THE EFFECT OF TIME DELAY ON THE ABILITY TO CONTROL UNSTABLE SYSTEMS

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This experiment examined the effect of time delays on the ability to stabilize an inverted pendulum. The results were determined by examining the amount of time the system's oscillations remained bounded before increasing exponentially. This was done for each combination of time delay and pendulum length covered in this experiment. The results showed that for a pendulum length of 49 meters, all time delays in the identified range were able to be stabilized. The results also showed that for a pendulum length of 2.45 meters, no time delays in the identified range were able to be stabilized.

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1.0 INTRODUCTION

For many years, research has been conducted regarding manual control. Many of these years have been spent studying manual control of unstable systems. The purpose of this past and present research is to understand the characteristics of human controllers so that these traits can be modeled and used in creation of robotics to assist with tasks that cannot be directly performed by humans.

Pioneers, like McRuer, Young, Gaines, and Smith led the way early on in the late 1950's and 1960's with the development of human controller modeling, with their work in References [6], [14], [2], and [11], respectively. The focus did not wane as Siegel and Wolf, and Miller and Swain studied the reliability of the human controller models in References [10] and [7], respectively.

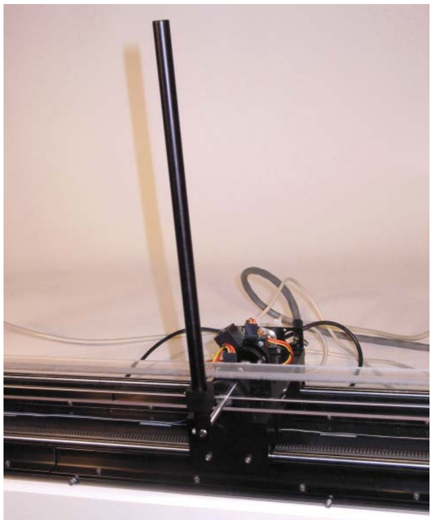

Figure 1. Inverted Pendulum

Previous research has identified numerous applications where robotics are already used, and many more where they would make the task easier to perform. In several research experiments, an inverted pendulum represents the unstable system, similar to the one shown above in Figure 1.

The results of previous and existing research have many real-life applications. For example, robotics that are human controlled have many uses in the medical field, including telesurgery and artificial limbs. If a surgeon can perform remote surgery, the diversity of patients that can be helped will dramatically increase. It is not unrealistic to believe that a surgeon in one hospital may be able to operate on a patient in any hospital. Additionally, human controlled artificial arms, legs, hands or feet can help amputees improve their quality of life by replacing a missing limb.

Existing research has examined the errors associated with delays internal to the human controller. The goal of this experiment is to determine the effect of the time delay on the ability to stabilize the inverted pendulum for different lengths of pendulums. This paper presents a definition of stability for the purpose of the experiment.

2.0 BACKGROUND

The following subsections will contain a summarized history of the research performed relative to the human controller, discussion of the wide range of applications where similar research has already benefited, and how this research can impact potential future applications.

2.1 HISTORICAL PERSPECTIVE

Reference [8] provides a historical summary of the development of human models. It begins discussing Tustin's research in 1947 attempting to integrate human control with servomechanisms, in the use of making gun turrets easier to use, described in Reference [13]. McRuer and Krendel, in Reference [6], present a transfer function representation of the human operator. In Reference [8], Pew summarizes McRuer's points, that for aircraft applications, the handling qualities of the aircraft could be predicted using control system representations of the human controller and the response of the aircraft.

In Reference [8], Pew describes the work of Miller and Swain, and of Siegel and Wolf regarding human reliability analyses. In Reference [7], Miller and Swain were attempting to provide evidence to back up predictions of the error of a human in normal operations. In Reference [10], Siegel and Wolf created a task network that described the behavior of a man-machine system where they calculated the probability of success and estimated completion times.

