A FACILITY LAYOUT DESIGN METHODOLOGY FOR RETAIL ENVIRONMENTS

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Based on an overall consideration of the principles and characteristics in designing a retail area layout, this research is the first work to integrate aisle structure design, department allocation, and detailed departmental layout, as a whole process. The main difference between previous research and this proposed research is the formulation of mathematical models that can be specifically applied in the retail sector. Unlike manufacturing, in retail environments, the design objective is profit maximization rather than minimizing material handling costs. The entire optimization process is completed by a series of sequential design problems, starting from aisle structure design, where aisle effects on merchandise exposure are taken into consideration, followed by department allocation design, which is modeled as a multiple knapsack problem with adjacency preferences in the objective, and finally the detailed departmental layout design. The optimization process is accomplished by maximizing the retail area exposure, optimizing the adjacency preference of all departments, and adjusting the detailed department layout and evaluating the effectiveness of the layout design.

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1.0 INTRODUCTION

Facility layout design problems have been studied for decades and there is a lot of literature regarding both theory and application. Most of the previous research on this topic focuses on manufacturing or distribution facilities for the objective of minimizing the material handling cost [#12, Botsali, 2005]. Recently, facility layout problems in service industries including hospitals and retail stores have received more attention because of the increasing need for improving customer satisfaction and the increasing competition in service industries. Since there are many differences between manufacturing and service settings, the previous models developed for manufacturing settings can't be directly applied to the service sector. New models need to be developed to fit the variety of applications and optimization required in the service sector. Therefore, this study will develop a solution method to optimize facility layout design in the retail sector.

1.1 STATEMENT OF THE RETAIL LAYOUT DESIGN PROBLEM

Usually the layout design problem in the retail sector is divided into three stages. The first one is facility site selection, which depends on the type of store, the size of its surrounding population and the geographical location of the store. This stage has been studied extensively in books, such as Retail Management [#44, Levy, 1998] and Facilities Planning [#60, Tompkins, 2003]. The second stage is the block layout design, which specifies the relative location of departments in a retail store. Several traditional approaches have been developed to model the block layout problem in manufacturing, such as quadratic assignment problem approaches, graph-theoretic approaches, and knapsack problem approaches. However, all of these models are NP-hard and cannot be solved to optimality with reasonable computational effort. In this research, the block layout problem is formulated using linear mixed integer programming models, with the help of symmetry breaking constraints and a reformulation-linearization technique to reduce the computation time, the global optimal solution can be obtained with reasonable computational effort. The third stage is detailed layout design. In manufacturing, it means adding material handling aisles, allocating resources to workstations and verifying machine and workstation placement. In a retail environment, this could include the detailed department design and inner aisle design.

Most previous research on retail management is from a strategy point of view, including situation analysis, targeting customers, choosing a store location, managing a retail business, developing customer service, and planning for the future. However, the design of the block layout and the detailed facility layout haven't received much attention. One exception is the work of Yapicioglu and Smith [#67, 2008] which developed a nonlinear model of the block layout problem using the departments' revenue and adjacency as the objective function. The nonlinear constraints arise in the model in order to enforce the departmental areas and aspect ratio limitations on the department shapes. It is difficult to solve the resulting non-linear program, Yapicioglu [#67, 2008] makes two assumptions to simplify the model – 1) that aisles occupy zero area and 2) that the department areas are known and fixed. Yapicioglu used three methods to solve the simplified problem: non-linear programming, a constructive heuristic, and tabu search. The non-linear programming approach could not find the optimal solution in a reasonable amount of time. Therefore emphasis was placed on using the constructive heuristic and tabu search approaches. More detailed contrasts of Yapicioglu's methodologies and those found in this research are discussed in detail in Chapter 3.

Establishing a retail image is an important step in communicating with customers and competing with other retail peers. The store layout is critical for creating and maintaining a store image to further maximize the profit of retail stores. The purpose of this research is to introduce a solution methodology to optimize the store layout in a given retail space.

1.2 CHARACTERISTICS OF STORE LAYOUT DESIGN

A successful store should keep a consumer interested and finally convert the consumer to a customer. From the customers' point of view, they would like the shopping process to be easy and satisfying. They prefer a pleasant shopping environment where the aisles are wide, the view of the merchandise is clear, the merchandise is easy to find and that there are sufficient items such that customers won't experience stock-outs. The retailer should have effective merchandising and displays in order to increase the satisfaction of customers.

To achieve this, several principles and characteristics of store layout design should be considered. Utilization of floor space has an effect on potential customers. The size of the store is constrained by budget, store type, merchandise assortment and the volume of sales. The aisle structure should be well designed to facilitate shoppers browsing and checking out the merchandise. The traffic flow density of the aisles should be balanced to provide a comfortable and safe shopping environment. Providing customers with a logical layout of merchandise, such as grouping similar and complementary products in distinct sections, insures customers can easily find what they want. Overcrowded displays can confuse and depress customers, however half full shelves can give the impression that the store is going out of business or the items are out of stock, which can lower customer goodwill. To effectively display the product, fixtures including stands, shelves, tables, bins and racks should be carefully chosen and organized, depending on the type of product sold and the customer demand. Overall there are both qualitative and quantitative criteria for store layout design.

1.3 CONTRIBUTIONS

Based on an overall consideration of the principles and characteristics in designing a retail area layout, this research is one of the first to integrate aisle structure design, block layout, meaning specific department allocation placement and final department area size specification. The main difference between previous research and this proposed research is the formulization of mathematical models that can be specifically applied in the retail sector. In retail environments, the layout design objective is to maximize profit based on having a floor plan that promotes sales rather than the minimization of material handling costs that is commonly seen in manufacturing. This is accomplished by maximizing the area exposure and optimizing the adjacency preference of all departments. The main contributions of the work in this research are the following:

- Establishing a hierarchical procedure to decompose the retail facility layout design process. The entire layout design problem is divided into four sub-problems to decrease the complexity of solving the original problem. Based on the results of the sub-problems, a new solution approach is presented for retailers to optimize the facility layout of their new stores or to redesign existing stores.
- The first sub-problem is the aisle configuration optimization problem. The potential combinations of aisle parameters are enumerated to find the set that maximizes the exposure of the entire retail area. Exposure is a measure of how likely a point in the retail area is to be noticed by customers. Several exposure functions are designed to reflect realistic considerations. Once the aisle structure, such as one main racetrack or an additional central aisle, is chosen the aisle configuration is tested by enumerating different aisle parameter values using the resultant exposure as the evaluation metric.
- The second sub-problem is the department allocation problem. Once the aisle configuration is fixed, the entire retail area is separated by aisles into smaller sub-areas. The objective in the second sub-problem is to allocate the departments into those smaller divisions. The problem can be modeled

using mathematical programming as a multiple knapsack problem with variable weight items (variable department areas). The multiple knapsack problem is NP-hard.

- The third sub-problem is to create the detailed assignment of departments within the sub-areas in the retail space. A model is developed to determine the actual allocation of departments. The departments should occupy a certain amount of adjacent strips in the retail sub-area considering the preference of exposure and adjacency with other departments.

Computer programs are developed using the C programming language to setup the mixed integer programming models and they are solved using CPLEX 11.0.

1.4 OVERVIEW OF THE DISSERTATION

The reminder of the dissertation is organized as follows. Section 2 introduces the background of this research and reviews the related previous research in retail facility layout design. Section 3 presents the methodologies for layout design in department stores. In Section 4, three sub-problems are discussed and mathematical models are developed to solve them. In Section 5, a case study is presented. Section 6 discusses extensions and potential future work based on this research. Section 7 summarizes the key contributions of this research.

2.0 BACKGROUND AND LITERATURE REVIEW

In the last twenty years, different aspects of retail management and customer behavior have been intensely studied. In general, these studies have been more qualitative in nature than quantitative. In this section qualitative achievements are reviewed such as studies concerning the physical attractiveness of a store, the type of store layout and impulse purchases based on exposure and adjacency, as well as previously applied models, including knapsack problems, shelf space allocation problems and space elasticity problems.

2.1 PHYSICAL ATTRACTIVENESS OF A STORE

Retailers spend millions of dollars each year in designing, building, and refurbishing stores [#3, Baker, 1992]. For instance, Neiman Marcus spent more than $200 million within five years to renovate its 23 stores [#42, Lawson, 1990]. In order to overcome competition, retailers must be sure that their stores are up-to-date and appeal to their customers, by providing a pleasant environment, choosing a convenient location as well as offering competitive merchandise and prices. In the academic environment, several factors have been shown to affect retail customers' decisions. Craig et al. [#2121, 1984] and Morey [#5050, 1980] indicate that customers' decisions are affected by location, service level, pricing policies, and merchandise assortment. Additionally researchers such as Darden et al. [#22, 1983] found that consumers' beliefs about the physical attractiveness of a store had a higher correlation with patronage intentions than merchandise quality, general price level, and seven other store beliefs.

Considerable research exists on the evaluation of department stores by consumers. Berry [#11, 1969] empirically identifies a number of attributes, which can be used to evaluate department stores, using a mall survey. May [#46, 1974] emphasizes the importance of the retail stores' image. Lindquist [#45, 1974] categorized store components into functional factors such as merchandise selection, price, store policies and store layout. His attributes list is a compilation from 26 researchers in this field. Lindquist distinguished store image components from functional qualities, such as merchandise selection, price range, credit policies and store layout as well as psychological attributes that were associated with the degree to which customers feel comfortable in the store.

These attributes can be categorized into four groups: merchandise, service, promotion, and navigation. Merchandise variables measure product selection assortment, quality, guarantees, and pricing. Service variables examine general service in the store and sales clerk service for merchandise return, credit policies, etc. Promotion variables record sales, advertising and appetizer features that attract customers. Navigation variables include store layout and organization features. Some features in these categories are usually the factors that researchers are interested in: store layout, image maps indices, aisle structure, number of floors in the store, number of store entrances and store outlets/branches, number of checkout cashiers, number of people entering the store and traffic patterns, etc.

An environmental psychology approach has been used to study store environments [#23, Donovan, 1982]. This approach supported the results of Mehrabian's model, which states that environmental stimuli affect the emotional states of pleasure and arousal, which, in turn, affect approach or avoidance behaviors [#47, Mehrabian, 1974, #57, Russel, 1980]. In Mehrabian's model on environmental reaction, they didn't classify the specific environmental features that affect customer behavior. Later, a framework was developed by Baker in 1986 to examine the effects of specific environmental factors. He classified environmental stimuli into three groups --- ambient, social, and design factors. Ambient factors are background conditions in the environment, such as temperature, scent, noise, music and lighting. Social factors represent the "people" component of the environment, including both store employees and customers. Design factors include functional and aesthetic elements such as architecture, style, and layout. Among the three factors, from the construction point of view, the ambient and social factors are relatively easy and less expensive to change for most retailers than the design factors. The design factors have equal importance, and are more difficult and expensive to change, thus it is important to put more effort into considering these factors when first designing those factors for retailers.

Due to the importance of these design factors, background and classification material related to facility layout is now introduced. The following sections discuss some of the results of previous studies in the retail sector.

2.2 LAYOUT DESIGN FOR A RETAIL SETTING

The facility layout problem has been studied in the manufacturing sector for a long time. There have been numerous papers presented in the last twenty years covering methodologies, objectives, algorithms, and extensions on this well-studied combinatorial optimization problem [#48, Meller, 1996].

Most of the current facility design research in the retail sector focuses on store location selection, without considering detailed design issues such as aisle structure and arrangement of racks. Store image is an important factor affecting customer behavior [#26, Erdem, 1999], and store layout design is a critical determinant affecting the creation of that store image [#7, Baker, 2002]. Furthermore, the selling floor layout can strongly influence in-store traffic patterns, shopping atmosphere, shopping behavior, and operational efficiency [#43, Lewison, 1994]. Furthermore, Merrilees and Miller [#49, 2001] state that store layout design is one of the most important determinants for store loyalty, and Simonson [#58, 1999] states that store layout design can play a key role not only in satisfying buyers' requirements but also in influencing their wants and preferences.

In conventional retailing, there are several common store layouts used, including grid, freeform, racetrack and serpentine layouts, as shown in Figures 1-5. By comparing the figures, one can point out the pros and cons in realistic applications of each type of layout. However in real life, the retail area would combine these types of layouts rather than being restricted to using only one type for the entire retail setting.

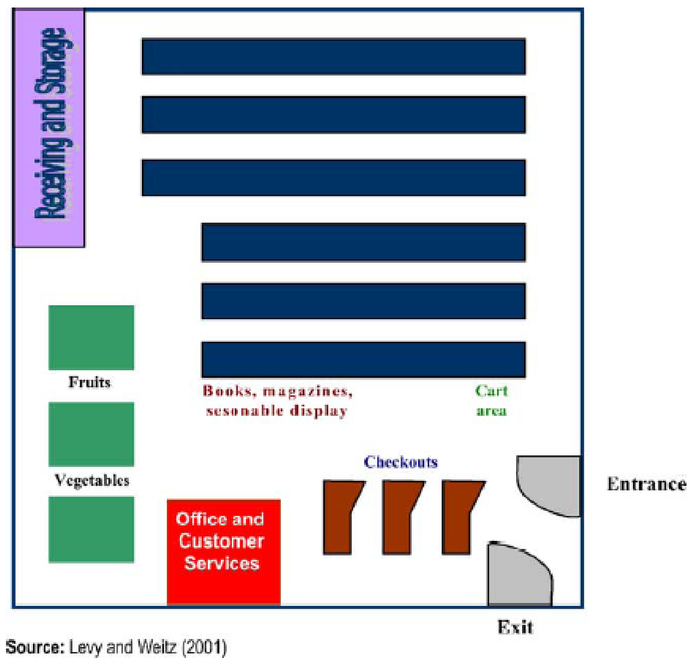


Figure 1. Grid store layout

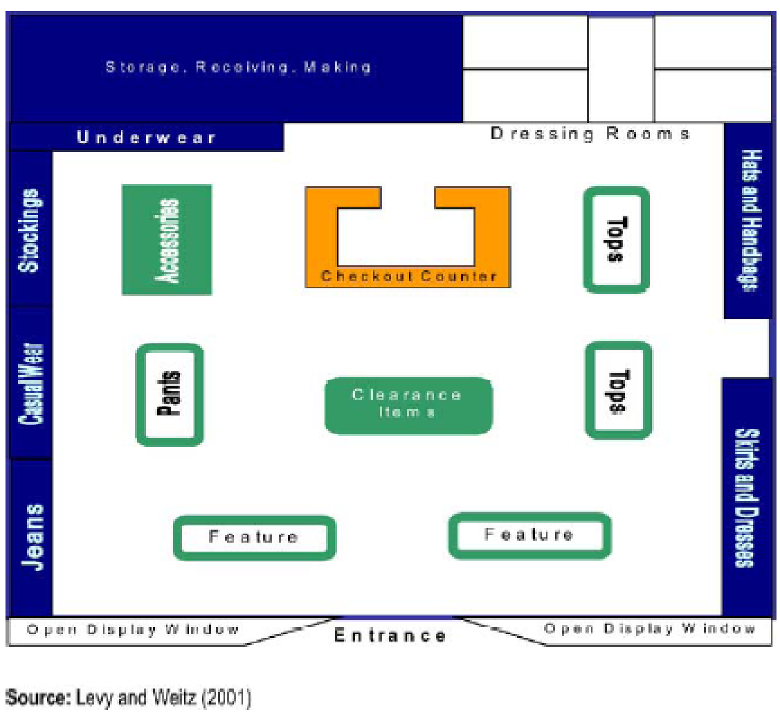
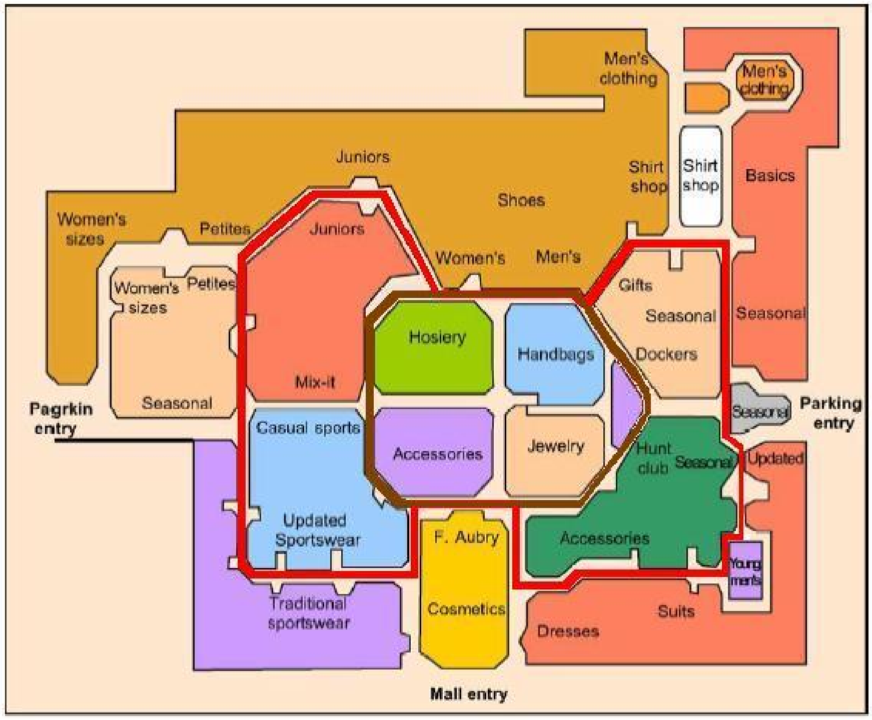


Figure 2. Freeform store layout



Source: Levy and Weitz (2001)

Figure 3. Racetrack layout illustration 1

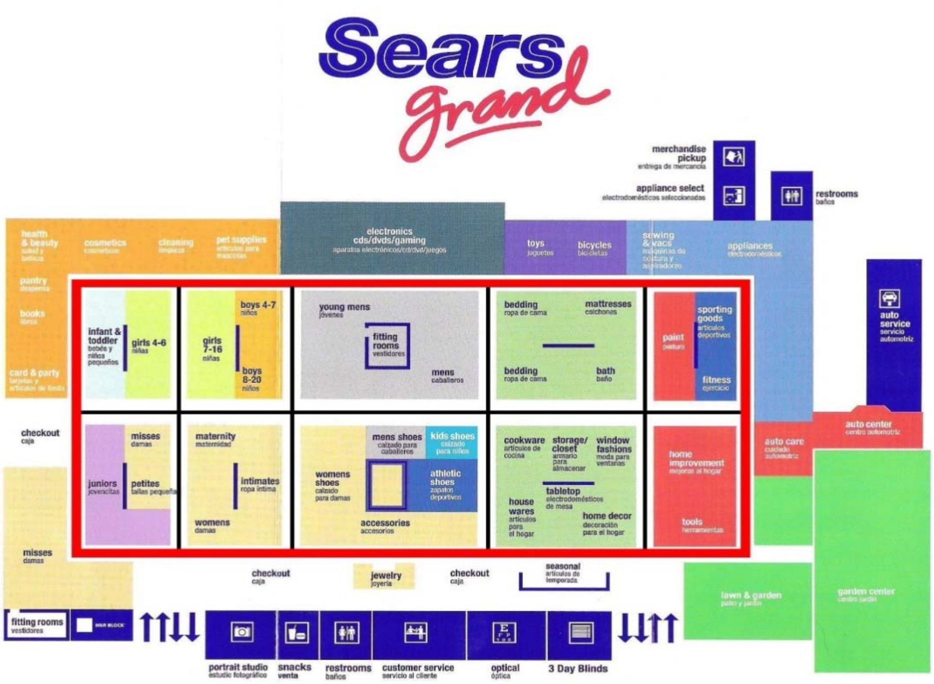


Figure 4. Racetrack layout illustration 2

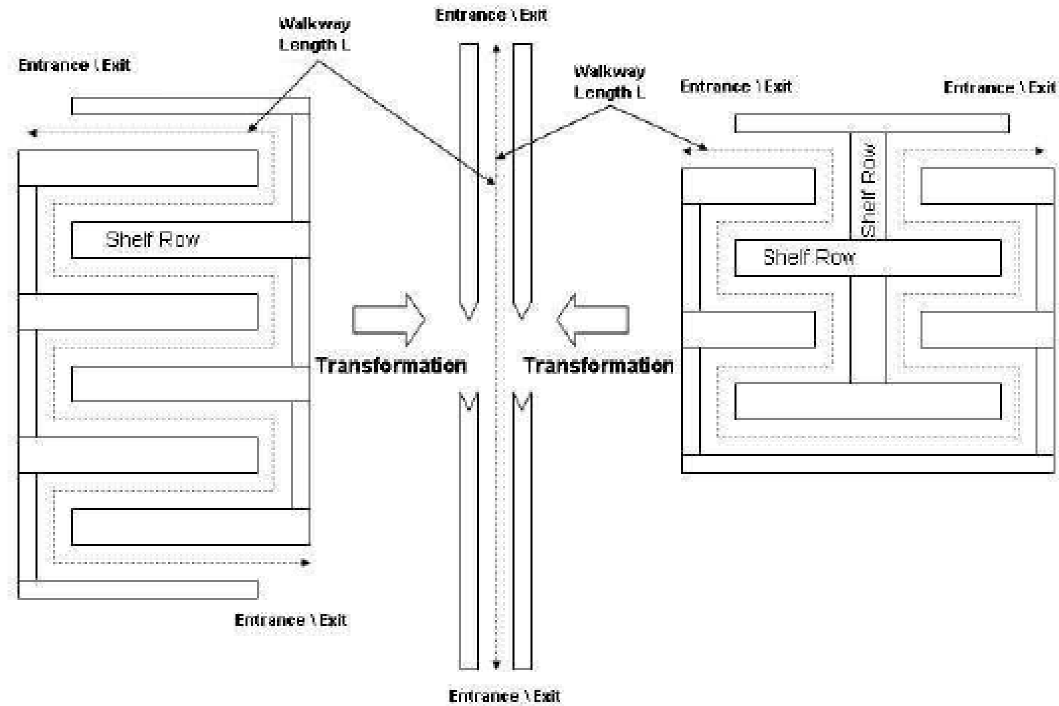


Figure 5. Serpentine layout and aisle representation

Grid store layout As shown in Figure 1, the grid layout is a rectangular arrangement of displays and long aisles that generally run parallel to one another. It has been shown that the grid layout facilitates routing and planned shopping behavior, providing consumers with flexibility and speed in identifying pre-selected products which appear on their shopping list [#44, Levy, 2001, #43, Lewison, 1994]. It is widely favored in the grocery sector because the majority of the customers visiting grocery stores have planned their purchases.

Freeform store layout As shown in Figure 2, the freeform layout is a free-flowing and asymmetric arrangement of displays and aisles, employing a variety of different sizes, shapes, and styles of display. In this pattern, the customer enjoys considerable freedom to move in any direction within the store. The freeform layout has been shown to increase the time that consumers are willing to spend in the store [#44, Levy, 2001, #43, Lewison, 1994]. It is mainly used by name brand stores, for example, fashion stores like American Eagle, Ann Taylor, etc.

Racetrack store layout Racetrack layouts are shown in Figure 3 and Figure 4. The selling floor is divided into individual areas along a circle or rectangular main aisle in the middle of the store (the red line shown in Figure 3 and Figure 4). Each individual area or sub-area is built for a particular shopping theme. The racetrack store layout leads the customers along specific paths to visit as many store sections or departments as possible because the main aisle facilitates customers moving through the store [#44, Levy, 2001, #43, Lewison, 1994]. It is mainly used in department stores, for example, Kohl's, and Sear's.

Serpentine layout There are some papers focusing on serpentine, hub and spoke layouts, which are variants of the grid store layout. Serpentine layouts and their corresponding aisle representation are shown in Figure 5. Using a serpentine layout design scheme, Botsali et al. [#12, 2005] developed a network based layout design model to analyze the performance of this layout in retail store settings. The advantage of the serpentine layout is that there is only one path for customers to follow that traverses all the floor space. Profit can be maximized by extending the shopping distance of the customer.

Kohl's Corporation credits its racetrack store design for a 15 percent same-store sales gain despite the economic downturn in the entire retail industry. Due to their success, this Midwestern discount department store chain opened 50 to 60 new stores in 2001 [DWC Magazine, May, 2001]. The advantage of the racetrack layout is shorter shopping distances for customers. Kohl's average store size of 86,000 square feet on a single floor is about half the size of most department stores. By walking just a quarter of a mile, a shopper covers the entire store while it takes twice that long at most competitors' stores. Thus, the objective here is opposite of that for the serpentine layout. Depending on the type of retail store and customer behaviors, store layout design goals and methods to attract customers are different and sometimes conflicting. It is important to find a compromise solution to achieve overall profit maximization.

For the freeform layout and racetrack store layout, since the shopping routes are more controlled by the customer, the store layout problem is more complicated and interesting. Few studies have been done in this domain. As a first step, this research considers racetrack layout design, which is credited by Kohl's store for much of their success.

Good aisle structure designs can affect impulse purchases by increasing the exposure of the merchandise in sales areas, and good department allocations can remind customers of merchandise which is not in their shopping lists. This is usually called impulse purchasing and is discussed more in the next section.

2.2.1 Impulse purchases and exposure

Impulse purchases are generally referred to as purchases that are made by customers without prior intention. In Kollat [#40, 1994], it is shown that there is a difference between the products purchased and the products planned to be purchased before entering into a store. Impulse purchases account for up to 38.7% of total purchases [#9, Bellenger, 1978] and varies by retail sector. In grocery shopping, customers usually have more planned shopping items (for example, a list) than in department store shopping.

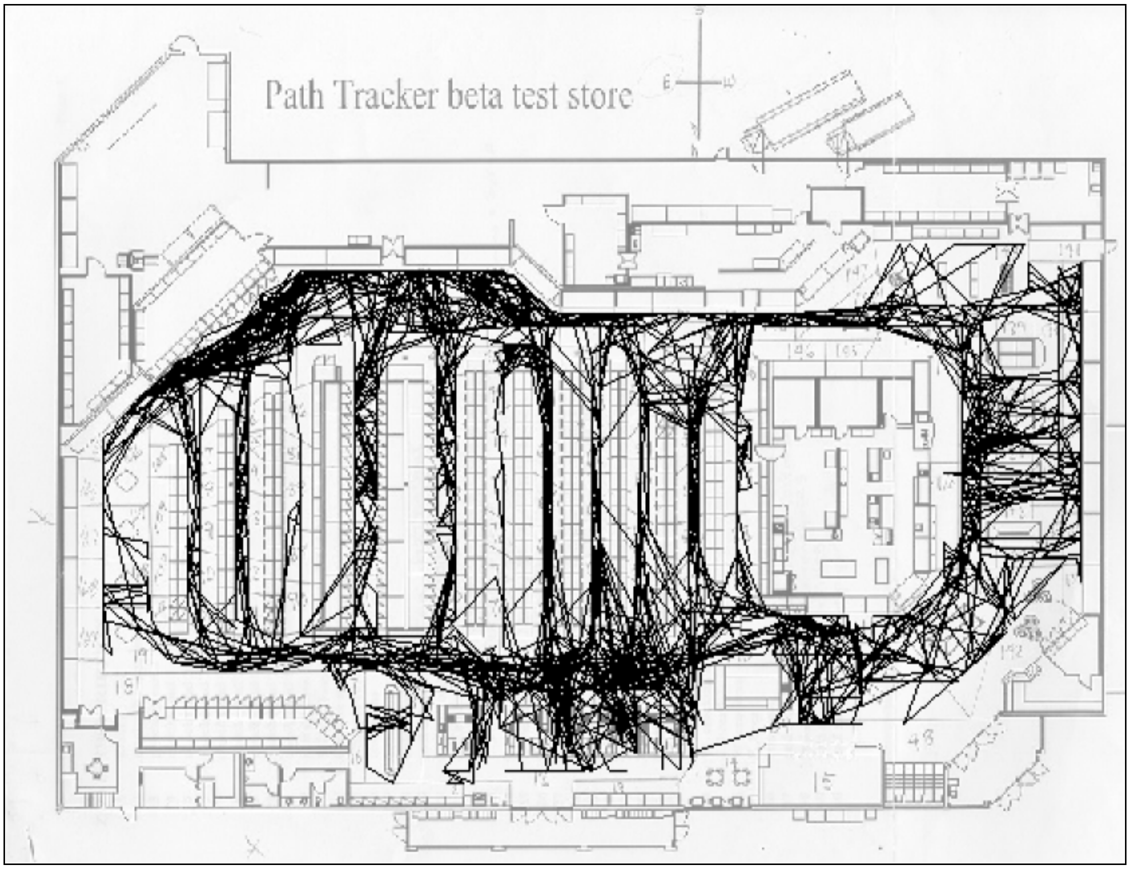


Figure 6. Path tracker data from 20 random customers

Figure 6 shows a subset of the PathTracker® data collected by Sorensen Associates, an in-store research firm, for the purpose of understanding shopper behavior in a supermarket [#41, Larson, 2005]. Figure 6 shows that in grocery shopping, customers tend to go back and forth from one department to another to find the items in their list, which results in a lot of impulse purchases. For department store shopping, customers usually have a target in mind instead of a detailed shopping list. They usually browse among departments until they are stopped by certain attractive items. They seldom do back and forth searching. Thus, department stores should be more careful in designing their retail floors to maximize product exposure in order to increase impulse purchases by customers.

To model merchandise exposure, Botsali et al. [#12, 2005] used a “visible factor” which is calculated by the length and the frequency of an edge of rack along the aisle. The expected impulse purchase revenue is affected by the visible factor, average impulse purchase rate and average revenue per purchase. Botsali et al. [#12, 2005] used a heuristic algorithm to find a good layout in reasonable time.

In this research, the exposure is modeled as a function of the distance the merchandise is from the aisles, the width of the aisles, the traffic in the surrounding areas and the choice of decay function representing the decreasing tendency of exposure as merchandise is located farther from the aisle. To maximize impulse purchases, the entire exposure of the retail area needs to be maximized.

Another important factor that stimulates impulse purchases is adjacency, which is related to customers’ tendencies to buy related items when they buy one item on their shopping lists. For example, a data mining analysis of market-baskets, supported by NSF and done by Stanford University, studied what items customers frequently purchase together. The goal was to understand the behavior of typical customers as they navigate the aisles of a store [#61, Tsur, D. et al., 1998]. By reviewing this data, retailers may decide to put related merchandise close together in order to stimulate impulse purchases. It may also be advantageous to locate related items a certain distance apart in order to stimulate impulse purchases from merchandise displays that are positioned between the pair of related items. In the next section, the relationship between impulse purchases and adjacency is discussed.

2.2.2 Impulse purchases and adjacency

Departmental adjacency is often considered in facility layout problems. In the manufacturing sector, facility designers want to locate departments with heavy traffic flows close to each other to minimize material handling cost or workload. In the retail sector, retailers want to put related merchandise close or far apart, depending on whether the objective is to shorten the customers' travel path or prolong the customers' travel distance.

In the manufacturing sector, adjacency in facility layout problems has been studied extensively. There are several methods used to optimize the layout by minimizing the material handling costs and shortening the distance between those departments with closer relationships. Three common methods for modeling adjacency are now discussed in more detail.

2.2.2.1 REL chart

One widely used approach involves the use of the REL chart. An REL chart, defined by Muther [#51, 1973], is a table that summarizes estimates of the desirability of locating facilities next to each other. Designers often attempt to maximize the sum of the REL chart scores of adjacent pairs of departments. However, in some situations, the preference could be negative, which means the two departments should not be adjacent. We denote a negative preference as adjacency avoidance. In our model, adjacency avoidance is also considered.

2.2.2.2 Adjacency representation in graph theory

Another approach to this problem utilizes graph theory. Usually researchers define a graph G = (V, E) with a vertex set V and an edge set E. In a facility design problem, the vertex set represents the set of given facilities (or departments), and the edge set represents the possible adjacency of the facilities (or departments). The edge weight is equal to the appropriate desirability rating. Foulds [#30, 1986] has shown that to find a planar subgraph with maximum total edge weight in the graph theoretic model is NP-Complete. The limitation of using graph theory is that it doesn't consider the shape or area of the individual facilities or departments, since graph theory considers the facilities (or department) as a set of nodes. It can be difficult to apply graph theoretic models to solve practical problems.

2.2.2.3 Adjacency representation in a quadratic assignment problem formulation

In the quadratic assignment problem, all the facilities are assumed to have the same size and there are *n* potential locations and *m* departments. The objective is to assign the departments to potential locations to maximize the summation of all pairs of departments' adjacency factors. In the quadratic assignment problem, the sizes and shapes of the departments are not considered, and the departments are assumed to be of equal sizes. In this study, each department can have a different area, thus it is not possible to ignore the actual area and various possible department shapes.

The three methods mentioned above can't be readily applied in the retail problem under investigation in this dissertation, because adjacency preference with the consideration of departmental area and shape is considered. Therefore a new model is needed to formulate the problem.

2.2.3 Knapsack problem

The next problem is how to assign the departments with different sizes and shapes to the different sub-areas in the retail area. After determining the aisle structure, the entire retail region is divided into several sub-areas by the aisles. It is necessary to allocate departments to those sub-areas. If each sub-area is treated as a knapsack, and each department is treated as an item to be put into a knapsack, then this part of the problem is similar to a knapsack problem. The knapsack problem is now discussed in more detail.

The knapsack problem is a classic combinatorial optimization problem, which has been studied extensively since the 1950's. There are many variants and extensions of the knapsack problem, for instance, the most fundamental ones are the one-dimensional, two-dimensional, multiple and stochastic knapsack problems. These problems have been studied and applied in many settings, such as in glass, steel, wood and paper industries, cargo loading in logistics and the service sector. The objectives of these problems may be to minimize the cost of the material used such as in cutting stock problems [#32, #33, #34, Gilmore 1961, 1963, 1964, #17, Chambers, 1976, #63, Wang, 1983]. The objective may also be to maximize the value or profit associated with a limited resource. The specific objective in the study is to maximize the total profit from a given retail area, which is classified as a two-dimensional knapsack problem. Because of their wide applications, two-dimensional knapsack problems are of great practical importance and many useful results have been produced by using them [#35, Gilmore, 1966, #27, Fayard, 1995, #38, Hadjiconstantinou, 1995]. Recently, some studies have investigated the stochastic knapsack problem, which has uncertainty in the resource constraints and/or the objective function. It has been studied in the application domains of investment and space allocation problems

To solve the knapsack problem, different types of approaches are used. For example dynamic programming is applied to solve the general knapsack problem containing a few side constraints [#53, Ozden, 1988]. Different types of knapsack problems which are relevant to this research are now discussed in more detail.

2.2.3.1 Multiple Knapsack problems

The multiple knapsack problem is a generalization of the standard knapsack problem from a single knapsack to multiple knapsacks with possibly different capacities [#56, Pisinger, 2004]. The objective is to assign each item to at most one of the knapsacks such that none of the capacity constraints are violated and the total profit of the items put into the knapsacks is maximized.

2.2.3.2 Quadratic Knapsack problems

In all the variants of the knapsack problems discussed so far, the profit of choosing a given item is independent of the other items chosen. In some real life applications, it is natural that the profit of a packing item is affected by the other items in the same knapsack. For example, the objective value may reflect how well the given items fit together. One possible formulation of such interdependence is the quadratic knapsack problem [#56, Pisinger, 2004], in which an item can bring a corresponding profit and an additional profit which is redeemed if the item is selected together with another item.

2.2.3.3 Stochastic Knapsack problems

A relatively new member in the knapsack problem family is the stochastic knapsack problem. In the standard problem, the profit and weight of the items, the upper and lower bound of the items to be chosen and the capacity of the knapsacks are given and constant. However in real life, the profit and weight may be uncertain and can be characterized probabilistically. Using probabilistic and stochastic models to formulate real problems may be more realistic in certain contexts [#56, Pisinger, 2004].

Among the many possible extensions of knapsack problems, multiple knapsack and quadratic problems get more attention due to their simple structures but difficulty in finding solutions. The retail layout problem in this research is formulated as a quadratic multiple knapsack problem with the consideration of adjacency and exposure, which is hard to solve for large scale data sets. Modeling techniques will be applied to generate formulations to reduce the complexity of the solution process.

2.2.4 Shelf allocation problem

In the detailed design stage of retail store design, most studies focus on very detailed shelf space allocation and merchandise assortment for shelf displays.

The shelf space allocation problem is presented by Yang [#64, #66, 1999, 2001] with an approach which is similar to the algorithm for solving a knapsack problem. It is an integer programming model that can help retailers optimally select products and allocate shelf space according to the chosen product categories and items. Retail shelf space is valuable real estate (Dreze [#24, 1994]). Store occupancy cost ranges from about $20/square foot for dry grocery shelf space to over $50/square foot for frozen foods.

Dreze finds that allocating too many facings is wasteful, while allocating too few results in lost sales due to stockouts. Buttle [#16, 1984] summarizes five aspects of space allocation which management can consider in pursuit of incremental sales and profit; these are fixture location, product category location, item location within categories, off-shelf display and point-of-sale promotional support.

Space elasticity has continued to be one of the hottest issues in retail management and is defined as the ratio of relative change in unit sales to relative change in shelf space [#66, Yang, 2001]. Dreze models the effect of space on sales using the Gompertz growth model. It implies that there is a decreasing marginal rate of return as the allocated area increases.

In this research, ideas similar to some of these shelf space concepts are applied in block layout design and department allocation. This study investigates the relationships between aisle settings and department space allocation, which no previous work has done. A decreasing marginal rate of return for increasing the unit area of a department is taken into consideration to allocate proper area to departments and to adjust aisle settings accordingly. To solve the problem, the robustness of the aisle structure is tested in terms of exposure and profit.

3.0 METHODOLOGY

3.1 INTRODUCTION

Several methods have been developed to solve facility layout problems and these can be classified into two broad groups: empirical and theoretical methods.

Empirical methods are used to develop initial or potential layouts based on experience and knowledge, evaluate the alternatives with quantitative criteria, and then revise the potential choices to find the optimal facility layout. These methods can be very ad hoc and difficult to apply to larger problems. One example is Pearson [#55, 1998] who developed a spreadsheet model to evaluate the arrangement of departments in retail stores using empirical methods to create the layout.

With theoretical methods, mathematical modeling techniques are commonly applied. In this research, one aspect of the retail store layout problem is modeled as a knapsack problem. For general knapsack problems, exact algorithms have been developed, such as branch-and-bound [#38, Hadjiconstantinou 1995]. Most knapsack problems can be solved by using dynamic programming in pseudopolynomial time. For more complex knapsack problems, heuristic search methods have been applied successfully [#31, Gendreau, 2004]. The heuristic algorithms that researchers are most interested in are genetic algorithms, simulated annealing, ant algorithms and other metaheuristic search methods [#13, #14, Burke, 1999, 2000]. Often the use of heuristics is motivated by the difficulty in solving models to optimality. For example, the layout model of Yapicioglu [#67, 2008] has nonlinear constraints to model the aspect ratios of the department shapes. The resulting nonlinear model is very difficult to solve and therefore some assumptions including fixing the departmental areas and ignoring the area of the aisles have been applied. However, even with these simplifying assumptions, the formulation is still nonlinear. The experimentation indicated that a provably optimal solution of the nonlinear model could not be found for a 16 department problem within 12 hours. Therefore, a constructive heuristic and a tabu search are applied to solve the problem [#67, Yapicioglu, 2008].

3.2 DATA COLLECTION

To apply a mathematical model it is necessary to gather the data that will be used in the model. This section describes some of the data that was collected for the retail layout problem.

3.2.1 Retail store attributes

When observing retail stores in a shopping center or shopping mall, one can notice that there are stores that represent the different classifications of traditional retail store layouts, i.e. grid, free form, racetrack and serpentine. The grid form is usually found in grocery stores, and stores such as Wal-Mart and Kmart. People can figure out the layout by following signage and learning the locations of the different functional areas. It is often relatively easy to find the departments because they are spacious and standard. The free form layout can be found in smaller fashion stores. Usually a map is not needed to walk around, since the themes and categories are limited. The serpentine form is a special case of the grid form, which has few entrances and exits, which is rarely seen in retail stores. The last one, the racetrack layout, is commonly found in department stores such as the anchor stores in shopping malls or in free standing department stores.

Since this research focuses on the distribution of departments in retail stores, several visits were made to local Kohl's, JCPenney's, Macy's and Sears to collect store data. Here are some example maps and directories from the stores that were visited.

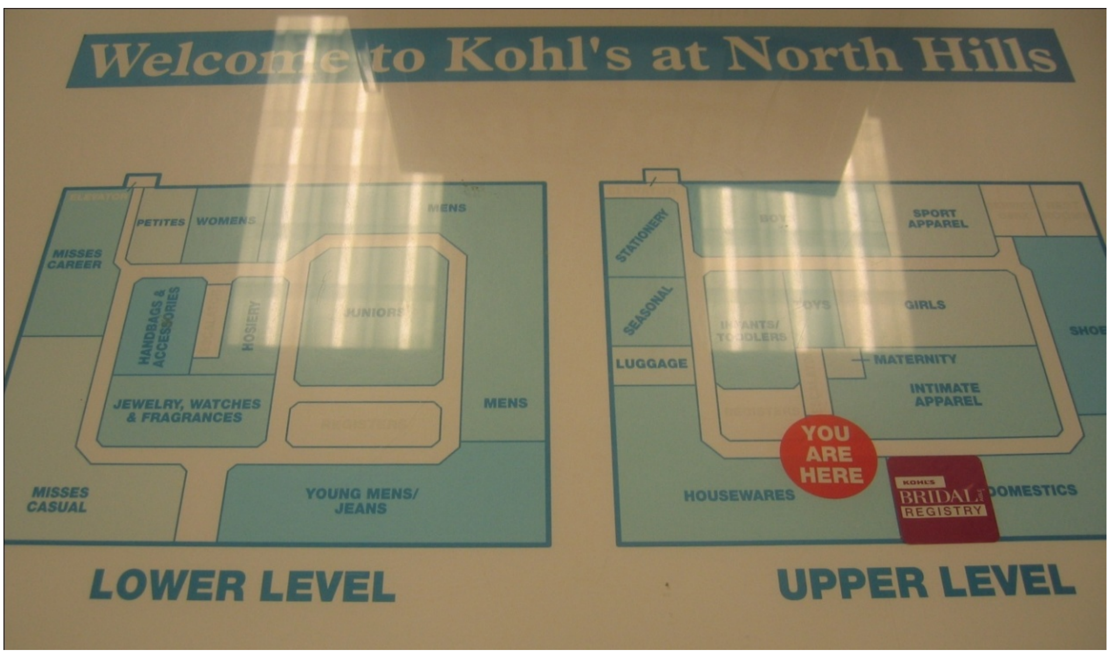


Figure 7. Kohl's department store in the North Hills section of Pittsburgh

Figure 8. JCPenney department store No. 1212, Pittsburgh

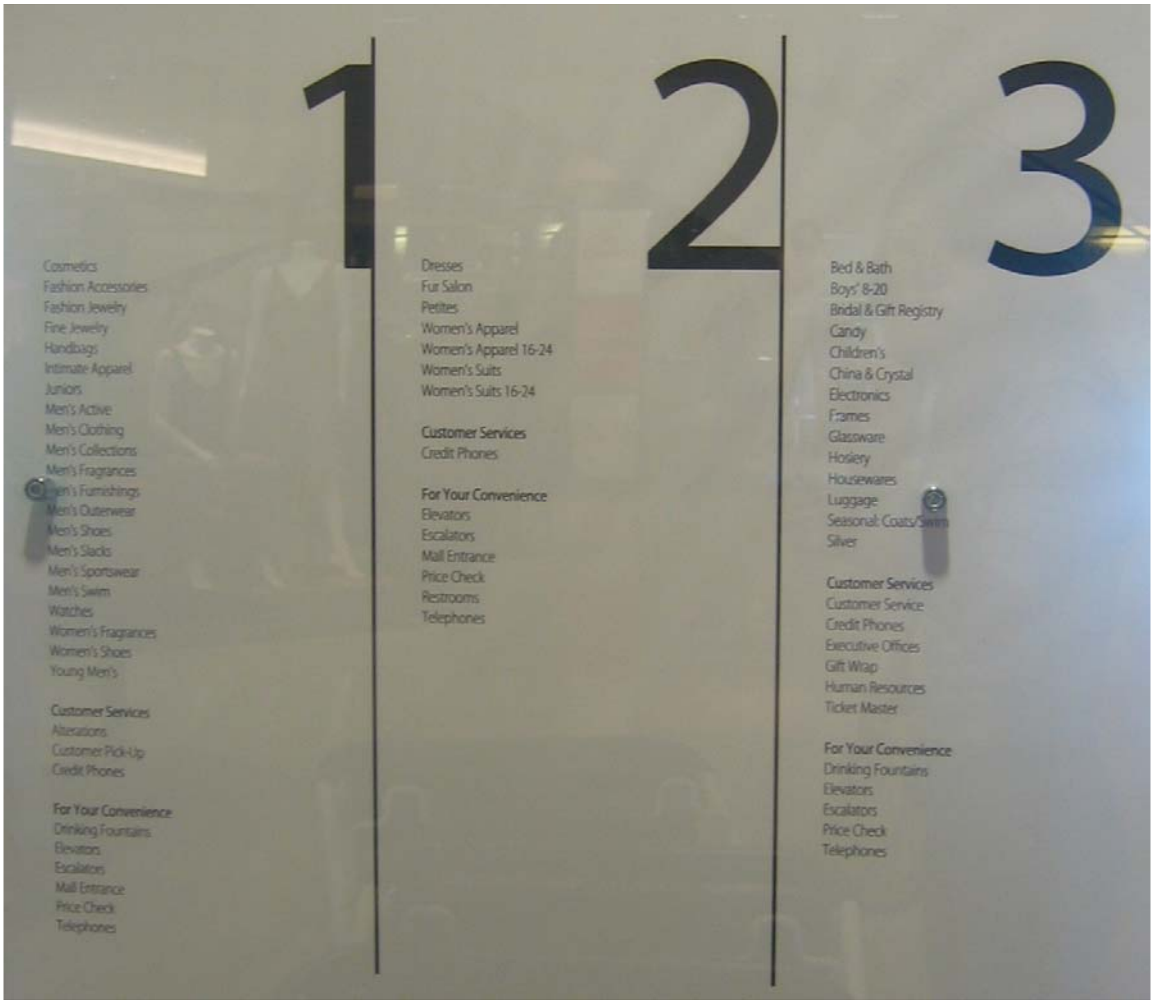


Figure 9. Macy's department store in Ross Park Mall, Pittsburgh

Figure 10. Sears department store in Ross Park Mall, Pittsburgh

Interestingly, the site visits showed that most of the department stores don't have maps like Kohl's has in its chain stores. The racetrack aisles are very easy to notice and follow in Kohl's stores. Other stores like Sears also have main racetrack aisles, however because of the larger retail area, branch aisles are also found in the stores.

