ARL																	
Normal Approx.	1.52	2.12	2.63	2.10	4.02	7.83	2.58	6.77	25.02	337.63	1.38	2.37	3.42	9.22	18.01	50.21	174.83
MNP chart	1.31	1.82	3.10	1.75	2.98	6.66	2.65	5.68	17.27	349.46	1.08	1.66	3.84	3.48	11.06	11.41	50.22
BPNN	1.30	1.77	2.97	1.73	2.87	6.39	2.63	5.46	16.82	353.82	1.09	1.73	4.02	3.65	12.58	13.21	58.40
ARL variance																	
Normal Approx.	0.86	2.59	4.80	2.55	14.16	71.85	4.73	52.20	886.77	189110.00	0.64	4.24	11.93	175.54	660.09	9200.40	60192.00
MNP chart	0.45	1.52	6.49	1.30	6.43	43.47	5.12	30.99	324.85	162130.00	0.08	1.10	12.42	9.87	150.32	148.76	3654.50
BPNN	0.42	1.42	6.51	1.28	5.87	47.37	4.90	29.54	397.69	260470.00	0.10	1.46	18.29	15.08	341.18	275.92	9017.80
Correct Classification of Shift Direction (Reps.)																	
MNP chart	2000	2000	2000	2000	2000	2000	2000	2000	2000	n/a	2000	2000	2000	2000	2000	2000	1996
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)		(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(99.80%)
BPNN	2000	2000	2000	2000	2000	2000	2000	2000	2000	n/a	2000	2000	2000	2000	2000	2000	1995
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)		(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(99.75%)

7.1.4.1 Comparing the BPNN to the Normal Approximation Techniques

Table 15 shows the process with correlation coefficient of 0.2 that BPNN outperformed normal approximation when the process is in-control. In general, the

BPNN technique will indicate a false alarm much later than the normal approximation technique (e.g. 16 samples later).

When the process is out of control and when both proportion nonconforming have shifted in the same direction with the same magnitude, performance of the two techniques depended largely on the extent of the shifts. Specifically, for large shifts (i.e. three standard deviations), the BPNN and the normal approximation techniques performed equally for shifts in both the positive and negative directions. The BPNN technique also detected small and medium shifts faster than the normal approximation technique in both positive and negative shift directions.

When the magnitude of the shifts were different and the magnitude of a shift was large and the other shift's magnitude was either medium or small (i.e. the proportion nonconforming of the first attribute shifted three standard deviations and the proportion nonconforming of the second attribute shifted either one or two standard deviations, or vice versa), the BPNN and the normal approximation techniques performed equally. In cases that the magnitudes of the shifts were different and the proportion nonconforming shifted with the magnitude of medium and small (i.e. the proportion nonconforming of the first attribute shifted two standard deviations and the proportion nonconforming of the second attribute shifted one standard deviation, or vice versa), the BPNN technique indicated an out-of-control process faster than the normal approximation technique. Above performances applied to both the positive and negative directions of shifts. Only weakly correlated processes were included in this particular experiment; therefore, the analyses excluded the strongly and moderately correlated processes.

7.1.4.2 Comparing the BPNN Technique to the MNP Chart

Table 15 shows the process with correlation coefficient of 0.2 that the BPNN technique outperformed the MNP chart when the process is in-control. In general, the BPNN technique will indicate a false alarm later than the MNP chart (e.g. 4 samples later).

When both proportion nonconforming shifted in positive direction and both shifts have either the same or different magnitudes, the BPNN technique and the MNP chart performed equally.

When both proportion nonconforming shifted in the negative direction and the first proportion nonconforming shifted three standard deviations and the second proportion nonconforming shifted from one to three standard deviations, the BPNN technique and the MNP chart performed equally. However, when both proportion nonconforming shifted one or two standard deviations, the MNP chart outperformed BPNN technique.

7.1.4.3 Comparing the MNP Chart to the Normal Approximation Technique

Table 15 shows that the MNP chart outperformed the normal approximation technique when the process is in-control. In general, the MNP chart will indicate a false alarm later than the normal approximation technique (e.g. 12 samples later).

When the process is out of control and when both proportion nonconforming shifted in the same direction with the same magnitude, performance of the two techniques depended largely on the extent of the shifts. Specifically, for large shifts (i.e. three standard deviations), the MNP chart and the normal approximation technique performed equally for shifts in both the positive and negative directions. The MNP chart also detected small and medium shifts faster than the normal approximation technique in both positive and negative shift directions.

When the magnitude of the shifts of proportion nonconforming were different and a shift magnitude was large and the other shift magnitude was either medium or small (i.e. the proportion nonconforming of the first attribute shifted three standard deviations and the proportion nonconforming of the second attribute shifted either one or two standard deviations, or vice versa), the MNP chart and the normal approximation technique performed equally. In cases where the magnitudes of the shift were different and they were medium and small (i.e. the proportion nonconforming of the first attribute shifted two standard deviations and the proportion nonconforming of the second attribute shifted one standard deviation, or vice versa), the MNP chart indicated an out-of-control process faster than the normal approximation technique. Above performances applied to both the positive and negative directions of shifts. Only weakly correlated processes were included in the experiment; therefore, the analyses excluded the strongly and moderately correlated processes.

7.2 Recommended Sample Size for the MNP Chart

The following three sub-sections presented for varied proportion nonconforming the pair-wise comparisons of the three techniques for sample size recommended for the MNP chart.

7.2.1 $p_1 = 0.3$, $p_2 = 0.3$, Sample Sizes = 10 (Levels of Correlation: 0.8, 0.5, and 0.2)

Table 16 through Table 21 present the ARLs for the normal approximation technique, the MNP chart, and the BPNN technique for three of the experimental subsets (proportion nonconforming $p_1 = 0.3$ and $p_2 = 0.3$; a sample size #2 = 10; and levels of correlation = 0.80, 0.50, and 0.20, respectively).

The results indicate that the in-control ARL for the normal approximation technique and the MNP chart are substantially different. To compare the BPNN technique to the normal approximation technique and the MNP chart, two different cut-off values were used for the BPNN technique in order to have the in-control ARL comparable to the ARL for each technique. As a result, there are two tables for an experimental subset. The first table (i.e. Table 16, Table 18, or Table 20)shows the comparison between the BPNN technique and the normal approximation technique and the second table (i.e. Table 17, Table 19, or Table 21)shows the comparison between the BPNN technique and the MNP chart.

Table 16 Performance of the BPNN and the Normal Approximation Techniques for Experimental Subset: p_1=0.3, p_2=0.3, Sample Sizes=10, Correlation Coefficient=0.8

	Shift							
	$+3s_1, +3s_2$	$+2s_1, +2s_2$	$+1s_1, +1s_2$	NoShift_NoShift	$-3s_1, -3s_2$	$-3s_1, -2s_2$	$-2s_1, -2s_2$	$-1s_1, -1s_2$
samples/replication	500	500	1000	10000	500	500	500	2000
Replication	4000	4000	4000	4000	4000	4000	4000	4000
Shifts Detected (Reps.)								
Normal Approx.	4000	4000	4000	4000	2	2	1	2710
(%)	(100%)	(100%)	(100%)	(100%)	(0.05%)	(0.05%)	(0.025%)	(67.75%)
BPNN	4000	4000	4000	3997	2868	2868	2868	2893
(%)	(100%)	(100%)	(100%)	(99.93%)	(71.70%)	(71.70%)	(71.70%)	(72.33%)
ARL								
Normal Approx.	2.44	8.16	40.54	248.66	255.00	255.00	81.00	589.65
BPNN	1.82	5.54	32.88	286.19	1.13	1.13	1.13	16.52
ARL variance								
Normal Approx.	5.11	95.51	3074.30	124840.00	60552.00	60552.00	0.00	278880.00
BPNN	1.64	32.51	1634.70	468180.00	0.15	0.15	0.15	12590.00

Table 17 Performance of the BPNN Technique and the MNP Chart for Experimental Subset: p_1=0.3, p_2=0.3, Sample Sizes=10, Correlation Coefficient=0.8

	Shift							
	$+3s_1, +3s_2$	$+2s_1, +2s_2$	$+1s_1, +1s_2$	NoShift_NoShift	$-3s_1, -3s_2$	$-3s_1, -2s_2$	$-2s_1, -2s_2$	$-1s_1, -1s_2$
samples/replication	500	500	1000	10000	500	500	500	2000
Replication	4000	4000	4000	4000	4000	4000	4000	4000
Shifts Detected (Reps.)								
MNP chart	4000	4000	4000	4000	4000	4000	4000	4000
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)
BPNN	4000	4000	4000	4000	3876	3876	3876	3878
(%)	(100%)	(100%)	(100%)	(100%)	(96.90%)	(96.90%)	(96.90%)	(96.95%)
ARL								
MNP chart	1.75	5.18	29.02	57.50	1.13	1.13	1.13	7.23
BPNN	1.46	3.52	16.45	60.06	1.12	1.12	1.12	7.73
ARL variance								
MNP chart	1.41	24.09	1061.80	3223.70	0.14	0.14	0.14	45.63
BPNN	0.75	10.88	455.53	23245.00	0.13	0.13	0.13	1805.90
Correct Classification of Shift Direction (Reps.)								
MNP chart	4000	3998	3854	n/a	4000	4000	4000	4000
(%)	(100%)	(99.95%)	(96.35%)		(100%)	(100%)	(100%)	(100%)
BPNN	4000	3999	3914	n/a	3876	3876	3876	3875
(%)	(100%)	(99.98%)	(97.85%)		(96.90%)	(96.90%)	(96.90%)	(96.88%)

Table 18 Performance of the BPNN and the Normal Approximation Techniques for Experimental Subset: p_1=0.3, p_2=0.3, Sample Sizes=10, Correlation Coefficient=0.5

	Shift										
	$+3s_1, +3s_2$	$+3s_1, +2s_2$	$+3s_1, +1s_2$	$+2s_1, +2s_2$	$+2s_1, +1s_2$	$+1s_1, +1s_2$	NoShift_NoShift	$-3s_1, -3s_2$	$-3s_1, -2s_2$	$-2s_1, -2s_2$	$-1s_1, -1s_2$
samples/replication	500	500	1000	500	1000	1000	10000	500	500	500	2000
Replication	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000
Shifts Detected (Reps.)											
Normal Approx.	4000	4000	4000	4000	4000	3999	3999	0	0	0	1229
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(99.98%)	(99.98%)	(0%)	(0%)	(0%)	(30.73%)
BPNN	4000	4000	4000	4000	4000	4000	3979	3334	3334	3334	3335
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(99.48%)	(83.35%)	(83.35%)	(83.35%)	(83.38%)
ARL											
Normal Approx.	1.82	2.60	2.61	6.10	9.38	35.60	394.41	No detection	No detection	No detection	879.48
BPNN	1.74	2.89	6.40	5.63	14.46	43.59	395.92	1.15	1.15	1.15	12.19
ARL variance											
Normal Approx.	1.79	5.48	5.39	56.88	132.77	2121.20	354160.00	No detection	No detection	No detection	321880.00
BPNN	1.43	6.69	52.03	32.88	294.19	3094.50	884590.00	0.17	0.17	0.17	527.31

Table 19 Performance of the BPNN Technique and the MNP Chart for Experimental Subset: p_1=0.3, p_2=0.3, Sample Sizes=10, Correlation Coefficient=0.5

	Shift										
	$+3s_1, +3s_2$	$+3s_1, +2s_2$	$+3s_1, +1s_2$	$+2s_1, +2s_2$	$+2s_1, +1s_2$	$+1s_1, +1s_2$	NoShift_NoShift	$-3s_1, -3s_2$	$-3s_1, -2s_2$	$-2s_1, -2s_2$	$-1s_1, -1s_2$
samples/replication	500	500	1000	500	1000	1000	10000	500	500	500	2000
Replication	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000
Shifts Detected (Reps.)											
MNP chart	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)
BPNN	4000	4000	4000	4000	4000	4000	3997	3873	3873	3873	3873
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(99.93%)	(96.83%)	(96.83%)	(96.83%)	(96.83%)
ARL											
MNP chart	1.44	2.15	4.03	3.84	8.92	24.03	124.00	1.16	1.16	1.16	11.91
BPNN	1.46	2.16	4.13	3.85	8.77	23.93	153.38	1.14	1.14	1.15	11.00
ARL variance											
MNP chart	0.64	2.69	14.70	12.92	84.88	787.96	15544.00	0.17	0.17	0.18	129.25
BPNN	0.71	2.97	19.77	15.74	96.35	931.23	166940.00	0.16	0.16	0.16	119.19
Correct Classification of Shift Direction (Reps.)											
MNP chart	4000	4000	4000	4000	4000	3981	n/a	4000	4000	4000	3999
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(99.53%)		(100%)	(100%)	(100%)	(99.98%)
BPNN	4000	4000	4000	4000	4000	3969	n/a	3873	3873	3873	3873
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(99.23%)		(96.83%)	(96.83%)	(96.83%)	(96.83%)

Table 20 Performance of the BPNN and the Normal Approximation Techniques for Experimental Subset: p_1=0.3, p_2=0.3, Sample Sizes=10, Correlation Coefficient=0.2

	Shift												
	$+3s_1, +3s_2$	$+3s_1, +2s_2$	$+3s_1, +1s_2$	$+2s_1, +2s_2$	$+2s_1, +1s_2$	$+1s_1, +1s_2$	NoShift_NoShift	$-3s_1, -3s_2$	$-3s_1, -2s_2$	$-3s_1, -1s_2$	$-2s_1, -2s_2$	$-2s_1, -1s_2$	$-1s_1, -1s_2$
samples/replication	500	500	1000	500	1000	1000	10000	500	500	1000	500	1000	2000
Replication	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000
Shifts Detected (Reps.)													
Normal Approx.	4000	4000	4000	4000	4000	4000	3998	2	2	882	2	893	582
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(99.95%)	(0.05%)	(0.05%)	(22.05%)	(0.05%)	(22.33%)	(14.55%)
BPNN	4000	4000	4000	4000	4000	3998	3925	3778	3778	3778	3778	3778	3778
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(99.95%)	(98.13%)	(94.45%)	(94.45%)	(94.45%)	(94.45%)	(94.45%)	(94.45%)
ARL													
Normal Approx.	1.45	2.06	2.65	4.24	8.20	28.45	461.98	2.00	2.00	433.65	2.50	435.09	962.69
BPNN	1.55	2.50	5.83	5.35	15.59	53.90	518.30	1.18	1.19	5.34	1.20	5.38	20.20
ARL variance													
Normal Approx.	0.80	2.88	5.70	18.73	85.43	1380.20	484180.00	0.00	0.00	81662.00	0.50	81530.00	340830.00
BPNN	1.01	4.65	45.07	36.64	421.42	5496.50	876140.00	0.22	0.23	23.41	0.23	24.25	421.54

Table 21 Performance of the BPNN Technique and the MNP Chart for Experimental Subset: p_1=0.3, p_2=0.3, Sample Sizes=10, Correlation Coefficient=0.2

	Shift												
	$+3s_1, +3s_2$	$+3s_1, +2s_2$	$+3s_1, +1s_2$	$+2s_1, +2s_2$	$+2s_1, +1s_2$	$+1s_1, +1s_2$	NoShift_NoShift	$-3s_1, -3s_2$	$-3s_1, -2s_2$	$-3s_1, -1s_2$	$-2s_1, -2s_2$	$-2s_1, -1s_2$	$-1s_1, -1s_2$
samples/replication	500	500	1000	500	1000	1000	10000	500	500	1000	500	1000	2000
Replication	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000
Shifts Detected (Reps.)													
MNP chart	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)
BPNN	4000	4000	4000	4000	4000	4000	3997	3974	3974	3974	3974	3974	3974
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(99.93%)	(99.35%)	(99.35%)	(99.35%)	(99.35%)	(99.35%)	(99.35%)
ARL													
MNP chart	1.23	1.70	3.03	2.93	6.43	18.26	237.87	1.19	1.20	5.47	1.20	5.52	20.86
BPNN	1.32	1.90	3.68	3.47	8.54	25.93	263.88	1.16	1.17	4.79	1.17	4.84	16.94
ARL variance													
MNP chart	0.31	1.35	7.79	6.43	46.07	466.14	70593.00	0.23	0.24	23.48	0.24	24.50	427.96
BPNN	0.48	1.94	13.23	10.25	116.68	1179.10	150690.00	0.18	0.19	21.29	0.20	22.20	354.39
Correct Classification of Shift Direction (Reps.)													
MNP chart	4000	4000	4000	4000	4000	3991	n/a	4000	4000	4000	4000	4000	3998
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(99.78%)		(100%)	(100%)	(100%)	(100%)	(100%)	(99.95%)
BPNN	4000	4000	4000	4000	4000	3988	n/a	3974	3974	3974	3974	3974	3974
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(99.70%)		(99.35%)	(99.35%)	(99.35%)	(99.35%)	(99.35%)	(99.35%)

7.2.1.1 Comparing the BPNN to the Normal Approximation Techniques

Table 16, Table 18, and Table 20 show for all the three correlation coefficients (0.8, 0.5, and 0.2, respectively) that the BPNN technique outperformed normal approximation when the process is in-control. In general, the BPNN technique will indicate a false alarm much later than the normal approximation technique (e.g. 1 to 57 samples later).

When the process is out of control and when both proportion nonconforming have shifted in the same direction with the same magnitude, performance of the two techniques depended largely on the extent and direction of the shifts. Specifically, for large shifts (i.e. three standard deviations), the BPNN and normal approximation techniques performed equally for shifts in the positive direction. However, when both proportion nonconforming shifted in the negative direction, the normal approximation technique could not detect any shifts; therefore, one may be able to conclude that the BPNN outperformed the normal approximation technique. When both proportion nonconforming had shifted two standard deviations (medium shifts) and in the positive direction, the BPNN technique outperformed the normal approximation technique for strongly correlated processes (i.e. 0.80). Both techniques performed equally for moderately correlated processes (i.e. 0.50). The normal approximation outperformed the BPNN technique for weakly correlated processes (i.e. 0.20).

When both proportion nonconforming had shifted two standard deviations (medium shifts) and in the negative direction, the normal approximation technique could not detect any shifts; therefore, the BPNN technique outperformed the normal approximation technique. When both proportion nonconforming had shifted one standard deviation (small shifts) and in the positive direction, BPNN technique outperformed the normal approximation technique for strongly correlated processes. The normal approximation technique outperformed the BPNN technique for moderately and weakly correlated processes. When both proportion nonconforming had shifted one standard deviation and in the negative direction, the BPNN technique outperformed the normal approximation technique.