This research led to the early applications described in Section 2.2.1. In Reference [14], Young and Meiry discuss manual control of high-order systems. Specifically, they investigate an on-off controller, which is examined in further detail in this paper. Reference [14] focuses on the comparison of a three-mode switch to a linear control stick and the error associated with manual control of high-order systems. The results and conclusions indicate that the operators exhibit better control when using the bang-bang method. For the on-off model replicated for use in the following experiment, Young and Meiry determined the switching lines and error trajectories. In Reference [2], Gaines describes the bang-bang model as a method for controlling unstable high-order systems. Gaines refers to the bang-bang model discussed by Young and Meiry in Reference [14].

Smith presents the limits that manual control can be used to control second order oscillatory systems in Reference [11]. The limits examined in Reference [11] correspond to the limits dependent on the relationship between damping and stiffness terms in the inverted pendulum. The stiffness term is related to the length of the inverted pendulum. MacKenzie and Ware state in Reference [4] that there is very little data that measures the effect of lag on human performance.

2.2 APPLICATIONS

2.2.1 Past Applications

Sheridan describes in Reference [9] many applications where the information presented by the authors previously discussed is used. Some of the applications discussed by Sheridan are summarized in the following subsections.

2.2.1.1 Space

Sheridan discusses a remote manipulator system (RMS) which is used for moving heavy loads outside the space shuttle in outer space. It is directly controlled by a human operator with two three-directional joysticks. One joystick controls three directions of translations while the other controls three directions of rotations. The human operator controls the system by watching the system through a window or over video.

Another example discussed by Sheridan is a flight telerobotic servicer (FTS), which is a general purpose manipulation device. At the time of publishing of Reference [9], final designs had not been finalized. McCain, Andary, Hewitt, and Spidaliere discuss the design and evolution in Reference [5]. The FTS has two "arms" with seven degrees of freedom and one "leg" with six degrees of freedom. At the time of initial design, most functions of the FTS are human-controlled with five video cameras to provide feedback. At the time of publishing Reference [5], plans existed to upgrade the hardware and software to automate more of the FTS functionality to ease the burden on the human operator. The FTS will be used to perform various maintenance tasks on Space Station Freedom.

2.2.1.2 Undersea

Remote operated vehicles (ROVs) are discussed by Sheridan in Reference [9] as the best example of submersible telerobotics. ROVs have been used in off-shore oil operations and in marine biologist research. In oil operations, ROVs have been used to monitor pipelines, place anodes and inspect welds on structures below the surface. Marine biologists use ROVs to conduct scientific investigations from the surface. ROVs used for these purposes require much more dexterity and maneuvering ability than in the oil operations.

2.2.1.3 Nuclear Power

Sheridan discusses the use for telerobotics in nuclear power applications. At the time of publication of Reference [9], telerobotics were just being introduced to the nuclear power field, but Sheridan does mention that teleoperators are becoming dexterous enough that they can be used for steam tube monitoring and maintenance.

2.2.2 Potential Future Applications

Sheridan describes in Reference [9] many applications in which telerobotics will be used in the future. The results presented in Section 4.0 of this experiment should prove useful in some of the following applications. It is likely that the below applications will involve unstable systems.

2.2.2.1 Surgery

In Reference [9], Sheridan discusses the beginning uses of telerobotics in surgical procedures. At the time of this publishing, endoscopes and arthroscopes were used to inspect and perform minor surgeries. Hokayem and Spong also discuss the telesurgery application in

Reference [3]. They acknowledge the fact that telesurgery allows physicians to provide medical expertise without traveling to the patient. At the time of the Reference [3] publication, remote telesurgery experiments had been reported between Italy and the USA.

2.2.2.2 Firefighting

At the time of Reference [9], Sheridan discusses that no use of telerobotics had been reported in firefighting applications. However, Sheridan estimates that firefighters would soon use telerobotics to extend their vision and hearing within a hazardous situation. With this additional information, firefighters could be more effective while taking fewer risks.

