ESSAYS ON STRATEGIC VOTING

by

Sun-Tak Kim

B. A. in English Language and Literature, Hankuk University of Foreign Studies, Seoul, Korea, 1998

Submitted to the Graduate Faculty of

the Kenneth P. Dietrich School of Arts and Sciences in partial fulfillment

of the requirements for the degree of

Doctor of Philosophy

University of Pittsburgh

2012

UNIVERSITY OF PITTSBURGH

ECONOMICS DEPARTMENT

This dissertation was presented

by

Sun-Tak Kim

It was defended on

May 8th 2012

and approved by

John Duffy, Professor, Economics Department

Andreas Blume, Professor, Economics Department

Luca Rigotti, Associate Professor, Economics Department

Jonathan Woon, Assistant Professor, Political Science Department

Sourav Bhattacharya, Assistant Professor, Economics Department

Dissertation Director: John Duffy, Professor, Economics Department

ESSAYS ON STRATEGIC VOTING

Sun-Tak Kim, PhD

University of Pittsburgh, 2012

This dissertation investigates strategic voting from two perspectives. The second chapter studies a theory of electoral competition in the presence of strategic forward-looking voters while the third chapter experimentally tests a rational voter model under alternative voting institutions that may be employed in jury trials.

In the second chapter, I study a spatial model of two-party electoral competition in which the final policy outcome can be different from electoral promises. The policy outcome depends in part on electoral promise, but also reflects the bargaining process between the winning and losing party whose outcome can be anticipated by strategic forward-looking voters. Unlike the prediction of the Median Voter Theorem which holds with the coincidence of electoral promises and policy outcomes, I find that parties have incentives to distinguish themselves from one another in the election with the consideration of policy concession that might result from post-electoral bargaining.

In the third chapter, I report on an experiment comparing compulsory and voluntary voting mechanisms. Theory predicts that these different mechanisms have important implications for strategic decisions in terms of both voting and abstention, and I find strong support for these theoretical predictions in the experimental data. Voters are able to adapt their strategic voting behavior or their participation decisions to the different voting mechanisms in such a way as to make the efficiency differences between these mechanisms negligible. I argue that this finding may account for the co-existence of these two voting mechanisms in nature.

In conclusion, I give a brief description of a way to extend the experimental study in the

third chapter by considering alternative mechanisms to obtain private information relevant to voting decisions.

Keywords: Strategic Voting, Spatial Model, Post-Electoral Bargaining, Platform Divergence, Mixed-strategy Equilibrium, Voting Behavior, Voting Mechanisms, Condorcet Jury Model, Information Aggregation, Information Acquisition, Laboratory Experiments.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

1.0 INTRODUCTION

This dissertation studies rational behavior and strategic interaction in collective decision-making problems. The applications are to elections or voting in committees. The focus is on understanding the implications of rationality for the outcomes of group decisions.

Various aspects of strategic voting are investigated in this dissertation. First, I study the theoretical model of spatial political competition in a rational expectations framework. Voters are postulated to anticipate correctly the possibility of post-electoral political bargaining and the resulting final policy outcome from the electoral choices of political parties, and political parties take this into account when they announce electoral promises. Second, I use laboratory experiments to compare (compulsory vs. voluntary) voting mechanisms in a situation where rational voters are assumed to vote only to affect the outcome (pivotal voter model). This implies that voters infer additional information from others' (equilibrium) behavior and voting rules, and weigh it against their private information. Finally, I discuss a way to extend this experimental works to study the incentives of rational voters to optimally invest in and use costly information.

In the second chapter titled "Policy Divergence with Post-Electoral Bargaining," I consider the spatial model of two-party (leftist vs. rightist) political competition with a departure that the final policy to be implemented by a winning party can be different from his announced platform in the election. The winning and opposing party are assumed to engage in bargaining over the final policy after the election. I model the post-electoral process as a Nash bargaining game in which the disagreement payoffs are determined by the electoral platforms and the opposing party's vote share. Voters are assumed to be rational in the sense that they take into account the implications of the announced platforms for the final policy. With this bargaining protocol and forward-looking voters, the two political parties are shown to choose extremely divergent electoral platforms in any symmetric pure-strategy equilibrium if the opposing party has sufficiently large bargaining power. In a political environment where the latter party cannot have large enough bargaining power, parties are shown to mix over separate sets of policies, again diverging in their electoral platforms at any (realized) strategy profile of mixed equilibrium.

In the third chapter titled "Compulsory versus Voluntary Voting: An Experimental Study," I consider two-state and two-alternative voting with private information. Voters have common values (like the jury who want to convict the guilty and acquit the innocent), and receive independent and noisy signals about the true state of nature (guilty vs. innocent). Here, we assume there are two signals one of which is regarded as *correct* in each state. Austen-Smith and Banks (1996) points out that voting according to signal (*sincere voting*) may not be strategically optimal if voting is mandatory (*compulsory voting*). However, Krishna and Morgan (2012) recently showed that sincere voting can be incentive compatible with endogenously determined participation rates under *voluntary voting*. Sincere voting is incentive compatible only if the probability of sincere voting is different between the groups with different signals (types). This necessarily implies that there exists a type whose probability of sincere voting is strictly less than one. Equilibrium analysis shows that this type mixes between sincere and insincere voting under compulsory voting while he mixes between sincere voting and abstention under voluntary voting. In the experimental data, I find strong evidence that subjects employ different mixing schemes under the different voting mechanisms (or treatments).

In the same setting of common-value jury trials, we can alternatively model that voters should expend cost to obtain (noisy) private signals instead of receiving them freely as in the third chapter. An interesting question in this case concerns the optimal voting mechanism. The standard model of jury voting with exogenous information predicts that the efficiency of group decision increases unambiguously with group size. However, once information acquisition becomes a costly decision, there is an important free-riding consideration that implies the existence of an optimal group size beyond which the efficiency of group decision declines. Future experimental tests of this hypothesis and the other about the optimal voting rule (e.g. majority rule) are discussed in the conclusion.

2.0 POLICY DIVERGENCE WITH POST-ELECTORAL BARGAINING

The Hotelling-Downs model of spatial (political) competition assumes that the platforms the politicians announce prior to an election will be the final policies they subsequently enact once in office. However, since voters have preferences not over electoral platforms but over final policy outcomes, the equivalence of electoral platforms and final policies is assumed for analytical tractability at the expense of realism, as pointed out by Banks (1990). In this chapter, we assume that the final policy outcome is determined not by the winner's electoral platform but by a bargaining process between the winning party and his opponent. The central question is how this concern for post-electoral bargaining affects the incentives of political parties competing in an election.

According to Ansolabehere (2006), the spatial theory of voting has been extremely successful because of its analytical simplicity. The simplicity of spatial models then follows from the very assumption of equivalence between electoral platforms and final policies. Although it seems to be realistic, eliminating the assumption of precommitment (to platforms) has proven to bring about a significant challenge for the development of alternative models of political competition. If politicians are not fully bound to their electoral promises, then can they say anything in political campaigns? What is the relationship between electoral platforms and final policies in the absence of full commitment to the former? At the other extreme, campaign promises can be alternatively modeled as cheap-talk as in Osborne and Slivinski (1996). However, it is equally unrealistic to postulate that politicians are completely unbeholden to their promises.

We therefore propose a model in which campaign promises are neither completely binding nor complete cheap-talk, but nevertheless serve as a basis for the determination of final policy outcomes. Once we allow the policy outcome to be different from the electoral platforms, an important question is how to define the policy outcome function, given electoral platforms and voting decisions. For this purpose, we introduce a stage of policy bargaining after the election. The benchmark is a one-dimensional spatial competition between two policy-motivated parties[1] (leftist vs. rightist) who are perfectly informed about the voter distribution (Wittman (1977), Calvert (1985), Roemer (1994)). In a departure, we assume that the losing party can obtain a policy concession from the winner at the bargaining stage and that the amount of this concession increases with the loser's vote share. One reason why the latter has bargaining power is that he can delay the policy-making process, for instance, by boycotting it in the legislature. More generally, we consider the loser as being able to impose costs on the winner, proportional to his bargaining power or vote share, e.g. as in a parliamentary system.

The bargaining outcome is given by a mapping from electoral platforms (and the implied vote share for the loser) to final policy outcomes. We can employ Nash bargaining to define this mapping.[2] Here, the winner's disagreement payoff is his utility at his own announced platform minus some utility cost that is proportional to the loser's vote share. The loser's disagreement payoff is his utility at the winner's platform. The resulting solution gives the final policy to be implemented by the winner. The final policy is to the left or to the right of the (rightist or leftist, respectively) winner's announced platform, depending on the identity of the winner. Hence, the final policy is more favorable for the loser than the winner's platform, but it is bounded by the loser's platform (i.e., the winner doesn't need to give more policy concessions than what is requested by the loser). Vote shares are determined under the assumption that rational voters would correctly anticipate the final policy from any given platforms and vote for the party whose (implemented) policy is closer to their ideal positions. Political parties are also assumed to be rational in the sense that they understand the implication of chosen platforms for vote shares and final policies.

An immediate consequence of our setup is non-convergence of equilibrium platforms. Our model thus presents a case in which the median voter theorem fails to hold.[3] The intuition

[1] We assume that the parties have single-peaked preferences over a given policy space and that the median of voter distribution is located between two parties' ideal positions.

[2] I also present later an axiomatic approach to policy outcome function which is an abstract version of Nash bargaining.

[3] The median voter theorem still holds in our benchmark case of policy-motivated parties with commitment

behind this result is simple. Suppose both parties announce the median in a campaign stage. Since their electoral stance is the same, there is nothing to bargain and the policy outcome remains to be the median. A policy-motivated party then finds it profitable to deviate towards his ideal policy and get policy concession from the winner (the resulting policy outcome is closer to his ideal position than is the median). Therefore, the possibility of bargaining significantly mitigates the motivation to win (the motivation to move towards the center) vis-à-vis policy motivation (the motivation to go to the extreme).

Next, political parties are shown to announce extremely divergent platforms at any symmetric *pure-strategy* equilibrium.[4] The losing party necessarily obtains a relatively large amount of policy concession at any symmetric equilibrium. But then, each party can change the policy outcome in his favor at any interior (symmetric) profile by deviating to a platform that is slightly closer to his ideal position. Thus, interior profiles can never be best responses and the parties will be located in the election at the boundary positions at which they no longer have an incentive to deviate. However, the final policy outcome will be the median no matter who wins the election at any pure-strategy equilibrium. Interestingly, De Sinopoli and Iannantuoni (2007) also obtains a Duvergerian two-party equilibrium in which voters vote only for the two extremist parties (or positions) in their model of multi-party election with proportional representation system.

When the political environment doesn't allow the loser to obtain sufficiently large policy concessions[5], there may not exist a pure-strategy equilibrium due to a discontinuity in parties' payoffs. However, a *mixed-strategy* equilibrium can still be shown to exist in this case. Since we have extreme divergence (in platforms) with large policy concession and perfect convergence with no concession, it is natural to think that parties will mix over platforms that lie between an extreme position and the median, with a relatively small but still positive concession. We establish the existence of a mixed equilibrium with continuous density strategies whose supports don't intersect with each other. In other words, the leftist

to platforms.

[4]A symmetric profile is defined to be a pair of platforms at which both parties get equal vote shares. A symmetry condition on the policy outcome function guarantees that both parties are equally distanced from the median at any symmetric profile.

[5]If the losing party can't get policy concessions, then we go back to our benchmark case where the final policy is equal to the winner's announced platform.

(rightist) mixes over the policies to the left (right) of the median, and hence, the two parties propose different policies in an election at any realization of mixed (equilibrium) strategies.

The main contribution of this chapter is an equilibrium analysis of platform choice game with a simple post-electoral bargaining structure. There are only a few models that incorporate both election and legislative bargaining although we have fairly well-developed (and separate) literature on both topics. Austen-Smith and Banks (1988) is the earliest full equilibrium model of both electoral and legislative process in a uni-dimensional policy space. Baron and Diermeier (2001) extend it to a two-dimensional setting and provide a tractable framework for studying such a wide range of topics as government formation, policy choice, election outcomes and parliamentary representation. However, their focus is on the account of multifarious aspects of government formation and less weight is put on the electoral choices of politicians.[6] De Sinopoli and Iannantuoni (2007, 2008) deliberately restrict their attention on the strategic voting stage and analyze a subgame where all party positions are fixed. In our model, the political parties are free to choose any platform in a given policy space and a bargaining outcome is defined for each pair of chosen platforms. This enables us to analyze the equilibrium effects of bargaining process on the electoral strategies of the parties.

The policy outcome function in our model is similar to that of De Sinopoli and Iannantuoni (2007, 2008). Their outcome function is given by a linear combination of party positions weighted with the share of votes that each party gets in the election. This compromise outcome function is a model of multiparty proportional representation systems and as such represents the bargaining outcome attained through the government formation process. We employ the same modeling strategy and summarize post-electoral bargaining process in a single outcome function, but we don't go further to model the details of such process.[7] The outcome function in our model can be derived as a Nash bargaining solution (Nash

[6]For example, bargaining in the Baron-Diermeier model takes place not over polices but over office-holding benefits to attain the efficient outcome of coalition government (there's a single efficient outcome for each possible government) and the electoral stage only decides which government will in fact emerge.

[7]This approach contrasts with the one taken by Austen-Smith and Banks (1988), Baron and Diermeier (2001) and Baron and Ferejohn (1989) who all build up an explicit game of legislative bargaining after the election. In particular, Baron and Diermeier (2001) derives the utilitarian solution of a bargaining process among three parties with a quadratic loss utility in a two-dimensional setting. The outcome function of De Sinopoli and Iannantuoni can be viewed as the Baron-Diermeier solution when the status quo is quite negative for the elected politicians (De Sinopoli and Iannantuoni 2007).

1950) as mentioned before. Nash bargaining with specific parameters gives rise to the De Sinopoli-Iannantuoni outcome function - the convex combination of party positions weighted by vote shares - in our example.

As Ansolabehere (2006) puts it, "the problem (of non-convergence to the median) has been perhaps the most fruitful for the development of a more robust economic theory of elections." The most well-known divergence result is that policy-motivated politicians do not locate at the same policy position when they are imperfectly informed about voter preferences (Wittman (1983), Calvert (1985), Roemer (1994)). Incomplete information or asymmetry in candidate characteristics often plays an important role in the recent theoretical development of candidate divergence (Aragones and Palfrey (2002), Bernhardt, Duggan and Squintani (2007, 2009a), Kartik and McAfee (2007), Callander (2008)).[8] Notable exceptions are Palfrey (1984) who derives a divergence result from the structure of political competition with strategic entry and Osborne and Slivinski (1996) which is a well-known citizen candidate model with non-binding campaign promises. Along a similar line, our divergence result is motivated by a purely institutional reason. Platform divergence follows as a consequence of the institutional structure of post-electoral bargaining.

In some sense, two-party competition may not be an adequate framework for post-electoral politics involving government formation and policy bargaining. One may argue that the outcome of two-party election is unambiguously given by the winner's platform and policy bargaining must be considered only under proportional systems with multi-party government formation. However, the US two-party presidential system is not free from policy bargaining and compromise. Korean politics has also witnessed occasional mass demonstrations against the military regimes in the late 70's and 80's which should have given the opposing party a footing for the negotiation with the ruling party even if the latter is the majority in the National Assembly. The Duvergerian extreme voting result of De Sinopoli and Iannantuoni (2007) gives a theoretical justification for the analysis of policy bargaining under two-party systems.

The chapter is organized as follows. The second section presents a model and an example

[8] For a broad categorization of the divergence results, see Ansolabehere (2006). The first footnote of Bernhardt, Duggan and Squintani (2009b) gives a succinct and up-to-date summary of the theoretical models that induce platform divergence.

about Nash bargaining. The third section derives extreme platform divergence as a necessary condition of the symmetric pure-strategy equilibrium. The fourth section shows the existence of mixed-strategy equilibrium and explores the possibility of a mixed equilibrium with non-overlapping supports. The fifth section concludes the chapter. The appendix at the back of the dissertation contains the proofs of all results.

2.1 MODEL

2.1.1 Preliminaries

The policy space is given by a closed and bounded interval $P = [a, b]$ of the real line.

Voters have single-peaked preferences and in particular they try to minimize the distance of their ideal policies from whatever policy is finally implemented - the policy outcome can be different from the electoral platforms, which is one of the main distinguishing features of our model. We assume a continuum of voters (or a single representative voter) whose ideal policies follow an atomless distribution F and F admits a density f which is strictly positive on the policy space P. We denote by m the median of the voter distribution F.

There are two political parties, denoted by A and B, who also have single-peaked preferences over P. In particular, we assume that both parties derive their preferences over the policy outcome y according to utility representation $v_j(y)$, $j = A, B$. Each v_j is assumed to be single-peaked with ideal policy θ_j,[9] $j = A, B$, in the policy space which are strictly different from the median and in conflict with each other in the sense that $\theta_A < m < \theta_B$. We will further require $v_j(y)$ to be continuously differentiable in y in Section 4.2 where we study "separating" mixed strategy equilibrium.

The game proceeds as follows. First, parties announce their electoral platforms $\mathbf{p} = (p_A, p_B)$. Next, voters cast their ballots after observing the chosen platforms. We denote the vote share for party j at $\mathbf{p}$ by $\alpha_j(\mathbf{p})$, $j = A, B$. Obviously, $\alpha_A(\mathbf{p}) + \alpha_B(\mathbf{p}) = 1$ as we don't allow abstention. We sometimes drop the subscript for A's vote share and express

[9] Single-peakedness implies each v_j is strictly increasing on $[a, \theta_j]$ and strictly decreasing on $[\theta_j, b]$.

$\alpha(\mathbf{p}) \equiv \alpha_A(\mathbf{p})$ so that $\alpha_B(\mathbf{p}) = 1 - \alpha(\mathbf{p})$. The election is decided by majority rule, so the winning party is the one who obtains a larger vote share. The final policy outcome is determined through bargaining between winner and loser. The bargaining is based on the electoral outcomes which can be summarized as $\{\mathbf{p}, \alpha(\mathbf{p})\}$.[10] In particular, the loser's bargaining power comes from his vote share.

Even though the policy outcome can differ from the electoral platforms, a rational voter who has knowledge about the entire voter distribution and the structure of policy bargaining institutions can form correct expectations about who will win the election and what the policy outcome will be just by observing the platforms announced by the parties. In this way, we view the vote share as a function of observed platforms. We also assume rationality of the political parties in the sense that they can predict voting behavior and the subsequent bargaining outcomes at any electoral strategy profiles.

2.1.2 Example

In this example, we show how the bargaining process can be modeled and which platforms will be chosen by the parties in an equilibrium of our electoral game with bargaining.

For simplicity, we assume the policy space is given by the unit interval $P = [0, 1]$ and voters' ideal policies are distributed uniformly on P. Parties' utilities are given by $v_j(y) = -|y - \theta_j|$ where y is the policy outcome and $\theta_A = 0$, $\theta_B = 1$. Voters decide whom to vote for after observing the platforms (p_A, p_B) and policy bargaining ensues based on the chosen platforms and the vote shares. In particular, each platform pair (p_A, p_B) induces a Nash bargaining problem in the subsequent post-electoral stage with the winner's disagreement payoff being his utility at his own platform minus the cost to be imposed by the losing party, $v_W(p_W) - c(d(\mathbf{p}), \alpha_L(\mathbf{p}))$, and the loser's disagreement payoff being his utility at the winner's platform, $v_L(p_W)$.[11] In this way, the bargaining power of the opposing party is modeled as a cost that he can impose in case the winner is not willing to bargain over policy

[10] In Austen-Smith and Banks (1988), the post-electoral legislative process is modeled as a noncooperative bargaining game between the parties in the elected legislature, and policy prediction is uniquely generated by the vote shares each party receives in the general election and the parties' electoral policy positions.

[11] W denotes the winning party and L, the losing party. Since the winner is determined by vote shares, and the vote shares are understood to change according to the platform $\mathbf{p}$, we can more precisely express winner and loser as $W(\mathbf{p})$ and $L(\mathbf{p})$, respectively.

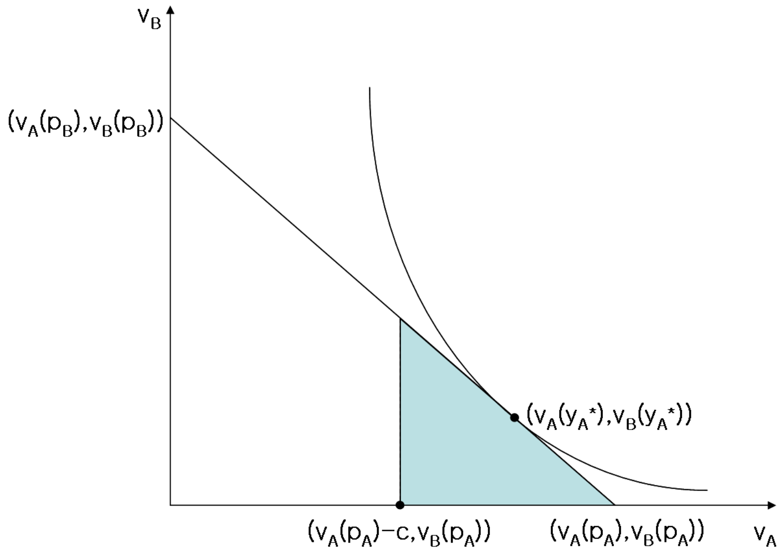


Figure 1: Nash Bargaining Solutions.

but tries to implement his own platform. Here, the cost is assumed to depend on the distance $d(\mathbf{p})(\equiv |p_A - p_B|)$ between platforms and the loser's vote share $\alpha_L(\mathbf{p})$.

Once the platforms (p_A, p_B) with $p_A < p_B$[12] are chosen, assuming party A wins, the bargaining set is given by

$$S = \{(v_A, v_B) : \; v_A \geq v_A(p_A) - c(d(\mathbf{p}), \alpha_B(\mathbf{p})), \; v_B \geq v_B(p_A), \; v_A + v_B \leq 1\},$$

which is a compact and convex set and expressed as the shaded triangular region in Figure 1. S contains a point at which both parties are strictly better off than at their disagreement payoffs. Therefore, this is a well-defined bargaining problem and the solution is found by maximizing the product of the utility differences. Formally, the final policy y_W^*, which will

[12]If $p_A \geq p_B$, there's nothing to bargain. We will come back to this later.

depend on the identity of the winner at any given platform $\mathbf{p}$, is obtained by solving the following problem:

$$\begin{aligned}
&y_W^*(\mathbf{p},\alpha(\mathbf{p}))\\
&\quad = argmax\{ln[v_W(y) - v_W(p_W) + c(d(\mathbf{p}), \alpha_L(\mathbf{p}))] + ln[v_L(y) - v_L(p_W)]\}\\
&\quad s.t.\ p_A \leq y \leq p_B\\
&\qquad v_W(y) \geq v_W(p_W) - c(d(\mathbf{p}), \alpha_L(\mathbf{p}))\\
&\qquad v_L(y) \geq v_L(p_W)
\end{aligned}$$

The resulting outcome depends on who wins the election;

$$\begin{aligned}
y_A^*(\mathbf{p}, \alpha(\mathbf{p})) = p_A + \frac{1}{2}c(d(\mathbf{p}), \alpha_B(\mathbf{p}))\\
y_B^*(\mathbf{p}, \alpha(\mathbf{p})) = p_B - \frac{1}{2}c(d(\mathbf{p}), \alpha_A(\mathbf{p}))
\end{aligned}$$

Specifying the form of cost as $c = 2d(\mathbf{p})\alpha_j(\mathbf{p})$, we get $y_A^* = y_B^* = \alpha(\mathbf{p})p_A + (1 - \alpha(\mathbf{p}))p_B$.[13] The vote share for A (voters vote for the party whose anticipated outcome is closer to their ideal policies) is

$$\alpha(\mathbf{p}) = \frac{1}{2}(y_A^* + y_B^*) = \frac{p_B}{1 + p_B - p_A},$$

hence,

$$y_A^* = y_B^* = \frac{p_B}{1 + p_B - p_A}(= y^*).$$

This specification leads to a unique pure-strategy equilibrium $(p_A, p_B) = (0, 1)$ and $y_A^* = y_B^* = \frac{1}{2}$ $(= y^*)$; i.e. the equilibrium platforms are as divergent as possible, but the policy outcome is the median no matter who wins. Thus, our model in this example predicts

[13]This is precisely the De Sinopoli-Iannantuoni policy outcome with two parties, which is also their equilibrium outcome. Thus, their outcome can be generated as a solution of the Nash bargaining problem -which models post-electoral bargaining - with the disagreement payoffs reflecting the loser's bargaining power.

divergence in platforms and convergence in final policies, which will be generalized later in a more abstract setting. The reason this is an equilibrium follows easily from the fact that the policy outcome is strictly increasing in platforms ($\frac{\partial y^*}{\partial p_A} > 0$ and $\frac{\partial y^*}{\partial p_B} > 0$), which implies that both parties have incentives to always deviate toward their own ideal policies at any interior platform profiles.

We finally note that there doesn't exist a pure-strategy equilibrium with a cost of the form $c(d(\mathbf{p}), \alpha_j(\mathbf{p})) = kd(\mathbf{p})\alpha_j(\mathbf{p})$ if $0 < k < 2$, which prompts us to search for mixed strategy equilibria (Section 4 will be devoted to the analysis of mixed equilibrium).

2.1.3 Abstract Bargaining Model

In general, the cost to be imposed by the opposing party is a function of the distance between platforms $d(\mathbf{p}) \equiv max\{p_B - p_A, 0\}$[14] and that party's vote share $\alpha_j(\mathbf{p})$. However, instead of cost functions taking specific forms, we consider a general class of the cost that satisfies the following assumptions which capture the basic characteristics of post-electoral bargaining:

A1. $c(0, \alpha_j) = 0 = c(d, 0)$ and $c(d, \alpha_j) > 0$ for $(d, \alpha_j) \gg 0$.

A2. $c(\cdot, \cdot)$ is a C^1 function with $c_d > 0$ and $c_\alpha > 0$;

A3. $c(d(\mathbf{p}), \alpha_A(\mathbf{p})) + c(d(\mathbf{p}), \alpha_B(\mathbf{p})) \leq d(\mathbf{p})$, $\forall \mathbf{p}$; and

A4. $c(d(\mathbf{p}), \alpha_i(\mathbf{p})) \geq c(d(\mathbf{p}), \alpha_j(\mathbf{p}))$ iff $|p_i - m| \leq |p_j - m|$ for $i \neq j$.

The first assumption says that the loser cannot impose costs if his vote share is zero or if the two parties announce the same platforms (or A announces a platform that is closer to B's ideal policy in which case B may not need to impose further cost since A's platform is already favorable enough for him relative to his own platform); hence we formally define that

[14] Our main interest lies in the case where party A's platform p_A is below party B's platform p_B, as the opposite case can easily be dismissed by strict dominance argument or by the fact that there's simply nothing to bargain. This is true especially when the median is in the middle of parties' ideal policies and the bargaining outcome is viewed as policy concession to be given by the winner. For completeness, we define here in the abstract model the distance between platforms to be zero if $p_A \geq p_B$ so that the cost is accordingly zero.

the distance between platforms is zero in this case. However, as long as parties announce distinct platforms in such a way that there is room for bargaining (i.e. $p_A < p_B$ so that the distance is positive) and their vote shares are positive, they have some bargaining power represented as a positive cost. In view of our definition of the distance between platforms, it is natural to assume that the cost is strictly increasing in the distance, and obviously, it must be increasing in the vote share of the opposing party, which is the second assumption. The third assumption guarantees that $y_A^* \leq y_B^*$ at any platform $\mathbf{p}$ with $p_A < p_B$ since without this assumption it is better for parties to lose *a priori* for some platforms, which is absurd. We have a formal derivation of this in the following lemma. We finally assume that a party can impose a larger cost if his platform is closer to the median. This assumption simplifies our analysis because the task of determining a winner at any platforms becomes very cumbersome without this. The reason is because vote shares are determined endogenously; i.e., given any platforms, they are determined by the midpoint of the anticipated outcomes y_A^*, y_B^* which depend crucially on the cost (and hence, vote shares) at the platforms, as is shown below.

Our model abstracts from any specific bargaining protocols and just assumes that, given the electoral outcomes, $\mathbf{p} = (p_A, p_B)$ and $(\alpha_A(\mathbf{p}), \alpha_B(\mathbf{p}))$, the bargaining outcomes are given, depending on the identity of the winner, by

$$y_A^*(\mathbf{p}) \equiv p_A + c(d(\mathbf{p}), \alpha_B(\mathbf{p}))$$
$$y_B^*(\mathbf{p}) \equiv p_B - c(d(\mathbf{p}), \alpha_A(\mathbf{p}))$$

As seen in Figure 2, when $p_A < p_B$, the bargaining outcome functions require the winner to move from his own platform in a direction that is favorable to his opponent. The extent of movement depends above all on the loser's vote share which represents his bargaining power. These are thus the simplest possible forms of the outcome functions under the policy concession interpretation of post-electoral bargaining. We also note that these forms are obviously motivated by the Nash bargaining solutions of our example.

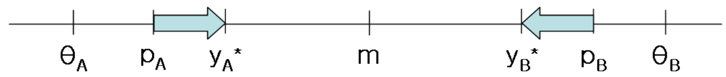


Figure 2: General Bargaining Outcomes.

