GAME THEORETIC FLOW AND ROUTING CONTROL FOR COMMUNICATION NETWORKS

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As the need to support high speed data exchange in modern communication networks grows rapidly, effective and fair sharing of the network resources becomes very important. Today's communication networks typically involve a large number of users that share the same network resources but may have different, and often competing, objectives. Advanced network protocols that are implemented to optimize the performance of such networks typically assume that the users are passive and are willing to accept compromising their own performance for the sake of optimizing the performance of the overall network. However, considering the trend towards more decentralization in the future, it is natural to assume that the users in a large network may take a more active approach and become more interested in optimizing their own individual performances without giving much consideration to the overall performance of the network. A similar situation occurs when the users are members of teams that are sharing the network resources. A user may find itself cooperating with other members of its team which itself is competing with the other teams in the network. Game theory appears to provide the necessary framework and mathematical tools for formulating and analyzing the strategic interactions among users, or teams of users, of such networks. In this thesis, we investigate networks in which users, or teams of users, either compete or cooperate for the same network resources. We considered two important network topologies and used many examples to illustrate the various solution concepts that we have investigated.. First we consider two-node

parallel link networks with non-cooperative users trying to optimally distribute their flows among the links. For these networks, we established a condition which guarantees the existence and uniqueness of a Nash equilibrium for the link flows. We derived an analytical expression for the Nash equilibrium and investigated its properties in terms of the network parameters and the users preferences. We showed that in a competitive environment users can achieve larger flow rates by properly emphasizing the corresponding term in their utility functions, but that this can only be done at the expense of an increase in the expected delay. Next, we considered a general network structure with multiple links, multiple nodes, and multiple competing users. We proved the existence of a unique Nash equilibrium. We also investigated many of its intuitive properties. We also extended the model to a network where multiple teams of users compete with each other while cooperating within the teams to optimize a team level performance. For this model, we studied the Noninferior Nash solution and compared its results with the standard Nash equilibrium solution.

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1.0 INTRODUCTION

This thesis is concerned with the optimization of multi-node multi-user communication networks by using concepts from game theory. This approach allows us to study cooperative and non-cooperative interactions among the users in the networks. The classical approach requires users to optimize the overall performance of the network [1,2]. However, a user in a large network such as the Internet may choose to optimize its own performance rather than the overall network performance [3]. Game theory becomes an important tool to model and analyze these types of network [4-6]. Problems in optimal routing, flow control, pricing policy, and bandwidth allocation have all been recently formulated and solved using cooperative and non-cooperative solution concepts from game theory.

Efficient use of network resources by a central control system can become difficult for the networks with large number of users. Therefore, decentralized control strategies have gained considerable importance as they remove the complexity of a central control architecture. Since users in a non-cooperative network make their own flow and routing control decisions, the iterative algorithms allowing these users to evaluate their optimal strategies can constitute the core of decentralized flow and routing control systems for next generation networks. These algorithms may conceivably be implemented in the Internet by using the capability of IPv4 and IPv6 (Internet Protocol version 4 and 6 respectively) which provide network users to route their flow on a specific path [1,2].

Competitive users decide on their flow rate and/or routing based on their utility functions which translate their needs and desires. Utility functions that have been used in most models typically represent a combination of two objectives: (1) maximizing the flow rate and (2) minimizing the congestion delay experienced by the user's data. There are numerous ways of combining these two objectives into a single optimization criterion for each user. One approach [7-12] is to consider a utility function for each user in the form of "benefit/cost", also known as a power criterion. In this approach, the benefit term represents the total throughput and the cost term represents a measure of the average expected delay. Maximizing the utility function in this case will achieve the desired two objectives. Another approach that has been considered is a utility function for each user in the form of "benefit – cost" [13-16]. Typically, the benefit term is a weighted measure of the throughput and the cost term is a weighted measure of the expected delay. The weights can be viewed as representing the user's preferences for one term over the other. Maximizing this utility function essentially means increasing the throughput while simultaneously reducing the expected delay. Since the users may differ in their throughput needs and tolerance for the expected delays, and they consequently can adjust the relative importance of the benefit and cost terms by selecting appropriate preference constants (i.e. weights) for each of these terms in their own utility function. In this thesis, we consider both "benefit/cost" and "benefit - cost" types of utility functions.

Since communication networks and game theory are focal points of this investigation, to facilitate the understanding of the concepts given later in this thesis, a brief review of these two topics is given in Chapter 1. After giving a short introduction of the principles of how today's communication networks work, we describe various solution concepts in game theory.

In Chapter 2, we consider a simple two-node parallel link network with multiple competing users. Each user decides on its total throughput and on the throughput sent on each link so as to maximize its own utility function which is in the 'benefit - cost' form. The model also provides the users the flexibility of choosing different preference parameters for different links. This allows them to adjust the amount of throughput sent on each link based on their previous satisfaction and experiences from these links. We give a condition under which the existence and uniqueness of a Nash equilibrium is guaranteed. We also study some intuitive properties of the Nash equilibrium and present two examples to demonstrate these properties.

In Chapter 3, we consider a general network environment with multiple nodes and multiple links. Users of this network can enter and exit from any node in the network. The route of each user is fixed; along a path in the network connecting a source node to a destination node specific for that user. Since users share the same network resources, they decide on the flow rate competitively by maximizing a power-criterion utility function. For this model, this utility function is more advantageous than the benefit-cost form of utility functions because it results in more analytically tractable solutions. We prove the existence of a unique Nash equilibrium and study some intuitive properties of this equilibrium. Next, we derive both synchronous and asynchronous numerical algorithms for the users so that they can evaluate their Nash equilibrium flow rates based on the information obtained from the network. As pointed out previously, these numerical algorithms are very desirable because they encourage the establishment of distributive networks.

In Chapter 4 to complete the analysis of multi-node multi-link networks, we also consider the same general network environment but with users that decide to *cooperate* with each other. The resultant solution is called Pareto dominant solution which usually gives larger level of satisfaction to every user than the level of satisfaction they would receive from Nash equilibrium flow rates. Next, we expand the model from a single team to multiple teams which compete with each other. We assume that each team has a leader which ensures cooperation within the team but competition with the other teams. In the new model, the optimization is performed at two level: (i) The leader of each team competes with the leaders of other teams in order to achieve the best utility for its team. Each team leader tries to optimize a social utility function which is the scaled sum of all the utility functions of the users in its team, and (ii) At the team level, the team members cooperate to achieve a team level satisfaction rather than an individual level satisfaction. The resultant equilibrium is called Noninferier Nash equilibrium [17, 18]. By means of two examples, we illustrate the Nash, Pareto, and Noninferior Nash equilibriums and compare their performances.

In Chapter 5, we summarize some important concluding remarks in this thesis and we also present some suggestions on possible extensions of the current research.

1.1 COMMUNICATION NETWORKS

Processing power of the early computers was limited and few of these computers could run calculation-dense applications. In the 1960's, the U.S. Department of Defense *Advanced Research Project Agency (ARPA)* decided to develop first data network, ARPANET, so that all research groups can share these highly powered computers [2]. The principle purpose was to design a reliable data network which can perform even though some parts of the network fail. This early network has evolved and eventually led to today's Internet, the global network interconnecting networks around the world.

Communication networks can be classified into three different networks, namely *circuit switched networks*, *packet switched networks,* and *virtual switched network* based on the switching type.. In *circuit switched networks*, a circuit between a source node and a destination node is set up before the communication starts. Here, "circuit" refers to all network resources such as switches, routers, communication links, etc. allocated for the communication between the source and the destination nodes. The main characteristic of this network is that network resources allocated for a communication between two nodes are dedicated to this communication. Therefore, these resources are not available to other users until the communication ends and the resources are released. However, dedicated resources can guarantee a quality of service in these networks. In *packet switched networks,* each source node splits its data into small blocks of information bits, called packets, then sends these packets to the destination. Since each packet has the source and destination addresses, a packet can be treated as an independent entity so that multiple sources can send their data through the same network resources. In the *virtual switched networks*, each communication session between two nodes starts with setting up a *virtual circuit* between these two nodes. Both nodes use this virtual path to send and receive packets until the end of communication. Virtual circuits support different level of quality services while many users still can share the same network resources.

Communication networks can also be classified based on the location. A network consisting of computers in a building or in a small geographical area is called *local area network (LAN)*. A *wide area network (WAN)* can contain computers in a very large geographical area. To understand how computer networks work, first we will introduce local area network, and then we will discuss how an internet works. We will try to keep the discussion as compact as possible by giving only the general underlying concepts and ignoring the details.

1.1.1 Local Area Networks (LANs)

Early communication networks used a connection scheme in which there was at least one link between any two nodes as illustrated in Figure 1.1. They are also called *point-to-point networks* or *mesh networks*.

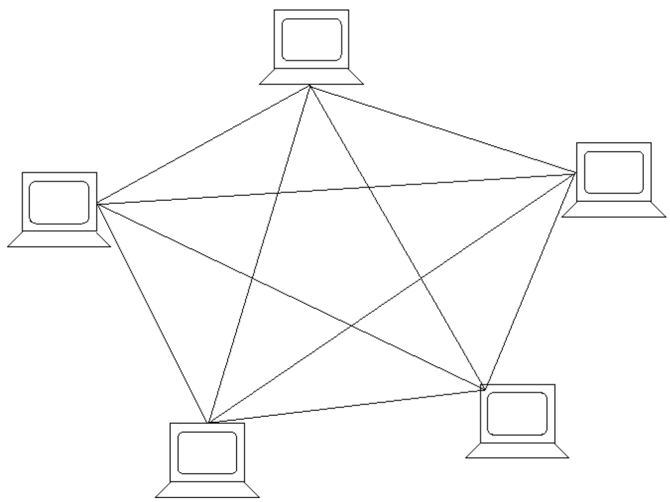

Figure 1.1. A Mesh Network

The number of links in mesh networks grows exponentially for each additional computer. Therefore, the construction cost of these networks can be prohibitively high. Another important shortcoming of these networks is that many links are not used most of the time. In modern LANs, almost all resources in the network are shared by all computers, therefore, the cost of building networks reduces and these resources are used more efficiently. Only one transmission at a time can take place in shared networks (in the medium) while others have to wait their turn.

There are many LAN technologies with their own specification about the topology of the network, the format of the packet, the modulation scheme, etc. Topology of the network specifies how to connect computers with each other. We use the term *"frame"* instead of *"packet"* for a given LAN technology since each LAN technology has its own packet specification. We will briefly explain how an *Ethernet* network, a well known LAN technology, operates.

A typical Ethernet network using 10Base-T wiring scheme is shown in Figure 1.2. Each node in the network needs a network interface card, called Ethernet card[1] which has a unique Ethernet address[2] that comes within its electronic circuitry. The Ethernet hub located at the center of the network is also a circuit that regulates the transmissions in the network. It allows only one user to transmit at a time, thus, mainly performing a multiplexing function.

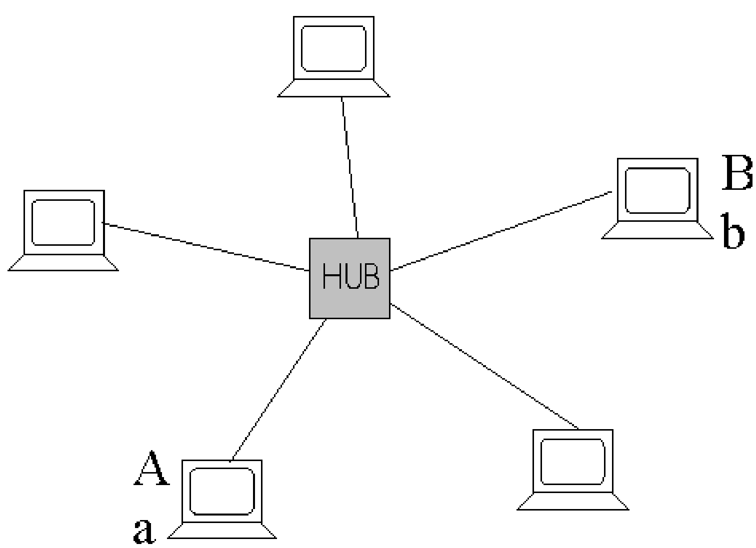

Figure 1.2. An Ethernet Network

[1] An Ethernet card is a collection of circuits printed on a board. It is plugged into the mother board of the computer.
[2] The *Ethernet address* is also called the *physical address* of the node.

Suppose that node A with Ethernet address **a** has some data to transmit to node B with Ethernet address **b** in Figure 1.2. First, node A checks the network for any on-going transmission. If there is no on-going transmission, node A, using the standard Ethernet frame format, puts the source and destination addresses, and its data into frames then it sends the frames to the hub. After receiving the frames, the hub copies each frame to all other links so that all computers can get the frames. Eventually, each computer extracts the destination Ethernet address from the frames it received and compares it with its own Ethernet address. They will match only for node B, thus, only node B will keep the frames while other nodes do not.

Two or more transmissions can not take place at the same time in an Ethernet since all links and the hub are used for only one transmission. Therefore, these Ethernets are called shared Ethernets. If two or more nodes start transmission at the same time, then the hub will inform all nodes that a collision happened. Being aware of the collision, the source nodes wait for random periods of time before re-transmitting. The mechanism of how to access to the medium, described very shortly the above, is an implementation of CSMA/CD (carrier sensitive multiple access / collision detection) protocol in Ethernet.

Due to performance concerns, there is a limit on the number of nodes that can be connected to the same Ethernet hub. Performance can be increased by interconnecting Ethernets with switches or routers.

1.1.2 The Internet

An internet consists of two or more networks such as Ethernets connected by routers. Each node in an internet has a unique network address and an Ethernet address. Unlike Ethernet addresses, a network address is usually based on the geographical position of the node. Routers make the decision of where to send the packet based on the network address not the Ethernet address. A simple internet illustrated in Figure 1.3 has five Ethernets which are interconnected by four routers: R1, R2, R3, and R4.

Suppose that node A wants to send some data to node B. The Ethernet addresses of node A and B are **a** and **b** and their network addresses are **A** and **B**, respectively. Node A sends its Ethernet frame [**a**, **r1** | **A**, **B** | data] to the router it is connected to. After receiving the frame, router R1 makes some modifications on the frame such as it throws away the Ethernet addresses and puts the information into a packet with IP (Internet Protocol) format. Then R1 checks its routing table to find out which router destination address B is connected to. In this case it is R4. R1 sends the IP packet [**A**, **B** | data] to R4. R4 checks its routing table and deliver the packet to R3. Finally, R3 finds out that the destination is in an Ethernet and sends the Ethernet frame [**r3**, **b** | **A**, **B** | data] to the Ethernet hub which copies the frame to its all output ports and eventually, the destination node **b** receives the frame.

The global Internet interconnects the networks around the world. Two important protocols, TCP (Transfer Control Protocol) and IP (Internet Protocol), usually referred together as TCP/IP, are used for exchanging data in the Internet. IP specifies how to assign a unique network address to each computer in the Internet and how to send a packet from a source node to a destination node. IP defines a universal packet format to be able to perform packet delivery in the Internet which contains various networks with different frame formats and technologies. IP supports best effort service for delivering packets from a source to a destination node in the Internet. It doesn't guarantee that the packets might be delayed, duplicated, lost, or corrupted with transmission errors.

TCP protocol has emerged to prevent these undesirable effects and provide a reliable transportation in the Internet. This protocol achieves reliability by means of an acknowledgment mechanism which requires a destination node to notify the source node about whether it received the transmitted packets successfully. This protocol also performs flow and congestion control based on information available to it so that network resources are used efficiently.

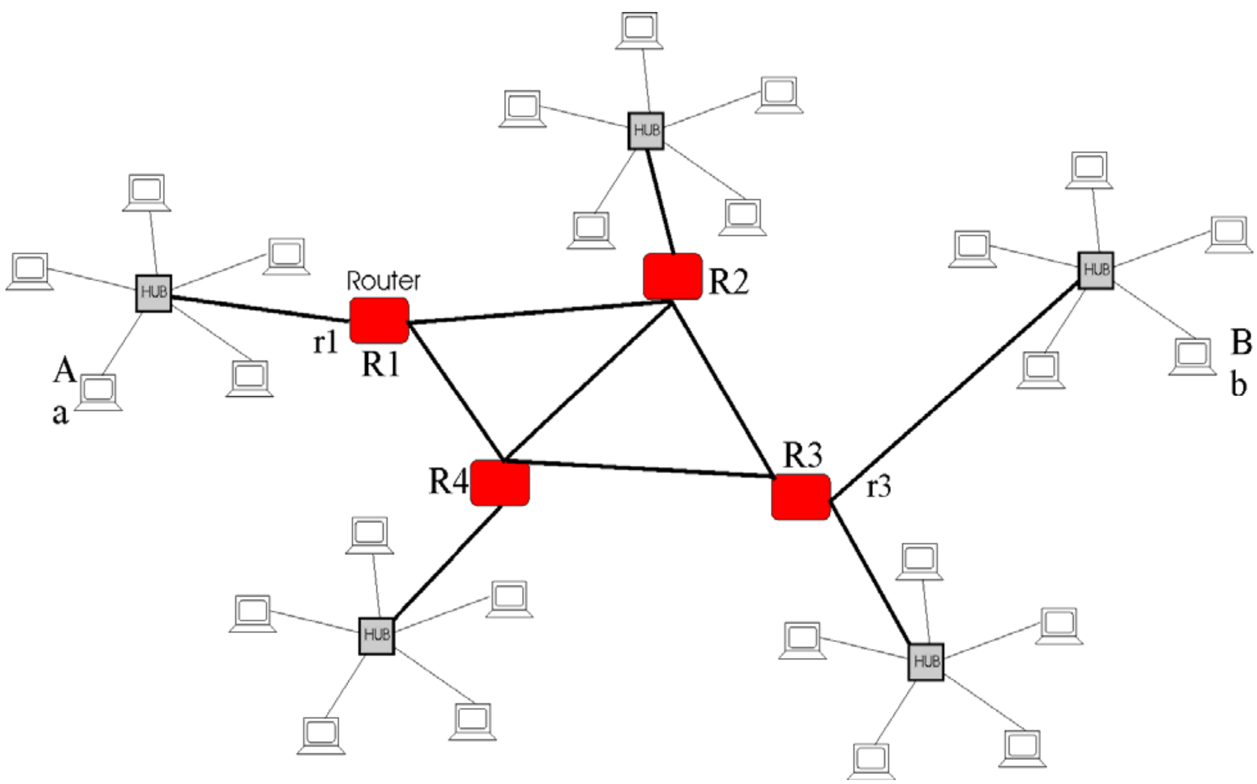

Figure 1.3. A small internet.

1.2 GAME THEORY

Game theory is an important mathematical tool to analyze many problems originating from various disciplines. Problems dealing with cooperative and competitive entities in engineering, economics, political science, and many other fields are modeled and analyzed using game theoretic principles. Two important solution concepts from game theory are Nash equilibrium and Pareto dominance which are usually used to investigate cooperative and competing entities respectively.

A *player*, also called *user* in the context of communication networks, is the basic entity in models of game theory. Each user has to make a decision about which action it[3] should take from a set of available actions. Each user optimally chooses its action among the set of all actions to maximize a utility function (payoff function) or to minimize a cost function. These functions translate the level of satisfaction associated with each possible action. In other works, these functions define a preference relation for the users. For instance, if x and y are two possible choices, a rational user will take action x if the utility of choosing x is higher than the utility of choosing y, or if the cost of choosing x is lower than the cost of choosing y.

Games arise in situations where there is interdependence among users. That is, in situations where one user's payoff is not only a function of its choice but also a function of all other users' choices. For instance, the amount of profit of a company depends on its price setting as well as on other companies' price settings. Therefore, to maximize its utility function or minimize its cost function, a user makes its decision by taking into account other users' possible decisions.

[3] A user can be thought as a software entity which makes optimizations and decisions in computer networks. So we will refer each of them as "it".

Even though games can be classified into many different categories, we will emphasize only three of them. **a)** Based on cooperation among the users, a game can be *cooperative* or *non-cooperative*. In non-cooperative games, each user is only concerned about its own payoff and doesn't pay attention to other's payoffs. On the other hand, in cooperative games, users collaborate to increase their individual as well their mutual (social) payoff functions. **b)** Games can be *strategic* (also called *static*) or *repeated* (also called *dynamic*). In strategic games, users simultaneously and only once make decisions. Differently, in repeated games, users interact more than once and play the game many times. In these games, users' future payoffs depend on their current strategic choices. **c)** In *nonhierarchical games*, no user enforces its strategy to other users. However, in *hierarchical games*, a user called the *leader*[4] can impose its strategy on another user which is called the *follower*. The solution concept for these games is called Stackelberg equilibrium [19]. The leader makes a choice to maximize its own utility ahead of the follower and then allow the follower to know its choice. Based on the leader's choice, the follower decides on its actions to maximize its own utility function. In this thesis, we will concentrate mainly on non-cooperative and cooperative strategic games.

1.2.1 Strategic Games

A strategic game, sometimes also called one shot game, has users who make their decisions only once, simultaneously, and independently [5,6]. "Once" means that users interact only one time and they finish the game by announcing their decisions simultaneously. Simultaneous decision making can be realized in many ways. In one scheme, each user sends its decision to a

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[4] There can be more than one leader in a hierarchical game. We focus on the games with only one leader.

central computer (not necessarily at the same time) and then the central computer publishes all users' decisions at once.