2.2.2.3 Military Operations

Similar to the uses in firefighting, Sheridan states in Reference [9] that the military has shown interest in telerobotics in order to assist soldiers in extending their vision in dangerous situations. Sheridan discusses that navigating mine fields, observation of enemy operations and remotely piloted aircraft as uses for telerobotics in different military operations.

3.0 METHODS

The following sections describe the methods used to develop the relationship of stability between the time delay in the human model and the length of the inverted pendulum. Reference [12] states that the inverted pendulum can be described by Equation (1).

$$(I + mL^2)\frac{d^2\theta}{dt^2} + mL\cos(\theta)\frac{d^2x}{dt^2} = mgL\sin(\theta) \tag{1}$$

For small angle disturbances, like the disturbance being investigated in this experiment, Equation (1) simplifies to Equation (2).

$$\frac{4}{3}L\frac{d^2\theta}{dt^2} - g\theta = -\frac{d^2x}{dt^2} \tag{2}$$

The transfer function of Equation (2) results in Equation (3).

$$\frac{\Theta(s)}{X(s)} = \frac{-s^2}{(4/3)Ls^2 - g} \tag{3}$$

When the length of the pendulum is related to the parameter ω in Equation (4) and substituted into the transfer function of Equation (3), the transfer function of the inverted pendulum is shown in Equation (5). This is the model that is used in this experiment and matches the model described by Young in Reference [14]. There is an extra K in the numerator of Young's model, which was described as the system gain, K = 2, to represent a maximum specific torque capability of $\pm 90\omega^2$ deg/sec^2. This system gain was maintained in this experiment.

$$L = \frac{3g}{4\omega^2} \tag{4}$$

$$\frac{\Theta(s)}{X(s)} = \frac{K\omega^2}{s^2 + \omega^2} \tag{5}$$

3.1 INVERTED PENDULUM SYSTEM

The model used for the human controller is the model of the on-off controller depicted in Figure 2, from Reference [14]. The human model consists of the lead time differentiator, on-off controller, noise function and time delay. The inverted pendulum is represented by the model in Equation (5).

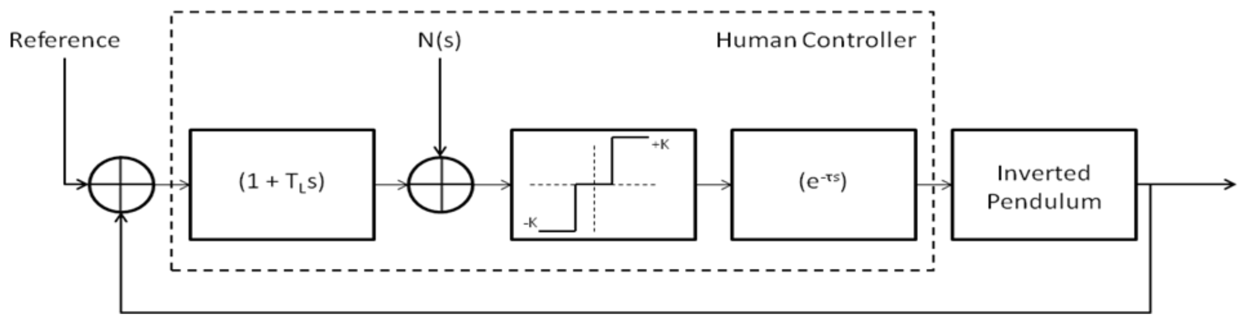

Figure 2. On-Off controller model

3.2 MATLAB SIMULINK DESCRIPTION

MATLAB's Simulink was used to simulate the On-Off controller depicted in Figure 2. The Simulink model is shown in Figure 3. There were some modifications made to the Figure 2 model in order to implement the system in MATLAB. The implementation of these modifications is described in the following subsections.

The reference angle used by the model is zero degrees, measured from the inverted pendulum, when the pendulum is perpendicular to the ground.