For completeness, we also consider what the above definition implies about the bargaining outcomes for $p_A \geq p_B$. When $p_A \geq p_B$, $d(\mathbf{p}) = 0$ by definition and hence $c(d(\mathbf{p}), \alpha_j(\mathbf{p})) = 0$ by assumption. Hence, if $p_A = p_B$,

$$y_A^*(\mathbf{p}, \alpha(\mathbf{p})) = p_A = p_B = y_B^*(\mathbf{p}, \alpha(\mathbf{p}))$$

and if $p_A > p_B$,

$$y_A^*(\mathbf{p}, \alpha(\mathbf{p})) = p_A \quad \text{and} \quad y_B^*(\mathbf{p}, \alpha(\mathbf{p})) = p_B.$$

We collect a couple of immediate consequences of our assumptions about cost and bargaining outcomes in the following lemma.

Lemma 1. *(1) If $p_A < p_B$, then $p_A < y_A^* \leq y_B^* < p_B$.*
(2) If $|p_i - m| < |p_j - m|$, then party i wins the election.

Voting behavior will be based not on the announced platforms but on the anticipation of the above bargaining outcomes. That is, voters will vote for the party whose anticipated outcome is closer to their ideal policies,[15] which can be viewed as a version of "sincere voting"

[15] Voters thus vote over *final policies*, not over *candidates*, in our model, which according to Austen-Smith and Banks (1988) is a correct specification of the choice set as what voters are ultimately interested in are policy outcomes, not policy promises.

under our modeling framework. Hence, with a continuum of voters, we determine the vote share for each party as follows;

$$\alpha_A(\mathbf{p}) \equiv F\Big(\frac{y_A^*(\mathbf{p}) + y_B^*(\mathbf{p})}{2}\Big) \quad \text{and} \quad \alpha_B(\mathbf{p}) = 1 - \alpha_A(\mathbf{p}).$$

For any given platform $\mathbf{p}$, party j wins if $\alpha_j(\mathbf{p})$ is greater than a half (majority rule) and ties are split evenly between the parties. The policy outcome y^* to be implemented is the winner's outcome; hence $y^* = y_j^*$ if $\alpha_j > \frac{1}{2}$, $j = A, B$, and y^* is equally likely to be y_A^* or y_B^* if $\alpha_j = \frac{1}{2}$.

2.2 PURE-STRATEGY EQUILIBRIUM

Following Nash, we define a pure strategy equilibrium as a platform pair (p_A^*, p_B^*) from which neither party can find a unilaterally profitable deviation. One thing we should keep in mind is that as a party changes his own platform, both bargaining outcomes y_A^*, y_B^* will change accordingly because the outcomes depend on the vote shares which are functions of *both* platforms. Since what ultimately matters is the final policy outcomes, it can be said that party j's deviation from his original platform changes in effect the opponent's ultimate position (y_k^*) as well as his own (y_j^*).

We first note the following result that states non-convergence in equilibrium platforms.

Proposition 1. *$p_A = p_B$ can never be an electoral equilibrium; in particular, both parties cannot choose the median with probability one in any equilibrium.*

This result is immediate from our modeling setup. As shown in Figure 3, when both parties locate at the median, it is better for party A to deviate toward his ideal policy since then B will win but A can get a policy concession and hence is better off at B's winning policy outcome.

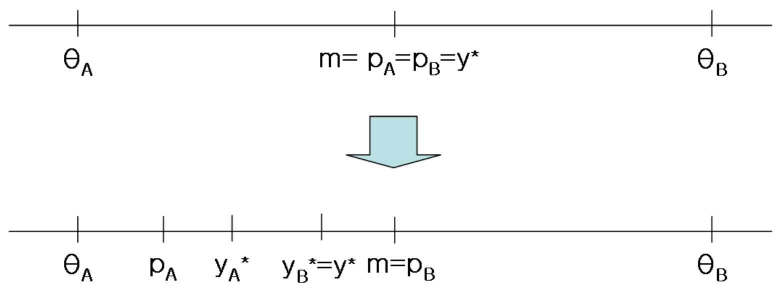


Figure 3: Non-convergence at equilibrium.

The parties in our model face two countervailing incentives of win motivation and utility maximization. The former incentive drives them to converge to the median whereas the latter one drives them in the opposite direction. When policy motivated political parties are required to commit themselves to platforms, the win motivation is so strong that they must converge to the median if voting behavior is deterministic; i.e. if the median is known with certainty (Wittman 1977; Calvert 1985; Roemer 1994). In our model, even if voting behavior is still deterministic, win motivation is substantially mitigated once we relax precommitment to electoral platforms and allow the losing party to have some degree of bargaining power over policy-making.

We next define a *symmetric* strategy profile as any pair (p_A, p_B) that satisfies

$$p_A < m < p_B \quad \text{and} \quad \alpha_A(\mathbf{p}) = \alpha_B(\mathbf{p}) = \frac{1}{2}$$

But then, by the definition of vote shares and strict monotonicity of F, we have $\frac{y_A^* + y_B^*}{2} = m$, which implies that $\frac{p_A + p_B}{2} = m$. Thus we conclude that the distances of the platforms from the median are the same at such a symmetric profile. Any other strategy profiles are defined to be *asymmetric*.

We also note that the only equilibrium candidates are the ones that satisfy $p_A < m < p_B$. By Proposition 1, we can disregard the case where $p_A = p_B$. The case $p_A > p_B$ can also be easily dismissed; if $m \leq p_B < p_A$, for example, then $\tilde{p}_A = p_B - \varepsilon$ is a profitable deviation for A. If $m \leq p_A < p_B$, then $m < y_A^* \leq y_B^*$, so $\tilde{p}_A = 2m - p_B$ is a profitable deviation for A as the parties will make a tie and the expected outcome is the median m at $(\tilde{p}_A, p_B)$. The case for $p_A < p_B \leq m$ is the same.

We need a couple of lemmas before we present the next main result that shows the necessity of a sufficiently large cost and extreme divergence in platforms at any symmetric pure strategy equilibrium.

Lemma 2. *Both policy outcomes $y_A^*(p_A, p_B)$ and $y_B^*(p_A, p_B)$ are continuously differentiable at any (p_A, p_B) with $p_A < p_B$.*

We obtain this lemma by viewing the expressions for outcomes y_A^*, y_B^* as implicitly defining these variables in terms of platforms p_A, p_B (note that the cost depends on the vote share which is a function of y_A^*, y_B^*). We also get as a consequence of the Implicit Function Theorem (IFT) the following lemma which is crucial in examining the profitability of a deviation.

Lemma 3. *At any symmetric strategy profile $\boldsymbol{p} = (p_A, p_B)$, party A's vote share $\alpha_A(\boldsymbol{p}) \equiv F\left(\frac{(y_A^* + y_B^*)(\boldsymbol{p})}{2}\right)$ is strictly increasing in p_j, $j = A, B$.*

With these two lemmas at hand, we are now ready to see what conditions necessarily hold at any symmetric pure strategy equilibrium.

Proposition 2. *(1) If $(\bar{p}_A, \bar{p}_B)$ is a symmetric pure-strategy equilibrium, then*

$$c(d(\bar{\boldsymbol{p}}), \frac{1}{2}) = \frac{d(\bar{\boldsymbol{p}})}{2}.$$

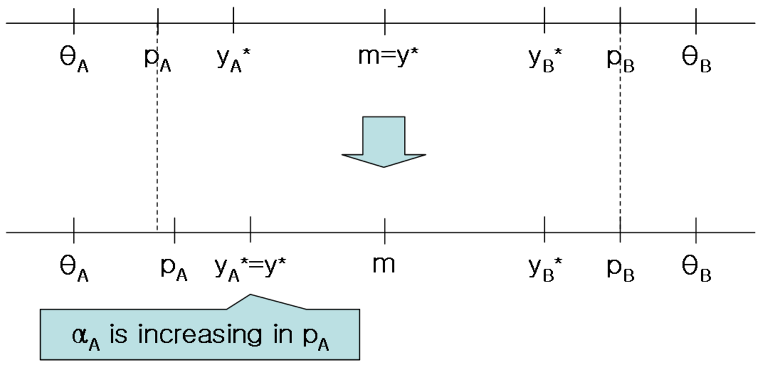


Figure 4: Necessity of large cost at pure equilibrium.

(2) Any interior symmetric profile cannot be a pure-strategy equilibrium; i.e. the only equilibrium candidate is the pair (a, b) of boundary positions, given (a, b) is symmetric.

By A3, we must have $c(d(\mathbf{p}), \frac{1}{2}) \leq \frac{d(\mathbf{p})}{2}$ for all $\mathbf{p}$, so (1) requires that the cost take its maximum possible value at a symmetric equilibrium. We can interpret the relative magnitude of cost as a characteristic of a particular post-electoral bargaining environment or political system. Thus, it is possible that the opposing party has a relatively large bargaining power with a given vote share if, for example, a political system is highly unstable and the winning party doesn't have full control over the military power of the polity to which it belongs.

But then, (2) requires that the parties announce their equilibrium platforms as extreme as possible anticipating the substantial policy concession that they must yield as a winner. Therefore, if each party's ideal policy is located at the boundary of the policy space, then announcing one's ideal policy may be an equilibrium as is the case in our previous example. This gives us an equilibrium support for the extremely differentiated campaign promises that might be observed in reality.

In proving (1), we will use the fact that the stated condition on cost is equivalent to $y_A^*(\bar{\mathbf{p}}) = y_B^*(\bar{\mathbf{p}}) = m$. Suppose the expected outcome (i.e., the midpoint of y_A^* and y_B^*)

at a symmetric profile is the median, but nevertheless the outcome pertaining to A, for example, is strictly below the median, as in Figure 4. Then, since Lemma 2 asserts that the expected outcome and hence A's vote share is increasing in A's platform, A can win for sure by announcing a platform slightly higher than his original platform while keeping his own outcome (which changes continuously and now becomes the policy outcome) below the median. In this way, A can find a profitable deviation.

We employ a similar argument to show the necessity of extreme divergence at any symmetric pure equilibrium. Figure 5 illustrates that any interior symmetric profile cannot be an equilibrium. If (1) holds, then we can show that all the outcomes y_j^* are increasing in each platform at any symmetric profile. Therefore, A can, for example, announce a slightly lower platform thereby making the winning outcome (y_B^*) below the median, which shows A can again find a profitable deviation.

The following is a sufficient condition for the existence of a pure strategy equilibrium that demands a large enough cost not only in symmetric but also in all possible strategy profiles. The failure of the conditions in Proposition 2 would lead to a local deviation while

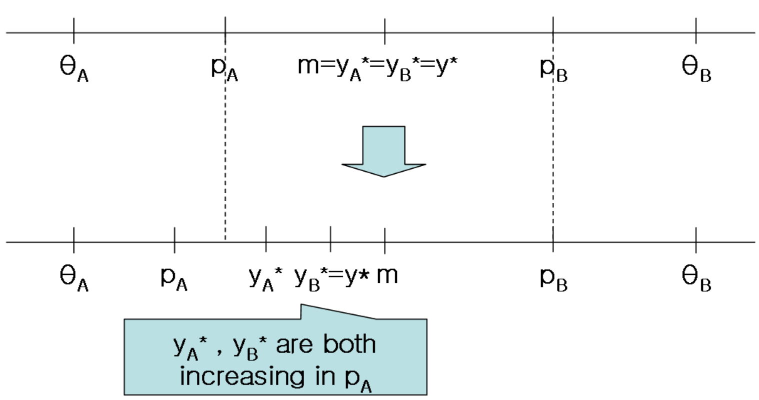


Figure 5: Incentive to diverge at interior symmetric profile.

the conditions in Proposition 3 guarantees global optimality of the suggested profile.

Proposition 3. *Suppose (a, b) is symmetric. Then, (a, b) is the unique symmetric equilibrium if*

$$c(d(p_A, b), \alpha_B(p_A, b)) \geq |p_A - m|, \quad \forall p_A \in [a, m),$$
$$c(d(a, p_B), \alpha_A(a, p_B)) \geq |p_B - m|, \quad \forall p_B \in (m, b].$$

The following result shows that the policy outcome must converge to the median at any pure equilibrium.

Proposition 4. *In any pure-strategy equilibrium, the final policy outcome is located at the median.*

Up to now, we have focused on symmetric equilibrium. However, we cannot exclude the possibility of asymmetric equilibrium without further modeling assumptions. The above result nevertheless shows that the median will be implemented in any (symmetric or asymmetric) pure equilibrium.[16]

2.3 MIXED-STRATEGY EQUILIBRIUM

Our analysis of pure strategy equilibrium suggests that the opposing parties should be able to impose a sufficiently large cost with a given vote share at any fixed equilibrium platforms. However, there may exist political environments in which the losing party can impose only

[16] In Austen-Smith and Banks (1988), parties' electoral platforms are symmetrically distributed about the median voter's ideal point, and the expected final policy outcome is at the median. However, the realized final outcome lies between the median and either the rightmost or the leftmost party's position, depending on which party gets the largest vote share (the middle party always gets the second largest vote share). Since our model involves two party competition, symmetric distribution of platforms always lead to the policy outcome at the median. Our equilibrium condition implies that the policy outcome should be the median even at any asymmetrically distributed profiles.

a small cost. In other words, the losing party may have relatively small bargaining power with any given vote share determined by the election. Thus, the relative magnitude of cost characterizes political environments in terms of the bargaining power that the losing party can derive from its vote share earned in the election.

Equilibrium analysis also depends on the relative magnitude of costs. The traditional Downsian or Wittman model of spatial competition can be viewed as a limiting case of our model where there exist discrete jumps in the cost that a party can impose as the vote share changes.[17] The traditional one-dimensional model can alternatively be specified as the one in which

$$
\begin{aligned}
c(d(\mathbf{p}), \alpha_j(\mathbf{p})) &= d(\mathbf{p}), \text{ if } \alpha_j(\mathbf{p}) > \frac{1}{2}; \\
&= \frac{d(\mathbf{p})}{2}, \text{ if } \alpha_j(\mathbf{p}) = \frac{1}{2}; \\
&= 0, \qquad \text{if } \alpha_j(\mathbf{p}) < \frac{1}{2}.
\end{aligned}
$$

Since the losing party whose vote share is less than a half can only impose zero costs, the final policy will always be the winner's platform and the equilibrium is in pure strategies by which both parties choose the median.

Another extreme is the case where the losing party can impose a sufficiently large cost so that we may have a pure strategy equilibrium. As we have seen before, the equilibrium platforms in this case involve the extreme policies lying on the boundary of the policy space. However, if the parties cannot impose sufficiently large costs but can impose positive costs with any positive vote share, we no longer have a pure strategy equilibrium.[18] Hence we direct our search for equilibrium to those in which the parties mix over a range of platforms between the median and the extreme policies.

[17] In our model, cost is assumed to vary continuously with vote shares.

[18] The policy outcome changes discontinuously at symmetric profiles where the winner changes, say, from A to B, which subsequently brings about discontinuity in the parties' payoffs.

2.3.1 General Existence of Mixed Equilibrium

We first consider the general existence of mixed strategy equilibrium in our platform choice game with bargaining. Suppose the pair (a, b) of boundary points is a symmetric profile and $c(d(\mathbf{p}), \frac{1}{2}) < \frac{d(\mathbf{p})}{2}$, for all $\mathbf{p} \in P^2 = [a, b]^2$ such that $\alpha_A(\mathbf{p}) = \alpha_B(\mathbf{p}) = \frac{1}{2}$. By Proposition 2 (1), we don't have a pure equilibrium in this case. The strategy space is given by $S_A = S_B = [a, b] = [\theta_A, \theta_B] \equiv P$; i.e. we assume that the ideal policies of the parties are located at the boundary of the policy space. We redefine parties' utilities $w_j(p_A, p_B) \equiv v_j(y^*(p_A, p_B))$, $j = A, B$, as a function of platforms. When the cost is not sufficiently large, our game becomes one with discontinuous payoffs, so we cannot apply the standard existence result of Debreu-Fan-Glicksberg.[19]

We shall apply Dasgupta and Maskin (1986)'s existence theorem (Theorem 5b) for mixed equilibrium.

Proposition 5. *(Dasgupta and Maskin) Suppose* $(a, b) = (\theta_A, \theta_B)$ *is symmetric and* $c(d(\boldsymbol{p}), \frac{1}{2}) < \frac{d(\boldsymbol{p})}{2}$, $\forall \boldsymbol{p}$. *Then, there exists a mixed-strategy equilibrium in the game* $[(S_j, w_j); j = A, B]$.

Theorem 5b (Dasgupta & Maskin) employs "compensating monotonicity" of both players' payoffs to show existence; roughly speaking, it applies to situations where at any point in which one player's payoff falls, the other's rises. Our game also shares this property once we restrict the parties' ideal policies to be located at the boundary (i.e. $P = [a, b] = [\theta_A, \theta_B]$).[20]

We first characterize the points at which the parties' utilities exhibit discontinuity.

[19] In particular, Glicksberg(1952) requires non-empty and compact strategy spaces and continuous utilities for the existence of a mixed strategy equilibrium.

[20] Alternatively, we can apply Dasgupta and Maskin's main theorem (Theorem 5) to guarantee the existence. In this case, the "compensating monotonicity" condition is replaced by upper semi-continuity of the sum of utilities, and weak lower semi-continuity of individual utilities. We can show the utility sum is upper semi-continuous, for example, by additionally restricting parties' utilities to be concave and symmetric in the sense that $v_A(y) = v_B(2m - y)$, $\forall y \in P$. Lower semi-continuity of individual utilities can be proved without such restrictions. In this case, we can have the parties' ideal policies in the interior of the policy space.

Fix $p_A \in [a, b]$ and examine how party B's utility $v_B(y^*(\mathbf{p}))$ changes as p_B changes. We fist consider $a < p_A < m$. If p_B approaches $2m - p_A$ from the left, then p_B is the winning platform, so y_B^* will be implemented. Hence,

$$\lim_{p_B \to (2m-p_A)^-} v_B(y^*(\mathbf{p})) = \lim_{p_B \to (2m-p_A)^-} v_B(p_B - c(p_B - p_A, \alpha_A)) = v_B(m_u)$$

where $m_u \equiv 2m - p_A - c(2m - 2p_A, \frac{1}{2})$.

On the other hand, if p_B approaches $2m - p_A$ from the right, y_A^* is implemented along the sequence, so

$$\lim_{p_B \to (2m-p_A)^+} v_B(y^*(\mathbf{p})) = \lim_{p_B \to (2m-p_A)^+} v_B(p_A + c(p_B - p_A, \alpha_B)) = v_B(m_l)$$

where $m_l \equiv p_A + c(2m - 2p_A, \frac{1}{2})$. Since $c(2m - 2p_A, \frac{1}{2}) < m - p_A$ by assumption,[21] we have $m_u > m > m_l$, implying

$$v_B(m_u) > v_B(m) = v_B(y^*(p_A, 2m - p_A)) > v_B(m_l).$$

But then, v_B is not continuous at $p_B = 2m - p_A$.

Similarly, if $m < p_A < b$, then

$$\begin{aligned} \lim_{p_B \to (2m-p_A)^-} v_B(y^*(\mathbf{p})) = v_B(p_A) > v_B(m) & \\ > v_B(2m - p_A) = & \lim_{p_B \to (2m-p_A)^+} v_B(y^*(\mathbf{p})) \end{aligned}$$

Hence, v_B is discontinuous again at $p_B = 2m - p_A$. It can easily be seen that v_B is continuous at $(p_A, 2m - p_A)$ if $p_A = m$, a, or b (since (a, b) is symmetric, $b = 2m - a$). Also, v_B is continuous at $(p_A, p_B) \neq (p, 2m - p)$.

We now formally state the assumptions of Dasgupta and Maskin (Theorem 5b);

1. $S_j = P = [a, b]$ for $j = A, B$, is a closed interval.

[21] This follows from the assumption that $c(d(\mathbf{p}), \frac{1}{2}) < \frac{d(\mathbf{p})}{2}$.

2. Each w_j is continuous except on a subset $S^{**}(j)$ of $S^*(j)$;

$$S^*(A) = \{(p_A, 2m - p_A) : a \le p_A \le b\} = S^*(B),$$
$$S^{**}(A) = \{(p_A, 2m - p_A) : a < p_A < m,\ m < p_A < b\} = S^{**}(B).$$

3. Each $|w_j(p_A, p_B)|$ is bounded;[22]

$$|w_j(p_A, p_B)| = |v_j(y^*(\mathbf{p}))| \le max\{|v_j(y_A^*(\mathbf{p}))|, |v_j(y_B^*(\mathbf{p}))|\}.$$

4. For each $p \in (a, m) \cup (m, b)$, w_A and w_B satisfy "compensating monotonicity"; i.e.

$$\begin{aligned}
\lim_{p_A \to p^-, p_B \to (2m-p)^-} w_A(p_A, p_B) &< w_A(p, 2m - p) \\
&< \lim_{p_A \to p^+, p_B \to (2m-p)^+} w_A(p_A, p_B) \\
\lim_{p_A \to p^-, p_B \to (2m-p)^-} w_B(p_A, p_B) &> w_B(p, 2m - p) \\
&> \lim_{p_A \to p^+, p_B \to (2m-p)^+} w_B(p_A, p_B)
\end{aligned}$$

We only need to verify the last assumption since the other assumptions clearly hold by the arguments up to now.

Lemma 4. *Assumption 4 in Dasgupta and Maskin (Theorem 5b) about "compensating monotonicity" is satisfied in our game* $[(S_j, w_j); j = A, B]$.

Dasgupta and Maskin gives an existence proof in their Theorem 5b for the case where the discontinuity occurs on the diagonal with a positive slope while the discontinuity in our model takes place on the diagonal with a negative slope. However, the existence in our case can be shown by a straightforward application of their proof which we reproduce in the appendix.

The idea is to modify the payoffs at the points of discontinuity in such a way that the game with modified payoffs satisfies the assumptions of Dasgupta and Maskin's main

[22] $y_A^*(\mathbf{p})$, $y_B^*(\mathbf{p})$ lie in the compact interval $[a, b]$ and v_j is continuous.

theorem (Theorem 5); i.e., upper semi-continuity of the sum of payoffs and weak lower semi-continuity of individual payoffs. We then show that the equilibrium of the modified game is also an equilibrium of the original game.

2.3.2 Separating Mixed Equilibrium

In this section, we try to understand what the equilibrium support would look like or what kind of equilibrium support is admissible. Here, we focus on the possibility of separating equilibrium with supports that don't intersect or intersect with measure zero.

Thus, we are led to explore the existence of a mixed equilibrium with continuous density strategies (g_A, g_B) that have the following features: (we assume in this section that each v_j is continuously differentiable.)

1. The supports of both equilibrium densities are symmetric around the median;

$$supp(g_A) = [\alpha, \beta], \quad supp(g_B) = [2m - \beta, 2m - \alpha]$$

2. Both supports are separated or overlap with measure zero;

$$\alpha < \beta \leq m$$

If we can find a separating mixed equilibrium, then platform divergence in varying degrees can be supported by a mixed equilibrium of the spatial model with post-electoral bargaining. This would provide a rational foundation for the divergence of campaign promises in a world where the parties' private payoff perturbation is not perfectly observed and hence their plays must be approximated by randomization over platforms.[23]

We begin with the condition that the parties must be indifferent between the platforms in their equilibrium support so that their expected payoffs must be constant on the support,

[23] This is Harsanyi's well-known "purification" interpretation of mixed strategy equilibrium.

given the opponent's equilibrium play.

$$\begin{aligned} V_A(p_A) &= \int_{2m-\beta}^{2m-p_A} v_A(y_B^*(p_A,p_B))g_B(p_B)dp_B \\ &\quad + \int_{2m-p_A}^{2m-\alpha} v_A(y_A^*(p_A,p_B))g_B(p_B)dp_B \;=\; k_A, \\ &\quad \forall p_A \in [\alpha,\beta] \end{aligned} \tag{2.1}$$

$$\begin{aligned} V_B(p_B) &= \int_{\alpha}^{2m-p_B} v_B(y_B^*(p_A,p_B))g_A(p_A)dp_A \\ &\quad + \int_{2m-p_B}^{\beta} v_B(y_A^*(p_A,p_B))g_A(p_A)dp_A \;=\; k_B, \\ &\quad \forall p_B \in [2m-\beta, 2m-\alpha] \end{aligned} \tag{2.2}$$

where k_A and k_B are constants. Using Leibniz Rule, we differentiate the expected payoff of party A with respect to his own platform to get a more tractable integral equations;

$$\begin{aligned} V_A'(p_A) &= [v_A(y_B^*(p_A,2m-p_A)) - v_A(y_A^*(p_A,2m-p_A))]g_B(2m-p_A) \\ &\quad - \int_{2m-\beta}^{2m-p_A} v_A'(y_B^*(p_A,p_B))\frac{\partial y_B^*}{\partial p_A}(p_A,p_B)g_B(p_B)dp_B \\ &\quad - \int_{2m-p_A}^{2m-\alpha} v_A'(y_A^*(p_A,p_B))\frac{\partial y_A^*}{\partial p_A}(p_A,p_B)g_B(p_B)dp_B \\ &= 0 \end{aligned}$$

So, we obtain, for all $p_A \in [\alpha,\beta]$,

$$\begin{aligned} g_B(2m-p_A) - \lambda(p_A)^{-1}\int_{2m-\beta}^{2m-p_A} v_A'(y_B^*(p_A,p_B))\frac{\partial y_B^*}{\partial p_A}(p_A,p_B)g_B(p_B)dp_B \\ - \lambda(p_A)^{-1}\int_{2m-p_A}^{2m-\alpha} v_A'(y_A^*(p_A,p_B))\frac{\partial y_A^*}{\partial p_A}(p_A,p_B)g_B(p_B)dp_B = 0 \end{aligned}$$

where

$$\lambda(p_A) \equiv v_A(y_B^*(p_A,2m-p_A)) - v_A(y_A^*(p_A,2m-p_A)).$$

Our goal is to turn this equation to a standard Fredholm or Volterra integral equation of the second kind.[24] Since $x(p_A) = 2m - p_A$ is invertible, we can define

$$\begin{aligned}
& v'_A(\hat{y}^*_B(x(p_A), t))\frac{\partial \hat{y}^*_B}{\partial p_A}(x(p_A), t) \\
& \quad \equiv v'_A(y^*_B(x^{-1}(x(p_A)), t))\frac{\partial y^*_B}{\partial p_A}(x^{-1}(x(p_A)), t) \\
& \quad = \; v'_A(y^*_B(p_A, t))\frac{\partial y^*_B}{\partial p_A}(p_A, t);
\end{aligned}$$

$$\begin{aligned}
& v'_A(\hat{y}^*_A(x(p_A), t))\frac{\partial \hat{y}^*_A}{\partial p_A}(x(p_A), t) \\
& \quad \equiv v'_A(y^*_A(x^{-1}(x(p_A)), t))\frac{\partial y^*_A}{\partial p_A}(x^{-1}(x(p_A)), t) \\
& \quad = \; v'_A(y^*_A(p_A, t))\frac{\partial y^*_A}{\partial p_A}(p_A, t); \text{ and}
\end{aligned}$$

$$\hat{\lambda}(x(p_A)) \equiv \lambda(x^{-1}(x(p_A))) = \lambda(p_A).$$

So, we finally get that, for all $x \in [2m - \beta, 2m - \alpha]$,

$$\begin{aligned}
g_B(x) - \hat{\lambda}(x)^{-1} \int_{2m-\beta}^{x} v'_A(\hat{y}^*_B(x, t))\frac{\partial \hat{y}^*_B}{\partial p_A}(x, t) g_B(t) dt \\
- \hat{\lambda}(x)^{-1} \int_{x}^{2m-\alpha} v'_A(\hat{y}^*_A(x, t))\frac{\partial \hat{y}^*_A}{\partial p_A}(x, t) g_B(t) dt = 0.
\end{aligned}$$

This is neither the Fredholm nor the Volterra equation in a standard sense, but is closer to the former one with its kernel $v'_A(\hat{y}^*(x, t))\frac{\partial \hat{y}^*}{\partial p_A}(x, t)$ having a discontinuity at x. We can still apply the Banach Fixed Point Theorem once $\hat{\lambda}(x)$ satisfies some condition that makes

[24] Fredholm integral equation of the second kind takes the form

$$x(t) - \mu \int_a^b k(t, \tau) x(\tau) d\tau = v(t),$$

where x is an unknown function on $[a, b]$, μ is a parameter, and the kernel k and v are given functions on $[a, b]^2$ and $[a, b]$, respectively. Volterra integral equation takes a similar form except for the upper limit of the integral being variable.

the integral operator a contraction mapping. Hence, there follows an existence result for the indifference conditions (9) and (10).

