A STUDY OF HIGH SCHOOL MATHEMATICS TEACHERS' ABILITY TO IDENTIFY AND CREATE QUESTIONS THAT SUPPORT STUDENTS' UNDERSTANDING OF MATHEMATICS

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This study analyzed changes in high school teachers' ability to identify and create questions that support mathematical understanding as they were participating in a professional development program focused on planning, teaching and reflecting on lessons featuring cognitively challenging tasks. The 35 participants were a subset of nearly 100 high school mathematics teachers from a large urban district participating in the professional development program.

Data related to questioning abilities was collected via a pre- and post-test that situated questioning within the practice of teaching Algebra. To account for changes in teachers' abilities related to questioning, demographic data describing the participants was collected. In addition, attendance sheets, agendas and materials from the professional development sessions and responses to two prompts at the conclusion of the program were collected

Analysis of data related to questioning indicated that participants significantly increased their abilities to identify and create questions that promote understanding of mathematics, particularly questions that prompt students to explore mathematical ideas

and connections. Asking this type of question has been linked to increased student achievement. However, most teachers rarely, if ever, ask this type of question.

An analysis of demographic data showed that the significant changes in teachers' questioning abilities were not associated with years of teaching experience or the high school or sub-district at which the teacher taught. In addition, analysis of data from the professional development program indicated that changes in teachers' questioning abilities were not associated with any one of the four facilitators of the professional development sessions.

Participants in the study, as well as teachers not participating in the study but participating in the professional development program, had a high attendance rate for the professional development sessions. During these sessions, teachers had a variety of opportunities to learn about and discuss aspects of questioning, including solving and discussing challenging mathematics tasks; analyzing and discussing episodes of teaching Algebra; analyzing and generating questions for student work from Algebra classrooms; and analyzing and planning lessons related to the Algebra curriculum.

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1. CHAPTER ONE: THE RESEARCH PROBLEM

1.1. INTRODUCTION

For the past decade, the poor mathematics performance of students in the U. S. as compared to other students internationally has been a source of concern, both in the educational community and in the political arena. The way in which mathematics is taught in the U. S. has been cited as a major contributing factor to this level of performance. When comparing mathematics instruction in the U.S. to that of other countries, Hiebert (1999, p. 12) noted, "Students learn what they have an opportunity to learn." Students' opportunities to learn mathematics in the U. S. consist primarily of memorizing procedures, terms, and definitions without the opportunity to discover or engage in considering why mathematical processes work. This form of instructional practice of mathematics teachers in the U.S. has remained quite consistent and predictable for nearly 100 years (Hiebert, 1999). Many authors (Hiebert, 1999; Stigler & Hiebert, 1999; USDE, 2000; Ball, Lubienski, & Mewborn, 2001; NCES, 2003) have described the typical mathematics lesson in the United States as being some form of the teacher reviewing the previous day's assignment, explaining or demonstrating new concepts or procedures, assigning practice problems to be worked on individually, and then assigning homework. Little, if any, time is spent developing conceptual understanding, connecting mathematical ideas, or engaging students in discussing mathematics. Thus, it is not surprising that U.S. students struggle to solve problems that require an understanding beyond memorization of procedures as is illustrated on national and international mathematics assessments (e.g. NCES, 2003).

The type of mathematics instruction that is lacking in many U. S. mathematics classrooms is also the type of instruction that has been associated with increased achievement in mathematics in other countries (Stigler & Hiebert, 1999). Recent research suggests that good instructional practice in mathematics focuses on student learning and understanding by engaging all students in challenging mathematics (Borasi & Fonzi, 2004). Such practice involves selecting cognitively demanding tasks and implementing them in ways that do not take over the thinking for students (Hiebert et. al., 1997; Stein, Smith, Henningsen & Silver, 2000; NCTM, 2000).   This type of instructional approach requires teachers to have a conceptual understanding of mathematical concepts, to possess the pedagogical knowledge of how mathematical understanding develops, and to know how students can best be supported in developing that mathematical understanding. Having such knowledge is what Shulman (1986) referred to as "the missing paradigm" in instruction.

Much research has been done, particularly at the elementary and middle school levels, on what pedagogical knowledge is needed for teaching mathematics, including the types of pedagogy that have been associated with students developing mathematical understanding and increasing their mathematical achievement (e.g. Yackel & Cobb, 1996; Carpenter et al., 2000; Stein et al., 2000; Ball & Bass, 2005). In all of the studies, multiple factors were associated with deeper mathematical understanding or higher achievement. However, the type of task that the teacher selected determined what students had the potential to learn. In addition, implementing cognitively demanding tasks well was related to asking questions that prompt students to reason, make connections, and justify their thinking and the thinking of others (Hiebert & Wearne, 1993; Martino & Maher, 1999; Boaler & Brodie, 2004).

In one particular study, Boaler and Staples (2005) found that high school students from one school, 'Railside', who were initially performing significantly lower in mathematics than students from two other high schools, were outperforming these same students after only two years. One reason cited for the improved achievement was the mathematics curriculum, written by the teachers at Railside, that consisted of a variety of cognitively demanding tasks containing multiple points of access and requiring the use of and connections among multiple representations. A second reason identified for the success of the Railside students was that "the teachers' questions significantly shaped the course of implementation" (Boaler & Staples, 2005, p. 19). Railside teachers consistently asked 'follow up' questions about the mathematics in the task on which students were working and asked students to justify their thinking and the thinking of others. Students were also expected to discuss mathematics with and ask questions of other students. Hence, engaging students in solving challenging tasks by asking them questions that require them to explore mathematics, explain or justify their thinking, and discuss mathematics with other students is one way to improve students' learning opportunities in mathematics classrooms and ultimately impact student achievement.

The study described herein focused on helping teachers improve their ability to ask questions that support students' understanding of and engagement in high level mathematics tasks. The following sections highlight the importance of selecting challenging mathematics tasks for use in the classroom and the nature of teachers' questions that support students' high level engagement when solving challenging tasks.

1.2. BACKGROUND

1.2.1. Selecting challenging mathematical tasks

High level mathematics tasks provide the greatest potential for improvement in students' learning of mathematics. Several studies report on the link between challenging tasks and increased student achievement (McCaffrey et al., 2001; NCES, 2003; Boaler & Staples, 2005). For example, Stein and Lane (1996) reported on Quantitative Understanding: Amplifying Student Achievement and Reasoning (QUASAR)[1] project which focused on assisting middle school mathematics teachers in disadvantaged schools with the implementation of reform-type mathematics programs. Greater gains in student achievement were documented in the areas of problem solving, reasoning, and communication for middle school students' whose teachers successfully implemented cognitively demanding mathematics tasks (Stein, Grover & Henningsen, 1996). These tasks required students to engage in thinking and reasoning about mathematics concepts. Many of the tasks could be represented in multiple ways and could be solved using a variety of strategies.

While selecting mathematics tasks that are cognitively demanding is a critical first step to mathematics instruction that promotes student understanding, implementing those tasks in ways that maintain the demands is the crucial second step.

[1] QUASAR (Quantitative Understanding : Amplifying Student Achievement and Reasoning) was based at the Learning Research and Development Center at the University of Pittsburgh and was directed by Edward A. Silver.

1.2.2. Implementing challenging tasks through teacher questioning

Enacting high level tasks at a level in which the cognitive demands are maintained is not an easy endeavor. Teachers must choose mathematical tasks that are challenging and relevant to all students. They must be able to reach all students while at the same time turning some of the authority over to the students. And they must monitor students as they work but not take over the process of thinking for them (Lappan, 1997).

While it is difficult for teachers to implement challenging mathematics tasks, researchers have identified a set of factors that come into play when high level tasks are implemented at a high level (Hiebert & Wearne, 1993; Henningsen & Stein, 1997; Weiss & Pasley, 2004; Boaler & Staples, 2005). Among these factors is the ability of teachers to ask students to explain and justify their thinking and reasoning, to ask questions that target important mathematics and to ask students to discuss mathematics with each other. For example, Hiebert and Wearne (1993) observed six second grade classrooms with a focus on the mathematical tasks that were used and the classroom discourse occurring around those tasks. In three of the classrooms, the vast majority of the tasks emphasized the use of written symbols and were to be solved by computation. The majority of the questions asked by the teachers in these classes were "recall" questions requiring students to recite facts, rules, or topics. In the other three classrooms, students solved fewer tasks but spent more time on the tasks they did solve. These tasks allowed for the use of alternative representations and many were situated within a context. Two of the three teachers in these classrooms asked fewer recall questions and more questions of other types such as: requiring students to describe the strategy they used or find a different strategy to solve the same problem; asking students to create a story to match a number sentence or create a problem to fit a description; requiring students to explain why they used a particular procedure or why the procedure works; or asking students to analyze the nature of a problem or strategy. The largest gains in achievement occurred in the two classrooms where teachers used a variety of mathematical tasks and asked questions that required students to explain their thinking, problem solving strategies and to make sense of mathematics procedures or problems.

In a five year longitudinal study of over 700 students in three high schools, Boaler and Staples (2005) reported that students in one high school who were initially performing significantly below the students in the other two schools at the beginning of 9^{th} grade, significantly outperformed those in the other two high schools on a test involving algebra and geometry at the end of the second year. The identified reasons for this increase in mathematics achievement included a curriculum consisting of cognitively demanding problems, an environment in which students worked collaboratively in heterogeneous groups and the variety of questions asked by teachers. These question types included those that required students to explain and justify their thinking, that promoted discussion of mathematics, and that prompted students to make sense of and connections among mathematical concepts.

When analyzing the questions asked by teachers, it was found that in two of the high schools, teachers used a traditional mathematics curriculum and presented new mathematical ideas by lecturing and having students work individually on short, close-ended problems. Over 95% of the questions asked by the teachers were procedural questions that required an immediate answer, the rehearsal of procedures, or the statement of fact (Boaler & Brodie, 2004). At the third school (Railside), however, teachers lectured only 4% of the time and students worked collaboratively in heterogeneous groups. The teachers at Railside posed longer, conceptual problems and combined student presentations with effective teacher questioning. Teachers at this school also asked procedural questions, but only 62% of the time. More importantly, teachers asked a variety of other types of questions. Three particular question types were highlighted as contributing to students' opportunities to engage in and learn mathematics. These three types of questions: 1.) prompted students to explore mathematical relationships and connect mathematical ideas, 2.) probed for students' understanding, and 3.) generated discussion of mathematics among the students. The researchers noted that the teachers' ability to ask these particular types of questions was likely related to the curriculum the teachers designed and the types of mathematical tasks in the curriculum. The types of questions asked by teachers at Railside, and absent in the other two schools, were considered to be an important factor that contributed to the significant increase in student achievement at the school.

In order to develop the capacity to ask questions that supports students' work on challenging mathematical tasks, teachers must have opportunities to learn about these instructional practices. One key vehicle that has the potential to provide such opportunities for in-service teachers is professional development.

1.2.3. The significance of professional development at the high school level

Adopting a practice of asking questions that support students' engagement on cognitively demanding tasks is not a trivial matter for teachers. They need professional development opportunities that allow them to build their knowledge and skill in teaching mathematics through experiences that make salient the complexity of enacting such a practice (Ball & Cohen, 1999; Smith, 2001).

A special challenge in the effort to reform instructional practice in mathematics is the case of America's high schools. The political climate of high schools has been described as "an atmosphere of distrust, affront, and impatience, and an expectation that they will be judged by the performance of their graduates (National Association of Secondary Principals, 2004, p. xi)." The success of professional development activities for high school teachers is dependent upon, among other things, "the ability to encourage the faculty to become collaborative learners in onsite professional development experiences" (Killion, 2002, p. 19). In addition, since most high school teachers' identities are determined by the subjects they teach, it is critical that professional development activities be grounded in the content background of the teacher (McLaughlin & Talbert, 2001).

In order to improve student achievement in mathematics, teachers must provide students with opportunities to engage in challenging mathematics and support their understanding by consistently asking questions that promote mathematical understanding. In turn, teachers must be provided with opportunities to build their knowledge and skills in these areas through professional development experiences grounded in the content and pedagogy of teaching mathematics.

1.3. PURPOSE OF THE STUDY

The purpose of this study was to examine changes in high school mathematics teachers' ability to identify and create questions that promote mathematical understanding as they participated in a professional development program focused on planning, teaching and reflecting on lessons that feature cognitively challenging tasks. Changes were determined in teachers' ability to identify and create questions of the following types:

- probing – questions that prompt students to explain or justify their mathematical thinking and reasoning;
- generating discussion – questions that prompt students to discuss challenging mathematics; and
- exploring – questions that prompt students to explore the meaning of and relationships among mathematical concepts and ideas.

Teachers participated in a set of professional development experiences in which they learned how to select challenging mathematics tasks and then implement those tasks at a level that maintained the challenges. This implementation included asking questions that promote understanding of the mathematics in the task. The professional development experiences included opportunities for teachers to: 1). learn new instructional practices focused on selecting and implementing challenging tasks, including asking questions that promote understanding; 2) plan lessons around challenging tasks with school and district colleagues that incorporate the new instructional practices; 3) enact lessons with a focus on implementing the new practices; and 4) reflect on their implementation of lessons with others from their schools and districts who were also implementing the new practices.

1.4. RESEARCH QUESTIONS

In order to determine if learning about particular question types during professional development experiences improved teachers' ability to identify and create particular types of questions, the research questions for this study included:

1. To what extent can high school mathematics teachers identify different question types that promote understanding of mathematics prior to and after participation in a professional development program focused on improving instructional practice in mathematics? In particular, to what extent can they identify probing, generating discussion, and exploring questions?

2. To what extent can high school mathematics teachers explain the reasons why different question types promote understanding of mathematics prior to and after participation in a professional development program focused on improving instructional practice in mathematics? In particular, to what extent can they explain why probing, generating discussion, and exploring questions promote understanding?

3. To what extent can high school teachers create questions that promote student understanding of mathematics prior to and after participation in a professional development program focused on improving instructional practice in mathematics? In particular, to what extent can they create probing, generating discussion, and exploring questions?

4. To what extent do high school teachers focus on promoting understanding of mathematics when identifying the purposes of the questions they ask their students prior to and after participation in a professional development program focused on improving instructional practice in mathematics?

5. What might account for changes in teachers' ability to identify and create questions that promote understanding of mathematics and to explain why such questions promote understanding?

The premise of this study was that providing teachers with opportunities to learn about question types would assist them in being able to: identify and create questions that prompt students to explain, justify, discuss and explore mathematical concepts in their classrooms; explain why particular question types promote mathematical understanding; and focus on mathematical understanding when identifying the purposes of their questions. It was hypothesized that these abilities would then transfer to the classroom and result in teachers asking a broader range of questions that ultimately provided their students with more opportunities to engage in rigorous mathematics.

1.5. SIGNIFICANCE OF THE STUDY

Although research has shown the relationship between maintaining the cognitive demands of tasks and student achievement and that questioning is a critical factor in maintaining the demands, no studies have focused explicitly on teachers' ability to learn about particular types of questions that support students' understanding of mathematics. Therefore, this study contributes to our understanding of the types of professional development experiences that are likely to impact teachers' knowledge related to questions that promote students' understanding of mathematics.

The study also has the potential to provide a valuable tool to the mathematics education research community - one that measures teachers' knowledge of questioning, a key instructional practice. Although this tool would be but a first step in measuring teachers' abilities related to particular types of questions that promote mathematical understanding, it could provide valuable insights into measuring an aspect of instruction on a large-scale without the challenge of classroom observations. It could also be used to measure the effectiveness of professional development programs designed to assist teachers in improving specific aspects of their instructional practice.

Finally, because much of the research on teaching mathematics has been conducted at the elementary and middle school levels, this study could also provide important insights into mathematics teaching at the high school level. Though a direct correlation between the professional development experiences and changes in teachers' knowledge of questioning practices was not studied, information obtained from the professional development program that might be linked to changes in teachers' knowledge could inform the field about the potential of professional development to change instructional practices at the high school level. Ultimately, the results from this study have the potential to provide the mathematics education community with information that could lead to better supporting students in learning high school mathematics. If high school teachers can improve their ability to identify and create good questions and change their perspective regarding the purposes of their questions, they will be better able to support their students' engagement in challenging tasks. Ideally, this could eventually lead to a diminishing of the high failure rates of students in high school mathematics, particularly in urban schools, resulting in more access to higher education, better career paths, and a more rewarding economic future.

1.6. LIMITATIONS

Although the goal of any professional development experience is to impact some aspect of the teachers' practice, this study focused only on expanding teachers' knowledge of questions that promote students' understanding of mathematics. Since teachers' ability to transfer the learning about questions to implementation in the classroom was not analyzed, the extent to which the results can be generalized to the classroom level was limited.

In addition, the investigator of this study was also one of four facilitators of the professional development program in which the teachers participated. As a result, bias in terms of teachers agreeing to participate in the study because they were in the investigators' professional development sessions could be a limitation.

1.7. OVERVIEW OF THIS DOCUMENT

This document consists of five chapters. Chapter 1 provided an introduction to the study, situating the study within existing research on reforming the teaching of mathematics, with a particular emphasis on questioning as a key instructional practice. Chapter 2 provides an in-depth review of the relevant literature pertaining to question types that have been associated with increased student achievement in mathematics and of the features of professional development that are necessary to engage high school teachers and impact their instructional practice. Chapter 3 explains the methodology of the study, including a description of the tools used for collecting data, the structure of the professional development in which the teachers participated, the types of data that were collected for the study, and the ways and methods in which the data was analyzed. Chapter 4 is a discussion and summary of the results of the study. Chapter 5 provides conclusions that can be drawn from the results of the study and discusses implications of the study for the teaching of mathematics, particularly at the high school level.

2. CHAPTER TWO: A REVIEW OF THE LITERATURE

2.1. INTRODUCTION

Improving the instructional practice of mathematics teachers is seen as a means for increasing student achievement in mathematics. However, the vast majority of practicing mathematics teachers learned to teach under a paradigm of teaching that consisted of having students practice and memorize facts and procedures. The type of mathematics instruction necessary for improving student learning and achievement entails "a comprehensive approach to mathematics instruction that is centered on teaching for understanding and enabling students to engage with meaningful problems and big ideas of mathematics" (Borasi & Fonzi, 2004, p. 9). The assumptions underlying such an instructional approach are that students construct knowledge through their interaction with others in a particular context with a particular purpose; learning results by making sense of something that builds on prior knowledge; and the purpose of teaching is to facilitate the learning process by setting up problem solving situations in a rich learning environment and providing support as students try to solve the problems (Borasi & Fonzi, 2004).

Two key components of such an instructional approach that have a research base to support them are selecting challenging tasks in which students will engage and supporting students as they solve the task by asking questions that assist students' learning without taking over the thinking for them (NCTM, 1991; Stein et al., 1996; Hiebert et al., 1997; Lappan, 1997).

In order to adopt a practice that incorporates these components, teachers need professional development experiences that allow them to build their knowledge and skill in teaching mathematics (Loucks-Horsly et. al, 1998). Providing teachers with opportunities to engage in challenging mathematics themselves and to learn about and discuss issues related to teaching mathematics with others in the same role is crucial if any change in instruction is expected to occur (Grossman & Stodolsky, 1995; Stein, Smith & Silver, 1999).

This chapter is comprised of two sections, each of which contains a review of literature related to this study. The initial section of this chapter reviews literature that discusses two key characteristics of instructional practice in mathematics that have been identified as contributing to enhanced learning opportunities for students – selecting challenging mathematics tasks and implementing those tasks through asking a variety of questions in order to promote students' understanding of mathematics. The second section highlights features of effective professional development for teachers of mathematics, with a particular focus on those that have resulted in increased student achievement in mathematics and those that have been successful in changing instructional practice, particularly at the high school level.

2.2. KEY INSTRUCTIONAL PRACTICES IN MATHEMATICS

The type of mathematics instruction that has been associated with increased achievement in mathematics is quite challenging to implement. Teachers must choose mathematical tasks that are challenging and relevant to all students. They must be able to reach all students while at the same time turning some of the authority over to the students. And they must monitor students as they work but not take over the process of thinking for them (Lappan, 1997). Researchers have identified features of lessons that result in such instruction and that have impacted student learning. For example, after observing over 350 mathematics and science lessons to determine their effectiveness, Weiss and Pasley (2004) identified characteristics of effective instruction that were common in these lessons. These characteristics included: a focus on significant and worthwhile content that was taught using various strategies that engaged the students and connected to their prior knowledge; a culture in which respect and rigor were evidenced by the teacher asking challenging questions and students freely contributing new ideas and questioning the ideas of others; active engagement of all students; and teacher questioning that not only monitored students' understanding and encouraged deeper thinking, but also ensured that students made sense of the key concepts.

Another research study conducted in six urban middle schools provides evidence of the impact of selecting and implementing worthwhile tasks on students' learning. The QUASAR Project was a reform project whose purpose was to foster and study how mathematics teachers implemented their mathematics programs at six, diverse and economically disadvantaged urban middle schools across the United States. One of the most significant findings of the project was that student achievement in mathematics increased significantly in the classrooms where teachers consistently selected cognitively challenging mathematics tasks and where the cognitive demands of the tasks were maintained when the tasks were implemented (Stein & Lane, 1996). A variety of factors were identified that contributed to teachers being able to maintain the demands. These factors, consistent with the characteristics identified by Weiss and Pasley (2004), included scaffolding student learning by asking questions that did not take over student thinking, consistently asking students to explain and justify their own thinking and reasoning and to explain the thinking and reasoning of others, and asking students to make sense of the mathematical ideas they were discussing (Henningsen & Stein, 1997).

The results of the Weiss and Pasley (2004) and Stein et al. (1996) studies suggest that effective instruction in the mathematics classroom must begin by selecting appropriate tasks that assist students in achieving the intended mathematics. Once the appropriate mathematics task has been selected to meet the goals of the lesson, the implementation of the lesson must take place so that the challenges of the task are maintained. Teacher questioning was identified as a key practice that supports the maintenance. The remainder of this section discusses research supporting the importance of task selection and teacher questioning when considering instructional practices in the mathematics classroom that impact student understanding.

2.2.1. Selecting and implementing worthwhile mathematics tasks

Choosing worthwhile tasks that provide opportunities for student exploration and the learning of important mathematical concepts is “a significant part of a teacher’s responsibility” (NCTM, 2000, p. 341). In fact, Lappan (1997) states, “no decision that teachers make has a greater impact on students' opportunity to learn…than the selection, adaptation, or creation of the tasks with which the teacher engages the students" (p. 213).

Common characteristics of these "worthwhile tasks" include having multiple entry points for students with different prior knowledge and experiences; being able to be approached in multiple ways using a variety of problems solving strategies; and providing opportunities to be represented in multiple ways, such as by symbols, tables, drawings, or graphs.

Hiebert et. al. (1997) describe three features necessary to classify a mathematics task as worthwhile which are consistent with the characteristics described above: 1.) The task must be problematic. This means students must see the task as something interesting they want to learn about and make sense of, but the mathematics necessary to solve the task poses a challenge or is "problematic"; 2.) The task must build on students' prior knowledge so that they have the knowledge and skills necessary to begin solving the task. The context of the problem must provide accessibility for all students; and 3.) The task must be grounded in important mathematical concepts that leave something of mathematical value with the student as a result of solving the problem. In addition, teachers must select tasks, not in isolation from each other, but that are sequenced over time so that solving the tasks will "add up to something important" for the student.

A major consideration in choosing worthwhile mathematical tasks in which students will engage is alignment with the teachers' mathematical goals for the lesson. This is influenced by the teacher's knowledge of mathematics as well as what the teacher believes to be the students' prior mathematical knowledge and experiences in mathematics (Simon, 1995, p. 138). At the high school level in particular, teachers' beliefs of what students' prior knowledge is and what they are capable of doing greatly impacts the choice of tasks they choose (McLaughlin & Talbert, 2001).

Unfortunately, many high school teachers do not believe their students come to them with sufficient prior mathematical knowledge and experiences. A longitudinal study of over 600 high school teachers in sixteen rural, suburban and urban districts in California and Michigan was conducted using iterative surveys and interviews. In discussing the results of the study, McLaughlin and Talbert (2001) reported that 65% of high school teachers felt their students were less prepared than students they had previously taught, and 73% believed students' habits and attitudes limited their chances for academic success. This resulted in teachers not giving certain students opportunities to engage in challenging tasks. The researchers were particularly concerned with mathematics teachers, who consistently viewed their content as sequential and constant and the instruction and the tasks that drive that instruction as being predetermined. As a result, "mathematics represents a worst case in terms of teachers' potential openness to rethinking traditional assumptions or developing new practices to engage nontraditional students in the discipline" (McLaughlin & Talbert, 2001, p. 57).

Results from a large scale survey provide additional supporting evidence as to the lack of selection of high level mathematics tasks for all students. In the "Status of High School Mathematics Teaching" report, Whittington (2002) analyzed survey responses on the 2000 National Survey of Science and Mathematics Education from a national probability sample that included 1300 high school mathematics teachers. The report indicates that 85% of high school mathematics teachers reported placing heavy emphasis on their students' learning mathematics concepts and over 70% reported emphasizing problem solving and mathematical reasoning. However, high school mathematics teachers who taught classes to high school students who had not yet taken or passed algebra 1, emphasized computational skills at a higher percentage and used tasks that required reasoning and problem solving at a lower rate than teachers whose students had passed algebra 1 and were taking subsequent algebra or geometry courses.

A growing body of evidence, however, challenges the traditional assumptions as to who can learn worthwhile mathematics. In the QUASAR study, a stratified random sample of 144 lessons was chosen from the 620 mathematics lessons that were observed in the four schools who had participated in the project for a full three-year period. Stein, Grover, and Henningsen (1996) categorized mathematical tasks that formed the core of those lessons as falling into one of 4 levels according to their cognitive demands – the types of thinking required as one engages in solving the tasks. The top two levels of tasks, considered to be cognitively demanding or having a "high level demand," were of two types - *procedures with connections to meaning* and *doing mathematics*. Procedures with connections tasks focus students' attention on the use of a particular procedure in order to develop a conceptual understanding of a mathematical concept. Doing mathematics tasks require students to explore and develop an understanding of a mathematical concept in a non-algorithmic way. In their analysis, Stein and her colleagues discovered that tasks considered to be cognitively demanding were the most difficult for teachers to implement. However, student achievement, as measured on an assessment of problem solving, reasoning, and communication, was greatest in classrooms where teachers set up and implemented high level tasks in ways that maintained the cognitive demands of the task (Stein & Lane, 1996). This study provides evidence that selection of worthwhile mathematics tasks is a critical, but insufficient, component of providing students with opportunities to increase their achievement in mathematics. Implementation of those tasks in ways that maintain the cognitive demands is the key to providing such opportunities.

2.2.1.1. Teachers' implementation of worthwhile tasks The impact of the selection and implementation of worthwhile tasks on student achievement has been noted in a large scale study on mathematics teaching and learning. The Third International Mathematics and Science Study (TIMSS) 1999 Video Study (NCES, 2003) reported on the analysis of videotaped mathematics lessons, as well as results from teacher questionnaires. The results suggest that in the two highest achieving countries in 8th grade mathematics, Japan and Hong Kong SAR, teachers emphasized learning new content through problem solving in their lessons, while U.S. teachers emphasized reviewing previous mathematical content. In addition, 17% of tasks used by 8th grade teachers in the U.S. were at a high level, about the same rate as most high performing countries (except for Japan). However, in the U.S. none of the tasks were implemented in a way that maintained the cognitive demands during the lessons. One implication of this result is that, in spite of the fact that 8th grade students in the U.S. scored higher on the assessment in 1999 than they did in 1995, they continued to score significantly lower than the other countries who participated in the study (NCES, 2003, p. 11). These results also indicate that task selection alone does not ensure greater student achievement. Maintaining the cognitive demands of the task throughout the lesson is a key factor that determines the student learning.

Another relevant finding from the TIMSS study (NCES, 2003) was that the 8th grade teachers in the U.S. reported consistently using the "current ideas" of teaching mathematics in their practice. Yet, the videotapes revealed that students in U.S. classrooms had limited opportunities to problem solve and use mathematical reasoning because teachers spent most of the time reviewing previously learned material. This raises the question as to what U.S. teachers view as the "current ideas" of teaching mathematics in terms of choosing and implementing worthwhile mathematics tasks.

Results from the "Status of High School Mathematics Teaching" report (Whittington, 2002) indicate that even though teachers reported placing heavy emphasis on their students' learning mathematics concepts and reported they emphasized problem solving and mathematical reasoning, teachers predominantly lectured and had students take notes or complete text or worksheet problems. In addition, over 70% of the lessons consisted of routine practice of algorithms and computations at least once a week. This also raises questions about teachers' conception of how to best implement challenging tasks with their students.

There is evidence that teachers' ability to successfully select and implement high level tasks with their students impacts student achievement. For example, the QUASAR Project documented greater student achievement in the areas of problem solving, reasoning, and communication for middle school students whose teachers selected high level tasks and then maintained the cognitive demands of the tasks during implementation (Stein & Lane, 1996). Researchers noted that three to five different factors typically contributed to teachers' ability to maintain the challenges in the task as students were solving the task (Henningsen & Stein, 1997). The five factors identified as having the most influence included: 1.) using tasks that build on students' prior knowledge; 2.) scaffolding student learning by "providing assistance that enables the student to complete the task alone, but that does not reduce the overall complexity or cognitive demands of the task (Henningsen & Stein, 1997, p. 527); 3.) allowing an appropriate amount of time for students to engage in solving the task; 4.) modeling high level performance; and 5.) consistently pressing students to explain and justify their thinking and reasoning.

There is also evidence of high school mathematics teachers successfully implementing cognitively demanding tasks with their students which resulted in increased student achievement. In the Boaler and Staples (2005) study of three high schools, over 600 hours of classroom observations, teacher and student questionnaires, teacher and student interviews, mathematical content tests and the state level assessment in mathematics were analyzed. Students at two of the high schools were enrolled in a traditional sequence of mathematics courses (i.e. Algebra 1, Geometry, Algebra 2) that were taught in a traditional manner.   The majority of students' time was spent individually watching and listening as their teachers presented or demonstrated new material. Students then practiced the new materials by working individually on short, closed problems from their textbooks.