When the shifts had different magnitudes and were in the same direction, the performance of the two techniques depends on the magnitude and direction of the shifts. The normal approximation technique outperformed the BPNN technique for the positive direction of shifts for all levels of correlation when the proportion nonconforming of the first attribute shifted three or two standard deviations and the proportion nonconforming of the second attribute shifted one standard deviation. However, when the proportion nonconforming of the first attribute shifted three standard deviations and the proportion nonconforming of the second attribute shifted two standard deviations and both proportion nonconforming shifted in the positive direction, the BPNN and the normal approximation techniques performed equally. When both proportion nonconforming shifted with different magnitudes and in the negative direction, the BPNN technique outperformed the normal approximation technique for all levels of correlation.

In summary, the BPNN technique outperformed the normal approximation technique when both proportion nonconforming shifted in the negative direction for any magnitude of shift. It is mentioned in section 7.1.2.1 that the normal approximation technique has an assumption that the covariance matrix remains the same when the process's proportion nonconforming shift. However, this assumption does not hold for this particular situation. As a result, the normal approximation technique does not operate as intended for shifts in the negative direction.

To provide further explanation, for processes with shifts in the negative direction and in which smaller samples sizes were applied, the covariance matrix was substantial smaller than the covariance matrix for in-control processes. (The proportion nonconforming were close to zero when they shifted in the negative direction; obviously, the data could not be generated for proportion non-conforming values less than zero. As a result, the data generated had a smaller covariance matrix.) Consequently, the inverse matrix of the covariance of negatively shifted processes was larger than the inverse matrix of the covariance of in-control processes. The *G*-square statistic for the normal approximation technique was calculated based on the inverse covariance matrix of the in-control processes (due to the equal covariance matrix assumption). Therefore, the *G*-square statistics for processes with shifts in the negative direction were smaller than what they should be. As a result, the normal approximation technique could not detect (or could detect for only a few replications) processes with negatively shifted proportion nonconforming.

Finally, as the correlation coefficient increased, the number of replications in which the BPNN technique detected negatively directed shifts decreased. This may be due to the limitations associated with generating data for smaller sample sizes. As discussed in Chapter 6, given strongly correlated processes with small sample sizes, the generated correlated coefficients for processes with both proportion nonconforming shifted in the negative direction were inconsistent with the desired correlation coefficients. As a result, the 100 samples used to train the network may not represent a sufficient number of patterns (or relationships of the two attributes) to adequately predict the test set data.

7.2.1.2 Comparing the BPNN Technique to the MNP Chart

Table 17, Table 19, and Table 21 show for all the three correlation coefficients (0.8, 0.5, and 0.2, respectively) that the BPNN technique outperformed the MNP chart when the process is in-control. In general, the BPNN technique will indicate a false alarm much later than the MNP chart (e.g. 3 to 30 samples later).

When the process is out of control and when both proportion nonconforming have shifted in the same direction with the same magnitude, performance of the two techniques depended largely on the extent and direction of the shifts. Specifically, for large shifts (i.e. three standard deviations), the BPNN technique and the MNP chart performed equally for shifts in the positive direction. When both proportion nonconforming had shifted two standard deviations (medium shifts) and in the positive direction, the BPNN technique outperformed the MNP chart for strongly correlated processes. Both techniques performed equally for moderately and weakly correlated processes. When both proportion nonconforming had shifted one standard deviation (small shifts) and in the positive direction, the BPNN technique outperformed the MNP chart for strongly correlated processes. Both techniques resulted in the same average run length for moderately correlated processes; however, the MNP chart identified the direction of shifts more correctly than the BPNN technique. This difference was considered negligible. The MNP chart outperformed the BPNN technique for weakly correlated processes. When both proportion nonconforming had shifted in the negative direction regardless of the magnitude, the MNP chart outperformed the BPNN technique. The BPNN technique could not indicate shifts for all the 4000 replications while the MNP chart could. The differences of the replications were considered significant.

When the shifts had different magnitudes and were in the positive direction, the BPNN technique and the MNP chart performed equally for moderately correlated processes. For weakly correlated processes, the MNP chart outperformed the BPNN technique when the proportion nonconforming of the first attribute shifted two standard deviations and the proportion nonconforming of the second attribute shifted one standard deviation. Both techniques performed equally for the other magnitudes of shift. When both proportion nonconforming shifted with different magnitudes and in the negative direction, the MNP chart outperformed the BPNN technique for all levels of correlation. The BPNN technique could not indicate shifts for all the 4000 replications while the MNP chart could. The differences of the replications were considered significant.

7.2.1.3 Comparing the MNP Chart to the Normal Approximation Technique

Table 16 through Table 21 show for all the three correlation coefficients (0.8, 0.5, and 0.2) that the normal approximation technique outperformed the MNP chart when the process is in-control. In general, the normal approximation technique will indicate a false alarm much later than the MNP chart (e.g. 191 to 224 samples later). The stronger the correlation coefficient, the smaller the in-control ARL for the MNP chart and the normal approximation technique.

When the process is out of control and when both proportion nonconforming have shifted in the same direction with the same magnitude, performance of the two techniques depended largely on the extent and direction of the shifts. Specifically, for large shifts (i.e. three standard deviations), the MNP chart and the normal approximation technique performed equally for shifts in the positive direction. However, when both proportion nonconforming shifted in the negative direction, the normal approximation technique could not detect any shifts; therefore, the MNP chart outperformed the normal approximation technique. When both proportion nonconforming had shifted two standard deviations (medium shifts) and in the positive direction, the MNP chart outperformed the normal approximation technique for all levels of correlation. When both proportion nonconforming had shifted two standard deviations (medium shifts) and in the negative direction, the normal approximation technique could not detect any shifts; therefore, the MNP chart outperformed the normal approximation technique. When both proportion nonconforming had shifted one standard deviation (small shifts) and in the positive direction, the MNP chart outperformed the normal approximation technique for all levels of correlation. When both proportion nonconforming had shifted one standard deviation and in the negative direction, the MNP chart outperformed the normal approximation technique.

When the magnitudes of the shifts were different, the level of correlation affected the results. For moderately correlated processes, the normal approximation technique outperformed the MNP chart when the proportion nonconforming of the first attribute shifted three standard deviations and the proportion nonconforming of the second attribute shifted one standard deviation and both proportion nonconforming shifted in the positive direction. Both techniques performed equally for other magnitudes of shifts. For weakly correlated processes, the MNP chart outperformed the normal approximation technique when the proportion nonconforming of the first attribute shifted two standard deviations and the proportion nonconforming of the second attribute shifted one standard deviation and both proportion nonconforming shifted in the positive direction. Both techniques performed equally for other magnitudes of shifts. When both proportion nonconforming shifted with different magnitudes and in the negative direction, the MNP chart outperformed the normal approximation technique for all levels of correlation.

7.2.2 p_1 = 0.1, p_2 = 0.1, Sample Sizes = 30 (Levels of Correlation: 0.8, 0.5, and 0.2)

Table 22 through Table 27 present the ARLs for the normal approximation technique, the MNP chart, and the BPNN technique for three of the experimental subsets (proportion nonconforming p_1 = 0.1 and p_2 = 0.1; a sample size #2 = 30; and level of correlation = 0.80, 0.50, and 0.20, respectively).

The results showed that the in-control ARL for the normal approximation technique and the MNP chart were substantial different. To compare the BPNN technique to the normal approximation technique and the MNP chart, two different cut-off values were used for the BPNN technique in order to have the in-control ARL comparable to the ARL for each technique. As a result, there were two tables for an experimental subset. The first table (i.e. Table 22, Table 24, or Table 26) showed the comparison between the BPNN technique and the normal approximation technique and the second table (i.e. Table 23, Table 25, or Table 27) showed the comparison between the BPNN technique and the MNP chart.

Table 22 Performance of the BPNN and the Normal Approximation Techniques for Experimental Subset: p_1=0.1, p_2=0.1, Sample Sizes=30, Correlation Coefficient=0.8

	Shift									
	$+3s_1, +3s_2$	$+3s_1, +2s_2$	$+2s_1, +2s_2$	$+2s_1, +1s_2$	$+1s_1, +1s_2$	NoShift_NoShift	$-3s_1, -3s_2$	$-3s_1, -2s_2$	$-2s_1, -2s_2$	$-1s_1, -1s_2$
samples/replication	500	500	500	1000	1000	10000	500	500	500	2000
Replication	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000
Shifts Detected (Reps.)										
Normal Approx.	4000	4000	4000	4000	4000	4000	0	0	0	1586
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(0%)	(0%)	(0%)	(39.65%)
BPNN	4000	4000	4000	4000	4000	3995	2637	2637	2637	2670
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(99.88%)	(65.93%)	(65.93%)	(65.93%)	(66.75%)
ARL										
Normal Approx.	2.13	2.28	4.66	4.49	18.23	158.59	No detection	No detection	No detection	851.97
BPNN	1.97	2.66	4.15	7.97	18.09	179.70	1.04	1.04	1.04	16.29
ARL variance										
Normal Approx.	2.58	3.17	22.05	17.02	430.96	38347.63	No detection	No detection	No detection	344660.00
BPNN	2.28	5.41	18.48	98.23	575.52	242420.00	0.04	0.04	0.04	13067.00

Table 23 Performance of the BPNN Technique and the MNP Chart for Experimental Subset: p_1=0.1, p_2=0.1, Sample Sizes=30, Correlation Coefficient=0.8

	Shift									
	$+3s_1, +3s_2$	$+3s_1, +2s_2$	$+2s_1, +2s_2$	$+2s_1, +1s_2$	$+1s_1, +1s_2$	NoShift_NoShift	$-3s_1, -3s_2$	$-3s_1, -2s_2$	$-2s_1, -2s_2$	$-1s_1, -1s_2$
samples/replication	500	500	500	1000	1000	10000	500	500	500	2000
Replication	5000	5000	5000	5000	5000	5000	5000	5000	5000	5000
Shifts Detected (Reps.)										
MNP chart	5000	5000	5000	5000	5000	5000	5000	5000	5000	5000
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)
BPNN	5000	5000	5000	5000	5000	5000	4502	4502	4502	4524
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(90.04%)	(90.04%)	(90.04%)	(90.48%)
ARL										
MNP chart	1.91	2.57	4.03	7.55	16.34	36.19	1.04	1.04	1.04	5.36
BPNN	1.67	2.18	3.15	5.72	11.52	55.99	1.03	1.03	1.03	10.27
ARL variance										
MNP chart	1.84	4.31	13.60	54.76	276.79	1276.80	0.04	0.04	0.04	23.84
BPNN	1.29	2.80	7.99	36.11	177.00	30733.00	0.04	0.04	0.04	6204.60
Correct Classification of Shift Direction (Reps.)										
MNP chart	4999	4997	4993	4977	4797	n/a	5000	5000	5000	5000
(%)	(99.98%)	(99.94%)	(99.86%)	(99.54%)	(95.94%)		(100%)	(100%)	(100%)	(100%)
BPNN	4999	4997	4994	4980	4875	n/a	4502	4502	4502	4501
(%)	(99.98%)	(99.94%)	(99.88%)	(99.60%)	(97.50%)		(90.04%)	(90.04%)	(90.04%)	(90.02%)

Table 24 Performance of the BPNN and the Normal Approximation Techniques for Experimental Subset: p_1=0.1, p_2=0.1, Sample Sizes=30, Correlation Coefficient=0.5

	Shift										
	$+3s_1, +3s_2$	$+3s_1, +2s_2$	$+3s_1, +1s_2$	$+2s_1, +2s_2$	$+2s_1, +1s_2$	$+1s_1, +1s_2$	NoShift_NoShift	$-3s_1, -3s_2$	$-3s_1, -2s_2$	$-2s_1, -2s_2$	$-1s_1, -1s_2$
samples/replication	500	500	1000	500	1000	1000	10000	500	500	1000	2000
Replication	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000
Shifts Detected (Reps.)											
Normal Approx.	4000	4000	4000	4000	4000	4000	4000	0	0	0	603
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(0%)	(0%)	(0%)	(15.08%)
BPNN	4000	4000	4000	4000	4000	4000	3993	3073	3073	3073	3077
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(99.83%)	(76.83%)	(76.83%)	(76.83%)	(76.93%)
ARL											
Normal Approx.	1.83	2.29	2.39	3.81	5.45	16.40	189.52	No detection	No detection	No detection	964.34
BPNN	1.84	2.56	4.36	4.09	8.29	19.59	225.97	1.04	1.04	1.04	9.31
ARL variance											
Normal Approx.	1.77	3.46	3.88	12.17	30.53	352.54	58980.00	No detection	No detection	No detection	342070.00
BPNN	1.71	4.93	25.44	14.88	115.37	548.03	321600.00	0.04	0.04	0.04	1484.30

Table 25 Performance of the BPNN Technique and the MNP Chart for Experimental Subset: p_1=0.1, p_2=0.1, Sample Sizes=30, Correlation Coefficient=0.5

	Shift										
	$+3s_1, +3s_2$	$+3s_1, +2s_2$	$+3s_1, +1s_2$	$+2s_1, +2s_2$	$+2s_1, +1s_2$	$+1s_1, +1s_2$	NoShift_NoShift	$-3s_1, -3s_2$	$-3s_1, -2s_2$	$-2s_1, -2s_2$	$-1s_1, -1s_2$
samples/replication	500	500	1000	500	1000	1000	10000	500	500	1000	2000
Replication	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000
Shifts Detected (Reps.)											
MNP chart	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)
BPNN	4000	4000	4000	4000	4000	4000	4000	3798	3798	3798	3800
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(94.95%)	(94.95%)	(94.95%)	(95%)
ARL											
MNP chart	1.65	2.23	3.45	3.40	6.31	14.48	72.76	1.04	1.04	1.04	8.02
BPNN	1.61	2.16	3.27	3.22	5.82	12.76	84.62	1.04	1.04	1.04	8.25
ARL variance											
MNP chart	1.13	3.00	10.60	8.52	40.88	233.35	5483.70	0.05	0.05	0.05	53.69
BPNN	1.12	3.01	9.23	7.66	36.88	190.24	34337.00	0.05	0.05	0.05	824.35
Correct Classification of Shift Direction (Reps.)											
MNP chart	4000	4000	4000	3999	3997	3960	n/a	4000	4000	4000	4000
(%)	(100%)	(100%)	(100%)	(99.98%)	(99.93%)	(99%)		(100%)	(100%)	(100%)	(100%)
BPNN	4000	4000	4000	3999	3998	3958	n/a	3798	3798	3798	3797
(%)	(100%)	(100%)	(100%)	(99.98%)	(99.95%)	(98.95%)		(94.95%)	(94.95%)	(94.95%)	(94.93%)

Table 26 Performance of the BPNN and the Normal Approximation Techniques for Experimental Subset: p_1=0.1, p_2=0.1, Sample Sizes=30, Correlation Coefficient=0.2

	Shift										
	$+3s_1, +3s_2$	$+3s_1, +2s_2$	$+3s_1, +1s_2$	$+2s_1, +2s_2$	$+2s_1, +1s_2$	$+1s_1, +1s_2$	NoShift_NoShift	$-3s_1, -3s_2$	$-3s_1, -2s_2$	$-2s_1, -2s_2$	$-1s_1, -1s_2$
samples/replication	500	500	1000	500	1000	1000	10000	500	500	500	2000
Replication	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000
Shifts Detected (Reps.)											
Normal Approx.	4000	4000	4000	4000	4000	4000	4000	0	0	0	369
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(0%)	(0%)	(0%)	(9.23%)
BPNN	4000	4000	4000	4000	4000	3999	3997	3851	3851	3851	3852
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(99.98%)	(99.93%)	(96.28%)	(96.28%)	(96.28%)	(96.30%)
ARL											
Normal Approx.	1.54	1.98	2.42	3.24	5.16	13.92	194.31	No detection	No detection	No detection	976.94
BPNN	1.55	2.13	3.40	3.35	6.74	17.38	198.61	1.06	1.06	1.06	12.10
ARL variance											
Normal Approx.	0.93	2.21	3.64	8.15	25.06	236.79	56485.23	No detection	No detection	No detection	318910.00
BPNN	1.00	3.05	10.24	9.95	93.28	495.13	243160.00	0.06	0.06	0.06	143.19

Table 27 Performance of the BPNN Technique and the MNP Chart for Experimental Subset: p_1=0.1, p_2=0.1, Sample Sizes=30, Correlation Coefficient=0.2

	Shift										
	$+3s_1, +3s_2$	$+3s_1, +2s_2$	$+3s_1, +1s_2$	$+2s_1, +2s_2$	$+2s_1, +1s_2$	$+1s_1, +1s_2$	NoShift_NoShift	$-3s_1, -3s_2$	$-3s_1, -2s_2$	$-2s_1, -2s_2$	$-1s_1, -1s_2$
samples/replication	500	500	1000	500	1000	1000	10000	500	500	500	2000
Replication	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000
Shifts Detected (Reps.)											
MNP chart	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)
BPNN	4000	4000	4000	4000	4000	4000	4000	3958	3958	3958	3958
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(98.95%)	(98.95%)	(98.95%)	(98.95%)
ARL											
MNP chart	1.40	1.84	2.83	2.78	5.16	12.65	124.22	1.06	1.06	1.06	12.31
BPNN	1.45	1.93	2.99	2.96	5.58	13.34	137.15	1.05	1.05	1.05	11.43
ARL variance											
MNP chart	0.57	1.66	5.50	5.12	23.55	190.19	17205.77	0.06	0.06	0.06	138.06
BPNN	0.74	2.23	7.20	6.60	33.51	225.72	70087.76	0.05	0.05	0.05	130.56
Correct Classification of Shift Direction (Reps.)											
MNP chart	4000	4000	4000	4000	4000	3991	n/a	4000	4000	4000	4000
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(99.78%)		(100%)	(100%)	(100%)	(100%)
BPNN	4000	4000	4000	4000	4000	3990	n/a	3958	3958	3958	3958
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(99.75%)		(98.95%)	(98.95%)	(98.95%)	(98.95%)

7.2.2.1 Comparing the BPNN to the Normal Approximation Techniques

Table 22, Table 24, and Table 26 show for all the three correlation coefficients (0.8, 0.5, and 0.2, respectively) that BPNN technique outperformed the normal approximation technique when the process is in-control. In general, the BPNN technique will indicate a false alarm much later than the normal approximation technique (e.g. 4 to 36 samples later).