3.2.2 Data collecting

It is easy to get a merchandise list in online retail stores. Usually internet retail stores and traditional local stores share many characteristics. For online retail stores, different categories of merchandise are listed under the icon of different groups like “beauty” “women”, “men”, “home” and “electronics”. For physical stores, the division of the retail area into different departments is often not as easy to follow as it is for online stores. Usually signs over the aisle or directions at the entrances are useful but are not always fully effective. Since this study is targeted on physical stores, real departmental layouts and department grouping data are collected from real stores as model input, and online merchandise grouping data is collected for reference purposes.

To get the store layout as well as the merchandise groups used in this research, data from over 16 local stores, including 5 Kohl’s, 5 JCPenney’s, 5 Macys and 1 Sears store was collected. From all those department stores, information on the grouping of departments and division of departments was collected. At a few of these stores, additional data was collected such as the locations, categories and areas of departments, entrances / exits, aisle structures of the stores, and traffic patterns of customers. By collecting information on the categories of departments, a common list of merchandise groups was determined and used as input for the models. By analyzing the department layouts, a department preference matrix was created. Department adjacency preference data could not be found in the literature. By measuring the real store sizes of departments, the nominal areas of the departments were found. A previous survey by POPAI [1963] was used to get the data for sales profit and exposure sensitivity of different departments.

To study customer behavior, an observation table to record the behavior of customers according to their gender and the number of customers in the shopping groups was designed. The data collection form is in Appendix A. The data is collected by individual observers and they follow an observation methodology to insure the data are collected in a standard manner. The customer observation methodology can be summarized as follows:

- Each store included in the study was observed simultaneously from all entrances. That is, the number of observers was equal to the number of entrances of the store.
- Each observation period lasted two hours.
- Male shoppers and female shoppers were recorded separately. Incoming and outgoing shoppers were recorded. In addition, the first direction of movement of the incoming shoppers was also recorded (e.g. left, center, or right).
- To identify whether the flow of customers through the different entrances at different time periods differed, at least two different observation sessions at each store were performed. The time intervals of these sessions did not overlap.

3.3 MATHEMATICAL MODELING

In the research, we use mathematical models to describe the optimization problems occurring in department store layout design. We modeled the problem as multiple knapsack problems and quadratic knapsack problems.

3.3.1 Multiple knapsack problem

In the multiple knapsack problem, instead of choosing from *n* items what items to place into one knapsack, it is choosing which of *n* items to place into *m* knapsacks with capacities W_i, to maximize the total value without violating the capacity limits of those knapsacks. The multiple knapsack problem is usually modeled as followed:

$$\max \sum_{i=1}^{m}\sum_{j=1}^{n} p_j x_{ij}$$

$$s.t. \quad \sum_{j=1}^{n} w_j x_{ij} \le W_i, \ \forall 1 \le i \le m$$

$$\sum_{i=1}^{m} x_{ij} = 1, \qquad \forall 1 \le j \le n$$

$$x_{ij} \in \{0,1\}, \ \ 1 \le j \le n \ and \ 1 \le i \le m$$

Similarly, in our problem, we have *n* departments and *m* knapsacks with area capacities S_i. For each department *j* we have the department profit p_j, and then we need to put those departments into these knapsacks to increase the total revenue.

We then add more considerations into this model. Since the locations of the knapsacks are different, the exposures of the knapsacks are not the same. However, the sensitivity of exposure of a certain department depends on the characteristics of the merchandise, therefore the profit of department *j* in different knapsacks varies. We take the exposure into consideration in determining the objective value by modifying the coefficient of x_{ij}.

Furthermore, the weight, or here, the area of department i can be changeable as well. Since the area of a department typically lies within a range of values, instead of being a fixed number, a constraint allowing department area to go up or down by 20% is formulated in the model to accommodate the changing area of departments.

3.3.2 Quadratic knapsack problem

The quadratic knapsack problem represents the following situation: among n items, some pairs of items benefit from having both items chosen for the same knapsack. The problem can be modeled as follows:.

$$\max \sum_{j=1}^{n} p_j x_j + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} p_{ij} x_i x_j$$

$$s.t. \quad \sum_{j=1}^{n} w_j x_j \leq W, \ \forall 1 \leq j \leq n$$

$$x_j \in \{0,1\}, 1 \leq j \leq n$$

In real department stores, impulse purchases happen and have been extensively studied. They include reminder purchases, such as putting binder splitters close to binders, comparison purchases, which is putting similar merchandise close together so consumers can make comparisons in order to encourage purchases, and grouping purchases, which could be affected by the clustering of merchandise groups. Retail stores exhibit adjacency preferences or grouping principles among departments. Although different chain stores have unique brands and classes of merchandise, the layout of departments should be reasonable and consistent with customer knowledge.

To properly consider adjacency, the necessary information for the adjacency preference matrix from the data collected in ten different department stores was gathered. Then we another term is placed in the objective function which considers the additional benefit of placing department *i* and department *j* close together. However it is a quadratic multiple knapsack problem, which is even harder to solve than the multiple knapsack or quadratic knapsack problem; therefore, it is necessary to apply some modeling techniques to make the resulting problem easier to solve.

3.3.3 Modeling techniques

The modeling techniques, used in this research, include

- Linearizing the quadratic models,
- RLT-reformulation linearization techniques,
- Reducing the number of integral variables,
- Relaxing integral constraints,
- Breaking symmetries in the model to reduce solution time,

These modeling techniques are introduced in detail when the mathematic model is introduced in Chapter 4.

4.0 FACILITY LAYOUT IN DEPARTMENT STORES

In this research, the facility layout problem in the retail sector is modeled as four sub-problems which are then solved sequentially. Several of these problems have a knapsack structure including multiple-knapsack and quadratic-knapsack problems with variable department area, and various profit functions based on considerations of adjacency and exposure.

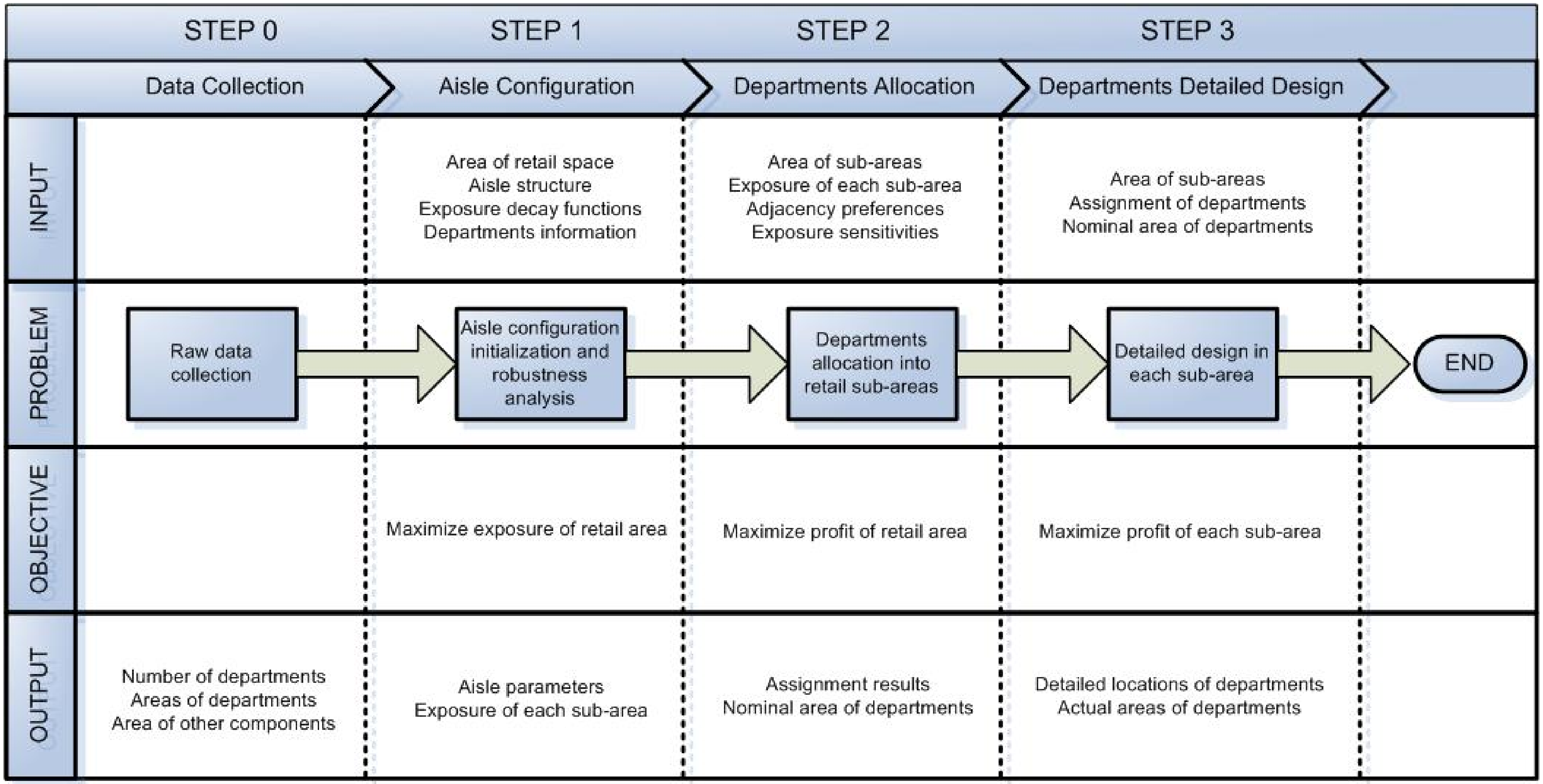


Figure 11. The framework of this research

Figure 11 shows the research framework and the connection of those sub-problems, where the input, main model, objective and output for each sub-problem are presented.

The initial step to design or optimize a layout for a retail store, which is shown in the figure as "Step 0", is to collect data. For example, the type of retail store, (grocery store, department store or fashion store), the number of departments, the size of the retail space and retail area, the area of departments and other components. The type of store determines many facets of the aisle structure and basic environment, and the number of departments and size of the retail area can affect the choice of aisle structure too. Clarification on the precise meaning of some terms in our context is as follows.

Retail space, in this research, refers to the whole space in the building, which includes retail area, aisle area, and area for other facilities (restrooms, etc.). Retail area includes the space for merchandise displays and sales (including cashier positions) with aisle area included. Aisle area includes central aisles and branch aisles, which divide the entire retail area into a number of retail sub-areas.

After collecting the raw data, in step 1, the aisle configuration is initialized. The objective of this step is to optimize the sum of the exposure within the entire retail area. The term exposure represents customer attention to a spot in the retail area and greater exposure leads to greater sales. Each spot in the store has an exposure value according to the distance from this point to the aisles. Points closer to aisles obtain more exposure. Points farther from aisles receive a lower exposure value. We define the proportion of full exposure obtained by a given point in the retail area as its decay function value. Decay functions are used to describe exposure as the distance from the aisle increases.

More discussion on this is in Section 4.1. Based on the collected data, the aisle structure chosen, (such as grid, freeform or racetrack) and the exposure decay function (linear, exponential or piecewise linear), the aisle configuration is initialized to maximize the exposure of the entire retail area. Then one can get the aisle parameters, including the width of the aisles, the location of each aisle, as well as the exposure distribution of each knapsack comprising the retail area.

Step 2 involves the assignment of the departments. With the aisle information obtained in the previous step, the entire retail area is divided into several sub-areas by the aisles. These sub-areas are treated as separate knapsacks, and the departments as items to be assigned to the knapsacks. During the assignment, adjacency avoidance and preference between each pair of departments are considered. When two departments are assigned to the same knapsack, the adjacency between the two departments is referred to as primary adjacency. When two departments are assigned to adjacent knapsacks, the adjacency between the two departments is secondary adjacency. If the adjacency preference of two departments is positive, they should be allocated as close as possible. If the adjacency preference of two departments is negative, it is important to avoid placing the two departments close to each other. Also, the exposure and profit maximization of the entire store is considered. This model distinguishes the merchandise as high-profit-low-exposure-sensitive, high-profit-high-exposure-sensitive, low-profit-low-exposure-sensitive, and low-profit-high-exposure-sensitive, and then maximizes both the exposure and the profit of all of the departments assigned in the knapsacks without violating the area constraints.

The next step is the detailed design of the department layout in each sub-area. A strip model is used to solve this problem. The input is detailed information about the departments, exposure of the retail area and the relative locations of each department in the entire store. The objective is to maximize the profit of each department and each sub-area to maximize the profit of the entire retail area.

4.1 AISLE CONFIGURATION PROBLEM

In this section, given a fixed retail area, the aisle structure is designed to maximize the exposure of the entire retail area. Depending on the specific environment and store scale, several alternative aisle structures are considered as shown in Figure 12. This research focuses on variations of a symmetric rectangular racetrack aisle configuration, though some other racetrack aisle shapes are shown in the figure.

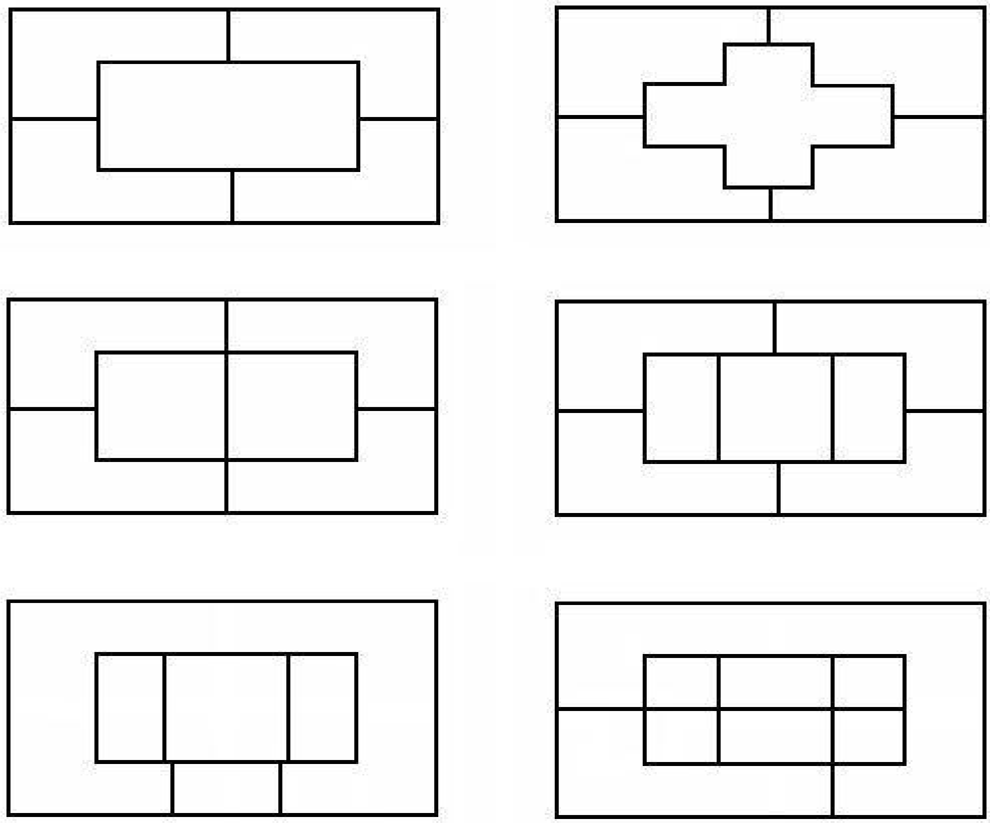


Figure 12. Retail area aisle structures

4.1.1 Introduction

As mentioned in the previous chapter, Kohl's department stores have accredited much of their profit success to their racetrack layout (the first aisle structure in Figure 12), which is one of the simplest aisle structures since it does not contain numerous vertical aisles as is the case in many department stores. For the purposes of illustration, this simplest race track layout is used to demonstrate the definition of exposure, the calculation of exposure, and the optimization objective of the design. Figure 13 gives more detailed information about the entire retail area, including a rectangular racetrack aisle and four entrance aisles. Note that the departments will be sequenced through the central sub-area (referred to as the inner bay) and along the perimeter, four sub-areas (referred to as the four outer bays).

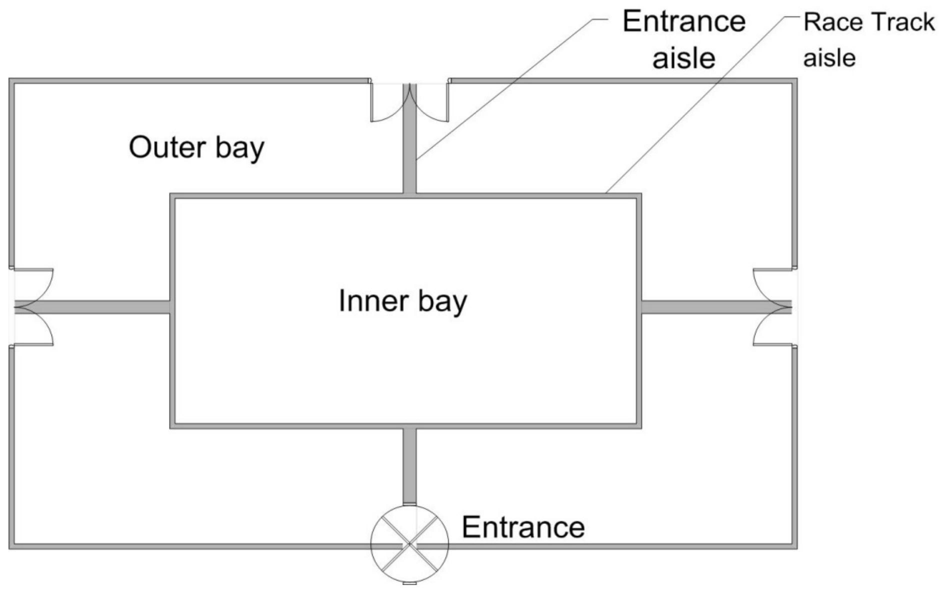


Figure 13. An example retail store

The parameter definitions are shown in Figure 14. The aisles divide the whole area into several sub-areas, which represent knapsacks in the following models. S_j is used to denote the nominal area of knapsack j. There are three types of aisles in the area. The racetrack aisle is located in the middle of the entire area. The aisle which connects the main entrance and the central racetrack aisle is defined as the main entrance aisle. The aisles connecting the branch entrances and the central racetrack are defined as the branch entrance aisles.

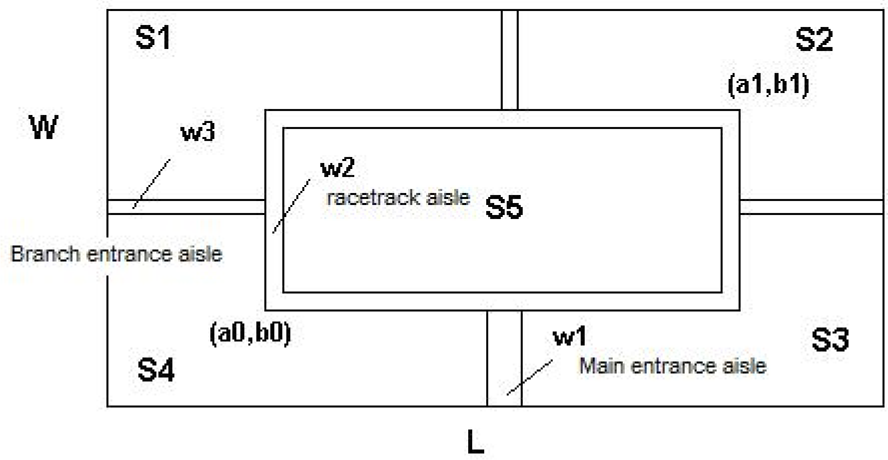


Figure 14. Illustration of the retail store parameters

4.1.2 Definition of Exposure

In the science of shopping, researchers believe the dynamics of shopping work by way of seeing, feeling, touching, and then buying. [#62, Underhill, 2000]. Retailers try their best to make retail environments user-friendly, in order to maximize customer purchasing. Paco Underhill, the founder, CEO and president of Envirosell, is concerned with understanding what motivates consumers to purchase in a marketplace. Their proprietary methodology utilizes small video cameras, qualitative observation techniques, mapping programs and attitudinal interviews to capture consumer behavior patterns. One of their recommendations for store designers is to maximize customers' view of the merchandise by chevroning. Chevroning involves "placing shelves or racks at an angle" instead of shelves being placed at the traditional ninety-degree angle to the aisle [#62, Underhill, 2000]. The logic behind the recommendation is that the more merchandise customers' see, the more merchandise they purchase.

Another study claims that shopping is an ongoing process rather than a predetermined activity. Appropriate and imaginative displays are very important in developing customer enjoyment of the retail process, in supporting the interaction with the atmosphere of stores and in enhancing the consumer experience. The first function, which is also the most important function, of display is to show customers what retailers have to offer. The retailer should use display equipment to enable the customer to see as much as possible of the available stock [#52, Newman, 2001].

An important question to answer is to what degree consumers can see products in a store. It is well known that how customers walk and where their eyes' sightlines go can determine to a great degree what they see. For example, if customers can't see a display from a distance---say ten or twenty feet---then they won't readily approach it. Customers need to walk off of the aisle to reach merchandise in the back. Also it is hard to get customers all the way to the back of stores. Figure 15 shows the depth at which customers walk into stores in percentages. The figure was provided by the Envirosell Consulting Company, which is an innovator in commercial research for shopping environments. As one of their research results, Figure 15 shows that 100 percent of customers go into the first quarter of stores, 83 percent of customers go into the second quarter of a store, 57 percent go up to three quarters of the way in and only 14 percent reach the back of a store. A key takeaway from this figure is that there is more traffic in the front of a store and less traffic in the back of a store if the entrance is only on the front side of the store.

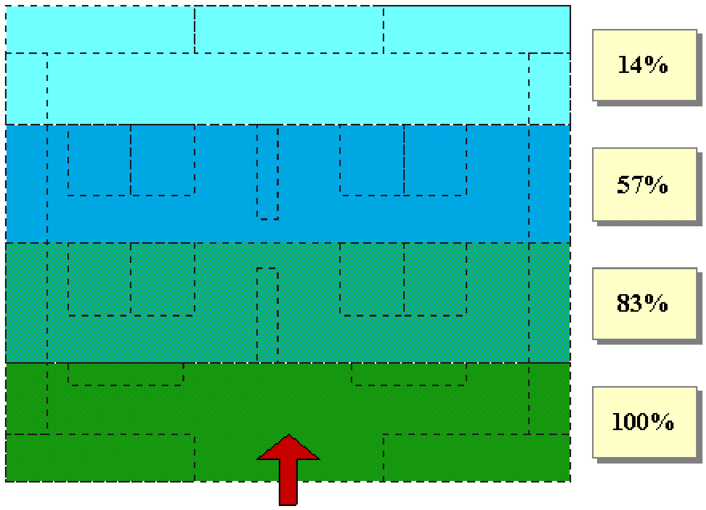


Figure 15. The depth customers walk into stores.

On the other hand, aisle structures can affect the way people walk through the retail area. Data show that the race track aisle can lead more people to the back wall of the store and therefore show more of the retail area to customers than other aisle structures. Figure 16, which is also one of the research results from Envirosell, shows the different densities of customer visits for various locations in a retail store with a loop aisle and central aisles. In the figure, one can see a higher density of customer visits in the front of the store. However, interestingly, there is a high exposure zone in the back. One explanation might be because of the racetrack aisles and the central aisles, more customers are lead to the back of the store. This figure confirms Figure 15 that in general the exposure of merchandise drops as the distance from the entrance increases.

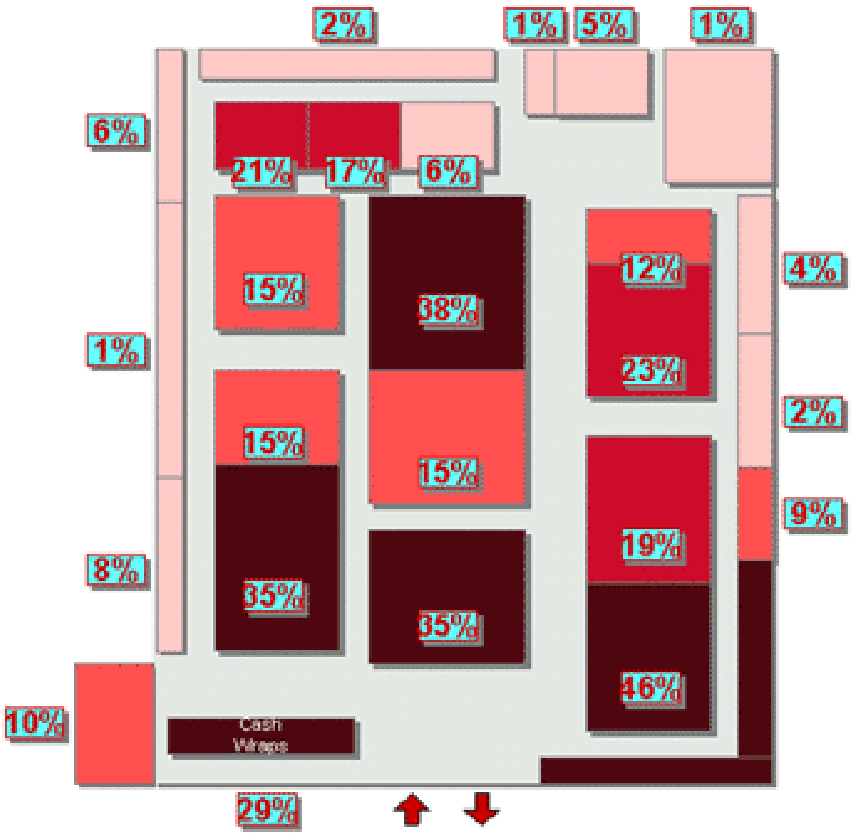


Figure 16. Hot spots in a retail store

4.1.2.1 Exposure decay function

The concept of exposure is used to quantify the extent to which merchandise is visible to customers. An exposure decay function *f(g)* is defined to quantify the extent of merchandise visibility for different store locations. Assume that portions of a department that are farther from the aisle receive a lower exposure value than those that are closer to the aisle (refer to this as exposure decay). Exposure falls as one moves farther from the aisle until a threshold distance, d, is met. Any points that are farther from the aisle than d have a constant exposure value of r, which is a fraction of the exposure that is obtained when one is directly on the aisle. The idea is that once an item is d units away from the aisle, further distance has little effect. A customer must be seeking this item in order to be exposed to it. The assumption in this case is that customers will travel as far as necessary to find an item. The rough relationship between exposure and distance is shown in Figure 15, in which darker colors indicate higher exposure. Furthermore, the exposure of a point in the retail area also depends on the aisle types of aisles that are close to it. For example, there is a larger customer flow in main aisles than in secondary aisles. (A main aisle might be one connecting a store and a shopping mall and secondary aisles might be ones that connect a store to its parking lots.)

According to Figure 15, the exposure of merchandise decreases as the distance between the merchandise and customers increases. The exposure decay curve is modeled by using the values and pattern shown in Figure 15. The possible exposure could be from 1 down to r, which denotes the lowest value of retained exposure.

No previous literature was found to quantify the exposure in this context, so different decay functions were fit to the data from Figure 15 representing both rapid exposure drops and slow exposure drops. Figure 17 shows the relationship between the exposure and the distance using 1) a linear decay function, 2) an exponential curve with slower decay(exponential decay 1), 3) an exponential curve with quicker decay (exponential decay 2), 4) a flat stage decay curve, 5) a piecewise linear decay and 6) a polynomial decay curve. In the next subsections, we will discuss these six decay functions in detail.

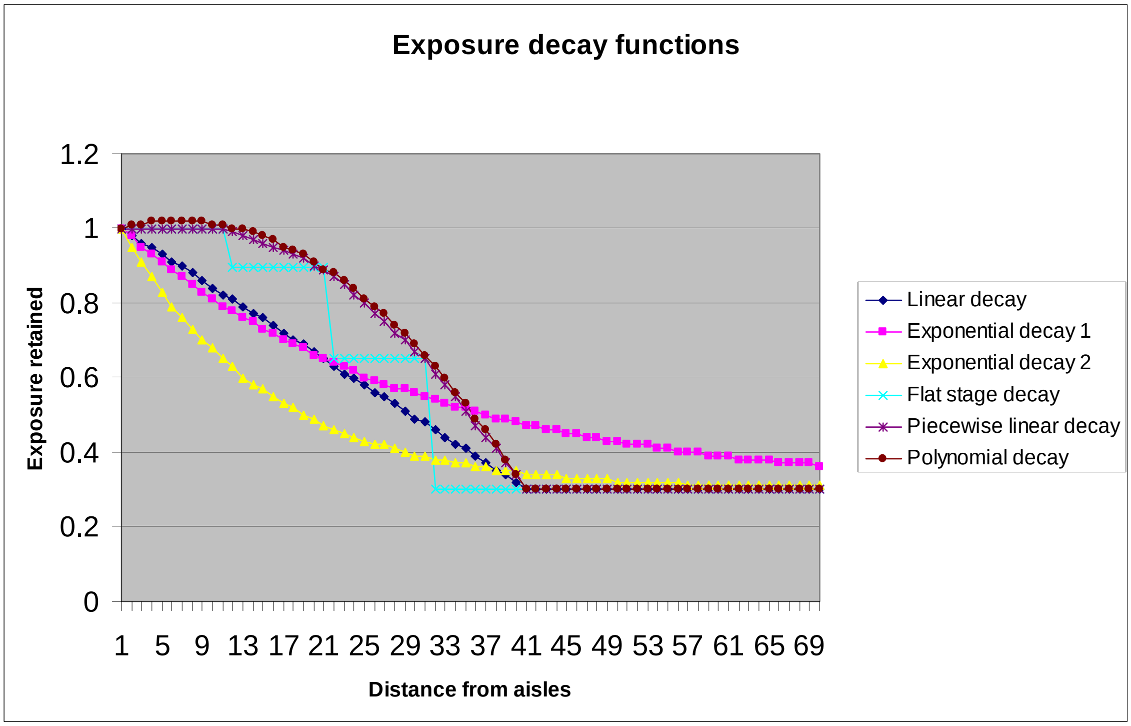


Figure 17. Different exposure decay functions

Linear exposure decay function

A linear decay function can be used to simulate the loss of exposure based on distances in the retail area. Since it is linear, the decay function is fixed when r and d are fixed, as shown in Equation (1).

$$f(g) = \begin{cases} -\dfrac{1-r}{d} g + 1, & g \le d \\ r, & g > d \end{cases} \quad (1)$$

An example is shown in Figure 17 where it is assumed that there is a 50 percent loss of total exposure at the threshold distance d, which is 50 feet. Thus, in this example, a point in the retail area which is 50 feet or more away from the aisle obtains half of the exposure of a point directly on the aisle. For this example, Equation (1) becomes:

$$f(g)=\begin{cases}-\dfrac{1-0.5}{50}g+1, & g\le 50\\ 0.5, & g>50\end{cases} \tag{2}$$

Exponential exposure decay function 1

An exponential decay function is also considered, which is shown in (3)

$$f(g)=(1-r)e^{-kg}+r \tag{3}$$

Where *r* is the retained exposure, and *k* is a control parameter, which is calculated based on the assumption of exposure loss. *k* controls the rate of decay. For example, Figure 17shows the results for two different values of *k*. The first case assumes that 50 percent of the exposure is retained at an infinite distance, which means r equals 0.5, and 75 percent of the exposure is retained at one-fourth of the threshold distance *d,* which is set at 50 feet. In this case equation (3) becomes

$$f(g)=(1-0.5)e^{-kg}+0.5 \tag{4}$$

Where $e^{-k(0.25d)}=0.5$ i.e., $k=4\ln 2/d$.

Exponential exposure decay function 2 (slower exposure decay function)

The second case assumes that 75 percent of the exposure is retained at one half of the threshold distance *d*, meaning one half of 50 feet, resulting in equation (4), where $e^{-k(0.5d)}=0.5$ i.e., $k=2\ln 2/d$.

Flat stage decay function

This function decreases at roughly the same level over a small range of distances, then decreases to the next level. This type of decay function is implied by Figure 15, which shows a descending visiting rate of customers related to the depth of a location within the store. Given a distance threshold *d* and retained exposure *r*, we obtain the decay function shown in equation (5)

$$f(g)=\begin{cases}1 & g\le 0.25d\\ 1-\frac{0.13}{0.86}(1-r) & 0.25d<g\le 0.5d\\ 1-\frac{0.43}{0.86}(1-r) & 0.5d<g\le 0.75d\\ r & g>0.75d\end{cases} \tag{5}$$

Piecewise linear decay function

Similar to the decay function above, a piecewise linear curve can be used to model the relationship between the exposure and the distance. The decrease of exposure is not to a certain level dramatically, but slowly and constantly, which is more realistic. The two exposure levels of each pair of adjacent break points in Figure 15 were connected to get the following equation (6)

$$f(g)=\begin{cases}1 & g\le 0.25d\\ \frac{\frac{0.13}{0.86}(1-r)}{-\frac{1}{4}d}(x-\frac{1}{4}d)+1 & 0.25d<g\le 0.5d\\ \frac{\frac{0.3}{0.86}(1-r)}{-\frac{1}{4}d}(x-\frac{1}{2}d)+1-\frac{0.13}{0.86}(1-r) & 0.5d<g\le 0.75d\\ \frac{\frac{0.43}{0.86}(1-r)}{-\frac{1}{4}d}(x-\frac{3}{4}d)+1-\frac{0.43}{0.86}(1-r) & 0.75d<g\le d\\ r & g>d\end{cases} \tag{6}$$

Polynomial decay function

A shortcoming of equations (5) and (6) is having too many break points. Here a polynomial relationship which simplifies the two exposure decay functions is shown.

$$f(g)=\begin{cases}-\frac{(1-r)}{0.86}(\frac{1.2}{d^2}x^2-\frac{0.34}{d}x)+1 & g\le d\\ r & g>d\end{cases} \tag{7}$$

There has not been any previous research done about how to quantify the exposure concept and therefore different decay functions are used to describe the relationship between exposure and distances from a point. The robustness of the different decay functions are determined for test problems.

4.1.2.2 Robustness of the optimal solution

It is difficult to choose an exposure decay function because there is no quantitative analysis of this in the literature. A key criteria used to evaluate the different exposure decay functions was to see if the solutions found by some methods were more robust than those found by other methods. Thus, robustness analysis is used to identify the best aisle configuration. Three commonly used robustness analysis methods are now described.

1). Absolute Robustness: the performance measure is used to evaluate the performance across all the scenarios. The objective is to minimize the maximum of the possible outcomes. In the proposed model, each potential solution is evaluated across all six different decay functions, and then the summations of the six objective function values are compared among all potential solutions.

2). Robust Deviation: the performance measure is used to evaluate the deviation of a single decision against the best possible decision for that scenario, and the maximum deviation is recorded for all scenarios. Then the smallest maximum deviation is considered the most robust.

3). Relative Robustness: similar to Robust Deviation, instead of recording the deviation of the current decision from the best possible decision, the percentage deviation from the optimal performance in a scenario is recorded.

By comparing with the actual location in the real store, we can also test the parameter settings of exposure decay function with the limited information from the literature.

Since there is uncertainty in the exposure decay function, six decay functions are developed to determine the optimal aisle location. Each decay function with different parameters is treated as a specific scenario, and then all the possible solutions are evaluated under all scenarios. By applying robustness analysis, one most robust solution will be picked up and the corresponding decay function will be used in the next three steps.

4.1.2.3 Effect of entrance, racetrack aisle and central aisle, etc

Besides the effect of the distance from the aisles, the entrances and the type of aisles around a location can also affect the exposure of a certain location in the retail area.

As people walk into stores, their eyes adjust to the retail environment and they slow down their pace and begin to look around for merchandise attractive to them. Usually merchandise facing the entrance aisle gains more attention than the displays along the entrance aisle. For example, as consumers enter stores, the first impression of the store is usually obtained from the display facing the entrances, instead of the display on the two sides of the entrance aisle. After looking at the facing display, consumers shift their attention to the side displays. Furthermore, a study has shown that every store has a transition zone or area where customers begin to slow their pace and make the transition from being outside of the store to being inside [#62, Underhill, 2000]. The author, Paco Underhill, refers to this area as “the shoppers’ landing strip”. What is important about the transition zone is whatever is located in the zone is pretty much lost on customers. If there is merchandise displayed in this area, customers are unlikely to see it.

The racetrack aisle can lead customers to go through the entire retail area. Most of the traffic is centered on the racetrack aisle and traffic then fans out to central aisles or entrance aisles. Central aisles are necessary to manage large retail spaces. They can lead to more exposure but sacrifice some of the retail area. Central aisles can also divide large spaces into several different departments to help guide customers through these sub-areas. Without central aisles, customers would feel lost in an endless display of merchandise and tire of paying attention to the huge amount of display racks.

4.1.2.4 Consideration of fixtures and customer service areas

All stores have non-profit generating areas supporting sales within their retail space. The areas for backroom storage and window displays are excluded in the proposed models. There are also other fixtures, such as escalators, elevators, fitting rooms, and customer service areas, rest rooms, and customer services desks that are not considered.

The areas listed above don't create profit and have to occupy some percentage of the selling space; however they can affect the aisle structure, the traffic pattern of customers and the display of merchandise. In large department stores, checkout counters might be spread over the retail space to save time and provide convenience to customers checking out. On the other hand, in small department stores, the cashiers are located by the entrances to check people out on their way out of the store.

In the case studies considered, since Kohl's and Sears are both one-floor retail stores, no spaces for escalators and elevators are considered. Cashiers are taken into consideration since they occupy large spaces and have to be near the entrances, which are high exposure areas.

4.1.3 Modeling the aisle configuration problem

The retail space is considered a discrete environment, which means the entire facility is represented using grids. The grids in the retail area are distinguished from the grids in the aisles, and the grids for the fixtures and customer services. Only the grids in the retail area have exposure values. The grids in the aisles and fixtures occupy space and are not counted in the calculation of exposure.

The way that the exposure value of a retail grid is calculated is shown in Figure 18. A retail grid is in a certain knapsack, which is enclosed by several aisles, such as entrance aisles, branch aisles, racetrack aisle and central aisles. There are several ways for one retail grid, for example, point A to obtain exposure, shown in Figure 18: 1) get exposure from the single closest aisle grid, in this example, it is *d1* for point A; 2) get exposure from the multiple aisle grids around that retail grid, in this example, *d1, d2, d3, d4* and *d5* for Point A; 3) get exposure from the two closest aisle grids considering the effect of entrance aisles, in this case, *d1* and *d2* should be considered; 4) from all aisle grids in the retail space, that is point A can obtain exposure from every single direction. Tests were done on the four options and the third option is considered the most representative, which gives a continuous contour illustration for exposure distribution within the entire area. Thus, the final exposure value for a grid in the retail area is the sum of the exposure from the two closest aisle grids. The exposure value of a retail grid from an aisle grid is calculated by the exposure decay function, which depends on the distance between the two grids.

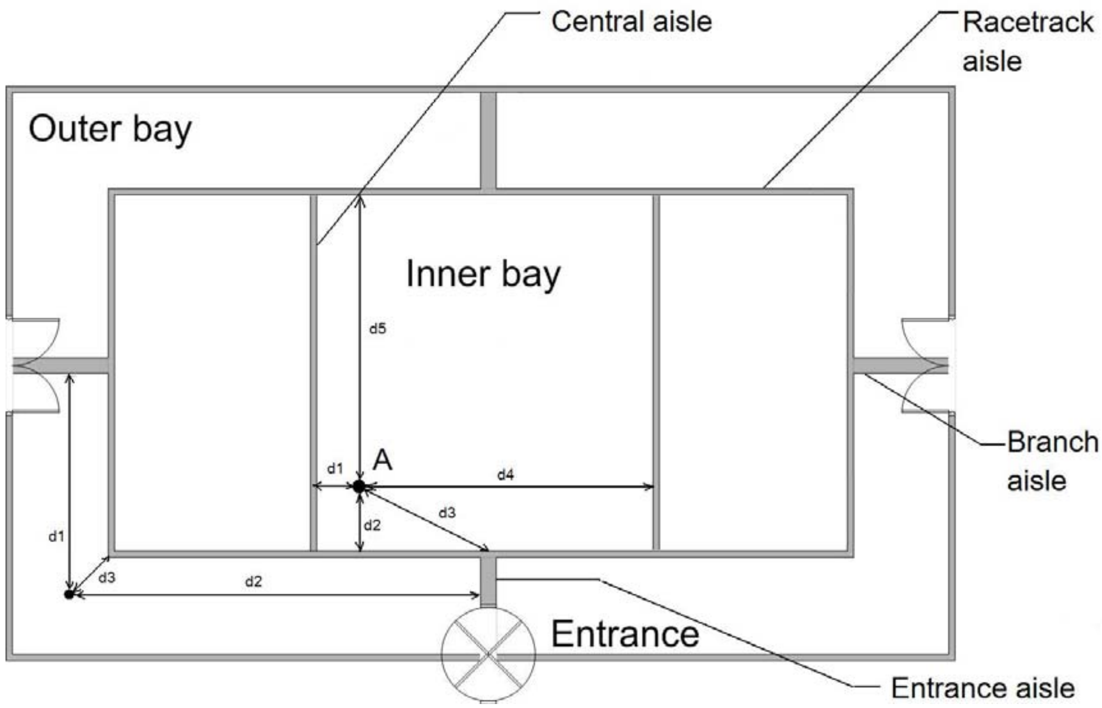


Figure 18. The illustration of exposure accumulation methods

Figure 18 shows the analytic version of the example retail area. The aisles divide the whole area into several sub-areas, which represent knapsacks in the following models. Recall that S_j is used to denote the nominal area of knapsack j. There are three types of aisles. The racetrack aisle is located in the middle of the entire area. The aisle connecting the main entrance and the racetrack aisle is defined as the main entrance aisle. The aisles connecting the branch entrances and the racetrack aisle are referred to as branch entrance aisles. One can also add vertical or horizontal central aisles if necessary. Some notation is introduced before formulating the problem.