Students at the third high school, Railside, an urban school with cultural and linguistic diversity, were taught a curriculum designed by their teachers. The Railside students were initially performing significantly below the students in the other two schools as measured by a test of middle school mathematics given at the beginning of their first year at the high school. By the end of the first year, there was no significant difference, however, between the performance of Railside students and the students from the other two schools on the end of year algebra test. At the end of their second year of high school, the Railside students significantly outperformed those in the other two high schools on a test that combined topics from algebra and geometry. All three tests were designed by the research team and incorporated the types of questions that were indicative of both a traditional approach to teaching mathematics and the approach developed by the Railside teachers.

Students at Railside also developed more positive attitudes about mathematics, took subsequent mathematics courses at a higher rate, and planned to pursue mathematics at the collegiate level in larger numbers. Perhaps most importantly, achievement differences between whites and ethnic groups were reduced, and in some cases, eliminated. Two of the reasons that researchers identified as contributing to this increase in mathematics achievement were the curriculum designed by the teachers and the way in which it was implemented. The curriculum was taught to all students at Railside who were grouped heterogeneously. Even though the developed curriculum followed the traditional sequence of courses (i.e. algebra, geometry, advanced algebra, etc.), it consisted of cognitively demanding problems that were worked on for longer periods of time in collaborative groups. Multiple points of entry into solving these challenging tasks was a key design feature and multiple representations were expected to be used by students, as were multiple strategies to solve the problems. The implementation of the curriculum by the teachers consisted of, among other factors, the use of a strategy called "complex instruction" in which students are grouped heterogeneously and a variety of practices are used to promote group interactions. These practices include: assigning roles to each group member that are interdependent and require collaboration among group members; assigning competence by publicly praising something a student has said or done that is mathematically valuable; and creating multi-dimensional classrooms in which there are multiple ways to be successful. Among these ways to be successful are solving problems using different strategies, explaining ones' thinking and the thinking of others, justifying solutions to problems, and asking questions of each other and of the teacher.

Conclusion A first step in improving mathematics instruction is selecting worthwhile tasks in which students will engage. However, implementing those tasks in ways that provide opportunities for students to develop and learn important mathematical concepts is a crucial, and difficult, second step of effective instruction. Several of the studies discussed in this section (e.g. QUASAR Project (1996), Boaler & Brodie (2004)) provide evidence that mathematics teachers are indeed capable of selecting challenging mathematics tasks for their classrooms and that effectively implementing such tasks pays off in terms of higher mathematics achievement for students. However, the TIMSS Video Study, the QUASAR Project, the Whittington report, and the McLaughlin and Talbert study also point to the fact that large numbers of teachers have difficulty in implementing such tasks. In addition, Weiss, Pasley, Smith, Banilower, and Heck (2003), who observed more than 350 mathematics and science lessons in grades K-12 using a structured observation protocol around lesson design, implementation, content and culture, report that, even though the majority of lessons they analyzed included important, worthwhile content, only 20% of those lessons were implemented in ways that engaged the students purposefully in the content.

A key component of implementing challenging tasks that purposefully engage the student in mathematics is asking questions that promote understanding of mathematics. Research literature highlighting the significance of asking a variety of questions that support and promote students' understanding and its impact on student understanding, as well as teachers' predominant questioning practices will be discussed in the next section.

2.2.2. Asking questions that support student understanding

Orchestrating classroom discourse has been identified as "the biggest problem for teachers" in the mathematics classroom (Hiebert et. al, 1997, p. 29). Classroom discourse consists of much more than just "discussing" or "communicating." Discourse can be described as the ways in which teachers and students make sense of mathematics by "representing, thinking, talking, agreeing and disagreeing" (NCTM, 1991, p. 36). It establishes how the teacher and students will work on a mathematical task, the importance of thinking and reasoning during engagement in a task, and the expectation that justifying and explaining one's thinking and reasoning is a valued norm. Asking a variety of questions that prompt these behaviors is a complex endeavor. All students must be supported as they participate in solving the task and the participation must be orchestrated in ways that maintain the challenges of the task. Teachers must decide when to share information to assist students' learning and how much of that information is "too much" to share. In short, the teacher's role in discourse is deciding when to provide information, when to ask for clarification, when to push for a deeper understanding, when to promote discussion, and when to let a student struggle with a difficulty (NCTM, 1991, p. 35).

2.2.2.1. Teachers' practice of asking questions Asking questions that prompt clarification, justification, meaning making, and discussion is a challenge for most mathematics teachers. They have not, for the most part, learned mathematics by publicly discussing, explaining, and justifying their thinking and reasoning through the exploration of single, challenging tasks and they do not know how to model or facilitate such a lesson (Lappan, 1997). In fact, research shows that most teachers engage in a three part discourse pattern in which the teacher initiates a conversation through asking questions that often require one word or simple answers, a student responds, and the teacher evaluates the response (Stodlosky, Ferguson, and Wimpelberg, 1981; Lemke, 1990; Cazden, 2001). This pattern, labeled IRE (initiate-respond-evaluate), occurs between the teacher and one student and usually results in students perceiving the curriculum being taught as a set of facts and believing that teacher questions have predetermined, correct answers.

In their study of K-12 mathematics and science lessons, Weiss, Pasley, Smith, Banilower & Heck (2003) provide evidence of the predominance of the IRE pattern in mathematics and science classrooms. In their observation and analysis of 350 lessons, they divided lesson quality into 5 levels, ranked from 1 through 5. Level 1 consisted of lessons that exhibited ineffective instruction, such as passive learning by students or doing activities without a purpose. Level 3 consisted of lessons that exhibited the beginning stages of effective instruction while level 5 consisted of lessons showing exemplary instruction. In rating lesson quality, the researchers considered lesson design, lesson implementation which included teacher questioning, quality of content, and classroom culture. Weiss and her colleagues report that only 16% of the lessons they observed consisted of high level questioning - the types of questions that "encourage students to think more deeply" (Weiss et al., 2003, p. 7). The predominant types of questions they observed included rapid series of questions that focused on a correct answer with no check for deeper understanding of the content, questions in which one student responded correctly with no follow up to determine if other students understood, or questions which the teacher answered herself. Teacher questioning was determined to be one of the weakest elements of instruction in the lessons they observed and was one of the major reasons that even well-developed and well-planned lessons fell in their ratings.

2.2.2.2. The impact of teacher questioning on student understanding For decades, Bloom's taxonomy has been used by many educators to classify student thinking (Bloom, Englehart, Furst, Hill & Krathwhol, 1956, p. 78). The taxonomy ranges from the lowest level of thinking, the knowledge level, that requires students to recite or remember facts, etc., to the highest levels of thinking, synthesizing and evaluating, that requires students to relate and connect different ideas, methods, etc. The assumption is made that higher order questions, those that ask students to synthesize or evaluate information, foster student thinking and contribute to better retention and problem solving skills, while the lowest level of questions, those that require recall or memorization, require little complex thinking and do not contribute to advancing students' thinking. Although this classification system suggests a way to consider the types of questions asked by teachers and the purposes they might serve, several more recent research studies that link student thinking and achievement to teacher questioning make salient the importance of teachers' asking students questions that require deep and complex thinking in order for students to develop deep conceptual understanding of mathematics.

Hiebert and Wearne (1993) observed six second grade classrooms on a weekly basis during 12 weeks of instruction in mathematics with a focus on the mathematical tasks that were used and the classroom discourse occurring around those tasks. All six teachers engaged students in the same mathematical topics in the same sequence but used different approaches to teach the topic. The researchers analyzed the tasks according to the number and types of tasks and time spent solving them and the discourse according to how much time teachers and students talked and the kinds of questions asked pertaining to mathematics. The questions were then grouped into four broad categories: 1. recall – requiring students to recite facts, rules, or topics; 2. describe strategies – requiring students to either describe the strategy they used or find a different strategy to solve the same problem; 3. generate problems – creating a story to match a number sentence or creating a problems to fit a description; and 4. examine underlying features – requiring students to explain why they used a particular procedure or why the procedure works or asking students to analyze the nature of a problem or strategy.

In three of the classrooms, students solved more problems per lesson and spent less time on problems. The vast majority of the problems required students to use written symbols and were solved by using computation. The teachers in these classrooms asked very few questions other than recall questions and only 10% of student responses were six words or longer. In the other three classrooms, students solved fewer tasks per lesson but spent more time on the tasks they did solve. In addition, their tasks included more alternative representations and many of the tasks were situated within a context. Two of the three teachers in these classrooms asked many more “nonrecall” questions that required students to explain and describe their thinking. Students in these two classrooms also had 25% to 35% of their responses being six words or longer.

Perhaps the most important result of the study was the impact on students' learning of mathematics. Prior to the study, four of the classrooms had been identified as being lower in mathematics achievement than two of the other classrooms. The largest gain in achievement occurred in one of the initially lower achieving classrooms. The teacher in this classroom was one of the two who asked a variety of questions and used different types of tasks. Students in this classroom ended the year with achievement levels nearly identical to the achievement levels in one of the higher achieving classrooms. In addition, the second initially higher achieving classroom also had significant gains in achievement and had a teacher who also asked a variety of questions and used a variety of tasks.

Martino and Maher (1999) also found a link between teachers' questioning and student learning. During a 10-year longitudinal study they conducted in three urban New Jersey school districts, they wanted to determine how children build ideas in mathematics. During the 1992-1993 school year they analyzed video transcripts, student work, and observer notes from 151 students in third, fourth, and fifth grade classrooms with a focus on the effects of teacher questioning. In their analysis of one of the classroom teachers, a strong relationship was discovered between the teacher's monitoring of students' problem solving methods and the teacher's questioning that helped students learn to justify their solutions, make connections between problems, and understand the strategies of other students. This resulted in students extending their own mathematical thinking and moving to deeper mathematical understanding.

In the Boaler and Brodie (2004) study of mathematics teaching and learning in three high schools, described in the previous section, the researchers coded questions asked by 6 teachers, each of whom were observed teaching 6 lessons. The types of questions asked were deemed to be an important indicator of the mathematics on which students and teachers worked and one of the factors that contributed to the significantly higher achievement of the students at Railside (Boaler & Staples, 2005).

Through the analysis of the questions asked by teachers, categories of types of questions emerged (see Figure 2.1). In the traditional mathematics classrooms of two of

Question Type	Description	Examples
1. Gathering information, leading students through a method	-Requires immediate answer -Rehearses known facts/procedures -Enables students to state facts/ procedures	"What is the value of x in the equation?" "How would you plot that point?"
2. Inserting terminology	-Once ideas are under discussion, enables correct mathematical language to be used to talk about them	"What is this called?" "How would we write this correctly?"
3. Exploring mathematical meanings and/or relationships	-Points to underlying mathematical relationships and meanings. -Makes links between mathematical ideas and representations.	"Where is the 'x' on the diagram?" "What does probability mean?"
4. Probing, getting students to explain their thinking	-Asks student to articulate, elaborate or clarify ideas	"How did you get 10?" "Can you explain your idea?"
5. Generating discussion	-Solicits contributions from other members of class.	"Is there another opinion of that?" "What did you say, Justin?"
6. Linking and applying	-Points to relationships among mathematical ideas and mathematics and other areas of study/life	"In what other situations could you apply this?" "Where else have we used this?"
7. Extending thinking	-Extends the situation under discussion to other situations where similar ideas may be used	"Would this work with other numbers?"
8. Orienting and focusing	-Helps students to focus on key elements or aspects of the situation in order to enable problem solving	"What is the problem asking you to do?" "What is important about this?"
9. Establishing context	-Talks about issues outside of math in order to enable links to be made with mathematics	"What is the lottery?" "How old do you have to be to play the lottery?"

Figure 2.1: Categories of question types, descriptions, and examples

the high schools, the teachers presented new mathematical methods through lectures and the students worked through short, closed problems. In particular, teachers in the traditional classrooms asked procedural questions more than 95% of the time. Procedural questions (type 1 questions in Figure 2.1) were those that required students to give an immediate answer, rehearse procedures, or state facts.

At Railside school, however, teachers lectured only 4% of the time and students worked collaboratively in heterogeneous groups. The teachers posed longer, conceptual problems and combined student presentations with effective teacher questioning. Teachers at Railside also asked procedural questions, but only 62% of the time. More importantly, it was noted that the Railside teachers asked a variety of other types of questions. In particular, 32% of the time, teachers at Railside asked questions that were considered to be either exploring or probing (types 3 and 4, respectively, in Figure 2.1). Exploring questions required students to identify mathematical relationships and connect mathematical ideas and representations. Probing questions required students to explain, justify, or clarify their thinking. Railside teachers also asked questions that prompted students to discuss mathematics with each other (type 5 in Figure 2.1). The researchers noted that the Railside teachers' practice of asking these particular types of questions was likely connected to the curriculum they designed and the types of mathematical tasks in the curriculum. The type of questioning used by teachers at Railside, which was absent in the other two schools, was considered to be an important factor that contributed to increased student achievement in mathematics at Railside.

Conclusion Although many teachers have difficulty implementing worthwhile mathematics tasks, asking particular types of questions around cognitively demanding mathematical content can lead to higher student learning and achievement. In particular, the Boaler and Brodie (2004) study makes a case for teacher questioning that goes beyond only procedural questions and includes exploring, probing and generating discussion questions as a key component in orchestrating effective discourse around high level tasks and in developing students' mathematical understanding.

Selecting a worthwhile mathematics task does not necessarily lead to teachers asking questions that support students' understanding of mathematics. In order to develop new knowledge about teaching that would support the implementation of worthwhile tasks through teacher questions, teachers must have opportunities to engage in activities that "are at the heart of a teachers' daily work" (Smith, 2001, pg. 2). These opportunities for in-service teachers occur primarily through professional development experiences. The next section describes features of professional development programs that have been shown to increase student achievement by changing the instructional practices of teachers and which will inform the design of the program for teachers in this study.

2.3. FEATURES OF EFFECTIVE PROFESSIONAL DEVELOPMENT

Students' opportunities to learn and understand mathematics are determined by the teachers who teach them. Instructional practices that provide students with opportunities to learn challenging mathematics are not the norm for mathematics teachers in the U. S. (Stigler & Hiebert, 1999). Therefore, teachers need multiple opportunities to learn about and try out new practices if they are expected to adopt them. For in-service teachers, professional development experiences have the potential to provide them with opportunities to learn about and try out these new practices.

Although existing research supporting a professional development focus on teacher questioning as a means to change practice and increase student achievement is very limited, several research studies have documented and described features of effective professional development programs that are linked to increased student achievement in mathematics and changes in teachers' instructional practice. These features will be described in the next section.

2.3.1. Professional development linked to increased achievement and changes in practice

The ultimate purpose of any professional development program for mathematics teachers should be to assist teachers in providing the best possible learning experiences for students in order to advance students' achievement in mathematics. Though research that illustrates a connection between professional development and increased student achievement is limited (see Loucks-Horsley & Matsumoto, 1999; Huffman & Thomas, 2003), it does exist. Carpenter et al. (1989) and Fennema et al. (1996) cite evidence of professional development impacting student achievement in mathematics. Their studies were conducted around the Cognitively Guided Instruction (CGI) program in which teachers learned about models of student thinking and applied these models to their own practice. The professional development program engaged teachers in learning about how children develop mathematical thinking and then assisted the teachers in creating frameworks of children's mathematical thinking by building on their existing content knowledge. As the authors note, the teachers learned how students used concrete materials to solve problems and then engaged in activities around how those materials are linked to more abstract and formal mathematical ideas. Teachers also had opportunities to learn how errors made by children allowed them to determine what conceptions and misconceptions students might have. And teachers became aware of how to assess their students' thinking and the strategies used by the children. For example, teachers analyzed videos of children solving problems and then connected what they saw to work produced by their own students.

The professional development opportunities for teachers involved in the CGI program took a variety of formats. Summer workshops focused on providing teachers with a research-base on children's mathematical thinking. When teachers were back at their schools, they then worked with colleagues within and across grade level in a variety of ways and had support from an experienced CGI teacher and a graduate student researcher.

In the initial study, 20 first grade teachers participated in a four week CGI workshop and another 20 first grade teachers participated in two 2 hour problem solving workshops. All 40 teachers were observed during the following school year and interviewed at the end of the school year. In addition, students were given a pre- and post-standardized mathematics test to determine their levels of achievement. The results showed that the teachers who participated in the CGI program implemented problem solving, encouraged the use of a variety of strategies, listened to their students thinking, and knew more about how each student approached problem solving significantly more than the other teachers. In addition, the students of the CGI teachers exhibited high achievement, not only in problem solving, but also in knowledge of number facts (Fennema, et al., 1996).

In a subsequent study, twenty-one teachers at grades 1-3 participated in the CGI program over a four year period. Based on data gathered via classroom observations and standardized tests, it was found that 90% of the teachers improved their level of instructional practice, in terms of the CGI model, over the four years. In addition, all teachers involved in the CGI program demonstrated an increased level of student achievement in terms of problem solving and conceptual knowledge from the first year of involvement to the fourth year. In a follow up study conducted fours year after the end of the CGI program, all of the teachers continued to implement principles of the program at some level and ten of the teachers continued to grow in their beliefs and practices around students' mathematical thinking (Carpenter, Fennema, Franke, Levi, & Empson, 2000).

At the high school level, a report from the National Staff Development Council (Killion, 2002) provides evidence of two cases of professional development programs that served to increase student achievement. The Houston Independent School District/Rice University School of Mathematics Project (RUSMP) provided intensive professional development for algebra teacher coordinators in all high schools in the Houston Independent School District. The professional development provided opportunities for coordinators to examine each concept taught in Algebra I and expand their repertoire of instructional strategies in order to meet the needs of their students. In particular, there was an emphasis on providing student-centered learning experiences, a focus on conceptual learning, and the integration of concepts across grades. Algebra coordinators met in weekly planning sessions with the Rice University partners to develop a blueprint, labeled a "Learning Plan," for organizing instruction around each algebra concept. The coordinators then returned to their schools and met with their own algebra teachers to adapt and expand the Learning Plan to meet the needs of their own algebra students. Over the 3 years of the project, the percent of students passing the end of course algebra exam increased from 23% to 43%. In addition, the passing rates for African-American, Hispanic, and economically disadvantaged students were higher than the statewide rate, which was not the case prior to the project (Killion, 2002, p. 82).

A second professional development project from the Rice University School of Mathematics Project that resulted in increased student achievement was a four week summer program in which Houston area mathematics teachers participated. Teachers were placed into groups according to grade level and worked with “master teachers” who had collaborated with Rice University staff to design the curriculum for the program. The master teachers modeled effective instructional practices and authentic assessment techniques. Teachers participated in cooperative learning groups, explorations and investigations of open-ended mathematics problems, and the use of technology and manipulatives. They also learned to use the Learning Plan template to organize their daily instruction. Ongoing support was provided to the teachers in their schools through the use of on-site leaders who had previously participated in the program.

Independent evaluations of student achievement were conducted and gains in achievement for students whose teachers participated in the program were shown. In addition, the Houston Independent School District noted higher student scores on the high school level state mathematics exam for students whose teachers participated in the program (Killion, 2002, p. 86).

Survey results from another study provide supporting evidence that links professional development to student achievement. Cohen and Hill (2000), in their survey study of teachers of grades 2–5 from 250 schools in California, noted that teachers who spent time in professional development activities centered around mathematics curriculum and instructional practices around the curriculum, were better able to implement those practices and had students who performed better on California's state mathematics assessment.

Conclusion Analyzing features of the professional development programs that were linked to increased student achievement across projects suggests that:

1. All of the professional development programs engaged groups of teachers in the context of teaching and learning mathematics.
2. The programs engaged teachers in working collaboratively with peers in a set of activities that were connected to each other and to the work of teaching.
3. The professional development activities in which the teachers engaged were tied directly to the teachers' day-to-day practice and involved both learning about mathematics and examining effective instructional practice.

The next sections describe in more depth the importance of these features. In addition, the significance of considering these features for professional development aimed at the high school level will be discussed.

2.3.2. A focus on the teaching and learning of mathematics

Teachers need opportunities to learn mathematics in the same ways they are expected to teach it. According to Mewborn (2003), "just as one cannot expect students to learn something simply by being told that it is so, one cannot expect teachers to change their teaching practice simply because they have been told to do so" (p. 49). And most mathematics teachers have never engaged in mathematics in a way that enables them to use instructional strategies that promote student learning of worthwhile and challenging mathematics (NCTM, 2000). Since most teachers of mathematics have experienced mathematics algorithmically and procedurally, they have not had opportunities to develop the conceptual understanding of the algorithms or procedures they use. And rarely have they ever approached mathematical concepts from different perspectives or representations. If teachers are to be enabled to change the way they teach, "then teachers must have opportunities to talk, think, try out, and hone new practices" (McLaughlin & Oberman, 1996, p. 189).

In the previous section, which linked effective professional development programs to increased student achievement, each of the programs was grounded within the content of mathematics. For example, the CGI program engaged teachers in learning about elementary students' thinking and problem solving in the area of numbers and operations. Likewise, the RUMSP program was focused on algebra concepts. The focus on mathematics content, when providing teachers with professional development opportunities, is particularly significant with high school mathematics teachers.

Implications for the high school level. Grossman and Stodolsky (1995, p. 5) note that secondary teachers define themselves by the subject they teach. "The nature of the parent discipline and features of the school subject, as well as teachers' beliefs regarding the subject, help create a conceptual context within which teachers work" at the high school level. In their study of 16 high schools, Grossman and Stodolsky (1995) discovered, through structured interviews and survey results, that each subject area at the high school level had its own subculture consisting of beliefs, norms, and accepted forms of practice. In mathematics, more than in any other content area, the culture supported a resistance to heterogeneous grouping and relied on grouping students by previous achievement, mainly because of a belief in the sequential and defined nature of mathematics. The researchers indicated that those responsible for professional development at the high school level must acknowledge and address these subcultures around content. Otherwise, teachers will disengage from the professional development activities, because they will conclude it does not support or address the needs of their particular content area.

Additional evidence supports the notion that "the most successful professional development activities are those that are extended over time and encourage the development of teachers' learning communities" (Bransford et al., 1999, p. 195) The next section will describe programs that illustrate this notion.

2.3.3. Conducted in collaborative learning communities

Stein, Silver and Smith (1998) cite building a community of collaboration and reflection in one of the schools involved in the QUASAR project as contributing to teachers' ability to sustain their growth and learning over a long period of time which resulted in improvement in their students' learning. Likewise, both the CGI and RUMSP professional development programs used learning communities within schools to support teachers in implementing elements of the program. Teachers who participated in the CGI Program attended the professional development sessions with colleagues from their own schools and received continued support when back at their schools through the use of mentors. Teachers also assisted each other at the schools by serving as "sounding boards" for each other as they applied what they learned in their professional development sessions to their own classroom practice (Carpenter, et. al, 2000). In the two programs at Rice University (RUMSP), teachers involved in the project met in their buildings on a weekly basis and collaboratively planned lessons and addressed other instructional issues such as assessment and pedagogy (Killion, 2002). Additional studies describe how collaborative learning communities assisted teachers in improving their instructional practice.

Importance for the high school level. McLaughlin and Talbert (2001), who studied 16 high schools in Michigan and California, determined that most high school teachers worked in weak teaching communities. However, they noted that strong teacher communities can have both positive and negative effects on teachers and their students. Two high school mathematics departments, with similar demographics including a majority of minority students, were considered to have a strong sense of community. At one of the schools, the math department responded to the increasing diversity in their student population by tracking students and subjecting students to competency tests in order to advance to the next level. When the district mandated a common pre-algebra course for all students, the teachers responded by failing a majority of students and then designing watered down algebra courses for the students in subsequent years. At the other high school, the mathematics department had a strong belief that their students could succeed in algebra and beyond and collaboratively changed their practice to accommodate their diverse student population's learning needs. As a result, they had high numbers of students enrolling in classes beyond algebra.

Structures for building professional learning communities are nearly nonexistent in U.S. high schools (Grossman, Weinburg & Woolworth, 2001). The norms of American high schools are a school day in which teachers' only interaction with other professionals is at lunch time, in the hall, or before and after the school day. Nearly all teacher learning opportunities occur outside of the school in one-shot workshops, generic inservice days, or summer or weekend sessions, leaving no opportunity to build a professional learning community and perpetuating the notion that teacher learning does not occur in the school.

Several researchers, however, have noted success in establishing learning communities in high school departments. Stodolsky and Grossman (2000) reported that English and mathematics teachers who worked in collaborative high school departments with a common focus on student learning were better able to equitably engage their students in learning. Gutierrez (2002) found similar results in her studies of successful urban high school mathematics departments. Teachers in those departments used rigorous mathematics curricula for all students, discussed their lessons, and observed each other teaching their diverse learners. The result was students who were more actively engaged in mathematics and who scored well on standardized tests.

When teachers do not have opportunities to participate in continuous learning about their practice in the setting in which they work, and continue to work in isolation of others, any attempts to improve instruction are fatal (Elmore, 2002, p. 29). This makes salient the final, and perhaps most important, feature of professional development – situating professional development experience in the actual work of teaching.

2.3.4. Situated in the day-to-day practice of teaching

In order to develop knowledge about teaching, teachers must have opportunities to engage in activities that "are at the heart of a teacher's daily work" (Smith, 2001, p. 2). Ball and Cohen (1999) promote the notion that if teachers participate in professional learning around inquiry into their own practice, their everyday work in the classroom would be the source for effective professional development and would result in increased student learning. They state that knowledge necessary for teaching is situated in the practice of teaching and must therefore be learned in the practice of teaching. In fact, "to propose otherwise would be like expecting someone to learn to swim on a sidewalk" (Ball & Cohen, 1999, p. 12). The professional development programs described previously that were linked to increased student achievement (i.e. CGI, QUASAR, and RUSMP) provide examples of what such professional development might entail.

Fennema et al. (1996) report that teachers who participated in the CGI program engaged in activities directly tied to what they would be doing in their classrooms. They viewed and analyzed videos of children solving problems, had their own students solve similar problems, and then shared the solutions with other participants in order to discuss what students understood mathematically. Teachers became significantly more successful in understanding not only the types of problems their students could solve, but also in understanding the strategies their students used, the misconceptions and errors their students made, and a deeper understanding of what individual students in their classrooms were thinking.

Many teachers who participated in the QUASAR project engaged in professional development activities directly related to their work around issues they identified (Brown & Smith, 1997). When attending university courses as part of their professional development, the courses addressed specific issues identified by the teachers and were taught by someone familiar with the teachers and their goals. The courses often drew on mathematical tasks the teachers used in their classrooms so as to inform their own instruction. Another major professional development activity in which the teachers participated was collaborating with their colleagues around activities that were "closely tied to the teaching process itself and to teachers' classroom practices" (Brown & Smith, 1997, p. 139). This often included planning and then reflecting on the implementation of lessons in their classrooms. During the summer, many of the teachers had opportunities to reflect on the previous years' instruction and then develop, refine, or review curriculum and other materials in order to provide better opportunities for their students to learn mathematics.

Cohen and Hill (2000) also discovered in their survey of elementary teachers that professional development must be aligned with the day-to-day work of teachers, particularly in terms of the mathematics curriculum. Professional development that had the greatest success in getting teachers to change their instructional practice and in improving student achievement was when the activities were aligned with the curriculum the teachers taught and which their students studied and in which student assessments were consistent with the curriculum. They report, "workshops that offered teachers an opportunity to learn about student math curriculum are associated with teacher reports of more reform-oriented practice" (p. 309). The majority of the workshops were either Marilyn Burns workshops or replacement unit workshops. In contrast, special topics or issues workshops (e.g. cooperative learning, classroom management, use of manipulatives) that were not specifically tied to the curriculum did not result in changing teachers' practice, either toward reform-oriented or toward traditional practice The authors conclude that professional development for teachers, in which teachers have more concrete, topic specific learning opportunities, may be effective in changing their practice, "because the workshops offered teachers elements of a student curriculum, which may have helped them to structure their teaching and support their practices when they left the workshop and returned to their classrooms" (p. 312.)

Importance for the high school level. Teachers involved in the RUMSP projects (Killion, 2002) made use of Learning Plans in their instruction to improve opportunities for their high school mathematics students. In one of the projects, teachers would meet with coordinators who had developed basic outlines for teaching each algebraic concept in collaboration with Rice University and then collaboratively adapt, expand, and implement the Learning Plan for each concept to meet the needs of their own students. The Learning Plans included, not only the key mathematical concepts addressed, but also instructional strategies and assessment techniques for that particular concept. In the second project, teachers attended a four week summer program in which master teachers, who collaborated with Rice University, modeled exemplary instruction and conducted authentic assessments which teachers then implemented during the school year in their own classrooms. Support for the teachers at the school level was provided by leaders who had previously participated in the RUMSP program so that the implementation of the instructional practices modeled at the summer session could be supported and sustained.

2.3.5. Conclusion

Situating professional development in the day-to-day work of teaching is a key component of effective professional development and has been shown to be successful in assisting teachers to improve their instructional practice (Smith & Brown, 1994; Fennema et al., 1996; Killion, 2002). In addition, materials that depict the day-to-day work of teaching such as mathematical tasks, records of teaching practice, and student work have all been shown to be successful resources for teachers as they engage in professional development experiences (Cohen & Hill, 2000; Stein et al., 2000; Kazemi & Franke, 2004). Finally, providing opportunities for teachers to collaborate with others is a key feature of professional development that has shown promise in engaging high school teachers in experiences intended to change practice (Stodolsky & Grossman, 2000; McLaughlin & Talbert, 2001; Gutierrez, 2002).

2.4. FRAMING THIS STUDY

Two key instructional practices that have been linked to promoting student understanding of mathematics and increased student achievement are selecting challenging tasks for students and supporting students' engagement in those tasks by prompting them to explore, explain and justify, and discuss mathematics. This type of instructional practice is not the norm in the U.S. Therefore, teachers need opportunities to learn about the importance of selecting challenging tasks for all students and then going beyond asking only procedural questions by asking a variety of other types of questions. Research suggests that the learning opportunities for teachers could be provided by professional development experiences grounded in the content of mathematics, conducted in a collaborative learning community and situated in the day-to-day practice of teaching mathematics. When working with teachers at the high school level, it is essential to consider these features relative to the context of teaching mathematics at the high school level.