Lemma 5. *Suppose $\alpha < \beta \leq m$. We have a unique pair (g_A, g_B) of continuous functions that satisfy the indifference conditions (9) and (10) if*

$$\begin{aligned}
\xi(\beta-\alpha) \; &< \; |v_A(y_B^*(p_A, 2m-p_A)) - v_A(y_A^*(p_A, 2m-p_A))| \; \equiv \; |\lambda(p_A)|, \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \forall p_A \in [\alpha, \beta] \\
\zeta(\beta-\alpha) \; &< \; |v_B(y_B^*(2m-p_B, p_B)) - v_B(y_A^*(2m-p_B, p_B))| \; \equiv \; |\mu(p_B)|, \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \forall p_B \in [2m-\beta, 2m-\alpha]
\end{aligned}$$

where $\xi \equiv max\{\xi_A, \xi_B\}$, $\zeta \equiv max\{\zeta_A, \zeta_B\}$,

$$\begin{aligned}
\xi_j &\equiv \max_{(p_A, p_B) \in R_j} \left| v_A'(y_j^*(p_A, p_B)) \frac{\partial y_j^*}{\partial p_A}(p_A, p_B) \right|, \\
\zeta_j &\equiv \max_{(p_A, p_B) \in R_j} \left| v_B'(y_j^*(p_A, p_B)) \frac{\partial y_j^*}{\partial p_B}(p_A, p_B) \right|, \quad j = A, B
\end{aligned}$$

$$\begin{aligned}
\textit{and} \quad R_A &\equiv \{(p_A, p_B) : \alpha \leq p_A \leq \beta, \, 2m - p_A \leq p_B \leq 2m - \alpha\} \\
R_B &\equiv \{(p_A, p_B) : \alpha \leq p_A \leq \beta, \, 2m - \beta \leq p_B \leq 2m - p_A\}.
\end{aligned}$$

We note that R_j is the set of platform pairs at which party j wins. Also, both $\lambda(p_A)$ and $\mu(p_B)$ are determined in terms of our primitives $v_j(\cdot)$ and $c(\cdot, \cdot)$ and strategies p_A, p_B since y_j^* is a function of platforms and cost:

$$\begin{aligned}
y_A^*(p_A, 2m-p_A) &= p_A + c(2m - 2p_A, \frac{1}{2}) \\
y_B^*(p_A, 2m-p_A) &= 2m - p_A - c(2m - 2p_A, \frac{1}{2}) \\
y_A^*(2m-p_B, p_B) &= 2m - p_B + c(2p_B - 2m, \frac{1}{2}) \\
y_B^*(2m-p_B, p_B) &= p_B - c(2p_B - 2m, \frac{1}{2})
\end{aligned}$$

Thus, Lemma 5 characterizes the endogenous quantities α, β in terms of primitives. Specifically, the sufficient condition requires that the length of the equilibrium support should be no greater than the ratio of the utility differences at any symmetric profiles that can arise by equilibrium play to the maximum possible rate of change in utilities with respect to the change in platforms within the equilibrium support.

The following is an immediate observation from Lemma 5.

Proposition 6. *If the equilibrium supports satisfy the sufficient conditions in Lemma 5, then the equilibrium supports supp*(g_A) *and supp*(g_B) *don't intersect; that is,* $\beta < m$.

Proof. The sufficient condition must hold for $p_A = \beta$ in particular. If $\beta = m$, then $y_B^*(m, m) = m = y_A^*(m, m)$, implying $\lambda(m) = 0$ and hence $\beta - \alpha < 0$, which is a contradiction. □

Proposition 6 suggests a fairly strong divergence result for our mixed equilibrium. It says that we can have a mixed equilibrium in which a platform that might be adopted by one party in the equilibrium can never be announced as the campaign platform of its opponent. The parties mix over some range of platforms below and above the median, respectively, but the boundaries of those ranges must be strictly away from the median in an equilibrium characterized by certain bounds on the length of the equilibrium supports.

2.3.3 Example

One immediate question is how restrictive are the sufficient conditions in Lemma 5. To get an idea about this, we next consider the environment in our earlier example where the policy space is given by the unit interval $P = [0, 1]$, the voter distribution F is uniform on $[0, 1]$ and the parties' utilities are linear $v_j(y) = -|y - \theta_j|$ with $\theta_A = 0$ and $\theta_B = 1$ (we can in this case represent without loss of generality the parties' utilities as $v_A(y) = -y$ and $v_B(y) = y$).

Suppose the cost is given by $c(d(\mathbf{p}), \alpha_j(\mathbf{p})) = \frac{1}{n} d(\mathbf{p})\alpha_j(\mathbf{p})$. Thus, this cost doesn't satisfy the necessary condition in Proposition 2(1) unless $n = 1$ (hence we don't have a pure equilibrium for $n \geq 2$) and indeed converges (uniformly) to zero as n tends to infinity. In this case,

$$\begin{aligned}
\lambda(p_A) &\equiv (1-\frac{1}{n})(1-2p_A) \geq (1-\frac{1}{n})(1-2\beta), \quad \forall p_A \in [\alpha, \beta] \\
\mu(p_B) &\equiv (1-\frac{1}{n})(2p_B-1) \geq (1-\frac{1}{n})(1-2\beta), \quad \forall p_A \in [1-\beta, 1-\alpha]
\end{aligned}$$

Hence, the sufficient condition of Lemma 5 becomes

$$\begin{aligned}
\xi(\beta-\alpha) &< (1-\frac{1}{n})(1-2\beta) \\
\zeta(\beta-\alpha) &< (1-\frac{1}{n})(1-2\beta)
\end{aligned}$$

from which it is clear that we must have $\beta < \frac{1}{2}$.

We maximize the first partial derivatives of the outcome functions to obtain[25]

$$\begin{aligned}
\xi = \xi_A = \frac{\partial y_A^*}{\partial p_A}(\beta, 1-\beta) &= 1-\frac{1}{2n}-\frac{1-2\beta}{2(n+1-2\beta)}, \\
\zeta = \zeta_B = \frac{\partial y_B^*}{\partial p_B}(\beta, 1-\beta) &= 1-\frac{1}{2n}-\frac{1-2\beta}{2(n+1-2\beta)}.
\end{aligned}$$

Therefore, our sufficient condition is equivalent to

$$\left(1-\frac{1}{2n}-\frac{1-2\beta}{2(n+1-2\beta)}\right)(\beta-\alpha) < (1-\frac{1}{n})(1-2\beta).$$

We can easily check that ξ (or ζ) is strictly greater than zero for all $\beta \geq 0$. If we define

$$\psi(x) \equiv 1-\frac{1}{2n}-\frac{1-2x}{2(n+1-2x)},$$

then,

[25]We can set up a standard constrained maximization problem and our calculation indicates that we have corner solutions at $(\beta, 1-\beta)$.

$$\psi'(x) = \frac{n}{(n+1-2x)^2} > 0 \quad \text{and} \quad \psi(0) = \frac{2n^2-1}{2n(n+1)} > 0.$$

We then consider a fixed sequence β^n that increases to $\frac{1}{2}$; from the sufficient condition, we know α^n must be bounded below, for each n, by

$$\beta^n - \frac{(1-\frac{1}{n})(1-2\beta^n)}{1-\frac{1}{2n}-\frac{1-2\beta^n}{2(n+1-2\beta^n)}} \;\rightarrow\; \frac{1}{2}$$

Here, the lower bound is strictly less than β^n for all n, hence the sufficient condition can be satisfied by letting α^n close enough to β^n. We also see that the lower bound converges to $\frac{1}{2}$. That is, α^n converges to the median for any given sequence β^n increasing to the median and thus we can say that the equilibrium support converges to the median along the sequence (α^n, β^n) on which our sufficient condition is satisfied.

The final issue to be resolved is to ascertain that the solution established by Lemma 5 is indeed a density. It is in general not an easy task to show that the solution to our integral equation exists as a density. We may proceed as in Meirowitz and Ramsay (2009) to construct a density solution in a simple example. However, our integral equation is somewhat more complicated than theirs, which prevents us from applying their method directly to our example. The problem of whether a density solution exists can be formulated as finding a solution function that satisfies the indifference conditions subject to the constraint that the solution must be integrated up to one. The problem can alternatively be formulated as one in which the constraint is given by our sufficient conditions and we must find a solution that attains a maximum norm (which is 1 in our case).

2.4 SUMMARY

We have a relatively well established literature about the spatial theories of elections and legislatures, but for the most part, theories of elections and theories of legislatures have developed independently of one another (Austen-Smith and Banks 1988). Therefore, studying the electoral implications of legislative outcomes can be an important research topic, and the game-theoretic literature on the topic is still in its inception. Even in two-party plurality elections, there is good reason to doubt the assumption that the winner's platform will be implemented as the policy outcome. That assumption is at best an approximation to the complicated post-electoral political process of policy-making as the opposing party can employ various governmental and non-governmental institutions to keep the ruling party in check.

This chapter thus extends the spatial model of Hotelling (1929) and Downs (1957) in a simple way to investigate the electoral stage and the subsequent policy-bargaining process at the same time. We model the bargaining process with a single policy outcome function that maps electoral platforms and vote results into a final policy outcome. Even if we don't consider an explicit noncooperative bargaining game to represent the post-electoral process, we require the outcome function to satisfy a certain set of assumptions that capture the idea that the losing party's bargaining power varies with his share of votes and enables him to get a policy compromise from the winner. Since the winner-takes-all scenario no longer holds in our case, parties' electoral incentives to converge to the center are substantially diminished and, when the parties retain relatively large bargaining power as losers with a given vote share, the equilibrium condition implies they must take extreme electoral positions, foreshadowing the subsequent policy concession to be made in favor of potential losers. On the other hand, if the amount of policy concession is not allowed to be sufficiently large under a political system, the parties will have an incentive to mix over a range of platforms. A boundedness condition on the length of equilibrium supports is sufficient to rationalize the mixed plays of political parties, and necessarily entails separation between the equilibrium supports.

The policy outcome function that reflects the preferences of the parties with both majority and minority supports changes the electoral results in a way that is contrary to the median voter theorem which is the single most important theoretical result in modern political science and at the same time is false by most accounts (Ansolabehere 2006). It would be interesting to study the various ways in which votes are translated into policies, which amounts to an alternative specification of the policy outcome function. The resulting models may have richer implications for mass elections involving campaign advertisement, political lobbying, information transmission through the media, etc.

3.0 COMPULSORY VERSUS VOLUNTARY VOTING: AN EXPERIMENTAL STUDY

Should voters be compelled to vote or should voting be voluntary? This question has been hotly debated for some time and has yielded many compelling arguments for both positions (see Birch (2009) for a history and review). Proponents of voluntary voting argue that the right to vote implies a right not to vote, that compulsion is at odds with democracy and may lead to inferior outcomes due to the inclusion of unwilling participants. Proponents of compulsory voting argue that many activities are compelled in democracies, (e.g., the paying of taxes, the completion of censuses) and that the larger turnout associated with compulsory voting conveys a greater legitimacy upon electoral outcomes.

The question as to whether voting should be compulsory or voluntary is of real world importance as both voting institutions coexist in nature. For instance, voting may be voluntary (abstention allowed) or compulsory in small committees or in jury deliberations. In U.S. federal court for example, juror abstention in a criminal trial is not allowed and the court can poll each juror about their vote after the verdict has been rendered (Rule 31, U.S. Federal Rules of Criminal Procedure). By contrast, juror abstention is allowed in certain U.S. state courts, e.g., for civil court cases where unanimity is not required. There are also differences in voting requirements for larger-scale, political elections. For instance, 29 countries, representing one-quarter of all democracies including Argentina, Australia and Belgium, currently compel their citizens to vote (more accurately, to show up to vote) in political elections (Birch 2009). Voluntary voting in political elections, as in the U.S., is the more commonly observed voting mechanism.

One approach to evaluating voting mechanisms is to focus on their ability to aggregate private information that is dispersed among the electorate. A standard assumption is that voters have *common values*, i.e., jury members wish to convict the guilty and acquit the innocent, or voters wish to elect the most suitable candidate or party given the true state of the world. In such an environment, the theoretical, rational-choice voting literature suggests that if voting is compulsory, rational voters may have incentives to vote *strategically*, i.e., sometimes voting *against* their private information (Austen-Smith and Banks 1996; Feddersen and Pesendorfer 1996, 1997, 1998; Myerson 1998). On the other hand, Krishna and Morgan (2011, henceforth K-M) have recently shown that under a voluntary voting mechanism, sincere voting, (i.e., always voting in accordance with one's private signal), can be optimal when voters face private costs of voting and can freely choose whether to vote or to abstain. While voting is sincere under the voluntary mechanism, participation decisions are strategic and will depend on costs to voting (if there are such costs).[1]

Under the assumption of common values, theory suggests that voters will adapt their behavior to the voting institution in place so that information aggregation is achieved and social welfare is maximized under either compulsory or voluntary voting mechanisms. In particular, if voting is costless, Feddersen and Pesendorfer (1997, 1999a) show that for large electorates, information aggregation is perfect under either voting mechanism. If voting involves privately observed voting costs, K-M show, under certain conditions on the distribution of voting costs[2] that information aggregation obtains for large electorates under the voluntary voting institution. Moreover, for certain group sizes, they show that voluntary voting is better at information aggregation than is compulsory voting, however these differences may be rather small and they disappear as the electorate gets large.

In essence, the debate over the merits of compulsory versus voluntary voting is one of quantity versus quality of information contained in the vote tally. Under compulsory voting, one obtains a high quantity of votes but if there is strategic voting, the quality may be worse than under voluntary voting, where sincere voting is more likely, therefore making the information of higher quality. If voting is costly, participation and therefore the quantity of

[1] Börgers (2004) compares compulsory versus voluntary voting under majority rule in a costly voting model with *private* values; as noted earlier, we study a common values framework. Börgers argues that voters ignore a negative externality generated by their own decision to vote: by voting they decrease the likelihood that other voters are pivotal. Consequently there is over-participation when voting is voluntary; making voting *compulsory* only serves to reduce welfare even further.

[2] Specifically, the lower bound for private voting costs is 0.

information can depend on the distribution of voting costs so it is necessary to also consider the case where voting is costly. Thus, the performance of each institution can depend on how the costs of voting are distributed in the electorate. However, as long as there exists individuals with zero costs of voting, K-M show that the welfare differences across voting mechanisms vanish for a large enough electorate size.

The goal of this study is to experimentally explore whether the institution of voluntary voting (the possibility of abstention) with or without voting costs does indeed suffice to induce sincere voting behavior in laboratory voting games relative to the case of compulsory voting, where insincere (strategic) voting is a possibility. We further explore the information aggregation consequences of these voting mechanisms with the aim of understanding how and why both compulsory and voluntary voting mechanisms can coexist in nature.

A laboratory experiment has several important advantages over field research for addressing these questions. First, we can carefully control the information signals that subjects receive prior to making their participation or voting decisions. Thus we can accurately determine if voters are voting sincerely, i.e., according to their signals, or if they are voting insincerely, i.e., against their signals. Second, we can carefully control and directly observe voting costs which is more difficult to do in the field. Third, in the laboratory, we can implement the theoretical requirement that subjects have identical preferences (common values) by inducing them to hold such preferences via the payoff function that determines their monetary earnings.[3] Finally, we note that all of our undergraduate subjects are voting-age adults (18 years of age or older); by contrast with many other laboratory studies, our "student subjects" may be regarded as "professional subjects" in that under U.S. law they are eligible to serve on juries or to vote in elections.

The experimental environment we study involves an abstract group decision-making task. All group members have identical preferences (the common value assumption) but each group member gets a noisy private signal regarding the unknown, binary state of the world (e.g., guilt or innocence). This is the environment of the Condorcet Jury Theorem (Condorcet

[3] Outside of the controlled conditions of the laboratory, preferences might differ greatly across voters; for example, jury members might have differing "thresholds of doubt," so that each requires a varying amount of evidence before s/he could vote to convict. Such a scenario can be modeled as each voter incurring a different magnitude of utility loss from an incorrect decision (as in Feddersen and Pesendorfer 1998, 1999b).

(1785)), which addresses the efficiency of various compulsory voting mechanisms in aggregating decentralized information. Condorcet assumed that voters would vote sincerely, i.e., according to their private information. However the validity of that assumption was first questioned by Austen-Smith and Banks (1996). In particular, they showed that, if agents are rational, the concern that an individual's vote may be pivotal can outweigh the information value of the signal he receives creating an incentive for the voter to vote strategically against his private signal. Here we fix the voting rule – majority rule – while using the Condorcet Jury environment to study the extent of sincere versus strategic voting when voter participation is either voluntary or compulsory.

The compulsory voting mechanism we study involves no voting cost.[4] Under our parameterization (discussed below) the unique compulsory voting equilibrium prediction is that one signal type always votes sincerely, according to their signal, but that a significant fraction (15.6%) of the other signal type votes against their signal. We refer to the latter behavior as strategic or insincere voting. Under the voluntary mechanism, we consider both the case where voting is costly and the case where there is no voting cost (costless). If voting is voluntary and costly, then the unique symmetric equilibrium prediction is that voters vote sincerely, conditional on choosing to vote (not abstaining). If voting is voluntary and costless, then there exist two symmetric, informative equilibria. In the Pareto superior equilibrium, conditional on choosing to vote, all voters vote sincerely (as in the voluntary but costly voting case). The other, less efficient equilibrium under the voluntary but costless voting mechanism is the same equilibrium that obtains under the compulsory mechanism; in this equilibrium there is full participation by all voters but 15.6% of one signal type vote insincerely against their signal, while the other signal type always votes sincerely. Thus under the voluntary but costless voting mechanism there is an interesting equilibrium selection issue that our experiment can address.

We further examine equilibrium predictions regarding participation rates under the two voluntary voting mechanisms. Under voluntary and costless voting, the participation rate of one signal type is predicted to be 54% while the participation rate for the other signal

[4] One could add a voting cost to the compulsory voting mechanism but since voting is compulsory, the addition of such a cost would not change the equilibrium prediction in any way.

type is predicted to be 100%; these type specific participation rates fall significantly to just 27% and 55%, respectively, under the voluntary but costly voting mechanism. Thus our design enables us to test the effects of voting mechanisms on the two important strategic dimensions (voting and participation) of the theory.

Finally, we also assess the efficiency of the groups in making collective decisions, in particular we ask to what extent groups reach the correct decision. For our parameterization of the model, the theory suggests that the voluntary but costless voting mechanism is the most efficient (accurate) followed by the compulsory mechanism and then by the voluntary but costly mechanism.

We report the following experimental findings. First, consistent with theoretical predictions, there is significantly more strategic voting under the compulsory voting mechanism than under either of the two voluntary voting mechanisms; under the latter two mechanisms, nearly all subjects are voting sincerely. Second, under the two voluntary voting mechanisms, there is over-participation in voting relative to theoretical predictions. However, the comparative static predictions of the theory find strong support in our data; in particular, consistent with the theory, participation rates are higher when voting is costless than when it is costly, and participation rates are always higher for one signal type than for the other. Finally, under both compulsory and voluntary voting mechanisms, groups achieve the correct outcome between 85 and 90 percent of the time and the ranking of the three mechanisms in terms of the accuracy of group decisions is in line with theoretical predictions. Still, the theoretical efficiency differences across the three mechanisms are small (under our parameterization of the model) and indeed, the observed differences in informational efficiency across the three voting mechanisms in our experimental data are not statistically significant from one another. Taken together, our findings suggest that individuals *do* adapt their behavior to the particular voting institution that is in place and thus provide an answer to the question posed at the beginning of the chapter as to why compulsory and voluntary voting mechanisms coexist in nature.

3.1 RELATED LITERATURE

Palfrey (2009) provides an up–to–date survey of experimental studies of voting behavior. Guarnaschelli, McKelvey and Palfrey (2000) is the earliest experimental study reporting evidence of strategic voting in the context of the same Condorcet jury model. Under the unanimity rule, a large percentage (between 30% and 50%) of subjects were observed voting against their signals, which is largely consistent with the equilibrium predictions of Feddersen and Pesendorfer (1998) for the model parameterization studied. Guarnaschelli, McKelvey and Palfrey (2000) also study behavior under a majority voting rule as we do in this chapter, but under their parameterization of the model, under majority rule, voters should always vote sincerely. By contrast, in the compulsory voting majority rule set-up that we study, the equilibrium prediction calls for some insincere voting.

Goeree and Yariv (2011) also report on an experiment using the Condorcet jury model where subjects are compelled to vote but where various voting rules are considered, preferences are varied so that jurors do not always have a common interest and most significantly, subjects are able to freely communicate with one another prior to voting. They report that absent communication, there is evidence that subjects vote strategically in accordance with equilibrium predictions under various voting rules, but that these institutional differences are diminished and efficiency is increased when subjects can communicate (deliberate) prior to voting. As with our study, the work of Goeree and Yariv provides further evidence that voters adapt their behavior to institutions, in this case, through the use of communication.

Importantly, neither Guarnaschelli, McKelvey and Palfrey (2000) nor Goeree and Yariv (2011) allow for abstention– they only study a compulsory and costless voting mechanism. If instead we allow voters to make participation decisions which can either be costless or costly prior to making their voting decisions as in K-M (2011), we can change the incentive structure of strategic voting decisions in such a way that sincere voting in the Condorcet Jury model no longer contradicts rationality.

A second, related experimental voting literature studies the team participation game model of voter turnout due to Palfrey and Rosenthal (1983, 1985); see, e.g., Schram and Sonnemans (1996), Cason and Mui (2005), Großer and Schram (2006), Levine and Palfrey

(2007) and Duffy and Tavits (2008). In this voluntary and costly voting game, two teams of players compete to win an election; for instance under majority rule, the team with the most votes wins. Experimental studies of this environment have typically involved no private information and have supposed that voters faced homogeneous costs to voting (abstention is free). Levine and Palfrey (2007) have designed experiments with heterogeneous voting costs to test several of the comparative statics predictions of the Palfrey and Rosenthal (1985) model. By contrast, the Condorcet jury environment that we study does not involve team competition, but does have private information (regarding the true state of the world) and we adopt Levine and Palfrey's (2007) design of having heterogeneous voting costs in our voluntary but costly voting treatment. Further, we are making the important comparison between the voluntary voting mechanism of the team participation game set-up and the compulsory voting mechanism that is more typically used in the Condorcet jury model. Thus, this chapter provides an important bridge between these two approaches.

Finally, we note that Battaglini et al. (2010) have recently reported on an experimental test of the "swing voter's curse" theory proposed by Feddersen and Pesendorfer (1996). They study the effects of asymmetric information on voter participation under a voluntary and costless voting mechanism; the swing voters are either informed or uninformed, and some fraction of the uninformed voters participate in voting to counterbalance votes by "partisans' while the remaining fraction of swing voters abstain so as to delegate their decisions to the informed.[5] We study a common interest situation with symmetric information, where abstention under the voluntary voting mechanism arises due to asymmetry in the precision of signals (and in part due to voting cost under the voluntary and costly voting mechanism), which has a direct impact on strategic voting behavior.

[5]The presence of partisans (whose preferences don't depend on the state) introduces a conflict of interest. By contrast, we study a common values setup where there is no conflict of interest after the state is realized.

3.2 MODEL

The experiments are based on the standard Condorcet Jury setup. We consider three different voting mechanisms: 1) compulsory and costless voting (C); 2) voluntary and costless voting (VN); 3) voluntary and costly voting (VC). In all three cases a group consisting of an odd number N of individuals faces a choice between two alternatives, labeled R (Red) and B (Blue). The group's choice is made in an election decided by simple majority rule. There are two equally likely states of nature, ρ and β. Alternative R is the better choice in state ρ while alternative B is the better choice in state β. Specifically, in state ρ each group member earns a payoff of $M(> 0)$ if R is the alternative chosen by the group and 0 if B is the chosen alternative. In state β the payoffs from R and B are reversed. Formally, we have

$$\begin{aligned} U(R|\rho) &= U(B|\beta) = M, \\ U(R|\beta) &= U(B|\rho) = 0. \end{aligned}$$

Prior to the voting decision, each individual receives a private signal regarding the true state of nature. The signal can take one of two values, r or b. The probability of receiving a particular signal depends on the true state of nature. Specifically, each subject receives a conditionally independent signal where

$$\Pr[r|\rho] = x_\rho \quad \text{and} \quad \Pr[b|\beta] = x_\beta.$$

We suppose that both x_ρ and x_β are greater than $\frac{1}{2}$ but less than 1 so that the signals are informative but noisy. Thus, the signal r is associated with state ρ while the signal b is associated with state β (we may say r is the correct signal in state ρ while b is the correct signal in state β). We shall assume that $x_\rho > x_\beta$, i.e., that the correct signal is more accurate in state ρ than in state β. This assumption is required for there to be some insincere voting under the compulsory voting mechanism and it yields sufficiently large differences in equilibrium predictions across the three voting mechanisms, facilitating our ability to identify such differences in the (possibly noisy) experimental data.

The posterior probabilities of the states after signals have been received are:

$$q(\rho|r) = \frac{x_\rho}{x_\rho + (1 - x_\beta)} \quad \text{and} \quad q(\beta|b) = \frac{x_\beta}{x_\beta + (1 - x_\rho)}.$$

Since $x_\rho > x_\beta$, we have $q(\rho|r) < q(\beta|b)$. Thus, b is a stronger signal in favor of state β than r is in favor of state ρ. The latter is a critical inference that individuals must make if they are to make rational voting decisions.

Having specified the preferences and information structure of the model, we discuss in the next three subsections, the strategies, equilibrium conditions and equilibrium predictions for each of the three voting mechanisms that we explore in our experiment. We restrict attention to symmetric equilibria in weakly undominated strategies, as these are the most relevant equilibrium predictions given the information that was available to subjects in our experiment.[6] In particular, we require that in equilibrium (*i*) all voters of the same signal type play the same strategies and (*ii*) no voter uses a weakly dominated strategy. In what follows we only discuss the equilibrium predictions and the conditions under which they are valid; a derivation of theses solutions is presented in the Appendix.

3.2.1 Compulsory Voting

When voting is compulsory, the strategy of a voter is a specification of two probabilities $\{v_r, v_b\}$ where v_r is the probability of voting for alternative R given an r signal and v_b is the probability of voting for alternative B given a b signal (that is, v_s is the probability of voting according to one's signal s, or voting *sincerely*). Under the compulsory voting mechanism, there exists a unique equilibrium in weakly undominated strategies. In this equilibrium for a large set of parameter values (including those of our experimental design) voters with signal b (i.e., signal type-b) always vote for B (i.e., $v_b^* = 1$) while those with signal r (i.e., signal type-r) mix between the two alternatives (i.e., $v_r^* \in (0, 1)$).

[6] There always exists an uninformative equilibrium in which everyone ignores their signal and votes for a fixed alternative. However, this kind of equilibrium involves the play of weakly dominated strategies, and for this reason we exclude consideration of such equilibria from our analysis.

Such mixing requires that the voter obtaining signal r be indifferent between voting for R or B conditioning on a tie vote (given play of equilibrium strategies by the other players), which gives the following equilibrium condition

$$U(R|r) - U(B|r) \equiv M\{q(\rho|r)\Pr[Piv|\rho] - q(\beta|r)\Pr[Piv|\beta]\} = 0,$$

where $U(A|s)$ is the payoff that a voter gets when alternative $A \in \{R, B\}$ is chosen and her signal (type) is $s \in \{r, b\}$; and $\Pr[Piv|\omega]$ is the probability that a vote is pivotal at state $\omega \in \{\rho, \beta\}$. Since voting is compulsory and N is chosen to be an odd number, a vote is pivotal only when exactly half of the other $N-1$ voters have voted for R and the other half have voted for B. Since the pivot probabilities depend on v_r, the above indifference condition determines v_r^*. Moreover, given this value for v_r^* and the fact that type-b voters strictly prefer to vote sincerely in equilibrium, we must have

$$U(B|b) - U(R|b) \equiv M\{q(\beta|b)\Pr[Piv|\beta] - q(\rho|b)\Pr[Piv|\rho]\} > 0.$$

The intuition for why type-b voters vote sincerely and type-r voters mix is as follows. If everyone votes her signal, the event where there is a tie vote among the other $N-1$ voters implies that there are an equal number of r and b signals. Since signals are less accurate in state β (i.e. $x_\rho > x_\beta$), an equal number of r and b signals is more likely to occur in state β than in state ρ. Conditioning on pivotality, the likelihood of state β is large enough that it swamps the information about states contained in the private signal, and the best response to a strategy profile with sincere voting is to vote for B irrespective of the signal. If, on the other hand, some type-r voters vote against their signals while all type-b voters vote sincerely, an equal number of votes for R and B implies a larger number of r signals than b signals: in particular, the information contained in the pivotal event is not strong enough to make the private signal irrelevant. In fact, the mixing probability is chosen in such a way that a private signal of r leads to the posterior likelihood of the two states being equal (conditioning on pivotality), thereby preserving the incentive to mix on obtaining an r signal. Clearly, a b signal leads to an inference of state β being more likely than state ρ in the event of a tie, and so the best response for a type-b voter is therefore to always vote for B (i.e., to always vote sincerely).

3.2.2 Voluntary and Costless Voting

When voting is voluntary, the action space includes three choices: a vote for R, a vote for B, or abstention, which we denote by ϕ. Thus, a voter's (mixed) strategy is a mapping from the signal type space $\{r, b\}$ to the set of all probability distributions over $\{R, B, \phi\}$. This set-up is exactly the same as that in K-M except that we have a fixed number, N, of voters (as this is easier to explain to subjects) while in K-M the number of voters is randomly drawn from a Poisson distribution.[7] In the K-M setting, all equilibria entail sincere voting: conditional on voting, type-b voters vote B and type-r voters vote R (K-M Theorem 1). This result does not automatically generalize to a set-up with fixed N; for arbitrary values of N there may be other kinds of equilibrium. Indeed, for any N, the unique symmetric equilibrium of the compulsory voting model, where there is full participation (no abstention) and type-b voters always vote sincerely while type-r voters mix with probability $v_r^* \in (0, 1)$, will also be an equilibrium under the voluntary and costless voting mechanism. Once we make voluntary voting costly, the latter insincere voting equilibrium disappears under the voluntary voting mechanism and, as discussed in the next section, we will have a unique symmetric sincere voting equilibrium.[8] To be consistent with K-M, we focus our attention in this section on the sincere voting equilibrium.