Decision variables:

a_0 x coordinate of the lower left corner of the racetrack aisle.

b_0 y coordinate of the lower left corner of the racetrack aisle.

Parameters:

X The x coordinate of the point whose exposure is calculated.

Y The y coordinate of the point whose exposure is calculated.

L: The length of the entire retail area.

W: The width of the entire retail area.

w_1: The width of the main entrance aisle

w_2: The width of the racetrack aisle

w_3: The width of the branch entrance aisle

β_i : The weight factor of the nearest aisle i

$f(g)$: The exposure decay function in terms of distance g

$g(x,y)$: The shortest rectilinear distance from the point (x,y) to any aisle.

r: Percent of exposure retained at infinite distance for the exponential decay function or at the distance threshold for the linear decay

d: The distance threshold in both decay functions.

Assume that the aisle structure is symmetric in the retail area. The objective is to maximize the exposure of the entire retail area by optimizing the aisle location.

$$\max_{(a_0,b_0)\in \text{retail area}} \{ \iint_{(x,y)\in retail\ area} \text{exposure}(g(x,y))dxdy \} \tag{8}$$

β_i, is the weight factor of aisle i. It is determined by the width of the nearest aisle from point (x, y). It is assumed that a wider aisle brings more customers to the adjacent area. Initially, a linear equation (9) is used to express the relationship between β_i and w_i. However, a linear function is not realistic because there should be a lower bound on the aisle width based on practical considerations and an infinitely wide aisle can't bring in an infinite number of customers. For these reasons, a nonlinear function is developed and shown in equation (10), and for the purpose of illustration, 5 feet is set as the lower limit of the aisle width.

$$\beta_i = pw_i + q \tag{9}$$

$$\beta_i = p * (1 - e^{-q(w_i - 5)}) + 1 \tag{10}$$

The exposure decay function *f(g)* was introduced in Section 4.1.2.1. *f(g)* equals 1 when a point is located directly on the aisle and it is between 0 and 1 when a point is located off of the aisle. *f(g)* is calculated using the six exposure decay functions. The exposure of a certain point *(x, y)* is calculated by using equation (11).

$$\text{exposure}(g(x, y)) = \beta_i \times f(g) \tag{11}$$

4.1.4 A case study: Kohl's department stores

In terms of classes of merchandise, department stores are often classified as upscale department stores, mid-range department stores, and discount department stores. A typical upscale department store usually has <u>multiple floors, a complicated aisle structure, and multiple checkout counters spread over the entire retail area</u>. Example upscale department stores include national chains like Saks Fifth Avenue, Nordstrom, Bloomingdales and Macy's. A typical mid-range department store, carrying more moderately-price brand names, usually has <u>one or two floors, a main loop aisle connecting the branch aisle and inside-loop aisle structures, and checkout counters near the entrances</u>. Example mid-range department stores include national chains like JCPenney, Sears and Target. However, Sears differs from most mid-range department store chains in its inclusion of departments for hardware, garden and outdoor equipment, and automotive service. Kohl's differs from other department stores in its centralized checkout counters. Kohl's has also pioneered the use of a "racetrack" aisle that circles the entire stores. The racetrack aisle structure is also used in some discount stores. Therefore, Kohl's is placed in a third class - discount stores having single floors with a racetrack aisle structure.

To study the optimal aisle location in a discount store with a racetrack as its main aisle structure, data was collected from Kohl's department stores. There are five Kohl's stores around Pittsburgh, and they have similar departments and racetrack aisle structures. In the following sections, the data collected from the stores is used to investigate the proposed design framework.

4.1.4.1 Background

Figure 19 is a picture taken in a store to show the floor plan to customers. From the picture, one can see a racetrack aisle and two vertical central aisles, two entrances, two registers and a customer service area. Since it is a one-floor store, there is no area reserved for elevators or escalators.

Figure 19. Layout of Kohl’s at West Mifflin, PA

The data collect from the store is used to make the floor plan. Two floor plans of Kohl’s stores are shown in Figure 20 and Figure 21, from which the similarities and differences between them are presented.

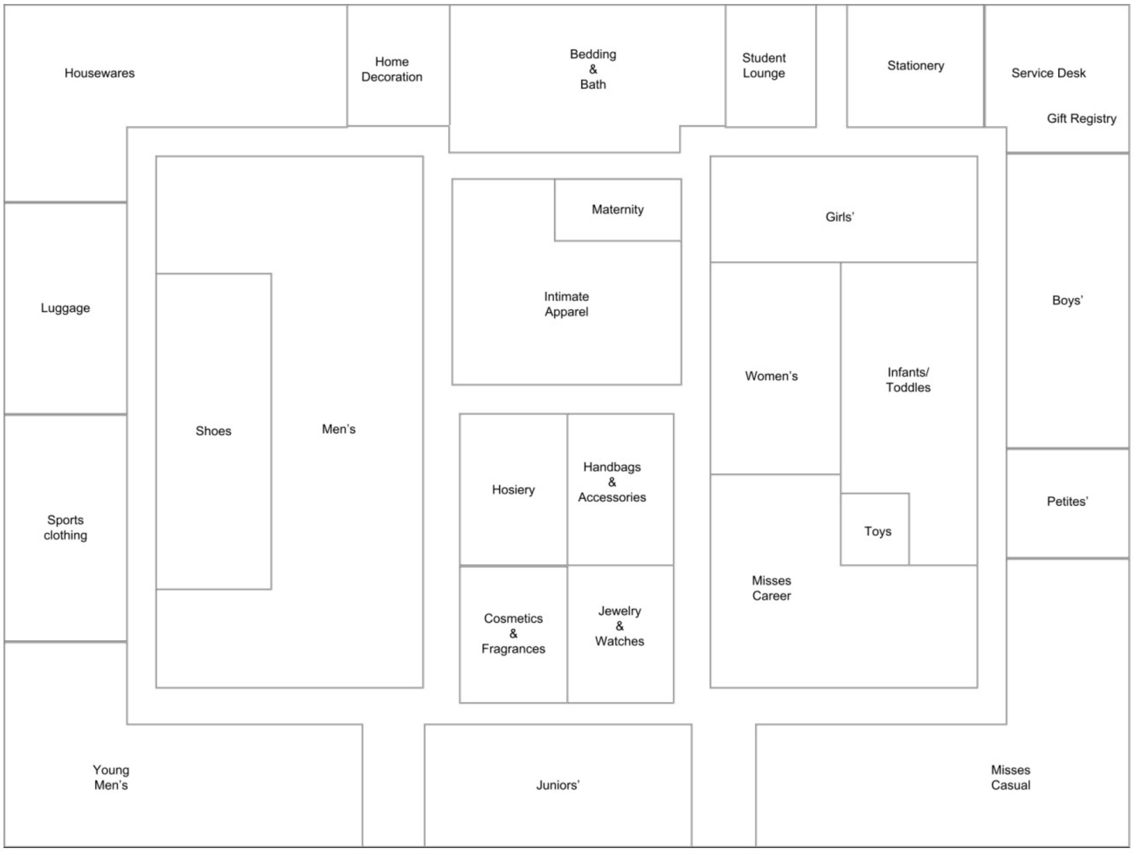


Figure 20. Measured Layout of Kohl's at West Mifflin, PA

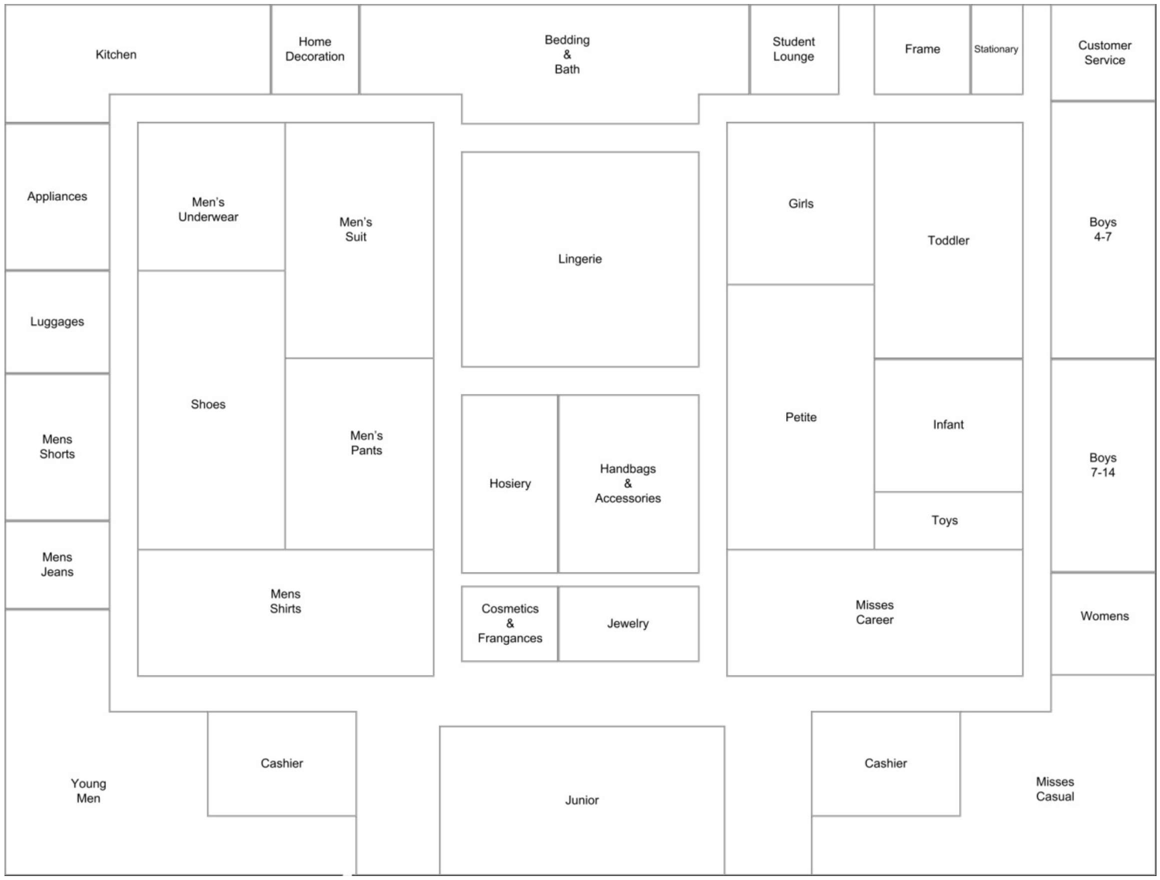


Figure 21. Measured Layout of Kohl's at Robinson Town Center, PA

Both stores have two entrances and two central aisles. Usually Kohl's stores are free standing stores, which are not attached to any other stores or malls. Thus, both entrances face the parking lot instead of having a mall entrance. The central aisles occupy potential retail space but increase the exposure of the retail area as a tradeoff. The cashiers are near the exits to facilitate the flow of customer traffic in the store. Customer services and the restrooms are located in the corner, where they don't take any of the best retail space, and require customers go to the back of the store for these services.

4.1.4.2 Model inputs

The first Kohl's store, which is in West Mifflin, Pittsburgh, is used as an example to illustrate the proposed retail layout modeling approach. The store has an area of 292 * 222 sq ft. The size of each checkout counter is 36 ft. * 26 ft. Six exposure decay functions are tested using this setting in order to find the optimal solution over all six possibilities.

In this example problem setting, the aisles widths are based on data collected in the West Mifflin Kohl's store. To be specific, the width of the main entrance aisle is 16 feet; the width of the racetrack aisle is 8 feet wide; no branch aisle is considered outside of the racetrack aisle. There are two central aisles within the area surrounded by the racetrack aisle, whose width is equal to 6 feet. Therefore the setting can be described as having

w_1 (the width of the main entrance aisle) = 16,

w_2 (the width of the racetrack aisle) = 8,

w_3 (the width of the branch aisle) = 0,

w_4 (the width of the horizontal central aisle) = 0,

w_5 (the width of the vertical central aisles) = 6.

As mentioned in Section 4.1.3, the objective function of this model is to maximize the total exposure of the retail area by searching all the possible locations of the central racetrack aisle. In the figures below two extreme cases, over squeezed and over expanded central racetrack aisles, are shown to compare different locations of the central racetrack aisles.

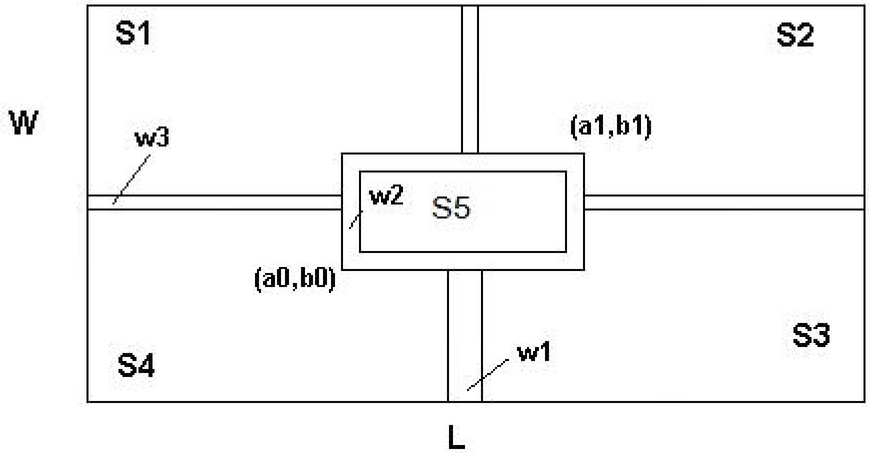


Figure 22. Over-squeezed racetrack aisle

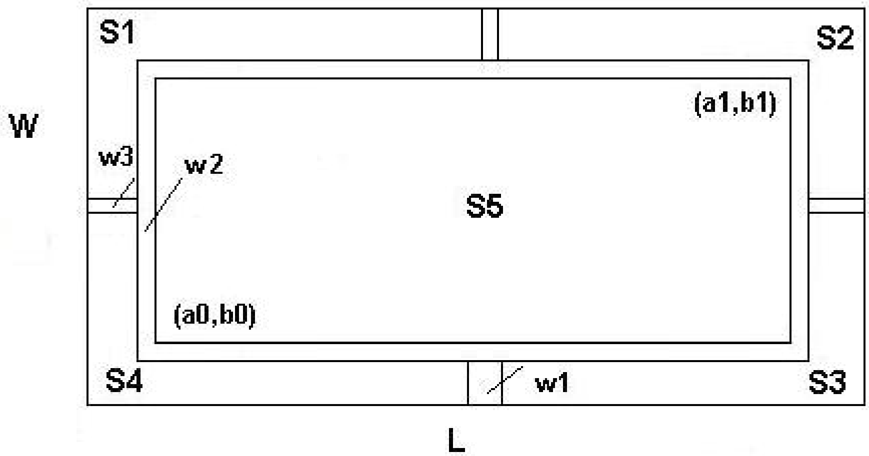


Figure 23. Over-expanded racetrack aisle

We assume that the central racetrack is symmetric. As discussed in Section 4.1.3, the exposure of each grid in the retail area is calculated by accumulating the effect of the two closest aisle grids (the second method listed). The six exposure decay functions are used for computing exposure for all possible locations of the central racetrack aisle. The optimized results of the central aisle location for the six exposure decay functions in each setting are shown in Section 4.1.4.3.

4.1.4.3 Optimized results

The optimized results are summarized in Table 1. A total of fifty-four combinations from having three threshold distance values, three different retained exposure values and six types of exposure decay functions are computed as shown in the table. The optimal locations for the lower, left corners of the racetrack are in a reasonable range for many of the combinations of parameters settings. The rows are shaded to show one robustness test group that share the same threshold distance and retained exposure values and only vary across the exposure decay functions.

Table 1. The optimal solution for the racetrack aisle location problem

Settings index	Setting of exposure decay function			Optimal solution		Objective
	Distance threshold	Retained exposure	Decay function	X coordinate	Y coordinate	Value of exposure
1	20	0.14	1	20	16	36878.80
2	20	0.14	2	28	18	43453.11
3	20	0.14	3	22	14	30222.64
4	20	0.14	4	20	14	41986.74
5	20	0.14	5	20	16	46538.51
6	20	0.14	6	20	16	47421.35
7	20	0.3	1	20	16	50688.14
8	20	0.3	2	32	20	56179.89
9	20	0.3	3	32	16	45331.61
10	20	0.3	4	20	16	54835.88
11	20	0.3	5	20	16	58550.69
12	20	0.3	6	20	18	59275.02
13	20	0.5	1	60	18	68272.1
14	20	0.5	2	40	24	72251.98
15	20	0.5	3	60	32	64782.61
16	20	0.5	4	56	16	71046.66
17	20	0.5	5	26	18	73605.45
18	20	0.5	6	20	18	74126.73

Table 1 (continued)

19	30	0.14	1	26	20	47288.17
20	30	0.14	2	30	20	53489.15
21	30	0.14	3	26	16	37257.89
22	30	0.14	4	22	22	54145.73
23	30	0.14	5	26	22	61203.71
24	30	0.14	6	26	22	62483.89
25	30	0.3	1	26	22	59286.94
26	30	0.3	2	32	22	64387.56
27	30	0.3	3	32	18	51102.35
28	30	0.3	4	22	22	64848.29
29	30	0.3	5	26	24	70640.73
30	30	0.3	6	26	24	71687.84
31	30	0.5	1	28	24	74346.6
32	30	0.5	2	38	26	78146.9
33	30	0.5	3	50	24	68658.53
34	30	0.5	4	22	22	78226.49
35	30	0.5	5	28	24	82481.14
36	30	0.5	6	28	24	83234.35
37	40	0.14	1	32	22	56646.18
38	40	0.14	2	32	20	61100.74
39	40	0.14	3	28	18	43453.11
40	40	0.14	4	30	20	65934.41
41	40	0.14	5	32	26	73995.8
42	40	0.14	6	32	26	75550.37
43	40	0.3	1	32	24	67008.59
44	40	0.3	2	34	22	70609.3
45	40	0.3	3	32	20	56179.89
46	40	0.3	4	30	26	74539.59
47	40	0.3	5	32	26	81166.07
48	40	0.3	6	32	28	82436.15
49	40	0.5	1	32	28	80006.13
50	40	0.5	2	40	26	82627.02
51	40	0.5	3	40	24	72251.98
52	40	0.5	4	30	30	85427.98
53	40	0.5	5	32	30	90172.62
54	40	0.5	6	32	28	91079.54

4.1.4.4 Robustness comparison

As mentioned in the previous section, there are many robustness analysis methods that can be used to test the robustness of a method or a solution. In the following tables, four robustness evaluations will be executed, 1) Absolute Robustness, 2) Robust Deviation, 3) Relative Robustness and 4) Average Solution. The robustness of the 54 optimal solutions are tested over 9 groups of settings (the three distance threshold values combined with the three exposure decay functions.).

Group One: The distance threshold equals 20, the retained exposure equals 0.14, and the exposure decay functions are from type 1 to type 6. The optimal solutions for the six exposure functions are shown in order 1 to 6 in the first column. For each solution, the objective values under the six scenarios are listed in each row. Obviously, the highest exposure values in every column are in the diagonal cells.

Table 2. The optimal solutions for each scenario

Optimal solution	Decay Function 1	Decay Function 2	Decay Function 3	Decay Function 4	Decay Function 5	Decay Function 6
20,16	36878.80	43269.41	30208.63	41974.60	46538.51	47421.35
28,18	36610.71	43453.11	30146.92	41493.00	46171.44	47057.68
22,14	36808.04	43268.95	30222.64	41925.06	46330.26	47202.59
20,14	36853.56	43192.42	30215.52	41986.74	46400.82	47275.49
20,16	36878.80	43269.41	30208.63	41974.60	46538.51	47421.35
20,16	36878.80	43269.41	30208.63	41974.60	46538.51	47421.35

For purposes of illustration, the robustness of Solution 1 (20, 16) is discussed in more detail. Absolute robustness shows the absolute values of the objective function when the solution (20, 16) is evaluated using decay functions 1-6 which gives values of 36878.80, 43269.41, 30208.63, 41974.60, 46538.51 and 47421.35. The summation of the objective values shows the absolute robustness of solution (20, 16). Given the best possible decision under that scenario, one can determine the absolute deviation and the relative deviation of the solution under each scenario. Therefore, we can compare the absolute robustness, robustness deviation and relative robustness. The following table shows the robustness analysis results for solution (20, 16) for the six decay functions for a threshold distance of 20, and retained exposure of 0.14.

Table 3. Robustness analysis of solution 1(20, 16)

	Absolute Robustness	Best Possible Decision	Robust Deviation	Relative Robustness
Scenario 1	36878.80	36878.8	0	0
Scenario 2	43269.41	43453.11	183.7	0.00423
Scenario 3	30208.63	30222.64	14.01	0.00046
Scenario 4	41974.60	41986.74	12.14	0.00029
Scenario 5	46538.51	46538.51	0	0
Scenario 6	47421.35	47421.35	0	0
Criteria	Sum(all values)		Max(all values)	Sum(all values)
Values	246291		183.7	0.00498

Similarly, we can calculate the values for the remaining 5 solutions for all 6 decay functions.

Solution 2 (28, 18)

Table 4. Robustness analysis of solution 2 (28, 18)

	Absolution Robustness	Best possible decision	Robust deviation	Relative Robustness
Scenario 1	36610.71	36878.80	268.09	0.007
Scenario 2	43453.11	43453.11	0.00	0.000
Scenario 3	30146.92	30222.64	75.72	0.003
Scenario 4	41493.00	41986.74	493.74	0.012
Scenario 5	46171.44	46538.51	367.07	0.008
Scenario 6	47057.68	47421.35	363.67	0.008
Criteria	Sum(all values)		Max(all values)	Sum(all values)
Values	244932.86		493.74	0.037

Solution 3 (22, 14)

Table 5. Robustness analysis of solution 3 (22, 14)

	Absolution Robustness	Best possible decision	Robust deviation	Relative Robustness
Scenario 1	36808.04	36878.80	70.76	0.002
Scenario 2	43268.95	43453.11	184.16	0.004
Scenario 3	30222.64	30222.64	0.00	0.000
Scenario 4	41925.06	41986.74	61.68	0.001
Scenario 5	46330.26	46538.51	208.25	0.004
Scenario 6	47202.59	47421.35	218.76	0.005
Criteria	Sum(all values)		Max(all values)	Sum(all values)
Values	245757.54		218.76	0.017

Solution 4 (20, 14)

Table 6. Robustness analysis of solution 4 (20, 14)

	Absolution Robustness	Best possible decision	Robust deviation	Relative Robustness
Scenario 1	36853.56	36878.80	25.24	0.001
Scenario 2	43192.42	43453.11	260.69	0.006
Scenario 3	30215.52	30222.64	7.12	0.000
Scenario 4	41986.74	41986.74	0.00	0.000
Scenario 5	46400.82	46538.51	137.69	0.003
Scenario 6	47275.49	47421.35	145.86	0.003
Criteria	Sum(all values)		Max(all values)	Sum(all values)
Values	245924.55		260.69	0.013

Solution 5 (20, 16) and Solution 6 (20, 16) are the same as Solution 1.

Another way to determine a robust solution is to take the average solution, meaning take the mean of the *x, y* coordinates of the six solutions. This yields the following results:

(20 + 28 + 22 + 20 + 20 + 20) / 6 = 21.7

(16 + 18 + 14 + 14 + 16 + 16) / 6 = 15.7,

which is close to (22, 16).

The problem under consideration is a multiple-scenario problem, so absolute robustness is not the best choice due to the range of objective function values across the different scenarios. Since there is a large relative difference with regard to the best solution across the scenarios, deviation robustness is not well suited to this situation. Relative robustness provides the most consistent results and therefore relative robustness is chosen to evaluate the solution performance. However, in the tables, all three robustness measures are recorded. For the first group, the most robust solution is (20, 16).

Other groups of settings, from group 2 to 9, where the decay distance threshold equals 20, 30 or 40 feet, and retained exposure equals 0.14, 0.30 and 0.5, are tested individually and the results are shown in Appendix B. The objective is to find the most robust solution as the potential optimal solution.

4.1.4.5 Conclusions and discussions

Based on the results from the previous section, one gets the following table indicating the most robust solution across the 9 settings. One can also see trends in how distance threshold and retained exposure affect the optimal solution.

Table 7. The most robust solutions for all scenarios

Settings	Distance threshold	Retained exposure	Optimal solution	
1	20	0.14	20	16
2		0.30	20	16/18
3		0.50	56	16
4	30	0.14	26	22
5		0.30	26	22
6		0.50	28	24
7	40	0.14	32	22
8		0.30	32	26/24
9		0.50	32	28

The table above shows the effect of distance threshold and retained exposure on the optimal solutions. These results are now discussed in more detail.

1). When the distance threshold increases, the aisles tend to be farther away from the walls to ensure that no location is too far from an aisle since the exposure curve does not become flat soon.

2). When the retained exposure increases, the racetrack aisles tend to move in towards the center. The reason is that with higher retained exposure, the impact of the aisles is smaller. The optimal solution of the racetrack tends to close to the center to shorten the distance of the entire racetrack, because the aisle grids have zero exposure, and the retail area grids have higher exposure. Thus, there is more benefit due to the aisles occupying less floor space than the gain from having the aisles closer to all of the retail grids.

In this particular Kohl's department store, the actual racetrack aisle corner is at (32, 26). By comparing with the real layout, setting the distance threshold at 40 ft. and the retained decay exposure at 0.3 will be used in the following steps.

For illustration purposes, the aisle structure including the locations of the entrance aisles, branch aisles, racetrack aisle and the width of those aisles is shown in Figure 24. From the figure, the light yellow part shows the main racetrack aisle, and dark blue part shows the vertical branch aisles inside the racetrack aisle. There are two main entrances in the front of the store, and two cashier counters are located near the entrances, accordingly.

Based on the choice of the exposure decay function type, the distance threshold and the retained decay exposure value, the exposure of each grid in the entire retail store is calculated. The exposure value of the retail area is shown in Figure 25. The exposure of the grid decays as the distance between the grid and the aisles increases. The entrances and branch aisles increase the exposure of adjacent grids. No exposure is calculated for aisles and cashier counters.

The next step, which is the allocation of departments, will be introduced in the next section. Specifically the departments in a Kohl's' store will be assigned to the retail area sub-areas that result from the chosen aisle structure.

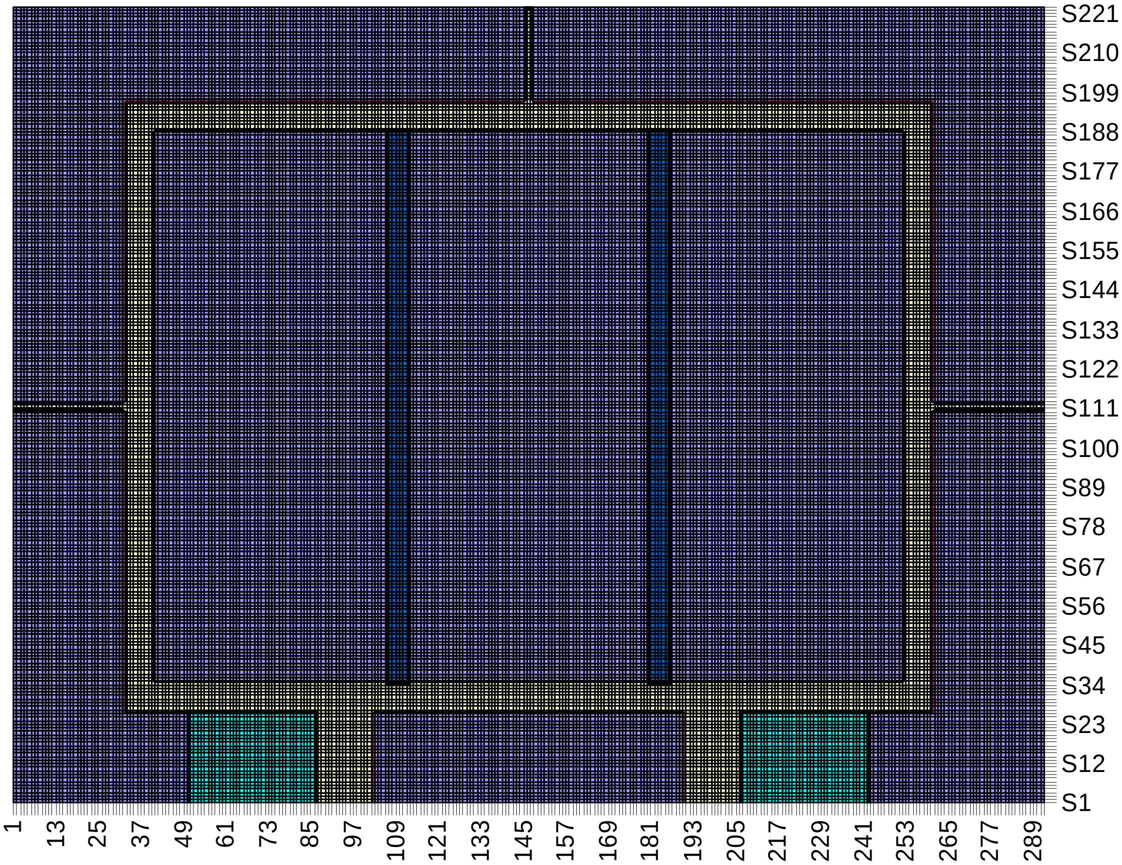


Figure 24. Optimal aisle structure in the Kohl's store example

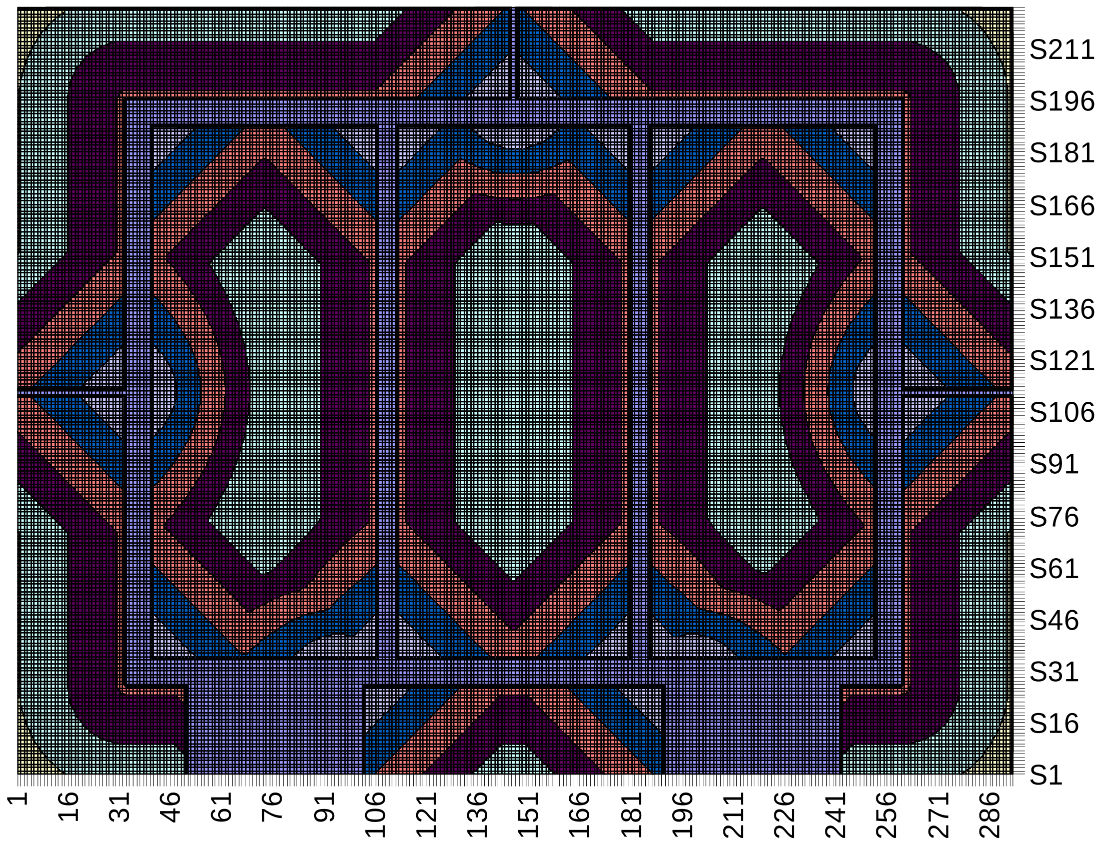


Figure 25. The exposure distribution of the Kohl's store

4.2 ALLOCATION OF DEPARTMENTS

After determining the aisle configuration which includes the specific location of the racetrack, the central aisles and the width of the aisles, the main aisle structure is fixed. With the aisles, the whole retail area is divided into several sub-areas, including inner sub-areas, which are inside the racetrack loop, and outer sub-areas, which are outside the racetrack loop. Each sub-area is treated as a knapsack. We assign departments into those knapsacks. The exposure sensitivity and adjacency preference of those departments is considered to optimize the allocation of those departments in the assignment. As a result, the goal to maximize sales by providing a pleasant and customer friendly retail environment is achieved.

4.2.1 Introduction

In this section, all the departments are assigned to predetermined knapsacks. In the example shown in Figure 26, twelve departments are assigned to five knapsacks (sub-areas). Each department has a certain lower bound of required retail area and an average revenue per unit of retail area. The areas of the five knapsacks are determined when the aisle structure is decided. The departments in these five areas are chosen to maximize the profit of the store, without violating the area limitations. The problem is formulated as a multiple knapsack problem with a number of items (departments) to be placed into the five knapsacks (sub-areas).

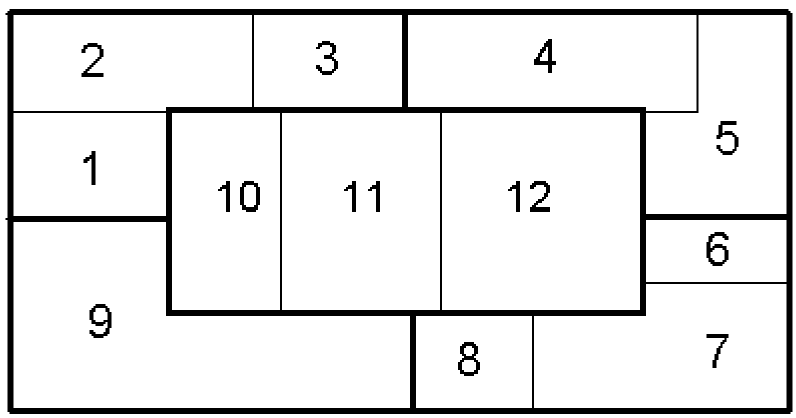


Figure 26. Sample of department allocation

During the assignment process, both the exposure sensitivity of each department and the adjacency interrelations between each pair of departments are considered. In the following sections, the definitions of exposure sensitivity and adjacency preference are introduced. Then the formulation of the model and its formulation variations are presented.

4.2.2 Definition of exposure sensitivity

Previous research showed that impulse purchasing is very common among the consumer population and across numerous product categories. One study found that between 27 and 62 percent of the purchases in department stores fell into the impulse category and that few product lines were unaffected by impulse buying. However, impulse buying varies across products and categories. For example, West [#64, 1951] showed that candies were purchased on impulse 65.8 percent of the time, bakery goods were purchased on impulse 70.1 percent of the time, cosmetics were purchased on impulse 41.8 percent of the time, and jewelry was purchased on impulse 49.5 percent of the time. From the data, those products' sales were highly dependent on impulse purchases, versus apparel where impulse purchases occur only 24.1 percent of the time. The probabilities of purchasing candies and bakery product on impulse are notably higher than those of many other products [#18, Chandon, 2009]. Therefore, stores often place bakery products in the front of their stores, where their smell and look also help attract customers to stop and come into the store. These candies and bakery products represent items that have high exposure sensitivities. If they are placed in the back of stores, they will probably be easily ignored by customers and lose their sales [#3, Baczewska, 2005].

On the contrary, household textiles, appliances, furniture and prescription drugs are less likely purchased on impulse and therefore have relatively low exposure sensitivities. For example, the study of POPAI (Point of Purchase Advertising International) in 1963 showed that prescription drugs were never purchased on impulse. Thus, it makes sense that, chain stores, such as CVS or Rite aid, always put the prescription counter at the very back of the store. Two reasons for this are that 1) the pharmacy is not exposure sensitive, people will find the counter no matter where it is, since they need the prescription and 2) it leads people to the very back of store, increasing the exposure of other products.

This study does not consider all products. Instead, those departments which are commonly found in stores are used to classify products. Although there are over thirty departments in many department stores, the focus in this study is on the 24 departments, which can be found in most department stores. Table 8 shows the rate of impulse purchasing for those 24 departments. The departments are classified with respect to consumer's impulse purchase likelihoods in these departments. For this purpose, the results of Bellenger et al.'s [#9, 1978] work are adopted. Unfortunately, their work does not cover all merchandise lines/product categories (i.e. departments). In order to determine the data of those departments for which data do not exist the following was done:

1) If there are departments with similar characteristics listed in Bellenger et al.'s work, use the similar ratings, where * will be listed.

2) If the department to be rated does not have a similar department in Bellenger et al.'s work, then its rating is evaluated by turning to other literature or using intuition, where ^ will be listed. When intuition is used, assumptions will be explained in Appendix C.

Table 8 divides departments into three categories based on the percentage of impulse purchases in the departments: departments where the likelihood of impulse purchase was low (below 33%), departments where the likelihood of impulse purchase was medium (between 33% and 50%), and departments where the likelihood of impulse purchase was high (above 50%).

Table 8. Impulse purchase sensitivity of departments

Dept No.	**Department Name**	**Impulse Purchase Percent**	**Impulse Purchase Sensitivity**
1	Cosmetics/Fragrances	41.8	M
2	Handbags	45*	M
3	Intimate Apparel	27	L

Table 8 (continued)

4	Jewelry	62	H
5	Accessories	44^	M
6	Infants and Toddlers	47*	M
7	Boys	47*	M
8	Girls	47	M
9	Juniors	47*	M
10	Young Men's	36	M
11	Men's	40	M
12	Misses Career	44	M
13	Misses Casual	54	H
14	Petites	47	M
15	Women's	47	M
16	Shoes	52	H
17	Hosiery	52*	H
18	Furniture	22	L
19	Home Dec (Window)	53	H
20	Bedding and Bath	49*	M
21	Luggage	10^	L
22	Frames/Stationary	39	M
23	Kitchen/Dining	53*	H
24	Toys	54^	H

For the profit data of the identified departments, the data provided on BizStats' web page is utilized (http://www.bizstats.com/spf.malls.htm). For the statistics that are not available it was assumed that the revenue generated in those departments can be approximated by referring to similar departments for which the statistics are provided. For example, as shown in Figure 19, Misses' and Women's clothing are placed in different departments. At BizStat the sales per square foot data for the Misses' clothing department is not available. Hence, an assumption is made that the unit profit for Misses' and Women's clothing are the same.

Table 9 summarizes the sales per square foot for the 24 departments used in this study.

Table 9. Departments' unit revenue data

Dept No.	Department Name	Sales per sq ft
1	Cosmetics/Fragrances	411
2	Handbags	478
3	Intimate Apparel	339
4	Jewelry	880
5	Accessories	478
6	Infants and Toddlers	399
7	Boys	393
8	Girls	393
9	Juniors	299
10	Young Men's	299
11	Men's	299

Table 9 (continued)

12	Misses Career	308
13	Misses Casual	308
14	Petites	308
15	Women's	308
16	Shoes	421
17	Hosiery	246
18	Furniture	286
19	Home Dec (Window)	333
20	Bedding and Bath	286
21	Luggage	286
22	Frames/Stationary	229
23	Kitchen/Dining	286
24	Toys	221

4.2.3 Definition of adjacency preference

Most previous research on departmental adjacency focuses on manufacturing settings. Surprisingly, little research has been done in the retail sector. The most famous case in the retail area is the diaper and beer case. The case is famous due to the successful application of data mining techniques on the study of customer behavior.

Beatty and Smith [#8, 1987] report that search is positively associated with purchase involvement; however, too much search will make customers frustrated and lose their patience. Park, Iyer and Smith [#54, 1989] report that consumers under time pressure tend to engage in less search. They also find that time-pressured consumers deliberate less and even fail to make some planned purchases. Too much search or time pressure therefore leads to less in-store decision-making and fewer purchases [#39, Inman and Winer, 1998].

There is some related research focusing on shelf space allocation based on adjacency considerations. Envirosell studied point of sale data to find out if binders and index dividers can stimulate each others' sales at OfficeMax.. First, they put binders and a small package of index dividers next to each other. The receipts showed that people were indeed buying index dividers in tandem with binders -- except that many people were not picking up the packages of dividers next to the binders, instead, the shoppers searched in a completely different part of the store for a larger package of dividers. OfficeMax moved the larger packages next to the binders, and sales of the index dividers increased. Chen, Chen and Tung [#18, 2006] considered the effect of spatial relationships, such as the shelf-space adjacencies of distinct items and used data mining techniques to discover the implicit relationship between the relative spatial distance of displayed products and the items' unit sales in a retail store. Compared to shelf-space allocation with adjacency consideration, little study has been done at the store level.

The merchandise allocation at the store level should consider the logical difference and similarity among the categories of the merchandise. Usually there is a rough partition of merchandise in a store, for example, apparel, jewelry, accessories, cosmetics and household items. Customers often want to know where these areas are when they step into a store. This is why there is sometimes a map of the sales floor in the front of the store.

To find the pattern of department allocation in department stores, data was collected from 10 department stores in Pittsburgh. The stores include free-standing stores and stores in malls, and the sizes of the stores vary greatly. The relationship of two departments can be 1) in the same sub-area and not separated by aisles, 2) next to each other with an aisle in between, 3) separated by another department, 4) separated by two or three departments, 5) far from each other or they are even on the different floors.

The layout of the department stores that were visited was analyzed yielding the adjacency matrix in Table 10 for the 24 departments.

Table 10. Relationship among 24 departments

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1		A	I	A	A	O	O	O	I	I	E	I	I	O	O	I	I	O	O	O	O	O	O	U
2	A		I	A	A	U	U	U	O	O	E	A	E	E	O	O	A	U	U	O	U	O	U	U
3	I	I		I	E	I	O	A	O	O	I	I	O	O	O	I	I	O	A	I	O	O	O	O
4	A	A	I		A	O	O	O	I	I	I	A	E	I	I	O	I	O	O	O	O	O	O	O
5	A	A	E	A		I	O	O	O	O	I	A	I	I	I	O	A	O	O	O	O	O	O	O
6	O	U	I	O	I		A	A	O	O	O	O	I	O	I	O	U	O	O	I	O	O	O	O
7	O	U	O	O	O	A		A	O	O	O	I	I	O	I	I	O	O	O	O	O	O	O	O
8	O	U	A	O	O	A	A		O	O	O	I	O	O	I	I	I	O	O	E	O	I	O	O
9	I	O	O	I	O	O	O	O		A	E	I	A	O	O	O	O	U	O	O	U	U	O	U
10	I	O	O	I	O	O	O	O	A		A	O	I	O	O	O	O	U	O	I	O	I	O	U
11	E	E	I	I	I	O	O	O	E	A		O	O	I	O	A	A	O	O	E	I	O	I	O
12	I	A	I	A	A	O	I	I	I	O	O		A	A	A	I	O	O	O	O	O	O	O	U
13	I	E	O	E	I	I	I	O	A	I	O	A		I	I	O	I	O	O	O	O	O	O	U
14	O	E	O	I	I	O	O	O	O	O	I	A	I		E	O	E	U	U	U	U	U	U	U
15	O	O	O	I	I	I	I	I	O	O	O	A	I	E		I	O	O	O	O	O	O	O	O
16	I	O	I	O	O	O	I	I	O	O	A	I	O	O	I		O	O	I	I	I	O	I	I
17	I	A	I	I	A	U	O	I	O	O	A	O	I	E	O	O		U	O	I	O	O	O	U
18	O	U	O	O	O	O	O	O	U	U	O	O	O	U	O	O	U		I	A	I	A	I	E
19	O	U	A	O	O	O	O	O	O	O	O	O	O	U	O	I	O	I		A	I	I	A	E
20	O	O	I	O	O	I	O	E	O	I	E	O	O	U	O	I	I	A	A		O	O	A	O
21	O	U	O	O	O	O	O	O	U	O	I	O	O	U	O	I	O	I	I	O		A	E	E
22	O	O	O	O	O	O	O	I	U	I	O	O	O	U	O	O	O	A	I	O	A		I	I
23	O	U	O	O	O	O	O	O	O	O	I	O	O	U	O	I	O	I	A	A	E	I		E
24	U	U	O	O	O	O	O	O	U	U	O	U	U	U	O	I	U	E	E	O	E	I	E	

In order to achieve the quantitative input data needed for this research, the REL values were converted to numbers using the scaling value w_{ij}, given in the third column of Table 11. Another set of scaling values related to the closeness rating, denoted by cij in the last column of Table 11, are determined using an exponential scale, as suggested by Askin and Standridge (1993). One could normalize the cij values but the result is that there is little difference among the normalized cij values for the ratings of I, O and U. In order to have the I and O adjacency values be more significant the values given in the wij column were used.