This study involved determining the extent to which high school teachers learned to: identify and create questions that promote student understanding of challenging mathematics; explain why particular types of questions promote mathematical understanding; and focus on mathematical understanding when identifying their purpose of asking questions as they engage in professional development experiences. These experiences were focused on: 1.) learning about the importance of and then practicing the selection and enactment of challenging tasks for their algebra students, and 2.) learning how to distinguish between question types and their purposes, and 3.) creating questions that promote student understanding. The teachers participated with other teachers from their schools as well as with a mathematics coach from the school. In addition to learning about the selection of challenging tasks and teacher questioning, teachers engaged in planning mathematics lessons which they then implemented in their own classrooms. Embedded within the planning process was a focus on selecting challenging tasks and the importance of planning to ask particular types of questions that promote understanding of mathematics. In the next chapter, the methodology for this study will be provided.

3. CHAPTER THREE: METHODOLOGY

3.1. INTRODUCTION

The purpose of this study was to examine changes in high school mathematics teachers' ability to: identify and create questions that promote student understanding of mathematics; explain why particular types of questions promote mathematical understanding; and focus on mathematical understanding when identifying their purposes for asking questions as they participated in a professional development program. The professional development program focused on planning, teaching and reflecting on lessons that featured cognitively demanding tasks. In particular, the study aimed to examine teachers' capacity to identify and create three types of questions that have been linked to promoting student understanding: 1.) probing; 2.) exploring mathematics; and 3.) generating discussion. Towards that end, 35 high school mathematics teachers completed a pre- and post-test designed to capture the extent to which their abilities related to teacher questioning changed over time.

The premise of this study was that providing teachers with ongoing opportunities to learn about questions of these three types and the ways such questions support students' engagement with cognitively challenging mathematics and promote mathematical understanding would assist them in identifying and creating questions of these types. It was also hypothesized that teachers would be less reliant on using only procedural questions. Although this study did not follow teachers into the classroom, it was hypothesized that building teachers' capacity to identify and create questions that promote student understanding of mathematics was necessary before the ability to ask such questions during instruction could occur.

This study utilized a pre-post test design with a voluntary sample of high school mathematics teachers. The pre-post test was used to determine changes in teachers' ability to: identify and create questions of the 3 types identified that promote understanding of mathematics; explain why these particular types of questions promote mathematical understanding; and focus on mathematical understanding when identifying their purposes asking questions. Data from the pre-post test was analyzed quantitatively to determine the extent to which the changes occurred and if any changes were significant. Particular responses from participants on the pre- and post-test were then used to illustrate the nature of the responses and changes in responses. Artifacts from the professional development sessions were analyzed quantitatively, when appropriate, and qualitatively to describe possible links between changes in teachers' ability to identify and create questions of the 3 types and the professional development experiences in which they engaged.

In this chapter, the methodology of the study is described including the context in which the professional development occurred, information about the participants, the data sources, and how the data was coded and analyzed. In addition, information about the rationale for and design of the pre-post instrument is described.

3.2. CONTEXT

This study was set in the context of a professional development project[2] that focused on providing high school algebra teachers from a large urban school district with opportunities to learn about instructional practices in mathematics, reflect on their implementation of the practices, and ultimately reform their own practice. The professional development in which teachers participated consisted of three components – large-group experiences, school-based study group experiences, and classroom based experiences. In the following sections, each of these three components -- and the potential opportunities they provided for teachers to learn about question types that promote student understanding of mathematics – is described.

Large-group sessions. The goal of the large-group professional development sessions was to provide opportunities for teachers to learn about effective instructional practices for teaching mathematics and to reflect with colleagues on issues related to planning for and implementing those practices. The large-group PD occurred in four sessions during the school year, each of which consisted of two days. The program involved 99 high school mathematics teachers from 17 high schools in two sub-districts within a large urban district. Each large-group session was conducted simultaneously in four rooms organized by sub-district and high school. The 4 facilitators of the sessions (one in each room) remained with the same group of teachers throughout the program. The facilitators

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[2] The project was a joint effort between a large university in eastern U.S. and a large urban district with a focus on reforming mathematics instruction at the high school level.

included the investigator and 3 other experienced mathematics educators all of whom were knowledgeable about best practices for teaching algebra at the high school level as well as the research base around which the professional development experiences were designed. The investigator and one of the other facilitators were responsible for the design of the experiences. They received input and feedback on the design from the other two facilitators as well as from two additional mathematics educators at the university.

The experiences in the large-group sessions focused on learning about best practices for teaching algebra, of which asking questions around cognitively demanding tasks was a key aspect. Teachers were expected to implement the practices they learned in their own classrooms and then reflect on the successes and challenges they experienced at the next large-group session. The professional development experiences, shown in Figure 3.1, were designed drawing on two bodies of research - the selection and implementation of cognitively demanding mathematical tasks (Stein, et al., 2000) and the relationship between questions asked during instruction and student understanding of mathematics (Boaler & Brodie, 2004).

3.2.1. Selection and implementation of cognitively demanding mathematics tasks

As noted in Chapter 2, choosing challenging mathematics tasks for students is a key component of effective mathematics instruction (NCTM, 1991; Lappan, 1997; Stein et al., 2000) and a prerequisite for asking questions that promote the understanding of mathematics. Therefore, teachers had multiple opportunities to learn about the importance of choosing cognitively demanding tasks and ways in which lessons can be implemented to maintain the demands of the tasks.

Session 1 Aug. 29-30, 2006	**Session 2** Oct. 12-13, 2006	**Session 3** Jan. 18-19, 2007	**Session 4** Mar. 28-29, 2007
Discuss challenges related to teaching mathematics	Reflect on implementation of T-Shirt task	Reflect on implementation of “Making the Team” task with a focus on questions asked during implementation	Reflect on implementation of “Shapes of Quadratics” task with a focus on questions asked during implementation
Engage in solving and discussing the “T-Shirt” task	Engage in solving and discussing the “Making the Team” task	Engage in solving and discussing the “Shapes of Quadratics” task	Engage in solving and discussing the “Multiplying Binomials” task
Sort a set of tasks according to cognitive demands	Read and discuss a vignette of a teacher implementing the “Making the Team” task with a focus on how understanding was supported or inhibited	Discuss strategies for assisting English Language Learners in understanding mathematics	Read and discuss a case of a teacher implementing the “Multiplying Binomials” task
Discuss importance of setting mathematical goals prior to planning a lesson	Analyze a set of student work from the “Making the Team” task and create questions that assess and advance student understanding	Read and discuss a vignette of a teacher implementing the “Shapes of Quadratics” task with a focus on types of questions asked	Analyze a set of student work from the “Multiplying Binomials” task and create questions to promote mathematical understanding
Analyze a vignette and identify potential student misconceptions or errors related to the T-Shirt task	Analyze the “Making the Team” lesson; Plan a related lesson with a focus on assessing and advancing questions.	Analyze the Boaler and Brodie framework of question types and compare to assessing and advancing questions	Analyze the “Multiplying Binomials” lesson; Plan a related lesson with a focus on asking a variety of questions
Plan a lesson around a task related to the T-Shirt task with a focus on goals and addressing misconceptions and errors		Generate questions that promote understanding for a set of student work from the “Shapes of Quadratics” task	
		Analyze the Shapes of Quads. Lesson; Plan a related lesson with a focus on asking a variety of questions	

Figure 3.1: Activities in the large-group professional development sessions

At each session, teachers engaged in solving and discussing an ‘adult version’ of a task they were asked to teach to their students as is noted in the rectangular shapes of Figure 3.1. ‘Adult’ versions of the tasks were adaptations of the tasks teachers would be asked to teach to their algebra students. These adaptations were made so that the tasks would be challenging for the teachers. For example, in Unit 3 (see Figure 3.2), teachers solved and discussed the “From Equations to Graphs” task (Appendix C.1.2), the adult version of the “Shapes of Quadratics” task (Appendix C.1.1). The adult version of the task was created to challenge teachers’ knowledge about the effect of the coefficient, b, on the graph of a parabola. The task used during a session aligned with the unit in the mathematics instructional guide for algebra in which teachers were currently working.

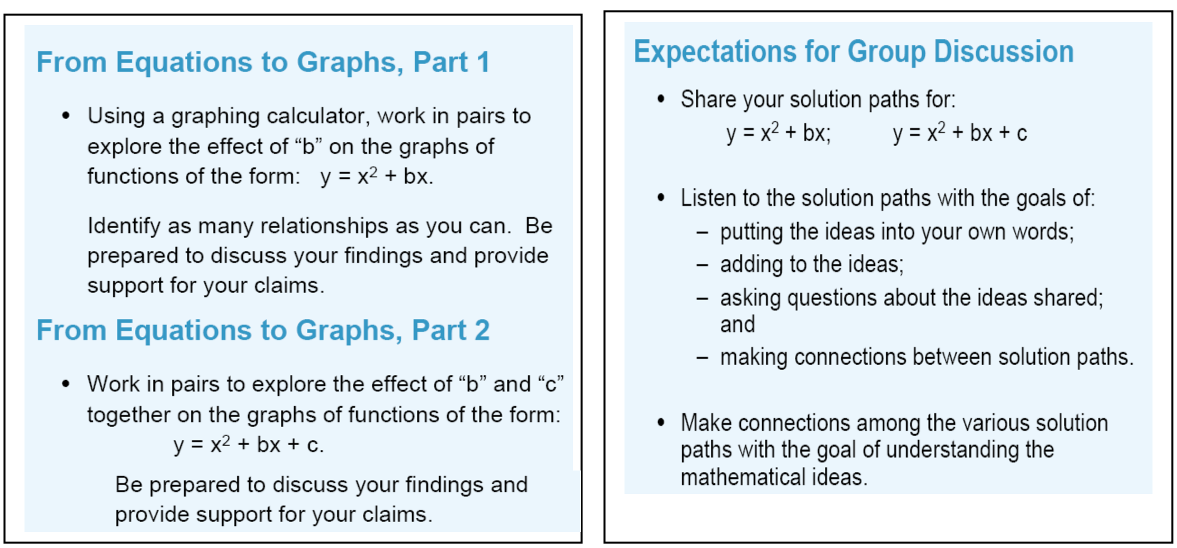


Figure 3.2: Sample ‘high level task’ activity from Session 3

A specific aspect of implementing lessons around cognitively demanding tasks was also addressed at each large-group session. At the first session (column 1 of Figure 3.1), teachers discussed how anticipating student misconceptions and errors prior to teaching the lesson could assist them in implementing high level tasks. Subsequent large-group sessions encompassed a variety of experiences which aimed to address supporting students' learning as they engage in solving challenging tasks.

For example, the two triangular shapes under Session 3 of Figure 3.1 indicate that teachers participated in two discussions – one concerning supporting English Learners as they engage in solving challenging mathematics tasks and the other concerning the Boaler and Brodie (2004) questioning framework and its impact on student achievement. Figure 3.3 shows the specific discussion activities in which teachers engaged during Session 3.

What's Ahead?

1. Introductions/Workshop Agenda and Objectives
2. Lecturette: *The Role of Language in Learning Mathematics*
3. Understanding Mathematics Vocabulary
   Reflecting and analyzing: Unit 3: Shapes of Quadratics
4. Why is reading in math different from reading other books?
5. Lecturette: Connecting reading, writing and math
6. Developing writing skills
   Reflecting and analyzing: Unit 3: Shapes of Quadratics
7. Integrating Second Language Strategies into Mathematical Tasks
8. Putting it All Together: Sheltered English instruction
9. Implementing Strategies
10. Closing/Reflection

Considering the Questions Teachers Generally Ask During Instruction

- Review the question types identified in the Boaler & Brodie article.
- Consider how the Boaler & Brodie categories match the characteristics of "good" questions we identified.

Figure 3.3: Sample 'discussion' activities from Session 3

Activities in the circular shapes were designed to engage teachers in analyzing and discussing an aspect of implementing high level tasks. These activities included reading vignettes (Session 2, Figure 3.1) or narrative cases (Sessions 3 and 4, Figure 3.1) of teachers implementing challenging tasks with their students and discussing various factors, include teacher questioning, that supported or inhibited students' understanding of mathematics. For example, in Session 3, after having solved the "From Equations to Graphs" task, teachers read the case of a teacher implementing the "Shapes of Quadratics" task with his students (Appendix C.2) and discussed factors related to the teacher's implementation that impacted students' learning. Figure 3.4 shows the specific activity in which teachers engaged.

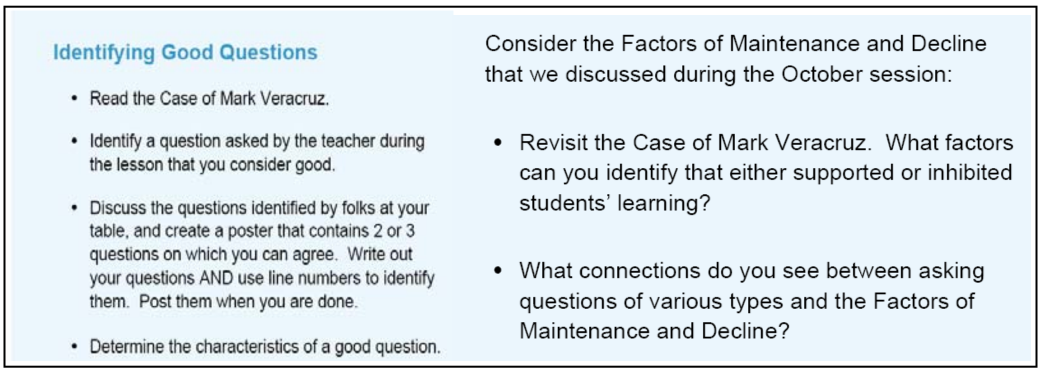


Figure 3.4: Sample 'factors related to implementation activity' from Session 3

The non-rectangular parallelogram shapes indicate activities that focused on analyzing student work samples and creating questions that support students' mathematical understanding. For example, in Session 2 of Figure 3.1, after having solved and discussed the "Multiplying Binomials" task (Appendix C.3), teachers were asked to generate questions that promote understanding for two scenarios that included student work (Appendix C.4) from the task. Figure 3.5 shows the specific activity in Session 2 for which teachers generated questions supporting mathematical understanding.

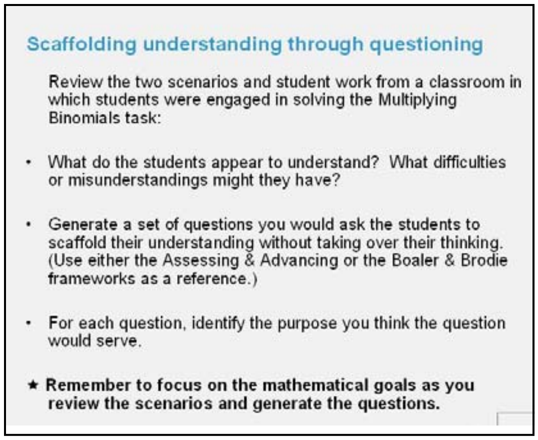
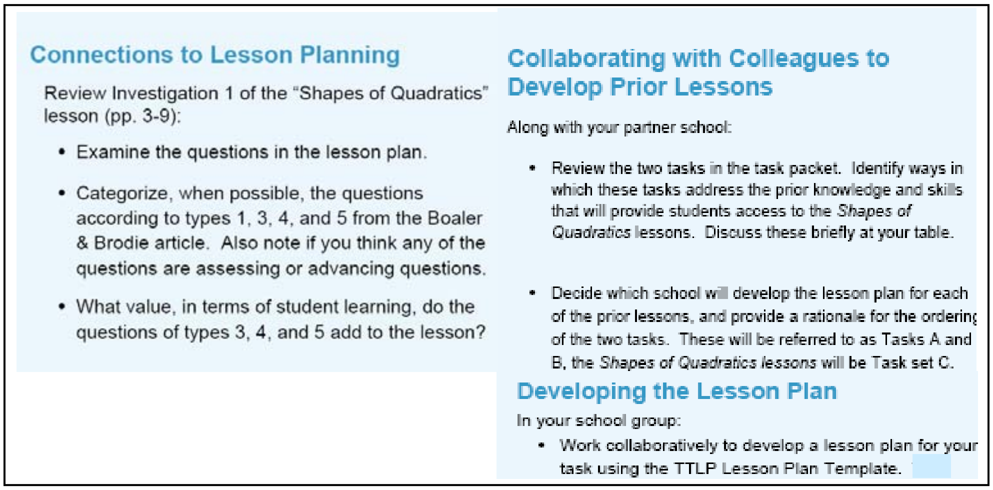


Figure 3.5: Sample 'analyzing and creating questions' activity from Session 4

At each session, teachers also analyzed a lesson plan based on a student version of the task in which they engaged. Teachers and their instructional coach then selected a challenging task that built on the concept addressed in the task for that unit and planned a lesson around that task. The hexagon shapes indicate opportunities teachers had to analyze and plan lessons with their colleagues using high level tasks related to the student version of the task featured in the lesson plan they had previously analyzed. The lessons planned by teachers focused on a particular aspect of instruction that was discussed during that session. For example, in Session 3 of Figure 3.1, teachers analyzed the "Shapes of Quadratics" lesson plan (Appendix C.5) and then planned a lesson around a task related to the "Shapes of Quadratics" task (Appendix C.6) with a focus on asking questions that assess and advance learning or are consistent with the Boaler and Brodie (2004) questioning framework. Figure 3.6 shows the specific activity in which teachers engaged related to analyzing the lesson plan in Session 3.



Connections to Lesson Planning

Review Investigation 1 of the "Shapes of Quadratics" lesson (pp. 3-9):

- Examine the questions in the lesson plan.
- Categorize, when possible, the questions according to types 1, 3, 4, and 5 from the Boaler & Brodie article. Also note if you think any of the questions are assessing or advancing questions.
- What value, in terms of student learning, do the questions of types 3, 4, and 5 add to the lesson?

Collaborating with Colleagues to Develop Prior Lessons

Along with your partner school:

- Review the two tasks in the task packet. Identify ways in which these tasks address the prior knowledge and skills that will provide students access to the *Shapes of Quadratics* lessons. Discuss these briefly at your table.
- Decide which school will develop the lesson plan for each of the prior lessons, and provide a rationale for the ordering of the two tasks. These will be referred to as Tasks A and B, the *Shapes of Quadratics lessons* will be Task set C.

Developing the Lesson Plan

In your school group:

- Work collaboratively to develop a lesson plan for your task using the TTLP Lesson Plan Template.

Figure 3.6: Sample 'analyzing and planning lessons' activities from Session 3

Finally, teachers were asked to implement and reflect on the implementation of the lesson they had analyzed and on the lesson designed by their school team. They also were asked to collect artifacts from one of the lessons. The artifacts and reflections were then shared with colleagues from other schools at the next large-group session (diamond shapes in Figure 3.1). For example, in Session 4, participants were asked to reflect on their implementation of the lesson they had planned related to the Shapes of Quadratics task. Figure 3.7 shows the implementation task teachers were given at the end of Session 3 which was the basis of the reflection in Session 4.

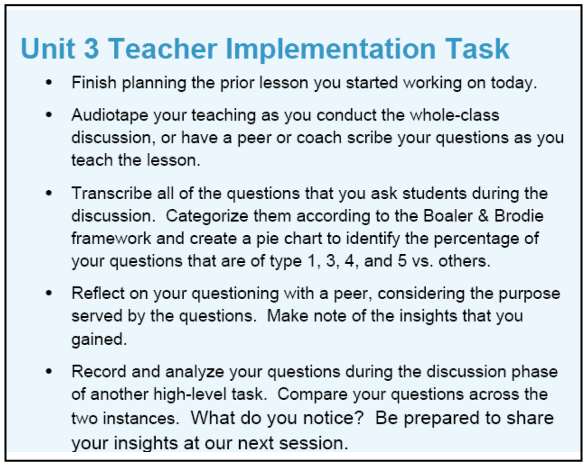


Figure 3.7: Sample 'reflecting on implementation' activity from Session 4

3.2.2. The importance of teacher questioning

Selecting a cognitively demanding task alone does not guarantee that the task will be implemented in a way that maintains the demands (Stein, et al., 2000). Teacher questioning has been identified as a key instructional practice that can either support teachers in maintaining the demands of a high level task or that can cause the demands of the task to decline (NCTM, 1991; Lappan, 1997; Boaler & Brodie, 2004). Therefore, teachers need opportunities to learn about questioning. The shaded shapes in Figure 3.1 indicate when teachers in the professional development program had such opportunities.

Beginning with the second session (column 2 of Figure 3.1) , teachers learned about questions that promote student understanding of mathematics by analyzing a vignette of teaching and discussing how the questions asked by the teacher supported or did not support students' understanding of mathematics (see the circular shape in column 2 of Figure 3.1). They also created questions to assess students' knowledge and advance their understanding of mathematics by analyzing various examples of student work representative of what students who engaged in the task were likely to produce (see the parallelogram shape in column 2 of Figure 3.1) At subsequent sessions, teachers participated in a text discussion about the article describing the Boaler and Brodie study (the shaded triangular shape in column 3 of Figure 3.1 and Figure 3.3), continued to analyze narrative cases or vignettes in terms of teacher questioning and its impact on student understanding (the shaded circular shapes in columns 3 and 4 of Figure 3.1 and Figure 3.4), and used student work as a vehicle to practice creating particular types of questions (the shaded parallelogram shapes in columns 3 and 4 of Figure 3.1 and Figure 3.5). In the planning of their lessons with colleagues, (the shaded hexagonal shapes in columns 3 and 4 of Figure 3.1 and Figure 3.6), teachers were asked to consider and discuss the questions they might ask in order to promote student understanding of mathematics. Teachers were also asked to reflect on their questioning during implementation of the lessons they had planned (diamond shapes in Figure 3.1 and Figure 3.7)

<u>School-based study group experiences</u>. A second component of the professional development program was the school-based study group experiences which were expected to occur a minimum of 2 hours every month. Sessions were to focus on planning for and reflecting on the implementation of lessons designed collaboratively around cognitively demanding tasks, and on issues associated with implementation, including teacher questioning. The experiences in the school-based study group sessions were intended to complement the large-group session and were to be planned and facilitated by the school-based coach who, in addition to attending the large-group PD

sessions with their teachers, received 2 additional days of training and assistance in conjunction with each large-group PD session.

Coach training sessions focused on supporting the coaches so that they were prepared to facilitate the school-based study group sessions with their teachers. This support included discussing facilitation moves and coaching practices, which were embedded within authentic coaching experiences; providing appropriate materials for the coaches' work; and assisting coaches in planning and reflecting on their study group sessions.

Between every two large-group sessions, coaches were asked to facilitate the school-based study group sessions with the teachers from their schools in which the lesson initially planned in the large-group session was to be discussed and finalized. They were also asked to orchestrate discussions in which teachers reflected on the implementation of the lesson they analyzed during the large-group session and the lesson they planned with their colleagues. These reflections were to include the analysis of student work from the teachers' classrooms for evidence of student understanding, the questions asked by teachers as they implemented the lessons, and the impact of the questions on students' understanding of mathematics (i.e. Did the questions support or inhibit understanding?).

The occurrence of the school-based study group sessions was dependent on a number of factors, including: administrators providing time for the teachers to meet; the coaches facilitating the sessions; and the teachers' willingness to attend the sessions if they do not occur during contractual time.

<u>Classroom based experiences</u>. The classroom based experiences were to focus on the individual implementation of and reflection on the mathematics lessons analyzed and planned in the large-group sessions.

At the conclusion of the first session, teachers were asked to implement and collect student work from the Custom T-Shirts lesson (session 1 in Figure 3.1). At the second large-group session, teachers and their colleagues reflected on the lesson implementation by analyzing the student work from their own classrooms and identifying evidence of mathematical understanding (diamond shape in column 2 of Figure 3.1). After the second large-group session, teachers were asked to implement the lesson in which they engaged at the large-group session, as well as the related lesson they planned with their colleagues. Teachers were asked to collect artifacts from these lessons and to reflect on their implementation of these lessons by focusing on the particular instructional practices learned in the large-group sessions (diamond shapes in columns 3 and 4 of Figure 3.1). This included reflecting on the successes and challenges in: asking questions that assess and advance student understanding of mathematics; asking questions that support English learners in understanding mathematics; and asking questions that probe for mathematical understanding, prompt students to explore mathematical relationships and connections, and generate discussion of mathematics. At both the school-based study group and large-group sessions, teachers had opportunities to share their artifacts and reflections on the successes and challenges related to implementation with other teachers who taught the same lessons.

Whether or not the classroom implementation occurred was dependent upon a number of factors, including the appropriateness of the lessons for teachers who were not on a traditional school schedule, teachers' willingness to implement the lessons, and support received from administrators and the coach.

3.3. PARTICIPANTS

A total of 99 high school mathematics teachers from 17 different high schools and two sub-districts within the large urban district participated in at least one day of the professional development program.

Selection of teachers and participants The criteria for participation in the professional development program was jointly established by the University of Pittsburgh and the Central Secondary Mathematics Team of the large urban district. The two partners also agreed on the selection of the two sub-districts from which the high schools and teachers would be selected. The criteria which impacted aspects of this study included:

- Principals from participating high schools would support the teachers and coaches involved in the program by providing time for them to meet between large-group sessions and by supporting attendance at the large-group sessions.
- At least 3 teachers from the same high school who taught Algebra 1 would participate in the program.
- Teachers from participating high schools would volunteer to participate in the professional development program.
- Teachers volunteering to participate in the program would be certified to teach high school mathematics.

Based on data gathered from 82 of the teachers who agreed to complete a survey from independent evaluators of the professional development program, 33 volunteered to take part in the program and 49 were told by their school administrators they were required or expected to participate. Therefore, over half of the teachers attending the professional development program did not do so on a voluntary basis. On the day that the pre-test was given, 50 of 63 teachers who were in attendance that day, approximately 79% of the attendees, agreed to take the pre-test. On the final day of the PD program, 50 of the 57 teachers in attendance, approximately 88% of the attendees, agreed to take the post-test. (Note: Not taking the pre-test did not exclude teachers from taking the post-test.) Therefore, over ¾ of the teachers in attendance agreed to participate in the study at the beginning of the program and nearly 90% agreed to participate at the conclusion of the program. Since this study used a pre-post test design, only the 35 teachers who took both the pre- and post-tests could comprise the sample for this study. Hereafter, the 35 teachers forming the sample will be described as 'participants.' The term, 'teachers,' will be used when discussing the 99 teachers who attended at least 1 day of the professional development program of whom the 35 participants were a subset.

Demographics of the participants and teachers Table 3.1 provides a summary of the available demographics describing the 35 participants as well as the 69 teachers in the PD program for whom demographic data was available. It should be noted that the 35 participants were a subset of the 69 teachers:

Table 3.1: Demographics - participants in the study and teachers in the PD program

	Participants (N = 35)	**Teachers (N = 69)**
Gender	57% male, 43% female	64% male, 36% female
Teaching experience:		
5 years or less	57%	69%
6-20 years	31%	23%
21 or more years	12%	8%
Race:		
Caucasian	51%	51%
African American	14%	13%
Hispanic	25%	26%
Asian	9%	9%
Certified to teach H.S. mathematics	94% (33 out of 35)*	88% (61 out of 69)*

*1 of 2 participants and 6 of 8 teachers indicated they were in the 1st or 2nd year of teaching and enrolled in a certification program

When considering the typical mathematics teacher in the U.S., as reported in results from the National Survey of Science and Mathematics Education (Horizon Research, Inc., 2000) and The Condition of Education 2000 – 2006 (NCES, 2006), mathematics teachers at the high school level have the following characteristics:

- 45% male, 55% female
- 91% are white, 4% are Hispanic, and 1% are Asian
- 28% have 5 or less years of teaching experience; 38% have 6-20 years of teaching experience; and 34% have more than 20 years of teaching experience.

In comparing both the teachers and the participants to the national sample, one can see that there was a slightly higher proportion of males (57% and 64% respectively) and a much higher proportion of minority teachers (39% African American and Hispanic) than in the national sample (49% male, 4% African American). The high percentage of African American and Hispanic teachers could be due to the fact that the urban district's student population included a high percentage of Hispanics and African Americans. The experience level of both the participants and teachers, over 50% having 5 or less years experience, was lower than the national average for which only 28% had 5 or less years experience. This could be due to the high turnover rates of teachers at the high school level in urban districts. When comparing the gender, race, and experience level of the 'participants' and 'teachers' for this study, however, it appears that the participants were fairly representative of the teachers from the district participating in the professional development program.

The certification of the participants and teachers was compared to data about urban school districts described in the *Condition of Education 2003* (NCES, 2003). The document reports that approximately 90% of America's high school mathematics teachers are certified to teach mathematics. In schools for which the majority of students are minorities and have high rates of poverty approximately 86% of the high school mathematics teachers are certified to teach mathematics. (Note: The report also notes that the certification rates for mathematics teachers at the middle school level are much lower. Nearly 23% are not certified.) The participants appear to be certified to teach mathematics at a higher rate than the national average while the teachers in the PD program appear similar to the national average. However, a 2006 report of teachers' credentials in the state and city in which the participants taught (Clark & Suckow, 2006) reported that, of the 2500 high school mathematics teacher in the district, less than 200, or 8%, were not certified to teach mathematics. Therefore, the certification level of the participants, although high, was not far from the reported rates for high school mathematics teachers in the district. In addition, the high rate of certification could be due to the fact that criteria for selection into the professional development program included being certified to teach mathematics. Thus, certification was an expected characteristic of the teachers participating in professional development program, and, ultimately, the subset of teachers who formed the sample for this study.

3.4. DATA SOURCES

The primary data source for this study was a pre- and post-test instrument (Appendix A) designed to assess teachers' ability to identify and create questions of particular types and explain why certain question types promote mathematical understanding. Various artifacts from large-group and school-based study group sessions were also collected in order to determine if a link could be made between changes in teachers' ability to identify and create the particular question types and the professional development experiences in which they participated. Table 3.2 indicates which data sources addressed each research question. The following sections describe in more detail, information related to the pre-post instrument and the artifacts that were collected.