When the process is out of control and when both proportion nonconforming have shifted in the same direction with the same magnitude, performance of the two techniques depended largely on the extent and direction of the shifts. Specifically, for large and medium shifts (i.e. three or two standard deviations), BPNN and normal approximation techniques performed equally for shifts in the positive direction. However, when both proportion nonconforming shifted in the negative direction, the normal approximation technique could not detect any shifts; therefore, the BPNN outperformed the normal approximation technique. When both proportion nonconforming had shifted one standard deviation (small shifts) and in the positive direction, the BPNN and the normal approximation technique performed equally for strongly correlated processes. The normal approximation technique outperformed the BPNN technique for moderately and weakly correlated processes. When both proportion nonconforming had shifted one standard deviation and in the negative direction, the BPNN technique outperformed the normal approximation technique.

When the shifts had different magnitudes and were in the positive direction, performance of the two techniques depended on the magnitude of the shifts. Specifically, for a process with the proportion nonconforming of the first attribute shifted three or two standard deviations and the proportion nonconforming of the second attribute shifted one standard deviation, the normal approximation technique outperformed the BPNN technique for all levels of correlation. However, when the proportion nonconforming of the first attribute shifted three standard deviations and the proportion nonconforming of the second attribute shifted two standard deviations, the BPNN and the normal approximation techniques performed equally. When both proportion nonconforming shifted with different magnitudes and in the negative direction, the BPNN technique outperformed the normal approximation technique for all levels of correlation.

7.2.2.2 Comparing the BPNN Technique to the MNP Chart

Table 23, Table 25, and Table 27 show for all the three correlation coefficients (0.8, 0.5, and 0.2, respectively) that the BPNN technique outperformed the MNP chart when the process is in-control. In general, the BPNN technique will indicate a false alarm much later than the MNP chart (e.g. 12 to 19 samples later).

When the process is out of control and when both proportion nonconforming have shifted in the same direction with the same magnitude, performance of the two techniques depended largely on the extent and direction of the shifts. Specifically, for large shifts (i.e. three standard deviations), the BPNN technique and the MNP chart performed equally for shifts in the positive direction. When both proportion nonconforming had shifted two standard deviations (medium shifts) and in the positive direction, both techniques had the same average run length (ARL) for all levels of correlation coefficient. However, the BPNN technique identified the direction of shifts more correctly than the MNP chart for strongly correlated processes. This difference was considered insignificant. When both proportion nonconforming had shifted one standard deviation (small shifts) and in the positive direction, the BPNN technique outperformed the MNP chart for strongly correlated processes. The BPNN detected shifts faster than the MNP chart (the BPNN had smaller ARL than the MNP chart) for moderately correlated processes; however, the MNP chart identified the direction of shifts more correctly than the BPNN technique. This difference was considered insignificant. The two techniques had the same average run length for weakly correlated processes, but the MNP chart identified the direction of shifts more correctly than the BPNN technique. Again, this difference was considered insignificant. When both proportion nonconforming had shifted in the negative direction with any magnitudes, the MNP chart could indicate shifts additional replications over the BPNN technique. These differences were considered significant.

When the shifts had different magnitudes and were in the positive direction, the performance depended on the magnitude of the shifts. For strongly correlated processes, the BPNN technique outperformed the MNP chart when the proportion nonconforming of the first attribute shifted two standard deviations and the proportion nonconforming of the second attribute shifted one standard deviation. Both techniques performed equally for the other magnitudes of shifts. For moderately and weakly correlated processes, both techniques performed equally. When both proportion nonconforming shifted with different magnitudes and in the negative direction, the MNP chart could indicate shifts additional replications over the BPNN technique for all levels of correlation. These differences were considered significant.

7.2.2.3 Comparing the MNP Chart to the Normal Approximation Technique

Table 22 through Table 27 are all used to compare the MNP chart and the normal approximation techniques (Note: the MNP chart and the normal approximation techniques have different in-control ARL. To compare them with the BPNN technique, different cut-values are used for the BPNN technique) In general, the normal approximation technique will indicate a false alarm much later than the MNP chart (e.g. 70 to 122 samples later). The stronger the correlation coefficient, the smaller the in-control ARL for the MNP chart and the normal approximation technique.

When the process is out of control and when both proportion nonconforming have shifted in the same direction with the same magnitude, performance of the two techniques depended largely on the extent and direction of the shifts. Specifically, for large and medium shifts, the MNP chart and the normal approximation technique performed equally for shifts in the positive direction. However, when both proportion nonconforming shifted in the negative direction, the normal approximation technique could not detect any shift; therefore, the MNP chart outperformed the normal approximation technique. When both proportion nonconforming had shifted one standard deviation (small shifts) and in the positive direction, the MNP chart outperformed the normal approximation technique for all levels of correlation. When both proportion nonconforming had shifted one standard deviation and in the negative direction, the MNP chart outperformed the normal approximation technique.

When the magnitudes of the shifts were different, the level of correlation affected the results. For strongly correlated processes, the normal approximation technique outperformed the MNP chart when the proportion nonconforming of the first attribute shifted two standard deviations and the proportion nonconforming of the second attribute shifted one standard deviation and both proportion nonconforming shifted in the positive direction. Both techniques performed equally for other magnitudes of shifts. For moderately correlated processes, the normal approximation technique outperformed the MNP chart when the proportion nonconforming of the first attribute shifted three standard deviations and the proportion nonconforming of the second attribute shifted one standard deviation and both proportion nonconforming shifted in the positive direction. Both techniques performed equally for other magnitudes of shifts. For weakly correlated processes, both techniques performed equally when both proportion nonconforming shifted with different magnitudes and in the positive direction. When both proportion nonconforming shifted with different magnitudes and in the negative direction, the MNP chart outperformed the normal approximation technique for all levels of correlation.

7.2.3 p_1 = 0.01, p_2 = 0.01, Sample Sizes = 810, 670, and 540 (Levels of Correlation: 0.8, 0.5, and 0.2)

Table 28 through Table 30 present the ARLs for the normal approximation technique, the MNP chart, and the BPNN technique for three of the experimental subsets (proportion nonconforming p_1 = 0.01 and p_2 = 0.01; a sample size #2 = 810, 670, and 540 respectively; and levels of correlation = 0.80, 0.50, and 0.20, respectively).

Table 28 Performance of the Three Techniques for Experimental Subset: p_1=0.01, p_2=0.01, Sample Sizes=810, Correlation Coefficient=0.8

	Shift									
	$+3s_1, +3s_2$	$+3s_1, +2s_2$	$+3s_1, +1s_2$	$+2s_1, +2s_2$	$+2s_1, +1s_2$	$+1s_1, +1s_2$	NoShift_NoShift	$-3s_1, -3s_2$	$-2s_1, -2s_2$	$-1s_1, -1s_2$
samples/replication	500	500	1000	500	1000	1000	10000	500	500	2000
Replication	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
Shifts Detected (Reps.)										
Normal Approx.	2000	2000	2000	2000	2000	2000	2000	76	131	1098
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(3.80%)	(6.55%)	(54.90%)
MNP chart	2000	2000	2000	2000	2000	2000	2000	2000	2000	1928
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(96.40%)
BPNN	2000	2000	2000	2000	2000	2000	2000	1996	1996	1996
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(99.80%)	(99.80%)	(99.80%)
ARL										
Normal Approx.	2.23	2.31	1.68	5.08	4.93	23.24	234.89	38.33	130.07	758.79
MNP chart	1.94	2.67	4.31	4.29	8.25	19.80	287.45	2.61	18.17	495.58
BPNN	2.04	2.86	4.86	4.66	9.46	22.31	292.69	1.52	5.64	94.52
ARL variance										
Normal Approx.	3.33	3.45	1.35	25.07	23.03	684.42	85589.00	11763.00	22568.00	326370.00
MNP chart	1.81	4.72	14.83	14.51	68.03	419.83	98453.00	4.34	315.31	187270.00
BPNN	2.17	6.08	22.22	18.74	105.34	622.81	141450.00	1.01	44.17	21310.00
Correct Classification of Shift Direction (Reps.)										
MNP chart	2000	2000	2000	2000	2000	2000	n/a	2000	2000	1896
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)		(100%)	(100%)	(94.80%)
BPNN	2000	2000	2000	2000	2000	2000	n/a	1996	1996	1990
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)		(99.80%)	(99.80%)	(99.50%)

Table 29 Performance of the Three Techniques for Experimental Subset: p_1=0.01, p_2=0.01, Sample Sizes=670, Correlation Coefficient=0.5

	Shift											
	$+3s_1, +3s_2$	$+3s_1, +2s_2$	$+3s_1, +1s_2$	$+2s_1, +2s_2$	$+2s_1, +1s_2$	$+1s_1, +1s_2$	NoShift_NoShift	$-3s_1, -3s_2$	$-3s_1, -2s_2$	$-2s_1, -2s_2$	$-2s_1, -1s_2$	$-1s_1, -1s_2$
samples/replication	500	500	1000	500	1000	1000	10000	500	500	500	1000	2000
Replication	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
Shifts Detected (Reps.)												
Normal Approx.	2000	2000	2000	2000	2000	2000	2000	44	51	46	1442	532
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(2.20%)	(2.55%)	(2.30%)	(72.10%)	(26.60%)
MNP chart	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	1967
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(98.35%)
BPNN	2000	2000	2000	2000	2000	2000	2000	1996	1996	1996	1996	1996
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(99.80%)	(99.80%)	(99.80%)	(99.80%)	(99.80%)
ARL												
Normal Approx.	1.76	2.26	2.50	4.01	5.97	18.72	238.16	2.91	57.02	18.46	299.86	851.87
MNP chart	1.62	2.26	3.49	3.54	6.56	16.21	270.15	2.72	5.62	10.01	86.49	458.48
BPNN	1.71	2.43	3.94	3.97	7.48	19.68	286.33	1.37	1.98	2.93	13.32	56.71
ARL variance												
Normal Approx.	1.29	3.02	4.22	12.38	37.36	430.14	96059.00	6.27	17526.22	2252.39	67841.92	314860.70
MNP chart	0.94	3.19	9.09	8.47	39.41	270.16	92361.00	4.80	25.68	86.20	6961.64	175513.90
BPNN	1.17	3.71	13.67	11.52	56.10	536.28	185050.00	0.64	2.58	8.66	369.94	7649.75
Correct Classification of Shift Direction (Reps.)												
MNP chart	2000	2000	2000	2000	2000	2000	n/a	2000	2000	2000	2000	1952
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)		(100%)	(100%)	(100%)	(100%)	(97.60%)
BPNN	2000	2000	2000	2000	2000	1999	n/a	1996	1996	1996	1996	1995
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(99.95%)		(99.80%)	(99.80%)	(99.80%)	(99.80%)	(99.75%)

Table 30 Performance of the Three Techniques for Experimental Subset: p_1=0.01, p_2=0.01, Sample Sizes=540, Correlation Coefficient=0.2

	Shift												
	$+3s_1, +3s_2$	$+3s_1, +2s_2$	$+3s_1, +1s_2$	$+2s_1, +2s_2$	$+2s_1, +1s_2$	$+1s_1, +1s_2$	NoShift_NoShift	$-3s_1, -3s_2$	$-3s_1, -2s_2$	$-3s_1, -1s_2$	$-2s_1, -2s_2$	$-2s_1, -1s_2$	$-1s_1, -1s_2$
samples/replication	500	500	1000	500	1000	1000	10000	500	500	1000	500	1000	2000
Replication	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
Shifts Detected (Reps.)													
Normal Approx.	2000	2000	2000	2000	2000	2000	2000	62	62	565	62	536	283
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(3.10%)	(3.10%)	(28.25%)	(3.10%)	(26.80%)	(14.15%)
MNP chart	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)
BPNN	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)
ARL													
Normal Approx.	1.56	2.00	2.46	3.27	5.28	13.89	213.88	3.32	3.24	371.12	3.65	412.50	844.36
MNP chart	1.45	1.83	2.74	2.81	5.16	12.57	246.23	2.59	3.28	29.03	3.96	34.18	252.39
BPNN	1.54	2.05	3.36	3.28	6.44	16.19	272.74	1.27	1.42	5.16	1.53	6.14	29.56
ARL variance													
Normal Approx.	0.88	2.12	4.23	8.44	28.96	204.15	79319.00	7.63	6.35	85343.12	8.13	90815.05	377905.66
MNP chart	0.68	1.57	4.93	5.19	24.05	171.97	91334.00	4.15	7.75	840.76	11.93	1183.90	66304.30
BPNN	0.90	2.48	11.38	8.24	49.43	311.52	167652.00	0.36	0.82	34.35	1.18	58.66	2016.18
Correct Classification of Shift Direction (Reps.)													
MNP chart	2000	2000	2000	2000	2000	2000	n/a	2000	2000	2000	2000	2000	1988
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)		(100%)	(100%)	(100%)	(100%)	(100%)	(99.40%)
BPNN	2000	2000	2000	2000	2000	1999	n/a	2000	2000	2000	2000	2000	1999
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(99.95%)		(100%)	(100%)	(100%)	(100%)	(100%)	(99.95%)

7.2.3.1 Comparing the BPNN to the Normal Approximation Techniques

Table 28 through Table 30 show for all the three correlation coefficients (0.8, 0.5, and 0.2) that the BPNN technique outperformed the normal approximation technique when the process is in-control. In general, the BPNN technique will indicate a false alarm much later than the normal approximation technique (e.g. 48 to 59 samples later).

When the process is out of control and when both proportion nonconforming have shifted in the same direction with the same magnitude, performance of the two techniques depended largely on the extent and direction of the shifts. Specifically, for large or medium shifts (i.e. three or two standard deviations), BPNN and normal approximation techniques performed equally for shifts in the positive direction. However, when both proportion nonconforming shifted in the negative direction, the BPNN technique outperformed the normal approximation technique. When both proportion nonconforming had shifted one standard deviation (small shifts) and in the positive direction, both techniques performed equally for strongly correlated processes. The normal approximation technique outperformed the BPNN technique for moderately and weakly correlated processes. When both proportion nonconforming had shifted one standard deviation and in the negative direction, the BPNN technique outperformed the normal approximation technique.

When the shifts had different magnitudes and were in the positive direction, performance of the two techniques depended on the magnitude of the shifts. Specifically, for a process with the proportion nonconforming of the first attribute shifted three or two standard deviations and the proportion nonconforming of the second attribute shifted one standard deviation, the normal approximation technique outperformed the BPNN technique for all levels of correlation. However, when the proportion nonconforming of the first attribute shifted three standard deviations and the proportion nonconforming of the second attribute shifted two standard deviations, the BPNN and the normal approximation techniques performed equally. When both proportion nonconforming shifted with different magnitudes and in the negative direction, the BPNN technique outperformed the normal approximation technique for all levels of correlation.

7.2.3.2 Comparing the BPNN Technique to the MNP Chart

Table 28 through Table 30 show for all the three correlation coefficients (0.8, 0.5, and 0.2) that the BPNN technique outperformed the MNP chart when the process is in-control. In general, the BPNN technique will indicate a false alarm much later than the MNP chart (e.g. 5 to 26 samples later).

When the process is out of control and when both proportion nonconforming have shifted in the same direction with the same magnitude, performance of the two techniques depended largely on the extent and direction of the shifts. Specifically, for large and medium shifts, the BPNN technique and the MNP chart performed equally for shifts in the positive direction. When both proportion nonconforming shifted in the negative direction with large magnitude, the BPNN technique detected shifts faster than the MNP chart (the BPNN technique had smaller ARL than the MNP chart) for all levels of correlation coefficient. However, for strongly and moderately correlated processes (Table 28 and Table 29) the BPNN technique could not indicate the shifts for four of the 2000 replications while the MNP chart indicated shifts for all replications. If one assumed that the BPNN technique detected the shifts for those four replications, one might conclude that the MNP chart performed either better than or equal to the BPNN technique.

When both proportion nonconforming shifted in the negative direction with medium magnitude, the BPNN technique detected shifts faster than the MNP chart (the BPNN technique had smaller ARL than the MNP chart) for all levels of correlation coefficient. However, for strongly and moderately correlated processes the BPNN technique could not indicate the shifts for four of the 2000 replications while the MNP chart indicated shifts for all replications. If one assumed that the BPNN technique detected the shifts for those four replications, one would still conclude that the BPNN technique outperformed the MNP chart. When both proportion nonconforming had shifted one standard deviation (small shifts) and in the positive direction, the MNP chart outperformed the BPNN for all levels of correlation coefficients. In contrary, the BPNN technique outperformed the MNP chart when both proportion nonconforming had shifted in the negative direction.

When the shifts had different magnitudes and were in the positive direction, the performance depended on the level of correlation coefficient and the magnitudes of the shifts. For strongly and weakly correlated processes, the MNP chart outperformed the BPNN technique when the proportion nonconforming of the first attribute shifted two standard deviations and the proportion nonconforming of the second attribute shifted one standard deviation. Both techniques performed equally for the other magnitudes of shifts. For moderately correlated processes, both techniques performed equally. When both proportion nonconforming shifted with different magnitudes and in the negative direction, the BPNN technique detected shifts faster than the MNP chart (the BPNN had smaller ARL than the MNP chart) for moderately correlated processes. However, the BPNN technique could not indicate the shifts for four of the 2000 replications while the MNP chart indicate shifts for all 2000 replications. If one assumed that the BPNN technique detected the shifts for those four replications, one would conclude that the BPNN technique outperformed the MNP chart. For weakly correlated processes, the BPNN outperformed the MNP chart.

7.2.3.3 Comparing the MNP Chart to the Normal Approximation Technique

Table 28 through Table 30 show for all the three correlation coefficients (0.8, 0.5, and 0.2) that the MNP chart will indicate a false alarm much later than the normal approximation technique (e.g. 32 to 53 samples later).

When the process is out of control and when both proportion nonconforming have shifted in the same direction with the same magnitude, performance of the two techniques depended largely on the extent and direction of the shifts. Specifically, for large and medium shifts, the MNP chart and the normal approximation technique performed equally for shifts in the positive direction. However, when both proportion nonconforming shifted in the negative direction, the MNP chart outperformed the normal approximation technique. When both proportion nonconforming had shifted one standard deviation (small shifts) and in the positive direction, the MNP chart outperformed the normal approximation technique for all levels of correlation. When both proportion nonconforming had shifted one standard deviation and in the negative direction, the MNP chart outperformed the normal approximation technique.