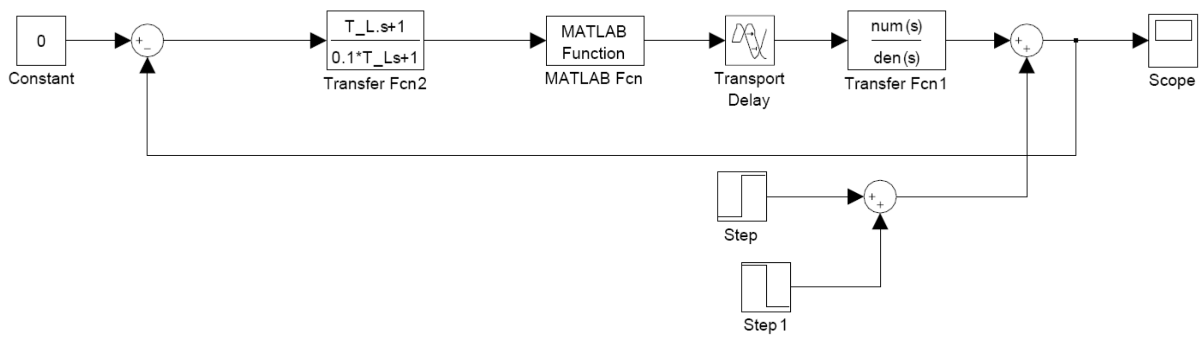

Figure 3. Simulink Model of On-Off Controller

3.2.1 Three-Level Controller

A MATLAB function was used to represent the three-level controller, with possible outputs +*K*, 0, and -*K*. The operation of this function is provided below. The three-level controller has a threshold where, for small inputs, the output will be 0. This threshold was chosen to be 0.1, based on the values of the other parameters in the experiment.

$$f(u) = \begin{cases} K, \text{ if } u > threshold \\ -K, \text{ if } u < -threshold \\ 0, \text{ otherwise} \end{cases}$$

3.2.2 Disturbance and Noise

The noise was ignored in this experiment and a disturbance was added at the output of the system to simulate a one-time disturbance as a pulse of magnitude 0.5 and duration 0.1 seconds. The implications of the disturbance and noise are discussed in Section 5.2.

3.2.3 Low-pass Filter on Lead-Time Parameter

A low-pass filter was added to the lead-time block because of MATLAB errors and warnings regarding differentiators. The low-pass filter has no effect on the system other than preventing the differentiator from becoming unbounded.

3.3 DETERMINING OPTIMAL LEAD TIME AND GAIN

The first step was to determine the optimal parameters for the lead time, T_L, and the gain, K. Different ranges of the lead time and gain were modified until finding the combination of parameters that resulted in the longest stable system. The longest stable system is defined by the amount of time that the system oscillates without becoming unbounded.

3.4 TIME DELAY VERSUS LENGTH OF PENDULUM

The optimal lead time and gain parameters were used when determining the relationship of stability between the time delay, τ, and the length of the inverted pendulum, L. The length of the inverted pendulum is related to ω in Equation (4) by the relationship displayed in Equation (5). Once determining the optimal parameters for the lead time and gain, the pendulum lengths listed in Table 1 were evaluated for length of stability across a range of time delays, τ.

Simulations were repeated to determine the duration of stability for each combination of length of pendulum, in Table 1, and each time delay, in Table 2. The time delays used for this experiment are defined in Reference [14], ranging from 0.095 to 1.1, which agrees with the delays used in most quasi-linear approximations.

Table 1. Pendulum Lengths

L (m)	ω^2
147	0.05
49	0.15
29.4	0.25
14.7	0.5
7.35	1.0
3.675	2.0
2.45	3.0
1.8375	4.0

The stability of the inverted pendulum system was then evaluated for each of the length and time delay combinations. As stated above, the stability was determined by the amount of time the simulation ran while the oscillations remained bounded. The system was considered more stable the longer the simulation ran.