Given the restriction to sincere voting, the strategy of a voter simplifies to two participation rates $\{p_r, p_b\}$, one for each signal type. In this case full participation (i.e., $p_r = p_b = 1$) cannot be an equilibrium for the same reason that sincere voting is not an equilibrium under the compulsory voting mechanism. In fact, following Lemma 1 in K-M, we can show that under voluntary and costless voting, $p_b > p_r$ in any equilibrium with sincere voting[9]. In our discussion of the unique symmetric equilibrium under compulsory voting, we observed that, in order to preserve the incentive for informative voting, the event where there is a tied vote among the other $N - 1$ players (i.e., equal number of votes for R and B) must indicate a signal profile where there are more r signals than b signals. Under sincere voting, this is

[7] K-M show that any difference between these two approaches disappears when the group size, N, is sufficiently large.

[8] A proof of the existence of two symmetric informative equilibria under the voluntary and costless voting mechanism is available on request.

[9] The statement and proof of Lemma 1 in K-M can be shown to apply to the fixed N environment that we study with only minor modifications.

achieved only if type-b voters vote with a higher probability than type-r voters. Therefore, while the compulsory voting mechanism addresses the pivotality concern by having type-r voters sometimes vote against their signal, under the voluntary voting mechanism the same concern is addressed by having type-r voters abstain from voting with a higher probability.

In the case with costless voting, in the equilibrium that involves sincere voting, we should have $p_b^* = 1$ and $p_r^* \in (0,1)$, i.e., type-b voters always participate and vote for B while type-r voters mix between abstaining and voting for R. The participation rate for type-r voters is determined by making the type-r voter indifferent between voting for R and abstaining, specifically by setting

$$U(R|r) - U(\phi|r) \equiv M\{q(\rho|r)\Pr[Piv_R|\rho] - q(\beta|r)\Pr[Piv_R|\beta]\} = 0,$$

where $\Pr[Piv_R|\rho]$ denotes, for example, the probability that a vote for R is pivotal in state ρ and this pivot probability is a function of the participation rate p_r of type-r.[10] Under our parameter specification, the above indifference condition identifies a unique value of p_r^*. Moreover, given p_r^*, since the type-b voter strictly prefers to vote for B rather than abstain, we must have that

$$U(B|b) - U(\phi|b) \equiv M\{q(\beta|b)\Pr[Piv_B|\beta] - q(\rho|b)\Pr[Piv_B|\rho]\} > 0.$$

Additionally, sincere voting by type-r voters requires that given equilibrium participation rates we must have

$$\begin{aligned} & U(R|r) - U(B|r) \geq 0 \\ \Leftrightarrow \quad & U(R|r) - U(\phi|r) \geq U(B|r) - U(\phi|r) \\ \Leftrightarrow \quad & q(\rho|r)\Pr[Piv_R|\rho] - q(\beta|r)\Pr[Piv_R|\beta] \geq q(\beta|r)\Pr[Piv_B|\beta] - q(\rho|r)\Pr[Piv_B|\rho], \end{aligned}$$

[10] Since we allow abstention under the voluntary voting mechanisms, a vote can either make or break a tie. If we denote by T, T_{-1}, and T_{+1} the events that the number of votes for R is the same as, one less than, and one more than the number of votes for B, respectively, then for each $\omega \in \{\rho, \beta\}$,

$$\Pr[Piv_R|\omega] = \Pr[T|\omega] + \Pr[T_{-1}|\omega] \qquad \text{and} \qquad \Pr[Piv_B|\omega] = \Pr[T|\omega] + \Pr[T_{+1}|\omega],$$

where the pivot probabilities depend on the participation rate p_r.

and similarly, sincere voting by type-b voters requires that

$$
\begin{aligned}
& U(B|b) - U(R|b) \geq 0 \\
\Leftrightarrow \quad & q(\beta|b)\Pr[Piv_B|\beta] - q(\rho|b)\Pr[Piv_B|\rho] \geq q(\rho|b)\Pr[Piv_R|\rho] - q(\beta|b)\Pr[Piv_R|\beta].
\end{aligned}
$$

These two conditions require that voting *sincerely* be incentive compatible. We check (in the Appendix) that both conditions hold given our solutions for p_r^* and p_b^*.

3.2.3 Voluntary and Costly Voting

Under the voluntary but costly voting mechanism, each voter faces a cost c to voting, so that his overall utility is $U(A|\omega) - c$ if he votes and $U(A|\omega)$ if he abstains, where $A \in \{R, B\}$ is the winning alternative and $\omega \in \{\rho, \beta\}$ is the state. The voting cost is a random variable drawn independently across individuals from a set $\mathcal{C} = [0, \bar{c}]$, $\bar{c} > 0$, according to an atomless distribution, F. We further assume that voting costs are drawn independently of signals. After observing their voting cost and signal, voters then decide whether to vote or to abstain. Thus, in this setting a player type consists of both a signal and a cost of voting. Generally, the (mixed) strategy of a voter is a mapping from the type space $\{r, b\} \times \mathcal{C}$ to the space of probability distributions over $\{R, B, \phi\}$. In order to replicate the results in K-M, we again restrict attention to equilibria with sincere voting, however, under certain conditions (that are satisfied by the parameters chosen in our experimental design), it can be shown that under costly, voluntary voting the insincere voting equilibrium of the compulsory voting mechanism can no longer be an equilibrium, and indeed, the unique symmetric equilibrium will involve sincere voting by all player types.[11] Therefore, the choice faced by each voter under the voluntary and costly voting mechanism is whether to vote sincerely or to abstain. If voting is costly, then there exists a positive threshold cost, c_s^*, for each signal $s \in \{r, b\}$ such that an agent whose signal is s votes only if her realized cost is below the threshold c_s^*. The equilibrium participation rate for each signal, $p_s^* = F(c_s^*)$, $s \in \{r, b\}$, are determined by

[11] We have verified that this is the case; a proof is available upon request.

the cost threshold at which a voter with signal s is indifferent between voting sincerely and abstaining, specifically

$$\begin{aligned} U(R|r) - U(\phi|r) &\equiv M\{q(\rho|r)\Pr[Piv_R|\rho] - q(\beta|r)\Pr[Piv_R|\beta]\} = F^{-1}(p_r), \\ U(B|b) - U(\phi|b) &\equiv M\{q(\beta|b)\Pr[Piv_B|\beta] - q(\rho|b)\Pr[Piv_B|\rho]\} = F^{-1}(p_b). \end{aligned}$$

These two equations require that the expected benefit from sincere voting must equal the realized costs for the cutoff cost types, c_s^*, given that all other voters adopt the same cutoff costs for participating in voting and that all those choosing to participate, also choose to vote sincerely. Here, the pivot probabilities are again functions of both types' participation rates (p_r, p_b).

The two equations above identity the equilibrium participation rates $\{p_r^*, p_b^*\}$ simultaneously (and uniquely for our parameter values and uniform cost distribution over $\mathcal{C}$). By the same logic used for the voluntary and costless voting mechanism, we must have $p_b^* > p_r^*$ to preserve the incentives for informative voting. In other words, we must have $c_b^* > c_r^*$. Furthermore, given the equilibrium participation rates, each participating voter must prefer to vote sincerely. Therefore, just as in the case with costless voluntary voting, we must have

$$\begin{aligned} & U(R|r) - c \geq U(B|r) - c \\ \Leftrightarrow \;& q(\rho|r)\Pr[Piv_R|\rho] - q(\beta|r)\Pr[Piv_R|\beta] \geq q(\beta|r)\Pr[Piv_B|\beta] - q(\rho|r)\Pr[Piv_B|\rho] \\ & U(B|b) - c \geq U(R|b) - c \\ \Leftrightarrow \;& q(\beta|b)\Pr[Piv_B|\beta] - q(\rho|b)\Pr[Piv_B|\rho] \geq q(\rho|b)\Pr[Piv_R|\rho] - q(\beta|b)\Pr[Piv_R|\beta]. \end{aligned}$$

We can again show (in the Appendix) that both of these inequalities hold given our solutions for p_r^* and p_b^*.

3.3 EXPERIMENTAL DESIGN

We consider two treatment variables: 1) the voting mechanism, compulsory or voluntary, and within the voluntary treatment alone we further consider 2) whether voting is costless or costly. We adopt a between subjects design so that in each session subjects only make decisions under one set of treatment conditions. Across the three treatments of our experiment all parameters of the voting model and all other dimensions of the experimental design, e.g., the group size, the number of repetitions, the history of play, the payoff function, etc., are held constant.

The experiment was presented to subjects as an abstract group decision–making task using neutral language that avoided any direct reference to voting, elections, jury deliberation, etc. so as not to trigger other (non-theoretical) motivations for voting (e.g., civic duty, the sanction of peers, etc.).

Each session consists of a group of 18 inexperienced subjects and 20 rounds. At the start of each round, the 18 subjects were randomly assigned to one of two groups of $N = 9$ subjects. One group is assigned to the red jar (state ρ) and the other group is assigned to the blue jar (state β) with equal probability, thus fixing the true state of nature for each group. No subject knows which group they have been assigned to and group assignments are determined randomly at the start of each new round so as to avoid possible repeated game dynamics. Subjects *do* know that it is equally likely that their group is assigned to the red jar or to the blue jar at the start of each round.

The red jar contained fraction x_ρ red balls (signal r) and fraction $1-x_\rho$ blue balls (signal b) while the blue jar contained fraction x_β blue balls and fraction $1-x_\beta$ red balls. We fixed the probabilities, x_ρ and x_β, at 0.9 and 0.6, respectively, across all sessions of our experiment, and these signal precisions were made public knowledge in the written instructions, which were also read aloud at the start of each session.[12] We chose values for x_ρ and x_β that provided stark differences in equilibrium predictions across our three treatments with the aim of facilitating identification of any treatment differences in the (possibly noisy) experimental data.

[12]A sample of the written instructions used in the experiment is provided in the Appendix.

The sequence of play in a round was as follows. First, each subject blindly and simultaneously draws a ball (with replacement) from her group's (randomly assigned) jar. This is done virtually in our computerized experiment; subjects click on one of 10 balls on their decision screen and the color of their chosen ball is revealed.[13] While the subject observes the color of the ball she has drawn, she does not observe the color of any other subject's selections or the color of the jar from which she has drawn a ball. A group's common and publicly known objective is to correctly determine the jar, "red" or "blue", that has been assigned to their group.

In the two treatments without voting costs, after subjects have drawn a ball (signal) and observed its color, they next make a voting decision. In the compulsory voting treatment (C), they must make a "choice" (i.e., vote) between "red" or "blue", with the understanding that their group's decision, either red or blue, will correspond to that of the majority of the 9 group members' choices and that the group aim is to correctly assess the jar (red or blue) that was assigned to the group. In the voluntary but costless voting treatment (VN), the only difference from the compulsory treatment is that subjects must make a "choice" between "red", "blue" or "no choice" (abstention). The group's decision in this case, "red" or "blue," will correspond to that of the majority of the group members who made a choice between "red" or "blue" i.e., who participated in voting. In the voluntary treatments (but not in the compulsory treatment) there is the possibility of ties in the voting outcome, i.e., equal numbers of votes for red and blue (including also the possibility that no one chooses to vote). In the event of a tie, the group's decision is labeled "indeterminate", otherwise it is labeled "red" or "blue" according to the majority choice of those who participated in voting.

In the voluntary but costly voting treatment (VC), after each subject i has drawn a ball, each gets a private draw of their cost of voting for that round, c_i, that is revealed to them *before* they face a voting/participation decision. After observing both the color of the ball drawn and the cost of voting, each group member privately votes for either the red jar or the blue jar or chooses to abstain ("no choice") as in the case where voting is voluntary and

[13] For each round and for each subject, the assignment of colors to the 10 ball choices the subject faced was made randomly according to whether the jar the subject was drawing from was the red jar (in which case percentage x_ρ of the balls were red) or the blue jar (in which case percentage x_β balls were blue).

costless. The group's decision is again made by majority rule among all group members who do not abstain and the color chosen by the majority is the group's decision. A tie is again regarded as an "indeterminate" outcome.

Payoffs each round are determined as follows. If the group's decision via majority rule is correct, i.e., the group's decision is red (blue) and the jar assigned to that group was in fact red (blue), then each of $N = 9$ members of a group, even those who abstained in the two voluntary voting treatments, receive 100 points ($M = 100$). If the group's decision is incorrect, then each of the 9 members of the group receive 0 points. If the group's decision is "indeterminate" i.e., there is a tied vote for "red" or "blue", then each of the 9 members of the group receive 50 points. This payoff function is the same across all three treatments.

In the voluntary and costly voting (VC) treatment only, the cost of voting is implemented using an "NC-bonus" payment where "NC" stands for "no choice". Thus, in the VC treatment, subject i gets $100 + c_i$ points if she abstains and her group decision is correct while she gets c_i points if she abstains but the group's decision is incorrect and $50 + c_i$ points if she abstains and the group's decision is indeterminate. A decision by subject i to vote in a round of the VC treatment means that she loses the NC-bonus for that round, receiving a payoff of either 100, 0 or 50 depending on whether the group's decision is correct, incorrect or indeterminate, respectively. Subjects are informed that the NC-bonus for each round (c_i) is an i.i.d. uniform random draw from the set $\{0, 1, ..., 10\}$[14] for each subject i and applies only to that round.[15]

Following 20 rounds of play, the session was over. Subjects' point totals from all 20 rounds of play were converted into dollars at the fixed and known rate of 1 point = \$0.01 and these dollar earnings were then paid to the subjects in cash. In addition, subjects were given a \$5 cash show–up payment. Thus, it was possible for each member of each group (red or blue) to earn up to \$1 in each of the 20 rounds of play and in the VC treatment only, subjects could earn or forego an additional NC bonus of up to \$0.10 per round. Average earnings for this 1-hour experiment (including the \$5 show-up payment) were \$22.51.

[14]The upper bound for c_i could have been set higher, up to 100, but we chose a low value to encourage voter participation.

[15]Our implementation of voting cost follows that of Levine and Palfrey (2007) and has the nature of an opportunity cost.

Session Numbers	No. of subjects per session	No. of rounds per session	Voting Mechanism	Voting Costly?
C1-4	18	20	compulsory	no
VN1-4	18	20	voluntary	no
VC1-4	18	20	voluntary	yes

Table 1: The Experimental Design

Table 1 summarizes our experimental design, which involved four sessions of each of our three treatments. As we have 18 subjects per session, we have collected data from a total of $4 \times 3 \times 18 = 216$ subjects. Subjects were recruited from the undergraduate population of the University of Pittsburgh and the experiment was conducted in the Pittsburgh Experimental Economics Laboratory. No subject participated in more than one session of this experiment.

3.4 RESEARCH HYPOTHESES

We first consider the equilibrium predictions for the compulsory voting mechanism (C). For our parameter values, there exists a unique symmetric equilibrium in weakly undominated strategies in which subjects with signal b always vote for Blue (vote sincerely) while those with signal r vote against their signal (vote for Blue) with strictly positive probability (i.e., there is some insincere or strategic voting). More precisely under our parameterization, voters receiving the red (r) signal are predicted to play a mixed strategy where they vote against their r-signal (they vote insincerely for Blue) 15.6% of the time and they vote sincerely according to their r-signal (they vote for Red), 84.4% of the time. Equivalently, we predict that an average of 15.6% of signal type-r subjects will vote against their signal each round.

The equilibrium predictions for the voluntary mechanism without voting costs (VN) are that participation rates should depend on the signal received, red (r) or blue (b). We

Voluntary Voting	p_r^*	p_b^*	c_r^*	c_b^*
VN (costless)	0.5387	1.000	n/a	n/a
VC (costly)	0.2700	0.5497	2.70	5.50

Table 2: Sincere Voting Equilibrium Predictions for the Voluntary Voting Treatments

denote these equilibrium participation rates by p_r^* and p_b^*. A further equilibrium prediction is that conditional on choosing to participate, all voters should vote sincerely, according to their signal. The same type of equilibrium behavior is predicted under the voluntary but costly voting mechanism (VC), but in the latter case the equilibrium predictions can be alternatively stated in terms of cut-off levels for the cost of voting for the two signal types, denoted by c_r^*, c_b^*. Table 2 summarizes the predicted values of these variables in the sincere voting equilibrium of our two voluntary voting treatments.

We can show (a proof is available on request) that the sincere voting equilibrium described above is unique in the case of the voluntary and *costly* (VC) voting mechanism. However, under the voluntary and *costless* voting mechanism (VN), the insincere voting equilibrium that is the unique symmetric equilibrium under the compulsory (C) voting mechanism is also an equilibrium under the VN mechanism. This insincere voting equilibrium would require full participation by all voters under the VN mechanism, i.e., $p_r^* = p_b^* = 1.0$, (even though voters are free to abstain under the voluntary mechanism) and would further predict that 15.6% of type-r voters vote insincerely. However, it is easily shown that under the VN mechanism, this insincere voting equilibrium is Pareto-dominated by the sincere voting equilibrium involving less than 100 percent participation by signal type-r players as described in Table 2. These two equilibria are the only symmetric equilibria in weakly undominated strategies under the voluntary and costless voting mechanism. Thus, for the VN treatment alone there is an open and interesting question of *equilibrium selection* that our experiment can address; for the other two treatments we have unique symmetric equilibrium predictions.

A final issue concerns the efficiency of group decisions. Let us denote by $W(\rho)$ and

Voting Mechanism	$W(\rho)$	$W(\beta)$	$\frac{1}{2}W(\rho)+\frac{1}{2}W(\beta)$
C	0.9582	0.8485	0.9033
VN	0.9513	0.9106	0.9309
VC	0.8572	0.8501	0.8536

Table 3: Efficiency Comparisons

$W(\beta)$ the probabilities of making a correct decision by the group assigned to the red and the blue jar, respectively (recall that the red jar corresponds to state ρ while the blue jar, to state β). The theory predicts that $W(\rho)$ is greater than $W(\beta)$ under all three mechanisms (compulsory, voluntary and costless, and voluntary and costly) although the difference is negligible under the voluntary and costly mechanism. $W(\rho)$ and $W(\beta)$ are measures of the informational efficiency of group decisions, hence the group assigned to the red jar (which entails more precise correct signals) is predicted to attain higher informational efficiency. Table 3 shows the predicted values for $W(\rho)$ and $W(\beta)$.

Furthermore, as shown in Table 3, if we take the equal weighted average of $W(\rho)$ and $W(\beta)$ as the overall efficiency measure for each voting mechanism (recall the equal prior over the two states), then the theory also gives us a ranking of the mechanisms in terms of the efficiency of group decisions; namely, the voluntary and costless mechanism is the best, the compulsory mechanism is second best and the voluntary and costly mechanism is the worst (if we consider the aggregate cost spent by those who participate in voting under the latter mechanism, then it is even worse).

Based on the equilibrium predictions, we now formally state our research hypotheses:

H1. The fraction of those who vote *against* their signals (insincerely) is significantly greater than zero (15.6% of subjects with signal r) when voting is compulsory while it is zero when voting is voluntary.

H2. Under the voluntary voting mechanisms, subjects with *b* signals (type-b) participate at a higher rate than subjects with r signals (type-r); $p_r^* < p_b^*$. Furthermore, the participation rate is higher under the voluntary and costless mechanism than under the voluntary and costly mechanism for each signal type.

H3. Under all three voting mechanisms, the probability of making a correct decision is strictly higher for the group assigned to the red jar than for the group assigned to the blue jar; $W(\rho) > W(\beta)$. Moreover, the three voting mechanisms can be ranked according to their ex-ante aggregate efficiency ($\frac{1}{2}W(\rho) + \frac{1}{2}W(\beta)$); $VN > C > VC$.

3.5 EXPERIMENTAL RESULTS

We report results from twelve experimental sessions (four sessions for each of the compulsory, voluntary and costless, and voluntary and costly treatments) with 18 subjects playing 20 rounds in each session. Overall, we find strong support for all three of our main research hypotheses. The next three sections discuss the support for each hypothesis in detail.

3.5.1 Sincerity/Insincerity of Voting Decisions

Finding 1. *Consistent with theoretical predictions, there is strong evidence of insincere voting by red-signal types under the compulsory voting mechanism. By contrast, nearly all voters of both signal types vote sincerely under both voluntary mechanisms (no cost and costly).*

Figure 6 shows the observed frequency of *in*sincere voting under the three treatments. In the compulsory treatment (C), the proportion of type-r voters (those who drew a red ball) who voted insincerely was greater than 10% (recall that red (r) signal types are the only type who are predicted to vote insincerely with positive probability). By contrast the frequency of insincere voting by type-b voters (those who drew a blue ball) under the compulsory (C) treatment as well as both signal types under the two other treatments (VN and VC) was always less than 5%. Thus Figure 6 suggests that there is a large difference in the sincerity of voting decisions between type-r voters in treatment C and all voters in all three treatments.

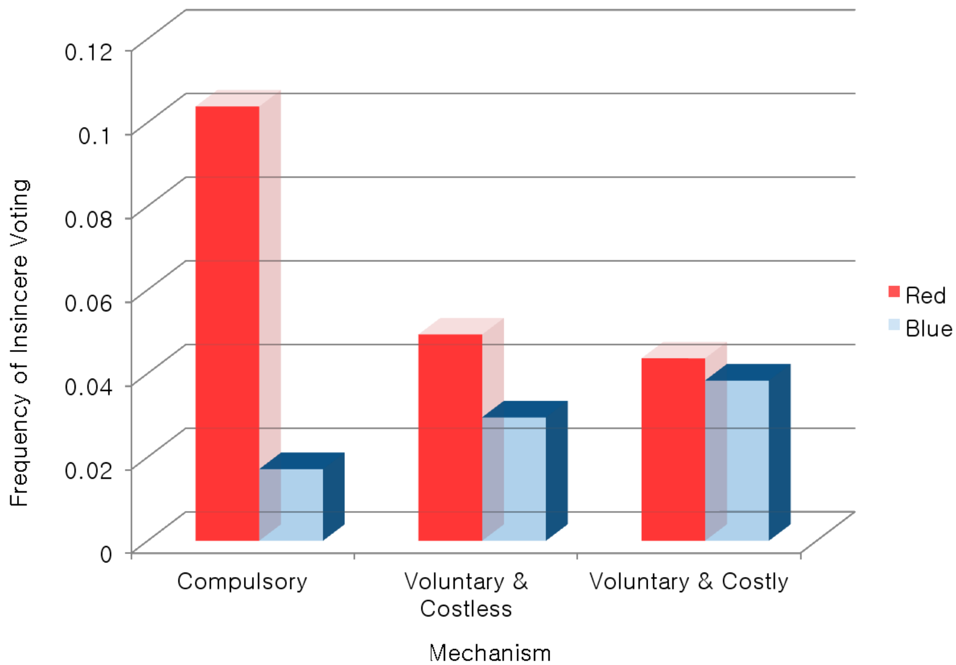


Figure 6: Overall Frequency of Insincere Voting. Pooled Data from All Rounds of All Sessions of Each of the Three Treatments

Table 4 shows disaggregated, session-level averages of the frequency of *sincere* voting in all 12 sessions by signal type. This table reveals that Nash equilibrium performs rather well in predicting the qualitative (if not the quantitative) results for our voting games of compulsory or voluntary participation. With a couple of exceptions, the frequency of sincere voting is close to 100% under the voluntary voting mechanisms. The decomposition of sincere voting behavior by signal types indicates that, consistent with theoretical predictions, subjects who participated in voting voted sincerely regardless of the signals drawn under both voluntary voting mechanisms. On the other hand, we do find evidence for insincere (or strategic) voting under the compulsory mechanism among subjects drawing a red ball; slightly more than 10% of type-r voters voted insincerely which is close to, though slightly lower than the equilibrium prediction of 15.6%. It is also interesting to note that the behavior of subjects under the compulsory mechanism was remarkably consistent across sessions in terms of the average frequencies of sincere voting between signal types. The data seem to confirm the prediction that the voting mechanism in place (compulsory vs. voluntary) affects the incentives for subjects to vote sincerely or insincerely.

Are the differences in voting behavior between mechanisms statistically significant? To answer this question, we conducted a Wilcoxon-Mann-Whitney (WMW) test using the session-level observations reported in Table 4. The null hypothesis is that the frequencies of sincere voting (4 session-level observations per treatment) from the two mechanisms under consideration come from the same distribution. Table 5 reports the rank sums as well as p-values for each pairwise treatment comparison.

First, consider the sincerity of voting by type-r subjects. The comparison between compulsory (C) and voluntary but costly (VC) treatments reveals a clear difference in the sincerity of voting.[16] Given the high frequency of sincere voting under the VC mechanism, we can say that subjects indeed behaved strategically under the C mechanism. We obtain the same result in the comparison between type-r subjects in the compulsory (C) treatment and type-r subjects in the combined voluntary treatments (V=VN+VC) as a group. Furthermore, we

[16] We report p-values from one-sided tests of the null of no difference in all pairwise comparisons (in Table 5) between treatment C and the 'V' treatments, VN, VC or V=VN+VC that involves voting behavior by type-r subjects. That is because we have a clear directional hypothesis that type-r subjects should have voted "less sincerely" in the C treatment versus the 'V' treatments. The same reasoning applies to all subsequent comparisons (in Table 6, Table 8, Table 9, and Table 11) for which one-sided tests and p-values are reported.

Treatment/ Session[a]	Red (v_r)[b]	Blue (v_b)
C1	0.8956 (249)[c]	0.9910 (111)
C2	0.8730 (244)	0.9914 (116)
C3	0.8970 (233)	0.9921 (127)
C4	0.9190 (247)	0.9558 (113)
C Overall	0.8962 (973)	0.9829 (467)
C Predicted	0.8440	1.0000
VN1	0.8871 (186)	0.9914 (116)
VN2	1.0000 (154)	0.9848 (132)
VN3	0.9752 (161)	0.9048 (105)
VN4	0.9524 (168)	0.9917 (121)
VN Overall	0.9507 (669)	0.9705 (474)
VN Predicted	1.0000	1.0000
VC1	0.9794 (97)	0.9600 (75)
VC2	0.9706 (102)	1.0000 (86)
VC3	0.9444 (108)	0.9574 (94)
VC4	0.9277 (83)	0.9286 (84)
VC Overall	0.9564 (390)	0.9617 (339)
VC Predicted	1.0000	1.0000

[a] C=Compulsory, VN=Voluntary & Costless, VC=Voluntary & Costly.

[b] v_s is the frequency of sincere voting by type-s.

[c] Number of observations is in parentheses.

Table 4: Observed Frequency of Sincere Voting by Signal Type

Red Signal	C vs. VN[a]	C vs. VC	VN vs. VC	C vs. V
Sum of ranks	$W_C = 13$	$W_C = 10$	$W_{VN} = 19$	$W_C = 13$
	$W_{VN} = 23$	$W_{VC} = 26$	$W_{VC} = 17$	$W_V = 65$
p-value	0.0745†	0.0105†	0.7728	0.0136†
Blue Signal	C vs. VN	C vs. VC	VN vs. VC	C vs. V
Sum of ranks	$W_C = 19.5$	$W_C = 20$	$W_{VN} = 19$	$W_C = 29.5$
	$W_{VN} = 16.5$	$W_{VC} = 16$	$W_{VC} = 17$	$W_V = 48.5$
p-value	0.6631	0.5637	0.7728	0.5515

[a] C=Compulsory, VN=Voluntary & Costless, VC=Voluntary & Costly.

† One-sided p-values.

Table 5: Wilcoxon-Mann-Whitney Test of Differences in the Sincerity of Voting Between Treatments by Signal Type

cannot reject the null hypothesis of the same frequency of (sincere) voting between both voluntary mechanisms for type-r subjects (VN versus VC).

We note that the evidence for a significant difference in sincere voting behavior by type-r subjects between the C and VN mechanisms is weak (p=.0745), suggesting that subjects under the voluntary but costless (VN) treatment have voted "less sincerely" as compared with the voluntary and costly (VC) treatment. According to the theory, the existence (or absence) of voting cost affects only participation decisions, and not voting decisions; hence, if subjects were playing in accordance with the sincere voting equilibrium they should have voted sincerely regardless of cost under both voluntary mechanisms. The weakly significant difference between the VN and C treatments has two possible explanations. First, recall that under the VN treatment, the symmetric *in*sincere voting equilibrium of the C treatment co-exists with the symmetric *sincere* voting equilibrium; the coexistence of these two symmetric equilibria may have resulted in a coordination problem for subjects. As a second explanation, we believe that subjects in the VN treatment may not think too seriously about their participation/abstention decisions because in the VN treatment participation is "free," and given that participation rates by type-r subjects are higher than the predicted rates (as we will show below), these type-r subjects might have been better off voting insincerely to raise the probability of reaching a correct decision in the event that their group is assigned to the blue jar. We will come back to the latter explanation later in the chapter when we attempt to rationalize the departures we observe from sincere voting using behavioral models.

As for the voting behavior of type-b subjects, we cannot reject the null hypothesis of no difference in the sincerity of voting for any of the four pairwise comparisons (C vs. VN, C vs. VC, VN vs. VC and C vs. V, where V again stands for the combined data from the costly and costless voluntary mechanisms). This leads to the conclusion that, consistent with all equilibrium predictions, the high sincerity of type-b subjects' voting decisions is constant across all treatments of our experiment. The test statistics also suggest that type-b subjects voted slightly "more sincerely" under the C treatment though that difference is not statistically significant at conventional levels.