When the shifts had different magnitudes and were in the positive direction, the level of correlation affected the results. For strongly correlated processes, the normal approximation technique outperformed the MNP chart when the proportion nonconforming of the first attribute shifted three or two standard deviations and the proportion nonconforming of the second attribute shifted one standard deviation. Both techniques performed equally for other magnitudes of shifts. For moderately correlated processes, the normal approximation technique outperformed the MNP chart when the proportion nonconforming of the first attribute shifted three standard deviations and the proportion nonconforming of the second attribute shifted one standard deviation. Both techniques performed equally for other magnitudes of shifts. For weakly correlated processes, the normal approximation technique and the MNP chart performed equally when both proportion nonconforming shifted with different magnitudes. When both proportion nonconforming shifted with different magnitudes and in the negative direction, the MNP chart detected shifts faster than the normal approximation technique for all levels of correlation.

7.3 Satisfying the Condition of Finding at Least One Nonconforming Item in a Sample

The following three sub-sections presented for varied proportion nonconforming the pair-wise comparisons of the three techniques for the sample size that satisfies the condition of finding at least one nonconforming item in a sample.

7.3.1 $p_1 = 0.3$, $p_2 = 0.3$, Sample Sizes = 10 (Levels of Correlation: 0.8, 0.5, and 0.2)

For a process with two attributes each with proportion non-conforming of 0.3, the sample size calculated to satisfy the condition of finding at least one nonconforming item in a sample was the same as the sample size recommended for the MNP chart. Thus, this sub-experiment is a duplication of the sub-experiment in section 7.2.1. See results in section 7.2.1.

7.3.2 $p_1 = 0.1$, $p_2 = 0.1$, Sample Sizes = 30 (Levels of Correlation: 0.8, 0.5, and 0.2)

For a process with two attributes each with proportion nonconforming of 0.1, the sample size calculated to satisfy the condition of finding at least one nonconforming item in a sample was the same as the sample size recommended for the MNP chart. Thus, this sub-experiment is a duplication of the sub-experiment in section 7.2.2. See results in section 7.2.2.

7.3.3 $p_1 = 0.01$, $p_2 = 0.01$, Sample Sizes = 300 (Levels of Correlation: 0.8, 0.5, and 0.2)

Table 31 through Table 36 present the ARLs for the normal approximation technique, the MNP chart, and the BPNN technique for three of the experimental subsets (proportion nonconforming p_1 = 0.01 and p_2 = 0.01; a sample size #3 = 300; and levels of correlation = 0.80, 0.50, and 0.20, respectively).

The results showed that the in-control ARL for the normal approximation technique and the MNP chart were substantially different. To compare the BPNN technique to the normal approximation technique and the MNP chart, two different cut-off values were used for the BPNN technique in order to have the in-control ARL comparable to the ARL for each technique. As a result, there were two tables for an experimental subset; the first table showed the comparison between the BPNN technique and the normal approximation technique and the second table showed the comparison between the BPNN technique and the MNP chart.

Table 31 Performance of the BPNN and the Normal Approximation Techniques for Experimental Subset: p_1=0.01, p_2=0.01, Sample Sizes=300, Correlation Coefficient=0.8

	Shift									
	$+3s_1, +3s_2$	$+3s_1, +2s_2$	$+2s_1, +2s_2$	$+2s_1, +1s_2$	$+1s_1, +1s_2$	NoShift_NoShift	$-3s_1, -3s_2$	$-3s_1, -2s_2$	$-2s_1, -2s_2$	$-1s_1, -1s_2$
samples/replication	500	500	500	1000	1000	10000	500	500	500	2000
Replication	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000
Shifts Detected (Reps.)										
Normal Approx.	10000	10000	10000	10000	10000	10000	9	9	9	3344
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(0.09%)	(0.09%)	(0.09%)	(33.44%)
BPNN	10000	10000	10000	10000	9999	9987	6844	6844	6844	6921
(%)	(100%)	(100%)	(100%)	(100%)	(99.99%)	(99.87%)	(68.44%)	(68.44%)	(68.44%)	(69.21%)
ARL										
Normal Approx.	2.06	2.26	4.19	4.18	14.72	135.62	271.78	271.78	271.78	894.43
BPNN	1.99	2.69	4.09	7.33	15.90	146.04	1.44	1.44	1.44	16.08
ARL variance										
Normal Approx.	2.29	3.14	15.56	15.20	260.42	30108.41	18016.19	18016.19	18016.19	334974.31
BPNN	2.45	5.88	18.15	81.45	585.14	194096.79	0.63	0.63	0.63	14872.98

Table 32 Performance of the BPNN Technique and the MNP Chart for Experimental Subset: p_1=0.01, p_2=0.01, Sample Sizes=300, Correlation Coefficient=0.8

	Shift									
	$+3s_1, +3s_2$	$+3s_1, +2s_2$	$+2s_1, +2s_2$	$+2s_1, +1s_2$	$+1s_1, +1s_2$	NoShift_NoShift	$-3s_1, -3s_2$	$-3s_1, -2s_2$	$-2s_1, -2s_2$	$-1s_1, -1s_2$
samples/replication	500	500	500	1000	1000	10000	500	500	500	2000
Replication	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000
Shifts Detected (Reps.)										
MNP chart	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)
BPNN	10000	10000	10000	10000	9999	9999	9625	9625	9625	9644
(%)	(100%)	(100%)	(100%)	(100%)	(99.99%)	(99.99%)	(96.25%)	(96.25%)	(96.25%)	(96.44%)
ARL										
MNP chart	1.95	2.60	3.92	6.78	14.00	30.67	1.45	1.45	1.45	4.71
BPNN	1.58	2.00	2.72	4.40	7.88	34.32	1.42	1.42	1.42	6.77
ARL variance										
MNP chart	1.95	4.26	12.28	40.82	213.99	902.68	0.63	0.63	0.63	17.57
BPNN	1.00	2.25	5.83	23.13	83.31	21711.67	0.60	0.60	0.60	3464.08
Correct Classification of Shift Direction (Reps.)										
MNP chart	10000	9995	9979	9919	9506	n/a	10000	10000	10000	10000
(%)	(100%)	(99.95%)	(99.79%)	(99.19%)	(95.06%)		(100%)	(100%)	(100%)	(100%)
BPNN	10000	9996	9974	9928	9661	n/a	9625	9625	9625	9618
(%)	(100%)	(99.96%)	(99.74%)	(99.28%)	(96.61%)		(96.25%)	(96.25%)	(96.25%)	(96.18%)

Table 33 Performance of the BPNN and the Normal Approximation Techniques for Experimental Subset: p_1=0.01, p_2=0.01, Sample Sizes=300, Correlation Coefficient=0.5

	Shift										
	$+3s_1, +3s_2$	$+3s_1, +2s_2$	$+3s_1, +1s_2$	$+2s_1, +2s_2$	$+2s_1, +1s_2$	$+1s_1, +1s_2$	NoShift_NoShift	$-3s_1, -3s_2$	$-3s_1, -2s_2$	$-2s_1, -2s_2$	$-1s_1, -1s_2$
samples/replication	500	500	1000	500	1000	1000	10000	500	500	500	2000
Replication	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000
Shifts Detected (Reps.)											
Normal Approx.	10000	10000	10000	10000	10000	10000	10000	0	0	0	1108
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(0%)	(0%)	(0%)	(11.08%)
BPNN	10000	10000	10000	10000	10000	10000	9993	8051	8051	8051	8064
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(99.93%)	(80.51%)	(80.51%)	(80.51%)	(80.64%)
ARL											
Normal Approx.	1.81	2.25	2.36	3.64	5.09	13.53	158.68	No detection	No detection	No detection	927.89
BPNN	1.89	2.68	4.16	4.11	7.79	18.03	169.25	1.60	1.60	1.60	7.99
ARL variance											
Normal Approx.	1.58	3.11	3.43	11.26	24.01	221.55	39388.00	No detection	No detection	No detection	345366.26
BPNN	1.81	5.35	17.23	15.59	72.99	499.85	210800.00	0.93	0.93	0.93	1762.37

Table 34 Performance of the BPNN Technique and the MNP Chart for Experimental Subset: p_1=0.01, p_2=0.01, Sample Sizes=300, Correlation Coefficient=0.5

	Shift										
	$+3s_1, +3s_2$	$+3s_1, +2s_2$	$+3s_1, +1s_2$	$+2s_1, +2s_2$	$+2s_1, +1s_2$	$+1s_1, +1s_2$	NoShift_NoShift	$-3s_1, -3s_2$	$-3s_1, -2s_2$	$-2s_1, -2s_2$	$-1s_1, -1s_2$
samples/replication	500	500	1000	5000	1000	1000	10000	500	500	500	2000
Replication	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000
Shifts Detected (Reps.)											
MNP chart	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)
BPNN	10000	10000	10000	10000	10000	10000	10000	9719	9719	9719	9722
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(97.19%)	(97.19%)	(97.19%)	(97.22%)
ARL											
MNP chart	1.71	2.29	3.34	3.38	5.95	12.86	60.05	1.61	1.61	1.61	6.73
BPNN	1.65	2.15	3.13	3.12	5.42	11.23	60.96	1.57	1.57	1.57	6.56
ARL variance											
MNP chart	1.24	3.17	8.30	8.76	30.69	183.41	3663.40	0.94	0.94	0.94	38.75
BPNN	1.10	2.72	7.74	7.72	28.88	149.29	19876.00	0.89	0.89	0.89	433.54
Correct Classification of Shift Direction (Reps.)											
MNP chart	10000	10000	9998	9999	9988	9879	n/a	10000	10000	10000	10000
(%)	(100%)	(100%)	(99.98%)	(99.99%)	(99.88%)	(98.79%)		(100%)	(100%)	(100%)	(100%)
BPNN	10000	10000	9997	9998	9983	9884	n/a	9719	9719	9719	9717
(%)	(100%)	(100%)	(99.97%)	(99.98%)	(99.83%)	(98.84%)		(97.19%)	(97.19%)	(97.19%)	(97.17%)

Table 35 Performance of the BPNN and the Normal Approximation Techniques for Experimental Subset: p_1=0.01, p_2=0.01, Sample Sizes=300, Correlation Coefficient=0.2

	Shift												
	$+3s_1, +3s_2$	$+3s_1, +2s_2$	$+3s_1, +1s_2$	$+2s_1, +2s_2$	$+2s_1, +1s_2$	$+1s_1, +1s_2$	NoShift_NoShift	$-3s_1, -3s_2$	$-3s_1, -2s_2$	$-3s_1, -1s_2$	$-2s_1, -2s_2$	$-2s_1, -1s_2$	$-1s_1, -1s_2$
samples/replication	500	500	1000	5000	1000	1000	10000	500	500	1000	500	1000	2000
Replication	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000
Shifts Detected (Reps.)													
Normal Approx.	10000	10000	10000	10000	10000	10000	10000	0	0	475	0	475	629
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(0%)	(0%)	(4.75%)	(0%)	(4.75%)	(6.29%)
BPNN	10000	10000	10000	10000	10000	10000	9997	9639	9639	9639	9639	9639	9639
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(99.97%)	(96.39%)	(96.39%)	(96.39%)	(96.39%)	(96.39%)	(96.39%)
ARL													
Normal Approx.	1.56	1.98	2.44	3.08	4.79	12.05	165.77	No detection	No detection	490.51	No detection	490.51	1008.48
BPNN	1.67	2.30	3.73	3.68	7.18	18.16	168.78	1.68	1.68	4.11	1.68	4.11	9.47
ARL variance													
Normal Approx.	0.95	2.14	3.99	7.01	20.41	166.86	40085.00	No detection	No detection	87023.52	0.00	87023.52	326596.72
BPNN	1.22	3.53	11.77	11.40	57.08	451.90	128230.00	1.20	1.20	13.14	1.20	13.14	84.19

Table 36 Performance of the BPNN Technique and the MNP Chart for Experimental Subset: p_1=0.01, p_2=0.01, Sample Sizes=300, Correlation Coefficient=0.2

	Shift												
	$+3s_1, +3s_2$	$+3s_1, +2s_2$	$+3s_1, +1s_2$	$+2s_1, +2s_2$	$+2s_1, +1s_2$	$+1s_1, +1s_2$	NoShift_NoShift	$-3s_1, -3s_2$	$-3s_1, -2s_2$	$-3s_1, -1s_2$	$-2s_1, -2s_2$	$-2s_1, -1s_2$	$-1s_1, -1s_2$
samples/replication	500	500	1000	5000	1000	1000	10000	500	500	1000	500	1000	2000
Replication	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000
Shifts Detected (Reps.)													
MNP chart	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)
BPNN	10000	10000	10000	10000	10000	10000	9999	9923	9923	9923	9923	9923	9923
(%)	(100%)	(100%)	(100%)	(100%)	(100%)	(100%)	(99.99%)	(99.23%)	(99.23%)	(99.23%)	(99.23%)	(99.23%)	(99.23%)
ARL													
MNP chart	1.46	1.87	2.79	2.79	4.92	11.40	103.67	1.72	1.72	4.25	1.72	4.25	9.96
BPNN	1.52	1.99	3.09	3.05	5.56	13.00	108.93	1.62	1.62	3.81	1.62	3.81	8.57
ARL variance													
MNP chart	0.71	1.77	5.23	5.13	21.34	141.29	12108.00	1.26	1.26	13.59	1.26	13.59	88.62
BPNN	0.87	2.22	7.60	7.35	30.71	211.04	29860.00	1.07	1.07	12.31	1.07	12.31	74.91
Correct Classification of Shift Direction (Reps.)													
MNP chart	10000	10000	10000	10000	9994	9982	n/a	10000	10000	10000	10000	10000	10000
(%)	(100%)	(100%)	(100%)	(100%)	(99.94%)	(99.82%)		(100%)	(100%)	(100%)	(100%)	(100%)	(100%)
BPNN	10000	9999	9999	10000	9993	9961	n/a	9923	9923	9923	9923	9923	9923
(%)	(100%)	(99.99%)	(99.99%)	(100%)	(99.93%)	(99.61%)		(99.23%)	(99.23%)	(99.23%)	(99.23%)	(99.23%)	(99.23%)

7.3.3.1 Comparing BPNN to the Normal Approximation Techniques

Table 31, Table 33, and Table 35 show for all the three correlation coefficients (0.8, 0.5, and 0.2) that BPNN technique outperformed the normal approximation technique when the process is in-control. In general, the BPNN technique will indicate a false alarm much later than the normal approximation technique (e.g. 3 to 11 samples later).

When the process is out of control and when both proportion nonconforming have shifted in the same direction with the same magnitude, performance of the two techniques depended largely on the extent and direction of the shifts. Specifically, for large or medium shifts (i.e. three or two standard deviations), BPNN and normal approximation techniques performed equally for shifts in the positive direction. However, when both proportion nonconforming shifted in the negative direction, the BPNN technique outperformed the normal approximation technique (the normal approximation technique could not detect any shift). When both proportion nonconforming had shifted one standard deviation (small shifts) and in the positive direction, the normal approximation outperformed the BPNN technique for all levels of correlation coefficient. When both proportion nonconforming had shifted one standard deviation and in the negative direction, the BPNN technique outperformed the normal approximation technique.

When the shifts had different magnitudes and they were in the positive direction, performance of the two techniques depended on the magnitude of shifts. Specifically, for a process with the proportion nonconforming of the first attribute shifted three or two standard deviations and the proportion nonconforming of the second attribute shifted one standard deviation, the normal approximation technique outperformed the BPNN technique for all levels of correlation. However, when the proportion nonconforming of the first attribute shifted three standard deviations and the proportion nonconforming of the second attribute shifted two standard deviations, the BPNN and the normal approximation technique performed equally. When both proportion nonconforming shifted with different magnitudes and in the negative direction, the BPNN technique outperformed the normal approximation technique for all levels of correlation.

7.3.3.2 Comparing BPNN to the MNP Chart

Table 32, Table 34, and Table 36 show for all the three correlation coefficients (0.8, 0.5, and 0.2) that the BPNN technique outperformed the MNP chart when the process is in-control. In general, the BPNN technique will indicate a false alarm much later than the MNP chart (e.g. 4 to 5 samples later).

When the process is out of control and when both proportion nonconforming have shifted in the same direction with the same magnitude, performance of the two techniques depended largely on the extent and direction of the shifts. Specifically, for large shifts, the BPNN technique and the MNP chart performed equally for shifts in the positive direction. When both proportion nonconforming shifted in the negative direction, both techniques had the same ARL for all levels of correlation coefficients. However, the BPNN technique could not indicate the shifts for all of the replications while the MNP chart could. If one assumed that the BPNN technique detected the shifts for those replications, one would conclude that the MNP chart outperformed the BPNN technique. For processes with medium shifts and both proportion nonconforming shifted in the positive direction, the BPNN technique detected shifts faster than the MNP chart (the

BPNN had smaller ARL than the MNP chart) for strongly correlated processes. However, the MNP chart identified the direction of shifts more correctly than the BPNN technique. This difference was considered insignificant. For processes with medium shifts and both proportion nonconforming shifted in the positive direction, the BPNN had the same ARL as the MNP chart for moderately and weakly correlated processes. However, the MNP chart identified the direction of shifts more correctly than the BPNN technique for moderately correlated processes. Again, this difference was considered insignificant. When both proportion nonconforming shifted in the negative direction, both techniques had the same ARL for all levels of correlation coefficients. However, the BPNN technique could not indicate the shifts for all of the replications while the MNP chart could. If one assumed that the BPNN technique detected the shifts for those replications, one would conclude that the MNP chart outperformed the BPNN technique. When both proportion nonconforming had shifted one standard deviation (small shifts) and in the positive direction, the BPNN technique outperformed the MNP chart for all strongly and moderately correlated processes. In contrary, the MNP chart outperformed the BPNN for weakly correlated processes. When both proportion nonconforming had shifted in the negative direction, the MNP chart outperformed the BPNN for strongly correlated processes. For moderately and weakly correlated processes, the BPNN technique had ARL equal to and smaller than the MNP chart respectively. However, the BPNN technique could not indicate the shifts for all of the replications while the MNP chart could. If one assumed that the BPNN technique detected the shifts for those replications, one would conclude that the MNP chart outperformed the BPNN technique.