Table 2. Time Delays

τ
0.095
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1

4.0 RESULTS

The results presented in the following subsections indicate that it is possible for a human to stabilize an unstable system, an inverted pendulum in this case, for some amount of time. The human model uses the on-off controller described above. It is evident that the time delay and length of pendulum are major factors in the amount of time that the system can be stabilized.

4.1 OPTIMAL LEAD TIME AND GAIN

As discussed in Section 3.2, the lead time, T_L, and gain, K, were varied to determine the optimal parameters in ideal conditions. Ideal conditions were defined as zero time delay ($\tau = 0$) and zero noise ($n(t) = 0$). The parameters that resulted in the longest stability were lead time of 15.2 and gain of 0.5.

Under these ideal conditions, Figure 4 plots the angle of the inverted pendulum versus time when the length of the pendulum is $L = 1.8375$ m (6.0285 ft). The system was able to oscillate within bounds until $t = 16$ seconds. Under these same conditions, the system was able to oscillate within bounds until $t = 55$ seconds when $L = 14.7$ m (48.228 ft), in Figure 5.

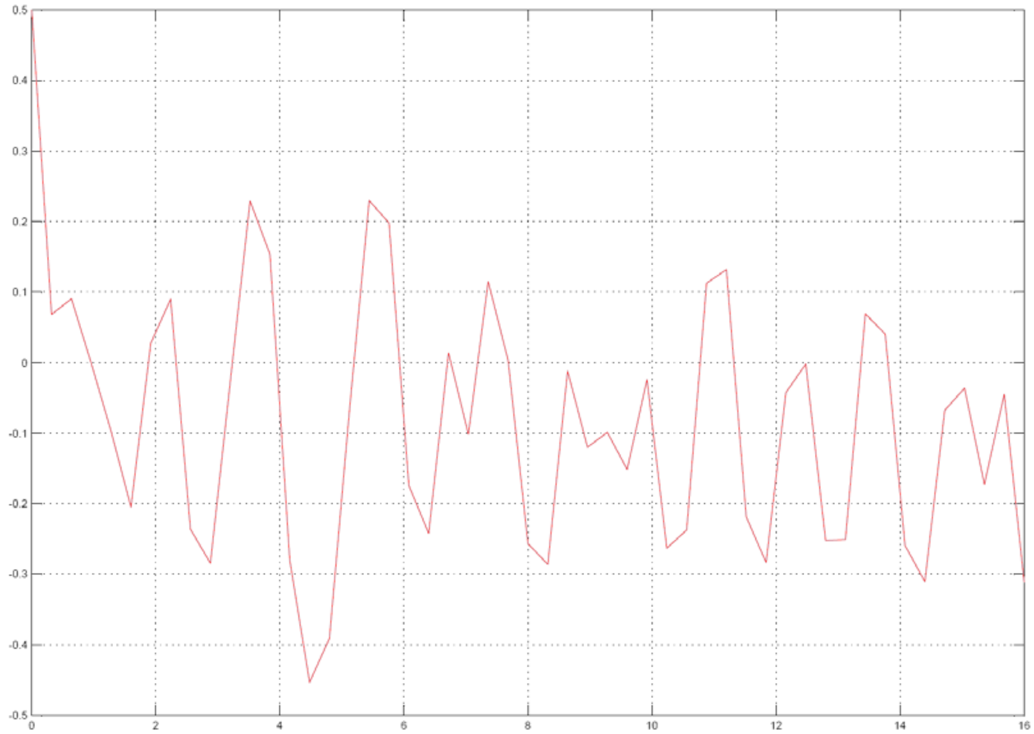

Figure 4. Response of Inverted Pendulum with Optimal Lead Time and Gain (L = 1.8375 m)