Table 3.2: Research questions addressed by data sources

Research Question	Item	Purpose of data source
1. To what extent can high school mathematics teachers identify different question types that promote understanding of mathematics prior to and after participation in a professional development program focused on improving instructional practice in mathematics? In particular, to what extent can they identify probing, generating discussion, and exploring questions?	Pre- and Post-test: Part 3	• Determine teachers' ability to identify questions that promote understanding of mathematics
2. To what extent can high school mathematics teachers explain the reasons why different question types promote understanding of mathematics prior to and after participation in a professional development program focused on improving instructional practice in mathematics? In particular, to what extent can they explain why probing, generating discussion, and exploring questions promote understanding?	Pre- and post-test: Part 3	• Determine teachers' ability to explain why particular types of questions promote understanding
3. To what extent can high school teachers create questions that promote student understanding of mathematics prior to and after participation in a professional development program focused on improving instructional practice in mathematics? In particular, to what extent can they create probing, generating discussion, and exploring questions?	Pre- and post-test: Part 2	• Determine teachers' ability to create questions that promote understanding of mathematics
4. To what extent do high school teachers focus on promoting understanding of mathematics when identifying the purposes of the questions they ask their students prior to and after participation in a professional development program focused on improving instructional practice in mathematics?	Pre- and post-test: Part 1	• Determine teachers' focus on student understanding when stating the purposes of questions they ask
5. What might account for changes in teachers' ability to identify and create questions that promote understanding of mathematics and to explain why such questions promote understanding?	Attendance sheets Agendas from PD sessions Teacher questionnaire Materials from PD sessions Responses to 2 prompts	• Determine what might account for changes in ability to identify and create questions that promote understanding of mathematics

3.4.1. The pre-post test instrument

The pre-test was given at the beginning of the second large-group session in October and the post-test was given at the end of the fourth and final large-group professional development session in March. The following sections describe the importance of and difficulties associated with measuring mathematics instruction and give detailed descriptions of how the instrument was developed and a rationale for its design.

3.4.1.1. The importance of and difficulties associated with measuring instruction

Ideally, one could follow a large number of teachers into their classrooms prior to and after the professional development program and analyze the discourse from multiple lessons in order to determine if they can, in fact, learn about and create particular types of questions that promote understanding of mathematics and then adopt a practice of asking those questions when appropriate. While extensive and numerous classroom observations may be the "gold standard" for measuring instruction, such a large-scale and detailed method that would provide valid and reliable results is a challenge, to say the least (Ball & Rowan, 2004). Using valid sampling methods, establishing validity in the measurement instrument, determining reliability among raters, and, even determining what constitutes good mathematics instruction are some of the many factors that contribute to the challenge of measuring instruction in a way that can be linked to student understanding (Ball & Rowan, 2004). Surveys and questionnaires, although much easier to design, utilize, and analyze, suffer from the inability to capture the complexities of what is actually occurring in the classroom and from the fact that self-report data is not necessarily consistent with real practice (Borko et al., 2003). Another approach to measuring instruction, in-depth case studies, has allowed researchers to explore the interactions occurring during instruction at an in-depth level. However, the inability to generalize the results from a small number of classrooms and the prohibitive cost of conducting case studies in a larger number of classrooms are limitations that have yet to be resolved in making case studies a viable method of measuring instruction on a large scale (Borko et al., 2003).

3.4.1.2. Alternatives to classroom observation. Alternative approaches to measuring classroom instruction have been evolving in recent years. For example, Matsumura et al. (2006) have studied the potential of collecting and analyzing student work as a proxy for classroom observations in order to measure the quality of mathematics instruction. Results from their study indicate there was a "statistically and qualitatively consistent" correlation between using student work and observing mathematics instruction (p. 45) and that the next steps might be to determine and measure instructional behaviors that affect student achievement. Borko et al.(2003) have studied collections of classroom artifacts for similar purposes and report that researchers had "a reasonable amount of agreement" in rating instructional practice based on the artifacts. Their study indicates that analyzing classroom artifacts has the potential to answer the question, "What is it like to learn science/mathematics in your classroom?" (p. 38) Finally, researchers in the University of Michigan's project, Learning Mathematics for Teaching, have reported a high correlation between a multiple choice test whose items reflect actual tasks performed by teachers of mathematics and a videotape of teachers teaching mathematics (Hill, Schilling & Ball, 2004).

Such alternative methodologies have become increasingly important as concerns about conducting large scale research in mathematics education, rather than studying individual schools or teachers, have emerged. However, implementing these alternatives to classroom observation has been a challenge and is a relatively new area of research in mathematics education (Borko, Stecher, Alonzo, Moncure & McClam, 2003; Hill, Ball, Bass & Schilling, 2006; Matsumara, Boston, Slater, Junker, Steele & Peterson, 2006). The pre- and post-test used in this study represents an effort to capture, without classroom observation, one aspect of instruction – teachers' ability to pose questions of particular types.

3.4.1.3. The design of the pre- and post-test instrument The purpose of the pre- and post-test was to determine any changes in teachers' ability to identify and create questions of particular types and to explain the purposes these question types serve in promoting mathematical understanding as they participated in the professional development program. The design of the instrument reflected an effort to situate it within the actual work of teaching mathematics.

3.4.1.4. Rationale for the design Although questions can be identified and described according to the potential they have to promote student understanding, in order to establish whether or not a particular question serves that purpose, one must also analyze the context in which the question was asked and, if possible, how the student responded, and what the teacher did after the response (Gall, 1970; Hiebert & Wearne, 1993; Boaler & Brodie, 2004). Thus, such an analysis requires more than just taking notice of the question itself. One must also examine the context in which the question was asked. Therefore, the instrument for this study was designed based on artifacts of authentic classroom practices.

3.4.1.5. Components in the design of the instrument The instrument was comprised of three types of items: Part 1--an open-ended prompt about the purposes of asking questions of students; Part 2--scenarios and accompanying student work for which teachers are asked to create questions that promote student understanding of mathematics; and Part 3--a transcript of a teaching episode which includes a set of embedded questions for which teachers are asked to identify those questions that promote student understanding and to explain why a chosen question promotes student understanding.

Part 1: The open-ended prompt asked teachers to identify what they believed were the purposes of asking students questions. The intent of the prompt was to determine the extent to which teachers perceived questioning as a vehicle for promoting student understanding of mathematics prior to and after the professional development program. The hypothesis was that the participants would not focus on student understanding as the primary purpose of questioning on the pre-test. On the post-test, it was expected that participants would refer to promoting student understanding when listing the purposes at a higher rate than on the pre-test.

Part 2: Before beginning part 2 of the pre- and post-test, participants were asked to solve a mathematics task. The second component of the test described 3 brief scenarios that might occur during the implementation of the mathematics task, including a teacher's mathematical goals and samples of student work for each scenario. Each scenario was based on authentic video footage and student work artifacts from four different classrooms in which the same task was taught to algebra students. For each scenario, the participants were asked to create 5 questions that they would ask to promote student understanding in terms of the mathematical goals of the lesson. The hypothesis was that the majority of teachers would create more procedural questions and fewer exploring mathematics, probing, or generating discussion questions on the pre-test and more of the three question types highlighted in the professional development on the post-test. In addition, it was hypothesized that the results of the Boaler & Brodie (2004) study would play out in the pre-test – few, if any, exploring mathematics questions would be created by the participants – and that more exploring mathematics questions would be created on the post-test.

Part 3: The third component of the instrument featured a transcript in which a teacher and her students are engaging in the task. The transcript was constructed by analyzing authentic video footage, transcripts and student work artifacts from one of the four classrooms. Participants were asked to read the transcript, select questions the teacher asked that promoted student understanding of mathematics, and explain why they thought the selected questions promoted understanding. The questions featured in the transcript were intended to exemplify the three question types identified for this study – probing questions, exploring questions, and generating discussion questions. Procedural questions were also included in the transcript since they are the most common type of question asked by teachers. It was hypothesized that in the pre-test, participants would choose to select fewer questions of the three types and more of the procedural type questions, and that the reverse would be true for the post-test. It was also hypothesized that participants' ability to identify reasons for choosing particular questions as promoting student understanding would increase and become more explicit on the post-test.

Because each scenario in part 2 and the transcript in part 3 were authentic and reflected what a teacher might encounter in a typical mathematics class, teachers should have had a sense of familiarity about each. Therefore, the context around which a particular question was to be identified or created had the potential to be somewhat representative of the teacher's own classroom allowing a connection to his/her own practice.

It was hoped that this instrument could serve as a beginning point for determining teachers' knowledge of particular question types and how that knowledge might change as they participated in a practice-based professional development program.

3.4.1.6. Development of the instrument An initial version of the pre- and post-test, developed by the investigator, went through several revisions based on feedback from four reviewers who are mathematics educators familiar with the Boaler and Brodie study.

For example, the initial version of the test included a section in which questions were identified in a vignette of teaching and participants were asked why a particular question promoted student understanding of mathematics. This section was eliminated when the investigator and reviewers agreed that the section would provide no additional information related to the research questions that would not already have been be provided in other parts of the test. In addition, phrases in the transcript that were not in the form of a question and yet served the same purpose as asking a question (e.g. *Explain what you mean, You said intersect because…*) were reworded at the suggestion of the reviewers so that they did not cause confusion to the participants or that add unnecessary complexity to the coding. Also, questions in the initial version of the transcript for which the reviewers and investigator did not agree as to the type of question or its purpose were reworded or eliminated. The final pilot version of the pre- and post-test reflects a version in which 3 of the reviewers agreed with the investigator as to how the types of question would be coded (i.e. procedural, probing, exploring, or generating discussion).

3.4.1.7. Pilot of the instrument The final draft version of the pre-post test was piloted with 6 high school mathematics teachers, all of whom were certified to teach secondary mathematics. Their experience ranged from 8 years to 30 years. This pilot served to inform the investigator as to potential responses to the various items and to determine any issues related to coding the responses. The pilot resulted in a revision to the prompt in part 1 of the pre- post-test which had initially asked teachers to list questions they would ask their students to promote understanding of mathematics and to state the purposes of asking questions of their students. Because the prompt did not provide a context in which the questions would be asked, it was difficult for teachers to list actual questions. However, they were able to list a variety of purposes for asking questions. Since the instrument includes a section in which participants are asked to create questions that promote understanding given particular scenarios, it was suggested by 2 of the reviewers and the investigator's advisor that the prompt only ask participants to state the purposes of asking questions of their students.

3.4.2. Artifacts from professional development sessions

In addition to the pre-post test instrument, artifacts from the large-group professional development sessions were collected. These consisted of:

- Attendance sheets from large-group sessions
- Agendas from large-group sessions
- Materials from large-group sessions
- Responses to two prompts at the conclusion of the program
- A teacher questionnaire requesting demographic information from the participants

These artifacts were used to determine if an association could be made between teachers' attendance at and participation in the professional development program and changes in their ability to identify and create questions that promote student understanding of mathematics. Although a direct cause-effect relationship could not be determined with these data, it was hoped that they might provide insight into the potential for professional development experiences to change instructional practice at the high school level which could then be studied in more depth with a different design in subsequent studies. It was hypothesized that greater changes would occur for teachers who attended the majority of the sessions.

3.5. CODING AND ANALYZING THE DATA

3.5.1. Coding and analyzing the pre- and post-test

The pre- and post-test were coded and analyzed by the investigator of the study. A subset of the data (20%) was also coded by a knowledgeable rater, who underwent 5 hours of training. The rater was not aware whether the data was from the pre- or the post-test. The next sections describe how each of the 3 parts of the test were coded and analyzed and report on the reliability.

3.5.1.1. Part 1 – Responding to open-ended prompt In part 1 of the test, participants were asked to identify the purposes of asking questions of their students.

Coding Each of the responses given by participants was coded using a two point scale as is shown in Table 3.3. A score of '1' was given if the stated purpose was consistent with the descriptions of 'exploring', 'probing' or 'generating discussion' questions as described in the questioning framework (Appendix B) and a score of ‘0’ if the stated purpose was consistent with ‘procedural’, ‘non-mathematical’ or ‘other’ questions.

Table 3.3: Rubric for scoring responses to Part 1 of pre-post test

Score	Description
1	Purpose refers to asking students to: • explain or clarify their thinking • justify their solutions to problems • identify mathematical relationships • link mathematical representations or ideas • explain the thinking and reasoning of others or to restate in their own words • contribute additional information to a discussion • agree or disagree and justify why
0	Purpose refers to asking students to: • give yes or no or one-word answers • recall facts or memorized procedures Other purposes related to learning mathematics Purposes unrelated to learning mathematics

<u>Reliability</u> On part 1 of the pre- and post-tests, the investigator and rater independently coded 40 responses from 14 participants. The rater was not aware which responses were from the pre-test and which were from the post-test. Only 3 of the 40 responses were scored differently by the raters.. In all 3 cases, the investigator coded the response as a ‘0’ while the rater coded it as a ‘1’. The coders achieved 92.5% reliability in their coding on the pre- and post-test.

<u>Analysis</u> Matched-pairs t-tests were conducted on the pre- and post-test data for participants’ average score (number of points received divided by the number of responses) to determine if there was a significant change from pre- to post in terms of explaining the purposes of questions as promoting student understanding of mathematics.

Because the hypothesis was that teachers would increase their ability to explain the purposes questions have in terms of promoting student understanding, a one-tailed test was conducted. Excerpts from the responses teachers gave were then used to describe the nature of the changes in their ability to identify the purpose of questions in promoting student understanding of mathematics.

The frequency of each score, 0 and 1, was also recorded for the pre- and post-tests. Matched pairs t-tests were also conducted on the percent of responses receiving each score to determine if there was a significant increase in responses receiving scores of ‘1’ and a significant decrease in responses receiving scores of ‘0’.

3.5.1.2. Part 2 – Creating questions that promote student learning of mathematics

In part 2 of the test, participants were asked to create 5 questions they would ask to promote student understanding of mathematics for each of 3 separate scenarios that included a piece or set of student work.

<u>Coding</u> The questions created were coded using a rubric consisting of 6 categories that were adapted from the Boaler and Brodie framework (Appendix B). The rubric used both the names and descriptions of three of the question types that were identified in the Boaler and Brodie study -- probing, exploring mathematics, and generating discussion. The fourth category, procedural questions (which Boaler and Brodie labeled ‘gathering information’ in their framework but also referred to as ‘procedural’ in their research) was included because of the prevalence of such questions in teachers’ practice. Two additional categories were used to describe other types of questions – those that dealt with the teaching and learning of mathematics but did not fit into one of the 4 previously

described categories (labeled ‘other’) and those that were labeled ‘non-mathematical’ because they did not address the teaching and learning of mathematics.

In several cases, two questions were bracketed together and counted as one question. Examples of when this occurred, as well as descriptions and examples of each of the question types are provided in the following paragraphs:

Generating discussion questions: Questions of this type, such as “How do you think the group made their graph?” and “Who can say more about the chart?”, would prompt students to participate in a discussion or to explain another student’s thinking. These types of questions tend to be directed to the entire class or to groups of students rather than to individual students.

Exploring mathematics questions: To be coded as an exploring question, the question needs to prompt students to explore mathematics beyond what was evident in the student work or to make connections to other representations or mathematical ideas, such as in the question “How can you see the cost per minute in the table?” for scenario A or scenario C. In this response, students would need to think about the relationship between the number of minutes and cost in the table to determine the rate. If, however, the same question was asked in both scenario A and scenario C, it would be coded as an exploring question in scenario A and a probing question in scenario C since the teacher would be probing for understanding when asking the question a second time.

Probing questions: Questions that would prompt students to explain their thinking or justify their answers were coded as ‘probing’ questions. These questions, such as “How did you graph the lines?” for scenario B ask students to explain or justify something that is present or seen in the student work.

Coding two questions as one: If a procedural question, such as “Where do the lines intersect?” was followed by another question, such as “How did you determine that?”, the two questions were bracketed together and counted as one probing question since the second question would not make sense without the first question.

Procedural questions: Procedural questions are those that require only one word answers or an explanation of a procedure. “Looking at the equations, what is the slope of each line?” and “By how much do the minutes increase in the chart”? for scenario C were coded as procedural questions because they prompt the student to only provide a one word answer based on information that is already given in the student work.

Other questions related to teaching or learning mathematics: This category was created to capture questions created by the teachers that did not fit into the previous 4 categories – e.g. “What other problems that we have solved does this problem remind you of?”, “What is the math term we use to talk about the rate at which the cost changes?” Although these questions deal with teaching or learning mathematics, they were not procedural, and yet, could not be coded as one of the 3 question types that promote student understanding as is defined in this study. In fact, the first question would be categorized as ‘Linking and Applying’ in the Boaler and Brodie framework. The second would be an example of ‘Inserting Terminology.’ Because this study focused only on three particular question types which are consistent with the PD in which teachers participated, the ‘other’ category acknowledged that there are additional question types related to teaching and learning mathematics but that are not targeted in this study.

Non-mathematical questions: “Why is your chart vertical instead of horizontal?” and “Why is your line crooked?” would be coded as ‘non-mathematical’ because they focused on aspects of the student work that were not related to the mathematics in the problem. Knowing why a student chose to make a vertical, rather than horizontal, table would not require the student to explain anything mathematical, nor would asking why a particular line was not drawn ‘straight’ for this particular problem.

Teachers received one point for each question they created that promoted understanding of mathematics (i.e. exploring mathematics, probing, and generating discussion). A total count of questions that promoted understanding was recorded as was the total number of questions created. Although teachers were asked to list 5 questions for each scenario, some teachers listed fewer. Counts of each type of question were also recorded.

Reliability On part 2 of the test, 75 responses from 14 participants were independently coded by the investigator and rater. The rater was not aware whether the responses were from the pre- or the post-test. The investigator and rater differed on their categorization of 7 of the responses, resulting in 91% reliability. The differences in coding did not follow a consistent pattern. In 3 cases, the investigator coded a question as ‘probing’ while the rater coded it as ‘exploring’ and in 3 cases the reverse was true. In one case the rater coded a question as ‘probing’ when the investigator coded it as ‘procedural’.

Analysis Several matched-pairs t-tests were conducted on the pre- and post-test results. Two of the t-tests compared, from pre- to post, the percent of questions each teacher created that promote student understanding of mathematics (i.e. combined number of exploring, probing, and generating discussion questions) and the percent of questions that were procedural, other, or non-mathematical. The average number of questions each participant created that promoted student understanding of mathematics (i.e. total number of questions created that promote understanding divided by total number of questions created) on the pre- and post and the average number of questions that were procedural, other, or non-mathematical were also compared using matched-pairs t-tests. Since the hypothesis was that participants would increase their ability to create questions that promote understanding and decrease the number of procedural or others type questions from pre- to post, one-tailed tests were used.

Additional t-tests compared from pre- to post the average number of each category of question each participant created and the percent of each question type created out of the total number of questions created. The hypothesis was that creating questions that prompt students to explore mathematics would be foreign to most participants and that there would be an increase in their ability to create such questions from pre- to post. It was also expected that there would be an increase in the number of probing and generating discussion questions and a decrease in the number of other types of questions. Thus, one-tailed tests were used.

3.5.1.3. Part 3 – Identifying questions that promote student understanding and explaining why selected questions promote student understanding

In the third part of the test, participants were given a transcript in which multiple question types were embedded. They were asked to select the questions they thought would promote student understanding of mathematics and then explain why the selected question promoted understanding. The responses were coded in terms of correctly selecting the questions and providing explanations.

Coding the identification of questions In terms of identifying the questions that promoted student understanding, there were 25 questions that the participants could have selected, 17 of which promoted student understanding of mathematics (i.e. generating discussion, probing, or exploring mathematics), 7 of which were procedural, and one which fell into the ‘other-mathematical’ category. (Note: Reviewers and the investigator reached 100 percent agreement on how a question would be identified.) Of the 17 questions that promoted understanding of mathematics, 4 were ‘generating discussion’ questions, 8 were ‘probing’ questions, and 7 were ‘exploring’ questions. Two of the questions could have been either ‘probing’ or ‘generating discussion’. For these 2 questions, the explanation given by the participant as to why the question promoted understanding determined whether the question was coded as ‘probing’ or ‘generating discussion.’ For example, line 4 contains the question, “So someone who wasn’t in group 1, how do you think they used the table to find the solution? Come up and show us.” If a participant provided an explanation similar to, “Get more students to talk about the solution,” it was recorded as a ‘generating discussion’ question. However, if a participant said, “Ask students to explain their thinking,” then the question was coded as a ‘probing’ question.

For the 17 questions that promoted student understanding of mathematics, participants could select the question or fail to select the question. For the 7 questions that were procedural, participants could either not select the question or incorrectly select such a question. Thus, there were two possibilities for being correct (selecting questions that promoted understanding and not selecting questions that did not promote understanding) and two possibilities for being incorrect (not selecting questions that promoted understanding and selecting questions that did not promote understanding.)

The number of questions that were correctly selected as promoting student understanding was recorded as was the number of procedural questions that were incorrectly selected as promoting student understanding. In addition, the number of each type of question that was correctly selected (i.e. exploring, probing, generating discussion) was also recorded.

Analysis One-tailed, matched pairs t-tests were conducted on the pre- and post-test results to determine if the number of questions correctly selected increased and the number of questions incorrectly identified decreased from pre- to post. In addition, one-tailed matched pairs t-tests were conducted to determine if the number of exploring, probing, and generating discussion questions correctly selected increased.

Coding the explanations A coding scheme that was based on the Boaler and Brodie framework, shown in Appendix B, was used to determine if an explanation provided was consistent with the descriptions of the three types of questions that promote understanding of mathematics as were identified for this study. The coding cheme, summarized in Table 3.4, provided for explanations to receive scores from 0 through 3 and provided examples from the test that corresponded with each code.

Reliability The investigator and rater independently coded 52 explanations from 14 participants on the test. The rater was not aware whether the explanations came from the pre- or the post-test. The coders' scores differed on 4 of the explanations resulting in over 92% agreement. All 4 scores differed by only 1 point and in all cases the investigator coded the explanation 1 point lower than the rater.

Table 3.4: Rubric for coding explanations for Part 3 of pre-post test

Code	Explanation of Code	Purpose of Question	
Exploring mathematical relationships	Correctly selects the question AND provides an explanation that:	• Require students to identify mathematical relationships • Require students to link mathematical representations • Require students to link mathematical ideas	
		Question Identified	**Explanation**
• **E3**	refers to exploring, connecting ideas or representations in a detailed way	(Line 8) How can we see the cost per minute in the table?	*Makes students think about how they got the entries in the chart and how the entries are related*
• **E2**	refers to exploring, connecting ideas or representations in a vague or surface level way	(Line 14) How do we see that cost per minute in the equations?	*Relationship between slope and cost per minute*
• **E1**	does not explicitly refer to promoting understanding	(Line 50) So, in terms of this problem, what does the y-intercept mean? What does it mean “they cross they-axis”?	*Analysis*
• **E0**	does not provide explanation		
Probing	Correctly selects the question AND provides an explanation that:	• Require students to explain or clarify their thinking • Require students to justify their solutions to problems	
• **Pr3**	refers to probing for student understanding in a detailed way	(Line 4) So someone who wasn’t in group 1, how do you think they used the table to find the solution?	*Analyze values in the table and justify why (50,7) is the answer*
• **Pr2**	refers to probing for student understanding in a vague or surface level way	(Line 44) What do you mean, “where each line started and ended?”	*Justify reasoning*
• **Pr1**	does not refer to promoting understanding	(Line 63) What do you mean by “intersect”?	*Vocabulary check*
• **Pr0**	does not provide explanation		
Generating Discussion	Correctly selects the question as promoting understanding AND provides an explanation that:	• Ask students to explain the thinking and reasoning of others or to restate in their own words • Ask students to contribute additional information to a discussion • Ask students to agree or disagree and justify why	
• **G3**	refers to student discussion, understanding others in a detailed way	(Line 4) So someone who wasn’t in group 1, how do you think they used the table to find the solution?	*Looking at someone else’s work and having to figure it out for themselves*
• **G2**	refers to student discussion, understanding others in a vague or surface level way	(Line 42) So I’d like someone who wasn’t in group 2 to explain, how do you think group 2 made their graph?	*Justify others’ thinking*
• **G1**	does not refer to promoting understanding	(Line 22) Can someone else add on to what Anita said?	*Make sure students are focused*
• **G0**	does not provide explanation		

Analysis Of the questions correctly selected as promoting student understanding of mathematics, the total score received for explanations was recorded, as was the total score for each type of question. Average scores were also determined, both for the score received for explanations of why a question promoted understanding (computed by dividing total score for explanations divided by number of questions correctly selected) and for scores received for explanations of each question type. One-tailed, matched-pairs t-tests were conducted to determine if there was an increase in the total and average scores, both overall and for each question type.

The frequency of each score, from 0 through 3, was also recorded for the pre- and post-tests. These frequencies were recorded both overall for explanations in part 3 and for explanations of each question type in part 3. One-tailed two-proportion z-tests were conducted on the percent of responses receiving each score from 0 through 3 to determine if there were significant increases in responses receiving scores of '2' and '3' and significant decreases in responses receiving scores of '0' and '1'.

3.5.1.4. Total score on pre- post-test A total score on both the pre- and post-test was computed so that any changes in teachers questioning abilities from pre- to post could be determined and analyzed. The total score was calculated by adding the percent of purposes stated in part 1 that focused on student understanding, the percent of questions created for each scenario in part 2 that promoted student understanding, the average score received for explanations in part 3 and the proportion of questions correctly selected in part 3. A 'difference' score was also calculated by subtracting the pre-test total from the post-test total.

Analysis Matched-pairs t-tests on the total pre-test and post-test scores were conducted to determine if there was a significant increase in participants' abilities related to questioning.

3.5.2. Artifacts from PD sessions

In order to determine a possible association between the professional development program and teachers' increased abilities related to questioning, written documents (e.g. attendance sheets; agendas and materials) from the large-group professional development sessions were collected.

3.5.2.1. Attendance sheets. Attendance sheets from each large-group session were collected so that participants' attendance could be calculated.

Coding For each participant in the study, the total number of days of large-group sessions they attended was recorded based on the attendance sheets from each session. In addition, the number of days they attended sessions in which an aspect of questioning was discussed (i.e. days 3, 4, 5, 6, 7 and 8 of sessions 2, 3 and 4) was recorded.

Analysis The intent was to compute a Pearson correlation coefficient to determine if there was a relationship between attendance at the large-group sessions and changes in teachers' questioning abilities. It was hypothesized that participants who significantly increased their abilities related to questioning would also have participated in most, if not all, of the sessions. However, almost all participants in the study attended the majority of sessions making the correlation unreliable in predicting a relationship between attendance and changes in questioning.

It was also intended that attendance at school-based study group sessions would be calculated. Because coaches were not consistent in submitting attendance sheets and agendas from their school-based study group sessions, the data that was collected was not reliable. For example, some coaches submitted attendance sheets for all study group sessions held at their schools (even those unrelated to the professional development program and those conducted on school time) while others only submitted attendance sheets for sessions not held on school time or for sessions that specifically were related to the PD project. It was not possible to determine the content of these sessions since they were not accompanied by agendas, reports, etc. In addition, several coaches did not submit any attendance sheets even though they verbally reported on the sessions they conducted. Because these data sources were unreliable but it was hypothesized that the school-base study group sessions might contribute to changes in teachers' abilities related to questioning, an ANOVA was conducted to compare the school at which the participant taught and the pre-test, post-test, and difference between pre- and post-test total scores for participants at each school.

3.5.2.2. Materials from professional development sessions. Materials from the large-group sessions were used to describe specific opportunities teachers had to learn about aspects of questioning. These materials included: 1. agendas from the sessions that indicated when teachers had opportunities to learn or discuss an aspect of questioning; 2. examples of activities in which teachers learned about different aspects of teacher questioning; and 3. written responses to two prompts at the conclusion of the program.

Analysis The agendas and materials from the professional development sessions were used to identify the opportunities participants had to learn about an aspect of teacher questioning and to describe the nature of the opportunities. Occurrences on the pre-post test of participants using language pertaining to questioning that was discussed in the large-group sessions (e.g. assessing, advancing, exploring, probing, etc.) were recorded and compared to determine changes from pre- to post. In addition, responses to the two written prompts were tallied to determine if an aspect of questioning was mentioned as something participants had learned about and/or planned to continue to work on in their practice . These analyses were used to provide a potential link between changes in teachers' questioning abilities and the professional development experiences.

3.5.2.3. Teacher questionnaire. The cover page of the pre- and post-test asked the participants to provide names, high schools, number of years' experience, and certification status. Each participant was assigned a number which was recorded and then noted on the pre- and post-test before the cover page containing the information was removed.

Analysis A Pearson correlation was computed to determine if there was a relationship between the number of years of teaching experience of participants and the level of change in questioning abilities. ANOVA tests were conducted to determine if the level of change in questioning abilities was different depending on the school at which the participants taught or the local sub-district in which the teacher taught. Since participants were grouped by high school for the professional development sessions, it was possible to disaggregate the pre-post test data according to the facilitator who lead the sessions. An ANOVA was calculated to determine if the changes in participants' questioning abilities differed by the facilitator of the professional development sessions.

3.5.3. Conclusion

After the data was coded and analyzed, the results were used to provide answers to each of the research questions. The synthesis of these results will be described in chapter 4 in order to draw conclusions and identify implications the results might have for mathematics education.

4, CHAPTER FOUR: RESULTS

4.1 INTRODUCTION

The purpose of this study was to examine changes in high school mathematics teachers' abilities to identify and create questions that promote understanding of mathematics as they participated in a professional development program focused on planning, teaching and reflecting on lessons that feature cognitively demanding tasks. In particular, the study aimed to examine teachers' abilities to identify and create three types of questions that are critical to promoting student understanding of mathematics: 1) probing; 2) exploring mathematics; and 3) generating discussion (Hiebert & Wearne, 1993; Martino & Maher, 1999; Boaler & Brodie, 2004).

The results of the study will be discussed in this chapter and situated within the research on the importance of asking particular types of questions when engaging students in solving challenging mathematics tasks (Boaler & Brodie, 2004) and the role practice-based professional development might play in assisting teachers in learning about and them implementing new instructional practices (Ball & Cohen, 1999; Smith, 2001).

The first section will present the results for the first four research questions which are related to teacher questioning. The second section will present the results related to the fifth research question pertaining to practice-based professional development.

4.2. TEACHERS' ABILITIES RELATED TO QUESTIONING

This section will report the results related to research questions 1, 2, 3 and 4:

1. To what extent can high school mathematics teachers identify different question types that promote understanding of mathematics prior to and after participation in a professional development program focused on improving instructional practice in mathematics? In particular, to what extent can they identify probing, generating discussion, and exploring questions?

2. To what extent can high school mathematics teachers explain the reasons why different question types promote understanding of mathematics prior to and after participation in a professional development program focused on improving instructional practice in mathematics? In particular, to what extent can they explain why probing, generating discussion, and exploring questions promote understanding?

3. To what extent can high school teachers create questions that promote student understanding of mathematics prior to and after participation in a professional development program focused on improving instructional practice in mathematics? In particular, to what extent can they create probing, generating discussion, and exploring questions?