As a further test of the equilibrium predictions, we also ask whether red and blue types behaved the same (in terms of sincere voting) under a given voting mechanism/treatment.

	C[a]	VN	VC	V (VN & VC)
Rank sum	positive - 0	positive - 4	positive - 3	positive - 14
	negative - 10	negative - 6	negative - 7	negative - 22
p-value	0.0340[†]	0.7150	0.4652	0.5754

[a] C=Compulsory, VN=Voluntary & Costless, VC=Voluntary & Costly.

[†] One-sided p-value.

Table 6: Wilcoxon Signed Ranks Test of Difference in the Sincerity of Voting Between Signal Types

Table 6 shows the results of a Wilcoxon signed-ranks test for matched pairs with the null hypothesis being that the frequencies of sincere voting are the same between signal types under a fixed voting mechanism. For the purpose of this test, we paired both types' observed frequencies of sincere voting in each session and generated 4 signed differences for each of the 3 treatments and 8 signed differences for the voluntary treatment as a group. Clearly, the only mechanism under which both types' behavior exhibits a significant difference was the compulsory voting mechanism. This finding again confirms our hypothesis regarding equilibrium voting behavior, which postulates that only the red signal type under the C treatment will vote insincerely. Under the two voluntary mechanisms individually or as a group, we never find any difference in the sincerity of voting decisions between signal types, which is consistent with equilibrium predictions.

3.5.2 Participation Decisions

Finding 2. *Under voluntary voting, the difference in participation rates by signal types are in accordance with the symmetric, sincere voting equilibrium predictions. However, subjects in both voluntary voting treatments and of both signal types over-participate relative to these equilibrium predictions.*

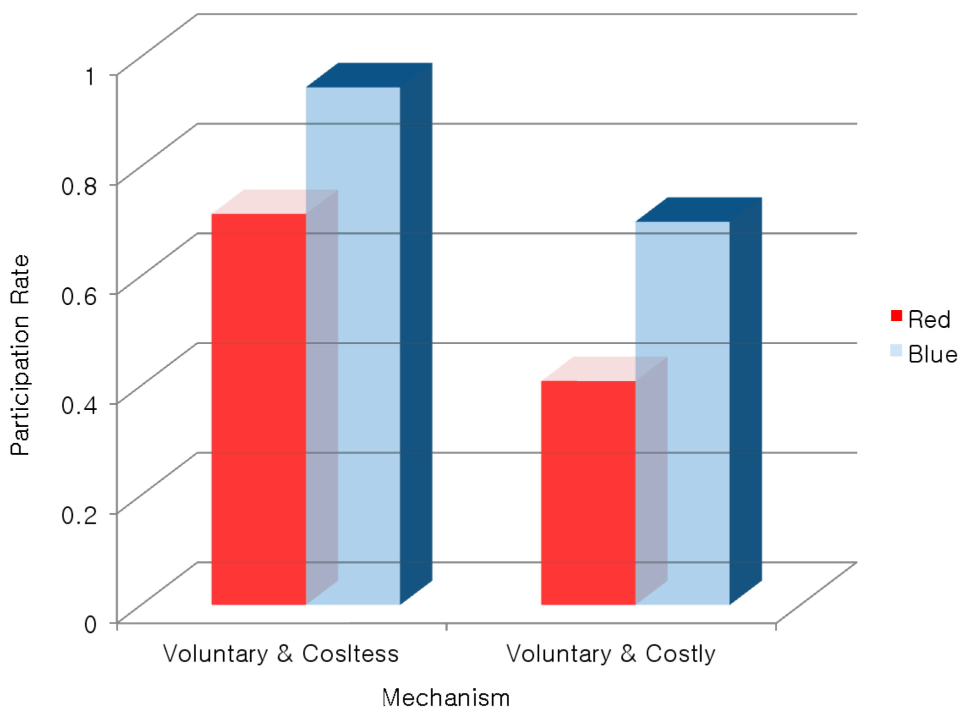


Figure 7: Overall Participation Rates, Pooled Data from All Rounds of All Sessions of Each of the Three Treatments

Treatment/ Session[a]	Red (p_r)[b]	Blue (p_b)
VN1	0.7815 (238)[c]	0.9508 (122)
VN2	0.6906 (223)	0.9635 (137)
VN3	0.6545 (246)	0.9211 (114)
VN4	0.7273 (231)	0.9380 (129)
VN Overall	0.7132 (938)	0.9442 (502)
VN Predicted	0.5397	1.0000
VC1	0.4128 (235)	0.6000 (125)
VC2	0.4250 (240)	0.7167 (120)
VC3	0.4519 (239)	0.7769 (121)
VC4	0.3444 (241)	0.7059 (119)
VC Overall	0.4084 (955)	0.6990 (485)
VC Predicted	0.2700	0.5497

[a] VN=Voluntary & Costless, VC=Voluntary & Costly.

[b] p_s is the participation rate of type-s.

[c] Number of observations is in parentheses.

Table 7: Observed Participation Rates by Signal Type in the Voluntary Treatments

	VN[a]	VC	V (VN & VC)
rank sum	positive - 0	positive - 0	positive - 0
	negative - 10	negative - 10	negative - 36
p-value	0.0340[b]	0.0340	0.0059

[a] VN=Voluntary & Costless, VC=Voluntary & Costly.

[b] All p-values are one-sided.

Table 8: Wilcoxon Signed Ranks Test of Differences in Participation Rates Between Signal Types

Support for Finding 2 comes from Figure 7 and Table 7, where we observe that, consistent with theoretical predictions the participation rate of type-b voters was substantially greater than that of type-r voters throughout all sessions of the voluntary treatments. Since blue balls were rare relative to red balls, type-b voters have more of an incentive to participate in voting decisions (and of course to vote sincerely). As reported in Table 8, Wilcoxon signed-rank tests (on the session level data shown in Table 7) lead us to reject the null hypothesis of no difference in participation rates at the lowest possible significance level given four observations for each of the two voluntary treatments (or eight observations for the voluntary treatments as a group). This finding is a natural consequence of the fact that the observed difference between participation rates ($\hat{p}_b - \hat{p}_r$) in each session was always positive without exception in both voluntary voting treatments.

We further observe that each signal type participated at a higher rate under the VN treatment than under the VC treatment, which is also consistent with the theoretical prediction that the introduction of voting costs will reduce participation incentives for all types. As Table 9 reveals, a Wilcoxon-Mann-Whitney test applied to the session–level data reported in Table 7 allows us to reject the null hypothesis of no difference in participation rates by signal type between the two voluntary treatments ($p < .05$) since all four participation observations in the VN treatment rank higher than those in the VC treatment for both signal types.

	VN vs. VC[a]
Red	$W_{VN} = 26$ $W_{VC} = 10$ p-value = 0.0105[b]
Blue	$W_{VN} = 26$ $W_{VC} = 10$ p-value = 0.0105

[a] VN=Voluntary & Costless, VC=Voluntary & Costly.

[b] Both p-values are one-sided.

Table 9: Wilcoxon-Mann-Whitney Test of Differences in Participation Rates Between Treatments

Therefore, the participation behavior observed in our data strongly supports the qualitative predictions of the Nash equilibrium.

However, as stated in Finding 2, we also observe that subjects tended to participate in voting at a higher rate than the equilibrium prediction, with the lone exception of type-b subjects under the VN treatment (the predicted participation rate is one for this type). This tendency for over-participation was also observed by Levine and Palfrey (2007), (when the electorate was sufficiently large, as in our case) with the rate of over-participation increasing with the group size. They explain such systematic tendency to over-participation using the notion of Quantal Response Equilibrium (QRE), an equilibrium concept that formalizes noisy best response. We will explore whether QRE estimates of both voting behavior and participation rates can help to explain the data from our experiment later in section 7. In particular, the participation by type-r voters was high under the VN mechanism to the point of changing their incentives with regard to voting decisions. Given such high participation rates, type-r players should have voted insincerely with a positive (but small) probability. We speculate that, despite our neutral framing of the problem (i.e, our avoidance of all references to voting), subjects may nevertheless have had a negative feeling about selecting the "No Choice" option and thus avoided choosing it when they should have. Offering a proper incentive to select No Choice, as in our costly voting treatment with its NC bonus, provides a better test of the importance of the voluntary voting mechanism in our opinion and it appears to have worked to reduce any stigma that might have been attached to choosing "No choice".

We further note that while the participation rate of type-r subjects in the VN treatment is high, it is still well below 100 percent (the average participation rate across all sessions of this treatment is 71.3 percent). Recall that the unique symmetric *in*sincere voting equilibrium under compulsory voting mechanism is an alternative symmetric equilibrium possibility under the VN mechanism. However, that insincere voting equilibrium would require 100 percent participation and more insincere voting by type-r subjects than we observe in the data from our VN treatment. Thus on the question of equilibrium selection, the data from our VN treatment seem closer to and more in accordance with the symmetric sincere voting equilibrium which, as noted earlier, payoff dominates the insincere voting equilibrium. We address this equilibrium selection issue in further detail later in section 3.6.1.

3.5.3 Accuracy of Group Decisions

Finding 3. *Consistent with theoretical predictions, the probability of making a correct decision is strictly higher for the group assigned to the red jar than for the group assigned to the blue jar, i.e.,* $W(\rho) > W(\beta)$*. Further the ranking of the voting mechanisms with respect to the ex-ante aggregate efficiency measure* $(\frac{1}{2}W(\rho) + \frac{1}{2}W(\beta))$*, is as predicted, with* $VN > C > VC$*. However, these efficiency differences are not statistically significant from one another in our experimental data.*

Recall that our measure of decision-making efficiency is the probability $W(\omega)$ of making the correct decision in each state $\omega \in \{\rho, \beta\}$. For notational convenience, let us denote the group that is assigned to the red jar as the ρ group and the group that is assigned to the blue jar as the β group. Consistent with theoretical predictions, Table 10 reveals that the ρ group made correct decisions significantly more frequently than did the β group across all treatments. We further observe that the frequencies of correct decisions by the ρ group tended to be higher than equilibrium predictions, while the frequency of correct decisions by the β group were generally lower than equilibrium predictions, with some exceptions in

Treatment/ Session[a]	$W(\rho)$[b]	$W(\beta)$	Aggregate[c]
C1	0.9500	0.6000	0.7750
C2	1.0000	0.8500	0.9250
C3	1.0000	0.7500	0.8750
C4	1.0000	0.7000	0.8500
C Overall	0.9875	0.7250	0.8563
C Predicted	0.9582	0.8485	0.9033
VN1	1.0000	0.8000	0.9000
VN2	1.0000	0.9250	0.9625
VN3	1.0000	0.6000	0.8000
VN4	0.9750	0.8750	0.9250
VN Overall	0.9938	0.8000	0.8969
VN Predicted	0.9513	0.9106	0.9309
VC1	0.8750	0.7250	0.8000
VC2	0.9000	0.7750	0.8375
VC3	0.9250	0.9000	0.9125
VC4	0.8250	0.8250	0.8250
VC Overall	0.8813	0.8063	0.8438
VC Predicted	0.8572	0.8501	0.8536

[a] C=Compulsory, VN=Voluntary & Costless, VC=Voluntary & Costly.

[b] $W(\omega)$ is the probability that group ω makes the correct decision.

[c] Aggregate efficiency $\equiv \frac{1}{2}W(\rho) + \frac{1}{2}W(\beta)$.

Table 10: Observed Efficiency by Group

	C vs. VN[a]	C vs. VC	VN vs. VC
rank sum	$W_C = 14.5$	$W_C = 20$	$W_{VN} = 21.5$
	$W_{VN} = 21.5$	$W_{VC} = 16$	$W_{VC} = 14.5$
p-value	0.1547[b]	0.2819	0.1547

[a] C=Compulsory, VN=Voluntary & Costless, VC=Voluntary & Costly.

[b] All p-values are one-sided.

Table 11: Wilcoxon-Mann-Whitney Test of Differences in Efficiency Between Treatments

several sessions.

These success frequencies are, of course, closely tied to participation decisions and voting behavior. The observed discrepancy follows from the higher than predicted rates of voter participation under the voluntary mechanisms and from the lower than predicted rates of insincere voting under the compulsory mechanism by type-r voters who drove up the success rates when they were in the ρ group, but drove up the error rate when they were in the β group, which explains the low success rates of the β group. This same finding continues to obtain in voluntary voting treatments where a much smaller fraction of type-r voters voted insincerely.

Finally, recall our prediction concerning the ranking of voting mechanisms in terms of ex-ante efficiency: groups were predicted to make correct decisions with the highest frequency under the voluntary and costless mechanism (VN), followed by the compulsory mechanism (C) and then by the voluntary and costly mechanism (VC). Our data produce this same ranking; the probability of correct decisions in the three regimes is, VN: 0.8969; C: 0.8563; and VC: 0.8438. These observed efficiency measures are lower than the predicted ones under all mechanisms/treatments. Table 11 shows the results of a test of whether the observed differences in efficiency are statistically significant between pairs of treatments. As the Table

11 reveals, we cannot reject the null hypothesis of no difference in any pairwise comparison ($p > .10$ for all three tests). This result might be due to our small number of observations (just four independent observations for each treatment) but it could also be due to the fact that the theoretically predicted differences are themselves very small for the group size of 9 that we have considered in our experiment. Since in the limit, information aggregation holds (i.e., the probability of making a correct group decision goes to one along all the informative equilibria as the size of the electorate goes to infinity) under all three mechanisms (see, e.g., Feddersen and Pesendorfer (1998), Krishna and Morgan (2011)), we would expect that the observed differences in efficiency would decrease as the size of the electorate was made even larger than in our experimental design.

3.5.4 Individual Behavior

Thus far we have only considered behavior at the aggregate group and signal type level. In this section we delve deeper and explore the behavior of individual subjects under the three voting mechanisms. Figure 8 provides pairwise comparisons of the cumulative distributions of the frequency of sincere voting by all subjects between different voting mechanisms for each signal type or between two different signal types for a given voting mechanism. Figure 9 provides similar pairwise comparisons of the cumulative distributions of voting participation rates for the voluntary treatments.

One implication of the theory is that the frequency of sincere voting by type-r players should be stochastically greater under the voluntary (VN or VC) mechanisms than under the compulsory (C) mechanism and that the same frequency for type-r players should be stochastically lower than that for type-b players under the compulsory (C) mechanism. This is the usual first-order stochastic dominance relationship, hence the cumulative distribution of a stochastically larger variable should lie everywhere below that of a stochastically smaller one. However, for all the other comparisons between mechanisms/types, the distributions are predicted to coincide. If we look at Figure 8, we can indeed find this relationship in our data; in particular, the main difference between the two distributions occurs in the neighborhood of the mixed equilibrium frequency, 0.844, of sincere voting by type-r voters in the C treatment

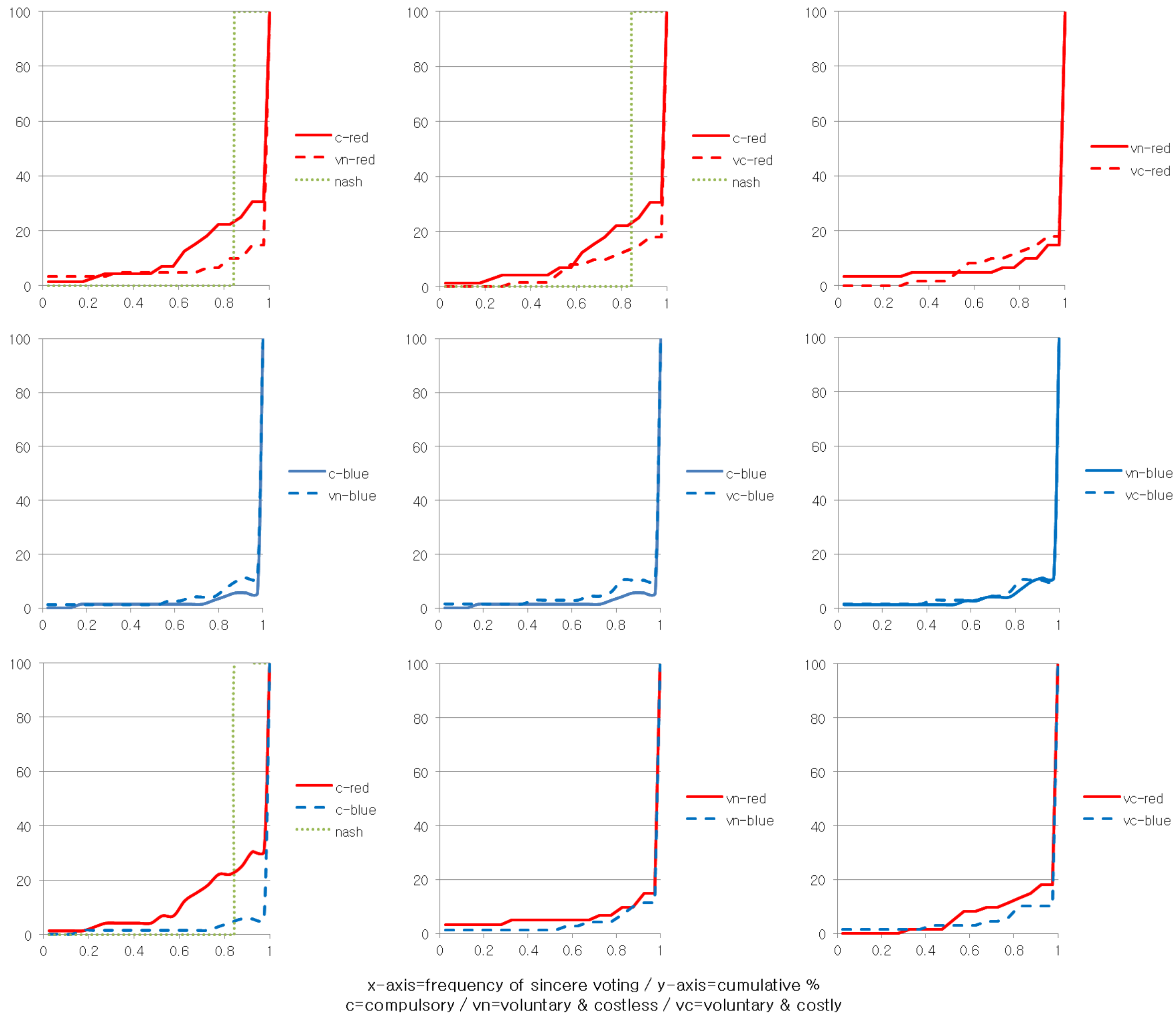


Figure 8: Distribution of the Frequency of Sincere Voting by Mechanism / Signal Type

(which is indicated by the dashed line labeled "Nash" in graphs depicting the cumulative frequency of sincere voting by type-r subjects in the C treatments). Consider the first two graphs in the first row of Figure 8 which compare the behavior of type-r subjects in the C vs. VN and C vs. VC treatments, respectively. Consider also the comparison between the two signal types (r and b) under the C mechanism alone (the first graph in the third row of Figure 8). In these three cases alone, there is a predicted stochastic-order relationship. In particular, the cumulative distribution of the frequency of sincere voting by type-r players in the C treatment should lie to the left of (or above) the cumulative distribution of the comparison group in these three graphs; more precisely the cumulative distribution of the frequency of sincere voting by type-r players in the C treatment should shift from 0% to 100% at the mixed equilibrium probability of .844. In all other pairwise comparisons the frequency of sincere voting is predicted to be 100% and so the cumulative distributions should coincide in those cases. Figure 8 reveals that, consistent with theoretical predictions, the cumulative frequency distribution of sincere voting by type-r players under the C voting mechanism is quite different from the cumulative frequency distribution of sincere voting by the comparison group. In particular, there is always a larger mass of type-r subjects voting insincerely under the C voting mechanism. Alternatively put, at 100% sincere voting, there is a large gap between the two cumulative frequencies, equal to 25% in the C/type-r vs. C/type-b comparison, 15.8% in the C/type-r vs. VN/type-r comparison or 12.6% in the C/type-r vs. VC/type-r comparison while the difference is relatively small in all other cases (precisely, it ranges from 1 to 7.7%).[17]

The theory also predicts stochastic-order relationships between the distributions of participation rates. Namely, the distribution of participation rates for type-r players should lie above the distribution of participation rates for type-b players under both voluntary mechanisms, and the distribution of participation rates for the VC mechanism should lie above the distribution of participation rates for the VN mechanism for both signal types. Pairwise comparisons of the cumulative frequency distributions of participation rates (and Nash

[17] Nevertheless, a Kolmogorov-Smirnov test of differences between the cumulative frequency distributions of sincere voting fails to detect a significant difference between C/type-r and VN/type-r or C/type-r and VC/type-r (the p-values are 0.191 and 0.339, respectively). However, the difference in cumulative frequency distributions of sincere voting between C/type-r and C/type-b is significant at 1% level (p-value=0.011) according to the same test.

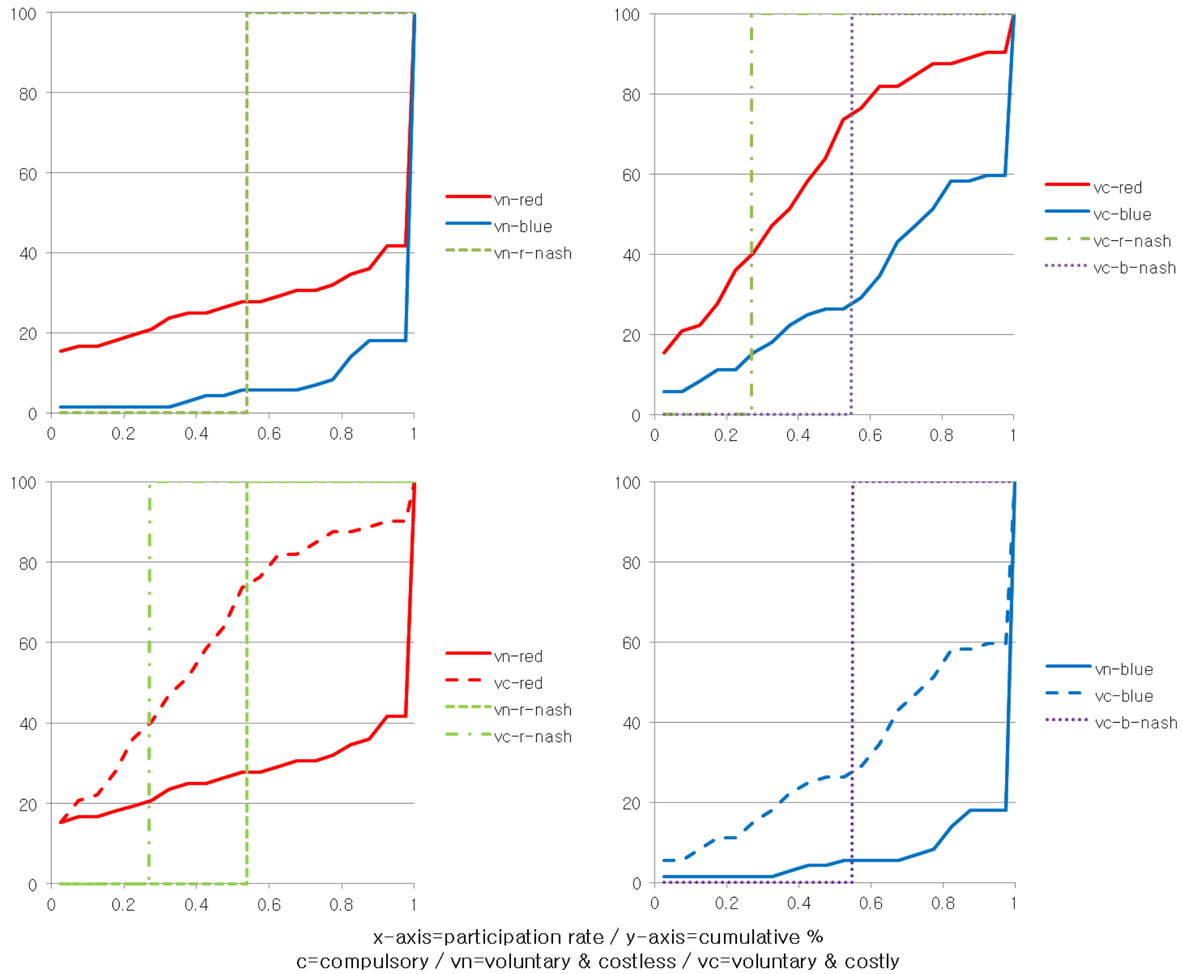


Figure 9: Distribution of Participation Rates by Mechanism / Signal Type

equilibrium predictions) are shown in Figure 9. As that figure makes clear, the observed differences in the distributions of participation decisions are all in the right direction providing strong support for the comparative statics hypotheses about participation rates even at the individual level of our experimental data.[18]

3.6 MODELS OF BOUNDED RATIONALITY

We have presented strong evidence in support of the comparative statics equilibrium predictions of the theory with respect to the impact of the various voting mechanisms on the sincerity of voting, participation decisions and the accuracy of group decisions. Nevertheless, we have also found some differences between the equilibrium point predictions and the experimental data, for example, over-participation relative to equilibrium predictions under the voluntary mechanisms. In this section we consider whether some models of boundedly rational behavior might help us to better account for these anomalous findings.

3.6.1 Equilibrium Plus Noise

Perhaps the simplest model of "noise" in the data is the so-called *equilibrium-plus-noise model*.[19] In this approach, the predicted choice probability $p(\eta)$ (sincere voting or participation choice) is a weighted average of the equilibrium prediction, p, and a purely random choice probability of $\frac{1}{2}$:

$$p(\eta) = \eta p + (1 - \eta)\frac{1}{2},$$

where $\eta \in [0, 1]$ and $p \in \{v_r, v_b, p_r, p_b\}$ with v_s and p_s, respectively, representing the equilibrium probability of sincere voting (given participation, in the voluntary treatments) and the probability of participation in voting by signal type $s \in \{r, b\}$. Here, η is a simple measure of the "closeness" of the data to equilibrium predictions; $\eta = 0$ corresponds to random choices

[18] Indeed, Kolmogorov-Smirnov tests indicate significant differences in the cumulative frequency distributions of participation rates in all four pairwise comparisons, - either at the 1% level (VN/type-r vs. VN/type-b) or at the 0.1% level (the other 3 comparisons).

[19] See, e.g., Blume et al. (2009).

whereas $\eta = 1$ corresponds to equilibrium play. We further impose the restriction that the weight η assigned to the choice probabilities is the same for both signal types and for both voting and participation decisions in any given treatment (however, we allow η to vary from treatment to treatment).

To construct a likelihood function, let ω_s denote the total number of signal type-s subjects; τ_s, the number of type-s subjects who participate in voting; and σ_s, the total number of type-s subjects who vote sincerely (among all type-s *subjects* in the compulsory treatment and among all type-s *participants* in the voluntary treatments). The likelihood function is then proportional to

$$\mathcal{L}(\eta) = v_r(\eta)^{\sigma_r}(1 - v_r(\eta))^{\omega_r - \sigma_r} v_b(\eta)^{\sigma_b}(1 - v_b(\eta))^{\omega_b - \sigma_b},$$

in case of the compulsory (C) treatment, and to

$$\begin{aligned}\mathcal{L}(\eta) = \quad & v_r(\eta)^{\sigma_r}(1 - v_r(\eta))^{\tau_r - \sigma_r} v_b(\eta)^{\sigma_b}(1 - v_b(\eta))^{\tau_b - \sigma_b} \\ & \times\, p_r(\eta)^{\tau_r}(1 - p_r(\eta))^{\omega_r - \tau_r} p_b(\eta)^{\tau_b}(1 - p_b(\eta))^{\omega_b - \tau_b},\end{aligned}$$

in case of the voluntary (VN or VC) treatments. Our restriction on η requires us to use pooled data from all sessions of a given treatment in maximizing the above likelihood functions.