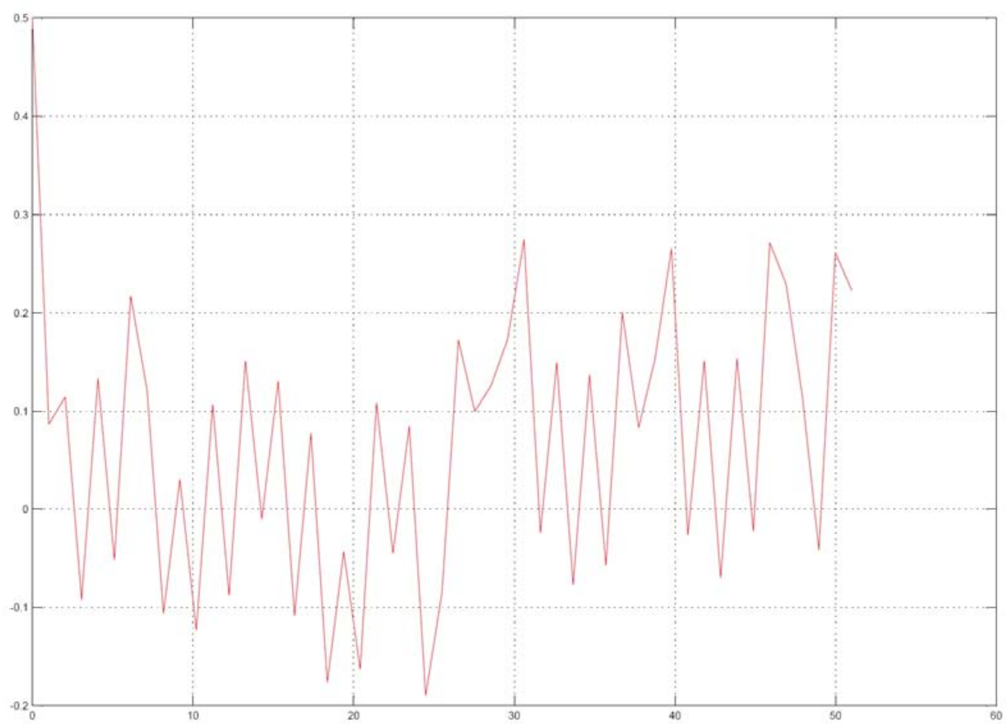

Figure 5. Response of Inverted Pendulum with Optimal Lead Time and Gain (L = 14.7 m)

4.2 TIME DELAY VERSUS LENGTH OF PENDULUM

As described by Section 3.4, the degree of stability was determined for different combinations of time delay, τ, and length of pendulum, L. Table 1 shows the different pendulum lengths (in meters) and Table 2 shows the different time delays used in this experiment. For each combination of time delay and length, Table 3 contains the amount of time the output remained bounded before becoming unstable. The entries in Table 3 that are labeled "N/A" were not recorded because the system was considered stable for those ranges of time delays, τ.

Table 3. Maximum time before becoming unstable

$\tau\downarrow$ $\omega^2\rightarrow$	0.05	0.15	0.25	0.5	1.0	2.0	3.0	4.0
0.095	96	42	30	27	18	10	5	6
0.1	N/A	N/A	N/A	25	15	9	5	6
0.2	N/A	N/A	N/A	20	11	8	5	3
0.3	N/A	N/A	N/A	19	10	4	1	1
0.4	N/A	N/A	N/A	16	8	1	1	1
0.5	N/A	N/A	20	12	7	1	1	1
0.6	N/A	N/A	19	11	1	1	1	1
0.7	N/A	N/A	17	9	1	1	1	1
0.8	N/A	N/A	14	1	1	1	1	1
0.9	N/A	N/A	12	1	1	1	1	1
1.0	N/A	N/A	9	1	1	1	1	1
1.1	67	22	9	1	1	1	1	1

4.3 DEGREE OF STABILITY

The degree of stability, as defined in Section 3.2, is the maximum amount of time that the oscillations remain bounded before the system becomes unstable. In order to consider the system stable, the minimum amount of time for the oscillations to remain stable was assumed to be t = 10 seconds. This threshold was established to determine which combinations of lengths and time delays would be considered stable. Other lengths of time were considered, but t = 10 seconds is a logical choice when considering a human controller of an unstable, inverted pendulum system.