In static games, users are aware of each others utility functions so they know the strategic interactions in a game. Let $\mathcal{N} = (1, 2, ..., N)$ be the set of users and N be the number of users in the game. A generic user i has to choose its action[5] a^i from its set of possible actions A^i. The outcome of the game is given by an action profile $a = (a^1, a^2, ..., a^N)$ which is a collection of choices made by all users. An action profile belongs to the set A which is the Cartesian product of action sets of all users, i.e. $A = \otimes_{i \in \mathcal{N}} A^i$. User i has the utility function $U^i : A \to R$ that defines the user's preference relations. Notice that the utility of user i does not only depend on its action $a^i \in A^i$ but also depends on the choices of all users $a \in A$. In summary, by using the notation given the above, a strategic game can be denoted by $\left\langle \mathcal{N}, \left(A^i\right), \left(U^i\right) \right\rangle$.

We interchangeably use $U^i(a)$ and $U^i(a^i, a^{-i})$ to stress the effect of all other users' actions on user i's utility. The decisions made by all users except user i is denoted by $a^{-i} \in A^{-i}$, to be precise, $a^{-i} = (a^1, a^2, ..., a^{i-1}, a^{i+1}, ..., a^N) = (a^k)_{k \in \mathcal{N} \setminus \{i\}}$. Similarly, A^{-i} denotes the set $A = \otimes_{k \in \mathcal{N} \setminus \{i\}} A^k$.

1.2.2 The Nash Equilibrium

Nash equilibrium is one of the most commonly used solution concepts when there is no cooperation among the users or when it is hard to impose cooperation. In these cases, regardless of what other users do, each user wants to maximize its own payoff. Nash equilibrium is safe

[5] The words "action" and "strategy" are used interchangeably even though they are not synonyms.

against greedy efforts of any user who wants to increase its payoff by deviating from it. A user can only have worse payoff if it changes its Nash strategy unilaterally. Therefore, the Nash strategy is an equilibrium and can be thought as a steady-state point of the game.

Non-cooperative games naturally lead to distributed control systems in which decision makers are individuals. A central authority that makes decisions for all users is not needed in distributed systems. This eliminates the complex control signaling schemes which are necessary for a central optimization. Therefore, the overall system design[6] is much simplified resulting into easily expandable communication network. Now, let us give a precise definition of the Nash equilibrium [5,6].

Definition 1: The action profile $a^* \in A$ is a Nash equilibrium of the strategic game $\left\langle \mathcal{N}, \left(A^i\right), \left(U^i\right) \right\rangle$ if it satisfies the following property for each user $i \in \mathcal{N}$:

$$U^i(a^{i*}, a^{-i*}) \geq U^i(a^i, a^{-i*}) \quad \text{for all } a^i \in A^i \tag{1}$$

It is clear from the definition that each user $i \in \mathcal{N}$ will only lose if it changes its Nash strategy a^{i*} to any other choice a^i. As a result, all users in the game will prefer not to change their Nash strategies. First, let us give the definition of the *best actions*[7] of a user and then give another definition of Nash equilibrium.

Definition 2: Given other users' actions $a^{-i} \in A^{-i}$, the set of *best actions* of user *i* is given by $B^i(a^{-i})$:

$$B^i(a^{-i}) = \left\{ a^i \in A^i : U^i(a^i, a^{-i}) \geq U^i(a^{i'}, a^{-i}) \text{ for all } a^{i'} \in A^i \right\} \tag{2}$$

[6] However the complexity of the cell phone might increase.
[7] *Best action* is also called *best response* or *rational response*.

Definition 3: A Nash equilibrium of the strategic game $\langle \mathcal{N}, (A^i), (U^i) \rangle$ is an action profile a with the property:

$$a^{i*} \in B^i(a^{-i*}) \text{ for all } i \in \mathcal{N} \quad (3)$$

Based on this definition we can evaluate a Nash equilibrium by obtaining the best responses of all users and then search for an action profile a such that $a^{i*} \in B^i(a^{-i*})$ for all $i \in \mathcal{N}$. The second step corresponds to finding the solution of the problem with N variable and N equations, if best response functions, B^i s, are singleton-valued functions.

1.2.3 The Stackelberg Strategy

In previous section, we considered games in which the users were able to make decisions independently and simultaneously. The Nash equilibrium is the appropriate solution concept for these games. But there are situations where one user has a dominant role over other users and affects their choices. Also, there are cases where there is no simple way for the users to make their decisions simultaneously and one user can declare its choice before other users [19]. These cases can be modeled and analyzed as Stackelberg games. In these games, as mentioned before, there are two types of users, namely, the *leader*[8] and the *follower*. The leader is the user which imposes its decision over the follower by declaring its decision before the follower. The follower is the user whose decision is affected by the leader's decision and it reacts to the leader's decision rationally [5,6]. Let A_1 and A_2 be the action (strategy) sets of user 1 and user 2 respectively. User 1 and user 2 would like to minimize their cost functions

[8] Sometimes, the *leader* is also called the *manager*.

$J_1(a_1,a_2)$ and $J_2(a_1,a_2)$, respectively. Let us give the formal definition of the Stackelberg equilibrium for user 2 to be the leader [19]:

Definition 4: $(a_1^{S_2}, a_2^{S_2})$ is called a Stackelberg strategy pair of the game if

1. There exists a mapping $T: A_2 \to A_1$, such that, for any fixed $a_2 \in A_2$, $J_1(Ta_2, a_2) \le J_1(a_1, a_2)$ for all $a_1 \in A_1$.
2. There exists a $a_2^{S_2} \in A_2$ such that $J_2(Ta_2^{S_2}, a_2^{S_2}) \le J_2(Ta_2, a_2)$ for all $a_2 \in A_2$.

Let us give another definition of the Stackelberg equilibrium with user 2 as the leader.

Definition 5: $(a_1^{S_2}, a_2^{S_2})$ is a Stackelberg strategy pair with player 2 as leader if and only if $(a_1^{S_2}, a_2^{S_2}) \in B^1$ and

$$J_2(a_1^{S_2}, a_2^{S_2}) \le J_2(a_1, a_2), \quad \text{for all } (a_1, a_2) \in B^1 \tag{4}$$

where B^1 is the best action set of user 1.

The leader evaluates its Stackelberg strategy by obtaining the best response of the follower for its all actions $a_2 \in A_2$ and then by performing a maximization based on its action and the follower's best response.

1.2.4 Pareto Dominance and Efficiency (Optimality)

It is well known that the Nash equilibrium is not usually efficient [20]. The inefficiency of a non-cooperative system can be interpreted as the cost of having a distributed control system.

In some cases, users can cooperate or they can be forced to cooperate to have a more efficient operating point so that some or all users can benefit from it. A *Pareto dominant* action profile increases some users' utility without hurting any users' utility. On the other hand, for any action profile $a \in A$, we can find an action profile a^* in *Pareto efficient (optimal)* action profile set $A^* \subset A$ that Pareto dominates the action profile a. That is, $a^* \in A^*$ is Pareto optimal if we cannot find any $a \in A$ that Pareto dominates a^*. Pareto dominance and optimality are formally given in the following definition.

Definition 6: An action profile $\hat{a}$ is said to be *Pareto dominant* over another action profile a if $U^i(\hat{a}) \geq U^i(a)$ for all $i \in \mathcal{N}$ and $U^i(\hat{a}) > U^i(a)$ for some $i \in \mathcal{N}$. Moreover, an action profile a^* is said to be *Pareto efficient (optimal)* if there is no other action profile a such that $U^i(a) \geq U^i(a^*)$ for all $i \in \mathcal{N}$ and $U^i(a) > U^i(a^*)$ for some $i \in \mathcal{N}$.

Let us give an example that demonstrates the game theoretical concepts described the above.

Example 1.1: Consider a static game where there are two users. The cost functions of user 1 and user 2 are

$$J_1(x_1, x_2) = (x_1 - 2.5)^2 + (x_2 - 0.5)^2 - x_1 x_2 + x_1 + x_2 \qquad (5)$$

$$J_2(x_1, x_2) = 3(x_1 - 1.2)^2 + (x_2 - 2.2)^2 - x_1 x_2 + x_1 + x_2 \qquad (6)$$

respectively. Let us consider the non-cooperative case in which each user wants to minimize its own cost function. The level curves of J_1 and J_2 can be seen in Figure 1.4. User 1 and user 2 choose actions $x_1 \in R$ and $x_2 \in R$ respectively. Notice that action sets of both users are the same, i.e. $X_1 \equiv X_2 \equiv R$. Let $X = X_1 \times X_2$ be the action profile set of the game. If user 1 had control on both actions x_1 and x_2, it would choose the action profile (2.66, 1.33) that corresponds to the minimum of J_1, denoted by J_1^* in Figure 1.4. Similarly, if user 2 had control on both actions it would choose the action profile (1.43, 2.41) that corresponds to the minimum of J_2, denoted by J_2^*.

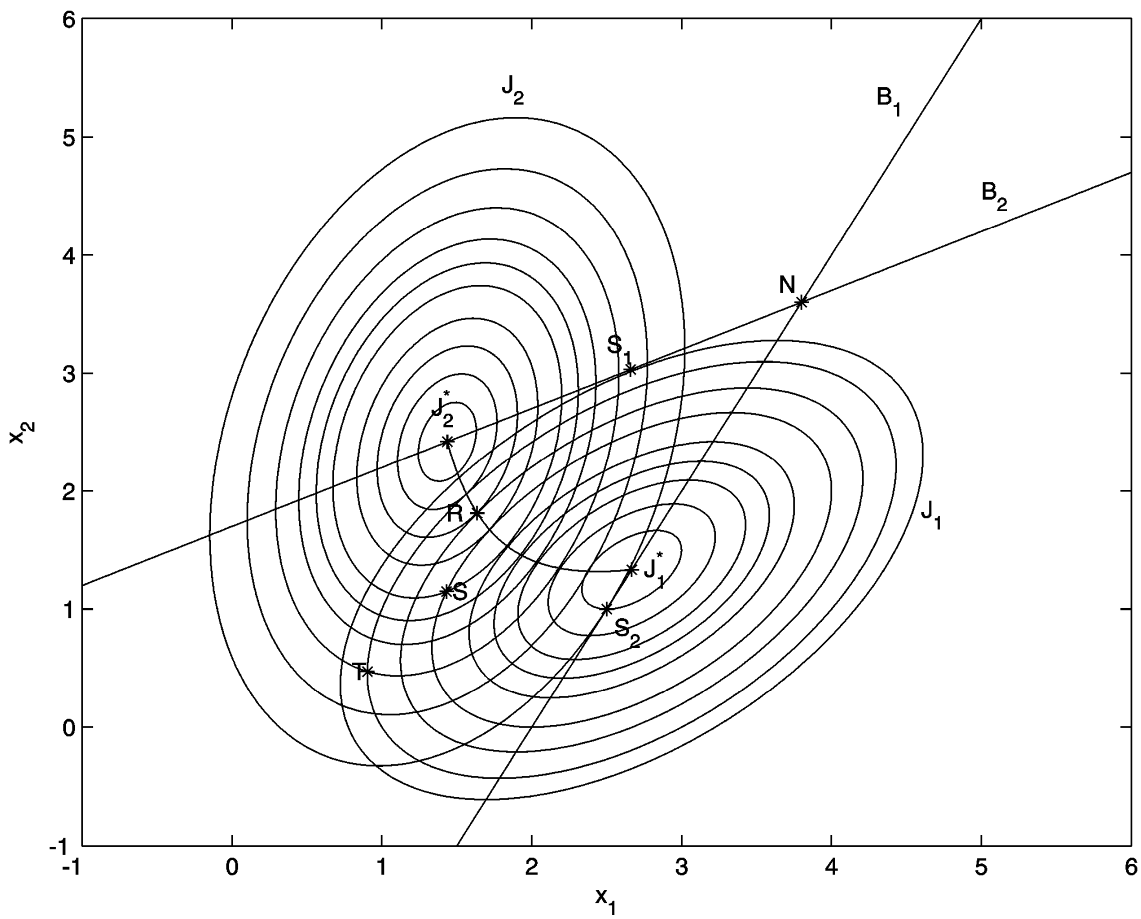


Figure 1.4. The level curves of J_1 and J_2

The best actions of user 1, $B_1(x_2)$, can be drawn by joining the points at which lines of constant x_2 are tangent to the level curves of J_1. Similarly, the best actions of user 2, $B_2(x_1)$, can be drawn by joining the points at which lines of constant x_1 are tangent to the level curves of J_2. Although the best actions can be found by means of level curves, their analytical expressions can be easily found for this example. We minimize J_1 for given x_2 to find $B_1(x_2)$. Therefore, J_1 will be only a function of x_1 when x_2 is fixed. The minimizer of J_1 satisfies the first order necessary condition (FONC):

$$\frac{dJ_1(x_1)}{dx_1} = 2(x_1 - 2.5) - x_2 + 1 = 0 \quad \Rightarrow \quad x_1^* = B_1(x_2) = 0.5x_2 + 2 \tag{7}$$

Second order necessary condition (SONC),

$$\frac{d^2 J_1(x_1)}{dx_1^2} = 2 > 0 \tag{8}$$

is also satisfied, thus, x_1^* is the minimizer for given x_2. In the same way, the best action set of user 2 can be found:

$$B_2(x_1) = 0.5x_1 + 1.7 \tag{9}$$

Finding best action sets are complete. Now, from the second definition of the Nash equilibrium, we have to find an action profile (x_1, x_2) such that

$$(x_1, x_2) \in B_1 \quad \text{and} \quad (x_1, x_2) \in B_2 \tag{10}$$

The intersection point of B_1 and B_2 satisfies this condition. Therefore, equate two best responses:

$$B_1 = B_2 \quad \Rightarrow \quad 2x_1 - 4 = 0.5x_1 + 1.7 \quad \Rightarrow \quad (x_1^N, x_2^N) = (3.8, 3.6).$$

In Figure 1.4 the point *N* corresponds to the Nash equilibrium of the game (x_1^N, x_2^N).

Now let us find the Stackelberg equilibrium when user 1 is the leader and user 2 is the follower. The leader knows the follower's best action for its each action. Therefore, the leader performs an optimization based on the follower's best action. We insert $B_2(x_1)$ into the leader's cost function $J_1(x_1, B_2(x_1))$ and minimize $J_1(x_1, B_2(x_1))$ with respect to x_1 to obtain the leader's strategy:

$$J_1(x_1, B_2(x_1)) = (x_1 - 2.5)^2 + \left((0.5x_1 + 1.7) - 0.5\right)^2 - x_1(0.5x_1 + 1.7) + x_1 + (0.5x_1 + 1.7) \quad (11)$$

$$\frac{dJ_1}{dx_1} = 1.5x_1 - 4 = 0 \quad \Rightarrow \quad x_1^{S_1} = 2.66 \quad and \quad x_2^{S_1} = B_2(x_1^{S_1}) = 3.03 \qquad (12)$$

The point indicated by S_1 corresponds to the Stackelberg equilibrium of the game, $(x_1^{S_1}, x_2^{S_1}) = (2.66, 3.03)$, when the first user is the leader. Similarly, for the case in which user 2 is the leader and user 1 is the follower, the Stackelberg equilibrium can be evaluated as $(x_1^{S_2}, x_2^{S_2}) = (2.5, 1)$, which is denoted by S_2 in Figure 1.4.

The curve joining the points of tangency between the level curves of J_1^* and J_2^* in Figure 1.4 corresponds to the Pareto optimal points of the game. In this case, the users cooperate to minimize a common cost function given as follows:

$$J(x_1, x_2) = \alpha J_1 + (1-\alpha) J_2 \qquad 0 \le \alpha \le 1 \qquad (13)$$

It is clear that if both users minimize *J,* indirectly, one user chooses its action profile that minimizes its own cost function as well another's cost function. Let us find an analytical expression for Pareto optimal action profiles:

$$\begin{aligned} J = \alpha &\left[(x_1 - 2.5)^2 + (x_2 - 0.5)^2 - x_1 x_2 + x_1 + x_2 \right] \\ &+ (1-\alpha)\left[3(x_1 - 1.2)^2 + (x_2 - 2.2)^2 - x_1 x_2 + x_1 + x_2 \right] \end{aligned} \qquad (14)$$

Write FONCs:

$$\frac{\partial J}{\partial x_1} = \alpha(-4x_1 + 2.2) + 6x_1 - x_2 - 6.2 = 0 \tag{15}$$

$$\frac{\partial J}{\partial x_2} = 3.4\alpha + 2x_2 - x_1 - 3.4 = 0 \tag{16}$$

Simultaneous solution of equations (15) and (16) results in:

$$x_1^P = \frac{-7.8\alpha + 15.8}{11 - 8\alpha} \quad \text{and} \quad x_2^P = \frac{13.6\alpha^2 - 36.2\alpha + 26.6}{11 - 8\alpha} \tag{17}$$

It is easy to check that (x_1^P, x_2^P) satisfies SONCs, therefore it is the minimizer of *J*. Let us write the Pareto optimal action set X^* formally becomes:

$$X^* = \left\{ (x_1, x_2) : x_1 = \frac{-7.8\alpha + 15.8}{11 - 8\alpha}, x_2 = \frac{13.6\alpha^2 - 36.2\alpha + 26.6}{11 - 8\alpha}, 0 \le \alpha \le 1 \right\} \tag{18}$$

Now consider the action profiles (1.65, 1.81), (1.43, 1.14), and (0.90, 0.47) indicated by *R*, *S*, and *T* in Figure 1.4, respectively. The costs of user 1 and user 2 for these action profiles are given in Table 1.1.

Table 1.1. Costs for user 1 and user 2 in Example 1.1 when action profiles *R*, *S*, and *T* are used.

	J^1	J^2
R	2.9	1.2
S	2.5	2.2
T	3.5	4.2

The action profile S Pareto dominates the action profile T because both users have smaller costs at profile S than at profile T. Similarly R also Pareto dominates T. A point in a Pareto optimal set doesn't necessarily mean to dominate all other points. For example, even $R \in X^*$, it does not dominate the profile S as the cost of user 1 is higher at profile R than profile S. It is also easy to see that there is no other profile that Pareto dominates R. Therefore, it is a point in Pareto optimal action profile set.

1.3 A BRIEF SURVEY OF PREVIOUS RESEARCH

Models of game theory have been extensively applied to a wide variety of problems arising from communication networks. Problems of routing control [3,10,15,17,18,21-33], flow control [7-9, 11-13, 34-42] , capacity allocation [43-46], and pricing[47-49] the network resources have received considerable attention in the control and communication literature. In a routing control problem, a user tries to find the path(s) through which it should send its throughput demand to have the best performance with respect to some performance criterion. In a flow control problem, a user asks how much flow it should send to the network to find its optimum flow control strategy. Differently, in the capacity allocation problems, users decide how much of the network resources they should allocate. Lastly, pricing of the network resources has been used to increase the use of network efficiency.

The existence and uniqueness of the Nash equilibrium constitutes one of the most important problems. If there exists a unique Nash equilibrium, then we can analytically or numerically obtain this equilibrium point. We may also try to tune the network parameters so that efficient use of the network resources is achieved at the Nash equilibrium.

There are few communication network models for which the uniqueness of the Nash equilibrium is established [50]. In [3], the authors provide a simple example to illustrate the difficulty of having a unique Nash equilibrium. In the example, a network with only four nodes has two Nash equilibriums. Therefore, usually, the uniqueness of the Nash equilibrium is established case by case.

Rosen's diagonal strict concavity (DSC) conditions are used to establish the uniqueness of the Nash equilibrium in some studies. These conditions guarantee the existence and uniqueness of the Nash equilibrium for convex games. DSC conditions are shown to be satisfied for general topology networks with polynomial costs in [21]. Also, these conditions are satisfied by the parallel link networks with two users and two links and by general topology networks under light traffic conditions as in [3].

A user in a competitive routing network makes its decision of how to split its given throughput demand into the available paths. Orda et al [3] study this problem for parallel link and general topology networks with *L* links and *I* users. In this game, each user decides on the amount of flow for each link to minimize its own cost function without paying attention to other users' performance. In other words, by using the notation in the paper, user *i* minimizes its own cost by adjusting its flow configuration vector $\mathrm{f}^{\mathrm{i}} = \left\{ f_1^i, f_2^i, ..., f_L^i \right\}$ where f_l^i denotes the amount of flow user *i* puts on link *l*. The outcome of the game is expressed by the system flow configuration that is the Nash equilibrium of the game given by $F = (\mathrm{f}^1, \mathrm{f}^2, ..., \mathrm{f}^{\mathrm{I}})$. The Nash equilibrium is investigated for a wide variety of cost functions that satisfy some mild convexity conditions. The cost function of each user is expressed as the sum of all link costs. The flow of a generic user through a link incurs some cost, called the link cost. The existence of Nash equilibrium is established for both networks for this cost function. The uniqueness of the Nash equilibrium for the network of parallel links is established under the assumption that all users send their flows over the same set of links. The uniqueness of the general topology networks demands even more restrictive conditions. It is established by means of Rosen's DSC conditions which hold for a lightly loaded general topology networks. The Nash equilibrium is also stable and unique for a special case of the parallel link networks with two users and two links. In addition to the existence and uniqueness of Nash equilibrium analysis, intuitive properties of the Nash equilibrium called monotonicity properties are also explored. For instance, the user with higher throughput demand uses a larger portion of each link than the user with lower throughput demand. They show that the general topology networks might not have these properties.

La and Anantharam consider the same competitive routing problem described above in a dynamic game context in [22]. The users are the Network Access Providers (NAPs) who compete with each other to support the best service to their individual network users. They interact many times with each other until the state of the network changes considerably due to the change of the number of the network users, or the topology of the network, or the load over the network. In practice, NAPs can communicate with each other before they make their decisions; so they can negotiate about the effective use of the network. It is natural to consider that each user wants to increase its own performance. Therefore, none of the users might desire to cooperate unless there is a reward for cooperation or might keep its agreement unless there is a punishment for deviating from it. These interactions among the users cannot be analyzed by static games. Dynamic games provide a better understanding of this situation. In this setting, it is possible to have a Nash equilibrium that yield a minimum total system cost and every user of the network has a cost that is not larger than that of the static game setting. That is the case for parallel link networks and the Nash equilibrium which is socially optimal is called subgame-perfect Nash equilibrium (SPNEP). SPNEP also exists for general topology networks if all users have the same source and destination nodes and if some light technical conditions hold for the network. It might not exist in a general topology network with users having different source and destination nodes.