Table 12 reports results from a maximum likelihood (ML) estimation of the *equilibrium-plus-noise model* using data from all 20 rounds or from the first or last 10 rounds of all sessions of a given treatment. The observed frequencies of sincere voting and participation (from the experimental data) are denoted by $\hat{v}_s$ and $\hat{p}_s$ and the corresponding estimates based on the equilibrium-plus-noise model are denoted by $v_s(\hat{\eta})$ and $p_s(\hat{\eta})$ and $\hat{\eta}$. The table also shows the results of likelihood ratio tests that compare the likelihood function for the unrestricted equilibrium-plus-noise model (with estimates $\hat{\eta}$) with those for a restricted version where $\eta = 0$ implying purely random choices. We use the same numbers of observations (ω_s, τ_s and σ_s) when evaluating the likelihood functions of both the restricted and unrestricted models. The last column of Table 12 in particular reports the likelihood ratio (LR) test statistics (LR Stat $\equiv -2\ ln\ l$, where l is the ratio of the restricted to the unrestricted likelihood functions) that can be evaluated under the null hypothesis (H_0) of no difference between the restricted and the unrestricted models. The LR test statistic follows a χ^2 distribution with

Treatment†	$\hat{v_r}$‡	$v_r(\hat{\eta})$	$\hat{v_b}$	$v_b(\hat{\eta})$	$\hat{p_r}$	$p_r(\hat{\eta})$	$\hat{p_b}$	$p_b(\hat{\eta})$	$\hat{\eta}$	LR Stat
C	0.896	0.837	0.983	0.989	n/a	n/a	n/a	n/a	0.979	1236.74
First 10 rounds	0.902	0.835	0.978	0.987	n/a	n/a	n/a	n/a	0.974	616.13
Last 10 rounds	0.890	0.838	0.987	0.991	n/a	n/a	n/a	n/a	0.983	620.99
Nash	0.844		1.00		n/a	n/a	n/a	n/a		
VN (Sincere)	0.951	0.956	0.970	0.956	0.713	0.536	0.944	0.956	0.912	1723.73
First 10 rounds	0.951	0.955	0.982	0.955	0.726	0.536	0.931	0.955	0.910	855.11
Last 10 rounds	0.951	0.956	0.959	0.956	0.700	0.536	0.957	0.956	0.913	868.64
Nash	1.00		1.00		0.540		1.00			
VN (Insincere)	0.951	0.753	0.970	0.867	0.713	0.867	0.944	0.867	0.734	1414.56
First 10 rounds	0.951	0.755	0.982	0.871	0.726	0.871	0.931	0.871	0.741	721.85
Last 10 rounds	0.951	0.750	0.959	0.864	0.700	0.864	0.957	0.864	0.727	692.97
Nash	0.844		1.00		1.00		1.00			
VC	0.956	0.944	0.962	0.944	0.408	0.296	0.699	0.544	0.888	764.96
First 10 rounds	0.946	0.928	0.951	0.928	0.440	0.303	0.723	0.542	0.856	364.91
Last 10 rounds	0.968	0.961	0.974	0.961	0.379	0.288	0.672	0.546	0.922	403.17
Nash	1.00		1.00		0.270		0.550			

† C=Compulsory, VN=Voluntary & Costless, VC=Voluntary & Costly.

‡ $\hat{v_s}$ is the observed frequency of sincere voting, and $\hat{p_s}$ is the observed participation rate, both by type-s; $(\cdot)(\hat{\eta})$ is the corresponding estimated frequency or rate.

Table 12: Equilibrium-Plus-Noise Model: Maximum Likelihood Estimates

degrees of freedom equal to the number of restrictions, in this case, 1. Finally, in the case of the VN treatment only, we assess the fit of the equilibrium-plus-noise model using two different symmetric equilibrium probability vectors: one corresponding to the sincere voting equilibrium vector (labeled 'Sincere') and the other corresponding to the insincere voting equilibrium (labeled 'Insincere').

We observe that our data are very close to the Nash equilibrium point predictions for all treatments, as indicated by the high estimated values for $\hat{\eta}$. We also observe that the data from the compulsory voting treatment are significantly closer to equilibrium predictions than are the data from the two voluntary voting treatments. This difference is largely due to the over-participation we observed in the voluntary treatments as reported in the previous section. Since we measure the closeness of both the voting and participation decisions to equilibrium predictions using a single estimate, $\hat{\eta}$, for each treatment (recall our restriction on η), a consequence is that we obtain lower values for $\hat{\eta}$ for the voluntary treatments.

We do find some improvement in the estimate of $\hat{\eta}$ for all voting mechanisms as we move from the first to the last 10 rounds (with the exception of $\hat{\eta}$ for the VN insincere equilibrium specification) meaning that subjects' behavior gets closer to the equilibrium predictions with experience.

Given the closeness of our data to the equilibrium predictions, it is perhaps not so surprising that we obtain the high likelihood ratio (LR) test statistics reported in Table 12. By construction, these statistics (and the corresponding p-values) measure the extent to which the equilibrium-plus-noise model outperforms a purely random choice model. Since all reported LR statistics are well above the critical value for the χ^2 statistic that corresponds to a p-value$= 0.001$ (which is 10.828 with d.f.=1), we can safely reject the null of random decision making in favor of the restricted model were subjects are close to playing the equilibrium predictions at the 0.1% level (or lower).

Regarding the issue of equilibrium selection under the VN mechanism, we can use our simple equilibrium-plus-noise model to assess which symmetric equilibrium provides a better characterization of the play of subjects in our VN treatment. As Table 12 reveals, when we use the symmetric *sincere* voting equilibrium probability vector as the benchmark, we obtain a much higher value for $\hat{\eta}$ (approximately .91) than we do if we use the symmetric *insincere* voting equilibrium probability vector as the benchmark (in which case the estimate of $\hat{\eta}$ is approximately .73). We thus conclude that, on the question of equilibrium selection, behavior in the VN sessions is better characterized by the symmetric sincere voting equilibrium than by the symmetric insincere voting equilibrium.

3.6.2 Quantal Response Equilibrium

A main drawback of the equilibrium-plus-noise model is that it does not rationally account for the possibility that subjects may be best responding to the noise they observe in the data. An equilibrium concept that formalizes this idea is the *quantal response equilibrium* or QRE, (McKelvey and Palfrey (1995) and Goeree, Holt and Palfrey (2005)) which we now apply to our experimental data. In particular, we consider the *logit quantal response equilibrium model* and assume that our subjects make decisions according to a stochastic, logistic choice rule.

In the quantal response equilibrium model, we calculate the choice probabilities as (quantal response) functions of the expected payoffs. Given the slope λ of the logistic quantal response function, the voting strategy of a subject can be written as:

$$v_r(\lambda) = \frac{1}{1 + \exp[-\lambda\{U(R|r) - U(B|r)\}]}, \tag{3.1}$$

$$v_b(\lambda) = \frac{1}{1 + \exp[-\lambda\{U(B|b) - U(R|b)\}]}. \tag{3.2}$$

where v_s is again defined as the probability of voting sincerely, given signal $s \in \{r, b\}$. Here, λ is understood to measure the "degree of rationality"; $\lambda = 0$ corresponds to random behavior whereas $\lambda = \infty$ corresponds to equilibrium behavior (perfect rationality). We can also specify participation strategies in a similar way. Under the voluntary and costless (VN) treatment, we have:

$$p_r(\lambda) = \frac{1}{1+\exp[-\lambda\{v_r(\lambda)(U(R|r)-U(\phi|r))+(1-v_r(\lambda))(U(B|r)-U(\phi|r))\}]}, \tag{3.3}$$

$$p_b(\lambda) = \frac{1}{1+\exp[-\lambda\{v_b(\lambda)(U(B|b)-U(\phi|b))+(1-v_b(\lambda))(U(R|b)-U(\phi|b))\}]}, \tag{3.4}$$

and under the voluntary and costly (VC) treatment we have,

$$p_r(\lambda) = \frac{1}{1+\exp[\lambda\{\frac{p_r(\lambda)}{10}-v_r(\lambda)(U(R|r)-U(\phi|r))-(1-v_r(\lambda))(U(B|r)-U(\phi|r))\}]}, \tag{3.5}$$

$$p_b(\lambda) = \frac{1}{1+\exp[\lambda\{\frac{p_b(\lambda)}{10}-v_b(\lambda)(U(B|b)-U(\phi|b))-(1-v_b(\lambda))(U(R|b)-U(\phi|b))\}]}, \tag{3.6}$$

where p_s is, as before, the rate of participation in voting, given signal $s \in \{r, b\}$. We treat the model parameter λ as a constant to be estimated. For the compulsory (C) treatment, we solve for $(v_r(\lambda), v_b(\lambda))$, the system of equations (1)-(2). For the voluntary treatments, we solve for $(v_r(\lambda), v_b(\lambda), p_r(\lambda), p_b(\lambda))$, the system of equations (1)-(4) for the VN mechanism and the system of equations (1)-(2) and (5)-(6) for the VC mechanism. We restrict λ to be the same for both signal types and for both voting and participation strategies in any given treatment (however, we allow λ to vary from treatment to treatment).

To construct the likelihood function, let ω_s denote the total number of type-s subjects; τ_s, the number of type-s subjects who participate in voting; and σ_s, the number of type-s subjects who vote sincerely (among all type-s *subjects* in the compulsory treatment and

Treatment[†]	$\hat{v_r}$[‡]	$v_r(\hat{\lambda})$	$\hat{v_b}$	$v_b(\hat{\lambda})$	$\hat{p_r}$	$p_r(\hat{\lambda})$	$\hat{p_b}$	$p_b(\hat{\lambda})$	$\hat{\lambda}$	LR Stat
C	0.896	0.797	0.983	0.994	n/a	n/a	n/a	n/a	42.33	1191.53
First 10 rounds	0.902	0.795	0.978	0.992	n/a	n/a	n/a	n/a	40.22	590.69
Last 10 rounds	0.890	0.800	0.987	0.996	n/a	n/a	n/a	n/a	45.16	601.35
Nash	0.844		1.00		n/a	n/a	n/a	n/a		
VN	0.951	0.944	0.970	0.997	0.713	0.531	0.944	0.877	48.59	1652.40
First 10 rounds	0.951	0.950	0.982	0.998	0.726	0.531	0.931	0.888	51.92	843.75
Last 10 rounds	0.951	0.939	0.959	0.996	0.700	0.531	0.957	0.868	45.69	809.67
Nash	1.00		1.00		0.540		1.00			
VC	0.956	0.909	0.962	0.984	0.408	0.361	0.699	0.554	20.75	798.82
First 10 rounds	0.946	0.888	0.951	0.976	0.440	0.371	0.723	0.550	18.37	387.61
Last 10 rounds	0.968	0.934	0.974	0.992	0.379	0.348	0.672	0.558	24.39	415.67
Nash	1.00		1.00		0.270		0.550			

[†] C=Compulsory, VN=Voluntary & Costless, VC=Voluntary & Costly.

[‡] $\hat{v_s}$ is the observed frequency of sincere voting, and $\hat{p_s}$ is the observed participation rate, both by type-s; $(\cdot)(\hat{\lambda})$ is the corresponding estimated frequency or rate.

Table 13: Quantal Response Equilibrium: Maximum Likelihood Estimates

among all type-s *participants* in the voluntary treatments). The likelihood function is then proportional to

$$\mathcal{L}(\lambda) = v_r(\lambda)^{\sigma_r}(1 - v_r(\lambda))^{\omega_r - \sigma_r} v_b(\lambda)^{\sigma_b}(1 - v_b(\lambda))^{\omega_b - \sigma_b}$$

in case of the compulsory (C) treatment, and to

$$\begin{aligned}\mathcal{L}(\lambda) = \quad & v_r(\lambda)^{\sigma_r}(1 - v_r(\lambda))^{\tau_r - \sigma_r} v_b(\lambda)^{\sigma_b}(1 - v_b(\lambda))^{\tau_b - \sigma_b} \\ & \times p_r(\lambda)^{\tau_r}(1 - p_r(\lambda))^{\omega_r - \tau_r} p_b(\lambda)^{\tau_b}(1 - p_b(\lambda))^{\omega_b - \tau_b}\end{aligned}$$

in case of the voluntary (VN or VC) treatments. Our restriction on λ requires us to use pooled data from all sessions of a given treatment in maximizing the above likelihood functions.

Table 13 reports the results from maximum likelihood (ML) estimation of the *quantal response equilibrium model*.[20] As in the previous subsection, $\hat{v}_s$ and $\hat{p}_s$ denote the observed

[20] Unlike Table 12 for the VN treatment we cannot use QRE estimates to compare between the two symmetric equilibrium possibilities that arise under the VN mechanism, as they involve different likelihood functions (one with participation choices and the other without participation choices) preventing us from making a fair comparison between the two types of equilibria. For this reason, we only report in Table 13 QRE estimates for the sincere voting equilibrium specification using the VN treatment data.

probabilities of sincere voting and participation while $v_s(\hat{\lambda})$ and $p_s(\hat{\lambda})$, denote the estimated probabilities. The table reports the estimates $v_s(\hat{\lambda})$, $p_s(\hat{\lambda})$ and $\hat{\lambda}$ (with the corresponding observed probabilities $\hat{v_s}$ and $\hat{p_s}$) from all rounds as well as from the first and the last 10 rounds of all sessions of each treatment. The table also shows the results of likelihood ratio (LR) tests that compare the unrestricted model with the restricted one, with the former being the quantal response equilibrium model and the restriction in the latter model being $\lambda = 0$ (purely random behavior). The details concerning the LR test statistics are exactly the same as in the previous subsection.

As Table 13 reveals, the estimated slope coefficients, $\hat{\lambda}$, of the quantal response function are quite high for all three treatments. In other words, subjects demonstrated a substantial degree of rationality in all three voting treatments. Similar evidence of rational voter behavior is also found in previous studies by Guarnaschelli, McKelvey and Palfrey (2000), Levine and Palfrey (2007) and Battaglini, Morton and Palfrey (2010); their estimated values for λ are also high. Notice further that the $\hat{\lambda}$ values are comparatively lower for the voluntary and costly (VC) mechanism, which intuitively makes sense as this mechanism entails the most complicated game that subjects in our experiment were asked to play.[21] This finding is also consistent with the findings of the previous section, i.e., the data from the VC treatment were found to be the furthest from the equilibrium predictions according to the estimates, $\hat{\eta}$. Finally, as in the equilibrium-plus-noise model, we again observe an improvement in $\hat{\lambda}$ as we move from estimates based on the first 10 rounds of data to estimates based on the last 10 rounds of data under both the C and VC mechanisms. While $\hat{\lambda}$ decreases with experience under the VN mechanism, from $\hat{\lambda} = 51.92$ to 45.69, both estimates still indicate a high degree of rationality; indeed, these estimates are higher than the $\hat{\lambda}$ estimates for the other two treatments.

Consider next the QRE predictions regarding the voting decisions and participation rates. Notice first that the QRE estimates for the frequency of sincere voting are lower for type-r players than for type-b players under both voluntary mechanisms. This stands in contrast to the Nash equilibrium prediction that both frequencies should be the same for both types.

[21]Subjects in the VC treatment have to process additional information concerning their private voting cost and must condition their participation decision on that cost. Hence, one can argue that the cognitive burden is higher under the VC mechanism.

This reflects the pattern in our data that type-b players tend to vote "more sincerely" than type-r players in these treatments. Second, the QRE estimates of participation rates are again consistent with the comparative statics prediction of the theory. As in our experimental data, the QRE predicts a higher participation rate for type-b players than for type-r players under each voluntary mechanism, and a higher participation rate under the VN mechanism than under VC mechanism for each type. Finally, the QRE predicts under-participation in the VN mechanism and over-participation in VC mechanism, relative to the Nash equilibrium predictions. However, our experimental data exhibit a strong tendency for over-participation in all cases except for type-b players under the VN mechanism.[22] This final observation suggests that QRE does not do a very good job of predicting the participation rates observed in our experimental data.

On the other hand, we again achieve very high likelihood ratio (LR) statistics for the comparison between the unrestricted QRE model and the restricted model of random behavior ($\lambda = 0$). The degree of freedom is the same as before (d.f.=1), and hence, the LR statistics reported in Table 13 exceeds to a high degree the critical value of the χ^2 statistic (=10.828) enabling us to reject at the 0.1% level, the null of no difference between the restricted and unrestricted QRE models.

To further investigate the relationship between our data, the equilibrium predictions and the two models of boundedly rational behavior, consider Figure 10 which illustrates the sincerity of voting decisions under all three voting mechanisms and Figure 11 which illustrates participation decisions under the two voluntary voting mechanisms. The circular dot in the middle represents random play in which the subjects mix between their two available actions (sincere/insincere voting or vote/abstain) with equal probability. The triangular dot in the upper right corner (Figure 10), or on the uppermost line or on the middle left (Figure 11) is the Nash equilibrium prediction. The straight line between these two dots corresponds to the predictions from the equilibrium-plus-noise model for various values of η. As we change the values of η from 0 to 1, we travel on the line from the point of random play toward the Nash equilibrium. Similarly, the curved line between these same two points represents the

[22]Of course type-b players cannot over-participate in the VN treatment as the Nash prediction for their participation rate is one.

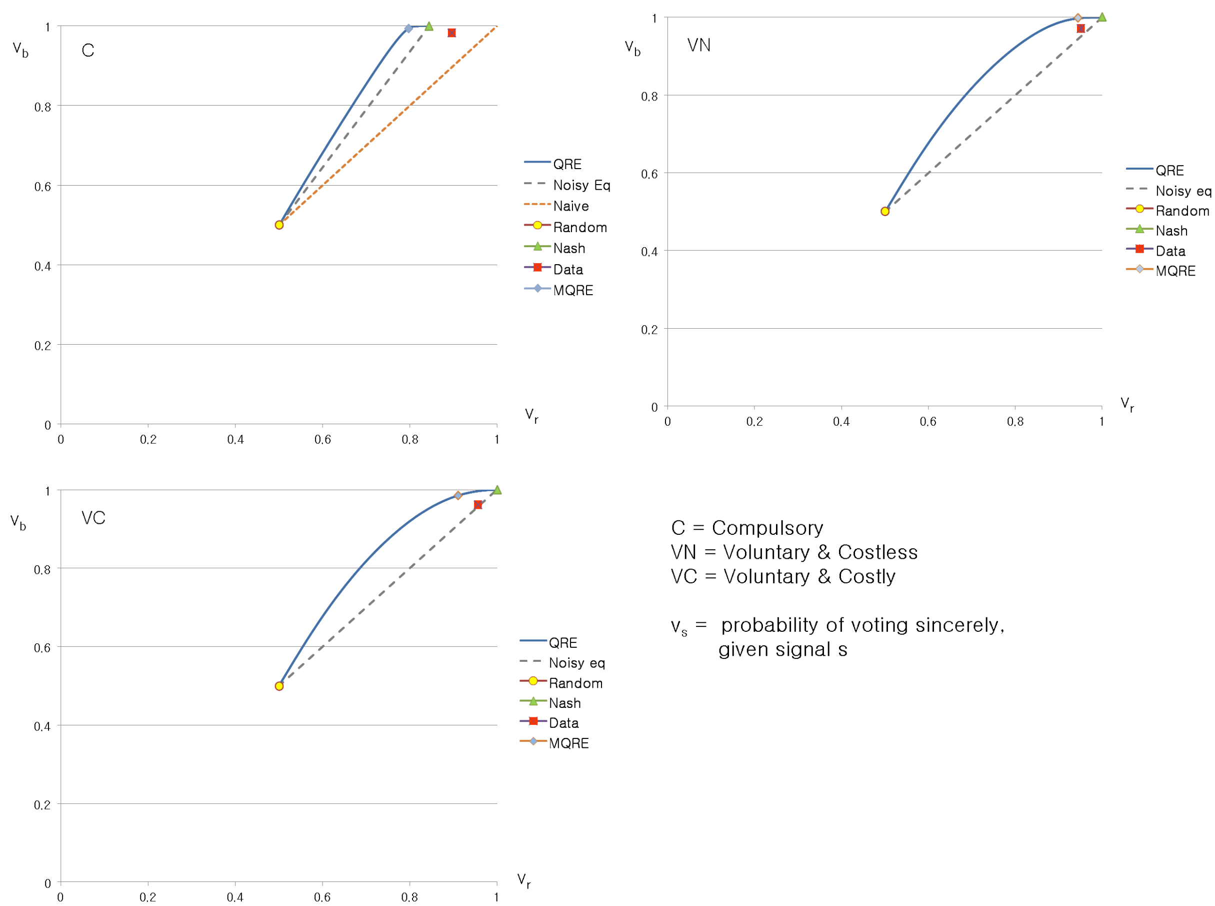


Figure 10: Data and Model Predictions Regarding the Sincere Voting Decisions, v_s, in Each Mechanism

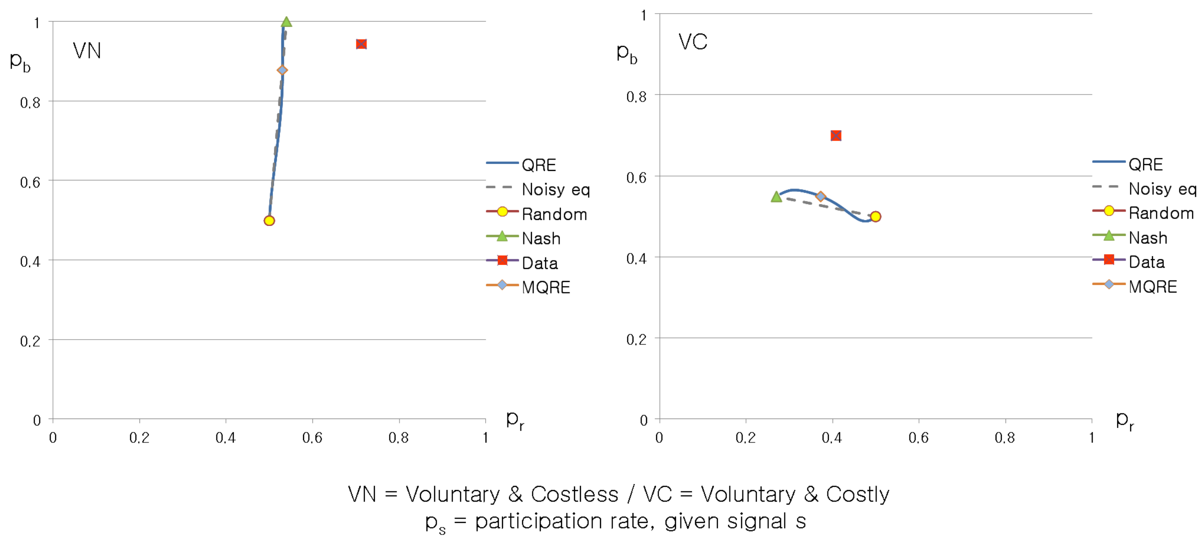


Figure 11: Data and Model Predictions Regarding Participation Decisions, p_s, in the Two Voluntary Voting Mechanisms

QRE predictions for various levels of λ. As we change the values of λ from 0 to ∞, we move from random play to the Nash equilibrium point. Finally, the square dot with the cross ($\times$) represents our data and the diamond dot on the QRE curve represents the maximum likelihood estimate for the QRE prediction (labeled MQRE).

If we just look at the sincerity of voting decisions by signal type, Figure 10 suggests that our data are pretty close to the Nash equilibrium predictions under all three voting mechanisms. This can be anticipated from the high estimated values for $\hat{\eta}$ and $\hat{\lambda}$ in Tables 12 and 13. We conclude that Nash equilibrium performs very well in making quantitative predictions of voting behavior. When we compare QRE with the equilibrium-plus-noise model, it seems that our data are somewhat closer to the predictions of the latter model.

On the other hand, as Figure 11 reveals, our participation data exhibit deviations from Nash equilibrium point predictions. For both voluntary voting mechanisms, over-participation by one signal type was too great to be justified by using either the Nash or QRE predictions. Specifically, in the VN treatment, type-r voters participated at rates greater than possible under any QRE parameter λ while in the VC treatment it was type-b voters who over-participated relative to QRE predictions. In the VC treatment, type-r voters also participated at a rate that is much higher than the Nash equilibrium prediction. These findings suggest that neither Nash nor QRE may yield good point predictions for the participation rates observed in our voluntary voting games. Nevertheless, as emphasized earlier, we do find strong support for the comparative statics predictions of the theory both in the data and in the estimated predictions using the two models of boundedly rational behavior.

3.7 LEARNING

Finally, it is of interest to consider whether there is any evidence of learning over the 20 repetitions of our voting games. In looking for evidence of learning, we compare the observations in the first 10 rounds with those in the last 10 rounds. Table 14 reports the decomposition of both the voting and participation choice data into the two halves, and also restates the Nash equilibrium predictions. Our data on voting and participation decisions both indicate movement toward equilibrium predictions as subjects gained experience; voting and participation decisions are always closer to Nash equilibrium predictions in the last 10 rounds as compared with the first 10 rounds, with the sole exception of voting behavior under the VN treatment. However, the frequencies of sincere voting by type-r voters remained largely the same between the two blocks of 10 rounds under the latter treatment. Hence the only instance in which there is some deviation away from Nash equilibrium predictions by experienced subjects is in the voting decisions of type-b players in the VN treatment.

Table 15 reports results from a signed ranks test examining whether there were any significant differences in the sincerity of voting decisions or in participation rates for each signal type between the first and the last 10 rounds. The results for voting behavior indicate that the differences are largely insignificant. However, the results for participation decisions indicate that there are significant learning effects on this dimension of voting behavior.

The evidence for learning in participation decisions is illustrated in Figure 12 which plots the participation rates of the two signal types, p_s, in the experimental data and relative to

Sincere Voting	Red (v_r)[a]	Blue (v_b)
C[b]- 1st 10 rounds	0.9022 (491)[c]	0.9782 (229)
C - 2nd 10 rounds	0.8900 (482)	0.9874 (238)
C Predicted	0.8440	1.0000
VN - 1st 10 rounds	0.9507 (345)	0.9825 (228)
VN - 2nd 10 rounds	0.9506 (324)	0.9593 (246)
VN Predicted	1.0000	1.0000
VC - 1st 10 rounds	0.9461 (204)	0.9514 (185)
VC - 2nd 10 rounds	0.9677 (186)	0.9740 (154)
VC Predicted	1.0000	1.0000

Participation Rates	Red (p_r)	Blue (p_b)
VN - 1st 10 rounds	0.7263 (475)	0.9306 (245)
VN - 2nd 10 rounds	0.6998 (463)	0.9572 (257)
VN Predicted	0.5397	1.0000
VC - 1st 10 rounds	0.4397 (464)	0.7227 (256)
VC - 2nd 10 rounds	0.3788 (491)	0.6725 (229)
VC Predicted	0.2700	0.5497

[a] v_s is the frequency of sincere voting, and p_s is the participation rate, both by type-s.

[b] C=Compulsory, VN=Voluntary & Costless, VC=Voluntary & Costly.

[c] Number of observations is in parentheses.

Table 14: Evidence of Learning Over Time

Sincere Voting	Red (type-r)	Blue (type-b)
C[a]	positive 6, negative 4 0.3575[b]	positive 2, negative 8 0.1367
VN	positive 5, negative 4 0.4264[†]	positive 9, negative 1 0.0721[†]
VC	positive 1, negative 9 0.0721	positive 2, negative 7 0.1766

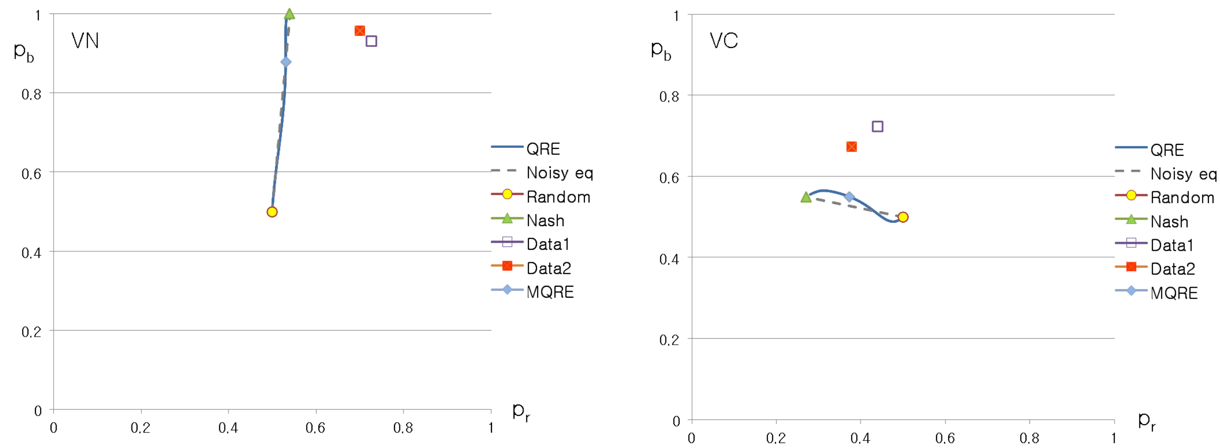
Participation Rates	Red (type-r)	Blue (type-b)
VN	positive 9, negative 1 0.0721	positive 0, negative 10 0.0340
VC	positive 10, negative 0 0.0340	positive 9, negative 1 0.0721

[a] C=Compulsory, VN=Voluntary & Costless, VC=Voluntary & Costly.

[b] All p-values are one-sided.

[†] Movement away from equilibrium predictions.

Table 15: Wilcoxon Signed Ranks Test: Learning



VN = Voluntary & Costless / VC = Voluntary & Costly
p_s = participation rate, given signal s

Figure 12: Learning-Participation

model predictions. Here, Data 1 (the unfilled dot) represents the data (average from all sessions of each voluntary treatment) from the first 10 rounds while Data 2 (the filled dot with the $\times$), represents the data from the last 10 rounds. As Figure 12 reveals, there is evidence of convergence toward the equilibrium from Data 1 to Data 2 under both voluntary treatments. The size of the learning effect is especially large under the VC mechanism. Since the game induced by the latter mechanism is rather complicated as reflected in the relatively low estimated values for the QRE parameter $\hat{\lambda}$, this evidence for learning in the VC treatment suggests that equilibration may take longer than the time frame allowed (20 repetitions) by our experiment.