Based on the assumption described above, the entries in Table 3 that are labeled "N/A" were not recorded because the system was considered stable for those ranges of time delays, τ.

Table 3 was evaluated to determine which combinations of time delay and length maintained bounded oscillations for at least 10 seconds. Table 4 displays the maximum amount of time delay that still allowed the system to stabilize for each length listed in Table 1. To display the relationship between the time delay and natural frequency, the information in Table 4 is plotted in Figure 6.

Table 4. Maximum amount of time delay

ω^2	Maximum τ for stable control of system
0.05	1.1
0.15	1.1
0.25	0.9
0.5	0.6
1.0	0.3
2.0	0.095
3.0	0
4.0	0

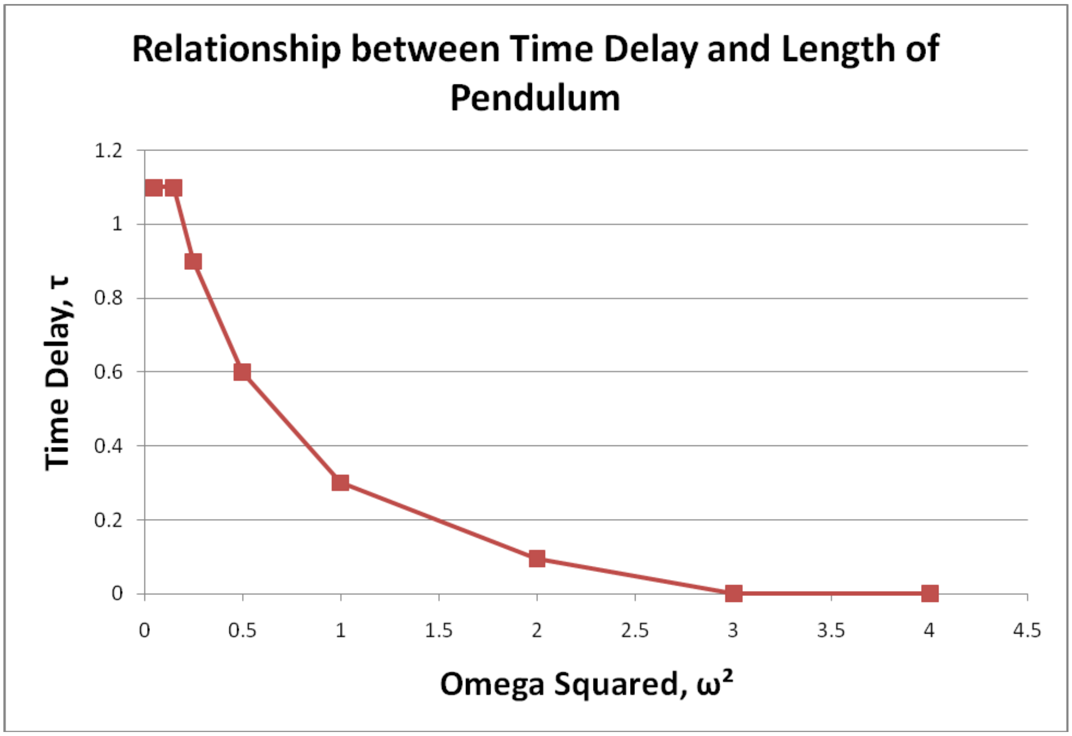

Figure 6. Relationship between Time Delay and Length

5.0 DISCUSSION

The results presented in Section 4.0 align with the expected behavior of an inverted pendulum system. When trying to stabilize an inverted pendulum, it has been proven that the task is easier when using a longer pendulum. As described by Equation (5), the length and the parameter ω^2 are inversely proportional, meaning that the inverted pendulum becomes easier to control with decreasing values of ω^2.