Routing in communication networks and transportation networks have some similarities in nature. In transportation networks, one driver has negligibly small effect on the other drivers and the solution concept is Wardrop equilibrium [50]. On the other hand, users' flows in the communication networks are not negligible. Altman et al [21] consider the routing control problem in general topology networks with polynomial costs. Users have a fixed amount of flow demand to send to the destination and they have polynomial cost function that is borrowed from the road traffic context. The cost function is defined by the US Bureau of Public Roads. The existence and uniqueness of the Nash equilibrium are established for the general topology networks. Moreover, it is shown that the Nash equilibrium does also result in a socially optimal network operating point at which the total cost of the network is minimum.

It is well known that the Nash equilibrium is not usually efficient. Therefore, different mechanisms are suggested to have an operating point which is more efficient then the Nash equilibrium [23,33,43]. Design parameters of the network and pricing the network resources are two examples of these mechanisms. Korilis et al [33] consider the problem of architecting a non-cooperative network with I users competitively routing their flows through the network just like the users in [3]. In this study, the inefficiency of the Nash equilibrium is overcome by means of two techniques. The first technique is employed in the provisioning phase, i.e., during the construction of the network. The designer of the network adjusts the capacity of each link so that the resultant network yields a Nash equilibrium which is system wide efficient. System wide efficiency means that the overall network performance is optimum with respect to some performance criteria, in this case the criterion is the minimum network cost. The designer is able to achieve this goal since it is assumed that users are rational and their performance criteria are known. A mapping, called Nash mapping, assigns each capacity configuration of the network to a unique Nash equilibrium (system flow configuration). The unique Nash mapping allows them to compare the Nash equilibriums to obtain the capacity configuration that yields the minimum total system cost. To be able to compare different Nash equilibriums, it is also assumed that all users send flows over the same set of links for different link capacity settings. For the network of parallel links, adding all available capacity to the link which has initially highest capacity is the user price optimal solution. A capacity configuration is the user price optimal if it minimizes all users' prices. Price of a user, called marginal cost in economy, is the partial derivative of cost function with respect to the user's own flow. The second technique for overcoming the inefficiency of the Nash equilibrium is employed in the run time. The network manager has control over some portion of the flow to route through the network. The manager adjusts the amount of flow for each link just like other users to lead to a Nash equilibrium at which the minimum average network delay is achieved. Thus, the manager plays a social role in this game to increase the efficiency of the system. Since the manager actively adjusts the available capacity to other users by its flow; this technique is similar to the capacity assignment technique during the provisioning phase. But, differently, the manager's flow consumes some resources and incurs some cost to the network. Authors show that the manager's flow demand must exceed a threshold to achieve its goal. Interestingly, the threshold decreases with increasing load in the network. In other words, the manager's job is easier for a heavily loaded network. Lastly, in this study, the authors also investigate Braess paradox. That is, the addition of some more capacity to a network can degrade all users' performances. The authors prove that this paradox does not occur in parallel link networks. One way to prevent this paradox from occurring in general topology networks is that the available capacity should be added to all links of the network uniformly.

The routing control problems we described earlier involves routing of a given flow demand through the network. The nature of the problem is completely different if the users are to send an unspecified amount of flow on the network and the problem becomes an optimal routing and flowing control problem. In other words, users simultaneously decide on their flow and routing strategies. Each user finds its optimum amount of flow for each link. Altman et al [10] consider this problem for parallel link networks for an arbitrarily large number of users. Users aim to maximize their utility function which is in the form of *benefit/cost*. As mentioned before, this form of the utility functions is recognized as the *power criterion*. A positive power of the total throughput of the user is considered to be the benefit term while the average expected delay experienced by the user's flow is considered to be the cost term. Maximizing this form of utility function can be interpreted as maximizing the throughput while minimizing the delay. The power of the total throughput, adjusting the tradeoff between throughput and delay, can be user specific. For this non-concave utility function the authors find an explicit expression of the solution for a single user that uses M parallel links. Interestingly, it is possible to have an optimal flow configuration for the user such that some links are not used. They also show that there exists a unique Nash equilibrium as the number of users becomes arbitrarily large. The large number of users results into the Nash equilibrium which has delay-equalizing property that delay for each link becomes the same.

As mentioned before, pricing mechanism is also widely used to utilize the resources effectively and increase the system performance. Korilis et al [47] consider a pricing mechanism that leads to minimum network congestion. The authors suggest that price of a link is directly proportional to the congestion level of the link. Therefore, highly congested links will have high prices that will produce an incentive for users to use the lightly congested links with low prices. The manager's objective is to have the network operate at a target point at which the minimum network congestion cost is achieved. A pricing vector of the network contains the prices for all links. The manager chooses the pricing vector and all network users use this pricing vector to decide on their flow rates by minimizing their cost functions. Flow rates of all users constitute the system flow configuration. The authors prove that there exists a unique price vector that induces a unique Nash equilibrium at which the system flow configuration and the manager's target system operating point matches up for parallel link networks. For a general network case such as the Internet where users' cost or utility functions are not known, they introduce an adaptive algorithm to find the price vector. They obtain the sufficient conditions for the algorithm to converge to the optimum price vector.

Rhee and Konstantopoulos [45] consider a bandwidth allocation problem for a parallel link network. Each user reserves the amount of bandwidth that maximizes its own utility function. Utility functions are assumed to be concave and smooth. In this model, users can have different utility functions and the users' throughput is bounded between a maximum and a minimum value. In previous works, each user had a fixed bandwidth demand or the users' demands were limited by the capacity of the resources. The existence and uniqueness of the Nash equilibrium is established for parallel link networks and also for general topology networks with users establishing virtual paths along a fixed route. That is, no user splits its flow over the links but each user sends its flow along one virtual path in the fixed routing scheme. Gauss-Seidel iterative method is shown to be converging to the unique Nash equilibrium for networks with one link and many users.

Lastly, in study [52], an elastic traffic model is studied. Traffic arising as a result of controlling flow rates with respect to available bandwidth within a network is referred as "elastic." In this model, each user chooses a price per unit time. Based on this price, the network assigns a flow rate to this user by optimizing a network performance criterion. For this network model, the authors establishes the stability of the algorithms which are based on additively increasing and multiplicatively decreasing flow rates. They generalize these results to large scale broadband communication systems in [53].

2.0 FLOW AND ROUTING CONTROL FOR PARALLEL-LINK NETWORKS

In this chapter, we consider a two-node parallel link network with multiple competing users. This simple type of network is encountered in many of today's communication networks in several different ways [43]. For instance, in a broadband network, users are assigned some pre-allocated network resources and independent virtual paths are created by splitting the available bandwidth. Each virtual path may be considered as a link in a parallel link network model. Similarly, to simplify routing in a complex communication network, users may be restricted to send their data flow on a specified number of paths between any two nodes in the network. Another example is that of an enterprise which is served by many ISPs. The connections to the ISPs can be modeled as parallel links and the enterprise may have different preferences for each connection based on the price and its previous satisfaction with that ISP.

Each user of the two-node parallel link network decides on its flow rate which maximizes a benefit-cost form of utility function. Since our network consists of several parallel links, we provide each user with the flexibility of using different preference constants for different links in their utility functions. That is, we allow the preference constants to be link-dependent. In doing this, our model gives each user the option to choose the preference constants not only to balance between throughput and delay but also to reduce the usage of links that they perceive to be defective and increase the usage of links that they perceive to best meet their needs.

In section 2.1, we present a mathematical formulation of the problem and we derive conditions on the link capacities and preference constants which guarantee the existence and uniqueness of a Nash equilibrium for this network. An analytic expression for the Nash control policy for each user is also derived. In section 2.2, we discuss six properties of the solution which characterize its dependence on the various network parameters. In particular, we show that, in general, allowing the preference constants to be link-dependent will lead to a control policy in which each user's flow rate on each link is directly correlated with the user's preference for that link. We also show that the resulting Nash equilibrium will be non-symmetric and characterized by the fact that all users are not necessarily required to have the same flow rate on each link. This can be especially useful as a mechanism for the network to offer different levels of quality of service to different users. In section 2.3, we present two illustrative examples, and in section 2.4, we provide some concluding remarks.

2.1 MATHEMATICAL FORMULATION AND DERIVATION OF THE NASH EQUILIBRIUM

Consider a data communication network with two nodes - a source node and a destination node - connected by M parallel links and shared by N competitive users. Let the set of links be denoted by $\mathcal{M} = \{1, 2, ..., M\}$ and the set of users by $\mathcal{N} = \{1, 2, ..., N\}$. Let c_m and λ_{mi} for $m \in \mathcal{M}$ and $i \in \mathcal{N}$ denote the capacity of link m and the flow (transmission) rate of user i on link m respectively. We note that the flow rate λ_{mi} is a static control variable chosen by user i. The total flow of all users on link m is therefore given by

$$\bar{\lambda}_m = \sum_{j \in \mathcal{N}} \lambda_{mj} \tag{19}$$

Let $\underset{\sim}{\lambda}_i = \left(\lambda_{1i}, \lambda_{2i}, \ldots\ldots, \lambda_{Mi}\right)$ be an M-dimensional vector denoting the flow configuration of user i on the network links $\{1, 2, \ldots, M\}$. This vector is chosen by user i from a control set:

$$\Omega_i = \left\{ \underset{\sim}{\lambda}_i \in R^M : 0 \le \lambda_{mi} < c_m \text{ for all } m \in \mathcal{M} \right\} \tag{20}$$

in such a way as to maximize a certain utility function. Clearly, a solution $\{\underset{\sim}{\lambda}_1^*, \underset{\sim}{\lambda}_2^*, \ldots, \underset{\sim}{\lambda}_i^*, \ldots, \underset{\sim}{\lambda}_N^*\}$ is feasible if the non-negativity condition $\lambda_{mi} \ge 0$ for all $i \in \mathcal{N}, m \in \mathcal{M}$ and the stability condition $\bar{\lambda}_m < c_m$ for all $m \in \mathcal{M}$ are both satisfied. Since a change in any one user's flow over the network affects all other users' flow configurations, and since each user wants to maximize its own utility function, this network problem is best analyzed using a non-cooperative game theoretic approach. In this chapter, we are interested in the Nash solution concept [4-6] which represents an equilibrium condition when the network reaches steady state operation.

We assume that the level of satisfaction of user $i \in \mathcal{N}$ with a set of flow configurations implemented by all users is measured by a utility function of the form:

$$U_i(\underset{\sim}{\lambda}_1, \underset{\sim}{\lambda}_2, \ldots\ldots, \underset{\sim}{\lambda}_N) = \sum_{m \in \mathcal{M}} \alpha_{mi} \lambda_{mi} - \sum_{m \in \mathcal{M}} \frac{\beta_{mi} \lambda_{mi}}{c_m - \bar{\lambda}_m} \tag{21}$$

As mentioned earlier, this function is in the form of "benefit – cost" where the benefit term corresponds to the i^{th} user's total weighted flow $\sum_{m \in \mathcal{M}} \alpha_{mi} \lambda_{mi}$, and the cost term corresponds to the i^{th} user's total weighted congestion cost $\sum_{m \in \mathcal{M}} \beta_{mi} \lambda_{mi} \Big/ \left(c_m - \bar{\lambda}_m\right)$. In this expression, the deterministic term $1 \Big/ \left(c_m - \bar{\lambda}_m\right)$ represents the expected congestion delay on link m for an M/M/1 delay function [3,22,33,43]. The parameters $\alpha_{mi}>0$ and $\beta_{mi}>0$ in (21) represent preferences for the relative importance that user i attaches to the benefit and cost terms on link m. We note that although all users have utility functions that are of the same form, they can (and most likely will) in general use different preference constants in these functions. Clearly, user i wants to implement a flow configuration $\underset{\sim}{\lambda}_i$ which maximizes U_i. However, U_i depends on the flow configurations implemented by all users. A Nash equilibrium solution for this multi-user game problem is defined as a set of feasible flow configuration vectors $\{\underset{\sim}{\lambda}_1^*, \underset{\sim}{\lambda}_2^*, ..., \underset{\sim}{\lambda}_i^*, ..., \underset{\sim}{\lambda}_N^*\}$ which satisfy the inequalities:

$$\begin{aligned} &U_i\left(\underset{\sim}{\lambda}_1^*, \underset{\sim}{\lambda}_2^*, ..., \underset{\sim}{\lambda}_{i-1}^*, \underset{\sim}{\lambda}_i^*, \underset{\sim}{\lambda}_{i+1}^*, ..., \underset{\sim}{\lambda}_N^*\right) \\ &\quad \geq U_i\left(\underset{\sim}{\lambda}_1^*, \underset{\sim}{\lambda}_2^*, ..., \underset{\sim}{\lambda}_{i-1}^*, \underset{\sim}{\lambda}_i, \underset{\sim}{\lambda}_{i+1}^*, ..., \underset{\sim}{\lambda}_N^*\right) \quad \text{for } \underset{\sim}{\lambda}_i \in \Omega_i \text{ and for } i = 1, 2, ..., N \end{aligned} \tag{22}$$

As we mentioned before, an important property of the Nash solution is that when implemented, no user will be able to benefit by unilaterally deviating (i.e. cheating) from it. In this sense, it represents an equilibrium condition for all network users.

We will now derive necessary and sufficient conditions for the existence and uniqueness of a Nash equilibrium solution for this network problem. For a solution in the interior of control set (20) for each user, these conditions[9] are:

$$\nabla_{\underset{\sim}{\lambda}_i} U_i(\underset{\sim}{\lambda}_1, \underset{\sim}{\lambda}_2, ..., \underset{\sim}{\lambda}_N) = 0 \quad \text{for all } i \in \mathcal{N} \tag{23}$$

and

$$\mathrm{H}_{\underset{\sim}{\lambda}_i} U_i(\underset{\sim}{\lambda}_1, \underset{\sim}{\lambda}_2, ..., \underset{\sim}{\lambda}_N) \text{ is negative definite for all } i \in \mathcal{N} \tag{24}$$

Condition (23) yields the following equations:

[9] The notations $\nabla_{\underset{\sim}{\lambda}_i}(U_i)$ and $\mathrm{H}_{\underset{\sim}{\lambda}_i}(U_i)$ denote the gradient vector and Hessian matrix of U_i with respect to $\underset{\sim}{\lambda}_i$

$$\alpha_{mi} - \beta_{mi} \frac{c_m - \overline{\lambda}_m + \lambda_{mi}}{\left(c_m - \overline{\lambda}_m\right)^2} = 0 \text{ for all } i \in \mathcal{N}, m \in \mathcal{M} \tag{25}$$

and condition (24) yields a Hessian matrix which is diagonal and whose m^{th} diagonal term is given by:

$$\left(\mathrm{H}_{\lambda_i} U_i\right)_{m,m} = -\frac{2\beta_{mi}(c_m - \overline{\lambda}_m + \lambda_{mi})}{(c_m - \overline{\lambda}_m)^3} \tag{26}$$

The equations in (25) have a unique Nash solution for the flow of user i on link m given by:

$$\lambda_{mi}^{\ *} = \left(c_m - \overline{\lambda}_m^*\right)^2 \gamma_{mi} - \left(c_m - \overline{\lambda}_m^*\right) \text{ for all } m \in \mathcal{M} \tag{27}$$

where $\gamma_{mi} = \alpha_{mi} / \beta_{mi}$. We will refer to the term γ_{mi} as the i^{th} user's tradeoff parameter for link m.

To determine $\overline{\lambda}_m^*$, insert (27) in (19) to obtain

$$\overline{\lambda}_m^* = \left(c_m - \overline{\lambda}_m^*\right)^2 \overline{\gamma}_m - N\left(c_m - \overline{\lambda}_m^*\right) \tag{28}$$

where $\overline{\gamma}_m = \sum_{i \in \mathcal{N}} \gamma_{mi}$. Equation (28) simplifies to the following quadratic expression in $\overline{\lambda}_m^*$:

$$\overline{\gamma}_m (\overline{\lambda}_m^*)^2 + (N - 2\overline{\gamma}_m c_m - 1)\overline{\lambda}_m^* + \overline{\gamma}_m c_m^{\ 2} - N c_m = 0 \tag{29}$$

The two roots of (29) are

$$\overline{\lambda}_m^{*\pm} = c_m + \frac{-\left(N-1\right) \pm \sqrt{\left(N-1\right)^2 + 4\overline{\gamma}_m c_m}}{2\overline{\gamma}_m} \tag{30}$$

The stability condition for the positive root $\overline{\lambda}_m^{*+} < c_m$ reduces to $4\overline{\gamma}_m c_m < 0$, and this inequality can not be satisfied since both $\overline{\gamma}_m$ and c_m are positive constants. Therefore, the root $\overline{\lambda}_m^{*+}$ can not be a feasible solution. On the other hand, the stability condition for the negative root $\overline{\lambda}_m^{*-} < c_m$ reduces to $-\left(N-1\right) < \sqrt{(N-1)^2 + 4\overline{\gamma}_m c_m}$ which is always satisfied. Therefore, the optimal total link flow is $\overline{\lambda}_m^* = \overline{\lambda}_m^{*-}$, or

$$\overline{\lambda}_m{}^* = c_m - \frac{(N-1) + \sqrt{(N-1)^2 + 4\overline{\gamma}_m c_m}}{2\overline{\gamma}_m} \tag{31}$$

The condition for non-negativity of total link flow $\overline{\lambda}_m^{*-} \geq 0$ can be reduced to $N/\overline{\gamma}_m \leq c_m$ for $m \in \mathcal{M}$. But the condition $N/\overline{\gamma}_m \leq c_m$ will be automatically satisfied if all individual flow rates are non-negative, i.e. $\lambda_{mi}{}^* \geq 0$ for all $i \in \mathcal{N}$ and $m \in \mathcal{M}$. By using (27), the inequality $\lambda_{mi}{}^* \geq 0$ yields

$$\overline{\lambda}_m^* \leq c_m - \frac{1}{\gamma_{mi}} = c_m - \frac{\beta_{mi}}{\alpha_{mi}} \tag{32}$$

Using $\overline{\lambda}_m^*$ as given by (31), condition (32) can be rewritten as:

$$\frac{1}{\gamma_{mi}}\left(\frac{\overline{\gamma}_m}{\gamma_{mi}} - (N-1)\right) \leq c_m \quad \text{for all } i \in \mathcal{N}, m \in \mathcal{M} \tag{33}$$

The sufficiency condition in (26) is clearly satisfied since (32) implies $\overline{\lambda}_m^* < c_m$ which guarantees that the expressions in (26) will all be negative. As a final remark, we note that for the Nash solution to be an interior point of the control set (20), the inequalities in (33) must all be strictly satisfied for all users which will then insure that all flows on all links will be positive. That is no link will be unused.

An expression for the residual capacity in link m can be derived from (31) as:

$$\begin{aligned} r_m &= c_m - \overline{\lambda}_m{}^* \\ &= \frac{(N-1) + \sqrt{(N-1)^2 + 4\overline{\gamma}_m c_m}}{2\overline{\gamma}_m} \end{aligned} \tag{34}$$

2.2 PROPERTIES OF THE NASH EQUILIBRIUM

The Nash solution derived in the previous section has certain properties that are consistent with what one would intuitively expect the network to satisfy. In this section we discuss six such properties. The first and second properties are unique to our utility function with link-dependent tradeoff parameters and the remaining four refer to the special case of a network with link-independent tradeoff parameters and are similar to the ones associated with the utility functions used in [3]. Throughout the discussion of these properties, the * superscript will be removed from the Nash solution to simplify the notation.

Property 1: For any link $m \in \mathcal{M}$, and arbitrary users $a, b \in \mathcal{N}$, $\gamma_{ma} > \gamma_{mb} \Leftrightarrow \lambda_{ma} > \lambda_{mb}$ and $\gamma_{ma} = \gamma_{mb} \Leftrightarrow \lambda_{ma} = \lambda_{mb}$.

Proof: Suppose $\gamma_{ma} > \gamma_{mb}$. Multiplying both sides by r_m^2 and subtracting r_m from both sides yields $\lambda_{ma} > \lambda_{mb}$. Now suppose $\lambda_{ma} > \lambda_{mb}$. Subtracting r_m from both sides and dividing by r_m^2 yields $\gamma_{ma} > \gamma_{mb}$. The case of equality can be proven similarly.

Essentially this property says that the Nash solution allows a user with a higher tradeoff parameter to send a higher flow than a user with a smaller tradeoff parameter on any link of the network. As a result, users with high throughput demand (such as in video applications) may choose large values for γ_{mi} while users with low throughput demand (such as in e-mail applications) may choose smaller values for γ_{mi}.

Property 2: For any link $m \in \mathcal{M}$, the total link flow $\bar{\lambda}_m$ increases monotonically with $\bar{\gamma}_m$.

Proof: If we rewrite equation (31) as:

$$\bar{\lambda}_m = c_m - \frac{1}{2}\left[\frac{(N-1)}{\bar{\gamma}_m} + \sqrt{\frac{(N-1)^2}{\bar{\gamma}_m{}^2} + \frac{4c_m}{\bar{\gamma}_m}}\right] \tag{35}$$

it becomes clear that as $\bar{\gamma}_m$ increases, the second term on the right hand side (which corresponds to r_m) decreases and as a result $\bar{\lambda}_m$ increases.

This property implies that an increase in the tradeoff parameters of all users for a link in the network will lead to a larger total flow on that link. This, for instance, might occur when all users choose to increase the preference constants for the benefit term (the throughput term) at the expense of the cost term (the congestion delay term). In doing so, the resulting Nash solution will allow each user to send more flow on the link, however at the expense that their flows will experience higher delays.