Table 11. Adjacency scores

Rating	Definition	w_{ij}	c_{ij}
A	Absolutely Necessary	1	125
E	Especially Important	0.8	25
I	Important	0.5	5
O	Ordinary Closeness	0.2	1
U	Unimportant	0	0
X	Undesirable	/	-25
XX	Prohibited	/	-125

In the table, there are scores for undesirable and prohibited adjacencies. They are useful in situations where two departments are undesirable to be close to each other. In the most extreme case, two departments might be prohibited from being next to each other and large negative values are applied. However, there are other situations, where using smaller negative adjacency preferences pushes departments away from each other. It is also possible that two departments could have a strong positive relationship and in order to stimulate traffic flow between two departments, and increase the exposure of items displayed along the path connecting the two departments, negative adjacency preferences might be used in the model to push the two departments away from each other. This research does not consider this last type of situation but rather assumes that if two departments have a strong positive relationship then it is desirable to locate them close to each other.

4.2.4 Modeling of the allocation of departments problem

In this section, the department assignment problem is formulated as a multiple knapsack problem. The objective is to maximize the profit of the entire retail space by allocating the known number of departments to a set of knapsacks (sub-areas).

The notation is shown below.

Decision variables:

x_{ij} If x_{ij}=1, then department i is in knapsack j; otherwise, x_{ij}=0.

a_{ij} The area of department i in knapsack j.

$A_{i_1,i_2,j}$ An indicator variable to signify primary adjacency of departments i_1 and i_2 in knapsack j. If both departments i_1 and i_2 are in knapsack j, then $A_{i_1,i_2,j} = 1$, otherwise it should be zero. $A_{i_1,i_2,j}$ is originally written as $A_{i_1,i_2,j} = x_{i_1,j} * x_{i_2,j}$

A'_{i_1,i_2,j_1,j_2} An indicator variable to signify secondary adjacency of departments i_1 and i_2. If both departments i_1 and i_2 are in two adjacent knapsacks j_1 and j_2 then $A'_{i_1,i_2,j_1,j_2} = 1$, otherwise it should be zero. Similarly, A'_{i_1,i_2,j_1,j_2} is originally written as $A'_{i_1,i_2,j_1,j_2} = x_{i_1,j_1} * x_{i_2,j_2}$

Parameters:

i Department index.

j Knapsack index.

s_i The nominal area of department i.

S_j The area of knapsack j, which is a sub-area in the retail area. Note that $\sum_{j=1}^{5} S_j = L * W$.

r_{ij} The revenue of department i that results from placing department i in knapsack j.

$r_{ij}^{'}$ The revenue per unit area of department i that results from placing department i in knapsack j. Note that $r_{ij}^{'} = r_{ij} / s_i$.

R_i The average revenue of department i. Note that $R_i = \sum_{j=1}^{J} r_{ij} / J$.

α_i The exposure sensitivity of department i.

E_j The average exposure of knapsack j.

w_{i_1, i_2} The adjacent preference between department i_1 and department i_2, shown as a matrix.

w Primary adjacency factor.

w' Secondary adjacency factor.

4.2.4.1 Standard multiple knapsack problem

The Multiple Knapsack Problem (also called the Multiple Loading Problem) is the problem of choosing a subset of n items to be loaded into m distinct containers, such that the total value of the selected items is maximized, without exceeding the capacity of each of the containers.

Initially, adjacency preferences are not considered in the model and the departments are allocated into knapsacks in order to maximize revenue based on the revenue per square foot of each individual department while insuring that there is no violation of the knapsack area constraints.

Formulation (1)

$$\max \sum_{i=1}^{I}\sum_{j=1}^{J} r_{ij}\, x_{ij}$$

$$st \quad \sum_{i=1}^{I} x_{ij} s_i \le S_j \qquad \forall j, \qquad (12)$$

$$\sum_{j=1}^{J} x_{ij} \le 1 \qquad \forall i, \qquad (13)$$

$$x_{ij} \in \{0,1\}$$

Formulation (1) is the standard formulation of the multiple knapsack problem. Note that in the current formulation each knapsack area is fixed. To make the formulation more general and flexible, area flexibility, letting each department expand or shrink up to 20 percent of its nominal size, is permitted. This results in Formulation (2).

Formulation (2)

$$\max \sum_{i=1}^{I}\sum_{j=1}^{J} r'_{ij} a_{ij}$$

$$st \quad \sum_{i=1}^{I} a_{ij} \le S_j \qquad \forall j, \qquad (14)$$

$$\sum_{j=1}^{J} x_{ij} \le 1 \qquad \forall i, \qquad (15)$$

$$0.8 s_i \le \sum_{j=1}^{J} a_{ij} \le 1.2 s_i \quad \forall i, \qquad (16)$$

$$S_j x_{ij} \ge a_{ij} \qquad \forall i, j, \qquad (17)$$

$$x_{ij} \in \{0,1\} \quad a_{ij} \in R^{+}$$

Constraint (14) ensures the area of all of the departments doesn't exceed the whole retail area. Constraint (15) ensures that each department is placed in at most one knapsack. Constraint (16) means that each department can be expanded or contracted within 20% of its nominal area. Constraint (17) requires that if a_{ij} is larger than zero, x_{ij}

has to be one, that is, if a department has revenue generating area then it must be located in a knapsack. By solving this problem, an integral solution is solved for the knapsack allocation. Note that the resulting layout may have some unused area. If constraint (15) were modeled as an equality then the model may be infeasible since it still may not be possible to pack all the departments into the knapsacks even with the given area flexibility. The model can be solved by CPLEX. For a small case, such as 12 departments (I=12) and 5 knapsacks (J=5), it can be solved in less than one second.

4.2.4.2 Considering primary adjacency

As mentioned in the literature review section, adjacency between departments can affect impulse purchases. During the department allocation, adjacency preference between each pair of departments should be considered. In this research, when two departments are allocated to the same sub-area (knapsack), we say that these two departments have primary adjacency.

In order to increase impulse purchases, department pairs with high adjacency preference should be allocated close to each other and department pairs with negative adjacency preference should not be placed close to each other. Therefore, the department allocation problem is formulated as a multiple knapsack problem with the consideration of both exposure and adjacency. The profit of each department is determined by the knapsack which it is located in and the relative locations of the other departments. Thus, the profit associated with placing department i in knapsack j, r_{ij}, includes two components - the knapsack exposure and the adjacency relationships, rather than only the knapsack exposure. First the case where the department area is fixed is considered which simplifies the model. The objective function is as follows:

$$\max \sum_{i_1=1}^{I}\left[R_{i_1}\left(\sum_{j=1}^{J}\alpha_i E_j x_{ij} + \sum_{i_2\neq i_1, i_2=1}^{I}\sum_{j=1}^{J} wW_{i_1,i_2}x_{i_1,j}x_{i_2,j}\right)\right]$$

Since it is a quadratic 0-1 problem, $A_{i_1,i_2,j}$ is used to linearize the objective function. The new formulation is as follows:

Formulation (3)

$$\max \sum_{i_1=1}^{I}\left[R_{i_1}\left(\sum_{j=1}^{J}\alpha_i E_j x_{ij} + \sum_{i_2\neq i_1, i_2=1}^{I}\sum_{j=1}^{J} wW_{i_1,i_2}A_{i_1,i_2,j}\right)\right]$$

$$st \quad \sum_{i=1}^{I} x_{ij}s_i \leq S_j, \qquad \forall j \tag{18}$$

$$\sum_{j=1}^{J} x_{ij} \leq 1, \qquad \forall i \tag{19}$$

$$A_{i_1,i_2,j} \leq x_{i_1,j}, \qquad \forall i_1, j \tag{20}$$

$$A_{i_1,i_2,j} \leq x_{i_2,j}, \qquad \forall i_2, j \tag{21}$$

$$x_{ij}, A_{i_1,i_2,j} \in \{0,1\}$$

Note that, there are two sets of binary variables in Formulation (3). However by adding equation (22), one set of binary variables can be relaxed to continuous variables, which results in less computation time being required to solve the model. Formulation (3) is revised is by adding constraint (22) and is referred to as Formulation (4).

$$0 \leq A_{i_1,i_2,j} \leq 1, \qquad \forall i_1, i_2, j \tag{22}$$

4.2.4.3 Considering primary and secondary adjacency

Sometimes even though two departments have a positive (tight) interaction it is not possible to place them in the same sub-area. To handle this situation, the concept of secondary adjacency is developed. Secondary adjacency arises when two departments are assigned to adjacent knapsacks instead of the same knapsack. Secondary adjacency results in gaining some of the benefits of having the two departments close together but less benefit than having them in the same knapsack. The following objective function considers both primary and secondary adjacency.

$$\max \sum_{i_1=1}^{I} \left[R_{i_1} \left(\sum_{j=1}^{J} \alpha_i E_j x_{ij} + \sum_{i_2 \neq i_1, i_2=1}^{I} \sum_{j=1}^{J} wW_{i_1,i_2} x_{i_1,j} x_{i_2,j} + \sum_{i_2 \neq i_1, i_2=1}^{I} \sum_{(j_1,j_2) \in \text{adjacent knapsacks}} w'W_{i_1,i_2} x_{i_1,j_1} x_{i_2,j_2} \right) \right]$$

Similarly, the function can be linearized by using $A_{i_1,i_2,j}$ and A'_{i_1,i_2,j_1,j_2}. Formulation (5) presents a model that considers both primary and secondary adjacency.

<u>Formulation (5)</u>

$$\max \sum_{i_1=1}^{I} \left[R_{i_1} \left(\sum_{j=1}^{J} \alpha_i E_j x_{ij} + \sum_{i_2 \neq i_1, i_2=1}^{I} \sum_{j=1}^{J} wW_{i_1,i_2} A_{i_1,i_2,j} + \sum_{i_2 \neq i_1, i_2=1}^{I} \sum_{(j_1,j_2) \in \text{adjacent knapsacks}} w'W_{i_1,i_2} A'_{i_1,i_2,j_1,j_2} \right) \right]$$

$$st. \quad \sum_{i=1}^{I} x_{ij} s_i \leq S_j, \qquad \forall j \tag{23}$$

$$\sum_{j=1}^{J} x_{ij} \leq 1, \qquad \forall i \tag{24}$$

$$A_{i_1,i_2,j} \leq x_{i_1,j}, \qquad \forall i_1, j \tag{25}$$

$$A_{i_1,i_2,j} \leq x_{i_2,j}, \qquad \forall i_2, j \tag{26}$$

$$A'_{i_1,i_2,j_1,j_2} \leq \left(x_{i_1,j_1} + x_{i_1,j_2} + x_{i_2,j_1} + x_{i_2,j_2} \right) / 2, \quad \forall (j_1, j_2) \in \text{adjacent knapsacks} \tag{27}$$

$$x_{ij}, A_{i_1,i_2,j}, A'_{i_1,i_2,j_1,j_2} \in \{0,1\}$$

The objective function is to maximize the profit of all of the departments based on exposure, primary adjacency and secondary adjacency. For each department i_1, the first term in the objective considers the exposure of the knapsack where department i_1 is located. The second term considers the primary adjacency effects of the other departments in the same knapsack as department i_1. The third term considers the secondary adjacency effects of the other departments in the adjacent knapsacks. Constraint (23) ensures that the total area of all departments in knapsack j does not exceed the knapsack area. Constraint (24) ensures that any department is in at most one knapsack. Constraints (25) and (26) ensure $A_{i_1,i_2,j}$ represents primary adjacency as defined above. Note that the binary adjacency indicator variable $A_{i_1,i_2,j}$ can only equal one if both department i_1 and i_2 are located in sub-area j – which is how primary adjacency is defined. Constraint (27) ensures A'_{i_1,i_2,j_1,j_2} represents secondary adjacency as defined above because the binary secondary adjacency indicator variable A'_{i_1,i_2,j_1,j_2} can only equal one if both department i_1 and i_2 are located in adjacent sub-areas – which is how secondary adjacency is defined.

There are three sets of binary variables in Formulation (5). Two integral sets are relaxed by adding constraints (32), (35) and (36). The result is Formulation (6) shown below.

<u>Formulation (6)</u>

$$\max \sum_{i_1=1}^{I}\left[R_{i_1}\left(\sum_{j=1}^{J}\alpha_i E_j x_{ij} + \sum_{i_2\neq i_1, i_2=1}^{I}\sum_{j=1}^{J} wW_{i_1,i_2}A_{i_1,i_2,j} + \sum_{i_2\neq i_1, i_2=1}^{I}\sum_{(j_1,j_2)\in\text{adjacent knapsacks}} w'W_{i_1,i_2}A'_{i_1,i_2,j_1,j_2}\right)\right]$$

$$st.\quad \sum_{i=1}^{I} x_{ij}s_i \leq S_j, \qquad \forall j \tag{28}$$

$$\sum_{j=1}^{J} x_{ij} = 1, \qquad \forall i \tag{29}$$

$$A_{i_1,i_2,j} \leq x_{i_1,j}, \qquad \forall i_1, j \tag{30}$$

$$A_{i_1,i_2,j} \leq x_{i_2,j}, \qquad \forall i_2, j \tag{31}$$

$$A'_{i_1,i_2,j_1,j_2} \leq 2-\left(x_{i_1,j_1}+x_{i_1,j_2}+x_{i_2,j_1}+x_{i_2,j_2}\right) \quad \forall (j_1,j_2)\in \text{not adjacent knapsacks} \tag{32}$$

$$A'_{i_1,i_2,j_1,j_2} \leq x_{i_1,j_1}, \qquad \forall i_1, j_1 \tag{33}$$

$$A'_{i_1,i_2,j_1,j_2} \leq x_{i_2,j_2}, \qquad \forall i_2, j_2 \tag{34}$$

$$0 \leq A_{i_1,i_2,j} \leq 1, \qquad \forall i_1, i_2, j \tag{35}$$

$$0 \leq A'_{i_1,i_2,j_1,j_2} \leq 1, \qquad \forall i_1, i_2, j_1, j_2 \tag{36}$$

$$x_{ij} \in \{0,1\}$$

To have more flexibility in the model, Formulation (6) was modified resulting in Formulation (7). Formulation (7) is similar to formulation (6) except that all departments are allowed to expand or shrink by 10 percent of their nominal area. In this case the decision variables a_{ij} represent the actual area of department i in knapsack j. This new formulation is more flexible and should permit the development of better layouts.

Formulation (7)

$$\max \sum_{i_1=1}^{I}\left[R_{i_1}\left(\sum_{j=1}^{J}\alpha_i e_j a_{ij} + \sum_{i_2\neq i_1, i_2=1}^{I}\sum_{j=1}^{J} wW_{i_1,i_2}A_{i_1,i_2,j} + \sum_{i_2\neq i_1,i_2=1}^{I}\sum_{(j_1,j_2)\in \text{adjacent knapsacks}} w'W_{i_1,i_2}A'_{i_1,i_2,j_1,j_2}\right)\right]$$

$$st. \quad \sum_{i=1}^{I} a_{ij} \leq S_j, \quad \forall j \tag{37}$$

$$0.9s_i \leq \sum_{j=1}^{J} a_{ij} \leq 1.1s_i, \quad \forall i \tag{38}$$

$$1.1s_i x_{ij} \geq a_{ij}, \quad \forall i, j \tag{39}$$

$$\sum_{j=1}^{J} x_{ij} = 1, \quad \forall i \tag{40}$$

$$A_{i_1,i_2,j} \leq x_{i_1,j}, \quad \forall i_1, j \tag{41}$$

$$A_{i_1,i_2,j} \leq x_{i_2,j}, \quad \forall i_2, j \tag{42}$$

$$A'_{i_1,i_2,j_1,j_2} \leq 2 - \left(x_{i_1,j_1} + x_{i_1,j_2} + x_{i_2,j_1} + x_{i_2,j_2}\right) \quad \forall (j_1, j_2) \in \text{not adjacent knapsacks} \tag{43}$$

$$A'_{i_1,i_2,j_1,j_2} \leq x_{i_1,j_1}, \quad \forall i_1, j_1 \tag{44}$$

$$A'_{i_1,i_2,j_1,j_2} \leq x_{i_2,j_2}, \quad \forall i_2, j_2 \tag{45}$$

$$0 \leq A_{i_1,i_2,j} \leq 1, \quad \forall i_1, i_2, j \tag{46}$$

$$0 \leq A'_{i_1,i_2,j_1,j_2} \leq 1, \quad \forall i_1, i_2, j_1, j_2 \tag{47}$$

$$x_{ij} \in \{0,1\}$$

4.2.4.4 RLT –reformulation-linearization technique

To improve the efficiency of solving Formulation (7) the reformulation-linearization technique (RLT) was applied to the model. RLT was introduced by Sherali and Adams, [#58, 1998], “The motivation of RLT is the role of tight linear programming representations or relaxations in solving such discrete and continuous nonconvex programming problems. The principal thrust is to commence with a model that affords a useful representation and structure, and then to further strengthen this representation through automatic reformulation and constraint generation techniques." Appling RLT results in Formulation (8), which can be solved more efficiently than Formulation (7).

Formulation (8)

$$\max \sum_{i_1=1}^{I}\left[R_{i_1}\left(\sum_{j=1}^{J}\alpha_i e_j a_{ij} + \sum_{i_2\neq i_1, i_2=1}^{I}\sum_{j=1}^{J} wW_{i_1,i_2}A'_{i_1,i_2,j,j} + \sum_{i_2\neq i_1, i_2=1}^{I}\sum_{(j_1,j_2)\in \text{adjacent knapsacks}} w'W_{i_1,i_2}A'_{i_1,i_2,j_1,j_2}\right)\right]$$

$$st. \quad \sum_{i=1}^{I} a_{ij} \leq S_j, \qquad \forall j \tag{48}$$

$$0.9s_i \leq \sum_{j=1}^{J} a_{ij} \leq 1.1s_i, \quad \forall i \tag{49}$$

$$1.1s_i x_{ij} \geq a_{ij}, \qquad \forall i, j \tag{50}$$

$$\sum_{j=1}^{J} x_{ij} = 1, \qquad \forall i \tag{51}$$

$$A'_{i_1,i_2,j_1,j_2} \leq 2 - \left(x_{i_1,j_1} + x_{i_1,j_2} + x_{i_2,j_1} + x_{i_2,j_2}\right) \quad \forall (j_1, j_2) \in \text{not adjacent knapsacks} \tag{52}$$

$$A'_{i_1,i_2,j_1,j_2} \leq x_{i_1,j_1}, \qquad \forall i_1, i_2, j_1, where\ i_1 \neq i_2 \tag{53}$$

$$A'_{i_1,i_2,j_1,j_2} \leq x_{i_2,j_2}, \qquad \forall i_1, i_2, j_2, where\ i_1 \neq i_2 \tag{54}$$

$$\sum_{j_1} A'_{i_1,i_2,j_1,j_2} = x_{i_2,j_2}, \qquad \forall i_1, i_2, j_2\ where\ i_1 \neq i_2 \tag{55}$$

$$\sum_{i_1} s_{i_1} x_{i_1,j_1} \leq S_{j_1}/0.9, \qquad \forall j_1 \tag{56}$$

$$\sum_{i1:i1\neq i2} s_{i1} A'_{i_1,i_2,j_1,j_1} \leq (S_{j1}/0.9)x_{i_2,j_2}, \qquad \forall i_2, j_1, j_2\ where i_1 \neq i_2,\ j_1 = j_2 \tag{57}$$

$$\sum_{i1:i1\neq i2} s_{i1} A'_{i_1,i_2,j_1,j_2} \leq (S_{j1}/0.9)x_{i_2,j_2}, \qquad \forall i_2, j_1, j_2\ where\ i_1 \neq i_2, j_1 \neq j_2 \tag{58}$$

$$0 \leq A'_{i_1,i_2,j_1,j_2} \leq 1, \qquad \forall i_1, i_2, j_1, j_2 \tag{59}$$

$$x_{ij} \in \{0,1\}$$

Constraints (37) to (40) and Constraints (43) to (47) are kept from Formulation (7), while Constraints (41) and (42) are deleted since the $A_{i_1,i_2,j}$ variables are replaced by A'_{i_1,i_2,j_1,j_1} variables.

Multiplying both sides of constraint (40), by x_{i_2,j_2} results in the following equation,

$$\sum_{j_1=1} x_{i_1,j_1} x_{i_2,j_2} = x_{i_2,j_2}, \qquad \forall i_1, i_2, j_2, \text{where} i_2 \neq i_1$$

Given that

$$A_{i_1,i_2,j} = x_{i_1,j_1} x_{i_2,j_2} \text{ if } j_1 = j_2 \text{ , where } i_2 \neq i_1$$
$$A'_{i_1,i_2,j_1,j_2} = x_{i_1,j_1} x_{i_2,j_2} \text{ if } j_1 \neq j_2 \text{ , where } i_2 \neq i_1$$

Then the result is

$$\sum_{j_1} A_{i_1,i_2,j_1} = x_{i_2,j_2}, \qquad \forall i_1, i_2, j_2 \; where i_2 \neq i_1, \; j_1 = j_2$$
$$\sum_{j_1} A'_{i_1,i_2,j_1,j_2} = x_{i_2,j_2}, \qquad \forall i_1, i_2, j_2 \; where i_2 \neq i_1, \; j_1 \neq j_2$$

Since the $A_{i_1,i_2,j}$ variables are replaced by A'_{i_1,i_2,j_1,j_1} variables, the two constraints above can be combined into one constraint, which is Constraint (55).

$$\sum_{j_1} A'_{i_1,i_2,j_1,j_2} = x_{i_2,j_2}, \qquad \forall i_1, i_2, j_2 \;\; where \;\; i_1 \neq i_2 \tag{55}$$

Similarly, from Constraints (37), (38) and (39), the following inequality is implied,

$$\sum_{i_1} s_{i_1} x_{i_1,j_1} \leq \sum_{i_1} a_{i_1,j_1} \leq S_{j_1} / 0.9, \qquad \forall j_1,$$

which is basically a knapsack type constraint. Applying RLT, similar to the previous, case generates another group of valid inequalities. Then multiplying both sides of the constraint by x_{i_2,j_2} and replace $A_{i_1,i_2,j}$ by A'_{i_1,i_2,j_1,j_2}, results in

$$\sum_{i_1} s_{i_1} x_{i_1,j_1} x_{i_2,j_2} \leq S_{j_1} / 0.9 x_{i_2,j_2}, \qquad \forall i_2, j_1, j_2 \tag{60}$$

There are several different inequalities derived from Equation (60).

First, cases are created by considering the different relationships between i_1 and i_2

Case 1, $i_1 = i_2$

When $i_1 = i_2$, meaning i_1 and i_2 refer to the same department, then necessarily $j_1 = j_2$. Then

$$x_{i_1,j_1} x_{i_2,j_2} = x_{i_1,j_1} x_{i_2,j_1} = x_{i_1,j_1},$$

And Equation (60) is transformed into the following expression

$$\sum_{i_1} s_{i_1} x_{i_1,j_1} \le S_{j_1} / 0.9, \qquad \forall j_1 \tag{56}$$

Case 2 $i_1 \ne i_2$,

This case is divided into the following two sub-cases.

(a) when $j_1 = j_2$,

Equation (60) becomes

$$\sum_{i_1:i_2 \ne i_1} s_{i_1} x_{i_1,j_1} x_{i_2,j_1} \le S_{j_1} / 0.9 x_{i_2,j_1}, \qquad \forall i_2, j_1$$

Because

$$A_{i_1,i_2,j} = x_{i_1,j_1} x_{i_2,j_2} \text{ if } j_1 = j_2 \text{, where } i_2 \ne i_1$$

Equation (60) can be rewritten as

$$\sum_{i_1:i_2 \ne i_1} s_{i_1} A_{i_1,i_2,j_1} \le S_{j_1} / 0.9 x_{i_2,j_1}, \qquad \forall i_2, j_1$$

Again the $A_{i_1,i_2,j}$ variables are replaced by A'_{i_1,i_2,j_1,j_1}, variables resulting in Constraint (57).

$$\sum_{i1:i1 \ne i2} s_{i1} A'_{i_1,i_2,j_1,j_1} \le (S_{j1} / 0.9) x_{i_2,j_2}, \qquad \forall i_2, j_1, j_2 \text{ where } i_1 \ne i_2,\ j_1 = j_2 \tag{57}$$

(b) when $j_1 \ne j_2$

Equation (60) becomes

$$\sum_{i_1:i_2\neq i_1} s_{i_1} x_{i_1,j_1} x_{i_2,j_2} \leq S_{j_1} / 0.9 x_{i_2,j_2}, \qquad \forall i_2, j_1, j_2$$

Because

$$A'_{i_1,i_2,j_1,j_2} = x_{i_1,j_1} x_{i_2,j_2} \text{ if } j_1 \neq j_2 \text{ , where } i_2 \neq i_1$$

Equation (60) can be rewritten as

$$\sum_{i_1:i_2\neq i_1} s_{i_1} A'_{i_1,i_2,j_1,j_2} \leq S_{j_1} / 0.9 x_{i_2,j_2}, \qquad \forall i_2, j_1, j_2 \text{ where } i_1 \neq i_2,\ j_1 \neq j_2 \qquad (58)$$

It is also possible to create cases based on considering j and branching accordingly. Then instead of Constraints (56), (57) and (58), Constraints (69) and (70) of Formulation (8’) can be derived. In Formulation (8’), Constraints (69) and (70) work similar to Constraints (56), (57) and (58), but requires a smaller number of constraints.

Formulation (8')

$$\max \sum_{i_1=1}^{I}\left[R_{i_1}\left(\sum_{j=1}^{J}\alpha_i e_j a_{ij} + \sum_{i_2\neq i_1, i_2=1}^{I}\sum_{j=1}^{J} wW_{i_1,i_2}A'_{i_1,i_2,j,j} + \sum_{i_2\neq i_1, i_2=1}^{I}\sum_{(j_1,j_2)\in \text{adjacent knapsacks}} w'W_{i_1,i_2}A'_{i_1,i_2,j_1,j_2}\right)\right]$$

$$st. \quad \sum_{i=1}^{I} a_{ij} \leq S_j, \qquad \forall j \tag{61}$$

$$0.9s_i \leq \sum_{j=1}^{J} a_{ij} \leq 1.1s_i, \quad \forall i \tag{62}$$

$$1.1s_i x_{ij} \geq a_{ij}, \qquad \forall i, j \tag{63}$$

$$\sum_{j=1}^{J} x_{ij} = 1, \qquad \forall i \tag{64}$$

$$A'_{i_1,i_2,j_1,j_2} \leq 2-\left(x_{i_1,j_1} + x_{i_1,j_2} + x_{i_2,j_1} + x_{i_2,j_2}\right) \quad \forall (j_1, j_2) \in \text{not adjacent knapsacks} \tag{65}$$

$$A'_{i_1,i_2,j_1,j_2} \leq x_{i_1,j_1}, \qquad \forall i_1, i_2, j_1, where\ i_1 \neq i_2 \tag{66}$$

$$A'_{i_1,i_2,j_1,j_2} \leq x_{i_2,j_2}, \qquad \forall i_1, i_2, j_2, where\ i_1 \neq i_2 \tag{67}$$

$$\sum_{j_1} A'_{i_1,i_2,j_1,j_2} = x_{i_2,j_2}, \quad \forall i_1, i_2, j_2\ where\ i_1 \neq i_2 \tag{68}$$

$$\sum_{i_1:i_1\neq i_2} s_{i_1} A'_{i_1,i_2,j_1,j_2} \leq (S_{j_1}/0.9)x_{i_2,j_2}, \qquad \forall i_2, j_1, j_2, \text{where } j_1 \neq j_2 \tag{69}$$

$$\sum_{i_1:i_1\neq i_2} s_{i_1} A'_{i_1,i_2,j_1,j_2} + s_{i_2}x_{i_2,j_2} \leq (S_{j_1}/0.9)x_{i_2,j_2}, \qquad \forall i_2, j_1, j_2, \text{where } j_1 = j_2 \tag{70}$$

$$0 \leq A'_{i_1,i_2,j_1,j_2} \leq 1, \qquad \forall i_1, i_2, j_1, j_2 \tag{71}$$

$$x_{ij} \in \{0,1\}$$

Only Constraints (69) and (70) are explained here, since all of the other constraints are the same as in Formulation (8).

(1) when $j_1 \neq j_2$, Constraint (60) changes to the following equation,

$$\sum_{i_1:i_1\neq i_2} s_{i_1} A'_{i_1,i_2,j_1,j_2} \leq (S_{j_1}/0.9)x_{i_2,j_2}, \qquad \forall i_2, j_1, j_2, \text{where } j_1 \neq j_2 \tag{69}$$

(2) when $j_1 = j_2$, Constraint (60) changes to the following equation,

$$\sum_{i_1:i_1\neq i_2} s_{i_1} A'_{i_1,i_2,j_1,j_2} + s_{i_2} A'_{i_2,i_2,j_2,j_2} \leq (S_{j_1}/0.9)x_{i_2,j_2}, \qquad \forall i_2, j_1, j_2, \text{where } j_1 = j_2$$

Which can be simplified to,

$$\sum_{i_1:i_1\neq i_2} s_{i_1} A'_{i_1,i_2,j_1,j_2} + s_{i_2}x_{i_2,j_2} \leq (S_{j_1}/0.9)x_{i_2,j_2}, \qquad \forall i_2, j_1, j_2, \text{where } j_1 = j_2 \tag{70}$$

Both Formulation (8) and Formulation (8’) apply the Reformulation-Linearization-Technique and give the same solution. However, Formulation (8’) runs slightly faster than Formulation (8) in almost all cases. The results are shown in the case study section.

4.2.5 Case study: Kohl’s department stores

4.2.5.1 Background

The inputs for the model developed above include revenue per square foot of a department, required area of each department and impulse purchase likelihoods. The departments taken into consideration were chosen based on Figure 20. The required area of each department can be obtained based on the layouts of the real department stores. The objective function formulation requires two sets of data concerning the knapsacks: (1) the average exposure of each knapsack, and (2) the area of each knapsack. This information can be obtained from the optimal results of the aisle configuration problem.

From the layout in Figure 20, twenty four distinct departments are identified. Of the twenty four departments, twenty three departments are in the summarized list. Similarly, there are twenty four departments in Figure 21 in the summarized list.

Table 12. Department area data

Dept No.	Department Name	Kohl's Store 1	
		Area (sq ft)	Percentage
1	Cosmetics/Fragrances	1008	1.534
2	Handbags	560	0.852
3	Intimate Apparel	2712	4.127
4	Jewelry	1008	1.534
5	Accessories	560	0.852
6	Infants and Toddlers	2538	3.862
7	Boys	2496	3.798
8	Girls	1960	2.983
9	Juniors	2240	3.409
10	Young Men's	2912	4.431
11	Men's	7310	11.124
12	Misses Career	3056	4.651
13	Misses Casual	3744	5.698
14	Petites	928	1.412
15	Women's	1904	2.897
16	Shoes	2490	3.789
17	Hosiery	1120	1.704
18	Furniture	752	1.144
19	Home Dec (Window)	864	1.315
20	Bedding and Bath	2336	3.555

Table 12 (continued)

21	Luggage	1792	2.727
22	Frames/Stationary	1152	1.753
23	Kitchen/Dining	3504	5.332
24	Toys	342	0.520
Others	Men's Sport	1920	2.922
	Customer Service	1440	2.191
	Cashier	1600	2.435
	Aisle area	11464	17.446
Total area		65712	100.000

For the layout in Figure 20, the total store area is 296 *222 = 65712 square feet. For the layout in Figure 21, the total store area is measured as 312 * 236 = 73632 square feet. The aisles occupy approximately 17~18% of the store area.

4.2.5.2 Example Model Application

First, the department relation adjacent matrix must be changed from A/E/I/O/U to numeric values.

Table 13. Adjacency preference matrix

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1		A	I	A	A	O	O	O	I	I	E	I	I	O	O	I	I	O	O	O	O	O	O	U
2	A		I	A	A	U	U	U	O	O	E	A	E	E	O	O	A	U	U	O	U	O	U	U
3	I	I		I	E	I	O	A	O	O	I	I	O	O	O	I	I	O	A	I	O	O	O	O
4	A	A	I		A	O	O	O	I	I	I	A	E	I	I	O	I	O	O	O	O	O	O	O
5	A	A	E	A		I	O	O	O	O	I	A	I	I	I	O	A	O	O	O	O	O	O	O
6	O	U	I	O	I		A	A	O	O	O	O	I	O	I	O	U	O	O	I	O	O	O	O
7	O	U	O	O	O	A		A	O	O	O	I	I	O	I	I	O	O	O	O	O	O	O	O
8	O	U	A	O	O	A	A		O	O	O	I	O	O	I	I	I	O	O	E	O	I	O	O
9	I	O	O	I	O	O	O	O		A	E	I	A	O	O	O	O	U	O	O	U	U	O	U
10	I	O	O	I	O	O	O	O	A		A	O	I	O	O	O	O	U	O	I	O	I	O	U
11	E	E	I	I	I	O	O	O	E	A		O	O	I	O	A	A	O	O	E	I	O	I	O
12	I	A	I	A	A	O	I	I	I	O	O		A	A	A	I	O	O	O	O	O	O	O	U
13	I	E	O	E	I	I	I	O	A	I	O	A		I	I	O	I	O	O	O	O	O	O	U
14	O	E	O	I	I	O	O	O	O	O	I	A	I		E	O	E	U	U	U	U	U	U	U
15	O	O	O	I	I	I	I	I	O	O	O	A	I	E		I	O	O	O	O	O	O	O	O
16	I	O	I	O	O	O	I	I	O	O	A	I	O	O	I		O	O	I	I	I	O	I	I
17	I	A	I	I	A	U	O	I	O	O	A	O	I	E	O	O		U	O	I	O	O	O	U
18	O	U	O	O	O	O	O	O	U	U	O	O	O	U	O	O	U		I	A	I	A	I	E
19	O	U	A	O	O	O	O	O	O	O	O	O	O	U	O	I	O	I		A	I	I	A	E
20	O	O	I	O	O	I	O	E	O	I	E	O	O	U	O	I	I	A	A		O	O	A	O
21	O	U	O	O	O	O	O	O	U	O	I	O	O	U	O	I	O	I	I	O		A	E	E
22	O	O	O	O	O	O	O	I	U	I	O	O	O	U	O	O	O	A	I	O	A		I	I
23	O	U	O	O	O	O	O	O	O	O	I	O	O	U	O	I	O	I	A	A	E	I		E
24	U	U	O	O	O	O	O	O	U	U	O	U	U	U	O	I	U	E	E	O	E	I	E	

To transfer the adjacent matrix into numbers, define: A = 1.0, E = 0.8, I = 0.5, O = 0.2 and U = 0. This results in the following numeric input matrix.

Table 14. Numeric department relation matrix

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	1	1	0.5	1	1	0.2	0.2	0.2	0.5	0.5	0.8	0.5	0.5	0.2	0.2	0.5	0.5	0.2	0.2	0.2	0.2	0.2	0.2	0
2	1	1	0.5	1	1	0	0	0	0.2	0.2	0.8	1	0.8	0.8	0.2	0.2	1	0	0	0.2	0	0.2	0	0
3	0.5	0.5	1	0.5	0.8	0.5	0.2	1	0.2	0.2	0.5	0.5	0.2	0.2	0.2	0.5	0.5	0.2	1	0.5	0.2	0.2	0.2	0.2
4	1	1	0.5	1	1	0.2	0.2	0.2	0.5	0.5	0.5	1	0.8	0.5	0.5	0.2	0.5	0.2	0.2	0.2	0.2	0.2	0.2	0.2
5	1	1	0.8	1	1	0.5	0.2	0.2	0.2	0.2	0.5	1	0.5	0.5	0.5	0.2	1	0.2	0.2	0.2	0.2	0.2	0.2	0.2
6	0.2	0	0.5	0.2	0.5	1	1	1	0.2	0.2	0.2	0.2	0.5	0.2	0.5	0.2	0	0.2	0.2	0.5	0.2	0.2	0.2	0.2
7	0.2	0	0.2	0.2	0.2	1	1	1	0.2	0.2	0.2	0.5	0.5	0.2	0.5	0.5	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
8	0.2	0	1	0.2	0.2	1	1	1	0.2	0.2	0.2	0.5	0.2	0.2	0.5	0.5	0.5	0.2	0.2	0.8	0.2	0.5	0.2	0.2
9	0.5	0.2	0.2	0.5	0.2	0.2	0.2	0.2	1	1	0.8	0.5	1	0.2	0.2	0.2	0.2	0	0.2	0.2	0	0	0.2	0
10	0.5	0.2	0.2	0.5	0.2	0.2	0.2	0.2	1	1	1	0.2	0.5	0.2	0.2	0.2	0.2	0	0.2	0.5	0.2	0.5	0.2	0
11	0.8	0.8	0.5	0.5	0.5	0.2	0.2	0.2	0.8	1	1	0.2	0.2	0.5	0.2	1	1	0.2	0.2	0.8	0.5	0.2	0.5	0.2
12	0.5	1	0.5	1	1	0.2	0.5	0.5	0.5	0.2	0.2	1	1	1	1	0.5	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0
13	0.5	0.8	0.2	0.8	0.5	0.5	0.5	0.2	1	0.5	0.2	1	1	0.5	0.5	0.2	0.5	0.2	0.2	0.2	0.2	0.2	0.2	0
14	0.2	0.8	0.2	0.5	0.5	0.2	0.2	0.2	0.2	0.2	0.5	1	0.5	1	0.8	0.2	0.8	0	0	0	0	0	0	0
15	0.2	0.2	0.2	0.5	0.5	0.5	0.5	0.5	0.2	0.2	0.2	1	0.5	0.8	1	0.5	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
16	0.5	0.2	0.5	0.2	0.2	0.2	0.5	0.5	0.2	0.2	1	0.5	0.2	0.2	0.5	1	0.2	0.2	0.5	0.5	0.5	0.2	0.5	0.5
17	0.5	1	0.5	0.5	1	0	0.2	0.5	0.2	0.2	1	0.2	0.5	0.8	0.2	0.2	1	0	0.2	0.5	0.2	0.2	0.2	0
18	0.2	0	0.2	0.2	0.2	0.2	0.2	0.2	0	0	0.2	0.2	0.2	0	0.2	0.2	0	1	0.5	1	0.5	1	0.5	0.8
19	0.2	0	1	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0	0.2	0.5	0.2	0.5	1	1	0.5	0.5	1	0.8
20	0.2	0.2	0.5	0.2	0.2	0.5	0.2	0.8	0.2	0.5	0.8	0.2	0.2	0	0.2	0.5	0.5	1	1	1	0.2	0.2	1	0.2
21	0.2	0	0.2	0.2	0.2	0.2	0.2	0.2	0	0.2	0.5	0.2	0.2	0	0.2	0.5	0.2	0.5	0.5	0.2	1	1	0.8	0.8
22	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.5	0	0.5	0.2	0.2	0.2	0	0.2	0.2	0.2	1	0.5	0.2	1	1	0.5	0.5
23	0.2	0	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.5	0.2	0.2	0	0.2	0.5	0.2	0.5	1	1	0.8	0.5	1	0.8
24	0	0	0.2	0.2	0.2	0.2	0.2	0.2	0	0	0.2	0	0	0	0.2	0.5	0	0.8	0.8	0.2	0.8	0.5	0.8	1

4.2.5.3 Optimized results

Three models were tested using CPLEX 9 and CPLEX 11. In the CPLEX 9 environment, Formulation 7 can be solved to a MIPGAP of 1.0% but requires over 100 hours. Formulation 8 can be solved to a MIPGAP of 1.0% in 17, 56, or 68 hours depending on the different preferences between exposure and adjacency that are used in the objective function. Formulation 8' can be solved to a MIPGAP of 1.0% in 30, 59, or 28 hours for the three preference settings.

Appling these models in the CPLEX 11.0 environment, results in dramatically better solution times. The optimization results of the Kohl's department allocation problem are shown in the following tables.

Formulation (8)

Exposure vs. Adjacency	Mipgap	Current MIP best bound	Solution time	Iterations	Nodes
4%	0.01%	7.90*10e6	47.1 hours	449100322	1456359
10%	0.01%	8.43*10e6	24.8 hours	193450466	443356
25%	0.01%	9.86*10e6	5.6 hours	44466981	98837
50%	0.01%	1.24*10e7	2.5 hours	19361430	31957
100%	0.01%	1.77*10e7	1.6 hours	11618872	15837

Formulation (8')

Exposure vs. Adjacency	Mipgap	Current MIP best bound	Solution time	Iterations	Nodes
4%	0.01%	7.90*10e6	35.2 hours	300330275	996709
10%	0.01%	8.43*10e6	12.9 hours	100088260	229582
25%	0.01%	9.86*10e6	4.1 hours	30823733	60851
50%	0.01%	1.24*10e7	1.6 hours	12536633	20179
100%	0.01%	1.77*10e7	1.0 hours	6670862	7025

Both formulations gave the same optimal solutions; however, formulation (8') gives the optimal solution in less time. To further shorten the solution time for all the cases, sparser adjacency matrixes can be used. The original adjacency matrix for the cases above has 42% non-zero values which represent the A, E and I rating in the 24 by 24 matrix In the following tests, a new adjacency matrix is tested that only keeps the A and E ratings. The non-zero ratio in the adjacency matrix decreases to 23% by eliminating the I ratings. Using the sparser adjacency matrix can shorten the solution time and keep the solution quality relatively high. In the following table, the solution time and solution quality of using the sparser adjacency matrix and the original adjacency matrix are compared.

Formulation (8')

Exposure vs. Adjacency	Mipgap	New solution time	New solution evaluated under original settings	Original MIP best bound	Optimal objective gap	Original solution time	Solution time shortened
4%	0.01%	3.8 hrs	7.82*10e6	7.90*10e6	1.0%	35.2 hrs	89.2%
10%	0.01%	3.2 hrs	8.14*10e6	8.43*10e6	3.4%	12.9 hrs	75.2%
25%	0.01%	0.4 hrs	9.16*10e6	9.86*10e6	7.1%	4.1 hrs	90.2%
50%	0.01%	4.0 mins	1.10*10e7	1.24*10e7	11.3%	1.6 hrs	95.9%
100%	0.01%	1.9 mins	1.47*10e7	1.77*10e7	17.0%	1.0 hrs	96.9%

The table shows that having a sparser adjacency matrix reduces the solution time since the simplified input generally reduces the number of non-zero values in the objective function. The compromise is the solution quality since the I relationship is ignored. If the solution time using the A, E, and I data is too long then the sparse adjacency matrix is a good alternative to consider. However, since CPLEX 11.0 can solve the A, E, and I problem data in a reasonable amount of time, the original adjacency matrix and derived optimal solutions are kept for this problem.

The table also shows that the solution time decreases as the relative weight of adjacency increases. Therefore the problem becomes easier to solve when the relative weight of adjacency increases.

The third discovery from the table is that the gaps between the two solutions obtained from using the regular and simplified inputs increases as the relative weight of adjacency increases. So the sparser adjacency matrix improves the solution time more when a lower adjacency weight rather than a larger adjacency weight is applied. Note that the solution time for an adjacency ratio of 25% using the less sparse matrix input is 3.2 hours, which is reasonable. Therefore the results based on using the I values in the input matrix and an adjacency ratio of 25% are used as the input for the next solution phase. Two problem solutions, one with a 4% ratio and one with a 25% ratio, are now shown to illustrate how the layout changes with different emphases on adjacency.