5.1 ANALYSIS OF THE RESULTS

5.1.1 Optimal Lead Time and Gain Parameters

The optimal lead time, T_L, and gain, K, were determined to be 15.2 and 0.5, respectively. The lead time parameter is the coefficient of the derivative term, and its magnitude is larger than expected. This is likely due to the small rates of change in the pendulum, which are easier to control than if the position of the pendulum was changing rapidly.

The gain approximated what was expected from the three-level controller. A larger gain would allow the controller to recover from larger disturbances, but it would be difficult to stabilize because the gain would force an overshoot past the equilibrium with each adjustment. A smaller gain would allow the controller to hone in on the equilibrium, but would not be able to recover from larger disturbances. The gain of 0.5 provided balance between the advantages of both larger and smaller gains.

A portion of the gain parameter that should not be overlooked is the threshold; a threshold value of 0.1 was used for this experiment, meaning that if the magnitude of the input to the three-level controller was less than 0.1, no action would be taken by the human and the output of the three-level controller would be 0. Larger and smaller values of threshold provide the same advantages and disadvantages as the gain.

5.1.2 Effect of Time Delay on Ability to Stabilize

Table 4 and Figure 6 prove that the inverted pendulum can be stabilized with larger time delays when the length of the pendulum is longer. To highlight the impact of the time delay parameter, the results indicate that a pendulum as long as 2.45 m (8.038 ft) cannot be stabilized with the minimum time delay, $\tau = 0.095$. When assuming a maximum time delay, $\tau = 1.1$, the pendulum length must be greater than 29.4 m (96.456 ft) in order to stabilize. Even at other definitions of stability, the pendulum length is required to be unbelievably large in order to stabilize the system for any length of time at this time delay.

Because the delay has so much impact on the ability to stabilize the inverted pendulum, it is important to try to minimize delays. It may be possible to reduce the internal delays through practice and repetition of the task, which may improve the lead time parameter and help to predict future movements.

5.2 OTHER CONSIDERATIONS

5.2.1 Range of Disturbance

The inverted pendulum has a limited range of disturbances that the control system can stabilize. If the disturbance is too large, the controller will not be able to recover and stabilize the system, even under ideal conditions, with no time delay and a pendulum of great length. The maximum magnitude of disturbance was not investigated during this experiment.

5.2.2 Noise Internal to Human Controller

As in most applications, even small magnitudes of noise handicap the ability of the controller to stabilize the pendulum. This experiment did not account for noise internal to the on-off human controller, as shown in Figure 2, Young's model in Reference [14]. However, Young may have assumed that the disturbances would have occurred internal to the human controller, which would be represented by the noise. This experiment moved the disturbance outside of the human controller and inserted it at the output of the system, to allow for external disturbances to affect the system. The disturbance used was a one-time pulse, as described in Section 3.1.

5.2.3 Extension beyond Human Controller

The time delays in this experiment were restricted to conventional time delays for a human controller, as stated in Reference [14]. It is possible that there will be other delays unrelated to the human control. If controlling a remote device, there could be communication link delays or processor delays if the device must process the received information. These external delays were not investigated, but could cause the time delay, τ, to extend beyond the range considered in this experiment.

6.0 CONCLUSION

This research determined the minimum length of pendulum that can be stabilized for given time delays. The delays examined ranged from 0.095 to 1.1 seconds, considered well-defined assumptions by Reference [14]. These results should prove useful in the development of telerobotics systems for the applications defined previously in this paper. In the field of telesurgery, the use of endoscopes reduces the risk of infection during surgery and decrease the post-surgery healing time. In addition, doctors may be able to perform remote surgeries from afar. This research was conducted to understand manual control so that it can be modeled and used in development of telerobotic systems to assist where humans cannot directly perform a particular task.

Future efforts in this area will most likely extend to the boundaries of this controller, investigating the maximum disturbance from which the controller can recover. Additional work could also investigate delays external to the human controller, such as communication links.

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