As mentioned earlier, the remaining four properties correspond to the special case where all users have the same tradeoff parameters on all links. That is $\gamma_{1i} = \gamma_{2i} = ... = \gamma_{Mi} = \gamma_i$ for all $i \in \mathcal{N}$, which implies that for any link $m \in \mathcal{M}$, $\bar{\gamma}_m = \bar{\gamma} = \sum_{i \in \mathcal{N}} \gamma_i$.

Property 3: For any pair of links $m, n \in \mathcal{M}$, $c_m > c_n \Leftrightarrow r_m > r_n$ and $c_m = c_n \Leftrightarrow r_m = r_n$.

Proof: It is clear from (34) that the residual capacity r_m of link *m* is monotonically increasing with c_m, which implies that $c_m > c_n$ yields $r_m > r_n$. Conversely, assume $r_m > r_n$, using simple cancellation of terms on both sides of (34) yields $c_m > c_n$. The same proof follows for the case of equality.

This property conveys the fact that the unused bandwidths of the links with larger capacities are greater than the unused bandwidths of links with smaller capacities.

Property 4: For any $i \in \mathcal{N}$ and $m,n \in \mathcal{M}$, $c_m > c_n \Rightarrow \lambda_{mi} > \lambda_{ni}$ and $c_m = c_n \Rightarrow \lambda_{mi} = \lambda_{ni}$.

Proof: Assume $c_m > c_n$, for $m,n \in \mathcal{M}$. Property 3 implies $r_m\gamma_i - 1 > r_n\gamma_i - 1$ since $\gamma_i > 0$. Then $r_m\left(r_m\gamma_i - 1\right) > r_m\left(r_n\gamma_i - 1\right) > r_n\left(r_n\gamma_i - 1\right)$ which yields that $\lambda_{mi} > \lambda_{ni}$. The same proof follows for the equality.

Intuitively, this property implies users will be allowed larger flow rates on links that have larger capacities and equal flow rates on links that have equal capacities.

Property 5: For any $i \in \mathcal{N}$ and $m,n \in \mathcal{M}$, $c_m > c_n \Leftrightarrow d_{mi} > d_{ni}$ and $c_m = c_n \Leftrightarrow d_{mi} = d_{ni}$, where d_{mi} is the residual capacity of link *m* seen by user *i*. That is:

$$\begin{aligned} d_{mi} &= c_m - \sum_{k \in \mathcal{N}, k \neq i} \lambda_{mk} \\ &= r_m + \lambda_{mi} \end{aligned} \tag{36}$$

Proof: Assume $c_m > c_n$ which implies $r_m > r_n$ by Property 3 and this implies $r_m^{\ 2}\gamma_i > r_n^{\ 2}\gamma_i$ which is equivalent to $d_{mi} > d_{ni}$. Conversely, assume $d_{mi} > d_{ni}$. Insert λ_{mi} given by (27) into (36) to obtain $d_{mi} = r_m^{\ 2}\gamma_i$. Then $d_{mi} > d_{ni}$ implies $r_m^{\ 2}\gamma_i > r_n^{\ 2}\gamma_i$ which, in turn, implies $r_m > r_n$ since $\gamma_i, r_m, r_n > 0$. The last inequality implies $c_m > c_n$ by Property 3. The same proof follows for the equality.

This property says that an arbitrary user sees larger unused bandwidths on those links that have larger capacities.

Property 6: For $m, n \in \mathcal{M}$, $c_m > c_n \Rightarrow \overline{\lambda}_m > \overline{\lambda}_n$ and $c_m = c_n \Rightarrow \overline{\lambda}_m = \overline{\lambda}_n$.

Proof: From Property 4, $c_m > c_n$ implies $\lambda_{mi} > \lambda_{ni}$ for all $i \in \mathcal{N}$. This implies that $\sum_{i \in \mathcal{N}} \lambda_{mi} > \sum_{i \in \mathcal{N}} \lambda_{ni}$ which is equivalent to $\overline{\lambda}_m > \overline{\lambda}_n$. The same proof follows for the equality.

This last property is a direct consequence of Property 4. That is, if all users are required to have larger flow rates on links with larger capacities then the overall total flow rate will be larger on the links with larger capacities than on links with smaller capacities. Similarly, in the limiting case where all links have the same capacities, the total flows in all links will be the same.

2.3 ILLUSTRATIVE EXAMPLES

In this section, we present two examples to illustrate the solution concept and properties derived in the previous two sections. For convenience, we will represent the tradeoff parameters of all users for all links in the network as an $N \times M$ matrix $G = \left(\gamma_m^i\right)_{\substack{i=1,\dots,N \\ m=1,\dots,M}}$ whose $(i,m)^{th}$ entry corresponds to user i's tradeoff parameter for link m. Similarly, we will write the Nash solution as an $N \times M$ flow matrix $\Lambda = \left(\lambda_m^i\right)_{\substack{i=1,\dots,N \\ m=1,\dots,M}}$ whose $(i,m)^{th}$ entry corresponds to the flow rate of user i on link m.

Example 2.1: Consider a simple network with two parallel links and two users. Let the link capacities be $c_1 = 10$ and $c_2 = 5$. First, we consider the case in which the users' tradeoff parameters are link-independent, i.e. $\gamma_1^i = \gamma_2^i = \gamma^i$ for i=1,2. For this case, the feasible region of tradeoff parameters for the existence and uniqueness of a Nash equilibrium as described in (33) is shown in Figure 2.1. As an illustration, the Nash equilibria for several arbitrary choices of tradeoff parameters in and out of this feasible region are given in Table 2.1 and also shown in Figure 2.1.

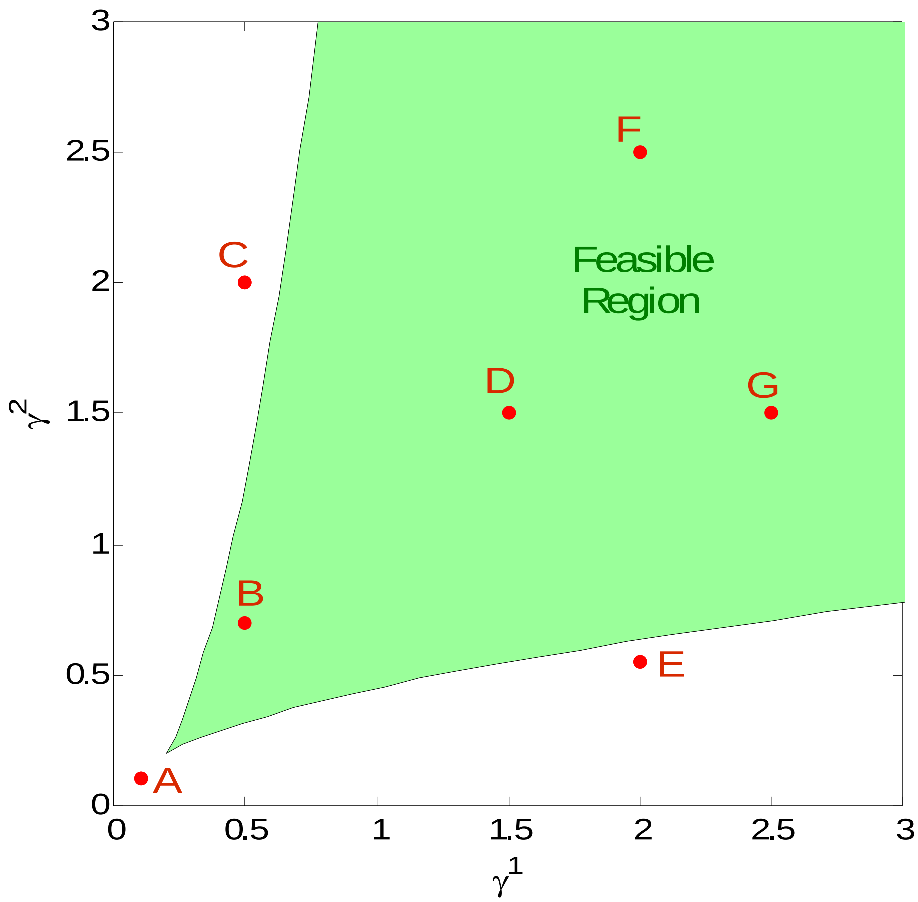


Figure 2.1. Region of feasibility for Example 1.1

when users have the same tradeoff parameters for all links. Feasible regions is defined by the set

$$\left\{(\gamma_1,\gamma_2): 5 \geq \frac{1}{\gamma_i}\left(\frac{\gamma_1+\gamma_2}{\gamma_i}-1\right) \text{ and } \gamma_i < \infty,\ i=1,2\right\}.$$

Notice that the tradeoff parameters of choices A, C, and E, which are outside the feasible region, produced Nash equilibria that are not feasible since at least one user has a negative flow on at least one link. In choice D, both users have the same tradeoff parameters and as a result the corresponding Nash flow rates are the same on both links (symmetric Nash equilibrium). Choices B, F, and G illustrate different feasible choices of parameters yielding feasible Nash solutions. Notice that in all cases, as expected; larger tradeoff parameters yield larger flows.

Table 2.1. Nash equilibria for seven different cases corresponding to seven different values of the pair (γ_1, γ_2) as shown in Figure 2.1.

	γ^1, γ^2	Λ^*	Feasible
A	0.1, 0.1	$\begin{bmatrix} 0 & -1.54 \\ 0 & -1.54 \end{bmatrix}$	No
B	0.5, 0.7	$\begin{bmatrix} 2.22 & 0.62 \\ 4.44 & 1.87 \end{bmatrix}$	Yes
C	0.5, 2	$\begin{bmatrix} 0.22 & -0.3 \\ 7.55 & 3.67 \end{bmatrix}$	No
D	1.5, 1.5	$\begin{bmatrix} 4 & 1.76 \\ 4 & 1.76 \end{bmatrix}$	Yes
E	2, 0.55	$\begin{bmatrix} 7.37 & 3.57 \\ 0.44 & -0.18 \end{bmatrix}$	No
F	2, 2.5	$\begin{bmatrix} 3.55 & 1.57 \\ 4.84 & 2.25 \end{bmatrix}$	Yes
G	2.5, 1.5	$\begin{bmatrix} 5.6 & 2.65 \\ 2.68 & 1.09 \end{bmatrix}$	Yes

In order to illustrate the dependence of the Nash flows on the tradeoff parameters, Figure 2.2 and Figure 2.3 are representative plots of the second user's Nash flow on link 2 and the total Nash flow on link 2, respectively, for varying γ^1 and γ^2. Notice that the flow rate is higher when γ^2 is larger than γ^1. This is in agreement with Property 1. In Figure 2.3, note that total flow on link 2 increases as $\gamma = \gamma^1 + \gamma^2$ increases, which is in agreement with Property 2.

Table 2.2. Nash equilibria for five different choices of link-dependent tradeoff parameters represented by the matrices G.

	Choice I	Choice II	Choice III	Choice IV	Choice V
G	$\begin{bmatrix} 11 & 1 \\ 1 & 2 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 \\ 1 & 6 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$	$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$	$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$
Λ^*	$\begin{bmatrix} 9.08 & 0.68 \\ -0.04 & 2.84 \end{bmatrix}$	$\begin{bmatrix} 3.75 & -0.07 \\ 3.75 & 4.15 \end{bmatrix}$	$\begin{bmatrix} 3.75 & 1.57 \\ 3.75 & 1.57 \end{bmatrix}$	$\begin{bmatrix} 6.0 & 1.57 \\ 2.0 & 1.57 \end{bmatrix}$	$\begin{bmatrix} 6.0 & 0.68 \\ 2.0 & 2.84 \end{bmatrix}$
	Not Feasible	Not Feasible	Feasible	Feasible	Feasible

Now, let us allow the users to have different tradeoff parameters for different links. That is, user i's tradeoff parameters γ_1^i and γ_2^i are now represented as the i^{th} row in the tradeoff configuration matrix G. Nash equilibria for five arbitrary choices of tradeoff parameters represented by the G matrix are illustrated in Table 2.2. Choices I and II are not feasible because the feasibility condition (33) is not satisfied. The tradeoff parameters in Choice I imply that user 1's desire to use link 1 is too high while those of Choice II imply that user 2's preference for link

2 is too high. Choice III corresponds to a situation where both users have equal preference for both links. The resulting Nash solution is symmetric and both users have the same flow on both links. Choices IV and V show that the user whose tradeoff parameter for a given link is higher is allowed a larger flow on that link. Also, it is clear that the Nash flows on one link do not depend on the capacities of, or tradeoff parameters for, the other link.

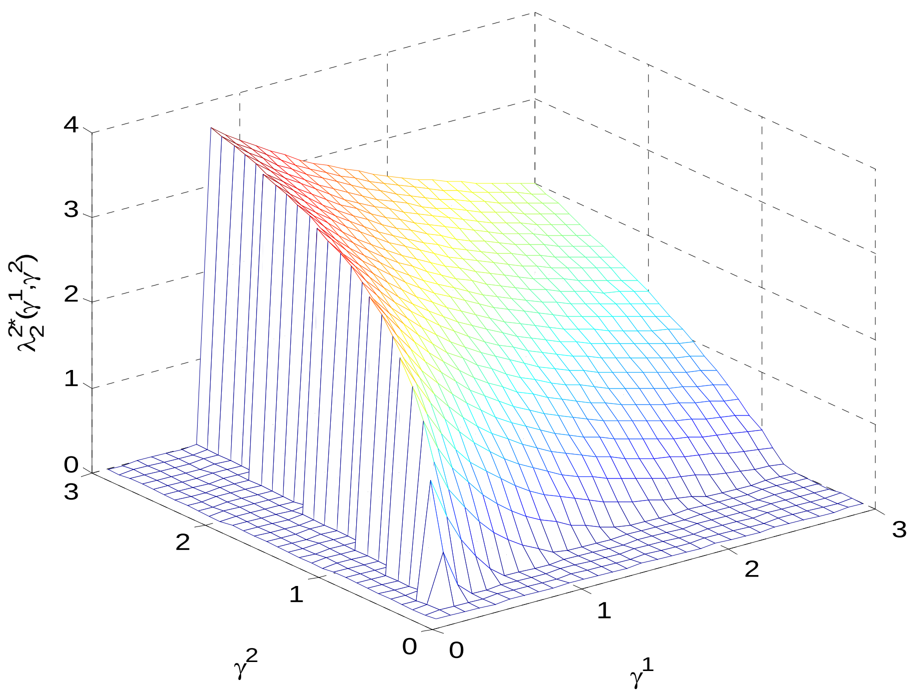


Figure 2.2. Flow rate of user 2 on link 2

as a function of γ_1 and γ_2

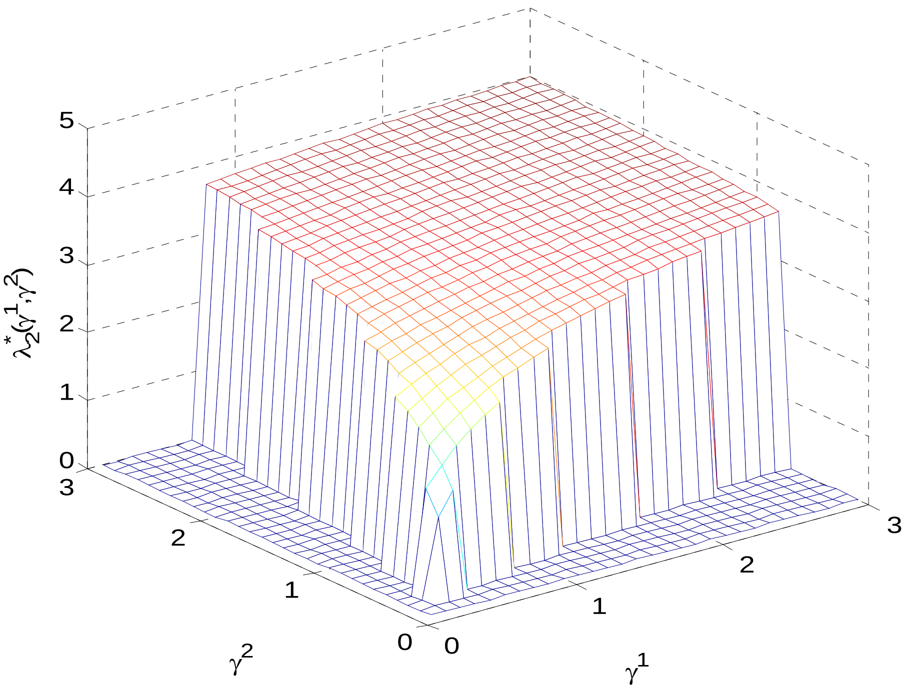


Figure 2.3. Total flow on link 2

Example 2.2: Consider a network with 3 users and 4 links, and let the network parameters be

$$c_1 = 10,\ c_2 = 8,\ c_3 = 8,\ c_4 = 6,\ \text{and}\ G = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 3 \\ 4 & 5 & 2 & 1 \end{bmatrix}$$

In this example, the four links can be considered as virtual paths connecting a source node to a destination node in a broadband network. We assume that the users are capable of sending data on all available links whose maximum available capacities are as specified above. Note that in this example, user 1 has equal preferences for all links, user 2 has highest preference for link 4, and user 3 has highest preference for link 2. It can be easily shown that this case satisfies the feasibility condition (16). The Nash equilibrium flow rates can be determined and are given by the matrix

$$\Lambda^* = \begin{bmatrix} 0.6878 & 0.1504 & 0.7116 & 0.4119 \\ 0.6878 & 1.4336 & 2.9039 & 3.8627 \\ 7.1561 & 5.2832 & 2.9039 & 0.4119 \end{bmatrix}.$$

Notice that the third user's preferences for the first and second links are larger than the other users'. Therefore, its flow rates on these links are also larger. This user might be using a video application. On the other hand, the first user, who is probably using an e-mail application and hence does not need a large flow rate, chooses smaller tradeoffs for all links, and as a result has smaller flow rates. Also, it is obvious that users with the same tradeoff parameter for the same link should have the same flow rate on that link. This occurs in the case of the flow rates of the first and second users on the first link. Note also that even though some users have the same tradeoff parameters for two or more links, they usually don't necessarily have the same flow rates on these links because of the difference in the link capacities and in the user preferences. For example, even though the first user's tradeoffs are the same for all links, it has different flow rates on these links. The Nash optimal total link flows are calculated as $\bar{\lambda}_1^* = 8.5316$, $\bar{\lambda}_2^* = 6.8672$, $\bar{\lambda}_3^* = 6.5194$, and $\bar{\lambda}_4^* = 4.6864$. It can be easily checked that all properties 1 through 6 are satisfied.

Now consider a scenario where all three users linearly increase their preferences for each link, i.e. $\tilde{G} = \sigma G$, where σ is some positive constant. Figure 2.4 illustrates the Nash equilibrium total flow rates and residual capacities in the network links as a function of σ. Notice that all total flows increase rapidly with σ for small values of σ such as $\sigma \in (0,1]$, and then level off as σ increases beyond a value of 2. This basically says that if all users decide to collectively

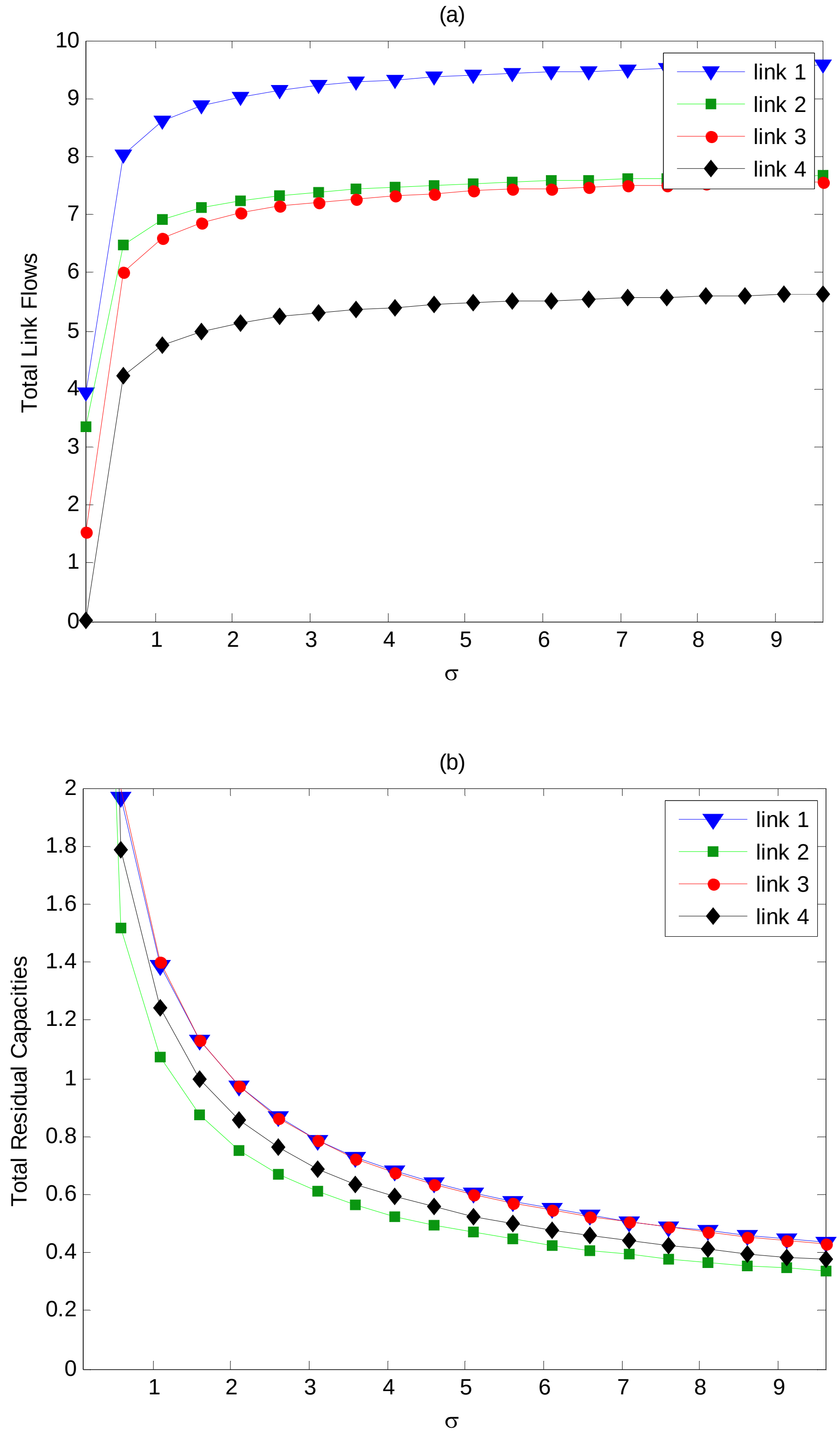


Figure 2.4. (a) Total flow and (b) residual capacity on each link

as a function of σ

increase their preferences in an attempt to increase the total flow in the network while at the same time preserving a Nash equilibrium, then the network will quickly saturate and will only allow for incrementally small increases in the total flows and correspondingly small decreases in the link residual capacities.