The layout corresponding to the optimal solution with a ratio of exposure vs. adjacency of 4% is shown in Figure 27.

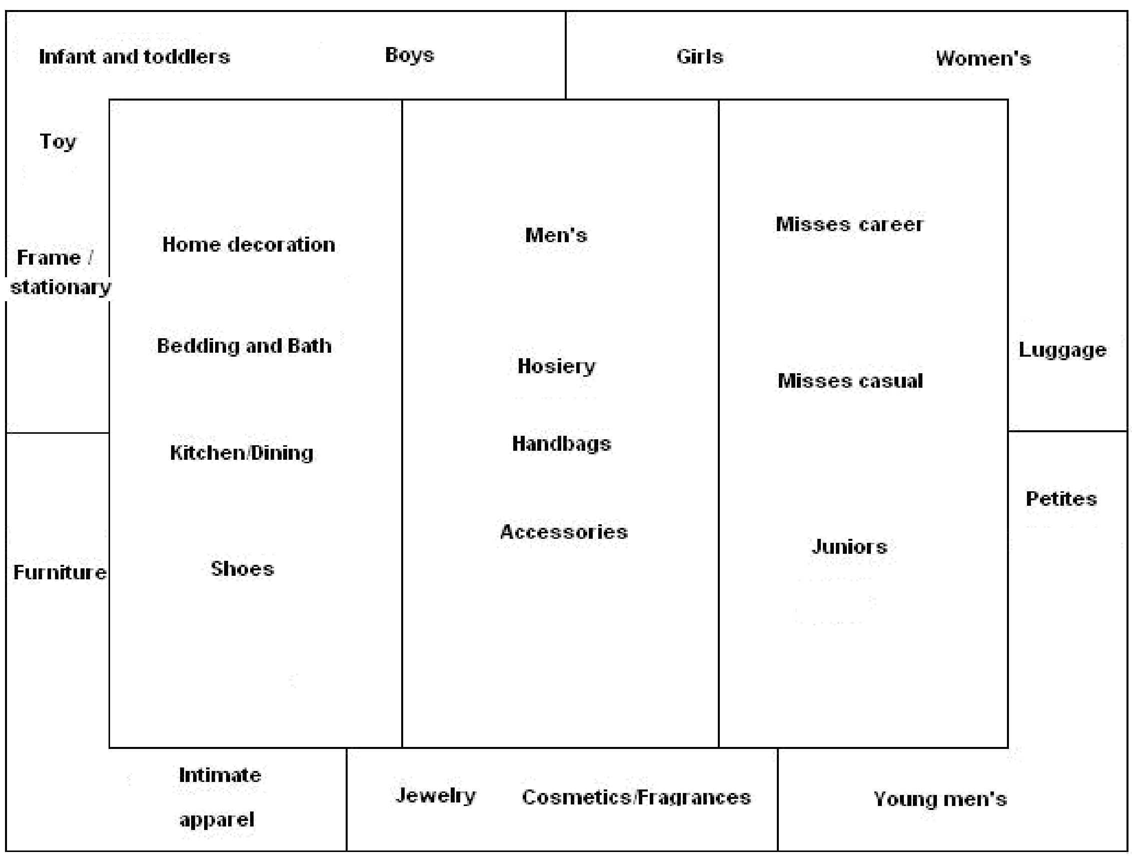


Figure 27. The optimal department allocation with relatively low emphasis on adjacency

The area of each department is listed in the following table, which is the input for the next step.

Variable Name	Solution Value		
a1_5	1108.800000	x1_5	1.000000
a2_7	616.000000	x2_7	1.000000
a3_1	2983.200000	x3_1	1.000000
a4_5	1108.800000	x4_5	1.000000
a5_7	616.000000	x5_7	1.000000
a6_2	2791.800000	x6_2	1.000000
a7_2	2321.600000	x7_2	1.000000
a8_3	2156.000000	x8_3	1.000000
a9_8	2464.000000	x9_8	1.000000
a10_4	2967.200000	x10_4	1.000000
a11_7	8008.000000	x11_7	1.000000
a12_8	3361.600000	x12_8	1.000000
a13_8	4118.400000	x13_8	1.000000
a14_4	1020.800000	x14_4	1.000000
a15_3	2094.400000	x15_3	1.000000
a16_6	2739.000000	x16_6	1.000000
a17_7	1232.000000	x17_7	1.000000
a18_1	827.200000	x18_1	1.000000
a19_6	950.400000	x19_6	1.000000
a20_6	2569.600000	x20_6	1.000000
a21_3	1971.200000	x21_3	1.000000
a22_2	1036.800000	x22_2	1.000000
a23_6	3854.400000	x23_6	1.000000
a24_2	307.800000	x24_2	1.000000

The layout corresponding to the optimal solution with a ratio of exposure vs. adjacency of 25% is shown in Figure 28.

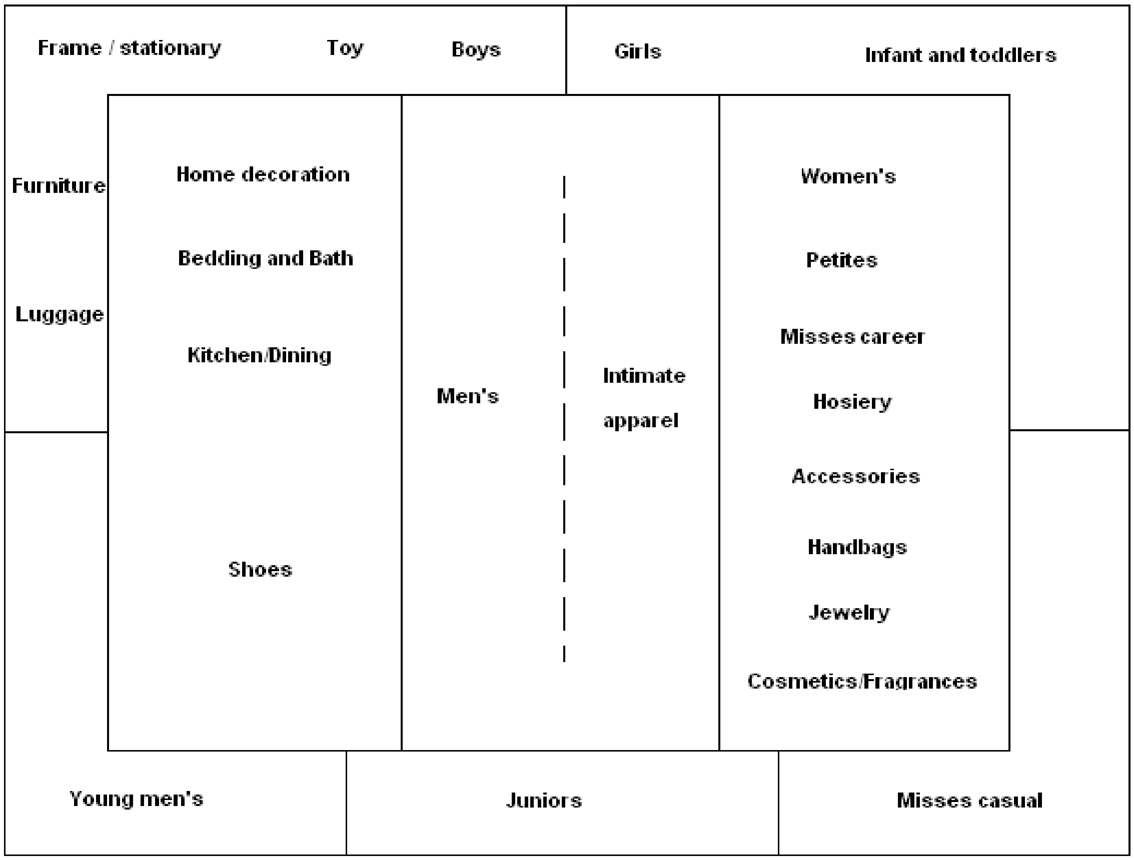


Figure 28. The optimal department allocation with more emphasis on adjacency

The area of each department is listed in the following table, which is the input for the next step.

Variable Name	Solution Value		
a1_8	1108.800000	x1_8	1.000000
a2_8	616.000000	x2_8	1.000000
a3_7	2440.800000	x3_7	1.000000
a4_8	1108.800000	x4_8	1.000000
a5_8	616.000000	x5_8	1.000000
a6_3	2791.800000	x6_3	1.000000
a7_2	2745.600000	x7_2	1.000000
a8_3	2156.000000	x8_3	1.000000
a9_5	2288.000000	x9_5	1.000000
a10_1	3203.200000	x10_1	1.000000
a11_7	8031.200000	x11_7	1.000000
a12_8	2750.400000	x12_8	1.000000
a13_4	3988.000000	x13_4	1.000000
a14_8	835.200000	x14_8	1.000000
a15_8	1896.800000	x15_8	1.000000
a16_6	2739.000000	x16_6	1.000000
a17_8	1232.000000	x17_8	1.000000
a18_2	686.600000	x18_2	1.000000
a19_6	950.400000	x19_6	1.000000
a20_6	2569.600000	x20_6	1.000000
a21_2	1612.800000	x21_2	1.000000
a22_2	1036.800000	x22_2	1.000000
a23_6	3854.400000	x23_6	1.000000
a24_2	376.200000	x24_2	1.000000

4.2.5.4 Summary and discussion

One aspect that needs further discussion is the relative weighting of exposure vs. adjacency in the objective function. If more emphasis is placed on exposure, then the result will tend to put higher exposure sensitive departments in locations with higher customer exposure. If the focus is more on adjacency, then the department placement is governed more by the departmental relationships rather than exposure. Then it might be more convenient for customers to find related items since they should be placed close together.

Another aspect to discuss is the difference between the nominal area of each department and the optimized result. Since we relax the area of each department by +/-10%, then the total area of the departments in each knapsack might be changed accordingly.

Table 15 shows the difference between the nominal size and the actual size for each knapsack and each department. The third column is the nominal size of the departments. The fourth column is the optimized department size. The sum of the nominal and optimized sizes for the departments in each knapsack is shown in the shaded rows.

Table 15. The comparison of nominal size and actual size of knapsacks

Knapsack index	Nominal size of knapsack	Nominal sizes of depts. (summed and separated)	Optimized sizes of depts. (summed and separated)
1	3988	2912	3203.2
	Young men	2912	3203.2
2	6458	6534	6458
	Boys	2496	2745.6
	Furniture	752	686.6
	Luggage	1792	1612.8
	Frame	1152	1036.8
	Toys	342	376.2

Table 15 (continued)

3	6458	4498	4947.8
	Infant	2538	2791.8
	Girls	1960	2156
4	3988	3744	3988
	Miss Casual	3744	3988
5	2288	2240	2288
	Junior	2240	2288
6	10164	9194	10113.4
	Shoes	2490	2739
	Home Dec	864	950.4
	Bedding	2336	2569.6
	Kitchen	3504	3854.4
7	10472	10022	10472
	Men	7310	8031.2
	Intimate	2712	2440.8
8	10164	10144	10164
	Cosmetics	1008	1108.8
	Handbag	560	616
	Jewelry	1008	1108.8
	Accessories	560	616
	Miss career	3056	2750.4
	Petite	928	835.2
	Women	1904	1896.8
	Hosiery	1120	1232

From the table above, the optimized sizes of the departments and the nominal sizes of the knapsacks are shown. The sizes of the departments are expanded or squeezed to fit in the knapsacks. However, in some cases, after the departments are expanded, there is still extra space in the knapsack, such as with Knapsacks 1 and 3. In this research, it assumed that if the department area is beyond 120 percent of its nominal area, there is no more value added by further expanding the area and the result is that there is unused space (or the space can be used by one of the departments but there will not be any additional accrued value). However, other assumptions can be made about what to do with the extra space and these are discussed as future research in section 6.2.3.

The current layout of the Kohl's store at West Mifflin, PA can be evaluated by the model. By using the current department allocation as given in Figure 19 as the model input, the objective function value is 9.06*10e6, compared with 9.86*10e6 found by the model. Thus, the current assignment has a gap of 8.1% compared to the optimal solution. Using the current areas of the departments in the figure in addition to their locations as an input, the objective function value decreases to 8.65*10e6, which is a gap of 12.3% relative to the optimal solution. If the objective function represents revenue per year, applying the optimization recommended by this process can increase the revenue by 1.21 million dollars.

Robustness tests were conducted considering the effects of changes to different elements of the input data. For each of four factors, unit price, nominal area, exposure sensitivity and adjacency preference, the input data has been randomly increased or decreased by 20%, and five cases have been tested for each factor. For example, in the first row of results found in Table 16 five test runs are made perturbing the unit price, which means the unit prices of the 24 departments are randomly increased or decreased by 20%. The second row shows the effect of perturbing the nominal area. The last row shows the effects of perturbing all four factors simultaneously. For comparison purposes, the base case, with no perturbation of the data, can be solved in 1412 seconds resulting in an objective function value of 9.16*10e6.

Table 16. Robustness test on input data

		Case 1	Case 2	Case 3	Case 4	Case 5	Max-Min
Unit price	Obj	1.02*10e7	9.49*10e6	7.99*10e6	9.09*10e6	9.77*10e6	2.21*10e6
	Time	1008	746	1379	839	2055	1309
Nominal Area	Obj	9.21*10e6	9.12*10e6	9.21*10e6	9.16*10e6	9.19*10e6	0.09*10e6
	Time	4599	1301	117	180	314	4482
Exposure sensitivity	Obj	9.30*10e6	9.02*10e6	8.95*10e6	8.67*10e6	8.52*10e6	0.78*10e6
	Time	1871	938	1841	732	1048	1139
Adjacency matrix	Obj	9.12*10e6	9.11*10e6	9.17*10e6	9.23*10e6	9.12*10e6	0.12*10e6
	Time	1099	864	1353	785	1266	568
Random	Obj	1.06*10e7	9.02*10e6	7.53*10e6	8.71*10e6	1.02*10e7	3.07*10e6
	Time	4929	91	520	262	4503	4838

From the table above, the most robust factor with regard to the objective is the nominal area. Recall also that the model allows the nominal area of departments to be changed up and down by 10%. On the other hand, the solution time varied a lot because certain nominal areas are harder to solve for than other settings. The adjacency has less influence than unit price and exposure sensitivity. In the objective function, the adjacency has a weight of 25% compared with the exposure effect. The adjacency values in the adjacency matrix are between 0.8 and 1, resulting in 20% non-zero density in the matrix. The perturbation doesn't affect the objective value and solution time very much. The exposure sensitivity plays an important role in the model by changing the objective function by 8.5%. The unit price is certainly the most important factor in determining the objective function. The last group of robustness experiments is shown in the last row of Table 16. Random perturbations are applied on all four factors simultaneously for five repeated runs. A large variance can be found in the five cases, whose gaps can reach 33.5% for the objective function values and require 3.5 times the original solution time. The robustness test can help us to know which factors need to be chosen most carefully and which factors have the most effect on the solution time and solution quality. Therefore, the most sensitive parameter is the unit price, which is the most important to estimate accurately. The next most sensitive parameters are the exposure sensitivity and adjacency preference. Depending on the weights of these two terms in the objective function one parameter could be more sensitive than the other.

To solve the problem, more formulations can be developed and further optimization can be done. The proposed approach can also be used to change the existing layout in a store if the current layout obtains extra space because a department is eliminated or becomes more space constrained because a new department is added. One solution could be to add or delete departments from the sub-areas. Alternatively, an analysis could also be done to rearrange the aisle configuration to increase the profit of the entire store without changing the distribution of departments. A preliminary study will be shown in the Section discussing extensions of this research. Another question is what to do with additional space that may exist in one of the sub-areas after the departments are assigned to the sub-areas. One solution is to push a department across an aisle and extend the adjacent knapsack. However, in this research, the extra spaces are left empty.

4.3 DETAILED DESIGN OF THE RETAIL AREA

From Section 4.2, a solution is obtained for the allocation of all the departments in all of the sub-areas. In this section, given the department and sub-area data, the exact position of each department in the corresponding knapsack is studied.

4.3.1 Introduction

There are two types of sub-areas that result from the racetrack aisle assumption. The first is an L-shaped polygon, which occurs in the four outer sub-areas. The second is a rectangle, which occurs in the inner sub-area and part of the outer sub-areas. Although the assignments of the departments to each knapsack are known; it is difficult to precisely locate each department within the knapsacks based on exposure optimization under the assumption of a continuous layout representation. This is because if the departments are located to optimize the exposure objective without any further constraints, the resulting layout will not be realistic because the most profitable department will occupy all the space along the aisle in order to maximize the overall profit. An example of this is shown in Figure 29 where part A of the figure shows the undesirable layout that will occur if no constraints are placed on the department shapes. Part B of this same figure shows the type of department placement that is desired.

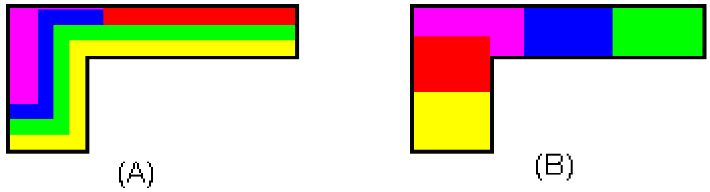


Figure 29. Undesirable layout (A) versus a desirable layout (B)

To avoid this problem a discrete representation is used in which the outer sub-areas and inner sub-areas are divided into strips where each department will cover one or more of the strips. An example outer sub-area is shown in Figure 30. Using this method, all the departments are forced to occupy a subset of the continuous strips, which helps create a reasonable layout. It will probably still be necessary to massage the final results from the strip layout. For example, one may want to ensure that every department has a part directly touching an aisle.

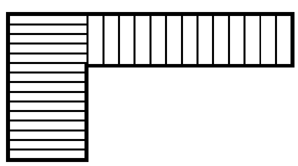


Figure 30. Example strip representation of an outer sub-area

Once the strip configuration is selected, one example being in Figure 30, then the exposure of each strip can be calculated. The result is that different strips in one knapsack have different exposure values as shown in Figure 31. In Figure 31, the two ends of the sub-area have high exposure since they border an entrance aisle (strip 1 is the horizontal strip at the bottom and strip 30 is the rightmost vertical strip). The other strips also have relatively high exposure, since one end of these strips is on the racetrack aisle. A mathematical formulation of the strip model is introduced in the next sub-section.

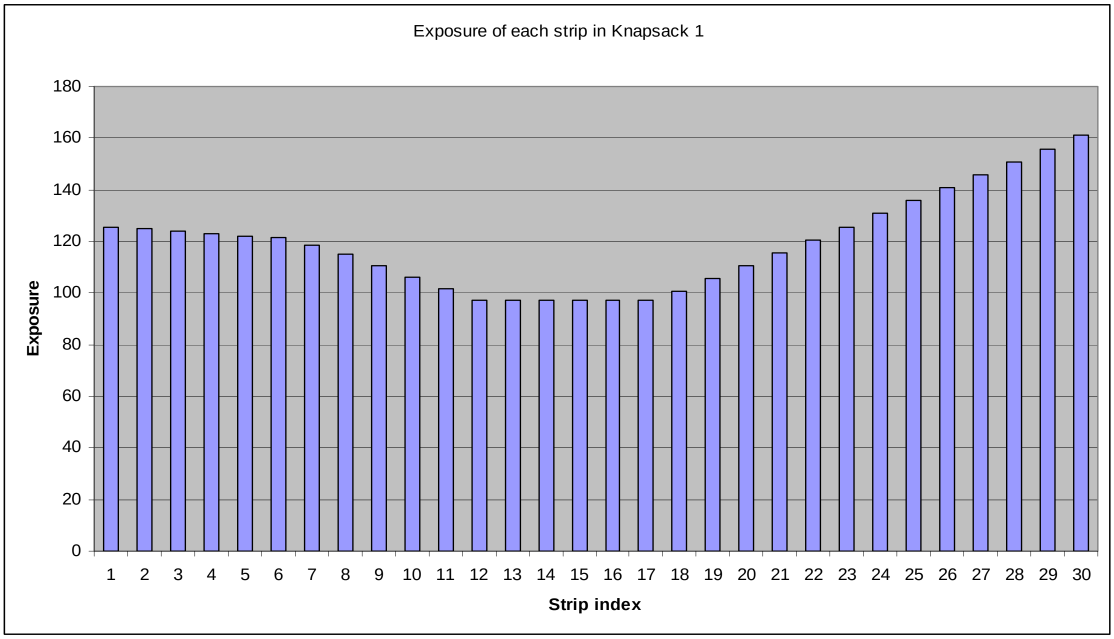


Figure 31. The exposure of strips in one knapsack

The exposure value of each strip is calculated based on the previous results. The area of each strip for a given knapsack is equal and pre-determined to achieve a compromise between solution time and solution quality. Using a smaller area for each strip can give a more precise solution for the problem, since the area of each knapsack is closer to the summation of the area of strips. However using a smaller area for each strip means a larger number of strips in the entire area. This results in a larger number of variables, which causes longer solution times. In the test problem experiments, strip areas of 200 or 100 square feet were used depending on the difficulty of solving the problems.

4.3.2 Initial mathematical strip models

Multiple strip models were developed and are now described in detail. Each knapsack is divided into strips of equal area. There are two ways to determine the area of the strips. First, one can use a standard amount, such as one hundred square feet. However, this might create too many decision variables in the formulation. Second, if one knows the area of each department, one can find the least common divisor of the department areas in the knapsack. The exposures of the strips are calculated by integration that can be done off-line. Then the departments are assigned to specific locations within the knapsack based on the exposure of the strips and the corresponding profit for each strip (which varies depending on the location of the strip).

4.3.2.1 Fixed department area

First the strip model is formulated with fixed department areas - meaning that the number of strips that one department occupies is fixed. Problem notation for this section is given below.

Decision Variables:

x_{it} If x_{it}=1, then department *i* occupies strip *t*; otherwise, x_{it}=0.

Parameters:

s_i: The nominal area of department *i*.

p_i: The profit that results from placing department *i*.

α_i The exposure sensitivity of department *i*.

e_t: The whole exposure of strip *t*.

S_j: The area of knapsack *j*, which is a sub-area of the whole retail area, where

$$\sum_{j=1}^{5} S_j = L*W \ .$$

a The area of each strip.

When the area of each department is fixed one can give the following mathematical formulation for the problem.

Formulation (9)

$$\max \sum_{t=1}^{T} \sum_{i=1}^{I} e_t \alpha_i p_i x_{it}$$

$$st. \quad a \sum_{t=1}^{T} x_{it} = s_i, \ \forall i \qquad (72)$$

$$\sum_{t=0}^{T} \left| x_{i,t} - x_{i,t+1} \right| \leq 2, \quad \forall i \qquad (73)$$

$$\sum_{i=1}^{I} x_{it} \leq 1, \ \forall t \qquad (74)$$

$$x_{it} \in \{0,1\}$$

Constraint (72) ensures the area allocated to department i equals the area that department i needs. Constraint (73) ensures that if all departments are contiguous then there are at most two transitions of departments for any department i. The way constraint (73) works is explained by the definition of x_{it}. The value of x_{i0} is set equal to zero for all departments and $x_{i,T+1}$ equals zero for all departments. The summation in equation (73) runs from t=0 to T, to ensure only two transitions occur for a department. The two transitions occur when another department changes to the current one, and the current one changes to another one, which is shown in the left part of Figure 32. If more than two transitions occur in a certain department, as shown in the right part of Figure 32, it indicates that the department is not allocated contiguously. Constraint (74) ensures each strip can be allocated to only one department.

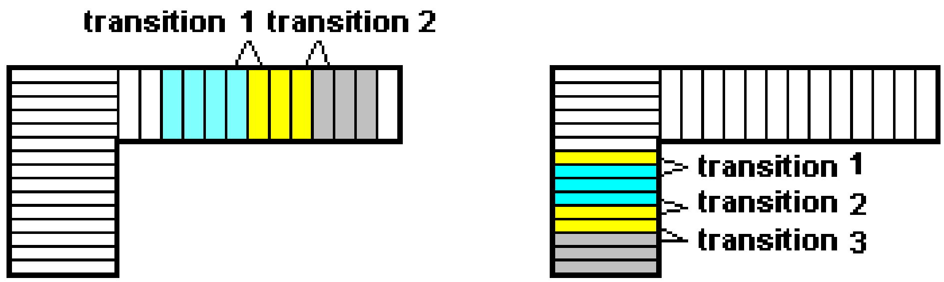


Figure 32. The explanation of constraint (73)

4.3.2.2 Variable department area

If the area of each department is variable with a nominal value represented by s_i, the following mathematical formulation is given for the problem.

Formulation (10)

$$\max \sum_{t=1}^{T} \sum_{i=1}^{I} e_t \alpha_i p_i x_{it}$$

$$st. \quad 0.8 s_i \le a \sum_{t=1}^{T} x_{it} \le 1.2 s_i, \ \forall i \qquad (75)$$

$$\sum_{t=0}^{T} \left| x_{i,t} - x_{i,t+1} \right| \le 2, \quad \forall i \qquad (76)$$

$$\sum_{i=1}^{I} x_{it} \le 1, \ \forall t \qquad (77)$$

$$x_{it} \in \{0,1\}$$

Constraint (75) allows department i to shrink or expand by 20 percent of the nominal area. Constraints (76) and (77) work the same as in the previous model.

4.3.2.3 Considering the adjacency preference between departments

An additional thought is to consider the outer loop knapsacks as one large entity. Then one can take adjacency preferences into consideration during detailed design. Additional notation is presented followed by the model formulation.

Decision Variable:

x_{it} If x_{it}=1, then department i is in strip t; otherwise, x_{it}=0. Specially $x_{i,0}$= $x_{i,t+1}$=0.

Z_{ijt} If Z_{ijt} =1, then department i is in strip t, j is in strip $t+1$; otherwise, Z_{ijt}=0, where t ranges from 1 to T.

Parameters:

$m_{i,k}$ A parameter that indicates if department i is in knapsack k.

$n_{t,k}$ A parameter that indicates if strip t is in knapsack k.

s_i The nominal area of department i.

p_i The profit that results from placing department i.

e_t The whole exposure of strip t.

So The sum of the area of the outer knapsacks, which is a sub-area of the entire retail area, where $S_o = L*W - S_c$. (S_c is the sum of the central knapsack area)

a The area of each strip.

$w_{i,j}$ The adjacency preference of department i and department j.

Formulation (11)

$$\max \sum_{t=1}^{T}\sum_{i=1}^{I} e_t \alpha_i p_i x_{it} + \sum_{t=1}^{T}\sum_{i=1}^{I}\sum_{j\neq i}^{I} w_{i,j} Z_{ijt}$$

$$st. \quad 0.8 s_i \le a\sum_{t=1}^{T} x_{it} \le 1.2 s_i, \forall i \qquad (78)$$

$$\sum_{1}^{I} s_i = S_o \qquad (79)$$

$$\sum_{t=0}^{T} \left| x_{i,t} - x_{i,t+1} \right| \le 2, \quad \forall i \qquad (80)$$

$$\sum_{i=1}^{I} x_{it} \le 1, \forall t \qquad (81)$$

$$2 * Z_{ijt} \le x_{i,t} + x_{j,t+1}, \forall i, j, t \qquad (82)$$

$$2 * x_{it} \le m_{ik} + n_{tk} \qquad (83)$$

x_{it}, Z_{ijt} binary

(m_{ik}, n_{tk} known binary)

4.3.2.4 Simplified formulation

Formulation (11) is an ideal situation, meaning the aisle location can shift in order to accommodate the various areas of the departments. But the problem is difficult to solve and if the aisles are allowed to move then the exposure of the strips change, which greatly complicates the model. Due to these difficulties, a simplified formulation is generated. If one assumes that the branch aisle location is fixed, which means the knapsack areas are predetermined, Formulation (11) can be simplified as Formulation (12). Assume that, for each knapsack *k*, the strip index inside knapsack *k* is from ts_k to te_k.

Formulation (12)

$$\max \sum_{i=1}^{I}\sum_{t=ts_i}^{te_i} e_t \alpha_i p_i x_{it} + \sum_{i=1}^{I}\sum_{j=1, j\neq i}^{I}\sum_{t=ts_i}^{te_i} w_{i,j} Z_{i,j,t}$$

$$st. \quad 0.8 s_i \le a \sum_{t=ts_i}^{te_i} x_{it} \le 1.2 s_i, \ \forall i \qquad (84)$$

$$\sum_{t=ts_i-1}^{te_i} \left| x_{i,t} - x_{i,t+1} \right| \le 2, \quad \forall i \qquad (85)$$

$$\sum_{i=1}^{I} x_{it} \le 1, \ \forall t \qquad (86)$$

$$2 * Z_{i,j,t} \le x_{i,t} + x_{j,t+1}, \forall i, j \neq i, t \qquad (87)$$

$$x_{it}, Z_{i,j,t} \text{ binary}$$

Since the aisle locations are predetermined and the area of each strip is fixed, it is possible to determine the starting and ending strips of each knapsack. Based on the results of the department allocation problem, the assignment of departments to knapsacks is known, so for each department, the potential starting or ending strip is the starting or ending strip for the particular knapsack the department belongs to rather than from 1 to *T*. Therefore, one can reduce the number of constraints and the number of variables in the model and the problem can be solved more efficiently.

In Formulation (12) the absolute value constraint (85) can be rewritten as constraints (88) and (89). They have the same function, except the absolute value constraints are not acceptable in CPLEX. By introducing new variables and one more set of constraints, the absolute value constraints can be rewritten as.

$$\sum_{t=ts_i-1}^{te_i} \left| x_{i,t} - x_{i,t+1} \right| \le 2,\ \forall i$$

$$\Leftrightarrow$$

$$x_{i,t} - x_{i,t+1} = pos_{i,t} - neg_{i,t} \tag{88}$$

$$\sum_{t=ts_i-1}^{te_i} (pos_{i,t} + neg_{i,t}) \le 2,\ \forall i \tag{89}$$

4.3.3 Revised mathematical strip models

The initial strip models require considerable computation time and therefore were revised to make them more computationally efficient. By introducing a first and last strip index as decision variables, a revised mathematical model can be formulated as follows. Two sets of new decision variables are introduced, a first and last strip index, f_{it} and l_{it}, respectively . If department i starts at strip t, then $f_{it} = 1$. Similarly, if department i ends at strip t, then $l_{it} = 1$. Problem notation for this section is given below.

Decision Variables:

y_{it} Occupancy index. If y_{it}=1, then department i is in strip t; otherwise, y_{it}=0.

f_{it} First strip index. If f_{it}=1, then department i starts from strip t, otherwise f_{it} =0.

l_{it} Last strip index. If l_{it}=1, then department i ends at strip t; otherwise l_{it} =0.

Z_{ijt} If Z_{ijt} =1, then department i is in strip t, j is in strip $t+1$; otherwise, Z_{ijt}=0, where t ranges from 1 to T.

Parameters:

s_i: The nominal area of department i.

p_i: The profit that results from placing department i.

α_i The exposure sensitivity of department i.

e_t: The whole exposure of strip t.

a The area of each strip.

Formulation (13)

$$\max \sum_{i=1}^{I}\sum_{t=ts_i}^{te_i} e_t \alpha_i p_i y_{it} + \sum_{i=1}^{I}\sum_{j=1, j\neq i}^{I}\sum_{t=ts_i}^{te_i} w_{ij} Z_{ijt}$$

$$st.\ 0.8 s_i \le a \sum_{t=ts_i}^{te_i} y_{it} \le 1.2 s_i,\ \forall i \quad (90)$$

$$\sum_{i=1}^{I} y_{it} \le 1, \forall t \quad (91)$$

$$\sum_{t=ts_1}^{te_i} f_{it} = 1,\ \forall i \quad (92)$$

$$\sum_{t=ts_1}^{te_i} l_{it} = 1,\ \forall i \quad (93)$$

$$f_{it} - y_{it} \le 0, \forall i, t \quad (94)$$

$$l_{it} - y_{it} \le 0, \forall i, t \quad (95)$$

$$y_{i,t+1} - y_{it} - f_{i,t+1} \le 0, \forall i, t \quad (96)$$

$$y_{it} - y_{i,t+1} - l_{it} \le 0, \forall i, t \quad (97)$$

$$y_{it} - y_{j,t+1} - 2Z_{ijt} \ge 0, \forall i, j, t \quad (98)$$

$$y_{i,ts-1} = 0, \forall i \quad (99)$$

$$y_{i,te+1} = 0, \forall i \quad (100)$$

$$y_{it}, Z_{ijt} \in \{0,1\}$$

In Formulation (13), Constraint (90) is the same as Constraint (84). Constraint (91) is the same as constraint (86). Constraints (92) and (93) limit a department so it can only start and end once it is in the knapsack it belongs. Thus, they insure that each department is placed in the proper knapsack. Constraints (94) and (95) insure that f_{it} and l_{it} can only equal 1 when y_{it} equals 1. Constraints (96) and (97) are derived from the definition of the first and last strip index. Constraint (98) is the same as constraint (87).

Constraint (99) and (100) are the boundary conditions. The new formulation gives the same solution as the previous formulation but is generally more efficient.

4.3.4 Case study: Kohl's department store

In this sub-section, the same case study used in the previous sections is investigated. From the results of the last two steps, one can calculate the accumulated exposure of each strip, depending on the strip area chosen. Also the real area of each knapsack and the nominal area of each department are known, and the exposure sensitivities and adjacency preference of the departments are the same as before. Formulation (12) and Formulation (13) will be used to solve the problem.

4.3.4.1 Background

In Section 4.1, the optimal aisle structure was obtained for the given Kohl's store size, so the outer-loop's knapsacks information is known, which includes their exposure distributions and their individual areas. From the results of Section 4.2, the optimal allocation of departments in the store was obtained. In this section, the strip model is developed and applied to calculate the specific location for each department. Formulation (12) and Formulation (13) will be applied to optimize the area of each department to maximize the profit of the entire store.

4.3.4.2 Model input

The same 24 department example Kohl's store is studied. There are 8 sub-areas, each containing one to eight departments. Recall that there were two optimal results from the last section, corresponding to different preferences of exposure versus adjacency. After analyzing the solutions and comparing them with actual department store layouts, the second result, which places more emphasis on adjacency, will be used as the input for this step.

Thus, Formulation (12) and Formulation (13) are used to solve the department location problem for the five knapsacks in the outer sub-areas. The Kohl's model is solved for 10 departments in knapsacks 1 to 5. The areas of the 10 departments are shown in the following table. The sub-area is divided into 233 strips each with an area of 100 ft^2, which is close to the least common divisor of the department areas. In the first column, the knapsack indexes are shown as well as the numbers of strips in knapsacks. The department profits per unit area are also shown in the table. The adjacency preference among departments is the same as that used in Sections 4.1 and 4.2. The possible starting strip and possible ending strip of each department are determined by the results of Section 4.2, which assigned each department to a knapsack. The exposures of the 233 strips are calculated off-line.

Table 17. Kohl's department store information for knapsacks 1 to 5

Knapsack Index (number of strips)	Dept. Index	Dept. Name	Dept. Area	# of Strips	Dept. Unit Profit	Dept Exposure Sensitivity
1(40)	10	Young Men's	2554	26	299	M
2 (65)	7	Boys	2971	22	393	M
	18	Furniture	602	6	286	L
	21	Luggage	1434	14	286	L
	22	Frames/Stationary	922	9	229	M
	24	Toys	410	4	221	H
3(65)	6	Infants and Toddles	3046	31	399	M
	8	Girls	2311	18	393	M
4(40)	13	Misses Casual	4493	34	308	H
5(23)	9	Juniors	2688	20	299	M

Similarly, the departments inside the racetrack aisle are optimized in the next step.

Table 18. Kohl's department store information for knapsacks 6 to 8

Knapsack Index	Dept. Index	Dept. Name	Dept. Area	# of Strips	Dept. Unit Profit	Dept Exposure Sensitivity
6 (77)	16	Shoes	2988	23	421	H
	19	Home Decoration	1037	11	333	H
	20	Bedding and Bath	2355	24	286	M
	23	Kitchen / Dining	4205	32	286	H
7 (77)	3	Intimate Apparel	2170	22	339	L
	11	Men's	8456	62	299	M
8 (77)	1	Cosmetics / Fragrances	806	8	411	H
	2	Handbags	672	5	478	M
	4	Jewelry	1210	12	880	H
	5	Accessories	672	5	478	M
	12	Misses Career	3412	34	308	M
	14	Petite	897	9	308	M
	15	Women's	2285	23	308	M
	17	Hosiery	1344	10	246	H

Figure 33 shows the knapsack numbers. The strip division pattern in each knapsack is shown in subsequent figures.

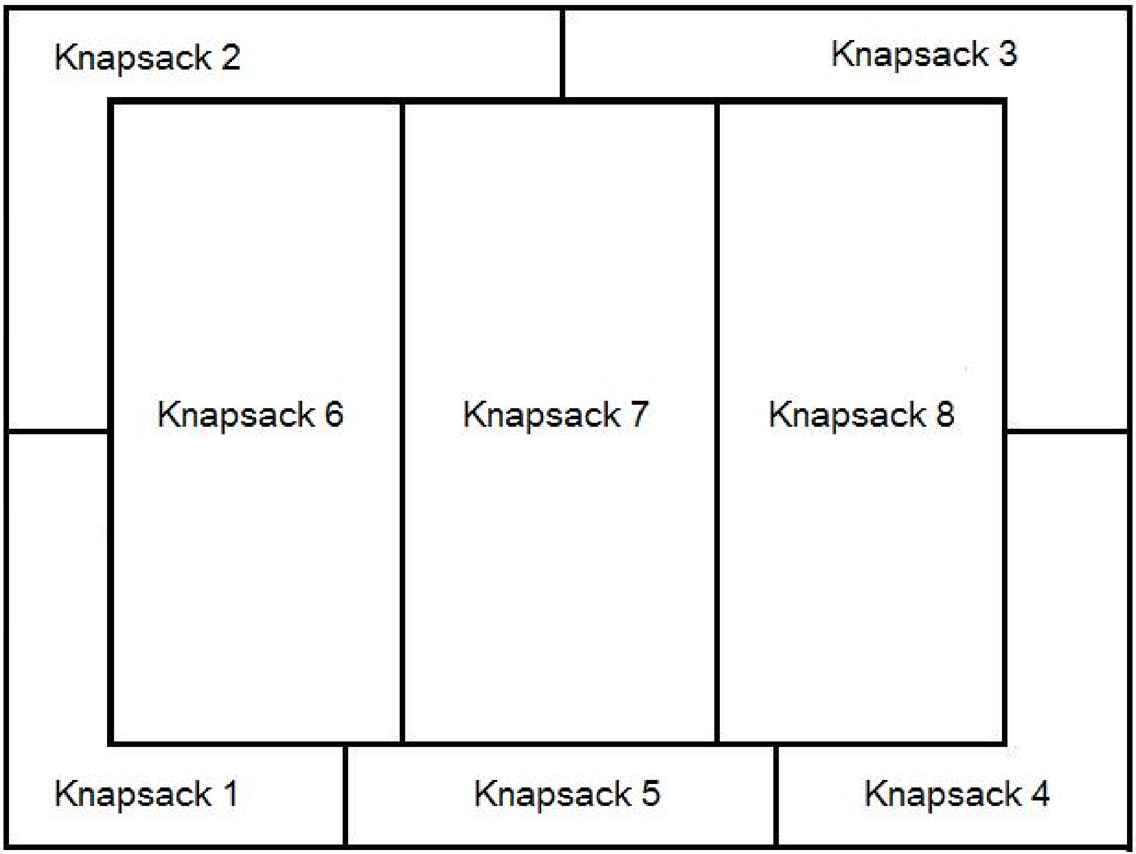


Figure 33. Numbered knapsack illustration

The strips for each subarea are divided as shown in the following figure. For knapsacks 1 to 5, the strips run from Knapsack 1 to Knapsack 5 with the same areas for every strip. The five knapsacks are optimized at one time and the exposure of each strip is calculated off-line.

Knapsacks 6 to 8 are handled a bit differently since they are in the center of the layout. The exposures of the center parts in Knapsacks 6 to 8 are relatively low compared to the edges of the knapsacks. There is a relatively long walking distance for a customer to reach the merchandise from the aisle. A good and practical solution, often seen in practice, is to build a central divider to minimize the visual distance from the aisles. Therefore, it is assumed that there will be walls running down the center of each knapsack that divide each knapsack into two separate areas. The walls may be required for building integrity and/or for displaying merchandise. Given the presence of these walls, the strips in the middle knapsacks run like a cycle as shown in Figure 34. The knapsack is divided into two sides and the strips run in the two sides with a cyclic pattern. Since at the two ends of central walls, there could be the same department or two different departments. The cyclic pattern works well to avoid departments' discontinuity. As in the outer knapsacks, the area of every strip in the inner knapsacks is the same. The three knapsacks are optimized individually and the exposure is calculated off-line.

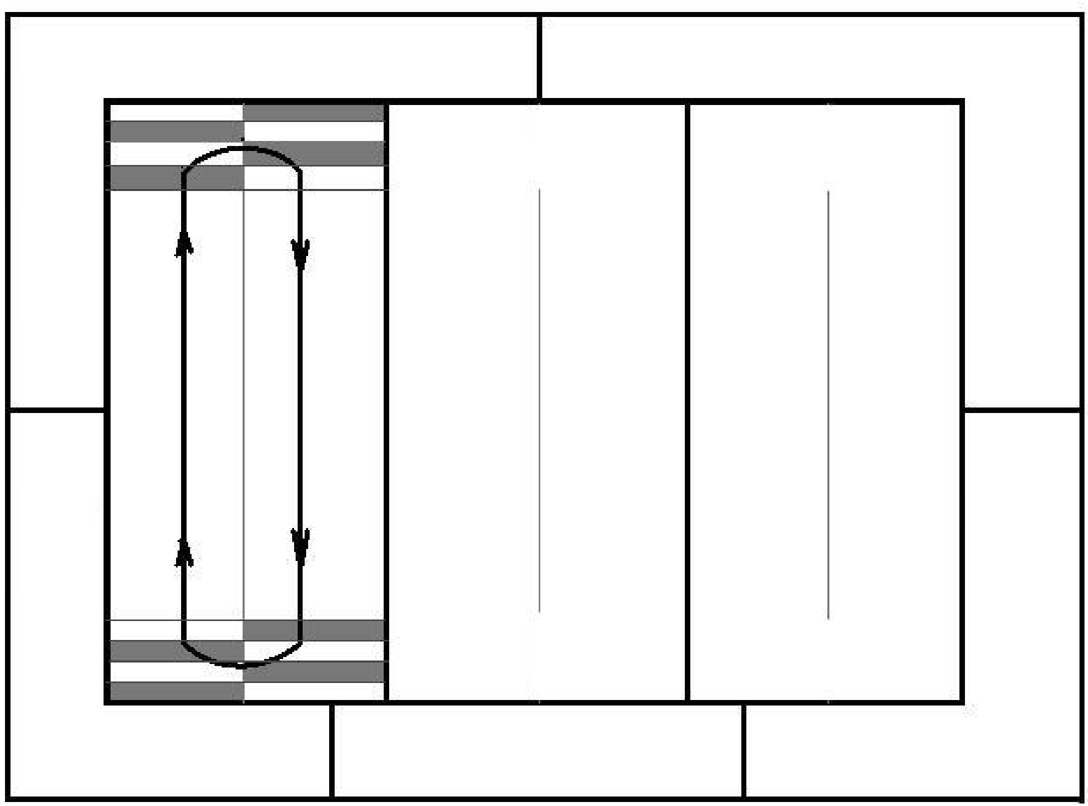


Figure 34. Strip pattern in Knapsack 6

Strip patterns in Knapsack 7 and 8 can be drawn similarly. The strip pattern for the entire area is shown in Figure 35.