When the shifts had different magnitudes and were shifted in the positive direction, the performance depended on the level of correlation coefficient and the magnitude of the shifts. For strongly correlated processes, the BPNN technique outperformed the MNP chart when the proportion nonconforming of the first attribute shifted two standard deviations and the proportion nonconforming of the second attribute shifted one standard deviation. Both techniques performed equally for other magnitudes of shifts. For moderately correlated processes, both techniques had the same ARL for all magnitudes of shifts. However, the MNP chart identified the direction of shifts more correctly than the BPNN technique when the proportion nonconforming of the first attribute shifted three or two standard deviations and the proportion nonconforming of the second attribute shifted one standard deviation. These differences were considered insignificant. For weakly correlated processes, both techniques had equal ARL but the MNP chart could identify the direction of shifts more correctly than the BPNN technique. Again, these differences were considered insignificant.

When both proportion nonconforming shifted with different magnitudes and in the negative direction, the MNP chart outperformed the BPNN technique. Both techniques had equal ARL but the BPNN technique could not indicate the shifts for all of the replications while the MNP chart could. If one assumed that the BPNN technique detected the shifts for those replications, one would conclude that the MNP chart outperformed the BPNN technique.

7.3.3.3 Comparing the MNP Chart to the Normal Approximation Technique

Table 31 through Table 36 show for all the three correlation coefficients (0.8, 0.5, and 0.2) that the normal approximation technique outperformed the MNP chart when the process is in-control. In general, the normal approximation technique will indicate a false alarm much later than the MNP chart (e.g. 62 to 105 samples later).

When the process is out of control and when both proportion nonconforming have shifted in the same direction with the same magnitude, performance of the two techniques depended largely on the direction of the shifts. Specifically, for large, medium, and small shifts, the MNP chart and the normal approximation technique performed equally when both proportion nonconforming shifted in the positive direction. However, when both proportion nonconforming shifted in the negative direction, the MNP chart outperformed the normal approximation technique.

When the shifts had different magnitudes and were in the positive direction, the level of correlation affected the results. For strongly correlated processes, the normal approximation technique outperformed the MNP chart when the proportion nonconforming of the first attribute shifted two standard deviations and the proportion nonconforming of the second attribute shifted one standard deviation. Both techniques performed equally for other magnitudes of shift. For moderately correlated processes, the normal approximation technique outperformed the MNP chart when the proportion nonconforming of the first attribute shifted three standard deviations and the proportion nonconforming of the second attribute shifted one standard deviation. Both techniques performed equally for other magnitudes of shift. For weakly correlated processes, the normal approximation technique and the MNP chart performed equally when both proportion nonconforming shifted with any magnitude. When both proportion nonconforming shifted with different magnitudes and in the negative direction, the MNP chart detected shifts faster than the normal approximation technique for all levels of correlation.

8.0 RECCOMENDATION FOR IMPLEMENTATION

This chapter provides summary of the results and how they may be implemented for practical use. Section 8.1 discusses guidelines for selecting a suitable technique for a particular process condition. General performances for the three multi-attribute process control techniques are summarized in section 8.2. Finally, the interpretation of out-of-control signals is discussed in section 8.3.

The results for large sample sizes (sample size #1: estimating multivariate normally distributed variables from a multivariate binomial distribution) discussed in section 7.1 are summarized in Table 37 through Table 39.

Table 37 Comparisons of the BPNN and Normal Approximation Techniques for Large Sample Sizes.

Status of Process's Proportion Nonconforming	Sample Size # 1								
	p1=0.3,p2=0.3,n=50			p1=0.1,p2=0.1,n=100			p1=0.01,p2=0.01,n=910		
In-control	B>N			B>N			B>N		
Out-of-control with the same magnitude of shifts									
large shift in the positive direction	B=N			B=N			B=N		
large shift in the negative direction	B>N(S.C.), B=N (M.C.&W.C.)			B>N (S.C.&M.C.), B=N (W.C.)			B>N		
medium shift in the positive direction	B>N			B=N			B=N		
medium shift in the negative direction	B>N			B>N			B>N		
small shift in the positive direction	B>N			B>N			B>N		
small shift in the negative direction	B>N			B>N			B>N		
Out-of-control with different magnitude of shifts	Strong	Mod.	Weak	Strong	Mod.	Weak	Strong	Mod.	Weak
3s,2s	B=N	B=N	B=N	B=N	B=N	B=N	B=N	B=N	B=N
3s,1s	n/a	N>B	B=N	n/a	N>B	B=N	N>B	N>B	B=N
2s,1s	N>B	B=N	B>N	N>B	B=N	B=N	N>B	B=N	B=N
-3s,-2s	n/a	B>N	B=N	n/a	n/a	B>N	n/a	n/a	n/a
-3s,-1s	n/a	N>B	B=N	n/a	n/a	B>N	n/a	n/a	n/a
-2s,-1s	N>B	B>N	B>N	n/a	B>N	B>N	n/a	B>N	B>N

Table 38 Comparisons of the BPNN Technique and the MNP chart for Large Sample Sizes.

Status of Process's Proportion Nonconforming	Sample Size # 1								
	p1=0.3,p2=0.3,n=50			p1=0.1,p2=0.1,n=100			p1=0.01,p2=0.01,n=910		
In-control	B>M			B>M			B>M		
Out-of-control with the same magnitude of shifts									
large shift in the positive direction	B=M			B=M			B=M		
large shift in the negative direction	B=M			B=M (S.C.$^{\#}$&M.C.$^{\#}$ &W.C.)			M>B(S.C.)$^{@}$, M=or>B(M.C.)$^{+}$, B=M (W.C.)		
medium shift in the positive direction	B=M			B=M			B=M		
medium shift in the negative direction	B=M			B>M(S.C.)*,M>B(M.C.),B=M(W.C.)			B>M (S.C.)*,M=or>B (M.C.)$^{+}$, M>B(W.C.)		
small shift in the positive direction	M>B(S.C&M.C.&W.C*)			M>B(S.C.&M.C.), B=M(W.C.)			B=M (S.C.$^{\#}$&M.C.&W.C.)		
small shift in the negative direction	B>M(S.C.),M>B(M.C.&W.C.)*			B>M(S.C.)*,M>B(M.C.&W.C.)			B>M(S.C.&M.C.*),M>B(W.C.)		
Out-of-control with different magnitude of shifts	Strong	Mod.	Weak	Strong	Mod.	Weak	Strong	Mod.	Weak
3s,2s	B=M	B=M	B=M	B=M	B=M	B=M	B=M	B=M	B=M
3s,1s	n/a	B=M	B=M	n/a	B=M	B=M	B=M	B=M	B=M
2s,1s	B=M	B=M	B=M	B=M	B=M	B=M	B=M	B=M	B=M
-3s,-2s	n/a	B=M	B=M	n/a	n/a	B=M	n/a	n/a	n/a
-3s,-1s	n/a	B=M	B=M	n/a	n/a	M>B	n/a	n/a	n/a
-2s,-1s	B=M$^{\#}$	B=M$^{\#}$	M>B	n/a	M>B	M>B	n/a	M>B	M>B

Table 39 Comparisons of the MNP Chart and the Normal Approximation Technique for Large Sample Sizes.

Status of Process's Proportion Nonconforming	Sample Size # 1								
	p1=0.3,p2=0.3,n=50			p1=0.1,p2=0.1,n=100			p1=0.01,p2=0.01,n=910		
In-control	M>N(S.C.),N>M(M.C.&W.C.)			M>N			M>N		
Out-of-control with the same magnitude of shifts									
large shift in the positive direction	M=N			M=N			M=N		
large shift in the negative direction	M>N(S.C.), M=N (M.C.&W.C.)			M>N (S.C.&M.C.), M=N (W.C.)			M>N		
medium shift in the positive direction	M>N			M=N			M=N		
medium shift in the negative direction	M>N			M>N			M>N		
small shift in the positive direction	M>N			M>N			M>N		
small shift in the negative direction	M>N			M>N			M>N		
Out-of-control with different magnitude of shifts	Strong	Mod.	Weak	Strong	Mod.	Weak	Strong	Mod.	Weak
3s,2s	M=N	M=N	M=N	M=N	M=N	M=N	M=N	M=N	M=N
3s,1s	n/a	N>M	M=N	n/a	N>M	M=N	N>M	N>M	M=N
2s,1s	N>M	M=N	M>N	N>M	M=N	M=N	N>M	M=N	M=N
-3s,-2s	n/a	M>N	M=N	n/a	n/a	M>N	n/a	n/a	n/a
-3s,-1s	n/a	N>M	M=N	n/a	n/a	M>N	n/a	n/a	n/a
-2s,-1s	N>M	M>N	M>N	n/a	M>N	M>N	n/a	M>N	M>N

Remark: S.C., M.C., and W.C. are strong, moderate, and weak correlation coefficients.

The symbol (>) shown in the tables indicates which technique "outperforms" the other technique (e.g. B>N represents the BPNN technique is better than the normal approximation technique). Some of the techniques outperform the other two for certain specified process conditions while they are inferior in other process conditions. To select a suitable technique, users must weigh the in-control against the out-of-control average run length. If stopping a process to investigate the out-of-control signal is critical, a technique with a large in-control ARL is recommended. For instance, the BPNN technique is preferred for a strongly correlated process with large proportion nonconforming when both proportion nonconforming only shift with the same magnitude and in the same direction (either positive or negative). In this condition, the BPNN technique detects a false alarm 61 and 20 samples later than the normal approximation technique and the MNP chart respectively, as shown in Table 6. In addition, the BPNN technique has an out-of-control ARL either smaller than or equal to the normal approximation technique. The out-of-control ARL for the BPNN and the MNP chart are equal when both proportion nonconforming shift from medium to large magnitude. The MNP chart detects shifts faster than the BPNN by only one ARL sample when both proportion nonconforming shift with a small magnitude and in the positive direction. The BPNN detects shifts faster than the MNP chart four ARL samples when both proportion nonconforming shift with a small magnitude and in the negative direction.

In contrary, if a process needs to be adjusted quickly from an out-of-control status to an in-control status, one might select a technique with a small out-of-control ARL for quick detection. For example, the MNP chart is preferred for a process with two weakly correlated attributes each having proportion nonconforming of 0.01. Both of the process's proportion nonconforming often shift with the same magnitude in the same direction, either positive or negative, and the sample sizes are large. The BPNN technique indicates a false alarm 22 and 53 samples later than the MNP chart and the normal approximation technique respectively as shown in Table 14. The MNP chart designates a false alarm 31 samples later than the normal approximation technique. All techniques performed equally when both proportion nonconforming shift from medium to large magnitudes and in the positive direction. The MNP chart and the BPNN technique outperform the normal approximation when both proportion nonconforming shift in the negative direction. The

MNP chart outperforms the BPNN when both proportion nonconforming shift with medium and small magnitudes in the negative directions.

In comparing the BPNN technique and the MNP chart, the two techniques did not always detect the shifts for all of the replications. As a result, it is unclear how to evaluate the performances of both techniques since the ARL results contradict the number of replications detected by each technique. For example, the BPNN technique had a smaller ARL than the MNP chart; however, the MNP chart detected the shifts for all of the replications while the BPNN technique failed to detect the shift for one replication. Furthermore, the ARL results contradict the number of correct classifications, i.e. the BPNN technique had a smaller out-of-control ARL than the MNP chart but the MNP chart correctly classified the direction of shifts more than the BPNN. The difference of the number of replications detected and the correct classification may or may not be significant to the ARL results.

Though not the majority, such contradictory results require decision rules to guide the user in determining which technique (BPNN or MNP) is best to apply. Table 40 provides the decision (i.e. yes or no) of whether or not there is a significant difference between the number of replications detected and the correct classification in comparison to the ARL results. The different situations are denoted by the superscripts, which are also shown in Table 38, Table 43, and Table 46.

Table 40 Meaning of Superscripts Used in the Comparisons between the BPNN technique and the MNP Chart.

Superscript	*Comparison Results*	*Out-of-Control ARL $_X$ compare to Out-of-Control ARL $_Y$*	*Number of Replications Detected*	*Correct Classification of Shift Direction*	*Significance of the Difference of the Number of Replications Detected and the Correct Classification of Shift Direction*
*	X>Y	ARL_X smaller than ARL_Y	Rep.$_X$ equal Rep.$_Y$	Y more correct than X	no
*	X>Y	ARL_X smaller than ARL_Y	Rep.$_X$ more than Rep.$_Y$	Y more correct than X	no
*	X>Y	ARL_X smaller than ARL_Y	Rep.$_X$ less than Rep.$_Y$	Y more correct than X	no
#	X=Y	ARL_X equal ARL_Y	Rep.$_X$ equal Rep.$_Y$	X more correct than Y	no
#	X=Y	ARL_X equal ARL_Y	Rep.$_X$ less than Rep.$_Y$	Y more correct than X	no
#	X=Y	ARL_X equal ARL_Y	Rep.$_X$ equal Rep.$_Y$	Y more correct than X	no
@	X>Y	ARL_X equal ARL_Y	Rep.$_X$ more than Rep.$_Y$	X more correct than Y	yes
@	X>Y	ARL_X larger than ARL_Y	Rep.$_X$ more than Rep.$_Y$	X more correct than Y	yes
+	X= or >Y	ARL_X larger than ARL_Y	Rep.$_X$ more than Rep.$_Y$	X more correct than Y	yes
+	X= or >Y	ARL_X equal ARL_Y	Rep.$_X$ more than Rep.$_Y$	X more correct than Y	yes

X and Y in Table 40 can be either BPNN technique or MNP chart. Rep.$_X$ and Rep.$_Y$ represent the number of replications that X and Y techniques detect shifts respectively.

None of the three techniques had equal in-control ARL for a number of reasons.[*] Although attempts were made to initialize the three techniques to have the same ARL, the initial process conditions (i.e. sample size, proportion nonconforming, level of correlation, etc.) often made this impossible. However, if one assumes equal in-control ARL, comparisons of the three techniques are shown in Table 41, which is a summary of Table 37 through Table 39.

[*] In some cases differences in the in-control ARL differed as much as 90 samples or as small as eight samples depending on the techniques compared and the process conditions.

Table 41 The BPNN, MNP Chart and the Normal Approximation Technique for Large Sample Sizes. (Best performing techniques in each situation are shown by their first letters)

Status of Process's Proportion Nonconforming	Sample Size # 1								
	p1=0.3,p2=0.3,n=50			p1=0.1,p2=0.1,n=100			p1=0.01,p2=0.01,n=910		
In-control	B,M,N			B,M,N			B,M,N		
Out-of-control with the same magnitude of shifts									
large shift in the positive direction	B,M,N			B,M,N			B,M,N		
large shift in the negative direction	B,M(S.C.), B,M,N(M.C.&W.C.)			B,M(S.C.&M.C.), B,M,N(W.C.)			M(S.C&M.C..),B,M,(W.C.)		
medium shift in the positive direction	B,M			B,M,N			B,M,N		
medium shift in the negative direction	B,M			B(S.C.),M(M.C.),B,M(W.C.)			B(S.C.), M(M.C.&W.C.)		
small shift in the positive direction	M			M(S.C.&M.C.),B,M(W.C.)			B,M		
small shift in the negative direction	B(S.C.),M(M.C.&W.C.)			B(S.C.),M(M.C.&W.C.)			B(S.C.&M.C.),M(W.C.)		
Out-of-control with different magnitude of shifts	Strong	Mod.	Weak	Strong	Mod.	Weak	Strong	Mod.	Weak
3s,2s	B,M,N	B,M,N	B,M,N	B,M,N	B,M,N	B,M,N	B,M,N	B,M,N	B,M,N
3s,1s	n/a	N	B,M,N	n/a	N	B,M,N	N	N	B,M,N
2s,1s	N	B,M,N	B,M	N	B,M,N	B,M,N	N	B,M,N	B,M,N
-3s,-2s	n/a	B,M	B,M,N	n/a	n/a	B,M	n/a	n/a	n/a
-3s,-1s	n/a	N	B,M,N	n/a	n/a	M	n/a	n/a	n/a
-2s,-1s	N	B,M	M	n/a	M	M	n/a	M	M

Table 41 presents the comparisons among the normal approximation technique, the MNP chart, and the BPNN technique for varied process conditions and when the sample sizes are large. One can use the table as a guideline to select the technique that most suitable to a particular process condition. For instance, the BPNN technique and the MNP chart are preferred for a process with two strongly correlated attributes, each having a large proportion nonconforming. The sample size is large and the proportion nonconforming are expected to shift from medium to large magnitude in either positive or negative direction.

The results also show that for any value of proportion nonconforming and level of correlation, all techniques performed equally when both of the process's proportion nonconforming shift with large magnitudes and in the positive direction.

The table further indicates that the MNP chart is preferred for a weakly correlated process for which both proportion nonconforming shift with the same magnitude. This applies to any level of proportion nonconforming.

It can be noticed that the normal approximation technique is preferred when both proportion nonconforming shift only in the positive direction and with different magnitudes. This is correct for strongly and moderately correlated processes for any level of in-control proportion nonconforming. For example, the normal approximation outperformed the other methods for moderately correlated processes when the proportion nonconforming of the first attribute shifts three standard deviations and the proportion nonconforming of the second attribute shifts one standard deviation. All three methods performed equally for the other magnitudes of shift. As a result, the normal approximation technique is recommended.

The results discussed in section 7.2 for sample sizes recommended for the MNP chart (sample size #2) are summarized in Table 42 through Table 44.

Table 42 Comparisons of the BPNN and Normal Approximation Techniques for the MNP Chart Sample Sizes.