4. To what extent do high school teachers focus on promoting understanding of mathematics when identifying the purposes of the questions they ask their students prior to and after participation in a professional development program focused on improving instructional practice in mathematics?

The purpose of the first four research questions was to determine teachers' growth in abilities to identify and create questions that promote understanding of mathematics. To that end, a pre- and post-test was administered to teachers prior to and after their participation in a professional development program that focused on teacher questioning as an important instructional practice.

4.2.1. Identifying questions that promote understanding of mathematics

To determine changes in teachers' ability to identify questions that promote understanding of mathematics, Part 3 of the pre- and post- test (Appendix A) was analyzed. Part 3 consisted of a transcript from a mathematics lesson in which a teacher asked her students a variety of types of questions. Participants were asked to select the questions asked by the teacher that promoted understanding of mathematics (i.e. probing, exploring, and generating discussion questions). Table 4.1 summarizes the results from a matched pairs t-test on the pre- and post-test data, indicating the mean number of promoting understanding questions correctly selected, as well as the mean number of procedural questions which were incorrectly selected. The table also includes the range of scores for each category.

Table 4.1: Number of questions selected on Part 3 of the pre-post test

<table>
<tr><th></th><th>N

Significance Level →</th><th colspan="3">Number of Promoting Understanding Questions Correctly Selected (Maximum score = 17)

p < .001* (Range)</th><th>Number of Procedural Questions Incorrectly Selected (Maximum Score = 7)
p = .400 (Range)</th></tr>
<tr><td>Pre-test</td><td>35</td><td colspan="3">8.03 (0-16)</td><td>1.63 (0-6)</td></tr>
<tr><td>Post-test</td><td>35</td><td colspan="3">11.34 (3-17)</td><td>1.54 (0-6)</td></tr>
<tr><td></td><td>Question Type → Significance Level →</td><td>Probing (Max.=8) p = .000* (Range)</td><td>Exploring (Max.=7) p = .013* (Range)</td><td>Gen.Dis. (Max.=4) p = .15 (Range)</td><td rowspan="3"></td></tr>
<tr><td>Pre-test</td><td>35</td><td>3.03 (0-8)</td><td>3.89 (0-7)</td><td>1.17 (0-4)</td></tr>
<tr><td>Post-test</td><td>35</td><td>4.97 (0-8)</td><td>5.06 (1-7)</td><td>1.40 (0-4)</td></tr>
</table>

* Statistically significant increase from pre- to post-test.
Note: Two questions could be classified as both 'probing' and 'generating discussion.' The question type was determined by the explanation given by the participant.

As can be seen in Table 4.1, there was a significant increase ($p < .001$) from pre to post in the number of questions selected that promoted understanding of mathematics (from 8.03 to 11.34). There was a decrease, though not significant, in the number of procedural questions incorrectly selected (from 1.63 to 1.54). The fact that this decrease was not significant could be due to the fact that there were not as many procedural questions that could have been selected as there were questions that promoted understanding (i.e. 7 procedural questions vs. 17 questions that promote understanding). The results suggest that teachers grew in their ability to identify questions that promote understanding of mathematics from pre- to post. When further analyzing the promoting understanding questions by type, there were also significant increases in the number of probing questions selected (from 3.03 to 4.97; $p < .001$) and the number of exploring questions selected (from 3.89 to 5.06; $p = .013$). There was virtually no change in the number of generating discussion questions selected. This was not surprising since there were only 4 possible opportunities to select generating discussion questions, two of which could also have been coded as 'probing'. These results suggest that participants improved their ability to identify probing and exploring questions as those that promote understanding of mathematics.

4.2.2. Explaining why certain question types promote understanding of mathematics

In part 3 of the pre- and post-test, participants were asked to select the questions asked by the teacher in the transcript that promoted understanding and to explain why the question they selected promoted understanding. This section presents the analysis of participants' explanations.

Each response that correctly identified a question that promoted understanding of mathematics received a score from 0, indicating no explanation given for a correctly selected question, to 3, indicating an explicit reference to promoting understanding of mathematics. Table 4.2 provides a summary of the results, indicating the percent of responses receiving each score from 0 to 3. Figure 4.1 displays the same information in a stacked bar graph so that the total make-up of the responses and how they changed from pre- to post-test can be observed.

Table 4.2: Percent of explanations receiving each score for Part 3 of pre-post test

	N **Significance Level →**	**3** **p < .001***	**2** **p < .001***	**1** **p < .001****	**0** **p < .001****
Pre-test	35	6%	56%	19%	19%
Post-test	35	15%	73%	10%	2%

* Statistically significant increase from pre- to post
** Statistically significant decrease from pre- to post

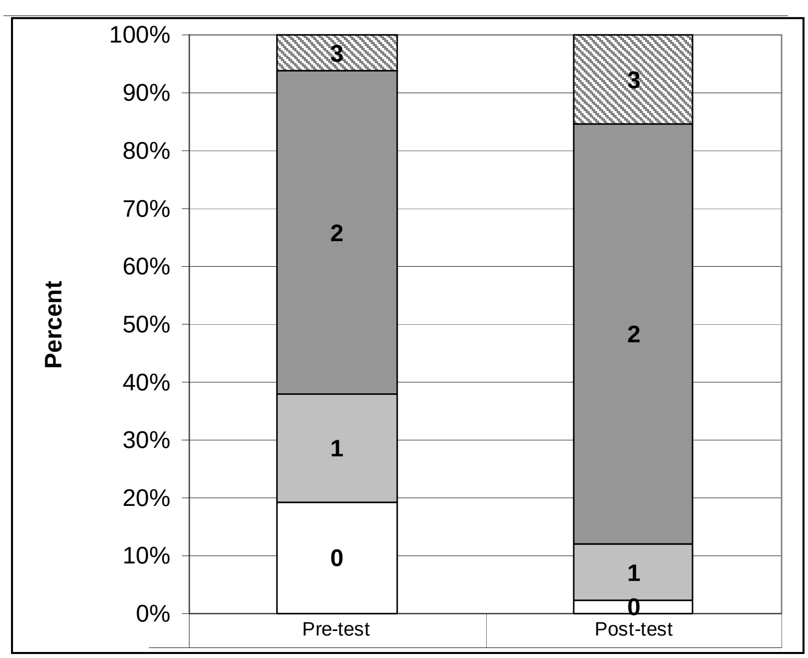


Figure 4.1: Percent of explanations receiving each score for Part 3 of pre-post test

As can be observed in Table 4.2 and Figure 4.1, there were significant increases from pre- to post in the percent of responses receiving scores of 2 and 3 and significant decreases in the percent of responses receiving scores of 0 or 1.

The total number of points received by each participant for their explanations was also computed. The maximum score a participant could receive was 51 (17 'correctly selected' questions x 3 point maximum for each explanation). The average score was then calculated by dividing the total points awarded for explanations by the number of questions correctly underlined. Results from two matched-pairs t-tests, one on the total points and one on the average score, are shown in Table 4.3 as are the range of values for each.

Table 4.3: Total points & average score for explanations on Part 3 of pre-post test

	N	**Total points (Maximum = 51)**		**Average (Maximum = 3)**	
	Significance Level →	**p .001***	**(Range)**	**p < .001***	**(Range)**
Pre-test	35	12.57	(0-29)	1.55	(0-2.67)
Post-test	35	22.83	(1-43)	1.98	(.25-2.80)

* Statistically significant increase from pre- to post-test

There was a significant increase ($p < .001$**)** in both the total points and average score participants received when explaining why particular question types promote understanding of mathematics. These results indicate that participants improved their ability to explain why particular types of questions promote understanding of mathematics from pre to post.

The hypothesis was that, prior to their participation in the professional development program, participants would be less aware of various question types, particularly exploring type questions, since they were asked rarely, if ever, in the Boaler and Brodie (2004) study. A further analysis, shown in Table 4.4, summarizes the results by question type for both the pre- and post-test. The table shows the percent of each question type awarded each score from 0 through 3, as well as the percent of total possible questions of each type that could have been selected. For example, in the pre-test, 114 exploring questions were selected by the 35 participants. However, there was a potential of 245 exploring questions that could have been selected if all 35 participants had selected all 7 exploring questions. Also, of the 114 exploring questions that were selected by the 35 participants on the pre-test, 10% had explanations receiving scores of 3, 49% had explanations receiving scores of 2, 22% received scores of ‘1’, and 20% received scores of ‘0’.

The table also adds to the results obtained for research question 1 in that participants significantly increased their ability to select exploring, probing, and generating discussion questions. For example, only 47% of the possible exploring questions were selected on the pre-test while 72% were selected on the post-test. Similarly, 33% of the probing questions and 24% of the generating discussion questions were selected on the pre-test while 62% of the probing and 35% of the generating discussion questions were selected on the post-test. The table also indicates that participants significantly increased their ability to explain why such questions promote understanding of mathematics. The percent of explanations receiving scores of ‘2’ and ‘3’ increased from 56% and 6% respectively on the pre-test to 73% and 15% on the post-test.. Furthermore, the percent of explanations receiving scores of ‘1’ and ‘0’ decreased from 19% each on the pre-test to 10% and 2% respectively on the post-test. In addition, the largest increase in scores of

‘2’ and ‘3’ and the largest decrease for scores of ‘0’ and ‘1’ occurred for explanations of selected exploring questions.

Table 4.4: No. of explanations by score and question type on Part 3 of pre-post test

	Score	Exploring		Probing		Gen. Disc.		Total
Pre-test	**3**	10 (9%)	58%	3 (3%)	65%	2 (6%)	67%	15 (6%)
	2	56 (49%)		57 (62%)		20(61%)		133 (56%)
	1	25 (22%)	42%	15 (16%)	35%	5 (15%)	33%	45 (19%)
	0	23 (20%)		17 (19%)		6 (18%)		46 (19%)
Total questions correctly selected		114		92		33		239
Total possible		35*7=245		35*8=280		35*4=140		665
Percent of total selected		47%		33%		24%		36%
Post-test	**3**	35 (20%)* (p < .001)	93%	14 (8%)	83%	12(25%)* (p=.01)	90%	61 (15%)* (p < .001)
	2	129 (73%)* (p < .001)		129(75%)* (p = .02)		32(65%)		290(73%)* (p < .001)
	1	9 (5%)** (p < .001)	7%	27(15%)	17%	3 (6%)	10%	39(10%)** (p < .001)
	0	4 (2%)** (p < .001)		3(2%)** (p<.001)		2(4%)** (p=.02)		9 (2%)** (p< .001)
Total questions correctly selected		177		173		49		399
Total possible		35*7=245		35*8=280		35*4=140		665
Percent of total selected		72%* (p < .001)		62%* (p < .001)		35%* (p = .02)		60%* (p <.001)

* Significant increase from pre- to post
** Significant decrease from pre- to post

Matched-pairs t-tests were also used to analyze changes in the total and average points by question type for Part 3. Table 4.5 shows the results for both number of points received for explanations and average explanation score received. (The maximum number of points possible for each question type was calculated by multiplying the number of questions of that particular type by a maximum score of 3.) The average was calculated by taking the total number of points received for each question type divided by the number of that question type that were selected. The range of values for each category is also included.

Table 4.5: Total and average number of points for explanations by question type on Part 3 of pre-post test

	N Significance Level →	Total Exploring (Maximum = 21) p < .001*(Range)	Total Probing (Maximum = 24) p < .001* (Range)	Total Generating Discussion (Maximum = 12) p = .007* (Range)
Pre-test	35	6.06 (0-18)	4.60 (0-15)	1.91 (0-8)
Post-test	35	10.57 (1-20)	9.26 (0-18)	3.00 (0-8)
	Significance Level →	**Avg. Exp. Pts. (Maximum = 3) p < .001* (Range)**	**Avg. Probing Pts. (Maximum = 3) p < .08 (Range)**	**Avg. Gen. Dis. Pts. (Maximum = 3) p = .006* (Range)**
Pre-test	35	1.58 (0-2.67)	1.61 (0-2.5)	1.62 (0-3)
Post-test	35	2.05 (.5-2.83)	1.83 (0-2.67)	2.21 (0-3)

* Statistically significant increase from pre- to post-test.

The results indicate that there were significant increases in the total number of points participants received for explanations associated with probing questions ($p < .001$), exploring questions ($p < .001$) and generating discussion questions ($p = .007$) and in the average number of points received for exploring question explanations ($p < .001$) and generating discussion question explanations ($p = .006$). There was an increase, though not significant, in the average number of points received for probing questions. This could be due to the fact that participants identified more questions that promote understanding on the post-test than on the pre-test resulting in higher point totals for all question types. However, participants had slightly higher scores for explanations of probing questions on the pre-test (65% of the scores were '2's or '3's) than exploring questions (58% were 2's or 3's) while the percent of explanations for probing questions receiving scores of 2 and 3 on the post-test was relatively lower than for exploring questions (83% vs. 93%). Hence, there was a greater increase in the percent of explanations receiving scores of ‘2’ or ‘3’ for exploring questions.

Table 4.6 provides sample responses from participants on both the pre- and post-tests to illustrate the nature of how their explanations changed. These particular responses were chosen because the participants provided explanations for the same line numbers on both the pre- and post-test thus making the contrast in the nature of their explanations more obvious.

Table 4.6: Sample participant explanations and scores on Part 3 of pre-post test

Participant	**Pre-test response(s)**			**Post-test response(s)**	
	Line no.	**Response**	**Score**	**Response**	**Score**
6	4	*Make sure students focus*	1	*Make certain all students can analyze the table 1 group made*	3
6	60	*Don’t miss the important part of the problem*	1	*Advancing question*	2
7	23	*Analysis*	1	*Stressing an understanding of how slope is represented in a table*	3
7	65	*Analysis*	1	*Meaning of the point and intersection*	2
49	6	*A got cheaper after 50 minutes*	1	*Assessing, asking student to explain*	2
49	8	*Student could reduce the interval of minutes*	1	*Advancing, further analyze the problem*	2
50	18	*Translate into precise language*	1	*Probing to see if they understand the meaning of that number in terms of the problem*	3
50	44	*To learn what a graph is*	1	*Check for understanding, ask them to explain the concept*	2

These examples show that participants moved from a focus on the mathematics in a general or non-specific way to a focus on students’ understanding of the mathematics. For example, line 23 of the transcript contained the exploring question, “What does the coefficient mean for this problem? Why is it important?” On the pre-test, participant 7 used the word, ‘*analysis*’, as an explanation. This was considered to be a vague reference to mathematics with no focus on understanding resulting in a score of ‘1’. On the post-test, however, the same participant focused on understanding and the meaning of mathematics by giving the following explanation which received a score of ‘3’: ‘*Stressing an understanding of how slope is represented in a table.*’ Likewise, on the pre-test, participant 49 focused on the correct answer to the question with no mention of student understanding on the explanation for the probing question on line 6: ‘*What do you mean, “A got cheaper”*? The participant wrote *A got cheaper after 50 minutes* which received a score of ‘1’. However, on the post test, the same participant made a reference to students’ understanding of mathematics in providing an explanation for the question on line 6 that stated, ‘*Assessing. Asking student to explain,*’ resulting in a score of 2. Participant 50’s pre- and post-test responses to the probing question on line 18, ‘*Jose, what do you mean, “it’s the point “O” 4 and the point “10”,* provide another example of how participants changed their explanations. On the pre-test, the participant responded that ‘*Translate into precise language*’ was the reason why the question promoted understanding. This response received a score of ‘1’. On the post-test, however, participant 50 provided a response that had an explicit focus on understanding the mathematics being discussed, ‘*Probing to see if they understand the meaning of that number in terms of the problem,*’ and received a score of 3.

The analysis of part 3 of the instrument indicates that participants improved their ability to identify questions that promote understanding and explain why particular question types promote understanding of mathematics. Of particular interest is the fact that this increased ability appeared to be greatest in the area which has been shown to be problematic for teachers in terms of their questioning – questions that promote exploring mathematical ideas.

4.2.3. Creating questions that promote understanding of mathematics

To determine teachers' ability to create questions that promote understanding of mathematics (i.e. probing, exploring, and generating discussion questions), participants were asked to create 5 questions they would ask students to promote understanding of mathematics for each of three scenarios depicting student work and a brief description of the teachers' mathematical goals. Each question created was coded according to the type of question: probing, exploring, generating discussion (the 3 types identified for this study that promote understanding of mathematics), procedural, other mathematical, and non-mathematical questions. No participants created 'generating discussion' questions for this part of the pre-post test. This could be due to the fact that the nature of the information provided in Part 2 was such that asking 'generating discussion' questions would not have been appropriate or particularly helpful in promoting understanding. Table 4.7 displays the number of questions of various types created by participants from pre- to post as well as the range in the numbers of questions created.

As is shown in Table 4.7, there was a significant increase in participants' ability to create questions that promote understanding of mathematics ($p < .001$) and a significant decrease ($p = .008$) in other types of questions created by participants. When analyzing the data by particular question type, there was a significant increase in the number of exploring questions created ($p < .001$) and a significant decrease in the number of procedural questions created ($p < .001$). There was also an increase, although not significant, in participants' ability to create probing questions. This could be due to the fact that participants initially were able to create probing questions at over twice the rate they created exploring questions (pre-test mean of 4.83 for probing; pre-test mean of 2.31 for exploring).

Table 4.7: Summary of types of questions created on Part 2 of pre-post test

	N Significance Level →	Number of promoting understanding questions (Note: No participants created Generating Discussion questions) (Maximum = 15) p < .001* (Range)		Number of procedural, other, non-mathematical questions (Maximum = 15) p = .008** (Range)
Pre-test	35	7.14 (0-15)		4.71 (0-11)
Post-test	35	9.26 (1-16)		3.54 (0-9)
	Significance Level →	**Probing p = .17 (Range)**	**Exploring p<.001* (Range)**	**Procedural p < .001** (Range)**
Pre-test	35	4.83 (0-9)	2.31 (0-7)	4.17 (0-11)
Post-test	35	5.34 (1-10)	3.91 (0-9)	2.49 (0-9)
	Significance Level →	**Percent Probing p = .41**	**Percent Exploring p < .001***	**Percent Procedural p < .001****
Pre-test	35	.41	.19	.34
Post-test	35	.42	.30	.19

*Statistically significant increase from pre- to post.
**Statistically significant decrease from pre- to post.

A final analysis of the data was conducted to determine how the percent of types of questions created might have changed. This was done because participants did not create the same number of questions (i.e. some created more than 15, some less than 15) and because questions were often bracketed together for coding reasons and counted as 1 question as was noted in chapter 3.

Matched-pairs t-test were conducted and showed there was no change in the percent of questions created that were probing questions (41% on the pre-test, 42% on the post-test). However, there was a significant increase in the percent of questions created that were exploring questions (from 19% to 30%) and a significant decrease in the percent of questions created that were procedural questions (from 34% to 19%).

These results indicate that participants improved their ability to create them in a context related to teaching mathematics. These results also indicate that participants were less dependent upon procedural type questions. Table 4.8 provides samples of questions generated by several participants in order to illustrate how their ability to create questions of particular types changed.

Table 4.8: Sample questions created on Part 2 of the pre-post test

Participant		Pre-test response(s)		Post-test response(s)	
No.	**Part**	**Response**	**Code**	**Response**	**Code**
5	A	*- What is the starting point in each column?* *- What is in the first column?*	Proc. Proc.	*- Explain the meaning of the numbers in the first row. How would these be represented on a graph?* *- What do the numbers in columns A and B represent?*	Explore Probe
11	A	*- How much do the minutes change from one row to the next?*	Proc.	*- What relationship is there between the number of minutes and the amount charged by the companies as you change the minutes?* *- What difference would there be in the table if there was no monthly fee?*	Explore Explore
12	B	*- Where did each line start?* *- Use rise/run to find the slope for each line.*	Proc. Proc.	*- What are the y-intercepts for each line and what do they represent for this problem?* *- Why does one line start at a lower point but then rises higher than the other line?*	Probe Explore
38	B	*- Which line goes up faster?* *- Where does each line cross 0 minutes?*	Proc. Proc.	*- Why is one line rising faster than the other?* *- What does each line represent in terms of this problem?*	Explore Explore
30	C	*- For every 5 minutes, how much does the cost increase?* *- Is the steepness of the line the same as the cost per minute?*	Proc. Proc.	*- In the table, what do the increments mean?* *- How can I see the cost per minute for each company in the table and in the graph?*	Probe Explore
41	C	*- In the equation, which number is the slope?* *- Show the rise/run for each line and find the slope.*	Proc. Proc.	*- What do the coefficients in the equation represent for this problem?* *- How do the coefficients in the equations affect the graphs?*	Probe Explore

These particular responses, which were representative of the all responses, were chosen because the question asked in the pre-test addressed the same particular mathematics concept as the question asked in the post-test, providing an opportunity to compare the type of question posed. It appears that participants initially focused on questions requiring one-word answers or the use of a known procedure. In the post-test, however, the questions were more focused on assessing or deepening student's mathematical knowledge. For example, participant 11's question on the pre-test dealt with looking at the given table and simply subtracting 2 given values to determine the change. On the post-test, this same participant focused on the relationship between the entries in the table and pushed students to go beyond the information given in the table. Likewise, on the pre-test, participant 41 asked students to find the slope in a given equation and a given graph. On the post-test, the participant asked students to interpret the meaning of slope and explain how it affects a graph.

The results from part 2 of the instrument provide evidence that participants grew in their ability to create questions that promote understanding of mathematics and decreased their use of procedural questions. In addition, participants' growth seemed to be greatest in terms of creating exploring questions, which, though not typically asked by teachers, have been shown to be important in improving student achievement in mathematics.

4.2.4. Focusing on understanding when stating the purpose of asking questions

In Part 1 of the pre- and post-test, participants were asked the following question: "Teachers ask hundreds of questions of their students every day. What are the purposes of the questions you ask your students?" They could respond with as few or as many responses as they chose. Each response received a score of either 0 if it did not refer to student understanding of mathematics or 1 if the stated purpose referred to student understanding of mathematics. Table 4.9 provides a summary of the results, indicating the percent of responses receiving each score and the average score for Part 1. The average score for each participant was computed by dividing the total number of points received for responses by the total number of responses listed. Figure 4.2 displays the same information in a stacked bar graph so that the total make-up of the responses and how they changed from pre- to post-test can be observed.

In observing Table 4.9 and Figure 4.2, it can be seen that there was a significant increase in responses receiving a score of 1 (from .58 to .86) and a significant decrease in responses receiving a score of 0 (from .42 to .16). This suggests that participants' focus on promoting understanding when identifying the purposes of their questions improved from pre- to post.

In analyzing the average score per response received by the participants, a matched-pairs t-test showed there was a significant increase ($p < .001$), from .64 to .89, in the average score participants received. This indicates that participants improved their ability to explain that the purposes of the questions they ask their students are to promote an understanding of mathematics.

Table 4.9: Percent of responses receiving each score and average score for Part 1 on pre- post test

	N **Significance Level →**	**Score of 1** **p < .001***	**Score of 0** **p< .001****	**Avg. score (Max.=1)** **p< .001***
Pre-test	35	.58	.42	.64
Post-test	35	.86	.14	.89

* Statistically significant increase from pre- to post
** Statistically significant decrease from pre- to post

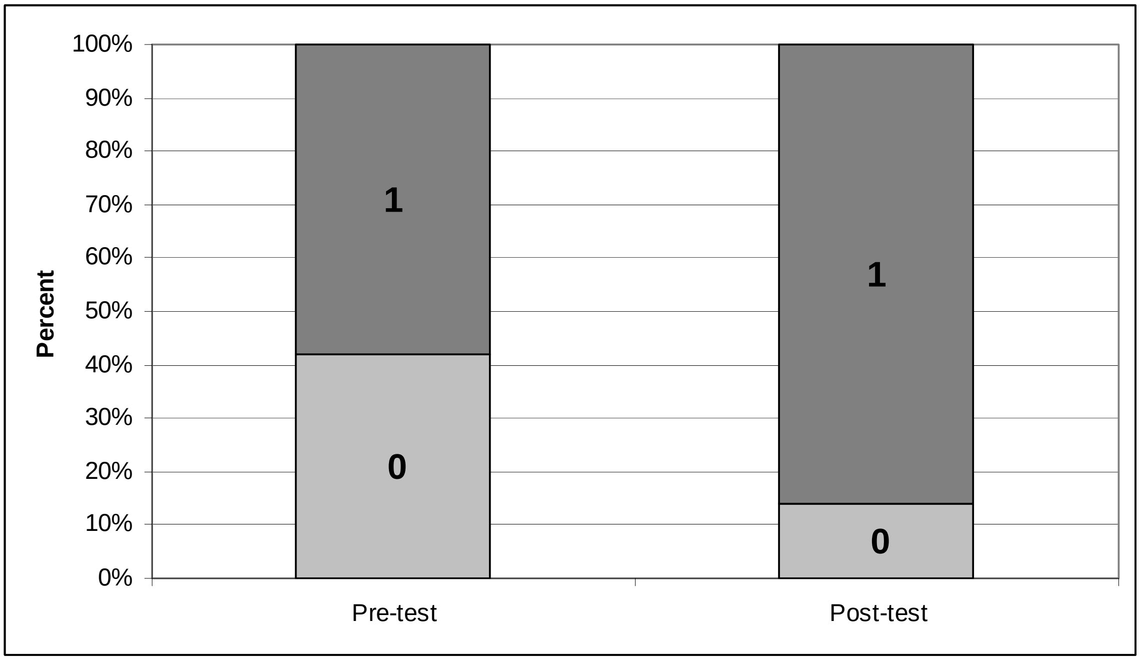


Figure 4.2: Percent of responses receiving each score for Part 1 on pre-post test

Table 4.10 presents examples of responses from particular participants to illustrate the ways in which their explanations changed from pre- to post. The responses are typical of all responses on Part 1 of the test.

In analyzing the participants' responses to Part 1 on both the pre- and post-test, it appears that teachers initially tended to focus on what they, as teachers, were doing. For example, over 40% of the responses on the pre-test made no mention of student understanding of mathematics (i.e. responses received a score of '0'). Specifically, responses tended to focus on students' prior skills or knowledge (e.g. responses from participants 7, 19, and 43 in column 2 of Table 4.10) or on getting students to the correct solution or main point of the lesson (e.g. response from participants 38 in column 2 of Table 4.10).

Table 4.10: Sample responses and scores on Part 1 of pre-post test

Participant	Pre-test response(s)		Post-test response(s)	
	Response	**Score**	**Response**	**Score**
7	*-Check their prior skills* *-Check their readiness* *-Check students' understanding of new material*	0 0 1 Avg.= .33	*- To assess students' prior knowledge* *- For scaffolding, each new question is based on the conclusion from the previous question* *- Elaborating questions, or questions which allow students to explore the concept and apply it*	1 1 1 Avg.= 1.00
19	*- Check prior knowledge in case I need to reteach pre-requisite concepts* *-Reinforce concepts taught*	0 0 Avg.= .00	*- To assess student understanding of the lesson taught* *- To scaffold their thinking* *- To advance their thinking*	1 1 1 Avg.= 1.00
38	*-Guide them to the right answer* *-Challenge them to make connections*	0 1 Avg.= .50	*- To guide them and push their thinking to higher levels*	1 Avg.= 1.00
42	*-To assess learners' understanding* *- To know learners' prior knowledge*	1 0 Avg.= .50	*- To assess their understanding. By knowing what they understand or don't understand I can plan a better lesson*	1 Avg.= 1.00
43	*- To stress main points, point to specific things I think are important*	0 Avg.= .00	*- To probe thought and advance student understanding. Students have a concept of what is going on but that thought needs to be directed and guided, like refining gold. Each question should allow the student to question his own thoughts and lead him to a deeper level of understanding. If you just keep giving statements, students will forget them. You need to allow them an opportunity to voice their understanding.*	1 Avg.= 1.00

Although over 50% of the responses on the pre-test did mention student understanding (i.e. received a score of '1'), they did so in a vague way such as the responses from participants 7, 38 and 42 which referred to 'assessing understanding' or 'making connections."

On the post-test, however, only 14% of the responses received a score of '0' and 86% of the responses made some reference to students' understanding of mathematics. In addition, participants' responses made much more explicit reference to students' understanding of mathematics on the post-test. Participants often referred to student understanding and then expanded on a response to describe the ways in which the purpose of the question impacted students. Fox example, participant 7 listed 'scaffolding' and 'elaborating questions' as the purposes and went on to describe what was meant by those terms. The same participant stated brief, vague phrases on the pre-test such as 'check their prior skills' and 'check their readiness.' Similarly, participant 42 referred to assessing students' understanding on both the pre- and post-test, but went on to explain that the assessment of understanding would inform his planning of lessons on the post-test.

In conclusion, participants in this study not only increased their ability to identify and create questions that promote understanding of mathematics and explain why certain questions promote understanding. It appears they also increased their focus on promoting student understanding when asked to state their purpose(s) of asking questions.

4.2.5. Summary of research questions 1, 2, 3 and 4

Results from the pre- and post-test instrument indicate that teachers significantly increased their abilities related to questions that promote understanding of mathematics. In particular, teachers grew in their ability to: identify exploring and probing questions as those that promote understanding, create exploring questions and explain why exploring questions promote understanding of mathematics, and focus on understanding of mathematics when identifying their purposes for asking questions. In addition, teachers significantly decreased the use of procedural questions when asked to create questions that promote mathematical understanding.

A key question raised by these results is what may have accounted for the significant changes. Some might argue that the experience of teaching between the pre- and post-tests in and of itself might cause teachers to increase their abilities to ask particular types of questions that promote understanding of mathematics. While this might be true for novice teachers, decades of research have documented that experienced teachers teach mathematics much in the same way they learned mathematics, which is also the same way it has been taught for nearly 100 years (Stigler & Hiebert, 1999; NCES, 2003; Weiss, et al., 2004) and that changes in instruction, though possible, are difficult to accomplish, particularly at the high school level (McLaughlin & Talbert, 2001; Stodlosky, S. & Grossman, P., 2000). However, particular types of professional development experiences that focus on the content taught by the teachers, that provide opportunities for collaboration, and that are situated within the day-to-day practice of teaching have shown evidence of impacting the instructional practices of mathematics teachers (Carpenter, et al., 2000; Stein, et al., 2000; Killion, 2002). In the next section, possible links between teachers' changes related to questioning and the professional development opportunities they experienced, as well as other potential factors, will be made.