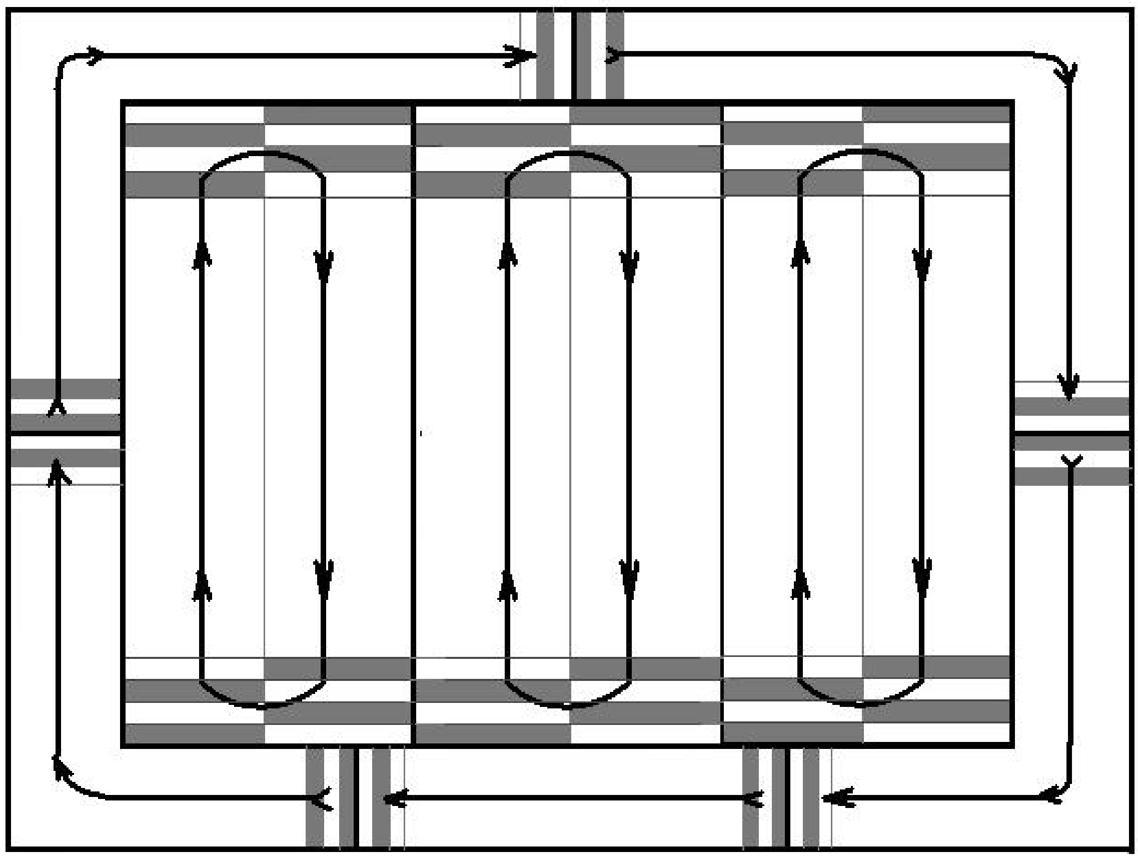


Figure 35. Strip illustrations of all knapsacks

4.3.4.3 Optimized results

For this case, Formulation (12) is used with a strip area of 100 square feet resulting in 233 strips for the 5 outer knapsacks and a strip area of 220 square feet resulting in 77 strips for Knapsack 6 to 8 individually. The optimal solutions for those settings are shown below.

Knapsack	Mipgap	MIP best bound	Solution time	Iterations	Nodes
1 to 5	0.01%	2.8*10e6	18042.23 sec	39211304	105352
6	0.01%	2.1*10e6	184.58 sec.	501333	6143
7	0.01%	9.7*10e5	0.08 sec.	564	0
8	20.52%	2.35*10e6	377977 sec.	521061923	739300

Formulation (13) has shorter solution times and the same solution quality.

Knapsack	Mipgap	MIP best bound	Solution time	Iterations	Nodes
1 to 5	0.01%	2.8*10e6	165.16 sec.	197497	644
6	0.01%	2.1*10e6	50.42 sec.	76749	530
7	0.01%	9.7*10e5	0.17 sec.	957	0
8	0.01%	2.4*10e6	1375.96 sec	971855	570

From the tables above a few conclusions can be drawn. Knapsack 8 has the longest solution times and this is because there are more departments in the knapsack. Formulation (13) is a better formulation in terms of solution time. The input for this model, which is the solution from the department allocation sub-problem, is shown Figure 36. The output of this model, which is the optimal solution of this sub-problem, is shown in Figure 37.

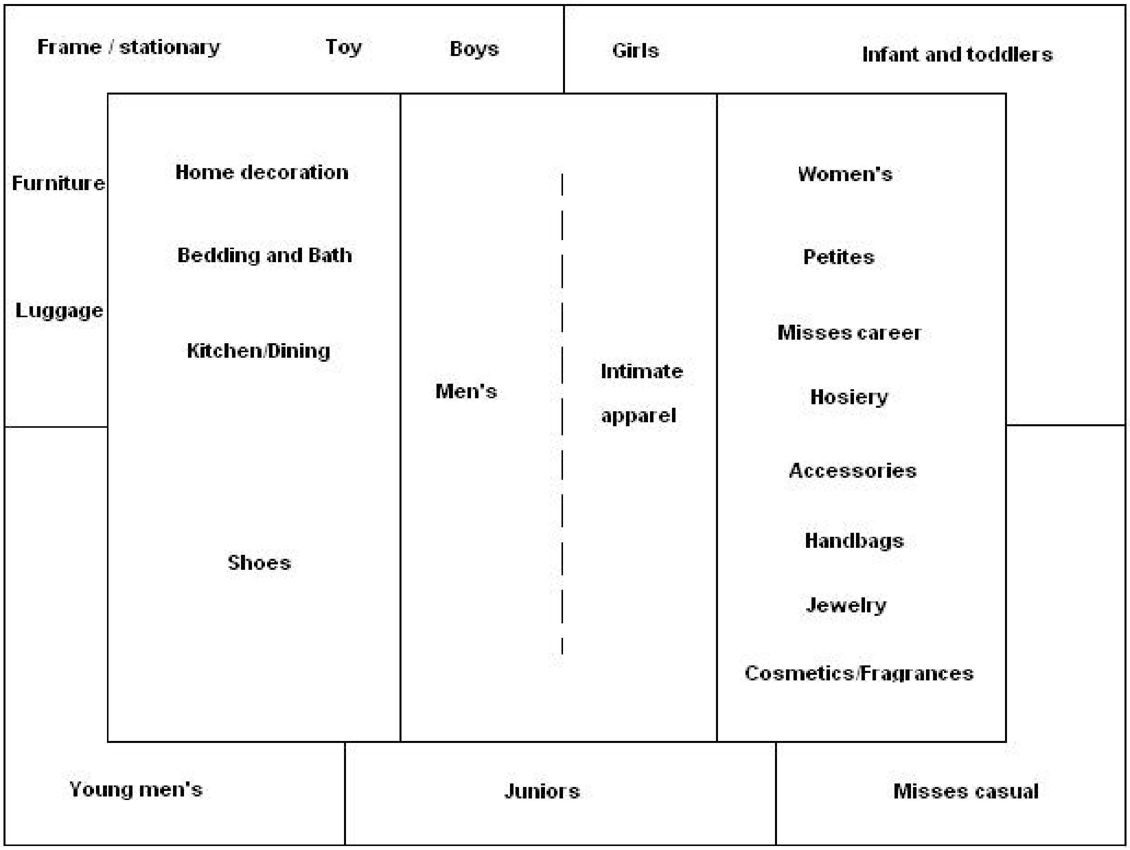


Figure 36. Input for the strip model

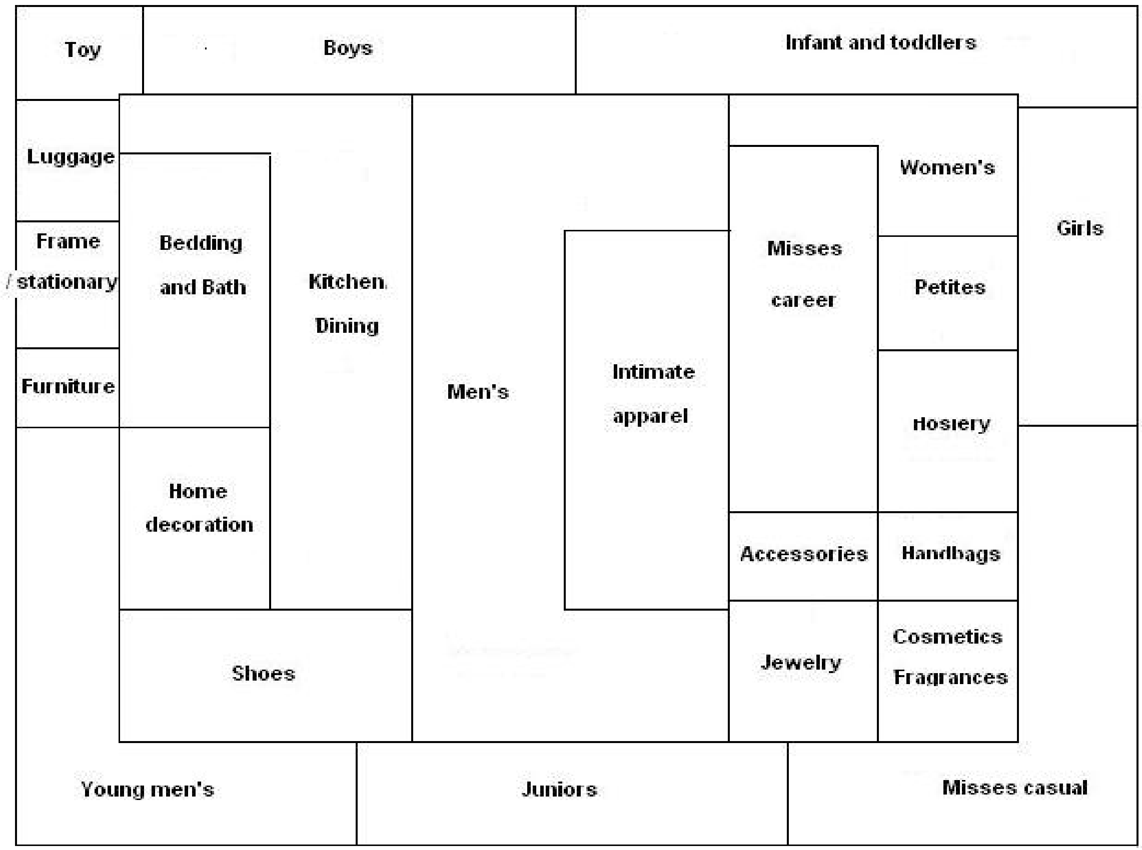


Figure 37. Optimized results for the strip model

The optimal result can be further massaged to get more regular department shapes by shifting the display wall in the middle of knapsacks 6 and 8. The result is the following figure.

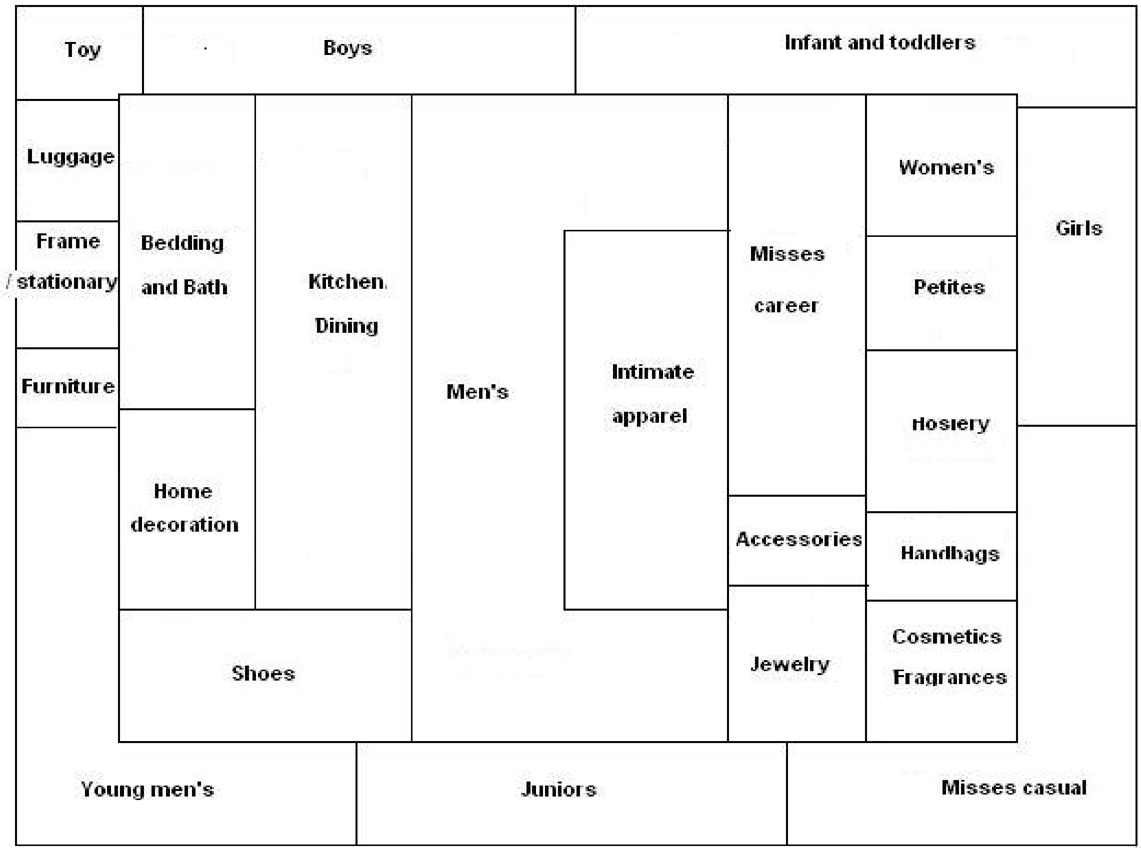


Figure 38. Massaged layout

4.3.4.4 Heuristic approach

As mentioned above, using a smaller area for each strip will give a more precise solution for the problem, but too many variables makes the problem hard to solve. Therefore, several examples were tested to compare the solution time and solution quality of different strip areas.

In the CPLEX 9 environment, for an instance of 20 departments and 5 knapsacks, if the area of each strip is 100 square feet, it took 274460 seconds to get the optimal solution. Next, strips with area equal to 200 square feet were used, and the original areas of the departments were rounded up or down to integral multiple times of 200. Since the number of constraints and variables decreases significantly, the optimal solution was found in 3909 seconds, which is 1 percent of the original solution time. However, the solution quality is not as good as the original problem, because of the approximation to the area of each department. To test the quality of the approximate solution, the department order found using the approximate problem was evaluated in the original problem and the objective value had a 1.7% gap between the new objective and the original optimal objective function values.

Although the approximate solution is only a couple of percent from the optimal solution, a heuristic can be developed to improve the solution quality of the approximate solution and increase the efficiency. After obtaining the optimal solution from the approximate problem, which used larger strip areas, a second round of optimization is applied. In this second round, the order of departments is re-optimized in one knapsack at a time, keeping the department order unchanged in the rest of the knapsacks. A strip area of 100 square feet is used as in the original problem. Since the order of most of the departments has been fixed, it is much easier to solve the problem. For the instances used to test the heuristic approach, the solution time for each knapsack took from less than 1 second to 5 minutes. After one or two rounds, the same optimal solution is found as with the original approach. In general, the total solution time was reduced to 2% of the original 76 hours for the same instance we mentioned at the beginning of this section. Thus, this approach looks promising in the event that Formulation (13) cannot solve larger knapsacks quickly enough.

4.3.4.5 Conclusions and discussion

There are several factors involved in the model which can affect the solution quality and solution time including: the area of each strip, the area change allowance of each department, the balance between the two objectives (exposure and adjacency) and the number of departments in the same knapsack.

Choosing the strip area requires careful consideration of the balance between solution time and solution quality. A smaller strip area provides more precise solutions to the problem. Facility layout decisions are made one time only or changed after a relatively long period of store operation. Therefore, the solution quality is more important than the solution time, as long as the problem can be solved within a reasonable amount of time.

A longer solution time is required when the department sizes are allowed to change within a larger range since more nodes will be branched on during the solution process. The balance between the two objectives of adjacency and exposure is similar to what has been stated in previous sections. If the decision maker would like to place more emphasis on adjacency then a larger weight can be put on adjacency in the objective function. Otherwise, the decision maker can emphasize the effect of impulse purchases by putting more emphasis on exposure.

From the solution time in the tables above, 2 or 4 departments in a knapsack can be easily solved in minutes. However, 8 departments in a knapsack makes the problem much harder in terms of solution time. For Formulation (12), the knapsack of size 8 could not be solved in 100 hours, which highlights the benefit of using Formulation (13). Clearly, the number of departments has a big effect on the solution time.

5.0 A COMPLETE APPLICATION

5.1 INTRODUCTION

In Chapter 4, a three-step approach to design and optimize the layout of department stores was developed. A Kohl's department store was used as an example to show the methodology. In this chapter, a Sears department store is used as an example to show a complete application of the methodology to a different type of store. Kohl's stores are usually free-standing stores, while Sears stores are often found in shopping malls as an anchor store. There are several differences between the two stores, including customer traffic flow, the assortment of departments found within the store, the facility areas and the aisle structures. From Figure 39, one can see there is a similar type of racetrack central aisle Also, because of the larger overall store area; there are several vertical central aisles and one horizontal central aisle. The Sears store has two main entrances and a side entrance into the mall. There are also more categories of merchandise in Sears than in a Kohl's store, including automotive, lawn and garden, sporting and fitness goods, etc. This case study considers the similarities and differences between the two store types.

Figure 39. Sears store layout, Pittsburgh

Based on the map and field observations, store information was collected, including the number of departments, the area of the departments, and fixture dimensions such as the entrances, side-entrances (including the mall entrance), as well as checkout counters and basic aisle structures.

As shown in the map in Figure 39, there are 34 departments (shown with different colors) in the store. However one can cluster the departments into several groups. For example,

1) Clothing, which includes infant and toddler, girls (4-6 and 7-16), boys (4-7 and 8-16), juniors, misses, women's, petites, maternity, intimates, men and young men's.

2) Shoes, which includes kids', men's , women's and athletic shoes.

3) Home, which includes bedding and bath, mattresses, cookware, housewares, etc.

4) Unique departments including paint, tools and auto care, pet supplies, books, bicycles, electronics, etc. Many of these are found in Sears but not in other department stores such as Kohl's, JCPenney and Macy's.

In the remainder of this chapter, the methodologies introduced in the previous chapters will be applied and the optimized result will be discussed.

5.2 INPUT DATA

5.2.1 The aisle structure sub-problem

In order to apply the proposed solution methodology it is first necessary to have the proper data. This section describes the data that was collected. Unlike Kohl's stores, which are usually free standing stores, Sears' stores are usually located in shopping malls as anchor stores. As a large department store, it is necessary for the financial stability of the store and the shopping mall, for the store to draw retail traffic that would result in visits not only to the Sears store but also to the smaller stores in the mall. Therefore, the store has four entrances, including two parking lot entrances, one side entrance which lets customer come and go through car services, and one mall entrance.

Several central aisles can be found on the layout. These aisles occupy retail space but increase the exposure of the retail area. The cashiers are near the exits to facilitate customer traffic through the store. There are two lanes for checkout near the mall entrance. A much larger checkout area is located near the parking lot entrances. The customer service area and the restrooms are in the corner, where they don't take any of the most valuable retail space, and require customers to travel all the way through the store to access these areas.

This Sears store, which is in the Pittsburgh Mills Mall, has a size of 450 * 254 sq ft., as shown in Figure 40. The width of the main entrance aisle, the mall entrance and side branch aisles are 40ft., 30 ft. and 6 ft., respectively. The race track aisle, one horizontal aisle and four central aisles are 8ft., 10ft., and 6ft., respectively. The size of the two cashier areas near the parking entrances are 80 ft. * 28 ft. each. The size of the two cashier stations near the mall entrance are 28 ft. *8 ft. each. Six exposure decay functions are tested to select the optimal aisle configuration. Each scenario uses the following data.

The width of the main entrance aisle = 40,

The width of the mall entrance aisle =30,

The width of the racetrack aisle = 8,

The width of the branch aisle or side entrance = 6,

The width of the horizontal central aisle = 10,

The width of the vertical central aisles = 6,

The cashier areas near the parking lot = 80 * 28,

The cashier areas near the mall entrance = 28 * 8.

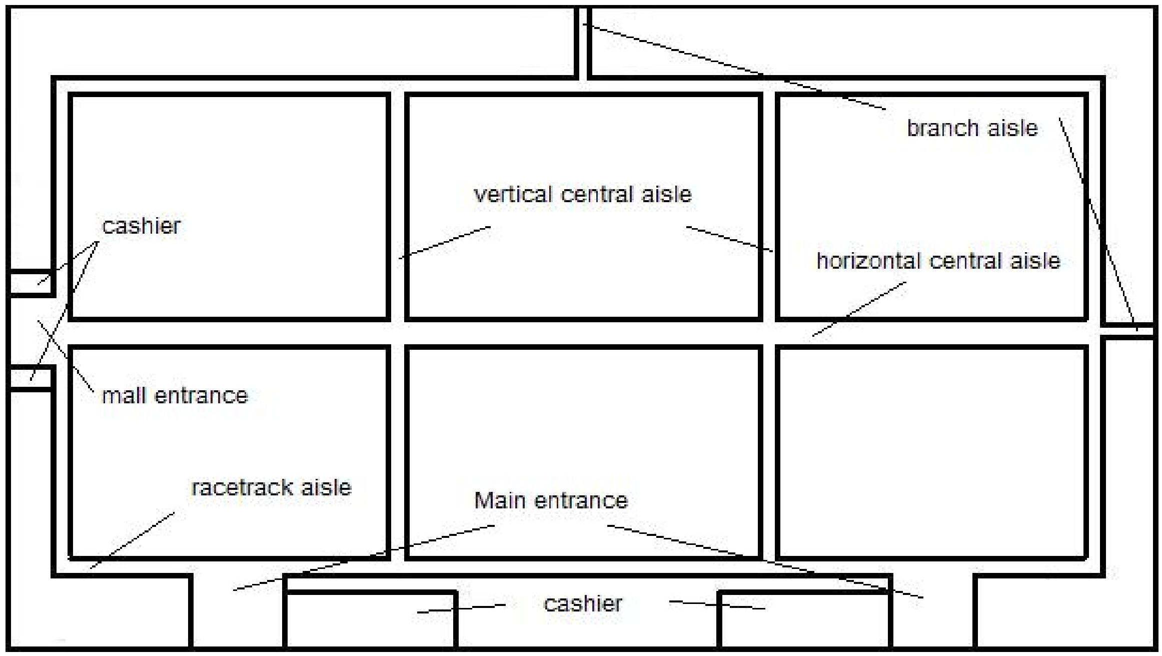


Figure 40. Sears layout illustration

5.2.2 The department allocation sub-problem

Recall that the model formulation requires three sets of data concerning the individual departments: revenue per square foot, nominal area and impulse purchase likelihoods. Also, data related to the relationships with other departments are also needed, e.g. the adjacency preferences between pairs of departments.

As discussed in the previous chapter, 24 departments are chosen as the common departments among all department stores. However there are several unique departments in Sears, which are not found in any Kohl's stores. Therefore, more departments are taken into consideration and involved in the optimization process. During the layout construction process, departments such as merchandise pickup, restrooms, auto service and the garden center (see Figure 39 the upper right corner through the lower right corner) are excluded. In addition, the facilities between the two entrance locations on the lower horizontal part of the layout (H&R Block, portrait studio, snacks, restrooms, customer service, optical and 3 day blinds) are also excluded. Finally there are seasonal or promotional items displayed in the aisle which are not included in the department list. The result is the 30 departments given in Table 19 with their associated areas.

Table 19. Sears department area information

Dept No.	Department Name	Area (sq ft)	Percentage
1	Cosmetics/Fragrances	2800	2.45%
2	Jewelry	800	0.70%
3	Accessories	2400	2.10%
4	Intimate Apparel	2400	2.10%
5	Infants and Toddlers	1600	1.40%
6	Toys	1200	1.05%
7	Boys	2000	1.75%
8	Girls	3400	2.97%
9	Juniors	2400	2.10%
10	Young Men's	2200	1.92%
11	Men's	4500	3.94%
12	Misses Career	3400	2.97%
13	Misses Casual	1000	0.87%
14	Petites	600	0.52%
15	Women's	2800	2.45%
16	Shoes	5600	4.90%
17	Hosiery	800	0.70%
18	Bedding and Bath	4000	3.50%
19	Furniture	1400	1.22%
20	Home Dec (Window)	4200	3.67%
21	Kitchen/Dining	2200	1.92%
22	Frames/Stationary	800	0.70%

Table 19 (continued)

23	Books	1400	1.22%
24	Tools	3600	3.15%
25	Paint	1600	1.40%
26	Appliances	6600	5.77%
27	Electronics	5200	4.55%
28	Auto	1000	0.87%
29	Lawn/garden	5600	4.90%
30	Sports goods	2000	1.75%
Else	Other departments	2600	2.27%
	Cashier	4200	3.67%
	Aisle area	28000	24.50%
Total area		114300	100.00%

For the layout in Figure 39, the total store area is measured as $450\text{ft} \times 254\text{ft}$, which is 114,300 sq. feet (excluding the customer service area, the order pick-up area, optical, etc). The sum of the total department areas is 82,100 sq. feet, which is about 71.83% of the store area. The cashier and aisle area is approximately 28.17%, which includes 3.67% for the cashier area and 24.5% for the aisle area.

For the sales figures of the identified departments, the data provided on BizStats' web page is utilized (http://www.bizstats.com/spf.malls.htm). Similarly, for the statistics that are not available it is assumed that the revenue generated by those departments can be approximated by similar departments for which statistics are provided. The table used for the 24 departments in Kohl's was expanded to 30 departments in Sears, which is shown in Table 20.

Table 20. Sears department profit information

Dept No.	Department Name	Unit Profit of
1	Cosmetics/Fragrances	411
2	Jewelry	880
3	Accessories	478
4	Intimate Apparel	339
5	Infants and Toddlers	399
6	Toys	221
7	Boys	393
8	Girls	393
9	Juniors	299
10	Young Men's	299
11	Men's	299
12	Misses Career	308
13	Misses Casual	308
14	Petites	308
15	Women's	308
16	Shoes	421
17	Hosiery	246
18	Bedding and Bath	286
19	Furniture	286
20	Home Dec (Window)	333
21	Kitchen/Dining	286
22	Frames/Stationary	229
23	Books	199
24	Tools	286
25	Paint	333
26	Appliances	286
27	Electronics	355
28	Auto	210
29	Lawn/garden	333
30	Sports goods	246

The next step is to classify the departments with respect to their impulse purchase likelihoods. This data is obtained from Bellenger et al.'s [#9, 1978] work.

Table 21 summarizes the department data based on the Sears layout, the revenue data from BizStat and Bellenger et al.'s work:

Table 21. Sears department exposure sensitivity information

Dept No.	Department Name	Impulse purchase	Impulse purchase
1	Cosmetics/Fragrances	33	H
2	Jewelry	62	H
3	Accessories	N.A.	M
4	Intimate Apparel	27	L
5	Infants and Toddlers	N.A.	M
6	Toys	N.A.	H
7	Boys	N.A.	M
8	Girls	47	M
9	Juniors	N.A.	H
10	Young Men's	36	L
11	Men's	40	M
12	Misses Career	44	M
13	Misses Casual	54	H
14	Petites	47	M
15	Women's	47	M
16	Shoes	52	H
17	Hosiery	N.A.	H
18	Bedding and Bath	N.A.	M
19	Furniture	N.A.	L
20	Home Dec (Window)	53	H
21	Kitchen/Dining	N.A.	H
22	Frames/Stationary	39	L
23	Books	N.A.	M
24	Tools	N.A.	L
25	Paint	N.A.	L
26	Appliances	N.A.	H
27	Electronics	N.A.	M
28	Auto	N.A.	L
29	Lawn/garden	N.A.	L
30	Sports goods	N.A.	M

The next step in the data collection process is determining the adjacency requirements among the departments. The previous matrix is expanded to account for the 30 departments in Sears. The data is shown in Table 22.

Table 22. Sears departments adjacency matrix

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1		A	A	I	O	U	O	O	I	I	E	I	I	O	O	I	I	O	O	O	O	O	I	U	U	U	O	U	U	U
2	A		A	I	O	O	O	O	I	I	I	A	E	I	I	O	I	O	O	O	O	O	U	U	U	U	U	U	U	U
3	A	A		E	I	O	O	O	O	O	I	A	I	I	I	O	A	O	O	O	O	O	U	U	U	U	U	U	U	U
4	I	I	E		I	O	O	A	O	O	I	I	O	O	O	I	I	I	O	A	O	O	U	U	U	U	U	U	U	U
5	O	O	I	I		O	A	A	O	O	O	O	I	O	I	O	U	I	O	O	O	O	O	U	U	U	U	U	U	U
6	U	O	O	O	O		O	O	U	U	O	U	U	U	O	I	U	O	E	E	E	I	O	O	O	O	I	O	U	O
7	O	O	O	O	A	O		A	O	O	O	I	I	O	I	I	O	O	O	O	O	O	I	U	U	U	U	U	U	U
8	O	O	O	A	A	O	A		O	O	O	I	O	O	I	I	I	E	O	O	O	I	I	U	U	U	U	U	U	U
9	I	I	O	O	O	U	O	O		A	E	I	A	O	O	O	O	O	U	O	O	U	I	U	U	U	U	U	U	U
10	I	I	O	O	O	U	O	O	A		A	O	I	O	O	O	O	I	U	O	O	I	I	U	U	U	I	U	U	O
11	E	I	I	I	O	O	O	O	E	A		O	O	I	O	A	A	E	O	O	I	O	I	U	U	U	I	U	U	O
12	I	A	A	I	O	U	I	I	I	O	O		A	A	A	I	O	O	O	O	O	O	I	U	U	U	O	U	U	U
13	I	E	I	O	I	U	I	O	A	I	O	A		I	I	O	I	O	O	O	O	O	I	U	U	U	O	U	U	U
14	O	I	I	O	O	U	O	O	O	O	I	A	I		E	O	E	U	U	U	U	U	I	U	U	U	O	U	U	U
15	O	I	I	O	I	O	I	I	O	O	O	A	I	E		I	O	O	O	O	O	O	I	U	U	U	O	U	U	U
16	I	O	O	I	O	I	I	I	O	O	A	I	O	O	I		O	I	O	I	I	O	O	O	U	U	U	U	U	U
17	I	I	A	I	U	U	O	I	O	O	A	O	I	E	O	O		I	U	O	O	O	O	O	U	U	U	U	U	U
18	O	O	O	I	I	O	O	E	O	I	E	O	O	U	O	I	I		A	A	A	O	O	I	I	I	O	O	O	O
19	O	O	O	O	O	E	O	O	U	U	O	O	O	U	O	O	U	A		I	I	A	O	I	I	I	I	I	I	I
20	O	O	O	A	O	E	O	O	O	O	O	O	O	U	O	I	O	A	I		A	I	O	I	E	I	I	O	I	O
21	O	O	O	O	O	E	O	O	O	O	I	O	O	U	O	I	O	A	I	A		I	O	I	I	I	I	O	O	O
22	O	O	O	O	O	I	O	I	U	I	O	O	O	U	O	O	O	O	A	I	I		A	O	O	O	U	O	U	U
23	I	U	U	U	O	O	I	I	I	I	I	I	I	I	I	O	O	O	O	O	O	A		O	O	O	I	U	O	O
24	U	U	U	U	U	O	U	U	U	U	U	U	U	U	U	O	O	I	I	E	I	O	O		A	E	I	A	A	I
25	U	U	U	U	U	O	U	U	U	U	U	U	U	U	U	U	U	E	E	A	I	O	O	A		O	I	E	E	O
26	U	U	U	U	U	O	U	U	U	U	U	U	U	U	U	U	U	I	I	I	I	O	O	E	O		E	E	I	I
27	O	U	U	U	U	I	U	U	U	I	I	O	O	O	O	U	U	O	I	I	I	U	I	I	I	E		I	O	O
28	U	U	U	U	U	O	U	U	U	U	U	U	U	U	U	U	U	O	I	O	O	O	U	A	E	E	I		E	I
29	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	O	I	I	O	U	O	A	E	I	O	E		I
30	U	U	U	U	U	O	U	U	U	O	O	U	U	U	U	U	U	O	I	O	O	U	O	I	O	I	O	I	I	

The relationship values are converted to numeric values using the same method as for the Kohl's case. Recall that an "A" relationship means the two departments prefer to be located in the same knapsack. An “E” relationship means the two departments are usually located in adjacent knapsacks. An “I” relationship means there is at most one other department between the two departments considered. An “O” relationship usually shows the two departments have little relationship to each other. A “U” relationship means the two departments do not have any relationship to each other. There are not any “X” or “XX” relationships in this example.

Table 23. Adjacency scores

Rating	Definition	w_{ij}	c_{ij}
A	Absolutely Necessary	1	125
E	Especially Important	0.8	25
I	Important	0.5	5
O	Ordinary Closeness	0.2	1
U	Unimportant	0	0
X	Undesirable	/	-25
XX	Prohibited	/	-125

5.2.3 The detail design sub-problem

Similar to the Kohl's problem, given the department data and knapsack allocations, the next step is to determine the exact position of each department. Given the specific shape and aisle structure of the Sears store, the five outer sub-areas can be considered as four L-shaped polygons and one rectangle which are considered as one outer loop. The inner sub-area can be divided into three individual rectangles.

Recall that for formulation (12), the input of the model is (1) the strip information including the exposure of each strip, the area of each strip and the location of the strips in the sub-areas, (2) the department information, including the profit and adjacency preferences, (3) the department information related to the starting and ending strips for the knapsack a department belongs to. The input data are determined based on the previously optimized results.

First, the outer sub-area is studied. From the results of the department allocation sub-problem, the departments in knapsacks 1 to 5 are known. The nominal area of the departments can be found in Table 19. The sub-area is divided into strips, each with an area of 100 $ft.^2$, which is close to the least common divisor of the department areas. The department profit per unit area and exposure sensitivity are also shown in Table 20 and Table 21.

Second, the inner sub-area, which includes knapsacks 6 to 8, is studied. Similarly, each knapsack is divided into strips. The main difference in aisle structure between the Kohl's and Sears department stores is the vertical and horizontal branch aisle inside each knapsack. Because Sears has a much larger sales area, more branch aisles are necessary. Because of this fact, serpentine style strip chains are introduced as shown in Figure 41. For example, one vertical and one horizontal branch aisle divide knapsack 6 into four pieces. The serpentine line shows how the strips connect to each other. Only one horizontal branch aisle is in the middle knapsack, 7, and therefore the serpentine line is just one loop. The aisle structure in knapsack 8 is the same as knapsack 6 and therefore a serpentine line is used as in knapsack 6 (though not shown in Figure 41).

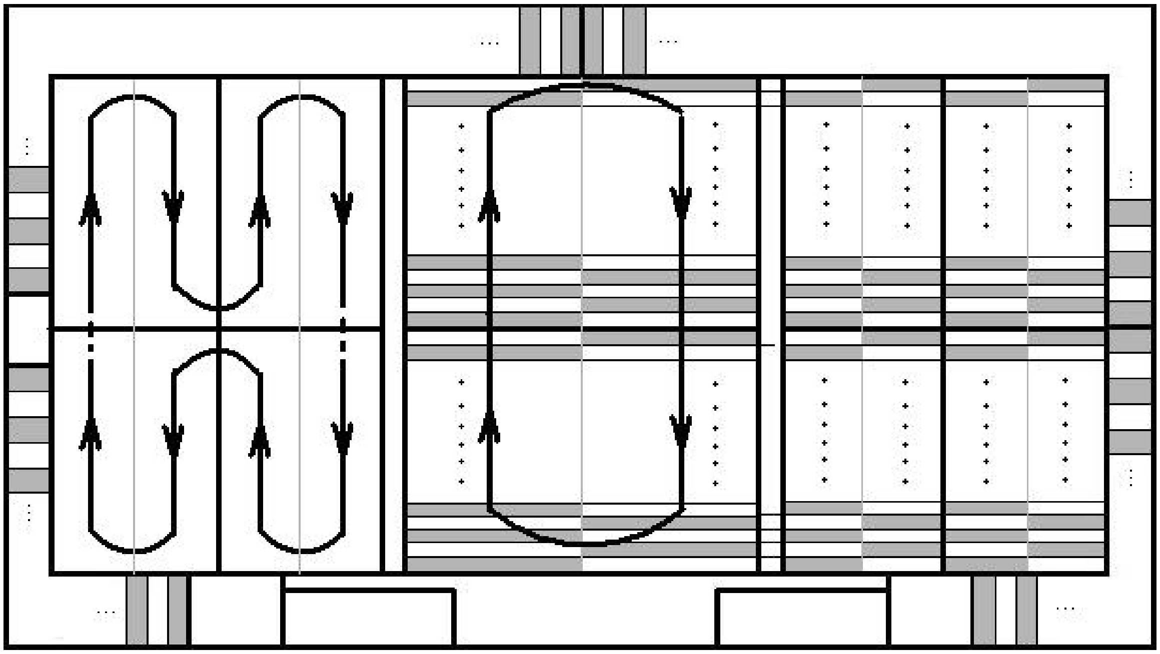


Figure 41. Inner sub-area serpentine strip chain

To complete the inputs, three more pieces of information are needed: (1) the area and exposure of the strips. The exposure of each strip is calculated off-line from the results of the first sub-problem; (2) the nominal area, unit profit and exposure sensitivity of the departments; and (3) the qualitative adjacency matrix.

As a summary, all the required input data are listed in this section. Then the three models generated in Chapter 4 are applied. In the next section, the optimized results are illustrated and the final detailed layout is shown.

5.3 OPTIMIZED RESULTS

5.3.1 The aisle structure sub-problem

Results for the aisle structure sub-problem are shown for different values of the three key parameters, which are distance threshold, retained exposure, and the six different exposure decay functions. These results are analyzed to find robust optimal solutions. Results are shown in Table 24.

Table 24. Sears optimal aisle structure solutions

Settings index	Setting of exposure decay			Optimal solution		
	Distance Threshold	Retained exposure	Decay function	X coordinate	Y coordinate	Exposure Value
1	20	0.14	1	16	20	67422.78
2	20	0.14	2	12	32	78994.99
3	20	0.14	3	12	32	55563.75
4	20	0.14	4	16	16	77021.74
5	20	0.14	5	16	20	84783.71
6	20	0.14	6	16	20	86384.47
7	20	0.3	1	16	20	88173.89
8	20	0.3	2	12	32	97521.69
9	20	0.3	3	12	32	78449.75
10	20	0.3	4	16	16	96004.86
11	20	0.3	5	16	20	102304.88
12	20	0.3	6	16	20	103607.83
13	20	0.5	1	16	20	114112.78
14	20	0.5	2	16	28	120734.59
15	20	0.5	3	20	16	107181.03
16	20	0.5	4	16	16	119733.76
17	20	0.5	5	16	20	124206.34
18	20	0.5	6	16	20	125137.02

Table 24 (continued)

19	30	0.14	1	16	28	86511.86
20	30	0.14	2	12	32	95869.24
21	30	0.14	3	12	28	68163.93
22	30	0.14	4	16	24	98405.15
23	30	0.14	5	20	28	111602.41
24	30	0.14	6	20	28	113965.73
25	30	0.3	1	20	28	103678.20
26	30	0.3	2	12	32	111256.55
27	30	0.3	3	16	28	88709.85
28	30	0.3	4	20	24	113404.57
29	30	0.3	5	20	28	124134.06
30	30	0.3	6	20	28	126057.68
31	30	0.5	1	20	28	125187.29
32	30	0.5	2	16	32	130523.57
33	30	0.5	3	16	28	114440.75
34	30	0.5	4	20	24	132162.12
35	30	0.5	5	20	28	139798.61
36	30	0.5	6	20	28	141172.63
37	40	0.14	1	16	32	103197.86
38	40	0.14	2	4	32	108105.77
39	40	0.14	3	12	32	78994.99
40	40	0.14	4	16	32	120643.94
41	40	0.14	5	4	36	133920.23
42	40	0.14	6	4	36	136749.99
43	40	0.3	1	16	32	117257.42
44	40	0.3	2	12	32	121174.73
45	40	0.3	3	12	32	97521.69
46	40	0.3	4	16	32	131457.72
47	40	0.3	5	4	36	142156.84
48	40	0.3	6	4	36	144460.13
49	40	0.5	1	16	32	134831.87
50	40	0.5	2	16	32	137585.05
51	40	0.5	3	16	28	120734.59
52	40	0.5	4	16	32	144974.94
53	40	0.5	5	16	32	152616.44
54	40	0.5	6	16	32	154209.88

The above 54 solutions are divided into 9 groups as in the Kohl's case. Similar robustness analysis was done for the 9 groups, and the following table shows the most robust solution for each setting

Table 25. The most robust solution for each scenario

Settings	Distance threshold	Retained exposure	Optimal solution	
			x	y
1	20	0.14	20	16
2		0.30	20	16
3		0.50	20	16
4	30	0.14	16	28
5		0.30	16	28
6		0.50	20	28
7	40	0.14	12	32
8		0.30	16	32
9		0.50	16	32

The corner location of (16, 32) with a distance threshold of 40 and retained exposure of 0.30 is chosen as the input for the next step because the actual layout has a corner location close to (16, 32).

5.3.2 The department allocation sub-problem

Two formulations are applied for this sub-problem. Depending on the balance of exposure and adjacency, three settings are tested. A review of the tables below indicates that all of the problems are solved within a reasonable time.

The results of Formulation (8) and Formulation (8') are listed in the tables below and show that Formulation (8') solved the problem in a slightly shorter time.

Formulation (8)

Adjacency weight	Mipgap	Current MIP best bound	Solution time	Iterations	Nodes
4%	0.01%	1.31*10e7	5902.44 sec	10250461	16340
10%	0.01%	1.37*10e7	18690.74 sec	31334829	42892
25%	0.01%	1.55*10e7	52712.22 sec	89265289	101774

Formulation (8')

Adjacency weight	Mipgap	Current MIP best bound	Solution time	Iterations	Nodes
4%	0.01%	1.31*10e7	3426.66 sec	5744883	11457
10%	0.01%	1.37*10e7	11074.64 sec	19617205	29986
25%	0.01%	1.55*10e7	51247.58 sec	87692821	99892

Two sample layouts are shown in Figure 42 and Figure 43.

If the adjacency weight is 4%, which means that the designer put less emphasis on adjacency and more emphasis on exposure improvement the result is the layout given in Figure 42.

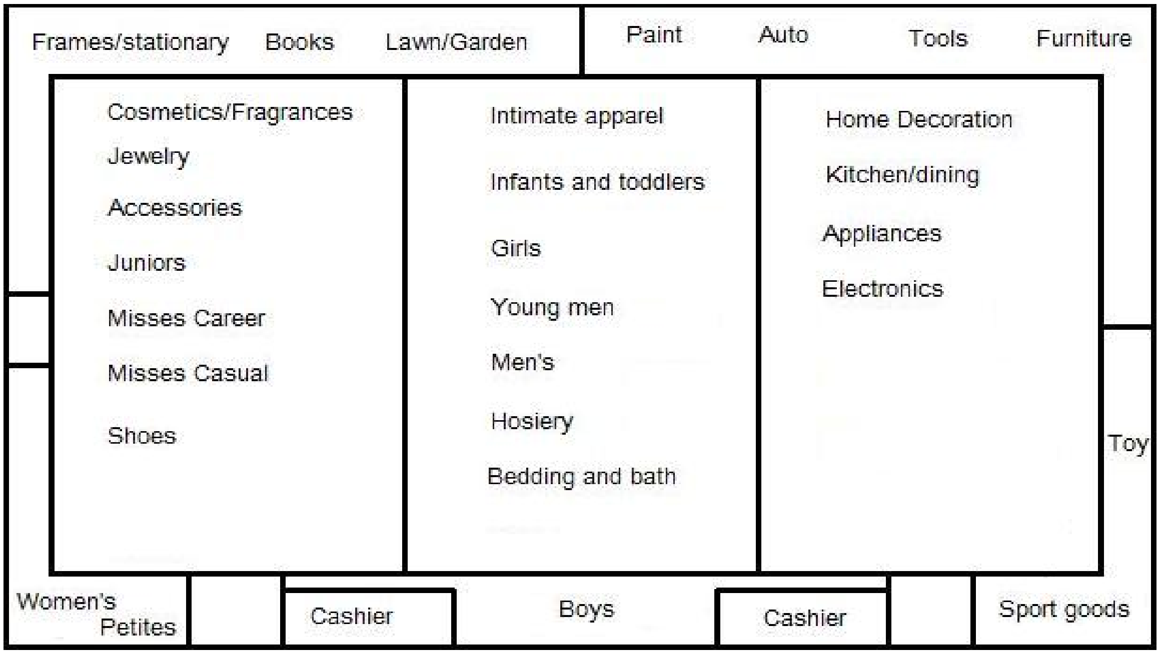


Figure 42. Less emphasis on adjacency

If the adjacency weight is 25%, which means that the designer put more emphasis on adjacency the result is the layout in Figure 43.