Status of Process's Proportion Nonconforming	Sample Size # 2								
	p1=0.3,p2=0.3,n=10			p1=0.1,p2=0.1,n=30			p1=0.01,p2=0.01,n=810,670,540		
In-control	B>N			B>N			B>N		
Out-of-control with the same magnitude of shifts									
large shift in the positive direction	B=N			B=N			B=N		
large shift in the negative direction	B>N			B>N			B>N		
medium shift in the positive direction	B>N(S.C.), B=N (M.C), N>B(W.C.)			B=N			B=N		
medium shift in the negative direction	B>N			B>N			B>N		
small shift in the positive direction	B>N(S.C.), N>B (M.C.&W.C.)			B=N(S.C.), N>B (M.C.&W.C.)			B=N(S.C.), N>B (M.C.&W.C.)		
small shift in the negative direction	B>N			B>N			B>N		
Out-of-control with different magnitude of shifts	Strong	Mod.	Weak	Strong	Mod.	Weak	Strong	Mod.	Weak
3s,2s	n/a	B=N	B=N	B=N	B=N	B=N	B=N	B=N	B=N
3s,1s	n/a	N>B	N>B	n/a	N>B	N>B	N>B	N>B	N>B
2s,1s	n/a	N>B	N>B	N>B	N>B	N>B	N>B	N>B	N>B
-3s,-2s	B>N	B>N	B>N	B>N	B>N	B>N	n/a	B>N	B>N
-3s,-1s	n/a	n/a	B>N	n/a	n/a	n/a	n/a	n/a	B>N
-2s,-1s	n/a	n/a	B>N	n/a	n/a	n/a	n/a	B>N	B>N

Table 43 Comparisons of the BPNN Technique and the MNP chart for the MNP Chart Sample Sizes.

Status of Process's Proportion Nonconforming	Sample Size # 2								
	p1=0.3,p2=0.3,n=10			p1=0.1,p2=0.1,n=30			p1=0.01,p2=0.01,n=810,670,540		
In-control	B>M			B>M			B>M		
Out-of-control with the same magnitude of shifts									
large shift in the positive direction	B=M			B=M			B=M		
large shift in the negative direction	M>B[@]			M>B[@]			M=or>B(S.C.&M.C.)[+], B>M (W.C.)		
medium shift in the positive direction	B>M(S.C.), B=M (M.C&W.C.)			B=M (S.C.[#]&M.C&W.C.)			B=M		
medium shift in the negative direction	M>B[@]			M>B[@]			B>M(S.C.*&M.C.*&W.C.)		
small shift in the positive direction	B>M (S.C.),B=M (M.C.)[#],M>B (W.C.)			B>M (S.C.&M.C.*),B=M (W.C.)[#]			M>B		
small shift in the negative direction	M>B[@]			M>B (S.C.&M.C.[@]&W.C.[@])			B>M		
Out-of-control with different magnitude of shifts	Strong	Mod.	Weak	Strong	Mod.	Weak	Strong	Mod.	Weak
3s,2s	n/a	B=M	B=M	B=M	B=M	B=M	B=M	B=M	B=M
3s,1s	n/a	B=M	B=M	n/a	B=M	B=M	B=M	B=M	B=M
2s,1s	n/a	B=M	M>B	B>M	B=M[#]	B=M	M>B	B=M	M>B
-3s,-2s	M>B[@]	M>B[@]	M>B[@]	M>B[@]	M>B[@]	M>B[@]	n/a	B>M*	B>M
-3s,-1s	n/a	n/a	M>B[@]	n/a	n/a	n/a	n/a	n/a	B>M
-2s,-1s	n/a	n/a	M>B[@]	n/a	n/a	n/a	n/a	B>M*	B>M

Table 44 Comparisons of the MNP chart and the Normal Approximation Technique for the MNP Chart Sample Sizes.

Status of Process's Proportion Nonconforming	Sample Size # 2								
	p1=0.3,p2=0.3,n=10			p1=0.1,p2=0.1,n=30			p1=0.01,p2=0.01,n=810,670,540		
In-control	N>M			N>M			M>N		
Out-of-control with the same magnitude of shifts									
large shift in the positive direction	M=N			M=N			M=N		
large shift in the negative direction	M>N			M>N			M>N		
medium shift in the positive direction	M>N			M=N			M=N		
medium shift in the negative direction	M>N			M>N			M>N		
small shift in the positive direction	M>N			M>N			M>N		
small shift in the negative direction	M>N			M>N			M>N		
Out-of-control with different magnitude of shifts	Strong	Mod.	Weak	Strong	Mod.	Weak	Strong	Mod.	Weak
3s,2s	n/a	M=N	M=N	M=N	M=N	M=N	M=N	M=N	M=N
3s,1s	n/a	N>M	M=N	n/a	N>M	M=N	N>M	N>M	M=N
2s,1s	n/a	M=N	M>N	N>M	M=N	M=N	N>M	M=N	M=N
-3s,-2s	M>N	M>N	M>N	M>N	M>N	M>N	n/a	M>N	M>N
-3s,-1s	n/a	n/a	M>N	n/a	n/a	n/a	n/a	n/a	M>N
-2s,-1s	n/a	n/a	M>N	n/a	n/a	n/a	n/a	M>N	M>N

The in-control ARL for the normal approximation technique and the MNP chart were substantially different, as mentioned in section 7.2. In order to compare the BPNN technique to the normal approximation technique and the MNP chart, two different cut-off values were used to obtain comparable in-control ARL. As a result, the BPNN in Table 42 and Table 43 have different in-control ARL and the three tables cannot be combined into a summary table.

For the sample sizes recommended for the MNP chart, Table 42 shows that the BPNN technique outperformed the normal approximation technique when shifts were in the negative direction for any level of proportion nonconforming and level of correlation coefficient. The normal approximation technique cannot detect shifts that are close to "0" and in the negative direction.

The normal approximation technique performed better than the BPNN technique when both proportion nonconforming shifted in the positive direction and with different magnitudes, i.e. the proportion nonconforming of the first attribute shifted with either three or two standard deviations and the proportion nonconforming of the second attribute shifted with one standard deviation. This applies to processes with all levels of proportion nonconforming and correlation coefficient.

Table 44 shows that the normal approximation technique indicates a false alarm later than the MNP chart when proportion nonconforming is either large or medium (0.3 or 0.1) for the sample sizes suggested by the MNP chart (sample size #2). This result is due to the fact that the in-control ARL for the MNP chart drops significantly when the sample sizes suggested for the MNP chart (sample size #2) are used instead of the large sample sizes (sample size #1). For instance, a process with two strongly correlated attributes each having proportion nonconforming of 0.3 will indicate a false alarm every 411 samples when the large sample size is used as shown in Table 6. Whereas the process will designate a false alarm every 58 samples when smaller sample sizes (as recommended by the MNP chart) are used as shown in Table 17. However, the MNP chart will indicate a false alarm later than the normal approximation technique for processes with small proportion nonconforming (0.01).

The MNP chart outperforms the normal approximation technique for all levels of proportion nonconforming and correlation coefficient when both proportion nonconforming shift with any magnitude in the negative direction. Also, the MNP chart performs better than the normal approximation technique for all levels of proportion nonconforming and level of correlation coefficient when both proportion nonconforming shift with the same magnitude in either the positive or the negative direction.

For a strongly correlated process with any level of proportion nonconforming, the normal approximation technique outperforms the MNP chart when the proportion nonconforming of the first attribute shifts with either three or two standard deviations and the proportion nonconforming of the second attribute shifts one standard deviation and both proportion nonconforming shift in the positive direction. For a moderately correlated process with any level of proportion nonconforming, the normal approximation technique outperforms the MNP chart when the proportion nonconforming of the first attribute shifts three standard deviations and the proportion nonconforming of the second attribute shifts one standard deviation and both proportion nonconforming shift in the positive direction.

When selecting between the MNP chart and the normal approximation technique one must weigh the criticality between stopping the process to investigate the out-of-control signal and adjusting the out-of-control process to get it back into in-control status as fast as possible. For example, if one is concerned about identifying the out-of-control signal as fast as possible, the MNP chart is preferred for a process with two strongly correlated attributes each having a large proportion nonconforming.

For this process condition, the in-control ARL for the MNP chart is approximately one-fourth the size (58 for the MNP chart as shown in Table 17 and 249 for the normal approximation technique as shown Table 16) of the normal approximation technique. Therefore, if a process requires substantial resources to stop and investigate potential out-of-control causes, the normal approximation is preferred. However, the normal approximation technique cannot detect shifts that are close to “0” and in the negative direction. For this particular situation, the BPNN technique is preferred to the normal approximation technique since the BPNN indicates a false alarm 38 samples later than the normal approximation technique (287 for the BPNN and 249 for the normal approximation technique as shown in Table 16). Furthermore, the BPNN technique detects shifts faster than the normal approximation technique when processes are out-of-control (proportion nonconforming shift with any magnitude and direction).

The results discussed in section 7.3 for sample sizes that satisfy the condition of finding at least one nonconforming item in a sample (sample size #3) are summarized in Table 45 through Table 47. Note, the first two columns of each table are the same as for Table 42 through Table 44 because the sample sizes were the same.

Table 45 Comparisons of the BPNN and Normal Approximation Techniques for the Small Sample Sizes.

Status of Process's Proportion Nonconforming	Sample Size # 3								
	p1=0.3,p2=0.3,n=10			p1=0.1,p2=0.1,n=30			p1=0.01,p2=0.01,n=300		
In-control	B>N			B>N			B>N		
Out-of-control with the same magnitude of shifts									
large shift in the positive direction	B=N			B=N			B=N		
large shift in the negative direction	B>N			B>N			B>N		
medium shift in the positive direction	B>N(S.C.), B=N (M.C), N>B(W.C.)			B=N			B=N		
medium shift in the negative direction	B>N			B>N			B>N		
small shift in the positive direction	B>N(S.C.), N>B (M.C.&W.C.)			B=N(S.C.), N>B (M.C.&W.C.)			N>B		
small shift in the negative direction	B>N			B>N			B>N		
Out-of-control with different magnitude of shifts	Strong	Mod.	Weak	Strong	Mod.	Weak	Strong	Mod.	Weak
3s,2s	n/a	B=N	B=N	B=N	B=N	B=N	B=N	B=N	B=N
3s,1s	n/a	N>B	N>B	n/a	N>B	N>B	n/a	N>B	N>B
2s,1s	n/a	N>B	N>B	N>B	N>B	N>B	N>B	N>B	N>B
-3s,-2s	B>N	B>N	B>N	B>N	B>N	B>N	B>N	B>N	B>N
-3s,-1s	n/a	n/a	B>N	n/a	n/a	n/a	n/a	n/a	B>N
-2s,-1s	n/a	n/a	B>N	n/a	n/a	n/a	n/a	n/a	B>N

Table 46 Comparisons of the BPNN Technique and the MNP chart for the Small Sample Sizes.

Status of Process's Proportion Nonconforming	Sample Size # 3								
	p1=0.3,p2=0.3,n=10			p1=0.1,p2=0.1,n=30			p1=0.01,p2=0.01,n=300		
In-control	B>M			B>M			B>M		
Out-of-control with the same magnitude of shifts									
large shift in the positive direction	B=M			B=M			B=M		
large shift in the negative direction	M>B[@]			M>B[@]			M>B[@]		
medium shift in the positive direction	B>M(S.C.), B=M (M.C&W.C.)			B=M (S.C.[#]&M.C&W.C.)			B>M(S.C.)*, B=M (M.C.[#]&W.C.)		
medium shift in the negative direction	M>B[@]			M>B[@]			M>B[@]		
small shift in the positive direction	B>M (S.C.),B=M (M.C.)[#],M>B (W.C.)			B>M (S.C.&M.C.*),B=M (W.C.)[#]			B>M (S.C.&M.C.),M>B (W.C.)		
small shift in the negative direction	M>B[@]			M>B (S.C.&M.C.[@]&W.C.[@])			M>B (S.C.&M.C.[@]&W.C.[@])		
Out-of-control with different magnitude of shifts	Strong	Mod.	Weak	Strong	Mod.	Weak	Strong	Mod.	Weak
3s,2s	n/a	B=M	B=M	B=M	B=M	B=M	B=M[#]	B=M	B=M[#]
3s,1s	n/a	B=M	B=M	n/a	B=M	B=M	n/a	B=M[#]	B=M[#]
2s,1s	n/a	B=M	M>B	B>M	B=M[#]	B=M	B>M	B=M[#]	B=M[#]
-3s,-2s	M>B[@]	M>B[@]	M>B[@]	M>B[@]	M>B[@]	M>B[@]	M>B[@]	M>B[@]	M>B[@]
-3s,-1s	n/a	n/a	M>B[@]	n/a	n/a	n/a	n/a	n/a	M>B[@]
-2s,-1s	n/a	n/a	M>B[@]	n/a	n/a	n/a	n/a	n/a	M>B[@]

Table 47 Comparisons of the MNP chart and the Normal Approximation Technique for the Small Sample Sizes.

Status of Process's Proportion Nonconforming	Sample Size # 3								
	p1=0.3,p2=0.3,n=10			p1=0.1,p2=0.1,n=30			p1=0.01,p2=0.01,n=300		
In-control		N>M			N>M			N>M	
Out-of-control with the same magnitude of shifts									
large shift in the positive direction		M=N			M=N			M=N	
large shift in the negative direction		M>N			M>N			M>N	
medium shift in the positive direction		M>N			M=N			M=N	
medium shift in the negative direction		M>N			M>N			M>N	
small shift in the positive direction		M>N			M>N			M=N	
small shift in the negative direction		M>N			M>N			M>N	
Out-of-control with different magnitude of shifts	Strong	Mod.	Weak	Strong	Mod.	Weak	Strong	Mod.	Weak
3s,2s	n/a	M=N	M=N	M=N	M=N	M=N	M=N	M=N	M=N
3s,1s	n/a	N>M	M=N	n/a	N>M	M=N	n/a	N>M	M=N
2s,1s	n/a	M=N	M>N	N>M	M=N	M=N	N>M	M=N	M=N
-3s,-2s	M>N	M>N	M>N	M>N	M>N	M>N	M>N	M>N	M>N
-3s,-1s	n/a	n/a	M>N	n/a	n/a	n/a	n/a	n/a	M>N
-2s,-1s	n/a	n/a	M>N	n/a	n/a	n/a	n/a	n/a	M>N

As mentioned in section 7.3 the in-control ARL for the normal approximation technique and the MNP chart were substantially different. To compare the BPNN technique to the normal approximation technique and the MNP chart, two different cut-off values were used for the BPNN technique in order to have comparable in-control ARL for each technique. As a result, the three tables cannot be combined into a summary table.

For small sample sizes, Table 45 shows that the BPNN outperformed the normal approximation technique when shifts were in the negative direction for any level of proportion nonconforming and level of correlation coefficient. The normal approximation technique cannot detect shifts that are close to "0" and in the negative direction.

The normal approximation technique performed better than the BPNN technique when both of the process's proportion nonconforming shifted in the positive direction and with different magnitudes, i.e. the proportion nonconforming of the first attribute shifted with either three or two standard deviations and the proportion nonconforming of the second attribute shifted with one standard deviation. This result applies to all levels of proportion nonconforming and correlation coefficient.

Table 46 indicates that the BPNN technique designates a false alarm later than the MNP chart. However, the MNP chart detects shifts faster than the BPNN technique for all levels of proportion nonconforming and correlation coefficient when both proportion nonconforming shift in negative direction.

Table 47 shows that the normal approximation technique indicates a false alarm later than the MNP chart when the proportion nonconforming varied from large to small for small sample sizes (sample size #3). This is due to the fact that the in-control ARL for the MNP drops significantly when small sample sizes (sample size #3) are used instead of sample sizes prescribed by the normal approximation (sample size #1).

The MNP chart outperforms the normal approximation technique for all levels of proportion nonconforming and correlation when both proportion nonconforming shift with any magnitude in the negative direction. Also, the MNP chart performs better than the normal approximation technique for all levels of proportion nonconforming and correlation coefficient when both proportion nonconforming shift with the same magnitude in either the positive or the negative direction.

For a strongly correlated process at any levels of proportion nonconforming, the normal approximation technique outperforms the MNP chart when the proportion nonconforming for the first attribute shifts two standard deviations and the proportion nonconforming for the second attribute shifts one standard deviation and both shifts are in the positive direction. For a moderately correlated process at any levels of proportion nonconforming, the normal approximation technique outperforms the MNP chart when the proportion nonconforming for the first attribute shifts three standard deviations and the proportion nonconforming for the second attribute shifts one standard deviation and both shifts are in the positive direction.

8.1 Guidelines for Selecting a Suitable Technique

The following key questions can serve as a guide to determine the most suitable technique for a particular process and its conditions.

1) What is the known proportion nonconforming for each attribute?
2) What is the known correlation coefficient of the process?
3) What is the most feasible sample size for the process (i.e. this may include cost considerations for collecting the samples)?
4) Is the user more concerned about Type I or II errors?
5) What is the preferable in-control ARL (i.e. how large should be the number of samples before detecting an in-control ARL)?
6) What are the magnitude and direction of shifts normally happen in the process?

Once these questions have been addressed, one should then make the following considerations in selecting a technique.

1) Compare in-control ARL for all techniques and see how are they different.
2) Compare out-of-control ARL for all techniques and see how are they different.
3) Weigh the differences between the in-control against the out-of-control ARLs to determine which technique yields the lowest risks associated with both ARLs.

Three scenarios are now presented how one might select a technique for their particular processes.

Scenario 1

A process engineer is looking for a technique to monitor a process with two weakly correlated attributes each having a proportion nonconforming of 0.3. Collecting large samples for this process are not a major concern as the costs associated with measuring for defects is minimal. Therefore, a sample size of 50 is preferred to a sample size of 10. The engineer is more concern about committing a Type II error than a Type I error since the product cost is high and it consumes several resources to correct nonconforming products. The engineer also has knowledge that the proportion nonconforming of both attributes naturally shift in the same direction (either in the positive or the negative direction) but often with different magnitudes.

Large sample sizes are selected; therefore, one should look at Table 37 through Table 39. They show that the BPNN has a larger in-control ARL than the normal approximation technique and the MNP chart. The normal approximation technique has a larger ARL than the MNP chart. The in-control ARL for the normal approximation technique, MNP chart, and the BPNN are 416, 386, and 451 respectively (as shown in Table 8). The BPNN technique and the MNP chart perform equally to the normal approximation technique when the proportion nonconforming of the first attribute shifts with three standard deviations and the proportion nonconforming of the second attribute shifts with either two or one standard deviation and both proportion nonconforming shift in the same direction (either in the positive to the negative direction).

In Table 8 section 7.1 both the BPNN technique and the MNP chart detect shifts faster than the normal approximation technique one ARL sample when the proportion nonconforming of the first attribute shifts with two standard deviations and the proportion nonconforming of the second attribute shifts with one standard deviation and both proportion nonconforming shift in the positive direction. When the proportion nonconforming of the first attribute shifts with two standard deviations and the proportion nonconforming of the second attribute shifts with one standard deviation and both proportion nonconforming shift in the negative direction, the BPNN technique and the MNP chart indicate the shifts faster than the normal approximation technique 10 and 12 ARL samples, respectively. The BPNN technique and the MNP chart perform equally in all magnitudes of shift except for the case where the proportion nonconforming of the first attribute shifts two standard deviations and the proportion nonconforming of the second attribute shift one standard deviation and both proportion nonconforming shift in the negative direction. In this situation, the MNP chart detects shifts faster than the BPNN technique 2 ARL samples.