4.3. POSSIBLE LINKS TO PROFESSIONAL DEVELOPMENT

As was reported in section 4.2, high school mathematics teachers who participated in this study significantly increased their abilities related to questioning. Because particular question types used by teachers during instruction have been linked to increased student achievement in mathematics (Boaler & Brodie, 2004; Martino & Maher, 1999; Hiebert & Wearne, 1993), an important question to consider becomes how and why these teachers were able to improve their abilities in this area. This section will report the results as they are related to research question 5:

5. What might account for changes in teachers' ability to identify and create questions that promote understanding of mathematics and to explain why such questions promote understanding?

Since the significant changes occurred while teachers were participating in a professional development program, aspects of the program were explored to determine if a link could be made between the PD experiences and the changes in questioning abilities.

Research has shown that professional development programs that resulted in changes in instructional practices have key features in common (Smith & Brown, 1994; Fennema et al., 1996; Killion, 2002; McLaughlin & Talbert, 2001): they are focused on the content which the teachers teach, they are conducted in collaborative learning communities, and the activities are situated within the day-to-day work of teaching. The professional development program in which teachers in this study were engaged incorporated these features. Therefore, several data sources were analyzed to determine what might account for changes in teachers' ability to identify and create questions of particular types. The results of the analysis of each data type will be discussed in the following sections.

4.3.1. Attendance at large-group professional development sessions

Prior to the beginning of the professional development program, it was hypothesized that attendance at the large-group professional development sessions might account for increases in teachers' abilities to identify and create questions that promote understanding. Although attendance at a professional development session does not guarantee participation in the activities of the session, attendance might be used as a first step in determining participation since most teachers volunteered for the program and were not required to attend.

Attendance at the 4 large-group sessions, each of which involved 2 days, was inconsistent. Table 4.11 displays the attendance data for all teachers in the four, 2-day large-group sessions.

Table 4.11 Attendance at large-group sessions by all teachers

Description	Number
Total number of teachers attending at least one day of any session	99
Total number of teachers attending at least one day in each of the four sessions	42
Total number of teachers attending all 8 days	31
Total number of teachers attending 1 or 2 days of first session	94
Total number of teachers attending 1 or 2 days of second session	83
Total number of teachers attending 1 or 2 days of third session	72
Total number of teachers attending 1 or 2 days of fourth session	60

Attrition and other factors affecting attendance A total of 99 teachers from 17 different high schools attended at least 1 day of the professional development program. Over the 7 month course of the program, 2 of the high schools, accounting for 7 teachers, withdrew from the program. For one high school, the Central Mathematics Team of the district requested the 3 teachers withdraw from the program after the third session since they had not volunteered for the program, they would attend only portions of the professional development sessions, and they were often considered to be disruptive or disrespectful to the facilitator as well as to other teachers. For the second high school, after the third session, the 4 teachers decided they were not benefiting from the program and chose to no longer participate.

Ten additional teachers withdrew from the program for unknown reasons, although it was known that at the change of the semester, several of these teachers schedules changed and they were no longer teaching Algebra 1. In two more cases, teachers became coaches during the course of the program and thus were not counted as ‘teachers’ after that time.

Nine teachers (not included in the above situations) from two high schools missed the first day of the first session because they had not been notified by their schools that they would be participating in the professional development program and 3 teachers from 3 different high schools (not already counted in the above situations) did not begin participating until the second session. At least 10 teachers from 4 high schools, who have not been accounted for in the previous situations, missed both days of the third session because they were on an alternate calendar and the third session occurred during their 3 month break. Finally, 4 teachers from one of the high schools (not already counted in the above numbers) were required to be at their schools rather than attend the final session because of a state audit of the school in terms of making Adequate Yearly Progress under the No Child Left Behind Act.

Since this study consisted of a pre-post test design, changes in abilities related to identifying and creating particular types of questions could only be determined for those who took both the pre- and post-test. This was the set of 35 “participants’ forming the sample whose attendance ranged from 5 to 8 days as can be seen in Table 4.12. Specific attendance data for 86 of the 99 ‘teachers’ (of whom the 35 participants were a subset) was available and is also noted in the table.

Table 4.12: Attendance over 8 days (4 sessions) for participants in study

Number of days attended	Number of participants	Number of teachers
Less than 5	0 (0%)	15* (10%)[1]
5	2 (6%)	11** (10%)[1]
6	7 (20%)	21*** (23%)[1]
7	4 (11%)	8 (12%)
8	22 (63%)	31 (45%)

*8 of the 15 teachers either dropped out of the program, started the program at Unit 2 or 3, or were required by their administrators to be at their schools rather than attend the last session
**4 of the 11 teachers either dropped out of the program, started the program at Unit 2 or 3, or were required by their administrators to be at their schools rather than attend the last session
***5 of the 21 teachers either dropped out of the program, started the program at Unit 2 or 3, or were required by their administrators to be at their schools rather than attend the last session
[1] Percents were computed by eliminating teachers who either dropped out of the program, started the program at Unit 2 or 3, or were required by their administrators to be at their schools rather than attend the last session

Nearly two thirds of the 35 participants in the study attended all 8 days of the 4 large-group sessions and over 90% of them attended at least three-fourths of the days of the sessions. Of the 69 teachers who could have attended all 8 days (i.e. started the program with unit 1; did not withdraw from the program; or were not required to attend school rather than the final PD session), 81% attended at least ¾ of the days. It appears that the participants in the study had a higher attendance rate than the group as a whole.

However, the group as a whole had a relatively high attendance rate during the program when accounting for factors such as when teachers started the program and whether or not they continued in the program. In addition, to be considered a ‘participant’ for the study, teachers would have had to attend both the 1st and 4th sessions which could account for the higher attendance rate of the participants. Therefore, participants were fairly representative of the teachers who participated in the program in terms of attendance.

Attendance was also calculated for the sessions in which questioning was explicitly discussed (all but the first session.) Table 4.13 displays this data.

Table 4.13: Attendance over 6 days at sessions 2, 3, 4

Number of days attended	Number of participants	Number of teachers
Less than 3	0	12* (9%)[1]
3	1 (2%)	9**(4%)[1]
4	8(23%)	22***(24%)[1]
5	2 (6%)	6 (9%)
6	24 (69%)	37 (54%)

*6 of the 12 had either dropped from the program or were required to be at their schools during the last session
** 6 of the 9 had either begun the program with unit 3, dropped from the program or were required to be at their schools during the last session
***5 of the 22 teachers dropped from the program or were required to be at their schools during the last session
[1] Percents were computed by eliminating teachers who either dropped out of the program, started the program at Unit 2 or 3, or were required by their administrators to be at their schools rather than attend the last session

Almost all (98%) of the participants and 87% of the teachers attended at least two-thirds of the sessions in which questioning was addressed.. Of the 8 participants and 22 teachers who missed two days, 7 of those participants and 10 of the teachers missed because they were on an alternative scheduling track and their 3 month ‘summer vacation’ occurred during the third session. When they returned to school, their school-based coaches provided opportunities for them at the school site to learn about what they had missed in the 2-day large-group session. There was no documentation available to discern whether or not any of the participants took advantage of these opportunities or the quality of those opportunities.

Because nearly all participants attended 4, 5, or 6 days of the 3 sessions in which questioning was addressed, a correlation between attendance at the sessions and changes from pre- to post could not be determined. One conclusion that might be drawn from these data, however, is that participants in the study attended the majority of the large-group professional development sessions (nearly 98% of these participants attended at least 2/3 of the days) and that they significantly changed their abilities related to teacher questioning. In addition, since 88% of the teachers in the professional development program attended at least 2/3 of the large-group sessions, the participants seemed to be fairly representative of the teachers in the program although participants' rate was somewhat higher. Again, this could be due to the fact that they had to be in attendance on the last day of the program in order to take the post-test.

Although a direct association could not be established between attendance at the professional development sessions and increases in various abilities related to questioning, it appears that attending the sessions had some impact on the participants. However, since the abilities increased differentially, other factors may have accounted for some of the increases. Therefore, other available data sources were analyzed.

4.3.2. Other possible factors

To determine if factors other than attendance at the large-group sessions could be connected to the changes in questioning abilities, statistical tests were computed on appropriate available data.

Teaching experience One potential link to changes in teachers' questioning abilities for which data was available was the number of years of teaching experience. A Pearson correlation was computed to determine if there was a correlation between teaching experience (in years) and changes in questioning abilities. The correlation coefficient of .265 was not significant ($p = .07$). This indicated that there was not a significant association between years of teaching and changes in teachers' questioning abilities. Because the significance level was close to being significant at the .05 level, however, further analyses were conducted on the pre-test average and the post-test average. An interesting finding occurred when analyzing the correlations for these averages. The Pearson correlation coefficient of -.143 was determined for the correlation between years of teaching and the post-test average and was not significant ($p = .210$). However, a Pearson correlation coefficient of -.352 was significant ($p = .022$) for the correlation between years of teaching and the pre-test average. Because the coefficient was negative, this indicated that on the pre-test, teachers with less experience actually performed better than experienced teachers. Because there were not significant correlations between years of teaching and the post-test average or the difference between the pre- and post-test averages, it appears that experienced teachers narrowed the performance gap related to questioning between newer and experienced teachers. Therefore, it appears that experienced teachers may have benefited from the learning experiences provided in the large-group sessions and grew in their abilities related to questioning.

Connections to the school at which teachers taught Another potential link to changes in teachers' questioning abilities could have been other PD opportunities in which the teachers participated. Since data on other professional development opportunities individual participants might have experienced was not collected, an ANOVA comparing the pre- and post-test scores by school was conducted to determine if changes in teachers' questioning abilities differed according to the schools at which they taught. The results are shown in Table 4.14.

Table 4.14: Average pre-, post-test & difference between pre-post scores by school

School	**N**	**Pre-test avg. (4.28) p = .841**	**Post-test avg. (5.70) p = .07**	**Diff. avg. (1.45) p = .999**
1	4	2.34	4.34	1.83
2	6	4.41	5.86	1.46
3	1	4.82	6.45	1.62
4	6	4.67	5.88	1.21
5	3	3.49	5.61	2.12
6	4	5.25	6.61	1.35
7	3	4.38	6.10	1.72
8	2	3.65	4.85	1.21
9	1	4.45	5.87	1.42
10	2	4.30	5.85	1.54
11	1	5.35	5.89	.55
12	2	4.50	5.70	.70

None of these averages proved to be significantly different by school. Because 3 of the schools had only 1 participant, the ANOVA was recalculated without these 3 schools. Again, the averages were not significant by school. This indicates that there appeared to be no association between changes in teachers' questioning abilities and the high school at which the teacher taught. These results suggest that, while there may have been differences in coaching support provided between sessions, this support did not appear to be associated with changes in teachers' ability to identify and create and create questions that promote understanding.

An additional analysis was done to account for district level professional development opportunities in which teachers may have participated. The results of an

ANOVA calculated on the pre-test average, the post-test average, and the difference between the pre- and post-test averages are displayed in Table 4.15.

Table 4.15: Average pre-test, post-test and difference scores by local district

District	Pre-test avg. p = .712	Post-test avg. p = .222	Difference avg. p =. 796
2 (N=14)	4.41	5.93	1.52
3 (N=21)	4.20	5.54	1.38

There were no differences by district between any of the scores indicating that changes in teachers' questioning abilities were not related to the local district in which they taught.

<u>Bias due to one facilitator also being the investigator</u> Because a number of the participants experienced professional development with the investigator of this study, it was possible this influenced their decision to volunteer to be a participant in the study. To determine if the rate of participation of teachers in the investigator's room differed from the rate of participation of teachers in the other facilitators' rooms, 2-proportion z-tests were conducted between the investigator's rate of participation and each of the other facilitator's rates of participation. The rate was determined by calculating the percent of teachers who were in the room at the time the test was given who took the test. The results, shown in Table 4.16, indicate that the participation rates for teachers in the other rooms did not differ significantly from the participation rate in the investigator's room.

Table 4.16: Rate of participation of teachers by facilitator

Facilitator	Pre-test participation	Pre-test rate	Post-test participation	Post-test rate
1[1]	15 out of 20	. 75	17 out of 19	.89
2	6 out of 13	.46 (p=.10)	7 out of 9	.78 (p=.86)
3	14 out of 14	1.00 (p=.85)	15 out of 15	1.00 (p=.94)
4	15 out of 16	.94 (p=.83)	11 out of 14	.79 (p=.85)

[1]investigator of this study

Differences among the facilitators of the large-group sessions To determine if changes in teachers' questioning abilities differed according to the facilitator who conducted the large-group sessions, an ANOVA was conducted on pre-test, post-test, and difference between pre- and post-test total scores for the 4 distinct groups. Table 4.17 shows the results of the test.

Table 4.17: Average pre-, post-, and difference between pre- and post-test by facilitator

Facilitator	N Significance level →	Pre-test total p = .762	Post-test total p = .186	Difference between pre-post p = .843
1	11	4.46	6.13	1.66
2	3	4.21	5.20	.98
3	11	3.83	5.34	1.59
4	10	4.56	5.74	1.44

As can be observed in the table, there were no significant differences in participants' scores according to the facilitator who conducted their sessions. Therefore, it appears that the facilitators did not account for differences in changes in participants' abilities related to questioning.

In conclusion, changes in teachers' ability to identify and create questions that promote understanding of mathematics did not appear to be influenced by participants' level of teaching experience, the high school in which they taught or the local sub-district containing the high school. In addition, bias toward the investigator or differences among the facilitators who conducted the large-group sessions did not appear to impact the changes. Therefore, based on available data, the only factor that appeared to be common for all participants was their attendance at the large-group PD sessions.

4.3.3. Features of the professional development program

If attendance at the large-group professional development sessions might account for changes in teachers' abilities related to questioning, then the content of those sessions should be examined to determine potential opportunities teachers had to learn about and discuss questions of particular types. Table 4.18 summarizes these opportunities, noting in which session (out of 4) each opportunity occurred and the approximate amount of time spent in the session on the activity. (Note: Each session consisted of 2 day of professional development.)

As can be seen in Table 4.18, participants had numerous and varied opportunities to learn about and discuss particular questions types. These opportunities occurred primarily through the analysis of student work; the discussion of vignettes or scenarios of teaching, the analysis and planning of lessons, and reflecting on practice. During these activities, facilitators discussed the following with participants: 'assessing questions', 'advancing questions', 'probing questions', 'generating discussion questions', 'exploring mathematical relationships questions' and 'scaffolding' student learning through questioning.

Table 4.18: Opportunities to learn about and discuss particular question types

Session	Activity	Description of activity	Time spent
2 (days 1 and 2)	*Supporting Students' Learning through Questioning*	Given a set of student work from the "Making the Team" task: -practice writing questions that assess students' understanding and advance their learning toward the mathematical goals. -discuss characteristics of assessing questions and characteristics of advancing questions	2 hours
2 (days 1 and 2)	*Analyzing a Lesson* *Planning a Lesson*	Analyze the "Making the Team" lesson plan in terms of assessing and advancing questions. Given a task related to the "Making the Team" task: -plan a lesson with a focus on asking assessing and advancing questions. - implement the lesson, collect student work, and note the questions asked of students.*	3 hours
3 (days 3 and 4)	*Reflecting on Practice*	Reflect on implementation of the lesson in terms of assessing and advancing questions.	1.5 hours
3 (days 3 and 4)	*Supporting Students' Learning through Questioning*	Read and discuss a vignette of the "Shapes of Quadratics" task with a focus on types of questions asked. Analyze and discuss the Boaler and Brodie framework. Generate a set of probing and exploring questions given a set of student work from "Shapes of Quadratics"	3 hours
3 (days 3 and 4)	*Analyzing a Lesson* *Planning a lesson*	Analyze the "Shapes of Quadratrics" lesson plan in terms of probing and exploring questions. Given a task related to the "Shapes of Quadratics" task: -plan a lesson with a focus on asking probing, exploring mathematical relationships, and generating discussion questions to move students towards the mathematical goals. -implement the lesson, collect student work, and note the questions asked of students.*	2.5 Hours
4	*Reflecting on Practice*	Reflect on implementation of the lesson in terms of probing, exploring mathematical relationships, and generating discussion questions.	1.5 hours
4	*Scaffolding Student Learning through Questioning*	Compare and discuss the Boaler and Brodie questioning framework and the Assessing and Advancing Questions framework. Given two scenarios and student work from the "Multiplying Binomials" task, choose one of the frameworks and generate questions that would scaffold students' learning. Discuss the various questions generated and identify the purposes each question would serve in terms of scaffolding student learning	2 hours
4	*Analyzing a Lesson* *Planning a Lesson*	Analyze the "Multiplying Binomials" lesson plan in terms of appropriate questions. Given a task related to the "Multiplying Binomials: -plan, a lesson with a focus on asking appropriate questions to move students towards the mathematical goals. -implement the lesson, collect student work, and note the questions asked of students.*	2.5 hours

*Data was not collected to determine if this occurred.

In session 2, after having engaged in the Making the Team task, teachers were given three samples of student work from the task. They were asked to create ‘assessing’ and ‘advancing’ questions for the work after a brief discussion of the definitions of each. At the end of Session 2, teachers were asked to plan, in their school groups, a lesson related to the Making the Team lesson and to consider the ‘assessing’ and ‘advancing’ questions they would ask their students

In session 3, after having solved and discussed solution paths to the From Equations to Graphs task (Appendix C.1.2), teachers read a vignette of a teacher implementing the same lesson with his students (Appendix C.2). As they read the vignette, they identified ‘good’ questions and discussed characteristics of the questions. They then read the Boaler and Brodie article (Boaler & Brodie, 2004) concerning the questioning framework and compared the framework to the identified characteristics. The focus of the discussion was on ‘probing’, ‘exploring’ and ‘generating discussion’ questions and the purpose they served. Also during session 3, teachers analyzed part of a lesson plan for the Shapes of Quadratics lesson (Appendix C.5) in terms of the embedded questions, their relationship to the Boaler and Brodie question types 3, 4, and 5 (i.e. ‘exploring’, ‘probing’, and ‘generating discussion), and ways in which they might add value to students’ learning. Teachers then generated the three types of questions for the remainder of the lesson plan.

In session 4, in addition to solving and discussing the Multiplying Binomials task (Appendix C.3) and reading and discussing the case of a teacher implementing the task (Stein et al., 2000), teachers discussed the meaning of ‘scaffolding’ and compared the “Assessing & Advancing” and “Boaler & Brodie” frameworks for questioning. They were then given several scenarios in which students were in the process of solving the

Multiplying Binomials task (Appendix C.4) and asked to generate questions that would scaffold the students' learning. Later in the session, they were asked to develop a lesson related to the Multiplying Binomials lessons and to focus on the questions they would ask during the discussion of the lesson

Evidence of learning about various types of questions from these opportunities can be seen in responses to Parts 1 and 3 of the pre- and post-test in terms of language used by participants who took both the pre- and post-tests. In Part 1, where participants were asked to state the purposes of the questions they ask their students, only 13 of the 35 participants used any of the above terminology in their responses on the pre-test. Of those 13 participants, 11 used the word 'assess' or 'assessment'. Only 3 participants used any of the other terms – 1 used the word 'probe', 1 used 'explore' and 1 used 'scaffold'). In contrast, 25 of the 35 participants used one or more of the terms on part 1 of the post-test. A form of the word 'assess' (i.e. 'assessing', 'assessing questions', 'assessment') was again the most predominantly used word with 18 participants choosing this term. However, other terminology discussed during the large-group sessions was also used more extensively on the post-test, including: 12 participants using the word 'advance' or 'advancing'; 4 used a form of 'probing'; 1 used 'exploring' and 4 used the term 'scaffolding.' Similar results occurred for part 3 of the test with a form of the word 'assess' being the most predominantly used word on the pre-test while use of forms of the words 'advance', 'probe', 'explore' and 'scaffold' having increased frequencies on the post-test. A summary of the number of participants using forms of each word is provided in Table 4.19.

Table 4.19: No. of participants using the term in parts 1 and 3 of pre- and post-test

Term	**Pre-test**		**Post-test**	
	Part 1	**Part 3**	**Part 1**	**Part 3**
Assess, assessing, etc.	11	8	18	15
Advance, advancing	0	0	12	12
Probe, probing	1	3	2	6
Explore, exploring	1	0	2	5
Scaffold	1	0	4	3

One might argue that being able to use the language of these particular question types indicates only a surface level appropriation of the vocabulary that does not necessarily indicate understanding the meaning or purpose of these questions. However further analysis shows that on the post-test, participants not only used the language of these question types more frequently, but went on to further describe what they meant by the language. On the pre-test, a total of only 3 of the 13 participants who used one of the words related to questioning, expanded on their use of the word 'assess' on parts 1 and 3. No participants expanded on the meaning of any of the other words related to questioning. On the post-test, however, 16 of the 25 participants who used the language around questioning, went on to further explain what they mean by the particular word. For example, on part 1 of the post-test, one of participant 54's purposes for asking questions was listed as 'to <u>advance</u> them, to go to a deeper understanding of the concepts." Likewise, one of participant 5's responses was, 'I incorporate <u>advancing</u> questions to stimulate my students' thinking process, to make connections to the problem, and to go beyond the problem." Participant 7 wrote, "For <u>scaffolding</u>. Each new question is based on the conclusion made for the previous question in terms of the concept we are considering," and "Questions which allow students to further <u>explore</u> the concept and apply it." A final example on part 1 of the post-test comes from participant 43 who wrote, "To probe thought and advance student understanding. Allowing them an opportunity to voice their understanding and giving them a chance to voice out what is on their mind helps them retain the understanding." Similar responses occurred on part 3 of the post-test. Participant 14's explanation for an exploring question she selected pertaining to the point of intersection said, "advancing students' understanding about real life applications and linear systems." A response to another exploring question, selected by participant 52, included the explanation, "advancing question – points to the underlying mathematical relationship." And participant 9 wrote, "scaffolding question into the meaning of a system of equations," in response to an exploring question selected. For probing questions selected, participant 22 wrote, "probing to check for understanding to see if they realize this is the point of intersection" as an explanation while participant 11 responded, "assessing to see if the student knows the relationship between the intercept and the solution." These responses seem to suggest that participants not only appropriately used the language about questioning that was discussed in the large-group sessions on post-test, but also were able to explain the language within the context of teacher questioning.

4.3.4. Reflections from participants

The final data source analyzed to determine what features of the professional development program might account for changes in teachers' ability to identify and create questions that promote understanding of mathematics and explain why such questions promote understanding were reflections to two prompts from teachers at the conclusion of program. All responses were submitted anonymously so that teachers would be more likely to respond and to respond honestly. Therefore, responses could not be connected to particular participants. Of the 53 teachers who attended the second day of the final session, 42 completed the reflections. Since the 35 participants in the study had to have been present at the end of the session in order to take the post-test and 42 of 53 teachers completed reflections at the end of the 4th session, between 24 and 35 participants in the study had to have completed the reflection and the majority of the reflections would have had to come from the participants in the study.

The two prompts on the reflection sheet asked participants to talk about how their participation in the professional development project impacted their teaching and what they would continue to work on in their practice. Following are summaries of each prompt (note: Some participants listed multiple responses. All responses were tabulated):

In what ways has your participation in [the professional development project] impacted your teaching this year? Out of the 42 participants who responded to the prompt, the highest number of responses dealt with an aspect of teacher questioning. There were 18 references to questioning, 11 references to planning lessons, and 10 responses related in a general way to an aspect (other than questioning or planning lessons) of teaching mathematics. Some of the responses regarding questioning include:

- *My participation…made me aware of the questions I ask my students.*
- *Not giving the answers right away, using different types of questions. Different types of questions have different purposes.*
- *I have focused more on the types of questions I ask the students. I don't answer the questions right away, but instead ask them questions so they can discover the answer.*

- *I believe tasks that are more open-ended help to create the necessity to practice questioning techniques. "Drill and kill" tasks are less effective in helping me develop and strengthen my questioning skills.*

- *I have a stronger focus on questioning and more patience in the classroom for letting the learning take the time it needs.*

- *I think more about my questioning so that it allows students to arrive at the solution. I have seen what takes rigor out of a lesson.*

<u>*What are you going to continue to work on?*</u> Once again, a focus on questioning was the most prevalent response. Fifteen participants noted an aspect of questioning in their responses. This was followed by 12 references to planning/use of cognitively demanding tasks, and 9 responses dealing with the use of technology. Some of the responses that referred to questioning included:

- *Work on my own questioning and train my students to ask meaningful questions of each other.*

- *Apply techniques like scaffolding and appropriate questioning to help students obtain the goals of the lesson.*

- *I want to improve on asking questions that allow for deeper student understanding.*

- *I will be doing more tasks that require group work, continue asking probing questions during my teaching, and model how to approach and solve problems in different ways.*

- *I would work on my questions and let the students come up with their own conclusions.*

Although these results cannot be directly connected to the participants who took both the pre- and post test, they do suggest that the components of the professional development which focused on aspects of questioning were salient to participants and appeared to influence their thinking about questioning.

4.3.5. Summary of research question 5

The 35 participants in this study were high school mathematics teachers from 12 high schools in two local sub-districts of a large urban district. Their teaching experience ranged from 1 to 22 years and the majority of them were certified to teach mathematics.

In analyzing the changes in participants' ability to identify and create questions that promote understanding of mathematics and explain why such questions promote understanding, no correlation could be found between level of teaching experience, local district or high school at which teachers taught and the changes in these abilities. The only common factor among the 35 participants appeared to be that they attended the majority of large-group professional development sessions. Although a direct association could not be made between the features of the professional development program and the significant changes in teachers' ability to identify and create particular types of questions, the number of professional development sessions that were attended, the content of those sessions, and the final reflections at the end of the program appear to suggest that an explicit focus on teacher questioning during the professional development program may account for those changes. In addition, since participants used terminology on the post-test which was not used on the pre-test but that was introduced, discussed and revisited during the last 6 days of the large-group sessions, it is likely the professional development program was connected to the changes.

In the next chapter, the results will be discussed and implications from this study will be given.

5. CHAPTER FIVE: DISCUSSION OF RESULTS AND IMPLICATIONS

The purpose of this study was to examine changes in high school mathematics teachers' ability to identify and create questions that promote student understanding of mathematics and explain why such questions promote understanding as they participated in a practice-based professional development program. This chapter will discuss possible explanations for the results of the study, the significance of the findings, and possible limitations that should be considered. The chapter concludes with potential contributions of these results for mathematics educators and for further research in mathematics education.

5.1. DISCUSSION OF RESULTS

The 35 participants in this study were all high school mathematics teachers from a large urban district. They represented 12 of the 17 high schools from two local districts participating in a four-session professional development program that took place over a 7 month period. In addition, 33 of the 35 participants were certified to teach mathematics. The make-up of this group was quite diverse. Teaching experience ranged from first year teachers to veteran teachers who had taught mathematics for 22 years and there was a mixture of ethnic groups represented, including Caucasian, African American, Hispanic, and Asian teachers, as well as a balance of males and females. The participants appeared to be representative of the teachers in the professional development program and, based on available data, fairly representative of high school mathematics teachers in the urban district.

The 12 high schools included schools in more affluent areas of the city as well as schools in low-socio economic areas. Students in these schools were also quite diverse with some schools having high concentrations of Hispanic students, others having a majority of African American students, and a few having a majority of white students. In some schools, students were performing well on the state assessment test while others were under corrective action due to poor student performance. In some of the high schools, teachers received a great deal of support from their principal and school-based coach to implement what they learned in the professional development program. In other schools teachers received some support, while in some schools the teachers received almost no support for their participation in the program. Finally, soon after the professional development program began, the district took on an additional initiative meant to complement the professional development program, but which often competed for the limited time teachers had to meet with each other to discuss their practice. In fact, in most cases, administrative support was given to the other initiative at the expense of the time teachers were supposed to have been given for the professional development program. In spite of these conditions, the teachers grew in their abilities related to questions that promote understanding of mathematics and growth was not limited to teachers in particular schools or local districts.

In light of research concerning the instructional practices of high school teachers and their willingness to engage in learning new practices (Grossman & Stodlosky, 1995;

McLaughlin & Talbert, 2001; Whittington, 2002), the results from this study are somewhat surprising. In addition to learning about aspects of questioning that promote understanding of mathematics, the largest growth participants showed was in the area of questioning that research has shown is often lacking or absent in the classroom – questions that prompt students to explore mathematical ideas and connections. The significant growth participants showed in learning about questioning and possible explanations for the growth will be discussed in the following sections.

5.1.1. Results related to teacher questioning

The analysis of the data indicated that the high school mathematics teachers in this study grew in five areas related to questioning. In the sections that follow, explanations of the results, the significance of the results, and possible limitations of the results will be discussed.

5.1.1.1. Explanations of results related to questions. This section discusses explanations of the results for each of the areas.

1. Participants significantly increased their ability to identify questions that promote the understanding of mathematics, particularly questions that probe for understanding and prompt student to explore mathematical relationships and connections. Participants also increased their ability to identify generating discussion questions but the increase was not significant. This could have due to the fact that there were fewer of this question type that could have been selected (4 as opposed to 7 and 8 of the other two types) and 2 of the generating discussion questions could have also been coded as probing questions.

2. Participants significantly increased their ability to explain why particular question types promote the understanding of mathematics, particularly questions that prompt

students to explore mathematical relationships and connections and generating discussion questions. There was also an increase in participants explaining why probing questions promote understanding but not at a significant level. A possible reason for this result was that participants' explanations related to probing questions on the pre-test had higher scores than the exploring or generating discussion questions;

3. Participants significantly increased their ability to <u>create questions that promote understanding of mathematics</u>, particularly questions that prompt students to explore mathematical relationships and connections. Participants also increased their ability to create probing questions, though not significantly. This was probably due to the fact that participants created more probing questions on the pre-test than exploring questions. Participants did not create any generating discussion questions on either the pre- or post-test. This could have been because of the nature of the information around which questions were to be created. Brief scenarios and samples of student work were given for which participants were to create questions that promote understanding. Generating discussion questions may not have been particularly useful in promoting understanding in the context of the scenarios since they are most often used in whole-group discussions.

4. Participants significantly increased their ability to <u>focus on promoting understanding when stating the purposes of their questions</u>; and

5. Participants significantly <u>decreased their use of procedural questions</u> when asked to create questions that promotes the understanding of mathematics. Although procedural questions are a necessary aspect of mathematics instruction, asking only procedural questions results in students having little, if any, opportunity to explore, explain and discuss mathematics.