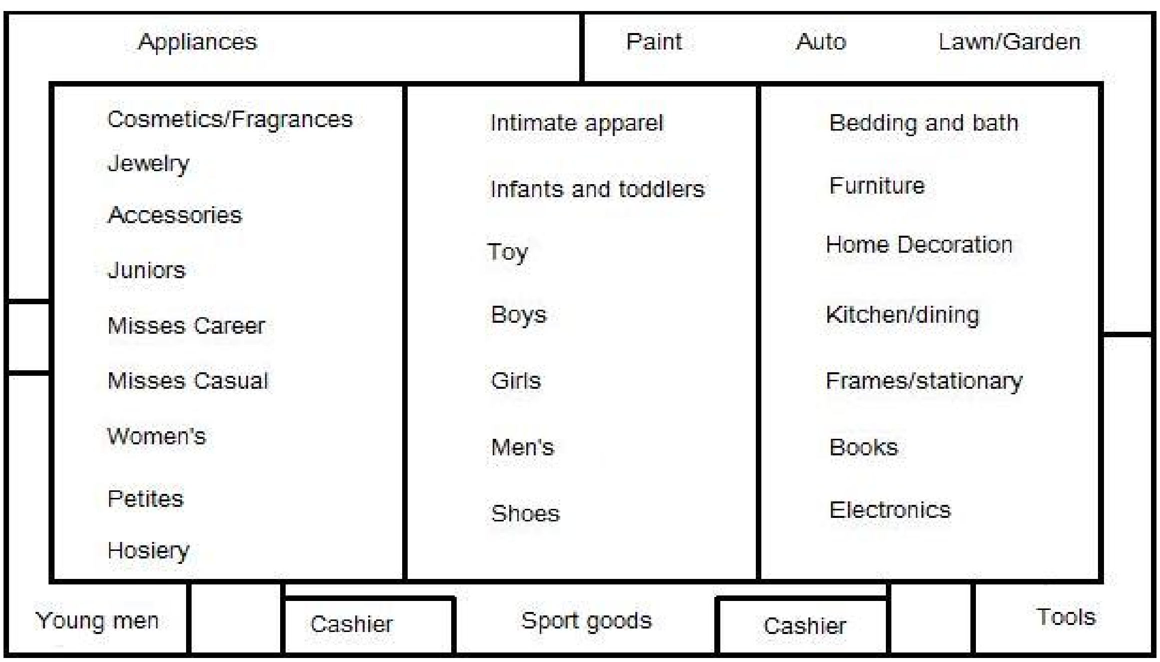


Figure 43. More emphasis on adjacency

The latter layout shows more logical connections among the locations of the departments. A customer might be more comfortable with the second layout. In this layout, the clothing departments are located in the left side part of the store, which is closer to the mall entrance. As an anchor store in a shopping mall, there is more traffic between the two parking lot entrances and the mall entrance, because on average 46% of customers turn left after they enter both parking entrances. Departments such as Lawn/Garden, Auto, Paint, and Furniture have low impulse purchase sensitivity. These departments are located on the right side of the store, close to the loading zone, which is very convenient for hardware shoppers and customers purchasing large items like furniture. Therefore, the second layout is chosen as a basis for conducting the detailed design process.

5.3.3 The detail design sub-problem

The optimal results in Section 5.3.2 and related input information are shown in the following table. Also, the number of strips for each knapsack is listed in the tables.

Table 26. Outer sub-areas' department information

Knapsack Index (# of strips)	Dept. Index	Dept. Name	Dept. nominal Area	Dept. optimized area	Dept. Unit Profit	Dept Exposure Sensitivity
1(8 strips)	10	Young men	2200	2420	299	L
2(17)	26	Appliances	6600	7260	286	H
3(17)	25	Paint	1600	1548	333	L
	28	Auto	1000	900	210	L
	29	Lawn/Garden	5600	6160	333	L
4(8)	24	Tools	3600	3960	286	L
5(4)	30	Sporting goods	2000	2200	246	M

Table 27. Inner sub-areas' department information

Knapsack	Dept.	Dept. Name	Dept.	Dept	Dept.	Dept.
6 (92)	1	Cosmetics/Fragrances	2800	3080	411	H
	2	Jewelry	800	880	880	H
	3	Accessories	2400	2640	478	M
	9	Juniors	2400	2640	299	H
	12	Misses Career	3400	3740	308	M
	13	Misses Casual	1000	1100	308	H
	14	Petites	600	660	308	M
	15	Women's	2800	3080	308	M
	17	Hosiery	800	880	246	H
7(82)	4	Intimate Apparel	2400	2342	339	L
	5	Infants and Toddlers	1600	1760	399	M
	6	Toys	1200	1080	221	H
	7	Boys	2000	2200	393	M
	8	Girls	3400	3740	393	M
	11	Men's	4500	4050	299	M
	16	Shoes	5600	6160	421	H
8(92)	18	Bedding and Bath	4000	4400	286	M
	19	Furniture	1400	1540	286	L
	20	Home Dec (Window)	4200	4620	333	H
	21	Kitchen/Dining	2200	2076	286	H
	22	Frames/Stationary	800	720	229	L
	23	Books	1400	1260	199	M
	27	Electronics	5200	5720	355	M

The strips follow the serpentine arrow line, shown in Figure 44 .

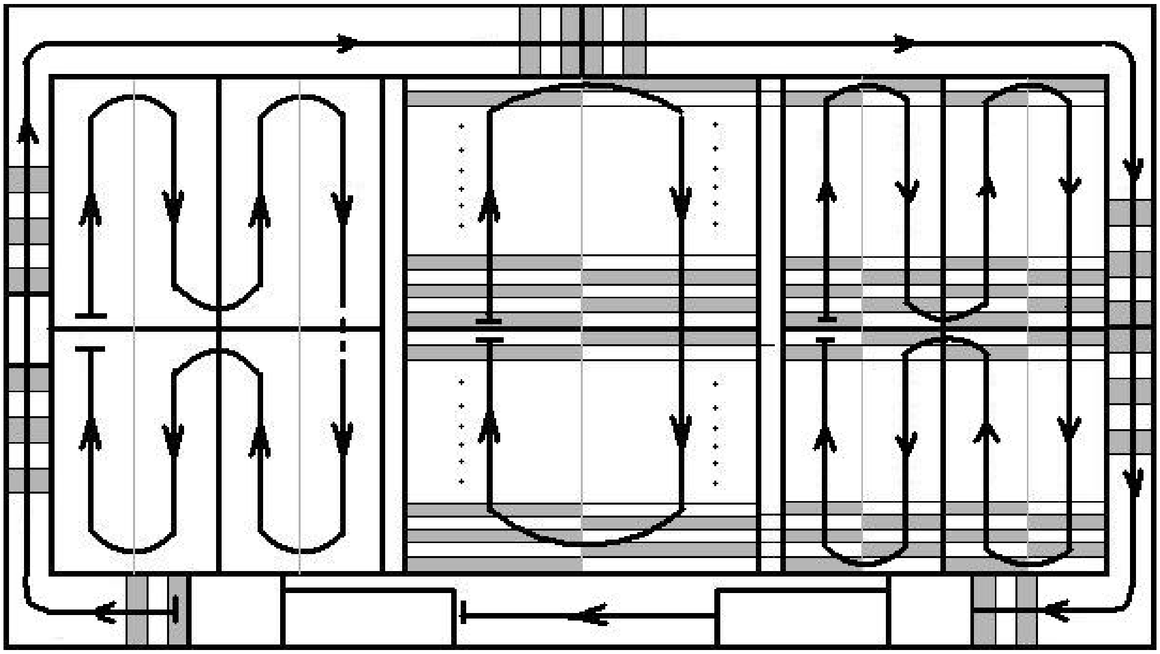


Figure 44. Serpentine lines in all knapsacks

Applying Formulation (13) yields the optimized detailed design results shown in the following tables.

Formulation (13)

Exposure vs. Adjacency	Mipgap	Current MIP best bound	Solution time	Iterations	Nodes
Knapsack 1 to 5	0.01%	2.29 *10e6	1.12 sec	2707	24
Knapsack 6	0.01%	4.76*10e6	35893.97 sec	14598467	13984
Knapsack 7	0.01%	3.87*10e6	667.48 sec	697334	1052
Knapsack 8	0.01%	3.63*10e6	1417.21 sec	962863	1755

The formulation had a hard time solving Knapsack 6. There are 9 departments in the knapsack, and the number of departments in the same knapsack greatly affects the solution time. Figure 45 shows the detailed design for the Sears model.

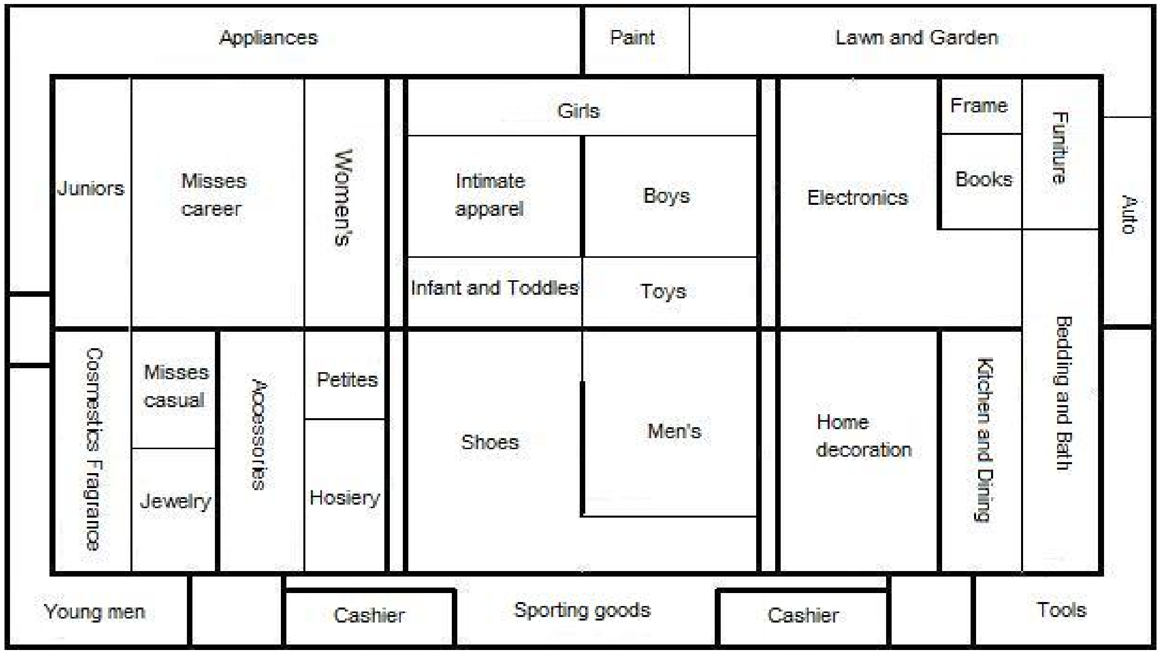


Figure 45. Detailed design of the Sears store

Then the aisle structure and entrances can be added into the layout illustration.

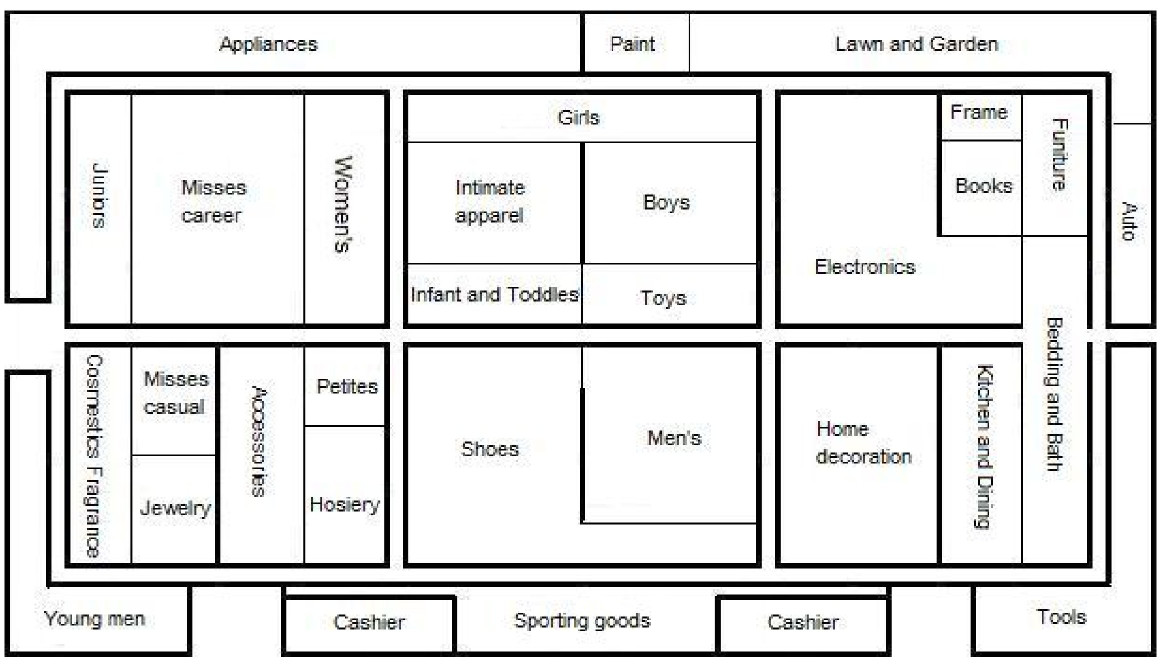


Figure 46. More detailed layout design

5.4 DISCUSSION AND CONCLUSIONS

In this chapter, a complete design application is discussed. The entire process of designing a department store layout was shown. The necessary input data was collected and then the three sub-problems are solved. The mixed-integer programming models solve the problem in a reasonable time frame. After applying all of the models a complete store layout design is obtained.

Compared with the Kohl’s department store layout, the special store characteristics of Sears include (1) the type of store, it is not free-standing but an anchor store in a shopping mall, (2) the large area of the store, (3) the complicated aisle structures, (4) the diversity of merchandise, and (5) the two parking lot entrances, one mall entrance and one loading exit. The methodology was successfully applied to both store types.

6.0 CONCLUSIONS AND FUTURE WORK

6.1 CONCLUSIONS

In summary, this research studies methodologies that can be applied in facility layout design for retail stores. The problem is decomposed into several models, which are solved by mathematic programming and heuristic algorithms.

In the first sub-problem, an aisle structure is decided for the facility. In this research, the racetrack layout is chosen as the generic layout configuration. The aisle configuration including aisle location are determined to maximize the overall exposure of the entire retail area.

In the second sub-problem, given the aisle configuration, the entire retail area is divided into several sub-areas. All store departments are assigned to these sub-areas subject to area constraints and with an objective of maximizing profit while considering the exposure of the area, the exposure sensitivity of the departments, and primary and secondary adjacency between pairs of departments. The problem is modeled as a multiple knapsack problem, which generates a mixed integer programming model. The solution time grows rapidly with an increase in problem size. To solve the model in a reasonable amount of time, efficient solution methods are required. Variable reduction, symmetry breaking constraints, and reformulation-linearization techniques are applied.

The third step is to specify the exact location of each department in each sub-area. To model this, each sub-area is divided into strips. Without the strip model (or some other constraints), the model will automatically put the most profitable departments along the aisles where they will have the most exposure to produce the highest profit-exposure objective value. However, the strip model can avoid the occurrence of long, skinny department allocations.

In the course of this research, several sub-problems are modeled mathematically. Linear programming and mixed-integer programming are applied to solve these problems. One important aspect to take into consideration is solving the problems efficiently. Therefore, specific modeling techniques and efficient solution methods are applied.

6.1.1 Contributions

This research makes several contributions in the following areas:

1. In the aisle structure stage, the racetrack aisle structure is emphasized. The merit of the racetrack aisle structure layout is that it facilitates customers passing through as many departments as possible to increase impulse purchases by customers. Aisles in a retail setting lead the customers to browse merchandise, affect the traffic patterns and affect the shopping atmosphere. This research provides a quantitative basis for determining the parameters for the racetrack aisle.
2. The concept of exposure is quantified by using weight factors for the aisles and an exposure decay function. The weight factor of an aisle, β, is used to

show the relationship between the width of an aisle and exposure of related grids in the retail area. Exposure decay functions are defined to show how points that are farther from the aisle receive a lower exposure value than those that are closer to the aisle. Six different types of exposure decay functions are introduced. Based on the exposure value, the potential aisle structures are evaluated and compared.

3. In the second step, the department allocation problem is modeled as a multiple knapsack problem with variable item weights. In the proposed approach, primary and secondary adjacency are defined, formulated with linear constraints and solved by mixed integer programming. The preference matrix used to create the primary and secondary adjacency information is generated by data mining from different classes of department stores and various retailers.

4. A strip model is developed to locate the departments within each sub-area. The constraints in the model ensure that one department won't be separated by other departments.

The study presents a comprehensive solution to retailers for the design of new stores and for aspects of redesign of existing facilities. The process considers the exposure of merchandise, having a logical department layout, rational traffic flow and easy-to-follow aisle structure design to provide a pleasant shopping environment and help to create and maintain a good store image.

6.1.2 Limitations and guidelines

There are several limitations of this research. First the entire optimizing process for the layout for department stores is divided into three steps. The reasons for this three step process are listed as follows. From the construction point of view, the store size is pre-determined. The entrances and aisle structure are also decided at the first stage of construction. Both are relatively difficult and expensive to change when the store is in operation. The department allocation problem can occur at any time, when new departments are introduced, existing departments are leaving or the department sizes change. So the allocation problem in this research has been studied separately with more emphasis on adjacency. The first aisle structure design step is only based on exposure considerations. From the modeling point of view, it is hard to capture the entire process in one mathematical model. If the knapsack size and department area are permitted to change at the same time, the model would be a quadratic model with a quadratic objective function and linear constraints. Furthermore, the detailed design is hard to combine with the second step because of the difficulty of modeling the relative locations among the departments. A limitation of dividing the problem into three steps is that the optimization solution of the entire process is not necessarily globally optimal. However, the same set of inputs such as exposure sensitivity values, unit prices of departments and adjacency preferences are used for all three steps. The consistency of the inputs helps the continuity of the entire optimization process.

The second limitation is the parameter values used in the models. Since little previous literature on the retail layout problem can be referenced, the parameters are collected from different resources. Additionally, some of the parameters can’t be found in the previous literature and this input data was evaluated by testing different values in order to find reasonable values to use in the models. However, this research aims to produce a methodology rather than focusing on the particular revenue values that are determined by the models.

To apply the process in this research, several guidelines should be followed.

1. Careful exposure decay function selection. Depending on the type of retail store under consideration, some exposure decay functions may be more appropriate to use than others. Clothing department stores have slower decay functions but smaller stores such as shoe and accessory stores (or departments) have faster decay functions due to the size of the merchandise and the racks used for display. Clothing departments usually take more space and require wider aisles than shoe and accessory departments.
2. Determine how large the aisle area is. Much of the traditional facility layout research considers that aisle space is a waste and adds no revenue. In the retail sector, the aisle serves two important purposes: to facilitate traffic and to display free-standing racks. Various aisle widths and aisle areas can affect the customer traffic pattern, and therefore affect the degree of exposure. Furthermore, a wider aisle can provide a more enjoyable shopping environment and provide space for free standing displays or seasonal promotions. However, any space in a retail environment is very valuable. So in the design stage, 20% to 30% of the retail area should be taken into consideration as aisle area.

3. How to balance exposure and adjacency. Since these are the two criteria in the objective function, the balance between them greatly affects the result. However the balance decision might vary depending on the layout objectives. If more emphasis is placed on exposure, then the model will tend to put higher exposure sensitive departments in locations with higher customer exposure. If the focus is more on adjacency, then the department placement is governed more by departmental relationships rather than exposure. In this case, it might be more convenient for customers to find related items since they would be placed close together. Either way, the model can used to serve retail decision makers.

6.2 FUTURE RESEARCH

This research is one of the first attempts to mathematically model the facility layout of a retail store. It was necessary to make many assumptions to devise the model and quantitatively define the parameters. It is hard to evaluate the preciseness of those parameters. For the sub problems with multiple objective function terms, the relative weighting of the objectives could be changed depending on a designer's or retailer's preferences. Additionally, future research can relax some of the assumptions in order to better address the needs and specification of retail store realities.

The research focuses on designing a layout from scratch, meaning designing a store from the very beginning. However, other interesting layout problems can also arise. For example, after a store has been operating for a while, a new department may be introduced, existing departments may be combined or eliminated, customer preferences may change or new customer behaviors may be found, or assumptions made during the original design may no longer be valid. For any of the situations just described, one may ask how the store layout can be improved without redesigning the entire store from scratch.

In this section, some methods and solutions for relaying existing departments are discussed. The solution approaches are not always mature, but can be studied in future work.

6.2.1 Departments areas change

After the departments are assigned to sub-areas, when new departments are introduced or if existing departments need to be eliminated, the areas of the departments no longer match the size of the knapsacks and department may need to be expanded or squeezed. In some knapsacks, extra space is wasted because the departments inside have already reached their 120 percent upper limit. On the other hand, there can be departments shrinking to their lower limit to fit in a knapsack. It may be possible that massaging the aisle configuration can lead to a better solution after the knapsack assignments are made. Also, this research assumed that the aisle layout is symmetric. If this constraint is relaxed and the central aisle can be shifted and reshaped, the store may be able to further increase the profit of the retail area. The next section discusses a new cubic nonlinear model, that relaxes the symmetric aisle structure constraint, but the model is difficult to solve due to its nonlinearity.

6.2.1.1 Cubic model with knapsack area variables

The formulation combines the optimization of aisle structure and department allocation together to maximize the profit of all of the departments based on the considerations of exposure, primary adjacency and secondary adjacency. The problem is formulated as a multiple knapsack problem.

The profit of each department is determined by the knapsack which it is located in and the relative locations of the other departments. Therefore, the profit associated with placing department i in knapsack j, p_{ij}, includes two components - the knapsack exposure and the adjacency relationships, rather than only the knapsack exposure.

Problem notation for this section is given below.

Decision variables:

x_{ij} — If department i is in knapsack j, then x_{ij}=1, otherwise, x_{ij}=0.

ΔS_j — The gap between the actual area of knapsack j and the nominal area of knapsack j.

$A_{i_1,i_2,j}$ — An indicator variable to signify primary adjacency of departments i_1 and i_2 in knapsack j. If both departments i_1 and i_2 are in knapsack j, then $A_{i_1,i_2,j} = 1$, otherwise it should be zero.

A'_{i_1,i_2,j_1,j_2} An indicator variable to signify secondary adjacency of departments i_1 and i_2. If both departments i_1 and i_2 are in two adjacent knapsacks j_1 and j_2 then $A'_{i_1,i_2,j_1,j_2} = 1$, otherwise it should be zero.

Parameters:

i Department index.

j Knapsack index.

R_i The revenue of department i.

r'_{ij} The revenue per unit area of department i that results from placing department i in knapsack j.

e_j The unit exposure of knapsack j.

W Primary adjacency factor.

W' Secondary adjacency factor.

s_i The nominal area of department i.

s'_i The actual area of department i .

S_j: The nominal area of knapsack j, which is a sub-area of the whole retail area, where $\sum_{j=1}^{5} S_j = L*W$.

Δe_j: The modification of the exposure of an area based on the actual area of knapsack j.

Now Formulation (14) is developed to optimize aisle location and department allocation at the same time.

Formulation (14)

$$\max \sum_{i_1=1}^{I} \left[r_{i_1,j} \sum_{j=1}^{J} (e_j + \Delta e_j) x_{ij} s_i' + R_i \sum_{i_2 \neq i_1, i_2=1}^{I} \sum_{j=1}^{J} wW_{i_1,i_2} A_{i_1,i_2,j} + R_i \sum_{i_2 \neq i_1, i_2=1}^{I} \sum_{\substack{(j_1,j_2)\in \\ \text{adjacent knapsacks}}} w' W_{i_1,i_2} A'_{i_1,i_2,j_1,j_2} \right]$$

$$st.\ 0.8 s_i \le \sum_{j=1}^{J} x_{ij} s_i' \le 1.2 s_i, \quad \forall i \tag{101}$$

$$\sum_{i=1}^{I} x_{i1} s_i' = S_1 + \Delta S_1, \tag{102}$$

$$\sum_{i=1}^{I} x_{i2} s_i' = S_2 + \Delta S_2, \tag{103}$$

$$\sum_{i=1}^{I} x_{i3} s_i' = S_3 + \Delta S_3, \tag{104}$$

$$\sum_{i=1}^{I} x_{i4} s_i' = S_4 + \Delta S_4, \tag{105}$$

$$\sum_{i=1}^{I} x_{i5} s_i' \le S_5 - (\Delta S_1 + \Delta S_2 + \Delta S_3 + \Delta S_4), \tag{106}$$

$$\Delta e_j = k_j \Delta S_j + b_j, \quad for\ i = 1,2,3,4 \tag{107}$$

$$\Delta e_5 = -k_5 (\Delta S_1 + \Delta S_2 + \Delta S_3 + \Delta S_4) + b_5, \tag{108}$$

$$\sum_{j=1}^{J} x_{ij} \le 1, \quad \forall i \tag{109}$$

$$A_{i_1,i_2,j} \le x_{i_1,j}, \quad \forall i_1, j \tag{110}$$

$$A_{i_1,i_2,j} \le x_{i_2,j}, \quad \forall i_2, j \tag{111}$$

$$A'_{i_1,i_2,j_1,j_2} \le \left(x_{i_1,j_1} + x_{i_1,j_2} + x_{i_2,j_1} + x_{i_2,j_2} \right)/2 \quad \forall (j_1, j_2) \in \text{adjacent knapsacks} \tag{112}$$

$$x_{ij}, A_{i_1,i_2,j}, A'_{i_1,i_2,j_1,j_2} \in \{0,1\}$$

Constraint (101) shows that the area of the departments can increase or decrease by as much as 20 percent of their nominal area. Constraints (102) to (106) show that the change of each knapsack area should be balanced. Constraint (107) and (108) show the relationship between area increases and exposure rate.

6.2.1.2 Conclusion and discussion

In this formulation, the objective function is cubic and nonlinear. Thus, one needs to apply nonlinear programming to solve the problem or revise the objective function. Since the formulation is so hard to solve, heuristic algorithms might be helpful in some steps. For example, if the values of the x_{ij} variables are known, then the A and A' variables can be determined which simplifies the model (though it is still not trivial to solve). It may be possible to start with an initial solution and use a swap/interchange type of heuristic to determine the x_{ij} variables if the resulting problem can be solved efficiently. The difficulty of solving for all of the variables in Formulation (8) is part of the reason that steps 1 and 2 were separated in the hierarchical model. In addition, it may be interesting to use this model or variations on it to conduct redesign analysis assuming there is an existing department allocation and layout. A possible relayout objective could be to consider relayout alternatives that would have minimal costs to implement.

Another one of the assumptions of this research is that the customer traffic density of every aisles segment is related to the aisle type, e.g. entrance aisle, branch aisle and racetrack aisle. There are only a few studies addressing the traffic patterns of customers within retail settings and those studies are limited to grocery stores. To investigate the relationship between traffic density of different parts of aisles and the location of those aisles, real store observations are made and recorded.

6.2.2 Customer traffic patterns

During department store operations, some interesting pattern of shopping behavior can be found, especially for returning customers, who know the store layout very well. Depending on the number and composition (family, male/female, etc.) of the group of consumers, the shopping times and shopping routes can be different. Previously, it was assumed that the same kind of aisle will give the same intensity of exposure; however, after conducting real world observations, it appears that the influences of all aisles are not the same. The merchandise displays should be adjusted to account for these differences. Observations made at several different department stores are now discussed.

6.2.2.1 Free-standing department store

A free-standing department store, such as Kohl's, usually has an entrance(s) on one or two sides facing the parking lot. Having too many entrances creates logistical problems such as needing more cashiers, and creating more potential for shoplifting.

Observations were made at three Kohl's stores in the Pittsburgh area. Data was collected for male and female shoppers. Below a diagram of the Kohl's at Century III mall is shown followed be the original data collection form that was used with results from the Kohl's at Century III mall.

Figure 47. Kohl's department store at Century III Mall

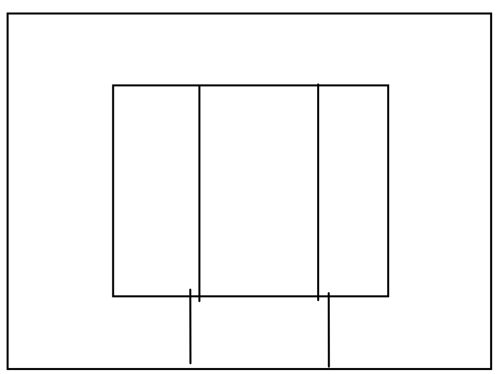


Figure 48. Aisle structure used for the Kohl's department store at Century III Mall

Left parking lot traffic pattern

Consumers	Left	Center	Right	**In Total**	Left	Center	Right	**Out Total**	
Female	26	34	31	91	66	16	2	84	
Male	18	13	8	39	36	4	0	40	
Total	44	47	39	**130**	102	20	2	124	**254**

Class	Left			Center		Right		**In Total**
# of Groups	27			30		28		85
Sub-class	F >M	F =M	F<M	Female dominant	Else	Female dominant	Else	
# of Group number	7	11	9	24	6	25	3	

From the table above, one can tell that the majority of the customers are female. The "In Total" represents the total number of consumers that enter the store, and the "Left", "Center", and "Right" indicate which direction they choose to travel. The "Out Total" indicates the percentage of consumers turning into customers, because the cashier is on one side of the exit aisle.

Depending on the layout, the ratios of male that turn left is higher than the ratio of female, for example, 18 vs. (13 + 8) for male and 26 vs. (34 + 31) for female. In terms of the number of groups, if the group is female dominant, meaning the number of females is greater than or equal to the number of males in the group, the group is more likely to turn right. For example, in the "Turn Left" class there are 50% "Female dominant" groups. In the "Turn to central aisle" class, there are 75% "Female dominant" groups. In the "Turn right" class, there are 89% "Female dominant" groups. One can tell that the layout is an important factor affecting consumers' behaviors, and consumer behaviors can also influence the layout design. Observations were also made of the other entrances of the same Kohl's location.

Right parking lot traffic pattern

	Left	Center	Right	**In Total**	Left	Center	Right	**Out Total**	
Female	23	8	20	51	18	3	77	98	
Male	6	4	5	15	4	2	21	27	
Total	29	12	25	**66**	22	5	98	125	**191**

Class	Left			Center		Right		**In Total**
# of Groups	20			11		19		50
Sub-class	F >M	F =M	F<M	Female dominant	Else	Female dominant	Else	
# of Group number	14	4	2	9	2	19	0	

Similarly, the ratios of male that turn left and center (since it is a right entrance) is higher than the ratio of females. The ratio of the "Female dominant" groups in the "Left", "Center", and "Right" categories decrease because the departments on the right part are all for ladies and misses.

Similar data is now shown for the Kohl's at Robinson Town Mall.

Figure 49. Kohl's department store at Robinson Town Mall

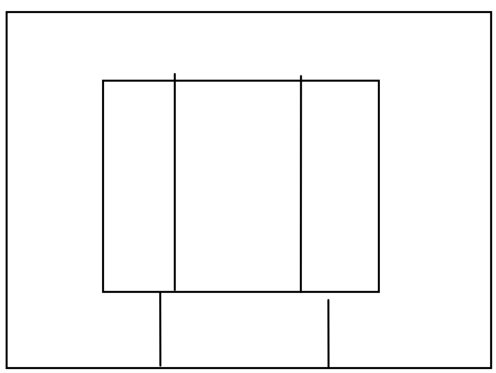


Figure 50. Aisle structure used for the Kohl's department store at Robinson Town Mall

Left parking lot traffic pattern

	Left	Center	Right	**In Total**	Left	Center	Right	**Out Total**	
Female	26	10	53	89	87	8	14	109	
Male	20	10	13	43	44	2	9	55	
Total	46	20	66	**132**	131	10	23	**164**	**296**

Class	Left		Center		Right		**In Total**
# of Groups	32		14		43		89
Sub-class	F >M	Male dominate	Female dominant	Else	Female dominant	Else	
# of Group number	17	15	9	5	42	1	

Right parking lot traffic pattern

	Left	Center	Right	**In Total**	Left	Center	Right	**Out Total**	
Female	18	24	66	108	4	14	98	116	
Male	12	9	10	31	1	6	16	23	
Total	30	33	76	**139**	5	20	114	**139**	**278**

Class	Left		Center		Right		**In Total**
# of Groups	19		23		54		96
Sub-class	F >M	Male dominate	Female dominant	Else	Female dominant	Else	
# of Group number	9	10	20	3	52	2	

Similar data is now shown for the Kohl's at the North Hills Shopping Center.

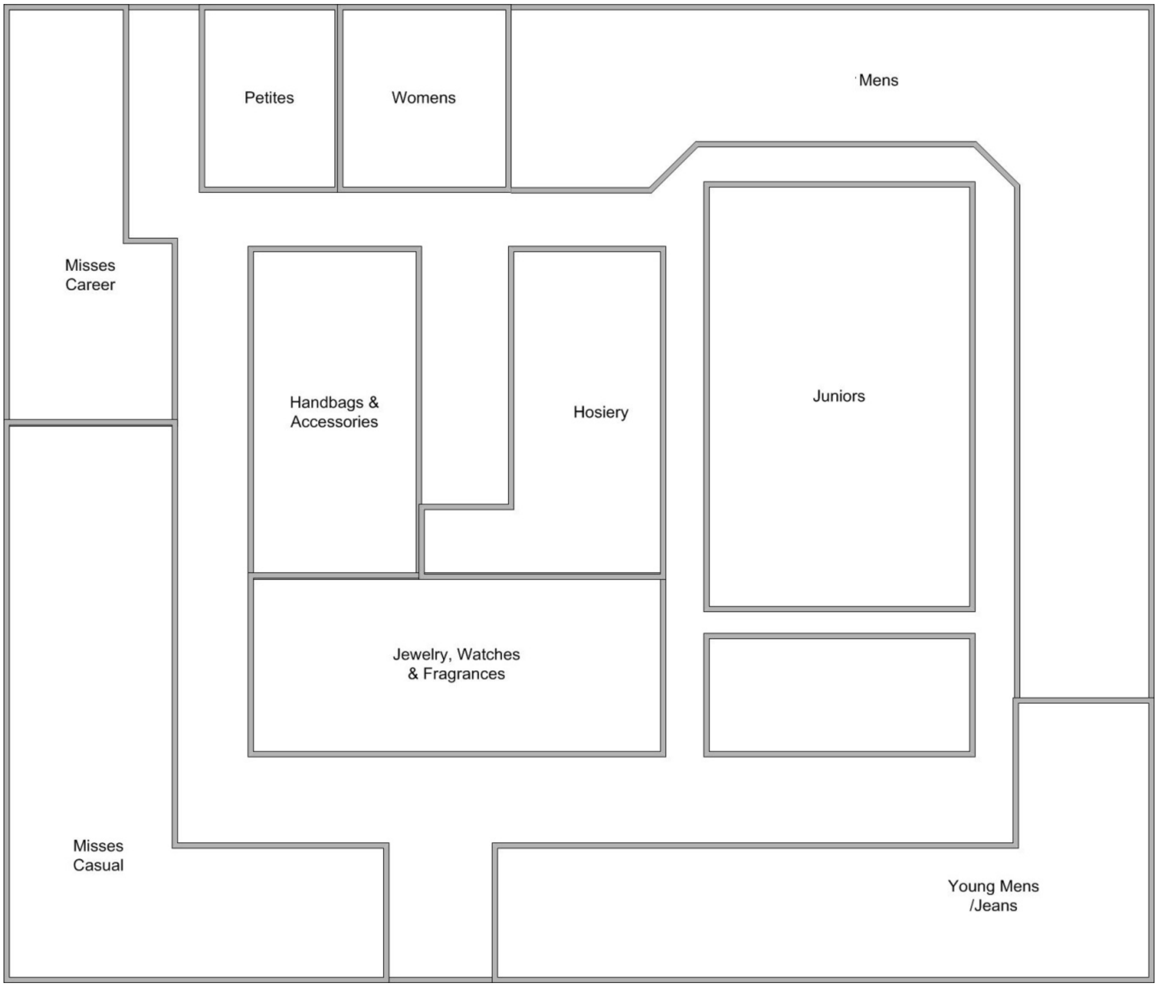


Figure 51. Kohl's department store at North Hills Shopping Center

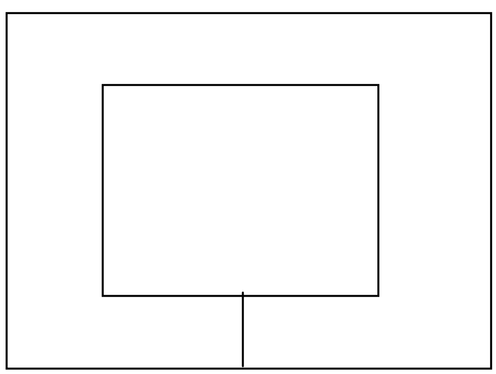


Figure 52. Aisle structure used for the Kohl's department store at the North Hills Shopping Center

	Left	Right	**In Total**	Left	Right	**Out Total**	
Female	41	70	111	10	157	167	
Male	13	33	46	4	58	62	
Total	54	103	**157**	14	215	**229**	**386**

Class	Left		Right		**In Total**
# of Groups	39		75		96
Sub-class	Female dominate	Else	Else	Male dominate	
# of Group number	34	5	46	18	

The ratios of females and males in the “Turn left” and “Turn right” categories are higher, which is conflicting with the results of the previous layout. The reason is that, in this Kohl’s layout, the Young Men’s and Men’s departments are on the right side and the Misses department is on the left.

The following conclusions can be drawn from the observations at the three facilities. For the two-entrance stores, Female consumers tend to turn right when they enter the stores, since the Women’s and Misses departments are on the right hand side. From the table, one can see the majority of the customers are female. During the observation periods groups that were entirely made up of males were seldom seen.

6.2.2.2 Mall-connecting department store

Data was also collected in department stores that are not free-standing, principally anchor stores in shopping malls. The same data collection form was used as for the free-standing stores.

Here is data for the Macy's store at the Pittsburgh Mills Mall. This Macy's store has two levels and a total of four entrances. Based on the location of the entrance, the traffic varies considerably. For example, within the same observation period, the mall entrance has a traffic flow of 497 vs. the upper level entrance with a traffic flow of 36. Even on the same level, the east entrance has a traffic flow of 255 vs. the south entrance with a traffic flow of 60. When the design processes are applied to the store, the customer traffic flow should be considered because the traffic volumes vary significantly across the store.

The directions that customers choose to move when entering the store can also affect the traffic flow along the aisles. This made the traffic flow between the mall entrance and the south entrance the densest. High exposure sensitivity merchandise should be considered for placement along these aisles.

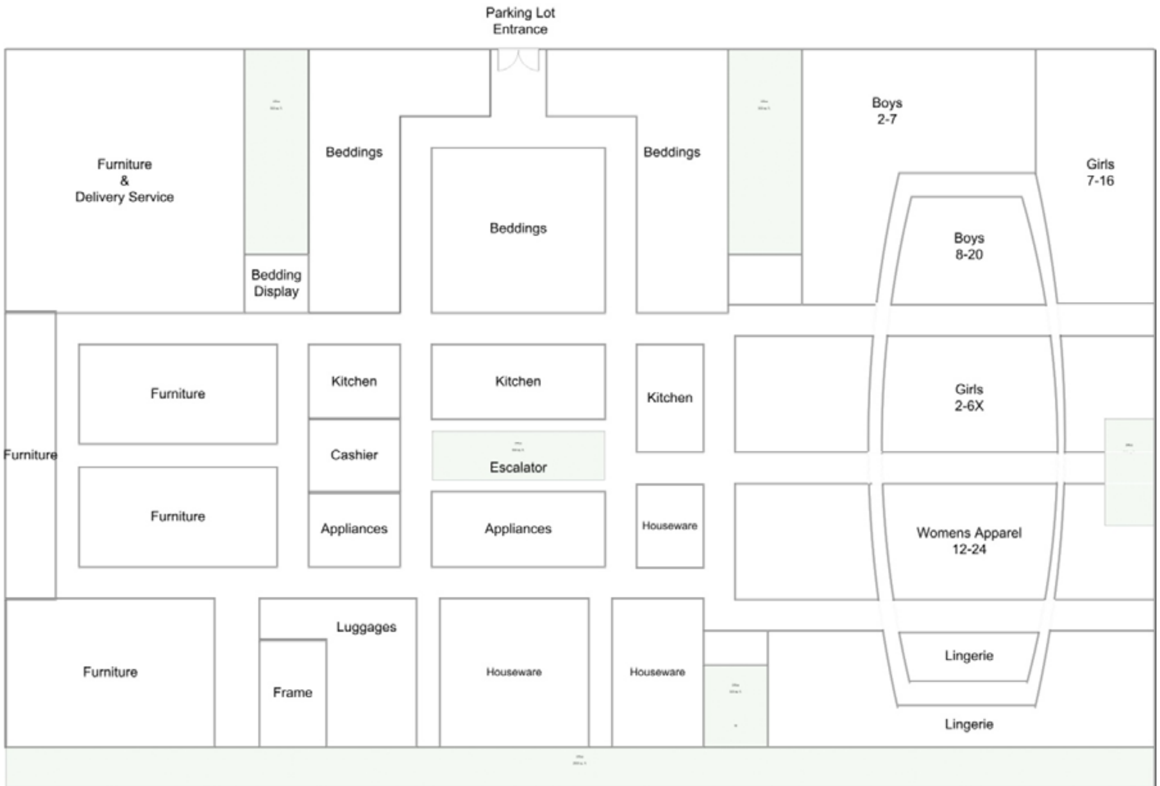


Figure 53. Macy's upper level at the Pittsburgh Mills Mall

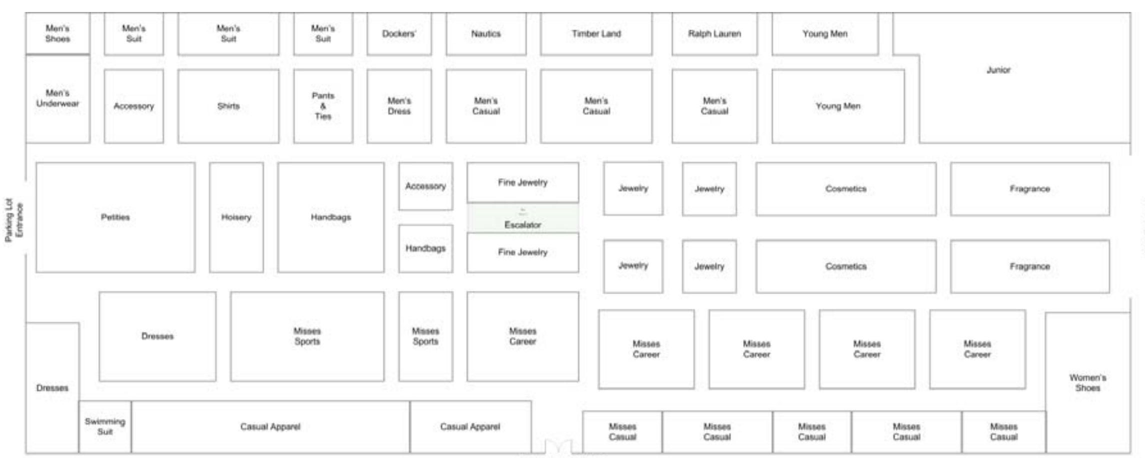


Figure 54. Macy's lower level at the Pittsburgh Mills Mall

EAST	In				Out				
	Left	Center	Right	In Total	Left	Center	Right	Out Total	
Female	29	8	79	116	27	0	51	78	
Male	10	1	19	30	14	0	17	31	
Total	39	9	98	146	41	0	68	109	255

SOUTH	In				Out				
	Left	Center	Right	In Total	Left	Center	Right	Out Total	
Female	10	3	12	25	8	1	4	13	
Male	5	2	8	15	4	1	2	7	
Total	15	5	20	40	12	2	6	20	60

WEST	In				Out				
	Left	Center	Right	In Total	Left	Center	Right	Out Total	
Female	8	1	3	12	11	0	6	17	
Male	2	1	1	4	3	0	0	3	
Total	10	2	4	16	14	0	6	20	36

MALL	In				Out				
	Left	Center	Right	In Total	Left	Center	Right	Out Total	
Female	81	15	67	163	94	40	67	201	
Male	22	7	38	67	16	8	42	66	
Total	103	22	105	230	110	48	109	267	497

Here is data for the JCPenney store in the Pittsburgh Mills Mall. It has only one level and three entrances. Similar to the previous example, the mall entrance has the densest traffic flow. The ratio of the traffic for the other two entrances is almost 2:1. The highest exposure zone is along the aisles between the mall entrance and the parking lot one entrance.