2.4 CONCLUSIONS

In this chapter, we considered two-node parallel link communication networks with competing users and derived a flow and routing control policy for each user which satisfies the Nash equilibrium condition of game theory. The network is characterized by each user having a utility function which combines, in a linear additive fashion, two objectives representing the user's desire to maximize its data throughput and minimize its expected delay. Preference constants are introduced in the utility functions to reflect each user's preferences not only with respect to the two objectives, as has been widely considered in the related literature, but also with respect to the links in the network. That is, each user is given the flexibility of choosing preference constants for the two objectives that are link-dependent. A closed form expression for the Nash solution has been derived and feasibility conditions on the link capacities and preference constants which guarantee existence and uniqueness of a Nash equilibrium have been established. Several properties of the resulting flow and routing policy have also been discussed. These properties demonstrate that the Nash solution concept provides a viable equilibrium condition for parallel link networks with competitive users. The resulting Nash flow and routing policies appear to be consistent with what would be intuitively expected as network behavior. The properties also demonstrate that the resulting flows are distributed over all links in the network in accordance with the users' preferences for these links, and that the flows will be higher on links with higher capacities. Finally, we should mention that although this network structure is simple, the results obtained can be considered as a first step in understanding the characteristics and properties of the Nash equilibrium in more complex multi-user networks which are in general extremely difficult to analyze analytically.

3.0 FLOW CONTROL FOR GENERAL MULTI-NODE MULTI-LINK COMMUNICATION NETWORKS

In this chapter, we consider a general network structure with many nodes and many links. We assume that each user has a pre-specified route on the network from a source node to a destination node and that its control variable is its data flow rate on this route. The model allows for each link between any two nodes to be shared by any number of users. Each user chooses its flow rate by maximizing a utility function that measures its level of satisfaction with the choices of flows made by all users. The utility function that we used in this chapter is the standard "power criterion" type that has been extensively used in the literature [7-12]. As mentioned in the Chapter 1, this function has the ability to combine the following two objectives: (i) maximizing the flow rate and (ii) minimizing the expected average delay experienced by the data flow. A user can adjust the importance of one objective with respect to the other by modifying a weight parameter in its own utility function. Using this utility function, a single link network model was considered in [12] and the convergence of synchronous and asynchronous algorithms was established. For a large number of users in a two-node parallel link network, an analytic expression for the asymptotic Nash equilibrium was derived in [10]. It was also shown that the Nash equilibrium flow rates tend to equalize the expected delay over the links. In this chapter, we will generalize the above results by considering a general network structure with many nodes and links. For the power criterion, we prove the existence and uniqueness of an interior Nash equilibrium and we establish the convergence of the synchronous Gauss-Seidel algorithm to this equilibrium. In this general network environment, we also derive some intuitive properties of the Nash equilibrium.

In the next section, we formulate this general multi-user, multi-node, multi-link network optimization problem as a non-cooperative game between the users. In section 3.2, we prove the existence and uniqueness of a Nash equilibrium, and in section 3.3, we give an illustrative example and explore some intuitive properties of the Nash equilibrium. In section 3.4, we present some concluding remarks.

3.1 MODEL FORMULATION

We consider general network topology with M links shared by N competitive users. Let the set of links be denoted by $\mathcal{M} = \{1, 2, ..., M\}$ and the set of users by $\mathcal{N} = \{1, 2, ..., N\}$. Let c_m denote the capacity of link $m \in \mathcal{M}$ and let λ_i denote the flow of user $i \in \mathcal{N}$. The flow rate λ_i which is the same on all links used by user i is a control variable for user i. Let θ_{mi} be a binary number defined as follows:

$$\theta_{mi} = \begin{cases} 1, & \text{if user } i \text{ sends data over link m} \\ 0, & \text{otherwise} \end{cases}. \tag{37}$$

Using vector notation, we define the vectors $\underset{\sim}{c}$, $\underset{\sim}{\lambda}$, and $\underset{\sim}{\theta}_i$ as:

$$\underset{\sim}{c} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_M \end{bmatrix}, \quad \underset{\sim}{\lambda} = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_N \end{bmatrix}, \quad \text{and} \quad \underset{\sim}{\theta}_i = \begin{bmatrix} \theta_{1i} \\ \theta_{2i} \\ \vdots \\ \theta_{Mi} \end{bmatrix} \tag{38}$$

where $\underset{\sim}{c} \in R^M$ denotes the network capacity vector, $\underset{\sim}{\lambda} \in R^N$ the flow control vector, and $\underset{\sim}{\theta}_i \in R^M$ the routing vector for user i (we assume that at least one entry in $\underset{\sim}{\theta}_i$ is nonzero) . We now define the routing matrix $\Theta \in R^{M \times N}$ for all users as:

$$\Theta = \begin{bmatrix} \theta_{11} & \theta_{12} & \cdots\cdots & \theta_{1N} \\ \theta_{21} & \theta_{22} & \cdots\cdots & \theta_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \theta_{M1} & \theta_{M2} & \cdots\cdots & \theta_{MN} \end{bmatrix}. \tag{39}$$

Note that this matrix has entries that are either zero or one and that its i^{th} column is the vector $\underset{\sim}{\theta}_i$. As an illustrative example, the routing matrix

$$\Theta = \left[\underset{\sim}{\theta}_1 \ \underset{\sim}{\theta}_2 \ \underset{\sim}{\theta}_3\right] = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

corresponds to the 3-user, 4-link, 4-node network shown in Figure 3.1 with user 1 sending data on links 1,2, and 4, user 2 sending data on links 1 and 3, and user 3 sending data on link 3.

The residual (unused) capacity of link m denoted by r_m is the difference between the total capacity of the link and the total flow rate over this link. Therefore, the residual capacity vector, denoted by $\underset{\sim}{r}$, is given as follows:

$$\underset{\sim}{r} = \underset{\sim}{c} - \Theta \underset{\sim}{\lambda} \tag{40}$$

A flow vector $\underset{\sim}{\lambda}$ is denoted as an interior point if

$$\underset{\sim}{\lambda} > 0 \quad \text{and} \quad \underset{\sim}{r} > 0 \tag{41}$$

Finally, we assume that each user chooses its flow rate λ_i to maximize a power criterion utility function of the form:

$$U_i\left(\lambda_1,\lambda_2,.......,\lambda_N\right)=\sum_{m\in\mathcal{M}}\theta_{mi}\left(\frac{\lambda_i^{\alpha_i}}{D_m}\right) \qquad (42)$$

where D_m denotes the expected delay on link m and $\alpha_i > 0$ is a preference parameter that user i chooses to adjust the relative importance of flow rate over the average delay of its data transmission. Clearly, the utility function of user i is the sum of the individual utilities over the links that it sends its data on.

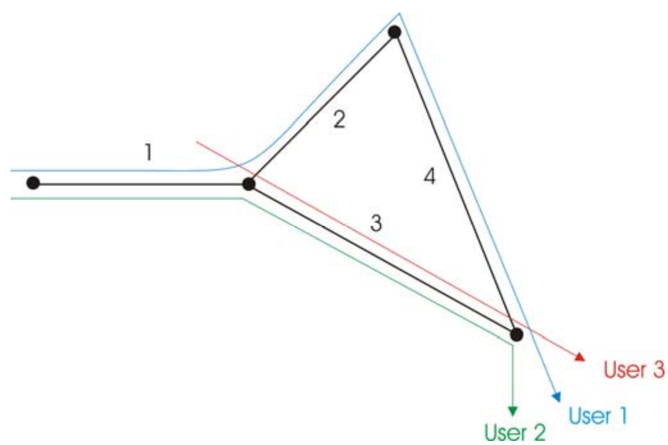


Figure 3.1. A four-link three-user network

A flow vector $\underset{\sim}{\lambda}^* = \left[\lambda_1^*\ \lambda_2^*\ ....\ \lambda_M^*\right]^T$ constitutes a Nash equilibrium if its entries satisfy the following inequalities:

$$U_i\left(\lambda_1^*,\lambda_2^*,..,\lambda_{i-1}^*,\lambda_i^*,\lambda_{i+1}^*,..,\lambda_N^*\right)\geq U_i\left(\lambda_1^*,\lambda_2^*,..,\lambda_{i-1}^*,\lambda_i,\lambda_{i+1}^*,..,\lambda_N^*\right) (43)$$

for $i=1,2,...,N$ and for all possible values of λ_i.

Assuming an M/M/1 queuing process for every link in the network, the expected delay becomes $D_m = 1/r_m$. Using this delay function and writing (42) in vector notation yields

$$U_i(\underset{\sim}{\lambda}) = \lambda_i^{\alpha_i} \underset{\sim i}{\theta}^T \underset{\sim}{r} \quad \text{for all } i \in \mathcal{N} . \tag{44}$$

3.2 EXISTENCE AND UNIQUENESS OF THE NASH EQUILIBRIUM

The following theorem establishes the existence and uniqueness of an interior Nash equilibrium for the general network environment discussed in the previous section.

Theorem: For the general network structure described in (37) through (42) and utility functions (44), there exists a unique Nash equilibrium.

Proof: A Nash equilibrium vector $\underset{\sim}{\lambda}^* = \left[\lambda_1^* \; \lambda_2^* \, ... \, \lambda_M^*\right]^T$, if it exists, must satisfy the following necessary and sufficient conditions:

$$\frac{\partial U_i}{\partial \lambda_i} = 0 \;\text{ and }\; \frac{\partial^2 U_i}{\partial \lambda_i^2} < 0 \;\text{ for }\; i = 1, 2, ..., N \tag{45}$$

When the necessary conditions are applied to (44) we get

$$\frac{\partial U_i}{\partial \lambda_i} = \alpha_i \underset{\sim i}{\theta}^T \underset{\sim}{r} - \lambda_i \underset{\sim i}{\theta}^T \underset{\sim i}{\theta} = 0 \quad \text{for } i = 1, 2, ..., N . \tag{46}$$

Inserting (40) in (46) and rearranging the resulting equation we obtain

$$\underset{\sim i}{\theta}^T \Theta \underset{\sim}{\lambda} + \frac{\lambda_i}{\alpha_i} \underset{\sim i}{\theta}^T \underset{\sim i}{\theta} = \underset{\sim i}{\theta}^T \underset{\sim}{c} \quad \text{for } i = 1, 2, ..., N . \tag{47}$$

Since $\Theta \underset{\sim}{\lambda} = \underset{\sim 1}{\theta} \lambda_1 + \underset{\sim 2}{\theta} \lambda_2 + ... + \underset{\sim N}{\theta} \lambda_N$, we can reduce (47) to the following:

$$\lambda_i\left(\frac{\alpha_i+1}{\alpha_i}\right)\underset{\sim}{\theta}_i^T\underset{\sim}{\theta}_i+\sum_{\substack{j\in\mathcal{N}\\ j\neq i}}\underset{\sim}{\theta}_i^T\underset{\sim}{\theta}_j\lambda_j=\underset{\sim}{\theta}_i^T\underset{\sim}{c} \tag{48}$$

for $i=1,2,\ldots N$. Equation (48) corresponds to a linear system of equations of the form $A\underset{\sim}{\lambda}=\underset{\sim}{b}$ where the matrix A and the vector $\underset{\sim}{b}$ are given by:

$$A=\begin{bmatrix}\frac{\alpha_1+1}{\alpha_1}\underset{\sim}{\theta}_1^T\underset{\sim}{\theta}_1 & \underset{\sim}{\theta}_1^T\underset{\sim}{\theta}_2 & \cdots & \underset{\sim}{\theta}_1^T\underset{\sim}{\theta}_N\\ \underset{\sim}{\theta}_2^T\underset{\sim}{\theta}_1 & \frac{\alpha_2+1}{\alpha_2}\underset{\sim}{\theta}_2^T\underset{\sim}{\theta}_2 & \cdots & \underset{\sim}{\theta}_2^T\underset{\sim}{\theta}_N\\ \vdots & \vdots & \ddots & \vdots\\ \underset{\sim}{\theta}_N^T\underset{\sim}{\theta}_1 & \underset{\sim}{\theta}_N^T\underset{\sim}{\theta}_2 & \cdots & \frac{\alpha_N+1}{\alpha_N}\underset{\sim}{\theta}_N^T\underset{\sim}{\theta}_N\end{bmatrix} \quad\text{and}\quad \underset{\sim}{b}=\begin{bmatrix}\underset{\sim}{\theta}_1^T\underset{\sim}{c}\\ \underset{\sim}{\theta}_2^T\underset{\sim}{c}\\ \vdots\\ \underset{\sim}{\theta}_N^T\underset{\sim}{c}\end{bmatrix} \tag{49}$$

Here, we note that the matrix A can be written in the form $A=B+C$ where $B=\Theta^{\mathrm{T}}\Theta$, which is a positive semidefinite matrix, and $C=\mathrm{diag}\left(\frac{1}{\alpha_1}\underset{\sim}{\theta}_1^T\underset{\sim}{\theta}_1,\frac{1}{\alpha_2}\underset{\sim}{\theta}_2^T\underset{\sim}{\theta}_2,\ldots.,\frac{1}{\alpha_N}\underset{\sim}{\theta}_N^T\underset{\sim}{\theta}_N\right)$, which is a positive definite matrix, since all of its diagonal entries are positive. Hence, A is positive definite which implies that it is also nonsingular, and the equation $A\underset{\sim}{\lambda}=\underset{\sim}{b}$ has a unique solution. Thus, A is invertible and the unique solution is:

$$\underset{\sim}{\lambda}^*=\left(\Theta^{\mathrm{T}}\Theta+\mathrm{C}\right)^{-1}\Theta^{\mathrm{T}}\underset{\sim}{c}\,. \tag{50}$$

When the sufficiency conditions are applied to (44) we get:

$$\frac{\partial^2U_i}{\partial\lambda_i^{\,2}}=\left(\alpha_i-1\right)\lambda_i^{\alpha_i-2}\left(\alpha_i\underset{\sim}{\theta}_i^T\underset{\sim}{r}-\lambda_i\underset{\sim}{\theta}_i^T\underset{\sim}{\theta}_i\right)-\lambda_i^{\alpha_i-1}\left(\alpha_i+1\right)\underset{\sim}{\theta}_i^T\underset{\sim}{\theta}_i<0 \qquad \text{for } i=1,2,\ldots,N \tag{51}$$

The first term on the right hand side in (51) becomes zero by(46). Since, for all $i\in\mathcal{N}$, λ_i and α_i are positive numbers and at least one θ_{mi} is nonzero for some $m\in\mathcal{M}$, we have

$\frac{\partial^2 U_i}{\partial \lambda_i^2} < 0$ for all $i \in \mathcal{N}$. Therefore, the flow rate vector given by (50) is the unique Nash equilibrium of this network. This completes the proof.

We note that if the network consists of only one link then the linear systems $A\underset{\sim}{\lambda} = \underset{\sim}{b}$ given by (49) reduces to the special case in [12] where $\underset{\sim}{\theta}_i^T \underset{\sim}{\theta}_j = 1$ for $i = 1, 2, \ldots N$ and the elements of vector $\underset{\sim}{b}$ are the same and equal to the capacity of the link.

From equation (50), it is clear that the Nash flow rates are linear in link capacities. In particular, if all link capacities are scaled by a real number then the flow rates of all users will also be scaled by the same number. Another important property of the Nash equilibrium is stated in the following proposition:

Property 1: Consider two network optimization problems X and X'. If all users have the same preference parameters and routing strategies in both X and X' except that user $i \in \mathcal{N}$ has a larger preference parameter in X' than in X, then this user's flow rate will be larger in X' than in X.

To prove this property, let us use the notation ' to denote the network parameters of the problem X'. Suppose $\alpha_i' > \alpha_i$. For simplicity of notation, we will drop the * notation from the Nash equilibrium solution. The Nash equilibriums $\underset{\sim}{\lambda}$ and $\underset{\sim}{\lambda}'$ of the problems X and X' satisfy the necessary conditions $A\underset{\sim}{\lambda} = \underset{\sim}{b}$ and $A'\underset{\sim}{\lambda}' = \underset{\sim}{b}$ respectively. Notice that the entries of matrices A and A' are the same except for the i^{th} diagonal entries which are $a_{ii} = (\alpha_i + 1)/\alpha_i$ and $a_{ii}' = (\alpha_i' + 1)/\alpha_i'$ respectively. Since both α_i and α_i' are positive numbers, $\alpha_i' > \alpha_i$ implies $a_{ii}' < a_{ii}$. Let us define an $N \times N$ matrix $D = A - A'$. Thus, all entries of D are zero except for the i^{th} diagonal entry $d_{ii} = a_{ii} - a_{ii}'$ which is a positive number. The necessary conditions $A\underset{\sim}{\lambda} = \underset{\sim}{b}$ of X can be written as $(A' + D)\underset{\sim}{\lambda} = \underset{\sim}{b} = A'\underset{\sim}{\lambda}'$. This is equivalent to $(\underset{\sim}{\lambda}' - \underset{\sim}{\lambda}) = (A')^{-1} D\underset{\sim}{\lambda}$. The vector $D\underset{\sim}{\lambda}$ has all zero entries except its i^{th} entry $(D\underset{\sim}{\lambda})_i = d_{ii}\lambda_i$. Let $\underset{\sim}{v}_i$ denote the i^{th} column of $(A')^{-1}$. Then the flow difference vector becomes $(\underset{\sim}{\lambda}' - \underset{\sim}{\lambda}) = d_{ii}\lambda_i \underset{\sim}{v}_i$. But the i^{th} entry of $\underset{\sim}{v}_i$ is a positive number because A' is positive definite, therefore its inverse $(A')^{-1}$ is also positive definite and all diagonal entries of a positive definite matrix $(A')^{-1}$ are also positive. Therefore, $(\underset{\sim}{\lambda}' - \underset{\sim}{\lambda})_i = d_{ii}\lambda_i (\underset{\sim}{v}_i)_i$ is a positive number indicating the amount of increase of flow rate of user i.

Property 2: In a realistic network with a large number of users, the enforcement of the Nash flow rates obtained in (48) may not be straightforward. Very likely, this may require a central entity in the network which has knowledge of the capacities of all links and the routing strategies and preference parameters of all users. This entity would then determine the Nash flow rates of all users by performing the computations required in (50) and transmitting the optimal flow rates to the users. This computation can be done in one step by computing the inverse $\left(\Theta^{\mathrm{T}}\Theta + \mathrm{C}\right)^{-1}$ or by an iterative Gauss-Seidel process depending on the state of the network. In the latter case, two possible algorithms known as synchronous and asynchronous algorithms can be implemented to converge to the Nash equilibrium. Both algorithms start with an initial flow rate vector $\lambda_j^{(0)}$ chosen arbitrarily but satisfying (41). In the synchronous algorithm, all users update their flow rates simultaneously at every iteration. In the asynchronous algorithm, a randomly selected subset of the users (could be one or more) update their flow rates simultaneously at every iteration. In either of these two algorithms, at the n[th] iteration the users that update their flow rates (i.e. all in the synchronous algorithm, or a subset in the asynchronous algorithm), use expression (48) to perform the following update:

$$\lambda_i^{(n+1)} = K_i \left(\underset{\sim}{\theta}_i^T \underset{\sim}{c} - \sum_{\substack{j \in \mathcal{N} \\ j \neq i}} \underset{\sim}{\theta}_i^T \underset{\sim}{\theta}_j \lambda_j^{(n)} \right) \tag{52}$$

where

$$K_i = \frac{\alpha_i}{\left(1+\alpha_i\right) \underset{\sim}{\theta}_i^T \underset{\sim}{\theta}_i} \tag{53}$$

We note that as a result of the positive definiteness of matrix $\left(\Theta^{\mathrm{T}}\Theta + \mathrm{C}\right)^{-1}$, convergence of the synchronous algorithm is guaranteed [51].

As a final remark, we note that after some simple manipulations, the iteration given by (52) can be reduced to:

$$\lambda_i^{(n+1)} = K_i \left(\sum_{l \in \mathcal{M}_i} r_l^{(n)} + M_i \lambda_i^{(n)} \right) \tag{54}$$

where $\mathcal{M}_i$ denotes the set of all links used by user i and M_i is the total number of links in $\mathcal{M}_i$. Note that K_i, M_i, and $\lambda_i^{(n)}$ are all specific to user i. Therefore, to update its flow rate, the only information user i needs to extract from the network is the sum of the residual capacities of the links which it uses. This is an important result because the users do not need to know each other's routing strategies or preference parameters to update their own flow rates. However, it is important to mention that in both cases (synchronous or asynchronous) the rate of convergence of the algorithm depends on the preference parameters of all users.