In this scenario the engineer is more concern about the out-of-control ARL; therefore, the normal approximation technique is disregarded. The engineer must weigh the differences between the in-control ARL and the out-of-control ARL for the BPNN technique and the MNP chart. The BPNN technique indicates a false alarm 65 ARL samples later than the MNP chart. However, the MNP chart detects an out-of-control signal 2 ARL samples faster than the BPNN technique when the proportion nonconforming of the first attribute shifts two standard deviations and the proportion nonconforming of the second attribute shifts one standard deviation and both proportion nonconforming shift in the negative direction. The engineer must consider how likely this condition will occur. If the engineer is not concerned, then he/she should select the BPNN technique; otherwise the MNP chart.

Scenario 2

A process has two strongly correlated attributes (0.80) and each attribute has proportion nonconforming of 0.01. Engineers are more concerned about committing a Type I error than a Type II error because substantial resources are required to stop and investigate the process. Furthermore, the cost associated with measuring product quality is high; therefore, small sample sizes are preferred. The process's proportion nonconforming are expected to shift with approximately the same magnitude in the same direction (either positive or negative).

Table 45 through Table 47 show for sample sizes of 300 that the BPNN technique has larger in-control ARL than the normal approximation technique and MNP chart for processes that have proportion nonconforming of 0.01. In general, the normal approximation technique has a larger in-control ARL than the MNP chart. When comparing the BPNN to the normal approximation technique as shown in Table 31, the in-control ARL for the BPNN and the normal approximation techniques are 147 and 136 respectively. The in-control ARL for the BPNN and the MNP chart are 35 and 31 respectively as shown in Table 32.

Since Type I error is more critical than Type II error, either BPNN or the normal approximation technique should be considered. From Table 45 the BPNN technique detects shifts equal to or faster than the normal approximation technique for all conditions of shifts except for one condition. Moreover, Table 31 describes that the normal approximation technique cannot detect shifts in the negative direction. Therefore, the BPNN technique is the most suitable to this process condition.

Scenario 3

A process has two attributes. Both attributes have proportion nonconforming of 0.1 and they are weakly correlated. Collecting large samples for this particular process is not a major concern as the costs associated with measuring for defects is minimal. The engineers are unclear as to which is more critical: committing a Type I or Type II error. The engineers also have no knowledge about the process with respect to how the process may shift as it goes out-of-control (direction and magnitude).

A sample size of 100 is selected. Table 37 and Table 39 show that the BPNN technique and the MNP chart have larger in-control ARL than the normal approximation technique. As shown in Table 11, the BPNN technique detects an in-control ARL 54 samples later than the normal approximation technique and the MNP chart detects an in-control ARL 40 samples later than the normal approximation technique. The BPNN, therefore, provides the best in-control ARL (14 samples later).

In general, the BPNN technique and the MNP chart perform equal to or better than the normal approximation technique for all conditions of shifts. It is obvious that the normal approximation technique is inferior to the other two techniques for this particular scenario in terms of in-control and out-of-control ARLs.

The MNP chart and the BPNN techniques perform equally for most shift combinations (i.e. direction and magnitude). There are three shift combinations in which the MNP chart outperforms the BPNN technique. First, both proportion nonconforming shift with small magnitude (one standard deviation) and in the negative direction. Table 11 shows that the MNP chart detects shifts 29 ARL samples faster than the BPNN technique. Second, the proportion nonconforming of the first attribute shifts with three standard deviations and the proportion nonconforming of the second attribute shifts with one standard deviation, and both proportion nonconforming shift in the negative direction. For this shift combination, the MNP chart detects shifts one sample faster than the BPNN technique. Third, the proportion nonconforming of the first attribute shifts with two standard deviations and the proportion nonconforming of the second attribute shifts with one standard deviation, and both proportion nonconforming shift in the negative direction. For this particular shift combination, the MNP chart indicates shifts faster than the BPNN technique seven ARL samples. If the time between samples is short, the differences in the results for these later two shift combinations may be considered negligible.

By weighing the in-control difference of the MNP chart and the BPNN technique (14 ARL samples) against the out-of-control difference of the MNP chart and the BPNN technique (29, 1, and 7 ARL samples for the three shift combinations, respectively), one should prefer the MNP chart since the conditions of shifts are vague in this process.

It should be noted that the more knowledge one knows about the process, a more informed decision can be made in choosing techniques that are most suitable. However, there may be some cases in which the users do not have much knowledge about the quality of their processes. For these situations, a tree diagram (shown in Figure 8) is provided to guide users in making a decision based on limited process knowledge.

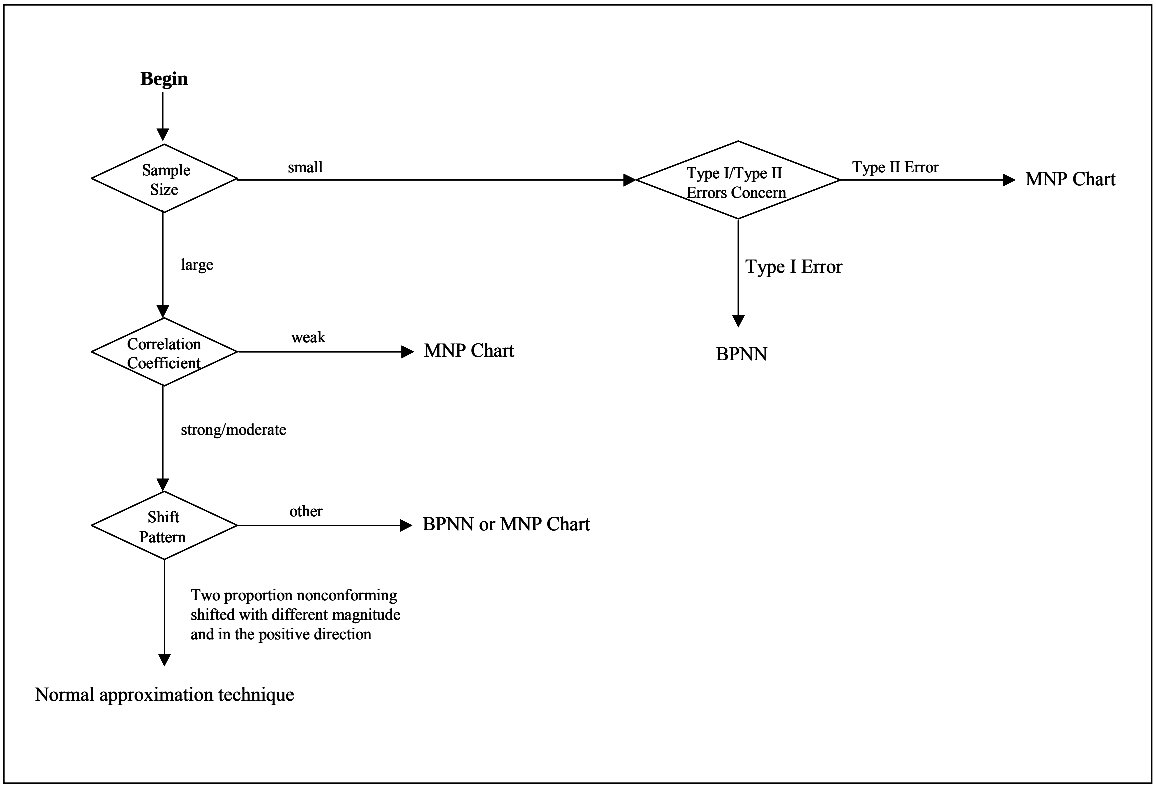

Figure 8 Decision tree diagram for process with limited knowledge

Once the proportion nonconforming of a process are known, a large sample size (estimating a multivariate normal variable from a multivariate binomial variable) can be calculated from equation (4-1). Users determine whether the sample size is appropriate to their process. If a large sample size is chosen, user must answer what the correlation coefficient of the process is. For weakly correlated processes, the MNP chart is the most suitable technique based on the information provided. More information about the magnitude and direction of shifts are required to make decision for strongly and moderately processes. If the process tends to have shifts with different magnitude and in the positive direction, the normal approximation technique is preferred. However, either the BPNN technique or MNP chart is preferred for other pattern of the shifts.

If a small sample size is selected, the BPNN technique is preferred when Type I error (false alarm rate) is more critical than Type II error. Otherwise, the MNP chart is preferable.

8.2. General Performances of Multi-Attribute Process Control Techniques

Following sections summarized the performances of each technique in general.

8.2.1 Normal Approximation Technique

The following conclusions can be made for the use of the normal approximation technique as a suitable technique for multi-attribute process control.

a) Could not adequately detect shifts in the negative direction (proportion nonconforming close to “0”) for smaller sample sizes (#2 and #3).
b) The stronger the level of correlation coefficient, the larger the out-of-control ARL when both proportion nonconforming shift with the same magnitude and in the positive direction as shown in Figure 9.

p_1 = 0.3, p_2 = 0.3, Sample Size = 50

Correlation Coefficient	+3s1, +3s2	+2s1, +2s2	+1s1, +1s2
Strong	2.287	7.568	49.436
Moderate	1.832	5.932	40.409
Weak	1.508	4.151	31.494

p_1 = 0.01, p_2 = 0.01, Sample Size = 910

Correlation Coefficient	+3s1, +3s2	+2s1, +2s2	+1s1, +1s2
Strong	2.166	5.237	24.257
Moderate	1.863	4.239	19.831
Weak	1.534	3.258	16.443

p_1 = 0.1, p_2 = 0.1, Sample Size = 30

Correlation Coefficient	+3s1, +3s2	+2s1, +2s2	+1s1, +1s2
Strong	2.128	4.661	18.227
Moderate	1.831	3.815	16.397
Weak	1.539	3.239	13.918

Figure 9 Performances of the normal approximation technique with different correlation coefficient

c) In general, normal approximation outperformed the other two techniques for strongly and moderately correlated processes when both proportion nonconforming shift with different magnitudes and in the positive direction.

8.2.2 MNP Chart

The following conclusions can be made for the use of the MNP chart as a suitable technique for multi-attribute process control.

a) The in-control ARL is small for sample sizes recommended for the MNP chart (sample size #2) and small sample size (sample size #3), especially for strong correlation coefficient.

b) The recommended sample size provided by the literature for the MNP chart for large and medium proportion nonconforming is not appropriate.

c) In general, MNP chart can detect shifts with the same magnitude faster than the normal approximation.

d) In general, MNP chart can detect negative shifts faster than the other two techniques for small sample sizes.

8.2.3 Backpropagation Neural Network Technique

The following conclusions can be made for the use of the BPNN technique as a suitable technique for multi-attribute process control.

a) In general, BPNN can detect shifts with the same magnitude faster than the normal approximation for large sample sizes.

b) In general, BPNN can outperform the normal approximation technique for shifts in the negative direction. BPNN is preferred for small sample sizes since it provids large in-control ARL and is able to detect negative shifts (while the normal approximation cannot).

8.3 Interpretation of Out-of-Control Signals

Once an out-of-control signal is indicated, process attributes that cause the process's proportion nonconforming shifts must be identified and adjusted to bring the process back to the in-control status. In addition to the MNP chart, which detects an out-of-control signal, Lu *et al.* discussed how to designate process attributes, which contribute to the out-of-control processes. A Z_{Di} score is calculated for each attribute as shown in equation (3-6) of section 3.1.2. The quality characteristic with the largest positive Z_{Di} score is considered the major contributor to an upward shift in the process.

Conversely, the smallest negative Z_{Di} score is considered the major contributor to a downward shift in the process.

As shown in section 2.5, there are several approaches suggested for multivariate control charts to interpret the out-of-control signals. One of the simplest approaches is plotting several univariate control charts accompanied by the multivariate control chart. Likewise, several uni-attribute control charts can be used together with a multi-attribute control chart to identify the attributes that cause the out-of-control signal. However, when implementing this, similar concerns arise as with multivariate processes. First, when a process includes several attributes, there are many uni-attribute control charts to interpret. Second, the uni-attribute control charts may not show any out-of-control signal when the multi-attribute control chart detects a signal since the signal may be a function of several correlated attributes. Third, the overall significance level of the simultaneous use of p uni-attribute control charts is difficult to determine.

Most of the approaches found in the literature for interpreting out-of-control signals for multivariate control charts are based on the T^2 control chart where the normality is assumed. Therefore, these techniques may be applicable to multi-attribute processes if sample sizes are large enough.

9.0 CONCLUSIONS, CONTRIBUTIONS, FUTURE WORK

A new technique using backpropagation (BPNN) was proposed for monitoring the quality of processes involving two correlated attributes. This new technique was compared to the current two techniques, the normal approximation technique and the MNP chart, for a large variety of process conditions. Five parameters, proportion nonconforming, level of correlation coefficient, sample size, and magnitude and direction of the shifts of proportion nonconforming, were varied. The proportion nonconforming contained three levels, large (0.3), medium (0.1), and small (0.01). The correlation coefficient included three levels, strong (0.8), moderate (0.5), and weak (0.2). Three different sample sizes were applied based on: (1) estimates of multivariate normally distributed variables from a multivariate binomial distribution, (2) recommendations for the MNP chart, and (3) satisfying the condition of finding at least one nonconforming item in a sample. Shifts of process's proportion nonconforming varied between one and three standard deviations. The process's proportion nonconforming also shifted in either the positive or the negative direction.

To compare the three techniques in-control average run length (ARL) and out-of-control ARL were used as performance measures. In addition, for each technique the number of replications that indicated out-of-control shifts were tracked, along with whether or not the direction of the shift was correct. In general, both the MNP chart and the BPNN technique are capable of correctly identifying the directions of shift; however depending on the process condition, one technique may be more favorable over the other method. By the nature of the method, the normal approximation technique could not identify the direction of shift. The number of replications of correctly identified direction of shifts (positive or negative) was also considered in the performance comparison between the MNP chart and the BPNN technique. All pair-wise comparisons for the different process conditions were summarized as previously shown in section 7.4.

The results showed that the normal approximation technique could not adequately detect a shift (or could detect but for only a few replications) in the negative direction (the shifted proportion nonconforming were close to “0”) when smaller sample sizes were applied. In addition, the stronger the level of correlation coefficient, the larger the out-of-control ARL for the normal approximation technique when both proportion nonconforming shifted with the same magnitude and in the positive direction. In general, the normal approximation technique detects shifts faster than the other two techniques for the strongly and moderately correlated processes when proportion nonconforming shifted with different magnitude and in the positive direction.

For most process conditions, especially when the sample sizes were small, the MNP chart detected negative shifts faster than the other two techniques. However, the in-control ARL for the MNP chart decreased substantially. For instance, for the process with two strongly correlated attributes, each having proportion nonconforming of 0.3, and employing the large sample size the in-control ARL was 411. The in-control ARL was reduced to 58 when the smaller sample size was used.

In general, the BPNN technique detected shifts faster than the normal approximation technique except when the proportion nonconforming shifted in the positive direction, but with different magnitudes. For most process conditions that involved small sample sizes, the BPNN was preferred as its in-control ARL is considerably larger than the MNP chart and its out-of-control ARL is smaller than the normal approximation technique (its in-control ARL is larger than the normal approximation technique). Unfortunately, for smaller sample sizes and as the level of correlation coefficient increased, the BPNN technique did not adequately detect shifts in the negative direction. As mentioned in the chapter 6 (validation), testing the hypothesis of strong correlation given small sample sizes, the data generated yielded inconsistent results. This may account for the inadequacy of the BPNN technique for this particular situation.

Selecting a technique for a process with correlated attributes is similar to that of selecting a conventional control chart. In conventional control charts, one must decide acceptable levels of the Type I and Type II errors and determine the costs associated with this decision. From this study, no one technique outperforms the other two techniques for all process conditions. A user must know their process conditions and concerns in order to select a most suitable technique.

This research has provided some recommendations for selecting a technique that is most suitable to one's process condition. Key questions were presented to serve as guidelines to determine an appropriate technique, as illustrated by the three scenarios. The scenarios provide a decision making process to follow to help satisfy process concerns (i.e. Type I and II errors, cost, and risks).

Once an out-of-control signal has been identified, process engineers must interpret the signal to determine which attribute(s) is delinquent so that an assignable cause may be investigated. Techniques, commonly used to interpret out-of-control signals in the multivariate control charts, were discussed. These same techniques may be applicable to the processes with multiple attributes.

In conducting this research other classification techniques potentially suitable for identifying shifts of the proportion nonconforming were investigated and evaluated. Concerns associated with employing these techniques were discussed.

Due to the limitation of the algorithms used to generate the data, process situations could not be studied where the proportion nonconforming varied (e.g. the first attribute had a proportion nonconforming of 0.3 and the second attribute had a proportion nonconforming of 0.1) and the correlation coefficient was strong or moderate. Furthermore, data cannot be generated for processes with two correlated attributes having substantially different proportion nonconforming (i.e. the proportion nonconforming of the first attribute is 0.3 and the proportion nonconforming of the second attribute is 0.01). If data can be generated or collected from a real process, future research may include the comparison of all the three techniques for this wider variety of process conditions.

The sample sizes, which estimate the multivariate normal distribution from the multivariate binomial distribution, are large when the proportion nonconforming is 0.1 or smaller. In a real environment, large sample sizes consume inspection time and money. In contrary, small sample sizes may not properly represent the relationship(s) between the attributes. Therefore, further research should be conducted on how to select appropriate sample sizes.

This study only investigated multi-attribute processes that consisted of two attributes. Certainly multiple attribute processes can and do involve more than two attributes. In the future, these three techniques should be evaluated across several attributes.

In this study shifts in the proportion nonconforming were investigated. It was assumed that the variance remained the same. Further research is needed to investigate multi-attribute processes in which there are changes in the variance-covariance of the attributes. A technique to identify changes in the variances of the attributes would be valuable.

Finally, the performance of the backpropagation neural network technique used to detect small shifts may improve by applying a sliding window procedure. The procedure allows prior sample(s) to be fed into the network simultaneously with the most recent sample. The number of samples fed to the networks depends on the window size. A small window size would shorten the out-of-control ARL, but it may result in a shorter in-control ARL. A large window size will most likely increase the time to detect shifts (thus longer out-of-control ARL).