3.8 SUMMARY

Voting mechanisms are often evaluated in terms of their ability to aggregate private information. As we have seen, in settings where voters have a common interest, rational choice theory predicts that voters will adopt mixed strategies that manifest themselves in different ways depending on whether voting is compulsory or voluntary (abstention is allowed). Under the compulsory, majority rule voting environment that we study, voters should play a mixed strategy with respect to whether they vote sincerely (according to their signal) when they receive an r signal, though they should always vote sincerely conditional on receiving the other b signal. Under the voluntary majority rule voting environment we study, voters should always vote sincerely, according to the signal they receive but they should play a mixed strategy with respect to their participation decision to vote or to abstain. We have designed the first ever experiment aimed at comparing these two different voting mechanisms and testing this important difference in the type of mixed strategy that rational players should adopt and we have found compelling evidence that voters do indeed adapt their behavior to the institutional voting mechanism that is in place in the manner predicted by theory. In particular, we find that signal type-r voters vote significantly more insincerely than signal type-b voters under the compulsory voting mechanism as well as by comparison with either signal type voters under both voluntary voting mechanisms. As for the voluntary voting mechanism, we find significant variations in voter participation rates, but sincere voting among those choosing to vote, all as predicted by the theory. We also observe that the differences in the efficiency of the three voting mechanisms in terms of generating the correct outcome are theoretically small. Under our parameterization of the voting model we predict and find that efficiency is highest on average under the voluntary and costless voting mechanism, followed by the compulsory voting model and that efficiency is lowest on average under the voluntary and costly mechanism. However, we do not find that these efficiency difference are statistically significant using our experimental data.

Taken together these findings help us to understand why both compulsory and voluntary voting mechanisms are observed to co-exist in nature, a question that we posed at the beginning of this chapter. The two institutions coexist because the informational efficiency differences between them are not very great and, most importantly, because voters can and do adapt their behavior to the institutional voting rules that have been put in place in a way that preserves full information aggregation.

4.0 CONCLUSIONS AND FUTURE RESEARCH

In the preceding two chapters, I considered the implications of rational voting and strategic choices either in elections or in committee decision-making. An interesting question related to the experimental study of the third chapter is how to optimally design voting mechanisms in the jury trail model in which voters must decide whether to acquire costly information about the true state of nature, in addition to making a (compulsory) voting decision. In this case, a voting mechanism, by definition, consists of a group size and a voting rule. The optimal voting mechanism is the one that maximizes the probability of group's making a correct decision (the efficiency of the group's decision). The optimality of a group size or a voting rule crucially depends on whether information is freely given, as in the voting environment studied in the third chapter, or is acquired at a (fixed) cost. We can consider which voting mechanism performs better in the sense of yielding the correct group decision under costless and costly information, and investigate whether simple theoretical models of voting games with alternative information structures can give us predictions that are useful to explaining laboratory data.

The basic setup is again the Condorcet jury model in which voters must make a decision as a group about whether to convict or acquit the accused, based on private noisy signals about whether the latter is guilty or innocent. When the signals are free information to the voters, they can do better - make a correct decision with a higher probability - with a larger group size. This is the standard information aggregation effect or a prediction of the celebrated Condorcet Jury Theorem (Condorcet 1785). Feddersen and Pesendorfer (1998) have shown that this result is robust to strategic voting. Even if people vote strategically against their signals, they do it in an optimal way, and as a consequence, we obtain information aggregation with increasing group size. An implication for the optimal voting mechanism is that we can always make the voting mechanism better by adding more voters when private signals are freely provided.

However, this is no longer the case when information is endogenous and its acquisition involves a costly decision. If people are asked to buy private signals at a fixed cost to be better informed about the true state, there is an important free-riding consideration that counterbalances the information aggregation effect mentioned above. As we add one more voter to a group, and as long as this voter still has an incentive to acquire information (with positive probability), the information aggregation effect implies a higher probability of making a correct group decision (a positive effect on the efficiency of group decision). On the other hand, the entire group of voters are less likely to acquire information as we add one more voter. Hence, the free-riding consideration entails a negative effect on efficiency. As we increase the group size with any fixed voting rule, the information aggregation effect is dominant at first and hence we have an increase in the efficiency of group decision up to a certain group size. Beyond this group size, the free-riding effect is dominant, resulting in a decrease in efficiency. Persico (2004) is the first study that establishes results about optimal voting mechanisms that are similar under the Condorcet Jury setup.

This gives us the first hypothesis that is testable against experimental data. For a fixed (majority) voting rule, we will increase the group size from, for instance, five to seven. Then we should observe an increase in efficiency when information is free while we should have a significant drop in efficiency when information is costly, under a suitable choice of parameter values. This results from a large fall in the frequency of information acquisition, which we also expect to see in our data. This broadly tests the hypothesis that the optimal group size is bounded from above with endogenous information (while it's unbounded with free information). In other words, the theory puts an upper bound on the optimal group size, and we examine whether this bound really works among laboratory subjects who are asked to make a decision about the purchase of costly information.

Endogeneity of information also has an implication about (sincere) voting behavior. When information is free and exogenous, voters have incentives to vote strategically against their signals, for instance, with the asymmetry in the precision of signals, along the line of Austen-Smith and Banks (1996). Under a broad range of parameter setups, voters of one signal type are predicted to vote against their private signals with positive probability (while voters of the other signal type are predicted to vote sincerely according to their signals).[1] This is in line with the mixed-strategy equilibrium of strategic voting game in Feddersen and Pesendofer (1998). However, when information is costly and endogenous, voters no longer have an incentive to vote strategically upon acquiring costly information. In other words, once voters decide to buy a costly signal, they will always vote according to their acquired signals.[2]

Similar tests about sincere voting behavior have been done in the previous chapter under the framework of compulsory versus voluntary voting. The compulsory voting game studied before is exactly the same as the voting game with free information in the present framework, and as such, induces a strategic (insincere) voting equilibrium. However, the voluntary voting game induces a sincere voting equilibrium (Krishna and Morgan 2010) in which all those who participate in voting, vote sincerely according to their signals (or abstain). As in the previous chapter, we test whether people properly adjust their (sincere) voting behavior as we change our voting institution from the one with free information to that with costly information.

The final consideration concerns the optimal voting rule. The maintained assumption is asymmetric precision of signals. In this case, for instance, the guilty signal is more precise at the guilty state than the innocent signal is at the innocent state, and hence, if we employed majority rule, then we would convict the subject more often than is desired. Therefore, majority rule cannot be the optimal voting rule no matter whether information is costly or not. If every voter gets a private signal for free (free information), then the voting rule may entail a relatively small difference in the efficiency of group decision. We obtain more interesting comparative statics predictions when information is endogenous and costly. Let's fix the group size. If we set the fixed cost of information large enough so that voters have no incentive to acquire it under majority rule, but small enough so that they *do* have an incentive to acquire it under the optimal rule, then the voting rule can make a great difference

[1] Which type will vote strategically or sincerely depends on the conditional probabilities with which the signals are realized and voting rules. For any group size, there exists a voting rule - a supermajority rule - under which both types vote sincerely in equilibrium.

[2] In typical mixed strategy equilibria of voting games with endogenous information, voters don't acquire information with positive probability, and they may randomize over the alternatives, not necessarily with equal probability especially when the voting rule itself is asymmetric (e.g. supermajority rule), upon not acquiring information.

in terms of efficiency. If no one acquire information, then the efficiency stays at 0.5 (with the assumption of an equal prior); however, when there is positive information acquisition, the group can do much better than a mere random decision.

Thus our final test is to see whether the voting rule alone is able to induce information acquisition large enough to make a significant difference in efficiency when other parameters of the voting environment are kept constant. We again compare the efficiency of voting mechanisms with free and costly information under the alternate voting rules and try to confirm whether the predicted role of the voting rule is relevant in explaining our laboratory data. The proposed experimental tests will shed light on how to design optimal voting mechanism - how to adjust the group size and the voting rule to achieve higher efficiency in group decisions - especially when the assumption of costly information acquisition is appropriate among voters with common preferences who are making a collective decision.

APPENDIX A

PROOFS FOR CHAPTER 2

This section collects the proofs for Chapter 2.

Proof of Lemma 1. (1) It follows directly from A3 that $y_A^* \le y_B^*$.

If $a < p_A < p_B < b$, then $\alpha_A \ge F(p_A) > 0$ and $\alpha_B \ge 1 - F(p_B) > 0$ as F is atomless. Hence, $c(d, \alpha_A) > 0$ and $c(d, \alpha_B) > 0$, implying $y_B^* < p_B$ and $y_A^* > p_A$. If $a = p_A < p_B < b$, then again, $\alpha_B \ge 1 - F(p_B) > 0$ implies $y_A^* > p_A$. But this implies $\alpha_A \ge F(y_A^*) > 0$, so $y_B^* < p_B$. The case $a < p_A < p_B = b$ is similar. Finally, if $a = p_A < p_B = b$, we cannot have $y_A^* = p_A$ and $y_B^* = p_B$ at the same time since the latter fact would imply $\alpha_A = \alpha_B = 0$ - a contradiction. Thus, we must have, say, $y_A^* > p_A$, but then, by the above argument, it follows $y_B^* < p_B$.

(2) Suppose p_A is closer to the median than p_B. Then, by A4, $c(d, \alpha_A) > c(d, \alpha_B)$, so by A2, $\alpha_A > \alpha_B$. $\square$

Proof of Lemma 2. This is a consequence of the Implicit Function Theorem(IFT). The bargaining outcomes constitute a system of equations that possibly define the outcomes as implicit functions of the platforms:

$$F_1(y_A^*, y_B^*; p_A, p_B) \equiv y_A^* - p_A - c(p_B - p_A, 1 - F\Big(\frac{y_A^* + y_B^*}{2}\Big)) = 0$$
$$F_2(y_A^*, y_B^*; p_A, p_B) \equiv y_B^* - p_B + c(p_B - p_A, F\Big(\frac{y_A^* + y_B^*}{2}\Big)) = 0$$

Both F_1, F_2 are C^1 functions as we assume that $c(\cdot,\cdot)$ is a C^1 function. If $p_A < p_B$, then since $c_\alpha > 0$,

$$det\frac{\partial(F_1, F_2)}{\partial(y_A^*, y_B^*)} = 1 + \frac{f(y_M^*)}{2}[c_\alpha(d, \alpha_A) + c_\alpha(d, \alpha_B)] > 0$$

where $y_M^* \equiv \frac{y_A^*+y_B^*}{2}$. Hence, there exist C^1 functions f_A, f_B such that $y_A^* = f_A(p_A, p_B)$ and $y_B^* = f_B(p_A, p_B)$. $\square$

Proof of Lemma 3. Since F is atomless, it suffices to check whether $(y_A^*+y_B^*)(\mathbf{p})$ is strictly increasing in p_j, $j = A, B$. IFT also tells us how the bargaining outcomes change according to the changes in the platforms;

$$\frac{\partial y_A^*}{\partial p_A} = \frac{1 - c_d(d, \alpha_B) + \frac{f(y_M^*)}{2}[c_\alpha(d, \alpha_A) - c_d(d, \alpha_A)c_\alpha(d, \alpha_B) - c_d(d, \alpha_B)c_\alpha(d, \alpha_A)]}{1 + \frac{f(y_M^*)}{2}[c_\alpha(d, \alpha_A) + c_\alpha(d, \alpha_B)]}$$
$$\frac{\partial y_A^*}{\partial p_B} = \frac{c_d(d, \alpha_B) - \frac{f(y_M^*)}{2}[c_\alpha(d, \alpha_B) - c_d(d, \alpha_A)c_\alpha(d, \alpha_B) - c_d(d, \alpha_B)c_\alpha(d, \alpha_A)]}{1 + \frac{f(y_M^*)}{2}[c_\alpha(d, \alpha_A) + c_\alpha(d, \alpha_B)]}$$
$$\frac{\partial y_B^*}{\partial p_A} = \frac{c_d(d, \alpha_A) - \frac{f(y_M^*)}{2}[c_\alpha(d, \alpha_A) - c_d(d, \alpha_A)c_\alpha(d, \alpha_B) - c_d(d, \alpha_B)c_\alpha(d, \alpha_A)]}{1 + \frac{f(y_M^*)}{2}[c_\alpha(d, \alpha_A) + c_\alpha(d, \alpha_B)]}$$
$$\frac{\partial y_B^*}{\partial p_B} = \frac{1 - c_d(d, \alpha_A) + \frac{f(y_M^*)}{2}[c_\alpha(d, \alpha_B) - c_d(d, \alpha_A)c_\alpha(d, \alpha_B) - c_d(d, \alpha_B)c_\alpha(d, \alpha_A)]}{1 + \frac{f(y_M^*)}{2}[c_\alpha(d, \alpha_A) + c_\alpha(d, \alpha_B)]}$$

where $y_M^* \equiv \frac{y_A^*+y_B^*}{2}$. If $\alpha_A = \alpha_B = \frac{1}{2}$, then $y_M^* = m$ and the above expressions for the partial derivatives of the outcomes imply, for $j = A, B$,

$$\frac{\partial(y_A^* + y_B^*)}{\partial p_j} = \frac{1}{1 + f(m)c_\alpha(d, \frac{1}{2})} > 0. \quad \square$$

Proof of Proposition 2.

(1) Let $(\bar{p}_A, \bar{p}_B)$ be a symmetric strategy profile. Note that

$$c(d(\bar{\mathbf{p}}), \frac{1}{2}) = \frac{d(\bar{\mathbf{p}})}{2} \quad \Leftrightarrow \quad \bar{y}_A^* = \bar{y}_B^* = m.$$

Suppose (toward a contradiction) that $c(d(\bar{\mathbf{p}}), \frac{1}{2}) < \frac{d(\bar{\mathbf{p}})}{2}$ so that $\bar{y}_A^* < m < \bar{y}_B^*$. In this case, the parties win with equal probability and the final outcome is still $\bar{y}^* = \frac{\bar{y}_A^*+\bar{y}_B^*}{2} = m$.

By Lemma 2, $\alpha_A \equiv F\Big(\frac{y_A^*+y_B^*}{2}\Big)$ is strictly increasing in p_A at the symmetric profile $(\bar{p}_A, \bar{p}_B)$. Since y_A^* is a C^1 function of (p_A, p_B), there exists a platform $\tilde{p}_A > \bar{p}_A$ such that $\tilde{y}_A^* < m$ and

$$\tilde{\alpha}_A \equiv F\Big(\frac{\tilde{y}_A^*+\tilde{y}_B^*}{2}\Big) > \frac{1}{2} = F\Big(\frac{\bar{y}_A^*+\bar{y}_B^*}{2}\Big) \equiv \bar{\alpha}_A.$$

Thus, as is depicted in Figure 4, A wins for sure and the final outcome is $\tilde{y}^* = \tilde{y}_A^* < m$, which means $\tilde{p}_A$ is a profitable deviation for A, given $\bar{p}_B$. $\square$

(2) Since we must have $c(d(\bar{\mathbf{p}}), \alpha_j(\bar{\mathbf{p}}) = \frac{1}{2}) = \frac{d(\bar{\mathbf{p}})}{2} = d(\bar{\mathbf{p}})\alpha_j(\bar{\mathbf{p}})$ at any symmetric equilibrium $\bar{\mathbf{p}}$, it follows $c_d(d(\bar{\mathbf{p}}), \alpha_j(\bar{\mathbf{p}})) = \frac{1}{2}$ and $c_\alpha(d(\bar{\mathbf{p}}), \alpha_j(\bar{\mathbf{p}})) = d(\bar{\mathbf{p}})$. But then, the bargaining outcomes are all strictly increasing in the platforms:

$$\frac{\partial y_A^*}{\partial p_A}(\bar{\mathbf{p}}) = \frac{\partial y_A^*}{\partial p_B}(\bar{\mathbf{p}}) = \frac{\partial y_B^*}{\partial p_A}(\bar{\mathbf{p}}) = \frac{\partial y_B^*}{\partial p_B}(\bar{\mathbf{p}}) = \frac{1}{2+2f(m)d(\bar{\mathbf{p}})} > 0.$$

Suppose $a < \bar{p}_A < \bar{p}_B < b$; (1) implies $\bar{y}_A^* = \bar{y}_B^* = m$. As p_A decreases, both y_A^*, y_B^* also decrease. Thus, strict monotonicity of the outcomes at $\bar{\mathbf{p}}$ ensures that any infinitesimal deviation $\tilde{p}_A < \bar{p}_A$ is profitable for A since

$$\tilde{\alpha}_A \equiv F\Big(\frac{\tilde{y}_A^*+\tilde{y}_B^*}{2}\Big) < F(m) = \frac{1}{2} \quad \text{but} \quad \tilde{y}^* = \tilde{y}_B^* < m.$$

The profitability of A's deviation is illustrated in Figure 5. $\square$

Proof of Proposition 3. Given $p_B = b$, $p_A \geq m$ cannot be a profitable deviation from a; similarly, $p_B \leq m$ is also ruled out. It suffices to note that the above conditions are equivalent to

$$y_A^*(p_A, b) = p_A + c(d(p_A, b), \alpha_B(p_A, b)) \geq m, \quad \forall p_A \in [a, m),$$
$$y_B^*(a, p_B) = p_B - c(d(a, p_B), \alpha_A(a, p_B)) \leq m, \quad \forall p_B \in (m, b].$$

which makes any deviations unprofitable. $\square$

Proof of Proposition 4. For the case of symmetric equilibria (where $\frac{y_A^*+y_B^*}{2} = m$ by definition), we must have $y^* = y_A^* = y_B^* = m$ by Proposition 2.

Let (p_A, p_B) be an equilibrium with $p_A < m < p_B$ but suppose it is not symmetric. We first show $y_A^* < m < y_B^*$ cannot occur at (p_A, p_B). We can have either $\frac{y_A^*+y_B^*}{2} < m$ or $\frac{y_A^*+y_B^*}{2} > m$, but then $\tilde{p}_A = 2m - p_B$ or $\tilde{p}_B = 2m - p_A$ is a profitable deviation. The reason is that it makes $(\tilde{p}_A, p_B)$ or $(p_A, \tilde{p}_B)$ symmetric.

If $m < y_A^* \leq y_B^*$ or $y_A^* \leq y_B^* < m$, then the parties prefer making a tie to winning, so again, $\tilde{p}_A = 2m - p_B$ or $\tilde{p}_B = 2m - p_A$ becomes a profitable deviation.

Thus, for the given equilibrium (p_A, p_B) that is not symmetric, we end up with two possibilities

$$y_A^* < m = y_B^* \quad \text{or} \quad y_A^* = m < y_B^*.$$

In any of these cases, the final bargaining outcome is $y^* = m$. $\square$

Proof of Lemma 4.

The arguments are similar to the ones given for payoff discontinuity. First, fix $a < p < m$. Then, as (p_A, p_B) converges to $(p, 2m - p)$ from below, the winning platform is p_B; while as (p_A, p_B) converges to $(p, 2m - p)$ from above, the winning platform is p_A. Thus,

$$\begin{aligned} &\lim_{p_A \to p^-, p_B \to (2m-p)^-} w_A(p_A, p_B) \\ &\qquad = \lim_{p_A \to p^-, p_B \to (2m-p)^-} v_A(p_B - c(p_B - p_A, \alpha_A)) = v_A(m_u) \end{aligned}$$

where $m_u \equiv 2m - p - c(2m - 2p, \frac{1}{2}) > m$; and

$$\begin{aligned} &\lim_{p_A \to p^+, p_B \to (2m-p)^+} w_A(p_A, p_B) \\ &\qquad = \lim_{p_A \to p^+, p_B \to (2m-p)^+} v_A(p_A + c(p_B - p_A, \alpha_B)) = v_A(m_l) \end{aligned}$$

where $m_l \equiv p + c(2m - 2p, \frac{1}{2}) < m$. However, $w_A(p, 2m - p) = v_A(m)$, hence A's utility is monotonic for $a < p < m$ since $v_A(m_u) < v_A(m) < v_A(m_l)$.

Similarly,

$$\lim_{p_A \to p^-, p_B \to (2m-p)^-} w_B(p_A, p_B) = v_B(m_u) > w_B(p, 2m-p) = v_B(m)$$

$$> v_B(m_l) = \lim_{p_A \to p^+, p_B \to (2m-p)^+} w_B(p_A, p_B).$$

Thus B's utility is also monotonic and compensates the monotonicity of A's utility.

Next, if we fix $m < p < b$, then the winner is A as the convergence is from below while it is B as the convergence is from above. Hence,

$$\lim_{p_A \to p^-, p_B \to (2m-p)^-} w_A(p_A, p_B) = v_A(p) < v_A(m)$$

$$< v_A(2m-p) = \lim_{p_A \to p^+, p_B \to (2m-p)^+} w_A(p_A, p_B)$$

and

$$\lim_{p_A \to p^-, p_B \to (2m-p)^-} w_B(p_A, p_B) = v_B(p) > v_B(m)$$

$$> v_B(2m-p) = \lim_{p_A \to p^+, p_B \to (2m-p)^+} w_B(p_A, p_B).$$

Therefore, in any cases, we've seen that the last assumption of Dasgupta and Maskin (Theorem 5b) holds. $\square$

Proof of Proposition 5.

For each $p \in (a, m) \cup (m, b)$, choose $\bar{w}_A(p)$, $\bar{w}_B(p)$ s.t.

$$\lim_{p_A \to p^+, p_B \to (2m-p)^+} w_A(p_A, p_B) \;>\; \bar{w}_A(p) \;>\; w_A(p, 2m-p) \tag{A.1}$$

$$\lim_{p_A \to p^-, p_B \to (2m-p)^-} w_B(p_A, p_B) \;>\; \bar{w}_B(p) \;>\; w_B(p, 2m-p) \tag{A.2}$$

and

$$\begin{aligned}\bar{w}_A(p) + \bar{w}_B(p) &\geq \lim_{p_A \to p^-, p_B \to (2m-p)^-} [w_A(p_A, p_B) + w_B(p_A, p_B)] \\ \bar{w}_A(p) + \bar{w}_B(p) &\geq \lim_{p_A \to p^+, p_B \to (2m-p)^+} [w_A(p_A, p_B) + w_B(p_A, p_B)]\end{aligned} \tag{A.3}$$

Define

$$\hat{w}_A(p, 2m-p) = \bar{w}_A(p) \tag{A.4}$$

$$\hat{w}_B(p, 2m-p) = \bar{w}_B(p) \tag{A.5}$$

For $(p_A, p_B) \neq (p, 2m-p)$ or $(p_A, p_B) \in \{(m, m), (a, b), (b, a)\}$, define

$$\hat{w}_A(p_A, p_B) \equiv w_A(p_A, p_B), \quad \hat{w}_B(p_A, p_B) \equiv w_B(p_A, p_B).$$

Since w_A, w_B are continuous except for $(p_A, p_B) = (p, 2m-p)$, $p \neq m, a, b$, $\hat{w}_A + \hat{w}_B$ is also continuous there.

By (3),(4) and (5), $\hat{w}_A + \hat{w}_B$ is upper semicontinuous at points $(p, 2m-p)$, $p \neq m, a, b$. From (1) and (4), $\hat{w}_A$ is weakly lower semicontinuous at $(p, 2m-p)$ and from (2) and (5), so is $\hat{w}_B$. Hence, the game $[(S_j, \hat{w}_j); j = A, B]$ possesses a mixed strategy equilibrium $(\hat{\mu}_A, \hat{\mu}_B)$ by Dasgupta and Maskin's Theorem 5.

It remains to show that $(\hat{\mu}_A, \hat{\mu}_B)$ is an equilibrium of the original game $[(S_j, w_j); j = A, B]$. Choose $\hat{p} \in supp(\hat{\mu}_A)$. Then

$$\int \hat{w}_A(\hat{p}, p_B) d\hat{\mu}_B \geq \int \hat{w}_A(p_A, p_B) d\hat{\mu}_B, \ \forall p_A \tag{A.6}$$

If $\hat{\mu}_B(2m - \hat{p}) > 0$ and if w_A is discontinuous at $(\hat{p}, 2m - \hat{p})$, then from (1) and (4), there exists a platform p' close to $\hat{p}$ s.t. $\int \hat{w}_A(p', p_B) d\hat{\mu}_B > \int \hat{w}_A(\hat{p}, p_B) d\hat{\mu}_B$, a contradiction of (6).

Hence,

$$\int \hat{w}_A(\hat{p}, p_B) d\hat{\mu}_B = \int w_A(\hat{p}, p_B) d\hat{\mu}_B \tag{A.7}$$

But from (1) and (4), $\hat{w}_A(p_A, p_B) \geq w_A(p_A, p_B)$ for all (p_A, p_B); hence

$$\int \hat{w}_A(p_A, p_B) d\hat{\mu}_B \;\geq\; \int w_A(p_A, p_B) d\hat{\mu}_B \tag{A.8}$$

Combining (6)-(8), $\int w_A(\hat{p}, p_B) d\hat{\mu}_B \geq \int w_A(p_A, p_B) d\hat{\mu}_B$, $\forall p_A$; i.e. $\hat{\mu}_A$ is a best response to $\hat{\mu}_B$. Similarly, $\hat{\mu}_B$ is a best response to $\hat{\mu}_A$. $\square$

Proof of Lemma 5. We define a linear integral operator $\Phi : C[2m-\beta, 2m-\alpha] \to C[2m-\beta, 2m-\alpha]$ by

$$\begin{aligned}
\Phi g(x) \;&\equiv \hat{\lambda}(x)^{-1} \int_{2m-\beta}^{x} v_A'(\hat{y}_B^*(x,t)) \frac{\partial \hat{y}_B^*}{\partial p_A}(x,t) g(t) dt \\
&+\; \hat{\lambda}(x)^{-1} \int_{x}^{2m-\alpha} v_A'(\hat{y}_A^*(x,t)) \frac{\partial \hat{y}_A^*}{\partial p_A}(x,t) g(t) dt
\end{aligned}$$

If the linear map Φ has a fixed point, then it must be a solution to our integral equation.

Since v_A, v_B, y_A^*, y_B^* are all C^1 functions, we can find some constants ξ_A, ξ_B s.t.

$$\begin{aligned}
\left| v_A'(\hat{y}_A^*(x,t)) \frac{\partial \hat{y}_A^*}{\partial p_A}(x,t) \right| &= \left| v_A'(y_A^*(p_A, p_B)) \frac{\partial y_A^*}{\partial p_A}(p_A, p_B) \right| \leq \xi_A, \;\forall (p_A, p_B) \in R_A, \\
\left| v_A'(\hat{y}_B^*(x,t)) \frac{\partial \hat{y}_B^*}{\partial p_A}(x,t) \right| &= \left| v_A'(y_B^*(p_A, p_B)) \frac{\partial y_B^*}{\partial p_A}(p_A, p_B) \right| \leq \xi_B, \;\forall (p_A, p_B) \in R_B.
\end{aligned}$$

$$\begin{aligned}
\text{where} \quad R_A &\equiv \{(p_A, p_B) : \alpha \leq p_A \leq \beta,\; 2m - p_A \leq p_B \leq 2m - \alpha\} \\
R_B &\equiv \{(p_A, p_B) : \alpha \leq p_A \leq \beta,\; 2m - \beta \leq p_B \leq 2m - p_A\}
\end{aligned}$$

Then, $\forall g,\; h \in C[2m-\beta, 2m-\alpha]$,

$$
\begin{aligned}
&\Big|\Phi g(x) - \Phi h(x)\Big| \\
&\quad = |\hat{\lambda}(x)|^{-1}\Big|\int_{2m-\beta}^{x} v_A'(\hat{y}_B^*(x,t))\frac{\partial \hat{y}_B^*}{\partial p_A}(x,t)[g(t)-h(t)]dt \\
&\qquad\qquad + \int_{x}^{2m-\alpha} v_A'(\hat{y}_A^*(x,t))\frac{\partial \hat{y}_A^*}{\partial p_A}(x,t)[g(t)-h(t)]dt\Big| \\
&\quad \le |\hat{\lambda}(x)|^{-1}\Big(\xi_B \max_{s\in[2m-\beta,2m-\alpha]}|g(s)-h(s)|\int_{2m-\beta}^{x} dt \\
&\qquad\qquad + \xi_A \max_{s\in[2m-\beta,2m-\alpha]}|g(s)-h(s)|\int_{x}^{2m-\alpha} dt\Big) \\
&\quad \le |\hat{\lambda}(x)|^{-1}\xi(\beta-\alpha)d(g,h),
\end{aligned}
$$

where $\xi \equiv max\{\xi_A, \xi_B\}$. Taking the maximum on the left-hand side,

$$
d(\Phi g, \Phi h) \le \frac{\xi(\beta-\alpha)}{|\hat{\lambda}(x)|}d(g,h) = \frac{\xi(\beta-\alpha)}{|\lambda(p_A)|}d(g,h).
$$

Hence, Φ is a contraction if

$$
\xi(\beta-\alpha) \; < \; |\lambda(p_A)|
$$

and we then apply the Banach Fixed Point Theorem to have a unique solution g for the original integral equation.

We can proceed similarly with party B's payoff $V_B(p_B)$ to get

$$
\zeta(\beta-\alpha) \; < \; |\mu(p_B)|, \quad \forall p_B \in [2m-\beta, 2m-\alpha]
$$

where $\zeta \equiv max\{\zeta_A, \zeta_B\}$,

$$
\begin{aligned}
\Big|v_B'(y_A^*(p_A,p_B))\frac{\partial y_A^*}{\partial p_B}(p_A,p_B)\Big| &\le \zeta_A, \quad \forall (p_A,p_B) \in R_A, \\
\Big|v_B'(y_B^*(p_A,p_B))\frac{\partial y_B^*}{\partial p_B}(p_A,p_B)\Big| &\le \zeta_B, \quad \forall (p_A,p_B) \in R_B
\end{aligned}
$$

and

$$
\mu(p_B) \equiv v_B(y_B^*(2m-p_B,p_B)) - v_B(y_A^*(2m-p_B,p_B)). \quad \square
$$

APPENDIX B

EQUILIBRIUM CALCULATIONS FOR CHAPTER 3

In deriving equilibrium predictions, we adopt the parameterization of our experimental design where the number of voters $N = 9$, $x_\rho = \Pr[r|\rho] = 0.9$ and $x_\beta = \Pr[b|\beta] = 0.6$. These choices imply that $q(\rho|r) = \frac{9}{13}$ and $q(\beta|b) = \frac{6}{7}$.