3.3 ILLUSTRATIVE EXAMPLE

In order to illustrate the results obtained in the previous section, consider the network with 5 users, 10 nodes, and 16 links shown in Figure 3.2. The routes for all 5 users are indicated on the network and assume that the preference parameters in their utilities are as follows $\alpha_1 = 0.6$, $\alpha_2 = 0.7$, $\alpha_3 = 0.8$, $\alpha_4 = 0.4$, and $\alpha_5 = 0.5$ respectively. Recall that the higher the preference parameter the more emphasis the user places on maximizing its flow rate over minimizing the expected delay of its data. Assume that all 16 links have the same capacity $c_k = 100$ Mbps for $k = 1,...,16$. Throughout the example, when we allow one parameter to vary, we will usually keep all other network parameters unchanged. Furthermore, we say that two users are "*direct competitors*" if they share at least one link on the network. In other words, users i and j are direct competitors if $\underset{\sim}{\theta}_i^T \underset{\sim}{\theta}_j \neq 0$. For example, in Figure 3.2, users 1, 3, and 5 are direct competitors because user 1 shares links 6 with user 3 and link 3 with user 5. On the other hand, users 1 and 4 are not direct competitors.

Using equation (50), the Nash equilibrium solution can be determined as follows:

$$\underset{\sim}{\lambda}^* = \begin{bmatrix} \lambda_1^* \\ \lambda_2^* \\ \lambda_3^* \\ \lambda_4^* \\ \lambda_5^* \end{bmatrix} = \begin{bmatrix} 30.5896 \\ 25.6984 \\ 31.8029 \\ 17.0026 \\ 23.4804 \end{bmatrix} \tag{55}$$

The total flow in each link can be determined by adding the flow rates of the users of that link. For example the total flow rate on link 4 which is used by users 2, 4 and 5 is 66/1814 Mbps and its residual capacity is 33.8186 Mbps. As mentioned earlier, the Nash flow rates are linear in the link capacities. This means that if all link capacities are reduced from 100 Mbps to 50 Mbps, then the flow rates given in (55) will also be reduced by 50%.

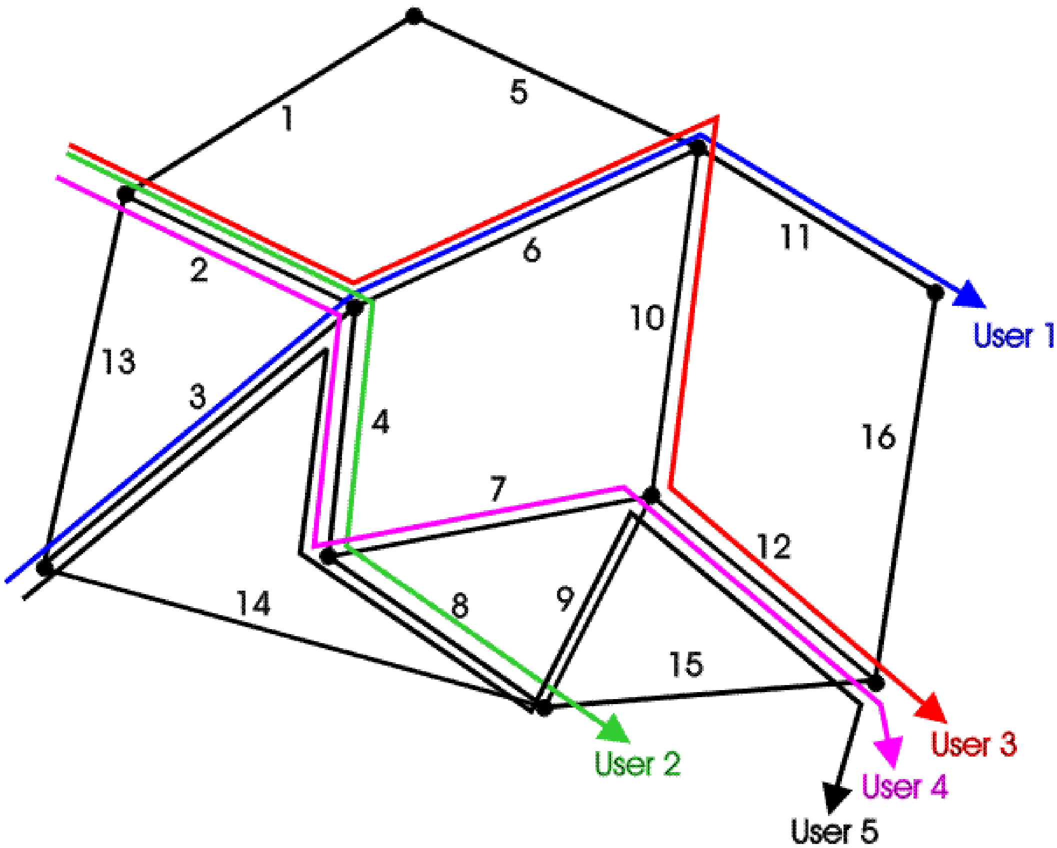


Figure 3.2. Network topology for the illustrative example
a 5-user data communication network.

There are several interesting observations that can be drawn from this example by examining how varying some of the parameters will affect the network equilibrium. We summarize these in the following six observations. We note that although these observations are stated here based on this single example, they have also been observed in many other examples with different network topologies, different routing strategies, and different number of users.

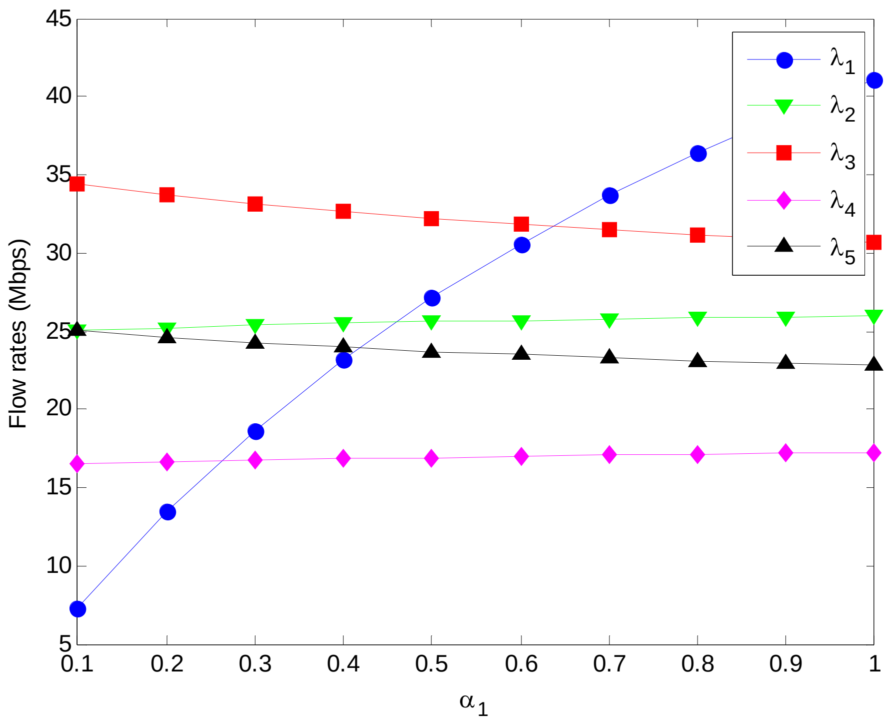


Figure 3.3. Flow rates of all users as a function of the preference parameter of user 1.

<u>Observation 1</u>**:** Let us assume that all network parameters are kept unchanged while the preference parameter of user 1 (α_1) becomes a variable. Figure 3.3 shows the flow rates of all users as α_1 increases from 0.1 to 1 (i.e. as user 1 puts more and more emphasis on maximizing its flow rate). Clearly, we see that the flow rate of user 1 increases with increasing α_1. This is consistent with Property 1. We also observe that the flow rates of the direct competitors of user 1 (i.e. users 3 and 5) decrease as α_1 increases. Note also that as the flow rates of users 3 and 5 decrease their direct competitors, which are users 2 and 4, obtain larger flow rates. Similar observations can be made if the preference parameters of any of the 5 users are varied while the other users parameters are kept constant. These remarks are summarized more formally in the following observation: *If a user's preference parameter increases, its flow rate will increase and the flow rates of all of its direct competitors will decrease.*

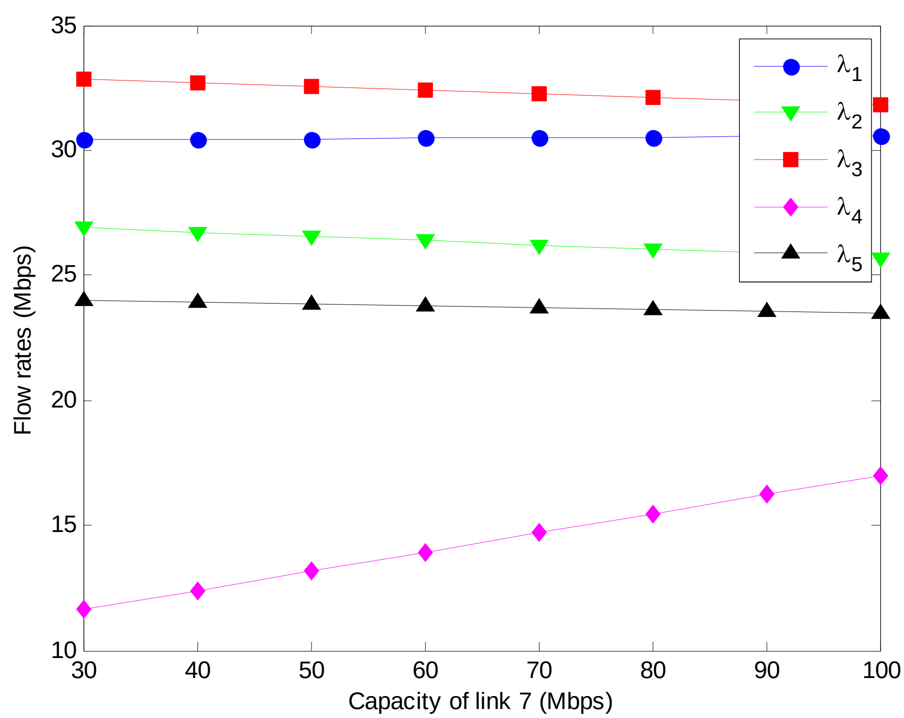


Figure 3.4. Flow rates of all users as a function of the capacity of link 7.

Observation 2: A similar remark can also be made when the capacity of a link used by only one user is increased. This has the same effect as increasing the preference parameter of this user. Figure 3.4 shows plots of the flow rates of all users if the capacity of link 7 is increased from 30 to 100Mbps. As can be seen from these plots, user 4, who is the only user of link 7, obtains a larger flow rate but the direct competitors of user 4, namely users 2, 3, and 5, obtain lower flow rates. This remark is summarized more formally in the following observation: *If the capacity of a link used by only one user is increased, the flow rate of that user will increase and the flow rates of all its direct competitors will decrease.*

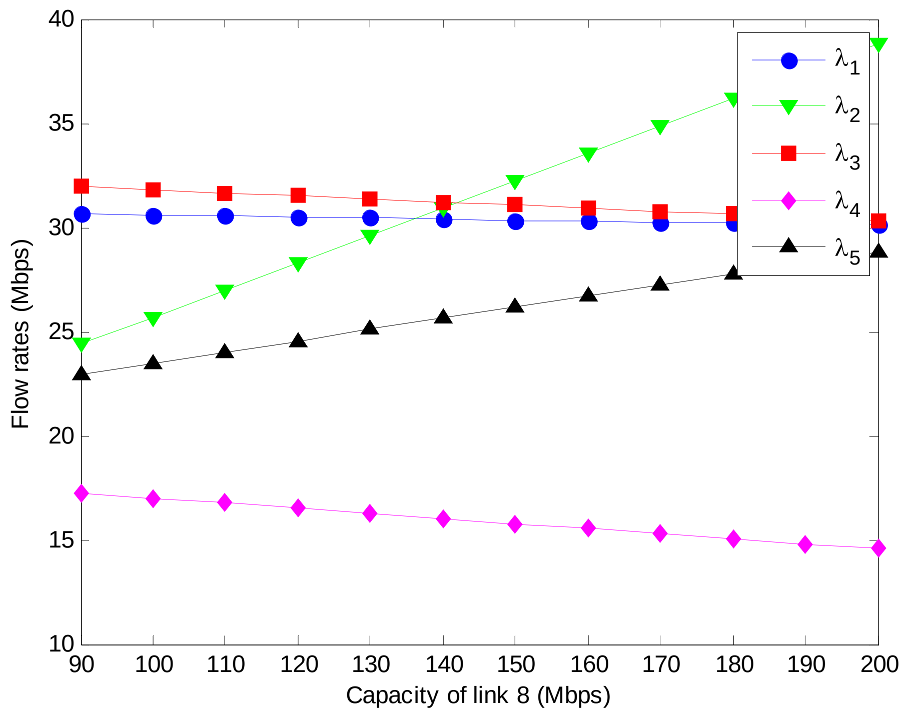


Figure 3.5. Flow rates of all users as a function of the capacity of link 8.

Observation 3: Another intuitive property of the Nash equilibrium is that an increase in the capacity of a link used by more than one user yields larger flow rates for all users of that link. Figure 3.5 shows plots of the flow rates of all users as the capacity of link 8 is increased from 90 to 200 Mbps. Observe that the users of link 8, namely users 2 and 5, obtain larger flow rates as the capacity of link 8 increases. This is summarized in the following observation: *If the capacity of a link used by more than one user is increased, the flow rate of all the user of that link will increase.*

Observation 4: We now examine the Nash equilibrium as a function of all users' preference parameters. To be able to do this and view the results graphically, let us assume that all users now have the same preference parameter $\alpha_i = \alpha$ for $i = 1,...,5$ and let us increase α from 0.1 to 1. This basically says that as α increases, all users are placing more emphasis on flow rates rather than expected delays. Figure 3.6 shows plots of the flow rates for all users as a function of α and Figure 3.7 shows plots of the residual capacities in some links (links 2, 6, 11 and 12) in the network. Observe that the flow rates of all 5 users increase, but that the rate of increase becomes less as α approaches 1. Similarly, the residual capacities decrease, but the rate of decrease becomes less as α approaches 1. These remarks are summarized in the following observation: *If all users have the same preference parameter then an increase in this parameter will result in an increase in the flow rates of all users and a decrease in the residual capacities of all links. In both cases, however, the rate of increase or decreases will be less for larger values of the parameter.*

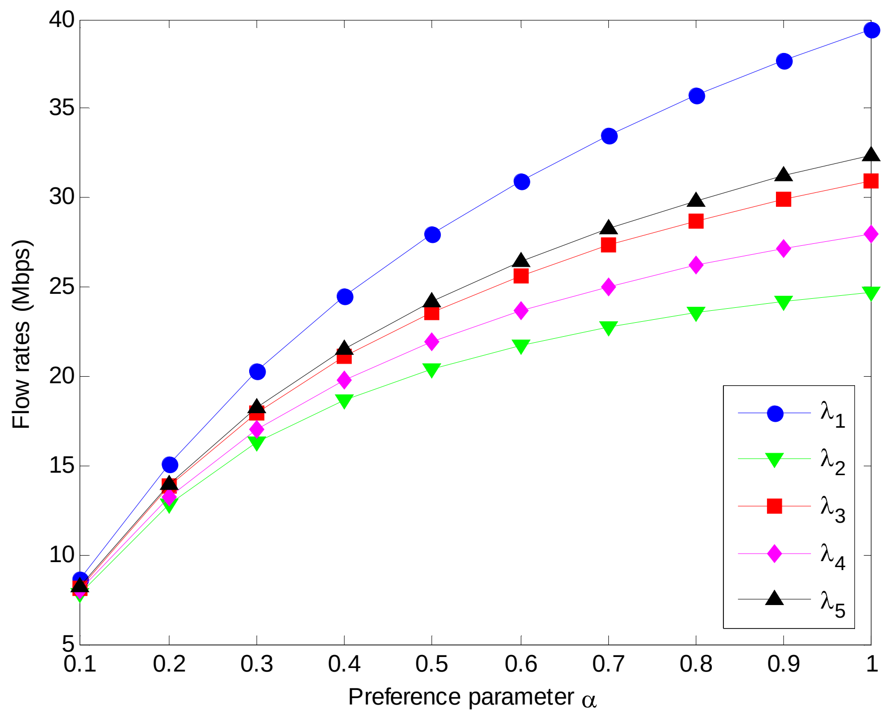


Figure 3.6. Flow rates for all users as the preference parameter of all users varies

Observation 5: We now examine how the Nash equilibrium changes when a new user enters the network. Assume a 6th user with a preference parameter $\alpha_6 = 0.6$ enters the network and follows the same route as user 1. The preference parameters of the initial 5 users are now back to their initial values: $\alpha_1 = 0.6$, $\alpha_2 = 0.7$, $\alpha_3 = 0.8$, $\alpha_4 = 0.4$, and $\alpha_5 = 0.5$ respectively. The Nash equilibrium for this new 6-user network can be determined as:

$$\underset{\sim}{\lambda}^* = \begin{bmatrix} \lambda_1^* \\ \lambda_2^* \\ \lambda_3^* \\ \lambda_4^* \\ \lambda_5^* \\ \lambda_6^* \end{bmatrix} = \begin{bmatrix} 22.4786 \\ 26.0934 \\ 30.1992 \\ 17.3102 \\ 22.5358 \\ 22.4786 \end{bmatrix} \tag{56}$$

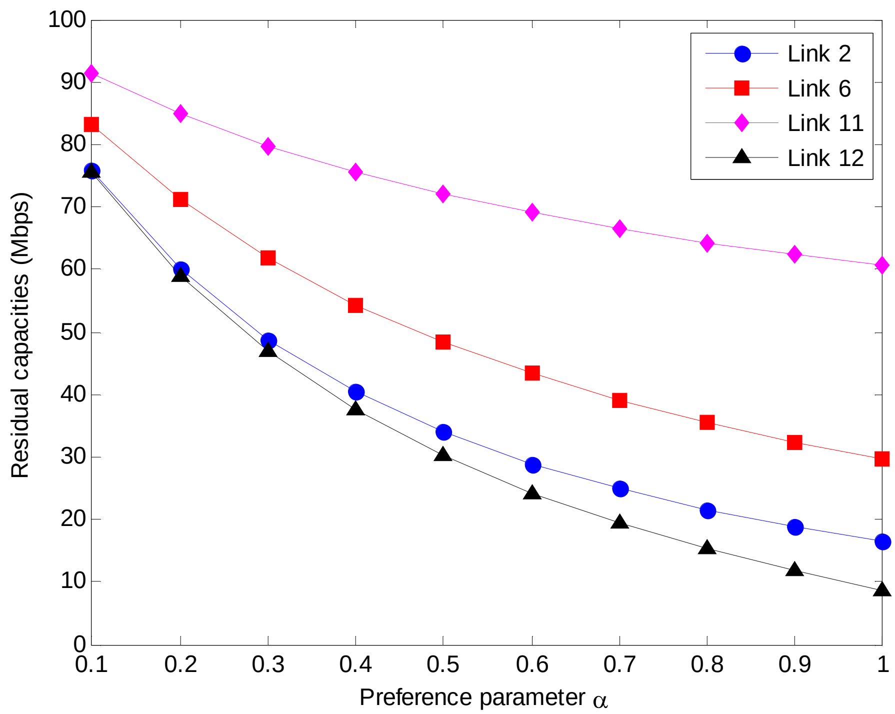


Figure 3.7. Residual capacities in some links as the preference parameter of all users varies

When we compare these flow rates with those of original network with 5 users as given in (55), we see that, as expected, the direct competitors of the new user, namely users 1, 3, and 5 obtain lower flow rates. Now let us examine the effect of increasing the preference parameter of the new user. Figure 3.8 shows plots of all users' flow rates as the preference parameter of the new user is increased from 0.5 to 0.9. Observe that the flow rates of the new user's direct competitors decrease monotonically as the new user's preference parameter increases. We summarize this in the following observation: *If a new user enters the network, then the flow rates of the direct competitors of the new user decrease as the preference parameter of the new user increases.*

Observation 6: Finally, using the same original 5 users network, we will examine the rate of convergence of the synchronous and asynchronous algorithms to the Nash equilibrium.