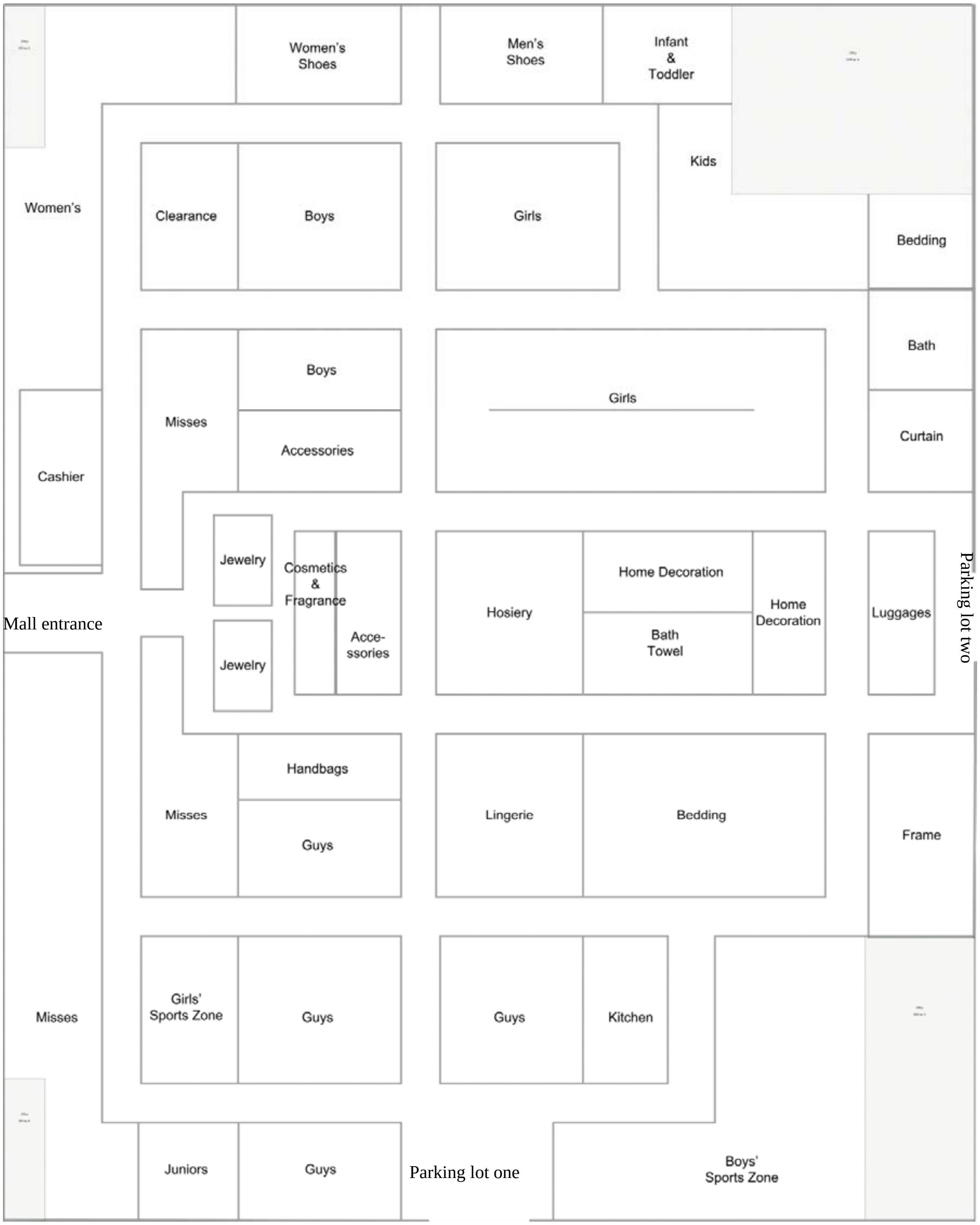


Figure 55. JCPenney store in the Pittsburgh Mills Mall

PARKING LOT ONE	In				Out				
	Left	Center	Right	In Total	Left	Center	Right	Out Total	
Female	55	1	63	119	92	2	45	139	
Male	28	0	20	48	35	0	16	51	
Total	83	1	83	167	127	2	61	190	357

PARKING LOT TWO	In				Out				
	Left	Center	Right	In Total	Left	Center	Right	Out Total	
Female	7	32	13	52	4	31	23	58	
Male	6	9	5	20	3	16	13	32	
Total	13	41	18	72	7	47	36	90	162

MALL	In				Out				
	Left	Center	Right	In Total	Left	Center	Right	Out Total	
Female	33	111	60	204	65	101	28	194	
Male	24	56	33	113	27	55	20	102	
Total	57	167	93	317	92	156	48	296	613

Here is data for the Sears store in the Pittsburgh Mills mall. The store has one level and three entrances. The one closer to the mall entrance has higher volume than the one further from the mall entrance. The department allocation should be adjusted according to this unbalanced traffic pattern.

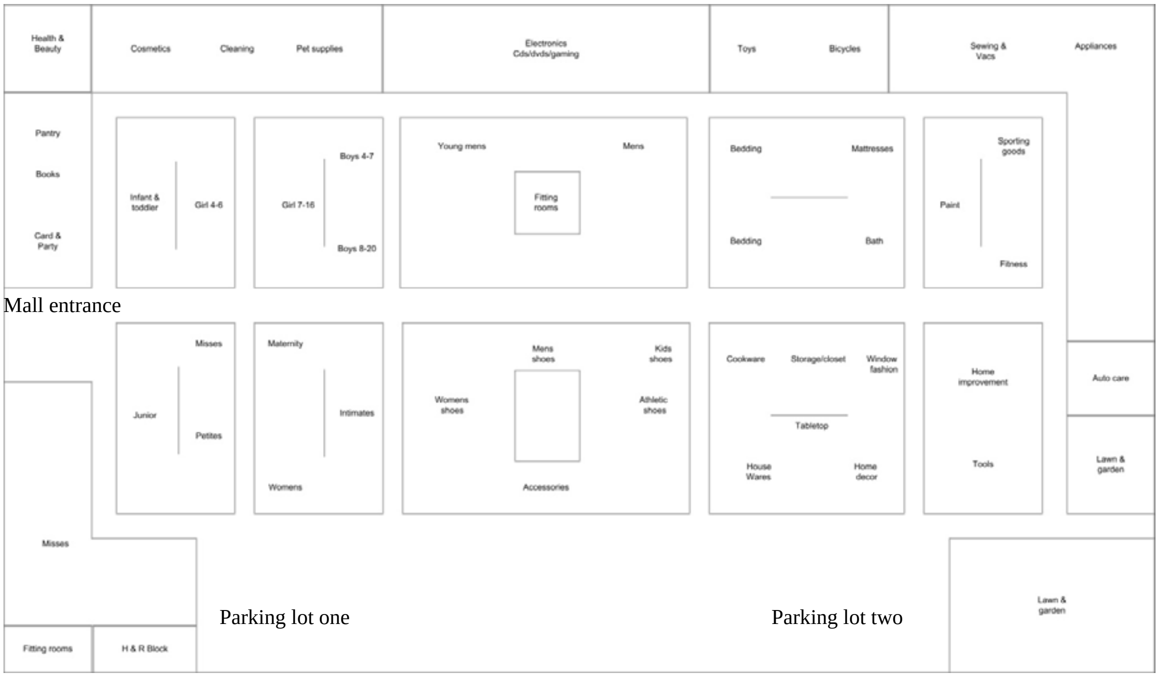


Figure 56. Sears store in the Pittsburgh Mills Mall

PARKING LOT ONE

	In				Out				
	Left	Center	Right	In Total	Left	Center	Right	Out Total	
Female	32	29	15	76	55	8	10	73	
Male	25	23	20	68	42	10	9	61	
Total	57	52	35	144	97	18	19	134	278

PARKING LOT TWO

	In				Out				
	Left	Center	Right	In Total	Left	Center	Right	Out Total	
Female	1	13	12	26	1	12	6	19	
Male	0	8	8	16	0	8	4	12	
Total	1	21	20	42	1	20	10	31	73

MALL	In				Out				
	Left	Center	Right	In Total	Left	Center	Right	Out Total	
Female	12	95	22	129	8	87	43	138	
Male	7	85	9	101	4	74	31	109	
Total	19	180	31	230	12	161	74	247	477

From the data collected from the three department stores in the mall, one finds there is a big difference between the traffic patterns at different doors. Usually the entrance connecting the store to the mall will have the highest traffic density. By this observation, the exposure of different retail areas should be adjusted based on the differences in the customer traffic patterns.

6.2.3 Marginal department profit function

In the previous sections, it is assumed that the profit per square foot of each department is fixed in the formulations. However, the relationship between the revenue and the area of each department is probably diminishing instead of fixed. The profit per square foot may change depending on the total area of the department. For example, the marginal rate of return for the profit per unit area of a department may decrease as the area of the department increases. An example is shown in Figure 57 where the marginal department profit function is \$200/ft.^{2} up to 300 ft.^{2}, \$150/ft.^{2} from 300 ft.^{2} to 500ft.^{2}, \$100/ft.^{2} from 500 to 700 ft.^{2}, and \$0 for up from 700 ft.^{2}

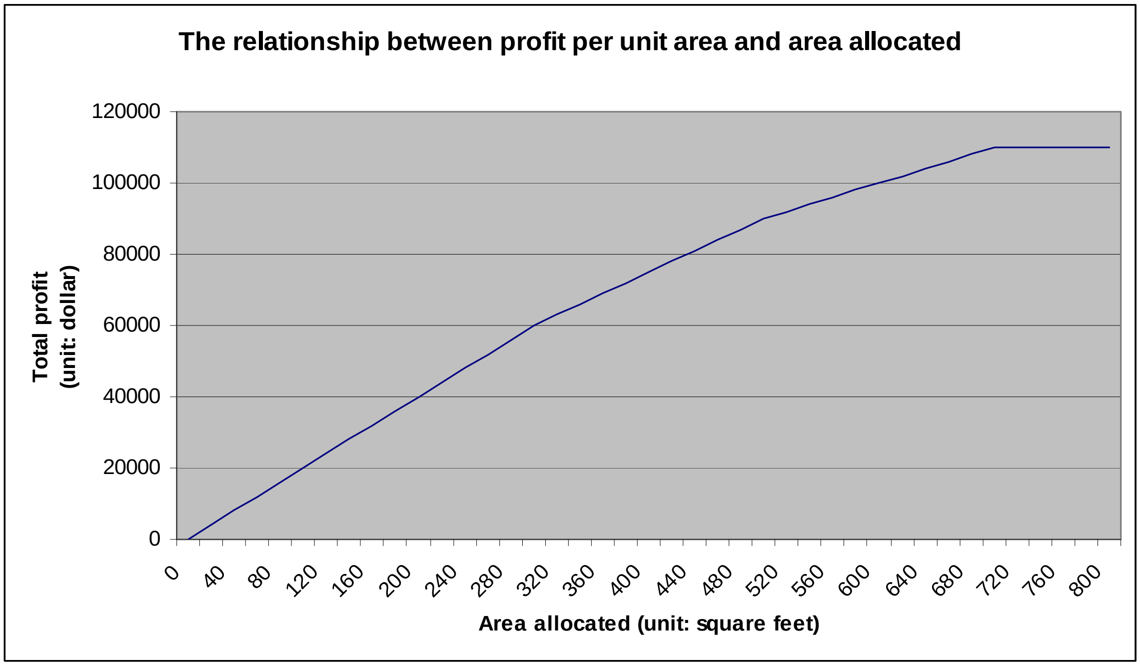


Figure 57. The relationship between profit and area

6.2.4 Other future research directions

The problems in this research are modeled as mixed integer programming models. Assumptions are made and techniques are applied, in order to solve the models in reasonable times. Besides the modeling approaches, it can also be interesting to study solution techniques. For large scale mixed integer programming models, techniques such as branch and bound and branch and price can be tailored to solve problems more efficiently than using standard solution techniques.

Additionally, in this research, the single retail store is emphasized. Actually, the exposure and adjacency concept can be extended into mall tenants' selection and allocation. In the mall facility layout, anchor stores, aisle structure, entrances, food court and escalators will affect stores' exposures and traffic patterns. An individual store has an adjacency preference for its neighborhood. For example, for a shoe store, having too many shoe stores located close to it will affect its profit and exposure. To a mall owner, the balance of the classes and categories of tenants are very important for the health and vibrancy of the mall as a whole.

APPENDIX A

DATA COLLECTION INSTRUCTIONS AND FORM

This page contains the store data that was collected. Note that the Layout and Traffic data will be based on site surveys and data collection. The profit data will come from annual reports, the Biz data, and other sources.

Layout

- Area of the entire store
- Store layout (take picture first, draw by MS Visio to scale)
- Department list (from the directories shown in stores, collect the data as detailed as possible, and aggregate the data later)
- Area of each department
- Aisle configuration
- Entrances - amount and location
- Adjacency data
- Single/multi-floor layout

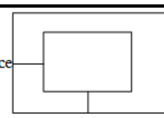
Traffic data

- In / out traffic
- Female / male traffic
- Tendency of turns after entering the store
- Density of different times
- Relative volume of different entrance during the same time period
- Mall entrances vs. parking lot entrances.
- Differences of different floor entrances

Profit

- Department unit profit (dollar per square foot)
- Profit of department stores (upscale, mid-end and low-end)
- Sale for different stores (upscale, mid-end and low-end)
- Maintenance cost and management cost (upscale, mid-end and low-end)
- Net revenue of department stores (upscale, mid-end and low-end)

Note that the time period during which data is collected should not be at the beginning of the day or the last hour of the day or a special day, such as Thanksgiving or Independence Day.

Store Name and Location:				Entrance Mall Parking Lot									Date:					
Observer's Name:				Department(s) Immediately Next to the Entrance									Day:		Total			
															In			Out
Group	Time														Left	Center	Right	
In/Out																		
Female																		
Male																		
L/R/C																		
In/Out																		
Female																		
Male																		
L/R/C																		
In/Out																		
Female																		
Male																		
L/R/C																		
In/Out																		
Female																		
Male																		
L/R/C																		
In/Out																		
Female																		
Male																		
L/R/C																		
In/Out																		
Female																		
Male																		
L/R/C																		
	Grand Total In:			Grand Total Out:			Grand Total Female:			Grand Total Male:			Grand Total Observed:					

APPENDIX B

ROBUSTNESS TEST FOR GROUPS

These are results for the other groups of settings, where the decay distance threshold equals 20 and the retained exposure equals 0.30. The six exposure decay functions are treated as six scenarios. The results are shown in the following tables.

Group Two: Setting 7-12: 20, 0.30. Scenarios (exposure decay functions): 1 – 6

Solution 1 (20, 16)

	Absolute Robustness	Best Possible Decision	Robust Deviation	Relative Robustness
Scenario 1	50688.14	50688.14	0.00	0.000
Scenario 2	55889.80	56179.89	290.09	0.005
Scenario 3	45258.93	45331.61	72.68	0.002
Scenario 4	54835.88	54835.88	0.00	0.000
Scenario 5	58550.69	58550.69	0.00	0.000
Scenario 6	59269.29	59275.02	5.73	0.000
Criteria	Sum(all values)		Max(all values)	Sum(all values)
Values	324492.73		290.09	0.007

Solution 2 (32, 20)

	Absolute Robustness	Best Possible Decision	Robust Deviation	Relative Robustness
Scenario 1	50441.67	50688.14	246.47	0.005
Scenario 2	56179.89	56179.89	0.00	0.000
Scenario 3	45306.15	45331.61	25.46	0.001
Scenario 4	54339.47	54835.88	496.41	0.009
Scenario 5	58118.15	58550.69	432.55	0.007
Scenario 6	58833.01	59275.02	442.01	0.007
Criteria	Sum(all values)		Max(all values)	Sum(all values)
Values	323218.34		496.41	0.029

Solution 3 (32, 16)

	Absolute Robustness	Best Possible Decision	Robust Deviation	Relative Robustness
Scenario 1	50573.02	50688.14	115.12	0.002
Scenario 2	56135.56	56179.89	44.33	0.001
Scenario 3	45331.61	45331.61	0.00	0.000
Scenario 4	54642.88	54835.88	193.00	0.004
Scenario 5	58295.44	58550.69	255.25	0.004
Scenario 6	59000.97	59275.02	274.05	0.005
Criteria	Sum(all values)		Max(all values)	Sum(all values)
Values	323979.47		274.05	0.016

Solution 4 (20, 16), similar to solution 1

Solution 5 (20, 16), similar to solution 1

Solution 6 (20, 18)

	Absolute Robustness	Best Possible Decision	Robust Deviation	Relative Robustness
Scenario 1	50663.36	50688.14	24.79	0.000
Scenario 2	55946.84	56179.89	233.05	0.004
Scenario 3	45260.49	45331.61	71.12	0.002
Scenario 4	54684.18	54835.88	151.70	0.003
Scenario 5	58543.82	58550.69	6.87	0.000
Scenario 6	59275.02	59275.02	0.00	0.000
Criteria	Sum(all values)		Max(all values)	Sum(all values)
Values	324373.70		233.05	0.009

Average solution

(20 + 32 + 32 + 20 + 20 + 20) / 6 = 24

(16 + 20 + 16 + 16 + 16 + 18) / 6 = 17

Close to (24, 17)

Given the three criteria, there are two potential optimal solutions, which are (20, 16) and (20, 18).

For the third group of settings, the decay distance threshold equals 20 and the retained exposure equals 0.50. The six exposure decay functions are treated as six scenarios. The table is recorded as follows.

Group Three: Setting 13-18: 20, 0.50. Scenarios: 1 – 6
Solution 1 (60, 18)

	Absolute Robustness	Best Possible Decision	Robust Deviation	Relative Robustness
Scenario 1	68272.10	68272.102	0.00	0.000
Scenario 2	72058.80	72251.982	193.19	0.003
Scenario 3	64721.27	64782.607	61.33	0.001
Scenario 4	70976.77	71046.657	69.88	0.001
Scenario 5	73500.93	73605.445	104.52	0.001
Scenario 6	73983.98	74126.728	142.75	0.002
Criteria	Sum(all values)		Max(all values)	Sum(all values)
Values	423513.85		193.19	0.008

Solution 2 (40, 24)

	Absolute Robustness	Best Possible Decision	Robust Deviation	Relative Robustness
Scenario 1	68037.96	68272.102	234.14	0.003
Scenario 2	72251.98	72251.982	0.00	0.000
Scenario 3	64532.50	64782.607	250.11	0.004
Scenario 4	70743.00	71046.657	303.66	0.004
Scenario 5	73340.73	73605.445	264.72	0.004

Scenario 6	73833.46	74126.728	293.26	0.004
Criteria	Sum(all values)		Max(all values)	Sum(all values)
Values	422739.64		303.66	0.019

Solution 3 (60, 32)

	Absolute Robustness	Best Possible Decision	Robust Deviation	Relative Robustness
Scenario 1	68014.62	68272.102	257.49	0.004
Scenario 2	71999.93	72251.982	252.05	0.003
Scenario 3	64782.61	64782.607	0.00	0.000
Scenario 4	70522.26	71046.657	524.40	0.007
Scenario 5	72895.45	73605.445	710.00	0.010
Scenario 6	73348.07	74126.728	778.66	0.011
Criteria	Sum(all values)		Max(all values)	Sum(all values)
Values	421562.93		778.66	0.035

Solution 4 (56, 16)

	Absolute Robustness	Best Possible Decision	Robust Deviation	Relative Robustness
Scenario 1	68246.56	68272.102	25.55	0.000
Scenario 2	72062.52	72251.982	189.46	0.003
Scenario 3	64646.10	64782.607	136.51	0.002
Scenario 4	71046.66	71046.657	0.00	0.000
Scenario 5	73530.77	73605.445	74.67	0.001

Scenario 6	74013.44	74126.728	113.29	0.002
Criteria	Sum(all values)		Max(all values)	Sum(all values)
Values	423546.05		189.46	0.008

Solution 5 (26, 18)

	Absolute Robustness	Best Possible Decision	Robust Deviation	Relative Robustness
Scenario 1	68028.51	68272.102	243.60	0.004
Scenario 2	71972.74	72251.982	279.25	0.004
Scenario 3	64250.76	64782.607	531.85	0.008
Scenario 4	70874.63	71046.657	172.03	0.002
Scenario 5	73605.45	73605.445	0.00	0.000
Scenario 6	74122.67	74126.728	4.06	0.000
Criteria	Sum(all values)		Max(all values)	Sum(all values)
Values	422854.74		531.85	0.018

Solution 6 (20, 18)

	Absolute Robustness	Best Possible Decision	Robust Deviation	Relative Robustness
Scenario 1	67975.54	68272.102	296.56	0.004
Scenario 2	71749.46	72251.982	502.53	0.007
Scenario 3	64116.35	64782.607	666.26	0.010
Scenario 4	70847.56	71046.657	199.10	0.003
Scenario 5	73604.44	73605.445	1.00	0.000

Scenario 6	74126.73	74126.728	0.00	0.000
Criteria	Sum(all values)		Max(all values)	Sum(all values)
Values	422420.07		666.26	0.024

Average solution

(60 + 40 + 60 + 56 + 26 + 20) / 6 = 43.7

(18 + 24 + 32 + 16 + 18 + 18) / 6 = 21

Close to (44, 21)

Given the three criteria, the optimal solution is the same - (56, 16).

In the next group of settings, the decay distance threshold equals 30 and the retained exposure equals 0.14. The xix exposure decay functions are treated as six scenarios. The table is recorded as follows.

Group Four: Setting 19-24: 30, 0.14. Scenarios: 1 – 6

Solution 1 (26, 20)

	Absolute Robustness	Best Possible Decision	Robust Deviation	Relative Robustness
Scenario 1	47288.17	47288.17	0.00	0.000
Scenario 2	53446.98	53489.154	42.17	0.001
Scenario 3	37215.70	37257.891	42.20	0.001
Scenario 4	53827.57	54145.73	318.16	0.006
Scenario 5	61125.93	61203.708	77.78	0.001
Scenario 6	62399.79	62483.887	84.10	0.001
Criteria	Sum(all values)		Max(all values)	Sum(all values)
Values	315304.13		318.16	0.010

Solution 2 (30, 20)

	Absolute Robustness	Best Possible Decision	Robust Deviation	Relative Robustness
Scenario 1	47152.28	47288.17	135.89	0.003
Scenario 2	53489.15	53489.154	0.00	0.000
Scenario 3	37180.64	37257.891	77.25	0.002
Scenario 4	53571.13	54145.73	574.60	0.011
Scenario 5	60942.29	61203.708	261.42	0.004

Scenario 6	62224.97	62483.887	258.92	0.004
Criteria	Sum(all values)		Max(all values)	Sum(all values)
Values	314560.46		574.60	0.024

Solution 3 (26, 16)

	Absolute Robustness	Best Possible Decision	Robust Deviation	Relative Robustness
Scenario 1	47155.81	47288.17	132.36	0.003
Scenario 2	53379.29	53489.154	109.86	0.002
Scenario 3	37257.89	37257.891	0.00	0.000
Scenario 4	53619.85	54145.73	525.88	0.010
Scenario 5	60704.86	61203.708	498.85	0.008
Scenario 6	61955.86	62483.887	528.03	0.008
Criteria	Sum(all values)		Max(all values)	Sum(all values)
Values	314073.56		528.03	0.031

Solution 4 (22, 22)

	Absolute Robustness	Best Possible Decision	Robust Deviation	Relative Robustness
Scenario 1	47193.42	47288.17	94.75	0.002
Scenario 2	53302.49	53489.154	186.66	0.003
Scenario 3	37136.69	37257.891	121.20	0.003
Scenario 4	54145.73	54145.73	0.00	0.000
Scenario 5	60982.70	61203.708	221.01	0.004

Scenario 6	62237.81	62483.887	246.08	0.004
Criteria	Sum(all values)		Max(all values)	Sum(all values)
Values	314998.84		246.08	0.016

Solution 5 (26, 22)

	Absolute Robustness	Best Possible Decision	Robust Deviation	Relative Robustness
Scenario 1	47262.01	47288.17	26.16	0.001
Scenario 2	53428.40	53489.154	60.75	0.001
Scenario 3	37154.09	37257.891	103.81	0.003
Scenario 4	53909.93	54145.73	235.80	0.004
Scenario 5	61203.71	61203.708	0.00	0.000
Scenario 6	62483.89	62483.887	0.00	0.000
Criteria	Sum(all values)		Max(all values)	Sum(all values)
Values	315442.02		235.80	0.009

Solution 6 (26, 22), similar to solution 5

Average solution

(26 + 30 + 26 + 22 + 26 + 26) / 6 = 26

(20 + 20 + 16 + 22 + 22 + 22) / 6 = 20.3

Close to (26, 20)

Given the three criteria, the optimal solution is the same - (26, 22).

For the next group of settings the decay distance threshold equals 30 and the retained exposure equals 0.30. The six exposure decay functions are treated as six scenarios. The table is recorded as follows.

Group Five: Setting 25-30: 30, 0.30. Scenarios: 1 – 6

Solution 1 (26, 22)

	Absolute Robustness	Best Possible Decision	Robust Deviation	Relative Robustness
Scenario 1	59286.94	59286.94	0.00	0.000
Scenario 2	64306.09	64387.558	81.46	0.001
Scenario 3	51059.56	51102.351	42.79	0.001
Scenario 4	64698.04	64848.292	150.26	0.002
Scenario 5	70634.83	70640.726	5.89	0.000
Scenario 6	71676.84	71687.843	11.00	0.000
Criteria	Sum(all values)		Max(all values)	Sum(all values)
Values	381662.30		150.26	0.005

Solution 2 (32, 22)

	Absolute Robustness	Best Possible Decision	Robust Deviation	Relative Robustness
Scenario 1	59168.16	59286.94	118.78	0.002
Scenario 2	64387.56	64387.558	0.00	0.000
Scenario 3	51067.95	51102.351	34.41	0.001
Scenario 4	64456.55	64848.292	391.74	0.006
Scenario 5	70439.78	70640.726	200.95	0.003

Scenario 6	71485.65	71687.843	202.19	0.003
Criteria	Sum(all values)		Max(all values)	Sum(all values)
Values	381005.65		391.74	0.014

Solution 3 (32, 18)

	Absolute Robustness	Best Possible Decision	Robust Deviation	Relative Robustness
Scenario 1	59127.68	59286.94	159.26	0.003
Scenario 2	64355.28	64387.558	32.28	0.001
Scenario 3	51102.35	51102.351	0.00	0.000
Scenario 4	64285.52	64848.292	562.77	0.009
Scenario 5	70228.82	70640.726	411.90	0.006
Scenario 6	71261.10	71687.843	426.74	0.006
Criteria	Sum(all values)		Max(all values)	Sum(all values)
Values	380360.75		562.77	0.024

Solution 4 (22, 22)

	Absolute Robustness	Best Possible Decision	Robust Deviation	Relative Robustness
Scenario 1	59189.43	59286.94	97.51	0.002
Scenario 2	64161.94	64387.558	225.62	0.004
Scenario 3	51003.73	51102.351	98.63	0.002
Scenario 4	64848.29	64848.292	0.00	0.000
Scenario 5	70413.27	70640.726	227.46	0.003

Scenario 6	71434.87	71687.843	252.98	0.004
Criteria	Sum(all values)		Max(all values)	Sum(all values)
Values	381051.52		252.98	0.014

Solution 5 (26, 24)

	Absolute Robustness	Best Possible Decision	Robust Deviation	Relative Robustness
Scenario 1	59244.32	59286.94	42.62	0.001
Scenario 2	64293.80	64387.558	93.76	0.001
Scenario 3	51020.38	51102.351	81.98	0.002
Scenario 4	64453.15	64848.292	395.14	0.006
Scenario 5	70640.73	70640.726	0.00	0.000
Scenario 6	71687.84	71687.843	0.00	0.000
Criteria	Sum(all values)		Max(all values)	Sum(all values)
Values	381340.21		395.14	0.010

Solution 6 (26, 24)

Average solution

(26 + 32 + 32 + 22 + 26 + 26) / 6 = 27.3

(22 + 22 + 18 + 22 + 24 + 24) / 6 = 22

Close to (27, 22)

Given the three criteria, the optimal solution is the same - (26, 22).

For the next group of settings the decay distance threshold equals 30 and the retained exposure equals 0.50. The six exposure decay functions are treated as six scenarios. The table is recorded as follows.

Group Six: Setting 31 - 36: 30, 0.50. Scenarios: 1 – 6

Solution 1 (28, 24)

	Absolute Robustness	Best Possible Decision	Robust Deviation	Relative Robustness
Scenario 1	74346.60	74346.601	0.00	0.000
Scenario 2	77997.53	78146.896	149.36	0.002
Scenario 3	68494.03	68658.534	164.50	0.002
Scenario 4	78020.52	78226.494	205.98	0.003
Scenario 5	82481.14	82481.141	0.00	0.000
Scenario 6	83234.35	83234.346	0.00	0.000
Criteria	Sum(all values)		Max(all values)	Sum(all values)
Values	464574.17		205.98	0.007

Solution 2 (38, 26)

	Absolute Robustness	Best Possible Decision	Robust Deviation	Relative Robustness
Scenario 1	74271.31	74346.601	75.30	0.001
Scenario 2	78146.90	78146.896	0.00	0.000
Scenario 3	68613.68	68658.534	44.85	0.001
Scenario 4	77764.72	78226.494	461.78	0.006
Scenario 5	82243.58	82481.141	237.57	0.003

Scenario 6	82990.49	83234.346	243.86	0.003
Criteria	Sum(all values)		Max(all values)	Sum(all values)
Values	464030.66		461.78	0.013

Solution 3 (50, 24)

	Absolute Robustness	Best Possible Decision	Robust Deviation	Relative Robustness
Scenario 1	74207.56	74346.601	139.04	0.002
Scenario 2	78038.39	78146.896	108.51	0.001
Scenario 3	68658.53	68658.534	0.00	0.000
Scenario 4	77725.92	78226.494	500.57	0.006
Scenario 5	81970.24	82481.141	510.90	0.006
Scenario 6	82690.54	83234.346	543.81	0.007
Criteria	Sum(all values)		Max(all values)	Sum(all values)
Values	463291.18		543.81	0.022

Solution 4 (22, 22)

	Absolute Robustness	Best Possible Decision	Robust Deviation	Relative Robustness
Scenario 1	74184.45	74346.601	162.15	0.002
Scenario 2	77736.24	78146.896	410.66	0.005
Scenario 3	68337.52	68658.534	321.02	0.005
Scenario 4	78226.49	78226.494	0.00	0.000
Scenario 5	82201.48	82481.141	279.66	0.003

Scenario 6	82931.19	83234.346	303.15	0.004
Criteria	Sum(all values)		Max(all values)	Sum(all values)
Values	463617.37		410.66	0.019

Solution 5 (28, 24), similar to solution 1

Solution 6 (28, 24), similar to solution 1

Average solution

(28 + 38 + 50 + 22 + 28 + 28) / 6 = 32.3

(24 + 26 + 24 + 22 + 24 + 24) / 6 = 24.3

Close to (32, 24)

Given the three criteria, the optimal solution is the same - (28, 24).

For the next group of settings the decay distance threshold equals 40 and the retained exposure equals 0.14. The six exposure decay functions are treated as six scenarios. The table is recorded as follows.

<u>Group Seven:</u> Setting 36 - 42: 40, 0.14. Scenarios: 1 – 6

Solution 1 (32, 22)

	Absolute Robustness	Best Possible Decision	Robust Deviation	Relative Robustness
Scenario 1	56646.18	56646.18	0.00	0.000
Scenario 2	61084.66	61100.74	16.08	0.000
Scenario 3	43348.37	43453.11	104.75	0.002
Scenario 4	65806.57	65934.41	127.84	0.002
Scenario 5	73853.01	73995.80	142.79	0.002
Scenario 6	75390.71	75550.37	159.66	0.002
Criteria	Sum(all values)		Max(all values)	Sum(all values)
Values	376129.50		159.66	0.009

Solution 2 (32, 20)

	Absolute Robustness	Best Possible Decision	Robust Deviation	Relative Robustness
Scenario 1	56608.80	56646.18	37.39	0.001
Scenario 2	61100.74	61100.74	0.00	0.000
Scenario 3	43402.72	43453.11	50.39	0.001
Scenario 4	65822.89	65934.41	111.52	0.002
Scenario 5	73680.52	73995.80	315.28	0.004

Scenario 6	75219.58	75550.37	330.79	0.004
Criteria	Sum(all values)		Max(all values)	Sum(all values)
Values	375835.25		330.79	0.012

Solution 3 (28, 18)

	Absolute Robustness	Best Possible Decision	Robust Deviation	Relative Robustness
Scenario 1	56468.11	56646.18	178.07	0.003
Scenario 2	61038.82	61100.74	61.92	0.001
Scenario 3	43453.11	43453.11	0.00	0.000
Scenario 4	65501.03	65934.41	433.38	0.007
Scenario 5	73220.32	73995.80	775.48	0.010
Scenario 6	74764.74	75550.37	785.64	0.010
Criteria	Sum(all values)		Max(all values)	Sum(all values)
Values	374446.13		785.64	0.032

Solution 4 (30, 20)

	Absolute Robustness	Best Possible Decision	Robust Deviation	Relative Robustness
Scenario 1	56599.44	56646.18	46.74	0.001
Scenario 2	61093.70	61100.74	7.04	0.000
Scenario 3	43422.99	43453.11	30.12	0.001
Scenario 4	65934.41	65934.41	0.00	0.000
Scenario 5	73606.41	73995.80	389.40	0.005

Scenario 6	75143.81	75550.37	406.56	0.005
Criteria	Sum(all values)		Max(all values)	Sum(all values)
Values	375800.76		405.56	0.012

Solution 5 (32, 26)

	Absolute Robustness	Best Possible Decision	Robust Deviation	Relative Robustness
Scenario 1	56580.46	56646.18	65.72	0.001
Scenario 2	60964.59	61100.74	136.15	0.002
Scenario 3	43163.99	43453.11	289.13	0.007
Scenario 4	65748.13	65934.41	186.28	0.003
Scenario 5	73995.80	73995.80	0.00	0.000
Scenario 6	75550.37	75550.37	0.00	0.000
Criteria	Sum(all values)		Max(all values)	Sum(all values)
Values	376003.34		289.13	0.013

Solution 6 (32, 26), similar to solution 5

Average solution

(32 + 32 + 28 + 30 + 32 + 32) / 6 = 31

(22 + 20 + 18 + 20 + 26 + 26) / 6 = 22

Close to (31, 22)

Given the three criteria, the optimal solution is the same - (32, 22).

For the next group of settings the decay distance threshold equals 40 and the retained exposure equals 0.30. The six exposure decay functions are treated as six scenarios. The table is recorded as follows.

Group Eight: Setting 43 - 48: 40, 0.30. Scenarios: 1 – 6

Solution 1 (32, 24)

	Absolute Robustness	Best Possible Decision	Robust Deviation	Relative Robustness
Scenario 1	67008.59	67008.59	0.00	0.000
Scenario 2	70591.32	70609.30	17.98	0.000
Scenario 3	56126.58	56179.89	53.31	0.001
Scenario 4	74453.26	74539.59	86.33	0.001
Scenario 5	81107.75	81166.07	58.32	0.001
Scenario 6	82365.24	82436.15	70.91	0.001
Criteria	Sum(all values)		Max(all values)	Sum(all values)
Values	431652.74		86.33	0.004

Solution 2 (34, 22)

	Absolute Robustness	Best Possible Decision	Robust Deviation	Relative Robustness
Scenario 1	66956.34	67008.59	52.25	0.001
Scenario 2	70609.30	70609.30	0.00	0.000
Scenario 3	56150.46	56179.89	29.43	0.001
Scenario 4	74312.37	74539.59	227.21	0.003
Scenario 5	80968.66	81166.07	197.41	0.002

Scenario 6	82227.16	82436.15	209.00	0.003
Criteria	Sum(all values)		Max(all values)	Sum(all values)
Values	431224.29		227.21	0.009

Solution 3 (32, 20)

	Absolute Robustness	Best Possible Decision	Robust Deviation	Relative Robustness
Scenario 1	66929.02	67008.59	79.57	0.001
Scenario 2	70585.25	70609.30	24.05	0.000
Scenario 3	56179.89	56179.89	0.00	0.000
Scenario 4	74428.86	74539.59	110.72	0.001
Scenario 5	80824.61	81166.07	341.46	0.004
Scenario 6	82077.33	82436.15	358.82	0.004
Criteria	Sum(all values)		Max(all values)	Sum(all values)
Values	431024.97		358.82	0.012

Solution 4 (30, 26)

	Absolute Robustness	Best Possible Decision	Robust Deviation	Relative Robustness
Scenario 1	66982.62	67008.59	25.98	0.000
Scenario 2	70540.36	70609.30	68.93	0.001
Scenario 3	56072.37	56179.89	107.52	0.002
Scenario 4	74539.59	74539.59	0.00	0.000
Scenario 5	81122.15	81166.07	43.92	0.001

Scenario 6	82387.33	82436.15	48.83	0.001
Criteria	Sum(all values)		Max(all values)	Sum(all values)
Values	431644.41		107.52	0.004

Solution 5 (32, 26)

	Absolute Robustness	Best Possible Decision	Robust Deviation	Relative Robustness
Scenario 1	66990.80	67008.59	17.79	0.000
Scenario 2	70559.27	70609.30	50.03	0.001
Scenario 3	56070.41	56179.89	109.48	0.002
Scenario 4	74452.85	74539.59	86.74	0.001
Scenario 5	81166.07	81166.07	0.00	0.000
Scenario 6	82431.42	82436.15	4.74	0.000
Criteria	Sum(all values)		Max(all values)	Sum(all values)
Values	431670.81		109.48	0.004

Solution 6 (32, 28)

	Absolute Robustness	Best Possible Decision	Robust Deviation	Relative Robustness
Scenario 1	66933.39	67008.59	75.21	0.001
Scenario 2	70505.59	70609.30	103.71	0.001
Scenario 3	55997.66	56179.89	182.23	0.003
Scenario 4	74442.65	74539.59	96.94	0.001
Scenario 5	81165.88	81166.07	0.19	0.000

Scenario 6	82436.15	82436.15	0.00	0.000
Criteria	Sum(all values)		Max(all values)	Sum(all values)
Values	431481.31		182.23	0.007

Average solution

(32 + 34 + 32 + 30 + 32 + 32) / 6 = 32

(24 + 22 + 20 + 26 + 26 + 28) / 6 = 24.3

Close to (32, 24)

Given the three criteria, there are two potential optimal solutions, which are (32, 26) and (32, 24).

For the next group of settings the decay distance threshold equals 40 and the retained exposure equals 0.50. The six exposure decay functions are treated as six scenarios. The table is recorded as follows.

Group Nine: Setting 49 - 54: 40, 0.50. Scenarios: 1 – 6

Solution 1 (32, 28)

	Absolute Robustness	Best Possible Decision	Robust Deviation	Relative Robustness
Scenario 1	80006.13	80006.13	0.00	0.000
Scenario 2	82557.71	82627.02	69.32	0.001
Scenario 3	72194.90	72251.98	57.08	0.001
Scenario 4	85369.89	85427.98	58.09	0.001
Scenario 5	90172.20	90172.62	0.42	0.000
Scenario 6	91079.54	91079.54	0.00	0.000
Criteria	Sum(all values)		Max(all values)	Sum(all values)
Values	501380.36		69.32	0.002

Solution 2 (40, 26)

	Absolute Robustness	Best Possible Decision	Robust Deviation	Relative Robustness
Scenario 1	79882.38	80006.13	123.76	0.002
Scenario 2	82627.02	82627.02	0.00	0.000
Scenario 3	72250.26	72251.98	1.72	0.000
Scenario 4	85092.30	85427.98	335.69	0.004
Scenario 5	89865.45	90172.62	307.17	0.003

Scenario 6	90770.06	91079.54	309.48	0.003
Criteria	Sum(all values)		Max(all values)	Sum(all values)
Values	500487.46		335.69	0.012

Solution 3 (40, 24)

	Absolute Robustness	Best Possible Decision	Robust Deviation	Relative Robustness
Scenario 1	79866.06	80006.13	140.07	0.002
Scenario 2	82613.43	82627.02	13.59	0.000
Scenario 3	72251.98	72251.98	0.00	0.000
Scenario 4	85065.16	85427.98	362.83	0.004
Scenario 5	89806.99	90172.62	365.64	0.004
Scenario 6	90707.49	91079.54	372.05	0.004
Criteria	Sum(all values)		Max(all values)	Sum(all values)
Values	500311.10		365.64	0.014

Solution 4 (30, 30)

	Absolute Robustness	Best Possible Decision	Robust Deviation	Relative Robustness
Scenario 1	79950.16	80006.13	55.98	0.001
Scenario 2	82505.43	82627.02	121.59	0.001
Scenario 3	72147.49	72251.98	104.49	0.001
Scenario 4	85427.98	85427.98	0.00	0.000
Scenario 5	90124.89	90172.62	47.73	0.001

Scenario 6	91031.12	91079.54	48.42	0.001
Criteria	Sum(all values)		Max(all values)	Sum(all values)
Values	501187.06		121.59	0.005

Solution 5 (32, 30)

	Absolute Robustness	Best Possible Decision	Robust Deviation	Relative Robustness
Scenario 1	79980.00	80006.13	26.13	0.000
Scenario 2	82548.22	82627.02	78.80	0.001
Scenario 3	72175.92	72251.98	76.06	0.001
Scenario 4	85390.03	85427.98	37.95	0.000
Scenario 5	90172.62	90172.62	0.00	0.000
Scenario 6	91078.32	91079.54	1.22	0.000
Criteria	Sum(all values)		Max(all values)	Sum(all values)
Values	501345.11		78.80	0.003

Solution 6 (32, 28), similar to solution 1

Average solution

(32 + 40 + 40 + 30 + 32 + 32) / 6 = 34.3

(28 + 26 + 24 + 30 + 30 + 28) / 6 = 27.7

Close to (34, 28)

Given the three criteria, the optimal solution is the same - (32, 28).

APPENDIX C

SOURCE OF DATA USED IN CHAPTER FOUR

The source of the data of “percent of purchase were impulse”.

Table 28. Impulse purchase sensitivity of departments

Dept No.	Department Name	Impulse Purchase Percentage	Source of Assumptions
1	Cosmetics/Fragrances	33/41.8	BizStats
2	Handbags	45*	Similar to Accessories
3	Intimate Apparel	27	BizStats
4	Jewelry	62	BizStats
5	Accessories	44^	Front-end focus
6	Infants and Toddlers	47*	Similar to Girls
7	Boys	47*	Similar to Girls
8	Girls	47	BizStats
9	Juniors	47*	Similar to Girls
10	Young Men's	36	BizStats
11	Men's	40	BizStats

Table 28 (continued)

12	Misses Career	44	BizStats
13	Misses Casual	54	BizStats
14	Petites	47	BizStats
15	Women's	47	BizStats
16	Shoes	52	BizStats
17	Hosiery	52*	Similar to shoes
18	Furniture	22	BizStats
19	Home Dec (Window)	53	BizStats
20	Bedding and Bath	47*	Refer to Furniture (20%) and Home Dec (80%)
21	Luggage	10^	Chandon
22	Frames/Stationary	39	BizStats
23	Kitchen/Dining	53*	Similar to Home Dec
24	Toy	54^	POPAI

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