APPENDIX A

SHIFTS OF PROPORTION NONCONFORMING FOR PROCESSES WITH TWO POSITIVELY CORRELATED ATTRIBUTES

Table A. 1 Shifts of Proportion Nonconforming for Process with In-control $p_1 = 0.3$, $p_2 = 0.3$

Correlation Coefficient = 0.8

Test Set Data Mean Shift (Sample Size = 50)		Test Set Data Mean Shift (Sample Size = 10)	
$+3s_1$	$+3s_2$	$+3s_1$	$+3s_2$
$+3s_1$	$+2s_2$	$+3s_1$	$+2s_2$
$+3s_1$	$+1s_2$	$+3s_1$	$+1s_2$
$+2s_1$	$+2s_2$	$+2s_1$	$+2s_2$
$+2s_1$	$+1s_2$	$+2s_1$	$+1s_2$
$+1s_1$	$+1s_2$	$+1s_1$	$+1s_2$
$-3s_1$	$-3s_2$	$-3s_1$ (.01)	$-3s_2$ (.01)
$-3s_1$	$-2s_2$	$-3s_1$ (.01)	$-2s_2$
$-3s_1$	$-1s_2$	$-3s_1$	$-1s_2$
$-2s_1$	$-2s_2$	$-2s_1$	$-2s_2$
$-2s_1$	$-1s_2$	$-2s_1$	$-1s_2$
$-1s_1$	$-1s_2$	$-1s_1$	$-1s_2$

Correlation Coefficient = 0.5

Test Set Data Mean Shift (Sample Size = 50)		Test Set Data Mean Shift (Sample Size = 10)	
$+3s_1$	$+3s_2$	$+3s_1$	$+3s_2$
$+3s_1$	$+2s_2$	$+3s_1$	$+2s_2$
$+3s_1$	$+1s_2$	$+3s_1$	$+1s_2$
$+2s_1$	$+2s_2$	$+2s_1$	$+2s_2$
$+2s_1$	$+1s_2$	$+2s_1$	$+1s_2$
$+1s_1$	$+1s_2$	$+1s_1$	$+1s_2$
$-3s_1$	$-3s_2$	$-3s_1$ (.01)	$-3s_2$ (.01)
$-3s_1$	$-2s_2$	$-3s_1$ (.01)	$-2s_2$
$-3s_1$	$-1s_2$	$-3s_1$	$-1s_2$
$-2s_1$	$-2s_2$	$-2s_1$	$-2s_2$
$-2s_1$	$-1s_2$	$-2s_1$	$-1s_2$
$-1s_1$	$-1s_2$	$-1s_1$	$-1s_2$

Correlation Coefficient = 0.2

Test Set Data Mean Shift (Sample Size = 50)		Test Set Data Mean Shift (Sample Size = 10)	
$+3s_1$	$+3s_2$	$+3s_1$	$+3s_2$
$+3s_1$	$+2s_2$	$+3s_1$	$+2s_2$
$+3s_1$	$+1s_2$	$+3s_1$	$+1s_2$
$+2s_1$	$+2s_2$	$+2s_1$	$+2s_2$
$+2s_1$	$+1s_2$	$+2s_1$	$+1s_2$
$+1s_1$	$+1s_2$	$+1s_1$	$+1s_2$
$-3s_1$	$-3s_2$	$-3s_1$ (.01)	$-3s_2$ (.01)
$-3s_1$	$-2s_2$	$-3s_1$ (.01)	$-2s_2$
$-3s_1$	$-1s_2$	$-3s_1$	$-1s_2$
$-2s_1$	$-2s_2$	$-2s_1$	$-2s_2$
$-2s_1$	$-1s_2$	$-2s_1$	$-1s_2$
$-1s_1$	$-1s_2$	$-1s_1$	$-1s_2$

Table A. 2 Shifts of Proportion Nonconforming for Process with In-control $p_1 = 0.3$, $p_2 = 0.1$

Correlation Coefficient = 0.2

Test Set Data Mean Shift (Sample Size = 100)		Test Set Data Mean Shift (Sample Size = 15)	
$+3s_1$	$+3s_2$	$+3s_1$	$+3s_2$
$+3s_1$	$+2s_2$	$+3s_1$	$+2s_2$
$+3s_1$	$+1s_2$	$+3s_1$	$+1s_2$
$+2s_2$	$+3s_2$	$+2s_2$	$+3s_2$
$+2s_2$	$+2s_2$	$+2s_2$	$+2s_2$
$+2s_2$	$+1s_2$	$+2s_2$	$+1s_2$
$+1s_2$	$+3s_2$	$+1s_2$	$+3s_2$
$+1s_2$	$+2s_2$	$+1s_2$	$+2s_2$
$+1s_2$	$+1s_2$	$+1s_2$	$+1s_2$
$-3s_1$	$-3s_2$	$-3s_1$ (.01)	$-3s_2$ (.001)
$-3s_1$	$-2s_2$	$-3s_1$ (.01)	$-2s_2$ (.001)
$-3s_1$	$-1s_2$	$-3s_1$ (.01)	$-1s_2$
$-2s_2$	$-3s_2$	$-2s_2$	$-3s_2$
$-2s_2$	$-2s_2$	$-2s_2$	$-2s_2$
$-2s_2$	$-1s_2$	$-2s_2$	$-1s_2$
$-1s_2$	$-3s_2$	$-1s_2$	$-3s_2$
$-1s_2$	$-2s_2$	$-1s_2$	$-2s_2$
$-1s_2$	$-1s_2$	$-1s_2$	$-1s_2$

Table A. 3 Shifts of Proportion Nonconforming for Process with In-control $p_1 = 0.1$, $p_2 = 0.1$

Correlation Coefficient = 0.8

Test Set Data Mean Shift (Sample Size = 100)		Test Set Data Mean Shift (Sample Size = 30)	
$+3s_1$	$+3s_2$	$+3s_1$	$+3s_2$
$+3s_1$	$+2s_2$	$+3s_1$	$+2s_2$
$+3s_1$	$+1s_2$	$+3s_1$	$+1s_2$
$+2s_1$	$+2s_2$	$+2s_1$	$+2s_2$
$+2s_1$	$+1s_2$	$+2s_1$	$+1s_2$
$+1s_1$	$+1s_2$	$+1s_1$	$+1s_2$
$-3s_1$	$-3s_2$	$-3s_1$ (.001)	$-3s_2$ (.001)
$-3s_1$	$-2s_2$	$-3s_1$ (.001)	$-2s_2$ (.001)
$-3s_1$	$-1s_2$	$-3s_1$	$-1s_2$
$-2s_1$	$-2s_2$	$-2s_1$ (.001)	$-2s_2$ (.001)
$-2s_1$	$-1s_2$	$-2s_1$	$-1s_2$
$-1s_1$	$-1s_2$	$-1s_1$	$-1s_2$

Correlation Coefficient = 0.5

Test Set Data Mean Shift (Sample Size = 100)		Test Set Data Mean Shift (Sample Size = 30)	
$+3s_1$	$+3s_2$	$+3s_1$	$+3s_2$
$+3s_1$	$+2s_2$	$+3s_1$	$+2s_2$
$+3s_1$	$+1s_2$	$+3s_1$	$+1s_2$
$+2s_1$	$+2s_2$	$+2s_1$	$+2s_2$
$+2s_1$	$+1s_2$	$+2s_1$	$+1s_2$
$+1s_1$	$+1s_2$	$+1s_1$	$+1s_2$
$-3s_1$	$-3s_2$	$-3s_1$ (.001)	$-3s_2$ (.001)
$-3s_1$	$-2s_2$	$-3s_1$ (.001)	$-2s_2$ (.001)
$-3s_1$	$-1s_2$	$-3s_1$	$-1s_2$
$-2s_1$	$-2s_2$	$-2s_1$ (.001)	$-2s_2$ (.001)
$-2s_1$	$-1s_2$	$-2s_1$	$-1s_2$
$-1s_1$	$-1s_2$	$-1s_1$	$-1s_2$

Correlation Coefficient = 0.2

Test Set Data Mean Shift (Sample Size = 100)		Test Set Data Mean Shift (Sample Size = 30)	
$+3s_1$	$+3s_2$	$+3s_1$	$+3s_2$
$+3s_1$	$+2s_2$	$+3s_1$	$+2s_2$
$+3s_1$	$+1s_2$	$+3s_1$	$+1s_2$
$+2s_1$	$+2s_2$	$+2s_1$	$+2s_2$
$+2s_1$	$+1s_2$	$+2s_1$	$+1s_2$
$+1s_1$	$+1s_2$	$+1s_1$	$+1s_2$
$-3s_1$	$-3s_2$	$-3s_1$ (.001)	$-3s_2$ (.001)
$-3s_1$	$-2s_2$	$-3s_1$ (.001)	$-2s_2$ (.001)
$-3s_1$	$-1s_2$	$-3s_1$	$-1s_2$
$-2s_1$	$-2s_2$	$-2s_1$ (.001)	$-2s_2$ (.001)
$-2s_1$	$-1s_2$	$-2s_1$	$-1s_2$
$-1s_1$	$-1s_2$	$-1s_1$	$-1s_2$

Table A. 4 Shifts of Proportion Nonconforming for Process with In-control $p_1 = 0.01$, $p_2 = 0.01$

Correlation Coefficient = 0.8

Test Set Data Mean Shift (Sample Size = 910)		Test Set Data Mean Shift (Sample Size = 810)		Test Set Data Mean Shift (Sample Size = 300)	
$+3s_1$	$+3s_2$	$+3s_1$	$+3s_2$	$+3s_1$	$+3s_2$
$+3s_1$	$+2s_2$	$+3s_1$	$+2s_2$	$+3s_1$	$+2s_2$
$+3s_1$	$+1s_2$	$+3s_1$	$+1s_2$	$+3s_1$	$+1s_2$
$+2s_1$	$+2s_2$	$+2s_1$	$+2s_2$	$+2s_1$	$+2s_2$
$+2s_1$	$+1s_2$	$+2s_1$	$+1s_2$	$+2s_1$	$+1s_2$
$+1s_1$	$+1s_2$	$+1s_1$	$+1s_2$	$+1s_1$	$+1s_2$
$-3s_1$	$-3s_2$	$-3s_1$ (.001)	$-3s_2$ (.001)	$-3s_1$ (.001)	$-3s_2$ (.001)
$-3s_1$	$-2s_2$	$-3s_1$	$-2s_2$	$-3s_1$ (.001)	$-2s_2$ (.001)
$-3s_1$	$-1s_2$	$-3s_1$	$-1s_2$	$-3s_1$	$-1s_2$
$-2s_1$	$-2s_2$	$-2s_1$	$-2s_2$	$-2s_1$ (.001)	$-2s_2$ (.001)
$-2s_1$	$-1s_2$	$-2s_1$	$-1s_2$	$-2s_1$	$-1s_2$
$-1s_1$	$-1s_2$	$-1s_1$	$-1s_2$	$-1s_1$	$-1s_2$

Correlation Coefficient = 0.5

Test Set Data Mean Shift (Sample Size = 910)		Test Set Data Mean Shift (Sample Size = 670)		Test Set Data Mean Shift (Sample Size = 300)	
$+3s_1$	$+3s_2$	$+3s_1$	$+3s_2$	$+3s_1$	$+3s_2$
$+3s_1$	$+2s_2$	$+3s_1$	$+2s_2$	$+3s_1$	$+2s_2$
$+3s_1$	$+1s_2$	$+3s_1$	$+1s_2$	$+3s_1$	$+1s_2$
$+2s_1$	$+2s_2$	$+2s_1$	$+2s_2$	$+2s_1$	$+2s_2$
$+2s_1$	$+1s_2$	$+2s_1$	$+1s_2$	$+2s_1$	$+1s_2$
$+1s_1$	$+1s_2$	$+1s_1$	$+1s_2$	$+1s_1$	$+1s_2$
$-3s_1$	$-3s_2$	$-3s_1$ (.001)	$-3s_2$ (.001)	$-3s_1$ (.001)	$-3s_2$ (.001)
$-3s_1$	$-2s_2$	$-3s_1$ (.001)	$-2s_2$	$-3s_1$ (.001)	$-2s_2$ (.001)
$-3s_1$	$-1s_2$	$-3s_1$	$-1s_2$	$-3s_1$	$-1s_2$
$-2s_1$	$-2s_2$	$-2s_1$	$-2s_2$	$-2s_1$ (.001)	$-2s_2$ (.001)
$-2s_1$	$-1s_2$	$-2s_1$	$-1s_2$	$-2s_1$	$-1s_2$
$-1s_1$	$-1s_2$	$-1s_1$	$-1s_2$	$-1s_1$	$-1s_2$

Correlation Coefficient = 0.2

Test Set Data Mean Shift (Sample Size = 910)		Test Set Data Mean Shift (Sample Size = 540)		Test Set Data Mean Shift (Sample Size = 300)	
$+3s_1$	$+3s_2$	$+3s_1$	$+3s_2$	$+3s_1$	$+3s_2$
$+3s_1$	$+2s_2$	$+3s_1$	$+2s_2$	$+3s_1$	$+2s_2$
$+3s_1$	$+1s_2$	$+3s_1$	$+1s_2$	$+3s_1$	$+1s_2$
$+2s_1$	$+2s_2$	$+2s_1$	$+2s_2$	$+2s_1$	$+2s_2$
$+2s_1$	$+1s_2$	$+2s_1$	$+1s_2$	$+2s_1$	$+1s_2$
$+1s_1$	$+1s_2$	$+1s_1$	$+1s_2$	$+1s_1$	$+1s_2$
$-3s_1$	$-3s_2$	$-3s_1$ (.001)	$-3s_2$ (.001)	$-3s_1$ (.001)	$-3s_2$ (.001)
$-3s_1$	$-2s_2$	$-3s_1$ (.001)	$-2s_2$	$-3s_1$ (.001)	$-2s_2$ (.001)
$-3s_1$	$-1s_2$	$-3s_1$ (.001)	$-1s_2$	$-3s_1$ (.001)	$-1s_2$
$-2s_1$	$-2s_2$	$-2s_1$	$-2s_2$	$-2s_2$ (.001)	$-2s_2$ (.001)
$-2s_1$	$-1s_2$	$-2s_1$	$-1s_2$	$-2s_2$ (.001)	$-1s_2$
$-1s_1$	$-1s_2$	$-1s_1$	$-1s_2$	$-1s_1$	$-1s_2$

APPENDIX B

VALIDATION OF DATA GENERATED

Table B. 1 Number of Null Hypothesis Accepted for Process Condition: p1=0.3, p2=0.3, Correlation Coefficient = 0.8, Sample Sizes = 50.

Data Set	Number of Null Hypothesis Accepted out of 10 Replications								
	In-control Status			Out-of-Control Status (Means shift 3σ in the positive direction.)			Out-of-Control Status (Means shift 3σ in the negative direction.)		
	$p_1=p_{10}$	$p_2=p_{20}$	Corr.=$Corr_0$	$p_1=p_{10}$	$p_2=p_{20}$	Corr.=$Corr._0$	$p_1=p_{10}$	$p_2=p_{20}$	Corr.=$Corr._0$
Training	10	10	10	10	10	10	10	10	9
Testing	10	10	10	10	10	9	10	10	10

Table B. 2 Number of Null Hypothesis Accepted for Process Condition: p1=0.01, p2=0.01, Correlation Coefficient = 0.8, Sample Sizes = 910.

Data Set	Number of Null Hypothesis Accepted out of 10 Replications								
	In-control Status			Out-of-Control Status (Means shift 3σ in the positive direction.)			Out-of-Control Status (Means shift 3σ in the negative direction.)		
	$p_1=p_{10}$	$p_2=p_{20}$	Corr.=$Corr_0$	$p_1=p_{10}$	$p_2=p_{20}$	Corr.=$Corr._0$	$p_1=p_{10}$	$p_2=p_{20}$	Corr.=$Corr._0$
Training	10	10	9	10	10	10	10	10	7
Testing	10	10	9	10	10	10	10	10	8

Table B. 3 Number of Null Hypothesis Accepted for Process Condition: p1=0.3, p2=0.1, Correlation Coefficient = 0.2, Sample Sizes = 100.

Data Set	Number of Null Hypothesis Accepted out of 10 Replications								
	In-control Status			Out-of-Control Status (Means shift 3σ in the positive direction.)			Out-of-Control Status (Means shift 3σ in the negative direction.)		
	$p_1=p_{10}$	$p_2=p_{20}$	Corr.=$Corr_0$	$p_1=p_{10}$	$p_2=p_{20}$	Corr.=$Corr._0$	$p_1=p_{10}$	$p_2=p_{20}$	Corr.=$Corr._0$
Training	10	10	10	10	10	10	10	10	10
Testing	10	10	10	10	10	9	10	10	10

Table B. 4 Number of Null Hypothesis Accepted for Process Condition: p1=0.3, p2=0.3, Correlation Coefficient = 0.8, Sample Sizes = 10.

Data Set	Number of Null Hypothesis Accepted out of 10 Replications								
	In-control Status			Out-of-Control Status (Means shift 3σ in the positive direction.)			Out-of-Control Status (Means shift 3σ in the negative direction.)		
	$p_1=p_{10}$	$p_2=p_{20}$	$Corr.=Corr_0$	$p_1=p_{10}$	$p_2=p_{20}$	$Corr.=Corr._0$	$p_1=p_{10}$	$p_2=p_{20}$	$Corr.=Corr._0$
Training	10	10	10	10	10	10	10	10	4
Testing	10	10	10	10	10	10	10	10	7

Table B. 5 Number of Null Hypothesis Accepted for Process Condition: p1=0.3, p2=0.3, Correlation Coefficient = 0.2, Sample Sizes = 10.

Data Set	Number of Null Hypothesis Accepted out of 10 Replications								
	In-control Status			Out-of-Control Status (Means shift 3σ in the positive direction.)			Out-of-Control Status (Means shift 3σ in the negative direction.)		
	$p_1=p_{10}$	$p_2=p_{20}$	$Corr.=Corr_0$	$p_1=p_{10}$	$p_2=p_{20}$	$Corr.=Corr._0$	$p_1=p_{10}$	$p_2=p_{20}$	$Corr.=Corr._0$
Training	10	10	9	10	10	10	10	10	8
Testing	10	10	9	10	10	10	7	10	10

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