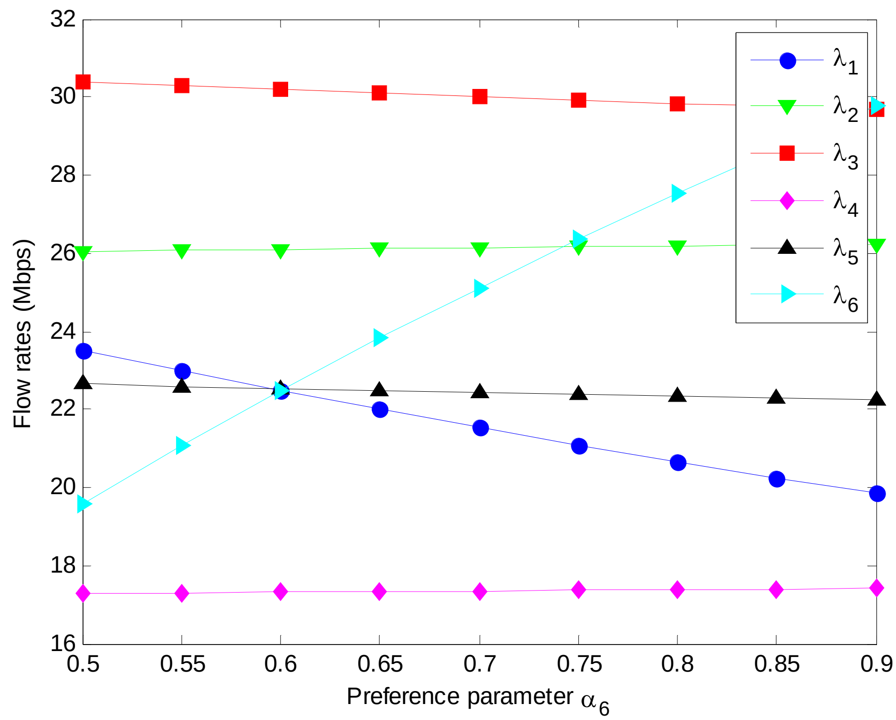


Figure 3.8. Flow rates of all users as the preference parameter of the new user varies

As mentioned earlier, in the synchronous algorithm all 5 users update their flow rates simultaneously at every iteration. For the asynchronous algorithm, we allowed, few users to update their flow rates at each iteration, while the flow rates of the remaining users were kept unchanged. We implemented this algorithm by randomly assigning to each user a real number from a uniform distribution over the interval [0, 1]. If the number assigned to a user is less than 0.4, then this user updates its flow rate; otherwise its flow rate is kept unchanged. This algorithm resulted in two users on average updating their flow rates at every iteration. Figure 3.9 and Figure 3.10 show the flow rates trajectories starting with an initial guess of $\lambda_i^{(0)} = 10$ Mbps for all users, versus iteration number for both the synchronous and asynchronous algorithms respectively. Notice that the synchronous algorithm converges faster to the Nash equilibrium than the asynchronous algorithm. The slower rate of convergence of the asynchronous algorithm can also be easily seen in Figure 3.11, which shows plots of the Euclidean error norms $\left\| e^{(n)} \right\| = \sqrt{\sum_{i=1}^{5} (\lambda_i^{(n)} - \lambda_i^*)^2}$ versus iteration number. The slower rate of convergence of the asynchronous algorithm can be attributed to two factors: 1) If it so happens in the asynchronous algorithm that the same users are selected to update their flow rates for two or more consecutive iterations, then their flow rates will not change resulting in no change in the flow rates of all other users, which will slow down the convergence rate; and 2) Users in the asynchronous algorithm do not update their flow rates as regularly as those in the synchronous algorithm, which will also slow down the convergence rate. These remarks are summarized in the following observation: *The asynchronous algorithm is slower than the synchronous algorithm in its convergence rate to the Nash equilibrium flow rates.*

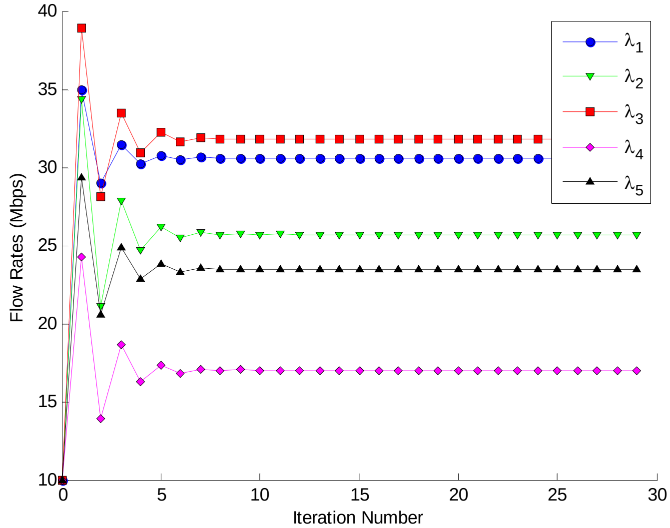


Figure 3.9. Flow rates trajectories for all users for the synchronous algorithm

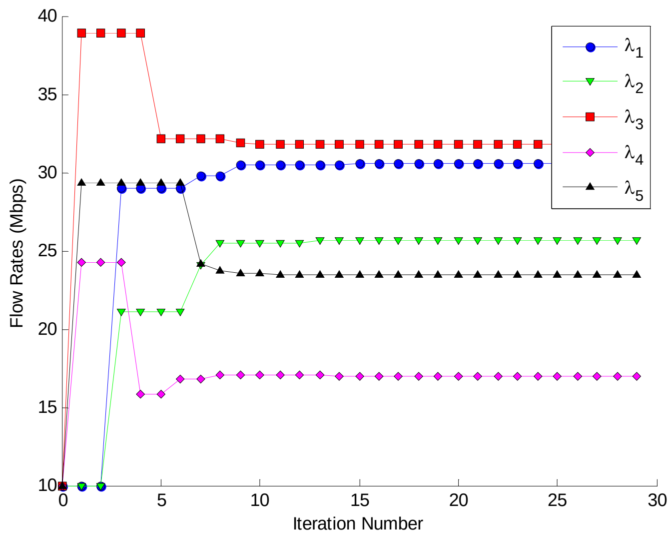


Figure 3.10. Flow rates trajectories for all users for the asynchronous algorithm.

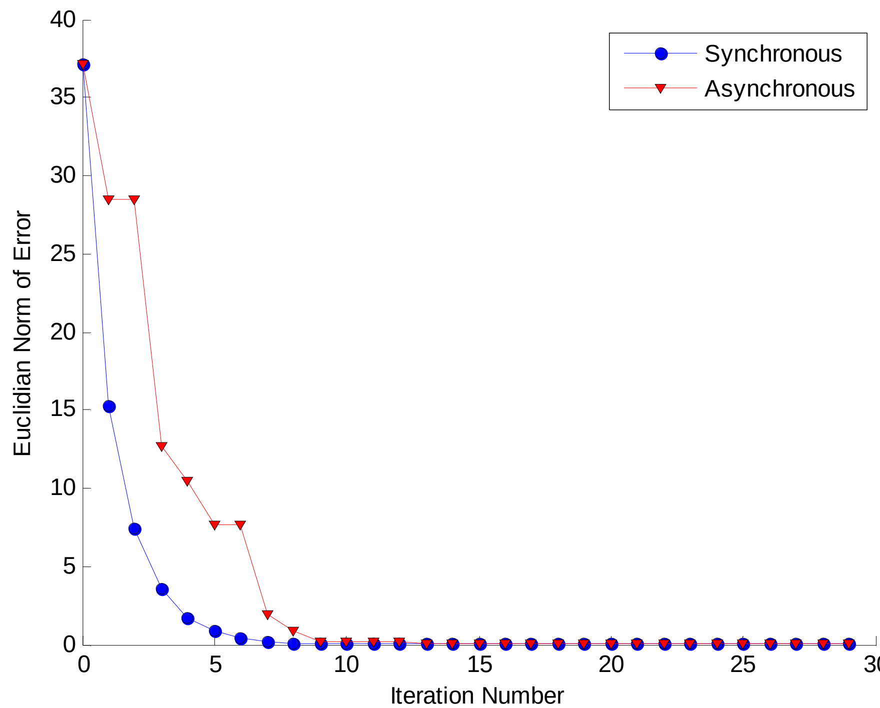


Figure 3.11. Plots of the Euclidean error norms
for the synchronous and asynchronous algorithms.

3.4 CONCLUSION

In this chapter, we investigated the flow control for a general multi-node multi-link communication network with multiple users competing for the same network resources. We assumed that each user's flow can enter and exit from any pair of nodes in the network, but that the route between these two nodes has already been determined. We further assumed that the flow rates for each user are determined in such a way as to optimize a utility function of the power criterion type that combines maximizing flow rate and minimizing expected delay. Although the possibility of cooperation among all users exist and may be feasible, in this chapter we considered only a situation where the users are competing and the possibility of cooperation and teaming does not exist. For this general network, we derived an analytical expression for the flow rates that satisfy the Nash equilibrium condition from game theory and established its uniqueness. This latter property is important because without it a mechanism will need to be added to the network management to guarantee that the Nash flow rates for all users are determined from the same equilibrium solution. Using an example, we illustrated the solution concept and derived some interesting observations that relate to the behavior of the Nash equilibrium in terms of various network parameters, including some convergence properties for the corresponding synchronous and asynchronous algorithms. We should note that although our model is quite general in nature, it still has some limitations. First, our analysis is mostly applicable to wired networks where data transmission error rates are much smaller than wireless networks, and hence can be ignored. Second, our approach is static in nature in that it assumes constant flow rates over fixed routes. Although modern networks are dynamic in nature, this static analysis can be viewed as corresponding to periods of time when steady state equilibrium conditions have been reached in the network. Third, we assumed that all users have the same type of utility function which indeed led to a unique Nash equilibrium. Finally, we assumed that the data routes for all users have already been determined.

4.0 COOPERATIVE FLOW CONTROL FOR GENERAL COMMUNICATION NETWORKS

In the previous chapter, we investigated general communication networks which consist of users, each competing with the others to maximize its own utility function. However, a group (team) of users which are related to each other through achieving the same goal may concern more about team level performance than individual performance. For instance, users of an organization using this organization's private communication network would like to cooperate to increase this network's performance instead of individual performances.

In this chapter, we investigate the same general network structure in the previous chapter but within a more general framework. We consider the network with only cooperative users which constitute a team and later we expand this model to multiple teams. In the network with only one team, each user tries to maximize a team level utility function which is the sum of weighted utilities of all users in the network. Each of these utility functions is in the form of power criterion given in the previous chapter [7-12]. This network has a leader which chooses the weight factors in the team level utility function so that the network users can achieve a larger utility value than their Nash equilibrium utility values. For the next case, rather than having only one team in the network, we consider a more general framework in which there are multiple teams competing with each other. Similar to single team case, members of each team cooperate to maximize their team's utility function which is again the sum of weighted utilities of all the users in this team. Similarly, the leader of each team decides on the weight factors in the team's utility function. For this two level optimization problem with competing teams and cooperating team members, the Noninferior Nash equilibrium [17,18] becomes a solution concept which guarantees the best achievable utilities for each team. We demonstrate this equilibrium with an example and compare its results with the standard Nash equilibrium.

4.1 MODEL AND FORMULATION

Even though most of the notation in this model is similar to the one given in the previous chapter, for convenience, we will give all notation used in this model. We consider the same general network topology but with M links shared by T competitive teams. Let the set of links be denoted by $\mathcal{M}=\{1,2,......,M\}$, the set of teams by $\mathcal{T}=\{1,2,........,T\}$, and the set of users in team(group) g by $\mathcal{G}_g=\{1,2,........,n_g\}$ where n_g denotes the number of users in team g. Therefore, the total number of users in the network, denoted by n_{tot}, becomes $n_{tot}=\sum_{g=1}^{T} n_g$. Let c_m denote the capacity of link $m \in \mathcal{M}$ and λ_i^g denote the flow of user i in team g. The flow rate λ_i^g which is the same on all links used by user i in team g is a control variable for user i in team g. Let θ_{mi}^g be a binary number defined as follows:

$$\theta_{mi}^g=\begin{cases}1, & \text{if user } i \text{ in team } g \text{ sends data over link m} \\ 0, & \text{otherwise}\end{cases}. \tag{57}$$

Using vector notation, we define the vectors $\underset{\sim}{c}$, $\underset{\sim}{\lambda}^g$, $\underline{\lambda}$, and $\underset{\sim}{\theta}_i^g$ as:

$$\underset{\sim}{c} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_M \end{bmatrix}, \quad \underset{\sim}{\lambda}^g = \begin{bmatrix} \lambda_1^g \\ \lambda_2^g \\ \vdots \\ \lambda_{n_g}^g \end{bmatrix}, \underline{\lambda} = \begin{bmatrix} \underset{\sim}{\lambda}^1 \\ \underset{\sim}{\lambda}^2 \\ \vdots \\ \underset{\sim}{\lambda}^T \end{bmatrix}, \text{ and } \quad \underset{\sim}{\theta}_i^g = \begin{bmatrix} \theta_{1i}^g \\ \theta_{2i}^g \\ \vdots \\ \theta_{Mi}^g \end{bmatrix} \tag{58}$$

where $\underset{\sim}{c} \in R^M$ denotes the network capacity vector, $\underset{\sim}{\lambda}^g \in R^{n_g}$ the flow control vector for team g, $\underline{\lambda}$ is system flow configuration vector (a $n_{tot} \times 1$ vector) which is constructed by placing individual team flow vectors one below another, and $\underset{\sim}{\theta}_i^g \in R^M$ the routing vector for user i in team g (we assume that at least one entry in $\underset{\sim}{\theta}_i^g$ is nonzero) . We now define the team routing matrix $\Theta^g \in R^{M \times n_g}$ for users in team g as:

$$\Theta^g = \begin{bmatrix} \theta_{11}^g & \theta_{12}^g & \cdots\cdots & \theta_{1n_g}^g \\ \theta_{21}^g & \theta_{22}^g & \cdots\cdots & \theta_{2n_g}^g \\ \vdots & \vdots & \ddots & \vdots \\ \theta_{M1}^g & \theta_{M2}^g & \cdots\cdots & \theta_{Mn_g}^g \end{bmatrix}. \tag{59}$$

The network routing matrix of $\Theta \in R^{M \times n_{tot}}$ is defined in the following:

$$\Theta = \left[\Theta^1 \middle| \Theta^2 \middle| \ldots\ldots \middle| \Theta^T \right] \tag{60}$$

where team routing matrices are placed next to each other. As an illustrative example, the routing matrix

$$\Theta = \left[\underset{\sim 1}{\theta}^1 \ \underset{\sim 2}{\theta}^1 \ \underset{\sim 1}{\theta}^2 \right] = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

corresponds to a 3-user, 4-link, 4-node network with user 1 in team 1 sending data on links 1,2, and 4, user 2 in team 1 sending data on links 1 and 3, and the only user of team 2 sending data on link 3.

The residual (unused) capacity of link k denoted by r_k:

$$\underset{\sim}{r} = \underset{\sim}{c} - \Theta \underline{\lambda} \tag{61}$$

As we defined in the previous chapter, a flow vector $\underset{\sim}{\lambda}$ is denoted as an interior point if

$$\underset{\sim}{\lambda} > 0 \quad \text{and} \quad \underset{\sim}{r} > 0 \tag{62}$$

Finally, we assume that each user chooses its flow rate λ_i^g to maximize a power criterion utility function [7-12] of the form:

$$U_i^g(\underline{\lambda}) = \sum_{k \in \mathcal{M}} \theta_{ki}^g \left(\frac{\left(\lambda_i^g\right)^{\alpha_i^g}}{D_k} \right) \tag{63}$$

where D_k denotes the expected delay on link k and $\alpha_i^g > 0$ is a preference parameter that user i in team g chooses to adjust the relative importance of flow rate over the average. The notation $U_i^g(\underline{\lambda})$ in (63) implies that utility of user i in team g is a function of all other users' flow rates.

Members of each team cooperatively decide on their flow rate to maximize the following team level utility function:

$$\hat{U}^g = \mu_1^g U_1^g + \mu_2^g U_2^g + ...+ \mu_{n_g}^g U_{n_g}^g \qquad \text{for all } g \in \mathcal{T} \tag{64}$$

where $\mu_i^g, i = 1, 2, ..., n_g$, denotes the weight factor for user i in team g and these weight factors satisfy $\sum_{i \in \mathcal{G}_g} \mu_i^g = 1$. The leader of team $g \in \mathcal{T}$ chooses these weight vector of its members $\underset{\sim}{\mu}^g = \left(\mu_1^g, \mu_2^g, ..., \mu_{n_g}^g\right)$ by performing the following optimization

$$\min_{\underset{\sim}{\mu}^g} \max_{\underset{\sim}{\lambda}^g} \hat{U}^g \tag{65}$$

for all possible values of $\underset{\sim}{\mu}^g$ and $\underset{\sim}{\lambda}^g$. Even though maximizing $\left(\max_{\underset{\sim}{\lambda}^g} \hat{U}^g\right)$ with respect to $\underset{\sim}{\mu}^g$ would result in a larger total utility for the team, usually the resultant flow rates gives very large utilities for some team members and very small utilities for the other team members. Therefore minimization of $\left(\max_{\underset{\sim}{\lambda}^g} \hat{U}^g\right)$ with respect to $\underset{\sim}{\mu}^g$ is more desirable by users because resultant utility levels of team members do not differ significantly.

4.2 SIMULATION RESULTS

In this section, we present two examples to investigate single and multi-team network flow rate optimization problems. We use the same network topology in the previous chapter but with different number of users. For both examples, all links of this network have the same capacity, i.e. $c_k = 100$ Mbps for $k = 1, 2, ..., 16$.

Example 4.1. In this example we consider the network given by Figure 4.1 where two users constitute a team by maximizing the following common utility function:

$$U = \mu U_1 + (1-\mu) U_2 \tag{66}$$

where $U_1 = (\lambda_1)^{\alpha_1} (\underset{\sim}{\theta}_1)^T \underset{\sim}{r}$ and $U_2 = (\lambda_2)^{\alpha_2} (\underset{\sim}{\theta}_2)^T \underset{\sim}{r}$. Since there is only one team in this example, we dropped the superscript referring to team number for simplicity. Let user 1 and 2 have the preference constants $\alpha_1 = 0.6$ and $\alpha_2 = 0.4$ respectively. Since the common utility function is the summation of weighted utilities of user 1 and 2 by the factors μ and $(1-\mu)$

respectively, for larger μ values, the maximization of U corresponds to the situation in which the maximization of the utility of user 1 becomes more significant than the maximization of the utility of user 2. Similarly, for smaller μ values the maximization of the utility of user 2 becomes more significant. By numerically maximizing equation (66) we plotted Figure 4.2, Figure 4.3, and Figure 4.4. In Figure 4.2, as μ gets larger, the utility of user 1 monotonically increases as we expect while the utility of user 2 monotonically decreases. As can be seen from Figure 4.3, flow rates of these users change correspondingly to their utility levels, that is, as μ increases the flow rate of user 1 increases while that of user 2 decrease.

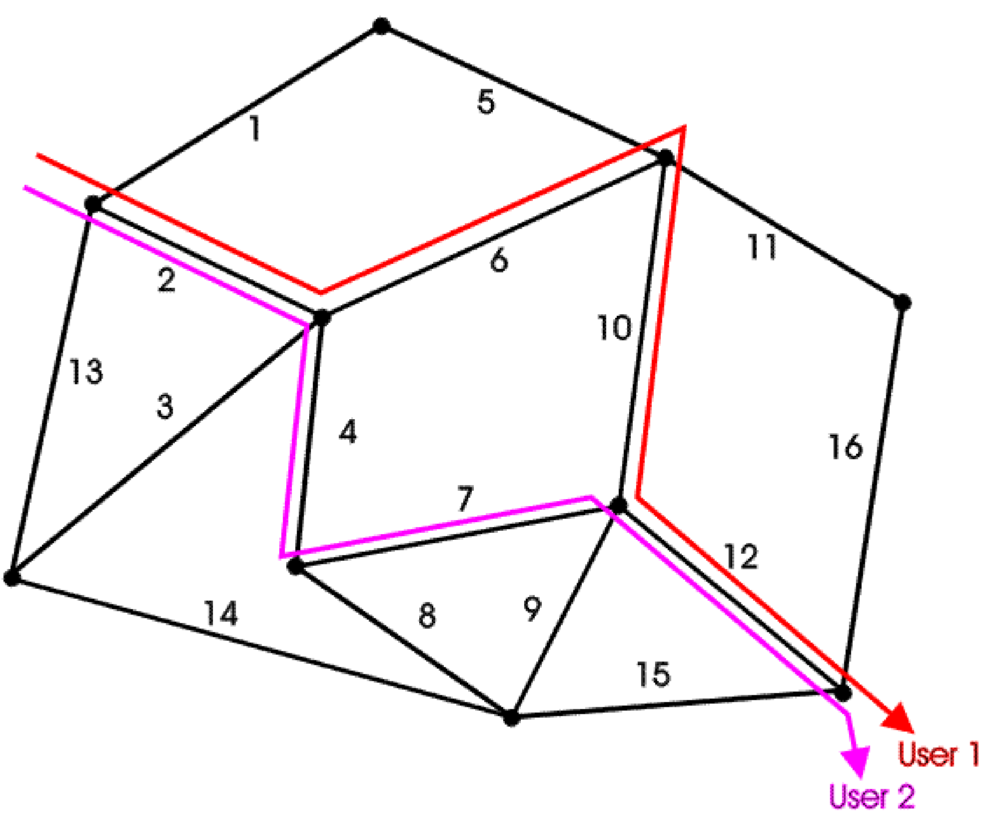


Figure 4.1. A network with 16 links and 2 users.

We plotted the contours of utilities of user 1 and 2, their Nash flow rate, the Pareto optimal set (green line) in Figure 4.4. For any given point in the strategy space of both users, the Pareto optimal set has the property of containing at least one point (a pair of flow rates belongs to user 1 and 2) which gives better utilities to both users than this given point. We also plotted the set of points as thick green line which have the property of yielding larger utility values than Nash utility values. This can be considered as the payoff of collaboration. Note that the Pareto optimal points connect the points at which the contours of U1 and U2 are tangent.