Consider first the compulsory voting mechanism (C). Let v_s denote the probability of voting sincerely given signal $s \in \{r, b\}$. A symmetric Bayesian Nash equilibrium is described by a strategy profile (v_r, v_s).

We begin by calculating the probability of pivotal events $\Pr[Piv|\omega]$. Suppose the probability of a randomly chosen voter voting for alternative A in state ω is denoted by $A(\omega)$. Then,

$$\begin{aligned} R(\rho) &= 0.9v_r + 0.1(1 - v_b), \\ B(\beta) &= 0.6v_b + 0.4(1 - v_r). \end{aligned}$$

Since only signal type-r mixes ($v_r \in (0, 1)$) while type-b plays a pure strategy of voting sincerely in our equilibrium, these expressions can be further simplified to $R(\rho) = 0.9v_r$ and $B(\beta) = 0.6 + 0.4(1 - v_r)$, i.e., the compulsory voting equilibrium is identified with a single number, v_r.

Let (j, k) denote the event that there are j votes for R and k votes for B. Under compulsory voting, the only pivotal event is $(4, 4)$, where a vote for either R or B is pivotal. The pivot probability in each state is given by

$$\begin{aligned}
\Pr[Piv|\rho] &= \Pr[(4,4)|\rho] &= \tbinom{8}{4}[R(\rho)]^4[1-R(\rho)]^4, \\
\Pr[Piv|\beta] &= \Pr[(4,4)|\beta] &= \tbinom{8}{4}[B(\beta)]^4[1-B(\beta)]^4.
\end{aligned}$$

Using these expressions for the pivot probabilities, we can calculate type-r's choice probability $v_r \in (0,1)$ by solving the following equation:

$$U(R|r) - U(B|r) = 0 \Rightarrow \frac{9}{13}\Pr[Piv|\rho] - \frac{4}{13}\Pr[Piv|\beta] = 0.$$

The equilibrium choice probability for type-r is $v_r = 0.8440$ which results in

$$U(B|b) - U(R|b) = M\left[\frac{6}{7}\Pr[Piv|\beta] - \frac{1}{7}\Pr[Piv|\rho]\right] = M \cdot 0.1389 > 0,$$

and this justifies type-b's choice of sincere voting, i.e., $v_b = 1$.

Consider next the voluntary and costless voting mechanism (VN). We focus here on the symmetric sincere voting equilibrium under this voting mechanism. Since we allow abstention, the event that a vote for R is pivotal may no longer coincide with the event that a vote for B is pivotal. Let us denote the former event by Piv_R and the latter event by Piv_B. We again need to calculate the pivot probabilities $\Pr[Piv_j|\omega]$, $j = R,B$.

As mentioned in footnote 10 if we denote by T, T_{-1}, and T_{+1} the events that the number of votes for R is the same as, one less than, and one more than the number of votes for B, respectively, then for each $\omega \in \{\rho, \beta\}$,

$$\begin{aligned}
\Pr[Piv_R|\omega] &= \Pr[T|\omega] + \Pr[T_{-1}|\omega], \\
\Pr[Piv_B|\omega] &= \Pr[T|\omega] + \Pr[T_{+1}|\omega],
\end{aligned}$$

where

$$\begin{aligned}
T &\equiv \{(k,k) : 0 \le k \le 4\}, \\
T_{-1} &\equiv \{(k-1,k) : 1 \le k \le 4\}, \\
T_{+1} &\equiv \{(k,k-1) : 1 \le k \le 4\}.
\end{aligned}$$

Next, let p_r and p_b denote the participation rates of type-r and type-b voters respectively. Since we have a sincere voting equilibrium under the two voluntary mechanisms, a symmetric

Bayesian Nash equilibrium is described by a pair of participation rates (p_r, p_b). $A(\omega)$ is analogously defined as the probability of a randomly chosen voter choosing alternative $A \in \{R, B, \phi\}$ in state $\omega \in \{\rho, \beta\}$. Assuming sincere voting, we have:

$$R(\rho) = 0.9p_r, \quad B(\rho) = 0.1p_b, \quad \phi(\rho) = 1 - R(\rho) - B(\rho),$$
$$R(\beta) = 0.4p_r, \quad B(\beta) = 0.6p_b, \quad \phi(\beta) = 1 - R(\beta) - B(\beta).$$

Under voluntary and costless voting (VN), type-r mixes between (sincere) voting and abstaining ($p_r \in (0,1)$) while type-b votes for certain ($p_b = 1$), hence $R(\rho) = 0.9p_r$, $B(\rho) = 0.1$, $R(\beta) = 0.4p_r$ and $B(\beta) = 0.6$ (the voluntary and costless voting equilibrium is again identified with a single number, p_r). Using the expressions for $A(\omega)$, we can write

$$\begin{aligned}
\Pr[T|\omega] &= \sum_{k=0}^{4} \binom{n}{2k}\binom{2k}{k} R(\omega)^k B(\omega)^k (1 - R(\omega) - B(\omega))^{n-2k}, \\
\Pr[T_{-1}|\omega] &= \sum_{k=1}^{4} \binom{n}{2k-1}\binom{2k-1}{k-1} R(\omega)^{k-1} B(\omega)^k (1 - R(\omega) - B(\omega))^{n-2k+1}, \\
\Pr[T_{+1}|\omega] &= \sum_{k=1}^{4} \binom{n}{2k-1}\binom{2k-1}{k} R(\omega)^{k} B(\omega)^{k-1} (1 - R(\omega) - B(\omega))^{n-2k+1}.
\end{aligned}$$

We now know how to express $\Pr[Piv_j|\omega]$ as a function of p_r. Type-r's equilibrium participation rate can then be obtained from

$$U(R|r) - U(\phi|r) = 0 \Rightarrow \frac{9}{13}\Pr[Piv_R|\rho] - \frac{4}{13}\Pr[Piv_R|\beta] = 0,$$

which yields $p_r = 0.5387$ and results in

$$U(B|b) - U(\phi|b) = M\left[\frac{6}{7}\Pr[Piv_B|\beta] - \frac{1}{7}\Pr[Piv_B|\rho]\right] = M \cdot (0.0342) > 0.$$

The latter condition again justifies type-b's full participation in voting ($p_b = 1$). Using the above solution for p_r, we can check that sincere voting is in fact incentive compatible. Specifically, we have:

$$
\begin{aligned}
U(R|r) - U(\phi|r) &= 0, \\
U(B|r) - U(\phi|r) &= M\left[\frac{4}{13}\Pr[Piv_B|\beta] - \frac{9}{13}\Pr[Piv_B|\rho]\right] = M \cdot (-0.0402) < 0 \\
&\Rightarrow U(R|r) > U(B|r). \\
U(B|b) - U(\phi|b) &= M \cdot (0.0342) > 0. \\
U(R|b) - U(\phi|b) &= M\left[\frac{1}{7}\Pr[Piv_R|\rho] - \frac{6}{7}\Pr[Piv_R|\beta]\right] = M \cdot (-0.0693) < 0 \\
&\Rightarrow U(B|b) > U(R|b).
\end{aligned}
$$

The final case of voluntary and costly voting (VC) is similar. We again have a sincere voting equilibrium and the expressions for $A(\omega)$ and the pivot probabilities $\Pr[Piv_j|\omega]$ are the same as those for the voluntary and costless voting case (VN) except that both participation rates for type-r and type-b voters are now less than 1; i.e., $p_r, p_b \in (0,1)$ (this means that the pivot probabilities $\Pr[Piv_j|\omega]$ are functions of both p_r and p_b). In the case of voluntary and costly voting, we have a cutoff-cost equilibrium with the cutoffs given by $F^{-1}(p_r)$, $F^{-1}(p_b)$, where F is the distribution of voting costs. In other words, a type-s voter participates in voting if and only if her realized voting cost is below $F^{-1}(p_s)$, $s = r,b$. A Bayesian Nash equilibrium is defined as a pair (p_r, p_b) that solves

$$
\begin{aligned}
U(R|r) - U(\phi|r) &\equiv M\left[\frac{9}{13}\Pr[Piv_R|\rho] - \frac{4}{13}\Pr[Piv_R|\beta]\right] = F^{-1}(p_r), \\
U(B|b) - U(\phi|b) &\equiv M\left[\frac{6}{7}\Pr[Piv_B|\beta] - \frac{1}{7}\Pr[Piv_B|\rho]\right] = F^{-1}(p_b).
\end{aligned}
$$

If F is the uniform distribution with the support $[0, \frac{M}{10}]$ as in our laboratory voting games, the resulting solutions are $p_r = 0.2700$, $p_b = 0.5497$ as reported in the text. These values again insure that sincere voting is incentive compatible. Specifically, we have:

$$
\begin{aligned}
U(R|r) - U(\phi|r) = M \cdot (0.0270) &> M \cdot (-0.1188) = U(B|r) - U(\phi|r) \\
U(B|b) - U(\phi|b) = M \cdot (0.0550) &> M \cdot (-0.1277) = U(R|b) - U(\phi|b)
\end{aligned}
$$

APPENDIX C

EXPERIMENTAL INSTRUCTIONS FOR CHAPTER 3

The following are the experimental instructions for the voluntary and costly voting (VC) treatment. The instructions for the other two treatments are similar, with the omission of the voting cost part for the voluntary and costless treatment and the further omission of the participation decision part for the compulsory and costless treatment. The complete set of instructions for all three treatments is available at http://www.pitt.edu/ jduffy/voting/

C.1 OVERVIEW

Welcome to this experiment in the economics of decision-making. Funding for this experiment has been provided by the University of Pittsburgh. We ask that you not talk with one another for the duration of the experiment.

For your participation in today's session you will be paid in cash, at the end of the experiment. Different participants may earn different amounts. The amount you earn depends partly on your decisions, partly on the decisions of others, and partly on chance. Thus it is important that you listen carefully and fully understand the instructions before we begin. There will be a short comprehension quiz following the reading of these instructions which you will all need to complete before we can begin the experimental session.

The experiment will make use of the computer workstations, and all interaction among you will take place through these computers. You will interact anonymously with one another and your data records will be stored only by your ID number; your name or the names of other participants will not be revealed in the session today or in any write-up of the findings from this experiment.

Today's session will involve 18 subjects and 20 rounds of a decision-making task. In each round you will view some information and make a decision. Your decision together with the decisions of others determine the amount of points you earn each round. Your dollar earnings are determined by multiplying your total points from all 20 rounds by a conversion rate. In this experiment, each point is worth 1 cent, so 100 points = $1.00. Following completion of the 20th round, you will be paid your total dollar earnings plus a show-up fee of $5.00. Everyone will be paid in private, and you are under no obligation to tell others how much you earned.

C.2 SPECIFIC DETAILS

At the start of each and every round, you will be randomly assigned to one of two groups, the R (Red) group or the B (Blue) group. Each group will consist of 9 members. All assignments of the 18 subjects to the two groups of size 9 at the start of each round are equally likely. Neither you nor any other member of your group or the other group will be informed of whether they are assigned to the R or to the B groups until the end of the round.

Imagine that there are two "jars", which we call the red jar and the blue jar. Each jar contains 10 balls; the red jar contains 9 red balls and 1 blue ball while the blue jar contains 6 blue balls and 4 red balls. The red jar is always assigned to the R (Red) group and the blue jar is always assigned to the B (Blue) group. However, recall that you do not know which group (Red or Blue) you have been assigned to; that is, you don't know the true color of your group's jar. Furthermore, your assignment to the R or B group is randomly determined at the start of every round.

To help you determine which jar is assigned to your group, each member of your group will be allowed to independently select one ball, at random, from your group's jar. You do this on the first stage screen on your computer by clicking on your choice of the ball to examine: the balls are numbered 1 to 10. Once you click on the number of a ball, you will be privately informed of the color of that ball. You will not be told the color of the balls drawn by the other members of your group, nor will they learn the color of the ball you chose, and it is possible for members of your group to draw the same ball as you do or any of the other 9 balls as well. Each member in your group selects one ball on their own, and only sees the color of their own ball. However, all members of your group (Red or Blue) will choose a ball from the *same* jar that contains the same number of red and blue balls. Recall again that if you are choosing a ball from the red jar, that jar contains 9 red balls and 1 blue ball while if you are choosing a ball from the blue jar, that jar contains 6 blue balls and 4 red balls.

After each individual has drawn a ball and observed the color of their chosen ball, each individual is asked to decide (1) whether they want to join in the group decision process and make a choice between “RED” or “BLUE” or (2) whether they do not want to join in the group decision process, corresponding to the option “NO CHOICE”.

Your group’s decision depends on both individual decisions.

Your 9-member group’s decision will be the color chosen by the majority of those who decided to join the group decision process. Suppose for example that 6 of your group members decided to join the group decision process (i.e., 3 members selected NO CHOICE). If 4 or more of the 6 who decided to make a choice choose RED, then the group decision is RED by the majority rule. Similarly, the group’s decision is BLUE if a majority of those who decided to make a choice chose BLUE. That is, your group’s decision will be whichever color receives more individual choices among the members of your group who decided to make a choice. In the case of a tie, where each color receives the same number of individual choices by members of your group (for example, 3 members chose RED and the other 3 chose BLUE), the group decision is INDETERMINATE. If the number of those who decided to make a choice is odd (for example, 5 members decided to make a choice while 4 members selected NO CHOICE), then your group’s decision can be either CORRECT or INCORRECT, as discussed below, but it cannot be INDETERMINATE.

If you decided not to join the group decision process, that is, you selected NO CHOICE, then you will get additional points, which we refer to as the NC BONUS. The amount of your NC BONUS is assigned randomly by the computer. In any given round, your NC bonus points for the round will be a number drawn randomly from the set $\{0, 1, 2, ..10\}$, with all numbers in that set being equally likely. Your NC BONUS in each round does not depend on your prior round NC BONUS or your decisions in any previous rounds, or on the NC BONUSes or decisions of other members. While you are told your own NC BONUS before you make any decision, you are never told the NC BONUSes of other participants. You only know that each of the other members has an NC BONUS that is some number between 0 and 10, inclusive.

The points you earn in any given round are determined as follows. Suppose you decided to join the group decision process and you then chose RED or BLUE. If your group's decision (via majority rule) is the same as the true color of the jar that is assigned to your group, then the group decision is CORRECT, and you will earn 100 points from the group's correct decision. If your group's decision is different from the true color of your group's jar, then the group decision is INCORRECT, and you will earn 0 points from the group's incorrect decision. If the group decision is INDETERMINATE, then you will earn 50 points from the group's indeterminate decision. Suppose instead that you selected NO CHOICE. In that case, if your group's decision is the same as the true color of the jar that is assigned to your group, then the group decision is CORRECT, and you will earn 100 points plus the NC BONUS assigned to you for that round. If your group's decision is different from the true color of your group's jar, then the group decision is INCORRECT, and you will earn the NC BONUS. If your group's decision is INDETERMINATE, then you will earn 50 points plus the NC BONUS. In other words, if you decide not to join the group decision-you select NO CHOICE-then your earnings will increase by the amount of the NC BONUS that is assigned to you in each round. Notice that both decisions, your decision to make a choice or not (NO CHOICE) and, if you decide to make a choice, your decision between RED or BLUE can affect whether the overall decision of your group is CORRECT, INCORRECT or INDETERMINATE.

If the final (20th) round has not yet been played, then at the start of each new round you and all of the other participants will be randomly assigned to a new 9-person group, R or B. You will not know which group, R or B you have been assigned to but you will have the opportunity to draw a new ball from your group's jar, to decide whether to make a choice or not (NO CHOICE) and if you have decided to make a choice to choose between RED or BLUE. In other words, the group you are in will change from round to round.

Following completion of the final round, your points earned from all rounds played will be converted into cash at the rate of 1 point = 1 cent. You will be paid these total earnings together with your \$5 show-up payment in cash and in private.

C.3 QUESTIONS?

Now is the time for questions. If you have a question about any aspect of these instructions, please raise your hand and an experimenter will answer your question in private.

APPENDIX D

QUIZ FOR CHAPTER 3

Before we start today's experiment we ask you to answer the following quiz questions that are intended to check your comprehension of the instructions. The numbers in these quiz questions are illustrative; the actual numbers in the experiment may be quite different. Before starting the experiment we will review each participant's answers. If there are any incorrect answers we will go over the relevant part of the instructions again.

1. I will be assigned to the same group, R or B in every round. Circle one: True False.
2. I will get a different NC Bonus in every round. Circle one: True False.
3. If I decide to make a choice I give up the NC Bonus Circle one: True False.
4. The red jar contains ______ red balls and ______ blue balls. The blue jar contains ______ red balls and _______ blue balls.
5. Consider the following scenario in a round. 5 members of your group decide to make a choice and 3 of these members choose RED.
   a. How many members of your group made NO CHOICE? _________
   b. What is your group's decision? _________
   c. If the jar of balls your group was drawing from was in fact the RED jar, how many points are earned by those who made a choice? _________
   d. If the jar of balls your group was drawing from was in fact the BLUE jar, how many points are earned by those who made a choice? _________

6. Consider the following scenario in a round. 4 members of your group decide to make a choice and 2 of these members choose RED.

a. How many members of your group made NO CHOICE? _________

b. What is your group's decision? _________

c. If the jar of balls your group was drawing from was in fact the RED jar, how many points are earned by those who made a choice? _________

d. If the jar of balls your group was drawing from was in fact the BLUE jar, how many points are earned by those who made a choice? _________

BIBLIOGRAPHY

[1] Alesina, A. (1988), "Credibility and Policy Convergence in a Two-Party System with Rational Voters," *American Economic Review*, 78(4), 796–805.

[2] Ansolabehere, S. (2006), "Voters, Candidates, and Parties," In *The Oxford Handbook of Political Economy*, B. Weingast and D. Wittman eds., New York, NY: Oxford University Press, Chapter 2, 29–49.

[3] Aragones, E. and T. Palfrey, (2002), "Mixed Equilibrium in a Downsian Model with a Favored Candidate," *Journal of Economic Theory*, 103(1), 131–161.

[4] Austen-Smith, D. and J. Banks (1988), "Elections, Coalitions, and Legislative Outcomes," *American Political Science Review*, 82(2), 405–422.

[5] Austen-Smith, D. and J. Banks (1996),"Information Aggregation, Rationality, and the Condorcet Jury Theorem," *American Political Science Review*, 90(1), 34–45.

[6] Austen-Smith, D. and J. Banks (1999), *Positive Political Theory I: Collective Preference*, Ann Arbor, MI: The University of Michigan Press.

[7] Banks, J. (1990), "A Model of Electoral Competition with Incomplete Information," *Journal of Economic Theory*, 50(2), 309–325.

[8] Baron, D. and D. Diermeier (2001), "Elections, Governments, and Parliaments in Proportional Representation Systems," *Quarterly Journal of Economics*, 116(3), 933–967.

[9] Baron, D. and J. Ferejohn (1989), "Bargaining in Legislatures." *American Political Science Review*, 83(4), 1181–1206.

[10] Battaglini, M., R. Morton and T. Palfrey (2010), "The Swing Voter's Curse in the Laboratory," *Review of Economic Studies*, 77(1), 61–89.

[11] Bernhardt, D., J. Duggan and F. Squintani (2007), "Electoral Competition with Privately Informed Candidates," *Games and Economic Behavior*, 58(1), 1–29.

[12] Bernhardt, D., J. Duggan and F. Squintani (2009a), "Private Polling and Voter Welfare," *Journal of Economic Theory*, 144(5), 2021–2056.

[13] Bernhardt, D., J. Duggan and F. Squintani (2009b), “The Case for Responsible Parties,” *American Political Science Review*, 103(4), 570–587.

[14] Birch, S. (2009), *Full Participation: A Comparative Study of Compulsory Voting*, New York: United Nations University Press.

[15] Blume, A., J. Duffy and A. Franco (2009), “Decentralized Organizational Learning: An Experimental Investigation,” *American Economic Review*, 99(4), 1178–1205.

[16] Börgers, T. (2004), “Costly Voting,” *American Economic Review*, 94(1), 57–66.

[17] Callander, S. (2008), “Political Motivations,” *Review of Economic Studies*, 75(3), 671–697.

[18] Calvert, R. (1985), “Robustness of the Multidimensional Voting Model: Candidate Motivations, Uncertainty, and Convergence,” *American Journal of Political Science*, 29(1), 69–95.

[19] Cason, T. and V. Mui (2005), “Uncertainty and Resistance to Reform in Laboratory Participation Games,” *European Journal of Political Economy*, 21(3), 708–737.

[20] Castellan, J. and S. Siegel (1988), *Nonparametric Statistics for the Behavioral Sciences*, 2nd ed., New York, McGraw-Hill.

[21] Condorcet, Marquis de (1785), *Essai sur l’application de l’analyse à la probabilité des décisions rendues à la probabilité des voix*, Paris: De l’imprimerie royale.

[22] Dasgupta, P. and E. Maskin, 1986, “The Existence of Equilibrium in Discontinuous Economic Games I: Theory,” *Review of Economic Studies*, 53(172), 1–26.

[23] De Sinopoli, F. and G. Iannantuoni (2007), “A Spatial Voting Model Where Proportional Rule Leads to Two-Party Equilibria,” *International Journal of Game Theory*, 35(2), 267–286.

[24] De Sinopoli, F. and G. Iannantuoni (2008), “Extreme Voting under Proportional Representation: the Multidimensional Case,” *Social Choice and Welfare*, 30(3), 401–417.

[25] Diermeier, D. (2006), “Coalition Government,” In *The Oxford Handbook of Political Economy*, B. Weingast and D. Wittman eds., New York, NY: Oxford University Press, Chapter 9, 162–179.

[26] Downs, A. (1957), *An Economic Theory of Democracy*, New York, NY: Harper and Row.

[27] Duffy, J. and M. Tavits (2008), “Beliefs and Voting Decisions: A Test of the Pivotal Voter Model,” *American Journal of Political Science*, 52(3), 603–618.

[28] Duggan, J. (2006), “Candidate Objectives and Electoral Equilibrium,” In *The Oxford Handbook of Political Economy*, B. Weingast and D. Wittman eds., New York, NY: Oxford University Press, Chapter 4, 64–83.

[29] Feddersen, T. (2004), “Rational Choice Theory and the Paradox of Not Voting,” *Journal of Economic Perspectives*, 18(1), 99–112.

[30] Feddersen, T. and W. Pesendorfer (1996), “The Swing Voter’s Curse,” *American Economic Review*, 86(3), 408–424.

[31] Feddersen, T. and W. Pesendorfer (1997), “Voting Behavior and Information Aggregation in Elections with Private Information,” *Econometrica*, 65(5), 1029–1058.

[32] Feddersen, T. and W. Pesendorfer (1998), “Convicting the Innocent: The Inferiority of Unanimous Jury Verdicts under Strategic Voting,” *American Political Science Review*, 92(1), 23–35.

[33] Feddersen, T. and W. Pesendorfer (1999a), “Abstention in Elections with Asymmetric Information and Diverse Preferences,” *American Political Science Review*, 93(2), 381–398.

[34] Feddersen, T. and W. Pesendorfer (1999b), “Elections, Information Aggregation, and Strategic Voting,” *Proceedings of the National Academy of Sciences*, 96, 10572–10574.

[35] Feddersen, T. and A. Sandroni (2006), “A Theory of Participation in Elections,” *American Economic Review*, 96(4), 1271-1282.

[36] Gerardi, D., M.A. McConnell, J. Romero and L. Yariv (2009), “Get Out the (Costly) Vote: Institutional Design for Greater Participation,” *Working Paper*.

[37] Glicksberg, L. (1952), “A Further Generalization of the Kakutani Fixed Point Theorem with Application to Nash Equilibrium Points,” *Proceedings of the American Mathematical Society*, 3(1), 170–174.

[38] Goeree, J., C. Holt and T. Palfrey (2005), “Regular Quantal Response Equilibrium,” *Experimental Economics*, 8(4), 347–367.

[39] Goeree, J., C. Holt and T. Palfrey (2008), “Quantal Response Equilibrium,” in *The New Palgrave: A Dictionary of Economics*, 2nd ed.

[40] Goeree, J. and L. Yariv (2011), “An Experimental Study of Collective Deliberation,” *Econometrica*, 79(3), 893–921.

[41] Großer, J. and A. Schram (2006), “Neighborhood Information Exchange and Voter Participation: An Experimental Study,” American Political Science Review, 100, 235–248.

[42] Guarnaschelli, S., R. McKelvey, and T. Palfrey (2000),"An Experimental Study of Jury Decision Rules" *American Political Science Review*, 94(2), 407–423.

[43] Harsanyi, J. (1973), "Games with Randomly Disturbed Payoffs: A New Rationale for Mixed-Strategy Equilibrium Points," *International Journal of Game Theory*, 2(1): 1–23.

[44] Hotelling, H. (1929), "Stability in Competition," *Economic Journal*, 39(153), 41–57.

[45] Kartik, N. and R.P. McAfee, (2007), "Signaling Character in Electoral Competition," *American Economic Review*, 97(3), 852–870.

[46] Kreyszig, E. (1978), *Introductory Functional Analysis with Applications*, New York, NY: Wiley and Sons, Chapter 5, 299–323.

[47] Krishna, V. and J. Morgan (2012),"Voluntary Voting: Costs and Benefits," *Working Paper*.

[48] Ledyard, J. (1984), "The Pure Theory of Two Candidate Elections," *Public Choice*, 44(1), 7–41.

[49] Levine, D. and T. Palfrey (2007),"The Paradox of Voter Participation: An Experimental Study," *American Political Science Review*, 101(1), 143–158.

[50] Liker, W. and P. Ordeshook (1968), "A Theory of the Calculus of Voting," *American Political Science Review*, 62(1), 25–42.

[51] McKelvey, R. and T. Palfrey (1995), "Quantal Response Equilibria for Normal Form Games," *Games and Economic Behavior*, 10(1), 6–38.

[52] McKelvey, R. and T. Palfrey (2008), "Quantal Response Equilibria: A Brief Synopsis," In *Handbook of Experimental Economics Results*, C. Plott and V. Smith eds., 541–548.

[53] Meirowitz, A. and K. Ramsay (2009), "Investing in Failure: Equilibrium Constraints on Prior Beliefs in Bargaining Problems," *Working Paper*.

[54] Mendenhall, W., R. Scheaffer and D. Wackerly (2002), *Mathematical Statistics with Applications*, 6th ed., Duxbury.

[55] Morris, S. (2008), "Purification," In *The New Palgrave Dictionary of Economics*, S. Durlauf and L. Blume eds., New York, NY: Palgrave Macmillan, 779–782.

[56] Myerson, R. (1998), "Extended Poisson Games and the Condorcet Jury Theorem," *Games and Economic Behavior*, 25(1), 111–131.

[57] Nash, J. (1950), "The Bargaining Problem," *Econometrica*, 18(2), 155–162.

[58] Osborne, M. (1995), "Spatial Models of Political Competition under Plurality Rule: A Survey of Some Explanations of the Number of Candidates and the Positions They Take," *Canadian Journal of Economics*, 28(2), 261–301.

[59] Osborne, M. and A. Rubinstein (1990), *Bargaining and Markets*, San Diego, CA: Academic Press, Chapter 2, 9–27.

[60] Osborne, M. and A. Slivinski (1996), "A Model of Political Competition with Citizen-Candidates," *Quarterly Journal of Economics*, 111(1), 65–96.

[61] Palfrey, T. (1984), "Spatial Equilibrium with Entry," *Review of Economic Studies*, 51(1), 139–156.

[62] Palfrey, T. (2009), "Laboratory Experiments in Political Economy," *Annual Review of Political Science*, 12, 379–388.

[63] Palfrey, T. and H. Rosenthal (1983), "A Strategic Calculus of Voting," *Public Choice*, 41(1), 7–53.

[64] Palfrey, T. and H. Rosenthal (1985), "Voter Participation and Strategic Uncertainty," *American Political Science Review*, 79(1), 62–78.

[65] Persico, N. (2004), "Committee Design with Endogenous Information," *Review of Economic Studies*, 71(1), 165–191.

[66] Roemer, J. (1994), "A Theory of Policy Differentiation in Single Issue Electoral Politics," *Social Choice and Welfare*, 11(4), 355–380.

[67] Roemer, J. (2006), "Modeling Party Competition in Elections," In *Oxford Handbook of Political Economy*, ed. B. Weingast and D. Wittman eds., New York, NY: Oxford University Press, Chapter 58, 1010–1030.

[68] Schram, A. and J. Sonnemans (1996), "Voter Turnout as a Participation Game: An Experimental Investigation," *International Journal of Game Theory*, 25(3), 385–406.

[69] Shaked, A. (1982), "Existence and Computation of Mixed Strategy Nash Equilibrium for 3-Firms Location Problem," *Journal of Industrial Economics*, 31(1/2), 93–96.

[70] Simon, C. and L. Blume (1994), *Mathematics for Economists*, New York, NY: W. W. Norton & Company, Chapter 18, 411–447.

[71] Wittman, D. (1977), "Candidates with Policy Preferences: A Dynamic Model," *Journal of Economic Theory*, 14(1), 180–189.

[72] Wittman, D. (1983), "Candidate Motivation: A Synthesis of Alternative Theories," *American Political Science Review*, 77(1), 142–157.