5.1.1.2. Significance of results related to questioning. Why might the above findings be important or relevant? Research has shown that teachers typically do not ask the types of questions that appeared to be learned by the participants in the study. Teachers tend to ask students questions requiring one word answers to which individual students respond. The teacher then evaluates the response as being correct or incorrect (Stodlosky, Ferguson & Wimpelberg, 1981; Lemke, 1990; Cazden, 2001). Observations of over 300 mathematics and science lessons showed that only 16% of the lessons consisted of high level questioning - the types of questions that "encourage students to think more deeply" (Weiss et al., 2003, p. 7). The predominant types of questions asked by teachers included those that focused on a correct answer in which one student responded or questions which the teacher answered herself. Teacher questioning was determined to be one of the weakest elements of instruction in the observed lessons.

Research has also identified the types of questions that have been linked to increased student achievement in mathematics (Hiebert & Wearne, 1993; Martino & Maher, 1999; Boaler & Staples, 2005). These question types - that probe for understanding, that promote discussion of mathematics and that prompt students to explore mathematical concepts – are seldom if ever asked in most mathematics classrooms. Yet, when teachers did ask these types of questions, along with procedural and other types of questions, students at the high school level were shown to significantly increase their achievement in mathematics. Not only did participants in this study learn about these types of questions, they had the largest gains in identifying and creating exploring mathematics questions. Exploring questions were the question type least observed in the study of teaching and learning high school mathematics and yet were considered key to increases in student achievement (Boaler & Brodie, 2004).

5.1.1.3. Possible limitations of the results related to questioning. When considering the results of this study in light of existing research related to teacher questioning, several questions emerge that point to possible limitations of the study:

1. Although participants increased their ability to identify and create questions that promote understanding of mathematics, to explain the purposes of various questions, and to explain why particular questions promote understanding, how do we know that this knowledge was not just a surface level appropriation of the language used in the professional development experiences? Analysis of the language used by participants on the pre- and post-tests indicated terms related to teacher questioning that were discussed in the professional development sessions were used by only a few participants on the pre-test. Most of these instances consisted of using a form of the word ‘assess’ to describe their purposes for asking questions or to explain why questions they selected promote understanding of mathematics. On the post-test, however, the majority of participants also appropriately used some form of the other terms (i.e. probe, explore, scaffold, advance) and used them with more frequency. In addition to using the terms, on the post-test participants also tended to expand on the term in the context for which it was being used. For example, in explaining their purposes for asking questions of their students, many participants used a form of the word ‘advance’ in a phrase that indicated deepening students’ understanding or pushing students to go further in their thinking. Similar occurrences were noted for participants’ explanations of why questions they selected

promote mathematical understanding. Thus it appears that participants gained more than a surface level understanding of questions that promote understanding.

2. Another question emerging from the results of the study concerns the role of the mathematics task relative to teachers' questioning abilities. Research has shown that selecting high level tasks for use in the classroom is a critical first step in promoting mathematical understanding (Stein & Lane, 1996; NCES, 2003; Boaler & Staples, 2005). As a result, the professional development program in which teachers participated focused specifically on selecting and implementing challenging mathematics tasks. In addition, the pre- and post-test instrument used to measure the changes in teachers' questioning abilities situated their ability to identify and create questions in the context of students solving and discussing a high level algebra task. However, what teachers in this study learned in terms of the relationship between selecting a challenging task and asking questions that promote mathematical understanding cannot be generalized to classroom practice. For example, Silver & Smith (1996, p. 25) noted that teachers who participated in the QUASAR project experienced challenges in centering discourse on appropriate tasks. The researchers provided an example of a teacher who understood the importance of asking students to explain and justify their thinking and proceeded to do so with his seventh grade students. However, the task around which the teacher asked the questions was a low level mathematics task which resulted in students explaining how to apply a particular procedure. The authors note that, while teachers may be aware of the importance of selecting worthwhile mathematics tasks as well as the importance of asking students to explain and justify their thinking, "teachers face additional challenges

as they seek to center … discourse on worthwhile tasks that engage students in thinking and reasoning about important mathematical ideas" (Silver & Smith, 1996, p. 24).

In terms of the current study, the extent to which teachers made a connection between asking questions that promote mathematical understanding and the fact that such questions are most effective when asked around challenging mathematics tasks is unclear and deserves further exploration.

3. A final question related to teachers' growth in questioning abilities concerns their ability to translate these abilities into their instructional practice. Even though participants increased their abilities related to identifying and creating questions that promote understanding of mathematics, we are left wondering what impact this learning had on teachers' instructional practice. Since teachers were not observed in their classrooms prior to and after their participation in the professional development program, there is no direct evidence as to whether their learning about questions that promote understanding of mathematics transferred to the classroom. However, recent research has shown that focusing on a particular aspect of instructional practice in professional development can lead to changes in teachers' knowledge, as well as impact their classroom practice (Boston, 2006). For example, a study was conducted with 18 mathematics teachers who participated in 6 professional development sessions over a 9 month period. These sessions focused on selecting and implementing high level mathematics tasks. Results showed that teachers significantly improved their knowledge in terms of identifying and describing the characteristics of tasks that provide students with opportunities for learning mathematics (i.e. high level tasks). In addition, the teachers in the study selected high level tasks for use in their classrooms more frequently

and were able to maintain the high level cognitive demands during the implementation of the tasks at a higher rate than a contrast group of teachers who did not participate in the professional development program (Boston, 2006). Thus participation in a professional development program that targeted a particular aspect of instructional practice in mathematics not only resulted in greater knowledge of the practice, but also impacted teachers' transfer of this practice to their own teaching.

There is no guarantee, however, that what teachers in this study learned about questioning in the professional development program was appropriated into teachers' instructional practice. While reflections at the conclusion of the professional development program indicate that participants were, at a minimum, making connections between the PD experiences in which they learned about questioning and their instructional practice, the ability to generalize these results to teachers' classroom practice is a major limitation of the study.

In spite of the above limitations, however, the fact that teachers did learn about a key instructional practice that promotes understanding of mathematics is an important finding. Perhaps even more important is that fact that the participants were high school mathematics teachers Potential reasons that account for this growth in knowledge will be explored in the next section.

5.1.2. Accounting for changes in teachers' questioning abilities

To determine what might have accounted for changes in teachers' abilities related to questioning, statistical analyses were done to determine if the professional development program, as well as additional factors, may have contributed to the changes. The following sections provide explanations for the results and discuss the significance as well as limitations pertaining to the results.

5.1.2.1. Explanation of results related to the professional development and other factors. Analyses to associate the additional factors – teaching experience of participants, high school at which participants taught, and local districts containing the high schools – with the changes in teachers' abilities related to questioning were conducted. None of these associations proved to be significant in terms of identifying which teachers had the greatest or least changes related to questioning. Teachers at all levels of experience had significant changes from pre- to post- in their questioning abilities. Also, it was initially hypothesized that school-based study group sessions might account for potential changes in teachers' questioning abilities since the sessions were intended to complement the large-group PD sessions. Since data collected from the schools proved to be unreliable, changes in teachers' questioning abilities were compared according to the high school at which teachers taught in an attempt to capture the effect of any potential learning that may have occurred at particular schools. This comparison also showed no significant association. To account for potential PD opportunities provided by the local district, the changes in teachers' questioning abilities were also compared by local district. Again, no significant association was found. Finally, an analysis of changes in teachers' questioning abilities was disaggregated according to each the four facilitators. There were no differences found among the four indicating that the changes could not be associated with the facilitator conducting the professional development sessions. Therefore, it seems reasonable that the professional development program contributed to the changes in teachers' questioning abilities.

The 35 participants attended the majority of the sessions in which an aspect of teacher questioning was discussed and showed evidence on the post-test and the final reflections that they had learned about teacher questioning. They were also using language consistent with what was discussed at the professional development sessions at a much higher rate on the post-test. In addition, the language used went beyond a surface level use of the words since participants often explained what they meant by the word in the context in which it was used.

In terms of making a connection between changes in teachers' questioning abilities and the professional development program, one should note that the 4 large-group sessions were designed to incorporate features of a practice-based professional development program (Smith, 2001; Ball & Cohen, 1999). And such programs have been linked to changes in teachers' instructional practice (Smith & Brown, 1994; Carpenter, et al, 2000; Killion, 2002.) Specific features of the professional development program discussed in this study include:

1. The content of the sessions consisted of the same content teachers were teaching in their classrooms and was designed to align with the curriculum they were using. For example, every unit of the professional development program consisted of engaging in a mathematics task that was aligned with the upcoming unit in the district's mathematics instructional guide. Teachers then analyzed and discussed a lesson plan they would be teaching to their students that included a version of the same task.

2. The activities in every session were situated within the day-to-day teaching mathematics. For example, teachers planned mathematics lessons they would be teaching, reflected on lessons they had already taught, and analyzed and discussed

episodes of other teachers implementing mathematics lessons. They also examined student work and engaged in generating a variety of questions they would ask students.

3. Participants were part of collaborative school teams that worked together during the sessions and were intended to work together at the school site. In all 8 sessions, teachers worked the majority of the time with their school team. All teachers on a school team were teaching the same content, in this case, Algebra 1. This allowed them to have focused discussions relevant to their particular situations.

5.1.2.2. Significance of the results. Although a direct correlation could not be made between the significant changes in teachers' questioning abilities and the professional development program, the results appear to support the notion that the practice-based professional development experiences can contribute to teachers' knowledge growth and abilities. In addition, since all participants in the study were high school mathematics teachers, this study contributes to the small, but growing, research base around the impact of professional development on the knowledge of high school mathematics teachers. In particular, the study suggests that high school mathematics teachers can learn new practices related to teaching mathematics if the professional development experiences in which they participate provides them with opportunities that are: grounded in their content area; conducted with colleagues from their own schools with whom they can collaborate; and situated within their day-to-day practice of teaching mathematics.

5.1.2.3. Possible limitations of the results related to the professional development program and other factors. Several limitations of this study related to the professional development program and other factors. One potential limitation was the fact that the investigator of this study was also one of the four facilitators of the professional development program. Therefore, it was possible that teachers in the investigator's sessions felt an obligation to volunteer for the study. However, statistical tests showed that the participation rate for teachers in the investigator's room was not different from the participation rate of the other facilitators. In fact, the investigator's rate of participation was neither the highest nor the lowest for all facilitators.

A second, perhaps more important, limitation to consider concerns the sample for this study. How representative were the participants in this study of the group of teachers who participated in the professional development program, of the high school mathematics teachers in the district, and of high school mathematics teachers in general? Although teachers were supposed to have volunteered to participate in the professional development program, survey data from an independent evaluator showed that over half of the teachers did not do so. In spite of this, however, over ¾ of the teachers would have participated in the study since 79% of the 63 teachers present when the pre-test was given took the pre-test and nearly 90% of the 57 teachers present when the post-test was given took the post-test. Therefore, the participants volunteering for the study did not do so at a much higher rate. In addition, the demographic information showed that the participants were quite similar to the teachers who took part in the program. Therefore it appears that the participants in this study were representative of the teachers who took part in the professional development program. Though both the participants and teachers were different along many dimensions when compared to a national sample of high school mathematics teachers, they were similar to teachers in their own district. Since available data did not allow for further comparison, the ability to generalize these results to high school mathematics at large is another limitation of the study.

Despite the acknowledged limitations associated with interpreting the results of this study, the findings also have the potential to contribute to the research base related to the teaching and learning of mathematics.

5.2. CONTRIBUTIONS OF THIS STUDY

The results reported from this study have several implications for the mathematics education community. These include possible contributions concerning the professional development of mathematics teachers and student achievement in mathematics, particularly at the high school level, and implications for measuring knowledge of instructional practice. Each of these will be described in the following sections.

5.2.1. Contributions related to professional development

This study suggests that teachers grew in their knowledge of questioning as they participated in a professional development program in which they had multiple opportunities to learn about teacher questioning. Even though teachers were not followed into the classroom to see if they had appropriated the new learning into their own practice, being able to identify and create questions that promote understanding of mathematics should be seen as a first step in changing teachers' instructional practice. The fact that the participants in the study were high school mathematics teachers also suggests that the experiences in the professional development sessions contributed to high school teachers learning a new instructional practice. Therefore, this study could add to the limited research base concerning changing instructional practice at the high school level.

Important follow-up research could also build on this study and follow teachers into their classrooms to observe the types and purposes of questions asked during instruction and to determine if this changes over time as teachers participate in professional development focused on this aspect of instruction. The Boaler and Brodie (2004) questioning framework could serve as a tool for categorizing the questions asked by teachers. In addition, the methodology used in their study could be used, or adapted, to collect and code data from classroom observations. This would allow for a comparison between their sample of teachers and a comparison sample of teachers prior to and after their participation in professional development focused on questioning.

Additional components of future research studies could examine the opportunities teachers have to learn about aspects of teacher questioning during professional development and their level of participation in the professional development sessions. This would allow direct connections to be made between changes in teachers' knowledge and abilities related to questioning and the professional development in which they engaged. The current study did not document the ways in which teachers participated in the professional development sessions or the ways in which the sessions were facilitated. Although attendance data was gathered, attendance does not necessarily imply active participation. And even though teachers were asked to bring artifacts of practice, such as student work, written reflections, and lists of questions they asked, to each 2 day session and the artifacts were reflected upon and discussed at each session, the artifacts were not collected or documented. The fact that a participant did or did not share an artifact and the quality of the artifact shared could have implications related to changes in teachers' knowledge and could possibly be used to predict whether or not instructional practice will change.

In addition, it could be beneficial to analyze video of professional development sessions, particularly the components that explicitly focus on questions that promote mathematical understanding to determine if changes in knowledge and abilities related to questioning could be connected to the level at which the participants engaged in the professional development or to particular ways in which the sessions were facilitated. For example, in the study conducted by Boston (2006), the professional development sessions were videotaped and artifacts (e.g. task solutions, charts generated during discussions, reflections) created during the sessions and during classroom implementation were collected. These two data sources provided a record of the potential opportunities teachers had to learn during the PD sessions and the ways and level at which they participated in the sessions or incorporated new learning into their practice. This allowed for an analysis of the impact of the professional development on changes in teachers' knowledge and practice to be made.

5.2.2. Contributions related to student achievement in mathematics

Although this study focused on teacher questioning, the premise is that improving teachers' ability to ask questions that promote mathematical understanding will result in better learning opportunities for students and, ultimately, increased student achievement in mathematics. This study showed that teachers can learn to identify and create the types of questions that are associated with increased student achievement in mathematics which is key to promoting mathematical understanding. Additional research studies that attempt to link professional development experiences to changes in teachers' knowledge and practice, as described in the previous section, could also provide for a measure of gains in student achievement. In particular, a study linking professional development, changes in teacher knowledge and practice of questioning, and student achievement at the high school level could be invaluable in expanding a very limited research base, most of which has been conducted at the elementary level.

5.2.3. Contributions to the measurement of instructional practice

A final implication from this study concerns the measurement of the instructional practices of mathematics teachers. A pre- and post-test instrument was used for this study and appears to have been successful in measuring changes in teachers' ability to identify and create questions that promote understanding of mathematics and explain why such questions promote understanding. Thus, the instrument could contribute to a small, but growing effort to measure teachers' knowledge related to teaching mathematics and to eventually use these measures to predict a teacher's instructional practices. For example, researchers at the University of Michigan (Ball, Hill & Bass, 2005) have designed paper and pencil measures to determine elementary teachers' ability to assess student work, represent mathematical ideas and explain the meaning of rules or procedures for particular areas of mathematics such as number and operations. They are currently focused on measuring how this knowledge is used in actual classrooms and on correlating the results with their paper and pencil measures of teachers' knowledge. The limited results that are currently available show evidence that such a correlation may exist. If such measures of teachers' knowledge for teaching can eventually be used to approximate the instructional practices of teachers, the instrument used in this current study has the potential to provide invaluable information to teachers, mathematics education researchers and professional developers relative to teacher questioning – a key practice for teaching mathematics that has been shown to impact student achievement.

5.3. CONCLUSION

This study examined changes in high school teachers' ability to identify and create questions that promote understanding of mathematics and explain why such questions promote understanding as they participated in a professional development program. Results suggest that teachers significantly increased their ability to identify and create questions that promote understanding of mathematics, particular exploring questions, which have been associated with increases in student achievement. Although a correlation between attendance and participation in the professional development program could not be established, it appears that the activities in the program, which provided ongoing opportunities to learn about teacher questioning in the context of teaching mathematics and in collaboration with colleagues, may account for these significant changes. In addition, since the participants in the study were high school mathematics teachers, the results are promising in terms of assisting teachers at this level to learn new instructional practices through professional development experiences.

This study can contribute to the limited, but ongoing research base being developed around professional development for high school mathematics teachers, mathematics teachers' instructional practice at the high school level and the measurement of teacher knowledge and instruction. This study also has the potential to further inform the mathematics education field as to the type of professional development that can impact teachers' knowledge for teaching mathematics in an effort to provide all students, especially high school students, with opportunities to learn challenging mathematics.

APPENDIX A
Pre- and Post-Test

IDENTIFYING AND CREATING QUESTIONS THAT PROMOTE STUDENT UNDERSTANDING OF MATHEMATICS

NOTE: Names will not be used to report results of this test. Names will only be used by me to match the pre- and post-tests for individuals. Each person's name will be assigned a number and then the cover page will be removed. The names and associated numbers will not be shared with any other person.

Name_______________________ School_____________________ Local District___

Number of years teaching mathematics____

Do you hold a credential to teach secondary mathematics (9-12) in the state of California? ___Y ___N

Part 1

Teachers ask hundreds of questions of their students every day. What are the purposes of the questions you ask your students?

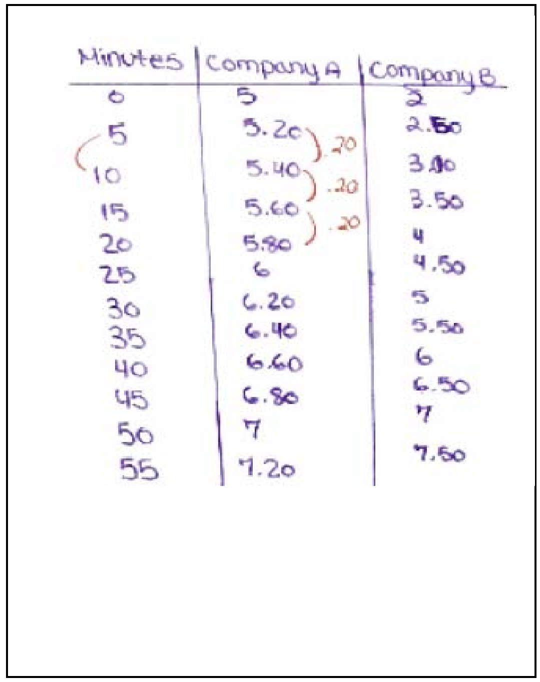
Parts 2-3

The following task was used by a teacher in an actual Algebra 1 classroom. Please solve the task and then complete the remaining parts of the test:

Calling Plans
Long-distance Company A charges a base rate of \$5 per month, plus 4 cents per minute that you are on the phone. Long-distance Company B charges a base rate of only \$2 per month, but they charge you 10 cents per minute used. How much time per month would you have to talk on the phone before subscribing to Company A would save you money?

Part 2 Students have worked in groups to solve the problem and have created posters that have been posted in the front of the room. Suppose you are conducting a whole group discussion of the solution to the task.

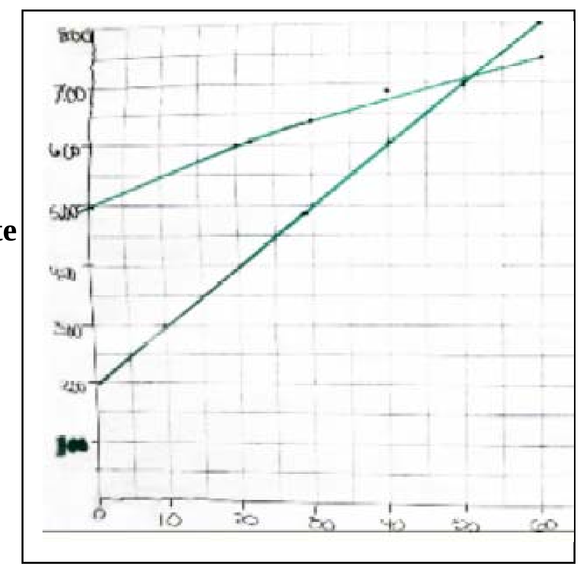
A One of your mathematical goals is for students to understand the slope and y-intercept in terms of the different plans. Group A has posted the table to the right in front of the class.



Minutes	Company A		Company B
0	5		2
5	5.20		2.50
10	5.40	.20	3.00
15	5.60	.20	3.50
20	5.80	.20	4
25	6		4.50
30	6.20		5
35	6.40		5.50
40	6.60		6
45	6.80		6.50
50	7		7
55	7.20		7.50

List 5 questions would you ask to promote students' understanding of mathematics:

B One of your mathematical goals is for students to understand the slope and y-intercept in terms of the different plans. Group B has posted the graph to the right in front of the class.



List 5 questions would you ask to promote students' understanding of mathematics:

C One of your goals is for students to understand the meaning of slope and y-intercept in different representations. The following solutions have been posted on the board.

equation

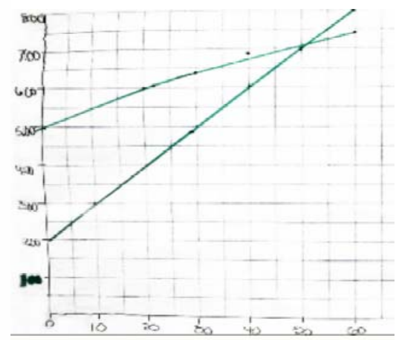
CA

$.04m+5=C$

CB

$.10m+2=C$

Minutes	Company A	Company B
0	5	2
5	5.20	2.50
10	5.40	3.00
15	5.60	3.50
20	5.80	4
25	6	4.50
30	6.20	5
35	6.40	5.50
40	6.60	6
45	6.80	6.50
50	7	7
55	7.20	7.50



List 5 questions you would ask to promote students' understanding of mathematics:

Part 3

Read the following transcript of a teacher discussing the Calling Plans task with her students. Select (underline) the questions that you think promote students' understanding of mathematics. Then explain why you think the question promotes understanding. The teacher's mathematical goals were to develop and strengthen students' understanding of the meaning of slope and y-intercept.

(T: teacher)

<table>
<tr><th colspan="2">Beginning of the discussion</th><th>Why does this question promote understanding of mathematics?</th></tr>
<tr><td>1 T:</td><td>Who thinks they got the correct table?
(Group 1 raises their hands.)</td><td></td></tr>
<tr><td>2 T:</td><td>Could you please post your table in front of the room?
(Group 1 posts the following table☺</td><td></td></tr>
<tr><td></td><td>
<table>
<tr><th>M</th><th>A</th><th>B</th></tr>
<tr><td>0</td><td>5.00</td><td>2.00</td></tr>
<tr><td>10</td><td>5.40</td><td>3.00</td></tr>
<tr><td>20</td><td>5.80</td><td>4.00</td></tr>
<tr><td>30</td><td>6.20</td><td>5.00</td></tr>
<tr><td>40</td><td>6.60</td><td>6.00</td></tr>
<tr><td>50</td><td>7.00</td><td>7.00</td></tr>
<tr><td>51</td><td>7.04</td><td>7.10</td></tr>
</table>
</td><td></td></tr>
<tr><td>3 T:</td><td>What did you get for the answer?
(A member of group 1 points to the row showing "51 7.04 7.10")</td><td></td></tr>
<tr><td>4 T:</td><td>So someone who wasn't in group 1, how do you think they used the table to find the solution? Come up and show us.
(Joe goes to the front of the room and points to chart.)</td><td></td></tr>
<tr><td>5 Joe:</td><td>Well, they went by 10's to find their answer. And they kept going until they got both numbers to be the same and then went 1 more. And that's when A got cheaper.</td><td></td></tr>
</table>

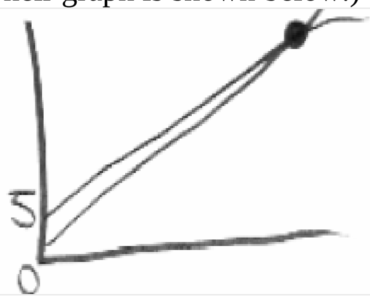
6 T:	What do you mean, “A got cheaper”?	
7 Joe:	Company A and B cost the same at 50 minutes and Company A got cheaper at 51 minutes.	
8 T:	OK. So I’m wondering. The problem said Company A charged 4 cents per minute and Company B charge 10 cents per minute. How can we see the cost per minute in the table?	

Later in the discussion

Why does this question promote understanding of mathematics?

11 T:	So, what did you get for the equations for Company A and Company B?	
12 Maria	C = .04 m + 5 and C = .10 m + 2.	
13 T:	Who else got the same equations? (Many students raise their hands.)	
14 T:	OK. Good. Now, we talked about the cost per minute in the table. How do we see that cost per minute in the equations? (Several students respond at once.)	
15 Jose:	Like, it’s the point “O” 4 and the point “10”.	
16 Anita:	It’s the coefficient.	
17 Joe:	Yeah, but you have to add on the monthly fee.	
18 T:	OK, wait a minute. Jose, what do you mean, “it’s the point “O” 4 and the point “10”?	
19 Jose:	The cost per minute… like for Company A it’s 4 cents per minute and there’s a .04 in front of m.	

20 T:	OK, and Anita, you said 'the coefficient.' What's a coefficient?	
21 Anita:	It's the number in front of x.	
22 T:	Can someone else add on to what Anita said? (No one responds.)	
23 T:	What does the coefficient mean for this problem? Why is it important?	

Occurs after the above discussion		Why does this question promote understanding of mathematics?
41 T:	Group 2, where is your graph? (Group 2 responds that their graph is under the table they constructed. Their graph is shown below.)	
		
42 T:	Oh, OK…… So I'd like someone who wasn't in group 2 to explain, how do you think group 2 made their graph?	
43 Jon:	I think they knew where each line started and ended.	
44 T:	What do you mean, "where each line started and ended?"	
45 Jon:	Like, Company A started at 2 and Company B started at 5.	
46 Marci:	And they both ended at 50…Well, they didn't really end at 50. That's where they both cost the same.	
47 T:	So let's start with what Jon said, what do you mean "they started at…"?	

48 Jon:	Well, it’s the y-intercept..	
49 Tony:	It’s where they cross the y-axis.	
50 T:	So, in terms of this problem, what does the y-intercept mean? What does it mean “they cross they-axis”?	
51 Deshay	It’s how much you have to pay just to have that plan.	
52 T:	Someone else, want to add on?	
53 Joe:	It’s what each plan costs at 0 minutes.	
54 Marci:	The y-intercept means when x is 0, that’s how much y is. So for this problem when we have 0 minutes, Company A costs $5 and B costs $2.	
55 T:	OK. We also looked at a table and an equation before. How could I tell what the y-intercept is in a table?	
	LATER IN THE DISCUSSION	
60 T:	I want to come back to something Marci said about where the two plans cost the same. Marci, can you tell us what you were thinking about that? Come up and show us.	
61 Marci:	(Marci points to the point of intersection) Well, I said the two plans cost the same at 50 minutes and you can see that right here.	
62 Tony:	It’s where they intersect.	
63 T:	What do you mean by “intersect”?	
64 Tony:	It’s where the two lines cross each other, they both go through the same point.	

65 T:	So what does that mean when two lines go through the same point? What does that tell me?	
66: Joe	It means both lines have a point, an x and a y, that's the same for both of them.	
67 T:	What is that point for this problem? Tami, what is the intersection point for this problem.	
68 Tami:	It's 50, 7.	
69 T:	So what does that tell me in terms of this problem? What does "50, 7" mean?	

APPENDIX B

Questions that Promote Student Understanding of Mathematics

Question Type	Purpose	Examples
Probing	• Require students to explain or clarify their thinking • Require students to justify their solutions to problems	• *"How did you get that answer?"* • *"Why did you use that scale for your graph?"* • *"Why did you use that formula to solve the problem?"* • *"Explain to me how you got that expression."*
Exploring mathematical relationships and connections	• Require students to identify mathematical relationships • Require students to link mathematical representations • Require students to link mathematical ideas	• *"What does 'n' represent in terms of the diagram?"* • *"How does the 'x' in your table related to the 'x' in your graph?"* • *"Will your expression work for any "function? Why?"* • *"What is staying the same in your equation? Why is it staying the same?"*
Generating discussion	• Ask students to explain the thinking and reasoning of others or to restate in their own words • Ask students to contribute additional information to a discussion • Ask students to agree or disagree and justify why	• *"Explain to me what John was saying."* • *"What else did you notice about the graph of the parabola?"* • *"Who agrees with what Sue said? Why do you agree?"*

Other common types of questions

Procedural	• Require yes or no answers • Require the recall of facts or memorized procedures	• *"What is the square root of 4?"* • *"What is the distance between the two points?"*
Other mathematical	• Relate to teaching or learning mathematics but are not procedural, probing, exploring, or generating discussion	• *"What is a coefficient?"* • *"How could you use this in the real world?"*
Non-mathematical	• Does not relate to teaching or learning mathematics	• *"Why didn't you use graph paper?"* • *"Who did their homework?"*

APPENDIX C.1.1

INVESTIGATING THE SHAPE OF QUADRATIC FUNCTIONS FROM EQUATIONS TO GRAPHS

INVESTIGATION 1: FROM EQUATIONS TO GRAPHS I

For graphs of each of the functions below, you will explore the appearance of the graph and its location on the coordinate plane, and how they are related to the graph of the basic function $y = x^2$.

- $y = ax^2$
- $y = x^2 + c$
- $y = ax^2 + c$

For ***each*** of the three equations:

Use your graphing calculator to graph different functions of each of the three forms. Substitute different values for a and/or c. Use a variety of values including ones that are greater than 1, between 0 and 1, positive and negative. Record your observations on the Investigation 1 Recording Sheet.

Divide the workload among members of your group. Graph a set of equations on the same screen by entering them in the calculator's "Y=" list. Keep $y = x^2$ as the first equation in the list. Use the standard viewing window.

As a group, use your graphs to answer these questions:

a) How are the graphs similar to, and different from, the graph of the basic function $y = x^2$?

b) How are these graphs similar to, and different from, each other?

c) How do changes in the values of a and c affect:

- the appearance of the graph and
- the location of the vertex of the graph in the coordinate plane?