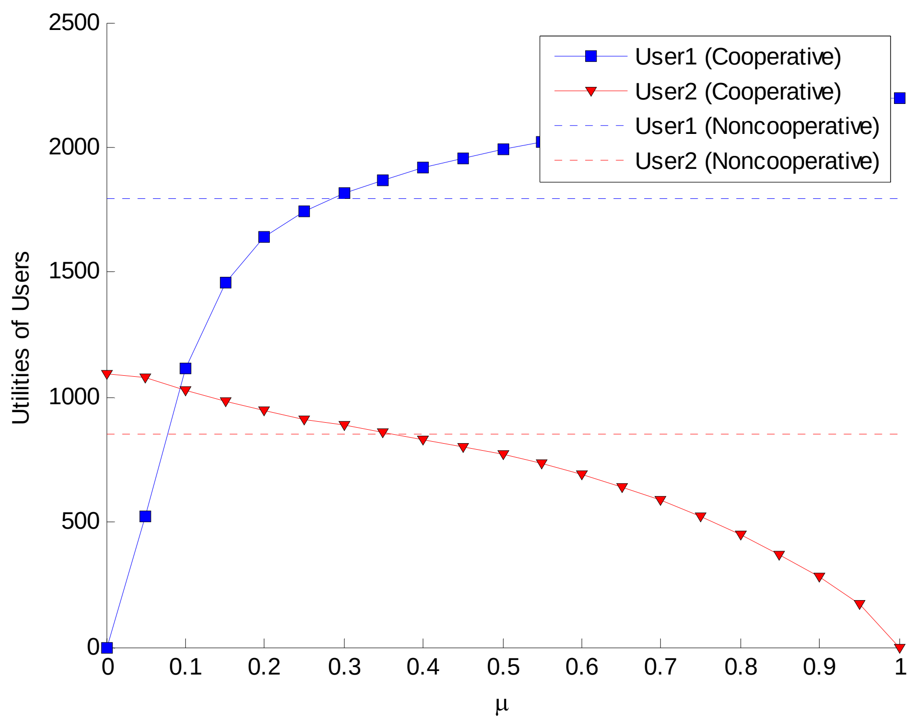


Figure 4.2. Utilities of the users when they are cooperative and non-cooperative

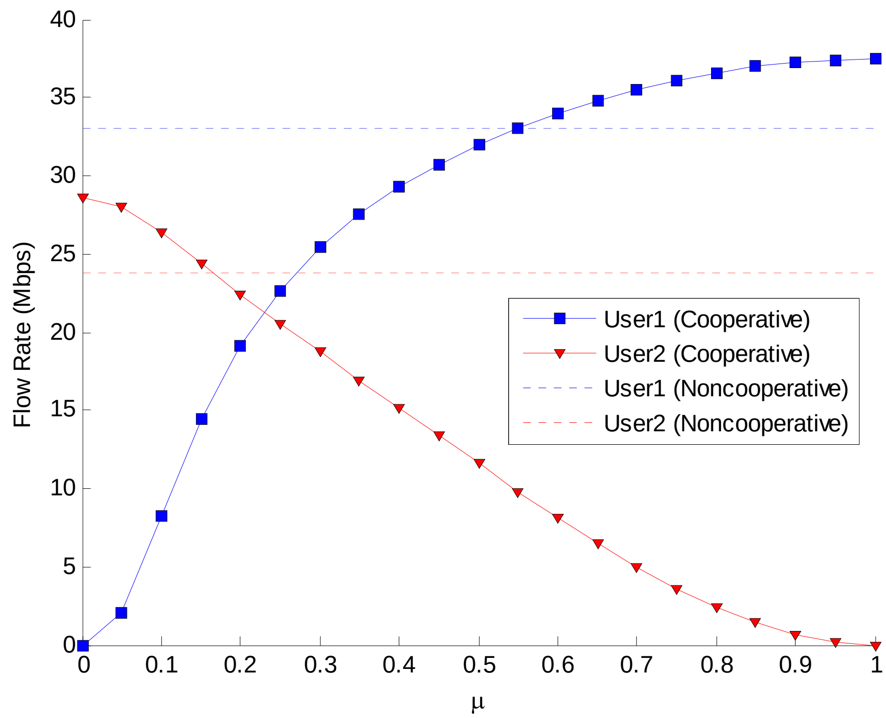


Figure 4.3. Cooperative and non-cooperative flow rates of users in Example 4.1

Example 4.2: In this example, we consider two teams, team A and B, which decide on their flow rates based on the network conditions. Users of each team cooperatively decide on their flow rate to achieve large utility values. For the illustration purposes, let each team have two users. The routing conditions for the network users are plotted in Figure 4.5. Utility functions of team A users become $U_1^A = \left(\lambda_1^A\right)^{\alpha_1^A} \left(\underset{\sim}{\theta}_1^A\right)^T \underset{\sim}{r}$ and $U_2^A = \left(\lambda_2^A\right)^{\alpha_2^A} \left(\underset{\sim}{\theta}_2^A\right)^T \underset{\sim}{r}$, and team B users become $U_1^B = \left(\lambda_1^B\right)^{\alpha_1^B} \left(\underset{\sim}{\theta}_1^B\right)^T \underset{\sim}{r}$ and $U_2^B = \left(\lambda_2^B\right)^{\alpha_2^B} \left(\underset{\sim}{\theta}_2^B\right)^T \underset{\sim}{r}$. Therefore, utility functions of team A and B can be written as $\hat{U}^A = \mu_1^A U_1^A + \mu_2^A U_2^A$ and $\hat{U}^B = \mu_1^B U_1^B + \mu_2^B U_2^B$, respectively, where $0 \le \mu_1^A, \mu_1^B \le 1$,

$\mu_1^A + \mu_2^A = 1$, and $\mu_1^B + \mu_2^B = 1$. Let users' preference parameters be $\alpha_1^A = 0.5$, $\alpha_2^A = 0.5$, $\alpha_1^B = 0.9$, and $\alpha_2^B = 0.4$. In order to obtain their optimal flow rates for given $\underset{\sim}{\mu}^A$ and $\underset{\sim}{\mu}^B$, users of team A and B perform the following optimizations:

$$\left(\max_{\underset{\sim}{\lambda}^A} \hat{U}^A\right) \quad \text{and} \quad \left(\max_{\underset{\sim}{\lambda}^B} \hat{U}^B\right). \tag{67}$$

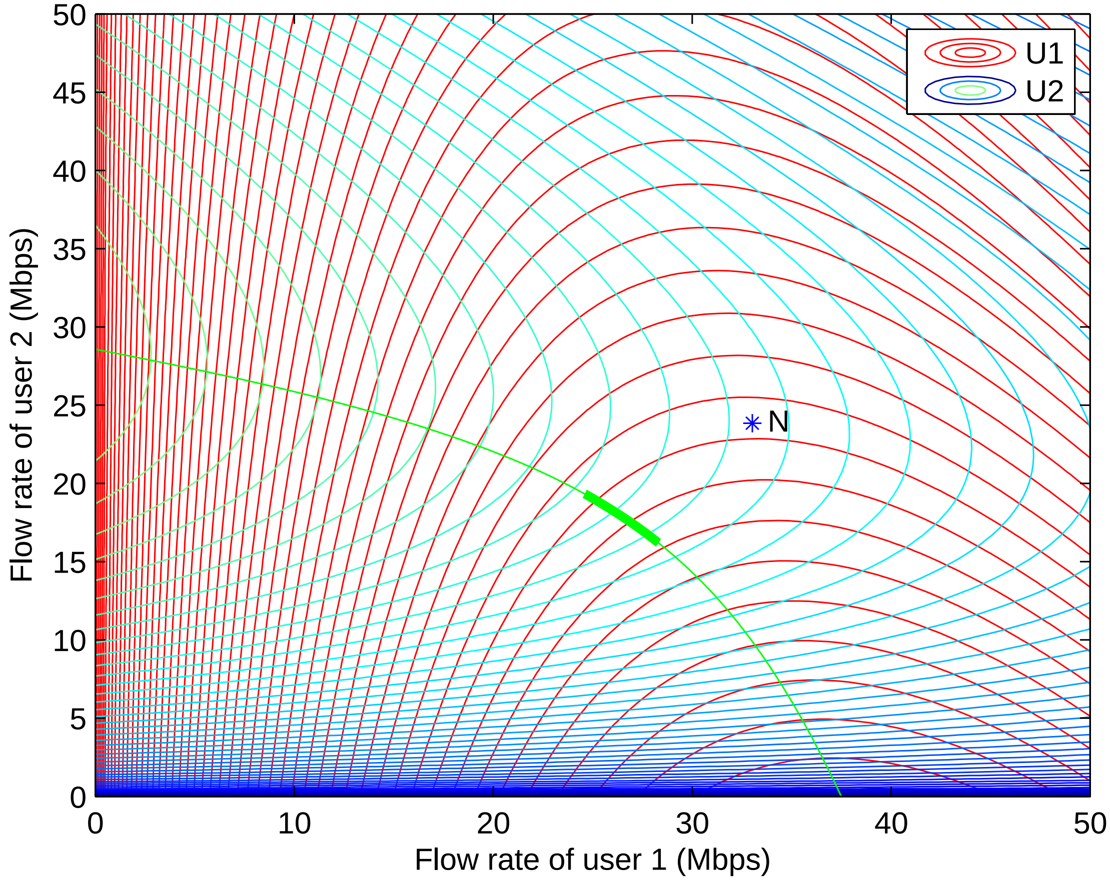


Figure 4.4. The contours of the utility functions of user 1 (red) and 2 (blue)
The Nash equilibrium is shown by a blue star with the letter N. The Pareto optimal set is plotted as green solid line. The set of Pareto optimal points which yield larger utilities for both users than their Nash utilities are plotted as thick green segment. The points on the thick green line are obtained when μ is between 0.285 and 0.37.

respectively. Due to nonlinear nature of utilities $\hat{U}^A$ and $\hat{U}^B$, we obtained numerical solutions of optimizations given in (67). We plotted the optimal flow rates of all users in Figure 4.6. Obviously, as μ_1^A increases λ_1^A increases and λ_2^A decreases. We also noticed that for a fixed μ_1^A value, the flow rates λ_1^A and λ_2^A do not change significantly even though μ_1^B varies. The similar trend is observable for μ_1^B and flow rates of the users in team B. Next we plotted the utilities of team A and B in Figure 4.7. The utility of team B monotonically increases

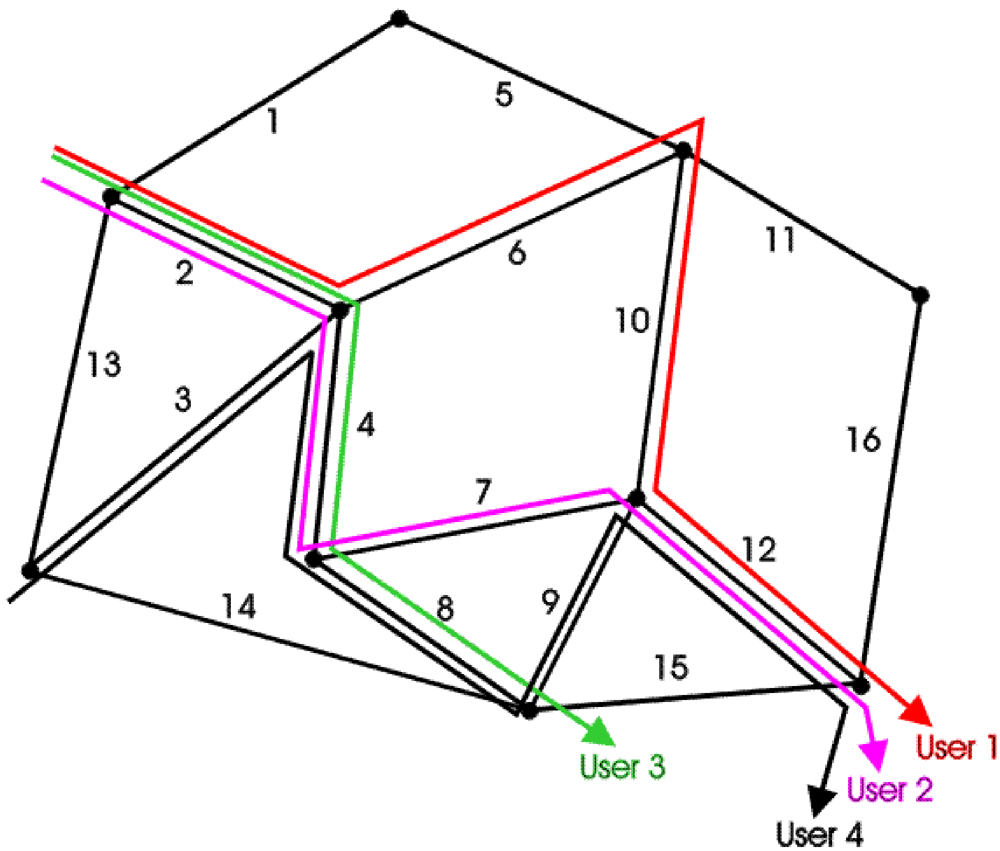
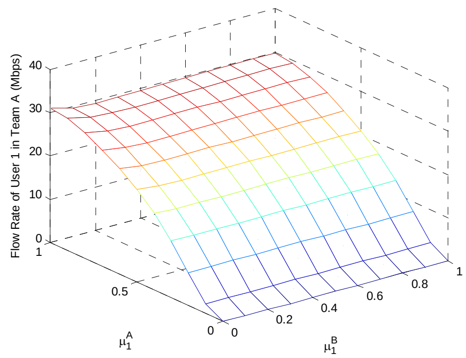


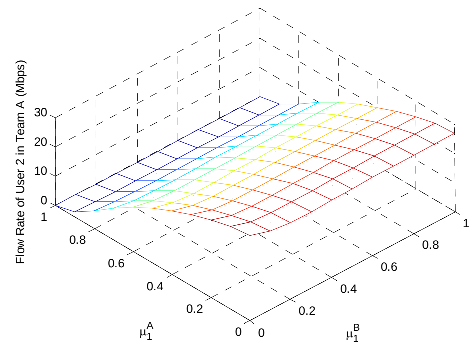
Figure 4.5. A network with 16 links and 4 users

whose routes are depicted as colored lines.

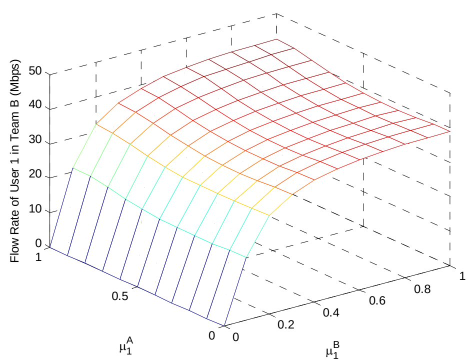
for $\mu_1^B > 0.055$. This may be the result of the fact that user 1 in team B has much larger preference parameter than user 2 in team B. Since users of A have the same preference parameter, we do not see the similar monotonic increase in the utility of team A.



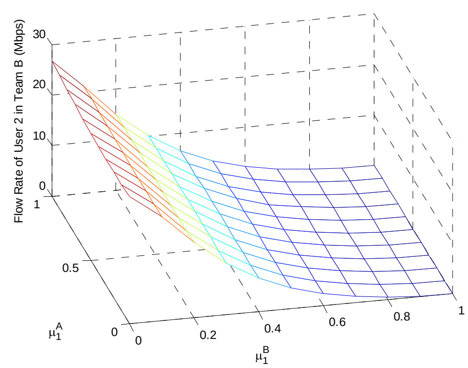
(a)



(b)



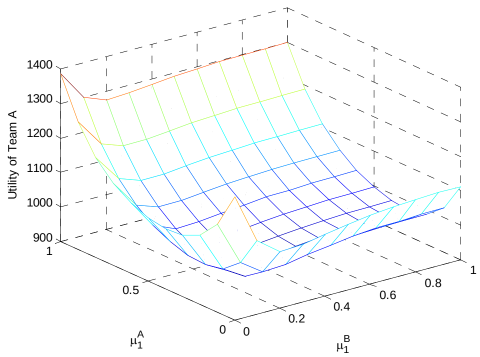
(c)



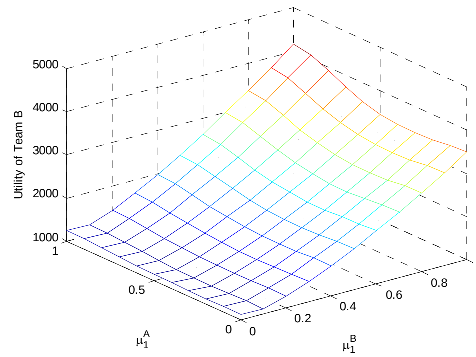
(d)

Figure 4.6. Flow rates of all users in Example 4.2.

(a) The flow rate of user 1 in team A, (b) The flow rate of user 2 in team A, (c) The flow rate of user 1 in team B, and (d) The flow rate of user 2 in team B.



(a)



(b)

Figure 4.7. Utilities of the teams in Example 4.2.
(a) the utility of team A and (b) the utility of team B.

The leader of team A evaluates the optimal weight vector $\underset{\sim}{\mu}^A = \left(\mu_1^A, \mu_2^A\right)$ which solves the following optimization problem:

$$\min_{\underset{\sim}{\mu}^A} \max_{\underset{\sim}{\lambda}^A} \hat{U}^A \tag{68}$$

The leader of team B performs a similar optimization. By means of numeric optimization, we evaluate the following unique Noninferior Nash equilibrium values: weight parameters: $\mu_1^{A*} = 0.359$, $\mu_2^{A*} = 0.641$, $\mu_1^{B*} = 0.055$, and $\mu_2^{B*} = 0.945$, users' utilities: $U_1^{A*} = 1048.4$, $U_2^{A*} = 1041.4$, $U_1^{B*} = 767.2$, $U_2^{B*} = 1102.3$, and teams' utilities: $U^{A*} = 1043.9$ and $U^{B*} = 1083.7$. On the other hand, if all four users non-cooperatively decide on their flow rates, then their Nash equilibrium flow rates become $\lambda_1^{A*} = 25.63$, $\lambda_2^{A*} = 20.48$, $\lambda_1^{B*} = 30.11$, and $\lambda_2^{B*} = 21.32$, and their Nash utilities $U_1^{A*} = 1038.2$, $U_2^{A*} = 741.9$, $U_1^{B*} = 2151$, and $U_2^{B*} = 906.4$. Obviously, both team A users obtain larger utilities when they cooperate than when they compete. However, user 1 in team B looses some utility while user 2 in the same team obtains larger utility. This may be a result of preference parameter of user 1 being much larger than that of user 2 in team B.

4.3 CONCLUSION

In this chapter, we investigated the same general network structure with the previous chapter but assumed that all users of the network cooperate with each other when there is only one team in the network. All network users try to maximize a network-level utility function which is the sum of their weighted utility functions. The leader of the network decides on weight factors in this network-level utility function. We showed that when all users cooperate, each user can obtain a utility value which is larger than its Nash utility value. Next we extended this model to a more general framework in which there exists multiple teams which compete with each other while members of each team cooperate with each other to maximize their team-level utility function. Again, the leader of each team decides on the weight factors of the team-level utility function so that the members of the team possibly get larger utility values than their corresponding Nash equilibrium. We illustrated the results of these two level network optimization with two examples and compared the standard Nash equilibrium with the Noninferior Nash equilibrium.

5.0 CONLUSION AND FUTURE WORK

Efficient flow and routing control of network resources becomes crucial as the need for accommodating ever growing number of new network users. In this thesis, we studied optimal flow and routing control problems using concepts from game theory. These concepts are useful whenever there is cooperation or conflict among network users. For several network configurations, we investigated the existence and uniqueness of a Nash equilibrium. We also investigated the Pareto optimal and Noninferior Nash strategies for networks with one team whose members cooperate and for with many teams which compete with each other respectively.

A two-node parallel link network provides a simple model to study the existence, uniqueness, and properties of a Nash equilibrium. Each user of this network competitively decides on its flow and routing strategies by maximizing a utility function which additively combines the objectives of obtaining larger flow rate and smaller network delay. Each user can choose the weight of the importance of one objective to the other by adjusting its preference parameters in its utility function. This model also provides these users the flexibility of choosing different preference parameters for different network links so that users can adjust their preferences for each link based on their previous experiences from these links. We established a necessary and sufficient condition which guarantees the existence and uniqueness of a Nash equilibrium for this network model and obtained an analytical expression for the Nash equilibrium. We also showed several intuitive properties of this equilibrium based on user preferences and other network parameters. One important property is that a user with a larger tradeoff parameter obtains larger flow rate than other users with smaller tradeoff parameters. Another intuitive property states that the larger links carry larger flow rates than the smaller links.

We extend our model to a general network environment where there is no constraint on the topology of the network such as links being parallel. In this network, there may be multiple nodes which are connected by multiple links. Multiple users may be sharing these links and each may have different source and destination nodes. We investigate this network using the power criterion utility function which provides a more tractable flow control solution. The routes of the users are fixed, that is, a user's flow can only follow one path from its source to its destination node without being split into many paths. In this model, we assume that user preference parameters are link-independent. We prove the existence of a unique Nash equilibrium and investigate some interesting properties of this equilibrium heuristically. An interesting Nash equilibrium property is that when a user increases its preference parameter, its flow rate increases while flow rates of its direct competitors decrease. Another intuitive property states that if the capacity of a link is increased, then the flow rates of all users using this link also increase. We also introduced synchronous and asynchronous numeric algorithms which become a distributive means of evaluating Nash flow rates. That is, these numeric schemes do not require user-specific information from network, they only require the knowledge of residual capacities of the links in a network. Since we heuristically observed properties of this Nash equilibrium, a future work may involve rigorous mathematical proof of these properties. Without fixing routes of network users, another challenging future research topic would be to investigate the simultaneous optimization of flow and routing controls for this model.

For the same general network structure, we investigated the case in which all users cooperative for the overall benefit of the network. In this solution, users can use one of Pareto optimal strategies which give larger utilities to every user than they would obtain from their Nash equilibrium strategies. Next, we expand this model by incorporating teams of users. Teams compete with each other while members (users) of each team cooperate to maximize a team-level utility function. The corresponding solution is called non-inferior Nash equilibrium. We compare these results with the standard Nash equilibrium and Pareto optimal flow rates. One extension of this study would be to investigate the properties of non-inferior Nash equilibrium for this general network model and compare their results with the ones obtained in this thesis. The investigation of numeric schemes for team members so that they could adjust their flow rate would be another interesting future research topic.

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