Check your conclusions. Create another function of the same form and predict what the graph will look like before graphing it on your calculator.

Be prepared to discuss your results with the class.

APPENDIX C.1.2

From Equations to Graphs: Quadratics

Part 1: Using a graphing calculator, explore graphs of equations of the form $y = x^2 + bx$. How does "b" affect the location of the graphs on the coordinate plane?

Identify as many relationships as you can. Be prepared to discuss your findings and provide support for your claims.

Part 2: Using a graphing calculator, explore graphs of equations of the form $y = x^2 + bx + c$. What effect do "b" and "c" together have on the location of the graph?

Be prepared to discuss your findings and provide support for your claims.

APPENDIX C.2
"Shapes of Quadratics" Vignette

The Case of Mark Veracruz

The Context

Mark Veracruz, a 19 year teaching veteran, started teaching mathematics at Dunbar High School 8 years ago when Dunbar adopted a reform mathematics curriculum. Although Mark felt he had strong mathematical content knowledge, he initially experienced some anxiety because the new curriculum required him to use more challenging task and to develop not only students' procedural fluency but their conceptual understanding. Since the curriculum adoption, Mark and his colleagues have met regularly to discuss implementation issues such as how to support students as they struggle with more open ended tasks without telling them what to do, what approaches students might use in solving particular problems and what errors might come up, and how to help students communicate their thinking and reasoning.

Mark's ninth and tenth grade algebra students have recently completed a unit in which they explored how changes in the parameters (i.e., m and b in $y = mx + b$) effected the graph of a linear function. using the TI-83 graphing calculator. Yesterday, Mark started a unit on quadratic functions by having students graph and informally discussed properties of $y = x^2$ without using their calculators. Today Mark planned to have his students use their calculators to explore the effects that coefficients and constants have on a quadratic function of the form $y = ax^2 + c$. He wanted students to begin to understand not only how, but why "a" causes the graph to become either wider or more narrow, why a negative value for 'a' causes the graph to be reflected, and why "c" causes the graph to move upward or downward. Mark also wanted his students to better articulate their mathematical thinking and reasoning so he planned to ask them questions that would help them explain what they were doing and why it made sense. .

The Lesson Set-Up

When students entered the classroom, they proceeded to their pre-assigned groups where copies of the task and a graphing calculator had been placed at each desk. Mark began the lesson by having students discuss what they had learned about the graph of $y = x^2$ from their previous exploration. They talked about the fact that the graph was a parabola and was symmetric to the y-axis, that it opened up, and that it had a vertex at the origin. He then asked the students to enter "$y=x^2$" into their calculators and explained that they would be exploring and comparing different forms of this equation. They read the task and briefly discussed what they would be doing in Part 1 of the first Investigation.

The Exploration

Mark explained to the class that they would have 10 minutes to work individually on the task after which time they could discuss their initial results with their group and continue to work on the task.. As students worked on the task, Mark walked around the classroom first making sure that everyone was getting started without any difficulty and then monitoring the progress of each group.

Part 1. From Equations to Graphs: Investigate the shape of quadratic functions.
Investigation 1: For each of the forms of quadratic functions below, you will explore the shape of the graph and its location on the coordinate plane, and how they are related to the graph of the basic function $y = x^2$.

1) $y = ax^2$ 2) $y = x^2 + c$ 3) $y = ax^2 + c$

For ***each*** of the three forms:
Use your graphing calculator to graph different functions of the form. Substitute different values for a and/or c. Use a variety of values including ones that are greater than 1, between 0 and 1, positive and negative.

Divide the workload among members of your group. Graph a set of equations on the same screen by entering them in the calculator's "Y=" list. Keep $y = x^2$ as the first equation in the list. Use the standard viewing window.

As a group, use your graphs to answer these questions:

1) How are the graphs similar to, and different from, the graph of the basic function $y = x^2$?
2) How are these graphs, similar to, and different from, each other?
3) How do changes in the values of a and c affect the shape of the graph and its location in the coordinate plane?

Check your conclusions. Create another function of the same form and predict what the graph will look like before graphing it on your calculator. Be prepared to discuss your results with the class.

Group 1: Mark stopped at Group 1 as students were discussing their results for $y = ax^2$. Charles was commenting that the graphs of the parabolas kept getting skinnier. Mark asked, "What do you mean by 'skinnier'?" Charles explained that when he made 'a' larger, the parabola became more narrow. Mark asked others in the group, "What do you think? Did you get the same results as Charles?" Audrey replied, "Yeah, and I also noticed that the parabola always opens up." Shala added, "Yep, that's what I found, too. Let's go on to the next one." "Whoa, wait a minute," Mark commented as he was looking at the group's recording sheet. "I have a couple of questions. Let's look at the values you used for 'a'."

Form	Value of a	Value of c	Equation graphed
1) $y = ax^2$		0	
	2	0	$y = 2x^2$
	5	0	$y = 5x^2$
	3	0	$y = 3x^2$
	10	0	$y = 10x^2$

Mark continued, "Hmm, you all agreed that the parabola became more narrow as 'a' became larger. So I'm wondering, what's the smallest 'a' value you could use?" Charles commented that he started with 2 since $y = x^2$ has an 'a' value of 1. Mark then decided to ask Roman, a student in the group who had been quiet, what he thought. Roman responded, "Well, I guess 'a' could be a negative number." Shala stated that they should start with -2 and see what happened." Mark replied, "OK. What do you think will happen to the graph when you use negative values for 'a'?" Several of the students said the graph would get wider while one thought the graph would move down. Mark told them to test their ideas and said, "Are there other values of 'a' you could try also? How do you think you could make the parabola wider?" as he moved on to the next group.

Group 2: Mark noticed that group 2 had completed the chart shown below:

Form	Value of a	Value of c	Equation graphed	Effect on shape of graph
1) $y = ax^2$		0		
	2	0	$y = 2x^2$	Skinnier
	-5	0	$y = -2x^2$	Skinnier and flipped over
	-3	0	$y = -3x^2$	Skinnier and flipped over
	1.5	0	$y = 1.5x^2$	Skinnier

Mark asked Nadia, "What does your group mean by 'skinnier' and 'flipped over'?" Nadia explained that 'flipped over' meant that the vertex stayed the same but the rest of the graph was like looking in a mirror. Darnell demonstrated with his hands that the graph became more narrow and Josh added, "you know, like squished together." Mark said, "OK. Let's talk about the 'flipped over' thing first. Nadia said it's like looking in a mirror. Does anyone know the math term we use to describe that?" Students did not respond so Mark told them it was called a 'reflection'. Nadia said, "Cool, like my reflection in a mirror!" Mark chuckled, "Right, it's like looking in a mirror. So when does the parabola get reflected? Look at your chart. What do you think causes the parabola to be reflected? Kayla, what do you think?" Kayla responded, "It seems like it flips over, I mean gets reflected when 'a' was negative." Mark asked, "What do the rest of you think about Kayla's conjecture?" Students were nodding in agreement so Mark said, "OK, it seems like you're agreeing. So maybe we should test the conjecture to see if it works. But before you do that, I have another question. Josh said, and Darnell showed us, that the graphs were 'squished together.' Were they all 'squished together' or 'more narrow' by the same amount?" The students replied that they all looked about the same. "So what do you think the graph of $y = 10x^2$ would look like," Mark inquired. Most of the students commented that it would look pretty much the same as the others. However, Belinda said that didn't make sense, "How can they all look the same when the front number is different?" Mark replied, "Belinda is making an interesting point. If the coefficient – remember, the number multiplied by the variable – changes, why wouldn't the graph change? So you guys have two things to think about – testing Kayla's conjecture about what happens when 'a' is negative and then answering Belinda's question about the coefficient. I'm also wondering if it is possible to make the parabola wider than $y = x^2$?"

Mark continued circulating around the room and stopped to ask each group questions about their graphs. He often had to prompt students to try positive and negative values for 'a' and to test values between -1 and 1. He made notes to himself as he visited each group to remind himself to address this during the whole group discussion.

Even though all students had not completed question 3 of the investigation (equations of the form $y = ax^2 + c$), Mark noticed that students could easily see and describe the effect 'c' had on the graph of the parabola. So, since all of the groups had at least tried out and discussed several examples of $y = ax^2 + c$, Mark decided it was time to pull the class together for a discussion.

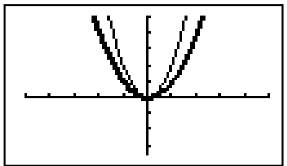
The Discussion

It had become a norm in his class, after many struggles and much persistence, for students to share and discuss their work publicly. When errors were made, Mark referred to them as "learning opportunities" and stressed that they often learned more about mathematics by discussing the errors. In fact, Mark modeled this by encouraging his students to point out "learning opportunities" he might provide during a lesson.

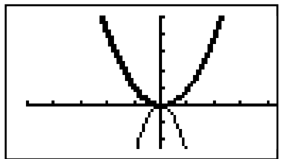
Also, prior to the discussion, Mark had asked students from several groups to enter their equations into the overhead graphing calculator. This would save time during the discussion since students would only need to highlight the equal sign and press the "graph" key to display a graph of the equation they were discussing. Mark also made certain that the equal sign for "$y=x^2$"

was highlighted and selected a heavier line for the graph so that it could be distinguished from the other graphs.

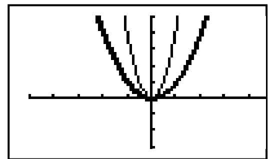
Mark began the whole group discussion by asking Hilary and Tamika to share and explain their group's graphs (shown below). Hilary explained that they graphed $y = 2x^2$, $y = -3x^2$, $y = 4x^2$ and $y = -10x^2$ and found that the larger they made 'a', the more narrow the parabola became. Tamika



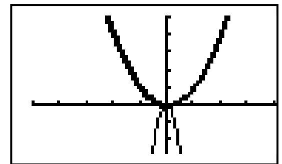
$y = 2x^2$



$y = -3x^2$



$y = 4x^2$



$y = -10x^2$

added that when they had 'a' values that were negative, the parabola was flipped. Mark asked if the negative coefficient affected how narrow the graph would be. Students responded that the negative only caused the graph to be "flipped." Mark took the opportunity to point out that since the negative sign did not affect the width of the graph, it would be appropriate to note that as |a| got larger, the graph got more narrow.

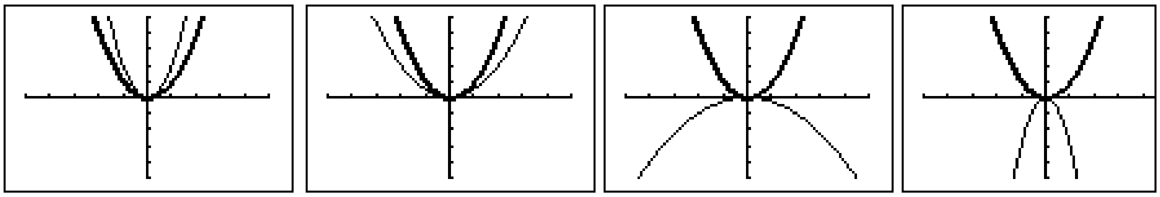
Mark then asked the class, "Do you agree with Hilary's group?" to which the majority of the students nodded. "Does anyone notice anything else about their graph?", Mark inquired. "Tyler, what do you notice?" Mark asked. Tyler stated, "Well, we found the same thing about 'a' but I just noticed that all of the graphs have the same vertex. The vertex didn't change even though the shape did." Mark stated, "Hmm, what do the rest of you think about what Tyler said? " Carlos said he noticed that, too. Veronica added, "We didn't notice that at first but in all of our graphs the vertex stayed the same, too." Mark pondered, "I wonder why the vertex didn't change when you were graphing $y = ax^2$?" He gave the students a few minutes to think about and discuss this in their groups and then asked if anyone thought they could explain. Renee commented, "I think it's because you're multiplying" as Jorge added "It's because of order of operations." Mark asked them to explain. "Let's start with Renee. What do you mean about multiplying?" Renee said, "Well, I think it's because no matter what 'a' is, you're multiplying it times 0 so you still get 0." Jorge added, "Yeah, I guess I was thinking of the same thing. The vertex is at (0,0) so you would first take 0 squared and then times 'a' so it's still 0."

Mark then refocused the discussion on a point that had been made by Tamika. "I want to come back to what Tamika said about the fact that when 'a' was negative, the parabola was flipped or reflected. Did any other groups arrive at that conclusion? Why do you think that occurred?", he continued. The students discussed that they thought it had something to do with what Renee and Jorge discussed earlier. Mark asked them to give examples of what they meant. Nadia raised her hand and explained, "I looked at our table (shown below) and when I substituted

Form	Value of a	Value of c	Equation graphed	Effect on shape of graph	Points on the graph (column added by student)
1) $y = ax^2$		0			
	2	0	$y = 2x^2$	Skinnier	(0, 0) (1, 2) (2, 8)
	-2	0	$y = -2x^2$	Skinnier and flipped over	(0, 0) (1, -2) (2, -8)

the same values in for x when 'a' was positive and 'a' was negative, the y-values were opposites – except for 0." "So how would that relate to the graph?," Mark asked. The class discussed the relationship between the graphs, the equations, and the coordinates by noting that, because of the order of operations, the x-value would first be squared and then multiplied by the coefficient. So for any given x-value, it would first be squared and then multiplied by 2 for the equation $y = 2x^2$. But it would be squared and then multiplied by -2 for the equation $y = -2x^2$. This would mean that for the same x-value, the corresponding y-values would be opposites for each equation. On the graph, for the same x-coordinate, the corresponding y-coordinates would be the same distances above and below the x-axis. But, because the product of 0 and any number is 0, the vertex would not change.

When Mark was satisfied that students understood why a negative value for ‘a’ resulted in a reflection, he decided to focus the discussion on how ‘a’ affected the width of the parabola. “So we’ve looked at parabolas that are more narrow than $y = x^2$. How would we make a parabola wider and why does it work that way?”, Mark asked. “Gina and Carlos, can you come up and show us your graphs?” Gina and Carlos showed their graphs of $y = 2x^2$, $y = ½\, x^2$, $y = -¼\, x^2$ and $y = -3x^2$ (shown below.) “We think that when ‘a’ is a fraction, the graph gets wider,” they explained.



$y = 2x^2$ $y = ½\, x^2$ $y = -¼\, x^2$ $y = -3x^2$

“See, when ‘a’ is ½ and -¼ the parabola gets wider.” Mark asked the class, “Do you agree with Gina and Carlos?” Students were nodding so Mark continued, “So they said when ‘a’ is a fraction the parabola gets wider. Is that always true?” Tyler remarked that they also used a value of $^1/_3$ for ‘a’ and that the parabola got wider so he thought it was probably always true. “Anyone else want to comment?”, Mark asked. Since no one volunteered to respond, Mark continued. “Something is confusing me. You agreed with Hilary and Tamika that as the |a| value got larger, the graph got more narrow and you agreed with Gina and Carlos that when ‘a’ is a fraction, the graph gets wider. What would happen if ‘a’ had a value of, say 2 ½? Would the graph be more narrow or wider than the graph of $y = x^2$?” Many students answered that the graph would be more narrow and Mark asked them to explain their thinking. “Because 2 ½ is more than 2 and we know the parabola gets more narrow the larger ‘a’ is,” Shala answered. “But ‘a’ is a fraction,” Mark responded. “I thought you said the graph got wider when ‘a’ was a fraction.” Many students responded that they meant fractions that were less than 1. “What about -3 ¼ ?” Mark inquired. “It’s less than 1.” “You know what we mean,” the students shouted. “You’re always making us say things so ‘mathy’!” Mark laughed to himself and continued with the discussion, prompting the students to use precise language and notation when answering questions. He made certain students discussed the fact that values of ‘a’ between -1 and 1 resulted in a parabola wider than $y = x^2$ and that values of ‘a’ greater than 1 or less than -1 resulted in more narrow parabolas. He also asked students to represent the relationship symbolically (i.e. If $-1 < a < 1$, the parabola is wider and if $a > 1$ or $a < -1$, the parabola is more narrow.)

Mark had hoped to also have students discuss WHY ‘a’ affected the graph of a parabola the way it did but time was running out. Since there were only a few minutes left in the class period, Mark asked the students to begin the discussion in their small groups and told them they would continue the discussion tomorrow. He also asked the students to think about the equations of the form $y = x^2 + c$ and $y = ax^2 + c$ that they had graphed and to be prepared to discuss their conclusions about the effects of ‘c’ and of both ‘a’ and ‘c’ on the graph of a parabola tomorrow.

APPENDIX C.3

Multiplying Binomials Task

Use algebra tiles to show these multiplications and make a sketch of your model. Write the product. **Complete problems 6, 1, 4, and 3 (in this order).**

1. $2x (x – 1)$
2. $(x + 1) (x + 2)$
3. $(x – 2) (3x + 3)$
4. $(x – 3) (x + 3)$
5. $(2x + 2) (2x – 2)$
6. $(x + 3) (x + 3)$

APPENDIX C.4

"Multiplying Binomials" Scenario 1

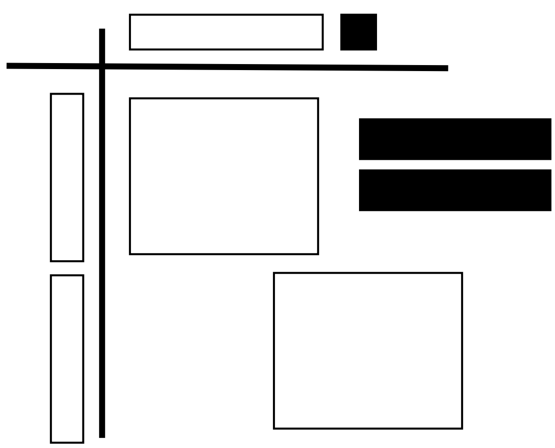
Prior to the lesson, the teacher had engaged the students in multiplying whole numbers using an area model and using algebra tiles to model multiplication of a constant and binomial.

The teacher set up the task by having students discuss what they had learned about modeling the multiplication of whole numbers and the multiplication of a constant and binomial. They also discussed the relationship between factors and products when multiplying.

Students have been introduced to the Multiplying Binomial task, including the expectation that they model each product with the algebra tiles, show pictorially how the product was determined, and then transform the area representation into a symbolic representation.

Students are now working in their groups. The group with Ben, Carla, Damon, and Erica had used their tiles to model 2x(x – 1) (see diagram below.) They were having difficulty making sense of the model and determining the product.

What questions would you ask the group to scaffold their understanding without taking over their thinking?

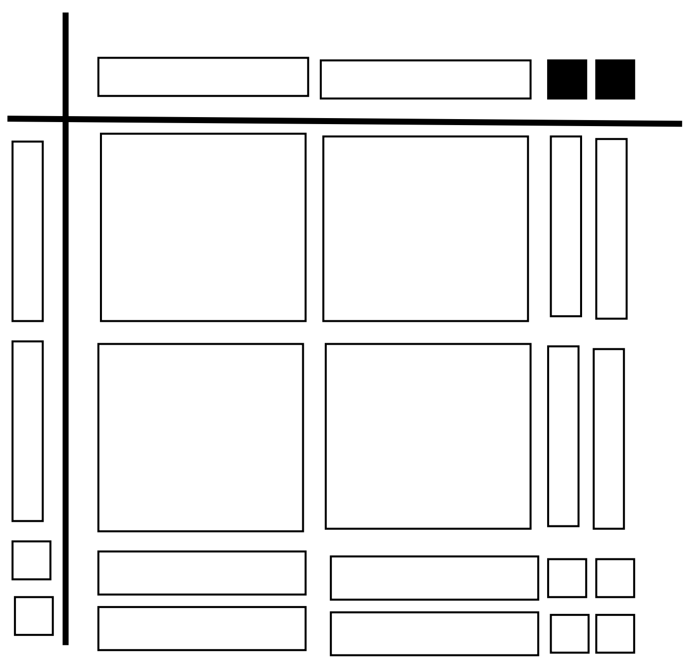


“Multiplying Binomials” Scenario 2

In the same classroom, Felicia, Gary, Hope, and Jamal are working on modeling (2x + 2)(2x – 2). They are arguing over the model they produced (see below).

Jamal states that the product is $4x^2 + 4x + 4$ according to their diagram. But Felicia is arguing that the model cannot be correct because her sister taught her a quicker way to do the multiplication. She proceeds to explain to her group how to use the FOIL method. So Hope suggests that it would be quicker to use the FOIL method on all of the problems and then just show the final answer using the tiles.

What questions would you ask the group to scaffold their understanding without taking over their thinking?



APPENDIX C.5

Shapes of Quadratics Lesson Plan

Phase	Investigation 1 Questions re: $y = ax^2$, $y = x^2 + c$ and $y = ax^2 + c$ 1. How are the graphs similar to, and different from, the graph of the basic function $y = x^2$? 2. How are these graphs, similar to, and different from, each other? 3. How do changes in the values of *a* and *c* affect: the appearance of the graph? the location of the vertex of the graph in the coordinate plane?*			
	Possible Solutions	**Possible Questions**	**Misconceptions/ Errors**	**Questions to Address Misconceptions/Errors**
E X P L O R E E X P L O R E E X P L O R E	Look for indictors of students' effective exploration: • Trying a range of values for *a* and *c*. • Graphing each equation and comparing the graph to that of the basic function. • Making conjectures about how the values of *a* and *c* affect the appearance and location of the vertex of the graph. Look for indicators of students' understanding: • Concluding that if *a* is negative, the graph opens down; if *a* is positive, the graph opens up. (Concave down; concave up) • Concluding that if $\|a\| > 1$, the graph is narrower; if $0 < \|a\| < 1$, the graph is wider. • Concluding that *c* produces a vertical translation of c units; a translation up when *c* is positive, down when c is negative.	**Ask questions such as:** • **What happens to the graph when *a* is negative?** • **What happens to the graph when *a* is between 0 and 1?** • **How can you tell if a parabola will open downward or open upward?** • **How can you make a parabola that is wider than the one you just made? How can you make a parabola that is more narrow? Try it out and see if your conjecture works.** • **How would you write the equation of a parabola whose vertex has a negative y-value?** • **What do parabolas with maximum (minimum) values look like? of the parabolas you graphed?**	• Values of *a* between 0 and 1 and 0 and -1 make a parabola narrower (i.e., smaller); larger values for *a* make the parabola wider (i.e., bigger). • When *a* is negative, the parabola has a minimum value (because negative numbers are smaller).	• **Show me a parabola that has a _____ value (*e.g., large value, value between 0 and 1*) for *a* (or c). What does it look like?** • **In what direction does the parabola open when *a* is negative? Does that give a maximum or minimum value?**

	Sequencing of Student Work	Rationale and Mathematical Ideas	Possible Questions and Student Responses
S H A R E D I S C U S S A N D A N A L Y Z E	Order of group presentations: 1. Conclusions from exploring equations of the form $y = ax^2$ 2. Conclusions from exploring equations of the form $y = x^2 + c$	This discussion will make explicit how values of *a* and *c* affect the graph of quadratic equations of forms $y = ax^2$, $y = x^2 + c$ and $y = ax^2 + c$ • If *a* is negative, the graph opens down; if *a* is positive, the graph opens up. (Concave down; concave up) When *a* is negative, the function has a maximum value; when *a* is positive, it has a minimum value. • If $\|a\| > 1$, the graph is narrower; if $0 < \|a\| < 1$, the graph is wider. Visually, this is a stretch of y values if $a > 1$; a compression if $0 < a < 1$. (Stretch or compression may help address the misconception that $a>1$ makes the graph wider; $0 < a < 1$ makes the graph narrower.) • *c* produces a vertical translation of c units; a translation up when *c* is positive, down when *c* is negative. • *c* is the y-intercept of the parabola. • The vertex of the parabola is (0, c)	What values did you use for *a*? (*c*?) • *We tried lots of different values: large numbers, negative numbers, and fractions.* What effect did different values of *a* have on the graphs? (If necessary, prompt by different values, e.g., what happened when *a* was positive? What direction did it open? What happened when *a* was negative? • *If a is negative, the graph opens down. If a is positive, the graph opens up.* Why does a negative *a* value cause the parabola to be reflected? Why does an *a* value between -1 and 1 cause the graph to become wider, while a value greater than 1 or less than -1 causes the graph to become more narrow? (After each group presentation, give other groups an opportunity to comment, ask questions, restate the conclusions given* or expand upon the conclusions presented. (E.g., ▪ ___, in your own words tell us what ___ concluded about the effect of 'a' on the graphs of parabolas. ▪ Do you agree with Group X's conclusions? Why or why not? Did you discover something different than what ____ did? Explain.) So we talked about positive, negative, and fractional values of *a*. Can *a* ever be 0? Why or why not? We talked about how the graph of the parabola changes when 'a' changes. Does anything stay the same? Why does it stay the same?

APPENDIX C.6

Tasks related to Shapes of Quadratics

Introduction to Graphing Quadratics

1. Graph the following equations using the same coordinate axis without using a graphing calculator. You might find it helpful to first create a table of values. Be sure to include both positive and negative values for x in your table.
   a. $y = x$

   b. $y = x^2$

2. Explain why the points on the graph $y = x^2$ should not be connected with a ruler.

3. Compare the two graphs and equations. List the similarities and differences that you observe.

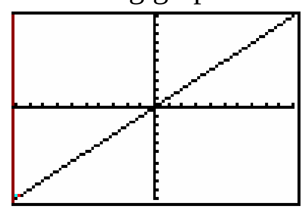
4. What is the minimum (lowest) value of the graph of $y = x^2$? Why does the graph of $y = x^2$ have a minimum value?

5. What is the line of symmetry of the graph of $y = x^2$? Why does the graph have a line of symmetry?

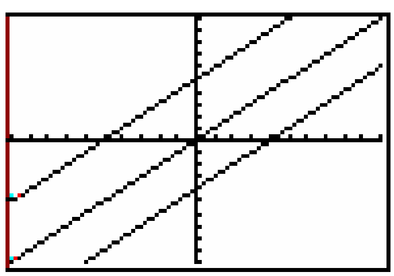
Analyzing Graphs of Linear Equations on a Graphing Calculator

Note: All calculator buttons will be **bold** and surrounded by brackets **[ ].**

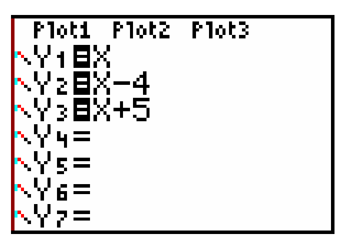
<table>
<tr><th>THE TASK</th><th>CALCULATOR INSTRUCTIONS</th></tr>
<tr><td>1. Graph the following equation into your calculator.
y = x</td><td>Press **[Y=] [X,T,θ,n] [GRAPH]**

Your calculator should display the following graph.

If not, press **[ZOOM] [6]**</td></tr>
<tr><td>2. Graph the next three equations in the same viewing window. Then match each equation with its corresponding graph.

a) y = x **b) y = x – 4** **c) y = x + 5**

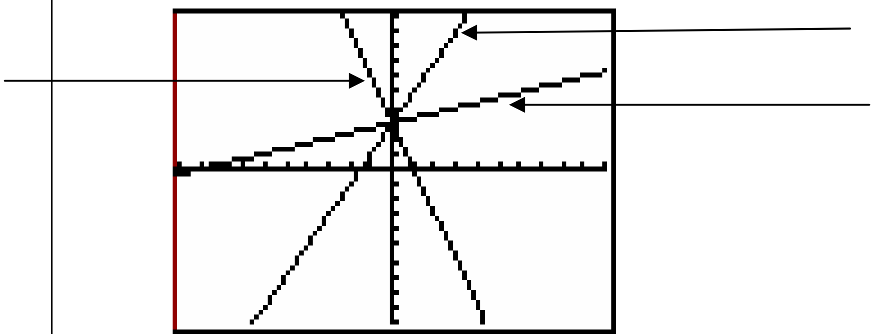
Write Y1, Y2 and Y3 on the graphs below that correspond to the equations.

Explain in words how you matched each equation with its graph. Use the terms **slope** and **y-intercept** in your explanation.</td><td>Press **[Y=]** and enter the first equation. Press the blue down arrow to enter the second equation. Repeat for the third equation.
For y = x – 4, press the following buttons.
[X,T,θ,n] [-] [4]

Common mistake:* **[-]** *is for subtraction. And* **[(-)]** *is a negative sign.

Before you press graph, your calculator screen should match the window below.
If your screen matches the window below, press **[GRAPH]**.

</td></tr>
</table>

3. Graph the next three equations in the same viewing window. Match each equation with its graph.

a) y = 2x + 3   b) y = (1/3) x + 3   c) y = -3x + 3



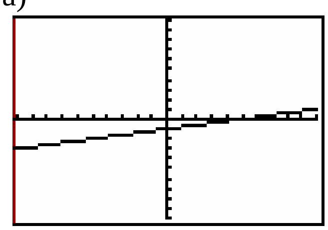
Explain in words how you matched each equation with its graph. Use the terms **slope** and **y-intercept** in your explanation.

***Remember to use the* **[(-)]** *button located next to the enter key at the bottom of the calculator to enter -3 for the third equation. Also use the division button* **[÷]** *for the fraction bar (/) in the second equation. ***

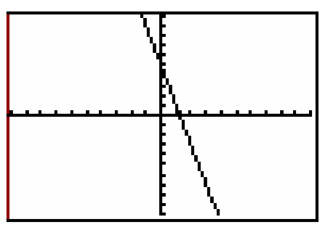
4. Without using your calculator, match each graph with its corresponding equation. Then use your calculator to check your predictions. Correct your responses if necessary.

i) y = -5x   ii) y = -4x + 5   iii) y = 1/5 x + 2   iv) y = 1/5 x – 1   v) y = 2x

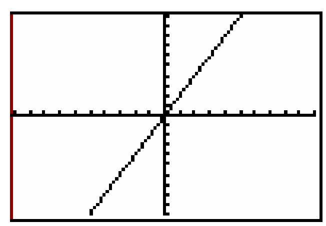
a)



b)



c)



5. y = mx + b is the slope-intercept form of a linear equation. Describe what “m” and “b” in the equation tell you about how graphs of this form will compare to the graph of y = x.

6. What do you think a graph of y = mx + b look like if “m” is zero?

Check your prediction with your graphing calculator. What equation did you graph?

y = ______________________.

What do you notice?

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